Shell-model project for Nuclear Talent course

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PART 1C

Dimensionality

The Hamiltonian matrix is calculated for N single-particle-states (sps) and n particles. Generally, (e.g. allowing pairs breaking) we have $\binom{N}{n} = \frac{N!}{n!(N-n)!}$ possibilities for ordering the particles.

If we look at k-particle states (e.g. pairs, 2-particle states) we will have $\binom{N}{k}$ possibilities for ordering (e.g. for $N = 8, k = 4$)

levels that should be filled by $n/2$ pairs (where N and n are both even integers), which yields $\frac{(N/2)!}{[n/2]![(N-n)/2]!}$ possibilities for ordering the pairs. Working in the pairs representation, this will generate a $\frac{(N/2)!}{[n/2]![(N-n)/2]!} \times \frac{(N/2)!}{[n/2]![(N-n)/2]!}$ Hamiltonian matrix.

As an example, for 8-sps and 4-particles in the pairs representation, we have a dimensionality of $\binom{4}{2} = 6$, whch yields a 6×6 Hamiltonian matrix.

Calculating the states

For a doubly-degenerate level with pair-particles (spin 1/2 for each and total spin 0), there are 6 possible orderings written as

$$|1,2,3,4\rangle \leftrightarrow |1,2\rangle\,, \tag{1a}$$
$$|1,2,5,6\rangle \leftrightarrow |1,3\rangle\,, \tag{1b}$$
$$|1,2,7,8\rangle \leftrightarrow |1,4\rangle\,, \tag{1c}$$
$$|3,4,5,6\rangle \leftrightarrow |2,3\rangle\,, \tag{1d}$$
$$|3,4,7,8\rangle \leftrightarrow |2,4\rangle\,, \tag{1e}$$
$$|5,6,7,8\rangle \leftrightarrow |3,4\rangle\,. \tag{1f}$$

The right column represents two pairs in two different levels (pair states), e.g. $|1,2\rangle$ represents a pair in level $P = 1$ and a pair in level $P = 2$. The left column represents the positions of all four particles (single particle states), e.g. $|1,2,3,4\rangle$ represents one particle in state number 1, one particle in state number 2, etc.

Hamiltonian Matrix

The Hamiltonian has two parts, the unperturbed Hamiltonian

$$\hat{H}_0 = \xi \sum_{p,\sigma} (p-1)\hat{a}^\dagger_{p,\sigma}\hat{a}_{p,\sigma}, \tag{2}$$

with $\xi = 1$, and the interacting part

$$\hat{V} = -\frac{1}{2} g \sum_{p,q} \hat{P}^+_p \hat{P}^-_q, \tag{3}$$

where $\hat{P}^+_p$ and $\hat{P}^-_p$ are the pair creation and annihilation operators

$$\hat{P}^+_p = \hat{a}^\dagger_{p,+}\hat{a}^\dagger_{p,-}, \qquad \hat{P}^-_p = \hat{a}_{p,-}\hat{a}_{p,+}. \tag{4}$$

The full Hamiltonian then reads $\hat{H} = \hat{H}_0 + \hat{V}$.

Next we calculate the Hamiltonian matrix. For the unperturbed part, we have

$$\hat{H}_0 = \sum_\sigma \left[\hat{a}^\dagger_{2,\sigma}\hat{a}_{2,\sigma} + 2\hat{a}^\dagger_{3,\sigma}\hat{a}_{3,\sigma} + 3\hat{a}^\dagger_{4,\sigma}\hat{a}_{4,\sigma}\right] \tag{5}$$