

1. A sample of wet soil in a metallic tin has a mass of 450g. After drying in an oven overnight, the sample and tin have a mass of 368g. The mass of the dish alone is 24g. Calculate the moisture content (w) of the sample.

$$w = \frac{M_w}{M_s} \cdot 100$$

$$M_w = (\text{mass of wet soil} + \text{tin}) - (\text{mass of dry soil} + \text{tin})$$

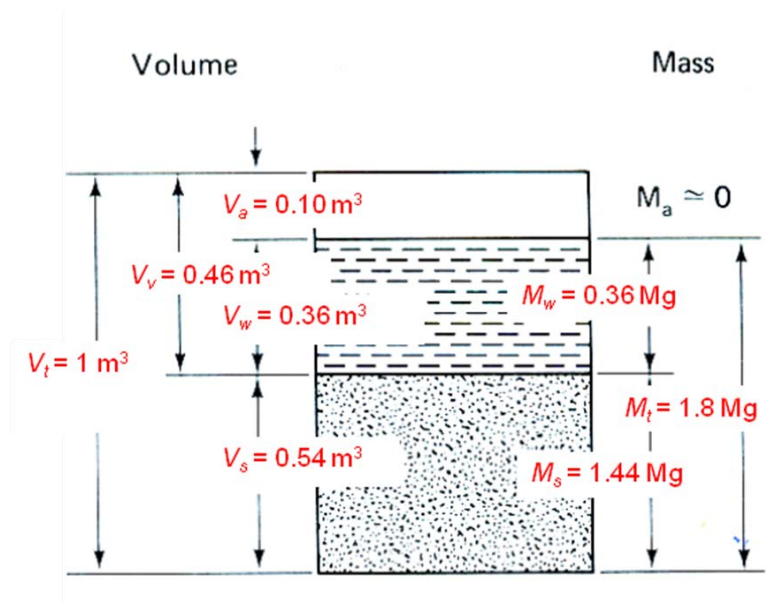
$$M_w = 450g - 368g = 82g$$

$$M_s = (\text{mass of dry soil} + \text{tin}) - (\text{mass of tin})$$

$$M_s = 368g - 24g = 344g$$

$$w = \frac{M_w}{M_s} \cdot 100 = \frac{82}{344} \cdot 100 = 23.8\%$$

2. A sample of sand has a bulk density (ρ_t) of 1.8 Mg/m³ and a moisture content (w) of 25%. Calculate e , n , S_r , ρ_d , ρ_{sat} and ρ' . Assume that $V_t = 1.0 \text{ m}^3$



Assume $V_t = 1.0 \text{ m}^3$

$$\rho_t = 1.8 \text{ Mg/m}^3$$

$$\frac{M_t}{V_t} = \frac{M_w + M_s}{V_t}$$

$$M_w = 0.25M_s$$

$$\rho_t = \frac{0.25M_s + M_s}{V_t}$$

$$1.8 \text{ Mg/m}^3 = \frac{0.25M_s + M_s}{1.0 \text{ m}^3}$$

$$1.8 \text{ Mg} = 1.25M_s$$

$$M_s = 1.44 \text{ Mg}$$

$$M_w = 0.25M_s$$

$$M_w = 0.25(1.44 \text{ Mg})$$

$$M_w = 0.36 \text{ Mg}$$

$$\rho_w = \frac{M_w}{V_w} = 1.0 \text{ Mg/m}^3$$

$$V_w = \frac{M_w}{\rho_w} = \frac{0.36 \text{ Mg}}{1.0 \text{ Mg/m}^3}$$

$$V_w = 0.36 \text{ m}^3$$

$$\text{Assume } G_s = 2.65$$

$$G_s = \frac{M_s}{V_s \rho_w} = \frac{\rho_s}{\rho_w} = 2.65$$

$$2.65 = \frac{\rho_s}{1.0 \text{ Mg/m}^3}$$

$$\rho_s = 2.65 \text{ Mg/m}^3$$

$$\rho_s = \frac{M_s}{V_s} = 2.65 \text{ Mg/m}^3$$

$$V_s = \frac{M_s}{\rho_s} = \frac{1.44 \text{ Mg}}{2.65 \text{ Mg/m}^3}$$

$$V_s = 0.54 \text{ m}^3$$

$$V_t = V_a + V_w + V_s$$

$$1.0 \text{ m}^3 = V_a + 0.36 \text{ m}^3 + 0.54 \text{ m}^3$$

$$V_a = 0.10 \text{ m}^3$$

$$e = \frac{V_v}{V_s} = \frac{0.46 \text{ m}^3}{0.54 \text{ m}^3} = 0.85$$

$$n = \frac{V_v}{V_t} = \frac{0.46 \text{ m}^3}{1.00 \text{ m}^3} \cdot 100 = 46\%$$

$$S_r = \frac{V_w}{V_v} \cdot 100$$

$$S_r = \frac{V_w}{V_a + V_w} \cdot 100$$

$$S_r = \frac{0.36 \text{ m}^3}{0.10 \text{ m}^3 + 0.36 \text{ m}^3} \cdot 100$$

$$S_r = 78 \%$$

$$\rho_d = \frac{M_s}{V_t} = \frac{1.44 \text{ Mg}}{1.00 \text{ m}^3}$$

$$\rho_d = 1.44 \frac{\text{Mg}}{\text{m}^3}$$

$$\rho_{sat} = \frac{M_s + M_w}{V_t}$$

$$\rho_{sat} = \frac{1.44 \text{ Mg} + (0.36 \text{ Mg} + 0.10 \text{ Mg})}{1.00 \text{ m}^3}$$

$$\rho_{sat} = 1.90 \frac{\text{Mg}}{\text{m}^3}$$

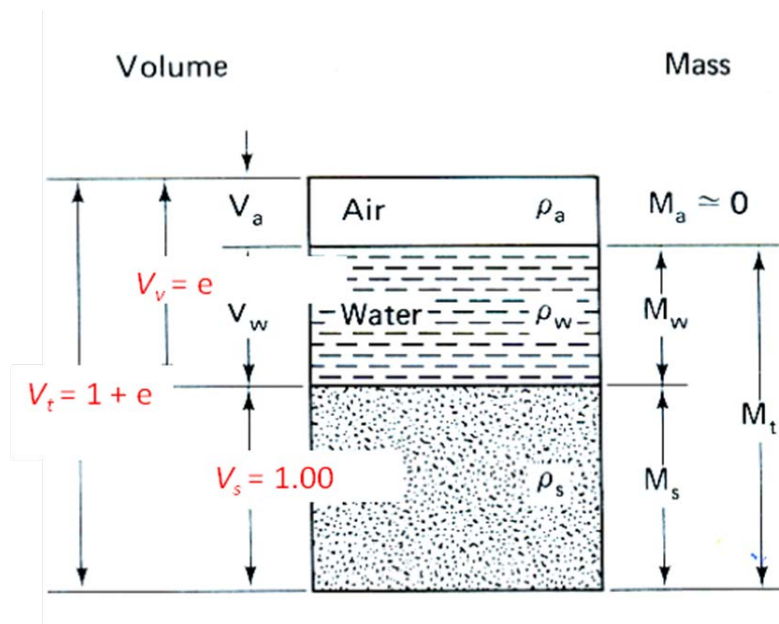
$$\rho'_{sat} = \rho_{sat} - \rho_w$$

$$\rho'_{sat} = 0.90 \frac{\text{Mg}}{\text{m}^3}$$

3. A sample of sand has a bulk density (ρ_t) of 1.8 Mg/m^3 and a moisture content (w) of 25%. Calculate e , n , S_r , ρ_d , ρ_{sat} and ρ' . Assume that $V_t = 1.0 \text{ m}^3$

Answers as above. Just different procedure.

4. Derive an expression for the porosity (n) in terms of the void ratio (e)



Assume $V_s = 1.0$

$$e = \frac{V_v}{V_s} = \frac{V_v}{1.0}$$

$$e = V_v$$

$$n = \frac{V_v}{V_t}$$

$$n = \frac{e}{1 + e}$$

5. A soil has a unit weight (γ_s) of 26.5 kN/m³ and a bulk unit weight (γ_t) of 18.0 kN/m³. The void ratio (e) was found to be 1.06 and the moisture content (w) was calculated as 40%. Is the soil saturated? **NOTE:** Use $g = 10 \text{ m/s}^2$ in your calculations.

$$\gamma_s = 26.5 \frac{\text{kN}}{\text{m}^3} = 26500 \frac{\text{N}}{\text{m}^3} = 26500 \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^3}$$

$$\gamma_s = \rho_s g$$

$$\rho_s = \frac{26.5 \frac{\text{Mg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^3}}{10 \frac{\text{m}}{\text{s}^2}} = 2.65 \frac{\text{Mg}}{\text{m}^3}$$

$$S_r e = w G_s$$

$$S_r = \frac{w G_s}{e} = \frac{0.40(2.65)}{1.06} = 1 \quad \Rightarrow \quad S_r = 100 \%$$

The soil is saturated

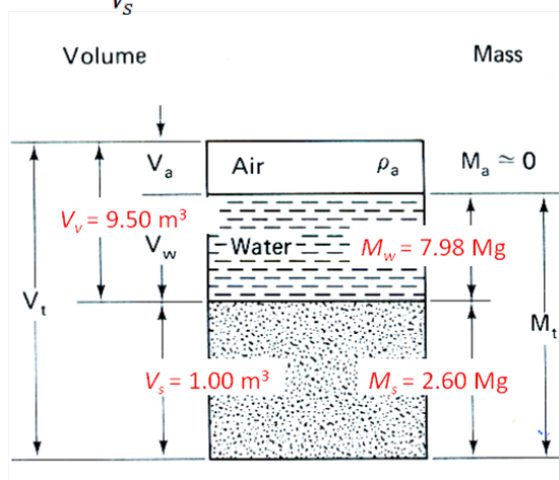
6. A sensitive volcanic clay soil tested in the laboratory was found to have the following properties:

- a) $\rho_s = 2.60 \text{ Mg/m}^3$
- b) $\rho_t = 1.35 \text{ Mg/m}^3$
- c) $w = 307 \%$
- d) $e = 9.5$
- e) $S_r = 84 \%$

After reviewing the values one was found to be inconsistent. Which one?

Assume $V_s = 1.0$

$$\rho_s = \frac{M_s}{V_s} \quad \Rightarrow \quad M_s = \rho_s V_s = 2.60 \text{ Mg/m}^3 (1.0 \text{ m}^3) = 2.60 \text{ Mg}$$



$$w = \frac{M_w}{M_s}$$

$$M_w = w M_s = 3.07 (2.60 \text{ Mg})$$

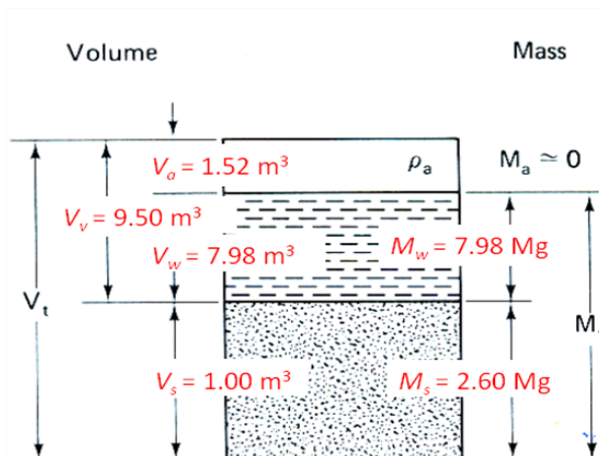
$$M_w = 7.98 \text{ Mg}$$

$$e = \frac{V_v}{V_s}$$

$$V_v = e V_s = 9.5 (1.0 \text{ m}^3)$$

$$V_v = 9.5 \text{ m}^3$$

$$\rho_w = \frac{M_w}{V_w} \quad \Rightarrow \quad V_w = \frac{M_w}{\rho_w} = \frac{7.98 \text{ Mg}}{1.0 \text{ Mg/m}^3} = 7.98 \text{ m}^3$$



$$V_v = V_w + V_a$$

$$9.50 \text{ m}^3 = 7.98 \text{ m}^3 + V_a$$

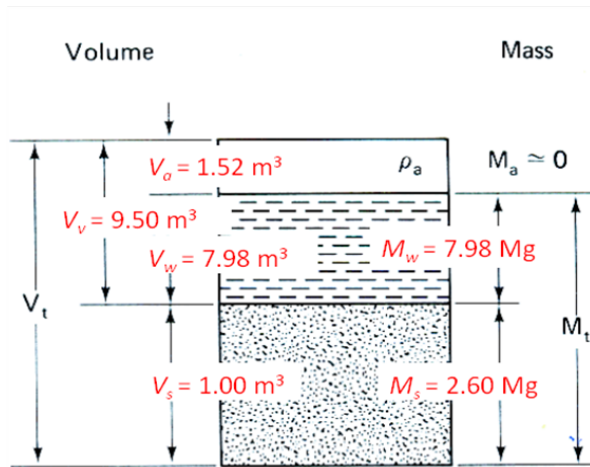
$$V_a = 1.52 \text{ m}^3$$

$$S_r = \frac{V_w}{V_v}$$

$$S_r = \frac{7.98 \text{ m}^3}{9.50 \text{ m}^3} = 0.84$$

$$S_r = 84 \%$$

$$\rho_t = \frac{M_t}{V_t} \Rightarrow \rho_t = \frac{M_w + M_s}{V_v + V_s} \Rightarrow \rho_t = \frac{7.98 \text{ Mg} + 2.60 \text{ Mg}}{9.50 \text{ m}^3 + 1.00 \text{ m}^3}$$



$$\rho_t = 1.01 \frac{\text{Mg}}{\text{m}^3}$$

$$1.35 \frac{\text{Mg}}{\text{m}^3} \neq 1.01 \frac{\text{Mg}}{\text{m}^3}$$

7. The *in-situ* dry density of a sand is 1.75 Mg/m^3 . The maximum and minimum dry densities, determined by standard laboratory tests, are 1.85 and 1.45 Mg/m^3 . Determine the relative density of the sand.

$$\rho_d = \frac{\rho_w G_s}{1 + e} \Rightarrow \rho_d + \rho_d e = \rho_w G_s \Rightarrow \rho_d e = \rho_w G_s - \rho_d$$

$$e = \frac{\rho_w G_s - \rho_d}{\rho_d} \Rightarrow e = \frac{\rho_w G_s}{\rho_d} - 1$$

$$e = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.75 \text{ Mg/m}^3} - 1 = 0.514$$

$$e_{min} = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.85 \text{ Mg/m}^3} - 1 = 0.432$$

$$e_{max} = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.45 \text{ Mg/m}^3} - 1 = 0.827$$

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.827 - 0.514}{0.827 - 0.432} = 0.79 \Rightarrow D_r = 79 \%$$

8. A soil of total volume 200 ml contains 25 ml air and 30 ml water. Calculate the void ratio and the degree of saturation.

$$V = 200 \text{ ml}, V_a = 25 \text{ ml}, V_w = 30 \text{ ml}$$

$$e = \frac{V_v}{V_s} = \frac{25 + 30}{200 - 25 - 30} = \underline{\underline{0.38}}$$

$$S_r = \frac{V_w}{V_v} = \frac{30}{55} = 0.55 = \underline{\underline{55\%}}$$

9. A soil has a porosity of 0.45. What is the void ratio?

$$e = \frac{n}{1-n}$$

$$e = 0.45/0.55$$

$$\Rightarrow \underline{e = 0.818}$$

10. A soil had a wet mass of 2.180 kg and occupied a volume of 1.2 litres. After oven drying the mass reduced to 1.890 kg. Calculate bulk density, moisture content and dry density.

$$\text{Bulk density} = 2.180/1.2 \times 10^{-3} \text{ m}^3 = 1816 \text{ kg/m}^3$$

$$\text{Moisture content} = (2.180-1.890)/1.890 = 15.3\%$$

$$\text{Dry density} = 1.890/1.2 \times 10^{-3} \text{ m}^3 = 1575 \text{ kg/m}^3$$

11. A sample of saturated clay has a volume of 245ml and after oven drying has a mass of 453g. If $G_s = 2.75$, determine the **dry** and **saturated unit weights** of the soil in its natural state.

$$\text{Dry unit weight} = 0.453/0.245 \times 9.81 = 18.13 \text{ kN/m}^3$$

$$\text{Dry volume} = 453[\text{g}]/2.75[\text{g/ml}] = 165 \text{ ml}$$

$$\text{Void volume} = 245 - 165 = 80 \text{ ml}$$

$$\text{Saturated moisture mass} = 80 \text{ g}$$

$$\text{Saturated unit weight} = (453 + 80)/245 \times 9.81 = 21.34 \text{ kN/m}^3$$

$$\text{Or} \quad \left(1 - \frac{1}{G_s}\right) \gamma_d + \gamma_w$$

12. A soil has a bulk density of 1.91 Mg/m³ and a moisture content of 9.5%. The value of G_s is 2.70. Calculate the void ratio and degrees of saturation of the soil. What would be the values of density and moisture content if the soil were fully saturated at the same void ratio?

$$\rho_d = \frac{\rho_b}{1+w} = 1.91/1.095 = 1.744 \text{ Mg/m}^3$$

$$V_s = 1.744 [\text{Mg/m}^3] / 2.70 [\text{# or Mg/m}^3] = 0.646 \text{ m}^3$$

$$V_v = 1 - 0.646 = 0.354 \text{ m}^3$$

$$e = 0.354/0.646 = 0.548$$

$$M_w = 1.744 \times 0.095 = 0.166 \text{ Mg}$$

$$V_w = 0.166 / 1.0 = 0.166 \text{ m}^3$$

$$S_r = 0.166/0.354 = 0.47$$

$$\text{Saturated density} = (1.744 + 0.354)/1 = 2.098 \text{ Mg/m}^3$$

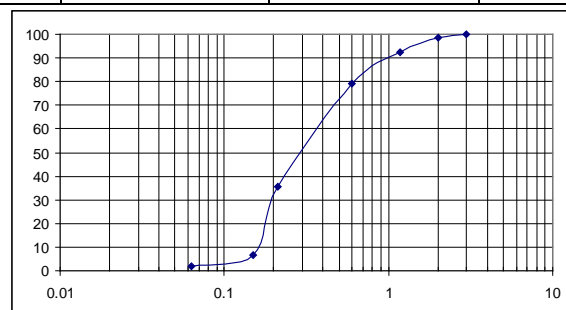
$$\text{Moisture content} = 0.354/1.744 = 20.3\%$$

13. A particle size distribution analysis on a 241 g sample of soil returned the following results.

sieve size (μm)	mass retained (g)
3350	0
2000	3
1180	15
600	32
212	105
150	70
63	11
<63	5

Plot the particle size distribution, calculate the coefficient of uniformity and describe the grading of the soil.

sieve size (mm)	mass retained (g)	%age retained	%age passing
3	0	0	100.0
2	3	1.2	98.8
1.18	15	6.2	92.6
0.6	32	13.3	79.3
.212	105	43.6	35.7
.150	70	29.0	6.7
.063	11	4.6	2.1
<63	5	2.1	0.0



Gravel = 1.2%, Sand (0.063 mm – 2 mm) = 96.7%, Fines = 2.1%

Slightly gravelly SAND,

$$\text{Uniformity coefficient, } C_u = \frac{d_{60}}{d_{10}} = \frac{0.36}{0.18} = 2.0 \quad (\text{soil is uniformly graded}).$$

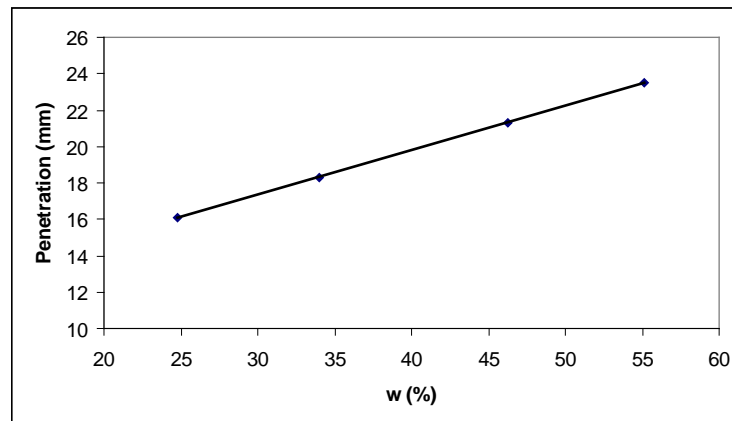
Uniformly graded slightly gravelly SAND

14. The following results were obtained during a liquid limit test on a soil. Determine the liquid limit.

Mass of wet soil (g)	Mass of dry soil (g)	Cone penetration (mm)
31.2	25.0	16.1
37.8	28.2	18.3
36.0	24.6	21.3
40.8	26.3	23.5

Solution:

Cone penetration(mm)	Water content(%)
16.1	24.8
18.3	34.0
21.3	46.3
23.5	55.1



Liquid limit is 41%

15. From the following falling cone test results:

Mass of tin [g]	18.2	19.1	17.7	18.6
Mass of tin + wet sample [g]	51.5	45.5	50.7	43.4
Mass of tin + dry sample [g]	37.8	35.6	39.7	36.3
Cone penetration [mm]	25.0	14.2	8.5	5.1

Determine moisture content of each sample. Plot graph of w against penetration and estimate liquid limit. If soil has a plastic limit of 22%, calculate plasticity index and classify the soil using the A-line chart.

Solution

At 20 mm penetration, moisture content is 63%; PI = 41%, hence from A-line, h=CH (high plasticity clay)

16. The following results were obtained for a fine-grained soil:

LL = 48%, PL = 26%, natural moisture content = 29%

Clay = 25%, Silt = 36%, Sand = 39%

Classify the soil.

The soil is predominantly fine-grained, with 61% passing the 63 micron sieve.

With LL = 48% and PI = 48-26 = 22%, it is CI, i.e. sandy CLAY of intermediate plasticity.

The clay has an activity = $22/25 = 0.88$, i.e. predominantly normal activity (illitic).
NMC is close to PL therefore clay is stiff.

17. The Atterberg limits of a soil are LL = 70% and PL = 35% and it contains 80% by weight of clay. The water content of the sample is 45%. Classify the soil.

$$PI = 70 - 35 = 35$$

$$LI = (45-35)/35 = 0.29$$

$$A = PI / \% \text{clay} = 35/80 = 0.44$$

From the Atterberg classification chart we see that this clay is a high to very high plasticity silt/clay. From its activity we see that it is an inactive clay, typically kaolinite.

18. Craig's Example 1.1

The results of particle size analyses of four soils A, B, C and D are shown in Table 1.4.
The results of limit tests on soil D are:

Liquid limit:

Cone penetration (mm)	15.5	18.0	19.4	22.2	24.9
Water content (%)	39.3	40.8	42.1	44.6	45.6

Plastic limit:

Water content (%)	23.9	24.3
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The fine fraction of soil C has a liquid limit of 26 and a plasticity index of 9. (a) Determine the coefficient of uniformity for soils A, B and C. (b) Classify with main and qualifying terms each soil.

Table 1.4

BS sieve	Particle size*	Percentage smaller			
		Soil A	Soil B	Soil C	Soil D
63mm		100		100	
20mm		64		76	
6.3mm		39	100	65	
2mm		24	98	59	
600µm		12	90	54	
212µm		5	9	47	100
63µm		0	3	34	95
	0.020mm			23	69
	0.006mm			14	46
	0.002mm			7	31

* From sedimentation test.

Solution:

The particle size distribution curves are plotted in [Figure 1.8](#). For soils A, B and C the sizes D_{10} , D_{30} and D_{60} are read from the curves and the values of C_u and C_z are calculated:

Soil	D_{10}	D_{30}	D_{60}	C_u	C_z
A	0.47	3.5	16	34	1.6
B	0.23	0.30	0.41	1.8	0.95
C	0.003	0.042	2.4	800	0.25

For soil D the liquid limit is obtained from [Figure 1.9](#), in which cone penetration is plotted against water content. The percentage water content, to the nearest integer, corresponding to a penetration of 20mm is the liquid limit and is 42. The plastic limit is the average of the two percentage water contents, again to the nearest integer, i.e. 24. The plasticity index is the difference between the liquid and plastic limits, i.e. 18.

Soil A consists of 100% coarse material (76% gravel size; 24% sand size) and is classified as GW: well-graded, very sandy GRAVEL.

Soil B consists of 97% coarse material (95% sand size; 2% gravel size) and 3% fines. It is classified as SPu: uniform, slightly silty, medium SAND.

Soil C comprises 66% coarse material (41% gravel size; 25% sand size) and 34% fines ($w_L = 26$, $I_P = 9$, plotting in the CL zone on the plasticity chart). The classification is GCL: very clayey GRAVEL (clay of low plasticity). This is a till, a glacial deposit having a large range of particle sizes.

Soil D contains 95% fine material: the liquid limit is 42 and the plasticity index is 18, plotting just above the A-line in the CI zone on the plasticity chart. The classification is thus CI: CLAY of intermediate plasticity.

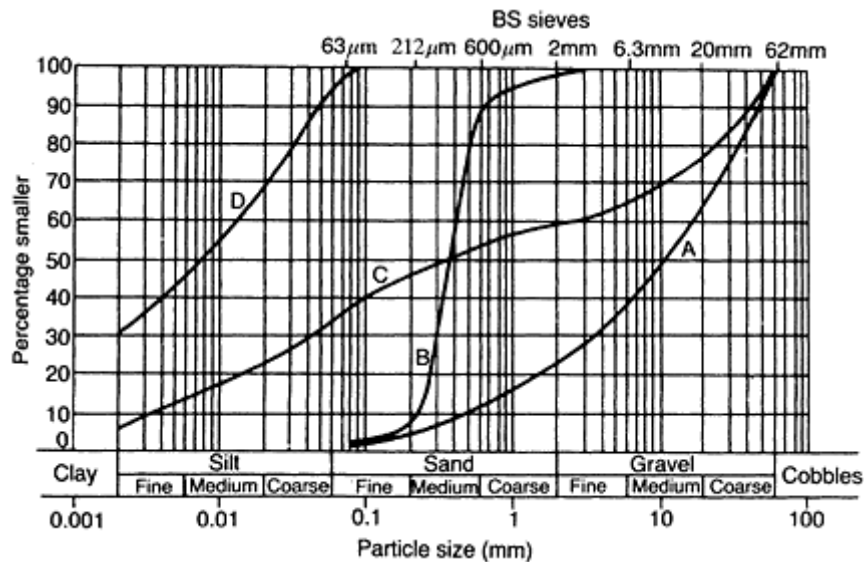


Figure 1.8 Particle size distribution curves

19. Craig's Example 1.2

In its natural condition a soil sample has a mass of 2290g and a volume of $1.15 \times 10^{-3} \text{ m}^3$. After being completely dried in an oven the mass of the sample is 2035g. The value of G_s for the soil is 2.68. Determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

$$\text{Bulk density, } \rho = \frac{M}{V} = \frac{2.290}{1.15 \times 10^{-3}} = 1990 \text{ kg/m}^3 \quad (1.99 \text{ Mg/m}^3)$$

$$\begin{aligned} \text{Unit weight, } \gamma &= \frac{Mg}{V} = 1990 \times 9.8 = 19\,500 \text{ N/m}^3 \\ &= 19.5 \text{ kN/m}^3 \end{aligned}$$

$$\text{Water content, } w = \frac{M_w}{M_s} = \frac{2290 - 2035}{2035} = 0.125 \text{ or } 12.5\%$$

From Equation 1.17

$$\begin{aligned}\text{Void ratio, } e &= G_s(1 + w) \frac{\rho_w}{\rho} - 1 \\ &= \left(2.68 \times 1.125 \times \frac{1000}{1990} \right) - 1 \\ &= 1.52 - 1 \\ &= 0.52\end{aligned}$$

$$\text{Porosity, } n = \frac{e}{1 + e} = \frac{0.52}{1.52} = 0.34 \text{ or } 34\%$$

$$\text{Degree of saturation, } S_r = \frac{wG_s}{e} = \frac{0.125 \times 2.68}{0.52} = 0.645 \text{ or } 64.5\%$$

$$\begin{aligned}\text{Air content, } A &= n(1 - S_r) = 0.34 \times 0.355 \\ &= 0.121 \text{ or } 12.1\%\end{aligned}$$

20. Craig's 1.1 The results of particle size analyses and, where appropriate, limit tests on samples of four soils are given in Table 1.5. Classify and give main and qualifying terms appropriate for each soil.

Table 1.5

BS sieve	Particle size	Percentage smaller			
		Soil E	Soil F	Soil G	Soil H
63mm					
20mm		100			
6.3mm		94	100		
2mm		69	98		
600µm		32	88	100	
212µm		13	67	95	100
63µm		2	37	73	99
	0.020mm		22	46	88
	0.006mm		11	25	71
	0.002mm		4	13	58
Liquid limit		Non-plastic		32	78
Plastic limit				24	31

Solution:

1.1 SW, MS, ML, CV

1. The natural water content of a soil is 10%. Assuming 2100 g of wet soil is used for laboratory compaction tests, calculate how much water is to be added to other 2100 g samples to bring their water contents up to 13, 17, 20 and 24%.

Solution:

First calculate the mass of soil in the initial sample:

$$M_t = M_w + M_s = 2100 \text{ g}$$

$$\text{Also } w = M_w/M_s = 0.10, \text{ then } M_s = 1909.1 \text{ g.}$$

The weight of water to be added is calculated from:

$$M_w = [M_s(w - w_0)]/100$$

	Test 1	Test 2	Test 3	Test 4	Test 5
w (%)	10	13	17	20	24
Ww (g)	0	57.27	133.64	190.91	267.27

2. Problem 1.7. from Craig (8th Edition). The following results were obtained from a standard compaction test on a soil.

Mass (g)	2010	2092	2114	2100	2055
Water content (%)	12.8	14.5	15.6	16.8	19.2

The value of G_s is 2.67. Plot the dry density–water content curve, and give the optimum water content and maximum dry density. The volume of the mould is 1000 cm³.

Solution:

In each case the bulk density (ρ) is equal to the mass of compacted soil divided by the volume of the mould. The corresponding value of dry density (ρ_d) is obtained from Equation 1.24.

Equation 1.26, with A equal, in turn, to 0, 0.05 and 0.10, is used to calculate values of dry density ($\rho_{d,0}$, $\rho_{d,5}$ and $\rho_{d,10}$ respectively) for use in plotting the air content curves. The experimental values of w have been used in these calculations; however, any series of w values within the relevant range could be used.

Mass (g)	ρ (Mg/m ³)	w	ρ_d (Mg/m ³)	$\rho_{d,0}$ (saturation line) (Mg/m ³)	$\rho_{d,5}$ (Mg/m ³)	$\rho_{d,10}$ (Mg/m ³)
2010	2.010	0.128	1.782	1.990	1.890	1.791
2092	2.092	0.145	1.827	1.925	1.829	1.733
2114	2.114	0.156	1.829	1.884	1.790	1.696
2100	2.100	0.168	1.798	1.843	1.751	1.658
2055	2.055	0.192	1.724	1.765	1.676	1.588

The dry density–water content curve is plotted (see figure below), from which the optimum water content (w_{opt}) and maximum dry density ($\rho_{d,max}$) can be read.

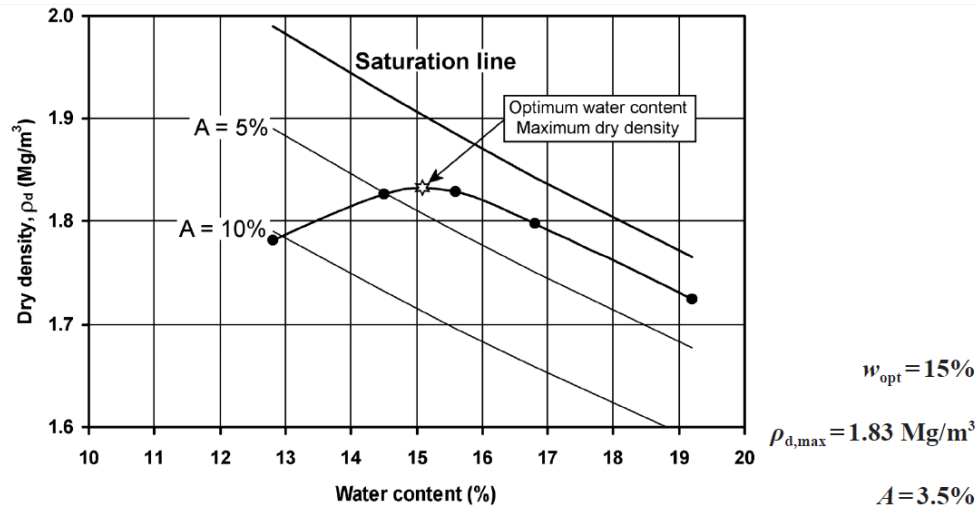


Figure Q1.7

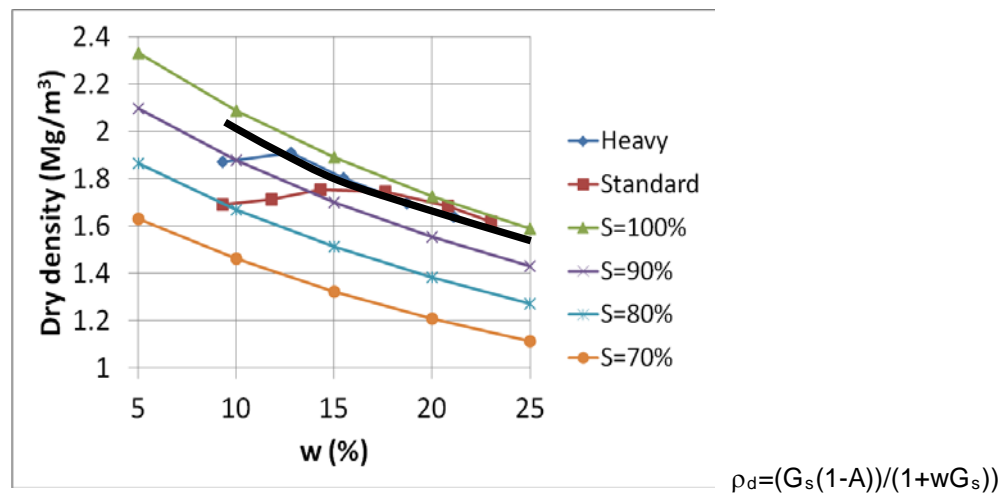
By inspection, the value of air content at maximum dry density is 3.5%.

3. For the data given below ($G_s = 2.64$):

- Plot the compaction curves.
- Establish the maximum dry density and optimum water content for each test.
- Plot the zero air voids line. Also plot the 70, 80 and 90% saturation curves.
- Plot the line of optimums.

Heavy compaction		Standard compaction	
Dry density (Mg/m ³)	w (%)	Dry density (Mg/m ³)	w (%)
1.873	9.3	1.691	9.3
1.91	12.8	1.715	11.8
1.803	15.5	1.755	14.3
1.699	18.7	1.747	17.6
1.641	21.1	1.685	20.8
		1.619	23

Solution:



1. The following results were obtained in a constant head permeameter test:

Duration = 4.0 mins

Quantity of water collected = 300ml

Head difference in manometer = 50mm

Distance between manometer tapings = 100mm

Diameter of test sample = 100mm

Determine k in m/sec.

Solution:

$$A = \frac{\pi D^2}{4} = \frac{\pi \times 100^2}{4} = 7854 \text{ mm}^2$$

$$k = \frac{Ql}{tAh} = \frac{300 \times 100}{(4 \times 60) \times 7854 \times 50} = 3.2 \times 10^{-4} \text{ m/s}$$

2. The following results were obtained in a falling head permeameter test:

Initial head of water in stand-pipe = 1.6 m Final head of water = 0.99 m

Duration of test = 480secs

Sample length = 100mm

Sample diameter = 100mm

Area of stand-pipe = $7.0 \times 10^{-5} \text{ m}^2$

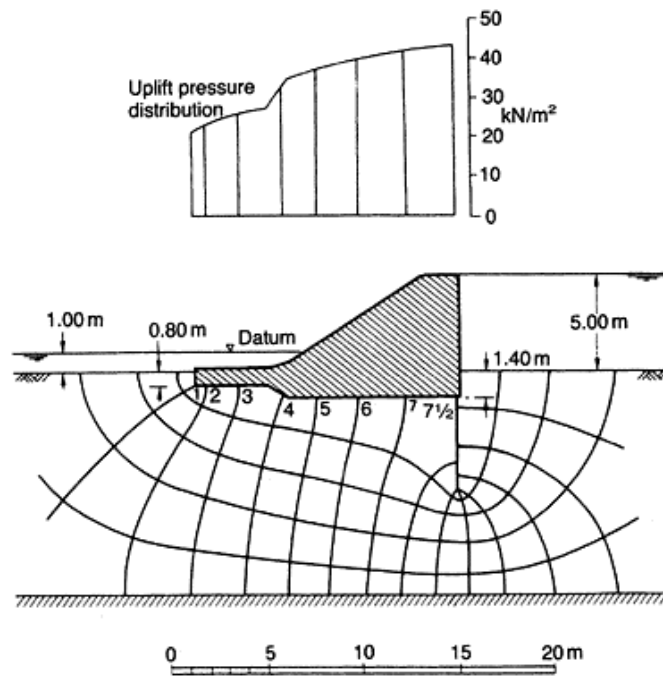
Determine k in m/sec.

Solution:

$$k = \frac{7.0 \times 10^{-5}}{0.00785} \frac{0.100}{480} \ln \frac{1.6}{0.99} = 8.9 \times 10^{-7} \approx 1 \times 10^{-6} \text{ m/s}$$

3. Craig's Example 2.2

The section through a dam is shown in Figure 2.9. Determine the quantity of seepage under the dam and plot the distribution of uplift pressure on the base of the dam. The coefficient of permeability of the foundation soil is 2.5×10^{-5} m/s



Solution:

The flow net is shown in the figure. The downstream water level is selected as datum. Between the upstream and downstream equipotentials the total head loss is 4.00m. In the flow net there are 4.7 flow channels and 15 equipotential drops (Note: As discussed with many of you, you can also use 14 equipotential drops and the result will not be significantly affected). The seepage is given by

$$q = kh \frac{N_f}{N_d} = 2.5 \times 10^{-5} \times 4.00 \times \frac{4.7}{15}$$

$$= 3.1 \times 10^{-5} \text{ m}^3/\text{s} \quad (\text{per m})$$

The pore water pressure is calculated at the points of intersection of the equipotentials with the base of the dam. The total head at each point is obtained from the flow net and the elevation head from the section. The calculations are shown in Table 2.3 and the pressure diagram is plotted in Figure 2.9.

Table 2.3

Point	h (m)	z (m)	$h-z$ (m)	$u = \gamma_w(h-z)$ (kN/m ²)
1	0.27	-1.80	2.07	20.3
2	0.53	-1.80	2.33	22.9
3	0.80	-1.80	2.60	25.5
4	1.07	-2.10	3.17	31.1
5	1.33	-2.40	3.73	36.6
6	1.60	-2.40	4.00	39.2

7	1.87	-2.40	4.27	41.9
7½	2.00	-2.40	4.40	43.1

4. Craig's Example 2.3

A river bed consists of a layer of sand 8.25m thick overlying impermeable rock; the depth of water is 2.50m. A long cofferdam 5.50m wide is formed by driving two lines of sheet piling to a depth of 6.00m below the level of the river bed and excavation to a depth of 2.00m below bed level is carried out within the cofferdam. The water level within the cofferdam is kept at excavation level by pumping. If the flow of water into the cofferdam is 0.25m³/h per unit length, what is the coefficient of permeability of the sand? What is the hydraulic gradient immediately below the excavated surface?

Solution:

The section and flow net appear in Figure 2.10. In the flow net there are 6.0 flow channels and 10 equipotential drops. The total head loss is 4.50m. The coefficient of permeability is given by

$$k = \frac{q}{h(N_f/N_d)}$$

$$= \frac{0.25}{4.50 \times 6/10 \times 60^2} = 2.6 \times 10^{-5} \text{ m/s}$$

The distance (Δs) between the last two equipotentials is measured as 0.9m. The required hydraulic gradient is given by

$$i = \frac{\Delta h}{\Delta s}$$

$$= \frac{4.50}{10 \times 0.9} = 0.50$$

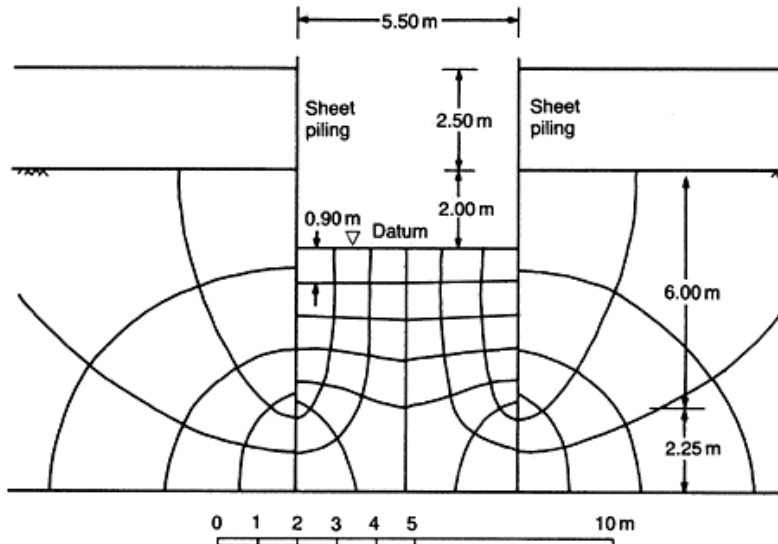


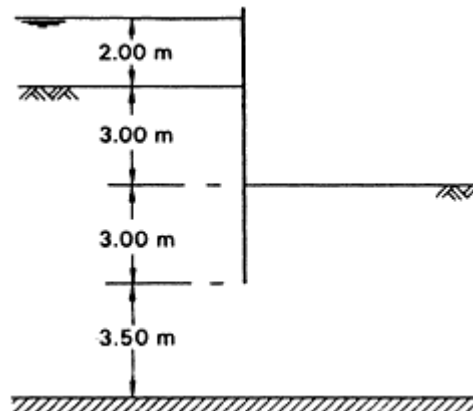
Figure 2

5. Craig's 2.1 In a falling-head permeability test the initial head of 1.00m dropped to 0.35m in 3h, the diameter of the standpipe being 5mm. The soil specimen was 200mm long by 100mm in diameter. Calculate the coefficient of permeability of the soil.

Solution:

$$4.9 \times 10^{-8} \text{ m/s}$$

6 Craig's 2.5 The section through part of a cofferdam is shown in Figure 2.25, the coefficient of permeability of the soil being 2.0×10^{-6} m/s. Draw the flow net and determine the quantity of seepage.



Solution:

4.7×10^{-6} m³/s (per m)

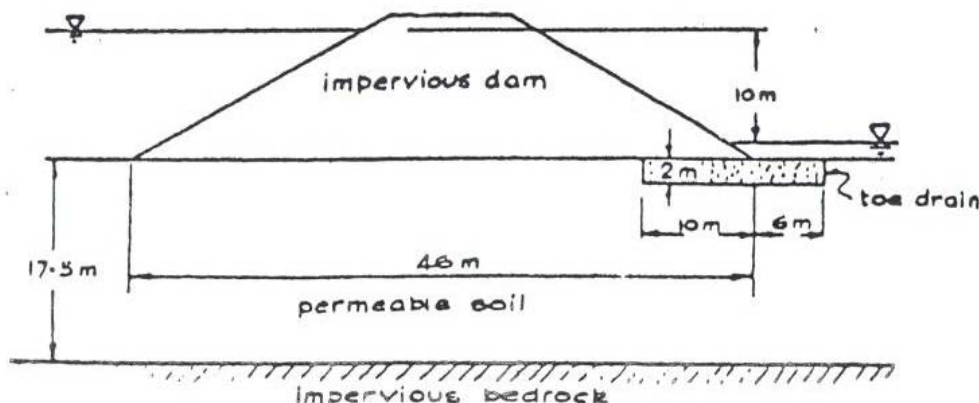
1. The flood-plain deposits of a river consist (from the surface downwards) of the following succession of soil layers: 1 m of coarse silt ($k = 3 \times 10^{-7}$ m/s), lying on 3 m of fine sand ($k = 3 \times 10^{-7}$ m/s), lying on 4 m of fine silty sand ($k = 1 \times 10^{-6}$ m/s). All these layers are underlain by a deep deposit of gravel. Upward flow is taking place as a consequence of artesian conditions in the gravel. Remembering that the flow velocity in all three layers must be the same, apply Darcy's Law to each layer and show that the hydraulic gradient will be the greatest in the upper layer. Then:

a) Determine the piezometric elevation in the underlying gravel which would just produce quick conditions in the layer above. Assume that the ground water level is at the surface of the flood plain, and that the unit weight of all layers are the same at $\gamma = 19.62$ kN/m³. Note that it is now possible to determine the water pressure variation that must have existed in the three uppermost layers, just prior to the establishment of quick conditions. What are these pressures?

b) What are the equivalent vertical and horizontal permeabilities of the three uppermost layers? Assume that each layer is isotropic and homogeneous within itself.

Answers: (a) 2.5 m; at surface ($d=0$), $u = 0$; at $d = 1$ m, $u = 19.6$ kN/m²; at $d = 4$, $u = 52.0$ kN/m²; at $d = 8$ m, $u = 103$ kN/m²; b) 9.6×10^{-7} m/s and 1.66×10^{-6} m/s

2. The impermeable dam shown in the figure below is 100 m long, and is founded on a permeable soil, which in turn overlies impermeable bedrock. If the soil has a horizontal permeability of 2.8×10^{-6} m/s and a vertical permeability of 7×10^{-7} m/s, what will be the total flow under the dam in one day? Assume that the same cross-section applies under the whole length of the dam. *Note: Allow adequate space both upstream and downstream of the dam when drawing your flow net.*



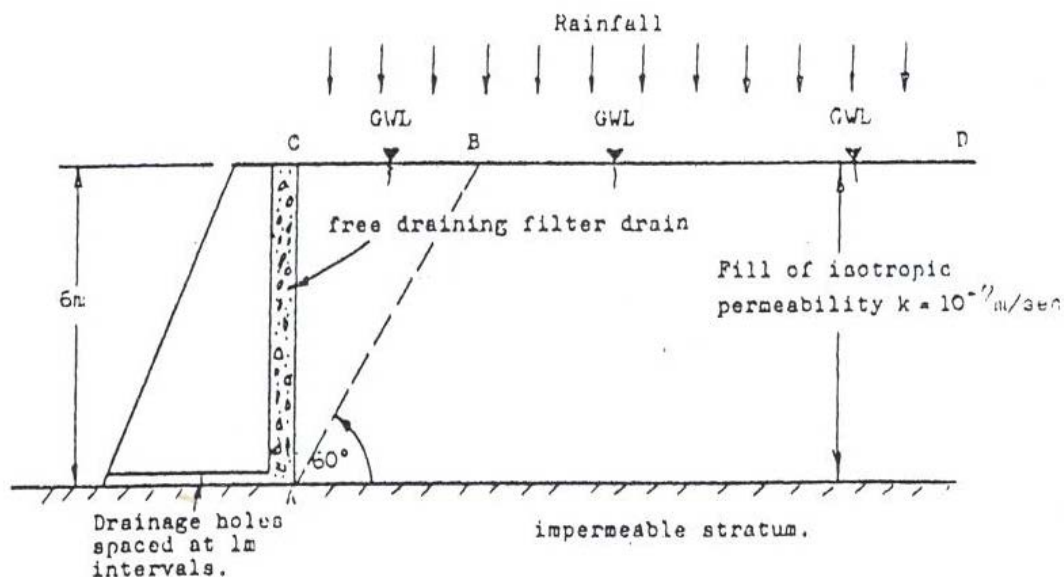
Answer ≈ 60 m³/day

3. The figure below shows a retaining wall 6 m high resting on an impermeable horizontal boundary. The back of the wall is vertical, and is provided with a vertical filter drain to intercept the flow of water from the fill. The vertical filter drain discharges through drainage holes spaced at 1 m intervals in the base of the wall.

a) Draw a flow net for the fill, assuming (i) that sufficient rain falls to maintain the ground-water level at the upper level (CBD) of the fill, and (ii) that any water in the vertical filter drain is at atmospheric pressure.

b) Determine from the flow the water pressure distribution along a surface AB that is inclined at 60° to the horizontal and passes through the toe of the fill.

c) Calculate the discharge from each drainage hole in m^3/day , assuming the permeability of the fill to be $k = 10^{-7} \text{ m/s}$



Answer $\approx 5 \times 10^{-2} \text{ m}^3/\text{day}$

Effective stress examples

1.a. Dry sand (1.5 kg) is poured into a 55 mm diameter measuring cylinder and occupies a volume of 0.95 litre. The particle density of the sand is 2.68 Mg/m³. Calculate the density, total vertical stress and effective vertical stress at the base of the cylinder.

$$\gamma = 1.5 \text{ kg} \times 9.81 / 0.95 \times 10^{-3} \text{ m}^3 = 15.5 \text{ kN/m}^3$$

$$\begin{aligned}\sigma_v &= \gamma \cdot V/A = 15.5 \times 0.95 \times 10^{-3} \text{ m}^3 / \pi 0.055^2 / 4 \\ &= 15.5 \times 0.4 = 6.2 \text{ kPa}\end{aligned}$$

$$u = 0$$

$$\sigma'_v = 6.2 - 0 = 6.2 \text{ kPa}$$

1.b Water is added to a height of 0.4 m to saturate the sand. Recalculate density and stresses.

$$V_w = V_t - V_s = 0.95 - 1.5 / 2.68 = 0.95 - 0.56 = 0.39 \text{ litre}$$

$$M_w = 0.39 \text{ kg}$$

$$\gamma = 1.89 \text{ kg} \times 9.81 / 0.95 \times 10^{-3} \text{ m}^3 = 19.5 \text{ kN/m}^3$$

$$\sigma_v = \gamma \cdot z = 19.5 \times 0.4 = 7.8 \text{ kPa}$$

$$u = 0.4 \times 9.81 = 3.9 \text{ kPa}$$

$$\sigma'_v = 7.8 - 3.9 = 3.9 \text{ kPa}$$

1.c. The cylinder is now tapped several times and the sand settles to occupy a volume of 0.87 litre. Recalculate density and stresses

$$\text{Height of saturated soil} = 0.4 \text{ m} - 0.87 \times 10^{-3} \text{ m}^3 / \pi 0.055^2 / 4 = 0.37 \text{ m}$$

$$V_w = V_t - V_s = 0.87 - 0.56 = 0.31 \text{ litre}$$

$$M_w = 0.31 \text{ kg}$$

$$\gamma = 1.81 \text{ kg} \times 9.81 \times 10^{-3} / 0.87 \times 10^{-3} \text{ m}^3 = 20.4 \text{ kN/m}^3$$

$$\sigma_v = \gamma \cdot z + \gamma_w \cdot z_w = 20.4 \times 0.4 + 9.81 \times 0.03 = 7.8 \text{ kPa}$$

$$u = 0.4 \times 9.81 = 3.9 \text{ kPa}$$

$$\sigma'_v = 7.8 - 3.9 = 3.9 \text{ kPa}$$

Although density increases, stresses are unchanged – because mass of material hasn't changed, simply redistributed about the cylinder.

2. A deep clay deposit with unit weight 20 kN/m^3 is subjected to a fluctuating ground water level. Calculate the effective stress at 3 m depth when groundwater is (a) 6 m below ground level and (b) 3 m above ground level.

a) $\sigma_v = \gamma \cdot z = 20 \times 3 = 60 \text{ kPa}$

$u = -3.0 \times 9.81 = -29.4 \text{ kPa}$

$\sigma'_v = 60 - (-29.4) = 89.4 \text{ kPa}$

b) $\sigma_v = \gamma \cdot z + \gamma_w \cdot z_w = 20 \times 3 + 3 \times 9.81 = 89.4 \text{ kPa}$

$u = 6.0 \times 9.81 = 58.9 \text{ kPa}$

$\sigma'_v = 60 - 58.9 = 31.1 \text{ kPa}$

3. A concrete bridge pier, 4 m tall with base area 10 m^2 , carries a load of 1 MN. Unit weight of concrete is 24 kN/m^3 . The pier is founded on the bed of a tidal river on 5 m of sand (unit weight = 20 kN/m^3). Low tide level is the river bed whilst at high tide there is 3 m depth of water. Calculate effective stress at 2 m depth below the base of the foundation at high and low tides.

$$\text{Foundation pressure} = 4 \text{ m} \times 24 \text{ kN/m}^3 + 1000 \text{ kN} / 10 \text{ m}^2 = 196 \text{ kPa}$$

$$\text{At low tide } \sigma_v = \gamma \cdot z + q = 20 \times 2 + 196 = 236 \text{ kPa}$$

$$u = 2 \times 9.81 = 19.6 \text{ kPa}$$

$$\sigma'_v = 236 - 19.6 = 216.4 \text{ kPa}$$

At high tide, there is uplift on base of foundation

$$\sigma_v = \gamma \cdot z + \gamma_w \cdot z_w + q - \text{uplift} = 20 \times 2 + 3 \times 9.81 + 196 - 3 \times 9.81 = 236 \text{ kPa}$$

$$u = 5 \times 9.81 = 49.1 \text{ kPa}$$

$$\sigma'_v = 236 - 49.1 = 187 \text{ kPa}$$

Effective stress is less due to uplift on foundation, NOT due to any soil related effects. Compare with example 2 above.

4. A ground profile comprises 4 m clay over 2 m sand over rock. Unit weights of all materials are 20 kN/m^3 and steady state ground water level is ground level. A wide embankment 4 m high is to be built from fill, unit weight = 15 kN/m^3 . Calculate total vertical stress and effective vertical stress (i) at the centre of the clay layer and (ii) at the centre of the sand layer: (a) before the embankment is built, (b) immediately after construction, (c) after a very long time.

$$\text{a-i) } \sigma_v = \gamma \cdot z = 20 \times 2 = 40 \text{ kPa}$$

$$u = 2 \times 9.81 = 19.6 \text{ kPa}$$

$$\sigma'_v = 40 - 19.6 = 20.4 \text{ kPa}$$

$$\text{ii) } \sigma_v = \gamma \cdot z = 20 \times 5 = 100 \text{ kPa}$$

$$u = 5 \times 9.81 = 49.1 \text{ kPa}$$

$$\sigma'_v = 100 - 49.1 = 50.9 \text{ kPa}$$

b-i) In short term, undrained condition, pore water pressures in clay are unchanged. In sands, pore pressure equilibrate instantaneously.

$$\sigma_v = \gamma \cdot z = 20 \times 2 + 4 \times 15 = 100 \text{ kPa}$$

$$u = \sigma_v - \sigma'_v = 100 - 20.4 = 79.6 \text{ kPa}$$

$$\sigma'_v = 20.4 \text{ kPa}$$

$$\text{ii) } \sigma_v = \gamma \cdot z = 20 \times 5 + 60 = 160 \text{ kPa}$$

$$u = 5 \times 9.81 = 49.1 \text{ kPa}$$

$$\sigma'_v = 160 - 49.1 = 110.9 \text{ kPa}$$

c-i) In long term, drained condition, excess pore water pressures in clay dissipate.

In sands, pore pressure are unchanged – already in equilibrium.

$$\sigma_v = \gamma \cdot z = 20 \times 2 + 4 \times 15 = 100 \text{ kPa}$$

ii) no change

$$u = 2 \times 9.81 = 19.6 \text{ kPa}$$

$$\sigma'_v = 80.4 \text{ kPa}$$

5. Craig's Example 3.1

A layer of saturated clay 4m thick is overlain by sand 5m deep, the water table being 3m below the surface. The saturated unit weights of the clay and sand are 19 and 20kN/m³, respectively; above the water table the unit weight of the sand is 17kN/m³. Plot the values of total vertical stress and effective vertical stress against depth. If sand to a height of 1m above the water table is saturated with capillary water, how are the above stresses affected?

The total vertical stress is the weight of all material (solids+water) per unit area above the depth in question. Pore water pressure is the hydrostatic pressure corresponding to the depth below the water table. The effective vertical stress is the difference between the total vertical stress and the pore water pressure at the same depth. Alternatively, effective vertical stress may be calculated directly using the buoyant unit weight of the soil below the water table. The stresses need to be calculated only at depths where there is a change in unit weight ([Table 3.1](#)).

Table 3.1

Depth (m)	σ_v (kN/m ²)	u (kN/m ²)	$\sigma'_v = \sigma_v - u$ (kN/m ²)
3	$3 \times 17 = 51.0$	0	51.0
5	$(3 \times 17) + (2 \times 20) = 91.0$	$2 \times 9.8 = 19.6$	71.4
9	$(3 \times 17) + (2 \times 20) + (4 \times 19) = 167.0$	$6 \times 9.8 = 58.8$	108.2

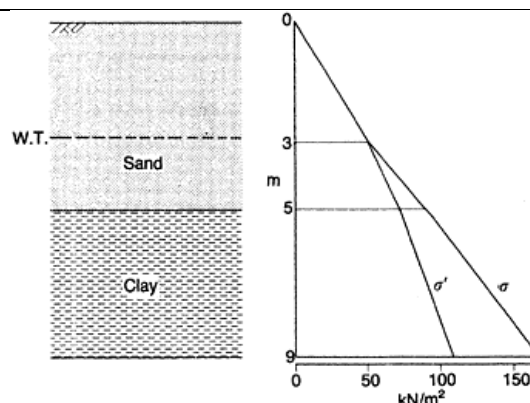


Figure 3.3

Effect of capillary rise

The water table is the level at which pore water pressure is atmospheric (i.e. $u = 0$). Above the water table, water is held under negative pressure and, even if the soil is saturated above the

water table, does not contribute to hydrostatic pressure below the water table. The only effect of the 1m capillary rise, therefore, is to increase the total unit weight of the sand between 2 and 3m depth from 17 to 20kN/m³, an increase of 3kN/m³. Both total and effective vertical stresses below 3m depth are therefore increased by the constant amount $3 \times 1 = 3.0 \text{ kN/m}^2$, pore water pressures being unchanged.

6. Craig's Example 3.2

A 5m depth of sand overlies a 6m layer of clay, the water table being at the surface; the permeability of the clay is very low. The saturated unit weight of the sand is 19kN/m³ and that of the clay is 20kN/m³. A 4m depth of fill material of unit weight 20kN/m³ is placed on the surface over an extensive area. Determine the effective vertical stress at the centre of the clay layer (a) immediately after the fill has been placed, assuming this to take place rapidly and (b) many years after the fill has been placed.

The soil profile is shown in [Figure 3.4](#). Since the fill covers an extensive area it can be assumed that the condition of zero lateral strain applies. As the permeability of the clay is very low, dissipation of excess pore water pressure will be very slow; immediately after the rapid placing of the fill, no appreciable dissipation will have taken place. Therefore, the effective vertical stress at the centre of the clay layer immediately after placing will be virtually unchanged from the original value, i.e.

$$\sigma'_v = (5 \times 9.2) + (3 \times 10.2) = 76.6 \text{ kN/m}^2$$

(the buoyant unit weights of the sand and the clay, respectively, being 9.2 and 10.2kN/m³).

Many years after the placing of the fill, dissipation of excess pore water pressure should be essentially complete and the effective vertical stress at the centre of the clay layer will be

$$\sigma'_v = (4 \times 20) + (5 \times 9.2) + (3 \times 10.2) = 156.6 \text{ kN/m}^2$$

Immediately after the fill has been placed, the total vertical stress at the centre of the clay increases by 80kN/m² due to the weight of the fill. Since the clay is saturated and there is no lateral strain there will be a corresponding increase in pore water pressure of 80kN/m² (the initial excess pore water pressure). The static pore water pressure is $(8 \times 9.8) = 78.4 \text{ kN/m}^2$. Immediately after placing, the pore water pressure increases from 78.4 to 158.4kN/m² and then during subsequent consolidation gradually decreases again to 78.4kN/m², accompanied by the gradual increase of effective vertical stress from 76.6 to 156.6kN/m².

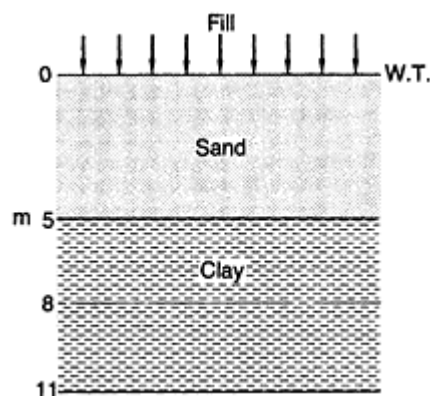


Figure 3.4 [Example 3.2](#).

7. The strata in the bottom of a flat valley consist of 3 m of coarse gravel overlying 12 m of clay. Beneath the clay is fissured sandstone of relatively high permeability.

The water table in the gravel is 0.6 m below ground water level. The water in the sandstone is under artesian pressure corresponding to a standpipe level of 6 m above ground level.

The unit weights are:

Gravel (above the water table):	16.8 kN/m ³
Gravel (below the water table - saturated):	20.8 kN/m ³
Clay (saturated):	21.6 kN/m ³
Water:	9.8 kN/m ³

Plot total and effective vertical stresses against depth for the following cases:

- The initial ground water levels
- Starting at case a), the water level in the gravel is lowered 1.8m by pumping.
- Starting at case a), the water level in the gravel is unchanged, but installation of relief wells lower the water pressure in the sandstone by 5.5 m.

Solution:

- a) For depth 0.6 m:

$$\begin{aligned}\sigma &= 0.6 \text{ m}(16.8 \text{ kN/m}^3) = 10.08 \text{ kPa} \\ u &= 0 \text{ kPa} \\ \sigma' &= 10.08 \text{ kPa} - 0.00 \text{ kPa} = 10.08 \text{ kPa}\end{aligned}$$

For depth 3.0 m:

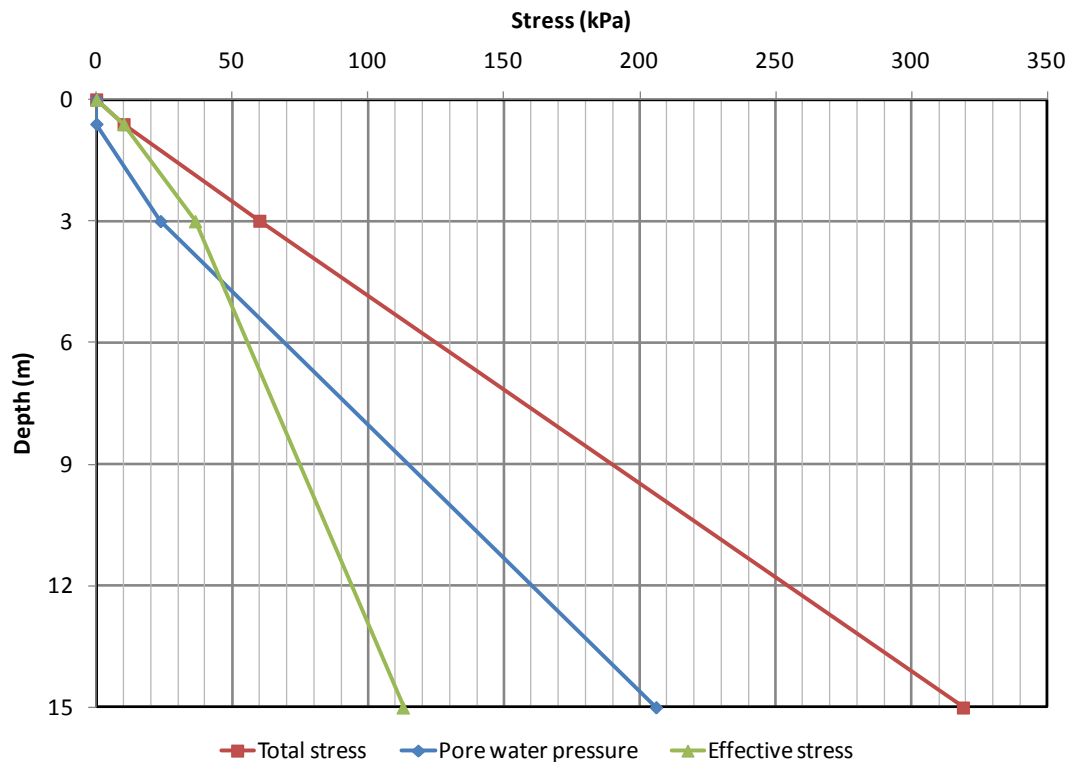
$$\begin{aligned}\sigma &= 10.08 \text{ kPa} + 2.4 \text{ m}(20.8 \text{ kN/m}^3) = 60.00 \text{ kPa} \\ u &= 2.4 \text{ m}(9.81 \text{ kN/m}^3) = 23.54 \text{ kPa} \\ \sigma' &= 60.00 \text{ kPa} - 23.54 \text{ kPa} = 36.46 \text{ kPa}\end{aligned}$$

For depth 15.0 m:

$$\begin{aligned}\sigma &= 60.00 \text{ kPa} + 12 \text{ m}(21.6 \text{ kN/m}^3) = 319.20 \text{ kPa} \\ u &= (15 \text{ m} + 6 \text{ m}^*)(9.81 \text{ kN/m}^3) = 206.01 \text{ kPa} \\ \sigma' &= 319.20 \text{ kPa} - 206.01 \text{ kPa} = 113.19 \text{ kPa}\end{aligned}$$

*Note the increment in pressure head due to the artesian pressure conditions

Example 1a)



b) For depth 2.4 m:

$$\sigma = 2.4 \text{ m}(16.8 \text{ kN/m}^3) = 40.32 \text{ kPa}$$

$$u = 0 \text{ kPa}$$

$$\sigma' = 40.32 \text{ kPa} - 0.00 \text{ kPa} = 40.32 \text{ kPa}$$

For depth 3.0 m:

$$\sigma = 40.32 \text{ kPa} + 0.6 \text{ m}(20.8 \text{ kN/m}^3) = 52.80 \text{ kPa}$$

$$u = 0.6 \text{ m}(9.81 \text{ kN/m}^3) = 5.89 \text{ kPa}$$

$$\sigma' = 52.80 \text{ kPa} - 5.89 \text{ kPa} = 46.91 \text{ kPa}$$

For depth 15.0 m:

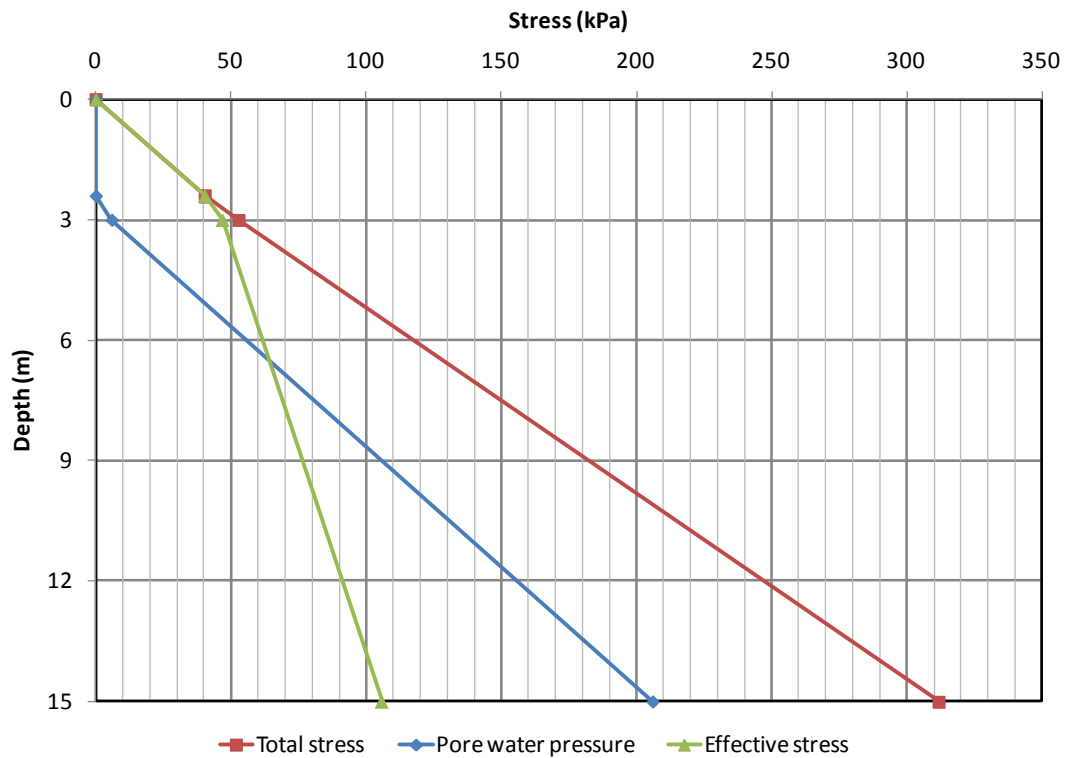
$$\sigma = 52.80 \text{ kPa} + 12 \text{ m}(21.6 \text{ kN/m}^3) = 312.00 \text{ kPa}$$

$$u = (15 \text{ m} + 6 \text{ m}^*)(9.81 \text{ kN/m}^3) = 206.01 \text{ kPa}$$

$$\sigma' = 312.00 \text{ kPa} - 206.01 \text{ kPa} = 105.99 \text{ kPa}$$

*Note the increment in pressure head due to the artesian pressure conditions

Example 1b



c) For depth 0.6 m:

$$\sigma = 0.6 \text{ m}(16.8 \text{ kN/m}^3) = 10.08 \text{ kPa}$$

$$u = 0 \text{ kPa}$$

$$\sigma' = 10.08 \text{ kPa} - 0.00 \text{ kPa} = 10.08 \text{ kPa}$$

For depth 3.0 m:

$$\sigma = 10.08 \text{ kPa} + 2.4 \text{ m}(20.8 \text{ kN/m}^3) = 60.00 \text{ kPa}$$

$$u = 2.4 \text{ m}(9.81 \text{ kN/m}^3) = 23.54 \text{ kPa}$$

$$\sigma' = 60.00 \text{ kPa} - 23.54 \text{ kPa} = 36.46 \text{ kPa}$$

For depth 15.0 m:

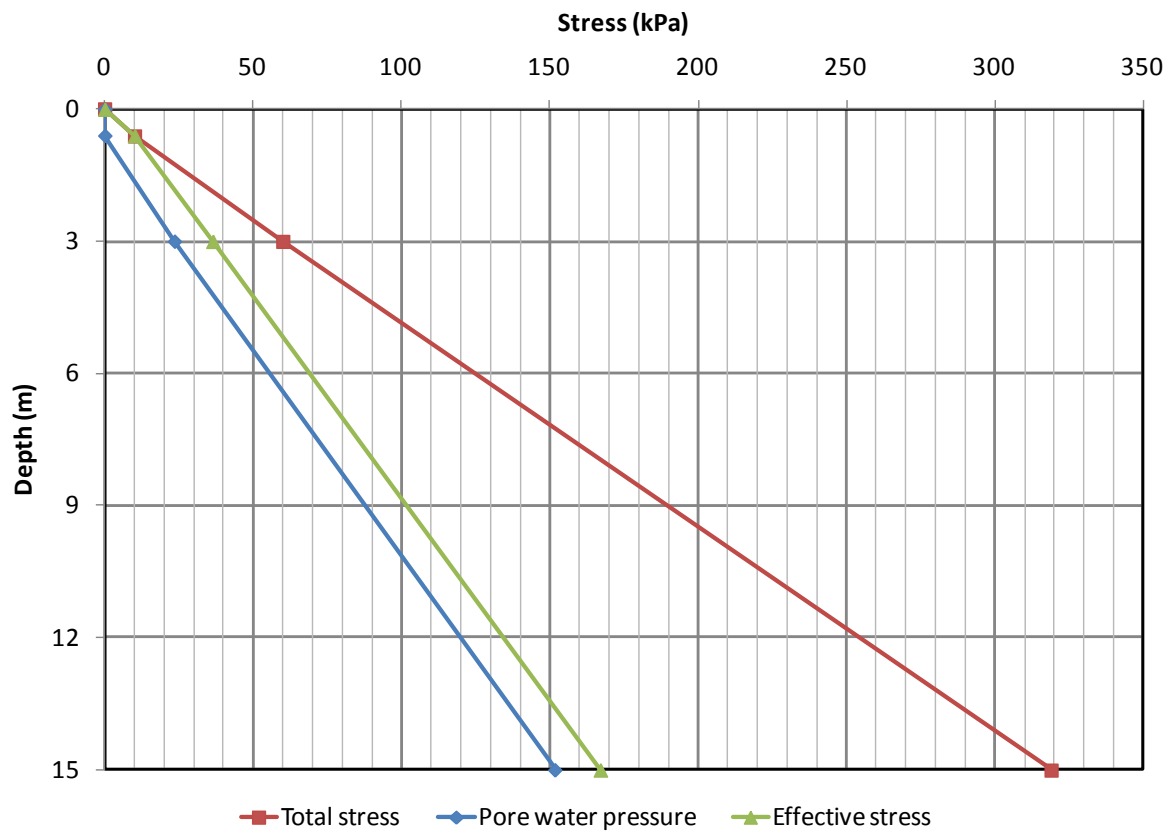
$$\sigma = 60.00 \text{ kPa} + 12 \text{ m}(21.6 \text{ kN/m}^3) = 319.20 \text{ kPa}$$

$$u = (15 \text{ m} + 0.50 \text{ m}^*)(9.81 \text{ kN/m}^3) = 152.06 \text{ kPa}$$

$$\sigma' = 319.20 \text{ kPa} - 152.06 \text{ kPa} = 167.15 \text{ kPa}$$

*Note that the additional pressure head caused by artesian pressure conditions is reduced due to the installation of pressure wells

Example 1c)



COMPRESSION

1. What is the vertical effective stress at the midpoint[1] of an 18 m surface clay layer, with bulk unit weight = 20.2 kN/m³ and ground water level 2 m below the clay surface?

$$\begin{aligned} & 9\text{m} \times 20.2 \text{ kN/m}^3 - 7\text{m} \times 9.81 \text{ kN/m}^3 \\ & = 113.13 \text{ kN/m}^2 \end{aligned}$$

2. What is the void ratio of a saturated soil sample with a moisture content of 28%? (Gs=2.7)

$$\begin{aligned} wG &= Se \\ e &= 0.28 \times 2.7 / 1.0 \\ &= 0.756 \end{aligned}$$

3. A soil sample loaded in an oedometer at 100 kPa has a height of 19.42 mm and a void ratio of 0.604. After loading to 200 kPa, the new sample height is 19.19 mm. Calculate the new void ratio.

$$\begin{aligned} \Delta h/h_0 &= \Delta e/(1+e_0) \\ \Delta e &= 0.23 / 19.42 \times (1.604) = 0.019 \\ e_1 &= 0.604 - 0.019 \\ &= 0.585 \end{aligned}$$

4. Calculate the volume compressibility of the soil above.

$$\begin{aligned} m_v &= (e_0 - e_1) / [(1+e_0) \Delta \sigma'_v] \\ &= 0.019 / [1.604 \times 100] \\ &= 0.118 \times 10^{-3} \text{ m}^2/\text{kN} \end{aligned}$$

5. What will be the settlement of a 7m clay layer with $m_v = 0.240 \times 10^{-3} \text{ m}^2/\text{kN}$ subjected to a uniform increase in vertical stress of 65 kN/m² ?

$$\begin{aligned} S &= m_v \cdot H \cdot \Delta \sigma'_v \\ &= 0.240 \times 10^{-3} \times 7 \times 65 \\ &= 109\text{mm} \text{ [say 110mm]} \end{aligned}$$

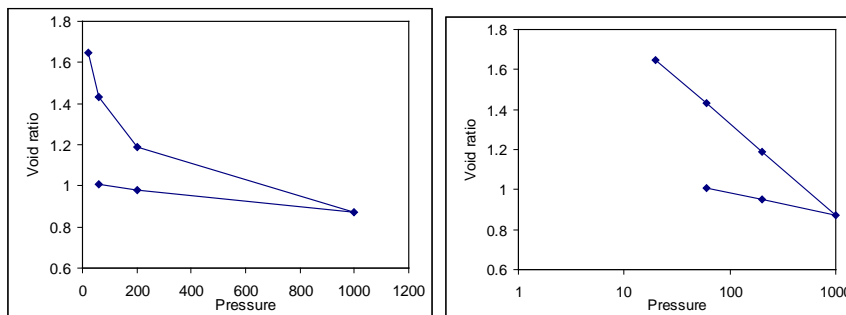
6. An oedometer test has produced the following results:

σ_v	e
20	1.65
60	1.43
200	1.19
1000	0.87
200	0.98
60	1.01

What would be the settlement in a 7 m thick clay deposit with an initial vertical effective stress of 200 kPa and subjected to a load increment of 200 kPa?

Determine the compression indices for the normal compression line and the unload line.

Solution



$$m_v = (1.19 - 1.11) / (2.19 \times 200) = 1.8 \times 10^{-4} \text{ m}^2/\text{kN}$$

$$s = \Delta\sigma_v \cdot H \cdot m_v = 200 \times 7 \times 1.8 \times 10^{-4} = 0.252 \text{ m}$$

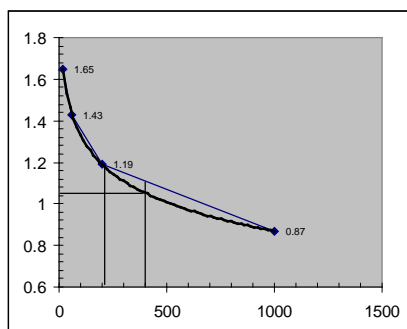
$$C_c = \Delta e / \log(\sigma_v / \sigma_v) = (1.19 - 0.87) / \log(1000/200) = 0.46 \text{ (NCL)}$$

$$= (1.01 - 0.98) / \log(200/60) = 0.057 \text{ (unload)}$$

$$s = C_c \cdot \log(\sigma_v / \sigma_v) \cdot H / (1 + e_0)$$

$$= 0.46 \times \log(400/200) \times 7 / 2.19 = 0.443 \text{ m (!)}$$

The discrepancy is due to the linearization of the graph between the 200 and 1000 kPa loads. A more accurate prediction from the graphical method would be obtained if a logarithmic curve were to replace the linear version. In this case the change in void ratio in the calculation is $1.19 - 1.05$, nearly double the original amount, and hence the discrepancy is resolved.



7. Craig's Example 7.1

The following compression readings were obtained in an oedometer test on a specimen of saturated clay ($G_s = 2.73$):

Pressure (kN/m ²)	0	54	107	214	429	858	1716	3432	0
Dial gauge after 24h (mm)	5.000	4.747	4.493	4.108	3.449	2.608	1.676	0.737	1.480

The initial thickness of the specimen was 19.0mm and at the end of the test the water content was 19.8%. Plot the e - $\log \sigma'$ curve and determine the preconsolidation pressure. Determine the values of m_v for the stress increments 100–200 and 1000–1500kN/m². What is the value of C_c for the latter increment?

$$\text{Void ratio at end of test} = e_1 = w_1 G_s = 0.198 \times 2.73 = 0.541$$

$$\text{Void ratio at start of test} = e_0 = e_1 + \Delta e$$

Now

$$\frac{\Delta e}{\Delta H} = \frac{1 + e_0}{H_0} = \frac{1 + e_1 + \Delta e}{H_0}$$

i.e.

$$\frac{\Delta e}{3.520} = \frac{1.541 + \Delta e}{19.0}$$

$$\Delta e = 0.350$$

$$e_0 = 0.541 + 0.350 = 0.891$$

In general, the relationship between Δe and ΔH is given by

$$\frac{\Delta e}{\Delta H} = \frac{1.891}{19.0}$$

i.e. $\Delta e = 0.0996 \Delta H$, and can be used to obtain the void ratio at the end of each increment period (see [Table 7.1](#)). The e - $\log \sigma'$ curve using these values is shown in [Figure 7.6](#). Using Casagrande's construction, the value of the preconsolidation pressure is 325kN/m².

$$m_v = \frac{1}{1 + e_0} \cdot \frac{e_0 - e_1}{\sigma'_1 - \sigma'_0}$$

Table 7.1

Pressure (kN/m ²)	ΔH (mm)	Δe	e
0	0	0	0.891
54	0.253	0.025	0.866
107	0.507	0.050	0.841
214	0.892	0.089	0.802
429	1.551	0.154	0.737
858	2.392	0.238	0.653
1716	3.324	0.331	0.560
3432	4.263	0.424	0.467

0 3.520 0.350 0.541

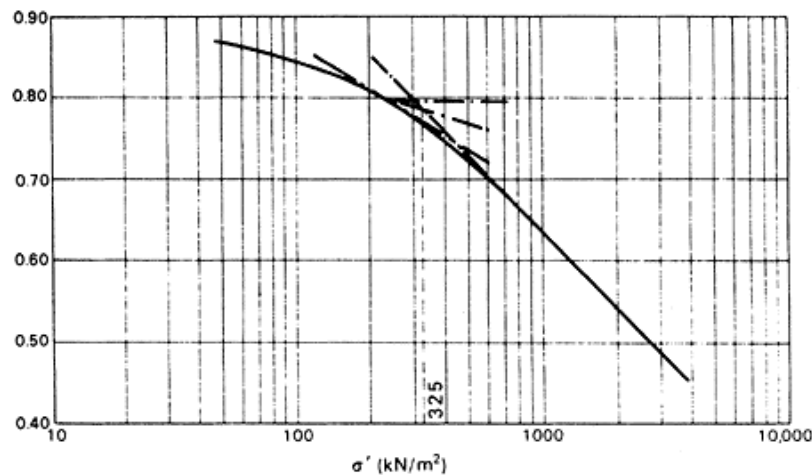


Figure 7.6

For $\sigma'_0 = 100 \text{ kN/m}^2$ and $\sigma'_1 = 200 \text{ kN/m}^2$,

$$e_0 = 0.845 \quad \text{and} \quad e_1 = 0.808$$

and therefore

$$m_v = \frac{1}{1.845} \times \frac{0.037}{100} = 2.0 \times 10^{-4} \text{ m}^2/\text{kN} = 0.20 \text{ m}^2/\text{MN}$$

For $\sigma'_0 = 1000 \text{ kN/m}^2$ and $\sigma'_1 = 1500 \text{ kN/m}^2$,

$$e_0 = 0.632 \quad \text{and} \quad e_1 = 0.577$$

and therefore

$$m_v = \frac{1}{1.632} \times \frac{0.055}{500} = 6.7 \times 10^{-5} \text{ m}^2/\text{kN} = 0.067 \text{ m}^2/\text{MN}$$

and

$$C_c = \frac{0.632 - 0.577}{\log(1500/1000)} = \frac{0.055}{0.176} = 0.31$$

Note that C_c will be the same for any stress range on the linear part of the e - $\log \sigma'$ curve; m_v will vary according to the stress range, even for ranges on the linear part of the curve.

8. Craig's 7.2 The following results were obtained from an oedometer test on a specimen of saturated clay. A layer of this clay 8m thick lies below a 4m depth of sand, the water table being at the surface. The saturated unit weight for both soils is 19 kN/m^3 . A 4m depth of fill of unit weight 21 kN/m^3 is placed on the sand over an extensive area. Determine the final settlement due to consolidation of the clay. If the fill were to be

removed some time after the completion of consolidation, what heave would eventually take place due to swelling of the clay?

Pressure (kN/m ²)	27	54	107	214	429	214	107	54
Void ratio	1.243	1.217	1.144	1.068	0.994	1.001	1.012	1.024

318 mm, 38 mm (four sublayers)

1. The predicted consolidation settlement of a soil layer due to the construction of a slab foundation and other structural loads is 160mm. Precision equipment to be situated on the slab foundation require foundation movement to be no more than 30 mm (maximum allowable settlement). Determine the degree of consolidation at which maximum allowable settlement condition is met.

$$U = (160-30)/160$$

$$= 0.81$$

2. What is the time factor for $U = 0.7$?

$$T_v = 0.405$$

3. A clay layer 6m thick, underlain by permeable sandstone, has a c_v of $0.34 \text{ m}^2/\text{yr}$. Determine the time to reach 50% consolidation.

$$c_v = T_v \cdot d^2 / t$$

$$t = 0.196 \times 3^2 / 0.34$$

$$= 5.2 \text{ yrs}$$

4. What effect would an impermeable layer at the base of the clay described in Q3 have on the predicted 50% consolidation time?

$$t = 0.196 \times 6^2 / 0.34$$

$$= 20.8 \text{ yrs}$$

$$\text{times } 4 \text{ (2}^2\text{)}$$

5. If a clay soil achieves 40% consolidation in 15 months, how long will it take to reach 90% consolidation?

$$\text{Assume } c_v \text{ constant, then } [T_v \cdot d^2 / t]_{40} = [T_v \cdot d^2 / t]_{90}$$

$$T_{v40} = 0.11$$

$$d \text{ is common, so } 0.11/1.25 = 0.848/t_{90}$$

$$t_{90} = 0.848 \times 1.25 / 0.11$$

$$= 9.6 \text{ yrs}$$

6. Craig's Example 7.4

The following compression readings were taken during an oedometer test on a saturated clay specimen ($G_s = 2.73$) when the applied pressure was increased from 214 to 429 kN/m²:

Time (min)	0	¼	½	1	2¼	4	9	16	25
Gauge (mm)	5.00	4.67	4.62	4.53	4.41	4.28	4.01	3.75	3.49
Time (min)	36	49	64	81	100	200	400	1440	
Gauge (mm)	3.28	3.15	3.06	3.00	2.96	2.84	2.76	2.61	

After 1440 min the thickness of the specimen was 13.60 mm and the water content was 35.9%. Determine the coefficient of consolidation from both the log time and the root time plots and the values of the three compression ratios. Determine also the value of the coefficient of permeability.

Total change in thickness during increment = $5.00 - 2.61 = 2.39$ mm

Average thickness during increment = $13.60 + \frac{2.39}{2} = 14.80$ mm

Length of drainage path, $d = \frac{14.80}{2} = 7.40$ mm

From the log time plot

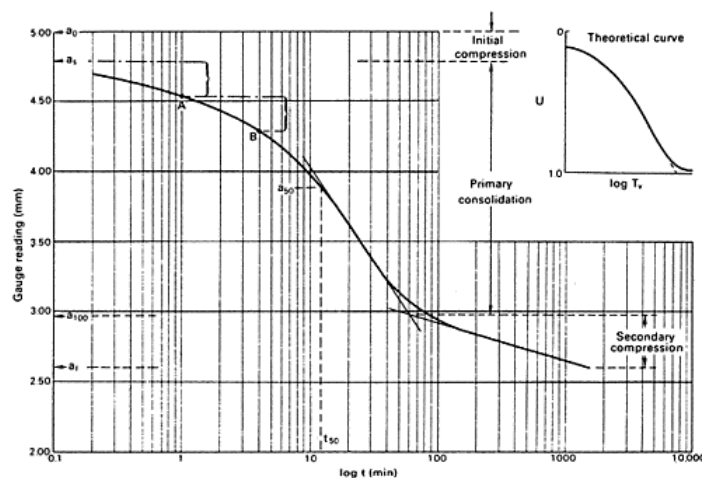


Figure 7.20 The log time method.

$$t_{50} = 12.5 \text{ min}$$

$$c_v = \frac{0.196d^2}{t_{50}} = \frac{0.196 \times 7.40^2}{12.5} \times \frac{1440 \times 365}{10^6} = 0.45 \text{ m}^2/\text{year}$$

$$r_0 = \frac{5.00 - 4.79}{5.00 - 2.61} = 0.088$$

$$r_p = \frac{4.79 - 2.98}{5.00 - 2.61} = 0.757$$

$$r_s = 1 - (0.088 + 0.757) = 0.155$$

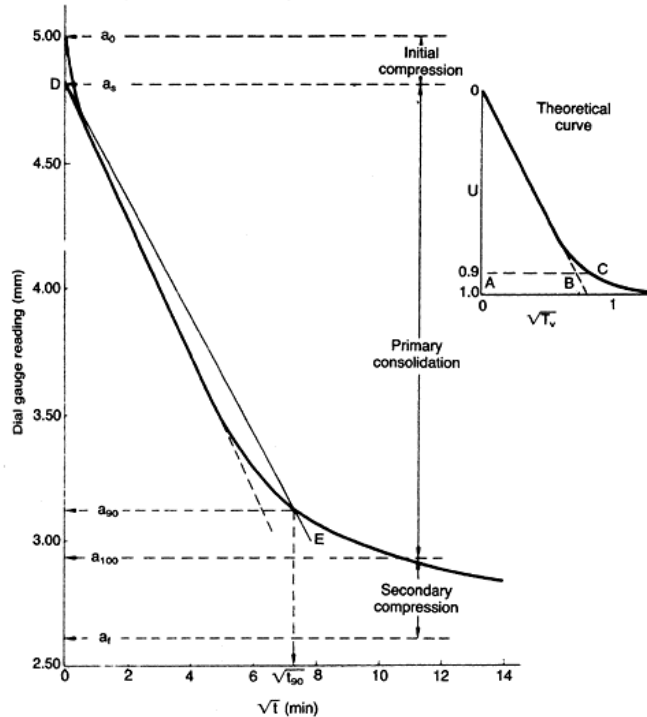


Figure 7.21 The root time method.

From the root time plot ([Figure 7.21](#)) $\sqrt{t_{90}} = 7.30$, and therefore

$$t_{90} = 53.3 \text{ min}$$

$$c_v = \frac{0.848d^2}{t_{90}} = \frac{0.848 \times 7.40^2}{53.3} \times \frac{1440 \times 365}{10^6} = 0.46 \text{ m}^2/\text{year}$$

$$r_0 = \frac{5.00 - 4.81}{5.00 - 2.61} = 0.080$$

$$r_p = \frac{10(4.81 - 3.12)}{9(5.00 - 2.61)} = 0.785$$

$$r_s = 1 - (0.080 + 0.785) = 0.135$$

In order to determine the permeability, the value of m_v must be calculated.

$$\text{Final void ratio, } e_1 = w_1 G_s = 0.359 \times 2.73 = 0.98$$

$$\text{Initial void ratio, } e_0 = e_1 + \Delta e$$

Now

$$\frac{\Delta e}{\Delta H} = \frac{1 + e_0}{H_0}$$

i.e.

$$\frac{\Delta e}{2.39} = \frac{1.98 + \Delta e}{15.99}$$

Therefore

$$\Delta e = 0.35 \quad \text{and} \quad e_0 = 1.33$$

Now

$$\begin{aligned} m_v &= \frac{1}{1 + e_0} \cdot \frac{e_0 - e_1}{\sigma'_1 - \sigma'_0} \\ &= \frac{1}{2.33} \times \frac{0.35}{215} = 7.0 \times 10^{-4} \text{ m}^2/\text{kN} \\ &= 0.70 \text{ m}^2/\text{MN} \end{aligned}$$

Coefficient of permeability:

$$\begin{aligned} k &= c_v m_v \gamma_w \\ &= \frac{0.45 \times 0.70 \times 9.8}{60 \times 1440 \times 365 \times 10^3} \\ &= 1.0 \times 10^{-10} \text{ m/s} \end{aligned}$$

7. Craig's 7.1 In an oedometer test on a specimen of saturated clay ($G_s = 2.72$) the applied pressure was increased from 107 to 214 kN/m² and the following compression readings recorded. After 1440 min the thickness of the specimen was 15.30 mm and the water content was 23.2%. Determine the values of the coefficient of consolidation and the compression ratios from (a) the root time plot and (b) the log time plot. Determine also the values of the coefficient of volume compressibility and the coefficient of permeability.

Time (min)	0	¼	½	1	2¼	4	6¼	9	16
Gauge (mm)	7.82	7.42	7.32	7.21	6.99	6.78	6.61	6.49	6.37
Time (min)	25	36	49	64	81	100	300	1440	
Gauge (mm)	6.29	6.24	6.21	6.18	6.16	6.15	6.10	6.02	

$$c_v = 2.7, 2.6 \text{ m}^2/\text{year}, m_v = 0.98 \text{ m}^2/\text{MN}, k = 8.1 \times 10^{-10} \text{ m/s}$$

Oedometer Test Laboratory Sheet

Aims

- To obtain compressibility data in the form of load-void ratio relationship.
- To obtain consolidation data in the form of compression-time relationships.

Some key features & considerations

- Specimens must be handled carefully to avoid disturbance or loss of moisture.
- Specimens are prepared from large undisturbed block samples (of clay) or sample tubes. They must be carefully cut to size then trimmed flush to the ring, typically 75mm diameter by 19mm height.
- Sample held between two porous discs, i.e. two way drainage, and subjected to vertical load applied to the upper cap.
- Loading and deformation are both vertical; confining ring ensures zero lateral strain.
- Initial seating load of about 2 kPa is followed by a sequence of load increments; usually loads are doubled to maximum value that comfortably embraces the anticipated project loading.
- Consolidation, i.e. settlement-time, data will be collected during each loading increment.
- Final moisture content vital.

General Test Details

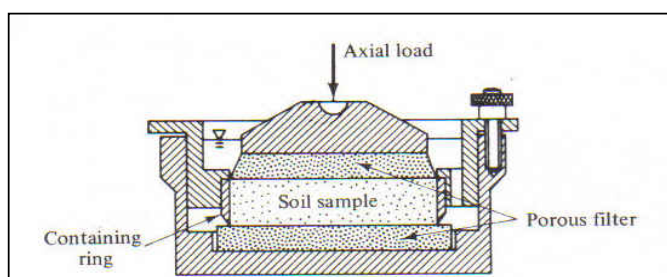
Ring Dimensions	
Diameter, mm	63.50
Area, mm ²	3166
Height, mm	19.05

Compression Data	
Initial dial reading, mm	4.485
Final dial reading, mm	2.811
Change in sample ht	-1.674

Initial Moisture Content (trimmings)	
Container mass, g	15.84
Container + wet soil mass, g	74.16
Container + dry soil mass, g	64.70
Moisture content, %	19.36

Initial Sample Calcs	
G _s	2.69
e ₀	w.G _s /S
H _s	M _s /(A.G _s ·ρ _w)
H _v	H-H _s
S	H _w /H _v

Sample Test Data		
	Before test	After test
Ring mass, g	75.55	75.55
Ring + wet soil mass, g	202.14	197.09
Ring + dry soil mass, g		181.49
Dry soil mass, g		105.94
Moisture content, %		14.73
Solids height, mm		12.44
Voids height, mm		4.94
Degree of saturation		1.00
Void ratio		0.396
Bulk density, Mg/m ³		2.209
Dry density, Mg/m ³		1.925

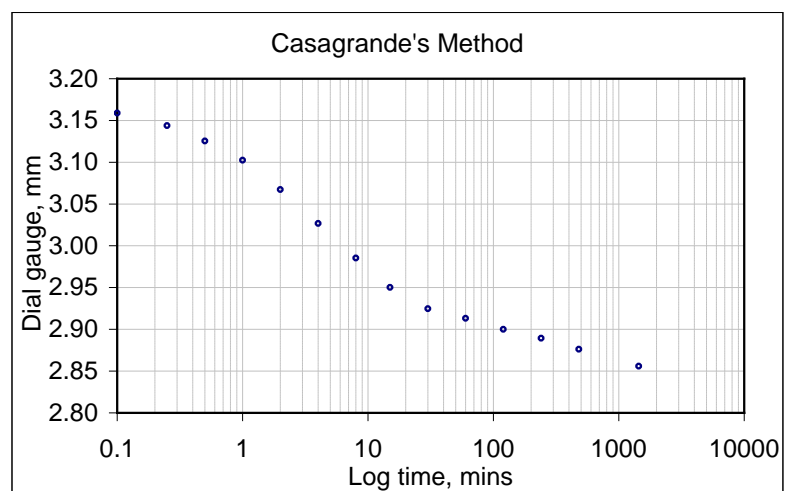
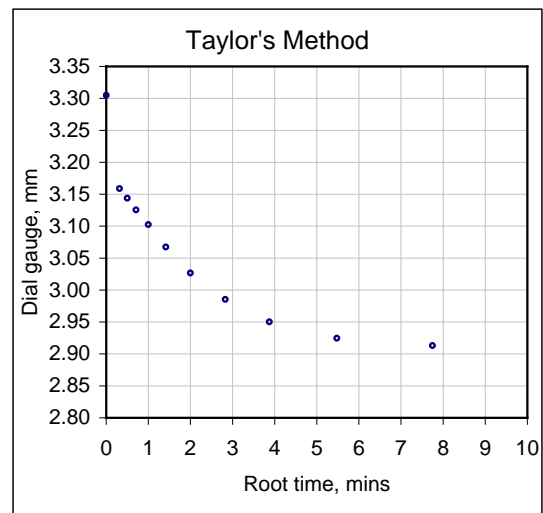
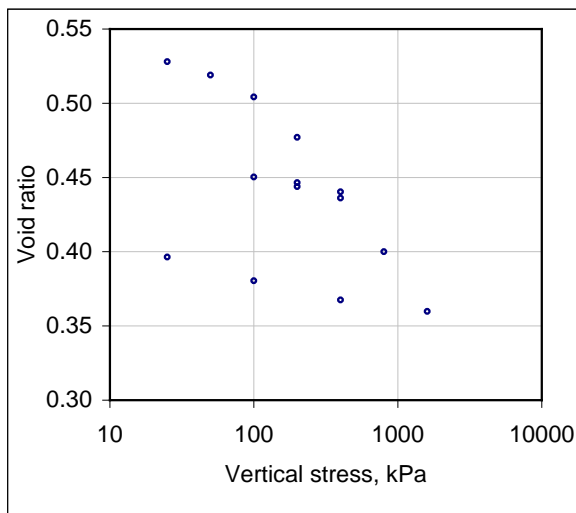


Compressibility: Displacement-load data

Load kPa	Dial gauge mm	ΔH mm	$\Delta e (= \Delta H/H_s)$	e
2	4.485	0	-	0.531
25	4.448	-0.037	-0.0030	0.528
50	4.336	-0.112	-0.0090	0.519
100	4.152		-0.0148	0.504
200	3.814			0.477
400	3.358			
200	3.401			
100	3.482			
200	3.434			
400	3.305			
800	2.856			
1600	2.356			
400	2.451			
100	2.612			
25	2.811			0.396

Consolidation: Displacement-time data

Time mins	Root time mins	\log_{10} time	Dial gauge mm
0	0.000		3.305
0.1	0.316	-1.000	3.159
0.25	0.500	-0.602	3.144
0.5	0.707	-0.301	3.125
1	1.000	0.000	3.103
2	1.414	0.301	3.067
4	2.000	0.602	3.027
8	2.828	0.903	2.985
15	3.873	1.176	2.950
30	5.477	1.477	2.925
60	7.746	1.778	2.913
120	10.954	2.079	2.900
240	15.492	2.380	2.889
480	21.909	2.681	2.876
1440	37.947	3.158	2.856



SOIL STRENGTH: Examples & self-check questions

1. Calculate the angle of shearing resistance of sand that shows a maximum resistance to shear of 135 kPa when subject to a normal stress of 200 kPa.
2. During shearing, the strength of a _____ soil rises rapidly to a peak then falls, whereas the strength of a _____ soil rises slowly to a maximum.
3. When sheared under the same vertical stress, the ultimate strength of dense and loose samples of the same soil, is _____
4. During shearing, the volume of a dense sample will _____, whilst that of a loose sample will _____
5. During shearing at the same _____, the volumetric state (void ratio) of two initially dense and initially loose samples, will become _____.
6. Shear box testing of a clay soil reveals a shear strength of 80 kPa, when the vertical stress is 100 kPa, and 145 kPa when the vertical stress is 200 kPa. Calculate the shear strength parameters for this soil.
7. What two terms are used to describe stress σ_1 ?
8. What two terms are used to describe stress σ_3 ?
9. What two triaxial stress components fix the position and size of a Mohr's circle?
10. Define the centre and the radius of a Mohr's circle in terms of the principal effective stresses.
11. The following table contains consolidated undrained triaxial test data obtained from a firm silty clay. Calculate the effective stress values at failure.

	A	B	C
Total cell pressure, σ_3	500	600	800
Deviator stress at failure, $(\sigma_1 - \sigma_3)_f$	109	223	424
Pore water pressure at failure, u_f	439	482	559
σ'_3			
σ'_1			

12. Using graph paper and pair of compasses, determine the shear strength parameters for the firm silty clay.
13. The deviator stress at failure in an undrained unconsolidated triaxial test is 140 kPa. What is the shear strength?

14. Use Skempton's (1957) method for estimating the undrained shear strength of a normally consolidated clay (plastic limit = 27%, liquid limit = 67%) at depth of 5m ($\sigma'_v = 48$ kPa) below ground surface

15. Craig's Example 4.1

The following results were obtained from direct shear tests on specimens of a sand compacted to the *in-situ* density. Determine the value of the shear strength parameter ϕ' .

Normal stress (kN/m ²)	50	100	200	300
Shear stress at failure (kN/m ²)	36	80	154	235

Would failure occur on a plane within a mass of this sand at a point where the shear stress is 122kN/m² and the effective normal stress 246kN/m²?

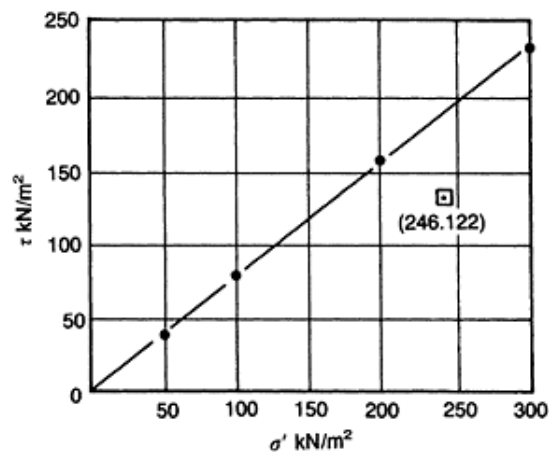


Figure 4.17

The values of shear stress at failure are plotted against the corresponding values of normal stress, as shown in Fig 4.17. The failure envelope is the line having the best fit to the plotted points; in this case a straight line through the origin. If the stress scales are the same, the value of ϕ' can be measured directly and is 38°.

The stress state $\tau = 122$ kN/m², $\sigma' = 246$ kN/m² plots below the failure envelope, and therefore would not produce failure.

Napier exam question

A shear box test is run on a sandy soil with the results given in Table Q2.1.

Table Q2.1

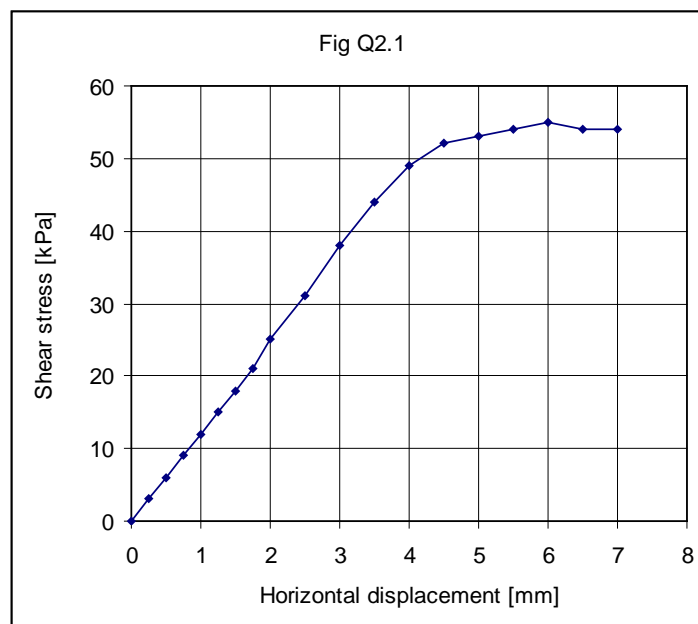
Test	Normal stress [kPa]	Shear stress at failure [kPa]
A	36	25
B	80	54
C	150	100

- a) Determine the shear strength parameters for the data in Table Q2.1.
(Graph paper provided)

6 marks

Graph or algebraic calculation.

Shear strength: $c = 0 \text{ kPa}$, $\phi = 34^\circ$



- b) From the shear stress and horizontal displacement data shown in Fig.Q2.1, identify and describe the volumetric state (i.e. relative density) of the soil at the start of the test.

4 marks

Gradual rise to maximum shear stress at failure = loose soil

- c) What are the advantages and disadvantages of soil strength testing in the direct shear box as compared with the triaxial test?

6 marks

Predefined plane of shearing, lack of pore water pressure measurement, unknown stress field, large samples, in-situ, large strains

- d) Sketch, compare and explain the relationships between (i) vertical displacement and horizontal displacement and (ii) void ratio and horizontal displacement, that are observed during the shearing of initially loose and initially dense sands.

10 marks

Diagrams with dense and loose displacement curves (i) showing divergence and long term trend of vertical displacement (ii) convergence of initial void ratios to long term volumetric state.

Discussion to highlight large strain condition dependent on normal stress alone, initial conditions lost by shearing.

- e) Peak strengths are often exhibited by dense sands. Explain the cause of peak strength in a soil and why it might be unsafe to use peak strength values in geotechnical design.

7 marks

Peak strength arises from combination of particle interlock and normal stress – dense soil, low stress.

Where straining is large enough to exceed peak strength.

Total 33 marks

16. Craig's Example 4.2

The results shown in Table 4.2 were obtained at failure in a series of triaxial tests on specimens of a saturated clay initially 38mm in diameter by 76mm long. Determine the values of the shear strength parameters with respect to (a) total stress and (b) effective stress.

Table 4.2

Type of test	All-round pressure (kN/m ²)	Axial load (N)	Axial deformation (mm)	Volume change (ml)
(a) Undrained	200	222	9.83	—
	400	215	10.06	—
	600	226	10.28	—
(b) Drained	200	403	10.81	6.6
	400	848	12.26	8.2
	600	1265	14.17	9.5

The principal stress difference at failure in each test is obtained by dividing the axial load by the cross-sectional area of the specimen at failure (Table 4.3). The corrected cross-sectional area is calculated from Equation 4.10. There is, of course, no volume change during an undrained test on a saturated clay. The initial values of length, area and volume for each specimen are:

$$l_0 = 76 \text{ mm}, \quad A_0 = 1135 \text{ mm}^2, \quad V_0 = 86 \times 10^3 \text{ mm}^3$$

Table 4.3

	σ_3 (kN/m ²)	$\Delta l/l_0$	$\Delta V/V_0$	Area (mm ²)	$\sigma_1 - \sigma_3$ (kN/m ²)	σ_1 (kN/m ²)
(a)	200	0.129	—	1304	170	370
	400	0.132	—	1309	164	564
	600	0.135	—	1312	172	772
(b)	200	0.142	0.077	1222	330	530
	400	0.161	0.095	1225	691	1091
	600	0.186	0.110	1240	1020	1620

The Mohr circles at failure and the corresponding failure envelopes for both series of tests are shown in Figure 4.18. In both cases the failure envelope is the line nearest to a common tangent to the Mohr circles. The total stress parameters, representing the undrained strength of the clay, are

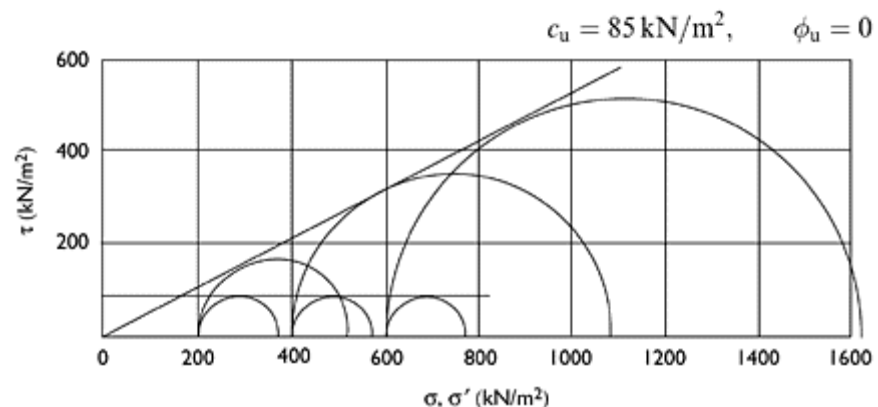


Figure 4.18 Example 4.2.

The effective stress parameters, representing the drained strength of the clay, are

$$c' = 0, \quad \phi' = 27^\circ$$

17. Craig's Example 4.3

The results shown in Table 4.4 were obtained for peak failure in a series of consolidated–undrained triaxial tests, with pore water pressure measurement, on specimens of a saturated clay. Determine the values of the effective stress parameters.

Table 4.4

All-round pressure (kN/m ²)	Principal stress difference (kN/m ²)	Pore water pressure (kN/m ²)
150	192	80
300	341	154
450	504	222

Values of effective principal stresses σ'_3 and σ'_1 at failure are calculated by subtracting pore water pressure at failure from the total principal stresses as shown in Table 4.5 (all stresses in kN/m²). The Mohr circles in terms of effective stress are drawn in Figure 4.19.

Table 4.5

σ_3	σ_1	σ'_3	σ'_1
150	342	70	262
300	641	146	487
450	954	228	732

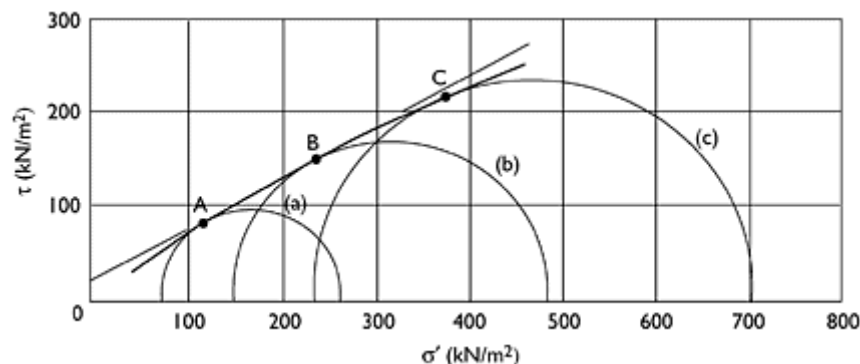


Figure 4.19

In Figure 4.19 a linear approximation has been drawn for the range of effective normal stress 200–300 kN/m², giving parameters $c' = 20$ kN/m² and $\phi' = 29^\circ$.

18. Craig's 4.1 What is the shear strength in terms of effective stress on a plane within a saturated soil mass at a point where the total normal stress is 295 kN/m² and the pore water pressure 120 kN/m²? The effective stress parameters of the soil for the appropriate stress range are $c' = 12$ kN/m² and $\phi' = 30^\circ$.

19. Craig's 4.2 A series of drained triaxial tests were carried out on specimens of a sand prepared at the same porosity and the following results were obtained at failure. Determine the value of the angle of shearing resistance ϕ' .

All-round pressure (kN/m ²)	100	200	400	800
Principal stress difference (kN/m ²)	452	908	1810	3624

20. Craig's 4.3 In a series of unconsolidated–undrained triaxial tests on specimens of a fully saturated clay the following results were obtained at failure. Determine the values of the shear strength parameters c_u and ϕ_u .

All-round pressure (kN/m ²)	200	400	600
Principal stress difference (kN/m ²)	222	218	220

21. Craig's 4.4 The effective stress parameters for a fully saturated clay are known to be $c' = 15 \text{ kN/m}^2$ and $\phi' = 29^\circ$. In an unconsolidated–undrained triaxial test on a specimen of the same clay the all-round pressure was 100 kN/m^2 and the principal stress difference at failure 170 kN/m^2 . Assuming that the above parameters are appropriate to the failure stress state of the test, what would be the expected value of pore water pressure in the specimen at failure?

Laboratory Sheet: SHEAR BOX

The direct shear box is used to investigate soil strength, i.e. the shearing resistance of a soil. It allows both the quantification of soil strength, as c' & ϕ' , and its broader interpretation as a peak or ultimate condition.

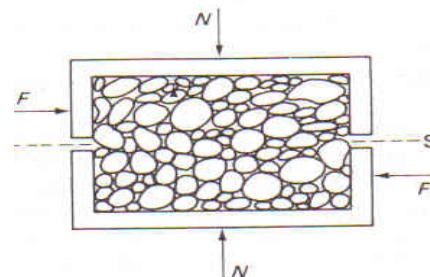
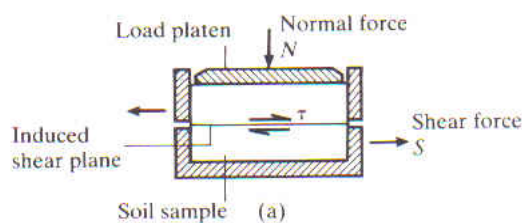
The raw data from which this information is obtained is:

Horizontal force vs. horizontal displacement, and
Vertical displacement vs. horizontal displacement.

The mobilised **shearing resistance** is a combination of **interparticle friction** and the work needed to overcome **particle interlock**. **Interparticle friction**, commonly reported as a **friction angle**, is a constant term which is dependent on the nature of the particle mineral surfaces. The work required to overcome **interlock** is dependent on the initial **packing arrangement**; available soil strength is thus highly dependent on the initial density.

The test has a number of limitations: pore water pressures are not measured and the failure surface is predefined. For free draining soils (sands) that do not contain pre-existing planes of weakness, the shear box is quite suitable and can be run at a displacement rate of about 1mm/min. A horizontal displacement of 8mm is adequate to test the soil, so this is both a quick and simple test.

In finer-grained materials, drainage is much slower. By running the shear box at slower rates, excess pore water pressure build up is avoided and total stresses are always equal to effective stresses. It is then possible to obtain effective strength parameters for silts (0.01mm/min) or clays (0.001mm/min). This does not, however, resolve the predefined failure surface issue.



	Dense	Loose
Mass of box, g	321.41	321.41
Mass of box & dry soil, g	449.26	442.00
Mass of soil, g	127.85	120.59
Dimension of box, mm	60.00	60.00
Dimension of box, mm	60.00	60.00
Sample height, mm	20.24	19.96
Sample area, cm ²	36.00	36.00
Sample volume, cm ³	72.86	71.86

Dry density, Mg/m ³	1.75	1.68
Gs	2.70	2.70
Volume solid, cm ³	47.35	44.66
Volume void, cm ³	25.51	27.19
Initial void ratio	0.539	0.609

Proving ring	3.68	N/div
Displacement	1	mm/min
Hanger wt	55	kg
Normal stress	150	kPa

Calculations	
Volume solid	$M_s / (G_s \cdot \rho_w)$
Volume void	$V_t - V_s$
Void ratio	V_v / V_s

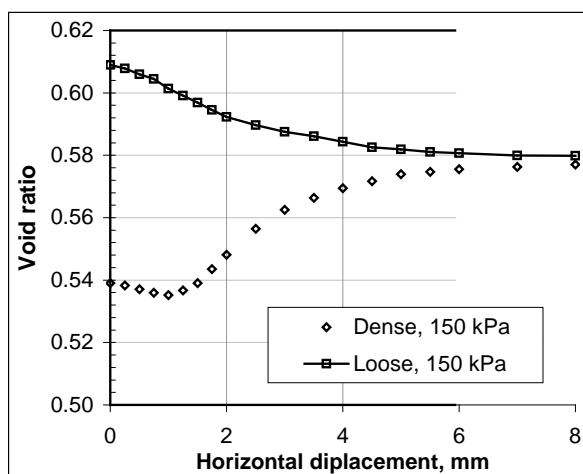
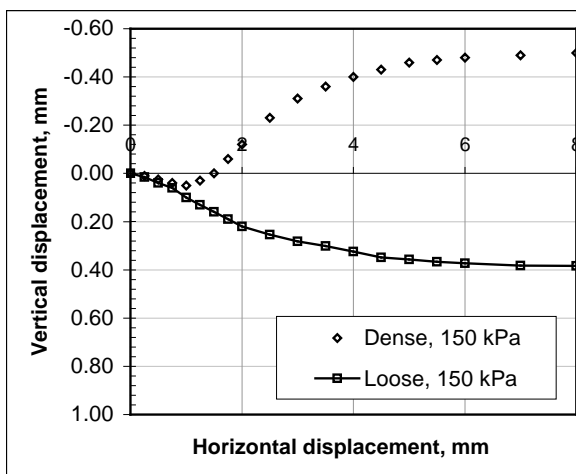
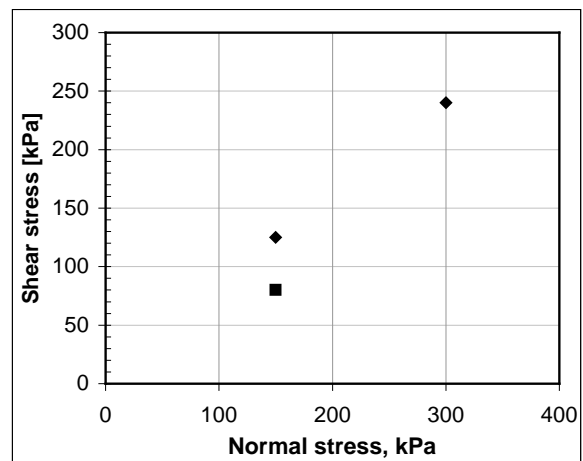
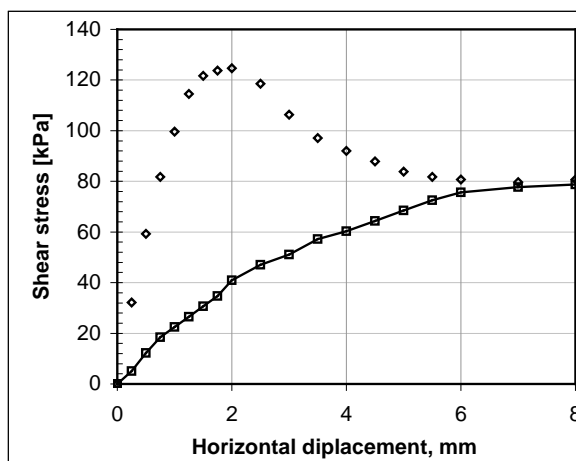
Normal stress	Loose	Dense
80	40	70
150	80	125
300	158	240

All stresses in kPa

Shear box test data

Dense, 150 kPa

Horizontal displacement, mm	Proving ring, divs	Horizontal load, N	Shear stress, kPa	Vertical displacement, mm	Void ratio
0.00	0.0	0.0	0.0	0.00	0.539
0.25	31.5	115.9	32.2	0.01	0.538
0.50	58.0	213.4	59.3	0.03	0.537
0.75	80.0	294.4	81.8	0.04	0.536
1.00	97.5	358.8	99.7	0.05	0.535
1.25	112.0	412.2	114.5	0.03	0.537
1.50	119.0	437.9	121.6	0.00	0.539
1.75	121.0	445.3	123.7	-0.06	0.544
2.00	122.0	449.0	124.7	-0.12	0.548
2.50	116.0	426.9	118.6	-0.23	0.556
3.00	104.0	382.7	106.3	-0.31	0.563
3.50	95.0	349.6	97.1	-0.36	0.566
4.00	90.0	331.2	92.0	-0.40	0.569
4.50	86.0	316.5	87.9	-0.43	0.572
5.00	82.0	301.8	83.8	-0.46	0.574
5.50	80.0	294.4	81.8	-0.47	0.575
6.00	79.0	290.7	80.8	-0.48	0.575
7.00	78.0	287.0	79.7	-0.49	0.576
8.00	79.0	290.7	80.8	-0.50	0.577



Laboratory Sheet: Triaxial Machine

The triaxial machine is used to investigate soil strength, i.e. the shearing resistance of a soil, under generalised states of stress. It allows for independent control of horizontal and vertical stress components to mimic in-situ conditions. With porewater pressure measurement, effective strength parameters c' & ϕ' , and their broader interpretation as a peak or ultimate conditions are obtained.

The raw data provide by the triaxial test are:

axial force vs. axial displacement, and
porewater pressure (or volumetric strain) vs. axial displacement.

The mobilised **shearing resistance** is a combination of **interparticle friction** and the work needed to overcome **particle interlock**, as elaborated in the shear box lab sheet.

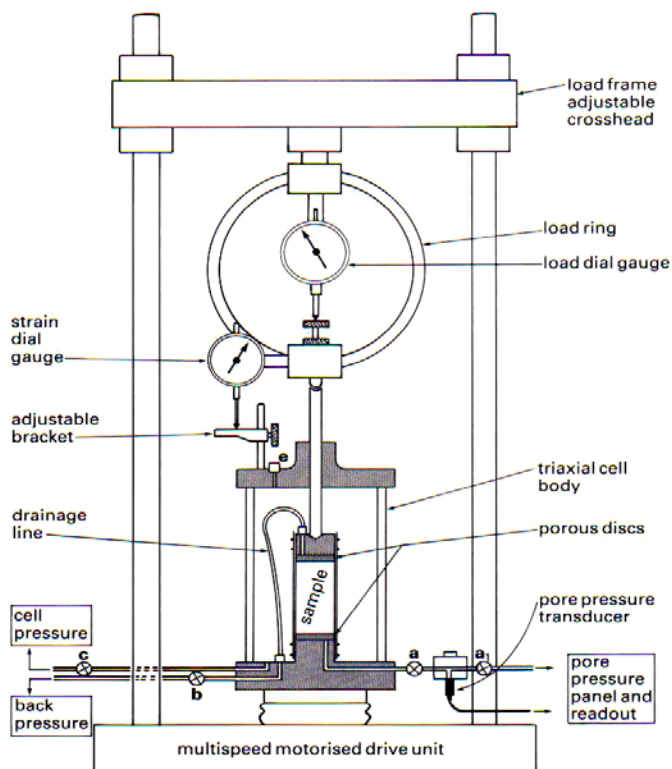
Tests are performed on cylindrical samples with a height to diameter ratio of 2:1. Samples from 38 mm up to 200 mm are common. Porous discs may be placed at each end to allow porewater drainage. In the case of very low permeability soils, filter paper drains may be wrapped around the sample. The sample, porous discs and end cap assembly is then sealed within a rubber membrane to avoid contact with the cell (pressure) water.

The test procedure comprises two main stages:

(i) consolidation - sample subject to isotropic cell pressure. Pressure is increased in steps during which checks for full saturation are made. This stage may take several days depending on the permeability of the sample

(ii) shear - sample subject to axial load from loading ram (cell pressure constant).

It is the permitted drainage conditions during each stage that distinguishes the three main types of triaxial test (see table below). Drainage options in conjunction with sample permeability dictate test duration, for example, (stepped) consolidation in a very fine-grained soil may take days. Undrained shearing is relatively quick - from 10 minutes - but when drained this stage may run to several days.



Sample drainage: triaxial test		
	Consol	Shear
D	open	open
CU	open	closed
UU	closed	closed

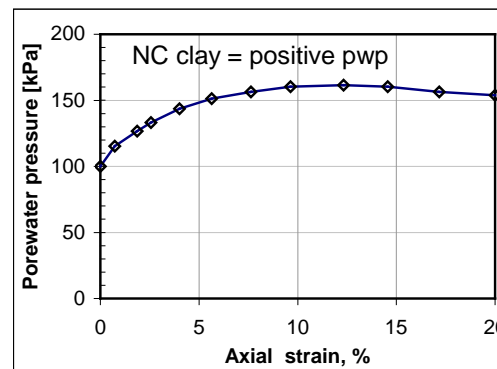
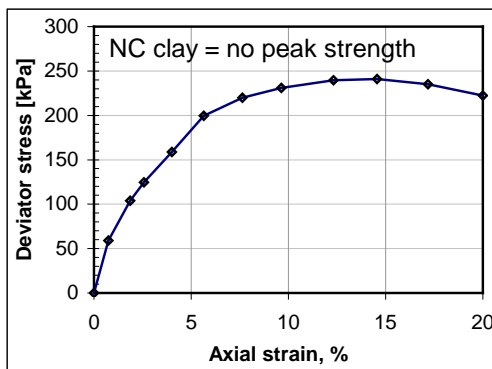
Open drain:
volume change possible
= DRAINED

Closed drain:
volume change NOT possible
= UNDRAINED

Consolidated Undrained (CU) Triaxial test data
NORMALLY CONSOLIDATED (NC) CLAY

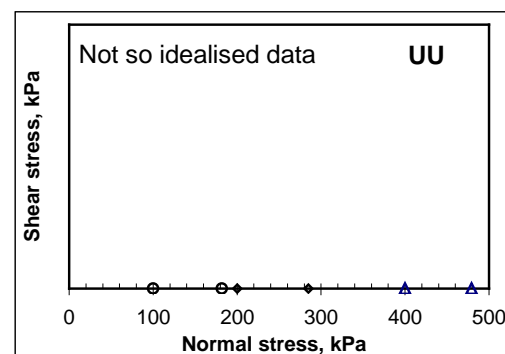
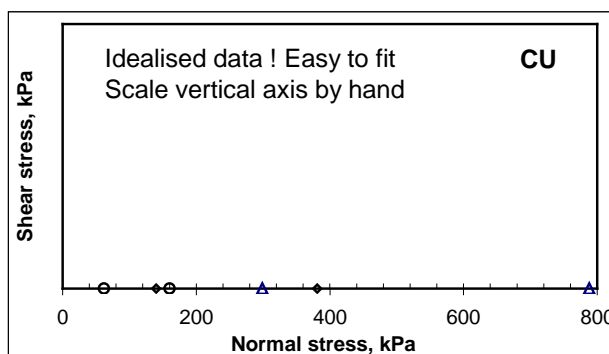
Cell pressure = 200 kPa

Axial displcmnt, mm	Axial strain, ε	Axial load, N	Corrected area, mm ² , $A_0/(1-\varepsilon)$	Deviator stress, kPa	Porewater pressure, kPa
0.0	0.0	0	1134	0	100
0.6	0.7	68	1143	59	115
1.4	1.9	120	1156	104	127
2.0	2.6	145	1164	125	133
3.1	4.0	188	1182	159	144
4.3	5.6	240	1202	200	151
5.8	7.6	270	1228	220	156
7.3	9.6	290	1255	231	160
9.4	12.3	310	1293	240	161
11.1	14.6	320	1327	241	160
13.1	17.2	322	1369	235	156
15.2	20.0	315	1418	222	154



Consolidated Undrained Summary Data			
Initial conditions	A	B	C
Cell pressure, kPa	200	300	500
Back pressure, kPa	100	100	100
Post consolidtn u, kPa	100	100	100
σ_3' , kPa	100	200	400
Failure			
Strain, %	9.8	14.6	18.2
$(\sigma_1' - \sigma_3')$ at failure, kPa	98	241	489
u, at failure, kPa	138	160	201
σ_3' at failure, kPa	62	140	299
σ_1' at failure, kPa	160	381	788

Unconsolidated Undrained			
	A	B	C
	100	200	400
	12.2	13.1	12.8
	82	85	79
σ_3	100	200	400
σ_1	182	285	479



1. Craig's Example 5.1

A load of 1500kN is carried on a foundation 2m square at a shallow depth in a soil mass. Determine the vertical stress at a point 5m below the centre of the foundation (a) assuming that the load is uniformly distributed over the foundation and (b) assuming that the load acts as a point load at the centre of the foundation.

a) Uniform pressure, $q = \frac{1500}{2^2} = 375 \text{ kN/m}^2$

The area must be considered as four quarters to enable Fadum's to be used.

In this case

$$mz = nz = 1 \text{ m}$$

Then, for

$$m = n = 0.2$$

From Fadums $I_r = 0.018$

Hence,

$$\sigma_z = 4qI_r = 4 \times 375 \times 0.018 = 27 \text{ kN/m}^2$$

b) From Table, $I_p = 0.478$ since $r/z = 0$ vertically below a point load.

Hence, $\sigma_z = \frac{Q}{z^2} I_p = \frac{1500}{5^2} \times 0.478 = 29 \text{ kN/m}^2$

The point load assumption should not be used if the depth to the point X is less than three times the larger dimension of the foundation.

2. Craig's Example 5.2

A rectangular foundation 6x3m carries a uniform pressure of 300kN/m² near the surface of a soil mass. Determine the vertical stress at a depth of 3m below a point (A) on the centre line 1.5m outside a long edge of the foundation (a) using influence factors and (b) using Newmark's influence chart.

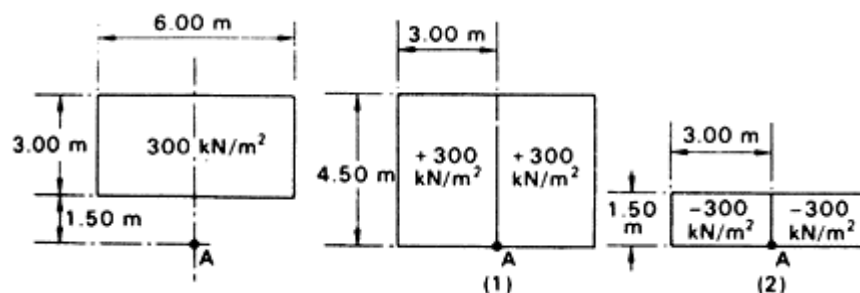


Figure 5.12

(a) Using the principle of superposition the problem is dealt with in the manner shown in [Figure 5.12](#). For the two rectangles (1) carrying a *positive* pressure of 300 kN/m^2 , $m = 1.00$ and $n = 1.50$, and therefore

$$I_r = 0.193$$

For the two rectangles (2) carrying a *negative* pressure of 300 kN/m^2 , $m = 1.00$ and $n = 0.50$, and therefore

$$I_r = 0.120$$

Hence,

$$\begin{aligned}\sigma_z &= (2 \times 300 \times 0.193) - (2 \times 300 \times 0.120) \\ &= 44 \text{ kN/m}^2\end{aligned}$$

(b) Using Newmark's influence chart ([Figure 5.11](#)) the scale line represents 3m, fixing the scale to which the rectangular area must be drawn. The area is positioned such that the point A is at the centre of the chart. The number of influence areas covered by the rectangle is approximately 30 (i.e. $N = 30$), hence

$$\begin{aligned}\sigma_z &= 0.005 \times 30 \times 300 \\ &= 45 \text{ kN/m}^2\end{aligned}$$

