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Fundamentals of Fuzzy Logic Control – Fuzzy Sets, Fuzzy Rules and Defuzzifications

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2.1 Introduction

Fuzzy logic idea is similar to the human being's feeling and inference process. Unlike classical control strategy, which is a point-to-point control, fuzzy logic control is a range-to-point or range-to-range control. The output of a fuzzy controller is derived from fuzzifications of both inputs and outputs using the associated membership functions. A crisp input will be converted to the different members of the associated membership functions based on its value. From this point of view, the output of a fuzzy logic controller is based on its memberships of the different membership functions, which can be considered as a range of inputs.

Fuzzy ideas and fuzzy logic are so often utilized in our routine life that nobody even pays attention to them. For instance, to answer some questions in certain surveys, most time one could answer with 'Not Very Satisfied' or 'Quite Satisfied', which are also fuzzy or ambiguous answers. Exactly to what degree is one satisfied or dissatisfied with some service or product for those surveys? These vague answers can only be created and implemented by human beings, but not machines. Is it possible for a computer to answer those survey questions directly as a human beings did? It is absolutely impossible. Computers can only understand either '0' or '1', and 'HIGH' or 'LOW'. Those data are called crisp or classic data and can be processed by all machines.

Is it possible to allow computers to handle those ambiguous data with the help of a human being? If so, how can computers and machines handle those vague data? The answer to the first question is yes. But to answer the second question, we need some fuzzy logic techniques and knowledge of fuzzy inference system.

The idea of fuzzy logic was invented by Professor L. A. Zadeh of the University of California at Berkeley in 1965 [1]. This invention was not well recognized until Dr. E. H. Mamdani, who is a professor at London University, applied the fuzzy logic in a practical application to control an automatic steam engine in 1974 [2], which is almost ten years after the fuzzy theory was invented. Then, in 1976, Blue Circle Cement and SIRA in Denmark developed an industrial application to control cement kilns [3]. That system began to operation in 1982. More and more fuzzy implementations have been reported since the 1980s, including those applications in industrial manufacturing, automatic control,

automobile production, banks, hospitals, libraries and academic education. Fuzzy logic techniques have been widely applied in all aspects in today's society.

To implement fuzzy logic technique to a real application requires the following three steps:

1. Fuzzification – convert classical data or crisp data into fuzzy data or Membership Functions (MFs)
2. Fuzzy Inference Process – combine membership functions with the control rules to derive the fuzzy output
3. Defuzzification – use different methods to calculate each associated output and put them into a table: the lookup table. Pick up the output from the lookup table based on the current input during an application

As mentioned before, all machines can process crisp or classical data such as either '0' or '1'. In order to enable machines to handle vague language input such as 'Somehow Satisfied', the crisp input and output must be converted to linguistic variables with fuzzy components. For instance, to control an air conditioner system, the input temperature and the output control variables must be converted to the associated linguistic variables such as 'HIGH', 'MEDIUM', 'LOW' and 'FAST', 'MEDIUM' or 'SLOW'. The former is corresponding to the input temperature and the latter is associated with the rotation speed of the operating motor. Besides those conversions, both the input and the output must also be converted from crisp data to fuzzy data. All of these jobs are performed by the first step – fuzzification.

In the second step, to begin the fuzzy inference process, one need combine the Membership Functions with the control rules to derive the control output, and arrange those outputs into a table called the lookup table. The control rule is the core of the fuzzy inference process, and those rules are directly related to a human being's intuition and feeling. For example, still in the air conditioner control system, if the temperature is too high, the heater should be turned off, or the heat driving motor should be slowed down, which is a human being's intuition or common sense. Different methods such as Center of Gravity (COG) or Mean of Maximum (MOM) are utilized to calculate the associated control output, and each control output should be arranged into a table called lookup table.

During an actual application, a control output should be selected from the lookup table developed from the last step based on the current input. Furthermore, that control output should be converted from the linguistic variable back to the crisp variable and output to the control operator. This process is called defuzzification or step 3.

In most cases the input variables are more than one dimension for real applications, and one needs to perform fuzzification or develop a Membership Function for each dimensional variable separately. Perform the same operation if the system has multiple output variables.

Summarily, a fuzzy process is a process of crisp-fuzzy-crisp for a real system. The original input and the terminal output must be crisp variables, but the intermediate process is a fuzzy inference process. The reason why one needs to change a crisp to a fuzzy variable is that, from the point of view of fuzzy control or a human being's intuition, no absolutely crisp variable is existed in our real world.

Any physical variable may contain some other components. For instance, if someone says: the temperature here is high. This high temperature contains some middle and even low temperature components. From this point of view, fuzzy control uses universal or global components, not just a limited range of components as the classical variables did.

With the rapid development of fuzzy technologies, different fuzzy control strategies have been developed based on different classical control methods, such as PID-fuzzy control [4], sliding-mode fuzzy control [5], neural fuzzy control, adaptor fuzzy control [6] and phase-plan mapping fuzzy control [7]. More and more new fuzzy control strategies or combined crisp and fuzzy control techniques are being developed and will be applied to many areas in our society in the future.

This chapter starts with an introduction to fuzzy logic control and fuzzy terminology. Following sections provide detailed description of the fuzzy inference process and the architecture of fuzzy logic control. Section 2 discusses fuzzy sets and crisp sets. Fuzzifications and the membership function are covered in Section 3. Fuzzy control rules and defuzzification are discussed in Sections 4 and 5, respectively. Some related combined fuzzy control methods are provided in the remaining sections of this chapter.

2.2 Fuzzy Sets

The concept of the fuzzy set is only an extension of the concept of a classical or crisp set. The fuzzy set is actually a fundamentally broader set compared with the classical or crisp set. The classical set only considers a limited number of degrees of membership such as '0' or '1', or a range of data with limited degrees of membership. For instance, if a temperature is defined as a crisp high, its range must be between 80 °F and higher and it has nothing to do with 70 °F or even 60 °F. But the fuzzy set will take care of a much broader range for this high temperature. In other words, the fuzzy set will consider a much larger temperature range such as from 0 °F to higher degrees as a high temperature. The exact degree to which the 0 °F can contribute to that high temperature depends on the membership function. This means that the fuzzy set uses a universe of discourse as its base and it considers an infinite number of degrees of membership in a set. In this way, the classical or crisp set can be considered as a subset of the fuzzy set.

2.2.1 Classical Sets and Operations

A classical set is a collection of objects in a given range with a sharp boundary. An object can either belong to the set or not belong to the set. For example, we assume to create a faculty set or a faculty collection F with ten-faculty members $x_1, x_2 \ldots \ldots x_{10}$, in a college:

$$F = \{ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \} \qquad (2.1)$$

In general, the entire object of discussion F is called a universe of discourse, and each member x_i is called an element. Assuming that elements $x_1 \sim x_4$ belong to the department of computer science, which can be considered as another set A. The elements $x_1 \sim x_3$ are under age 40, which can be considered as a set B. Therefore the following relations exist:

$$\begin{aligned} & X \in F \\ & A = \{x_1, x_2, x_3, x_4\} \in F \\ & B = \{x_1, x_2, x_3\} \subset A \end{aligned} \qquad (2.2)$$

It can be seen that all elements in set B belong to set A, or the set A contains set B. In this case, set B can be considered as a subset of set A and can be expressed as $B \subset A$.

The relationship among different sets we discussed above can be described in Figure 2.1. The basic classical set operations include complement, intersection and union, which are represented as

Complement of A (A^C) $\qquad A^C(x) = 1 - A$ $\qquad\qquad$ (2.3)

Intersection of A and B $(A \cap B)$ $\quad A \cap B = A(x) \cap B(x)$ $\qquad$ (2.4)

Union of A and B $(A \cup B)$ $\qquad A \cup B = A(x) \cup B(x)$ $\qquad$ (2.5)
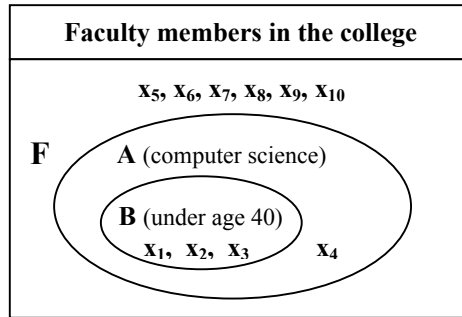

Figure 2.1. The classical set of faculty members in a college

The representations of those classical set operations are shown in Figure 2.2.

It is clear that an element either belongs to a set or does not belong to that set in the classical set and its operation. There is a sharp boundary between different elements for different sets and these cannot be mixed with each other. However, for the fuzzy set, it has different laws.

2.2.2 Mapping of Classical Sets to Functions

The classical set we discussed in the last section can be represented or mapped into some functions. This means that we can relate set-theoretic forms to function-theoretic representations, and map elements on one universe of discourse to elements or sets in another universe. For the classical or crisp sets, this mapping is

very easy and straightforward. Assume that X and Y are two different universes of discourse. If an element x belongs to X and it corresponds to an element y belonging to Y, the mapping between them can be expressed as

$$\mu_A(x) = \begin{cases} 1, & (x \in A) \\ 0, & (x \notin A) \end{cases} \tag{2.6}$$

where μ_A represents 'membership' in a set A for the element x in the universe. This membership idea is exactly a mapping from the element x in the universe X to one of two elements in universe Y, or to elements 0 or 1.
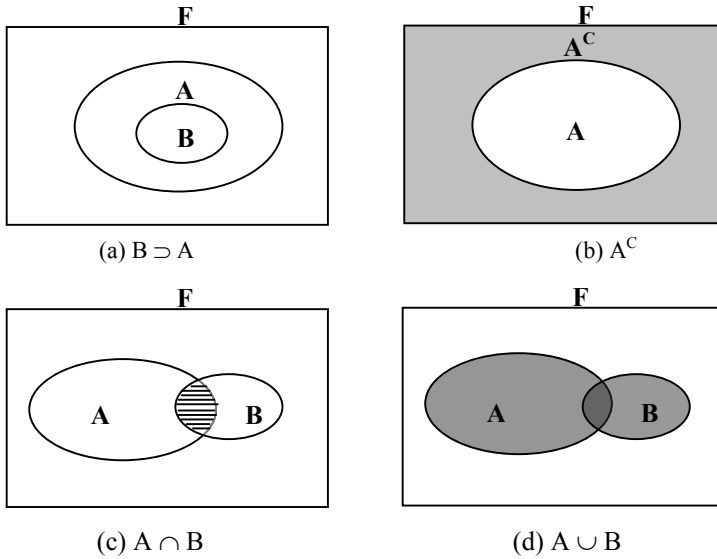

Figure 2.2. The classical set and its operations

Still using the example in the last section, a set of faculty members in a college that contains two subsets A and B, we want to define a new set P(x) as

$$P(x) = \{ A, B \} \tag{2.7}$$

and mapping these two sets from universe of discourse F to another universe Y with two elements, 0 or 1.

$$M(P(x)) = \{ (1, 1, 1, 1, 0, 0, 0, 0, 0, 0), (1, 1, 1, 0, 0, 0, 0, 0, 0, 0) \} \tag{2.8}$$

Now we can define operations of two classical sets, says A and B, on the universe F as follows:

Union: $A \cup B = \mu_A(x) \cup \mu_B(x) = \max (\mu_A(x), \mu_B(x))$

Intersection: $A \cap B = \mu_A(x) \cap \mu_B(x) = \min (\mu_A(x), \mu_B(x))$
Complement: $A^C = F \setminus A$ (2.9)

2.2.3 Fuzzy Sets and Operations

As we discussed in the previous section, the classical set has a sharp boundary, which means that a member either belongs to that set or does not. Also, this classical set can be mapped to a function with two elements, 0 or 1. For example, in the previous section, we defined the faculty member in the department of computer science as set A. A faculty either fully belongs to this set ($\mu_A(x) = 1$) if one is a faculty of the computer science department or has nothing to do with set A ($\mu_A(x) = 0$) if he is not a faculty in that department. This mapping is straightforward with sharp boundary without any ambiguity. In other words, this 'fully belonging to' can be mapped as a member of set A with degree of 1, and 'not belong to' can be mapped as a member of set A with degree of 0. This mapping is similar to a black-and-white binary categorization.

Compared with a classical set, a fuzzy set allows members to have a smooth boundary. In other words, a fuzzy set allows a member to belong to a set to some partial degree. For instance, still using the temperature as an example, the temperature can be divided into three categories: LOW (0 ~30 °F), MEDIUM (30 °F ~ 70 °F) and HIGH (70 ~ 120 °F) from the point of view of the classical set, which is shown in Figure 2.3a.
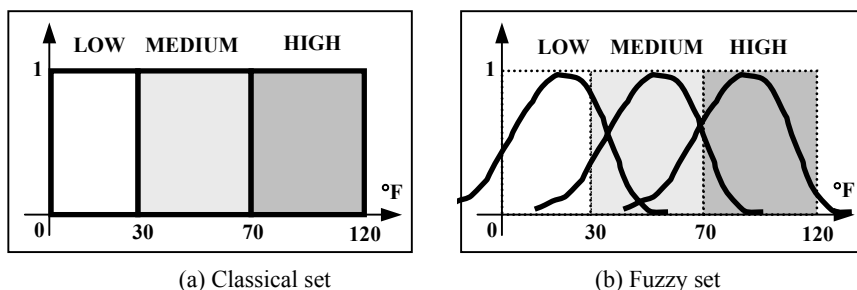

(a) Classical set   (b) Fuzzy set

Figure 2.3. Representations of classical and fuzzy sets

In the classical set, any temperature can only be categorized into one subset, either LOW, MEDIUM or HIGH, and the boundary is crystal clear. But in the fuzzy set such as shown in Figure 2.3b, these boundaries becomes vague or smooth. One temperature can be categorized into two or maybe even three subsets simultaneously. For example, the temperature 40 °F can be considered to belong to LOW to a certain degree, say 0.5 degree, but at the same time it can belong to MEDIUM to about 0.7 degree. Another interesting thing is the temperature 50 °F, which can be considered to belong to LOW and HIGH to around 0.2 degree and belong to MEDIUM to almost 1 degree. The dash-line in Figure 2.3b represents the classical set boundary.

It is clear that a fuzzy set contains elements which have varying degrees of membership in the set, and this is contrasted with the classical or crisp sets because members of a classical set cannot be members unless their membership is full or complete in that set. A fuzzy set allows a member to have a partial degree of membership and this partial degree membership can be mapped into a function or a universe of membership values. Assume that we have a fuzzy set A, and if an element x is a member of this fuzzy set A, this mapping can be denoted as

$$\mu_A(x) \in [0, 1] \quad (A = (x, \mu_A(x)| x \in X) \quad (2.10)$$

A fuzzy subset A with an element x has a membership function of $\mu_A(x)$. When the universe of discourse X is discrete and finite, this mapping can be expressed as

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \cdots\cdots = \sum_i \frac{\mu_A(x_i)}{x_i} \quad (2.11)$$

When the universe X is continuous and infinite, the fuzzy set A can be represented as

$$A = \int \frac{\mu_A(x)}{x} \quad (2.12)$$

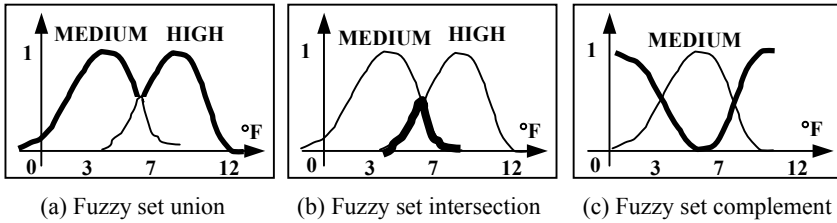(a) Fuzzy set union   (b) Fuzzy set intersection   (c) Fuzzy set complement

Figure 2.4. Fuzzy set operations

As we discussed in the previous section for the classical set operations, the basic fuzzy set operations also include intersection, union and complement, and those operations are defined as

Union: $A \cup B = \mu_A(x) \cup \mu_B(x) = \max (\mu_A(x), \mu_B(x))$
Intersection: $A \cap B = \mu_A(x) \cap \mu_B(x) = \min (\mu_A(x), \mu_B(x))$
Complement: $A^C = F \setminus A$ (2.13)

where A and B are two fuzzy sets and x is an element in the universe of discourse, X. The graphical illustrations of those operations are shown in Figure 2.4.

According to the definition of fuzzy set operations, a fuzzy set union operation is exactly equivalent to select the maximum member from those members in the sets, and the intersection operation is selecting the minimum member from the sets.

2.2.4 Comparison Between Classical and Fuzzy Sets

Recall that in Section 2.2.1 we defined a college faculty set F, a faculty set for the computer science department, set A, and a set of faculty members who are under age 40, or set B. Now we can make a clear comparison for the classical and fuzzy sets using this example.

According to classical or crisp set theory, set B includes only three members who are under age 40 (x_1, x_2, x_3), so the boundary between the faculty who are under age 40 and the faculty who are above age 40 is crystal clear (see solid-line in Figure 2.5). However, for the fuzzy set theory, set B contains not only those three members, x_1, x_2, x_3, but also some other members to some varying degree. The actual age distribution of all ten faculty members is displayed in the x-axis in Figure 2.5.

It can be found from Figure 2.5 that the member x_4 and x_5 do not belong to the set B in the view of the crisp set, but they can be considered as partial members in a partial membership of 0.7 and 0.3 degrees, respectively. Mapping this to a membership function, it can be expressed as

$$\mu_B(x) = \frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{0.7}{x_4} + \frac{0.3}{x_5} \tag{2.14}$$
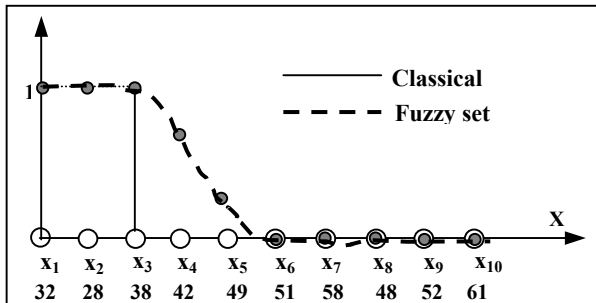

Figure 2.5. The comparison between classical and fuzzy sets

where the division operator is called a separator, and '+' is the or operator. All other members whose membership is 0 are omitted from this function.

2.3 Fuzzification and Membership Functions

The fuzzy set is a powerful tool and allows us to represent objects or members in a vague or ambiguous way. The fuzzy set also provides a way that is similar to a human being's concepts and thought process. However, just the fuzzy set itself cannot lead to any useful and practical products until the fuzzy inference process is applied. To implement fuzzy inference to a real product or to solve an actual

problem, as we discussed before, three consecutive steps are needed, which are: Fuzzification, fuzzy inference and defuzzification.

Fuzzification is the first step to apply a fuzzy inference system. Most variables existing in the real world are crisp or classical variables. One needs to convert those crisp variables (both input and output) to fuzzy variables, and then apply fuzzy inference to process those data to obtain the desired output. Finally, in most cases, those fuzzy outputs need to be converted back to crisp variables to complete the desired control objectives.

Generally, fuzzification involves two processes: derive the membership functions for input and output variables and represent them with linguistic variables. This process is equivalent to converting or mapping classical set to fuzzy set to varying degrees.

In practice, membership functions can have multiple different types, such as the triangular waveform, trapezoidal waveform, Gaussian waveform, bell-shaped waveform, sigmoidal waveform and S-curve waveform. The exact type depends on the actual applications. For those systems that need significant dynamic variation in a short period of time, a triangular or trapezoidal waveform should be utilized. For those system that need very high control accuracy, a Gaussian or S-curve waveform should be selected.

To illustrate the process of fuzzification, we still use the air conditioner example developed in the previous section. Assume that we have an air conditioner control system that is under the control of only a heater. If the temperature is high, the heater control motor should be turned off, and if the temperature is low, that heater motor should be turned on, which are common sense.

Regularly, the normal temperature range is from 20 °F to 90 °F. This range can be further divided into three sub-range or subset, which are

|	
Low temperature:	20 °F ~ 40 °F, 30 °F is center
Medium temperature:	30 °F ~ 80 °F, 55 °F is center
High temperature:	60 °F ~ 90 °F, 75 °F is center

Next those three ranges need to be converted to linguistic variables: LOW, MEDIUM and HIGH, which correspond to the three temperature ranges listed above.

The membership function of these temperatures is shown in Figure 2.6. To make thing simple, a trapezoidal waveform is utilized for this type of membership function. A crisp low temperature can be considered as a medium temperature to some degree in this fuzzy membership function representation. For instance, 35 °F will belong to LOW and MEDIUM to 0.5 degree. Some terminologies used for the membership function are also shown in Figure 2.6.
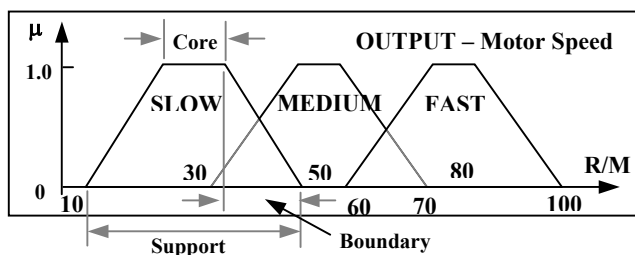
The support of a fuzzy set, says LOW, is the set of elements whose degree of membership in LOW is greater than 0. This support can be expressed in a function form as

$$\text{Support(LOW)} = \{ \, x \in T \mid \mu_{LOW}(x) > 0 \, \} \qquad (2.15)$$

It can be shown that the support of a fuzzy set is a classical set.
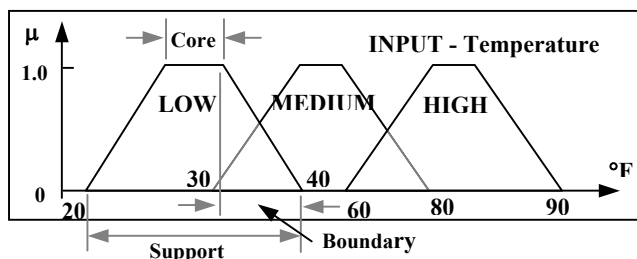
(a) Membership function of the input

(b) Membership function of the output

Figure 2.6. Membership function of input and output

The core of a fuzzy set is the set of elements whose degree of membership in that set is equal to 1, which is equivalent to a crisp set. The boundary of a fuzzy set indicates the range in which all elements whose degree of membership in that set is between 0 and 1 (0 and 1 are excluded).

After the membership functions are defined for both input and output, the next step is to define the fuzzy control rule.

2.4 Fuzzy Control Rules

Fuzzy control rule can be considered as the knowledge of an expert in any related field of application. The fuzzy rule is represented by a sequence of the form IF-THEN, leading to algorithms describing what action or output should be taken in terms of the currently observed information, which includes both input and feedback if a closed-loop control system is applied. The law to design or build a set of fuzzy rules is based on a human being's knowledge or experience, which is dependent on each different actual application.

A fuzzy IF-THEN rule associates a condition described using linguistic variables and fuzzy sets to an output or a conclusion. The IF part is mainly used to capture knowledge by using the elastic conditions, and the THEN part can be utilized to give the conclusion or output in linguistic variable form. This IF-THEN rule is widely used by the fuzzy inference system to compute the degree to which the input data matches the condition of a rule. Figure 2.7 illustrates a way to

calculate the degree between a fuzzy input T (temperature) and a fuzzy condition LOW. Here we still use the air conditioner system as an example.
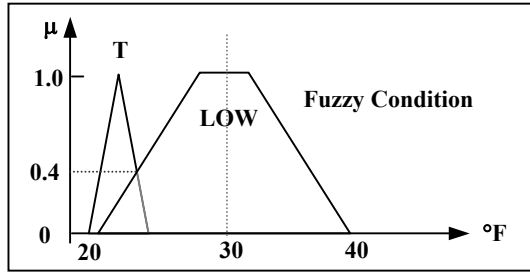

Figure 2.7. Matching a fuzzy input with a fuzzy condition

This calculation can also be represented by the function

$$M(T, LOW) = \text{Support } min(\mu_T(x), \mu_{LOW}(x)) \qquad (2.16)$$

Two types of fuzzy control rules are widely utilized for most real applications. One is fuzzy mapping rules and the other is called fuzzy implication rules.

2.4.1 Fuzzy Mapping Rules

Fuzzy mapping rules provide a functional mapping between the input and the output using linguistic variables. The foundation of a fuzzy mapping rule is a fuzzy graph, which describes the relationship between the fuzzy input and the fuzzy output. Sometimes, in real applications, it is very hard to derive a certain relationship between the input and the output, or the relationship between those inputs and outputs are very complicated even when that relationship is developed. Fuzzy mapping rules are a good solution for those situations.

Fuzzy mapping rules work in a similar way to human intuition or insight, and each fuzzy mapping rule only approximates a limited number of elements of the function, so the entire function should be approximated by a set of fuzzy mapping rules. Still using our air conditioner system as an example, a fuzzy mapping rule can be derived as

IF the temperature is LOW, THEN the heater motor should be rotated FAST.

For other input temperatures, different rules should be developed.

For most actual applications, the input variables are commonly more than one dimension. For example, in our air conditioner system, the inputs include both current temperature and the change rate of the temperature. The fuzzy control rules should also be extended to allow multiple inputs to be considered to derive the output. Table 2.1 is an example of fuzzy control rules applied in our air conditioner system.

Table 2.1. An example of fuzzy rules

$\dot{T}$ \ T	LOW	MEDIUM	HIGH
LOW	FAST	MEDIUM	MEDIUM
MEDIUM	FAST	SLOW	SLOW
HIGH	MEDIUM	SLOW	SLOW

The rows and columns represent two inputs, the temperature input and the change rate of the temperature input, and those inputs are related to IF parts in IF-THEN rules. The conclusion or control output can be considered as a third dimensional variable that is located at the cross point of each row (temperature) and each column (change rate of the temperature), and that conclusion is associated with the THEN part in IF-THEN rules. For example, when the current temperature is LOW, and the current change rate of the temperature is also LOW, the heater motor's speed should be FAST to increase the temperature as soon as possible. This can be represented by the IF-THEN rule as

IF the temperature is LOW, and the change rate of the temperature is LOW,
THEN the conclusion or output (heater motor speed) should be FAST

All other rules follow a similar strategy, which is very similar to a human being's intuition.

In this air conditioner example, a total of nine rules are developed. For those applications that need high control accuracy, the input and output should be divided into more small segments, and more fuzzy rules should be applied.

2.4.2 Fuzzy Implication Rules

A fuzzy implication rule describes a generalized logic implication relationship between inputs and outputs. The foundation of a fuzzy implication rule is the narrow sense of fuzzy logic [8]. Fuzzy implication rules are related to classical two-valued logic and multiple valued logic.

Still using the air conditioner system as an example, the implication is

IF the temperature is LOW, THEN the heater motor should be FAST.

Based on this implication and a fact: the temperature is HIGH. The result that the heater motor should slow down or the SLOW can be inferred.

2.5 Defuzzification and the Lookup Table

The conclusion or control output derived from the combination of input, output membership functions and fuzzy rules is still a vague or fuzzy element, and this process in called fuzzy inference. To make that conclusion or fuzzy output

available to real applications, a defuzzification process is needed. The defuzzification process is meant to convert the fuzzy output back to the crisp or classical output to the control objective. Remember, the fuzzy conclusion or output is still a linguistic variable, and this linguistic variable needs to be converted to the crisp variable via the defuzzification process. Three defuzzification techniques are commonly used, which are: Mean of Maximum method, Center of Gravity method and the Height method.

2.5.1 Mean of Maximum (MOM) Method

The Mean of Maximum (MOM) defuzzification method computes the average of those fuzzy conclusions or outputs that have the highest degrees. For example, the fuzzy conclusion is: the heater motor x is rotated FAST. By using the MOM method, this defuzzification can be expressed as

$$\text{MOM(FAST)} = \frac{\sum_{x' \in T} x'}{T} \qquad T = \{x' \,|\, \mu_{\text{FAST}}(x') = \text{Support } \mu_{\text{FAST}}(x)\} \quad (2.17)$$
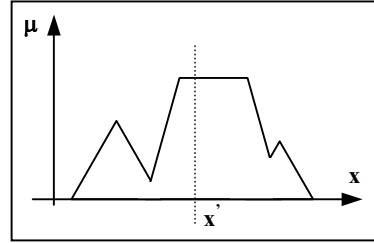
where T is the set of output x that has the highest degrees in the set FAST.

A graphic representation of the MOM method is shown in Figure 2.8a.

A shortcoming of the MOM method is that it does not consider the entire shape of the output membership function, and it only takes care of the points that have the highest degrees in that function. For those membership functions that have different shapes but the same highest degrees, this method will produce the same result.
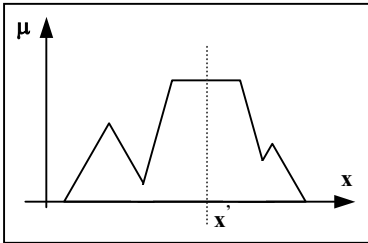
(a) MOM method example   (b) COG method example

Figure 2.8. Graphic representation of defuzzification techniques

2.5.2 Center of Gravity (COG) Method

The Center of Gravity method (COG) is the most popular defuzzification technique and is widely utilized in actual applications. This method is similar to the formula

for calculating the center of gravity in physics. The weighted average of the membership function or the center of the gravity of the area bounded by the membership function curve is computed to be the most crisp value of the fuzzy quantity. For example, for the conclusion: the heater motor x is rotated FAST. The COG output can be represented as

$$\text{COG(FAST)} = \frac{\sum_{x} \mu_{FAST}(x) \times x}{\sum_{x} \mu_{FAST}(x)} \tag{2.18}$$

If x is a continuous variable, this defuzzification result is

$$\text{COG(FAST)} = \frac{\int \mu_{FAST}(x)x\,dx}{\int \mu_{FAST}(x)\,dx} \tag{2.19}$$

A graphic representation of the COG method is shown in Figure 2.8b.

2.5.3 The Height Method (HM)

This defuzzification method is valid only for the case where the output membership function is an aggregated union result of symmetrical functions [9]. This method can be divided into two steps. First, the consequent membership function F_i can be converted into a crisp consequent $x = f_i$ where f_i is the center of gravity of F_i. Then the COG method is applied to the rules with crisp consequents, which can be expressed as

$$x = \frac{\sum_{i=1}^{M} w_i f_i}{\sum_{i=1}^{M} w_i} \tag{2.20}$$

where w_i is the degree to which the ith rule matches the input data. The advantage of this method is its simplicity. Therefore many neuro-fuzzy models use this defuzzification method to reduce the complex of calculations.

2.5.4 The Lookup Table

The terminal product of defuzzification is the lookup table. Defuzzification needs to be performed for each subset of a membership function, both inputs and outputs. For instance, in the air conditioner system, one needs to perform defuzzification for each subset of temperature input such as LOW, MEDIUM and HIGH based on the associated fuzzy rules. The defuzzification result for each subset needs to be stored

in the associated location in the lookup table according to the current temperature and temperature change rate. In the following we use the air conditioner system as an example to illustrate the defuzzification process and the creation of the lookup table.

To make this illustration simple, we make two assumptions: I. assume that the membership function of the change rate of the temperature can be described as in Figure 2.9; II. only four rules are applied to this air conditioner system, which are

1) IF the temperature is LOW, and the change rate of the temperature is LOW, THEN the heater motor speed should be FAST
2) IF the temperature is MEDIUM, and the change rate of the temperature is MEDIUM, THEN the heater motor speed should be SLOW
3) IF the temperature is LOW, and the change rate of the temperature is MEDIUM, THEN the heater motor speed should be FAST
4) IF the temperature is MEDIUM, and the change rate of the temperature is LOW, THEN the heater motor speed should be MEDIUM
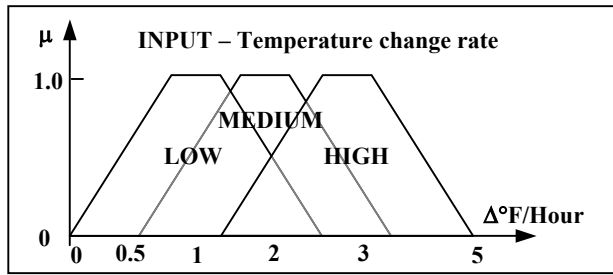

Figure 2.9. The membership function of the change rate of temperature

Based on the assumption made for the membership function and fuzzy rules, we can illustrate this defuzzification process using a graph. Four fuzzy rules can be interpreted as functional diagrams, as shown in Figure 2.10.

As an example, consider the current input temperature is 35 °F and the change rate of the temperature is 1 °F per hour.

From Figure 2.10, it can be found that the points of intersection between the temperature values of 35 °F and the graph in the first column (temperature input T) have the membership functions of 0.6, 0.8, 0.5 and 0.8. Likewise, the second column (temperature change rate ΔT) shows that a temperature change rate of 1 °F per hour has the membership functions of 1.0, 0.4, 0.4 and 1.0. The fuzzy output for the four rules is the intersection of the paired values obtained from the graph, or the AND result between the temperature input and the temperature change rate input. According to Equation (2.13), this operation result should be: min (0.6, 1.0), min (0.8, 0.4), min (0.5, 0.4) and min (0.8, 1.0), which produces to 0.6, 0.4, 0.4 and 0.8, respectively.
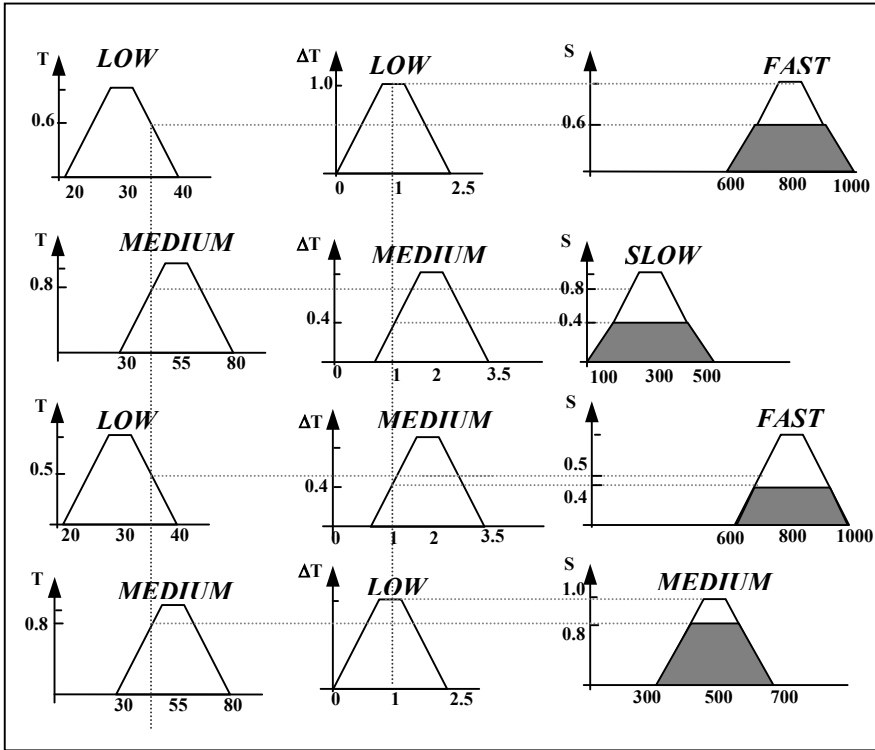
Figure 2.10.   An illustration of fuzzy output calculation

The membership functions representing the control adjustment are weighted according to the input change and the different control contributions as shown in Figure 2.11.

Now, for a pair of temperature and temperature change rate, four sets of fuzzy outputs exist. To determine the crisp value of action to be taken from these contributions, one can either choose the maximum value using the MOM method or use the Center of Gravity method (COG). In this example, the COG method is used and the action is given by the center of the summed area, which is contributed by the different fuzzy outputs. Furthermore, the COG method gives a more reliable lookup table compared with the MOM operation.

Thus, for a temperature of 35 °F and a change rate of temperature is 1 °F per hour, the fuzzy output element y for this input pair is

$$y = \frac{0.6 \times 800 + 0.4 \times 300 + 0.4 \times 800 + 0.8 \times 500}{0.6 + 0.4 + 0.4 + 0.8} = 600 \text{ R/M} \qquad (2.21)$$
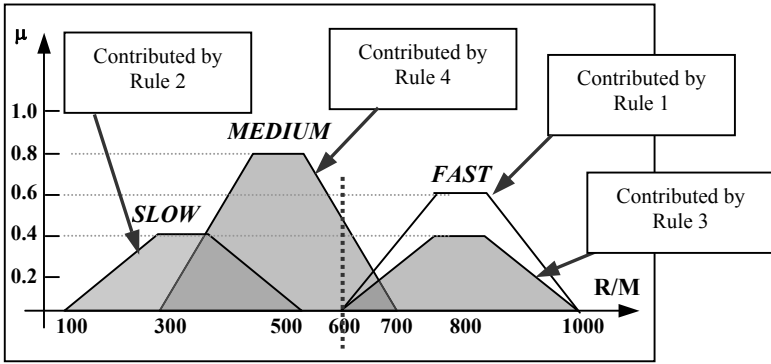
Figure 2.11. Determination of fuzzy output by the center of gravity method

This defuzzified fuzzy output is a crisp or classical value, and should be entered into a certain location in a table called the lookup table. Since this fuzzy output element is associated with a temperature input pair with a temperature of 35 °F (belonging to LOW in the temperature membership function) and a change rate of temperature of 1 °F/h (also belonging to LOW in the membership function of change rate of temperature), this output value should be located in the cross point between the LOW temperature T (row) and the LOW change rate $\dot{T}$ (column) as shown in Table 2.2.

Table 2.2 shows an example of a lookup table.

Table 2.2. An example of a lookup table

$\dot{T}$ \ T	LOW	MEDIUM	HIGH
LOW	600	?	?
MEDIUM	?	?	?
HIGH	?	?	?

To fill this table, one needs to use the defuzzification technique to calculate all other fuzzy output values and locate them in the associated positions in the lookup table as we did in this example. Generally, the dimensions of the fuzzy rules should be identical with the dimensions of the lookup table as shown in this example. To obtain more accurate control accuracy or fine fuzzy output element values, one can divide the inputs, say the temperature input and change rate of the temperature, into multiple smaller subsets to get finer membership functions. For instance, in this example, one can define a VERY LOW subset that covers 20 ~ 30 °F for the temperature input, a LOW subset that contains a range of 30 ~ 40 °F, and so on. Performing the same process to the change rate of temperature and the heater motor output, one can get a much finer lookup table.

For higher control accuracy applications, an interpolation process can be added after the lookup table to obtain finer output [10].

When implementing a fuzzy logic technique to a real system, the lookup table can be stored in a computer's memory, and the fuzzy output can be obtained based on the current inputs. In practice, there are two ways to calculate the lookup table using fuzzy inference process in the real control applications: off-line and on-line methods.

2.5.5 Off-line and On-line Defuzzification

As mentioned , the defuzzification process is to derive the desired crisp output value using a defuzzification technique that combines the membership functions with the fuzzy rules. The defuzzification process can be further divided into two categories: off-line defuzzification and on-line defuzzification.

So-called off-line defuzzification means that all input and output membership functions, fuzzy rules and the lookup table should be developed based on the estimations of the real application prior to the implementation of the fuzzy logic technique to the actual application. This means that all input and output membership functions are developed based on the actual experience or the input and output parameter ranges of a specified application, and the lookup table is calculated in terms of those definitions of input and output membership functions. The advantage of this method is that most fuzzy inference related calculations are performed prior to the real implementation and therefore the fuzzy process is less time consuming. The disadvantage of this technique is that the fuzzy output is only based on the estimation of input and output parameters, so the control accuracy is not as high as that of on-line method.

The on-line method has real-time controllability. Both input and output membership functions are developed during the real-time processing of an actual application. Also the lookup table elements are calculated in real-time based on the current actual inputs and outputs. In this method, only fuzzy rules are developed prior to the real applications. The advantage of this method is that higher control accuracy can be obtained for a process and the fuzzy output can be computed in real-time. The disadvantage of this method is that a longer processing time is needed and it is a somewhat time-consuming process. However, with the development of new computer technologies, today much faster CPUs are available, and processing time is no longer a big deal for this method.

2.6 Architectures of Fuzzy Logic Controls

Combining the discussions we made in the previous sections, a structure or architecture of fuzzy logic control system is given here.
As shown in Figure 2.12, which is a typical fuzzy closed-loop control system, the inputs are error and error rate, which are combined by block M to input to the fuzzy inference system. The lookup table is derived based on the membership function of inputs, the output and the fuzzy control rules. A control gain factor G is used to tune the output of the lookup table to obtain different output values. The

interpolation block S is used to smooth the output element of the lookup table. A feedback signal is obtained from the output of the system.
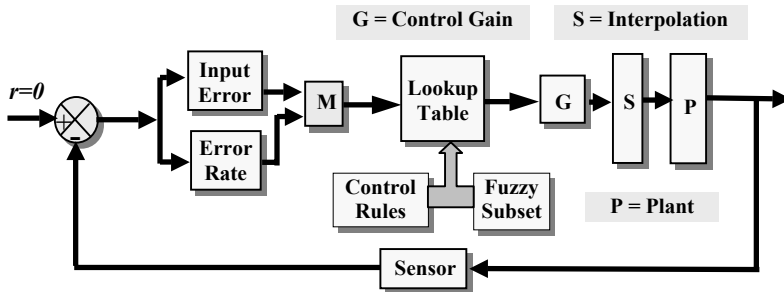

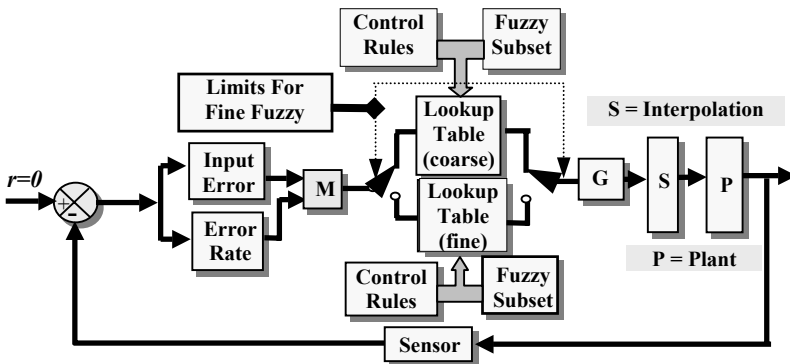**Figure 2.12.** Block diagram of a fuzzy control system

Figure 2.13. Block diagram of an accurate fuzzy control system

For a system that needs higher control accuracy, a multiple lookup tables fuzzy control system is needed, which is shown in Figure 2.13.

Two lookup tables are developed in this control system, a coarse and a fine table. During the application, the switch between the coarse and the fine table is under the control of the input error limit. This limit value can be defined by the user based on the real application. Two-set membership functions and control rules are utilized in this system to satisfy the requirement of higher control accuracy. When the system needs quick responses or quick actions, the coarse table is used. When the system needs high control accuracy or small control errors, the fine lookup table is selected. The sacrifice for this method is that more memory is needed to store both coarse and fine tables, and a little longer time is needed to make decision in selecting table in terms of the input error limit value.

2.7 Summary

A review of the fundamentals of fuzzy sets, fuzzy rules and fuzzy inference systems is provided in this chapter. Beginning with crisp or classical sets and their operations, we derived fuzzy sets and their operations. Classical set membership functions and fuzzy membership functions are discussed in detail following set theory. Fuzzy rules are described using an air conditioner control example. The different defuzzification techniques and their processes are discussed with the same example step by step. Finally, some other fuzzy techniques are discussed such as off-line and on-line fuzzy control systems as well as a fuzzy closed-loop control system including multiple lookup tables.

Because of space limitations, the author cannot cover all different fuzzy systems and their applications such as sliding mode fuzzy, adaptor fuzzy and neural fuzzy control systems in one chapter. With the development of new fuzzy techniques, fuzzy control will play a more and more important role in our society.

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