1 Simple affine transformations in 3D

1.1 Translate

by Δ (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta x \\ 0 & 1 & 0 & \Delta y \\ 0 & 0 & 1 & \Delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{1}$$

1.2 Scale

about origin by S (x, y, z)

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{2}$$

1.3 Rotate

about O_z by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{3}$$

about O_x by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{4}$$

about O_y by θ

$$\begin{bmatrix} x' \\ y' \\ z' \\ \_ \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \tag{5}$$

2 Axonometrix projections

$T_{axonometric_{plane}} = T_{rot_1} * T_{rot_2} * T_{orto_{plane}}$
e.g.

$$T_{axonometric_{z=0}} = T_{rot_y} * T_{rot_x} * T_{orto_{z=0}} \tag{6}$$

$$T_{axonometric_{y=0}} = T_{rot_x} * T_{rot_z} * T_{orto_{y=0}} \tag{7}$$

$$T_{axonometric_{x=0}} = T_{rot_z} * T_{rot_y} * T_{orto_{x=0}} \tag{8}$$

where $T_{rot_{axis}}$ is one of matrices from Section 1.3
and $T_{orto_{plane}}$ is ortographic projection onto a plane
e.g. $T_{orto_{z=0}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
so we get $T_{axonometric_{z=0}} = \begin{bmatrix} \cos\theta & \sin\theta * \cos\phi & 0 & 0 \\ 0 & \cos\phi & 0 & 0 \\ \sin\theta & -\cos\theta * \sin\phi & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
where θ and ϕ are rotation angles around O_x and O_y accordingly
trimetric, isometric and dimetric have different ratios of distortion coefficients (K_x, K_y, K_z)

2.1 Isometric

Isometric projections are commonly used in technical drawings and used to be used in some computer game graphics. In an isometric projection the three axes appear 120° drawings and used to from each other and are equally foreshortened. It can be achieved by rotating an object 45° in the plane of the screen and $\sim 35.3°(\arctan(1/\sqrt{2}))$ through the horizontal axis

2.2 Dimetric