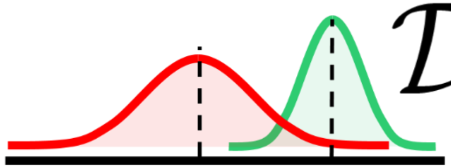
Probabilistic Methods-II

CS771: Introduction to Machine Learning

Purushottam Kar

Recap

The Generative Story



$\overset{?}{\mathcal{D}_{\boldsymbol{\theta}^*}} \Leftarrow \mathbf{Z}$

$\mathbb{P}\left[\mathbf{Z} \mid \boldsymbol{\theta}\right]$	Likelihood
$\mathbb{P}\left[\boldsymbol{\theta}\right]$	Prior
$\mathbb{P}\left[\boldsymbol{\theta} \mid \mathbf{Z}\right]$	Posterior

Defined for all parameters $\boldsymbol{\theta}$

Prediction with learnt models

 $= ?$

$\mathbf{Z} = \mathbf{y} =$ 

$\mathbb{P}\left[p\right]$

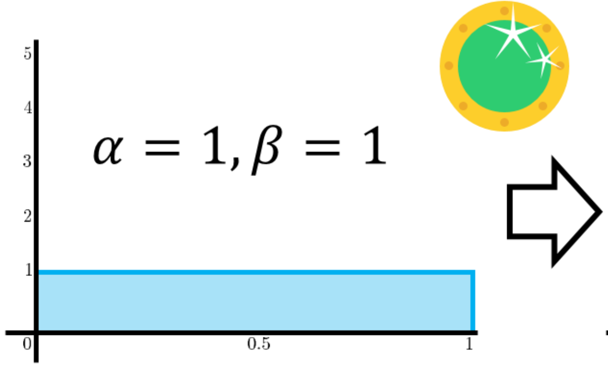
$$\mathbb{P}\left[y^i \mid p\right] = p^{y^i}(1-p)^{1-y^i}$$

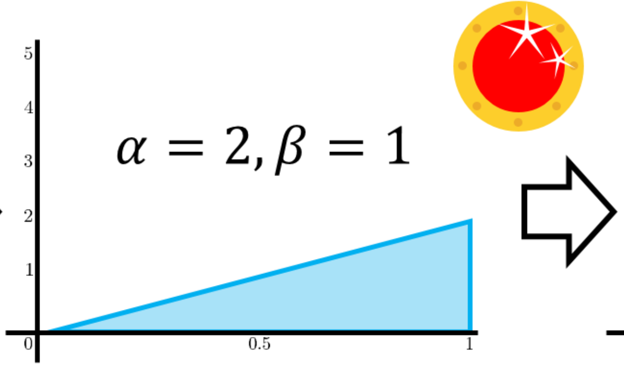
$\mathbb{P}[$  $= H \mid \mathbf{y}] = \int_p \mathbb{P}[$  $= H \mid p]\, \mathbb{P}[p \mid \mathbf{y}]\, dp$

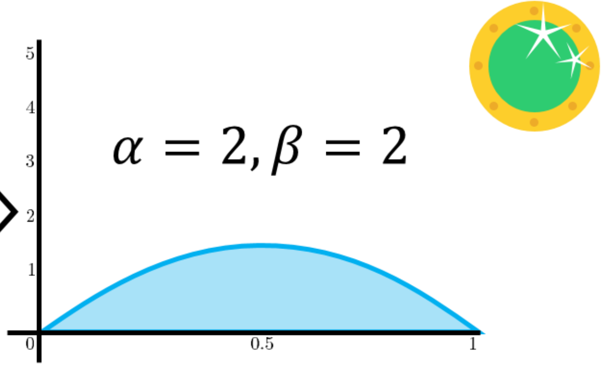
$\approx \mathbb{P}[$  $= H \mid \hat{p}_{\mathrm{MLE}}] = \frac{n_H}{n}$

$\approx \mathbb{P}[$  $= H \mid \hat{p}_{\mathrm{MAP}}] = \frac{n_H + \alpha - 1}{n + \alpha + \beta - 2}$

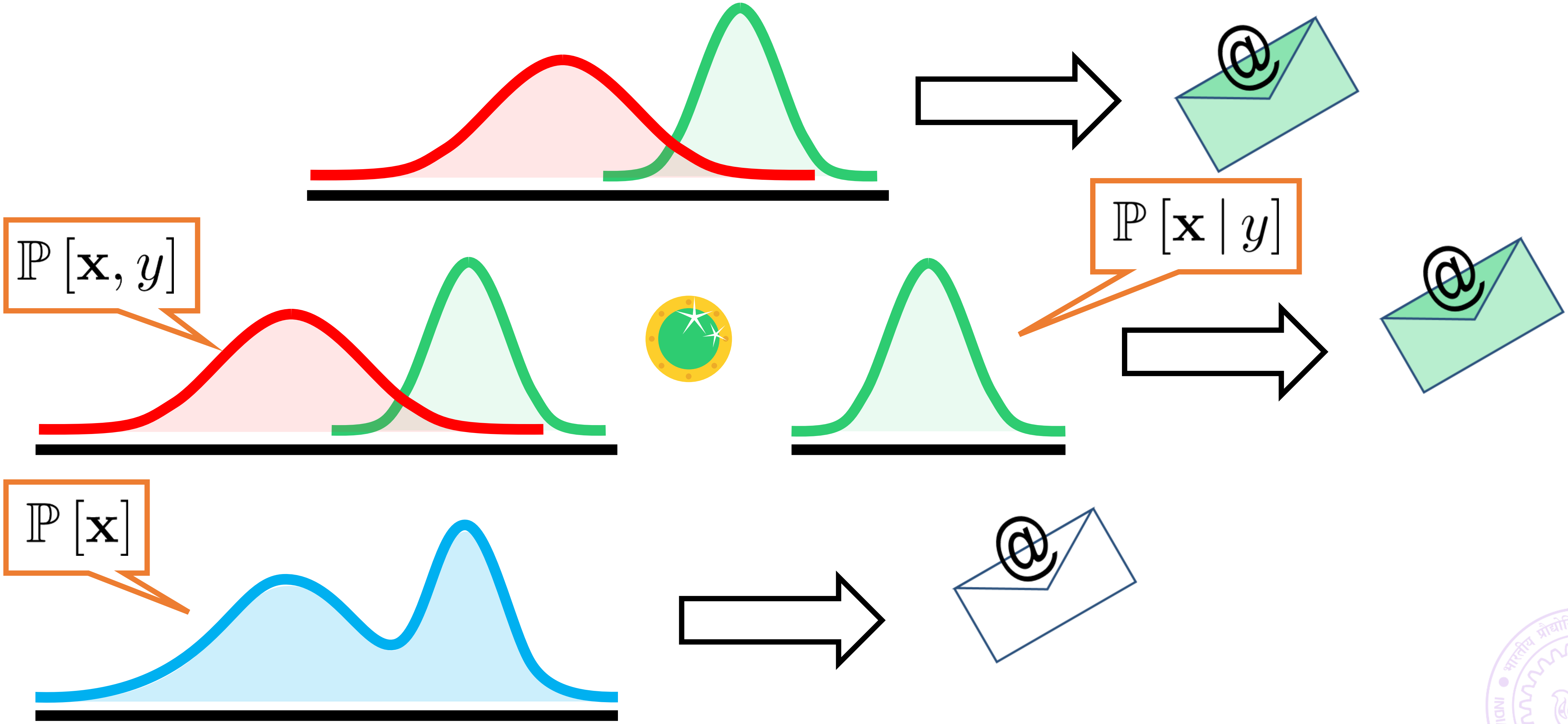
Online MAP!







Generative Models for Labeled Data



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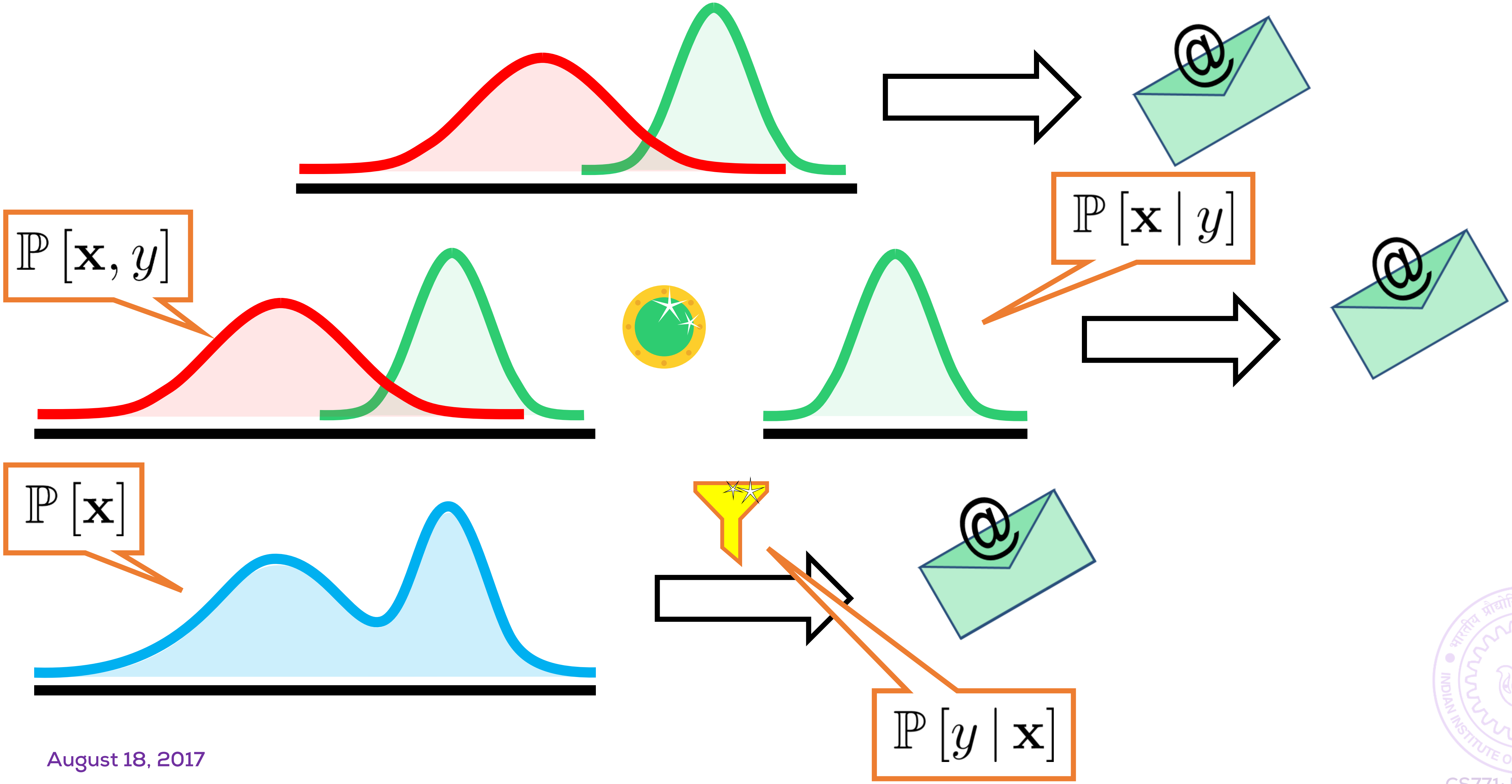
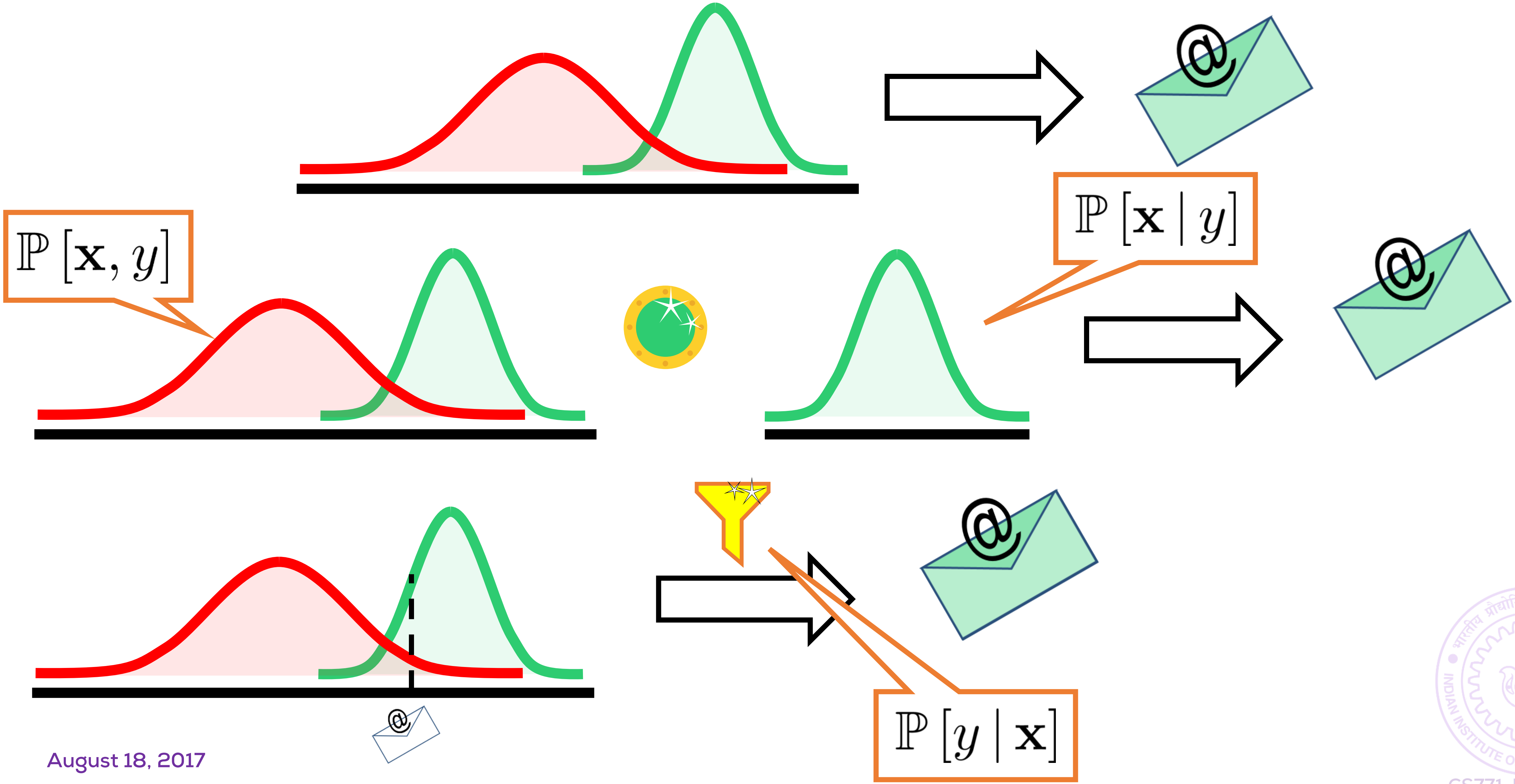
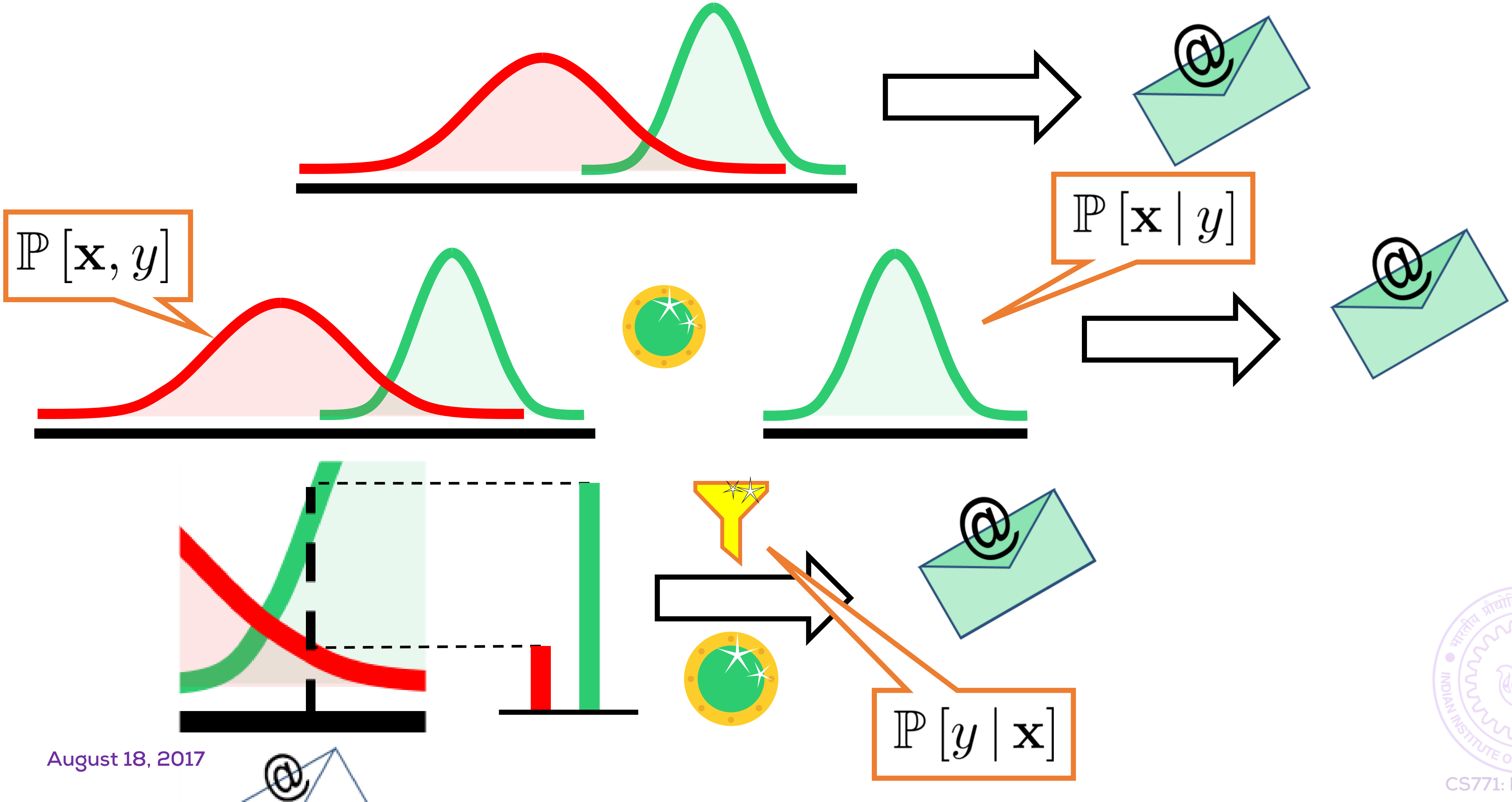


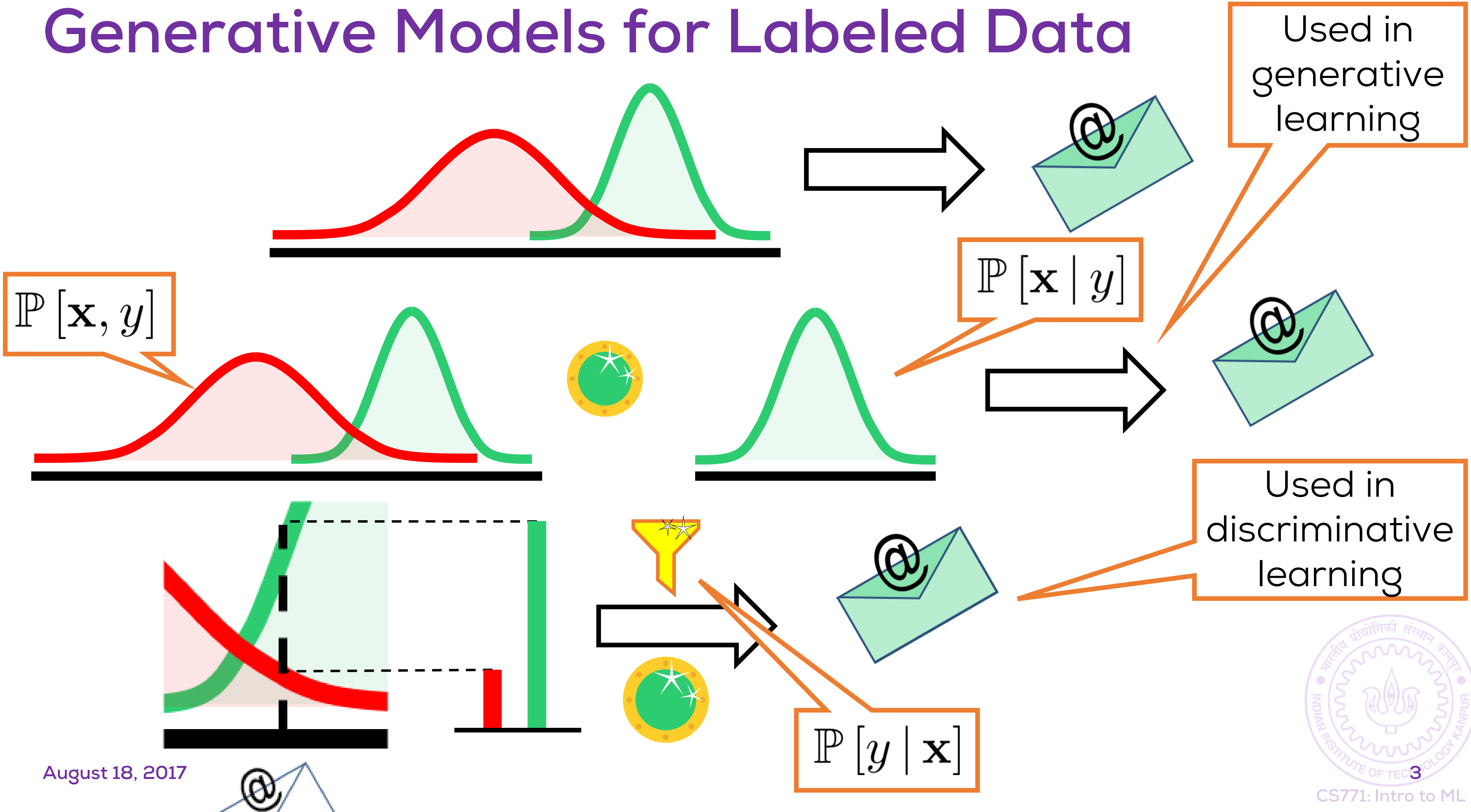
Generative Models for Labeled Data



Generative Models for Labeled Data

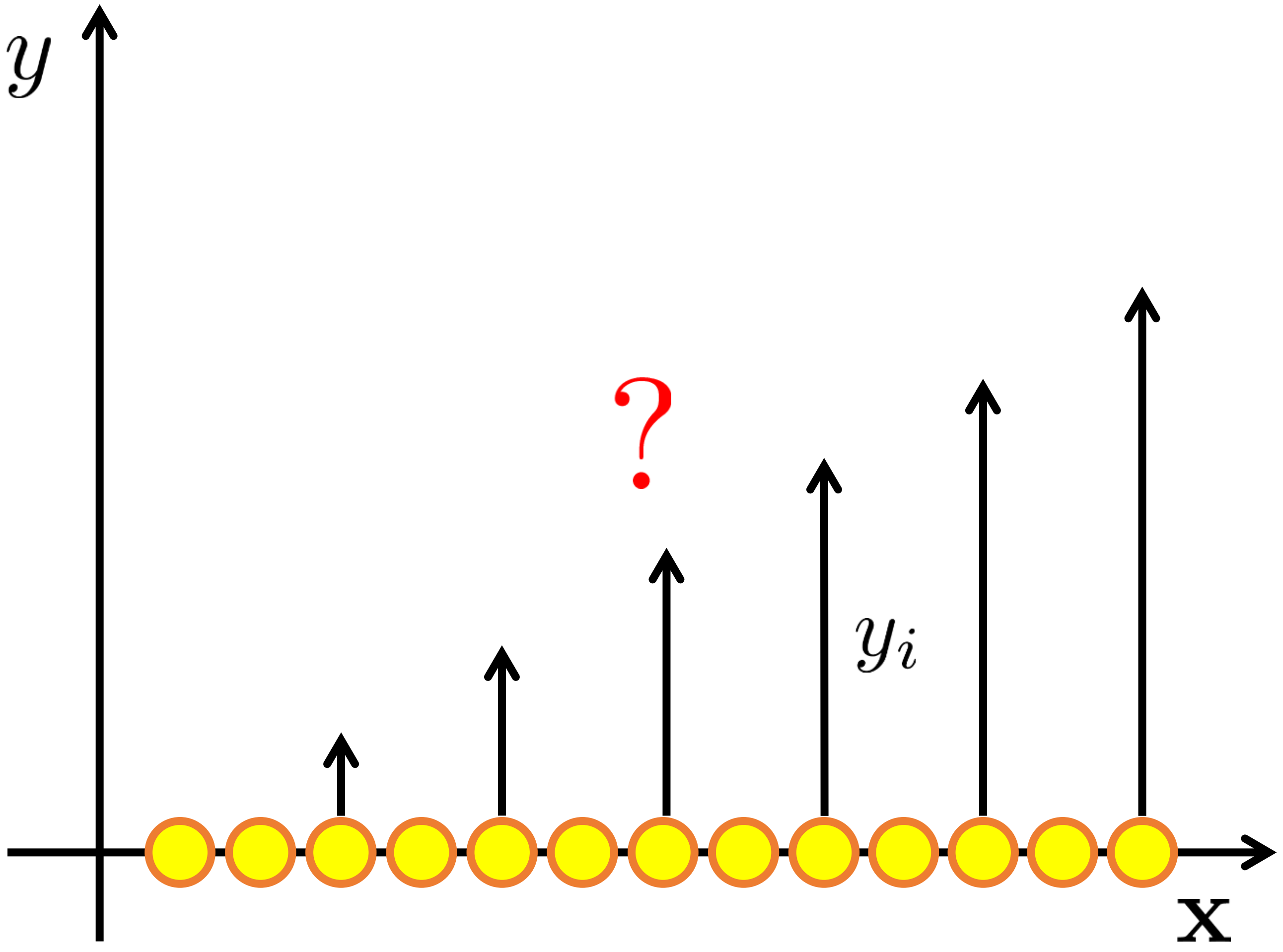


Generative Models for Labeled Data



Regression using PML

Linear Regression



Data: $\mathbf{x}^1, \mathbf{x}^2, \ldots, \mathbf{x}^n \in \mathbb{R}^d$

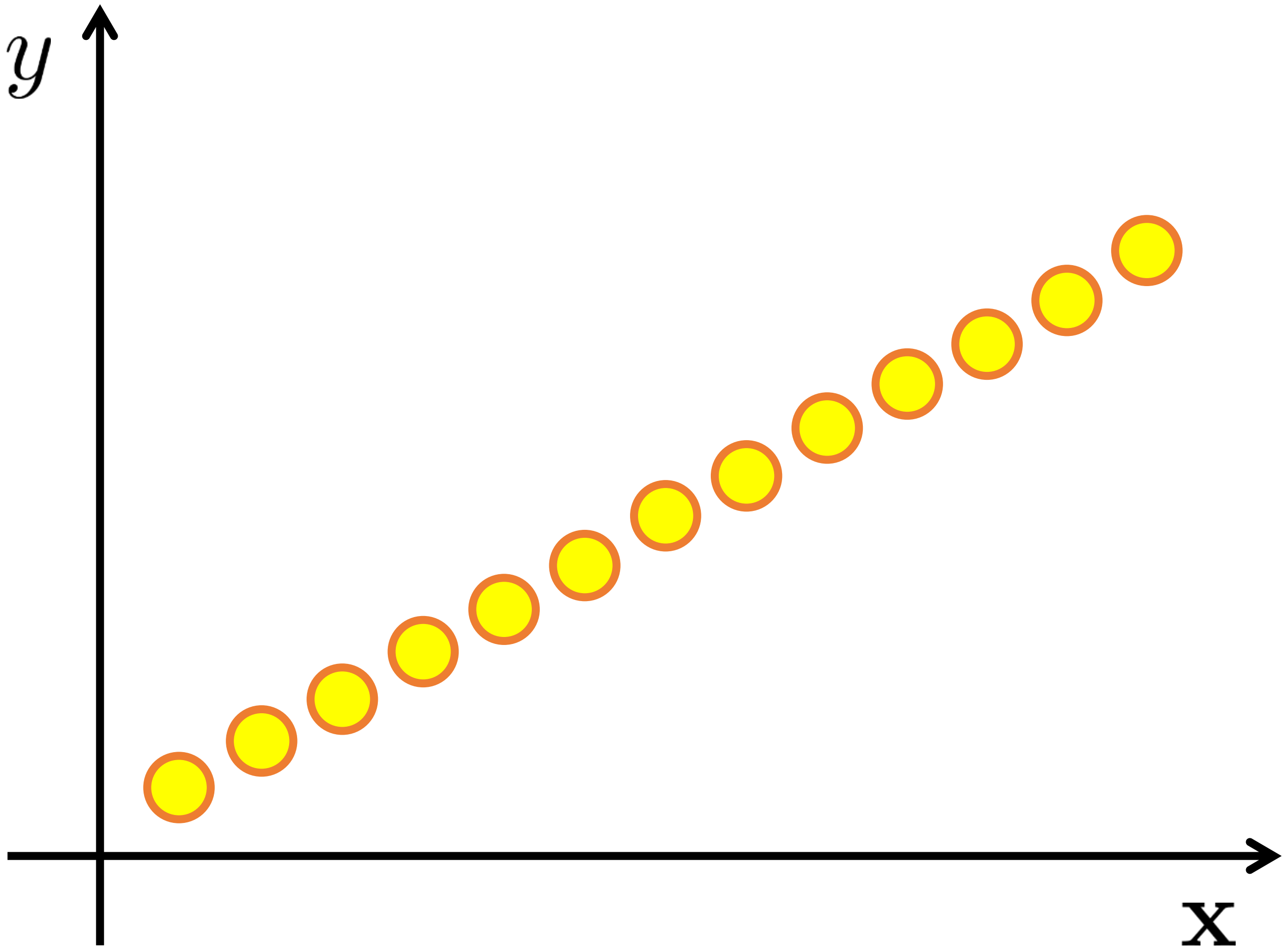
Model: $\mathbf{w}^*$ (hidden)

$$y^i = \langle \mathbf{w}^*, \mathbf{x}^i \rangle$$

Given: $(\mathbf{x}^1, y^1), \ldots, (\mathbf{x}^n, y^n)$

Recover $\mathbf{w}^*$?

Linear Regression



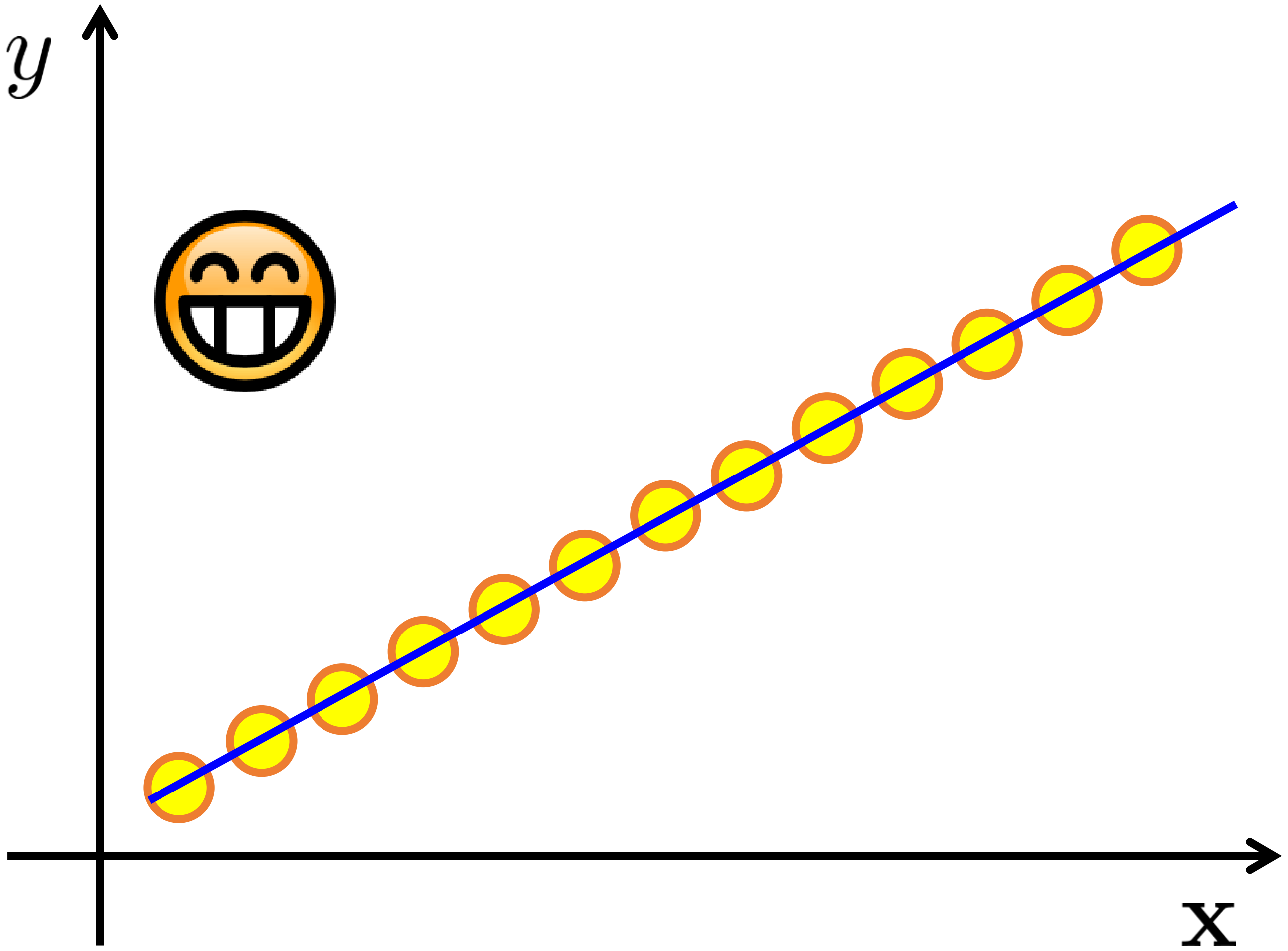
Given: $(\mathbf{x}^1, y^1), \ldots, (\mathbf{x}^n, y^n)$

$$y^i = \langle \mathbf{w}^*, \mathbf{x}^i \rangle$$

$$n \quad \underbrace{\begin{bmatrix} y^1 \\ y^2 \\ y^3 \\ \vdots \\ y^n \end{bmatrix}}_{\mathbf{y}} = \underbrace{\overbrace{\begin{bmatrix} \text{---} & \mathbf{x}^1 & \text{---} \\ \text{---} & \mathbf{x}^2 & \text{---} \\ \text{---} & \mathbf{x}^3 & \text{---} \\ & \vdots & \\ \text{---} & \mathbf{x}^n & \text{---} \end{bmatrix}}^{d}}_{\mathbf{X}^\top} \begin{bmatrix} | \\ \mathbf{w} \\ | \end{bmatrix} \quad d$$

?

Linear Regression



Given: $(\mathbf{x}^1, y^1), \ldots, (\mathbf{x}^n, y^n)$

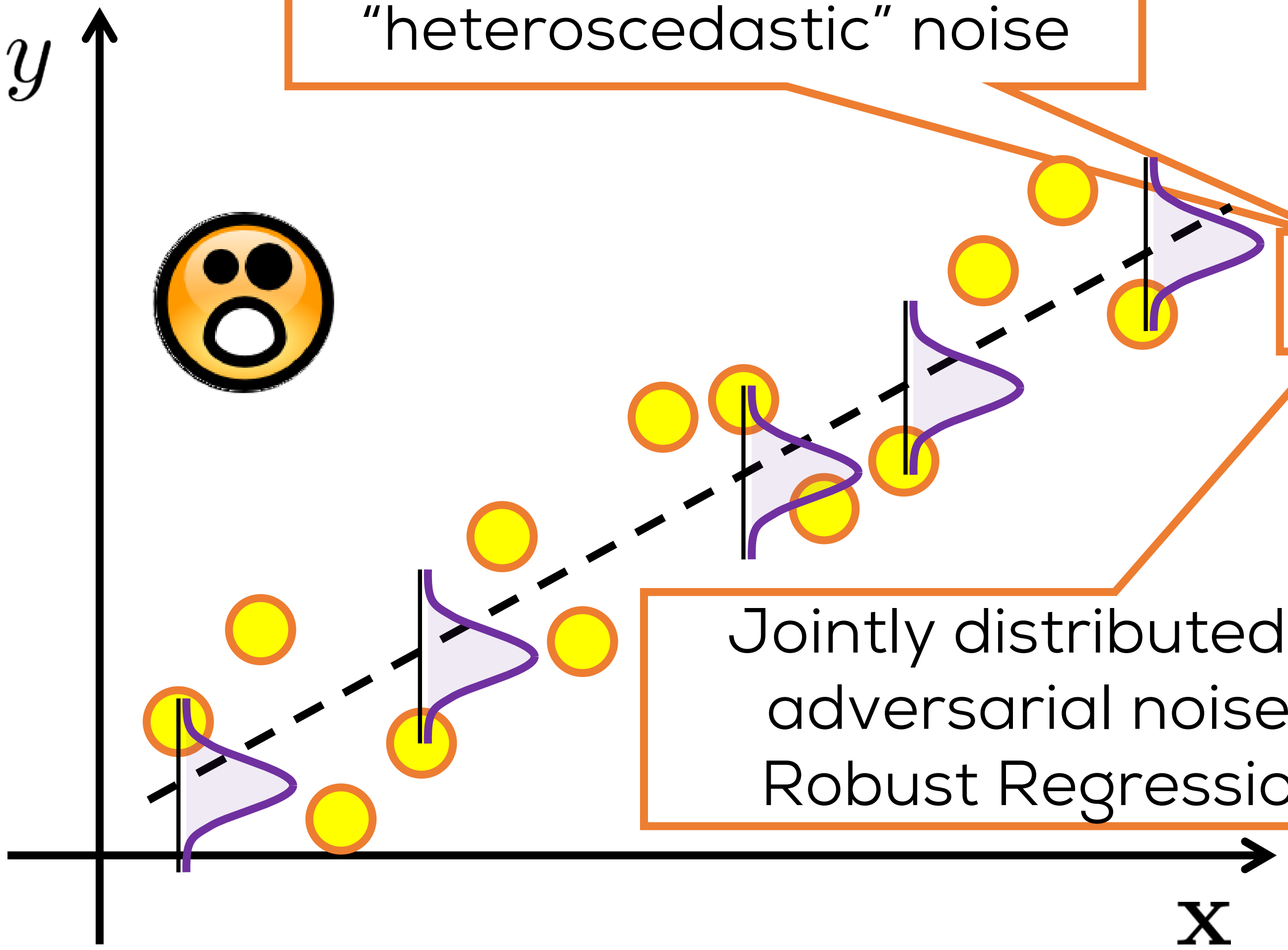
$$y^i = \langle \mathbf{w}^*, \mathbf{x}^i \rangle$$

$$\mathbf{y} = \mathbf{X}^\top \mathbf{w}$$

Linear system!!

$\mathbf{w}^*$ Recovered!!

Linear Regression with Noise



Non-identically distributed "heteroscedastic" noise

Given: $(\mathbf{x}^1, y^1), \ldots, (\mathbf{x}^n, y^n)$

$$y^i = \langle \mathbf{w}^*, \mathbf{x}^i \rangle + e^i$$

i.i.d noise

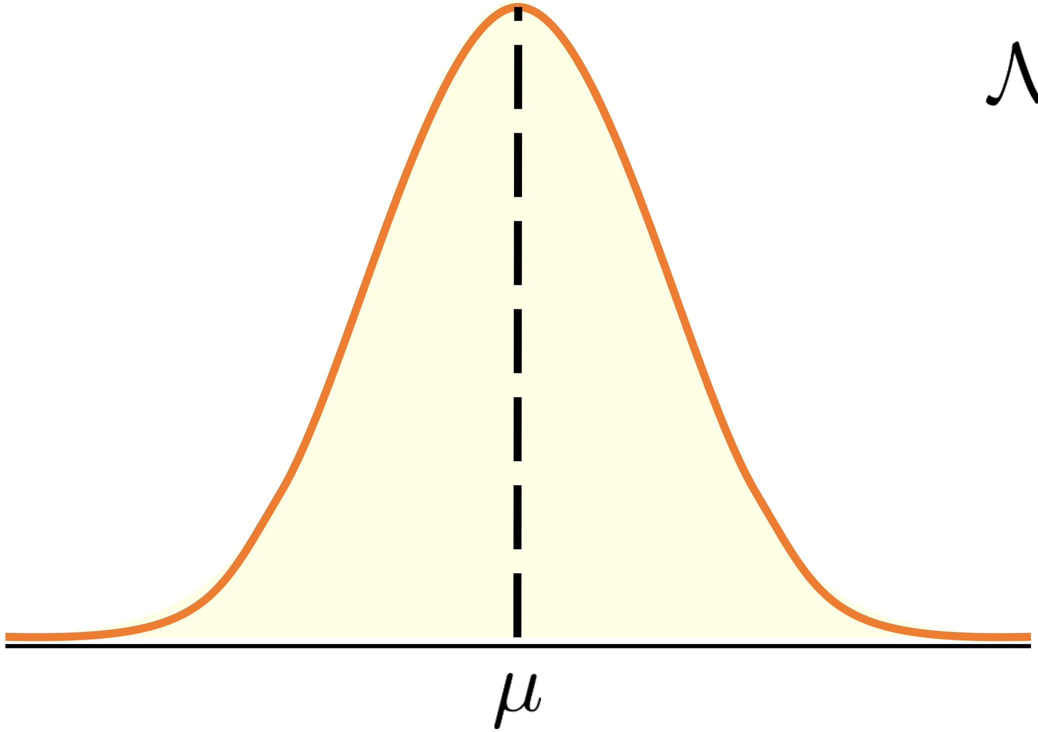
$$e^i \sim \mathcal{N}(0, \sigma^2)$$

$$y^i \sim \mathcal{N}(\langle \mathbf{w}^*, \mathbf{x}^i \rangle, \sigma^2)$$

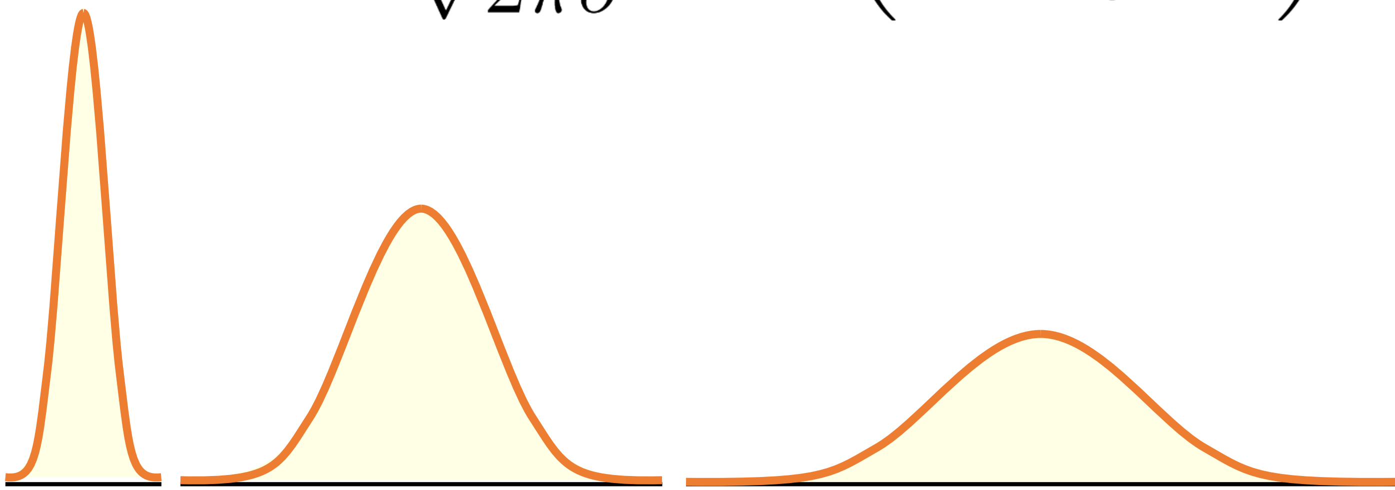
Jointly distributed or adversarial noise?
Robust Regression

Non-Gaussian noise?
Generalized models (GLM)!

The Gaussian Distribution



$$\mathcal{N}(t;\mu,\sigma^2)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(t-\mu)^2}{2\sigma^2}\right)$$



Multivariate Gaussian

$$\mathcal{N}(\mathbf{v};\boldsymbol{\mu},\Sigma)=\frac{1}{\sqrt{(2\pi)^d\,|\Sigma|}}\exp\left(-\frac{1}{2}(\mathbf{v}-\boldsymbol{\mu})^\top\Sigma^{-1}(\mathbf{v}-\boldsymbol{\mu})\right)$$

Linear Regression using MLE

$$\mathbb{P}\left[y \,|\, \mathbf{x}^i, \mathbf{w}\right] = \mathcal{N}(\langle \mathbf{w}, \mathbf{x}^i \rangle, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y - \langle \mathbf{w}, \mathbf{x}^i \rangle)^2}{2\sigma^2}\right)$$

Linear function

Why isn't $\mathbf{x}$ getting modelled too?

Generative model

Likelihood

Log-likelihood

$$\log \mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] = C - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2$$

Least Squares!

$$\hat{\mathbf{w}}_{\mathrm{MLE}} = \arg\min \sum_{i=1}^{n} \left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 = (\mathbf{X}\mathbf{X}^\top)^\dagger \mathbf{X}\mathbf{y}$$

$\log \mathbb{P}\left[\mathbf{y}, \mathbf{X} \,|\, \boldsymbol{\theta}\right]$ Generative MLE??

Exercise

Linear Regression using MAP

$$\mathbb{P}\left[\mathbf{w}\right] = \mathcal{N}(\mathbf{0}, \rho^2 \cdot I_d) = \frac{1}{\sqrt{(2\pi)^d \rho^2}} \exp\left(-\frac{\|\mathbf{w}\|_2^2}{2\rho^2}\right)$$

Sparsity inducing priors

$$\log \mathbb{P}\left[\mathbf{w} \mid \mathbf{X}, \mathbf{y}\right] = C - \frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 - \frac{1}{2\rho^2}\|\mathbf{w}\|_2^2$$

$$\hat{\mathbf{w}}_{\text{MAP}} = \arg\min \sum_{i=1}^{n}\left(y^i - \langle \mathbf{w}, \mathbf{x}^i \rangle\right)^2 + \frac{\sigma^2}{\rho^2}\|\mathbf{w}\|_2^2 = (\mathbf{X}\mathbf{X}^\top + \lambda I)^{-1}\mathbf{X}\mathbf{y}$$

$\log \mathbb{P}\left[\boldsymbol{\theta} \mid \mathbf{y}, \mathbf{X}\right]$
Generative MAP??

Ridge Regression

Exercise

Bayesian Linear Regression?

$$\mathbb{P}\left[\mathbf{w} \,|\, \mathbf{X}, \mathbf{y}\right] = \mathcal{N}(\boldsymbol{\nu}, \Lambda)$$

Cool! Posterior is Gaussian like the prior!

Wait! MAP?

$$\boldsymbol{\nu} = \left(\mathbf{X}\mathbf{X}^\top + \frac{\sigma^2}{\rho^2}\cdot I\right)^{-1}\mathbf{X}\mathbf{y}$$

$$\Lambda = \left(\frac{1}{\sigma^2}\mathbf{X}\mathbf{X}^\top + \frac{1}{\rho^2}\cdot I\right)^{-1}$$

By definition, MAP is the mode of the posterior

$$\mathbb{P}\left[y \,|\, \mathbf{x}, \mathbf{X}, \mathbf{y}\right] = \int_{\mathbf{w}} \mathbb{P}\left[y \,|\, \mathbf{x}, \mathbf{w}\right]\mathbb{P}\left[\mathbf{w} \,|\, \mathbf{X}, \mathbf{y}\right] d\mathbf{w}$$

Predictive Posterior

$$= \mathcal{N}\left(\langle \boldsymbol{\nu}, \mathbf{x}\rangle, \sigma^2 + \mathbf{x}^\top\Lambda\mathbf{x}\right)$$

Extra Information

A few Thoughts

- Do I have to use these very forms for likelihood and prior?

$$\mathbb{P}\left[y \,|\, \mathbf{x}^i, \boldsymbol{\theta}\right] = \mathcal{D}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

  - No, however some choices make sense and make estimation easier
  - Conjugacy makes life very simple if possible to achieve
  - Look at your application and take these decisions
    - If your know your model is sparse, use a Laplacian prior
    - If you know your noise is not Gaussian, use a GLM
- Can I build probabilistic models for other ML tasks as well?
  - Yes, of course. We will look at classification problems next.
  - We can do probabilistic clustering, dim. redn., ranking
  - There are entire courses devoted to PML techniques: CS772, CS698S

A few Thoughts

- What about non-linear regression?

$$\mathbb{P}\left[y \,|\, \mathbf{x}^i, \boldsymbol{\theta}\right] = \mathcal{N}(f(\mathbf{x}^i, \boldsymbol{\theta}), \sigma^2)$$

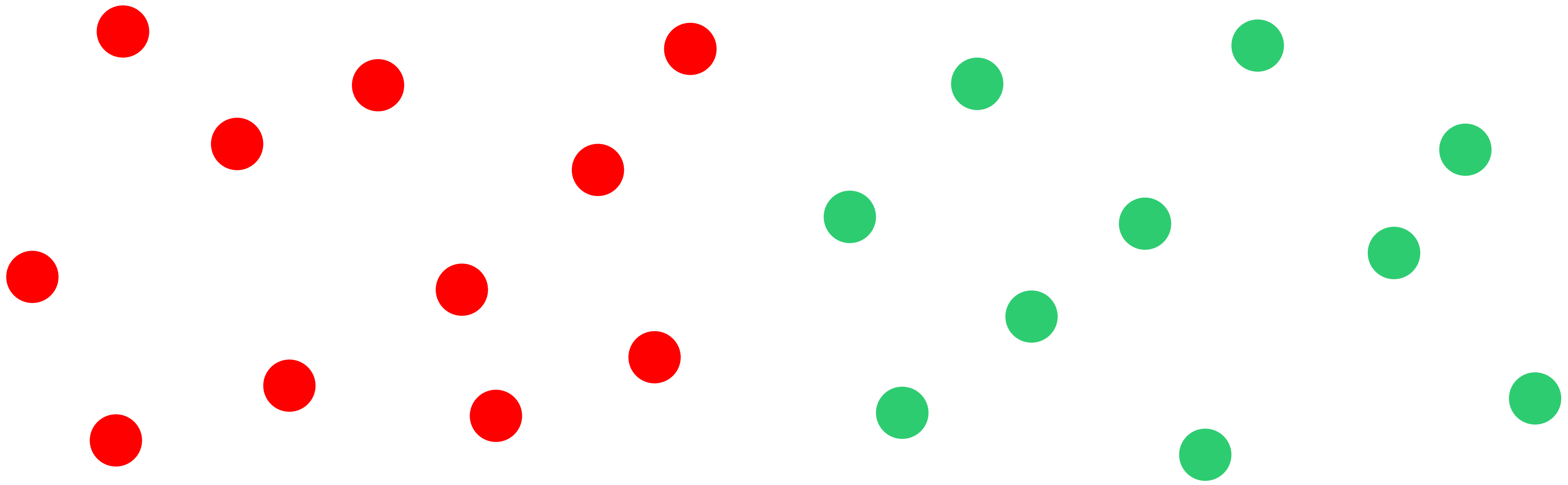
  - MLE, MAP estimation more challenging - kernel methods, deep learning
  - Gaussian processes more suited for non-linear PML – beyond the scope!
- Are Bayesian models better than non-Bayesian ones?
  - Bayesian models more informative $\mathcal{N}\left(\langle \boldsymbol{\nu}, \mathbf{x} \rangle, \sigma^2 + \mathbf{x}^\top \Lambda \mathbf{x}\right)$
  - Useful in settings like active learning, anomaly detection
  - Can be more expensive at training and prediction time
  - Ask your doctor if your application needs Bayesian reasoning or not!
- Bayesian ≠ Generative

A few Thoughts

- How do I solve all these optimization problems that MLE, MAP procedures keep throwing up?
  - We are in safe hands – optimization theory is very evolved
  - Wait for the next set of lectures on function approximation methods
- Are generatively trained models better or discriminative ones?
  - Discriminative models reason about $\mathbb{P}[y|\boldsymbol{x}]$
  - Generative models reason about $\mathbb{P}[y, \boldsymbol{x}]$
  - For prediction, all we need is $\mathbb{P}[y|\boldsymbol{x}]$
  - Generative models handle missing, erroneous data easily but bulky
  - Discriminative models are much lighter, easier to train
  - Models such GMMs, deep nets, can be trained in gen. and disc. manner

Classification using PML

Revisiting Binary Classification



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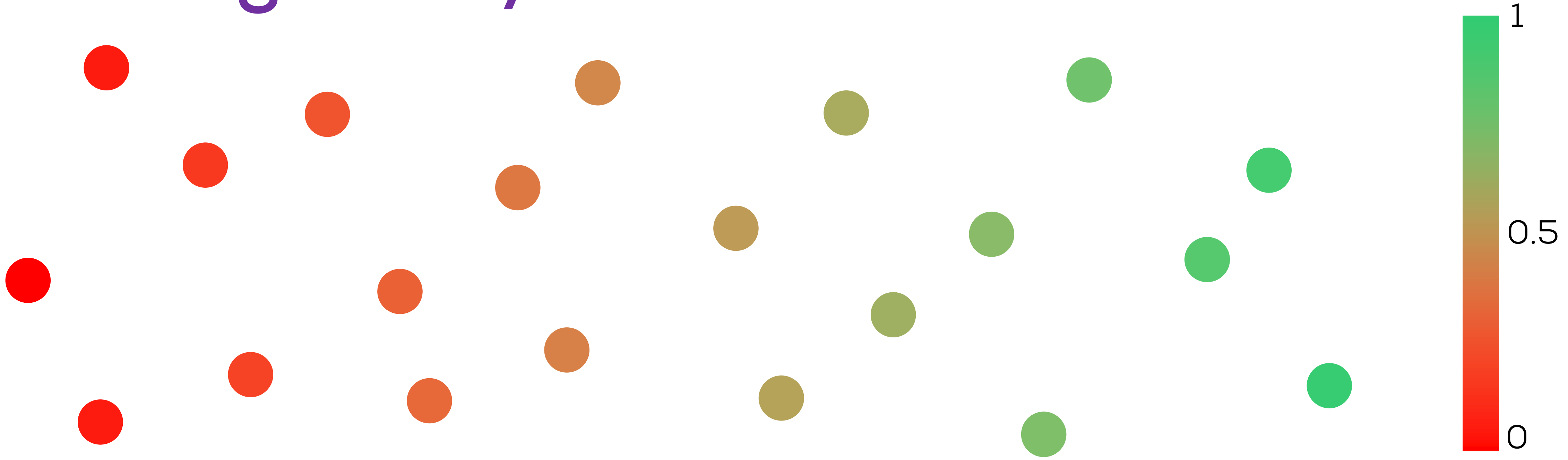
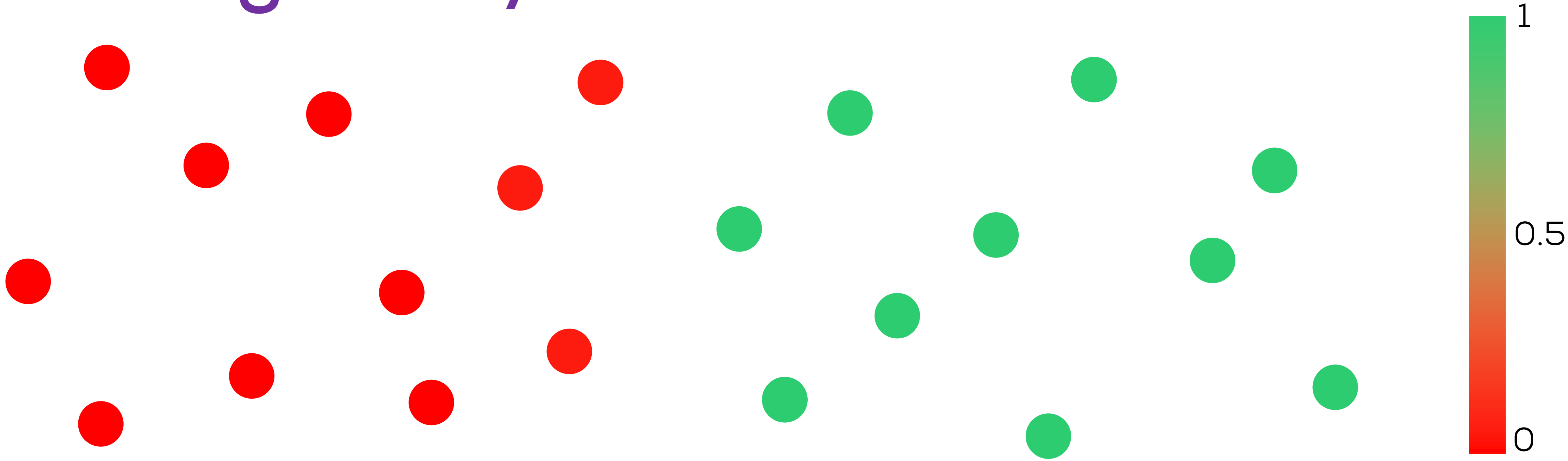


$$\eta(\mathbf{x}) = \mathbb{P}\left[y = 1 \,|\, \mathbf{x}\right]$$

Revisiting Binary Classification



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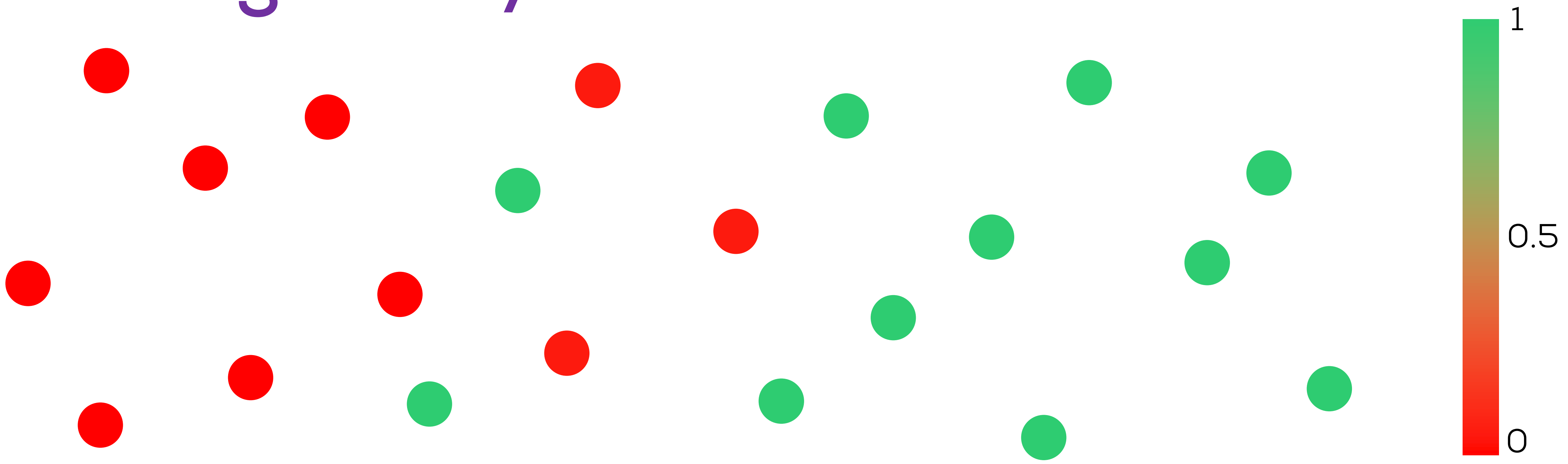

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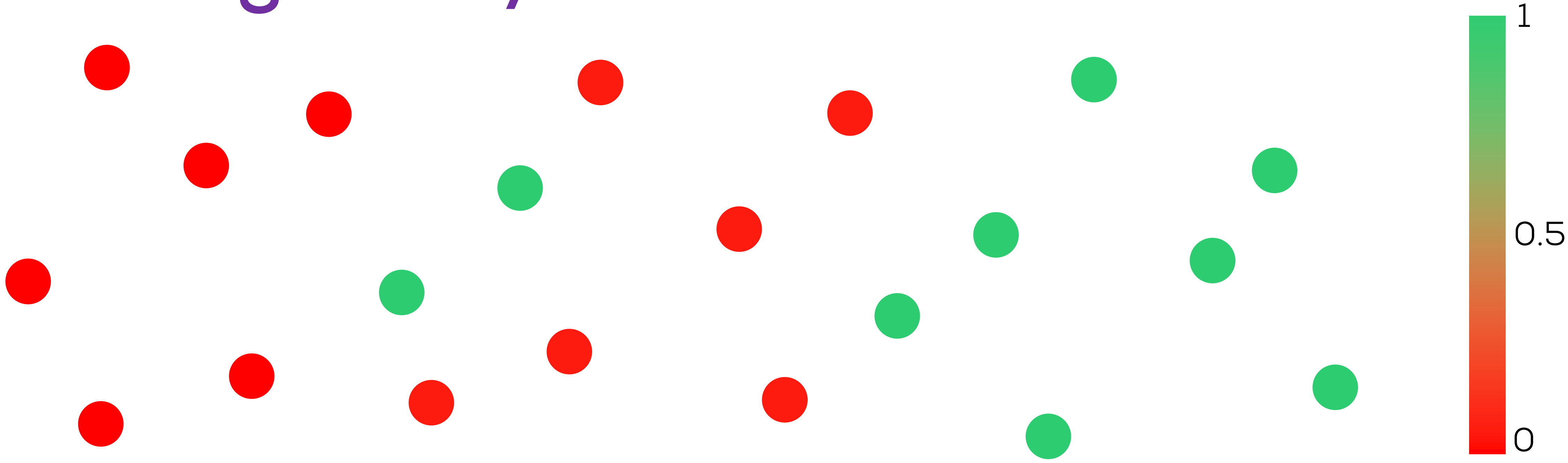

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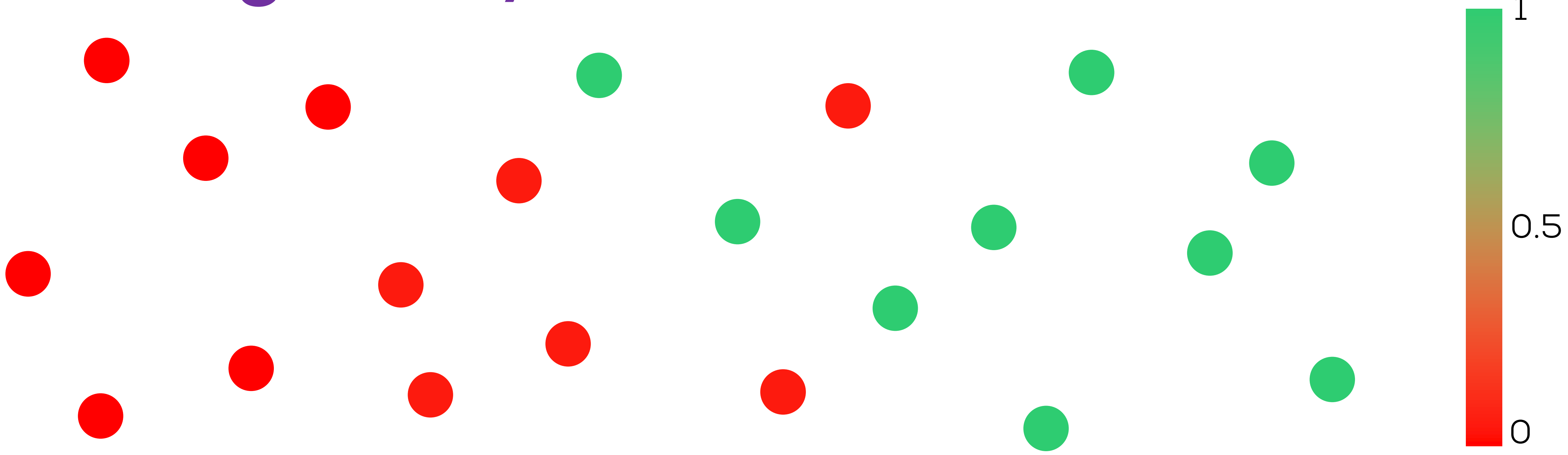

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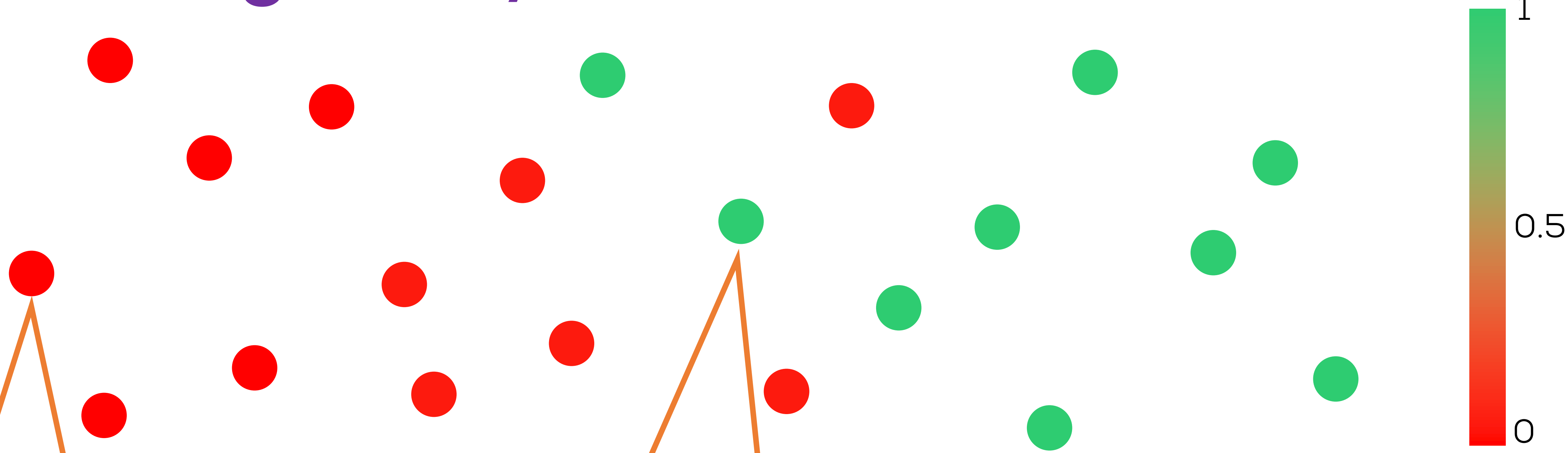
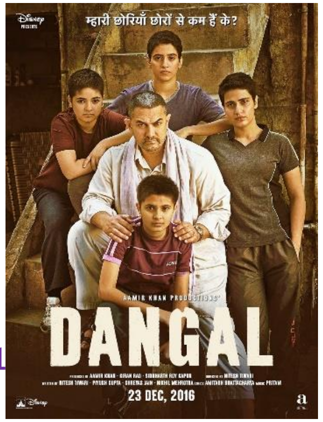


$$\eta(\mathbf{x}) = \mathbb{P}\left[y = 1 \mid \mathbf{x}\right]$$

Others are not

Some decisions are hard



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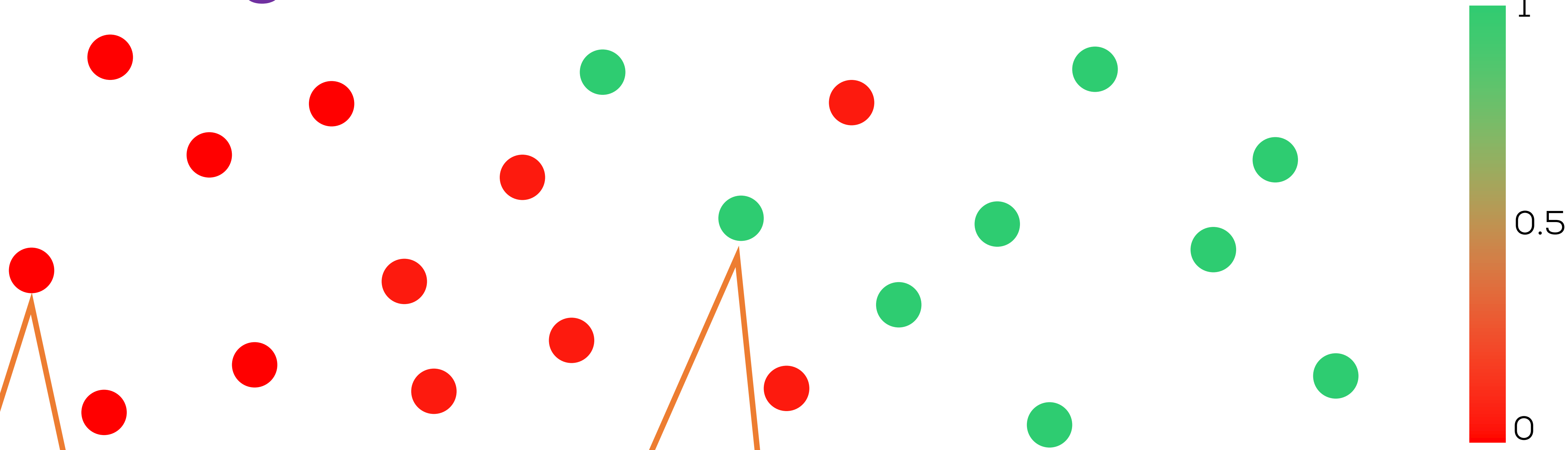


Others are not

Some decisions are hard

Gives us a prediction rule!

$$\eta(\mathbf{x}) = \mathbb{P}\left[y = 1 \,|\, \mathbf{x}\right]$$

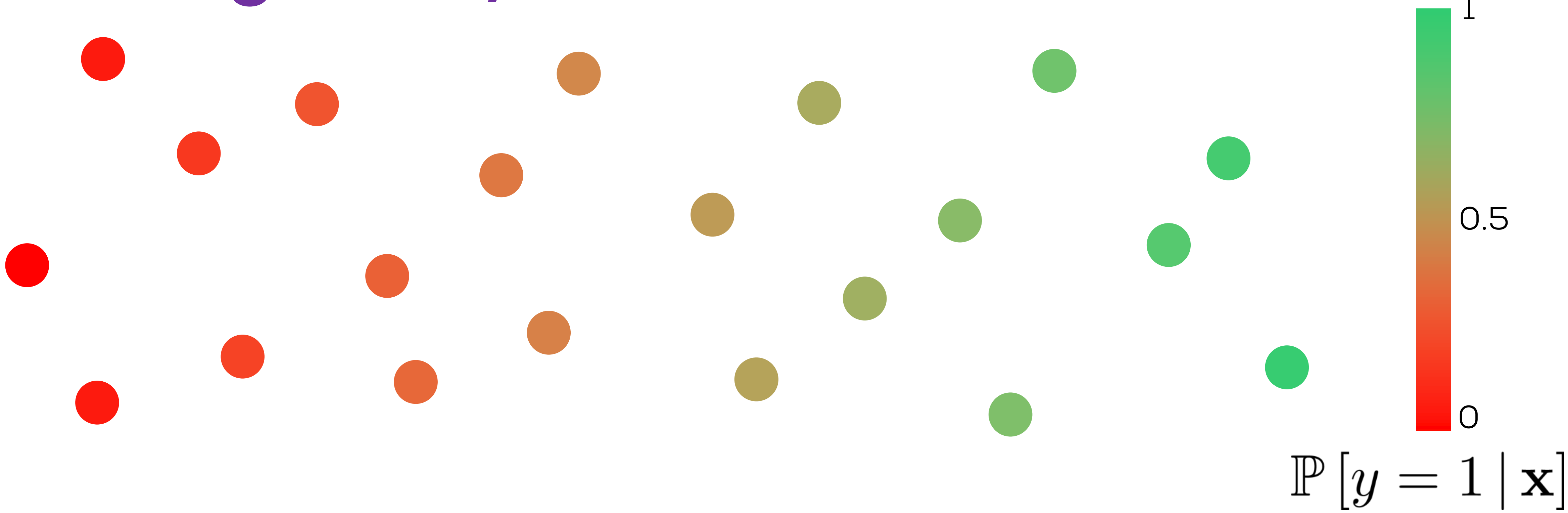
$$\mathbb{P}\left[y = 1 \,|\, \mathbf{x}\right] \geq 0.5 \quad \bullet$$

$$\mathbb{P}\left[\bullet \,|\, \mathbf{x}\right] > \mathbb{P}\left[\bullet \,|\, \mathbf{x}\right]$$

$$\mathbb{P}\left[y = -1 \,|\, \mathbf{x}\right] > 0.5 \quad \bullet$$

ost.com, wikipedia.com, wikia.com

Revisiting Binary Classification



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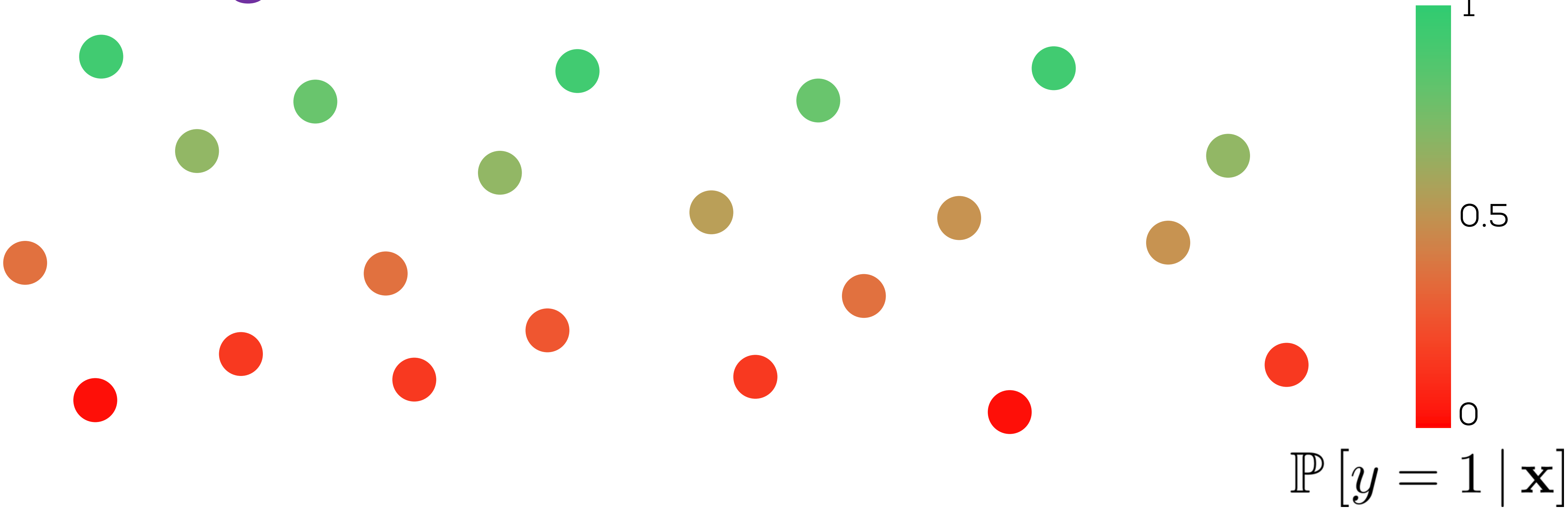

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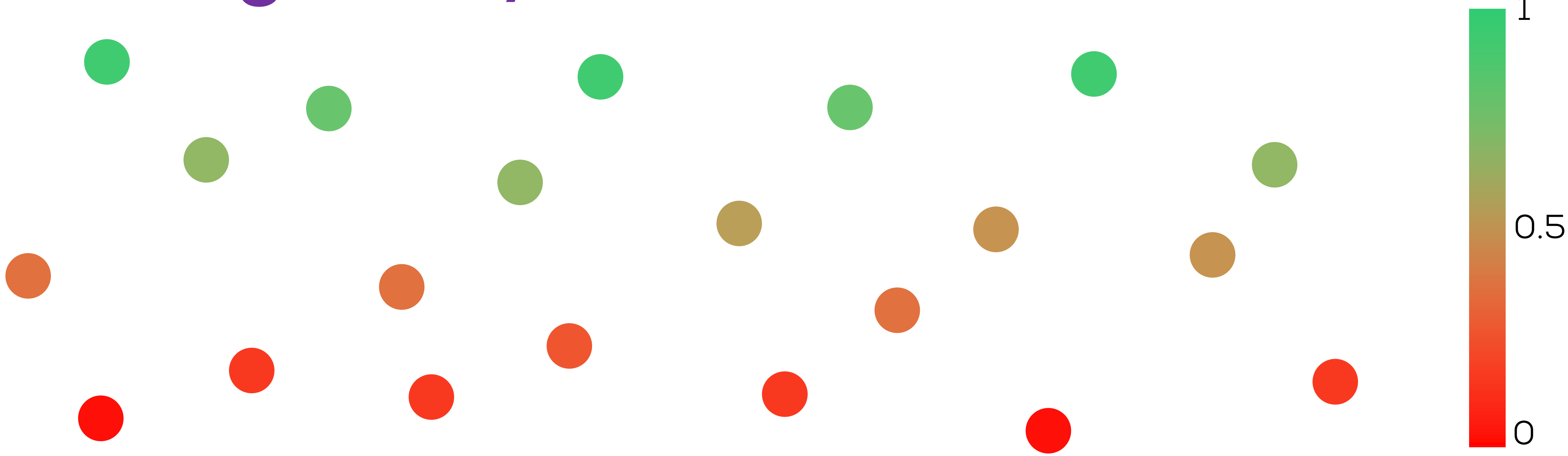
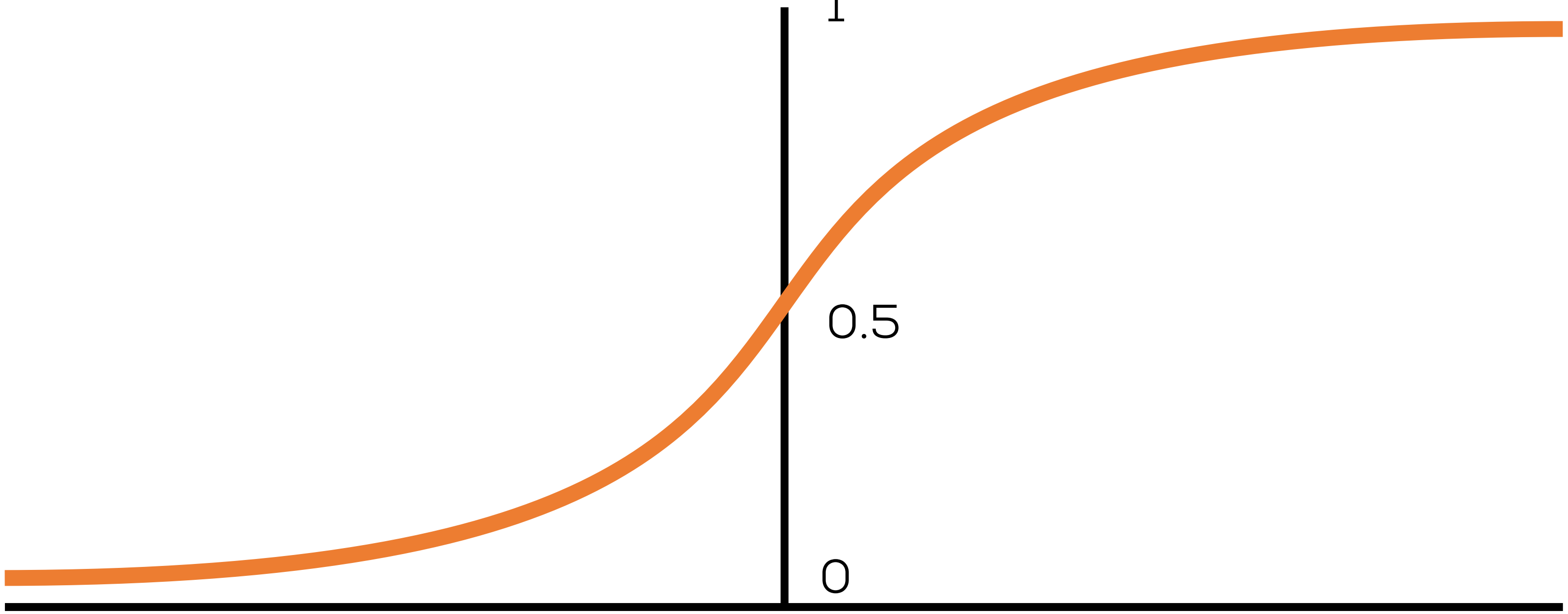


$\mathbb{P}\left[y = 1 \,|\, \mathbf{x}\right]$

Features Matter!

The Sigmoid Function



$$\sigma(t) = \frac{1}{1 + \exp(-t)} = \frac{\exp(t)}{\exp(t) + 1}$$

Classification using MLE

Shorthand
$\mathbb{P}[y^i = y] =: \mathbb{P}[y]$

$$\mathbb{P}\left[y \,|\, \mathbf{x}^i, \mathbf{w}\right] = \sigma\left(y\left\langle \mathbf{w}, \mathbf{x}^i \right\rangle\right) = \frac{1}{1 + \exp(-y\left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)}$$

Likelihood

$$\mathbb{P}\left[y^i = 1 \,|\, \mathbf{x}^i, \mathbf{w}\right] + \mathbb{P}\left[y^i = -1 \,|\, \mathbf{x}^i, \mathbf{w}\right] = 1$$

$$\log \frac{\mathbb{P}\left[y^i = 1 \,|\, \mathbf{x}^i, \mathbf{w}\right]}{\mathbb{P}\left[y^i = -1 \,|\, \mathbf{x}^i, \mathbf{w}\right]} = \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle$$

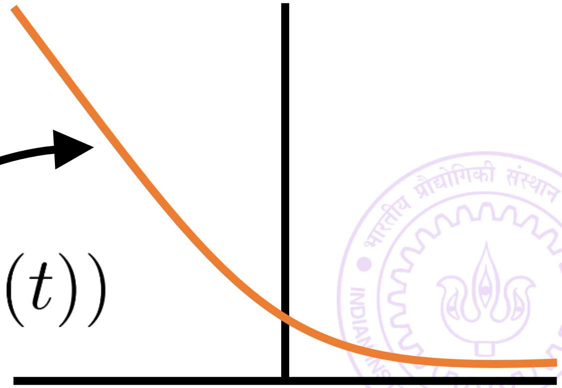
Log-odds

Log-likelihood

$$\log \mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] = \sum_{i=1}^{n} \log\left(\frac{1}{1 + \exp(-y^i\left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)}\right)$$

$$\hat{\mathbf{w}}_{\mathrm{MLE}} = ?$$

Wait for learning with FA



$$f(t) = -\log(\sigma(t))$$

Another possibility

- What happens if $y_i \in \{0,1\}$ instead of $y_i \in \{-1,1\}$?
- Binomial instead of Rademacher
- How to redefine likelihood?
- Need to ensure $\mathbb{P}\left[y^i = 1 \,|\, \mathbf{x}^i, \mathbf{w}\right] + \mathbb{P}\left[y^i = 0 \,|\, \mathbf{x}^i, \mathbf{w}\right] = 1$
- One way is the following

$$\hat{\eta}_i = \sigma(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

$$\mathbb{P}\left[y^i \,|\, \mathbf{x}^i, \mathbf{w}\right] = (\hat{\eta}_i)^{y^i} \cdot (1 - \hat{\eta}_i)^{1-y^i}$$

- Do you get the same MLE problem?
- What about MAP estimates? $\mathbb{P}\left[\mathbf{w}\right] = \mathcal{N}(\mathbf{0}, \rho^2 \cdot I_d)$

Exercise

Bayesian Logistic Regression?

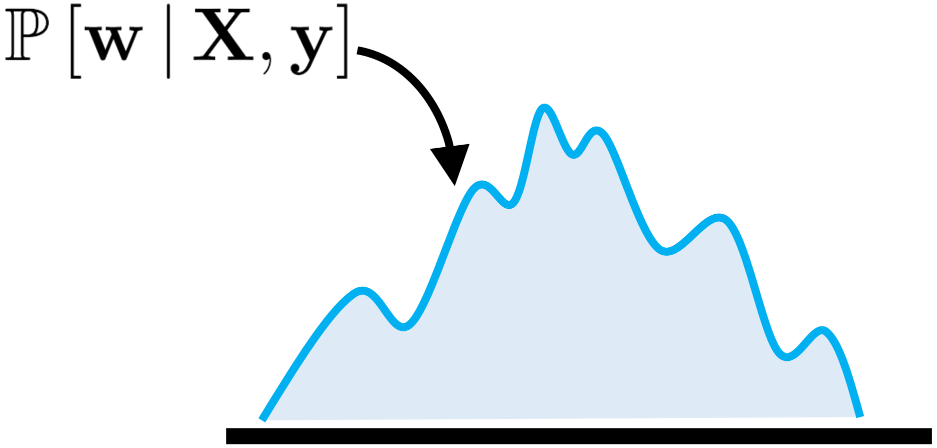
$$\mathbb{P}\left[y \,|\, \mathbf{x}, \mathbf{w}\right] = \sigma\left(y \left\langle \mathbf{w}, \mathbf{x} \right\rangle\right)$$

$$\mathbb{P}\left[\mathbf{w}\right] = \mathcal{N}(\mathbf{0}, \rho^2 \cdot I_d)$$

$$\mathbb{P}\left[\mathbf{w} \,|\, \mathbf{X}, \mathbf{y}\right] = \frac{\mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] \mathbb{P}\left[\mathbf{w}\right]}{\int_{\mathbf{w}'} \mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}'\right] \mathbb{P}\left[\mathbf{w}'\right] d\mathbf{w}'} = \frac{\prod_{i=1}^{n} \sigma(y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)}{\int_{\mathbf{w}'} \prod_{i=1}^{n} \sigma(y^i \left\langle \mathbf{w}', \mathbf{x}^i \right\rangle) \exp\left(\frac{-\|\mathbf{w}'\|_2^2}{2\sigma^2}\right) d\mathbf{w}'}$$

Posterior Approximation

MCMC, Variational Inference,
Laplace approx



Bayesian Logistic Regression?

$$\mathbb{P}\left[y \,|\, \mathbf{x}, \mathbf{w}\right] = \sigma\left(y \left\langle \mathbf{w}, \mathbf{x} \right\rangle\right)$$

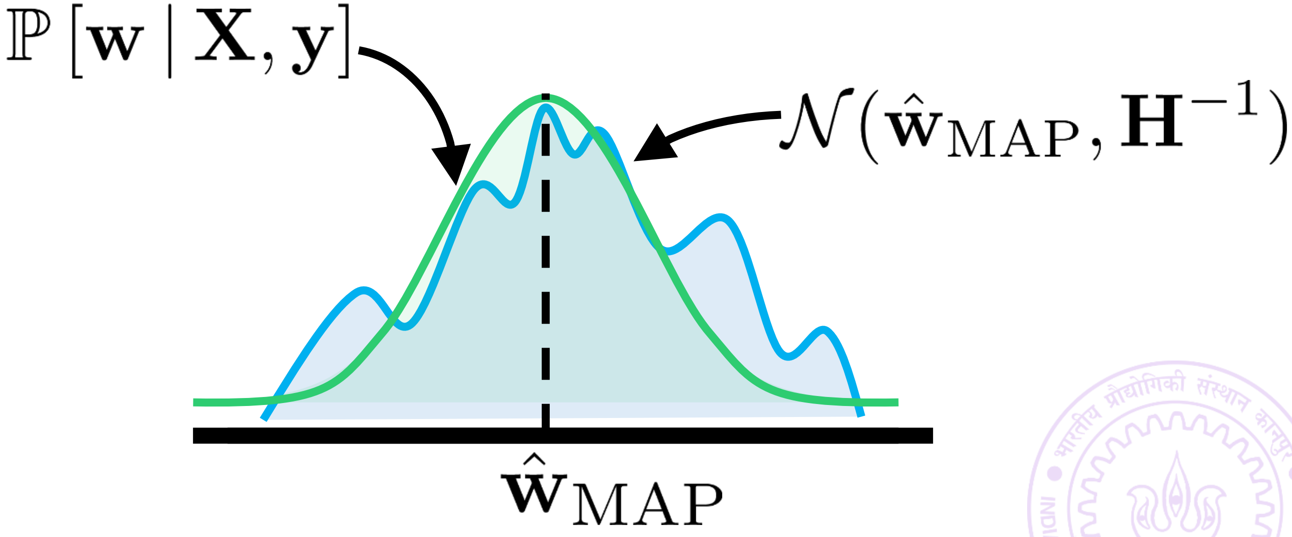
$$\mathbb{P}\left[\mathbf{w}\right] = \mathcal{N}(\mathbf{0}, \rho^2 \cdot I_d)$$

$$\mathbb{P}\left[\mathbf{w} \,|\, \mathbf{X}, \mathbf{y}\right] = \frac{\mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] \mathbb{P}\left[\mathbf{w}\right]}{\int_{\mathbf{w}'} \mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}'\right] \mathbb{P}\left[\mathbf{w}'\right] d\mathbf{w}'} = \frac{\prod_{i=1}^{n} \sigma(y^i \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle)}{\int_{\mathbf{w}'} \prod_{i=1}^{n} \sigma(y^i \left\langle \mathbf{w}', \mathbf{x}^i \right\rangle) \exp\left(\frac{-\|\mathbf{w}'\|_2^2}{2\sigma^2}\right) d\mathbf{w}'}$$

Posterior Approximation

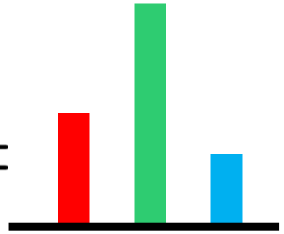
MCMC, Variational Inference, Laplace approx

However, even the Laplace predictive posterior is intractable ☹



Multi-class and Multi-label Classification using PML

Multi-classification using MLE

- $K > 2$ classes – need more detailed parameters
- For each point, its label profile is a vector $\boldsymbol{\eta}(\mathbf{x}) =$ 

$$\mathbb{P}\left[y^i = k \,|\, \mathbf{x}^i, \{\mathbf{w}^l\}_{1,\ldots,K}\right] \propto \exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)$$

$$\mathbb{P}\left[y^i = k \,|\, \mathbf{x}^i, \{\mathbf{w}^l\}_{1,\ldots,K}\right] = \frac{\exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)}{\sum_{l=1}^{K} \exp(\langle \mathbf{w}^l, \mathbf{x}^i \rangle)}$$

- Likelihood function is multinomial instead of binomial

$$\mathbb{P}\left[\mathbf{y} \,|\, \mathbf{X}, \mathbf{w}\right] = \prod_{i=1}^{n} \hat{\boldsymbol{\eta}}^i_{y^i}(\mathbf{x}) \qquad \hat{\boldsymbol{\eta}}^i_k(\mathbf{x}) = \frac{\exp(\langle \mathbf{w}^k, \mathbf{x}^i \rangle)}{\sum_{l=1}^{K} \exp(\langle \mathbf{w}^l, \mathbf{x}^i \rangle)}$$

Softmax Regression

Binary vs Multi-Classification using Linear LR

- Binary

$$\underset{y \in \{-1,1\}}{\arg\max}\ \mathbb{P}\left[y \mid \mathbf{x}\right]$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$\mathbb{P}\left[y \mid \mathbf{x}\right] \geq 0.5$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$\mathbb{P}\left[y \,|\, \mathbf{x}\right] \geq 0.5$$

$$\mathbb{P}\left[y \,|\, \mathbf{x}, \mathbf{w}\right] = \sigma\left(y \left\langle \mathbf{w}, \mathbf{x} \right\rangle\right)$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$\sigma(y \langle \mathbf{w}, \mathbf{x}^i \rangle) \geq 0.5$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$y \left\langle \mathbf{w}, \mathbf{x}^i \right\rangle \geq 0$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \operatorname{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \mathrm{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \mathrm{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

$$\arg\max_{y \in [K]} \mathbb{P}\left[y \mid \mathbf{x}\right]$$

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \text{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

$$\arg\max_{y \in [K]} \mathbb{P}\left[y \,|\, \mathbf{x}\right]$$

$$\mathbb{P}\left[y = k \,|\, \mathbf{x}, \mathbf{w}\right] = \sigma\left(\langle \mathbf{w}^k, \mathbf{x} \rangle\right)$$

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \text{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

$$\arg\max_{y \in [K]} \sigma\left(\langle \mathbf{w}^k, \mathbf{x} \rangle\right)$$

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \text{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

$$\arg\max_{y \in [K]} \langle \mathbf{w}^k, \mathbf{x} \rangle$$

Binary vs Multi-Classification using Linear LR

- Binary

$$y = \operatorname{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

$$\underset{y \in [K]}{\arg\max} \; \langle \mathbf{w}^k, \mathbf{x} \rangle$$

Binary vs Multi-Classification using Linear LR

- Binary

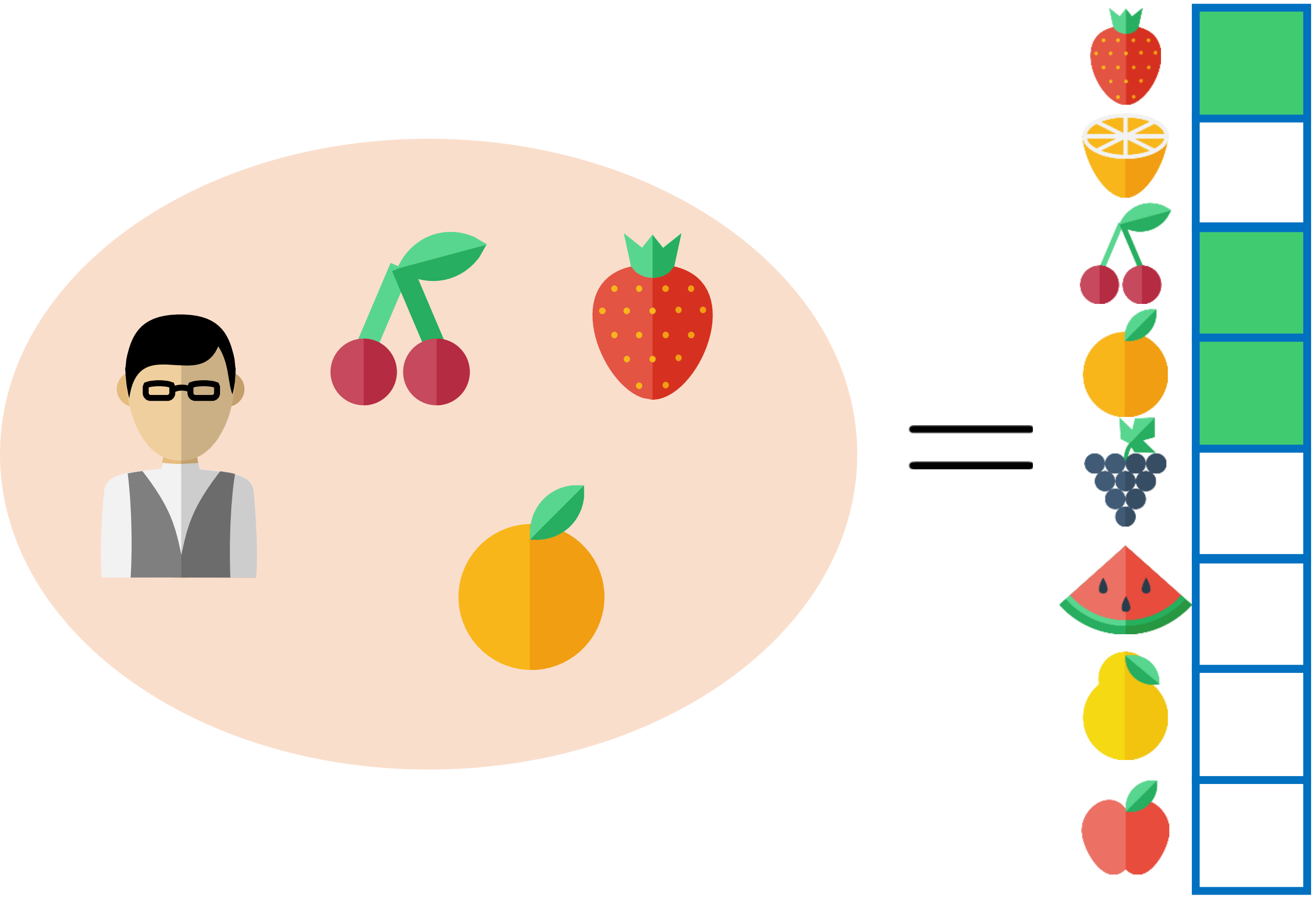
$$y = \text{sign}(\langle \mathbf{w}, \mathbf{x}^i \rangle)$$

- Multiclass

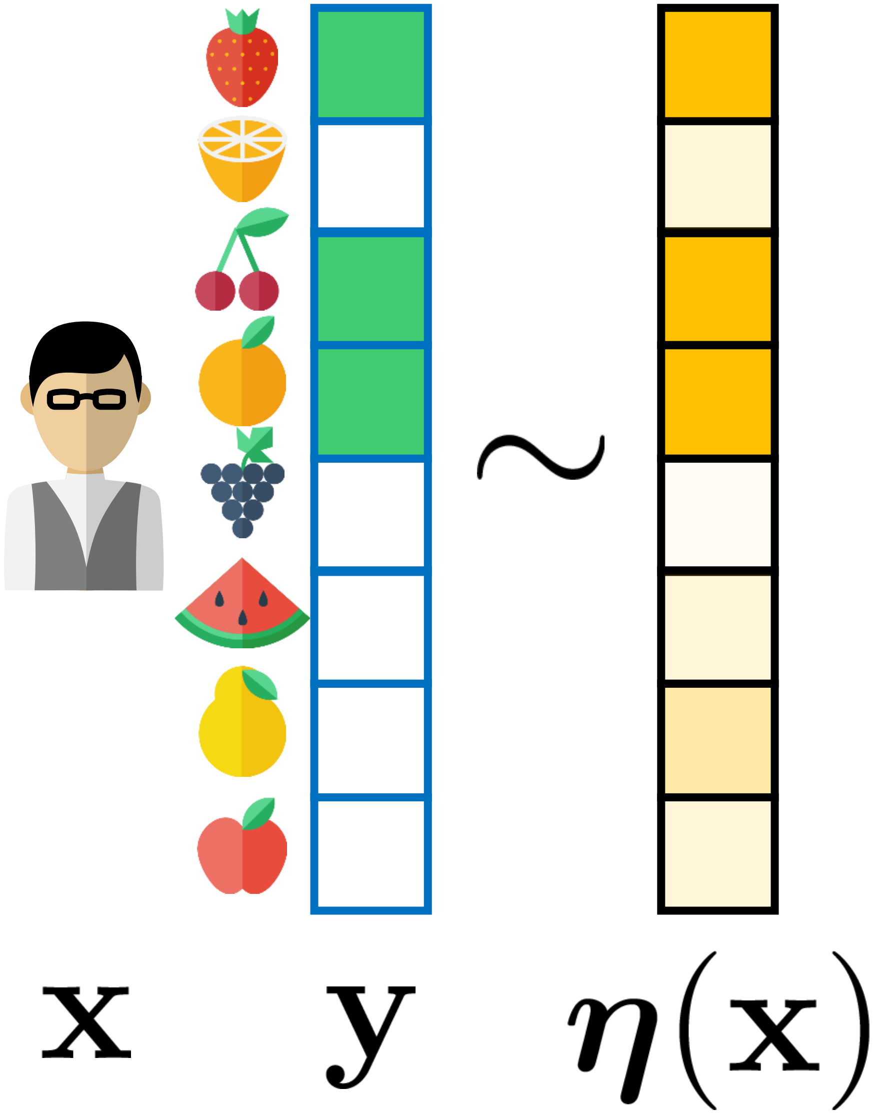
$$\arg\max_{y \in [K]} \langle \mathbf{w}^k, \mathbf{x} \rangle$$

Linear Classifier

Multi-label Learning using PML



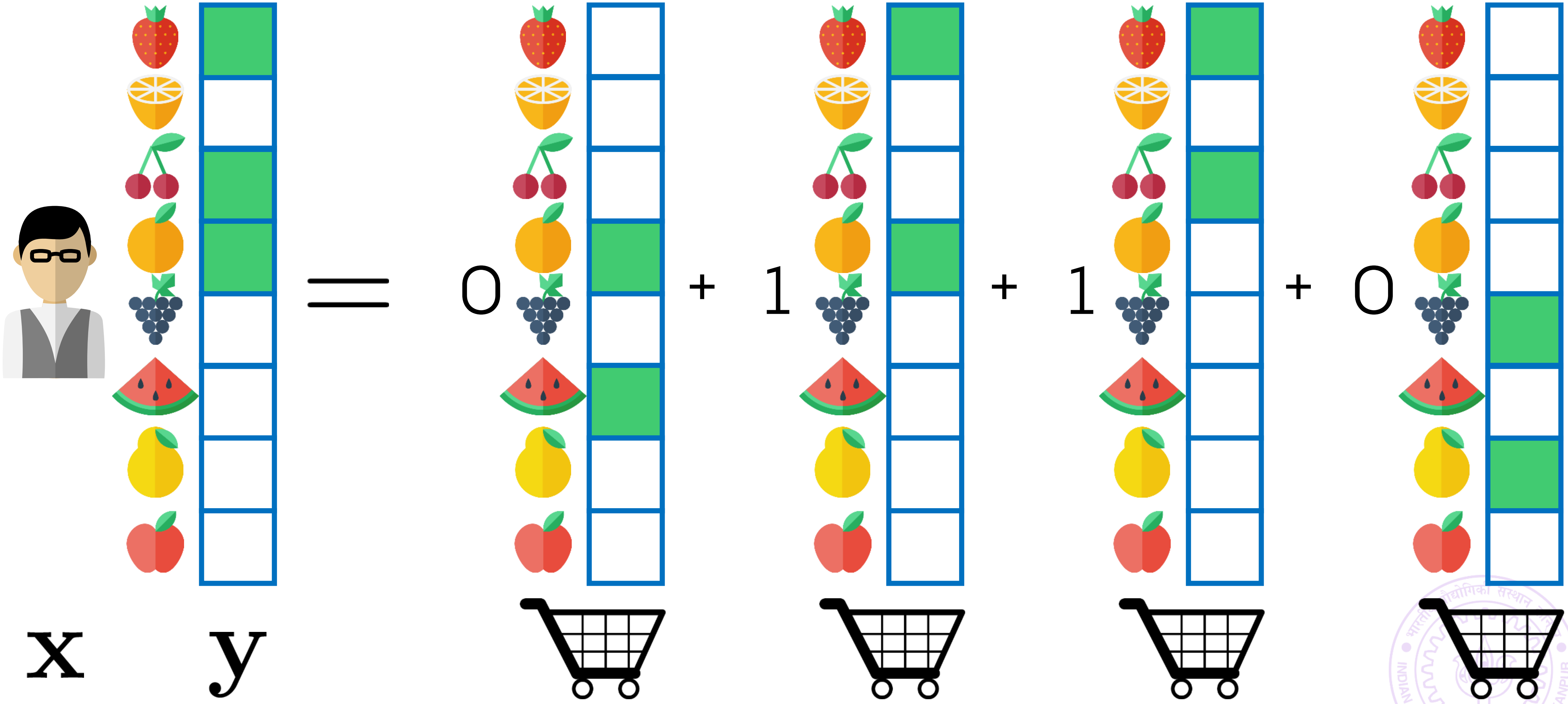
Multi-label Learning using PML



Solve as L independent Binary problems using Logistic regression!

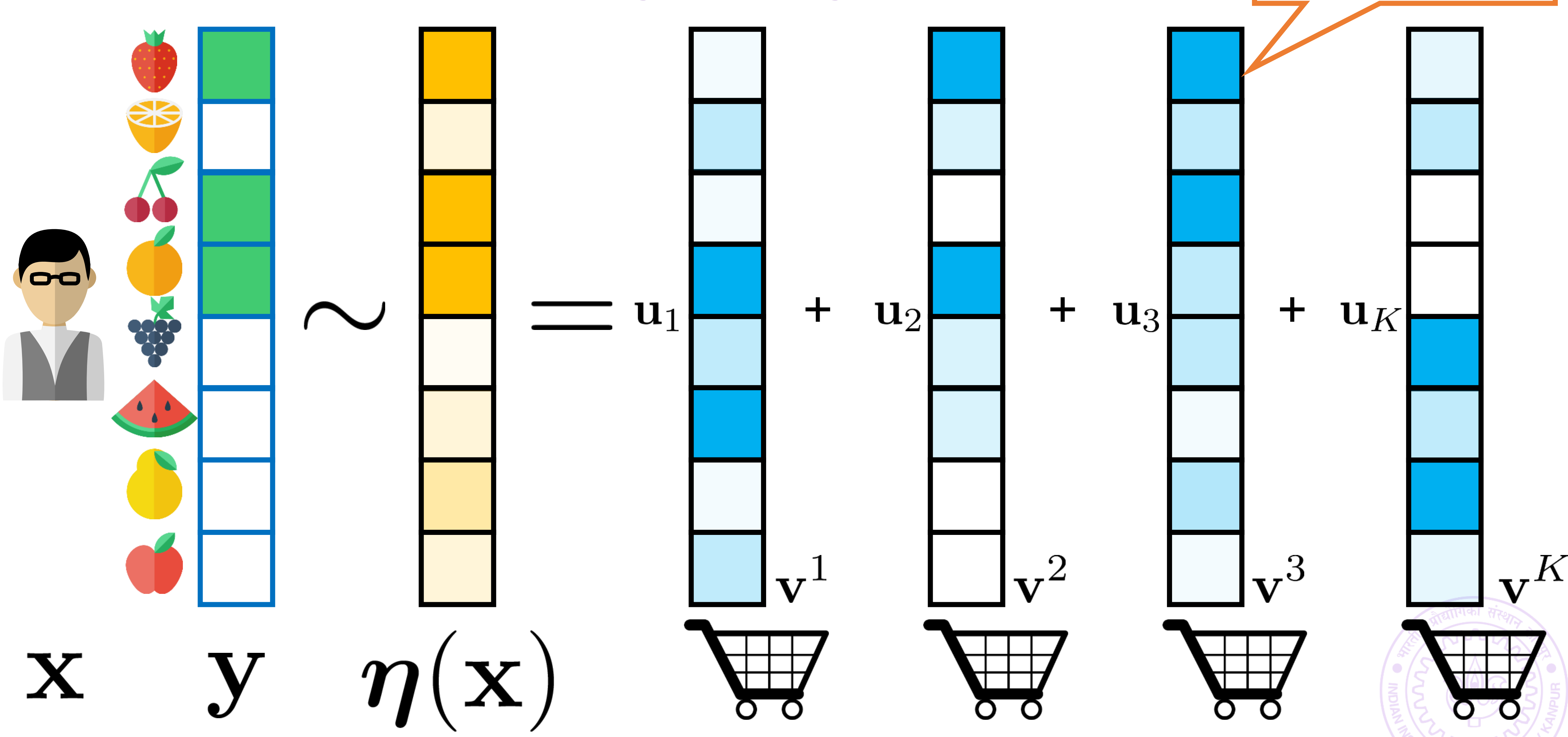
Does not scale!

Multi-label Learning using PML



Rai et al. Large-Scale Bayesian Multi-Label Learning via Topic-Based Label Embeddings, NIPS 2015

Multi-label Learning using PML



Rai et al. Large-Scale Bayesian Multi-Label Learning via Topic-Based Label Embeddings, NIPS 2015

Multi-label Learning using PML

$$\mathbf{y}^i \sim \text{Bernoulli}(\boldsymbol{\eta}^i)$$
$$\boldsymbol{\eta}^i = \mathbf{V}\mathbf{u}^i, \mathbf{V} = [\mathbf{v}^1, \mathbf{v}^2, \ldots, \mathbf{v}^K]$$
$$\mathbf{u}^i_j \sim \mathcal{N}(\langle \mathbf{w}^j, \mathbf{x}^i \rangle, \sigma^2), j = 1, \ldots, K$$

Can fix $\boldsymbol{V}$ or else …

$$\mathbf{v}^j \sim \text{Dirichlet}(\boldsymbol{\alpha})$$
$$\mathbf{w}^j \sim \mathcal{N}(\mathbf{0}, \rho^2 \cdot I), j = 1, \ldots, K$$

Generalization of Beta distribution

$$\mathbb{P}[\mathbf{v}; \boldsymbol{\alpha}] = \frac{1}{B(\boldsymbol{\alpha})} \prod_{i=1}^{L} \mathbf{v}_i^{\boldsymbol{\alpha}_i - 1}$$
$$\mathbb{P}[p; \alpha, \beta] = \frac{1}{B(\alpha, \beta)} p^{\alpha-1}(1-p)^{\beta-1}$$

Only K models, not L

Have fun posterioring!

Please give your Feedback

http://tinyurl.com/ml17-18afb