Control of a heavy duty vehicle powered by a hydrostatic power-train and diesel engine.

DTL

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Chapter 1

Introduction

1.1 Scope of the document

The following document reports the description of some control systems used in the management of an heavy duty vehicle. The set of control systems specification can be summarized as follows

- Limit load control: the electronic control system regulates the engine speed and controls the driving speed depending on the required thrust force.
- Maximum speed limiter: controls the travel drive (speed) so that the set travel speed is not exceeded (e.g. downhill)
- Tracks synchronization control: adapts both sides of the tracks so that, for example, when driving straight ahead, the machine actually drives straight ahead.
- Pressure control: automatically reduces the pump adjustment when the high pressure limits are exceeded in order to prevent unnecessary energy loss via the pressure relief valves (e.g. in plowing operations).

As first step a mathematical model of the hydrostatic power-train has been presented. The reason of the mathematical model has different scopes

- find the fundamental motion equations of the system
- find the interconnection among the different dynamic equations
- find a possible model representation for model predictive control implementation

Chapter 2

Vehicle Control architecture

2.1 Nomenclature

- v_{track}^{R}: speed of the right track in $\mathrm{km\,h^{-1}}$.
- v_{track}^{L}: speed of the right track in $\mathrm{km\,h^{-1}}$.
- $v_{track}^{sum} = v_{track}^{R} + v_{track}^{L}$: sum of the speed of the right and left track in $\mathrm{km\,h^{-1}}$.
- $v_{track}^{diff} = v_{track}^{R} - v_{track}^{L}$: sum of the difference between the right and left track $\mathrm{km\,h^{-1}}$.
- $v_{track}^{ref}\Big|_{R}$: speed set-point of the right track in $\mathrm{km\,h^{-1}}$.
- $v_{track}^{ref}\Big|_{L}$: speed set-point of the left track in $\mathrm{km\,h^{-1}}$.
- $v_{track}^{ref}\Big|_{R}^{max}$: maximum speed set-point of the right track in $\mathrm{km\,h^{-1}}$.
- $v_{track}^{ref}\Big|_{L}^{max}$: maximum speed set-point of the right track in $\mathrm{km\,h^{-1}}$.
- ω_{l}^{R}: rotational speed of the follower (side which is connected to the track) of the right driver gear in $\mathrm{rad\,s^{-1}}$.
- ω_{l}^{L}: rotational speed of the follower (side which is connected to the track) of the left driver gear in $\mathrm{rad\,s^{-1}}$.
- ω_{m}^{R}: rotational speed of the driver (side which is connected to the hydraulic motor) of the right driver gear in $\mathrm{rad\,s^{-1}}$.
- ω_{m}^{L}: rotational speed of the follower (side which is connected to the hydraulic motor) of the left driver gear in $\mathrm{rad\,s^{-1}}$.
- ω_{m}^{ref}: hydraulic motor rotational speed set-point.
- n_{tg}: drive gear ration.
- R_{tg}: radius of the sprocket wheel connected to the track.
- V_{m}^{nom}: nominal capacity of the hydraulic motor.
- V_{p}^{nom}: nominal capacity of the hydraulic pump.

- η_m^v: volumetric efficiency of the hydraulic motor.
- η_m^m: mechanical efficiency of the hydraulic motor.
- η_p^v: volumetric efficiency of the hydraulic pump.
- η_p^m: mechanical efficiency of the hydraulic pump.
- d_p^R: right drive-line,per unit pump volumetric displacement, where $d_p \in [-d_p^{\max},\ d_p^{\max}]$, $d_p^{\max}$ is in per unit.
- d_m^R: right drive-line, per unit motor volumetric displacement, where $d_m \in [d_m^{\min},\ d_m^{\max}]$, $d_m^{\max}$ and $d_m^{\min}$ are in per unit.
- d_L: left drive-line, per unit global volumetric displacement, where $d \in [-2d_p^{\max}+d_m^{\min},\ 2d_p^{\max}-d_m^{\min}]$, $d_p^{\max}$ and $d_m^{\min}$ are in per unit.
- d_p^L: left drive-line, per unit pump volumetric displacement, where $d_p \in [-d_p^{\max},\ d_p^{\max}]$, $d_p^{\max}$ is in per unit.
- d_m^L: left drive-line, per unit motor volumetric displacement, where $d_m \in [d_m^{\min},\ d_m^{\max}]$, $d_m^{\max}$ and $d_m^{\min}$ are in per unit.
- ω_e: engine speed (also pump speed).
- ω_e^{ref}: engine rotational speed set-point.
- τ_e: engine torque in N m.
- dir_R: direction of the right track (1 or -1).
- dir_L: direction of the left track (1 or -1).
- dir_S: direction of the sum of the right and left track (1 or -1).

2.2 Introduction

In this document we propose a first draft of heavy duty vehicle control architecture based on independent feedback control loops which can be summarized as follows

- A control loop which limits the maximum speed reached by the (*sum*) sum of the tracks speed, where for *sum* tracks speed it is intended $v_{track}^{sum} = v_{track}^{R} + v_{track}^{L}$. Supposing $v_{track}^{ref}\Big|_{sum}^{max} = v_{track}^{ref}\Big|_{R}^{max} + v_{track}^{ref}\Big|_{L}^{max}$ is the sum of the right and left maximum speed track reference (set-point). This control loop limits the maximum tracks speed sum which e.g. we can assume the following value $v_{track}^{ref}\Big|_{sum}^{max} = 11\,\mathrm{km\,h^{-1}} + 11\,\mathrm{km\,h^{-1}} = 22\,\mathrm{km\,h^{-1}}$.
- A (*diff*) difference tracks speed control or steering. Supposing $v_{track}^{ref}\Big|_{diff} = v_{track}^{ref}\Big|_{R} - v_{track}^{ref}\Big|_{L}$ the difference between the right and the left speed track reference (set-point) which results into a steering set-point. This control loop keeps the difference of the tracks speed to a given reference value and moreover saturates to a maximum value the steering speed.

- An *engine anti-stall.* This control loop enables an automatic total volumetric displacement reduction in order to keep the engine at the given value of speed set-point. The effect of the anti-stall is to reduce both (for each side left and right) the current volumetric displacement according the maximum engine torque available at a given engine speed set-point.
- Two *pressure limitation* control loops. These two regulators automatically reduce the corresponding volumetric displacement in order to keep the relative driveline pressure below a given maximum value. E.g. we can assume a delta pressure limit value of $\Delta p^{lim} = 420\,\text{bar}$.

2.3 Control architecture

2.3.1 Introduction

The power-train control which we are going to describe in next sections can be summarized as shown in Figure 2.1. Three groups of quantities can be distinguished

- References
- Measures
- Control outputs

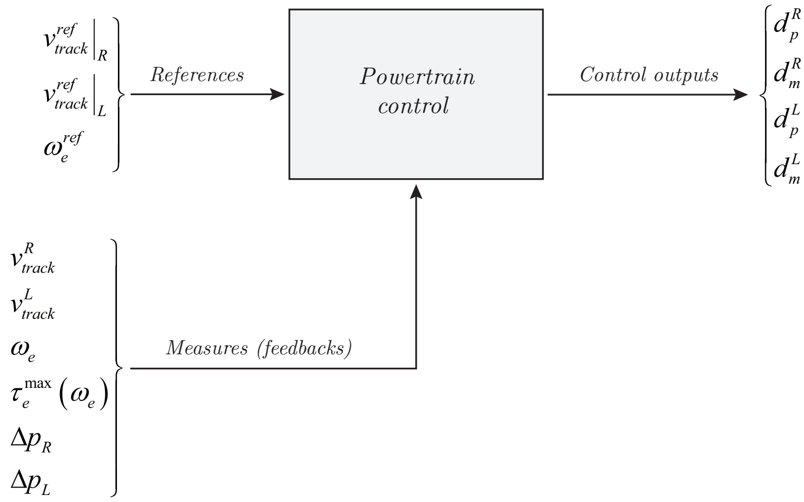


Figure 2.1: Power-train control overview.

As *References* are intended the right and left track speed targets, in $\left[\mathrm{km\,h^{-1}}\right]$, and the engine rotational speed target, in $\left[\mathrm{min^{-1}}\right]$.

As *Measures* are intended the right and left track speeds, in $\left[\mathrm{km\,h^{-1}}\right]$, the engine rotational speed, in $\left[\mathrm{min^{-1}}\right]$, the right and left drive-line delta pressure, in $\left[\mathrm{bar}\right]$, and the available engine torque, in $\left[\mathrm{N\,m}\right]$, derived from engine speed.

As *Control outputs* are intended the volumetric displacements of the hydraulic pumps and hydraulic motors.

In order to simplify the control architecture the following mathematical objects are defined:

- A **global** volumetric displacement (an object which include both motor and pump volumetric displacement) is created (see also Figure 2.2 and Figure 2.3). Let d be the **global** volumetric displacement, the terms d_m and d_p are derived as follows

$$\begin{cases} d_m = d_m^{max} - \Big[|d| - |d_p| \Big] \quad \Rightarrow \quad d_m^{min} \leq d_m \leq d_m^{max} \\ d_p \quad \Rightarrow \quad -d_p^{max} \leq d \leq d_p^{max} \end{cases} \tag{2.3.1}$$

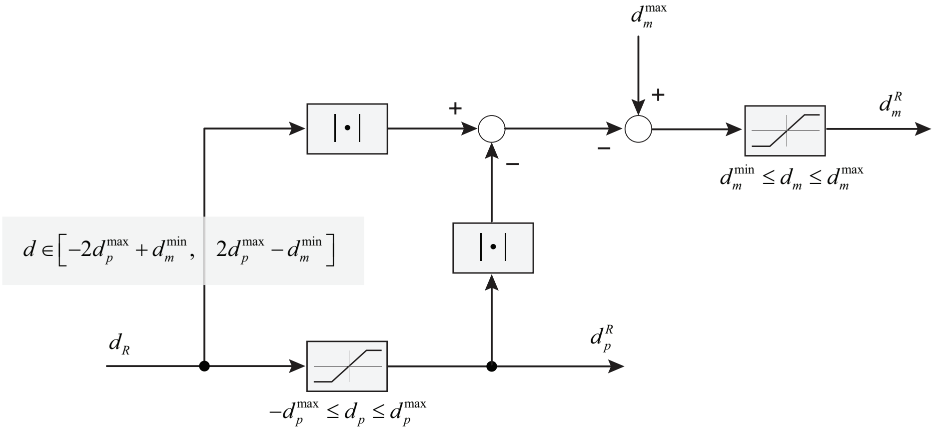


Figure 2.2: Total volumetric displacement for the right drive-line.

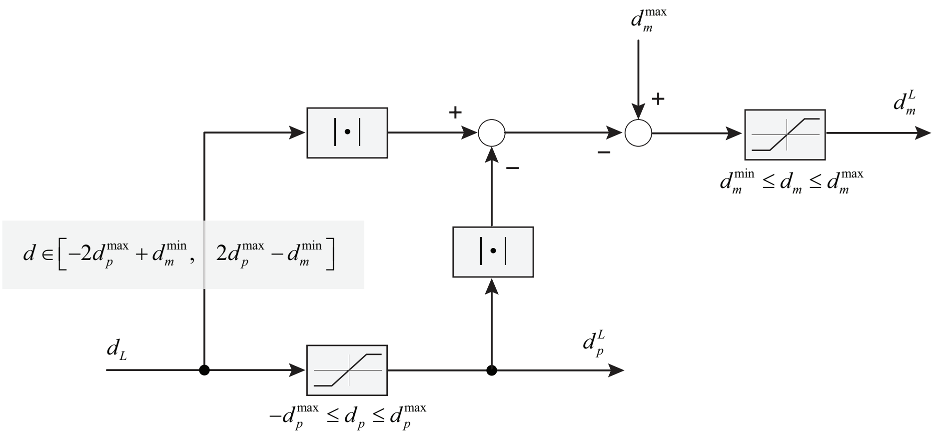


Figure 2.3: Total volumetric displacement for the left drive-line.

The construction of the **global** volumetric displacement is shown in Figure 2.6 Figure 2.8 and Figure 2.9.

Let d_R^{ctrl} and d_L^{ctrl} be the volumetric displacements which are generated by the *sum* and *diff* tracks speed loops.

Let d_R^{ff} and d_L^{ff} be the volumetric displacements which are generated as feed-forward from the tracks speed set-points as $v_{track}^{ref}\Big|_R$ and $v_{track}^{ref}\Big|_L$.

The **global** volumetric displacements d_R and d_L are derived as follows

$$\begin{cases} d_R = d_R^{ctrl} + d_R^{ff} \\ d_L = d_L^{ctrl} + d_L^{ff} \end{cases} \tag{2.3.2}$$

- The **sum** and **diff** regulators are PI-based control loops, which require feedback defined as follows

$$\begin{aligned} v_{track}^{sum} &= v_{track}^{R} + v_{track}^{L} \\ v_{track}^{diff} &= v_{track}^{R} - v_{track}^{L} \end{aligned} \tag{2.3.3}$$

  where

$$\begin{aligned} v_{track}^{R} &= \frac{1}{2}\left[v_{track}^{sum} + v_{track}^{diff}\right] \\ v_{track}^{L} &= \frac{1}{2}\left[v_{track}^{sum} - v_{track}^{diff}\right] \end{aligned} \tag{2.3.4}$$

- Two **global** volumetric displacement feed-forward as d_R^{ff} and d_L^{ff}. The construction of the feed-forward terms are here reported. Let v_{track}^{ref} be the speed track reference in $\mathrm{m\,s^{-1}}$ the equivalent hydraulic motor speed reference is given as follows

$$\omega_m^{ref} = \frac{v_{track}^{ref}}{R_{tg}} n_{tg} \tag{2.3.5}$$

  Let α_{ff} be the preliminary per unit volumetric displacement is given as follows

$$\alpha_{ff} = \frac{V_m^{nom}}{V_p^{nom}} \frac{1}{\eta_m^v \eta_p^v} \frac{\omega_m^{ref}}{\omega_e^{ref}} \tag{2.3.6}$$

  or also

$$\alpha_{ff} = \left| \frac{1}{\eta_m^v \eta_p^v} \frac{1}{\omega_e^{ref}} \frac{V_m^{nom}}{V_p^{nom}} \frac{n_{tg}}{R_{tg}} v_{track}^{ref} \right| \tag{2.3.7}$$

  the final formulation of the feed-forward d^{ff} can be represented as follows

$$\boxed{d^{ff} = \begin{cases} 2\,d_p^{max} - \dfrac{1}{\alpha_{ff}} & \text{if } \alpha_{ff} > d_p^{max} \\ \alpha_{ff} \quad \text{if } \alpha_{ff} \leq d_p^{max} \end{cases}} \tag{2.3.8}$$

2.3.2 Control layout

In this section the global control layout is depicted. Some fundamental points are here reported.

- The main speed level is achieved by a feed-forward inputs d_R^{ff} and d_L^{ff}, that means the main control is a open loop control. The external loops control just apply adjustment in order to avoid engine stall, over pressure and perform a synchronization among the two tracks. See Figure 2.6, Figure 2.8 and Figure 2.9.

- The speed track measure is not performed, but in its place, the driver gear (follower) rotational speeds ω_l^R and ω_l^L are measured. That means the overall quantities regarded v_{track} will be converted (in algebraic way) into quantities of the form ω_l, as follows
  - The $\omega_l^{sum} = \omega_l^R + \omega_l^L$ control loop limits the maximum vehicle speed.
  - The $\omega_l^{diff} = \omega_l^R - \omega_l^L$ control loop maintains both tracks speed at the same level when not steering is required, and adjust the ω_l^{diff} in case of steering demand.

  During straight driving the $\omega_l^{ref}\Big|_{diff}$ reference is set to zero.

- An engine anti-stall control loop is implemented, where the actual engine torque is compared with the torque limit curve $\tau_e(\omega_e)$ and the total volumetric displacement is properly compensated, see also Figure 2.8, Figure 2.9 and Figure 2.4.

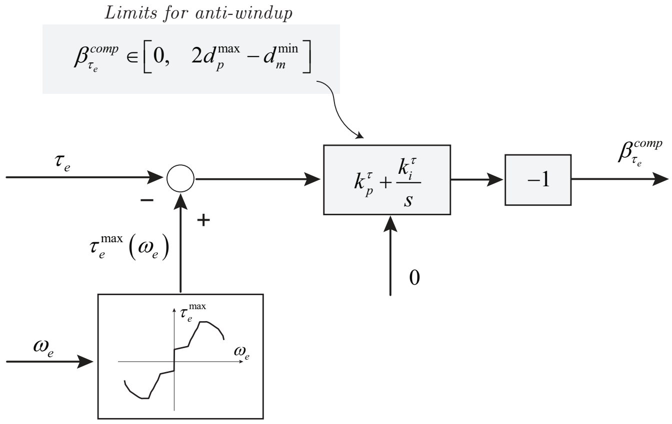


Figure 2.4: Engine anti-stall implementation.

- The limit drive line delta pressure is also constrained by proper compensation of the total volumetric displacement, in order to limit its maximum value, when operative field condition permits it, see also Figure 2.8, Figure 2.9 and Figure 2.5.

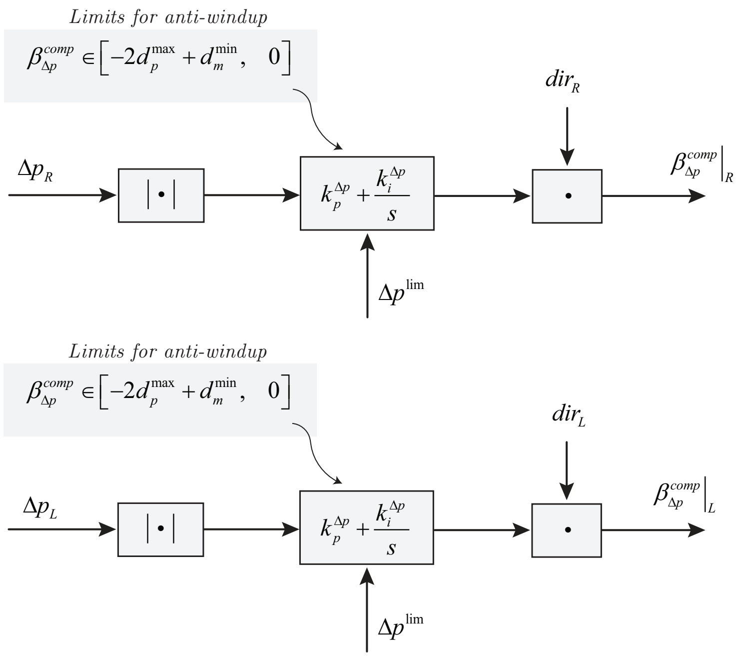


Figure 2.5: Drive line delta pressure limitation.

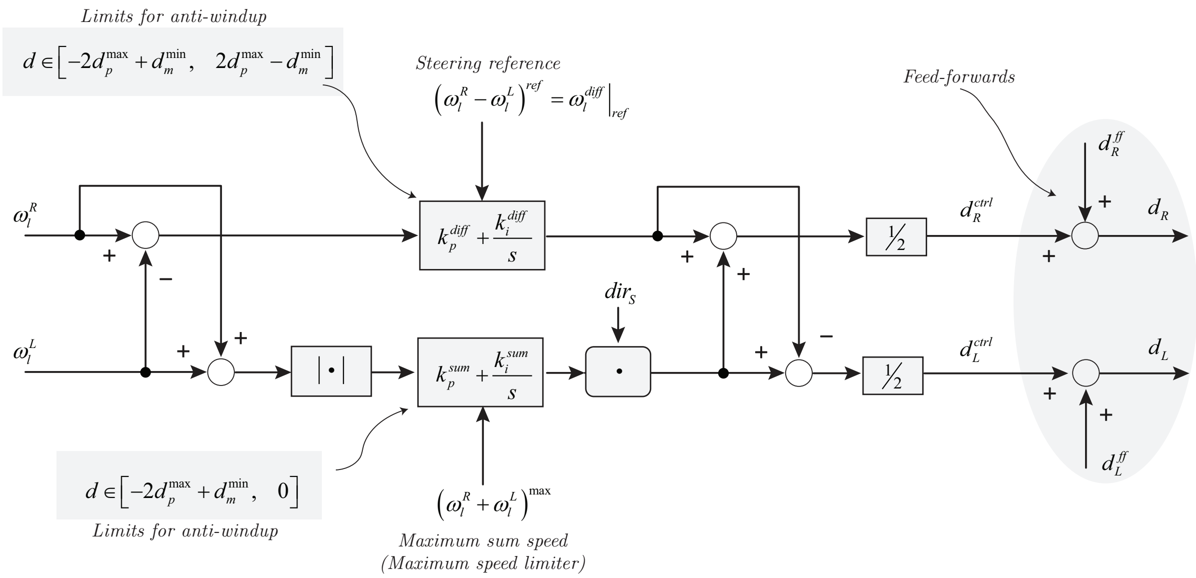


Figure 2.6: Steering and maximum speed limitation.

The global volumetric displacement is derived adding to the feed-forward and to the *sum* and *diff* control loops the *field* compensations. For field here the engine anti-stall and over-pressure compensation is intended. Figure 2.7 shows the global volumetric displacement architecture comprehensive of the *field* compensations.

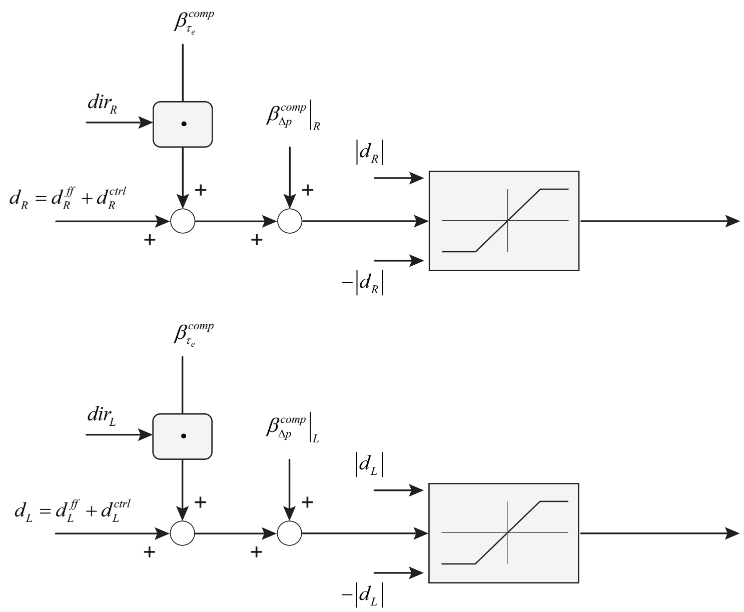


Figure 2.7: Global volumetric displacement construction comprehensives of the compensations.

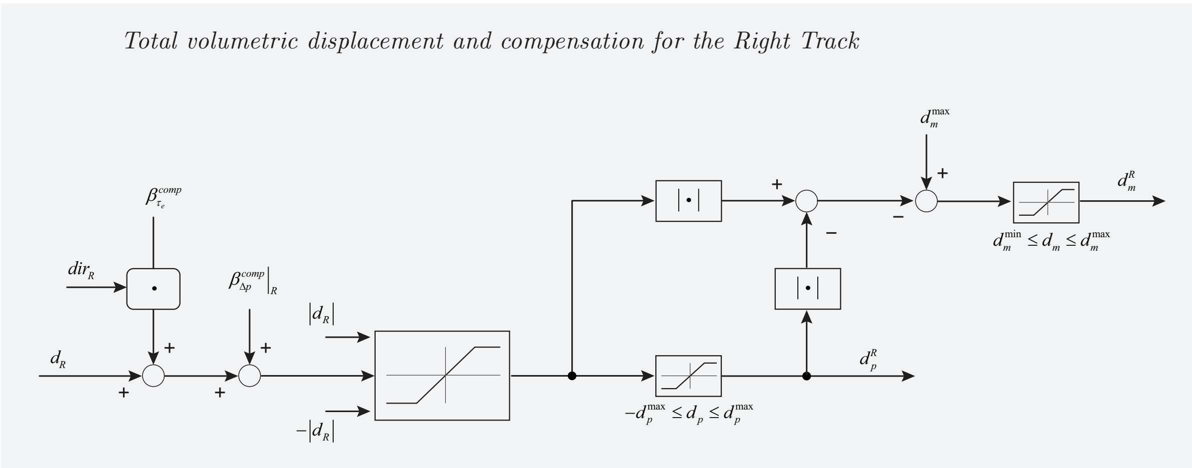


Figure 2.8: Architecture of the final volumetric displacement containing the engine stall and over-pressure compensation for right track.

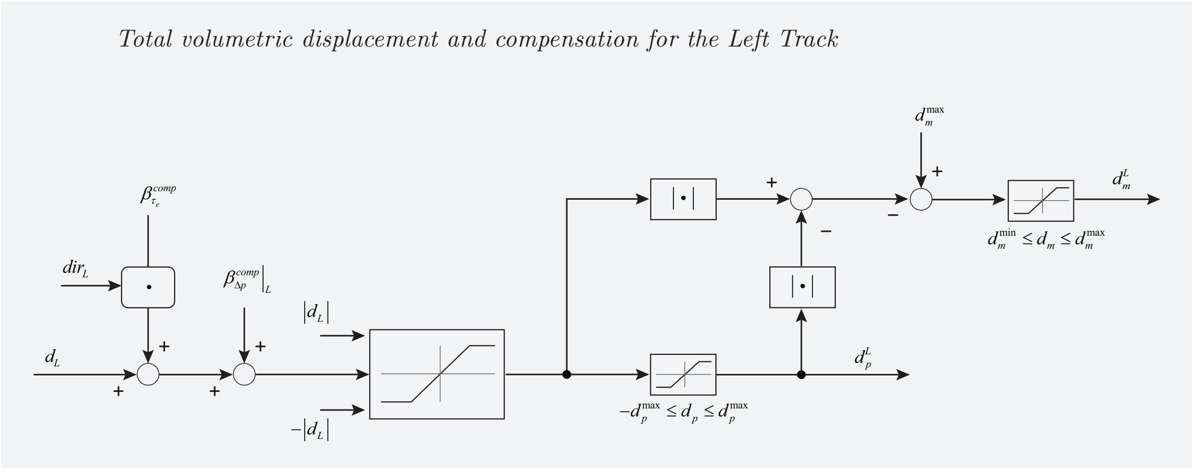


Figure 2.9: Architecture of the final volumetric displacement containing the engine stall and over-pressure compensation for left track.

2.4 Case study

The case study we are going to consider is a drive-train with the following data characteristics

- $V_m^{nom} = 252.8\,\mathrm{cm}^3$
- $V_p^{nom} = 147.2\,\mathrm{cm}^3$
- $R = 0.440\,56\,\mathrm{m}$
- $n_{tg} = n_1/n_2 = 41.4$
- $v_{tr}^{max} = 11.0\,\mathrm{km\,h^{-1}}$
- $\eta_p^m = 0.909$
- $\eta_p^v = 0.959$
- $\eta_m^m = 0.939$
- $\eta_m^v = 0.943$

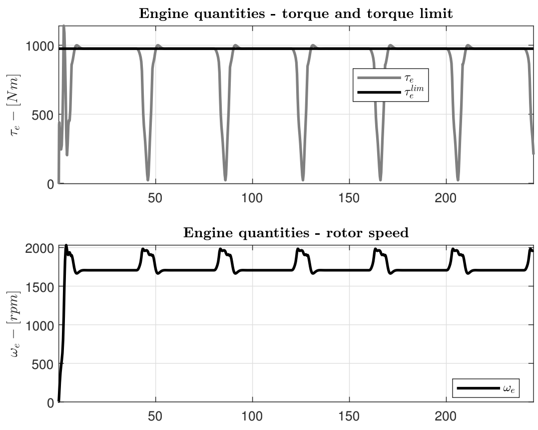
2.4.1 Simulation results

Scenario 1

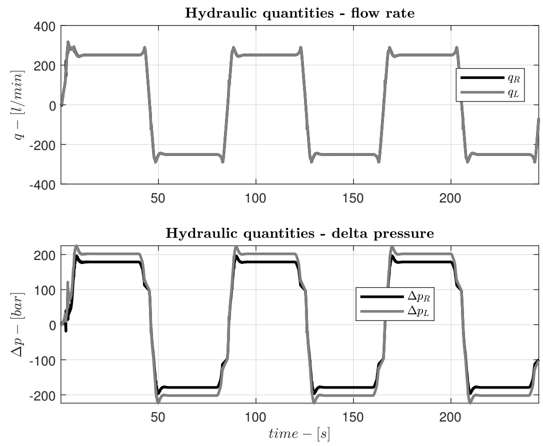
In this scenario we are considering the following case

- Straight driving.
- Not homogeneous viscosity load on tracks.
- The total amount of load is enough to stall the engine.
- The maximum per drive-line load doesn't exceed the maximum admissible delta pressure.

Figure 2.10: Description of the test scenario 1.

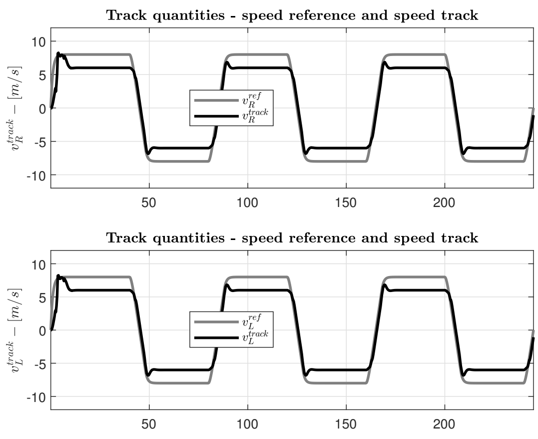


(a) Engine data.

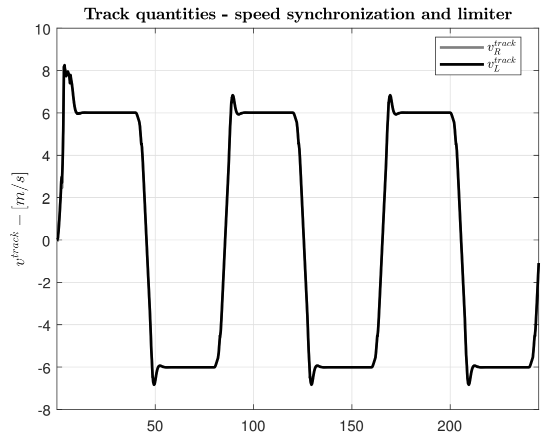


(b) Hydrostatic drive-line data.

Figure 2.11: Simulation results scenario 1.



(a) Speed tracks tracking performance.



(b) Speed tracks synchronization and maximum speed limiter.

Figure 2.12: Simulation results scenario 1.

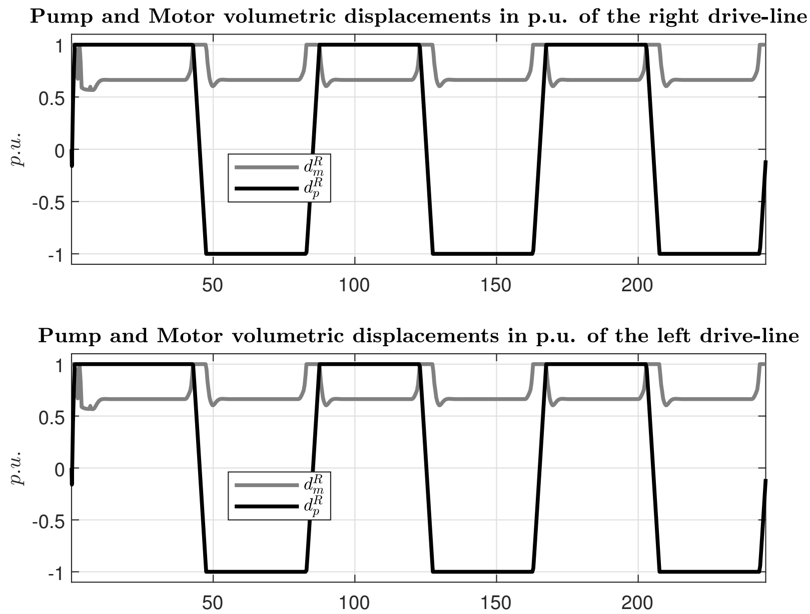


Figure 2.13: Volumetric displacements.

Scenario 2

In this scenario we are considering the following case

- Straight driving.
- Not homogeneous viscosity load on tracks.
- The total amount of load is enough to stall the engine.
- The maximum load on drive-line one (Right) exceed the maximum admissible delta pressure.

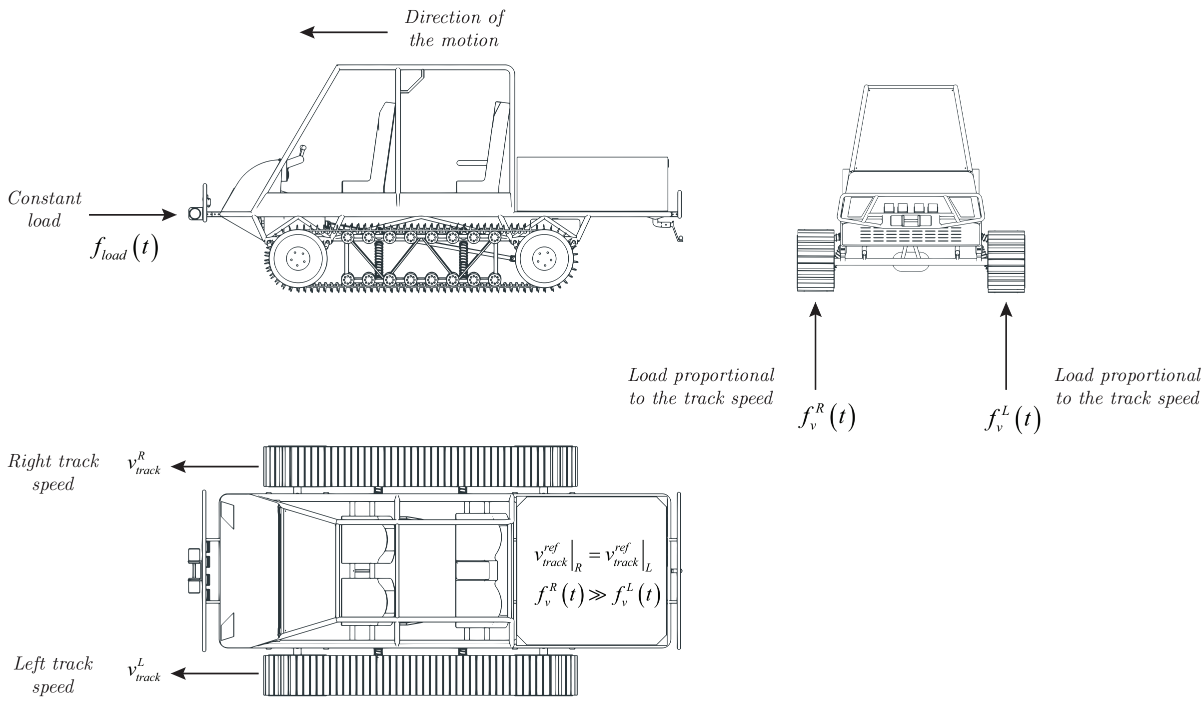
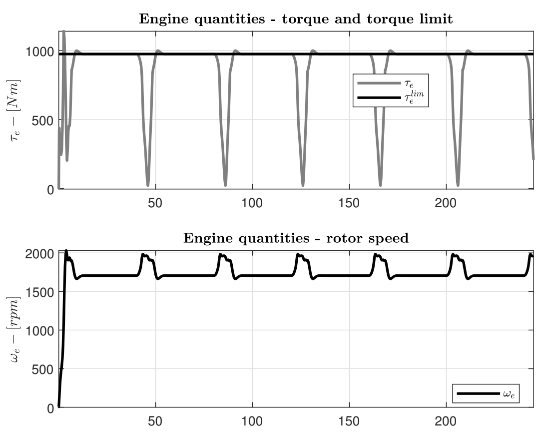
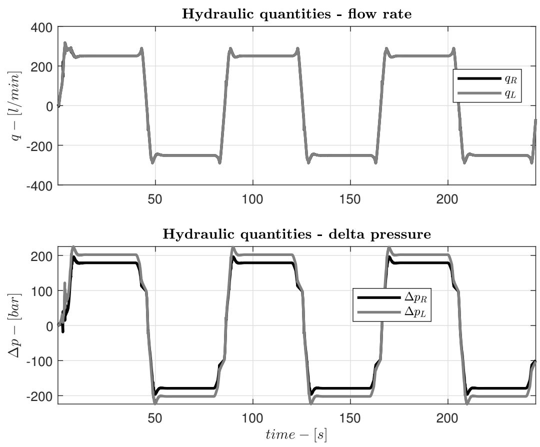


Figure 2.14: Description of the test scenario 2.

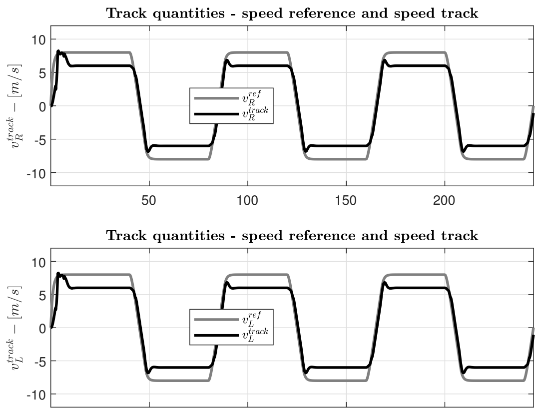


(a) Engine data.

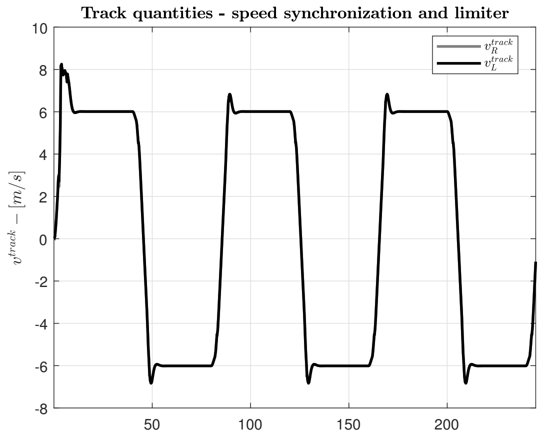


(b) Hydrostatic drive-line data.

Figure 2.15: Simulation results scenario 2.



(a) Speed tracks tracking performance.



(b) Speed tracks synchronization and maximum speed limiter.

Figure 2.16: Simulation results scenario 2.

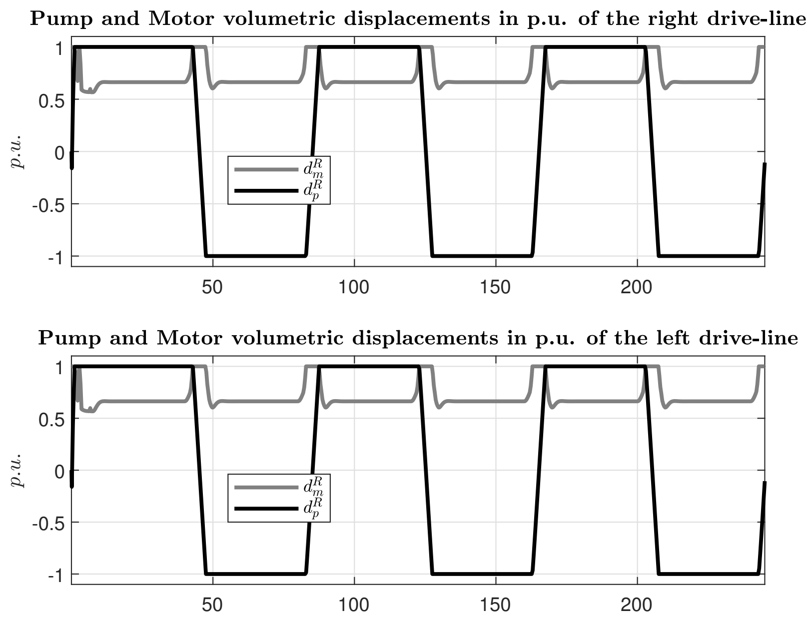


Figure 2.17: Volumetric displacements.

Scenario 3

In this scenario we are considering the following case

- Straight driving.
- Homogeneous negative load on both track (downhill scenario).
- The maximum negative load brings the vehicle in over-speed condition (over-speed management).

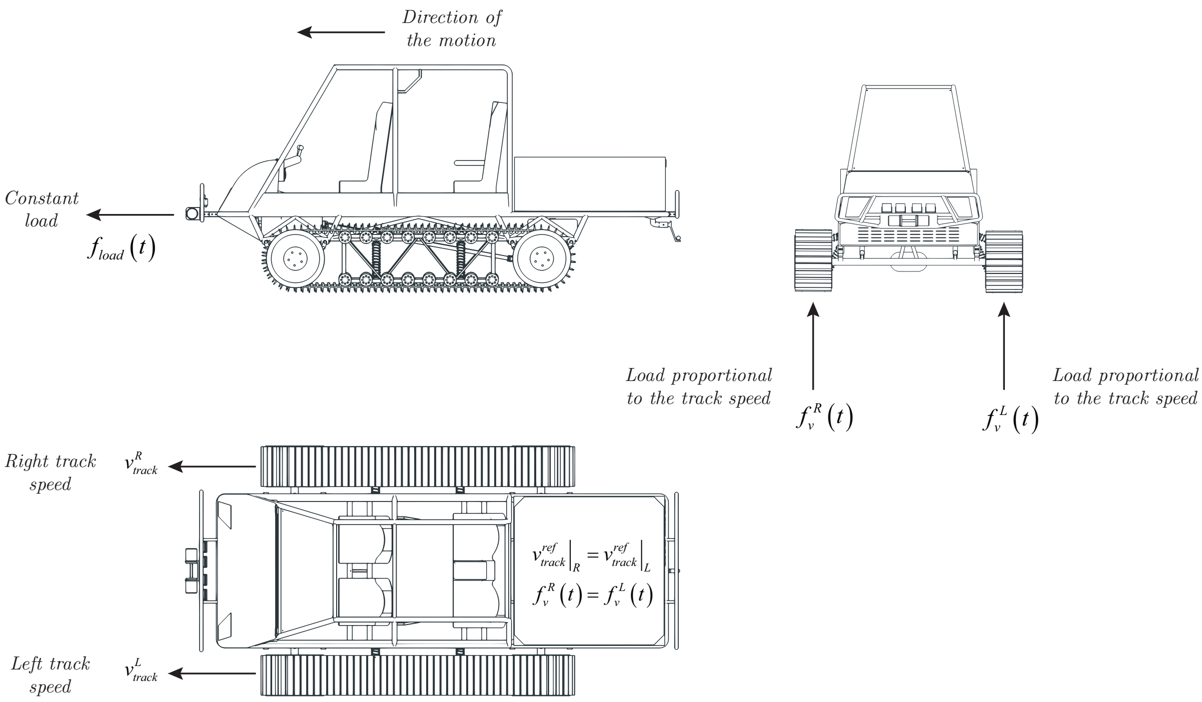
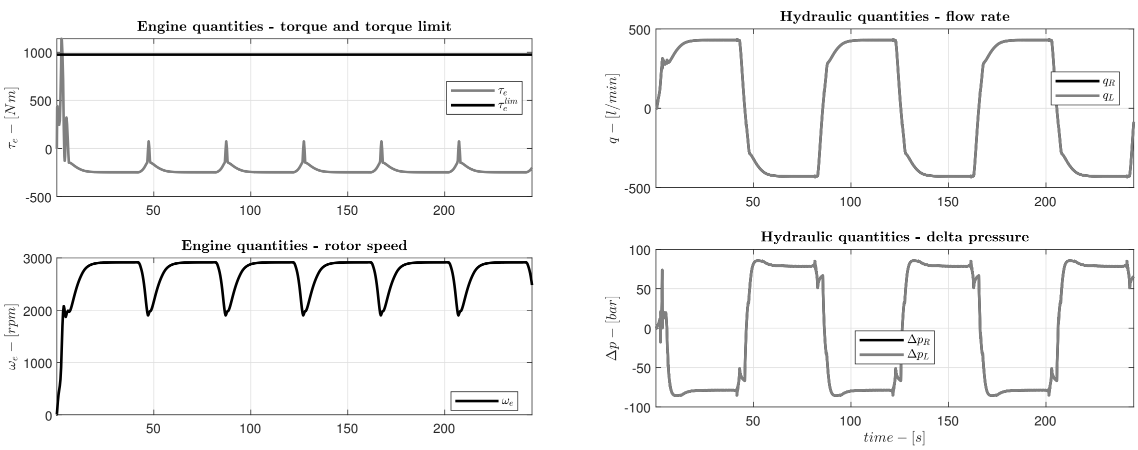


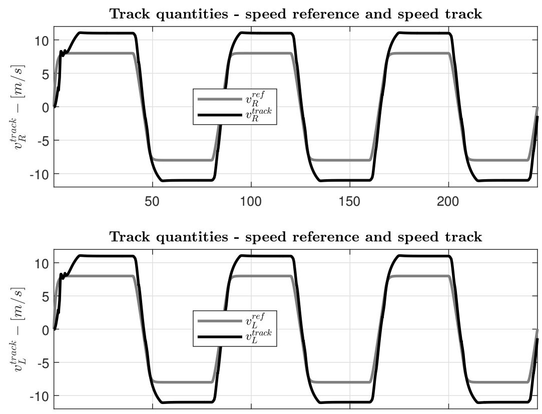
Figure 2.18: Description of the test scenario 3.



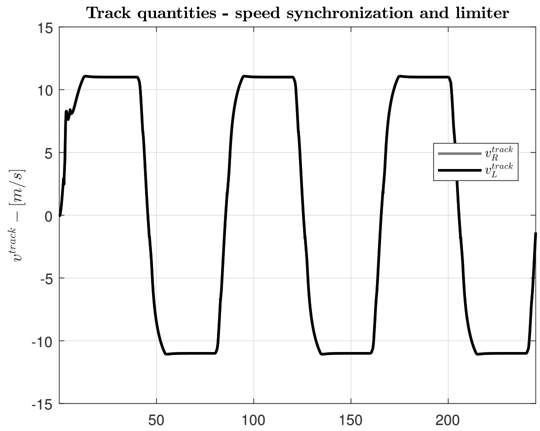
(a) Engine data.

(b) Hydrostatic drive-line data.

Figure 2.19: Simulation results scenario 3.



(a) Speed tracks tracking performance.



(b) Speed tracks synchronization and maximum speed limiter.

Figure 2.20: Simulation results scenario 3.

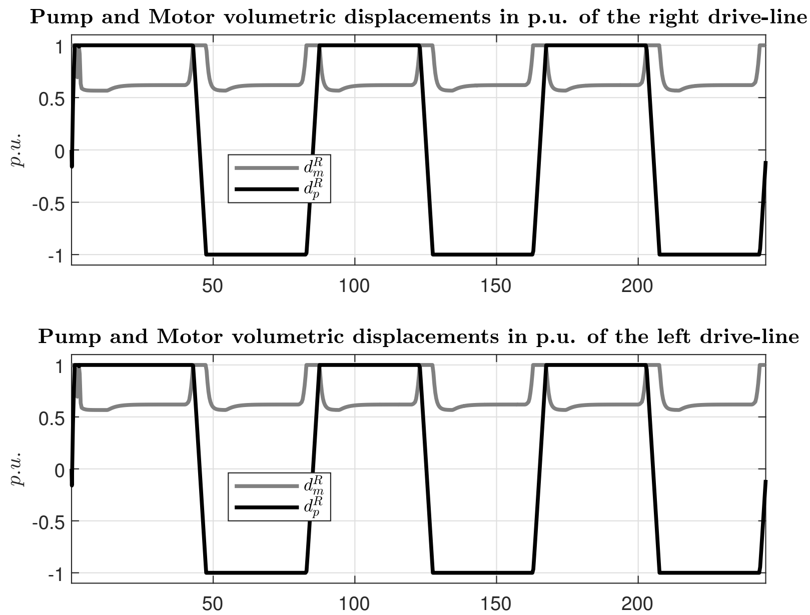


Figure 2.21: Volumetric displacements.

Scenario 4

In this scenario we are considering the following case

- Steering driving.
- Not homogeneous viscosity load on tracks.
- The total amount of load is enough to stall the engine.
- The maximum load on drive-line one (Right) exceed the maximum admissible delta pressure.

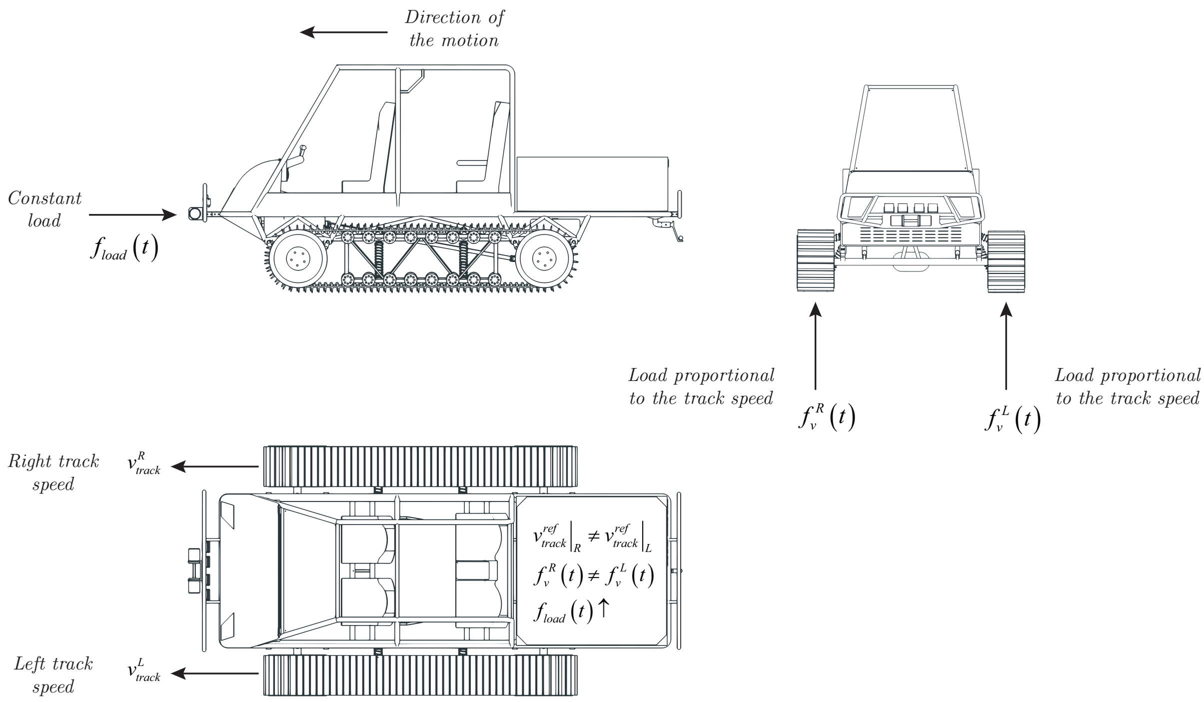
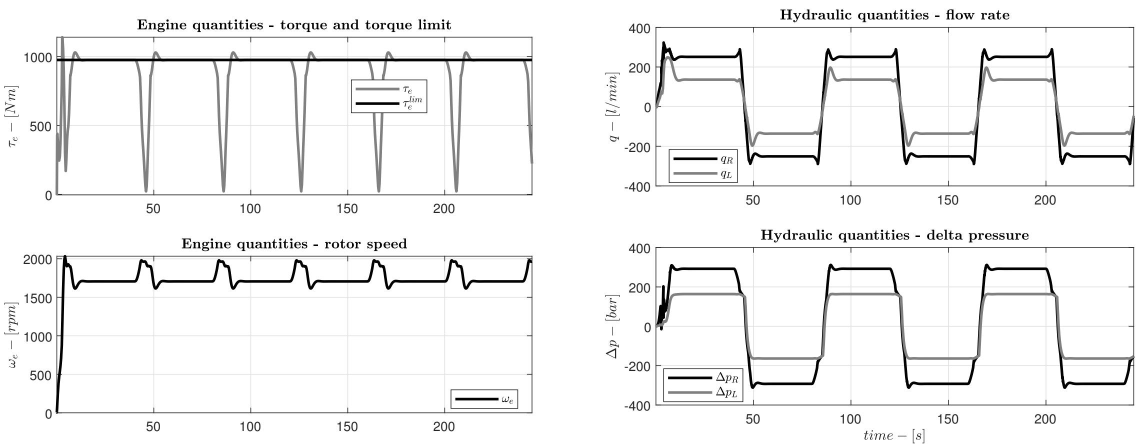


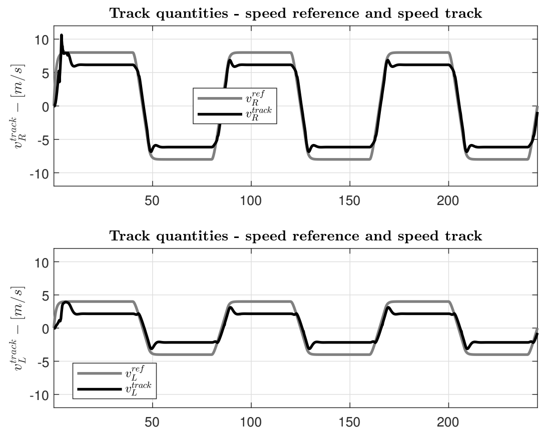
Figure 2.22: Description of the test scenario 4.



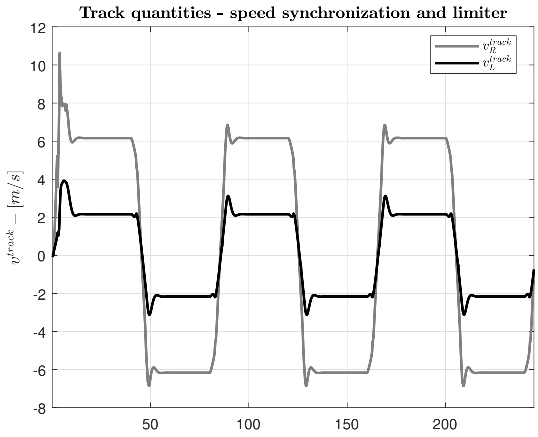
(a) Engine data.

(b) Hydrostatic drive-line data.

Figure 2.23: Simulation results scenario 3.



(a) Speed tracks tracking performance.



(b) Speed tracks synchronization and maximum speed limiter.

Figure 2.24: Simulation results scenario 3.

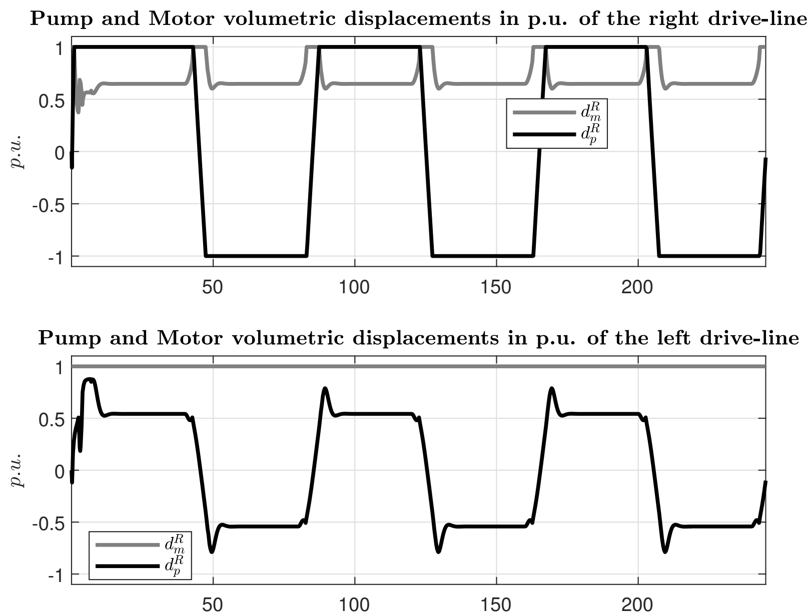


Figure 2.25: Volumetric displacements.

Chapter 3

Appendix - Mathematical models

3.1 Driveline mathematical model

The heavy duty vehicle power-train is, in general, performed by a diesel engine followed by an hydrostatic transmission, which is composed by a variable displacement hydraulic pump and by a variable (or fixed) displacement hydraulic motor. In the following, we will consider the case where both pump and motor are equipped with a adjustable displacements (which means variable volumetric flow).

The use of a both adjustable volumetric displacement pump and motor permits to extend the working operating area, in terms of speed and torque, of the whole transmission line. Basically, this kind of transmission is often identified as a continuous variable gear-ratio. Figure 3.1 depicts a typical hydrostatic power train, where the dynamic equations can be represented as follows

Section *R*.

$$\begin{cases} \dot{\Delta p}_R = \dfrac{1}{\beta}\Big[D_p^R\omega_p^R - D_m^R\omega_m^R - q_d^R\Big] \\ \dot{\omega}_m^R = \dfrac{1}{J_m}\Big[\tau_m^R - \tau_\vartheta^R - b_m\omega_m^R\Big] \qquad \text{where} \qquad \tau_m^R = D_m^R \Delta p_R \\ \dot{\omega}_l^R = \dfrac{1}{J_l}\Big[\tau_\vartheta^R n_1/n_2 - \tau_l^R - b_l\omega_l^R\Big] \\ \dot{\tau}_\vartheta^R = k_\vartheta\Big[\omega_m^R - \omega_l^R n_1/n_2\Big] \end{cases} \tag{3.1.1}$$

where $D_m^R = d_m^R V_m^{nom}$, $D_p^R = d_p^R V_p^{nom}$ and where V^{nom} means the nominal volumetric displacement of the pump or motor.

The term $D_p\omega_p$ represents the total flow at the output of the pump, while the term $D_m\omega_m$ represents the total flow at the input of the motor. The equation $\dot{\Delta p} = \frac{1}{\beta}\Big[D_p\omega_p - D_m\omega_m - q_d\Big]$ represents the continuity's law of the hydrostatic driveline.

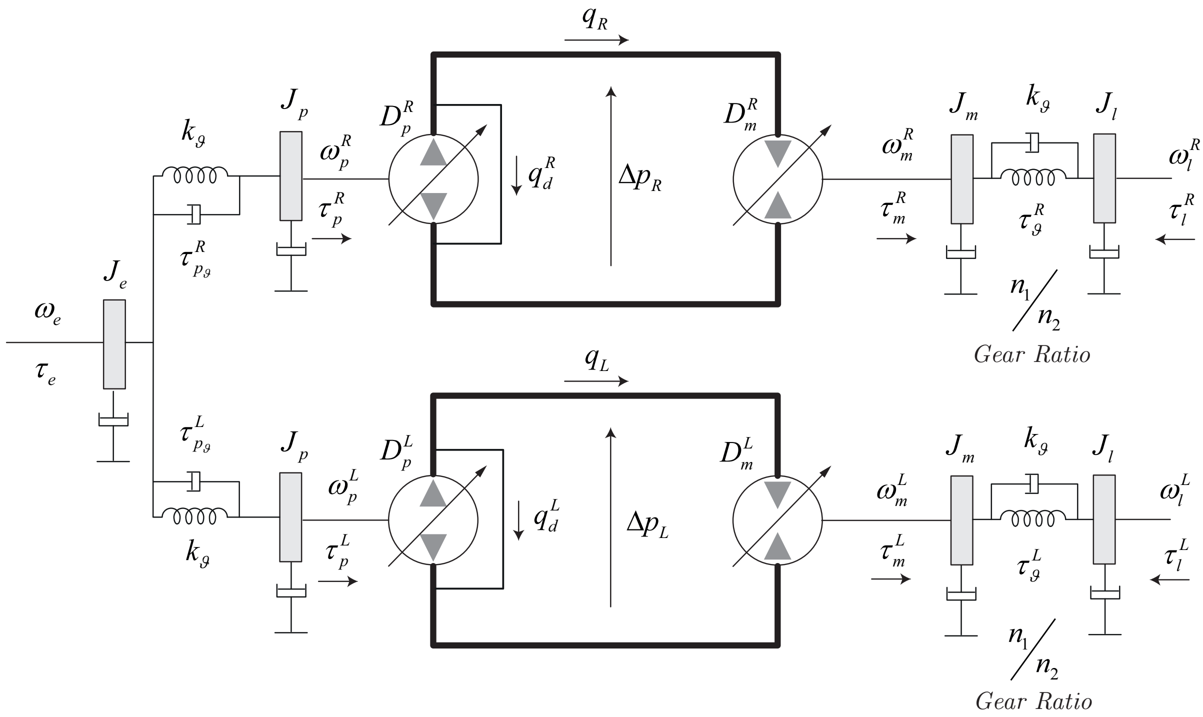


Figure 3.1: Hydrostatic transmission - the whole transmission power-train is made two, independent lines, one for the left side (L) and one for the right side (R) as follows.

Section *L*.

$$\begin{cases} \dot{\Delta p}_L = \dfrac{1}{\beta}\Big[D_p^L\omega_p^L - D_m^L\omega_m^L - q_d^L\Big] \\[2ex] \dot{\omega}_m^L = \dfrac{1}{J_m}\Big[\tau_m^L - \tau_\vartheta^L - b_m\omega_m^L\Big] \qquad \text{where} \qquad \tau_m^L = D_m^L\Delta p_L \\[2ex] \dot{\omega}_l^L = \dfrac{1}{J_l}\Big[\tau_\vartheta^L n_m/n_l - \tau_l^L - b_l\omega_l^L\Big] \\[2ex] \dot{\tau}_\vartheta^L = k_\vartheta\Big[\omega_m^L - \omega_l^L n_m/n_l\Big] \end{cases} \tag{3.1.2}$$

where $D_m^L = d_m^L V_m^{nom}$, $D_p^L = d_p^L V_p^{nom}$ and where V^{nom} means the nominal volumetric displacement of the pump or motor.

Equations of the source side (engine) can be written as follows

$$\begin{cases} \dot{\omega}_e = \dfrac{1}{J_e}\Big[\tau_e - \tau^R_{p_\vartheta} - \tau^L_{p_\vartheta} - b_p\omega_e\Big] \\ \dot{\omega}^R_p = \dfrac{1}{J_p}\Big[\tau^R_{p_\vartheta} - \tau^R_p - b_p\omega^R_p\Big] \\ \dot{\omega}^L_p = \dfrac{1}{J_p}\Big[\tau^L_{p_\vartheta} - \tau^L_p - b_p\omega^L_p\Big] \\ \dot{\tau}^R_{p_\vartheta} = k_\vartheta\Big[\omega_e - \omega^R_p\Big] \\ \dot{\tau}^L_{p_\vartheta} = k_\vartheta\Big[\omega_e - \omega^L_p\Big] \end{cases} \tag{3.1.3}$$

This final group of equations represent the Newton's law at the engine side,

3.1.1 Steady state equations

An important aspect, which has to be taken into account, is the evaluation of the hydrostatic transmission in steady state condition and the relations among torques and rotational speeds, as follows

$$\eta^v_p D_p\omega_p = \frac{D_m\omega_m}{\eta^v_p} \tag{3.1.4}$$

Eq. (3.1.4) represent the steady state flow balance between pump and motor. The terms η^v_p and η^v_m are the volumetric efficiency of the pump and motor respectively. Eq. (3.1.4) can be also represented as follows

$$\boxed{\omega_m = \left(\frac{D_p}{D_m}\right)\omega_p\left(\eta^v_p\eta^v_m\right)} \tag{3.1.5}$$

where eq. (3.1.5) shows that the hydraulic motor speed is proportional to the pump volumetric displacement and inversely proportional to the motor volumetric displacement.

The equation Eq. (3.1.6) represents a balance of power among pump and motor

$$\frac{\tau_m\omega_m}{\eta^m_m} = \eta^m_p\tau_p\omega_p \tag{3.1.6}$$

and can also be represented as shown in Eq. (3.1.7)

$$\boxed{\tau_p = \frac{1}{\eta^m_p\eta^m_m}\left(\frac{D_p}{D_m}\right)\tau_m} \tag{3.1.7}$$

where η^m_p and η^m_m are the mechanical efficiency of the pump and motor respectively, $D_m = V^{nom}_m d_m$ where

$$d_m \in \big[0.36,\ 1\big] = \big[d^{min}_m,\ d^{max}_m\big]$$

and $D_p = V^{nom}_p d_p$ where

$$d_p \in \big[-1,\ 1\big] = \big[-d^{max}_p,\ d^{max}_p\big].$$

The torque which the engine has to sustain (for steady-state condition) is proportional to the load and to the pump volumetric displacement and inversely proportional to the motor volumetric displacement.

3.2 Driver gear model

The hydraulic motor is connected to the track (undercarriages) via a gear device which is also called *drive-gear*, see also Figure 3.2.

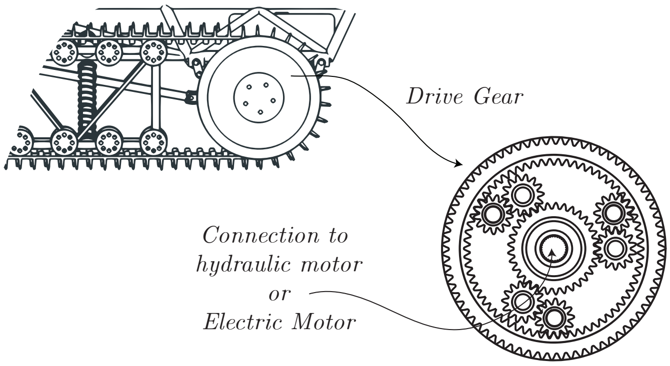


Figure 3.2: Drive-gear representation.

Along this document we suppose that the rotational low speed side has been measured for condition monitoring and control.

The mathematical model of the gear is here reported in a very simplified form:

$$\omega_l = \omega_m / n_{tg} \tag{3.2.1}$$

where ω_l is the low speed high torque side connected to the track and ω_m is the high speed low torque connected to the hydraulic motor. The term n_{tg} represents the ratio between high speed and low speed of the gear.

The planetary gear or *driver gear* is here modelled as a flexible shaft (as already reported),

$$\begin{cases} \dot{\omega}_m = \dfrac{1}{J_m}\left[\tau_m - \tau_\vartheta^m - b_m\omega_m\right] \\ \dot{\omega}_l = \dfrac{1}{J_l}\left[\tau_\vartheta^l - \tau_l - b_l\omega_l\right] \\ \dot{\tau}_\vartheta = k_\vartheta\left[\omega_m - \omega_l n_{tg}\right] \\ n_{tg} = \dfrac{n_m}{n_l} \\ \tau_\vartheta = \tau_\vartheta^m = \dfrac{\tau_\vartheta^l}{n_{tg}} \end{cases} \tag{3.2.2}$$

where τ_ϑ^m is the torsional torque seen at high speed gear side as well as τ_ϑ^l is the torsional torque seen at low speed gear side, see also Figure 3.3.

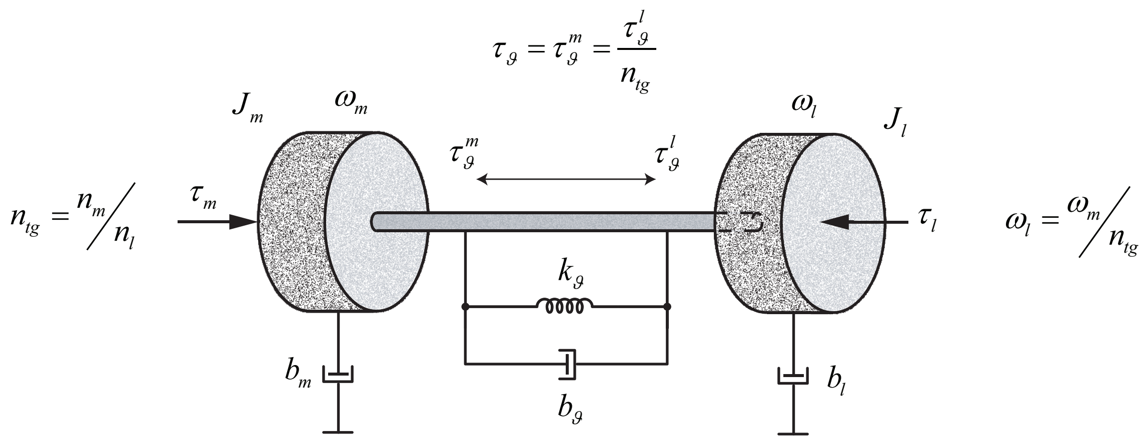


Figure 3.3: Drive-gear representation as flexible shaft model.

A practical example can be as follows.

Let $n_{tg} = 41.4$ be the gear ratio of the *drive-gear* and let $R = 0.440\,56\,\mathrm{m}$ be its radius. Considering a maximum (nominal) vehicle speed of $v_{tr} = 11\,\mathrm{km\,h^{-1}}$ we obtain a maximum (nominal) hydraulic motor speed of $\omega_m^{nom} = 2742\,\mathrm{min^{-1}}$, see also Figure 3.4.

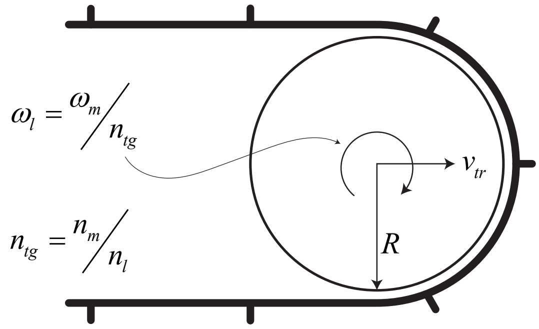


Figure 3.4: Track representation.

3.3 Pump and motor volumetric displacement actuator

Hydrostatic pump is a device which converts a mechanical power into a hydraulic power. We assume that the pressure in each chamber is everywhere the same and does not saturate or cavitate, fluid velocities in the chambers are small, temperature and density are constant, see also Figure 3.5. Application of the continuity equation to each chamber yields

$$\begin{cases} \text{Chamber 1: } \rho_1 q_1 = \dfrac{d(V_1\rho_1)}{dt} = \rho_1 \dfrac{dV_1}{dt} + V_1 \dfrac{d\rho_1}{dt} \\ \text{Chamber 2: } -\rho_2 q_2 = \dfrac{d(V_2\rho_2)}{dt} = \rho_2 \dfrac{dV_2}{dt} + V_2 \dfrac{d\rho_2}{dt} \end{cases} \tag{3.3.1}$$

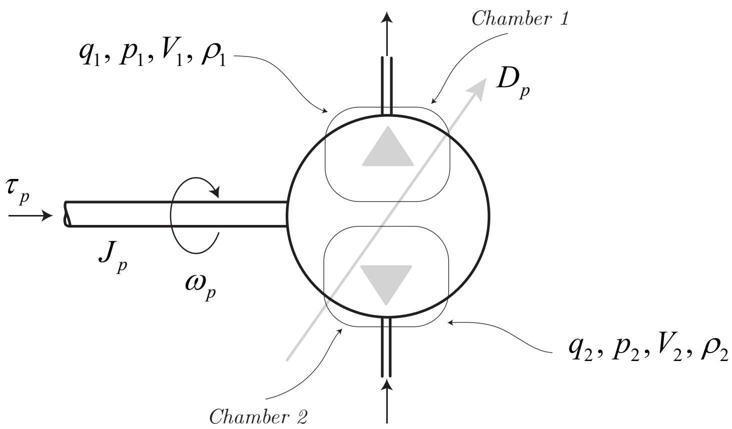


Figure 3.5: Hydrostatic pump model for analysis.

The volume in each pump chamber is not constant but varies in a discontinuous saw-toothed fashion with shaft rotation, as illustrated in Figure (3.6). This is a characteristic of all types of pumps and motors and may be deduced from an examination of the chamber. The chamber volume changes at each instants because the piston completing its power stroke has accumulated a small head volume, and it is replaced by a piston starting a power stroke with a large head volume. Hence there is an instantaneous change in volume equal to the area of one piston times its stroke. These discontinuities occur each time a piston undergoes transition from low pressure chamber to high pressure chamber so that the frequency is equal to the number of pistons times the motor speed. Hence we have now identical change in volume: one in the low pressure chamber and one in the high pressure chamber which are simultaneous. The two chamber volumes may be expressed by

$$V_1 = V_0 + f_v(\vartheta_p) \tag{3.3.2}$$

$$V_2 = V_0 - f_v(\vartheta_p) \tag{3.3.3}$$

where

- V_0 is the average contained volume of each chamber (include case and pipes) $\left[\mathrm{m}^3\right]$,
- $f_v(\vartheta_p)$ is the saw-toothed variation in each chamber $\left[\mathrm{m}^3\right]$,
- ϑ_p is the angular position of the pump shaft $\left[\mathrm{rad}\right]$,
- D_p is the volumetric displacement of the pump $\left[\mathrm{m}^3\,\mathrm{rad}^{-1}\right]$

It should be apparent that the volume of a chamber must depend on shaft rotation. Because there is no direct connection between the chambers, a continuous flow through the pump is achieved on if the volume on one chamber steadily increase while the other chamber volume decreases with shaft position. This is physically possible because a piston cylinder is filled with fluid, and the trapped fluid is then transported to the other chamber where it empties. A succession of such pistons forms a sort of *Archimedes' screw* which permits continuous flow. The rate at which the volume increase (or decrease) with shaft position is therefore the pump displacement. Hence the time derivatives of Eq. (3.3.2) and Eq. (3.3.3) are given by

$$\begin{cases} \dfrac{dV_1}{dt} = \dfrac{df_v(\vartheta_p)}{dt} = D_p\omega_p \\ \dfrac{dV_2}{dt} = -\dfrac{df_v(\vartheta_p)}{dt} = -D_p\omega_p \\ \dfrac{d\vartheta_p}{dt} = \omega_p \end{cases} \tag{3.3.4}$$

and represent the theoretical flow to and from the pump (motor). respectively. Eq. (3.3.4) overlooks the discontinuities in Figure 3.6 and assumes constant pump displacement. Adding Eq. (3.3.2) and Eq. (3.3.3), we obtain

$$V_t = V_1 + V_2 = 2V_0 \tag{3.3.5}$$

V_t is the total volume of oil under pressure p_1 and p_2 and therefore is often called the *total compressed volume*.

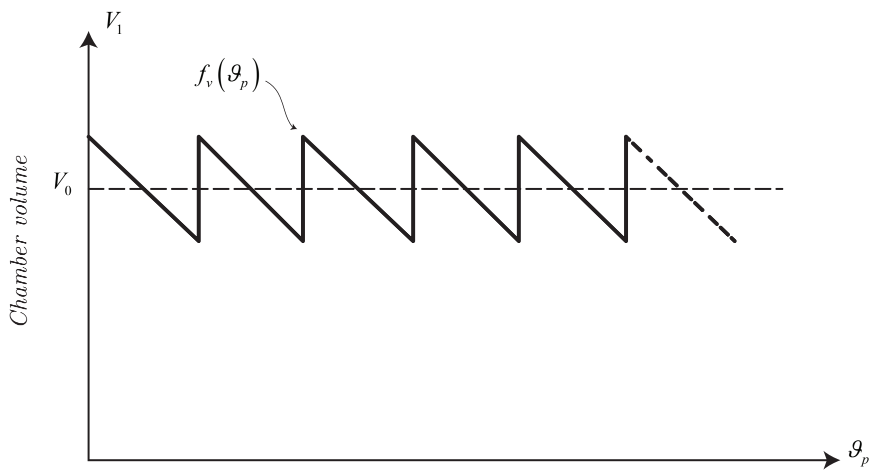


Figure 3.6: Volume of a pump chamber V_1 versus the shaft angle ϑ_p.

Applying the relation of Eq. (3.3.6) to the system of Eq. (3.3.1)

$$\frac{d\rho}{dt} = \frac{d}{dt}\frac{1}{\nu} = -\frac{1}{\nu^2}\frac{d\nu}{dp}\frac{dp}{dt} = -\frac{1}{\nu^2}\Big(-\frac{\nu}{\beta_E}\Big)\frac{dp}{dt} = \Big(\frac{\rho}{\beta_E}\Big)\frac{dp}{dt} \tag{3.3.6}$$

where $\rho = \nu^{-1}$.

Eq. (3.3.1) can be written as follows

$$\begin{cases} q_1 = \dfrac{dV_1}{dt} + \dfrac{V_1}{\beta_E}\dfrac{dp_1}{dt} \\ -q_2 = \dfrac{dV_2}{dt} + \dfrac{V_2}{\beta_E}\dfrac{dp_2}{dt} \end{cases} \tag{3.3.7}$$

Applying the assumption of Eq. (3.3.2) and Eq. (3.3.3), the equation (3.3.7) can be written as follows

$$\begin{cases} q_1 = \dfrac{dV_1}{dt} + \dfrac{V_1}{\beta_E}\dfrac{dp_1}{dt} = D_p\omega_p + \dfrac{V_0 + f_v(\vartheta_p)}{\beta_E}\dfrac{dp_1}{dt} \\ -q_2 = \dfrac{dV_2}{dt} + \dfrac{V_2}{\beta_E}\dfrac{dp_2}{dt} = -D_p\omega_p + \dfrac{V_0 - f_v(\vartheta_p)}{\beta_E}\dfrac{dp_2}{dt} \end{cases} \tag{3.3.8}$$

Now we calculate the quantity $q_{pt} = \frac{1}{2}\Big(q_1 + q_2\Big)$ resulting in the following equation

$$q_{pt} = D_p\omega_p + \frac{V_0}{2\beta_E}\frac{d}{dt}\Big(p_1 - p_2\Big) + \frac{f_v(\vartheta_p)}{2\beta_E}\Big(\frac{dp_1}{dt} + \frac{dp_2}{dt}\Big) \approx D_p\omega_p + \frac{V_0}{2\beta_E}\frac{d}{dt}\Big(p_1 - p_2\Big) \qquad (3.3.9)$$

where, due to symmetry of the system, it is assumed that

$$\Big(\frac{dp_1}{dt} + \frac{dp_2}{dt}\Big) \approx 0 \qquad (3.3.10)$$

setting $\Delta p = p_1 - p_2$ the continuity equation of the pump written as follows

$$q_{pt} = D_p\omega_p + \frac{V_0}{2\beta_E}\frac{d}{dt}\Delta p \qquad (3.3.11)$$

The final fundamental equation for the pump model is the torque balance which can be expressed as follows

$$J_p\frac{d}{dt}\omega_p = \tau_p - \Delta p D_p - b_p\omega_p \qquad (3.3.12)$$

where b_b is a viscosity coefficient. In steady state condition Eq. (3.3.12) will reduce to

$$\tau_p = D_p\Delta p + b_p\omega_p \qquad (3.3.13)$$

For more general case we can now consider the system in Figure 3.7

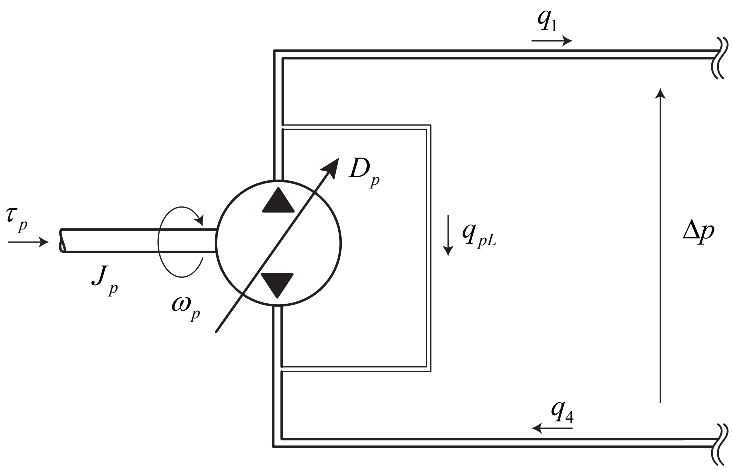


Figure 3.7: Hydrostatic pump with leakage.

where the term q_{pL} represents an internal flow leakage. The mathematical model of the system depicted in Figure 3.7 can be represented as follows

Mathematical model of the hydrostatic pump

$$\begin{cases} \dfrac{V_0}{2\beta_E}\dfrac{d}{dt}\Delta p = \omega_p D_p - q - q_{pL} \\ \\ J_p \dfrac{d}{dt}\omega_p = \tau_p - \Delta p D_p - b_p \omega_p \end{cases} \tag{3.3.14}$$

where

$$q_{pL} = (l_k)^2 \sqrt{2\rho}\sqrt{|\Delta_p|}\text{sign}(\Delta_p) \tag{3.3.15}$$

and $q = q_1 = q_4$. The term l_k is the equivalent leakage orifice correlated to the volumetric efficiency of the pump.

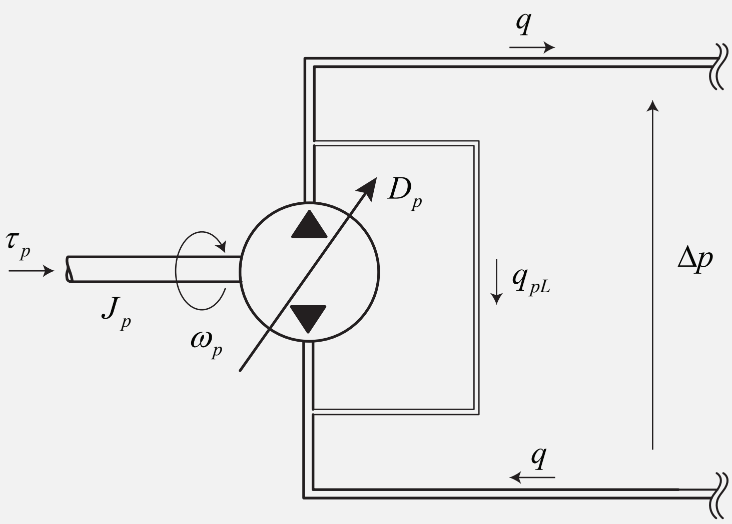


Figure 3.8: Hydrostatic pump with leakage.

3.4 Diesel engine model

In this document we describe the Liebherr diesel engine D934 represented in the Digital Twin Library. The model is based on the main dynamical equation of the crank-shaft and doesn't take into account the internal combustion thermodynamic. Torque curve as well as consumption contour map is taken from supplier and implemented as lookup table.

3.4.1 Technical data

The following technical data has been taken into account during the modelization of the engine.

Rated Power	175 kW
at engine speed	1600 min^{-1}
Torque max	1250 N m
at engine speed	1300 min^{-1}
Engine speed	
low idle speed, standard	600 min^{-1}
maximum no load speed, max	2140 min^{-1}
nominal engine speed	1900 min^{-1}
Fuel consumption	
at rated power	196 g/kWh
at full load, min	192 g/kWh
at characteristic map best point	191 g/kWh

Table 3.1: Liebherr diesel engine D934 data.

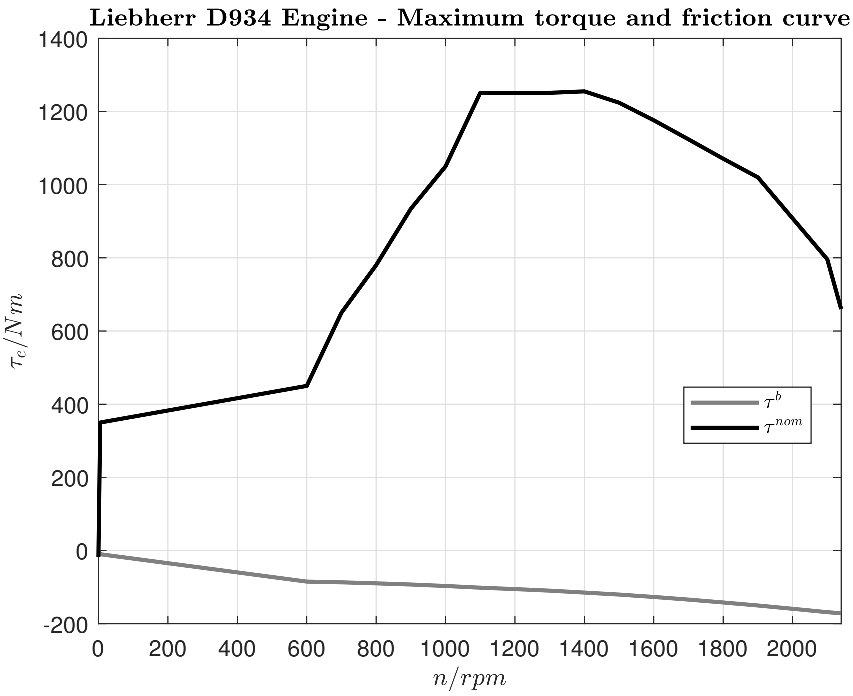


Figure 3.9: Liebherr diesel engine D934 - maximum torque and friction curve.

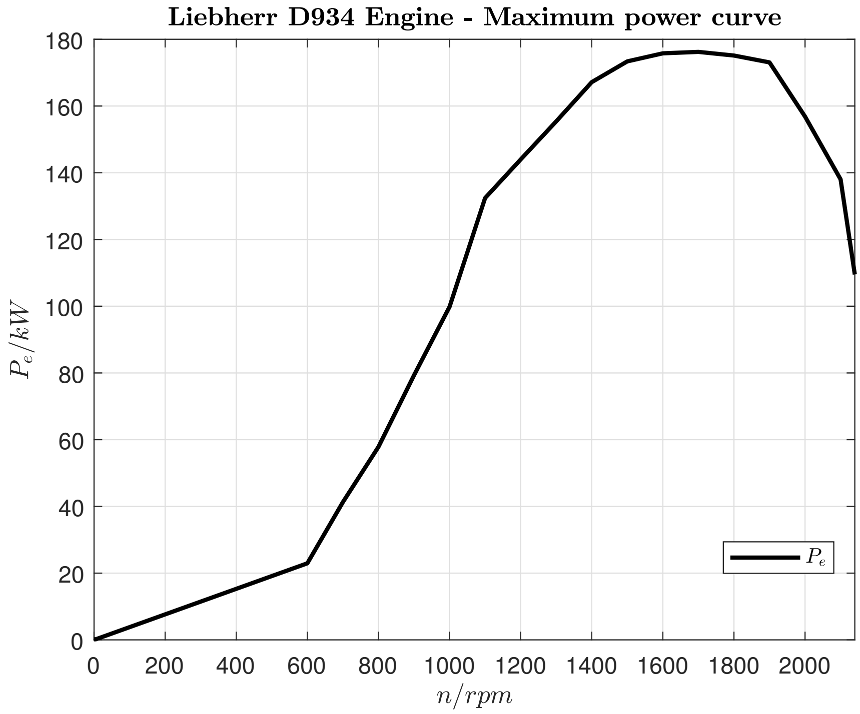


Figure 3.10: Liebherr diesel engine D934 - maximum available power at the crankshaft.

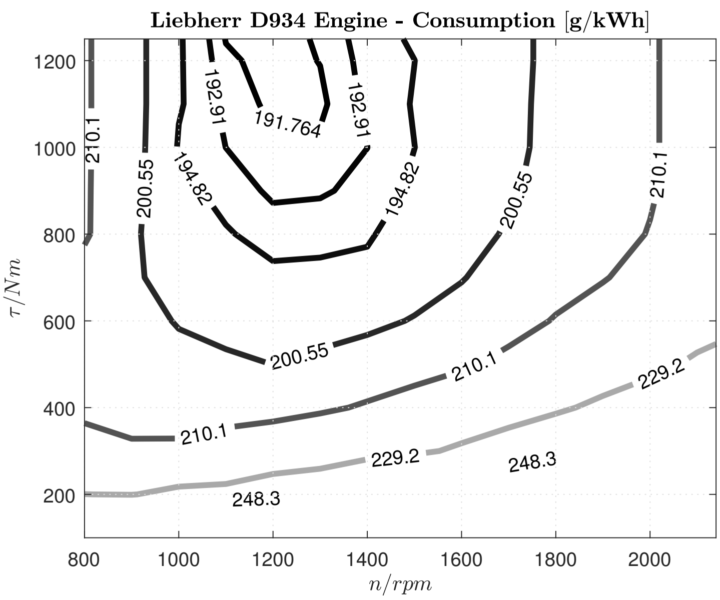


Figure 3.11: Liebherr diesel engine D934 - Specific consumption map in [g/kWh].

In the next figures the torque step responses at different rotor speed have been shown. Figure 3.12 shows the torque step response of the engine at rotor speed of $n = 1400\,\mathrm{min}^{-1}$

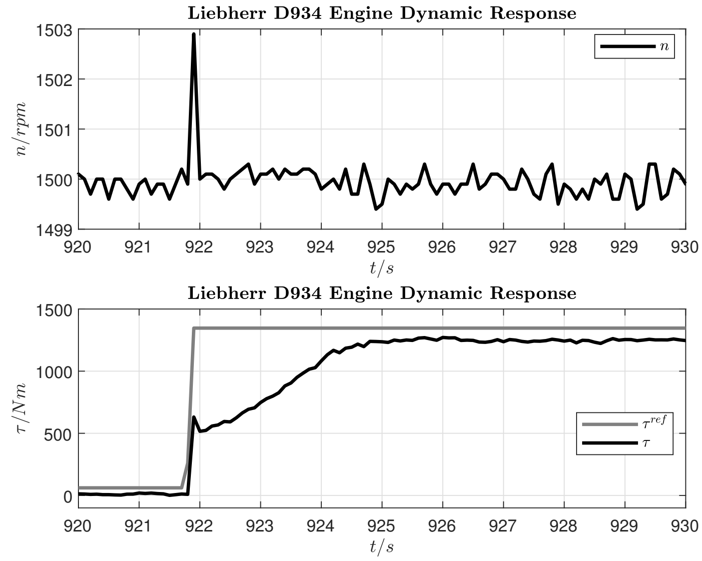


Figure 3.12: Liebherr diesel engine D934 - step response.

Figure 3.13 shows the torque step response of the engine at rotor speed of $n = 1800\,\mathrm{min}^{-1}$

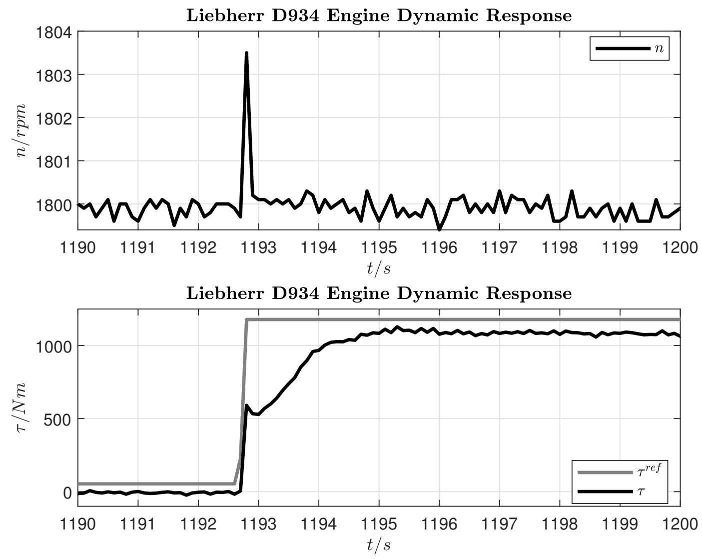


Figure 3.13: Liebherr diesel engine D934 - step response.

Figure 3.14 shows the torque step response of the engine at rotor speed of $n = 2100\,\mathrm{min}^{-1}$

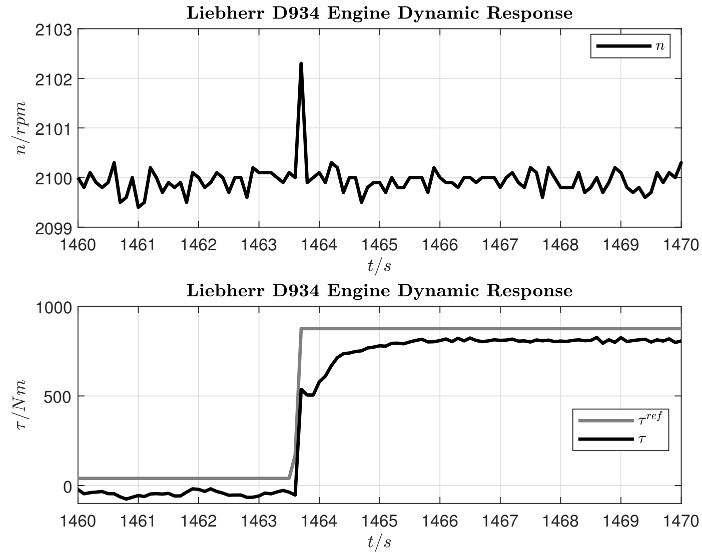


Figure 3.14: Liebherr diesel engine D934 - step response.

In the next section the equivalent mathematical model of the Liebherr D934 diesel engine will be proposed.

3.4.2 Equivalent mathematical model

In this section the description of the IC-engine is reported. The knowledge of the torque and power curve of the engine plays an important role in the performance of the engine anti-stall control.

In the following, we describe how the IC-engine has been modelized. We start to consider two curves:

- Torque curve: maximum torque available for a given rotor speed and maximum displacement of the throttle.
- Torque friction curve: torque generated by the friction for a given rotor speed. This friction is considered always present, for any value of the throttle. See also Figure 3.9

The mechanical model of the engine can be described as follows

$$J\frac{d\omega_e}{dt} = \theta_f(t)\tau^{nom}(\omega_e) + \tau^b(\omega_e) - \tau_{load} - |\tau_{ebs}| \tag{3.4.1}$$

where $\tau^{nom}(\omega_e)$, $\tau^b(\omega_e)$ are shown in Figure 3.9. $\theta_f(t)$ is the throttle which is generated by the external speed loop control as shown in Figure 3.15 and $J = 7.5\,\mathrm{kg\,m^{-2}}$.

The speed control is performed by a PI-controller and it adapts the throttle displacement in order to keep the request speed tracked.

An additional second order filter is taken into account in order to modelize additional dynamics.

The controller can be described as follows

$$\begin{cases} \tilde{\omega}_e(t) = \dfrac{1}{\omega_e^{\text{nom}}}\left(\omega_e^{\text{ref}} - \omega_e(t)\right) \\ \theta(t) = k_p\,\tilde{\omega}_e(t) + \theta^i(t) \\ \dfrac{d\theta^i}{dt}(t) = k_i\,\tilde{\omega}_e(t) \end{cases} \tag{3.4.2}$$

The control output θ is passed through a second order filter:

$$\theta_f(s) = \frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}\theta(s) \tag{3.4.3}$$

where $\zeta = 1$ and $\omega_0 = 2\pi 25\,\text{Hz}$.

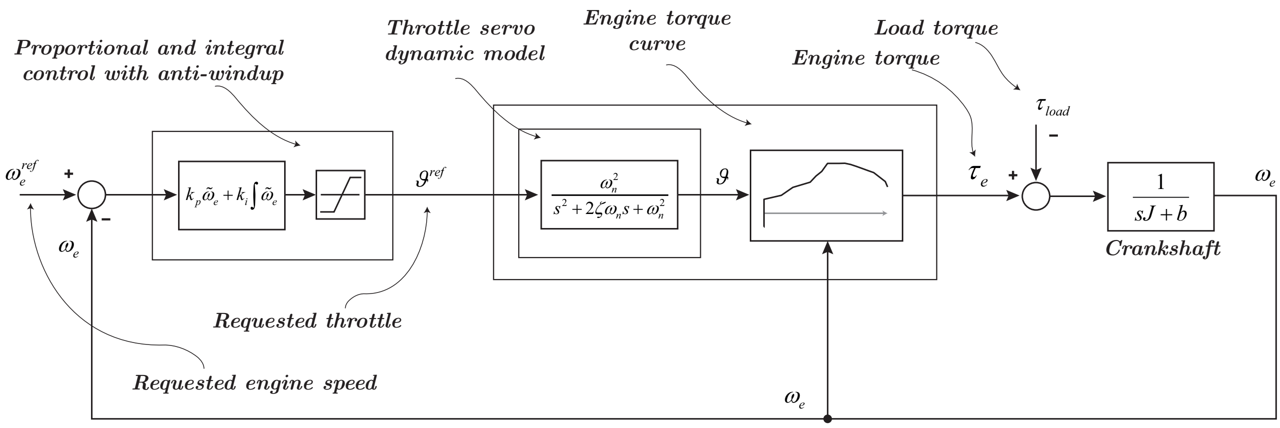


Figure 3.15: Engine model and control architecture.

3.4.3 Model parameters settings

Parameters setting consist of two parts

- Engine parameters
- Control parameters

Engine model parameters consist of the following data

- **Inertia**: This parameter sets the total inertia which is comprehensive of the crank-shaft and (if included) the wheel.

$$J \text{ in } \left[\text{kg}\,\text{m}^2\right]$$

- $\boldsymbol{\omega_0}$: This parameter sets the initial engine rotation speed

$$\omega_0 \text{ in } \left[\text{min}^{-1}\right]$$

Control parameters consist of the parameters which affect the PID speed control

- k_p This parameter sets the proportional gain of the PID control.
- k_i This parameter sets the integral gain of the PID control.
- k_d This parameter sets the derivative gain of the PID control (typically set to zero).

3.4.4 Model output variables

The output bus channel **ENGINE data** includes the following data output

- P_e: Engine power (power correlated to the consumption) $\left[\mathrm{kW}\right]$.
- ω_e: Engine speed $\left[\mathrm{min}^{-1}\right]$.
- τ_e: Engine torque $\left[\mathrm{N\,m}\right]$.
- fc^s: Specific fuel consumption $\left[\mathrm{g/kWh}\right]$.
- fc: Fuel consumption $\left[\mathrm{g}\right]$.
- ϑ: throttle in %.

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