Single Phase Transformer

DTL

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In this document we describe the model derivation of the two-winding real transformer comprehensive of the power losses and thermal equivalent model.

Chapter 1

Fundamental of Magnetic Devices

1.1 Introduction

Many electronic circuits require the use of inductors and transformers. These are usually the largest, heaviest, and most expensive components in a circuit. They are defined by their electromagnetic (EM) behavior. The main feature of an inductor is its ability to store magnetic energy in the form of magnetic field. The important feature of a transformer is its ability to couple magnetic fluxes of different windings and transfer AC energy from the input to the output through the magnetic field. The amount of energy transferred is determined by the operating frequency, flux density, and temperature. Transformers are used to change the AC voltage and current levels as well as to provide DC isolation while transmitting AC signals. They can combine energy from many AC sources by the addition of the magnetic flux and deliver the energy from all the inputs to one or multiple outputs simultaneously. Power losses in inductors and transformers are due to DC current flow, AC current flow, and associated skin and proximity effects in windings, as well as due to eddy currents and hysteresis in magnetic cores. In addition, there are dielectric losses in materials used to insulate the core and the windings. Failure mechanism in magnetic components are mostly due to excessive temperature rise. Therefore, these devices should satisfy both magnetic requirements and thermal limitations.

1.2 Fields

A *field* is defined as a spatial distribution of quantity everywhere in a region. There are two categories of fields: scalar fields and vector fields. A *scalar field* is a set of scalars assigned at individual points in space. A scalar quantity has a magnitude only. Example of scalar fields are time, temperature, humidity, pressure, mass, sound intensity, altitude of terrain, energy, power density, electrical charge density, and electrical potential. The scalar field may be described by a real or a complex function. The intensity of scalar field may be represented graphically by undirected field lines. A higher density of the field lines indicates a stronger field in the area.

A *vector field* is a set of vectors assigned a every point in space. A vector quantity has both magnitude and direction. Examples of vector fields are velocity $\vec{v}$, the Earth's gravitational force field $\vec{f}$, electric current density field $\vec{J}$, magnetic field intensity $\vec{H}$, and magnetic flux density $\vec{B}$. The vector field may be represented graphically by directed field lines. The density of field lines indicates the field intensity, and the direction of the field lines indicates the direction of the vector at each point. In general, fields are function of position and time, for example, $\rho_v(x, y, z, t)$. The rate of charge of a scalar field with distance is a vector.

1.3 Magnetic Relationships

The magnetic field is characterized by magnetomotive force (MMF) $\mathscr{F}$, magnetic field intensity $\vec{H}$, magnetic flux density $\vec{B}$, magnetic flux ϕ, and magnetic flux linkage λ (in rotating machines is represented as ψ).

1.3.1 Magnetomotive Force

An inductor with N turns carrying an AC current i produces the MMF, which is also called the magnetomotance. The MMF is given by

$$\mathscr{F} = Ni \quad \left[\mathrm{A} \cdot \mathrm{turns}\right] \tag{1.3.1}$$

Its descriptive unit is ampere-turns. However, the approved SI unit of the MMF is the ampere (A), where $1\,\mathrm{A} = 6.25 \times 10^{18}\,\mathrm{e}^{-}/\mathrm{s}$. The MMF is a *source* in magnetic circuit. The magnetic flux ϕ is forced to flow in a magnetic circuit by the MMF $\mathscr{F} = Ni$, driving a magnetic circuit.

The MMF between any two point P_1 and P_2 produced by a magnetic field $\vec{H}$ is determined by a line integral of the magnetic field intensity $\vec{H}$ present between these two points

$$\mathscr{F} = \int_{P_1}^{P_2} \vec{H} \cdot d\vec{l} = \int_{P_1}^{P_2} H \cos\vartheta dl \tag{1.3.2}$$

where $d\vec{l}$ is the incremental vector at a point located on the path l. The MMF depends only on the endpoints, and it is independent of the path between points P_1 and P_2. Any path can be chosen. If the path is broken up into segments parallel and perpendicular to H, only parallel segments contribute to $\mathscr{F}$. The contribution from the perpendicular segments are zero.

For a uniform magnetic field and parallel to path l, the MMF is given by

$$\mathscr{F} = Hl \tag{1.3.3}$$

Thus

$$\mathscr{F} = Hl = Ni \tag{1.3.4}$$

The MMF forces a magnetic flux ϕ to flow.

1.3.2 Magnetic Field intensity

The *magnetic field intensity* (or *magnetic field strength*) is defined as the MMF $\mathscr{F}$ per unit length

$$H = \frac{\mathscr{F}}{l} = \frac{Hl}{l} = \frac{N}{l}i \quad \left[\mathrm{A\,m}\right] \tag{1.3.5}$$

where l is the inductor length and N is the number of turns. Magnetic fields are produced by moving charges. Therefore, magnetic field intensity H is directly proportional to the amount of current i and the number of turns per unit length N/l. If a conductor conducts current i (which is a moving charge), it produces a magnetic field H. Thus, the source of the magnetic field H is a conductor carrying a current i. The magnetic field intensity $\vec{H}$ is a vector field. It is described by a magnitude and a direction at any given point. The lines of magnetic field H always form closed loops. By Ampere's law, the magnetic field produced by a straight conductor carrying current i is given by

$$\vec{H}(r) = \frac{i}{2\pi r}\vec{a}_{\phi}. \tag{1.3.6}$$

The magnetic field intensity H is directly proportional to current i and inversely proportional to the radial distance from the conductor r. The Earth's magnetic field intensity is approximately $50\,\mu\mathrm{T}$.

1.3.3 Magnetic Flux

The amount of the *magnetic flux* passing through an open surface $\mathcal{S}$ is determined by a surface integral of the magnetic flux density $\vec{B}$

$$\phi = \iint_{\mathcal{S}} \vec{B} \cdot d\vec{\mathcal{S}} = \iint_{\mathcal{S}} \vec{B} \cdot \vec{n} d\mathcal{S} \quad \left[\mathrm{Wb}\right] \tag{1.3.7}$$

where $\vec{n}$ is the unit vector normal to the incremental surface area $d\mathcal{S}$ at a given position, $d\vec{\mathcal{S}} = vecnd\mathcal{S}$ is the incremental surface vector normal to the local surface $d\mathcal{S}$ at a given position. The magnetic flux is a scalar.

If the magnetic flux density B is uniform and forms a angle ϑ_B with the vector perpendicular to the surface $\mathcal{S}$, the amount of the magnetic flux passing through the surface $\mathcal{S}$ is

$$\phi = \vec{B} \cdot \vec{\mathcal{S}} = B\mathcal{S} \cos \vartheta_B. \tag{1.3.8}$$

If the magnetic flux density B is uniform and perpendicular to the surface $\mathcal{S}$, the angle between vectors $\vec{B}$ and $d\vec{\mathcal{S}}$ is $\vartheta_B = 0$ and the amount of magnetic flux passing through the surface $\mathcal{S}$ is $B\mathcal{S}$.

For an inductor, the amount of magnetic flux ϕ may be increased by increasing the surface area of a single turn A, the number of turns in the layer N_{tl}, and the number of layers N_{l}. Hence, $\mathcal{S} = N_{\mathrm{tl}} N_{\mathrm{l}} A = NA$, where $N = N_{\mathrm{tl}} N_{\mathrm{l}}$ is the total number of turns.

1.3.4 Magnetic Flux Density

The *magnet flux density*, or *induction*, is the magnetic flux perunit area given by

$$B = \frac{\phi}{\mathcal{S}} \quad \left[\mathrm{T}\right]. \tag{1.3.9}$$

The unit of magnetic flux density B in Tesla. The magnetic flux density is a vector field and it can be represented by magnetic lines. The density of magnetic lines indicates the magnetic flux density B, and the direction of the magnetic lines indicates the direction of the magnetic flux density at a given point.

The relationship between the magnetic flux density B and the magnetic field intensity H is given by

$$B = \mu H = \mu_r \mu_0 H = \mu \frac{Ni}{l_c} = \mu \frac{\mathcal{F}}{l_c} < B_s \tag{1.3.10}$$

where the permeability of the free space is

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}} \tag{1.3.11}$$

$\mu = \mu_r \mu_0$ is the permeability, $\mu_r = \mu/\mu_0$ is the relative permeability (i.e. relative to that of free space), and l_c is the length of the core. The permeability is the measure of the ability of a material to conduct magnetic flux ϕ. It describes how easily a material can be magnetized. For a large value of μ_r, a small current i produces a large magnetic flux density B. The magnetic flux ϕ takes the path of the highest permeability.

The magnetic flux density field is a vector field. For example, the vector of the magnetic flux density produced by a straight conductor carrying current i is given by

$$\vec{B}(\vec{r}) = \mu \vec{H}(\vec{r}) = \frac{\mu i}{2\pi r} \vec{a}_\phi. \tag{1.3.12}$$

For ferromagnetic materials, the relationship between B and H is nonlinear because the relative permeability μ_r depends on the magnetic field intensity H. Figure 1.1 shows simplified plots of the magnetic flux density B as a function of the magnetic field intensity H for air-core inductors (straight line) and for ferromagnetic core inductors. The straight line describes the air-core inductor and has a slope μ_0 for all values of H. These inductor are linear, The piecewise linear approximation corresponds to the ferromagnetic core inductors, where B_s is the saturation magnetic flux density and $H_s = B_s/(\mu_r \mu_0)$ is the magnetic field intensity corresponding to B_s. At low values of the magnetic flux density $B < B_s$, the relative permeability μ_r is high and the slope of the $B - H$ curve $\mu_e \mu_0$ is also high. For $B > B_s$, the core saturates and $\mu_r = 1$, reducing the slope of the $B - H$ curve to μ_0.

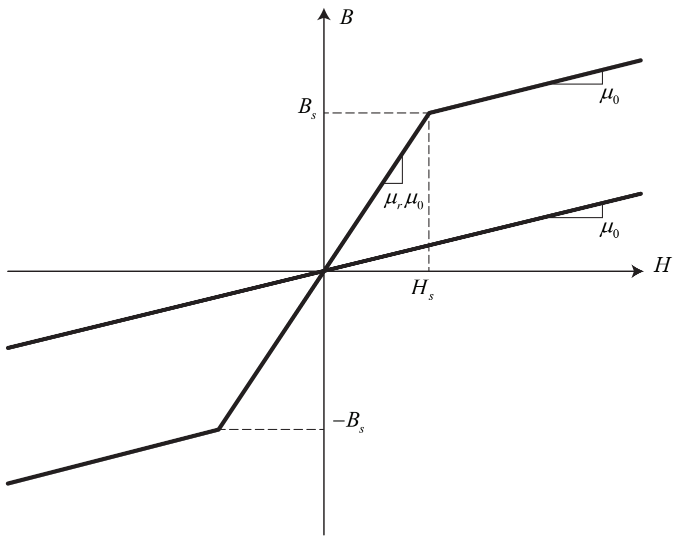


Figure 1.1: Simplified plots of magnetic flux density B as a function of magnetic field intensity H for air-core inductors (strait line) and ferromagnetic core inductors (piecewise linear approximation)

The total peak magnetic flux density B_{pk}, which in general consists of both the DC component B_{DC} and the amplitude of AC component B_m, should be lower than the saturation flux density B_s of a magnetic core at the highest operating temperature $T_{\max}$.

1.3.5 Magnetic Flux Linkage

The *magnetic flux linkage* is the sum of the flux enclosed by each turn of the wire wound around the core

$$\lambda = N \iint_{\mathscr{S}} \vec{B} \cdot d\mathscr{S} = \int v dt \tag{1.3.13}$$

For the uniform magnetic flux density, the magnetic flux linkage is the magnetic flux linking N turns and is described by

$$\lambda = N\phi = NA_c B = NA_c \mu H = \frac{\mu A_c N^2 i}{l_c} = \frac{N^2}{\mathscr{R}} i = Li \quad \left[\mathrm{V\,s}\right] \tag{1.3.14}$$

where $\mathscr{R}$ is the core reluctance and NA_c is the effective area through which the magnetic flux ϕ passes.

1.4 Magnetic Circuit

1.4.1 Reluctance

The *reluctance* $\mathscr{R}$ is the resistance of the core to the flow of magnetic flux ϕ. It opposes the magnetic flux flow, in the same way as the resistance opposes the electric current flow. An element of a magnetic circuit can be called *reluctor*. The concept of the reluctance is illustrated in Figure 1.2 The reluctance of a linear, isotropic, and homogeneous magnetic material is given by

$$\mathscr{R} = \frac{1}{\mathscr{P}} = \frac{l_c}{\mu A_c} = \frac{l_c}{\mu_0 \mu_r A_c} \quad \left[\mathrm{A\,Wb^{-1}}\right] \tag{1.4.1}$$

where A_c is the cross-sectional area of the core (i.e., the area through which the magnetic flux flows) and l_c is the mean magnetic path (MPL), which is the mean length of the closed path that the magnetic flux flows around a magnetic circuit. The reluctance is directly proportional to the length of the magnetic path l_c and is inversely proportional to the cross-sectional area A_c through which the magnetic flux ϕ flows. The *permeance* of the basic magnetic circuit element is

$$\mathscr{P} = \frac{1}{\mathscr{R}} = \frac{\mu A_c}{l_c} \quad \left[\mathrm{Wb\,A^{-1}}\right] \tag{1.4.2}$$

When the number of turns $N = 1$, $L = \mathscr{P}$. The reluctance is the magnetic resistance because it opposes the establishment and the flow of the magnetic flux ϕ in a medium. Magnetic Ohm's law is expressed as

$$\phi = \frac{\mathscr{F}}{\mathscr{R}} = \mathscr{P}\mathscr{F} = \frac{\mu_r \mu_0 A_c N i}{l_c}. \tag{1.4.3}$$

Magnetic flux always takes the path with the highest permeability μ. The reluctance $\mathscr{R}$ in magnetic circuits is analogous to the resistance R in a electrical circuit as shown in Figure 1.3.

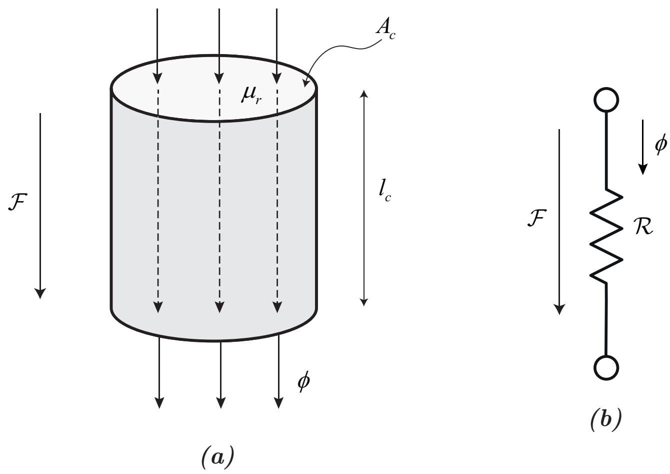


Figure 1.2: Reluctance. (a) Basic magnet circuit element conducting magnetic flux ϕ. (b) Equivalent magnetic circuit.

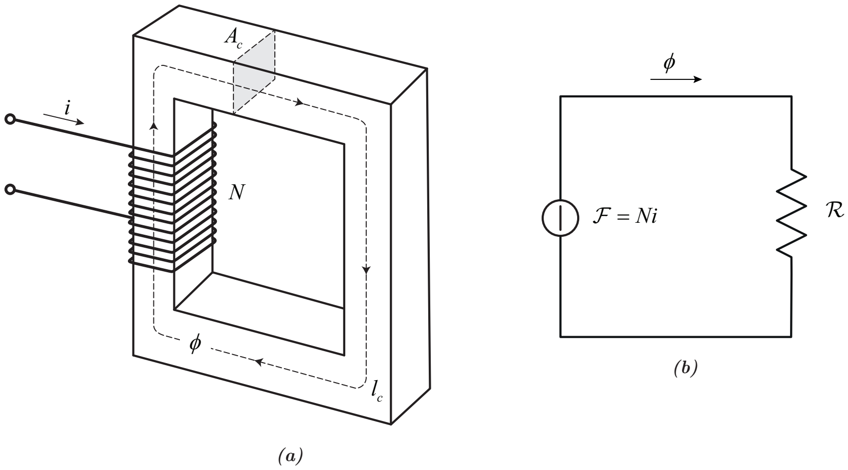


Figure 1.3: Magnetic circuit. (a) An inductor composed of a core and a winding. (b) Equivalent magnetic circuit.

1.4.2 Magnetic KVL

Physical structures, which are made of magnetic devices, such as inductors and transformers, can be analyzed just like electric circuit. The magnetic law, analogous to Kirchhoff's voltage law (KVL), states that the sum of the MMFs $\sum_{k=1}^{n} \mathscr{F}_k$ and the magnet potential differences $\sum_{k=1}^{m} \mathscr{R}_k \phi_k$ around the closed magnetic loop is zero

$$\sum_{k=1}^{n} \mathscr{F}_k - \sum_{k=1}^{m} \mathscr{R}_k \phi_k = 0. \tag{1.4.4}$$

For instance, an inductor with a simple core having an air gap as illustrated in Figure 1.4 is given by

$$Ni = \mathscr{F} = \mathscr{F}_c + \mathscr{F}_g = \phi\Big(\mathscr{R}_c + \mathscr{R}_g\Big), \tag{1.4.5}$$

where the reluctance of the core is

$$\mathscr{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} \tag{1.4.6}$$

the reluctance of the air gap is

$$\mathscr{R}_g = \frac{g}{\mu_0 A_c} \tag{1.4.7}$$

and it is assumed that $\phi_c = \phi_g = \phi$. This means that fringing flux is neglected. If $\mu_r \gg 1$, the magnetic flux is confined to the magnetic material, reducing the leakage flux. The ratio of the air gap reluctance to the core reluctance is

$$\frac{\mathscr{R}_g}{\mathscr{R}_c} = \mu_r \frac{g}{l_c}. \tag{1.4.8}$$

The reluctance of the air gap $\mathscr{R}_g$ is much higher than the reluctance of the core $\mathscr{R}_c$ if $\mu_r \gg l_c/g$.

The magnetic potential difference between points a and b is

$$\mathscr{F}_{\text{ab}} = \int_a^b \vec{H} \cdot d\vec{l} = \mathscr{R}_{\text{ab}} \phi \tag{1.4.9}$$

where $\mathscr{R}_{\text{ab}}$ is the reluctance between point a and b.

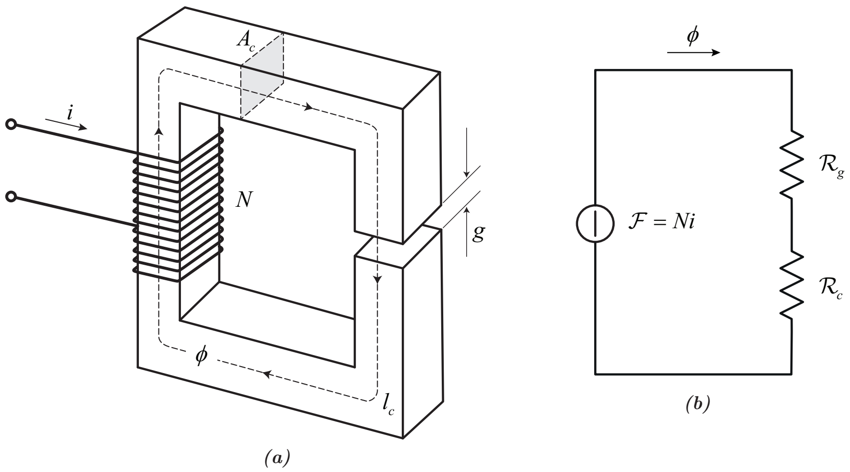


Figure 1.4: Magnetic circuit illustrating the magnetic KVL. (a) An inductor composed of a core with an air gap and a winding. (b) Equivalent magnetic circuit.

1.4.3 Magnetic Flux Continuity

The continuity of the magnetic flux law states that the net magnetic flux through any closed surface is always zero

$$\phi = \oiint_{\mathcal{S}} B d\mathcal{S} = 0 \tag{1.4.10}$$

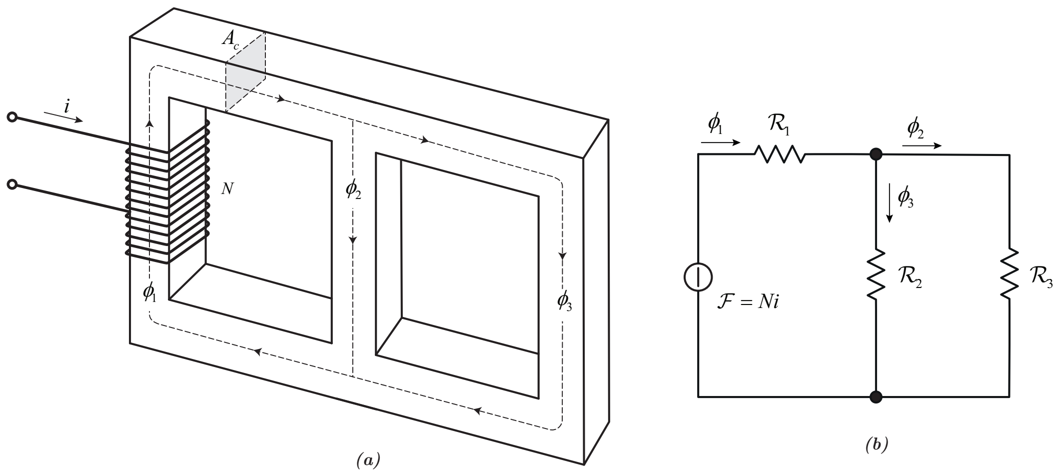


Figure 1.5: Magnetic circuit illustrating the continuity of the magnetic flux for EE core. (a) An inductor composed of a core and a winding. (b) Equivalent magnetic circuit.

or the magnetic flux entering and exiting the node is zero

$$\sum_{k=1}^{n} \phi_k = \sum_{k=1}^{n} \mathcal{S}_k B_k = 0. \tag{1.4.11}$$

This law is analogous to Kirchhoff's current law (KCL) introduced by Gauss and can be called Kirchhoff's flux law (KFL). Figure 1.5 illustrates the continuity of the magnetic flux law. For example when three core legs meet at a node,

$$-\phi_1 + \phi_2 1 + \phi_3 = 0 \tag{1.4.12}$$

which can be expressed by

$$-\frac{\mathscr{F}_1}{\mathscr{R}_1} + \frac{\mathscr{F}_2}{\mathscr{R}_2} + \frac{\mathscr{F}_3}{\mathscr{R}_3} = 0. \tag{1.4.13}$$

if all the three legs of the core have windings, then we have

$$-\frac{N_1 i_1}{\mathscr{R}_1} + \frac{N_2 i_2}{\mathscr{R}_2} + \frac{N_3 i_3}{\mathscr{R}_3} = 0. \tag{1.4.14}$$

1.5 Magnetic Laws

1.5.1 Ampere's Laws

Ampere's law relates the magnetic field intensity H inside a closed loop to the current passing through the loop. A magnetic field can be produced by a current and a current can be produced by a magnetic field. Ampere's law is illustrated in Figure 1.6. A magnetic field is presented around a current carrying conductor or conductors. The integral form of Ampere's circuital law, or simply Ampere's law, describes the relationship between the (conduction, convection and/or displacement) current and the magnetic field produced by this current. It states that the closed line integral of the magnetic field intensity $\vec{H}$ around a closed path $\mathscr{C}$ is equal to the total current i_{enc} enclosed by that path and passing through the interior of the closed path bounding the open surface $\mathscr{S}$

$$\oint_{\mathscr{C}} \vec{H} \cdot d\vec{l} = \iint_{\mathscr{S}} \vec{J} \cdot d\mathscr{S} = \sum_{k=1}^{N} i_k = i_{\text{enc}}, \tag{1.5.1}$$

where $d\vec{l}$ is the vector length element pointing in the direction of the path $\mathscr{C}$ and $\vec{J}$ is the current density. The current i_{enc} enclosed by the path $\mathscr{C}$ is given by the surface integral of the normal component $\vec{J}$ over the open surface $\mathscr{S}$. The surface integral of the current density $\vec{J}$ is equal to the current I flowing through the surface $\mathscr{S}$.

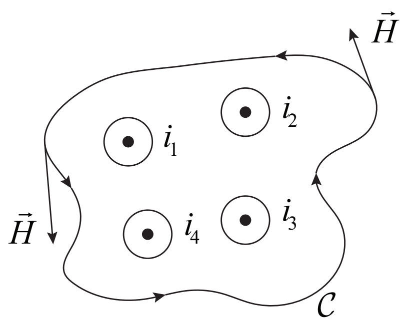


Figure 1.6: Illustration of Ampere's law.

For example, consider a long, straight, round conductor that carries current I. The line integral about a circular path of radius r centered on the axis of the round wire is equal to the product of the circumference and the magnetic field intensity H_ϕ

$$\oint_{\mathscr{C}} \vec{H} \cdot d\vec{l} = 2\pi H_\phi = I, \tag{1.5.2}$$

yielding the magnetic field intensity

$$H_\phi = \frac{I}{2\pi r}. \tag{1.5.3}$$

thus, the magnetic field decreases in the radial direction away from the conductor.

For an inductor with N turns, Ampere's law is

$$\oint_{\mathscr{C}} \vec{H} \cdot d\vec{l} = Ni. \tag{1.5.4}$$

Ampere's law in the discrete form can be expressed as

$$\sum_{k=1}^{n} H_k l_k = \sum_{k=1}^{m} N_k i_k. \tag{1.5.5}$$

For example, Ampere's law for an inductor with an air gap os given by

$$H_c l_c + H_g g = Ni. \tag{1.5.6}$$

1.5.2 Faraday's Laws

A time-varying current produces a time-varying magnetic field, and a time varying magnetic field can produce a time-varying electric current. In 1831, Michael Faraday discovered that a current can be produced by an alternating magnetic field and that a time varying magnetic field can induce a voltage, or an EMF, in an adjacent circuit. This voltage is proportional to the rate of change of magnetic flux linkage λ, or magnetic flux ϕ, or current i producing the magnetic field.

Faraday's law of induction states that a time varying magnetic flux $\phi(t)$ passing through a closed stationary loop, such as an inductor turn, generates a voltage $v(t)$ in the loop and for a linear inductor is expressed by

$$\begin{aligned} v(t) &= \frac{d\lambda}{dt} = N\frac{d\phi}{dt} = NA\frac{dB}{dt} = NA\mu\frac{dH}{dt} = \frac{\mu A N^2}{l}\frac{di}{dt} \\ &= N\frac{d}{dt}\left(\frac{\mathscr{F}}{\mathscr{R}}\right) = N\frac{d}{dt}\left(\frac{Ni}{\mathscr{R}}\right) = \frac{N^2}{\mathscr{R}}\frac{di}{dt} = \mathscr{P}N^2\frac{di}{dt} = L\frac{di}{dt} \end{aligned} \tag{1.5.7}$$

This voltage, in turn, may be produced a current $i(t)$. The voltage $v(t)$ is proportional to the rate of change of the magnetic linkage $d\lambda/dt$, or to the rate of change of the magnetic flux density dB/dt and the effective area NA through which the flux is passing. The inductance L relates the induced voltage $v(t)$ to the current $i(t)$. The voltage $v(t)$ across the terminals of an inductor L is proportional to the time rate of change of the current $i(t)$ in the inductor and the inductance L. If the inductor current is constant, the voltage across an ideal inductor is zero. The inductor behaves as a short circuit for DC current. The inductor current cannot change instantaneously.

For nonlinear, time varying inductors, the relationship are

$$d\lambda(t) = L(i)di(t) \tag{1.5.8}$$

and

$$\begin{aligned} v(t) &= \frac{d\lambda}{dt} = L(i)\frac{di(t)}{dt} + i(t)\frac{dL(i)}{dt} = L(i)\frac{di(t)}{dt} + i(t)\frac{dL(i)}{di}\frac{di(t)}{dt} \\ &= \left[L(i) + i(t)\frac{dL(i)}{di}\right]\frac{di(t)}{dt} = L_{\text{eq}}\frac{di(t)}{dt} \end{aligned} \tag{1.5.9}$$

where

$$L_{\text{eq}} = L(i) + i(t)\frac{dL(i)}{di}. \tag{1.5.10}$$

Chapter 2

Theory of the Transformer

2.1 Ideal Transformer

A basic two-winding transformer is shown in Figure 2.1, where the windings are wound on a magnetic core.

A sinusoidal excitation is applied to the input winding and the second winding is on open circuit. These windings are usually referred to as the primary and secondary windings respectively. The primary winding has an inductance L_m called the magnetizing inductance. This is given by:

$$L_m = \frac{N_1^2}{\mathscr{R}_c} \tag{2.1.1}$$

and the reluctance of the core is

$$\mathscr{R}_c = \frac{l_c}{\mu_r \mu_0 A_c} \tag{2.1.2}$$

where l_c is the mean length of the magnetic path around the closed core and A_c is the cross sectional area of the core.

Evidently, as the relative permeability of the core increases the reluctance becomes smaller, this in turn means that the mmf ($N_1 I_1$) required to establish the flux in the core also becomes smaller. For this reason, it is normally assumed that the magnetization current to establish the flux in the core is infinitesimally small.

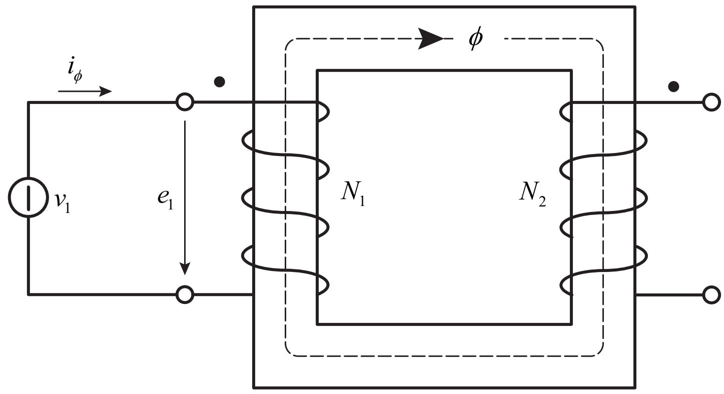


Figure 2.1: Two-winding transformer: no load conditions.

2.1.1 No Load Conditions

The secondary winding in Figure 2.1 is in open circuit under no load conditions. A magnetizing current i_ϕ flows in the primary winding, which establishes the alternating flux ϕ in the magnetic core. The basic relationship between the applied voltage and the flux in the core follows from Ampere's law and Faraday's law:

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_1}{dt} \tag{2.1.3}$$

By Lenz's law, e_1 is a counter-emf to v_1 and, in accordance with Kirchhoff's voltage law, $v_1 = e_1$.

At this point, we shall assume that we are dealing with sinusoidal excitation at frequency f $\big(\omega = 2\pi f\big)$ and the amplitude of the flux is $\phi_{\max}$:

$$\phi(t) = \phi_{\max} \sin \omega t \tag{2.1.4}$$

$$e_1 = N_1 \frac{d\phi}{dt} = \omega N_1 \phi_{\max} \cos \omega t \tag{2.1.5}$$

The amplitude of the primary EMF is

$$E_{1_{\max}} = 2\pi f N_1 \phi_{\max} \tag{2.1.6}$$

The magnetic flux may be expressed in terms of the flux density:

$$\phi_{\max} = B_{\max} A_c \tag{2.1.7}$$

and it follows that:

$$V_{1_{\mathrm{rms}}} = 4.44 f N_1 B_{\max} A_c \tag{2.1.8}$$

2.1.2 Load Conditions

A load at secondary winding with N_2 turns, causing a current i_2 to flow, is applied as shown in Figure 2.2.

The voltage v_1 applied to the primary winding establishes the flux ϕ as before:

$$v_1 = e_1 = N_1 \frac{d\phi}{dt} \tag{2.1.9}$$

The common core flux links the secondary winding and induces an emf e_2 in the secondary winding and a voltage v_2 across the load:

$$v_2 = e_2 = N_2 \frac{d\phi}{dt} \tag{2.1.10}$$

Taking the ratio v_1/v_2 from Eq. (2.1.9) and (2.1.10) yields:

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \tag{2.1.11}$$

The mmf corresponding to the load current is $N_2 i_2$. Ampere's law dictates that the integral of the magnetic field intensity around a closed loop that links the primary and secondary windings is equal to the net mmf.

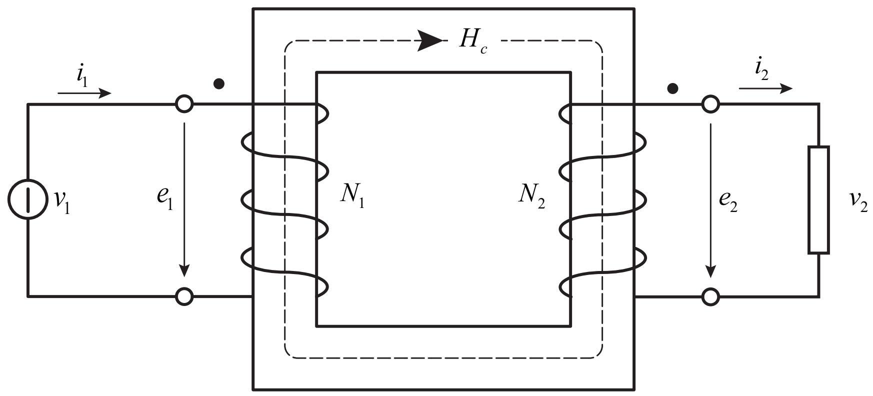


Figure 2.2: Two-winding transformer: load conditions.

The direction of mmf is given by the right hand rule, then referring to Figure 2.2, we have:

$$H_c l_c = N_1 i_1 - N_2 i_2 \tag{2.1.12}$$

The negative sign of $N_2 i_2$ arises because the current in the secondary winding opposes the flux ϕ by the right hand screw rule convention.

The flux density B_c inside the core is related to H_c by the magnetic permeability:

$$B_c = \mu_r \mu_0 H_c = \mu_r \mu_0 \frac{N_1 i_1 - N_2 i_2}{l_c} \tag{2.1.13}$$

The flux inside the core is

$$\phi_c = B_c A_c \tag{2.1.14}$$

Combining Eqs. (2.1.2), (2.1.12), (2.1.13) and (2.1.14) yields:

$$N_1 i_1 - N_2 i_2 = \phi_c \cdot \mathscr{R}_c \tag{2.1.15}$$

In the ideal transformer, we assume the core has infinite permeability ($\mu_r \to \infty$), that the resistance of the windings is negligible and that there is no core loss ($\sigma_c \to 0$). Infinite permeability in the core means that the magnetic reluctance is negligible, which, in turn, means that the mmf required to establish the flux is negligibly small. Thus, if the secondary mmf $N_2 i_2$ is established by the load, it must be countered by an mmf $N_1 i_1$ in the primary to satisfy Eq. 2.1.15. In the hypothesis of ideal transformer, Eq. 2.1.15 becomes:

$$N_1 i_1 - N_2 i_2 = 0 \tag{2.1.16}$$

2.1.3 Dot Convention

The windings in Figure 2.2 show the input current into the positive voltage terminal of the primary winding and the load current out ot the positive voltage terminal of the secondary winding. This conveniently meets the conditions imposed on the mmf by Ampere's law. However, we could just as easily draw the windings as shown in Figure 2.3, and again we can judiciously select the positive voltage terminal and the positive direction of current, so the relationship in Eq. (2.1.12) holds.

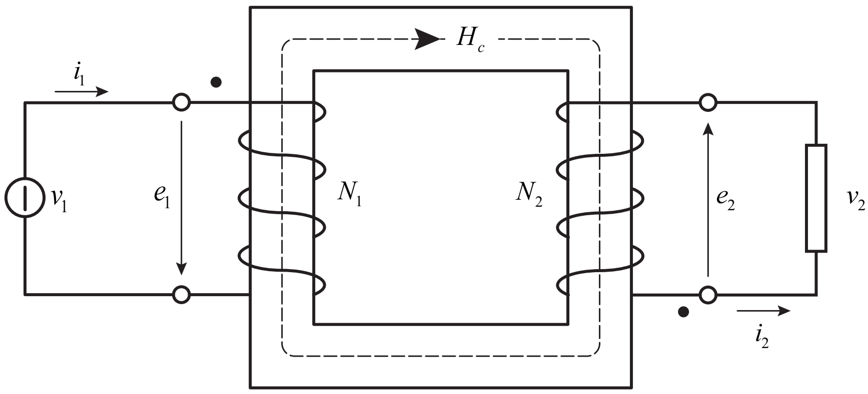


Figure 2.3: Two-winding transformer: load conditions, alternative winding.

Obviously, great care must be taken in drawing the windings and in selecting the voltage and current polarities; adding more windings makes the situation more complex. In reality, most transformers are enclosed and it is not possible by inspection to tell the direction in which each coil is wound. In order to avoid any confusion, the dot convention is adopted.

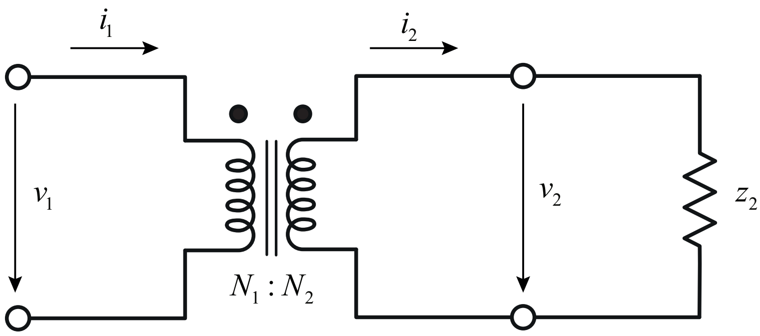
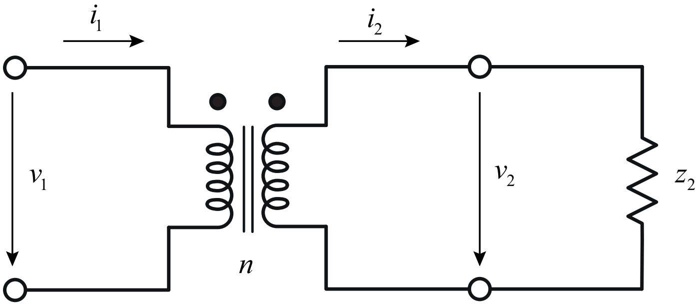


Figure 2.4: Electrical circuit symbol for a transformer.

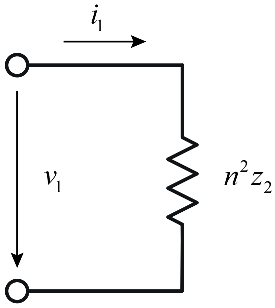
The dot markings in Figure 2.2 and 2.3 indicate terminals of corresponding polarity. If one follows through either windings, beginning at the dotted terminal, both windings encircle the core in the same direction with respect to flux (in accordance with the right hand screw rule). Using this convention, the voltage at the dotted terminals are at the same instantaneous polarity for the primary and secondary windings. Similarly, the currents as shown are in phase. According to the convention, the instantaneous currents are in opposite directions through the windings, so therefore their mmfs cancel. In the electrical circuit symbol for a transformer, we can deduce the physical direction of the winding from the dot convention. Thus, the transformers of Figure 2.2 and 2.3 are represented by the electrical circuit symbol in Figure 2.4. The parallel bars between the two windings represent a common ferromagnetic core.

2.1.4 Reflected Impedance

When signals are transmitted in a circuit, the maximum power transfer theorem dictates that maximum power is transferred from a source to a load when the source impedance is equal to the load impedance. The transformer may be used to match the impedances between the source and the load.



(a) Real circuit.



(b) Equivalent circuit.

Figure 2.5: Reflected impedance in a transformer winding.

The ratio v_1/i_1 is the impedance seen by the input terminals of the transformer, recalling:

$$v_1 = nv_2 \tag{2.1.17}$$

$$i1 = \frac{1}{n} i_2 \tag{2.1.18}$$

$$z_2^1 = \frac{v_1}{i_1} = n^2 \frac{v_2}{i_2} = n^2 z_2 \tag{2.1.19}$$

where z_2 is the impedance of the load. Thus, the impedance z_2 in the secondary may be replaced by the equivalent impedance z_2^1, as seen from the primary terminals. The transformer equivalent circuit of Figure 2.5a is shown in Figure 2.5b.

2.1.5 Summary

In an ideal transformer

1. Voltage are transformed in the direct ratio of turns

$$v_1 = \frac{N_1}{N_2} v_2 = nv_2 \tag{2.1.20}$$

2. Current are transformed in the inverse ratio of turns

$$i_1 = \frac{N_2}{N_1} i_2 = \frac{1}{n} i_2 \tag{2.1.21}$$

3. Impedance are transformed in the direct ratio squared

$$z_2^1 = \left(\frac{N_1}{N_2}\right)^2 z_2 = n^2 z_2 \tag{2.1.22}$$

The notation z_2^1 means the secondary impedance z_2 reflected in the primary.

The voltage impressed on a winding is related to the frequency, the number of turns, the maximum flux density and the core cross-section area.

$$V_{1_{\text{rms}}} = 4.44 f N_1 B_{\max} A_c \tag{2.1.23}$$

In building up the equivalent electrical circuit for the transformer, we can refer quantities in one winding to another winding so that the secondary voltage reflected into the primary winding is v_2^1 and the secondary voltage reflected into the primary winding is v_1^2.

The relationship are

$$v_2^1 = \frac{N_1}{N_2} v_2 = n v_2 \tag{2.1.24}$$

$$v_1^2 = \frac{N_2}{N_1} v_1 = \frac{1}{n} v_1 \tag{2.1.25}$$

2.2 Practical Transformer

In practical transformer, the following factor must be taken into account:

- magnetizing current and core loss;
- winding resistance;
- magnetic leakage flux;

In power electronics applications, winding capacitance may be an issue because a resonance condition can occur at high frequency.

2.2.1 Magnetizing Current and Core Loss

The current in the primary winding of a transformer plays two roles:

- It sets up the mutual flux in accordance with Ampere's law.
- It balances the demagnetizing effect of the load current in the secondary winding.

The net mmf is $N_1 i_1 - N_2 i_2$ and, in terms of magnetic circuit law, this may be related to the reluctance of the transformer core:

$$N_1 i_1 - N_2 i_2 = \phi_m \cdot \mathscr{R} \tag{2.2.1}$$

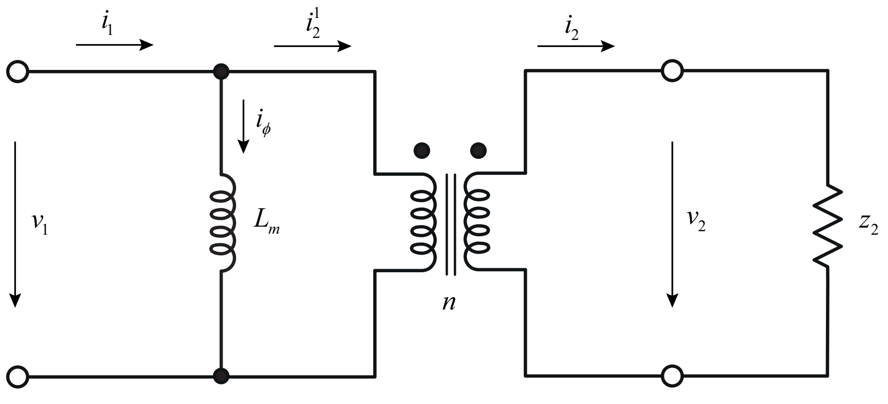


Figure 2.6: Electrical circuit for a transformer with the magnetizing inductance.

We had previously assumed that, in the ideal transformer, the core had infinite permeability. In reality, however, there is a finite permeability and there is an inductance associated with the reluctance of the core. We call this the magnetizing inductance L_m as shown in Figure 2.6.

Thus, the primary current has two components: the magnetizing component i_ϕ and a load component reflected into the primary i_2^1:

$$i_2^1 = \frac{1}{n} i_2 \tag{2.2.2}$$

The instantaneous magnetizing current, i_ϕ, which establishes the flux in the ferromagnetic core, is determined by the magnetic properties of the core. Let us examine the establishment of the core flux in more detail. Returning to no-load conditions and assuming as before that the winding resistance are negligible, according to Eq. (2.1.4) and (2.1.5), the applied voltage leads the flux in the core by 90°.

At this point, we need to turn our attention to the magnetizing current. To simplify the construction of the magnetizing current curve, we will use a single value normal magnetization curve for flux versus current, as shown in Figure 2.7. This assumption, in effect, neglects hysteresis. The flux corresponding to the current in the time domain graph on the right of Figure 2.7 is derived by the corresponding magnetization curve reported in current domain on the left of Figure 2.7. The magnetization current shape reflects the effects of the saturation. On additional observation is that the magnetizing current and flux are in phase as expected, because mmf is the product of flux and reluctance.

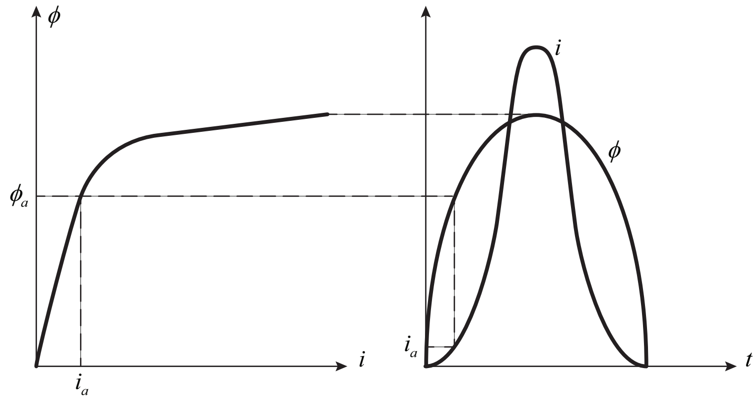


Figure 2.7: Magnetizing current wave shape.

The second observation is that the distorted magnetizing current contains harmonics, and Fourier analysis shows that these are odd harmonics. This further shows that the percentage pf third and fifth harmonics will increase with increased distortion as the core goes further into saturation. Evidently, the peak value of the magnetizing current will increase rapidly as the transformer goes further into saturation. We assumed a single valued magnetization curve and neglected the hysteresis loss; the construction in Figure 2.7 may be repeated for hysteresis by noting the rising and falling value of flux, which introduces further distortion of the magnetization curve, but the overall effect on harmonics is not radically altered. Hysteresis is a power loss in the core and, therefore, it will introduce a component of magnetizing current that is in phase with the applied voltage. In power electronics application, eddy current loss in the core will also add to the hysteresis loss, and the current representing these losses will also be in phase with the applied voltage.

As first approximation, the magnetizing current can therefore be split into two components: one in phase with the applied voltage for the core loss i_c; and the other in phase with the flux i_m. This approach allows us to construct phasor diagrams for the transformer. The harmonic components of the magnetizing current could, in some circumstances, lead to resonant conditions with capacitive components of connected circuits. The magnetizing inductance L_m may represent the flux in the core, and R_c may represent the core loss; the components of the current through these circuit elements combine to form the magnetizing current. The magnetizing current is now represented by a shunt branch connected across v_1, consisting of R_c and L_m in parallel as shown in Figure 2.8

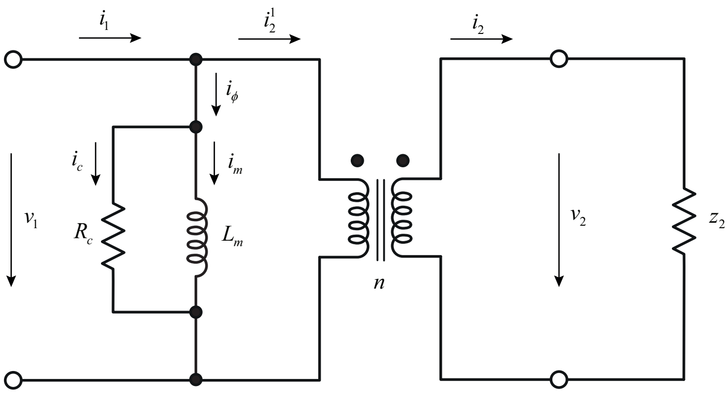


Figure 2.8: Electrical circuit for a transformer with the magnetizing branch.

2.2.2 Winding Resistance

Winding resistance can be represented by the resistances of the wires used in the windings, R_1 and R_2 for primary and secondary, respectively. The AC resistance due to the internal flux in the conductor may be approximated by

$$R_{\mathrm{ac}} = R_{\mathrm{dc}}\left[1 + \frac{\left(\frac{r_0}{\delta}\right)^4}{48 + 0.8\left(\frac{r_0}{\delta}\right)^4}\right] \tag{2.2.3}$$

Where δ is the skin depth in the conductor and r_0 is the radius of the conductor. For high frequency operation, we have to take AC loss in the form of skin effect and proximity effects into account.

2.2.3 Magnetic Leakage

In the ideal transformer, the same flux links both the primary and the secondary circuits. However, in practice, there is always some leakages flux which links only one winding.

Leakage inductance is a property of one winding relative to another. If there is a third winding on the transformer core, the primary-secondary leakage will be distinctly different from the primary-tertiary leakage and so on. Consider the two elementary coils in air presented in Figure 2.9, which constitute an elementary transformer. In Figure 2.9a, coil 1_1 has an alternating current i_1 applied and coil 2 is open-circuited. This produces a magnetic field described by the flux lines in the diagram. Some of this flux links the second coil ans is thus termed the mutual flux.

The remaining flux does not link the secondary, and is termed leakage flux. In Figure 2.9b, coil 1 is open circuited and a current i_2 is applied to coil 2. Again, leakage flux is represented by solid flux lines and mutual flux is denoted by dotted lines. Clearly, the nature of the two leakages fields is quite different, Transformer action occurs when current flows in both coils.

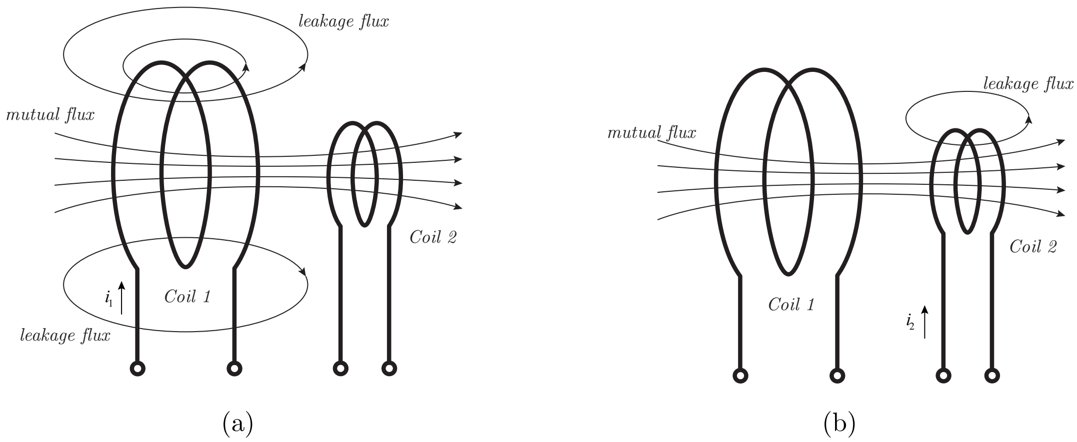


Figure 2.9: Leakage inductance in a transformer.

In this case, the flux linking both coils has three components. If ϕ_{11} is the total flux linking coil 1 and ϕ_{22} is the total flux coil 2, then

$$\phi_{11} = \phi_{l1} + \phi_{21} + \phi_{12} \tag{2.2.4}$$

$$\phi_{22} = \phi_{l2} + \phi_{12} + \phi_{21} \tag{2.2.5}$$

Where ϕ_{l1} is the leakage flux associated with coil 1 due to i_1 in coil 1, ϕ_{21} is the flux linking both coils due to the current i_1, ϕ_{12} is the mutual flux due to current i_2 in the coil 2 and ϕ_{l2} is the leakage flux of coil 2. Each of these flux elements represents an inductance. From the flux equations, we can derive equations for the voltages on each coil:

$$v_1 = \left[L_{l1} + L_1\right]\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} \tag{2.2.6}$$

$$v_2 = \left[L_{l2} + L_2\right]\frac{di_2}{dt} + M_{21}\frac{di_1}{dt} \tag{2.2.7}$$

From the form of these equations, we can extract the self inductances of coil 1 and coil 2 as L_{11} and L_{22} respectively, where:

$$L_{11} = L_{l1} + L_1 \tag{2.2.8}$$

$$L_{22} = L_{l2} + L_2 \tag{2.2.9}$$

Of course, as is always the case with mutual inductance, $M_{12} = M_{21} = M$.

Current in coil 1 sets up flux, some of which links coil 2. By definition, the mutual inductance is the ratio of the flux linking one coil due to the current in the other coil, so that:

$$L_1 = \frac{N_1}{N_2}M \tag{2.2.10}$$

$$L_2 = \frac{N_2}{N_1}M \tag{2.2.11}$$

Thus

$$M = \sqrt{L_1 L_2} \tag{2.2.12}$$

The leakage inductances terms in Eqs. (2.2.8) and (2.2.9) may be obtained using Eqs. (2.2.10) and (2.2.11)

$$L_{l1} = L_{11} - \frac{N_1}{N_2} M \tag{2.2.13}$$

$$L_{l2} = L_{22} - \frac{N_2}{N_1} M \tag{2.2.14}$$

Define

$$k_1 = \frac{L_1}{L_{11}} = 1 - \frac{L_{l1}}{L_{11}} \tag{2.2.15}$$

and

$$k_2 = \frac{L_2}{L_{22}} = 1 - \frac{L_{l2}}{L_{22}} \tag{2.2.16}$$

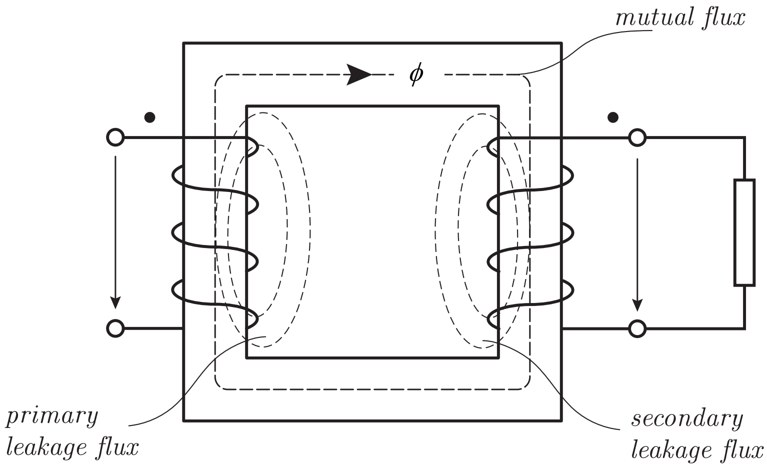


Figure 2.10: Leakage inductance in a transformer.

It follows that

$$k = \sqrt{k_1 k_2} \tag{2.2.17}$$

and

$$M = k\sqrt{L_{11} L_{22}} \tag{2.2.18}$$

k is called the coupling coefficient. Taking this definition of k and the circuit relationships in Eqs. (2.2.6) and (2.2.7), with the appropriate dot convention, yields the classical equivalent electrical circuit representation of the transformer - the model that is normally used in circuit simulations of the transformer. Figure 2.10 shows the physical layout of the winding with the dot convention and the equivalent electrical circuit with coupled inductors. Leakage inductance is affected by high frequency operation.

The leakage affects can be represented by primary and secondary leakage inductors.

2.2.4 Equivalent Circuit

The equivalent circuit model is now complete and shown in Figure 2.11a.

For an ideal transformer:

$$n = \frac{e_1}{e_2} \tag{2.2.19}$$

A further step, we can refer all quantities in the secondary to primary in order to obtain the equivalent circuit of Figure 2.11b.

The leakage inductance of the secondary winding referred to the primary winding is:

$$L_{l2}^{1} = n^2 L_{l2} \tag{2.2.20}$$

The resistance of the secondary winding referred to the primary winding is

$$R_2^1 = n^2 R_2 \tag{2.2.21}$$

The voltage across the secondary winding referred to the primary is given by Eq. (2.1.24)

$$v_2^1 = n v_2 \tag{2.2.22}$$

and the current in the secondary referred to the primary is

$$i_2^1 = \frac{1}{n} i_2 \tag{2.2.23}$$

Finally, we can combine corresponding quantities such as winding resistance and leakage inductances

$$R_{\mathrm{eq}} = R_1 + n^2 R_2 \tag{2.2.24}$$

$$L_{\mathrm{eq}} = L_{l1} + n^2 L_{l2} \tag{2.2.25}$$

The shunt branch representing the core loss and the core magnetization may be moved to the input terminals with little loss of accuracy to obtain Figure 2.11c. This is an approximation to Figure 2.11b, since i_ϕ will change slighly, but it is very small, so the error is negligible. However, it greatly simplifies circuit calculation.

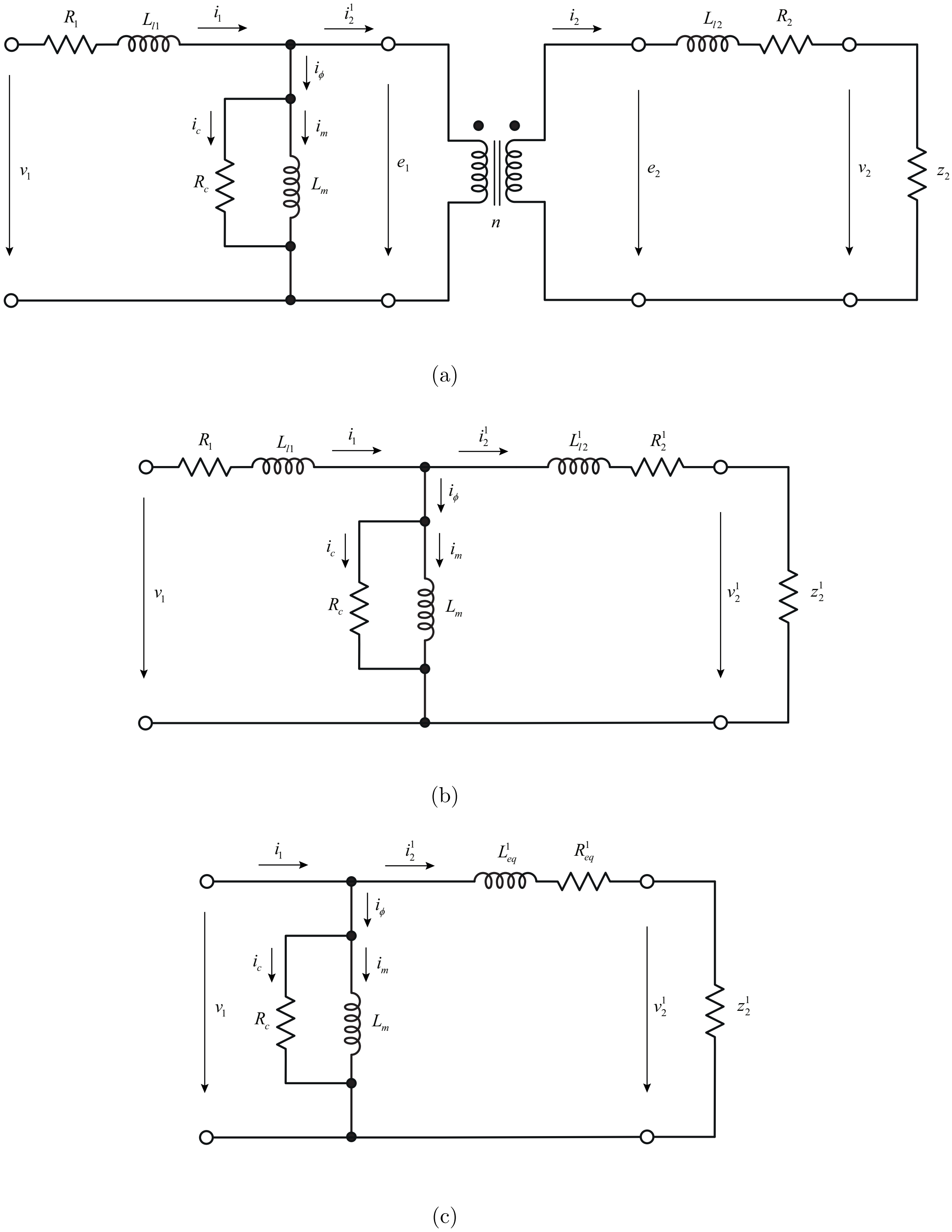


Figure 2.11: Transformer equivalent circuits.

2.3 General Transformer Equations

The traditional treatment of electrical transform considers the case of sinusoidal excitation. For a general treatment which may cover application on power electronics an expanded analysis has to be taken into account including non-sinusoidal excitation and deal with frequencies above the typical mains frequencies. We will begin by generalizing the equations for voltage, power and losses. The dissipation of the losses will determine the temperature rise in the winding, which will lead to an optimization of the transformer core size.

2.3.1 The Voltage Equation

Faraday's law relates the impressed voltage v on a winding to the rate of change of flux density B

$$v = N\frac{d\phi}{dt} = NA_m\frac{dB}{dt} \tag{2.3.1}$$

where N is the number of turns and A_m is the effective cross-sectional area of the magnetic core. In the case of laminated and tape-wound cores this is less than the physical area, A_c, due to interlamination space and insulation.The layout of a typical transformer is shown in Figure 2.12 and the physical parameters are illustrated. The two areas are related by the core stacking factor, k_f ($A_m = k_f A_c$).

Typically, k_f is 0.95 for laminated cores.

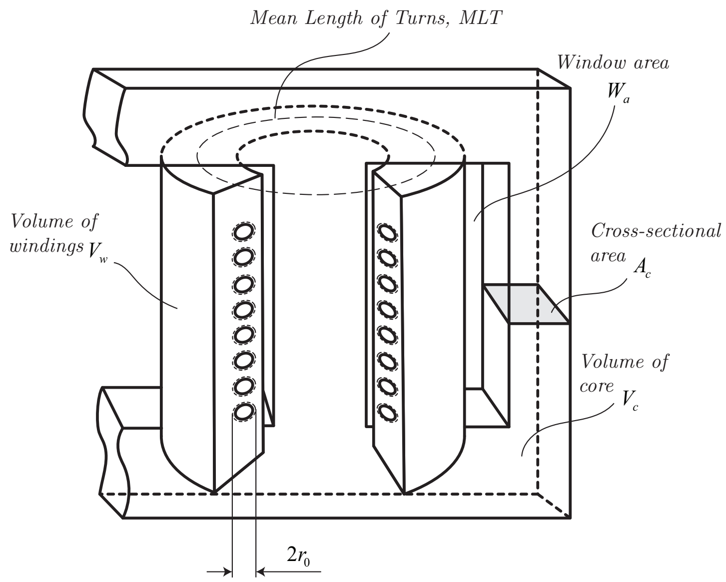


Figure 2.12: Typical layout of a transformer.

The average value of the applied voltage during the interval τ from the point where the flux density is zero to the point where it is at its maximum value ($B_{\max}$) is $\langle v\rangle$. This may be found by integrating Eq. (2.3.1):

$$
\begin{aligned}
\langle v \rangle &= \frac{1}{\tau}\int_0^{\tau} v(t)dt \\
&= \frac{1}{\tau} N A_m \int_0^{B_{\max}} dB \\
&= \frac{1}{\tau} N A_m B_{\max}
\end{aligned}
\tag{2.3.2}
$$

We want to relate this to the rms value of the applied voltage waveform. The form factor k is defined as the ratio of the rms value of the applied voltage waveform to the average value $\langle v \rangle$:

$$k = \frac{V_{\text{rms}}}{\langle v \rangle} \tag{2.3.3}$$

Combining Eqs. (2.3.2) and (2.3.3) yields:

$$V_{\text{rms}} = \frac{k}{\tau f} f N B_{\max} A_m = K_v f B_{\max} A_m \tag{2.3.4}$$

with

$$K_v = \frac{k}{\tau f} \tag{2.3.5}$$

where f is the frequency of the periodic applied voltage $v(t)$.

Eq. (2.3.4) has the same form as the classical transformer voltage equation, as given in Eq. (2.1.6), with K_v, the waveform factor, defined by k, τ and f. For a sinusoidal waveform, $K_v = 4.44$ and, for a square waveform, $K_v = 4.0$. The calculation of K_v for typical power electronic applications will be given in a example.

Example 2.3.1. *Establish the value of K_v for a square waveform.*

Figure 2.13 shows the voltage and flux distribution in a transformer winding with a square wave of voltage applied to the winding. The flux rises from 0 to B_{max} in time $\tau = T/4$ and, therefore, $\tau/T = 0.25$. The form factor for a square wave is 1 since the average value over the time τ is V_{dc} and the rms value of the waveform is V_{dc}. K_v is $1/0.25 = 4.0$.

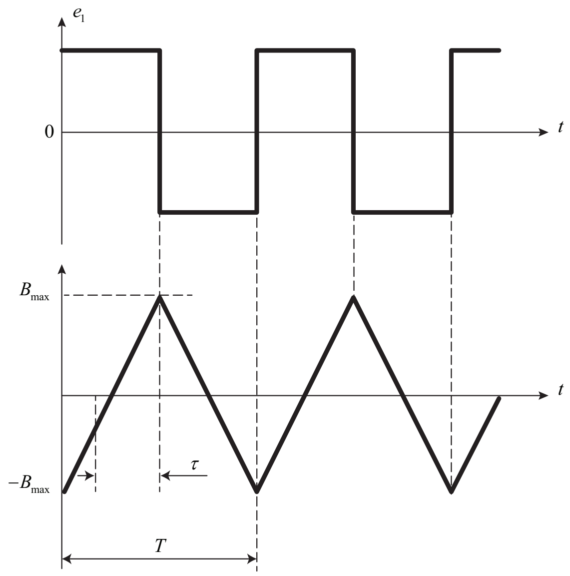


Figure 2.13: Square-wave voltage and flux waveforms.

Example 2.3.2. *Establish the value of K_v for the input voltage waveform in a forward converter.*

The input voltage and flux waveforms for a forward converter are shown in Figure 2.14. The ratio (N_p/N_t) is fixed such that:

$$\frac{N_p}{N_t} = \frac{D}{1-D}$$

so that the volt-seconds balance in the winding is maintained.

The flux density increases from 0 to its maximum value in the time $\tau = DT$. The rms value of the voltage waveform in Figure 2.14 is

$$V_{rms} = \sqrt{\frac{D}{1-D}} V_{dc}$$

The average value of the voltage waveform during the time τ is V_{dc}

$$\langle v \rangle = V_{dc}$$

and thus, from Eq. (2.3.3)

$$k = \frac{V_{rms}}{\langle v \rangle} = \sqrt{\frac{D}{1-D}}$$

and from Eq. (2.3.5)

$$K_v = \frac{1}{\sqrt{D(1-D)}}$$

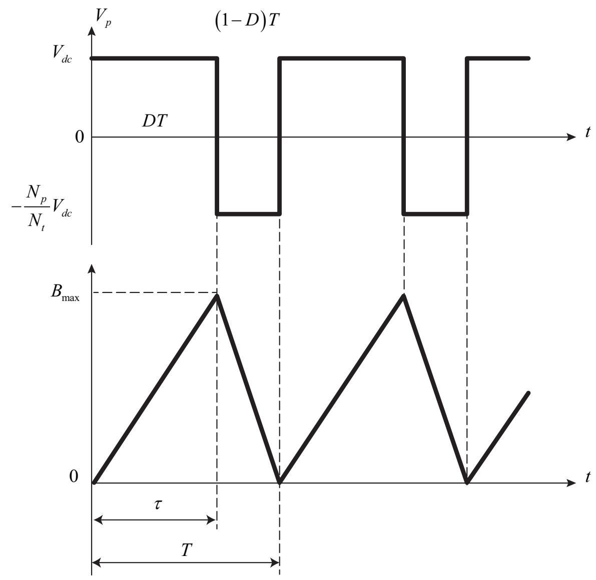


Figure 2.14: Voltage and flux waveform with duty cycle D.

2.3.2 The Power Equation Loss

Equation (2.3.4) applies to each winding of the transformer. It is straightforward to calculate the voltage × current product or VA rating of each winding in the transformer. Taking the sum of the VA products in an n winding transformer and taking the voltage given by Eq. (2.3.4)

$$\sum VA = K_v f B_{\max} A_m \sum_{i=1}^{n} N_i i_i \tag{2.3.6}$$

N_i is the number of turns in winding i that carries a current with rms value i_i.

The window utilization factor k_u is the ratio of the total conduction area W_c for all conductors in all windings to the total window winding area W_a of the core

$$k_u = \frac{W_c}{W_a} \tag{2.3.7}$$

The total conduction area is related to the number of conductors (turns) and the area of each conductor summed over all the windings

$$W_c = \sum_{i=1}^{n} N_i A_{\text{wi}} \tag{2.3.8}$$

where A_{wi} is the conducting area of the wire in winding i. Substituting Eq. (2.3.8) into Eq. (2.3.7)

$$k_u = \frac{\sum_{i=1}^{n} N_i A_{\text{wi}}}{W_a} \tag{2.3.9}$$

Thus

$$\sum_{i=1}^{n} N_i A_{\text{wi}} = k_u W_a \tag{2.3.10}$$

The current density in each winding is $J_i = i_i / A_{\text{wi}}$. Normally, the wire area and the conduction area are taken as the area of the bare conductor. However, we can account for skin effect in a conductor and proximity effect between conductors by noting that the increase in resistance due to these effects is manifested by reducing the effective conduction area. The skin effect, and likewise for the proximity effect factor, k_x

$$k_s = \frac{R_{\text{ac}}}{R_{\text{dc}}} \tag{2.3.11}$$

$$k_x = \frac{R'_{\text{ac}}}{R_{\text{dc}}} \tag{2.3.12}$$

Incorporating these definitions into the window utilization factor

$$k_u = \frac{k_b}{k_s k_x} \tag{2.3.13}$$

where k_b is the ratio of bare conductor total area to the winding area.

The definition in Eq. (2.3.13) makes allowance in the window utilization factor for skin and proximity effects. At this point, we do not have analytical expression for skin and proximity effects. Typically, $k_b = 0.7$, $k_s = 1.3$ and $k_x = 1.3$, giving $k_u = 0.4$.

Combining Eq. (2.3.6) and (2.3.10), with the same current density J_0 in each winding, yields the total VA for all the windings. The optimum distribution of current between multiple windings is achieved when the same current density is applied to each winding

$$\sum VA = K_v f B_{\max} k_f A_c J_0 k_u W_a \tag{2.3.14}$$

The product of the core cross-sectional area ans the window winding area $A_c W_a$ appears in Eq. (2.3.14) and is an indication of the core size, and is designated window-cross-section product A_p. Rearranging Eq. (2.3.14) relates the summation of the VA ratings of all the windings to the physical, electrical magnetic properties of the transformer

$$\sum VA = K_v f B_{\max} J_0 k_f k_u W_a A_p \tag{2.3.15}$$

2.3.3 Winding Loss

The ohmic or I^2R loss in any of the winding is

$$I^2R = \rho_w \frac{l_{\mathrm{wi}}}{A_{\mathrm{wi}}} I_i^2 = \rho_w \frac{N_i MLT (J_0 A_{\mathrm{wi}})^2}{A_{\mathrm{wi}}} \tag{2.3.16}$$

The electrical resistivity of the conductor is ρ_w and the length of the conductor in the winding is l_{wi}, i.e. the product of the number of turns N_i and the mean length of a turn (MILT). The current in the winding is expressed in terms of the current density. The total resistive loss for all the windings is:

$$P_{\mathrm{Cu}} = \sum RI^2 = \rho_w \sum_{i=1}^{n} \frac{N_i MLT (J_0 A_{\mathrm{wi}})^2}{A_{\mathrm{wi}}} \tag{2.3.17}$$

Incorporating the definition od window utilization factor, k_u, and noting that the volume of the windings (fully wound $k_u = 1$) is $V_w = MLT \times W_a$, then

$$P_{\mathrm{Cu}} = \rho_w V_w k_u J_0^2 \tag{2.3.18}$$

2.3.4 Core Loss

In general, the core loss per unit volume are given in $\mathrm{W\,m^{-3}}$ in accordance with the general Steinmetz equation

$$P_{\mathrm{fe}} = K_c f^{\alpha} B_{\max}^{\beta} \tag{2.3.19}$$

where K_c, α and β are constants. Typical values are given in Table 2.1 and Table 2.2. The core loss includes hysteresis and eddy current losses. The manufacturer's data is normally measured for sinusoidal excitation. In the absence of the test data on the design core, the manufacturer's data must be used in establishing the constants in Eq. (2.3.19). The constants may also be deduced from measurements of the core loss.

Materials	Ferrites	Nanocrystalline	Amorphous
Model	Epcos N87	Vitroperm 500F	Metglass 2605
Permeability, μ_i	2200	15000	10000-150000
B_{peak},T	0.49	1.2	1.56
ρ, µΩ m	10×10^6	1.15	1.3
Curie temp. T_c, °C	210	600	399
P_{fe}, mW cm^{-3}	288 (0.2 T 50 kHz)	312 (0.2 T 100 kHz)	366 (0.2 T 25 kHz)
K_c	16.9	2.3	1.377
α	1.25	1.32	1.51
β	2.35	2.1	1.74

Table 2.1: Soft magnetic materials.

Materials	Ni-Fe (permalloy)	Powdered iron	Si iron
Model	Magnetics Permalloy 80	Micro-metals 75μ	Unisil 23M3
Permeability, μ_i	20000-50000	75	5000-10000
B_{peak},T	0.82	0.6-1.3	2.0
ρ, µΩ m	0.57	1×10^6	0.48
Curie temp. T_c, °C	460	665	745
P_{fe}, mW cm^{-3}	12.6 (0.2 T 5 kHz)	1032 (0.2 T 10 kHz)	5.66 (1.5 T 50 kHz)
K_c	0.448	1798	3.388
α	1.56	1.02	1.70
β	1.89	1.89	1.9

Table 2.2: Soft magnetic materials.

Chapter 3

Transformer Design

3.0.1 High Frequency Effects in the Windings

3.0.2 High Frequency Effects in the Core

Chapter 4

Measurements

Traditionally, measurements for transformers and inductors have been focused on power frequency operation. These approaches remain relevant today, but they must be modified or replaced to take account of the high frequencies encountered in power electronics. Measurement of inductance provides several challenges, since it is not always a single value, particularly when saturation is involved. Two methods for inductance measurement are treated - one involving DC current and another involving AC signals.

Traditionally, the measurement of losses in transformers have been carried out by the established open-circuit and short-circuit tests. However, these tests must be adapted to the field of power electronics in a manner that allows us to measure the core loss over a wide frequency range and to present the data in a format compatible with manufacturer's data sheets.

The measurement of $B - H$ loop is described for the core materials. Knowledge of the winding capacitance is important, particularly when there are resonance frequencies involved. Winding capacitance also plays a role in the dynamic response when a step change in voltage is shorted by winding capacitance. A suitable measurement of capacitance in transformer windings is presented, along with some basic formulas to estimate the capacitance in typical winding configurations.

4.1 Measurement of Inductance

We may divide inductance measurement into two categories: the quiescent or DC value; and the incremental or AC value. The DC measurement of inductance is easily obtained from the response of an applied step function of voltage. The incremental value may be found from the application of an AC signal.

4.1.1 Step Voltage Method

The inductor may be modeled by a series combination of resistance and inductance, as shown in Figure 4.1.1.

The simplest way to apply the step voltage is to charge a large capacitor - an ultra-capacitor is ideal. The capacitor is initially charged from a power supply and the switch S is closed at $t = 0$. The capacitor provides a constant voltage, and the current rises in the inductor and may be measured by a small non-inductive precision sampling resistor.

The general solution for the setup in Figure 4.1 is

$$v = Ri + \frac{d\lambda}{dt} \tag{4.1.1}$$

In discretized form becomes

$$v = Ri(k) + \frac{1}{t_s}\Big(\lambda(k) - \lambda(k-1)\Big) \tag{4.1.2}$$

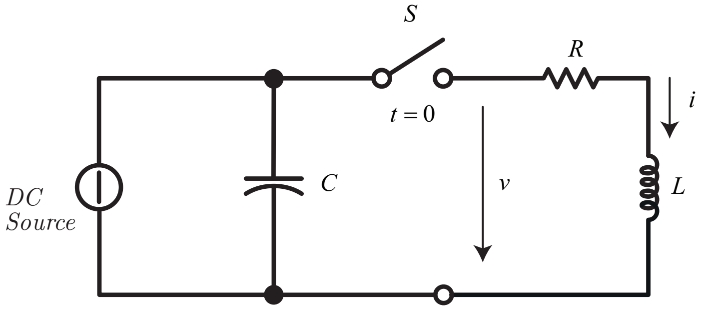


Figure 4.1: Inductance measurement by step voltage method.

4.2 Measurement of B-H Loop

The $B-H$ loop is of interest because we need knowledge of the magnetic parameters such that B_{sat}, the saturation flux density, the coercive force H_c and the residual flux B_r.

The simplest set up is a coil of N turns on a toroidal core, as illustrated in Figure 4.2. Recalling Ampere's circuital law

$$\oint_{\mathscr{C}} = \vec{H} \cdot d\vec{l} = Ni \tag{4.2.1}$$

The magnetic field intensity may be obtained directly from the current measurement.

$$H_c = \frac{N}{l_c} i \tag{4.2.2}$$

where l_c is the mean length of the magnetic path in the test specimen.

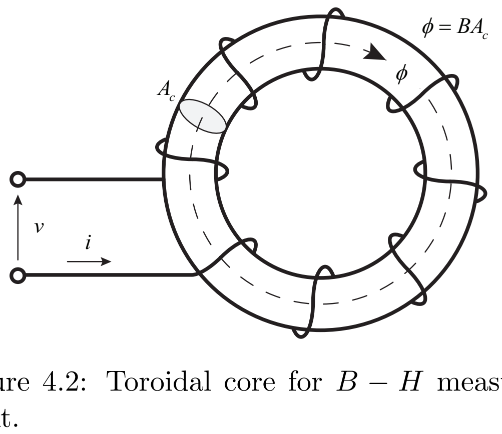


Figure 4.2: Toroidal core for $B-H$ measurement.

The flux density in the coil is found from Faraday's law, assuming negligible winding resistance:

$$B = \frac{1}{NA_c}\int v dt \tag{4.2.3}$$

where v is the terminal voltage of the coil.

The number of turns must be selected to ensure that the correct value of B_{sat} and H_c are correctly included in the measurement.

For a sinusoidal voltage input at frequency f, the maximum flux density is related to the peak of the applied voltage, from Eq. (2.5)

$$B_{\max} = \frac{V_{\text{peak}}}{2\pi f N A_c} \tag{4.2.4}$$

For practical measure we have to considering a more realistic configuration as what shown in Figure 4.3 where we can find the following measure method

- The system is excited applying a sinusoidal voltage v
- The current i flowing into the excitation winding is measured
- The voltage e available at the measure winding is measured

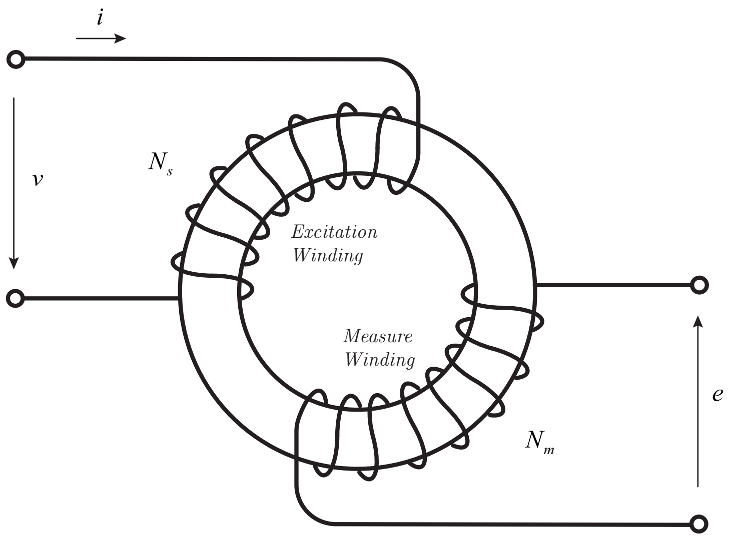


Figure 4.3: Measurement setup for $B-H$ curve tracing.

The three quantities are correlated by the following equations

$$H = \frac{N_s i}{l_c} \tag{4.2.5}$$

$$B = \frac{1}{A_c}\int e dt \tag{4.2.6}$$

4.3 Measurement of Losses in a Transformer

The main parameters in a transformer may be measured by two simple tests, namely the short circuit test that forces rated current through the windings at a low voltage, and the open-circuit test is carried out at rated voltage to include the magnetizing current. These test are traditionally associated with 50 Hz power transformers. In power electronics applications, the transformers, operate at hundreds of kHz, hence, the core loss shall be evaluated at high frequency.

4.3.1 Short-circuit test

With one winding short circuited, typically 5-6% of rated voltage on the other winding is sufficient to establish rated full load current. For convenience, we will short the secondary winding, take measurements in the primary winding and refer the secondary quantities as appropriate. The core loss is negligible, since the input voltage is very low. For that reason, the core circuit parameters are shown dotted in Figure 4.4.

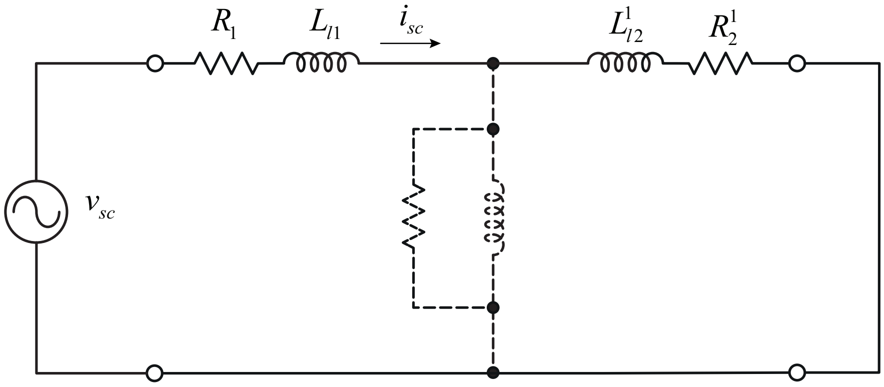


Figure 4.4: Transformer short-circuit test.

The equivalent impedance z_{eq} looking into the terminals of the transformer is given by the short-circuit impedance z_{sc}

$$z_{\mathrm{eq}} = z_{\mathrm{sc}} = \frac{v_{\mathrm{sc}}}{i_{\mathrm{sc}}} \tag{4.3.1}$$

As a first approximation, it is reasonable to assume that $R_1 = n^2 R_2$, $L_{\mathrm{l1}} = n^2 L_{\mathrm{l2}}$ in a well designed transformer. A more realistic approach is to take the ratios of R_1/R_2 and $L_{\mathrm{l1}}/L_{\mathrm{l2}}$ as the ratio of the DC resistance of the individual windings, which may be easily measured.

4.3.2 Open-circuit test

With rated voltage on the primary winding and with the secondary winding open-circuited, the magnetizing current flows in the primary winding. The voltage drops in R_{eq} and Z_{eq} are very small due to small magnetization current and the power input is very nearly equal to the core loss.

The equivalent circuit for this test is shown in Figure 4.5. The core reactance is given

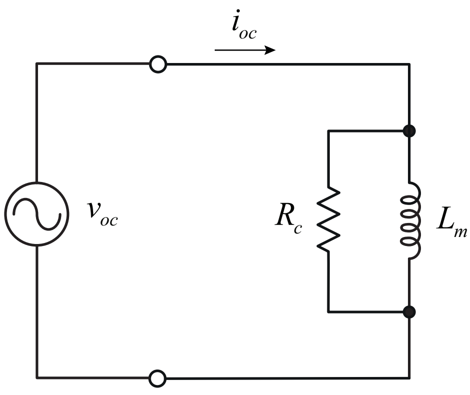


Figure 4.5: Transformer open-circuit test.

4.3.3 Core loss at high frequencies

Chapter 5

Planar Magnetics

5.1 Self and Mutual Inductance of Circular Coils

Chapter 6

Theory of Electromechanical Energy Conversion

6.1 Mutual Inductance

Magnetically coupled electric circuits are central to the operation of transformers and electric machines. In the case of transformers, stationary circuit are magnetically coupled for the purpose of changing the voltage and current levels. In the case of electric machines, circuits in relative motion are magnetically coupled for the purpose of transferring energy between mechanical and electrical systems. Since magnetically coupled circuits such an important role in power transmission and conversion, it is important to establish the equations that describe their behaviour and to express these equations in a form convenient for analysis. These goals may be achieved by starting with two stationary electric circuits that are magnetically coupled as shown in Figure 6.1. The two coils consist of turns N_1 and N_2, respectively, and they are wound on a common core that is generally a ferromagnetic material with permeability large relative to that of air. The permeability of free space, μ_0, is $4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$. The permeability of other materials is expressed as $\mu = \mu_r \mu_0$, where μ_r is the relative permeability. In the case of transformer steel the relative permeability may be as high as $2000 \div 4000$.

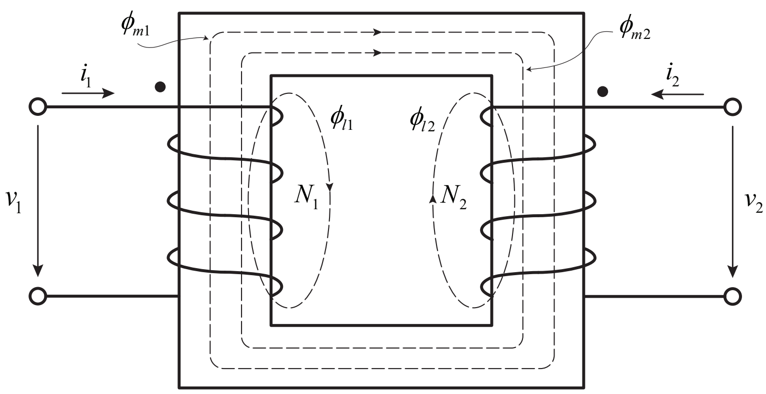


Figure 6.1: Magnetically coupled circuits.

In general, the flux produced by each coil can be separated into two components. A leakage component is denoted with an l subscript and a magnetizing component is denoted by m subscript. Each of these components is depicted by a single streamline with the positive direction determined by applying the right hand rule to the direction of current flow in the coil, Often, in transformer analysis, i_2 is selected positive out of the top of coil 2 and a dot placed at that terminal.

The flux linking each coil may be expressed

$$\phi_1 = \phi_{l1} + \phi_{m1} + \phi_{m2} \tag{6.1.1}$$

$$\phi_2 = \phi_{l2} + \phi_{m2} + \phi_{m1} \tag{6.1.2}$$

The leakage flux ϕ_{l1} is produced by current flowing in coil 1, and it links only the turns of coil 1. Likewise, the leakage flux ϕ_{l2} is produced by current flowing in coil 2, and it links only the turns of coil 2. The magnetization flux ϕ_{m1} is produced by current flowing in coil 1, and it links all turns of coil 1 and coil 2. Similarly, the magnetizing flux ϕ_{m2} is produced by current flowing in coil 2, and it also links all turns of coil 1 and 2. With the selected positive direction of current flow and the manner in that the coils are wound Figure 6.1, magnetizing flux produced by positive current in one coil adds to the magnetizing flux produced by positive current in the other coil. In other words, if both currents are flowing in the same direction, the magnetizing fluxes produced by each coil are in the same direction, making the total magnetizing flux or the total core flux the sum of the instantaneous magnitudes of the individual magnetizing fluxes. If the currents are in opposite directions, the magnetizing fluxes are in opposite directions. In this case, one coil is said to be magnetizing the core, the other demagnetizing.

The voltage equations may be expressed in matrix form as

$$\vec{v} = \mathbf{R}\vec{i} + \frac{d\vec{\lambda}}{dt} \tag{6.1.3}$$

where $\mathbf{R} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$, and

$$\begin{aligned} \vec{v} &= \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T \\ \vec{\lambda} &= \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}^T \\ \vec{i} &= \begin{bmatrix} i_1 & i_2 \end{bmatrix}^T \end{aligned} \tag{6.1.4}$$

Since it is assumed that ϕ_1 links the equivalent turns of coil 1 and ϕ_2 links the equivalent turns of coil 2, the flux linkages may be written

$$\lambda_1 = N_1\phi_1 \tag{6.1.5}$$

$$\lambda_2 = N_2\phi_2 \tag{6.1.6}$$

where ϕ_1 and ϕ_2 are given by Eq. (6.1.1) and (6.1.2), respectively.

If saturation is neglected, the system is linear and the fluxes may be expressed as

$$\phi_{l1} = \frac{N_1 i_1}{\mathscr{R}_{l1}} \tag{6.1.7}$$

$$\phi_{m1} = \frac{N_1 i_1}{\mathscr{R}_m} \tag{6.1.8}$$

$$\phi_{l2} = \frac{N_2 i_2}{\mathscr{R}_{l2}} \tag{6.1.9}$$

$$\phi_{m2} = \frac{N_2 i_2}{\mathscr{R}_m} \tag{6.1.10}$$

where $\mathscr{R}_{l1}$ and $\mathscr{R}_{l2}$ are the reluctances of the leakage path and $\mathscr{R}_m$ is the reluctance of the path of the magnetizing fluxes. The product of N times i is the magnetomotive force (MMF), which is determined by the application of Ampere's law. The reluctance of the leakage paths is difficult to express and measure. A unique determination of the inductances associated with the leakage flux is typically either calculated or approximated from design considerations. The reluctance of the magnetizing path of the core shown in Figure 6.1 may be computed with sufficient accuracy from the well-known relationship

$$\mathscr{R} = \frac{l}{\mu A} \tag{6.1.11}$$

where l is the mean of equivalent length of the magnetic path, A the cross-section area and μ the permeability.

Substituting Eq. (6.1.7) – (6.1.10) into Eq. (6.1.1) and (6.1.2) yields

$$\phi_1 = \frac{N_1}{\mathscr{R}_{l1}} i_1 + \frac{N_1}{\mathscr{R}_m} i_1 + \frac{N_2}{\mathscr{R}_m} i_2 \tag{6.1.12}$$

$$\phi_2 = \frac{N_2}{\mathscr{R}_{l2}} i_2 + \frac{N_2}{\mathscr{R}_m} i_2 + \frac{N_1}{\mathscr{R}_m} i_1 \tag{6.1.13}$$

Substituting Eq. (6.1.12) – (6.1.13) into Eq. (6.1.5) and (6.1.6) yields

$$\lambda_1 = \frac{N_1^2}{\mathscr{R}_{l1}} i_1 + \frac{N_1^2}{\mathscr{R}_m} i_1 + \frac{N_1 N_2}{\mathscr{R}_m} i_2 \tag{6.1.14}$$

$$\lambda_2 = \frac{N_2^2}{\mathscr{R}_{l2}} i_2 + \frac{N_2^2}{\mathscr{R}_m} i_2 + \frac{N_2 N_1}{\mathscr{R}_m} i_1 \tag{6.1.15}$$

When the magnetic system is linear, the flux linkage are generally expressed in terms of inductance and currents. We see that the coefficients of the first two terms on the right-hand side of Eq. (6.1.14) depend upon the turns of coil 1 and the reluctance of the magnetic system, independent of the existence of coil 2. An analogous statement may be made regarding Eq. (6.1.15). Hence, the self-inductances are defined as

$$L_1 = \frac{N_1^2}{\mathscr{R}_{l1}} + \frac{N_1^2}{\mathscr{R}_m} = L_{l1} + L_{m1} \tag{6.1.16}$$

$$L_2 = \frac{N_2^2}{\mathscr{R}_{l2}} + \frac{N_2^2}{\mathscr{R}_m} = L_{l2} + L_{m2} \tag{6.1.17}$$

where L_{l1} and L_{l2} are the leakage inductances and L_{m1} and L_{m2} the magnetization inductances of coil 1 and 2, respectively. From Eq, (6.1.16) and (6.1.17), it follows that the magnetizing inductances may be related as

$$\frac{L_{m2}}{N_2^2} = \frac{L_{m1}}{N_1^2}. \tag{6.1.18}$$

The mutual inductances are defined as the coefficient of the third term of Eq (6.1.14) and (6.1.15).

$$M_{12} = M = \frac{N_1 N_2}{\mathscr{R}_m} \tag{6.1.19}$$

$$M_{21} = M = \frac{N_2 N_1}{\mathscr{R}_m}. \tag{6.1.20}$$

The mutual inductances may be related to the magnetizing inductances. In particular,

$$M = \frac{N_2}{N_1} L_{m1} = \frac{N_1}{N_2} L_{m2}. \tag{6.1.21}$$

The flux linkages may be written as

$$\vec{\lambda} = \mathbf{L}\vec{i} \tag{6.1.22}$$

where

$$\mathbf{L} = \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} = \begin{bmatrix} L_{l1} + L_{m1} & \frac{N_2}{N_1} L_{m1} \\ \frac{N_1}{N_2} L_{m2} & L_{l2} + L_{m2} \end{bmatrix}. \tag{6.1.23}$$

6.2 Electromechanical Energy Conversion

Chapter 7

Thermal Model

7.1 Thermal Model of a Transformer

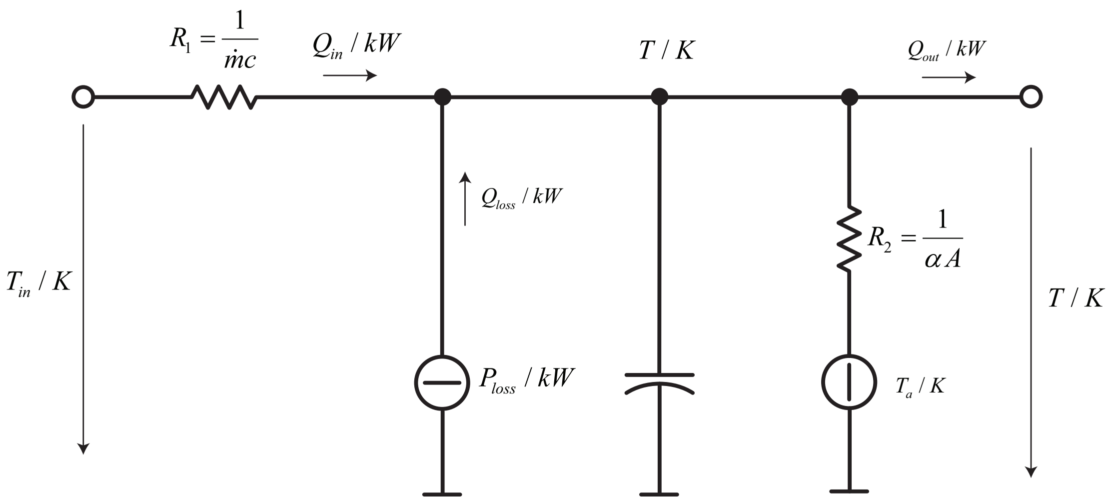


Figure 7.1: Generic thermal model with input flow.

Bibliography

[**W.G. Hurley, 2013**] W.G. Hurley, W.H. Wölfle - *Transformers and Inductors for Power Electronics: Theory, Design and Applications*. Wiley 2013.

[**M.K.Kazimierczuk, 2014**] Marian K. Kazimierczuk - *High-Frequency Magnetic Components*. Wiley 2014.