Permanent magnet synchronous motor with control: digital twin model description

DTL

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In this document we describe the permanent magnet synchronous motor (pmsm) and its control as represented in the Digital Twin Library. The pmsm has been implemented using the DQ (rotating) reference frame representation, which consents to represent the sin-wave quantities as pure fundamental amplitude signals. The whole model is comprehensive of the motor and its control, in particular MTPA (max torque per ampere) and FW (field weakening) are available. Subsequently the following arguments will be presented:

- **Model derivation in different reference frames.**

- **Control description.**

  - Normalization of the motor quantities.
  - Speed control.
  - Torque (vector current) control.
  - Max torque per ampere.
  - Field weakening.

- **Parameters identification from common data-sheet.**

- **Parameters setting.**

- **Model output variables.**

1 PMSM control based model derivation

In this section the control based PMSM model is derived. The developed model is used as physical model in the *Simscape* environment as well as reference model for observer model based.

We can summarize the PMSM as a rotating permanent magnet coupled to three 120° shifted coils. During the rotation the time derivative of the magnetic flux linkages to the coils induces an electromagnetic force (emf) into the coils as

$$e(t) = -\frac{d\psi^r(t)}{dt}$$

. hence we can write the following generalized Kirchhoff's equations

$$\begin{cases} \vec{u}_{abc} - \mathbf{R}\vec{i}_{abc} - \dfrac{d\vec{\psi}^s_{abc}}{dt} = 0 \\[2mm] J\dot{\omega}_m = \tau_m - \tau_l \\[2mm] \dot{\theta} = \omega = p\omega_m \end{cases} \tag{1.1}$$

where $\vec{u}_{abc} = \begin{bmatrix} u_a(t) & u_b(t) & u_c(t) \end{bmatrix}^T, \vec{i}_{abc} = \begin{bmatrix} i_a(t) & i_b(t) & i_c(t) \end{bmatrix}^T$ are the three phase voltage and current vector. The magnetic flux $\vec{\psi}^s_{abc} = \begin{bmatrix} \psi^s_a(t) & \psi^s_b(t) & \psi^s_c(t) \end{bmatrix}^T$ represents the stator magnetic flux linkages and p is the number of pole-pairs.

Recall that

$$\vec{u}^s_{abc} - \mathbf{R}\vec{i}^s_{abc} - \frac{d\vec{\psi}^s_{abc}}{dt} = 0 \leftrightarrow \vec{u}_{abc} - \mathbf{R}\vec{i}_{abc} - \frac{d\vec{\psi}^s_{abc}}{dt} = 0$$

The flux linkages may be written as

$$\vec{\psi}_{abc}^s = \mathbf{L}_s \vec{i}_{abc} + \vec{\psi}_{abc}^r \tag{1.2}$$

where $\vec{\psi}_{abc}^r$ are the rotor flux linked to the stator generated by the rotor permanent magnets and where

$$\mathbf{L}_s = \begin{bmatrix} L_\mu + L_a - L_b \cos 2\theta & -\frac{1}{2}L_a - L_b \cos 2(\theta - \frac{\pi}{3}) & -\frac{1}{2}L_a - L_b \cos 2(\theta + \frac{\pi}{3}) \\ -\frac{1}{2}L_a - L_b \cos 2(\theta - \frac{\pi}{3}) & L_\mu + L_a - L_b \cos 2(\theta - \frac{2\pi}{3}) & -\frac{1}{2}L_a - L_b \cos 2(\theta + \pi) \\ -\frac{1}{2}L_a - L_b \cos 2(\theta + \frac{\pi}{3}) & -\frac{1}{2}L_a - L_b \cos 2(\theta + \pi) & L_\mu + L_a - L_b \cos 2(\theta + \frac{2\pi}{3}) \end{bmatrix} \tag{1.3}$$

is the stator inductance matrix. L_b is the anisotropic inductance term which is characteristic of internal permanent magnet machines.
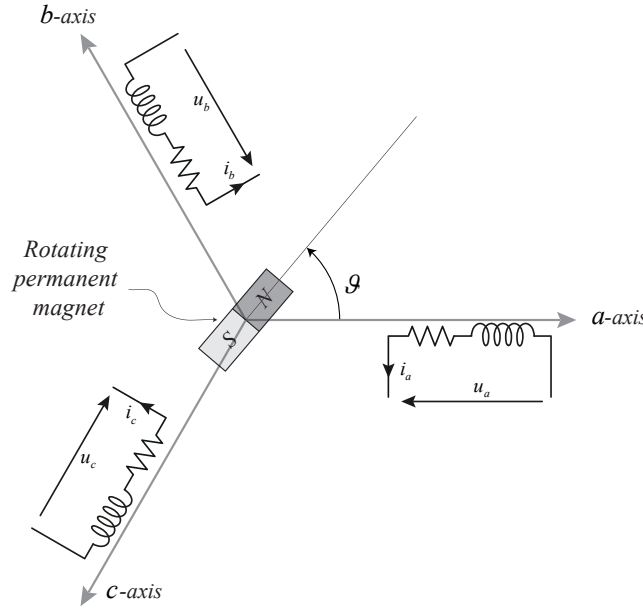
Figure 1: Stationary and rotating reference frame

In case of isotropic machine construction this term is null and the matrix of Eq. (1.3) becomes

$$\mathbf{L}_s = \begin{bmatrix} L_\mu + L_a & -\frac{1}{2}L_a & -\frac{1}{2}L_a \\ -\frac{1}{2}L_a & L_\mu + L_a & -\frac{1}{2}L_a \\ -\frac{1}{2}L_a & -\frac{1}{2}L_a & L_\mu + L_a \end{bmatrix} = \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ L_m & L_m & L_s \end{bmatrix} \tag{1.4}$$

Where the term L_μ is the leakage inductance and in general can be considered between one tenth and one twentieth of the synchronous inductance $L_s \circ L_a$ and where L_m is the mutual inductance and is defined as negative value.

$$\begin{cases} \vec{u}_{abc} - \mathbf{R}\vec{i}_{abc} - \omega \dfrac{d\mathbf{L}_s}{d\theta}\vec{i}_{abc} - \mathbf{L}_s \dfrac{d\vec{i}_{abc}}{dt} - \dfrac{d\vec{\psi}_{abc}^r}{dt} = 0 \\[2ex] J\dot{\omega}_m = \tau_m - b\omega_m - \tau_l \\[2ex] \dot{\vartheta} = \omega = p\omega_m \end{cases} \tag{1.5}$$

where

$$
\begin{cases}
\psi_a^r(t) = \Psi^M \cos(\theta) \\[2mm]
\psi_b^r(t) = \Psi^M \cos(\theta - \dfrac{2\pi}{3}) \\[2mm]
\psi_c^r(t) = \Psi^M \cos(\theta + \dfrac{2\pi}{3})
\end{cases}
\tag{1.6}
$$

where Ψ^M is the flux linkages generated by the permanent magnet.

1.1 PMSM model respect to a stationary reference frame

The system which we are considering is a 3-wire circuit that means we can apply the Kirchhoff condition $x_a + x_b + x_c = 0$ that means we have three (not independent) vectors which lay in a plane (2-dimensions), hence the whole three phase system can be represented by the use of only one independent vector and can be represented by two coordinates (α, β).

Applying the coordinates transformation $\mathbf{T}_1$ defined as follows

$$
\mathbf{T}_1 = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\[2mm] 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}
\qquad
\mathbf{T}_1^{-1} = \begin{bmatrix} 1 & 0 \\[2mm] -\frac{1}{2} & \frac{\sqrt{3}}{2} \\[2mm] -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}
\tag{1.7}
$$

$$
\mathbf{T}_1\mathbf{T}_1^{-1} = \mathbf{I}_2
\tag{1.8}
$$

where

$$
\vec{x}_{\alpha\beta} = \mathbf{T}_1\vec{x}_{abc}
\tag{1.9}
$$

and

$$
\mathbf{T}_1^{-1}\vec{x}_{\alpha\beta} = \vec{x}_{abc}
\tag{1.10}
$$

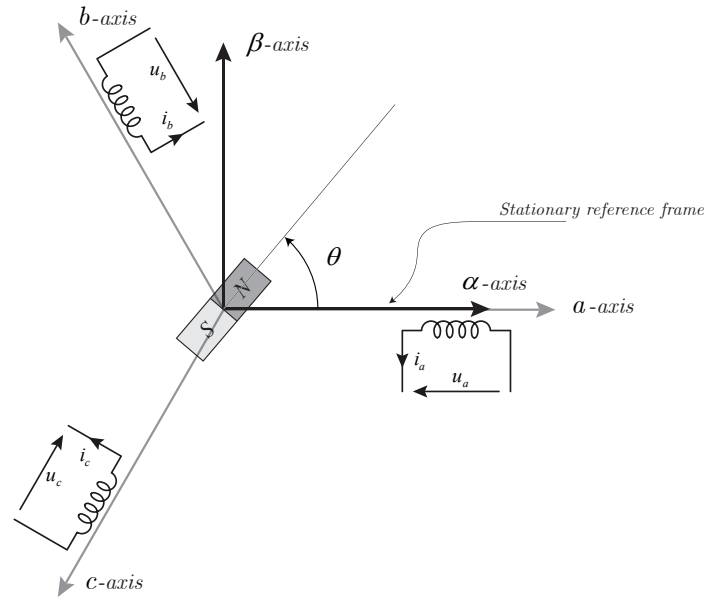A graphical representation of the $\alpha\beta$ reference frame is depicted in Figure 2.

Figure 2: Stationary $\alpha\beta$ reference frame

Applying the relation of Eq. (1.10) to the Eq. (1.2) we obtain

$$\mathbf{T}_1^{-1}\vec{u}_{\alpha\beta} - \mathbf{R}\mathbf{T}_1^{-1}\vec{i}_{\alpha\beta} - \omega\frac{d\mathbf{L}_s}{d\theta}\mathbf{T}_1^{-1}\vec{i}_{\alpha\beta} - \mathbf{L}_s\mathbf{T}_1^{-1}\frac{d\vec{i}_{\alpha\beta}}{dt} - \mathbf{T}_1^{-1}\frac{d\vec{\psi}_{\alpha\beta}^r}{dt} = 0$$

Multiplying left side for $\mathbf{T}_1$ we obtain

$$\vec{u}_{\alpha\beta} - \mathbf{T}_1\mathbf{R}\mathbf{T}_1^{-1}\vec{i}_{\alpha\beta} - \omega\frac{d}{d\theta}\Big[\mathbf{T}_1\mathbf{L}_s\mathbf{T}_1^{-1}\Big]\vec{i}_{\alpha\beta} - \mathbf{T}_1\mathbf{L}_s\mathbf{T}_1^{-1}\frac{d\vec{i}_{\alpha\beta}}{dt} - \frac{d\vec{\psi}_{\alpha\beta}^r}{dt} = 0$$

which results as follows

$$\vec{u}_{\alpha\beta} - \mathbf{R}^{\alpha\beta}\vec{i}_{\alpha\beta} - \omega\frac{d\mathbf{L}_s^{\alpha\beta}}{d\theta}\vec{i}_{\alpha\beta} - \mathbf{L}_s^{\alpha\beta}\frac{d\vec{i}_{\alpha\beta}}{dt} - \frac{d\vec{\psi}_{\alpha\beta}^r}{dt} = 0 \tag{1.11}$$

where

$$\frac{d\vec{\psi}_{\alpha\beta}^r}{dt} = \omega\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{\psi}_{\alpha\beta}^r$$

in fact from Eq. (1.6) we can write $\vec{\psi}_{\alpha\beta}^r = \begin{bmatrix} \psi_\alpha^r(t) & \psi_\beta^r(t) \end{bmatrix}^T$ as follows

$$\begin{cases} \psi_\alpha^r(t) = \Psi^M\cos\theta \\ \psi_\beta^r(t) = \Psi^M\sin\theta \end{cases} \tag{1.12}$$

which results in the following set of equations

$$\begin{cases} \vec{u}_{\alpha\beta} - \mathbf{R}^{\alpha\beta}\,\vec{i}_{\alpha\beta} - \omega\dfrac{d\mathbf{L}_s^{\alpha\beta}}{d\theta}\vec{i}_{\alpha\beta} - \mathbf{L}^{\alpha\beta}\,\dfrac{d\vec{i}_{\alpha\beta}}{dt} - \vec{\psi}_{\alpha\beta}^r = 0 \\[2mm] \dfrac{d\vec{\psi}_{\alpha\beta}^r}{dt} = \omega\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{\psi}_{\alpha\beta}^r \\[2mm] J\dot{\omega}_m = \tau_m - b\omega_m - \tau_l \\[2mm] \dot{\vartheta} = \omega = p\omega_m \end{cases} \tag{1.13}$$

where

$$\mathbf{L}_s^{\alpha\beta} = \begin{bmatrix} L_\mu + \frac{3}{2}(L_a - L_b\cos 2\theta) & -\frac{3}{2}L_b\sin 2\theta \\ -\frac{3}{2}L_b\sin 2\theta & L_\mu + \frac{3}{2}(L_a + L_b\cos 2\theta) \end{bmatrix} \tag{1.14}$$

and

$$\frac{d\mathbf{L}_s^{\alpha\beta}}{d\theta} = 3L_b\begin{bmatrix} \sin 2\theta & -\cos 2\theta \\ -\cos 2\theta & -\sin 2\theta \end{bmatrix} \tag{1.15}$$

The isotropic $(L_b = 0 \rightarrow d\mathbf{L}_s^{\alpha\beta}/d\theta = \mathbf{0})$ PMSM model respect to the stationary reference

frame results as follows

$$
\begin{cases}
\vec{u}_{\alpha\beta}(t) - \mathbf{R}^{\alpha\beta} \, \vec{i}_{\alpha\beta}(t) - \mathbf{L}^{\alpha\beta} \, \dfrac{d\vec{i}_{\alpha\beta}(t)}{dt} - \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}^r_{\alpha\beta} = 0 \\[3ex]
\dfrac{d\vec{\psi}^r_{\alpha\beta}(t)}{dt} = \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}^r_{\alpha\beta}(t) \\[3ex]
\vec{\psi}^r_{\alpha\beta}(0) = \begin{bmatrix} \Psi^M & 0 \end{bmatrix}^T \\[2ex]
J\dot{\omega}_m(t) = \tau_m(t) - b\omega_m(t) - \tau_l(t) \\[2ex]
\dot{\theta}(t) = \omega(t) = p\omega_m(t)
\end{cases}
\tag{1.16}
$$

where $\vec{\psi}^r_{\alpha\beta} = \begin{bmatrix} \Psi^M \cos(\theta) & \Psi^M \sin(\theta) \end{bmatrix}^T$ and

$$
\mathbf{R}^{\alpha\beta} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}
$$

$$
\mathbf{L}^{\alpha\beta} = \begin{bmatrix} L_\alpha & 0 \\ 0 & L_\beta \end{bmatrix} = \frac{3}{2} \begin{bmatrix} L_\mu + L_a & 0 \\ 0 & L_\mu + L_a \end{bmatrix}
$$

Ψ^M is the amplitude of the permanent magnet flux as viewed from the stator phase winding and we assume constant.

1.2   PMSM model respect to a rotating reference frame

Consider now a new change of the reference frame, in particular, we are going to consider the a rotating reference frame. Let θ_0 the phase of a not stationary reference frame here called (dq), the set of equations reported in Eq. 1.13 can now be written respect the rotating reference frame as follows
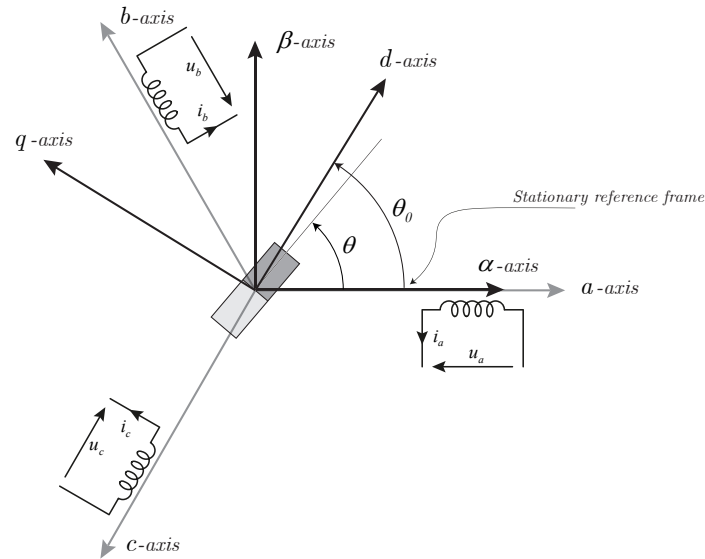

Figure 3: Rotating dq reference frame

$$\vec{x}_{dq} = \mathbf{T}_2 \vec{x}_{\alpha\beta} \tag{1.17}$$

$$\mathbf{T}_2^{-1} \vec{x}_{dq} = \vec{x}_{\alpha\beta} \tag{1.18}$$

where

$$\mathbf{T}_2(\theta_0) = \begin{bmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{bmatrix}$$

$$\mathbf{T}_2^{-1}(\theta_0) = \begin{bmatrix} \cos(\theta_0) & -\sin(\theta_0) \\ \sin(\theta_0) & \cos(\theta_0) \end{bmatrix} \tag{1.19}$$

$$\mathbf{T}_2(\theta_0) \ \mathbf{T}_2^{-1}(\theta_0) = \mathbf{I}_2$$

$$\dot{\theta}_0 = \omega_0$$

Note that $d\mathbf{T}_2(\theta_0)/dt \neq 0$

Hence applying the transformation $\mathbf{T}_2(\vartheta_0)$ to the system represented by Eq. (1.13) results as follows

$$\mathbf{T}_2^{-1}\vec{u}_{dq} - \mathbf{R}^{\alpha\beta} \ \mathbf{T}_2^{-1}\vec{i}_{dq} \ - \omega \frac{d\mathbf{L}_s^{\alpha\beta}}{d\theta}\mathbf{T}_2^{-1}\vec{i}_{dq} \ - \mathbf{L}^{\alpha\beta} \ \frac{d\left(\mathbf{T}_2^{-1}\vec{i}_{dq}\right)}{dt} - \frac{d\left(\mathbf{T}_2^{-1}\vec{\psi}_{dq}^r\right)}{dt} = 0$$

where

$$\frac{d\mathbf{T}_2^{-1}(\vartheta_0)}{dt} = -\omega_0 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{T}_2^{-1}(\vartheta_0) \tag{1.20}$$

left multiplying by $\mathbf{T}_2$ we obtain the following terms

$$- \mathbf{T}_2\mathbf{L}^{\alpha\beta} \ \frac{d\left(\mathbf{T}_2^{-1}\vec{i}_{dq}\right)}{dt} =$$

$$= \omega_0 \ \mathbf{T}_2\mathbf{L}^{\alpha\beta} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{T}_2^{-1}\vec{i}_{dq} \ + \mathbf{T}_2\mathbf{L}^{\alpha\beta}\mathbf{T}_2^{-1}\frac{d\vec{i}_{dq}}{dt} \tag{1.21}$$

$$\xrightarrow{\theta_0 \to \theta} \omega_0 \begin{pmatrix} 0 & L_d \\ -L_q & 0 \end{pmatrix} \vec{i}_{dq} \ + \mathbf{L}^{dq}\frac{d\vec{i}_{dq}}{dt}$$

where

$$\mathbf{L}^{dq} \xrightarrow{\theta_0 \to \theta} \begin{pmatrix} L_d & 0 \\ 0 & L_q \end{pmatrix} = \begin{pmatrix} L_\mu + \frac{3}{2}(L_a - L_b) & 0 \\ 0 & L_\mu + \frac{3}{2}(L_a + L_b) \end{pmatrix} \tag{1.22}$$

and

$$- \omega\mathbf{T}_2\frac{d\mathbf{L}_s^{\alpha\beta}}{d\theta}\mathbf{T}_2^{-1}\vec{i}_{dq} = -\omega\mathbf{L}_m^{dq}\vec{i}_{dq} \xrightarrow{\theta_0 \to \theta} \omega \begin{pmatrix} 0 & 3L_b \\ 3L_b & 0 \end{pmatrix} \vec{i}_{dq} \tag{1.23}$$

Which results in the following set of equations

$$\begin{cases} \vec{u}_{dq} - \mathbf{R}^{dq} \ \vec{i}_{dq} - \omega\mathbf{L}_m^{dq}\vec{i}_{dq} + \omega_0 \begin{pmatrix} 0 & L_d \\ -L_q & 0 \end{pmatrix} \vec{i}_{dq}+ \\[3mm] \qquad - \mathbf{L}^{dq} \ \frac{d\vec{i}_{dq}}{dt} + \omega_0 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{\psi}_{dq} + (\omega - \omega_0) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{\psi}_{dq}^r = 0 \\[3mm] \dot{\psi}_d^r = - (\omega - \omega_0) \, \psi_q^r \\[2mm] \dot{\psi}_q^r = (\omega - \omega_0) \, \psi_d^r \end{cases}$$

rearranging and approximating the term $\omega \mathbf{L}_m^{dq} \vec{i}_{dq}$ to

$$-\omega \mathbf{L}_m^{dq} \vec{i}_{dq} \xrightarrow{\theta_0 \to \theta} \omega \begin{pmatrix} 0 & 3L_b \\ 3L_b & 0 \end{pmatrix} \vec{i}_{dq} = \omega \begin{pmatrix} 0 & L_q - L_d \\ L_q - L_d & 0 \end{pmatrix} \vec{i}_{dq}$$

we obtain the following set of equations

$$
\begin{cases}
\vec{u}_{dq} - \mathbf{R}^{dq} \, \vec{i}_{dq} + \omega \begin{pmatrix} 0 & L_q - L_d \\ L_q - L_d & 0 \end{pmatrix} \vec{i}_{dq} + \omega_0 \begin{pmatrix} 0 & L_d \\ -L_q & 0 \end{pmatrix} \vec{i}_{dq} + \\[2ex]
\qquad - \mathbf{L}^{dq} \dfrac{d\vec{i}_{dq}}{dt} + \omega \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{\psi}_{dq}^r = 0 \\[2ex]
\dot{\psi}_d^r = -\left(\omega - \omega_0\right) \psi_q^r \\[2ex]
\dot{\psi}_q^r = \left(\omega - \omega_0\right) \psi_d^r
\end{cases}
\tag{1.24}
$$

recall that $3L_b = (L_q - L_d)$.

The flux equations in Eq. (1.24) are derived as follows

$$\frac{d\vec{\psi}_{\alpha\beta}^r(t)}{dt} = \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_{\alpha\beta}^r(t) \Rightarrow$$

$$\frac{d\mathbf{T}_2^{-1}\vec{\psi}_{dq}^r(t)}{dt} = \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{T}_2^{-1}\vec{\psi}_{dq}^r(t) \Rightarrow \tag{1.25}$$

$$-\omega_0 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{T}_2^{-1}\vec{\psi}_{dq} + \mathbf{T}_2^{-1}\frac{d\vec{\psi}_{dq}^r(t)}{dt} = \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{T}_2^{-1}\vec{\psi}_{dq}^r(t)$$

Multiplying both terms per $\mathbf{T}_2$ we obtain

$$\frac{d\vec{\psi}_{dq}^r(t)}{dt} = \left(\omega - \omega_0\right) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}_{dq}^r(t) \tag{1.26}$$

where $\vec{\psi}_{dq}^r(t)$ is given as follows

$$
\begin{aligned}
\vec{\psi}_{dq}^r(t) = \mathbf{T}_2 \vec{\psi}_{\alpha\beta}^r &= \begin{bmatrix} \cos(\theta_0) & \sin(\theta_0) \\ -\sin(\theta_0) & \cos(\theta_0) \end{bmatrix} \begin{bmatrix} \Psi^M \cos(\theta_0) \\ \Psi^M \sin(\theta_0) \end{bmatrix} = \\[2ex]
&= \Psi^M \begin{bmatrix} \cos\theta_0 \cos\theta + \sin\theta_0 \sin\theta \\ -\sin\theta_0 \cos\theta + \cos\theta_0 \sin\theta \end{bmatrix} = \Psi^M \begin{bmatrix} \cos\left(\theta - \theta_0\right) \\ \sin\left(\theta - \theta_0\right) \end{bmatrix}
\end{aligned}
\tag{1.27}
$$

Considering the *DQ*-classical control, we can observe that, when the rotating reference frame (which is the frame where the controller is implemented) and the rotor reference frame are aligned ($\theta_0 \to \theta$ and $\omega_0 \to \omega$), all the inductance matrices become time-invariant and the flux components become $\psi_q \to 0$, $\psi_d \to \Psi^M$.

The DQ-PMSM model for a generic rotating reference frame is here reported

$$
\begin{cases}
\vec{u}_{dq} - \mathbf{R}^{dq}\,\vec{i}_{dq} + \omega \begin{pmatrix} 0 & L_q - L_d \\ L_q - L_d & 0 \end{pmatrix} \vec{i}_{dq} + \omega_0 \begin{pmatrix} 0 & L_d \\ -L_q & 0 \end{pmatrix} \vec{i}_{dq} \\
\qquad\qquad\qquad - \mathbf{L}^{dq}\,\dfrac{d\vec{i}_{dq}}{dt} + \omega \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{\psi}_{dq}^{\,r} = 0 \\[4pt]
\dot{\psi}_d^r = -\left(\omega - \omega_0\right)\psi_q^r \\[4pt]
\dot{\psi}_q^r = \left(\omega - \omega_0\right)\psi_d^r \\[4pt]
\vec{\psi}_{dq}^{\,r}(0) = \begin{bmatrix} \Psi^M & 0 \end{bmatrix}^T \\[4pt]
J\dot{\omega}_m(t) = \tau_m(t) - b\omega_m - \tau_l(t) \\[4pt]
\dot{\vartheta}(t) = \omega(t) = p\omega_m(t)
\end{cases}
\tag{1.28}
$$

where $\vec{\psi}_{dq}^{\,r} = \begin{bmatrix} \Psi^M \cos(\theta - \theta_0) & \Psi^M \sin(\theta - \theta_0) \end{bmatrix}^T$ and

$$
\mathbf{R}^{dq} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}
$$

$$
\mathbf{L}^{dq} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} = \begin{bmatrix} L_\mu + \frac{3}{2}L_s - \frac{L_b}{2} & 0 \\ 0 & L_\mu + \frac{3}{2}L_s + \frac{L_b}{2} \end{bmatrix}
$$

Ψ^M is the amplitude of the permanent magnet flux as viewed from the stator phase winding and we assume constant.

The DQ-PMSM model, when rotating reference frame coincides with the rotor reference frame $\omega_0 \to \omega$, can be written as follows

$$
\begin{cases}
\vec{u}_{dq} - \mathbf{R}^{dq}\,\vec{i}_{dq} + \omega \begin{pmatrix} 0 & L_q \\ -L_d & 0 \end{pmatrix} \vec{i}_{dq} - \mathbf{L}^{dq}\,\dfrac{d\vec{i}_{dq}}{dt} + \omega \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \vec{\psi}_{dq}^{\,r} = 0 \\[6pt]
\dot{\psi}_d^r = 0 \\[4pt]
\dot{\psi}_q^r = 0 \\[4pt]
\vec{\psi}_{dq}^{\,r}(0) = \begin{bmatrix} \Psi^M & 0 \end{bmatrix}^T \\[4pt]
J\dot{\omega}_m(t) = \tau_m(t) - b\omega_m - \tau_l(t) \\[4pt]
\dot{\theta}(t) = \omega(t) = p\omega_m(t)
\end{cases}
\tag{1.29}
$$

where

$$\mathbf{R}^{dq} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix}$$

$$\mathbf{L}^{dq} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} = \begin{bmatrix} L_\mu + \frac{3}{2}(L_a - L_b) & 0 \\ 0 & L_\mu + \frac{3}{2}(L_a + L_b) \end{bmatrix}$$

Ψ^M is the amplitude of the permanent magnet flux as viewed from the stator phase winding and we assume constant.

1.3 PMSM torque derivation

Torque equation of the isotropic PMSM - stationary reference frame
Let E be the stored energy inside of the PMSM, it can be described as follows

$$E = \frac{3}{2}\left(\frac{1}{2}\, \vec{i}_{\alpha\beta}^{\,T}\, \mathbf{L}^{\alpha\beta}\, \vec{i}_{\alpha\beta} \right) + \frac{1}{2}J\omega_m^2$$

Applying the principle of energy conservation we can write

$$\frac{dE}{dt} = 0$$

$$\frac{dE}{dt} = \frac{3}{2}\left(\vec{i}_{\alpha\beta}^{\,T}\, \mathbf{L}^{\alpha\beta}\, \frac{d\vec{i}_{\alpha\beta}}{dt} \right) + J\omega_m\dot{\omega}_m = P_{in} - P_{out} - P_{loss} = 0$$

We can write

$$\frac{3}{2}\left(\vec{i}_{\alpha\beta}^{\,T}\, \mathbf{L}^{\alpha\beta}\, \frac{d\vec{i}_{\alpha\beta}}{dt} \right) + J\omega_m\dot{\omega}_m =$$

$$= \frac{3}{2}\left(\underbrace{u_\alpha i_\alpha + u_\beta i_\beta}_{= P_{in}} - \underbrace{Ri_\alpha^2 - Ri_\beta^2}_{= P_{loss}} - \underbrace{\omega \begin{bmatrix} i_\alpha & i_\beta \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \psi_\alpha^r & \psi_\beta^r \end{bmatrix}^T}_{= \omega_m \tau_m} \right) + \omega_m\tau_m - \omega_m\tau_l$$

But

$$\frac{3}{2}\left(\vec{i}_{\alpha\beta}^{\,T}\, \mathbf{L}^{\alpha\beta}\, \frac{d\vec{i}_{\alpha\beta}}{dt} \right) + J\omega_m\dot{\omega}_m = P_{in} - P_{out} - P_{loss}$$

Hence

$$P_{in} = \frac{3}{2}\left(u_\alpha i_\alpha + u_\beta i_\beta \right)$$

$$P_{out} = \omega_m\,\tau_l$$

$$P_{loss} = \frac{3}{2}\left(Ri_\alpha^2 + Ri_\beta^2 \right)$$

The expression of the torque τ_m becomes as follows

$$\tau_m = \frac{3}{2}p\left(\psi_\alpha^r i_\beta - \psi_\beta^r i_\alpha \right) \tag{1.30}$$

The full PMSM model respect to the stationary reference frame is as follows

$$
\begin{cases}
\vec{u}_{\alpha\beta}(t) - \mathbf{R}^{\alpha\beta}\,\vec{i}_{\alpha\beta}(t) - \mathbf{L}^{\alpha\beta}\,\dfrac{d\vec{i}_{\alpha\beta}(t)}{dt} - \omega(t)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{\psi}^r_{\alpha\beta} = 0 \\[4mm]
\dfrac{d\vec{\psi}^r_{\alpha\beta}(t)}{dt} = \omega(t)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\vec{\psi}^r_{\alpha\beta}(t) \qquad \vec{\psi}^r_{\alpha\beta}(0) = \begin{bmatrix} \Psi^M & 0 \end{bmatrix}^T \\[4mm]
J\dot{\omega}_m(t) = \dfrac{3}{2}p\Big[\psi^r_\alpha(t)i_\beta(t) - \psi^r_\beta(t)i_\alpha(t)\Big] - b\omega_m(t) - \tau_l(t) \\[4mm]
\dot{\theta}(t) = \omega(t) = p\omega_m(t)
\end{cases}
\tag{1.31}
$$

Torque equation of the anisotropic PMSM - rotor reference frame

Let E be the stored energy inside of the PMSM, it can be described as follows

$$
E = \frac{3}{2}\left(\frac{1}{2}\,\vec{i}^T_{dq}\,\mathbf{L}^{dq}\,\vec{i}_{dq}\right) + \frac{1}{2}J\omega^2_m
$$

Applying the principle of energy conservation we can write

$$
\frac{dE}{dt} = 0
$$

$$
\frac{dE}{dt} = \frac{3}{2}\left(\vec{i}^T_{dq}\,\mathbf{L}^{dq}\,\frac{d\vec{i}_{dq}}{dt}\right) + J\omega_m\dot{\omega}_m = P_{in} - P_{out} - P_{loss} = 0
$$

We can write as follows

$$
\frac{3}{2}\left(\vec{i}^T_{dq}\,\mathbf{L}^{dq}\,\frac{d\vec{i}_{dq}}{dt}\right) + J\omega_m\dot{\omega}_m =
$$

$$
\underbrace{\frac{3}{2}\Big(u_d i_d + u_q i_q\Big)}_{=P_{in}} - \underbrace{\frac{3}{2}\Big(Ri^2_d - Ri^2_q\Big)}_{=P_{loss}} +
$$

$$
\underbrace{-\frac{3}{2}\left(\omega\begin{bmatrix} i_d & i_q \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \psi^r_d & \psi^r_q \end{bmatrix}^T + \omega\begin{bmatrix} i_d & i_q \end{bmatrix}\begin{bmatrix} 0 & -L_q \\ L_d & 0 \end{bmatrix}\begin{bmatrix} i_d & i_q \end{bmatrix}^T\right)}_{=\omega_m\tau_m} +
$$

$$
+\,\omega_m\tau_m - \omega_m\tau_l
$$

But

$$
\frac{3}{2}\left(\vec{i}^T_{dq}\,\mathbf{L}^{dq}\,\frac{d\vec{i}_{dq}}{dt}\right) + J\omega\dot{\omega} = P_{in} - P_{out} - P_{loss}
$$

Hence

$$
P_{in} = \frac{3}{2}\left(u_d i_d + u_q i_q\right)
$$

$$
P_{out} = \omega_m\,\tau_l
$$

$$
P_{loss} = \frac{3}{2}\left(Ri^2_d + Ri^2_q\right)
$$

The expression of the torque τ_m becomes as follows

$$
\tau_m = \frac{3}{2}p\left[\psi^r_d i_q - \psi^r_q i_d + \left(L_d - L_q\right)i_d i_q\right] \xrightarrow{\theta_0 \to \theta} \frac{3}{2}p\left[\Psi^M i_q + \left(L_d - L_q\right)i_d i_q\right]
\tag{1.32}
$$

The full PMSM model respect to the rotor reference frame is as follows

$$
\begin{cases}
\vec{u}_{dq}(t) - \mathbf{R}^{dq}\,\vec{i}_{dq}(t) + \omega \begin{bmatrix} 0 & L_q \\ -L_d & 0 \end{bmatrix} \vec{i}_{dq} - \mathbf{L}^{dq}\,\dfrac{d\vec{i}_{dq}(t)}{dt} - \omega(t) \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \vec{\psi}^{r}_{dq} = 0 \\[2em]
\dfrac{d\vec{\psi}^{r}_{dq}(t)}{dt} = 0 \qquad \vec{\psi}^{r}_{dq}(0) = \begin{bmatrix} \Psi^{M} & 0 \end{bmatrix}^{T} \\[2em]
J\dot{\omega}_m(t) = \dfrac{3}{2}\,p\left[\Psi^{M} i_q + \left(L_d - L_q\right) i_d i_q \right] - b\omega_m(t) - \tau_l(t) \\[1.5em]
\dot{\theta}(t) = \omega(t) = p\omega_m(t)
\end{cases}
\qquad (1.33)
$$

1.4 Equivalent circuit of a PMSM

The DQ equivalent circuits of the PMSM is shown for both transient and steady state conditions.

(a) PMSM equivalent circuit for the generic rotating reference frame case.

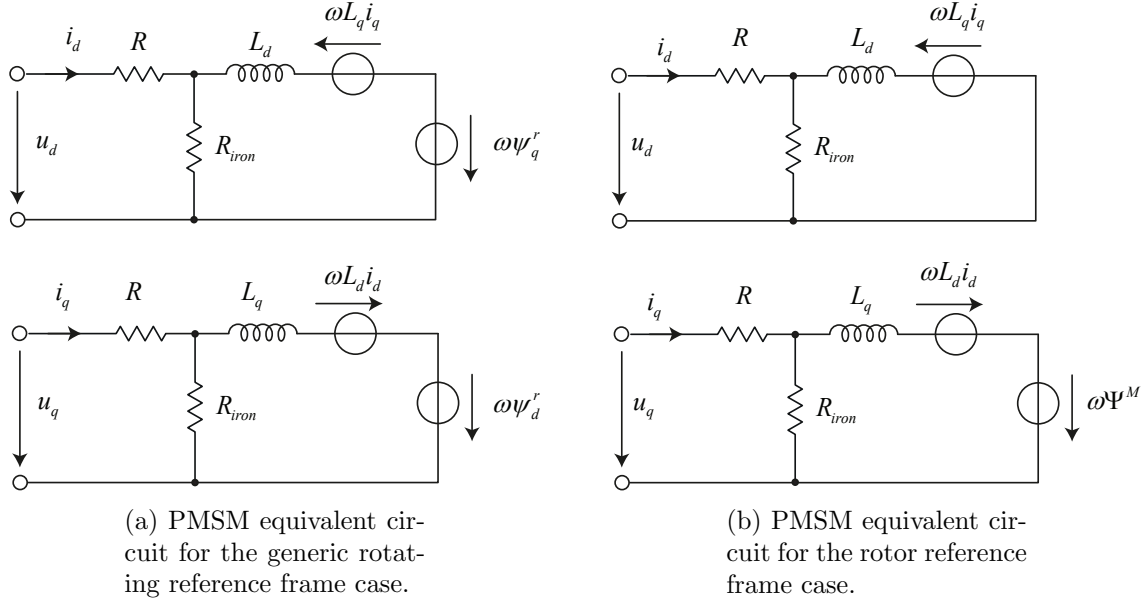(b) PMSM equivalent circuit for the rotor reference frame case.

Figure 4: Rotating and rotor reference frame representation of the pmsm equivalent circuit.

2 Control of a PMSM

The structure of the PMSM control is based on the following parts:

- Speed control.

- Torque control based on current vector control in DQ reference frame.

- MTPA

- Field weakening

2.1   Classical Field Oriented Control of a PMSM

The classical field oriented control (FOC) DQ-control of PMSMs in Figure 6 is depicted.
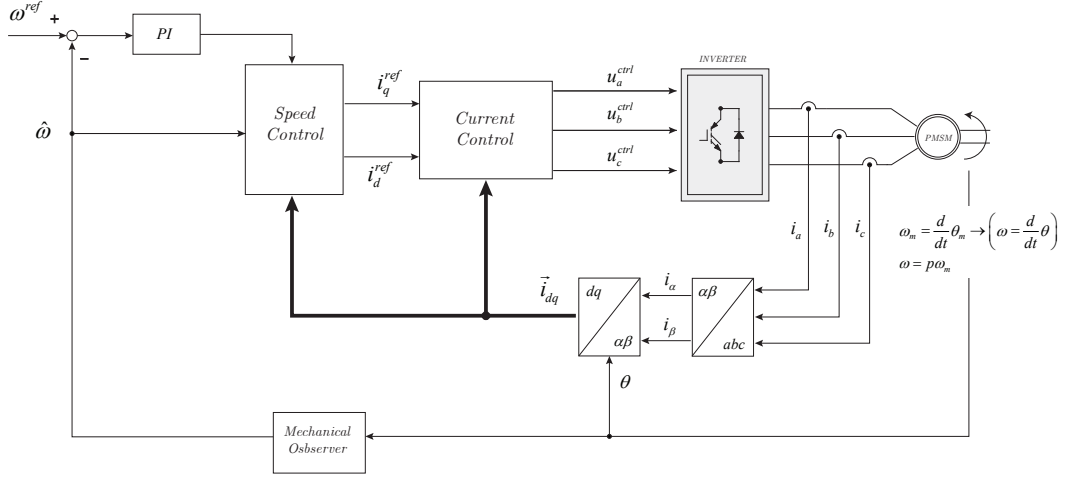

Figure 5: Global vector control of the classical
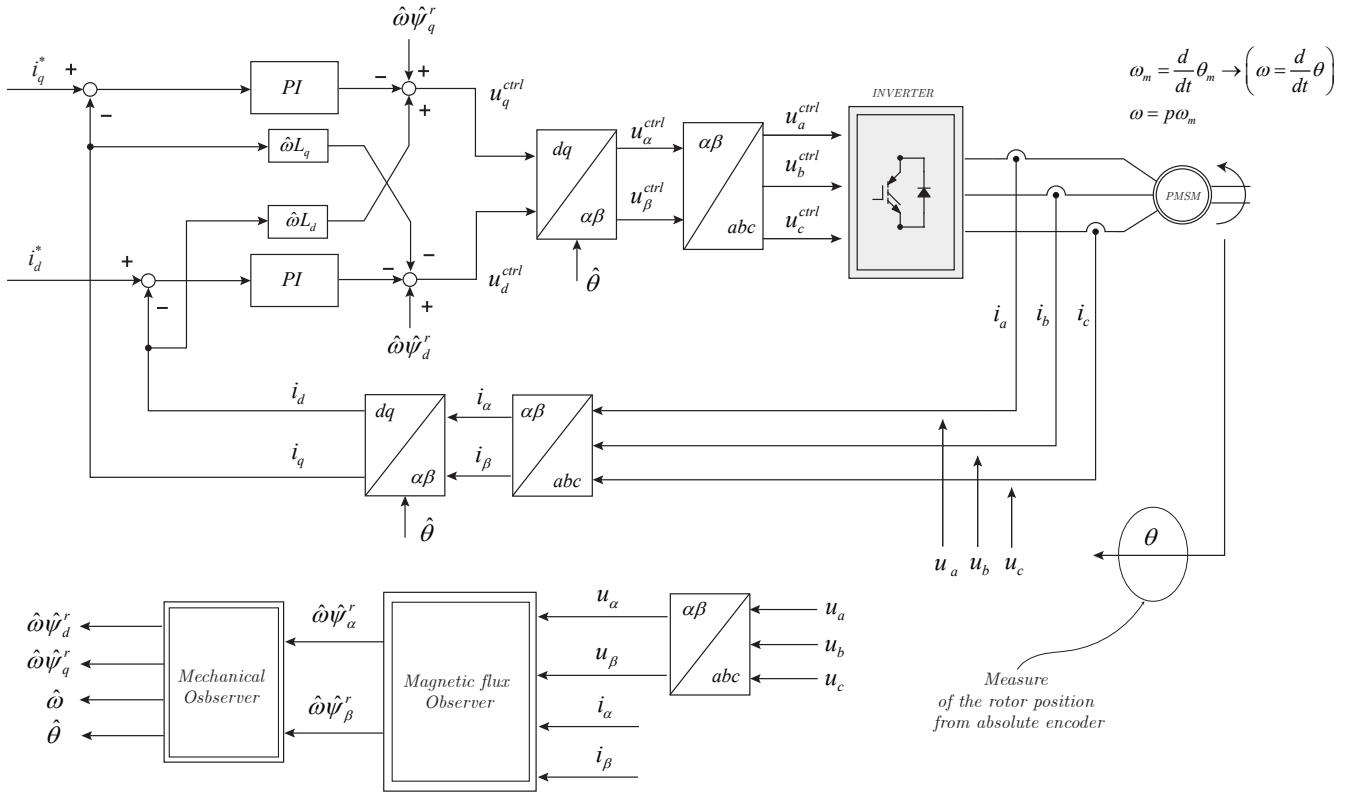of a PMSM.

Figure 6: Internal current control of the classical
FOC of a PMSM.

The **FOC** control strategy is based on the control of two current components. The two
current components (i_d, i_q) are derived by a proper transformation of the PMSM three phase

current system into the *dq* reference frame. The knowledge of the rotor phase θ plays a fundamental role in the performance of the control system. A perfect knowledge of the rotor position θ permits to decouple the control system into two current component where the Q axis concerns the generation of the torque and the D axis affect the value of the terminal voltage and can be properly set in order to extend the working operative are of the PMSMs as shown in Figure 7.
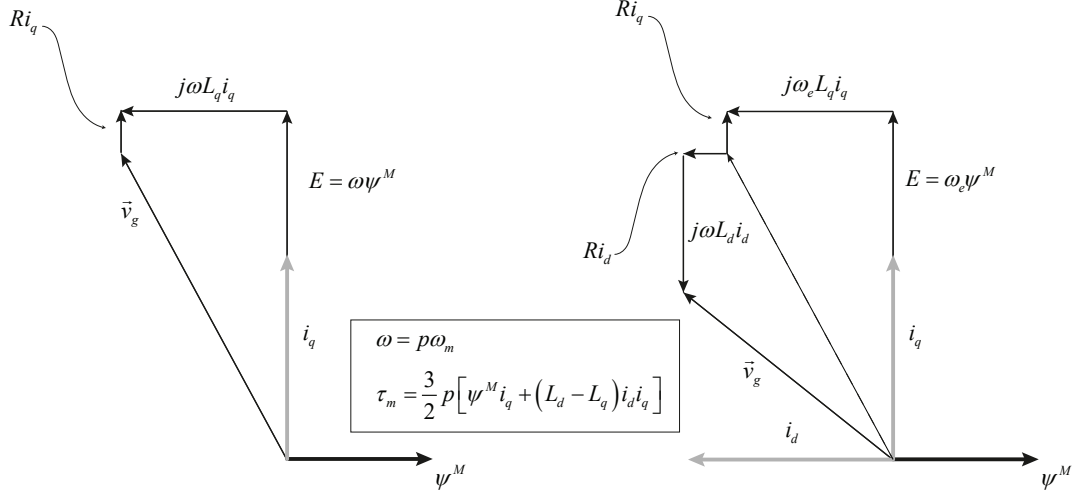

Figure 7: Vector diagram of the PMSM and effect of the i_d current component on the terminal voltage $\vec{v}_g$.

2.2 Normalization and control implementation.

Concerning the model representation and the control implementation we in front of two completely different approach. A model implementation is, in general, represented in SI unit and often in continuous time domain. On the other hand a control system as well as the state space representation of the dynamical system of the plant is, often, represented in per unit and in discrete time domain. The representation in per unit given many advantages in term of usability and *control gain* stability among different plants implementations.

The per unit representation of the model pass through the *normalization*, as follows

– Reference quantities

– $u_{bez} =$ is the peak phase voltage of the no-load voltage at nominal rotor speed ω_m^{nom} of the motor in V and can be derived from parameter K_e (Voltage Constant) as well as from parameter K_t (Torque Constant) as follows

$$u_{bez} = \Psi^M \omega_{bez} \tag{2.1}$$

where

$$\Psi^M = K_e \quad \text{or}$$

$$\Psi^M = K_t \frac{\sqrt{2}}{3p} \qquad \text{we consider } i_d = 0 \text{ control} \tag{2.2}$$

– $i_{bez} =$ is the peak phase nominal current of the motor in A and can be derived from parameter τ_m^{nom} (Nominal Torque) as follows

$$i_{bez} = \frac{\tau_m^{nom}}{K_t} \sqrt{2} \qquad \text{we consider } i_d = 0 \text{ control} \tag{2.3}$$

– $\omega_m^{nom} =$ is the nominal mechanical rotor speed of the motor in $\mathrm{rad\,s^{-1}}$

– $\omega_{bez} = p\omega_m^{nom}$ is the nominal electrical speed of the motor in $\mathrm{rad\,s^{-1}}$

– $X_{bez} = u_{bez}/i_{bez} =$ is the reference electrical impedance of the motor in Ω

– $L_{bez} = X_{bez}/\omega_{bez} =$ is the reference inductance of the motor in H

– $\psi_{bez} = u_{bez}/\omega_{bez} =$ is the reference flux of the motor in Wb

– Per unit quantities

– $R^{norm} = R/X_{bez}$ is the per unit phase resistance.

– $L_d^{norm} = L_d/L_{bez}$ is the per unit phase L_d inductance.

– $L_q^{norm} = L_q/L_{bez}$ is the per unit phase L_q inductance.

– $\omega^{norm} = p\omega_m/\omega_{bez}$ is the per unit electrical speed.

– $i_d^{norm} = i_d/i_{bez}$ is the per unit i_d current.

– $i_q^{norm} = i_q/i_{bez}$ is the per unit i_q current.

– $u_d^{norm} = u_d/u_{bez}$ is the per unit u_d voltage.

– $u_q^{norm} = u_q/u_{bez}$ is the per unit u_q voltage.

– $\psi_d^{norm} = \psi_d/\psi_{bez}$ is the per unit D-flux.

– $\psi_q^{norm} = \psi_q/\psi_{bez}$ is the per unit Q-flux.

– $\psi_M^{norm} = \Psi^M/\psi_{bez}$ is the per unit M-flux.

And the FOC control implementation become (in continuous time domain) - in the following we omit the term $\binom{norm}{}$ and we consider all as per unit quantities

$$
\begin{cases}
i_q^{ref} = k_p^w\left(\omega^{ref} - \omega\right) + i_q^i \\[2mm]
\dfrac{d}{dt}i_q^i = k_i^w\left(\omega^{ref} - \omega\right) \\[2mm]
u_d = k_p^i\left(i_d^{ref} - i_d\right) + u_d^i - \omega\hat{\psi}_q - \omega L_q i_q \\[2mm]
\dfrac{d}{dt}u_d^i = k_i^i\left(i_d^{ref} - i_d\right) \\[2mm]
u_q = k_p^i\left(i_q^{ref} - i_q\right) + u_d^i + \omega\hat{\psi}_d - \omega L_d i_d \\[2mm]
\dfrac{d}{dt}u_q^i = k_i^i\left(i_q^{ref} - i_q\right)
\end{cases}
\tag{2.4}
$$

2.3   PMSM current loci and over-speed management

The control of a PMSM and in particular of an IPMSM where L_d and L_q are strongly different, has some degree of freedom concerning the set-point of the current i_d. As we will see, the current i_d can be used to coercive the terminal voltage (flux weakening, FW) of PMSM (which is subjected to the inverter constraints) to a given value. Moreover the a proper set of i_d current can be selected to optimize some properties of the PMSM like the maximum torque per ampere

(MTPA) or the maximum torque per voltage (MTPV), or to minimize the copper power losses, or in flux weakening, etc.

Field Weakening Principle

$$\omega = p\omega_m$$

$$\tau_m = \frac{3}{2} p \left[\psi^M i_q + \left(L_d - L_q \right) i_d i_q \right]$$
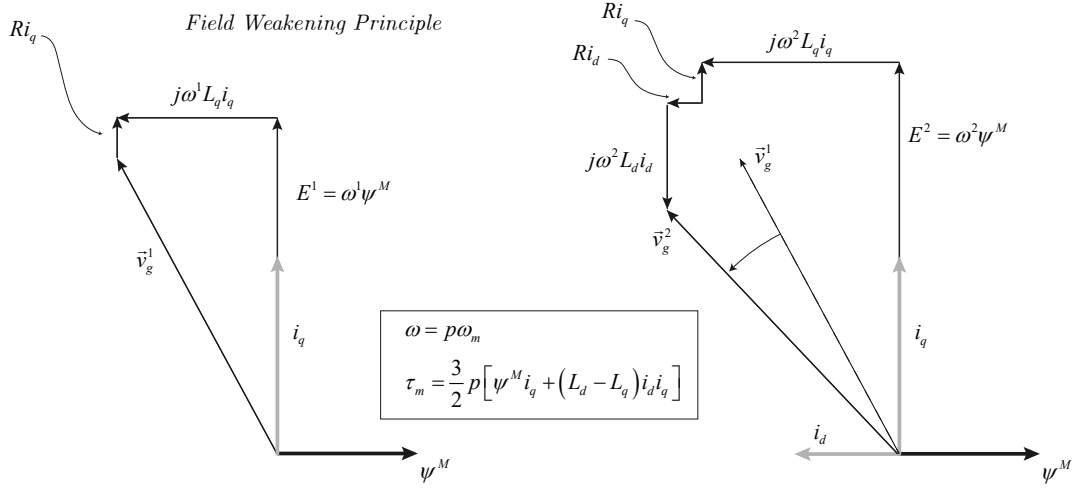
Figure 8: Vector diagram of the PMSM and effect of the i_d current component on the terminal voltage $\vec{v}_g$ - Field weakening case.

In general an IPMSM is controlled by the MTPA and when it reach the maximum voltage (due to over-speed) the current loci $i_d - i_q$ can be kept along a maximum voltage curve until a maximum current (circle - $i_d - i_q$) is reached. When the current loci lays in a maximum current curve, the control can move along the limit curve until a different control strategy, like MTPV is adopted.
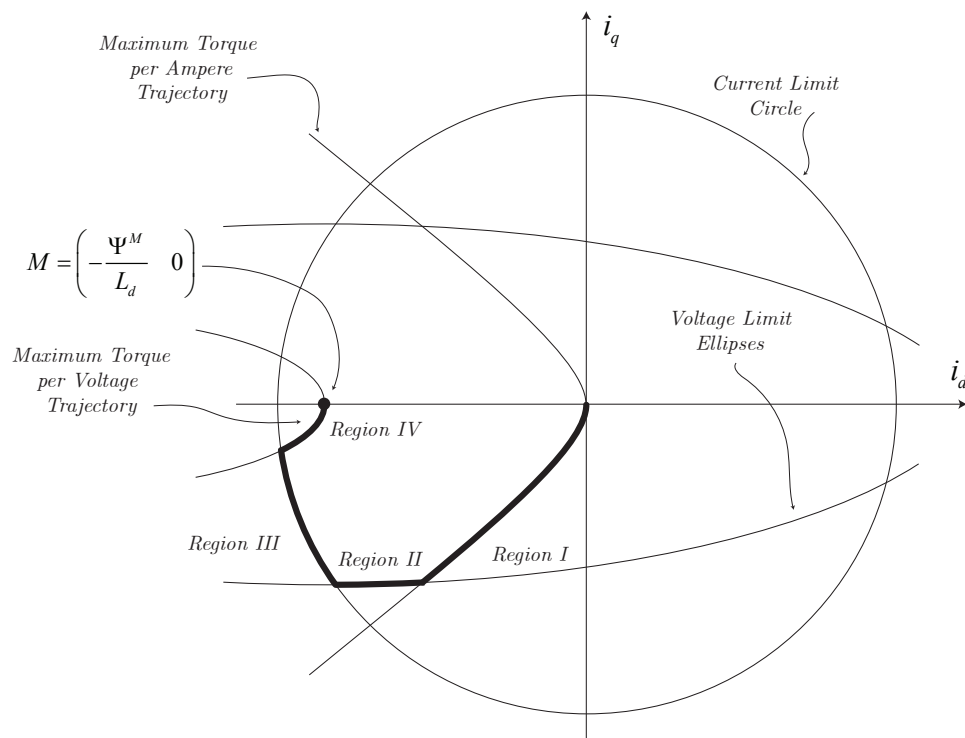
Figure 9: Current limit circle and voltage limit ellipses in $i_d - i_q$ plane.

Another very common control strategy is the $i_d = 0$ control, obviously this control is naturally the best choice for isotropic PMSM ($L_d = L_q$), even if is widely used in IPMSM. Using $i_d = 0$ control in a IPMSM de-rate the effective efficiency of the system, requiring more current than what can be used in the case of a MTPA control.

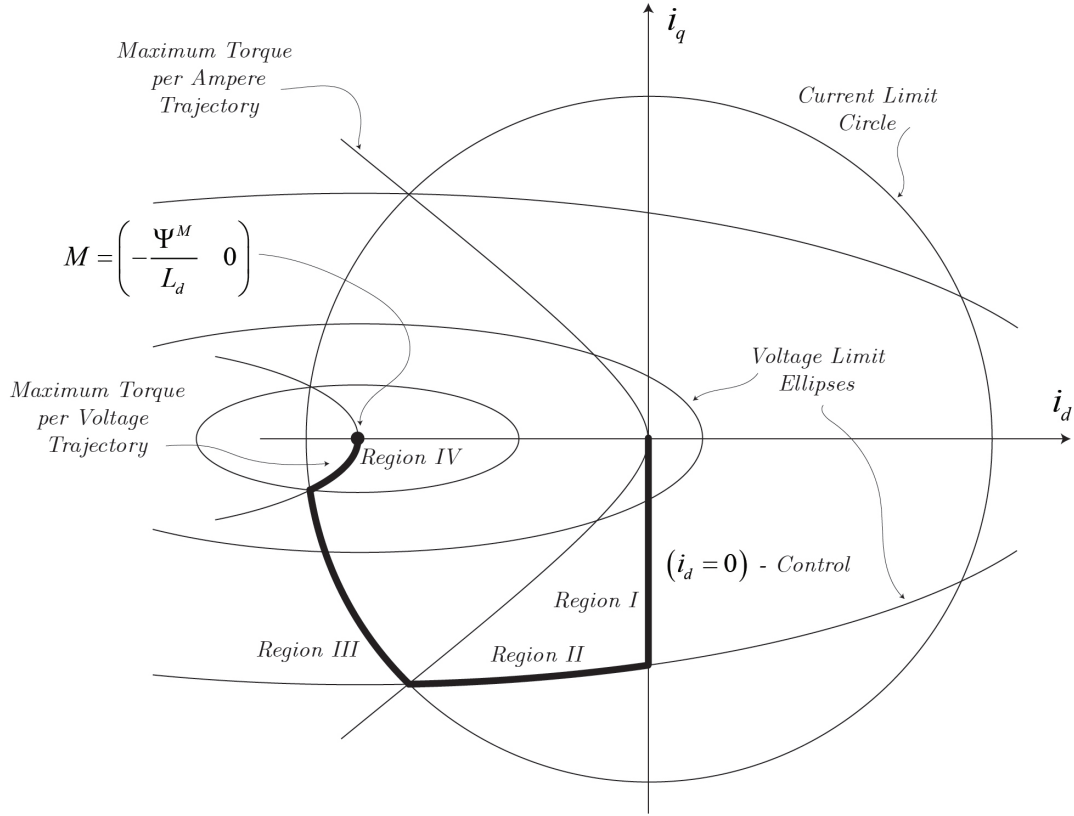Some current loci are shown in Figures 9 and 10

Figure 10: Some possible current control strategies and their properties.

The relation between *d*-axis and *q*-axis currents for the **maximum torque per ampere** (MTPA) condition is given as follows

$$
\begin{cases}
i_d^{ref} = \dfrac{\Psi^M}{2\left(Lq - Ld\right)} - \sqrt{\dfrac{\left(\Psi^M\right)^2}{4\left(L_q - L_d\right)^2} + i_q^2} \\[4mm]
i_d^{ref} = \quad \text{as per torque}
\end{cases}
\tag{2.5}
$$

The optimal current vector by which the torque per flux linkage $\psi_0 = \sqrt{\left(\Psi^M + L_d i_d\right)^+ \left(L_q i_q\right)^2}$ become maximal is given as follows

$$
\begin{cases}
i_d = -\dfrac{\Psi^M + \Delta\psi_d^r}{L_d} \\[4mm]
i_q = \dfrac{1}{L_q}\sqrt{\left(\psi_0\right)^2 - \left(\Delta\psi_d^r\right)^2}
\end{cases}
\tag{2.6}
$$

where

$$
\Delta\psi_d^r = \frac{-L_q\Psi^M + \sqrt{\left(L_q\Psi^M\right)^2 + 8\left(L_q - L_d\right)^2\psi_0^2}}{4\left(L_q - L_d\right)}
\tag{2.7}
$$

The vector control based on Eq. (2.6) and 2.7 can be called *the maximum torque per flux linkage (MTPF) control*. This condition is equal to maximizing the torque to the voltage v_g, where v_g is defined as follows

$$v_g = \sqrt{\left(\omega \Psi^M + Ri_q + \omega L_d i_d\right)^2 + \left(Ri_d - \omega L_q i_q\right)^2} \tag{2.8}$$

thus this vector control is named *the maximum torque per voltage (MTPV) control*.

One of the most interesting and important $i_d - i_q$ current loci is the *constant voltage* locus, where in Figure 10 is described like a ellipses. This current loci is very important and widely used in **generator mode** where the speed motor is imposed by the external system. Along this region (*region II*) the output voltage of the inverter (or of the terminals of the PMSM) is kept constant (in modulus) and the current vector is not more controlled but is free to rotate or move along this ellipses. This region is often used as **self field weakening control**, in fact, we can observe that in this condition the inverter is able to keep loaded the PMSM by automatically rotation of the motor current (the current is rotating out of control) in order to maintain the modulus of the terminal voltage constant and modifying the actual torque.

2.4   Robust field weakening control architecture

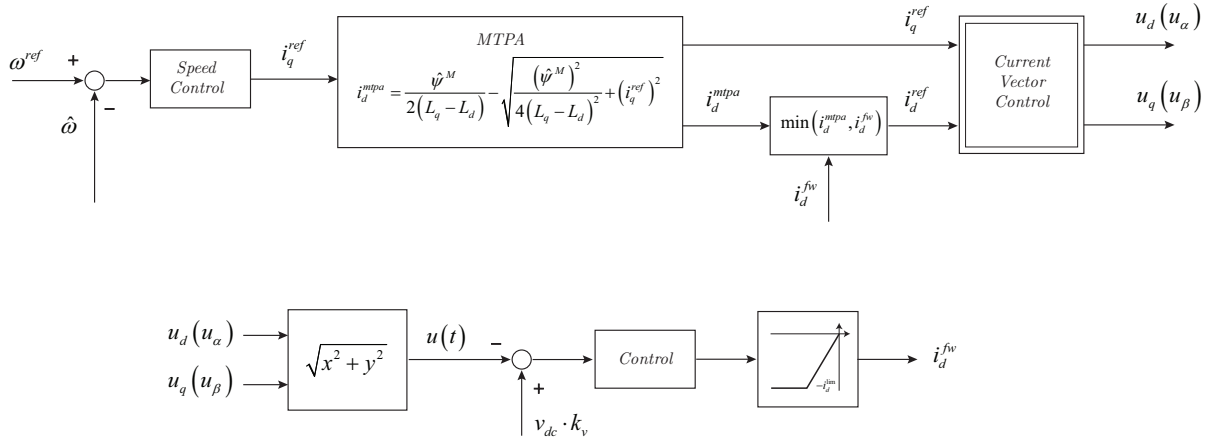In this section a feedback (closed loop) field weakening control structure is proposed.

Figure 11: Robust flux weakening control architecture.

3   Parameters identification

PMSM parameters can be determined from datasheet and factory tests which in general are available from supplier. In order to characterize the motor the following data shall be collected:

- **Number of poles.**

- **Nominal data.**

    - Nominal speed.
    - Nominal current.
    - Nominal voltage.

– Nominal torque.

– **No-load voltage test.**

– **Terminals short circuit test.**

– **Measure of the stator phase resistance.**

In order to identify correctly the values of L_d and L_q which can affect considerably the performance of MTPA (for anisotropic permanent magnet motor as IPMSM) additional test can be carried out as in the next will be explained.

3.1 No-load test

The **no-load test** in one of the most important test in order to characterize the model of the PMSM. The no-load test consists to run the motor without any load at nominal speed. In this condition the load current is very low and completely negligible. In this condition the terminal phase voltages correspond to the back emf

$$\vec{E} = -\frac{d\psi^r(t)}{dt} = \begin{bmatrix} \omega\psi^M \sin(\vartheta t) \\ \omega\psi^M \sin(\vartheta t - 2\pi/3) \\ \omega\psi^M \sin(\vartheta t + 2\pi/3) \end{bmatrix} \tag{3.1}$$

That means by the no-load test permanent magnet flux linkage $\psi^M(t)$ can be determined. During the no-load test the voltage vector $\vec{E}$ is measured and the frequency ω is known.

3.2 Terminals short circuit test

From **terminals short circuit test** the value of L_d can approximately determined. During these test the equivalent model of the PMSM can represented as shown in Figure 12 where the voltage drop across the phase resistance has been considered negligible.
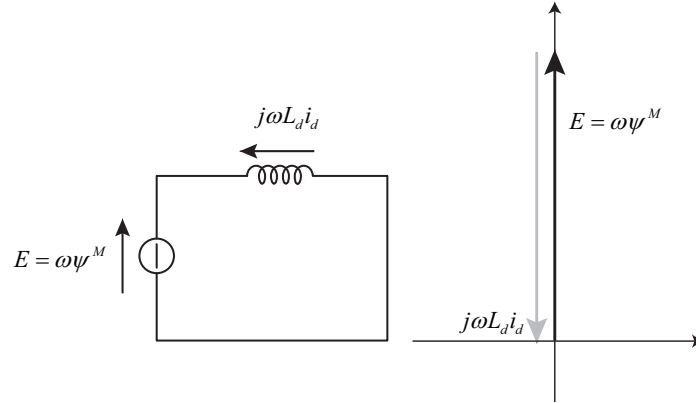

Figure 12: Terminals short circuit test from electrical point of view.

Measuring the phase current $i_{ph} = i_d$ and knowing the back emf voltage from no-load test is it possible to determine the value of inductance $L_d = \frac{\psi^M}{i_d}$.

3.3 Load test

Load test is one of the mandatory test which have to be performed. This test results in a very wide range of information results, from thermodynamic behavior to internal machine behavior.

For MTPA the knowledge of both L_d and L_q must be known. The inductance value L_q can be determined from the comparison of the terminal voltage at different loads. In this contest the load test (at different loads) can be used for this purpose.
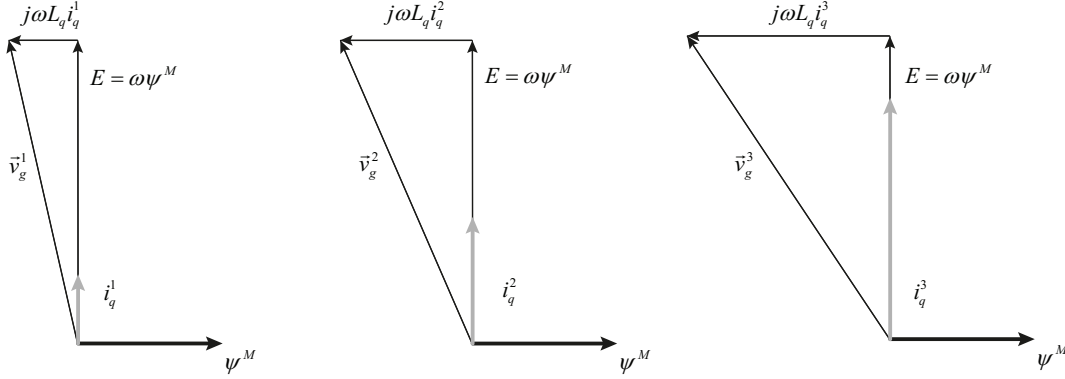

Figure 13: Load test and method to determine the L_q inductance.

From Figure 13 we can se that the terminal voltage DQ components $\vec{v}_g = \begin{bmatrix} v_d^g & v_q^g \end{bmatrix}^T$ can be written as follows

$$
\begin{cases}
v_d^g = \omega L_q i_q \\
v_q^g = \omega \psi^M
\end{cases}
\tag{3.2}
$$

by the knowledge of the rotor position ϑ is it possible to measure the voltage components $\begin{bmatrix} v_d^g & v_q^g \end{bmatrix}^T$ and hence the value of L_q at different load conditions.

4   Model parameters settings

 – Mode of operation

 – Motor parameters

 – Control parameters

 Mode of operation consists of the following parameters

 – **Enable Speed Mode**

  – **Speed Mode**: Motor is controlled in speed mode. A speed set-point has to be set.

  – **Torque Mode**: Motor is controlled in torque mode. A torque set-point has to be. set.

 – **Enable Speed Mode**: if speed mode is enabled

  – **Torque limited from torque reference**: A maximum torque set-point has to be provide.

– **Torque unlimited**: Motor behaves as an ideal speed source.

Motor parameters consists of the following parameters

– **Number of motor pole pairs**: is the number of motor pole pairs.

– **Maximum Speed** ω_m^{max}[min^{-1}]: is the motor maximum speed (mechanical).

– **Nominal Speed** ω_m^{nom}[min^{-1}]: is the motor nominal speed (mechanical).

– **Nominal Torque** τ_m^{nom}[N m]: is the motor nominal torque.

– **Phase Resistance** R_s[Ω]: is the nominal stator phase resistance.

– **D-inductance** L_d[H]: is the direct axis inductance.

– **Q-inductance** L_q[H]: is the quadrature axis inductance.

– **Voltage Constant** K_e[V s rad^{-1}]: Neutral-phase voltage peak per electrical pulsation.

– **Torque Constant** K_t[Nm/Arms]: Torque per ampere rms.

– **Rotor Inertia** J_m[kg m^2]: is the nominal rotor mechanical inertia.

– **Rotor Friction** b[Nm/rad/s]: is the bearing viscosity.

Motor parameters consists of the following parameters

– **Proportional Speed Ctrl Gain** k_p^ω

– **Integral Speed Ctrl Gain** k_i^ω

– **Proportional Current Ctrl Gain** k_p^i

– **Integral Current Ctrl Gain** k_i^i

– **Proportional Field Weakening Ctrl Gain** k_p^{fw}

– **Integral Field Weakening Ctrl Gain** k_i^{fw}

5   Model output variables

The output bus channel **PMSM data** includes the following data output

– ω_m: mechanical speed $\left[\text{min}^{-1}\right]$.

– τ_m: electromagnetic torque $\left[\text{N m}\right]$.

– i_g: phase current in rms $\left[\text{Arms}\right]$.

– i_d: direct current $\left[\text{A}\right]$.

– i_q: quadrature current $\left[\text{A}\right]$.

- $\psi_d^s = \psi_d^r + i_d L_d$: direct flux $\left[\text{Wb}\right]$.

- $\psi_q^s = \psi_q^r + i_q L_q$: quadrature flux $\left[\text{Wb}\right]$.

- e_d: direct back EMF component $\left[\text{V}\right]$.

- e_q: quadrature back EMF component $\left[\text{V}\right]$.

- v_g: phase to phase terminal voltage in rms $\left[\text{Vrms}\right]$.

- u_d: direct terminal voltage component $\left[\text{V}\right]$.

- u_q: quadrature terminal voltage component $\left[\text{V}\right]$.

- P_{in}: Active power at motor terminal $\left[\text{kW}\right]$.

References

[**P. Krause, 2013**] P. Krause, O. Wasynczuk, S. Sudhoff, S. Pekarek - *Analysis of Electric Machinery and Drive Systems*. Wiley 2013.

[**N. Bianchi, 2004**] N. Bianchi, T. Jahns - *Design, analysis, and control of interior PM synchronous machines*. CLEUP 2004.