

## Master Thesis

# Spatial Summarization of Image Collections

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This dissertation is submitted for the degree of *Master of Science ETH in Computer Science*

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# Preface

Acknowledgments and such ...



# **Abstract**

The Abstract ...



# Symbols

## Symbols

$\eta$	Learning rate
$\gamma$	Model parameter
$\kappa$	Largest subset size
$\nu$	Noise-to-data ratio
$\sigma$	Gaussian distribution's standard deviation
$\mathcal{D}$	Set of data samples
$K$	Latent coherence dimensions
$L$	Latent diversity dimensions
$M$	Number of features
$\mathcal{N}$	Set of noise samples
$\mathbb{R}$	Real numbers
$S, T$	Subset
$S_t$	Sequence
$V$	Ground set
$Z$	Normalization constant

## Matrices

$\mathbf{B}$	Feature diversity weights
$\mathbf{E}$	Feature coherence weights
$\mathcal{I}$	Identity
$\mathbf{W}^b$	Coherence weights
$\mathbf{W}^e$	Diversity weights
$\mathbf{X}$	Features

## Vectors

$\theta$	Model parameters
$\mathbf{a}$	Utility weights for features
$\mathbf{u}$	Utility weights
$\mathbf{G}$	Accumulated gradient

## Indices

$i, j$	Element
$k$	Feature
$c$	Coherence dimension
$d$	Diversity dimension

## Acronyms and Abbreviations

ETH	Eidgenössische Technische Hochschule
FLID	Facility Location Diversity
FLDC	Facility Location Diversity and Coherence
FFLDC	Featurized Facility Location Diversity and Coherence
MLE	Maximum Likelihood Estimation
NCE	Noise Contrastive Estimation
LAS	Learning & Adaptive Systems
SGD	Stochastic Gradient Descent

# **Chapter 1**

## **Introduction**

Motivation (tourist routes and summarization), and quick idea of the methodology.

### **1.1 Contributions**

Summarization of contributions.



# **Chapter 2**

## **Related Work**

### **2.1 Probabilistic Sub/super-modular models**

Talk about the importance of these models and the recent interest in them, e.g. DPPs, Sampling.

### **2.2 Image collection summarization**

Generally about the problem and the different approaches.

### **2.3 Tourist routes recommendation**

The initial papers I found on this, basically Markov chains and clustering.



# Chapter 3

## Background

This chapter presents the supporting concepts and methods that will be used throughout this work.

### 3.1 Submodular Probability Set Functions

As described before, probability models over sets are of great interest and can be applied to diverse settings. In particular, submodular probability set functions have been utilized in multiple domains, such as document summarization and information gathering (Krause and Golovin, 2014).

Probability set functions are a class of distributions  $P$  over subsets  $S$  of a ground set  $V$ , i.e. functions that satisfy  $\forall S \subseteq V, P : 2^V \rightarrow \mathbb{R} | 0 \leq P(S) \leq 1$  and  $\sum_{S \subseteq V} P(S) = 1$ . These distributions can also be understood as probabilities over binary sequences  $[a_1, \dots, a_n]$  of fixed size  $n = |V|$  (Bruno et al., 1999). Hereby, w.l.o.g., let  $V = \{1, \dots, n\}$ .

A set function  $F : 2^V \rightarrow \mathbb{R}$  is submodular if it exhibits a "diminishing returns" property (Krause and Golovin, 2014), namely if it satisfies:

$$\forall S, T \subseteq V : S \subseteq T, i \notin T \mid F(S \cup i) - F(S) \geq F(T \cup i) - F(T) \quad (3.1)$$

Intuitively this indicates that adding an element to a smaller set yields a larger effect, i.e. in terms of the set function  $F$  value, than adding it to a larger one. This is a natural property in the context of summarization where adding more information to a large summary can be less effective than adding it to a smaller one. This property also makes them good candidates for modeling the concept of diversity (Tschiatschek et al., 2016).

### 3.2 Supermodularity and Modularity

Analogous to the use of submodular functions to model diversity, supermodular functions can be used to model complementarity or coherence between elements.

A set function is supermodular if it satisfies the condition in (3.1) with the inequality sign reversed, i.e. a set function  $F(S)$  is supermodular iff  $-F(S)$  is submodular. Supermodularity is used extensively in economics to model complementarity between strategies in games (Amir, 2005).

Finally, if a function  $F$  is both submodular and supermodular, i.e. it satisfies condition (3.1) with equality, then it is said to be a modular function. These are the simplest examples of submodular or supermodular functions and are akin to linear functions but in a discrete domain (Krause and Golovin, 2014).

### 3.3 Probabilistic Log-sub/supermodular Models

This work is interested in a particular class of probability set functions, namely probabilistic log-submodular and log-supermodular models, which are probabilities  $P(S)$  of the form (Djolonga and Krause, 2014):

$$P(S) \propto \exp(F(S)) \quad (3.2)$$

Where  $F(S)$  is a submodular or supermodular function, respectively. These models encompass many practical probabilistic models such as repulsive Ising models, Deterministic Point Processes (DPPs) and binary pairwise Markov Random Fields (MRFs) (Djolonga and Krause, 2014, 2015).

This model class is of interest because the contributions of this work focus on extensions to a novel probabilistic log-submodular model proposed by Tschiatschek et al. (2016) which will be described in the next section.

#### 3.3.1 Partition Function

In log-submodular models the normalization constant  $Z$  is known as the *partition function* (Djolonga and Krause, 2014).  $Z$  is necessary to fully determine the normalized model and compute quantities such as marginal probabilities, however its exact computation is known to be #P-complete (Jerrum and Sinclair, 1990). This makes it impossible in practice to perform such computations except for special cases of the model.

## 3.4 Facility Location Diversity Model (FLID)

Every modular function can be written as a sum of individual weights for each element  $i \in V$  assuming a normalization such that  $F(\emptyset) = 0$  (Krause and Golovin, 2014), i.e.

$$F(S) = \sum_{i \in S} u(i) \quad (3.3)$$

Where  $u(i)$  is some function  $u : V \rightarrow \mathbb{R}$ . For convenience, denote  $\mathbf{u}$  as the vector of modular weights where  $u_i = u(i)$ . Therefore any log-modular function can be written as:

$$P(S) \propto \exp\left(\sum_{i \in S} u_i\right) = \prod_{i \in S} \exp(u_i) \quad (3.4)$$

This is the simplest log-submodular probability function and is the modular part for the FLID model proposed by Tschiatschek et al. (2016). In this model,  $\mathbf{u}$  can be thought of as modeling the relevance or utility of each element, for example in the context of spatial summarization this utility could be proportional to the popularity of a location or to how many times it has been photographed.

However, these utilities alone can not capture interactions between the elements in a set. To address this, Tschiatschek et al. (2016) propose an additional term that models set diversity, its objective is to characterize redundant elements and assign lower probabilities to sets that contain them.

This term is based on representing each element  $i$  with an  $L$ -dimensional vector  $\mathbf{w}^b \in \mathbb{R}_{\geq 0}^L$ , where each dimension  $1 \leq d \leq L$  captures a concept related to set diversity and  $w_{i,d}^b$  quantifies the relevance of each element  $i$  for each concept  $d$  (Tschiatschek et al., 2016). Hereby, define  $\mathbf{W}^b \in \mathbb{R}_{\geq 0}^{|V| \times L}$  as the matrix where the  $i$ -th row is the representation  $\mathbf{w}^b$  of element  $i$ .

For each dimension  $d$ , the diversity of a set  $S$  is quantified by  $\max_{i \in S} w_{i,d}^b - \sum_{i \in S} w_{i,d}^b$ . This assigns a negative value for sets that contain more than one element with nonzero weight

Table 3.1: FLID probability distribution for the scenario in Example 3.4.2.

$S$	$P(S)$
$\{h,s\}, \{h,f\}$	$\approx 0.41$
$\{h\}, \{s\}, \{f\}$	$\approx 0.06$
$\{\}, \{s,f\}, \{h,s,f\}$	$\approx 0.00$

$w_{i,d}^b$  in that dimension (Tschiatschek et al., 2016). Equation (3.5) shows the complete term, this sums over all  $L$  dimensions to account the diversity in each concept.

$$\text{div}(S) = \sum_{d=1}^L \left( \max_{i \in S} w_{i,d}^b - \sum_{i \in S} w_{i,d}^b \right) \quad (3.5)$$

Finally, the complete FLID model proposed by Tschiatschek et al. (2016) combines these two terms and is given by:

$$P(S) = \frac{1}{Z} \exp \left( \sum_{i \in S} u_i + \sum_{d=1}^L \left( \max_{i \in S} w_{i,d}^b - \sum_{i \in S} w_{i,d}^b \right) \right) \quad (\text{FLID})$$

### 3.4.1 Partition Function

As mentioned before, the computation of the partition function is generally intractable for log-submodular models. However, for FLID this can be computed in time  $O(|V|^{L+1})$  which is polynomial on the size of the ground set and significantly better than the cost of enumerating the powerset of  $V$ , i.e.  $O(2^{|V|})$  (Tschiatschek et al., 2016). Nevertheless, it is worth noting that this complexity is exponential in  $L$  which means that the partition function computation is only practical for a limited range of FLID models, namely those with  $L \ll |V|$ .

### 3.4.2 Example: Two Landmarks

In order to better illustrate the model, consider a town with 3 popular locations: A town hall ( $h$ ), a statue ( $s$ ) and a fountain ( $f$ ). Assume that historic data shows that visitors only take photos at either the town hall and the statue, or at the town hall and the fountain. This can be modeled with FLID by introducing a latent dimension representing some concept that is present in both the statue and the fountain but not in the town hall, e.g. "is not a building".

Concretely, let  $V = \{h,s,f\}$  and  $\mathbf{u} = (2, 2, 2)^\top$ , indicating that all locations are equally popular. A suitable diversity weight matrix would then be  $\mathbf{W}^b = (0, 20, 20)^\top$ . Table 3.1 shows the resulting probabilities of the subsets, accurately representing the aforementioned problem. Note that the probabilities are not exactly 0.5/0.5 for the sets of interest but this could be easily fixed by increasing the utilities to ensure that sets of size 2 have a larger unnormalized magnitude compared to individual ones.

## 3.5 Learning from Data

Much of the interest in log-submodular models is concentrated on performing inference for known functions (Tschiatschek et al., 2016). However, Tschiatschek et al. (2016) explore how to learn such models from data, i.e. estimate the model parameters from observations with an unknown distribution.

### 3.5.1 Learning Log-modular Models

A special case of log-submodular models where the parameters can be estimated efficiently using Maximum Likelihood Estimation (MLE) is the log-modular model presented in Equation (3.4). For this basic model, it is possible to estimate the utilities  $\mathbf{u}$  through the maximum likelihood estimator for the marginals  $\hat{P}(i \in S)$ . These marginals are given by<sup>1</sup>:

$$P(i \in S) = \frac{1}{1 + \exp(-u_i)} \quad (3.6)$$

And its maximum likelihood estimator for a dataset  $\mathcal{D}$  is:

$$\hat{P}(i \in S) = \frac{N(\#i \in S)}{|\mathcal{D}|} \quad (3.7)$$

Therefore, the utilities can be estimated as:

$$u_i = -\log \left( \frac{1}{\hat{P}(i \in S)} - 1 \right) \quad (3.8)$$

This model also allows computing the partition function in closed form, this is given by:

$$Z = \prod_{i \in S} (1 + \exp(u_i)) \quad (3.9)$$

Because these quantities can be computed in closed form, it is possible to efficiently estimate the parameters of a log-modular model and sample from it. This is useful for producing noise samples and constructing baselines for the experiments.

Even though MLE is the commonly used method for the task of parameter estimation from data, it only works efficiently for normalized models (Gutmann and Hyvärinen, 2012). This makes it intractable for the general class of log-submodular models, and also a wide range of FLID models because the computation of  $Z$  is exponential on  $L$ . Gutmann and Hyvärinen (2012) propose an alternative method for parameter estimation in unnormalized models known as Noise Contrastive Estimation (NCE) and this is the method used by Tschiatschek et al. (2016) to learn the FLID model.

### 3.5.2 NCE Learning

The idea behind NCE is to transform the unsupervised learning task of estimating a probability density from data into a supervised classification task. In order to do this, the observed data  $\mathcal{D}$ , assumed to be drawn from an unknown distribution  $P_d$ , is compared to an artificially generated set of noise samples  $\mathcal{N}$  drawn from a known distribution  $P_n$  that can be efficiently normalized. The classification task is to maximize the likelihood of correctly discriminating each sample as either observed data or artificial noise.

Formally, denote  $\mathcal{A}$  as the complete set of labeled samples, i.e.  $\mathcal{A} = \{(S, Y_s) : S \in \mathcal{D} \cup \mathcal{N}\}$  where  $Y_s = 1$  for  $S \in \mathcal{D}$  and  $Y_s = 0$  for  $S \in \mathcal{N}$ . Additionally, let  $v$  be the noise-to-data ratio, i.e.  $v = |\mathcal{N}|/|\mathcal{D}|$ .

The goal is to estimate the posterior probabilities  $P(Y_s = 1 | S; \theta)$  and  $P(Y_s = 0 | S; \theta)$ , in order to discriminate noise from data samples. These probabilities are given by equations (3.10) and (3.11).

$$P(Y_s = 1 | S; \theta) = \frac{\hat{P}_d(S; \theta)}{\hat{P}_d(S; \theta) + vP_n(S)} \quad (3.10)$$

$$P(Y_s = 0 | S; \theta) = \frac{vP_n(S)}{\hat{P}_d(S; \theta) + vP_n(S)} \quad (3.11)$$

---

<sup>1</sup>Refer to the appendix for the derivation of these equations.

It is worth noting that  $\hat{P}_d$  is used instead of  $P_d$ , because the real density is not known. As indicated by Gutmann and Hyvärinen (2012),  $\hat{P}_d$  can be an unnormalized distribution for NCE where the partition function  $Z$  is included in the set of parameters  $\theta$  as  $\hat{Z}$ , hence resulting in an approximately normalized distribution after the optimization.

In order to obtain these posterior probabilities, the following conditional log-likelihood objective is maximized (Gutmann and Hyvärinen, 2012):

$$g(\theta) = \sum_{S \in \mathcal{D}} \log P(Y_s = 1 | S; \theta) + \sum_{S \in \mathcal{N}} \log P(Y_s = 0 | S; \theta) \quad (3.12)$$

This maximization can be performed using a gradient-based optimization method such as Stochastic Gradient Descent (SGD).

### Practical Considerations

A couple of considerations are necessary for obtaining good estimates with NCE (Gutmann and Hyvärinen, 2012):

1. The parameterized probability function  $\hat{P}_d(S; \theta)$  must be of the same family as the real distribution  $P_d$ , i.e.  $\exists \theta^* | \hat{P}_d(S; \theta^*) = P_d$ .
2. The noise distribution  $P_n$  is nonzero whenever  $P_d$  is nonzero.

Additionally, the following statements must be considered when choosing the noise distribution  $P_n$  (Gutmann and Hyvärinen, 2012):

1. A distribution that can be sampled easily.
2. Noise that is as similar as possible to the data, otherwise the classification problem could be too easy.
3. A noise sample as large as possible.

### 3.5.3 Learning FLID via NCE and SGD

Stochastic Gradient Descent was used by Tschiatschek et al. (2016) to learn the FLID model through NCE. SGD is a gradient-based method that has proven effective in large-scale learning tasks due to its efficiency when the computation time is a limiting factor (Bottou, 2010; Zhang, 2004), hence making it appropriate for the scale of data sourced from the internet, such as public user photos in Flickr.

In each iteration, the gradient  $\nabla \log P(Y_s = y | S; \theta)$  must be computed. For FLID, the parameter vector  $\theta$  is composed by  $\mathbf{u}, \mathbf{W}^b$  and  $\hat{Z}$ , and the corresponding gradients are given by:

$$\nabla \log P(Y_s = y | S; \theta) = \left( y - \frac{1}{1 + \nu \frac{P_n(S)}{\hat{P}_d(S; \theta)}} \right) \nabla \log \hat{P}_d(S; \theta) \quad (3.13)$$

$$\hat{P}_d(S; \theta) = \frac{1}{\hat{Z}} P(S; \mathbf{u}, \mathbf{W}^b) \quad (3.14)$$

$$(\nabla_{\mathbf{u}} \log \hat{P}_d(S; \hat{Z}, \mathbf{u}, \mathbf{W}^b))_i = \begin{cases} 1 & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases} \quad (3.15)$$

$$(\nabla_{\mathbf{W}^b} \log \hat{P}_d(S; \hat{Z}, \mathbf{u}, \mathbf{W}^b))_{i,d} = \begin{cases} -1 & \text{if } i \in S \text{ and } i \neq \text{argmax}_{j \in S} w_{j,d}^b \\ 0 & \text{otherwise} \end{cases} \quad (3.16)$$

$$\nabla_{\hat{Z}} \log \hat{P}_d(S; \hat{Z}, \mathbf{u}, \mathbf{W}^b) = \frac{-1}{\hat{Z}} \quad (3.17)$$

Where  $P(S; \mathbf{u}, \mathbf{W}^b)$  is the unnormalized (FLID) equation,  $(\nabla_{\mathbf{u}} \cdot)_i$  represents the  $i$ -th entry of the gradient with respect to  $\mathbf{u}$  and  $(\nabla_{\mathbf{W}^b} \cdot)_{i,d}$  represents the  $(i, d)$ -th entry of the sub-gradient with respect to  $\mathbf{W}^b$  (Tschiatschek et al., 2016).

### Computational Complexity

As described by Tschiatschek et al. (2016), the running time requirement for calculating the sub-gradient for FLID is  $O(L|S|)$ , and for a complete pass over data and noise samples is then  $O(|\mathcal{D} \cup \mathcal{N}| \kappa L)$  where  $\kappa = \max_{S \in \mathcal{D} \cup \mathcal{N}} |S|$ . This shows that learning is efficient as it is only linear on the size of the data and the noise, assuming small values for  $\kappa$  and  $L$ .

#### 3.5.4 AdaGrad

A commonly used learning rate function for SGD is  $\eta(t) = \eta_0 t^{-p}$ , which offers the best convergence speed in theory and also does it often in practice (Bottou, 2012), however it can lead to poor performance due to slow rates of convergence to the solution (Darken and Moody, 1992). By contrast, choosing a larger  $\eta_0$  may not lead to better results due to the instability in the parameters for small  $t$  (Darken and Moody, 1990).

This behavior was observed during the experiments with real data to be presented in later sections, therefore an alternative for the learning rate was sought. In particular, Adaptive Gradient (AdaGrad) was implemented.

AdaGrad was proposed by Duchi et al. (2011), the idea of this method is to adapt the learning rate for each parameter in  $\theta$  individually in a way that frequently updated parameters have slower learning rates while infrequent parameters have larger ones. The intuition is that an update to a rare parameter is more informative than one to a parameter that is frequently updated.

Concretely, with AdaGrad the update step for each parameter  $\gamma$  in  $\theta$  is given by:

$$\theta_\gamma^{\tau+1} \leftarrow \theta_\gamma^\tau - \frac{\eta}{\sqrt{G_\gamma}} \nabla g(\theta)_\gamma^\tau \quad (3.18)$$

Where  $\mathbf{G}^\tau$  is the vector of accumulated gradients at time  $\tau$  and each component  $G_\gamma^\tau$  is given by:

$$G_\gamma^\tau = \sum_{t=1}^{\tau} (\nabla g(\theta)_\gamma^t)^2 \quad (3.19)$$

Where  $\nabla g(\theta)_\gamma^t$  denotes the gradient of the  $\gamma$  parameter at time step  $t$ .

It is worth noting that AdaGrad does not incur in a significant increase in running time or memory requirements for the training which makes it inexpensive to include in the implementation of NCE.

## 3.6 Mean-shift Clustering

TODO

# Chapter 4

## Extending FLID

### 4.1 Beyond Diversity: FLDC

Diversity is an important property in the context of summarization (Tschiatschek et al., 2016), however it is not the only one. Coherence is also a desired property of summaries, particularly for structured summaries (Yan et al., 2011). Balancing coherence and diversity is considered a challenge and maximizing diversity alone may lead to disconnected summaries, while focusing on coherence can hinder the breadth of the resulting summaries (Shahaf et al., 2012).

To better understand why coherence is important, consider the construction of a spatial summary for photos taken in a city. A model based on spatial diversity would create sets containing photos taking at distant locations in the city, however another possibility is to have summaries that contain photos that were taken close to some starting point, in this case the desired property is not spatial diversity but rather coherence.

Therefore, a natural extension to the FLID model is to add the capacity of modeling coherence along with diversity. I propose the addition of a log-supermodular term analogous to the log-submodular term (3.5) in order to model the complementarity between elements. This supermodular term introduces a representation of the elements using  $K$ -dimensional vectors  $\mathbf{w}^K \in \mathbb{R}_{\geq 0}^K$ , where each dimension  $1 \leq c \leq K$  captures a concept that relates to set coherence and  $w_{i,c}$  quantifies the contribution of each element  $i$  to each coherence concept  $c$ . The coherence of a set  $S$  with respect to each concept  $c$  is quantified as  $\sum_{i \in S} w_{i,c}^e - \max_{i \in S} w_{i,c}^e$  which is a supermodular term mirroring the submodular term in FLID. Summing over all the concepts results in the proposed coherence term. Hereby, define  $\mathbf{W}^e \in \mathbb{R}_{\geq 0}^{|V| \times K}$  as the matrix where the  $i$ -th row is the representation  $\mathbf{w}^e$  of element  $i$ .

Adding the coherence term to FLID results in the extended model, which I refer to as the Facility Location Diversity and Coherence (FLDC) model. Its probability distribution is defined as:

$$P(S) = \frac{1}{Z} \exp \left( \sum_{i \in S} u_i + \text{div}(S) + \sum_{c=1}^K \left( \sum_{i \in S} w_{i,c}^e - \max_{i \in S} w_{i,c}^e \right) \right) \quad (\text{FLDC})$$

#### 4.1.1 Partition Function

For the FLDC model, the partition function can be computed in  $O(|V|^{L+K+1})$  time. This complexity can be derived using a similar algorithm as the one presented by Tschiatschek et al. (2016) for FLID, the key observation is that the algorithm they present only requires the ordering of weights to determine eligible element for a set given a set of maximum weights per dimension, therefore this can be easily extended to include the ordering

Table 4.1: Probability distribution for Example 4.1.3.

$S$	$P(S)$
{1, 2}	0.5
{3, 4}	0.5
Otherwise	0.0

for the coherence weights as well. This requires evaluating  $K$  more dimensions which results in the added term to the exponent of the complexity expression, which makes the computation of the partition function more impractical than in the FLID case.

### 4.1.2 Learning FLDC Models

As with FLID, the parameters for an FLDC model can be estimated using NCE. For FLDC the vector  $\theta$  is composed of  $[\mathbf{u}, \mathbf{W}^b, \mathbf{W}^e, \hat{\mathbf{Z}}]$  and its gradient is the same as in equations (3.13)-(3.17) with the addition of a gradient with respect to the new parameter,  $\mathbf{W}^e$ , which is given by:

$$\left( \nabla_{\mathbf{W}^e} \log \hat{P}_d(S; \hat{\mathbf{Z}}, \mathbf{u}, \mathbf{W}^b, \mathbf{W}^e) \right)_{i,c} = \begin{cases} 1 & \text{if } i \in S \text{ and } i \neq \operatorname{argmax}_{j \in S} w_{j,c}^e \\ 0 & \text{otherwise} \end{cases} \quad (4.1)$$

Where  $(\nabla_{\mathbf{W}^e} \cdot)_{i,c}$  represents the  $(i, c)$ -th entry in the gradient with respect to  $\mathbf{W}^e$ .

This also affects the overall complexity of the learning algorithm. Computing the gradient takes  $O(L + K)|S|$  for FLDC at each step, therefore the running time of a full pass over the data and noise samples takes  $O(|\mathcal{D} \cup \mathcal{N}| \kappa(L+K))$ . This is not significantly more expensive than learning FLID under the assumption that  $K \approx L$ .

### 4.1.3 Example: Two Non-overlapping Clusters

As an example of the extended model, consider the distribution presented in Table 4.1 for  $V = \{1, 2, 3, 4\}$ , representing a set of two non-overlapping clusters with 2 elements each. Intuitively, this can be modeled by considering that there is diversity between elements in different clusters while inside a cluster there is a coherence component that ties them together.

Concretely, the weight matrices  $\mathbf{W}^b, \mathbf{W}^e$  in Figure 4.1 illustrate one possible instance of the model. The corresponding utility vector is  $\mathbf{u} = \vec{0}$ , because there is no indication that an individual element is favored over the pairs. This model is easily interpretable and accurately realizes the distribution.

## 4.2 Generalizing through Features: FFLDC

An important characteristic of the FLID model, and by extension the FLDC one, is that it is agnostic to the type of elements in the ground set, this allows its application to a wide range of problems without prior knowledge. However the downside is that the model has no capability to make use of information about the elements, if available, to improve the modeling of the data. Moreover, if a new element is added to the set there is no way to generalize the existing knowledge about similar elements.

In order to solve these problems, I propose a further extension to the model. Firstly, an element ( $i \in V$ ) is represented as a vector  $\mathbf{x}_i \in \mathbb{R}^M$  where each entry is a feature, e.g. for venues one feature could be its aggregated rating while another indicates whether it is

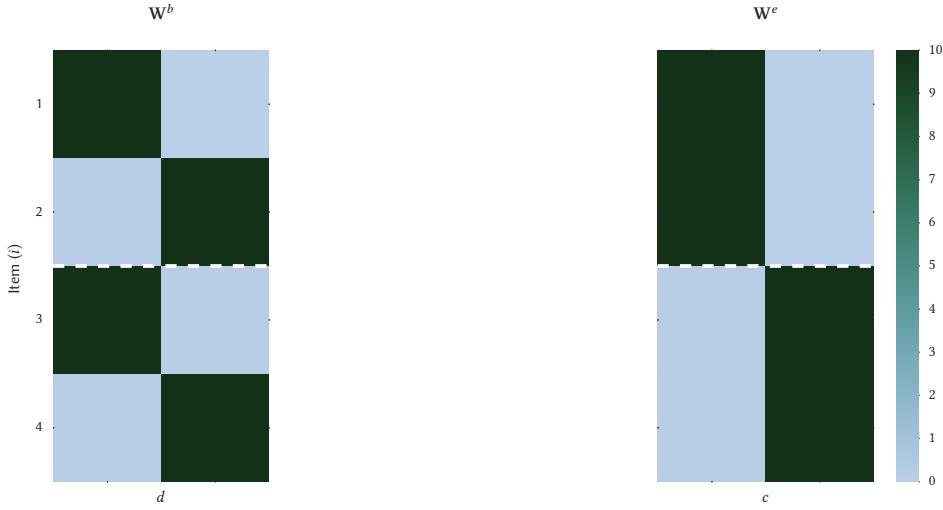


Figure 4.1: Diversity and coherence weights for FLDC model in Example 4.1.3. The dotted line divides the clusters.

indoors or outdoors. The representations of all elements is collected by matrix  $\mathbf{X} \in \mathbb{R}^{|V| \times M}$  where each row  $i$  corresponds to the representation  $\mathbf{x}_i$  of element  $i$ .

Then, the utility vector  $\mathbf{u}$  and weight matrices  $\mathbf{W}^b, \mathbf{W}^e$  are factorized to include this feature matrix  $\mathbf{X}$  as follows:

$$\mathbf{u} = \mathbf{X}\mathbf{a} \quad (4.2)$$

$$\mathbf{W}^b = \mathbf{X}\mathbf{B} \quad (4.3)$$

$$\mathbf{W}^e = \mathbf{X}\mathbf{E} \quad (4.4)$$

Where  $\mathbf{a} \in \mathbb{R}^M$  represents the contribution of each feature to the total utility of an item, while  $\mathbf{B} \in \mathbb{R}^{M \times L}$  and  $\mathbf{E} \in \mathbb{R}^{M \times K}$  encode the contribution of each feature to each latent diversity and coherence dimension, respectively. The intuition behind this factorization is that the information about the items can enhance the latent representations, hence producing a richer model. For example if  $M < |V|$  and the features are sufficient to represent the items, it is expected that learning will be easier because of the reduced number of parameters with respect to learning the equivalent FLDC model.

I refer to the extended model as the Featurized Facility Location Diversity and Coherence (FFLDC) model and its probability distribution is defined as:

$$P(S) = \frac{1}{Z} \exp \left( \sum_{i \in S} \sum_{k=1}^M x_{i,k} a_k + fdiv(S) + fcoh(S) \right) \quad (\text{FFLDC})$$

$$fdiv(S) = \sum_{d=1}^L \left( \max_{i \in S} \sum_{k=1}^M x_{i,k} b_{k,d} - \sum_{i \in S} \sum_{k=1}^M x_{i,k} b_{k,d} \right) \quad (4.5)$$

$$fcoh(S) = \sum_{c=1}^K \left( \sum_{i \in S} \sum_{k=1}^M x_{i,k} e_{k,c} - \max_{i \in S} \sum_{k=1}^M x_{i,k} e_{k,c} \right) \quad (4.6)$$

Where  $a_i$  is the  $i$ -th entry of  $\mathbf{a}$ ,  $b_{k,d}$  is the  $(k,d)$ -th entry of  $\mathbf{B}$ ,  $e_{k,c}$  is the  $(k,c)$ -th entry of  $\mathbf{E}$  and  $x_{i,k}$  is the  $(i,k)$ -th entry of  $\mathbf{X}$ .

*Remark.* If  $\mathbf{X} = \mathcal{I}$ , then FFLDC is equivalent to FLDC, with  $\mathbf{a} = \mathbf{u}$ ,  $\mathbf{W}^b = \mathbf{B}$  and  $\mathbf{W}^e = \mathbf{E}$ .

The use of features also allows the application of the model to previously unseen elements, hence solving the aforementioned problem of generalization. This is possible because the model is defined on the space of features and not items, and for a previously unseen element  $j$  only its representation  $\mathbf{x}_j$  is required in order to evaluate the probability of sets containing this element.

### 4.2.1 Partition Function

The computation of the partition function for this model can be done using the fact that any FFLDC model can be transformed into an equivalent FLDC model through the factorizations in equations (4.2)-(4.4). This transformation requires  $O(|V|M(L+K))$  time for the matrix multiplications, therefore the overall complexity of calculating the partition function for an FFLDC model is  $O(|V|M(L+K) + |V|^{L+K+1})$ . This expression can be simplified to  $O(|V|^{L+K+1})$  because the exponential term is significantly larger assuming sensible values for  $M$ . Given that the complexity of computing the partition function for FFLDC is equivalent to the FLDC model, it is also intractable in practice.

### 4.2.2 Learning FFLDC Models

For FFLDC, the parameter vector  $\theta$  is different from the previous models, it is composed by  $[\mathbf{a}, \mathbf{B}, \mathbf{C}, \hat{\mathbf{Z}}]$  and the gradient of the NCE objective  $g(\theta)$  is given by:

$$\left( \nabla_{\mathbf{a}} \log \hat{P}_d(S; \hat{\mathbf{Z}}, \mathbf{a}, \mathbf{B}, \mathbf{E}) \right)_m = \sum_{i \in S} x_{i,m} \quad (4.7)$$

$$\left( \nabla_{\mathbf{B}} \log \hat{P}_d(S; \hat{\mathbf{Z}}, \mathbf{a}, \mathbf{B}, \mathbf{E}) \right)_{m,d} = x_{i^d, m} - \sum_{i \in S} x_{i,m} \quad (4.8)$$

$$\left( \nabla_{\mathbf{E}} \log \hat{P}_d(S; \hat{\mathbf{Z}}, \mathbf{a}, \mathbf{B}, \mathbf{E}) \right)_{m,c} = x_{i^c, m} - \sum_{i \in S} x_{i,m} \quad (4.9)$$

Where  $i^d = \text{argmax}_{i \in S} \sum_{k=1}^M x_{i,k} b_{k,d}$  and  $i^c = \text{argmax}_{i \in S} \sum_{k=1}^M x_{i,k} e_{k,c}$ .  $(\nabla_{\mathbf{a}} \cdot)_m$  represents the  $m$ -th entry of the sub-gradient with respect to  $a$ ,  $(\nabla_{\mathbf{B}} \cdot)_{m,d}$  represents the  $(m,d)$ -th entry of the sub-gradient with respect to  $B$  and  $(\nabla_{\mathbf{E}} \cdot)_{m,c}$  represents the  $(m,c)$ -th entry of the sub-gradient with respect to  $E$ .

*Remark.* For  $\mathbf{X} = \mathcal{I}$ , the sub-gradients in 4.7-4.9 are equivalent to the FLDC sub-gradients. Because  $x_{i,m} = 1$  only when  $i = m$ .

Regarding running time complexity, the FFLDC model requires more operations for the gradient due to the inclusion of features. Concretely, computing the gradient for a set  $S$  takes  $O(|S|M(L+K))$  which makes the running time of a single pass over data and noise samples  $O(|\mathcal{D} \cup \mathcal{N}|M(L+K)\kappa)$ . Even though this is larger than the complexity of FLID or FLDC, it is still linear on the data.

### 4.2.3 Example: Rated Locations

A small town has 6 popular locations, each of them has been rated from 0 to 5, where 0 represents a bad review and 5 an excellent review. Collected data shows that a typical visitor is more likely to visit and take photos at places with high ratings, however this is not the only factor driving their behavior. For example, an average visitor does not visit two places that serve food on the same day instead the data shows that visitors usually go to one place that serves food and then to an outdoor one, or they just visit one that has both characteristics. This data is summarized in Table 4.2, the task is to model this behavior using the FFLDC model.

In this example there is knowledge about the items and what features are relevant for the data. These features are summarized in equation (4.10). The first column corresponds to

Table 4.2: Synthetic data for Example 4.2.3.

Locations visited ( $S$ )	$P(S)$
{1, 3}	0.30
{3, 4}	0.25
{3, 6}	0.15
{2}	0.10
{1}	0.06
{3}, {4}	0.04
{5}, {6}	0.03
Otherwise	0.00

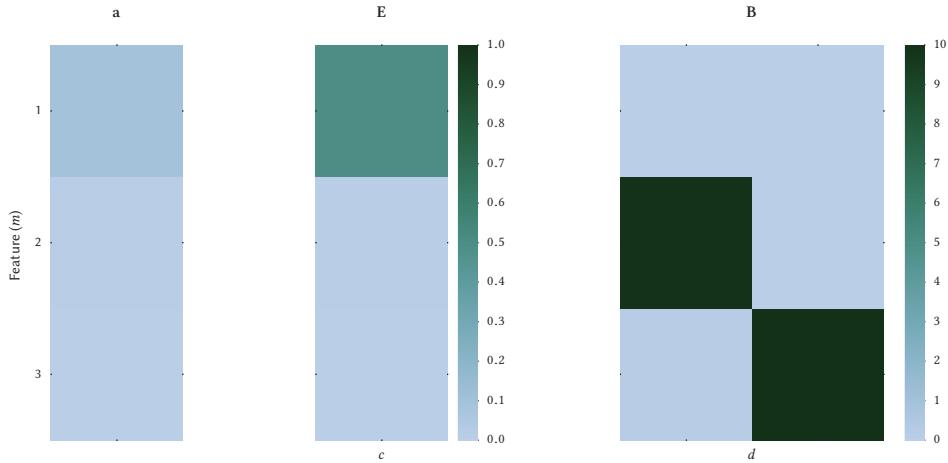


Figure 4.2: FFLDC sample model for Example 4.2.3.

the aforementioned rating, the second and third are binary features indicating whether the location is an outdoor one and whether it serves food, respectively.

$$\mathbf{X} = \begin{pmatrix} 4 & 1 & 0 \\ 4 & 1 & 1 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 0 \end{pmatrix} \quad (4.10)$$

A possibility is an FFLDC model that encourages diversity on the second and third feature, i.e. models the idea that visitors do not go to more than one place that serves food or is outdoor, while assigning a positive utility and coherence value to the first feature, i.e. modeling the preference for places with high ratings. One such model is presented in Figure 4.2, this approximates distribution from Table 4.2 and illustrates the type of model that is useful in this example.

If a new location  $j$  is considered then it is possible to apply the model and estimate the probability of sets that include it. For example, consider a seventh location with the following features  $\mathbf{x}_7 = [5, 1, 0]$ , using the same model parameters a new distribution can be computed without changing the model parameters, as it would be the case with FLID or

Table 4.3: Synthetic data for Example 4.2.3 after adding a new item.

Locations visited ( $S$ )	$P(S)$
{3, 7}	0.24
{1, 3}	0.21
{3, 4}	0.19
{3, 6}	0.10
{7}	0.06
{1}, {2}	0.04
{3}, {4}	0.03
{5}, {6}	0.02
Otherwise	0.00

FLDC. The updated distribution is shown in Table 4.3 and it can be considered a sensible distribution given the problem description and the high rating of the new location, hence showing the model's capacity to generalize.

# Chapter 5

## Synthetic Experiments

This chapter describes experiments done on synthetic data to explore the learning performance of the methods described in the background section.

### 5.1 Learning Setup

This section describes implementation details of NCE and SGD, these are applicable for both the experiments in this chapter as well as those performed on real data, which will be presented in subsequent chapters.

#### 5.1.1 Noise Distribution

As mentioned in section 3.5.2, there are several considerations when choosing the noise distribution. A log-modular distribution is a natural choice for the noise when learning the FLID model (Tschiatschek et al., 2016), and this distribution will also be used for the noise with FLDC and FFLDC models.

The parameters of a log-modular distribution can be estimated efficiently from data through MLE, as given by Equation (3.8). Also the partition function can be computed in closed form, as shown in Equation (3.9). This makes the distribution a good candidate because it can be used efficiently for sampling as well as for computing the normalized probability of sets, i.e.  $P_n(S)$  during the learning procedure.

#### 5.1.2 Initialization

The log-modular model also provides a sensible initialization for vector  $\mathbf{u}$  in the FLID and FLDC models. For FFLDC, the initialization for  $\mathbf{a}$  is done by solving the linear system derived from the factorization in Equation (4.2), this linear system is given by:

$$\mathbf{a} = \mathbf{X}^{-1}\mathbf{u} \tag{5.1}$$

Unfortunately, this system can be under/over-determined in many cases so an approximated solution must be used. The chosen method for finding this solution is Ridge regression which minimizes the squared error  $\sum_{i \in S} (u_i - (\mathbf{X}\mathbf{a})_i)^2$  with a regularization term on the  $l_2$  norm of  $\mathbf{a}$ .

The weight matrices, e.g.  $\mathbf{W}^e, \mathbf{B}$ , are initialized with random values drawn from a uniform distribution  $\mathcal{U} \sim [0, 0.001]$ . These are the same initialization settings proposed by Tschiatschek et al. (2016).

In the cases where AdaGrad is used, the accumulated gradient  $\mathbf{G}$  is initialized with a constant value for each parameter  $\gamma$ , namely  $G_\gamma^0 = 0.01$ .

### 5.1.3 Learning Rate

When not using AdaGrad, the learning rate used in the experiments follows the power law mentioned in section 3.5.4, i.e.

$$\eta(t) = \frac{\eta_0}{t^p} \quad (5.2)$$

### 5.1.4 Projection in SGD

The weight matrices, e.g.  $\mathbf{W}^e, \mathbf{B}$ , are defined only for positive values. Therefore, they must be projected in each gradient step to ensure that the solution stays in the feasible space. This is done following the procedure proposed by Tschiatschek et al. (2016) which produces better results than simply clipping the values to 0. Let  $\theta_h$  be an element of these matrices, e.g.  $\theta_h = w_{i,d}^e$ , then the projection is defined as:

$$\theta_h = \begin{cases} \theta'_h & \text{if } \theta'_h \geq 0 \\ \mathcal{U} \sim [0, 0.001] & \text{otherwise} \end{cases} \quad (5.3)$$

Where  $\theta'_h$  is the unnormalized parameter after the gradient update and  $\mathcal{U} \sim [0, 0.001]$  means that the value is drawn from an uniform distribution between  $[0, 0.001]$  as in the initialization.

### 5.1.5 Termination Criteria

The termination criteria for learning was the number of epochs, i.e. full passes over the data and noise samples.

## 5.2 Example Datasets

This section presents the results obtained after learning the datasets from the examples given for each model. For these experiments, the settings were:

- 1000 samples of data drawn from the corresponding distribution, i.e.  $|\mathcal{D}| = 1000$ .
- 2000 noise samples, i.e.  $\nu = 2$ .
- 100 epochs.
- $\eta_0$  is 0.005 and  $p = 0.1$  for the learning rate, without AdaGrad.

It is worth noting that the presented results for these experiments correspond to the best of multiple runs. It was observed that the learning algorithm does not always arrive to a good solution. This is because  $g(\theta)$  is not a concave function and some initializations may lead to local maxima.

### 5.2.1 FLID: Two Landmarks

Recall the following characteristics about the dataset in Example 3.4.2:

- There are 3 elements,  $h, s, f$ .
- There are only 2 possible sets,  $\{h, s\}$  and  $\{h, f\}$ . Both equally likely, i.e.  $P(S) = 0.5$ .
- A model can be constructed with a single diversity dimension, i.e.  $L = 1$ .

Table 5.1: Learned distribution for Example 4.2.3

$S$	$P(S)$
$\{h, s\}$	0.48
$\{h, f\}$	0.47
$\{h\}$	0.03
$\{h, s, f\}$	0.02

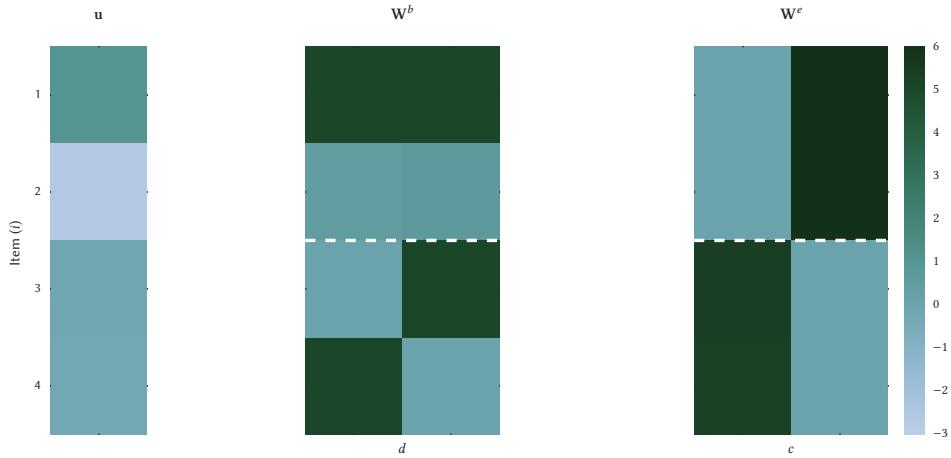


Figure 5.1: Learned model for example 4.1.3. The white line divides the clusters described in the example.

After estimating the parameters from this data, the resulting utility vector and weight matrix are:

$$\mathbf{u} = (5.42, 2.82, 2.79)^\top$$

$$\mathbf{W}^b = (0.02, 6.10, 6.10)^\top$$

This is a slightly different model from the one suggested in section 3.4.2, however the normalized probability distribution for this model presented in Table 5.1 shows that it closely approximates the distribution.

### 5.2.2 FLDC: Two Non-overlapping Clusters

Recall the following characteristics about the dataset in Example 4.1.3:

- There are 4 items, i.e.  $V = \{1, 2, 3, 4\}$ .
- There are only 2 possible sets,  $\{1, 2\}$  and  $\{3, 4\}$ . Both equally likely, i.e.  $P(S) = 0.5$ .
- A model can be constructed with two diversity and two coherence dimensions, i.e.  $L = 2, K = 2$ .

Figure 5.1 shows the resulting utility vector  $\mathbf{u}$  and weight matrices  $\mathbf{W}^b, \mathbf{W}^e$  after the learning procedure.

The model shown in the figure displays the desired properties of diversity between the clusters while having coherence between the items in each one. Despite being different from the proposed model in section 4.1.3 it also approximately realizes the desired distribution.

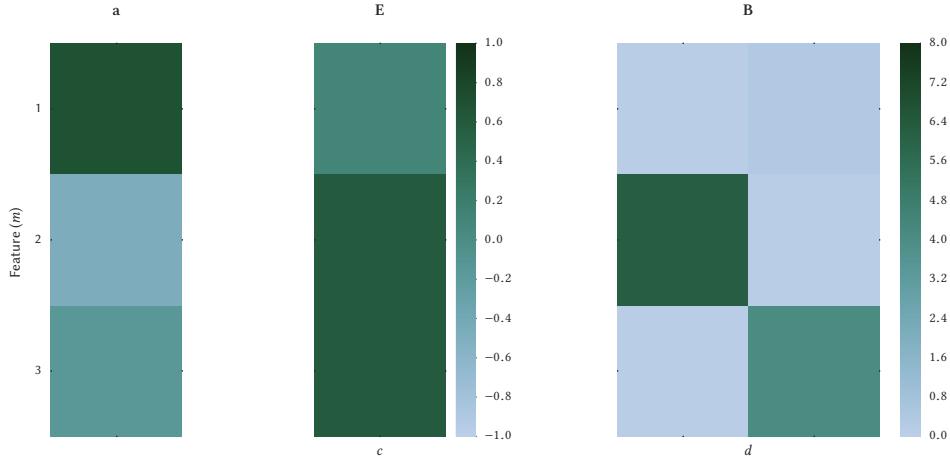


Figure 5.2: Learned model for example 4.2.3

### FFLDC: Rated Locations

Recall the following characteristics about the dataset in Example 4.2.3:

- There are 6 items, i.e.  $V = \{1, 2, 3, 4, 5, 6\}$ .
- The full distribution is related to the features and is given in Table 4.2.
- The proposed model has 2 diversity dimensions and one coherence dimension.

Figure 5.2 shows the resulting model after learning. Unfortunately, this model can be not as easily interpreted as the one proposed in figure 4.2, nevertheless it accurately models the distribution from the example thus showing the learning algorithm's ability to estimate a distribution using features.

## 5.3 Learning Rate Sensitivity

This section explores the learning rate's effect on the learning performance for the synthetic dataset from Example 4.2.3.

The first experiment considers the stability of the optimization without AdaGrad as the learning rate increases. The settings are the same as for the previous experiments with the following additions:

- $\eta_0$  is increased between 0.0005 and 0.02
- The learning is performed independently 8 times to obtain statistics on the results.

Figure 5.3 shows  $g(\theta)$  during the optimization, each point corresponds to the mean value calculated after a given number of epochs and the error bars show the corresponding 95% confidence interval for the mean.

The results in figure 5.3 show that for small values of  $\eta_0$  the learning can be significantly slow and that large values of  $\eta_0$  may lead to instability, as discussed in section 3.5.4. Concretely, for  $\eta_0 \leq 0.01$  the learning is slow but stable, while for  $\eta_0 \geq 0.01$  the objective value converges quickly but oscillates significantly. Fortunately, in the particular case of this synthetic dataset there exists a learning rate for which the objective function converges and is stable, i.e.  $\eta_0 = 0.005$  however this is not always the case for real data.

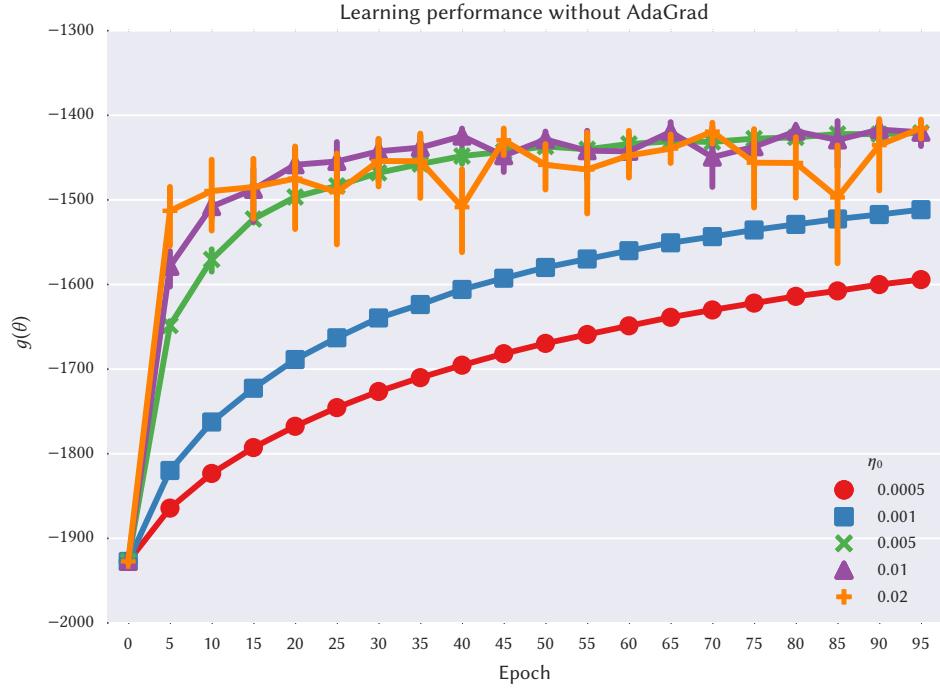


Figure 5.3: Objective function for various values of  $\eta_0$  without using AdaGrad.

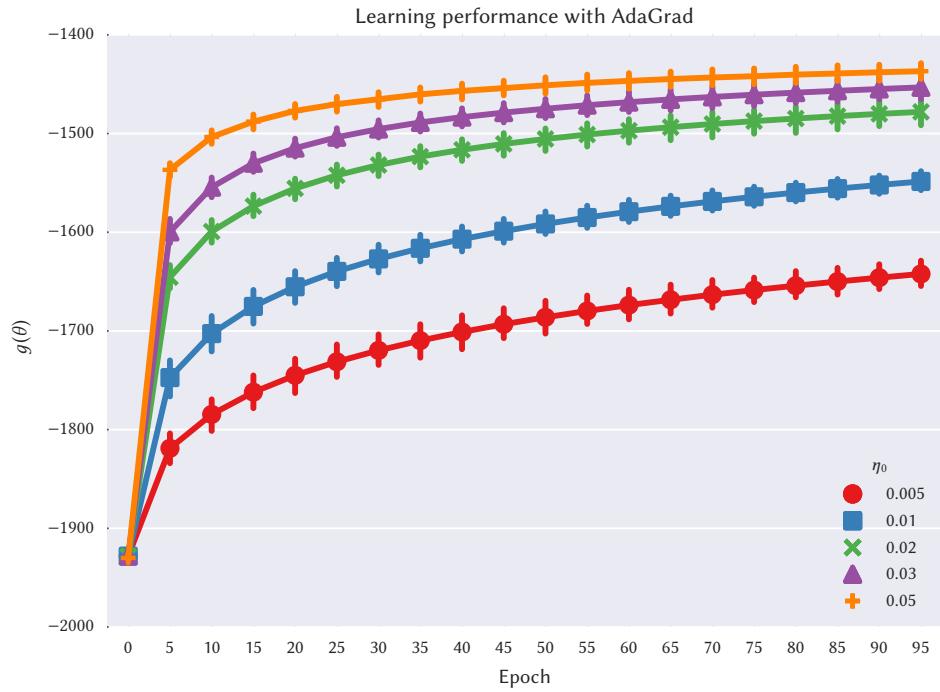


Figure 5.4: Objective function during learning for various values of  $\eta_0$  with AdaGrad.

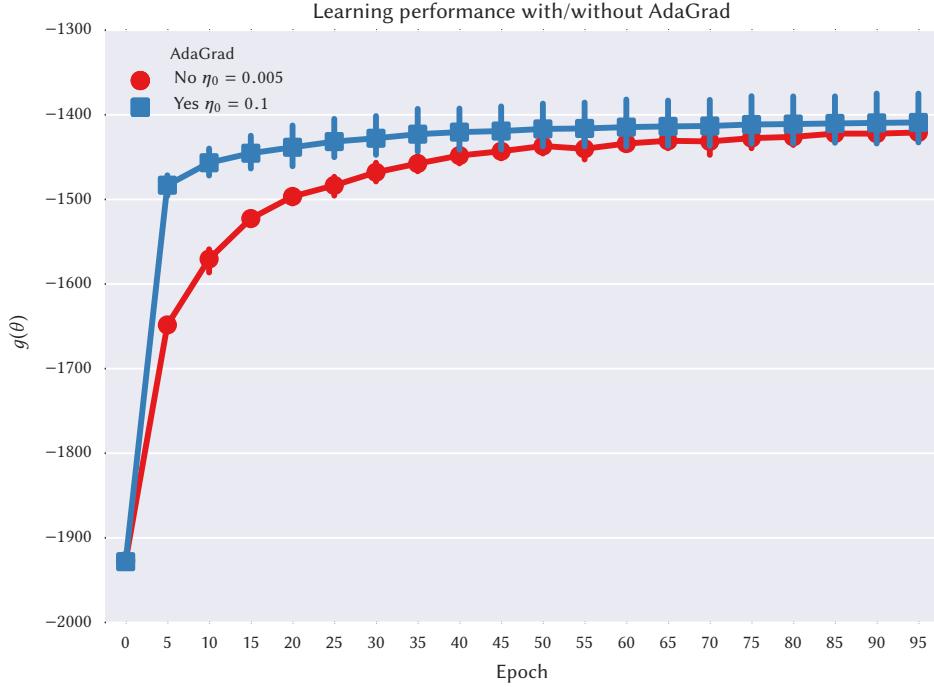


Figure 5.5: Comparison of learning performance with and without AdaGrad.

By contrast, figure 5.4 shows the objective function for  $\eta_0$  between 0.005 and 0.05 after incorporating AdaGrad to the implementation. These results show that training with AdaGrad is stable for a larger range of  $\eta_0$ .

Finally, figure 5.5 shows a direct comparison between training with and without AdaGrad using the best values of  $\eta_0$  for each configuration. In this figure both algorithms converge to an optimal value, however it also shows that training with AdaGrad converges significantly faster while maintaining a stable mean. These combined results indicate that using AdaGrad is the best strategy for learning the FFLDC model.

## 5.4 Noise-to-data Ratio

The noise-to-data ratio  $v$  is another hyperparameter for NCE learning, the recommendation is to have as many noise samples as computationally possible (Gutmann and Hyvärinen, 2012). In order to better understand the effect of  $v$ , an experiment was performed where different models for the dataset from Example 4.2.3 are learned with different values of  $v$ . In order to compare the models, the root mean squared error between the target distribution  $P_d$ , from Table 4.2, and the learned distribution  $\hat{P}_d$  will be used. This is defined as:

$$e_{rms} = \sqrt{\frac{\sum_{S \subseteq V} (P_d(S) - \hat{P}_d(S))^2}{2|V|}} \quad (5.4)$$

The setting is the same as for the learning rate experiments, but with  $\eta_0 = 0.05$  and always using AdaGrad. The noise parameter  $v$  was varied between 0.05 and 3.

The results for this experiment are shown in Figure 5.6. It shows that increasing the number of noise samples reduces the approximation error of the resulting model as expected, interestingly the results suggest that there is a threshold after which adding more noise

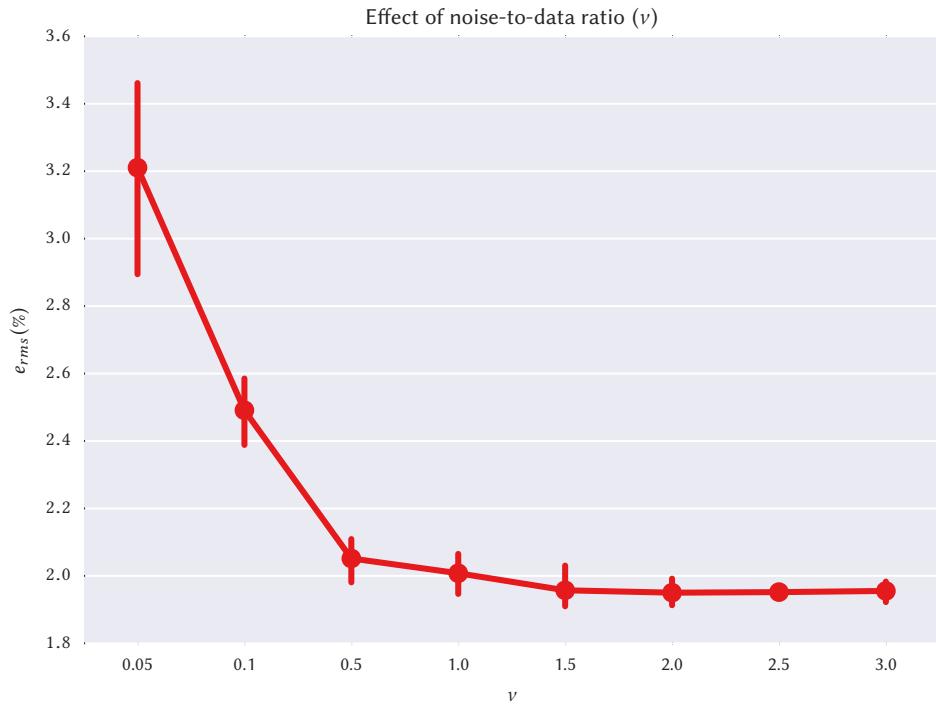


Figure 5.6: Approximation error for different values of  $v$ .

does not have a significant effect on the results. Additionally, it can be seen that adding more noise samples reduces the error variability even though the mean remains the same.



# Chapter 6

## Modeling an Image Collection - Setup

This chapter describes an application of the models presented in the Chapter 4 to a real dataset. As mentioned in the introduction, finding and recommending paths for tourists is an interesting research topic. These paths can be understood as spatial summaries of a city, representing the most important locations to visit and photograph in a city.

These tourist paths can be discovered from a large collection of geotagged images by using the accompanying timestamps as explained in section 6.2. Even though these paths are naturally ordered, they can also be used as unordered sets of locations that represent the behavior of different visitors taking photographs in the city, which can be modeled using the FLID, FLDC and FFLDC models. The modeling quality of these proposed models was compared to other existing methods from the literature as explained in section 6.4. The procedure used for evaluating the results are described in section 6.5.

### 6.1 Flickr Dataset

For the experiments on real data, public geotagged photos from Flickr were used. There are close to two million geo-tagged photos available on Flickr<sup>1</sup>, this offers an unique and rich dataset for the problem of spatial summarization. But as described before, this work focuses on the city level. Therefore, the experiments were performed on a subset of the Flickr data, namely on public geotagged photos taken within Zürich, Switzerland.

The Flickr API<sup>2</sup> provides multiple functions to explore and use the data, in particular the search endpoint allows developers to retrieve photo records according to a specific criteria. Each photo record contains the following information:

**Owner** The ID of the user who uploaded and owns the photo.

**Title** A title given by the user.

**Description** A description given by the user.

**Visibility** Indicates if a photo is public, private or semi-private, i.e. shared only with friends.

**Dates** Dates when the photo was taken, uploaded and last modified.

**Geo** Indicates if the photo is geotagged, the geotag's precision, latitude and longitude.

---

<sup>1</sup><https://www.flickr.com/map> reports 1,833,307 geotagged photos as of April 9th, 2016.

<sup>2</sup><https://www.flickr.com/services/api/>



Figure 6.1: Bounding box used for filtering photos taken in the Zürich area.

**License** Copyright license.

**Tags** Text tags set by users.

**Machine tags** Automatic text tags set by Flickr.

**URL** The URL to download the photo.

Most of these properties can be used to filter the search results, for collecting the desired data for Zürich the following criteria was used:

- The photo was taken between 2005 and 2015.
- The photo is marked as public and has no copyright restrictions.
- The photo is geotagged and has street level accuracy. The location of the photo must be within the area shown in Figure 6.1.

The API sets some limitations on the search function, namely it does not allow more than 3600 requests per hour and it restricts the search results to the first 4000 records. This means that it was not possible to issue a single search request with the outlined criteria, instead the retrieval was done over the course of several hours though single month queries. This ensured that each query had less than 4000 results, hence retrieving all the available records.

This search yielded 168608 photos taken by 6159 users. Figure 6.2 shows a random sample of these photos.

Figure 6.3 shows the distribution of photos taken in 2015. The majority of them is concentrated at the city center also around locations of interest outside the center like Uetliberg, Oerlikon and the Zoo. This indicates that the dataset contains good information regarding the places to visit and take photographs in Zürich. Figure 6.4 shows zoomed in areas

## TODO: COLLAGE

Figure 6.2: A random sample of the photos available in the dataset for Zürich.

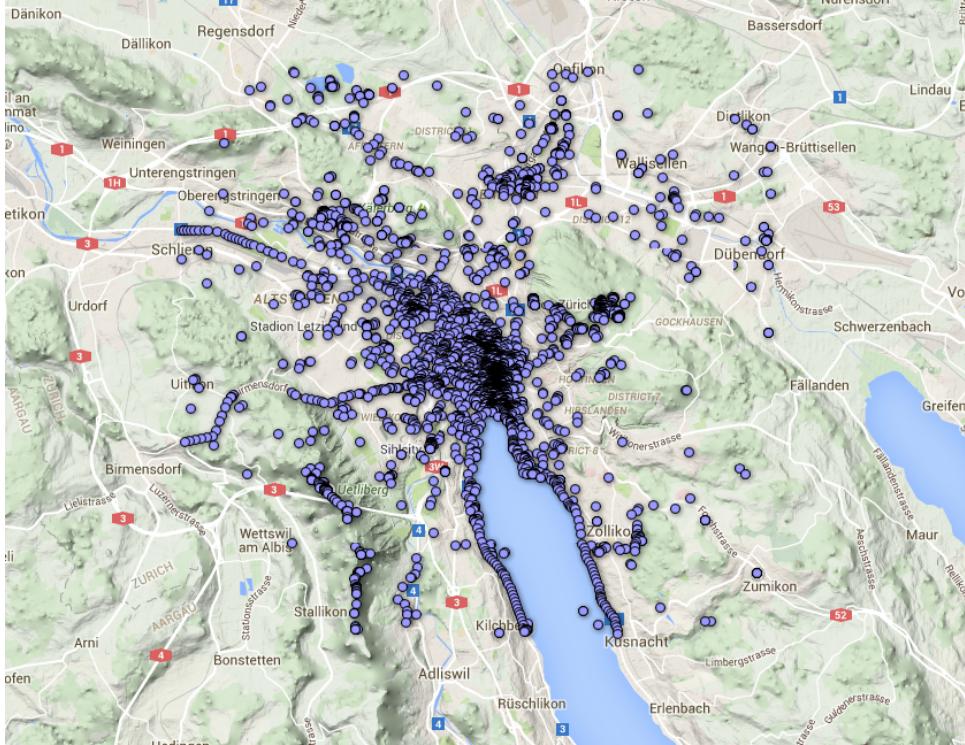


Figure 6.3: Feature map with the photos from the Zürich dataset taken in 2015.

around the city showing that the data is mostly concentrated in touristic places like the historic center, where Fraumünster, Grossmünster and the city hall are.

Additional to the spatial coverage of the dataset, one interesting property to explore is how the photos are distributed across users. The histogram of photos taken by each user is shown in figure 6.5, this is cut off at 100 photos because the distribution has a heavy tail that is unimportant. This heavy tail is due to professional users like newspapers and event venues that have uploaded hundreds of photos under a single user, these users are not of particular interest for the use case of this work because they do not reflect the interests of a typical visitor. The histogram shows that most of the users uploaded one or two pictures which indicates that most of the photos come from very active users that take hundred of photos, this should be taken into account when analyzing the results.

## 6.2 Path Finding

The photos collected in the previous section were used to identify the important locations that are often photographed by many people. These locations define the ground set  $V$  for the models. In order to identify the popular locations automatically, the method proposed by Kleinberg et al. (2009) was used.

Kleinberg et al. (2009) define the problem of finding highly-photographed locations as a clustering problem in a two-dimensional feature space, namely the space defined by the spatial coordinates of each photo. The clustering method used for this problem is mean-shift clustering, which was described in the background section.

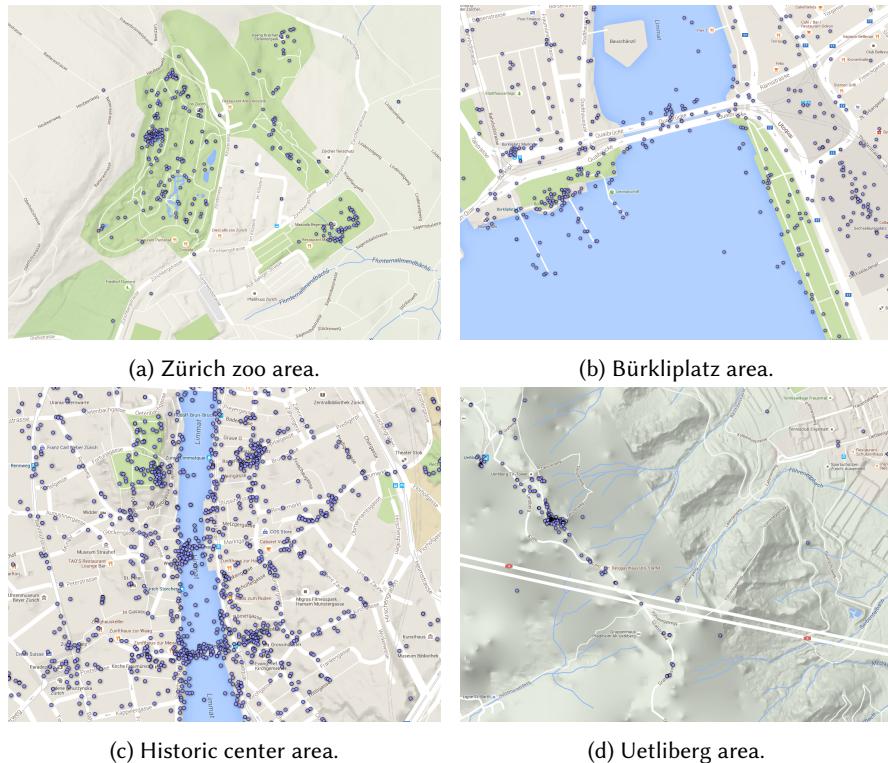


Figure 6.4: Zoomed in maps displaying popular locations in Zürich.

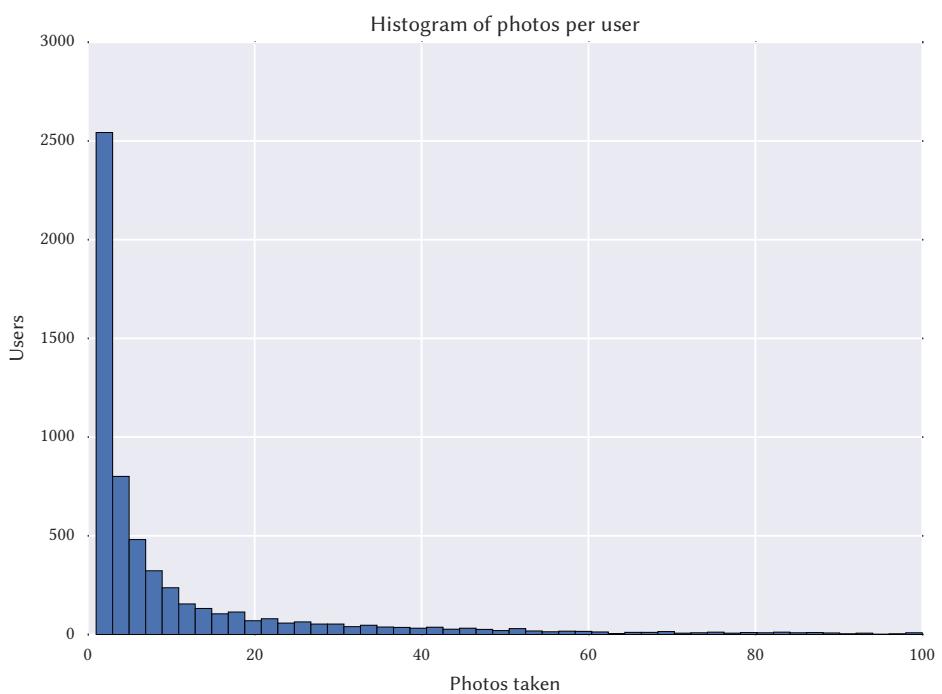


Figure 6.5: Histogram of photos taken per user.

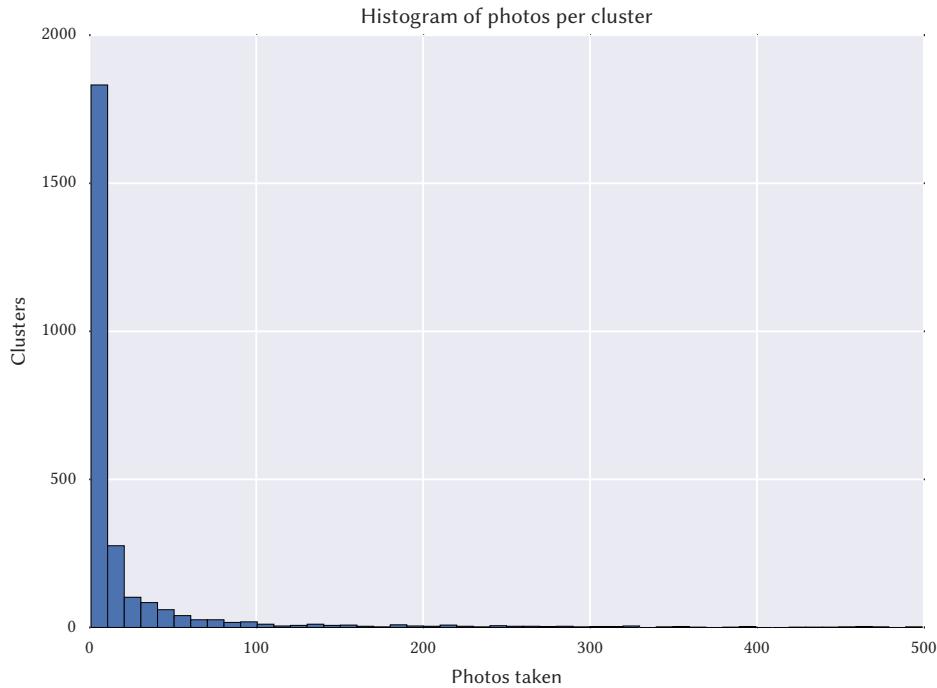


Figure 6.6: Histogram of the number of photos per cluster. The graph is cut off at 500 because the distribution has a heavy tail.

Scikit<sup>3</sup> was used for the mean-shift clustering implementation. The clustering was performed on the complete photo dataset with the unnormalized latitude and longitude features using a uniform disc as the kernel. The kernel bandwidth was 0.001 degrees which approximately represents a radius of 100m without significant errors for the geographic scale of Zürich.

The clustering procedure yielded 2686 clusters covering 137019 photos, the left over photos represent outliers that are not inside any of the resulting kernels. Figure 6.6 displays a histogram of photos per cluster, it shows that over half of the clusters only contain one or two photos. This indicates that many of the clusters are actually not landmarks or highly photographed places, so in order to concentrate the model around the most interesting locations only the top  $N$  clusters sorted by the number of photos were used.

Two datasets were constructed, a small one with the top 10 clusters and a large one with the top 100 clusters. The small dataset was heavily used during the development of the models to test the implementation and try out different settings. On the other hand, the large dataset was mainly used for assessing the quality of the final models and comparing it against the baseline models.

Figures 6.7 and 6.8 show the selected clusters in the small and large setting, respectively. Additionally, Table 6.1 presents additional information about the top 10 clusters in the small dataset.

To extract the paths taken by individual users from the photo dataset, the procedure was as follows:

1. The photos were grouped by user and date when they were taken.
2. The photos in each user and date group represent a single path, ordered by date and time.

<sup>3</sup><http://scikit-learn.org/stable/>

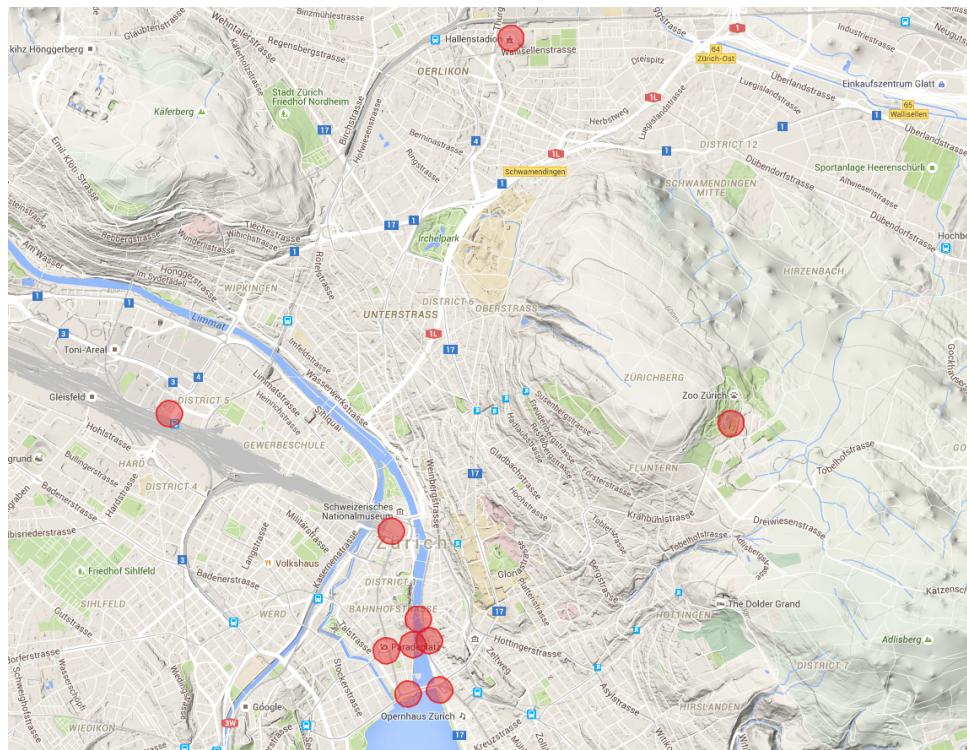


Figure 6.7: Top 10 clusters in Zürich according to the photo count.

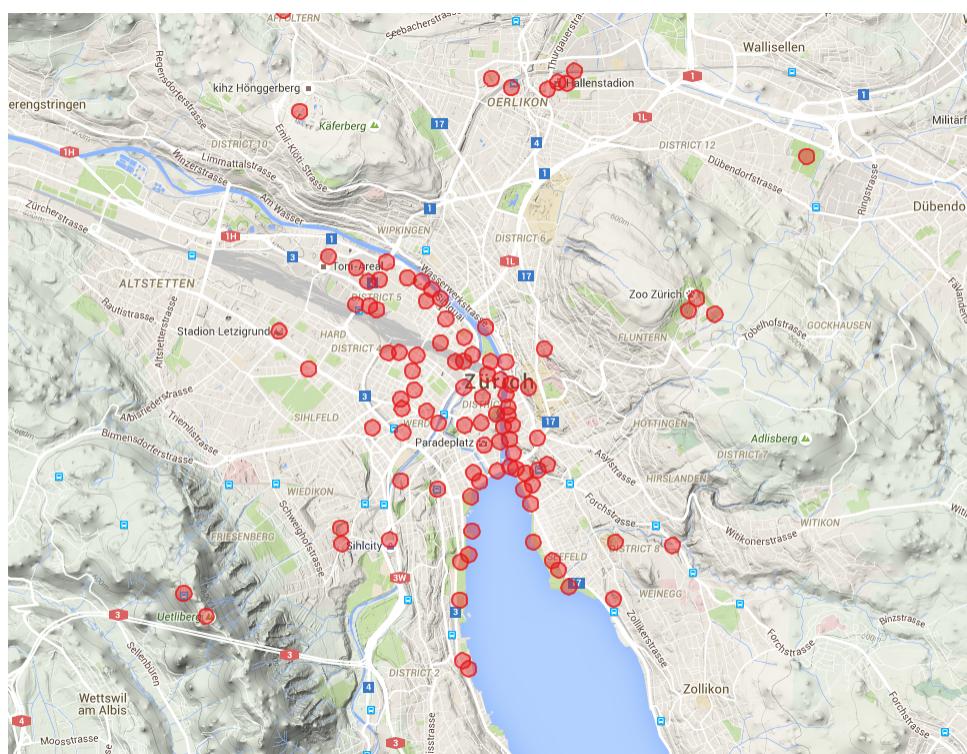


Figure 6.8: Top 100 clusters in Zürich according to the photo count.

Table 6.1: Top 10 clusters for Zürich.

Landmark	Photos taken	Unique users
Hauptbahnhof (Main train station)	4224	932
Grossmünster (Church)	3797	951
Bürkliplatz (Outdoors)	3728	612
Hallenstadion (Event venue)	3575	82
Fraumünster (Church)	3414	889
Bellevueplatz (Outdoors)	2584	437
Rathaus (Town hall)	2569	690
Zoo	2462	151
Paradeplatz (Outdoors)	2372	438
Prime Tower	1930	257

Table 6.2: Path dataset statistics for Zürich.

	Small ( $ V  = 10$ )	Large ( $ V  = 100$ )
Included photos (% of total)	30655 (18%)	91661 (54%)
Paths extracted, i.e. $ \mathcal{D} $	6781	16439
Singleton paths, i.e. $ S  = 1$	5396	12927
Pair paths, i.e. $ S  = 2$	799	1980

3. The photos were replaced in the sequences by their corresponding clusters, including the cluster only once. If the photo did not belong to one of the top clusters, then the photo would be discarded.
4. The resulting sequence of clusters represent the paths taken by users through the city.

These sequences were used as the unordered sets for the set models, e.g. FLID, FLDC, or as ordered sequences for the Markov model baseline. Table 6.2 show the statistics for the final datasets.

Additionally, Figures 6.9 and 6.10 show the histograms of the length of the sets in the small and large datasets, respectively. This shows that most of the sets are either singletons or pairs, which matches the fact that many users just take a few photos as it was shown in Figure 6.5.

### 6.3 Models

For the experiments, the FLID, FLDC and FFLDC models were used. The settings for these experiments will be presented in the next chapter along with the results. However, for all experiments the following implementation details were fixed:

- The data was split in 8 random folds, with 90% of the data used for training and 10% for testing in each.
- The noise was generated using the log-modular model as described for the synthetic experiments.

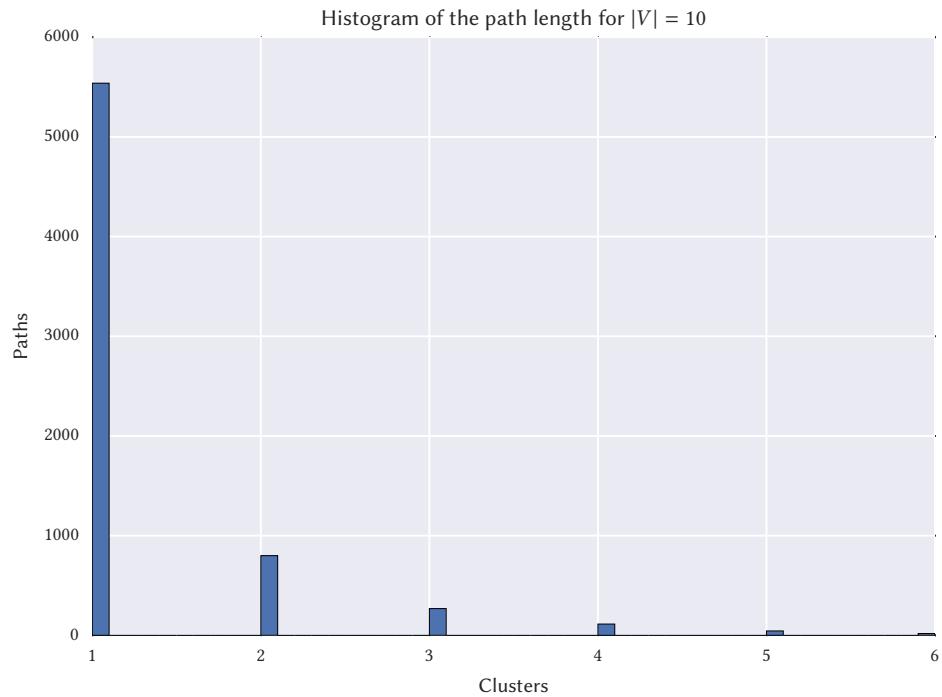


Figure 6.9: Histogram of set/path length for the small dataset.

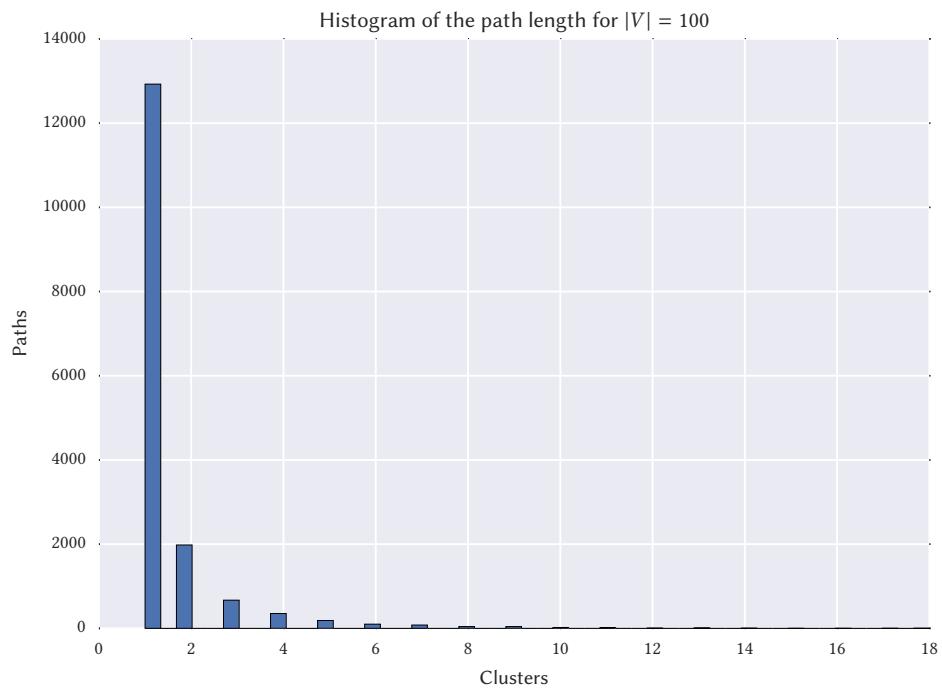


Figure 6.10: Histogram of set/path length for the large dataset.

## 6.4 Baselines

This section describes the baseline models that were used as a comparison for the proposed models.

### 6.4.1 Markov Chain

A Markov chain can be used to model the behavior of Flickr users, assuming that each user selects the next location to visit and photograph only based on the current location, i.e. the Markov property. This model depends on the transition probabilities between the different landmarks in the dataset, concretely  $P(l_{t+1} = i \mid l_t = j)$  where  $l_{t+1}$  represents the next location and  $l_t$  the current one. These probabilities can be estimated by counting all the occurrence of the transitions in the dataset, i.e.

$$\hat{P}(l_{t+1} = i \mid l_t = j) = \frac{N(i,j)}{N(j)} \quad (6.1)$$

Where  $N(i,j)$  is the number of times that the location  $i$  was photographed immediately after location  $j$ , and  $N(j)$  is the number of times that location  $N(j)$  was photographed. This model was used successfully by Kurashima et al. (2010) with good results for recommending tourists routes for a city based on Flickr data, therefore it is a natural choice for comparison against the newly proposed models. A full path can be constructed by selecting a starting location and choosing the next one according to the highest transition probability, then repeating the procedure until a desired length is reached.

Note that this model can be estimated efficiently from data, its running time is  $O(|\mathcal{D}|)$ .

### 6.4.2 Proximity Model

Another, possibly naive, model is to assume that users photograph the next closest location to their current one. This can be useful to model paths that are concentrated in certain areas, for example in the Zürich dataset many of the paths only visit locations in the center and users seldom take photographs at the Zoo and a landmark in the center on the same day.

Formally, this model defines a series of transition probabilities  $P(l_{t+1} = i \mid l_{1\dots t})$  which are inversely proportional to the distance of the last visited location, i.e.

$$P(l_{t+1} = i \mid l_{1\dots t}) \propto \frac{1}{d(i, l_t)} \quad (6.2)$$

Where  $d(i,j)$  is the distance between locations  $i$  and  $j$ . Contrary to the Markov chain model, the proximity model takes into account the complete history of locations, this information is used to avoid going back to a previously visited location. The complete model is then defined by:

$$P(l_{t+1} = i \mid l_{1\dots t}) \propto \begin{cases} 0 & \text{if } i \in l_{1\dots t} \\ \frac{1}{d(i, l_t)} & \text{otherwise} \end{cases} \quad (6.3)$$

This is a natural choice for a baseline model because it provides a very quick model that is not completely random, the probabilities in this model can be computed in  $O(|V|^2)$  time, which is less than  $|\mathcal{D}|$  for both datasets. To construct a path, the same procedure as for the Markov chain can be used.

### 6.4.3 Log-modular Distribution

As described before, a log-modular model is the simplest log-submodular model which makes it a natural comparison for the FLID and FLDC models. The log-modular model

can also be used with features by solving the linear system in Equation (5.1) to make it comparable to the FFLDC model.

The log-modular can be estimated from data following the procedure defined in Section 3.5.1, which only requires  $O(|\mathcal{D}|)$  time.

This model is significantly different from the two previous baselines because it does not model the order of the elements, simply the membership. However, it is also possible to build a path by first constructing a set of fixed size  $n$  and then ordering the elements in order to minimize the total travel distance. A set of size  $n$  can be constructed by selecting the  $n$  locations with highest utility, i.e.  $\text{argmax}_i u_i$ .

## 6.5 Evaluation

The evaluation of the models aims to assess their quality for the task of recommending additional locations to visit based on an initial set of locations that the user wants to visit. This recommendation can be structured as a path either directly from the output of the model, e.g. for the Markov chain case, or as a post-processing step, i.e. for the set based models.

The evaluation task is to predict the next location in a sequence given its first  $n$  elements, this can be constructed by creating partial paths of increasing size from the paths in the test dataset and withholding the last element in the partial sequence. Thus each path of length  $k$  generates  $k - 1$  partial paths for testing. For example the path 1, 5, 6, generates 2 partial paths, namely 1 and 1, 5, and the elements to predict are 5 and 6, respectively. Note that only paths of at least size 2 are used in the task.

After each model is trained with the corresponding training dataset, the predictions for each partial path  $S_t = [s_1, \dots, s_t]$  are made as follows:

**FLID, FLDC, FFLDC, Log-modular** The element with maximum probability, that is not already in the partial path is predicted, i.e.  $\text{argmax}_{i \notin S} P(S \cup \{i\})$  where  $S$  is the set of the elements in the partial path  $S_t$ .

**Markov Chain** The element with the highest transition probability with respect to the last element in the partial path is predicted, i.e.  $\text{argmax}_i P(i | s_t)$ .

**Proximity** The element with the highest transition probability, i.e. minimum distance, with respect to the last element in the partial path is predicted, ignoring elements already in the sequence as defined in Equation (6.3).

The metric used for the quality of the predictions is overall accuracy, i.e. the percentage of correct predictions. Formally,

$$Acc_{model} = \frac{1}{|\mathcal{T}|} \sum_{S_t \in \mathcal{T}} (\mathbb{I}_{\hat{s}_{t+1}, s_{t+1}}) \quad (6.4)$$

Where  $\mathcal{T}$  is the set of all partial paths available for testing and  $\mathbb{I}_{\hat{s}_{t+1}}$  is an indicator function that evaluates to 1 when the predicted element  $\hat{s}_{t+1}$  is equal to the next element in the sequence  $s_{t+1}$  and 0 otherwise.

Accuracy is the most used metric for recommender systems (Herlocker et al., 2004) and it was the metric used by Kurashima et al. (2010) for evaluating single step recommendations in tourist routes. This accuracy metric also is similar to the leave-one-out procedure used by Tschiatschek et al. (2016) to evaluate the FLID model for product recommendation. Therefore, it is expected to provide a good assessment on the quality of the proposed models.

## **Chapter 7**

# **Modeling an Image Collection - Results**

### **7.1 Small dataset**

Results on the small dataset. Difference between FLID, FLDC, FFLDC models. Different feature sets.

### **7.2 Large dataset**

Results on the large dataset. Difference between FLID, FLDC, FFLDC models. Different feature sets.



## **Chapter 8**

# **Conclusion**



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