

# Generate some random channels

- we are using a methodology from Ruskai and Rudnicki (see e.g. <https://arxiv.org/abs/quant-ph/0101003> and 1707.06926) Start off with a Pauli Liou Superoperator matrix (PL rep) of the form (general introduction eg in 1707.06926 preliminaries)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \tau_1 & \lambda_1 & 0 & 0 \\ \tau_2 & 0 & \lambda_2 & 0 \\ \tau_3 & 0 & 0 & \lambda_3 \end{pmatrix}$$

- We want high fidelity therefore in the PL rep we know that the diagonal elements have to average close to 1
- So we can use a normal distribution, with an average and a sigma we can keep generating random  $\lambda$ s numbers until they lie between min and max (function genTruncated)
- this gives us three  $\lambda$ s. Then we know that  $1 + \lambda_3 < |(\lambda_1 + \lambda_2)|$  and  $1 - \lambda_3 < |(\lambda_2 - \lambda_1)|$  for a completely positive map so we keep generating  $\lambda_3$  until this is the case.
- finally we need a non-unital part  $\tau$ . (You can skip this part if you don't care about your map being unital).
- Each element of  $\tau_x$  has to be smaller than  $1 - |\lambda_x|$
- Then  $||\tau||^2 > 1 - \sum_{x \in \lambda_1 \lambda_2 \lambda_3} (x^2) + 2 * \lambda_1 * \lambda_2 * \lambda_3$
- Finally  $Z\eta(\tau, [\lambda_1 \lambda_2 \lambda_3]) < 0$ , where for  $q(l)$  and  $Z\eta(t,l)$  See equations 20 and 21 of 1707.06926 (Rudnicki et al) (or the code below)

Once we have this, its just a matter of applying a couple of random rotations either side of it. i.e

$$\mathcal{U} \begin{pmatrix} 1 & 0 & 0 & 0 \\ \tau_1 & \lambda_1 & 0 & 0 \\ \tau_2 & 0 & \lambda_2 & 0 \\ \tau_3 & 0 & 0 & \lambda_3 \end{pmatrix} \mathcal{V}^\dagger$$

For the rotations, we know that any rotation can be made up from a 'z' rotation followed by an 'x' rotation followed by 'z' rotation.

For a random rotation, generate three random (small) angles, the rotation in PL form is then:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 & 0 \\ 0 & -\sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos \theta_2 & \sin \theta_2 \\ 0 & 0 & -\sin \theta_2 & \cos \theta_2 \end{pmatrix} * \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta_3 & \sin \theta_3 & 0 \\ 0 & -\sin \theta_3 & \cos \theta_3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$