

1. A sample of wet soil in a metallic tin has a mass of 450g. After drying in an oven overnight, the sample and tin have a mass of 368g. The mass of the dish alone is 24g. Calculate the moisture content ( $w$ ) of the sample.

$$w = \frac{M_w}{M_s} \cdot 100$$

$$M_w = (\text{mass of wet soil} + \text{tin}) - (\text{mass of dry soil} + \text{tin})$$

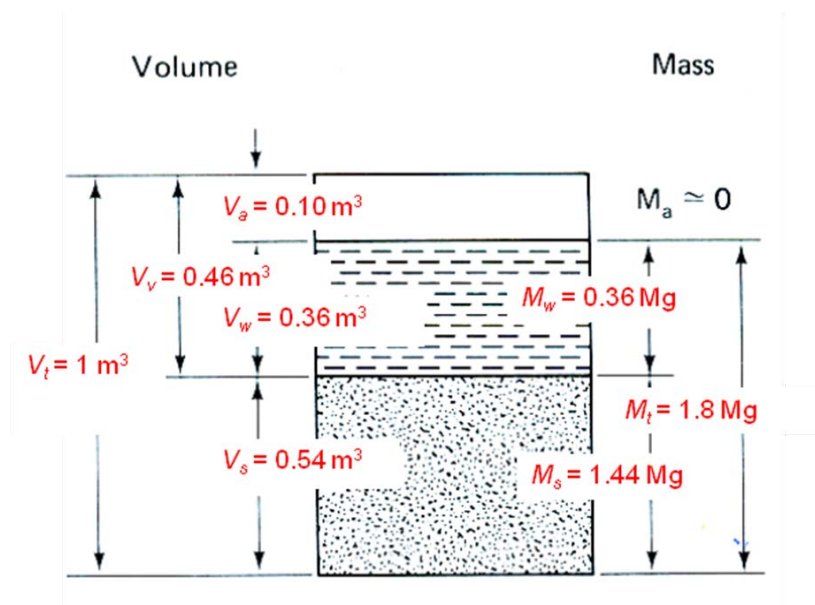
$$M_w = 450g - 368g = 82g$$

$$M_s = (\text{mass of dry soil} + \text{tin}) - (\text{mass of tin})$$

$$M_s = 368g - 24g = 344g$$

$$w = \frac{M_w}{M_s} \cdot 100 = \frac{82}{344} \cdot 100 = 23.8\%$$

2. A sample of sand has a bulk density ( $\rho_t$ ) of 1.8 Mg/m<sup>3</sup> and a moisture content ( $w$ ) of 25%. Calculate  $e$ ,  $n$ ,  $S_r$ ,  $\rho_d$ ,  $\rho_{sat}$  and  $\rho'$ . Assume that  $V_s = 1.0 \text{ m}^3$



Assume  $V_t = 1.0 \text{ m}^3$

$$\rho_t = 1.8 \text{ Mg/m}^3$$

$$\frac{M_t}{V_t} = \frac{M_w + M_s}{V_t}$$

$$M_w = 0.25M_s$$

$$\rho_t = \frac{0.25M_s + M_s}{V_t}$$

$$1.8 \text{ Mg/m}^3 = \frac{0.25M_s + M_s}{1.0 \text{ m}^3}$$

$$1.8 \text{ Mg} = 1.25M_s$$

$$M_s = 1.44 \text{ Mg}$$

$$M_w = 0.25M_s$$

$$M_w = 0.25(1.44 \text{ Mg})$$

$$M_w = 0.36 \text{ Mg}$$

$$\rho_w = \frac{M_w}{V_w} = 1.0 \text{ Mg/m}^3$$

$$V_w = \frac{M_w}{\rho_w} = \frac{0.36 \text{ Mg}}{1.0 \text{ Mg/m}^3}$$

$$V_w = 0.36 \text{ m}^3$$

$$\text{Assume } G_s = 2.65$$

$$G_s = \frac{M_s}{V_s \rho_w} = \frac{\rho_s}{\rho_w} = 2.65$$

$$2.65 = \frac{\rho_s}{1.0 \text{ Mg/m}^3}$$

$$\rho_s = 2.65 \text{ Mg/m}^3$$

$$\rho_s = \frac{M_s}{V_s} = 2.65 \text{ Mg/m}^3$$

$$V_s = \frac{M_s}{\rho_s} = \frac{1.44 \text{ Mg}}{2.65 \text{ Mg/m}^3}$$

$$V_s = 0.54 \text{ m}^3$$

$$V_t = V_a + V_w + V_s$$

$$1.0 \text{ m}^3 = V_a + 0.36 \text{ m}^3 + 0.54 \text{ m}^3$$

$$V_a = 0.10 \text{ m}^3$$

$$e = \frac{V_v}{V_s} = \frac{0.46 \text{ m}^3}{0.54 \text{ m}^3} = 0.85$$

$$n = \frac{V_v}{V_t} = \frac{0.46 \text{ m}^3}{1.00 \text{ m}^3} \cdot 100 = 46\%$$

$$S_r = \frac{V_w}{V_v} \cdot 100$$

$$S_r = \frac{V_w}{V_a + V_w} \cdot 100$$

$$S_r = \frac{0.36 \text{ m}^3}{0.10 \text{ m}^3 + 0.36 \text{ m}^3} \cdot 100$$

$$S_r = 78 \%$$

$$\rho_d = \frac{M_s}{V_t} = \frac{1.44 \text{ Mg}}{1.00 \text{ m}^3}$$

$$\rho_d = 1.44 \frac{\text{Mg}}{\text{m}^3}$$

$$\rho_{sat} = \frac{M_s + M_w}{V_t}$$

$$\rho_{sat} = \frac{1.44 \text{ Mg} + (0.36 \text{ Mg} + 0.10 \text{ Mg})}{1.00 \text{ m}^3}$$

$$\rho_{sat} = 1.90 \frac{\text{Mg}}{\text{m}^3}$$

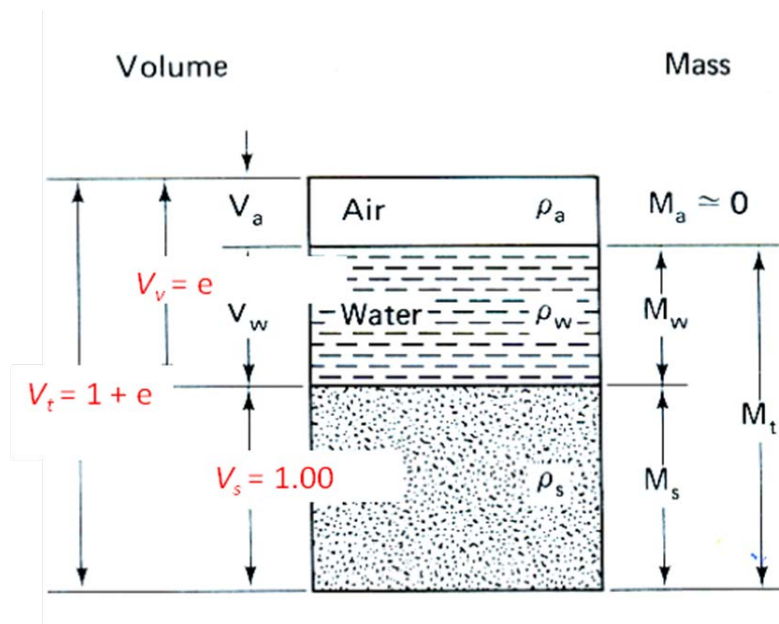
$$\rho'_{sat} = \rho_{sat} - \rho_w$$

$$\rho'_{sat} = 0.90 \frac{\text{Mg}}{\text{m}^3}$$

3. A sample of sand has a bulk density ( $\rho_t$ ) of  $1.8 \text{ Mg/m}^3$  and a moisture content ( $w$ ) of 25%. Calculate  $e$ ,  $n$ ,  $S_r$ ,  $\rho_d$ ,  $\rho_{sat}$  and  $\rho'$ . Assume that  $V_t = 1.0 \text{ m}^3$

Answers as above. Just different procedure.

4. Derive an expression for the porosity ( $n$ ) in terms of the void ratio ( $e$ )



Assume  $V_s = 1.0$

$$e = \frac{V_v}{V_s} = \frac{V_v}{1.0}$$

$$e = V_v$$

$$n = \frac{V_v}{V_t}$$

$$n = \frac{e}{1 + e}$$

5. A soil has a unit weight ( $\gamma_s$ ) of 26.5 kN/m<sup>3</sup> and a bulk unit weight ( $\gamma_t$ ) of 18.0 kN/m<sup>3</sup>. The void ratio ( $e$ ) was found to be 1.06 and the moisture content ( $w$ ) was calculated as 40%. Is the soil saturated? **NOTE:** Use  $g = 10 \text{ m/s}^2$  in your calculations.

$$\gamma_s = 26.5 \frac{\text{kN}}{\text{m}^3} = 26500 \frac{\text{N}}{\text{m}^3} = 26500 \frac{\text{kg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^3}$$

$$\gamma_s = \rho_s g$$

$$\rho_s = \frac{26.5 \frac{\text{Mg} \cdot \text{m}}{\text{s}^2 \cdot \text{m}^3}}{10 \frac{\text{m}}{\text{s}^2}} = 2.65 \frac{\text{Mg}}{\text{m}^3}$$

$$S_r e = w G_s$$

$$S_r = \frac{w G_s}{e} = \frac{0.40(2.65)}{1.06} = 1 \quad \Rightarrow \quad S_r = 100 \%$$

**The soil is saturated**

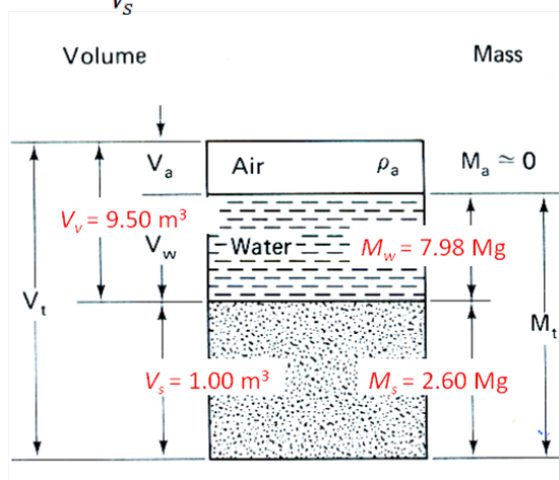
6. A sensitive volcanic clay soil tested in the laboratory was found to have the following properties:

- a)  $\rho_s = 2.60 \text{ Mg/m}^3$
- b)  $\rho_t = 1.35 \text{ Mg/m}^3$
- c)  $w = 307 \%$
- d)  $e = 9.5$
- e)  $S_r = 84 \%$

After reviewing the values one was found to be inconsistent. Which one?

Assume  $V_s = 1.0$

$$\rho_s = \frac{M_s}{V_s} \quad \Rightarrow \quad M_s = \rho_s V_s = 2.60 \text{ Mg/m}^3 (1.0 \text{ m}^3) = 2.60 \text{ Mg}$$



$$w = \frac{M_w}{M_s}$$

$$M_w = w M_s = 3.07 (2.60 \text{ Mg})$$

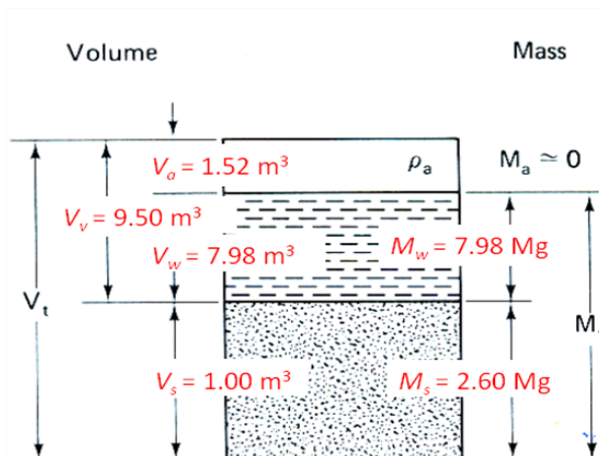
$$M_w = 7.98 \text{ Mg}$$

$$e = \frac{V_v}{V_s}$$

$$V_v = e V_s = 9.5 (1.0 \text{ m}^3)$$

$$V_v = 9.5 \text{ m}^3$$

$$\rho_w = \frac{M_w}{V_w} \quad \Rightarrow \quad V_w = \frac{M_w}{\rho_w} = \frac{7.98 \text{ Mg}}{1.0 \text{ Mg/m}^3} = 7.98 \text{ m}^3$$



$$V_v = V_w + V_a$$

$$9.50 \text{ m}^3 = 7.98 \text{ m}^3 + V_a$$

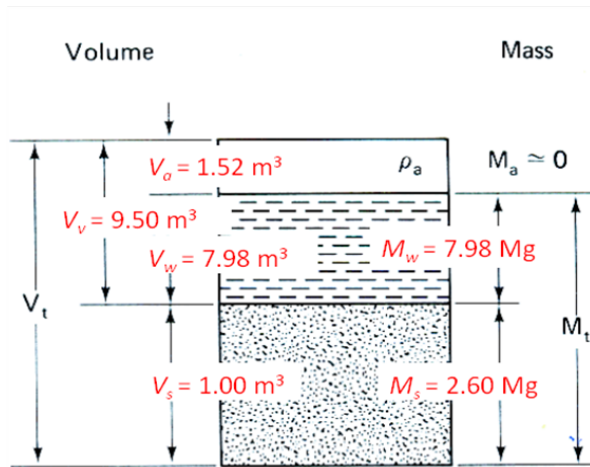
$$V_a = 1.52 \text{ m}^3$$

$$S_r = \frac{V_w}{V_v}$$

$$S_r = \frac{7.98 \text{ m}^3}{9.50 \text{ m}^3} = 0.84$$

$$S_r = 84 \%$$

$$\rho_t = \frac{M_t}{V_t} \Rightarrow \rho_t = \frac{M_w + M_s}{V_v + V_s} \Rightarrow \rho_t = \frac{7.98 \text{ Mg} + 2.60 \text{ Mg}}{9.50 \text{ m}^3 + 1.00 \text{ m}^3}$$



$$\rho_t = 1.01 \frac{\text{Mg}}{\text{m}^3}$$

$$1.35 \frac{\text{Mg}}{\text{m}^3} \neq 1.01 \frac{\text{Mg}}{\text{m}^3}$$

7. The *in-situ* dry density of a sand is  $1.75 \text{ Mg/m}^3$ . The maximum and minimum dry densities, determined by standard laboratory tests, are  $1.85$  and  $1.45 \text{ Mg/m}^3$ . Determine the relative density of the sand.

$$\rho_d = \frac{\rho_w G_s}{1 + e} \Rightarrow \rho_d + \rho_d e = \rho_w G_s \Rightarrow \rho_d e = \rho_w G_s - \rho_d$$

$$e = \frac{\rho_w G_s - \rho_d}{\rho_d} \Rightarrow e = \frac{\rho_w G_s}{\rho_d} - 1$$

$$e = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.75 \text{ Mg/m}^3} - 1 = 0.514$$

$$e_{min} = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.85 \text{ Mg/m}^3} - 1 = 0.432$$

$$e_{max} = \frac{1.00 \text{ Mg/m}^3 (2.65)}{1.45 \text{ Mg/m}^3} - 1 = 0.827$$

$$D_r = \frac{e_{max} - e}{e_{max} - e_{min}} = \frac{0.827 - 0.514}{0.827 - 0.432} = 0.79 \Rightarrow D_r = 79 \%$$

8. A soil of total volume  $200 \text{ ml}$  contains  $25 \text{ ml}$  air and  $30 \text{ ml}$  water. Calculate the void ratio and the degree of saturation.

$$V = 200 \text{ ml}, V_a = 25 \text{ ml}, V_w = 30 \text{ ml}$$

$$e = \frac{V_v}{V_s} = \frac{25 + 30}{200 - 25 - 30} = \underline{\underline{0.38}}$$

$$S_r = \frac{V_w}{V_v} = \frac{30}{55} = 0.55 = \underline{\underline{55\%}}$$

9. A soil has a porosity of 0.45. What is the void ratio?

$$e = \frac{n}{1-n}$$

$$e = 0.45/0.55$$

$$\Rightarrow \underline{e = 0.818}$$

10. A soil had a wet mass of 2.180 kg and occupied a volume of 1.2 litres. After oven drying the mass reduced to 1.890 kg. Calculate bulk density, moisture content and dry density.

$$\text{Bulk density} = 2.180/1.2 \times 10^{-3} \text{ m}^3 = 1816 \text{ kg/m}^3$$

$$\text{Moisture content} = (2.180-1.890)/1.890 = 15.3\%$$

$$\text{Dry density} = 1.890/1.2 \times 10^{-3} \text{ m}^3 = 1575 \text{ kg/m}^3$$

11. A sample of saturated clay has a volume of 245ml and after oven drying has a mass of 453g. If  $G_s = 2.75$ , determine the **dry** and **saturated unit weights** of the soil in its natural state.

$$\text{Dry unit weight} = 0.453/0.245 \times 9.81 = 18.13 \text{ kN/m}^3$$

$$\text{Dry volume} = 453[\text{g}]/2.75[\text{g/ml}] = 165 \text{ ml}$$

$$\text{Void volume} = 245 - 165 = 80 \text{ ml}$$

$$\text{Saturated moisture mass} = 80 \text{ g}$$

$$\text{Saturated unit weight} = (453 + 80)/245 \times 9.81 = 21.34 \text{ kN/m}^3$$

$$\text{Or} \quad \left(1 - \frac{1}{G_s}\right) \gamma_d + \gamma_w$$

12. A soil has a bulk density of 1.91 Mg/m<sup>3</sup> and a moisture content of 9.5%. The value of  $G_s$  is 2.70. Calculate the void ratio and degrees of saturation of the soil. What would be the values of density and moisture content if the soil were fully saturated at the same void ratio?

$$\rho_d = \frac{\rho_b}{1+w} = 1.91/1.095 = 1.744 \text{ Mg/m}^3$$

$$V_s = 1.744 [\text{Mg/m}^3] / 2.70 [\text{# or Mg/m}^3] = 0.646 \text{ m}^3$$

$$V_v = 1 - 0.646 = 0.354 \text{ m}^3$$

$$e = 0.354/0.646 = 0.548$$

$$M_w = 1.744 \times 0.095 = 0.166 \text{ Mg}$$

$$V_w = 0.166 / 1.0 = 0.166 \text{ m}^3$$

$$S_r = 0.166/0.354 = 0.47$$

$$\text{Saturated density} = (1.744 + 0.354)/1 = 2.098 \text{ Mg/m}^3$$

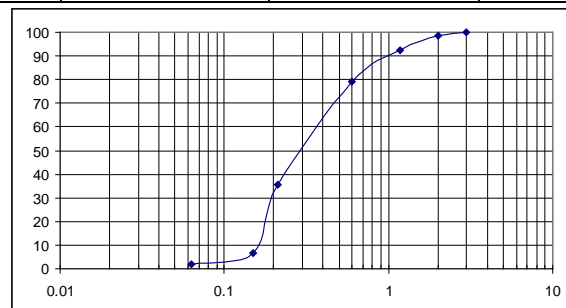
$$\text{Moisture content} = 0.354/1.744 = 20.3\%$$

13. A particle size distribution analysis on a 241 g sample of soil returned the following results.

sieve size ( $\mu\text{m}$ )	mass retained (g)
3350	0
2000	3
1180	15
600	32
212	105
150	70
63	11
<63	5

Plot the particle size distribution, calculate the coefficient of uniformity and describe the grading of the soil.

sieve size (mm)	mass retained (g)	%age retained	%age passing
3	0	0	100.0
2	3	1.2	98.8
1.18	15	6.2	92.6
0.6	32	13.3	79.3
.212	105	43.6	35.7
.150	70	29.0	6.7
.063	11	4.6	2.1
<63	5	2.1	0.0



Gravel = 1.2%, Sand (0.063 mm – 2 mm) = 96.7%, Fines = 2.1%

Slightly gravelly SAND,

$$\text{Uniformity coefficient, } C_u = \frac{d_{60}}{d_{10}} = \frac{0.36}{0.18} = 2.0 \quad (\text{soil is uniformly graded}).$$

Uniformly graded slightly gravelly SAND

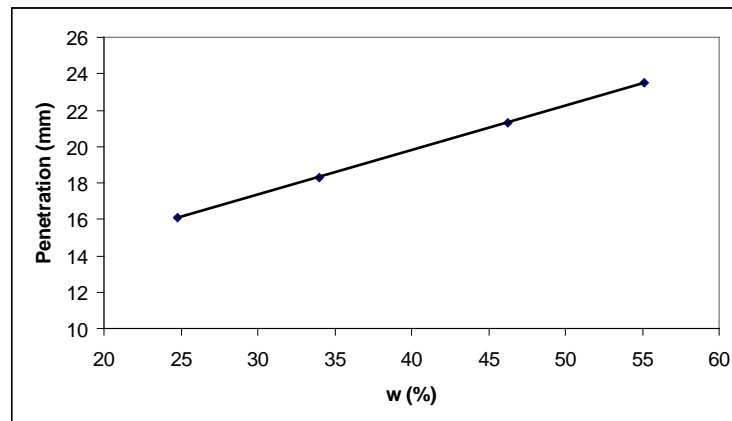


14. The following results were obtained during a liquid limit test on a soil. Determine the liquid limit.

Mass of wet soil (g)	Mass of dry soil (g)	Cone penetration (mm)
31.2	25.0	16.1
37.8	28.2	18.3
36.0	24.6	21.3
40.8	26.3	23.5

**Solution:**

Cone penetration(mm)	Water content(%)
16.1	24.8
18.3	34.0
21.3	46.3
23.5	55.1



Liquid limit is 41%

15. From the following falling cone test results:

Mass of tin [g]	18.2	19.1	17.7	18.6
Mass of tin + wet sample [g]	51.5	45.5	50.7	43.4
Mass of tin + dry sample [g]	37.8	35.6	39.7	36.3
Cone penetration [mm]	25.0	14.2	8.5	5.1

Determine moisture content of each sample. Plot graph of w against penetration and estimate liquid limit. If soil has a plastic limit of 22%, calculate plasticity index and classify the soil using the A-line chart.

**Solution**

At 20 mm penetration, moisture content is 63%; PI = 41%, hence from A-line, h=CH (high plasticity clay)

16. The following results were obtained for a fine-grained soil:

$$\begin{aligned} \text{LL} &= 48\%, & \text{PL} &= 26\%, & \text{natural moisture content} &= 29\% \\ \text{Clay} &= 25\%, & \text{Silt} &= 36\%, & \text{Sand} &= 39\% \end{aligned}$$

Classify the soil.

The soil is predominantly fine-grained, with 61% passing the 63 micron sieve.

With  $\text{LL} = 48\%$  and  $\text{PI} = 48 - 26 = 22\%$ , it is CI, i.e. sandy CLAY of intermediate plasticity.

The clay has an activity  $= 22/25 = 0.88$ , i.e. predominantly normal activity (illitic).  
NMC is close to PL therefore clay is stiff.

17. The Atterberg limits of a soil are  $\text{LL} = 70\%$  and  $\text{PL} = 35\%$  and it contains 80% by weight of clay. The water content of the sample is 45%. Classify the soil.

$$\text{PI} = 70 - 35 = 35$$

$$\text{LI} = (45 - 35)/35 = 0.29$$

$$A = \text{PI} / \% \text{clay} = 35/80 = 0.44$$

From the Atterberg classification chart we see that this clay is a high to very high plasticity silt/clay. From its activity we see that it is an inactive clay, typically kaolinite.

18. Craig's Example 1.1

The results of particle size analyses of four soils A, B, C and D are shown in Table 1.4.  
The results of limit tests on soil D are:

Liquid limit:

Cone penetration (mm)	15.5	18.0	19.4	22.2	24.9
Water content (%)	39.3	40.8	42.1	44.6	45.6

Plastic limit:

Water content (%)	23.9	24.3
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The fine fraction of soil C has a liquid limit of 26 and a plasticity index of 9. (a) Determine the coefficient of uniformity for soils A, B and C. (b) Classify with main and qualifying terms each soil.

Table 1.4

BS sieve	Particle size*	Percentage smaller			
		Soil A	Soil B	Soil C	Soil D
63mm		100		100	
20mm		64		76	
6.3mm		39	100	65	
2mm		24	98	59	
600µm		12	90	54	
212µm		5	9	47	100
63µm		0	3	34	95
	0.020mm			23	69
	0.006mm			14	46
	0.002mm			7	31

\* From sedimentation test.

Solution:

The particle size distribution curves are plotted in [Figure 1.8](#). For soils A, B and C the sizes  $D_{10}$ ,  $D_{30}$  and  $D_{60}$  are read from the curves and the values of  $C_u$  and  $C_z$  are calculated:

Soil	$D_{10}$	$D_{30}$	$D_{60}$	$C_u$	$C_z$
A	0.47	3.5	16	34	1.6
B	0.23	0.30	0.41	1.8	0.95
C	0.003	0.042	2.4	800	0.25

For soil D the liquid limit is obtained from [Figure 1.9](#), in which cone penetration is plotted against water content. The percentage water content, to the nearest integer, corresponding to a penetration of 20mm is the liquid limit and is 42. The plastic limit is the average of the two percentage water contents, again to the nearest integer, i.e. 24. The plasticity index is the difference between the liquid and plastic limits, i.e. 18.

Soil A consists of 100% coarse material (76% gravel size; 24% sand size) and is classified as GW: well-graded, very sandy GRAVEL.

Soil B consists of 97% coarse material (95% sand size; 2% gravel size) and 3% fines. It is classified as SPu: uniform, slightly silty, medium SAND.

Soil C comprises 66% coarse material (41% gravel size; 25% sand size) and 34% fines ( $w_L = 26$ ,  $I_P = 9$ , plotting in the CL zone on the plasticity chart). The classification is GCL: very clayey GRAVEL (clay of low plasticity). This is a till, a glacial deposit having a large range of particle sizes.

Soil D contains 95% fine material: the liquid limit is 42 and the plasticity index is 18, plotting just above the A-line in the CI zone on the plasticity chart. The classification is thus CI: CLAY of intermediate plasticity.

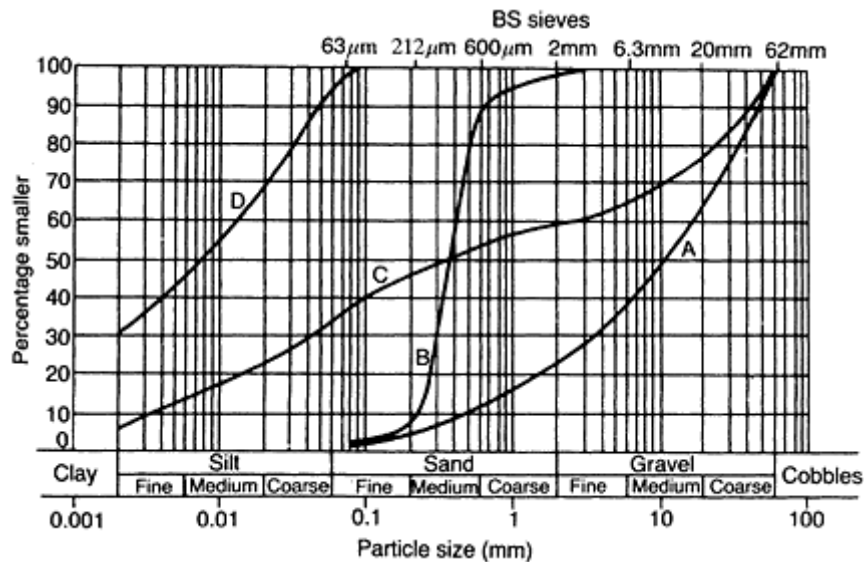


Figure 1.8 Particle size distribution curves

### 19. Craig's Example 1.2

In its natural condition a soil sample has a mass of 2290g and a volume of  $1.15 \times 10^{-3} \text{ m}^3$ . After being completely dried in an oven the mass of the sample is 2035g. The value of  $G_s$  for the soil is 2.68. Determine the bulk density, unit weight, water content, void ratio, porosity, degree of saturation and air content.

$$\text{Bulk density, } \rho = \frac{M}{V} = \frac{2.290}{1.15 \times 10^{-3}} = 1990 \text{ kg/m}^3 \quad (1.99 \text{ Mg/m}^3)$$

$$\begin{aligned} \text{Unit weight, } \gamma &= \frac{Mg}{V} = 1990 \times 9.8 = 19\,500 \text{ N/m}^3 \\ &= 19.5 \text{ kN/m}^3 \end{aligned}$$

$$\text{Water content, } w = \frac{M_w}{M_s} = \frac{2290 - 2035}{2035} = 0.125 \text{ or } 12.5\%$$

From Equation 1.17

$$\begin{aligned}\text{Void ratio, } e &= G_s(1 + w) \frac{\rho_w}{\rho} - 1 \\ &= \left( 2.68 \times 1.125 \times \frac{1000}{1990} \right) - 1 \\ &= 1.52 - 1 \\ &= 0.52\end{aligned}$$

$$\text{Porosity, } n = \frac{e}{1 + e} = \frac{0.52}{1.52} = 0.34 \text{ or } 34\%$$

$$\text{Degree of saturation, } S_r = \frac{wG_s}{e} = \frac{0.125 \times 2.68}{0.52} = 0.645 \text{ or } 64.5\%$$

$$\begin{aligned}\text{Air content, } A &= n(1 - S_r) = 0.34 \times 0.355 \\ &= 0.121 \text{ or } 12.1\%\end{aligned}$$

20. Craig's 1.1 The results of particle size analyses and, where appropriate, limit tests on samples of four soils are given in Table 1.5. Classify and give main and qualifying terms appropriate for each soil.

Table 1.5

BS sieve	Particle size	Percentage smaller			
		Soil E	Soil F	Soil G	Soil H
63mm					
20mm		100			
6.3mm		94	100		
2mm		69	98		
600µm		32	88	100	
212µm		13	67	95	100
63µm		2	37	73	99
	0.020mm		22	46	88
	0.006mm		11	25	71
	0.002mm		4	13	58
Liquid limit		Non-plastic		32	78
Plastic limit				24	31

**Solution:**

1.1 SW, MS, ML, CV