Fachhochschule Nordwestschweiz

Exersise sheet 2

Ex 1

Statement:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Base case $n=1$:

$$\sum_{i=1}^{1} i = \underline{1} = \frac{1(1+1)}{2} = \frac{2}{2} = \underline{1} \checkmark$$

Inductive step $n+1$:

$$\sum_{i=1}^{n+1} i = \underbrace{\sum_{i=1}^{n} i + \underline{n+1}} = \frac{(n+1)(n+1+1)}{2}$$

$$= \frac{n^2+3n+2}{2}$$

$$= \frac{n(n+3)+2}{2}$$

$$= \frac{n(n+1)+2n+2}{2}$$

$$= \underbrace{\frac{n(n+1)}{2} + \underline{n+1}} \checkmark$$

Ex 2

Statement:

$$\sum_{i=1}^{d} (2^{d-i} \cdot i) \leq 2^{d+1} - d - 2$$

Base Case $d=1$

$$\sum_{i=1}^{1} (2^{1-i} \cdot i) = 2^{1-1} \cdot 1 = \underline{1} \leq 2^{1+1} - 1 - 2 = \underline{1}$$ ✓

Inductive Step $d \mapsto d+1$

$$\sum_{i=1}^{d+1} (2^{d+1-i} \cdot i) = \sum_{i=1}^{d} (2^{d+1-i} \cdot i) + \underbrace{2^{\overbrace{d+1-(d+1)}^{0}}}_{1}(d+1)$$

$$= \underline{2 \sum_{i=1}^{d} (2^{d-i} \cdot i) + d + 1}$$

$$\leq 2^{d+1+1} - (d+1) - 2$$

$$= 2^{d+2} - d - 3$$

$$= 2(2^{d+1}) - d - 3$$

$$= 2(2^{d+1}) - 2(d+2) + d + 1$$

$$= \underline{2(2^{d+1} - d - 2) + d + 1}$$ ✓