Modelization and control of a front-blade for a heavy duty vehicle

Davide Bagnara

May 18, 2021

Contents

List of Figures

List of Tables

1 Introduction

The following document shows a possible modelization of a front-blade (also called front-shield) for a heavy duty vehicle. At first, we assume a completely decoupling of the servo actuators among the three rotational axes: the rotation of the blade along one axis doesn't affect the position of the blade along the other axes.

Two axes are governed by two parallelized servo-piston actuators as shown in Figure 1 - Figure 3.

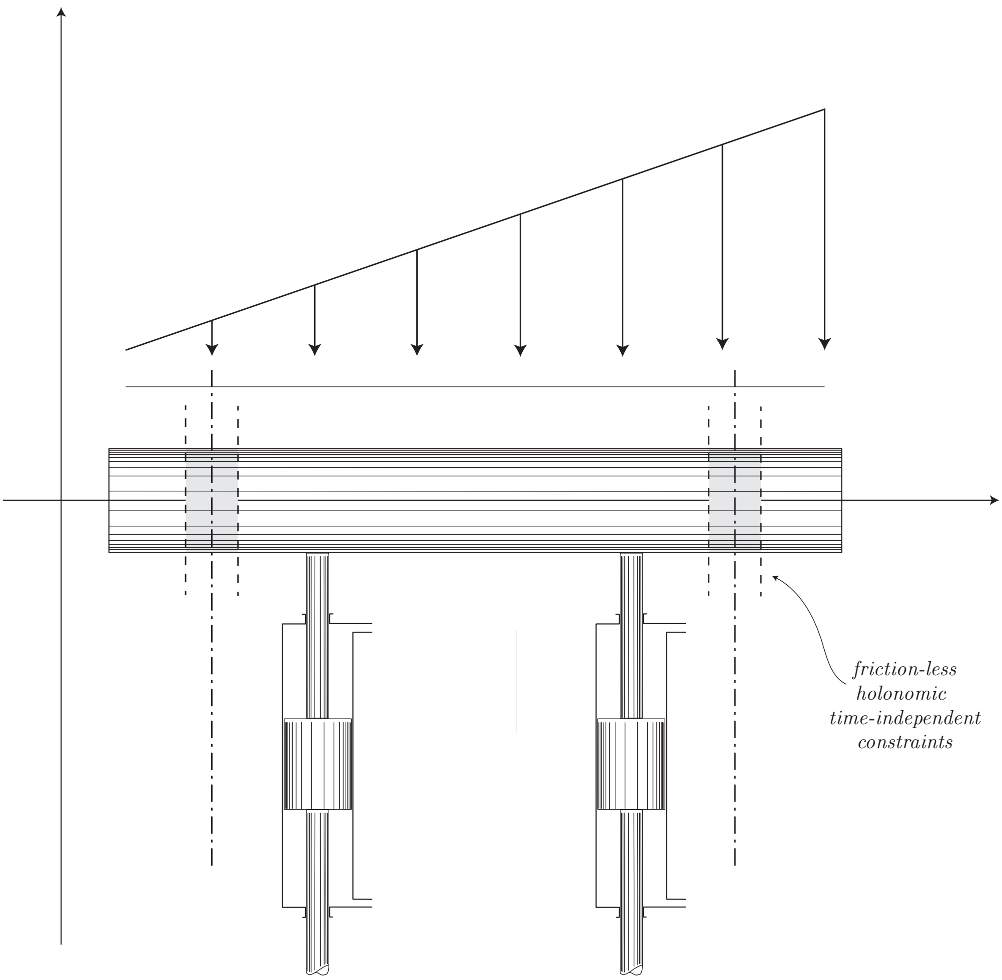

Figure 1: Lift system.

Figure 1 show the presence of the friction-less holomonic time-independent constraints which permit to share equally among the two pistons any unsymmetrical load, see Figure 1 - Figure 2.

The pitch axis can be approximated by two servo-pistons which move across a pivot. This configuration force the speed of the piston to be in opposite direction and identical magnitude.

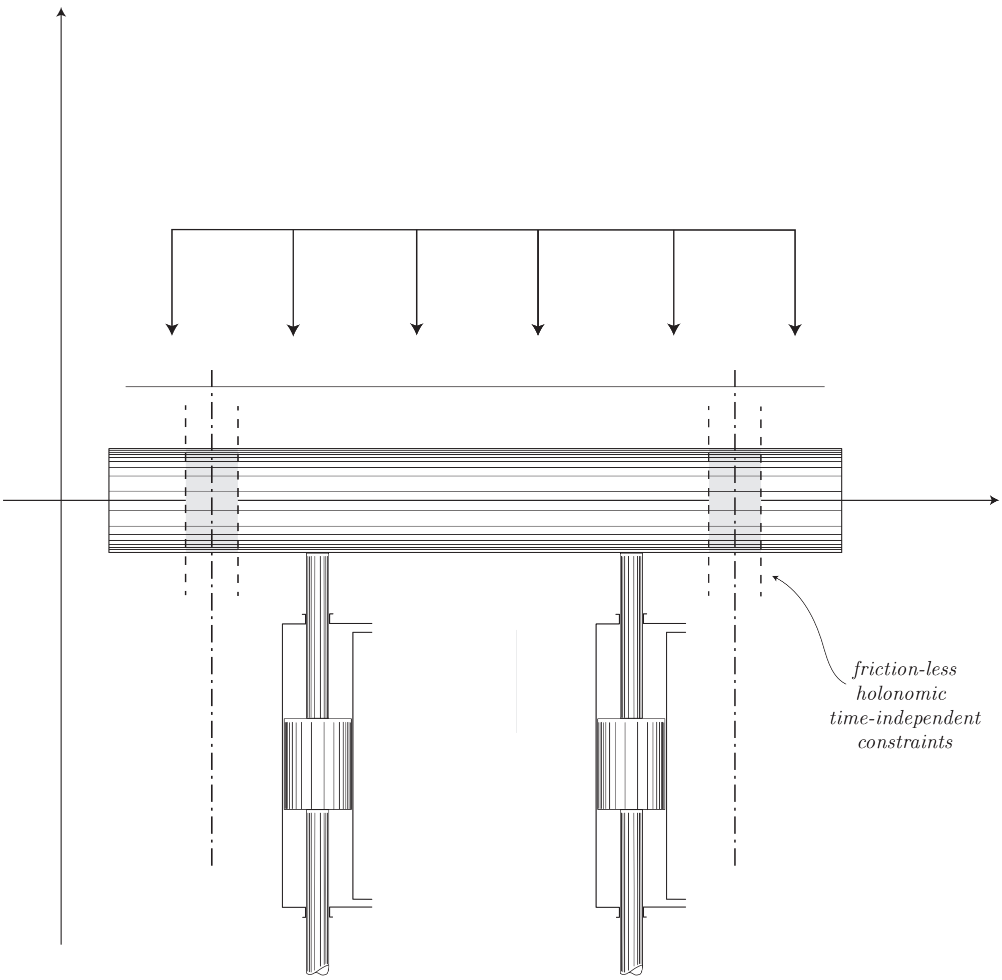

Figure 2: Lift system.

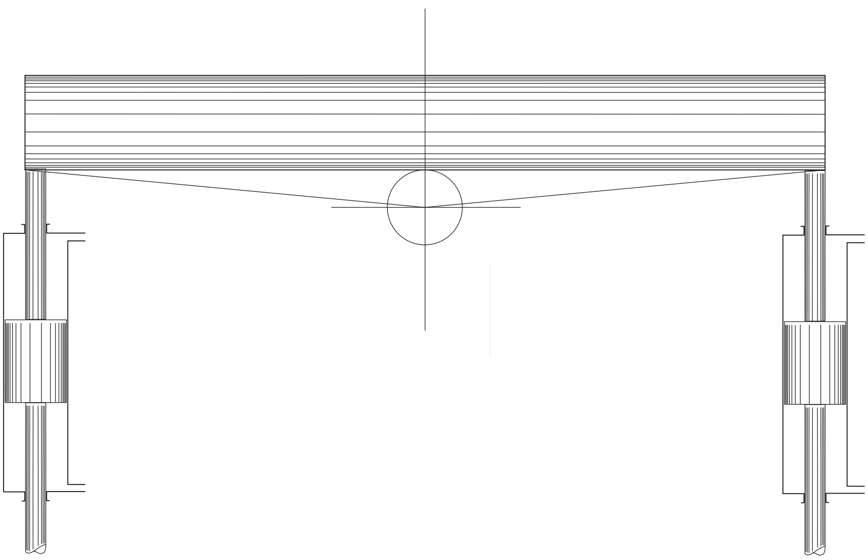

Figure 3: Pitch system.

2 Lift system

2.1 Model description

The lift system has been modelized as two servo-piston actuators where each double-chamber is connected to a *four-way four-land spool valve* as shown in Figure 4. The control is implemented as a *position control* in order to emulate the operator-functionality which works as a position controller. One valve-piston system is governed by a closed loop controller while the second one is governed in open loop control, see also Figure 7.

The whole valve-piston system is shown in Figure 5.

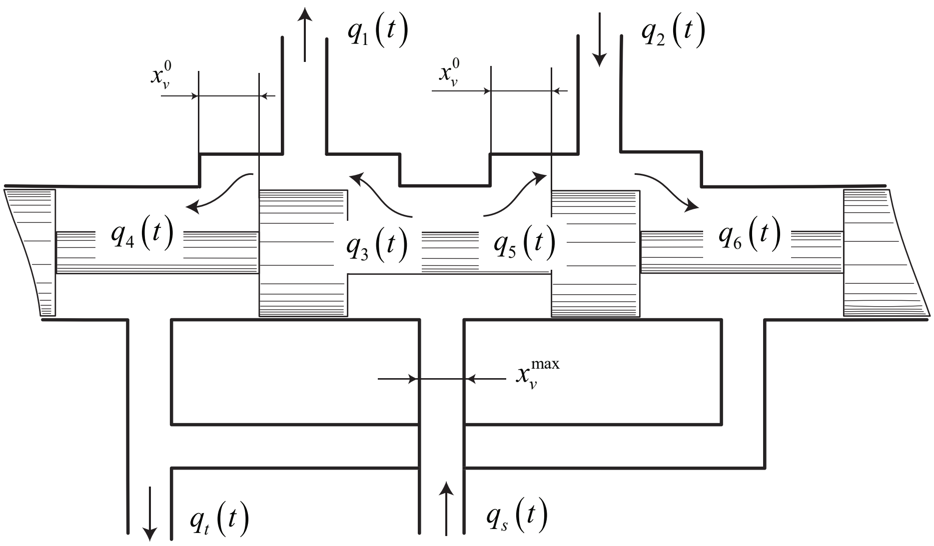

Figure 4: Underlapped configuration of the four-way valve.

Consider the four-way spool shown in Figure 4. When the valve is centered, the underlap of the supply and return ports are identical with a value of x_v^0. It is assumed the valve is matched and symmetrical and that its operation remains within the underlap region, that is $|x_v| \leq x_v^0$. The orifice areas therefore are

$$A_4 = x_v^{\max}\left(x_v^0 + x_v\right) = A_5 \tag{2.1}$$

$$A_3 = x_v^{\max}\left(x_v^0 - x_v\right) = A_6 \tag{2.2}$$

The continuity conditions relays in the following constraints

$$q_1 = q_3 - q_4 \tag{2.3}$$

$$q_2 = q_6 - q_5 \tag{2.4}$$

$$q_s = q_3 + q_5 \tag{2.5}$$

$$q_t = q_4 + q_6 \tag{2.6}$$

Now we consider the case where the open center spool drivers a piston as shown in Figure 5

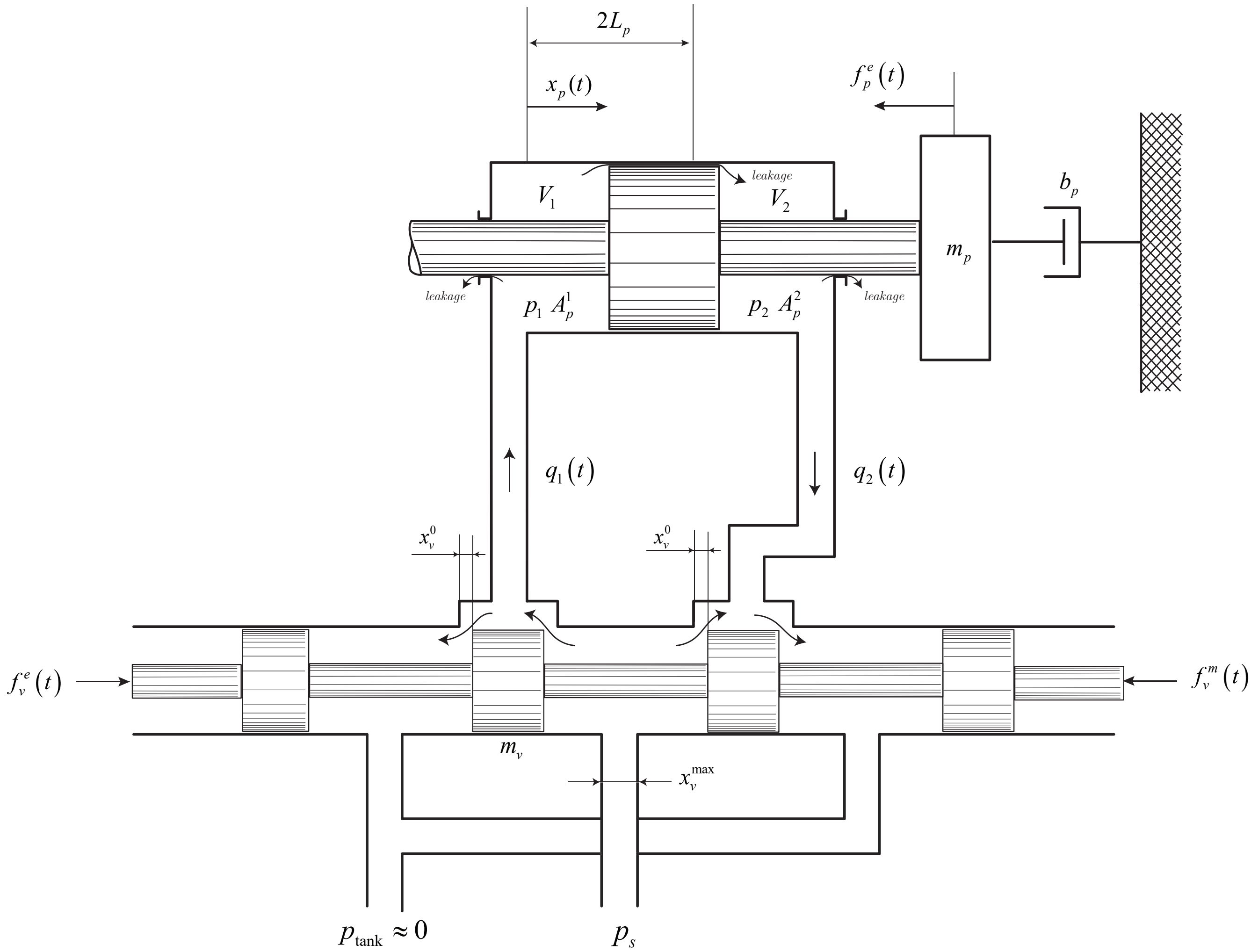

Figure 5: Four-way four-land valve bi-chamber piston system.

The dynamical equations become as follows:
Case - 1

$$\begin{cases} x_v > x_v^0 \\ q_3 = (x_v^0 + x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_s - p_1} \\ q_4 = 0 \\ q_5 = 0 \\ q_6 = (x_v^0 + x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_2 - p_t} \end{cases} \tag{2.7}$$

Case - 2

$$\begin{cases} x_v < -x_v^0 \\ q_3 = 0 \\ q_4 = (x_v^0 - x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_1 - p_t} \\ q_5 = (x_v^0 - x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_s - p_2} \\ q_6 = 0 \end{cases} \tag{2.8}$$

Case - 3

$$\begin{cases} x_v^0 \le x_v \le -x_v^0 \\ q_3 = (x_v^0 + x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_s - p_1} \\ q_4 = (x_v^0 - x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_1 - p_t} \\ q_5 = (x_v^0 - x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_s - p_2} \\ q_6 = (x_v^0 + x_v)x_v^{\max}\sqrt{\dfrac{2}{\rho}}\sqrt{p_2 - p_t} \end{cases} \tag{2.9}$$

where the dynamical equation of the piston become

$$\begin{cases} \dfrac{d}{dt}p_1 = \beta\left[\dfrac{q_1 - A_p^1 v_p}{V_0 + A_p^1 x_p}\right] \\ \dfrac{d}{dt}p_2 = \beta\left[\dfrac{-q_2 + A_p^2 v_p}{V_0 - A_p^2 x_p}\right] \\ m_p\dfrac{dv_p}{dt} = \left(A_p^1 p_1 - A_p^2 p_2\right) - b_p v_p - f_p \\ \dfrac{dx_p}{dt} = v_p \end{cases} \tag{2.10}$$

2.2 Control layout

The plant consisting of the valve and the double chamber piston can be modelized as simple single integrator, as follows

$$H_1(s) = \frac{X_p(s)}{X_v(s)} = k_1\frac{1}{s} \tag{2.11}$$

Due to the fact that the plant contains an integrator, a simple proportional control can be adopted, see Figure 6

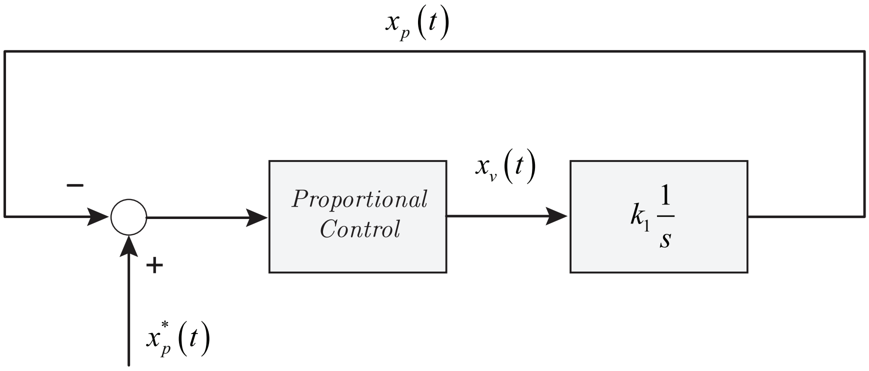

Figure 6: Control block layout for the lift blade system.

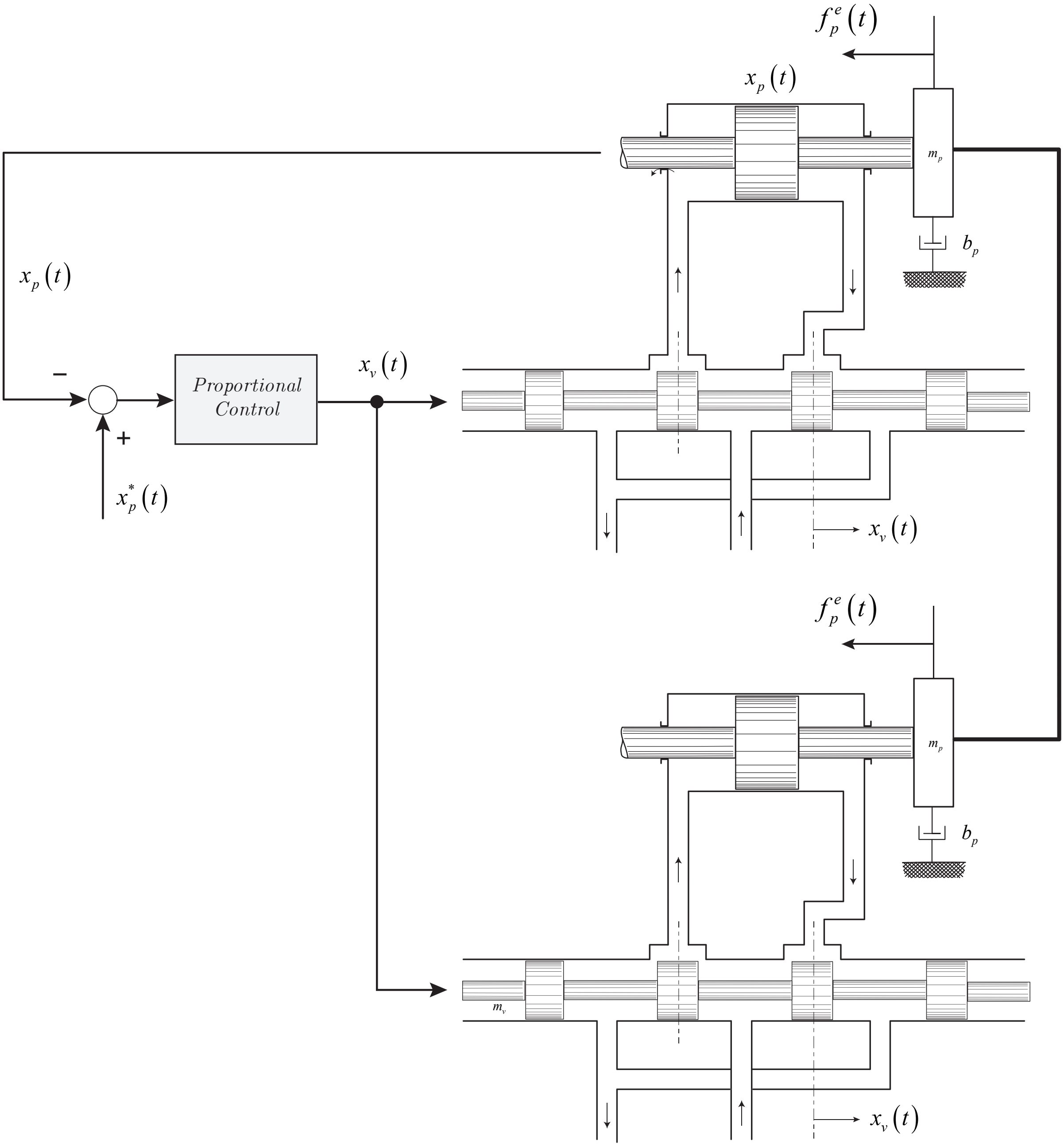

Figure 7: Control architecture for the lift blade system.

3 Pitch system

3.1 Model description

The pitch system has been modelized as two servo-piston actuators where each double-chamber is connected to a *four-way three-land spool valve* as shown in Figure 8. The system control is implemented as a *position control* in order to emulate the operator-functionality which works as a position controller. One valve-piston system is governed by a closed loop controller while the second one is governed in open loop control, see also Figure 10. **The two piston are driven in opposite way**.

The whole valve-piston system is shown in Figure 9.

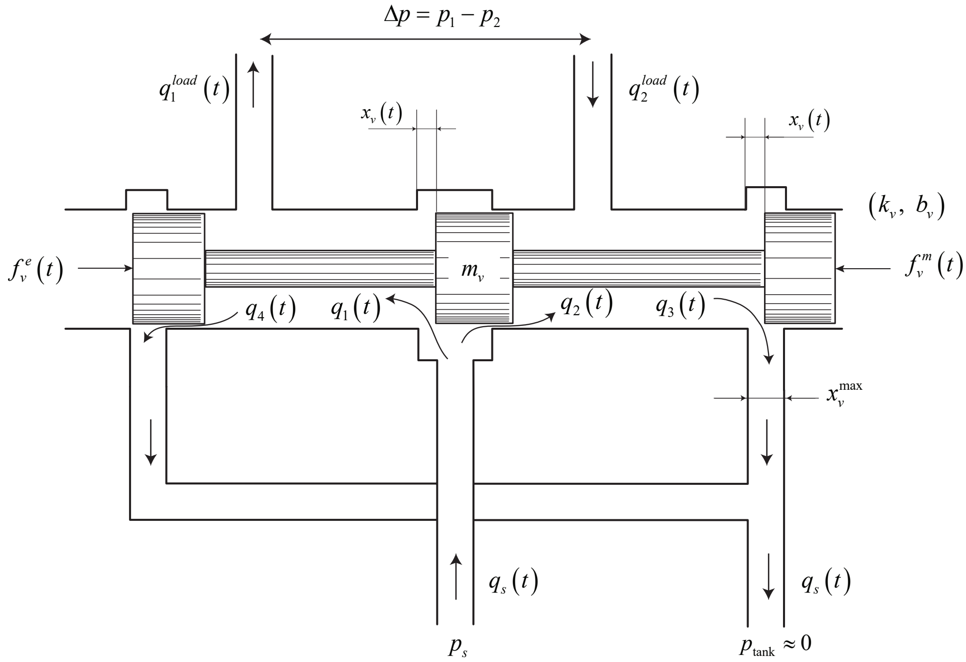

Figure 8: Four-way three-lands valve.

As reported in Figure 9 the system we are going to describe is made by the coupling of a four-way valve and a double chambers piston. For model representation we are considering the overall leakage and offset orifices to be zero, then, we can write the following set of continuity equations

$$\begin{cases} q_1(t) = \dfrac{dV_1(t)}{dt} + \dfrac{V_1(t)}{\beta}\dfrac{dp_1(t)}{dt} \\[2ex] -q_2(t) = \dfrac{dV_2(t)}{dt} + \dfrac{V_2(t)}{\beta}\dfrac{dp_2(t)}{dt} \end{cases} \tag{3.1}$$

The volume of the piston chambers may be written as follows (we are assuming the piston is centered)

$$\begin{cases} V_1(t) = V_0 + A_1 x_p(t) \\ V_2(t) = V_0 - A_2 x_p(t) \end{cases} \tag{3.2}$$

and

$$\begin{cases} \dfrac{dV_1(t)}{dt} = A_1 v_p(t) \\[2ex] \dfrac{dV_2(t)}{dt} = -A_2 v_p(t) \end{cases} \tag{3.3}$$

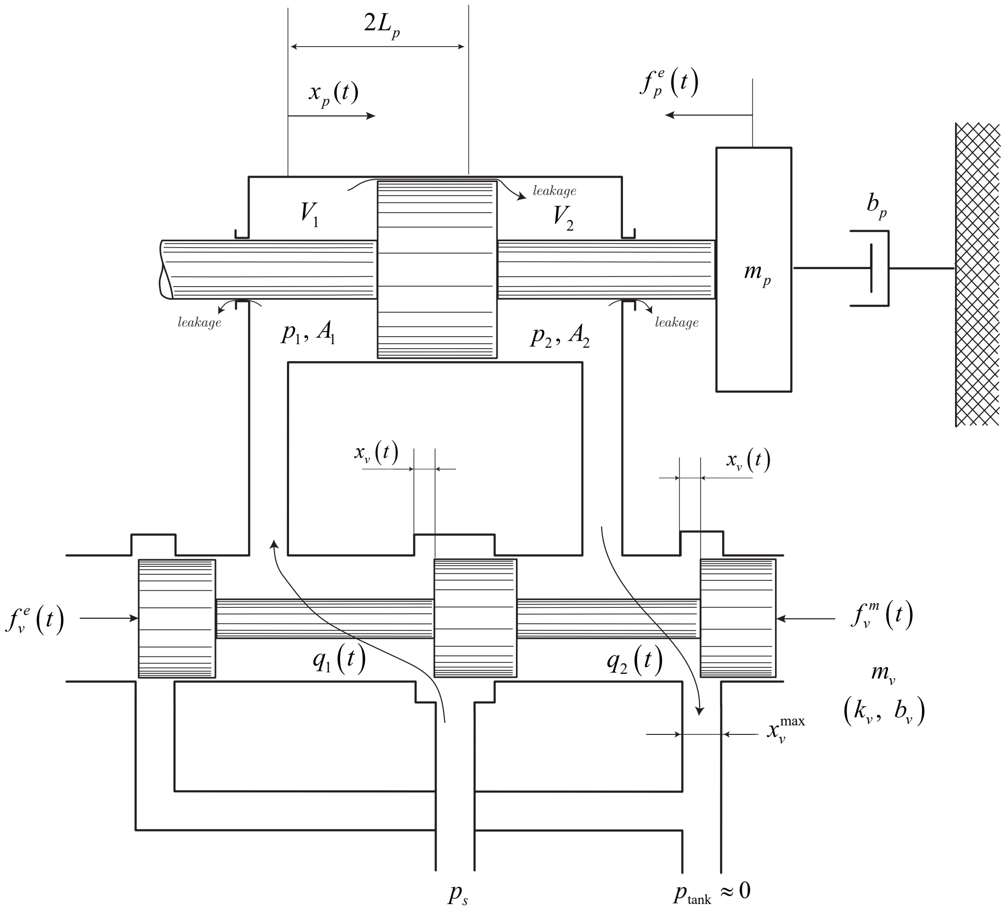

Figure 9: Four-way valve actuated piston during position changing.

which bring to the following state equations

$$\begin{cases} \dfrac{dp_1(t)}{dt} = \beta \left[\dfrac{q_1(t) - A_1 v_p(t)}{V_0 + A_1 x_p(t)} \right] \\ \dfrac{dp_2(t)}{dt} = \beta \left[\dfrac{A_2 v_p(t) - q_2(t)}{V_0 + A_2\big(-x_p(t)\big)} \right] \\ \dfrac{dv_p(t)}{dt} = \dfrac{1}{m_p} \left[\big(A_1 p_1(t) - A_2 p_2(t)\big) - b_p v_p(t) - k_p x_p(t) - f_p^e(t) \right] \\ \dfrac{dx_p(t)}{dt} = v_p(t) \end{cases} \tag{3.4}$$

where $\rho = 850\,\mathrm{kg\,m^{-3}}$ is the density of the fluid, $\beta = 1.2 \times 10^9\,\mathrm{Pa}$ is the bulk modulus, x_v is the valve orifice where we suppose the orifice area is $x_v \cdot x_v^{\max}$, k_p is the spring coefficient, b_p the damping coefficient and m_p the mass of the piston. Like in most of the hydraulic system the ratio between force and mass is very high that means the inertia effects are in general less significant.

Due to compressibility effect the flow $q_1(t)$ and the flow $q_2(t)$ are different as consequence of the conservation of the mass flow between chamber 1 and chamber 2.

The first two equations of Eq. (3.4) represent the state equation of the pressure of the chambers. The flow $q_1(t)$ is function of the delta pressure between a orifice, hence we can write the following relations

$$q_1(t) = \begin{cases} x_v x_v^{\max} \sqrt{\dfrac{2}{\rho}} \sqrt{|p_s - p_1|}\,\mathrm{sign}\big(p_s - p_1\big) \quad \text{for} \quad x_v > 0 \\ x_v x_v^{\max} \sqrt{\dfrac{2}{\rho}} \sqrt{|p_1 - p_{\text{tank}}|}\,\mathrm{sign}\big(p_1 - p_{\text{tank}}\big) \quad \text{for} \quad x_v < 0 \end{cases} \tag{3.5}$$

while $q_2(t)$ is given the following relations

$$q_2(t) = \begin{cases} x_v x_v^{\max} \sqrt{\dfrac{2}{\rho}} \sqrt{|p_2 - p_{\text{tank}}|}\,\mathrm{sign}\big(p_2 - p_{\text{tank}}\big) \quad \text{for} \quad x_v > 0 \\ x_v x_v^{\max} \sqrt{\dfrac{2}{\rho}} \sqrt{|p_2 - p_s|}\,\mathrm{sign}\big(p_s - p_2\big) \quad \text{for} \quad x_v < 0 \end{cases} \tag{3.6}$$

Moreover, according Figure 9 we have to consider the following relations:

$$x_v > 0 \Rightarrow \begin{cases} q_s(t) = q_1(t) \\ q_t(t) = q_2(t) \end{cases} \tag{3.7}$$

$$x_v < 0 \Rightarrow \begin{cases} q_s(t) = -q_2(t) \\ q_t(t) = -q_1(t) \end{cases} \tag{3.8}$$

and

$$x_v = 0 \Rightarrow \begin{cases} q_s(t) = 0 \\ q_t(t) = 0 \end{cases} \tag{3.9}$$

The pressure source p_s is generated by a pressure controlled pump.

3.2 Control layout

A fundamental point of the blade pitch system is the interconnection of the two servo-pistons. In fact, they are connected in a way which forces both to work in opposite way. The control layout is fundamentally the same of the lift system except for the control of the open loop part. in more details, one piston is controlled in closed loop and the corresponding output control signal (valve orifice) is sent in opposite way to the open loop controlled piston, see Figure 10.

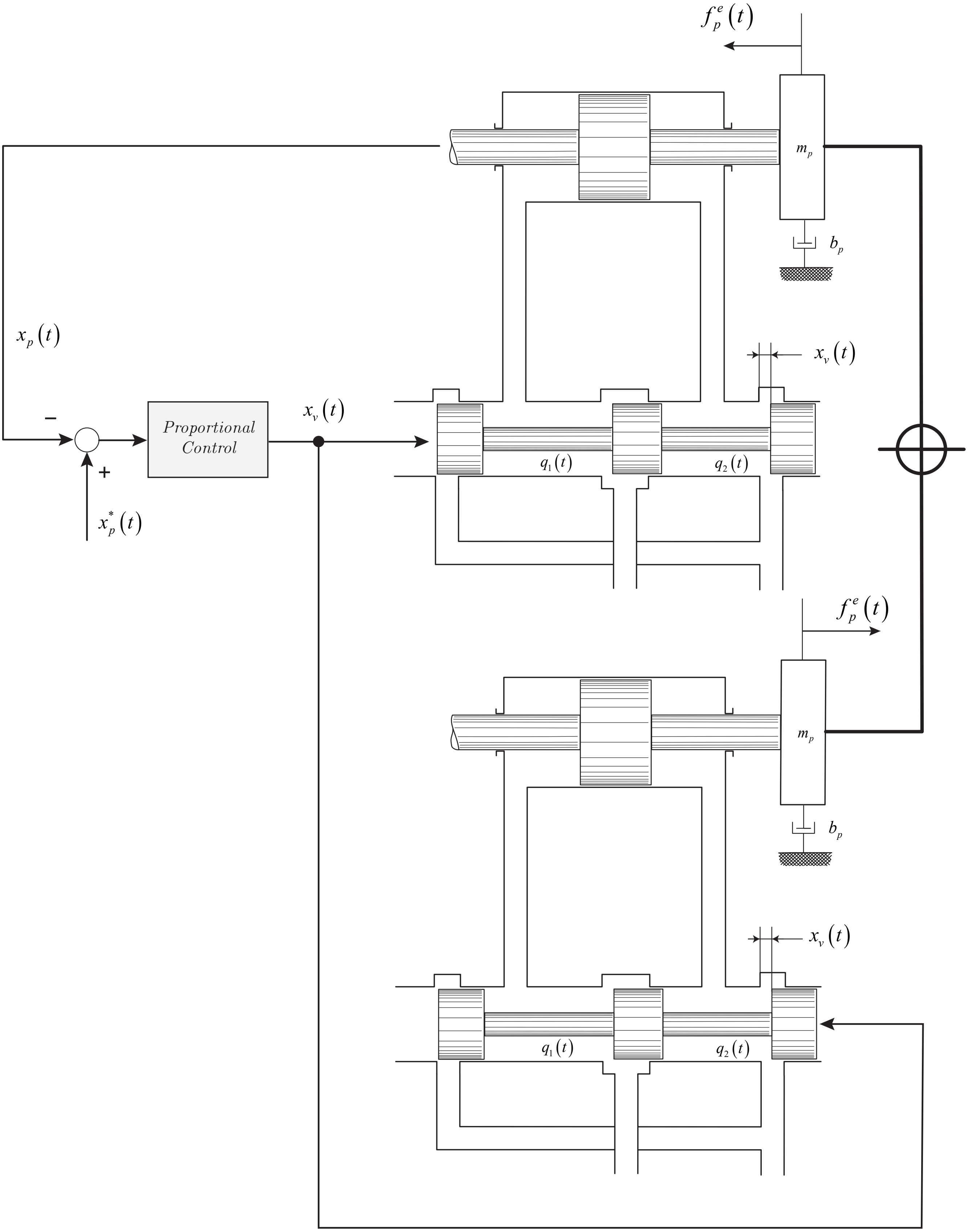

Figure 10: Control system layout of the blade pitch system.

4 Pressure controlled pump

4.1 Model derivation

Pressure controlled pump is a hydraulic equipment used to generate a constant pressure source. The pump is represent by a variable wash plate displacement pump and is governed by the output of a pressure regulator where in our modelization is a proportional-integral control (PI-control).

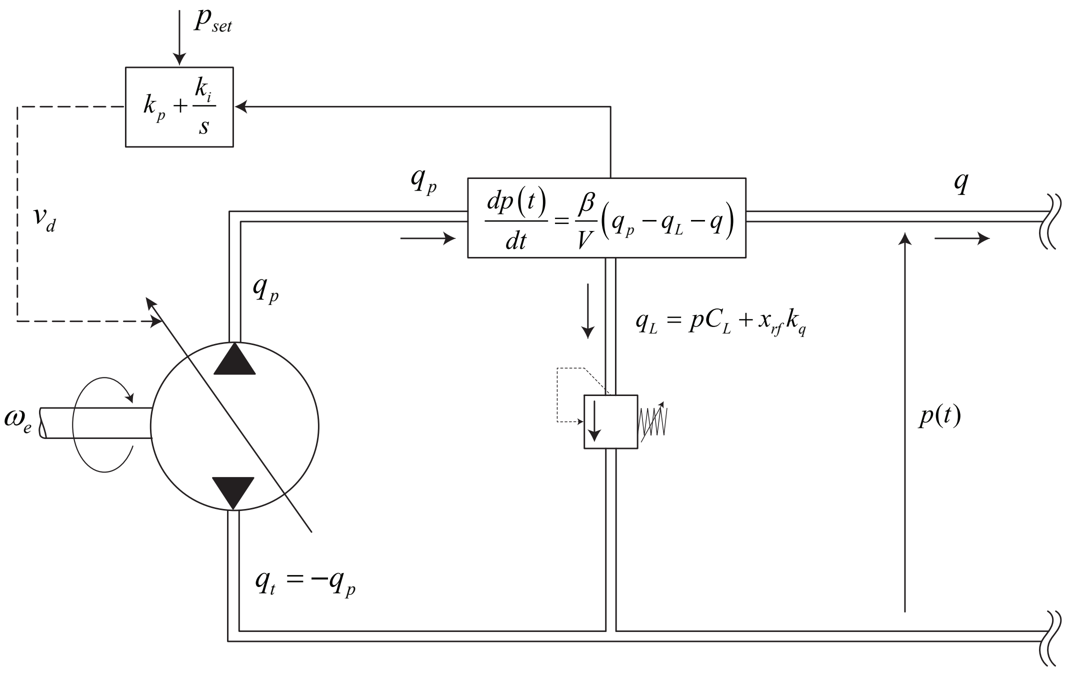

Figure 11: Pressure controlled pump.

The pump is connected via mechanical shaft to the primary source speed which in our case is a IC-engine. In our modelization we have considered a pump capable of two quadrant operative working range coupled with a relief valve. The pump can be selected to work also at one quadrant. Here the mathematical model

The flow through an orifice is here modelized with first order linearization

$$k_q = x_{rf}^{\max} \sqrt{\frac{2}{\rho \, p_{\mathrm{nom}}}} \tag{4.1}$$

where $x_{rf}^{\max}$ is the maximum orifice opening of the relief valve, $\rho\left[\mathrm{kg\,m^{-3}}\right]$ is the density of the fluid.

The PI-controller can be represented as follows

$$\begin{cases} \tilde{p} = \dfrac{1}{p^{\mathrm{nom}}}\left(p^{\mathrm{ref}} - p\right) \\ v_d = k_p \, \tilde{p} + v_d^i \\ \dfrac{dv_d^i}{dt} = k_i \, \tilde{p} \end{cases} \tag{4.2}$$

The output of the controller $v_d(t)$ is a per-unit volumetric displacement. p^{nom} is the nominal pressure of the system which is set to $p^{\mathrm{nom}} = 250\,\mathrm{bar}$.

Concerning the mathematical model of the system we can start from the mechanical part as follows

$$J\frac{d\omega_e}{dt} = \tau_e - \omega_e b - p v_d v_d^{\mathrm{nom}} \tag{4.3}$$

where ω_e is the pump rotational speed and v_d^{nom} is the nominal volumetric displacement of the pump, set to $v_d^{\text{nom}} = 62.3\,\text{cm}^3\,\text{s}^{-1}$ and b is the friction factor extrapolate from the mechanical efficiency. Concerning the hydraulic model we can write the following equations

$$\begin{cases} \dfrac{dp(t)}{dt} = \dfrac{\beta}{V}\Big[q_p(t) - q_L(t) - q(t)\Big] \\ q_p(t) = \omega_e(t) v_d^{\text{nom}} v_d(t) \\ q_L(t) = p(t) C_L + x_{rf}(t) k_q \end{cases} \tag{4.4}$$

where

$$\begin{cases} x_{rf} = x_{rf}^{\max} \quad \text{if } p > p^{\max} \\ x_{rf} = 0 \quad \text{if } p < p^{\text{set}} \\ x_{rf} = \dfrac{x_{rf}^{\max}}{p^{\text{reg}}}\left(p - p^{\text{set}}\right) \quad \text{if } p^{\text{set}} < p < p^{\max} \\ p^{\max} = p^{\text{set}} + p^{\text{reg}} \end{cases} \tag{4.5}$$

and where C_L is the leakage coefficient.

The whole model results as follows

$$\begin{cases} \tilde{p} = \dfrac{1}{p^{\text{nom}}}\Big(p^{\text{ref}} - p\Big) \\ v_d = k_p\,\tilde{p} + v_d^i \\ \dfrac{dv_d^i}{dt} = k_i\,\tilde{p} \\ J\dfrac{d\omega_e}{dt} = \tau_e - \omega_e b - p v_d v_d^{\text{nom}} \\ \dfrac{dp(t)}{dt} = \dfrac{\beta}{V}\Big[q_p(t) - q_L(t) - q(t)\Big] \\ q_p(t) = \omega_e(t) v_d^{\text{nom}} v_d(t) \\ q_L(t) = p(t) C_L + x_{rf}(t) k_q \end{cases} \tag{4.6}$$

The leakage coefficient C_L and the friction coefficient b has been derived as follows

1. The leakage coefficient is calculated considering a flow leakage $q_{\text{leak}} = 5\,\text{L/min}$ at the nominal pressure of $p^{\text{nom}} = 250\,\text{bar}$, which results in

$$C_L = 4 \times 10^{-12}\,\text{m}^3\,\text{s}^{-1}\,\text{Pa}^{-1} \tag{4.7}$$

2. The coefficient friction b has been calculated considering a mechanical efficiency of $\eta_{\text{mech}} = 0.9$ at a nominal speed which has been considered $\omega_e^{\text{nom}} = 1900\,/\,\text{min}$, which means

$$b = \frac{P^{\text{nom}}\Big(1 - \eta_{\text{mech}}\Big)}{\left(\omega_e^{\text{nom}}\right)^2} = 0.124\,\text{N}\,\text{m}\,\text{rad}^{-1}\,\text{s} \tag{4.8}$$

References

[**A. Mehrle, 2020**] Andreas Mehrle - *Advanced Control Engineering*. MCI 2020.

[**B. Gibb, 2011**] B. P. Gibb - *Advanced Kalman Filtering, Least-Squares and Modeling*. J. Wiley 2011.

[**D. Simon 2006**] Dan Simon - *Optimal State Estimation*. Wiley 2006.