Chapter 2, Problem 4

Let $\{e_t\}$ be a zero-mean white noise process. Suppose that the observed process is $Y_t = e_t + \theta e_{t-1}$, where θ is either 3 or 1/3.

(a) Find the autocorrelation function $\{Y_t\}$ both when $\theta = 3$ and when $\theta = 1/3$.

(b) You should have discovered that the time series is stationary regardless of the value of θ and that the autocorrelation functions are the same for $\theta = 3$ and $\theta = 1/3$. For simplicity, suppose that the process mean is known to be zero and the variance of Y_t is known to be 1. You observe the series $\{Y_t\}$ for $t = 1, 2, ..., n$ and suppose that you can produce good estimates of the autocorrelations ρ_k. Do you think that you could determine which value of θ is correct (3 or 1/3) based on the estimate of ρ_k? Why or why not?

Chapter 2, Problem 6

Let $\{X_t\}$ be a stationary time series, and define $Y_t = \begin{cases} X_t & \text{for } t \text{ odd.} \\ X_t + 3 & \text{for } t \text{ even.} \end{cases}$

(a) Show that $Cov(Y_t, Y_{t-k})$ is free of t for all lags k.

(b) Is $\{Y_t\}$ stationary?

Chapter 2, Problem 11

Suppose $Cov(X_t, X_{t-k}) = \gamma_k$ is free of t but that $E(X_t) = 3t$.

(a) Is $\{X_t\}$ stationary?

(b) Let $Y_t = 7 - 3t + X_t$. Is $\{Y_t\}$ stationary?

Chapter 2, Problem 21

For a random walk with random starting value, let $Y_t = Y_0 + e_t + e_{t-1} + \cdots + e_1$ for $t > 0$ where Y_0 has a distribution with mean μ_0 and variance σ_0^2. Suppose further that $Y_0, e_1, ..., e_t$ are independent.

(a) Show that $E(Y_t) = \mu_0$ for all t.

(b) Show that $Var(Y_t) = t\sigma_e^2 + \sigma_0^2$.

(c) Show that $Cov(Y_t, Y_s) = \min(t, s)\sigma_e^2 + \sigma_0^2$.

(d) Show that $Corr(Y_t, Y_s) = \sqrt{\dfrac{t\sigma_a^2 + \sigma_0^2}{s\sigma_a^2 + \sigma_0^2}}$ for $0 \leq t \leq s$.

Chapter 2, Problem 23

Two processes $\{Z_t\}$ and $\{Y_t\}$ are said to be **independent** if for any time points $t_1, t_2, ..., t_m$ and $s_1, s_2, ..., s_n$ the random variables $\{Z_{t_1}, Z_{t_2}, ..., Z_{t_m}\}$ are independent of the random variables $\{Y_{s_1}, Y_{s_2}, ..., Y_{s_n}\}$. Show that if $\{Z_t\}$ and $\{Y_t\}$ are independent stationary processes, then $W_t = Z_t + Y_t$ is stationary.
