Multivariate Statistical Methods (TAMS39)
– Computer Problems 1 –

1. Given the observation matrix $\boldsymbol{X} \sim N_{4,10000}(\boldsymbol{\mu}\mathbf{1}', \boldsymbol{\Sigma}, \boldsymbol{I})$ estimate the mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ using the first 10, 100, 1000 and all 10000 observations, respectively.

2. The quality-control department of a manufacturer of microwave ovens is required by the federal government to monitor the amount of radiation emitted when the doors of the ovens are closed and open. Assume that $\boldsymbol{x} \sim N_2(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Observations of the radiation emitted of $n = 42$ randomly selected ovens were made and the following were found (after some transformation)

$$\bar{\boldsymbol{x}} = \begin{pmatrix} 0.564 \\ 0.603 \end{pmatrix}, \quad \widehat{\boldsymbol{\Sigma}} = \begin{pmatrix} 0.0144 & 0.0117 \\ 0.0177 & 0.0146 \end{pmatrix}, \quad \text{and} \quad \widehat{\boldsymbol{\Sigma}}^{-1} = \begin{pmatrix} 203.018 & -163.391 \\ -163.391 & 200.228 \end{pmatrix}.$$

   a) Test the hypothesis $H : \boldsymbol{\mu} = \frac{1}{2}\mathbf{1}_2$ vs. $A \neq H$ at level $\alpha = 0.05$.

   b) Derive and plot the confidence region for the mean vector $\boldsymbol{\mu} = (\mu_1, \mu_2)'$ with $\alpha = 0.05$.

   c) Compute the simultaneous 95% T^2-intervals for μ_1 and μ_2 and the 95% Bonferroni intervals for μ_1 and μ_2.

3. **6.21 a-c)**

4. (Greenhouse and Geyser, 1959) Five groups of mothers, classified into their groups according to some external criteria, were given a maternal attitude questionnaire containing 23 scales. For purposes of this illustration, six of these scales have been selected.

   Mean profiles for five groups of mothers on selected scales of a maternal attitude questionnaire

	No. of	Scale						Group
Groups	Mothers	1	3	6	9	13	14	Mean
A	59	17.02	10.97	13.24	11.47	9.80	15.44	12.99
B	13	17.92	13.85	17.23	14.00	12.23	17.38	15.44
C	15	18.87	11.60	14.13	8.93	8.27	17.73	13.26
D	32	16.75	14.47	15.41	11.78	9.91	15.94	14.04
E	9	18.33	10.78	13.89	14.44	12.11	18.78	14.72
All groups	128	17.35	12.20	14.34	11.72	10.05	16.27	13.65

   An estimate of the matrix $\boldsymbol{\Sigma}$ is given by the pooled variance-covariance matrix

$$\boldsymbol{S} = \begin{pmatrix} 3.100 & .101 & -.279 & -.083 & -.009 & 1.557 \\ .101 & 5.780 & 1.013 & -.114 & -1.014 & .039 \\ -.279 & 1.013 & 5.560 & 1.039 & 1.366 & -.169 \\ -.083 & -.114 & 1.039 & 5.600 & 3.080 & .258 \\ -.009 & -1.014 & 1.366 & 3.080 & 6.820 & .222 \\ 1.557 & .039 & -.169 & .258 & .222 & 5.170 \end{pmatrix}$$

   a) Test the hypothesis that this is an intraclass covariance matrix.

   b) Estimate the two parameters in the intraclass covariance matrix.