

Chapter 3, Problem 11

(Continuation of Exercise 3.5) Return to the WAGES series.

- (a) Consider the residuals from a least squares fit of a quadratic time trend.
 - (b) Perform a runs test on the standardized residuals and interpret the results.
 - (c) Calculate and interpret the sample autocorrelations for the standardized residuals.
 - (d) Investigate the normality of the standardized residuals (error terms). Consider histograms and normal probability plots. Interpret the plots.
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Chapter 3, Problem 13

(Continuation of Exercise 3.7) Return to the WINNEBAGO time series.

- (a) Calculate the least squares residuals from a seasonal-means plus linear time trend model on the logarithms of the sales time series.
 - (b) Perform a runs test on the standardized residuals and interpret the results.
 - (c) Calculate and interpret the sample autocorrelations for the standardized residuals.
 - (d) Investigate the normality of the standardized residuals (error terms). Consider histograms and normal probability plots. Interpret the plots.
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Chapter 3, Problem 16

Suppose that a stationary time series $\{Y_t\}$, has an autocorrelation function of the form $\rho_k = \phi^k$ for $k > 0$, where ϕ is a constant in the range $(-1, +1)$.

- (a) Show that $Var(\bar{y}) = \frac{\gamma_0}{n} \left[\frac{1+\phi}{1-\phi} - \frac{(1-\phi^n)}{(1-\phi)^2} \right]$. (Hint: Use Equation (3.2.3) on page 28, the finite geometric sum

$$\sum_{k=0}^n \phi^k = \frac{1-\phi^{n+1}}{1-\phi}, \text{ and the related sum } \sum_{k=0}^n k\phi^{k-1} = \frac{d}{d\phi} \left[\sum_{k=0}^n \phi^k \right].$$

- (b) If n is large, argue that $Var(\bar{Y}) \approx \frac{\gamma_0}{n} \left[\frac{1+\phi}{1-\phi} \right]$.
- (c) Plot $(1+\phi)/(1-\phi)$ for ϕ over the range -1 to $+1$. Interpret the plot in terms of the precision in estimating the process mean.
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Chapter 4, Problem 1

Use first principles to find the autocorrelation function for the stationary process defined by

$$Y_t = 5 + e_t - \frac{1}{2}e_{t-1} + \frac{1}{4}e_{t-2}$$

Chapter 4, Problem 2

Sketch the autocorrelation functions for the following MA(2) models with parameters as specified:

(a) $\theta_1 = 0.5$ and $\theta_2 = 0.4$.

(b) $\theta_1 = 1.2$ and $\theta_2 = -0.7$.

(c) $\theta_1 = -1$ and $\theta_2 = -0.6$.

Chapter 4, Problem 5

Calculate and sketch the autocorrelation functions for each of the following AR(1) models. Plot for sufficient lags that the autocorrelation function has nearly died out.

- (a) $\phi_1 = 0.6$.
 - (b) $\phi_1 = -0.6$.
 - (c) $\phi_1 = 0.95$. (Do out to 20 lags.)
 - (d) $\phi_1 = 0.3$.
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