Chapter 4, Problem 6

Suppose that $\{Y_t\}$ is an AR(1) process with $-1 < \phi < +1$.

(a) Find the autocovariance function for $W_t = \nabla Y_t = Y_t - Y_{t-1}$ in terms of ϕ and σ_e^2.

(b) In particular, show that $Var(W_T) = 2\sigma_e^2/(1+\phi)$.

Chapter 4, Problem 7

Describe the important characteristics of the autocorrelation function for the following models: (a) MA(1), (b) MA(2), (c) AR(1), (d) AR(2), and (e) ARMA(1,1).

Chapter 4, Problem 9

Use the recursive formula of Equation (4.3.13) to calculate and then sketch the autocorrelation functions for the following AR(2) models with parameters as specified. In each case, specify whether the roots of the characteristic equation are real or complex. If the roots are complex, find the damping factor, R, and frequency, Θ, for the corresponding autocorrelation function when expressed as in Equation (4.3.17), on page 73.

(a) $\phi_1 = 0.6$ and $\phi_2 = 0.3$.

(b) $\phi_1 = -0.4$ and $\phi_2 = 0.5$.

(c) $\phi_1 = 1.2$ and $\phi_2 = -0.7$.

(d) $\phi_1 = -1$ and $\phi_2 = -0.6$.

(e) $\phi_1 = 0.5$ and $\phi_2 = -0.9$.

(f) $\phi_1 = -0.5$ and $\phi_2 = -0.6$.

Chapter 4, Problem 13

Let $\{Y_t\}$ be a stationary process with $\rho_k = 0$ for $k > 1$. Show that we must have $\mid \rho_1 \mid \leq \frac{1}{2}$. (Hint: Consider $Var(Y_{n+1} + Y_n + \cdots + Y_1)$ and then $Var(Y_{n+1} - Y_n + Y_{n-1} - \cdots \pm Y_1)$. Use the fact that both of these must be nonnegative for *all* n.)

Chapter 4, Problem 23

Suppose that $\{Y_t\}$ is an AR(1) process with $\rho_1 = \phi$. Define the sequence $\{b_t\}$ as $b_1 = Y_t - \phi Y_{t+1}$.

(a) Show that $Cov(b_1, b_{t-k}) = 0$ for all t and k.

(b) Show that $Cov(b_t, Y_{t+k}) = 0$ for all t and $k > 0$.