SOIL MECHANICS

Dr Daniel Barreto
d.barreto@napier.ac.uk

Unit 10 – Shear Strength

May 29th and 30th, 2025

Day	08:00-09:30	09:45-11:15	13:00-14:30	14:45-16:15
19/05/25	Introduction	Programming	Phase Rel.	Tutorial
20/05/25	Classification	Tutorial	LAB	LAB
21/05/25	Compaction	Tutorial		
22/05/25	Groundwater	Tutorial	LAB	LAB
23/05/25	Groundwater	Tutorial		
26/05/25	Effective Str.	Tutorial	Stress Incr.	Tutorial
27/05/25	Compressib.	Tutorial	LAB	LAB
28/05/25	Consolidation	Tutorial		
29/05/25	Shear Str.	Tutorial	Shear.Str.	Tutorial
30/05/25	Shear Str.	Review		

1. Stress & strain in soils. Failure of soils
2. Coulomb failure criterion
3. The direct shear box test

If the most recent lectures have considered serviceability (i.e. magnitude and time for settlement), the following lectures deal with the ultimate limit state of soils (i.e. failure)

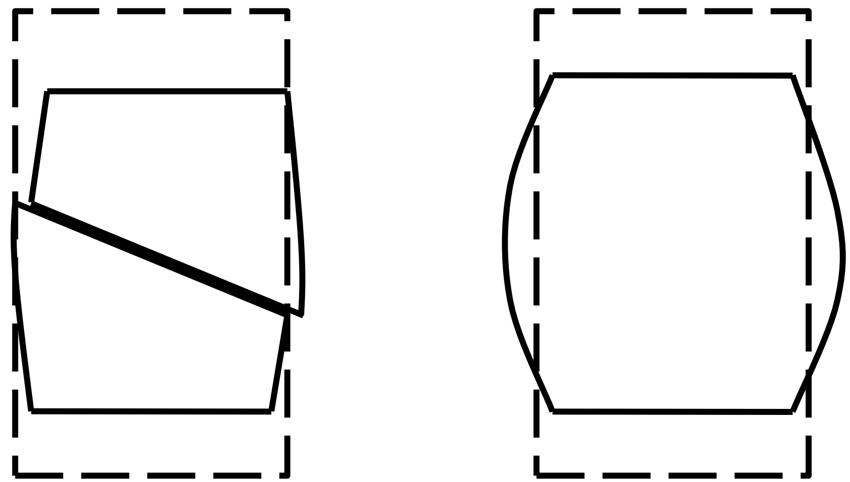
You should by now appreciate that failure needs to be avoided at all costs. This lecture introduces you to the concept of failure in soils and then shows you an experimental test which helps you to interpret failure states.

The importance of this lecture…

We have already learned that adequate stiffness and adequate strength must be assured for any geotechnical structure. It is only by quantifying the strength of a soil that questions such as: will it stand up?, or is it safe?, can be answered.

The strength of a material describes the ultimate state of stress that the material can sustain before 'failure' occurs. But the notion of failure in soils can be difficult to define; there can be very large displacements before 'failure' occurs. In this case failure is defined by deformation or serviceability criteria, i.e. by excessive settlement, rather than by a distinct rupture or collapse or **shearing** of a soil. In the triaxial machine, the failure of a normally consolidated clay that 'barrels' is deemed to have been reached at 20% axial strain.

The concept of failure in soils



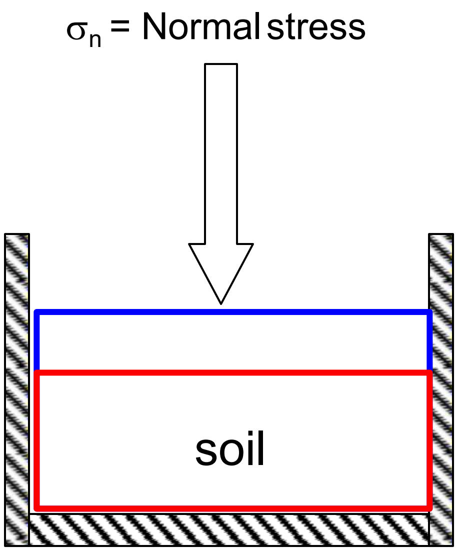
Triaxial test

Overconsolidated clay
Dense sand

Normally consolidated clay
Loose sand

The concept of failure in soils

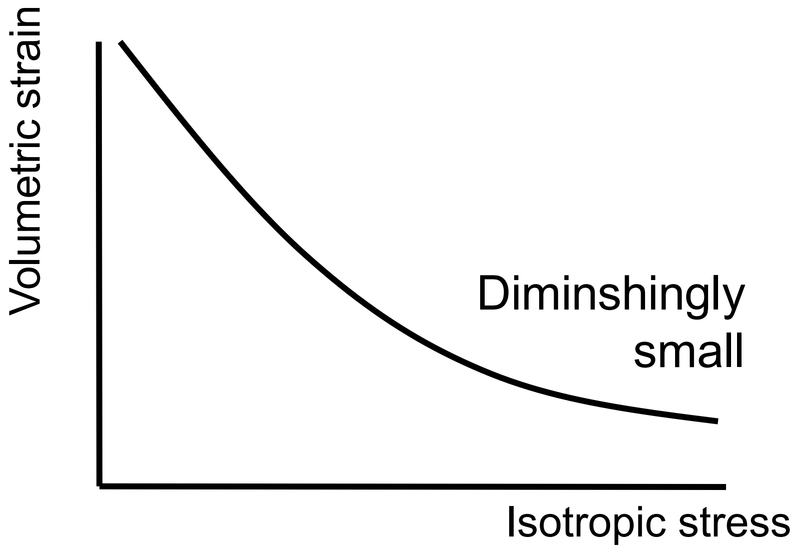
Before we pursue the idea of failure let us review the form of stress and strain shown by geotechnical test equipment. Firstly, in ‘compression’, where the stress strain behaviour is physically limited by the compression of the voids. Increasing increments of stress are required to continue compression of the soil.



Oedometric

Strains are finite & reducing

Volumetric strain

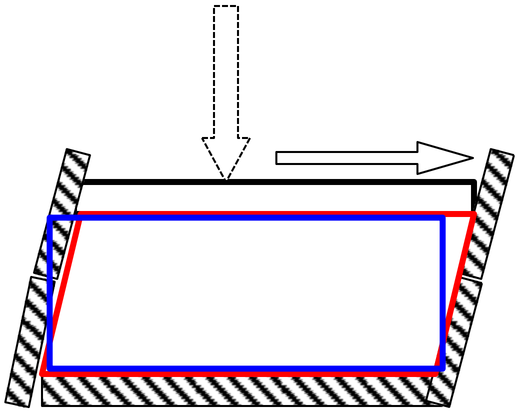


We know this very well by now

Oedometric (strictly isotropic) stress: volumetric strain relationship

Volumetric strain

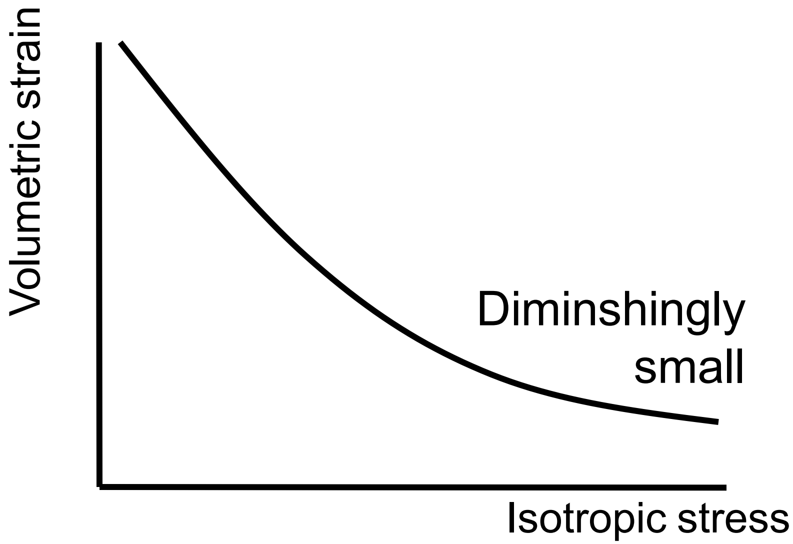
In contrast, when the strength of a soil is approached, the additional stress required to continue shearing reduces until shearing can be maintained under constant stress.

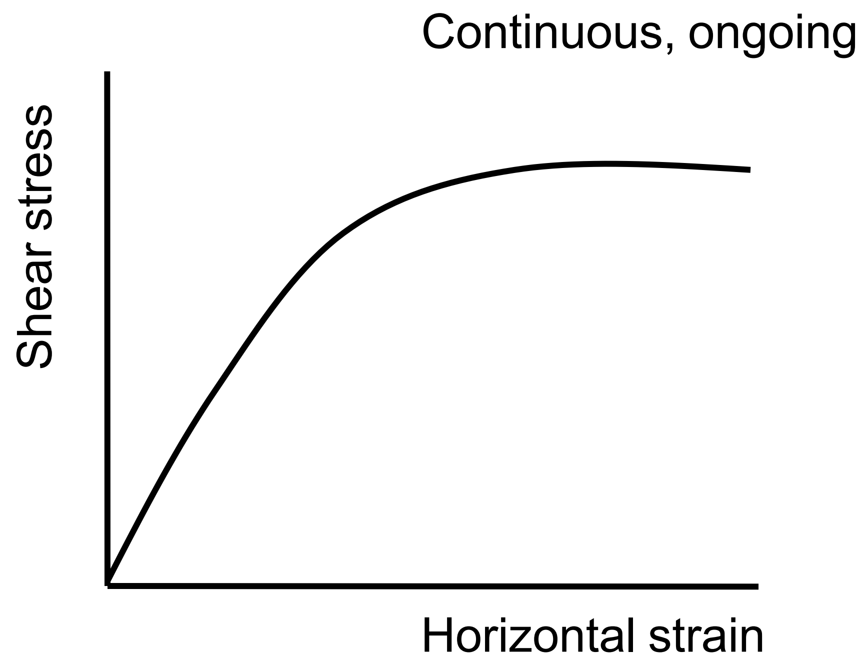


Shear

Strains may be large and 'continuous'

Shear strain



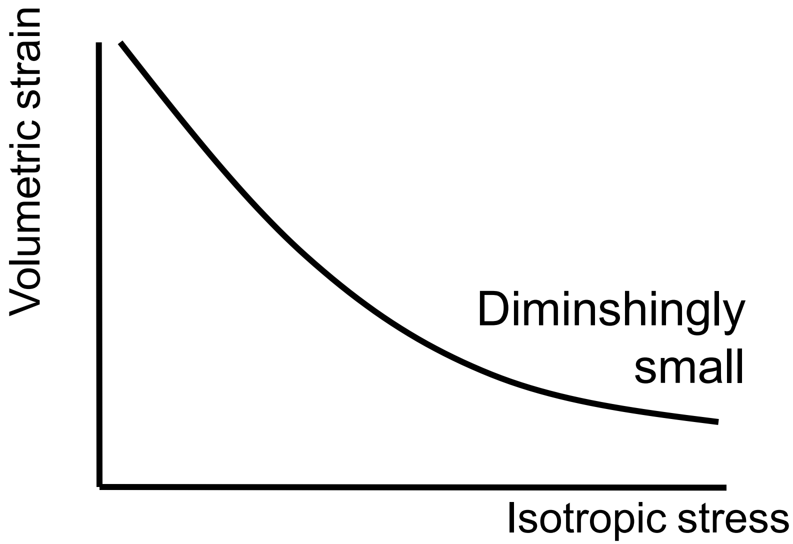


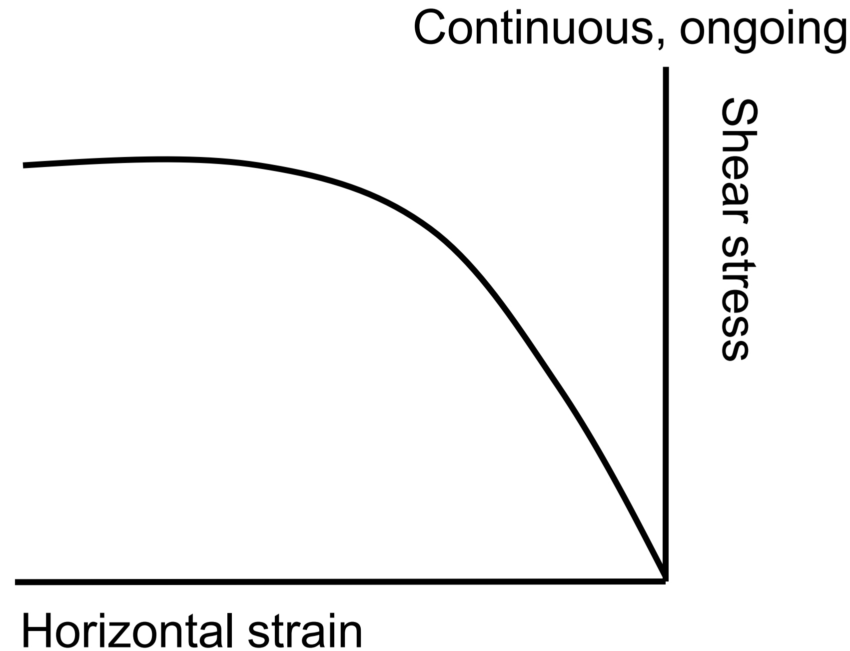
Oedometric or isotropic stress:
volumetric strain relationship

Shear stress: shear strain
relationship

Let us rotate and flip the shear stress:strain curve to emphasise the point.

Shear strain & shear stress



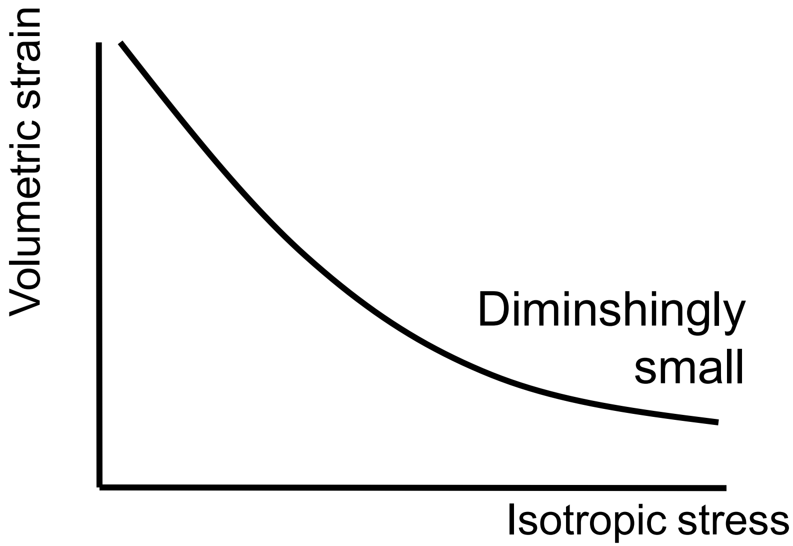


Oedometric or isotropic stress: volumetric strain relationship

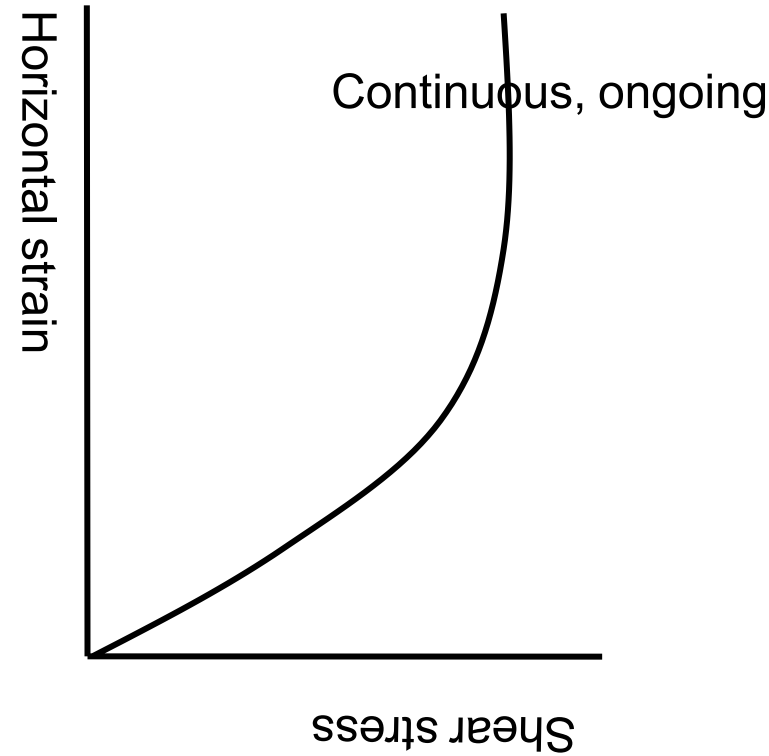
Shear stress: shear strain relationship

Let us rotate and flip the shear stress:strain curve to emphasise the point.

Shear strain & shear stress



Oedometric or isotropic stress:
volumetric strain relationship



Shear stress: shear strain
relationship

In contrast, when the strength of a soil is approached, the additional stress required to shear the soil reduces until shearing occurs under constant stress.

Shear strain & shear stress

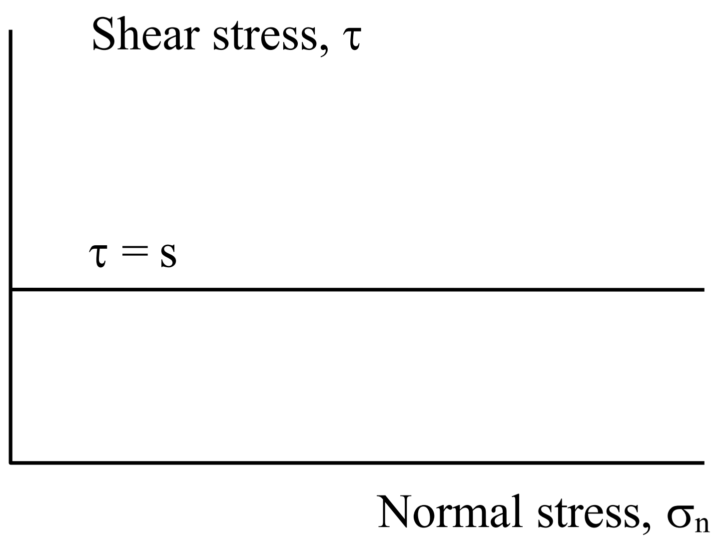
Distinct failure or rupture in soils is associated with a maximum shear stress. Stress points or Mohr's circles provide an excellent and visual way of comparing the mobilisation of shear stress in relation to the maximum or limiting conditions. Failure envelopes can be drawn on stress plots to reflect the strength characteristics of the soil. In some cases the envelope depicts a constant strength, as in the undrained case. Failure occurs when a Mohr's stress circle contacts an envelope given by,

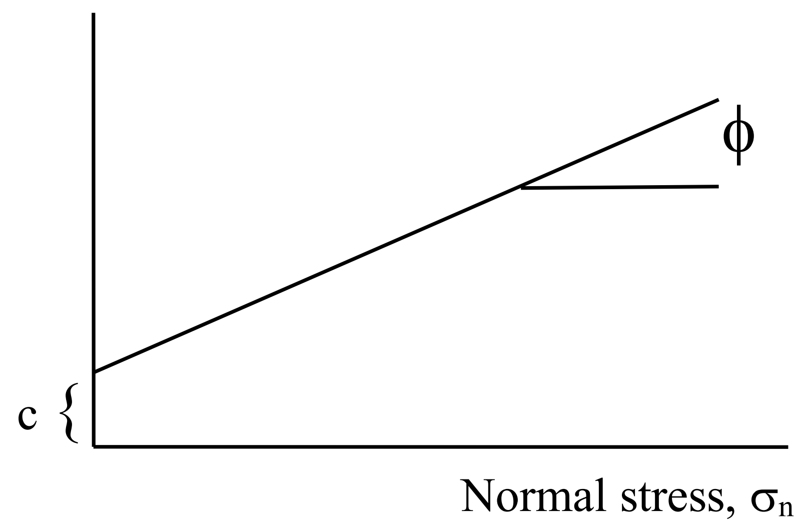
$$\tau = s$$

where *s* is some constant shear strength (in fact this is a Tresca failure criterion).

Analysis of failure in soils

But soils commonly behave more like frictional materials and soil strength is then dependent on the stress at right angles or normal to the plane of 'shearing'. Soil strength increases with normal stress; the failure envelope is not constant but inclined at angle ϕ to the horizontal – the Coulomb failure criterion.





Let's return to the concept of failure in soils

Originally expressed by Coulomb in 1776 in terms of total stresses, the shear strength of a soil was defined as a function of cohesion and normal stress.

$$\tau = c + \sigma_n \tan\phi$$

where,

τ = shear strength along failure surface

c = cohesion

σ_n = normal stress

ϕ = angle of friction

Cohesion – a component of soil strength that is 'regarded' as independent of any normal or confining stress. "Evident" in clay soils (it is never associated with sands), it is interpreted by some as the inherent stick(iness) of a soil. Hence "cohesive" & "cohesionless" soils. Note however that c has no physical meaning. It is the result of fitting data to a linear regression from limited experimental data!

Coulomb failure criterion

We know (after Terzaghi, 1936) that the pore water is unable to support any shear stress. This, and any other, failure criterion must therefore be expressed in terms of those stresses seated in the soil skeleton, i.e. effective stresses, so

$$\tau = c' + \sigma_n' \tan\phi'$$

where,

τ = shear strength along failure surface

c' = “apparent” effective cohesion

σ'_n = effective normal stress

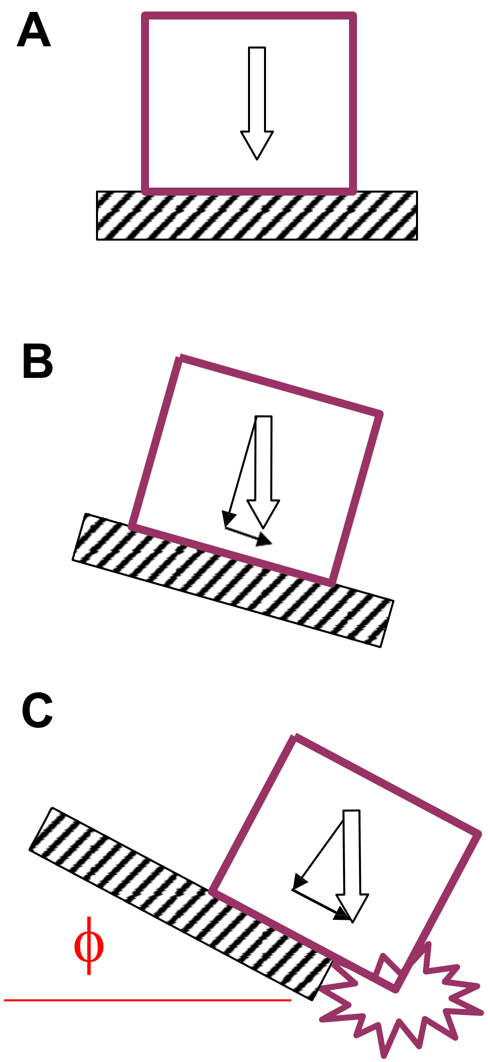
ϕ' = effective angle of friction

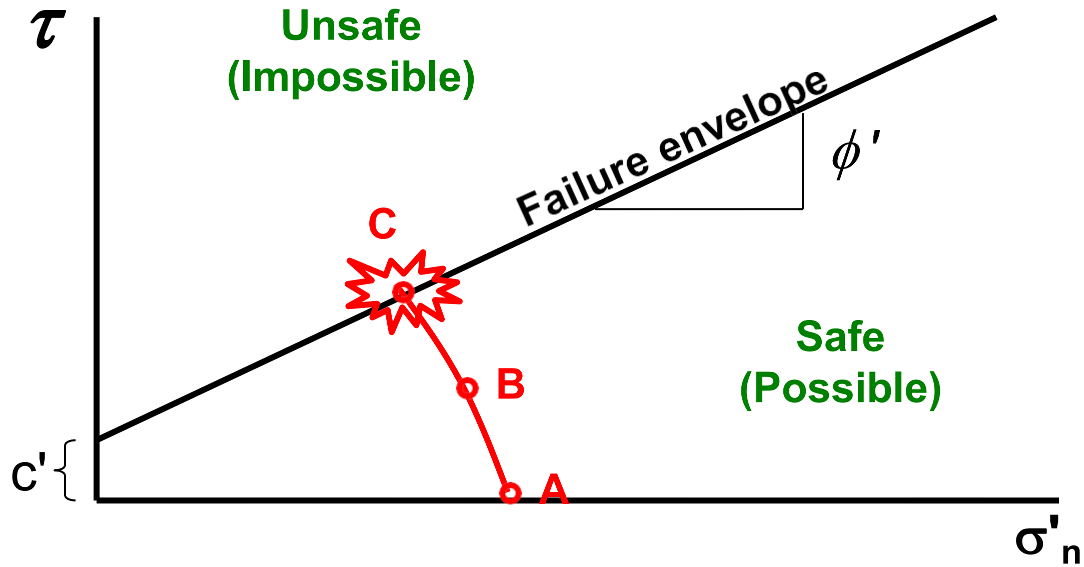
Coulomb failure criterion

How do we use it?

Compare 'mobilised' strength with available soil strength.

Failure occurs when mobilised strength/stress combination touches the envelope.



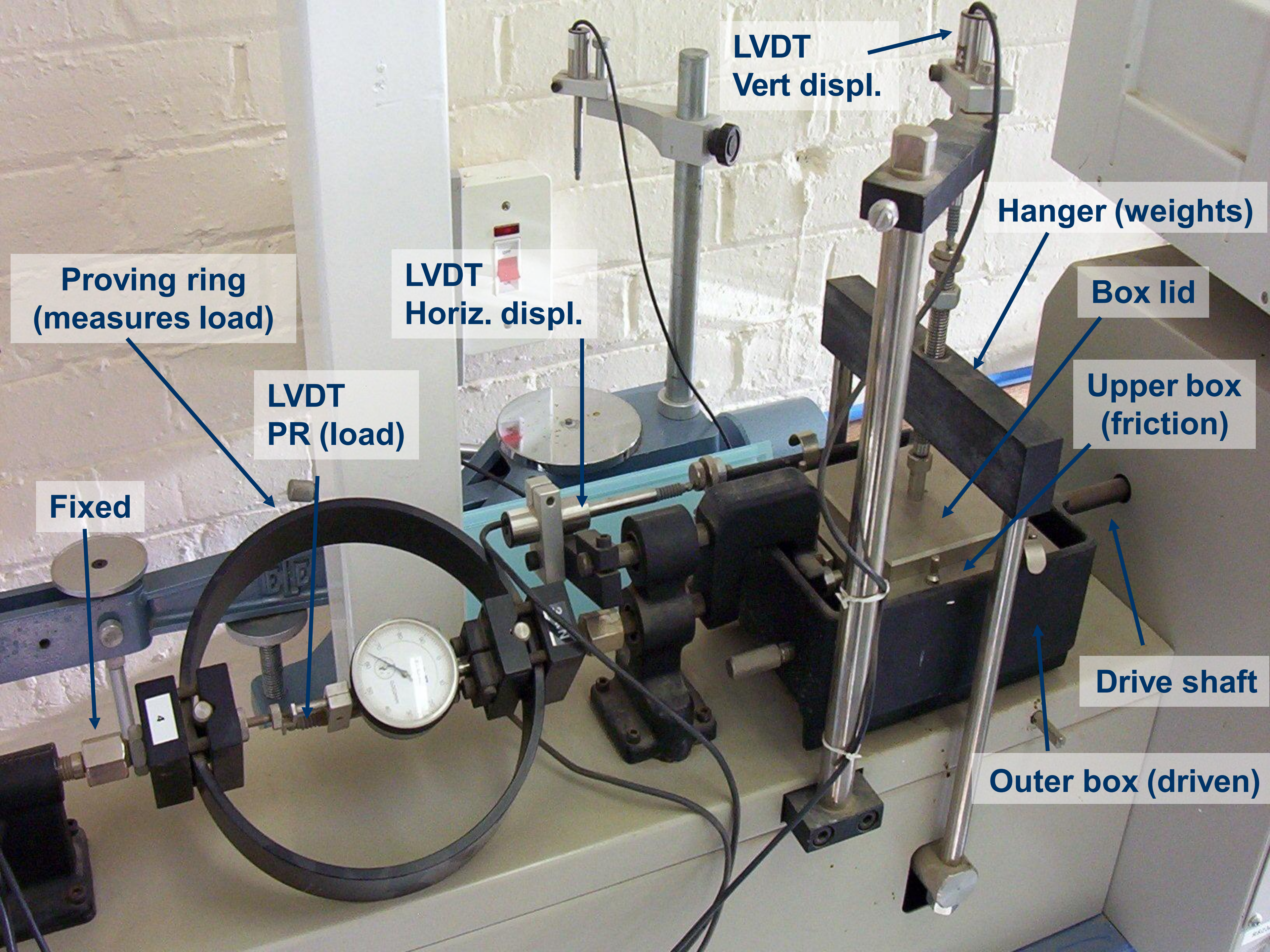


Remember! Sliding blocks show none of the volumetric behaviour that accompanies shearing in a particulate material

Coulomb failure – graphical interpretation

SHEAR BOX

Shear box with automatic data collection

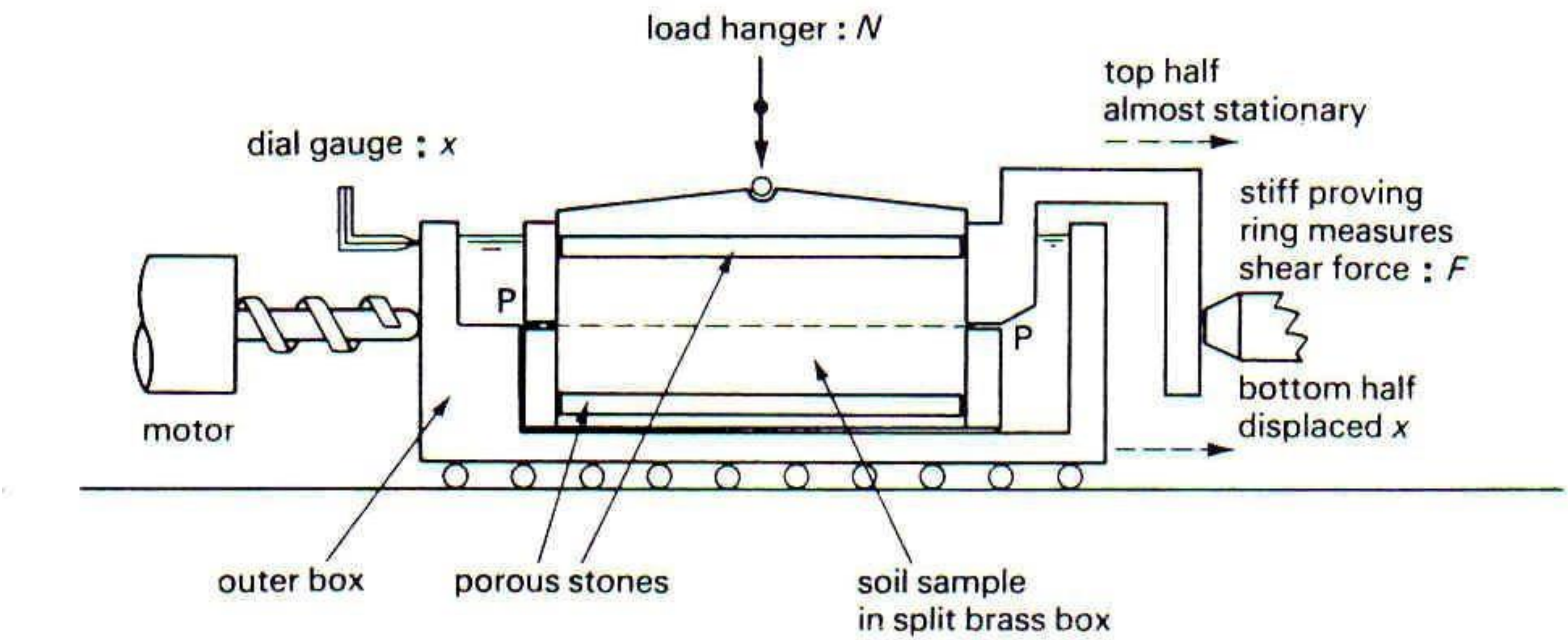


LVDT
Vert displ.
Hanger (weights)
Proving ring
(measures load)
LVDT
Horiz. displ.
Box lid
Upper box
(friction)
LVDT
PR (load)
Fixed
Drive shaft
Outer box (driven)

Shear box is generically a “drained” test – no excess pore water pressure – which dictates the rate of strain.

For sands – 10 mins; for clays – days

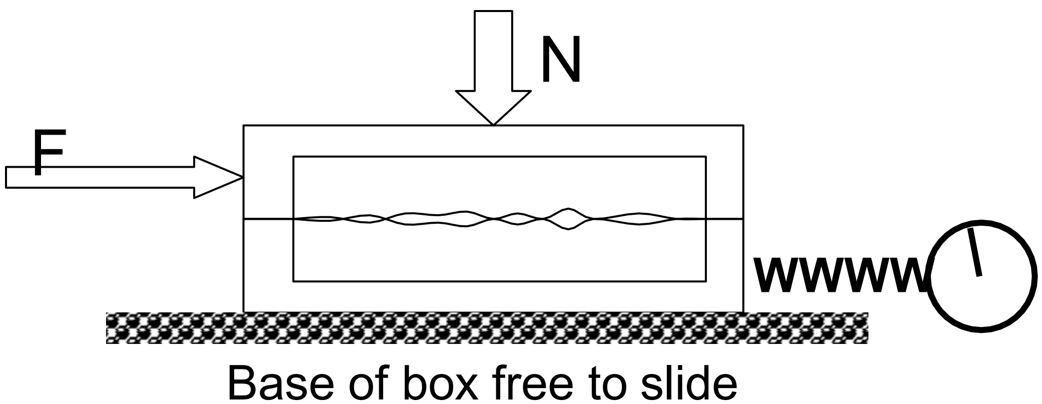
How could you perform an undrained test in the shear box?

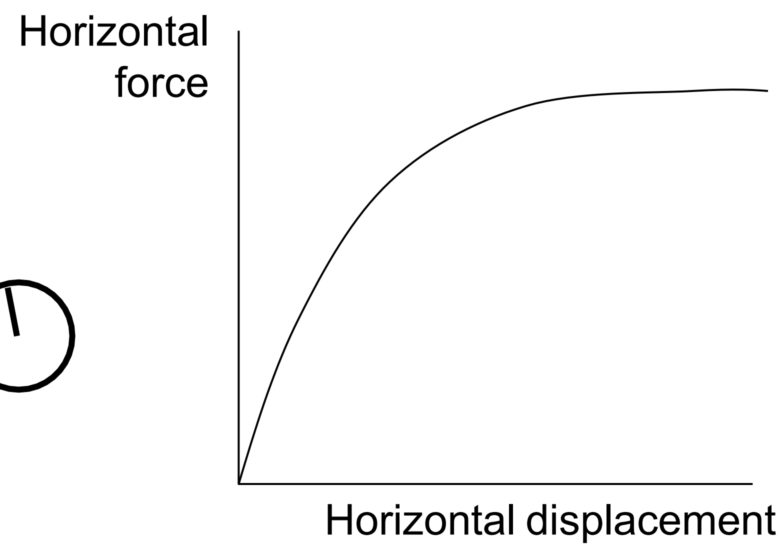


Shear box – schematic view

Shear box – schematic view

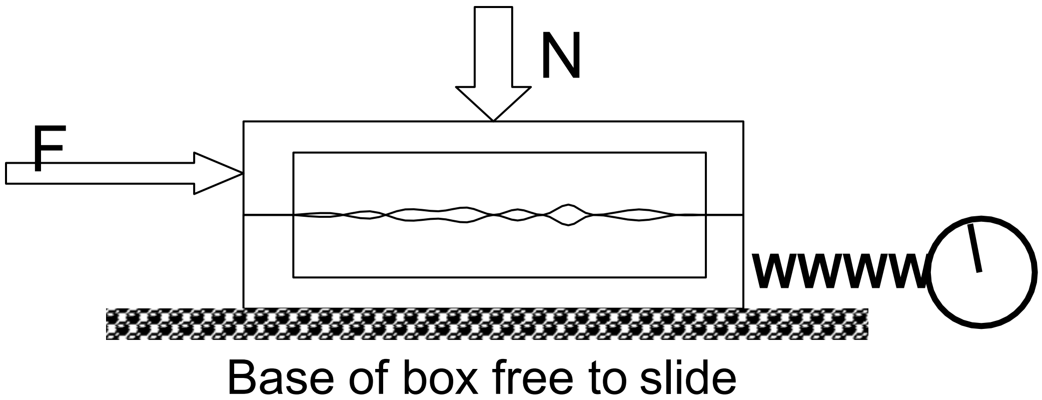
A relative displacement is produced between the two halves of the box. The resistance to shearing with displacement is recorded by a proving ring.

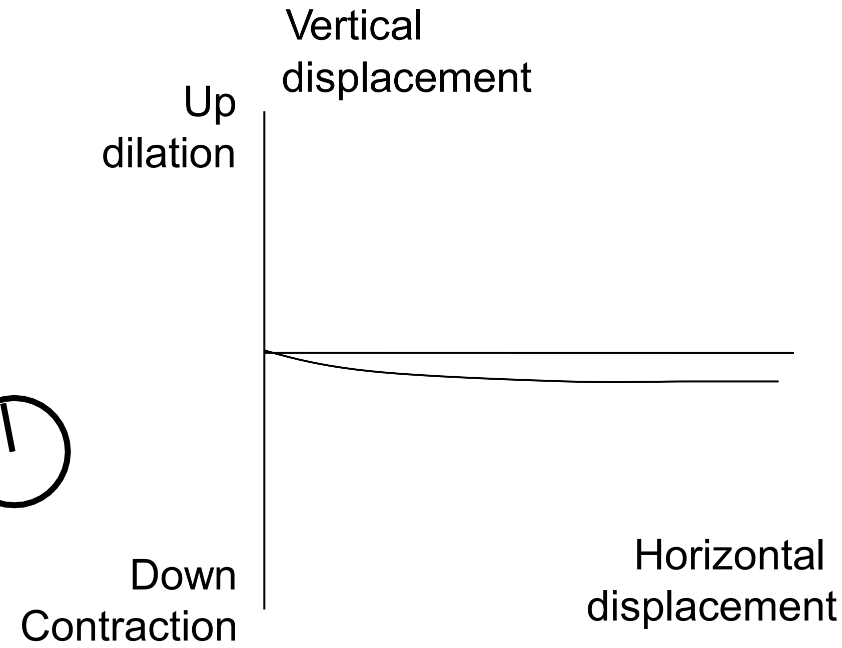




And …

… the vertical displacement is recorded by a dial gauge or transducer. In this case there is a small amount of ‘contraction’ (volume reduction) in the sample





Shear box – schematic animation

Usually three tests would be performed at different normal stresses.

The raw data (for any one test) is:

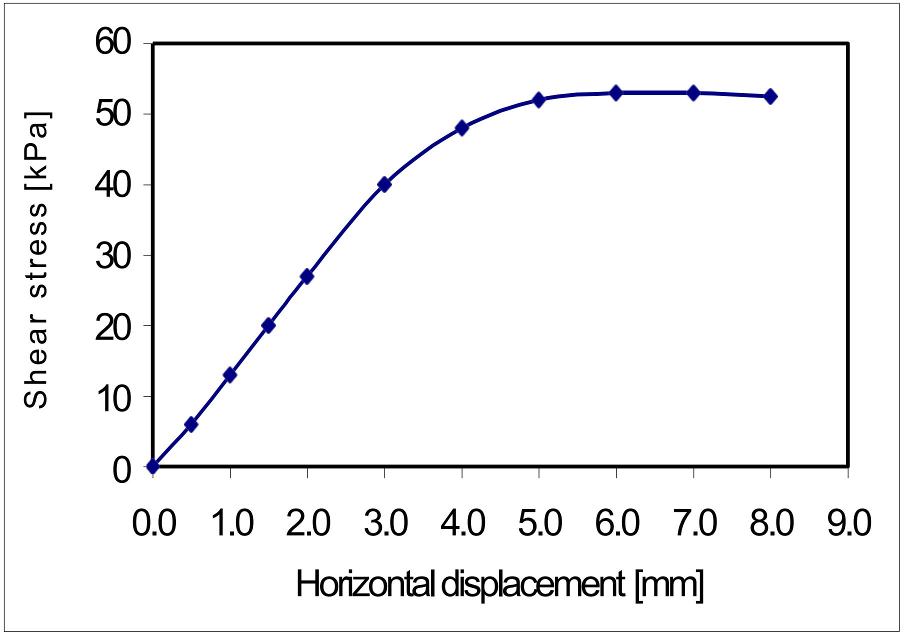
Horizontal displacement	Horizontal force	Horizontal stress	Vertical displacement
mm	N	kPa	mm
0.0	0	0	0.000
0.5	22	6	-0.150
1.0	47	13	-0.300
1.5	72	20	-0.450
2.0	97	27	-0.550
3.0	144	40	-0.700
4.0	173	48	-0.780
5.0	187	52	-0.860
6.0	191	53	-0.900
7.0	191	53	-0.940
8.0	189	52.5	-0.945

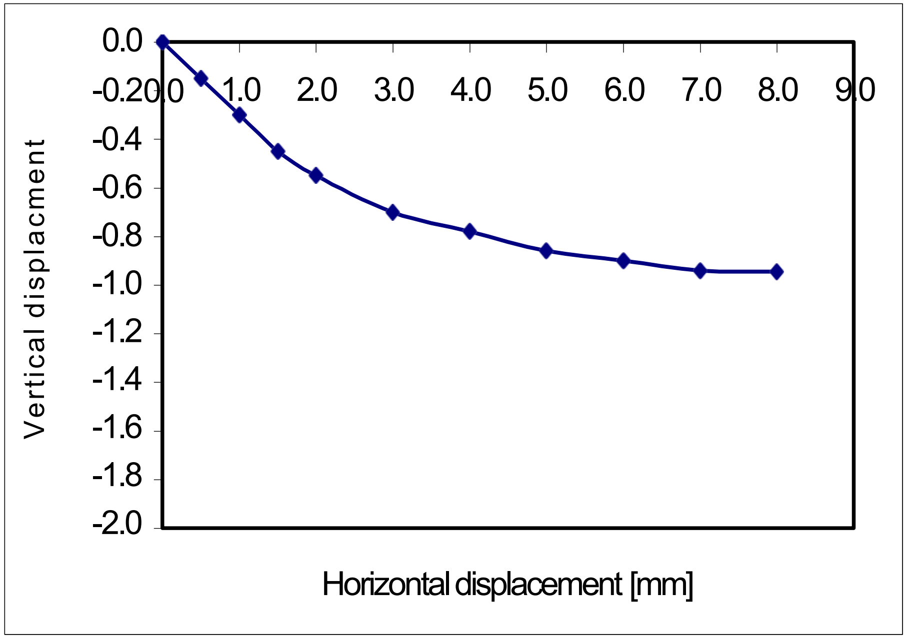
Hanger load = 36kg
Shear box dimensions
60mmx60mm
Normal stress = 98 kPa

From which we can calculate:

- shear stress
- volume change/void ratio change

Shear box data





So in this test, under a normal stress of 98 kPa, a maximum shear stress of 53 kPa was recorded. Two other tests run at different vertical loads produced the following results:

σ_n = 196 kPa; τ = 106 kPa and σ_n = 395 kPa; τ = 208 kPa

By plotting the shear stress and normal stress combinations to all three test we obtain the (Mohr-Coulomb) failure envelope and strength parameters, c' & ϕ'.

Shear box data

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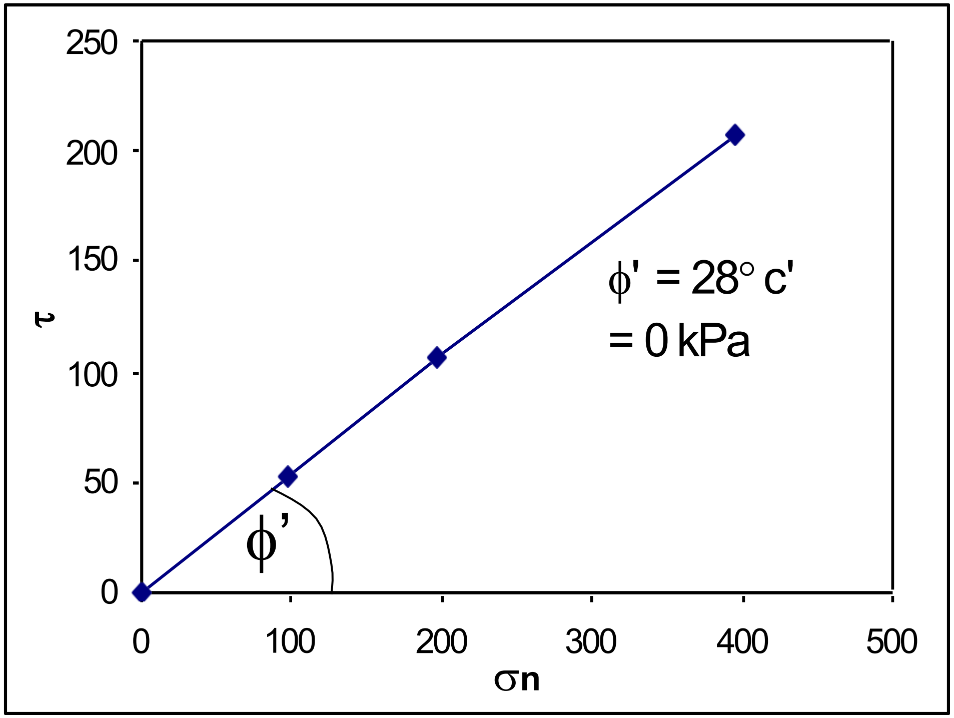


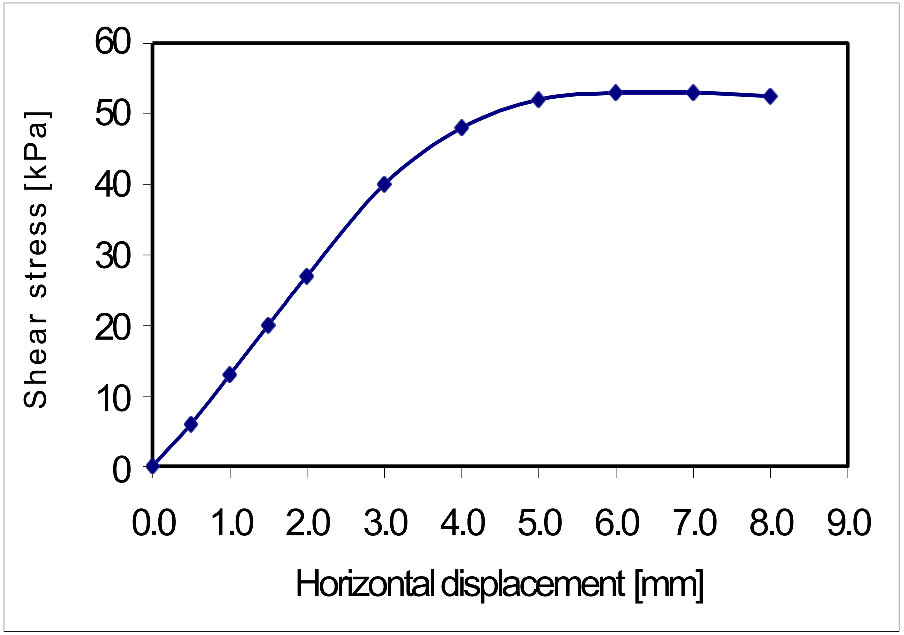
Shear box data

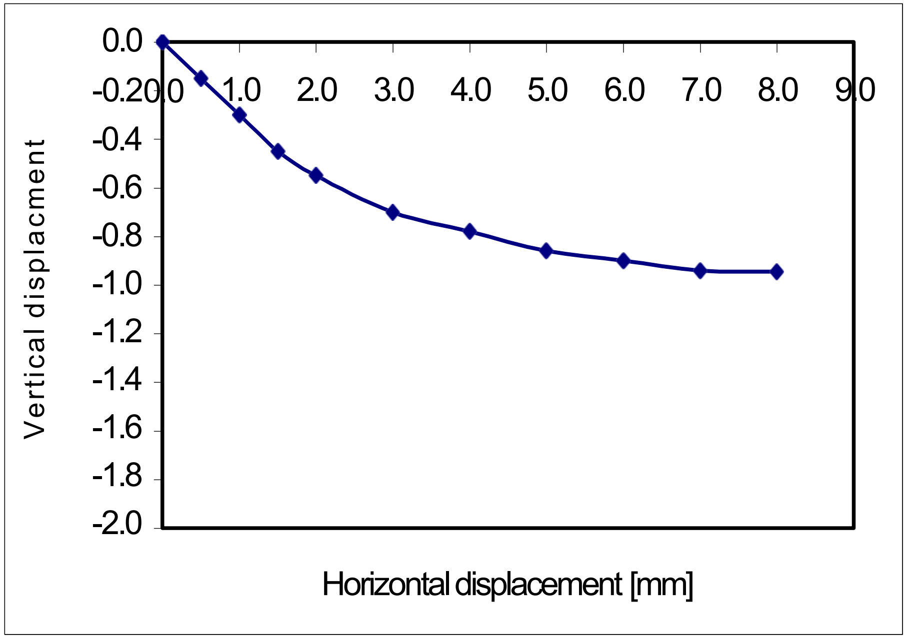
These results are typical for a soft or normally consolidated clay, or a loose sand. There are two features to note:

•a gradual rise to maximum strength and

•a contraction in the volume of the sample.

Beyond this point the rise in stress as strength is mobilised begins to fall as the cross-sectional contact area of the two halves of the direct shear box reduces.



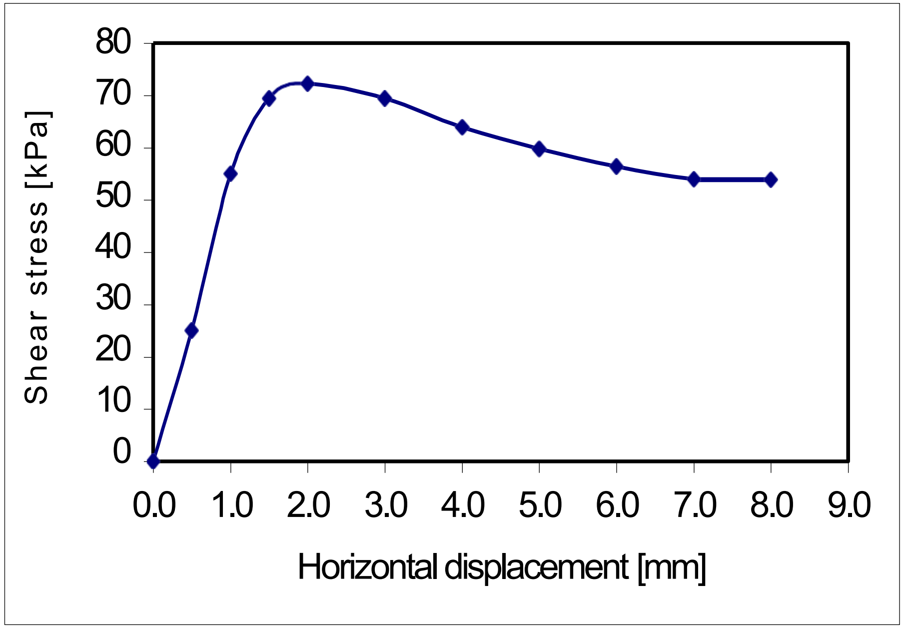


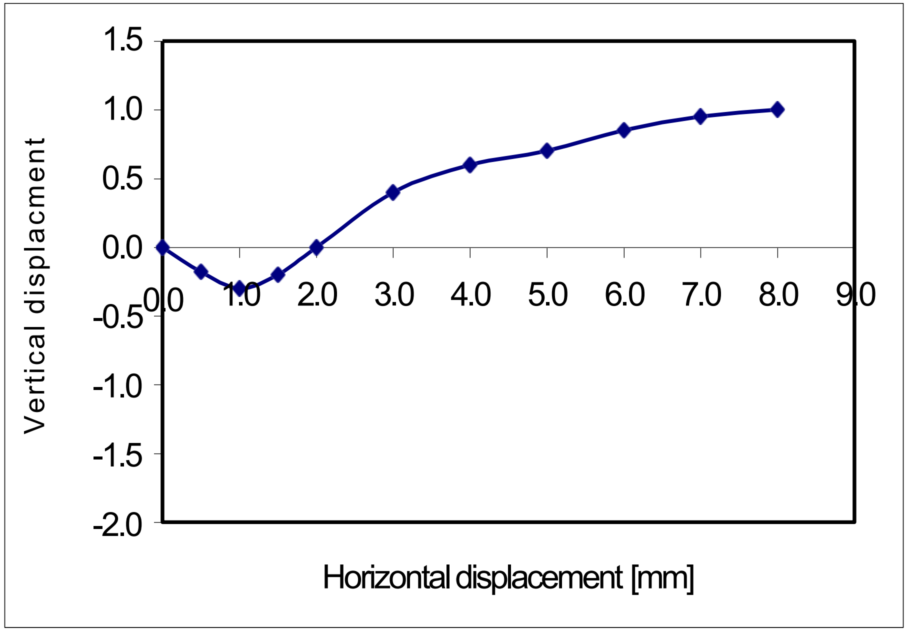
Shear box data for a loose sand/soft clay

Horizontal displacement	Horizontal force	Horizontal stress	Vertical displacement
mm	N	kPa	mm
0.0	0	0	0.000
0.5	90	25	-0.180
1.0	198	55	-0.300
1.5	250	69	-0.200
2.0	260	72	0.000
3.0	250	69	0.400
4.0	230	64	0.600
5.0	215	60	0.700
6.0	203	56	0.850
7.0	194	54	0.950
8.0	194	54	1.000

Here, under a normal stress of 98 kPa, a maximum shear stress of 72 kPa was recorded. But note shear stress shows a reduction after the 72 kPa maximum.

Shear box data for a dense sand/stiff clay

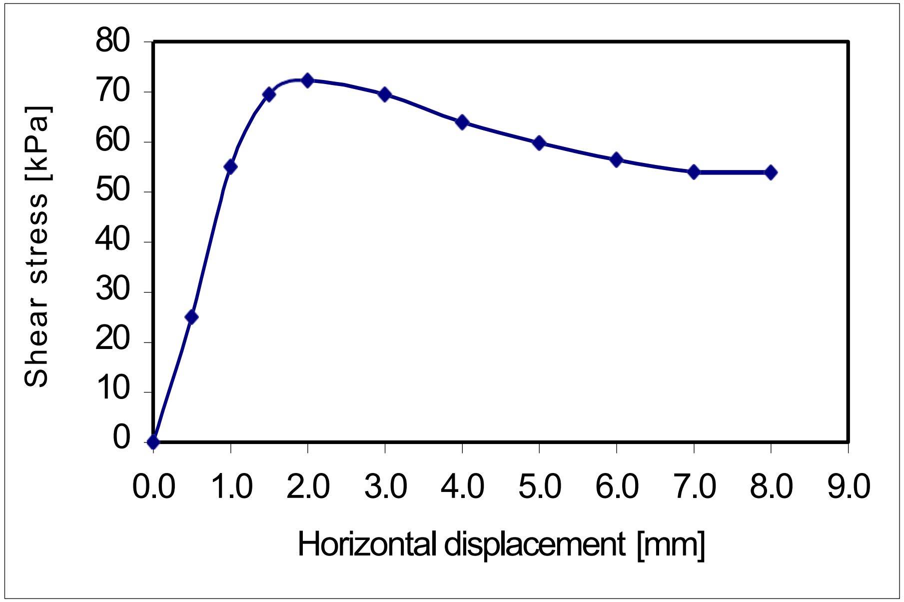


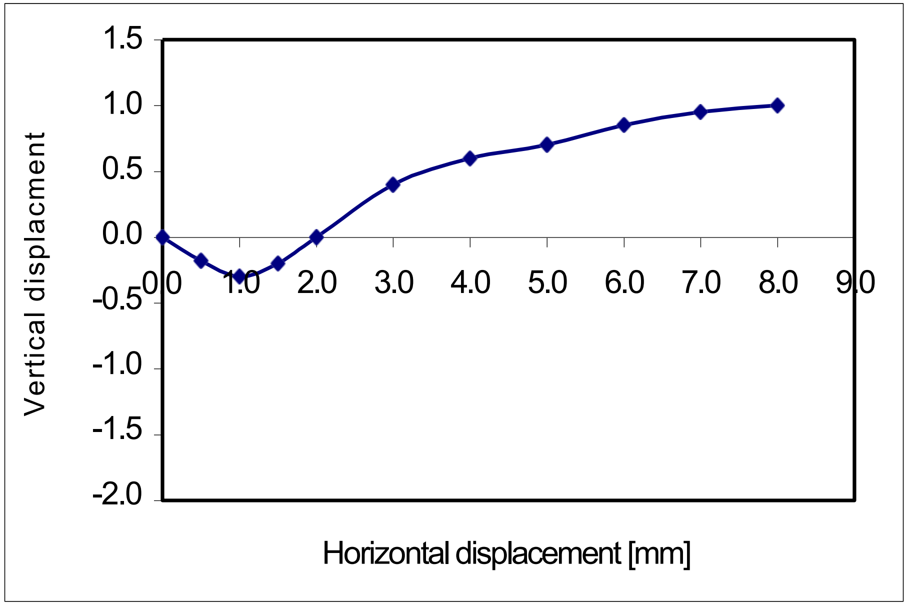


Here, under a normal stress of 98 kPa, a maximum shear stress of 72 kPa was recorded. But note shear stress shows a reduction after the 72 kPa maximum…

… and the vertical displacement is one of volume increase.

Shear box data for a dense sand/stiff clay





Here, under a normal stress of 98 kPa, a maximum shear stress of 72 kPa was recorded. But note shear stress shows a reduction after the 72 kPa maximum…

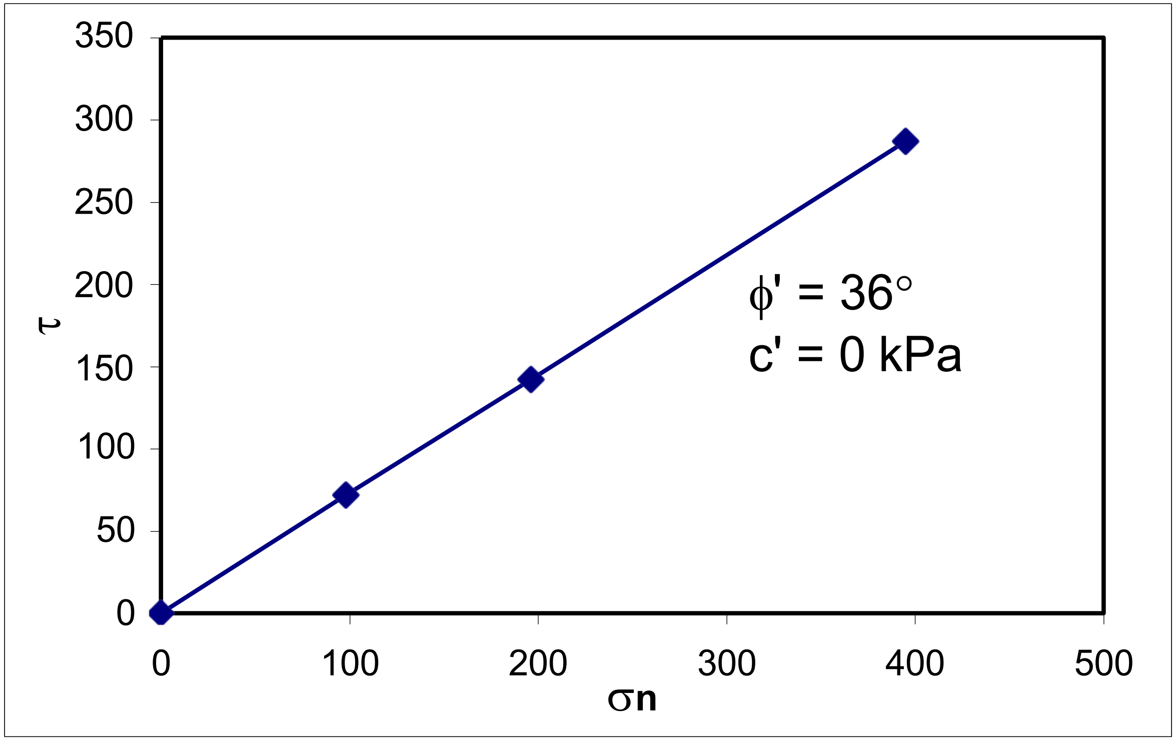
… and the vertical displacement is one of volume increase.

We will return to this matter shortly but first let's calculate the shearing resistance or angle of friction.

Shear box data for a dense sand/stiff clay

Here, two other tests run at different vertical loads produced the following results:

σ_n = 196 kPa, τ = 142 kPa; σ_n = 395 kPa, τ = 287 kPa



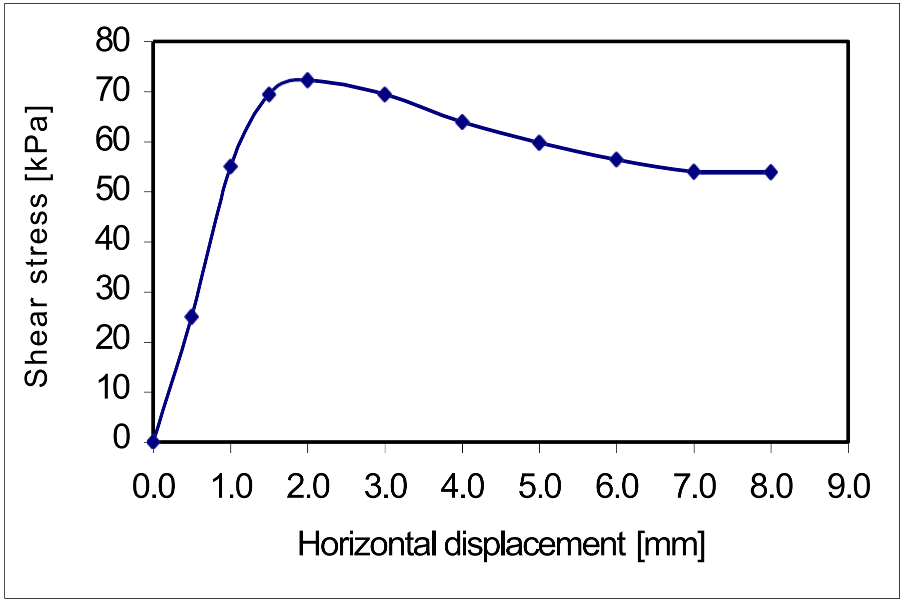
Shear box data for a dense sand/stiff clay

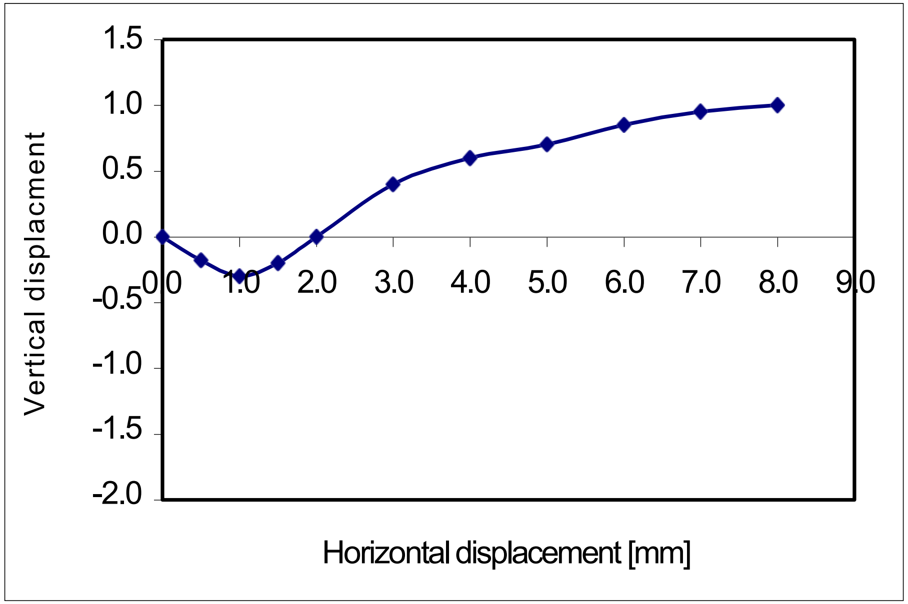
These results are typical for a stiff or overconsolidated clay, or indeed a dense sand. There are two features to note:

• shear strength rises sharply to a peak and subsequently reduces to some long term value [compare long term values in both tests]

• the sample first contracts then dilates.

The difference in response is significant and is due to the different initial density.





Shear box data for a dense sand/stiff clay

You surely can see how this lecture is very important for your understanding of soil behaviour.

We however have just started starting to understand the complexity of shear behaviour.

So I will provide a summary only when I believe things are clearer for you (i.e. after the next two lectures when this topic concludes)

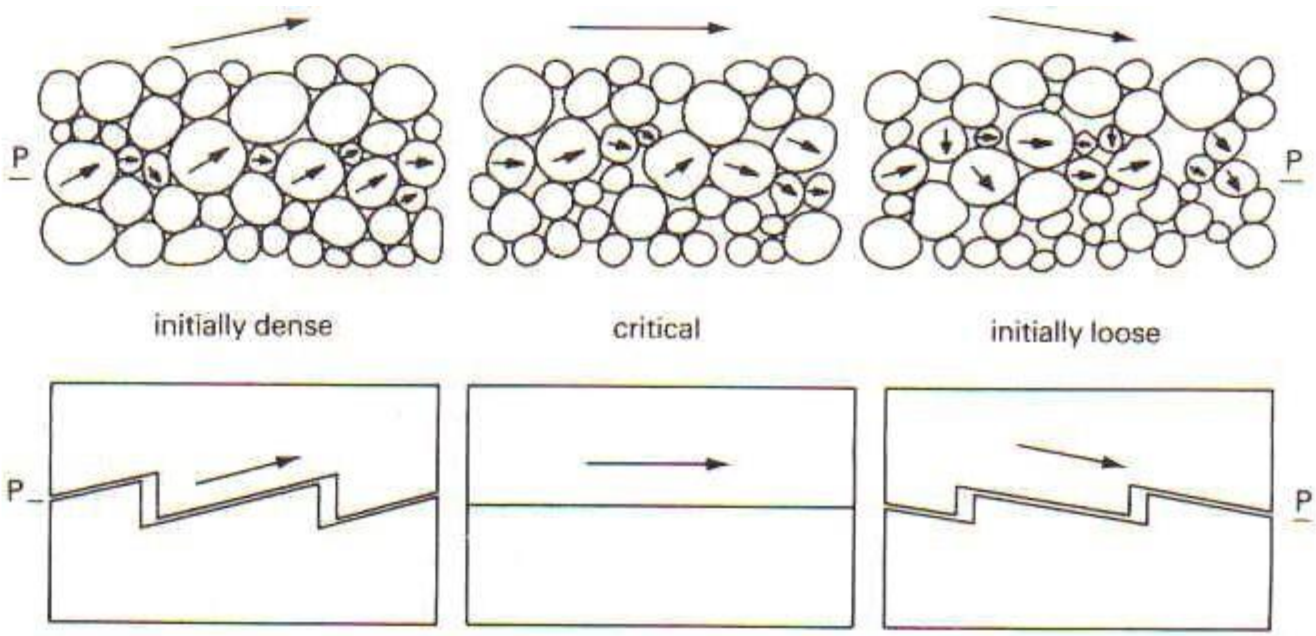
Summary


SHEAR
STRENGTH 2
ELE
ARTHUR
McKAY
RS232

1. Density effects on shear strength
2. Residual strength
3. Mohr-Coulomb failure criterion
4. Mohr’s circle and the pole of planes

But remember ***density effects***

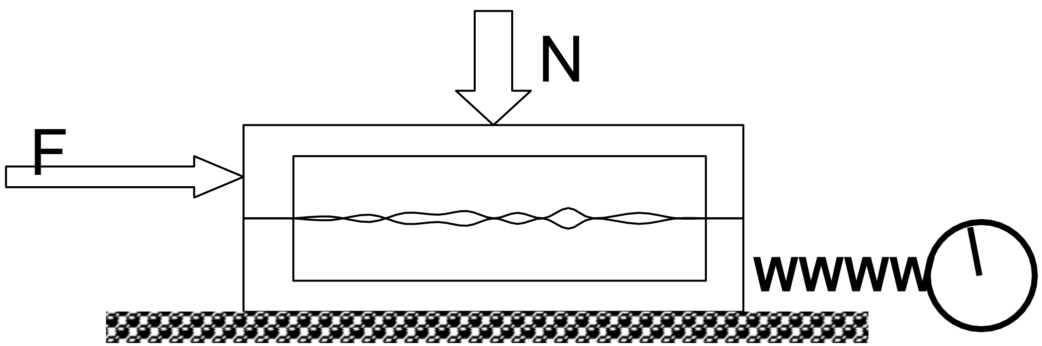
Void ratio = low medium high



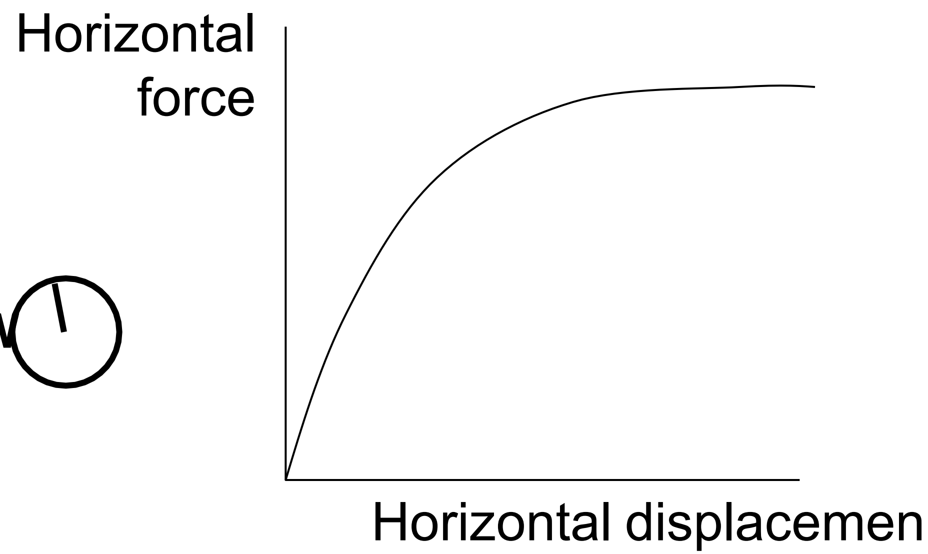
So that when sheared soil may: dilate be at ‘critical state’ contract

Shearing response and density

A relative displacement is produced between the two halves of the box. The resistance to shearing with displacement is recorded by a proving ring.



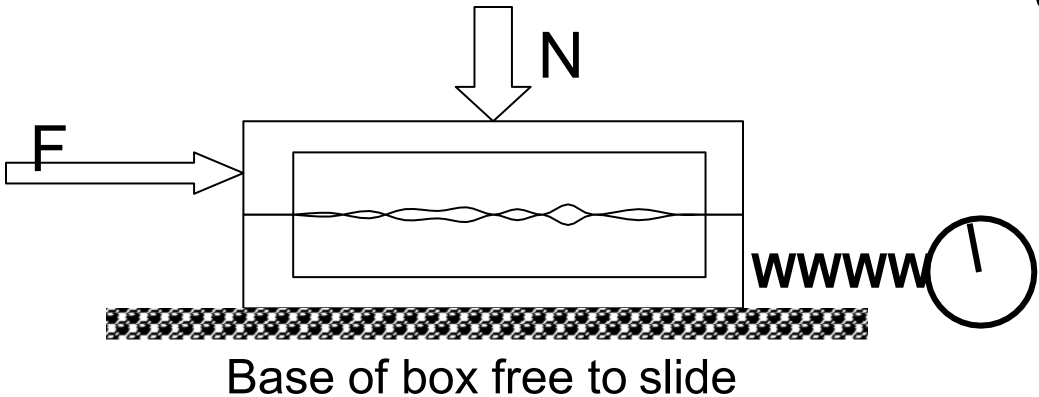
Base of box free to slide

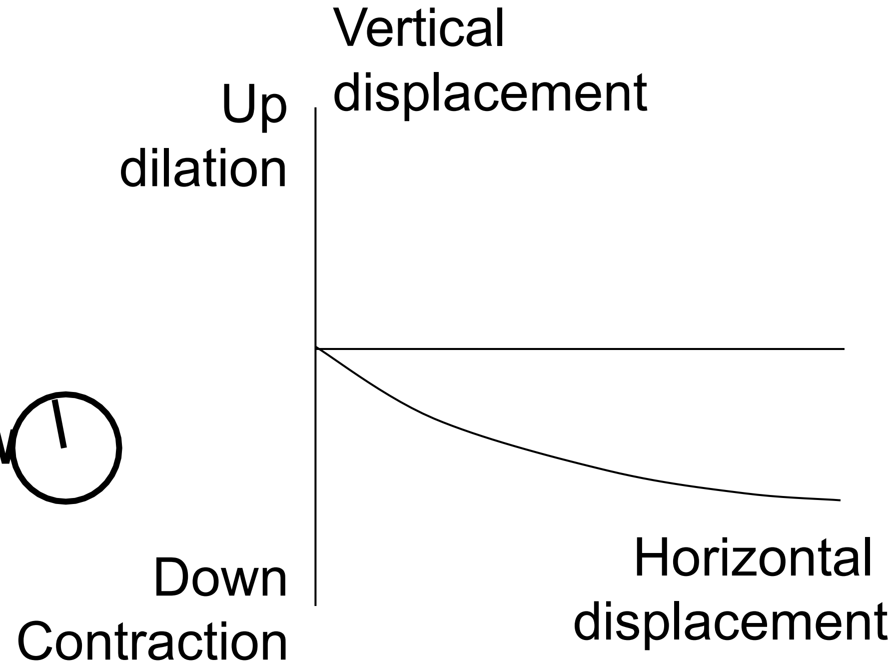


And …

Shear box – schematic view loose sample

… the vertical displacement is recorded by a dial gauge or transducer. In this case there is a small amount of ‘contraction’ (volume reduction) in the sample





Shear box – schematic animation loose sample

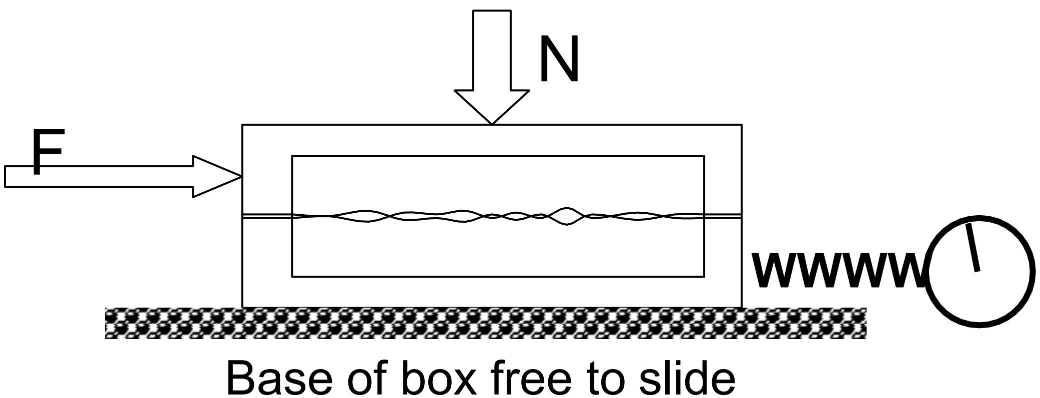
In the initially loose sample, resistance to shearing is due to friction with the orientation of the sliding surface actually helping the shearing process. In soil, neighbouring particles will tumble into adjacent void spaces on shearing. In this case the resistance to shearing is relatively low.

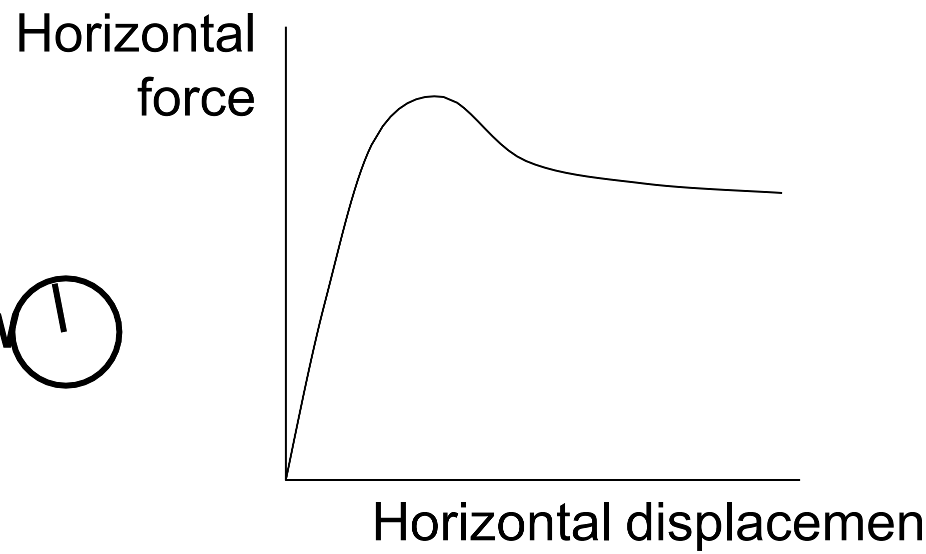
In between lies a condition known as the **critical state**, where the two surfaces are horizontal and their orientation adds nothing to, nor diminishes, shearing resistance. In soils, neighbouring particles are in a kind of turbulent motion where some ride up, some tumble down and others just 'stay on the level'. In this case, with particle interlock effects balanced, we can think of the resistance to shearing as being due to friction; it is really only in this condition that the term angle of friction is synonymous with soil strength. In general it is better to use term shearing resistance where,

Shearing resistance = frictional resistance + initial density effects.

Shear box – schematic view: dense sample

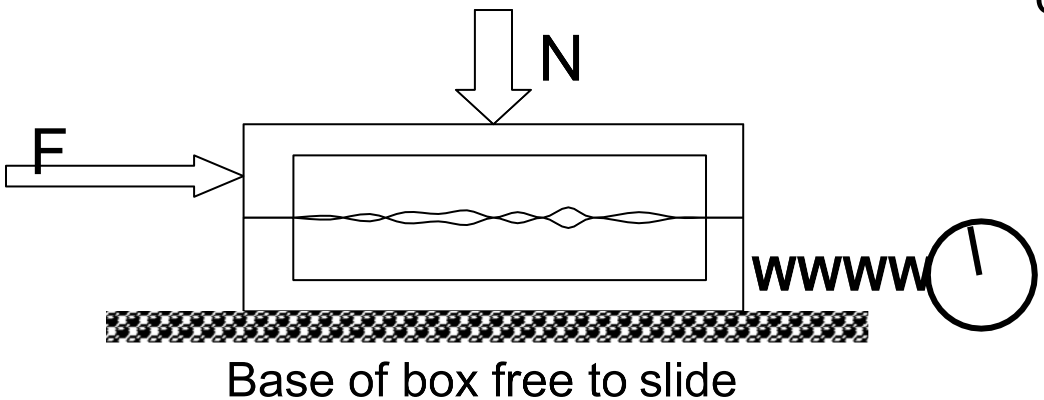
A relative displacement is produced between the two halves of the box. The resistance to shearing with displacement is recorded by a proving ring.

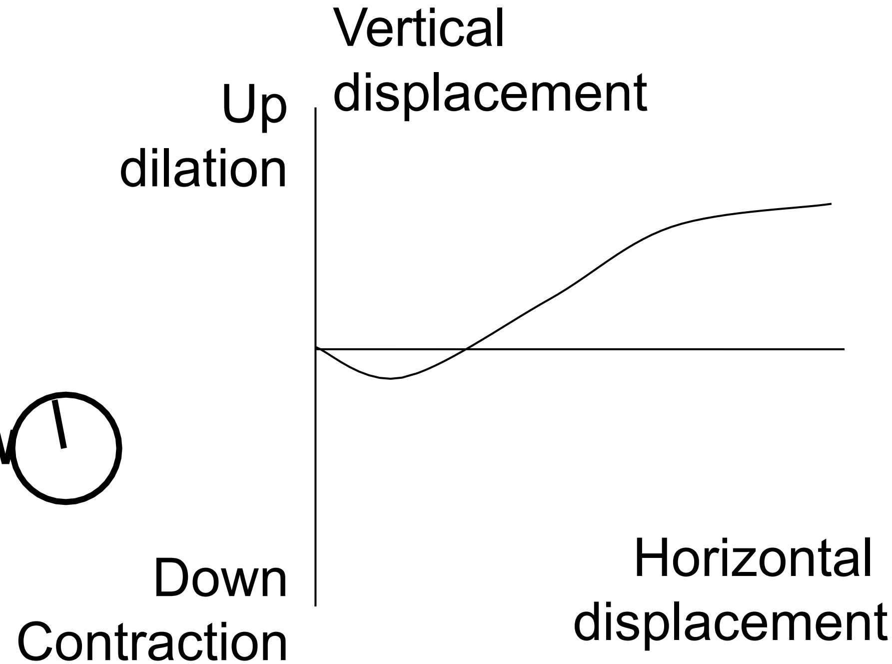




And …

… the vertical displacement is recorded by a dial gauge or transducer. In this case there is a small amount of contraction then 'dilation' (volume increase) of the sample





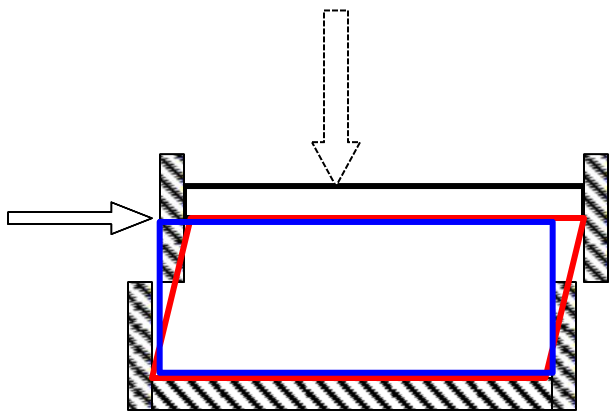
Shear box – schematic animation: dense sample

In the initially dense sample, the resistance to shearing is a combination of friction between the two surfaces and the work needed to push the upper block up the inclined plane. In soil, neighbouring particles must ride up and over one another in order to trigger shearing. Shearing resistance is relatively high.

In between dense and loose lies a condition known as the **critical state**, where the two surfaces are horizontal and their orientation adds nothing to, nor diminishes, shearing resistance. In soils, neighbouring particles are in a kind of turbulent motion where some ride up, some tumble down and others just ‘stay on the level’. In this case, with particle interlock effects balanced, we can think of the resistance to shearing as being due to friction; it is really only in this condition that the term angle of friction is synonymous with soil strength. In general it is better to use term shearing resistance where,

Shearing resistance = frictional resistance + initial density effects.

Shearing produces an indicative (predictable) volume change that is dependent on the initial density of the sample.



Direct shear

Sample may:

Referred to as critical state

Shear strain with volume change

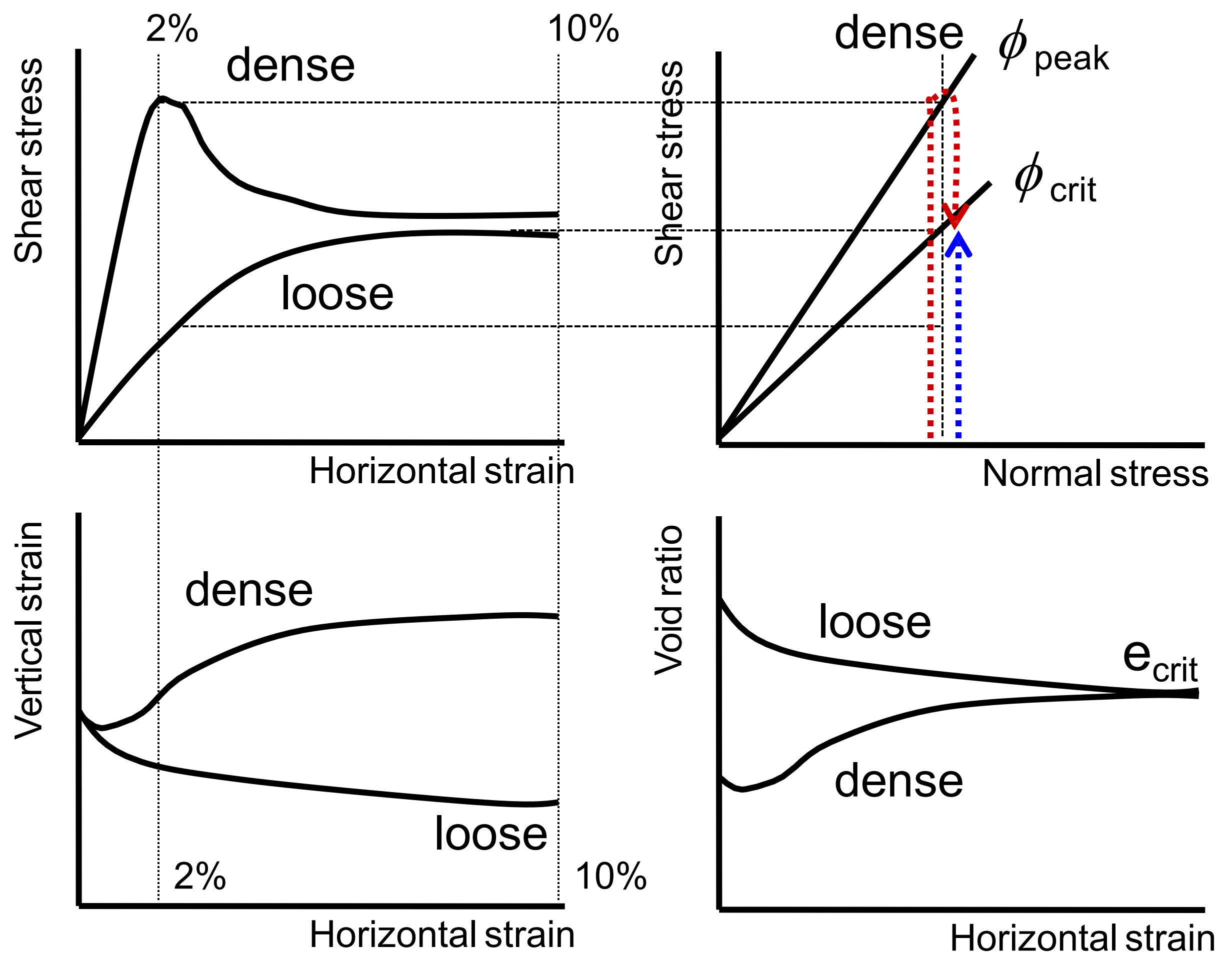
The dilation or contraction that accompanies shearing moves initially dense and initially loose soils towards a common volumetric state. Indeed test data clearly support the hypothesis of a common **critical state**, which has been recognised since the 1960s and has spawned a generation of soil models, the most famous family of which is the critical state models (see Schofield & Wroth, 1963 or Atkinson & Bransby, 1976).

The critical state is characterised by soil undergoing continuous shearing at constant stress and constant volume. It further implies that if two different samples of the same soil under the same vertical stress, but of quite different initial densities, were taken to the critical state, the strength of the two samples would be the same. We can see this in the data that follows.

Critical state

Shear box test results

… which reveal the importance of selecting a friction angle appropriate to both stress and strain levels …



Graphical interpretation of density effects

Note that when two samples of the same sand, one initially loose and one initially dense, are tested at the same normal stress:

- Strength of loose sample rises steadily to maximum value
- Peak shear stress occurs in dense sample at relatively small strains <2% and is followed by 'post peak softening'.
- Ultimate shear strength is SAME for both initially dense and initially loose samples

- Volume of loose sample gradually reduces.
- Dense soil contracts at very small strains then dilates to the ultimate state.
- Ultimate void ratio is SAME for both initially dense and initially loose samples

Density effects on soil behaviour during shearing: a summary

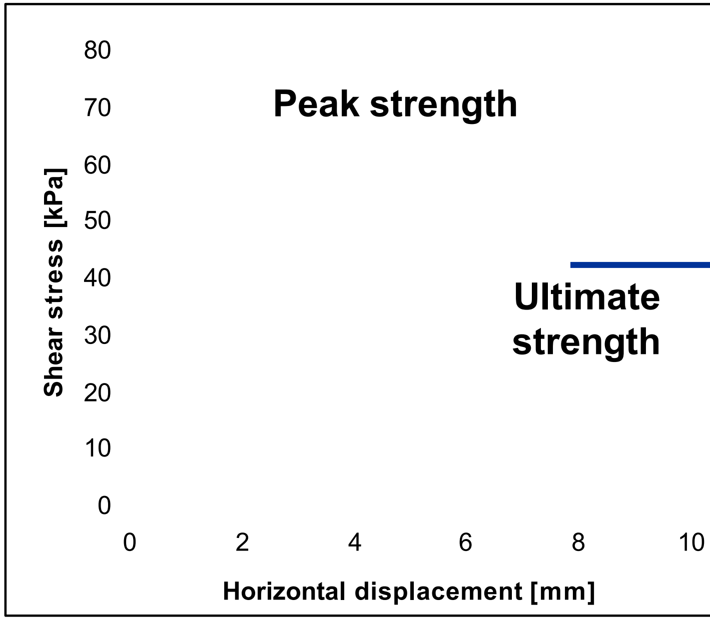
SELF-CHECK QUESTIONS – SHEAR STRENGTH 1

1. Calculate the angle of shearing resistance of sand that shows a maximum resistance to shear of 135 kPa when subject to a normal stress of 200 kPa.
2. During shearing, the strength of a _dense _ soil rises rapidly to a peak then falls, whereas the strength of a _ loose _ soil rises slowly to a maximum.
3. When sheared under the same vertical stress, the ultimate strength of dense and loose samples of the same soil, is _ the same_
4. During shearing, the volume of a dense sample will _ dilate _ , whilst that of a loose sample will _ contract _
5. During shearing at the same _ vertical stress _, the volumetric state (void ratio) of initially dense and initially loose samples, will become _ _ the same.
6. Shear box testing of a clay soil reveals a shear strength of 80 kPa, when the vertical stress is 100 kPa, and 145 kPa when the vertical stress is 200 kPa. Calculate the shear strength parameters for this soil.

Residual strength

In clays, soil strength can fall below ultimate/critical state values.

Normal stress = 98 kPa



Residual strength

Why?

Due to:

- Softening
- Re-orientation of clay platelets (slickensides)

Slickenside surface

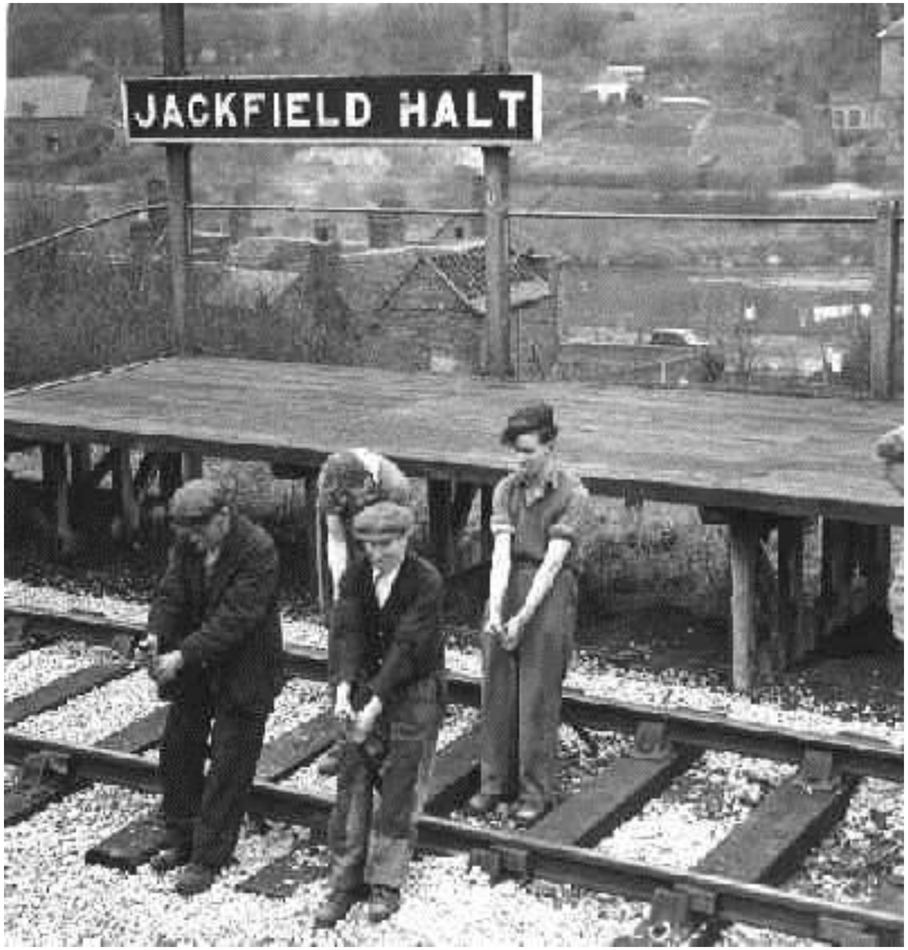
A real problem for slopes. The presence of pre-existing shear surfaces (evident as slickensides) can allow movement.

Eventually, relative displacement is sufficient to push soil strength below the ultimate or critical state and cause failure.

How? Actually 2 mechanisms:

- Dilation & softening in thin shear band
- Reorientation of platelets: $\phi' > 9^\circ$

Residual strength



Jackfield slide, Shropshire, 1952

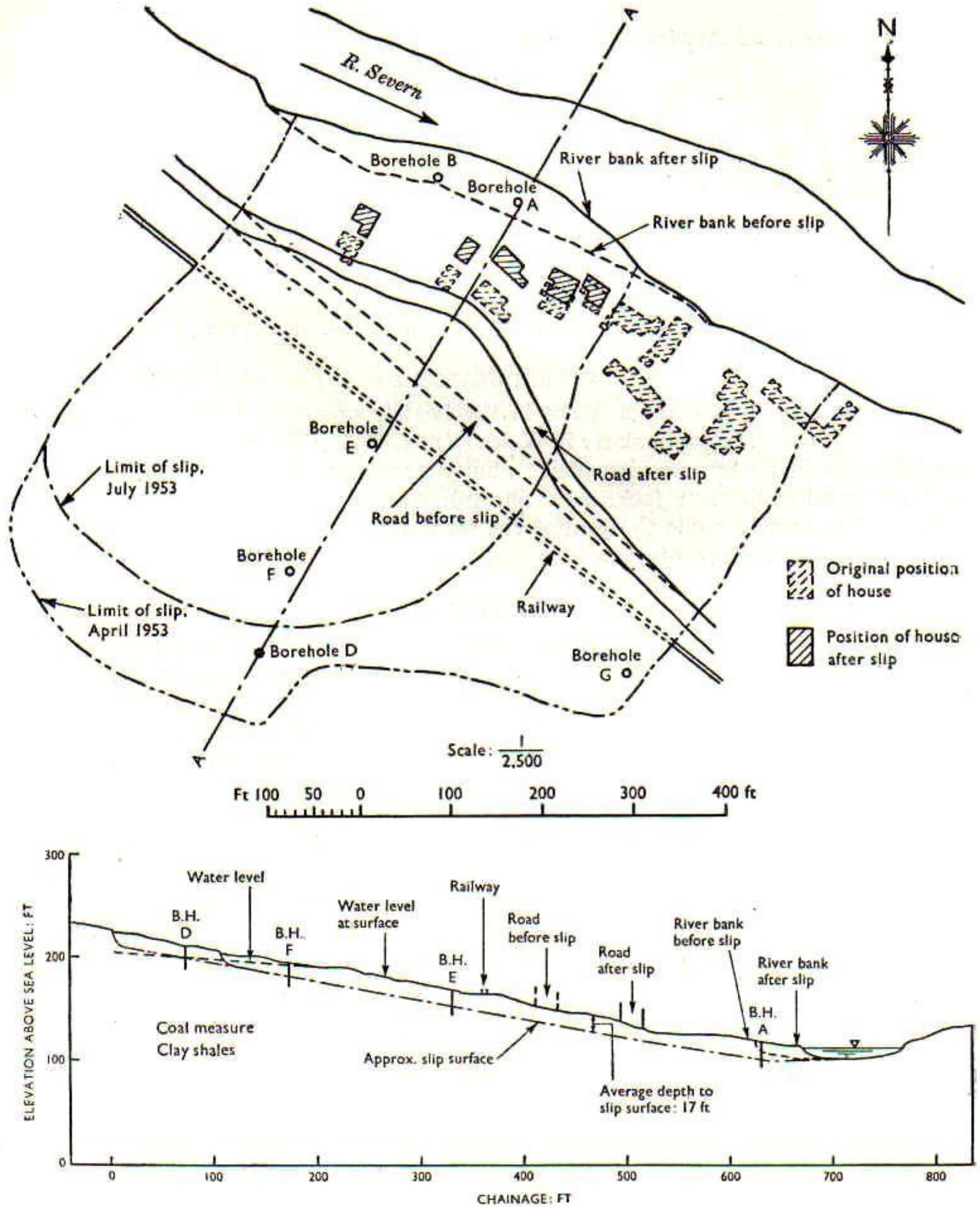


Fig. 69. Jackfield landslip: site plan, and cross-section A-A

Jackfield

Jackfield, Shropshire

From drained shear box

Peak strength c\` = 10.5 kN/m^2; ϕ'= 25° ; w = 30%

Estimated FoS = 2.04

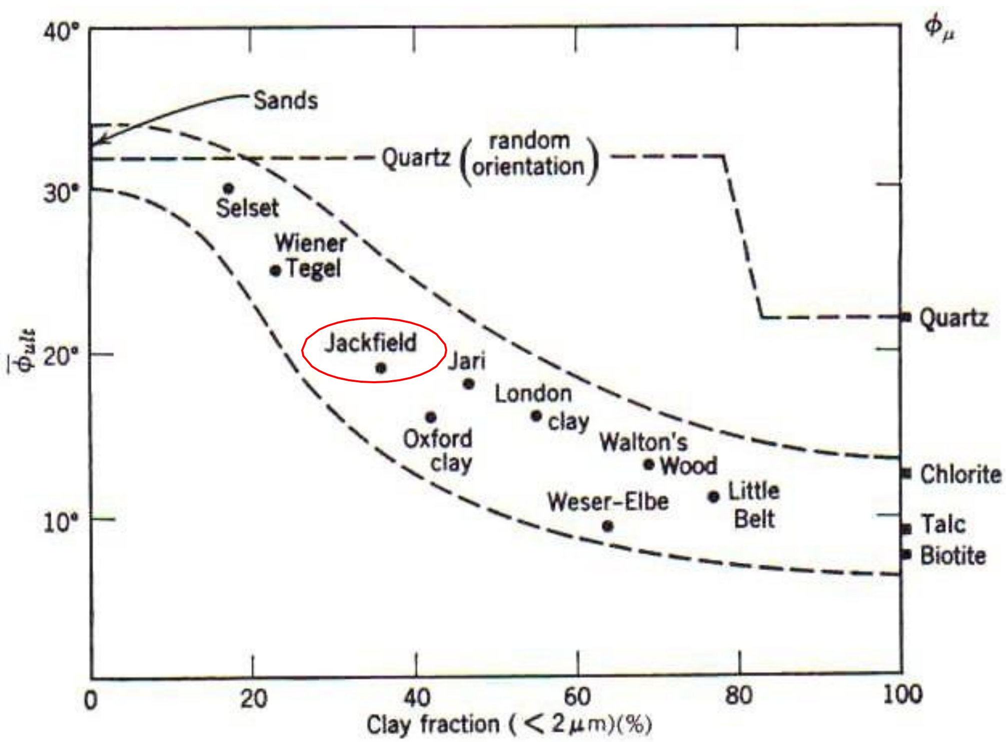
Residual strength c\` = 0 kN/m^2; ϕ'= 19° ; w = 30%

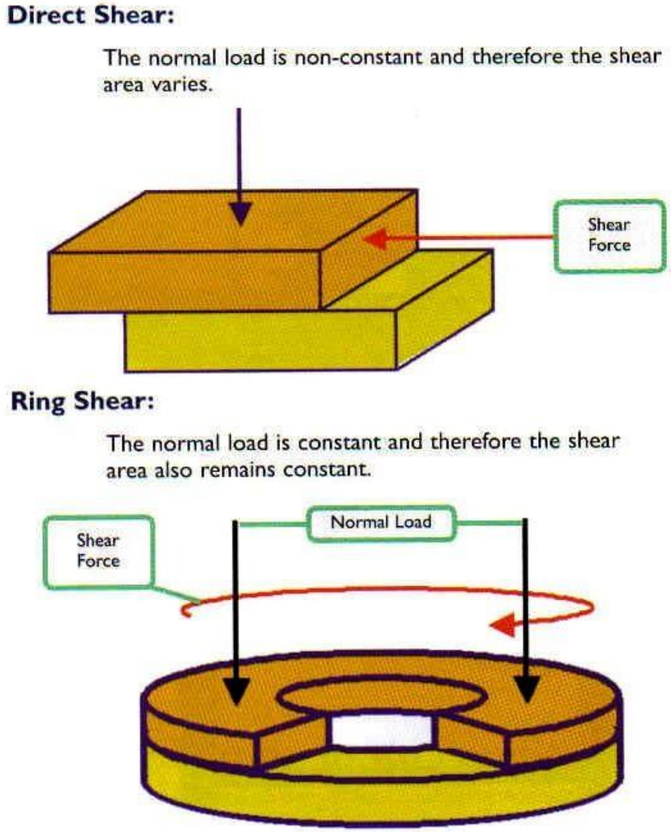
Estimated FoS = 1.08

Jackfield

In case of Jackfield, the pre-existing weakness was located and tested.

In other cases it is necessary to subject the sample to large displacements… RING SHEAR apparatus





Residual strength

15. Craig's Example 4.1 The following results were obtained from direct shear tests on specimens of a sand compacted to the *in-situ* density.

Determine the value of the shear strength parameter .

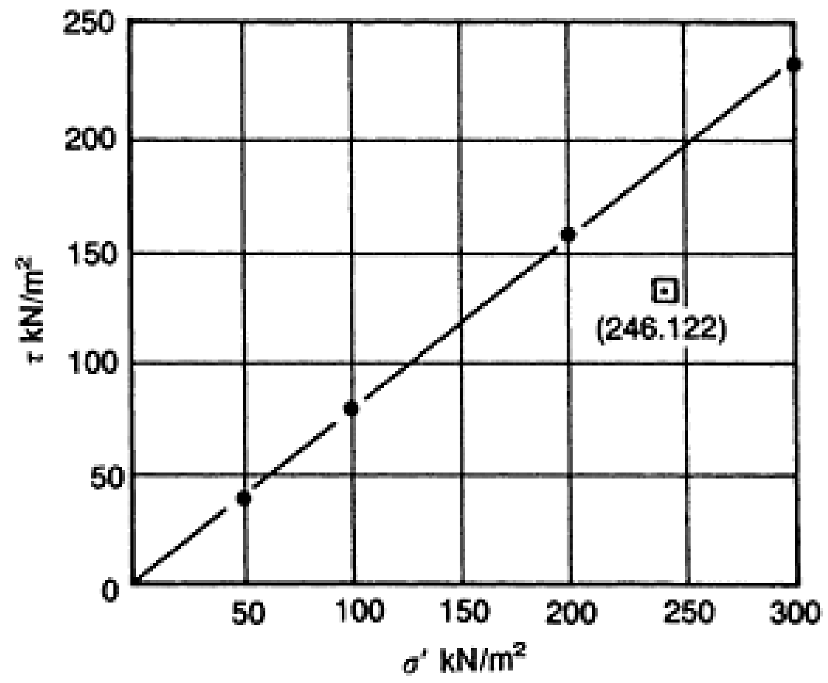
Normal stress (kN/m^2)	50	100	200	300
Shear stress at failure (kN/m^2)	36	80	154	235

Would failure occur on a plane within a mass of this sand at a point where the shear stress is 122kN/m^2 and the effective normal stress 246kN/m^2?

Direct shearbox: worked example

Determine the value of the shear strength parameter .

The values of shear stress at failure are plotted against the corresponding values of normal stress, as shown. The failure envelope is the line having the best fit to the plotted points; in this case a straight line through the origin. If the stress scales are the same, the value of can be measured directly and is 38° .



The stress state σ_n = 246 kPa, τ = 122 kPa, plots below the failure envelope, and therefore would not produce failure.

Direct shearbox: worked example

e) Sketch and label a plot of shear stress vs. horizontal displacement for shear box tests performed on a loose and a dense sand under the same vertical stress. Explain the shape of the two curves and comment on the soil condition reached at large strains.

Napier Exam Question (8/33 marks)

Direct shearbox: worked example

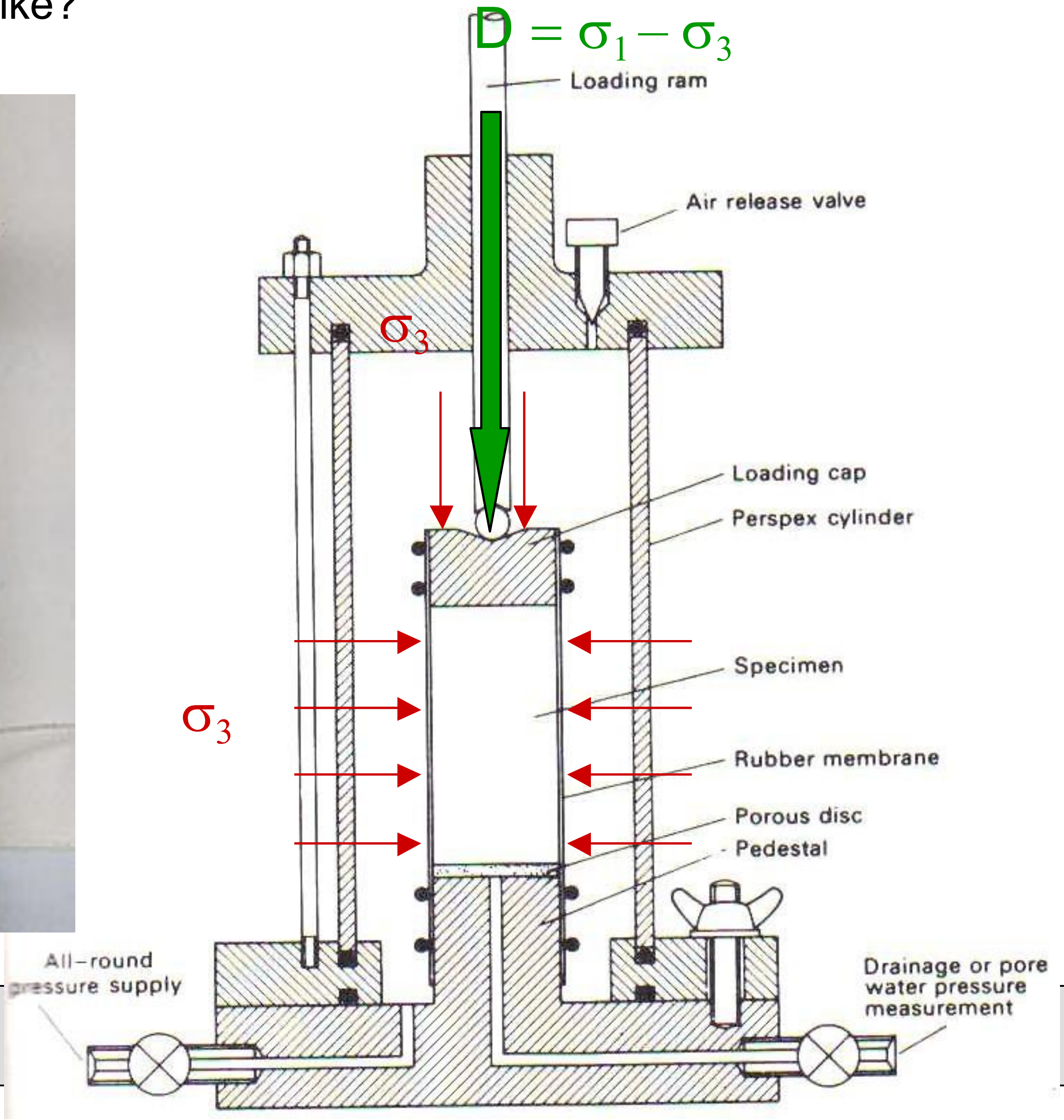
TRIAXIAL STRENGTH TESTING

The direct shear box provides much useful information and insight into the shearing behaviour of soils. However, it suffers from not being able to provide information on the pore pressure within the soil nor on the principal stress conditions along the failure surface.

Additionally the failure surface is restricted to the horizontal surface between the upper and lower parts of the shear box.

Bear in mind that such limitations are not a major problem for sands, where an undisturbed samples are not possible and pore pressures equilibrate quickly anyway. But for clays these issues are significant and the triaxial test becomes the test procedure of choice.

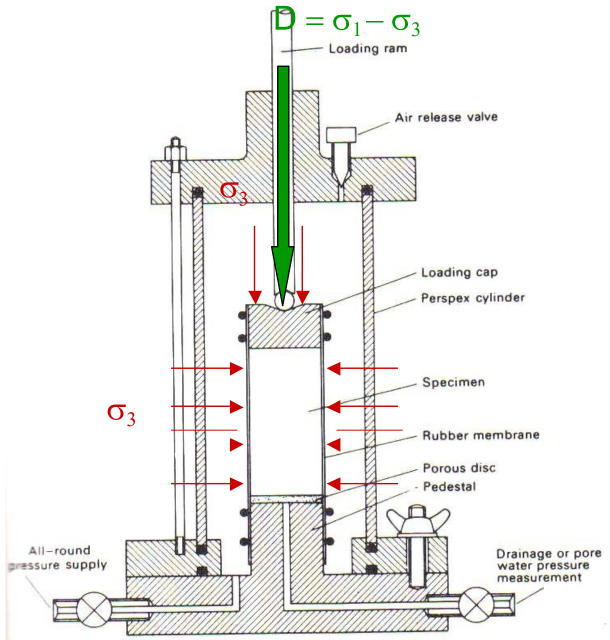
What does a triaxial rig look like?



General Test Procedure

Cylindrical samples with height to diameter ratio of 2:1 – from 38 mm to 200 mm diameter.

Sample is placed within an impermeable membrane to isolate if from the water filling the cell (perspex cylinder).



Drainage of the sample is controlled – drained or undrained

Two main stages:

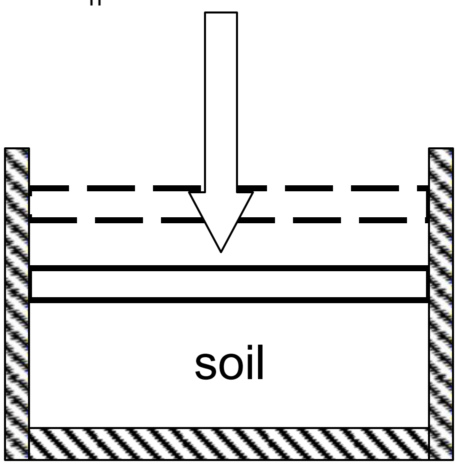
1. Apply an all-round or cell pressure (σ_3) and allow sample to consolidate – or not as case may be (UU).
2. Apply an axial (deviatoric) load to deform and shear the sample. Again drainage may or may not be permitted.

- Triaxial stress
  - Triaxial vs. shear box – selection
  - From the boundary to a plane (any plane!): stress states & Mohr's circle
  - Mohr-Coulomb Failure criterion
- Triaxial testing
  - Drained
  - Consolidated Undrained
  - Unconsolidated Undrained
  - Practical considerations controlling test type – D, CU, UU ?

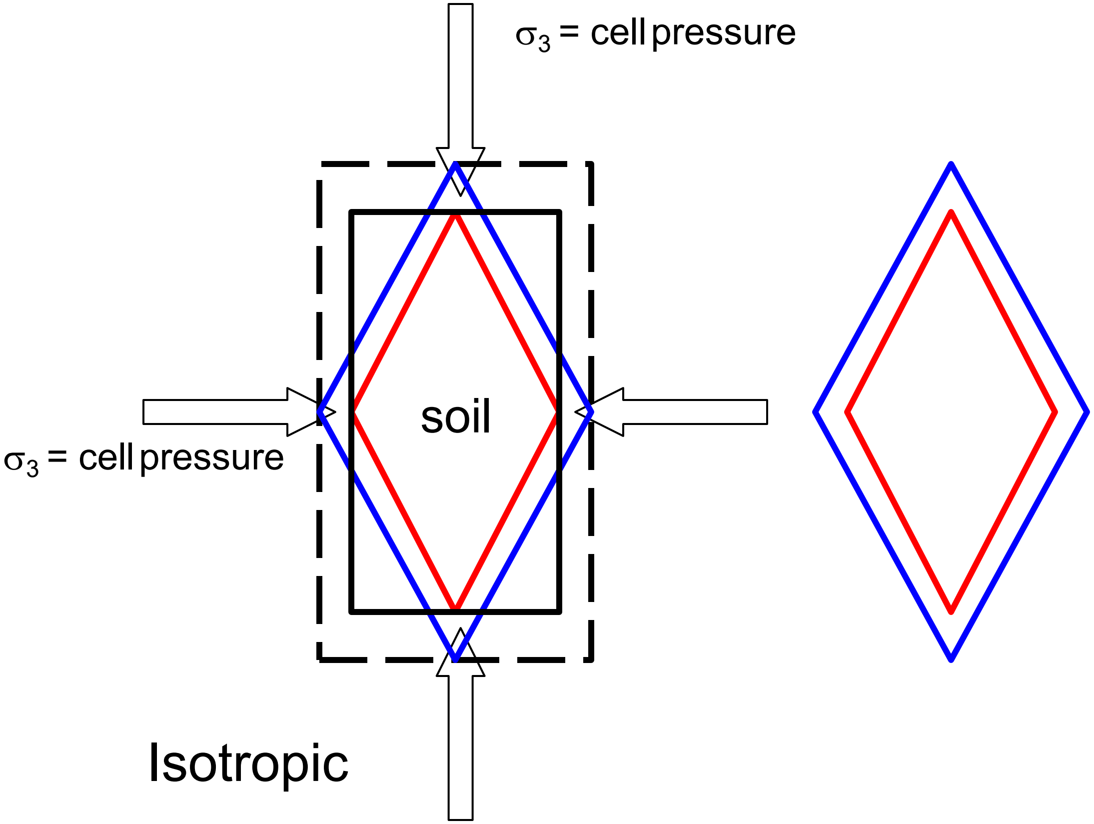
Shear strength; triaxial testing

Triaxial vs direct shearbox loading and strain response:

oedometric or isotropic loading - change in volume



Oedometric



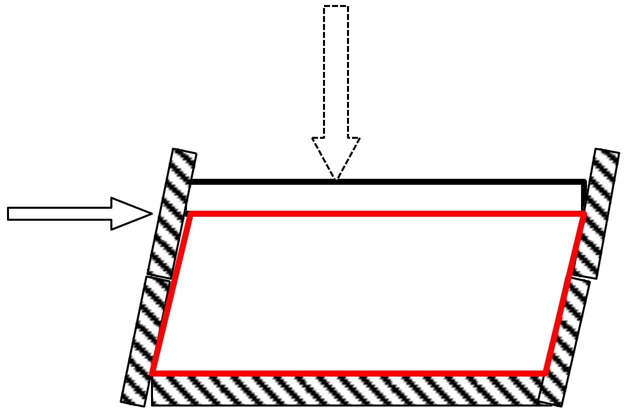
Isotropic

Strains are finite & reducing

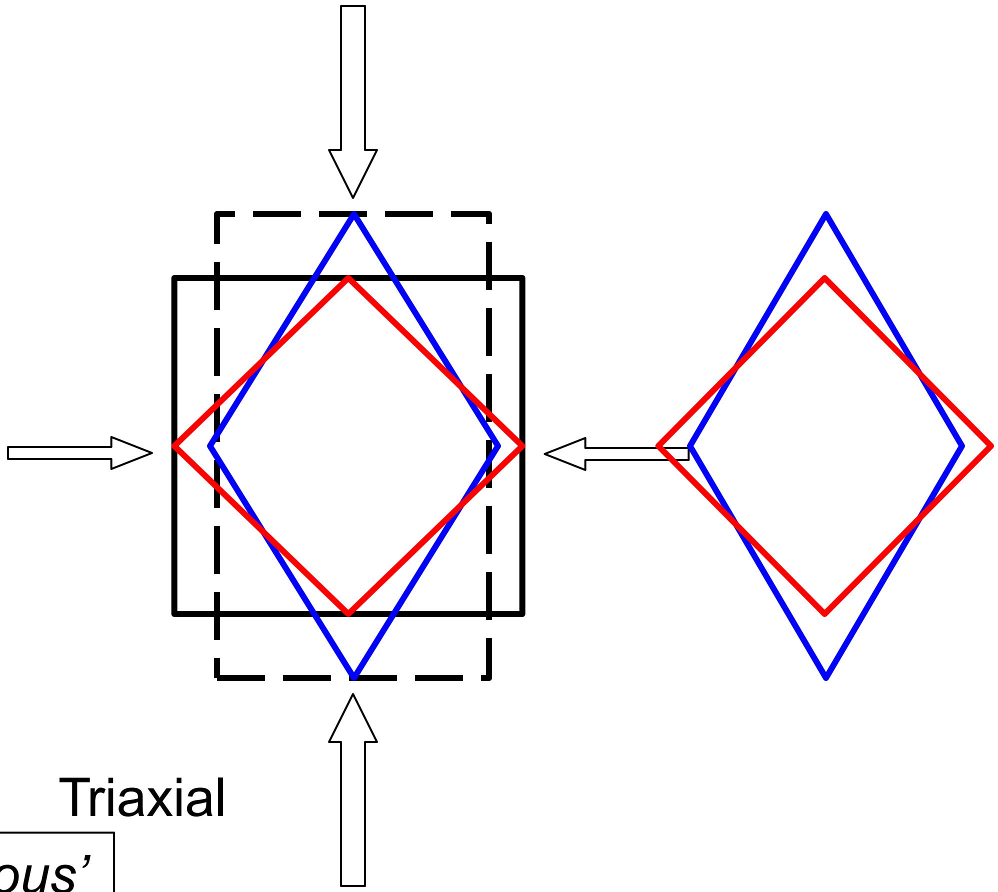
Volumetric strain

Loading and load response:

shear stresses - change in shape



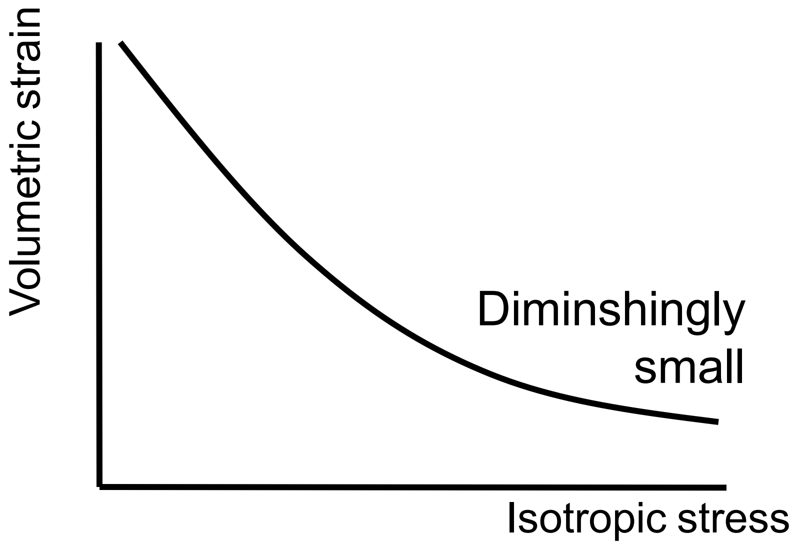
Simple shear

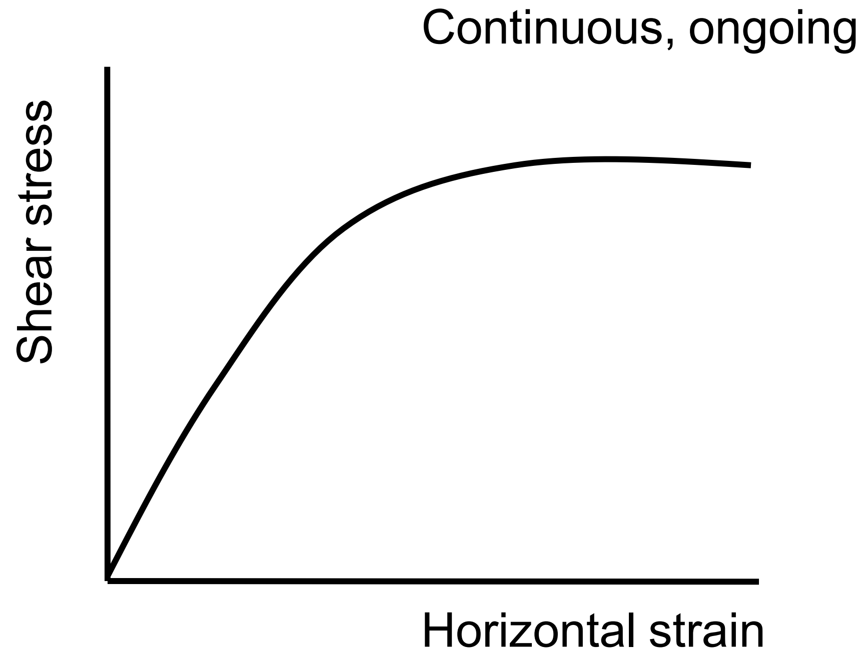


Triaxial

Strains may be large and 'continuous'

Shear strain



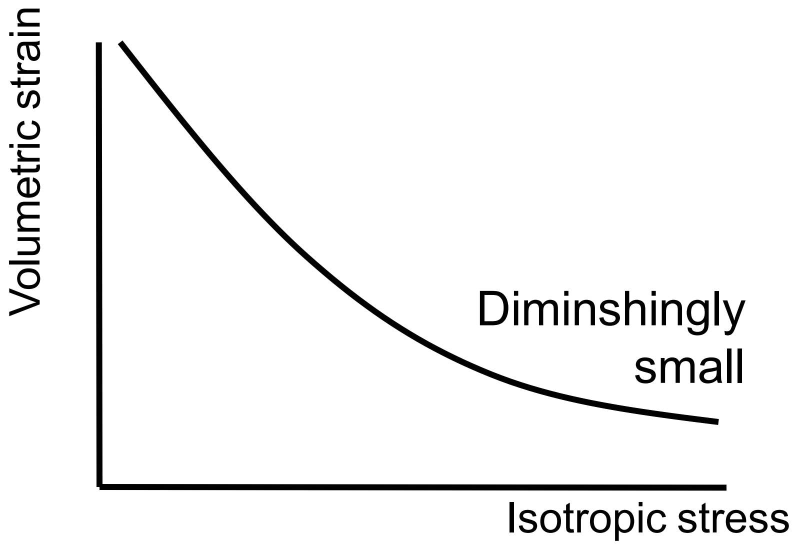


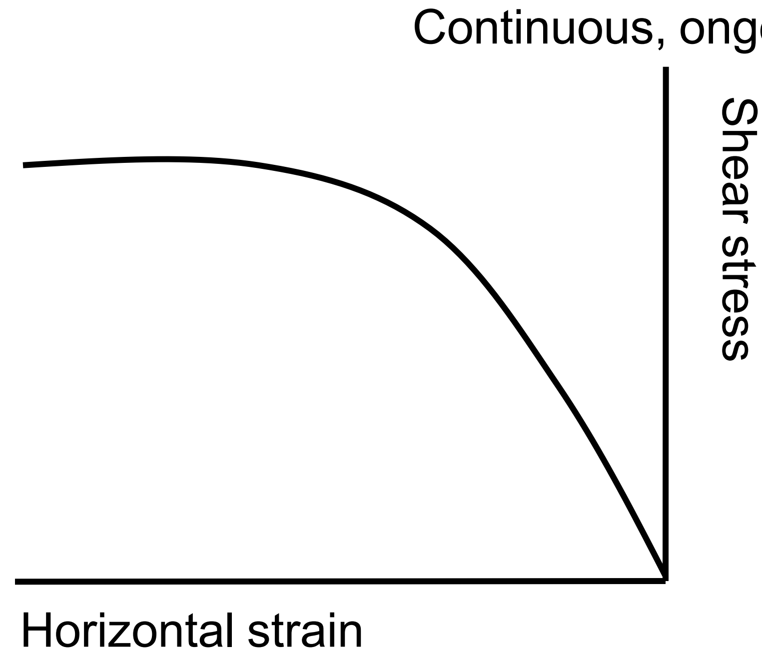
Oedometric or isotropic stress: volumetric strain relationship

Shear stress: shear strain relationship

Let us rotate and flip the shear stress:strain curve to emphasise the point.

Shear strain & shear stress



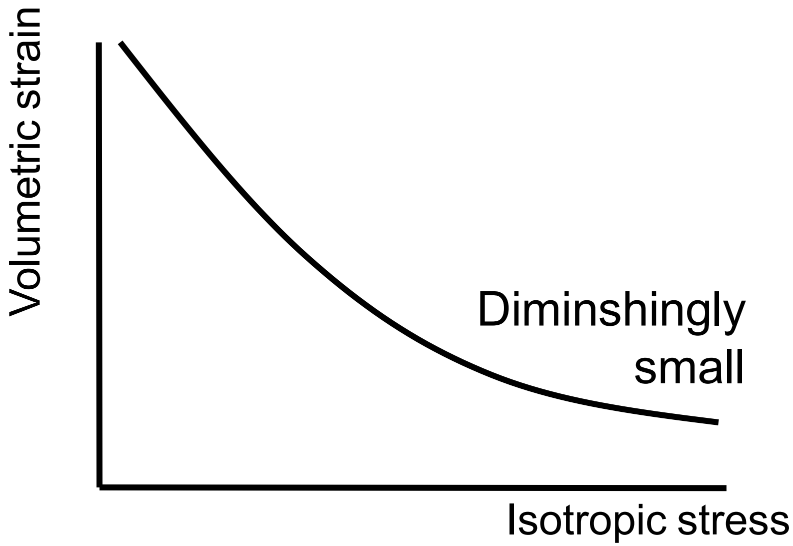


Oedometric or isotropic stress: volumetric strain relationship

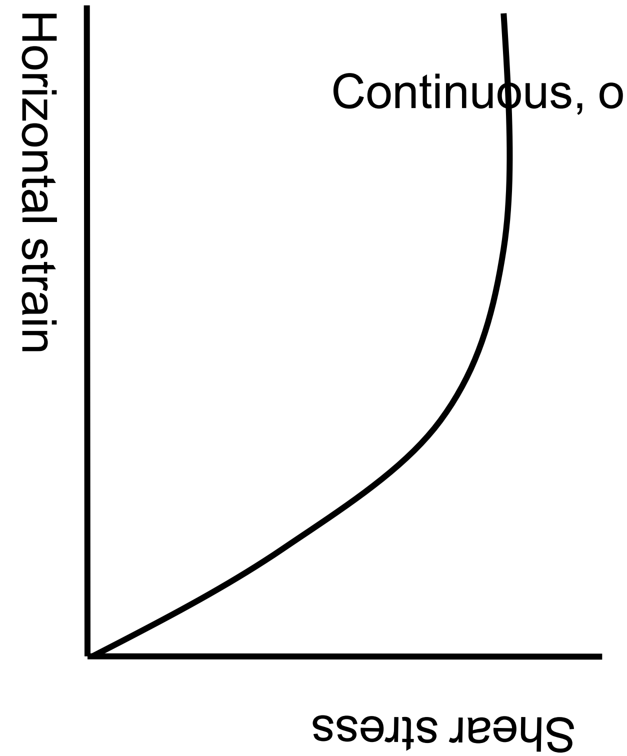
Shear stress: shear strain relationship

Let us rotate and flip the shear stress:strain curve to emphasise the point.

Shear strain & shear stress



Oedometric or isotropic stress: volumetric strain relationship



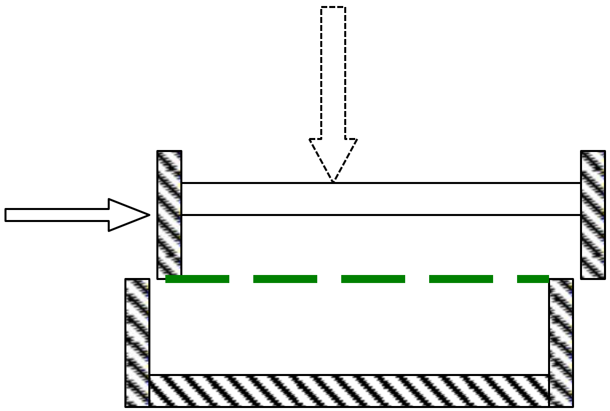
Shear stress: shear strain relationship

In contrast, when the strength of a soil is approached, the additional stress required to shear the soil reduces until shearing occurs under constant stress.

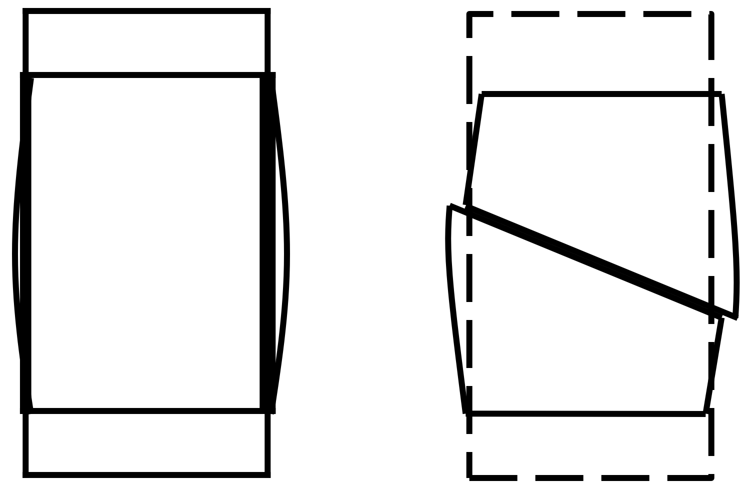
Shear strain & shear stress

Loading and load response at failure:

shear stresses - change in shape

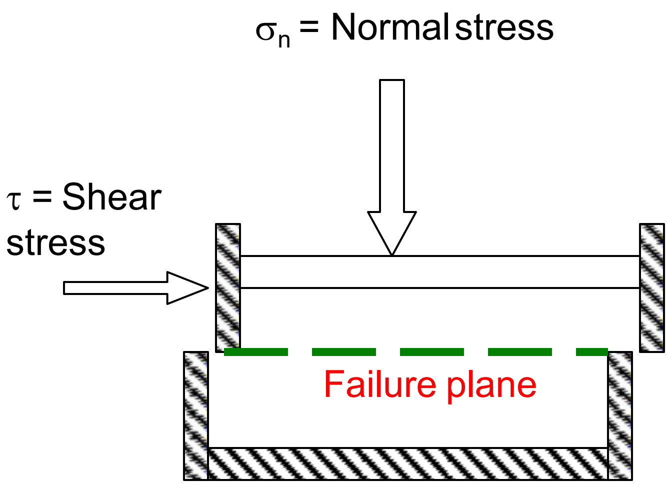


Direct shear

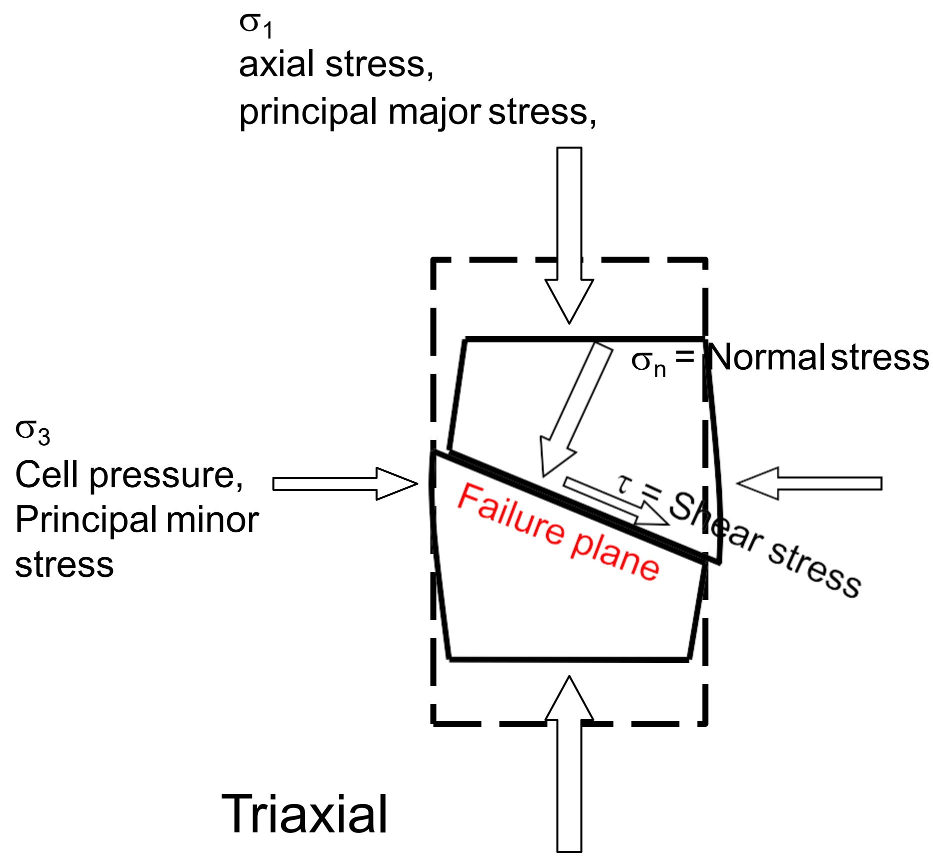


Triaxial: by barrelling in shear

Modes of failure



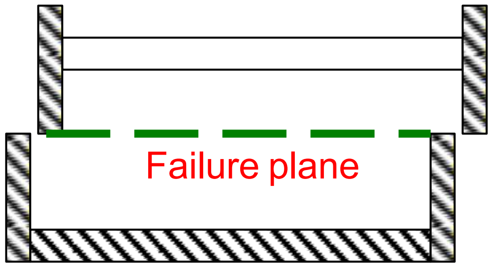
Simple shear



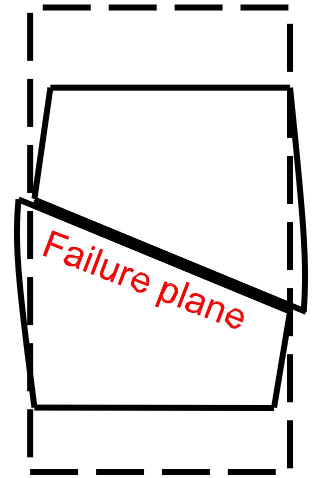
Triaxial

Deviator stress = σ_1-σ_3

Terminology & notation



Simple shear



Triaxial

Failure surface is forced, confined to predetermined path

Mode of failure is unconstrained – barrelling or shear surface.
Plane of sliding is not forced – pre-existing planes of weakness can be followed.

Some comments on two test methods…

H

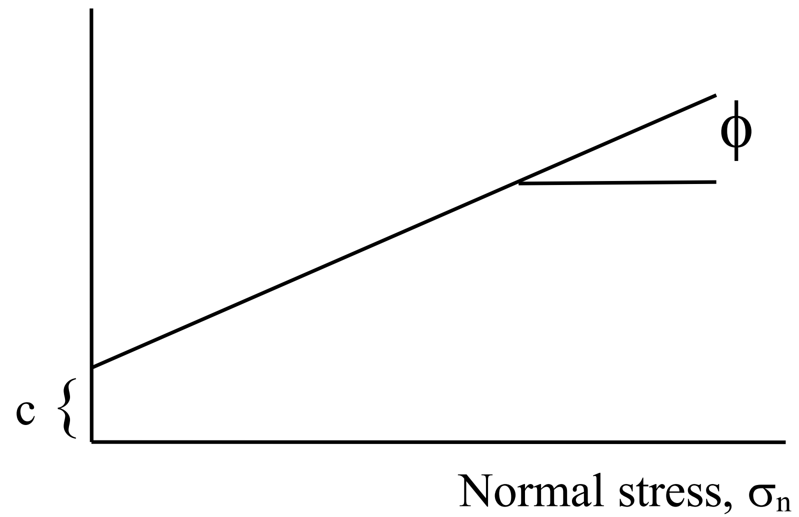
Defining failure

Remind ourselves of the Coulomb failure criterion

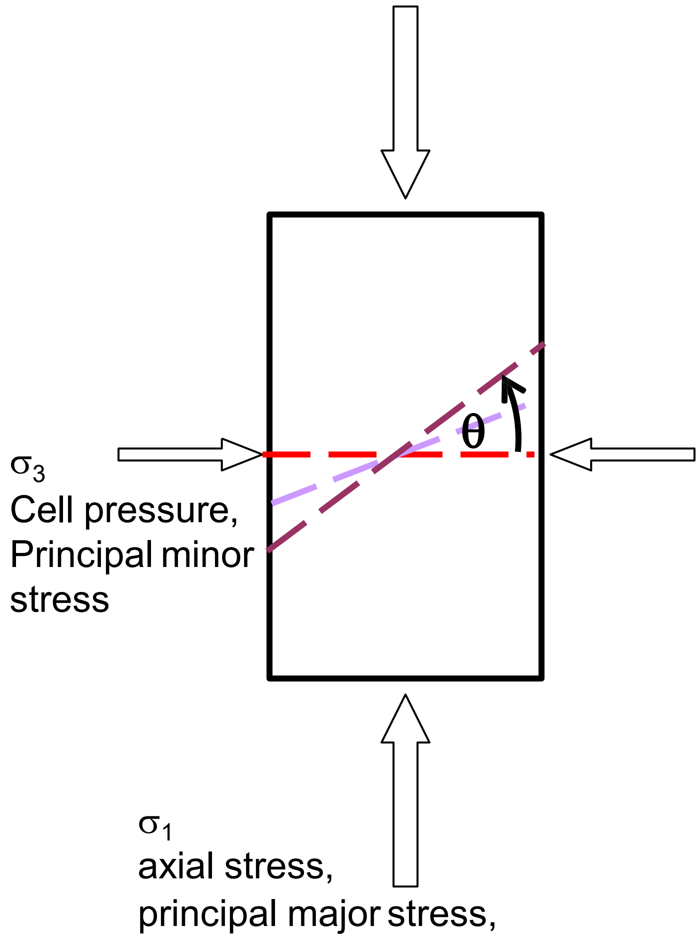
$$\tau = c' + \sigma_n' \tan\phi'$$

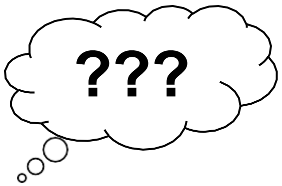
where,

τ = shear strength along failure surface
c' = cohesion
σ'_n = normal stress
ϕ' = angle of friction



Coulomb failure





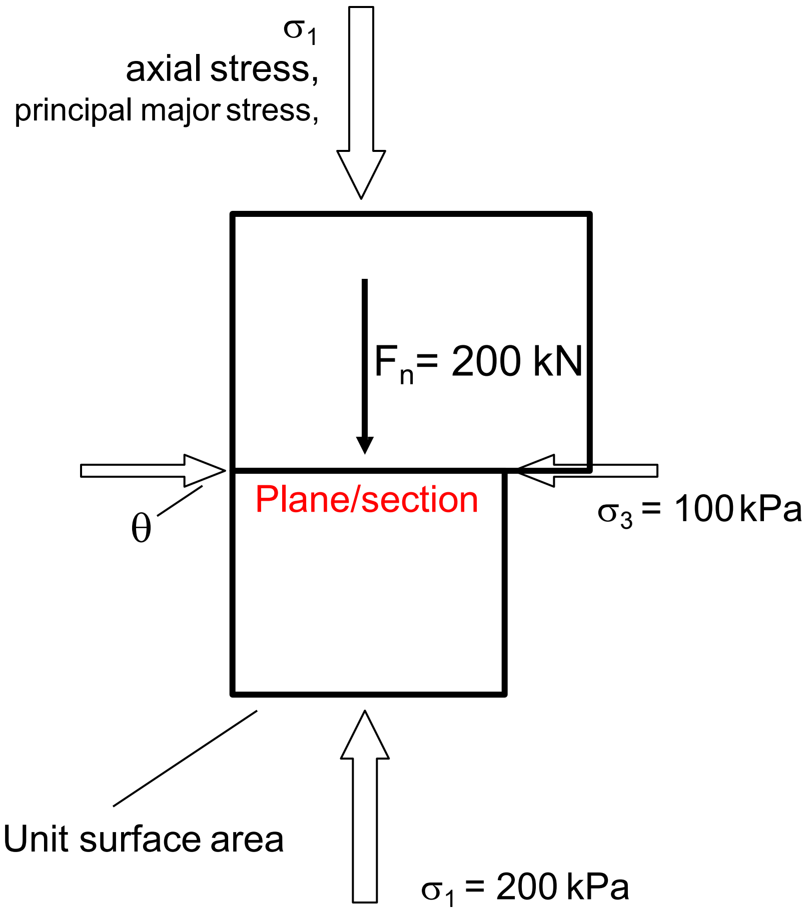
Problem

How can we use a Coulomb failure criterion?

How can we define shear strength as a function of normal stress, if orientation/location of failure surface on which the shear and normal stresses act is not known?

Coulomb failure criterion in triaxial stress state

Answer! … In 1914, Otto Mohr introduced a useful device – a circle – by which the stress state on a surface oriented at any angle within a body could be determined.



Consider a body sectioned along a surface and the resulting combination of normal and tangential (shear) forces required to maintain equilibrium of the body.

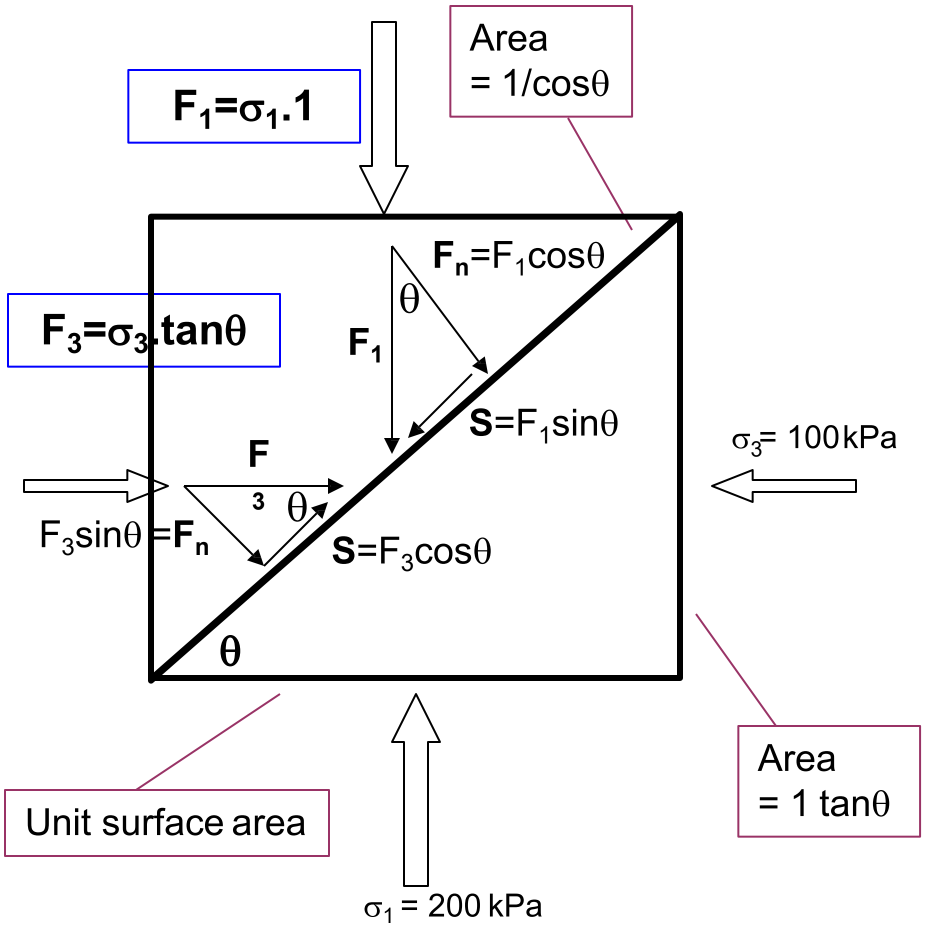
In this simple example, where the plane is at right angles to the principal major stress σ_1 (i.e. θ = 0), the equilibrating forces (from simple trigonometry) are:

$F_n = \sigma_1.\cos\theta.\ A = \sigma_1 x\ 1\ x\ 1 = 200$ kN

$S = \sigma_1.\sin\theta.\ A = \sigma_1 x\ 0\ x 1 = 0$ kN

Mohr …

A more general definition of the normal and shear forces acting on a plane is necessary to progress any further.



Collect the normal force components:

$$F_n = F_1 \cos\theta + F_3 \sin\theta$$

$$F_n = \sigma_1 \cos\theta + \sigma_3 \tan\theta.\sin\theta$$

The normal stress on the inclined plane is

$$\sigma_n = F_n/A = \sigma_1 \cos^2\theta + \sigma_3 \tan\theta.\sin\theta.\cos\theta$$

which from trigonometry gives

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

Collection of shear forces gives:

$$S = F_1 \sin\theta - F_3 \cos\theta$$

$$S = \sigma_1 \sin\theta - \sigma_3 \tan\theta.\cos\theta$$

$$\tau = \sigma_1 \sin\theta.\cos\theta - \sigma_3 \tan\theta.\cos^2\theta$$

$$\tau = \sigma_1 \sin\theta.\cos\theta - \sigma_3 \sin\theta.\cos\theta$$

$$\tau = (\sigma_1 - \sigma_3) \sin\theta.\cos\theta$$

More on Mohr …

Using equations for normal and shear stresses on a plane, in conjunction with trigonometric identities, expressions for the normal and shear stresses on a plane orientated at θ from horizontal are given by:

$\sigma_n = ½(\sigma_1+\sigma_3)+½(\sigma_1-\sigma_3)\cos(2\theta)$ and $\tau = ½(\sigma_1-\sigma_3)\sin(2\theta)$

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \sin^2\theta$$

$$Identity: \cos 2\theta = \cos^2\theta - \sin^2\theta$$

So

$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_3 \cos^2\theta - \sigma_3 \cos 2\theta$$

$$\sigma_n = (\sigma_1 + \sigma_3)\cos^2\theta - \sigma_3 \cos 2\theta$$

$$Identity: \cos 2\theta = 2\cos^2\theta - 1$$

$$\sigma_n = (\sigma_1 + \sigma_3)\frac{1}{2}(\cos 2\theta + 1) - \sigma_3 \cos 2\theta$$

$$\sigma_n = \frac{1}{2}(\sigma_1 \cos 2\theta + \sigma_3 \cos 2\theta + \sigma_1 + \sigma_3) - \sigma_3 \cos 2\theta$$

$$\sigma_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau = (\sigma_1 - \sigma_3)\sin\theta\cos\theta$$

$$Identity: \sin 2\theta = 2\sin\theta\cos\theta$$

So

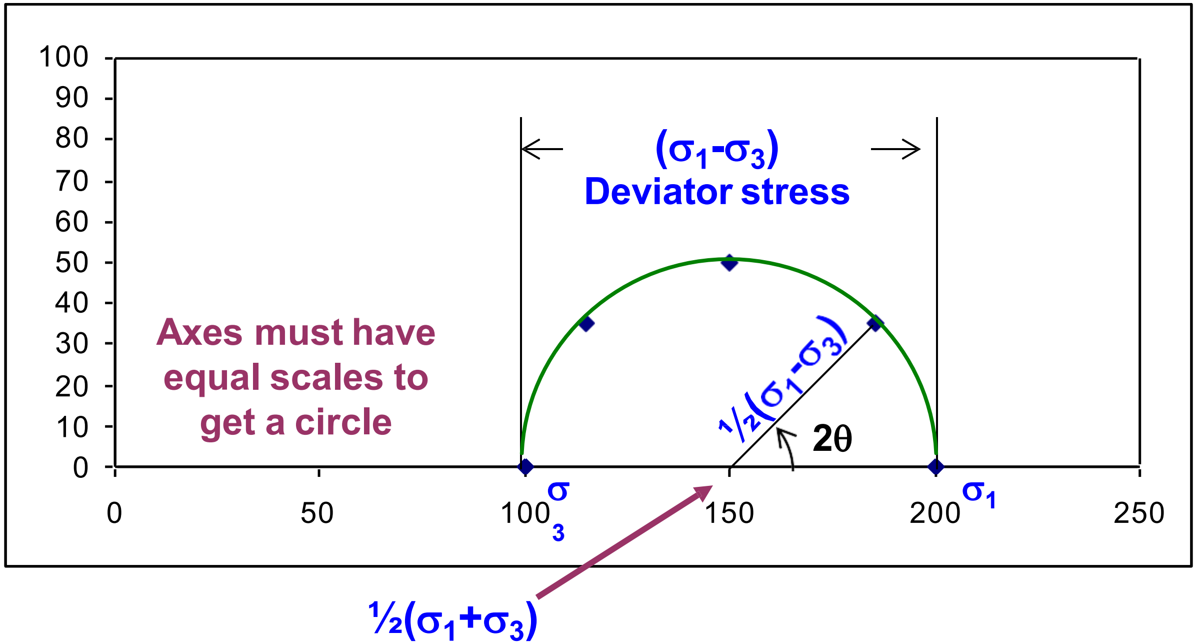
$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

Using equations for normal and shear stresses on a plane, in conjunction with trigonometric identities, expressions for the normal and shear stresses on a plane orientated at θ from horizontal are given by:

$$\sigma_n = \tfrac{1}{2}(\sigma_1+\sigma_3)+\tfrac{1}{2}(\sigma_1-\sigma_3)\cos(2\theta) \quad \text{and} \quad \tau = \tfrac{1}{2}(\sigma_1-\sigma_3)\sin(2\theta)$$

Consider now the stress regime σ_1 = 200 kPa & σ_3 = 100 kPa

θ	σ_n	τ
0	200	0
22.5	185.36	35.36
45	150	50
67.5	114.64	35.36
90	100	0



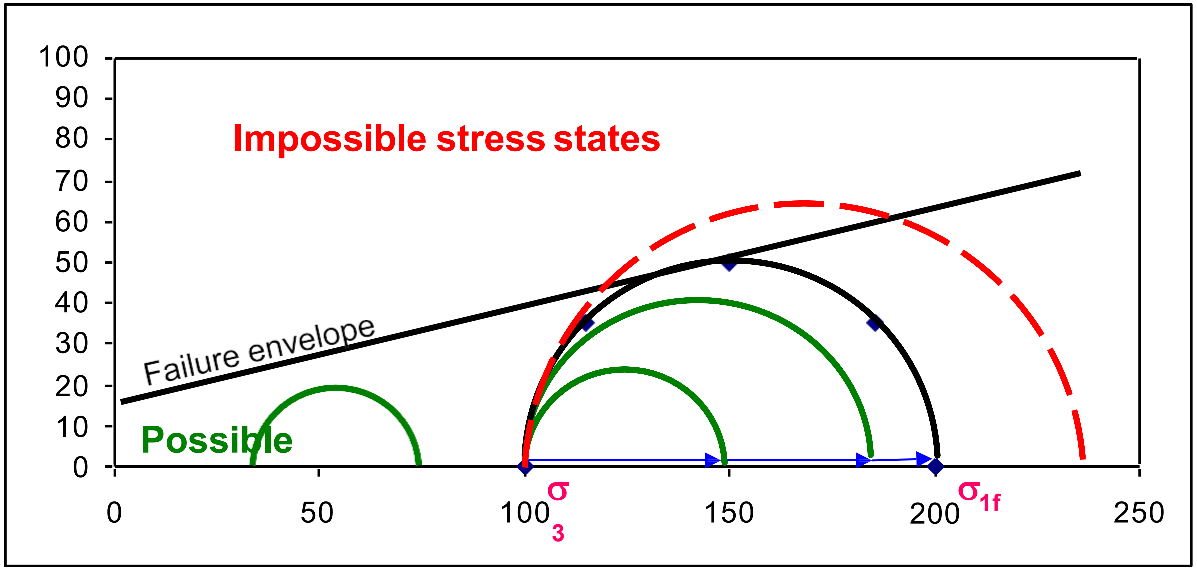
Centre of Mohr's circle = ½($\sigma_1+\sigma_3$) = *mean stress*

Radius of Mohr's circle = ½($\sigma_1-\sigma_3$) = max *shear stress*

When Mohr's circle is combined with a failure condition, such as the Coulomb condition, we are able to define failure in the triaxial test.
If Mohr's circle for a stress regime:

- Lies beneath the failure condition – 'safe'; sustainable loading condition
- Intersects the failure condition – 'unsafe'; unsustainable, failure will occur
- Tangent to failure condition – 'trigger state' to which factors of safety refer.

The failure condition is an envelope separating possible from impossible stress states.

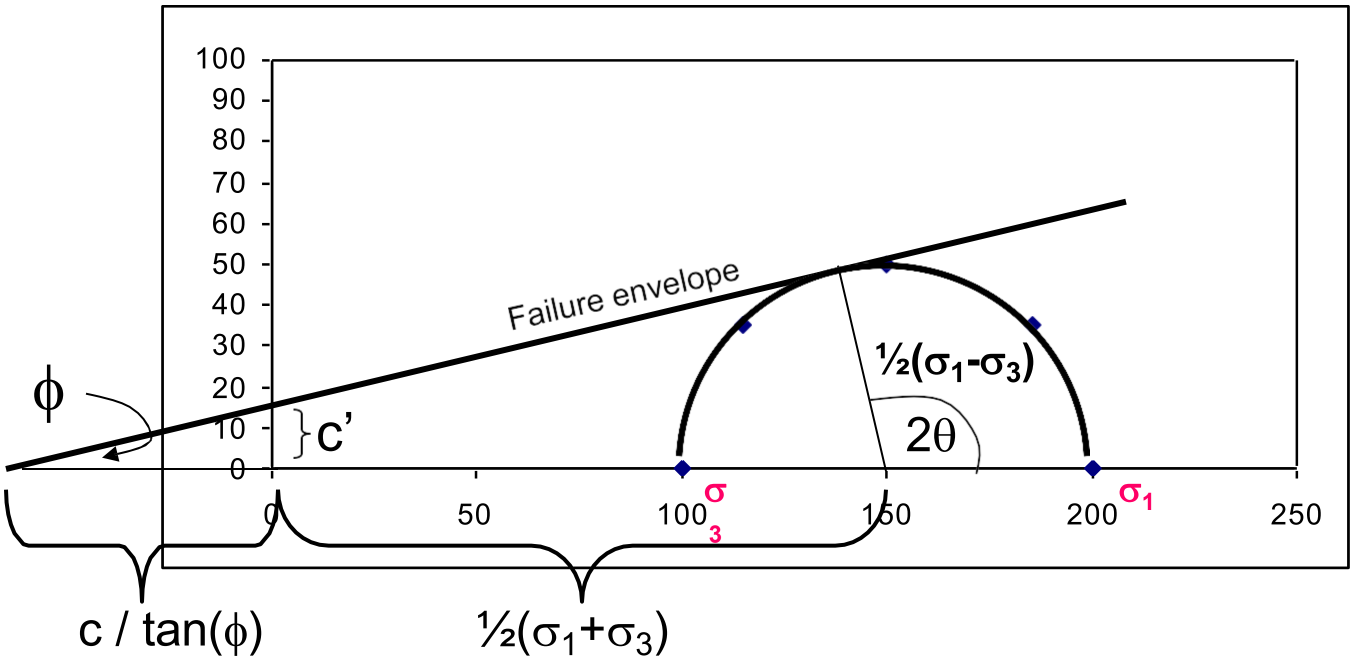


Mohr-Coulomb Failure Criterion

Consideration of the geometry of a Mohr's circle which is tangential to a Coulomb failure envelope, shows that Mohr Coulomb failure criterion is:

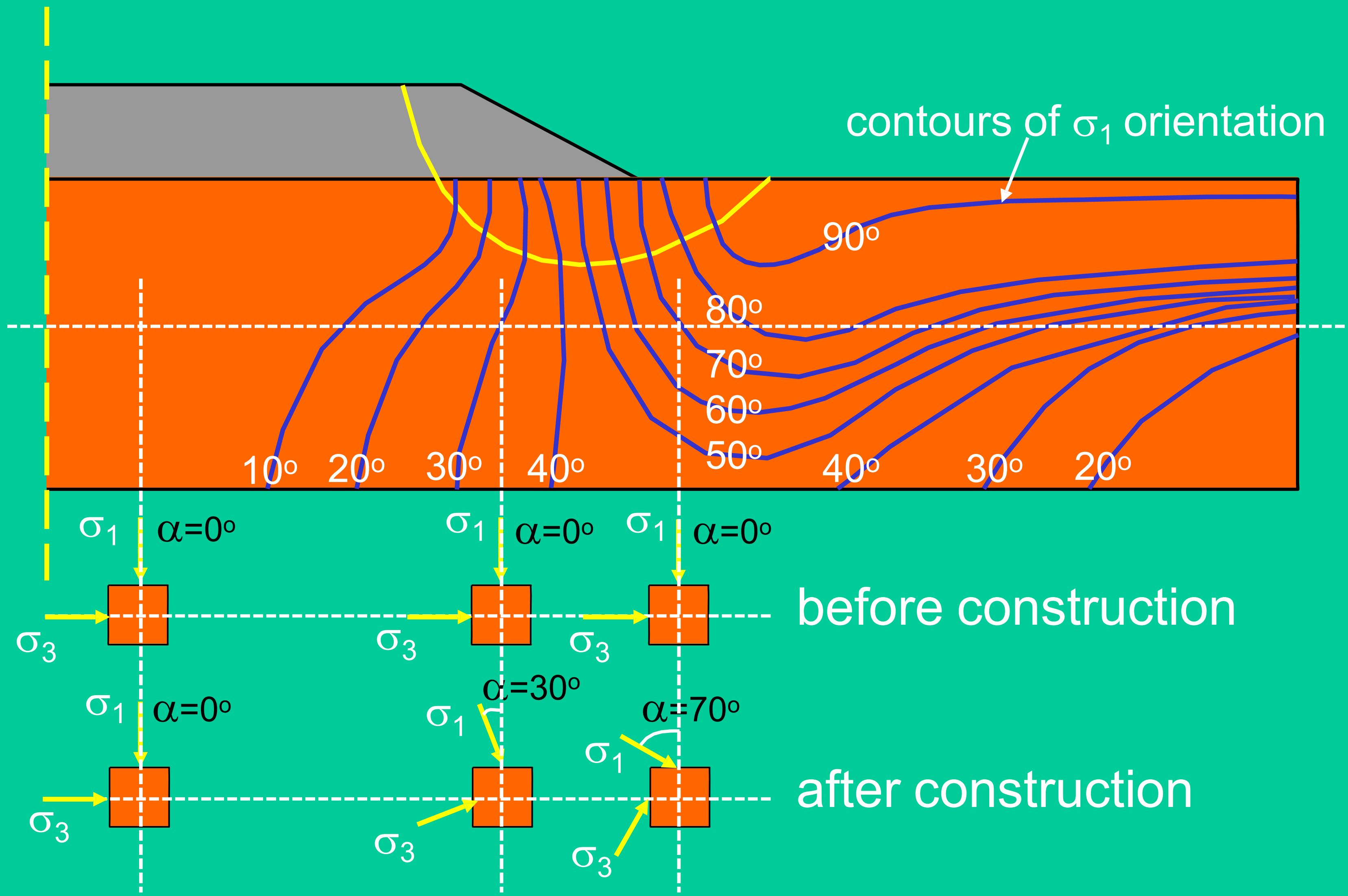
Mohr-Coulomb failure condition: $\frac{1}{2}(\sigma_1 - \sigma_3) = c' \cos \phi' + \frac{1}{2}(\sigma_1 + \sigma_3) \sin \phi'$

With orientation of failure plane: $\theta = 45^\circ + \frac{1}{2}\phi'$



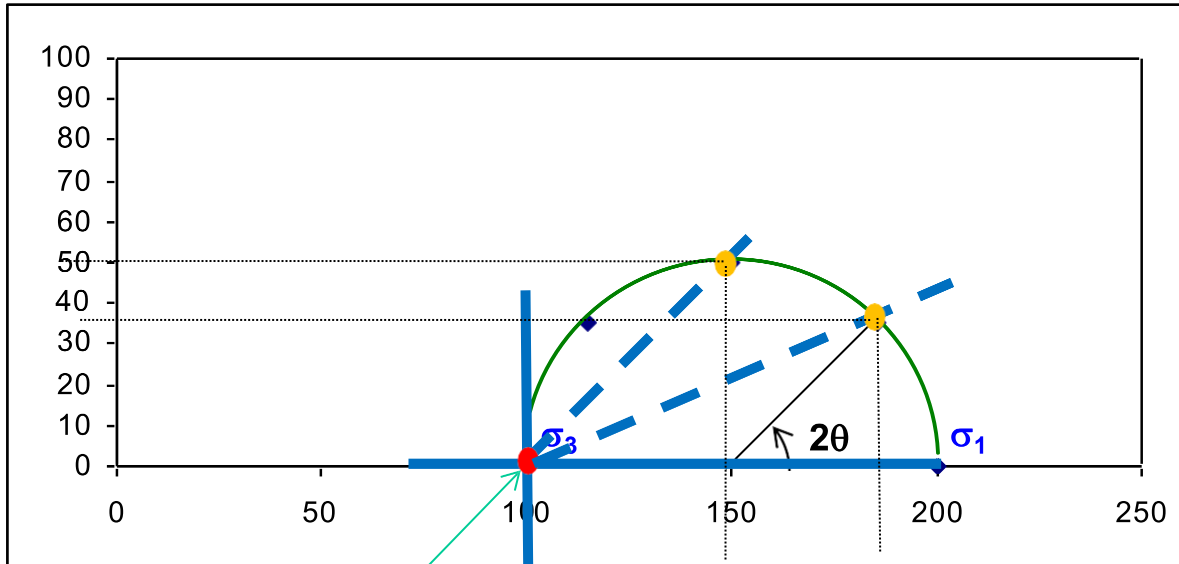
Mohr-Coulomb Failure Criterion

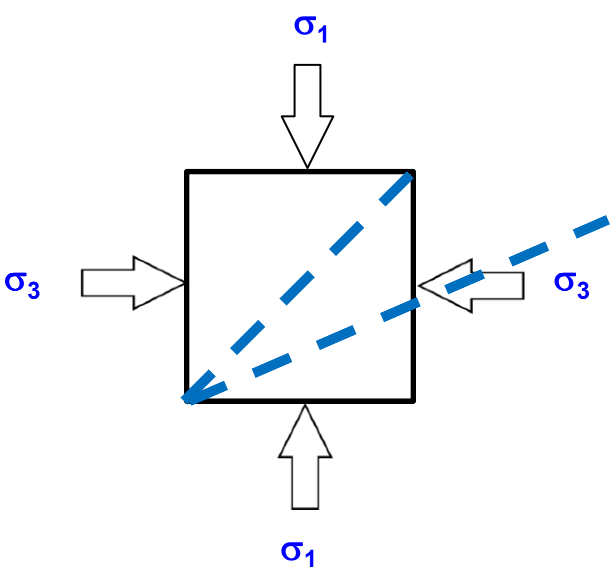
STRESSES AND STRESS CHANGES



Consider this stress regime:

σ_1 = 200 kPa
σ_3 = 100 kPa





POLE OF PLANES

θ	σ_n	τ
0	200	0
22.5	185.36	35.36
45	150	50
67.5	114.64	35.36
90	100	0

A useful concept... The pole of planes

You surely can see how this lecture is very important for your understanding of soil behaviour.

We however have just started starting to understand the complexity of shear behaviour.

So I will provide a summary only when I believe things are clearer for you (i.e. after the next lecture when this topic concludes)

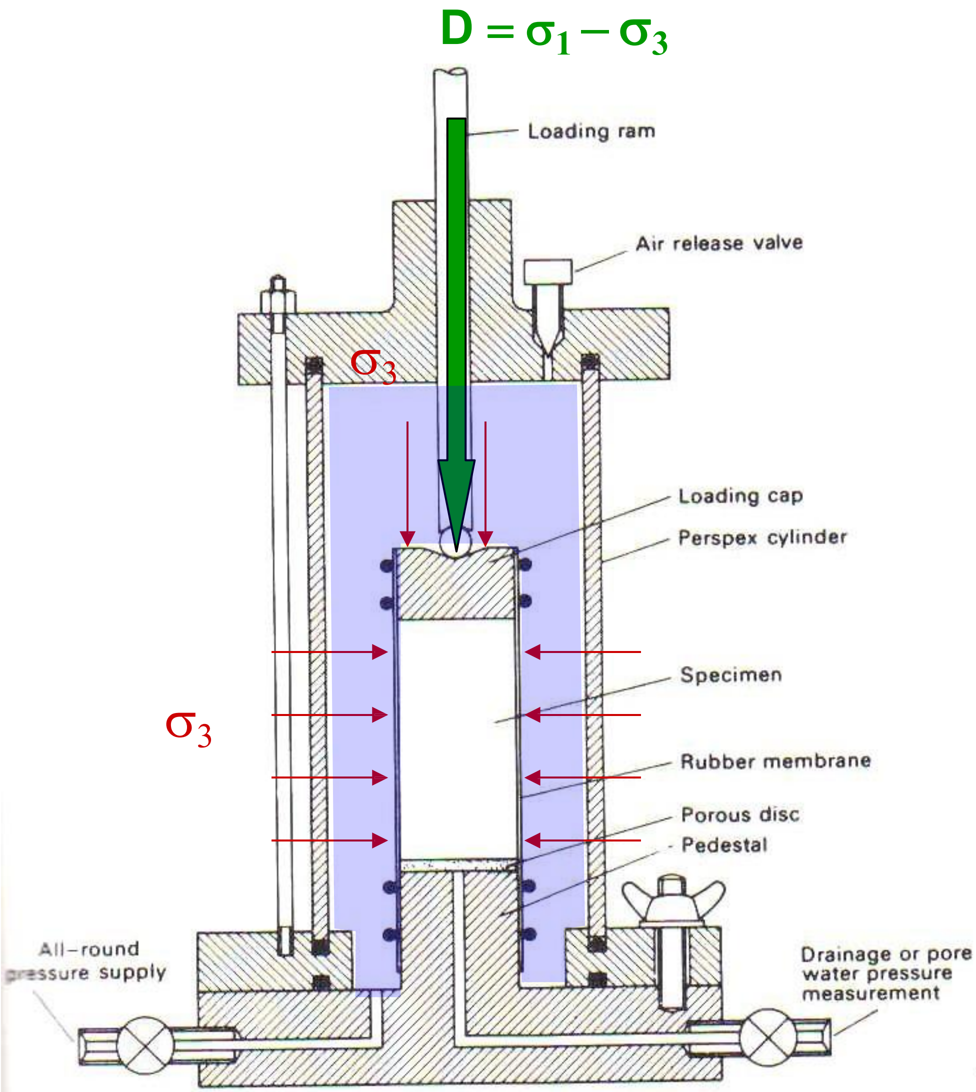
Summary

A triaxial test machine

Contents 目录

1. The triaxial test

(and we will spend the whole lecture talking about it!)



For each of **<u>three</u>** soil samples.

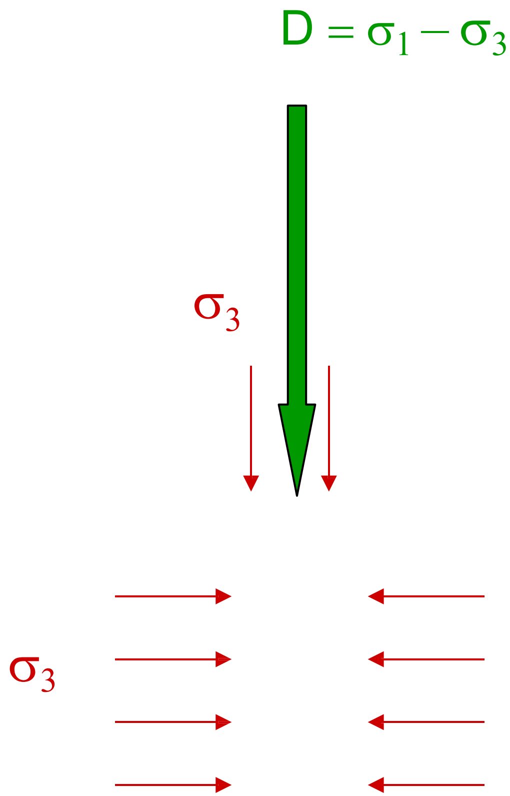
1. Apply an all-round or cell pressure.

   Drainage and volumetric strain is allowed (D & CU) or not (UU).

2. Apply an axial (deviatoric) load to deform and shear the sample.

   Drainage and volumetric strain is allowed (D) or not (CU & UU)

A triaxial test procedure



After consolidation (DRAINED) to a predetermined cell pressure,

the sample is sheared whilst drainage line is open (DRAINED = volumetric strain during shearing).

Test is performed slowly (days!) so no excess porewater pressure; applied boundary pressures are equivalent to effective stresses.

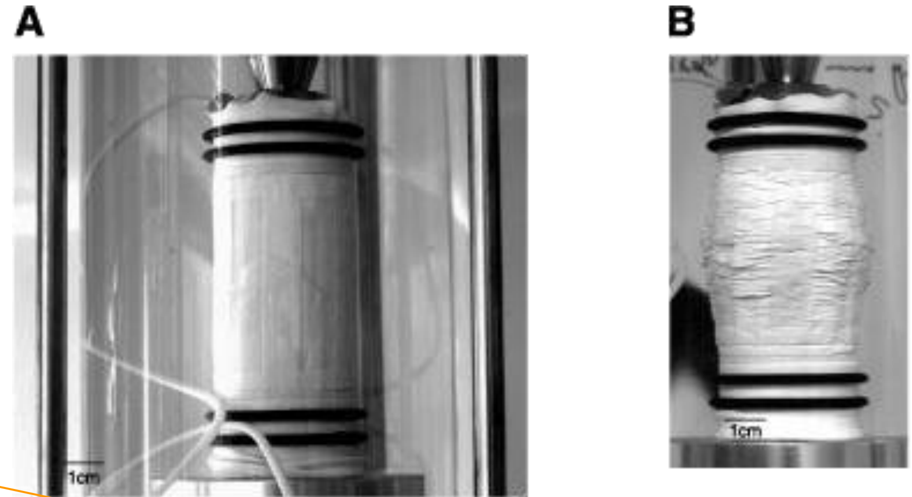
From each test, note the cellpressure and deviator stress at failure

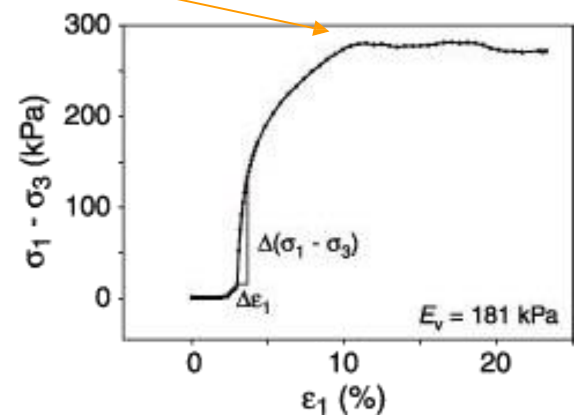
Drainage open

A drained triaxial test

i.e. volumetric strain during shearing

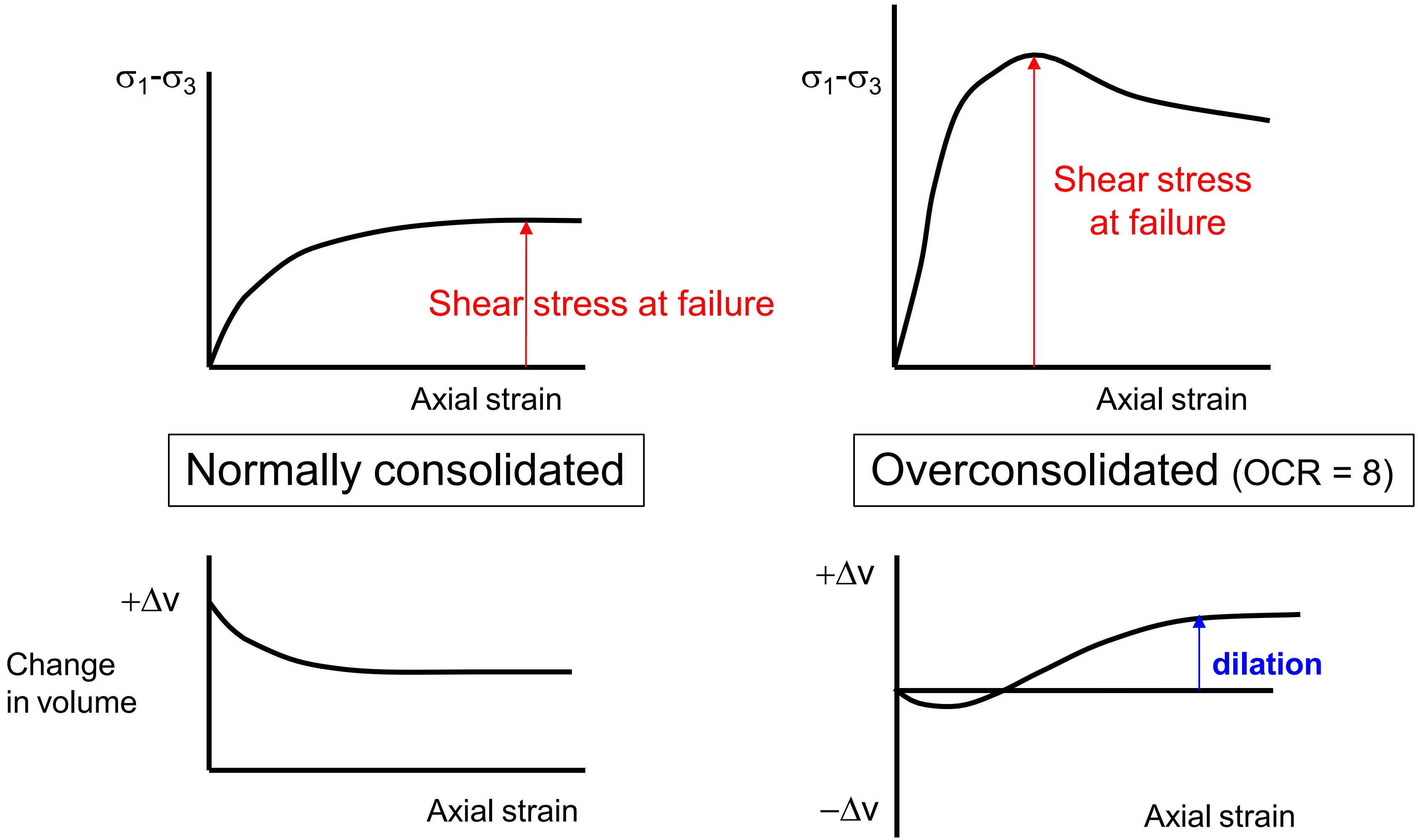
1. Note maximum deviator stress and cell pressure.
2. Plot Mohr's circles for each test.
3. Fit a tangent to the circles





Analysis of triaxial test

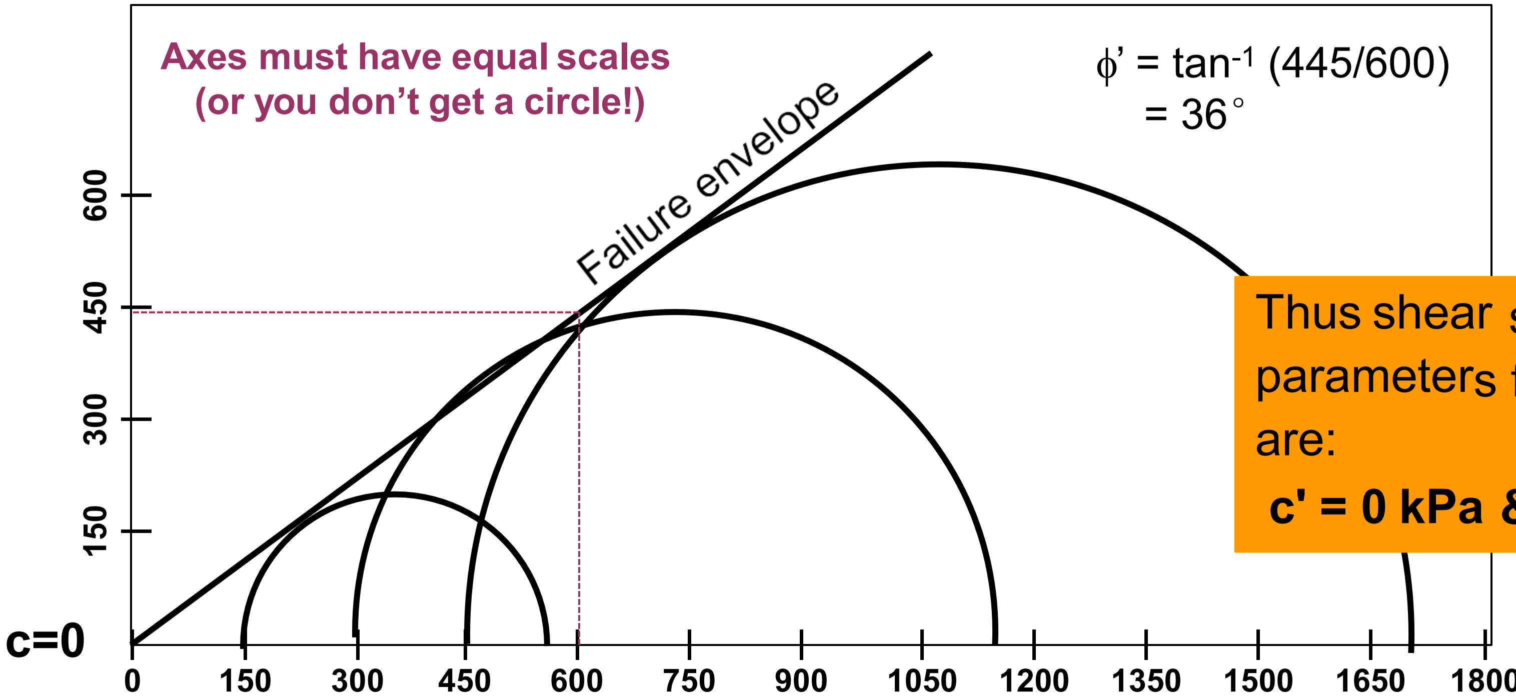
Typical shearing results – with echoes of shear box:



Drained triaxial - D

		Test 1	Test 2	Test 3
Cell pressure (kPa)	σ_3	150	300	450
Deviator stress (kPa)	D	426	848	1256
Principal major effective stress	σ_1	576	1148	1706

$\sigma_1 = \sigma_3 + D$

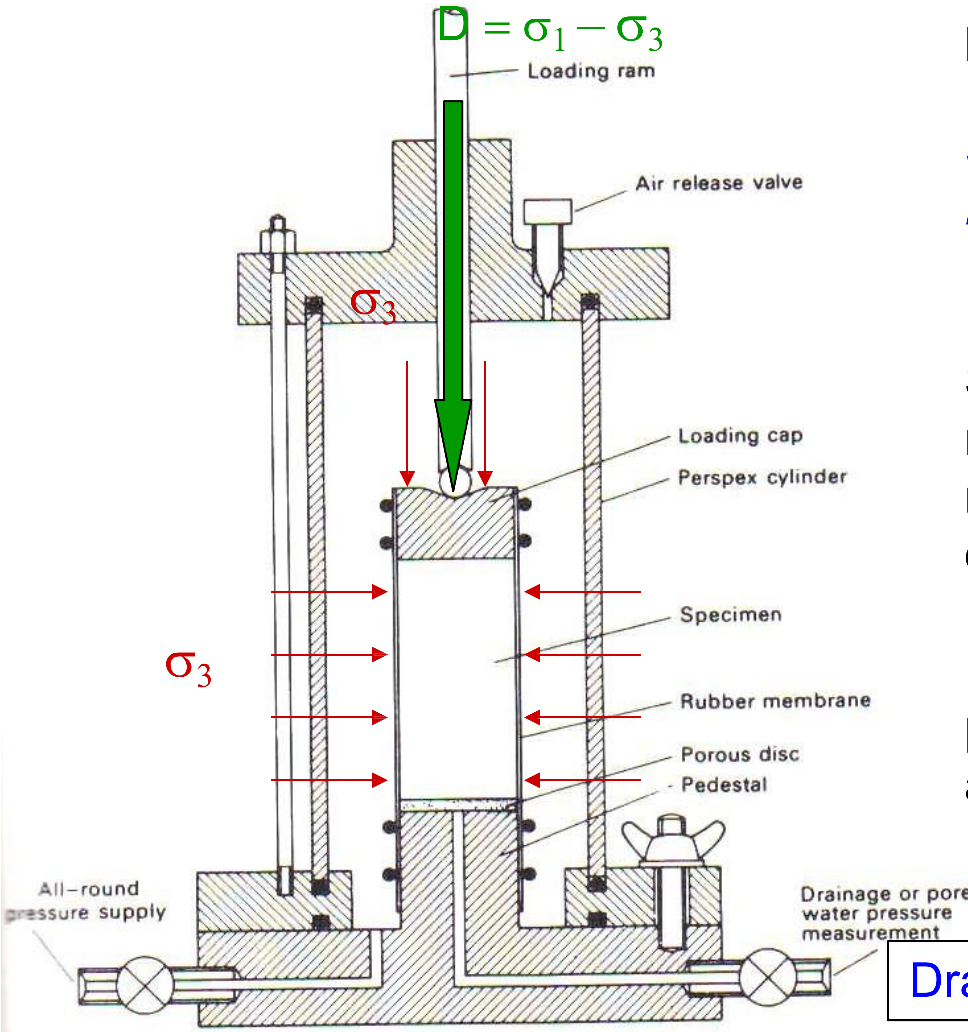


No pwp, $\Delta u = 0$

Thus shear strength parameters for this soil are:

$c' = 0$ kPa & $\phi' = 36^\circ$

D Triaxial analysis – Example 1



After consolidation (DRAINED) to a predetermined cell pressure

Shearing takes place UNDRAINED, i.e. drainage line is closed and is used to measure porewater pressure.

Shearing stage can now be performed more quickly – hours – but we need to measure the porewater pressure to determine effective stress conditions

From each test we note the cell pressure, pore water pressure and deviator stress at failure.

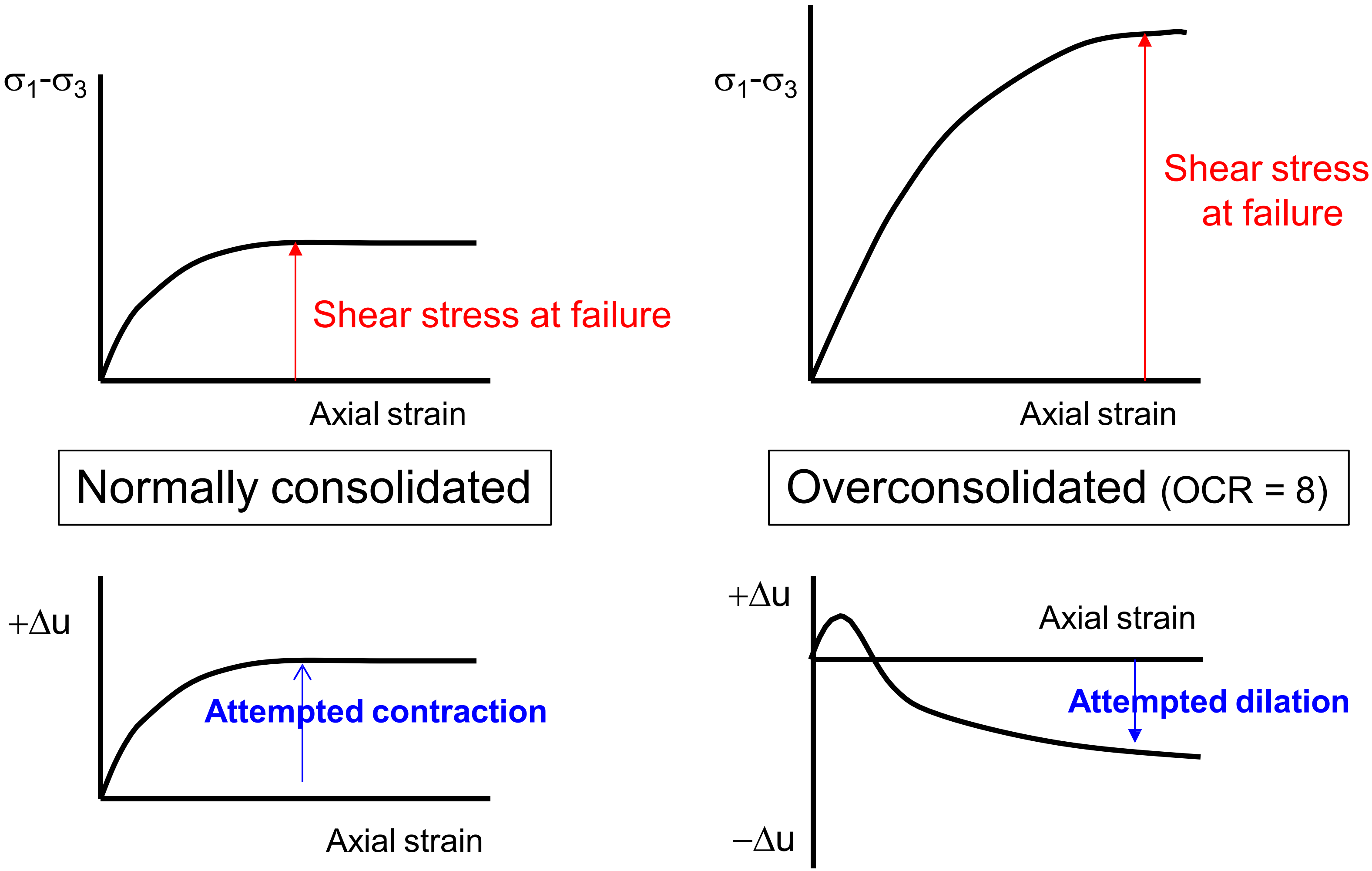
Drainage open

Drainage closed/ pwp measured

Consolidated undrained triaxial test

i.e. no volumetric strain during shearing

Typical shearing results – with echoes of shear box:

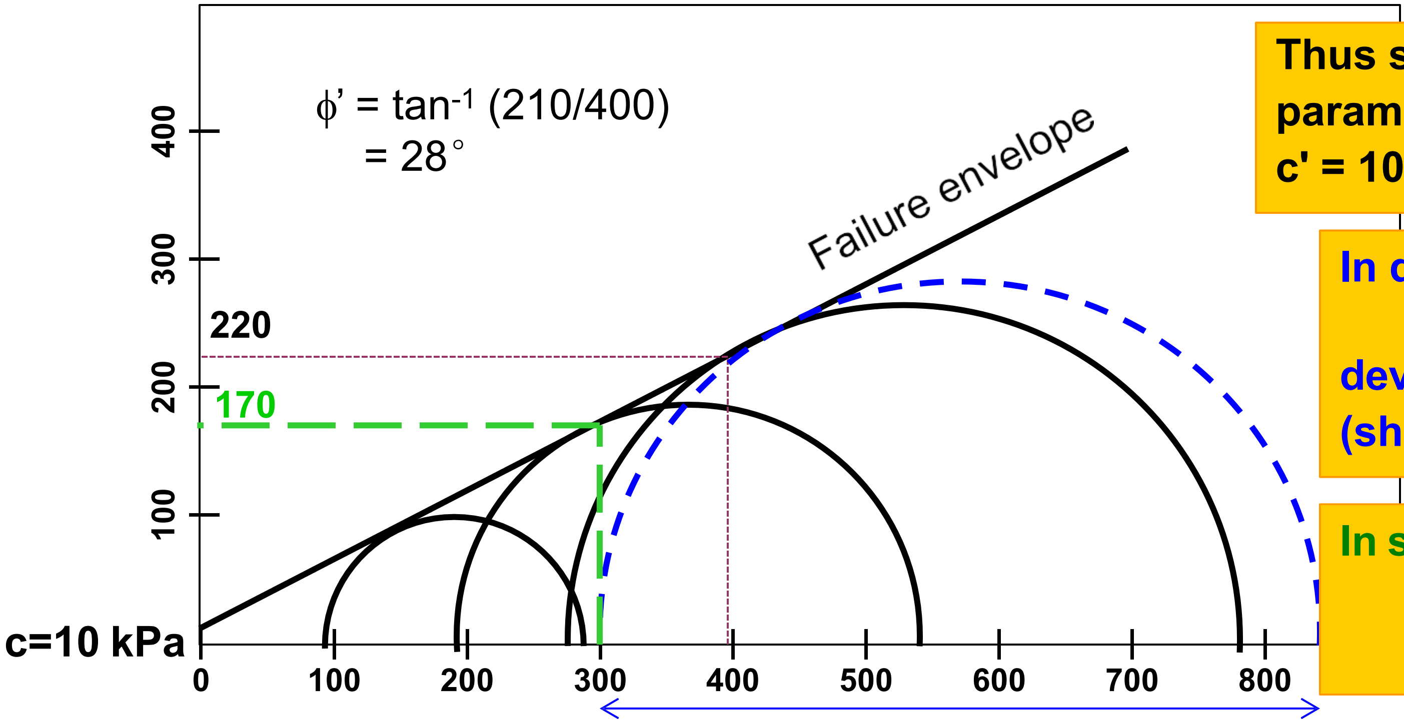


Consolidated undrained traixial - CU

	Test 1	Test 2	Test 3
Cell pressure (kPa)	100	200	300
Deviator stress (kPa)	196	356	516
Pore water pressure (kPa)	8	18	24
σ'_3	92	182	276
σ'_1	288	538	792

$\sigma'_3 = \sigma_3 - u$

$\sigma'_1 = \sigma'_3 + D$

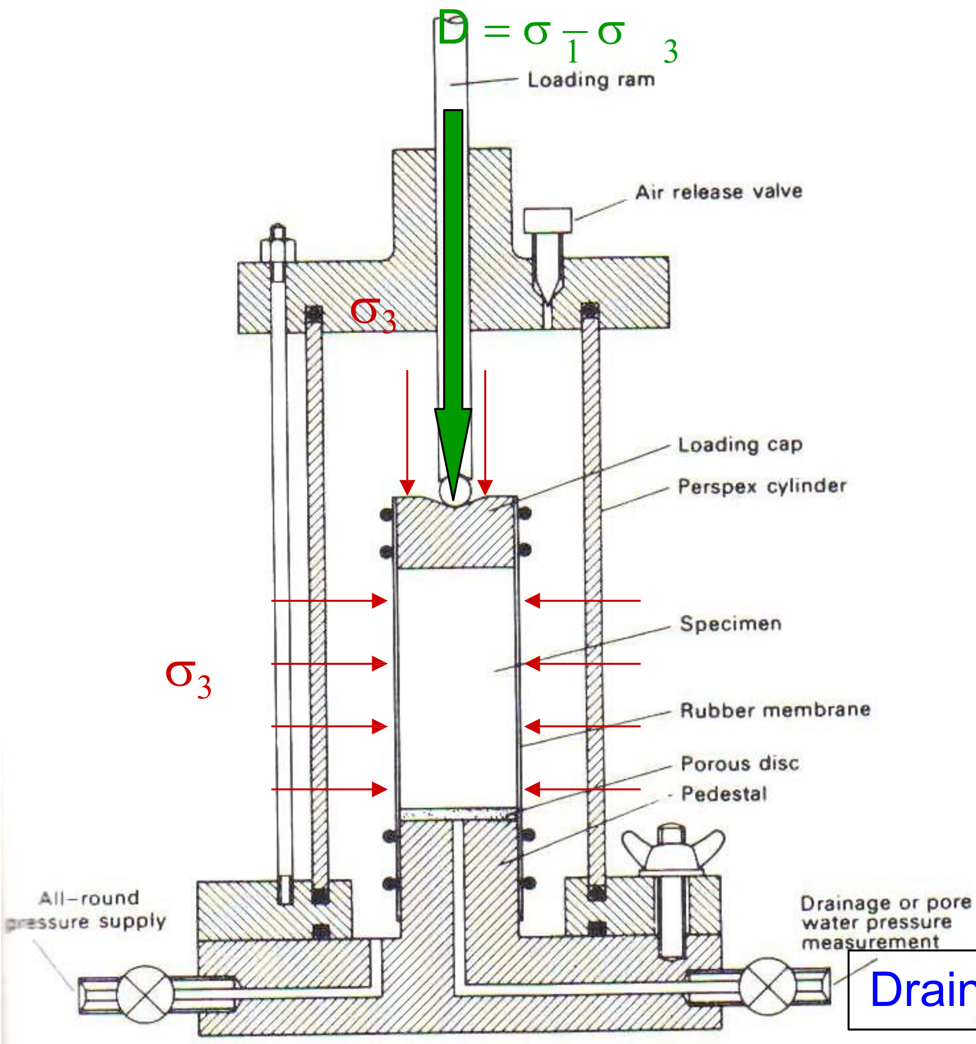


Thus shear strength parameters for this soil are: c' = 10 kPa & ϕ' = 28°

In drained test with σ_3 = 300 kPa, deviator stress = 540 kPa (shear stress = 270 kPa)

In shear box with σ_n = 300 kPa, τ = 170 kPa

CU Triaxial analysis – Example 2



Cell pressures are applied but because the drainage valve is closed, the sample is unable to consolidate ***(UNDRAINED)***.

Shearing takes place UNDRAINED, i.e. drainage line is closed but is NOT used to measure porewater pressure.

The entire test can be performed very quickly – minutes.

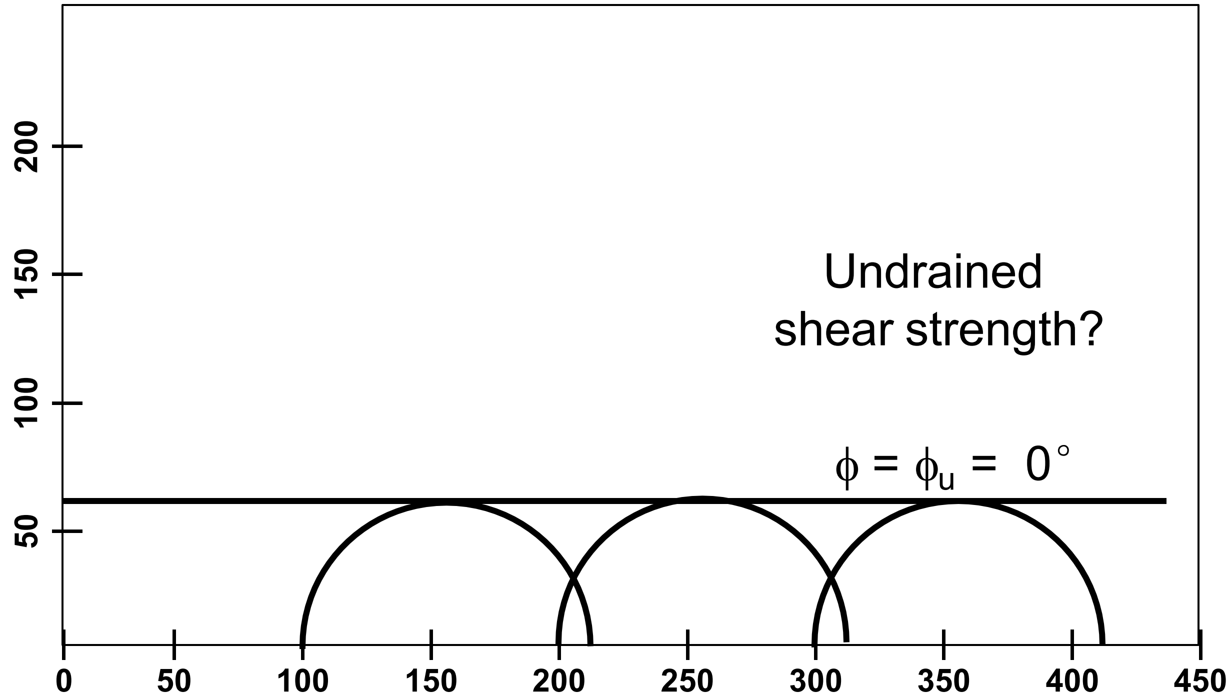
From each test we note the load at failure … interestingly … ?

Drainage permanently closed

Unconsolidated undrained triaxial test

i.e. no volumetric strain AT ALL

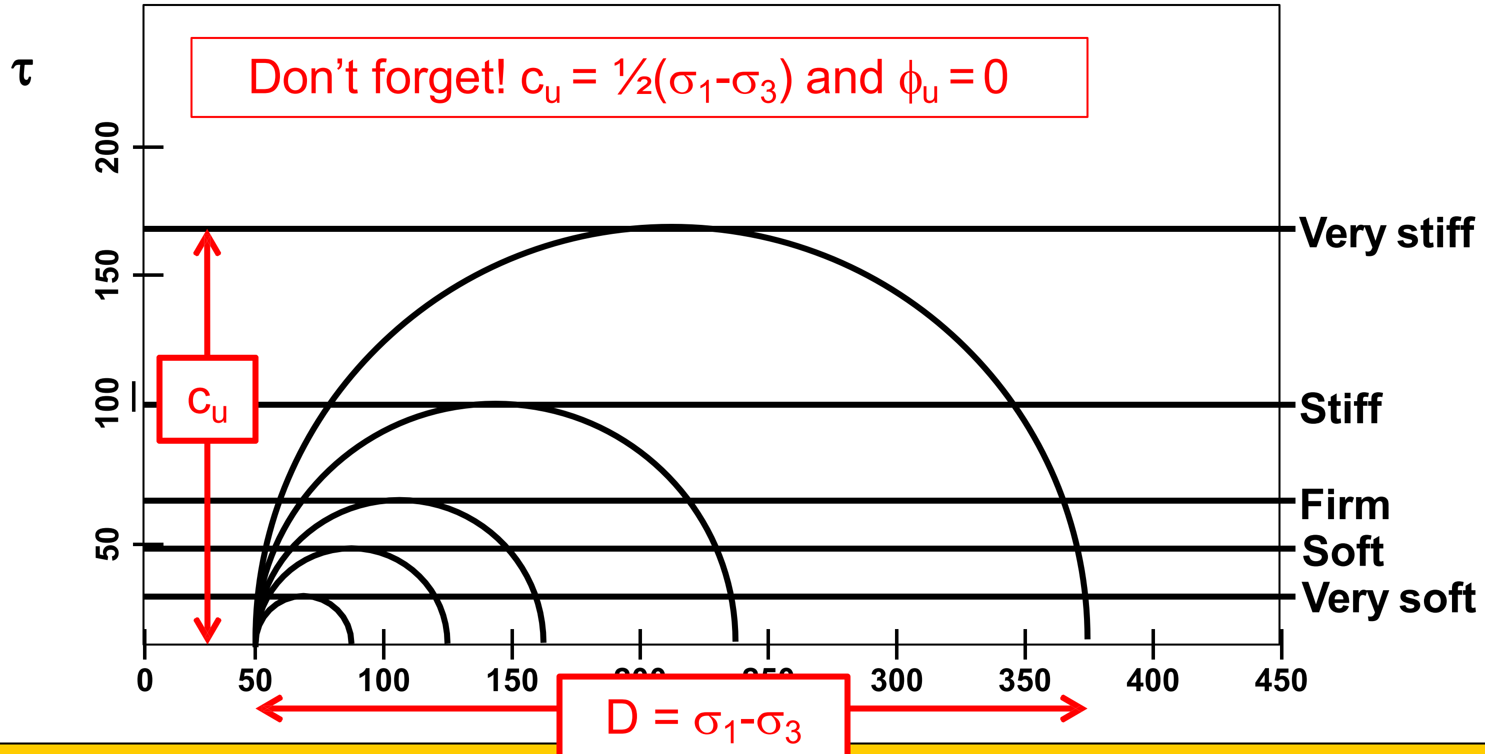
	Test 1	Test 2	Test 3
Cell pressure (kPa)	100	200	300
Deviator stress (kPa)	120	122	118



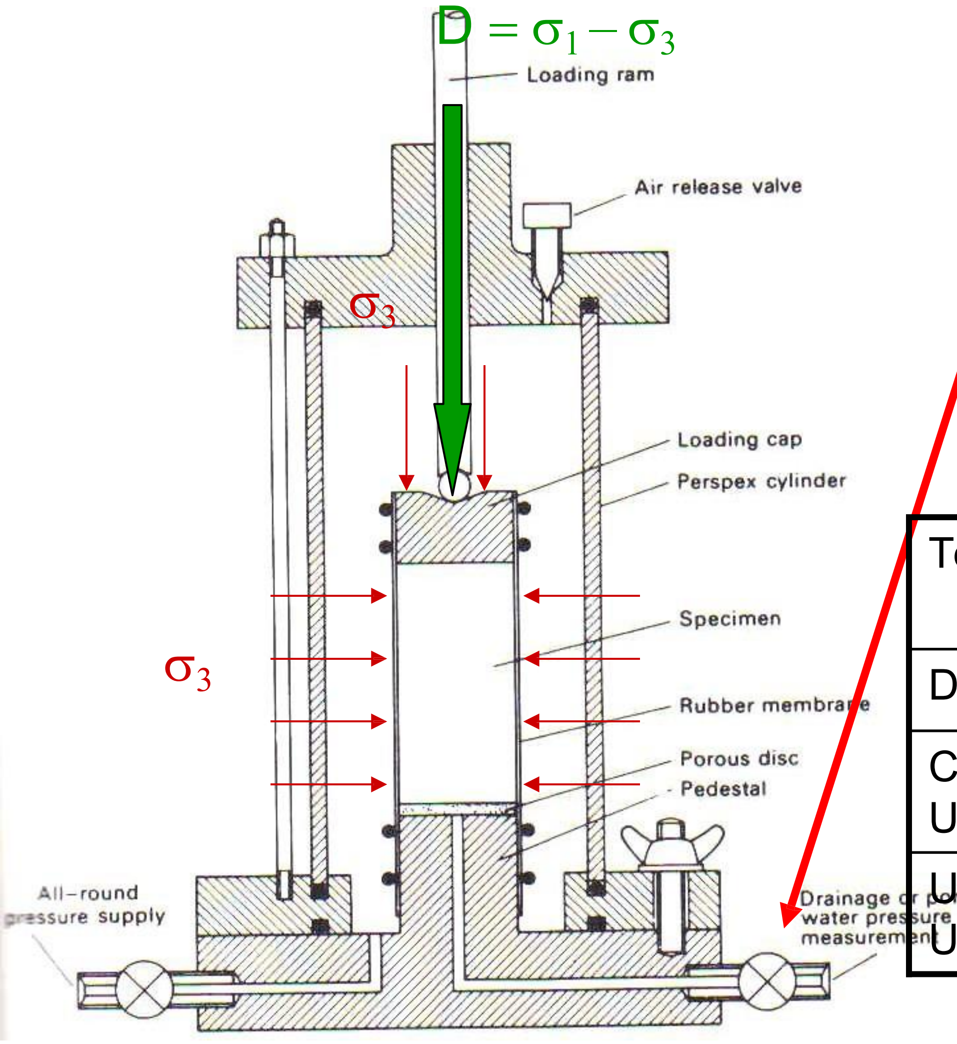
c_u= 60 kPa

Unconsolidated undrained triaxial test results -

Description (BS 5930)	Texture	τ (= c_u) (kPa)
Very soft clay	squeezeable	<20
Soft clay	thumb	20 – 40
Firm clay		40 – 75
Stiff clay	Finger nail	75 – 150
Very stiff	knife	>150



Unconsolidated undrained – typical values

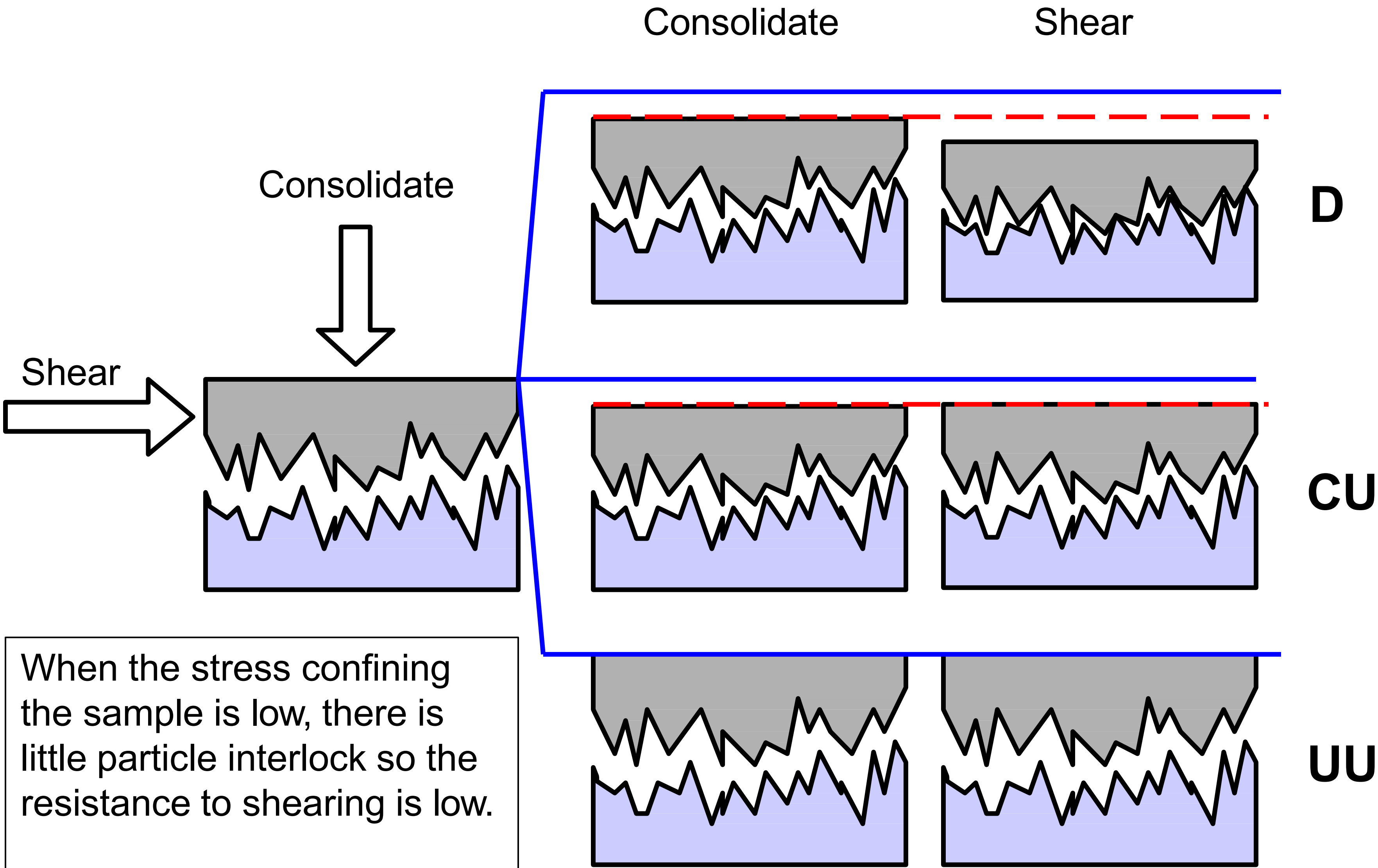


During each stage of triaxial test procedure, the sample drain port/valve is either:

open = drained condition
closed = undrained condition

Test Type	Consolidation Stage	Shear Stage
Drained (D) c' ϕ'	Open	Open
Consolidated Undrained (CU) c' ϕ'	Open	Closed
Unconsolidated Undrained (UU) c_u ϕ_u	Closed	Closed

Summary of drainage conditions in triaxial tests



When the stress confining the sample is low, there is little particle interlock so the resistance to shearing is low.

Consolidation, shearing and drainage

<u>Soil strength is a function of the effective stress state</u>

Shear box, D & CU triaxial - effective stress parameters.

UU triaxial - undrained or total stress parameter c_u.

<u>Why use total stress?</u>

- If the most critical condition occurs immediately after loading, i.e. before any drainage has occurred, and porewater pressure dissipation leads to an increase in soil strength, total stress analysis is appropriate.

- It is quicker to perform than effective stress tests and avoids difficulty of defining the induced change in porewater pressures.

- We also call this a short-term analysis.

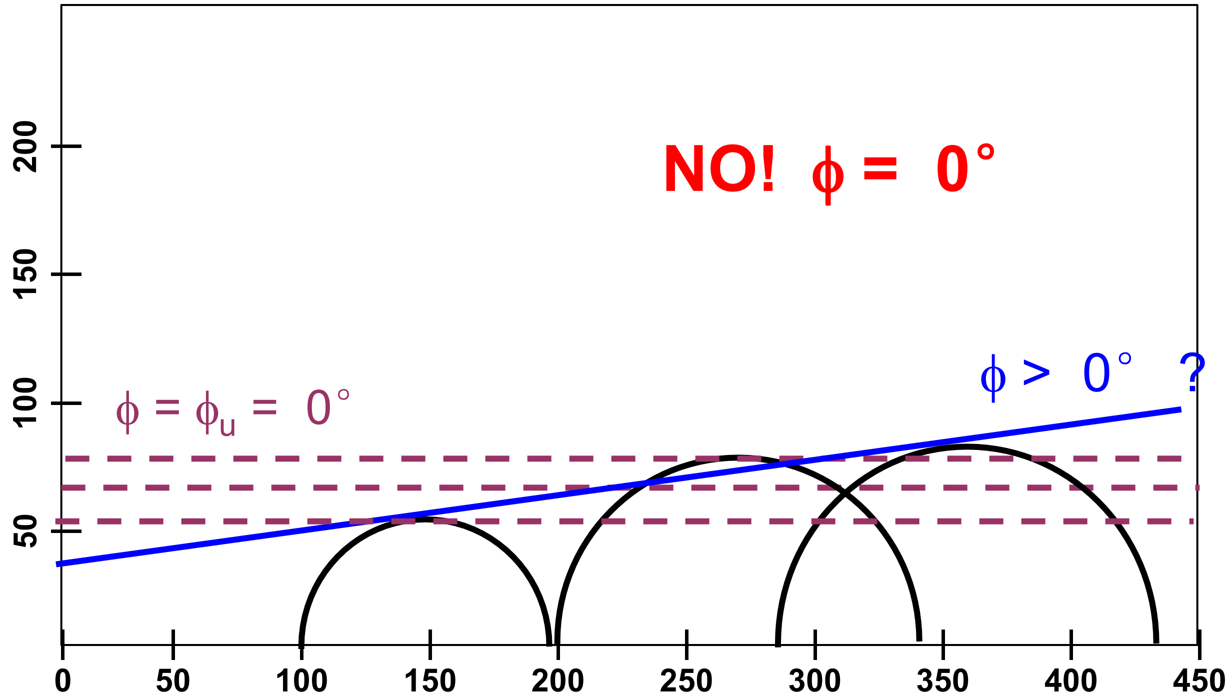
Which strength test?

An unconsolidated undrained triaxial test?

Cu = max?

Cu = average?

Cu = min (fractured clay)

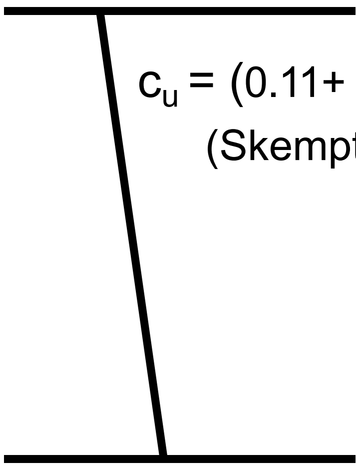


More tests ? What of depth ?

Less accommodating test data? – a reminder

Undrained strength in clays

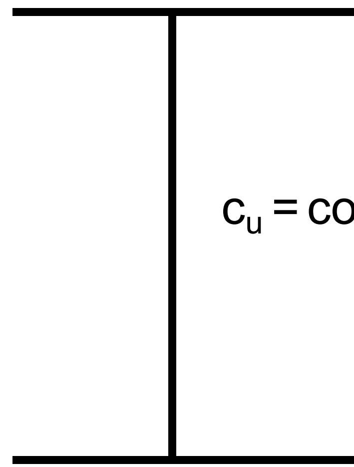
Undrained strength of a normally consolidated clay increases linearly with depth (or overburden), and is related to soil mineralogy …



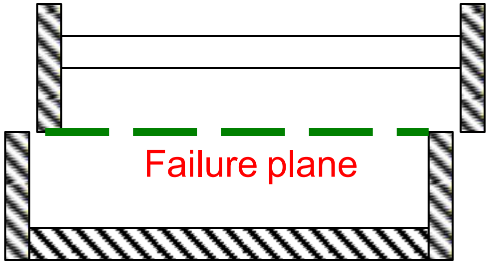
$c_u = (0.11 + 0.0037\, I_p)\, \sigma'_v$

(Skempton, 1957)

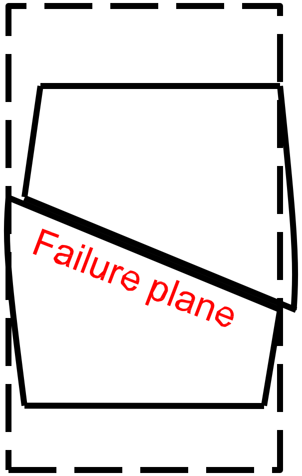
… whereas in an overconsolidated clay, undrained strength is usually constant with depth



$c_u = \text{constant}$



Direct shear



Triaxial

✓	Simple to perform	✗
✓	Large displacements possible	✗
1m	Large specimens possible	250mm
✓	Can be done in-situ	✗
✗	Control of drainage	✓
✗	‘Uniform’(ish) stress distribution	✓
✗	Unconstrained failure surface	✓

Triaxial vs. shear box: final comment

17. Craig's Example 4.3

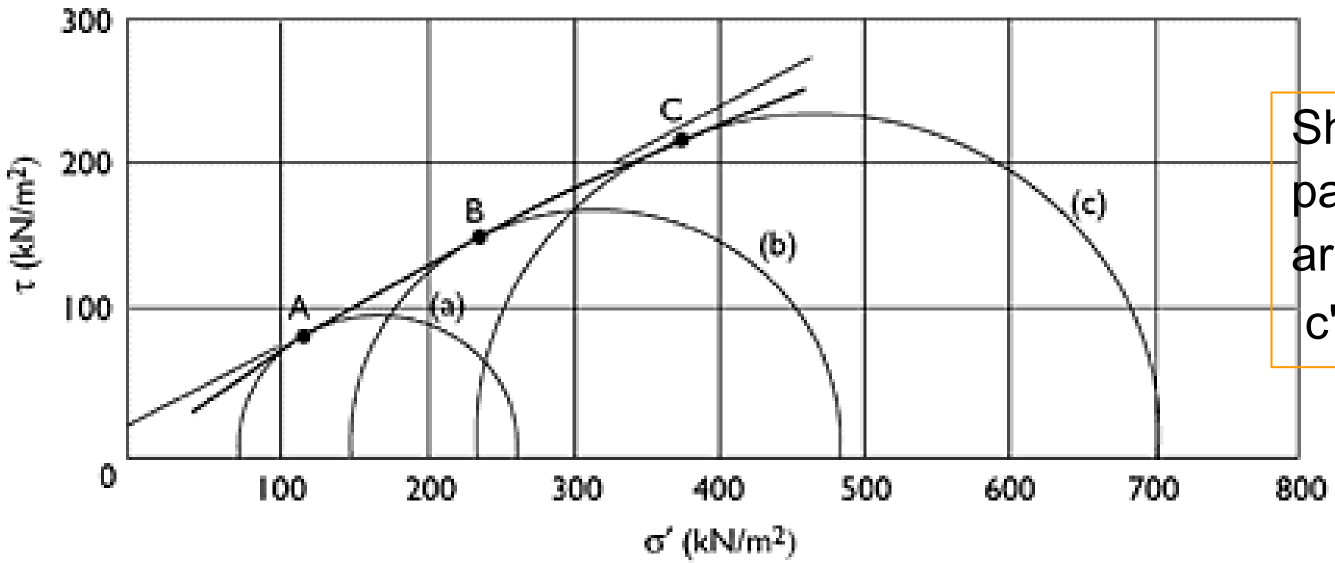
The results shown in Table 4.4 were obtained for peak failure in a series of consolidated–undrained triaxial tests, with pore water pressure measurement, on specimens of a saturated clay. Determine the values of the effective stress parameters.

All-round pressure (kN/m^2)	Principal stress difference (kN/m^2)	Pore water pressure (kN/m^2)
150	192	80
300	341	154
450	504	222

Effective cell pressure (kN/m^2)	Effective principal major stress(kN/m^2)
70	262
146	487
228	732

Worked example

σ_3	σ_1	σ'_3	σ'_1
150	342	70	262
300	641	146	487
450	954	228	732



Shear strength parameters for this soil are:
$c' = 30$ kPa & $\phi' = 28^\circ$

Worked example

After these lectures you should:

- Appreciate the difficulties of measuring strength in soils
- Know the main testing techniques – direct shear and triaxial machines
- Process data obtained from both tests
- Interpret data and recognise the different forms of shear stress and shear strain behaviour caused by differences in initial sample density
- Appreciate the existence of ultimate or critical states.

Shear strength: summary

SELF-CHECK QUESTIONS – SHEAR STRENGTH 3

1. What two terms are used to describe stress σ_1?

2. What two terms are used to describe stress σ_3?

3. What two triaxial stress components fix the position and size of a Mohr's circle?

4. Define the centre and the radius of a Mohr's circle in terms of the principal effective stresses.

5. The following table contains consolidated undrained triaxial test data obtained from a firm silty clay. Calculate the effective stress values at failure.

	A	B	C
Total cell pressure, σ_3	500	600	800
Deviator stress, $\sigma_1 - \sigma_3$	109	223	424
Pore water pressure at failure, u_f	439	482	559
σ'_3			
σ'_1			

6. Using graph paper and pair of compasses, determine the shear strength parameters for the firm silty clay.

7. The deviator stress at failure in an undrained unconsolidated triaxial test is 140 kPa. What is the shear strength?

8. Use Skempton's (1957) method for estimating the undrained shear strength of a normally consolidated clay (plastic limit = 27%, liquid limit = 67%) at depth of 5m (σ'_v = 48 kPa) below ground surface