COMP425 Computational Logic (2016)
Proof of a key result

This note discusses how we might prove the following result, which is required for A2:

$(\forall i.1 \leq i \leq k \Rightarrow P(i)) \wedge P(k+1) \Rightarrow (\forall i.1 \leq i \leq k+1 \Rightarrow P(i))$

I would write a pen and paper proof ofthis as follows:

$$
\begin{array}{ll}
 & (\forall i \ . \ 1 \leq i \leq k \Rightarrow P(i)) \wedge P(k+1) \\
\Longrightarrow & \\
 & (\forall i \ . \ 1 \leq i \leq k \Rightarrow P(i)) \wedge (\forall i \ . \ i = k+1 \Rightarrow P(i)) \\
\Longrightarrow & \\
 & (\forall i \ . \ (1 \leq i \leq k \Rightarrow P(i)) \wedge (i = k+1 \Rightarrow P(k+1))) \\
\Longrightarrow & \\
 & (\forall i \ . \ 1 \leq i \leq k \vee i = k+1 \Rightarrow P(i)) \\
\Longrightarrow & \\
 & (\forall i \ . \ 1 \leq i \leq k+1 \Rightarrow P(i))
\end{array}
$$

These steps are justified as follows:

1. One-point rule for universal quantification: $(\forall x \ . \ x = t \Rightarrow p) \equiv p[t/x]$, where $p[t/x]$ is p with t substituted for x.

   There is a similar rule for exisitential quantificatioin.

2. Merge quantifers quantifiers: $(\forall i \ . \ P) \wedge (\forall i \ . \ Q) \equiv (\forall i \ . \ P \wedge Q)$.

3. Distribute $\wedge$ over $\Rightarrow$ (or perhaps the other way around): $(P \Rightarrow R) \wedge (Q \Rightarrow R) \equiv P \vee Q \Rightarrow R$.

4. Extend range: $1 \leq i \leq k \vee i = k+1 \equiv 1 \leq i \leq k+1$. I haven't worked out exactly how to justify this step yet. Any thoughts?