

1 **The Sensitivity to Oscillation Parameters**
2 **from a Simultaneous Beam and**
3 **Atmospheric Neutrino Analysis that**
4 **combines the T2K and SK Experiments**



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10 *Doctor of Philosophy*
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Abstract

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Acknowledgements

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Introduction

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Neutrino Oscillation Physics

¹²⁹ When first proposed, neutrinos were expected to be approximately massless
¹³⁰ fermions that only interact through weak and gravitational forces. This meant
¹³¹ they were very difficult to detect as they can pass through significant amounts
¹³² of matter without interacting. Despite this, experimental neutrino physics has
¹³³ developed many different detection techniques and observed neutrinos from
¹³⁴ both natural and artificial sources. In direct tension with Standard Model physics,
¹³⁵ neutrinos have been determined to oscillate between different lepton flavours,
¹³⁶ requiring them to have mass.

¹³⁷ The observation techniques which led to the discovery of the neutrino are doc-
¹³⁸ umented in section 2.1. The theory underpinning neutrino oscillation is described
¹³⁹ in section 2.2 and includes the approximations which can be made to simplify
¹⁴⁰ the understanding of neutrino oscillation in the two-flavour approximation. Past,
¹⁴¹ current, and future neutrino experiments are detailed in section 2.3, including the
¹⁴² reactor, atmospheric, and long-baseline accelerator neutrino sources that have
¹⁴³ been used to successfully constrain oscillation parameters. Finally, the current
¹⁴⁴ state of oscillation parameter measurements are summarised in section 2.4.

2.1 Discovery of Neutrinos

At the start of the 20th century, the electrons emitted from the β -decay of the nucleus were found to have a continuous energy spectrum [1, 2]. This observation seemingly broke the energy conservation invoked within that period's nuclear models. In 1930, Pauli provided a solution to this problem in the form of a new particle, the neutrino (originally termed "neutron"). It was theorized to be an electrically neutral spin-1/2 fermion with a mass smaller than that of the electron [3]. This neutrino was emitted with the electron in β -decay to alleviate the apparent breaking of energy conservation. As a predecessor of today's weak interaction model, Fermi's theory of β -decay developed the understanding by coupling the four constituent particles: electron, proton, neutron, and neutrino, into a quantitative model [4].

Whilst Pauli was not convinced of the ability to detect neutrinos, the first observations of the particle were made in the mid-1950s when neutrinos from a reactor were observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [5, 6]. The detector consisted of two parts: a neutrino interaction medium and a liquid scintillator. The interaction medium was built from two water tanks, loaded with cadmium chloride to allow for increased efficiency in the detection of neutron capture. The positron emitted from IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured on the cadmium via $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$, producing a delayed signal. An increase in the coincidence rate was observed when the reactor was operating which was interpreted as interactions from neutrinos generated in the reactor.

After the discovery of the ν_e , the question of how many flavours of neutrino exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven National Laboratory [7]. A proton beam was directed at a beryllium target, generating pions which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the subsequent interactions of the ν_μ were observed. As the subsequent interaction of the neutrino generated muons rather than electrons, it was determined that

¹⁷⁴ the ν_μ was fundamentally different from ν_e . The final observation to be made
¹⁷⁵ was that of the ν_τ from the DONUT experiment [8]. Three neutrinos seem the
¹⁷⁶ obvious solution as it mirrors the known number of charged leptons (as they form
¹⁷⁷ weak isospin doublets) but there could be evidence of more. Several neutrino
¹⁷⁸ experiments have found anomalous results [9, 10] which could be attributed
¹⁷⁹ to “sterile” neutrinos. These hypothesised particles are not affected by gauge
¹⁸⁰ interactions in the Standard Model so their presence can only be inferred through
¹⁸¹ the observation of non-standard oscillation modes. However, cosmological
¹⁸² observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [11], as
¹⁸³ measured from the cosmic microwave background power spectrum. LEP also
¹⁸⁴ measured the number of active neutrino flavours to be $N_\nu = 2.9840 \pm 0.0082$ [12]
¹⁸⁵ from measurements of the Z-decay width, but this does not strongly constrain
¹⁸⁶ the number of sterile neutrinos.

¹⁸⁷ 2.2 Theory of Neutrino Oscillation

¹⁸⁸ A neutrino generated with lepton flavour α can change into a different lepton
¹⁸⁹ flavour β after propagating some distance. This phenomenon is called neutrino
¹⁹⁰ oscillation and requires that neutrinos must have a non-zero mass. This behaviour
¹⁹¹ has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13–
¹⁹² 15] mixing matrix which describes how the flavour and mass of neutrinos are
¹⁹³ associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [16]
¹⁹⁴ matrix measured in quark physics.

¹⁹⁵ 2.2.1 Three Flavour Oscillations

¹⁹⁶ The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ
¹⁹⁷ (indexed ν_α), which are eigenstates of the weak interaction and three mass
¹⁹⁸ eigenstates, ν_1 , ν_2 and ν_3 (indexed ν_i). Each mass eigenstate is the superposition

¹⁹⁹ of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

²⁰⁰ Where U is the 3×3 PMNS matrix which is unitary and connects the mass
²⁰¹ and flavour eigenstates.

²⁰² The weak interaction, when interacting via a W^{\pm} boson, couples to flavour
²⁰³ eigenstates so neutrinos interact with leptons of the same flavour. The prop-
²⁰⁴ agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a
²⁰⁵ plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

²⁰⁶ The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
²⁰⁷ three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
²⁰⁸ coordinates). The probability of observing a neutrino of flavour eigenstate β from
²⁰⁹ one which originated as flavour α can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)}. \quad (2.3)$$

²¹⁰ The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

²¹¹ For a relativistic particle, $E_i \gg m_i$, a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

²¹² Making the approximations that neutrinos are relativistic, the mass eigenstates
²¹³ were created with the same energy and that $x = L$, where L is the distance
²¹⁴ travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This, combined with further use of unitarity relations results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \end{aligned} \quad (2.7)$$

Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last term is included for the oscillation probability of antineutrinos. As an important point to note, the observation of oscillation probability requires a non-zero value of Δm_{ij}^2 , which in turn requires that neutrinos have differing masses.

Typically, the PMNS matrix is parameterised into three mixing angles, a charge parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many neutrino experiments aim to measure the PMNS parameters from a wide array of origins, as is the purpose of this thesis.

The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8 is only included for completeness. For an oscillation analysis experiment, any terms containing this phase disappear due to taking the expectation value of the PMNS matrix. Measurements of these phases can be performed by experiments searching for neutrino-less double β -decay [17].

232 A two-flavour approximation can be obtained when one assumes the third
233 mass eigenstate is degenerate with another. This results in the two-flavour
234 approximation being reasonable for understanding the features of the oscillation.
235 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

236 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

237 Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is
238 a sinusoidal function depending upon the distance over which the neutrino
239 propagates. The frequency and amplitude of oscillation are dependent upon
240 $\Delta m^2/4E$ and $\sin^2 2\theta$, respectively. The oscillation probabilities presented thus far
241 assume $c = 1$, where c is the speed of light in a vacuum. In more familiar units, the
242 maximum oscillation probability for a fixed value of θ is given at $L[\text{km}] / E[\text{GeV}] \sim$
243 $1.27/\Delta m^2$. It is this calculation that determines the best L/E value for a given
244 experiment to be designed around for measurements of a specific value of Δm^2 .

245 2.2.2 The MSW Effect

246 The theory of neutrino oscillation in a vacuum has been described in subsec-
247 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating
248 from below the horizon propagate through the matter in the Earth. The coherent
249 scattering of neutrinos from a material target modifies the Hamiltonian of the
250 system which results in a change in the oscillation probability. This modification
251 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [18–20]. This occurs
252 because charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$, propagated by a W boson)
253 only affects electron neutrinos whereas the neutral current scattering ($\nu_l + l^- \rightarrow$

²⁵⁴ $\nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino flavours equally.
²⁵⁵ In the two-flavour approximation, the effective mixing parameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

²⁵⁶ where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F
²⁵⁷ is Fermi's constant. It is clear that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for
²⁵⁸ $\Delta m^2 > 0$, which results in a divergent mixing parameter, colloquially called the
²⁵⁹ matter resonance. This resonance regenerates the electron neutrino component of
²⁶⁰ the neutrino flux [18–20]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

²⁶¹ At densities lower than this critical value, the oscillation probability will
²⁶² be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$
²⁶³ [21]. The resonance occurring from the MSW effect depends on the sign of Δm^2 .
²⁶⁴ Therefore, any neutrino oscillation experiment which observes neutrinos and
²⁶⁵ antineutrinos which have propagated through matter can have some sensitivity
²⁶⁶ to the ordering of the neutrino mass eigenstates.

²⁶⁷ 2.3 Neutrino Oscillation Measurements

²⁶⁸ As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics
²⁶⁹ was awarded to the Super-Kamiokande (SK) [22] and Sudbury Neutrino Ob-
²⁷⁰ servatory (SNO) [23] collaborations for the first definitive observation of solar
²⁷¹ and atmospheric neutrino oscillation [24]. Since then, the field has seen a wide
²⁷² array of oscillation measurements from a variety of neutrino sources. As seen
²⁷³ in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio
²⁷⁴ of the propagation baseline, L , to the neutrino energy, E . It is this ratio that
²⁷⁵ determines the type of neutrino oscillation a particular experiment is sensitive to.

²⁷⁶ As illustrated in Figure 2.1, there are many neutrino sources that span a
²⁷⁷ wide range of energies. The least energetic neutrinos are from reactor and

²⁷⁸ terrestrial sources at $O(1)$ MeV whereas the most energetic neutrinos originate
²⁷⁹ from atmospheric and galactic neutrinos of $> O(1)$ TeV.



Figure 2.1: The electro-weak cross-section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [25]

²⁸⁰ 2.3.1 Solar Neutrinos

²⁸¹ Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun.
²⁸² The solar neutrino flux, given as a function of neutrino energy for different
²⁸³ fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion
²⁸⁴ generates the largest flux of neutrinos, the neutrinos are low energy and are
²⁸⁵ difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [26].
²⁸⁶ Consequently, most experiments focus on the neutrinos from the decay of 8B
²⁸⁷ (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher energy.

²⁸⁸ The first measurements of solar neutrinos observed a significant reduction in
²⁸⁹ the event rate compared to predictions from the Standard Solar Model [28, 29]. A
²⁹⁰ proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a



Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [27].

291 precursory version of the PMNS model [30]. The Kamiokande [31], Gallex [32]
 292 and Sage [33] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

293 The conclusive solution to this problem was determined by the SNO col-
 294 laboration [34]. Using a deuterium water target to observe 8B neutrinos, the
 295 event rate of charged current (CC), neutral current (NC), and elastic scattering
 296 (ES) interactions (Given in Equation 2.13) was simultaneously measured. CC
 297 events can only occur for electron neutrinos, whereas the NC channel is agnostic
 298 to neutrino flavour, and the ES reaction has a small excess sensitivity for the
 299 detection of electron neutrino interactions. This meant that there were direct
 300 measurements of the ν_e and ν_x neutrino flux. It was concluded that the CC and
 301 ES interaction rates were consistent with the deficit previously observed. Most
 302 importantly, the NC reaction rate was only consistent with the others under the

303 hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (CC) \\ \nu_x + d &\rightarrow p + n + \nu_x & (NC) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (ES) \end{aligned} \quad (2.13)$$

304 Since the SNO measurement, many experiments have since measured the
 305 neutrino flux of different interaction chains within the sun [35–37]. The most
 306 recent measurement was that of CNO-cycle neutrinos which were recently
 307 observed with 5σ significance by the Borexino collaboration [35].

308 2.3.2 Accelerator Neutrinos

309 The concept of using an artificial “neutrino beam” was first realised in 1962 [38].
 310 Since then, many experiments have adopted the same fundamental concepts.
 311 Typically, a proton beam is aimed at a target producing charged mesons that
 312 decay to neutrinos. The mesons can be sign-selected by the use of magnetic
 313 focusing horns to generate a neutrino or antineutrino beam. Pions are the primary
 314 mesons that decay and depending on the orientation of the magnetic field, a
 315 muon (anti-)neutrino beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$.
 316 The decay of muons and kaons results in an irreducible intrinsic electron neutrino
 317 background. In T2K, this background contamination is $O(< 1\%)$ [39]. There is
 318 also an approximately $\sim 5\%$ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated
 319 via the same decays. As the beam is generated by proton interactions (rather
 320 than anti-proton interactions), the wrong-sign component in the antineutrino
 321 beam is larger when operating in neutrino mode.

322 Tuning the proton energy in the beam and using beam focusing techniques
 323 allows the neutrino energy to be set to a value that maximises the disappear-
 324 ance oscillation probability in the L/E term in Equation 2.10. This means that
 325 accelerator experiments are typically more sensitive to the mixing parameters as
 326 compared to a natural neutrino source. However, the disadvantage compared
 327 to atmospheric neutrino experiments is the cost of building a facility to provide

328 high-energy neutrinos, with a high flux, which is required for longer baselines.
 329 Consequently, there is typically less sensitivity to matter effects and the ordering
 330 of the neutrino mass eigenstates.

331 A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

332 where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of
 333 neutrinos with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and
 334 $\epsilon(\vec{x})$ is the efficiency and resolution of the detector. In order to leverage the most
 335 out of an accelerator neutrino experiment, the flux and cross-section systematics
 336 need to be constrained. This is typically done via the use of a “near detector”,
 337 situated at a baseline of $O(1)$ km. This detector observes the unoscillated neutrino
 338 flux and constrains the parameters used within the flux and cross-section model.

339 The first accelerator experiments to precisely measure oscillation parameters
 340 were MINOS [40] and K2K [41]. These experiments confirmed the ν_μ disappear-
 341 ance seen in atmospheric neutrino experiments by finding consistent parameter
 342 values for $\sin^2(\theta_{23})$ and Δm_{32}^2 . The current generation of accelerator neutrino
 343 experiments, T2K and NO ν A extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead
 344 the sensitivity to atmospheric mixing parameters as seen in Figure 2.6 [42]. The
 345 two experiments differ in their peak neutrino energy, baseline, and detection tech-
 346 nique. The NO ν A experiment is situated at a baseline of 810km from the NuMI
 347 beamline which delivers 2GeV neutrinos. The T2K neutrino beam is peaked
 348 around 0.6GeV and propagates 295km. Additionally, the NO ν A experiment
 349 uses functionally identical detectors (near and far) whereas T2K uses a plastic
 350 scintillator technique at the near detector and a water Cherenkov far detector.
 351 The future generation experiments DUNE [43] and Hyper-Kamiokande [44]
 352 will succeed these experiments as the high-precision era of neutrino oscillation
 353 parameter measurements develops.

354 Several anomalous results have been observed in the LSND [9] and Mini-
 355 BooNE [10] detectors which were designed with purposefully short baselines.

356 Parts of the neutrino community attributed these results to oscillations induced
357 by a fourth “sterile” neutrino [45] but several searches in other experiments,
358 MicroBooNE [46] and KARMEN [47], found no hints of additional neutrino
359 species. The solution to the anomalous results is still being determined.

360 2.3.3 Atmospheric Neutrinos

361 The interactions of primary cosmic ray protons in the Earth’s upper atmosphere
362 generate showers of energetic hadrons. These are mostly pions and kaons that
363 decay to produce a natural source of neutrinos spanning energies of MeV to
364 TeV [48]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu), \end{aligned} \tag{2.15}$$

365 such that for a single pion decay, three neutrinos can be produced. The
366 atmospheric neutrino flux energy spectra as predicted by the Bartol [49], Honda
367 [50–52], and FLUKA [53] models are illustrated in Figure 2.3. The flux distribution
368 peaks at an energy of $O(10)$ GeV. The uncertainties associated with these models
369 are dominated by the hadronic production of kaon and pions as well as the
370 primary cosmic flux.

371 Unlike long-baseline experiments which have a fixed baseline, the distance
372 atmospheric neutrinos propagate is dependent upon the zenith angle at which
373 they interact. This is illustrated in Figure 2.4. Neutrinos that are generated
374 directly above the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the
375 height of the atmosphere, whereas neutrinos that interact directly below the
376 detector ($\cos(\theta) = -1.0$) have to travel a length equal to the diameter of the Earth.
377 This means atmospheric neutrinos have a baseline that varies from $O(20)$ km to
378 $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will be subject
379 to MSW matter resonance as they propagate through the Earth.

380 Figure 2.5 highlights the neutrino flux as a function of the zenith angle for
381 different slices of neutrino energy. For medium to high-energy neutrinos (and to



Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [50], the 2004 Honda model (“HKKM04”)[51], the Bartol model [49] and the FLUKA model [53]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [50].



Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [54].

382 a lesser degree for low-energy neutrinos), the flux is approximately symmetric
 383 around $\cos(\theta) = 0$. To the accuracy of this approximation, the systematic
 384 uncertainties associated with atmospheric flux for comparing upward-going
 385 and down-going neutrino cancels. This allows the down-going events, which are

386 mostly insensitive to oscillation probabilities, to act as an unoscillated prediction
 387 (similar to a near detector in an accelerator neutrino experiment).



Figure 2.5: Prediction of ν_e , $\bar{\nu}_e$, ν_μ , $\bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [52]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [49], Honda [50] and FLUKA [53] are given in [54].

388 Precursory hints of atmospheric neutrinos were observed in the mid-1960s
 389 searching for $\nu_\mu + X \rightarrow X^* + \mu^\pm$ [55]. This was succeeded by the IMB-3 [56]
 390 and Kamiokande [57] experiments which measured the double ratio of muon
 391 to electron neutrinos in data to Monte Carlo, $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$.
 392 Both experiments were found to have a consistent deficit of muon neutrinos,
 393 with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$, respectively.
 394 Super-Kamiokande (SK) [54] extended this analysis by fitting oscillation pa-
 395 rameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and
 396 $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}\text{eV}^2$.

397 Since then, atmospheric neutrino experiments have been making precision
 398 measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric
 399 neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ
 400 discovery of ν_τ appearance [58]. Figure 2.6 illustrates the current estimates on
 401 the atmospheric mixing parameters, from a wide range of atmospheric and
 402 accelerator neutrino observatories.



Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long-baseline experiments: SK [59], T2K [60], NOvA [61], IceCube [62] and MINOS [63]. Figure taken from [64].

2.3.4 Reactor Neutrinos

As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very useful artificial source of electron antineutrinos. For reactors that use low-enriched uranium ^{235}U as fuel, the antineutrino flux is dominated by the β -decay fission of ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu [65] as illustrated in Figure 2.7.

Due to their low energy, reactor electron antineutrinos predominantly interact via the inverse β -decay (IBD) interaction. The typical signature contains two signals delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihilation, and secondly the photon emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [67].

There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured the $\sin^2(\theta_{13})$ and Δm_{32}^2 oscillation parameters. Daya Bay [68], RENO [69] and Double Chooz [70] have all provided precise measurements, with the first discovery of a non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K [70]). The constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field. They



Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [66].

are often used as external inputs to accelerator neutrino experiments to improve their sensitivity to δ_{CP} and mass hierarchy determination. JUNO-TAO [71], a small collaboration within the larger JUNO experiment, is a next-generation reactor experiment that aims to precisely measure the isotopic antineutrino yields from the different fission chains.

Kamland [72] is the only experiment to have observed reactor neutrinos using a long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have sensitivity to Δm_{21}^2 . Combined with the SK solar neutrino experiment, the combined analysis puts the most stringent constraint on Δm_{21}^2 [73].

428 2.4 Summary Of Oscillation Parameter Measurements

429 Since the first evidence of neutrino oscillations, numerous measurements of the
 430 mixing parameters have been made. Many experiments use neutrinos as a tool
 431 for the discovery of new physics (diffuse supernova background, neutrinoless
 432 double beta decay and others) so the PMNS parameters are summarised in the
 433 Particle Data Group (PDG) review tables. The analysis presented in this thesis
 434 focuses on the 2020 T2K oscillation analysis presented in [74] which the 2020
 435 PDG constraints [75] were used. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	0.547 ± 0.021
$\sin^2(\theta_{23})$ (N.H.)	0.545 ± 0.021
Δm_{32}^2 (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2020 Particle Data Group constraints of the oscillation parameters taken from [75]. The value of Δm_{32}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

436 The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance,
 437 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is taken as the average best-fit from the combination of
 438 Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within
 439 other neutrino oscillation experiments, typically termed the reactor constraint.
 440 The $\sin^2(\theta_{12})$ parameter is predominantly measured through electron neutrino
 441 disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline
 442 reactor neutrino experiment Kamland also has a sensitivity to this parameter and
 443 is used in a joint fit to solar data from SNO and SK, using the reactor constraint.
 444 Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric neutrino
 445 experiments. The PDG value is a joint fit of T2K, NO ν A, MINOS and IceCube
 446 DeepCore experiments. The latest T2K-only measurement, provided at Neu-
 447 trino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [74].

The PDG constraint on Δm_{21}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{32}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [74]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analyses have reduced the credible intervals to 90%. Since the 2020 PDG publication, there has been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [76], alongside updated Δm_{32}^2 and $\sin^2(\theta_{23})$ measurements.

Throughout this thesis, several sample spectra predictions and contours are presented, which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [77] and “Asimov B” being CP-conserving and further from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

3

466

467

T2K and SK Experiment Overview

468 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)
469 collaboration has been leading atmospheric neutrino oscillation analyses for
470 over two decades. The detector has provided some of the strongest constraints
471 on proton decay and the first precise measurements of the Δm_{32}^2 and $\sin^2(\theta_{23})$
472 neutrino oscillation parameters. The history, detection technique, and operation
473 of the SK detector is described in section 3.1.

474 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline
475 experiments to use both neutrino and antineutrino beams to precisely measure
476 charge parity violation within the neutrino sector. The T2K experiment observed
477 the first hints of a non-zero $\sin^2(\theta_{13})$ measurement and continues to lead the
478 field with the constraints it provides on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{32}^2 and δ_{CP} . In
479 section 3.2, the techniques that T2K use to generate the neutrino beam and
480 constrain systematic parameter through near detector constraints are described.

481 3.1 The Super-Kamiokande Experiment

482 The SK experiment began taking data in 1996 [78] and has had many modifi-
483 cations throughout its operation. There have been seven defined periods of
484 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

485 years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were
 486 damaged during maintenance [79]. Those that survived were equally distributed
 487 throughout the detector in the SK-II era, which resulted in a reduced 19% photo-
 488 coverage. From SK-III onwards, repairs to the detector meant the full suite of
 489 PMTs was operational recovering the 40% photo-coverage. Before the start of
 490 SK-IV, the data acquisition and electronic systems were upgraded. Between
 491 SK-IV and SK-V, a significant effort was placed into tank open maintenance and
 492 repair/replacement of defective PMTs in preparation for the Gadolinium upgrade;
 493 a task for which the author of this thesis was required. Consequently, the detector
 494 conditions were significantly changed from this point. SK-VI marked the start of
 495 the SK-Gd era, with the detector being doped with gadolinium at a concentration
 496 of 0.01% by concentration. SK-VII, which started during the writing of this thesis,
 497 has increased the gadolinium concentration to 0.03% for continued operation [80].

498 The oscillation analysis presented within this thesis focuses on the SK-IV
 499 period of running and the data taken within it. This follows from the recent
 500 SK analysis presented in [81]. Therefore, the information presented within this
 501 section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

502 **Table 3.1:** The various SK periods and their respective live-time. The SK-VI live-time is
 503 calculated until 1st April 2022. SK-VII started during the writing of this thesis.

502 3.1.1 The SK Detector

503 The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank
 504 with a diameter 39.3m and height 41.1m filled with ultrapure water [79]. A
 505 diagram of the significant components of the SK detector is given in Figure 3.1.

506 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine
 507 is underground with roughly 1km rock overburden (2.7km water equivalent
 508 overburden) [82]. At this depth, the rate of cosmic ray muons is significantly
 509 decreased to a value of $\sim 2\text{Hz}$ (net rate) [83]. The top of the tank is covered
 510 with stainless steel which is designed as a working platform for maintenance,
 511 calibration, and location for high voltage and data acquisition electronics.



Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [84].

512 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated
 513 inside the tank, with an approximate 2m gap between this structure and the outer
 514 tank wall. The purpose of this structure is to support the photomultiplier tubes
 515 (PMTs). The volume inside and outside the support structure is referred to as the
 516 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the
 517 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs respectively
 518 [79]. The ID contains a 32kton mass of water. Many analyses performed at SK
 519 use a “fiducial volume” defined by the volume of water inside the ID excluding
 520 some distance to the ID wall. This reduces the volume of the detector which is
 521 sensitive to neutrino events but reduces radioactive backgrounds and allows for

522 better reconstruction performance. The nominal fiducial volume is defined as the
523 area contained inside 2m from the ID wall for a total of 22.5kton water [85].

524 The two regions of the detector (ID and OD) are optically separated with
525 opaque black plastic hung from the support structure. The purpose of this is
526 to determine whether an event entered or exited the ID. This allows cosmic ray
527 muons and partially contained events to be tagged and separated from neutrino
528 events entirely contained within the ID. This black plastic is also used to cover
529 the area between the ID PMTs to reduce photon reflection from the ID walls.
530 Opposite to this, the OD is lined with a reflective material to allow photons to
531 reflect around inside the OD until collected by one of the PMTs. Furthermore,
532 each OD PMT is optically coupled with $50 \times 50\text{cm}$ plates of wavelength shifting
533 acrylic which increases the efficiency of light collection [82].

534 In the SK-IV data-taking period, the photocathode coverage of the detector, or
535 the fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [82]. The PMTs have
536 a quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$
537 for photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$ [86, 87]. The proportion
538 of photoelectrons that produce a signal in the dynode of a PMT, termed the
539 collection efficiency, is $> 70\%$ [82]. The PMTs used within SK are most sensitive
540 to photons with wavelength $300\text{nm} \leq \lambda \leq 600\text{nm}$ [82]. One disadvantage of
541 using PMTs as the detection media is that the Earth's geomagnetic field can
542 modify its response. Therefore, a set of compensation coils is built around the
543 inner surface of the detector to mitigate this effect [83].

544 The SK detector is filled with ultrapure water, which in a perfect world, con-
545 tains no impurities. However, bacteria and organic compounds can significantly
546 degrade the water quality. This decreases the attenuation length, which reduces
547 the total number of photons that hit a PMT. To combat this, a sophisticated water
548 treatment system has been developed [82, 88]. UV lights, mechanical filters, and
549 membrane degasifiers are used to reduce the bacteria, suspended particulates,
550 and radioactive materials from the water. The flow of water within the tank
551 is also critical as it can remove stagnant bacterial growth or build-up of dust

552 on the surfaces within the tank. Gravity drifts impurities in the water towards
553 the bottom of the tank which, if left uncontrolled, can create asymmetric water
554 conditions between the top and bottom of the tank. Typically, the water entering
555 the tank is cooled below the ambient temperature of the tank to control convection
556 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs
557 is sensitive to the PMT temperature [89]. Therefore controlling the temperature
558 gradients within the tank is beneficial for stable measurements.

559 SK-VI is the first phase of the SK experiment to use gadolinium dopants
560 within the ultrapure water [80]. As such, the SK water system had to be replaced
561 to avoid removing the gadolinium concentrate from the ultrapure water [90]. For
562 an inverse β -decay (IBD) interaction on a water target, the emitted neutron is
563 thermally captured on hydrogen. This process releases a 2.2MeV γ ray which is
564 difficult to detect as the resulting Compton scattered electrons are very close to the
565 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons
566 on gadolinium generates γ rays with higher energy (8MeV [67]) meaning they
567 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%
568 gadolinium sulphate by mass) which causes \approx 50% of neutrons emitted by IBD to
569 be captured on gadolinium[91, 92] . Whilst predominantly useful for low energy
570 analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino
571 event selections [93]. Efforts are currently in place to increase the gadolinium
572 concentrate to 0.03% for \approx 75% neutron capture efficiency on gadolinium [94].
573 The final stage of loading targets 0.1% concentrate for \approx 90% neutron capture
574 efficiency on gadolinium.

575 3.1.2 Calibration

576 The calibration of the SK detector is documented in [79] and summarised below.
577 The analysis presented within this thesis is dependent upon ‘high energy events’
578 (Charged particles with $O(> 100)$ MeV momenta). These are events that are
579 expected to generate a larger number of photons such that each PMT will
580 be hit with multiple photons. The reconstruction of these events depends

upon the charge deposited within each PMT and the timing response of each individual PMT. Therefore, the most relevant calibration techniques to this thesis are outlined.

Before installation, 420 PMTs were calibrated to have identical charge responses and then distributed throughout the tank in a cross-shape pattern (As illustrated by Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed at similar geometric positions within SK to be calibrated against. To perform this calibration, a xenon lamp is located at the center of the SK tank which flashes uniform light at 1Hz. This allows for geometrical effects, water quality variation, and timing effects to be measured in situ throughout normal data-taking periods.



Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [79].

When specifically performing calibration of the detector (in out-of-data taking mode), the water in the tank was circulated to avoid top/bottom asymmetric water quality. Any non-uniformity within the tank significantly affects the PMT hit probability through scattering or absorption. This becomes a dominant effect for very low-intensity light sources that are designed such that only one photon is incident upon a given PMT.

598 The gain of a PMT is defined as the ratio of the total charge of the signal
 599 produced compared to the charge of photoelectrons emitted by the photocathodes
 600 within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”
 601 gain values are measured. The relative gain is the variation of gain among each
 602 of the PMTs whereas the absolute gain is the average gain of all PMTs.

603 The relative gain is calibrated as follows. A laser is used to generate two
 604 measurements: a high-intensity flash that illuminates every PMT with a sufficient
 605 number of photons, and a low-intensity flash in which only a small number
 606 of PMTs collect light. The first measurement creates an average charge, $Q_{obs}(i)$
 607 on PMT i , whereas the second measurement ensures that each hit PMT only
 608 generates a single photoelectron. For the low-intensity measurement, the number
 609 of times each PMT records a charge larger than 1/4 photoelectrons, $N_{obs}(i)$, is
 610 counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i). \end{aligned} \tag{3.1}$$

611 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the
 612 acceptance efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and
 613 collection efficiency of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative
 614 gain for each PMT can be determined by taking the ratio of these quantities.

615 The absolute gain calibration is performed by observing fixed energy γ -rays
 616 of $E_\gamma \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source
 617 situated at the center of the detector. This generates a photon yield of about 0.004
 618 photoelectrons/PMT/event, meaning that $> 99\%$ of PMT signals are generated
 619 from single photoelectrons. A charge distribution is generated by performing
 620 this calibration over all PMTs, and the average value of this distribution is taken
 621 to be the absolute gain value.

622 As mentioned in subsection 3.1.1, the average quantum and collection effi-
 623 ciency for the SK detector PMTs is $\sim 21\%$ and $> 70\%$ respectively. However,
 624 these values do differ between each PMT and need to be calibrated accordingly.

625 Consequently, the NiCf source is also used to calibrate the “quantum \times collection”
626 efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is
627 used as the PMT hit probability is proportional to the QE ($N_{obs}(i) \propto \epsilon(i)$ in
628 Equation 3.1). A Monte Carlo prediction which includes photon absorption,
629 scattering, and reflection is made to estimate the number of photons incident on
630 each PMT and the ratio of the number of predicted to observed hits is calculated.
631 The difference is attributed to the QE efficiency of that PMT. This technique is
632 extended to calculate the relative QE efficiency by normalizing the average of
633 all PMTs which removes the dependence on the light intensity.

634 Due to differing cable lengths and readout electronics, the timing response
635 between a photon hitting the PMT and the signal being captured by the data
636 acquisition can be different between each PMT. Due to threshold triggers (De-
637 scribed in subsection 3.1.3), the time at which a pulse reaches a threshold is
638 dependent upon the size of the pulse. This is known as the ‘time-walk’ effect
639 and also needs to be accounted for in each PMT. To calibrate the timing response,
640 a pulse of light with width 0.2ns is emitted into the detector through a diffuser.
641 Two-dimensional distributions of time and pulse height (or charge) are made
642 for each PMT and are used to calibrate the timing response. This is performed
643 in-situ during data taking with the light source pulsing at 0.03Hz.

644 The top/bottom water quality asymmetry is measured using the NiCf calibra-
645 tion data and cross-referencing these results to the “standard PMTs”. The water
646 attenuation length is continuously measured by the rate of vertically-downgoing
647 cosmic-ray muons which enter via the top of the tank.

648 Dark noise is where a PMT registers a pulse that is consistent with a single
649 photoelectron emitted from photon detection despite the PMT being in complete
650 darkness. This is predominately caused by two processes. Firstly there is
651 intrinsic dark noise which is where photoelectrons gain enough thermal energy
652 to be emitted from the photocathode, and secondly, the radioactive decay of
653 contaminants inside the structure of the PMT. Typical dark noise rate for PMTs
654 used within SK are $O(3)\text{kHz}$ [82]. This is lower than the expected number of

655 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)
656 but instability in this value can cause biases in reconstruction. Dark noise is
657 related to the gain of a PMT and is calibrated using hits inside a time window
658 recorded before an event trigger [95].

659 3.1.3 Data Acquisition and Triggering

660 As the analysis presented in this thesis will only use the SK-IV period of the
661 SK experiment so this subsection focuses on the relevant points of the data
662 acquisition and triggering systems to that SK period. The earlier data acquisition
663 and triggering systems are documented in [96, 97].

664 Before the SK-IV period started, the existing front-end electronics were re-
665 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [98]. When
666 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-
667 time (QTC) converter generates a rectangular pulse. The start of the rectangular
668 pulse indicates the time at which the analog photoelectron signal was received
669 and the width of the pulse indicates the total charge integrated throughout the
670 signal. This is then digitized by time-to-digital converters and sent to the “front-
671 end” PCs. The digitized signal from every QBEE is then chronologically ordered
672 and sent to the “merger” PCs. It is the merger PCs that apply the software trigger.
673 Any triggered events are passed to the “organizer” PC. This sorts the data stream
674 of multiple merger PCs into chronologically ordered events, which are then saved
675 to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

676 The software trigger (described in [100]) operates by determining the number
677 of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the
678 maximum time that a Cherenkov photon would take to traverse the length of the
679 SK tank [97]. For lower energy events that generate fewer photons, this technique
680 is useful for eliminating background processes like dark noise and radioactive
681 decay which would be expected to be separated in time. When the value of
682 N_{200} exceeds some pre-defined threshold, a software trigger is issued. There are
683 several trigger thresholds used within the SK-IV period which are detailed in

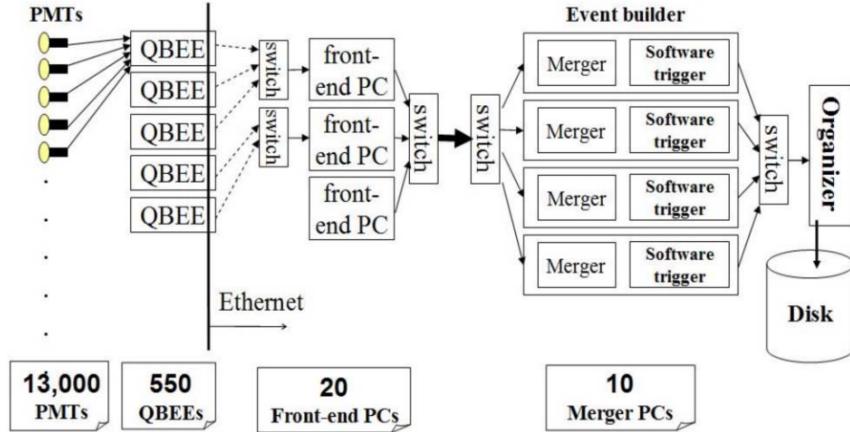


Figure 3.3: Schematic view of the data flow through the data acquisition and online system. Taken from [99].

684 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time
 685 window are also read out and saved to disk. In the special case of an event that
 686 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks
 687 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in
 688 a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband
 689 Intelligent Trigger (WIT)” has been deployed and is described in [101].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

690 3.1.4 Cherenkov Radiation

691 Cherenkov light is emitted from any highly energetic charged particle traveling
 692 with relativistic velocity, β , greater than the local speed of light in a medium [102].

693 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

694 Where n is the refractive index of the medium. Consequently, the Cherenkov
 695 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged
 696 particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}}. \quad (3.3)$$

697 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy
 698 for various particles are given in Table 3.3. In contrast, γ -rays are detected
 699 indirectly via the combination of photons generated by Compton scattering
 700 and pair production. The threshold for detection in the SK detector is typically
 701 higher than the threshold for photon production. This is due to the fact that the
 702 attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov
 703 photons reach the ID PMTs. Then the collection and quantum efficiencies
 704 described in subsection 3.1.1 result in the number of detected photons being
 705 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

706 The Frank-Tamm equation [103] describes the relationship between the num-
 707 ber of Cherenkov photons generated per unit length, dN/dx , the wavelength of
 708 the photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

709 where α is the fine structure constant. For a 100MeV momentum electron,
 710 approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq$
 711 700nm region which the ID PMTs are most sensitive to [82].

712 3.2 The Tokai to Kamioka Experiment

713 The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation
 714 experiment located in Japan. Proposed in the early 2000s [104, 105] to replace
 715 K2K [106], T2K was designed to observe electron neutrino appearance whilst
 716 precisely measuring the oscillation parameters associated with muon neutrino
 717 disappearance [107]. The experiment consists of a neutrino beam generated
 718 at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near
 719 detectors situated 280m from the beam target, and the Super Kamiokande far
 720 detector positioned at a 295km baseline. The cross-section view of the T2K
 721 experiment is drawn in Figure 3.4.

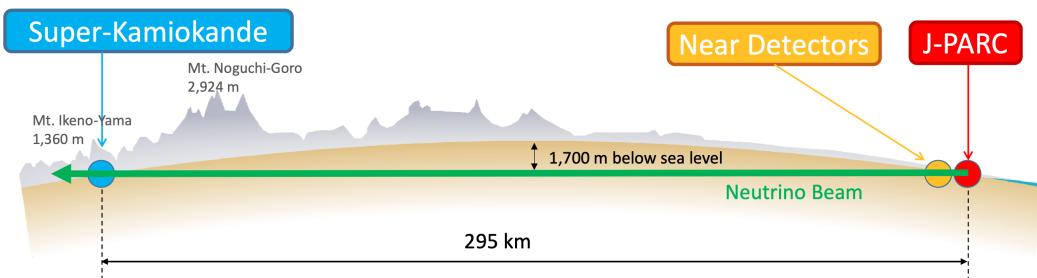


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

722 The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$,
 723 Δm_{32}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accu-
 724 racy of parameter estimates are still being made by including new data samples
 725 and developing the models which describe the neutrino interactions and detector
 726 responses [108]. Electron neutrino appearance was first observed at T2K in 2014
 727 [109] with 7.3σ significance.

728 The near detectors provide constraints on the beam flux and cross-section
 729 model parameters used within the oscillation analysis by observing the unoscil-
 730 lated neutrino beam. There are a host of detectors situated in the near detector hall
 731 (As illustrated in Figure 3.5): ND280 (subsection 3.2.3), INGRID (subsection 3.2.4),
 732 NINJA [110], WAGASCI [111], and Baby-MIND [112]. The latter three are not
 733 currently used within the oscillation analysis presented in this thesis.

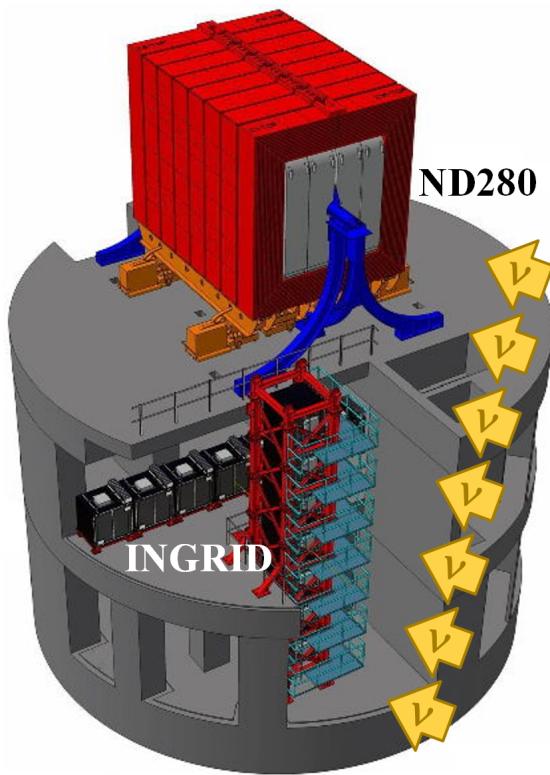


Figure 3.5: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

734 Whilst this thesis presents the ND280 in terms of its purpose for the oscillation
 735 analysis, the detector can also make many cross-section measurements at neutrino
 736 energies of $O(1)$ GeV for the different targets within the detector [113, 114]. These
 737 measurements are of equal importance as they can lead the way in determining
 738 the model parameters used in the interaction models for the future high-precision
 739 era of neutrino physics.

3.2.1 Analysis Overview

There are two independent fitters, MaCh3 and BANFF, which perform the near detector fit. MaCh3 uses a bayesian Markov Chain Monte Carlo fitting technique, whereas BANFF uses a frequentist gradient descent technique. The output of each fitter is compared as a method of cross-checking the behaviour of the two fitters. This is done by comparing: the Monte Carlo predictions using various tunes, the likelihood that is calculated in each fitter and the post-fit constraint associated with every parameter used in the fit. Once validated, the output converted into a covariance matrix to describe the error and correlations between all the flux and cross-section parameters. This is then propagated to the far-detector oscillation analysis group.

The far detector group has three independent fitters: P-Theta, VALOR and MaCh3. The first two fitters use a hybrid frequentist fitting technique where the likelihood is minimised with respect to the parameters of interest and marginalised over all other parameters. These fitters use the covariance provided by the near detector fitters as a basis for implementing the near detector constraints. The MaCh3 fitter uses a simultaneous fit of all near and far detector samples. This removes any Gaussian assumptions when making the covariance matrix from the near detector results. The results for all three fitters are compares using a technique similar to the validation of the near detector fitters.

There are three particular tunes of the T2K flux and low energy cross section model typically considered. Firstly, the “generated” tune which is the set of dial values with which the Monte Carlo was generated. Secondly, the set of dial values which are taken from external data measurements and used as inputs. These are the “pre-fit” dial values. The reason these two sets of dial values are different is that the external data measurements are continually updated but it is very computationally intensive to regenerate a Monte Carlo prediction after each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial values. These are the values taken from the constraints provided by the near detector.

3.2.2 The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [39, 115] and summarised below. The accelerator facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [39],

$$\begin{array}{ll}
\pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
\rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
\rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
\rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e.
\end{array}$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

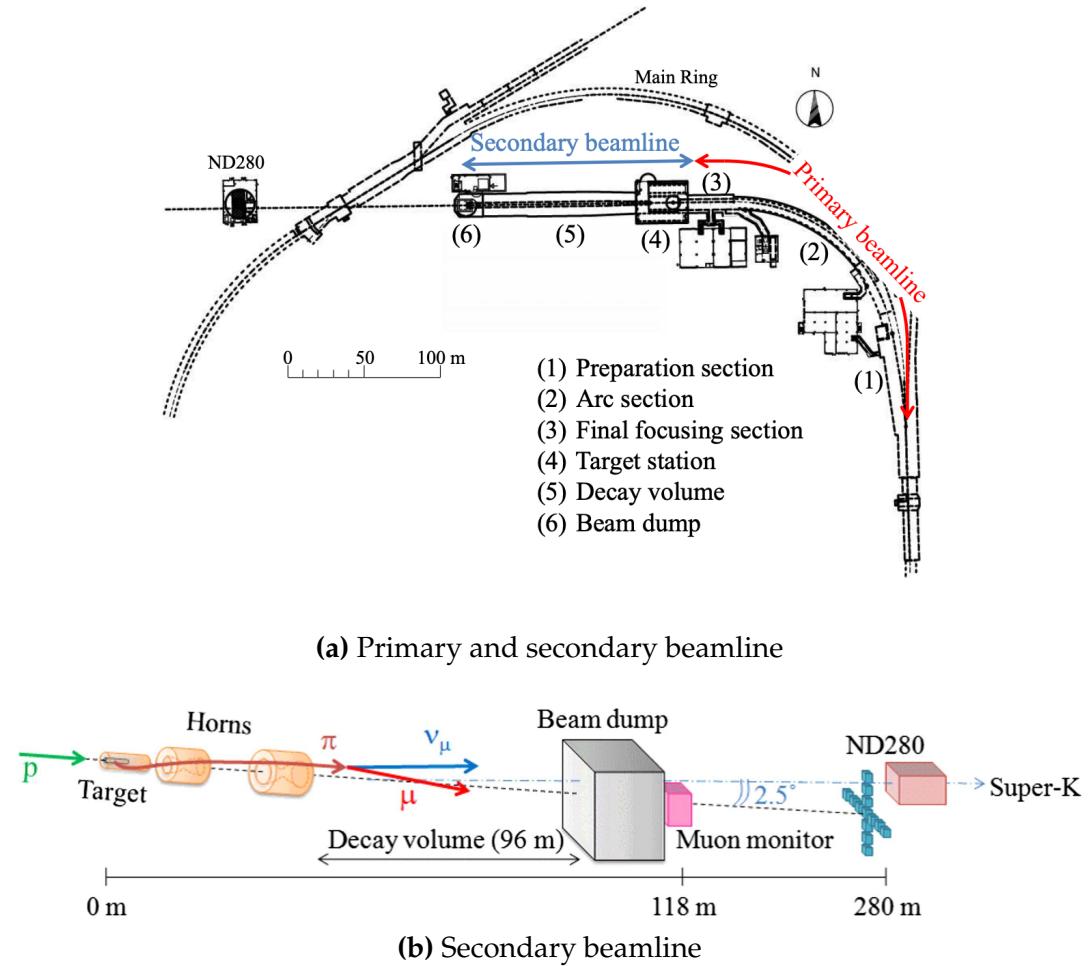


Figure 3.6: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [115]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [116].

795 Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino
 796 flux. The low energy flux is dominated by the decay of pions whereas kaon
 797 decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-
 798 sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic
 799 irreducible ν_e background, are predominantly due to muon decay for $E_\nu <$
 800 2GeV . As the antineutrino production cross-section is smaller than the neutrino
 801 cross-section, the wrong-sign component is more dominant in the RHC beam
 802 as compared to that in the FHC beam.

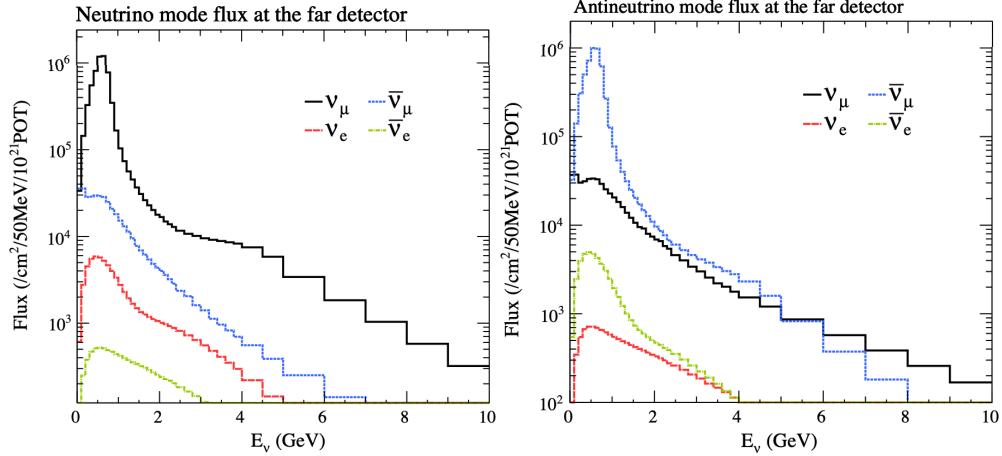


Figure 3.7: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [117].

803 The beam dump, situated at the end of the decay volume, stops all charged
 804 particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector
 805 monitors the penetrating muons to determine the beam direction and inten-
 806 sity which is used to constrain some of the beam flux systematics within the
 807 analysis [116, 118].

808 The T2K experiment uses an off-axis beam to narrow the neutrino energy
 809 distribution. This was the first implementation of this technique in a long-
 810 baseline neutrino oscillation experiment after its original proposal [119]. Pion
 811 decay, $\pi \rightarrow \mu + \nu_\mu$, is a two-body decay. Consequently, the neutrino energy,
 812 E_ν , can be determined based on the pion energy, E_π , and the angle at which
 813 the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

814 where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
 815 energy pion, the neutrino energy distribution is dependent upon the angle at
 816 which the neutrinos are observed from the initial pion beam direction. For the
 817 295km baseline at T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance
 818 probability, $P(\nu_\mu \rightarrow \nu_e)$, whilst minimising the muon disappearance probability,

819 $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8 illustrates the neutrino energy distribution for a range of
 820 off-axis angles, as well as the oscillation probabilities most relevant to T2K.

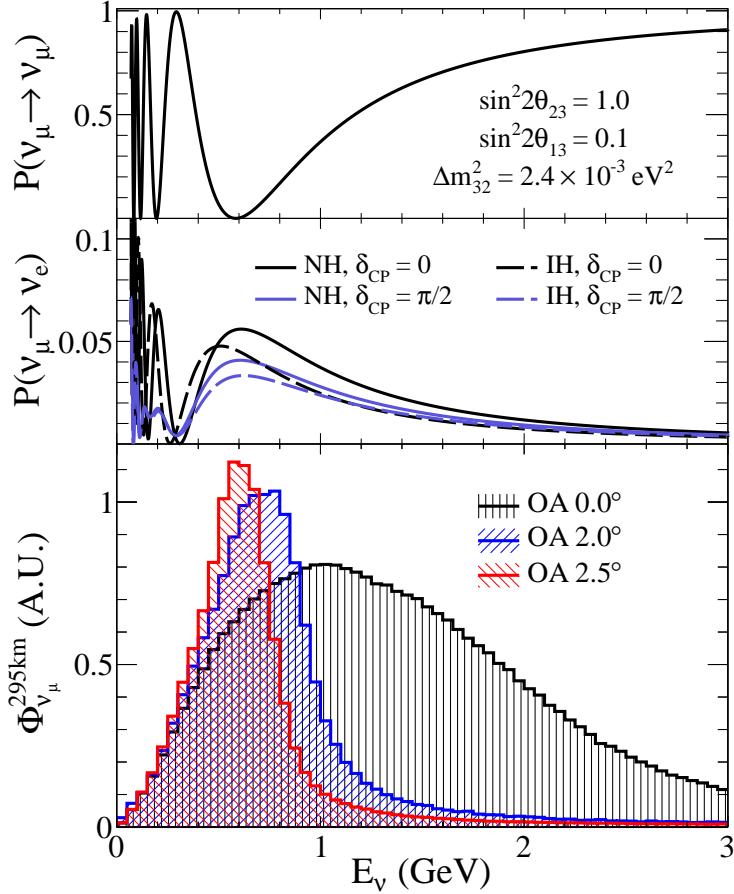


Figure 3.8: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

821 3.2.3 The Near Detector at 280m

822 Whilst all the near detectors are situated in the same “pit” located at 280m from
 823 the beamline, the “ND280” detector is the off-axis detector which is situated at
 824 the same off-axis angle as the Super-Kamiokande far detector. It has two primary
 825 functions; firstly it measures the neutrino flux and secondly, it counts the event
 826 rates of different types of neutrino interactions. Both of these constrain the flux
 827 and cross-section systematics invoked within the model for a more accurate
 828 prediction of the expected event rate at the far detector.

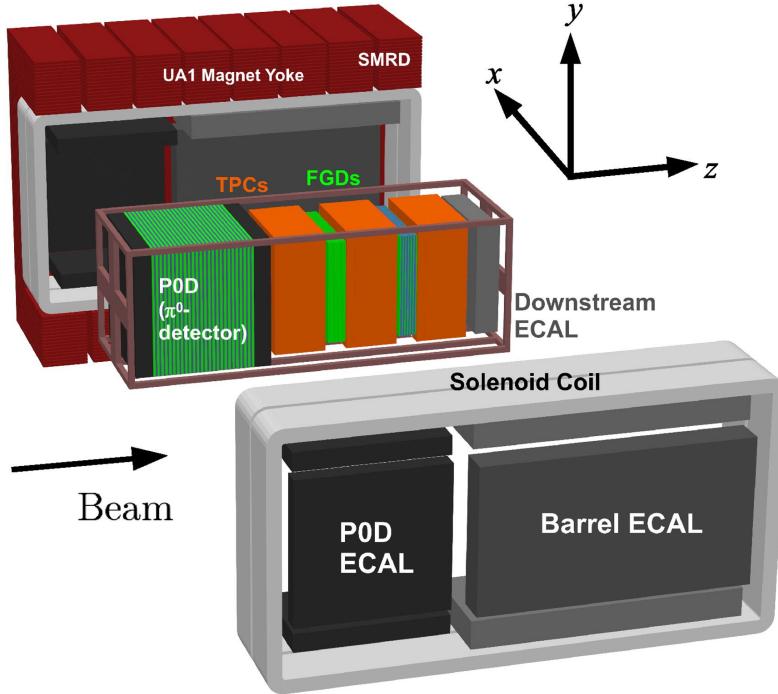


Figure 3.9: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [115].

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two-time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.3.1 and subsubsection 3.2.3.2 respectively. The electromagnetic calorimeter (ECAL) encapsulates the tracker region alongside the π^0 detector (P0D). The ECAL measures the deposited energy from photons emitted from interactions within the FGD. The P0D constrains the cross-section of neutral current interactions which generate neutral pions, which is one of the largest backgrounds in the electron neutrino appearance oscillation channel. The P0D and ECAL detectors are detailed in subsubsection 3.2.3.3 and subsubsection 3.2.3.4 respectively. The entire detector is located within a large yoke magnet which produces a 0.2T magnetic field.

845 field. This design of the magnet also includes a scintillating detector called the
846 side muon range detector (SMRD), which is used to track high-angle muons as
847 well as acting as a cosmic veto. The SMRD is described in subsubsection 3.2.3.5.

848 **3.2.3.1 Fine Grained Detectors**

849 The T2K tracker region is comprised of two fine-grained detectors (FGD) and
850 three Time Projection Chambers (TPC). A detailed description of the FGD design,
851 construction, and assembly is found in [120] and summarised below. The FGDS
852 are the primary target for neutrino interactions with a mass of 1.1 tonnes per FGD.
853 Alongside this, the FGDS are designed to be able to track short-range particles
854 which do not exit the FGD. Typically, short-range particles are low momentum
855 and are observed as tracks that deposit a large amount of energy per unit length.
856 This means the FGD needs good granularity to resolve these particles. The
857 FGDS have the best timing resolution ($\sim 3\text{ns}$) of any of the sub-detectors of the
858 ND280 detector. As such, the FGDS are used for time of flight measurements
859 to distinguish forward-going positively charged particles from backward-going
860 negatively charged particles. Finally, any tracks which pass through multiple
861 sub-detectors are required to be track matched to the FGD.

862 Both FGDS are made from square scintillator planes of side length 186cm and
863 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an
864 X or Y orientation. A wavelength-shifting fiber is threaded through the center
865 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is
866 the most upstream of the two FGDS and contains 15 planes of carbon plastic
867 scintillator which is a common target in external neutrino scattering data. As
868 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2
869 have been replaced with a hybrid water-scintillator target. Due to the complexity
870 of the nucleus, nuclear effects can not be extrapolated between different nuclei.
871 Therefore having the ability to take data on one target which is the same as
872 external data and another target which is the same as the far detector target is
873 beneficial for reliable model parameter estimates.

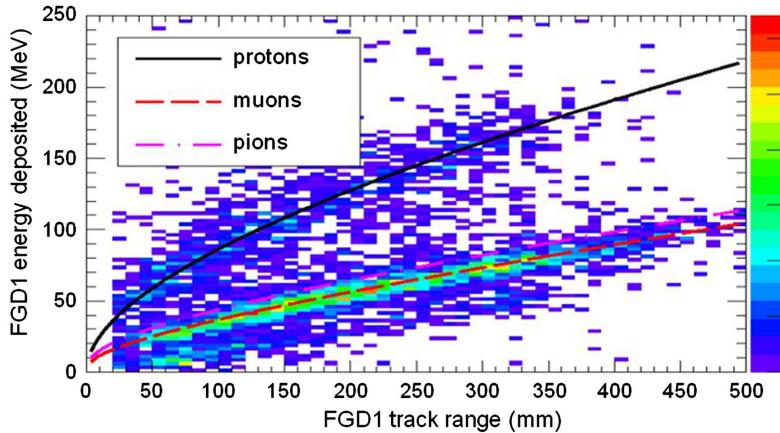


Figure 3.10: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [120].

874 The integrated deposited energy is used for particle identification. The FGD
875 can distinguish protons from other charged particles by comparing the integrated
876 deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

877 3.2.3.2 Time Projection Chambers

878 The majority of particle identification and momentum measurements within
879 ND280 are provided by three Time Projection Chambers (TPCs) [121]. The
880 TPCs are located on either side of the FGDs. They are located inside of the
881 magnetic field meaning the momentum of a charged particle can be determined
882 from the bending of the track.

883 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11,
884 which are made of non-magnetic material. The outer box is filled with CO₂ which
885 acts as an electrical insulator between the inner box and the ground. The inner box
886 forms the field cage which produces a uniform electric drift field of $\sim 275\text{V/cm}$
887 and is filled with an argon gas mixture. Charged particles moving through this
888 gas mixture ionize the gas and the ionised charge is drifted towards micromegas
889 detectors which measure the ionization charge. The time and position information
890 in the readout allows a three-dimensional image of the neutrino interaction.

891 The particle identification of tracks that pass through the TPCs is performed
892 using dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo



Figure 3.11: Schematic design of a Time Projection Chamber detector. Taken from [121].

893 distributions of the energy lost by a charged particle passing through the TPC as
 894 a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$
 895 meaning that electrons and muons can be distinguished. This allows reliable
 896 measurements of the intrinsic ν_e component of the beam.

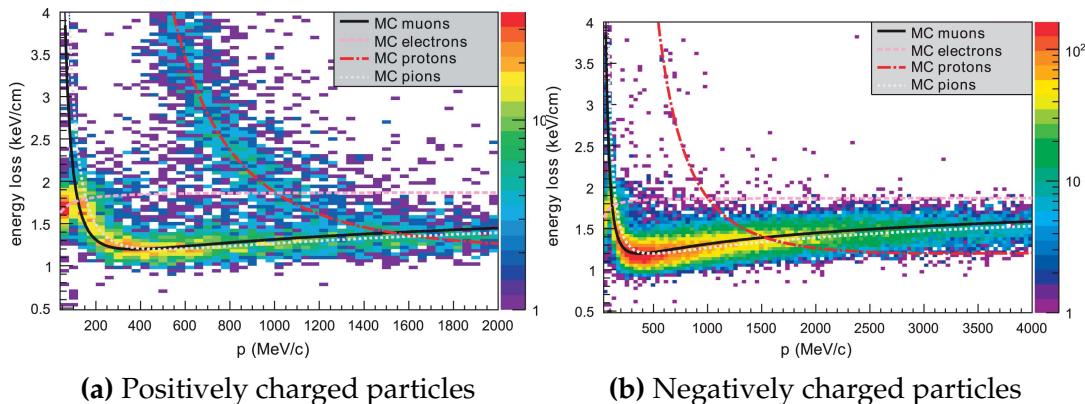


Figure 3.12: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [121].

897 **3.2.3.3 π^0 Detector**

898 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the
 899 reconstruction will determine that event to be a charge current ν_e -like event.
 900 This is one of the main backgrounds hindering the electron neutrino appearance

901 searches. The π^0 detector (P0D) measures the cross-section of the neutral current
 902 induced neutral pion production on a water target to constrain this background.

903 The P0D is a cube of approximately 2.5m length consisting of layers of scin-
 904 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.13.
 905 Two electromagnetic calorimeters are positioned at the most upstream and most
 906 downstream position in the sub-detector and the water target is situated in
 907 between them. The scintillator layers are built from two triangular bars orientated
 908 in opposite directions to form a rectangular layer. Each triangular scintillator bar
 909 is threaded with optical fiber which is read out by MPPCs. The high-Z brass and
 910 lead regions produce electron showers from the photons emitted in π^0 decay.

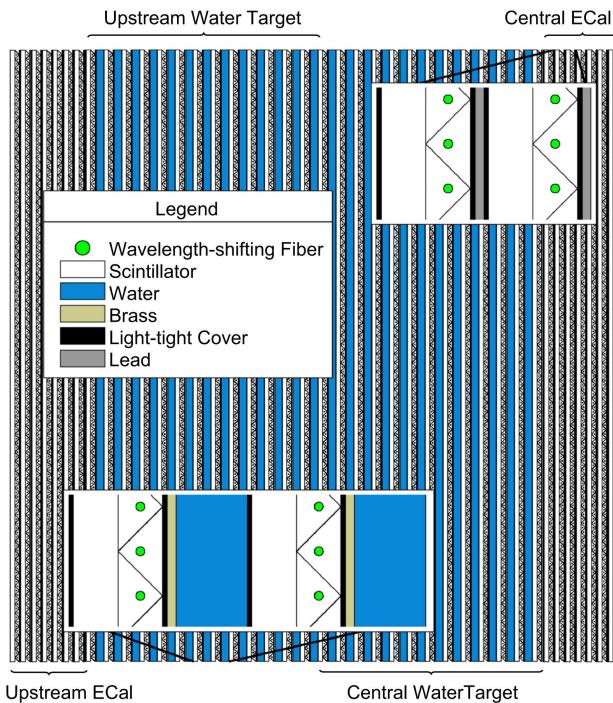


Figure 3.13: A schematic of the P0D side-view. Taken from [122].

911 The sub-detector can generate measurements of NC1 π^0 cross-sections on a
 912 water target by measuring the event rate both with and without the water target,
 913 with the cross-section on a water target being determined as the difference. The to-
 914 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

915 **3.2.3.4 Electromagnetic Calorimeter**

916 The electromagnetic calorimeter [123] (ECal) encapsulates the P0D and tracking
 917 sub-detectors. Its primary purpose is to aid π^0 reconstruction from any interac-
 918 tion in the tracker. To do this, it measures the energy and direction of photon
 919 showers from $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks
 920 depending on the shape of the photon shower deposited.

921 The ECal is comprised of three sections; the P0D ECal which surrounds the
 922 P0D, the barrel ECal which encompasses the tracking region, and the downstream
 923 ECal which is situated downstream of the tracker region. The barrel and down-
 924 stream ECals are tracking calorimeters that focus on electromagnetic showers
 925 from high-angle particles emitted from the tracking sub-detectors. Particularly in
 926 the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis)
 927 can travel along a single scintillator bar resulting in very few hits. The width of
 928 the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths
 929 to ensure a significant amount of the π^0 energy is contained. As the P0D has
 930 its own calorimetry which reconstructs showers, the P0D ECal determines the
 931 energy which escapes the P0D.

932 Each ECal is constructed of multiple layers of scintillating bars sandwiched
 933 between lead sheets. The scintillating bars are threaded with optical fiber and read
 934 out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular
 935 to the previous which allows a three-dimensional event reconstruction. The
 936 target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80,
 937 and 6.62 tonnes respectively.

938 **3.2.3.5 Side Muon Range Detector**

939 As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within
 940 the UA1 magnet. Reconditioned after use in the UA1 [124] and NOMAD [125]
 941 experiments, this magnet provides a uniform horizontal magnetic field of 0.2T
 942 with an uncertainty of 2×10^{-4} T.

943 Built into the UA1 magnet, the side muon range detector (SMRD)[126] monitors
944 high-energy muons which leave the tracking region and permeate through
945 the ECal. It additionally acts as a cosmic muon veto and trigger.

946 **3.2.4 The Interactive Neutrino GRID**

947 The Interactive Neutrino GRID (INGRID) detector is situated within the same
948 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”
949 position and measures the beam direction, spread, and intensity. The detector
950 was originally designed with 16 identical modules [115] (two modules have since
951 been decommissioned) and a “proton” module. The design of the detector is 14
952 modules oriented in a cross with length and height $10\text{m} \times 10\text{m}$, as illustrated
953 in Figure 3.14.

954 Each module is composed of iron sheets interlaced with eleven tracking
955 scintillator planes for a total target mass of 7.1 tonnes per module. The scintillator
956 design is an X-Y pattern of 24 bars in both orientations, where each bar contains
957 wave-length shifting fibers which are connected to multi-pixel photon counters
958 (MPPCs). Each module is encapsulated inside veto planes to aid the rejection
959 of charged particles entering the module.

960 The proton module is different from the other modules in that it consists
961 of entirely scintillator planes with no iron target. The scintillator bars are also
962 smaller than those used in the other modules to increase the granularity of
963 the detector and improve tracking capabilities. The module sits in the center
964 of the beamline and is designed to give precise measurements of quasi-elastic
965 charged current interactions to evaluate the performance of the Monte Carlo
966 simulation of the beamline.

967 The INGRID detector can measure the beam direction to an uncertainty of
968 0.4mrad and the beam centre within a resolution of 10cm [115]. The beam
969 direction in both the vertical and horizontal directions is discussed in [127]
970 and it is found to be in good agreement with the MUMON monitor described
971 in subsection 3.2.2.

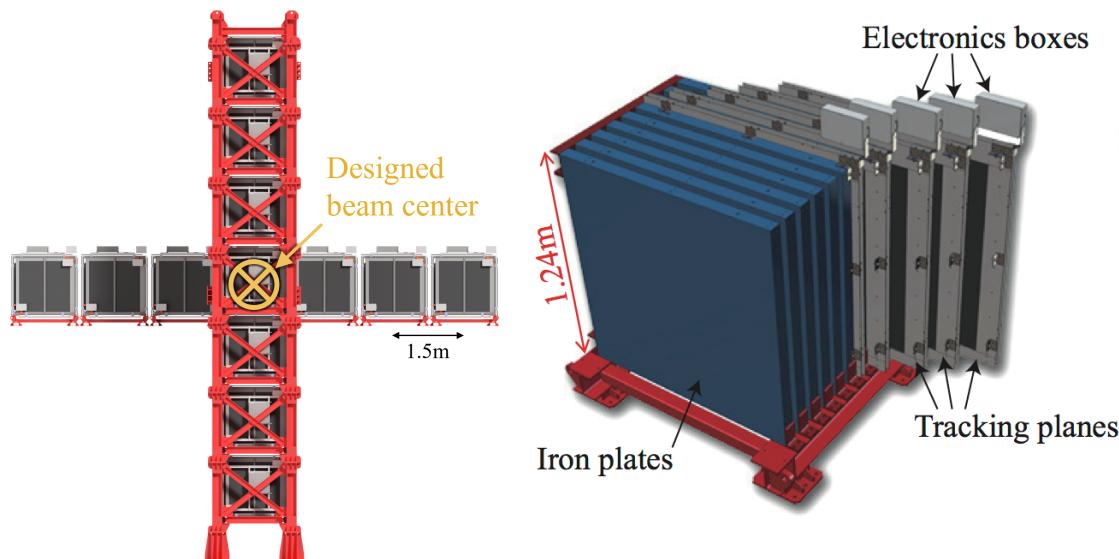


Figure 3.14: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [115].

4

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974

Bayesian Statistics and Markov Chain Monte Carlo Techniques

975 This thesis presents a Bayesian oscillation analysis. To extract the oscillation
976 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter
977 explains the theory of how parameter estimates can be determined using this
978 technique and condenses the material found in the literature [128–131].

979 The oscillation parameter determination presented here is built upon a si-
980 multaneous fit to neutrino beam data in the near detector, beam data at SK, and
981 atmospheric data at SK. In total, there are four oscillation parameters of interest
982 ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP}), two oscillation parameters to which this
983 study will not be sensitive ($\sin^2(\theta_{12})$, Δm_{21}^2) and many nuisance parameters that
984 control the systematic uncertainty models.

985 This analysis uses a Monte Carlo technique to generate a multi-dimensional
986 probability distribution across all of the model parameters used in the fit. To
987 determine an estimate for each parameter, this multi-dimensional object is in-
988 tegrated over all other parameters. This process is called Marginalisation and
989 is described in subsection 4.3.1. Monte Carlo techniques approximate the prob-
990 ability distribution of each parameter within the limit of generating infinite
991 samples. As ever, generating a large number of samples is time and resource-

992 dependent. Therefore, an MCMC technique is utilised within this analysis to
993 reduce the required number of steps to sufficiently sample the parameter space.
994 This technique is described in further detail in subsection 4.2.1.

995 The Bayesian analysis techniques used within this thesis are built within the
996 MaCh3 framework [132]. This uses a custom MCMC library package exclusively
997 supported and developed by the MaCh3 collaborators (which includes the author
998 of this thesis).

999 4.1 Bayesian Statistics

1000 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
1001 footing such that a probability model of both data and parameters is required.
1002 This is the joint probability distribution $P(D, \vec{\theta})$ and can be described by the
1003 prior distribution for model parameters $P(\vec{\theta})$ and the likelihood of the data given
1004 the model parameters $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

1005 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the
1006 parameters within the model. For example, if the risk of developing health
1007 problems is known to increase with age, the prior distribution would describe the
1008 increase. For the purpose of this analysis, the prior distribution is typically
1009 the best-fit values taken from external data measurements with a Gaussian
1010 uncertainty. The prior distribution can also contain correlations between model
1011 parameters. In an analysis using Monte Carlo techniques, the likelihood of
1012 measuring some data assuming some set of model parameters is calculated
1013 by comparing the Monte Carlo prediction generated at that particular set of
1014 model parameters to the data.

1015 It is parameter estimation that is important for this analysis and as such, Bayes'
1016 theorem [133] is applied to calculate the probability for each parameter to have a

1017 certain value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior
1018 distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

1019 The denominator in Equation 4.2 is the integral of the joint probability distri-
1020 bution over all values of all parameters used within the fit. For brevity, the
1021 posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

1022 For the purposes of this analysis, it is acceptable to neglect the normalisation
1023 term and focus on this proportional relationship.

1024 4.1.1 Application of Prior Knowledge

1025 The posterior distribution is proportional to the prior uncertainty applied to
1026 each parameter, as illustrated by Equation 4.3. This means that it is possible
1027 to change the prior after the posterior distribution has been determined. The
1028 prior uncertainty of a particular parameter can be ‘divided’ out of the posterior
1029 distribution and the resulting distribution can be reweighted using the new
1030 prior uncertainty that is to be applied. The methodology and implementation
1031 of changing the prior follows that described in [134].

1032 An example implementation that is useful for this analysis is the application
1033 of the “reactor constraint”. As discussed in section 2.4, an external constraint
1034 on $\sin^2(\theta_{13})$ is determined from measurements taken from reactor experiments.
1035 However, the sensitivities from just using the T2K and SK samples is equally
1036 as important. Without this technique, two fits would have to be run, doubling
1037 the required resources. Therefore, the key benefit for this analysis is the fact that
1038 only a single ‘fit’ has to be performed and can be used to build the two posterior
1039 distributions of the with and without reactor constraint applied.

1040 4.2 Monte Carlo Simulation

1041 Monte Carlo techniques are used to numerically solve a complex problem that
1042 does not necessarily have an analytical solution. These techniques rely on
1043 building a large ensemble of samples from an unknown distribution and then
1044 using the ensemble to approximate the properties of the distribution.

1045 An example that uses Monte Carlo techniques is to calculate the area under-
1046 neath a curve. For example, take the problem of calculating the area under a
1047 straight line with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can
1048 calculate the area under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte
1049 Carlo techniques, one can calculate the area under this line by throwing many
1050 random values for the x and y components of each sample and then calculating
1051 whether that point falls below the line. The area can then be calculated by the
1052 ratio of points below the line to the total number of samples thrown multiplied by
1053 the total area in which samples were scattered. The study is shown in Figure 4.1
1054 highlights this technique and finds the area under the curve to be 29.9 compared
1055 to an analytical solution of 30.0. The deviation of the numerical to analytical
1056 solution can be attributed to the number of samples used in the study. The
1057 accuracy of the approximation in which the properties of the Monte Carlo samples
1058 replicate those of the desired distribution is dependent on the number of samples
1059 used. Replicating this study with a differing number of Monte Carlo samples
1060 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo
1061 techniques are only accurate within the limit of a high number of samples.

1062 Whilst the above example has an analytical solution, these techniques are just
1063 as applicable to complex solutions. Clearly, any numerical solution is only as
1064 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is
1065 dependent upon the number of samples generated to approximate the properties
1066 of the distribution. Furthermore, if the positions at which the samples are
1067 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique
1068 significantly drops. Given the example in Figure 4.1, if the region in which the

1069 samples are scattered significantly extends passed the region of interest, many
1070 calculations will be calculated but do not add to the ability of the Monte Carlo
1071 technique to achieve the correct result. For instance, any sample evaluated at
1072 a $y \geq 5$ could be removed without affecting the final result. This does bring in
1073 an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,
1074 one needs to know the distribution beforehand.

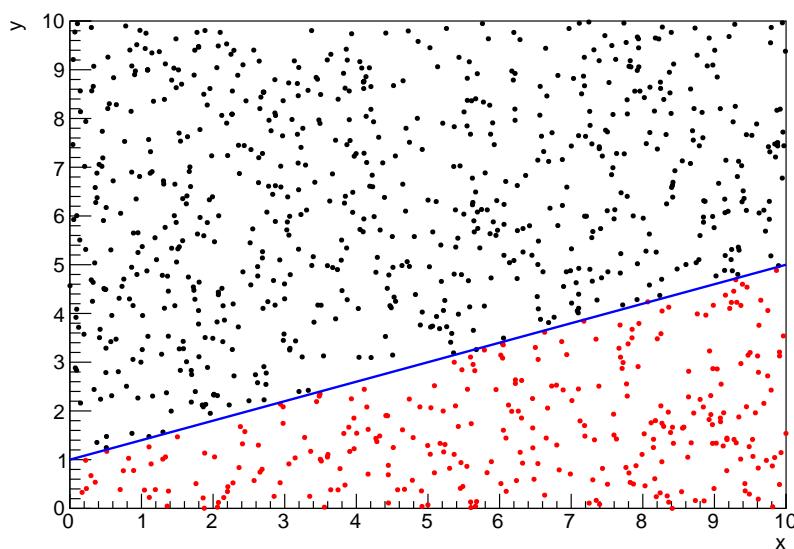


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

1075 4.2.1 Markov Chain Monte Carlo

1076 This analysis utilises a multi-dimensional probability distribution, with some
1077 dimensions being significantly more constrained than others. These constraints
1078 can be from prior knowledge of parameter distributions from external data or
1079 un-physical regions in which parameters can not exist. To maximise the efficiency
1080 of building the posterior distribution, a Markov Chain Monte Carlo (MCMC)
1081 technique is used. This employs a Markov chain to select the points at which
1082 to sample the posterior distribution. It performs a semi-random stochastic walk
1083 through the allowable parameter space. This builds a posterior distribution



Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1084 which has the property that the density of sampled points is proportional to the
 1085 probability density of that parameter. This means that the samples produced by
 1086 this technique are not statistically independent but they will cover the space
 1087 of the distribution.

1088 A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the
 1089 position of \vec{x}_i . The space in which the Markov chain selects samples is dependent
 1090 upon the total number of parameters utilised within the fit, where a discrete point
 1091 in this space is described by the N-dimensional space \vec{x} . In a perfectly operating
 1092 Markov chain, the position of the next step depends solely on the previous step
 1093 and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving
 1094 the multi-dimensionality of the fit used within this analysis, each step becomes
 1095 correlated with several of the steps preceding itself. Providing the MCMC chain is
 1096 well optimised, it will begin to converge towards a unique stationary distribution.
 1097 The period between the chain's initial starting point and the convergence to the
 1098 unique stationary distribution is colloquially known as the burn-in period. Once
 1099 the chain reaches the stationary distribution, all points sampled after that point

1100 will look like samples from that distribution.

1101 Further details of the theories underpinning MCMC techniques are discussed
1102 in [129] but can be summarised by the requirement that the chain satisfies the
1103 three ‘regularity conditions’:

- 1104 • Irreducibility: From every position in the parameter space \vec{x} , there must
1105 exist a non-zero probability for every other position in the parameter space
1106 to be reached.
- 1107 • Recurrence: Once the chain arrives at the stationary distribution, every step
1108 following from that position must be samples from the same stationary
1109 distribution.
- 1110 • Aperiodicity: The chain must not repeat the same sequence of steps at any
1111 point throughout the sampling period.

1112 The output of the chain after burn-in (i.e. the sampled points after the chain
1113 has reached the stationary distribution) can be used to approximate the posterior
1114 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1115 stationary distribution found by the chain be the posterior distribution, one
1116 can use the Metropolis-Hastings algorithm. This guides the stochastic process
1117 depending on the likelihood of the current proposed step compared to that
1118 of the previous step.

1119 4.2.2 Metropolis-Hastings Algorithm

1120 As a requirement for MCMCs, the Markov chain implemented in this technique
1121 must have a unique stationary distribution that is equivalent to the posterior
1122 distribution. To ensure this requirement and that the regularity conditions are
1123 met, this analysis utilises the Metropolis-Hastings (MH) algorithm [135, 136].
1124 For the i^{th} step in the chain, the MH algorithm determines the position in the
1125 parameter space to which the chain moves to based on the current step, \vec{x}_i , and
1126 the proposed step, \vec{y}_{i+1} . The proposed step is randomly selected from some

1127 proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely on the current step (ie. not
1128 the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the
1129 current step or the proposed step determined by whether the proposed step is
1130 accepted or rejected. To decide if the proposed step is selected, the acceptance
1131 probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)}\right). \quad (4.4)$$

1132 Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To
1133 simplify this calculation, the proposal function is required to be symmetric such
1134 that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution is
1135 used to throw parameter proposals. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)}\right). \quad (4.5)$$

1136 DB: Figure out what Giles means

1137 After calculating this quantity, a random number, β , is generated uniformly
1138 between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise,
1139 the chain sets the next step equal to the current step. This procedure is repeated
1140 for subsequent steps. This can be interpreted as if the posterior probability
1141 of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq$
1142 $P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true,
1143 ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability
1144 $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped
1145 in any local minima in the potentially non-Gaussian posterior distribution. The
1146 outcome of this technique is that the density of steps taken in a discrete region
1147 is directly proportional to the probability density in that region.

1148 4.2.3 MCMC Optimisation

1149 As discussed in subsection 4.2.2, the proposal function invoked within the MH
1150 algorithm can take any form and the chain will still converge to the stationary
1151 distribution. At each set of proposed parameter values, a prediction of the same

1152 spectra has to be generated which requires significant computational resources.
1153 Therefore, the number of steps taken before the unique stationary distribution
1154 is found should be minimised as only steps after convergence add information
1155 to the oscillation analysis. Furthermore, the chain should entirely cover the
1156 allowable parameter space to ensure that all values have been considered. Tuning
1157 the distance that the proposal function jumps between steps on a parameter-by-
1158 parameter basis can both minimise the length of the burn-in period and ensure
1159 that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently small.

1160 The effect of changing the width of the proposal function is highlighted in
1161 Figure 4.3. Three scenarios, each with the same underlying stationary distribution
1162 (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between
1163 the three scenarios is the width of the proposal function, colloquially known as
1164 the ‘step size σ ’. Each scenario starts at an initial parameter value of 10.0 which
1165 would be considered an extreme variation. For the case where $\sigma = 0.1$, it is
1166 clear to see that the chain takes a long time to reach the expected region of the
1167 parameter. This indicates that this chain would have a large burn-in period and
1168 does not converge to the stationary distribution until step ~ 500 . Furthermore,
1169 whilst the chain does move towards the expected region, each step is significantly
1170 correlated with the previous. Considering the case where $\sigma = 5.0$, the chain
1171 approaches the expected parameter region almost instantly meaning that the
1172 burn-in period is not significant. However, there are clearly large regions of steps
1173 where the chain does not move. This is likely due to the chain proposing steps
1174 in the tails of the distribution which have a low probability of being accepted.
1175 Consequently, this chain would take a significant number of steps to fully span
1176 the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can
1177 see a relatively small burn-in period of approximately 100 steps. Once the chain
1178 reaches the stationary distribution, it moves throughout the expected region of
1179 parameter values many times, sufficiently sampling the full parameter region.
1180 This example is a single parameter varying across a continuous distribution and
1181 does not fully reflect the difficulties in the many-hundred multi-variate parameter

1182 distribution used within this analysis. However, it does give a conceptual idea of
1183 the importance of selecting the proposal function and associated step size.

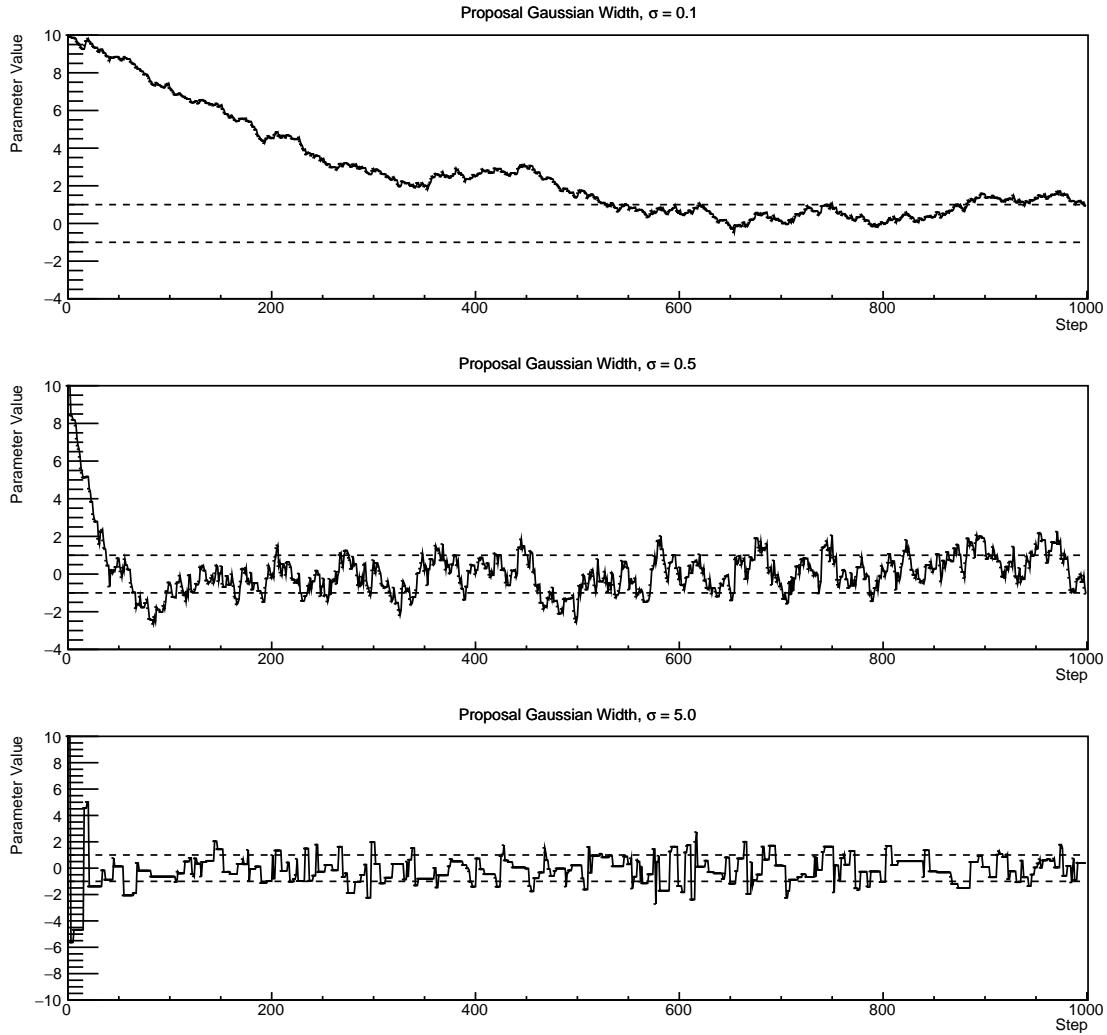


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1184 As discussed, step size tuning directly correlates to the average step accep-
1185 tance rate. If the step size is too small, many steps will be accepted but the
1186 chain moves slowly. If the opposite is true, many steps will be rejected as the
1187 chain proposes steps in the tails of the distribution. Discussion in [137] suggests
1188 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be

¹¹⁸⁹ approximately $\sim 25\%$. An “ideal” step size [137] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

¹¹⁹⁰ where N_p is the number of parameters included in the MCMC fit. However,
¹¹⁹¹ the complex correlations between systematics mean that some parameters have
¹¹⁹² to be hand-tuned and many efforts have been taken to select a set of parameter-
¹¹⁹³ by-parameter step sizes to approximately reach the ideal acceptance rate.

¹¹⁹⁴ Figure 4.4 highlights the likelihood as calculated by the fit in DB: [Link to](#)
¹¹⁹⁵ [AsimovA Sensitivity Section](#) as a function of the number of steps in each chain. In
¹¹⁹⁶ practice, many independent MCMC chains are run simultaneously to parallelise
¹¹⁹⁷ the task of performing the fit. This figure overlays the distribution found in each
¹¹⁹⁸ chain. As seen, the likelihood decreases from its initial value and converges
¹¹⁹⁹ towards a stationary distribution after $\sim 1 \times 10^5$ steps.

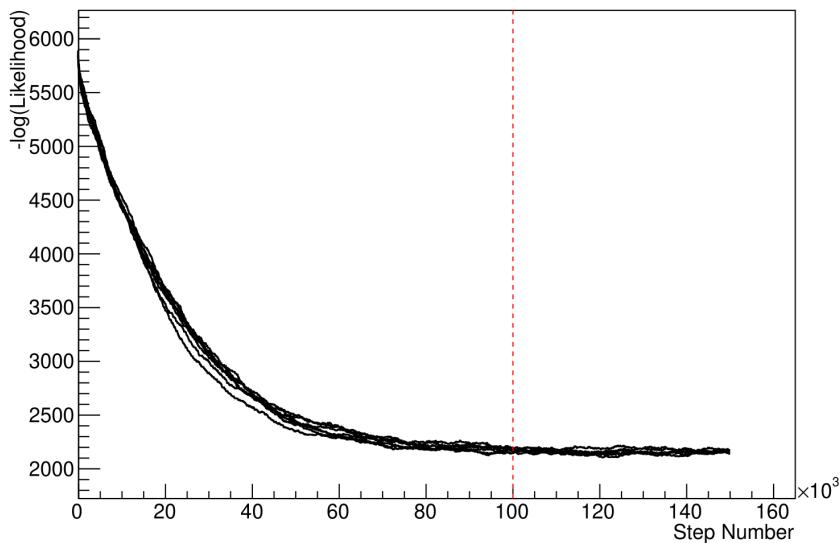


Figure 4.4: The log-likelihood from the fit detailed in DB: [Link to AsimovA Sensitivity Section](#) as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

¹²⁰⁰ Multiple configurations of this analysis have been performed throughout this
¹²⁰¹ thesis where different samples or systematics have been used. For all of these con-
¹²⁰² figurations, it was found that a burnin period of 1×10^5 was sufficient in all cases.

1203 4.3 Understanding the MCMC Results

1204 The previous sections have described how to generate the posterior probability
1205 distribution using Bayesian MCMC techniques. However, this analysis focuses
1206 on oscillation parameter determination. The posterior distribution output from
1207 the chain is a high-dimension object, with as many dimensions as there are
1208 parameters included in the oscillation analysis. However, this multi-dimensional
1209 object is difficult to conceptualize so parameter estimations are often presented
1210 in one or two-dimensional projections of this probability distribution. To do
1211 this, marginalisation techniques are invoked.

1212 4.3.1 Marginalisation

1213 The output of the MCMC chain is a highly dimensional probability distribution
1214 which is very difficult to interpret. From the standpoint of an oscillation analysis
1215 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters
1216 of interest are most relevant. Despite this, the best fit values and uncertainties on
1217 the oscillation parameters of interest should correctly encapsulate the correlations
1218 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).
1219 For this joint beam and atmospheric analysis, the oscillation parameters of
1220 interest are $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . All other parameters (includ-
1221 ing the oscillation parameters this fit is insensitive to) are deemed nuisance
1222 parameters. To generate these projections, the posterior distribution is integrated
1223 over all nuisance parameters. This is called marginalisation. This technique
1224 also explains why it is acceptable to neglect the normalisation constant of the
1225 posterior distribution, which was discussed in section 4.1.

1226 A simple example of the marginalisation technique is to imagine the scenario
1227 where two coins are flipped. To determine the probability that the first coin
1228 returned a ‘head’, the exact result of the second coin flip is disregarded and
1229 simply integrated over. For the parameters of interest, $\vec{\theta}_i$, the marginalised

₁₂₃₀ posterior is calculated by integrating over the nuisance parameters, $\vec{\theta}_n$. In this
₁₂₃₁ case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.7)$$

₁₂₃₂ Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and
₁₂₃₃ correlations between the parameters of interest and the nuisance parameters.
₁₂₃₄ In practice, this is simply taking the one or two-dimensional projection of the
₁₂₃₅ multi-dimensional probability distribution.

₁₂₃₆ While in principle an easy solution to a complex problem, correlations be-
₁₂₃₇ tween the interesting and nuisance parameters can bias the marginalised results.
₁₂₃₈ A similar effect is found when the parameters being marginalised over have
₁₂₃₉ non-Gaussian probability distributions. For example, Figure 4.5 highlights the
₁₂₄₀ marginalisation bias in the probability distribution found for a parameter when
₁₂₄₁ requiring a correlated parameter to have a positive parameter value. Due to
₁₂₄₂ the complex nature of the oscillation parameter fit presented in this thesis, there
₁₂₄₃ are correlations occurring between the oscillation parameters of interest and the
₁₂₄₄ other nuisance parameters included in the fit.

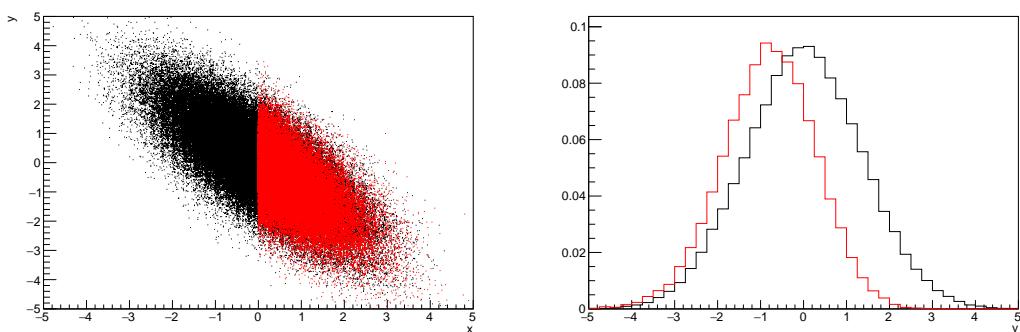


Figure 4.5: Left: The two-dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two-dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

1245 4.3.2 Parameter Estimation and Credible Intervals

1246 The purpose of this analysis is to determine the best fit values for the oscillation
 1247 parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$,
 1248 $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . The posterior probability density, taken from the output
 1249 MCMC chain, is binned in these parameters. The parameter best-fit point is then
 1250 taken to be the value that has the highest posterior probability. This is performed
 1251 in both one and two-dimensional projections.

1252 However, the single best-fit point in a given parameter is not of much use on its
 1253 own. The uncertainty on the best-fit point must also be presented using credible
 1254 intervals. The definition of the 1σ credible interval is that there is 68% belief
 1255 that the parameter is within those bounds. For a more generalised definition,
 1256 the credible interval is the region, R , of the posterior distribution that contains
 1257 a specific fraction of the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha. \quad (4.8)$$

1258 Where θ is the parameter being evaluated. This technique then calculates
 1259 the $\alpha \times 100\%$ credible interval.

1260 In practice, this analysis uses the highest posterior density (HPD) credible in-
 1261 tervals which are calculated through the following method. First, the probability
 1262 distribution is area-normalised such that it has an integrated area equal to 1.0.
 1263 The bins of probability are then summed from the highest to lowest until the sum
 1264 exceeds the 1σ level (0.68 in this example). This process is repeated for a range of
 1265 credible intervals, notably the 1σ , 2σ and 3σ along with other levels where the
 1266 critical values for each level can be found in [75]. This process can be repeated
 1267 for the two-dimensional probability distributions by creating two-dimensional
 1268 contours of credible intervals rather than a one-dimensional result.

1269 4.3.3 Bayesian Model Comparisons

1270 Due to the matter resonance, this analysis has some sensitivity to the mass
 1271 hierarchy of neutrino states (whether Δm_{32}^2 is positive or negative) and the
 1272 octant of $\sin^2(\theta_{23})$. The Bayesian approach utilised within this analysis gives an
 1273 intuitive method of model comparison by determining which hypothesis is most
 1274 favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal
 1275 hierarchy, NH , and inverted hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

1276 The middle term defines the Bayes factor, $B(NH/IH)$, which is a data-driven
 1277 interpretation of how strong the data prefers one hierarchy to the other. For this
 1278 analysis, equal priors on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) =$
 1279 $P(\vec{\theta}_{IH}) = 0.5$). In practice, the MCMC chain proposes a value of $|\Delta m_{32}^2|$ and
 1280 then applies a 50% probability that the value is sign flipped. Consequently,
 1281 the Bayes factor can be calculated from the ratio of the probability density in
 1282 either hypothesis. This equates to counting the number of steps taken in the
 1283 normal and inverted hierarchies and taking the ratio. The same approach can be
 1284 taken to compare the upper octant (UO) compared to the lower octant (LO)
 1285 hypothesis of $\sin^2(\theta_{23})$.

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [138]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

1286 Whilst the value of the Bayes factor should always be shown, the Jeffreys scale
 1287 [138] (highlighted in Table 4.1) gives an indication of the strength of preference

1288 for one model compared to the other. Other interpretations of the strength of
1289 preference of a model exist, e.g. the Kass and Raferty Scale [139].

1290 4.3.4 Comparison of MCMC Output to Expectation

1291 To ensure the fit is performing well, a best-fit spectrum is produced using the
1292 posterior probability distribution and compared with the data, allowing easy
1293 by-eye comparisons to be made. A simple method of doing this is to perform a
1294 comparison in the fitting parameters (For instance, the reconstructed neutrino
1295 energy and lepton direction for T2K far detector beam samples) of the spectra
1296 generated by the MCMC chain to ‘data’. This ‘data’ could be true data or some
1297 variation of Monte Carlo prediction. This allows easy comparison of the MCMC
1298 probability distribution to the data. To perform this, N steps from the post-burnin
1299 MCMC chain are randomly selected. From these, the Monte Carlo prediction
1300 at each step is generated by reweighting the model parameters to the values
1301 specified at that step. Due to the probability density being directly correlated
1302 with the density of steps in a certain region, parameter values close to the best
1303 fit value are most likely to be selected.

1304 In practice, for each bin of the fitting parameters has a probability distribution
1305 of event rates, with one entry per sampled MCMC step. This distribution is
1306 binned where the bin with the highest probability is selected as the mean and an
1307 error on the width of this probability distribution is calculated using the approach
1308 highlighted in subsection 4.3.2. Consequently, the best fit distribution in the fit
1309 parameter is not necessarily that which would be attained by reweighting the
1310 Monte Carlo prediction to the most probable parameter values.

1311 A similar study can be performed to illustrate the freedom of the model
1312 parameter space prior to the fit. This can be done by throwing parameter values
1313 from the prior uncertainty of each parameter.

5

1314

1315

1316

Simulation, Reconstruction, and Event Reduction

1317 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1318 neutrino spectrum at the far detector is required. This includes modeling the
1319 flux generation, neutrino interactions, and detector effects. All of the simulation
1320 packages required to do this are briefly described in section 5.1. The reconstruc-
1321 tion of neutrino events in the far detector, including the `fitQun` algorithm, is
1322 documented in section 5.2. This also includes data quality checks of the SK-V
1323 data which the author performed for the T2K oscillation analysis presented at the
1324 Neutrino 2020 conference [74]. Finally, section 5.3 describes the steps taken in the
1325 SK detector to trigger on events of interest whilst removing the comparatively
1326 large rate of cosmic ray muon events.

1327 5.1 Simulation

1328 In order to generate a Monte Carlo prediction of the expected event rate at
1329 the far detector, all the processes in the beam and atmospheric fluxes, neutrino
1330 interaction, and detector need to be modeled.

1331 5.1.1 Neutrino Flux

1332 The beamline simulation consists of three distinct parts: the initial hadron
1333 interaction modeled by FLUKA [140], the target station geometry and particle
1334 tracking performed by JNUBEAM, [141, 142] and any hadronic re-interactions
1335 simulated by GCALOR [143]. The primary hadronic interactions are $O(10)\text{GeV}$,
1336 where FLUKA matches external cross-section data better than GCALOR [144].
1337 However, FLUKA is not very adaptable so a small simulation is built to model
1338 the interactions in the target and the output is then passed to JNUBEAM and
1339 GCALOR for propagation. The hadronic interactions are tuned to data from the
1340 NA61/SHINE [145–147] and HARP [148] experiments. The tuning is done by
1341 reweighting the FLUKA and GCALOR predictions to match the external data
1342 multiplicity and cross-section measurements, based on final state particle kine-
1343 matics [144]. The culmination of this simulation package generates the predicted
1344 flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.7.

1345 The atmospheric neutrino flux is simulated by the HKKM model [50, 52]. The
1346 primary cosmic ray flux is tuned to AMS [149] and BESS [150] data assuming
1347 the US-standard atmosphere '76 [151] density profile and includes geomagnetic
1348 field effects. The primary cosmic rays interact to generate pions and muons.
1349 The interaction of these secondary particles to generate neutrinos is handled by
1350 DPMJET-III [152] for energies above 32GeV and JAM [52, 153] for energies below
1351 that value **DB: Question for Giles: Why different generators for above/below**
1352 **32GeV?**. These hadronic interactions are tuned to BESS and L3 data [154, 155]
1353 using the same methodology as the tuning of the beamline simulation. The
1354 energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3
1355 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked
1356 around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions
1357 and kaons can travel further than their vertically-going counterparts resulting
1358 in a larger probability of decaying to neutrinos. The symmetry is broken in
1359 lower-energy neutrinos due to geomagnetic effects, which modify the track of the
1360 primary cosmic rays. Updates to the HKKM model are currently ongoing [156].

5.1.2 Neutrino Interaction

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [157, 158] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a W^\pm boson in Charged Current (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. This is the reason why neutrinos that interact through NC modes are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events.

As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of neutrino interactions around $E_\nu \sim 0.5\text{GeV}$. The NEUT implementation adopts the Llewellyn Smith [159] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [160] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions is controlled by vector and axial-vector form factors parameterised by the BBBA05 [161] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data [162], respectively. NEUT implements the Valencia [163] model to simulate MEC events, where two nucleons and two holes in the nuclear target are produced (often called 2p2h interactions).

For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant. These predominantly produce charged and neutral pions although γ , kaon,



Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [157].

and η production is also considered. To simulate these interactions, the Berger-Sehgal [164] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [165] model to simulate these COH interactions.

DIS and multi- π producing interactions become the most dominant for energies $> O(5)\text{GeV}$. PYTHIA [166] is used to simulate any interaction with invariant mass $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces at least two mesons but has $W < 2\text{GeV}/c^2$, the

₁₄₀₁ Bronner model is used [167]. Both of these models use Parton distribution
₁₄₀₂ functions based on the Bodek-Yang model [168–170].

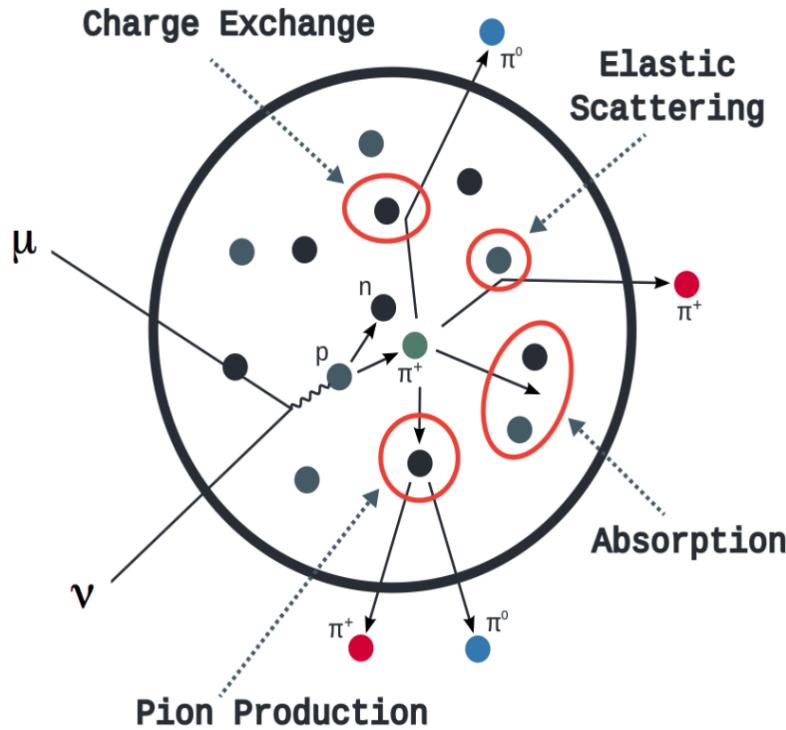


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [171].

₁₄₀₃ Any pion that is produced within the nucleus can re-interact through final
₁₄₀₄ state interactions before it exits, as illustrated by the scattering, absorption,
₁₄₀₅ production, and exchange interactions in Figure 5.2. These re-interactions alter
₁₄₀₆ the observable particles within the detector. For instance, if the charged pion
₁₄₀₇ from a CC PROD interaction is absorbed, the observables would mimic a CC QE
₁₄₀₈ interaction. To simulate these effects, NEUT uses a semi-classical intranuclear
₁₄₀₉ cascade model [157]. This cascade functions by stepping the pion through the
₁₄₁₀ nucleus in fixed-length steps equivalent to $dx = R_N/100$, where R_N is the radius
₁₄₁₁ of the nucleus. At each step, the simulation allows the pion to interact through
₁₄₁₂ scattering, charged exchange, absorption, or production with an interaction-
₁₄₁₃ dependent probability calculated from a fit to external data [172]. This cascade
₁₄₁₄ continues until the pion is absorbed or exits the nucleus.

1415 5.1.3 Detector

1416 Once the final state particle kinematics have been determined by NEUT, they
1417 are passed into the detector simulation. The near detectors, ND280 and INGRID,
1418 are simulated using a GEANT4 package [115, 173] to simulate the detector geom-
1419 etry, particle tracking, and energy deposition. The response of the detectors is
1420 simulated using the elecSim package [115]. The far detector simulation is based
1421 upon the original Kamiokande experiment software which uses the GEANT3-based
1422 SKDETSIM [115, 174] package. This simulates the interactions of particles in
1423 the water as well as Cherenkov light production. The water quality and PMT
1424 calibration measurements detailed in subsection 3.1.2 are also used within this
1425 simulation to make accurate predictions of the detector response.

1426 **DB: Make sure the rest of this section flows and is relevant to simulation**

1427 Any event which generates optical photons that occurs in SK will be ob-
1428 served by the PMT array, where each PMT records the time and accumulated
1429 charge. This recorded information is shown in event displays similar to those
1430 illustrated in Figure 5.3. To be useful for physics analyses, this series of PMT
1431 hit information needs to be reconstructed to determine the number and identity
1432 of particles and their kinematics (or track parameters): four-vertex, direction,
1433 and momentum. The reconstruction uses the fact that the charge and timing
1434 distribution of photons generated by a particular particle in an event is dependent
1435 upon its initial kinematics. Electron and muon rings are distinguished by their
1436 “fuzziness”. Muons are heavier and less affected by scattering or showering
1437 meaning they typically produce “crisp” rings. Electrons are more likely to interact
1438 via electromagnetic showering or scattering which results in larger variations
1439 of their direction from the initial direction. Consequently, electrons typically
1440 produce “fuzzier” rings compared to muons.

1441 5.2 Event Reconstruction at SK

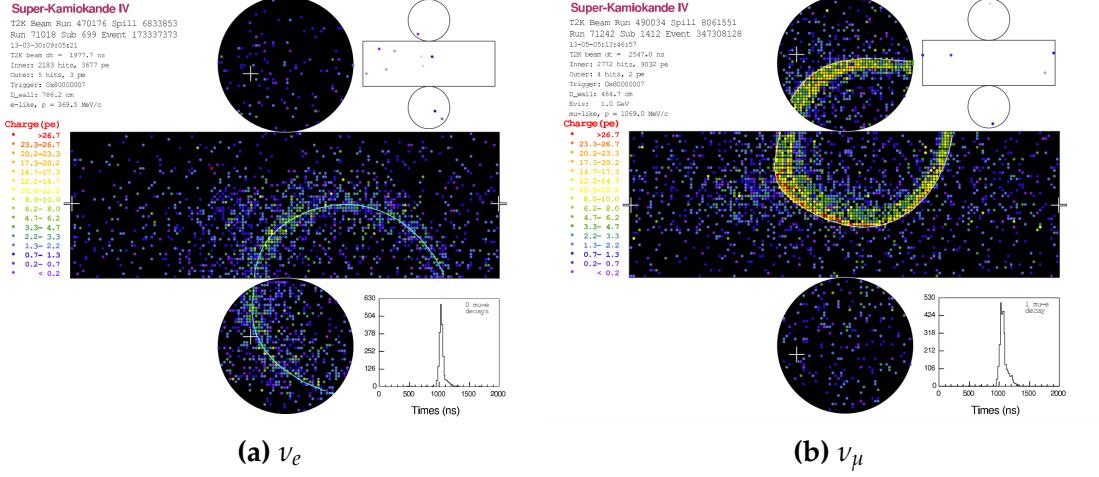


Figure 5.3: Event displays from Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzier” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [175].

DB: Better separate FiTQun and APFit from the below paragraph, and add a little more information about APFit - Hough transform For the purposes of this analysis, the `fitQun` reconstruction algorithm [176] is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by maximising a likelihood function (or minimising a log-likelihood function) which includes information from PMTs which were hit and those that were not hit. The `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [177]. It improves upon the previous `APFit` reconstruction algorithm which has been used for many previous SK analyses. `APFit` fits the vertex from timing information and then fits the momentum and direction of the particle from PMT hits within a 43 deg Cherenkov cone (assuming an ultra-relativistic particle). Due to the additional information being used in the fit, `fitQun` improves both the accuracy of the fit parameters and the rejection of neutral current π^0 events as compared to `APFit` [178, 179].

An event in SK can consist of prompt (or primary) and decay (or secondary) particles. For example, a charged current muon neutrino interaction can gen-

1460 erate two particles that have the potential of generating Cherenkov photons
 1461 (assuming the proton is below the Cherenkov threshold): the prompt muon,
 1462 and the secondary decay-electron from the muon, approximately $2\mu\text{s}$ later. To
 1463 reconstruct all particles within an event, it is divided into time clusters which are
 1464 called “subevents”. Subevents after the primary subevent are considered to
 1465 be decay electrons.

1466 The main steps of the `fitQun` reconstruction algorithm are:

- 1467 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit
 1468 metric based on PMT hit times
- 1469 • **Peak finding:** The initial time of each subevent is determined by clustering
 1470 events by time residuals
- 1471 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,
 1472 a maximum likelihood technique searches for a single particle generating
 1473 light. Electron, muon, charged pion, and proton hypotheses are considered
- 1474 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple
 1475 light-producing particles are considered using the same maximum likeli-
 1476 hood technique. Electron-like or charged pion-like rings are added until
 1477 the likelihood stops improving

1478 To find all the subevents in an event, a vertex goodness metric is calculated
 1479 for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right), \quad (5.1)$$

1480 where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n, \quad (5.2)$$

1481 is the residual hit time. It is the difference in time between the PMT hit time
 1482 t^i , of the i^{th} PMT, and the expected time of the PMT hit if the photon was at

1483 the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in water
 1484 and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1485 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends
 1486 to zero resulting in subevents appearing as spikes in the goodness metric. The
 1487 proposed fit vertex and time are grid-scanned, and the values which maximise
 1488 the goodness metric are selected as the “pre-fit vertex”. Whilst this predicts a
 1489 vertex for use in the clustering algorithm, the final vertex is fit using the higher-
 1490 precision maximum likelihood method described below.

1491 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1492 a function of t to determine the number of subevents. A peak-finding algorithm
 1493 is then used on the goodness metric, requiring the goodness metric to exceed
 1494 some threshold and drop below a reduced threshold before any subsequent
 1495 additional peaks are considered. The thresholds are set such that the rate of
 1496 false peak finding is minimised while still attaining good data to Monte Carlo
 1497 agreement. To improve performance, the pre-fit vertex for each delayed subevent
 1498 is re-calculated after PMT hits from the previous subevent are masked. This
 1499 improves the decay-electron tagging performance. Once all subevents have
 1500 been determined, the time window around each subevent is then defined by the
 1501 earliest and latest time which satisfies $-180 < T_{Res}^i < 800\text{ns}$. The subevents and
 1502 associated time windows are then used as seeds for further reconstruction.

1503 For a given subevent, the `fitQun` algorithm constructs a likelihood based on
 1504 the accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}). \quad (5.3)$$

1505 Where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and
 1506 momenta, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the proba-
 1507 bility of the i^{th} tube to not register a hit given the track parameters and particle
 1508 hypothesis. The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$,

represents the probability density function of observing charge q_i and time t_i on the i^{th} PMT given the specified track parameters and particle hypothesis.

The predicted charge is calculated based on contributions from both the direct light and the scattered light. The direct light contribution is determined based on the integration of the Cherenkov photon profile along the track. PMT angular acceptance, water quality, and calibration measurements discussed in subsection 3.1.2 are included to accurately predict the charge probability density at each PMT. The scattered and reflected light is calculated in a similar way, although it includes a scattering function that depends on the vertex of the particle and the position of the PMT. The charge likelihood is calculated by comparing the prediction to the observed charge in the PMT which is tuned to the PMT simulation.

The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} , and time t of the track as well as the particle hypothesis. The expected time for PMT hits is calculated by assuming unscattered photons being emitted from the midpoint of the track, S_{mid} ,

$$t_{exp}^i = t + S_{mid}/c + |R_{PMT}^i - \vec{x} - S_{mid}\vec{d}|/c_n, \quad (5.4)$$

where c is the speed of light in a vacuum. The time likelihood is then expressed in terms of the residual difference between the PMT hit time and the expected hit time, $t_{Res}^i = t^i - t_{exp}^i$. The particle hypothesis and momentum also affect the Cherenkov photon distribution. These parameters modify the shape of the time likelihood density since in reality not all photons are emitted at the midpoint of the track. As with the charge likelihood, the contributions from both the direct and scattered light to the time likelihood density are calculated separately, which are both calculated from particle gun Monte Carlo studies.

The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined as the best-fit parameters. In practice MINUIT [180] is used to minimise the value of $-\ln L(\Gamma, \vec{\theta})$. The `fitQun` algorithm considers an electron-like, muon-like, and charged pion-like hypothesis for events with a single final state particle, denoted

“single-ring events”. The particle’s identity is determined by taking the ratio of the likelihood of each of the hypotheses. For instance, electrons and muons are distinguished by considering the value of $\ln \left(L(e, \vec{\theta}_e) / L(\mu, \vec{\theta}_\mu) \right)$ in comparison to the reconstructed momentum of the electron hypothesis, as illustrated by Figure 5.4. The coefficients of the discriminator between electron-like and muon-like events are determined from Monte Carlo studies [176]. Similar distributions exist for distinguishing electron-like events from π^0 -like events, and muon-like events from pion-like events. The cuts are defined as,

$$\begin{aligned} \text{Electron/Muon} : & \ln(L_e / L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 : & \ln(L_e / L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} : & \ln(L_\mu / L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

as taken from [181], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed invariant mass of the two photons emitted from π^0 decay. Typically, the distance between a particular entry in these two-dimensional distributions and the cut-line is termed the PID parameter and is illustrated in Figure 5.5.

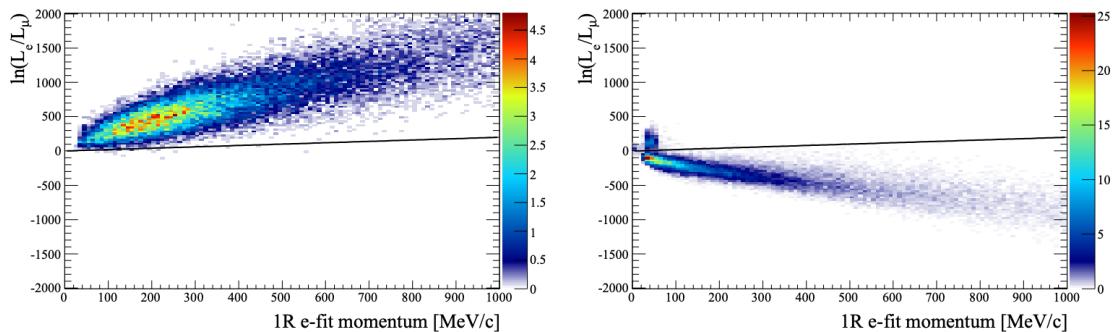


Figure 5.4: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, with coefficients obtained from Monte Carlo studies. Figures from [176].

The `fitQun` algorithm also considers a π^0 hypothesis. To do this, it performs a fit looking for two standard electron-hypothesis tracks which point to the

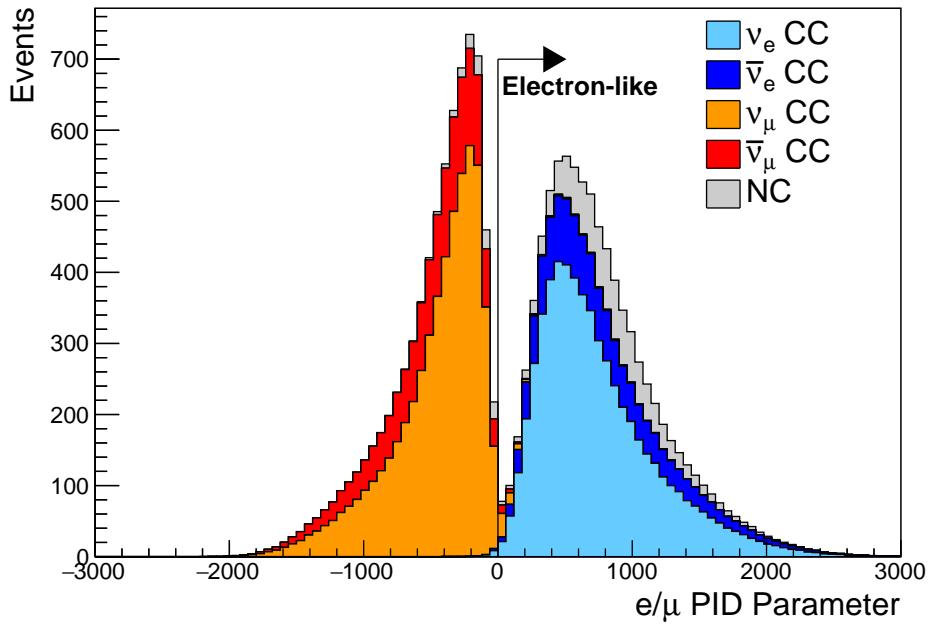


Figure 5.5: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1552 same four-vertex. This assumes the electron tracks are generated from photon-
 1553 conversion so the electron tracks actually appear offset from the proposed π^0
 1554 vertex. For these fits, the conversion length, direction, and momentum of each
 1555 photon are also considered as track parameters which are then fit in the same
 1556 methodology as the standard single-ring hypotheses.

1557 Whilst lower energy events are predominantly single-ring events, higher
 1558 energy neutrino events can generate final states with multiple particles which
 1559 generate Cherenkov photons. These “multi-ring” hypotheses are also considered
 1560 in the `fitQun` algorithm. When calculating the charge likelihood density, the
 1561 predicted charge associated with each ring is calculated separately and then
 1562 summed to calculate the total accumulated charge on each PMT. Similarly,
 1563 the time likelihood for the multi-ring hypothesis is calculated assuming each
 1564 ring is independent. Each track is time-ordered based on the time of flight
 1565 from the center of the track to the PMT and the direct light from any ring

1566 incident on the PMT is assumed to arrive before any scattered light. To reduce
1567 computational resource usage, the multi-ring fits only consider electron-like
1568 and charged pion-like rings as the pion fit can be used as a proxy for a muon
1569 fit due to their similar mass.

1570 **DB: Figure out how pion-like rings are different from the muon hypothesis**

1571 Multi-ring fits proceed by proposing another ring to the previous fit and
1572 then fitting the parameters in the method described above. Typically, multi-ring
1573 fits have the largest likelihood because of the additional degrees of freedom
1574 introduced. A likelihood value is calculated for the n -ring and $(n + 1)$ -ring
1575 hypotheses, where the additional ring is only included if the likelihood value
1576 is above 9.35, based on Monte Carlo studies in [182].

1577 5.2.1 Validation of Reconstruction in SK-V

1578 **DB: Link this into the previous section**

1579 As an example of how the reconstruction depends on the detector conditions,
1580 the author of this thesis assessed the quality of `fitQun` event reconstruction for
1581 SK-V data. The detector systematics used in the T2K-only oscillation analysis
1582 are determined using data-to-Monte Carlo comparisons of the SK-IV data [183].
1583 Due to tank-open maintenance occurring between SK-IV and SK-V, the dark
1584 rate of each PMT was observed to increase in SK-V due to light exposure for a
1585 significant time during the repairs. This increase can be seen in Figure 5.6. Run-10
1586 of the T2K experiment was conducted in the SK-V period, so the consistency
1587 of SK-IV and SK-V data needs to be studied to determine whether the SK-IV-
1588 defined systematics can be applied to the run-10 data. This comparison study was
1589 performed using the stopping muon data set for both the SK-IV and SK-V periods.
1590 This data sample is used due to the high rate of interactions ($O(200)$ events per
1591 hour) as well as having similar energies to muons from CCQE ν_μ interactions from
1592 beam interactions. The rate of cosmic muons does depend on the solar activity
1593 cycle [184] but has been neglected in this comparison study. This is because the
1594 shape of the distributions is most important for the purposes of being compared

1595 to the detector systematics. The SK-IV and SK-V data samples consist of 2398.42
1596 and 626.719 hours of data which equates to 686k and 192k events respectively.
1597 These samples do not correspond to the full data sets of either period but do
1598 contain enough events to be systematics limited rather than statistics limited.

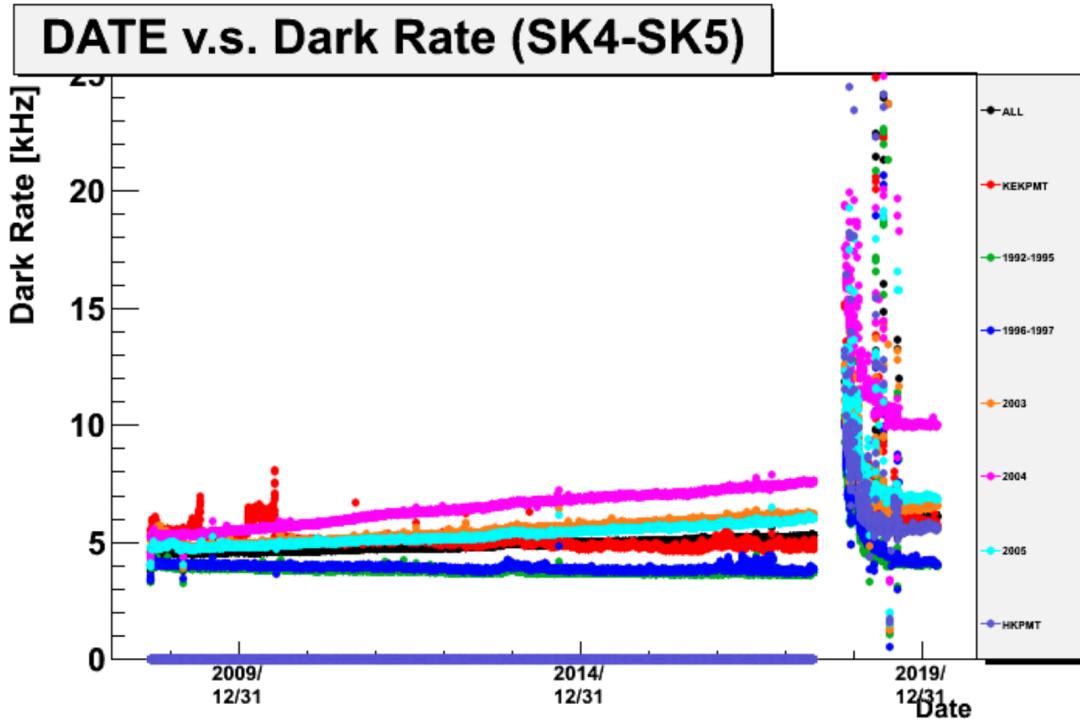


Figure 5.6: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [183].

1599 The predicted charge calculated in the `fitQun` algorithm includes a contribu-
1600 tion from the photoelectron emission due to dark noise. Therefore, the increase
1601 in the SK-V dark rate needs to be accounted for. In practice, the average dark rate
1602 in each SK period is calculated and used as an input in the reconstruction. This is
1603 calculated by averaging the dark rate per run for each period separately, using
1604 the calibration measurements detailed in subsection 3.1.2. The average dark rate
1605 from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The
1606 charges associated with the muon and decay electron subevents are illustrated in
1607 Figure 5.7. The photoelectron emission from dark noise is more significant for

events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in lower-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent is mostly unaffected by the increase in dark rate, whilst the charge associated with the decay-electron is clearly affected.

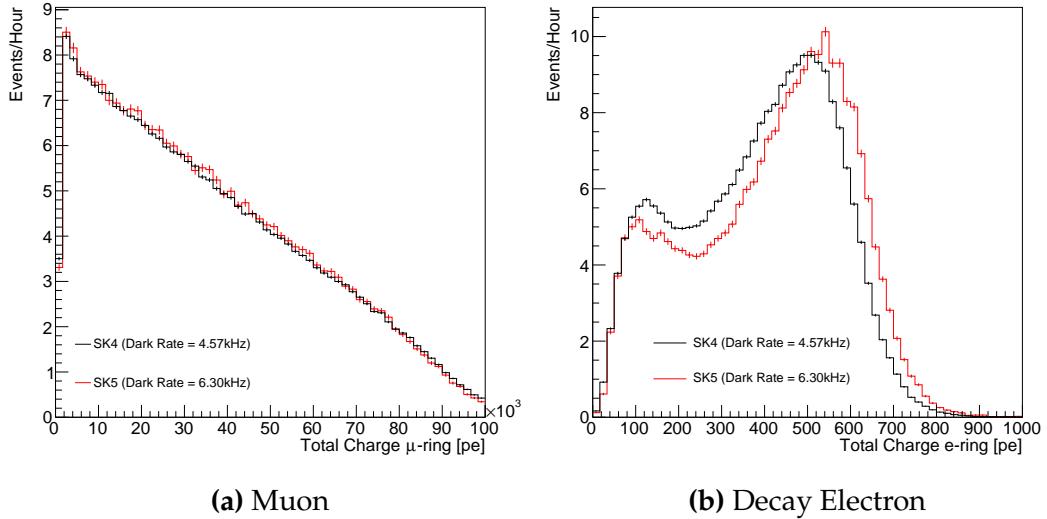


Figure 5.7: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent (left) and the associated decay electron subevent (right).

The energy scale systematic is estimated from data-to-Monte Carlo differences in the stopping muon sample in [185] and found to be 2.1%. To determine the consistency of SK-IV and SK-V with respect to the energy scale systematic, the muon momentum distribution is compared between the two SK periods. As the total number of Cherenkov photons is integrated across the track length, the reconstructed momentum divided by track length (or range) is compared between SK-IV and SK-V as illustrated in Figure 5.8.

The consistency between these muon distributions has been computed in two ways. Firstly, a Gaussian is fit to the peak of each distribution separately, whose mean is found to be $(2.272 \pm 0.003)\text{MeV/cm}$ and $(2.267 \pm 0.006)\text{MeV/cm}$ for SK-IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of the Gaussian fits are consistent with the expected stopping power of a minimum

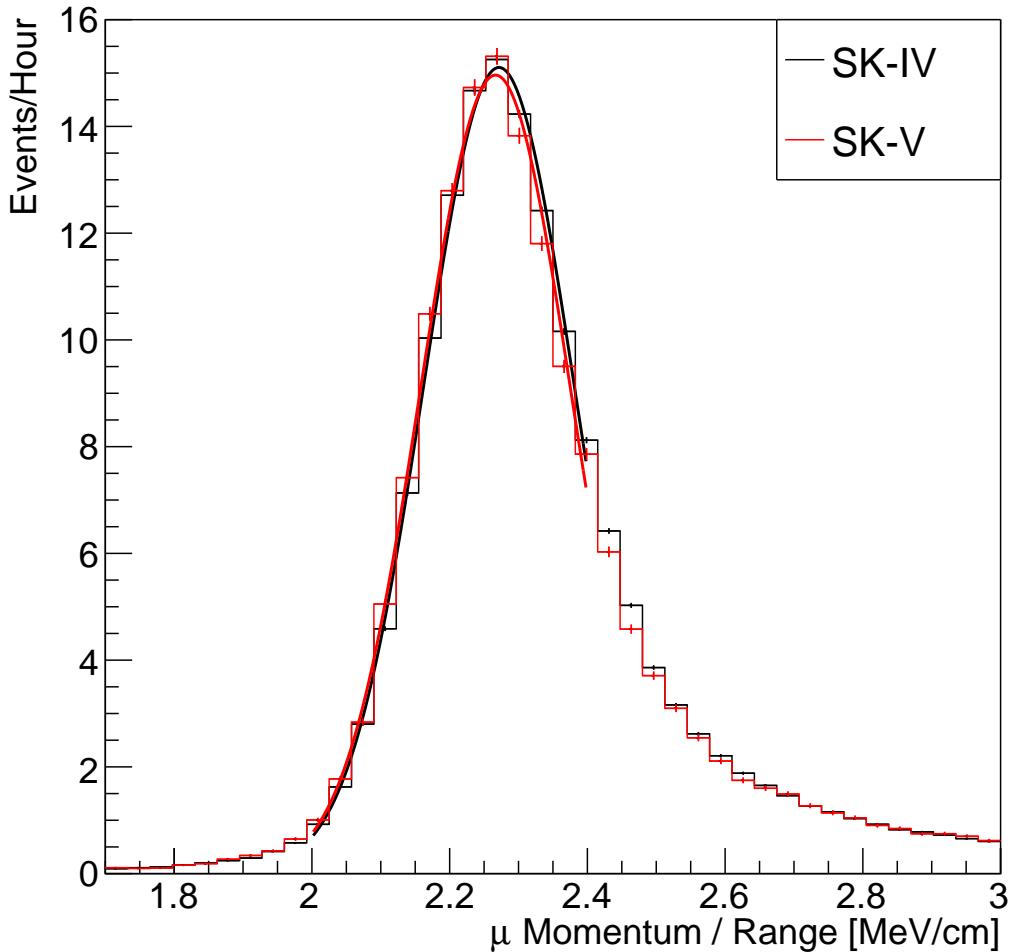


Figure 5.8: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-V (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range $[2.0, 2.4]$ MeV/cm and illustrated as the solid curve.

1625 ionising muon for a target material (water) with $Z/A \sim 0.5$ [186]. The second
 1626 consistency check is performed by introducing a nuisance parameter, α , which
 1627 modifies the SK-V distribution. The value of α which minimises the χ^2 value
 1628 between the SK-IV and SK-V is determined by scanning across a range of values.
 1629 This is repeated by applying the nuisance parameter as both a multiplicative
 1630 factor and an additive shift. The χ^2 distributions for different values of α is
 1631 illustrated in Figure 5.9. The values which minimise the χ^2 are found to be 0.0052

¹⁶³² and 1.0024 for the additive and multiplicative implementations, respectively. No
¹⁶³³ evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic
¹⁶³⁴ has been found in the reconstructed momentum distribution of SK-IV and SK-V.

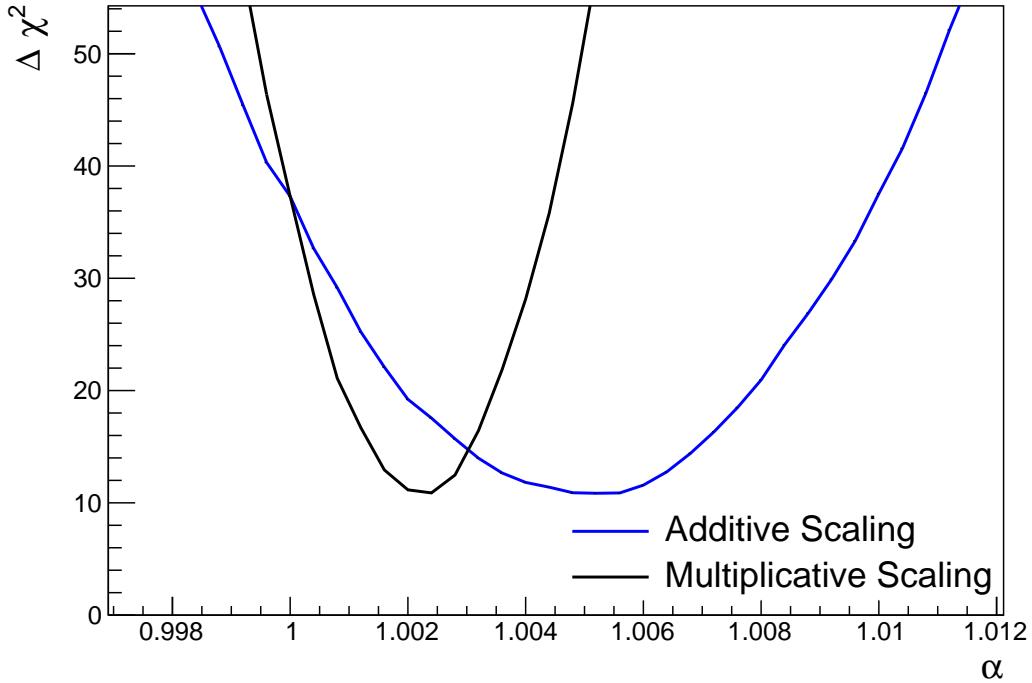


Figure 5.9: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

5.3 Event Reduction at SK

¹⁶³⁵ In normal data-taking operations, the SK detector observes many background
¹⁶³⁶ events alongside the beam and atmospheric neutrino signal events of physics
¹⁶³⁷ interest for this thesis. Cosmic ray muons and flasher events, which are the spon-
¹⁶³⁸ taneous discharge of a given PMT, contribute the largest amount of background
¹⁶³⁹ events in the energy range relevant to this thesis. Therefore the data recorded
¹⁶⁴⁰ is reduced with the aim of removing these background events. The reduction
¹⁶⁴¹ process is detailed in [54, 85] and briefly summarised below.
¹⁶⁴²

Atmospheric neutrino events observed in the SK detector are categorised into three different types of samples: fully contained (FC), partially contained (PC) and up-going muon (Up- μ), using PMT hit signatures in the inner and outer detector (ID and OD, respectively). To identify FC neutrino events, it is required that the neutrino interacts inside the fiducial volume of the ID and that no significant OD activity is observed. For this analysis, an event is defined to be in the fiducial volume provided the event vertex is at least 0.5m away from the ID walls. PC events have the same ID requirements but can have a larger signal present inside the OD. Typically, only high energy muons from ν_μ interactions can penetrate the ID wall. The Up- μ sample contains events where muons are created from neutrino interactions in the OD water or rock below the tank. They then propagate upwards through the detector. Downward-going muons generated from neutrino interactions above the tank are neglected because of the difficulty in separating their signature from the cosmic muon shower background. The sample categories are visually depicted in Figure 5.10.

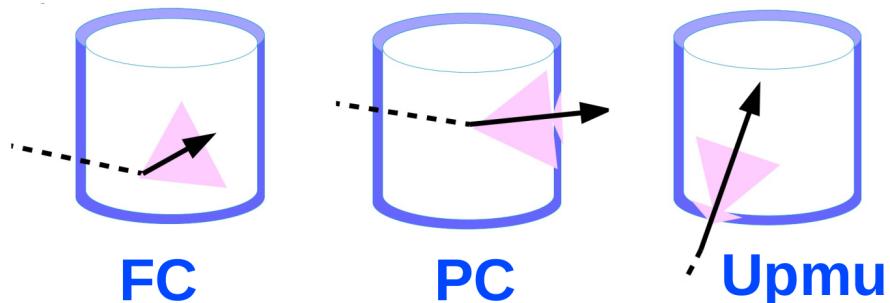


Figure 5.10: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC), and up-going muon (Up- μ) samples included in this analysis.

Based on the event characteristics, as defined by the `f1TQun` event reconstruction software, the FC events are categorised by

- **Visible Energy:** equal to the sum of the reconstructed kinetic energy of particles above the Cerenkov threshold for all rings present in the event. The purpose is to separate events into sub-GeV and multi-GeV categories.

- **Number of observed Cerenkov rings.** The purpose is to separate single-ring and multi-ring events, where single-ring events predominantly consist of quasi-elastic interactions and multi-ring events are typically resonant pion production or deep inelastic scattering events.
- **Particle identification parameter of the most energetic ring:** A value determined from the maximum likelihood value based on `fitQun`'s electron, muon, or pion hypothesis. The purpose is to separate electron-like and muon-like events.
- **Number of decay electrons:** The purpose is to separate quasi-elastic events (which have one decay electron emitted from the muon decay) and resonant pion production events (which have two decay electrons emitted from the muon and pion).

The PC and Up- μ categories are broken down into “through-going” and “stopping” samples depending on whether the muon leaves the detector. This is because the PC stopping events deposit the entire energy of the interaction into the detector, resulting in better reconstruction. The energy of events that exit the detector has to be estimated, with a typically worse resolution, which introduces much larger systematic uncertainties. Through-going Up- μ samples are further broken down by whether any hadronic showering was observed in the event which typically indicates DIS interactions. The expected neutrino energy for the different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events peak around 0.7GeV and 3GeV respectively, with slightly different peak energies for ν_e and ν_μ oscillation channels. PC and Up- μ are almost entirely comprised of ν_μ events and peak around 7GeV and 100GeV, respectively.

The first two steps in the FC reconstruction remove the majority of cosmic ray muons by requiring a significant amount of ID activity compared to that measured in the OD. Events that pass this cut are typically very high momentum muons or events that leave very little activity in the OD. Consequently, a third reduction step is then applied to select cosmic-ray muons that pass the initial

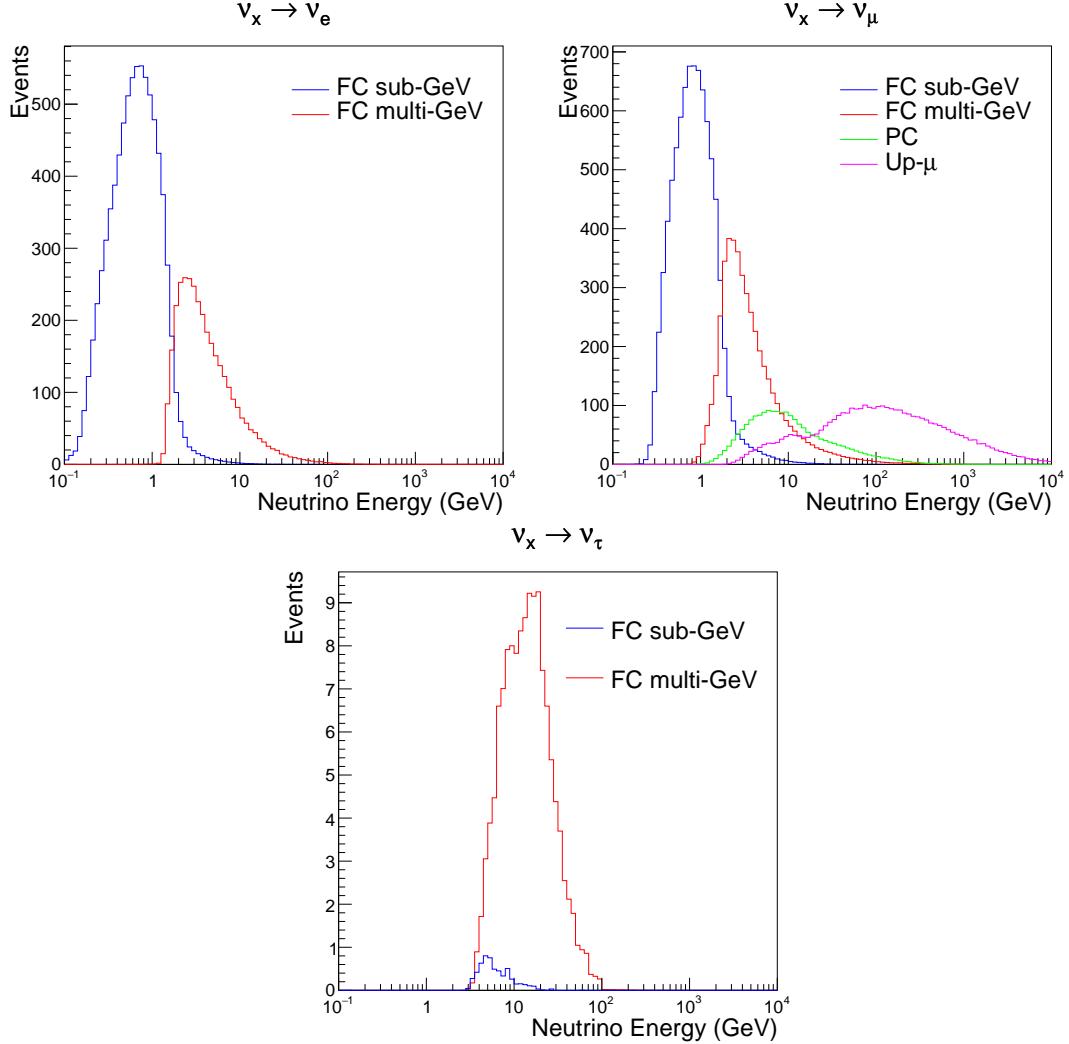


Figure 5.11: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). ν_x represents the flavours of neutrinos produced in the cosmic ray showers (electron and muon). Asimov A oscillation parameters are assumed (given in Table 2.2).

reduction step. A purpose-built cosmic muon fitter is used to determine the entrance (or exit) position of the muon and a cut is applied to OD activity contained within 8m of this position. Flasher events are removed in the fourth reduction step which is based on the close proximity of PMT hits surrounding the PMT producing the flash. Events that pass all these reduction steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses information from the more precise fitter to repeat the previous two steps with

1699 tighter cuts. Muons below the Cherenkov threshold can not generate optical
1700 photons in the ID but the associated decay electron can due to its lower mass.
1701 These are the types of events targeted in the fifth reduction step. The final
1702 cuts require the event vertex to be within the fiducial volume (0.5m from the
1703 wall although the nominal distance is 2.0m), visible energy $E_{vis} > 30\text{MeV}$ and
1704 fewer than 16 hits within the higher energy OD cluster. The culmination of
1705 the fully contained reduction results in 8.09 events/day in the nominal fiducial
1706 volume [81]. The uncertainty in the reconstruction is calculated by comparing
1707 Monte Carlo prediction to data. The largest discrepancy is found to be 1.3%
1708 in the fourth reduction step.

1709 The PC and Up- μ events are processed through their own reduction processes
1710 detailed in [54]. Both of these samples are reconstructed with the APFit algorithm
1711 rather than fiTQun. This is because the efficiency of reconstructing events that
1712 leave the detector has not been sufficiently studied for reliable systematic uncer-
1713 tainties with fiTQun. The PC and Up- μ samples acquire events at approximately
1714 0.66 and 1.44 events/day.

1715 Beam neutrinos events undergo the same reduction steps as FC events and are
1716 then subject to further cuts [187]. The GPS system that links the timing between
1717 the beam facility and SK needs to be operating correctly and there should be no
1718 activity within the detector in the previous $100\mu\text{s}$ before the trigger. The events
1719 then need to triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1720 The beam neutrino samples are not split by visible energy since their energy
1721 range is smaller than the atmospheric neutrino events. Following the T2K
1722 analysis in [74], only single-ring beam neutrino events are considered. Similar to
1723 atmospheric event selection, the number of decay electrons is used as a proxy for
1724 distinguishing CCQE and CCRES events. The expected neutrino energy, broken
1725 down by the number of decay electrons, is given in Figure 5.12.

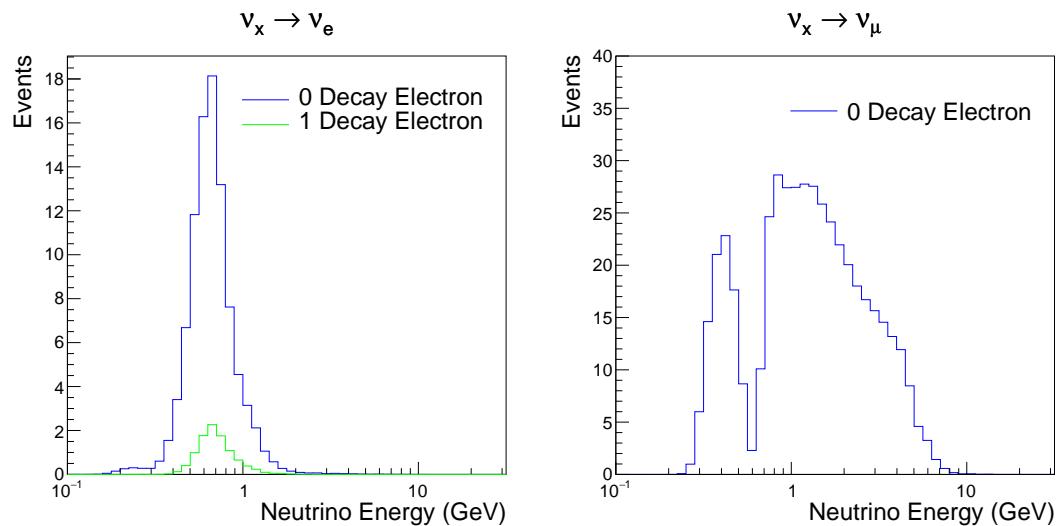


Figure 5.12: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

6

1726

1727

Sample Selections and Systematics

1728 The oscillation analysis presented within this thesis is built upon a simultaneous
1729 fit to atmospheric data at SK, neutrino beam data in the near detector, and
1730 beam data measured at SK. The definitions of these samples are documented
1731 in section 6.1, section 6.2, and section 6.3, respectively. The data collected and
1732 used within this analysis is detailed in Table 6.1. The near and far detector data
1733 corresponds to T2K runs 2-9 and runs 1-10, respectively. The accumulated POT
1734 and beam power for runs 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

1735 The difference in POT recorded at the near and far detector is due to the
1736 difference in downtime. The SK detector is very stable with almost 100% of
1737 data recorded during beam operation. Due to various technical and operational

¹⁷³⁸ issues, the downtime of the near detector is significantly higher due to its more
¹⁷³⁹ complex design and operating requirements.

¹⁷⁴⁰ The systematic parameters invoked within the flux, detector, and interaction
¹⁷⁴¹ models used within this analysis are documented in section 6.4. The standard
¹⁷⁴² configuration of the joint beam and atmospheric data fit utilises far detector sys-
¹⁷⁴³ tematics provided in the official inputs from the two experiments. Additionally,
¹⁷⁴⁴ a correlated detector model which fits the parameters used in sample selections
¹⁷⁴⁵ to data has been developed and documented in subsection 6.4.5.

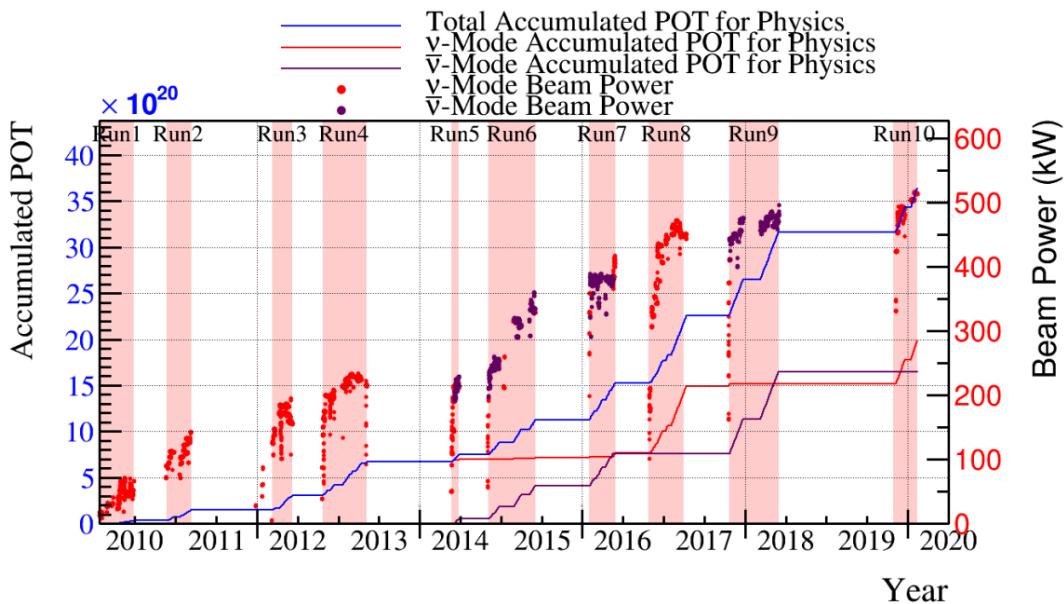


Figure 6.1: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

¹⁷⁴⁶ DB: Say that I have merged T2K and SK which hadn't previously been done

1747 6.1 Atmospheric Samples

1748 The atmospheric event selection follows the official SK-IV analysis presented
1749 in [85] and is documented below. The Monte Carlo prediction used within this
1750 analysis corresponds to 500 years worth of neutrino events, which is scaled down
1751 to match the SK-IV livetime of 3244.4 days.

1752 The fully contained (FC), partially contained (PC), and upward going muon
1753 events ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further
1754 broken down into different samples based on reconstruction information. This
1755 section details the samples used within this oscillation analysis, alongside the
1756 chosen binning.

1757 FC events are first separated by the visible energy deposited within the
1758 detector. This is calculated as the sum of the reconstructed kinetic energy
1759 above the Cherenkov threshold for all rings present in the event. Events are
1760 separated by whether they were above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates
1761 “subGeV” and “multiGeV” events. Typically, lower energy events consist of
1762 charged current quasi-elastic (CCQE) interactions which are better understood
1763 and simpler to reconstruct resulting in smaller systematic uncertainties. Events
1764 are further separated by the number of rings associated with the event due to
1765 similar reasoning. As the oscillation probability is dependant upon the flavour
1766 of neutrino, electron and muon events are separated using a similar likelihood
1767 method to that discussed in section 5.2. To reduce computational resources
1768 required for the reconstruction, only electron and pion hypotheses are considered
1769 so this separation cut depends on the ratio of the electron to pion likelihoods,
1770 $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1771 Charged current resonant pion production (CCRES) interactions generate a final-
1772 state pion. This can decay, mostly likely through a muon, into a decay electron.
1773 Therefore any electron-like event with one decay electron or muon-like event
1774 with two decay electrons was most likely produced by a CCRES interaction.
1775 Consequently, the number of decay electrons can be used to distinguish CCQE

₁₇₇₆ and CCRES interaction modes. Ultimately, FC subGeV events are separated
₁₇₇₇ into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

₁₇₇₈ In addition to the cuts discussed above, multiGeV samples also have addi-
₁₇₇₉ tional cuts to separate samples which target neutrino and antineutrino events.
₁₇₈₀ As discussed in section 7.1, the matter resonance only occurs for neutrinos in the
₁₇₈₁ normal hierarchy and antineutrinos in the inverted mass hierarchy. Therefore,
₁₇₈₂ having flavour-enriched samples aids in the determination of the mass hierarchy.
₁₇₈₃ For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

₁₇₈₄ The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be captured by an
₁₇₈₅ oxygen nucleus than the π^+ from ν_e interactions [188]. These pions then decay,
₁₇₈₆ mostly through muons, to electrons. Therefore the number of tagged decay
₁₇₈₇ electrons associated with an event gives an indication of whether the interaction
₁₇₈₈ was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
₁₇₈₉ The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
₁₇₉₀ where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with
₁₇₉₁ only 7% antineutrino background, the rest consisting of NC backgrounds.

1792 The number of decay electrons discriminator works reasonably well for single-
 1793 ring events. However, this is not the case for multi-ring events. A multiGeV
 1794 multiring electron-like (MME) likelihood cut was introduced in [189, 190]. This
 1795 is a two-stage likelihood selection cut. Four observables are used in the first
 1796 likelihood cut to distinguish $CC\nu_e$ and $CC\bar{\nu}_e$ events from background:

- 1797 • The number of decay electrons
 1798 • The maximum distance between the vertex of the neutrino and the decay
 1799 electrons
 1800 • The energy deposited by the highest energy ring
 1801 • The particle identification of that highest energy ring

1802 Background events consist of $CC\nu_\mu$ and NC interactions. Typically, the
 1803 majority of the energy in these background events is carried by the hadronic
 1804 system. Additionally, muons tend to travel further than the pions from $CC\nu_e$
 1805 before decaying. Thus, the parameters used within the likelihood cut target these
 1806 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring e -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring e -like events with one or more decay electrons
MultiGeV-mulike	Single ring μ -like events
MultiRing-elike-nue	Two or more ring events with leading energy e -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy e -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy μ -like ring and only requires $E_{vis} > 0.6\text{GeV}$ DB: Why is this not }1.33\text{GeV}
MultiRing-Other1	Two or more ring events with leading energy e -like ring and failed the MME likelihood cut

Table 6.3: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1807 Neutrino and antineutrino events are then separated by a second likelihood
 1808 method ($\nu/\bar{\nu}$ separation) detailed in [59]. This uses the number of decay electrons,

1809 the number of reconstructed rings, and the event’s transverse momentum. The
1810 last two parameters are used because higher-energy samples tend to have more
1811 pions produced above the Cherenkov threshold which results in more rings
1812 compared to an antineutrino interaction. Furthermore, the angular distribution
1813 also tends to be more forward peaked in antineutrino interactions as compared
1814 to neutrino interactions [85]. These FC multiGeV sample definitions are de-
1815 tailed in Table 6.3.

1816 The PC and up- μ samples are split by the amount of energy deposited within
1817 the outer detector, into “stopping” and “through-going” samples. If an event
1818 leaves the detector, the energy it takes with it has to be estimated which increases
1819 the systematic uncertainty compared to events entirely contained within the
1820 inner detector. This estimation is particularly poor at high energies, thus the
1821 up- μ through-going events are not binned in reconstructed momentum. The
1822 through-going up- μ are further separated by the presence of any electromagnetic
1823 showering in the event, as the assumption of non-showering muon does not give
1824 reliable reconstruction for these types of events [54]. In total, 13 FC, 2 PC, and
1825 3 up- μ atmospheric samples are included within this analysis.

1826 The atmospheric samples are binned in direct observables: reconstructed
1827 lepton momentum and direction, as given by Table A.1. The distribution of
1828 the reconstructed lepton momentum (for samples that only have one bin in
1829 reconstructed zenith angle) and reconstructed direction for each atmospheric
1830 sample used within this analysis is illustrated in Figure 6.2. The by-mode
1831 breakdown of each of the atmospheric samples is given in Appendix A.

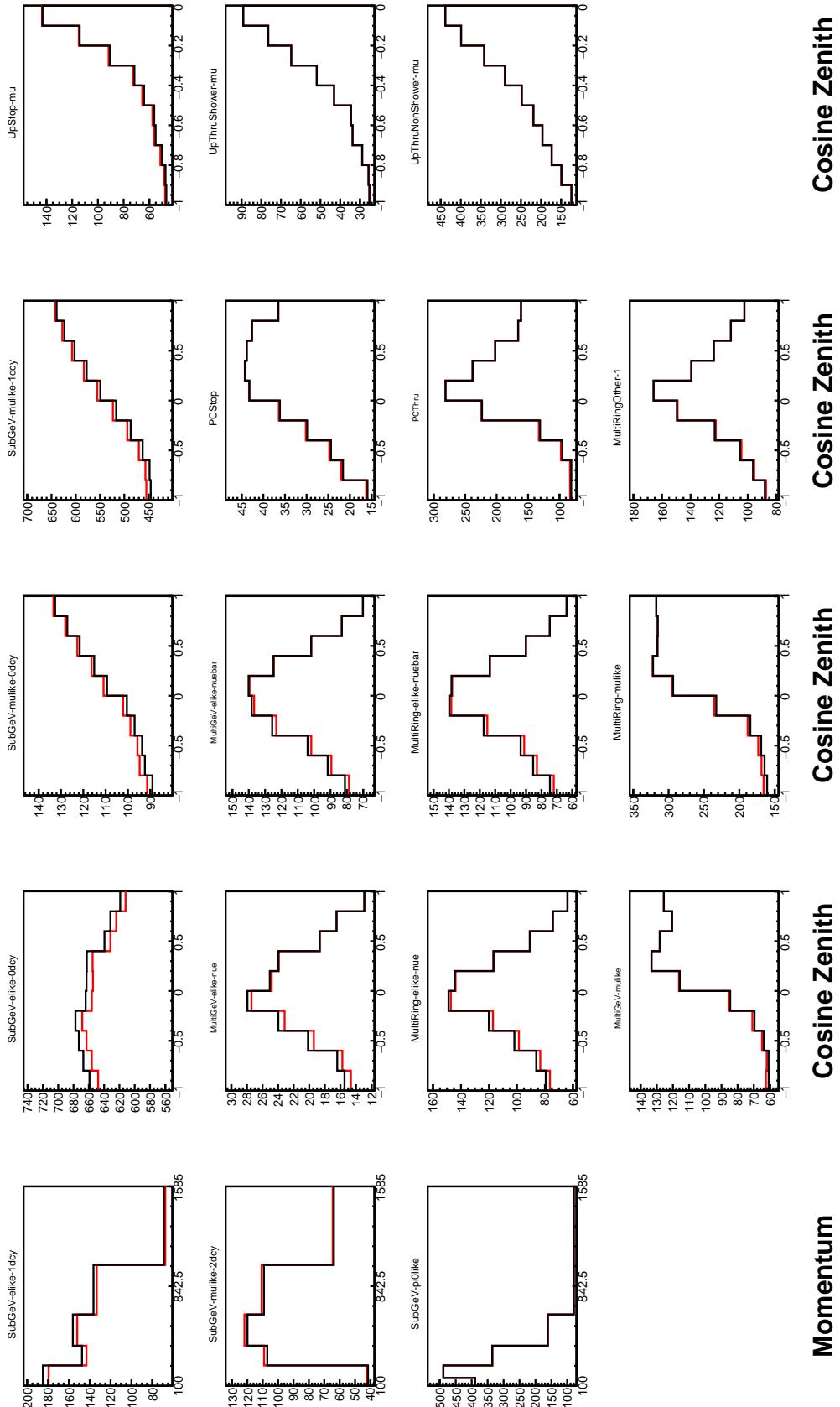


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThru	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Each row sums to 100% by definition. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

1832 6.2 Near Detector Beam Samples

1833 The near detector sample selections are documented in detail within [191] and
1834 summarised below. Samples are selected based upon which of the two Fine
1835 Grained Detector (FGD) the vertex is reconstructed in as well as the operating
1836 mode of the beam: FHC or RHC. Wrong-sign neutrino background samples are
1837 considered in the RHC mode in order to add additional constraints on model
1838 parameters. Samples from the wrong-sign component of the FHC beam mode
1839 are not included as they are statistically insignificant compared to those samples
1840 already listed.

1841 The reconstruction algorithm uses a clustering algorithm to group hits within
1842 the TPC. It then adds information from the upstream FGD to form a track
1843 that passes through both sub-detectors. In FHC(RHC), the highest momentum
1844 negative(positive) curvature track is defined as the muon candidate. Before
1845 being assigned a sample, these candidate muon events must pass CC-inclusive
1846 cuts, as defined in [192]:

- 1847 • Event Timing: The DAQ must be operational and the event must occur
1848 within the expected beam time window consistent with the beam spill
- 1849 • TPC Requirement: The muon-candidate track path must intercept one or
1850 more TPCs
- 1851 • Fiducial volume: The event must originate from within the fiducial volume
1852 defined in [193]
- 1853 • Upstream Background: Remove events that have muon tracks that originate
1854 upstream of the FGDs by requiring no high-momentum tracks within
1855 150mm upstream of the candidate vertex. Additionally, events that occur
1856 within the downstream FGD are vetoed if a secondary track starts within
1857 the upstream FGD

- 1858 • Broken track removal: All candidates where the muon candidate is broken
1859 in two are removed

- 1860 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-
1861 like events, from electron-like or proton-like, using a likelihood cut

1862 In addition to these cuts, RHC neutrino events also have to undergo the
1863 following cuts to aid in the separation of neutrino and antineutrino [194]:

- 1864 • TPC Requirement: The track path must intercept TPC2

- 1865 • Positive Track: The highest momentum track must have a positive recon-
1866 structed charge

- 1867 • TPC1 Veto: Remove any events originating upstream of TPC1

1868 Once all CC-inclusive events have been determined, they are further split
1869 by pion multiplicity: CC0 π , CC1 π , and CCOther. This breakdown targets the
1870 specific interaction modes CCQE, CCRES, and other CC background interactions,
1871 respectively. Pions in the TPCs and FGDs are selected by requiring a second track
1872 to be observed, which is separate from the muon track and is in the same beam
1873 spill window and sub-detector. If the pion originated within a FGD, it must also
1874 pass through the sequential downstream TPC (TPC2 for FGD1, TPC3 for FGD2).

1875 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

1876 **DB: Understand pion cuts at ND**

- 1877 • ν_μ CC0 π **Selection:** No electrons in TPC and no charged pions or decay
1878 electrons within the TPC or FGD

- 1879 • ν_μ CC1 π **Selection:** Exactly one charged pion in either the TPC or FGD,
1880 where the number of charged pions in the FGD is equal to the number of
1881 decay electrons

- 1882 • ν_μ CCOther **Selection:** All events which are not classified into the above
1883 two selections

Counting the three selections for each FGD in FHC and RHC running, including the wrong-sign background in RHC, 18 near detector samples are used within this analysis. These samples are binned in reconstructed lepton momentum (illustrated in Figure 6.3) and direction with respect to the beam. The binning is chosen such that each event has at least 20 Monte Carlo events in each bin [193]. This is to ensure that the bins are coarse enough to ensure the reduction of statistical errors, whilst also being fine enough to sample the high-resolution peak regions. The exact binning is detailed in [193].

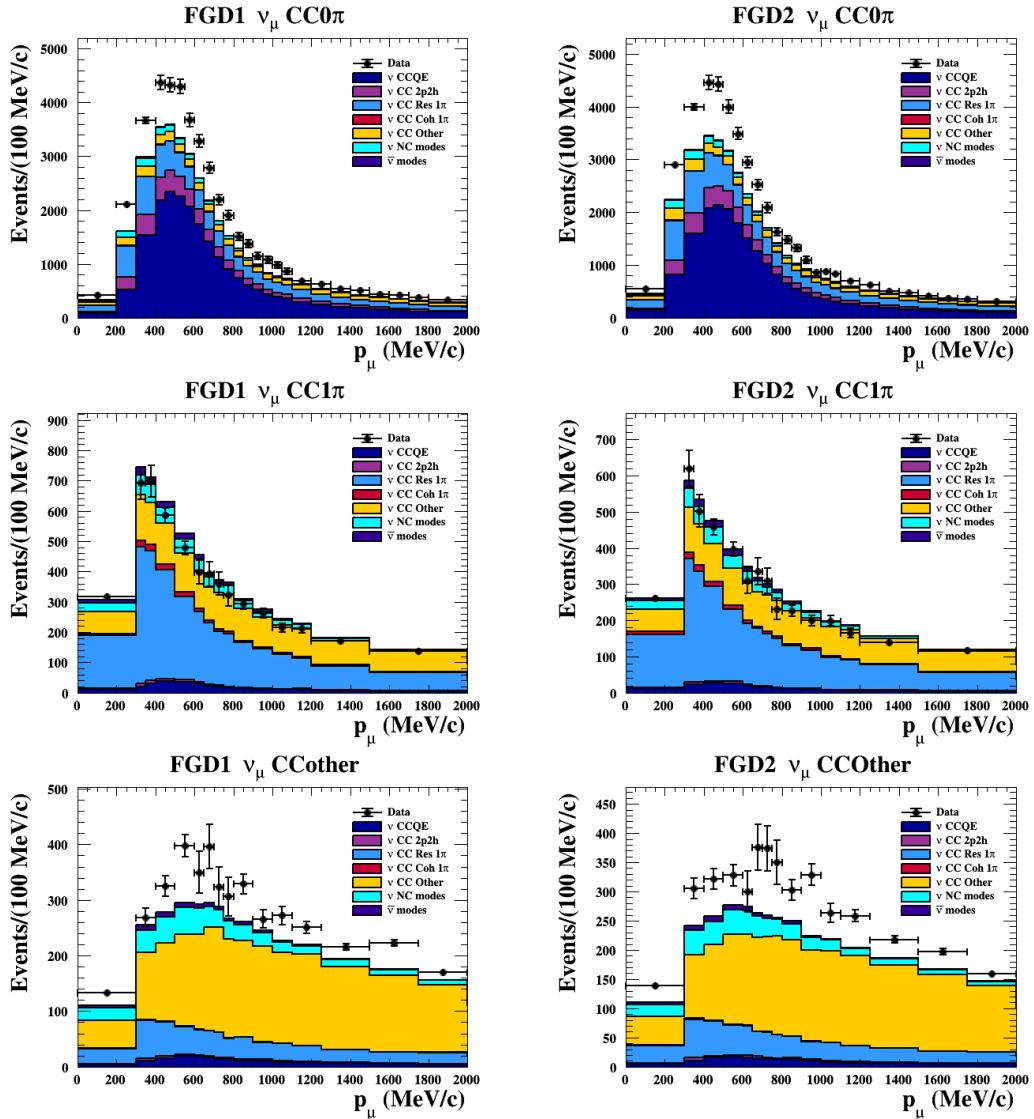


Figure 6.3: The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $CC\nu_\mu 0\pi$, $CC\nu_\mu 1\pi$ and $CC\nu_\mu$ Other categories. Figures taken from [191].

6.3 Far Detector Beam Samples

The beam neutrino events which occur at the SK detector, which pass the reduction cuts detailed in section 5.3, are separated based on whether the beam was operating in FHC or RHC mode. The events are then separated into three samples: electron-like ($1Re$), muon-like ($1R\mu$), and $CC1\pi^+$ -like ($1Re1de$) which are observed as electron-like events with an associated decay electron [183]. As discussed in section 6.1, positively charged pions emitted from neutrino interactions are more likely to produce decay electrons than negatively charged pions. Consequently, the $CC1\pi^+$ -like sample is only selected when the beam is operating in FHC mode. Therefore, five beam samples measured at SK are used in this analysis.

The fiducial volume definition for beam samples is slightly different from that used for the atmospheric samples. It uses both the distance to the closest wall (`dWall`) and the distance to the wall along the trajectory of the particle (`toWall`). This allows events that originate close to the wall but are facing into the tank to be included within the analysis, which would have otherwise been removed. These additional events are beneficial for a statistics-limited experiment. The exact cut values for both `dWall` and `toWall` are different for each of the three types of sample and are optimised based on T2K sensitivity to δ_{CP} [181, 195]. They are:

1Re event selection For an event to be classified as a $1Re$ -like, the event must satisfy:

- Fully-contained and have $dWall > 80\text{cm}$ and $toWall > 170\text{cm}$
- Total of one ring which is reconstructed as electron-like with reconstructed momentum $P_e > 100\text{MeV}$
- Zero decay electrons are associated with the event
- Passes π^0 rejection cut discussed in section 5.2

1918 The zero decay electron cut removes non-CCQE interactions and the π^0
 1919 rejection cut is designed to remove neutral current π^0 background events which
 1920 can be easily reconstructed as 1Re-like events.

1921 The zero decay electron cut removes non-CCQE interactions and the π^0
 1922 rejection cut is designed to remove neutral current π^0 background events which
 1923 can be easily reconstructed as 1Re-like events.

1924 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re
 1925 sample. The only differences are that the `dWall` and `toWall` criteria are changed
 1926 to $> 50\text{cm}$ and $> 270\text{cm}$, respectively, and exactly one decay electron is required
 1927 from the π^+ decay.

1928 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

- 1929 • Fully-contained and have `dWall` $> 50\text{cm}$ and `toWall` $> 250\text{cm}$
- 1930 • Total of one ring which is reconstructed as muon-like with reconstructed
 1931 momentum $P_\mu > 200\text{MeV}$
- 1932 • Fewer than two decay electrons are associated with the event
- 1933 • Passes π^+ rejection cut discussed in section 5.2

1934 All of these samples are binned in reconstructed neutrino energy. This is
 1935 possible under a particular interaction mode assumption, as the direction from
 1936 the source is known extremely well. For the 1Re-like and 1R μ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})}. \quad (6.2)$$

1937 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing
 1938 lepton, respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nucleus
 1939 [183], θ_{beam} is the angle between the beam and the direction of the outgoing
 1940 lepton, and E_l and P_l are the energy and momentum of that outgoing lepton.

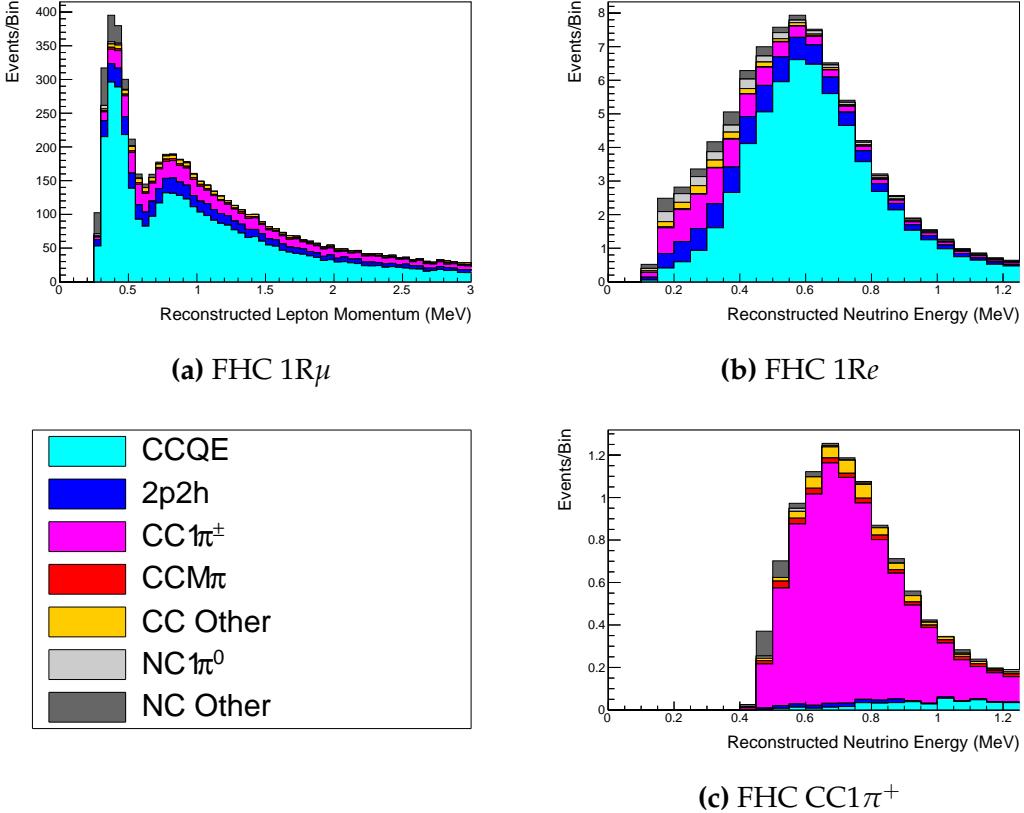


Figure 6.4: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1Re-like, and CC1 π^+ -like samples. The AsimovA oscillation parameters are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0 \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV.

1941 The reconstructed neutrino energy of the CC1 π^+ -like events also accounts
 1942 for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))}. \quad (6.3)$$

1943 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
 1944 considered as a two-body process, with the delta baryon, is assumed. This follows
 1945 the T2K oscillation analysis presented in [74], although recent developments of
 1946 the interaction model in the latest T2K oscillation analysis do include effects
 1947 from binding energy in this calculation [196].

1948 The reconstructed neutrino energy for the FHC samples is illustrated in
 1949 Figure 6.4. As expected, the 1R μ -like and 1Re-like samples are heavily dominated

by CCQE interactions, with smaller contributions from 2p2h meson exchange and resonant pion production interactions. The CC $1\pi^+$ -like sample predominantly consists of charged current resonant pion production interactions. The 1Re-like and CC $1\pi^+$ -like samples are also binned by the angle between the neutrino beam and the reconstructed lepton momentum. This is to aid in charged current and neutral current separation, as indicated in Figure 6.5. This is because the neutral current backgrounds are predominantly due to π^0 -decays, which decay into two γ rays. The opening angle of which (alongside the different final state kinematics) can produce a slightly broader angular distribution compared to the final state particles originating from charged current ν_e interactions.

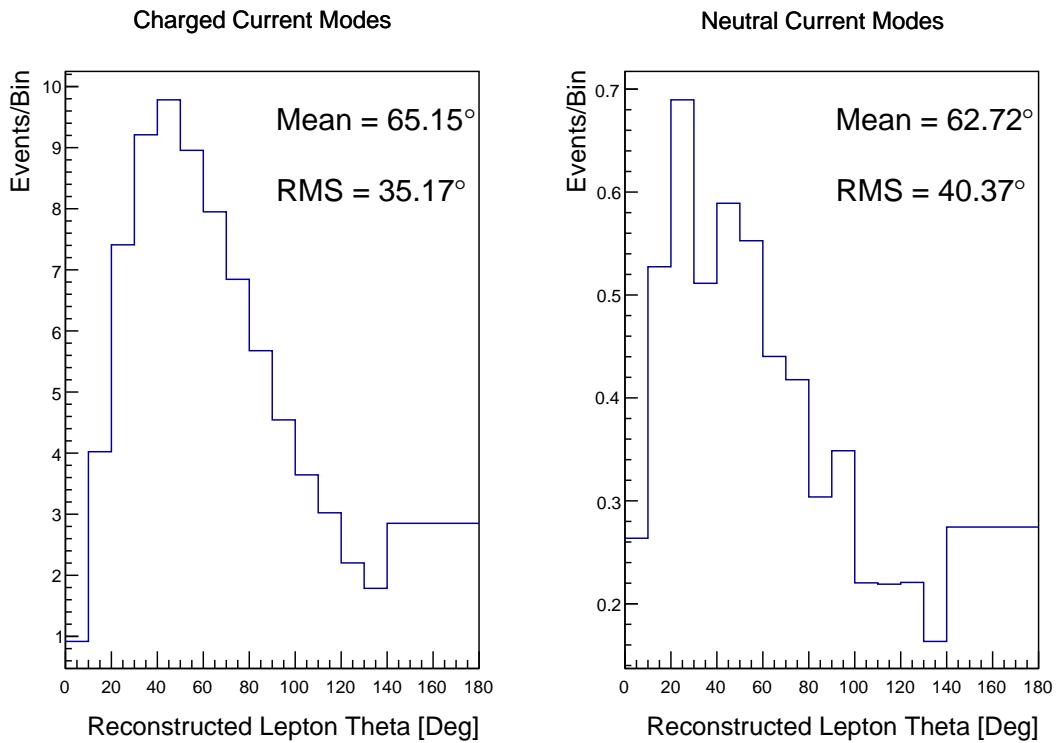


Figure 6.5: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are 35.17° and 40.37° , respectively.

1960 6.4 Systematic Uncertainties

1961 The systematic model parameters for this analysis are split into groups, or blocks,
 1962 depending on their purpose. They consist of flux uncertainties, neutrino-matter
 1963 interaction systematics, and detector efficiencies. There are also uncertainties on
 1964 the oscillation parameters to which this analysis is not sensitive, namely Δm_{21}^2
 1965 and $\sin^2(\theta_{12})$. These oscillation parameter uncertainties are taken from the 2020
 1966 PDG measurements [75]. As described in chapter 4, each model parameter used
 1967 within this analysis requires a prior uncertainty. This is provided via separate
 1968 covariance matrices for each block. The covariance matrices can include prior
 1969 correlations between parameters within a single block, but the separate treatment
 1970 means prior correlations can not be included for parameters in different groups.
 1971 Some parameters in these models have no reasonably motivated uncertainties
 1972 and are assigned flat priors which do not modify the likelihood penalty. In
 1973 practice, these flat prior parameters are actually assigned a Gaussian with a
 1974 very large width to ensure the covariance matrix is positive definite. They are
 1975 then checked at run time to determine if they contribute to the likelihood. The
 1976 flux, neutrino interaction, and detector modeling simulations have already been
 1977 discussed in section 5.1 and section 5.2. The uncertainties invoked within each
 1978 of these models are described below.

1979 6.4.1 Beam Flux

1980 The neutrino beam flux systematics are based upon the uncertainty in the mod-
 1981 eling of the components of the beam simulation. This includes the model of
 1982 hadron productions and reinteractions, the shape, intensity, and alignment of
 1983 the beam with respect to the target, and the uniformity of the magnetic field
 1984 produced by the horn, alongside other effects. The uncertainty, as a function
 1985 of neutrino energy, is illustrated in Figure 6.6 which includes a depiction of
 1986 the total uncertainty as well as the contribution from individual components.
 1987 The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$) is

1988 dominated by uncertainties in the beam profile and alignment. Outside of this
1989 region, uncertainties on hadron production dominate the error.

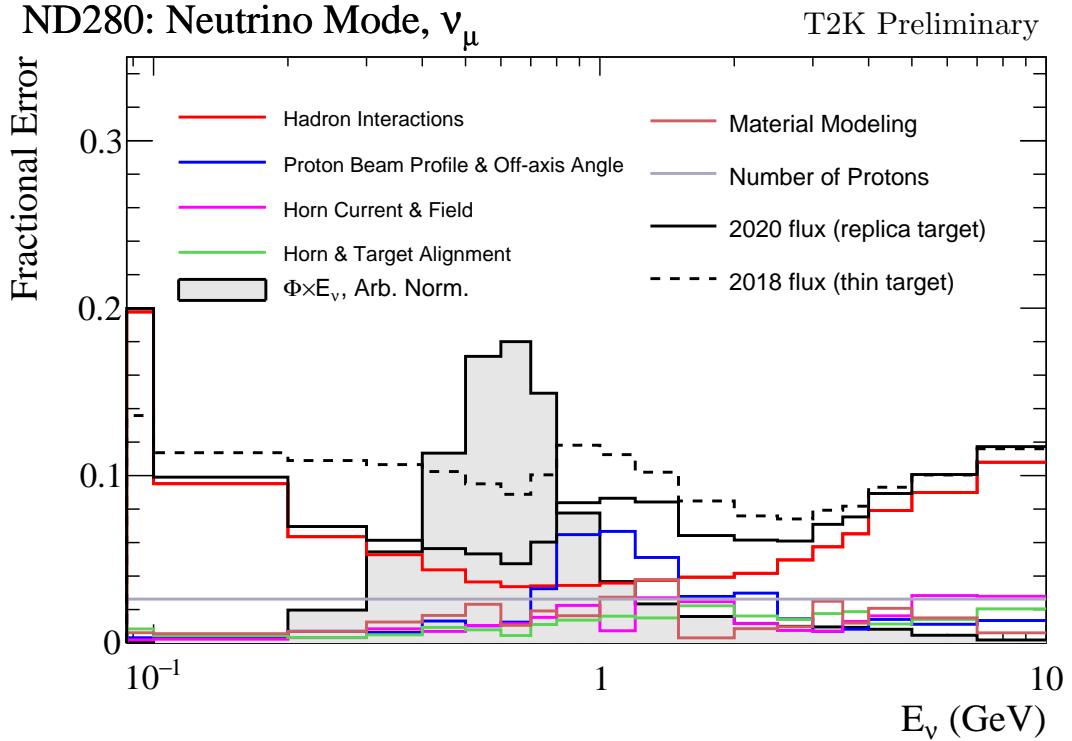


Figure 6.6: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [197]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [198].

1990 The beam flux uncertainties are described by one hundred parameters. They
1991 are split between the ND280 and SK detectors and binned by neutrino flavour:
1992 ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. The response is then broken down as a function of neutrino
1993 energy. The bin density in the neutrino energy is the same for the ν_μ in FHC
1994 and $\bar{\nu}_\mu$ in RHC beams, and narrows for neutrino energies close to the oscillation
1995 maximum of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.5. All of these
1996 systematic uncertainties are applied as normalisation parameters with Gaussian
1997 priors centered at 1.0 and error specified from a covariance matrix provided
1998 by the T2K beam group [198].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.5: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

1999 6.4.2 Atmospheric Flux

2000 The atmospheric neutrino flux is modeled by the HKKM model [50]. 16 systematic
 2001 uncertainties are applied to control the normalisation of each neutrino flavour,
 2002 energy, and direction. They are summarised below:

- 2003 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour
 2004 is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$
 2005 and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction
 2006 uncertainties for the production of pions and kaons [50]. The strength of
 2007 the response is dependent upon the neutrino energy. The uncertainty is
 2008 parameterized following Figure 11 in [50].
- 2009 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$
 2010 are controlled by the difference between the HKKM model [50], FLUKA
 2011 [53] and Bartol models [49]. Three independent parameters are applied in
 2012 the energy ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 2013 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent)
 2014 production uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using
 2015 the same methodology as the relative normalisation parameters.
- 2016 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-
 2017 tematics, the difference between the HKKM, FLUKA, and Bartol model

2018 predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of
 2019 events as a function of zenith angle.

- 2020 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) mostly originate in
 2021 kaon decay. Measurements of the ratio of K/π production [199] are used to
 2022 control the systematic uncertainty of the expected ratio of pion and kaon
 2023 production.
- 2024 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic
 2025 field, the flux of primary cosmic rays varies across the same period. The
 2026 uncertainty is calculated by taking a ± 1 year variation, equating to a 10%
 2027 uncertainty for the SK-IV period.
- 2028 • **Atmospheric Density:** The height of the interaction of the primary cosmic
 2029 rays is dependent upon the atmospheric density. The HKKM assumes the
 2030 US standard 1976 [151] profile. This systematic controls the uncertainty in
 2031 that model.

2032 The total uncertainty is dominated by the absolute and relative normalisation
 2033 parameters. The effect of which is illustrated in Figure 6.7. Generally, the
 2034 uncertainty is large at low energy, reducing to $O(10\%)$ around the peak of the
 2035 flux distribution and then increasing once the neutrino energy exceeds 10GeV.

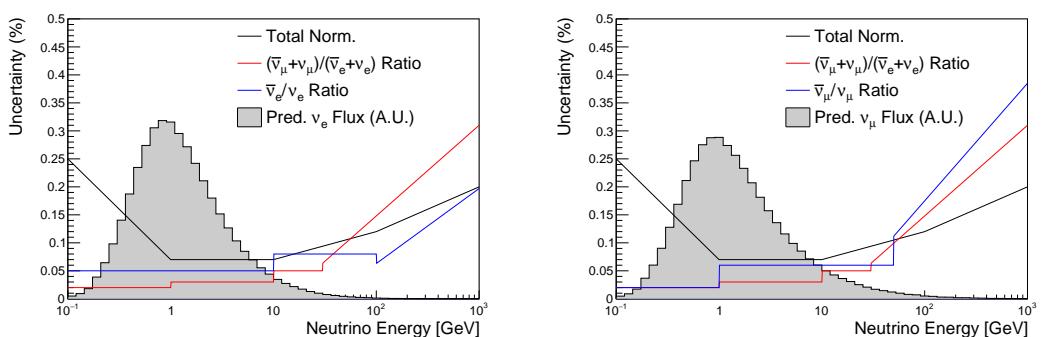


Figure 6.7: The uncertainty evaluated on the atmospheric ν_e (left) and ν_μ (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2036 Updates to the HKKM and Bartol models are underway [156] to use a similar
2037 tuning technique to that used in the beam flux predictions. After those updates,
2038 it may be possible to include correlations in the hadron production uncertainty
2039 systematics for beam and atmospheric flux predictions.

2040 6.4.3 Neutrino Interaction

2041 Neutrino interactions in the detectors are modeled by NEUT. The two indepen-
2042 dent oscillation analyses, T2K-only [200] and the SK-only [59], have developed
2043 separate interaction models. To maximise sensitivity out of this simultaneous
2044 beam and atmospheric analysis, a correlated interaction model has been defined
2045 in [201]. Where applicable, correlations allow the systematic uncertainties applied
2046 to the atmospheric samples to be constrained by near detector neutrino beam
2047 measurements. This can lead to stronger sensitivity to oscillation parameters
2048 as compared to an uncorrelated model.

2049 The low-energy T2K systematic model has a more sophisticated treatment
2050 of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of
2051 this model have been performed to external data [200]. However, the model
2052 is not designed for high-energy atmospheric events, like those illustrated in
2053 Figure 5.11. Therefore the high energy systematic model from the SK-only
2054 analysis is implemented for the relevant multi-GeV, PC, and up- μ samples.
2055 The T2K CCQE model is more sophisticated so it has been implemented for
2056 all samples within this analysis, where separate low-energy and high-energy
2057 dials have been implemented. The low-energy dials are constrained by the near
2058 detector measurements and are uncorrelated to their high-energy counterparts.
2059 The author of this thesis was responsible for implementing and validating the
2060 combined cross-section model as documented in [201, 202].

2061 The high energy systematic model includes parameters developed from
2062 comparisons of Nieves and Rein-Seghal models which affect resonant pion
2063 producing interactions, comparisons of the GRV98 and CKMT models which
2064 control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the ν_τ cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.8 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.9 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

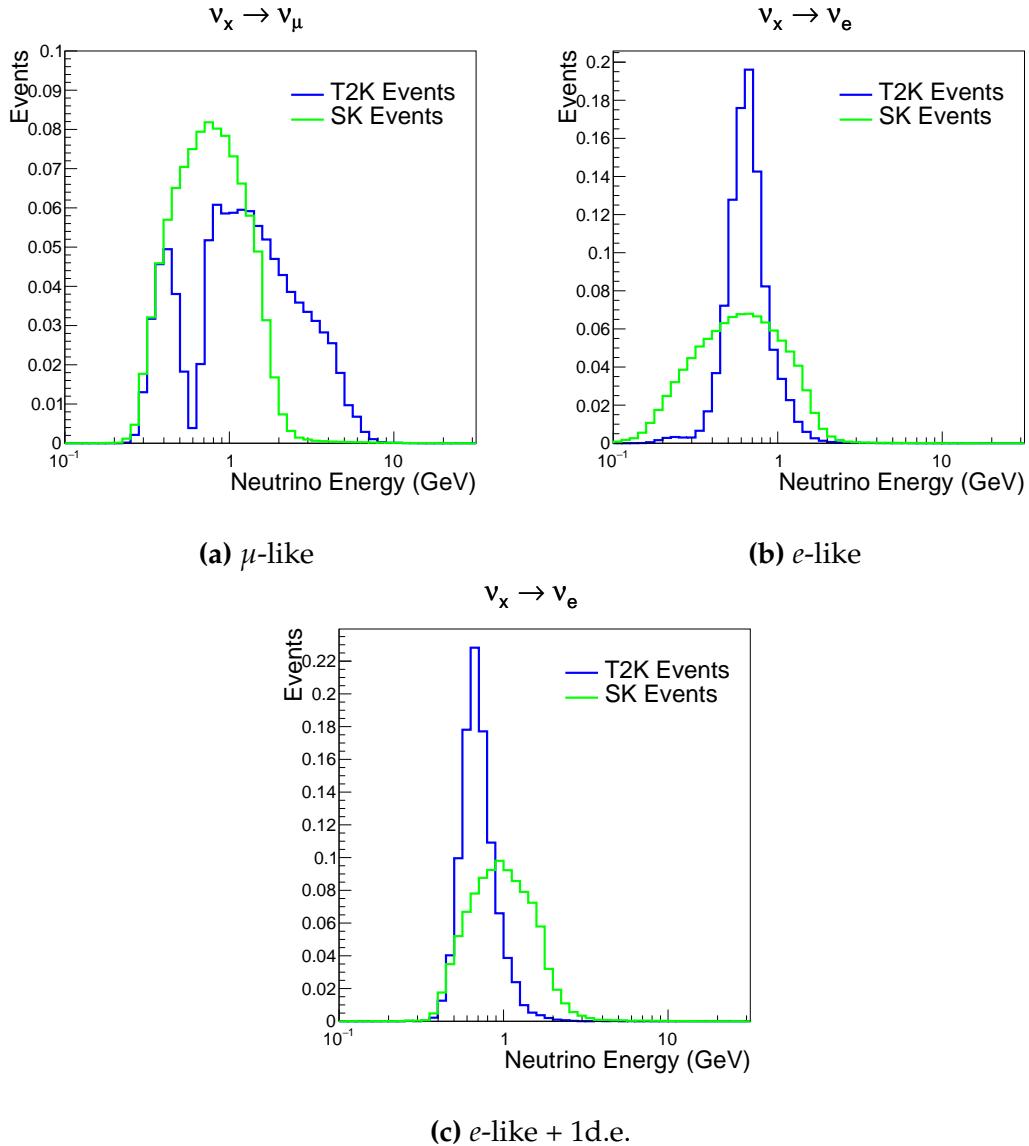


Figure 6.8: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

2095 The T2K systematic model [200] is applied in a similar methodology to the
 2096 SK model parameters. It consists of 19 shape parameters and 24 normalisation
 2097 parameters. Four additional parameters, which model the uncertainty in the
 2098 binding energy, are applied in a way to shift the momentum of the lepton emitted
 2099 from a nucleus. This controls the uncertainty specified on the 27MeV binding
 2100 energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.9: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

2101 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated
 2102 uncertainty, or those which have not been fit to external data, are assigned a
 2103 flat prior which does not affect the penalty term.

2104 On top of the combination of the SK and T2K interaction models, several
 2105 other parameters have been specifically developed for the joint oscillation anal-
 2106 ysis. The majority of the atmospheric samples' δ_{CP} sensitivity comes from the
 2107 normalisation of subGeV electron-like events. These are modeled using a spectral
 2108 function to approximate the nuclear ground state. However, the near detector is
 2109 not able to constrain the model so an additional systematic is introduced which
 2110 models an alternative Continuous Random Phase Approximation (CRPA) nuclear
 2111 ground state. This dial approximates the event weights if a CRPA model had

been assumed rather than a spectral function. This dial only applies to ν_e and $\bar{\nu}_e$ as the near detector does not constraint ν_e cross-section measurements. It is applied as a shape parameter.

Further additions to the model have been introduced due to the inclusion of the subGeV π^0 atmospheric sample. This particularly targets charged current and neutral current π^0 producing interactions to help constrain the systematic uncertainties. Therefore, an uncertainty that affects neutral current resonant π^0 production is incorporated into this analysis. Comparisons of NEUT's NC resonant pion production predictions have been made to MiniBooNE [203] data and a consistent 16% to 21% underprediction is observed [201]. Consequently, a conservative 30% normalisation parameter is invoked.

Down-going events are mostly insensitive to oscillation parameters and can act similar to the near detector within an accelerator experiment (Details will be discussed in chapter 7). This region of phase space can act as a sideband and allows the cross-section model and near detector constraint to be studied. The distribution of events in this region is calculated using the technique outlined in subsection 4.3.4. The results are illustrated in Figure 6.10. For CCQE-targeting samples, the application of the near detector constraint is well within the statistical fluctuation of the down-going data. This means there is no significant tension is observed between the data and the Monte Carlo prediction after the near detector constraint is applied. This is not the case for samples with target CCRES interactions. The electron-like data is consistent with the constrained prediction at high reconstructed momenta but diverges at lower momentum, whereas the muon-like sample is under-predicted throughout the range of momenta. To combat this disagreement, an additional cross-section systematic dial, specifically designed to inflate the low pion momentum systematics was developed in [201]. This is a shape parameter implemented through a splined response.

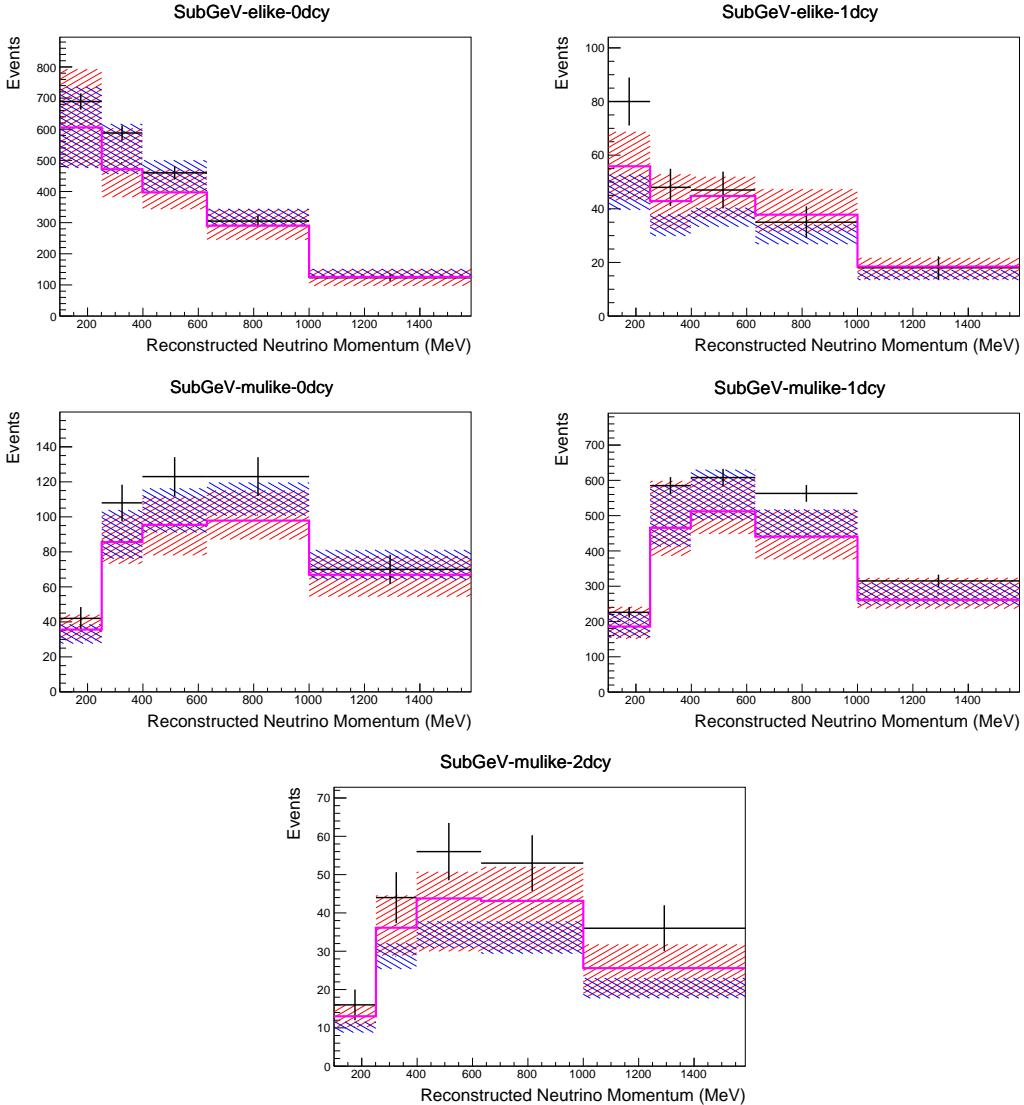


Figure 6.10: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

2139 6.4.4 Near Detector

2140 The systematics applied due to uncertainties arising from the response of the near
 2141 detector is documented in [130]. The response is described by 574 normalisation
 2142 parameters binned in the selected sample as well as momentum and angle,

2143 P_μ and $\cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance
2144 matrix with each parameter being assigned a Gaussian prior from that covariance
2145 matrix. These normalisation parameters are built from underlying systematics,
2146 e.g. pion secondary interaction systematics, which are randomly thrown and
2147 the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. Two thousand throws are
2148 evaluated and a covariance matrix response is created. This allows significant
2149 correlations between FGD1 and FGD2 samples, as well as adjacent $P_\mu \times \cos(\theta_\mu)$
2150 bins. Statistical uncertainties are accounted for by including fluctuations of each
2151 event's weight from a Poisson distribution.

2152 Similar to the cross-section systematics, MaCh3 and BANFF are used to
2153 constrain the uncertainty of these systematics through independent validations.
2154 Each fitter generates a post-fit covariance matrix which is compared and passed
2155 to the far-detector oscillation analysis working group. As the analysis presented
2156 within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all
2157 three sets of samples and their respective systematics is performed.

2158 6.4.5 Far Detector

2159 Two configurations of the far detector systematic model implementation have
2160 been considered. Firstly, the far detector systematic uncertainties for beam and
2161 atmospheric samples are taken from their respective analysis inputs, denoted
2162 “official inputs” analysis, with no correlations assumed between the beam and at-
2163 mospheric samples. The beam- and atmospheric-specific inputs are documented
2164 in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative
2165 detector model has been developed which correlates the response of the SK
2166 detector systematics between the beam and atmospheric samples. Here, the
2167 distribution of parameters used for applying event cuts (e.g. electron-muon
2168 PID separation) is modified within the fit. It follows a similar methodology to
2169 the beam far detector systematics implementation but performs a joint fit of
2170 the beam and atmospheric data. This alternative implementation is detailed
2171 in subsubsection 6.4.5.3.

2172 **6.4.5.1 Beam Samples**

2173 There are 45 systematics which describe the response of the far detector to
2174 beam events [183], split into 44 normalisation parameters and one energy scale
2175 systematic. The energy scale systematic is applied as a multiplicative scaling
2176 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo
2177 differences in the stopping muon sample in [185] and found to be 2.1%. The
2178 normalisation parameters are assigned a Gaussian error centered at one with
2179 width taken from a covariance matrix. A detailed breakdown of the generation
2180 of the covariance matrix is found in [195]. To build the covariance matrix, a fit
2181 is performed on atmospheric data which has been selected using beam sample
2182 selection cuts. These cuts use the variables, L^i , where the index i is detailed in
2183 Table 6.6. Each L^i is a smear, α , and shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i \quad (6.4)$$

2184 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in
2185 Table 6.6. The shift and smear parameters are nuisance parameters with no prior
2186 constraints. They are binned by final-state topology, j , where the binning is given
2187 in Table 6.7. The final-state topology binning is because the detector will respond
2188 differently to events that have one or multiple rings. For example, the detector
2189 will be able to distinguish single-ring events better than two overlapping ring
2190 events, resulting in different systematic uncertainty for one-ring events compared
2191 to two-ring events. This approach is used to allow the cut parameter distributions
2192 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.6: List of cut variables that are included within the shift/smear fit documented in [195].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 μ	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 μ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 π^0	Only one π^0 in the final state
1 π^\pm or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.7: Reconstructed event topology categories on which the SK detector systematics [195] are based.

2193 The mis-modeling of π^0 events is also considered. If one of the two rings
 2194 from a π^0 event is missed, this will be reconstructed as a CC ν_e -like event. This
 2195 is one of the largest systematics hindering the electron neutrino appearance
 2196 analyses. Consequently, additional systematics have been introduced to con-
 2197 strain the mis-modeling of π^0 events in SK, binned by reconstructed neutrino
 2198 energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples is
 2199 constructed. These events are built by overlaying one electron-like ring from
 2200 the SK atmospheric neutrino samples or decay electron ring from a stopping
 2201 cosmic ray muon with one simulated photon ring. Both rings are chosen so
 2202 that momenta and opening angle follow the decay kinematics of NC π^0 events
 2203 from the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the
 2204 SK Monte Carlo are produced to compare with the hybrid- π^0 data samples and
 2205 the difference in the fraction of events that pass the ν_e selection criteria is used
 2206 to assign the systematic errors. In order to investigate any data to Monte Carlo
 2207 differences that may originate from either the higher energy ring or lower energy
 2208 ring, two samples are built; a sample in which the electron constitutes the higher
 2209 energy ring from the π^0 decay (called the primary sample) and another one in
 2210 which it constitutes the lower energy ring (called the secondary sample). The
 2211 standard T2K ν_e fitQun event selection criteria are used to select events.

2212 Final contributions to the covariance matrix are determined by supplemen-
 2213 tary uncertainties obtained by comparing stopping muon data to Monte Carlo

prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data to Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or fewer decay electrons. Similarly, the rate at which fake decay electrons are reconstructed by `fitQun` is estimated by comparing the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events is also applied. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [183, 204, 205].

Two additional sources of uncertainty are included: secondary and photoneuclear interactions. These are estimated by varying the underlying parameters are building a distribution of sample event rates. These contributions are then added in quadrature to the above covariance matrix.

DB: Matrix Diagonals from SK Det Matrix

6.4.5.2 Atmospheric Samples

The detector systematics for atmospheric samples, documented in [85], are split into two sub-groups: those which are related to particle identification and ring counting systematics, and those which are related to calibration, separation, and reduction uncertainties.

The particle identification systematics consist of five parameters. The ring separation systematic enforces an anti-correlated response between the single-ring and multi-ring samples. This is implemented as a fractional increase/decrease in the overall normalisation of each sample, depending on the distance to the nearest wall from an event's vertex. The coefficients of the normalisation are estimated prior to the fit and depend on the particular atmospheric sample. Two electron-muon separation systematics are included within this model which

2243 anti-correlates the response of the electron-like and muon-like samples: one for
2244 single-ring events and another for multi-ring events.

2245 The multi-ring electron-like separation likelihood, discussed in section 6.1,
2246 encodes the ability of the detector to separate neutrino from anti-neutrino events.
2247 Two normalisation parameters vary the relative normalisation of multi-ring ν_e
2248 and $\bar{\nu}_e$ samples whilst keeping a consistent overall event rate.

2249 There are 22 systematics related to calibration measurements, including effects
2250 from backgrounds, reduction, and showering effects. They are documented in
2251 [85] and are briefly summarised in Table 6.8. They are applied via normalisation
2252 parameters, with the separation systematics requiring the conservation of event
2253 rate across all samples.

2254 6.4.5.3 Correlated Detector Model

2255 A complete uncertainty model of the SK detector would be able to determine
2256 the systematic shift on the sample spectra for a variation of the underlying
2257 parameters, e.g. PMT angular acceptance. However, this is computationally
2258 intensive, requiring Monte Carlo predictions to be made for each plausible
2259 variation. Consequently, an effective parameter model has been utilised for
2260 a correlated detector model following from the T2K-only model implementation
2261 documented in subsubsection 6.4.5.1. It correlates the detector systematics
2262 between the far-detector beam and subGeV atmospheric samples due to their
2263 similar energies and interaction types. As there are no equivalent beam samples,
2264 the multi-GeV, multiring, PC, and Up- μ samples will be subject to the particle
2265 identification systematics implementation as described in subsubsection 6.4.5.2
2266 rather than using this correlated detector model. The calibration systematics also
2267 described in the aforementioned chapter still apply to all atmospheric samples.

2268 The correlated detector model utilises the same smear and shift parameters
2269 documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,
2270 the shift and smear parameters are split by visible energy deposited within the
2271 detector, with binning specified in Table 6.9. This is because atmospheric events

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

Table 6.8: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

are categorised by subGeV and multi-GeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where k is the visible energy bin.

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

Table 6.9: Visible energy binning for which the correlated SK detector systematics are based

parameter for that event, and then re-applies event selection. This causes event migration, which is a new feature incorporated into the MaCh3 framework which is only achievable due to the event-by-event reweighting scheme.

Particular care has to be taken when varying the ring counting parameter. This is because the number of rings is a finite value (one-ring, two-ring, etc.) which can not be continuously varied through this shift and smear technique. Consequently a continuous ring counting parameter, RC_i , is calculated for the i^{th} event, following the definition in [182]: the preferred likelihoods from all considered one-ring (L_{1R}) and two-ring (L_{2R}) fits are determined. The difference is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an intermediate likelihood value used within the `fitQun` algorithm to decide the number of rings associated with a particular event. However, fake-ring merging algorithms are applied after this likelihood value is used. Consequently, this ring counting parameter does not always exactly correspond to the number of reconstructed rings. This can be seen in Figure 6.11.

As the `fitQun` algorithm does not provide a likelihood value after the fake-ring algorithms have been applied, the ring counting parameter distribution is correlated to the final number of reconstructed rings through “maps”. These are two-dimensional distributions of the ring counting parameter and the final number of reconstructed rings. An example is illustrated in Figure 6.12. In

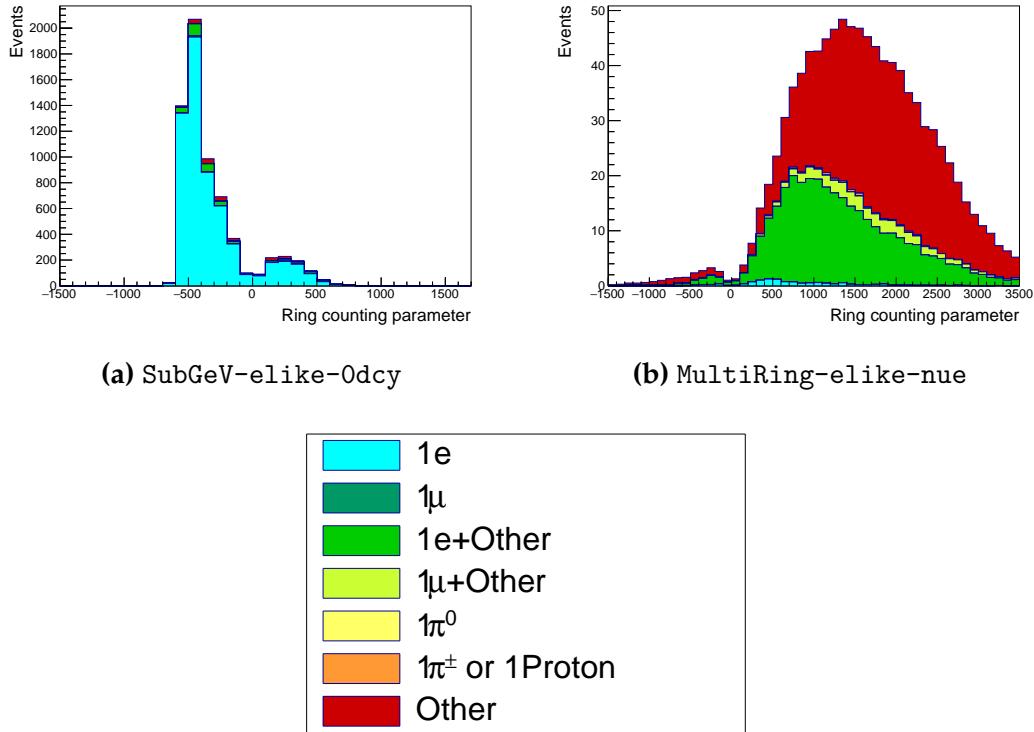


Figure 6.11: The ring counting parameter as defined in Equation 6.6 for the SubGeV-elike-0dcy and MultiRing-elike-nue samples.

principle, the `fitQun` reconstruction algorithm should be re-run after the variation in the ring counting parameter. However, this is not computationally viable. Therefore the “maps” are used as a reweighting template.

The maps are split by final state topology and true neutrino flavour and all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps are row-normalised to represent the probability of X rings for a given RC_i value. Prior to the oscillation fit, an event's nominal weight is calculated as $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th} event and $W^i(x, y)$ is the bin content in map associated with the i^{th} event, where x number of rings and y is ring counting parameter. Then during the fit, the value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as the event weight for the i^{th} event. This is the only cut variable that uses a reweighting technique rather than event migration.

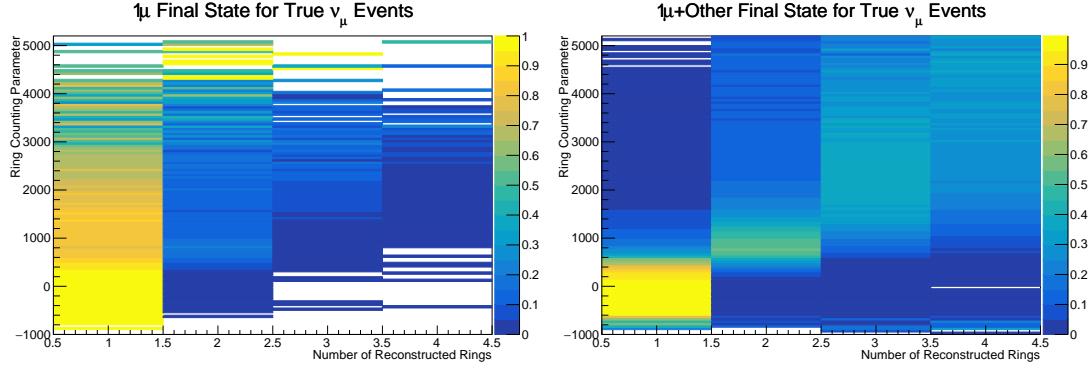


Figure 6.12: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitTQu` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2315 The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance
 2316 matrix. This is not possible in the alternative model as no covariance matrix
 2317 is used. Thus, the implementation of the π^0 systematics has been modified.
 2318 The inputs from the hybrid π^0 sample are included via the use of “ χ^2 maps”,
 2319 which are two-dimensional histograms in α_{jk}^i and β_{jk}^i parameters over some
 2320 range. Illustrative examples of the χ^2 maps are given in Figure 6.13. Due to
 2321 their nature, the shift and smear parameters are typically very correlated. A
 2322 map is produced for each cut parameter given in Table 6.6 and for each visible
 2323 energy bin given in Table 6.9.

2324 The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo
 2325 and data in the particle identification parameters documented in Table 6.6. The
 2326 Monte Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-
 2327 section and flux nuisance parameters are thrown from their prior uncertainties.
 2328 The χ^2 between the scaled Monte Carlo and data is calculated and the relevant
 2329 point in the χ^2 map is filled.

2330 The implementation within this alternative detector model is to add the bin
 2331 contents of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to
 2332 the likelihood penalty. Only 1 π^0 final state topology shift and smear parameters
 2333 use this prior uncertainty.

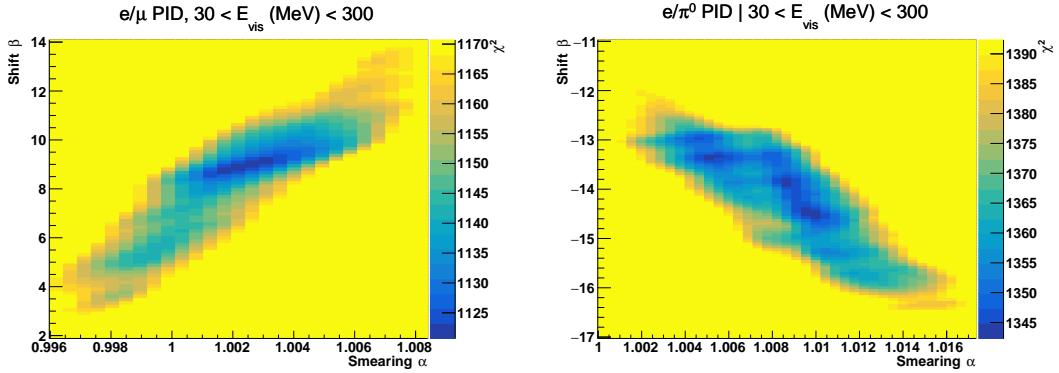


Figure 6.13: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$.

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [206] was built in tandem with the T2K-SK working group [183] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular interaction mode, number of tagged decay electrons, and whether the primary particle generated Cherenkov light. They are assigned Gaussian uncertainties with widths described by a covariance matrix. Furthermore, the secondary interaction and photo-nuclear effects need to be accounted for in this detector model using a different implementation than that in subsubsection 6.4.5.1. This was done by including a shape parameter for each of the secondary interactions and the photo-nuclear systematic parameters.

There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior constraints from the hybrid π^0 samples.

One final complexity of this correlated detector model is that the two sets of samples, beam and subGeV atmospheric, use slightly different parameters to distinguish electron and muon-like events. The T2K samples use the value of $\log(L_e/L_\mu)$ whereas the atmospheric samples use the value of $\log(L_e/L_\pi)$, where L_X is the likelihood for hypothesis X. This is because the T2K fits use single-ring fiTQun fitting techniques, whereas multi-ring fits are applied to the atmospheric samples where only the electron and pion hypothesis are considered.

2354 The correlation between the two likelihood ratios is illustrated in Figure 6.14. As
 2355 discussed in section 5.2, the pion hypothesis is a very good approximation of the
 2356 muon hypothesis due to their similar mass. Consequently, using the same shift
 2357 and smear parameters correlated between the beam and subGeV atmospheric
 2358 samples is deemed a good approximation.

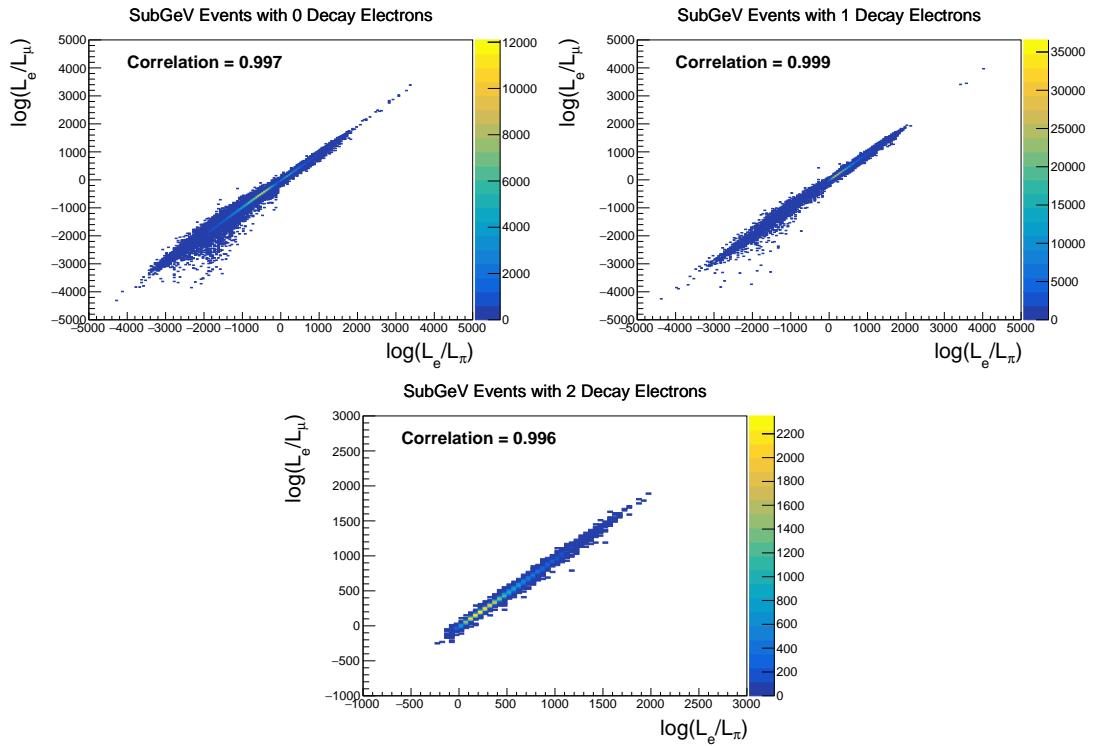


Figure 6.14: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

7

2359

2360

Oscillation Probability Calculation

2361 It is important to understand how and where the sensitivity to the oscillation parameters comes from for both atmospheric and beam samples. An
2362 overview of how these samples respond to changes in δ_{CP} , Δm_{32}^2 , and $\sin^2(\theta_{23})$
2363 is given in section 7.1. This section also explains the additional complexities
2364 involved when performing an atmospheric neutrino analysis as compared to
2365 a beam-only analysis.

2366 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to δ_{CP}
2367 due to the quickly varying oscillation probability in this region. Therefore, a
2368 “sub-sampling” approach has been developed to reduce these biases ensuring
2369 accurate and reliable sensitivity measurements. This technique ensures that small-
2370 scale unresolvable features of the oscillation probability have been averaged over
2371 whilst the large-scale features in the oscillation probability are unaffected. The
2372 documentation and validation of this technique are found in section 7.2. The
2373 oscillation probability calculation is computationally intensive due to the large
2374 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-
2375 mentation choice made within the fitting framework, as detailed in section 7.3,
2376 ensures that the analysis can be done in a timely manner.
2377

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.4. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrino oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation.

This implementation of an external technique is described in section 7.5.

7.1 Overview

DB: Should this be moved into an earlier chapter? The selections chapter references the matter resonance which has not yet been explained at that point

The analysis presented within this thesis focuses on the determination of oscillation parameters from atmospheric and beam neutrinos. Whilst subject to the same oscillation formalism, the way in which the two samples have sensitivity to the different oscillation parameters differs significantly.

Atmospheric neutrinos have a varying baseline, or “path length” L , such that the distance each neutrino travels before interacting is dependent upon the zenith angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s surface and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (7.1)$$

2404 DB: Ask Giles what he means about the horizontal issue and Layering in
 2405 the ProdH section

2406 Where $R_E = 6,371\text{km}$ is the Earth's radius. Consequently, the oscillation
 2407 probability is dependent upon two parameters, $\cos(\theta_Z)$ and E_ν .

2408 The oscillation probability used within this analysis is based on [21]. The
 2409 neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of
 2410 constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (7.2)$$

2411 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
 2412 energy, G_F is Fermi's constant, N_e is the electron number density and U is the
 2413 PMNS matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied
 2414 for antineutrino propagation. Thus, a model of the Earth's density is required
 2415 for neutrino propagation. Following the official SK-only methodology [207],
 2416 this analysis uses the Preliminary Reference Earth Model (PREM) [208] which
 2417 provides piecewise cubic polynomials as a function of the Earth's radius. This
 2418 density profile is illustrated in Figure 7.1. As the propagator requires layers of
 2419 constant density, the SK methodology approximates the PREM model by using
 2420 four layers of constant density [207], detailed in Table 7.1.

Layer	Outer Radius [km]	Density [g/cm^3]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 7.1: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [208].

2421 The atmospheric neutrino oscillation probabilities can be presented as two di-
 2422 mensional “oscillograms” as illustrated in Figure 7.2. The distinct discontinuities,
 2423 as a function of $\cos(\theta_Z)$, are due to the discontinuous density in the PREM model.

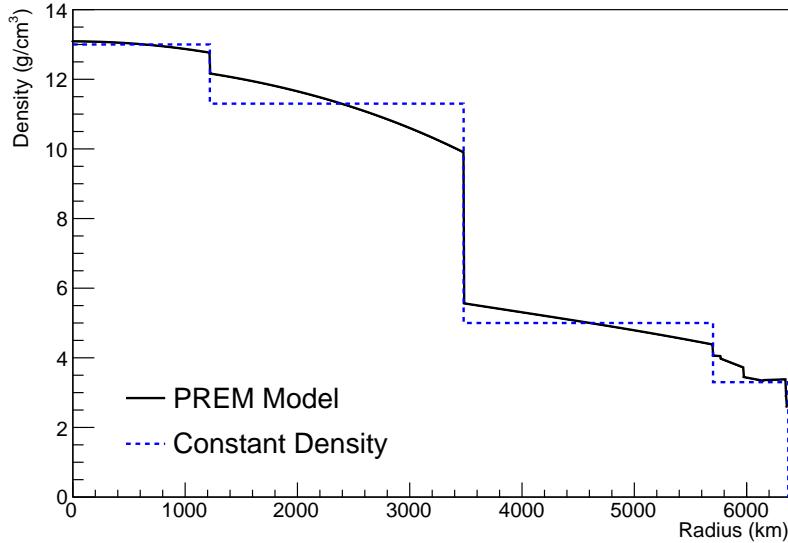


Figure 7.1: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

Atmospheric neutrinos have sensitivity to δ_{CP} through the overall event rate. Figure 7.3 illustrates the difference in oscillation probability between CP-conserving ($\delta_{CP} = 0.$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation parameter set (Table 2.2). The result is a complicated oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric neutrinos comes via the overall normalisation of these events.

The presence of matter means that the effect δ_{CP} has on the oscillation probability is not equal between neutrinos and antineutrinos. Furthermore, the interaction cross-section for neutrinos is larger than for antineutrinos so the two effects have to be disentangled. These effects are further convoluted by detector efficiencies as SK cannot distinguish neutrinos and antineutrinos well. Furthermore, the sample selections discussed in section 6.1 have different efficiencies for neutrino-antineutrino selections. All of these effects lead to a difference in the number of neutrinos detected compared to antineutrinos. This changes how the δ_{CP}

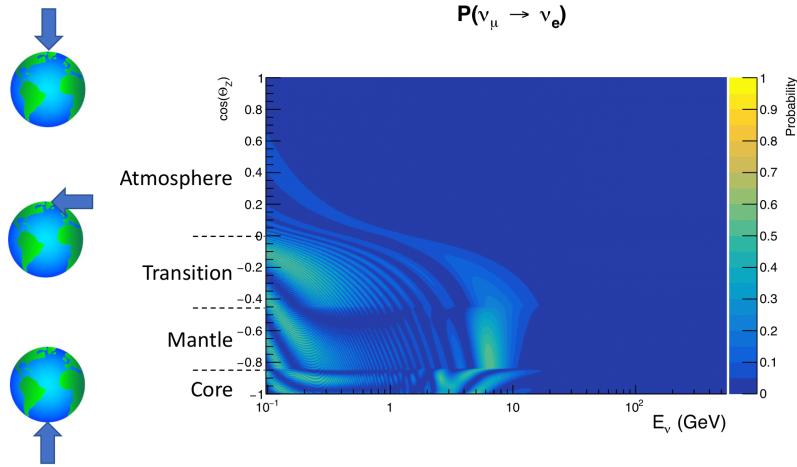


Figure 7.2: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

normalisation term is observed, resulting in a very complex sensitivity to δ_{CP} .

The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ are presented in Figure 7.4, where the PREM model has been assumed. The oscillation probability for both neutrinos and antineutrinos is affected in the presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only occur for neutrinos in the normal mass hierarchy and antineutrinos in the inverse mass hierarchy. The exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$, further increasing the atmospheric neutrinos’ sensitivity to the parameter.

As the T2K beam flux is centered at the first oscillation maximum ($E_\nu = 0.6\text{GeV}$), the sensitivity to δ_{CP} is predominantly observed as a change in the event-rate of e-like samples in $\nu/\bar{\nu}$ modes. Figure 7.5 illustrates the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a range of δ_{CP} values. A circular modulation of the first oscillation peak (in both magnitude and position) is observed when varying throughout the allowable values of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation maximum than the CP-violating values of $\delta_{CP} = -\pi/2$ ($\delta_{CP} = \pi/2$). A sub-dominant shift in the energy of the oscillation peak is

$$\mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = -1.601) - \mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = 0.)$$

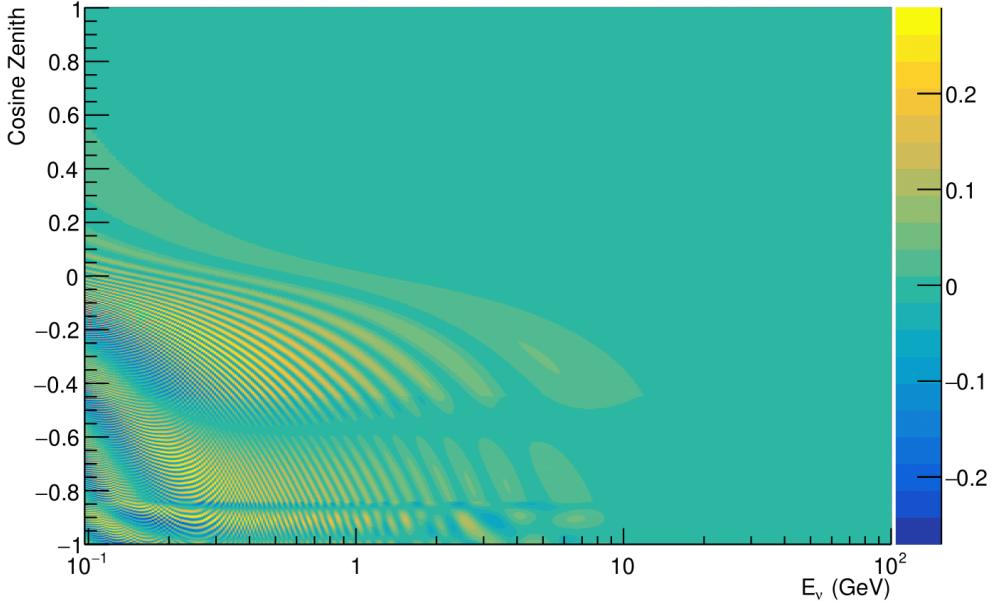


Figure 7.3: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

2456 also present, which aids in separating the two CP-conserving values of δ_{CP} .
 2457 T2K's sensitivity to $\sin^2(\theta_{23})$ and Δm_{32}^2 is observed as a shape-based variation
 2458 of the muon-like samples, as illustrated in Figure 7.5. The value of Δm_{32}^2 laterally
 2459 shifts the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$
 2460 oscillation probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as
 2461 a vertical shift of the oscillation dip with second-order horizontal shifts being
 2462 due to matter effects. The beam neutrinos have limited sensitivity to matter
 2463 effects due to the relatively shorter baseline as well as the Earth's mantle being
 2464 a relatively low-density material (as compared to the Earth's core). For some
 2465 values of δ_{CP} , the degeneracy in the number of e-like events allows the mass
 2466 hierarchy to be broken. This leads to a δ_{CP} -dependent mass hierarchy sensitivity
 2467 which can be seen in Figure 7.6.

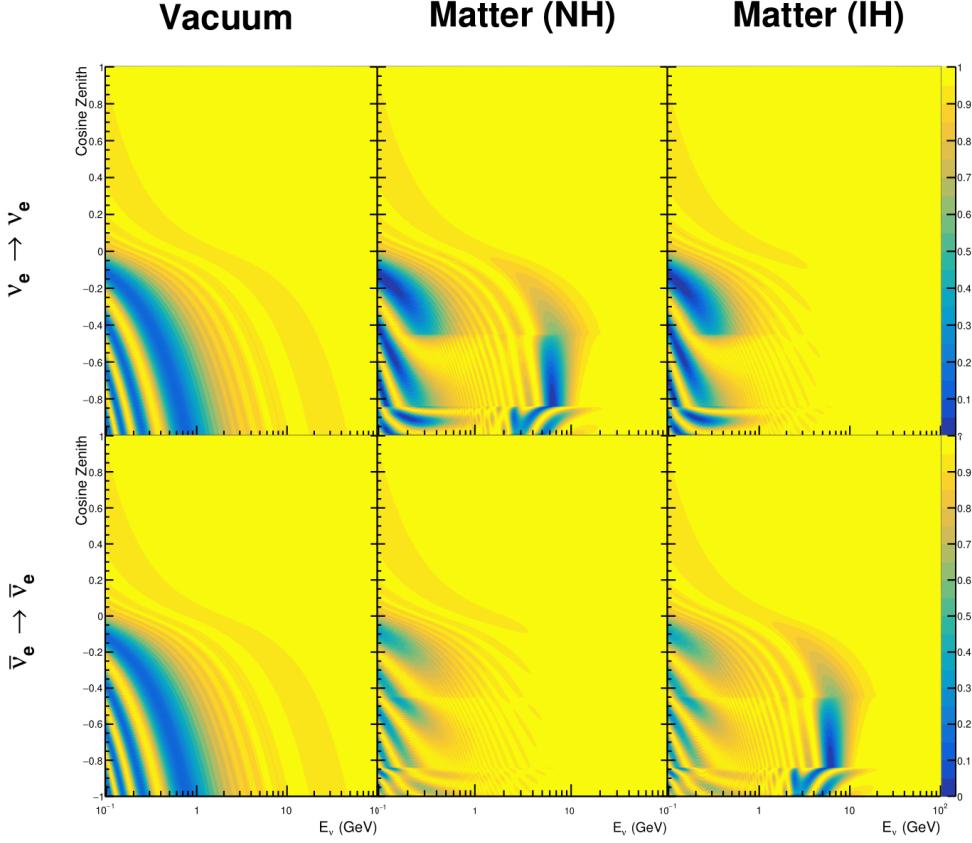


Figure 7.4: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations which reduce the number of oscillation probability calculations that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux. Furthermore, as the peak neutrino energy of the beam is well below the

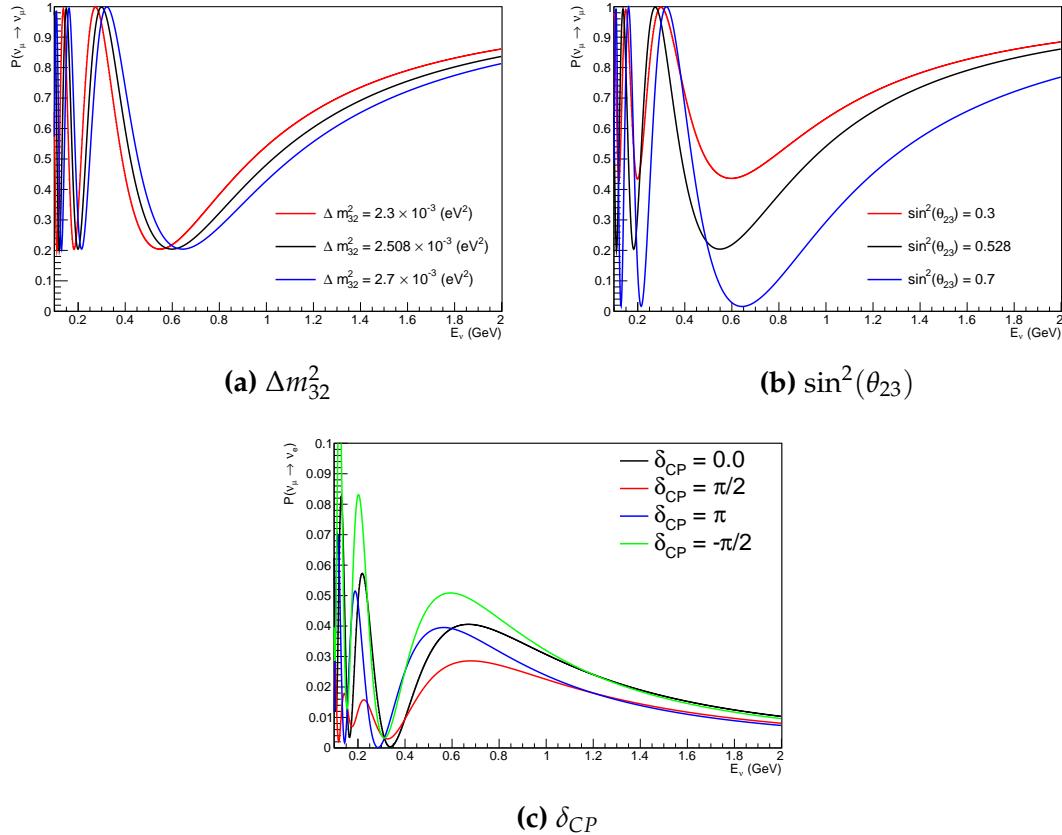


Figure 7.5: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [58]), only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very small due to their energy being well above the oscillation maximum (small value of L/E). Whilst these approximations have been made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold. These events can have non-negligible oscillation probabilities due to the further distance they travel.

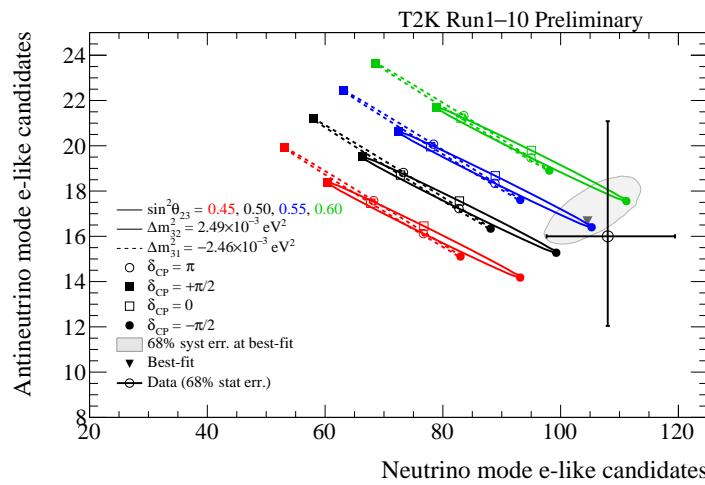


Figure 7.6: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{32}^2 are given.

2484 7.2 Treatment of Fast Oscillations

2485 As shown in Figure 7.7, atmospheric neutrino oscillations have a significantly
 2486 more complex structure for upgoing neutrinos with energy below 1GeV. This is
 2487 because the L/E dependence of the oscillation probability in this region induces
 2488 rapid variations for small changes in L or E . As discussed in section 7.1, this is also
 2489 the region in which atmospheric neutrinos have sensitivity to δ_{CP} . In practice,
 2490 the direction of the neutrino is inferred from the direction of the final state
 2491 particles traveling in the detector. The correlation between these two directions
 2492 can be particularly weak for low-energy neutrino interactions. This creates a
 2493 distinct difference from the beam neutrinos where the position of the source
 2494 is very precisely known.

2495 As a consequence of the unresolvable structure, an event rate consistent with
 2496 the averaged oscillation probability is observed in the subGeV upgoing region.
 2497 This creates a computational problem: A significantly large amount of Monte
 2498 Carlo statistics would be required to accurately predict the number of events
 2499 if Monte Carlo averaging was the only technique used. This section describes
 2500 the ‘sub-sampling’ approach developed for this analysis and compares it to the
 2501 methodology used within the SK-only analysis.

2502 **DB: Figure out what this actually means** The official SK-only analysis uses the
 2503 osc3++ oscillation parameter fitter [207]. To perform the fast oscillation averaging,
 2504 it uses a ‘nearest-neighbour’ technique. For a given Monte Carlo neutrino event,
 2505 the nearest twenty Monte Carlo neighbours in reconstructed lepton momentum
 2506 and zenith angle are found and a distribution of their neutrino energies is built.
 2507 The RMS, σ , of this distribution is then used to compute an average oscillation
 2508 probability for the given neutrino Monte Carlo event.

2509 For the i^{th} event, the oscillation weight is calculated as

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1,-0.5,0.5,1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.3)$$

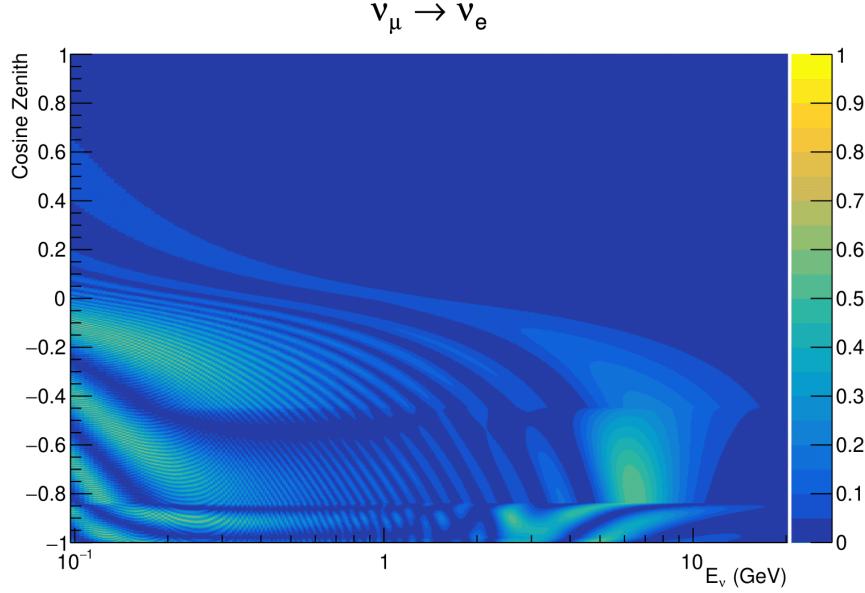


Figure 7.7: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2510 where $P(E, L)$ is the oscillation probability calculation for neutrino energy
 2511 E and path length L and the two path lengths, \bar{L}_i and L_β are discussed below.
 2512 All of the oscillation probability calculations are performed with a fixed zenith
 2513 angle such that the same density profile is used.

2514 The uncertainty in the production height is controlled by using an “average”
 2515 production height, \bar{L}_i , which represents the average path length computed using
 2516 twenty production heights taken from the Honda flux model’s prediction [52].
 2517 For a given event, the production heights are sampled in steps of 5% of their
 2518 cumulative distribution function. L_β values are similarly calculated but instead
 2519 use different combinations of four production heights,

$$\begin{aligned}
 L_{-1.0} &= \frac{1}{4}L(45, 50, 55, 60), \\
 L_{-0.5} &= \frac{1}{4}L(35, 40, 65, 70), \\
 L_{+0.5} &= \frac{1}{4}L(25, 30, 75, 68), \\
 L_{+1.0} &= \frac{1}{4}L(15, 20, 85, 89).
 \end{aligned} \tag{7.4}$$

This averaging technique works because of the inference between the zenith angle and the reconstructed direction of final state particles in the detector. For low-energy neutrinos, where the resolution of the true neutrino direction is poor, σ_i will be large, resulting in significant averaging effects. Contrary to this, the inferred direction of high-energy neutrinos will be much closer to the true value, meaning that σ_i will be smaller, culminating in small averaging effects.

In practice, these calculations are performed prior to the fit as only oscillation parameters at fixed points are considered. The MCMC technique used in this thesis requires oscillation probabilities to be evaluated at arbitrary parameter values, not known *a priori*. Calculating the five oscillation probabilities per event required by the SK technique is computationally infeasible, so a different averaging technique is used. However, the concept of the averaging technique can be taken from it.

To perform a similar averaging as the SK analysis, a sub-sampling approach using binned oscillograms has been devised. A coarsely binned oscillogram is defined in $\cos(\theta_Z)$ and E_ν . For a given set of oscillation parameters, a single oscillation probability will be assigned to each coarse bin. This value will then apply to all Monte Carlo events which fall into that bin. To assign these oscillation probabilities, the probability is calculated at $N \times N$ points on a grid within a particular bin. This ensemble of oscillation probabilities is averaged to define the coarse bin's oscillation probability, assuming a flat prior in E_ν and $\cos(\theta_Z)$ within the bin. Figure 7.8 illustrates the $N = 2$ example where the assigned value to a coarse bin is the average of the four fine bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear on either axis, the sub-division of the fine bins is linear within the range of a coarse bin.

The coarse binning is defined with 67×52 bins in true neutrino energy \times cosine zenith. It is picked to be identical to that provided in [209]. In general, the binning is logarithmically spaced in neutrino energy but has some hand-picked bin edges around the matter resonance to smoothly increased the bin density. This is to avoid smearing this region which can be well sampled by the Monte

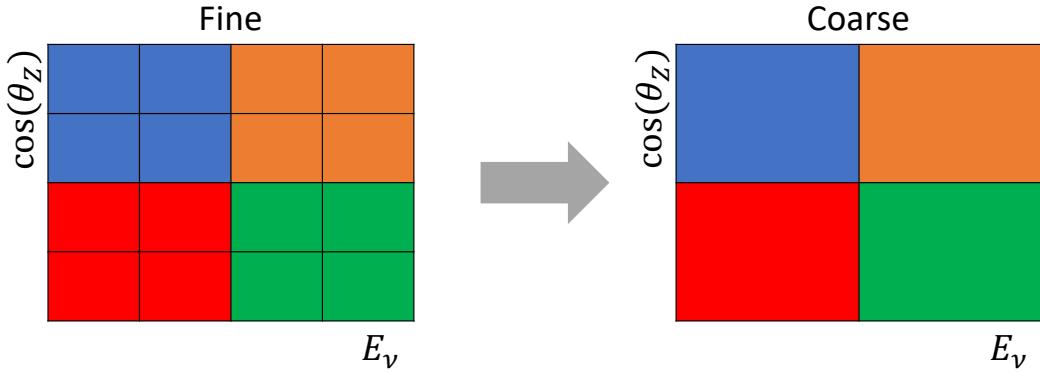


Figure 7.8: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2550 Carlo. The cosine zenith binning is approximately linearly spaced across the
 2551 allowable range but the values of layer transitions are hit precisely: -0.8376 (core-
 2552 mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for
 2553 downgoing events as this is a region unaffected by the fast oscillation wavelengths
 2554 and reduces the total number of calculations required to perform the calculation.

2555 The choice of N is justified based on two studies. Firstly, the variation of event
 2556 rates of each sample is studied as a function of N . For a given set of oscillation
 2557 parameters thrown from the PDG prior constraints (detailed in Table 2.1), the
 2558 oscillation probabilities are calculated using a given value of N . Each sample
 2559 is re-weighted and the event rate is stored. The value of N is scanned from
 2560 1, which corresponds to no averaging, to 19, which corresponds to the largest
 2561 computationally viable subdivision binning. The event rate of each sample at
 2562 large N is expected to converge to a stationary value due to the fine binning fully
 2563 sampling the small-scale structure. Figure 7.9 illustrates this behaviour for the
 2564 SubGeV_elike_0dcy sample for 9 different throws of the oscillation parameters.

2565 Denoting the event rate for one sample for a given throw t at each N by λ_t^N ,
 2566 the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The

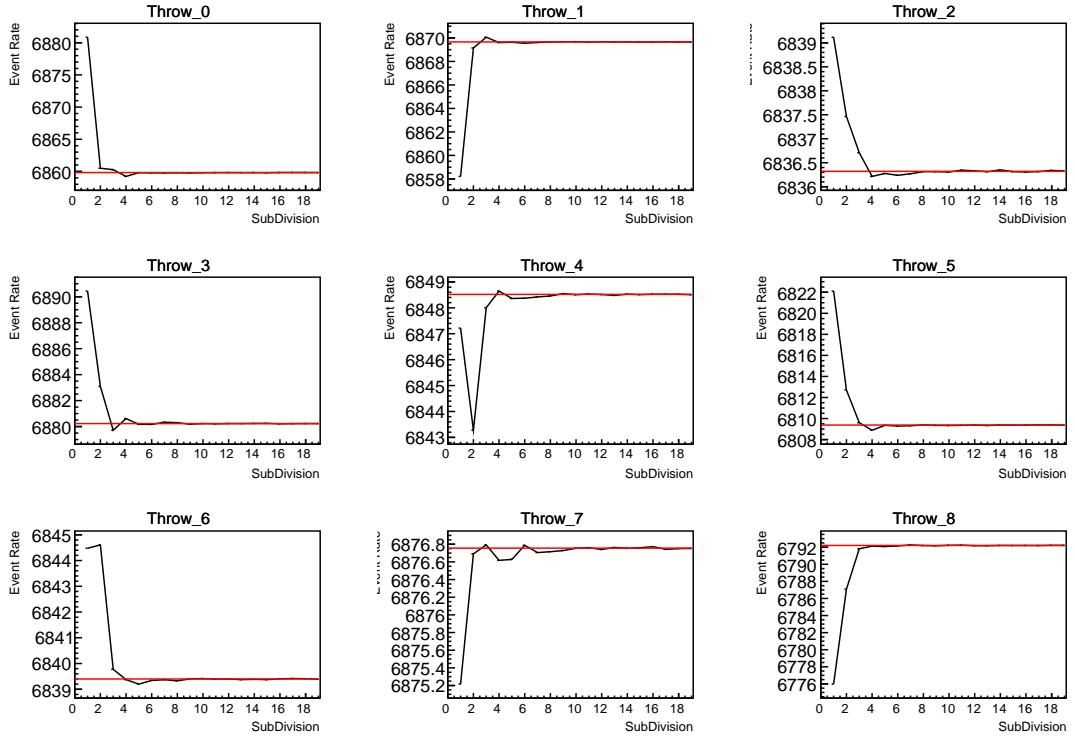


Figure 7.9: Event rate of the SubGeV_elike_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

variance in the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t) \right]^2. \quad (7.5)$$

In practice, the following procedure is undertaken. For a particular throw, the difference between the event rate at a particular choice of N and the mean of the distribution is calculated. This is illustrated in Figure 7.10. This value is then calculated for all the 2000 throws, generating a distribution of $\lambda_t^N - \bar{\lambda}_t$. This is repeated for each of the values of N considered within this study. The distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.11. As expected, the distribution gets narrower and tends towards zero for the higher values of N .

The aim of the study is to find the lowest value of N such that this variance is below 0.001. This utilises the width of the distributions given in Figure 7.11.

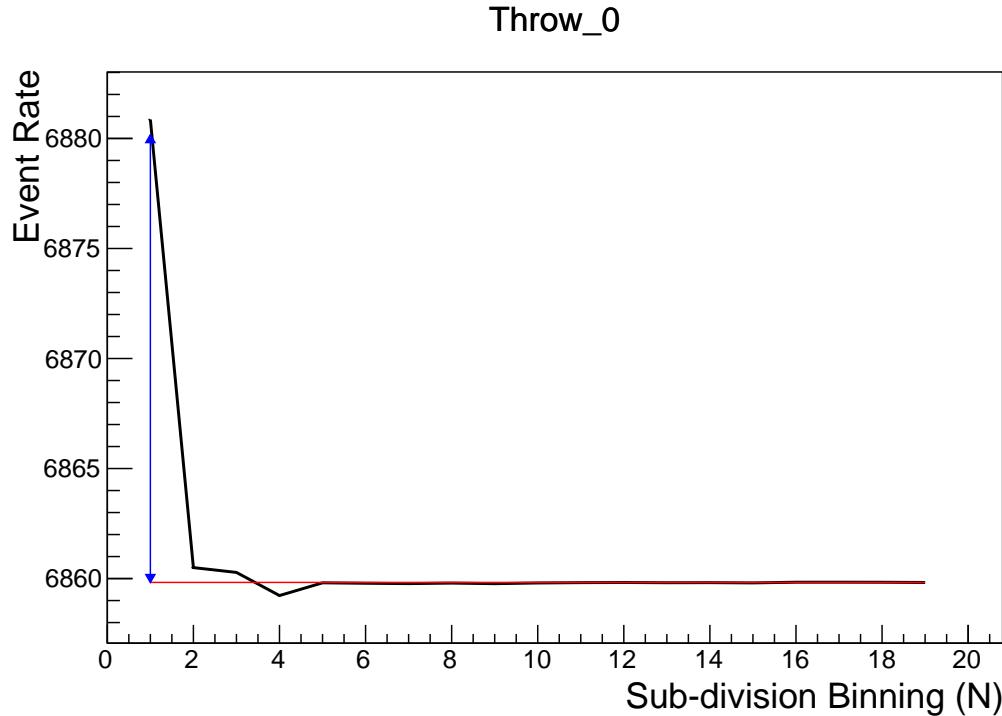


Figure 7.10: Event rate of the SubGeV_elike_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions, N , per coarse bin. The difference between the mean event rate (red), $\bar{\lambda}$, and the event rate at $N = 1$, $\lambda^{N=1}$ is defined as $\lambda^N - \bar{\lambda}$ and illustrated by the blue arrow.

2577 This is the typical threshold used by T2K fitters to validate systematic imple-
 2578 mentation so has been set as the same criteria. The results of this study for
 2579 each atmospheric sample used within this thesis are illustrated in Figure 7.12 for
 2580 2000 throws of the oscillation parameters. As can be seen, the variance is below
 2581 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy
 2582 and SubGeV_elike_0dcy samples.

2583 The second study to determine the value of N is as follows. The likelihood
 2584 for each sample is computed against an Asimov data set created with Asimov
 2585 A oscillation parameters (Table 2.2). Following Equation 7.5, the variance of
 2586 the log-likelihood over all considered N is computed. The results are shown
 2587 in Figure 7.13.

2588 A choice of $N = 10$ sub-divisions per coarse bin has a variance in both
 2589 event rate and log-likelihood residuals less than the required threshold of 0.001.

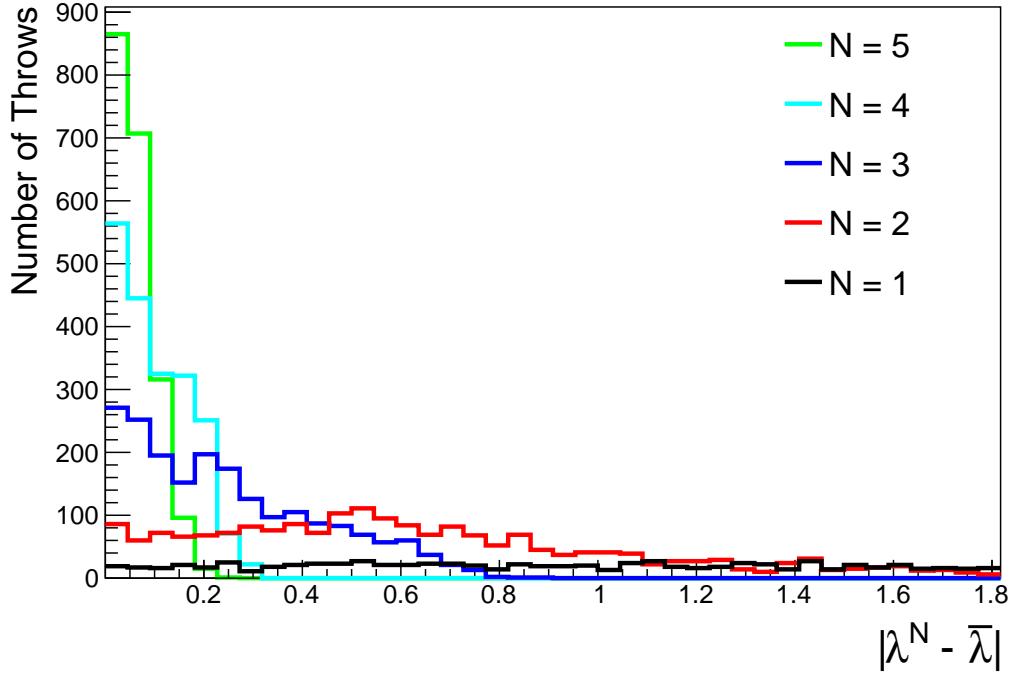


Figure 7.11: The distribution of $\lambda^N - \bar{\lambda}$ for various values of N . As expected, the distribution gets narrower for larger values of N .

2590 The largest value of the likelihood variance is of order 10^{-7} , corresponding to
 2591 an error on the log-likelihood of about 3×10^{-4} which is small enough to be
 2592 negligible for the oscillation analysis.

2593 Figure 7.14 illustrates the effect of the smearing using $N = 10$. The fast oscil-
 2594 lations in the sub-GeV upgoing region have been replaced with a normalisation
 2595 effect whilst the large matter resonance structure remains.

2596 7.3 Calculation Engine

2597 As previously discussed in section 7.2, the calculation of oscillation probabilities
 2598 is performed at run-time. Consequently, the time per calculation is crucial for fit
 2599 performance. The initial fitting framework used for this analysis was developed
 2600 with ProbGPU [210]. This is a GPU-only implementation of the prob3 engine
 2601 [211]. It is primarily designed for neutrino propagation in a beam experiment

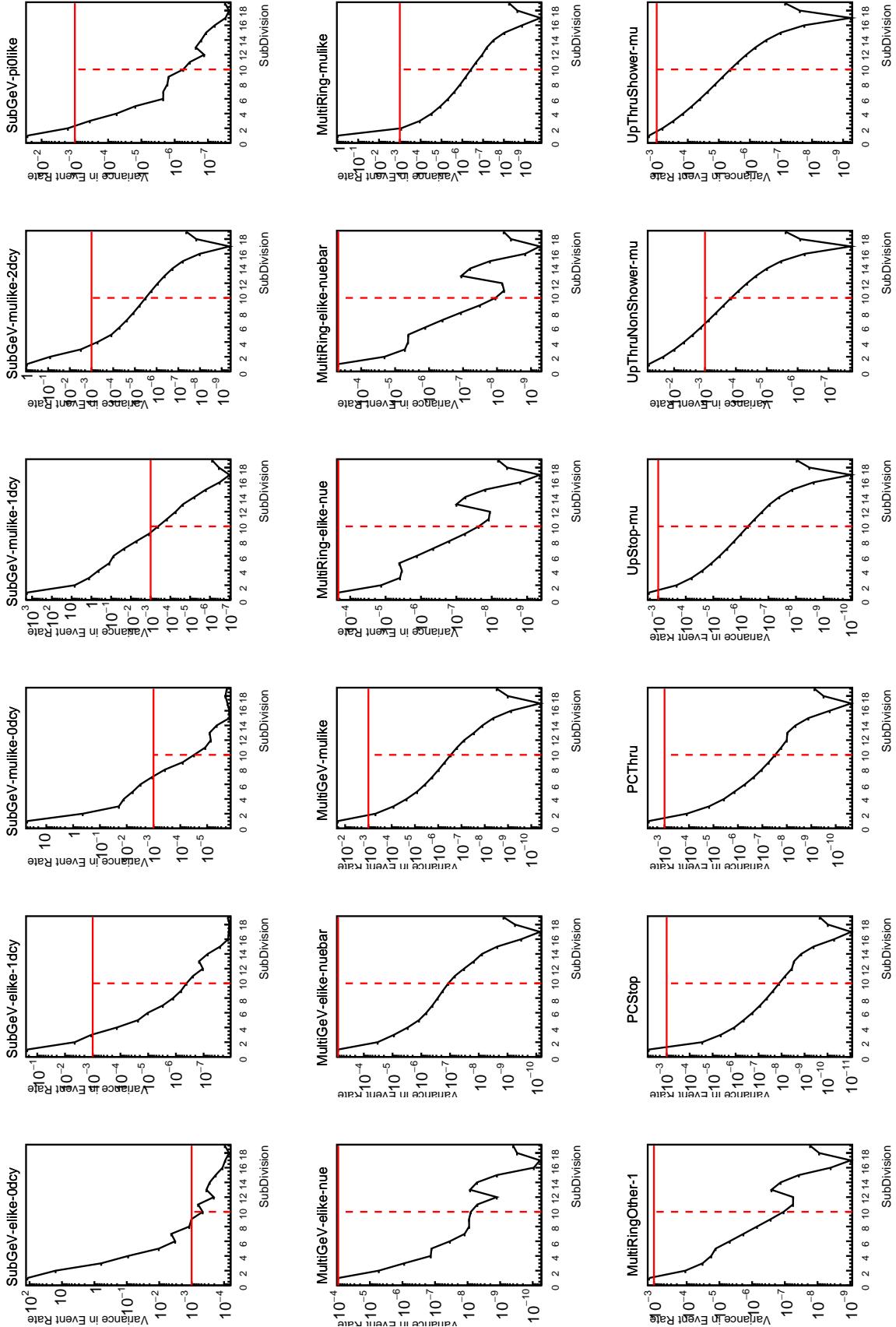


Figure 7.12: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

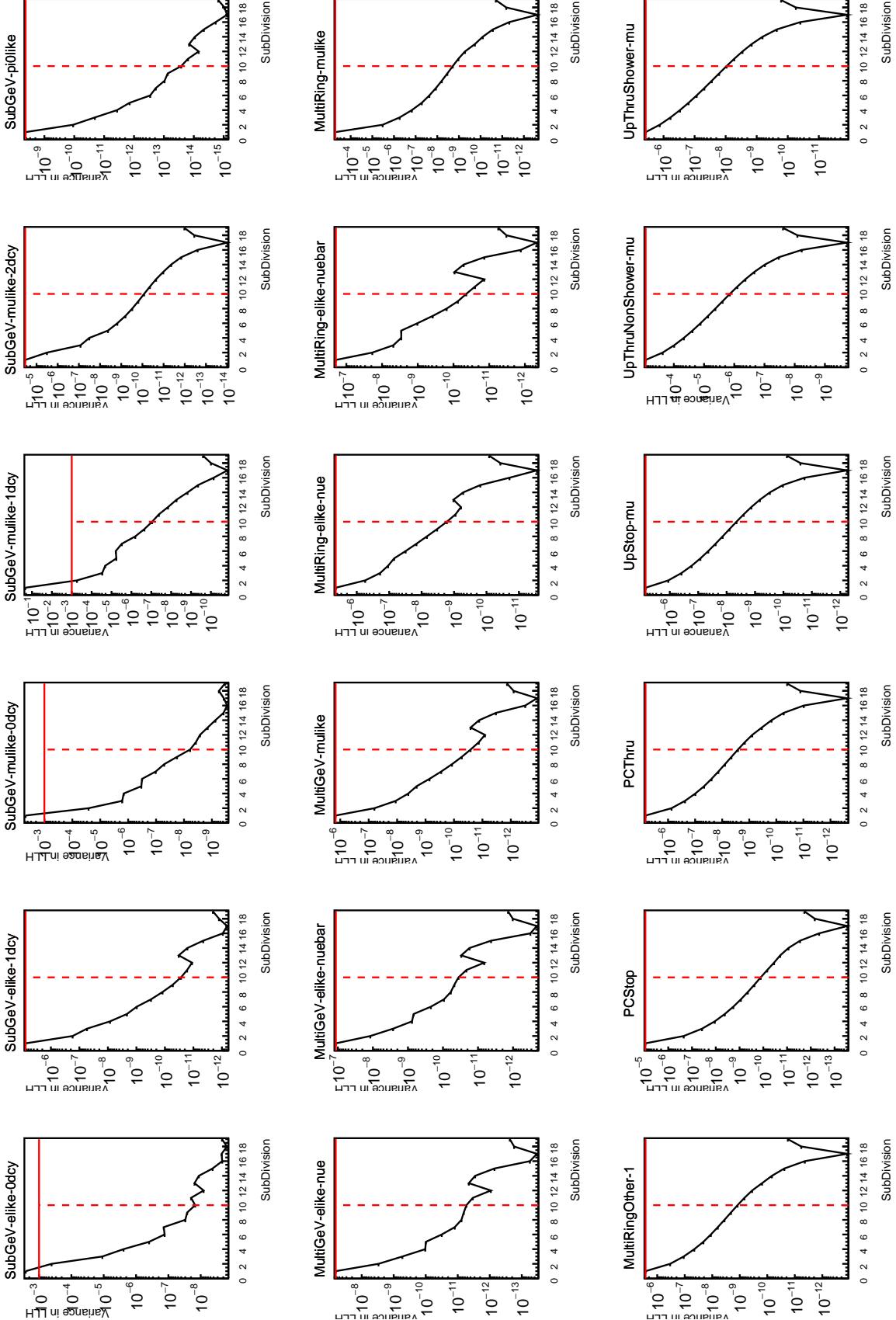


Figure 7.13: Variance of sample likelihood, when compared to ‘Asimov data’ set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

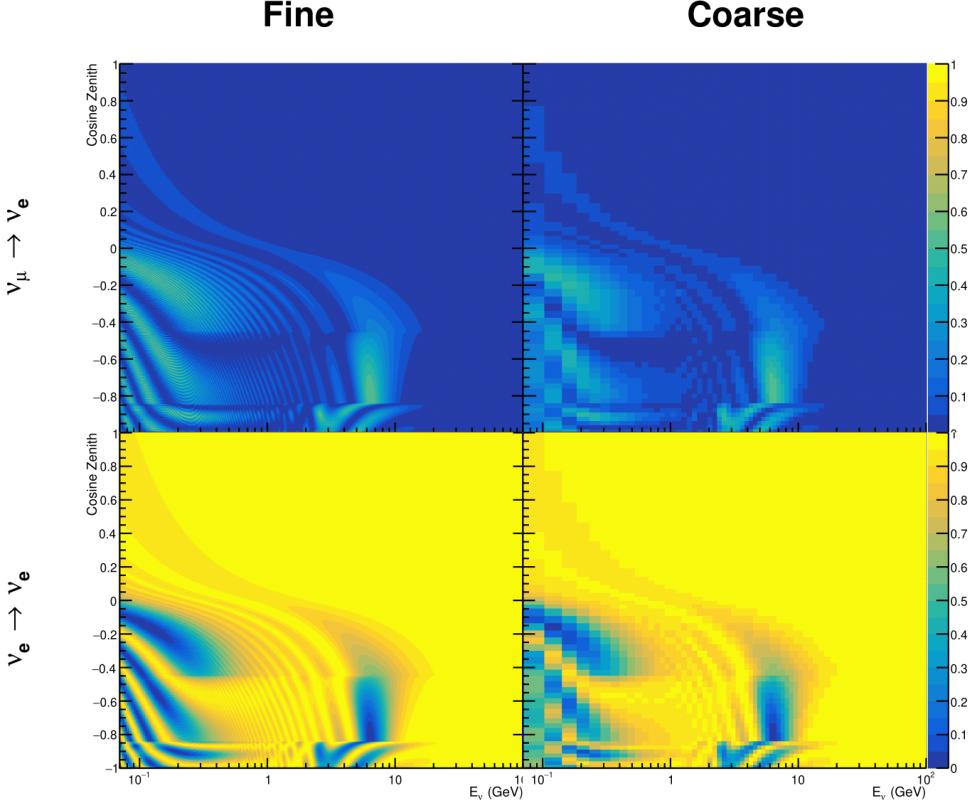


Figure 7.14: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.12 and Figure 7.13.

2602 (single layer of constant density) with the atmospheric propagation code not
 2603 being used prior to the analysis in this thesis.

2604 Another engine, CUDAProb3 [212], has been interfaced with the fitting frame-
 2605 work used in this analysis. This interfacing was done by the author of this
 2606 thesis. It has been specifically optimised for atmospheric neutrino oscillation
 2607 calculation so does not contain the code to replace the beam oscillation calculation.
 2608 The engine utilises object-orientated techniques as compared to the functional
 2609 implementation of ProbGPU. This allows the energy and cosine zenith arrays to
 2610 be kept on GPU memory, rather than having to load these arrays onto GPU

2611 memory for each calculation. Reducing the memory transfer between CPU and
 2612 GPU significantly reduces the time required for calculation. This can be seen
 2613 in Figure 7.15, where the GPU implementation of CUDAProb3 is approximately
 2614 three times faster than the ProbGPU engine.

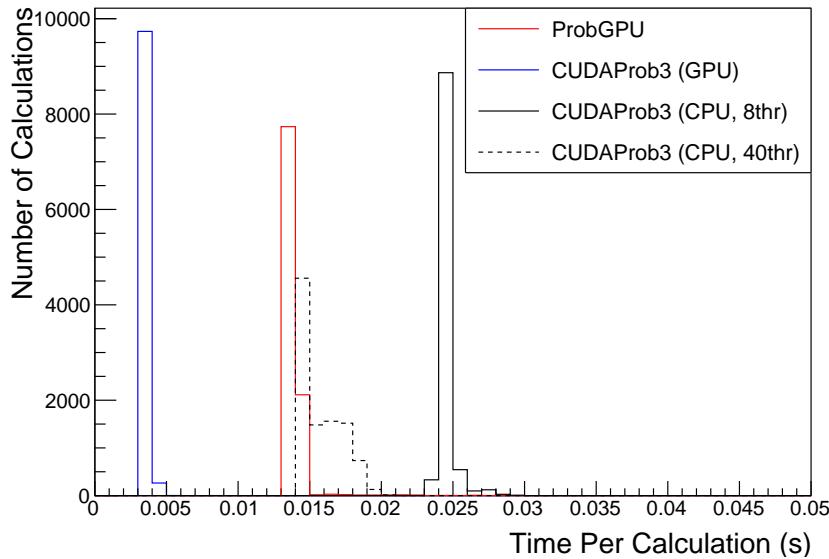


Figure 7.15: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.2, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multi-threaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

2615 Another significant advantage of CUDAProb3 is that it contains a CPU mul-
 2616 tithreaded implementation which is not possible with the ProbGPU or prob3
 2617 engines. This eliminates the requirement for GPU resources when submitting
 2618 jobs to batch systems. As illustrated in Figure 7.15, the calculation speed depends
 2619 on the number of available threads. Using 8 threads (which is typical of the
 2620 batch systems being used) is approximately twice as slow as the ProbGPU engine
 2621 implementation, but would allow the fitting framework to be run on many more
 2622 resources. This fact is utilised for any SK-only fits but GPU resources are required
 2623 for any fits which include beam samples due to the ProbGPU requirement. Based
 2624 on the benefits shown by the implementation in this section, efforts are being

2625 placed into including linear propagation for beam neutrino propagation into
2626 the engine [213].

2627 7.4 Matter Density Profile

2628 For an experiment observing neutrinos propagating through the Earth, a model
2629 of the Earth's density profile is required. The model used within this analysis is
2630 based on the Preliminary Reference Earth Model (PREM) [208], as illustrated in
2631 Figure 7.1. Table 7.1 documents the density and radii of the layers used within
2632 the constant density approximation used by the SK-only analysis [207]. The
2633 density measurements provided in the PREM model are provided in terms of
2634 mass density, whereas neutrino oscillations are sensitive to the electron number
2635 density. This value can be computed as the product of the chemical composition,
2636 or the Z/A value, and the mass density of each layer. Currently, the only way
2637 to measure the chemical composition value for layers close to the Earth's core
2638 is through neutrino oscillations. The chemical composition of the upper layers
2639 of the Earth's Mantle and the Transition zone is well known due to it being
2640 predominantly pyrolite which has a chemical composition value of 0.496 [214].
2641 The chemical composition dial for the core layers is set to a value of 0.468, as
2642 calculated in [215]. As this value is less well known, it is assigned a Gaussian error
2643 with a standard deviation equivalent to the difference in chemical composition
2644 in core and mantle layers. Figure 7.16 illustrates the effect of moving from
2645 the $Z/A = 0.5$ method which is used in the official SK-only analysis to these
2646 more precise values.

2647 The beam oscillation probability in this thesis uses a baseline of 295km, density
2648 $2.6\text{g}/\text{cm}^3$, and chemical composition 0.5 as is done by the official T2K-only
2649 analysis [216].

2650 For a neutrino with given E_ν , $\cos(\theta_Z)$, the oscillation probability calculation
2651 engine must be passed a list of the matter regions that the neutrino traversed,
2652 with the path length and fixed density in each region. However, a neutrino
2653 passing through the earth experiences a range of radii, and thus a range of

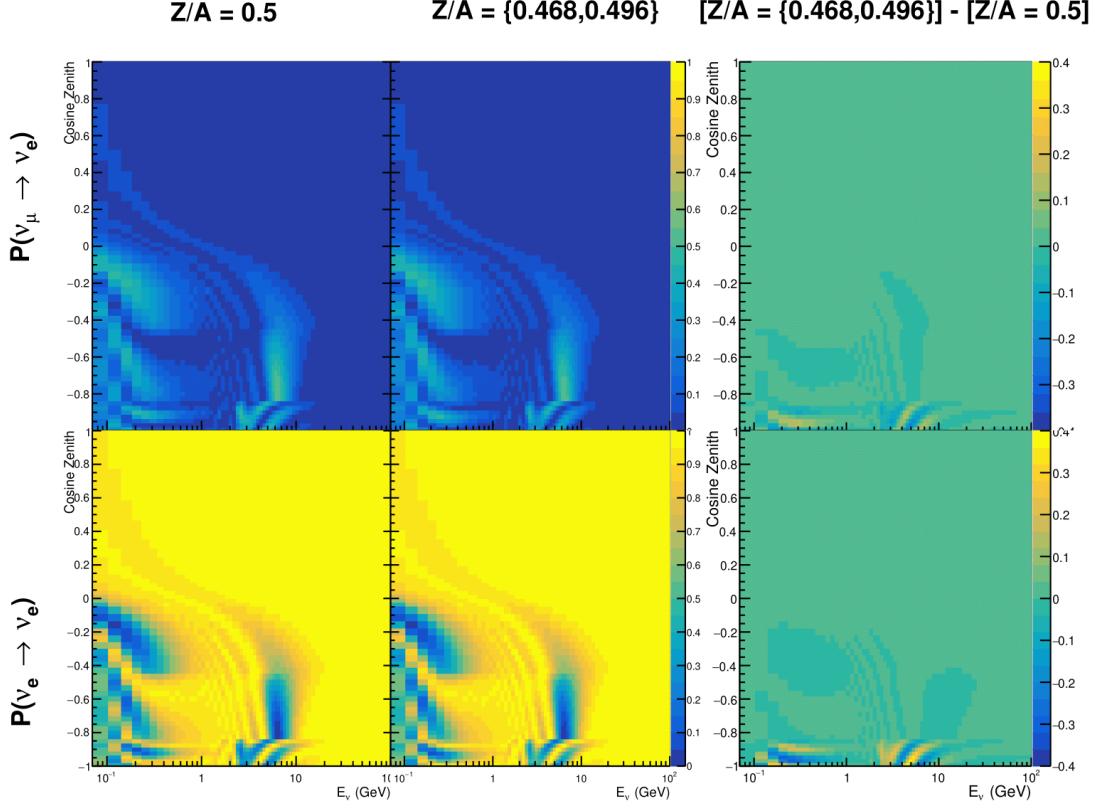


Figure 7.16: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 7.1. The right column illustrates the difference in oscillation probability between the two different techniques.

2654 densities, in each region. In the SK-only analysis, the earth density model used
 2655 is piecewise-constant, thereby ignoring this effect. For this thesis, the density
 2656 values for the calculation engine are found by averaging the earth density along
 2657 the neutrino's path in each layer,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt \quad (7.6)$$

2658 where t_i are the intersection points between each layer and t is the path length
 2659 of the trajectory across the layer. This leads to an improved approximation.
 2660 For this averaging, the simplification of the PREM model developed in [217] is

used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [209]. The coefficients of the quadratic fit to each layer are given in Table 7.2 with the final distribution illustrated in Figure 7.17. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

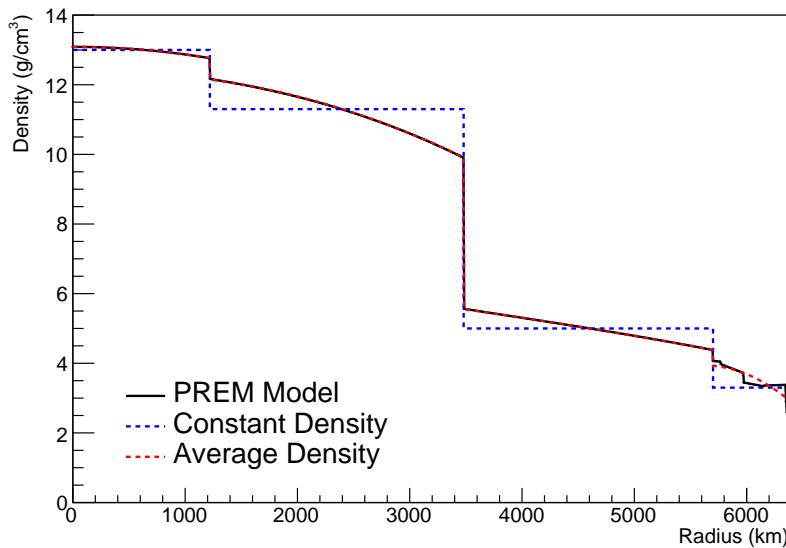


Figure 7.17: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.2: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [209], where $x = R/R_{Earth}$.

The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in Figure 7.18. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -0.45$ in the fixed density model, which is due to the transition to mantle layer boundary, has been reduced. This is expected as the difference in

the density across this boundary is significantly smaller in the quadratic density model as compared to the constant density model. Whilst the difference in density across the other layer transitions is reduced, there is still a significant difference. This means the discontinuities in the oscillation probabilities remain but are significantly reduced. However, as the quadratic density approximation matches the PREM model well in this region, these discontinuities are due to the Earth model rather than an artifact of the oscillation calculation.

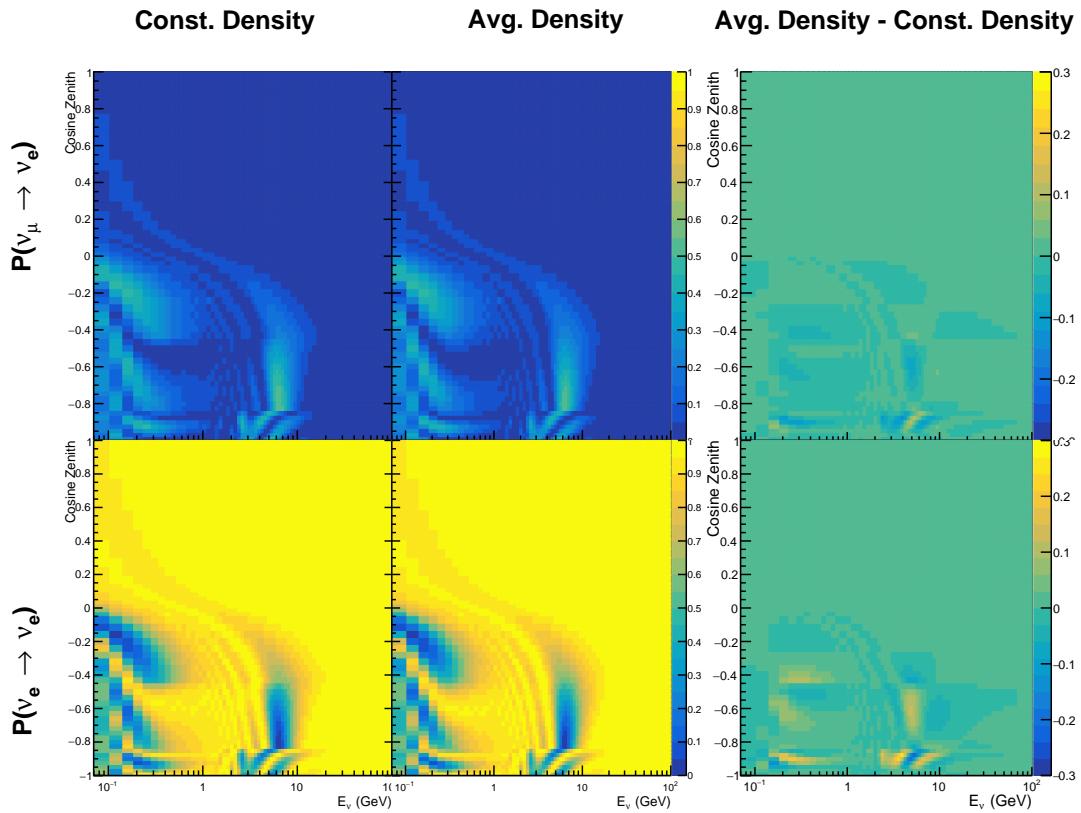


Figure 7.18: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2678 7.5 Production Height Averaging

2679 As discussed in section 7.1, the height at which the cosmic ray flux interacts
 2680 in the atmosphere is not known on an event-by-event basis. The production
 2681 height can vary from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only
 2682 analysis methodology (described in section 7.2) for including the uncertainty
 2683 on the production height is to include variations from the Honda model when
 2684 pre-calculating the oscillation probabilities prior to the fit. This technique is not
 2685 possible for this analysis which uses continuous oscillation parameters that can
 2686 not be known prior to the fit. Consequently, an analytical averaging technique
 2687 was developed in [209]. The author of this thesis was not responsible for the
 2688 derivation of the technique but has performed the implementation and validation
 2689 of the technique for this analysis alone.

2690 Using the 20 production heights per Monte Carlo neutrino event, provided
 2691 as 5% percentiles from the Honda flux model, a production height distribution
 2692 $p_j(h|E_\nu, \cos \theta_Z)$ is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a
 2693 histogram is filled with 20 evenly spaced bins in production height h between
 2694 0 and 50km. The neutrino energy and cosine zenith binning of the histogram
 2695 are the same as that provided in section 7.2. The average production height,
 2696 $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. This assumes a linear average over
 2697 the four flavours of neutrino which are considered to be generated in cosmic ray
 2698 showers. The production height binning of this histogram is then translated into
 2699 $\delta t(h) = t(\bar{h}) - t(h)$, where $t(h)$ is the distance travelled along the trajectory.

2700 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed.
 2701 The time-ordered product of these is then used as the overall transition amplitude
 2702 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.7)$$

2703 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.8)$$

2704 is expressed as a diagonalised time-dependent solution to the Schrodinger
 2705 equation. The 0th layer is the propagation through the atmosphere and is the
 2706 only term that depends on the production height. Using the substitution $t_0 =$
 2707 $t(\bar{h}) - \delta t(h)$, it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.9)$$

2708 Thus Equation 7.7 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.10)$$

2709 The oscillation probability averaged over production height is then calculated
 2710 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)^*_{ij} \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m) \delta t] \end{aligned} \quad (7.11)$$

2711 It is important to note that the exact value of \bar{h} used does not matter as the
 2712 values of δt would change to compensate for any modification to the value of \bar{h} .

2713 In practice, implementation in CUDAProb3 [212] is relatively straightforward
 2714 as the majority of these terms are already calculated in the standard oscillation
 2715 calculation. Figure 7.19 illustrates the results of the production height averaging.

²⁷¹⁶ As expected, the main effect is observed in the low-energy downward-going
²⁷¹⁷ and horizontal-going events. Upward-going events have to travel the radius
²⁷¹⁸ of the Earth, $R_E = 6371\text{km}$, where the production height uncertainty is a small
²⁷¹⁹ fraction of the total path length.

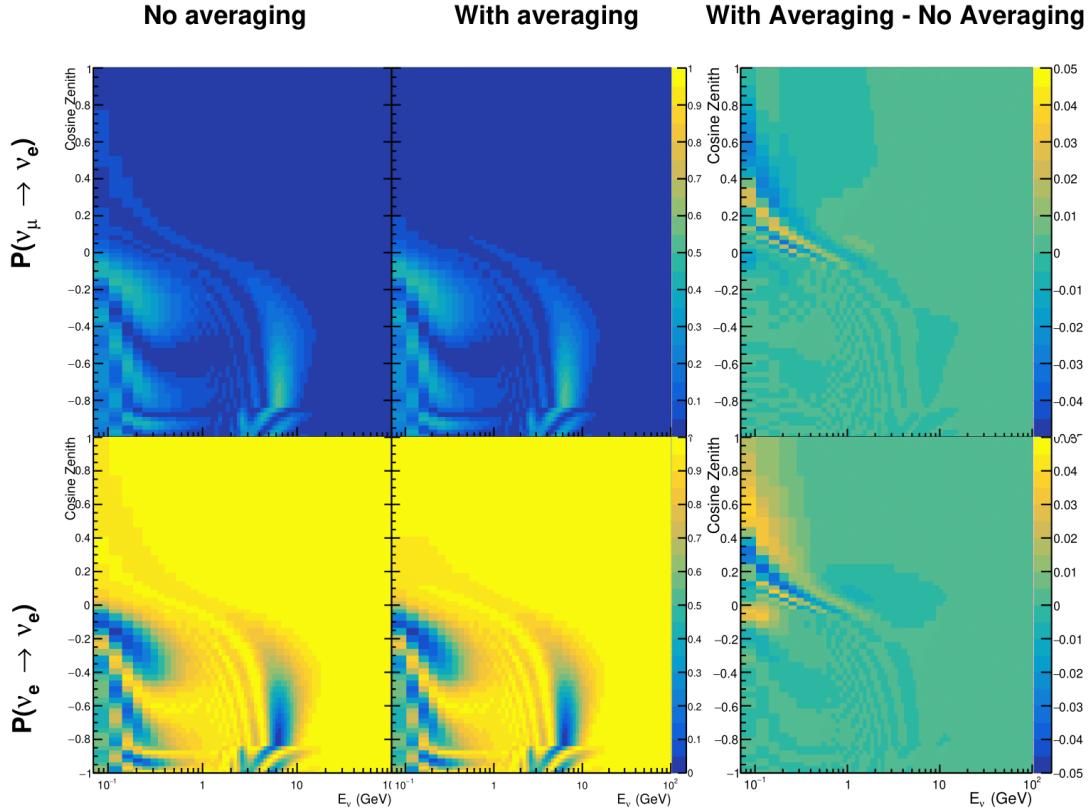


Figure 7.19: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

8

2720

2721

Oscillation Analysis

2722 Using the samples and systematics defined in chapter 6, this chapter documents
2723 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK
2724 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4
2725 is used for all studies performed within this thesis.

2726 The MaCh3 framework used throughout this thesis has been validated through
2727 many tests. The code which handles the beam far detector samples was developed
2728 by the author and validated by comparison to the 2020 T2K analysis [74]. The
2729 sample event rates and likelihood evaluations of beam samples generated by
2730 the framework used within this thesis were compared to those from the T2K
2731 analysis by the author of this thesis. Variations of the sample predictions were
2732 compared at $\pm 1\sigma$ and $\pm 3\sigma$ and good agreement was found in all cases. A similar
2733 study, led by Dr. C. Wret was used to validate the near detector portion of
2734 the code [202]. The implementation of the atmospheric samples within MaCh3
2735 was completed and cross-checked by the author of this thesis against the P-
2736 Theta framework (Introduced in section 3.2). Both fitters are provided with
2737 the same inputs and can therefore cross-validate each other. These validations
2738 compared the event rate and likelihood calculation. Documentation of all the
2739 above validations can be found in [202].

2740 8.1 Monte Carlo Prediction

2741 Using the three sets of dial values (generated, pre-BANFF, and post-BANFF tunes)
 2742 defined in subsection 6.4.3, the predicted event rates for each sample are defined
 2743 in Table 8.1. The oscillated (AsimovA defined in Table 2.2) and un-oscillated
 2744 event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThru	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1: The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

2745 Generally, the samples which target CCQE interaction modes observe a
 2746 decrease in prediction when using the pre-fit dial values. This is in accordance
 2747 with the Monte Carlo being produced assumed $M_A^{QE} = 1.21\text{GeV}$ [162] whilst

2748 the pre-fit dial value is set to $M_A^{QE} = 1.03\text{GeV}$ as suggested by [200]. Further-
 2749 more, the predicted event rates of samples that target CCRES interaction modes
 2750 are significantly reduced when considering the post-BANFF fit. This follows
 2751 the observations in subsection 6.4.3. The strength of the accelerator neutrino
 2752 experiment can be seen in the remarkable difference between the oscillated and
 2753 unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very
 2754 clear decrease in the expected event rate between the oscillated and un-oscillated
 2755 predictions which is not as obvious as in the atmospheric samples. This is due to
 2756 the fact that the beam energy is tuned to the maximum disappearance probability
 2757 which is not the case for the naturally generated atmospheric neutrinos.

2758 8.2 Likelihood Calculation

2759 DB: Should this section be moved?

2760 This analysis performs a joint oscillation parameter fit of the ND280 beam
 2761 samples, the T2K far detector beam samples, and the SK atmospheric samples.

2762 Once the Monte Carlo predictions of each beam and atmospheric sample
 2763 have been built (see chapter 6) a likelihood needs to be constructed. This is done
 2764 by comparing the binned Monte Carlo prediction to binned data. The Monte
 2765 Carlo prediction is calculated at a particular point, $\vec{\theta}$, in the model parameter
 2766 space such that $N_i^{MC} = N_i^{MC}(\vec{\theta})$, where N_i represents the bin content of the
 2767 i^{th} bin. The data and Monte Carlo spectra are represented by N_i^D and N_i^{MC} ,
 2768 respectively. The bin contents for the beam near detector, beam far detector and
 2769 atmospheric samples are denoted with ND , FD , and Atm , respectively. The
 2770 binning index, i , runs over all the bins within a sample. Taking the FHC1Rmu far
 2771 detector sample as an example, the binning index runs over all the reconstructed
 2772 neutrino energy bins. The likelihood calculation between the data and the Monte
 2773 Carlo prediction for a particular bin follows a Poisson distribution, where the
 2774 data is treated as a fluctuation of the simulation.

2775 The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction,
 2776 which is typically used for sensitivity studies and denoted ‘Asimov data’. The

process for building Asimov data is as follows. The Monte Carlo prediction is reweighted using a particular set of oscillation parameters (potentially those listed in Table 2.2) and systematic parameter tune. The resulting spectra for each sample is then defined to be the Asimov data for that sample. Whilst this results in unphysical non-integer data predictions, it eliminates statistical fluctuations from the data. Therefore, the results of a fit to Asimov data should not include any biases from statistical fluctuations. Furthermore, these results should produce posterior probability distributions consistent with the parameters which were used to make the data prediction. That is to say, the fit results should return the known parameters. Any biases seen would be attributed to correlations between each oscillation parameter and correlations between oscillation and systematic parameters. Consequently, Asimov fit results present the maximum precision at which the oscillation parameters could be measured to.

Following the T2K analysis presented in [74], the likelihood contribution for the near detector samples also includes a Monte Carlo statistical uncertainty term, derived from the Barlow and Beeston statistical treatment [218, 219]. In addition to treating the data as a Poisson fluctuation of the Monte Carlo prediction, it includes a contribution to the likelihood that which treats the generated Monte Carlo prediction as a statistical fluctuation of the actual true simulation assuming an infinite amount of statistics had been created. The technical implementation of this additional likelihood term is documented in [191] and briefly summarised as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (8.1)$$

where β_i represents a scaling parameter for the i^{th} bin that relates the bin content for the amount of Monte Carlo actually generated N_i^{MC} to the bin content if an infinite amount of Monte Carlo statistics had been generated $N_{i,true}^{MC}$, such that $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$. In the case where a sufficient amount of Monte Carlo statistics had been generated, $\beta_i = 1$. An analytical solution for β_i is given in

[191]. Additionally, $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2} / N_i^{MC}$ where $\sqrt{\sum_i w_i^2}$ represents the sum of the square of the weights of the Monte Carlo events which fall into bin i .

An additional contribution to the likelihood comes from the variation of the systematic model parameters. For those parameters with well-motivated uncertainty estimates, a covariance matrix, V , describes the prior knowledge of each parameter as well as any correlations between the parameters. Due to a technical implementation, a single covariance matrix describes each “block” of model parameters, e.g. beam flux systematics. The covariance matrix associated with the k^{th} block is denoted V^k . This substitution results in $\vec{\theta} = \sum_k^{N_b} \vec{\theta}^k$ and $V = \sum_k^{N_b} V^k$ where N_b denotes the number of blocks. A single covariance matrix is provided for: the oscillation parameters, the beam flux parameters, the atmospheric flux parameters, the neutrino interaction systematics, the near detector parameters, the beam far detector systematics, and the atmospheric far detector systematics. All of these are detailed in section 6.4. The number of parameters in the k^{th} block is defined as $n(k)$.

The equation for the likelihood \mathcal{L} includes all the terms discussed above.

It is defined as,

$$\begin{aligned}
 -\ln(\mathcal{L}) = & \\
 & \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
 & + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
 & + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
 & + \frac{1}{2} \sum_k^{N_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
 \end{aligned} \tag{8.2}$$

The negative log-likelihood value is determined at each step of the MCMC to build the posterior distribution defined in chapter 4. This value is minimised when the Monte Carlo prediction tends towards the data spectrum.

8.2.1 Likelihood Scans

Using the definition of the likelihood presented in section 8.2, the response of each sample to a variation of a particular parameter can be studied. Figure 8.1 presents the variation of all the samples (beam and atmospheric) at the far detector to the oscillation parameters of interest: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . These plots are colloquially called ‘likelihood scan’ (or ‘log-likelihood scans’). The process of making these plots is as follows. An Asimov data set (following technique detailed in section 8.2) is built using the AsimovA oscillation parameters and pre-fit systematic tune. The Monte Carlo is then reweighted using the value of the oscillation parameter at each point on the x-axis of the scan. The likelihood is then calculated between the Asimov data and Monte Carlo prediction and plotted. This process identifies which samples drive the determination of the oscillation parameters in the joint fit.

Due to the caveat of fixed systematic parameters and the correlations between oscillation parameters being ignored when creating these likelihood scans, the value of $\chi^2 = 1$ (or $-2 \times \ln(\text{Likelihood}) = 1$) does not equate to the typical 1σ sensitivity. However, it does give an indication of which samples respond the strongest to a variation in a particular oscillation parameter. The point at which the likelihood tends to zero illustrates the value of the parameter used to build the Asimov data prediction.

The sensitivity to $\sin^2(\theta_{23})$ is mostly dominated by the beam muon-like samples. The response of an individual atmospheric sample is small but non-negligible such that the summed response over all atmospheric samples becomes comparable to that of the muon-like beam samples. Consequently, the sensitivity of the joint fit to $\sin^2(\theta_{23})$ would be expected to be greater than the beam-only analysis. The only sample which respond to the $\sin^2(\theta_{13})$ oscillation parameter is the electron-like beam samples. Consequently, no increase in sensitivity beyond that of the T2K-only analysis would be expected from the joint fit. Regardless, the sensitivity of the beam sample is significantly weaker than the external reactor constraint so prior knowledge will dominate any measurement that is included

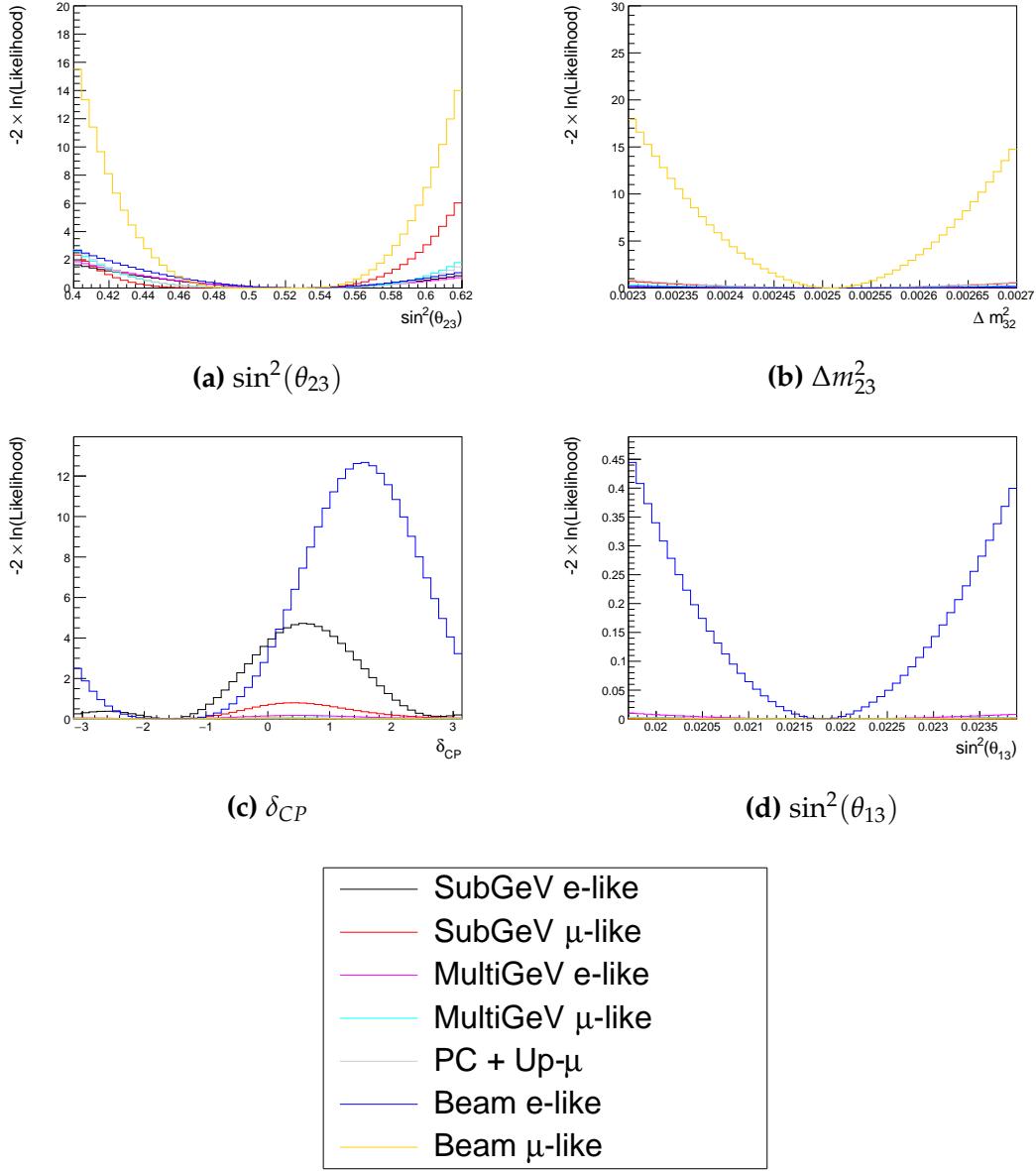


Figure 8.1: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to a variation of an oscillation parameter.

within this thesis. The Δm_{21}^2 and $\sin^2(\theta_{12})$ parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to Δm_{32}^2 is completely dominated by the beam muon-like samples. This is because the beam neutrino energy can be specifically tuned to match the maximal disappearance probability. Despite this, improvements to the $|\Delta m_{32}^2|$ sensitivity may be expected due to additional mass hierarchy determination

2860 added by the atmospheric samples.

2861 Two-dimensional scans of the appearance ($\sin^2(\theta_{13})$ - δ_{CP}) and disappearance
2862 ($\sin^2(\theta_{23})$ - Δm_{32}^2) parameters are illustrated in Figure 8.2 and Figure 8.3, respec-
2863 tively. The caveat of fixed systematic parameters and correlations between other
2864 oscillation parameters being neglected still apply.

2865 The appearance log-likelihood scans show the distinct difference in how
2866 the beam and atmospheric samples respond. The beam samples have an ap-
2867 proximately constant width of the 2σ and 3σ contours, throughout all ranges
2868 of δ_{CP} . The response of the atmospheric samples to $\sin^2(\theta_{13})$ is very strongly
2869 correlated to the value of δ_{CP} being evaluated, with the strongest constraints
2870 around $\delta_{CP} \sim 1$. Consequently, this difference allows some of the degeneracy in
2871 a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood
2872 scans, the 2σ continuous contour in δ_{CP} for beam samples is broken when the
2873 atmospheric samples are added. This may result in a stronger sensitivity to
2874 δ_{CP} . Similarly, the width of the 3σ contours also becomes dependent upon the
2875 value of δ_{CP} . Whilst these are encouraging results for the joint fit, these are
2876 not sensitivity measurements as the systematic parameters are fixed and the
2877 correlation between oscillation parameters is neglected. An interesting point to
2878 note is that the atmospheric samples have little sensitivity to $\sin^2(\theta_{13})$ on their
2879 own, as evidenced in Figure 8.1, but can improve sensitivity to the parameter
2880 when combined within the simultaneous fit.

2881 The response of the atmospheric samples in Figure 8.2 shows an interesting
2882 behaviour when considering the application of the reactor constraint. At higher
2883 values of $\sin^2(\theta_{13})$, two lobes appear around $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2.4$. If this
2884 distribution was projected onto the δ_{CP} axis, these lobes would mean the posterior
2885 distribution would have a significant dip between these values. However, the
2886 region of $\sin^2(\theta_{13})$ near the reactor constraint ($\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$)
2887 is flatter across the range of δ_{CP} . Therefore, if we were to project only this
2888 region onto the δ_{CP} axis, the dip between the peaks would not be as significant.
2889 If this behaviour was to be seen in the results of a fit, these marginalisation

2890 effects would actually conspire to reduce the sensitivity to δ_{CP} if the reactor
2891 constraint was to be applied.

2892 The disappearance log-likelihood scans in $\sin^2(\theta_{23})$ - Δm_{32}^2 space (Figure 8.3)
2893 show the expected behaviour when considering the one-dimensional scans
2894 already discussed. The uncertainty on the width of $|\Delta m_{32}^2|$ is mostly driven
2895 by the beam-only sensitivities. However, the width of this contour in the
2896 inverted mass region ($\Delta m_{32}^2 < 0$) is significantly reduced due to the ability
2897 of the atmospheric samples to select the correct (normal) mass hierarchy. The
2898 width of the uncertainty in $\sin^2(\theta_{23})$ is also reduced compared to the beam-only
2899 sensitivities, with a further decrease in the inverted hierarchy region due to the
2900 better mass hierarchy determination.



Figure 8.2: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})$ - δ_{CP}) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

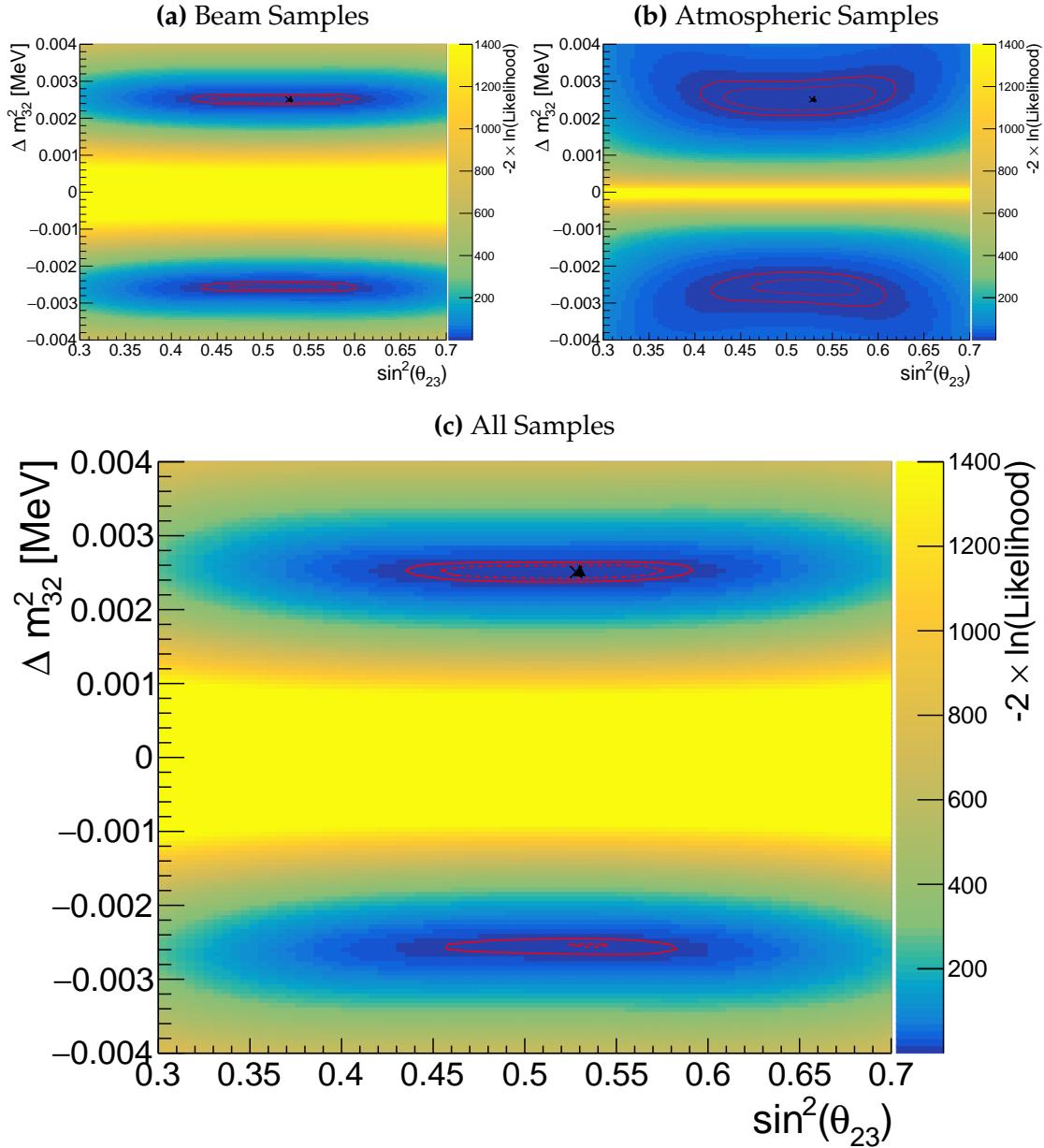


Figure 8.3: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23})$)- Δm_{32}^2) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

The likelihood scans illustrated thus far only consider the sensitivity of this analysis for a fixed set of true oscillation parameters, namely Asimov A defined in Table 2.2. Whilst computational infeasible to run many fits at different parameter sets, it is possible to calculate the likelihood response to different Asimov data sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing true values of δ_{CP} and $\sin^2(\theta_{23})$, respectively. For both of these plots, the other oscillation parameters are fixed at their Asimov A values. Consequently, the caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still applies.

To explain how these plots are made, consider Figure 8.4. This plot is built by considering multiple one-dimensional log-likelihood scans, each creating an Asimov data with the value of δ_{CP} taken from the x-axis. The likelihood to a particular Asimov data set is calculated after reweighting the Monte Carlo prediction to each value of δ_{CP} on the y-axis.

Figure 8.4 illustrates the sensitivity to δ_{CP} . Notably, the 1σ intervals contain regions in the off-diagonal for which the beam and atmospheric samples have broken and discontinuous contours. This indicates that there are regions of δ_{CP} which are degenerate. For example, for the x-axis value of $\delta_{CP} = 0$, the beam samples sensitivity would include two discontinuous regions excluded from the 1σ interval: $\delta_{CP} \sim 0$ and $\delta_{CP} \sim \pi$. The offset in δ_{CP} between the beam and atmospheric samples allows the joint fit to have increased sensitivity in these regions. Consequently, the difference between the beam-only and joint beam-atmospheric fit should be studied using multiple Asimov data sets.

Despite the increased sensitivity at 1σ , the 2σ intervals from the joint fit are more similar to the two independent sensitivities and the off-diagonal degeneracies mostly remain. This indicates that the joint fit has the strength to aid parameter determination but can not entirely break the degeneracies in δ_{CP} at higher confidence levels.

Figure 8.5 illustrates a similar analysis as above, although the value of $\sin^2(\theta_{23})$ is varied and δ_{CP} is fixed to the Asimov A parameter value. Due to the beam

parameters and baseline being tuned to specifically target this oscillation parameter, the average sensitivity of the beam samples is stronger than the atmospheric samples. However, the degeneracy around maximal mixing ($\sin^2(\theta_{23}) = 0.5$) is significantly more peaked in the beam samples compared to the atmospheric samples. This behaviour is strengthened when considering the 2σ intervals, to the point where two distinct discontinuous regions of the 2σ intervals exist around the Asimov point $\sin^2(\theta_{23}) \sim 0.41, 0.6$. Given the caveat of only considering likelihood scans, the joint analysis would mostly eliminate the discontinuous intervals in these regions. This means that the joint fit could feasibly have an increased preference for the correct octant hypothesis.

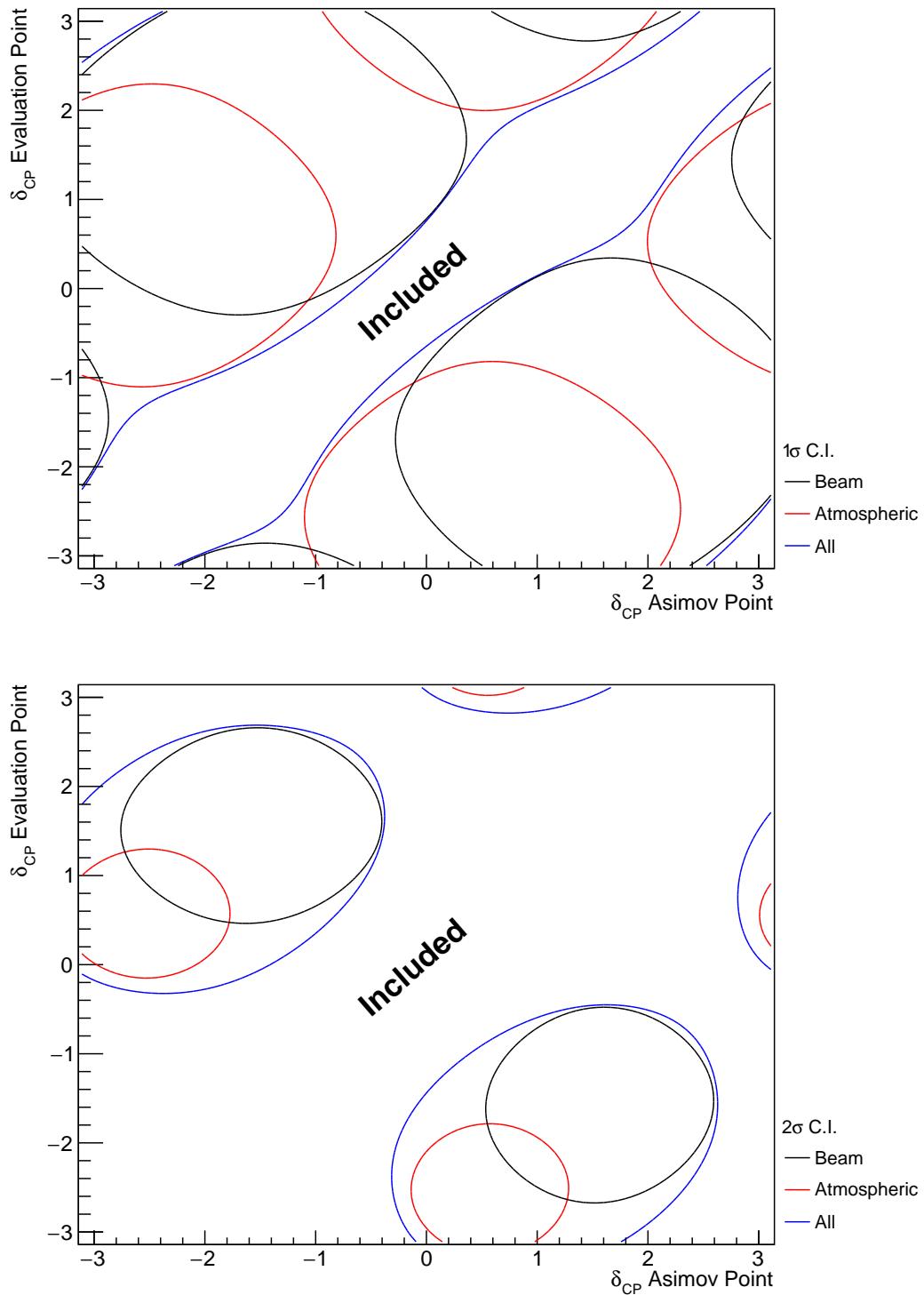


Figure 8.4: A series of one-dimensional likelihood scans over δ_{CP} , where an Asimov data set is built for each value of δ_{CP} on the x-axis and the likelihood is evaluated for each value of δ_{CP} on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

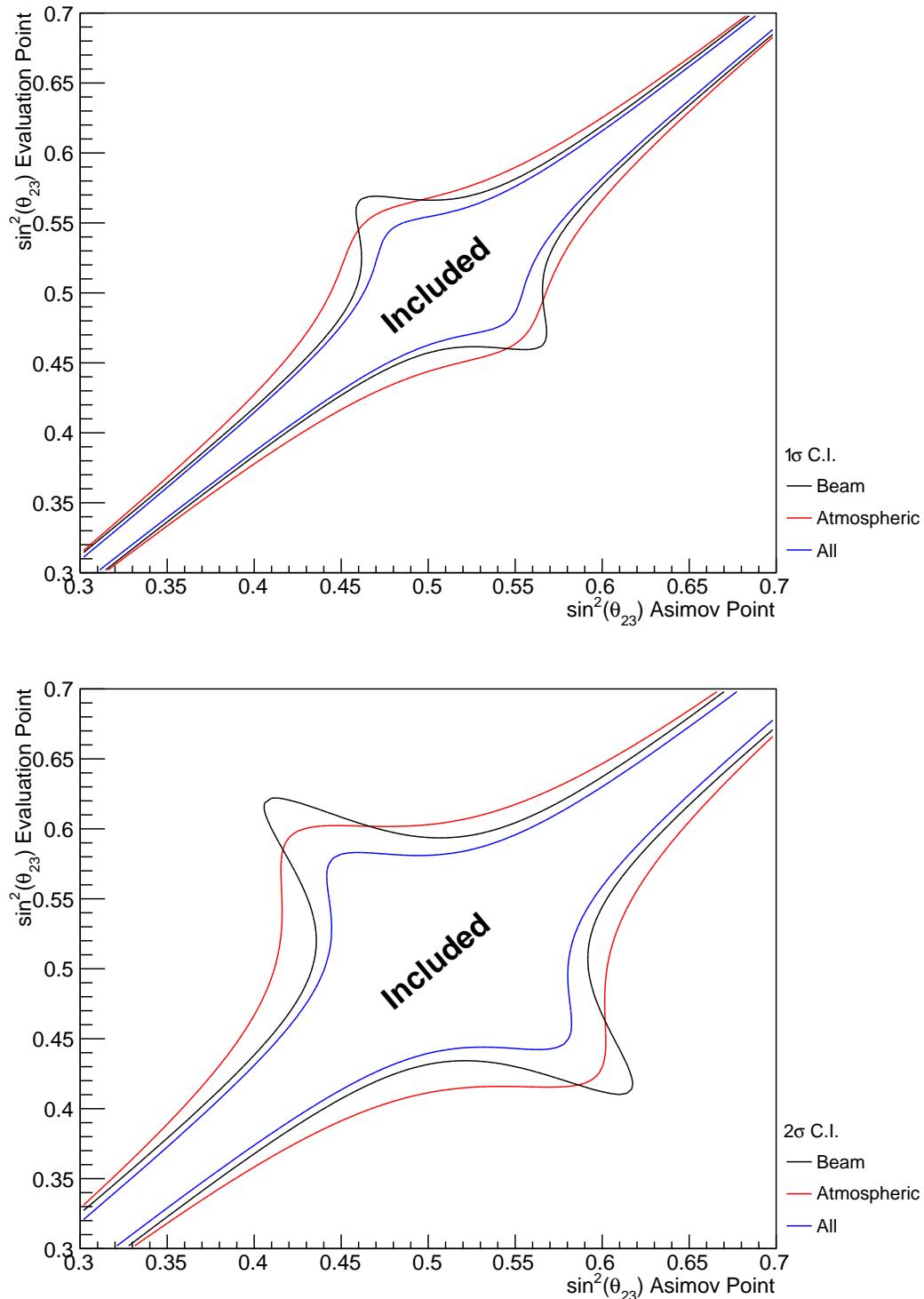


Figure 8.5: A series of one-dimensional likelihood scans over $\sin^2(\theta_{23})$, where an Asimov data set is built for each value of $\sin^2(\theta_{23})$ on the x-axis and the likelihood is evaluated for each value of $\sin^2(\theta_{23})$ on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than what they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions, M_A^{QE} , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h_Cto0 normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics which have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric samples are significantly more sensitive to these systematics than the beam samples due to their similar interaction contributions but relatively higher statistics (Table 8.1). As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be considerably more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.

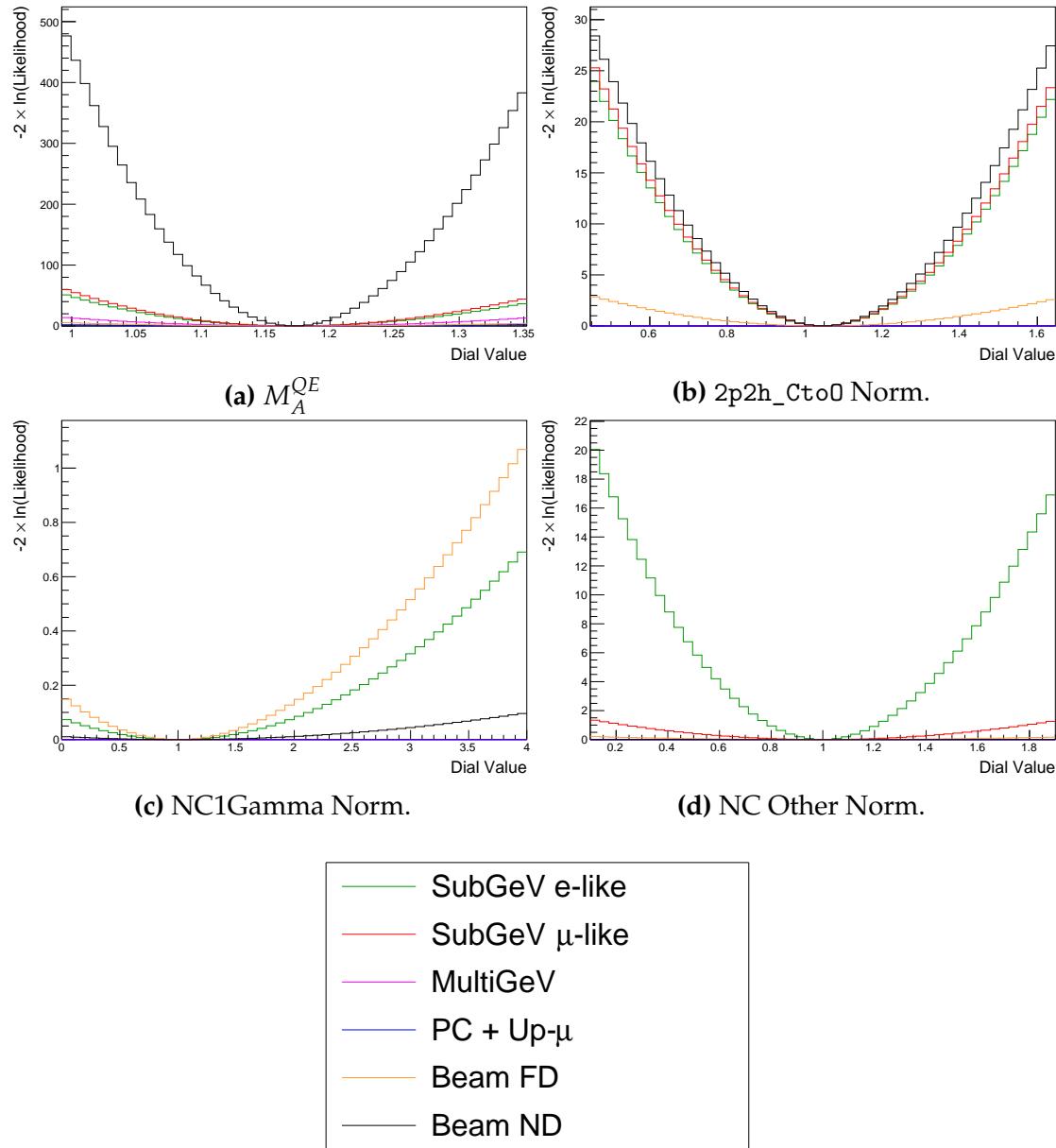


Figure 8.6: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

2966 8.3 Sensitivities

2967 The sensitivities of the joint T2K and SK oscillation analysis are presented in
2968 the form of Asimov fits. This technique builds an Asimov data set (follow-
2969 ing section 8.2) using the AsimovA oscillation parameters and post-BANFF
2970 systematic tune.

2971 In practice, the Asimov fits presented within this analysis are modified from
2972 the above definition. An Asimov prediction of both beam and atmospheric far
2973 detector samples is fit whilst the true data is used for near detector samples.
2974 The Asimov predictions at the far detector are built using the BANFF tuning (as
2975 discussed in section 3.2). These modifications mean that the results are equivalent
2976 to performing a far detector Asimov fit using inputs from the BANFF data fit.
2977 Consequently, this allows the results to be cross-checked with the results from
2978 the P-Theta analysis. The comparison has been performed and is documented in
2979 [220]. No significant discrepancies were found between the fitters.

2980 This section proceeds with the following studies. Firstly, the sensitivity of
2981 the atmospheric samples after the T2K cross-section has been applied to the low-
2982 energy events is detailed in subsection 8.3.1. This includes studying the choice of
2983 applying the 2020 PDG reactor constraint [75] to the atmospheric samples, which
2984 is documented in subsection 8.3.2. Additionally, the effect of applying the near-
2985 detector constraints onto the atmospheric samples is discussed in subsection 8.3.3.
2986 The main result is the sensitivity of the simultaneous beam and atmospheric fit.
2987 The sensitivities, both with and without the application of the reactor constraint,
2988 are presented in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate
2989 the benefit of the joint analysis, the sensitivities are compared to the 2020 T2K
2990 sensitivities [74, 183] in subsection 8.3.6 and subsection 8.3.7. As shown in
2991 subsection 8.2.1, the response of the beam and atmospheric samples change
2992 depending upon the true set of oscillation parameters assumed. Therefore,
2993 subsection 8.3.8 documents the sensitivities at an alternative oscillation parameter

²⁹⁹⁴ set. It is important to note that these results have been published at the Neutrino
²⁹⁹⁵ 2022 conference on behalf of the T2K and SK collaborations [108].

2996 8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

2997 This section presents the results of an Asimov fit using samples from the near
 2998 detector and only atmospheric samples from the far detector. The results are
 2999 presented as one-dimensional or two-dimensional histograms which have been
 3000 marginalised over all other parameters using the technique outlined in sub-
 3001 section 4.3.1. Each histogram displays the posterior probability density and
 3002 illustrates the credible intervals, calculated using the technique in subsection 4.3.2.
 3003 For these fits in this subsection, a flat penalty term is used for $\sin^2(\theta_{13})$ such the
 3004 reactor constraint is not applied. The Asimov data is generated assuming the
 3005 AsimovA oscillation parameter set defined in Table 2.2 and the post-BANFF
 3006 systematic parameter tune.

Without Reactor Constraint, Both Hierarchies

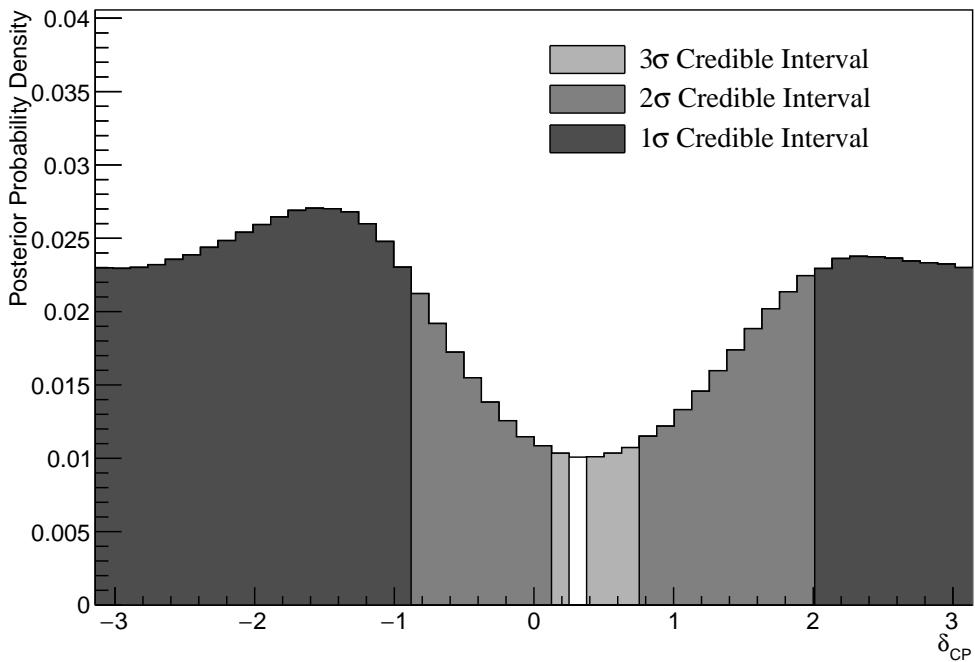


Figure 8.7: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3007 Figure 8.7 illustrates the posterior probability density for δ_{CP} , marginalised
 3008 over both hierarchies. If instead, only steps in the normal hierarchy were
 3009 considered, the shape of the contours would change. The fit favours the known

3010 oscillation parameter ($\delta_{CP} = -1.601$) although the posterior probability is very
 3011 flat through the range of $-\pi < \delta_{CP} < -1$ and $2 < \delta_{CP} < \pi$. There is also a region
 3012 around $\delta_{CP} \sim 0.4$ which is disfavoured at 2σ . This indicates that the SK samples
 3013 can rule out some parts of the CP conserving parameter space reasonably well,
 3014 near $\delta_{CP} \sim 0.4$, when the true value of $\delta_{CP} \sim \pi/2$.

Without Reactor Constraint, Both Hierarchies



Figure 8.8: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3015 The posterior probability density in Δm_{32}^2 is given in Figure 8.8. This dis-
 3016 tribution includes steps in both the normal hierarchy (NH, $\Delta m_{32}^2 > 0$) and the
 3017 inverse hierarchy (IH, $\Delta m_{32}^2 < 0$). The highest posterior probability density is
 3018 found within the NH, which agrees with the known oscillation parameter value.
 3019 However, all of the credible intervals span both of the hierarchies hypotheses.
 3020 If instead, only steps in the normal hierarchy were considered, the shape of the
 3021 contours would change. The known oscillation parameter is $2.509 \times 10^{-3} \text{ eV}^2$,
 3022 which is contained within the 1σ credible interval.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.17	0.40	0.58
IH ($\Delta m_{32}^2 < 0$)	0.13	0.29	0.42
Sum	0.31	0.69	1.00

Table 8.2: The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.37$ and $B(\text{UO}/\text{LO}) = 2.24$.

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO, $\sin^2 \theta_{23} < 0.5$) or upper octant (UO, $\sin^2 \theta_{23} > 0.5$). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy model is $B(\text{NH}/\text{IH}) = 1.37$. Table 4.1 states this value of the Bayes factor indicates a weak preference for the normal hierarchy model. The Bayes factor for choice of octant is $B(\text{UO}/\text{LO}) = 2.24$. This is also classified as a weak preference for the UO. Both of these show that the fit is returning the correct choice of models (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The 1σ credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. For the known Asimov value of $\delta_{CP} = -1.601$, the 1σ credible interval rules out a region between $\delta_{CP} = -0.86$ and $\delta_{CP} = 1.96$, when marginalising over both hierarchies. The position of the highest posterior density is $\delta_{CP} = -1.57$ which is clearly compatible with the known oscillation parameter value.

The sensitivity of the atmospheric samples to $\sin^2(\theta_{13})$ is presented in Figure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -0.86], [1.96, \pi]$	-1.57
δ_{CP} , (NH)	$[-\pi, -0.86], [1.88, \pi]$	-1.57
δ_{CP} , (IH)	$[-\pi, -0.94], [1.96, \pi]$	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-3.00, -2.50], [2.35, 3.15]$	2.65
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.39, 3.04]$	2.64
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-3.15, -2.45]$	-2.70
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.59]$	0.542
$\sin^2(\theta_{23})$ (NH)	$[0.476, 0.59]$	0.554
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.59]$	0.542

Table 8.3: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3045 to $\sin^2(\theta_{13})$ will be small. This behaviour is also seen in the fit results, where the
 3046 width of the 1σ credible intervals span the region of $\sin^2(\theta_{13}) = [0.008, 0.08]$. This
 3047 is more than an order of magnitude worse than the constraint from reactor
 3048 experiments [75].

3049 As previously discussed, the correlations between oscillation parameters are
 3050 also important to understand how the atmospheric samples respond. Figure 8.10
 3051 illustrates the two dimensional $\sin^2(\theta_{13}) - \delta_{CP}$ sensitivity, marginalised over all
 3052 other parameters. The displayed contours are calculated by marginalising over
 3053 both hierarchies. The shape of the 1σ credible interval shows that the constraining
 3054 power of the fit on δ_{CP} is dependent upon the value of $\sin^2(\theta_{13})$. Furthermore,
 3055 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.
 3056 Whilst the atmospheric samples do not strongly constrain the value of $\sin^2(\theta_{13})$,
 3057 the value of $\sin^2(\theta_{13})$ does impact the atmospheric sensitivity to δ_{CP} . A value of
 3058 $\sin^2(\theta_{13}) \sim 0.02$ would select a continuous contour over all values of δ_{CP} . This
 3059 shows the effect of the marginalisation effect previously described.

Without Reactor Constraint, Both Hierarchies



Figure 8.9: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3060 The $\sin^2(\theta_{23}) - \Delta m_{32}^2$ disappearance contours are illustrated in Figure 8.11.

3061 As expected, the area contained in the inverted hierarchy 1σ credible interval is
 3062 slightly smaller than that in the normal hierarchy. This follows from the Bayes
 3063 factor showing a weak preference for NH meaning that more of the steps will exist
 3064 in the $\Delta m_{32}^2 > 0$ region. The known oscillation parameters of $\sin^2(\theta_{23}) = 0.528$
 3065 and $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ are contained within the 1σ credible interval.

3066 Figure 8.12 illustrates the two-dimensional projections for each permutation of
 3067 oscillation parameters which this analysis is sensitive to: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$,
 3068 and Δm_{32}^2 . The purpose of this plot is to illustrate the correlations between the
 3069 oscillation parameters. The contours are calculated whilst marginalising over
 3070 both hierarchies, however, only the NH is illustrated when plotting the Δm_{32}^2
 3071 parameter. As expected the correlations play a significant role in these sensitivity
 3072 measurements, especially the choice of the $\sin^2(\theta_{13})$ constraint. The application
 3073 of reactor constraint would be expected to alter both the width and position of the

Without Reactor Constraint, Both Hierarchies



Figure 8.10: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3074 Δm_{32}^2 , δ_{CP} , and $\sin^2(\theta_{23})$ constraints. The majority of the octant model preference
 3075 comes from the region of $\sin^2(\theta_{13}) \sim 0.03$ such that the application of the reactor
 3076 constraint would not be expected to significantly change the octant preference.
 3077 The reactor constraint would result in lower values of $|\Delta m_{32}^2|$. Interestingly, the
 3078 distribution of steps in the δ_{CP} - $\sin^2(\theta_{13})$ plot is slightly flatter in the region of the
 3079 reactor constraint. Both the posterior distribution from this fit and the distribution
 3080 in Figure 8.2 show a region of low negative log-likelihood extending out towards
 3081 higher values of $\sin^2(\theta_{13})$ in the $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2$ region. Consequently,
 3082 the reactor constraint could feasibly reduce the sensitivity of the atmospheric
 3083 samples to δ_{CP} , due to the previously discussed marginalisation effects.

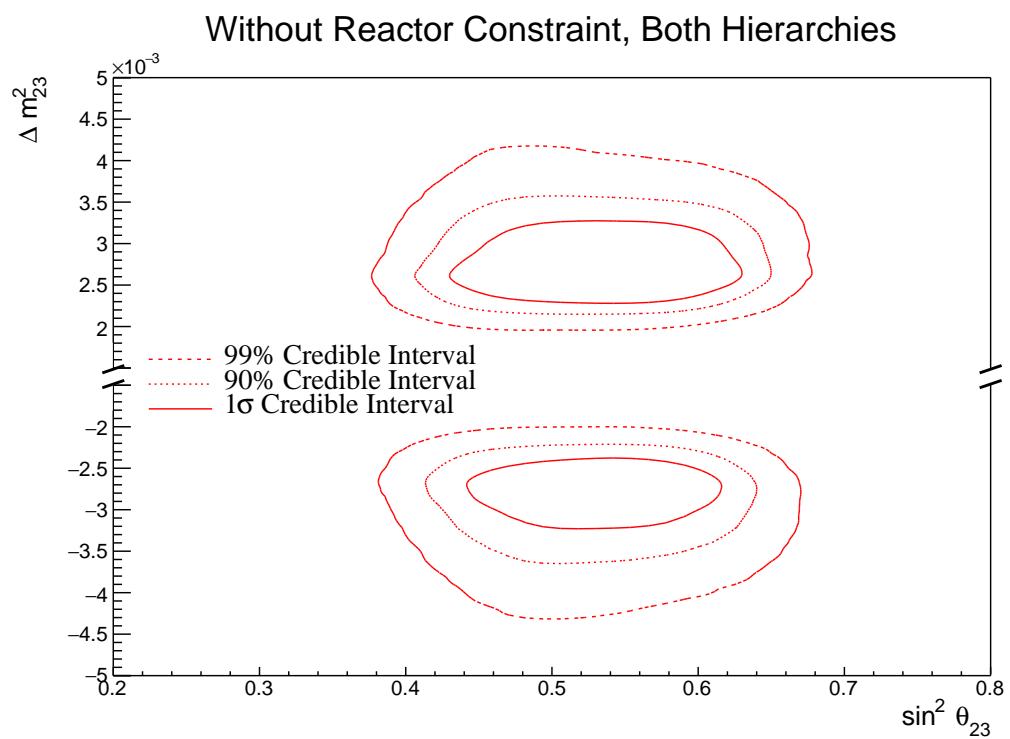


Figure 8.11: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

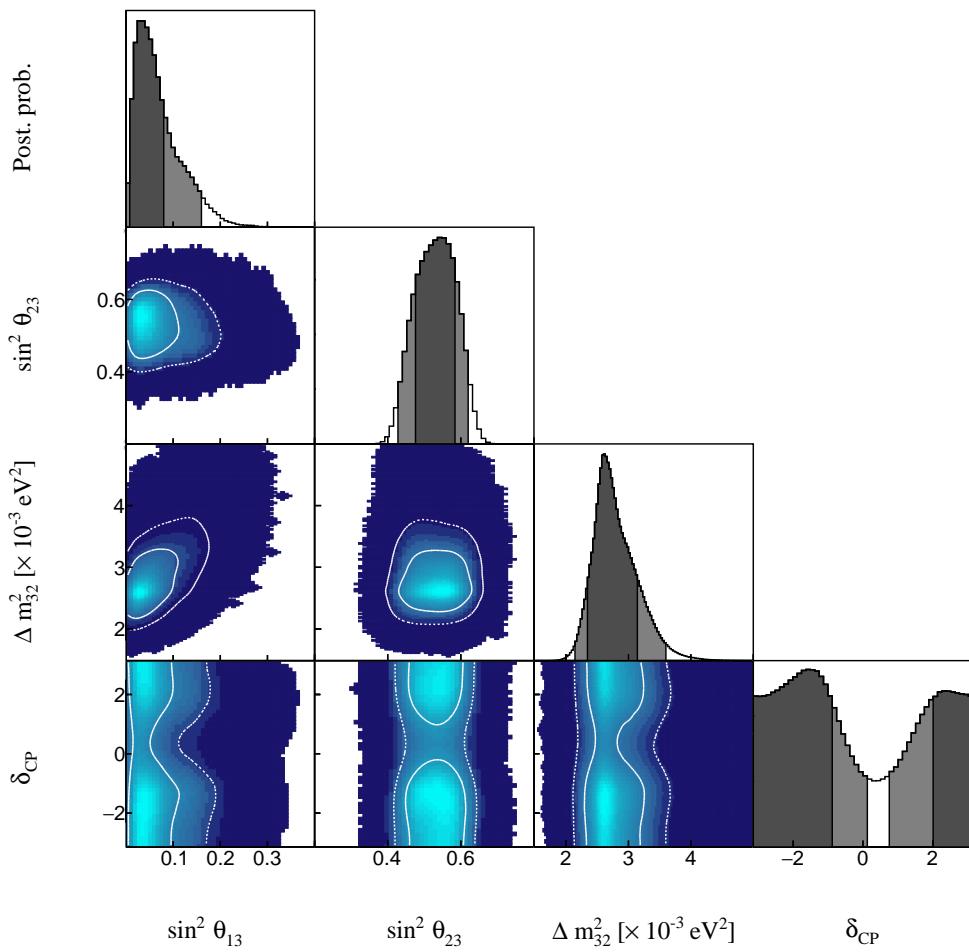


Figure 8.12: The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3084 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

3085 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor
 3086 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that
 3087 the marginalisation effects could contribute to differing sensitivities when the
 3088 external reactor constraint is applied. Using the technique discussed in subsec-
 3089 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted
 3090 to include the reactor constraint of $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ [75].

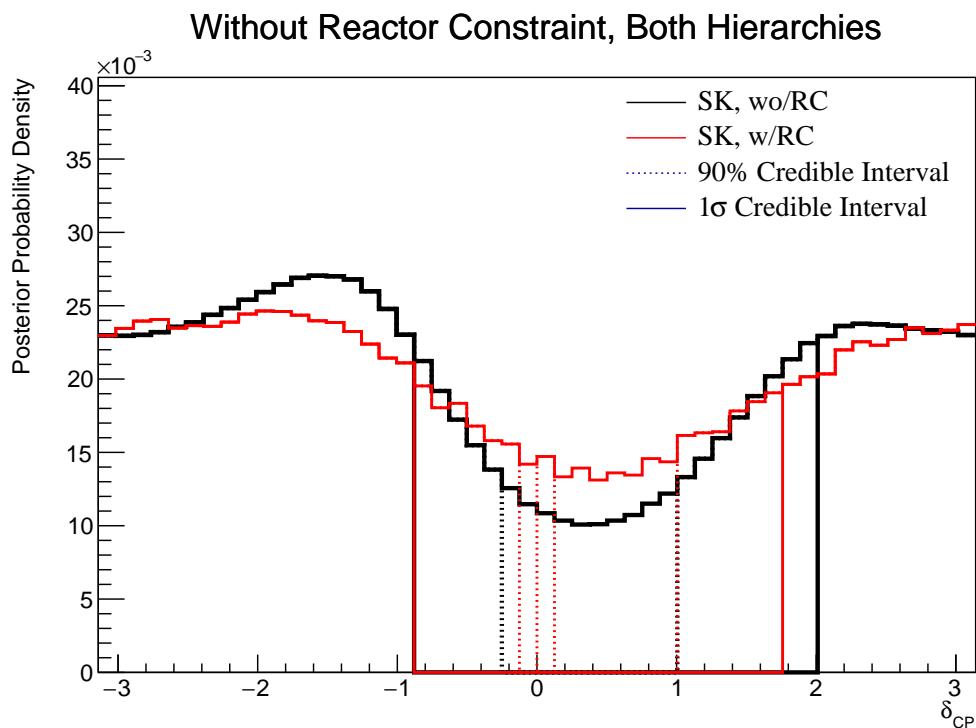


Figure 8.13: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

3091 Figure 8.13 illustrates the sensitivity to δ_{CP} of the atmospheric fit with reactor
 3092 constraint applied. The distribution is less peaked than the previous results.
 3093 This is due to the expected marginalisation effect previously discussed. The
 3094 width of the 1σ credible interval is increased when the reactor constraint is
 3095 applied, indicating less sensitivity to δ_{CP} in the region of $\sin^2(\theta_{13})$ preferred
 3096 by the reactor constraint.

Without Reactor Constraint, Both Hierarchies

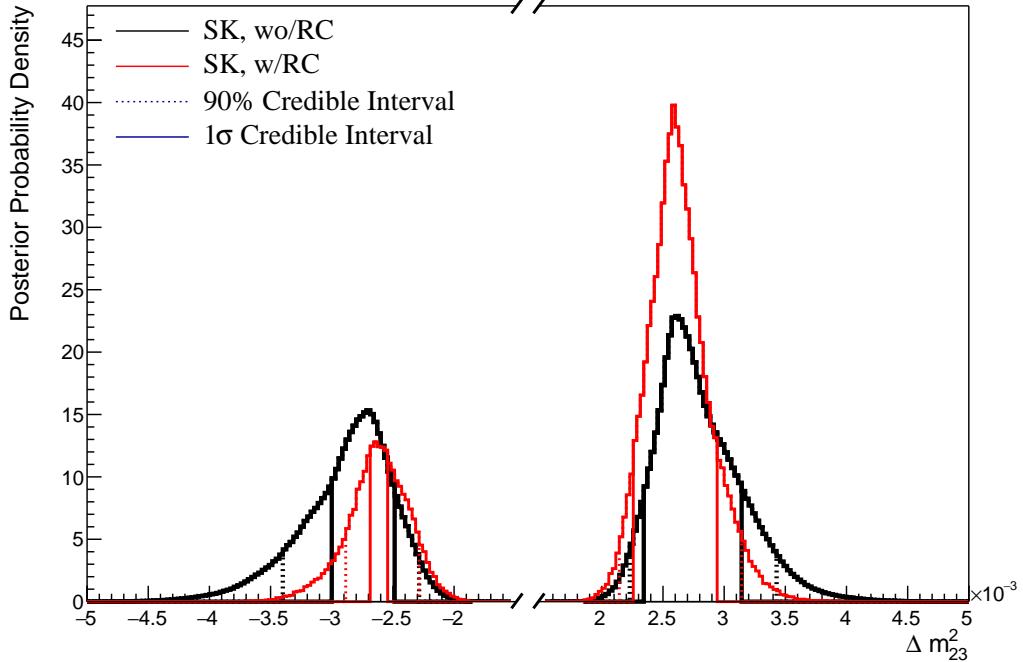


Figure 8.14: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

The reactor constraint increases the sensitivity of the atmospheric samples to Δm_{32}^2 as illustrated in Figure 8.14. The 1σ credible interval in Δm_{32}^2 is determined to be $[-2.70, -2.55] \times 10^{-3}\text{eV}^2$ and $[2.25, 2.95] \times 10^{-3}\text{eV}^2$. The width of the IH credible interval is reduced by $\sim 70\%$ when the reactor constraint is applied. Due to the marginalisation effects observed in Figure 8.12, the favoured region of Δm_{32}^2 moves closer to zero for both hierarchies. A clear explanation of this behaviour is illustrated in Figure 8.15 which illustrates the posterior distribution in the $\Delta m_{32}^2 - \sin^2(\theta_{13})$ parameters, marginalised over both hierarchies. The correlation between Δm_{32}^2 and $\sin^2(\theta_{13})$ is such that lower values of $\sin^2(\theta_{13})$ tend towards lower values of $|\Delta m_{32}^2|$. This moves the posterior distribution towards the known oscillation parameter $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$.

Table 8.4 presents the fraction of steps in each hierarchy and octant model for the fit after the reactor constraint has been applied. The reactor constraint significantly increases the NH preference, increasing the Bayes factor from



Figure 8.15: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$ compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.21	0.53	0.74
IH ($\Delta m_{32}^2 < 0$)	0.08	0.18	0.26
Sum	0.29	0.71	1.00

Table 8.4: The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 2.86$ and $B(\text{UO}/\text{LO}) = 2.39$.

³¹¹¹ $B(\text{NH}/\text{IH}) = 1.37$ to $B(\text{NH}/\text{IH}) = 2.86$ when the reactor constraint is applied.

³¹¹² This is still defined as a weak preference for NH hypothesis according to Jeffrey's

³¹¹³ scale (see Table 4.1), however, it is a stronger preference than when the constraint

³¹¹⁴ is not applied. The preference for the correct octant model is slightly increased

³¹¹⁵ by the application of the reactor constraint which is consistent with expectation.

³¹¹⁶ However, the conclusion that would be made does not significantly change.

3117 8.3.3 Application of Near Detector Constraints for Atmospheric 3118 Samples

3119 The choice of applying the near detector constraints to the low-energy atmo-
3120 spheric samples was introduced in subsection 6.4.3. This subsection illustrates
3121 the effect of that choice on the sensitivities of the atmospheric samples to the
3122 oscillation parameters. This Asimov data was generated assuming the ‘AsimovA’
3123 oscillation parameter set defined in Table 2.2 and the post-BANFF systematic
3124 parameter tune.

3125 The change in sensitivity on δ_{CP} is given in Figure 8.16. The reactor constraint
3126 is not applied in either of the fits within this comparison. The shape of the
3127 posterior is similar although less peaked at the Asimov point ($\delta_{CP} = -1.601$)
3128 and more symmetric between the regions of $\delta_{CP} = -1.601$ and $\delta_{CP} \sim 2.5$. The
3129 width of the 1σ credible intervals are approximately the same (identical to within
3130 a bin width) and the same conclusion holds for the higher credible intervals. The
3131 change in sensitivity to other oscillation parameters has been studied and no
3132 significant discrepancies were found. As expected, the sensitivities are statistics
3133 dominated and the exact choice of systematic model and constraint does not
3134 significantly affect the physics conclusions one would make from this analysis.



Figure 8.16: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies.

3135 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3136 straint

3137 This section presents the sensitivities of the simultaneous beam and atmospheric
 3138 analysis where the reactor constraint is not applied. Similar to the previous
 3139 studies, the Asimov data is built assuming the post-BANFF cross-section tune
 3140 and Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near
 3141 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.
 3142 The sensitivity to δ_{CP} , marginalised over both hierarchies, is given in Figure 8.17.

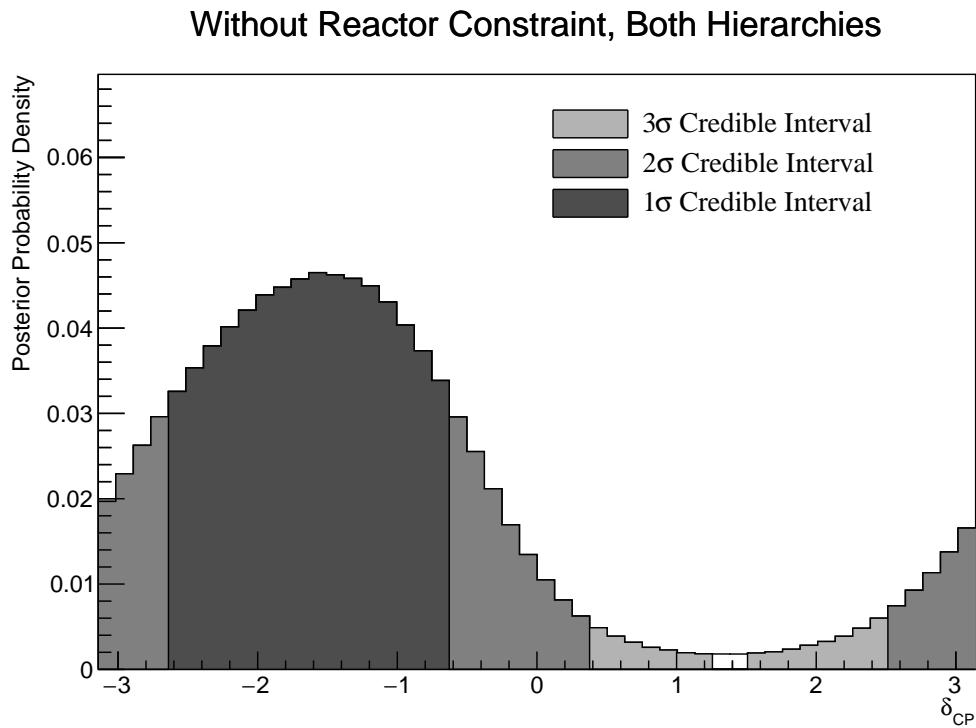


Figure 8.17: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

3143 The credible intervals and highest posterior distribution for each oscillation
 3144 parameter is given in Table 8.5. The highest posterior probability density is
 3145 $\delta_{CP} = -1.58$ and is compatible with the known Asimov A value of $\delta_{CP} = -1.601$.
 3146 The CP-conserving values of $\delta_{CP} = 0, \pi, -\pi$ are disfavoured at 1σ credible
 3147 interval. There is also a region around $\delta_{CP} = 1.4$ which is disfavoured at more
 3148 than 3σ . Whilst these conclusions can only be made at this particular Asimov

3149 point, it does show that if the true value of δ_{CP} was CP-violating, this joint
3150 analysis would be able to disfavour CP conserving values at over 1σ without
3151 any external constraints. The highest posterior probability density does move
3152 further away from the Asimov point when only steps in the NH region are
3153 considered. This is due to the correlations between the value of δ_{CP} and the
3154 mass hierarchy, as will be later discussed.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-2.64, -0.63]$	-1.57
δ_{CP} , (NH)	$[-2.76, -0.63]$	-1.45
δ_{CP} , (IH)	$[-2.39, -0.88]$	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.46, 2.58]	2.49
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.48, 2.56]	2.51
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.52]$	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (NH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (IH)	[0.48, 0.55]	0.521

Table 8.5: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam and atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3155 The sensitivity to Δm_{32}^2 is illustrated in Figure 8.18, marginalised over both
3156 hierarchies. Notably, the 1σ credible interval is entirely contained within the
3157 normal hierarchy region, as illustrated in Table 8.5. This is illustrates reasonable
3158 sensitivity to the mass hierarchy model. This is also reflected in the 1σ credible
3159 intervals being approximately the same when they are made considering both
3160 hierarchies and when considering only the NH. The known oscillaton parameter
3161 is $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$. The normal hierarchy distribution favours this value
3162 with the highest posterior probability density of $\Delta m_{32}^2 = 2.51 \times 10^{-3}\text{eV}^2$.

3163 The fraction of steps in each of the mass hierarchy regions and octants of
3164 $\sin^2(\theta_{23})$ is given in Table 8.6. The Bayes factors are determined to be $B(\text{NH}/\text{IH}) =$

3165 3.67 and $B(\text{UO}/\text{LO}) = 1.74$. Jeffrey's scale (presented in Table 4.1) states that
 3166 this value of the hierarchy Bayes factor illustrates substantial evidence for the
 3167 normal hierarchy hypothesis. This corresponds to the correct hypothesis given
 3168 the known oscillation parameters. It is a stronger statement than the atmospheric-
 3169 only analysis can provide. It is important to note that this is a substantial
 3170 preference that requires no external constraints required. The Bayes factor for
 3171 octant determination represents a weak preference for the upper octant but does
 3172 select the correct octant model.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.29	0.50	0.79
IH ($\Delta m_{32}^2 < 0$)	0.08	0.13	0.21
Sum	0.37	0.63	1.00

Table 8.6: The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 3.67$ and $B(\text{UO}/\text{LO}) = 1.74$.

3173 The sensitivity to $\sin^2(\theta_{23})$ is presented in Figure 8.19. There is a clear
 3174 preference for the upper octant but the peak of the distribution is relatively
 3175 flat. It peaks at $\sin^2(\theta_{23}) = 0.509$ which is in the region of the known value of
 3176 $\sin^2(\theta_{23}) = 0.528$. The difference in the highest posterior distribution and the
 3177 width of the credible interval is relatively unchanged when considering different
 3178 hierarchy models showing no strong correlation between $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.
 3179 The sensitivity presented as a function of the appearance parameters ($\sin^2(\theta_{13}) -$
 3180 δ_{CP}) is given in Figure 8.20. As expected, the contours follow that given in
 3181 Figure 8.2, where the 2σ credible intervals have a closed contour excluding the
 3182 region around $\delta_{CP} \sim 1.2$. The width of the 3σ credible interval is also clearly
 3183 dependent upon the value of δ_{CP} . Close to the Asimov point, $\delta_{CP} = -1.601$, the
 3184 width of the 3σ credible interval approximately spans $\sin^2(\theta_{13}) = [0.013, 0.04]$.
 3185 This is reduced to a region of $\sin^2(\theta_{13}) = [0.023, 0.042]$ at the most disfavoured
 3186 value of δ_{CP} . This follows the behaviour shown in the likelihood scans. The 1σ

Without Reactor Constraint, Both Hierarchies

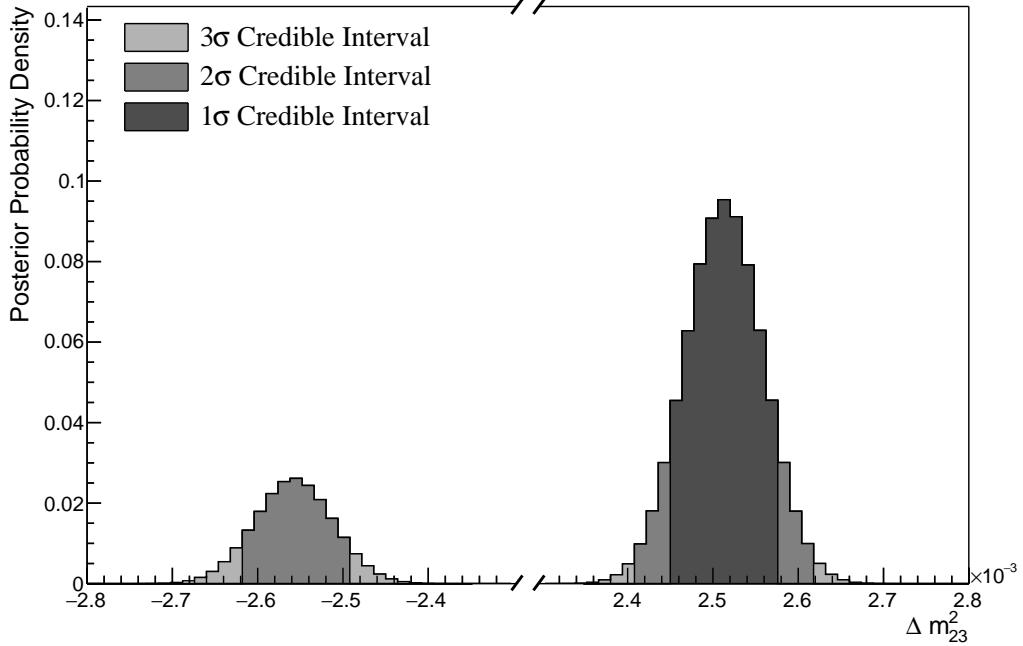


Figure 8.18: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

credible interval is consistent with both the known oscillation parameter and the reactor constraint ($\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$). Application of the reactor constraint would be expected to decrease the width of the 1 σ credible intervals of δ_{CP} due to the triangular shape of the posterior probability.

The sensitivity in terms of the ‘disappearance’ parameters marginalised over both hierarchies is given in Figure 8.21. The area contained within the IH credible intervals is significantly smaller than those in the NH region. This is reflected in the IH credible intervals being tighter in the $\sin^2(\theta_{23})$ dimension. No significant correlation is observed between the value of $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

The two-dimensional posterior distribution for each permutation of the oscillation parameters of interest is given in Figure 8.22. The most notable observation is that the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ are anti-correlated. If the value of $\sin^2(\theta_{13})$ was known to be closer to the known oscillation parameter value, the preferred value of $\sin^2(\theta_{23})$ would increase furthering the preference for the UO. That

Without Reactor Constraint, Both Hierarchies

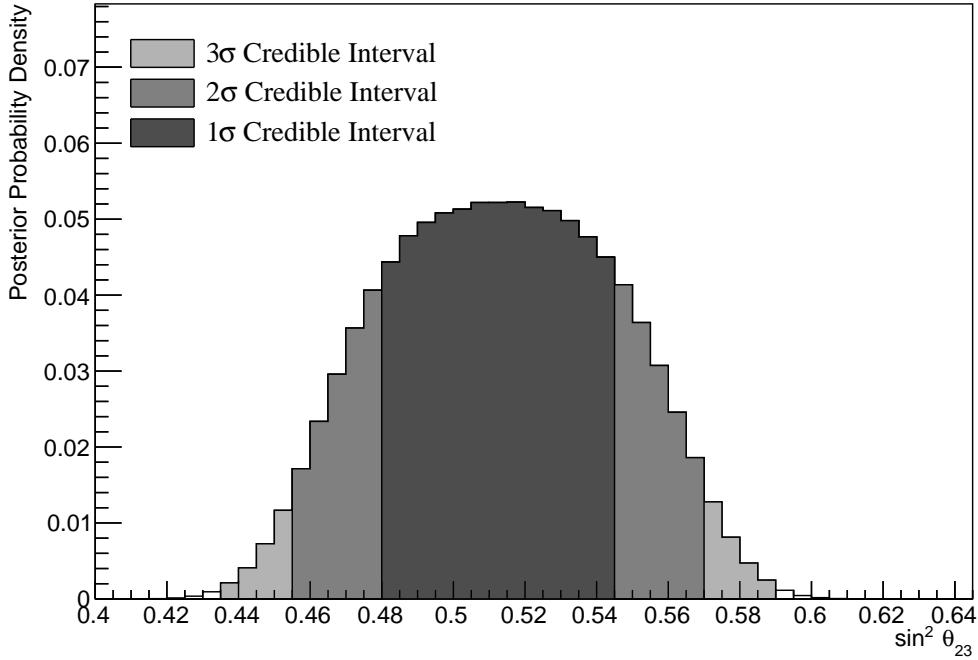


Figure 8.19: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied.

would move the highest posterior probability closer in line with the Asimov value. This also means that the preference for the UO would be increased if the reactor constraint was to be applied.

Furthermore, the δ_{CP} and $|\Delta m_{32}^2|$ oscillation parameters are anti-correlated, such that higher values of $|\Delta m_{32}^2|$ prefer lower values of δ_{CP} . Whilst this is an interesting result on its own, the width of the Δm_{32}^2 contours also depend on $\sin^2(\theta_{13})$. This introduces another correlation effect that could modify the sensitivity to δ_{CP} once the reactor constraint is applied.

The correlation between $\sin^2(\theta_{13})$ and Δm_{32}^2 can be seen in Figure 8.23. A much larger fraction of the posterior distribution is contained in the NH for lower values of $\sin^2(\theta_{13})$. Consequently, the application of the reactor constraint would be expected to significantly increase the preference for NH.

Without Reactor Constraint, Both Hierarchies

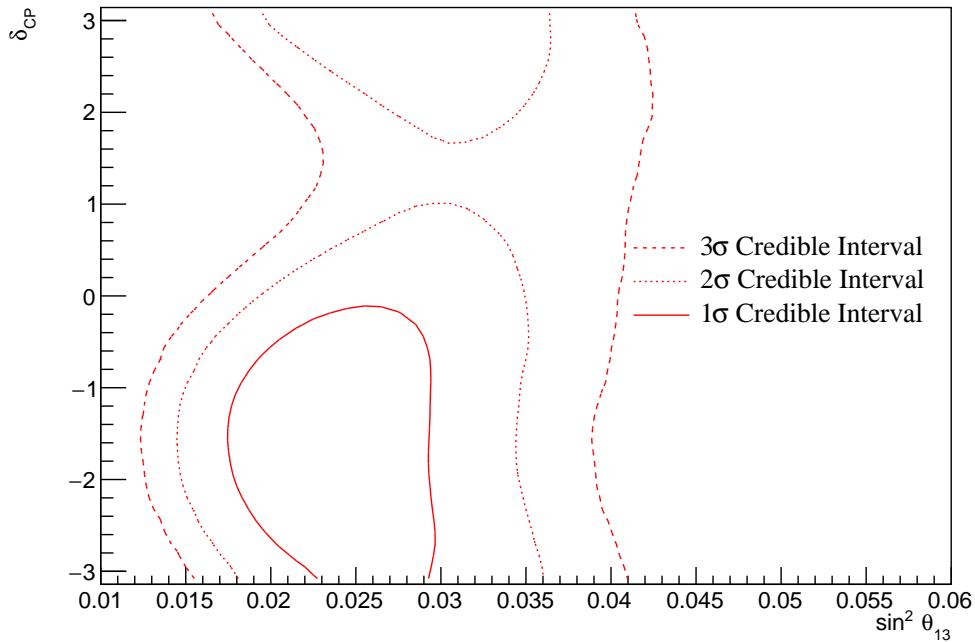


Figure 8.20: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

Without Reactor Constraint, Both Hierarchies

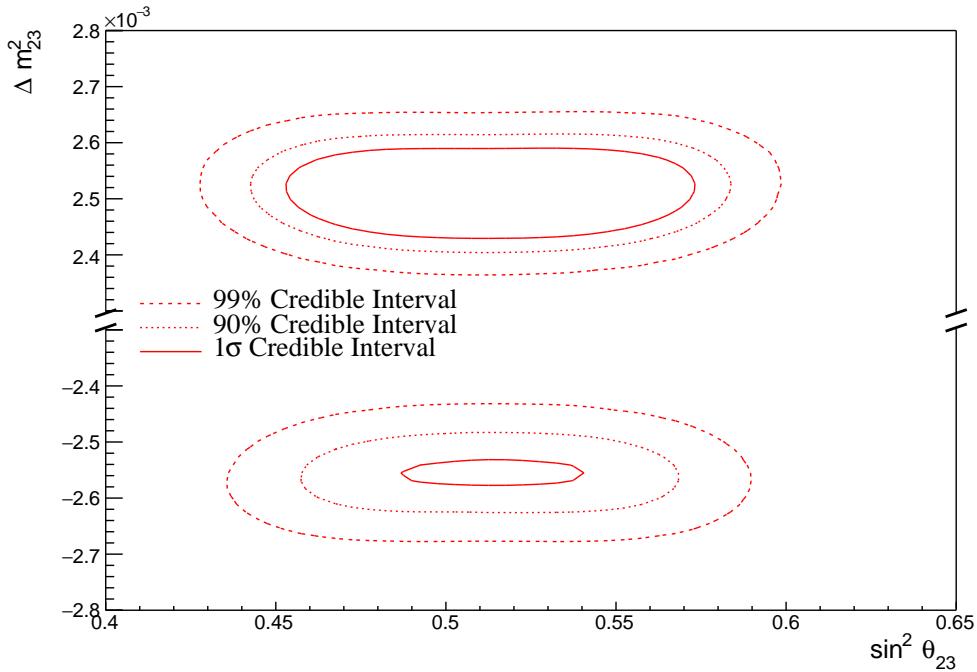


Figure 8.21: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

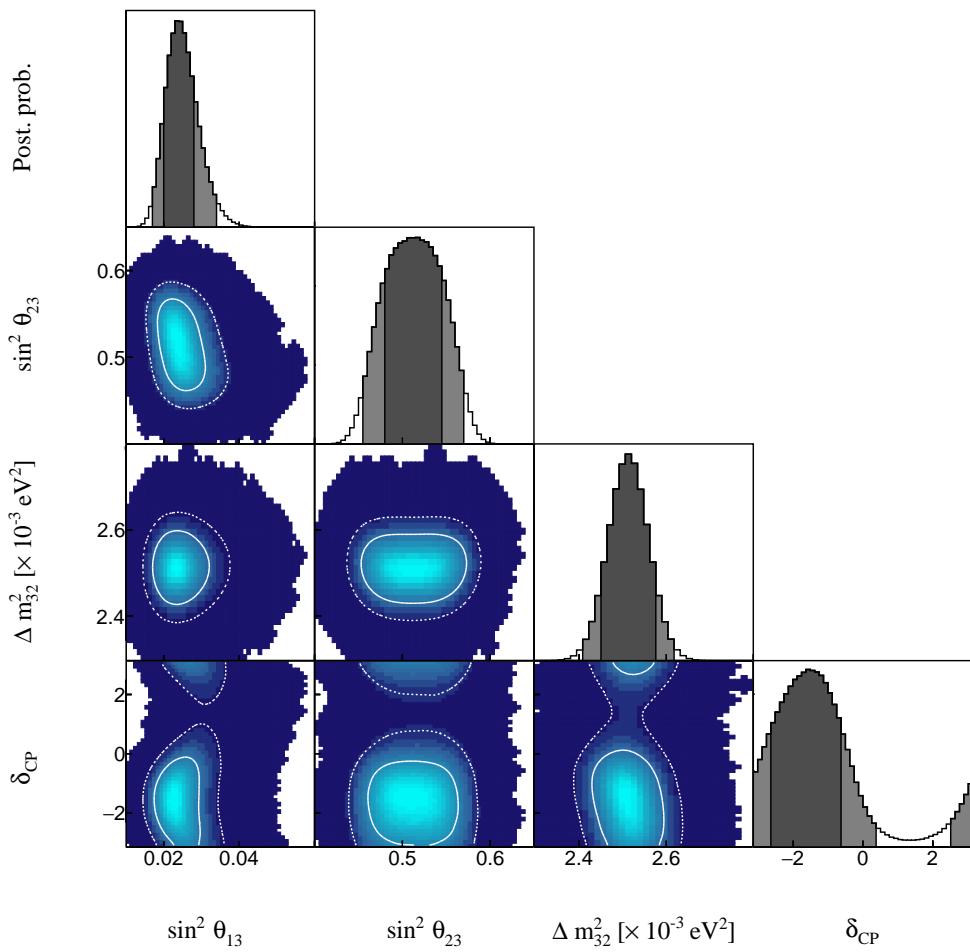


Figure 8.22: The posterior probability density distribution from the joint beam and atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

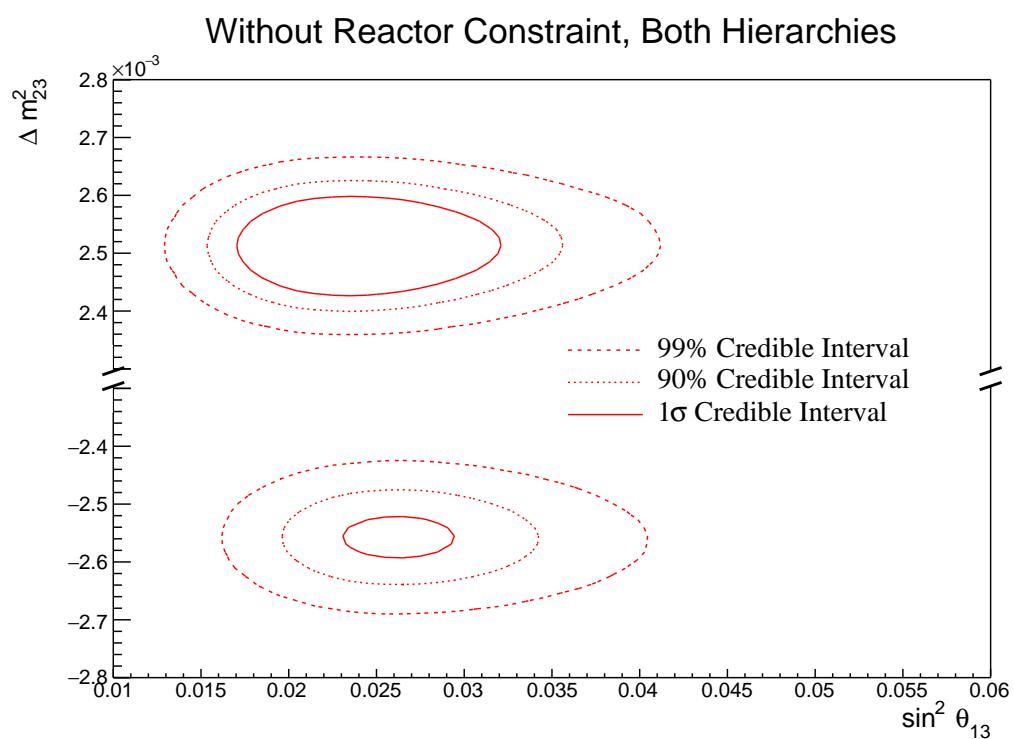


Figure 8.23: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

3213 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3214 This section presents the sensitivities of the joint beam and atmospheric fit when
 3215 the reactor constraint is applied to $\sin^2(\theta_{13})$. As with the previous studies, the
 3216 Asimov data is made using the AsimovA oscillation parameter set defined in
 3217 Table 2.2 and the post-BANFF systematic parameter tune.

With Reactor Constraint, Both Hierarchies

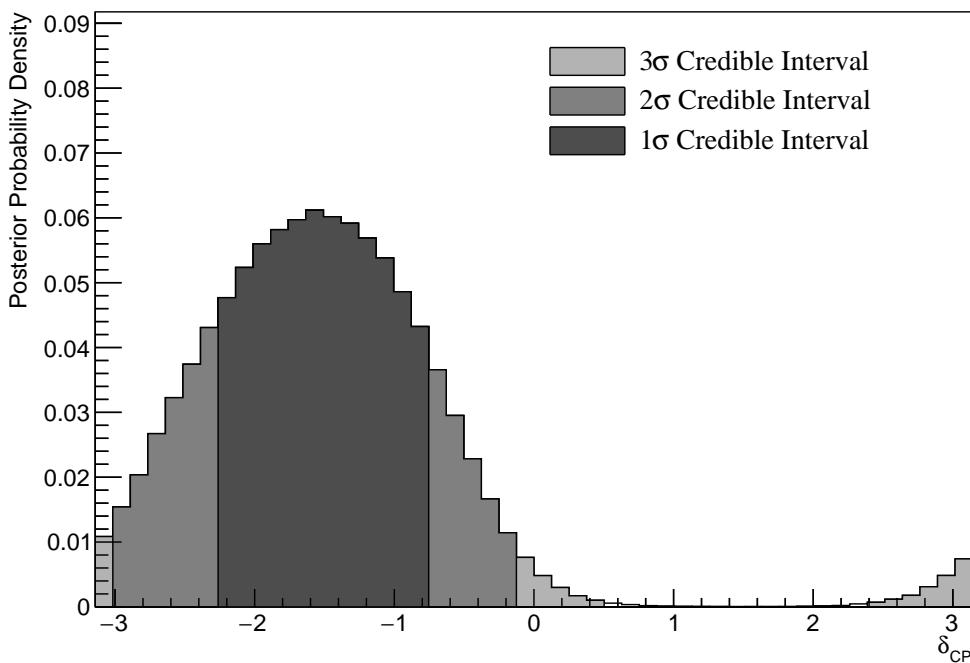


Figure 8.24: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3218 Figure 8.24 illustrates the sensitivity to δ_{CP} , marginalised over both hierarchies.
 3219 The CP-conserving values of $\delta_{CP} = -\pi, 0, \pi$ are disfavoured at 2σ . Furthermore,
 3220 the 3σ credible interval excludes the region of $\delta_{CP} = [0.50, 2.39]$. Thus clearly
 3221 disfavouring the region of $\delta_{CP} = \pi/2$ at more than 3σ for this particular set
 3222 of known oscillation parameters. The width of the 1σ credible intervals and
 3223 the position of the highest posterior probability density is given in Table 8.7.
 3224 The highest posterior probability density in δ_{CP} is calculated as $\delta_{CP} = -1.57$
 3225 showing no significant biases in the determination of the known oscillation

parameters. The posterior distribution is more peaked around the known oscillation parameter value of $\delta_{CP} = -1.601$, as compared to the sensitivities when the reactor constraint is not applied (subsection 8.3.4). This follows from the correlations shown in Figure 8.20, where a lower value of $\sin^2(\theta_{13})$ results in tighter constraints on δ_{CP} .

The effect of applying the reactor constraint for δ_{CP} in the joint beam-atmospheric fit is presented in Figure 8.25. The posterior distribution from the two fits are marginalised over both hierarchies. Clearly, the reactor constraint improves the ability of the fit to select the known oscillation parameter as the shape of the distribution is much more peaked. This is also evidenced by the tightening of the 1σ and 90% credible intervals. Additionally, the disfavoured region of $1 < \delta_{CP} < 2$ is wider when the reactor constraint is applied.

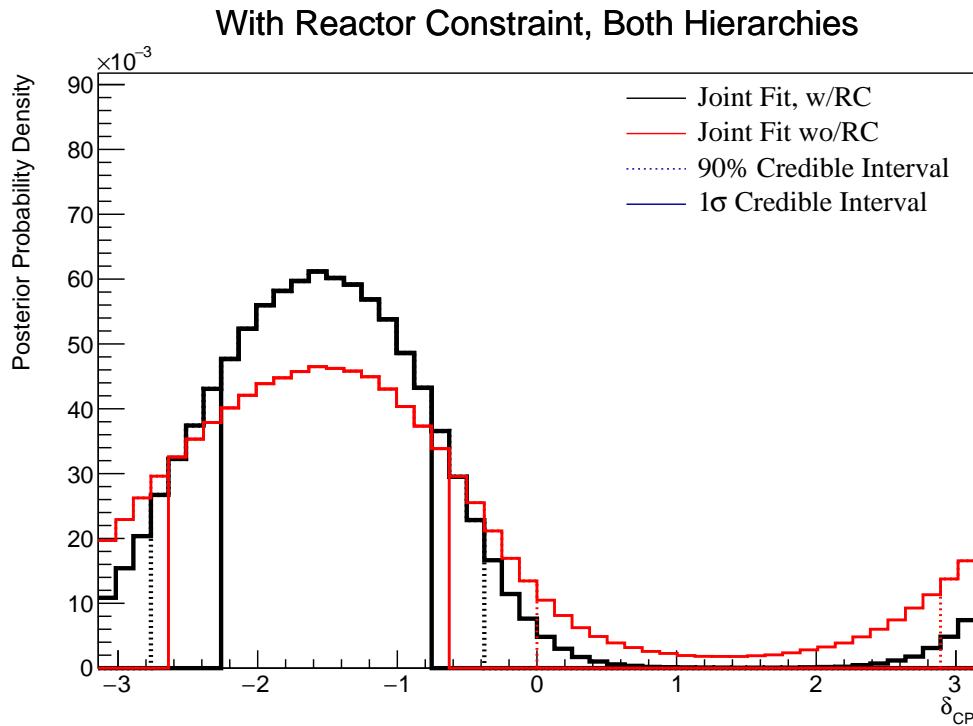


Figure 8.25: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, is given in Figure 8.26. The highest posterior probability density is located at $\sin^2(\theta_{23}) = 0.527$

Parameter	Interval	HPD
δ_{CP} , (BH)	[-2.26, -0.75]	-1.57
δ_{CP} , (NH)	[-2.26, -0.75]	-1.57
δ_{CP} , (IH)	[-2.13, -1.00]	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.46, 2.52]	2.49
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.48, 2.56]	2.51
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	[-2.60, -2.52]	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (NH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (IH)	[0.50, 0.56]	0.539

Table 8.7: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam and atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

which agrees with the known value of $\sin^2(\theta_{23}) = 0.528$. The distribution clearly favours the UO with almost the entirety of the 1σ credible interval contained in the region. Figure 8.27 highlights the sensitivity of the joint fit both with and without the reactor constraint. The fit where the reactor constraint is applied selects the known value much better ($\sin^2(\theta_{23}) = 0.528$). Furthermore, the reactor constraint increases the UO preference which is evidenced by the distribution moving further away from the octant boundary. This indicates that there are marginalisation effects between the two mixing parameters. This follows from the correlation illustrated between $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$ in Figure 8.22. The posterior distribution of the fit with reactor constraint is more peaked compared to the flatter distribution when the reactor constraint is not applied.

The fraction of steps contained within the two hierarchy and two octant models is given in Table 8.8. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.02, whilst significantly increasing the fraction of steps within the NH-UO region from 0.53 to 0.64. The application of the reactor constraint increases the Bayes factor

With Reactor Constraint, Both Hierarchies



Figure 8.26: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

from $B(\text{NH}/\text{IH}) = 3.67$ to $B(\text{NH}/\text{IH}) = 7.29$. There is a very clear preference for the NH, with the Jeffreys scale stating a substantial preference for both fits (see subsection 4.3.3). The Bayes factor for UO preference is calculated as $B(\text{UO}/\text{LO}) = 2.86$. Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.24	0.64	0.88
IH ($\Delta m_{32}^2 < 0$)	0.02	0.10	0.12
Sum	0.26	0.74	1.00

Table 8.8: The distribution of steps in a joint beam and atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 7.29$ and $B(\text{UO}/\text{LO}) = 2.86$.

The sensitivity to Δm_{32}^2 , with the reactor constraint applied, is presented in

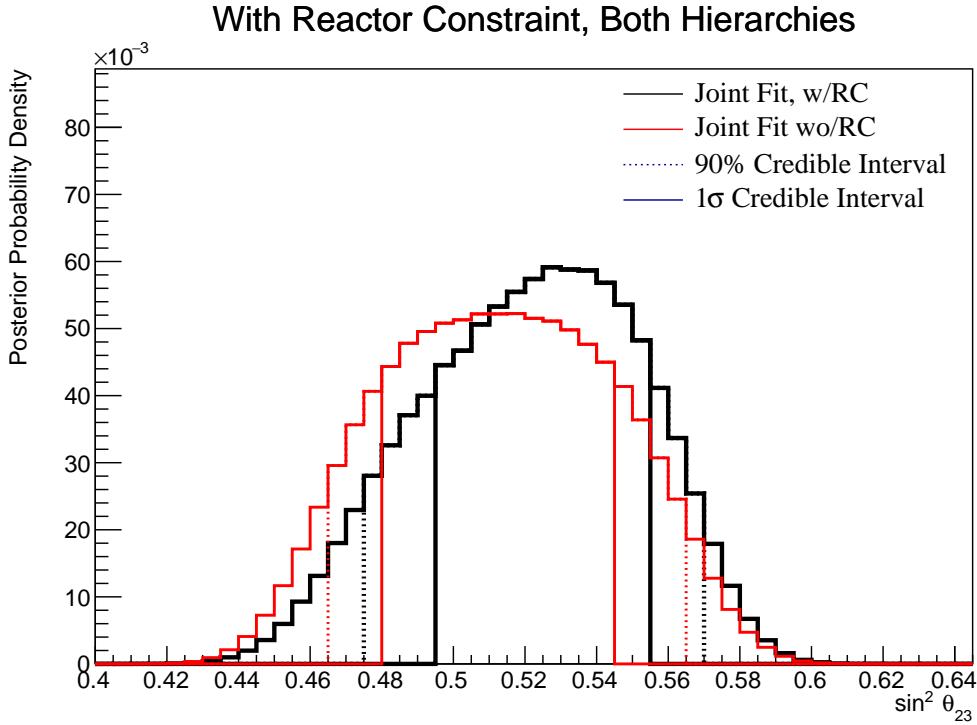


Figure 8.27: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

3262 Figure 8.28. The posterior distribution is marginalised over both hierarchies. As
 3263 expected, the 1σ credible interval is entirely contained within the NH region. The
 3264 position of the highest posterior probability density is given as $2.49 \times 10^{-3} \text{ eV}^2$,
 3265 illustrating no significant bias between the fit results and the known oscillation
 3266 parameters. The application of the reactor constraint does move significantly the
 3267 position of the credible intervals but does reduce their width.

3268 The sensitivity to the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$) is given in
 3269 Figure 8.29. The distribution is mostly uncorrelated between the two parameters
 3270 and is centered at the known oscillation parameters. The 1σ credible interval
 3271 excludes $\delta_{CP} = 0$ and $\delta_{CP} = (-)\pi$. Furthermore, the 3σ credible intervals
 3272 exclude the region of $\delta_{CP} = \pi/2$.

3273 The sensitivity to the disappearance parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) is illus-
 3274 trated in Figure 8.30. As expected from the one-dimensional distribution, the 1σ
 3275 credible interval is entirely contained within the NH region. Both the NH and

With Reactor Constraint, Both Hierarchies

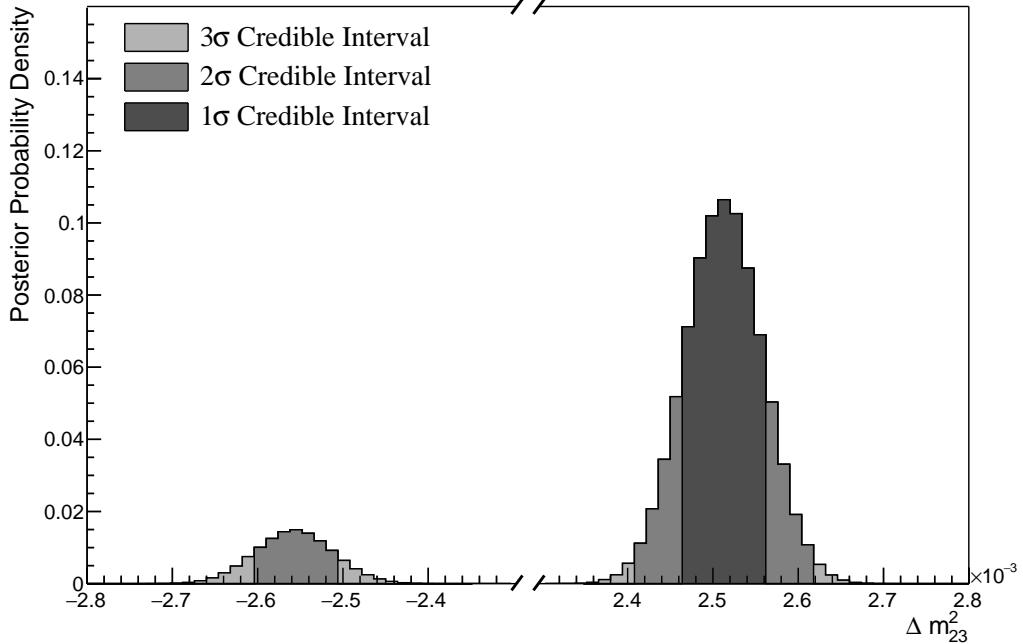


Figure 8.28: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3276 IH regions favour the UO, with a visually similar preference in both hierarchies.
 3277 The width of the Δm_{32}^2 1 σ credible interval does not significantly depend upon
 3278 the value or octant of $\sin^2(\theta_{23})$. This shows that there are no strong correlations
 3279 between these two parameters.

3280 Figure 8.31 illustrates the posterior distribution for each permutation of
 3281 two oscillation parameters of interest. The application of the reactor constraint
 3282 significantly reduces the correlations previously seen in Figure 8.22. There is
 3283 still a small correlation between δ_{CP} and Δm_{32}^2 . The application of the reactor
 3284 constraint has not significantly affected this correlation. The width of the 1 σ
 3285 credible interval in Δm_{32}^2 is wider for a value of $\delta_{CP} = 0$ as compared to a value
 3286 of $\delta_{CP} = \pi$. Similarly, the width of the 1 σ credible interval in δ_{CP} is smaller
 3287 for lower values of $\sin^2(\theta_{23})$.

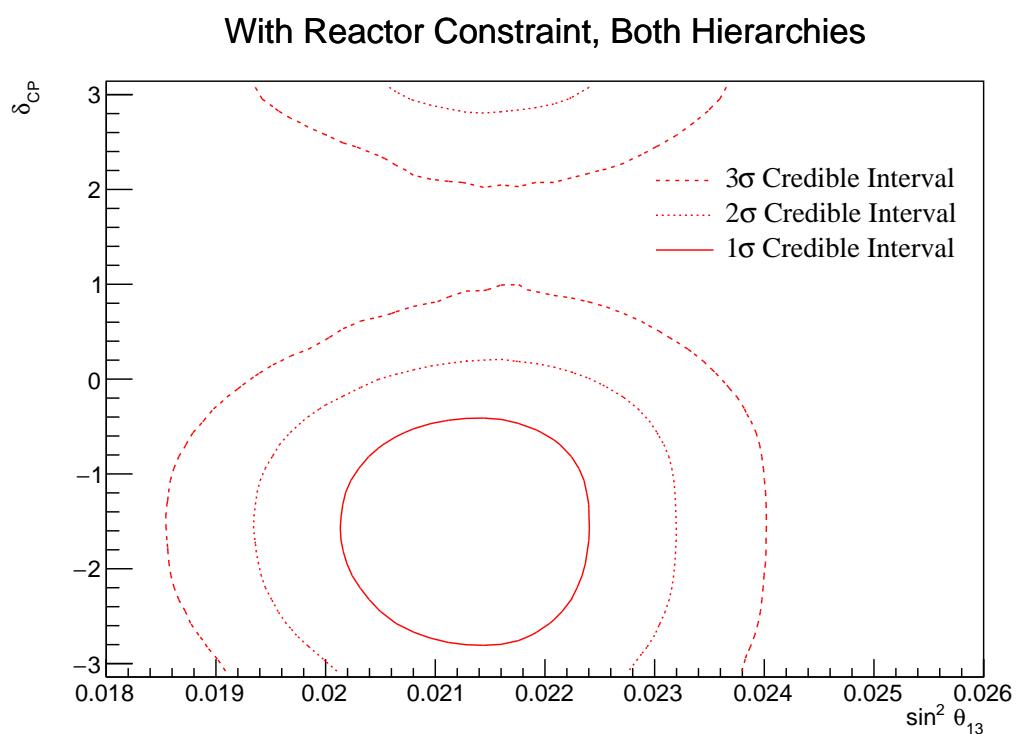


Figure 8.29: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

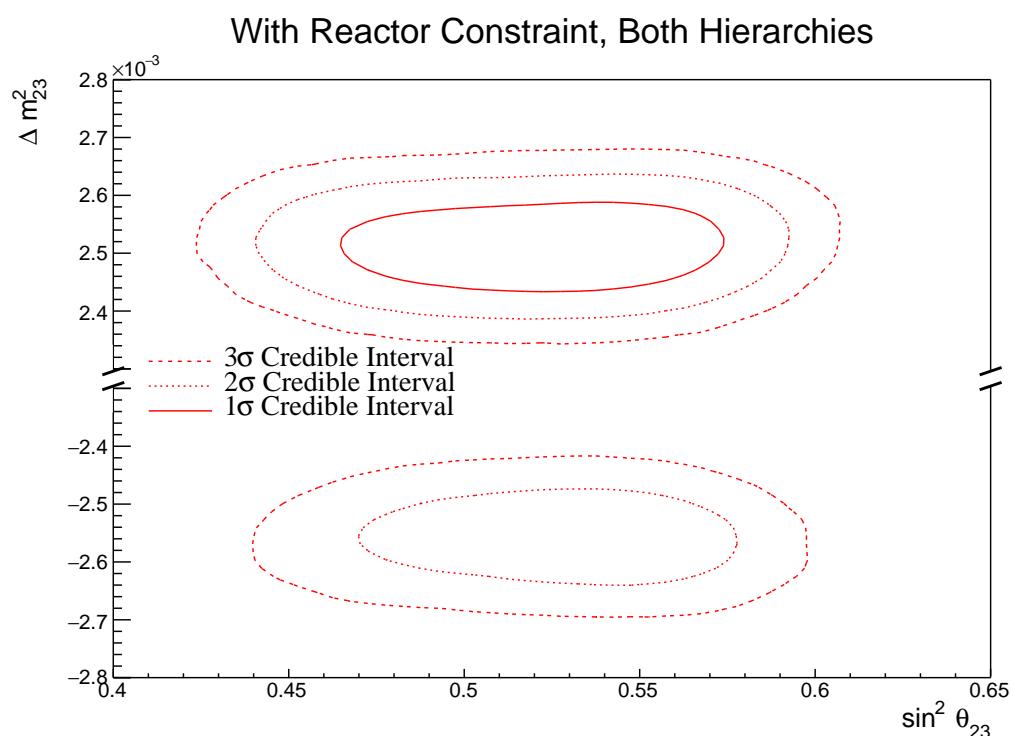


Figure 8.30: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.



Figure 8.31: The posterior probability density distribution from the joint beam and atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3288 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3289 Constraint

3290 The benefits of the joint beam and atmospheric analysis can be determined by
 3291 comparing the sensitivities to the beam-only analysis. This section presents those
 3292 comparisons for sensitivities built using the Asimov A oscillation parameters
 3293 defined in Table 2.2 and the post-BANFF systematic tune. The reactor constraint
 3294 is not applied within either of the fits used in these comparisons.

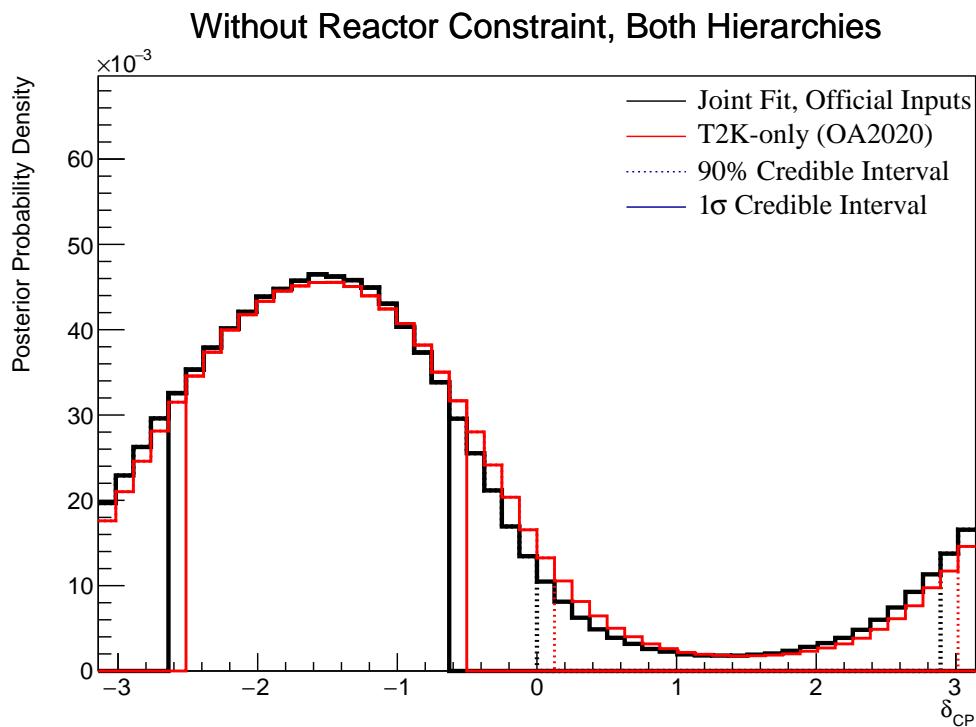


Figure 8.32: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3295 The sensitivity, marginalised over both hierarchies, to δ_{CP} from the joint
 3296 beam-atmospheric and beam-only fits is presented in Figure 8.32. As expected
 3297 from the likelihood scans (Figure 8.4), the sensitivity to δ_{CP} is not significantly
 3298 increased. This is because the known oscillation parameter value ($\delta_{CP} = -1.601$)
 3299 lies at the position where the beam samples dominate the sensitivity compared
 3300 to the SK samples.

The sensitivity to Δm_{32}^2 of the joint beam-atmospheric fit is illustrated in Figure 8.33, where the posterior distribution has been marginalised over both hierarchies. The 1σ credible interval of the joint beam and atmospheric fit is entirely contained within the NH region. This shows the significant increase in the ability of the fit to determine the correct mass hierarchy, as compared to the beam-only analysis. This is further evidenced by the fact that the 90% credible intervals from the joint fit are also tighter in the IH region as compared to the beam-only analysis. The Bayes factor for mass hierarchy determination for the beam-only and joint beam and atmospheric are $B(\text{NH}/\text{IH}) = 1.91$ and $B(\text{NH}/\text{IH}) = 3.67$, respectively. According to Jeffrey's scale (Table 4.1), the beam-only analysis represents a weak preference for the NH hypothesis whereas the joint fit returns a substantial preference for the NH hypothesis. To summarise, the joint beam-atmospheric fit has a substantial preference for the correct hierarchy without the requirement of external constraints.



Figure 8.33: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3315 The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, for both the
 3316 beam-only and joint beam and atmospheric analysis are presented in Figure 8.34.
 3317 The peak of the posterior distribution from the joint analysis is more aligned
 3318 with the known value of $\sin^2(\theta_{23}) = 0.528$ as compared to the beam-only
 3319 analysis. This indicates that the marginalisation effects from other oscillation
 3320 parameters ($\sin^2(\theta_{13}) - \sin^2(\theta_{23})$) presented in Figure 8.22) are less prevalent in
 3321 the projection of this parameter. The Bayes factors for the beam-only and joint
 3322 beam-atmospheric fit are $B(\text{UO}/\text{LO}) = 1.56$ and $B(\text{UO}/\text{LO}) = 1.74$, respectively.
 3323 Therefore, the joint beam-atmospheric fit does prefer the UO more strongly than
 3324 the beam-only analysis, albeit slightly.

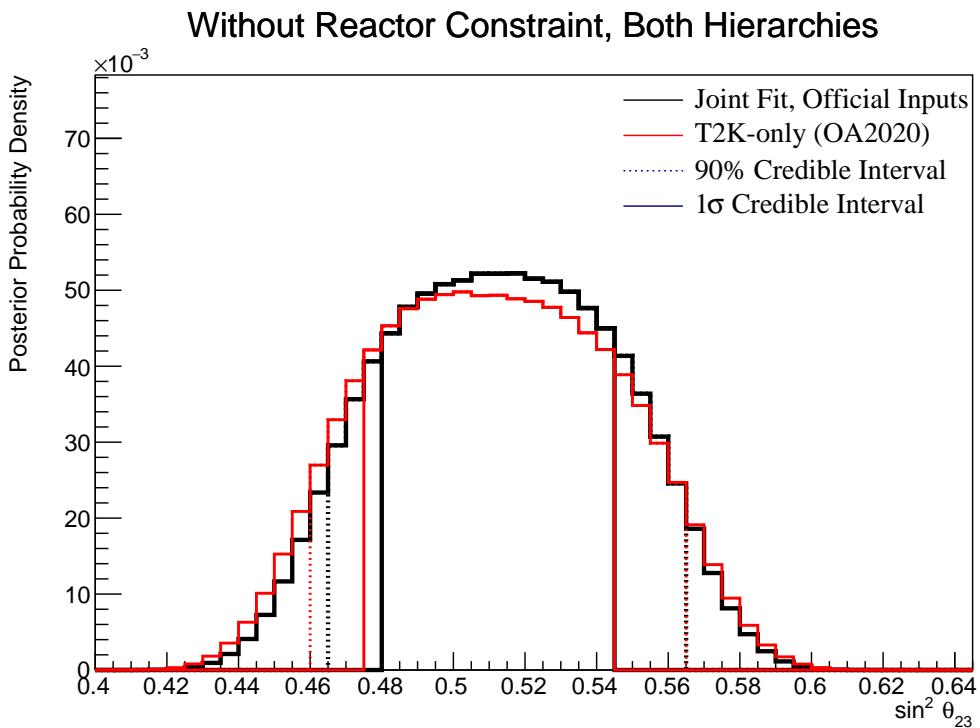


Figure 8.34: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3325 Whilst the beam-only and joint beam-atmospheric fits have similar sensitivity
 3326 to δ_{CP} and $\sin^2(\theta_{23})$ when projected in one-dimension, the benefit of the joint
 3327 analysis becomes more obvious when the sensitivities are presented in two-

dimensions. The sensitivity of the two fits to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$) are illustrated in Figure 8.35.

The width of the 99% joint fit credible interval in $\sin^2(\theta_{13})$ is squeezed in the region of $\delta_{CP} \sim 0$ compared to the beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The 1σ and 90% also exhibit slightly tighter constraints on δ_{CP} . This is most prevalent in the region of $\delta_{CP} \sim 0$ and $\sin^2(\theta_{13}) \sim 0.03$. Whilst the atmospheric samples do not have significant sensitivity to $\sin^2(\theta_{13})$ (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters $\sin^2(\theta_{23}) - \Delta m_{32}^2$, marginalised over both hierarchies, is presented in Figure 8.36 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. The 1σ joint fit credible interval in the IH region more strongly favours the UO as compared to the beam-only fit. Even in the NH region, the width of the credible intervals in $\sin^2(\theta_{23})$ decrease, albeit to a smaller extent.

The change in sensitivity to $\delta_{CP} - \Delta m_{32}^2$ is illustrated in Figure 8.37. As expected, the contours presented within the IH region are much smaller in the joint fit due to the increased sensitivity to mass hierarchy determination. This culminates in a region around $\delta_{CP} \sim \pi/2$ which is excluded at 3σ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to Δm_{32}^2 and $\sin^2(\theta_{23})$, as a function of $\sin^2(\theta_{13})$, is presented in Figure 8.38 and Figure 8.39, respectively. These sensitivities are marginalised

Without Reactor Constraint, Both Hierarchies



Figure 8.35: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

over both hierarchies. As expected from the previous observations, the Δm_{32}^2 contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH 1σ credible intervals exclude the region around the reactor constraint. This is not a bias from the fit as the known value for Δm_{32}^2 is in the NH region. This does suggest that the application of the reactor constraint would further increase the preference for NH in the joint fit as compared to its effect on the beam-only analysis.

The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as illustrated by Figure 8.39. The joint analysis disfavours the wrong octant hypothesis more strongly in the region of high $\sin^2(\theta_{13})$. This suggests that the application of the reactor constraint will favour the UO more strongly in the joint analysis compared to the beam-only analysis.



Figure 8.36: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

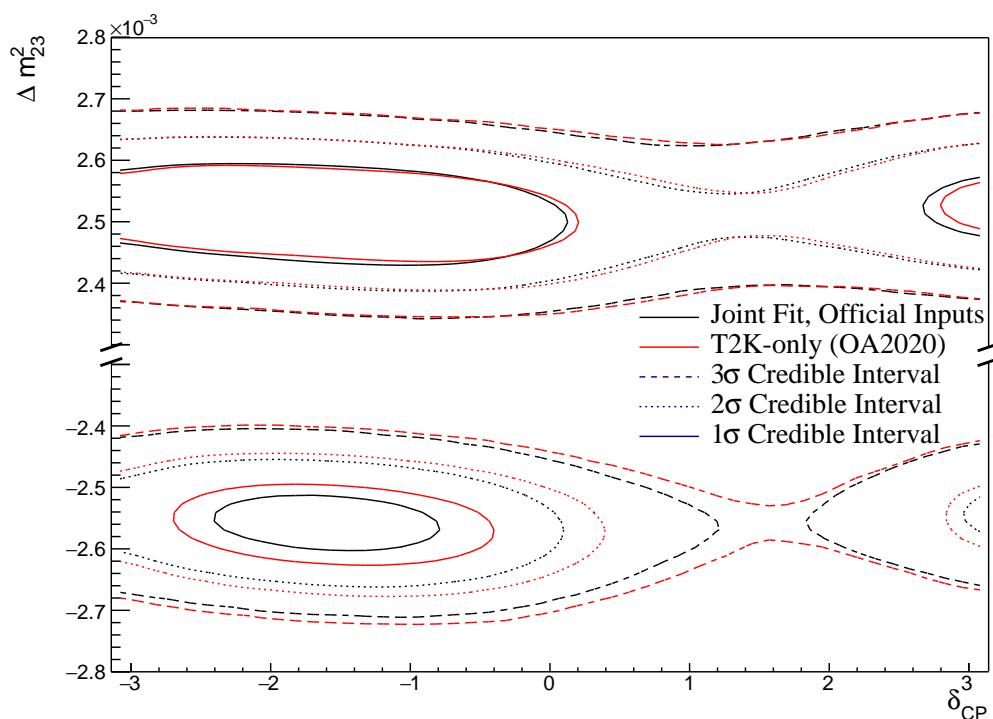


Figure 8.37: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \Delta_{CP}$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

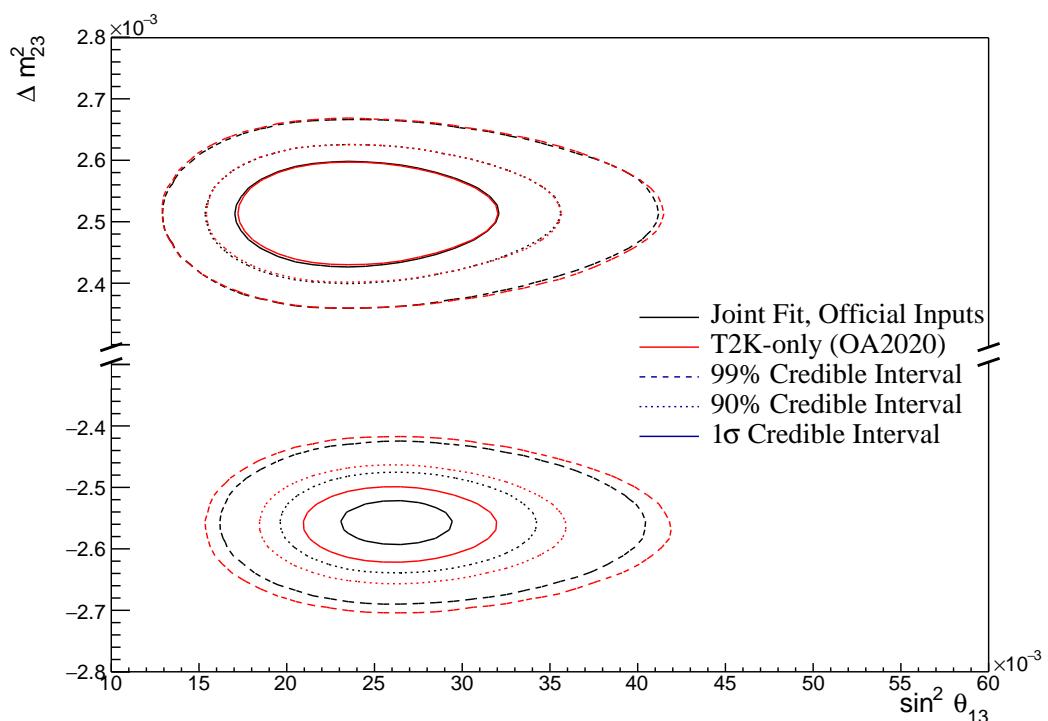


Figure 8.38: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

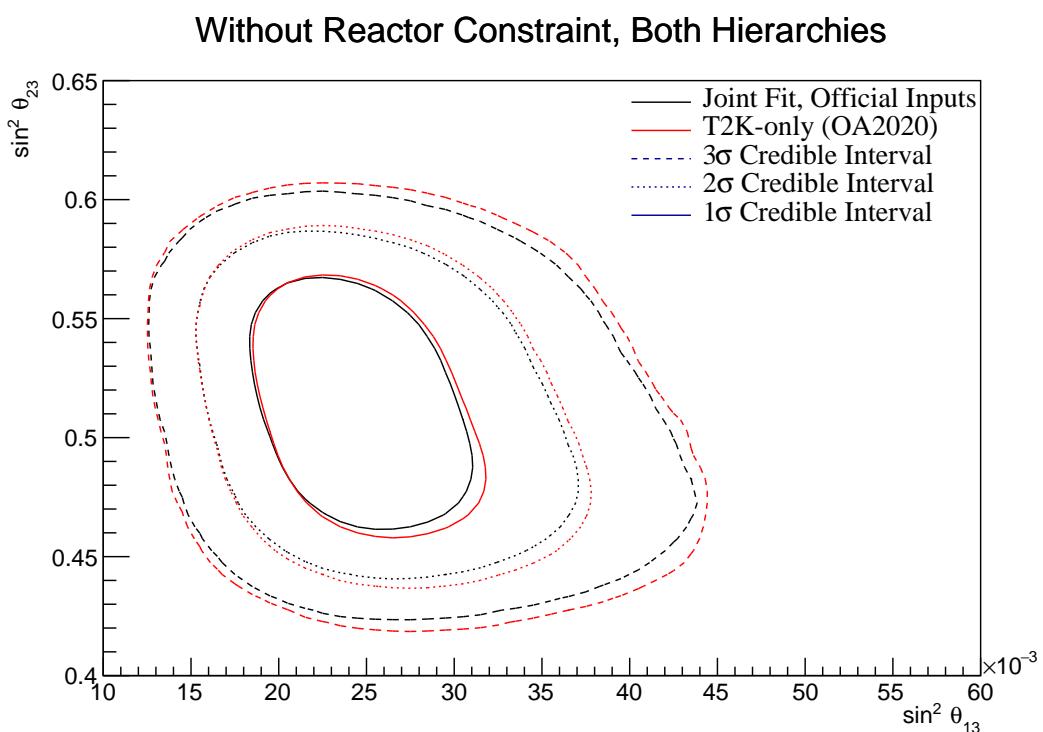


Figure 8.39: The two-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3371 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3372 straint

3373 The comparison between the beam-only and joint beam-atmospheric fits are
 3374 compared in subsection 8.3.6. Those comparisons were made with the reactor
 3375 constraint not applied to either of the fits. This section illustrates the com-
 3376 parison when the reactor constraint is applied. As shown in Figure 8.38, the
 3377 application of the reactor constraint is expected to significantly increase the
 3378 joint fit's preference for the NH hypothesis, as compared to the beam-only
 3379 analysis. Figure 8.40 illustrates the sensitivities of the two fits to the disappearance
 3380 parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) marginalised over both hierarchies and with the
 3381 reactor constraint applied. This plot clearly illustrates the benefit of the joint
 3382 beam and atmospheric analysis. The 1σ credible interval in the IH region is
 3383 entirely removed in the joint analysis, illustrating the improved NH preference.

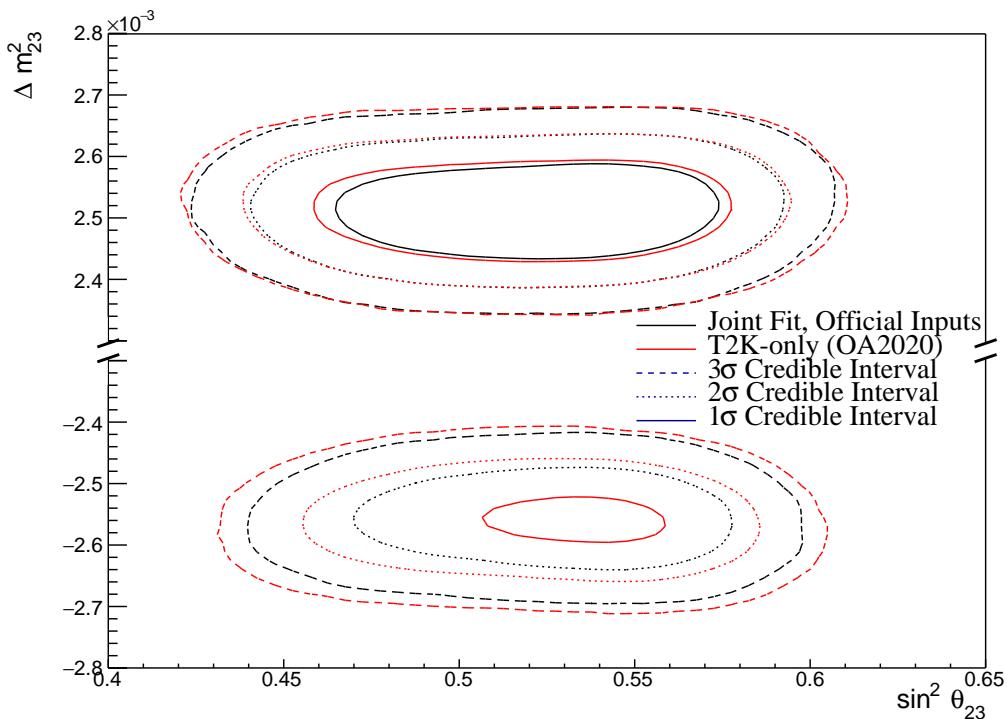


Figure 8.40: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

3384 The credible intervals of the joint fit are also tighter in the $\sin^2(\theta_{23})$ dimension
3385 than the beam-only analysis in both mass hierarchy regions. This shows that
3386 beyond the ability of the joint fit to prefer the NH more strongly than the beam-
3387 only analysis, the precision to which it can measure $\sin^2(\theta_{23})$ is also improved.
3388 The Bayes factor for NH preference is calculated as $B(\text{NH}/\text{IH}) = 7.29$ and
3389 $B(\text{NH}/\text{IH}) = 3.41$ for the joint beam-atmospheric and beam-only analysis,
3390 respectively. Whilst both present a significant preference for the NH hypothesis
3391 (Table 4.1), the joint fit's preference is much stronger. A similar conclusion can be
3392 made regarding the Bayes factors for UO preference which are $B(\text{UO}/\text{LO}) = 2.86$
3393 and $B(\text{UO}/\text{LO}) = 2.67$ for the joint beam-atmospheric and beam-only analysis,
3394 respectively. Both of these represent a mild preference for the UO but there is
3395 a stronger preference observed in the joint analysis.

3396 The sensitivity of the beam-only and joint beam-atmospheric analyses, to the
3397 appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$), are compared in Figure 8.41. These
3398 results are marginalised over both hierarchies and include the reactor constraint
3399 on $\sin^2(\theta_{13})$. For this particular set of known oscillation parameters (AsimovA
3400 defined in Table 2.2), the beam-only analysis dominates the sensitivity. The
3401 joint fit does slightly increase the sensitivity to δ_{CP} but it does not change any
3402 conclusions that would be made.



Figure 8.41: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

3403 8.3.8 Effect of Asimov Parameter Set

3404 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the
 3405 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters
 3406 are defined at a region of δ_{CP} which is dominated by the T2K experiment. This
 3407 explains why the addition of the atmospheric samples does not significantly in-
 3408 crease the sensitivity to δ_{CP} , as illustrated in subsection 8.3.6 and subsection 8.3.7.
 3409 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,
 3410 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when
 3411 building the Asimov data.

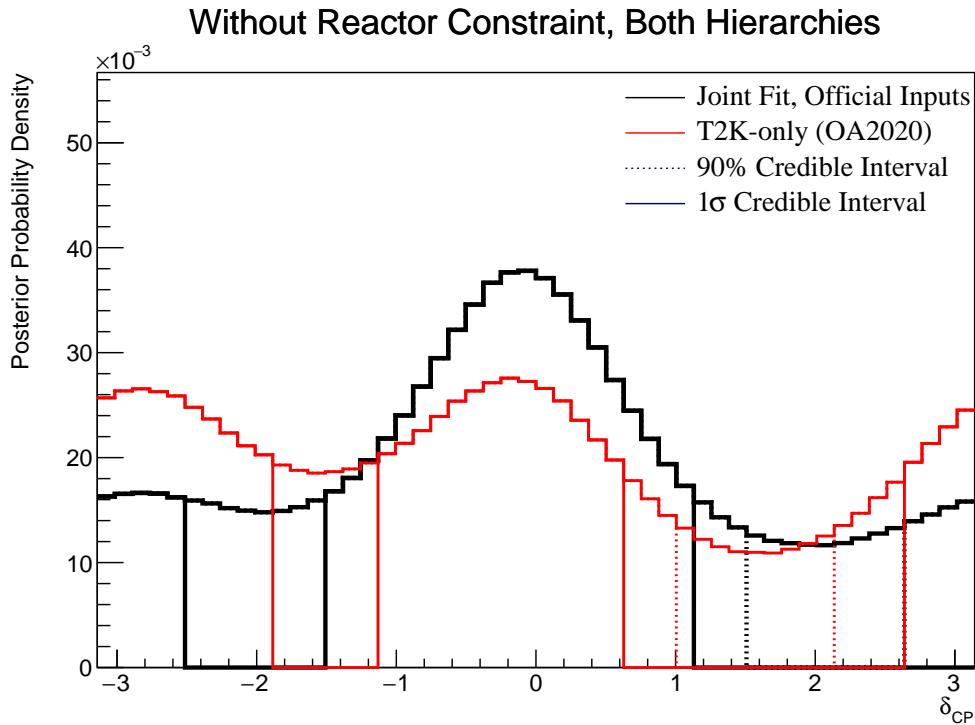


Figure 8.42: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3412 The sensitivity to δ_{CP} for the joint beam and atmospheric fit is presented
 3413 in Figure 8.42. The results are compared to those from the beam-only analysis
 3414 in [183]. The reactor constraint is not applied in either of the fits. The known
 3415 oscillation parameter value is $\delta_{CP} = 0$. The shape of the posterior distribution

3416 from the joint analysis is more peaked at $\delta_{CP} = 0$ as compared to the beam-only
3417 analysis which has approximately the same posterior probability density at $\delta_{CP} =$
3418 0 and $\delta_{CP} = \pi$. This shows the ability of the joint analysis to better determine the
3419 correct phase of δ_{CP} if the true value was CP-conserving. The 1σ credible intervals
3420 and the position of the highest posterior probability density are given in Table 8.9.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -2.51], [-1.51, 1.31]$	-0.06
δ_{CP} , (NH)	$[-1.13, 1.63]$	0.06
δ_{CP} , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.49], [2.46, 2.59]$	2.51
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.47, 2.56]$	2.52
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.61, -2.52]$	-2.57
$\sin^2(\theta_{23})$ (BH)	$[0.43, 0.48], [0.55, 0.59]$	0.45
$\sin^2(\theta_{23})$ (NH)	$[0.43, 0.49], [0.55, 0.58]$	0.45
$\sin^2(\theta_{23})$ (IH)	$[0.44, 0.48], [0.54, 0.59]$	0.57

Table 8.9: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

3421 Naively, if just the 1σ credible interval were considered without observing
3422 the shape of the distribution, it would appear that the joint analysis would
3423 have a worse sensitivity to δ_{CP} due to the larger interval around δ_{CP} . The
3424 1σ credible interval for the beam-only analysis is given as the range $\delta_{CP} =$
3425 $[-\pi, -1.88], [-1.13, 0.63]$ and $[2.64, \pi]$ which contains 56% of all values of δ_{CP} .
3426 The joint beam and atmospheric analysis contains 52% of all δ_{CP} values within
3427 the 1σ credible interval. Therefore, if the area within the 1σ credible interval
3428 were to be compared between the two fits, the joint analysis would be shown
3429 to have better precision.

3430 This contradiction stems from the methodology in which the credible interval
3431 is calculated. The technique used in this analysis (documented in subsection 4.3.2)

3432 fills the credible interval by selecting bins in order of magnitude until 68% of the
3433 posterior density is contained. If instead, the credible interval was calculated
3434 by expanding around the highest posterior probability, the benefits of the joint
3435 fit would be more obvious. In the case where the shape of the posterior was
3436 Gaussian, these two techniques would be equivalent.

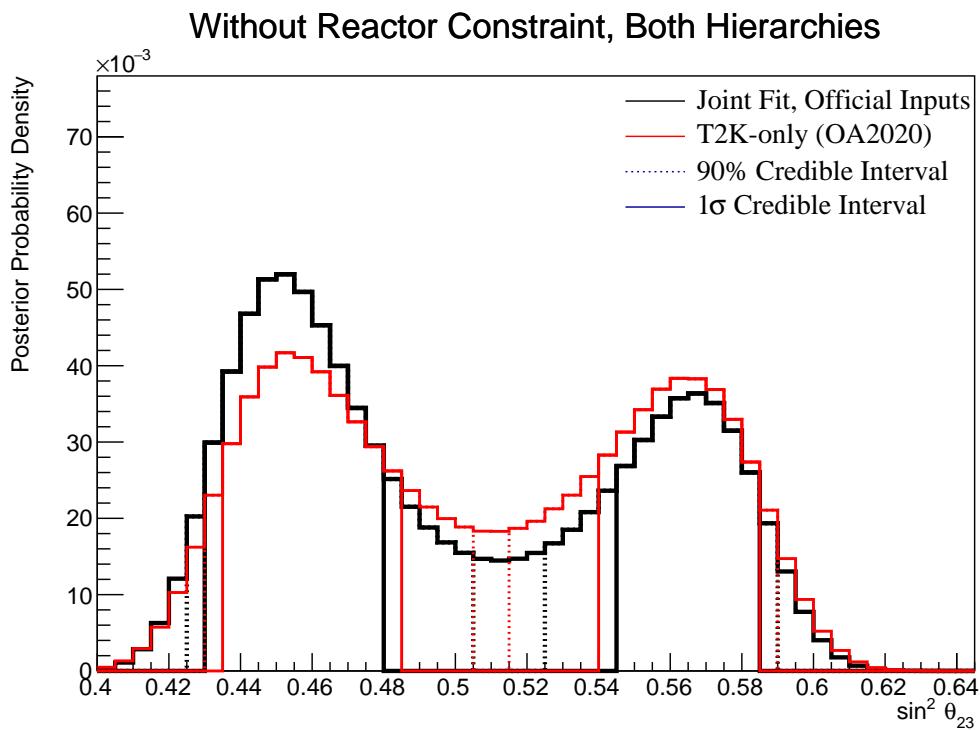


Figure 8.43: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3437 The sensitivity of the joint beam and atmospheric fit to $\sin^2(\theta_{23})$ is presented
3438 in Figure 8.43. The sensitivity is compared to that of the beam-only analysis
3439 in [183]. The reactor constraint is not applied in either of the fits being com-
3440 pared. The Asimov parameter value is $\sin^2(\theta_{23}) = 0.45$ and the sensitivities are
3441 marginalised over both hierarchies. Clearly, the joint beam and atmospheric
3442 fit has a much larger probability density in the region surrounding the known
3443 oscillation parameters. This shows the better octant determination of the joint
3444 analysis compared to the beam-only fit. The ratio of the posterior density at

the peak of the lower octant to the peak of the upper octant from the joint fit is 1.43 compared to 1.09 from the beam-only analysis. This shows further support for the joint analysis in correctly selecting the lower octant, which is the correct hypothesis given the known oscillation parameters.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.35	0.24	0.59
IH ($\Delta m_{32}^2 < 0$)	0.19	0.22	0.41
Sum	0.54	0.46	1.00

Table 8.10: The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$.

The distribution of steps, split by hierarchy and octant hypothesis, is presented in Table 8.10. The Bayes factor for hierarchy and octant determination are $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$, respectively. The octant Bayes factor is now presented as LO/UO as the known oscillation parameter is contained within the lower octant. These values compare to $B(\text{NH}/\text{IH}) = 1.08$ and $B(\text{LO}/\text{UO}) = 0.91$ from the beam-only analysis. This shows additional evidence of the joint analysis's preference for selecting the correct octant and hierarchy hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6 show how the preference for the correct octant and hierarchy depend on the true value of δ_{CP} and $\sin^2(\theta_{23})$.

The sensitivity of the beam-only and joint beam-atmospheric analysis to Δm_{32}^2 is given in Figure 8.44. Both of the results are marginalised over both hierarchies and the reactor constraint is not applied in either analysis. The joint analysis has a stronger preference for the correct hierarchy (NH) which is shown by the higher Bayes factor ($B(\text{NH}/\text{IH}) = 1.43$) compared to the beam-only analysis ($B(\text{NH}/\text{IH}) = 1.08$).

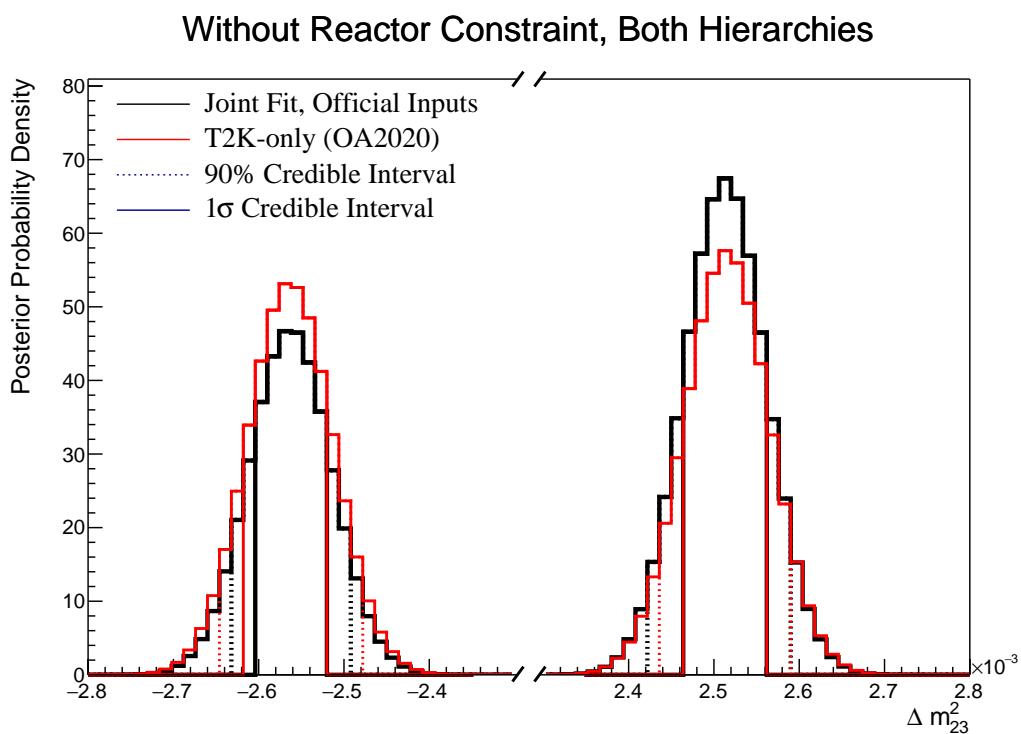


Figure 8.44: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [183]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

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Conclusions and Outlook

Appendices

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A

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Atmospheric Sample Spectra

3470 This appendix documents the interaction mode breakdown of all the atmospheric
3471 samples used within the analysis. The generated tune of the model parameters
3472 and the Asimov A oscillation parameter set (defined in Table 2.2) are assumed.
3473 The livetime of SK-IV is taken to be 3244.4 days.

3474 A.1 Binning

3475 The lepton momentum and cosine zenith binning edges for the atmospheric
3476 samples used within this analysis are defined in Table A.1.

3477 A.2 Fully Contained Sub-GeV Samples

3478 The interaction mode breakdown of the fully contained Sub-GeV samples are
3479 shown in Figure A.1 and Figure A.2, for the samples with enriched CC0 π and
3480 CC1 π^\pm respectively.

3481 The CC0 π sample are dominated by CCQE events ($\sim 70\%$) with smaller
3482 contributions of 2p2h ($\sim 12\%$) and CC1 π ($\sim 10\%$) components. The energy peaks
3483 around 300 MeV, which is slightly below that of the T2K samples but still has
3484 significant contribution upto 1 GeV which overlaps the T2K sample energy range.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV-elike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-elike-1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-pi0like	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV-elike-nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-elike-nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-mulike	10	3.0, 3.4, 5.0
MultiRing-elike-nue	10	3.0, 3.4, 3.7, 5.0
MultiRing-elike-nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing-mulike	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing-Other1	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table A.1: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples.

3485 The one-ring CC1 π samples, where the pion is tagged via its decay electron,
 3486 are dominated by CC1 π events ($\sim 75\%$) with a small contribution of CCM π
 3487 ($\sim 10\%$). The two-ring pion sample is mostly dominated by the NC1 π^0 via
 3488 resonances, and has several equally-sized contributions from CCQE, NC1 π^\pm via
 3489 resonances, and NC coherent pion production, where the π^0 likely comes from
 3490 nucleon and π^\pm final state interactions in the nucleus.

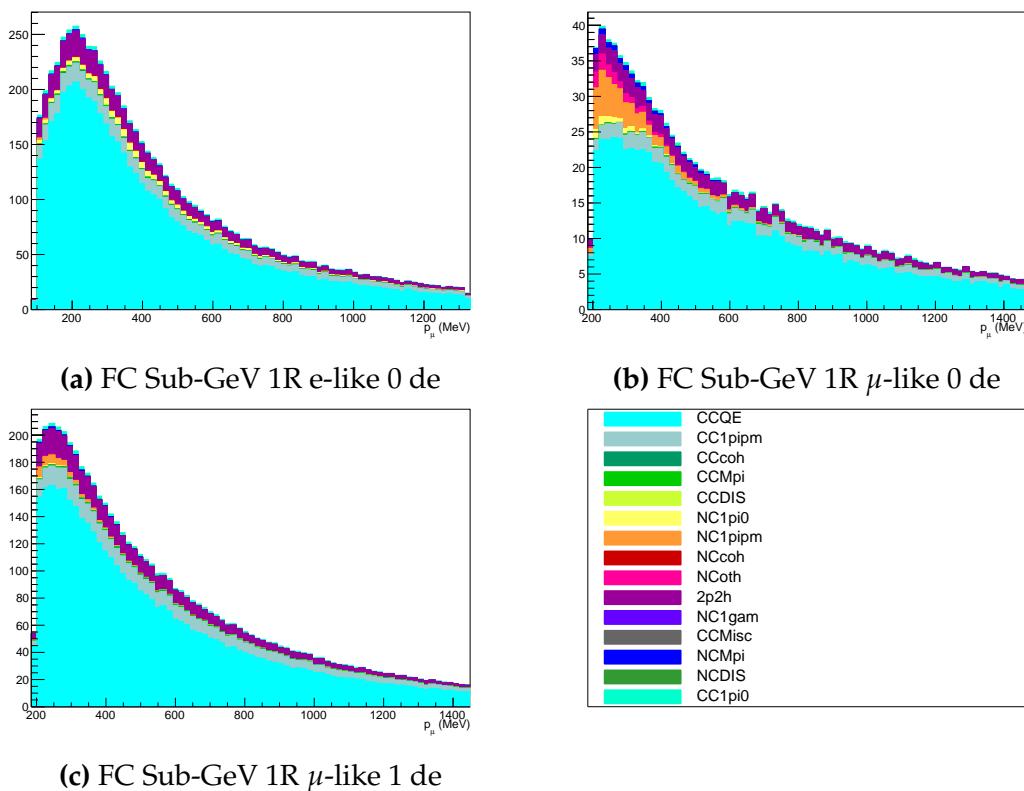


Figure A.1: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting CC 0π events.

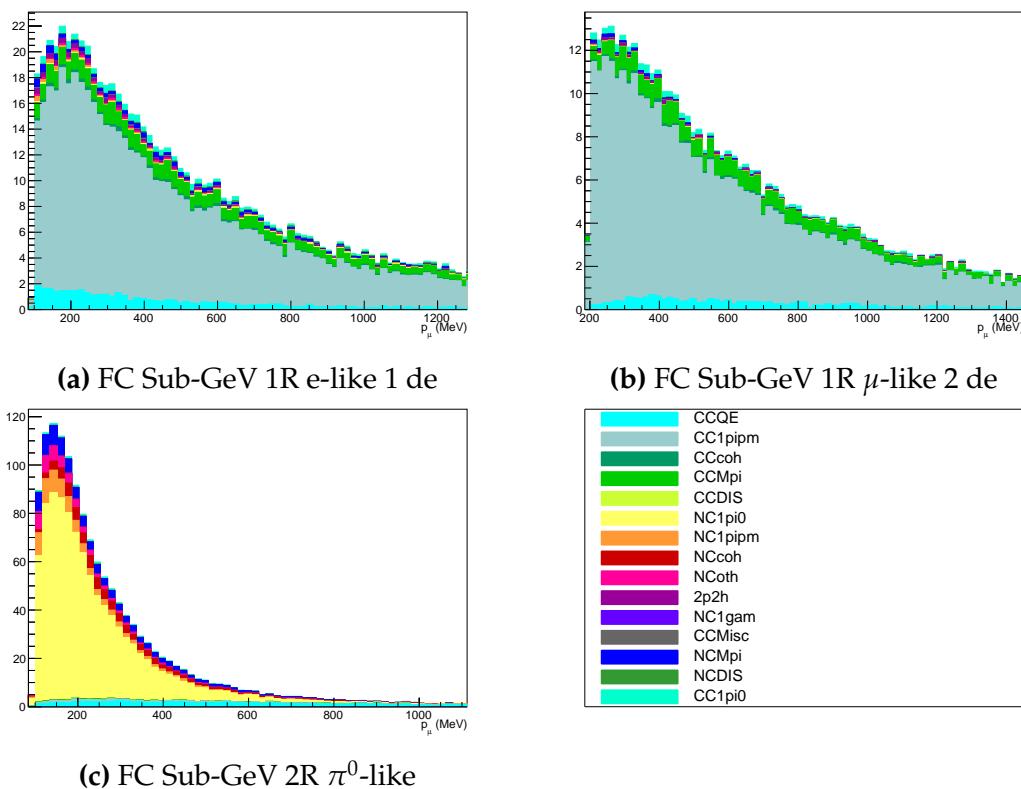


Figure A.2: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting single pion events.

3491 A.3 Fully Contained Multi-GeV Samples

3492 The interaction mode breakdown of fully contained multi-GeV samples is high-
 3493 lighted in Figure A.3. Due to the event selection applied in SK which targets π^+
 3494 and π^- separation, the ν_e sample mainly consists of events with pions (single pion
 3495 production or multi-pion/DIS interactions). The pion separation is explained in
 3496 Section section 6.1. This reasoning also explains the significant CCQE contribution
 3497 of the $\bar{\nu}_e$ sample. The muon-like sample is dominated by CCQE interactions with
 3498 $\sim 10 - 15\%$ 2p2h and CC1 π contribution of events.

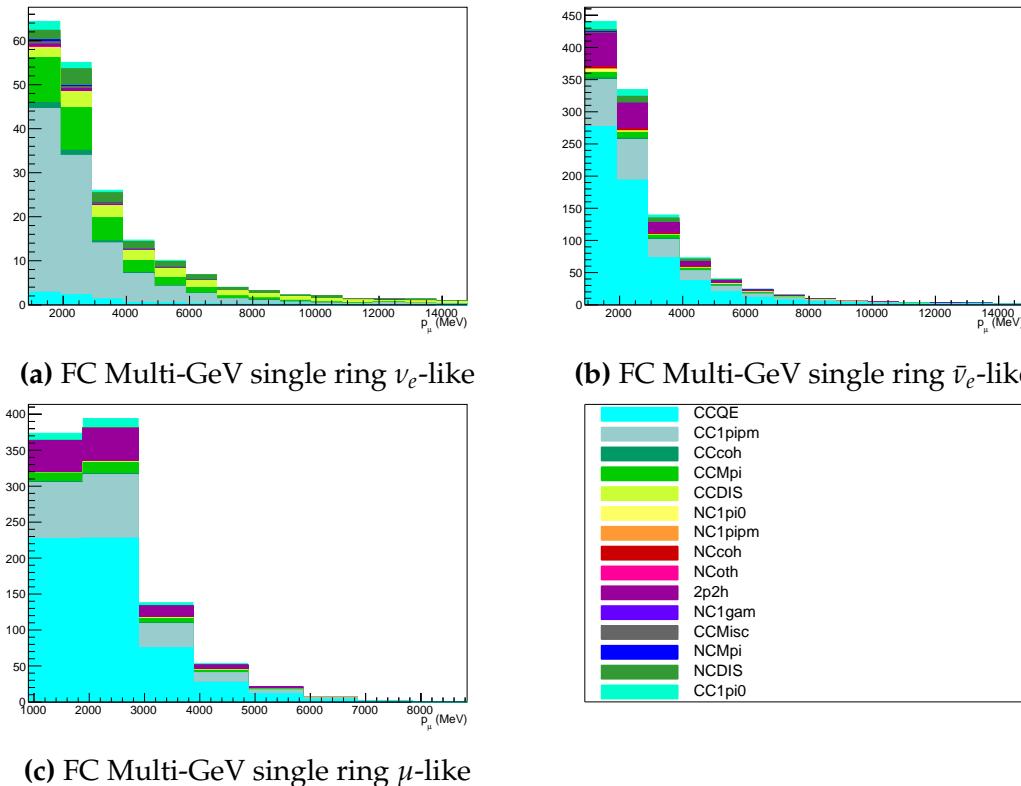


Figure A.3: Breakdown by interaction mode of the FC Multi-GeV single ring atmospheric samples.

3499 A.4 Fully Contained Multi-Ring Samples

3500 The interaction mode breakdown of fully contained multi-ring events is shown
 3501 in Figure A.4. These samples see more interaction modes contributing in general,
 3502 and there is a much larger contribution from multi-pion and DIS interaction
 3503 modes, compared to the other samples.



Figure A.4: Breakdown by interaction mode of the FC Multi-GeV multi-ring atmospheric samples.

3504 A.5 Partially Contained Samples

3505 The breakdown for partially contained samples is highlighted in Figure A.5.
 3506 As with the multi-ring samples, there is no dominating interaction mode. The
 3507 neutrino energies of events in this sample extend into the tens of GeV and become
 3508 dominated by DIS interaction modes in the high energy limit.

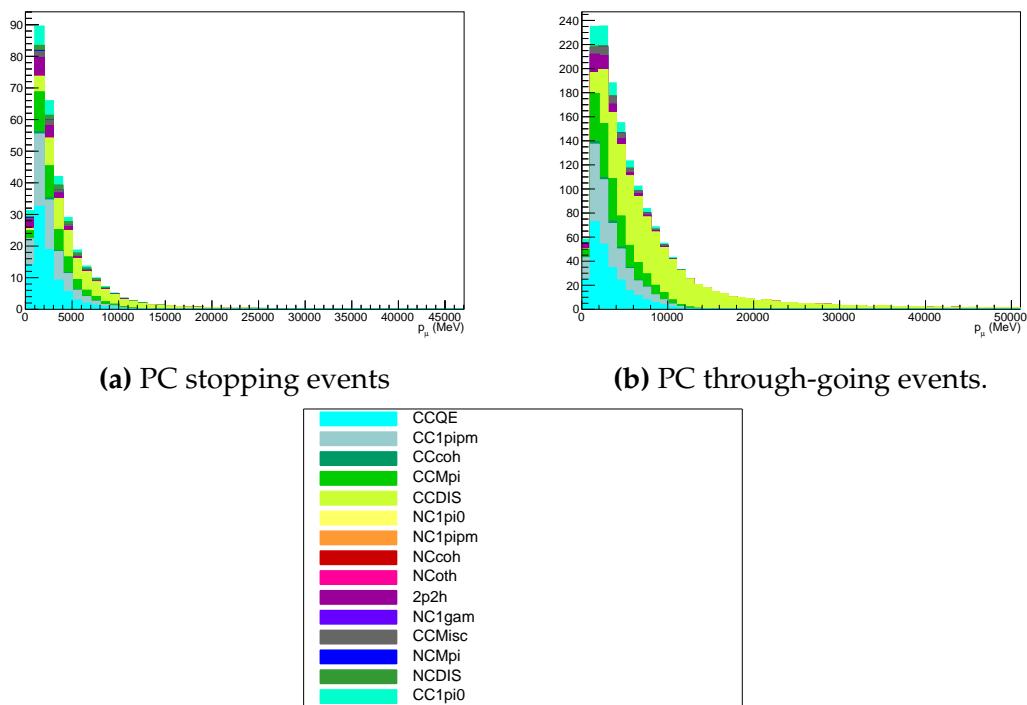


Figure A.5: Breakdown by interaction mode of the PC atmospheric samples.

3509 A.6 Upward-Going Muon Samples

3510 The breakdown for upward-going muons is illustrated in Figure A.6. These
 3511 samples are significantly dominated by DIS interactions with energies extending
 3512 up into the hundreds of GeV.

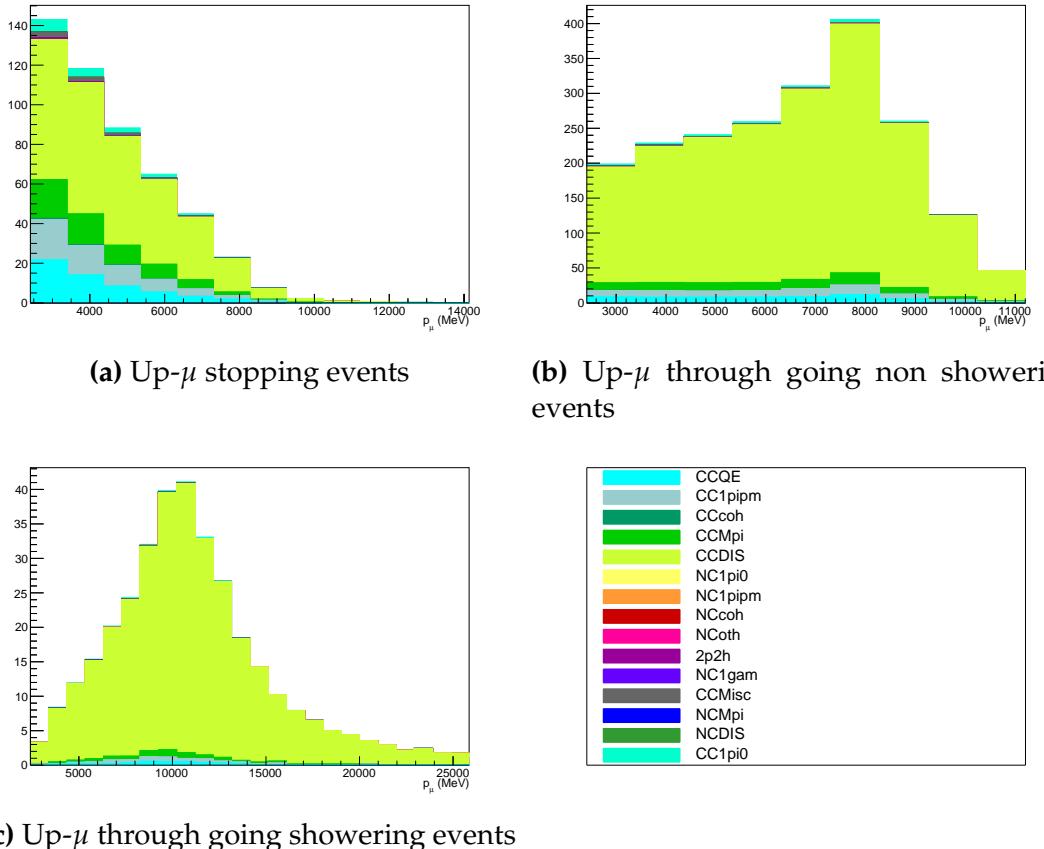


Figure A.6: Breakdown by interaction mode of the atmospheric upward going muon samples.

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