Chapter 4, Problem 14

Suppose that $\{Y_t\}$ is a zero mean, stationary process with $\mid \rho_1 \mid < 0.5$ and $\rho_k = 0$ for $k > 1$. Show that $\{Y_t\}$ must be representable as an MA(1) process. That is, show that there is a white noise sequence $\{e_t\}$ such that $Y_t = e_t - \theta e_{t-1}$, where ρ_1 is correct and e_t is uncorrelated with Y_{t-k} for $k > 0$. (Hint: Choose θ such that $\mid \theta \mid < 1$ and $\rho_1 = -\theta/(1+\theta^2)$; then let $e_t = \sum_{j=0}^{\infty} \theta^j Y_{t-j}$. If we assume that $\{Y_t\}$ is a normal process, e_t will also be normal, and zero correlation is equivalent to independence.)

Chapter 4, Problem 16

Consider the "nonstationary" AR(1) model $Y_t = 3Y_{t-1} + e_t$.

(a) Show that $Y_t = -\sum_{j=1}^{\infty} \left(\frac{1}{3}\right)^j e_{t+j}$ satisfies the AR(1) equation.

(b) Show that the process defined in part (a) is stationary.

(c) In what way is this solution unsatisfactory?

Chapter 4, Problem 19

Consider an MA(6) model with $\theta_1 = 0.5$, $\theta_2 = -0.25$, $\theta_3 = 0.125$, $\theta_4 = -0.0625$, $\theta_5 = 0.003125$, and $\theta_6 = -0.015625$. Find a much simpler model that has nearly the same Φ-weights.

Chapter 5, Problem 1

Identify the following as specific ARIMA models. That is, what are p, d, and q and what are the values of the parameters (the ϕ's and θ's)?

(a) $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$.

(b) $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$.

(c) $Y_t = 0.5Y_{t-1} - 0.5Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$.

Chapter 5, Problem 2

For each of the ARIMA models below, give the values for $E(\nabla Y_t)$ and $Var(\nabla Y_t)$.

(a) $Y_t = 4 + Y_{t-1} + e_t - 0.75e_{t-1}$.

(b) $Y_t = 10 + 1.25Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$.

(c) $Y_t = 5 + 2Y_{t-1} - 1.7Y_{t-2} + 0.7Y_{t-3} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$.

Chapter 5, Problem 11

The data file WINNEBAGO contains monthly unit sales of recreational vehicle (RVs) from Winnebago, Inc., from November 1966 through February 1972.

(a) Display and interpret the time series plot for these data.

(b) Now take natural logarithms of the monthly sales figures and display the time series plot of the transformed values. Describe the effect of the logarithms on the behavior of the series.

(c) Calculate the fractional relative changes, $(Y_t - Y_{t-1})/Y_{t-1}$, and compare them with the differences of (natural) logarithms, $\nabla \log(Y_t) = \log(Y_t) - \log(Y_{t-1})$. How do they compare for smaller values and for larger values?

Chapter 5, Problem 15

Quarterly earnings per share for the Johnson & Johnson Company are given in the data file named JJ. The data cover the years from 1960 through 1980.

(a) Display a time series plot of the data. Interpret the interesting features in the plot.

(b) Use software to produce a plot similar to exhibit 5.11, on page 102, and determine the "best" value of λ for a power transformation of the data.

(c) Display a time series plot of the transformed values. Does this plot suggest that a stationary model might be appropriate?

(d) Display a time series plot of the differences of the transformed values. Does this plot suggest that a stationary model might be appropriate for the differences?