SOIL MECHANICS

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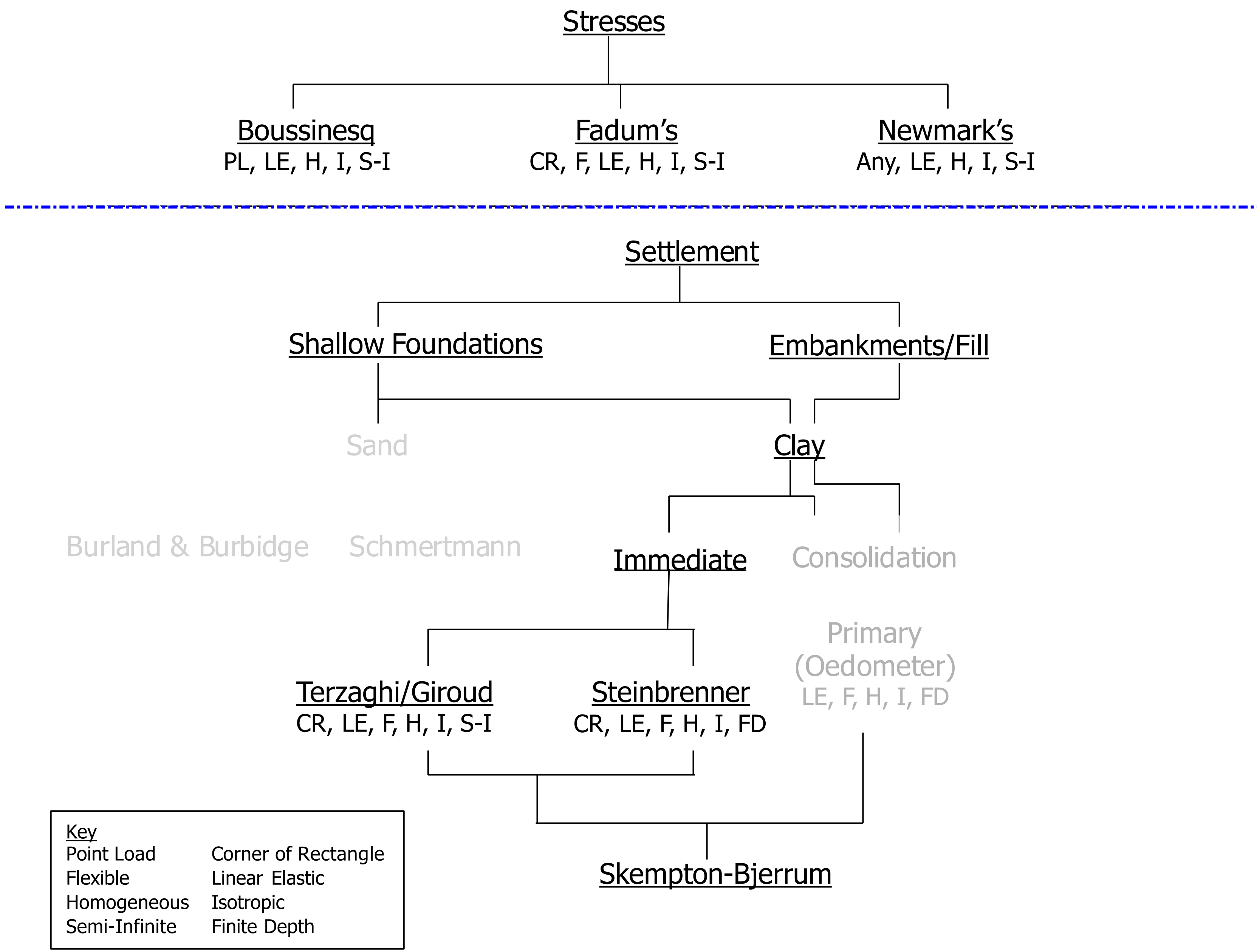
Unit 8 - Stress Increments using Elasticity Theory

May 26th, 2025

Day	08:00-09:30	09:45-11:15	13:00-14:30	14:45-16:15
19/05/25	Introduction	Programming	Phase Rel.	Tutorial
20/05/25	Classification	Tutorial	LAB	LAB
21/05/25	Compaction	Tutorial		
22/05/25	Groundwater	Tutorial	LAB	LAB
23/05/25	Groundwater	Tutorial		
26/05/25	Effective Str.	Tutorial	Stress Incr.	Tutorial
27/05/25	Compressib.	Tutorial	LAB	LAB
28/05/25	Consolidation	Tutorial		
29/05/25	Shear Str.	Tutorial	Shear.Str.	Tutorial
30/05/25	Shear Str.	Review		


STRESSES IN THE GROUND

Summary of Geotechnical Settlement Analysis

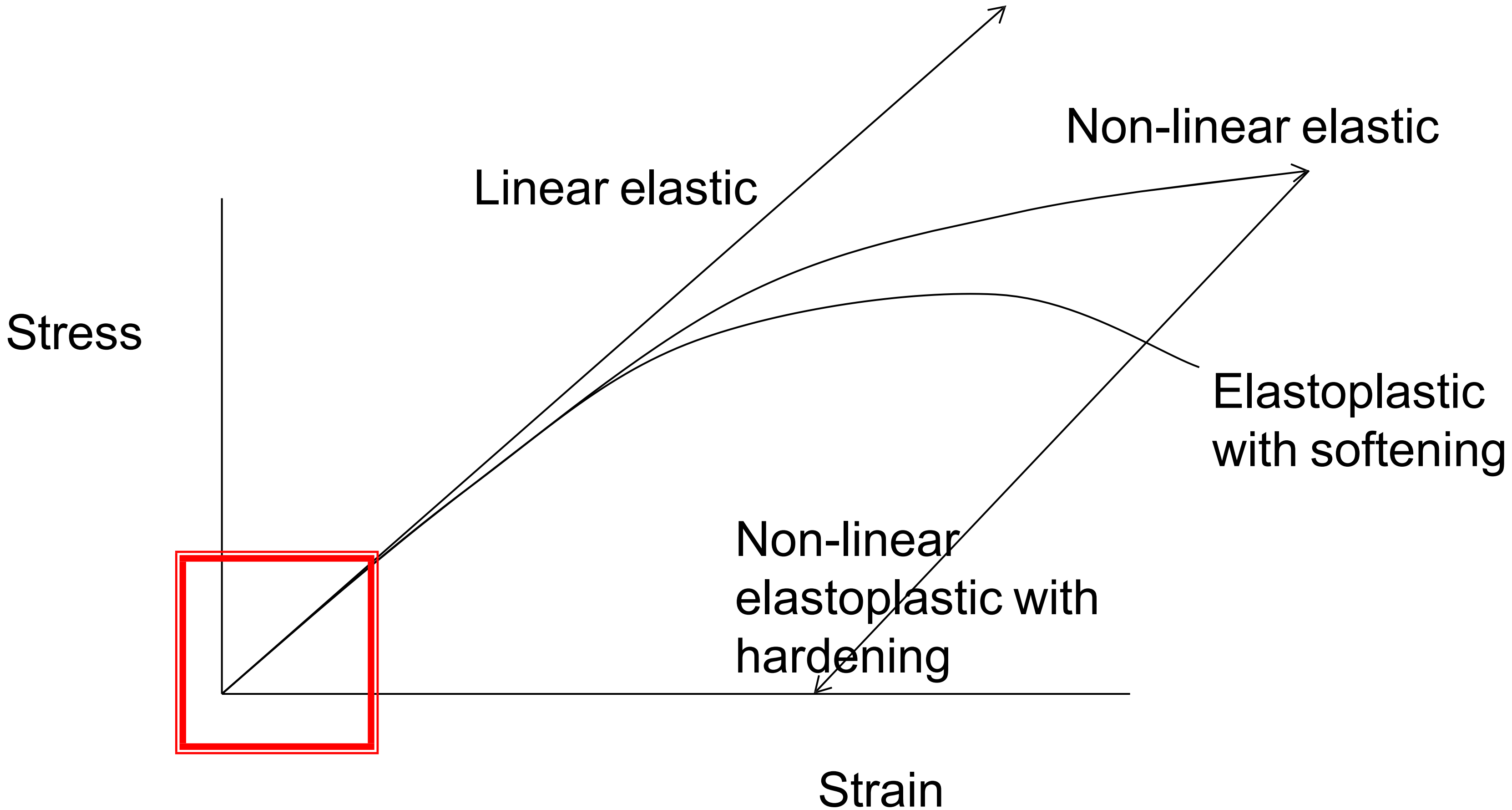


Hookes Law: (1-d) $\varepsilon = \dfrac{\Delta\sigma}{E}$

Sometimes serves well, e.g. $S = \Delta\sigma . m_v . H$

as long as we select a modulus value that is appropriate for the in situ stress state.

But we need to be clear that the soil behaviour described by elastic theory does not admit the possibility of failure. If the working stress state approaches the failure condition or envelope, then elastic methods are not likely to give reliable results, especially in the case of settlement.



Rheology of soil models

Hookes Law: $$\varepsilon = \frac{\Delta\sigma}{E}$$

Sometimes serves well, e.g. $$S = \Delta\sigma . m_v . H$$

For the two (strip) and three (pad) dimensional cases.

Consider the generalised case of Hooke's Law,

$$\varepsilon_1 = \frac{1}{E}\left(\Delta\sigma_1 - \nu.\Delta\sigma_2 - \nu.\Delta\sigma_3\right)$$

Elastic Theory

We may have confidence in our determination of σ_1 and m_v (which is related to E, actually = 1/E) but under a two- or three-dimensional stress regime, what of σ_2, σ_3, ν, and E?

Fortunately,a large amount of work (e.g. Burland, Broms & de Mello, 1977) has shown that elastic methods give a reasonable prediction of ***stresses*** in the ground.

So let’s look at a few methods for predicting stress increments.

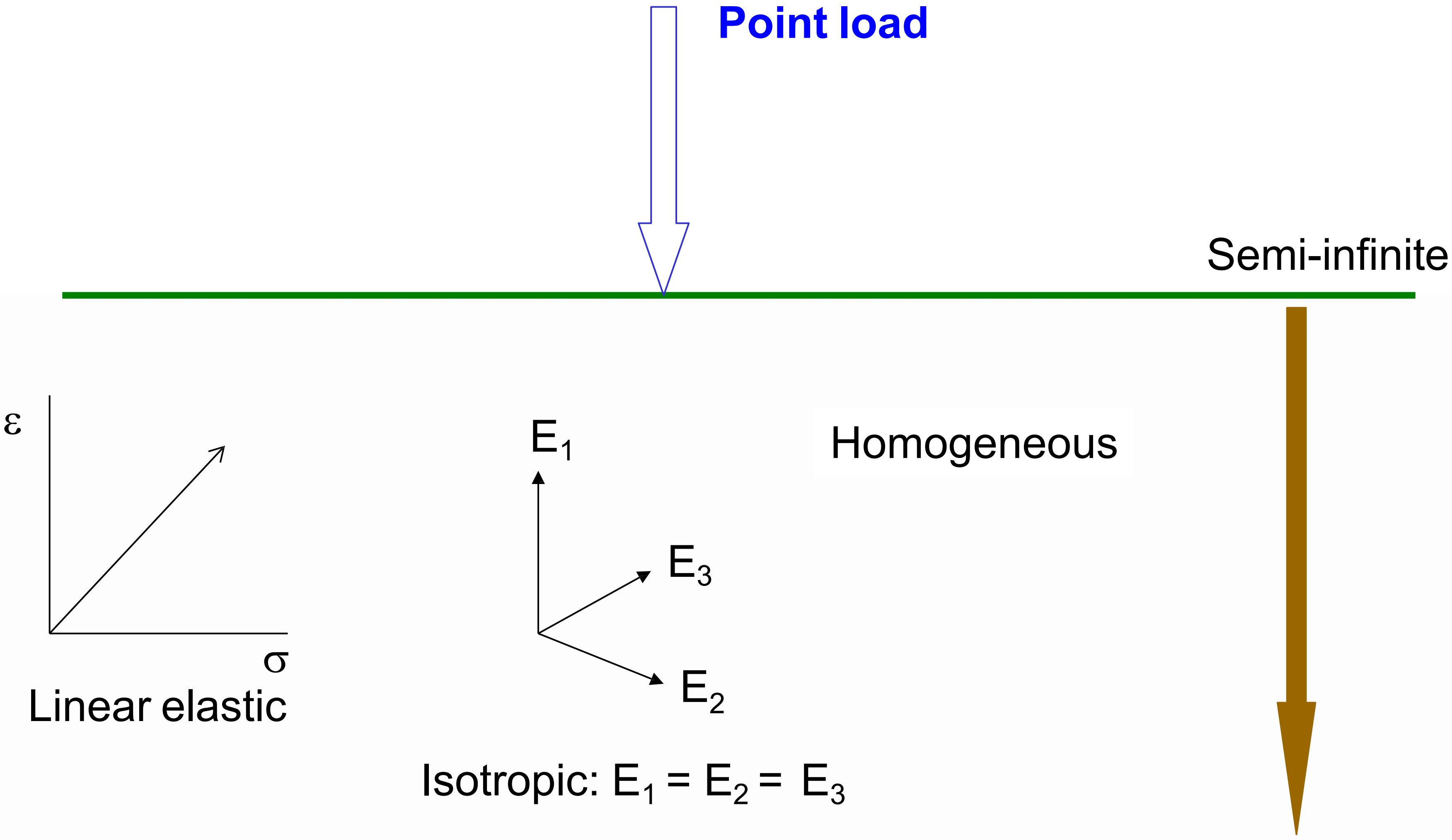
Elastic Theory

Boussinesq

[stresses – point load, linear elastic, homogeneous, isotropic, semi-infinite soil space]

Elastic theory was first used by Boussinesq in 1885 to determine stress increments in the ground under a point load.

Elastic methods usually make a number of simplifying assumptions; in this case the soil is taken to be linearly elastic, homogeneous, isotropic and occupies a semi-infinite half space.

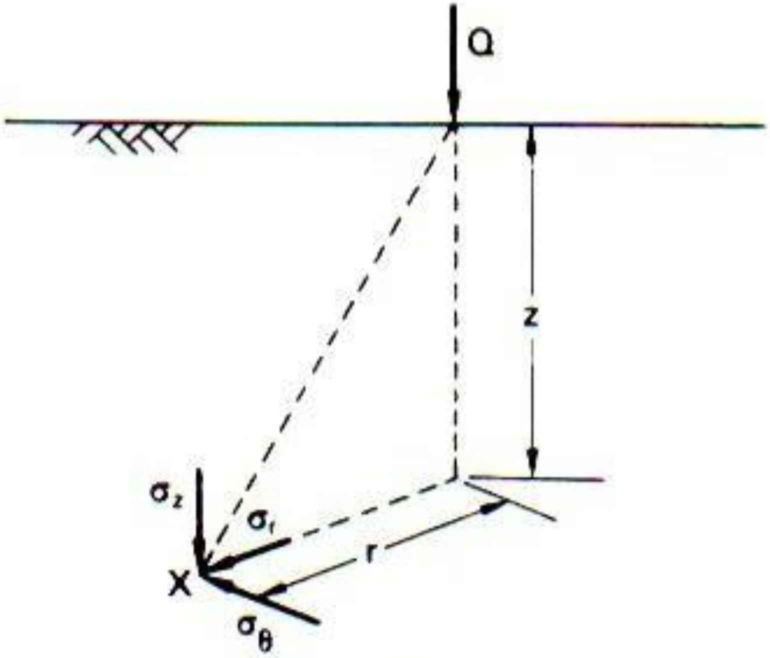


Simplifying assumptions: point load, linear elastic, homogeneous, isotropic, semi-infinite]

For vertical stress at some depth below and some horizontal distance from the point of application, denoted by z and r respectively,

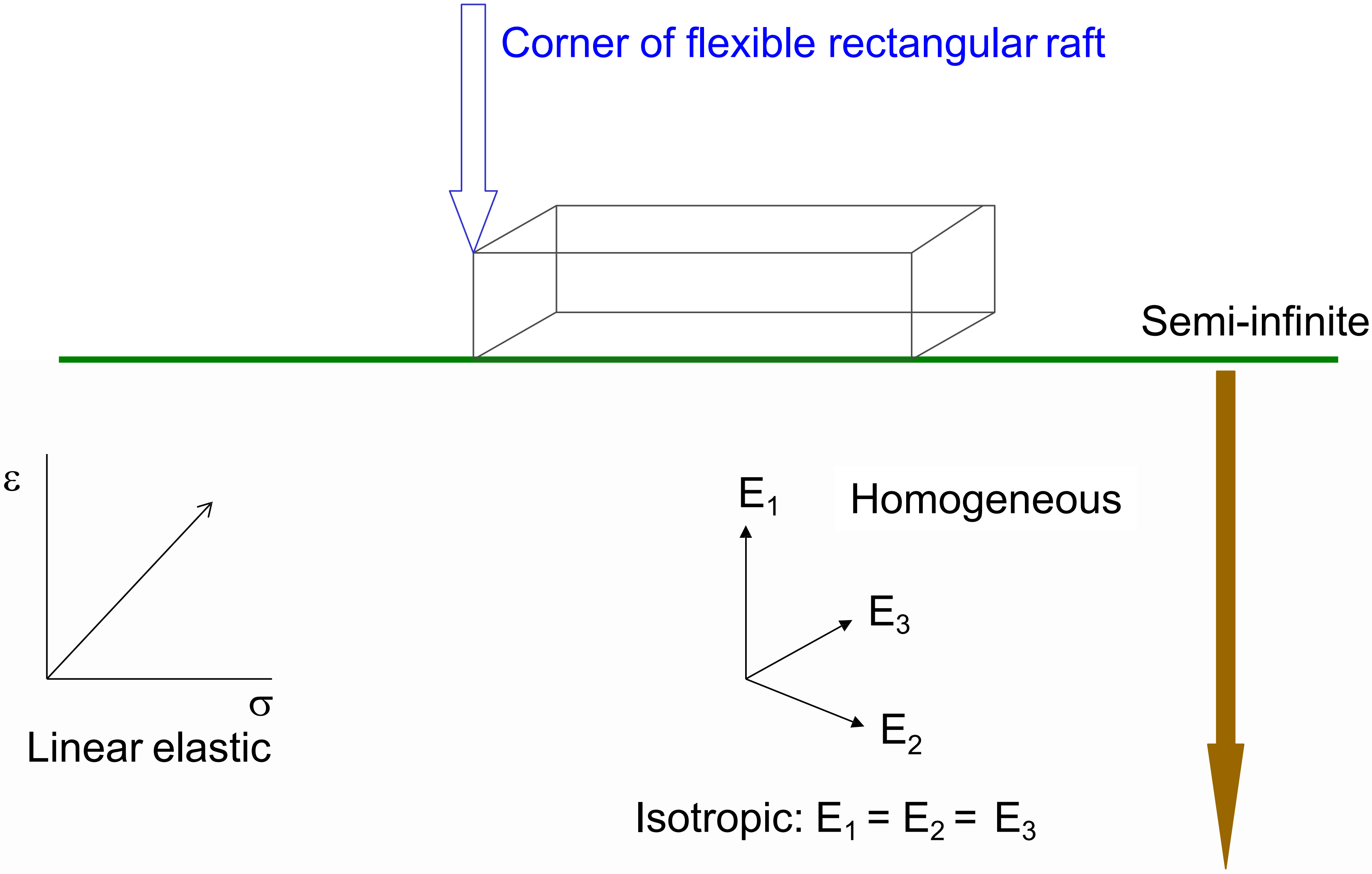
$$\Delta\sigma_z = \frac{3Q}{2\pi z^2}\left\{\frac{1}{\left[1+\left(r/z\right)^2\right]}\right\}^{5/2}$$

or $$\Delta\sigma_z = \frac{3Q}{2\pi}\frac{z^3}{\left(z^2+r^2\right)^{5/2}}$$



where Q the point load and r & z are as indicated. The latter {bracketed} part of which is often obtained from a table of influence factors, *Ip*, calculated as functions of r/z.

Boussinesq [stresses – point load, linear elastic, homogeneous, isotropic, semi-infinite]



Simplifying assumptions: corner flexible rectangular raft, linear elastic, homogeneous, isotropic, semi-infinite

Fadum's Chart

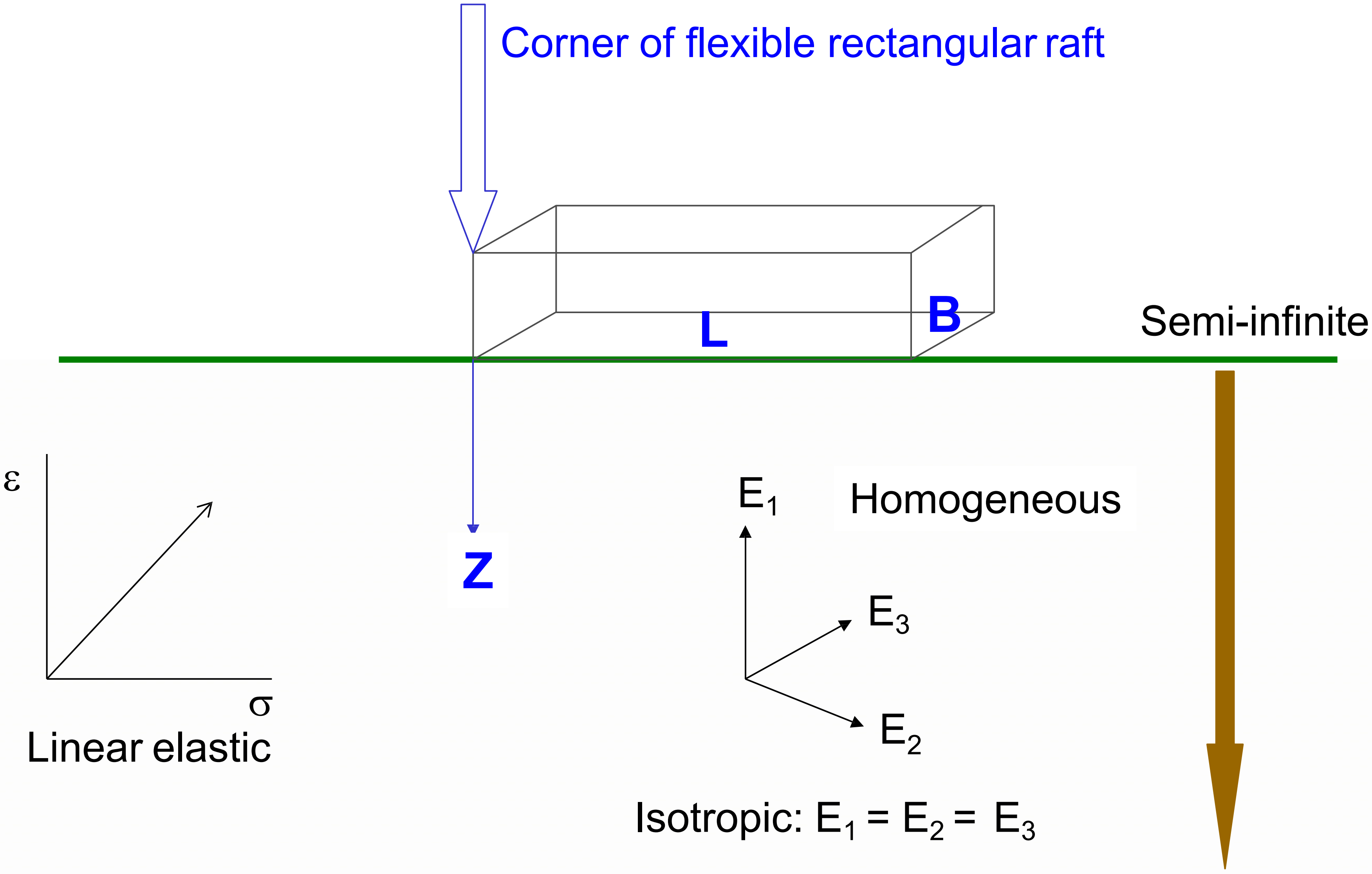
[stress – corner, rectangle, flexible, linear elastic, homogeneous, infinite]

Integration of Boussinesq's equations gives analytical expressions for stress increments at depth under loads distributed over circular or rectangular loaded areas.

For the rectangular case,

$$\Delta\sigma_z = q\, I_p$$

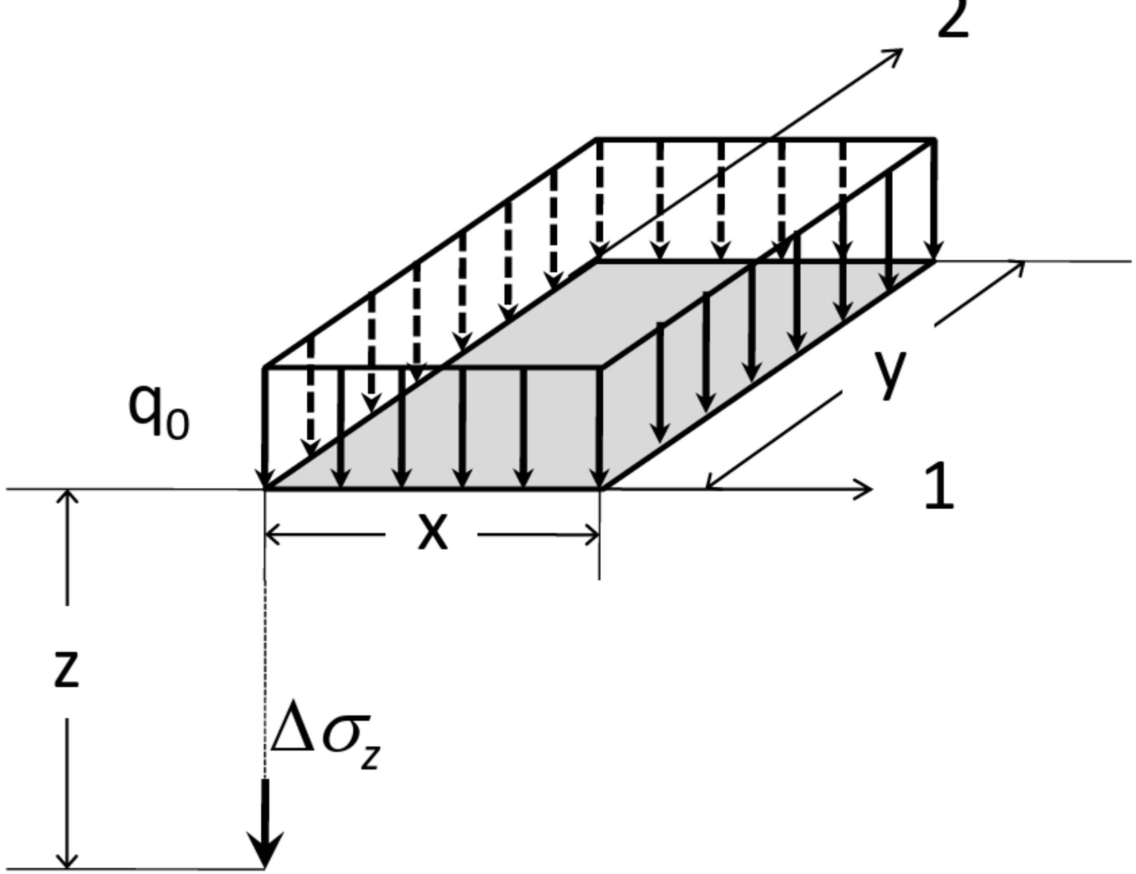
with influence factor I_p being obtained from Fadum (1948).



Simplifying assumptions: corner flexible rectangular raft, linear elastic, homogeneous, isotropic, semi-infinite

UNIFORM RECTANGULAR LOAD

$$\Delta\sigma_z = q_0 \frac{1}{4\pi}\left[\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1+m^2n^2}\cdot\frac{(m^2+n^2+2)}{(m^2+n^2+1)} + \tan^{-1}\frac{2mn(m^2+n^2+1)^{1/2}}{m^2+n^2+1-m^2n^2}\right]$$

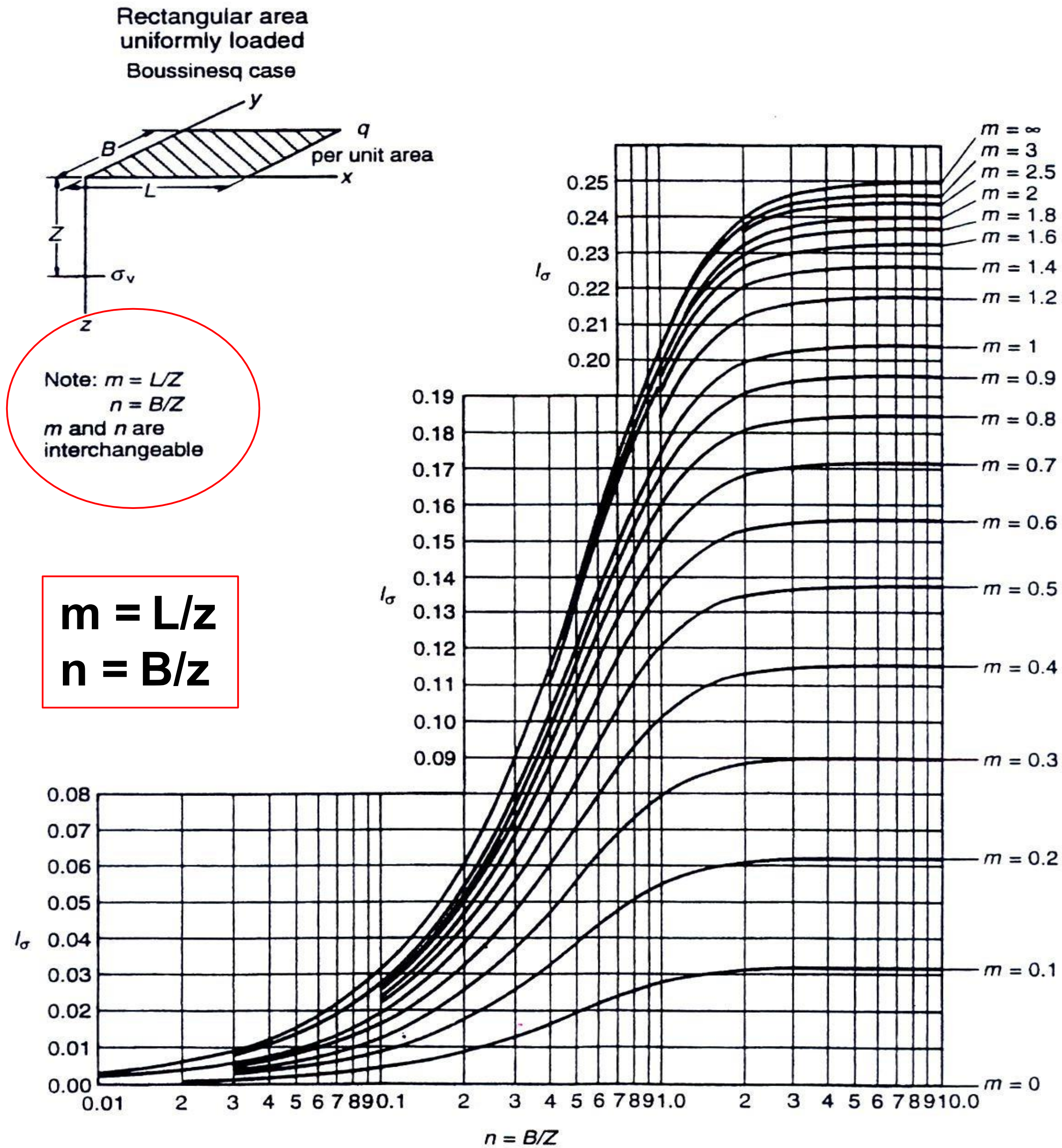


$$\boxed{\Delta\sigma_z = q_0 I}$$

where...

$m = x/z$

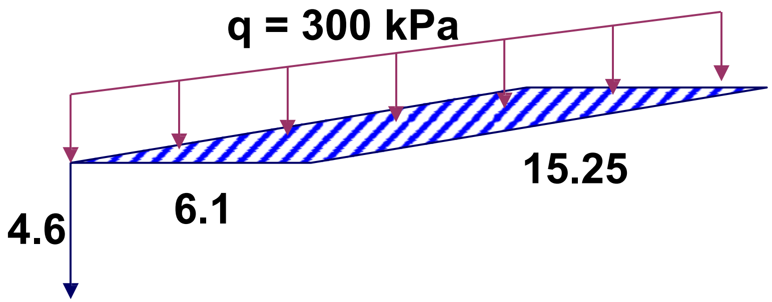
$n = y/z$

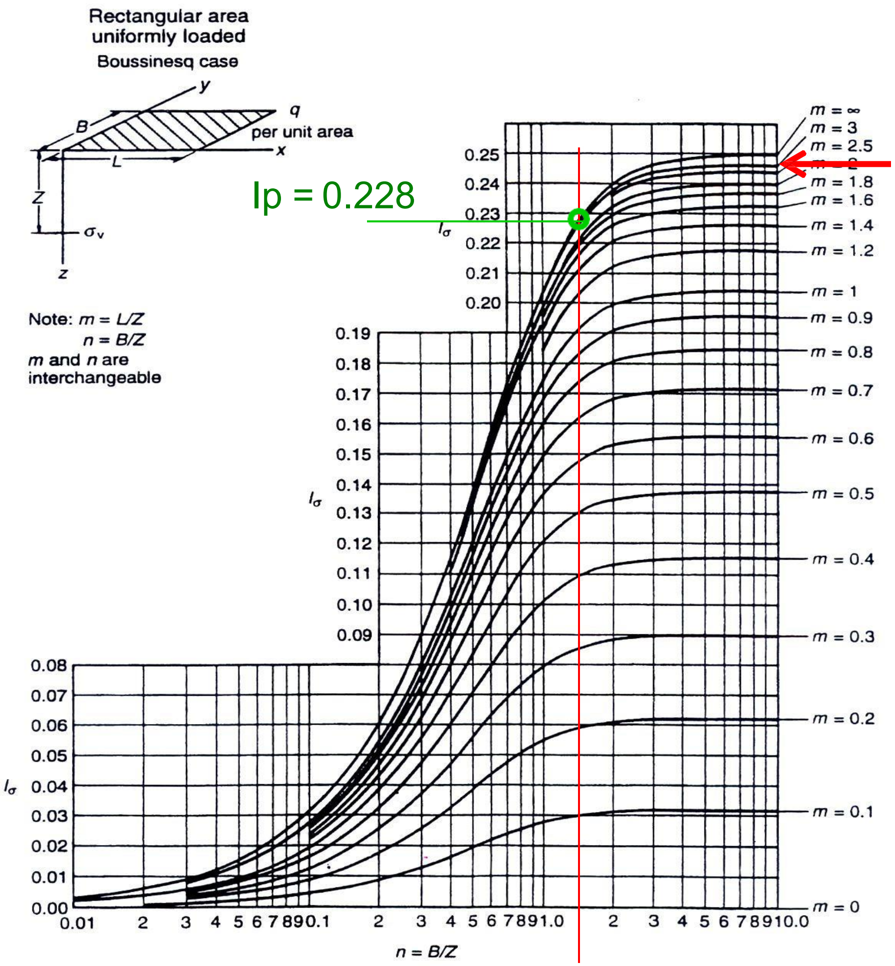


Influence factors for the increase in vertical stress below the corner of a uniform rectangular surcharge. (Redrawn from Fadum, 1948.)

Example 1a

A raft foundation, 6.1 m by 15.25 m, carries a uniform pressure of 300 kN/m^2. Determine the vertical stress increments due to the raft at a depth of 4.6 m below a corner





Influence factors for the increase in vertical stress below the corner of a uniform rectangular surcharge. (Redrawn from Fadum, 1948.)

m = L/z = 15.25/4.6 = 3.3

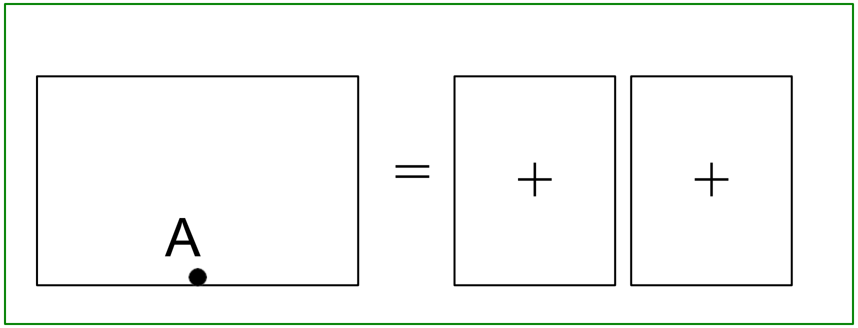
n = B/z = 6.1/4.6 = 1.3

$$\Delta\sigma_v = q.I_p$$

$$= 300 \times 0.228$$

$$= \underline{\mathbf{68\ kPa}}$$

Elastic methods allow the use of **superposition**, i.e. the simple summation of stress contributions thereby making this class of methods very powerful.

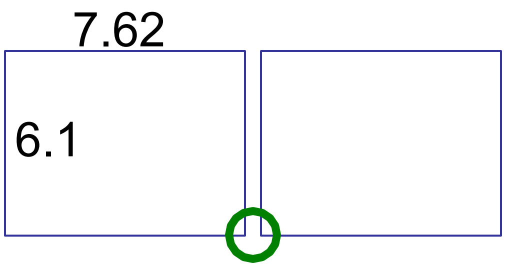


Decompose the loaded area into sub-rectangles (preferably of equal size to reduce calculation steps) then SUM the co-located rectangle corners to obtain stress increment at location within the foundation area.

Superposition

Example 1b

A raft foundation, 6.1 m by 15.25 m, carries a uniform pressure of 300 kN/m². Determine the vertical stress increments due to the raft at a depth of 4.6 m below (b) the centre of a long edge.



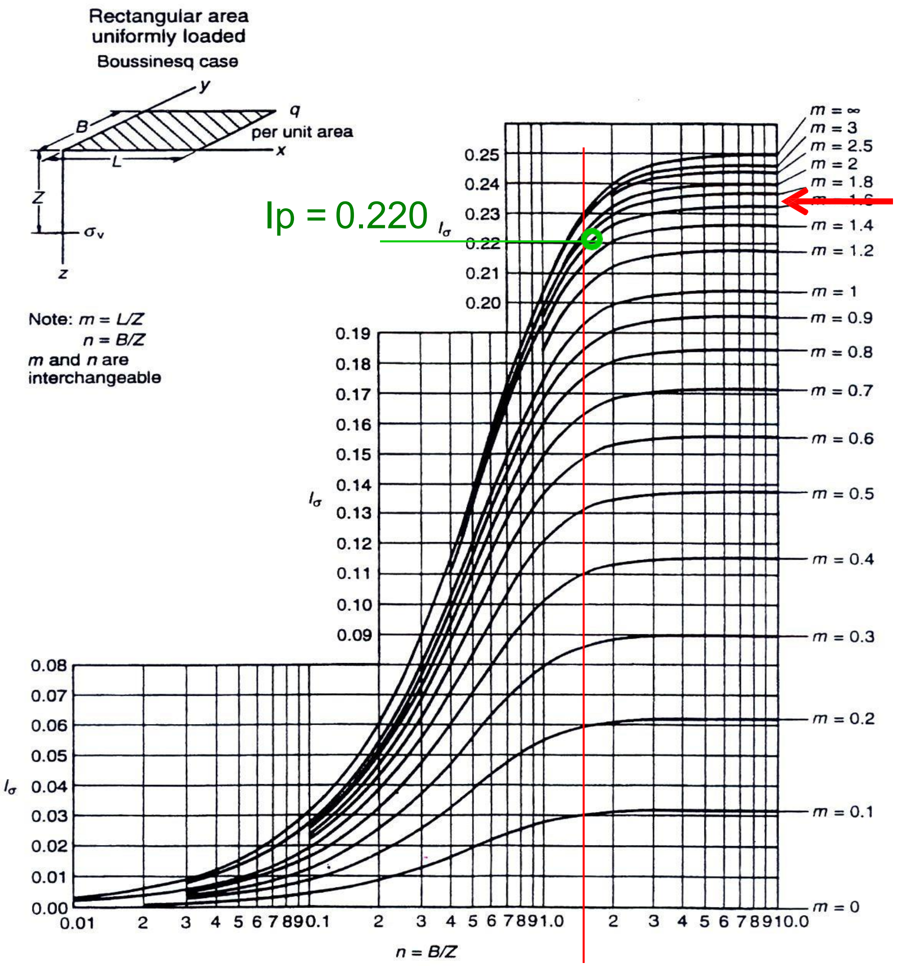
$m = L/z = 7.62/4.6 = 1.66$

$n = B/z = 6.1/4.6 = 1.3$

$\Delta\sigma = q.I_p \times 2$

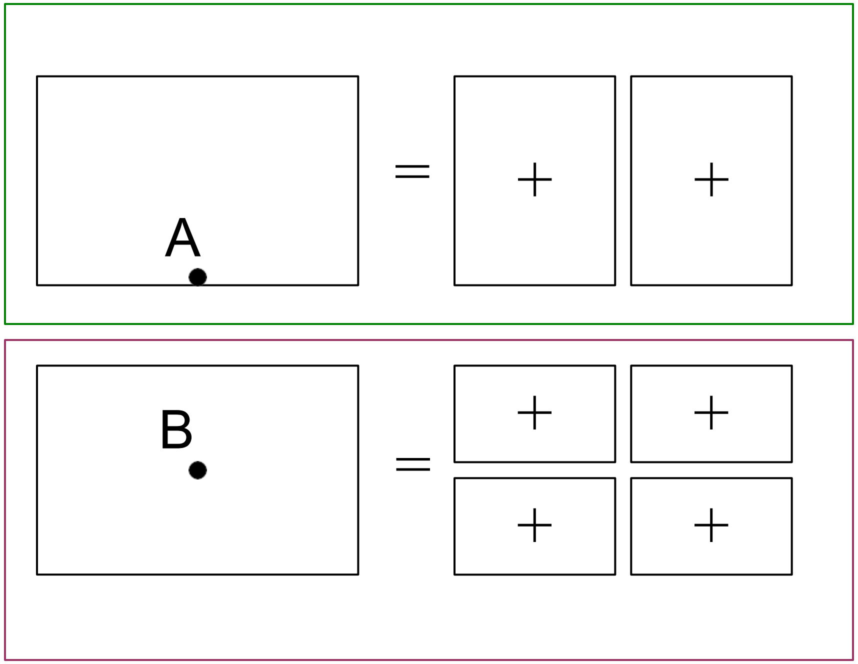
$= 300 \times 0.220 \times 2$

$= \underline{\mathbf{132\ kPa}}$



Influence factors for the increase in vertical stress below the corner of a uniform rectangular surcharge. (Redrawn from Fadum, 1948.)

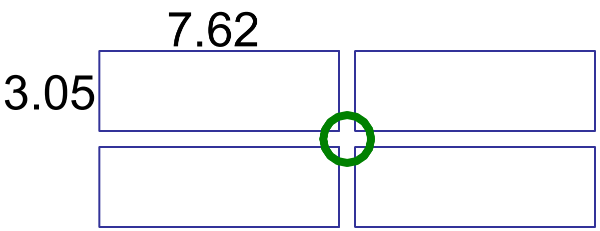
Elastic methods allow the use of superposition, i.e. the simple summation of stress contributions thereby making this class of methods very powerful.

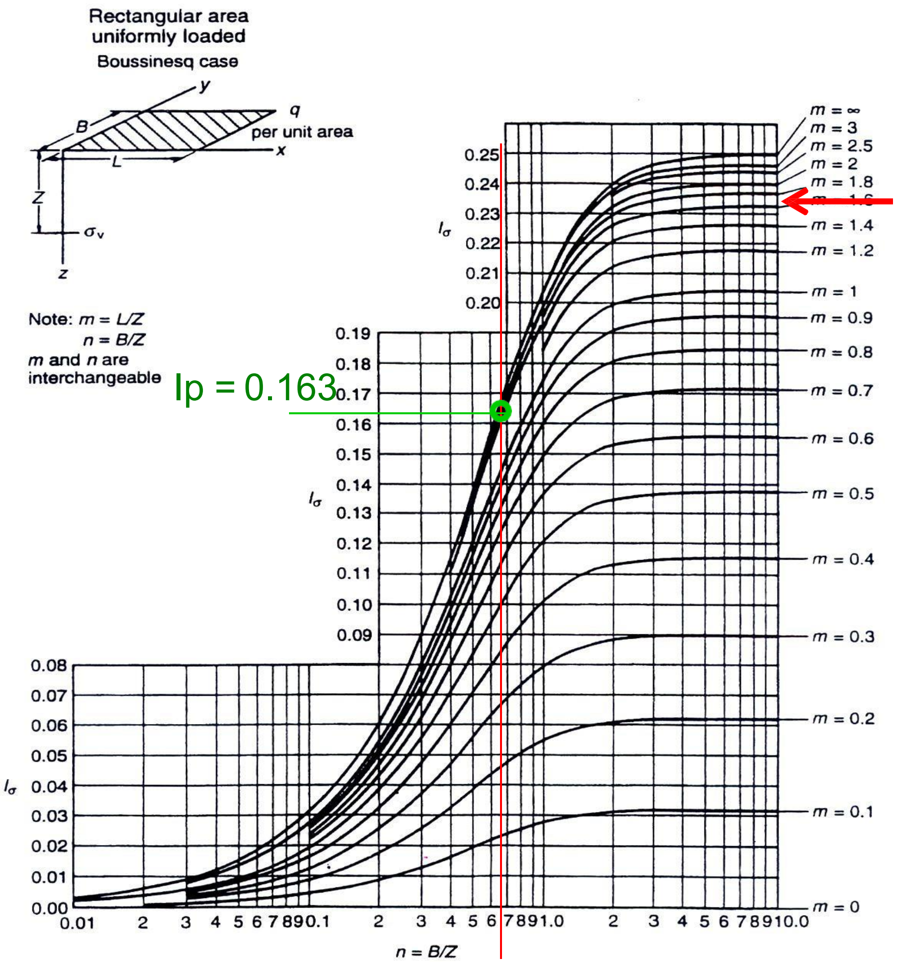


Superposition

Example 1c

A raft foundation, 6.1 m by 15.25 m, carries a uniform pressure of 300 kN/m². Determine the vertical stress increments due to the raft at a depth of 4.6 m below (c) the centre of the raft.





Influence factors for the increase in vertical stress below the corner of a uniform rectangular surcharge. (Redrawn from Fadum, 1948.)

$m = L/z = 7.62/4.6 = 1.66$

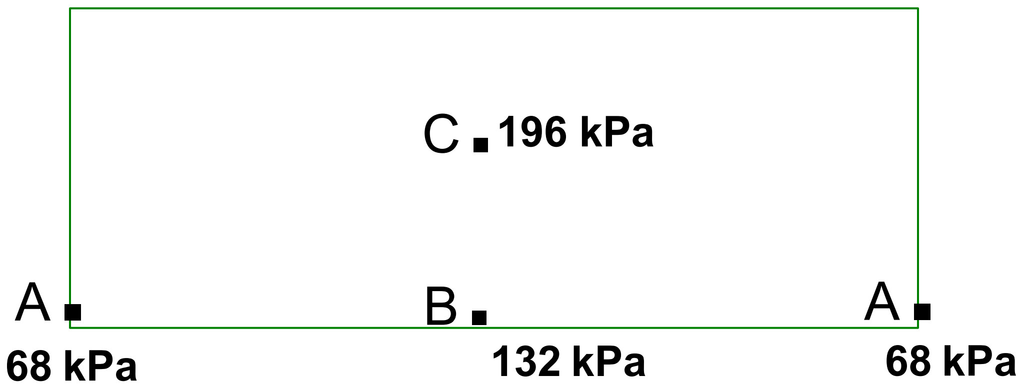
$n = B/z = 3.05/4.6 = 0.66$

$\sigma = q.I_p.4$

$= 300 \times 0.163 \times 4$

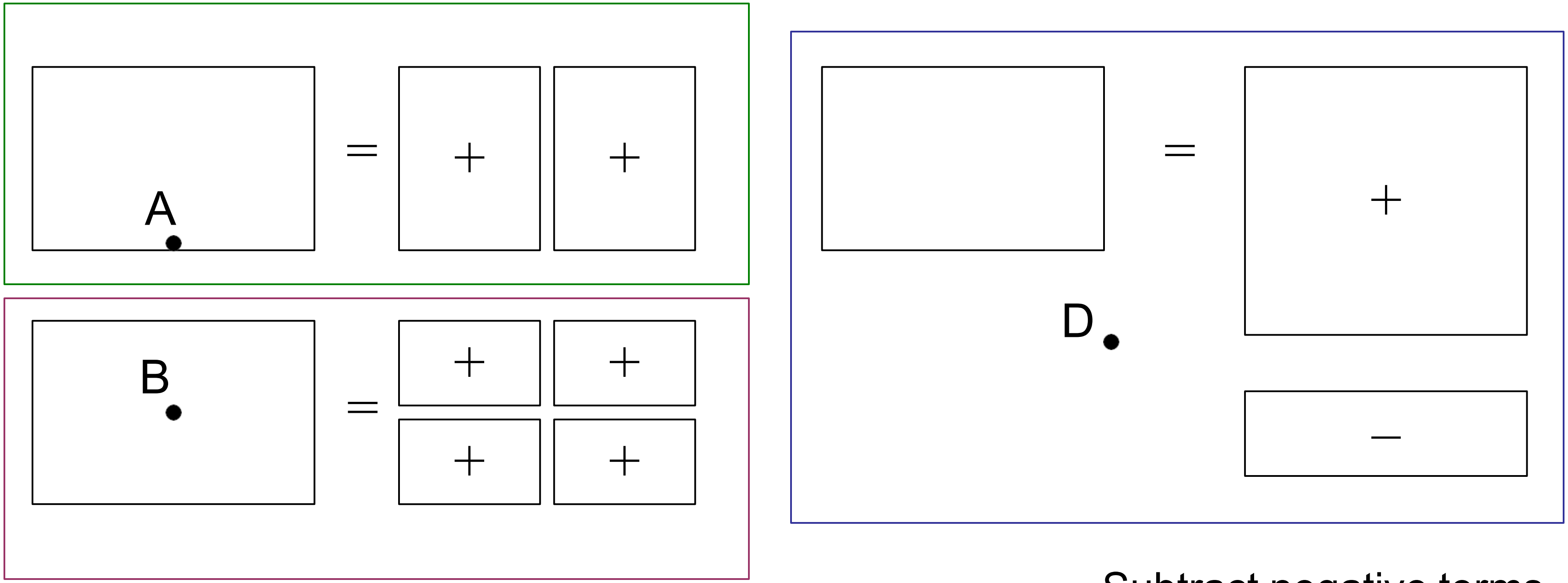
$= \underline{196 \text{ kPa}}$

Prediction of stress increments/contact pressure distribution under flexible raft



Superposition

Elastic methods allow the use of superposition, i.e. the simple summation of stress contributions thereby making this class of methods very powerful.

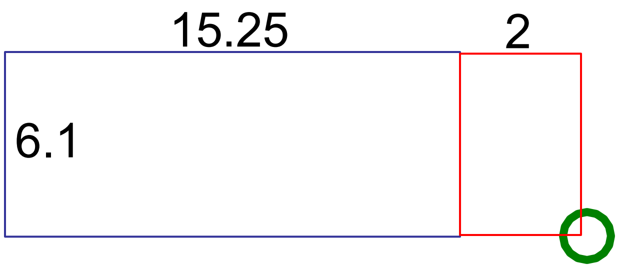


Subtract negative terms

Superposition

Example 1d

A raft foundation, 6.1 m by 15.25 m, carries a uniform pressure of 300 kN/m². Determine the vertical stress increments due to the raft at a depth of 4.6 m below a point 2 m outside and along the line of a long edge, i.e. point **d**.



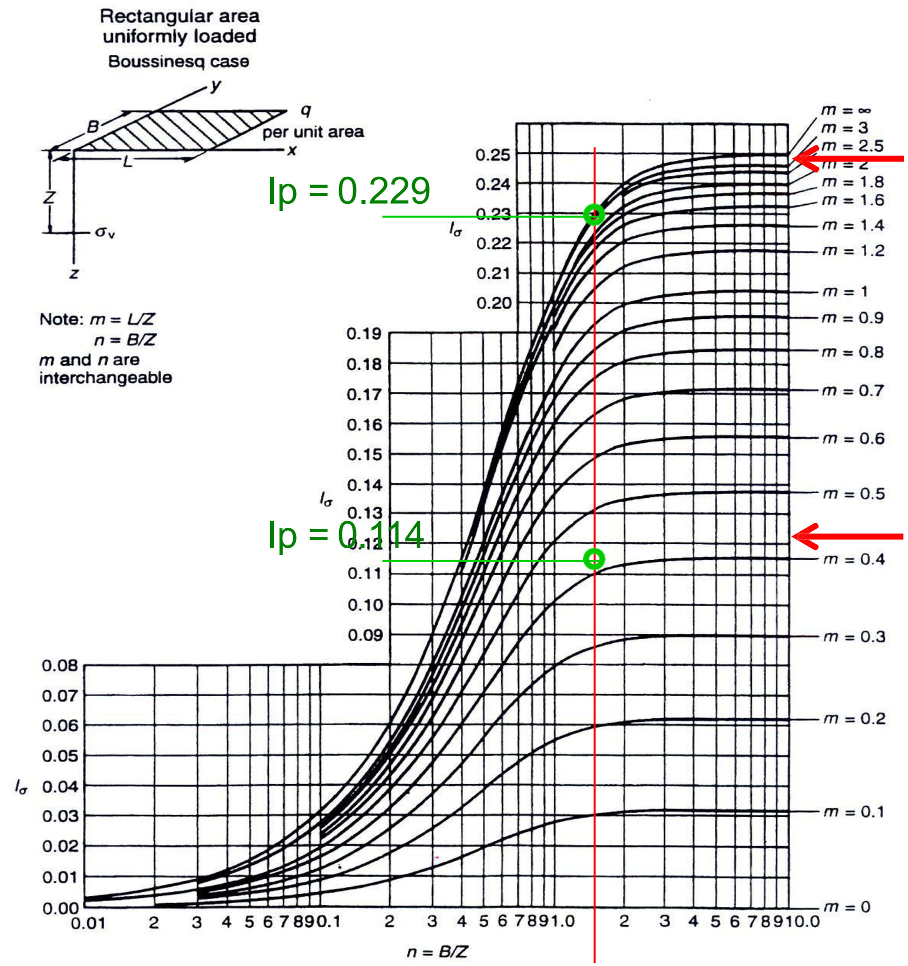
m = L/z = 17.25/4.6 = 3.75
n = B/z = 6.1/4.6 = 1.3
Ip = 0.229

m = L/z = 2/4.6 = 0.43
n = B/z = 6.1/4.6 = 1.3
Ip = 0.114

$$\Delta\sigma_v = q.I_p^*$$

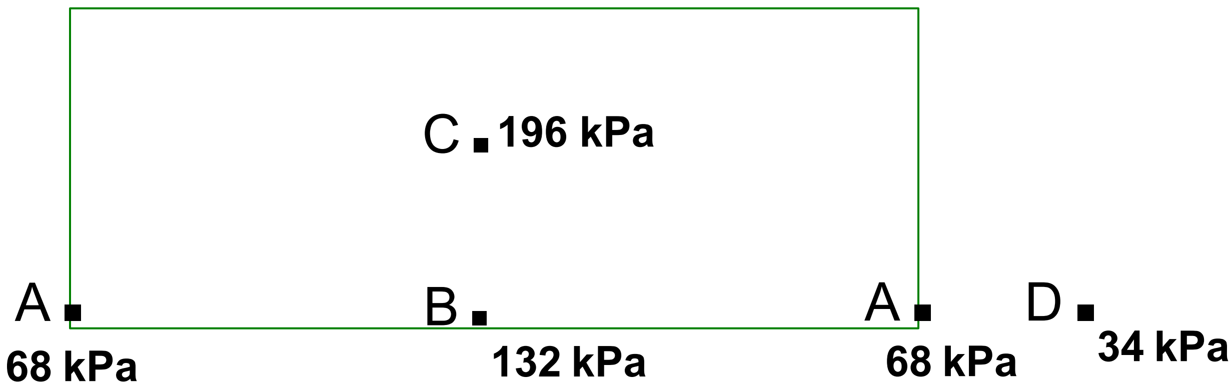
$$= 300 \times (0.229 - 0.114)$$

= **<u>34 kPa</u>**



Influence factors for the increase in vertical stress below the corner of a uniform rectangular surcharge. (Redrawn from Fadum, 1948.)

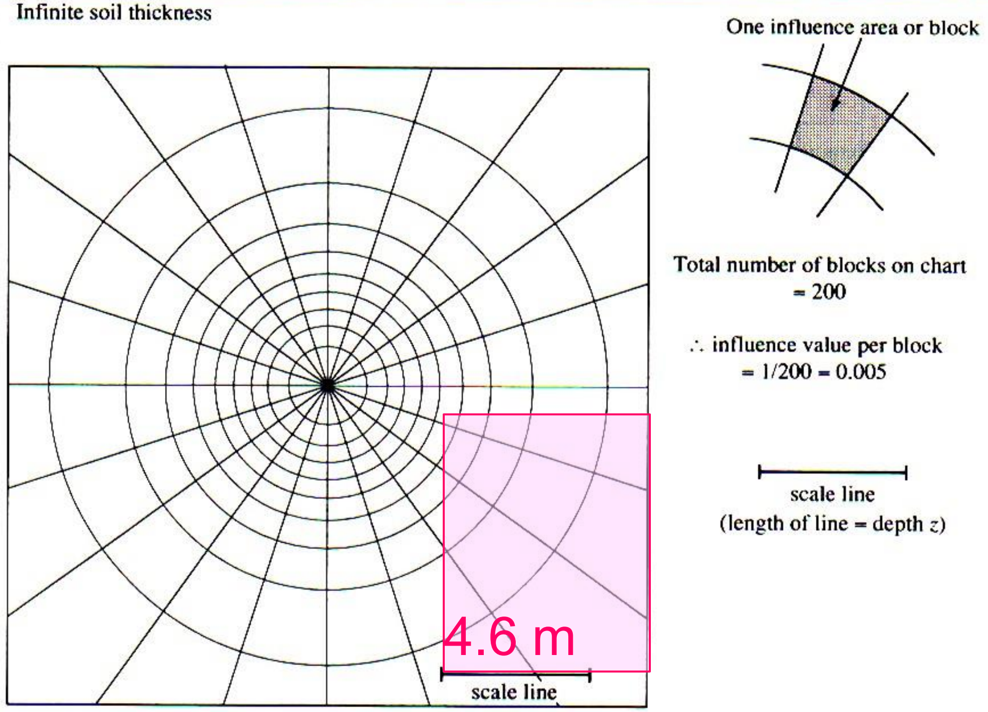
Prediction of stress increments/contact pressure distribution under flexible raft



Superposition

Stresses beneath flexible area of any shape (Newmark's chart)

Infinite soil thickness



For the vertical stress σ_v at a depth z beneath any point × on or outside a loaded area:

1. Draw a plan sketch of the building outline on tracing paper such that the length of the scale line equals the depth z where the stress is required.
2. Place the scale drawing on the chart with the point × at the centre of the chart.
3. Count the number of blocks N covered by the scale drawing. Group together part blocks.
4. The vertical stress at the depth z and beneath the point × is given by $\sigma_v = 0.005\, N\, q$
5. The tracing can then be moved to other locations to obtain the stress beneath other points.

Figure 5.9 *Stress beneath flexible area of any shape (Newmark's chart)*

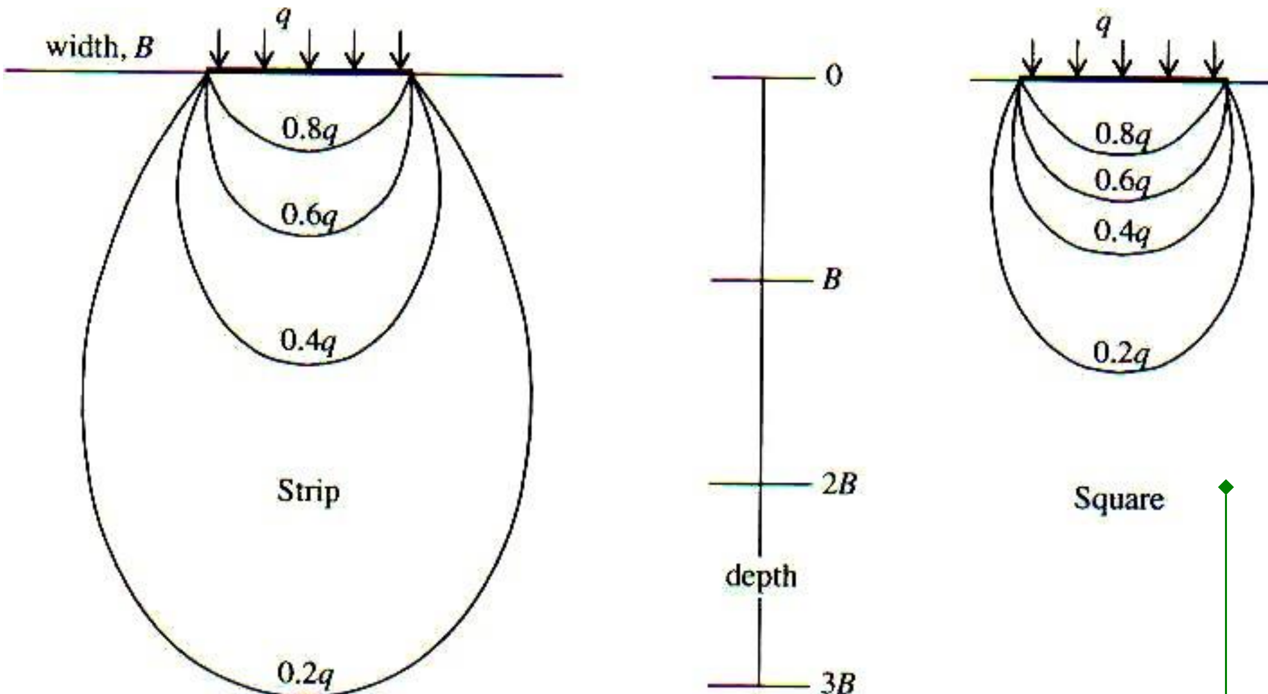
This is a graphical procedure for the determination of stress increment at any point under an irregular shaped foundation. It requires scaled drawing of the foundation to be superimposed on the appropriate Newmark Chart (for vertical stress, horizontal stress, etc) but is time consuming and fiddly!

Remember the 6.1 m x 7.62 m raft for stress increment at 4.6 m depth?
Ip = 0.220.

For corner of rectangle, N = 44
$\Delta\sigma = 0.005\ N\ q$
i.e. $\Delta\sigma = 0.220\ q$

Newmark's Chart [stress – any position, any shape, flexible, linear, homogeneous, infinite]

From Boussinesq equations, contours of vertical stress increment beneath shallow foundations can be drawn. These are known as bulbs of pressure.



10%, c2B

10%, c4B

Site investigation – depths of interest

Bulbs of pressure