

Normality Assumption (on ε)

$$y = \underset{\substack{+ \text{ True} \\ \text{True}}}{X\beta} + \underset{\text{True}}{\varepsilon}$$

Normality Assumption

$$\varepsilon_i \sim N(0, \sigma^2)$$

\Rightarrow Probability Model \Rightarrow

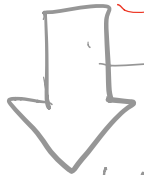
important properties of ε :

ε_i independent

$$E(\varepsilon_i) = 0$$

$$\text{Var}(\varepsilon_i) = \sigma^2$$

equals to $\sum \varepsilon_i^2 = \sigma^2 \mathbf{I}_n$



more facts

All, Normal Distribution

$$\vec{\varepsilon} \sim N(\vec{0}, \sigma^2 \mathbf{I}_n)$$

$$A\vec{X} + c \sim N(\vec{c}, A(\sigma^2 \mathbf{I}_n)A')$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2(\vec{\mu}, \Sigma)$$

$$\text{Cov}(X, Y) = 0$$

Linearity of Normal Distribution

① $X \sim N(\mu, \sigma^2)$

$$(ax+b) \sim N(a\mu+b, a^2\sigma^2)$$

② $X \sim N(\mu_1, \sigma_1^2)$

$$Y \sim N(\mu_2, \sigma_2^2)$$

$$(a+bX+cY) \sim N(a+b\mu_1+c\mu_2, b^2\sigma_1^2+c^2\sigma_2^2 + 2bc \text{Cov}(X,Y))$$

⚠ covariance!

How about \vec{y} & $\vec{\beta}$?



$$(\vec{y}|x) \sim N_n(X\beta, \sigma^2 \mathbf{I}_n)$$

why?

$$\vec{y} = X\beta + \varepsilon$$

conditioned $X \rightarrow X$ settled
parameter $\beta \rightarrow$ settled value

$$(\vec{\beta}|x) \sim N_{p+1}(\beta, \sigma^2 (X^T X)^{-1})$$

In estimate model, only β & ε are R.V.

why?

$$\beta = \frac{(X^T X)^{-1} X^T y}{\text{scalar}}$$

\therefore linearity of Normal Distr.

equals to $(\beta_i|x) \sim N(\beta_i, \sigma^2 V_{ii})$

$$\text{Var}(\beta) = (X^T X)^{-1} X^T \text{Var}(y) X (X^T X)^{-1}$$

the diagonal entries of $(X^T X)^{-1}$

$$= \sim \sigma^2 \mathbf{I} \sim$$

$$= \sigma^2 (X^T X)^{-1} X^T X (X^T X)^{-1}$$

$$= \sigma^2 (X^T X)^{-1}$$