Three Phase Inductors

Mathematical Model and Construction

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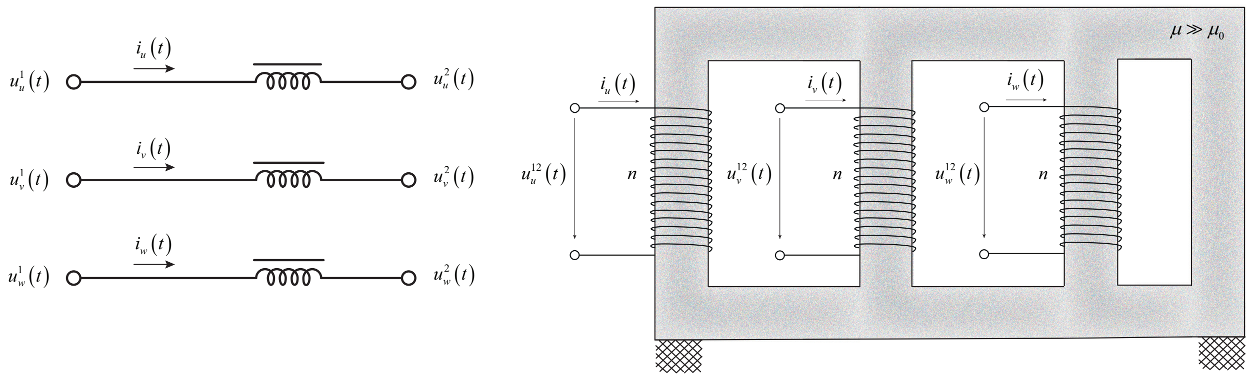

Figure 1: Three-phase inductance: differential and common operational mode with four magnetic limbs.

Three-phase inductance with three limbs - pure differential mode:

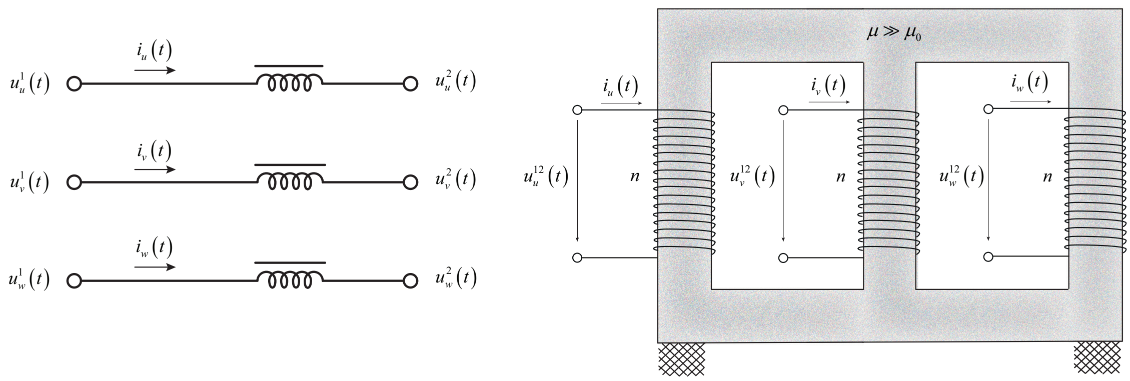

Figure 2: Three-phase inductance: pure differential operative mode with three magnetic limbs.

Three-phase inductance with one limb - pure common mode:

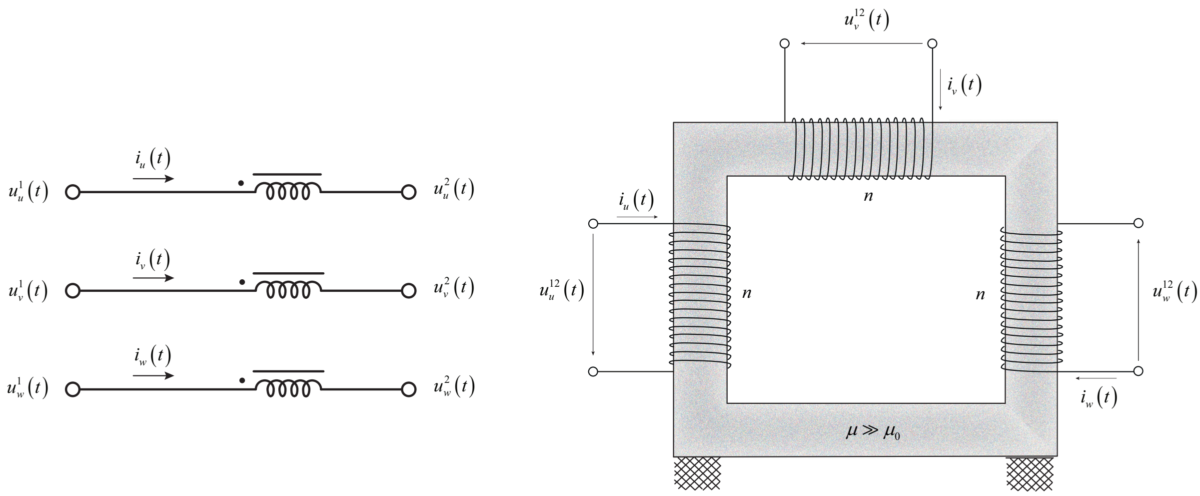

Figure 3: Three-phase inductance: pure common operative mode with one magnetic limbs (classical toroidal core is used for low power application).

Mathematical model:

$$\begin{cases} u_u^{12}(t) = u_u^{cm}(t) + u_u^{dm}(t) \\ u_v^{12}(t) = u_v^{cm}(t) + u_v^{dm}(t) \\ u_w^{12}(t) = u_w^{cm}(t) + u_w^{dm}(t) \end{cases} \tag{1.1}$$

where

$$\begin{cases} u_u^{dm}(t) + u_v^{dm}(t) + u_w^{dm}(t) = 0 \\ \dfrac{1}{3}\Big(u_u^{cm}(t) + u_v^{cm}(t) + u_w^{cm}(t)\Big) = u_{cm}(t) \end{cases} \tag{1.2}$$

$$\begin{cases} u_u^{12}(t) - Ri_u(t) - L_a\dfrac{d\,i_u}{dt}(t) + L_m\dfrac{d\,i_v}{dt}(t) + L_m\dfrac{d\,i_w}{dt}(t) = 0 \\ u_v^{12}(t) - Ri_v(t) - L_a\dfrac{d\,i_v}{dt}(t) + L_m\dfrac{d\,i_u}{dt}(t) + L_m\dfrac{d\,i_w}{dt}(t) = 0 \\ u_w^{12}(t) - Ri_w(t) - L_a\dfrac{d\,i_w}{dt}(t) + L_m\dfrac{d\,i_u}{dt}(t) + L_m\dfrac{d\,i_v}{dt}(t) = 0 \end{cases} \tag{1.3}$$

which results in matrix form

$$\vec{u}_{uvw}^{12}(t) - \mathbf{R}\vec{i}_{uvw}(t) - \mathbf{L}\frac{d}{dt}\vec{i}_{uvw}(t) = 0 \tag{1.4}$$

where

$$\mathbf{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} L_a & -L_m & -L_m \\ -L_m & L_a & -L_m \\ -L_m & -L_m & L_a \end{bmatrix}. \tag{1.5}$$

in order to derive the equivalent *differential* and *common* mode equations the following transformation matrix is accounted

$$\mathbf{T} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}. \tag{1.6}$$

where

$$\vec{u}^{12}_{uvw} = \begin{bmatrix} u^{12}_u(t) \\ u^{12}_v(t) \\ u^{12}_w(t) \end{bmatrix} = \mathbf{T}^{-1} \begin{bmatrix} u^{dm}_{uv}(t) \\ u^{dm}_{vw}(t) \\ u^{dm}_{wu}(t) \\ u_{cm}(t) \end{bmatrix}. \tag{1.7}$$

where

$$\begin{bmatrix} u^{dm}_{uv}(t) \\ u^{dm}_{vw}(t) \\ u^{dm}_{wu}(t) \\ u_{cm}(t) \end{bmatrix} - \begin{bmatrix} R & -R & 0 \\ 0 & R & -R \\ -R & 0 & R \\ R & R & R \end{bmatrix} \begin{bmatrix} i_u(t) \\ i_v(t) \\ i_w(t) \end{bmatrix} - \begin{bmatrix} L_a + L_m & -(L_a + L_m) & 0 \\ 0 & L_a + L_m & -(L_a + L_m) \\ -(L_a + L_m) & 0 & L_a + L_m \\ L_a - 2L_m & L_a - 2L_m & L_a - 2L_m \end{bmatrix} \begin{bmatrix} \frac{di_u}{dt}(t) \\ \frac{di_v}{dt}(t) \\ \frac{di_w}{dt}(t) \end{bmatrix} = 0 \tag{1.8}$$

expressing in terms of $\begin{bmatrix} u^{12}_{dmu}(t) & u^{12}_{dmv}(t) & u^{12}_{dmu}(t) \end{bmatrix}^T$ and $u_{cm}(t)$ we obtain the following system

$$\vec{u}^{12}_{duvw}(t) - \mathbf{R}\vec{i}_{uvw}(t) - \mathbf{L}_{dm}\frac{d}{dt}\vec{i}_{uvw}(t) = 0 \tag{1.9}$$

where

$$\mathbf{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} L_a + L_m & 0 & 0 \\ 0 & L_a + L_m & 0 \\ 0 & 0 & L_a + L_m \end{bmatrix}. \tag{1.10}$$

and

$$u_{cm}(t) - Ri_{cm}(t) - (L_a - 2L_m)\frac{d}{dt}i_{cm}(t) = 0 \tag{1.11}$$

where $i_{cm}(t) = \frac{1}{3}\Big(i_u(t) + i_v(t) + i_w(t)\Big)$. Hence, in a three-phase inductance we have two inductive contributions

- differential mode (also called *positive sequence inductance*) given by $L_p = L_a + L_m$;
- common mode (also called *zero sequence inductance*) given by $L_0 = L_a - 2L_m$

where L_a is the self-inductance, and L_m is the mutual-inductance.

The relation between sequence values and machine values can be summarized as follows

$$
\begin{cases}
L_p = L_a + L_m \\
L_0 = L_a - 2L_m \\
L_m = \dfrac{1}{3}\Big(L_p - L_0\Big) \\
L_a = \dfrac{1}{2}\Big(L_p + L_0 + L_m\Big)
\end{cases}
\tag{1.12}
$$

Commercially speaking three-phase inductor can be either *common mode* or *differential mode*:

- Typically three-phase three-limbs inductors present only differential inductance. The residual common mode inductance is associated to the leakage inductance, see Figure 2.
- Typically three-phase common mode inductors are built around a common single limb core. The residual differential mode inductance is associated to the leakage inductance, see Figure 3.

2 Saturation effects

The saturation effects can be modelized assuming L_a and L_m as function of the current, as follows

$$
\begin{cases}
L_{au} = L_a^{nom} k_{sat}^a(i_u) + L_{a0} \to L_{a0} \quad \text{for } i \gg i_{sat} \\
L_{av} = L_a^{nom} k_{sat}^a(i_v) + L_{a0} \to L_{a0} \quad \text{for } i \gg i_{sat} \\
L_{aw} = L_a^{nom} k_{sat}^a(i_w) + L_{a0} \to L_{a0} \quad \text{for } i \gg i_{sat} \\
L_{mu} = L_m^{nom} k_{sat}^m(i_u) + L_{m0} \to L_{m0} \quad \text{for } i \gg i_{sat} \\
L_{mv} = L_m^{nom} k_{sat}^m(i_v) + L_{m0} \to L_{m0} \quad \text{for } i \gg i_{sat} \\
L_{mw} = L_m^{nom} k_{sat}^m(i_w) + L_{m0} \to L_{m0} \quad \text{for } i \gg i_{sat}
\end{cases}
\tag{2.1}
$$

where L_{a0} and L_{m0} are the equivalent self and mutual inductance in air, condition which raises e.g. during short circuit current.

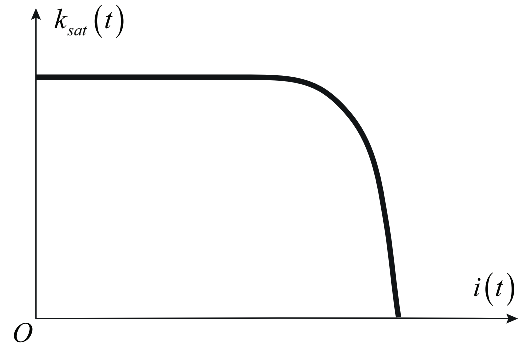

Figure 4: Saturation curve applied to L_a and L_m, self and mutual inductance respectively.

$$\mathbf{L}(\vec{i}_{uvw}) = \begin{bmatrix} L_{ls} + L_a^{nom} k_{sat}^a(i_u) + L_{a0} & -\left[L_m^{nom} k_{sat}^m(i_v) + L_{m0}\right] & -\left[L_m^{nom} k_{sat}^m(i_w) + L_{m0}\right] \\ -\left[L_m^{nom} k_{sat}^m(i_u) + L_{m0}\right] & L_{ls} + L_a^{nom} k_{sat}^a(i_v) + L_{a0} & -\left[L_m^{nom} k_{sat}^m(i_w) + L_{m0}\right] \\ -\left[L_m^{nom} k_{sat}^m(i_u) + L_{m0}\right] & -\left[L_m^{nom} k_{sat}^m(i_v) + L_{m0}\right] & L_{ls} + L_a^{nom} k_{sat}^a(i_w) + L_{a0} \end{bmatrix} \tag{2.2}$$

References

[1] M. L. Heldwein, L. Dalessandro, J. W. Kolar - *The Three-Phase Common-Mode Inductor: Modeling Design Issues.* IEEE Transactions on industrial electronics, vol. 58, no. 8, August 2011.

[2] W. G. Hurley, W. H. Woelfle - *Transformers and Inductors for Power Electronics. Theory design and applications.* J. Wiley 2013.