Normality Assumption (on ε)

$$y = \underset{\text{True}}{X\beta} + \underset{\text{True}}{\varepsilon}$$

+

Normality Assumption

$\varepsilon_i \sim N(0, \sigma^2)$

$\Rightarrow$ **Probability Model** $\Rightarrow$

important properties of ε:

ε_i independent

$E(\varepsilon_i) = 0$ $\quad$ equals to $\quad = \quad \Sigma_{\varepsilon\varepsilon} = \sigma^2 I_n$

$Var(\varepsilon_i) = \sigma^2$

(all about "True Model")

more facts

All, Normal Distribution

$\vec{\varepsilon} \sim N(\vec{0}, \sigma^2 I_n)$

$A\vec{X} + c \sim N(\vec{c}, A(\sigma^2 I_n)A^T)$

$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2(\vec{\mu}, \Sigma) \rightarrow Cov(X, Y) = 0$

Linearity of Normal Distribution

① $X \sim N(\mu, \sigma^2)$

$(aX+b) \sim N(a\mu+b, a^2\sigma^2)$

② $X \sim N(\mu_1, \sigma_1^2)$

$Y \sim N(\mu_2, \sigma_2^2)$

$(a + bX + cY) \sim N(a + b\mu_1 + c\mu_2,\ b^2\sigma_1^2 + c^2\sigma_2^2 + 2bc\, Cov(X,Y))$

⚠ Covariance!

How about $\vec{y}$ & $\vec{\beta}$?

$(\vec{y} \mid X) \sim N_n(X\beta, \sigma^2 I_n)$ $\rightarrow$ why? $\quad \vec{y} = X\beta + \varepsilon$ $\quad$ conditioned X $\rightarrow$ X setted; parameter β $\rightarrow$ setted value.

$(\hat{\beta} \mid X) \sim N_{p+1}(\beta, \sigma^2 (X^TX)^{-1})$ $\rightarrow$ why? $\quad \hat{\beta} = \underbrace{(X^TX)^{-1}X^T}_{\text{scalar}} y$ $\quad$ ∵ linearity of Normal Distr

In estimate model, only $\hat{\beta}$s & $\hat{\varepsilon}$ are R.V.

equals to $(\hat{\beta}_i \mid X) \sim N(\beta_i, \sigma^2 V_{ii})$

V_{ii}: the diagonal entries of $(X^TX)^{-1}$

$Var(\hat{\beta}) = (X^TX)^{-1}X^T\, Var(y)\, X((X^TX)^{-1})^T$

$= \quad \sim \quad \sigma^2 I \quad \sim$

$= \sigma^2 \cancel{(X^TX)^{-1}}\cancel{X^TX}(X^TX)^{-1}$

$= \sigma^2 (X^TX)^{-1}$