Function Approximation Methods-III

CS771: Introduction to Machine Learning

Purushottam Kar

Discussion Session

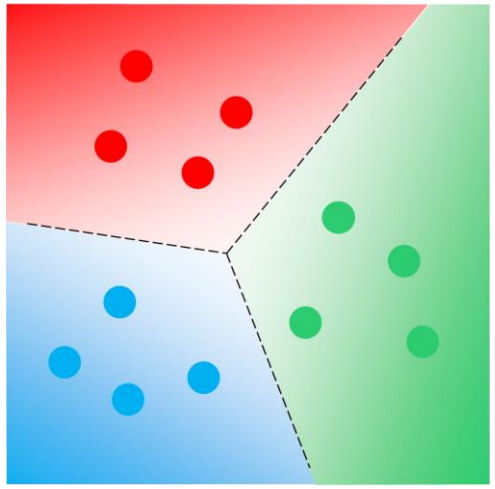
Sunday, 03 September, 2017
1800-1930 hrs, RM101, CSE department
Post query on http://tinyurl.com/ml17-18ads1 before coming!

Recap

Multi-classification Loss Functions

One-vs-All (OVA)

$$\hat{y}^i = \arg\max_{j \in [K]} \langle \mathbf{w}^j, \mathbf{x}^i \rangle$$



$$\mathbf{W} = [\mathbf{w}^1, \mathbf{w}^2, \ldots, \mathbf{w}^K]$$

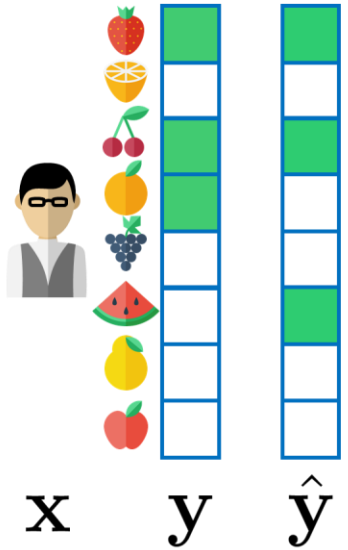
$$\widehat{\mathbf{W}} = \arg\min_{\mathbf{W}} \sum_{k=1}^{K} \|\mathbf{w}^k\|_2^2 + \sum_{i=1}^{n} \ell_{\text{cs}}(y^i, \boldsymbol{\eta}^i)$$

$$\boldsymbol{\eta}^i = \langle \mathbf{W}, \mathbf{x}^i \rangle = [\boldsymbol{\eta}_1^i, \boldsymbol{\eta}_2^i, \ldots, \boldsymbol{\eta}_K^i]$$

$$\ell_{\text{cs}}(y, \boldsymbol{\eta}) = [1 + \max_{k \neq y} \boldsymbol{\eta}_k - \boldsymbol{\eta}_y]_+$$

Crammer-Singer loss function

Multi-label Classification Loss Functions



$$\begin{array}{c|c} \left|\left\{i : \begin{matrix}\hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = 1\end{matrix}\right\}\right| & \left|\left\{i : \begin{matrix}\hat{\mathbf{y}}_i = 1 \\ \mathbf{y}_i = -1\end{matrix}\right\}\right| \\ \hline \left|\left\{i : \begin{matrix}\hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = 1\end{matrix}\right\}\right| & \left|\left\{i : \begin{matrix}\hat{\mathbf{y}}_i = -1 \\ \mathbf{y}_i = -1\end{matrix}\right\}\right| \end{array}$$

a b c d

$$\ell_{\text{Hamming}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{b+c}{a+b+c+d}$$

$$r_{\text{Precision}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+b} \qquad r_{\text{Recall}}(\mathbf{y}, \hat{\mathbf{y}}) = \frac{a}{a+c}$$

$$F(\mathbf{y}, \hat{\mathbf{y}}) = \frac{2 r_{\text{Prec}} \cdot r_{\text{Rec}}}{r_{\text{Prec}} + r_{\text{Rec}}} = \frac{2a}{2a+b+c}$$

Can be used for evaluation and training

Historically

More recent

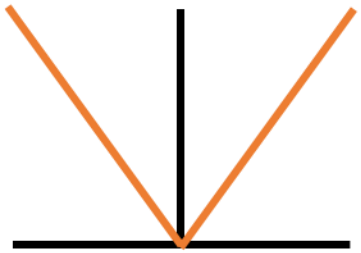
First-order Optimality

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w} \in \mathbb{R}^d} f(\mathbf{w}) + r(\mathbf{w})$$

$$\nabla f(\mathbf{w}) + \nabla r(\mathbf{w}) = \mathbf{0}$$

$$f(\mathbf{w}) = \sum_{i=1}^{n} \left(y^i - \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle\right)^2 \qquad f(\mathbf{w}) = \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right)$$

$$r(\mathbf{w}) = \lambda \cdot \|\mathbf{w}\|_2^2 \qquad r(\mathbf{w}) = \lambda \cdot \|\mathbf{w}\|_2^2$$



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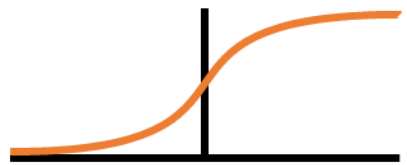

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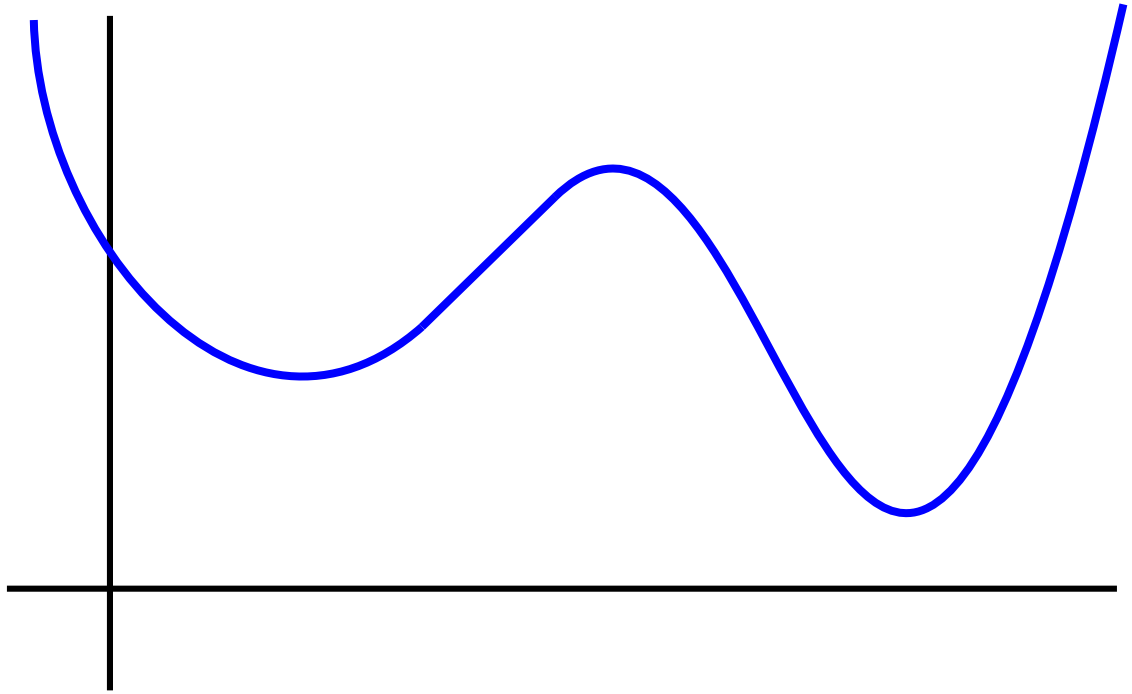
Gradient Descent



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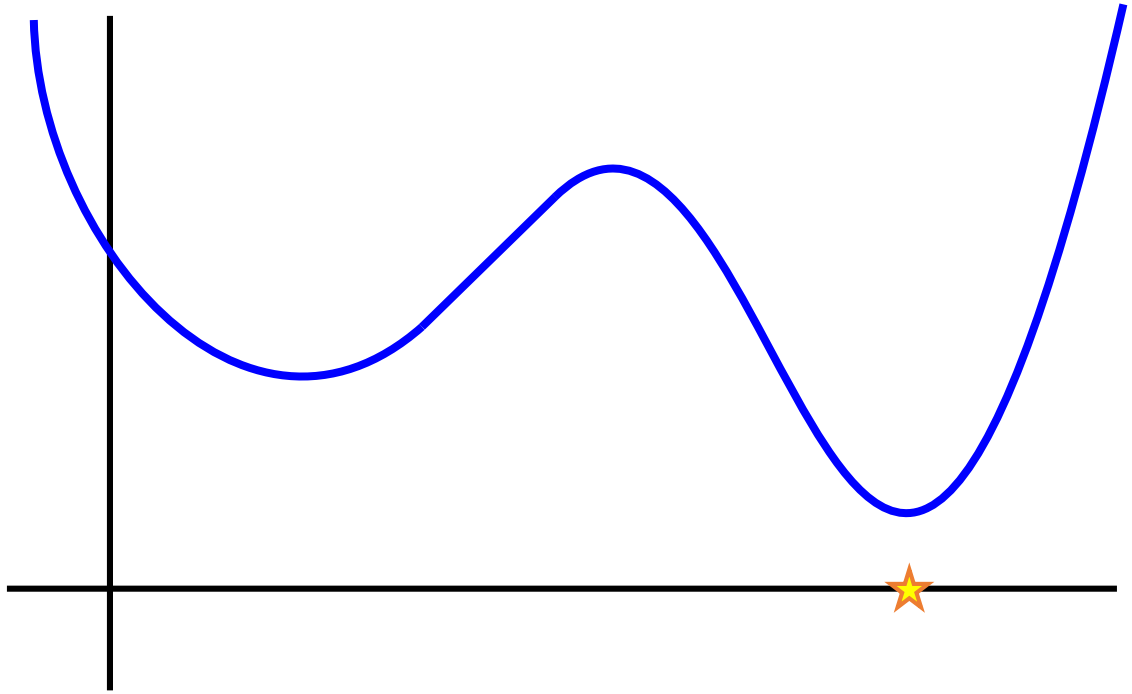

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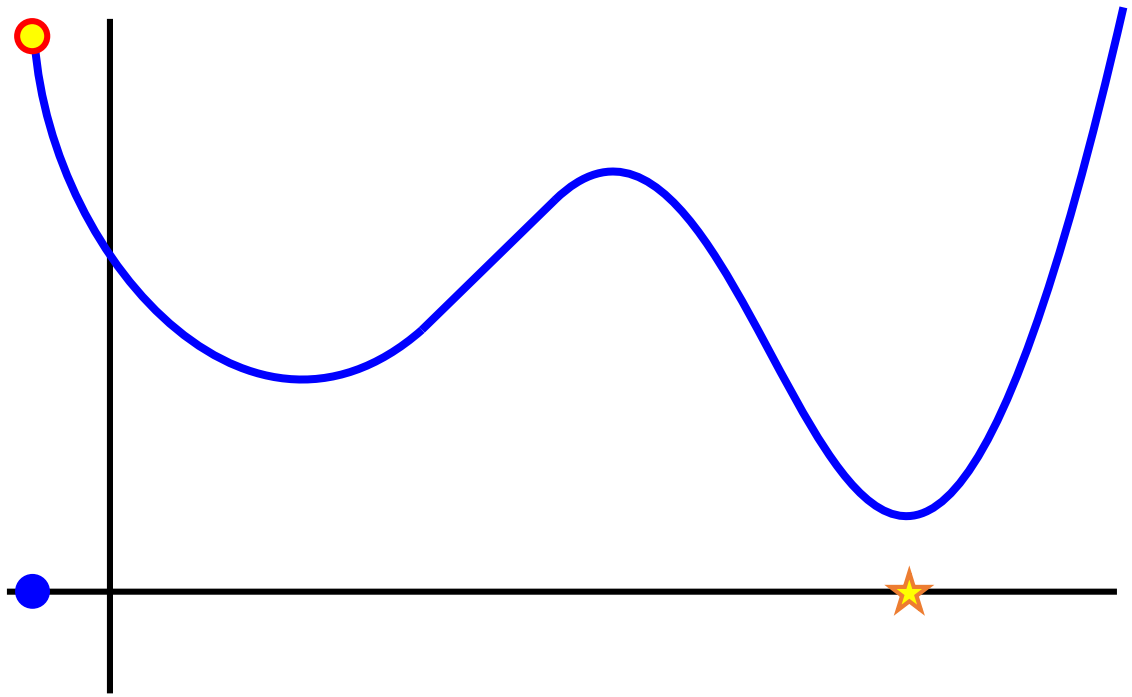

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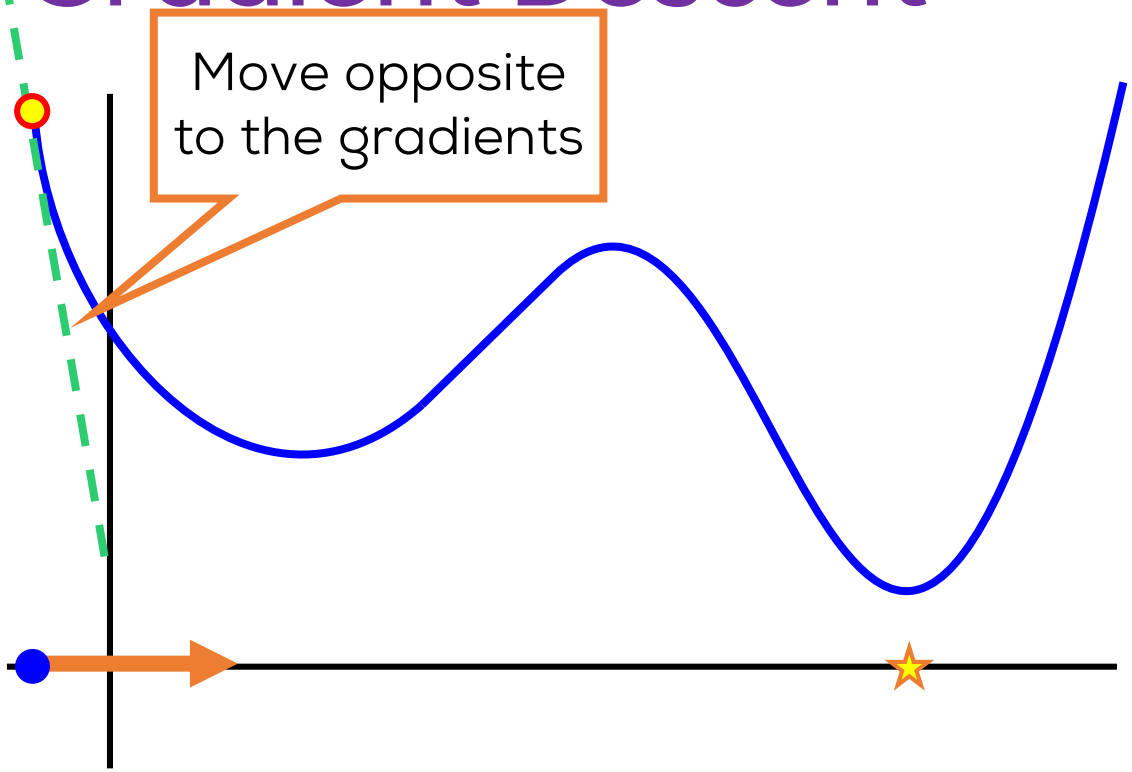

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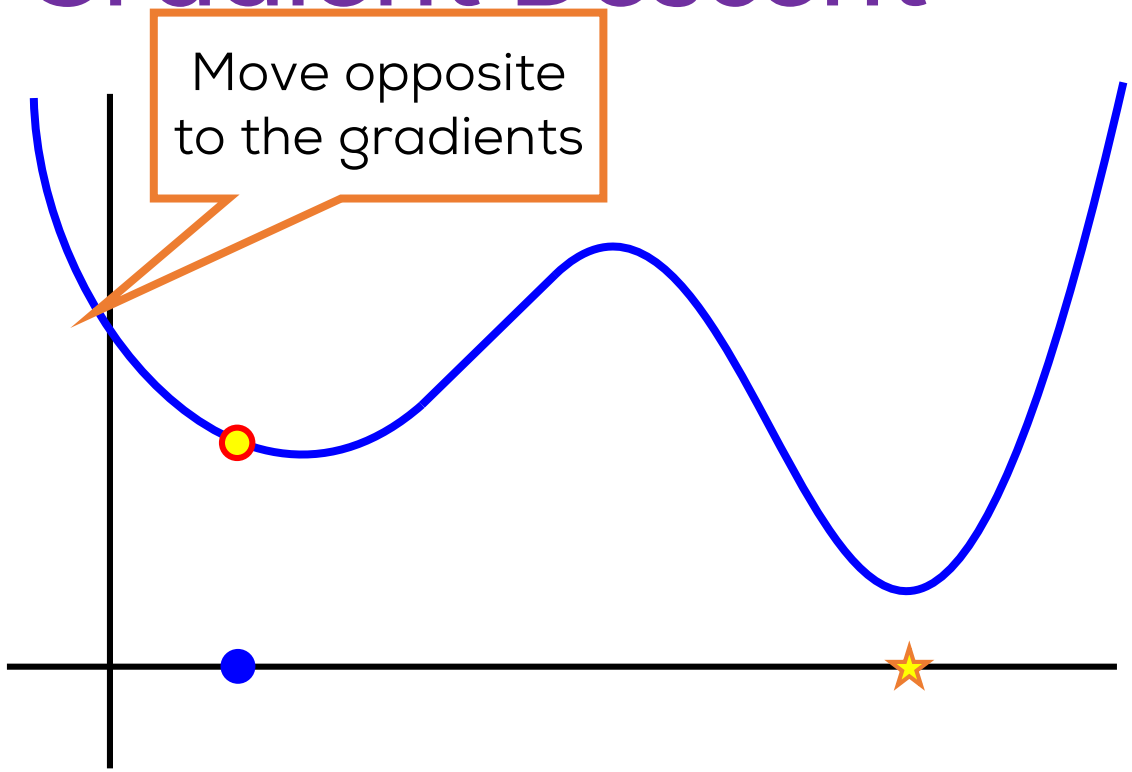

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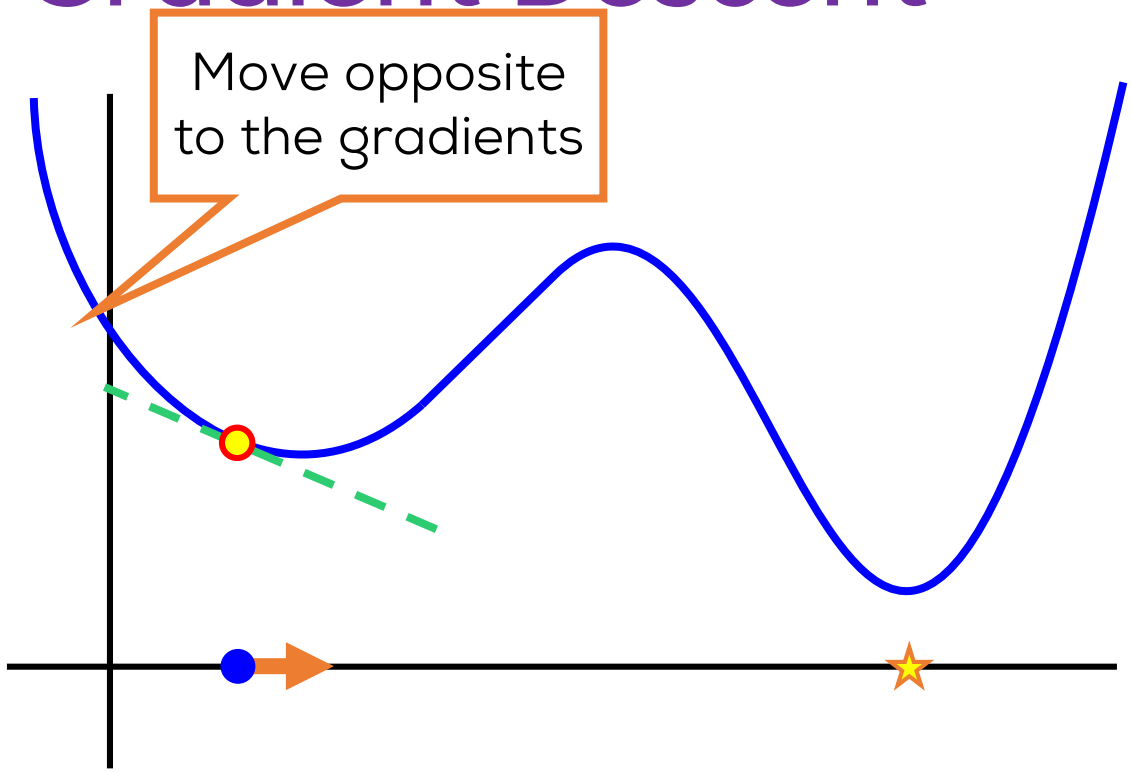

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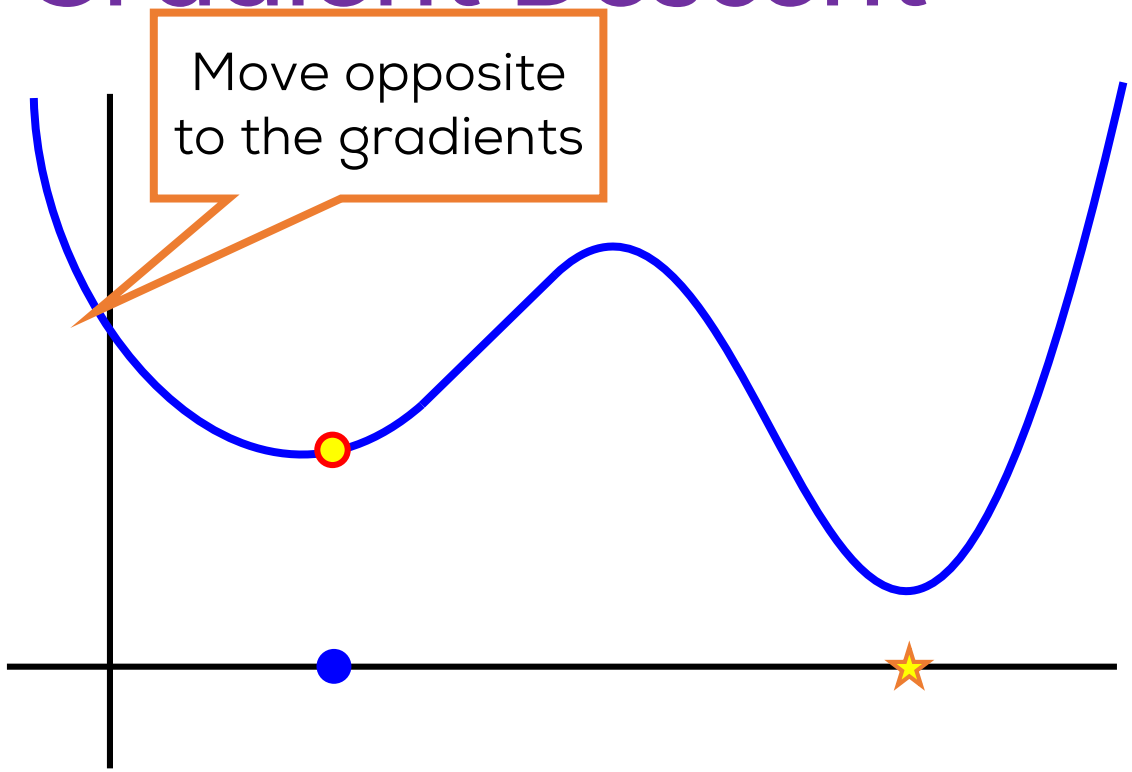

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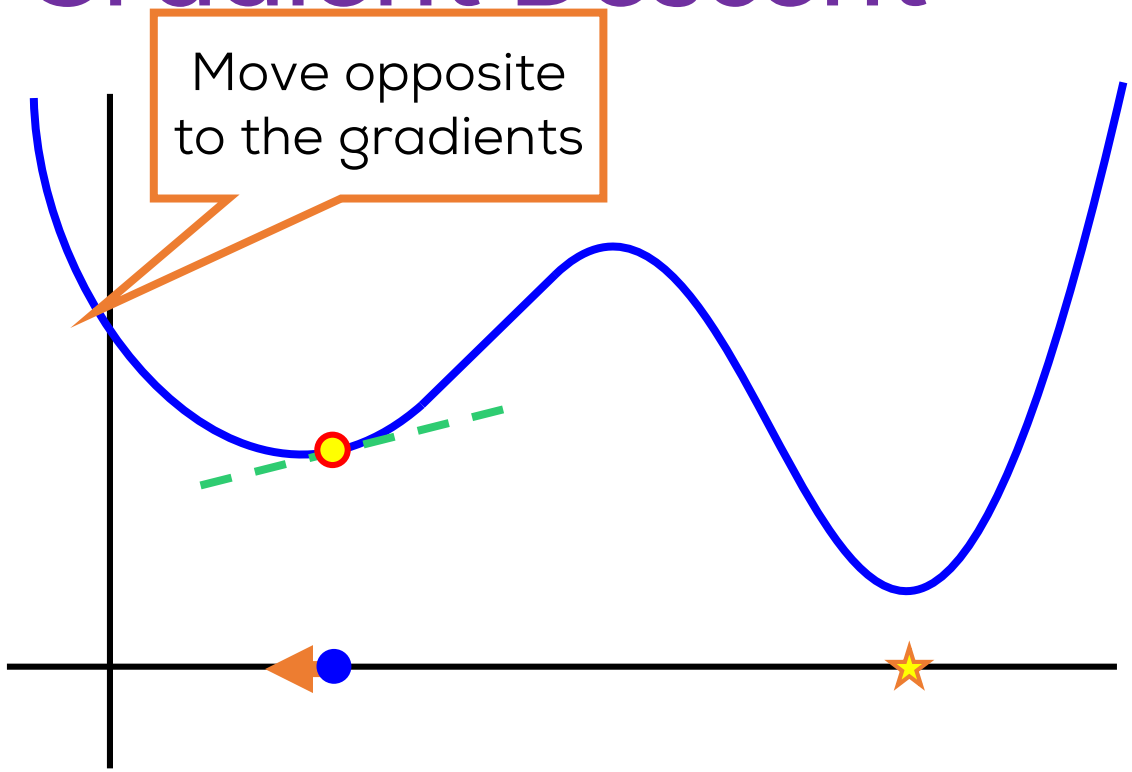

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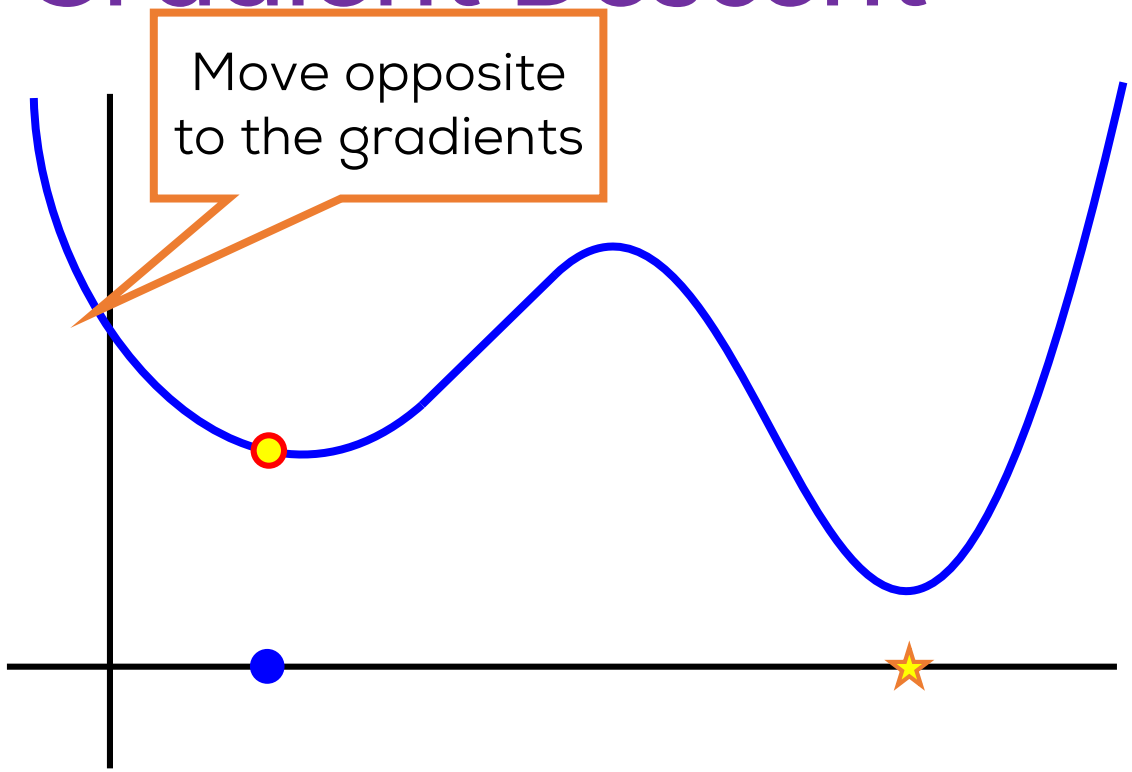

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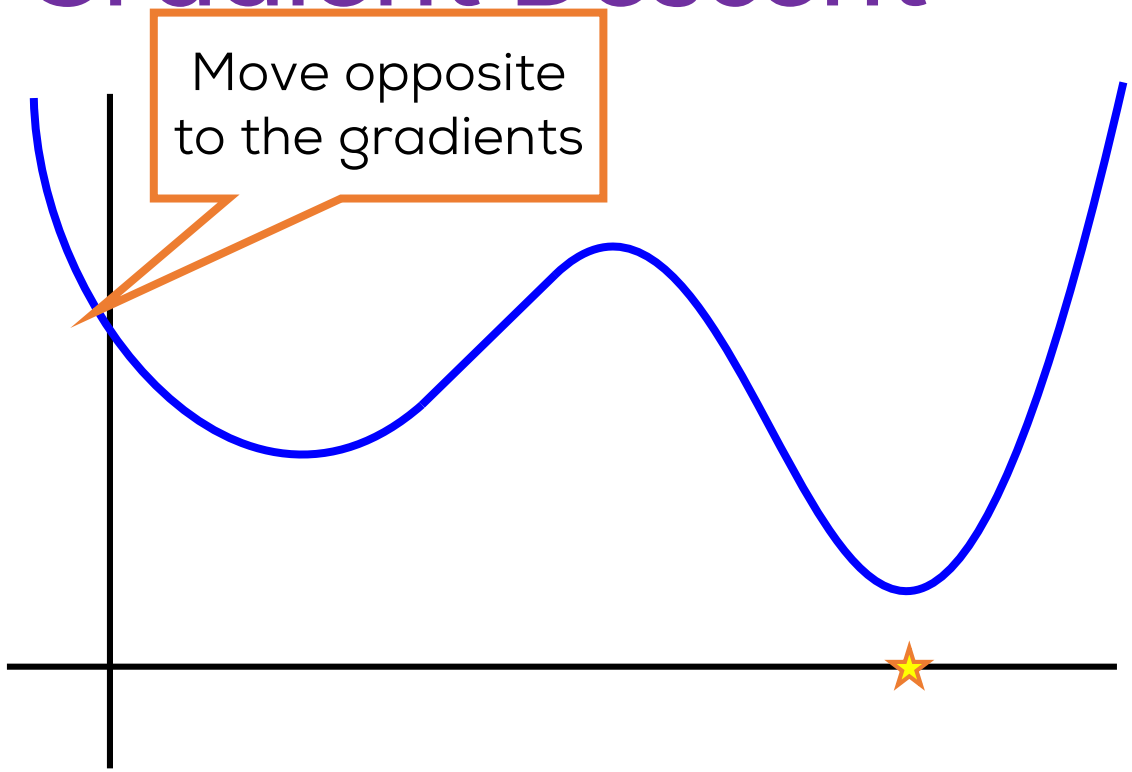

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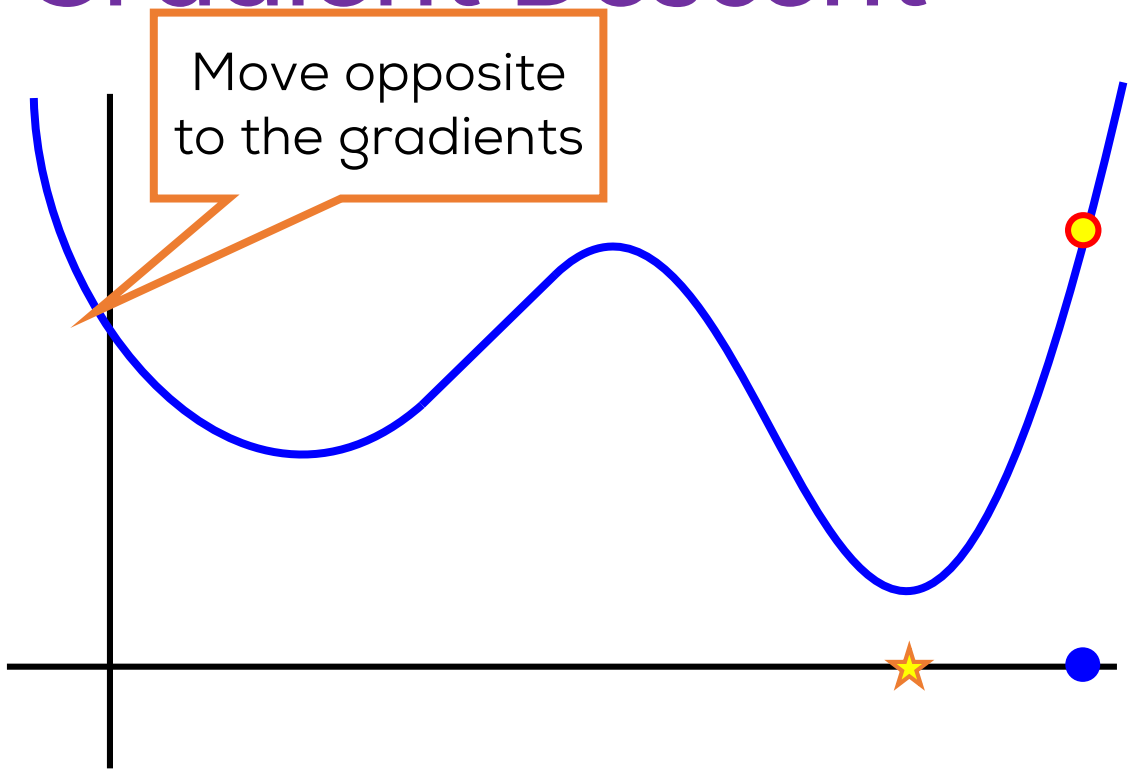

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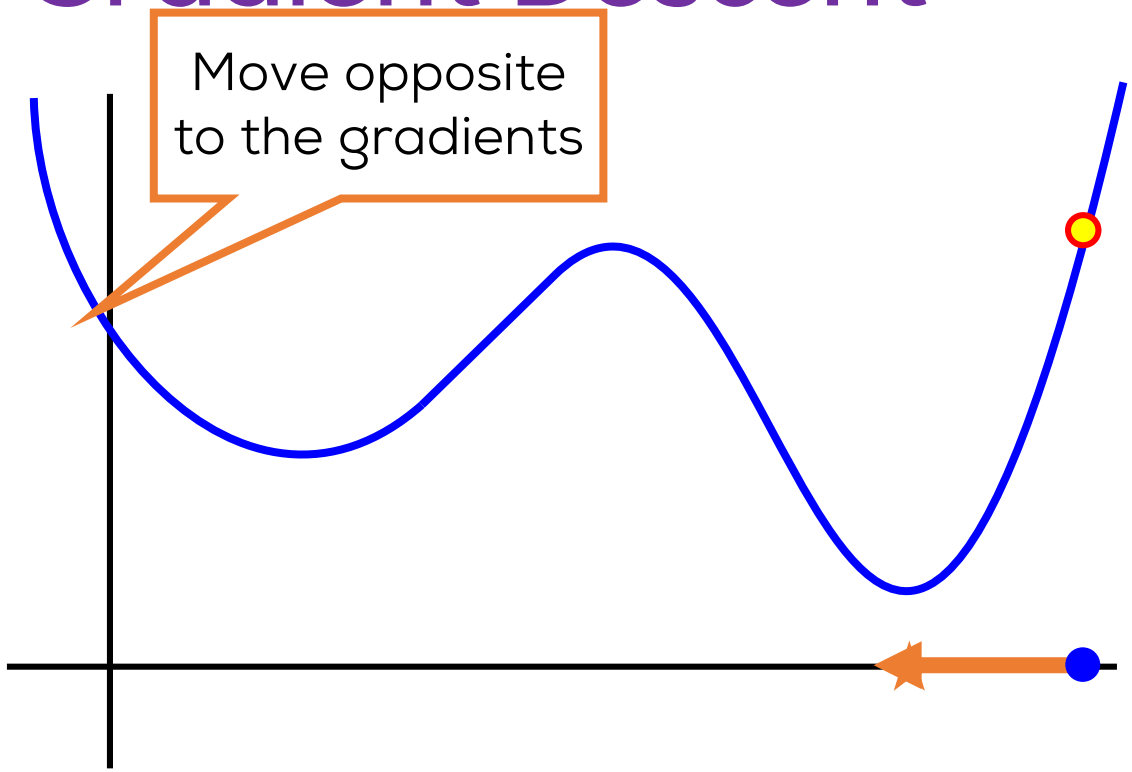

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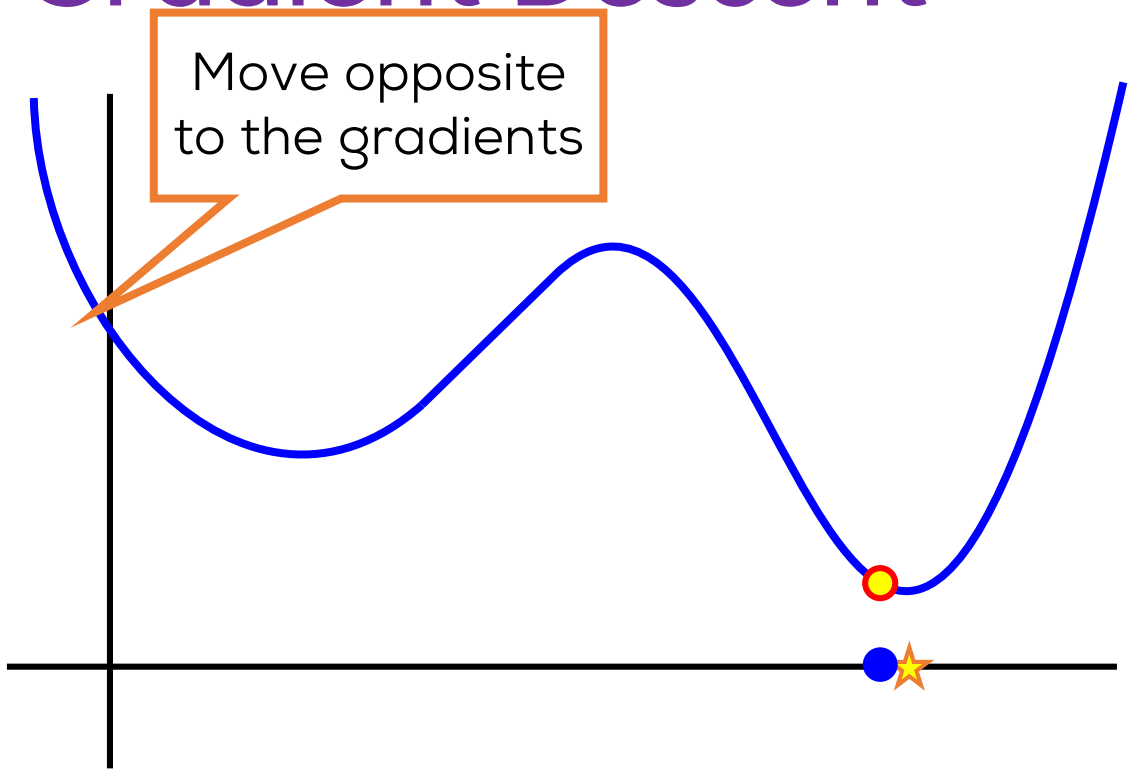

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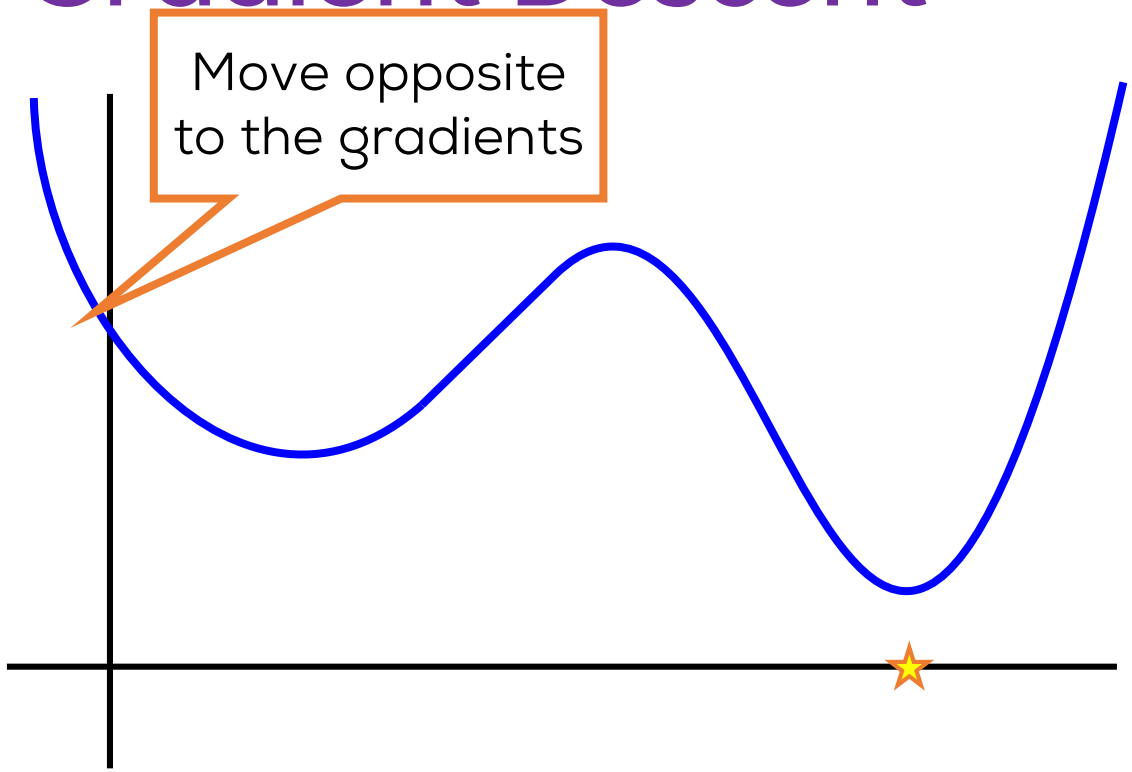

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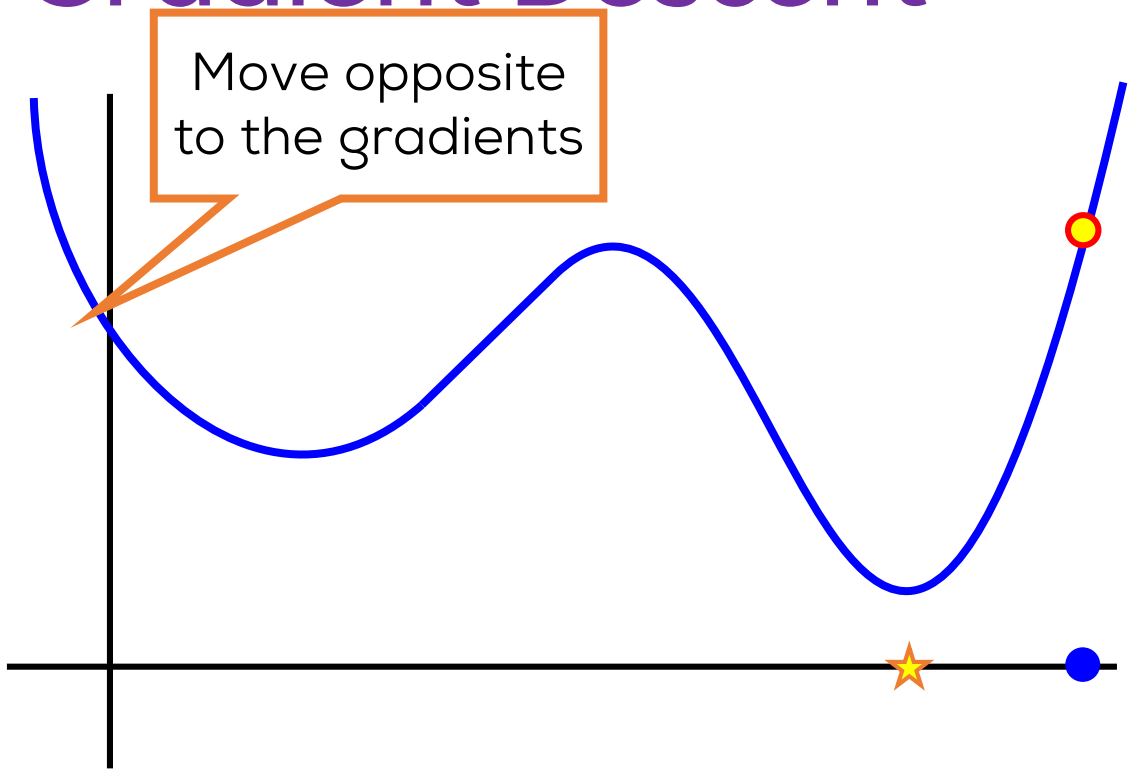

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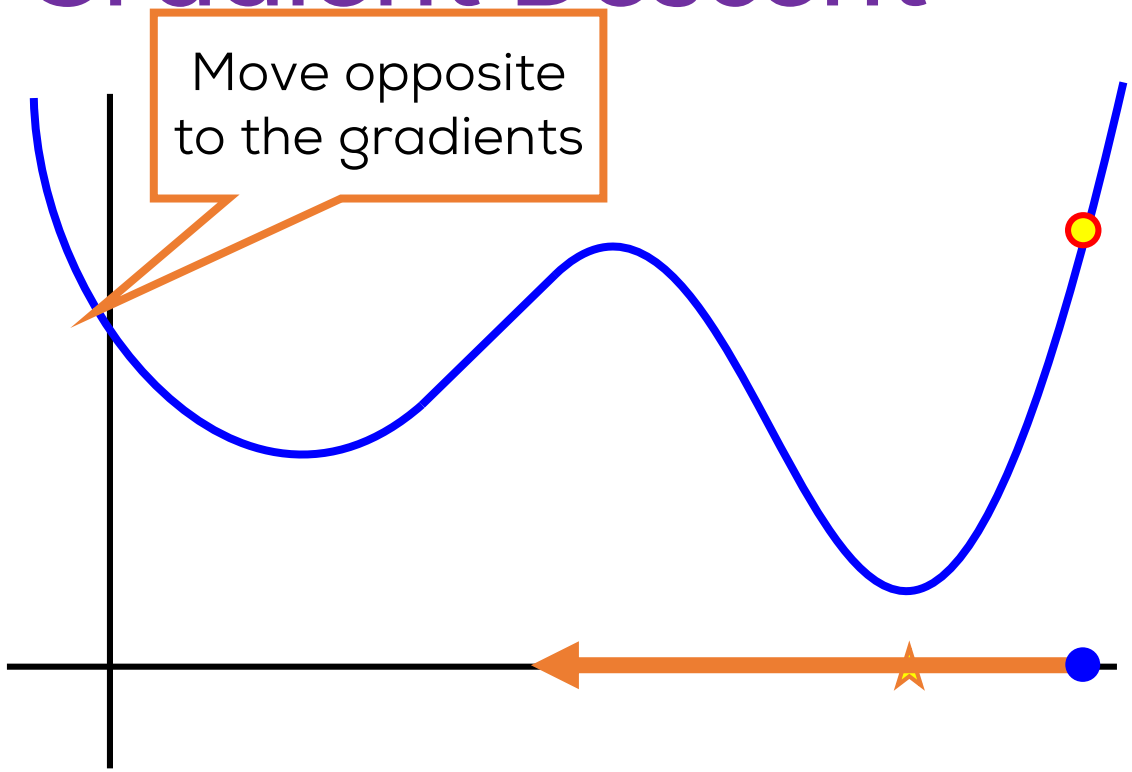

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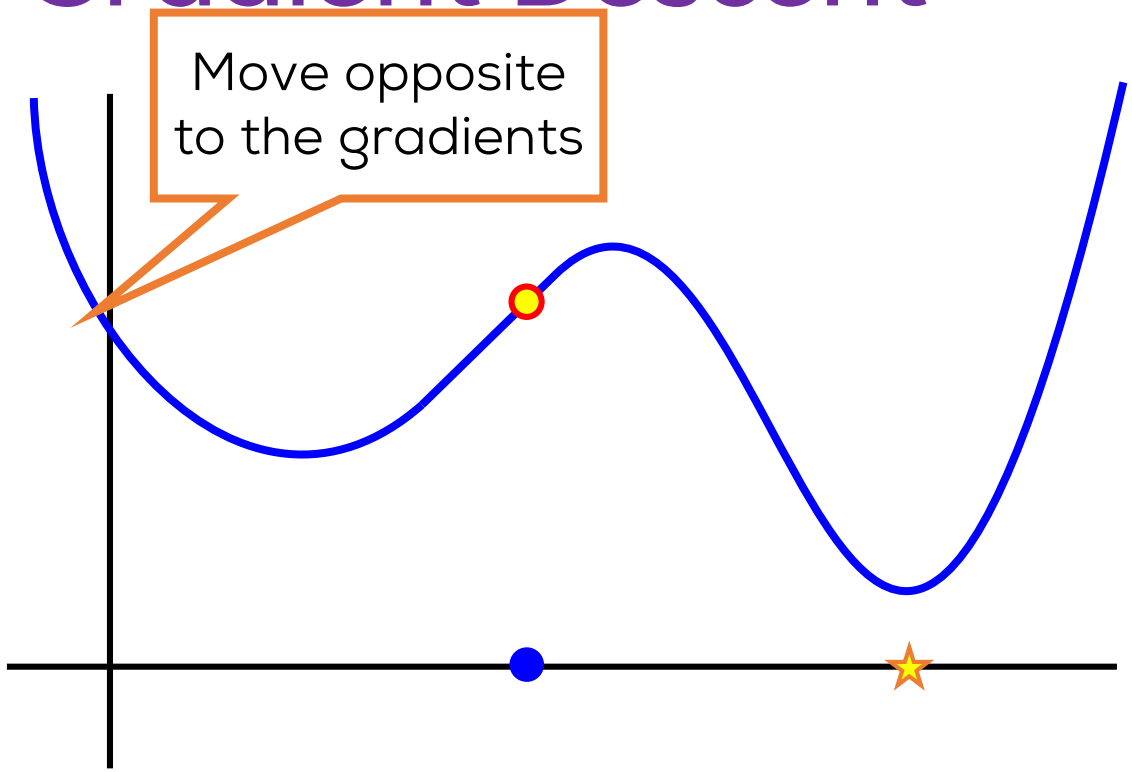

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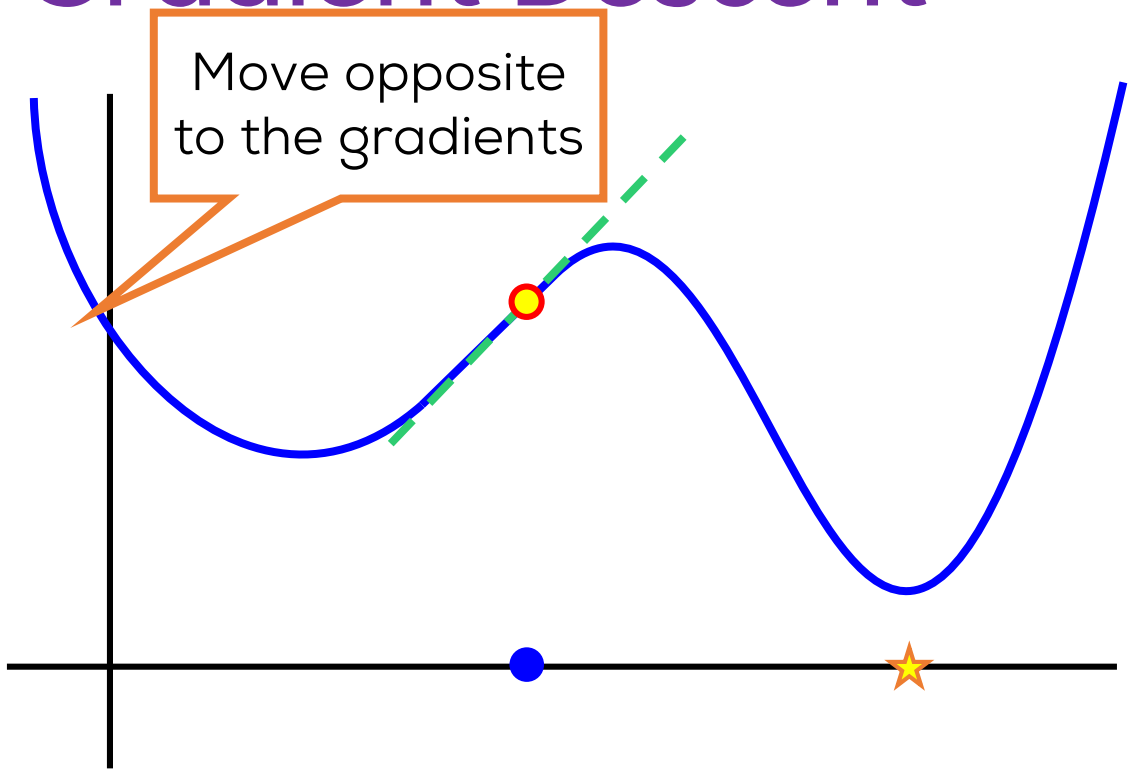

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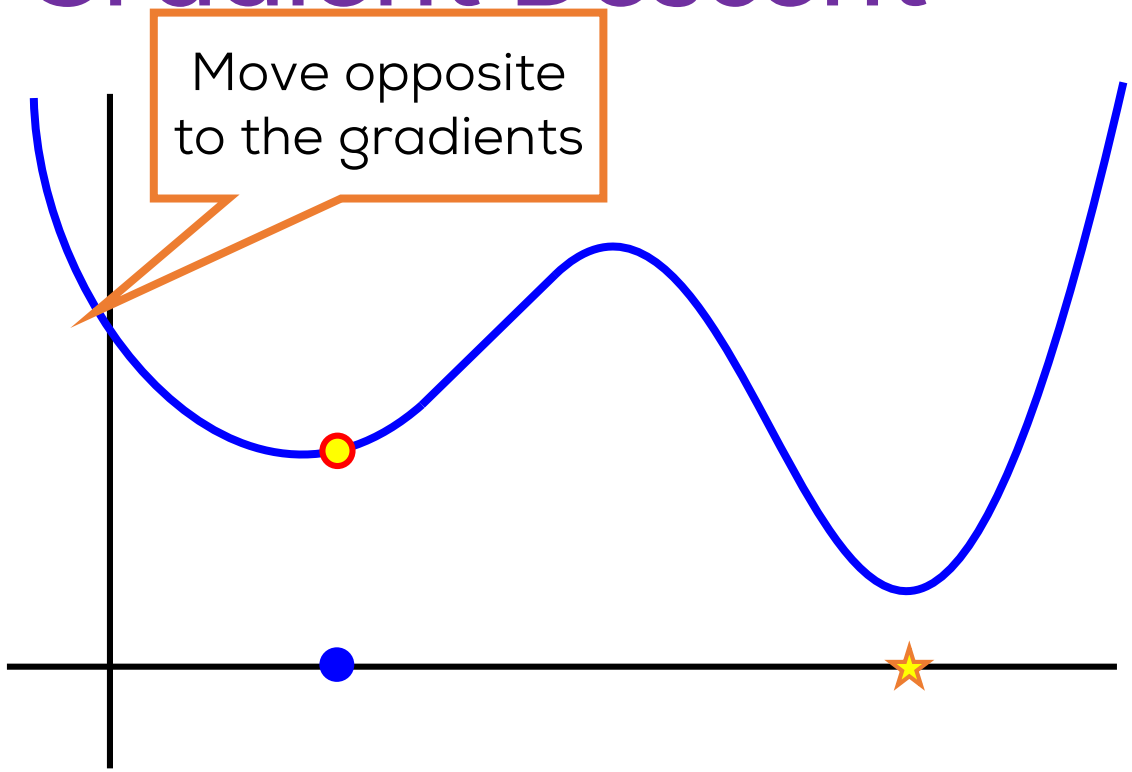

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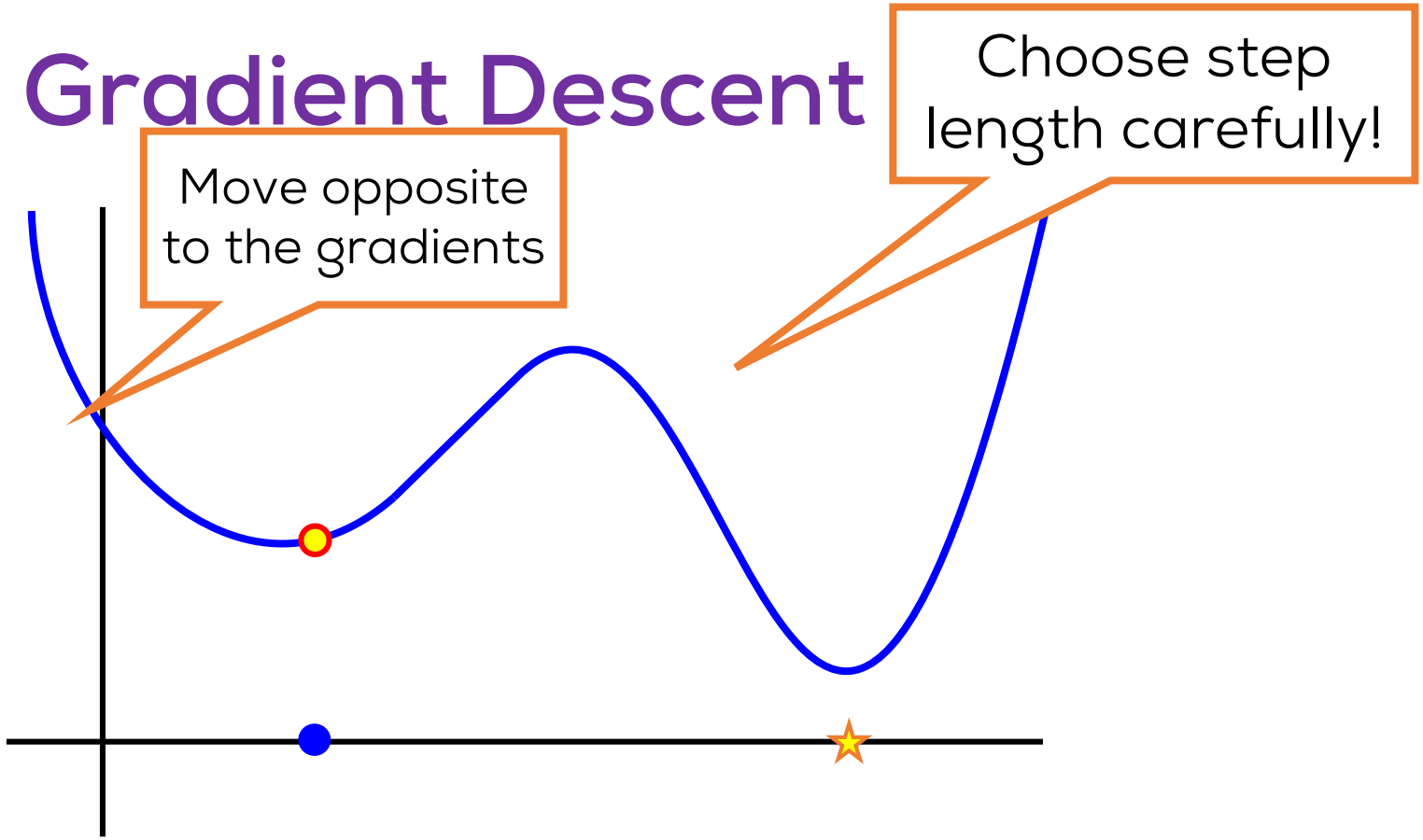

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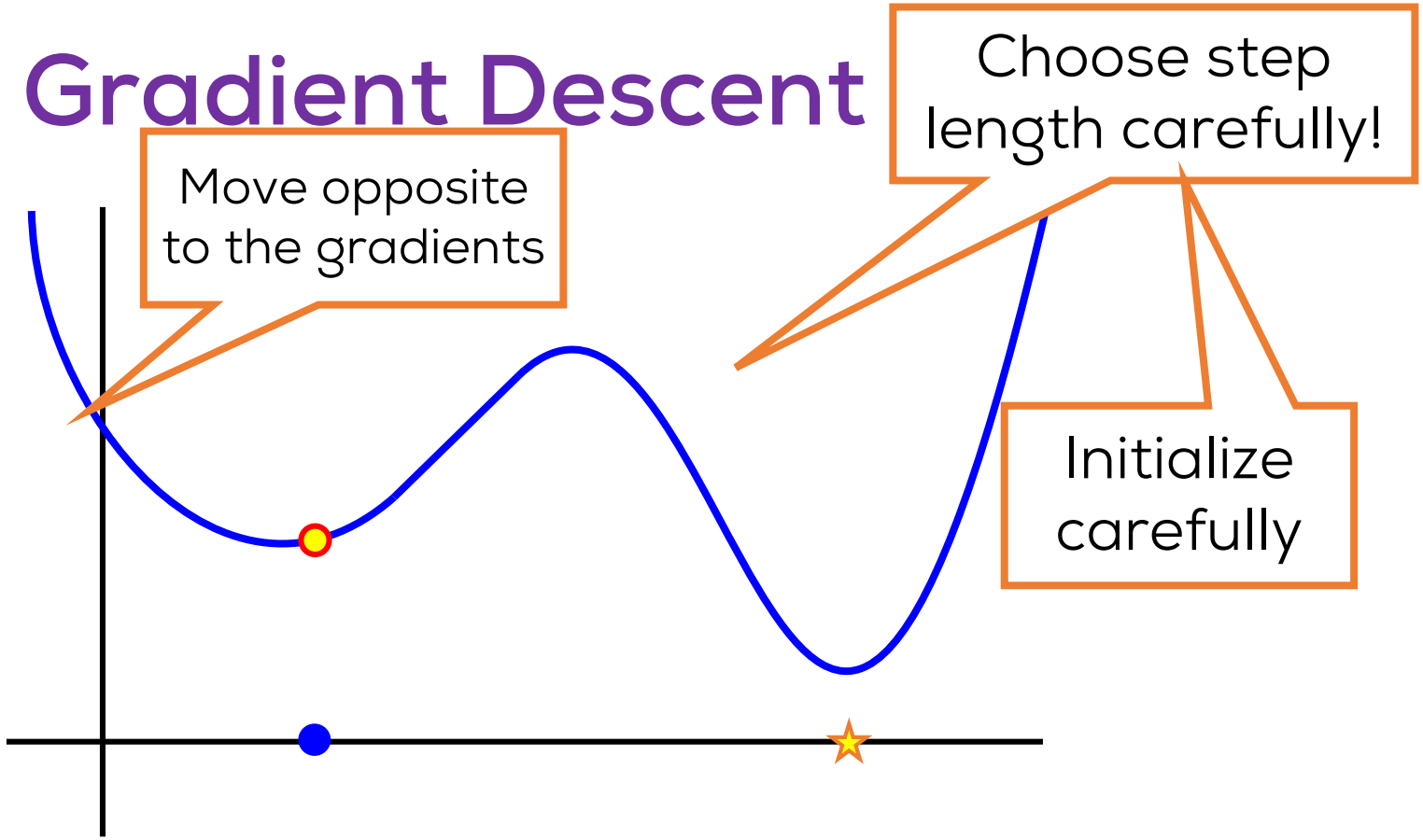

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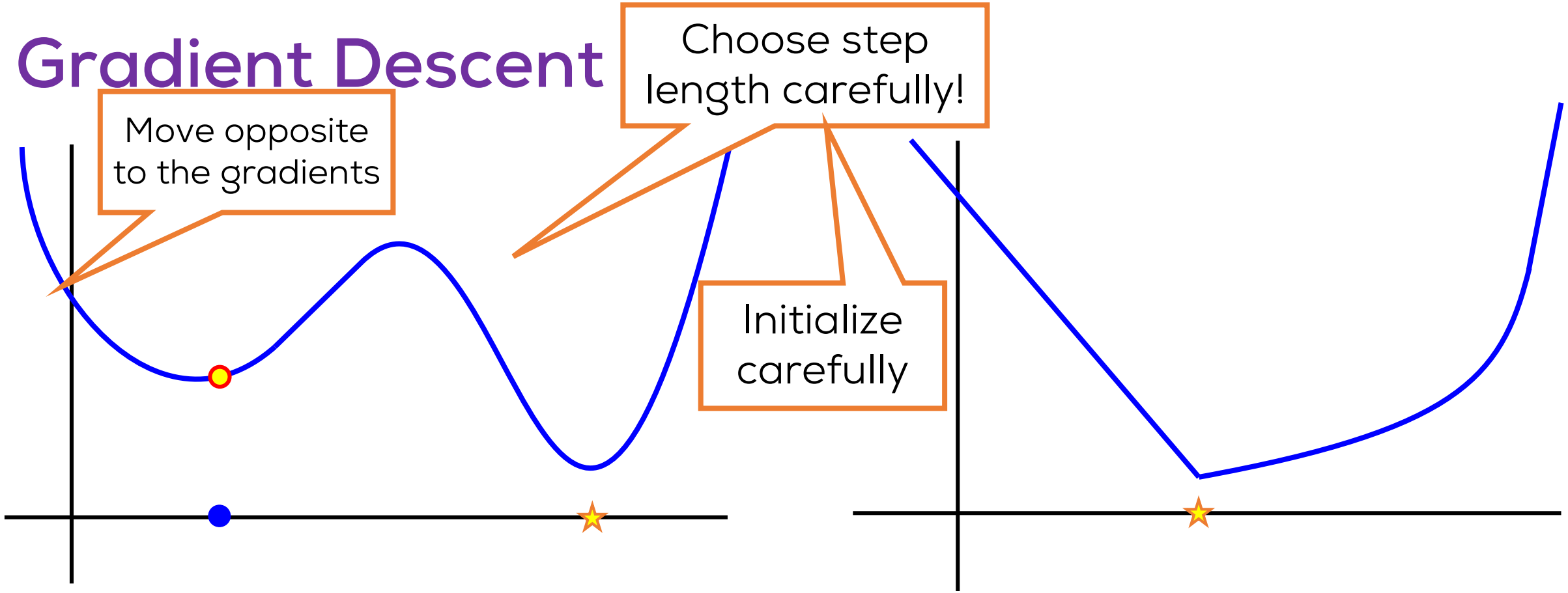

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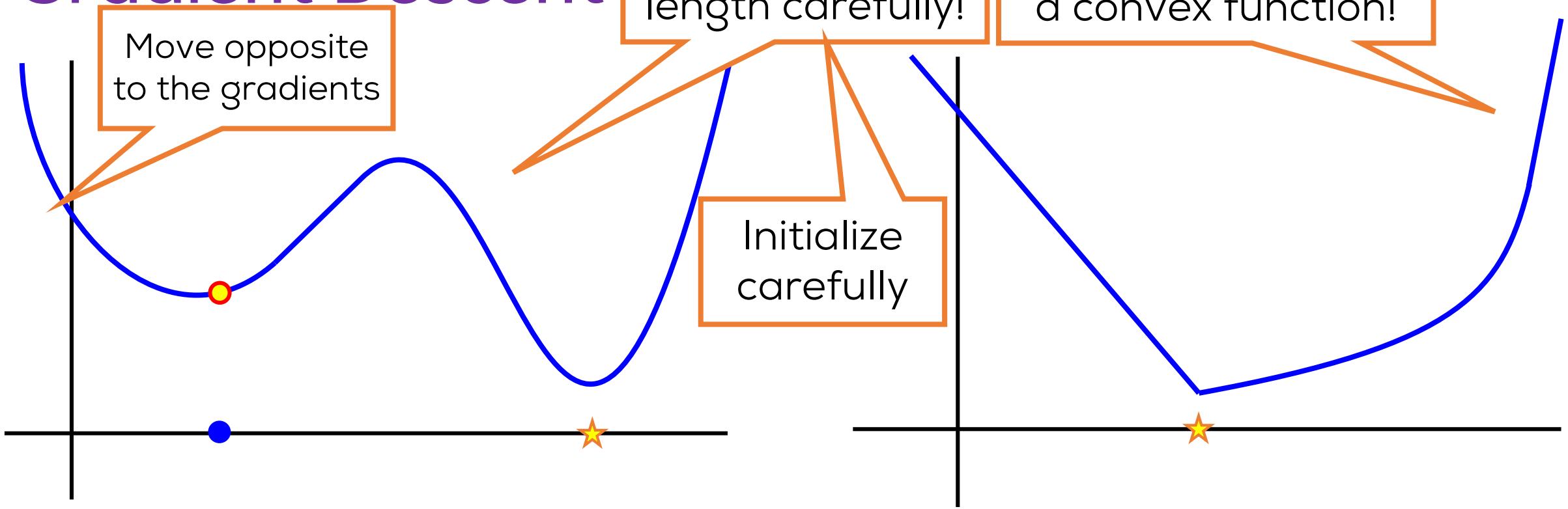

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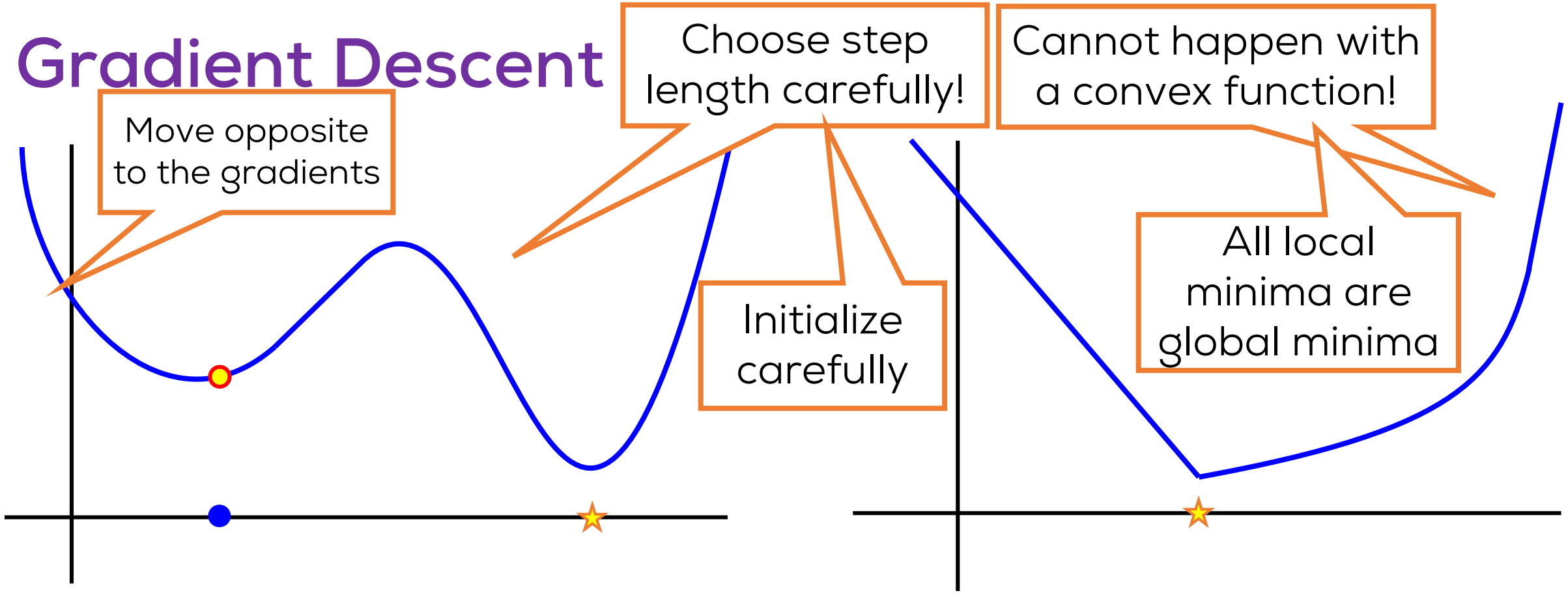

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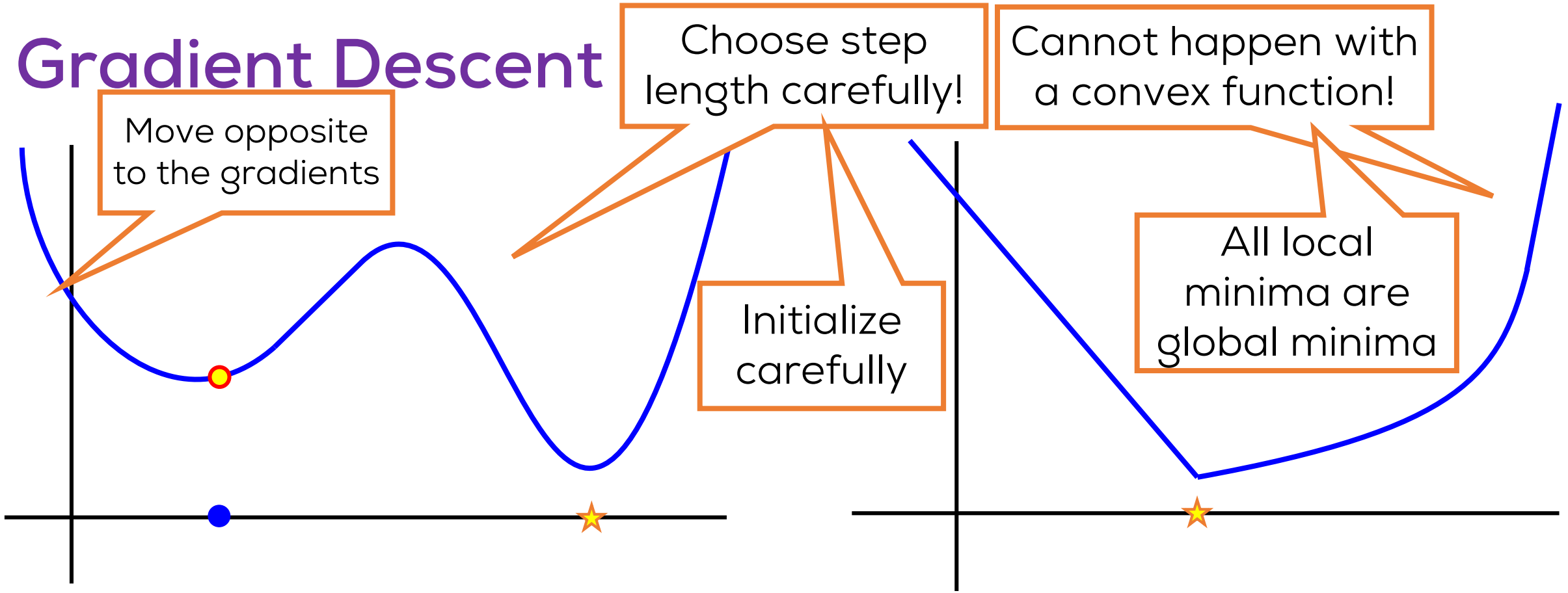
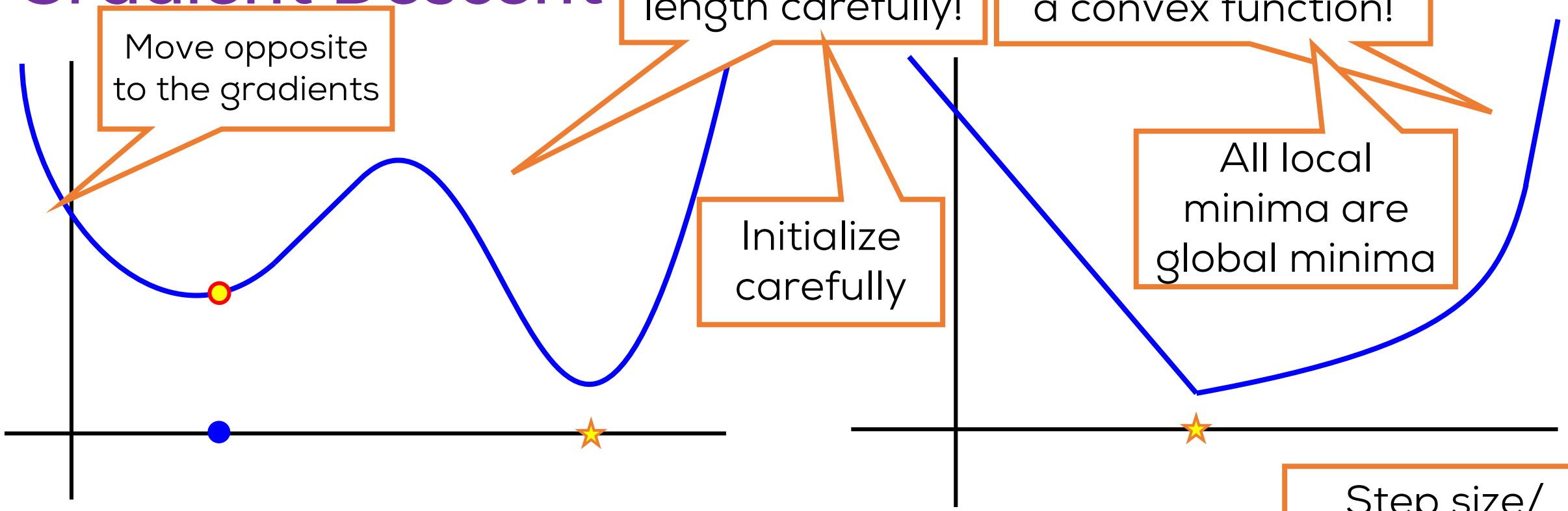


GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
3. Repeat until convergence

Gradient Descent



Step size/ learning rate

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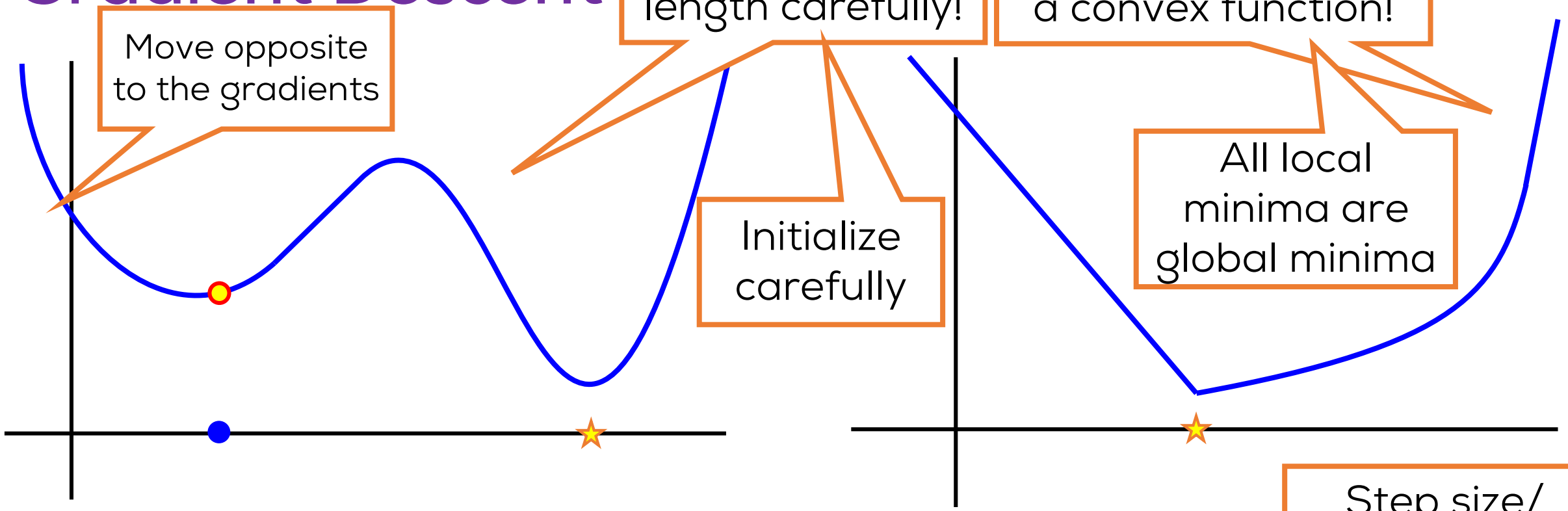

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Tuned carefully CV, Adam, Adagrad

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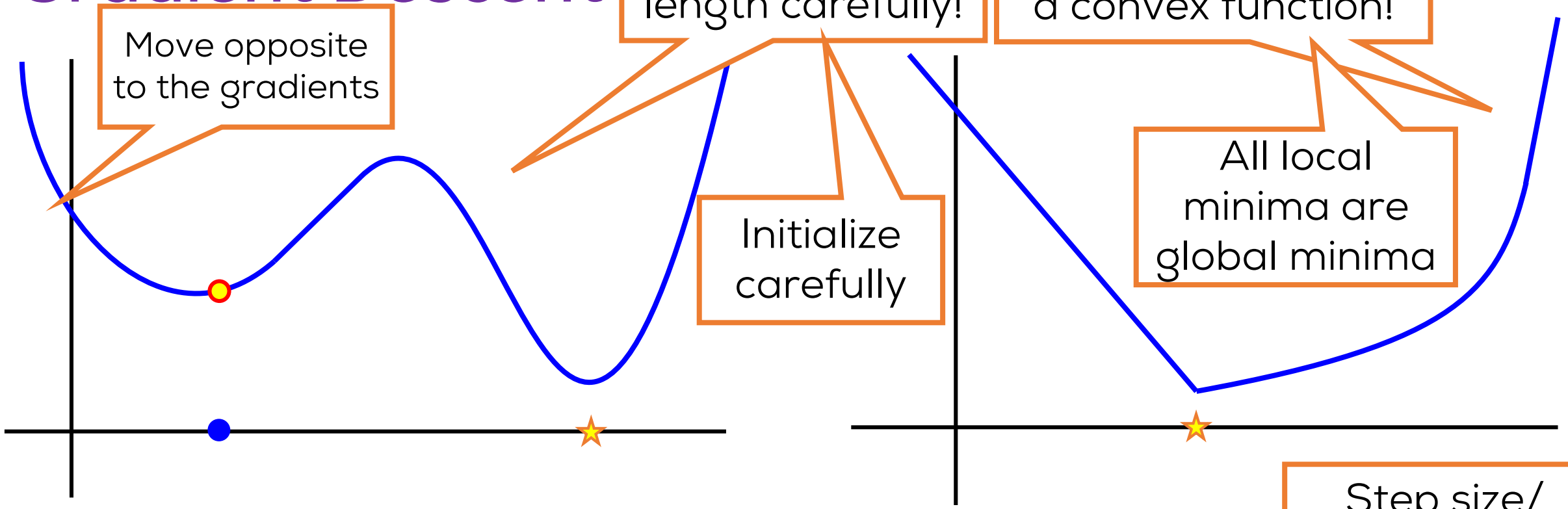

Many convergence criteria – length of gradient, performance threshold, dual criteria

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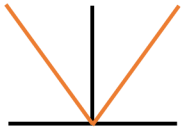

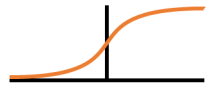

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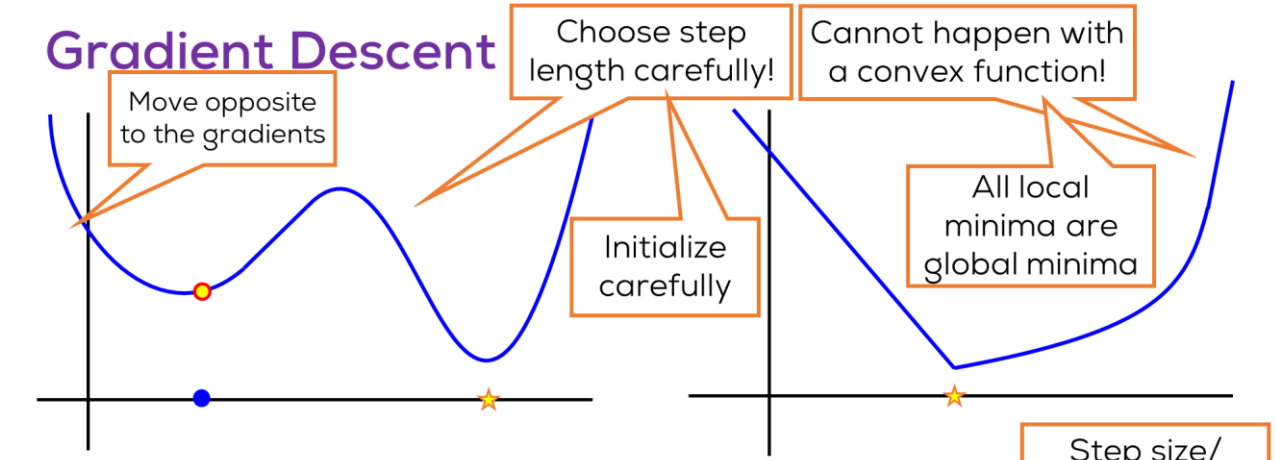

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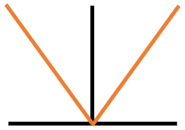

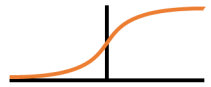

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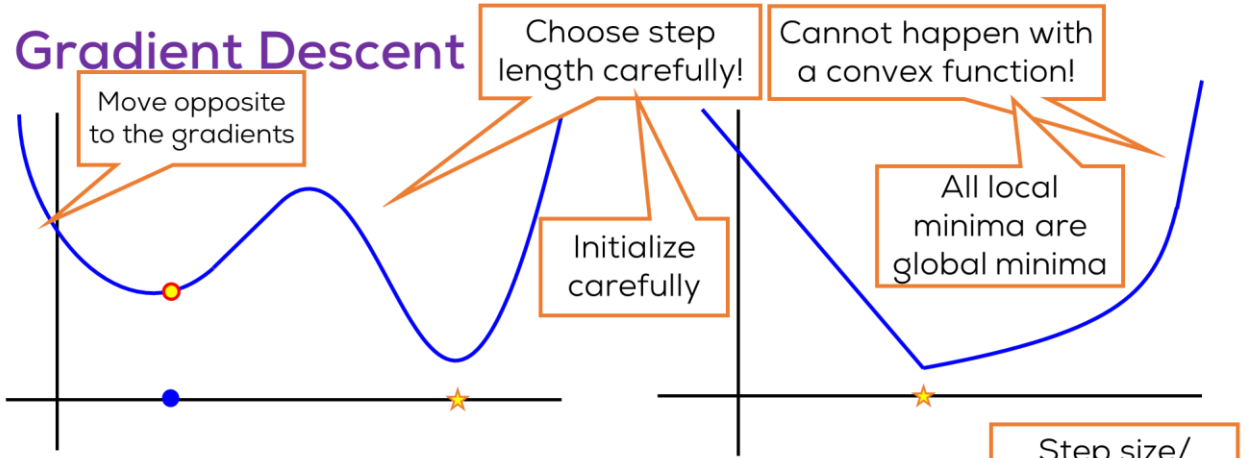

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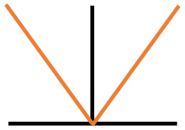

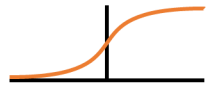

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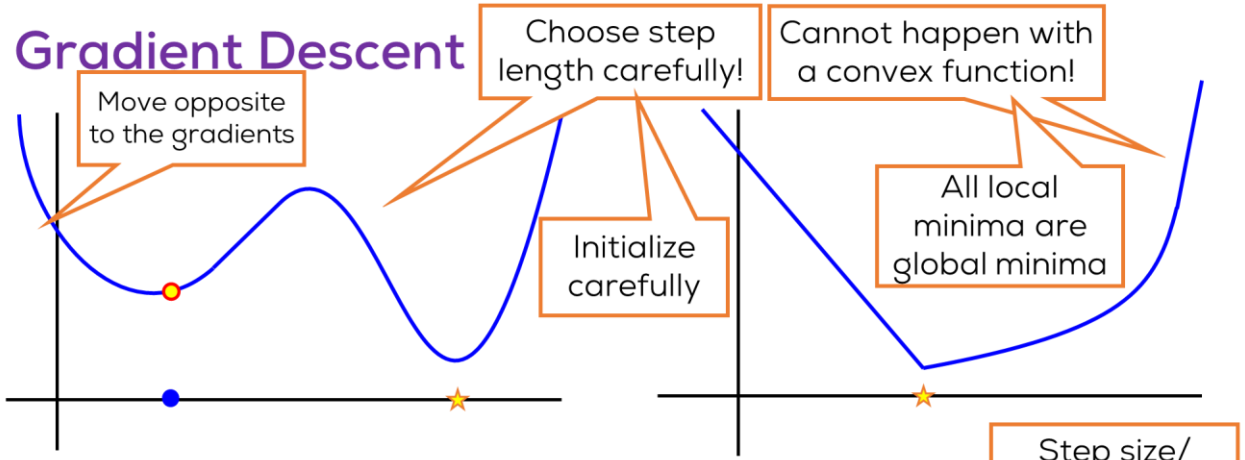

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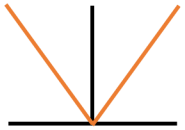

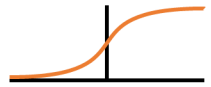

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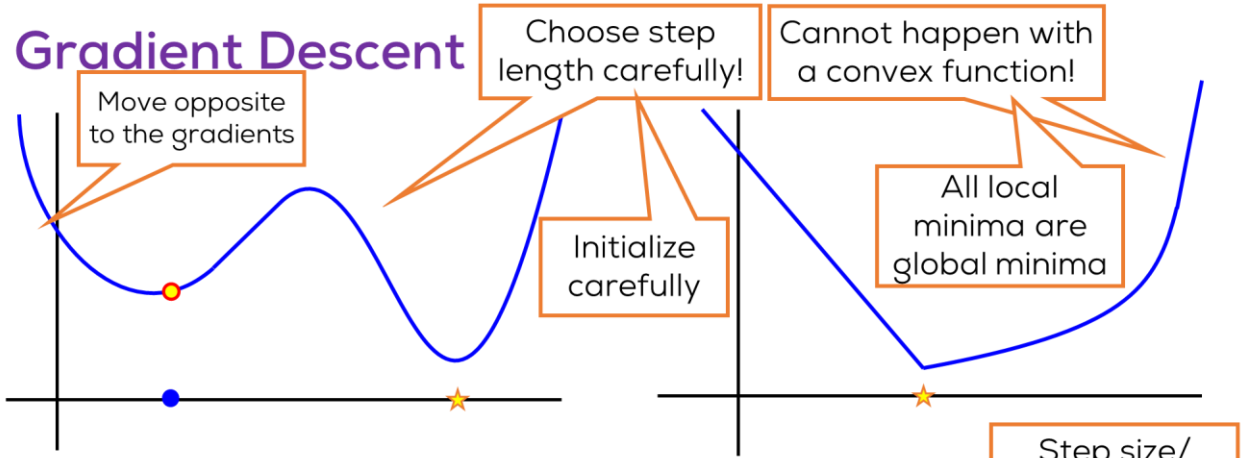

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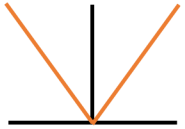

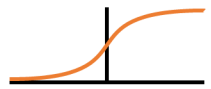

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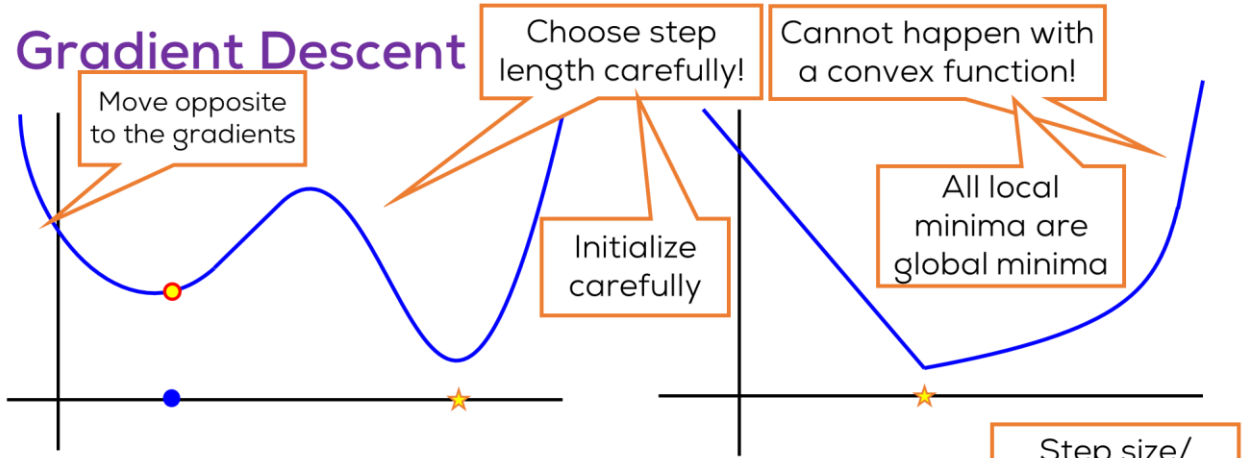

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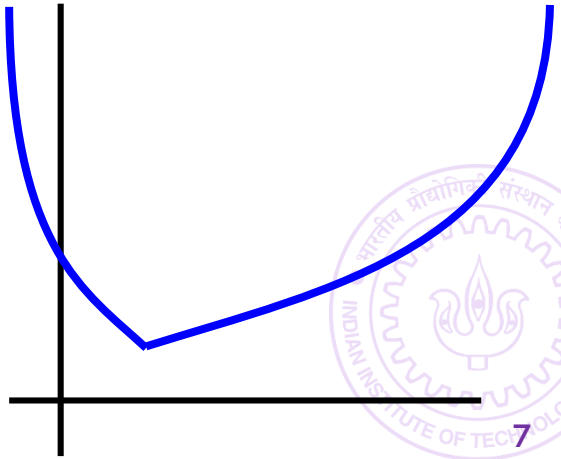

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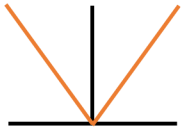

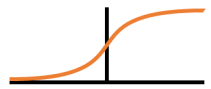

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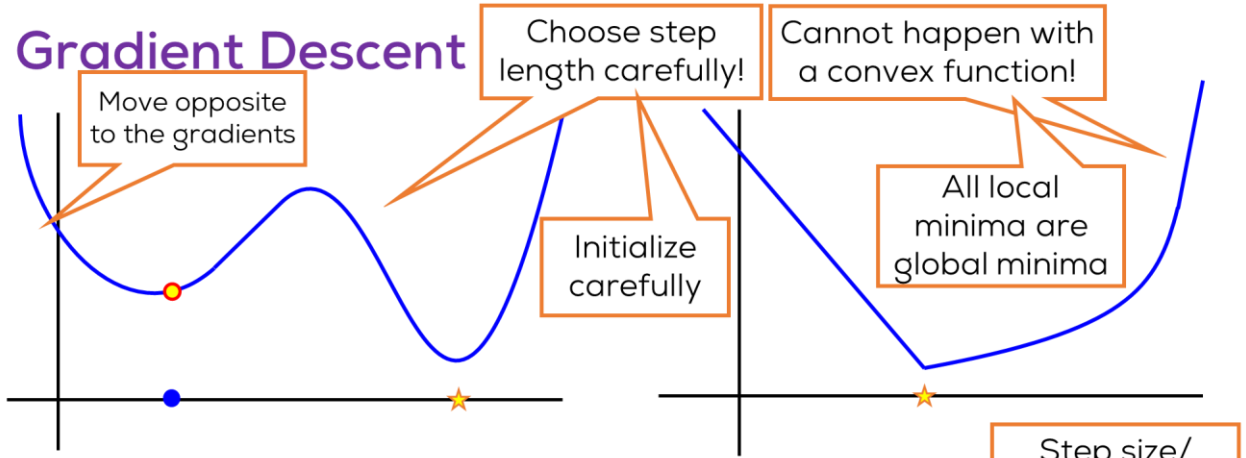

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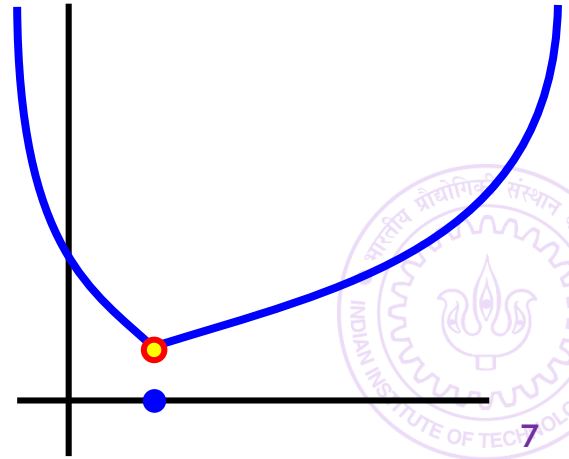

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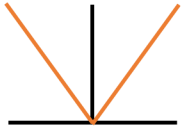

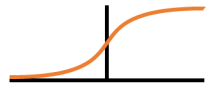

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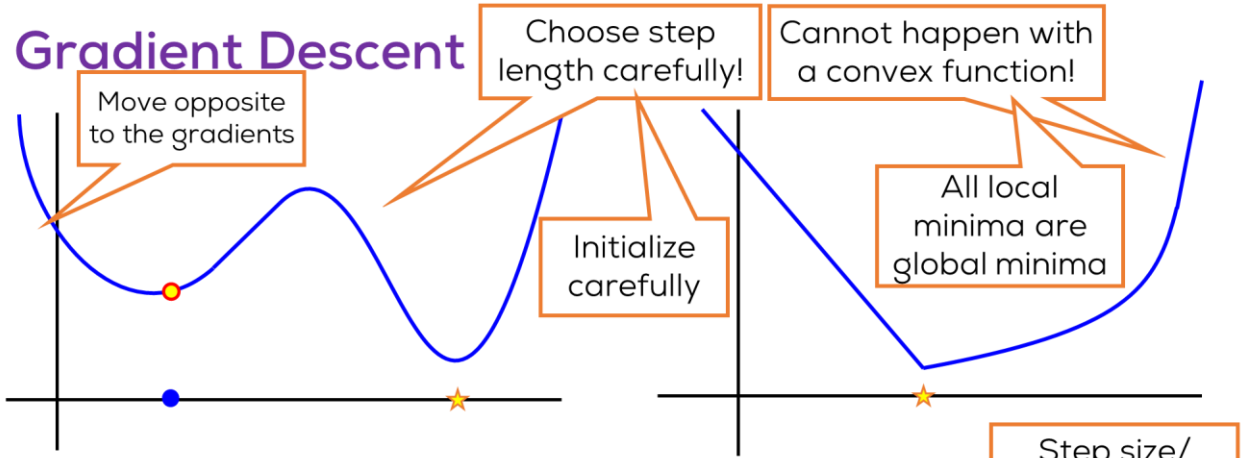

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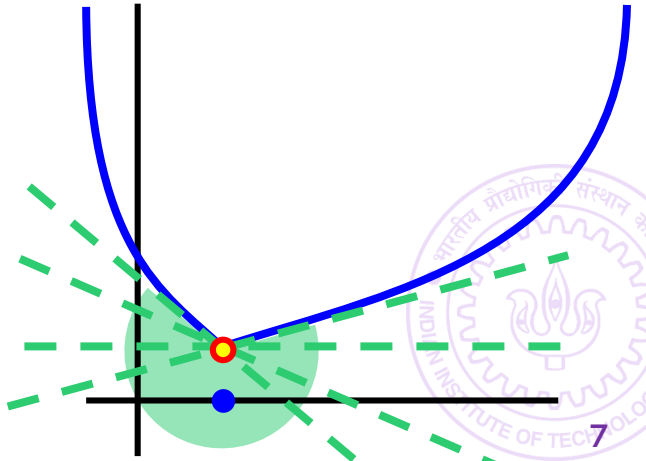

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Step length
$\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

Only $O(nd)$ time per iter!

$O\left(\frac{1}{\epsilon^2}\right)$ iterations suffice to reach ϵ-optimal solution

$$\mathbf{g}^t = \nabla f(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$
$$= 2\sum_{i=1}^{n} (\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}^i + 2\lambda \cdot \mathbf{w}^t$$

For convex problems

$f(\mathbf{w}^t) + r(\mathbf{w}^t)$
$\leq f(\mathbf{w}^*) + r(\mathbf{w}^*) + \epsilon$

A Closer look at the GD Update

A Closer look at the GD Update

$$\mathbf{w}^{t+1} = (1 - 2\lambda\eta_t) \cdot \mathbf{w}^t - 2\eta_t \cdot \sum_{i=1}^{n} (\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}^i$$

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Does not let model change too much!

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Does not let model change too much!

What if $\lambda = 0$?
What if $\eta_t = 0$?
What if no $f(\cdot)$?

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What if no $f(\cdot)$?

If $\langle \mathbf{w}^t, \mathbf{x}^i \rangle > y^i$
If $\langle \mathbf{w}^t, \mathbf{x}^i \rangle = y^i$
If $\langle \mathbf{w}^t, \mathbf{x}^i \rangle < y^i$

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What if $\eta_t = 0$?
What if no r(·)?

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What if $\eta_t = 0$?
What if no r(·)?

Exercise: for clarity, see what happens when $d = 1$

A Closer look at the GD Update

Lets see the case $n = 1, \lambda = 0$

$$\mathbf{w}^{t+1} = \mathbf{w}^t - 2\eta_t \cdot (\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}^i$$

Does not let model change too much!

What if $\lambda = 0$?
What if $\eta_t = 0$?
What if no $f(\cdot)$?

Push $\mathbf{w}^t$ "away" from $\mathbf{x}^i$

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A Closer look at the GD Update

Lets see the case $n = 1, \lambda = 0$

$$\mathbf{w}^{t+1} = \mathbf{w}^t - 2\eta_t \cdot \left(\left\langle \mathbf{w}^t, \mathbf{x}^i \right\rangle - y^i\right) \cdot \mathbf{x}^i$$

Does not let model change too much!

What if $\lambda = 0$?
What if $\eta_t = 0$?
What if no $f(\cdot)$?

$\left\langle w^{t+1}, x^i \right\rangle < \left\langle w^t, x^i \right\rangle$

Push $\mathbf{w}^t$ "away" from $\mathbf{x}^i$

No need to care about $\mathbf{x}^i$

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$\langle w^{t+1}, x^i \rangle < \langle w^t, x^i \rangle$

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What if $\lambda = 0$?
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$\langle w^{t+1}, x^i \rangle < \langle w^t, x^i \rangle$ — Push $\mathbf{w}^t$ "away" from $\mathbf{x}^i$ — If $\langle \mathbf{w}^t, \mathbf{x}^i \rangle > y^i$

$\langle w^{t+1}, x^i \rangle = \langle w^t, x^i \rangle$ — No need to care about $\mathbf{x}^i$ — If $\langle \mathbf{w}^t, \mathbf{x}^i \rangle = y^i$

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What if $\eta_t = 0$?
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A Closer look at the GD Update

Lets see the case $n = 1, \lambda = 0$

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What if $\eta_t = 0$?
What if no r(·)?

We are making corrective updates!

Subgradient Calculus

Do not always hold ☹

Should hold in nice settings ☺

	MTH101 Calculus	Subgradient Calculus
	$\mathbf{x} \in \mathbb{R}^d, \mathbf{A} \in \mathbb{R}^{k \times d}, \mathbf{b} \in \mathbb{R}^k, c \in \mathbb{R}$	
Scaling	• $\nabla(c \cdot f)(\mathbf{x}) = c \cdot \nabla f(\mathbf{x})$	• $\partial(c \cdot f)(\mathbf{x}) = c \cdot \partial f(\mathbf{x}) = \{c \cdot \mathbf{v} : \mathbf{v} \in \partial f(\mathbf{x})\}$
Sum Rule	• $\nabla(f + g)(x) = \nabla f(x) + \nabla g(x)$	• $\partial(f + g)(\mathbf{x}) = \partial f(\mathbf{x}) + \partial g(\mathbf{x}) = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in \partial f(\mathbf{x}), \mathbf{v} \in \partial g(\mathbf{x})\}$
Chain Rule	• $\nabla f(\mathbf{A}\mathbf{x} + \mathbf{b}) = \mathbf{A}^\top \nabla f(\mathbf{A}\mathbf{x} + \mathbf{b})$	• $\partial f(\mathbf{A}\mathbf{x} + \mathbf{b}) = \mathbf{A}^\top \partial f(\mathbf{A}\mathbf{x} + \mathbf{b}) = \{\mathbf{A}^\top \mathbf{v} : \mathbf{v} \in \partial f(\mathbf{x})\}$
Max Rule	$\ell_{\text{cs}}(y, \{\boldsymbol{\eta}_j\}) = [1 + \max_{k \neq y} \boldsymbol{\eta}_k - \boldsymbol{\eta}_y]_+$	$f(\mathbf{x}) = \max_i f_i(\mathbf{x})$

$$\partial f(\mathbf{x}) = \left\{\sum \alpha_i \mathbf{v}^i : \mathbf{v}^i \in \partial f_i(\mathbf{x}), \sum \alpha_i = 1, \alpha_i \geq 0\right\}$$

Subgradient Calculus

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	MTH101 Calculus	Subgradient Calculus
	$\mathbf{x} \in \mathbb{R}^d, \mathbf{A} \in \mathbb{R}^{k \times d}, \mathbf{b} \in \mathbb{R}^k, c \in \mathbb{R}$	
Scaling	• $\nabla(c \cdot f)(\mathbf{x}) = c \cdot \nabla f(\mathbf{x})$	• $\partial(c \cdot f)(\mathbf{x}) = c \cdot \partial f(\mathbf{x}) = \{c \cdot \mathbf{v} : \mathbf{v} \in \partial f(\mathbf{x})\}$
Sum Rule	• $\nabla(f + g)(x) = \nabla f(x) + \nabla g(x)$	• $\partial(f + g)(\mathbf{x}) = \partial f(\mathbf{x}) + \partial g(\mathbf{x}) = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in \partial f(\mathbf{x}), \mathbf{v} \in \partial g(\mathbf{x})\}$
Chain Rule	• $\nabla f(\mathbf{Ax} + \mathbf{b}) = \mathbf{A}^\top \nabla f(\mathbf{Ax} + \mathbf{b})$	• $\partial f(\mathbf{Ax} + \mathbf{b}) = \mathbf{A}^\top \partial f(\mathbf{Ax} + \mathbf{b}) = \{\mathbf{A}^\top \mathbf{v} : \mathbf{v} \in \partial f(\mathbf{x})\}$
Max Rule	?	$f(\mathbf{x}) = \max_i f_i(\mathbf{x})$

$$\partial f(\mathbf{x}) = \left\{\sum \alpha_i \mathbf{v}^i : \mathbf{v}^i \in \partial f_i(\mathbf{x}), \sum \alpha_i = 1, \alpha_i \geq 0\right\}$$

Application: SVM via subGD

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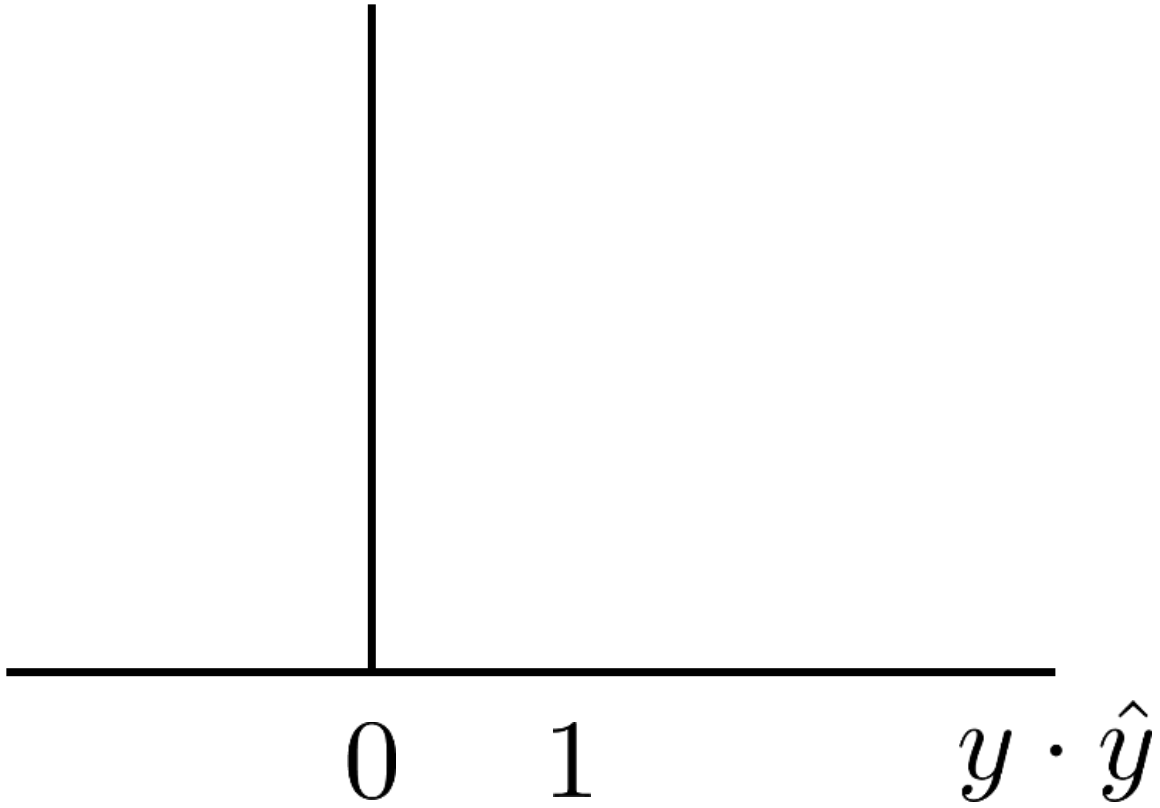
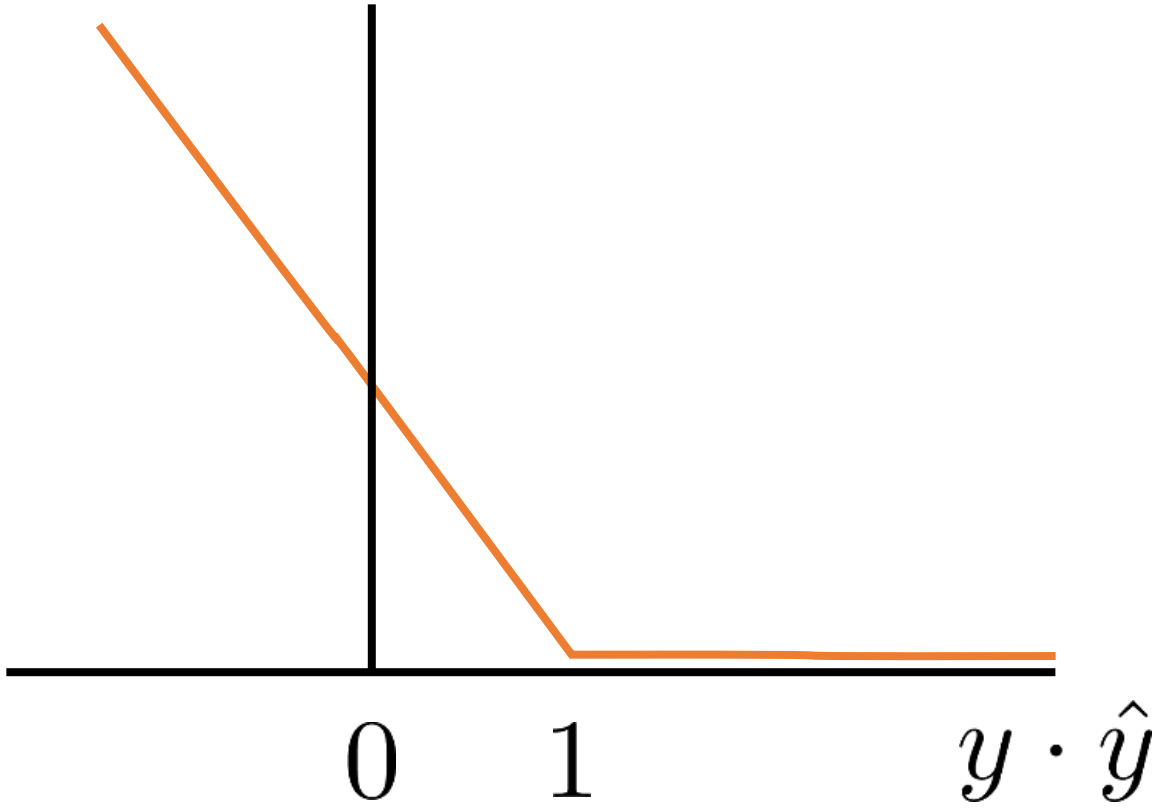
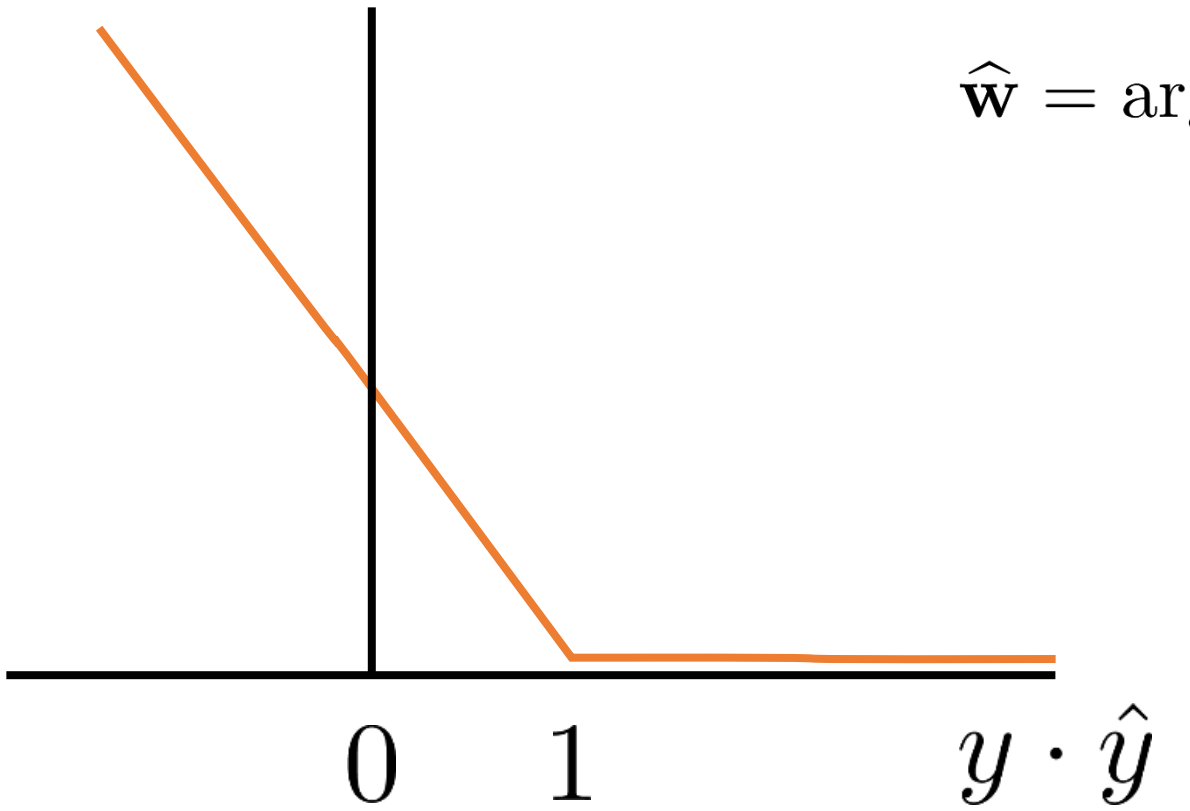


Application: SVM via subGD



Application: SVM via subGD



$$\widehat{\mathbf{w}} = \arg\min_{\mathbf{w}} \sum_{i=1}^{n} \ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle) + \frac{\lambda}{2} \|\mathbf{w}\|_2^2$$

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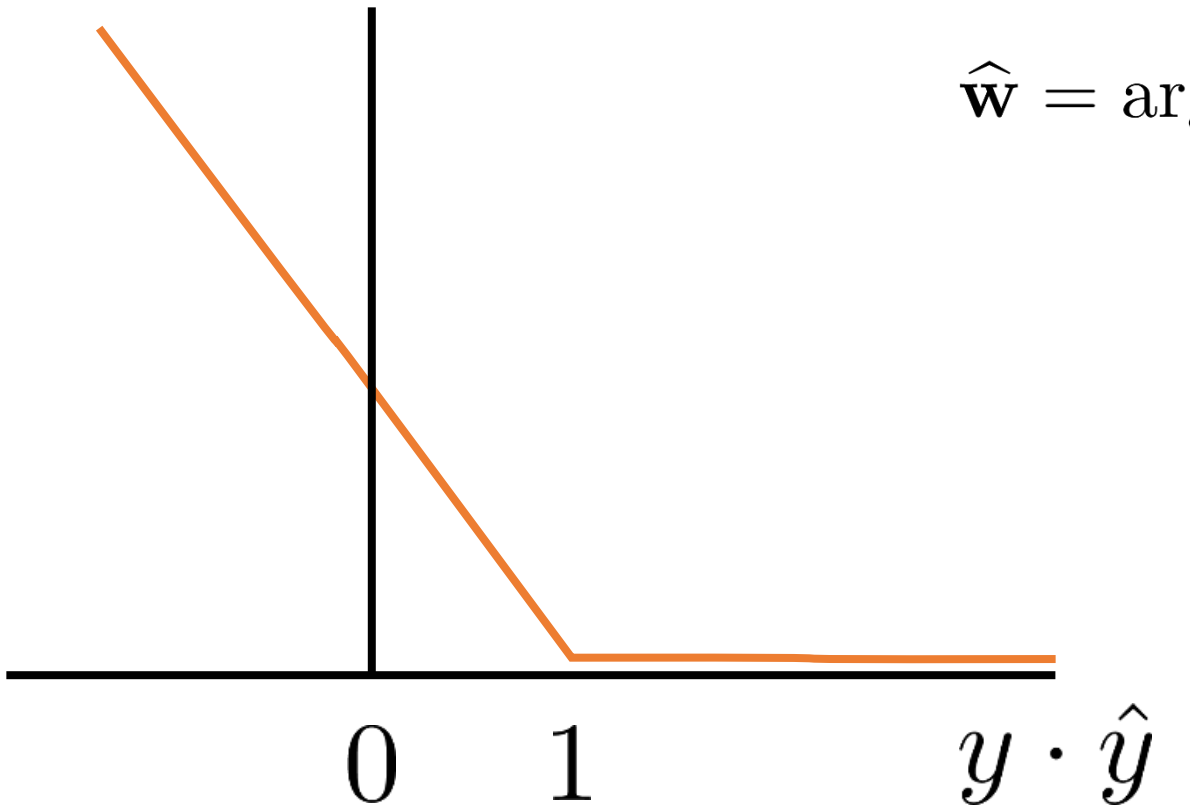

$$\widehat{\mathbf{w}} = \arg\min_{\mathbf{w}} \underbrace{\sum_{i=1}^{n} \ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)}_{f(\mathbf{w})} + \frac{\lambda}{2} \|\mathbf{w}\|_2^2$$

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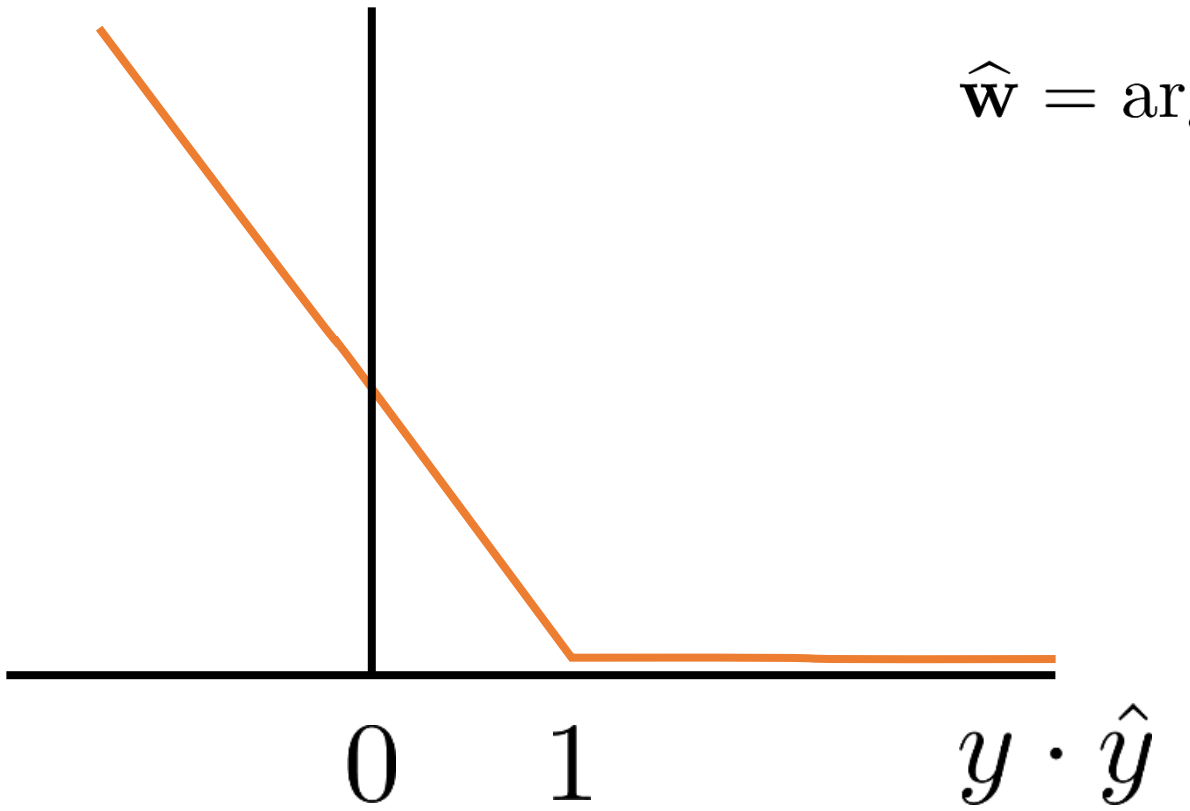

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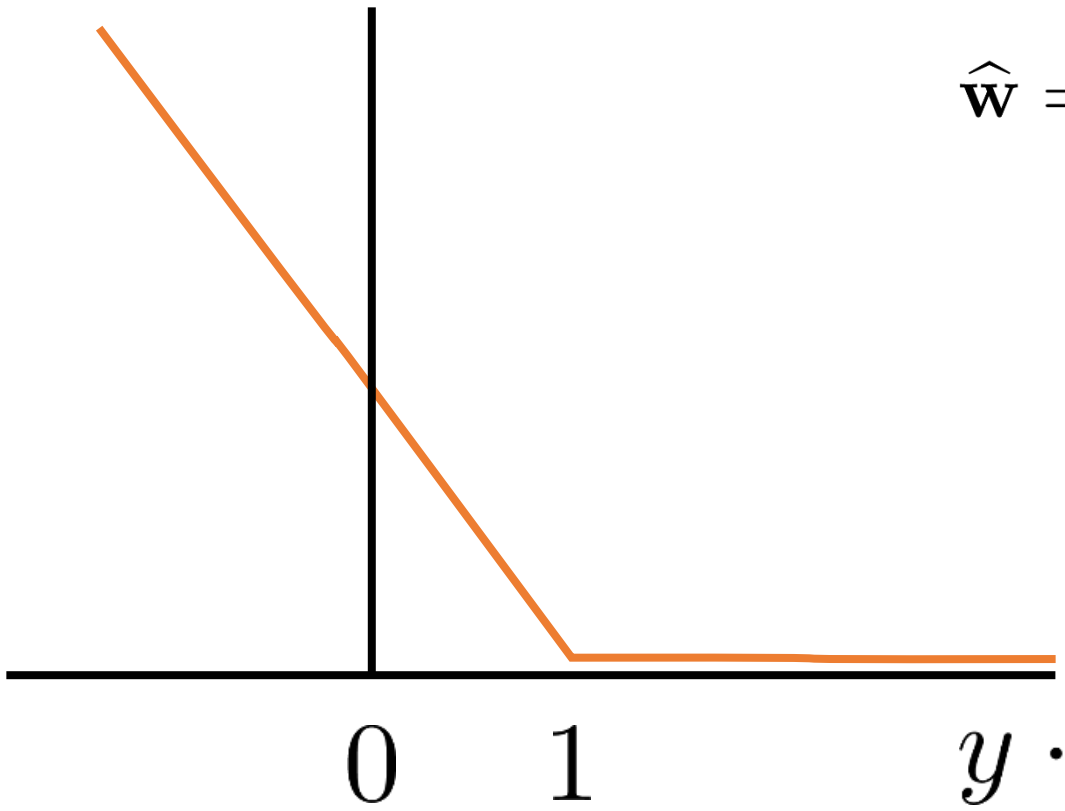
Application: SVM via subGD



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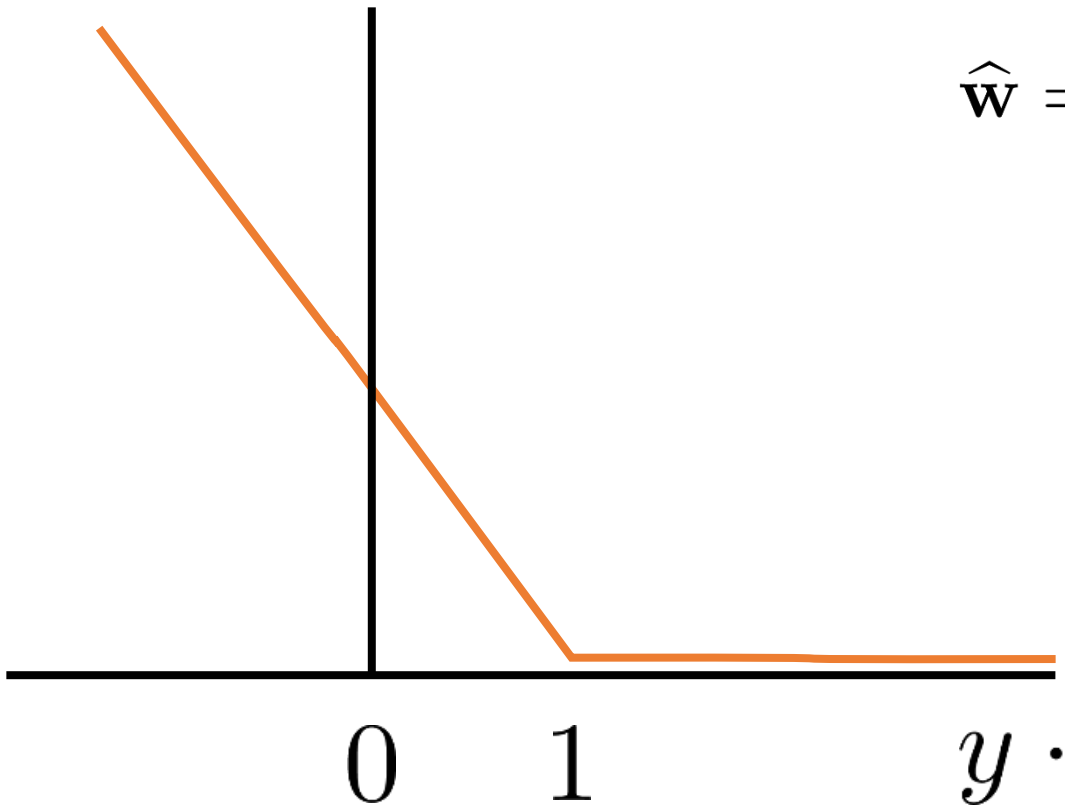
$$\nabla r(\mathbf{w}) = \lambda \cdot \mathbf{w}$$

Application: SVM via subGD



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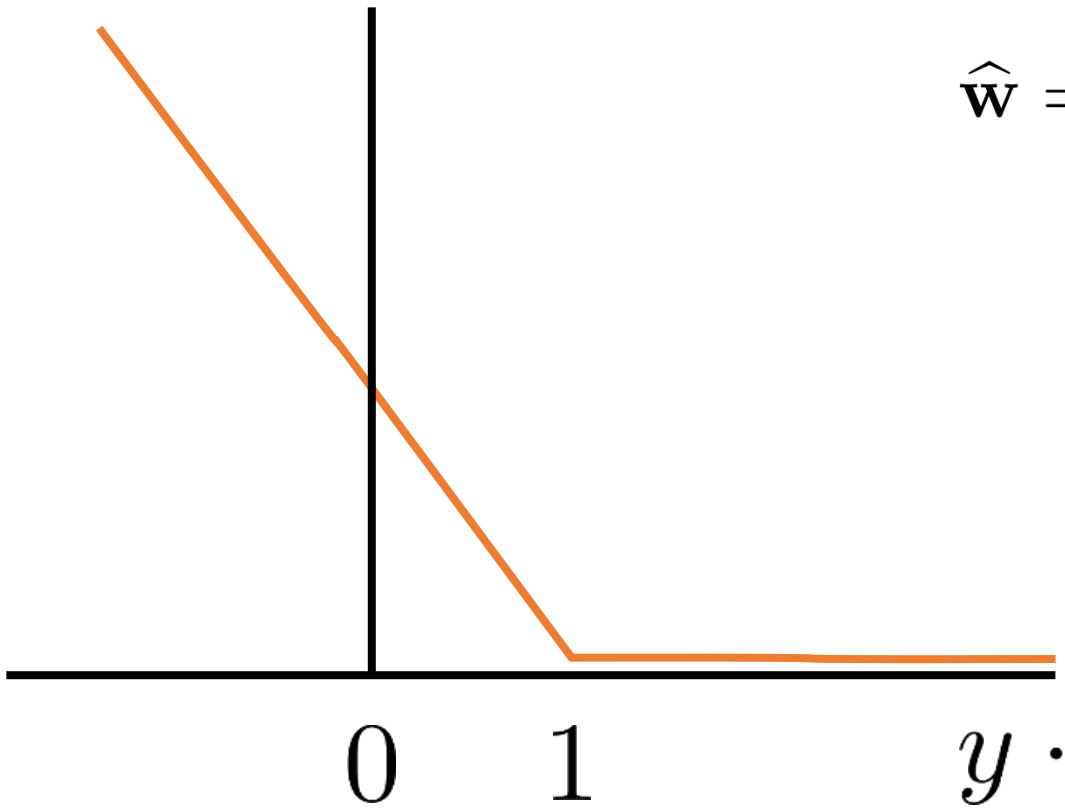
$$\text{Need } \mathbf{v}^i \in \partial \ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

Application: SVM via subGD



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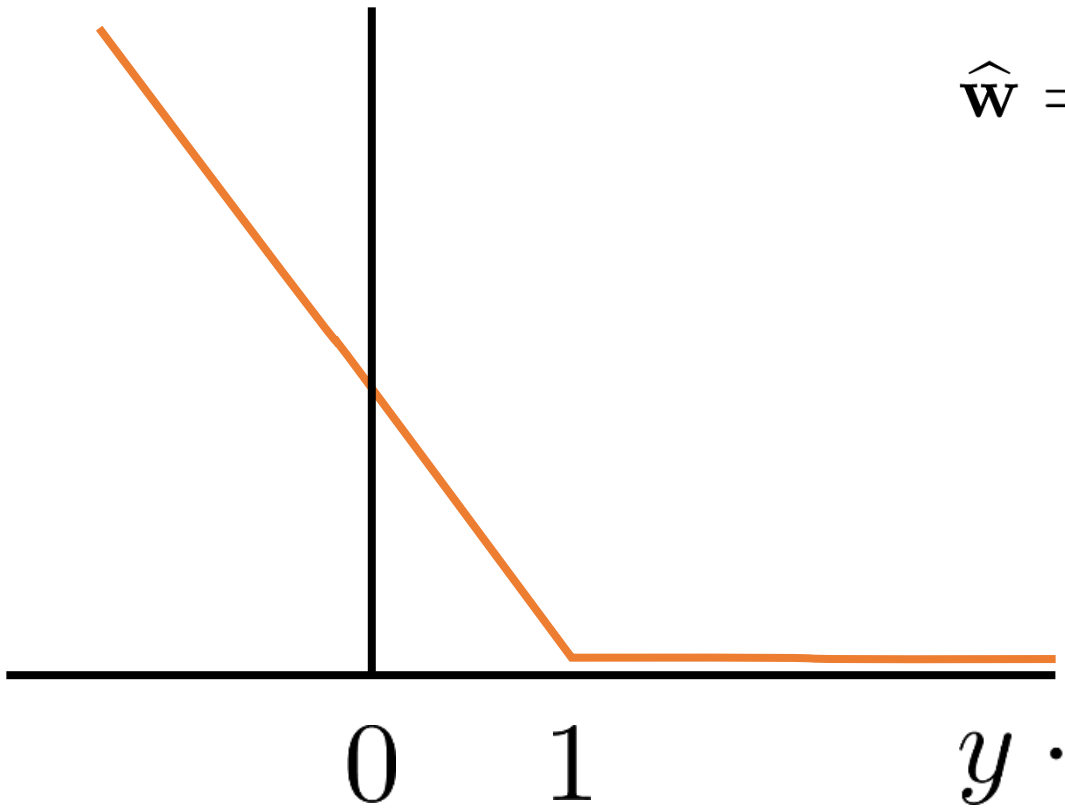
$$\ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle) = [1 - y^i \langle \mathbf{w}, \mathbf{x}^i \rangle]_+$$

Application: SVM via subGD



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$$\nabla r(\mathbf{w}) = \lambda \cdot \mathbf{w}$$

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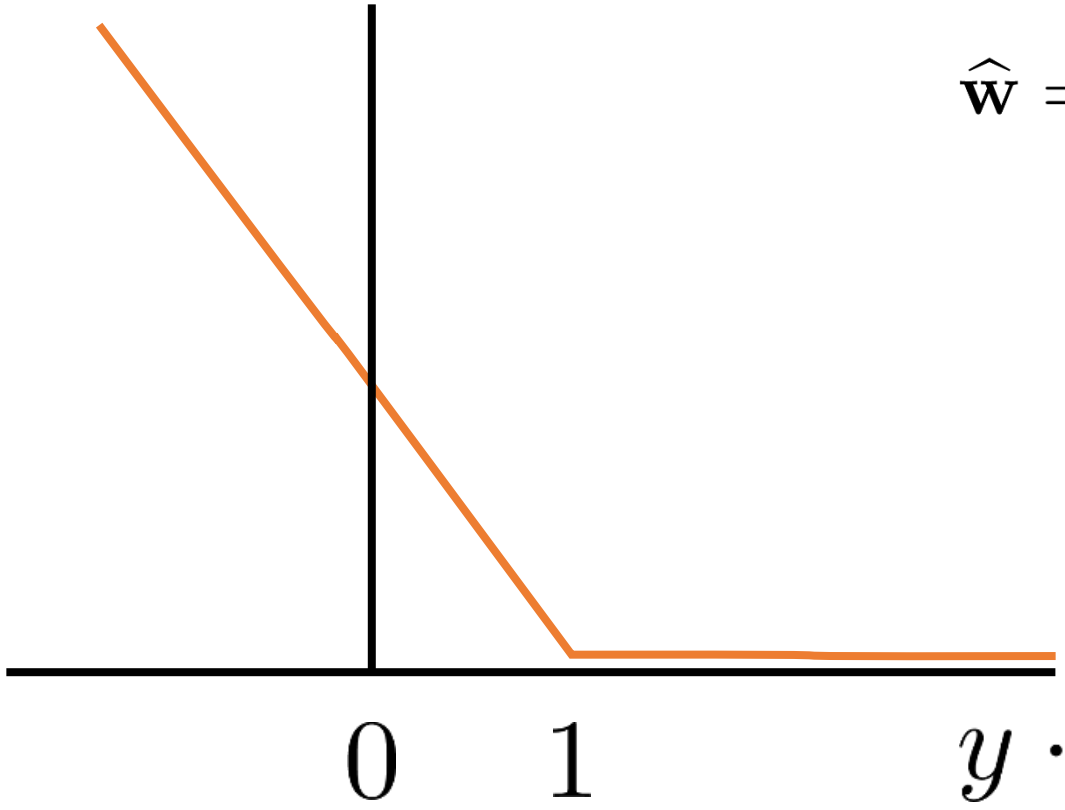
$$\ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle) = [1 - y^i \langle \mathbf{w}, \mathbf{x}^i \rangle]_+$$

$$\mathbf{v}^i = \begin{cases} \mathbf{0} & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle > 1 \\ -y^i \cdot \mathbf{x}^i & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle < 1 \\ c \cdot y^i \cdot \mathbf{x}^i & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle = 1 \\ c \in [-1, 0] \end{cases}$$

Application: SVM via subGD



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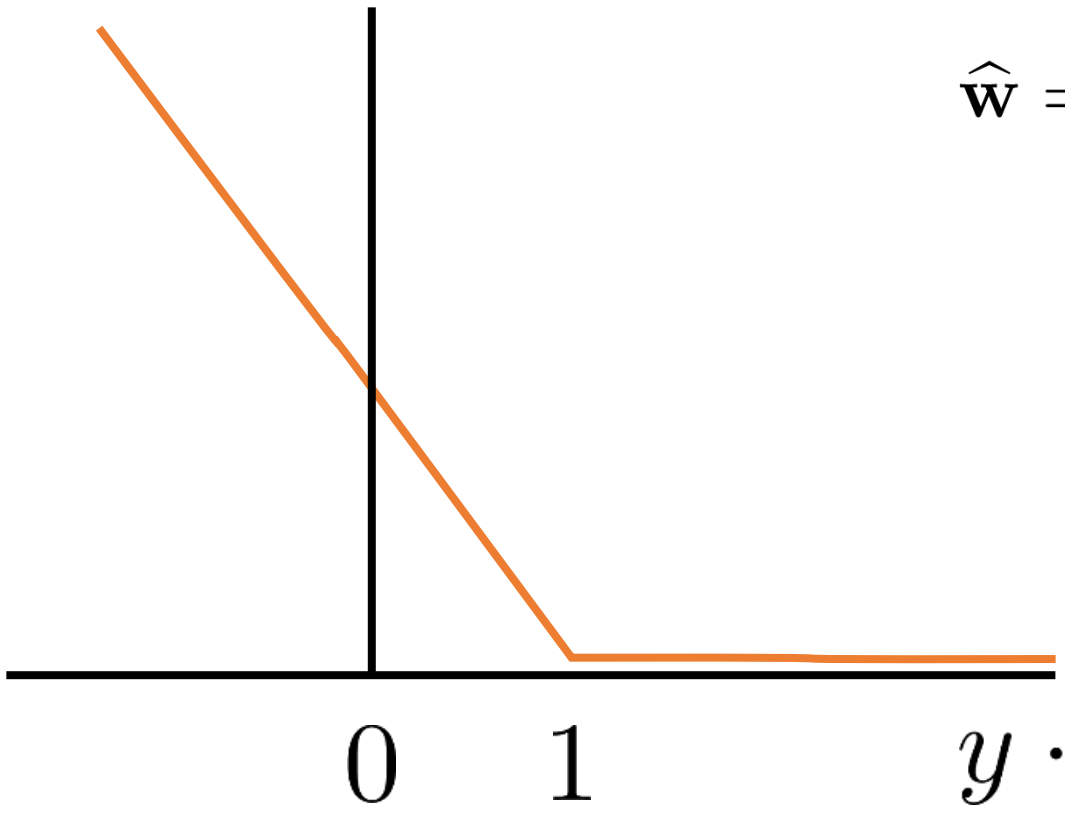
$O(d)$ time per data point!

Application: SVM via subGD



$$\widehat{\mathbf{w}} = \arg\min_{\mathbf{w}} \underbrace{\sum_{i=1}^{n} \ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)}_{f(\mathbf{w})} + \underbrace{\frac{\lambda}{2} \|\mathbf{w}\|_2^2}_{r(\mathbf{w})}$$

$$\nabla r(\mathbf{w}) = \lambda \cdot \mathbf{w}$$

$$\text{Need } \mathbf{v}^i \in \partial \ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\ell_{\text{hinge}}(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle) = [1 - y^i \langle \mathbf{w}, \mathbf{x}^i \rangle]_+$$

$$\mathbf{v}^i = \begin{cases} \mathbf{0} & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle > 1 \\ -y^i \cdot \mathbf{x}^i & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle < 1 \\ c \cdot y^i \cdot \mathbf{x}^i & \text{if } y^i \langle \mathbf{w}, \mathbf{x}^i \rangle = 1 \\ c \in [-1, 0] \end{cases}$$

$O(nd)$ time per iter

$O(d)$ time per data point!

App: Sparse Regression via subGD

App: Sparse Regression via subGD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_1$$

App: Sparse Regression via sul

Convex?

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \left(y^i - \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle\right)^2 + \lambda \cdot \|\mathbf{w}\|_1$$

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Convex?

Non-differentiable

$$\mathbf{g}^t \in \nabla f(\mathbf{w}) + \partial r(\mathbf{w})$$

App: Sparse Regression via sul

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$$\mathbf{g}^t \in \nabla f(\mathbf{w}) + \partial r(\mathbf{w})$$

$$\mathbf{g}^t = 2 \sum_{i=1}^{n} (\left\langle \mathbf{w}^t, \mathbf{x}^i \right\rangle - y^i) \cdot \mathbf{x}^i + 2\lambda \cdot \boldsymbol{\rho}^t$$

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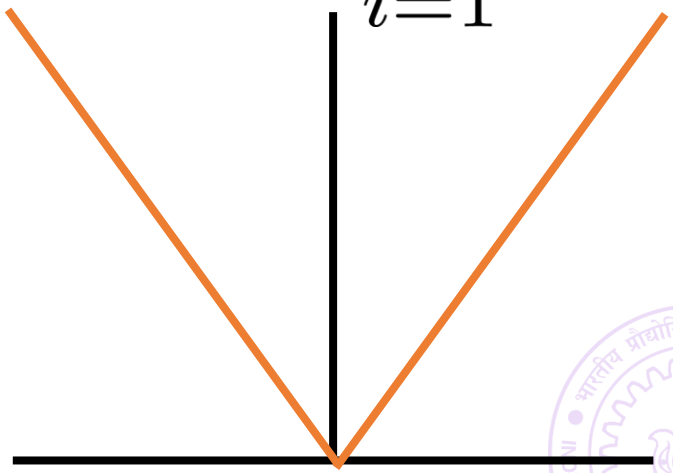

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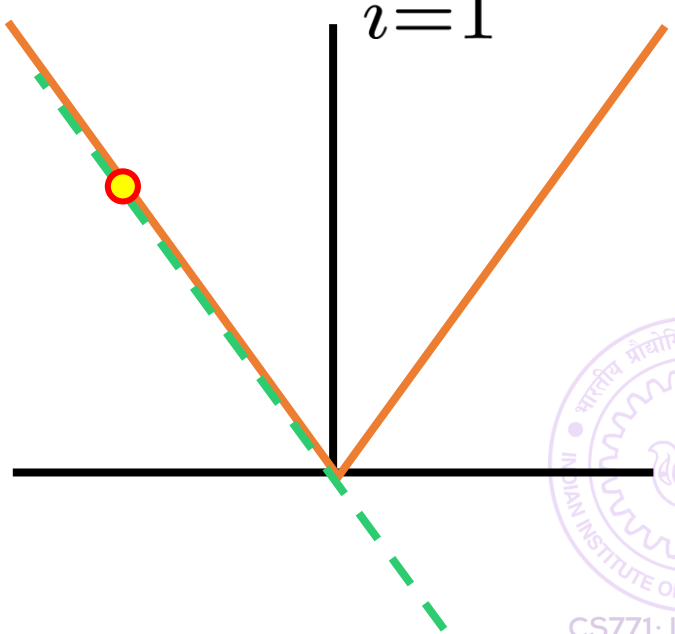

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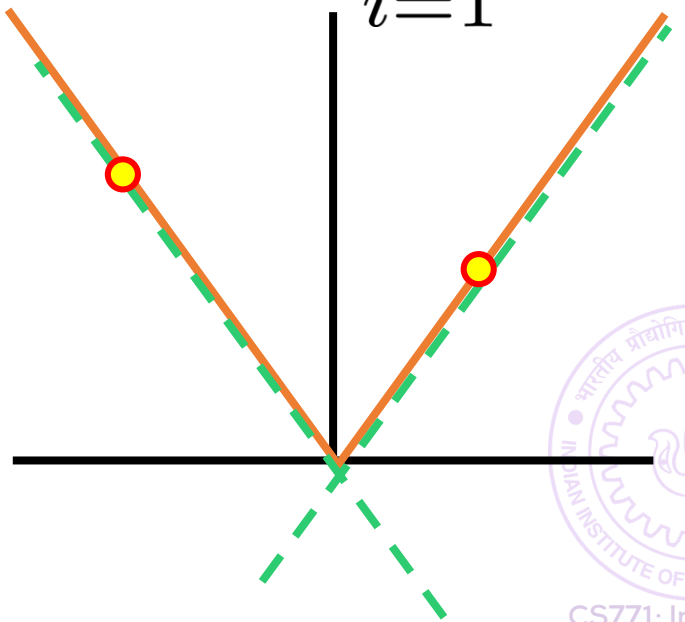

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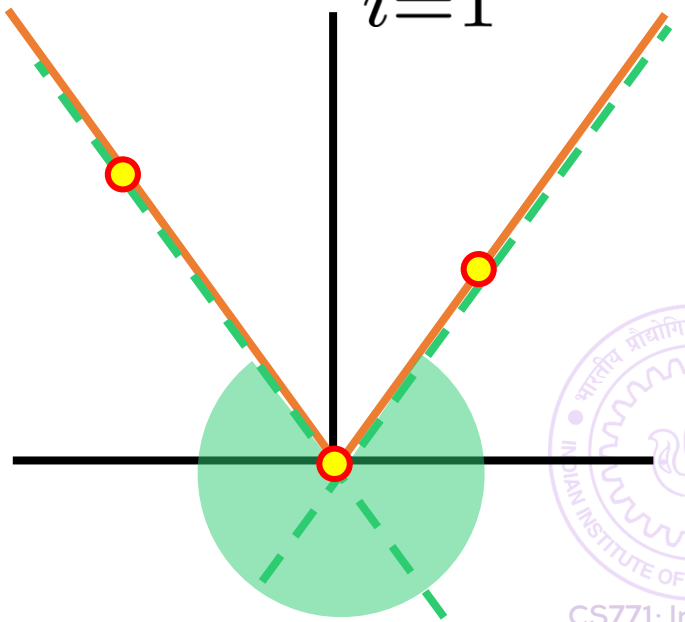

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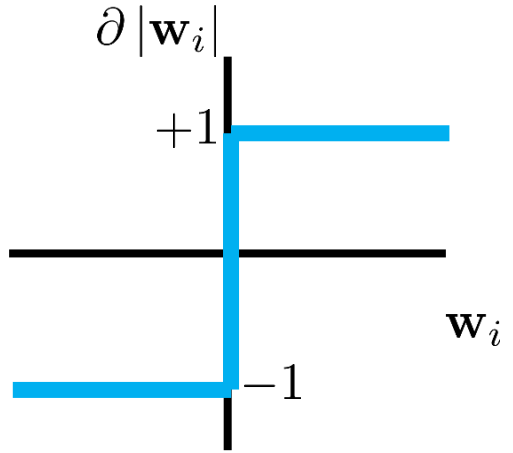
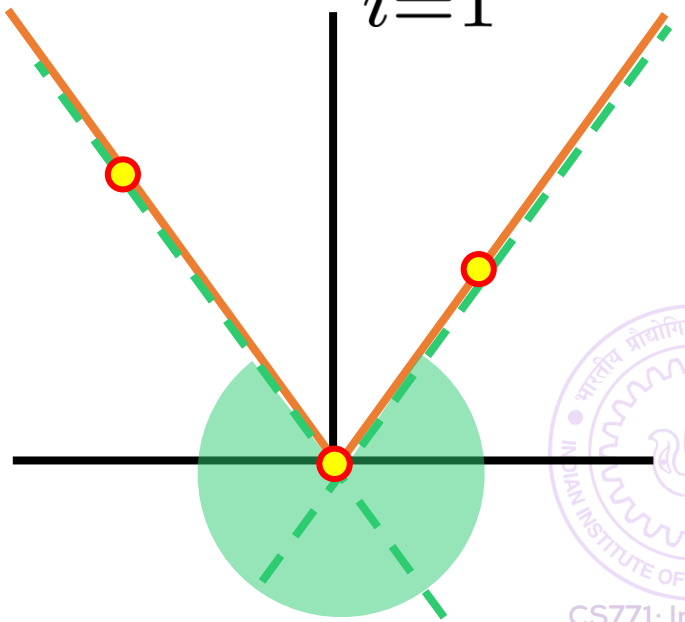
$$\|\mathbf{w}^t\|_1 = \sum_{i=1}^{d} |\mathbf{w}_i^t|$$

$$\boldsymbol{\rho}_i^t = \begin{cases} +1 & \text{if } \mathbf{w}_i^t > 0 \\ [-1, 1] & \text{if } \mathbf{w}_i^t = 0 \\ -1 & \text{if } \mathbf{w}_i^t < 0 \end{cases}$$

e.g. $\boldsymbol{\rho}_i^t = \text{sign}(\mathbf{w}_i^t)$





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$$\mathbf{g}^t \in \nabla f(\mathbf{w}) + \partial r(\mathbf{w})$$

$$\mathbf{g}^t = 2\sum_{i=1}^{n}(\langle \mathbf{w}^t, \mathbf{x}^i \rangle - y^i) \cdot \mathbf{x}^i + 2\lambda \cdot \boldsymbol{\rho}^t$$

$O(nd)$ time per iter!

$$\|\mathbf{w}\|_1 = \sum_{i=1}^{d} |\mathbf{w}_i^t|$$

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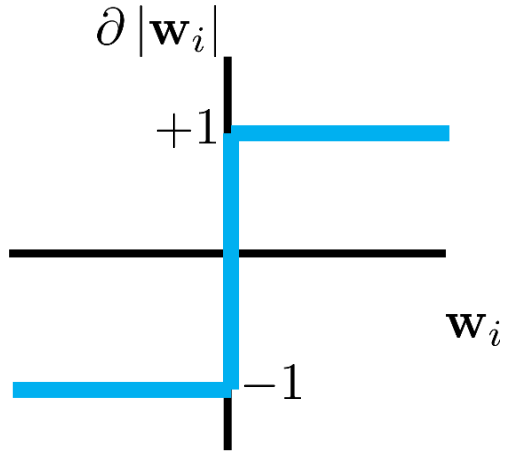

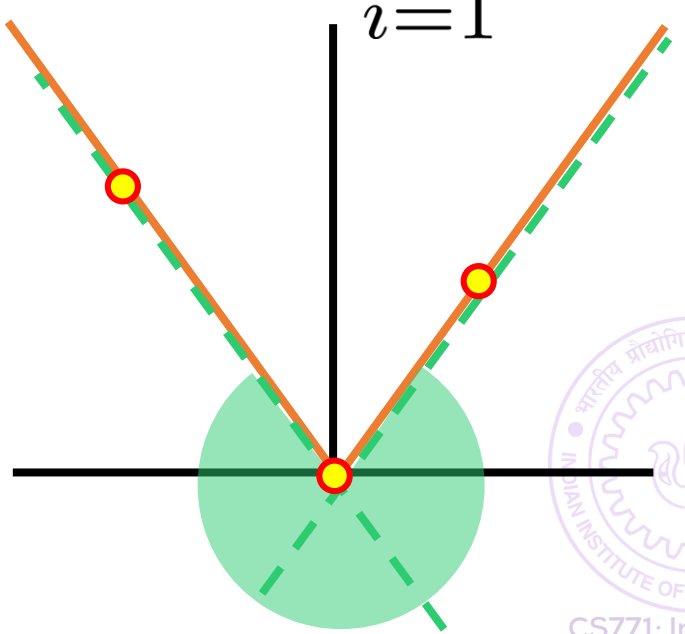

App: Logistic Regression via GD

App: Logistic Regression via GD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right) + \lambda \cdot \|\mathbf{w}\|_2^2$$

App: Logistic Regression via GD

Convex?

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right) + \lambda \cdot \|\mathbf{w}\|_2^2$$

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$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right) + \lambda \cdot \|\mathbf{w}\|_2^2$$

$$\mathbf{g}^t = \sum_{i=1}^{n} (1 - \sigma(y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)) y^i \cdot \mathbf{x}^i + 2\lambda \cdot \mathbf{w}$$

App: Logistic Regression via GD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right) + \lambda \cdot \|\mathbf{w}\|_2^2$$

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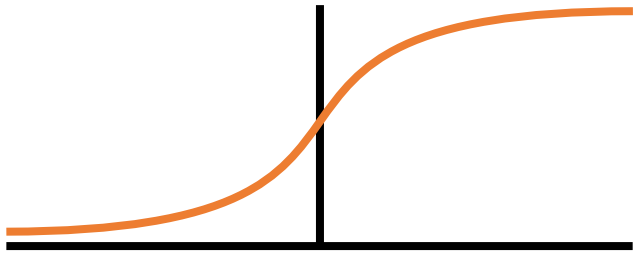
$O(nd)$ time per iter!



Whats going on here?

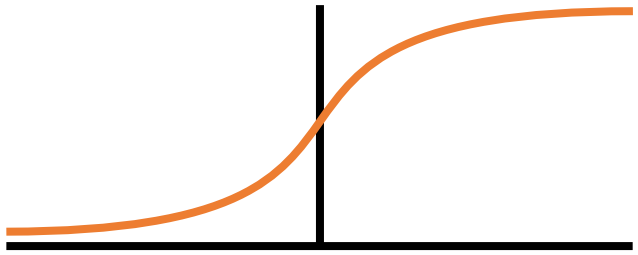
App: Logistic Regression via GD

$$\hat{\mathbf{w}} = \arg\min \sum_{i=1}^{n} \log\left(1 + \exp(-y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)\right) + \lambda \cdot \|\mathbf{w}\|_2^2$$

Convex?

$$\mathbf{g}^t = \sum_{i=1}^{n} (1 - \sigma(y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)) y^i \cdot \mathbf{x}^i + 2\lambda \cdot \mathbf{w}$$

$O(nd)$ time per iter!



Whats going on here?

Exercise

Stochastic Gradient Method

$$\hat{\mathbf{w}} = \underset{\mathbf{w} \in \mathbb{R}^d}{\arg\min} \; f(\mathbf{w}) + r(\mathbf{w})$$

Stochastic Gradient Method

$$f(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

Stochastic Gradient Method

$$f(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

Stochastic Gradient Method

$O(nd)$ time

$$\nabla f(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n} \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

Stochastic Gradient Method

$O(nd)$ time

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

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$O(d)$ time!!

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Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \quad \nabla f_i(\mathbf{w}) \approx \quad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

STOCHASTIC GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Select a unif. random point $I_t \in [n]$
3. Let $\mathbf{g}^t \leftarrow \nabla f_{I_t}(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$
4. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
5. Repeat until convergence

Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

$$\mathbb{E}[\mathbf{g}^t | \mathbf{w}^t] = \nabla f(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

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Stochastic Gradient Method

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Only $O(d)$ time per iter!

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Stochastic Gradient Method

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5. Repeat until convergence

Step length

$$\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$$

Stochastic Gradient Method

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$O(d)$ time!!

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

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Step length $\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

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Only $O(d)$ time per iter!

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w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

$f(\mathbf{w}^t) + r(\mathbf{w}^t) \leq f(\mathbf{w}^*) + r(\mathbf{w}^*) + \epsilon$

Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \qquad \nabla f_i(\mathbf{w}) \approx \qquad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

Only $O(d)$ time per iter!

ONLINE GRADIENT DESCENT

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2. Select a unif. random point $I_t \in [n]$
3. Let $\mathbf{g}^t \leftarrow \nabla f_{I_t}(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$
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5. Repeat until convergence

Step length $\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

$f(\mathbf{w}^t) + r(\mathbf{w}^t) \leq f(\mathbf{w}^*) + r(\mathbf{w}^*) + \epsilon$

Stochastic Gradient Method

$O(d)$ time!!

$$\nabla f(\mathbf{w}) \approx \quad \nabla f_i(\mathbf{w}) \approx \quad \nabla \ell(y^i, \langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbf{g}^t = \nabla f_i(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$$

Only $O(d)$ time per iter!

ONLINE GRADIENT DESCENT

1. Initialize $\mathbf{w}^0$
2. Receive a data point $\mathbf{z}^t = (y^t, \mathbf{x}^t)$
3. Let $\mathbf{g}^t \leftarrow \nabla f_{I_t}(\mathbf{w}^t) + \nabla r(\mathbf{w}^t)$
4. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
5. Repeat until convergence

Step length $\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

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3. Let $\mathbf{g}^t \leftarrow \nabla f(\mathbf{w}^t, \mathbf{z}^t) + \nabla r(\mathbf{w}^t)$
4. Update $\mathbf{w}^{t+1} \leftarrow \mathbf{w}^t - \eta_t \cdot \mathbf{g}^t$
5. Repeat until convergence

Step length $\eta_t = C, \frac{C}{t}, \frac{C}{\sqrt{t}}$

w.h.p. ϵ-optimal solution in $O\left(\frac{1}{\epsilon^2}\right)$ iterations

$f(\mathbf{w}^t) + r(\mathbf{w}^t) \leq f(\mathbf{w}^*) + r(\mathbf{w}^*) + \epsilon$

Please give your Feedback

http://tinyurl.com/ml17-18afb