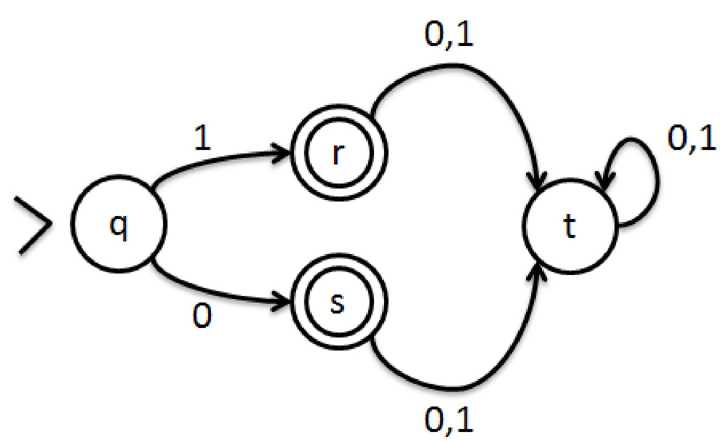
COMP 455
Models of Languages and Computation
Fall 2019
Mid Semester Exam.
Tuesday, November 5, 2019
Closed Book - Closed Notes
This exam has four pages
Don't forget to write your name or ID and pledge on the exam sheet.

1. (4 points) Recall that $x \approx_L y$ is a relation on strings x and y where L is a language, $x \sim_M y$ is a relation on strings x and y where M is a deterministic finite-state automaton, and $s \equiv t$ is a relation on states s and t of a deterministic finite-state automaton. Consider the deterministic finite automaton M with states $\{q, r, s, t\}$, input alphabet $\{0, 1\}$, start state q, accepting states r and s, and the following transitions:



True or false:

1.1) $q \equiv t$. ________
1.2) $1 \sim_M 0$. ________
1.3) $1 \approx_{L(M)} 0$. ________
1.4) $1 \approx_{L(M)} 10$. ________

2. (6 points) Suppose L is a regular language. True or false:

2.1) The intersection of L with a context-free language will always be context free. ________

2.2) The union of L with a regular language will always be context free. ________

2.3) L must be finite. ________

2.4) L must be infinite. ________

2.5) L must not be context-free. ________

2.6) The relation $\approx_L$ must have infinitely many equivalence classes. ________

3. (4 points) Which of the following is a method for showing that a language L is not regular?

a) Showing that the relation $\approx_L$ has finitely many equivalence classes.
b) Showing that you can always win the regular expression game for L.
c) Constructing a push-down automaton recognizing L
d) Constructing a finite state automaton recognizing L

4. (5 points) Consider the context free grammar $G = (V, \Sigma, R, S)$ where V is $\{S, A, B, a, b\}$, Σ is $\{a, b\}$, and R consists of the following rules:

$$S \to AB \qquad A \to AB \qquad B \to AA$$
$$A \to a \qquad A \to b$$

Is this grammar ambiguous? ________________________ Justify your answer.

5. (6 points) Consider the language $L = \{(ab)^m(cd)^n : m, n \geq 0\}$. This language contains the strings $abcd, ababcd, ababcdcdcd$, et cetera. Consider the following theorem and proof:

Theorem: L is not regular.

Proof: We show that in the regular expression game, we can always win. Suppose the opponent picks the integer N, and we pick the string $(ab)^N(cd)^N$, which is of length $4N$. Then, the opponent splits the string into xyz such that $y \neq e$ and $|xy| \leq N$. Thus, y will not have any c's or d's. So, we pick $i = 2$, and $xy^iz = xy^2z$ will have more a's or b's than c's and d's, and therefore xy^2z is not in L. Therefore, we can always win, so L is not regular.

Is the theorem correct? Justify your answer.

Is the proof correct? That is, does it show that the theorem is true? Justify your answer.

6. (4 points) Among the following statements, write the letters of all correct statements here:________________________

a) The union of finitely many context-free languages is context-free.
b) The language $\{a^i b^j c^k d c^k b^j a^i : i, j, k \geq 0\}$ is context free.
c) The language $\{a^n b c^n : n > 0\}$ is regular.
d) The language $\{a^m b^n c^p d^q : m + n = p + q\}$ is context-free.

7. (8 points) Consider the following context-free grammars and languages. In each grammar, S is the start symbol, capital letters are nonterminals, lower case letters are terminals, and e is the empty string. Match up the grammars with the languages they generate. The grammar should generate all strings in the language and no other strings. Include the letter of the language after each grammar. A given language may be used for more than one grammar. Each grammar should match one language.

Grammars:
7.1.) $S \to aSbS$, $S \to bSaS$, $S \to e$. ________
7.2.) $S \to aSb$, $S \to bSa$, $S \to aSa$, $S \to bSb$, $S \to e$.________
7.3.) $S \to aS$, $S \to bS$, $S \to Sa$, $S \to Sb$, $S \to e$. ________
7.4.) $S \to aSa$, $S \to b$. ________
7.5.) $S \to aSb$, $S \to e$. ________
7.6.) $S \to SaS$, $S \to b$. ________
7.7.) $S \to aSa$, $S \to bSb$, $S \to e$. ________
7.8.) $S \to aSb$, $S \to bSa$, $S \to SS$, $S \to e$. ________

Languages:
A. $\{a, b\}^*$
B. Strings over $\{a, b\}$ of even length.
C. Strings over $\{a, b\}$ with the same number of a's as b's.
D. $\{a^n b^n : n \geq 0\}$
E. $(ba)^* b$
F. $\{a^n b a^n : n \geq 0\}$
G. $\{ww^R : w \in \{a, b\}^*\}$

8. (6 points) Consider the push-down automaton $M = (K, \Sigma, \Gamma, \Delta, s, F)$ where $K = \{s, f\}$, $\Sigma = \{a, b\}$, $\Gamma = \{a, b\}$, $F = \{f\}$, and Δ consists of the following transitions:

$$((s, a, e), (s, b)),\ ((s, b, e), (s, a)),\ ((s, e, e), (f, e)),\ ((f, a, a), (f, e)),\ ((f, b, b), (f, e)).$$

Which of the following strings are in $L(M)$? Write the letters of all strings that are in $L(M)$ here: ________________________

a) $aabaabb$
b) $abaaba$
c) $aabb$
d) $abab$
e) ba
f) $bbbaaabbb$

9. EXTRA CREDIT (6 points) Recall that $|x|$ is the length of a string x. Let $L_{3,5}$ be $\{x \in \{0, 1\}^* : |x| \text{ is a multiple of 3 or a multiple of 5}\}$. Thus $L_{3,5}$ contains the strings 001, 11011, 111000, 1111010011, and all strings of length 3, 5, 6, 9, 10, et cetera. For this, consider 0 to be a multiple of 3 and a multiple of 5.

a) Show that $L_{3,5}$ is regular.

b) How many states are there in a minimal deterministic finite state automaton recognizing $L_{3,5}$? Justify your answer.

c) How many states are there in a minimal nondeterministic finite state automaton recognizing $L_{3,5}$? Try to justify your answer to some extent.