Edinburgh Napier UNIVERSITY

SOIL MECHANICS

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Unit 6 - Effective Stress

May 26th, 2025

Day	**08:00-09:30**	**09:45-11:15**	**13:00-14:30**	**14:45-16:15**
19/05/25	Introduction	Programming	Phase Rel.	Tutorial
20/05/25	Classification	Tutorial	LAB	LAB
21/05/25	Compaction	Tutorial		
22/05/25	Groundwater	Tutorial	LAB	LAB
23/05/25	Groundwater	Tutorial		
26/05/25	Effective Str.	Tutorial	Stress Incr.	Tutorial
27/05/25	Compressib.	Tutorial	LAB	LAB
28/05/25	Consolidation	Tutorial		
29/05/25	Shear Str.	Tutorial	Shear.Str.	Tutorial
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2. Total stress and unit weight
3. Pore water pressure
4. Effective stress
5. Consequences: Undrained and drained behaviour

The principle of effective stress is the single most important concept in soil mechanics.

*"All **measurable** effects of a change of stress, such as compression and distortion are due **exclusively** to changes in effective stress."*

Karl Terzaghi

Why is this lecture important?

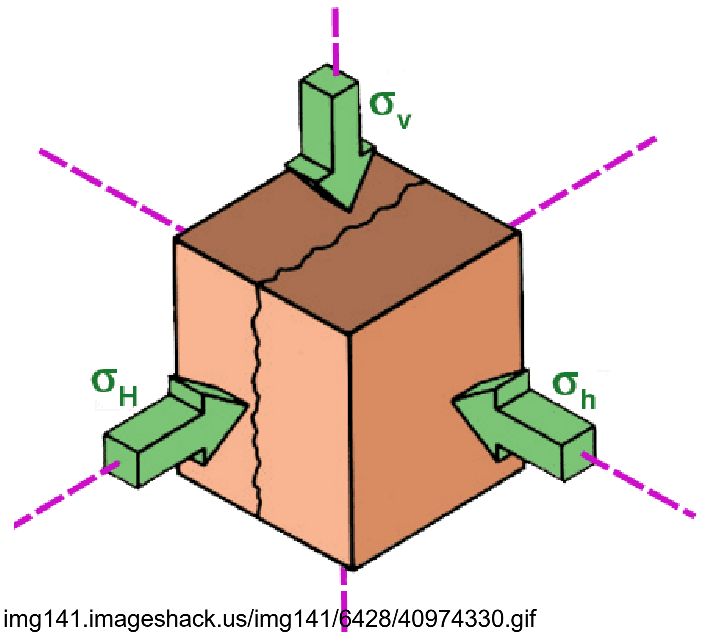
STRESSES IN THE GROUND: TOTAL AND EFFECTIVE

The first step in geotechnical engineering calculations, e.g. foundation design, wall design, slope stability, is the determination of the stress state of the ground.

The magnitude of the stresses will change with surface loading or excavation so we must consider the change in stress, i.e. the stress increment.

Often, vertical stresses are sufficient to allow analysis to proceed. But for the design of walls, tunnels, culverts, etc., horizontal stresses will also be required. In this lecture, we restrict our attention to vertical stresses only.

We begin our study of stresses in the ground with calculation of total stress. Total stress calculations are based on the unit weight of the soil (rather like pore water pressures).



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Stress in the ground: total & effective

Total Stress and Unit Weight

The total vertical stress acting at a point below ground surface is due to the weight of everything lying above, e.g. soil, water, surface loading.

<u>a) Total stress in a homogeneous soil mass</u>

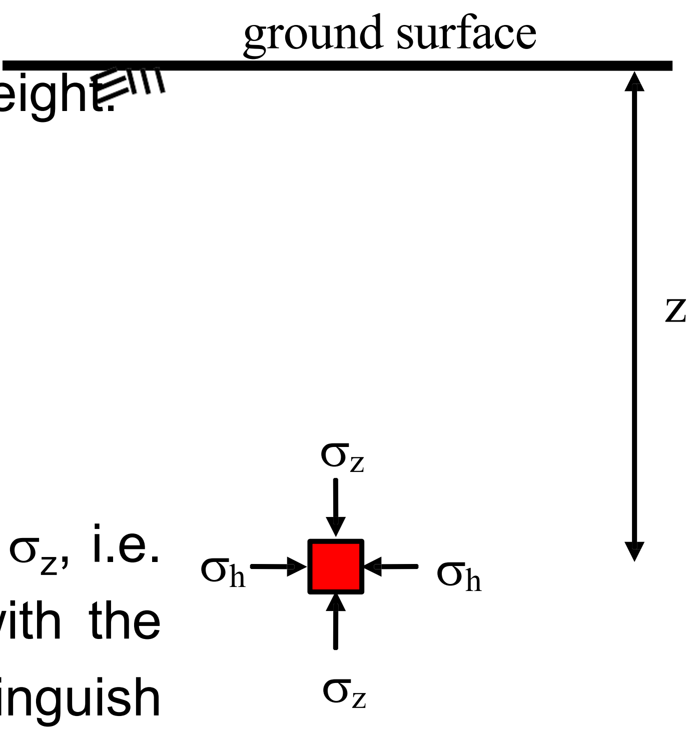
Total stress increases with depth and with unit weight.

Vertical total stress at depth z,

$$\sigma_v = \gamma . z$$

where γ is the unit weight of the soil

The symbol for total stress may also be written σ_z, i.e. related to depth z. The unit weight will vary with the water content of the soil and we may distinguish between unit weight above and below the water table.

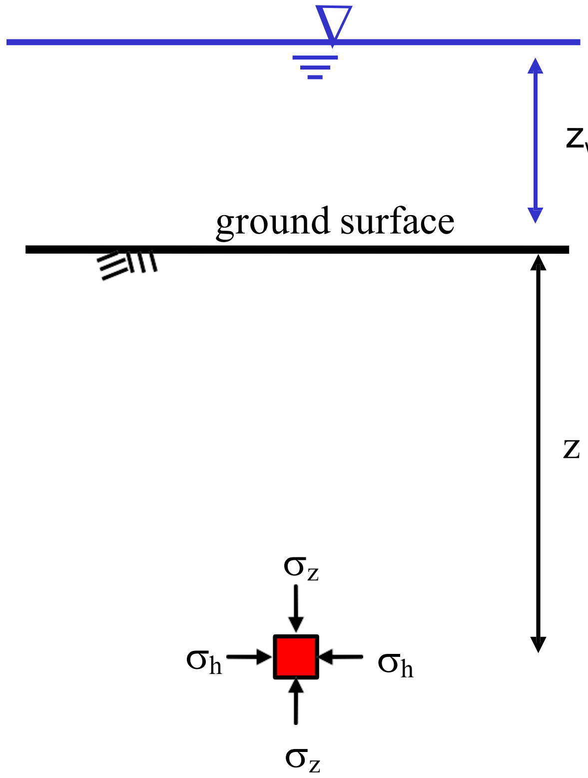


b) Total stress in a soil mass below a river or lake

The total stress is the sum of the weight of the soil up to the surface and the weight of water above this:

Vertical total stress at depth z,

$$\sigma_z = \gamma . z + \gamma_w z_w$$



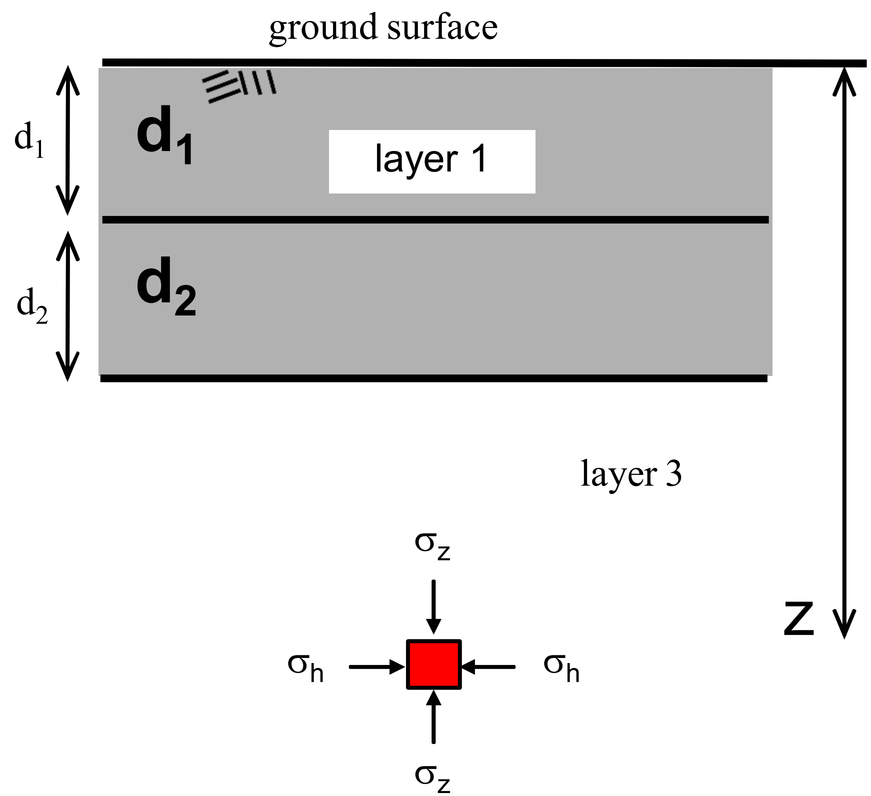
where γ is the unit weight of the *saturated* soil, i.e. the total weight of soil grains and water and γ_w is the unit weight of water.

The vertical total stress will change with changes in water level and with excavation.

Total stress and unit weight

(c) Total stress in a layered soil mass

The total stress at depth z is the sum of the weights of soil in each layer above.



Vertical total stress at depth z,

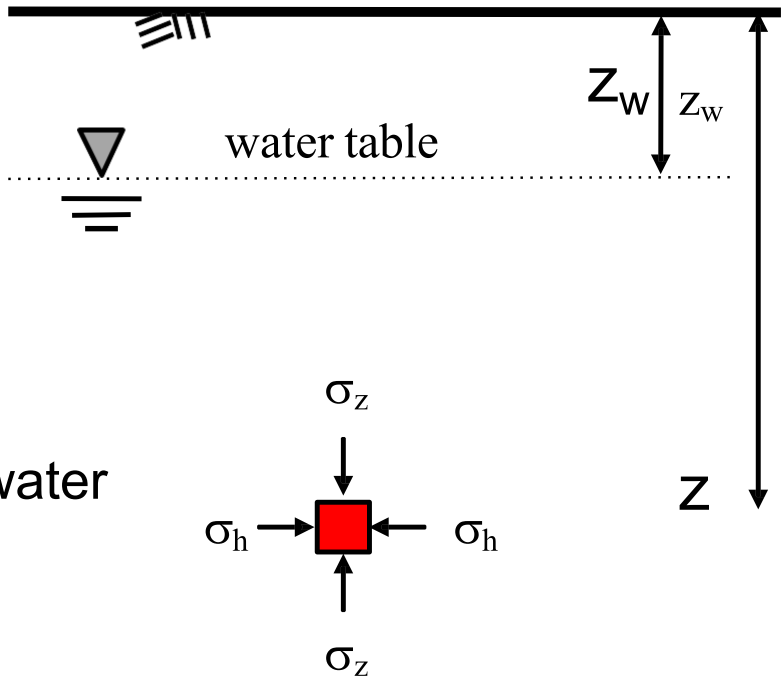
$$\sigma_z = \gamma_1 d_1 + \gamma_2 d_2 + \gamma_3(z - d_1 - d_2)$$

where γ_1, γ_2, γ_3, etc. are unit weights of soil layers 1, 2 , 3, etc respectively.

If a new layer is placed on the surface the total stresses at all points below will increase.

(d) Total stress in a soil mass with water table below ground level

The soils above and below the water table may be treated separately:



Below the GWT: the soil is saturated:

$\gamma = \gamma_{sat}$

Above the GWT: the soil can vary from dry to saturated:

1. sands and gravels soils may become dry
2. clays may remain saturated with capillary water

Vertical total stress at depth z,

$$\sigma_z = \gamma z_w + \gamma_{sat}(z - z_w)$$

where γ is the unit weight of the soil lying above the water table.

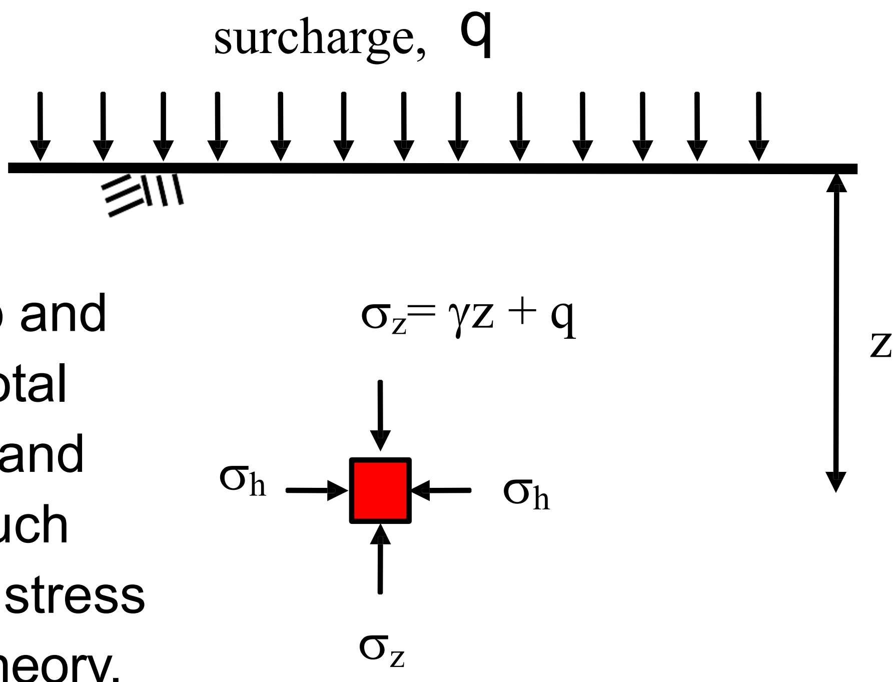
If the water table level changes and the soil above it is not saturated, the total stresses below it will also change.

(e) Total stress in a soil mass with a surface surcharge load

The addition of a surface surcharge load will increase the total stresses below it. If the surcharge loading is extensive (wide), the increase in vertical total stress below it may be considered constant with depth and equal to the magnitude of the surcharge

Vertical total stress at depth z,

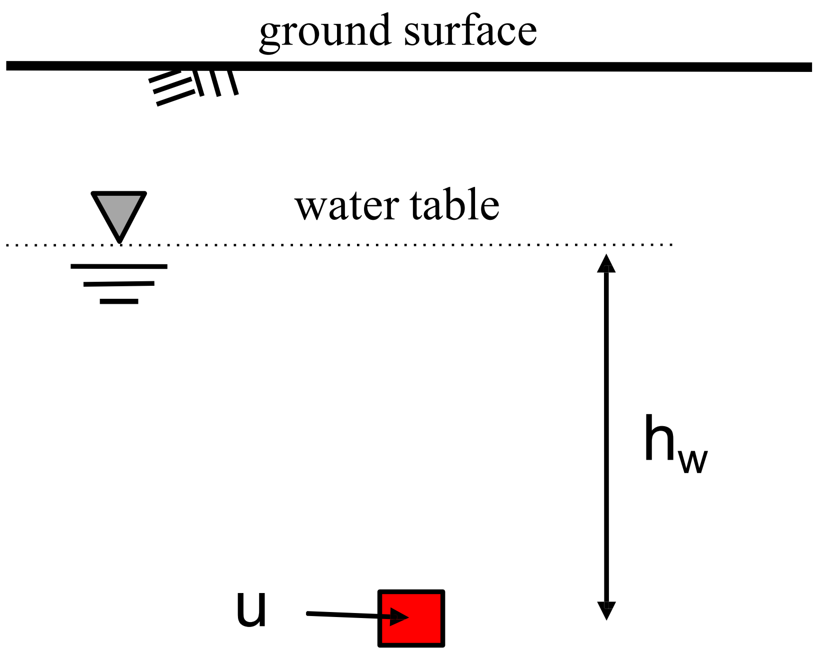
$$\sigma_z = \gamma . z + q$$



For narrow surcharges, e.g. under strip and pad foundations, the induced vertical total stresses will decrease both with depth and horizontal distance from the load. In such cases, it is necessary to use a suitable stress distribution theory (e.g. Boussinesq's theory, covered later).

Let us remind ourselves about pore water pressure. It is going to be a very important part of what follows.

$$u = \gamma_w h_w$$



Pore water pressure

And now for the single most important concept in soil mechanics and geotechnical engineering …

Effective Stress

Ground movements and instability can be caused by changes in total stress; e.g. loading due to foundations, unloading due to excavations.

They can also be caused by changes in pore water pressures; e.g. slope failures which occur after rainfall due to increasing pore water pressures.

It is the combined effect of total stress and pore water pressure that controls soil behaviours such as shear strength, compression and distortion.

The combined effect of load and pore water pressure is captured by the concept of effective stress (σ'), i.e.

Effective stress = total stress – pore water pressure

$$\sigma' = \sigma - u$$

Effective stress

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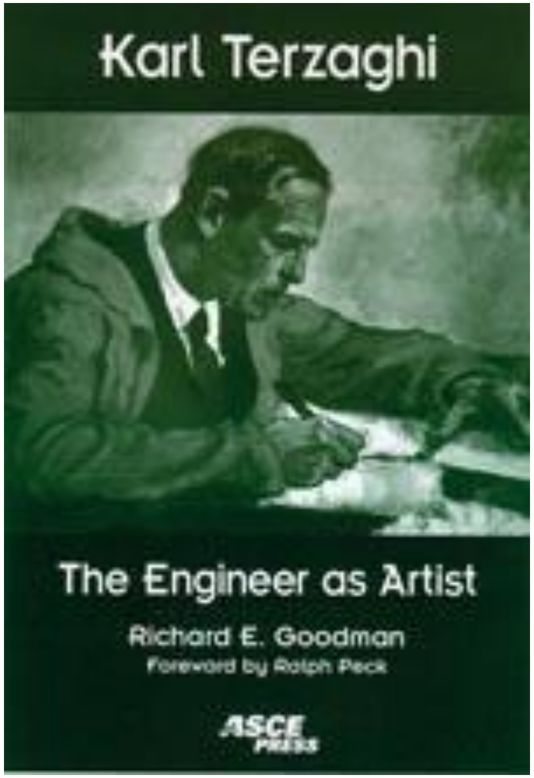
Karl Terzaghi was born in Vienna and subsequently became professor of soil mechanics in the USA (at MIT and Harvard). In 1936 he proposed the relationship for effective stress.



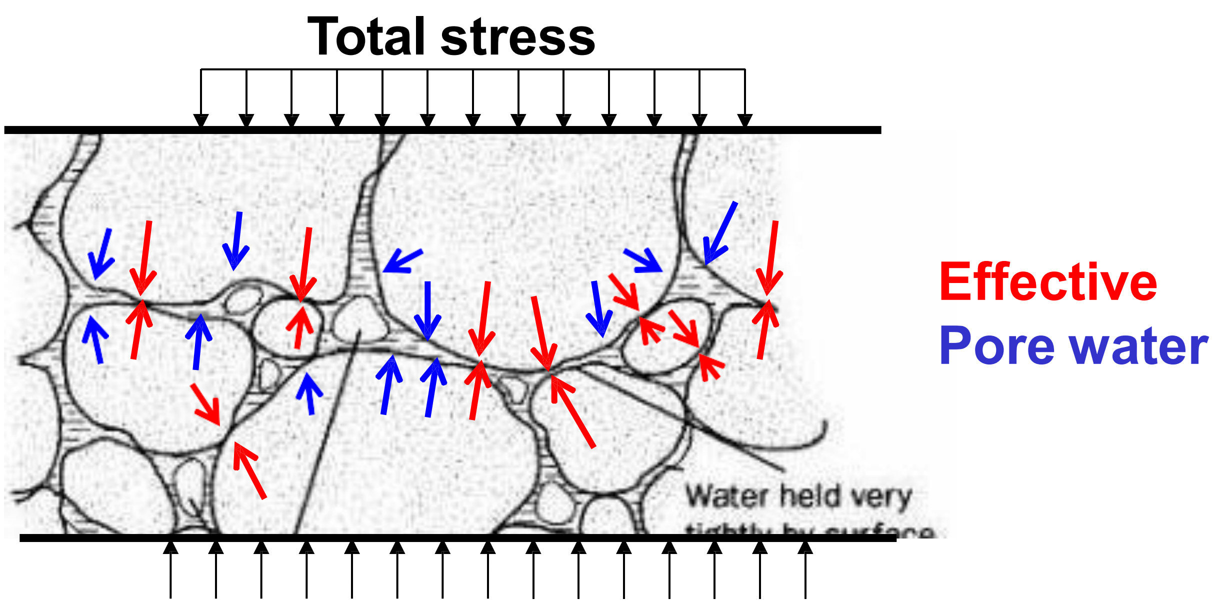
*All **measurable** effects of a change of stress, such as compression and distortion are due **exclusively** to changes in effective stress.*

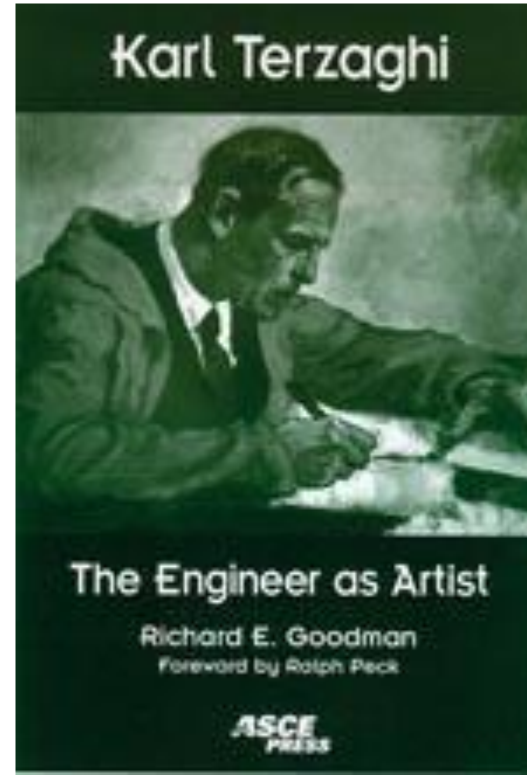
The adjective *effective* is particularly apt, because it is effective stress that is 'effective' in causing important changes: changes in strength, changes in volume, changes in shape.

Effective stress

Terzaghi's principle of effective stress

You can think of effective stress as that part of the total stress that is carried by the soil skeleton – the interparticle stress. The remaining part of the total stress is carried by the pore water pressure.





Case (a): water level below ground level

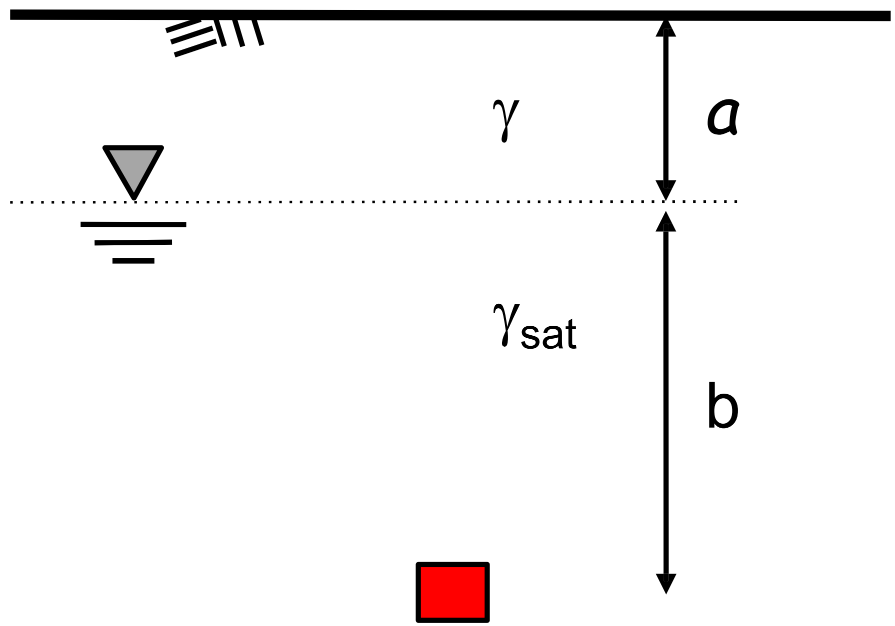
$$o = \gamma.a + \gamma_{sat} b$$

$$u = \gamma_w b$$

$$\sigma' = \sigma - u = \gamma.a + \gamma_{sat} b - \gamma_w b$$

$$= \gamma.a + (\gamma_{sat} - \gamma_w) b$$

$$\sigma' = \gamma.a + \gamma'.b$$



which means that changes in GWL cause changes in σ'

N.B. γ & γ_{sat} may or may not be the same: clays and sands

Case (b): water level above ground level

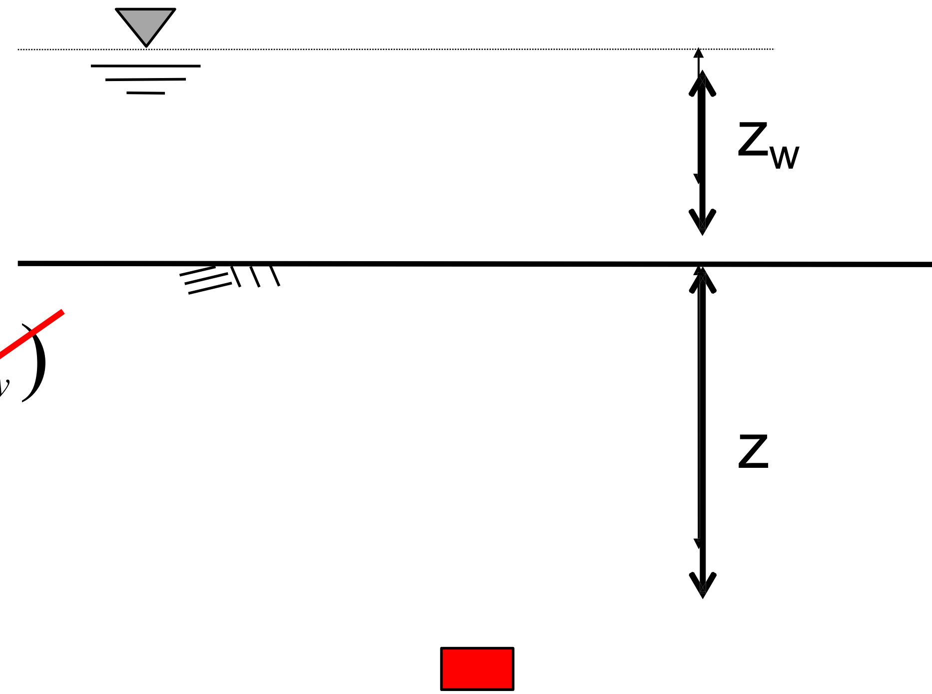
$$\sigma = \gamma . z + \gamma_w z_w$$

$$u = \gamma_w \left(z + z_w \right)$$

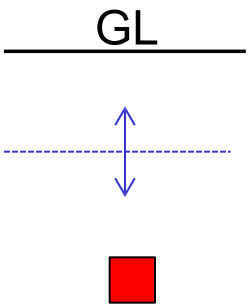
$$\sigma' = \sigma - u = \gamma . z + \cancel{\gamma_w z_w} - \gamma_w \left(z + \cancel{z_w} \right)$$

$$= \left(\gamma - \gamma_w \right) z$$

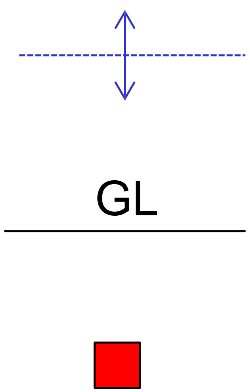
$$\sigma' = \gamma' . z$$



which means changes in GWL do not cause changes in σ‘



In other words, changes in water level below ground level result in changes in effective stresses below the water table as the depth of submerged soil and partially saturated soil lying above the depth of measurement changes.



But changes in water level above ground (e.g. in lakes, rivers, etc.) do not cause changes in effective stresses in the ground below as the depth of submerged material is unchanged.

Effective stress

Example 1 Total and effective stresses

The figure shows 3 soil layers at a site.

Unit weights are:

dry sand, γ_d = 16 kN/m^3

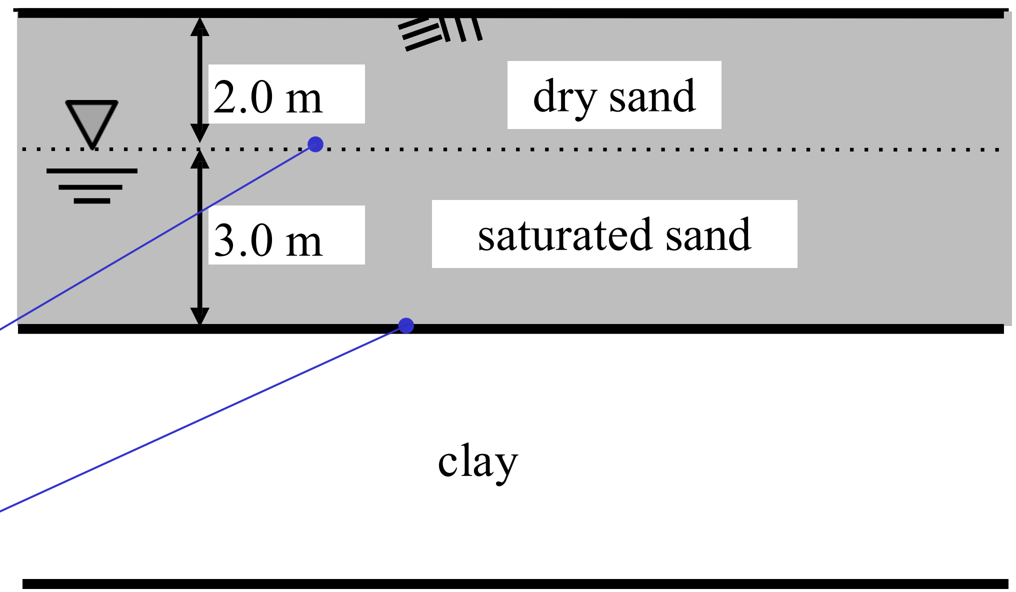
saturated sand, γ_{sat} = 20 kN/m^3

Determine:

(a) the vertical total stress,

(b) pore water pressure

(c) vertical effective stress at depths of 2.0 m and 5.0 m.



Effective stress: Example 1

Example 1 Total and effective stresses

Solution:

At the top of the saturated sand (z = 2.0 m):

Vertical total stress,

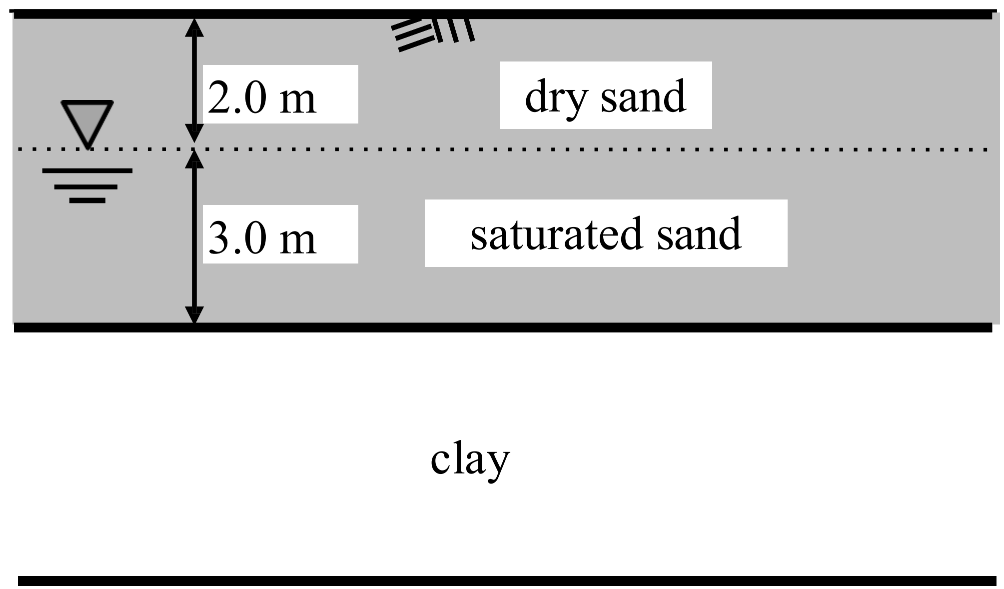
σ_z = 16.0 x 2.0 = 32.0 kN/m^2

Pore water pressure,

u = 0

Vertical effective stress,

$\sigma'_z = \sigma_z - u$ = 32.0 kN/m^2



Example 1 Total and effective stresses

At the top of the clay (z = 5.0 m):

Vertical total stress,

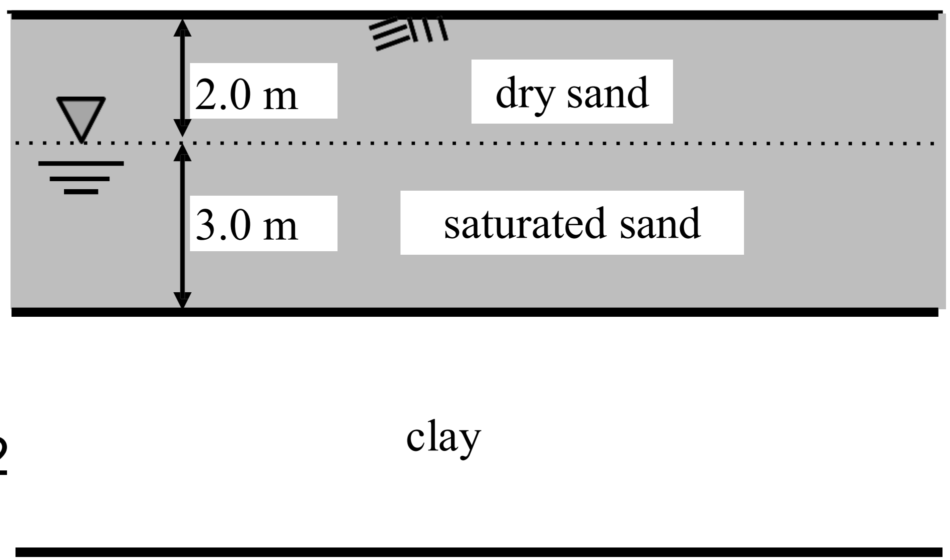
σ_z = 32.0+20.0 x 3.0=92.0 kN/m^2

Pore water pressure,

u = 9.81 x 3.0 = 29.4 kN/m^2

Vertical effective stress,

$\sigma'_z = \sigma_z - u$ = 92.0 -29.4 = 62.6 kN/m2



Example 2: Total and effective stresses

A soil profile consists of 5m sand overlying 4m gravel, resting on bedrock. GWL is 2m below ground surface.

ρ_b sand above GWL = 1.7 Mg/m^3

ρ_{sat} sand below GWL = 2.05 Mg/m^3

ρ_{sat} gravel = 2.15 Mg/m^3

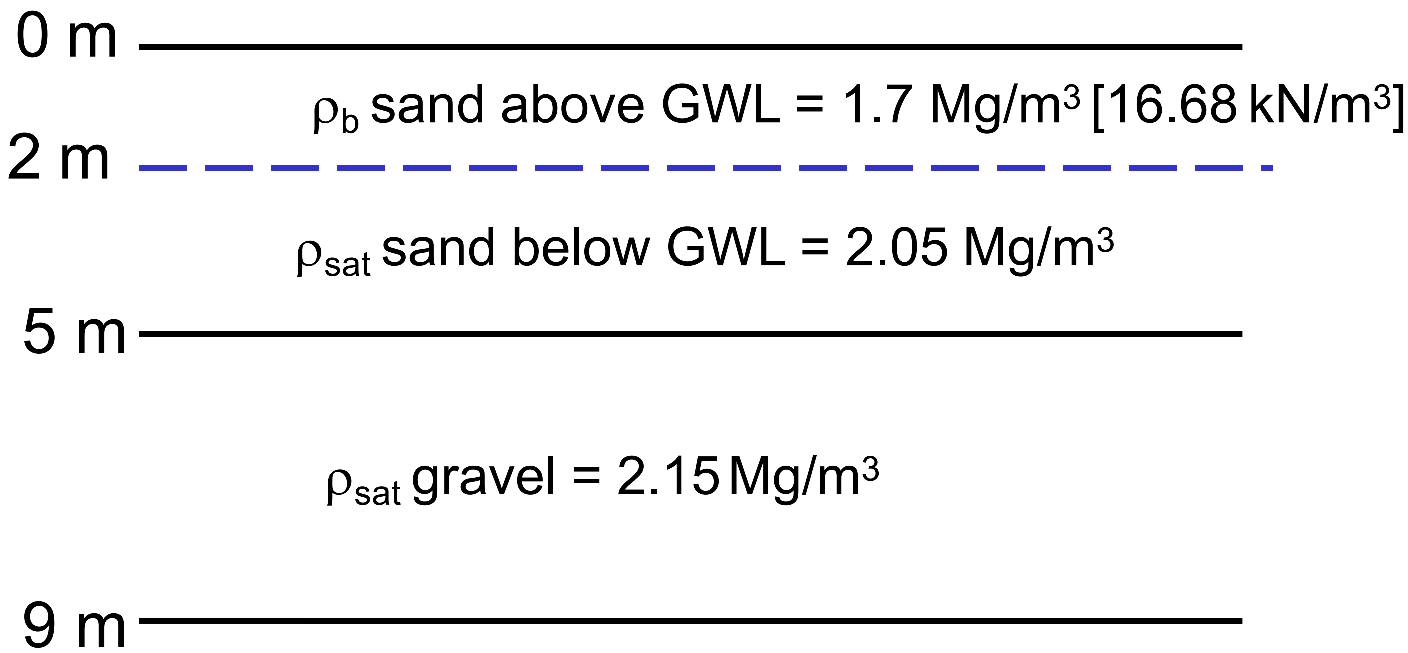
(a) Determine distributions of vertical total stress, pore water pressure, and vertical effective stress with depth down to bedrock.

(ρ_w = 1.0 Mg/m^3)

(b) How do the distributions change if the GWL is lowered to the sand/gravel interface?

Effective stress: Example 2

A soil profile consists of 5m sand overlying 4m gravel, resting on bedrock. GWL is 2m below ground surface.



Effective stress: Example 2

A comment on converting densities to unit weights

Remember,

$$\rho.g = \gamma$$

With units: $\rho.[kg/m^3] \times g\ [m/s^2] = \gamma\ [N/m^3]$

Hence, $1000\ [kg/m^3] \times 9.81\ [m/s^2] = 9810\ [N/m^3]$

then divide by 1000 to obtain the familiar 9.81 $[kN/m^3]$

So giving densities in Mg/m^3 enables unit weights in kN/m^3 to be directly obtained:

$$\rho_b \text{ sand} = 1.7\ Mg/m^3$$

$$\gamma \text{ sand} = 1.7 \times 9.81 = 16.68\ kN/m^3$$

Effective stress: Example 2

A soil profile consists of 5m sand overlying 4m gravel, resting on bedrock. GWL is 2m below ground surface.

0 m

ρ_b sand above GWL = 1.7 Mg/m^3 [16.68 kN/m^3]

2 m

ρ_{sat} sand below GWL = 2.05 Mg/m^3

5 m

ρ_{sat} gravel = 2.15 Mg/m^3

9 m

Calculations (a)

Depth = 0m σ_z = 0; u = 0; σ' = 0 kN/m^2

Depth = 2m σ_z = 1.70 x 9.81 x 2 = 33.35 kN/m^2

u = 0

σ'_z = 33.35 – 0 = 33.35 kN/m^2

Depth = 5m σ_z = 33.35 + (2.05 x 9.81 x 3) = 93.69 kN/m^2

u = 1 x 9.81 x 3 = 29.43 kN/m^2

σ'_z = 93.69 – 29.43 = 64.26 kN/m^2

Depth = 9m σ_z = 93.69 + (2.15 x 9.81 x 4) = 178.05 kN/m^2

u = 1 x 9.81 x 7 = 68.67 kN/m^2

σ'_z = 178.05 – 68.67 = 109.38 kN/m^2

Depth = 0m σ_z = 0; u = 0; σ' =

Depth = 2m σ_z = 33.35 kN/m^2

u = 0

σ'_z = 33.35 kN/m

Depth = 5m σ_z = 93.69 kN/m^2

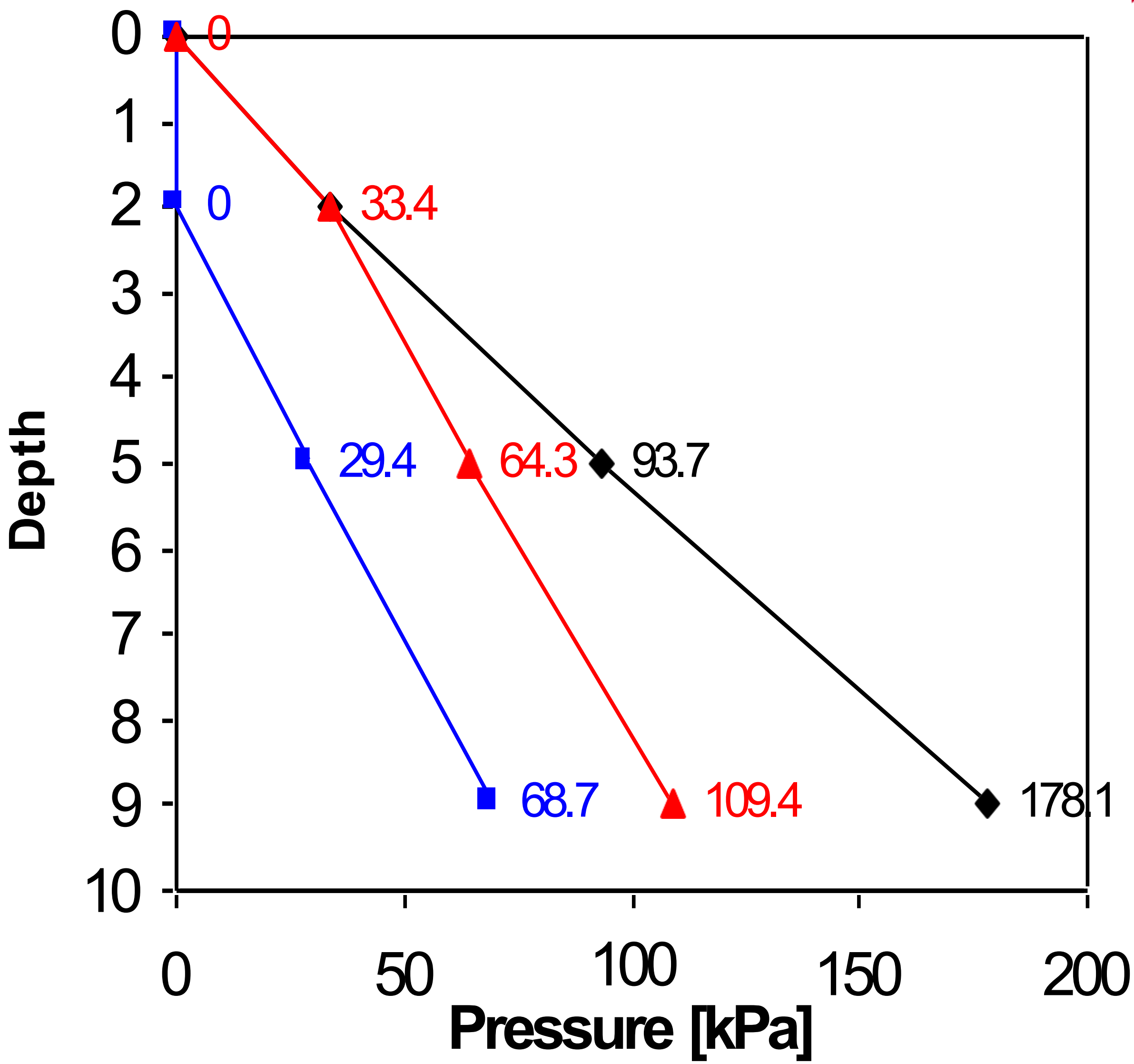
u = 29.43 kN/m^2

σ'_z = 64.26 kN/m

Depth = 9m σ_z = 178.05 kN/m^2

u = 68.67 kN/m^2

σ'_z = 109.38 kN/m



Effective stress

A soil profile consists of 5m sand overlying 4m gravel, resting on bedrock. GWL is 2m below ground surface.

0 m

ρ_b sand above GWL = 1.7 Mg/m^3 [16.68 kN/m^3]

ρ_{sat} sand below GWL = 2.05 Mg/m^3

5 m

ρ_{sat} gravel = 2.15 Mg/m^3

9 m

Effective stress: Example 2

Calculations (b)

<u>Depth = 0m</u> σ_z = 0; u = 0; σ' = 0

<u>Depth = 5m</u> σ_z = 1.70 x 9.81 x 5 = 83.39 kN/m^2

u = 0

σ'_z = 83.39 – 0 = <u>83.39</u> kN/m^2

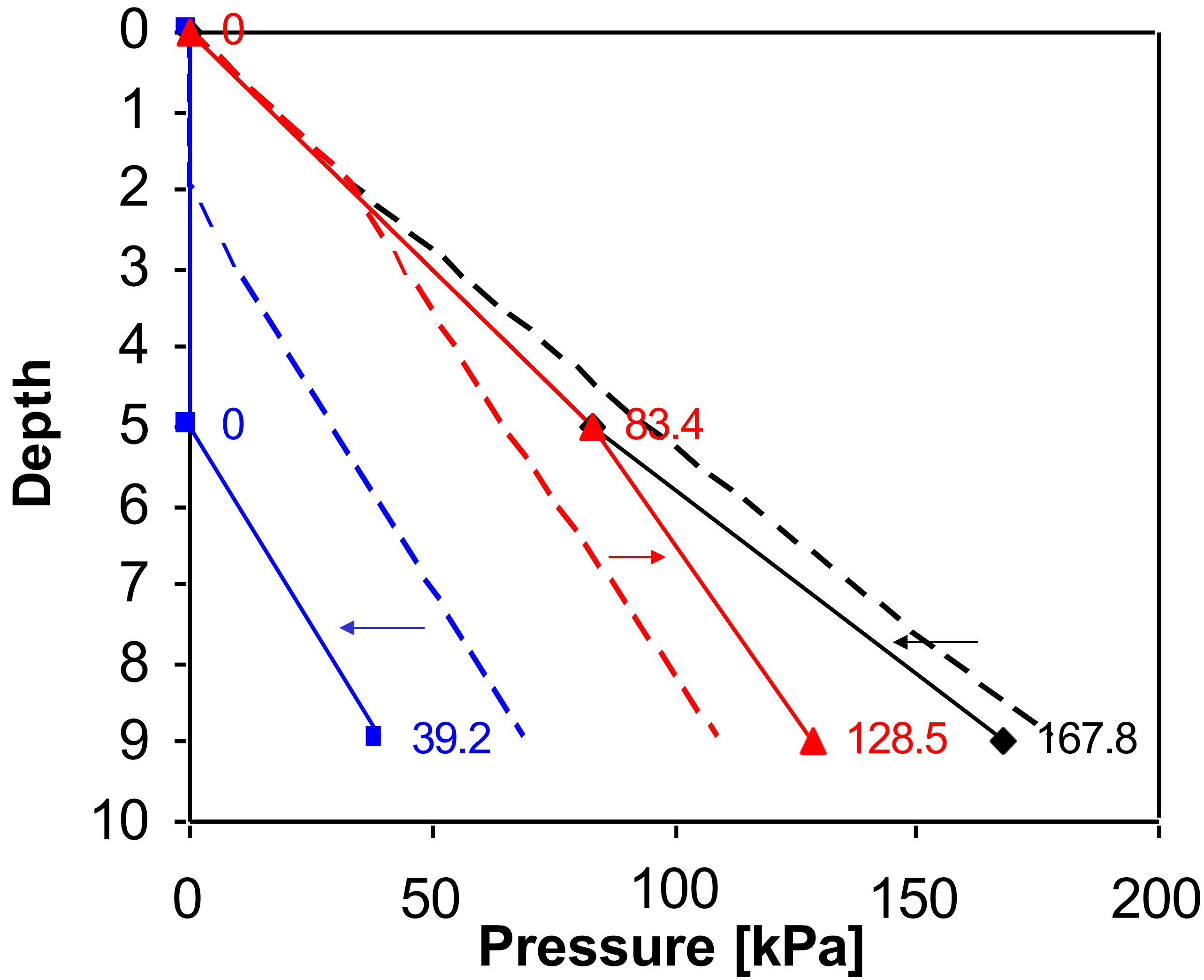
<u>Depth = 9m</u> σ_z = 83.39 + (2.15 x 9.81 x 4) = 167.75 kN/m^2

u = 1 x 9.81 x 4 = 39.24 kN/m^2

σ'_z = 167.75 – 39.24 = <u>128.51</u> kN/m^2

Effective stress

Lowering of GWL has reduced pore water pressure and total stress whilst effective stress is increased – exactly as indicated in case (a) above



Effective stress

Consequences of the principle of effective stress (Terzaghi):

- The behaviour of two soils having same mineralogy and structure will be the same if they have the same effective stress.

- If a soil is loaded or unloaded without a change of volume [as in an undrained (??) test] and without any distortion [under cell pressure control only], there will be no change in effective stress.

- Soil will swell in volume (and weaken) or compress (and strengthen) if pore pressure alone is raised or lowered.

Sometimes it is better work in changes of quantity, rather than in absolute quantities. The effective stress expression then becomes:

$$\Delta\sigma' = \Delta\sigma - \Delta u$$

A change in total stress with pore water pressures constant, i.e. $\Delta u = 0$, means $\Delta\sigma' = \Delta\sigma$ and there will be a change in volume – it could be compression due to an increase in effective stress or swelling due to a decrease.

If both total stress and pore water pressure change by the same amount, i.e. $\Delta\sigma = \Delta u$, then $\Delta\sigma' = 0$ and the effective stress is constant. There is no change in condition (strength or volume) of the soil.

Changes in effective stress: drained & undrained conditions

Now consider a change in ground water level due to say pumping, i.e. $\Delta\sigma = 0$, then $\Delta\sigma' = \Delta u$.

In this case total stress remains constant and pore water pressure changes. As above, a change in effective stress occurs and the soil state will change accordingly.

You should now be clear that any change in the state of a soil is dependent on a change in the effective stress state, which in turn is dependent on both total stress and pore water pressure changes.

$$\Delta\sigma' = \Delta\sigma - \Delta u$$

Changes may be additive or they may cancel each other out.

Changes in effective stress: drained & undrained conditions

The aforegoing analysis has implicitly assumed the soil will drain freely; the soil immediately adjusts to a change in loading or groundwater level.

In practice, the speed of adjustment is dependent on the permeability of the soil. We have assumed a sandy soil, i.e. one of high permeability. On loading the soil skeleton and pore water is compressed. But in a clay, with low permeability, the pore water is trapped. The immediate effect in a clay is for the pore water pressure to increase by the amount of additional load. This temporary condition, during which time the pore water is trapped and no volume change can occur, is know as the UNDRAINED condition.

Undrained condition

The undrained condition is an important concept in soil mechanics. In the undrained state, the existing soil strength and volume cannot change. There is no change in effective stress as $\Delta\sigma = \Delta u$. Any additional load must be carried by the existing soil strength.

Undrained condition

As the pore water is squeezed out and pore water pressure dissipates, the clay soil compresses. In so doing it mobilises additional strength. The (temporary) undrained condition is eventually given up in favour of some new long term condition in which both volume and strength can change, indeed they equilibrate with the externally applied load. This is the **DRAINED** condition.

Drained condition

The drained state may result in an increase in soil strength – as in the case of dissipation of positive excess pore water pressures. Or indeed it may lead to a decrease in soil strength, if the initial loading was one of unloading and triggered an initial reduction in pore water pressure – as can happen in ground adjacent to newly formed cuttings. There are cases of failures of cuttings in stiff London clay that have occurred many years after construction and have been attributable to loss of construction- induced negative water pressures.

Drained condition

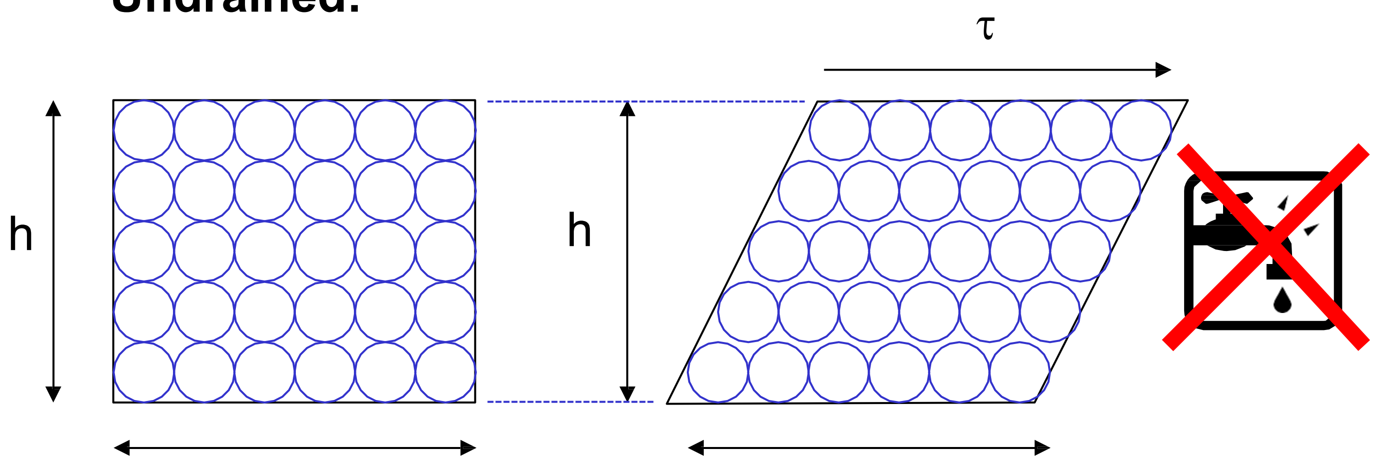
Of course the distinction between undrained and drained conditions depends not only on the permeability of the soil but also on the rate of loading; a clay soil can be loaded in a drained manner if the rate of loading is sufficiently slow to allow pore water pressure build up. Conversely, a silty sandy soil could experience undrained conditions if loading were sufficiently quick – as would be the case with earthquake loading.

We will return to the idea of rate of loading and pore water dissipation in the study of consolidation.

Drained condition

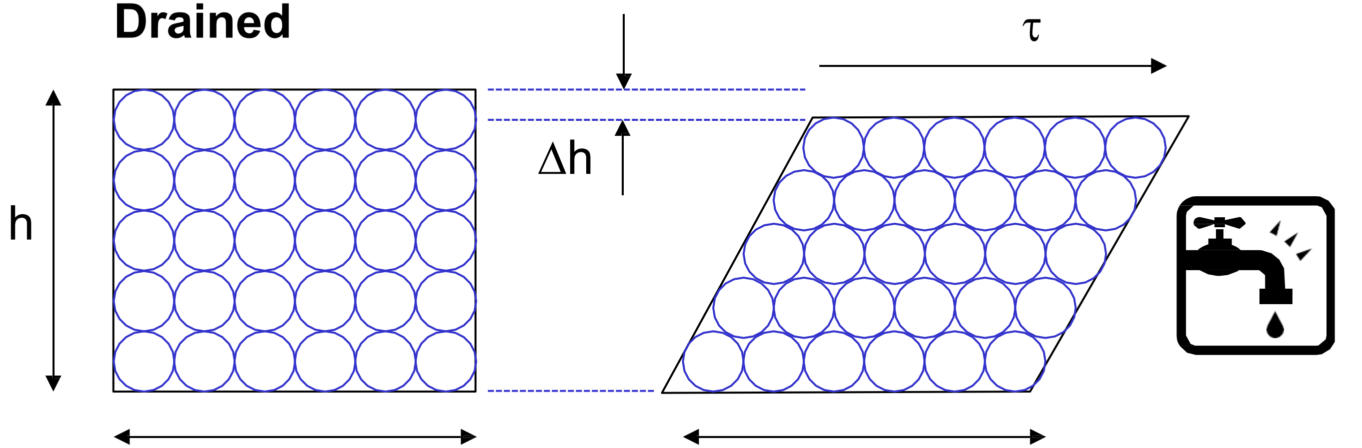
Effect of drainage on pore water pressure and effective stress

Undrained:



No drainage
No volume change
No densification
PWP increase
σ' no change
Short term

Drained



Drainage
Volume change
Densification
PWP no change
σ' increase
Long term

At this stage you should:

- Recognise the influence of soil unit weight and water on total stress
- Recognise the concept of effective stress
- Understand the influence of pore water pressure on the effective stress.
- Appreciate the impact of effective stress on the volumetric state of a soil
- Appreciate the concepts of undrained and drained conditions

STRESS IN THE GROUND: Summary