1 Distribution Problem for Project Work on Course EMGT 6952

1.1 Problem Statement 1

A company has two factories, one at Atlanta(GA) and one at Charlotte (NC). In addition,it has four warehouses with storage facilities at Birmingham (AL), Greenville(SC), Gastonia (NC), and Augusta(GA). The company sells its product to six customers C1, C2, C3,C4,C5 , C6. Customers can be supplied from either a warehouse or the factory directly. Distribution costs are given in the following table (in $ per ton delivered).

Table 1: Distribution Costs

	Atlanta	Charlotte	Birmingham	Greenville	Gastonia	Augusta
Birmingham	80	-				
Greenville	60	60				
Gastonia	120	30				
Augusta	80	80				
C1	160	320	-	160	-	-
C2	-	-	240	80	240	-
C3	240	-	80	80	320	40
C4	320	-	240	160	-	240
C5	-	-	-	80	80	80
C6	160	-	160	-	240	240

Dash indicates the impossibility of certain suppliers for certain warehouses or customers.

Certain customers have expressed preferences for being supplied from factories or warehouses, which they are used to. The preferred suppliers are as follows:

C1: Atlanta

C2: Birmingham

C3: No preferences

C4: No preferences

C5: Greenville

C6: Gastonia or Augusta.

Each factory has a monthly capacity given as follows, which cannot be exceeded:

Atlanta: 150,000 tons

Charlotte: 200,000 tons.

Each warehousehas a maximum monthly throughput given as follows, which cannot be exceeded:

Birmingham:70,000 tons

Greenville:50,000 tons

Gastonia:100,000 tons

Augusta:40,000 tons.

Each customer has a monthly requirement given as follows, which must be met:

C1:50,000 tons

C2:10,000 tons

C3:40,000 tons

C4:35,000 tons

C5:60,000 tons

C6:20,000 tons

1.2 Project Question 1

The company would like to determine the following:

1. What distribution pattern would minimize overall cost?

2. What the effect of increasing factory and warehouse capacities would be ondistribution costs?

3. What the effects of small changes in costs, capacities and requirements would be on the distribution pattern?

4. Would it be possible to meet all customer preferences regarding suppliers,and if so what would the extra cost of doing this be?

1.3 Problem Statement 2

There is a possibility of opening new warehouses at Knoxville (TN) and Asheville (NC), as well as of enlarging the Greenville warehouse.It is not considered desirable to have more than four warehouses and if necessary Birmingham or Augusta(or both) can be closed down.The monthly costs (in interest charges) of the possible new warehouses and expansion at Greenville are given in the following table together with the potential monthly throughputs.

	Cost($)	Throughput (1000 tons)
Knoxville	24,000	30
Asheville	10,000	25
Greenville	8,000	20

The monthly savings of closing down the Birmingham and Augusta warehouses are $18,000 and $12,000, respectively. The distribution costs involving with the new warehouses are given in the following table.

	Atlanta	Charlotte	Knoxville	Asheville
Knoxville	100	-		
Asheville	64	50		
C1			180	
C2			90	
C3			80	160
C4			-	80
C5			50	90
C6			130	150

1.4 Project Question 2

Which new warehouses should be built? Should Greenville be expanded? Should Birmingham or Augusta be closed down? What would be the best resultant distribution pattern to minimize overall costs?

2 Solution Formulation

2.1 Decision Variables

x_{AB}: Quantity of product moving from location A to location B (in tons). e.g x_{CGa} in table below refers to the quantity of products being transferred from Charlotte to Gastonia.

Table 2: Decision Variables

	Atl.(A)	Clt(C)	Bgm(B)	Grv(G)	Gst(Ga)	Aug(Au)
Bgm(B)	x_{AB}/x^1	-				
Grv(G)	x_{AG}/x^2	x_{CG}/x^9				
Gst(Ga)	x_{AGa}/x^3	x_{CGa}/x^{10}				
Aug(Au)	x_{AAu}/x^4	x_{CAu}/x^{11}				
C1	x_{AC1}/x^5	x_{CC1}/x^{12}	-	x_{GC1}/x^{17}	-	-
C2	-	-	x_{BC2}/x^{13}	x_{GC2}/x^{18}	x_{GaC2}/x^{22}	-
C3	x_{AC3}/x^6	-	x_{BC3}/x^{14}	x_{GC3}/x^{19}	x_{GaC3}/x^{23}	x_{AuC3}/x^{26}
C4	x_{AC4}/x^7	-	x_{BC4}/x^{15}	x_{GC4}/x^{20}	-	x_{AuC4}/x^{27}
C5	-	-	-	x_{GC5}/x^{21}	x_{GaC5}/x^{24}	x_{AuC5}/x^{28}
C6	x_{AC6}/x^8	-	x_{BC6}/x^{16}	-	x_{GaC6}/x^{25}	x_{AuC6}/x^{29}

where, x^i is the index linked with a particular decision variable. e.g. x^{10} refers to the quantity of products being transferred from Charlotte to Gastonia.

It can be observed that there 29 decision variables in total.

2.2 Objective Function

Using Table 1 and Table 2, the following objective function to minimize the overall distribution cost is formulated:

Minimize:

Z = $80x^1$+$60x^2$+$120x^3$+$80x^4$+$160x^5$+$240x^6$+$320x^7$+$160x^8$+$60x^9$+$30x^{10}$+$80x^{11}$+$320x^{12}$+ $240x^{13}$+$80x^{14}$+$240x^{15}$+$160x^{16}$+$160x^{17}$+$80x^{18}$+$80x^{19}$+$160x^{20}$+$80x^{21}$+$240x^{22}$+$320x^{23}$+ $80x^{24}$+$240x^{25}$+$40x^{26}$+$240x^{27}$+$80x^{28}$+$240x^{29}$

2.3 Inequality Constraints

Factory Production Constraints:

1. x^1+x^2+x^3+x^4+x^5+x^6+x^7+x^8 $\leq$ 150,000 (Atlanta Monthly Production Capacity)

2. x^9+x^{10}+x^{11}+x^{12} $\leq$ 200,000 (Charlotte Monthly Production Capacity)

Warehouse throughput constraints: This is considered as: the quantity moving into the warehouse should be less than its maximum monthly throughput and later it is also considered that items delivered out of a warehouse should be equal to the item coming into the warehouse as an equality constraint.

3. x^1 $\leq$ 70,000 (Birmingham maximum monthly throughput)
4. x^2+x^9 $\leq$ 50,000 (Greenville maximum monthly throughput)
5. x^3+x^{10} $\leq$ 100,000 (Gastonia maximum monthly throughput)
6. x^4+x^{11} $\leq$ 40,000 (Augusta maximum monthly throughput)

2.4 Equality Constraints

Warehouse input output balance

1. x^1-x^{13}-x^{14}-x^{15}-x^{16} = 0 (Input Output balance of birmingham warehouse)
2. x^2+x^9-x^{17}-x^{18}-x^{19}-x^{20}-x^{21} = 0 (Input Output balance of Greenville warehouse)
3. x^3+x^{10}-x^{22}-x^{23}-x^{24}-x^{25} = 0 (Input Output balance of Gastonia warehouse)
4. x^4+x^{11}-x^{26}-x^{27}-x^{28}-x^{29} = 0 (Input Output balance of Augusta warehouse)

Consumer Demand

5. x^5+x^{12}+x^{17} = 50,000 (Consumer C1 demand)
6. x^13+x^{18}+x^{22} = 10,000 (Consumer C2 demand)
7. x^6+x^{14}+x^{19}+x^{23}+x^{26} = 40,000 (Consumer C3 demand)
8. x^7+x^{15}+x^{20}+x^{27} = 35,000 (Consumer C4 demand)
9. x^{21}+x^{24}+x^{28} = 60,000 (Consumer C5 demand)

10. x^8+x^{16}+x^{25}+x^{29} = 20,000 (Consumer C6 demand)

2.5 Non-negativity constraints

$x^i \geq 0$ i = 1,2,3,$\cdots$,28,29

2.6 Solution Obtained from MATLAB Linprog solver

Based on the problem formulation above, MATLAB's linear programming solver was used to get an optimal solution. Based on MATLAB's solver, the minimum cost that will incur in this distribution problem is equal to \$ 31,700,000, with the different variable value as :

x^2 = 40,000, x^4 = 40,000, x^5 = 50,000 , x^8 = 20,000, x^9 = 5,000, x^{10} = 60,000, x^{18} = 10,000, x^{20} = 35,000, x^{24} = 60,000, x^{26} = 40,000 and rest are all zeros.

3 Preferences of the customer considered

Certain customers have expressed preferences for being supplied from factories or warehouses, which they are used to. The preferred suppliers are as follows:

C1: Atlanta

C2: Birmingham

C3: No preferences

C4: No preferences

C5: Greenville

C6: Gastonia or Augusta.

When the preference of the customer is considered, it is assumed that only the preferred route is available to reach to the customer. For example, as customer C1 prefers the product to be delivered from Atlanta, the possibilities of the product being delivered from Charlotte or Greenville is ignored. i.e. the variable x^{12} and x^{17} in Table 2 are ignored.

Likewise, considering the preference for the other customers, variables x^{18}, x^{22}, x^{24},x^{28},x^8 and x^{16} in Table 2 are ignored. Note that: intuitively, the transportation cost is going to go higher when the customer preference is taken into consideration as preferred route from customers are not necessarily cost effective. route.

With this the decision variables will be as follows (Note the variable numbers are updated):

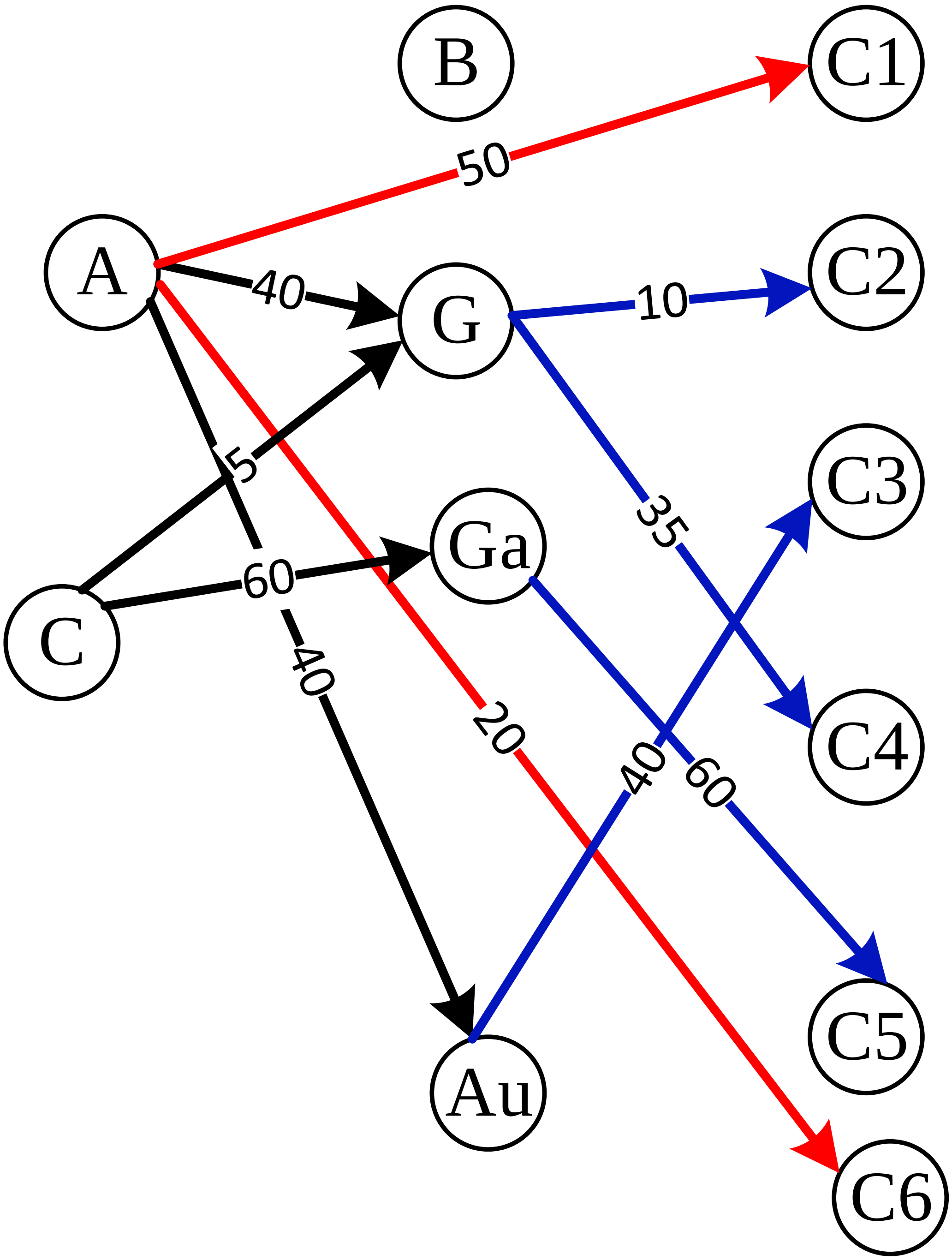

Figure 1: Optimal cost distribution pattern when no consumer preference is included.

Table 3: Decision variables with customer preference considered

	Atl.(A)	Clt(C)	Bgm(B)	Grv(G)	Gst(Ga)	Aug(Au)
Bgm(B)	x_{AB}/x^1	-				
Grv(G)	x_{AG}/x^2	x_{CG}/x^8				
Gst(Ga)	x_{AGa}/x^3	x_{CGa}/x^9				
Aug(Au)	x_{AAu}/x^4	x_{CAu}/x^{10}				
C1	x_{AC1}/x^5	-	-	-	-	-
C2	-	-	x_{BC2}/x^{11}	-	-	-
C3	x_{AC3}/x^6	-	x_{BC3}/x^{12}	x_{GC3}/x^{14}	x_{GaC3}/x^{17}	x_{AuC3}/x^{19}
C4	x_{AC4}/x^7	-	x_{BC4}/x^{13}	x_{GC4}/x^{15}	-	x_{AuC4}/x^{20}
C5	-	-	-	x_{GC5}/x^{16}	-	-
C6	-	-	-	-	x_{GaC6}/x^{18}	x_{AuC6}/x^{21}

It can be observed that the decision variables are now reduced from 29 variables to 21 in total.

3.1 Objective Function

Using the updated preference from the customer. the following objective function to minimize the overall distribution cost is formulated:

Minimize:

Z = $80x^1$+$60x^2$+$120x^3$+$80x^4$+$160x^5$+$240x^6$+$320x^7$+$60x^8$+$30x^9$+$80x^{10}$

$240x^{11}$+$80x^{12}$+$240x^{13}$+$80x^{14}$+$160x^{15}$+$80x^{16}$+$320x^{17}$+$240x^{18}$+$40x^{19}$+$240x^{20}$+$240x^{21}$

3.2 Inequality Constraints

Factory Production Constraints:

1. x^1+x^2+x^3+x^4+x^5+x^6+x^7 $\leq$ 150,000 (Atlanta Monthly Production Capacity)

2. x^8+x^9+x^{10} $\leq$ 200,000 (Charlotte Monthly Production Capacity)

Warehouse throughput constraints: This is considered as: the quantity moving into the warehouse should be less than its maximum monthly throughput and later it is also considered that items delivered out of a warehouse should be equal to the item coming into the warehouse as an equality constraint.

3. x^1 $\leq$ 70,000 (Birmingham maximum monthly throughput)
4. x^2+x^8 $\leq$ 50,000 (Greenville maximum monthly throughput)
5. x^3+x^9 $\leq$ 100,000 (Gastonia maximum monthly throughput)
6. x^4+x^{10} $\leq$ 40,000 (Augusta maximum monthly throughput)

3.3 Equality Constraints

Warehouse input output balance

1. x^1-x^{11}-x^{12}-x^{13} $= 0$ (Input Output balance of birmingham warehouse)
2. x^2+x^8-x^{14}-x^{15}-x^{16} $= 0$ (Input Output balance of Greenville warehouse)
3. x^3+x^9-x^{17}-x^{18} $= 0$ (Input Output balance of Gastonia warehouse)
4. x^4+x^{10}-x^{19}-x^{20}-x^{21} $= 0$ (Input Output balance of Augusta warehouse)

Consumer Demand

5. x^5 $=$ 50,000 (Consumer C1 demand)
6. x^{11} $=$ 10,000 (Consumer C2 demand)
7. x^6+x^{12}+x^{14}+x^{17}+x^{19} $=$ 40,000 (Consumer C3 demand)
8. x^7+x^{13}+x^{15}+x^{20} $=$ 35,000 (Consumer C4 demand)
9. x^{16} $=$ 60,000 (Consumer C5 demand)
10. x^{18}+x^{21} $=$ 20,000 (Consumer C6 demand)

3.4 Non-negativity constraints

$x^i \geq 0$ $i =$ 1,2,3,$\cdots$,20,21

3.5 Issue with the above formulation

One of the issue that is directly evident with the problem formulation above is that the demand of consumer C5 cannot be met through the supply from Greenville warehouse alone. This is because the maximum throughput of Greenville warehouse is 50,000 tons and the consumer C5 has a demand of 60,000 tons. Thus, it is clear that the demand of consumer C5 cannot be met alone from its preferred suppliers. This was observed when the above problem formulation was passed through MATLAB's linear programming solver was printed the statement:

"No feasible solution found."

One of the ways to fix this issue of preference of consumer C5 is to assume that out of the demand of 60,000 tons, 50,000 tons will be supplied from its preferred location: Greenville. And, the remaining 10,000 can be supplied from either Gastonia or Augusta warehouse. With the assumption regarding the preference of consumer C5, the problem formulation then changes to:

Table 4: Decision variables with customer preference updated

	Atl.(A)	Clt(C)	Bgm(B)	Grv(G)	Gst(Ga)	Aug(Au)
Bgm(B)	x_{AB}/x^1	-				
Grv(G)	x_{AG}/x^2	x_{CG}/x^8				
Gst(Ga)	x_{AGa}/x^3	x_{CGa}/x^9				
Aug(Au)	x_{AAu}/x^4	x_{CAu}/x^{10}				
C1	x_{AC1}/x^5	-	-	-	-	-
C2	-	-	x_{BC2}/x^{11}	-	-	-
C3	x_{AC3}/x^6	-	x_{BC3}/x^{12}	x_{GC3}/x^{14}	x_{GaC3}/x^{17}	x_{AuC3}/x^{20}
C4	x_{AC4}/x^7	-	x_{BC4}/x^{13}	x_{GC4}/x^{15}	-	x_{AuC4}/x^{21}
C5	-	-	-	x_{GC5}/x^{16}	x_{GaC6}/x^{18}	x_{AuC6}/x^{22}
C6	-	-	-	-	x_{GaC6}/x^{19}	x_{AuC6}/x^{23}

It can be observed that the decision variables have now increased from 21 variables to 23 in total.

3.6 Objective Function so that consumer C5 gets as much as it can from Greenville warehouse

Using the updated preference from the customer. the following objective function to minimize the overall distribution cost is formulated:

Minimize:

Z = $80x^1$+$60x^2$+$120x^3$+$80x^4$+$160x^5$+$240x^6$+$320x^7$+$60x^8$+$30x^9$+$80x^{10}$

$240x^{11}$+$80x^{12}$+$240x^{13}$+$80x^{14}$+$160x^{15}$+$80x^{16}$+$320x^{17}$+$80x^{18}$+$240x^{19}$+$40x^{20}$+$240x^{21}$+$80x^{22}$+$240x^{23}$

3.7 Inequality Constraints

Factory Production Constraints:

1. x^1+x^2+x^3+x^4+x^5+x^6+x^7 $\leq$ 150,000 (Atlanta Monthly Production Capacity)

2. x^8+x^9+x^{10} $\leq$ 200,000 (Charlotte Monthly Production Capacity)

Warehouse throughput constraints: This is considered as: the quantity moving into the warehouse should be less than its maximum monthly throughput and later it is also considered that items delivered out of a warehouse should be equal to the item coming into the warehouse as an equality constraint.

3. x^1 $\leq$ 70,000 (Birmingham maximum monthly throughput)
4. x^2+x^8 $\leq$ 50,000 (Greenville maximum monthly throughput)
5. x^3+x^9 $\leq$ 100,000 (Gastonia maximum monthly throughput)

6. x^4+x^{10} $\leq$ 40,000 (Augusta maximum monthly throughput)

3.8 Equality Constraints

Warehouse input output balance

1. x^1-x^{11}-x^{12}-x^{13} = 0 (Input Output balance of birmingham warehouse)
2. x^2+x^8-x^{14}-x^{15}-x^{16} = 0 (Input Output balance of Greenville warehouse)
3. x^3+x^9-x^{17}-x^{18}-x^{19} = 0 (Input Output balance of Gastonia warehouse)
4. x^4+x^{10}-x^{20}-x^{21}-x^{22}-x^{23} = 0 (Input Output balance of Augusta warehouse)

Consumer Demand

5. x^5 = 50,000 (Consumer C1 demand)
6. x^{11} = 10,000 (Consumer C2 demand)
7. x^6+x^{12}+x^{14}+x^{17}+x^{20} = 40,000 (Consumer C3 demand)
8. x^7+x^{13}+x^{15}+x^{21} = 35,000 (Consumer C4 demand)
9. x^{16}+x^{18}+x^{22} = 60,000 (Consumer C5 demand)
10. x^{19}+x^{23} = 20,000 (Consumer C6 demand)

Consumer C5 preference

11. x^{16} = 50,000 (supplying as much as possible from greenville for consumer C5)

3.9 Non-negativity constraints

$x^i \geq 0$ i = 1,2,3,$\cdots$,22,23

3.10 Solution Obtained from MATLAB Linprog solver

Based on the problem formulation above, MATLAB's linear programming solver was used to get an optimal solution. Based on MATLAB's solver, the minimum cost that will incur in this distribution problem is equal to \$ 40,700,000, with the different variable value as :

x^1 = 10,000, x^2 = 50,000, x^5 = 50,000, x^7 = 35,000 , x^9 = 30,000, x^{10} =40,000, x^{11} = 10,000, x^{16} = 50,000, x^{18} = 10,000, x^{19} = 20,000, x^{20} = 40,000 and rest are all zeros.

Thus, it can be observed that with the consumer preference considered, the cost of the shipping increased from 31,700,000 to 40,700,000.

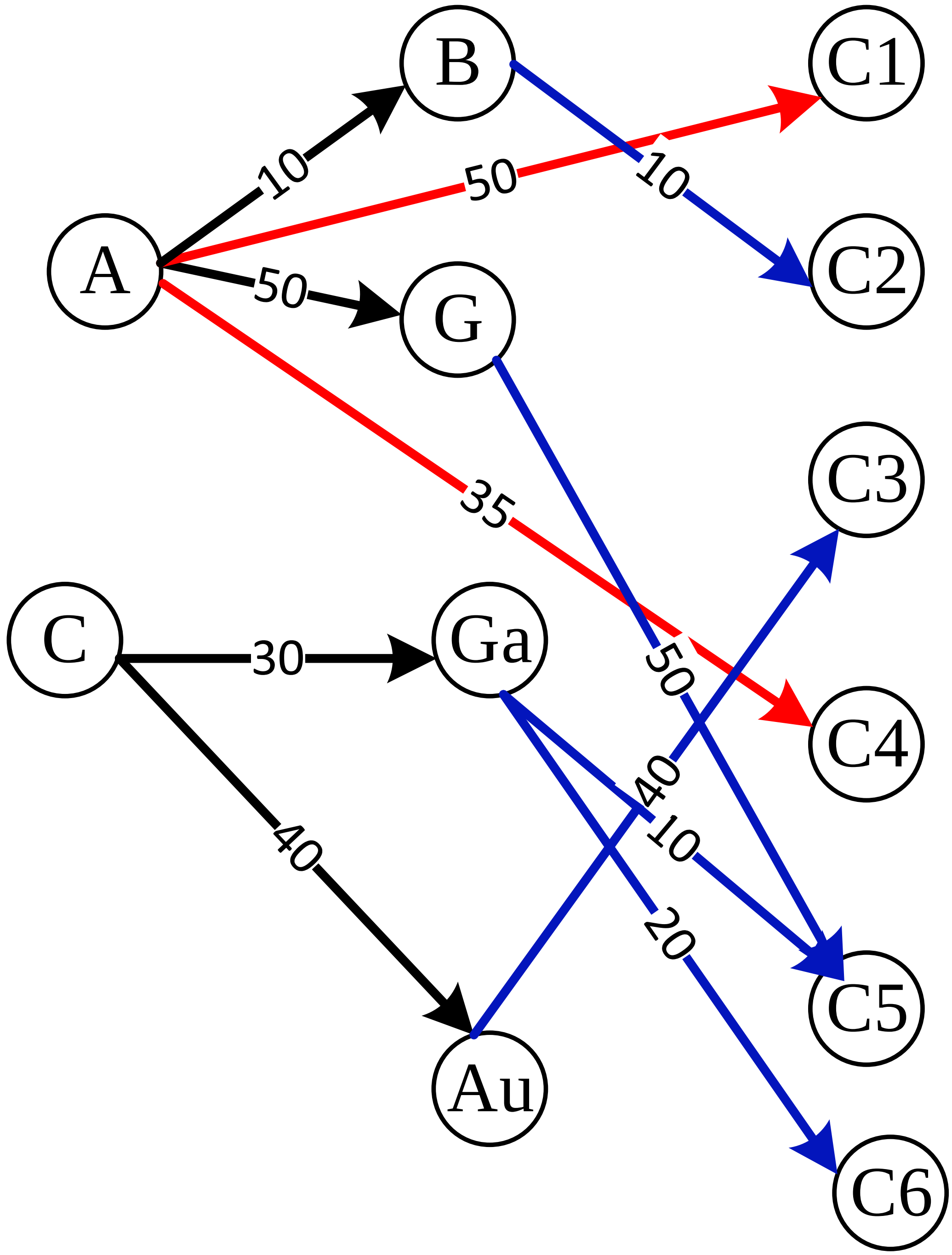

Figure 2: Optimal cost distribution pattern when consumer preference is included.

3.11 What the effect of increasing factory and warehouse capacities would be on distribution costs?

Since in both the preferred case and in the case where the preference is not considered, the factory capacity at the Charlotte factory is within the limits i.e. $\leq 200,000$, increasing the production capacity at Charlotte would have no effect on the overall distribution cost.

For case where no consumer preference is considered, it can be observed that the factory at Atlanta is utilized to its limit of 150,000 tons. Intuitively, it seems like increasing the capacity of Atlanta factory would change the distribution costs. However, this increment on Atlanta's factory capacity does not change the distribution cost as delivering to Greenville from either Charlotte/Atlanta costs the same price and delivering to Gastonia is cheaper from Charlotte when compared to Atlanta, so it does not intuitively feels that increase in the production capacity of Atlanta Factory would change in the distribution cost.

In both the cases, it can be observed that the warehouse at Augusta is increased to its full limits, with this it is necessary to examine the effect of expansion of the warehouse at Augusta has on the overall distribution costs. However, it should also be noted that Augusta is the cheapest option to deliver to consumer C3 only (for C5, it matches with Greenville and Gastonia) and as the optimal solution has consumer C3's demand met from Augusta warehouse alone, until the demand in consumer C3 increases, increment in warehouse capacity at Augusta would have no impact on the distribution cost.

In the case with consumer preference considered, it can be observed that consumer C5 is limited to getting the products from the desired location because of the limitation of warehouse at Greenville. As, the warehouse at Greenville is increased in it capacity, the price of the distribution cost increases.

3.12 What the effects of small changes in costs, capacities and requirements would be on the distribution pattern?

The cost related to variables $x1$ through x_{23} can be changed in the following range without affecting the optimal cost function.

As warehouse in Birmingham and Gastonia are underutilized , the constraint related to them are non-binding so increasing their capacity has no impact on the distribution pattern. However, as Greenville and Augusta are utilized to their limits the changes in their capacity will have an impact on the distribution cost.

Table 5: Sensitivity Analysis

Variables	Reduction in Cost Allowed without changing optimal solution	Increment in Cost Allowed without changing optimal solution
x^1	0	0
x^2	0	∞
x^3	90	∞
x^4	∞	0
x^5	∞	∞
x^6	80	∞
x^7	80	0
x^8	∞	0
x^9	190	90
x^{10}	0	∞
x^{11}	∞	∞
x^{12}	40	80
x^{13}	0	∞
x^{14}	80	∞
x^{15}	∞	80
x^{16}	∞	∞
x^{17}	190	∞
x^{18}	∞	90
x^{19}	∞	90
x^{20}	∞	40
x^{21}	40	∞
x^{22}	90	∞
x^{23}	90	∞

Table 6: Constraint Analysis

Constraint	Constraint Type	Slack
Atlanta Production Constraint	Non binding	15000
Charlotte Production Constraint	Non binding	120000
Birmingham Throughput Constraint	Non binding	60000
Greenville Throughput Constraint	Binding	0
Gastonia Throughput Constraint	Non binding	70000
Augusta Throughput Constraint	Binding	0

4 Would it be possible to meet all customer preferences regarding suppliers,and if so what would the extra cost of doing this be?

It was observed that not all the customer preference regarding suppliers was met, the extra cost incurred to meet the preference requirement is $ 9,000,000.

5 Part II (Preference Not Considered)

The new constraints that are added are:

Table 7: Distribution Costs

	Atlanta	Charlotte	Birmingham	Greenville	Gastonia	Augusta	Knoxville	Asheville
Birmingham	80	-						
Greenville	60	60						
Gastonia	120	30						
Augusta	80	80						
C1	160	320	-	160	-	-	180	
C2	-	-	240	80	240	-	90	
C3	240	-	80	80	320	40	80	160
C4	320	-	240	160	-	240	-	80
C5	-	-	-	80	80	80	50	90
C6	160	-	160	-	240	240	130	150
Knoxville	100	-						
Asheville	64	50						

a. It is not considered desirable to have more than four warehouses, so the number of warehouses should be limited to 4.

b. Throughput of Knoxville : 30,000 tons

c. Throughput of Asheville : 25,000 tons

d. Throughput of Greenville after expansion: 70,000

The following binary variables are now introduced in the problem formulation:

$$b_1 = \begin{cases} 1 & \text{if Birmingham warehouse is not closed} \\ 0 & \text{if Birmingham warehouse is closed} \end{cases}$$

$$b_2 = \begin{cases} 1 & \text{if Greenville warehouse is expanded} \\ 0 & \text{if Greenville warehouse is not expanded} \end{cases}$$

$$b_4 = \begin{cases} 1 & \text{if Augusta warehouse is not closed} \\ 0 & \text{if Augusta warehouse is closed} \end{cases}$$

$$b_5 = \begin{cases} 1 & \text{if Knoxville warehouse is built} \\ 0 & \text{if Knoxville warehouse is not built} \end{cases}$$

$$b_6 = \begin{cases} 1 & \text{if Asheville warehouse is built} \\ 0 & \text{if Asheville warehouse is not built} \end{cases}$$

Now, the decision variable table will be:

Table 8: Decision Variable Table

	Atlanta	Charlotte	Birmingham	Greenville	Gastonia	Augusta	Knoxville	Asheville
Birmingham	x^1	-						
Greenville	x^2	x^{11}						
Gastonia	x^3	x^{12}						
Augusta	x^4	x^{13}						
C1	x^5	x^{14}	-	x^{20}	-	-	x^{33}	
C2	-	-	x^{16}	x^{21}	x^{25}	-	x^{34}	
C3	x^6	-	x^{17}	x^{22}	x^{26}	x^{29}	x^{35}	x^{38}
C4	x^7	-	x^{18}	x^{23}	-	x^{30}	-	x^{39}
C5	-	-	-	x^{24}	x^{27}	x^{31}	x^{36}	x^{40}
C6	x^8	-	x^{19}	-	x^{28}	x^{32}	x^{37}	x^{41}
Knoxville	x^9	-						
Asheville	x^{10}	x^{15}						

5.1 Objective Function

Using the updated preference from the customer. the following objective function to minimize the overall distribution cost is formulated:

Minimize:

Z = $80x^1+60x^2+120x^3+80x^4+160x^5+240x^6+320x^7+160x^8+100x^9+64x^{10}$

$60x^{11}+30x^{12}+80x^{13}+320x^{14}+50x^{15}+240x^{16}+80x^{17}+240x^{18}+160x^{19}+160x^{20}+80x^{21}+80x^{22}+160x^{23}+80x^{24}+240x^{25}+$

$320x^{26}+80x^{27}+240x^{28}+40x^{29}+240x^{30}+80x^{31}+240x^{32}+180x^{33}+90x^{34}+80x^{35}+50x^{36}+130x^{37}+160x^{38}+80x^{39}+90x^{40}+150x^{41}$

+18,000(1-b_1)+12,000(1-b_4)+24,000b_5+10,000b_6+8,000b_2

5.2 Inequality Constraints

Factory Production Constraints:

1. $x^1+x^2+x^3+x^4+x^5+x^6+x^7+x^8+x^9+x^{10} \leq$ 150,000 (Atlanta Monthly Production Capacity)

2. $x^{11}+x^{12}+x^{13}+x^{14}+x^{15} \leq$ 200,000 (Charlotte Monthly Production Capacity)

Warehouse throughput constraints: This is considered as: the quantity moving into the warehouse should be less than its maximum monthly throughput and later it is also considered that items delivered out of a warehouse should be equal to the item coming into the warehouse as an equality constraint.

3. $x^1 \leq$ 70,000b_1 (Birmingham maximum monthly throughput)

4. x^2+x^{11} $\leq$ 50,000+20,000*b_2 (Greenville maximum monthly throughput)
5. x^3+x^{12} $\leq$ 100,000 (Gastonia maximum monthly throughput)
6. x^4+x^{13} $\leq$ 40,000*b_4 (Augusta maximum monthly throughput)
7. x^9 $\leq$ 30,000*b_5 (Knoxville maximum monthly throughput)
8. x^{10}+x^{15} $\leq$ 25,000*b_6 (Asheville maximum monthly throughput)

Number of warehouse constraint:

9. b_1+b_4+b_5+b_6 $\leq$ 2 (As Greenville and Gastonia remains operational)

5.3 Equality Constraints

Warehouse input output balance

1. x^1-x^{16}-x^{17}-x^{18}-x^{19} = 0 (Input Output balance of Birmingham warehouse)
2. x^2+x^{11}-x^{20}-x^{21}-x^{22}-x^{23}-x^{24} = 0 (Input Output balance of Greenville warehouse)
3. x^3+x^{12}-x^{25}-x^{26}-x^{27}-x^{28} = 0 (Input Output balance of Gastonia warehouse)
4. x^4+x^{13}-x^{29}-x^{30}-x^{31}-x^{32} = 0 (Input Output balance of Augusta warehouse)
5. x^9-x^{33}-x^{34}-x^{35}-x^{36}-x^{37} = 0 (Input Output balance of Knoxville warehouse)
6. x^{10}+x^{15}-x^{38}-x^{39}-x^{40}-x^{41} = 0 (Input Output balance of Asheville warehouse)

Consumer Demand

7. x^5+x^{14}+x^{20}+x^{33} = 50,000 (Consumer C1 demand)
8. x^{16}+x^{21}+x^{25}+x^{34} = 10,000 (Consumer C2 demand)
9. x^6+x^{17}+x^{22}+x^{26}+x^{29}+x^{35}+x^{38} = 40,000 (Consumer C3 demand)
10. x^7+x^{18}+x^{23}+x^{30}+x^{39} = 35,000 (Consumer C4 demand)
11. x^{24}+x^{27}+x^{31}+x^{36}+x^{40} = 60,000 (Consumer C5 demand)
12. x^8+x^{19}+x^{28}+x^{32}+x^{37}+x^{41} = 20,000 (Consumer C6 demand)

5.4 Non-negativity constraints

$x^i \geq 0$ i = 1,2,3,$\cdots$,40,41

$0 \geq b_1, b_2, b_4, b_5, b_6 \leq 1$

5.5 Solution Obtained from MATLAB Linprog solver

Based on the problem formulation above, MATLAB's integer linear programming solver was used to get an optimal solution. Based on MATLAB's solver, the minimum cost that will incur in this distribution problem

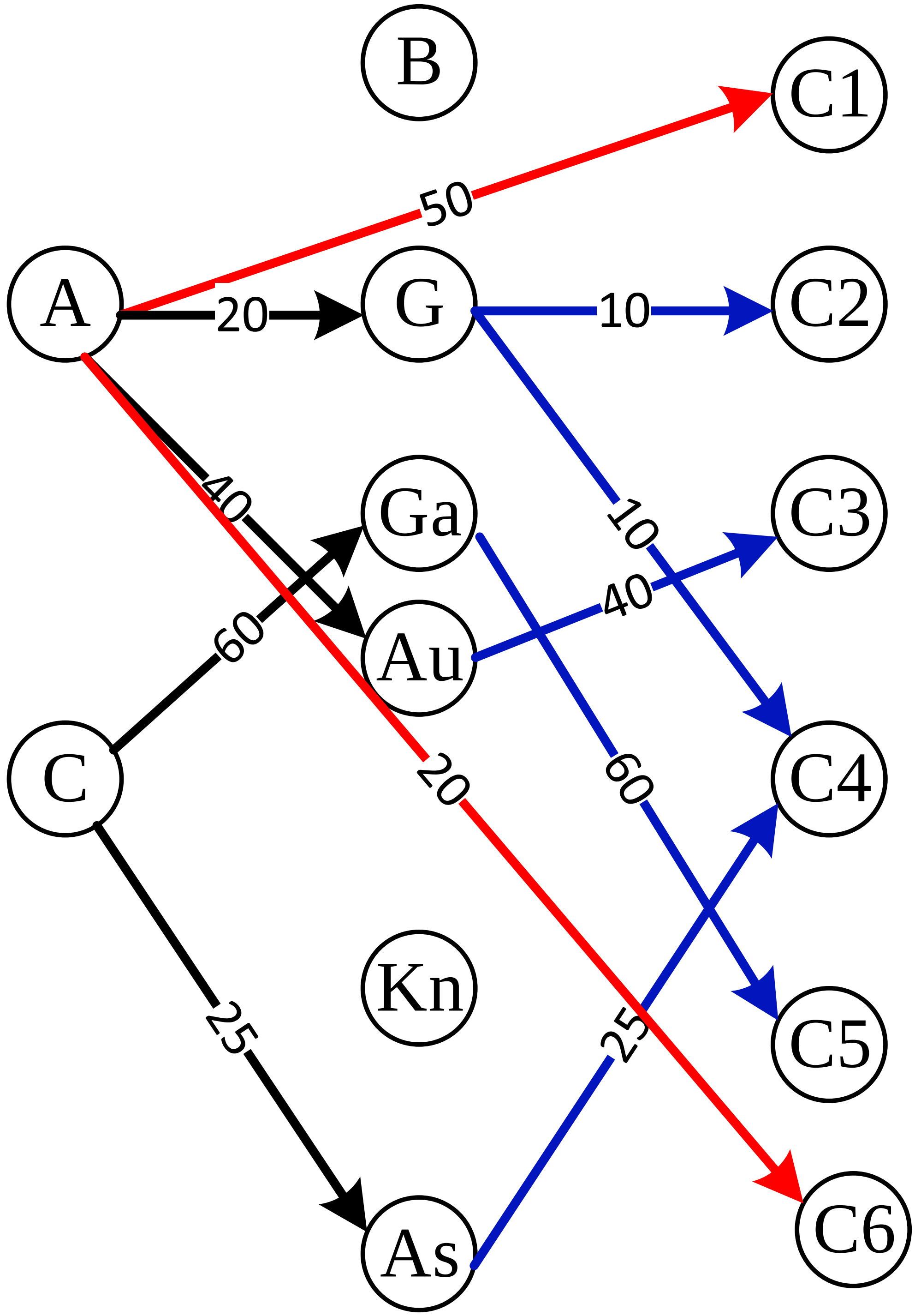

Figure 3: Optimal cost distribution pattern when consumer preference is not included for part 2.

is equal to \$ 26,070,000, with the different variable value as :

x^2 = 20,000, x^4 = 40,000, x^5 = 50,000, x^8 = 20,000 , x^{12} = 60,000, x^{15} =25,000, x^{21} = 10,000, x^{23} = 10,000, x^{27} = 60,000, x^{29} = 40,000, x^{39} = 20,000 and rest are all zeros except $b_4 = 1$, and $b_6 = 1$.

And, the optimal objective value is \$ 29,472,000.

New warehouse should be built in Asheville. The Greenville warehouse need not be expanded. Birmingham warehouse needs to be closed down. And the best resultant distribution is shown in Figure 3.