COMP 455
Models of Languages and Computation
Fall 2019
Mid Semester Exam
Thursday, September 26, 2019
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.

This exam has three pages

1. (8 points) Among the following statements, write the letters of all correct (true) statements here: ______________________________

a) For all languages A over Σ^*, $(A^*)^* = A^*$.

b) If L and M are regular languages then $LM \cup (M^*)$ is also a regular language.

c) For every n state nondeterministic finite automaton there is an equivalent deterministic finite automaton having at most 2^n states.

d) If A and B are regular languages then $A - B$, which equals $\{x : x \in A, x \notin B\}$, is also a regular language.

e) All subsets of a regular language are regular.

f) For every nondeterministic finite automaton having 2^n states there is an equivalent deterministic finite automaton having n states.

g) For every regular expression E there is a deterministic finite automaton M such that $\mathcal{L}(E) = L(M)$, where $\mathcal{L}(E)$ is the language represented by the regular expression E.

h) If E and F are regular expressions, then there is a regular expression for the set of strings in $\mathcal{L}(E) \cup \mathcal{L}(F)$ where $\mathcal{L}$ gives the set of strings represented by a regular expression.

2. (4 points)
Suppose R is the relation $\{(2,1),(3,1)\}$ and S is the relation $\{(1,4),(1,5)\}$. What is the relation $R \circ S$ obtained by composing R and S? List all ordered pairs in the relation. Do not put set brackets around them, and do not add spaces when you enter this answer into Sakai.

3. (6 points) Suppose $L_1 = \{01, 10\}$ and $L_2 = \{00, 11\}$.

3.1) Is 010001 in $L_1^* L_2^*$?

3.2) List all strings in $L_1 \cup L_2$.
3.3) List all strings in $L_2 L_1$.
3.4) Is the empty string in L_2^*?
3.5) Is 110001 in $L_1^* L_2$?
3.6) Is the string 011001 in L_1^*?

4. (6 points) Which of the following regular expressions represent the set of all strings in {0,1}* that have at most two 0's? There may be more than one correct answer. List the letters of all correct answers here: ______________

a. $1^*(0 \cup \phi)1^*(0 \cup \phi)1^*$
b. $1^*(0 \cup \phi^*)1^*(0 \cup \phi^*)1^*$
c. $1^*01^*01^*$
d. $1^*(0 \cup 1)1^*(0 \cup 1)1^*$
e. $1^* \cup 1^*01^*01^* \cup 1^*01^*$
f. $1^* \cup 1^*01^* \cup 1^*01^*01^*$

5. (5 points) Which of the following regular expressions represent the set of all expressions containing at least one digit (0 through 9), followed by a decimal point (.), followed by at least one digit? Thus 23.1415 is an example of such an expression. There may be more than one correct answer. List the letters of all correct answers here: ______________

a. $(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9).(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)$

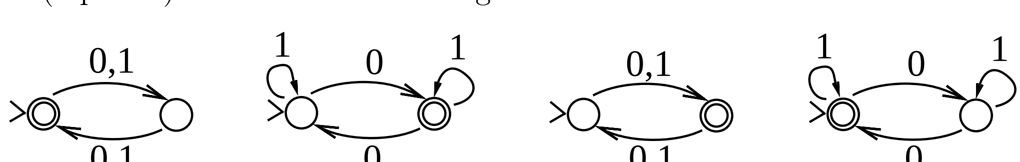
b. $(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*.(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*$

c. $(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9) \cup . \cup (0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*$

d. $(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*.(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*$

e. $(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9).(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)^*(0 \cup 1 \cup 2 \cup 3 \cup 4 \cup 5 \cup 6 \cup 7 \cup 8 \cup 9)$

6. (6 points) Consider the following finite automata:



A. B. C. D.

Let them be named A,B,C,D from left to right. For which of these automata M is $L(M)$ exactly the following sets of strings? There may not be any correct answers for some of the questions.

After each part (6.1, 6.2, 6.3, 6.4, 6.5, 6.6) write the name (letter) of the automaton that recognizes the given set of strings, or write “none” if none of the automata fit the description.

6.1) Strings having an even number of zeroes.
6.2) Strings having an odd number of ones.
6.3) Strings having an even number of ones.
6.4) Strings of odd length.
6.5) Strings of even length.
6.6) Strings having an odd number of zeroes.

7. (5 points) Consider the following theorem:

Theorem. Suppose $M = (K, \Sigma, \Delta, s, F)$ is a nondeterministic finite automaton. Suppose that all states of M are accepting states. Suppose $K = \{q_0, q_1, q_2, \cdots, q_n\}$ for some n and that for all states $q_i \in K$, if there is an arrow from q_i to a state q_j, $q_j \in K$, then $j > i$. This is true for arrows labeled with elements of Σ and also for ϵ arrows. Then

1. M accepts at least one string of length $n + 1$.

2. M accepts all strings of length $n + 1$.

Say which of conclusions 1. and 2. are correct. It is not necessarily true that one is correct and one is false. Justify your answers.

8. Extra Credit (5 points)
Consider the following theorem:

Theorem. Suppose a deterministic finite automaton M has n states, $n > 2$, and M accepts at least one string of length 2^n. Then M accepts at least one string of length n^2 or less.

Either prove that the theorem is true or give a counter example.