1 Simple affine transformations

1.1 Translate

by Δ(x, y)

$$\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1.2 Rotate

about origin by θ

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1.3 Scale

about origin by (w, h)

$$\begin{bmatrix} w & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2 Complex affine transformations

2.1 Rotate

about p(x, y) by θ

$$\begin{bmatrix} 1 & 0 & px \\ 0 & 1 & py \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -px \\ 0 & 1 & -py \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2.2 Scale

about p(x, y) by (w, h)

$$\begin{bmatrix} 1 & 0 & px \\ 0 & 1 & py \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -px \\ 0 & 1 & -py \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$