Chapter 3, Problem 11

(Continuation of Exercise 3.5) Return to the WAGES series.

(a) Consider the residuals from a least squares fit of a quadratic time trend.

(b) Perform a runs test on the standardized residuals and interpret the results.

(c) Calculate and interpret the sample autocorrelations for the standardized residuals.

(d) Investigate the normality of the standardized residuals (error terms). Consider histograms and normal probability plots. Interpret the plots.

Chapter 3, Problem 13

(Continuation of Exercise 3.7) Return to the WINNEBAGO time series.

(a) Calculate the least squares residuals from a seasonal-means plus linear time trend model on the logarithms of the sales time series.

(b) Perform a runs test on the standardized residuals and interpret the results.

(c) Calculate and interpret the sample autocorrelations for the standardized residuals.

(d) Investigate the normality of the standardized residuals (error terms). Consider histograms and normal probability plots. Interpret the plots.

Chapter 3, Problem 16

Suppose that a stationary time series $\{Y_t\}$, has an autocorrelation function of the form $\rho_k = \phi^k$ for $k > 0$, where ϕ is a constant in the range(-1,+1).

(a) Show that $Var(\bar{Y}) = \frac{\gamma_0}{n}\left[\frac{1+\phi}{1-\phi} - \frac{2\phi}{n}\frac{(1-\phi^n)}{(1-\phi)^2}\right]$. (Hint: Use Equation (3.2.3) on page 28, the finite geometric sum

$$\sum_{k=0}^{n} \phi^k = \frac{1-\phi^{n+1}}{1-\phi}, \text{ and the related sum } \sum_{k=0}^{n} k\phi^{k-1} = \frac{d}{d\phi}\left[\sum_{k=0}^{n} \phi^k\right].$$

(b) If n is large, argue that $Var(\bar{Y}) \approx \frac{\gamma_0}{n}\left[\frac{1+\phi}{1-\phi}\right]$.

(c) Plot $(1+\phi)/(1-\phi)$ for ϕ over the range -1 to +1. Interpret the plot in terms of the precision in estimating the process mean.

Chapter 4, Problem 1

Use first principles to find the autocorrelation function for the stationary process defined by

$$Y_t = 5 + e_t - \frac{1}{2}e_{t-1} + \frac{1}{4}e_{t-2}$$

Chapter 4, Problem 2

Sketch the autocorrelation functions for the following MA(2) models with parameters as specified:

(a) $\theta_1 = 0.5$ and $\theta_2 = 0.4$.

(b) $\theta_1 = 1.2$ and $\theta_2 = -0.7$.

(c) $\theta_1 = -1$ and $\theta_2 = -0.6$.

Chapter 4, Problem 5

Calculate and sketch the autocorrelation functions for each of the following AR(1) models. Plot for sufficient lags that the autocorrelation function has nearly died out.

(a) $\phi_1 = 0.6$.

(b) $\phi_1 = -0.6$.

(c) $\phi_1 = 0.95$. (Do out to 20 lags.)

(d) $\phi_1 = 0.3$.