Foundations of Algorithms Homework 6 SU24

Strategies for solving ***Collaborative Problem*** problems are to be formed by your assigned group, with each member *participating and contributing fully*. All answers must be your own work, in your own words, even on collaborative problems—collaborate on strategies only. Individual participation will be evaluated through collaborative assessments, submitted when the homework is due.

1. [30 points total] (0-1 Integer Linear Programming)

   Multi-object tracking within optical video is a common problem in machine learning. Tracking is extremely difficult in general as the number, sizes, and dynamics of objects can be large. Further, objects can by occluded by other objects and scenery, causing variable time gaps between one detection and the next.

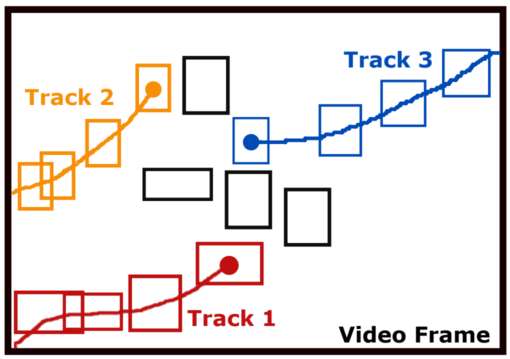

Figure 1: A tracking system that has three active tracks and has received four detections (black boxes) from the *Detector*. Note that the time intervals between detections assigned to tracks varies, as does the shape of their bounding boxes.

Tracking systems require several key components:

- **Detector**– A model that signals when an object of interest is apparently present, and returns a bounding box in pixel coordinates.
- **Tracker**– Software module that stores information regarding active tracks, and assigns object detections to tracks using a similarity score matrix produced by the *Evaluator*. Optimally, the *Tracker* assigns detections to tracks in a manner that maximizes the sum of the similarity scores from each track/detection pair.
- **Evaluator**– Given N active tracks and M detections, produces a score matrix $\boldsymbol{S} \in \mathbb{R}^{M \times N}$ such that element $s_{ij} \in \boldsymbol{S}$ indicates the similarity of detection i to track j under some similarity measure. Note that this definition leaves the actual definition of "track" fairly abstract. In the simplest case, for example, the *Evaluator* may define s_{ij} as the Euclidean distance between the center of the final detection within track j and the new detection i. More sophisticated models may uses multiple past detections within a track, or model the dynamics of an objects (e.g., with a Kalman filter).

In this problem, assume that you'd like to produce a novel tracking system. The *Detector* and *Evaluator* have already been created, and you must now describe the *0-1 integer linear program* that the *Tracker* will solve to assign detections to tracks. **Integer linear programs are constraint problems where the variables are restricted to be integers, and are typically NP-hard. In particular, 0-1 integer linear programs restrict the variables further by requiring them to be Boolean (i.e., zero or one).**

Assume that you are provided with the score matrix S, detections can only be matched to a single track, and that tracks cannot be assigned more than once.

   (a) [4 points] What do the variables in this problem represent? How many are there? Note: if there is a matrix of variables involved, count each matrix entry as one variable.

(b) [8 points] Define the objective for this 0-1 integer linear program. HINT: we are looking for a summation here, among other things.

(c) [18 points] Define the entire 0-1 integer linear program, including constraints, in standard form. How many constraints are there in the program, total?

2. [30 points total] CLRS 34.3-2 (4th ed): Show that the $\leq_P$ relation is a transitive relation on languages. That is, show that if $L_1 \leq_P L_2$ and $L_2 \leq_P L_3$, then $L_1 \leq_P L_3$.

3. [40 points total] ***Collaborative Problem:***

NP Completeness Proof Requirements

For all problems proving that a problem belongs to the set NP, you are **required** to use the method in the NP Completeness Short Guide available in your course materials. Also, I urge you to work the collaborative problem first. Your group will help you build the skills needed to solve the other problem individually.

Suppose you're organizing a summer sports camp. The camp is supposed to have at least one counselor who's skilled at each of the n sports covered by the camp (baseball, volleyball, and so on). They have received job applications from m potential counselors. For each of the n sports, there is some subset of the m applicants qualified in that sport. The question is "For a given number $k < m$, is is possible to hire at most k of the counselors and have at least one counselor qualified in each of the n-sports?" We'll call this the *Efficient Recruiting Problem*. Prove that Efficient Recruiting is NP-complete. For this problem, use **Vertex Cover** as your reduction problem.