Generic Thermal Model for Electromechanical Apparatus

DTL

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In this document we describe the Liebherr diesel engine D9508 represented in the Digital Twin Library. The model is based on the main dynamical equation of the crank-shaft and doesn't take into account the internal combustion thermodynamic. Torque curve as well as consumption contour map is taken from supplier and implemented as lookup table.

1 Equivalent mathematical model

The mass-spring-mass model is a valid approximation of the flexible shaft. The aim of the control, we are going to develop, is to modify the natural frequency and damping of the original system in order to keep both rotor speeds (ω_1 and ω_2) as close as possible. Moreover a zero steady-state error from a given reference speed must be achieved despite a load disturbance applied to the system, see also Figure 1 where τ_1 is the torque actuated by the control and τ_2 a disturbance (or load torque).

1.1 Model Derivation

In this example a simplified model of a transmission shaft is presented. The model is carried out implementing two masses (J_1, J_2) connected by a torsional spring (k_θ) which include a friction term (b_θ). Both two masses are connected to the chassis by two virtual bearings modeled with a simple viscosity coefficient (b).

Mathematical model can be derived using Euler-Lagrange equations as follows

$$\frac{d}{dt}\left(\frac{\partial E_{kin}}{\partial \dot{q}_1}\right) - \frac{\partial E_{kin}}{\partial q_1} + \frac{\partial E_{pot}}{\partial q_1} = Q_{diss}^{(12)} + Q_1 \tag{1.1}$$

$$\frac{d}{dt}\left(\frac{\partial E_{kin}}{\partial \dot{q}_2}\right) - \frac{\partial E_{kin}}{\partial q_2} + \frac{\partial E_{pot}}{\partial q_2} = Q_{diss}^{(21)} + Q_2 \tag{1.2}$$

Where Q_1, Q_2, $Q_{diss}^{(12)}$ and $Q_{diss}^{(21)}$ are generalized force, like electromagnetic force, unknown load, disturbance and friction.

The kinematic and potential energy becomes as follows

$$E_{kin} = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 \tag{1.3}$$

$$E_{pot} = \frac{1}{2}k_\theta\left(\theta_1 - \theta_2\right)^2 \tag{1.4}$$

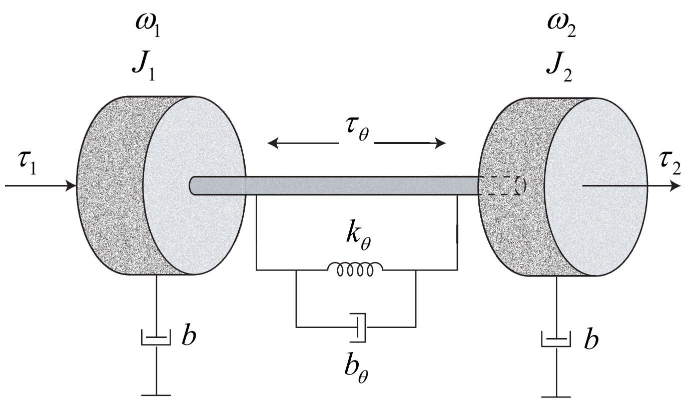

Figure 1: Simplified crankshaft model.

Lagrange equations become as follows

$$J_1\ddot{\theta}_1 + k_\theta\left(\theta_1 - \theta_2\right) = -b\dot{\theta}_1 - b_\theta\left(\dot{\theta}_1 - \dot{\theta}_2\right) + \tau_1 \tag{1.5}$$

$$J_2\ddot{\theta}_1 + k_\theta\left(\theta_1 - \theta_2\right) = -b\dot{\theta}_2 - b_\theta\left(\dot{\theta}_2 - \dot{\theta}_1\right) + \tau_2 \tag{1.6}$$

we can also write as follows

$$\begin{cases} \dot{\theta}_1 = \omega_1 \\ \dot{\omega}_1 = -\dfrac{k_\theta}{J_1}\theta_1 - \dfrac{b+b_\theta}{J_1}\omega_1 + \dfrac{k_\theta}{J_1}\theta_2 + \dfrac{b_\theta}{J_1}\omega_2 + \dfrac{1}{J_1}\tau_1 \\ \dot{\theta}_2 = \omega_2 \\ \dot{\omega}_2 = \dfrac{k_\theta}{J_2}\theta_1 + \dfrac{b_\theta}{J_2}\omega_1 - \dfrac{k_\theta}{J_2}\theta_2 - \dfrac{b+b_\theta}{J_2}\omega_2 + \dfrac{1}{J_2}\tau_2 \end{cases} \tag{1.7}$$

The state space representation become

$$\dot{\vec{z}} = \tilde{\mathbf{G}}\vec{z} + \tilde{\mathbf{H}}\ \tau_1 + \tilde{\mathbf{M}}\ \tau_2 \tag{1.8}$$

where $\vec{z} = \begin{bmatrix}\theta_1 & \omega_1 & \theta_2 & \omega_2\end{bmatrix}^T$ and

$$\tilde{\mathbf{G}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_1} & -\frac{b+b_\theta}{J_1} & \frac{k_\theta}{J_1} & \frac{b_\theta}{J_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{J_2} & +\frac{b_\theta}{J_2} & -\frac{k_\theta}{J_2} & -\frac{b+b_\theta}{J_2} \end{bmatrix} \tag{1.9}$$

and $\tilde{\mathbf{H}} = \begin{bmatrix}0 & \frac{1}{J_1} & 0 & 0\end{bmatrix}^T$, $\tilde{\mathbf{M}} = \begin{bmatrix}0 & 0 & 0 & -\frac{1}{J_2}\end{bmatrix}^T$.

For our purpose we want to change the state representation using a new state vector $\vec{x} = \mathbf{T}\vec{z} = \begin{bmatrix}\omega_1 & \omega_2 & \tau_\theta\end{bmatrix}^T$ where $\mathbf{T}$ is given as follows

$$\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ k_\theta & 0 & -k_\theta & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{T}^{-1} = \begin{bmatrix} 0 & 0 & \frac{1}{2k_\theta} \\ 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2k_\theta} \\ 0 & 1 & 0 \end{bmatrix} \tag{1.10}$$

The new state space representation becomes

$$\begin{aligned} \dot{\vec{z}} &= \tilde{\mathbf{G}}\vec{z} + \tilde{\mathbf{H}}\ \tau_1 + \tilde{\mathbf{M}}\ \tau_2 \\ \mathbf{T}^{-1}\dot{\vec{x}} &= \tilde{\mathbf{G}}\mathbf{T}^{-1}\vec{x} + \tilde{\mathbf{H}}\ \tau_1 + \tilde{\mathbf{M}}\ \tau_2 \\ \dot{\vec{x}} &= \mathbf{T}\tilde{\mathbf{G}}\mathbf{T}^{-1}\vec{x} + \mathbf{T}\tilde{\mathbf{H}}\ \tau_1 + \mathbf{T}\tilde{\mathbf{M}}\ \tau_2 \end{aligned}$$

or

$$\begin{bmatrix}\dot{\omega}_1\\\dot{\omega}_2\\\dot{\tau}_\theta\end{bmatrix} = \begin{bmatrix}-\frac{b+b_\theta}{J_1} & \frac{b_\theta}{J_1} & -\frac{1}{J_1}\\ \frac{b_\theta}{J_2} & -\frac{b+b_\theta}{J_2} & \frac{1}{J_2}\\ k_\theta & -k_\theta & 0\end{bmatrix}\begin{bmatrix}\omega_1\\\omega_2\\\tau_\theta\end{bmatrix} + \begin{bmatrix}\frac{1}{J_1}\\0\\0\end{bmatrix}\tau_1 + \begin{bmatrix}0\\\frac{1}{J_2}\\0\end{bmatrix}\tau_2 \tag{1.11}$$

where

$$\vec{x} = \begin{bmatrix}\omega_1 & \omega_2 & \tau_\theta\end{bmatrix}^T$$

$$\tilde{\mathbf{A}} = \mathbf{T}\tilde{\mathbf{G}}\mathbf{T}^{-1} = \begin{bmatrix}-\frac{b+b_\theta}{J_1} & \frac{b_\theta}{J_1} & -\frac{1}{J_1}\\ \frac{b_\theta}{J_2} & -\frac{b+b_\theta}{J_2} & \frac{1}{J_2}\\ k_\theta & -k_\theta & 0\end{bmatrix}$$

$$\tilde{\mathbf{B}} = \mathbf{T}\tilde{\mathbf{H}} = \begin{bmatrix}\frac{1}{J_1}\\0\\0\end{bmatrix} \quad \tilde{\mathbf{E}} = \mathbf{T}\tilde{\mathbf{M}} = \begin{bmatrix}0\\\frac{1}{J_2}\\0\end{bmatrix}$$

which results in

$$\dot{\vec{x}}(t) = \tilde{\mathbf{A}}\ \vec{x}(t) + \tilde{\mathbf{B}}\ \tau_1 + \tilde{\mathbf{E}}\ \tau_2$$

Consider the load term τ_2 as a disturbance which perturbs the system.

For a more physical viewpoint equation can also be written as follows:

$$\begin{cases} J_1\dot{\omega}_1 + b\omega_1 + b_\theta(\omega_1 - \omega_2) = \tau_1 - \tau_\theta \\ J_2\dot{\omega}_2 - b\omega_2 + b_\theta(\omega_2 - \omega_1) = \tau_\theta + \tau_2 \\ \dot{\tau}_\theta = k_\theta\,(\omega_1 - \omega_2) \end{cases} \tag{1.12}$$

2 Model parameters settings

Parameters setting consist of

- J_1 $\left[\mathrm{kg\,m^2}\right]$: Inertia of the first rotating mass.
- J_2 $\left[\mathrm{kg\,m^2}\right]$: Inertia of the second rotating mass.
- b_1 $\left[\mathrm{kg\,m^2}\right]$: Inertia of the first rotating mass.
- b_2 $\left[\mathrm{kg\,m^2}\right]$: Inertia of the second rotating mass.
- k_θ $\left[\mathrm{kg\,m^2}\right]$: Inertia of the second rotating mass.
- b_θ $\left[\mathrm{kg\,m^2}\right]$: Inertia of the second rotating mass.