

1 Simple affine transformations in 3D

1.1 Translate

by Δ (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ - \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (1)$$

1.2 Scale

about origin by S (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ - \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (2)$$

1.3 Rotate

about O_z by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ - \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (3)$$

about O_x by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ - \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (4)$$

about O_y by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ - \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \quad (5)$$

2 Axonometrix projections

$T_{axonometric_{axis}} = T_{rot_1} * T_{rot_2} * T_{orto_{axis}}$
e.g. $T_{axonometric_z} = T_{rot_y} * T_{rot_x} * T_{orto_z}$
where $T_{rot_{axis}}$ is one of matrices from Section 1.3

2.1 Isometric

Isometric projections are commonly used in technical drawings and used to be used in some computer game graphics. In an isometric projection the three axes appear 120° drawings and used to from each other and are equally foreshortened. It can be achieved by rotating an object 45° in the plane of the screen and $\sim 35.3^\circ (\arctan(1/\sqrt{2}))$ through the horizontal axis

2.2 Dimetric