Soil Mechanics
Unit 5 - Groundwater

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Programme

Day	08:00-09:30	09:45-11:15	13:00-14:30	14:45-16:15
19/05/25	Introduction	Programming	Phase Rel.	Tutorial
20/05/25	Classification	Tutorial	LAB	LAB
21/05/25	Compaction	Tutorial		
22/05/25	Groundwater	Tutorial	LAB	LAB
23/05/25	Groundwater	Tutorial		
26/05/25	Effective Str.	Tutorial	Stress Incr.	Tutorial
27/05/25	Compressib.	Tutorial	LAB	LAB
28/05/25	Consolidation	Tutorial		
29/05/25	Shear Str.	Tutorial	Shear.Str.	Tutorial
30/05/25	Shear Str.	Review		

Overview

Why is this lecture important?

Water significantly affects the mechanical behaviour of soil. In fact, many of the reported legal cases associated with construction failures in projects are associated with a lack of understanding of how water flows through and how it affects soil behaviour.

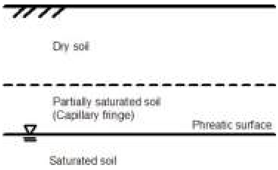
In simple terms, within geotechnical engineering, water means trouble.

... unless you follow and understand this unit!

Water in soils

Groundwater within a soil usually reveals two distinct regimes – a saturated (pores contain only water) and a partially saturated zone (pores contain both water and air).

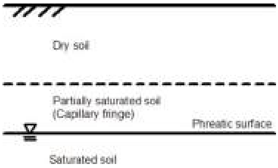
A well or borehole reveals the ground water level (or phreatic surface). The phreatic surface may or may not be the boundary between the saturated and partially saturated states.



Under the groundwater level (table), pore pressures are positive and increase with depth. Above, they may become negative if the soil is able to sustain suction (i.e. negative pore water pressures). At the phreatic level, pore pressure is zero.

Water in soils

Water is held in a soil against the pull of gravity by capillary forces. These forces are not present in a borehole but in a fine-grained soil they can generate a zone of fully saturated soil several metres above the phreatic surface.



In sands, the saturated zone rises only centimetres and the saturated zone in the soil and the water table in the borehole are more or less coincident.

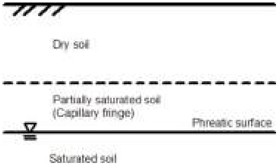
Regardless of the extent of capillary rise, the groundwater table or phreatic surface marks the zero water pressure contour. Below ground water level pressures are positive, above it they are negative.

Water in soils

Pore water pressure is calculated from the bulk unit weight of the pore water and depth below the free (or phreatic) surface,

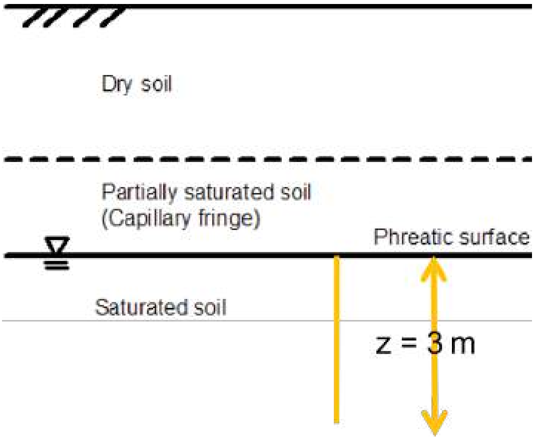
$$u = \gamma_w z$$

where u is pore water pressure [kPa], and z is depth **below** the free surface.



Pore water pressure is very significant in soil mechanics; it controls the effective stress, which governs soil strength and compressibility.

Water in soils - Pore pressures



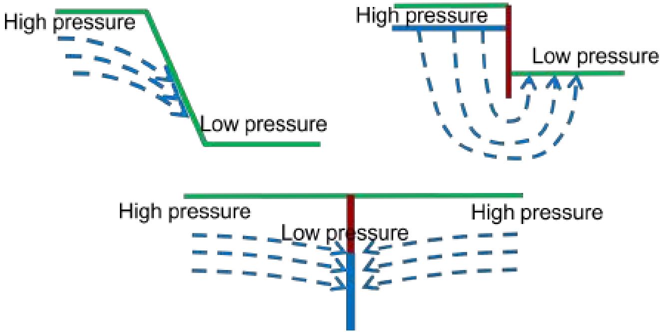
$$u = \gamma_w z$$

$$u = 9.81 \left(\frac{\text{kg}}{\text{m}^3}\right) \times 3 \text{ m}$$

$$u = 29.4 \text{ kPa}$$

If the pore water pressure is hydrostatic, there is no flow. This does not mean that pore water pressures are constant; it means that they vary uniformly with depth below the surface.

If there is groundwater flow, then there must be a pressure difference over and above that associated with the self weight of the fluid below the surface.



REMEMBER: If the pore water pressure (u) is hydrostatic, there is no flow. **If there is groundwater flow, then there is a pressure difference or gradient.**

Hydraulic head

We use the concept of hydraulic head to define the driving force of groundwater flow. Hydraulic head can be divided into three parts, as per Bernoulli's equation. At any point the hydraulic head, h [metres], is given by:

$$h = \frac{v^2}{2g} + \frac{u}{\gamma_w} + z$$

where v is the flow velocity, u is the pore water pressure, and z is the elevation above some arbitrary datum. Also:

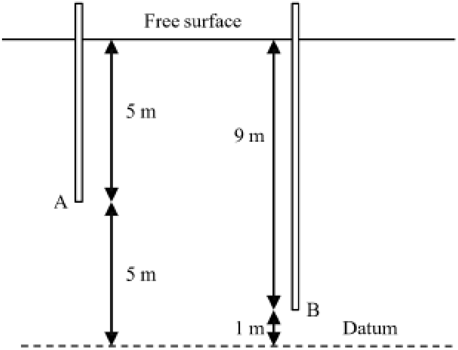
First term: **Velocity** head
Second term: **Pressure** head
Third term: **Position** (potential) head)

In soils, v is (usually) negligible, hence:

$$h = \frac{u}{\gamma_w} + z$$

Hydraulic head

According to Bernoulli's equation, for the circumstances shown, the hydraulic head at the base of each standpipe is the same:



$h = \frac{u}{\gamma_w} + z$

$h_A = \frac{9.81 \times 5}{9.81} + 5 = 10$ m and

$h_B = \frac{9.81 \times 9}{9.81} + 1 = 10$ m

i.e. there is no hydraulic head difference so there is no flow.

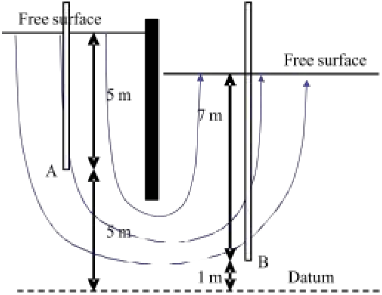
These are **hydrostatic** conditions.

Hydraulic head

Here, a difference in free surfaces either side of a sheet pile wall (or cofferdam), a head difference exists. The hydraulic head at A is the same as previously (= 10 m), but at B,

$h_B = \frac{9.81 \times 7}{9.81} + 1 = 8$ m

There is a head difference (of 2 m) and **flow will occur** from the location of high hydraulic head (A) to the location of low hydraulic head (B) (even though the pore water pressure is higher at B than at A). The existence of a hydraulic head difference gives rise to the concept of a hydraulic gradient.



Hydraulic gradient

Water flows due to a difference of hydraulic head (Δh). Hydraulic gradient is the difference in hydraulic head per unit length. It is directional and is a dimensionless term. So,

$$i = \frac{\Delta h}{\Delta x} \text{ or}$$

$$i = \frac{\Delta h}{l}$$

where i is the hydraulic gradient and x (or l) is a linear dimension

Darcy's law

Darcy showed experimentally that the velocity of a fluid through a saturated porous medium is directly proportional to the hydraulic gradient causing the flow. In mathematical terms,

$$v = ki$$

where v is the velocity [m/s] and k is the coefficient of permeability (or hydraulic conductivity) [m/s]

The velocity in question is the 'Darcy' velocity, which cannot be measured directly because of the tortuous nature of the flow paths in a soil, but can be determined by measuring the discharge over unit cross section of a soil, i.e.

$$v = \frac{Q}{A}$$

where Q is the quantity flow rate or volume discharge [m^3/s] and A is the cross-sectional area [m^2] normal to the direction of flow.

Darcy's law

Combining $v = ki$ and $v = \frac{Q}{A}$ gives

$$v = kAi = kA\frac{\Delta h}{\Delta x}$$

The Darcy velocity relates to flow occurring uniformly over unit cross-sectional area whereas in reality flow only occurs within the pore space network, which is a much smaller flow domain.

The intrinsic flow velocity, i.e. the speed at which liquid moves through the pores (sometimes called the seepage velocity), is greater than the Darcy velocity. In fact the two are related by the cross-sectional area occupied by pore space, which is broadly equivalent to the porosity.

Coefficient of permeability

Taking values ranging over 7 or 8 orders of magnitude, k is the most wide ranging soil parameter. As a consequence, laboratory determination of k is only indicative. We will, however, consider laboratory tests but note that for more reliable measurement, field tests are required.

k [m/s] =	$>10^{-2}$	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}	10^{-10}
	gravel			sand			silt		clay

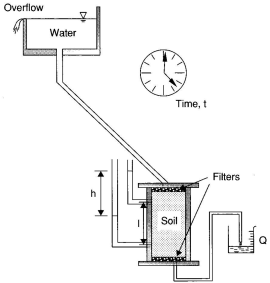
For the purposes of this lecture course we will restrict our attention to flow in one and two dimensions but it should be kept in mind that the value of k is likely to be anisotropic, i.e. to vary between horizontal and vertical directions.

Due to the depositional characteristics of a soil, the horizontal permeability is often greater than the vertical permeability by an order of magnitude or more

Constant head test

If the soil is coarse-grained, and can be formed into a tube sample, we can establish the rate of flow under a constant head. This test is only suitable for soils having a coefficient of permeability between 10^{-2} and 10^{-5} m/s. Note that the soil is disturbed and does not retain the in situ condition.

The test considers the head drop or head difference and quantity discharge at steady state. In a 100 mm diameter permeameter, with constant head drop of 210 mm and length between manometer ports of 250 mm, the steady state discharge is 136 ml/min. The permeability of the soil being tested is (in m/s),

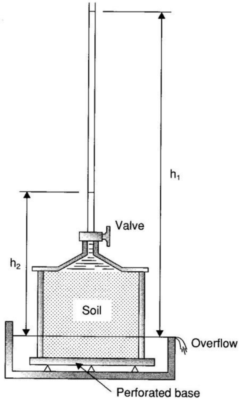


$$Q = kA\frac{\Delta h}{l}$$

$$k = \frac{Ql}{A\Delta h} = \frac{\left(\frac{136\times10^{-6}}{60}\right)\times0.250}{0.00785\times0.210} = 3.4\times10^{-4}$$

Falling head test

Where flow rates are small, as in fine-grained soils, the falling head permeameter is used. In this test a graduated fine bore tube is connected to a much larger diameter sample. The fall of the water level in the fine bore tube is monitored over time and analysed graphically.



The governing equation is based on the velocity of the falling head in the fine bore tube,

$$v = \frac{dH}{dt} = \frac{Q}{a}$$

where a is the area of the fine bore tube. Recalling that

$$Q = kA\frac{\Delta h}{l}$$

$$\frac{dh}{dt} = k\frac{A}{a}\frac{h}{l}$$

where h is the height of the falling head from the top of the sample and l is the distance across the sample.

Falling head test

Integrating $\frac{dh}{dt} = k\frac{A}{a}\frac{h}{l}$ between $h = h_0$ at $t = t_0$ and $h = h_1$ at $t = t_1$,

$$\int_{h_0}^{h_1} \frac{dh}{h} = \int_{t_0}^{t_1} \frac{kA}{la} dt \rightarrow \ln\frac{h_1}{h_0} = \frac{kA}{al}(t_1 - t_0) \rightarrow k = \frac{al}{At}\ln\frac{h_0}{h_1}$$

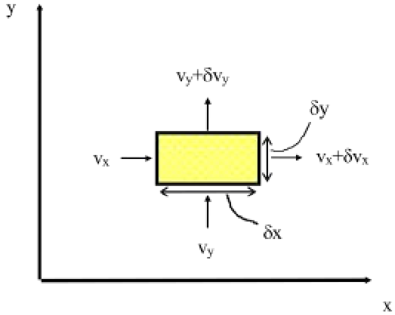
For example, an undisturbed sample of clay tested in a falling head permeameter gave the following results:

Initial head of water in tube = 1500 mm
Duration of test = 281 secs
Sample diameter = 100 mm, A = $\pi \times 0.05^2 = 0.00785$ m^2
Final head of water = 605 mm
Sample length = 150 mm
Tube diameter = 5 mm, a = $\pi \times 0.0025^2 = 1.96 \times 10^{-5}$ m^2

$$k = \frac{1.96\times10^{-5}}{0.00785}\frac{0.150}{281}\ln\frac{1.5}{0.605} = 1.2 \times 10^{-6} \approx 1.0 \times 10^{-6} \text{ m/s}$$

Groundwater flow in two dimensions - Governing equations

Two-dimensional steady saturated flow of an incompressible pore fluid in a rigid soil is described by Laplace's equation. This differential equation refers to an element of the soil and states that under steady state conditions, any flow into the element must be balanced by a flow out. These flows are defined by Darcy's Law and can be applied to each of the three orthogonal dimensions of a volume element, or as we shall examine below, to two orthogonal faces of a plane element.



Groundwater flow in two dimensions - Governing equations

For a rectangular element with dimensions dx, dy and unit thickness, velocity of flow into the element in the x direction is (according to Darcy's Law)

$$v_x = -k\frac{\delta h}{\delta x}$$

(The negative sign is required because there is a head loss in the direction of flow.)

The velocity of flow out of the element is:

$$v_x + \delta v_x = \left(v_x + \frac{\delta v_x}{\delta x}\delta x\right)$$

Similar expressions can be written for the y direction.

Groundwater flow in two dimensions - Governing equations

Balance of flow requires that $Q_{in} = Q_{out}$, so

$$v_x A_x + v_y A_y = \left(v_x + \frac{\delta v_x}{\delta x}\delta x\right) A_x + \left(v_y + \frac{\delta v_y}{\delta y}\delta y\right) A_x$$

After cancelling $\rightarrow \frac{\delta v_x}{\delta x}\delta x A_x + \frac{\delta v_y}{\delta y}\delta y A_y = 0$

Inserting Darcy's Law for the flow velocity

$$-k_x \frac{\delta h_x^2}{\delta x^2}\delta x \delta y \delta z - k_y \frac{\delta h_y^2}{\delta y^2}\delta x \delta y \delta z = 0$$

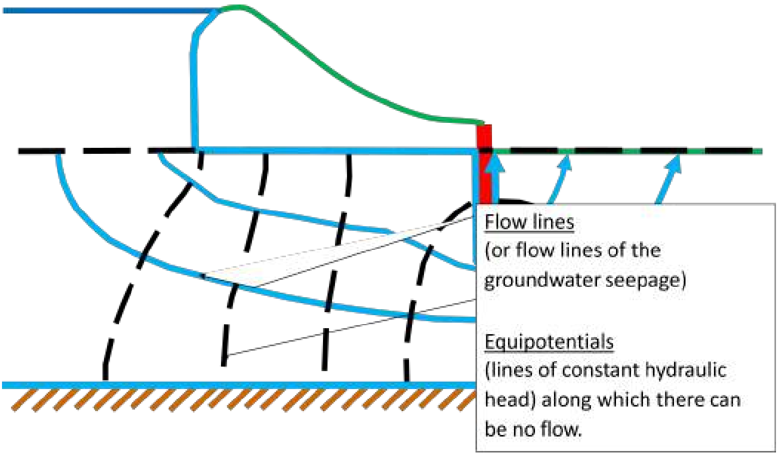
$$k_x \frac{\delta^2 h}{\delta x^2} + k_y \frac{\delta^2 h}{\delta y^2} = 0$$

...and when permeability is isotropic, i.e. $k_x = k_y$, we obtain Laplace's equation:

$$k_x \frac{\delta h^2 x}{\delta x^2} = 0$$

Flow nets

Solutions to Laplace's equation for two- dimensional seepage can be presented as sketched flow nets.



Drawing flow nets

- Flow is along flow lines, by definition, and at right angles to equipotentials. There need be only 4 or 5 of these.
- Flow lines cannot cross – water paths cannot occupy the same space at the same time, which is what would happen if flow lines crossed.
- No flow along an equipotential – there is no hydraulic gradient in this direction.
- Impermeable boundaries are flowlines.
- Free surfaces and bodies of water are equipotentials.
- The flow lines define channels along which the volume flow rate is constant.

Note: If standpipe piezometers were inserted into the ground with their tips on a single equipotential then the water would rise to the same level in each standpipe. (The pore pressures would be different because of their different elevations.)

Using flow nets to calculate flow

Consider an element of a flow channel of length l, between equipotentials which indicate a fall in total head Δh and between flow lines b apart.

The average hydraulic gradient is $i = \frac{\Delta h}{l}$

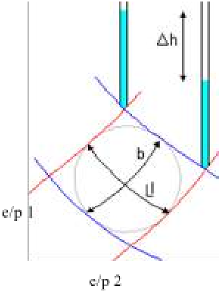
and within any flow tube the volume flow rate is

$Q = kb\frac{\Delta h}{l}$

There is an advantage in sketching flow nets in the form of curvilinear 'squares' so that a circle can be inscribed within each four-sided figure bounded by two equipotentials and two flow lines. Then $b = l$ and

$$Q = k\Delta h$$

so the flow rate through the flow channel is the permeability multiplied by the uniform interval Δh between equipotentials.



Using flow nets to calculate flow

The flow net is drawn with the overall head loss, h divided into N_d equal intervals with N_f flow channels, then across any 'square':

$$\Delta h = h/N_d$$

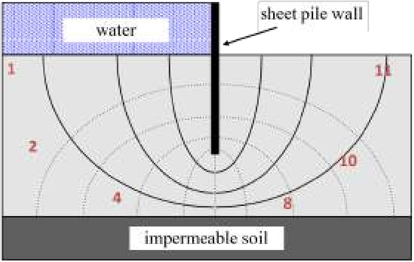
Then the total flow rate through the ground is $Q = N_f \frac{kh}{N_d} = \frac{N_f}{N_d} kh$

The number of potential drops (N_d) is simply equal to the number of equipotentials minus one.

The number of flow channels (N_f) is easily counted from the flow net.

Here, $N_d = 10,\ N_f = 3.5$

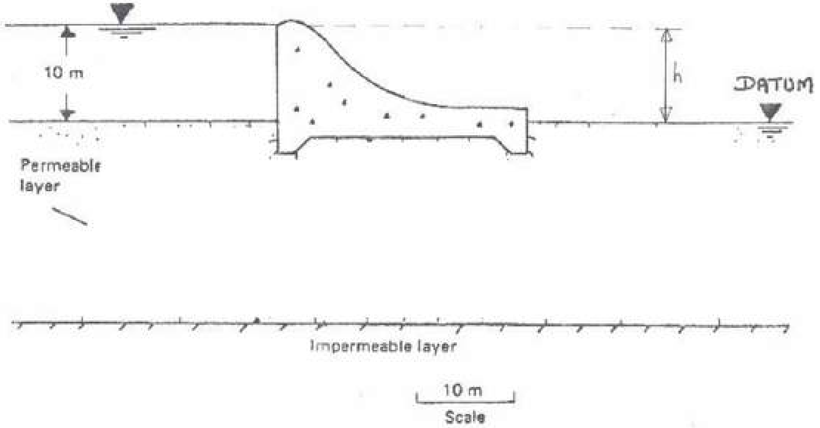
so $Q = 0.35kh$.



Flow nets - An example

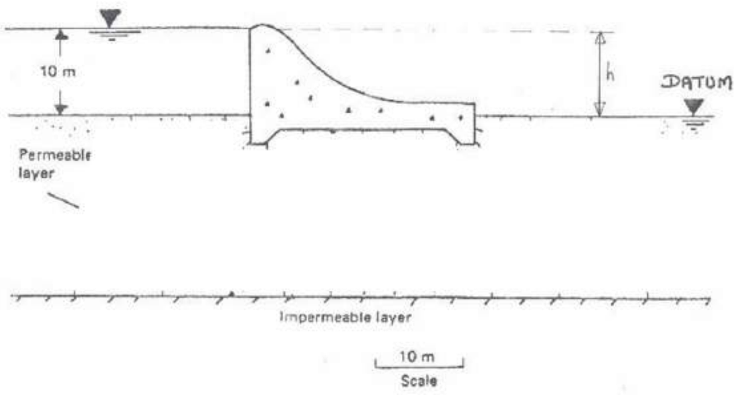
Seepage beneath concrete dam

a) If $k = 0.01$ mm/s, determine the seepage loss under the dam in m^3/day.
b) What is the total head at (i) A, (ii) B, and (iii) C?



First sketch the flow net. Remember

- curved' squares with right-angled intersections;
- free surfaces are equipotentials and;
- impermeable boundaries are flow lines



Flow nets - An example

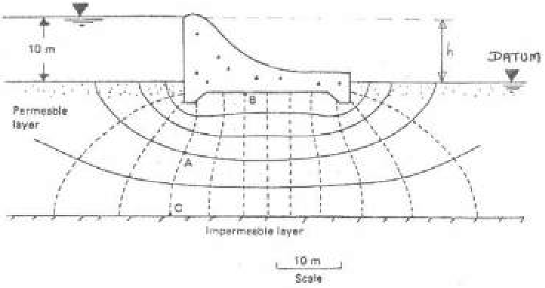
Part a)

$N_f = 5$
$N_d = 12$
$k = 0.01 \text{ mm/s} = (0.01) \times 10^{-3)} \times 3600 \times 24 \text{ m/day} = 0.864 \text{ m/day}$

$Q = \frac{N_f}{N_d} kh$

$Q = \frac{5}{12}(0.864)(10)$

$Q = 3.6 \text{ m}^3/\text{day}$

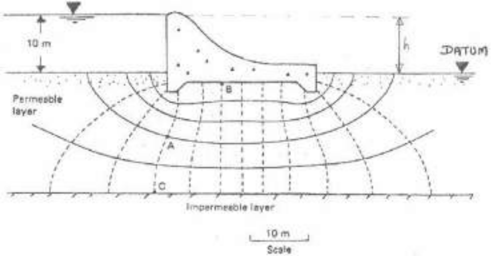


Flow nets - An example

Part b) $\Delta h = \frac{h}{N_d} = \frac{10}{12} = 0.833$ m

(i) To reach A, water has to go through three potential drops
$\rightarrow$ head loss is $3 \times 0.833 = 2.5$ m
$\rightarrow h_A = 10 - 2.5 = 7.5$ m

(ii) To reach B, water has to go through five potential drops
$\rightarrow 5 \times 0.833 = 4.166$ m
$\rightarrow h_B = 10 - 4.166$
$\rightarrow h_B = 5.83$ m



(iii) Points A and C are located on the same equipotential line, hence, $h_C = h_A = 7.5$ m

What is the pore water pressure at B? (Assume B is 1 m below datum)

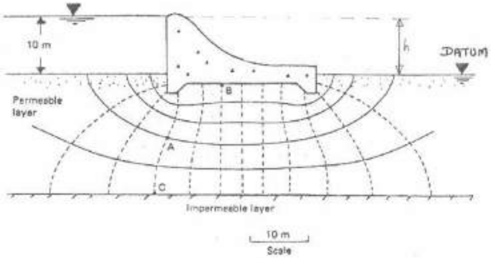
Recall the total head is $h = \frac{u}{\gamma_w} + z$

So at B, $u = (h - z)\gamma_w = (5.83–(–1)) \times 9.81 = 67.0$ kPa

What is the pore water pressure at the 8^{th} equipotential?

$H = 10–(8 \times 0.833)$
$H = 3.333$ m
$u = (3.333–(–1)) \times 9.81$
$u = 42.5$ kPa

Uplift pressures?



Critical hydraulic gradient

The viscous drag of water flowing through a soil imposes a seepage force on the soil in the direction of flow. If flow is upwards these forces can offset the self-weight of a (sandy) soil and lead to a boiling or quick sand condition. The seepage force, J, due to the head loss, in the direction of flow of a flow net element is given by

$$J = \Delta h.b.t.\gamma_w = i.l.b.t.\gamma_w$$

Consider now the soil submerged in a flow net element. The soil exerts a downward force, F, based on its buoyant unit weight

$$F = V\gamma'$$

where V is the volume of the soil ($= l \times b \times t$). When the upward seepage force just balances the submerged weight of the soil, a quick condition exists.

$$F = J$$
$$i.l.b.t.\gamma_w = l.b.t.\gamma'$$

Critical hydraulic gradient

The hydraulic gradient at which the quick condition occurs is referred to as the critical hydraulic gradient, i_c, i.e.,

$$i_c = \frac{\gamma'}{\gamma_w}$$

which for most soils implies i_c is approximately unity. Any increase in hydraulic gradient or seepage force will cause the soil to boil. An increase in seepage force may be caused by either in increase in Δh or by a reduction in l. Thus, in the case of the latter, an excavation into sands below the water table could result in a quick condition.

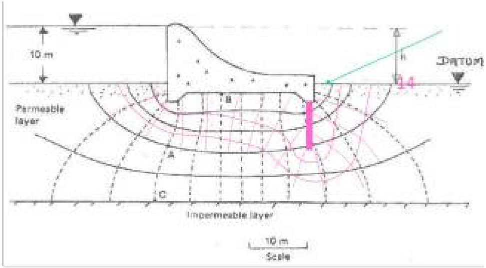
Flow nets - An example

Estimate the critical hydraulic gradient adjacent to the downstream toe of the dam. (Assume the penultimate equipotential is 1 m below datum)

$\Delta h = 0.833$ m
$l = 1$ m
$i = 0.833$
(<1.0 ... just)

What could be done to reduce the critical hydraulic gradient?

A cut off curtain?
Extends the flow path and hence reduces the hydraulic gradient



Anisotropic soil conditions

Most in-situ soils are anisotropic (they behave differently in all directions). Normally natural soils have the maximum coefficient of permeability in the direction of stratification (or bedding) and the minimum coefficient of permeability in the direction perpendicular to that of stratification. Therefore...

$k_x = k_{max}$ and $k_z = k_{min}$ and using Darcy's law again

$$v_x = k_x i_x = -k_x \frac{\delta h}{\delta x}$$
$$v_z = k_z i_z = -k_z \frac{\delta h}{\delta z}$$

Also, in any direction s, inclined at an angle α to the x direction, the coefficient of permeability can be defined as:

$$v_s = -k_s \frac{\delta h}{\delta s}$$

Anisotropic soil conditions

Also, in any direction s, inclined at an angle α to the x direction, the coefficient of permeability can be defined as:

$v_s = -k_s \frac{\delta h}{\delta s}$. Now...

$\frac{\delta h}{\delta s} = \frac{\delta h}{\delta x}\frac{\delta x}{\delta s} + \frac{\delta h}{\delta z}\frac{\delta z}{\delta s}$ i.e. $\frac{v_s}{k_s} = \frac{v_x}{k_x}\cos\alpha + \frac{v_z}{k_z}\sin\alpha$

The components of discharge velocity are also related as follows:

$v_x = v_s \cos\alpha$

$v_z = v_s \sin\alpha$. Hence

$\frac{1}{k_s} = \frac{\cos^2\alpha}{k_x} + \frac{\sin^2\alpha}{k_z}$ or $\frac{s^2}{k_s} = \frac{x^2}{k_x} + \frac{z^2}{k_z}$

Conclusion: You can define the directional variation of permeability using the equation for a ellipse.

Anisotropic soil conditions

Given Darcy's law, the equation of continuity can be written as:

$$k_x \frac{\delta^2 h}{\delta x^2} + k_z \frac{\delta^2 h}{\delta z^2} = 0 \text{ or } \frac{\delta^2 h}{(k_z/k_x)\delta x^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

Substituing $x_t = x\sqrt{\frac{k_z}{k_x}}$ the equation of continuity becomes:

$$\frac{\delta^2 h}{\delta x_t^2} + \frac{\delta^2 h}{\delta z^2} = 0$$

which is the equation of continuity for an isotropic soil (i.e. that behaves equally in all directions) in a $x_t - z$ plane.

Anisotropic soil conditions

So how do you do a flow net for anisotropic soil conditions?

$$x_t = x\sqrt{\frac{k_z}{k_x}}$$

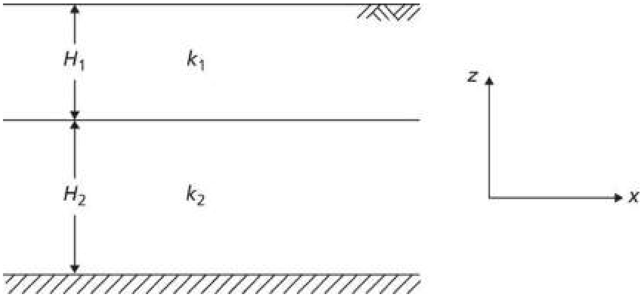
x_t defines a scale factor which can be applied in the x direction to transform a given anisotropic flow region into a fictitious isotropic flow region in which the Laplace equation is valid.

Once the flow net (which represents the solution of the Laplace equation) has been drawn for the transformed section, the flow net for the natural section can be obtained by applying the inverse of the scaling factor. Essential data, however can normally be obtained from the transformed section. The necessary transformation could also be made in the z direction. The value of the coefficient of permeability applying to the transformed section, referred to as the equivalent isotropic coefficient is:

$$k' = \sqrt{k_x k_z}$$

Non-homogeneous soil conditions

Two isotropic soil layers of thicknesses H_1 and H_2 are shown below, the respective coefficients of permeability being k_1 and k_2; the boundary between the layers is horizontal. (If the layers are anisotropic, k_1 and k_2 represent the equivalent isotropic coefficients for the layers). The two layers can be approximated as a single homogeneous anisotropic layer of thickness $H_1 + H_2$ in which the coefficients in the directions parallel and normal to that of stratification are $\overline{k_x}$ and $\overline{k_z}$, respectively.



Non-homogeneous soil conditions - Horizontal seepage

For one-dimensional seepage in the horizontal direction, the equipotentials in each layer are vertical. If h_1 and h_2 represent total head at any point in the respective layers, then for a common point on the boundary $h_1 = h_2$. Therefore, any vertical line through the two layers represents a common equipotential. Thus the hydraulic gradients in the two layers, and in the equivalent single layer, are equal; the equal hydraulic gradients are denoted by i_x. The total horizontal flow per unit time is then given by:

$$\overline{q_x} = (H_1 + H_2)\overline{k_x} i_x = (H_1 k_1 + H_2 k_2) i_x$$

$$\overline{k_x} = \frac{(H_1 k_1 + H_2 k_2)}{H_1 + H_2}$$

Non-homogeneous soil conditions - Vertical seepage

For one-dimensional seepage in the vertical direction, the discharge velocities in each layer, and in the equivalent single layer, must be equal if continuity is to be satisfied. Then,

$$v_z = \overline{k_z}\,\overline{i_z} = k_1 i_1 = k_2 i_2$$

where $\overline{i_z}$ is the average hydraulic gradient over the depth $(H1 + H2)$. Therefore

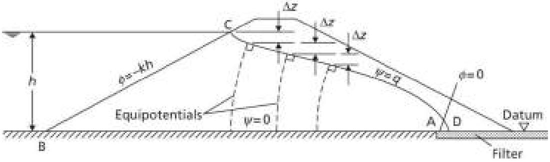
$$i_1 = \frac{\overline{k_z}}{k_1}\overline{i_z} \text{ and } i_2 = \frac{\overline{k_z}}{k_2}\overline{i_z}$$

Now the loss in total head over the depth $(H1 + H2)$ is equal to the sum of losses in total head in the individual layers, i.e.

$$\overline{i_z}(H_1 + H_2) = i_1 H_1 + i_2 H_2 = \overline{k_z}\,\overline{i_z}\left(\frac{H_1}{k_1} + \frac{H_2}{k_2}\right)$$

$$\overline{k_z} = \frac{H_1 + H_2}{\frac{H_1}{k_1} + \frac{H_2}{k_2}}$$

Seepage through embankment dams



The main issue here is how to determine the top flow line (i.e. phreatic surface). Note:

- At every point in segment BC the total head is constant (i.e. an equipotential)
- Segment BA is a flow line
- Discharge surface AD is the equipotential for zero head.
- At every point of the CD line the pressure is 0, so total head is equal to elevation head and there must be equal vertical intervals Δz between the points of intersection between successive equipotentials and the top flow line.

Recap

- What is the phreatic surface? What does it separate in terms of degree of saturation? How are the pressures above, in and below this surface?
- How are pore water pressures calculated?
- Is there groundwater flow if pore water pressures are hydrostatic? What does this mean?
- Why is a gradient necessary for groundwater flow to occur?
- What is the hydraulic head? What are the three components of Bernoulli's equation?
- How is the hydraulic gradient defined?
- What is Darcy's law? Why is it important?
- What is the coefficient of permeability and how do you determinate it?
- What is the difference between the constant head test and the variable head test?
- What is Laplace's equation used for?

Recap

- What is the relationship between Laplace's equation and a flow net?
- What is a flow line?
- What is an equipotential line?
- What is the critical hydraulic gradient? Why is it important to know about it?
- You need to know how to calculate flow using flow nets. You have learnt to do this for embankment dams, and various soil conditions including anisotropic and non-homogenous soils.

Before the break...

Are there any questions?