Final Project - TALENT 2017 at ECT*

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INTRODUCTION OF OUR THEORETICAL FRAMEWORK

We investigate the structural changes of the $^{18-28}$O isotopes, both even and odd, by examining their low-lying states. This is done by developing our own shell model program and comparing the energies it generates to the NushellX@MSU program [1]. We use an ^{16}O core, having 8 protons and 8 neutron at the $s-p$-shells $(0s_{1/2}, 0p_{3/2}, 0p_{1/2})$. More neutrons are then excited in the sd-shell $(0d_{5/2}, 1s_{1/2}, 0d_{3/2})$, which serves as our model space. We use all possible configurations in these orbits and work in a harmonic oscillator basis with spin-orbit splitting. The Hamiltonian reads

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \tag{1}$$

for

$$\hat{H}_0 = \sum_{p,q} \epsilon_p \hat{a}_p^\dagger \hat{a}_q, \tag{2}$$

the one-body Hamiltonian, where $\hat{n}_p = \hat{a}_p^\dagger \hat{a}_p$ is the number operator for the spherical orbit p with quantum numbers (n_p, ℓ_p, j_p, m_p) and $\epsilon_p = \langle p|h_0|p\rangle$ are the single-particle energies (SPE) and

$$\hat{H}_I = \sum_{p<q=1}^{N} \sum_{r<s=1}^{N} V(p,q;r,s)\hat{T}(p,q;r,s), \tag{3}$$

with

$$\hat{T}(p,q;r,s) = \hat{P}^+_{p,q}\hat{P}^-_{r,s}, \tag{4}$$

where $\hat{P}^+_{p,q} = \sum_{p,q} \hat{a}_p^\dagger \hat{a}_q^\dagger, \qquad \hat{P}^-_{r,s} = \sum_{r,s} \hat{a}_r \hat{a}_s.$

$\hat{H}_I$ is given in M-scheme. It is the two-body density operator for nucleon pairs in orbits p,q and r,s coupled to the total spin projection M, where N is the number of particles in the configuration. In J-scheme, $\hat{H}_I$ reads

$$\hat{H}_I = \sum_{a\le b, c\le d} \sum_{JT} V_{J,T}(p,q;r,s)\hat{T}_{J,T}(p,q;r,s), \tag{5}$$

where $\hat{T}_{J,T}(p,q;r,s)$ is the scalar two-body density operator for nucleon pairs in orbits p,q and r,s coupled to spin quantum numbers J,M and isospin quantum numbers T,T_z [2]. Here the appropriate quantum numbers are (n_i,ℓ_i,j_i), $i \in \{a,b,c,d\}$. The transformation between the two-body matrix elements (TBME) from J- to M-scheme reads

$$\begin{aligned} V(p,q;r,s) &= \langle j_p, m_p; j_q, m_q|V|j_r, m_r; j_s, m_s\rangle \\ &= \sum_{J,M} \langle j_p, m_p; j_q, m_q|JM\rangle \langle j_r, m_r; j_s, m_s|JM\rangle \\ &\times \langle (j_p, j_q)JM|V|(j_r, j_s)JM\rangle , \end{aligned} \tag{6}$$

where $\langle j_a, m_a; j_b, m_b|JM\rangle$ is a Clebsch-Gordan coefficient and other quantum numbers are implicitly implied.

We use the SPE and TBME of the USDB interaction [2] and work in M-scheme. The SPE values and order is given in Table I. The TBME for $A = 18$ are given in [2] in J-scheme for $T = 1, 0$ in Tables I and II, respectively. As was done for the USD interaction [3], the SPE are taken to be mass independent and for the TBME we employ a mass dependence of the form

$$V(p,q;r,s)(A) = \left(\frac{18}{A}\right)^p V(p,q;r,s)(A=18), \tag{7}$$

with $p = 0.3$. This qualitative mass dependence is expected from the evaluation of a medium-range interaction with harmonic-oscillator radial wave functions. It also defines TBME for other A values.

Using the SPS we construct the appropriate slater determinants, $|\psi\rangle$, according to the number of particles which we place in the sd-shell.

TABLE I. Single particle energies of the sd-shell in the M-scheme basis with their corresponding quantum numbers: N, the principle quantum number; ℓ, the orbital angular momentum; J, the total angular momentum; M_j, the total angular momentum projection on the z axis.

index	N	ℓ	J	M_j	SPE
1	1	0	1	$-1/2$	-3.20790
2	1	0	1	$+1/2$	-3.20790
3	0	2	3	$-3/2$	2.11170
4	0	2	3	$-1/2$	2.11170
5	0	2	3	$+1/2$	2.11170
6	0	2	3	$+3/2$	2.11170
7	0	2	5	$-5/2$	-3.92570
8	0	2	5	$-3/2$	-3.92570
9	0	2	5	$-1/2$	-3.92570
10	0	2	5	$+1/2$	-3.92570
11	0	2	5	$+3/2$	-3.92570
12	0	2	5	$+5/2$	-3.92570

STRUCTURE OF THE OXYGEN ISOTOPES WAVE FUNCTIONS

The even-even Oxygen isotopes have been widely investigated (see [4] for a review). To describe their low lying spectrum within the shell model, different model spaces are used, starting from an ^{16}O core with extra neutrons in the *sd*-shell [2, 3, 5, 6], adding the lower *p*-shell for core-excitation (intruder states) with particle-hole configurations [7] and even incorporating the *pf*-shell as well [8]. The structure of the wave functions of the different levels in these isotopes are interesting to examine since they can reveal the role of different configurations and orbits. Thus one can examine, on top of low lying energies, electromagnetic transitions rates. Here we will only investigate the $E2$ between the first 2^+ and the first 0^+, i.e. $B(E2; 2_1^+ \rightarrow 0_1^+)$ using NushellX@MSU, given in Fig. 1. For the experimental data there are only three values, however it is notable that they become closer to the USDB interaction values as neutrons are added. The ^{18}O isotope has the largest discrepancy with the USDB interaction. The reason might be due to the fact we are neglecting *p*-shell core-excitations when using only the USDB interaction. It was found in [7] that the 0_2^+ should have a dominant $4p - 2h$ component. However there are indications that larger mixing with the 0_1^+ should arise. The problem for mixing of these states due to the truncation in the np–mh sequence is discussed in [9]. Therefore, when close to the ^{16}O core, e.g. at ^{18}O, a large part of the *p*-shell is missing in the wave function rendering less components in the wave functions to connect between the 0_1^+ and the 2_1^+ and hence yielding a smaller $B(E2)$ value. As neutrons are added to the ^{16}O core, the *p*-shell core-excitations move to higher energy and mix less with the low-lying states, then the USDB interaction becomes a better approximation.

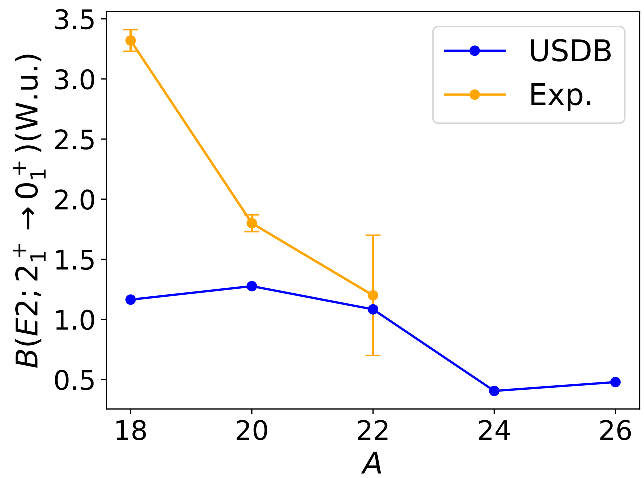

FIG. 1. $B(E2; 2^+ \rightarrow 0^+)$ of experimental (orange) and the USDB interaction (blue) for $^{18-26}$O.

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