Problem 2: Hill-Climbing [20 points]

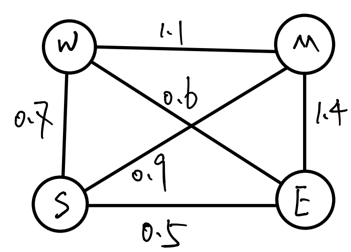
Given a set of locations and distances between them, the goal of the Traveling Salesperson Problem (TSP) is to find the shortest tour that visits each location exactly once. Assume that you do *not* return to the start location after visiting the last location in a tour. We would like to solve the TSP problem using a greedy hill-climbing algorithm. Each state corresponds to a permutation of all the locations (called a *tour*). The operator *neighbors*(*s*) generates all neighboring states of state *s* by swapping two locations. For example, if *s* = <*A-B-C*> is a tour, then <*B-A-C*>, <*C-B-A*> and <*A-C-B*> are the three neighbors generated by *neighbors*(*s*). Use as the evaluation function for a state the length of the tour, where each pairwise distance is given in a distance matrix. For example, if *s* = <*A-B-C*> is a tour, then its total length is *d*(*A*, *B*) + *d*(*B*, *C*) where *d*(*A*, *B*) is the distance between location *A* and *B*.

(a) [3] If there are *n* locations, how many neighboring states does the *neighbors*(*s*) function produce? we need to swapping two nodes that is $\binom{n}{2} = \frac{n(n-1)}{2}$

(b) [3] What is the *total size* of the entire search space, i.e., the total number of states, when there are *n* locations? for location i in the path, there are $(n-i)$ ways because the first location is 0, that is $n(n-1)(n-2)\cdots = n!$

(c) [14] Imagine that a student wants to hand out fliers about an upcoming programming contest. The student wants to visit the Memorial Union (M), Wisconsin Institute of Discovery (W), Computer Sciences Building (S), and Engineering Hall (E) to deliver the fliers. The goal is to find a tour as short as possible. The distance matrix between these locations is given as follows:

	M	W	E	S
M	0	1.1	1.4	0.9
W	1.1	0	0.6	0.7
E	1.4	0.6	0	0.5
S	0.9	0.7	0.5	0



The student starts applying the hill-climbing algorithm from the initial state: <*W-M-E-S*>

(i) [2] What is the length of the tour associated with the initial state?

the length is $1.1 + 1.4 + 0.5 = 3$

(ii) [4] What are the possible neighboring states of <*W-M-E-S*> and what are each of their tour lengths?

neighboring	length		length
MWES	$1.1 + 0.6 + 0.5 = 2.2$	<WEMS>,	$0.6 + 1.4 + 0.9 = 2.9$
EMWS	$1.4 + 1.1 + 0.7 = 3.2$	<WSEM>	$0.7 + 0.5 + 1.4 = 2.6$
SMEW	$0.9 + 1.4 + 0.6 = 2.9$	<WMSE>	$1.1 + 0.9 + 0.5 = 2.5$

(iii) [2] What is the *next* state reached by hill-climbing? Or explain why there is no next state, if there isn't one.

Let's range these values: $2.2 < 2.5 < 2.6 < 2.9 < 3.2$, so the next state is <MWES>, because it is the shortest distance compare to others.

(iv) [6] If (iii) had a next state, continue the algorithm until it terminates, and list the sequence of states found, from initial state to final state. What is the tour length associated with the final state?

<W-M-E-S> $\longrightarrow$ <M-W-E-S> $\longrightarrow$ <M-S-E-W>
3 2.2 2

Length of <M-S-E-W> $= 0.9 + 0.5 + 0.6 = 2$

<M-S-E-W> is the final state because we can not find any shorter distance from its neighbors.