A Review on Single Qubit Randomised Benchmarking of non-Clifford Gates

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Optimising and characterising error rates of quantum gates is undoubtedly critical towards the actualisation of a universal quantum computer. The randomised benchmarking (RB) protocol is now the *de facto* standard when it comes to the partial characterisation of quantum gates' performance, since it has time and time again consolidated itself to be an efficient, scalable method and yet robust against state preparation and measurement errors. Here, a review of said method performed over Clifford gates is presented, since they satisfy the precise condition of forming a unitary 2-design; a requirement for RB. Finally, the future work of non-Clifford RB is discussed.

CONTENTS

I. INTRODUCTION

Ever since its initial conceptualisation by Feynman [1], there has been a growing interest in the physical realisation of a universal quantum computer due to its potential ability to considerably outperform classical computers, whether it is on tasks such as factoring integers using the notable Shor's algorithm, or on unstructured searching using Grover's algorithm [2 – 4]. In practice, a quantum computer can never be a perfectly closed system, since it will inevitably be affected by the intrinsic presence of random noise under its external environment. Therefore, it was not so surprising to find that quantum computers are only efficient as far as its quality of error control are allowed during a computation [5]. The result of combating limitations imposed by decoherence instigated a plethora of efforts on improving quantum error control and developing quantum error correction codes [6 – 9]. Despite the long-standing progress that had already been made in these fields, they nevertheless remain a challenging endeavour of utmost importance in heralding the era of *fault-tolerant* quantum computing [10 – 13].

In order to improve the quality of a quantum channel, it must first be characterised in terms of the level of coherent control over which it is possible. Although such a method has long existed - namely Quantum Process Tomography (QPT) [14]- and with the full characterisation of a quantum process over different candidates of small number of qubits had already been demonstrated [15 – 19], the shortcomings that it would suffer from became inevitably apparent. Firstly, without being able to differentiate the errors accrued from state preparation and measurement (SPAM), the method cannot ascertain that the error analysis obtained is solely due to the quantum logic gates themselves. Secondly, it lacks experimental scalability; QPT becomes practically infeasible for higher number of qubits as the amount of experiments that must be performed grows exponentially [20, 21].

The randomised benchmarking (RB) protocol [22 – 24] is an efficient and scalable method which supersedes QPT by allowing for the partial characterisation of quantum transformations by a specific gate set. By accumulating noise models represented by quantum operations in between long sequences of random unitary operators U followed by their inverse $U^\dagger$, it extracts an estimate of tomographic data from the targeted gate set, all while being uninfluenced by SPAM errors. It is important to note that due to its original rigorous analyses, in order for RB to work it is required to assume that the noise models are not gate-dependent (i.e. independent of the choice of unitaries) and time-dependent, though crucial work has been done on the relaxation of these assumptions [25]. The variations of RB protocol, especially Clifford-group RB, have become the *de facto* means across multiple research groups on benchmarking and optimising error rates related to quantum operations [11, 26 – 28], with a notable example as recently shown by Google's ostensible demonstration of *quantum supremacy* [29].

Going forward, the project will work on the benchmark of non-Clifford gates using RB protocol. The review will first lay its mathematical preliminaries of quantum information processing in Sec. II, along with the general representation of noise in quantum computation. Section III will set the specific notations required in stone and proceed with the details of the protocol. Finally, the discussion of generating and benchmarking non-Clifford gates is presented in Sec. IV.

II. BACKGROUND

A. Density Matrices and Quantum Channels

The Hilbert space comes with an association of a *state space*, such that a normalised unit vector, $|\psi\rangle$, living in such space can best provide the *complete description* of a quantum system. Since the outcome of measurements in quantum mechanics is inherently random in nature, the complete description of a quantum system is then only attainable as far as the best prediction of a system the state vector can allow one to make. A quantum state then serves as a mathematical entity which allows one to compute the statistical properties of a physical system, something that one can only obtain by repeated measurement of an experiment under identical initial conditions.

However, sometimes a quantum system cannot be described completely only by a single state vector, rather it could be in one of the number of states $|\psi_i\rangle$ with corresponding probabilities p_i, where $\{p_i, |\psi_i\rangle\}$ is called *an ensemble of pure states*. Therefore, a new type of formalism is better introduced for the convenience of these cases. The density matrix or density operator is introduced as

$$\rho = \sum_{i=1}^{n} p_i |\psi_i\rangle\langle\psi_i|, \tag{1}$$

obeying the normalisation condition $\sum_i^n p_i = 1$. Note that the states $|\psi_i\rangle$ are not required to be orthogonal. A density matrix where its exact state is known, written as $|\psi\rangle\langle\psi|$, is usually referred to as a *pure state* whereas an ensemble of pure states are called a *mixed state*.

In an open quantum system, the transformation of an initial state ρ to a final state ρ' through a quantum channel $\mathcal{E}$ under some dynamical process can be described by a superoperator, such that the evolution is

$$\rho \to \rho' = \mathcal{E}(\rho). \tag{2}$$

The channel $\mathcal{E}$ is generally a linear, *completely positive trace preserving* (CPTP) map such that it must fulfil the following requirements [30]:

1. $\mathcal{E}$ is linear in density operator, meaning for a mixed state such as Eq.(1), its operation is given by
$$\mathcal{E}\left(\sum_{i=1}^{n} p_i |\psi_i\rangle\langle\psi_i|\right) = \sum_{i=1}^{n} p_i \mathcal{E}(|\psi_i\rangle\langle\psi_i|) \tag{3}$$
2. $\mathcal{E}$ is trace preserving, i.e. $\mathrm{tr}(\mathcal{E}(\rho)) = \mathrm{tr}(\rho) = 1$.
3. $\mathcal{E}$ is positive semidefinite (positive); it has no negative eigenvalues and is Hermitian. Hence if ρ is positive, then $\mathcal{E}(\rho)$ must also be positive.
4. $\mathcal{E}$ is completely positive. For a compound system of A and B described by the joint state ρ_{AB}, there exist superoperators where it is possible to yield negative eigenvalues when acting on a reduced state $\mathcal{E}(\rho_A)$, despite never having a negative eigenvalue when acting on its full state $\mathcal{E}(\rho_{AB})$. A superoperator is thus completely positive if it can never obtain any negative eigenvalues when it acts on a full system or even on a sub-system alone.

The conditions (1-3) ensure that $\mathcal{E}$ maps a normalised density operator ρ to a normalised density operator ρ'. The complete positivity requirement however does not only satisfy the former, it also crucially allows for the extended mapping of density operators to density operators on a larger composite system. This is due to the fact that *a priori* it is not possible to rule out if a principal system is initially entangled with another faraway unknown system [31].

In order for quantum evolution to be physical, its dynamical process must be described by a CPTP map. Under this definition, a quantum channel is said to be equivalent to a quantum gate and also a noise map. It has been shown that any superoperator that is CPTP can be written in its operator-sum representation, or more commonly known as Kraus representation [32], denoted by

$$\mathcal{E}(\rho) = \sum_i K_i \rho K_i^\dagger, \tag{4}$$

where $\{K_i\}$ are called Kraus operators. It can be shown that Kraus operators satisfy the completeness relation

$$\sum_i K_i^\dagger K_i = I, \tag{5}$$

due to the requirement of Kraus operations being trace preserving [33]. In quantum computation, information are being processed using quantum circuits which make use of unitary gates for operations. It can be seen later that the Kraus representation of superoperator serves as a great mathematical framework for representing a unitary quantum channel and its noisy counterpart.

B. Noise in Quantum Computation

For a particular principal system, the Kraus representation provides for the description of its dynamical process without requiring the explicit consideration of the surrounding environment's attributes; they are all incorporated within the Kraus operators K_i. The operators act solely on the principal system, and therefore serve as an intrinsic means for the characterisation of its transformation. As such, the Kraus operators remain quantities of interest to be determined by experimentalists, i.e. by using QPT [34].

Mathematically, noise in quantum computation can be described by an arbitrary CPTP map in the Kraus representation. One of the most common noise model is the *depolarising channel*, which either has the probability p

of changing an input state into a maximally mixed state $\frac{I}{2}$, or a probability of $1-p$ doing nothing, i.e.

$$\mathcal{E}(\rho) = (1-p)\rho + p\frac{I}{2} \tag{6a}$$
$$= (1-p)\rho + \frac{p}{3}(X\rho X + Y\rho Y + Z\rho Z) \tag{6b}$$

One of its common Kraus decomposition is given in Eq.(6b), with Kraus operators $\sqrt{\frac{p}{3}}X, \sqrt{\frac{p}{3}}Y, \sqrt{\frac{p}{3}}Z$ and $\sqrt{1-p}I$, where the operators are Hermitian and comprised of the identity matrix and Pauli matrices $\{I, X, Y, Z\}$, given by

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{7}$$

III. THE PROTOCOL

A. Specific Notations

Consider a 2-dimensional Hilbert space $\mathcal{H} = \mathbb{C}^2$ where the RB for one qubit is concerned, the action of an ideal unitary quantum channel on a density matrix ρ can be described by a superoperator $\mathcal{U}$, defined as

$$\mathcal{U}(\rho) = U\rho U^\dagger, \tag{8}$$

where U is a Kraus operator representing a unitary operation on ρ. The Hermitian conjugated and repeated application of the channel are then intuitively described by $\mathcal{U}^\dagger(\rho) = U^\dagger\rho U$ and $\mathcal{U}^m(\rho) = U^m\rho U^{m\dagger}$ respectively. The noisy approximation of this channel $\tilde{\mathcal{U}}$ is implemented by the ideal target operation $\mathcal{U}$ in composition with a CPTP map $\mathcal{D}$ - which is not necessarily unitary - such that $\tilde{\mathcal{U}} = \mathcal{D}\circ\mathcal{U}$, with "$\circ$" denoting the channel composition. To be specific, if $\mathcal{D}$ is a depolarising channel in terms of its Kraus decomposition of the Pauli matrices for example, then

$$\begin{aligned}\tilde{\mathcal{U}}(\rho) &= \mathcal{D}\circ\mathcal{U}(\rho) \\ &= \mathcal{D}(U\rho U^\dagger) \\ &= (\sqrt{1-p})^2 U\rho U^\dagger + \left(\sqrt{\frac{p}{3}}\right)^2 (XU\rho U^\dagger X + YU\rho U^\dagger Y + ZU\rho U^\dagger Z) \\ &= (1-p)\mathcal{U}(\rho) + \frac{p}{3}[X\mathcal{U}(\rho)X + Y\mathcal{U}(\rho)Y + Z\mathcal{U}(\rho)Z]\end{aligned} \tag{9}$$

In general, a sequential composition of n channels is denoted as

$$\bigcirc_{i=1}^{n} \mathcal{E}_i(\rho) = \mathcal{E}_n \circ \cdots \circ \mathcal{E}_1(\rho), \tag{10}$$

where $\mathcal{E}_i$ can be a non-unitary superoperator, applied in a right to left manner.

The crux of RB lies on its ability to efficiently quantify the "distance" between an ideal target operation $\mathcal{U}$ and its noisy approximation $\tilde{\mathcal{U}}$ while being immune to SPAM errors. Traditionally, a conventional distance known as *fidelity* is used due to its ability to supply for a "worst case" measure of closeness between the distinguishability of two quantum states ρ and σ under certain conditions (i.e. the maximum overlap between purification of two mixed states) [35]. It is defined by the quantity

$$F(\rho,\sigma) = \left(\mathrm{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}}\right)^2, \tag{11}$$

where tr is the trace operation. Moreover, one can then draw a similar comparison between a noisy implementation of channel $\tilde{\mathcal{U}}$ and its ideal unitary channel $\mathcal{U}$, or just two quantum channel in general, by their *average gate fidelity*

$$F(\mathcal{U},\tilde{\mathcal{U}}) = \int \mathrm{d}\phi\ \mathrm{tr}(\mathcal{U}(\phi)\tilde{\mathcal{U}}(\phi)), \tag{12}$$

where the integral is performed over the unitarily invariant Haar measure $\mathrm{d}\phi$ on state space containing uniformly distributed pure states ϕ [36].

In the case where a gate belongs to a gate set, it is more useful to consider their behaviour with noise as a whole. For a gate set $\mathbf{G}$ containing a total of $|\mathbf{G}|$ ideal unitary gates U_r, one can talk about *gate set fidelity* which is the statistical mean of gate fidelity for all $U_r \in \mathbf{G}$, denoted as $\bar{\mathbf{F}}_\mathbf{G}$, given in a simple form of

$$\bar{\mathbf{F}}_\mathbf{G} = \frac{1}{|\mathbf{G}|}\sum_{r=1}^{|\mathbf{G}|} F(\mathcal{U}_r,\tilde{\mathcal{U}}_r), \tag{13}$$

where $\mathcal{U}_r$ and $\tilde{\mathcal{U}}_r$ are the ideal and noisy unitary superoperator for each unitary gate U_r in $\mathbf{G}$, respectively. Moreover, it has been shown that so long as $\mathbf{G}$ forms a *unitary 2-design*, a scalable and efficient experimental method in estimating the quantity $\bar{\mathbf{F}}_\mathbf{G}$ arises [37], which in essence

is the RB protocol. A gate set $\mathbf{G}$ containing unitary gates $\{U_r\}_{r=1,\dots,|\mathbf{G}|}$ is defined to be a unitary 2-design [38] if the action of the *twirl* over the entire n-qubit unitary group $U(d)$ on the d-dimensional Hilbert space $\mathcal{H} = \mathbb{C}^d$ is equivalent to the twirling of an arbitrary quantum channel $\mathcal{E}$ on a quantum state ρ over $\mathbf{G}$:

$$\begin{aligned}\int_{U(d)} dU \tilde{\mathcal{U}} \circ \mathcal{E} \circ \mathcal{U}(\rho) &:= \frac{1}{|\mathbf{G}|}\sum_{r=1}^{|\mathbf{G}|} \mathcal{U}_r^\dagger \circ \mathcal{E} \circ \mathcal{U}_r(\rho) \\ &= \frac{1}{|\mathbf{G}|}\sum_{r=1}^{|\mathbf{G}|} U_r^\dagger \mathcal{E}(U_r \rho U_r^\dagger) U_r.\end{aligned} \tag{14}$$

B. Randomised Benchmarking

In this section the specific RB protocol described by Alexander *et al.* [39] will be reviewed. The protocol is laid out in detail as follows:

1 For a unitary gate set $\mathbf{G}$ which forms a 2-design, draw from it a random sequence of gates $\{U_r\}_{r=1,\dots,s}$ with sequence length s independently and uniformly at random.

2 Repeat step **1** K_s times to obtain K_s sequences of sequence length s. The jth unitary gate of the ith sequence will be denoted as U_j^i, where $1 \leq i \leq K_s$ and $1 \leq j \leq s$, with $U_j^i \in \mathbf{G}$. Compute the inverse element of every sequence, such that $U_{s+1}^i := (U_s^i U_{s-1}^i \dots U_1^i)^\dagger$. This can be done efficiently if U_j^i is a single qubit gate or if $\mathbf{G}$ forms a group (e.g., a Clifford group)[40].

3 Prepare an input qubit as the ideal state $\rho := |\psi\rangle\langle\psi|$, denoting its noisy preparation as $\tilde{\rho}$.

4 Apply the sampled gates from step **1** and finally their sequence reversal gate U_{s+1}^i. Every noisy channel is described by the aforementioned channel composition:

$$\tilde{\mathcal{U}}_j^i(\rho) = \mathcal{D}_j^i \circ \mathcal{U}_j^i(\rho) \tag{15}$$

where $\mathcal{D}_j^i$ represents a realistic noise model in the form of an arbitrary CPTP map. For a particular ith sequence, the initially prepared mixed state $\tilde{\rho}$ evolves noisily under

$$\tilde{\mathcal{U}}^i(\tilde{\rho}) := \bigcirc_{j=1}^{s+1} \tilde{\mathcal{U}}_j^i(\tilde{\rho}) = \bigcirc_{j=1}^{s+1} [\mathcal{D}_j^i \circ \mathcal{U}_j^i](\tilde{\rho}) \tag{16}$$

5 At the end of the sequence, measure with projectors $\{\tilde{E}_\rho, I - \tilde{E}_\rho\}$ - the noisy implementation of the ideal projective measurement $\{\rho, I - \rho\}$ - giving the ith *survival probability*

$$\mathrm{tr}\Big[\tilde{E}\tilde{\mathcal{U}}^i(\tilde{\rho})\Big]. \tag{17}$$

Subsequently, the *sequence fidelity* can now be obtained as the average of survival probabilities across all K_s random sequences $\tilde{U}^i$ as

$$F_{\mathbf{G}}(s, K_s) := \frac{1}{K_s}\sum_{i=1}^{K_s} \mathrm{tr}\Big[\tilde{E}\tilde{\mathcal{U}}^i(\tilde{\rho})\Big]. \tag{18}$$

It has been shown that in the limit of large number of sequences K_s, asymptotically $F_{\mathbf{G}}(s, K_s)$ converges rapidly to $F_{\mathbf{G}}(s)$, with

$$F_{\mathbf{G}}(s) = \frac{1}{|\mathbf{G}|^s}\sum_{i=1}^{|\mathbf{G}|^s} \mathrm{tr}\Big[\tilde{E}\tilde{\mathcal{U}}^i(\tilde{\rho})\Big] \tag{19}$$

as a uniform average over total number of $|\mathbf{G}|^s$ possible sequences. As a result this asymptotic makes RB a protocol that is efficiently scalable with higher number of qubits and sequence length s [24, 41].

From the original paper Ref. [24], it was derived rigorously that the estimation of $F_{\mathbf{G}}(s)$ over increasing sequence lengths s produces an exponential decay of the sequence fidelity with respect to it. Its decay equation,

$$F_{\mathbf{G}}(s) \approx A_0(2\bar{F}_{\mathbf{G}} - 1)^s + B_0, \tag{20}$$

can be fitted and thus the average gate fidelity $\bar{F}_{\mathbf{G}}$ can be approximated. As aforementioned, RB excels by being able to differentiate errors due to SPAM, and it does so by isolating them into the parameters A_0 and B_0, effectively rendering them "nuisance", providing that the noise are gate-independent and time-independent. Such derivation has also been reviewed in Appendix A of Ref. [39].

IV. THE GATE SET

By the *Gottesman-Knill* theorem, which asserts that any quantum circuit which is composed of gates from the Clifford group (i.e. a normaliser of the qubit Pauli group) can be efficiently simulated by a classical computer [42]. Therefore, Clifford gates had long been a subject of interest in the study of quantum information processing and especially in quantum error correction [43]. However, this crucially implies that for universal quantum computation, the Clifford gates are simply not enough. Moreover, the roles of non-Clifford gates and their possibilities in quantum computation are still not well explored.

The future project will seek to perform an elaborate numerical analysis, addressing towards a particular family of infinite non-Clifford gate sets - which include the Clifford gates in its simplest case - using the standard RB technique for a single qubit with realistic noise models according to modern experiments. Albeit the target

gate set are non-Clifford, they nevertheless still form a 2-design, and therefore are suitable for the RB protocol.

Here the mathematical method of generating the gates are provided, which are built upon prior work done by the supervisor; no derivations however will be given. The targeted non-Clifford gate set $\mathbf{J}$ of size $4n$, such that $\mathbf{J} = \{U_1, U_2, \ldots, U_{4n}, \forall n \geq 3\}$, where $U_i \in \mathbf{J}$ are unitary gates, can be constructed by

$$\mathbf{J} = \{\mathcal{P} \times B \times \{I, C_n, C_n^2, \ldots, C_n^{n-1}\} \times B^\dagger\} \tag{21}$$

where C_n and B are given by

$$C_n = \begin{pmatrix} e^{\frac{2\pi i}{n}} & 0 \\ 0 & e^{-\frac{2\pi i}{n}} \end{pmatrix}, \tag{22a}$$

$$B = \frac{1}{\sqrt{6-2\sqrt{3}}} \begin{pmatrix} 1-i & \sqrt{3}-1 \\ \sqrt{3}-1 & -1-i \end{pmatrix}, \tag{22b}$$

and $\mathcal{P} = \{I, X, Y, Z\}$ is once again the Pauli group containing the identity I and the Pauli matrices $\{X, Y, Z\}$. Note that here only $n \geq 3$ will be concerned since that is the condition for forming a unitary 2-design, i.e. it satisfies Eq.(14). In the simplest case where $n = 3$, $\mathbf{J}$ simply reduces down to a total of 12 gates, i.e.

$$\mathbf{J} = \{\{I, X, Y, Z\} \times B \times \{I, C_3, C_3^2\} \times B^\dagger\} \tag{23}$$

which does form the Clifford group.

V. CONCLUSION

The robust estimation of error rates in quantum gates by the RB protocol has been provided in detail. It was shown when under the assumption that noise model are not gate and time-dependent, the sequence fidelity decay exponentially with respect to its sequence length when the gates are sampled from a unitary 2-design. This defines the characterisation of error rates as aforementioned. It then allowed for the extraction of an average gate fidelity estimate, which is only dependent on the error model's underlying gate-independence, as a parameter $\bar{F}_{\mathbf{G}}$ all while discarding irrelevant figure of merits into other nuisance parameters.

A potential protocol improvement would be to implement that of Helsen *et al.* [44], where the same sequence applied to a down spin input state is also applied to an up spin input state, allowing for the subtraction between their survival probabilities. In doing so, it effectively removes the nuisance parameter B_0, allowing for a better fit of the exponential and hence a more precise estimation of average gate fidelity.

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