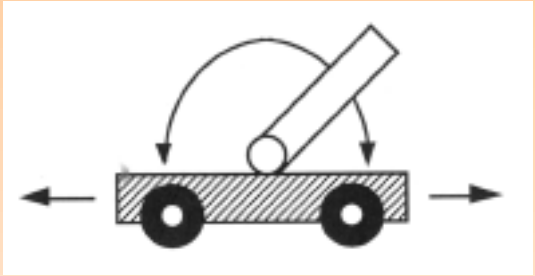
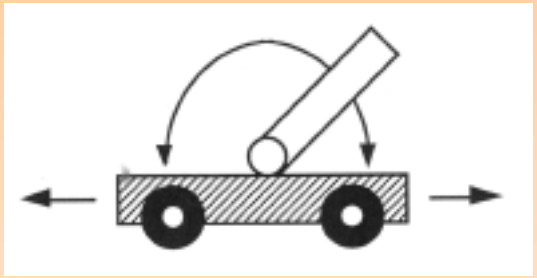
Fuzzy Application



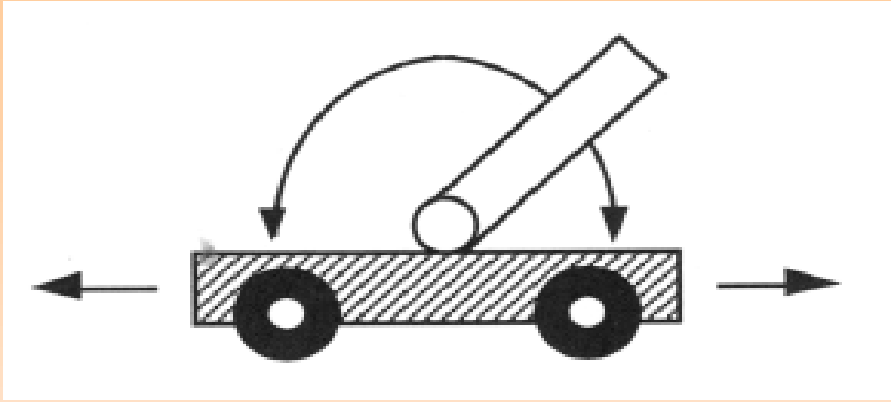
Inverted Pendulum

Control: Inverted Pendulum



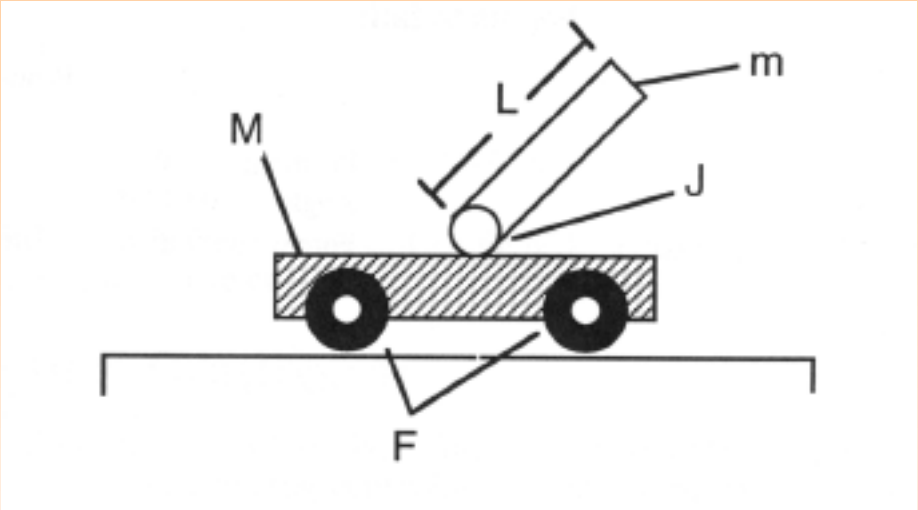
- **Statement of Problem**
  - **Design a controller that will balance an inverted pendulum on a cart, while at the same time keeping the cart centered at a desired horizontal position.**
  - **One can apply a force in either direction to accomplish the balancing act and center the cart.**

Control: Inverted Pendulum



- The pendulum is attached to the center of the cart with a joint that allows it to have 180 degree of freedom.
- The cart can move to the right or left.

Control: Inverted Pendulum



The cart has a mass M, a pendulum mass m, wheel friction F, Pendulum friction J, and pendulum length L

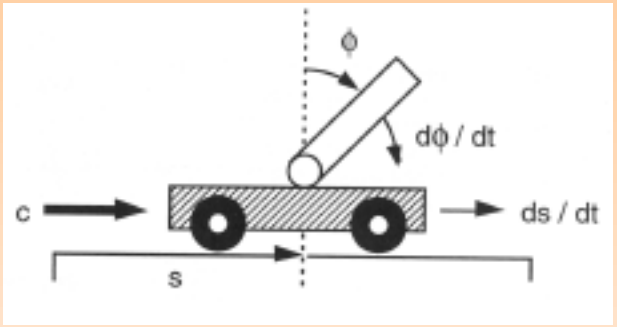
Inverted Pendulum: Equations

$$H = m\frac{d^2 s}{dt^2} + mL\left(\frac{d^2\phi}{dt^2}\cos(\phi) - \frac{d\phi}{dt}\sin(\phi)\right)$$

$$V = mg - mL\left(\frac{d^2\phi}{dt^2}\sin(\phi) - \frac{d\phi}{dt}\cos(\phi)\right)$$

$$\frac{J}{L}\frac{d^2\phi}{dt^2} = V\sin(\phi) - H\cos(\phi)$$

$$(M+m)\frac{d^2 s}{dt^2} = c - F\frac{ds}{dt} - H$$



First equation: Pendulum-Horizontal forces
Second equation:Pendulum-Vertical forces
Third equation: PendulumAngular forces
Fourth equation:Cart-Control force

Inverted Pendulum: Equations

```
function ds=pendmod(s,u)
% PENDMOD Dynamical model of inverted pendulum.
%   This function is referred to in Chapter 5.
%
% PENDMOD(X,U)
%   X - Current state of pendulum.
%   U - Force to be applied to pendulum base.
% Returns the derivative of state dX.
%
% The state X is = [base position;
%                   base velocity;
%                   pendulum angle;
%                   pendulum angular velocity]
%
% All angles are in radians.
```

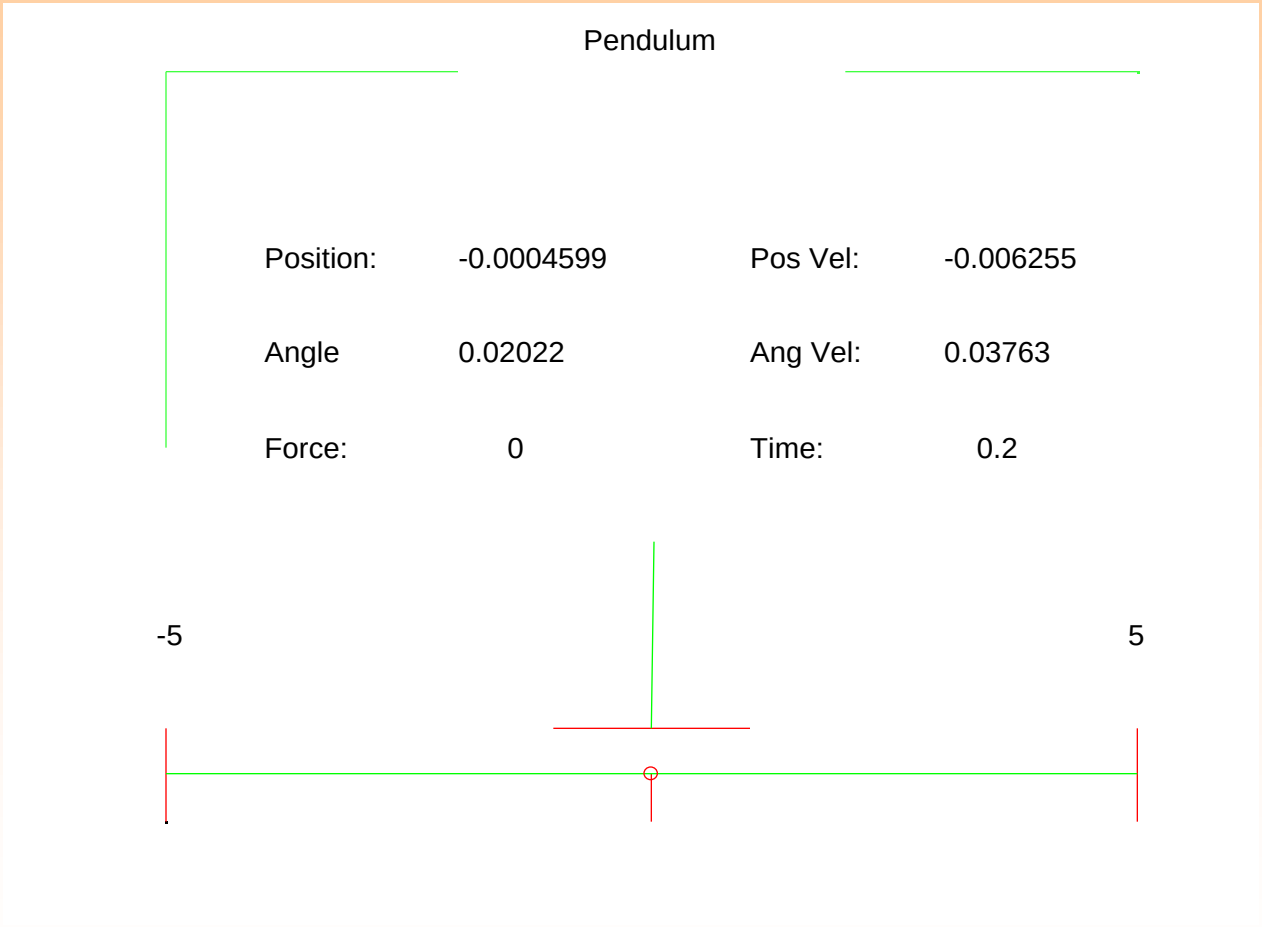
Inverted Pendulum: Equations

```
% Mark Beale 6-24-93
% Copyright (c) 1993 by PWS Publishing Company

% CONSTANTS
L = 1;              % Length of pendulum
M = 5;              % Mass of base
m = 1;              % Mass of pendulum
F = 0.1;            % Friction for base wheels
J = 0.1;            % Friction in pendulum joint
g = 9.81;           % Gravity
deg = pi/180;       % Used to convert degrees to radians

% STATE
x = s(1);
dx = s(2);
phi = s(3);
dphi = s(4);
```

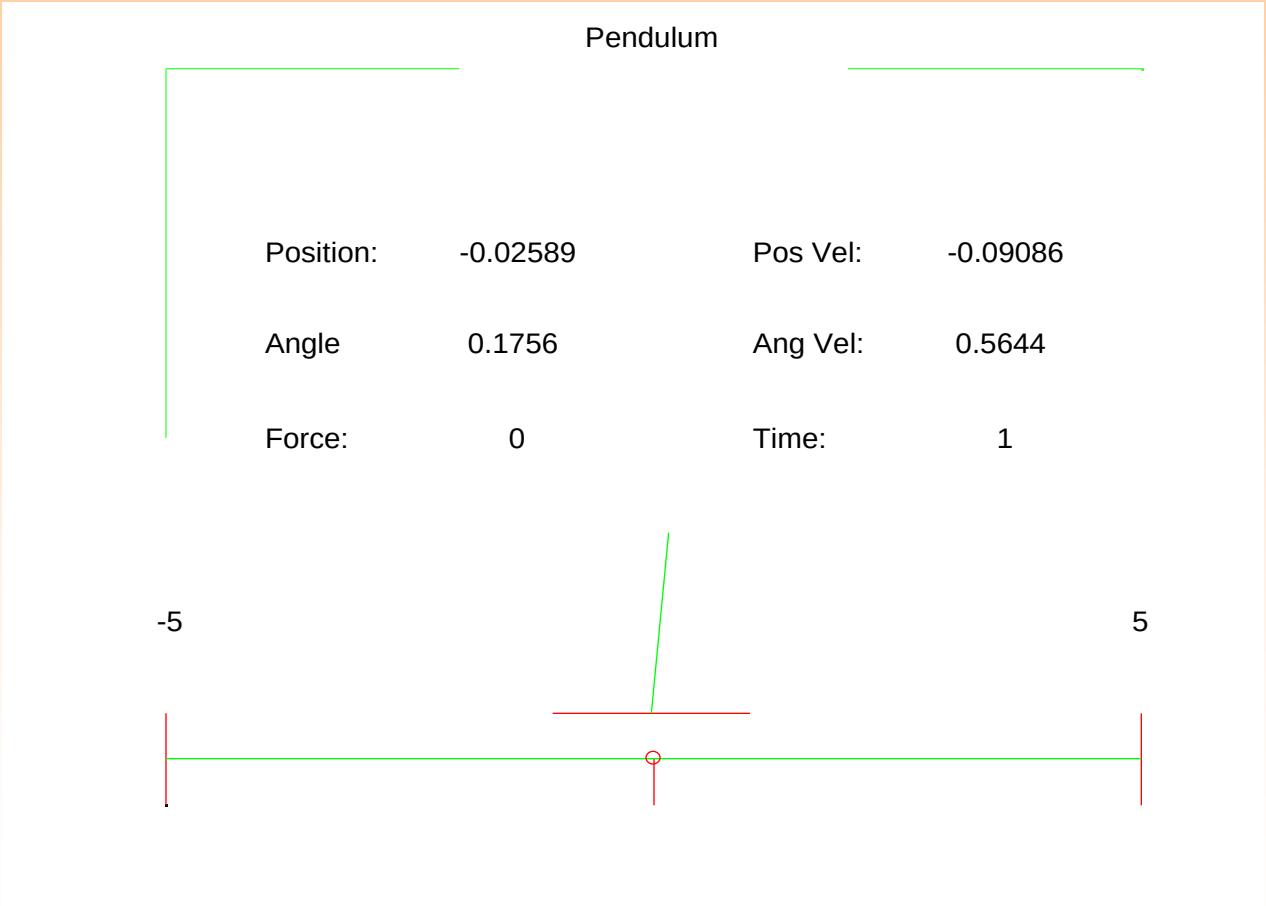
Inverted Pendulum: Equations

```
% INTERMEDIATE VALUES
c1 = dphi*sin(phi);
c2 = dphi*cos(phi);
c3 = u-F*dx;
c4 = (J/L)+(sin(phi)^2)*m*L;
c5 = sin(phi)*m*(g-L*c2);
% CALCULATE PENDULUM ACCELERATION
if abs(phi) > (89*deg)
        dphi=0; ddphi=0; c1=0; phi=90;
else
        ddphi = c5 + cos(phi)*(M/(M+m)*(c3+m*L*c1)-c3);
        ddphi = ddphi / (c4 + (cos(phi)^2)*M*m*L/(M+m));
end
% CALCULATE BASE ACCELERATION
ddx = (c3 - m*L*(ddphi*cos(phi) - c1))/(M+m);
% STATE DERIVATIVES
ds = [dx; ddx; dphi; ddphi];
```

Inverted Pendulum: Uncontrolled



Inverted Pendulum: Uncontrolled

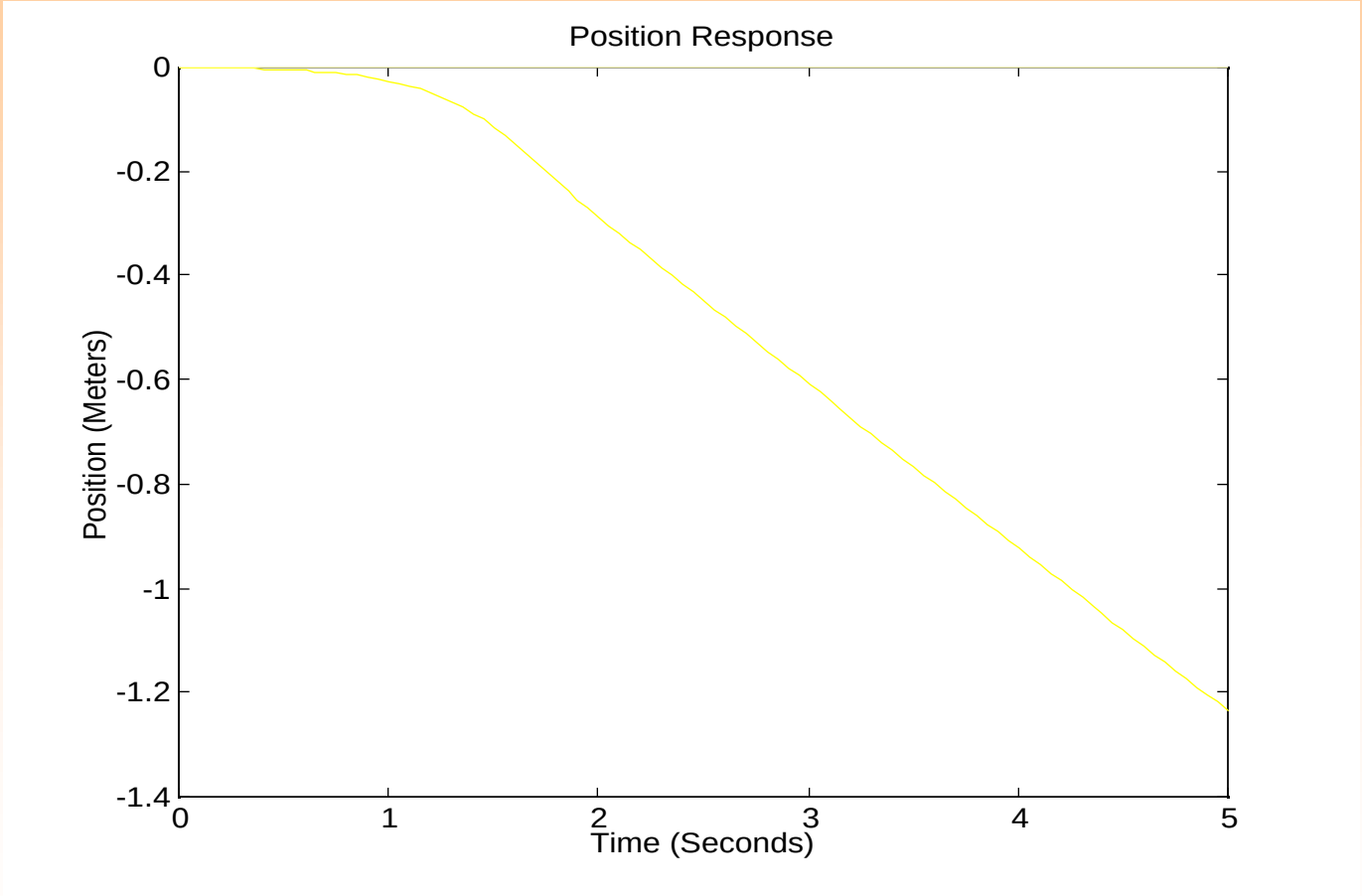


Inverted Pendulum: Uncontrolled

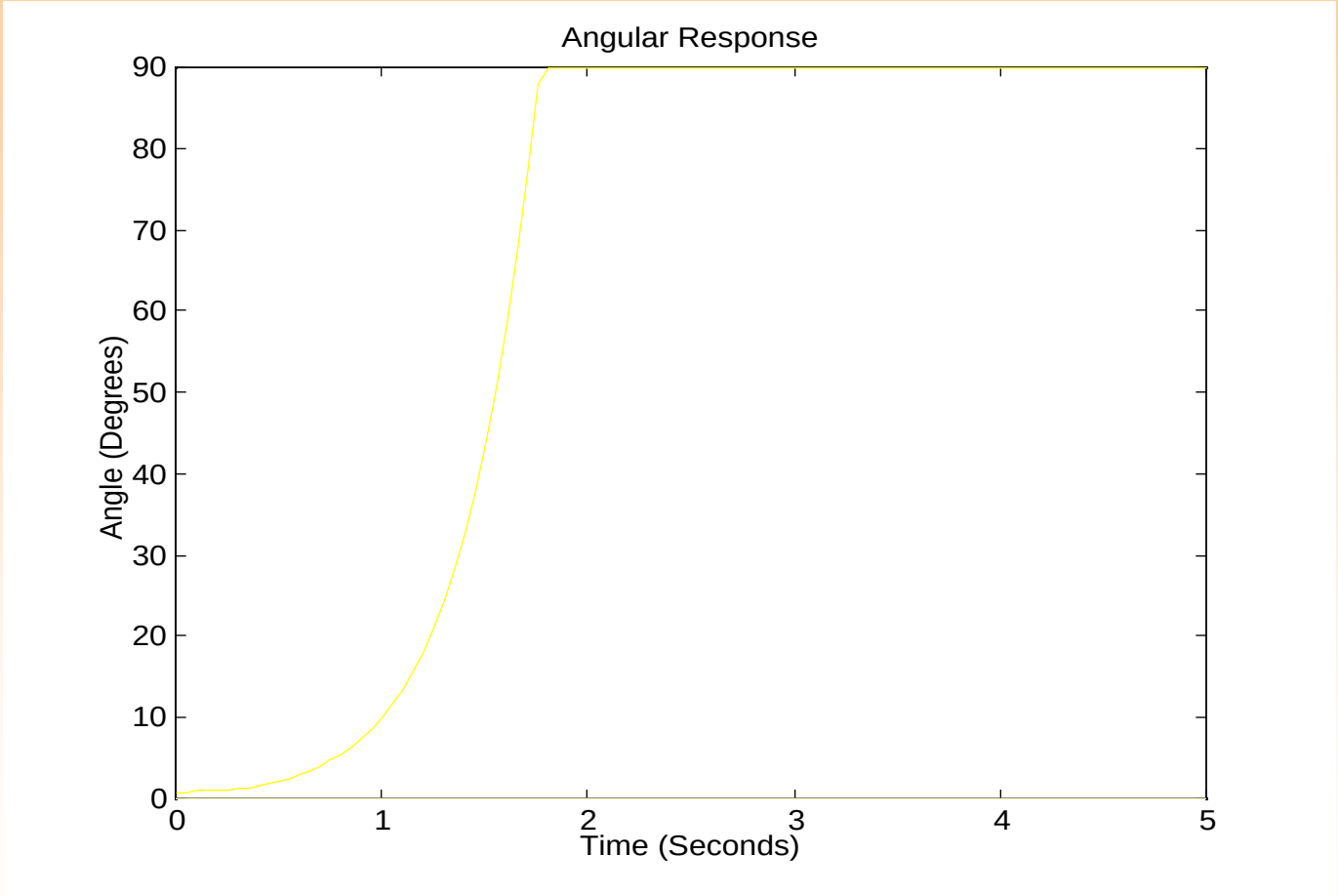
Inverted Pendulum: Uncontrolled

Inverted Pendulum: Uncontrolled

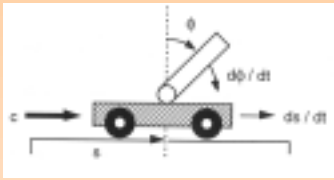
Inverted Pendulum: Uncontrolled



Inverted Pendulum: Uncontrolled



Inverted Pendulum: Controller



Pendulum Angular Velocity

Pendulum Angle

+

Σ

-

Fuzzy
Balance controller*

Motor control

*Produces the force to move the pendulum angle to its zero error position

Cart Velocity

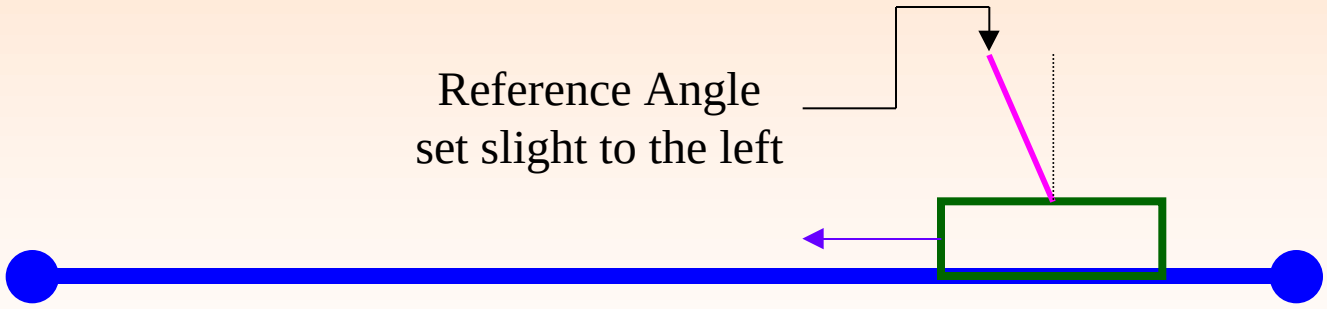
Cart Position

Fuzzy
Reference Controller~

~produces the reference angle for the pendulum

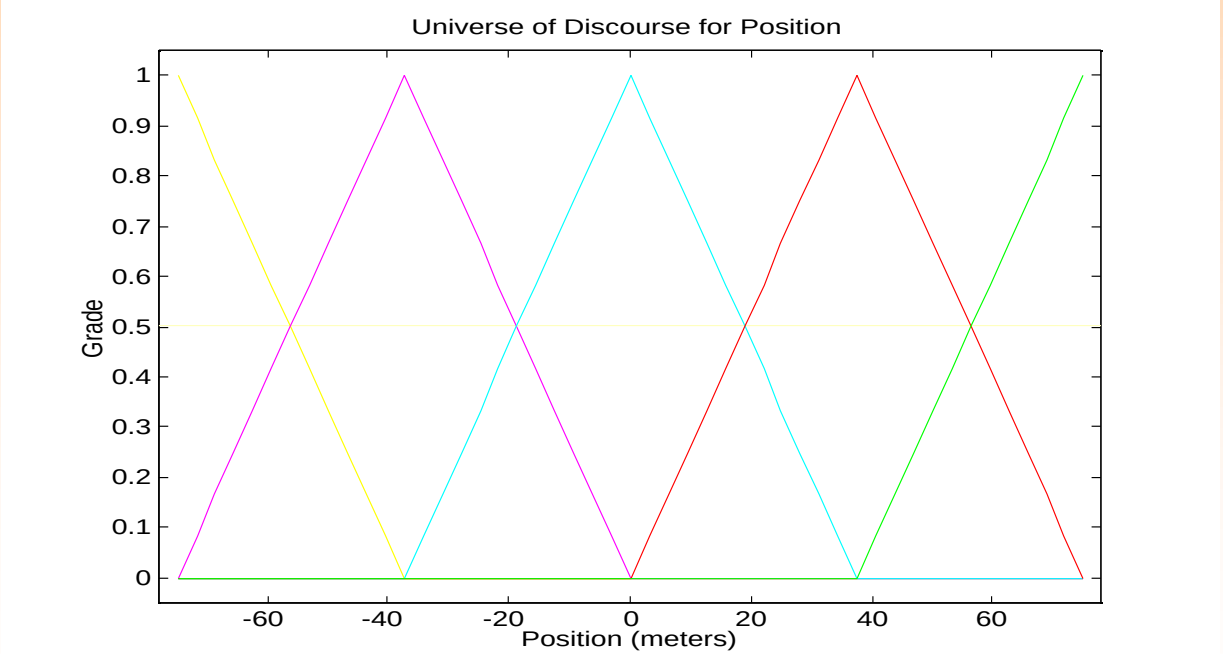
Why: Reference Controller?

Example: If the cart is far to the right of center, the reference angle controller sets the reference angle to be slightly to the left of center, i.e. the balancing controller has to move the cart left, toward center, to keep the pendulum balanced at such an angle!



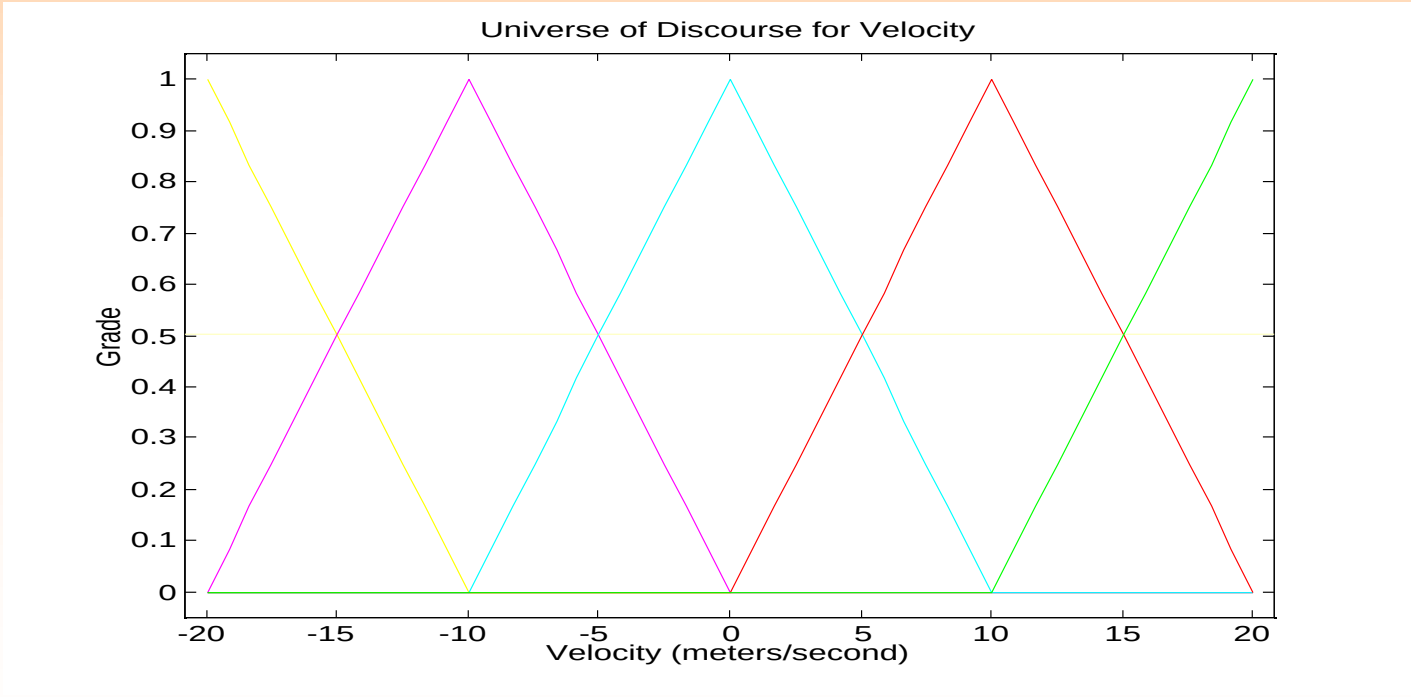
Reference Controller

- **INPUT**
  - **Cart Position Fuzzy Sets**



Reference Controller

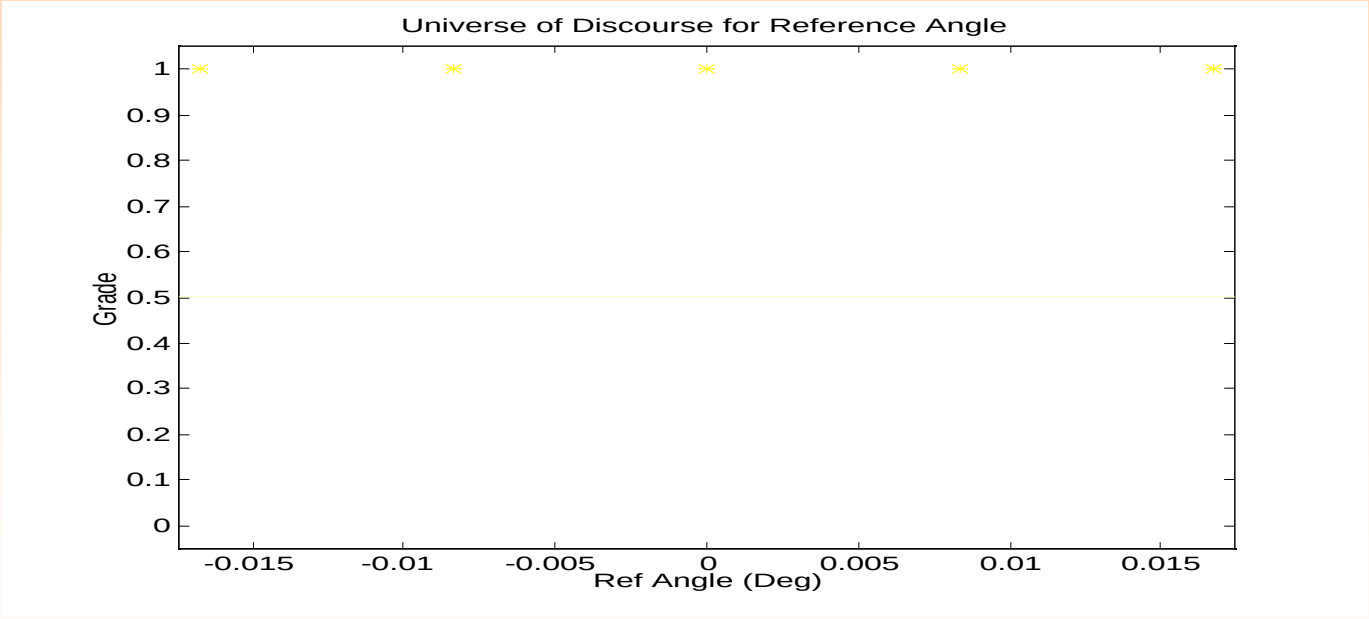
- **INPUT**
  - **Cart Velocity Fuzzy Sets**



Fuzzy Systems ToolBox, Mark Beale and Howard Demuth

Reference Controller

- **Output**
  - **Reference Angle Fuzzy Sets**



Reference Controller

- 25 Fuzzy Rules
  - Antecedents

    A1 = Table[1:5,1:5]
  - Consequences

    W1 = RefS([5 5 5 4 3 . . .

    5 5 4 3 2 . . .

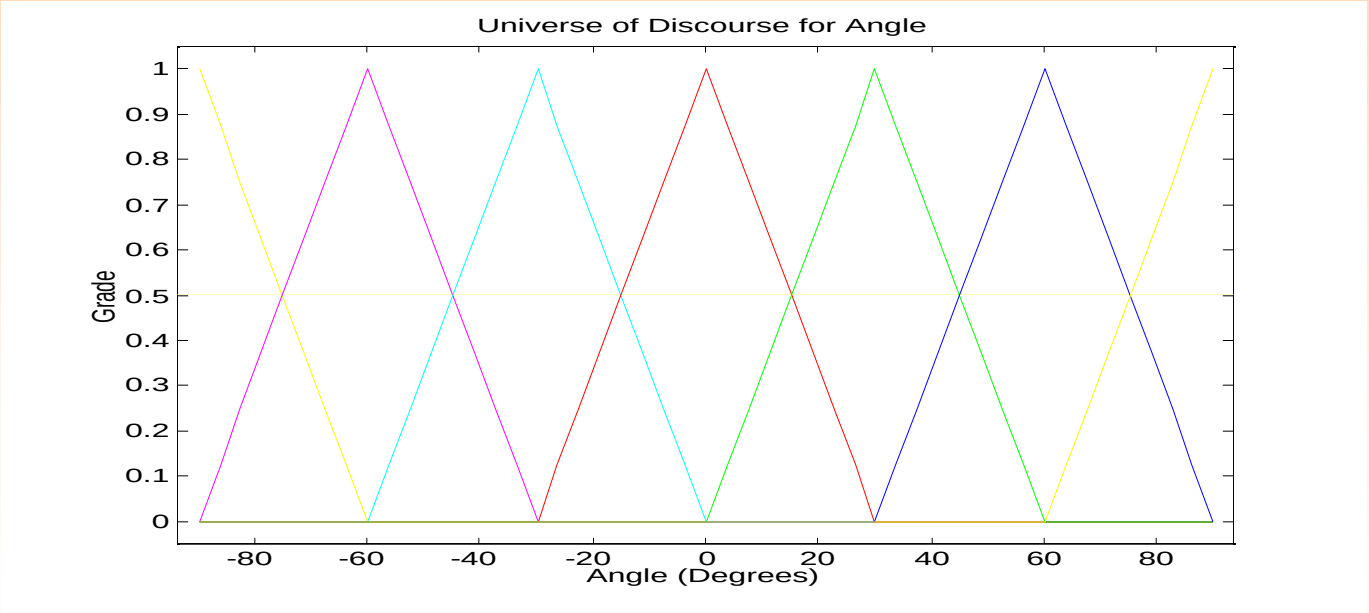
    5 4 3 2 1 . . .

    4 3 2 1 1 . . .

    3 2 1 1 1])';

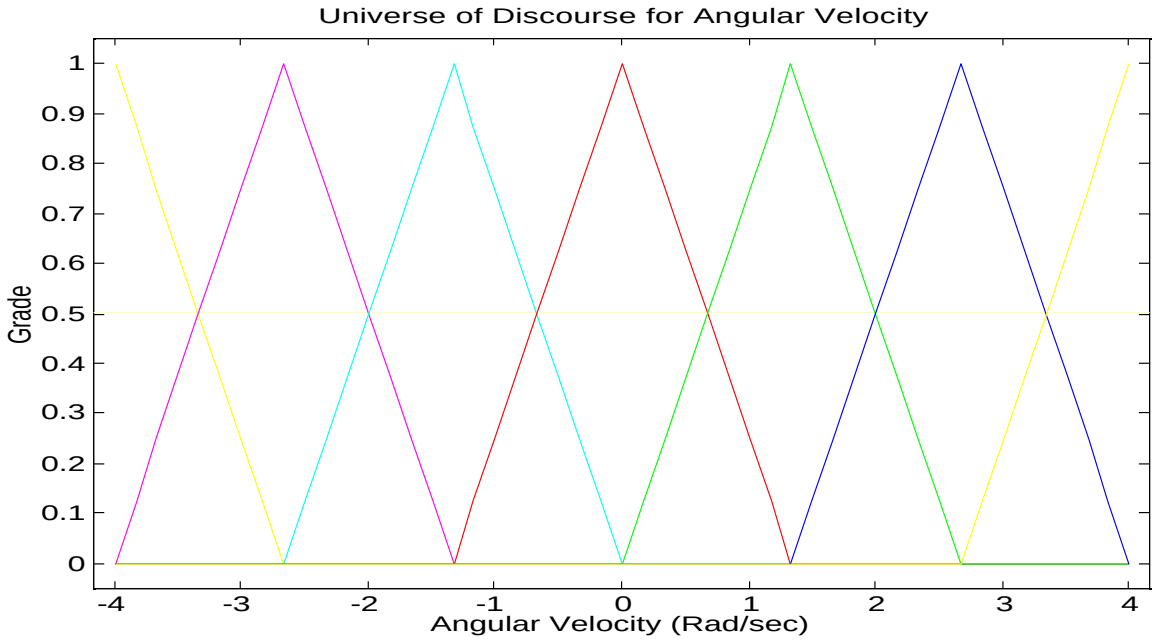
Balance Controller

- **INPUT**
  - **Pendulum Angle Fuzzy Sets**



Balance Controller

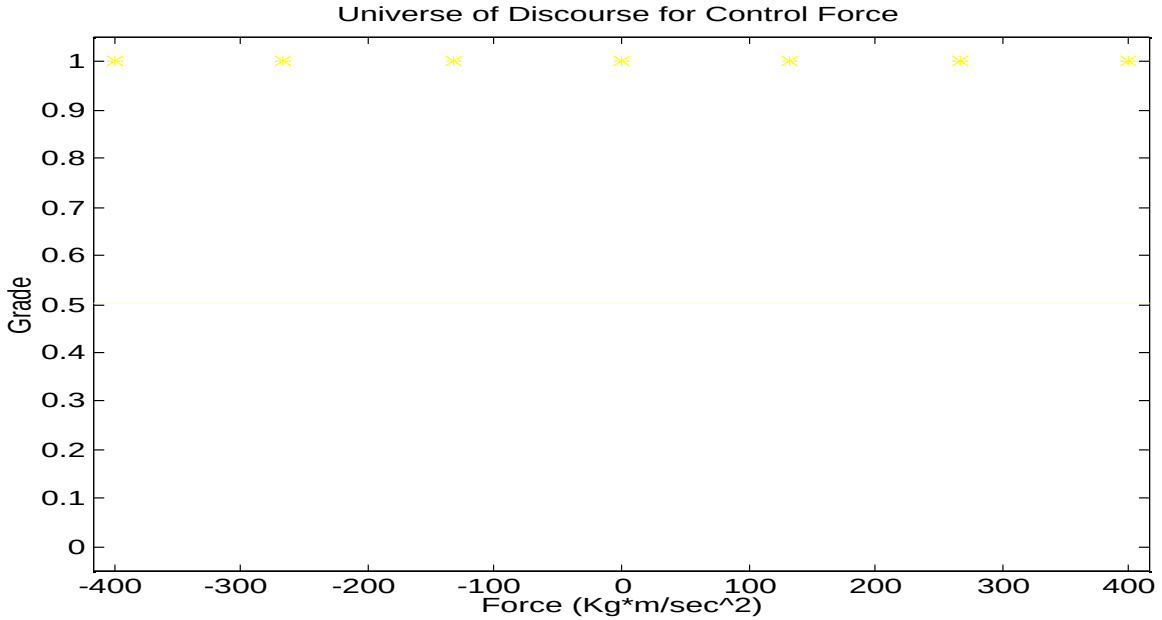
- **INPUT**
  - **Pendulum Angular Velocity Fuzzy Sets**



Fuzzy Systems ToolBox, Mark Beale and Howard Demuth

Balance Controller

- **Output**
  - **Control Force Fuzzy Sets**



Balance Controller

- **49 Fuzzy Rules**
  - **Antecedents**

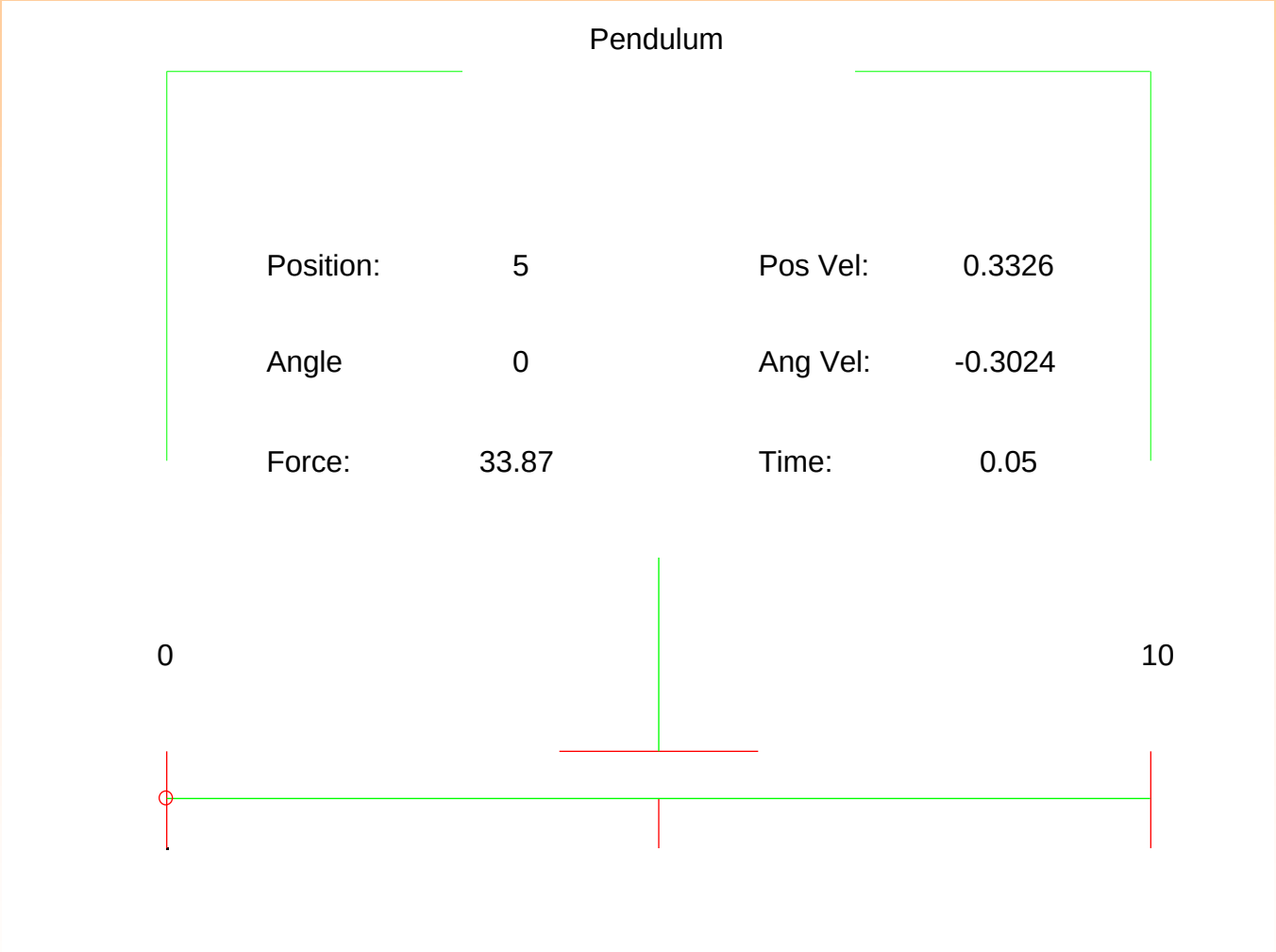
    A2 = Table[1:7,1:7]
  - **Consequences**

    W2 = ForceS([1 1 1 1 2 3 4 . . .
    1 1 1 2 3 4 5 . . .
    1 1 2 2 4 5 6 . . .
    1 2 3 4 5 6 7 . . .
    2 3 4 5 6 7 7 . . .
    3 4 5 6 7 7 7 . . .
    4 5 6 7 7 7])';

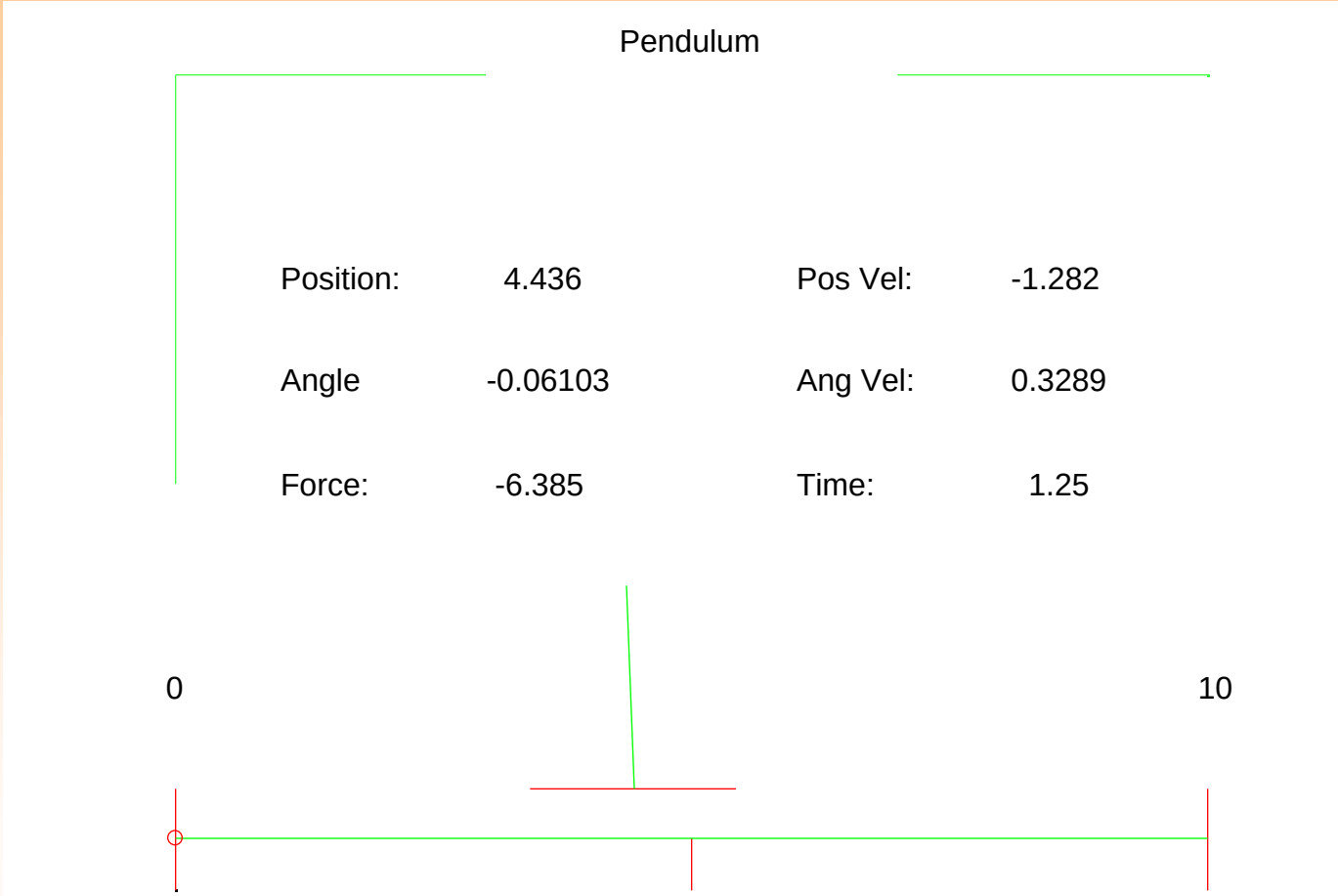
Simulating the Controlled Pendulum

```
Z1 = group(PosL, VelL);
Z2 = group(AngleVelL, AngleL);
deg = pi/180    %a degree in radians
X0 = [5; 0; 0*deg; 0*deg];
dt = 0.05
X = x0;
For I=1:100
    if rem(I,3) ==1
            r = frules(A1,W1,Z1,[x(1); x(2)]);
            u = frulles(A2,W2,Z2,[x(4); x(3)-r]);
    end
    x = x + dt*pendmod(x,u);
end
```

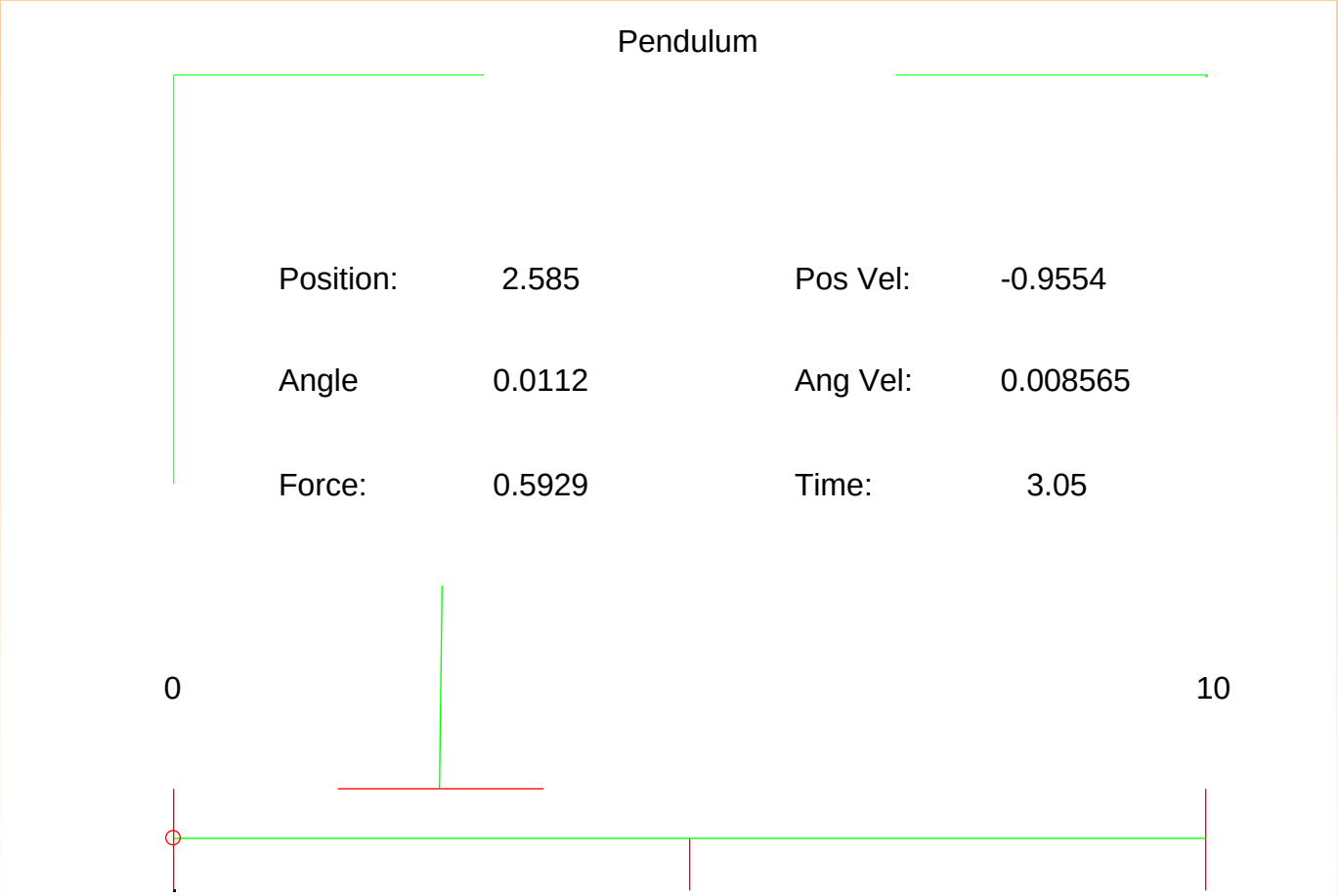
Simulating the Controlled Pendulum



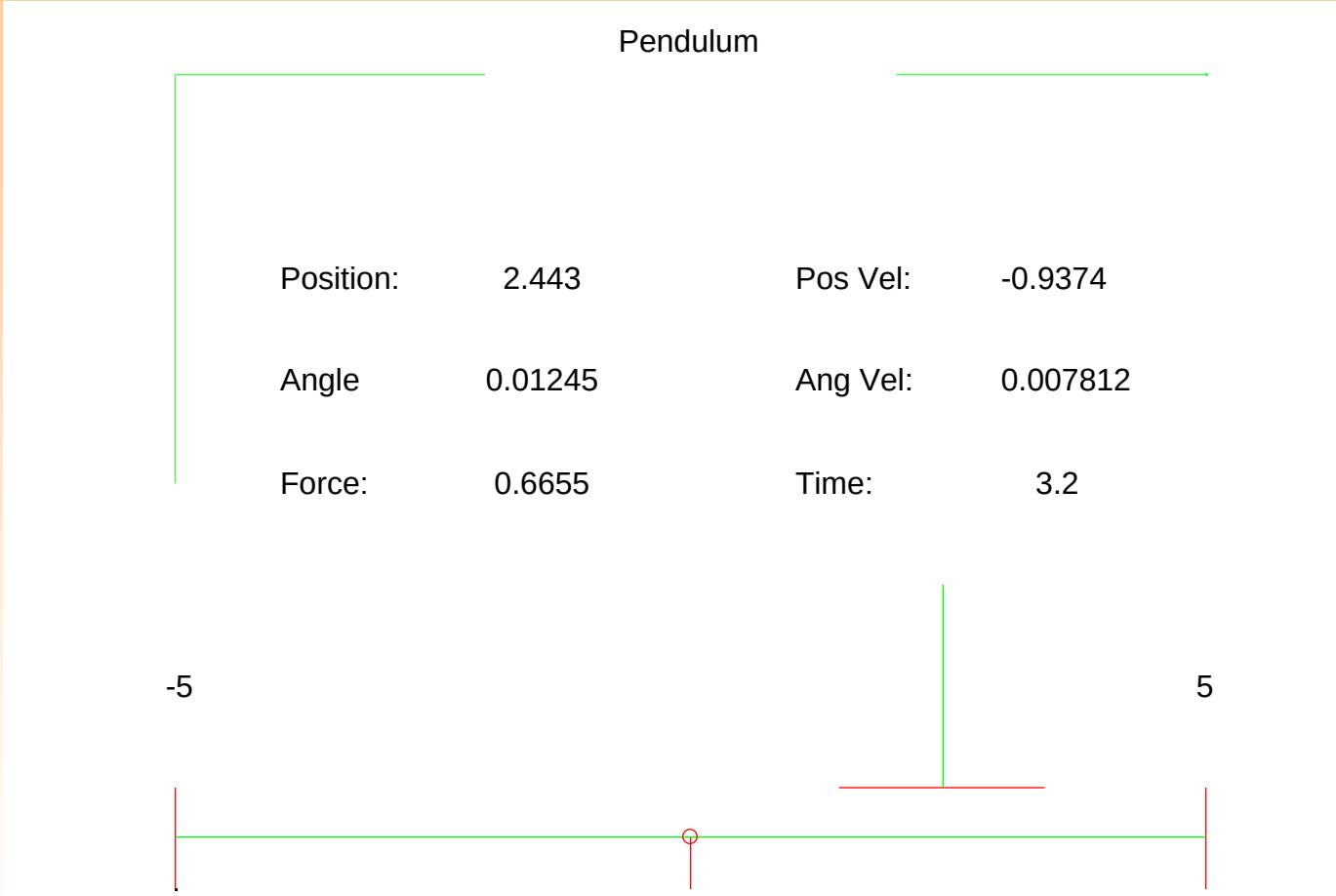
Simulating the Controlled Pendulum



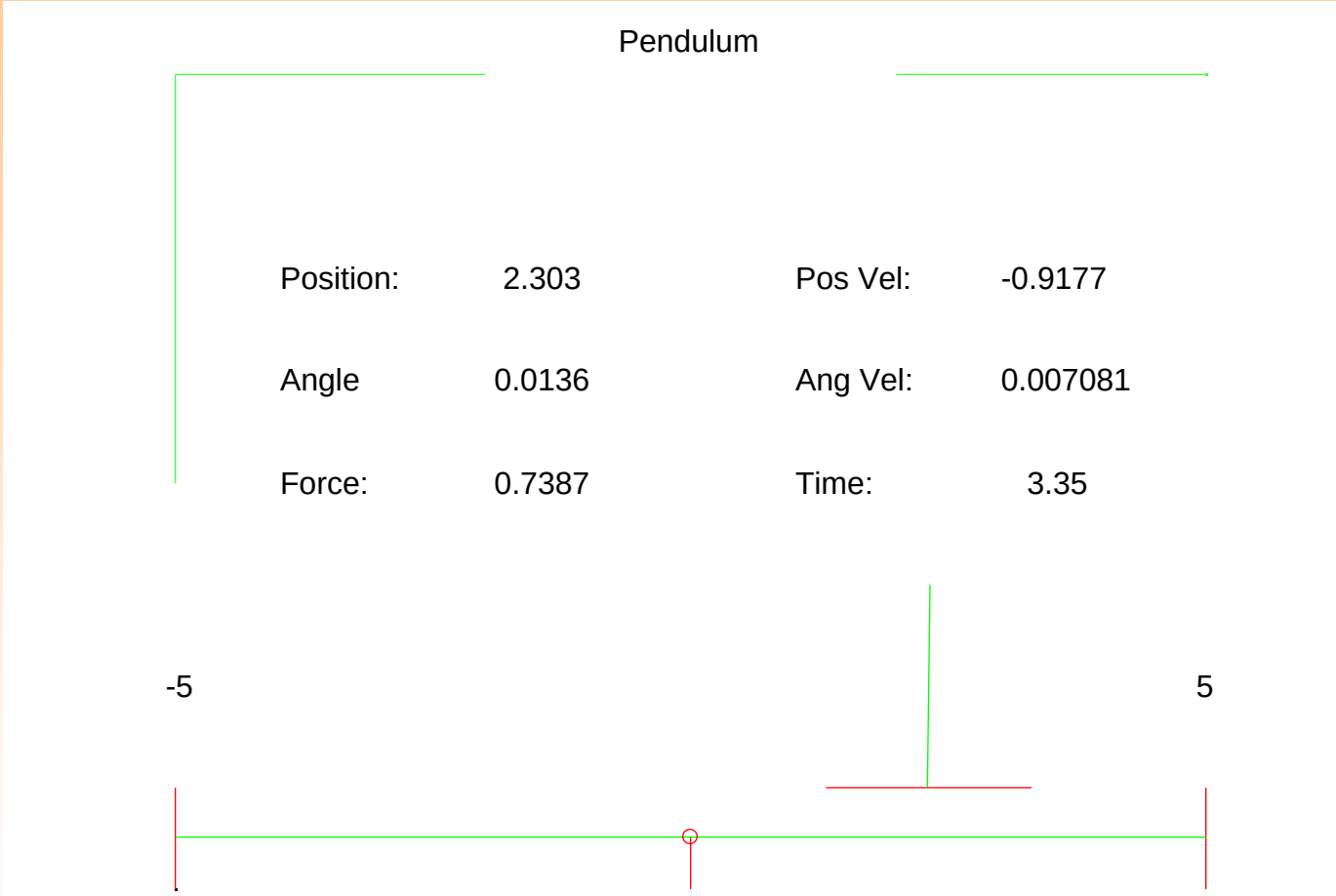
Simulating the Controlled Pendulum



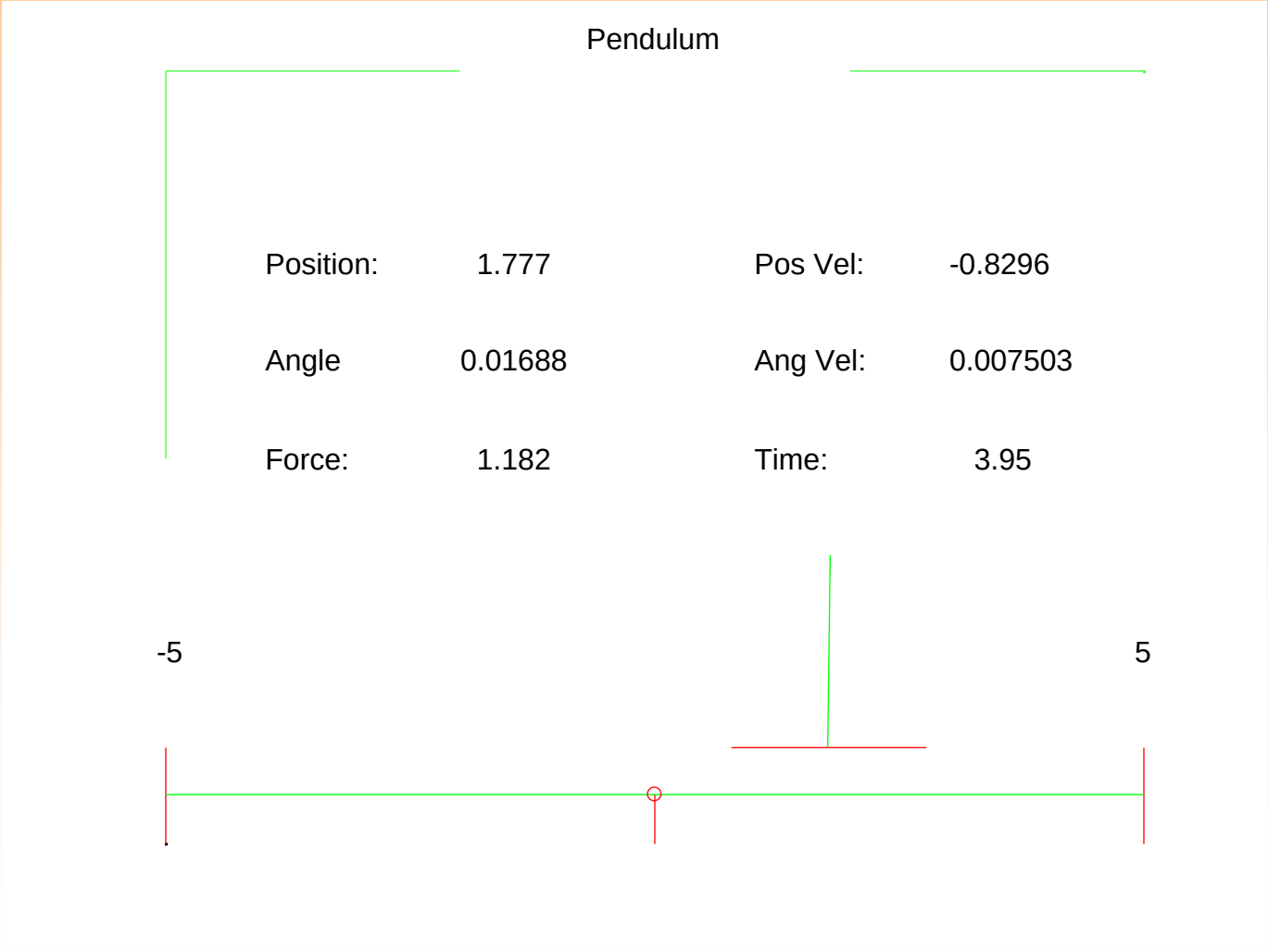
Simulating the Controlled Pendulum



Simulating the Controlled Pendulum

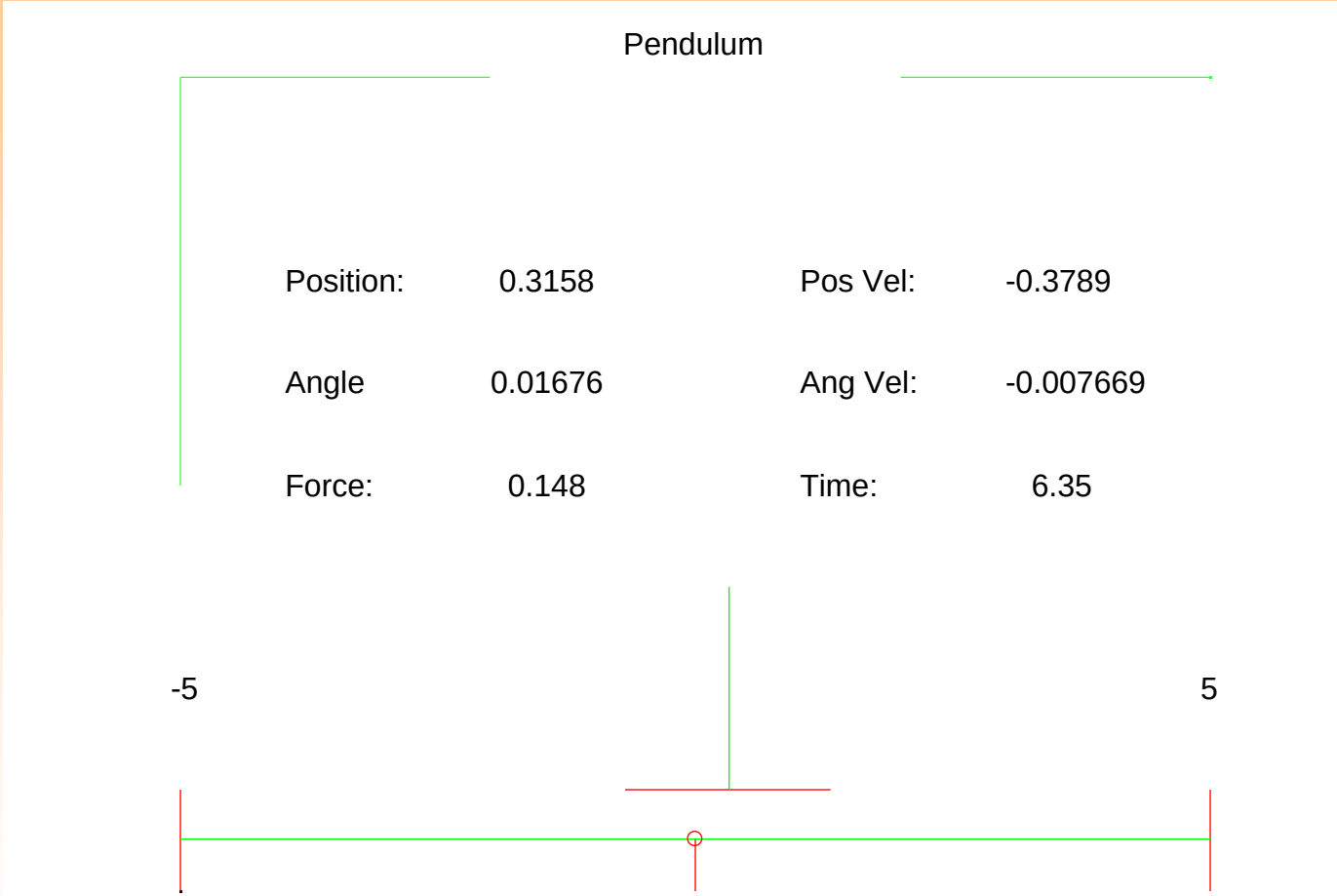


Simulating the Controlled Pendulum

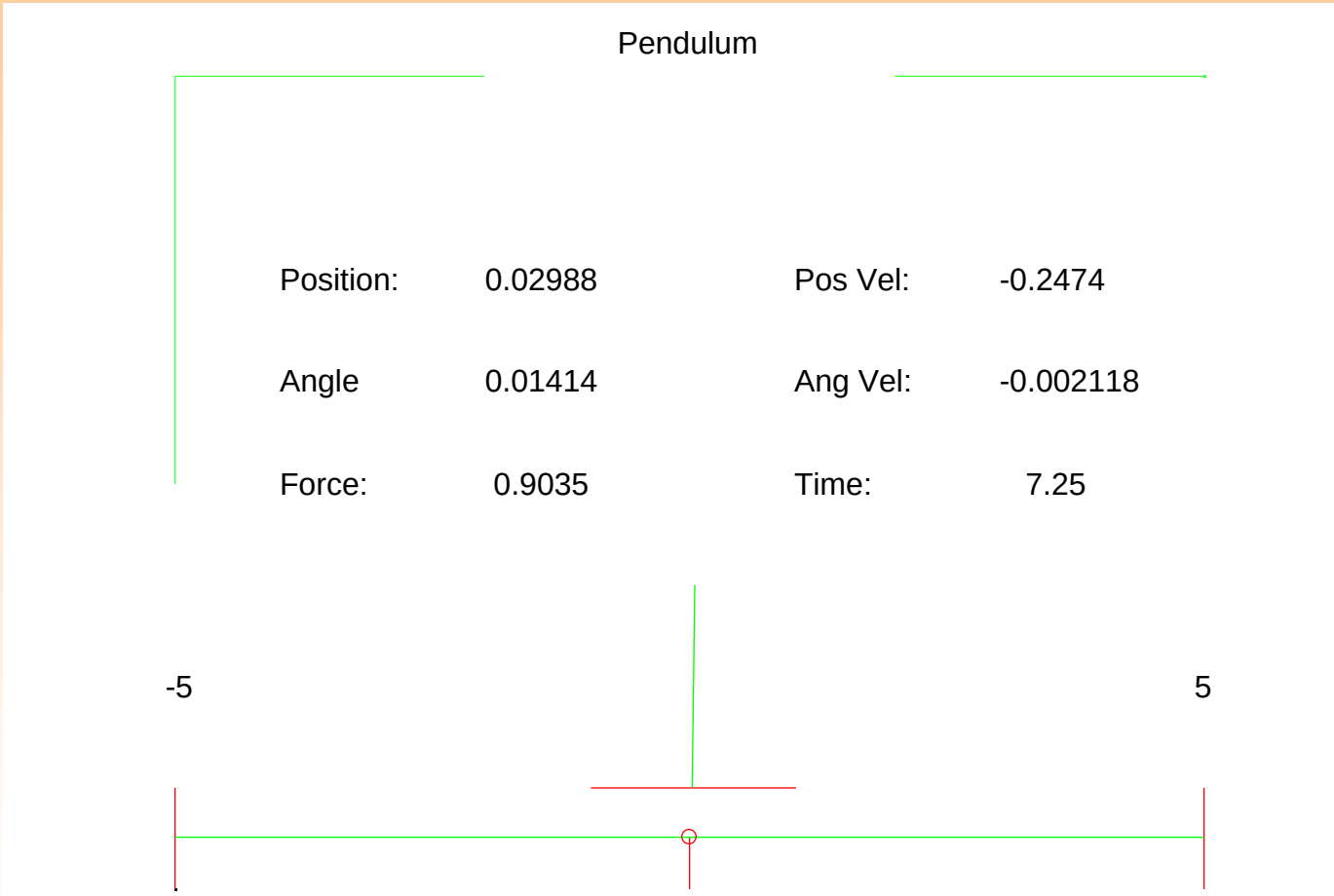


Simulating the Controlled Pendulum

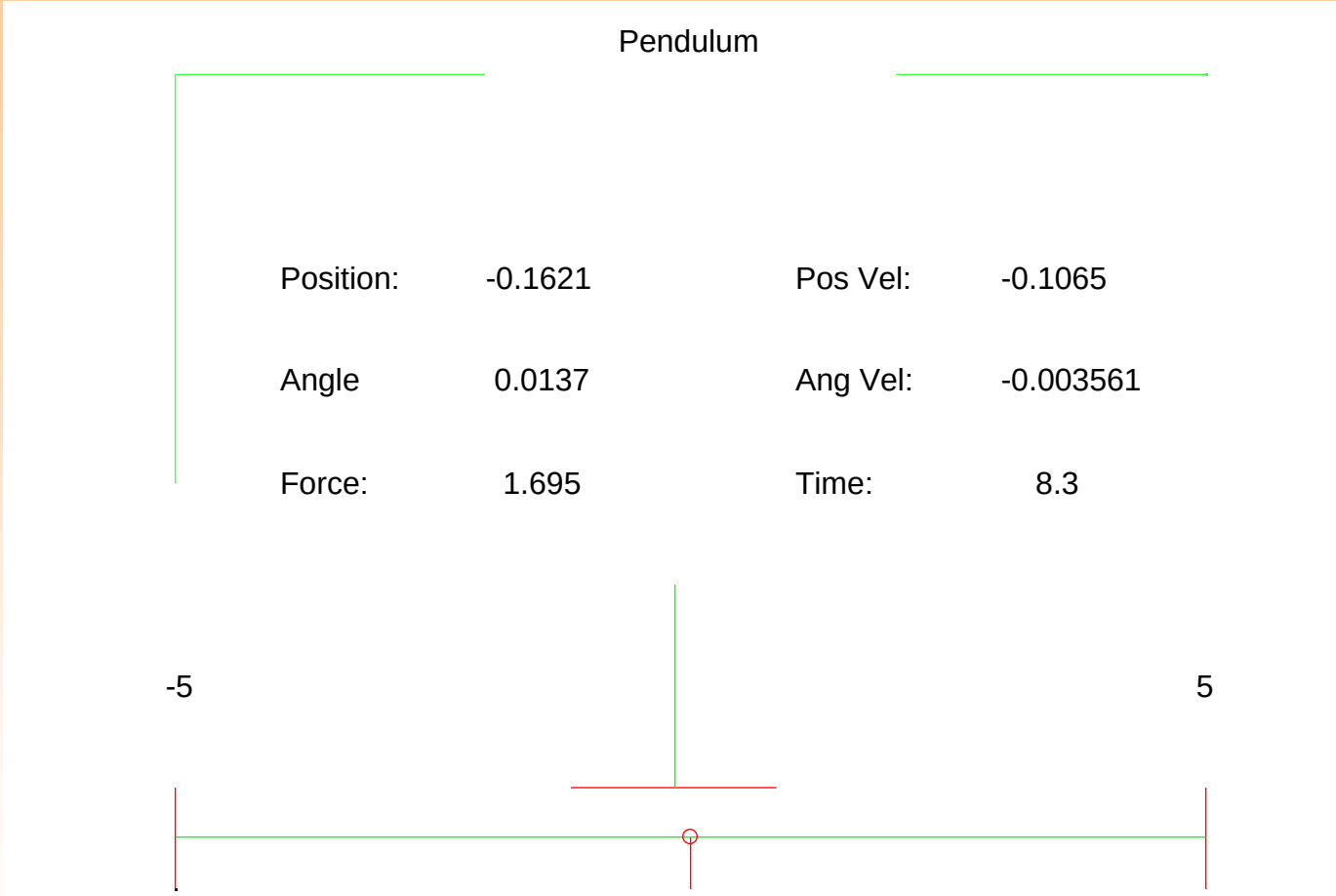
Simulating the Controlled Pendulum



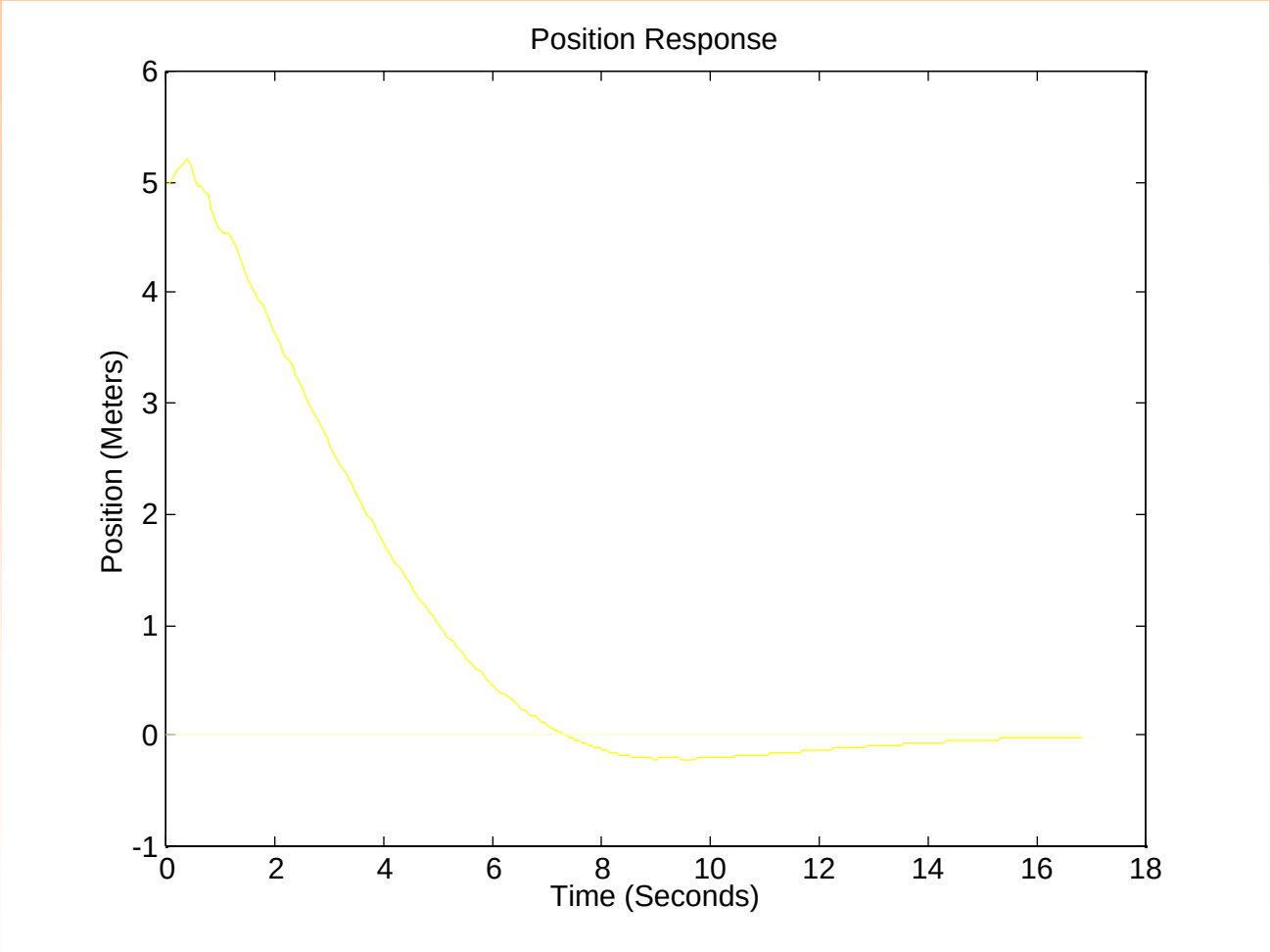
Simulating the Controlled Pendulum



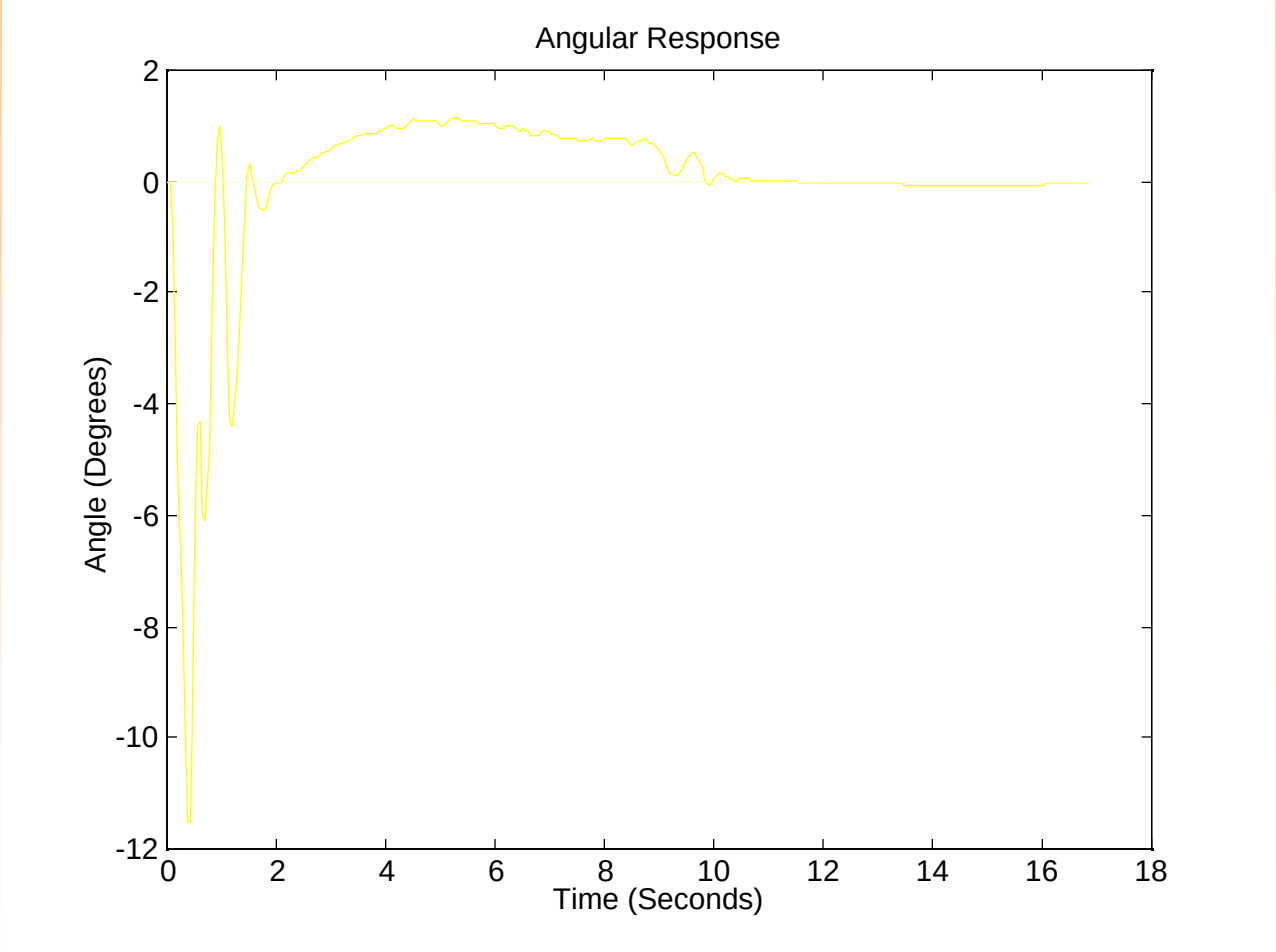
Simulating the Controlled Pendulum



Simulating the Controlled Pendulum



Simulating the Controlled Pendulum



Simulating the Controlled Pendulum