CAT Language Specification

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Introduction

CAT (Calculus and Algebra Tutor) is a domain-specific language designed to assist individuals with mathematical calculations. More specifically, the language allows users to input mathematical expressions and operations to perform on the expressions. Running a program in this language will then produce a textual represenation of the ensuing calculation as well as an optional LaTeX file and PDF that displays the calculation in a pretty manner.

While there exist computational tools for generalized mathematical expressions (for example Mathematica), such tools tend to focus only on getting to an answer. The purpose of CAT is to assist its users in understanding *how* to get the answer by outputting the intermediate steps of the calculation. This addresses another point of annoyance with tools like Mathematica that will sometimes output, seemingly arbitrary, restrictions on the parameters alongside its final expression. By working through the intermediate steps of the calculation, CAT should allow its users to understand why such restrictions were imposed and what the limits of the calculation would be if such assumptions were not made.

Design Principles

The primary goal of CAT is to produce easily understandable calculations. This goal corresponds with a prioritization of simpler techniques that are easier to follow over more powerful techniques that might be able to find more general answers but are less decipherable. This likely also means a tradeoff in terms of speed as it is necessary to work through the intermediate details of a calculation sufficiently to make them human readable.

Examples

Delete this TODO and replace with 3+ examples and accompanying descriptions.

Language Concepts

At it's core, CAT is a way of expressing and manipulating mathematical expressions. Each line of a CAT program constitutes an expression that will be evaluated. Evaluating an expression involves reducing it to the simplist form possible by applying operators and performing algebraic manipulations. The primitives of CAT are variables (i.e. x, y, z) and numbers (i.e. $1, -10, 3.141592$). These primitives can be combined with mathematical operations such as +, -, *, /, ^, sin, cos, log, sqrt, Differentiate, and Integrate in order to form expressions. Additionally, users are able to define variables (i.e. $a = \text{sqrt}(2)$, or $f = x + 1$) that can be used in later lines.

Formal Syntax

```
<expr>     ::=   <number>
<number>   ::=   <digits>
             |   <digits>.<digits>
<digits>   ::=   <d><digits>
             |   <d>
<d>        ::=   0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

Semantics

Delete this TODO and replace with as much text as is needed.