

Exercise sheet 2Ex 1

Statement:

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

Base case  $n=1$ :

$$\sum_{i=1}^1 i = 1 = \frac{1(1+1)}{2} = \frac{2}{2} = 1 \quad \checkmark$$

Inductive step  $n+1$ :

$$\begin{aligned} \sum_{i=1}^{n+1} i &= \sum_{i=1}^n i + n+1 = \frac{(n+1)(n+1+1)}{2} \\ &= \frac{n^2 + 3n + 2}{2} \\ &= \frac{n(n+3) + 2}{2} \\ &= \frac{n(n+1) + 2n+2}{2} \\ &= \frac{n(n+1)}{2} + \frac{n+1}{1} \quad \checkmark \end{aligned}$$



Ex 2

Statement:

$$\sum_{i=1}^d (2^{d-i} \cdot i) \leq 2^{d+1} - d - 2$$

Base Case  $d=1$

$$\sum_{i=1}^1 (2^{1-i} \cdot i) = 2^{1-1} \cdot 1 = 1 \leq 2^{1+1} - 1 - 2 = 1 \quad \checkmark$$

Inductive Step  $d \mapsto d+1$

$$\begin{aligned} \sum_{i=1}^{d+1} (2^{d+1-i} \cdot i) &= \sum_{i=1}^d (2^{d+1-i} \cdot i) + \overbrace{2^{d+1-(d+1)} \cdot (d+1)}^0 \\ &= 2 \sum_{i=1}^d (2^{d-i} \cdot i) + \underbrace{d+1} \\ &\leq 2^{d+1+1} - (d+1) - 2 \\ &= 2^{d+2} - d - 3 \\ &= 2(2^{d+1}) - d - 3 \\ &= 2(2^{d+1}) - 2(d+2) + d + 1 \\ &= \underbrace{2(2^{d+1} - d - 2)} + \underbrace{d + 1} \quad \checkmark \end{aligned}$$