1 Simple affine transformations in 3D

1.1 Translate

by Δ (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{1}$$

1.2 Scale

about origin by S (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{2}$$

1.3 Rotate

about O_z by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{3}$$

about O_x by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{4}$$

about O_y by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{5}$$

2 Axonometric projections

$T_{axonometric_{plane}} = T_{rot_1} * T_{rot_2} * T_{orto_{plane}}$
e.g.

$$T_{axonometric_{z=0}} = T_{rot_y} * T_{rot_x} * T_{orto_{z=0}} \tag{6}$$

$$T_{axonometric_{y=0}} = T_{rot_x} * T_{rot_z} * T_{orto_{y=0}} \tag{7}$$

$$T_{axonometric_{x=0}} = T_{rot_z} * T_{rot_y} * T_{orto_{x=0}} \tag{8}$$

where $T_{rot_{axis}}$ is one of matrices from Section 1.3
and $T_{orto_{plane}}$ is ortographic projection onto a plane
e.g. $T_{orto_{z=0}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
so we get $T_{axonometric_{z=0}} = \begin{bmatrix} \cos\theta & \sin\theta * \cos\phi & 0 & 0 \\ 0 & \cos\phi & 0 & 0 \\ \sin\theta & -\cos\theta * \sin\phi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
where θ and ϕ are rotation angles around O_y and O_x accordingly
trimetric, isometric and dimetric have different ratios of distortion coefficients (K_x, K_y, K_z)
where
$K_{xx=0} = \sqrt{\sin^2\psi - \cos^2\psi * \sin^2\theta}$
$K_{yx=0} = \sqrt{\cos^2\psi + \sin^2\psi * \sin^2\theta}$
$K_{zx=0} = \sqrt{\cos^2\theta}$

$K_{xy=0} = \sqrt{\cos^2\psi}$
$K_{yy=0} = \sqrt{\sin^2\phi - \cos^2\phi * \sin^2\psi}$
$K_{zy=0} = \sqrt{\cos^2\phi + \sin^2\phi * \sin^2\psi}$

$K_{xz=0} = \sqrt{\cos^2\theta + \sin^2\theta * \sin^2\phi}$
$K_{yz=0} = \sqrt{\cos^2\phi}$
$K_{zz=0} = \sqrt{\sin^2\theta - \cos^2\theta * \sin^2\phi}$

2.1 Isometric

Isometric projections are commonly used in technical drawings and used to be used in some computer game graphics. In an isometric projection the three axes appear 120° drawings and used to from each other and are equally foreshortened. It can be achieved by rotating an object 45° in the plane of the screen and $\sim 35.3°(\arctan(1/\sqrt{2}))$ through the horizontal axis.
$K_x = K_y = K_z$

2.2 Dimetric

$K_x = K_y \neq \ K_z$
$K_y = K_z \neq \ K_x$
$K_z = K_x \neq \ K_y$
right angle dimetric projection: $K_x = K_z = 0.94; K_y = 0.47$
$K_x = K_z = 1; K_y = 0.5$

$$\begin{bmatrix} x' \\ y' \\ \text{-} \\ \text{-} \end{bmatrix} = T_{dimetric\ right\ angle} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{9}$$

$$T_{dimetric\ right\ angle} \approx \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

3 Parametric cubic curve surfaces

3.1 Bezier form

$$x(s,t) = SM_bP_xM_b{}^TT^T \tag{11}$$

$$y(s,t) = SM_bP_yM_b{}^TT^T \tag{12}$$

$$z(s,t) = SM_bP_zM_b{}^TT^T \tag{13}$$

$$S = \begin{bmatrix} s^3 & s^2 & s & 1 \end{bmatrix} \tag{14}$$

$$M_b = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{15}$$

$$P - inputdata \tag{16}$$

$$T = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \tag{17}$$