

1      **The Sensitivity to Oscillation Parameters**  
2      **from a Simultaneous Beam and**  
3      **Atmospheric Neutrino Analysis that**  
4      **combines the T2K and SK Experiments**



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9      A thesis submitted for the degree of  
10     *Doctor of Philosophy*  
11     Michaelmas 2022

# Abstract

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# Acknowledgements

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# 1

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## Introduction

# 2

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## Neutrino Oscillation Physics

129 When first proposed, neutrinos were expected to be approximately massless  
130 fermions that only interact through weak and gravitational forces. This meant  
131 they were very difficult to detect as they can pass through significant amounts  
132 of matter without interacting. Despite this, experimental neutrino physics has  
133 developed many different detection techniques and observed neutrinos from  
134 both natural and artificial sources. In direct tension with Standard Model physics,  
135 neutrinos have been determined to oscillate between different lepton flavours,  
136 requiring them to have mass.

137 The observation techniques which led to the discovery of the neutrino are doc-  
138 umented in section 2.1. The theory underpinning neutrino oscillation is described  
139 in section 2.2 and includes the approximations which can be made to simplify  
140 the understanding of neutrino oscillation in the two-flavour approximation. Past,  
141 current, and future neutrino experiments are detailed in section 2.3, including the  
142 reactor, atmospheric, and long-baseline accelerator neutrino sources that have  
143 been used to successfully constrain oscillation parameters. Finally, the current  
144 state of oscillation parameter measurements are summarised in section 2.4.

## 2.1 Discovery of Neutrinos

At the start of the 20<sup>th</sup> century, the electrons emitted from the  $\beta$ -decay of the nucleus were found to have a continuous energy spectrum [1, 2]. This observation seemingly broke the energy conservation invoked within that period's nuclear models. In 1930, Pauli provided a solution to this problem in the form of a new particle, the neutrino (originally termed "neutron"). It was theorized to be an electrically neutral spin-1/2 fermion with a mass smaller than that of the electron [3]. This neutrino was emitted with the electron in  $\beta$ -decay to alleviate the apparent breaking of energy conservation. As a predecessor of today's weak interaction model, Fermi's theory of  $\beta$ -decay developed the understanding by coupling the four constituent particles: electron, proton, neutron, and neutrino, into a quantitative model [4].

Whilst Pauli was not convinced of the ability to detect neutrinos, the first observations of the particle were made in the mid-1950s when neutrinos from a reactor were observed via the inverse  $\beta$ -decay (IBD) process,  $\bar{\nu}_e + p \rightarrow n + e^+$  [5, 6]. The detector consisted of two parts: a neutrino interaction medium and a liquid scintillator. The interaction medium was built from two water tanks, loaded with cadmium chloride to allow for increased efficiency in the detection of neutron capture. The positron emitted from IBD annihilates,  $e^+ + e^- \rightarrow 2\gamma$ , generating a prompt signal and the neutron is captured on the cadmium via  $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$ , producing a delayed signal. An increase in the coincidence rate was observed when the reactor was operating which was interpreted as interactions from neutrinos generated in the reactor.

After the discovery of the  $\nu_e$ , the question of how many flavours of neutrino exist was asked. In 1962, a measurement of the  $\nu_\mu$  was conducted at the Brookhaven National Laboratory [7]. A proton beam was directed at a beryllium target, generating pions which then decayed via  $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$ , and the subsequent interactions of the  $\nu_\mu$  were observed. As the subsequent interaction of the neutrino generated muons rather than electrons, it was determined that

<sup>174</sup> the  $\nu_\mu$  was fundamentally different from  $\nu_e$ . The final observation to be made  
<sup>175</sup> was that of the  $\nu_\tau$  from the DONUT experiment [8]. Three neutrinos seem the  
<sup>176</sup> obvious solution as it mirrors the known number of charged leptons (as they form  
<sup>177</sup> weak isospin doublets) but there could be evidence of more. Several neutrino  
<sup>178</sup> experiments have found anomalous results [9, 10] which could be attributed  
<sup>179</sup> to “sterile” neutrinos. These hypothesised particles are not affected by gauge  
<sup>180</sup> interactions in the Standard Model so their presence can only be inferred through  
<sup>181</sup> the observation of non-standard oscillation modes. However, cosmological  
<sup>182</sup> observations indicate the number of neutrino species  $N_{eff} = 2.99 \pm 0.17$  [11], as  
<sup>183</sup> measured from the cosmic microwave background power spectrum. LEP also  
<sup>184</sup> measured the number of active neutrino flavours to be  $N_\nu = 2.9840 \pm 0.0082$  [12]  
<sup>185</sup> from measurements of the Z-decay width, but this does not strongly constrain  
<sup>186</sup> the number of sterile neutrinos.

## <sup>187</sup> 2.2 Theory of Neutrino Oscillation

<sup>188</sup> A neutrino generated with lepton flavour  $\alpha$  can change into a different lepton  
<sup>189</sup> flavour  $\beta$  after propagating some distance. This phenomenon is called neutrino  
<sup>190</sup> oscillation and requires that neutrinos must have a non-zero mass. This behaviour  
<sup>191</sup> has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13–  
<sup>192</sup> 15] mixing matrix which describes how the flavour and mass of neutrinos are  
<sup>193</sup> associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [16]  
<sup>194</sup> matrix measured in quark physics.

### <sup>195</sup> 2.2.1 Three Flavour Oscillations

<sup>196</sup> The PMNS parameterisation defines three flavour eigenstates,  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$   
<sup>197</sup> (indexed  $\nu_\alpha$ ), which are eigenstates of the weak interaction and three mass  
<sup>198</sup> eigenstates,  $\nu_1$ ,  $\nu_2$  and  $\nu_3$  (indexed  $\nu_i$ ). Each mass eigenstate is the superposition

<sup>199</sup> of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

<sup>200</sup> Where  $U$  is the  $3 \times 3$  PMNS matrix which is unitary and connects the mass  
<sup>201</sup> and flavour eigenstates.

<sup>202</sup> The weak interaction, when interacting via a  $W^{\pm}$  boson, couples to flavour  
<sup>203</sup> eigenstates so neutrinos interact with leptons of the same flavour. The prop-  
<sup>204</sup> agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a  
<sup>205</sup> plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

<sup>206</sup> The  $\phi_i$  term can be expressed in terms of the energy,  $E_i$ , and magnitude of the  
<sup>207</sup> three momenta,  $p_i$ , of the neutrino,  $\phi_i = E_i t - p_i x$  ( $t$  and  $x$  being time and position  
<sup>208</sup> coordinates). The probability of observing a neutrino of flavour eigenstate  $\beta$  from  
<sup>209</sup> one which originated as flavour  $\alpha$  can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)} \quad (2.3)$$

<sup>210</sup> The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

<sup>211</sup> For a relativistic particle,  $E_i \gg m_i$ , a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

<sup>212</sup> Making the approximations that neutrinos are relativistic, the mass eigenstates  
<sup>213</sup> were created with the same energy and that  $x = L$ , where  $L$  is the distance  
<sup>214</sup> travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

where  $\Delta m_{ij}^2 = m_i^2 - m_j^2$ . This, combined with further use of unitarity relations results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left( U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left( \frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left( U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left( \frac{\Delta m_{ij}^2 L}{2E} \right) \end{aligned} \quad (2.7)$$

Where  $\delta_{\alpha\beta}$  is the Kronecker delta function and the negative sign on the last term is included for the oscillation probability of antineutrinos. As an important point to note, the observation of oscillation probability requires a non-zero value of  $\Delta m_{ij}^2$ , which in turn requires that neutrinos have differing masses.

Typically, the PMNS matrix is parameterised into three mixing angles, a charge parity (CP) violating phase  $\delta_{CP}$ , and two Majorana phases  $\alpha_{1,2}$ ,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

Where  $s_{ij} = \sin(\theta_{ij})$  and  $c_{ij} = \cos(\theta_{ij})$ . The oscillation parameters are often grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many neutrino experiments aim to measure the PMNS parameters from a wide array of origins, as is the purpose of this thesis.

The Majorana phase,  $\alpha_{1,2}$ , included within the fourth matrix in Equation 2.8 is only included for completeness. For an oscillation analysis experiment, any terms containing this phase disappear due to taking the expectation value of the PMNS matrix. Measurements of these phases can be performed by experiments searching for neutrino-less double  $\beta$ -decay [17].

232 A two-flavour approximation can be obtained when one assumes the third  
233 mass eigenstate is degenerate with another. This results in the two-flavour  
234 approximation being reasonable for understanding the features of the oscillation.  
235 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

236 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

237 Where  $\alpha \neq \beta$ . For a fixed neutrino energy, the oscillation probability is  
238 a sinusoidal function depending upon the distance over which the neutrino  
239 propagates. The frequency and amplitude of oscillation are dependent upon  
240  $\Delta m^2/4E$  and  $\sin^2 2\theta$ , respectively. The oscillation probabilities presented thus far  
241 assume  $c = 1$ , where  $c$  is the speed of light in a vacuum. In more familiar units, the  
242 maximum oscillation probability for a fixed value of  $\theta$  is given at  $L[\text{km}] / E[\text{GeV}] \sim$   
243  $1.27/\Delta m^2$ . It is this calculation that determines the best  $L/E$  value for a given  
244 experiment to be designed around for measurements of a specific value of  $\Delta m^2$ .

### 245 2.2.2 The MSW Effect

246 The theory of neutrino oscillation in a vacuum has been described in subsec-  
247 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating  
248 from below the horizon propagate through the matter in the Earth. The coherent  
249 scattering of neutrinos from a material target modifies the Hamiltonian of the  
250 system which results in a change in the oscillation probability. This modification  
251 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [18–20]. This occurs  
252 because charged current scattering ( $\nu_e + e^- \rightarrow \nu_e + e^-$ , propagated by a W boson)  
253 only affects electron neutrinos whereas the neutral current scattering ( $\nu_l + l^- \rightarrow$

<sup>254</sup>  $\nu_l + l^-$ , propagated by a  $Z^0$  boson) interacts through all neutrino flavours equally.  
<sup>255</sup> In the two-flavour approximation, the effective mixing parameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

<sup>256</sup> where  $A = 2\sqrt{2}G_F N_e E$ ,  $N_e$  is the electron density of the medium and  $G_F$   
<sup>257</sup> is Fermi's constant. It is clear that there exists a value of  $A = \Delta m^2 \cos(2\theta)$  for  
<sup>258</sup>  $\Delta m^2 > 0$ , which results in a divergent mixing parameter, colloquially called the  
<sup>259</sup> matter resonance. This resonance regenerates the electron neutrino component of  
<sup>260</sup> the neutrino flux [18–20]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

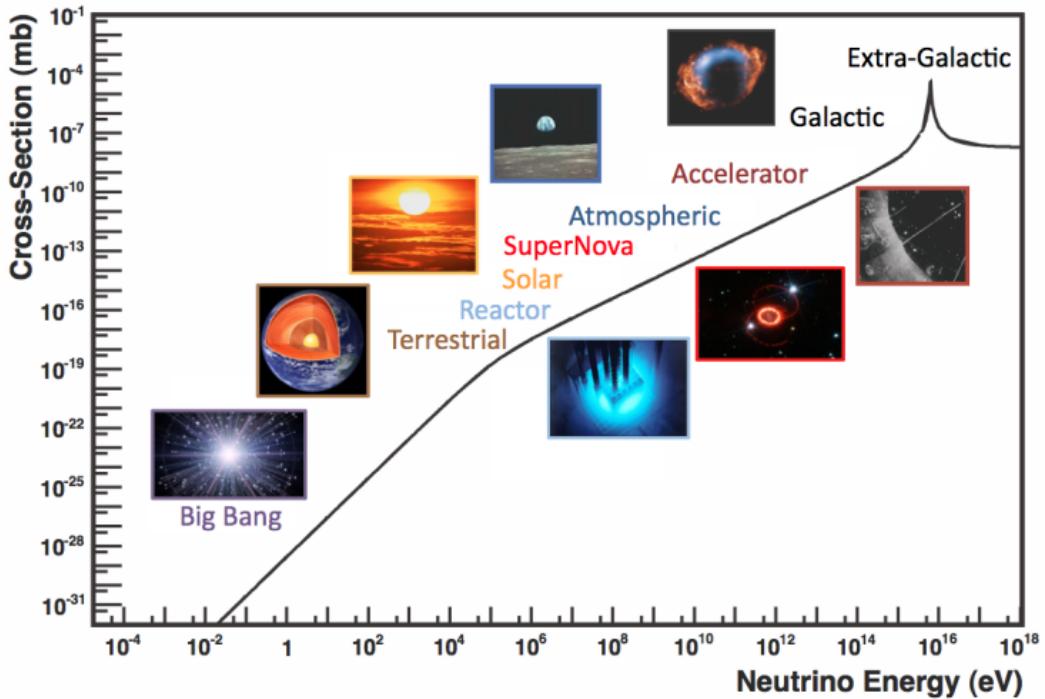
<sup>261</sup> At densities lower than this critical value, the oscillation probability will  
<sup>262</sup> be much closer to that of vacuum oscillation. For antineutrinos,  $N_e \rightarrow -N_e$   
<sup>263</sup> [21]. The resonance occurring from the MSW effect depends on the sign of  $\Delta m^2$ .  
<sup>264</sup> Therefore, any neutrino oscillation experiment which observes neutrinos and  
<sup>265</sup> antineutrinos which have propagated through matter can have some sensitivity  
<sup>266</sup> to the ordering of the neutrino mass eigenstates.

## <sup>267</sup> 2.3 Neutrino Oscillation Measurements

<sup>268</sup> As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics  
<sup>269</sup> was awarded to the Super-Kamiokande (SK) [22] and Sudbury Neutrino Ob-  
<sup>270</sup> servatory (SNO) [23] collaborations for the first definitive observation of solar  
<sup>271</sup> and atmospheric neutrino oscillation [24]. Since then, the field has seen a wide  
<sup>272</sup> array of oscillation measurements from a variety of neutrino sources. As seen  
<sup>273</sup> in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio  
<sup>274</sup> of the propagation baseline,  $L$ , to the neutrino energy,  $E$ . It is this ratio that  
<sup>275</sup> determines the type of neutrino oscillation a particular experiment is sensitive to.

<sup>276</sup> As illustrated in Figure 2.1, there are many neutrino sources that span a  
<sup>277</sup> wide range of energies. The least energetic neutrinos are from reactor and

<sup>278</sup> terrestrial sources at  $O(1)$ MeV whereas the most energetic neutrinos originate  
<sup>279</sup> from atmospheric and galactic neutrinos of  $> O(1)$ TeV.



**Figure 2.1:** The electro-weak cross-section for  $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$  scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [25]

### <sup>280</sup> 2.3.1 Solar Neutrinos

<sup>281</sup> Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun.  
<sup>282</sup> The solar neutrino flux, given as a function of neutrino energy for different  
<sup>283</sup> fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion  
<sup>284</sup> generates the largest flux of neutrinos, the neutrinos are low energy and are  
<sup>285</sup> difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [26].  
<sup>286</sup> Consequently, most experiments focus on the neutrinos from the decay of  $^8B$   
<sup>287</sup> (via  $^8B \rightarrow ^8Be^* + e^+ + \nu_e$ ), which are higher energy.

<sup>288</sup> The first measurements of solar neutrinos observed a significant reduction in  
<sup>289</sup> the event rate compared to predictions from the Standard Solar Model [28, 29]. A  
<sup>290</sup> proposed solution to this “solar neutrino problem” was  $\nu_e \leftrightarrow \nu_\mu$  oscillations in a



**Figure 2.2:** The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [27].

291 precursory version of the PMNS model [30]. The Kamiokande [31], Gallex [32]  
 292 and Sage [33] experiments confirmed the  $\sim 0.5$  factor deficit of solar neutrinos.

293 The conclusive solution to this problem was determined by the SNO col-  
 294 laboration [34]. Using a deuterium water target to observe  ${}^8B$  neutrinos, the  
 295 event rate of charged current (CC), neutral current (NC), and elastic scattering  
 296 (ES) interactions (Given in Equation 2.13) was simultaneously measured. CC  
 297 events can only occur for electron neutrinos, whereas the NC channel is agnostic  
 298 to neutrino flavour, and the ES reaction has a small excess sensitivity for the  
 299 detection of electron neutrino interactions. This meant that there were direct  
 300 measurements of the  $\nu_e$  and  $\nu_x$  neutrino flux. It was concluded that the CC and  
 301 ES interaction rates were consistent with the deficit previously observed. Most  
 302 importantly, the NC reaction rate was only consistent with the others under the

303 hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (CC) \\ \nu_x + d &\rightarrow p + n + \nu_x & (NC) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (ES) \end{aligned} \quad (2.13)$$

304 Since the SNO measurement, many experiments have since measured the  
 305 neutrino flux of different interaction chains within the sun [35–37]. The most  
 306 recent measurement was that of CNO-cycle neutrinos which were recently  
 307 observed with  $5\sigma$  significance by the Borexino collaboration [35].

### 308 2.3.2 Accelerator Neutrinos

309 The concept of using an artificial “neutrino beam” was first realised in 1962 [38].  
 310 Since then, many experiments have adopted the same fundamental concepts.  
 311 Typically, a proton beam is aimed at a target producing charged mesons that  
 312 decay to neutrinos. The mesons can be sign-selected by the use of magnetic  
 313 focusing horns to generate a neutrino or antineutrino beam. Pions are the primary  
 314 mesons that decay and depending on the orientation of the magnetic field, a  
 315 muon (anti-)neutrino beam is generated via  $\pi^+ \rightarrow \mu^+ + \nu_\mu$  or  $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ .  
 316 The decay of muons and kaons results in an irreducible intrinsic electron neutrino  
 317 background. In T2K, this background contamination is  $O(< 1\%)$  [39]. There is  
 318 also an approximately  $\sim 5\%$  “wrong-sign” neutrino background of  $\bar{\nu}_\mu$  generated  
 319 via the same decays. As the beam is generated by proton interactions (rather  
 320 than anti-proton interactions), the wrong-sign component in the antineutrino  
 321 beam is larger when operating in neutrino mode.

322 Tuning the proton energy in the beam and using beam focusing techniques  
 323 allows the neutrino energy to be set to a value that maximises the disappear-  
 324 ance oscillation probability in the  $L/E$  term in Equation 2.10. This means that  
 325 accelerator experiments are typically more sensitive to the mixing parameters as  
 326 compared to a natural neutrino source. However, the disadvantage compared  
 327 to atmospheric neutrino experiments is the cost of building a facility to provide

328 high-energy neutrinos, with a high flux, which is required for longer baselines.  
 329 Consequently, there is typically less sensitivity to matter effects and the ordering  
 330 of the neutrino mass eigenstates.

331 A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

332 where  $R(\vec{x})$  is the event rate of neutrinos at position  $\vec{x}$ ,  $\Phi(E_\nu)$  is the flux of  
 333 neutrinos with energy  $E_\nu$ ,  $\sigma(E_\nu)$  is the cross-section of the neutrino interaction and  
 334  $\epsilon(\vec{x})$  is the efficiency and resolution of the detector. In order to leverage the most  
 335 out of an accelerator neutrino experiment, the flux and cross-section systematics  
 336 need to be constrained. This is typically done via the use of a “near detector”,  
 337 situated at a baseline of  $O(1)$ km. This detector observes the unoscillated neutrino  
 338 flux and constrains the parameters used within the flux and cross-section model.

339 The first accelerator experiments to precisely measure oscillation parameters  
 340 were MINOS [40] and K2K [41]. These experiments confirmed the  $\nu_\mu$  disappear-  
 341 ance seen in atmospheric neutrino experiments by finding consistent parameter  
 342 values for  $\sin^2(\theta_{23})$  and  $\Delta m_{32}^2$ . The current generation of accelerator neutrino  
 343 experiments, T2K and NO $\nu$ A extended this field by observing  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  and lead  
 344 the sensitivity to atmospheric mixing parameters as seen in Figure 2.6 [42]. The  
 345 two experiments differ in their peak neutrino energy, baseline, and detection tech-  
 346 nique. The NO $\nu$ A experiment is situated at a baseline of 810km from the NuMI  
 347 beamline which delivers 2GeV neutrinos. The T2K neutrino beam is peaked  
 348 around 0.6GeV and propagates 295km. Additionally, the NO $\nu$ A experiment  
 349 uses functionally identical detectors (near and far) whereas T2K uses a plastic  
 350 scintillator technique at the near detector and a water Cherenkov far detector.  
 351 The future generation experiments DUNE [43] and Hyper-Kamiokande [44]  
 352 will succeed these experiments as the high-precision era of neutrino oscillation  
 353 parameter measurements develops.

354 Several anomalous results have been observed in the LSND [9] and Mini-  
 355 BooNE [10] detectors which were designed with purposefully short baselines.

356 Parts of the neutrino community attributed these results to oscillations induced  
357 by a fourth “sterile” neutrino [45] but several searches in other experiments,  
358 MicroBooNE [46] and KARMEN [47], found no hints of additional neutrino  
359 species. The solution to the anomalous results is still being determined.

### 360 2.3.3 Atmospheric Neutrinos

361 The interactions of primary cosmic ray protons in the Earth’s upper atmosphere  
362 generate showers of energetic hadrons. These are mostly pions and kaons that  
363 decay to produce a natural source of neutrinos spanning energies of MeV to  
364 TeV [48]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu) \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu) \end{aligned} \tag{2.15}$$

365 such that for a single pion decay, three neutrinos can be produced. The  
366 atmospheric neutrino flux energy spectra as predicted by the Bartol [49], Honda  
367 [50–52], and FLUKA [53] models are illustrated in Figure 2.3. The flux distribution  
368 peaks at an energy of  $O(10)$ GeV. The uncertainties associated with these models  
369 are dominated by the hadronic production of kaon and pions as well as the  
370 primary cosmic flux.

371 Unlike long-baseline experiments which have a fixed baseline, the distance  
372 atmospheric neutrinos propagate is dependent upon the zenith angle at which  
373 they interact. This is illustrated in Figure 2.4. Neutrinos that are generated  
374 directly above the detector ( $\cos(\theta) = 1.0$ ) have a baseline equivalent to the  
375 height of the atmosphere whereas neutrinos that interact directly below the  
376 detector ( $\cos(\theta) = -1.0$ ) have to travel a length equal to the diameter of the Earth.  
377 This means atmospheric neutrinos have a baseline that varies from  $O(20)$ km to  
378  $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will be subject  
379 to MSW matter resonance as they propagate through the Earth.

380 Figure 2.5 highlights the neutrino flux as a function of the zenith angle for  
381 different slices of neutrino energy. For medium to high-energy neutrinos (and to



**Figure 2.3:** Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [50], the 2004 Honda model (“HKKM04”)[51], the Bartol model [49] and the FLUKA model [53]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [50].



**Figure 2.4:** A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [54].

382 a lesser degree for low-energy neutrinos), the flux is approximately symmetric  
 383 around  $\cos(\theta) = 0$ . To the accuracy of this approximation, the systematic  
 384 uncertainties associated with atmospheric flux for comparing upward-going  
 385 and down-going neutrino cancels. This allows the down-going events, which are

- 386 mostly insensitive to oscillation probabilities, to act as an unoscillated prediction  
 387 (similar to a near detector in an accelerator neutrino experiment).



**Figure 2.5:** Prediction of  $\nu_e$ ,  $\bar{\nu}_e$ ,  $\nu_\mu$ ,  $\bar{\nu}_\mu$  fluxes as a function of zenith angle as calculated by the HKKM model [52]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [49], Honda [50] and FLUKA [53] are given in [54].

388 Precursory hints of atmospheric neutrinos were observed in the mid-1960s  
 389 searching for  $\nu_\mu + X \rightarrow X^* + \mu^\pm$  [55]. This was succeeded by the IMB-3 [56]  
 390 and Kamiokande [57] experiments which measured the double ratio of muon  
 391 to electron neutrinos in data to Monte Carlo,  $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$ .  
 392 Both experiments were found to have a consistent deficit of muon neutrinos,  
 393 with  $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$  and  $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$ , respectively.  
 394 Super-Kamiokande (SK) [54] extended this analysis by fitting oscillation pa-  
 395 rameters in  $P(\nu_\mu \rightarrow \nu_\tau)$  which found best fit parameters  $\sin^2(2\theta) > 0.92$  and  
 396  $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3} \text{ eV}^2$ .

397 Since then, atmospheric neutrino experiments have been making precision  
 398 measurements of the  $\sin^2(\theta_{23})$  and  $\Delta m^2_{32}$  oscillation parameters. Atmospheric  
 399 neutrino oscillation is dominated by  $P(\nu_\mu \rightarrow \nu_\tau)$ , where SK observed a  $4.6\sigma$   
 400 discovery of  $\nu_\tau$  appearance [58]. Figure 2.6 illustrates the current estimates  
 401 on the atmospheric mixing parameters from a wide range of atmospheric and  
 402 accelerator neutrino observatories.



**Figure 2.6:** Constraints on the atmospheric oscillation parameters,  $\sin^2(\theta_{23})$  and  $\Delta m_{32}^2$ , from atmospheric and long-baseline experiments: SK [59], T2K [60], NOvA [61], IceCube [62] and MINOS [63]. Figure taken from [64].

### 2.3.4 Reactor Neutrinos

As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very useful artificial source of electron antineutrinos. For reactors that use low-enriched uranium  $^{235}\text{U}$  as fuel, the antineutrino flux is dominated by the  $\beta$ -decay fission of  $^{235}\text{U}$ ,  $^{238}\text{U}$ ,  $^{239}\text{Pu}$  and  $^{241}\text{Pu}$  [65] as illustrated in Figure 2.7.

Due to their low energy, reactor electron antineutrinos predominantly interact via the inverse  $\beta$ -decay (IBD) interaction. The typical signature contains two signals delayed by  $O(200)\mu\text{s}$ ; firstly the prompt photons from positron annihilation, and secondly the photon emitted ( $E_{tot}^\gamma = 2.2\text{MeV}$ ) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [67].

There are many short baseline experiments ( $L \sim O(1)\text{km}$ ) that have measured the  $\sin^2(\theta_{13})$  and  $\Delta m_{32}^2$  oscillation parameters. Daya Bay [68], RENO [69] and Double Chooz [70] have all provided precise measurements, with the first discovery of a non-zero  $\theta_{13}$  made by Daya Bay and RENO (and complemented by T2K [70]). The constraints on  $\sin^2(\theta_{13})$  by the reactor experiments lead the field and



**Figure 2.7:** Reactor electron antineutrino fluxes for  $^{235}\text{U}$  (Black),  $^{238}\text{U}$  (Green),  $^{239}\text{Pu}$  (Purple), and  $^{241}\text{Pu}$  (Orange) isotopes. The inverse  $\beta$ -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse  $\beta$ -decay interaction including the eventual capture of the emitted neutron. This capture emits a  $\gamma$ -ray which provides a second signal of the event. Taken from [66].

are often used as external inputs to accelerator neutrino experiments to improve their sensitivity to  $\delta_{CP}$  and mass hierarchy determination. JUNO-TAO [71], a small collaboration within the larger JUNO experiment, is a next-generation reactor experiment that aims to precisely measure the isotopic antineutrino yields from the different fission chains.

Kamland [72] is the only experiment to have observed reactor neutrinos using a long baseline (flux weighted averaged baseline of  $L \sim 180\text{km}$ ) which allows it to have sensitivity to  $\Delta m_{21}^2$ . Combined with the SK solar neutrino experiment, the combined analysis puts the most stringent constraint on  $\Delta m_{21}^2$  [73].

## 2.4 Summary Of Oscillation Parameter Measurements

Since the first evidence of neutrino oscillations, numerous measurements of the mixing parameters have been made. Many experiments use neutrinos as a tool for the discovery of new physics (diffuse supernova background, neutrinoless double beta decay and others) so the PMNS parameters are summarised in the Particle Data Group (PDG) review tables. The analysis presented in this thesis focuses on the 2020 T2K oscillation analysis presented in [74] which the 2020 PDG constraints [75] were used. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	$0.307 \pm 0.013$
$\Delta m_{21}^2$	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	$0.547 \pm 0.021$
$\sin^2(\theta_{23})$ (N.H.)	$0.545 \pm 0.021$
$\Delta m_{32}^2$ (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
$\Delta m_{32}^2$ (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

**Table 2.1:** The 2020 Particle Data Group constraints of the oscillation parameters taken from [75]. The value of  $\Delta m_{32}^2$  is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and  $\sin^2(\theta_{23})$  is broken down by whether its value is below (Q1) or above (Q2) 0.5.

The  $\sin^2(\theta_{13})$  measurement stems from the electron antineutrino disappearance,  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ , and is take as the average best-fit from the combination of Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within other neutrino oscillation experiments, typically termed the reactor constraint. The  $\sin^2(\theta_{12})$  parameter is predominantly measured through electron neutrino disappearance,  $P(\nu_e \rightarrow \nu_{\mu,\tau})$ , in solar neutrino experiments. The long-baseline reactor neutrino experiment Kamland also has a sensitivity to this parameter and is used in a joint fit to solar data from SNO and SK, using the reactor constraint. Measurements of  $\sin^2(\theta_{23})$  are made by long-baseline and atmospheric neutrino experiments. The PDG value is a joint fit of T2K, NO $\nu$ A , MINOS and IceCube DeepCore experiments. The latest T2K-only measurement, provided at Neutrino2020 and is the basis of this thesis, is given as  $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$  [74].

The PDG constraint on  $\Delta m_{21}^2$  is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a  $\sin^2(\theta_{13})$  constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on  $\Delta m_{32}^2$  although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is  $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$  favouring normal hierarchy [74]. The value of  $\delta_{CP}$  is largely undetermined. CP-conserving values of 0 and  $\pi$  were rejected with  $\sim 2\sigma$  intervals, as published in Nature, although more recent analyses have reduced the credible intervals to 90%. Since the 2020 PDG publication, there has been a new measurement of  $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$  [76], alongside updated  $\Delta m_{32}^2$  and  $\sin^2(\theta_{23})$  measurements.

Throughout this thesis, several sample spectra predictions and contours are presented which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [77] and “Asimov B” being CP-conserving and further from maximal  $\theta_{23}$  mixing.

Parameter	Asimov A	Asimov B
$\Delta m_{12}^2$	$7.53 \times 10^{-5} \text{ eV}^2$	
$\Delta m_{32}^2$	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
$\delta_{CP}$	-1.601	0.0

**Table 2.2:** Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

# 3

466

467

## T2K and SK Experiment Overview

468 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)  
469 collaboration has been leading atmospheric neutrino oscillation analyses for  
470 over two decades. The detector has provided some of the strongest constraints  
471 on proton decay and the first precise measurements of the  $\Delta m_{23}^2$  and  $\sin^2(\theta_{23})$   
472 neutrino oscillation parameters. The history, detection technique, and operation  
473 of the SK detector is described in section 3.1.

474 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline  
475 experiments to use both neutrino and antineutrino beams to precisely measure  
476 charge parity violation within the neutrino sector. The T2K experiment observed  
477 the first hints of a non-zero  $\sin^2(\theta_{13})$  measurement and continues to lead the  
478 field with the constraints it provides on  $\sin^2(\theta_{13})$ ,  $\sin^2(\theta_{23})$ ,  $\Delta m_{23}^2$  and  $\delta_{CP}$ .  
479 In section 3.2, the techniques that T2K use to generate the neutrino beam and  
480 constrain systematic parameter through near detector constraints are described.

### 481 3.1 The Super-Kamiokande Experiment

482 The SK experiment began taking data in 1996 [78] and has had many modifi-  
483 cations throughout its operation. There have been seven defined periods of  
484 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

485 years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were  
 486 damaged during maintenance [79]. Those that survived were equally distributed  
 487 throughout the detector in the SK-II era, which resulted in a reduced 19% photo-  
 488 coverage. From SK-III onwards, repairs to the detector meant the full suite of  
 489 PMTs was operational recovering the 40% photo-coverage. Before the start of  
 490 SK-IV, the data acquisition and electronic systems were upgraded. Between  
 491 SK-IV and SK-V, a significant effort was placed into tank open maintenance and  
 492 repair/replacement of defective PMTs in preparation for the Gadolinium upgrade,  
 493 a task for which the author of this thesis was required. Consequently, the detector  
 494 conditions were significantly changed from this point. SK-VI marked the start of  
 495 the SK-Gd era, with the detector being doped with gadolinium at a concentration  
 496 of 0.01% by concentration. SK-VII, which started during the writing of this thesis,  
 497 has increased the gadolinium concentration to 0.03% for continued operation [80].

498 The oscillation analysis presented within this thesis focuses on the SK-IV  
 499 period of running and the data taken within it. This follows from the recent  
 500 SK analysis presented in [81]. Therefore, the information presented within this  
 501 section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

502 **Table 3.1:** The various SK periods and their respective live-time. The SK-VI live-time is  
 503 calculated until 1<sup>st</sup> April 2022. SK-VII started during the writing of this thesis.

### 502 3.1.1 The SK Detector

503 The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank  
 504 with a diameter 39.3m and height 41.1m filled with ultrapure water [79]. A  
 505 diagram of the significant components of the SK detector is given in Figure 3.1.

506 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine  
 507 is underground with roughly 1km rock overburden (2.7km water equivalent  
 508 overburden) [82]. At this depth, the rate of cosmic ray muons is significantly  
 509 decreased to a value of  $\sim 2\text{Hz}$  (net rate) [83]. The top of the tank is covered  
 510 with stainless steel which is designed as a working platform for maintenance,  
 511 calibration, and location for high voltage and data acquisition electronics.



**Figure 3.1:** A schematic diagram of the Super-Kamiokande Detector. Taken from [84].

512 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated  
 513 inside the tank, with an approximate 2m gap between this structure and the outer  
 514 tank wall. The purpose of this structure is to support the photomultiplier tubes  
 515 (PMTs). The volume inside and outside the support structure is referred to as the  
 516 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the  
 517 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs respectively  
 518 [79]. The ID contains a 32kton mass of water. Many analyses performed at SK  
 519 use a “fiducial volume” defined by the volume of water inside the ID excluding  
 520 some distance to the ID wall. This reduces the volume of the detector which is  
 521 sensitive to neutrino events but reduces radioactive backgrounds and allows for

522 better reconstruction performance. The nominal fiducial volume is defined as the  
523 area contained inside 2m from the ID wall for a total of 22.5kton water [85].

524 The two regions of the detector (ID and OD) are optically separated with  
525 opaque black plastic hung from the support structure. The purpose of this is  
526 to determine whether an event entered or exited the ID. This allows cosmic ray  
527 muons and partially contained events to be tagged and separated from neutrino  
528 events entirely contained within the ID. This black plastic is also used to cover  
529 the area between the ID PMTs to reduce photon reflection from the ID walls.  
530 Opposite to this, the OD is lined with a reflective material to allow photons to  
531 reflect around inside the OD until collected by one of the PMTs. Furthermore,  
532 each OD PMT is optically coupled with  $50 \times 50\text{cm}$  plates of wavelength shifting  
533 acrylic which increases the efficiency of light collection [82].

534 In the SK-IV data-taking period, the photocathode coverage of the detector, or  
535 the fraction of the ID wall instrumented with PMTs, is  $\sim 40\%$  [82]. The PMTs have  
536 a quantum efficiency (the ratio of detected electrons to incident photons) of  $\sim 21\%$   
537 for photons with wavelengths of  $360\text{nm} < \lambda < 390\text{nm}$  [86, 87]. The proportion  
538 of photoelectrons that produce a signal in the dynode of a PMT, termed the  
539 collection efficiency, is  $> 70\%$  [82]. The PMTs used within SK are most sensitive  
540 to photons with wavelength  $300\text{nm} \leq \lambda \leq 600\text{nm}$  [82]. One disadvantage of  
541 using PMTs as the detection media is that the Earth's geomagnetic field can  
542 modify its response. Therefore, a set of compensation coils is built around the  
543 inner surface of the detector to mitigate this effect [83].

544 The SK detector is filled with ultrapure water, which in a perfect world, con-  
545 tains no impurities. However, bacteria and organic compounds can significantly  
546 degrade the water quality. This decreases the attenuation length, which reduces  
547 the total number of photons that hit a PMT. To combat this, a sophisticated water  
548 treatment system has been developed [82, 88]. UV lights, mechanical filters, and  
549 membrane degasifiers are used to reduce the bacteria, suspended particulates,  
550 and radioactive materials from the water. The flow of water within the tank  
551 is also critical as it can remove stagnant bacterial growth or build-up of dust

552 on the surfaces within the tank. Gravity drifts impurities in the water towards  
553 the bottom of the tank which, if left uncontrolled, can create asymmetric water  
554 conditions between the top and bottom of the tank. Typically, the water entering  
555 the tank is cooled below the ambient temperature of the tank to control convection  
556 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs  
557 is sensitive to the PMT temperature [89] so controlling the temperature gradients  
558 within the tank is beneficial for stable measurements.

559 SK-VI is the first phase of the SK experiment to use gadolinium dopants  
560 within the ultrapure water [80]. As such, the SK water system had to be replaced  
561 to avoid removing the gadolinium concentrate from the ultrapure water [90]. For  
562 an inverse  $\beta$ -decay (IBD) interaction on a water target, the emitted neutron is  
563 thermally captured on hydrogen. This process releases a 2.2MeV  $\gamma$  ray which is  
564 difficult to detect as the resulting Compton scattered electrons are very close to the  
565 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons  
566 on gadolinium generates  $\gamma$  rays with higher energy (8MeV [67]) meaning they  
567 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%  
568 gadolinium sulphate by mass) which causes  $\approx$  50% of neutrons emitted by IBD to  
569 be captured on gadolinium[91, 92] . Whilst predominantly useful for low energy  
570 analyses, Gd loading allows better  $\nu/\bar{\nu}$  separation for atmospheric neutrino  
571 event selections [93]. Efforts are currently in place to increase the gadolinium  
572 concentrate to 0.03% for  $\approx$  75% neutron capture efficiency on gadolinium [94].  
573 The final stage of loading targets 0.1% concentrate for  $\approx$  90% neutron capture  
574 efficiency on gadolinium.

### 575 3.1.2 Calibration

576 The calibration of the SK detector is documented in [79] and summarised below.  
577 The analysis presented within this thesis is dependent upon ‘high energy events’  
578 (Charged particles with  $O(> 100)$ MeV momenta). These are events that are  
579 expected to generate a larger number of photons such that each PMT will  
580 be hit with multiple photons. The reconstruction of these events depends

upon the charge deposited within each PMT and the timing response of each individual PMT. Therefore, the most relevant calibration techniques to this thesis are outlined.

Before installation, 420 PMTs were calibrated to have identical charge responses and then distributed throughout the tank in a cross-shape pattern (As illustrated by Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed at similar geometric positions within SK to be calibrated against. To perform this calibration, a xenon lamp is located at the center of the SK tank which flashes uniform light at 1Hz. This allows for geometrical effects, water quality variation, and timing effects to be measured in situ throughout normal data-taking periods.



**Figure 3.2:** The location of “standard PMTs” (red) inside the SK detector. Taken from [79].

When specifically performing calibration of the detector (in out-of-data taking mode), the water in the tank was circulated to avoid top/bottom asymmetric water quality. Any non-uniformity within the tank significantly affects the PMT hit probability through scattering or absorption. This becomes a dominant effect for the very low-intensity light sources discussed later which are designed such that only one photon is incident upon a given PMT.

598      The gain of a PMT is defined as the ratio of the total charge of the signal  
 599      produced compared to the charge of photoelectrons emitted by the photocathodes  
 600      within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”  
 601      gain values are measured. The relative gain is the variation of gain among each  
 602      of the PMTs whereas the absolute gain is the average gain of all PMTs.

603      The relative gain is calibrated as follows. A laser is used to generate two  
 604      measurements: a high-intensity flash that illuminates every PMT with a sufficient  
 605      number of photons, and a low-intensity flash in which only a small number  
 606      of PMTs collect light. The first measurement creates an average charge,  $Q_{obs}(i)$   
 607      on PMT  $i$ , whereas the second measurement ensures that each hit PMT only  
 608      generates a single photoelectron. For the low-intensity measurement, the number  
 609      of times each PMT records a charge larger than 1/4 photoelectrons,  $N_{obs}(i)$ , is  
 610      counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i), \end{aligned} \tag{3.1}$$

611      Where  $I_H$  and  $I_L$  is the intensity of the high and low flashes,  $f(i)$  is the  
 612      acceptance efficiency of the  $i^{\text{th}}$  PMT,  $\epsilon(i)$  is the product of the quantum and  
 613      collection efficiency of the  $i^{\text{th}}$  PMT and  $G(i)$  is the gain of the  $i^{\text{th}}$  PMT. The relative  
 614      gain for each PMT can be determined by taking the ratio of these quantities.

615      The absolute gain calibration is performed by observing fixed energy  $\gamma$ -rays  
 616      of  $E_\gamma \sim 9\text{MeV}$  emitted isotropically from neutron capture on a NiCf source  
 617      situated at the center of the detector. This generates a photon yield of about 0.004  
 618      photoelectrons/PMT/event, meaning that  $> 99\%$  of PMT signals are generated  
 619      from single photoelectrons. A charge distribution is generated by performing  
 620      this calibration over all PMTs, and the average value of this distribution is taken  
 621      to be the absolute gain value.

622      As mentioned in subsection 3.1.1, the average quantum and collection effi-  
 623      ciency for the SK detector PMTs is  $\sim 21\%$  and  $> 70\%$  respectively. However,  
 624      these values do differ between each PMT and need to be calibrated accordingly.

625 Consequently, the NiCf source is also used to calibrate the “quantum  $\times$  collection”  
626 efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is  
627 used as the PMT hit probability is proportional to the QE ( $N_{obs}(i) \propto \epsilon(i)$  in  
628 Equation 3.1). A Monte Carlo prediction which includes photon absorption,  
629 scattering, and reflection is made to estimate the number of photons incident on  
630 each PMT and the ratio of the number of predicted to observed hits is calculated.  
631 The difference is attributed to the QE efficiency of that PMT. This technique is  
632 extended to calculate the relative QE efficiency by normalizing the average of  
633 all PMTs which removes the dependence on the light intensity.

634 Due to differing cable lengths and readout electronics, the timing response  
635 between a photon hitting the PMT and the signal being captured by the data  
636 acquisition can be different between each PMT. Due to threshold triggers (De-  
637 scribed in subsection 3.1.3), the time at which a pulse reaches a threshold is  
638 dependent upon the size of the pulse. This is known as the ‘time-walk’ effect  
639 and also needs to be accounted for in each PMT. To calibrate the timing response,  
640 a pulse of light with width 0.2ns is emitted into the detector through a diffuser.  
641 Two-dimensional distributions of time and pulse height (or charge) are made  
642 for each PMT and are used to calibrate the timing response. This is performed  
643 in-situ during data taking with the light source pulsing at 0.03Hz.

644 The top/bottom water quality asymmetry is measured using the NiCf calibra-  
645 tion data and cross-referencing these results to the “standard PMTs”. The water  
646 attenuation length is continuously measured by the rate of vertically-downgoing  
647 cosmic-ray muons which enter via the top of the tank.

648 Dark noise is where a PMT registers a pulse that is consistent with a single  
649 photoelectron emitted from photon detection despite the PMT being in complete  
650 darkness. This is predominately caused by two processes. Firstly there is  
651 intrinsic dark noise which is where photoelectrons gain enough thermal energy  
652 to be emitted from the photocathode, and secondly, the radioactive decay of  
653 contaminants inside the structure of the PMT. Typical dark noise rate for PMTs  
654 used within SK are  $O(3)\text{kHz}$  [82]. This is lower than the expected number of

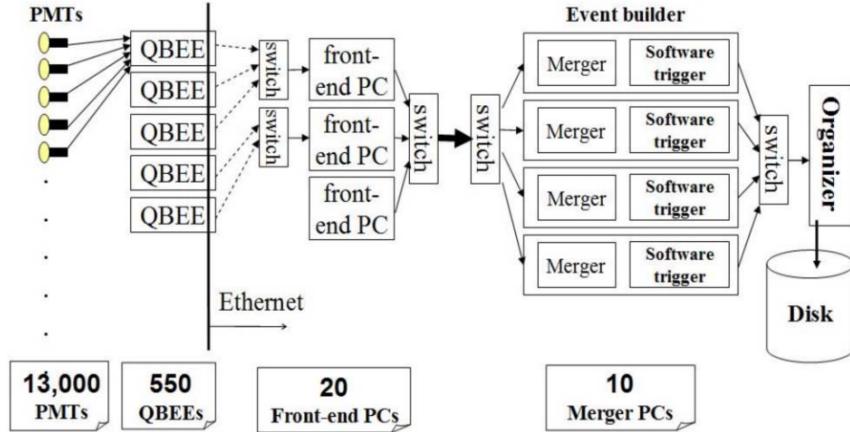
655 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)  
656 but instability in this value can cause biases in reconstruction. Dark noise is  
657 related to the gain of a PMT and is calibrated using hits inside a time window  
658 recorded before an event trigger [95].

### 659 3.1.3 Data Acquisition and Triggering

660 As the analysis presented in this thesis will only use the SK-IV period of the  
661 SK experiment so this subsection focuses on the relevant points of the data  
662 acquisition and triggering systems to that SK period. The earlier data acquisition  
663 and triggering systems are documented in [96, 97].

664 Before the SK-IV period started, the existing front-end electronics were re-  
665 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [98]. When  
666 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-  
667 time (QTC) converter generates a rectangular pulse. The start of the rectangular  
668 pulse indicates the time at which the analog photoelectron signal was received  
669 and the width of the pulse indicates the total charge integrated throughout the  
670 signal. This is then digitized by time-to-digital converters and sent to the “front-  
671 end” PCs. The digitized signal from every QBEE is then chronologically ordered  
672 and sent to the “merger” PCs. It is the merger PCs that apply the software trigger.  
673 Any triggered events are passed to the “organizer” PC. This sorts the data stream  
674 of multiple merger PCs into chronologically ordered events which are then saved  
675 to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

676 The software trigger (described in [100]) operates by determining the number  
677 of PMT hits within a 200ns sliding window,  $N_{200}$ . This window coincides with the  
678 maximum time that a Cherenkov photon would take to traverse the length of the  
679 SK tank [97]. For lower energy events that generate fewer photons, this technique  
680 is useful for eliminating background processes like dark noise and radioactive  
681 decay which would be expected to be separated in time. When the value of  
682  $N_{200}$  exceeds some pre-defined threshold, a software trigger is issued. There are  
683 several trigger thresholds used within the SK-IV period which are detailed in



**Figure 3.3:** Schematic view of the data flow through the data acquisition and online system. Taken from [99].

684 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time  
 685 window are also read out and saved to disk. In the special case of an event that  
 686 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks  
 687 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in  
 688 a  $535\mu\text{s}$  window after the SHE trigger. A similar but more complex “Wideband  
 689 Intelligent Trigger (WIT)” has been deployed and is described in [101].

Trigger	Acronym	Condition	Extended time window ( $\mu\text{s}$ )
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

**Table 3.2:** The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

### 690 3.1.4 Cherenkov Radiation

691 Cherenkov light is emitted from any highly energetic charged particle traveling  
 692 with relativistic velocity,  $\beta$ , greater than the local speed of light in a medium [102].

693 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

694 where  $n$  is the refractive index of the medium. Consequently, the Cherenkov  
 695 momentum threshold,  $P_{thres}$ , is dependent upon the mass,  $m$ , of the charged  
 696 particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}} \quad (3.3)$$

697 For water, where  $n = 1.33$ , the Cherenkov threshold momentum and energy  
 698 for various particles are given in Table 3.3. In contrast,  $\gamma$ -rays are detected  
 699 indirectly via the combination of photons generated by Compton scattering  
 700 and pair production. The threshold for detection in the SK detector is typically  
 701 higher than the threshold for photon production. This is due to the fact that the  
 702 attenuation of photons in the water means that typically  $\sim 75\%$  of Cherenkov  
 703 photons reach the ID PMTs. Then the collection and quantum efficiencies  
 704 described in subsection 3.1.1 result in the number of detected photons being  
 705 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

**Table 3.3:** The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index  $n = 1.33$ .

706 The Frank-Tamm equation [103] describes the relationship between the num-  
 707 ber of Cherenkov photons generated per unit length,  $dN/dx$ , the wavelength of  
 708 the photons generated,  $\lambda$ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

709 where  $\alpha$  is the fine structure constant. For a 100MeV momentum electron,  
 710 approximately 330 photons will be produced per centimeter in the  $300\text{nm} \leq \lambda \leq$   
 711  $700\text{nm}$  region which the ID PMTs are most sensitive to [82].

## 712 3.2 The Tokai to Kamioka Experiment

713 The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation  
 714 experiment located in Japan. Proposed in the early 2000s [104, 105] to replace  
 715 K2K [106], T2K was designed to observe electron neutrino appearance whilst  
 716 precisely measuring the oscillation parameters associated with muon neutrino  
 717 disappearance [107]. The experiment consists of a neutrino beam generated  
 718 at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near  
 719 detectors situated 280m from the beam target, and the Super Kamiokande far  
 720 detector positioned at a 295km baseline. The cross-section view of the T2K  
 721 experiment is drawn in Figure 3.4.

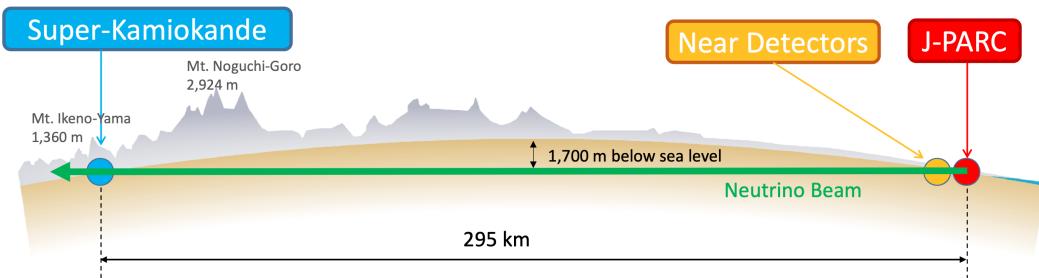
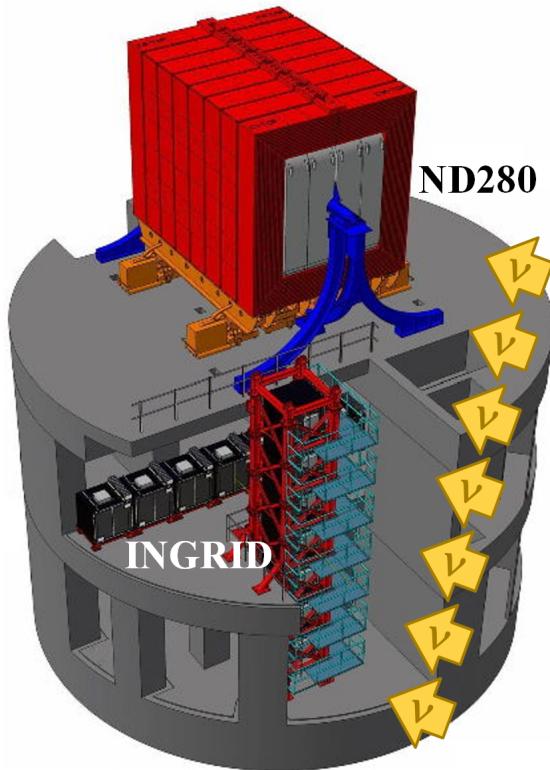


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

722 The T2K collaboration makes world-leading measurements of the  $\sin^2(\theta_{23})$   
 723 ,  $\Delta m_{23}^2$  , and  $\delta_{CP}$  oscillation parameters. Improvements in the precision and  
 724 accuracy of parameter estimates are still being made by including new data  
 725 samples and developing the models which describe the neutrino interactions  
 726 and detector responses [108]. Electron neutrino appearance was first observed  
 727 at T2K in 2014 [109] with  $7.3\sigma$  significance.

The near detectors provide constraints on the beam flux and cross-section model parameters used within the oscillation analysis by observing the unoscillated neutrino beam. There are a host of detectors situated in the near detector hall (As illustrated in Figure 3.5): ND280 (subsection 3.2.2), INGRID (subsection 3.2.3), NINJA [110], WAGASCI [111], and Baby-MIND [112]. The latter three are not currently used within the oscillation analysis presented in this thesis.



**Figure 3.5:** The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

Whilst this thesis presents the ND280 in terms of its purpose for the oscillation analysis, the detector can also make many cross-section measurements at neutrino energies of  $O(1)$ GeV for the different targets within the detector [113, 114]. These measurements are of equal importance as they can lead the way in determining the model parameters used in the interaction models for the future high-precision era of neutrino physics.

There are two independent fitters, MaCh3 and BANFF, which perform the near detector fit. MaCh3 is the basis of this analysis and uses a bayesian Markov

742 Chain Monte Carlo fitting technique, whereas BANFF uses a frequentist gradient  
743 descent technique. The output of each fitter is converted into a covariance matrix  
744 to describe the error and correlations between all the flux and cross-section  
745 parameters. This is then propagated to the far-detector oscillation analysis group  
746 for use in the P-Theta and VALOR fitting framework. As MaCh3 can handle both  
747 near and far detector samples, it does not use this covariance matrix and instead  
748 opts for a simultaneous fit of the two detector measurements. This is an analysis  
749 choice that removes the assumption of Gaussian posterior distributions required  
750 when building the post-fit covariance matrix.

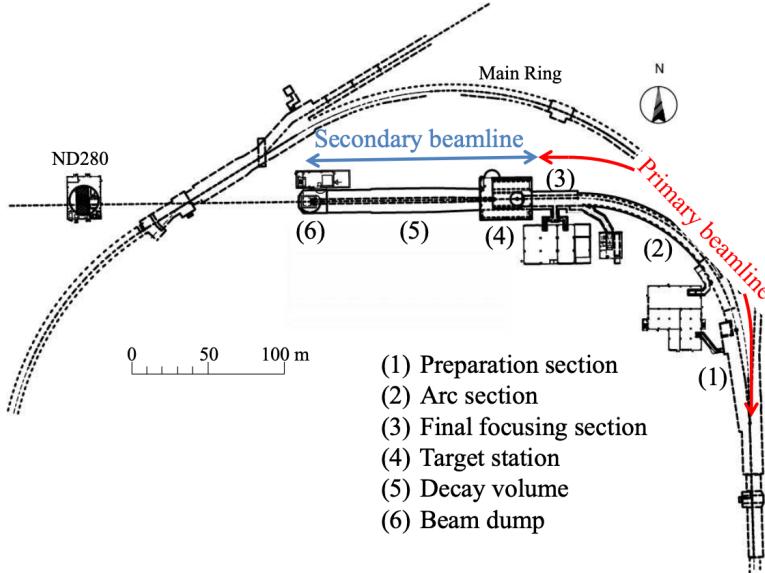
751 **DB: MaCh3 vs PTheta and Valor**

752 There are three particular tunes of the T2K flux and low energy cross section  
753 model typically considered. Firstly, the “generated” tune which is the set of  
754 dial values with which the Monte Carlo was generated. Secondly, the set of dial  
755 values which are taken from external data measurements and used as inputs.  
756 These are the “pre-fit” dial values. The reason these two sets of dial values are  
757 different is that the external data measurements are continually updated but it  
758 is very computationally intensive to regenerate a Monte Carlo prediction after  
759 each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial  
760 values. These are the values taken from the fit to the beam near detector data.

761 **3.2.1 The Neutrino Beam**

762 The neutrino beam used within the T2K experiment is described in [39, 115]  
763 and summarised below. The accelerator facility at J-PARC is composed of two  
764 sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic  
765 of the beamline, focusing mostly on the components of the secondary beamline.  
766 The primary beamline has three accelerators that progressively accelerate pro-  
767 tons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR)  
768 synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy  
769 of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted  
770 per “spill” from the MR and directed towards a graphite target (a rod of length

91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with  $\sim 3 \times 10^{14}$  protons contained per spill.



(a) Primary and secondary beamline



(b) Secondary beamline

**Figure 3.6:** Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [115]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [116].

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [39],

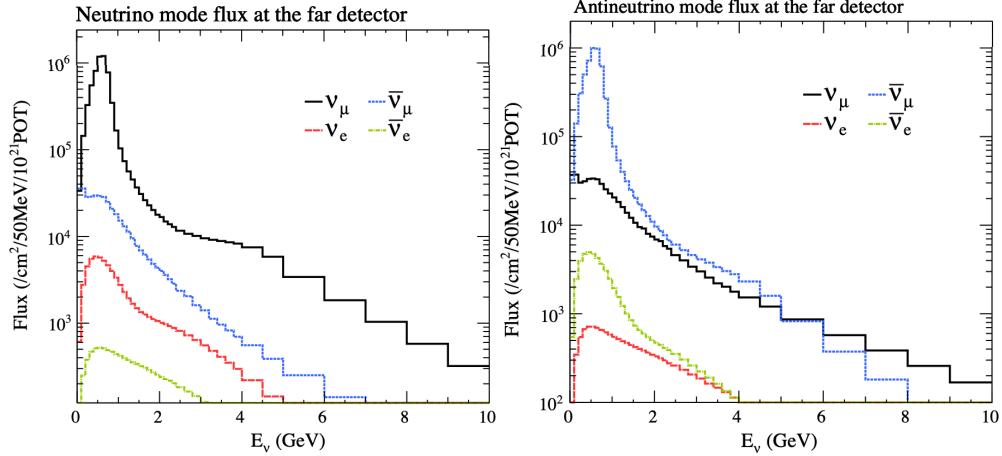
$$\begin{array}{ll}
\pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
\rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
\rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
779 \quad K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
\rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e
\end{array}$$

780        The electrically charged component of the secondary beam is focused towards  
 781        the far detector by the three magnetic horns. These horns direct charged particles  
 782        of a particular polarity towards SK whilst defocusing the oppositely charged  
 783        particles. This allows a mostly neutrino or mostly antineutrino beam to be used  
 784        within the experiment, denoted as “forward horn current (FHC)” or “reverse  
 785        horn current (RHC)” respectively.

786        Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino  
 787        flux. The low energy flux is dominated by the decay of pions whereas kaon  
 788        decay becomes the dominant source of neutrinos for  $E_\nu > 3\text{GeV}$ . The “wrong-  
 789        sign” component, which is the  $\bar{\nu}_\mu$  background in a  $\nu_\mu$  beam, and the intrinsic  
 790        irreducible  $\nu_e$  background, are predominantly due to muon decay for  $E_\nu <$   
 791         $2\text{GeV}$ . As the antineutrino production cross-section is smaller than the neutrino  
 792        cross-section, the wrong-sign component is more dominant in the RHC beam  
 793        as compared to that in the FHC beam.

794        The beam dump, situated at the end of the decay volume, stops all charged  
 795        particles other than highly energetic muons ( $p_\mu > 5\text{GeV}$ ). The MuMon detector  
 796        monitors the penetrating muons to determine the beam direction and inten-  
 797        sity which is used to constrain some of the beam flux systematics within the  
 798        analysis [116, 118].

799        The T2K experiment uses an off-axis beam to narrow the neutrino energy  
 800        distribution. This was the first implementation of this technique in a long-  
 801        baseline neutrino oscillation experiment after its original proposal [119]. Pion  
 802        decay,  $\pi \rightarrow \mu + \nu_\mu$ , is a two-body decay. Consequently, the neutrino energy,



**Figure 3.7:** The Monte Carlo prediction of the energy spectrum for each flavour of neutrino ( $\nu_e$ ,  $\bar{\nu}_e$ ,  $\nu_\mu$  and  $\bar{\nu}_\mu$ ) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [117].

803  $E_\nu$ , can be determined based on the pion energy,  $E_\pi$ , and the angle at which  
 804 the neutrino is emitted,  $\theta$ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

805 where  $m_\pi$  and  $m_\mu$  are the mass of the pion and muon respectively. For a fixed  
 806 energy pion, the neutrino energy distribution is dependent upon the angle at  
 807 which the neutrinos are observed from the initial pion beam direction. For the  
 808 295km baseline at T2K,  $E_\nu = 0.6\text{GeV}$  maximises the electron neutrino appearance  
 809 probability,  $P(\nu_\mu \rightarrow \nu_e)$ , whilst minimising the muon disappearance probability,  
 810  $P(\nu_\mu \rightarrow \nu_\mu)$ . Figure 3.8 illustrates the neutrino energy distribution for a range of  
 811 off-axis angles, as well as the oscillation probabilities most relevant to T2K.

### 812 3.2.2 The Near Detector at 280m

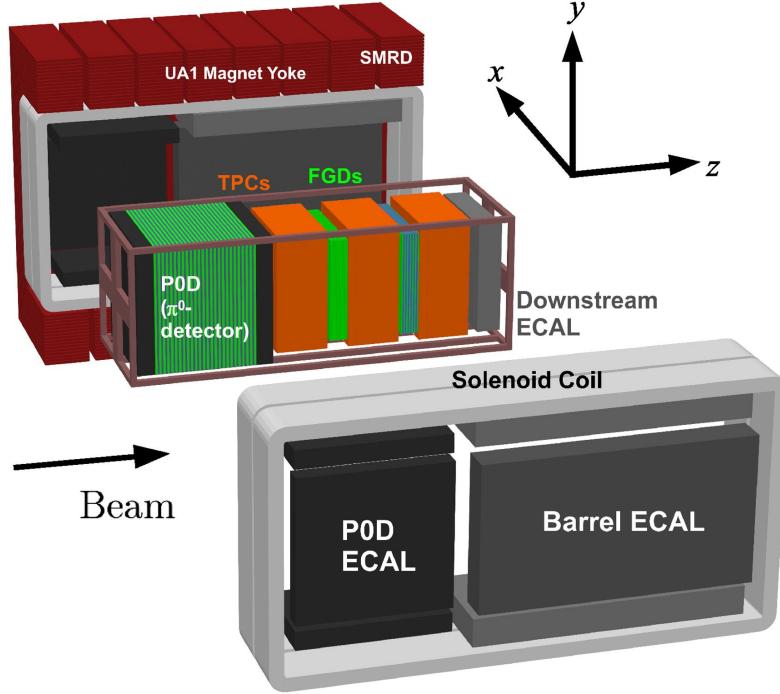
813 Whilst all the near detectors are situated in the same “pit” located at 280m from  
 814 the beamline, the “ND280” detector is the off-axis detector which is situated at  
 815 the same off-axis angle as the Super-Kamiokande far detector. It has two primary  
 816 functions; firstly it measures the neutrino flux and secondly, it counts the event  
 817 rates of different types of neutrino interactions. Both of these constrain the flux



**Figure 3.8:** Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

and cross-section systematics invoked within the model for a more accurate prediction of the expected event rate at the far detector.

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two-time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.2.1 and subsubsection 3.2.2.2 respectively. The electromagnetic calorimeter (ECAL)



**Figure 3.9:** The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [115].

encapsulates the tracker region alongside the  $\pi^0$  detector (P0D). The ECAL measures the deposited energy from photons emitted from interactions within the FGD. The P0D constrains the cross-section of neutral current interactions which generate neutral pions, which is one of the largest backgrounds in the electron neutrino appearance oscillation channel. The P0D and ECAL detectors are detailed in subsubsection 3.2.2.3 and subsubsection 3.2.2.4 respectively. The entire detector is located within a large yoke magnet which produces a 0.2T magnetic field. This design of the magnet also includes a scintillating detector called the side muon range detector (SMRD) which is used to track high-angle muons as well as acting as a cosmic veto. The SMRD is described in subsubsection 3.2.2.5.

### 3.2.2.1 Fine Grained Detectors

The T2K tracker region is comprised of two fine-grained detectors (FGD) and three Time Projection Chambers (TPC). A detailed description of the FGD design, construction, and assembly is found in [120] and summarised below. The FGDs are the primary target for neutrino interactions with a mass of 1.1 tonnes per FGD.

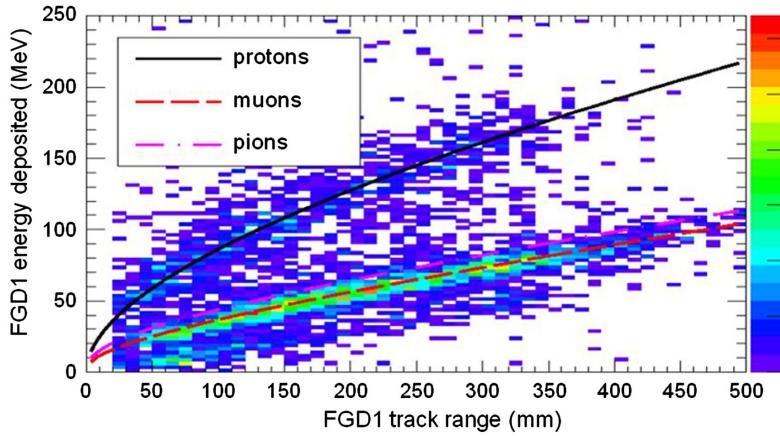
844 Alongside this, the FGDs are designed to be able to track short-range particles  
845 which do not exit the FGD. Typically, short-range particles are low momentum  
846 and are observed as tracks that deposit a large amount of energy per unit length.  
847 This means the FGD needs good granularity to resolve these particles. The  
848 FGDs have the best timing resolution ( $\sim 3\text{ns}$ ) of any of the sub-detectors of the  
849 ND280 detector. As such, the FGDs are used for time of flight measurements  
850 to distinguish forward-going positively charged particles from backward-going  
851 negatively charged particles. Finally, any tracks which pass through multiple  
852 sub-detectors are required to be track matched to the FGD.

853 Both FGDs are made from square scintillator planes of side length 186cm and  
854 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an  
855 X or Y orientation. A wavelength-shifting fiber is threaded through the center  
856 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is  
857 the most upstream of the two FGDs and contains 15 planes of carbon plastic  
858 scintillator which is a common target in external neutrino scattering data. As  
859 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2  
860 have been replaced with a hybrid water-scintillator target. Due to the complexity  
861 of the nucleus, nuclear effects can not be extrapolated between different nuclei.  
862 Therefore having the ability to take data on one target which is the same as  
863 external data and another target which is the same as the far detector target is  
864 beneficial for reliable model parameter estimates.

865 The integrated deposited energy is used for particle identification. The FGD  
866 can distinguish protons from other charged particles by comparing the integrated  
867 deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

### 868 3.2.2.2 Time Projection Chambers

869 The majority of particle identification and momentum measurements within  
870 ND280 are provided by three Time Projection Chambers (TPCs) [121]. The  
871 TPCs are located on either side of the FGDs. They are located inside of the



**Figure 3.10:** Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [120].

872 magnetic field meaning the momentum of a charged particle can be determined  
873 from the bending of the track.

874 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11,  
875 which are made of non-magnetic material. The outer box is filled with CO<sub>2</sub> which  
876 acts as an electrical insulator between the inner box and the ground. The inner box  
877 forms the field cage which produces a uniform electric drift field of  $\sim 275\text{V/cm}$   
878 and is filled with an argon gas mixture. Charged particles moving through this  
879 gas mixture ionize the gas and the ionised charge is drifted towards micromegas  
880 detectors which measure the ionization charge. The time and position information  
881 in the readout allows a three-dimensional image of the neutrino interaction.

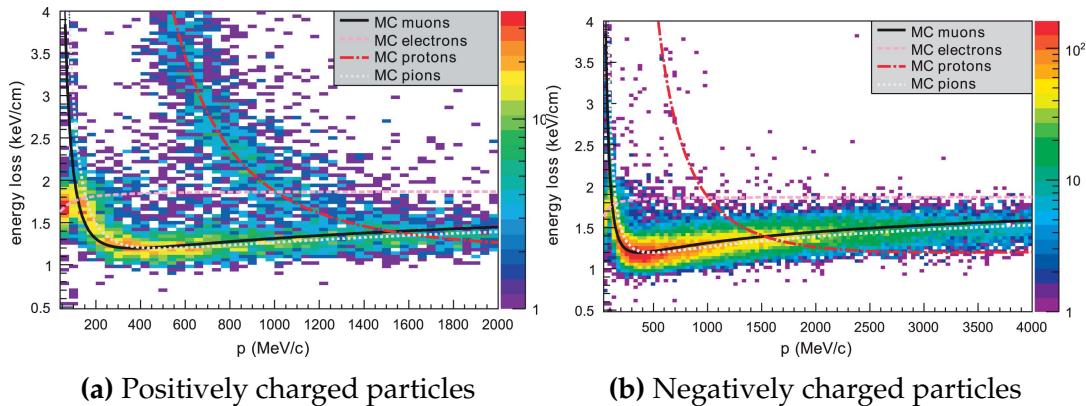
882 The particle identification of tracks that pass through the TPCs is performed  
883 using dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo  
884 distributions of the energy lost by a charged particle passing through the TPC as  
885 a function of the reconstructed particle momentum. The resolution is  $7.8 \pm 0.2\%$   
886 meaning that electrons and muons can be distinguished. This allows reliable  
887 measurements of the intrinsic  $\nu_e$  component of the beam.

### 888 3.2.2.3 $\pi^0$ Detector

889 If one of the  $\gamma$ -rays from a  $\pi^0 \rightarrow 2\gamma$  decay is missed at the far detector, the  
890 reconstruction will determine that event to be a charge current  $\nu_e$ -like event.



**Figure 3.11:** Schematic design of a Time Projection Chamber detector. Taken from [121].



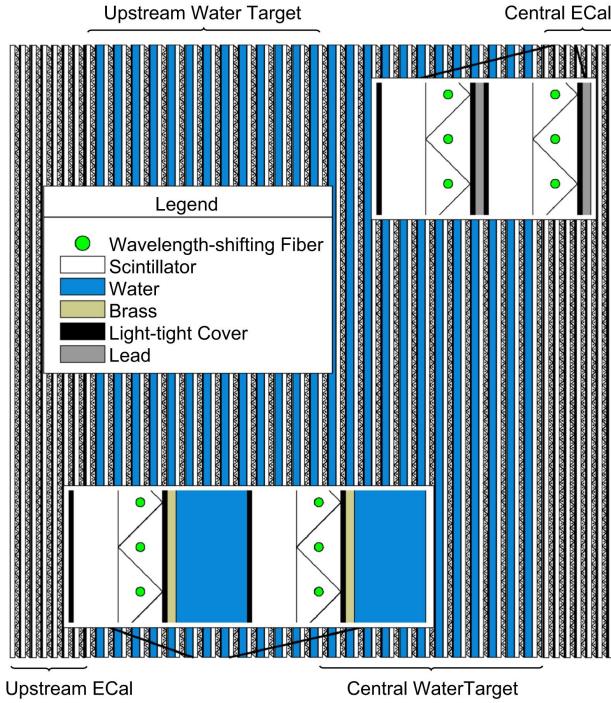
**Figure 3.12:** The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [121].

891 This is one of the main backgrounds hindering the electron neutrino appearance  
 892 searches. The  $\pi^0$  detector (P0D) measures the cross-section of the neutral current  
 893 induced neutral pion production on a water target to constrain this background.

894 The P0D is a cube of approximately 2.5m length consisting of layers of scin-  
 895 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.13.

896 Two electromagnetic calorimeters are positioned at the most upstream and most  
 897 downstream position in the sub-detector and the water target is situated in  
 898 between them. The scintillator layers are built from two triangular bars orientated  
 899 in opposite directions to form a rectangular layer. Each triangular scintillator bar  
 900 is threaded with optical fiber which is read out by MPPCs. The high-Z brass and

901 lead regions produce electron showers from the photons emitted in  $\pi^0$  decay.



**Figure 3.13:** A schematic of the P0D side-view. Taken from [122].

902 The sub-detector can generate measurements of NC1 $\pi^0$  cross-sections on a  
 903 water target by measuring the event rate both with and without the water target,  
 904 with the cross-section on a water target being determined as the difference. The to-  
 905 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

### 906 3.2.2.4 Electromagnetic Calorimeter

907 The electromagnetic calorimeter [123] (ECal) encapsulates the P0D and tracking  
 908 sub-detectors. Its primary purpose is to aid  $\pi^0$  reconstruction from any interac-  
 909 tion in the tracker. To do this, it measures the energy and direction of photon  
 910 showers from  $\pi^0 \rightarrow 2\gamma$  decay. It can also distinguish pion and muon tracks  
 911 depending on the shape of the photon shower deposited.

912 The ECal is comprised of three sections; the P0D ECal which surrounds the  
 913 P0D, the barrel ECal which encompasses the tracking region, and the downstream  
 914 ECal which is situated downstream of the tracker region. The barrel and down-  
 915 stream ECals are tracking calorimeters that focus on electromagnetic showers

916 from high-angle particles emitted from the tracking sub-detectors. Particularly in  
917 the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis)  
918 can travel along a single scintillator bar resulting in very few hits. The width of  
919 the barrel and downstream ECal corresponds to  $\sim 11$  electron radiation lengths  
920 to ensure a significant amount of the  $\pi^0$  energy is contained. As the P0D has  
921 its own calorimetry which reconstructs showers, the P0D ECal determines the  
922 energy which escapes the P0D.

923 Each ECal is constructed of multiple layers of scintillating bars sandwiched  
924 between lead sheets. The scintillating bars are threaded with optical fiber and read  
925 out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular  
926 to the previous which allows a three-dimensional event reconstruction. The  
927 target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80,  
928 and 6.62 tonnes respectively.

### 929 3.2.2.5 Side Muon Range Detector

930 As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within  
931 the UA1 magnet. Reconditioned after use in the UA1 [124] and NOMAD [125]  
932 experiments, this magnet provides a uniform horizontal magnetic field of 0.2T  
933 with an uncertainty of  $2 \times 10^{-4}$ T.

934 Built into the UA1 magnet, the side muon range detector (SMRD)[126] moni-  
935 tors high-energy muons which leave the tracking region and permeate through  
936 the ECal. It additionally acts as a cosmic muon veto and trigger.

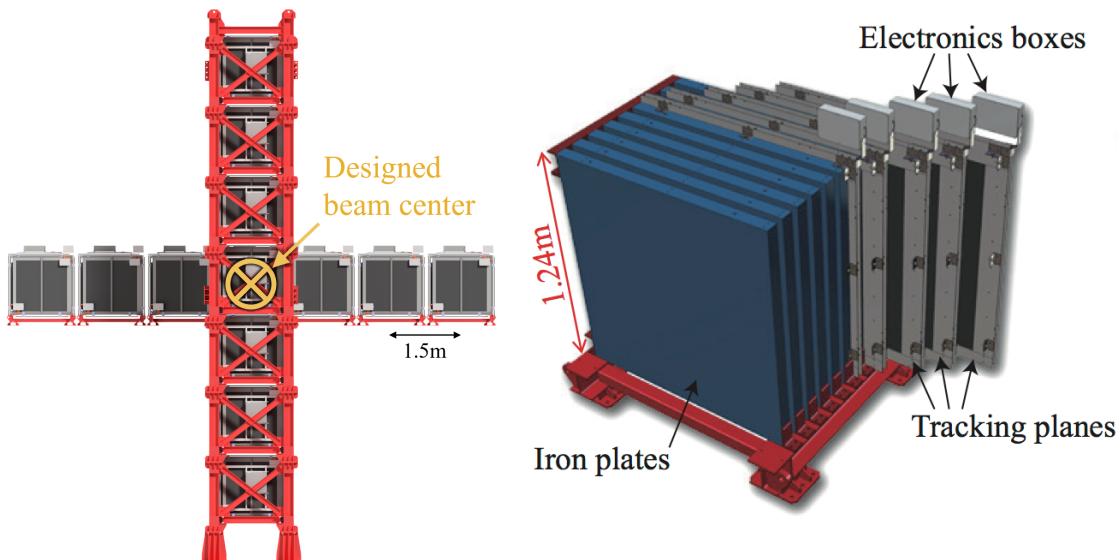
### 937 3.2.3 The Interactive Neutrino GRID

938 The Interactive Neutrino GRID (INGRID) detector is situated within the same  
939 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”  
940 position and measures the beam direction, spread, and intensity. The detector  
941 was originally designed with 16 identical modules [115] (two modules have since  
942 been decommissioned) and a “proton” module. The design of the detector is 14

modules oriented in a cross with length and height  $10\text{m} \times 10\text{m}$ , as illustrated in Figure 3.14.

Each module is composed of iron sheets interlaced with eleven tracking scintillator planes for a total target mass of 7.1 tonnes per module. The scintillator design is an X-Y pattern of 24 bars in both orientations, where each bar contains wave-length shifting fibers which are connected to multi-pixel photon counters (MPPCs). Each module is encapsulated inside veto planes to aid the rejection of charged particles entering the module.

The proton module is different from the other modules in that it consists of entirely scintillator planes with no iron target. The scintillator bars are also smaller than those used in the other modules to increase the granularity of the detector and improve tracking capabilities. The module sits in the center of the beamline and is designed to give precise measurements of quasi-elastic charged current interactions to evaluate the performance of the Monte Carlo simulation of the beamline.



**Figure 3.14:** Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [115].

The INGRID detector can measure the beam direction to an uncertainty of 0.4mrad and the beam centre within a resolution of 10cm [115]. The beam

<sup>960</sup> direction in both the vertical and horizontal directions is discussed in [127]  
<sup>961</sup> and it is found to be in good agreement with the MUMON monitor described  
<sup>962</sup> in subsection 3.2.1.

# 4

963

964

965

## Bayesian Statistics and Markov Chain Monte Carlo Techniques

966 This thesis presents a Bayesian oscillation analysis. To extract the oscillation  
967 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter  
968 explains the theory of how parameter estimates can be determined using this  
969 technique and condenses the material found in the literature [128–131].

970 The oscillation parameter determination presented here is built upon a si-  
971 multaneous fit to neutrino beam data in the near detector, beam data at SK and  
972 atmospheric data at SK. In total, there are four oscillation parameters of interest  
973 ( $\sin^2(\theta_{23})$ ,  $\sin^2(\theta_{13})$ ,  $\Delta m_{23}^2$ , and  $\delta_{CP}$ ), two oscillation parameters to which this  
974 study will not be sensitive ( $\sin^2(\theta_{12})$ ,  $\Delta m_{12}^2$ ) and many nuisance parameters  
975 that control the systematic uncertainty models.

976 The MCMC technique generates a multi-dimensional probability distribution  
977 across all of the model parameters used in the fit. To determine the parameter  
978 estimate of a single parameter, this multi-dimensional object is integrated over all  
979 other parameters. This process is called Marginalisation and is further described  
980 in subsection 4.3.1. Monte Carlo techniques approximate the probability distribu-  
981 tion of each parameter within the limit of generating infinite samples. As ever,  
982 generating a large number of samples is time and resource-dependent. Therefore,

983 an MCMC technique is utilised within this analysis to reduce the required number  
 984 of steps to sufficiently sample the parameter space. This technique is described  
 985 in further detail in subsection 4.2.1.

986 DB: Introduce MaCh3 and say what I did on it

987 **4.1 Bayesian Statistics**

988 Bayesian inference treats observable data,  $D$ , and model parameters,  $\vec{\theta}$ , on equal  
 989 footing such that a probability model of both data and parameters is required.  
 990 This is the joint probability distribution  $P(D, \vec{\theta})$  and can be described by the  
 991 prior distribution for model parameters  $P(\vec{\theta})$  and the likelihood of the data given  
 992 the model parameters  $P(D|\vec{\theta})$ ,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

993 The prior distribution,  $P(\vec{\theta})$ , describes all previous knowledge about the  
 994 parameters within the model. For example, if the risk of developing health  
 995 problems is known to increase with age, the prior distribution would describe the  
 996 increase. For the purpose of this analysis, the prior distribution is typically  
 997 the best-fit values taken from external data measurements with a Gaussian  
 998 uncertainty. The prior distribution can also contain correlations between model  
 999 parameters. In an analysis using Monte Carlo techniques, the likelihood of  
 1000 measuring some data assuming some set of model parameters is calculated  
 1001 by comparing the Monte Carlo prediction generated at that particular set of  
 1002 model parameters to the data.

1003 It is parameter estimation that is important for this analysis and as such, we  
 1004 apply Bayes' theorem [132] to calculate the probability for each parameter to have  
 1005 a certain value given the observed data,  $P(\vec{\theta}|D)$ , which is known as the posterior  
 1006 distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

1007     The denominator in Equation 4.2 is the integral of the joint probability dis-  
 1008     tribution over all values of all parameters used within the fit. For brevity, we  
 1009     say that the posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

1010     In subsection 4.3.1, we see that for the cases used within this analysis, it is  
 1011     reasonable to know the posterior to some normalisation constant.

### 1012     4.1.1 Application of Prior Knowledge

1013     The posterior distribution is proportional to the prior uncertainty applied on  
 1014     each parameter, as illustrated by Equation 4.3. This means that it is possible  
 1015     to change the prior after the posterior distribution has been determined. The  
 1016     prior uncertainty of a particular parameter can be ‘divided’ out of the posterior  
 1017     distribution and the resulting distribution can be reweighted using the new  
 1018     prior uncertainty that is to be applied. The methodology and implementation  
 1019     of changing the prior follows that described in [133].

1020     An example implementation that is useful for this analysis is the application  
 1021     of the “reactor constraint”. As discussed in section 2.4, an external constraint  
 1022     on  $\sin^2(\theta_{13})$  is determined from measurements taken from reactor experiments.  
 1023     However, the sensitivities from just using the T2K and SK samples is equally  
 1024     as important. Without this technique, two fits would have to be run, doubling  
 1025     the required resources. Therefore, the key benefit for this analysis is the fact that  
 1026     only a single ‘fit’ has to be performed and can be used to build the two posterior  
 1027     distributions of the with and without reactor constraint applied.

## 1028     4.2 Monte Carlo Simulation

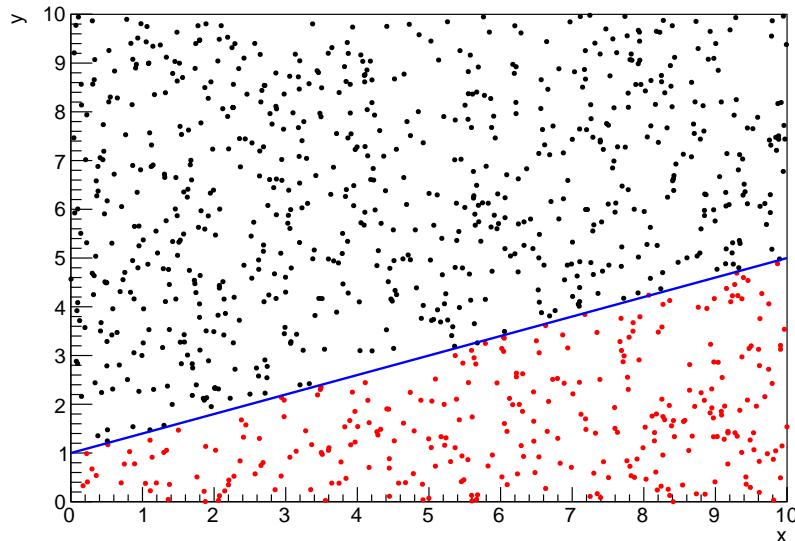
1029     Monte Carlo techniques are used to numerically solve a complex problem that  
 1030     does not necessarily have an analytical solution. These techniques rely on

1031 building a large ensemble of samples from an unknown distribution and then  
1032 using the ensemble to approximate the properties of the distribution.

1033 An example that uses Monte Carlo techniques is to calculate the area under-  
1034 neath a curve. For example, take the problem of calculating the area under a  
1035 straight line with gradient  $M = 0.4$  and intercept  $C = 1.0$ . Analytically, one can  
1036 calculate the area under the line is equal to 30 units for  $0 \leq x \leq 10$ . Using Monte  
1037 Carlo techniques, one can calculate the area under this line by throwing many  
1038 random values for the  $x$  and  $y$  components of each sample and then calculating  
1039 whether that point falls below the line. The area can then be calculated by the  
1040 ratio of points below the line to the total number of samples thrown multiplied by  
1041 the total area in which samples were scattered. The study is shown in Figure 4.1  
1042 highlights this technique and finds the area under the curve to be 29.9 compared  
1043 to an analytical solution of 30.0. The deviation of the numerical to analytical  
1044 solution can be attributed to the number of samples used in the study. The  
1045 accuracy of the approximation in which the properties of the Monte Carlo samples  
1046 replicate those of the desired distribution is dependent on the number of samples  
1047 used. Replicating this study with a differing number of Monte Carlo samples  
1048 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo  
1049 techniques are only accurate within the limit of a high number of samples.

1050 Whilst the above example has an analytical solution, these techniques are just  
1051 as applicable to complex solutions. Clearly, any numerical solution is only as  
1052 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is  
1053 dependent upon the number of samples generated to approximate the properties  
1054 of the distribution. Furthermore, if the positions at which the samples are  
1055 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique  
1056 significantly drops. Given the example in Figure 4.1, if the region in which the  
1057 samples are scattered significantly extends passed the region of interest, many  
1058 calculations will be calculated but do not add to the ability of the Monte Carlo  
1059 technique to achieve the correct result. For instance, any sample evaluated at  
1060 a  $y \geq 5$  could be removed without affecting the final result. This does bring in

- 1061 an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,  
1062 one needs to know the distribution beforehand.



**Figure 4.1:** Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

### 4.2.1 Markov Chain Monte Carlo

- 1063 **4.2.1 Markov Chain Monte Carlo**
- 1064 This analysis utilises a multi-dimensional probability distribution, with some  
1065 dimensions being significantly more constrained than others. This could be  
1066 from prior knowledge of parameter distributions from external data or un-  
1067 physical regions in which parameters can not exist. Consequently, the Monte  
1068 Carlo techniques used need to be as efficient as possible. For this analysis, the  
1069 Markov Chain Monte Carlo (MCMC) technique is chosen. An MCMC technique  
1070 is a Monte Carlo technique that uses a Markov chain to select which points at  
1071 which to sample the parameter distribution. This technique performs a semi-  
1072 random stochastic walk through the allowable parameter space. This builds a  
1073 posterior distribution which has the property that the density of sampled points  
1074 is proportional to the probability density of that parameter. This does mean that



**Figure 4.2:** The area under a line of gradient 0.4 and intercept 1.0 for the range  $0 \leq x \leq 10$  as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1075 the samples produced by this technique are not statistically independent but  
 1076 they will cover the space of the distribution.

1077 A Markov chain functions by selecting the position of step  $\vec{x}_{i+1}$  based on the  
 1078 position of  $\vec{x}_i$ . The space in which the Markov chain selects samples is dependent  
 1079 upon the total number of parameters utilised within the fit, where a discrete point  
 1080 in this space is described by the N-dimensional space  $\vec{x}$ . In a perfectly operating  
 1081 Markov chain, the position of the next step depends solely on the previous step  
 1082 and not on the further history of the chain ( $\vec{x}_0$ ,  $\vec{x}_1$ , etc.). However, in solving  
 1083 the multi-dimensionality of the fit used within this analysis, each step becomes  
 1084 correlated with several of the steps preceding itself. This behaviour is further  
 1085 explained in subsection 4.2.3. Providing the MCMC chain is well optimised, it will  
 1086 begin to converge towards a unique stationary distribution. The period between  
 1087 the chain's initial starting point and the convergence to the unique stationary  
 1088 distribution is colloquially known as the burn-in period. This is discussed further  
 1089 in subsection 4.2.3. Once the chain reaches the stationary distribution, all points  
 1090 sampled after that point will look like samples from that distribution.

1091     Further details of the theories underpinning MCMC techniques are discussed  
1092     in [129] but can be summarised by the requirement that the chain satisfies the  
1093     three ‘regularity conditions’:

- 1094     • Irreducibility: From every position in the parameter space  $\vec{x}$ , there must  
1095         exist a non-zero probability for every other position in the parameter space  
1096         to be reached.
- 1097     • Recurrence: Once the chain arrives at the stationary distribution, every step  
1098         following from that position must be samples from the same stationary  
1099         distribution.
- 1100     • Aperiodicity: The chain must not repeat the same sequence of steps at any  
1101         point throughout the sampling period.

1102     The output of the chain after burn-in (ie. the sampled points after the chain  
1103     has reached the stationary distribution) can be used to approximate the posterior  
1104     distribution and model parameters  $\vec{\theta}$ . To achieve the requirement that the unique  
1105     stationary distribution found by the chain be the posterior distribution, one  
1106     can use the Metropolis-Hastings algorithm. This guides the stochastic process  
1107     depending on the likelihood of the current proposed step compared to that  
1108     of the previous step. Implementation and other details of this technique are  
1109     discussed in subsection 4.2.2.

### 1110     **4.2.2 Metropolis-Hastings Algorithm**

1111     As a requirement for MCMCs, the Markov chain implemented in this technique  
1112     must have a unique stationary distribution that is equivalent to the posterior  
1113     distribution. To ensure this requirement and that the regularity conditions are  
1114     met, this analysis utilises the Metropolis-Hastings (MH) algorithm [134, 135].  
1115     For the  $i^{th}$  step in the chain, the MH algorithm determines the position in the  
1116     parameter space to which the chain moves to based on the current step,  $\vec{x}_i$ , and  
1117     the proposed step,  $\vec{y}_{i+1}$ . The proposed step is randomly selected from some

1118 proposal function  $f(\vec{x}_{i+1}|\vec{x}_i)$ , which depends solely on the current step (ie. not  
1119 the further history of the chain). The next step in the chain  $\vec{x}_{i+1}$  can be either the  
1120 current step or the proposed step determined by whether the proposed step is  
1121 accepted or rejected. To decide if the proposed step is selected, the acceptance  
1122 probability,  $\alpha(\vec{x}_i, \vec{y}_i)$ , is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)}\right). \quad (4.4)$$

1123 Where  $P(\vec{y}_{i+1}|D)$  is the posterior distribution as introduced in section 4.1. To  
1124 simplify this calculation, the proposal function is required to be symmetric such  
1125 that  $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$ . In practice, a multi-variate Gaussian distribution is  
1126 used to throw parameter proposals from. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)}\right). \quad (4.5)$$

1127 DB: Figure out what Giles means

1128 After calculating this quantity, a random number,  $\beta$ , is generated uniformly  
1129 between 0 and 1. If  $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$ , the proposed step is accepted. Otherwise,  
1130 the chain sets the next step equal to the current step. This procedure is repeated  
1131 for subsequent steps. This can be interpreted as if the posterior probability  
1132 of the proposed step is greater than that of the current step, ( $P(\vec{y}_{i+1}|D) \geq$   
1133  $P(\vec{x}_i|D)$ ), the proposed step will always be accepted. If the opposite is true,  
1134 ( $P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$ ), the proposed step will be accepted with probability  
1135  $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$ . This ensures that the Markov chain does not get trapped  
1136 in any local minima in the potentially non-Gaussian posterior distribution. The  
1137 outcome of this technique is that the density of steps taken in a discrete region  
1138 is directly proportional to the probability density in that region.

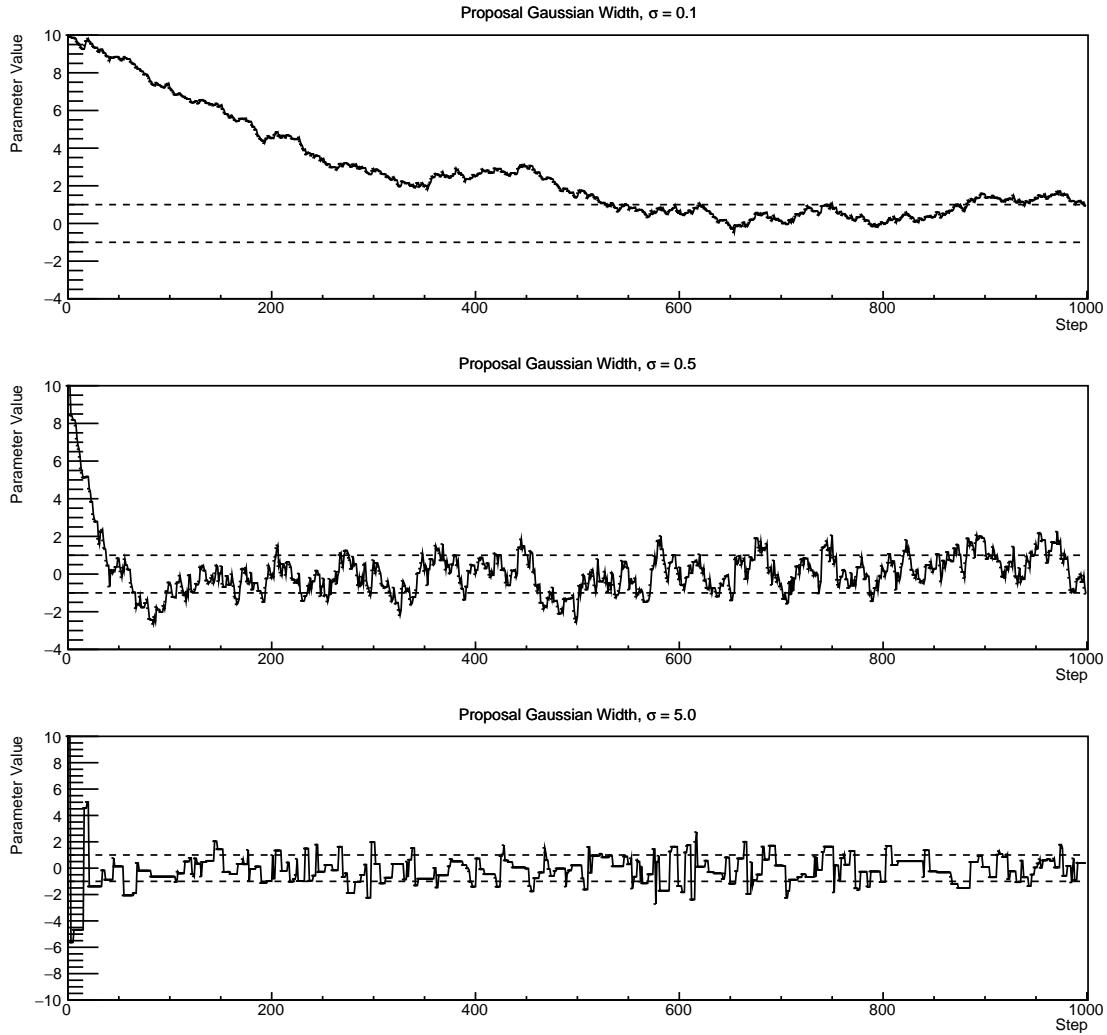
### 1139 4.2.3 MCMC Optimisation

1140 As discussed in subsection 4.2.2, the proposal function invoked within the MH  
1141 algorithm can take any form and the chain will still converge to the stationary  
1142 distribution. At each set of proposed parameter values, a prediction of the same

1143 spectra has to be generated which requires significant computational resources.  
1144 Therefore, the number of steps taken before the unique stationary distribution  
1145 is found should be minimised as only steps after convergence add information  
1146 to the oscillation analysis. Furthermore, the chain should entirely cover the  
1147 allowable parameter space to ensure that all values have been considered. Tuning  
1148 the distance that the proposal function jumps between steps on a parameter-by-  
1149 parameter basis can both minimise the length of the burn-in period and ensure  
1150 that the correlation between step  $\vec{x}_i$  and  $\vec{x}_j$  is sufficiently small.

1151 The effect of changing the width of the proposal function is highlighted in  
1152 Figure 4.3. Three scenarios, each with the same underlying stationary distribution  
1153 (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between  
1154 the three scenarios is the width of the proposal function, colloquially known as  
1155 the ‘step size  $\sigma$ ’. Each scenario starts at an initial parameter value of 10.0 which  
1156 would be considered an extreme variation. For the case where  $\sigma = 0.1$ , it is  
1157 clear to see that the chain takes a long time to reach the expected region of the  
1158 parameter. This indicates that this chain would have a large burn-in period and  
1159 does not converge to the stationary distribution until step  $\sim 500$ . Furthermore,  
1160 whilst the chain does move towards the expected region, each step is significantly  
1161 correlated with the previous. Considering the case where  $\sigma = 5.0$ , the chain  
1162 approaches the expected parameter region almost instantly meaning that the  
1163 burn-in period is not significant. However, there are clearly large regions of steps  
1164 where the chain does not move. This is likely due to the chain proposing steps  
1165 in the tails of the distribution which have a low probability of being accepted.  
1166 Consequently, this chain would take a significant number of steps to fully span  
1167 the allowable parameter region. For the final scenario, where  $\sigma = 0.5$ , you can  
1168 see a relatively small burn-in period of approximately 100 steps. Once the chain  
1169 reaches the stationary distribution, it moves throughout the expected region of  
1170 parameter values many times, sufficiently sampling the full parameter region.  
1171 This example is a single parameter varying across a continuous distribution and  
1172 does not fully reflect the difficulties in the many-hundred multi-variate parameter

1173 distribution used within this analysis. However, it does give a conceptual idea of  
1174 the importance of selecting the proposal function and associated step size.



**Figure 4.3:** Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size  $\sigma$ ’). The top panel has  $\sigma = 0.1$ , middle panel has  $\sigma = 0.5$  and the bottom panel has  $\sigma = 5.0$ .

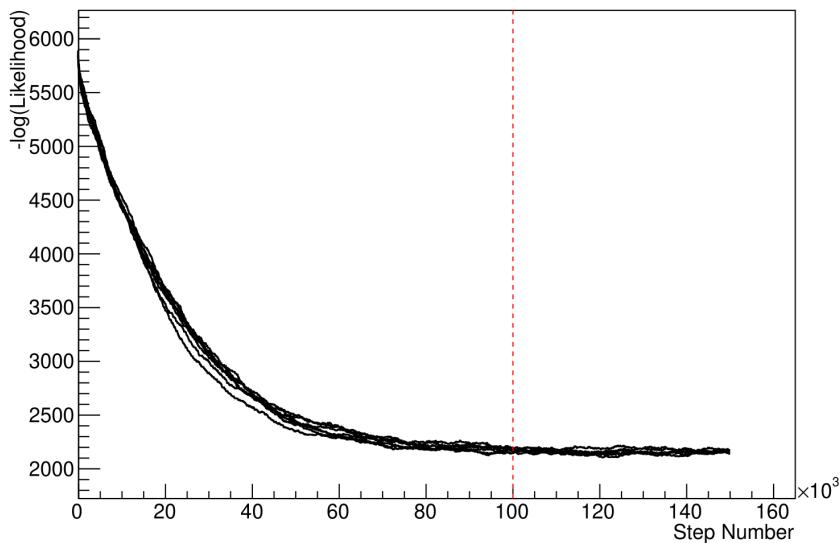
1175 As discussed, step size tuning directly correlates to the average step accep-  
1176 tance rate. If the step size is too small, many steps will be accepted but the  
1177 chain moves slowly. If the opposite is true, many steps will be rejected as the  
1178 chain proposes steps in the tails of the distribution. Discussion in [136] suggests  
1179 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be

<sub>1180</sub> approximately  $\sim 25\%$ . An “ideal” step size [136] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

<sub>1181</sub> where  $N_p$  is the number of parameters included in the MCMC fit. However,  
<sub>1182</sub> the complex correlations between systematics mean that some parameters have  
<sub>1183</sub> to be hand tuned and many efforts have been taken to select a set of parameter-  
<sub>1184</sub> by-parameter step sizes to approximately reach the ideal acceptance rate.

<sub>1185</sub> Figure 4.3 highlights the likelihood as calculated by the fit in DB: [Link to](#)  
<sub>1186</sub> [AsimovA Sensitivity Section](#) as a function of the number of steps in each chain. In  
<sub>1187</sub> practice, many independent MCMC chains are run simultaneously to parallelise  
<sub>1188</sub> the task of performing the fit. This figure overlays the distribution found in each  
<sub>1189</sub> chain. As seen, the likelihood decreases from its initial value and converges  
<sub>1190</sub> towards a stationary distribution after  $\sim 1 \times 10^5$  steps.



**Figure 4.4:** The log-likelihood from the fit detailed in DB: [Link to AsimovA Sensitivity Section](#) as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the  $1 \times 10^5$  step burn-in period after which the log-likelihood becomes stable.

<sub>1191</sub> Multiple configurations of this analysis have been performed throughout this  
<sub>1192</sub> thesis where different samples or systematics have been used. For all of these con-  
<sub>1193</sub> figurations, it was found that a burnin period of  $1 \times 10^5$  was sufficient in all cases.

## 1194 4.3 Understanding the MCMC Results

1195 The previous sections have described how to generate the posterior probability  
1196 distribution using Bayesian MCMC techniques. However, this analysis focuses  
1197 on oscillation parameter determination. The posterior distribution output from  
1198 the chain is a high dimension object, with as many dimensions as there are  
1199 parameters included in the oscillation analysis. However, this multi-dimensional  
1200 object is difficult to conceptualize so parameter estimations are often presented  
1201 in one or two-dimensional projections of this probability distribution. To do this,  
1202 we invoke the marginalisation technique highlighted in subsection 4.3.1.

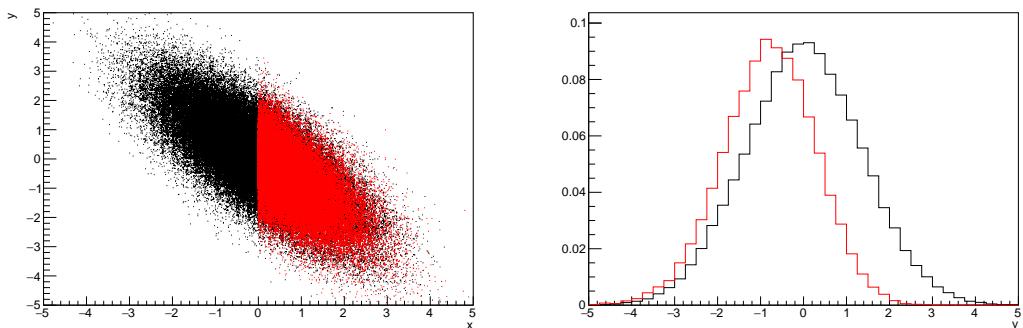
### 1203 4.3.1 Marginalisation

1204 The output of the MCMC chain is a highly dimensional probability distribution  
1205 which is very difficult to interpret. From the standpoint of an oscillation analysis  
1206 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters  
1207 of interest are most relevant. Despite this, the best fit values and uncertainties on  
1208 the oscillation parameters of interest should correctly encapsulate the correlations  
1209 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).  
1210 For this joint beam and atmospheric analysis, the oscillation parameters of interest  
1211 are  $\sin^2(\theta_{23})$ ,  $\sin^2(\theta_{13})$ ,  $\Delta m_{23}^2$ , and  $\delta_{CP}$ . All other parameters (Including the  
1212 oscillation parameter this fit is insensitive to) are deemed nuisance parameters.  
1213 To generate these projections, we rely upon integrating the posterior distribution  
1214 over all nuisance parameters. This is called marginalisation. A simple example  
1215 of this technique is to imagine the scenario where two coins are flipped. To  
1216 determine the probability that the first coin returned a ‘head’, the exact result of  
1217 the second coin flip is disregarded and simply integrated over. For the parameters  
1218 of interest,  $\vec{\theta}_i$ , we can calculate the marginalised posterior by integrating over  
1219 the nuisance parameters,  $\vec{\theta}_n$ . In this case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}} \quad (4.7)$$

1220 Where  $P(\vec{\theta}_i, \vec{\theta}_n)$  encodes the prior knowledge about the uncertainty and  
 1221 correlations between the parameters of interest and the nuisance parameters.  
 1222 In practice, this is simply taking the one or two-dimensional projection of the  
 1223 multi-dimensional probability distribution.

1224 Whilst in principle an easy solution to a complex problem, correlations  
 1225 between the interesting and nuisance parameters can bias the marginalised  
 1226 results. A similar effect is found when the parameters being marginalised over  
 1227 have non-Gaussian probability distributions. For example, Figure 4.5 highlights  
 1228 the marginalisation bias in the probability distribution found for a parameter  
 1229 when requiring a correlated parameter to have a positive parameter value. Due  
 1230 to the complex nature of the oscillation parameter fit presented in this thesis,  
 1231 there are correlations occurring between the oscillation parameters of interest  
 1232 and the other nuisance parameters included in the fit.



**Figure 4.5:** Left: The two dimensional probability distribution for two correlated parameters  $x$  and  $y$ . The red distribution shows the two dimensional probability distribution when  $0 \leq x \leq 5$ . Right: The marginalised probability distribution for the  $y$  parameter found when requiring the  $x$  to be bound between  $-5 \leq x \leq 5$  and  $0 \leq x \leq 5$  for the black and red distribution, respectively.

### 1233 4.3.2 Parameter Estimation and Credible Intervals

1234 The purpose of this analysis is to determine the best fit values for the oscillation  
 1235 parameters that the beam and atmospheric samples are sensitive to:  $\sin^2(\theta_{23})$   
 1236 ,  $\sin^2(\theta_{13})$  ,  $\Delta m_{23}^2$  , and  $\delta_{CP}$  . Typically, the results presented take the form of  
 1237 one or two-dimension marginalised probability distributions for the appearance

1238 ( $\sin^2(\theta_{13})$  and  $\delta_{CP}$ ) and disappearance ( $\sin^2(\theta_{23})$  and  $\Delta m_{23}^2$ ) parameters. The  
1239 posterior probability density taken from the output MCMC chain is binned in  
1240 these parameters. The parameter best-fit point is then taken to be the value  
1241 that has the highest posterior probability. This is performed in both one and  
1242 two-dimensional projections.

1243 However, the single best-fit point in a given parameter is not of much use on  
1244 its own. We would also like to determine the uncertainty, or credible interval,  
1245 on that best-fit point. The definition of the  $1\sigma$  credible interval is that we have  
1246 68% belief that the parameter is within those bounds. For a more generalised  
1247 definition, the credible interval is the region,  $R$ , of the posterior distribution that  
1248 contains a specific fraction of the total probability, such that

$$\int_R P(\theta|D) d\theta = \alpha \quad (4.8)$$

1249 Where  $\theta$  is the parameter on which we calculate the credible interval. This  
1250 technique then calculates the  $\alpha \times 100\%$  credible interval.

1251 In practice, this analysis uses the highest posterior density (HPD) credible in-  
1252 tervals which are calculated through the following method. First, the probability  
1253 distribution is area-normalised such that it has an integrated area equal to 1.0.  
1254 The bins of probability are then summed from the highest to lowest until the sum  
1255 exceeds the  $1\sigma$  level (0.68 in this example). This process is repeated for a range of  
1256 credible intervals, notably the  $1\sigma$ ,  $2\sigma$  and  $3\sigma$  along with other levels where the  
1257 critical values for each level can be found in [75]. This process can be repeated  
1258 for the two-dimensional probability distributions by creating two-dimensional  
1259 contours of credible intervals rather than a one-dimensional result.

### 1260 4.3.3 Bayesian Model Comparisons

1261 Due to the matter resonance, this analysis has some sensitivity to the mass  
1262 hierarchy of neutrino states (whether  $\Delta m_{23}^2$  is positive or negative) and the  
1263 octant of  $\sin^2(\theta_{23})$ . The Bayesian approach utilised within this analysis gives an  
1264 intuitive method of model comparison by determining which hypothesis is most

<sub>1265</sub> favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal  
<sub>1266</sub> hierarchy,  $NH$ , and inverted hierarchy,  $IH$ , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

<sub>1267</sub> The middle term defines the Bayes factor,  $B(NH/IH)$ , which is a data-driven  
<sub>1268</sub> interpretation of how strong the data prefers one hierarchy to the other. For this  
<sub>1269</sub> analysis, equal priors on both mass hierarchy hypotheses are chosen ( $P(\vec{\theta}_{NH}) =$   
<sub>1270</sub>  $P(\vec{\theta}_{IH}) = 0.5$ ). In practice, the MCMC chain proposes a value of  $|\Delta m_{23}^2|$  and  
<sub>1271</sub> then applies a 50% probability that the value is sign flipped. Consequently,  
<sub>1272</sub> the Bayes factor can be calculated from the ratio of the probability density in  
<sub>1273</sub> either hypothesis. This equates to counting the number of steps taken in the  
<sub>1274</sub> normal and inverted hierarchies and taking the ratio. The same approach can be  
<sub>1275</sub> taken to compare the upper octant (UO) compared to the lower octant (LO)  
<sub>1276</sub> hypothesis of  $\sin^2(\theta_{23})$ .

<sub>1277</sub> Whilst the value of the Bayes factor should always be shown, the Jeffreys scale  
<sub>1278</sub> [137] (highlighted in Table 4.1) gives an indication of the strength of preference  
<sub>1279</sub> for one model compared to the other. Other interpretations of the strength of  
<sub>1280</sub> preference of a model exist, e.g. the Kass and Raferty Scale [138].

$\log_{10}(B_{AB})$	$B_{AB}$	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

**Table 4.1:** Jeffreys scale for strength of preference for two models  $A$  and  $B$  as a function of the calculated Bayes factor ( $B_{AB} = B(A/B)$ ) between the two models [137]. The original scale is given in terms of  $\log_{10}(B(A/B))$  but converted to linear scale for easy comparison throughout this thesis.

### 1281 4.3.4 Comparison of MCMC Output to Expectation

1282 To ensure the fit is performing well, a best-fit spectrum is produced using the  
1283 posterior probability distribution and compared with the data, allowing easy  
1284 by-eye comparisons to be made. A simple method of doing this is to perform a  
1285 comparison in the fitting parameters (For instance, the reconstructed neutrino  
1286 energy and lepton direction for T2K far detector beam samples) of the spectra  
1287 generated by the MCMC chain to ‘data’. This ‘data’ could be true data or some  
1288 variation of Monte Carlo prediction. This allows easy comparison of the MCMC  
1289 probability distribution to the data. To perform this,  $N$  steps from the post burn-in  
1290 MCMC chain are randomly selected (Where for all plots of this style in this thesis,  
1291  $N = 3000$ ). From these, the Monte Carlo prediction at each step is generated by  
1292 reweighting the model parameters to the values specified at that step. Due to the  
1293 probability density being directly correlated with the density of steps in a certain  
1294 region, parameter values close to the best fit value are most likely to be selected.

1295 In practice, for each bin of the fitting parameters has a probability distribution  
1296 of event rates, with one entry per sampled MCMC step. This distribution is  
1297 binned where the bin with the highest probability is selected as the mean and an  
1298 error on the width of this probability distribution is calculated using the approach  
1299 highlighted in subsection 4.3.2. Consequently, the best fit distribution in the fit  
1300 parameter is not necessarily that which would be attained by reweighting the  
1301 Monte Carlo prediction to the most probable parameter values.

1302 A similar study can be performed to illustrate the freedom of the model  
1303 parameter space prior to the fit. This can be done by throwing parameter values  
1304 from the prior uncertainty of each parameter. This becomes troublesome for  
1305 parameters with no prior uncertainty as the range is technically infinite. Where  
1306 applicable solutions to remove these have been addressed.

# 5

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## Simulation, Reconstruction, and Event Reduction

1310 As a crucial part of the oscillation analysis, an accurate prediction of the expected  
1311 neutrino spectrum at the far detector is required. This includes modeling the  
1312 flux generation, neutrino interactions, and detector effects. All of the simulation  
1313 packages required to do this are briefly described in section 5.1. The reconstruc-  
1314 tion of neutrino events inside the far detector, including the `fitQun` algorithm,  
1315 is documented in section 5.2. This also includes data quality checks of the SK-  
1316 V data which the author performed for the T2K oscillation analysis presented  
1317 at Neutrino 2020 [74]. Finally, section 5.3 describes the steps taken in the SK  
1318 detector to trigger on events of interest whilst removing the comparatively large  
1319 rate of cosmic ray muon events.

### 1320 5.1 Simulation

1321 In order to generate a Monte Carlo prediction of the expected event rate at the far  
1322 detector, all the processes in the beam and atmospheric flux, neutrino interaction,  
1323 and detector need to be modeled. Each of these parts is individually modeled  
1324 and each of them is detailed below.

1325 The beamline simulation consists of three distinct parts: the initial hadron  
1326 interaction modeled by FLUKA [139], the target station geometry and particle  
1327 tracking performed by JNUBEAM, [140, 141] and any hadronic re-interactions  
1328 simulated by GCALOR [142]. The primary hadronic interactions are  $O(10)\text{GeV}$ ,  
1329 where FLUKA matches external cross-section data better than GCALOR [143].  
1330 However, FLUKA is not very adaptable so a small simulation is built to model  
1331 the interactions in the target and the output is then passed to JNUBEAM and  
1332 GCALOR for propagation. The hadronic interactions are tuned to data from the  
1333 NA61/SHINE [144–146] and HARP [147] experiments. The tuning is done by  
1334 reweighting the FLUKA and GCALOR predictions to match the external data  
1335 multiplicity and cross-section measurements, based on final state particle kine-  
1336 matics [143]. The culmination of this simulation package generates the predicted  
1337 flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.7.

1338 The atmospheric neutrino flux is simulated by the HKKM model [50, 52]. The  
1339 primary cosmic ray flux is tuned to AMS [148] and BESS [149] data assuming  
1340 the US-standard atmosphere '76 [150] density profile and includes geomagnetic  
1341 field effects. The primary cosmic rays interact to generate pions and muons.  
1342 The interaction of these secondary particles to generate neutrinos is handled by  
1343 DPMJET-III [151] for energies above 32GeV and JAM [52, 152] for energies below  
1344 that value **DB: Question for Giles: Why different generators for above/below**  
1345 **32GeV?**. These hadronic interactions are tuned to BESS and L3 data [153, 154]  
1346 using the same methodology as the tuning of the beamline simulation. The  
1347 energy and cosine zenith predictions of  $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$  flux are given in Figure 2.3  
1348 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked  
1349 around the horizon ( $\cos(\theta_Z) = 0.0$ ). This is because horizontally-going pions  
1350 and kaons can travel further than their vertically-going counterparts resulting  
1351 in a larger probability of decaying to neutrinos. The symmetry is broken in  
1352 lower-energy neutrinos due to geomagnetic effects, which modify the track of the  
1353 primary cosmic rays. Updates to the HKKM model are currently ongoing [155].

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [156, 157] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a  $W^\pm$  boson in Charged Current (CC) interactions or via a  $Z^0$  boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. This is the reason why neutrinos which interact through NC modes are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events.

As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of neutrino interactions around  $E_\nu \sim 0.5\text{GeV}$ . The NEUT implementation adopts the Llewellyn Smith [158] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [159] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions is controlled by vector and axial-vector form factors parameterised by the BBBA05 [160] model and a dipole form factor with  $M_A^{QE} = 1.21\text{GeV}$  fit to external data [161], respectively. NEUT implements the Valencia [162] model to simulate MEC events, where two nucleons and two holes in the nuclear target are produced (Often called 2p2h interactions).

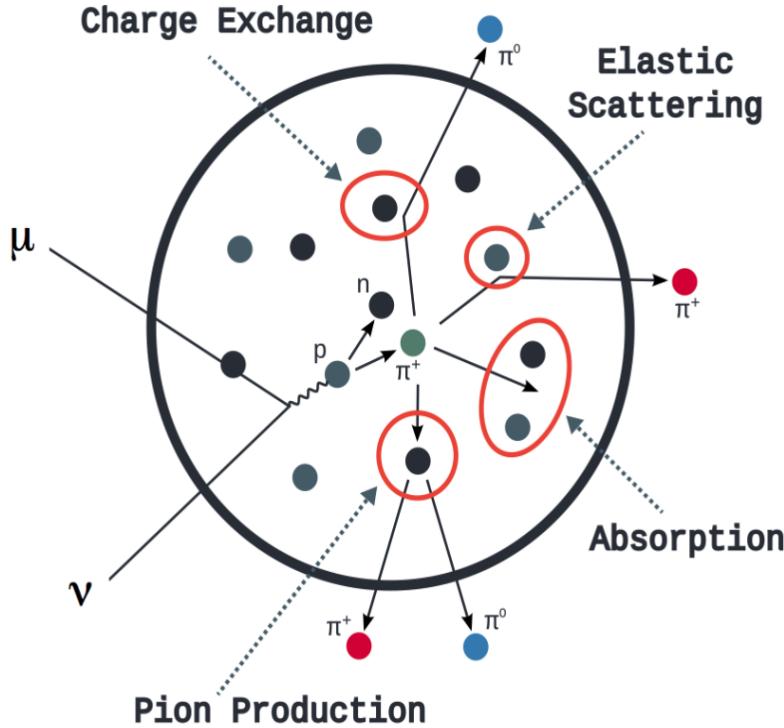
For neutrinos of energy  $O(1)\text{GeV}$ , PROD interactions become dominant. These predominantly produce charged and neutral pions although  $\gamma$ , kaon, and  $\eta$  production is also considered. To simulate these interactions, the Berger-Sehgal [163] model is implemented within NEUT. It simulates the excitation of a



**Figure 5.1:** The NEUT prediction of the  $\nu_\mu$ -H<sub>2</sub>O cross-section overlaid on the T2K  $\nu_\mu$  flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- $\pi$  and DIS (Purple) cross-sections are illustrated. Figure taken from [156].

nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or  $\gamma$ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [164] model to simulate these COH interactions.

DIS and multi- $\pi$  producing interactions become the most dominant for energies  $> O(5)\text{GeV}$ . PYTHIA [165] is used to simulate any interaction with invariant mass  $W > 2\text{GeV}/c^2$ , which produces at least one meson. For any interaction which produces at least two mesons but has  $W < 2\text{GeV}/c^2$ , the Bronner model is used [166]. Both of these models use parton distribution functions based on the Bodek-Yang model [167–169].



**Figure 5.2:** Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [170].

1395 Any pion which is produced within the nucleus can re-interact through  
 1396 final state interactions before it exits, as illustrated by the scattering, absorption,  
 1397 production, and exchange interactions in Figure 5.2. These re-interactions alter  
 1398 the observable particles within the detector. For instance, if the charged pion  
 1399 from a CC PROD interaction is absorbed, the observables would mimic a CC QE  
 1400 interaction. To simulate these effects, NEUT uses a semi-classical intranuclear  
 1401 cascade model [156]. This cascade functions by stepping the pion through the  
 1402 nucleus in fixed-length steps equivalent to  $dx = R_N/100$ , where  $R_N$  is the radius  
 1403 of the nucleus. At each step, the simulation allows the pion to interact through  
 1404 scattering, charged exchange, absorption, or production with an interaction-  
 1405 dependent probability calculated from a fit to external data [171]. This cascade  
 1406 continues until the pion is absorbed or exits the nucleus.

1407 Once the final state particle kinematics have been determined by NEUT,  
 1408 they are passed into the detector simulation. The near detectors, ND280 and

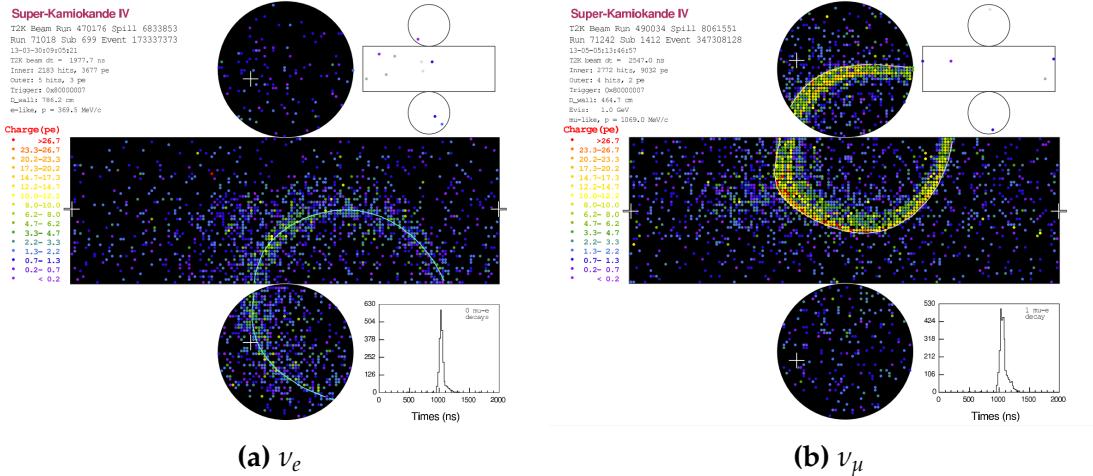
1409 INGRID, are simulated using a GEANT4 package [115, 172] to simulate the detector  
1410 geometry, particle tracking, and energy deposition. The response of the detectors  
1411 is simulated using the elecSim package [115]. The far detector simulation is based  
1412 upon the original Kamiokande experiment software which uses the GEANT3-based  
1413 SKDETSIM [115, 173] package. This simulates the interactions of particles in  
1414 the water as well as Cherenkov light production. The water quality and PMT  
1415 calibration measurements detailed in subsection 3.1.2 are also used within this  
1416 simulation to make accurate predictions of the detector response.

## 1417 5.2 Event Reconstruction at SK

1418 Any event which generates optical photons that occurs in SK will be observed by  
1419 the PMT array, where each PMT records the time and accumulated charge. This  
1420 recorded information is shown in event displays similar to those illustrated in  
1421 Figure 5.3. To be useful for physics analyses, this series of PMT hit information  
1422 needs to be reconstructed to determine the particle’s identity and kinematics (or  
1423 track parameters): four-vertex, direction, and momenta. The reconstruction  
1424 uses the fact that the charge and timing distribution of photons generated  
1425 by a particular particle in an event is dependent upon its initial kinematics.  
1426 Electron and muon rings are distinguished by their “fuzziness”. Muons are  
1427 heavier and less affected by scattering or showering meaning they typically  
1428 produce “crisp” rings. Electrons are more likely to interact via electromagnetic  
1429 showering or scattering which results in larger variations of their direction from  
1430 the initial direction. Consequently, electrons typically produce “fuzzier” rings  
1431 compared to muons.

1432 For the purposes of this analysis, the `fiTQun` reconstruction algorithm is  
1433 utilised. Its core function is to compare a prediction of the accumulated charged  
1434 and timing distribution from each PMT, generated for a particular particle identity  
1435 and track parameters, to that observed in the neutrino event. It determines the  
1436 preferred values by minimising a likelihood function which includes information  
1437 from PMTs which were hit and those that were not hit. `fiTQun` performs a

1438 simultaneous fit of particle kinematics and identity, improving both the accuracy  
 1439 of the fit parameters and the rejection of neutral current  $\pi^0$  events [174, 175]. The  
 1440 `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction  
 1441 algorithm [176] and is described in [177] which is summarised below. The  
 1442 `fitQun` algorithm improves upon the `APFit` reconstruction algorithm which has  
 1443 been used for many previous SK analyses. `APFit` fits the vertex from timing  
 1444 information and then fits the momentum and direction of the particle from PMT  
 1445 hits within a 43 deg Cherenkov cone (which assumes an ultra-relativistic particle).  
 1446 It then fits the particle identity once the track parameters have been fit.



**Figure 5.3:** Event displays from Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [178].

1447 The `fitQun` reconstruction algorithm proceeds by:

- 1448 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit  
 1449 metric
- 1450 • **Peak finding:** The initial time of the event is determined by clustering  
 1451 events by time residuals
- 1452 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,  
 1453 a maximum likelihood technique searches for a single particle generating  
 1454 light

- 1455     • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple  
 1456       light-producing particles are considered using the same maximum likeli-  
 1457       hood technique

1458     An event in SK can consist of a primary and decay particles. For example,  
 1459     a charged current muon neutrino interaction can generate two particles that  
 1460     have the potential of generating Cherenkov photons: the primary muon, and  
 1461     the secondary decay-electron from the muon. To ensure the particles are recon-  
 1462     structed separately, each event is divided into time clusters which are called  
 1463     “subevents”. Subevents after the primary subevent are considered to be decay  
 1464     electrons. To find all the subevents in an event, a vertex goodness metric is  
 1465     calculated for some vertex position  $\vec{x}$  and time  $t$ ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left( -\frac{1}{2} \left( \frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right) \quad (5.1)$$

1466     where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n \quad (5.2)$$

1467     is the residual hit time. It is the difference in time between the PMT hit time,  
 1468      $t^i$ , of the  $i^{th}$  PMT and the expected time of the PMT hit if the photon was emitted  
 1469     at the start of the vertex.  $R_{PMT}^i$  is the position of the  $i^{th}$  PMT,  $c_n$  is the speed of  
 1470     light in water and  $\sigma = 4\text{ns}$  which is comparable to the time resolution of the  
 1471     PMT. When the proposed fit values of time and vertex are close to the true values,  
 1472      $T_{Res}^i(\vec{x}, t)$  tends to zero resulting in subevents appearing as spikes in the goodness  
 1473     metric. The proposed fit vertex and time are grid-scanned, and the values which  
 1474     maximise the goodness metric are selected as the “pre-fit vertex”. Whilst this  
 1475     predicts a vertex for use in the clustering algorithm, the final vertex is fit using  
 1476     the higher-precision maximum likelihood method described below.

1477     Once the pre-fit vertex has been determined, the goodness metric is scanned as  
 1478     a function of  $t$  to determine the number of subevents. A peak-finding algorithm  
 1479     is then used on the goodness metric, requiring the goodness metric to exceed

some threshold and drop below a reduced threshold before any subsequent additional peaks are considered. The thresholds are set such that the rate of false peak finding is minimised while still attaining good data to Monte Carlo agreement. To improve performance, the pre-fit vertex for each delayed subevent is re-calculated after PMT hits from the previous subevent are masked. This improves the decay-electron tagging performance. Once all subevents have been determined, the time window around each subevent is then defined by the earliest and latest time which satisfies  $-180 < T_{Res}^i < 800\text{ns}$ . The subevents and associated time windows are then used as seeds for further reconstruction.

For a given subevent, the `fitQun` algorithm constructs a likelihood based on the accumulated charge  $q_i$  and time information  $t_i$  from the  $i^{th}$  PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}), \quad (5.3)$$

where  $\vec{\theta}$  defines the track parameters; vertex position, direction vector and momenta, and  $\Gamma$  represents the particle hypothesis.  $P_i(\text{unhit}|\Gamma, \vec{\theta})$  is the probability of the  $i^{th}$  tube to not register a hit given the track parameters and particle hypothesis. The charge likelihood,  $f_q(q_i|\Gamma, \vec{\theta})$ , and time likelihood,  $f_t(t_i|\Gamma, \vec{\theta})$ , represents the probability density function of observing charge  $q_i$  and time  $t_i$  on the  $i^{th}$  PMT given the specified track parameters and particle hypothesis.

The predicted charge is calculated based on contributions from both the direct light and the scattered light. The direct light contribution is determined based on the integration of the Cherenkov photon profile along the track. PMT angular acceptance, water quality, and calibration measurements discussed in subsection 3.1.2 are included to accurately predict the charge probability density at each PMT. The scattered light is calculated in a similar way, although it includes a scattering function that depends on the vertex of the particle and the position of the PMT. The charge likelihood is calculated by comparing the prediction to the observed charge in the PMT.

1506     The time likelihood is approximated to depend on the vertex  $\vec{x}$ , direction  $\vec{d}$ ,  
 1507     and time  $t$  of the track as well as the particle hypothesis. The expected time  
 1508     for PMT hits is calculated by assuming unscattered photons being emitted from  
 1509     the midpoint of the track,  $S_{mid}$ ,

$$t_{exp}^i = t + S_{mid}/c + |R_{PMT}^i - \vec{x} - S_{mid}\vec{d}|/c_n, \quad (5.4)$$

1510     where  $c$  is the speed of light in a vacuum. The time likelihood is then expressed  
 1511     in terms of the residual difference between the PMT hit time and the expected  
 1512     hit time,  $t_{Res}^i = t^i - t_{exp}^i$ . The particle hypothesis and momentum also affect the  
 1513     Cherenkov photon distribution. These parameters modify the shape of the time  
 1514     likelihood density since in reality not all photons are emitted at the midpoint of  
 1515     the track. As with the charge likelihood, the contributions from both the direct  
 1516     and scattered light to the time likelihood density are calculated separately, which  
 1517     are both calculated from particle gun studies.

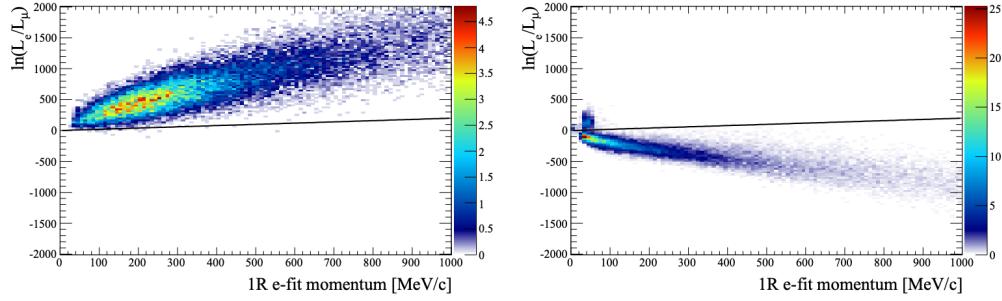
1518     The track parameters and particle identity which maximise  $L(\Gamma, \vec{\theta})$  are defined  
 1519     as the best-fit parameters. In practice MINUIT [179] is used to minimise the value  
 1520     of  $-\ln L(\Gamma, \vec{\theta})$ . The `fiTQun` algorithm considers an electron-like, muon-like, and  
 1521     charged pion-like hypothesis for events with a single final state particle, denoted  
 1522     “single-ring events”. The particle’s identity is determined by taking the ratio of  
 1523     the likelihood of each of the hypotheses. For instance, electrons and muons are  
 1524     distinguished by considering the value of  $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$  in comparison  
 1525     to the reconstructed momentum of the electron hypothesis, as illustrated by  
 1526     Figure 5.4. The coefficients of the discriminator between electron-like and muon-  
 1527     like events are determined from Monte Carlo studies [177]. Similar distributions  
 1528     exist for distinguishing electron-like events from  $\pi^0$ -like events, and muon-like  
 1529     events from pion-like events. The cuts are defined as,

$$\text{Electron/Muon} : \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}],$$

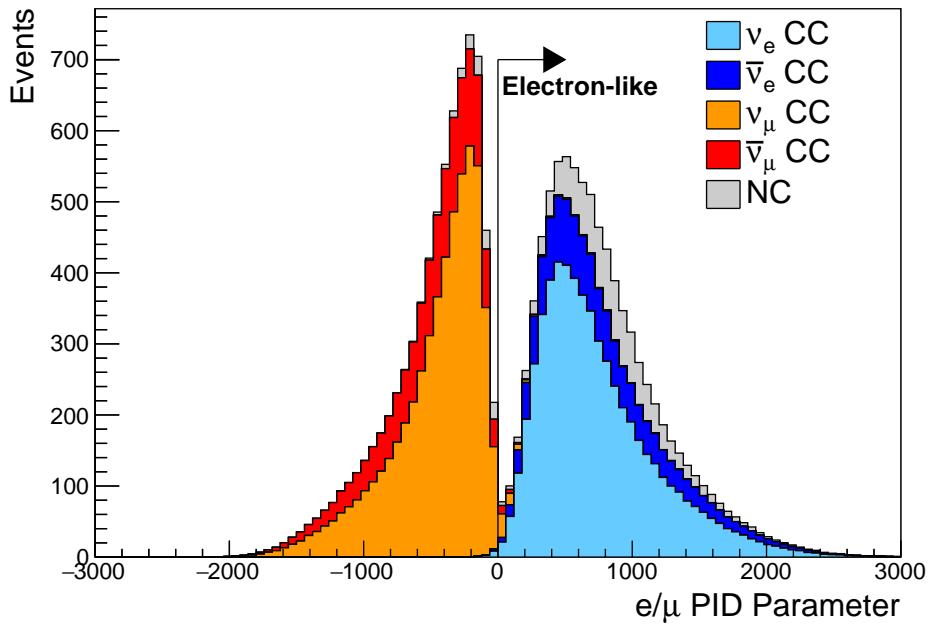
$$\text{Electron}/\pi^0 : \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \quad (5.5)$$

$$\text{Muon/Pion} : \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}],$$

as taken from [180], where  $p_e^{rec}$  and  $p_\mu^{rec}$  are the reconstructed momentum of the single-ring electron and muon fits, respectively.  $m_{\gamma\gamma}$  represents the reconstructed invariant mass of the two photons emitted from  $\pi^0$  decay. Typically, the distance between a particular entry in these two-dimensional distributions and the cut-line is termed the PID parameter and is illustrated in Figure 5.5.



**Figure 5.4:** The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric  $\nu_e$  (left) and  $\nu_\mu$  (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, which coefficients obtained from Monte Carlo studies. Figures taken from [177].



**Figure 5.5:** The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino ( $\nu_\mu$ ,  $\bar{\nu}_\mu$ ,  $\nu_e$  and  $\bar{\nu}_e$ ). Events with positive values of the parameter are determined to be electron-like.

1535     The `fitQun` algorithm also considers a  $\pi^0$  hypothesis. To do this, it performs  
1536     a fit looking for two standard electron-hypothesis tracks which point to the  
1537     same four-vertex. This assumes the electron tracks are generated from photon-  
1538     conversion so the electron tracks actually appear offset from the proposed  $\pi^0$   
1539     vertex. For these fits, the conversion length, direction, and momentum of each  
1540     photon are also considered as track parameters which are then fit in the same  
1541     methodology as the standard single-ring hypotheses.

1542     Whilst lower energy events are predominantly single-ring events, higher  
1543     energy neutrino events can generate final states with multiple particles which  
1544     generate Cherenkov photons. These “multi-ring” hypotheses are also considered  
1545     in the `fitQun` algorithm. When calculating the charge likelihood density, the  
1546     predicted charge associated with each ring is calculated separately and then  
1547     summed to calculate the total accumulated charge on each PMT. Similarly,  
1548     the time likelihood for the multi-ring hypothesis is calculated assuming each  
1549     ring is independent. Each track is time-ordered based on the time of flight  
1550     from the center of the track to the PMT and the direct light from any ring  
1551     incident on the PMT is assumed to arrive before any scattered light. To reduce  
1552     computational resource usage, the multi-ring fits only consider electron-like  
1553     and charged pion-like rings as the pion fit can be used as a proxy for a muon  
1554     fit due to their similar mass.

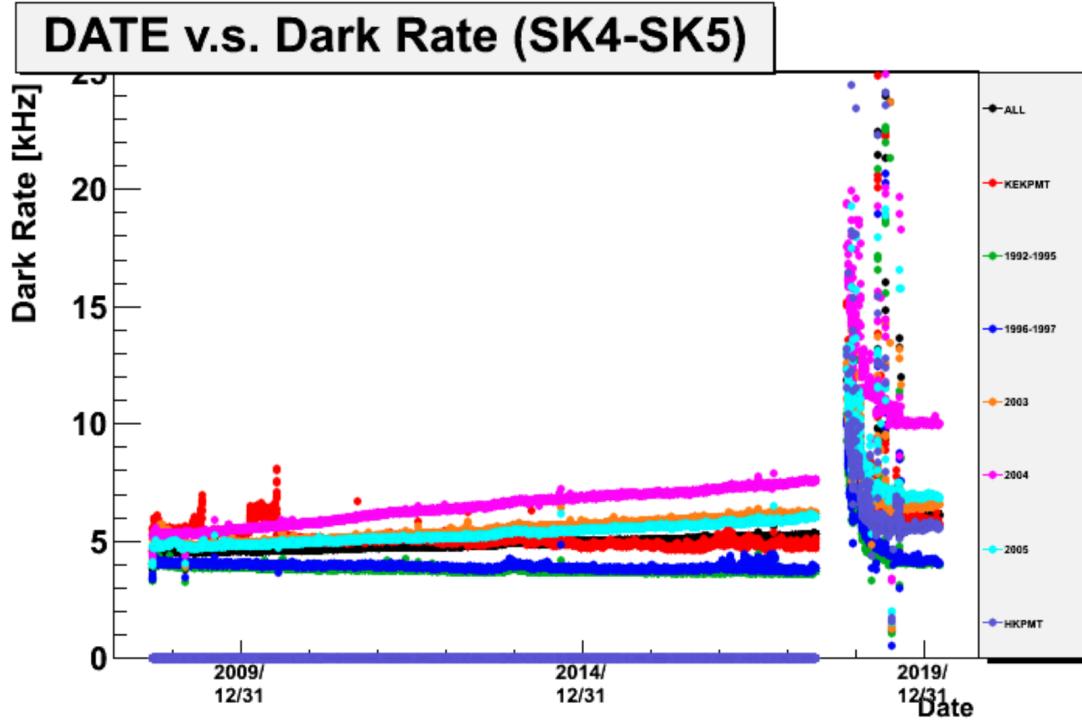
1555     Multi-ring fits proceed by proposing another ring to the previous fit and  
1556     then fitting the parameters in the method described above. Typically, multi-ring  
1557     fits have the largest likelihood because of the additional degrees of freedom  
1558     introduced. A likelihood value is calculated for the  $n$ -ring and  $(n + 1)$ -ring  
1559     hypotheses, where the additional ring is only included if the likelihood value  
1560     is above 9.35, based on Monte Carlo studies in [181].

### 1561     5.2.1 Validation of Reconstruction in SK-V

1562     As an example of how the reconstruction depends on the detector conditions, the  
1563     author of this thesis assessed the quality of event reconstruction for SK-V data.

1564 The detector systematics used in the T2K-only oscillation analysis are determined  
1565 using data-to-Monte Carlo comparisons of the SK-IV data [182]. Due to tank-open  
1566 maintenance occurring between SK-IV and SK-V, the dark rate of each PMT was  
1567 observed to increase in SK-V due to light exposure for a significant time during  
1568 the repairs. This increase can be seen in Figure 5.6. Run-10 of the T2K experiment  
1569 was conducted in the SK-V period, so the consistency of SK-IV and SK-V data  
1570 needs to be studied to determine whether the SK-IV-defined systematics can  
1571 be applied to the run-10 data. This comparison study was performed using  
1572 the stopping muon data set for both the SK-IV and SK-V periods. This data  
1573 sample is used due to the high rate of interactions ( $O(200)$  events per hour) as  
1574 well as having similar energies to muons from CCQE  $\nu_\mu$  interactions from beam  
1575 interactions. The rate of cosmic muons does depend on the solar activity cycle  
1576 [183] but has been neglected in this comparison study. This is because the shape  
1577 of the distributions is most important for the purposes of being compared to the  
1578 detector systematics. The SK-IV and SK-V data samples consist of 2398.42 and  
1579 626.719 hours of data which equates to 686k and 192k events respectively.

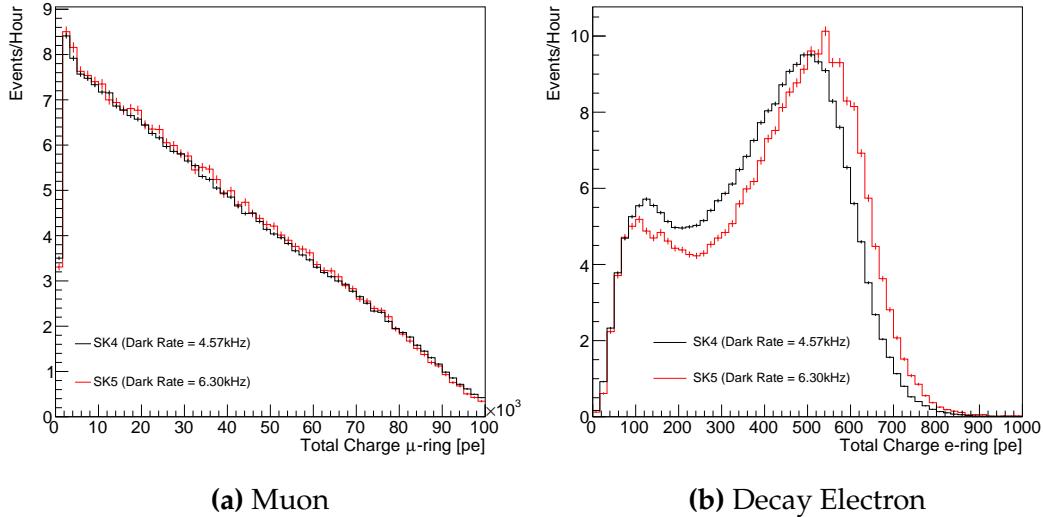
1580 The predicted charge calculated in the `fitQun` algorithm includes a contribu-  
1581 tion from the photoelectron emission due to dark noise. Therefore, the increase  
1582 in the SK-V dark rate needs to be accounted for. In practice, the average dark rate  
1583 in each SK period is calculated and used as an input in the reconstruction. This is  
1584 calculated by averaging the dark rate per run for each period separately, using  
1585 the calibration measurements detailed in subsection 3.1.2. The average dark rate  
1586 from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The  
1587 charges associated with the muon and decay electron subevents are illustrated in  
1588 Figure 5.7. The photoelectron emission from dark noise is more significant for  
1589 events that have lower energy. This is because this contribution becomes more  
1590 comparable to the number of photoelectrons emitted from incident photons in  
1591 lower-energy events. This behaviour is observed in the data, where the charge  
1592 deposited by the muon subevent is mostly unaffected by the increase in dark rate,  
1593 whilst the charge associated with the decay-electron is clearly affected.



**Figure 5.6:** The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [182].

1594        The energy scale systematic is estimated from data-to-Monte Carlo differences  
 1595        in the stopping muon sample in [184] and found to be 2.1%. To determine the  
 1596        consistency of SK-IV and SK-V with respect to the energy scale systematic, the  
 1597        muon momentum distribution is compared between the two SK periods. As  
 1598        the total number of Cherenkov photons is integrated across the track length,  
 1599        the reconstructed momentum divided by track length (or range) is compared  
 1600        between SK-IV and SK-V as illustrated in Figure 5.8.

1601        The consistency between these distributions has been computed in two ways.  
 1602        Firstly, a Gaussian is fit to the peak of each distribution separately, whose mean  
 1603        is found to be  $(2.272 \pm 0.003)\text{MeV/cm}$  and  $(2.267 \pm 0.006)\text{MeV/cm}$  for SK-IV  
 1604        and SK-V respectively. The ratio of these is equal to  $1.002 \pm 0.003$ . The means of  
 1605        the Gaussian fits are consistent with the expected stopping power of a minimum  
 1606        ionising muon for a target material (water) with  $Z/A \sim 0.5$  [185]. The second

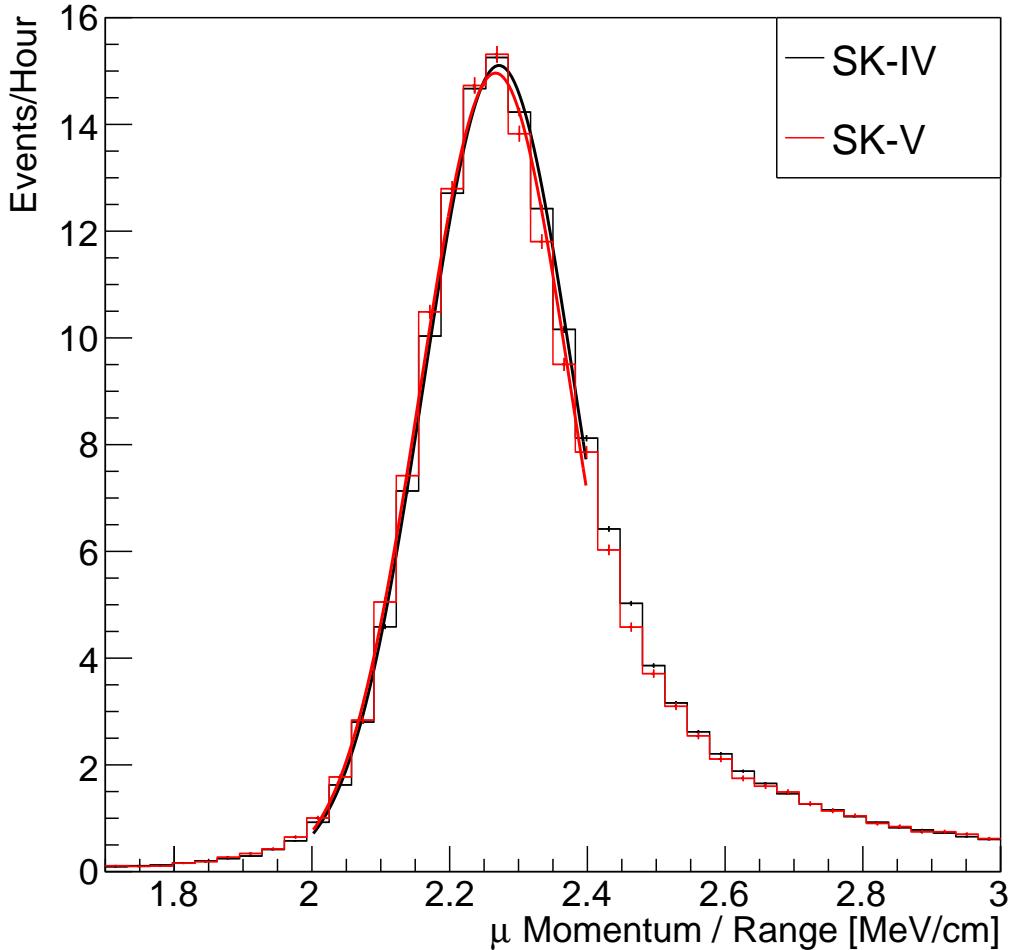


**Figure 5.7:** Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent and the associated decay electron subevent.

1607 consistency check is performed by introducing a nuisance parameter,  $\alpha$ , which  
 1608 modifies the SK-V distribution. The value of  $\alpha$  which minimises the  $\chi^2$  value  
 1609 between the SK-IV and SK-V is determined by scanning across a range of values.  
 1610 This is repeated by applying the nuisance parameter as both a multiplicative  
 1611 factor and an additive shift. The  $\chi^2$  distributions for different values of  $\alpha$  is  
 1612 illustrated in Figure 5.9. The values which minimise the  $\chi^2$  are found to be 0.0052  
 1613 and 1.0024 for the additive and multiplicative implementations, respectively. No  
 1614 evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic  
 1615 has been found in the reconstructed momentum distribution of SK-IV and SK-V.

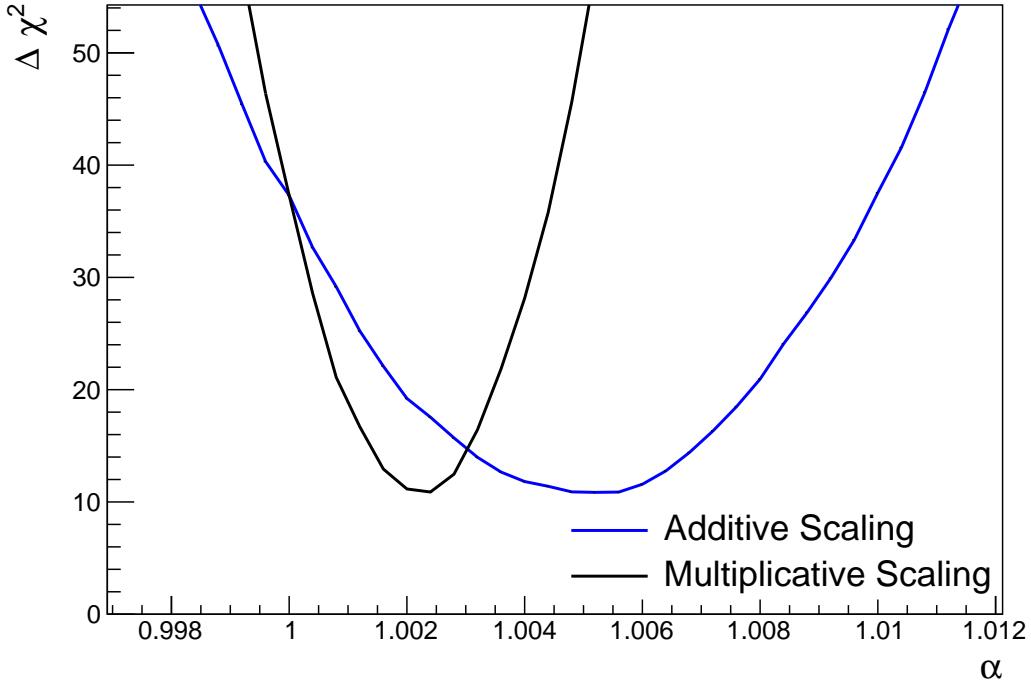
### 1616 5.3 Event Reduction at SK

1617 Atmospheric neutrino events observed in the SK detector are categorised into  
 1618 three different types of samples: fully contained (FC), partially contained (PC)  
 1619 and up-going muon (Up- $\mu$ ), using PMT hit signatures in the inner and outer  
 1620 detector (ID and OD, respectively). To identify FC neutrino events, it is re-  
 1621 quired that the neutrino interacts inside the fiducial volume of the ID and that  
 1622 no significant OD activity is observed. For this analysis, an event is defined



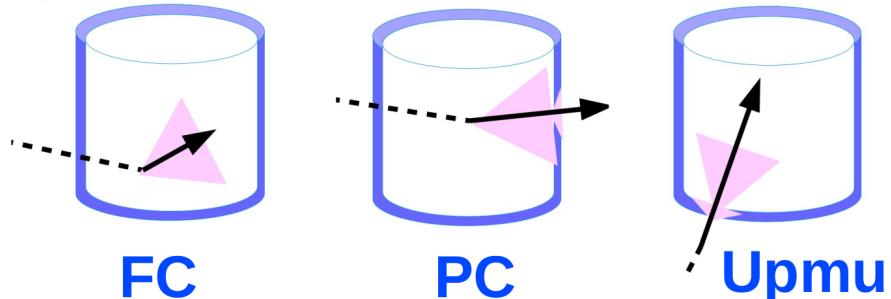
**Figure 5.8:** The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range [2.0, 2.4] MeV/cm and illustrated as the solid curve.

1623 to be in the fiducial volume provided the event vertex is at least 0.5m away  
 1624 from the ID walls. PC events have the same ID requirements but can have a  
 1625 larger signal present inside the OD. Typically, only high energy muons from  
 1626  $\nu_\mu$  interactions can penetrate the ID wall. The Up- $\mu$  sample contains events where  
 1627 muons are created from neutrino interactions in the OD water or rock below the  
 1628 tank. They then propagate upwards through the detector. Downward-going  
 1629 muons generated from neutrino interactions above the tank are neglected is



**Figure 5.9:** The  $\chi^2$  difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter  $\alpha$ . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of  $(\alpha - 1.0)$  but is illustrated like this so the results can be shown on the same axis range.

<sup>1630</sup> because of the difficulty in separating their signature from the cosmic muon  
<sup>1631</sup> shower background. The sample categories are visually depicted in Figure 5.10.

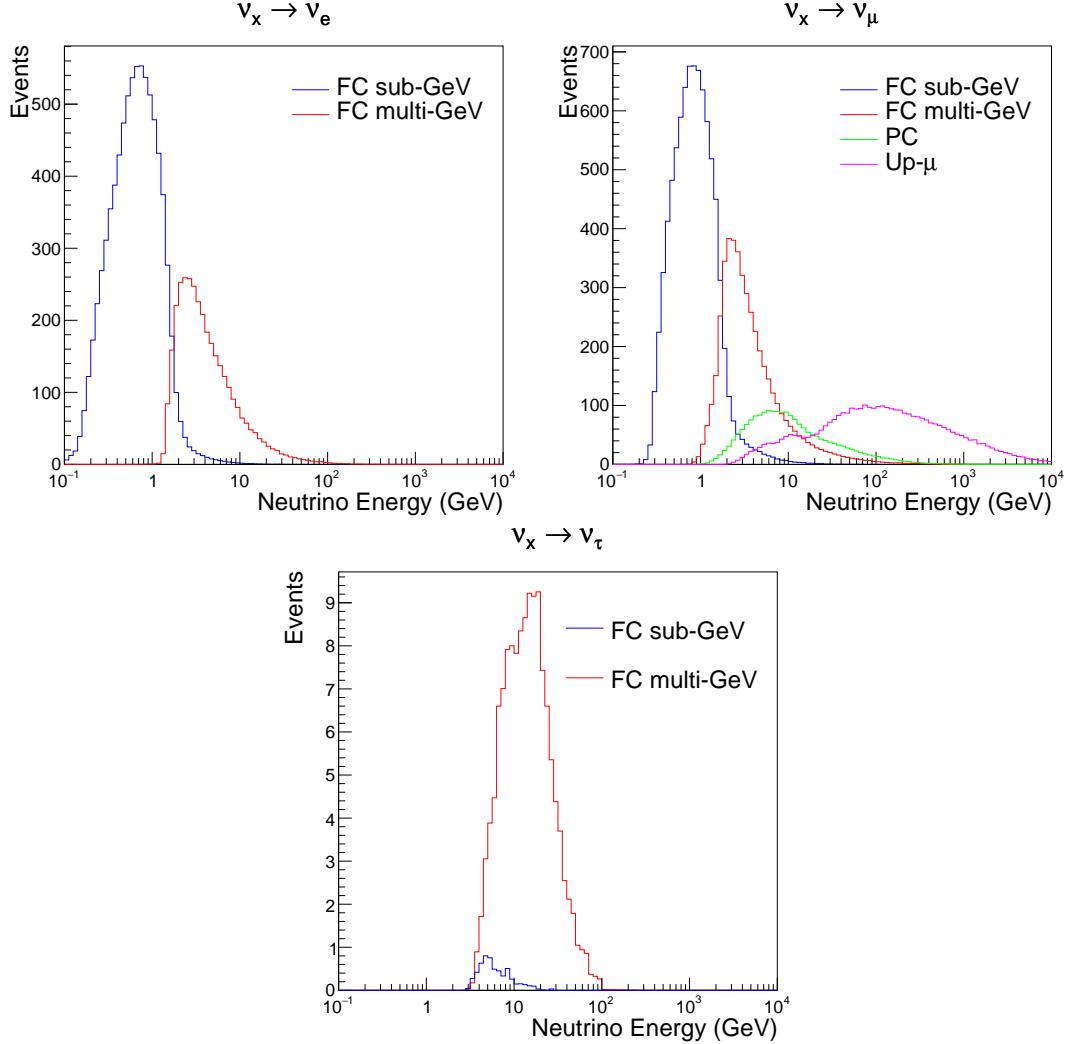


**Figure 5.10:** A depiction of the topology patterns for fully-contained (FC), partially-contained (PC) and up-going muon ( $Up-\mu$ ) samples included in this analysis.

<sup>1632</sup> Based on the event characteristics, as defined by the `fitQun` event reconstruction software, the FC events are categorised by  
<sup>1633</sup>

- 1634     • **Visible Energy:** equal to the sum of the reconstructed kinetic energy of  
1635        particles above the Cerenkov threshold for all rings present in the event.  
1636        The purpose is to separate events into sub-GeV and multi-GeV categories.
- 1637     • **Number of observed Cerenkov rings.** The purpose is to separate single-  
1638        ring and multi-ring events, where single-ring events predominantly consist  
1639        of quasi-elastic interactions and multi-ring events are typically resonant  
1640        pion production or deep inelastic scattering events.
- 1641     • **Particle identification parameter of the most energetic ring:** A value de-  
1642        termined from the maximum likelihood value based on `fitQun`'s electron,  
1643        muon, or pion hypothesis. The purpose is to separate electron-like and  
1644        muon-like events.
- 1645     • **Number of decay electrons:** The purpose is to separate quasi-elastic events  
1646        (which have one decay electron emitted from the muon decay) and resonant  
1647        pion production events (which have two decay electrons emitted from the  
1648        muon and pion).

1649     The PC and Up- $\mu$  categories are broken down into “through-going” and  
1650     “stopping” samples depending on whether the muon leaves the detector. This is  
1651     because the PC stopping events deposit the entire energy of the interaction into  
1652     the detector, resulting in better reconstruction. The energy of events that exit the  
1653     detector has to be estimated, with typically worse resolution, which introduces  
1654     much larger systematic uncertainties. Through-going Up- $\mu$  samples are further  
1655     broken down by whether any hadronic showering was observed in the event  
1656     which typically indicates DIS interactions. The expected neutrino energy for the  
1657     different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events  
1658     peak around 0.7GeV and 3GeV respectively, with slightly different peak energies  
1659     for  $\nu_e$  and  $n\nu_\mu$  oscillation channels. PC and Up- $\mu$  are almost entirely comprised  
1660     of  $\nu_\mu$  events and peak around 7GeV and 100GeV, respectively.



**Figure 5.11:** The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- $\mu$ ) events. The prediction is broken down by the  $\nu_x \rightarrow \nu_e$  prediction (top left),  $\nu_x \rightarrow \nu_\mu$  prediction (top right) and  $\nu_x \rightarrow \nu_\tau$  prediction (bottom). Asimov A oscillation parameters are assumed (given in Table 2.2).

In normal data-taking operations, the SK detector observes many background events alongside the beam and atmospheric neutrino signal events of physics interest for this thesis. Cosmic ray muons and flasher events, which are the spontaneous discharge of a given PMT, contribute the largest amount of background events in the energy range relevant to this thesis. Lower energy analyses like DSNB searches are also subject to radioactive backgrounds [186]. Therefore the data recorded is reduced with the aim of removing these background events. The reduction process is detailed in [54, 85] and briefly summarised below.

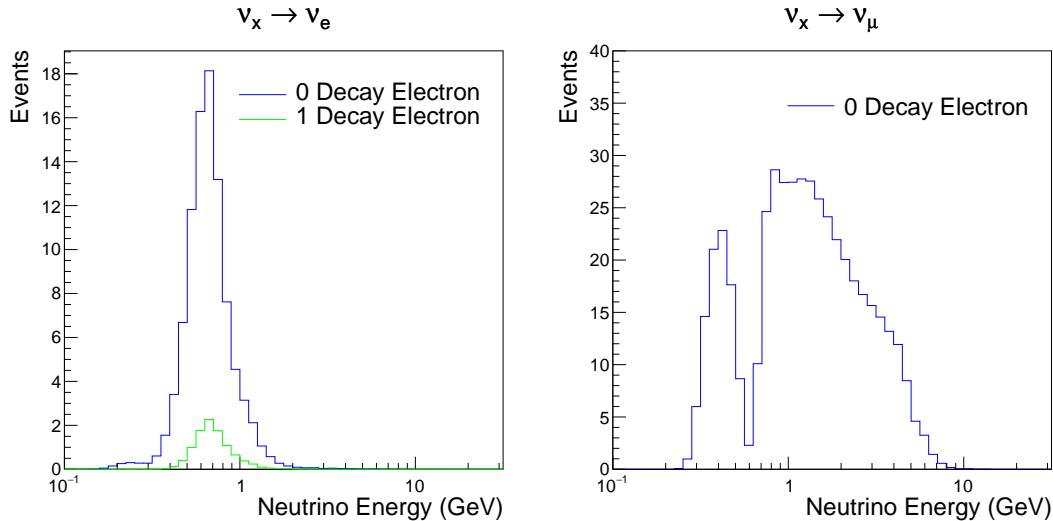
1669        The first two steps in the FC reconstruction remove the majority of cosmic  
1670        ray muons by requiring a significant amount of ID activity compared to that  
1671        measured in the OD. Events that pass this cut are typically very high momentum  
1672        muons or events that leave very little activity in the OD. Consequently, a third  
1673        reduction step is then applied to select cosmic-ray muons that pass the initial  
1674        reduction step. A purpose-built cosmic muon fitter is used to determine the  
1675        entrance (or exit) position of the muon and a cut is applied to OD activity  
1676        contained within 8m of this position. Flasher events are removed in the fourth  
1677        reduction step which is based on the close proximity of PMT hits surrounding  
1678        the PMT producing the flash. Events that pass all these reduction steps are  
1679        reconstructed with the APFit algorithm. The fifth step of the reduction uses  
1680        information from the more precise fitter to repeat the previous two steps with  
1681        tighter cuts. Muons below the Cherenkov threshold can not generate optical  
1682        photons in the ID but the associated decay electron can due to its lower mass.  
1683        These are the types of events targeted in the fifth reduction step. The final  
1684        cuts require the event vertex to be within the fiducial volume (0.5m from the  
1685        wall although the nominal distance is 2.0m), visible energy  $E_{vis} > 30\text{MeV}$  and  
1686        fewer than 16 hits within the higher energy OD cluster. The culmination of  
1687        the fully contained reduction results in 8.09 events/day in the nominal fiducial  
1688        volume [81]. The uncertainty in the reconstruction is calculated by comparing  
1689        Monte Carlo prediction to data. The largest discrepancy is found to be 1.3%  
1690        in the fourth reduction step.

1691        The PC and Up- $\mu$  events are processed through their own reduction processes  
1692        detailed in [54]. Both of these samples are reconstructed with the APFit algorithm  
1693        rather than `fiTQun`. This is because the efficiency of reconstructing events that  
1694        leave the detector has not been sufficiently studied for reliable systematic uncer-  
1695        tainties wth `fiTQun`. The PC and Up- $\mu$  samples acquire events at approximately  
1696        0.66 and 1.44 events/day.

1697        Events due to beam neutrinos undergo the same reduction steps as FC events  
1698        and are then subject to further cuts [187]. The GPS system which links the timing

<sub>1699</sub> between the beam facility and SK needs to be operating correctly and there should  
<sub>1700</sub> be no activity within the detector in the previous  $100\mu\text{s}$  before the trigger. The  
<sub>1701</sub> events then need to triggered between  $-2\mu\text{s}$  and  $10\mu\text{s}$  of the expected spill timing.

<sub>1702</sub> The beam neutrino samples are not split by visible energy since their energy  
<sub>1703</sub> range is smaller than the atmospheric neutrino events. Following the T2K  
<sub>1704</sub> analysis in [74], only single-ring beam neutrino events are considered. Similar to  
<sub>1705</sub> atmospheric event selection, the number of decay electrons is used as a proxy for  
<sub>1706</sub> distinguishing CCQE and CCRES events. The expected neutrino energy, broken  
<sub>1707</sub> down by number of decay electrons, is given in Figure 5.12.



**Figure 5.12:** The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

# 6

1708

1709

## Sample Selections and Systematics

1710 The oscillation analysis presented within this thesis is built upon a simultaneous  
1711 fit to atmospheric data at SK, neutrino beam data in the near detector, and  
1712 beam data measured at SK. The definitions of these samples are documented  
1713 in section 6.1, section 6.2, and section 6.3, respectively. The data collected and  
1714 used within this analysis is detailed in Table 6.1. The near and far detector data  
1715 corresponds to T2K runs 2-9 and runs 1-10, respectively. The accumulated POT  
1716 and beam power for runs 1 – 10 are illustrated in Figure 6.1.

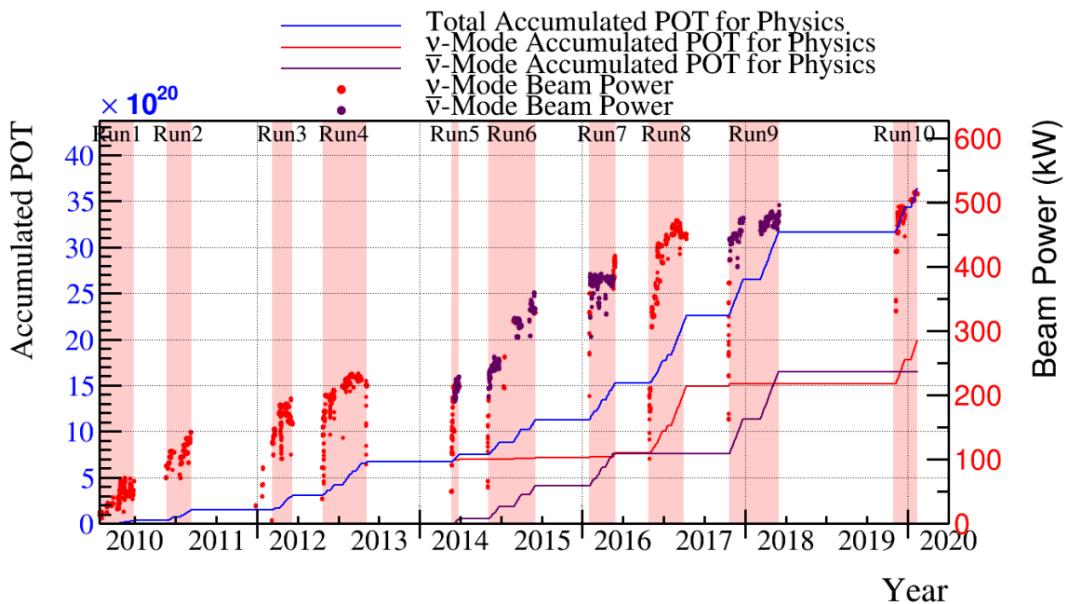
Data Type	Total
Near Detector FHC	$1.15 \times 10^{21}$ POT
Near Detector RHC	$8.34 \times 10^{20}$ POT
Far Detector FHC	$1.97 \times 10^{21}$ POT
Far Detector RHC	$1.63 \times 10^{21}$ POT
Atmospheric SK-IV	3244.4 days

**Table 6.1:** The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

1717 The difference in POT recorded at the near and far detector is due to the  
1718 difference in downtime. The SK detector is very stable with almost 100% of  
1719 data recorded during beam operation. Due to various technical and operational

1720 issues, the downtime of the near detector is significantly higher due to its more  
1721 complex design and operating requirements.

1722 The systematic parameters invoked within the flux, detector, and interaction  
1723 models used within this analysis are documented in section 6.4. The standard  
1724 configuration of the joint beam and atmospheric data fit utilises far detector sys-  
1725 tematics provided in the official inputs from the two experiments. Additionally,  
1726 a correlated detector model which fits the parameters used in sample selections  
1727 to data has been developed and documented in subsection 6.4.5.



**Figure 6.1:** The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

## 1728 6.1 Atmospheric Samples

1729 The atmospheric event selection follows the official SK-IV analysis presented  
1730 in [85] and is documented below. The Monte Carlo prediction used within this  
1731 analysis corresponds to 500 years worth of neutrino events, which is scaled down  
1732 to match the SK-IV livetime of 3244.4 days.

1733 The fully contained (FC), partially contained (PC), and upward going muon  
1734 events ( $\text{up-}\mu$ ) which pass the reduction cuts discussed in section 5.3 are further  
1735 broken down into different samples based on reconstruction information. This  
1736 section details the samples used within this oscillation analysis, alongside the  
1737 chosen binning.

1738 FC events are first separated by the visible energy deposited within the  
1739 detector. This is calculated as the sum of the reconstructed kinetic energy  
1740 above the Cherenkov threshold for all rings present in the event. Events are  
1741 separated by whether they were above or below  $E_{\text{vis}} = 1.33\text{GeV}$ . This separates  
1742 “subGeV” and “multiGeV” events. Typically, lower energy events consist of  
1743 charged current quasi-elastic (CCQE) interactions which are better understood  
1744 and simpler to reconstruct resulting in smaller systematic uncertainties. Events  
1745 are further separated by the number of rings associated with the event due to  
1746 similar reasoning. As the oscillation probability is dependent upon the flavour  
1747 of neutrino, electron and muon events are separated using a similar likelihood  
1748 method to that discussed in section 5.2. To reduce computational resources  
1749 required for the reconstruction, only electron and pion hypotheses are considered  
1750 so this separation cut depends on the ratio of the electron to pion likelihoods,  
1751  $\log(L_e/L_\pi)$ . Finally, the number of decay electrons is used to classify events.  
1752 Charged current resonant pion production (CCRES) interactions generate a final-  
1753 state pion. This can decay, mostly likely through a muon, into a decay electron.  
1754 Therefore any electron-like event with one decay electron or muon-like event  
1755 with two decay electrons was most likely produced by a CCRES interaction.  
1756 Consequently, the number of decay electrons can be used to distinguish CCQE

<sup>1757</sup> and CCRES interaction modes. Ultimately, FC subGeV events are separated  
<sup>1758</sup> into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed $\pi^0$ mass $85 \leq m_{\pi^0} < 215$ MeV

**Table 6.2:** The fully contained subGeV samples, defined as events with visible energy  $E_{vis} < 1.33$  GeV, used within this oscillation analysis.

<sup>1759</sup> In addition to the cuts discussed above, multiGeV samples also have addi-  
<sup>1760</sup> tional cuts to separate samples which target neutrino and antineutrino events.  
<sup>1761</sup> As discussed in section 7.1, the matter resonance only occurs for neutrinos in  
<sup>1762</sup> normal hierarchy and antineutrinos in an inverted mass hierarchy. Therefore,  
<sup>1763</sup> having flavour-enriched samples aids in the determination of the mass hierarchy.  
<sup>1764</sup> For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

<sup>1765</sup> The  $\pi^-$  emitted from a  $\bar{\nu}_e$  interaction is more likely to be absorbed within the  
<sup>1766</sup> oxygen nucleus than the  $\pi^+$  from  $\nu_e$  interactions [188]. These pions then decay,  
<sup>1767</sup> mostly through muons, to electrons. Therefore the number of tagged decay  
<sup>1768</sup> electrons associated with an event gives an indication of whether the interaction  
<sup>1769</sup> was due to a neutrino or antineutrino: zero for  $\bar{\nu}_e$  events, and one for  $\nu_e$  events.  
<sup>1770</sup> The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,  
<sup>1771</sup> where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with  
<sup>1772</sup> only 7% antineutrino background, the rest consisting of NC backgrounds.

1773 The number of decay electrons discriminator works reasonably well for single-  
1774 ring events. However, this is not the case for multi-ring events. A multiGeV  
1775 multiring electron-like (MME) likelihood cut was introduced in [189, 190]. This  
1776 is a two-stage likelihood selection cut. Four observables are used in the first  
1777 likelihood cut to distinguish  $CC\nu_e$  and  $CC\bar{\nu}_e$  events from background:

- 1778 • The number of decay electrons
- 1779 • The maximum distance between the vertex of the neutrino and the decay  
1780 electrons
- 1781 • The energy deposited by the highest energy ring
- 1782 • The particle identification of that highest energy ring

1783 Background events consist of  $CC\nu_\mu$  and NC interactions. Typically, the  
1784 majority of the energy in these background events is carried by the hadronic  
1785 system. Additionally, muons tend to travel further than the pions from  $CC\nu_e$   
1786 before decaying. Thus, the parameters used within the likelihood cut target these  
1787 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring $e$ -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring $e$ -like events with one or more decay electrons
MultiGeV-mulike	Single ring $\mu$ -like events
MultiRing-elike-nue	Two or more ring events with leading energy $e$ -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy $e$ -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy $\mu$ -like ring and only requires $E_{vis} > 0.6\text{GeV}$ <b>DB: Why is this not }1.33\text{GeV</b>
MultiRing-Other1	Two or more ring events with leading energy $e$ -like ring and failed the MME likelihood cut

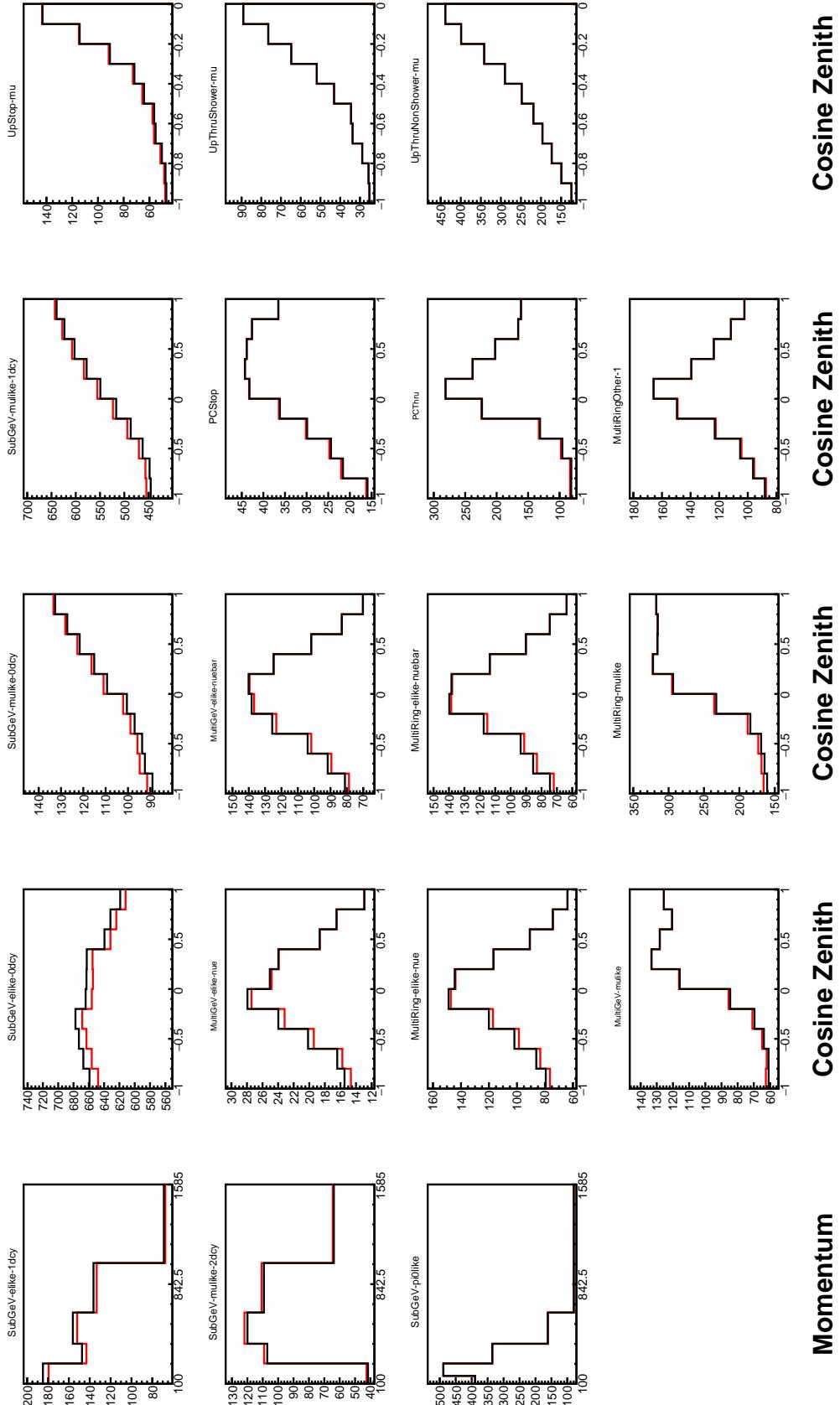
**Table 6.3:** The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1788 Neutrino and antineutrino events are then separated by a second likelihood  
1789 method ( $\nu/\bar{\nu}$  separation) detailed in [59]. This uses the number of decay electrons,

1790 the number of reconstructed rings, and the event’s transverse momentum. The  
1791 last two parameters are used because higher-energy samples tend to have more  
1792 pions produced above the Cherenkov threshold which results in more rings  
1793 compared to an antineutrino interaction. Furthermore, the angular distribution  
1794 also tends to be more forward peaked in antineutrino interactions as compared  
1795 to neutrino interactions [85]. These FC multiGeV sample definitions are de-  
1796 tailed in Table 6.3.

1797 The PC and up- $\mu$  samples are split by the amount of energy deposited within  
1798 the outer detector, into “stopping” and “through-going” samples. If an event  
1799 leaves the detector, the energy it takes with it has to be estimated which increases  
1800 the systematic uncertainty compared to events entirely contained within the  
1801 inner detector. This estimation is particularly poor at high energies, thus the  
1802 up- $\mu$  through-going events are not binned in reconstructed momentum. The  
1803 through-going up- $\mu$  are further separated by the presence of any electromagnetic  
1804 showering in the event, as the assumption of non-showering muon does not give  
1805 reliable reconstruction for these types of events [54]. In total, 13 FC, 2 PC, and  
1806 3 up- $\mu$  atmospheric samples are included within this analysis.

1807 The atmospheric samples are binned in direct observables: reconstructed  
1808 lepton momentum and direction, as given by Table A.1. The distribution of  
1809 the reconstructed lepton momentum (for samples that only have one bin in  
1810 reconstructed zenith angle) and reconstructed direction for each atmospheric  
1811 sample used within this analysis is illustrated in Figure 6.2. The by-mode  
1812 breakdown of each of the atmospheric samples is given in Appendix A.



**Figure 6.2:** Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and  $\pi^0$ -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThrus	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

**Table 6.4:** The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

## 1813 6.2 Near Detector Beam Samples

1814 The near detector sample selections are documented in detail within [191] and  
1815 summarised below. Samples are selected based upon the which Fine Grained  
1816 Detector (FGD) that the vertex is reconstructed in as well as the operating mode  
1817 of the beam: FHC or RHC. Wrong-sign neutrino samples are considered in the  
1818 RHC mode in order to add additional constraints on model parameters. Samples  
1819 from the wrong-sign component of the FHC beam mode are not included as they  
1820 are statistically insignificant compared to those samples already listed.

1821 For additional constraints on model parameters, wrong-sign neutrino samples  
1822 are also considered when the beam is operating in RHC mode.

1823 The reconstruction algorithm uses a clustering algorithm to group hits within  
1824 the TPC. It then adds information from the upstream FGD to form a track  
1825 which passes through both sub-detectors. In FHC(RHC), the highest momentum  
1826 negative(positive) curvature track is defined as the muon candidate. Before  
1827 being assigned a sample, these candidate muon events must pass CC-inclusive  
1828 cuts, as defined in [192]:

- 1829 • Event Timing: The DAQ must be operational and the event must occur  
1830 within the expected beam time window consistent with the beam spill
- 1831 • TPC Requirement: The muon-candidate track path must intercept one or  
1832 more TPCs
- 1833 • Fiducial volume: The event must originate from within the fiducial volume  
1834 defined in [193].
- 1835 • Upstream Background: Remove events that have muon tracks that originate  
1836 upstream of the FGDs by requiring no high-momentum tracks within  
1837 150mm upstream of the candidate vertex. Additionally, events that occur  
1838 within the downstream FGD are vetoed if a secondary track starts within  
1839 the upstream FGD

- 1840 • Broken track removal: All candidates where the muon candidate is broken  
1841 in two are removed

- 1842 • Muon PID: Measurements of  $dE/dx$  in a TPC are used to distinguish muon-  
1843 like events, from electron-like or proton-like, using a likelihood cut

1844 In addition to these cuts, RHC neutrino events also have to undergo the  
1845 following cuts to aid in the separation of neutrino and antineutrino [194]:

- 1846 • TPC Requirement: The track path must intercept TPC2

- 1847 • Positive Track: The highest momentum track must have positive recon-  
1848 structed charge

- 1849 • TPC1 Veto: Remove any events originating upstream of TPC1

1850 Once all CC-inclusive events have been determined, they are further split by  
1851 pion multiplicity: CC0 $\pi$ , CC1 $\pi$ , and CCOther. These target specific interaction  
1852 modes CCQE, CCRES, and other CC background interactions, respectively. Pions  
1853 in the TPCs and FGDs are selected by requiring a second track to be observed,  
1854 which is separate from the muon track and is in the same beam spill window and  
1855 sub-detector. If the pion originated within a FGD, it must also pass through the  
1856 sequential downstream TPC (TPC2 for FGD1, TPC3 for FGD2).

1857 CC0 $\pi$ , CC1 $\pi$ , and CCOther samples are defined with the following cuts:

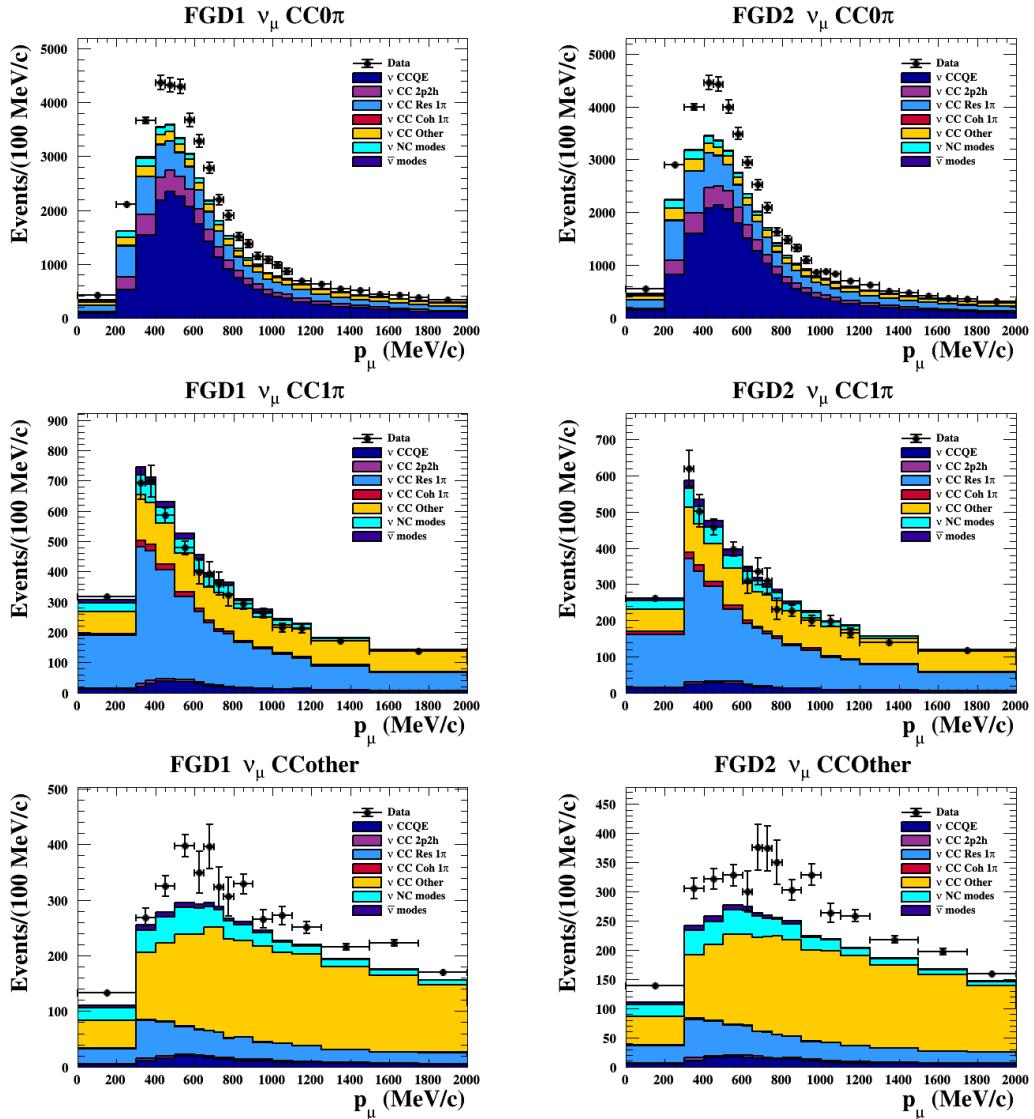
1858 **DB: Understand pion cuts at ND**

- 1859 •  $\nu_\mu$ CC0 $\pi$  **Selection:** No electrons in TPC and no charged pions or decay  
1860 electrons within the TPC or FGD

- 1861 •  $\nu_\mu$ CC1 $\pi$  **Selection:** Exactly one charged pion in either the TPC or FGD,  
1862 where the number of charged pions in the FGD is equal to the number of  
1863 decay electrons

- 1864 •  $\nu_\mu$ CCOther **Selection:** All events which are not classified into the above  
1865 two selections.

Counting the three selections for each FGD in FHC and RHC running, including the wrong-sign background in RHC, 18 near detector samples are used within this analysis. These samples are binned in reconstructed lepton momentum (illustrated in Figure 6.3) and direction with respect to the beam. The binning is chosen such that each event has at least 20 Monte Carlo events in each bin [193]. This is to ensure that the bins are coarse enough to ensure the reduction of statistical errors, whilst also being fine enough to sample the high-resolution peak regions. The exact binning is detailed in [193].



**Figure 6.3:** The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the  $CC\nu_\mu 0\pi$ ,  $CC\nu_\mu 1\pi$  and  $CC\nu_\mu$  Other categories. Figures taken from [191].

### **1874 6.3 Far Detector Beam Samples**

1875 The beam neutrino events which occur at the SK detector, which pass the  
 1876 reduction cuts detailed in section 5.3, are separated based on whether the beam  
 1877 was operating in FHC or RHC mode. The events are then separated into three  
 1878 samples: electron-like ( $1Re$ ), muon-like ( $1R\mu$ ), and  $CC1\pi^+$ -like ( $1Re1de$ ) which  
 1879 are observed as electron-like events with an associated decay electron [182].  
 1880 As discussed in section 6.1, positively charged pions emitted from neutrino  
 1881 interactions are more likely to produce decay electrons than negatively charged  
 1882 pions. Consequently, the  $CC1\pi^+$ -like sample is only selected when the beam is  
 1883 operating in FHC mode. Therefore, five beam samples measured at SK are  
 1884 used in this analysis.

1885 The fiducial volume definition for beam samples is slightly different from that  
 1886 used for the atmospheric samples. It uses both the distance to the closest wall  
 1887 (`dWall`) and the distance to the wall along the trajectory of the particle (`toWall`).  
 1888 This allows events that originate close to the wall but are facing into the tank to be  
 1889 included within the analysis, which would have otherwise been removed. These  
 1890 additional events are beneficial for a statistics-limited experiment. The exact  
 1891 cut values for both `dWall` and `toWall` are different for each of the three types of  
 1892 sample and are optimised based on T2K sensitivity to  $\delta_{CP}$  [180, 195]. They are:

1893 **1Re event selection** For an event to be classified as a  $1Re$ -like, the event must sat-  
 1894 isfy:

- 1895 • Fully-contained and have  $dWall > 80\text{cm}$  and  $toWall > 170\text{cm}$
- 1896 • Total of one ring which is reconstructed as electron-like with reconstructed  
   1897 momentum  $P_e > 100\text{MeV}$
- 1898 • Zero decay electrons are associated with the event
- 1899 • Passes  $\pi^0$  rejection cut discussed in section 5.2

1900      The zero decay electron cut removes non-CCQE interactions and the  $\pi^0$   
 1901     rejection cut is designed to remove neutral current  $\pi^0$  background events which  
 1902     can be easily reconstructed as 1Re-like events.

1903      The zero decay electron cut removes non-CCQE interactions and the  $\pi^0$   
 1904     rejection cut is designed to remove neutral current  $\pi^0$  background events which  
 1905     can be easily reconstructed as 1Re-like events.

1906   **CC1 $\pi^+$  event selection** This event selection is very similar to that of the 1Re  
 1907   sample. The only differences are that the `dWall` and `toWall` criteria are changed  
 1908   to  $> 50\text{cm}$  and  $> 270\text{cm}$ , respectively, and exactly one decay electron is required  
 1909   from the  $\pi^+$  decay.

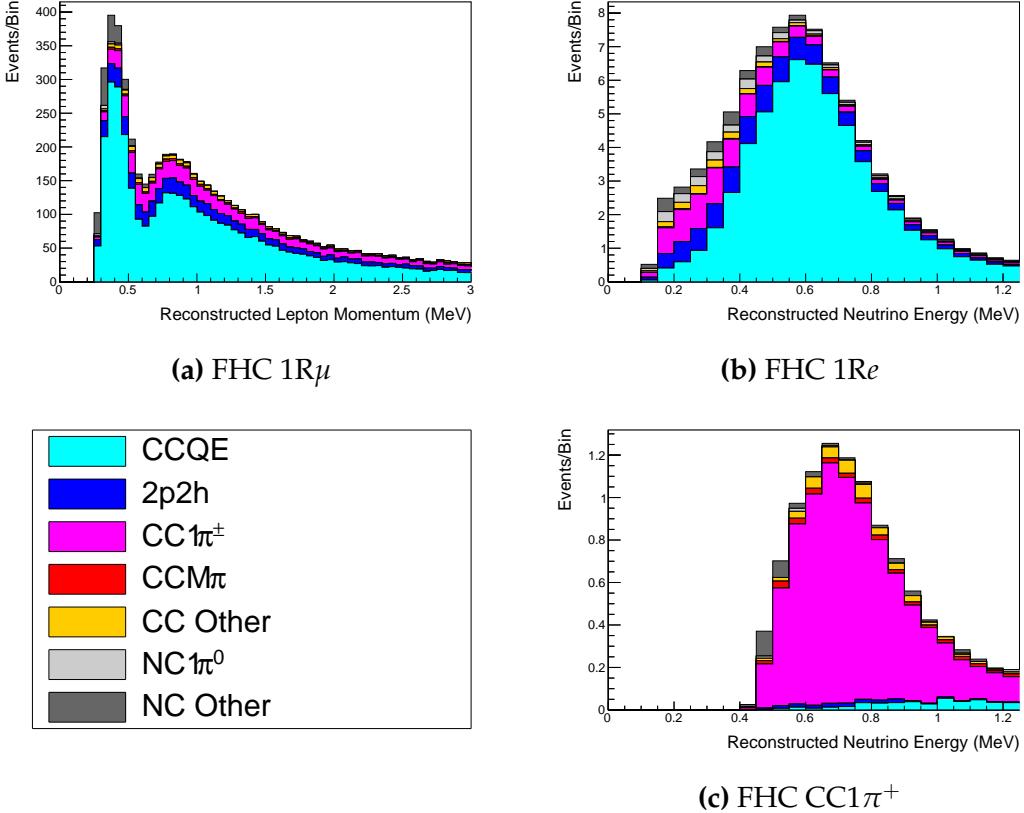
1910   **1R $\mu$  event selection** A 1R $\mu$ -like event is determined by the following cuts:

- 1911      • Fully-contained and have `dWall`  $> 50\text{cm}$  and `toWall`  $> 250\text{cm}$
- 1912      • Total of one ring which is reconstructed as muon-like with reconstructed  
               momentum  $P_\mu > 200\text{MeV}$
- 1914      • Fewer than two decay electrons are associated with the event
- 1915      • Passes  $\pi^+$  rejection cut discussed in section 5.2

1916      All of these samples are binned in reconstructed neutrino energy. This is  
 1917   possible under a particular interaction mode assumption, as the direction from  
 1918   the source is known extremely well. For the 1Re-like and 1R $\mu$ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})} \quad (6.2)$$

1919      Where  $M_N$ ,  $M_P$  and  $m_l$  are the masses of the neutron, proton and outgoing  
 1920   lepton, respectively.  $V_{nuc} = 27\text{MeV}$  is the binding energy of the oxygen nucleus  
 1921   [182],  $\theta_{beam}$  is the angle between the beam and the direction of the outgoing  
 1922   lepton, and  $E_l$  and  $P_l$  are the energy and momentum of that outgoing lepton.



**Figure 6.4:** The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R $\mu$ -like, 1Re-like and CC1 $\pi^+$ -like samples. Asimov A oscillation parameter sets are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R $\mu$  sample only shows the  $0 \leq E_\nu^{rec} < 3.0\text{GeV}$  but the binning extends to 30.0GeV.

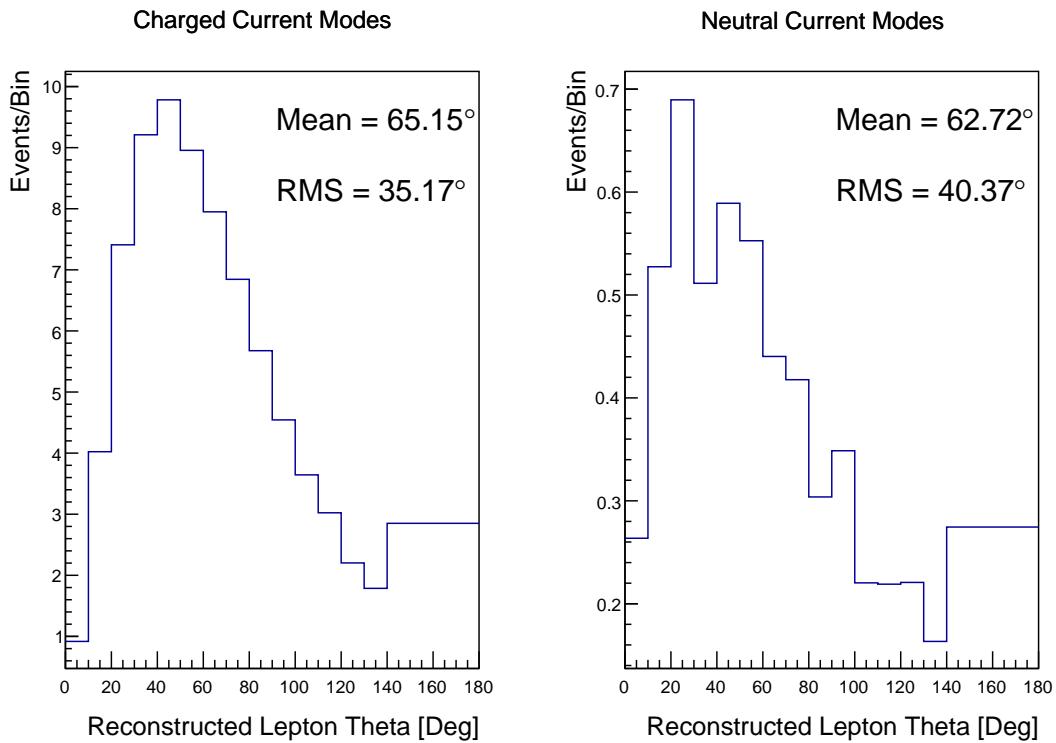
1923      The reconstructed neutrino energy of the CC1 $\pi^+$ -like events also accounts  
 1924      for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))} \quad (6.3)$$

1925      Where  $M_{\Delta^{++}}$  is the mass of the delta baryon. Binding energy effects are not  
 1926      considered as a two-body process with the delta baryon is assumed. This follows  
 1927      the T2K oscillation analysis presented in [74], although recent developments  
 1928      of the interaction model in the latest T2K oscillation analysis do include effects  
 1929      from binding energy in this calculation [196].

1930      The reconstructed neutrino energy for the FHC samples is illustrated in  
 1931      Figure 6.4. As expected, the 1R $\mu$ -like and 1Re-like samples are heavily dominated

<sup>1932</sup> by CCQE interactions, with smaller contributions from 2p2h meson exchange and  
<sup>1933</sup> resonant pion production interactions. The CC1 $\pi^+$ -like sample predominantly  
<sup>1934</sup> consists of charged current resonant pion production interactions. The 1Re-like  
<sup>1935</sup> and CC1 $\pi^+$ -like samples are also binned by the angle between the neutrino  
<sup>1936</sup> beam and the reconstructed lepton momentum. This is to aid in charged current  
<sup>1937</sup> and neutral current separation, as indicated in Figure 6.5. This is because the  
<sup>1938</sup> neutral current backgrounds are predominantly due to  $\pi^0$ -decays, where the  
<sup>1939</sup> opening angle of the two gammas alongside the different final state kinematics  
<sup>1940</sup> produces a slightly broader angular distribution compared to the final state  
<sup>1941</sup> particles originating from charged current  $\nu_e$  interactions.



**Figure 6.5:** The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are  $35.17^\circ$  and  $40.37^\circ$ , respectively.

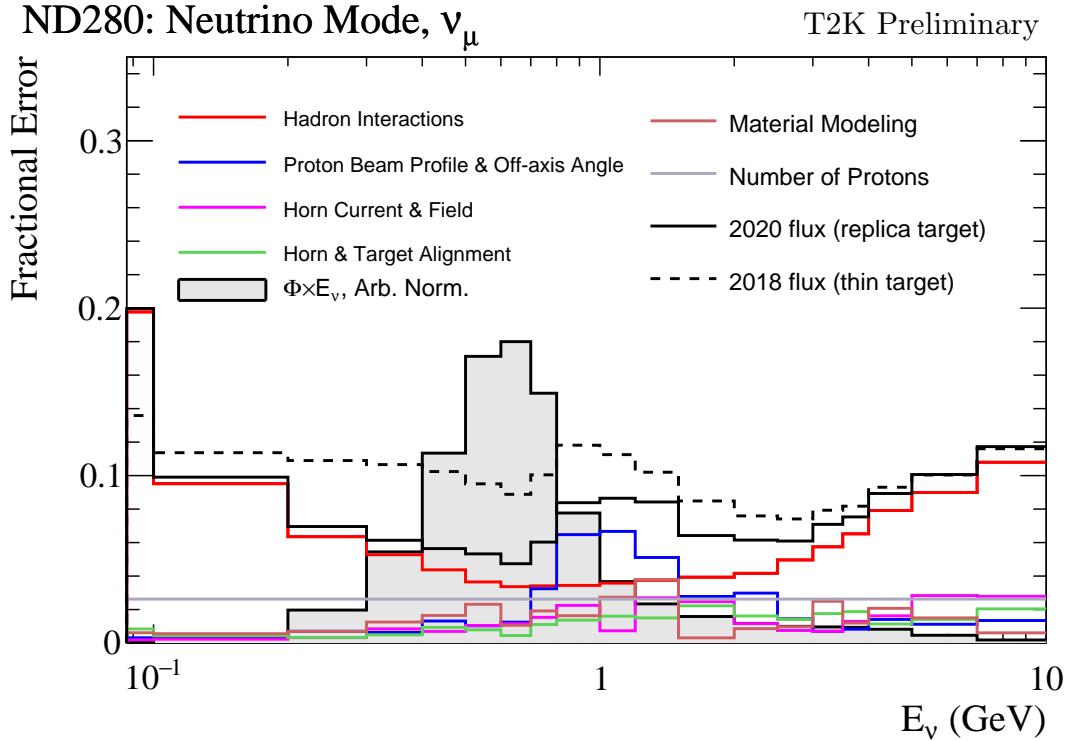
## 6.4 Systematic Uncertainties

The systematic model parameters for this analysis are split into groups, or blocks, depending on their purpose. They consist of flux uncertainties, neutrino-matter interaction systematics, and detector efficiencies. There are also uncertainties on the oscillation parameters which this analysis is not sensitive to, namely  $\Delta m_{12}^2$  and  $\sin^2(\theta_{12})$ . These oscillation parameters uncertainties are taken from the 2020 PDG measurements [75]. As described in chapter 4, each model parameter used within this analysis requires a prior uncertainty. This is provided via separate covariance matrices for each block. The covariance matrices can include prior correlations between parameters within a single block, but the separate treatment means prior correlations can not be included for parameters in different groups. Some parameters in these models have no reasonably motivated uncertainties and are assigned flat priors which do not modify the likelihood penalty. In practice, these flat prior parameters are actually assigned a Gaussian with a very large width to ensure the covariance matrix is positive definite. They are then checked at run time to determine if they contribute to the likelihood. The flux, neutrino interaction, and detector modeling simulations have already been discussed in section 5.1 and section 5.2. The uncertainties invoked within each of these models are described below.

### 6.4.1 Beam Flux

The neutrino beam flux systematics are based upon the uncertainty in the modeling of the components of the beam simulation. This includes the model of hadron productions and reinteractions, the shape, intensity, and alignment of the beam with respect to the target, and the uniformity of the magnetic field produced by the horn, alongside other effects. The uncertainty, as a function of neutrino energy, is illustrated in Figure 6.6 which includes a depiction of the total uncertainty as well as the contribution from individual components. The uncertainty around the peak of the energy distribution ( $E_\nu \sim 0.6\text{GeV}$ ) is

1970 dominated by uncertainties in the beam profile and alignment. Outside of this  
1971 region, uncertainties on hadron production dominate the error.



**Figure 6.6:** The total uncertainty evaluated on the near detector  $\nu_\mu$  flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [197]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [198].

1972 The beam flux uncertainties are described by one hundred parameters. They  
1973 are split between the ND280 and SK detectors and binned by neutrino flavour:  
1974  $\nu_\mu$ ,  $\bar{\nu}_\mu$ ,  $\nu_e$  and  $\bar{\nu}_e$ . The response is then broken down as a function of neutrino  
1975 energy. The bin density in the neutrino energy is the same for the  $\nu_\mu$  in FHC  
1976 and  $\bar{\nu}_\mu$  in RHC beams, and narrows for neutrino energies close to the oscillation  
1977 maximum of  $E_\nu = 0.6\text{GeV}$ . This binning is specified in Table 6.5. All of these  
1978 systematic uncertainties are applied as normalisation parameters with Gaussian  
1979 priors centered at 1.0 and error specified from a covariance matrix provided  
1980 by the T2K beam group [198].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
$\mu$	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
$\mu$	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
$e$	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
$e$	Wrong	0., 2.5, 30.

**Table 6.5:** The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

## 1981 6.4.2 Atmospheric Flux

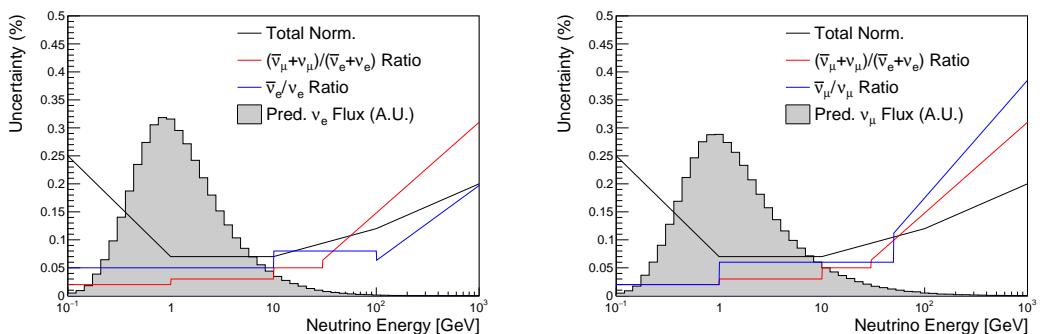
1982 The atmospheric neutrino flux is modeled by the HKKM model [50]. 16 systematic  
 1983 uncertainties are applied to control the normalisation of each neutrino flavour,  
 1984 energy, and direction. They are summarised below:

- 1985 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour  
 1986 is controlled by two independent systematic uncertainties, for  $E_\nu < 1\text{GeV}$   
 1987 and  $E_\nu > 1\text{GeV}$ , respectively. This is driven mostly by hadronic interaction  
 1988 uncertainties for the production of pions and kaons [50]. The strength of  
 1989 the response is dependent upon the neutrino energy. The uncertainty is  
 1990 parameterized following Figure 11 in [50].
- 1991 • **Relative Normalisation:** Uncertainties on the ratio of  $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$   
 1992 are controlled by the difference between the HKKM model [50], FLUKA  
 1993 [53] and Bartol models [49]. Three independent parameters are applied in  
 1994 the energy ranges:  $E_\nu < 1\text{GeV}$ ,  $1\text{GeV} < E_\nu < 10\text{GeV}$ , and  $E_\nu > 10\text{GeV}$ .
- 1995 •  **$\nu/\bar{\nu}$  Normalisation:** The uncertainties in the  $\pi^+/\pi^-$  (and kaon equivalent)  
 1996 production uncertainties in the flux of  $\nu/\bar{\nu}$ . The response is applied using  
 1997 the same methodology as the relative normalisation parameters.
- 1998 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-  
 1999 tematics, the difference between the HKKM, FLUKA, and Bartol model

2000 predictions, as a function of  $\cos(\theta_Z)$ , is used to control the normalisation of  
 2001 events as a function of zenith angle.

- 2002 •  **$K/\pi$  Ratio:** Higher energy neutrinos ( $E_\nu > 10\text{GeV}$ ) mostly originate in  
 2003 kaon decay. Measurements of the ratio of  $K/\pi$  production [199] are used to  
 2004 control the systematic uncertainty of the expected ratio of pion and kaon  
 2005 production.
- 2006 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic  
 2007 field, the flux of primary cosmic rays varies across the same period. The  
 2008 uncertainty is calculated by taking a  $\pm 1$  year variation, equating to a 10%  
 2009 uncertainty for the SK-IV period.
- 2010 • **Atmospheric Density:** The height of the interaction of the primary cosmic  
 2011 rays is dependent upon the atmospheric density. The HKKM assumes the  
 2012 US standard 1976 [150] profile. This systematic controls the uncertainty in  
 2013 that model.

2014 The total uncertainty is dominated by the absolute and relative normalisation  
 2015 parameters. The effect of which is illustrated in Figure 6.7. Generally, the  
 2016 uncertainty is large at low energy, reducing to  $O(10\%)$  around the peak of the  
 2017 flux distribution and then increasing once the neutrino energy exceeds 10GeV.



**Figure 6.7:** The uncertainty evaluated on the atmospheric  $\nu_e$  (left) and  $\nu_\mu$  (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2018      Updates to the HKKM and Bartol models are underway [155] to use a similar  
2019      tuning technique to that used in the beam flux predictions. After those updates,  
2020      it may be possible to include correlations in the hadron production uncertainty  
2021      systematics for beam and atmospheric flux predictions.

### 2022    6.4.3 Neutrino Interaction

2023    Neutrino interactions in the detectors are modeled by NEUT. The two indepen-  
2024    dent oscillation analyses, T2K-only [200] and the SK-only [59], have developed  
2025    separate interaction models. To maximise sensitivity out of this simultaneous  
2026    beam and atmospheric analysis, a correlated interaction model has been defined  
2027    in [201]. Where applicable, correlations allow the systematic uncertainties applied  
2028    to the atmospheric samples to be constrained by near detector neutrino beam  
2029    measurements. This can lead to stronger sensitivity to oscillation parameters  
2030    as compared to an uncorrelated model.

2031    The low-energy T2K systematic model has a more sophisticated treatment  
2032    of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of  
2033    this model have been performed to external data [200]. However, the model  
2034    is not designed for high-energy atmospheric events, like those illustrated in  
2035    Figure 5.11. Therefore the high energy systematic model from the SK-only  
2036    analysis is implemented for the relevant multiGeV, PC, and up- $\mu$  samples. The  
2037    T2K CCQE model is more sophisticated so it has been implemented for all  
2038    samples within this analysis, where separate low-energy and high-energy dials  
2039    have been implemented. The low-energy dials are constrained by the near  
2040    detector measurements and are uncorrelated to their high-energy counterparts.  
2041    The author of this thesis was responsible for implementing and validating the  
2042    combined cross section model as documented in [201, 202].

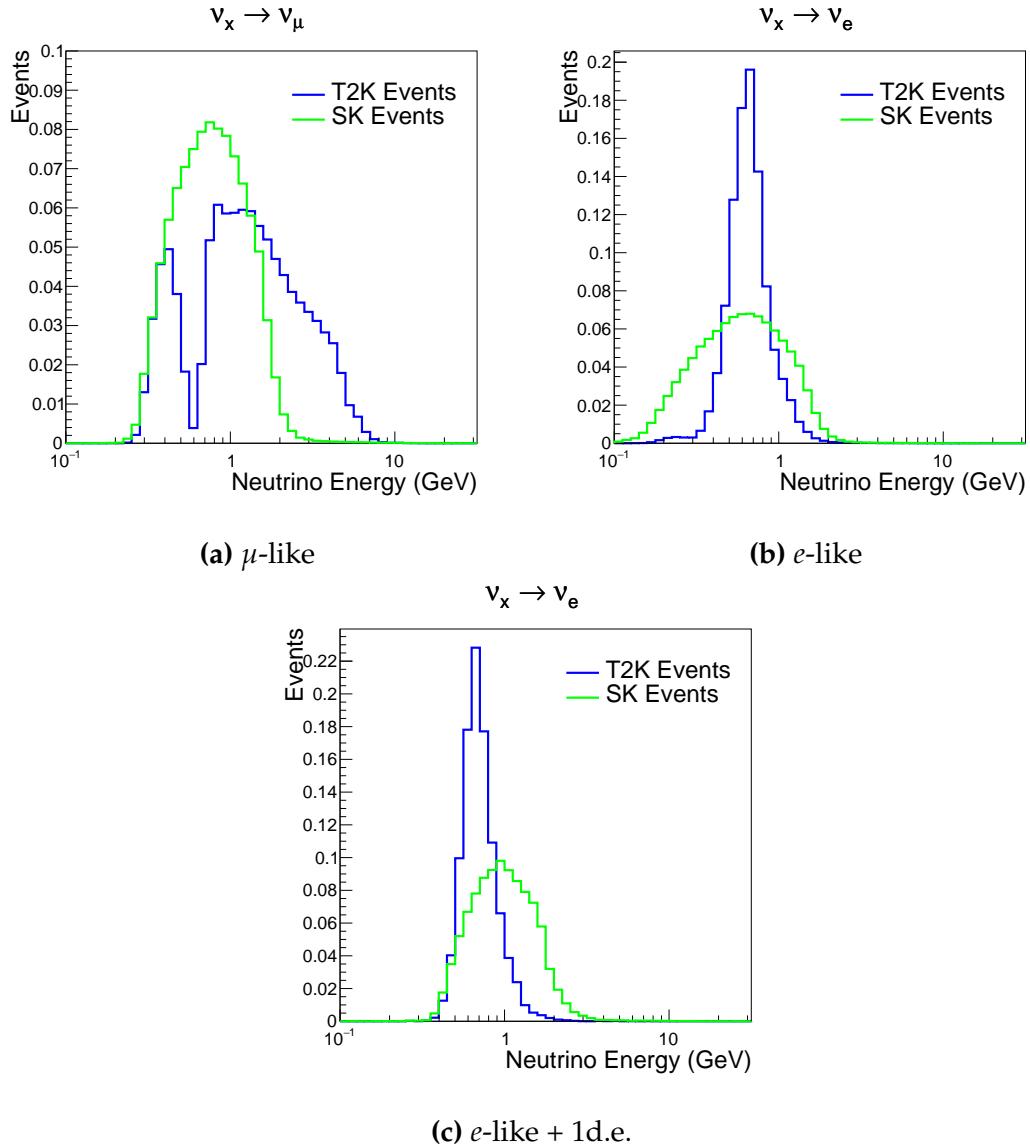
2043    The high energy systematic model includes parameters developed from  
2044    comparisons of Nieves and Rein-Seghal models which affect resonant pion  
2045    producing interactions, comparisons of the GRV98 and CKMT models which  
2046    control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the  $\nu_\tau$  cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.8 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.9 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G.  $\mu$ -like [0,1]de, [FHC,RHC] 1R  $\mu$ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 $\pi^\pm$  interactions. The samples which target CC1 $\pi^\pm$  interactions, (S.G. e-like 0de, S.G.  $\mu$ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.



**Figure 6.8:** The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

The T2K systematic model [200] is applied in a similar methodology to the SK model parameters. It consists of 19 shape parameters and 24 normalisation parameters. Four additional parameters, which model the uncertainty in the binding energy, are applied in a way to shift the momentum of the lepton emitted from a nucleus. This controls the uncertainty specified on the 27MeV binding energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	<b>0.04</b>	<b>0.02</b>	<b>0.83</b>	<b>0.03</b>	<b>0.04</b>	<b>0.01</b>	<b>0.01</b>	<b>0.01</b>	<b>0.00</b>	<b>0.01</b>
RHC 1R e-like	<b>0.62</b>	<b>0.12</b>	<b>0.11</b>	<b>0.01</b>	<b>0.02</b>	<b>0.06</b>	<b>0.01</b>	<b>0.01</b>	<b>0.01</b>	<b>0.04</b>
FHC 1R e-like	<b>0.68</b>	<b>0.12</b>	<b>0.10</b>	<b>0.00</b>	<b>0.02</b>	<b>0.04</b>	<b>0.01</b>	<b>0.00</b>	<b>0.00</b>	<b>0.02</b>
RHC 1R $\mu$ -like	<b>0.62</b>	<b>0.13</b>	<b>0.17</b>	<b>0.02</b>	<b>0.03</b>	<b>0.00</b>	<b>0.02</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
FHC 1R $\mu$ -like	<b>0.62</b>	<b>0.12</b>	<b>0.16</b>	<b>0.02</b>	<b>0.03</b>	<b>0.00</b>	<b>0.03</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
S.G. $\pi^0$ -like	<b>0.05</b>	<b>0.01</b>	<b>0.02</b>	<b>0.00</b>	<b>0.01</b>	<b>0.68</b>	<b>0.06</b>	<b>0.07</b>	<b>0.06</b>	<b>0.04</b>
S.G. $\mu$ -like 2de	<b>0.04</b>	<b>0.01</b>	<b>0.80</b>	<b>0.10</b>	<b>0.04</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
S.G. $\mu$ -like 1de	<b>0.72</b>	<b>0.11</b>	<b>0.12</b>	<b>0.01</b>	<b>0.02</b>	<b>0.00</b>	<b>0.01</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>
S.G. $\mu$ -like 0de	<b>0.68</b>	<b>0.11</b>	<b>0.10</b>	<b>0.01</b>	<b>0.02</b>	<b>0.01</b>	<b>0.05</b>	<b>0.01</b>	<b>0.00</b>	<b>0.02</b>
S.G. e-like 1de	<b>0.05</b>	<b>0.01</b>	<b>0.75</b>	<b>0.10</b>	<b>0.05</b>	<b>0.00</b>	<b>0.01</b>	<b>0.02</b>	<b>0.00</b>	<b>0.01</b>
S.G. e-like 0de	<b>0.73</b>	<b>0.11</b>	<b>0.10</b>	<b>0.01</b>	<b>0.02</b>	<b>0.02</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>	<b>0.00</b>

**Figure 6.9:** The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ( $1\pi^\pm$ ), multi-pion production ( $M\pi$ ), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories:  $\pi^0$  production, resonant charged pion production, multi-pion production, and others.

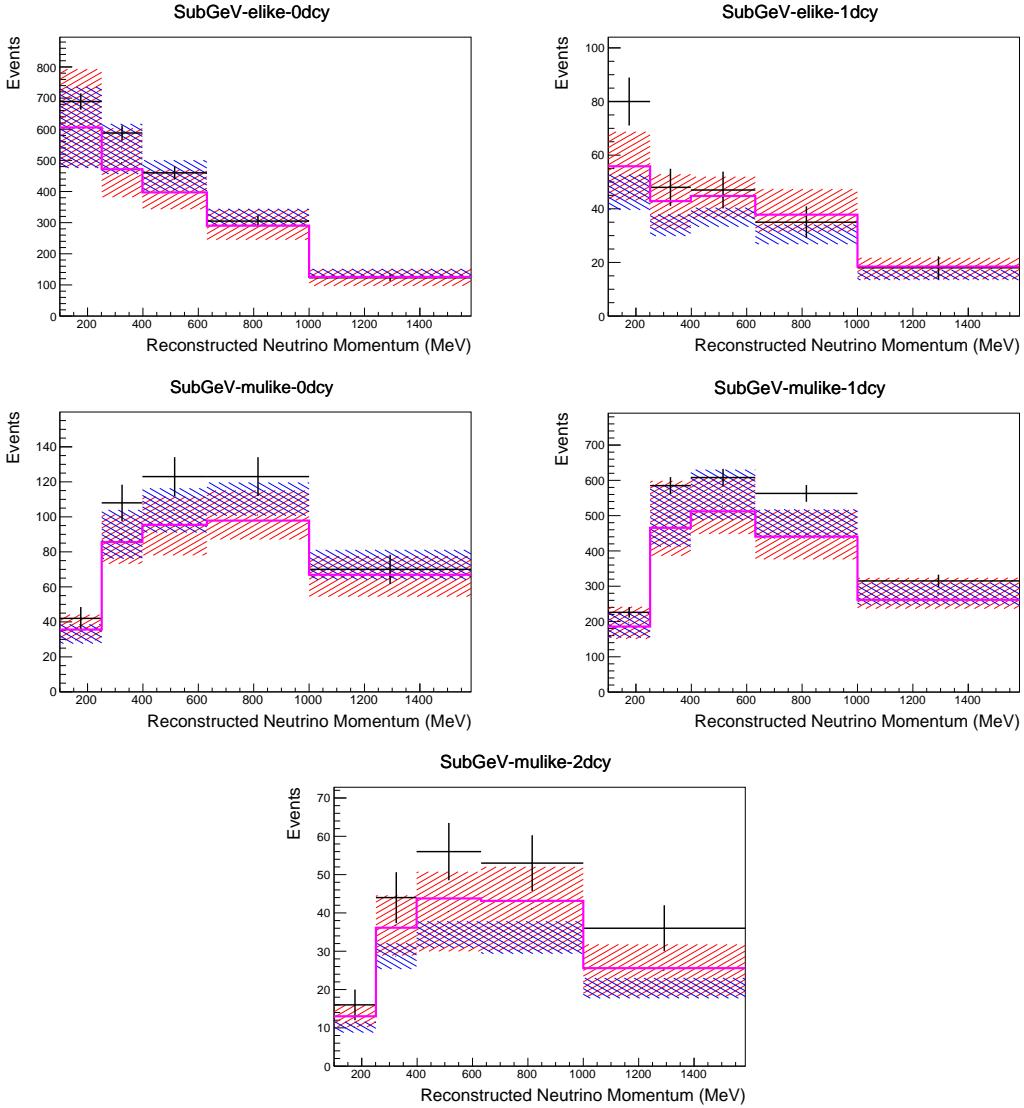
2083 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated  
 2084 uncertainty, or those which have not been fit to external data, are assigned a  
 2085 flat prior which does not affect the penalty term.

2086 On top of the combination of the SK and T2K interaction models, several  
 2087 other parameters have been specifically developed for the joint oscillation anal-  
 2088 ysis. The majority of the atmospheric samples'  $\delta_{CP}$  sensitivity comes from the  
 2089 normalisation of subGeV electron-like events. These are modeled using a spectral  
 2090 function to approximate the nuclear ground state. However, the near detector is  
 2091 not able to constrain the model so an additional systematic is introduced which  
 2092 models applies an alternative Continuous Random Phase Approximation (CRPA)  
 2093 nuclear ground state. This dial approximates the event weights if a CRPA model

2094 had been assumed rather than a spectral function. This dial only applies to  $\nu_e$   
2095 and  $\bar{\nu}_e$  as the near detector does not constraint  $\nu_e$  cross-section measurements.  
2096 It is applied as a shape parameter.

2097 Further additions to the model have been introduced due to the inclusion of  
2098 the subGeV  $\pi^0$  atmospheric sample. This particularly targets charged current  
2099 and neutral current  $\pi^0$  producing interactions to help constrain the systematic  
2100 uncertainties. Therefore, an uncertainty that affects neutral current resonant  
2101  $\pi^0$  production is incorporated into this analysis. Comparisons of NEUT's NC  
2102 resonant pion production predictions have been made to MiniBooNE [203] data  
2103 and a consistent 16% to 21% underprediction is observed [201]. Consequently,  
2104 a conservative 30% normalisation parameter is invoked.

2105 Down-going events are mostly insensitive to oscillation parameters and can  
2106 act similar to the near detector within an accelerator experiment (Details will  
2107 be discussed in chapter 7). This region of phase space can act as a sideband  
2108 and allows the cross-section model and near detector constraint to be studied.  
2109 The distribution of events in this region is calculated using the technique out-  
2110 lined in subsection 4.3.4. The results are illustrated in Figure 6.10. For CCQE-  
2111 targeting samples, the application of the near detector constraint is well within  
2112 the statistical fluctuation of the down-going data such that no significant tension  
2113 is observed between the data and the Monte Carlo prediction after the near  
2114 detector constraint is applied. This is not the case for samples with target CCRES  
2115 interactions. The electron-like data is consistent with the constrained prediction  
2116 at high reconstructed momenta but diverges at lower momentum, whereas the  
2117 muon-like sample is under-predicted throughout the range of momenta. To  
2118 combat this disagreement, an additional cross-section systematic dial, specifically  
2119 designed to inflate the low pion momentum systematics was developed in [201].  
2120 This is a shape parameter implemented through a splined response.



**Figure 6.10:** Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

#### 2121 6.4.4 Near Detector

2122 The systematics applied due to uncertainties arising from the response of the near  
 2123 detector is documented in [130]. The response is described by 574 normalisation  
 2124 parameters binned in the selected sample as well as momentum and angle,

$P_\mu$  and  $\cos(\theta_\mu)$ , of the final-state muon. These are applied via a covariance matrix with each parameter being assigned a Gaussian prior from that covariance matrix. These normalisation parameters are built from underlying systematics, e.g. pion secondary interaction systematics, which are randomly thrown and the variation in each  $P_\mu \times \cos(\theta_\mu)$  bin is determined. Two thousand throws are evaluated and a covariance matrix response is created. This allows significant correlations between FGD1 and FGD2 samples, as well as adjacent  $P_\mu \times \cos(\theta_\mu)$  bins. Statistical uncertainties are accounted for by including fluctuations of each event's weight from a Poisson distribution.

Similar to the cross-section systematics, MaCh3 and BANFF are used to constrain the uncertainty of these systematics through independent validations. Each fitter generates a post-fit covariance matrix which is compared and passed to the far-detector oscillation analysis working group. As the analysis presented within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all three sets of samples and their respective systematics is performed.

#### 6.4.5 Far Detector

Two configurations of the far detector systematic model implementation have been considered. Firstly, the far detector systematic uncertainties for beam and atmospheric samples are taken from their respective analysis inputs, denoted “official inputs” analysis, with no correlations assumed between the beam and atmospheric samples. The beam- and atmospheric-specific inputs are documented in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative detector model has been developed which correlates the response of the SK detector systematics between the beam and atmospheric samples. Here, the distribution of parameters used for applying event cuts (e.g. electron-muon PID separation) is modified within the fit. It follows a similar methodology to the beam far detector systematics implementation but performs a joint fit of the beam and atmospheric data. This alternative implementation is detailed in subsubsection 6.4.5.3.

2154 **6.4.5.1 Beam Samples**

2155 There are 45 systematics which describe the response of the far detector to  
2156 beam events [182], split into 44 normalisation parameters and one energy scale  
2157 systematic. The energy scale systematic is applied as a multiplicative scaling  
2158 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo  
2159 differences in the stopping muon sample in [184] and found to be 2.1%. The  
2160 normalisation parameters are assigned a Gaussian error centred at one with  
2161 width taken from a covariance matrix. A detailed breakdown of the generation  
2162 of the covariance matrix is found in [195]. To build the covariance matrix, a fit  
2163 is performed on atmospheric data which has been selected using beam sample  
2164 selection cuts. These cuts use the variables,  $L^i$ , where the index  $i$  is detailed in  
2165 Table 6.6. Each  $L^i$  is a smear,  $\alpha$ , and shift,  $\beta$  parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i \quad (6.4)$$

2166 Where  $L_j^i$  ( $\bar{L}_j^i$ ) correspond to nominal(varied) PID cut parameters given in  
2167 Table 6.6. The shift and smear parameters are nuisance parameters with no prior  
2168 constraints. They are binned by final-state topology,  $j$ , where the binning is given  
2169 in Table 6.7. The final-state topology binning is because the detector will respond  
2170 differently to events that have one or multiple rings. For example, the detector  
2171 will be able to distinguish single-ring events better than two overlapping ring  
2172 events, resulting in different systematic uncertainty for one-ring events compared  
2173 to two-ring events. This approach is used to allow the cut parameter distributions  
2174 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

**Table 6.6:** List of cut variables that are included within the shift/smear fit documented in [195].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 $\mu$	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 $\mu$ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 $\pi^0$	Only one $\pi^0$ in the final state
1 $\pi^\pm$ or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

**Table 6.7:** Reconstructed event topology categories on which the SK detector systematics [195] are based.

2175        The mis-modeling of  $\pi^0$  events is also considered. If one of the two rings  
 2176        from a  $\pi^0$  event is missed, this will be reconstructed as a CC $\nu_e$ -like event. This  
 2177        is one of the largest systematics hindering the electron neutrino appearance  
 2178        analyses. Consequently, additional systematics have been introduced to con-  
 2179        strain the mis-modeling of  $\pi^0$  events in SK, binned by reconstructed neutrino  
 2180        energy. To evaluate this systematic uncertainty, a set of “hybrid- $\pi^0$ ” samples is  
 2181        constructed. These events are built by overlaying one electron-like ring from  
 2182        the SK atmospheric neutrino samples or decay electron ring from a stopping  
 2183        cosmic ray muon with one simulated photon ring. Both rings are chosen so  
 2184        that momenta and opening angle follow the decay kinematics of NC  $\pi^0$  events  
 2185        from the T2K-MC. Hybrid- $\pi^0$  Monte Carlo samples with both rings from the  
 2186        SK Monte Carlo are produced to compare with the hybrid- $\pi^0$  data samples and  
 2187        the difference in the fraction of events that pass the  $\nu_e$  selection criteria is used  
 2188        to assign the systematic errors. In order to investigate any data to Monte Carlo  
 2189        differences that may originate from either the higher energy ring or lower energy  
 2190        ring, two samples are built; a sample in which the electron constitutes the higher  
 2191        energy ring from the  $\pi^0$  decay (called the primary sample) and another one in  
 2192        which it constitutes the lower energy ring (called the secondary sample). The  
 2193        standard T2K  $\nu_e$  fitQun event selection criteria are used to select events.

2194        Final contributions to the covariance matrix are determined by supplemen-  
 2195        tary uncertainties obtained by comparing stopping muon data to Monte Carlo

2196 prediction, as first introduced in section 5.2. The efficiency of tagging decay  
2197 electrons is estimated by the stopping muon data to Monte Carlo differences by  
2198 comparing the number of one decay electron events to the number of events with  
2199 one or fewer decay electrons. Similarly, the rate at which fake decay electrons  
2200 are reconstructed by `f iTQun` is estimated by comparing the number of two decay  
2201 electron events to the number of events with one or two reconstructed decay  
2202 electrons. The two sources of systematics are added in quadrature weighted by  
2203 the number of events with one true decay electron yielding a 0.2% systematic  
2204 uncertainty. A fiducial volume systematic of  $\pm 2.5\text{cm}$  which corresponds to a 0.5%  
2205 shift in the normalisation of events is also applied. Additional normalisation  
2206 uncertainties based on neutrino flavour and interaction mode are also defined  
2207 in [182, 204, 205].

2208 Two additional sources of uncertainty are included: secondary and photo-  
2209 nuclear interactions. These are estimated by varying the underlying parameters  
2210 are building a distribution of sample event rates. These contributions are then  
2211 added in quadrature to the above covariance matrix.

2212 **DB: Matrix Diagonals from SK Det Matrix**

#### 2213 6.4.5.2 Atmospheric Samples

2214 The detector systematics for atmospheric samples, documented in [85], are split  
2215 into two sub-groups: those which are related to particle identification and ring  
2216 counting systematics, and those which are related to calibration, separation,  
2217 and reduction uncertainties.

2218 The particle identification systematics consist of five parameters. The ring sep-  
2219 aration systematic enforces an anti-correlated response between the single-ring  
2220 and multi-ring samples. This is implemented as a fractional increase/decrease  
2221 in the overall normalisation of each sample, depending on the distance to the  
2222 nearest wall from an event's vertex. The coefficients of the normalisation are  
2223 estimated prior to the fit and depend on the particular atmospheric sample. Two  
2224 electron-muon separation systematics are included within this model which

2225 anti-correlate the response of the electron-like and muon-like samples: one for  
2226 single-ring events and another for multi-ring events.

2227 The multi-ring electron-like separation likelihood, discussed in section 6.1,  
2228 encodes the ability of the detector to separate neutrino from anti-neutrino events.  
2229 Two normalisation parameters vary the relative normalisation of multi-ring  $\nu_e$   
2230 and  $\bar{\nu}_e$  samples whilst keeping a consistent overall event rate.

2231 There are 22 systematics related to calibration measurements, including effects  
2232 from backgrounds, reduction, and showering effects. They are documented in  
2233 [85] and are briefly summarised in Table 6.8. They are applied via normalisation  
2234 parameters, with the separation systematics requiring the conservation of event  
2235 rate across all samples.

#### 2236 6.4.5.3 Correlated Detector Model

2237 A complete uncertainty model of the SK detector would be able to determine  
2238 the systematic shift on the sample spectra for a variation of the underlying  
2239 parameters, e.g. PMT angular acceptance. However, this is computationally  
2240 intensive, requiring Monte Carlo predictions to be made for each plausible  
2241 variation. Consequently, an effective parameter model has been utilised for  
2242 a correlated detector model following from the T2K-only model implementation  
2243 documented in subsubsection 6.4.5.1. It correlates the detector systematics  
2244 between the far-detector beam and subGeV atmospheric samples due to their  
2245 similar energies and interaction types. As there are no equivalent beam samples,  
2246 the multiGeV, multiring, PC, and Up- $\mu$  samples will be subject to the particle  
2247 identification systematics implementation as described in subsubsection 6.4.5.2  
2248 rather than using this correlated detector model. The calibration systematics also  
2249 described in the aforementioned chapter still apply to all atmospheric samples.

2250 The correlated detector model utilises the same smear and shift parameters  
2251 documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,  
2252 the shift and smear parameters are split by visible energy deposited within the  
2253 detector, with binning specified in Table 6.9. This is because atmospheric events

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from $\pi^0$ decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

**Table 6.8:** Sources of systematic errors specified within the grouped into the “calibration” systematics model.

are categorised by subGeV and multiGeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where  $k$  is the visible energy bin.

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

**Table 6.9:** Visible energy binning for which the correlated SK detector systematics are based

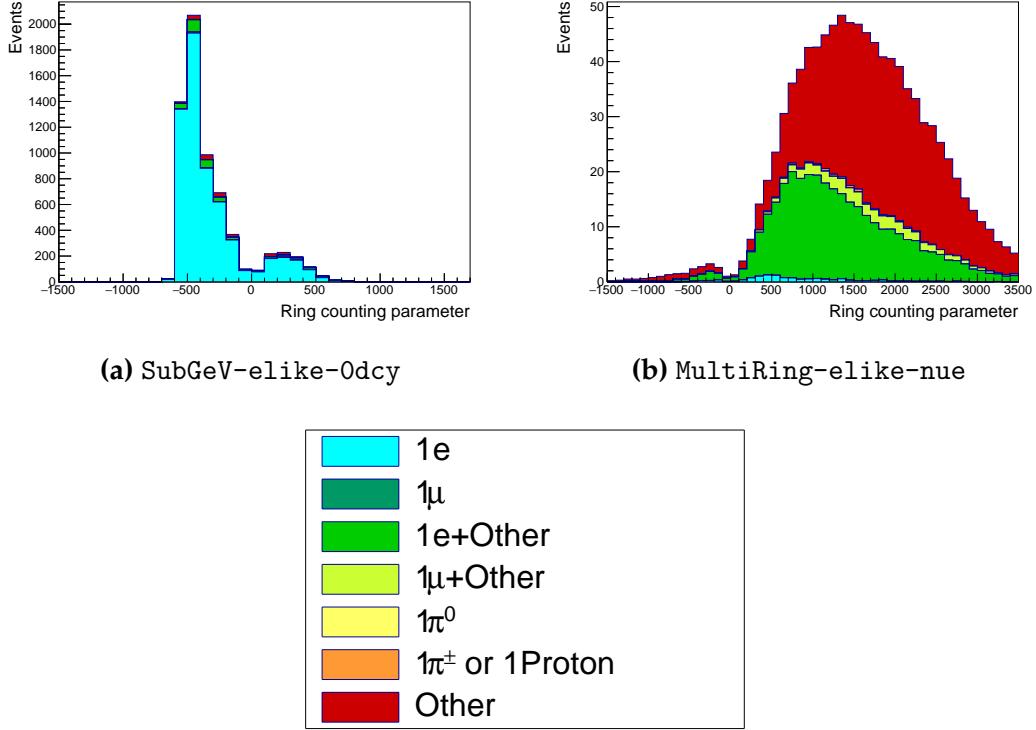
parameter for that event, and then re-applies event selection. This causes event migration, which is a new feature incorporated into the MaCh3 framework which is only achievable due to the event-by-event reweighting scheme.

Particular care has to be taken when varying the ring counting parameter. This is because the number of rings is a finite value (one-ring, two-rings, etc.) which can not be continuously varied through this shift and smear technique. Consequently a continuous ring counting parameter,  $RC_i$ , is calculated for the  $i^{th}$  event, following the definition in [181]: the preferred likelihoods from all considered one-ring ( $L_{1R}$ ) and two-ring ( $L_{2R}$ ) fits are determined. The difference is computed as  $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$ . The ring counting parameter is then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

where  $\text{sgn}(x) = x/|x|$ . This ring counting parameter corresponds to an intermediate likelihood value used within the `fitQun` algorithm to decide the number of rings associated with a particular event. However, fake-ring merging algorithms are applied after this likelihood value is used. Consequently, this ring counting parameter does not always exactly correspond to the number of reconstructed rings. This can be seen in Figure 6.11.

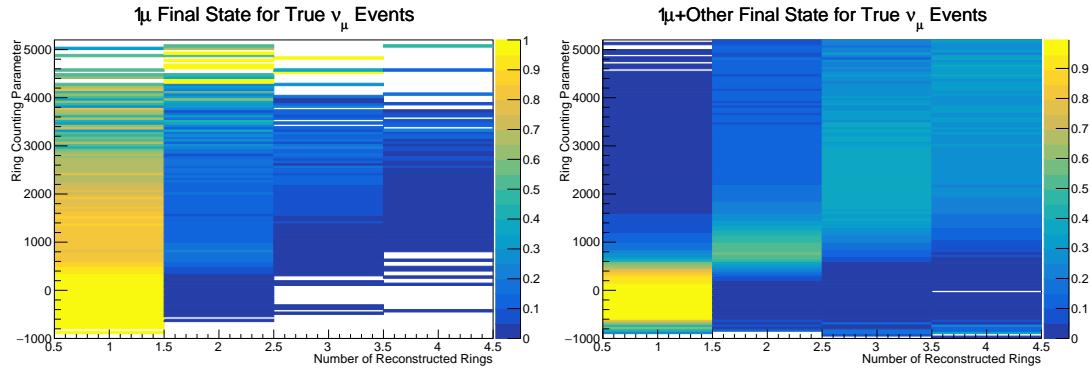
As the `fitQun` algorithm does not provide a likelihood value after the fake-ring algorithms have been applied, the ring counting parameter distribution is correlated to the final number of reconstructed rings through “maps”. These are two-dimensional distributions of the ring counting parameter and the final number of reconstructed rings. An example is illustrated in Figure 6.12. In principle,



**Figure 6.11:** The ring counting parameter as defined in Equation 6.6 for the SubGeV-else-like-0dcy and MultiRing-else-like-nue samples.

the `fitQun` reconstruction algorithm should be re-run after the variation in the ring counting parameter. However, this is not computationally viable. Therefore the “maps” are used as a reweighting template.

The maps are split by final state topology and true neutrino flavour and all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps are row-normalised to represent the probability of  $X$  rings for a given  $RC_i$  value. Prior to the oscillation fit, an event’s nominal weight is calculated as  $W^i(N_{Rings}^i, L_{jk}^i)$ , where  $N_{Rings}^i$  is the reconstructed number of rings for the  $i^{th}$  event and  $W^i(x, y)$  is the bin content in map associated with the  $i^{th}$  event, where  $x$  is the number of rings and  $y$  is the ring counting parameter. Then during the fit, the value of  $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$  is calculated as the event weight for the  $i^{th}$  event. This is the only cut variable that uses a reweighting technique rather than event migration.

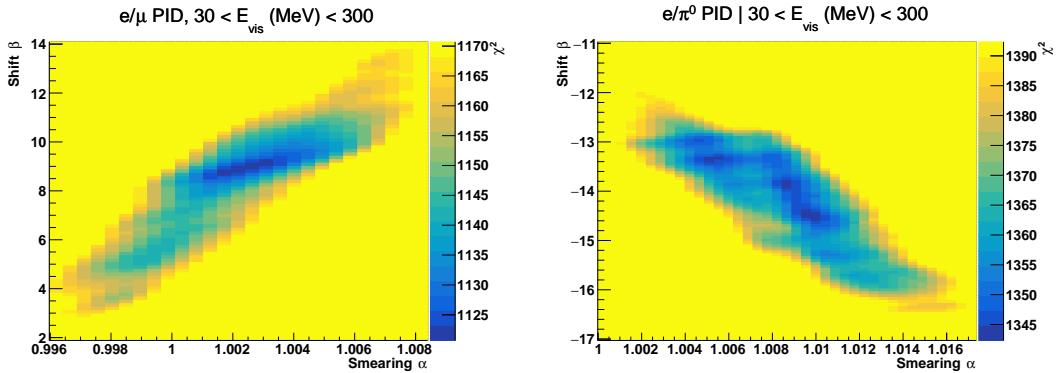


**Figure 6.12:** The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true  $\nu_\mu$  events with only one muon above the Cherenkov threshold in the final state. Right: true  $\nu_\mu$  events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2297     The  $\pi^0$  systematics introduced in subsection 6.4.4 are applied via a covariance  
 2298     matrix. This is not possible in the alternative model as no covariance matrix  
 2299     is used. Thus, the implementation of the  $\pi^0$  systematics has been modified.  
 2300     The inputs from the hybrid  $\pi^0$  sample are included via the use of “ $\chi^2$  maps”,  
 2301     which are two-dimensional histograms in  $\alpha_{jk}^i$  and  $\beta_{jk}^i$  parameters over some  
 2302     range. Illustrative examples of the  $\chi^2$  maps are given in Figure 6.13. Due to  
 2303     their nature, the shift and smear parameters are typically very correlated. A  
 2304     map is produced for each cut parameter given in Table 6.6 and for each visible  
 2305     energy bin given in Table 6.9.

2306     The maps are filled through the  $\chi^2$  comparison of the hybrid  $\pi^0$  Monte Carlo  
 2307     and data in the particle identification parameters documented in Table 6.6. The  
 2308     Monte Carlo distribution is modified by the  $\alpha_{jk}^i$  and  $\beta_{jk}^i$  scaling, whilst cross-  
 2309     section and flux nuisance parameters are thrown from their prior uncertainties.  
 2310     The  $\chi^2$  between the scaled Monte Carlo and data is calculated and the relevant  
 2311     point in the  $\chi^2$  map is filled.

2312     The implementation within this alternative detector model is to add the bin  
 2313     contents of the maps, for the relevant values of the  $\alpha_{jk}^i$  and  $\beta_{jk}^i$  parameters, to  
 2314     the likelihood penalty. Only 1 $\pi^0$  final state topology shift and smear parameters  
 2315     use this prior uncertainty.



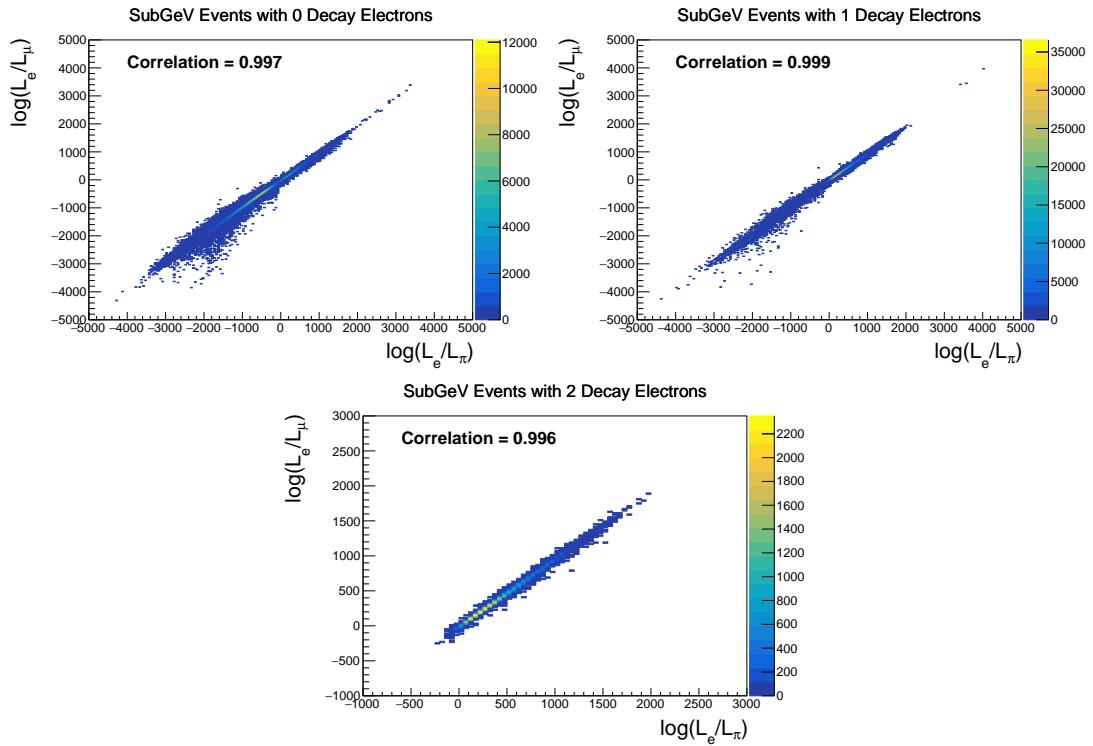
**Figure 6.13:** The  $\chi^2$  between the hybrid- $\pi^0$  Monte Carlo and data samples, as a function of smear ( $\alpha$ ) and shift ( $\beta$ ) parameters, for events which have  $1\pi^0$  final state topology. Left: Electron-muon separation PID parameter for events with  $30 \leq E_{\text{vis}}(\text{MeV}) < 300$ . Right: Electron- $\pi^0$  separation PID parameter for events with  $30 \leq E_{\text{vis}}(\text{MeV}) < 300$ .

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [206] was built in tandem with the T2K-SK working group [182] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular interaction mode, number of tagged decay electrons, and whether the primary particle generated Cherenkov light. They are assigned Gaussian uncertainties with widths described by a covariance matrix. Furthermore, the secondary interaction and photo-nuclear effects need to be accounted for in this detector model using a different implementation than that in subsubsection 6.4.5.1. This was done by including a shape parameter for each of the secondary interactions and the photo-nuclear systematic parameters.

There are a total of 224  $\alpha_{jk}^i$  and  $\beta_{jk}^i$  parameters, of which 32 have prior constraints from the hybrid  $\pi^0$  samples.

One final complexity of this correlated detector model is that the two sets of samples, beam and subGeV atmospheric, use slightly different parameters to distinguish electron and muon-like events. The T2K samples use the value of  $\log(L_e/L_\mu)$  whereas the atmospheric samples use the value of  $\log(L_e/L_\pi)$ , where  $L_X$  is the likelihood for hypothesis X. This is because the T2K fits use single-ring fiTQun fitting techniques, whereas multi-ring fits are applied to the atmospheric samples where only the electron and pion hypothesis are considered.

2336 The correlation between the two likelihood ratios is illustrated in Figure 6.14. As  
 2337 discussed in section 5.2, the pion hypothesis is a very good approximation of the  
 2338 muon hypothesis due to their similar mass. Consequently, using the same shift  
 2339 and smear parameters correlated between the beam and subGeV atmospheric  
 2340 samples is deemed a good approximation.



**Figure 6.14:** The distribution of  $\log(L_e/L_\mu)$  compared to  $\log(L_e/L_\pi)$  for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

# 7

2341

2342

## Oscillation Probability Calculation

2343 It is important to understand how and where the sensitivity to the oscillation  
2344 parameters comes from for both atmospheric and beam samples. An overview  
2345 of how these samples observe changes in  $\delta_{CP}$ ,  $\Delta m^2_{23}$ , and  $\sin^2(\theta_{23})$  affect  
2346 these samples is given in section 7.1. This section also explains the additional  
2347 complexities involved when performing an atmospheric neutrino analysis as  
2348 compared to a beam-only analysis.

2349 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ( $E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$ ) can artificially inflate the sensitivity to  $\delta_{CP}$   
2350 due to the quickly varying oscillation probability in this region. Therefore, a  
2351 “sub-sampling” approach has been developed to reduce these biases ensuring  
2352 accurate and reliable sensitivity measurements. This technique ensures that small-  
2353 scale unresolvable features of the oscillation probability have been averaged over  
2354 whilst the large-scale features in the oscillation probability are unaffected. The  
2355 documentation and validation of this technique are found in section 7.2. The  
2356 oscillation probability calculation is computationally intensive due to the large  
2357 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-  
2358 mentation choice made within the fitting framework, as detailed in section 7.3,  
2359 ensures that the analysis can be done in a timely manner.  
2360

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.4. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrinos oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation.

This implementation of an external technique is illustrated in section 7.5.

## 7.1 Overview

**DB: Should this be moved into an earlier chapter? The selections chapter references the matter resonance which has not yet been explained at that point**

The analysis presented within this thesis focuses on the determination of oscillation parameters from atmospheric and beam neutrinos. Whilst subject to the same oscillation formalism, the way in which the two samples have sensitivity to the different oscillation parameters differs significantly.

Atmospheric neutrinos have a varying baseline, or “path length”,  $L$ , such that the distance each neutrino travels before interacting is dependent upon the zenith angle,  $\theta_Z$ . As primary cosmic rays can interact anywhere between the Earth’s surface and  $\sim 50\text{km}$  above that, the height,  $h$ , in the atmosphere at which the neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (7.1)$$

2386 Where  $R_E = 6,371\text{km}$  is the Earth's radius. Consequently, the oscillation  
 2387 probability is dependent upon two parameters,  $\cos(\theta_Z)$  and  $E_\nu$ .

2388 The oscillation probability used within this analysis is based on [21]. The  
 2389 neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of  
 2390 constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (7.2)$$

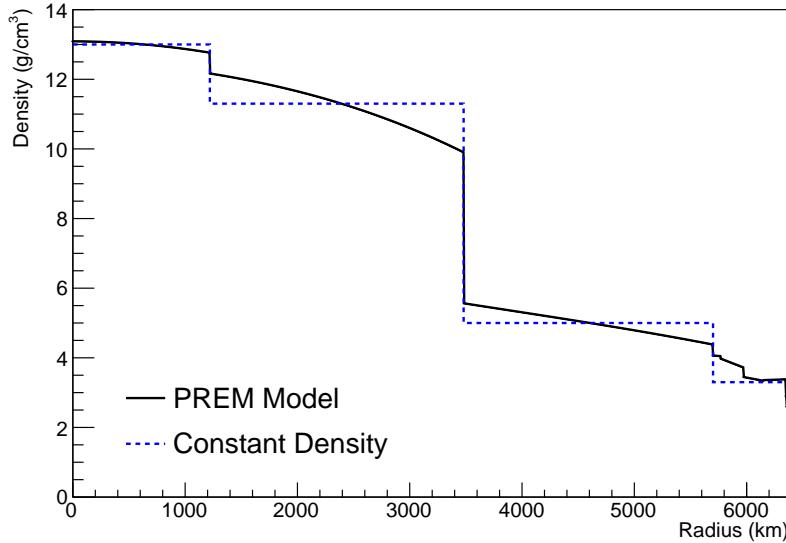
2391 where  $m_j^2$  is the square of the  $j^{th}$  vacuum eigenstate mass,  $E_\nu$  is the neutrino  
 2392 energy,  $G_F$  is Fermi's constant,  $N_e$  is the electron number density and  $U$  is the  
 2393 PMNS matrix. The transformation  $N_e \rightarrow -N_e$  and  $\delta_{CP} \rightarrow -\delta_{CP}$  is applied  
 2394 for antineutrino propagation. Thus, a model of the Earth's density is required  
 2395 for neutrino propagation. Following the official SK-only methodology [207],  
 2396 this analysis uses the Preliminary Reference Earth Model (PREM) [208] which  
 2397 provides piecewise cubic polynomials as a function of the Earth's radius. This  
 2398 density profile is illustrated in Figure 7.1. As the propagator requires layers of  
 2399 constant density, the SK methodology approximates the PREM model by using  
 2400 four layers of constant density [207], detailed in Table 7.1.

Layer	Outer Radius [km]	Density [g/cm <sup>3</sup> ]	Chemical composition (Z/A)
Inner Core	1220	13	$0.468 \pm 0.029$
Outer Core	3480	11.3	$0.468 \pm 0.029$
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

**Table 7.1:** Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [208].

2401 The atmospheric neutrino oscillation probabilities can be presented as two di-  
 2402 mensional “oscillograms” as illustrated in Figure 7.2. The distinct discontinuities,  
 2403 as a function of  $\cos(\theta_Z)$ , are due to the discontinuous density in the PREM model.

2404 Atmospheric neutrinos have sensitivity to  $\delta_{CP}$  through the overall event  
 2405 rate. Figure 7.3 illustrates the difference in oscillation probability between CP-  
 2406 conserving ( $\delta_{CP} = 0$ ) and a CP-violating ( $\delta_{CP} = -1.601$ ) value taken from

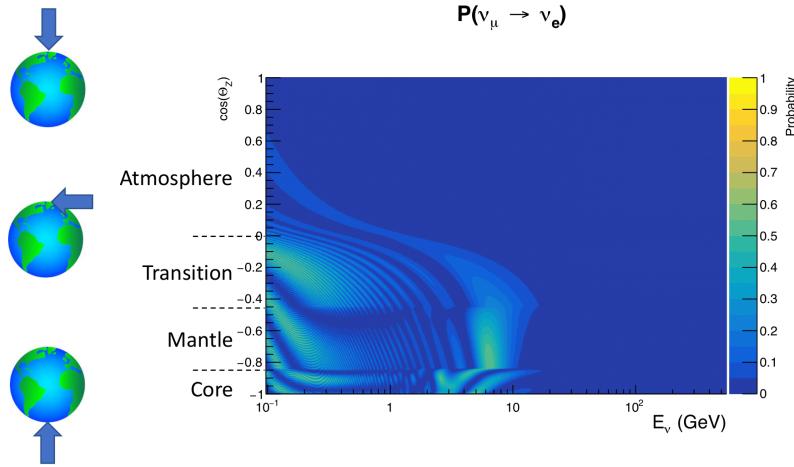


**Figure 7.1:** The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

2407 Asimov A oscillation parameter set (Table 2.2). The result is a complicated  
 2408 oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos.  
 2409 The detector does not have sufficient resolution to resolve these individual  
 2410 patterns so the sensitivity to  $\delta_{CP}$  for atmospheric neutrinos comes via the overall  
 2411 normalisation of these events.

2412 The presence of matter means that the effect  $\delta_{CP}$  has on the oscillation  
 2413 probability is not equal between neutrinos and antineutrinos. Furthermore,  
 2414 the interaction cross-section for neutrinos is larger than antineutrinos so the  
 2415 two effects have to be disentangled. These effects are further convoluted by  
 2416 detector efficiencies as SK cannot distinguish neutrinos and antineutrinos well.  
 2417 Furthermore, the sample selections discussed in section 6.1 have difference  
 2418 efficiencies for neutrino-antineutrino selections. All of these effects lead to  
 2419 a difference in the number of neutrinos detected compared to antineutrinos.  
 2420 This changes how the  $\delta_{CP}$  normalisation term is observed, resulting in a very  
 2421 complex sensitivity to  $\delta_{CP}$ .

2422 The vacuum and matter oscillation probabilities for  $P(\nu_e \rightarrow \nu_e)$  and  $P(\bar{\nu}_e \rightarrow$



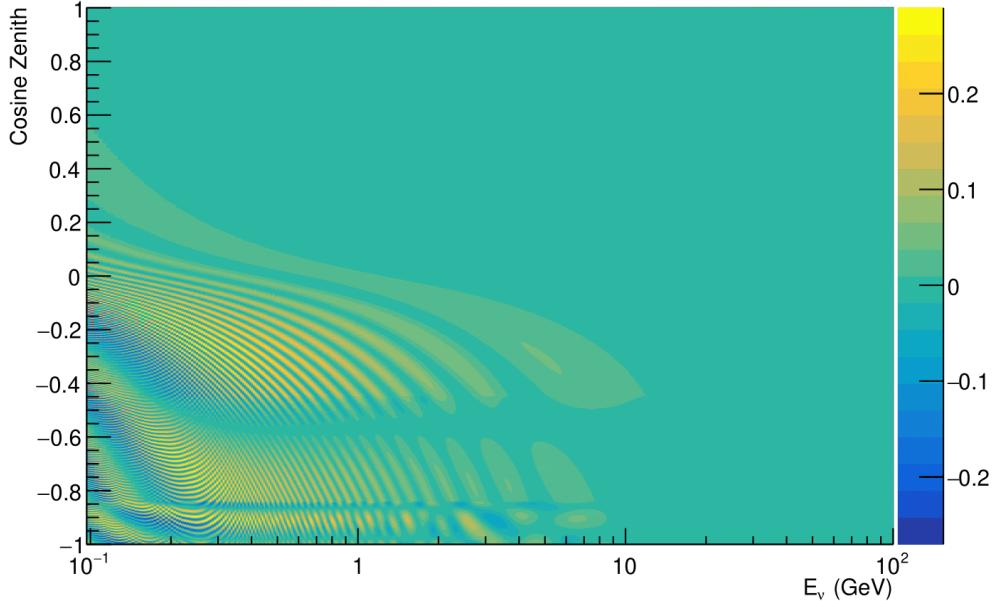
**Figure 7.2:** An “oscillogram” that depicts the  $P(\nu_\mu \rightarrow \nu_e)$  oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that  $\cos(\theta_Z) = 1.0$  represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

$\bar{\nu}_e$ ) are presented in Figure 7.4, where the PREM model has been assumed.  
 2423 The oscillation probability for both neutrinos and antineutrinos is affected in  
 2424 the presence of matter. However, the resonance effects around  $O(5)\text{GeV}$  only  
 2425 occur for neutrinos in normal mass hierarchy and antineutrinos in inverse mass  
 2426 hierarchy. The exact position and amplitude of the resonance depend on  $\sin^2(\theta_{23})$   
 2427 , further increasing the atmospheric neutrinos’ sensitivity to the parameter.  
 2428

$\bar{\nu}_e$ ) are presented in Figure 7.4, where the PREM model has been assumed.  
 2423 The oscillation probability for both neutrinos and antineutrinos is affected in  
 2424 the presence of matter. However, the resonance effects around  $O(5)\text{GeV}$  only  
 2425 occur for neutrinos in normal mass hierarchy and antineutrinos in inverse mass  
 2426 hierarchy. The exact position and amplitude of the resonance depend on  $\sin^2(\theta_{23})$   
 2427 , further increasing the atmospheric neutrinos’ sensitivity to the parameter.  
 2428 As the T2K beam flux is centered at the first oscillation maximum ( $E_\nu =$   
 2429  $0.6\text{GeV}$ ), the sensitivity to  $\delta_{CP}$  is predominantly observed as a change in the  
 2430 event-rate of e-like samples in  $\nu/\bar{\nu}$  modes. Figure 7.5 illustrates the  $P(\nu_\mu \rightarrow \nu_e)$   
 2431 oscillation probability for a range of  $\delta_{CP}$  values. A circular modulation of the  
 2432 first oscillation peak (in both magnitude and position) is observed when varying  
 2433 throughout the allowable values of  $\delta_{CP}$ . The CP-conserving values of  $\delta_{CP} = 0, \pi$   
 2434 have a lower(higher) oscillation maximum than the CP-violating values of  $\delta_{CP} =$   
 2435  $-\pi/2(\delta_{CP} = \pi/2)$ . A sub-dominant shift in the energy of the oscillation peak is  
 2436 also present, which aids in separating the two CP-conserving values of  $\delta_{CP}$ .  
 2437

$\bar{\nu}_e$ ) are presented in Figure 7.4, where the PREM model has been assumed.  
 2438 T2K’s sensitivity to  $\sin^2(\theta_{23})$  and  $\Delta m_{23}^2$  is observed as a shape-based variation

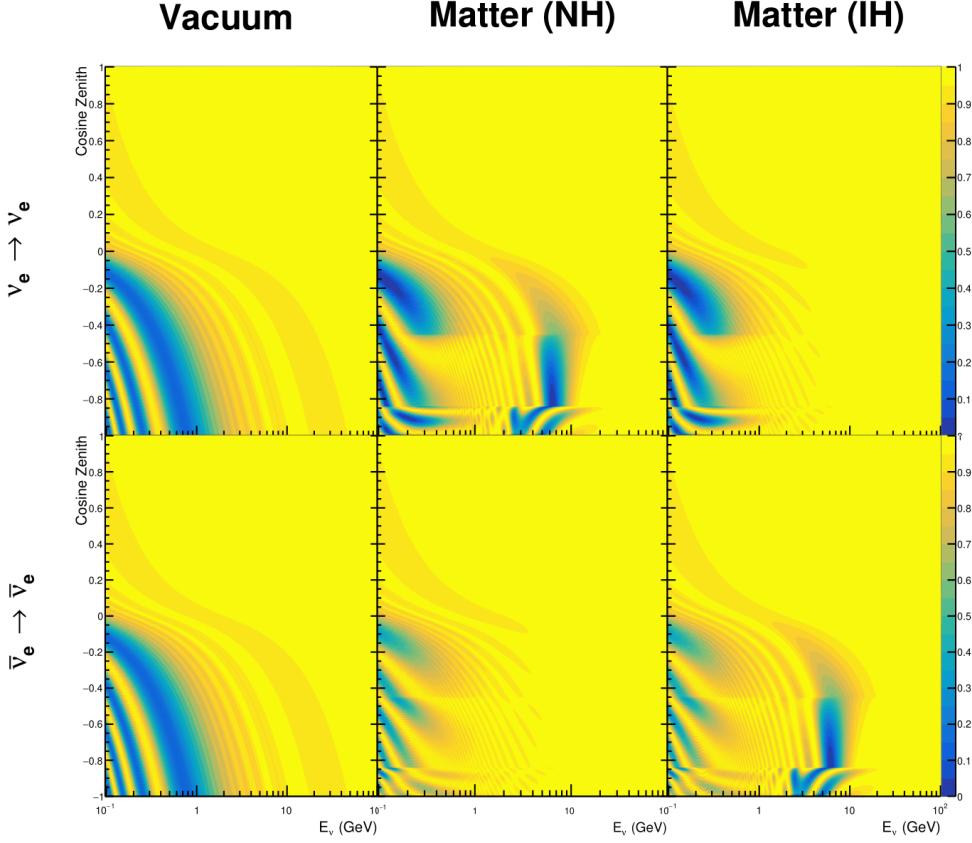
$$\mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = -1.601) - \mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = 0.)$$



**Figure 7.3:** The effect of  $\delta_{CP}$  for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the  $P(\nu_\mu \rightarrow \nu_e)$  oscillation probability for a CP conserving ( $\delta_{CP} = 0.0$ ) and a CP violating ( $\delta_{CP} = -1.601$ ) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

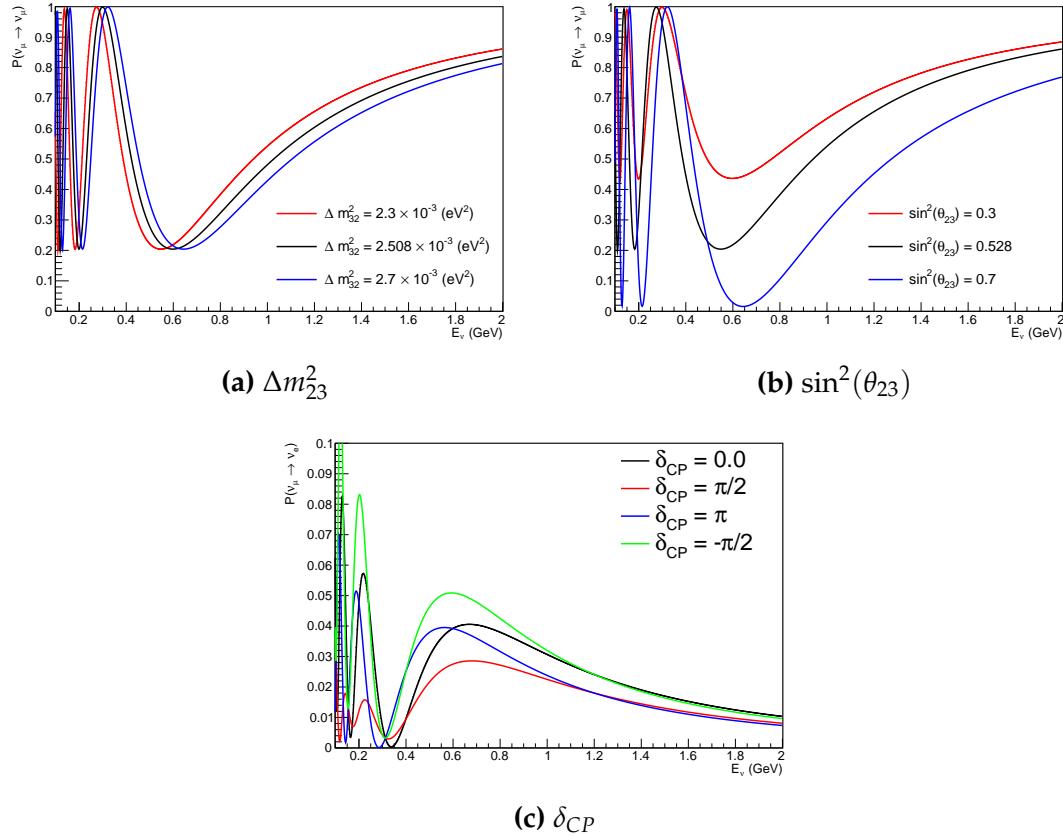
of the muon-like samples, as illustrated in Figure 7.5. The value of  $\Delta m_{32}^2$  laterally shifts the position of the oscillation dip (around  $E_\nu \sim 0.6\text{GeV}$ ) in the  $P(\nu_\mu \rightarrow \nu_\mu)$  oscillation probability. A variation of  $\sin^2(\theta_{23})$  is predominantly observed as a vertical shift of the oscillation dip with second-order horizontal shifts being due to matter effects. The beam neutrinos have limited sensitivity to matter effects due to the relatively shorter baseline as well as the Earth's mantle being a relatively low-density material (as compared to the Earth's core). For some values of  $\delta_{CP}$ , the degeneracy in the number of e-like events allows the mass hierarchy to be broken. This leads to a  $\delta_{CP}$ -dependent mass hierarchy sensitivity which can be seen in Figure 7.6.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations which reduce the number of oscillation probability calculations



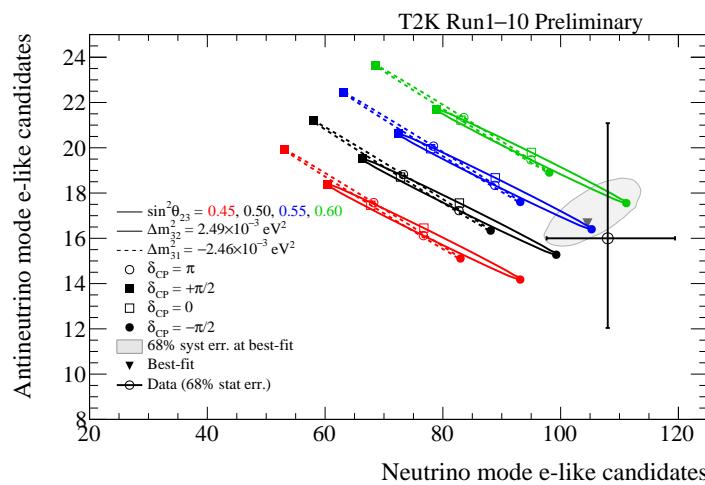
**Figure 7.4:** An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the  $P(\nu_e \rightarrow \nu_e)$  oscillation probability and the bottom row illustrates the  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at  $E_\nu \sim 5\text{GeV}$  can be seen in the  $P(\nu_e \rightarrow \nu_e)$  for normal mass hierarchy and  $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$  for inverted hierarchy.

that need to be made should be applied. The  $\nu_e \rightarrow \nu_{e,\mu,\tau}$  (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the  $\nu_e/\bar{\nu}_e$  fluxes are approximately two orders of magnitude smaller than the corresponding  $\nu_\mu/\bar{\nu}_\mu$  flux. Furthermore, as the peak neutrino energy of the beam is well below the threshold for charged current tau production ( $E_\nu = 3.5\text{GeV}$  [58], only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very



**Figure 7.5:** The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

2459 small due to the short baseline. Whilst these approximations have been made for  
 2460 the beam neutrinos, the atmospheric flux of  $\nu_e$  is of the same order of magnitude  
 2461 as the  $\nu_\mu$  flux and the energy distribution of atmospheric neutrinos extends well  
 2462 above the tau production threshold.



**Figure 7.6:** The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of  $\Delta m_{23}^2$  are given.

## 2463 7.2 Treatment of Fast Oscillations

2464 As shown in Figure 7.7, atmospheric neutrino oscillations have a significantly  
 2465 more complex structure for upgoing neutrinos with energy below 1GeV. This is  
 2466 because the  $L/E$  dependence of the oscillation probability in this region induces  
 2467 rapid variations for small changes in  $L$  or  $E$ . As discussed in section 7.1, this  
 2468 is also the region in which atmospheric neutrinos have sensitivity to  $\delta_{CP}$ . In  
 2469 practice, the direction of the neutrino is inferred from the direction of the final  
 2470 state particles traveling in the detector, which can be poor for low-energy neutrino  
 2471 interactions. This creates a distinct difference from the beam neutrinos where  
 2472 the position of the source is very precisely known.

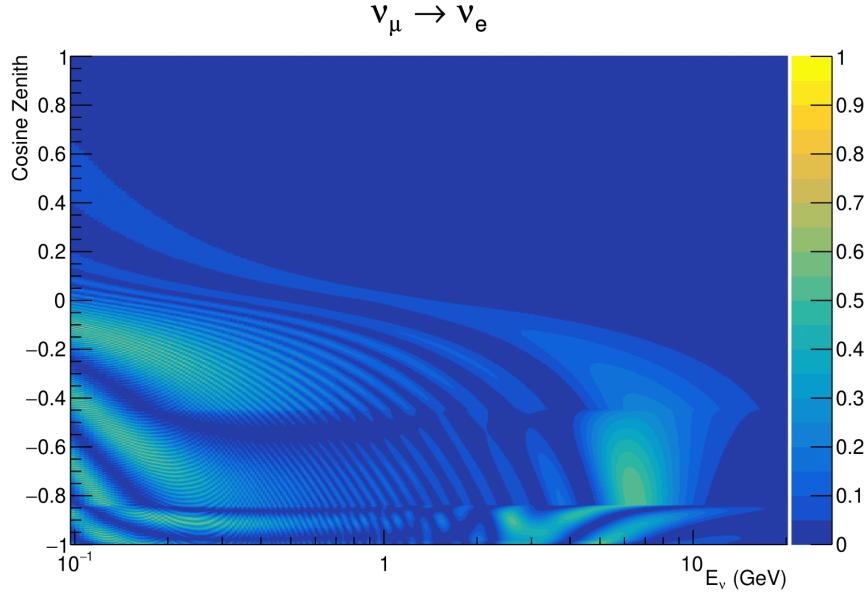
2473 As a consequence of the unresolvable structure, an event rate consistent with  
 2474 the averaged oscillation probability is observed in the subGeV upgoing region.  
 2475 This creates a computational problem: A significantly large amount of Monte  
 2476 Carlo statistics would be required to accurately predict the number of events  
 2477 if Monte Carlo averaging was the only technique used. This section describes  
 2478 the ‘sub-sampling’ approach developed for this analysis and compares it to the  
 2479 methodology used within the SK-only analysis.

2480 The official SK-only analysis uses the osc3++ oscillation parameter fitter  
 2481 [207]. To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’  
 2482 technique. For a given Monte Carlo neutrino event, the nearest twenty Monte  
 2483 Carlo neighbours in reconstructed lepton momentum and zenith angle are  
 2484 found and a distribution of their neutrino energies is built. The RMS,  $\sigma$ , of  
 2485 this distribution is then used to compute an average oscillation probability for  
 2486 the given neutrino Monte Carlo event.

2487 For the  $i^{th}$  event, the oscillation weight is calculated as

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1, -0.5, 0.5, 1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.3)$$

2488 where  $P(E, L)$  is the oscillation probability calculation for neutrino energy  
 2489  $E$  and path length  $L$  and the two path lengths,  $\bar{L}_i$  and  $L_\beta$  are discussed below.



**Figure 7.7:** The oscillation probability  $P(\nu_\mu \rightarrow \nu_e)$ , given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2490 All of the oscillation probability calculations are performed with a fixed zenith  
 2491 angle such that the same density profile is used.

2492 The uncertainty in the production height is controlled by using an “average”  
 2493 production height,  $\bar{L}_i$ , which represents the average path length computed using  
 2494 twenty production heights taken from the Honda flux model’s prediction [52].  
 2495 For a given event, the production heights are sampled in steps of 5% of their  
 2496 cumulative distribution function.  $L_\beta$  values are similarly calculated but instead  
 2497 use different combinations of four production heights,

$$\begin{aligned}
 L_{-1.0} &= \frac{1}{4}L(45, 50, 55, 60), \\
 L_{-0.5} &= \frac{1}{4}L(35, 40, 65, 70), \\
 L_{+0.5} &= \frac{1}{4}L(25, 30, 75, 68), \\
 L_{+1.0} &= \frac{1}{4}L(15, 20, 85, 89).
 \end{aligned} \tag{7.4}$$

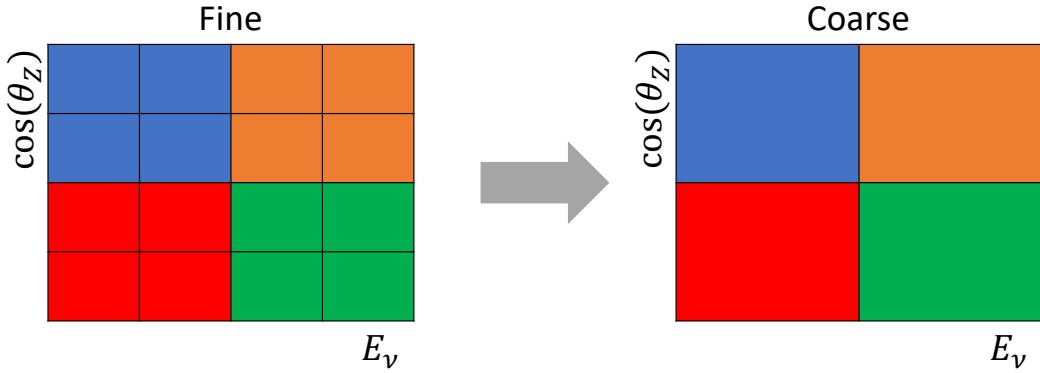
2498 This averaging technique works because of the inference between the zenith  
 2499 angle and the reconstructed direction of final state particles in the detector. For

2500 low-energy neutrinos, where the resolution of the true neutrino direction is poor,  
2501  $\sigma_i$  will be large, resulting in significant averaging effects. Contrary to this, the  
2502 inferred direction of high-energy neutrinos will be much closer to the true value,  
2503 meaning that  $\sigma_i$  will be smaller, culminating in small averaging effects.

2504 In practice, these calculations are performed prior to the fit as only oscillation  
2505 parameters at fixed points are considered. The MCMC technique used in this  
2506 thesis requires oscillation probabilities to be evaluated at arbitrary parameter  
2507 values, not known *a priori*. Calculating the five oscillation probabilities per  
2508 event required by the SK technique is computationally infeasible, so a differencct  
2509 averaging technique is used. However, the concept of the averaging technique  
2510 can be taken from it.

2511 To perform a similar averaging as the SK analysis, a sub-sampling approach  
2512 using binned oscillograms has been devised. A coarsly binned oscillogram is  
2513 defined in  $\cos(\theta_Z)$  and  $E_\nu$ . For a given set of oscillation parameters, a single  
2514 oscillation probability will be assigned to each coarse bin. This value will then  
2515 apply to all Monte Carlo events which fall into that bin. To assign these oscillation  
2516 probabilities, the probability is calculated at  $N \times N$  points on a grid within a  
2517 particular bin. This ensemble of oscillation probabilities is averaged to define  
2518 the coarse bin's oscillation probability, assuming a flat prior in  $E_\nu$  and  $\cos(\theta_Z)$ .  
2519 Figure 7.8 illustrates the  $N = 2$  example where the assigned value to a coarse  
2520 bin is the average of the four fine bins which fall in that coarse bin. Whilst the  
2521 coarse bin edges do not have to be linear on either axis, the sub-division of the  
2522 fine bins is linear over the range of a coarse bin.

2523 The coarse binning is defined with  $67 \times 52$  bins in true neutrino energy  $\times$   
2524 cosine zenith. It is picked to be identical to that provided in [209]. In general, the  
2525 binning is logarithmically spaced in neutrino energy but has some hand-picked  
2526 bin edges around the matter resonance to smoothly increased the bin density.  
2527 This is to avoid smearing this region which can be well sampled by the Monte  
2528 Carlo. The cosine zenith binning is approximately linearly spaced across the



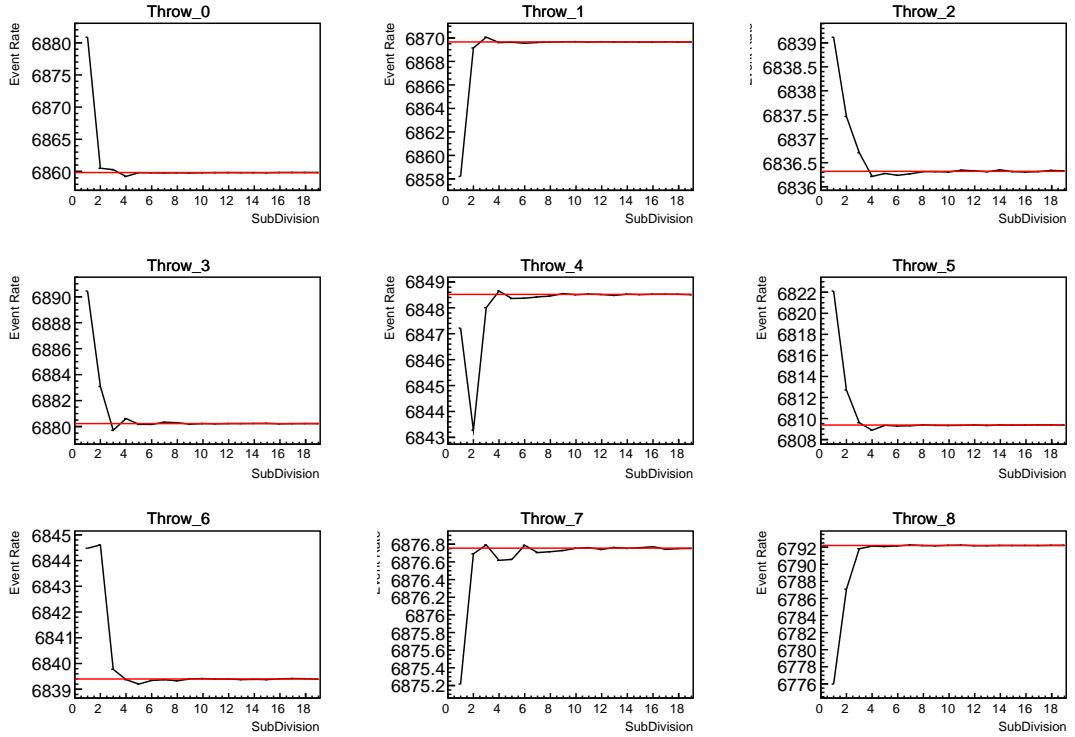
**Figure 7.8:** Illustration of the averaging procedure for  $N = 2$ . The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

allowable range but the values of layer transitions are hit precisely:  $-0.8376$  (core-mantle) and  $-0.4464$  (mantle/transition zone). Bins are spread further apart for downgoing events as this is a region unaffected by the fast oscillation wavelengths and reduces the total number of calculations required to perform the calculation.

The choice of  $N$  is justified based on two studies. Firstly, the variation of event rates of each sample is studied as a function of  $N$ . For a given set of oscillation parameters thrown from the PDG prior constraints (detailed in Table 2.1), the oscillation probabilities are calculated using a given value of  $N$ . Each sample is re-weighted and the event rate is stored. The value of  $N$  is scanned from 1, which corresponds to no averaging, to 19, which corresponds to the largest computationally viable subdivision binning. The event rate of each sample at large  $N$  is expected to converge to a stationary value due to the fine binning fully sampling the small-scale structure. Figure 7.9 illustrates this behaviour for the SubGeV\_elike\_0dcy sample for 9 different throws of the oscillation parameters.

Denoting the event rate for one sample for a given throw  $t$  at each  $N$  by  $\lambda_t^N$ , the average over all considered  $N$  values ( $\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$ ) is computed. The variance in the event rate at each  $N$  is then calculated as

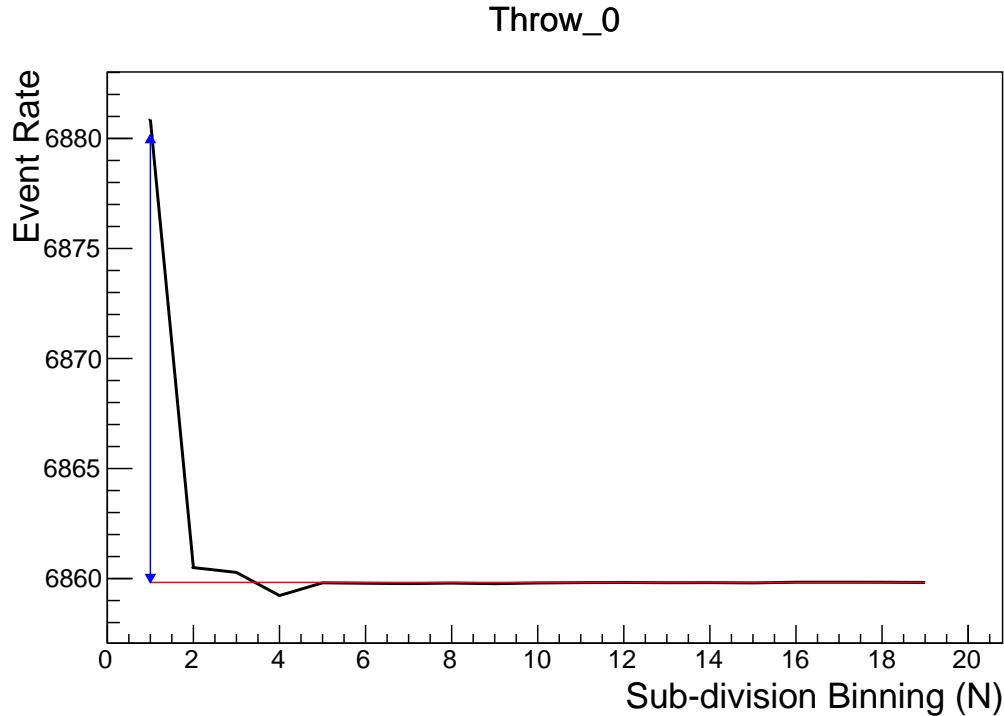
$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t)^2 - \left[ \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t) \right]^2. \quad (7.5)$$



**Figure 7.9:** Event rate of the SubGeV\_elike\_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2546     In practice the following procedure is undertaken. For a particular throw,  
 2547     the difference between the event rate at a particular choice of  $N$  and the mean  
 2548     of the distribution is calculated. This is illustrated in Figure 7.10. This value  
 2549     is then calculated for all the 2000 throws, generating a distribution of  $\lambda_t^N - \bar{\lambda}_t$ .  
 2550     This is repeated for each of the values of  $N$  considered within this study. The  
 2551     distributions of this value, for  $N = \{1, 5\}$ , are given in Figure 7.11. As expected,  
 2552     the distribution gets narrower and tends towards zero for the higher values of  $N$ .

2553     The aim of the study is to find the lowest value of  $N$  such that this variance  
 2554     is below 0.001. This utilises the width of the distributions given in Figure 7.11.  
 2555     This is the typical threshold used by T2K fitters to validate systematic imple-  
 2556     mentation so has been set as the same criteria. The results of this study for  
 2557     each atmospheric sample used within this thesis are illustrated in Figure 7.12 for  
 2558     2000 throws of the oscillation parameters. As can be seen, the variance is below



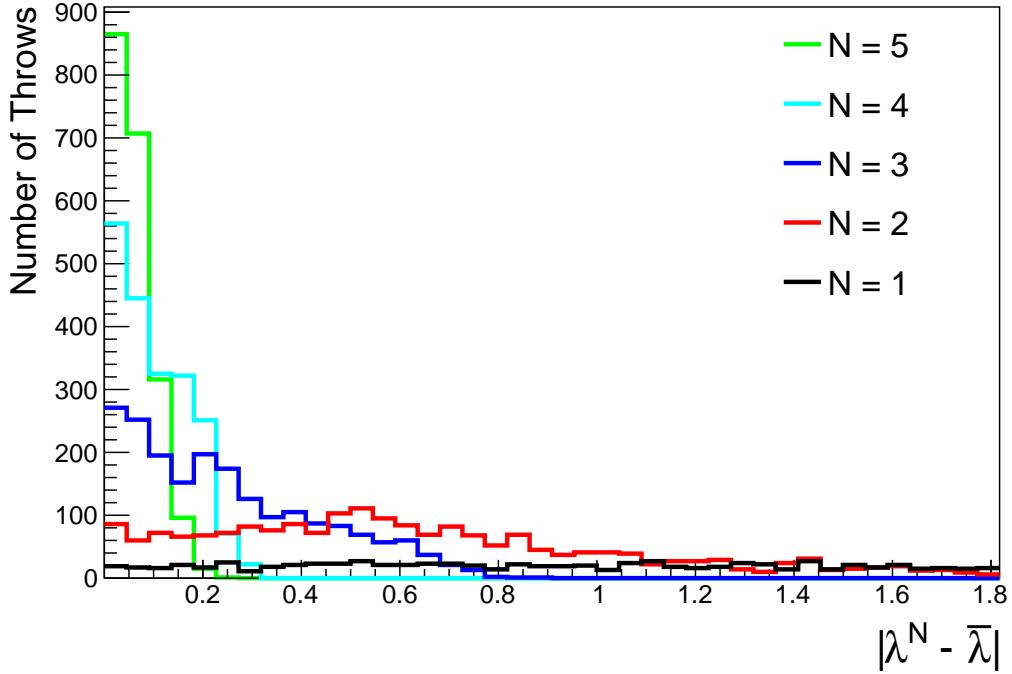
**Figure 7.10:** Event rate of the SubGeV\_elike\_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions,  $N$ , per coarse bin. The difference between the mean event rate (red),  $\bar{\lambda}$ , and the event rate at  $N = 1$ ,  $\lambda^{N=1}$  is defined as  $\lambda^N - \bar{\lambda}$  and illustrated by the blue arrow.

2559 the threshold at  $N = 10$ , and is driven primarily by the SubGeV\_mulike\_1dcy  
 2560 and SubGeV\_elike\_0dcy samples.

2561 The second study to determine the value of  $N$  is as follows. The likelihood  
 2562 for each sample is computed against an Asimov data set created with Asimov  
 2563 A oscillation parameters (Table 2.2). Following Equation 7.5, the variance of  
 2564 the log-likelihood over all considered  $N$  is computed. The results are shown  
 2565 in Figure 7.13.

2566 A choice of  $N = 10$  sub-divisions per coarse bin has a variance in both  
 2567 event rate and log-likelihood residuals less than the required threshold of 0.001.  
 2568 The largest value of the likelihood variance is of order  $10^{-7}$ , corresponding to  
 2569 an error on the log-likelihood of about  $3 \times 10^{-4}$  which is small enough to be  
 2570 negligible for the oscillation analysis.

2571 Figure 7.14 illustrates the effect of the smearing using  $N = 10$ . The fast oscil-



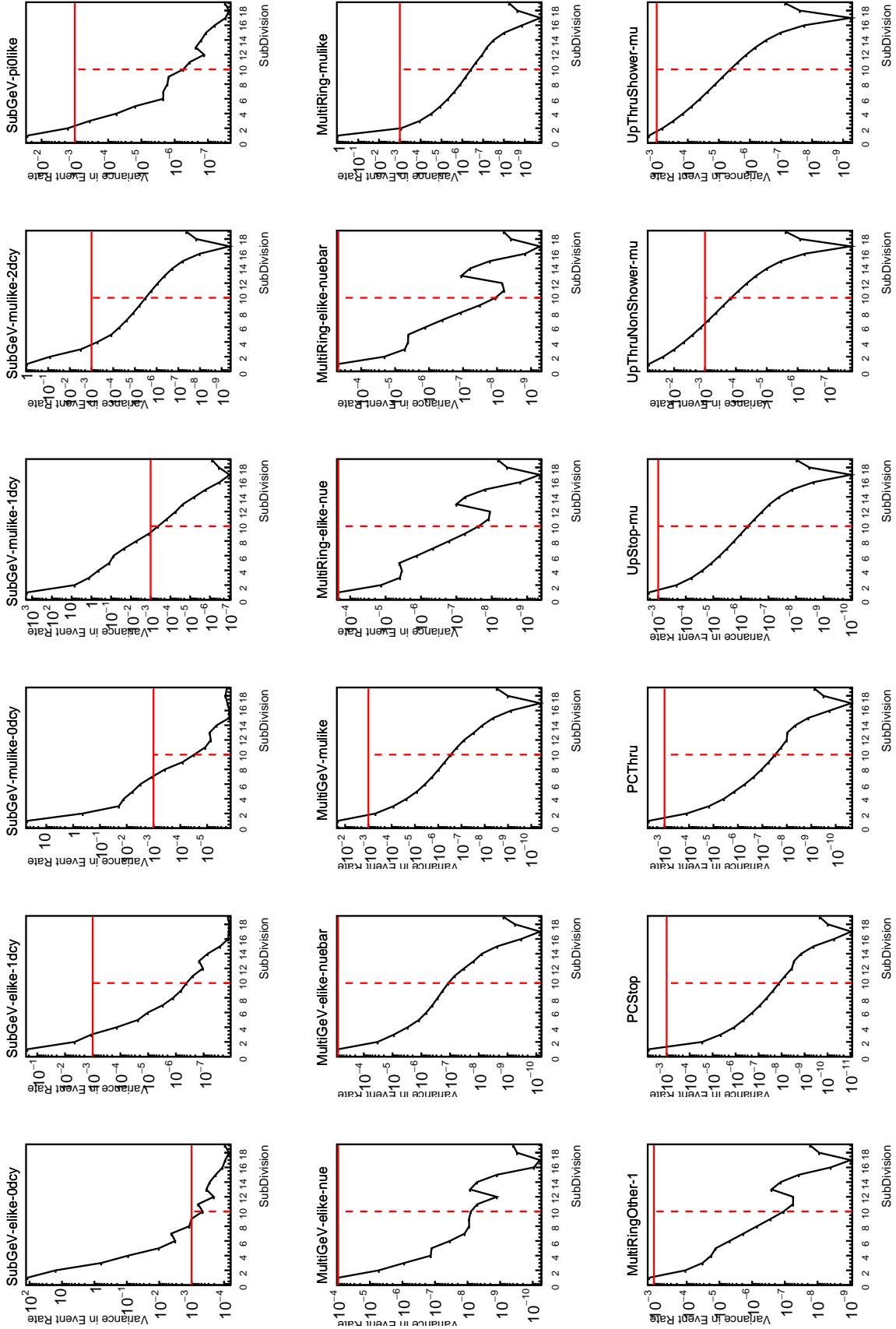
**Figure 7.11:** The distribution of  $\lambda^N - \bar{\lambda}$  for various values of  $N$ . As expected, the distribution gets narrower for larger values of  $N$ .

2572 lations in the sub-GeV upgoing region have been replaced with a normalisation  
 2573 effect whilst the large matter resonance structure remains.

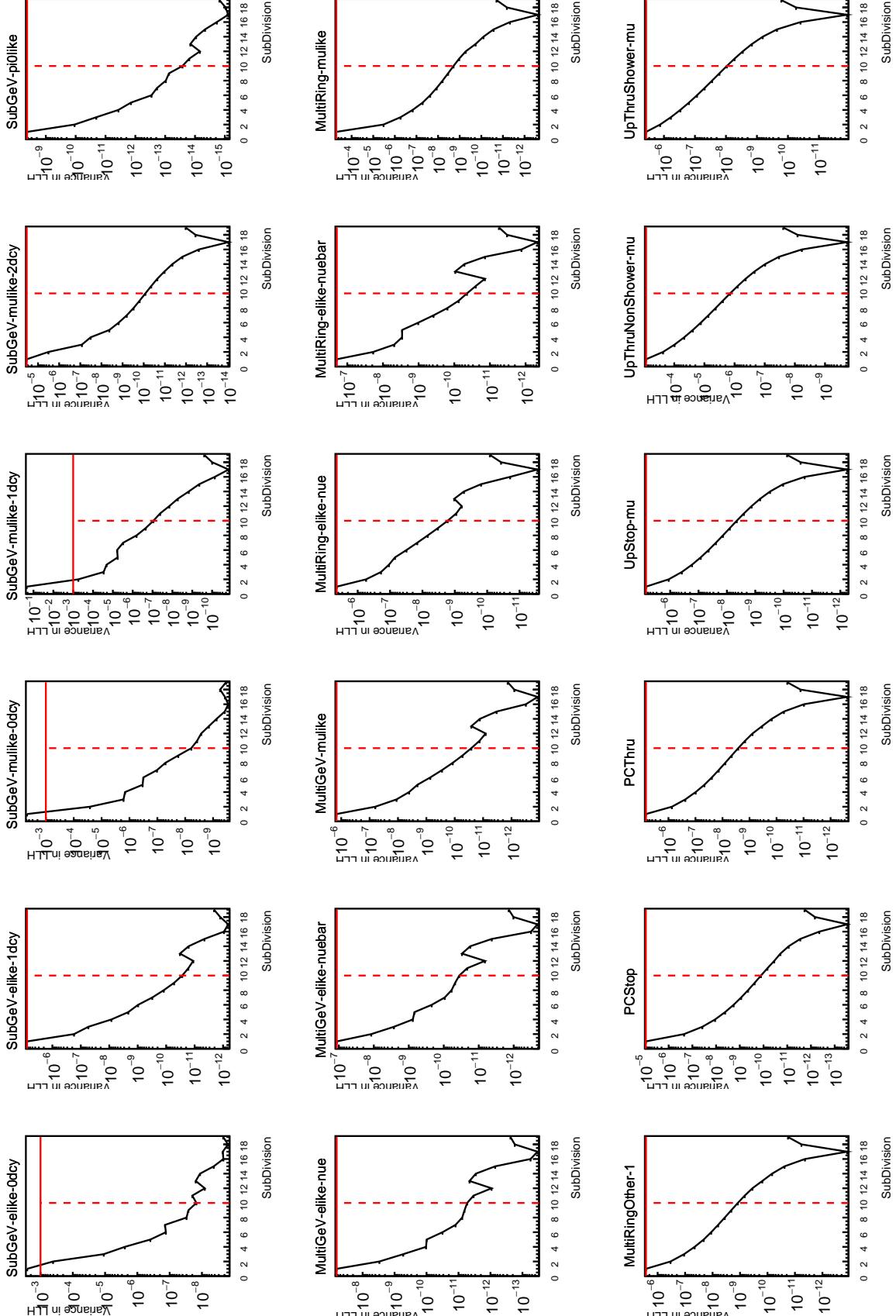
### 2574 7.3 Calculation Engine

2575 As previously discussed in section 7.2, the calculation of oscillation probabilities  
 2576 is performed at run-time. Consequently, the time per calculation is crucial for fit  
 2577 performance. The initial fitting framework used for this analysis was developed  
 2578 with ProbGPU [210]. This is a GPU-only implementation of the prob3 engine  
 2579 [211]. It is primarily designed for neutrino propagation in a beam experiment  
 2580 (single layer of constant density) with the atmospheric propagation code not  
 2581 being used prior to the analysis in this thesis.

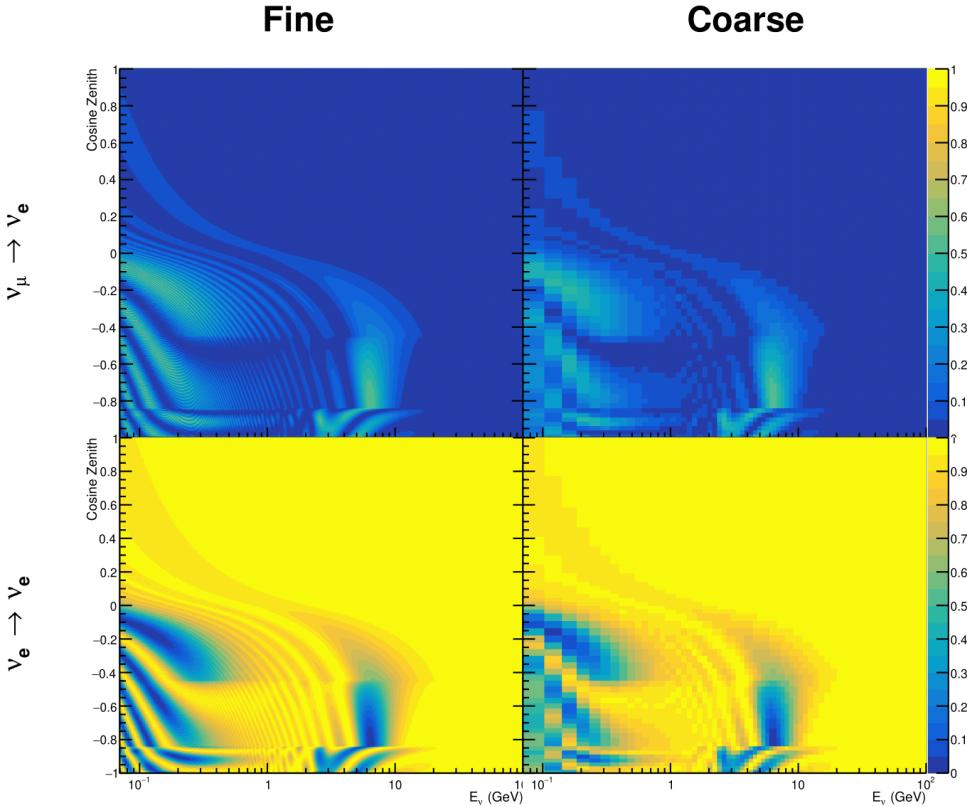
2582 Another engine, CUDAProb3 [212], has been interfaced with the fitting frame-  
 2583 work used in this analysis. It has been specifically optimised for atmospheric  
 2584 neutrino oscillation calculation so does not contain the code to replace the



**Figure 7.12:** Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division  $N = 10$ .



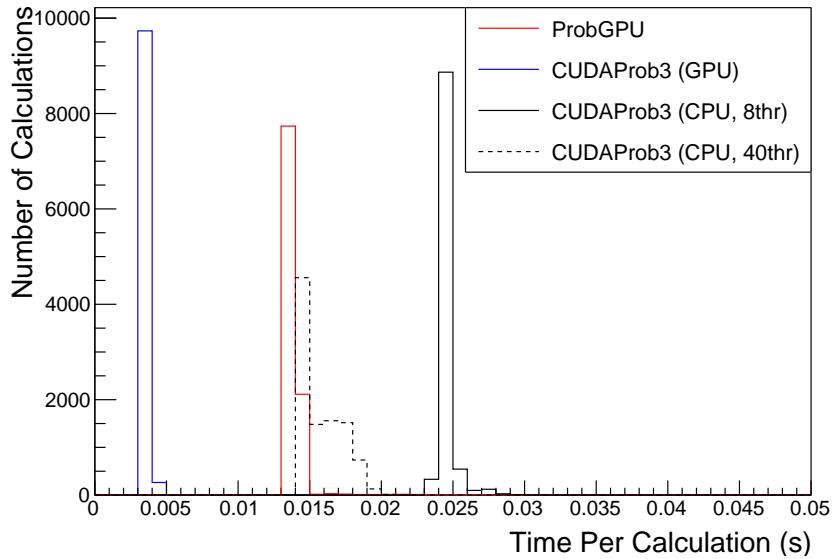
**Figure 7.13:** Variance of sample likelihood, when compared to 'Asimov data' set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division  $N = 10$ .



**Figure 7.14:** The oscillation probability,  $P(\nu_\mu \rightarrow \nu_e)$  (top row) and  $P(\nu_e \rightarrow \nu_e)$  (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with  $N = 10$ , which was determined to be below the threshold from Figure 7.12 and Figure 7.13.

beam oscillation calculation. The engine utilises object-orientated techniques as compared to the functional implementation of ProbGPU. This allows the energy and cosine zenith arrays to be kept on GPU memory, rather than having to load these arrays onto GPU memory for each calculation. Reducing the memory transfer between CPU and GPU significantly reduces the time required for calculation. This can be seen in Figure 7.15, where the GPU implementation of CUDAProb3 is approximately three times faster than the ProbGPU engine.

Another significant advantage of CUDAProb3 is that it contains a CPU multithreaded implementation which is not possible with the ProbGPU or prob3



**Figure 7.15:** The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.2, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multithreaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

2594 engines. This eliminates the requirement for GPU resources when submitting  
 2595 jobs to batch systems. As illustrated in Figure 7.15, the calculation speed depends  
 2596 on the number of available threads. Using 8 threads (which is typical of the  
 2597 batch systems being used) is approximately twice as slow as the ProbGPU engine  
 2598 implementation, but would allow the fitting framework to be run on many more  
 2599 resources. This fact is utilised for any SK-only fits but GPU resources are required  
 2600 for any fits which include beam samples due to the ProbGPU requirement. Based  
 2601 on the benefits shown by the implementation in this section, efforts are being  
 2602 placed into including linear propagation for beam neutrino propagation into  
 2603 the engine [213].

## 2604 7.4 Matter Density Profile

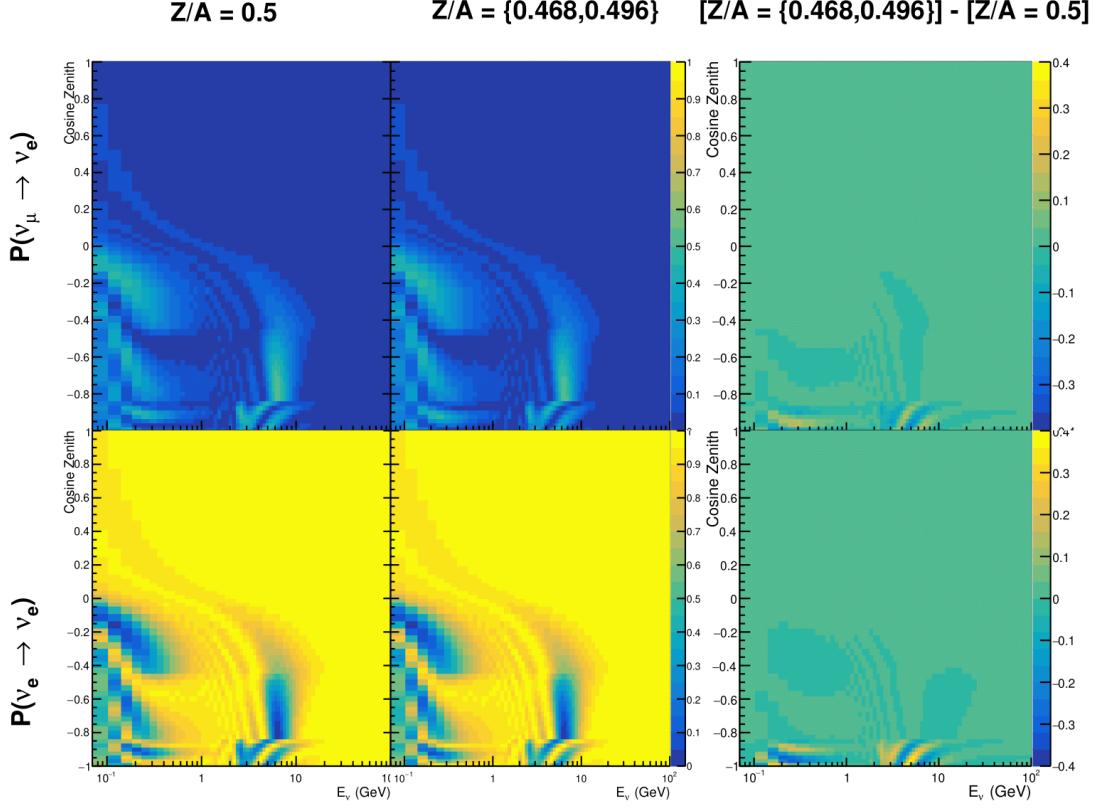
2605 For an experiment observing neutrinos propagating through the Earth, a model  
 2606 of the Earth’s density profile is required. The model used within this analysis is

2607 based on the Preliminary Reference Earth Model (PREM) [208], as illustrated in  
 2608 Figure 7.1. Table 7.1 documents the density and radii of the layers used within  
 2609 the constant density approximaton used by the SK-only analysis [207]. The  
 2610 density measurements provided in the PREM model are provided in terms of  
 2611 mass density, whereas neutrino oscillations are sensitive to the electron number  
 2612 density. This value can be computed as the product of the chemical composition,  
 2613 or the  $Z/A$  value, and the mass density of each layer. Currently, the only  
 2614 way to measure the chemical composition value for layers close to the Earth's  
 2615 core is through neutrino oscillations. The chemical composition of the upper  
 2616 layers of the Earth's Mantle and the Transition zone is well known due to it  
 2617 being predominantly pyrolite which has a chemical composition value of 0.496  
 2618 [214]. The chemical composition dial for the core layers is set to a value of  
 2619 0.468, as calculated in [215]. As this value is lesss well known, it is assigned a  
 2620 Gaussian error with a standard deviation equivalent to the difference in chemical  
 2621 composition in core and mantle layers. Figure 7.16 illustrates the effect of moving  
 2622 from the  $Z/A = 0.5$  method which is used in the official SK-only analysis  
 2623 to these more precise values.

2624 The beam oscillation probability in this thesis uses a baseline of 295km, density  
 2625  $2.6\text{g}/\text{cm}^3$ , and chemical composition 0.5 as is done by the official T2K-only  
 2626 analysis [216].

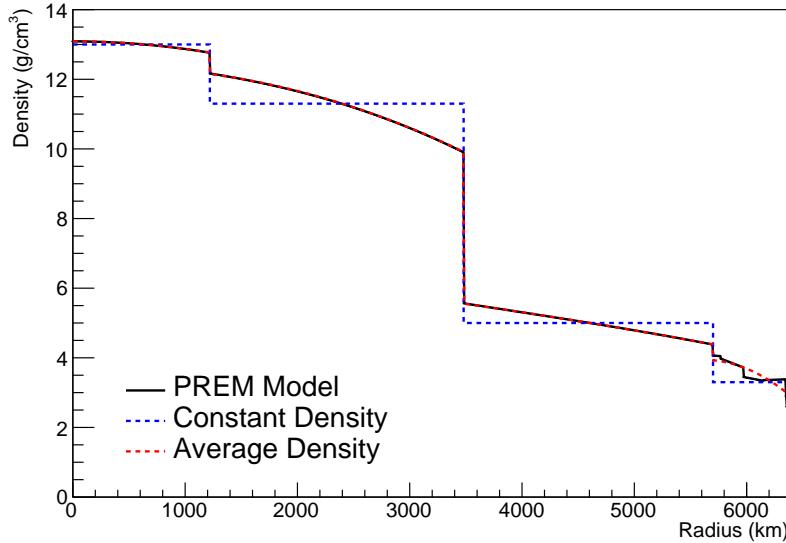
2627 For a neutrino with given  $E_\nu$ ,  $\cos(\theta_Z)$ , the oscillation probability calculation  
 2628 engine must be passed a list of the matter regions that the neutrino traversed,  
 2629 with the path length and fixed density in each region. However, a neutrino  
 2630 passing through the earth experiences a range of radii, and thus a range of  
 2631 densities, in each region. In the SK-only analysis, the earth density model used  
 2632 is piecewise-constant, thereby ignoring this effect. For this thesis, the density  
 2633 values for the calculation engine are found by averaging the earth density along  
 2634 the neutrino's path,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt \quad (7.6)$$



**Figure 7.16:** The oscillation probability,  $P(\nu_\mu \rightarrow \nu_e)$  (top row) and  $P(\nu_e \rightarrow \nu_e)$  (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant  $Z/A = 0.5$  approximation which is used in the official SK-only analysis. The middle column gives the probabilities where  $Z/A = [0.468, 0.498]$  values are used, as given in Table 7.1. The right column illustrates the difference in oscillation probability between the two different techniques.

where  $t_i$  are the intersection points between each layer and  $t$  is the path length of the trajectory across the layer. This leads to an improved approximation. For this averaging, the simplification of the PREM model developed in [217] is used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [209]. The coefficients of the quadratic fit to each layer are given in Table 7.2 with the final distribution illustrated in Figure 7.17. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

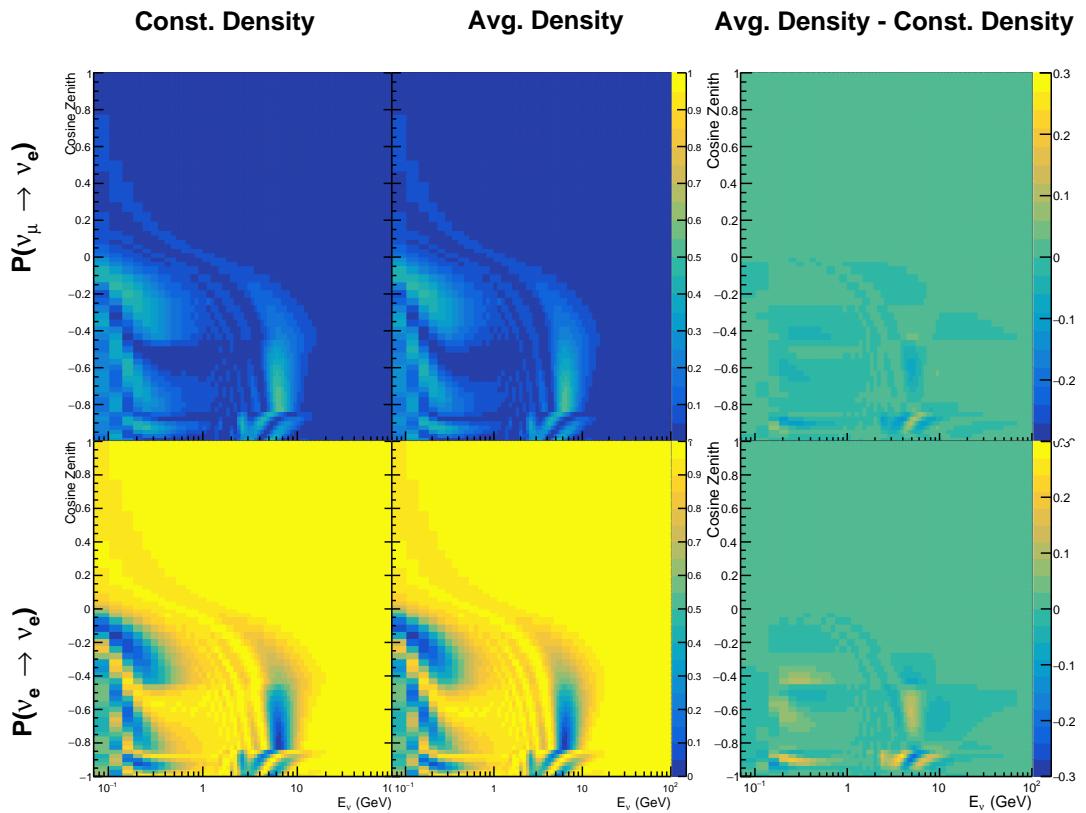


**Figure 7.17:** The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm <sup>3</sup> ]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

**Table 7.2:** The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [209], where  $x = R / R_{\text{Earth}}$ .

2644     The effect of using the quadratic density per  $\cos(\theta_Z)$  model is highlighted  
 2645     in Figure 7.18. The slight discontinuity in the oscillation probability around  
 2646      $\cos(\theta_Z) \sim -0.45$  in the fixed density model, which is due to the transition to  
 2647     mantle layer boundary, has been reduced. This is expected as the difference in  
 2648     the density across this boundary is significantly smaller in the quadratic density  
 2649     model as compared to the constant density model. Whilst the difference in  
 2650     density across the other layer transitions is reduced, there is still a significant  
 2651     difference. This means the discontinuities in the oscillation probabilities remain  
 2652     but are significantly reduced. However, as the quadratic density approximation  
 2653     matches the PREM model well in this region, these discontinuities are due to the  
 2654     Earth model rather than an artifact of the oscillation calculation.



**Figure 7.18:** The oscillation probability,  $P(\nu_\mu \rightarrow \nu_e)$  (top row) and  $P(\nu_e \rightarrow \nu_e)$  (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each  $\cos(\theta_Z)$  is used. The right column illustrates the difference in oscillation probability between the two different techniques.

## 2655 7.5 Production Height Averaging

2656 As discussed in section 7.1, the height at which the cosmic ray flux interacts  
 2657 in the atmosphere is not known on an event-by-event basis. The production  
 2658 height can vary from the Earth’s surface to  $\sim 50\text{km}$  above that. The SK-only  
 2659 analysis methodology (described in section 7.2) for including the uncertainty  
 2660 on the production height is to include variations from the Honda model when  
 2661 pre-calculating the oscillation probabilities prior to the fit. This technique is not  
 2662 possible for this analysis which uses continuous oscillation parameters that can  
 2663 not be known prior to the fit. Consequently, an analytical averaging technique  
 2664 was developed in [209]. The author of this thesis was not responsible for the  
 2665 derivation of the technique but has performed the implementation and validation  
 2666 of the technique for this analysis alone.

2667 Using the 20 production heights per Monte Carlo neutrino event, provided  
 2668 as 5% percentiles from the Honda flux model, a production height distribution  
 2669  $p_j(h|E_\nu, \cos \theta_Z)$  is built for each neutrino flavour  $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ . In practice, a  
 2670 histogram is filled with 20 evenly spaced bins in production height  $h$  between  
 2671 0 and 50km. The neutrino energy and cosine zenith binning of the histogram  
 2672 is the same as that provided in section 7.2. The average production height,  
 2673  $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$ , is calculated. The production height binning  
 2674 of this histogram is then translated into  $\delta t(h) = t(\bar{h}) - t(h)$ , where  $t(h)$  is the  
 2675 distance travelled along the trajectory.

2676 For the  $i^{th}$  traversed layer, the transition amplitude,  $D_i(t_{i+1}, t_i)$ , is computed.  
 2677 The time-ordered product of these is then used as the overall transition amplitude  
 2678 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.7)$$

2679 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.8)$$

2680 is expressed as a diagonalised time-dependent solution to the Schrodinger  
 2681 equation. The 0<sup>th</sup> layer is the propagation through the atmosphere and is the  
 2682 only term that depends on the production height. Using the substitution  $t_0 =$   
 2683  $t(\bar{h}) - \delta t(h)$ , it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.9)$$

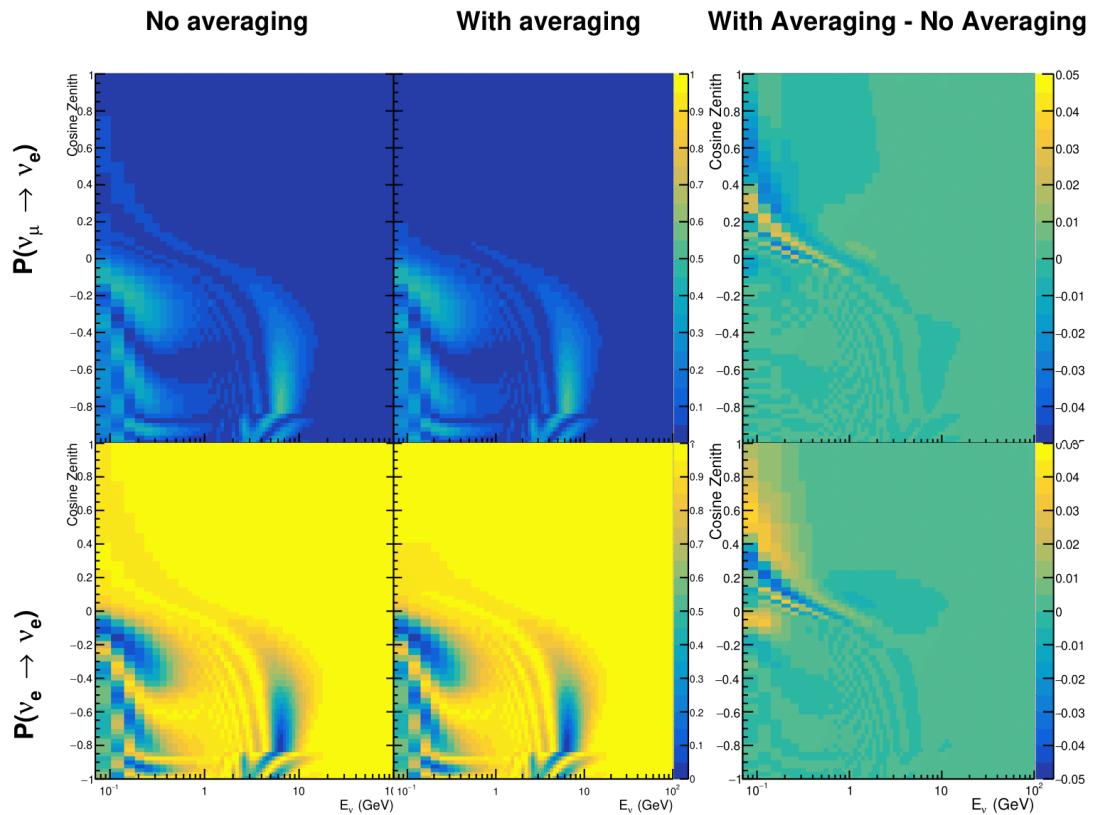
2684 Thus Equation 7.7 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, \bar{h}) D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.10)$$

2685 The oscillation probability averaged over production height is then calculated  
 2686 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)^*_{ij} \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m)\delta t] \end{aligned} \quad (7.11)$$

2687 In practice, implementation in CUDAProb3 [212] is relatively straightforward  
 2688 as the majority of these terms are already calculated in the standard oscillation  
 2689 calculation. Figure 7.19 illustrates the results of the production height averaging.  
 2690 As expected, the main effect is observed in the low-energy downward-going  
 2691 and horizontal-going events. Upward-going events have to travel the radius  
 2692 of the Earth,  $R_E = 6371\text{km}$ , where the production height uncertainty is a small  
 2693 fraction of the total path length.



**Figure 7.19:** The oscillation probability,  $P(\nu_\mu \rightarrow \nu_e)$  (top row) and  $P(\nu_e \rightarrow \nu_e)$  (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

# 8

2694

2695

## Oscillation Analysis

2696 Using the samples and systematics defined in chapter 6, this chapter documents  
2697 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK  
2698 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4  
2699 is used for all studies performed within this thesis.

2700 The MaCh3 framework used throughout this thesis has been validated through  
2701 many tests. The code which handles the beam far detector samples was developed  
2702 by the author and validated by comparison to the 2020 T2K analysis [74]. The  
2703 sample event rates and likelihood evaluations of beam samples generated by  
2704 the framework used within this thesis were compared to those from the T2K  
2705 analysis by the author of this thesis. Variations of the sample predictions were  
2706 compared at  $\pm 1\sigma$  and  $\pm 3\sigma$  and good agreement was found in all cases. A similar  
2707 study, led by Dr. C. Wret was used to validate the near detector portion of  
2708 the code [202]. The implementation of the atmospheric samples within MaCh3  
2709 was completed and cross-checked by the author of this thesis against the P-  
2710 Theta framework (Introduced in section 3.2). Both fitters are provided with  
2711 the same inputs and can therefore cross-validate each other. These validations  
2712 compared the event rate and likelihood calculation. Documentation of all the  
2713 above validations can be found in [202].

## 2714 8.1 Monte Carlo Prediction

2715 Using the three sets of dial values (generated, pre-BANFF, and post-BANFF tunes)  
 2716 defined in subsection 6.4.3, the predicted event rates for each sample are defined  
 2717 in Table 8.1. The oscillated (AsimovA defined in Table 2.2) and un-oscillated  
 2718 event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThru	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

**Table 8.1:** The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

2719 Generally, the samples which target CCQE interaction modes observe a  
 2720 decrease in prediction when using the pre-fit dial values. This is in accordance  
 2721 with the Monte Carlo being produced assumed  $M_A^{QE} = 1.21\text{GeV}$  [161] whilst

2722 the pre-fit dial value is set to  $M_A^{QE} = 1.03\text{GeV}$  as suggested by [200]. Further-  
 2723 more, the predicted event rates of samples that target CCRES interaction modes  
 2724 are significantly reduced when considering the post-BANFF fit. This follows  
 2725 the observations in subsection 6.4.3. The strength of the accelerator neutrino  
 2726 experiment can be seen in the remarkable difference between the oscillated and  
 2727 unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very  
 2728 clear decrease in the expected event rate between the oscillated and un-oscillated  
 2729 predictions which is not as obvious as in the atmospheric samples. This is due to  
 2730 the fact that the beam energy is tuned to the maximum disappearance probability  
 2731 which is not the case for the naturally generated atmospheric neutrinos.

## 2732 8.2 Likelihood Calculation

2733 This analysis performs a joint oscillation parameter fit of the ND280 beam  
 2734 samples, the T2K far detector beam samples, and the SK atmospheric samples.

2735 Once the Monte Carlo predictions of each beam and atmospheric sample  
 2736 have been built (see chapter 6) a likelihood needs to be constructed. This is done  
 2737 by comparing the binned Monte Carlo prediction to binned data. The Monte  
 2738 Carlo prediction is calculated at a particular point,  $\vec{\theta}$ , in the model parameter  
 2739 space such that  $N_i^{MC} = N_i^{MC}(\vec{\theta})$ , where  $N_i$  represents the bin content of the  
 2740  $i^{th}$  bin. The data and Monte Carlo spectra are represented by  $N_i^D$  and  $N_i^{MC}$ ,  
 2741 respectively. The bin contents for the beam near detector, beam far detector and  
 2742 atmospheric samples are denoted with *ND*, *FD*, and *Atm*, respectively. The  
 2743 binning index,  $i$ , runs over all the bins within a sample. Taking the FHC1Rmu far  
 2744 detector sample as an example, the binning index runs over all the reconstructed  
 2745 neutrino energy bins. The likelihood calculation between the data and the Monte  
 2746 Carlo prediction for a particular bin follows a Poisson distribution, where the  
 2747 data is treated as a fluctuation of the simulation.

2748 The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction,  
 2749 which is typically used for sensitivity studies and denoted ‘Asimov data’. The  
 2750 process for building Asimov data is as follows. The Monte Carlo prediction is

2751 reweighted using a particular set of oscillation parameters (potentially those  
 2752 listed in Table 2.2) and systematic parameter tune. The resulting spectra for each  
 2753 sample is then defined to be the Asimov data for that sample. Whilst this results  
 2754 in unphysical non-integer data predictions, it eliminates statistical fluctuations  
 2755 from the data. Therefore, the results of a fit to Asimov data should not include any  
 2756 biases from statistical fluctuations. Furthermore, these results should produce  
 2757 posterior probability distributions consistent with the parameters which were  
 2758 used to make the data prediction. That is to say, the fit results should return the  
 2759 known parameters. Any biases seen would be attributed to correlations between  
 2760 each oscillation parameter and correlations between oscillation and systematic  
 2761 parameters. Consequently, Asimov fit results present the maximum precision  
 2762 at which the oscillation parameters could be measured to.

2763 Following the T2K analysis presented in [74], the likelihood contribution for  
 2764 the near detector samples also includes a Monte Carlo statistical uncertainty term,  
 2765 derived from the Barlow and Beeston statistical treatment [218, 219]. In addition  
 2766 to treating the data as a Poisson fluctuation of the Monte Carlo prediction, it  
 2767 includes a contribution to the likelihood that which treats the generated Monte  
 2768 Carlo prediction as a statistical fluctuation of the actual true simulation assuming  
 2769 an infinite amount of statistics had been created. The technical implementation  
 2770 of this additional likelihood term is documented in [191] and briefly summarised  
 2771 as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (8.1)$$

2772 where  $\beta_i$  represents a scaling parameter for the  $i^{th}$  bin that relates the bin  
 2773 content for the amount of Monte Carlo actually generated  $N_i^{MC}$  to the bin content  
 2774 if an infinite amount of Monte Carlo statistics had been generated  $N_{i,true}^{MC}$ , such  
 2775 that  $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$ . In the case where a sufficient amount of Monte Carlo  
 2776 statistics had been generated,  $\beta_i = 1$ . An analytical solution for  $\beta_i$  is given in  
 2777 [191]. Additionally,  $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2 / N_i^{MC}}$  where  $\sqrt{\sum_i w_i^2}$  represents the sum of  
 2778 the square of the weights of the Monte Carlo events which fall into bin  $i$ .

An additional contribution to the likelihood comes from the variation of the systematic model parameters. For those parameters with well-motivated uncertainty estimates, a covariance matrix,  $V$ , describes the prior knowledge of each parameter as well as any correlations between the parameters. Due to a technical implementation, a single covariance matrix describes each “block” of model parameters, e.g. beam flux systematics. The covariance matrix associated with the  $k^{th}$  block is denoted  $V^k$ . This substitution results in  $\vec{\theta} = \sum_k^{N_b} \vec{\theta}^k$  and  $V = \sum_k^{N_b} V^k$  where  $N_b$  denotes the number of blocks. A single covariance matrix is provided for: the oscillation parameters, the beam flux parameters, the atmospheric flux parameters, the neutrino interaction systematics, the near detector parameters, the beam far detector systematics, and the atmospheric far detector systematics. All of these are detailed in section 6.4. The number of parameters in the  $k^{th}$  block is defined as  $n(k)$ .

The equation for the likelihood  $\mathcal{L}$  includes all the terms discussed above. It is defined as,

$$\begin{aligned}
-\ln(\mathcal{L}) = & \\
& \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[ N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
& + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[ N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
& + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[ N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
& + \frac{1}{2} \sum_k^{N_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
\end{aligned} \tag{8.2}$$

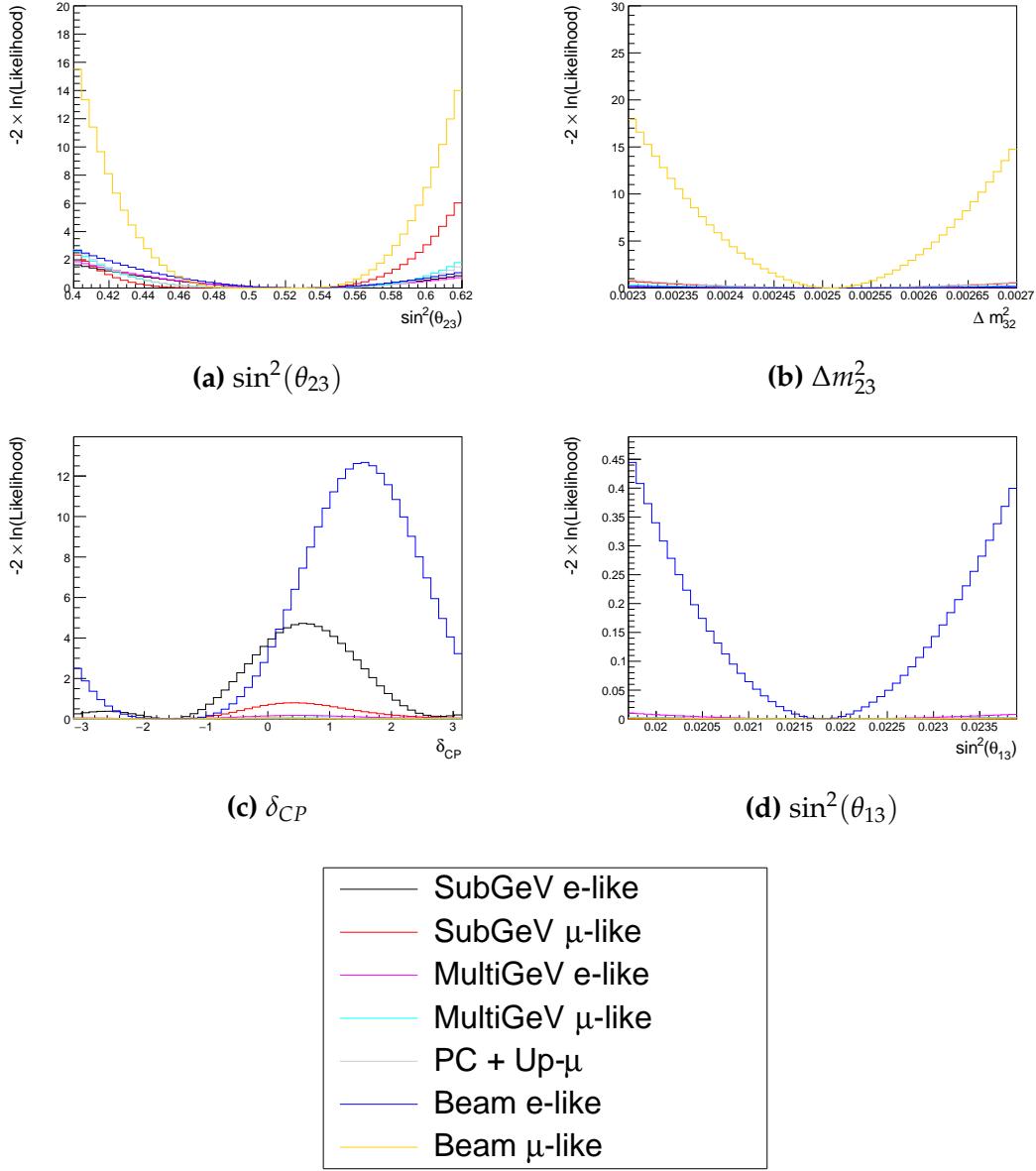
The negative log-likelihood value is determined at each step of the MCMC to build the posterior distribution defined in chapter 4. This value is minimised when the Monte Carlo prediction tends towards the data spectrum.

### 2797 8.2.1 Likelihood Scans

2798 Using the definition of the likelihood presented in section 8.2, the response of each  
2799 sample to a variation of a particular parameter can be studied. Figure 8.1 presents  
2800 the variation of all the samples (beam and atmospheric) at the far detector to the  
2801 oscillation parameters of interest:  $\delta_{CP}$ ,  $\sin^2(\theta_{13})$ ,  $\sin^2(\theta_{23})$ , and  $\Delta m_{32}^2$ . These plots  
2802 are colloquially called ‘likelihood scan’ (or ‘log-likelihood scans’). The process  
2803 of making these plots is as follows. An Asimov data set (following technique  
2804 detailed in section 8.2) is built using the AsimovA oscillation parameters and  
2805 pre-fit systematic tune. The Monte Carlo is then reweighted using the value of  
2806 the oscillation parameter at each point on the x-axis of the scan. The likelihood  
2807 is then calculated between the Asimov data and Monte Carlo prediction and  
2808 plotted. This process identifies which samples drive the determination of the  
2809 oscillation parameters in the joint fit.

2810 Due to the caveat of fixed systematic parameters and the correlations between  
2811 oscillation parameters being ignored when creating these likelihood scans, the  
2812 value of  $\chi^2 = 1$  (or  $-2 \times \ln(\text{Likelihood}) = 1$ ) does not equate to the typical  
2813  $1\sigma$  sensitivity. However, it does give an indication of which samples respond  
2814 the strongest to a variation in a particular oscillation parameter. The point at  
2815 which the likelihood tends to zero illustrates the value of the parameter used  
2816 to build the Asimov data prediction.

2817 The sensitivity to  $\sin^2(\theta_{23})$  is mostly dominated by the beam muon-like  
2818 samples. The response of an individual atmospheric sample is small but non-  
2819 negligible such that the summed response over all atmospheric samples becomes  
2820 comparable to that of the muon-like beam samples. Consequently, the sensitivity  
2821 of the joint fit to  $\sin^2(\theta_{23})$  would be expected to be greater than the beam-only  
2822 analysis. The only sample which respond to the  $\sin^2(\theta_{13})$  oscillation parameter is  
2823 the electron-like beam samples. Consequently, no increase in sensitivity beyond  
2824 that of the T2K-only analysis would be expected from the joint fit. Regardless, the  
2825 sensitivity of the beam sample is significantly weaker than the external reactor  
2826 constraint so prior knowledge will dominate any measurement that is included



**Figure 8.1:** The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to a variation of an oscillation parameter.

within this thesis. The  $\Delta m_{21}^2$  and  $\sin^2(\theta_{12})$  parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to  $\Delta m_{32}^2$  is completely dominated by the beam muon-like samples. This is because the beam neutrino energy can be specifically tuned to match the maximal disappearance probability. Despite this, improvements to the  $|\Delta m_{32}^2|$  sensitivity may be expected due to additional mass hierarchy determination

2833 added by the atmospheric samples.

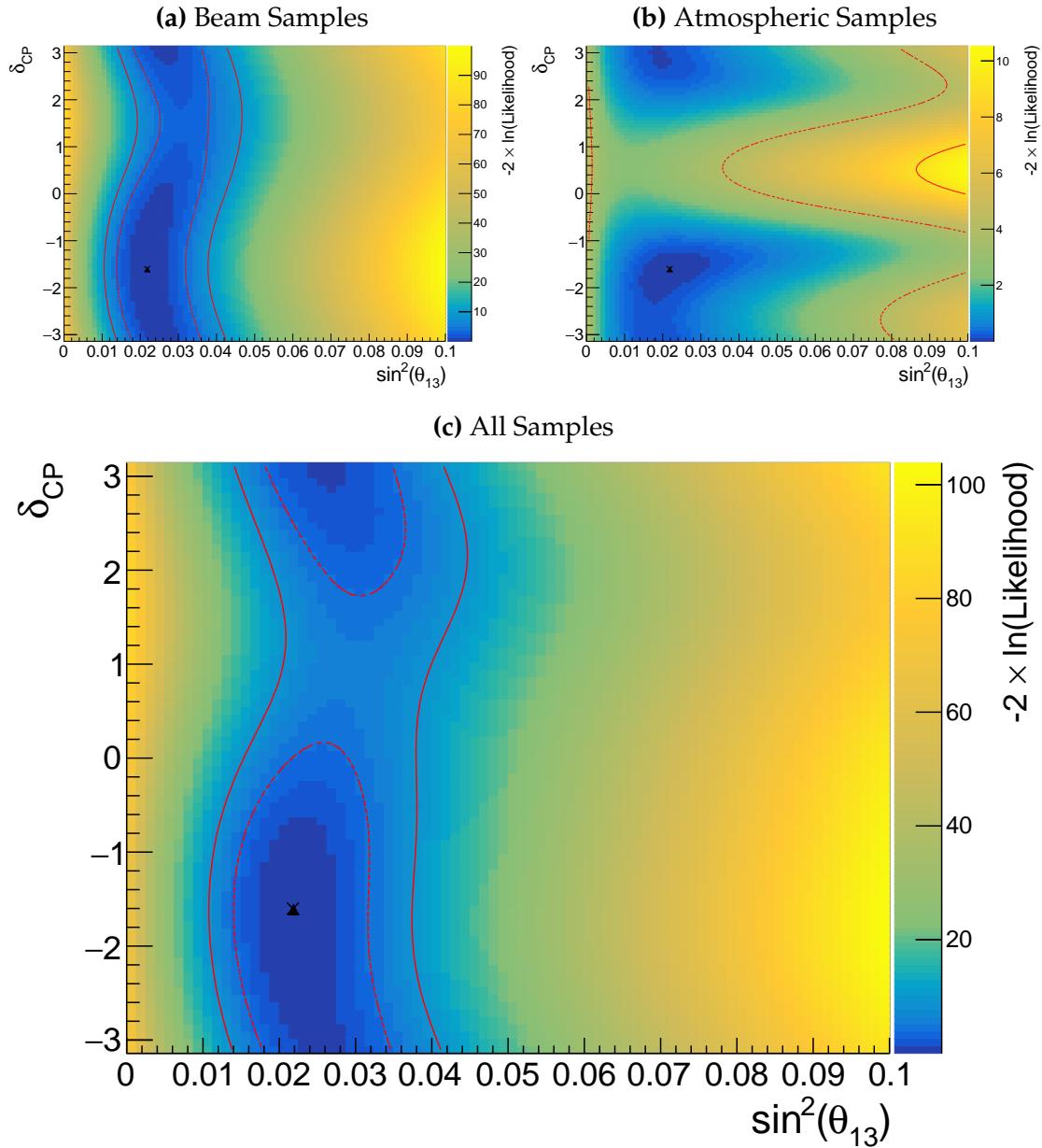
2834 Two-dimensional scans of the appearance ( $\sin^2(\theta_{13})-\delta_{CP}$ ) and disappearance  
2835 ( $\sin^2(\theta_{23})-\Delta m_{32}^2$ ) parameters are illustrated in Figure 8.2 and Figure 8.3, respec-  
2836 tively. The caveat of fixed systematic parameters and correlations between other  
2837 oscillation parameters being neglected still apply.

2838 The appearance log-likelihood scans show the distinct difference in how  
2839 the beam and atmospheric samples respond. The beam samples have an ap-  
2840 proximately constant width of the  $2\sigma$  and  $3\sigma$  contours, throughout all ranges  
2841 of  $\delta_{CP}$ . The response of the atmospheric samples to  $\sin^2(\theta_{13})$  is very strongly  
2842 correlated to the value of  $\delta_{CP}$  being evaluated, with the strongest constraints  
2843 around  $\delta_{CP} \sim 1$ . Consequently, this difference allows some of the degeneracy in  
2844 a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood  
2845 scans, the  $2\sigma$  continuous contour in  $\delta_{CP}$  for beam samples is broken when the  
2846 atmospheric samples are added. This may result in a stronger sensitivity to  
2847  $\delta_{CP}$ . Similarly, the width of the  $3\sigma$  contours also becomes dependent upon the  
2848 value of  $\delta_{CP}$ . Whilst these are encouraging results for the joint fit, these are  
2849 not sensitivity measurements as the systematic parameters are fixed and the  
2850 correlation between oscillation parameters is neglected. An interesting point to  
2851 note is that the atmospheric samples have little sensitivity to  $\sin^2(\theta_{13})$  on their  
2852 own, as evidenced in Figure 8.1, but can improve sensitivity to the parameter  
2853 when combined within the simultaneous fit.

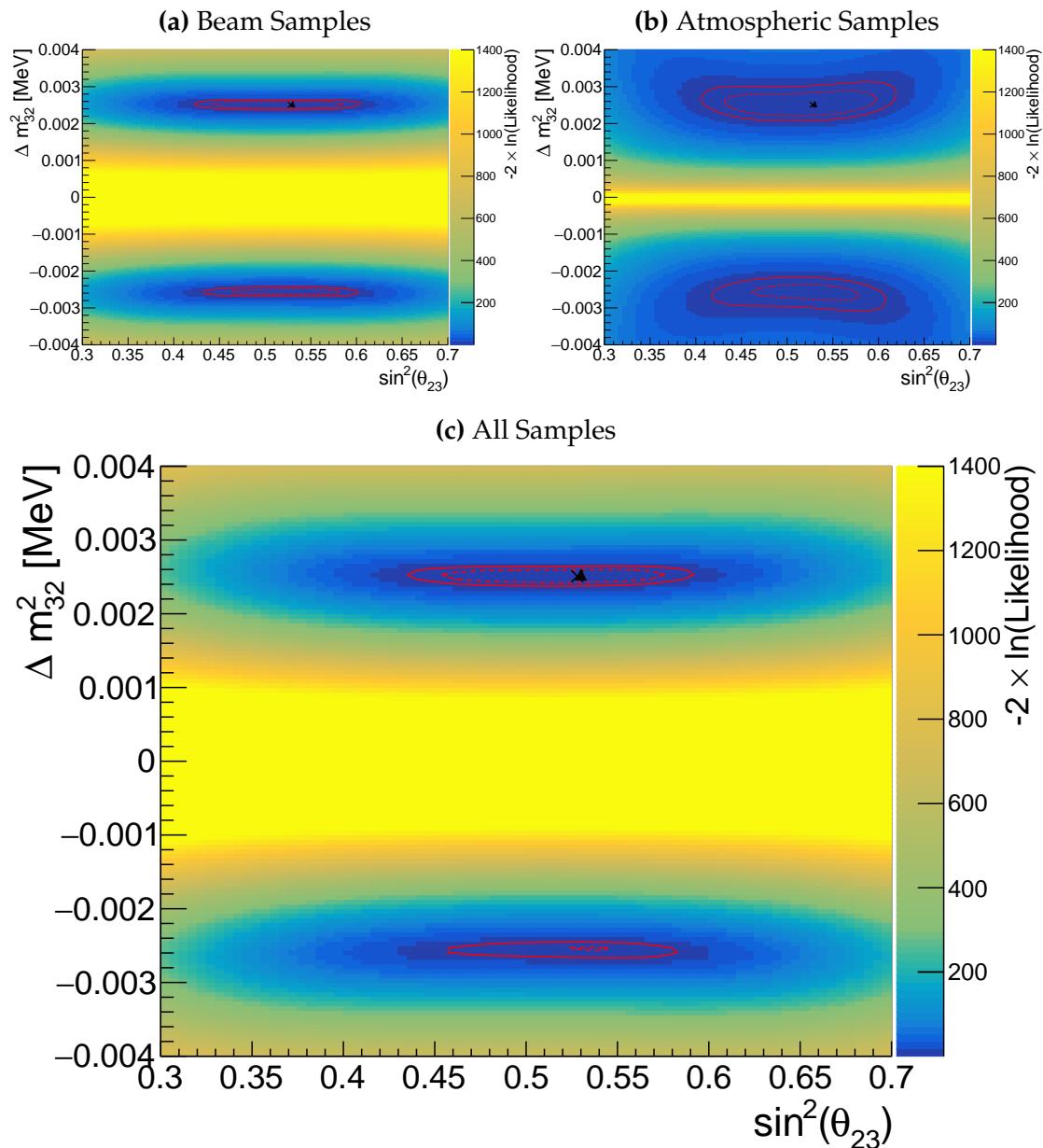
2854 The response of the atmospheric samples in Figure 8.2 shows an interesting  
2855 behaviour when considering the application of the reactor constraint. At higher  
2856 values of  $\sin^2(\theta_{13})$ , two lobes appear around  $\delta_{CP} \sim -\pi/2$  and  $\delta_{CP} \sim 2.4$ . If this  
2857 distribution was projected onto the  $\delta_{CP}$  axis, these lobes would mean the posterior  
2858 distribution would have a significant dip between these values. However, the  
2859 region of  $\sin^2(\theta_{13})$  near the reactor constraint ( $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ )  
2860 is flatter across the range of  $\delta_{CP}$ . Therefore, if we were to project only this  
2861 region onto the  $\delta_{CP}$  axis, the dip between the peaks would not be as significant.  
2862 If this behaviour was to be seen in the results of a fit, these marginalisation

2863 effects would actually conspire to reduce the sensitivity to  $\delta_{CP}$  if the reactor  
2864 constraint was to be applied.

2865 The disappearance log-likelihood scans in  $\sin^2(\theta_{23})$ - $\Delta m_{32}^2$  space (Figure 8.3)  
2866 show the expected behaviour when considering the one-dimensional scans  
2867 already discussed. The uncertainty on the width of  $|\Delta m_{32}^2|$  is mostly driven  
2868 by the beam-only sensitivities. However, the width of this contour in the  
2869 inverted mass region ( $\Delta m_{32}^2 < 0$ ) is significantly reduced due to the ability  
2870 of the atmospheric samples to select the correct (normal) mass hierarchy. The  
2871 width of the uncertainty in  $\sin^2(\theta_{23})$  is also reduced compared to the beam-only  
2872 sensitivities, with a further decrease in the inverted hierarchy region due to the  
2873 better mass hierarchy determination.



**Figure 8.2:** Two-dimensional log-likelihood scan of the appearance ( $\sin^2(\theta_{13})$ - $\delta_{CP}$ ) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.



**Figure 8.3:** Two-dimensional log-likelihood scan of the disappearance ( $\sin^2(\theta_{23})$ )- $\Delta m_{32}^2$ ) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

2874 The likelihood scans illustrated thus far only consider the sensitivity of this  
2875 analysis for a fixed set of true oscillation parameters, namely Asimov A defined in  
2876 Table 2.2. Whilst computational infeasible to run many fits at different parameter  
2877 sets, it is possible to calculate the likelihood response to different Asimov data  
2878 sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing  
2879 true values of  $\delta_{CP}$  and  $\sin^2(\theta_{23})$ , respectively. For both of these plots, the other  
2880 oscillation parameters are fixed at their Asimov A values. Consequently, the  
2881 caveat of fixed systematic parameters and correlations between other oscillation  
2882 parameters being neglected still applies.

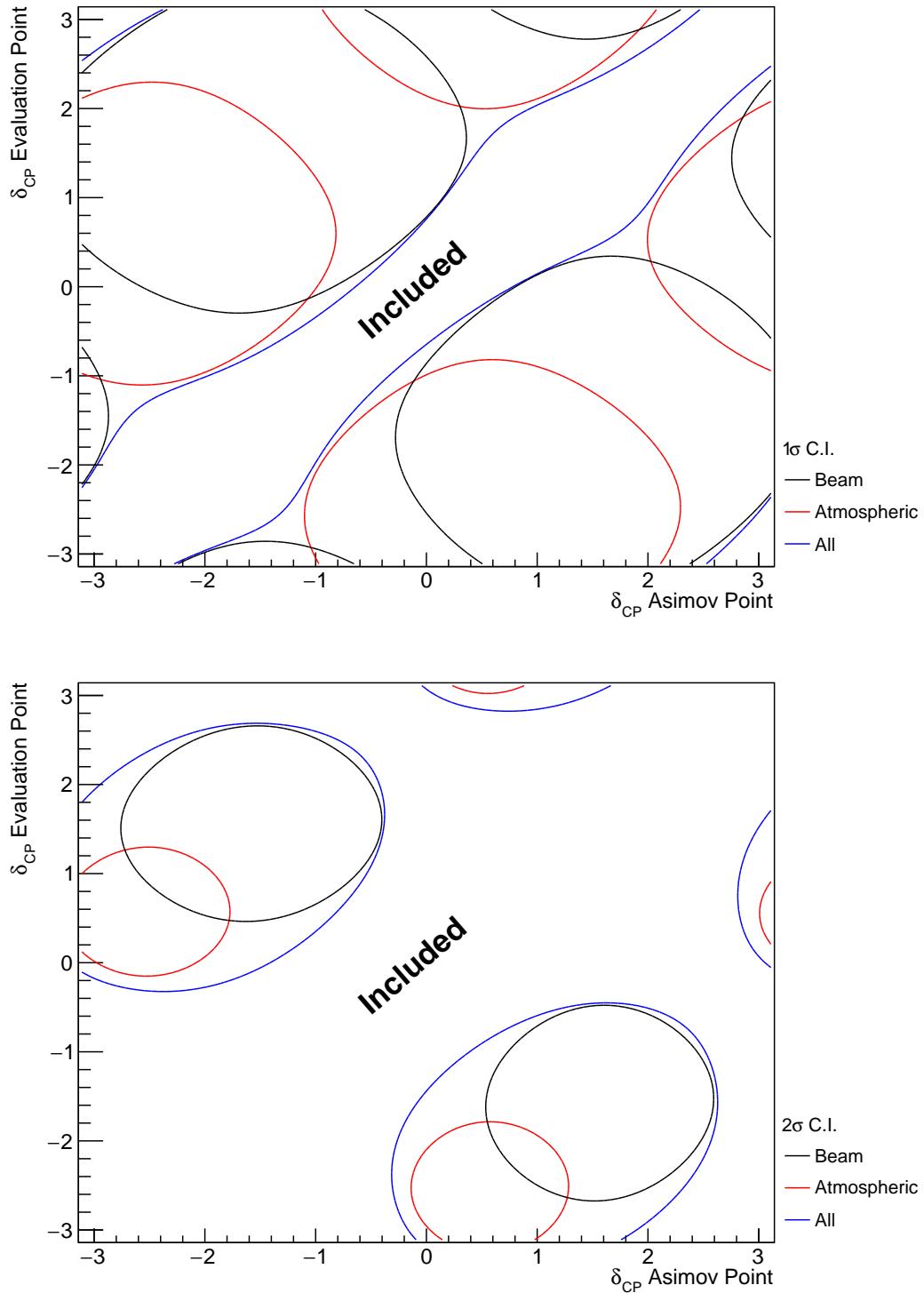
2883 To explain how these plots are made, consider Figure 8.4. This plot is built  
2884 by considering multiple one-dimensional log-likelihood scans, each creating  
2885 an Asimov data with the value of  $\delta_{CP}$  taken from the x-axis. The likelihood to  
2886 a particular Asimov data set is calculated after reweighting the Monte Carlo  
2887 prediction to each value of  $\delta_{CP}$  on the y-axis.

2888 Figure 8.4 illustrates the sensitivity to  $\delta_{CP}$ . Notably, the  $1\sigma$  intervals contain  
2889 regions in the off-diagonal for which the beam and atmospheric samples have  
2890 broken and discontinuous contours. This indicates that there are regions of  $\delta_{CP}$   
2891 which are degenerate. For example, for the x-axis value of  $\delta_{CP} = 0$ , the beam  
2892 samples sensitivity would include two discontinuous regions excluded from  
2893 the  $1\sigma$  interval:  $\delta_{CP} \sim 0$  and  $\delta_{CP} \sim \pi$ . The offset in  $\delta_{CP}$  between the beam  
2894 and atmospheric samples allows the joint fit to have increased sensitivity in  
2895 these regions. Consequently, the difference between the beam-only and joint  
2896 beam-atmospheric fit should be studied using multiple Asimov data sets.

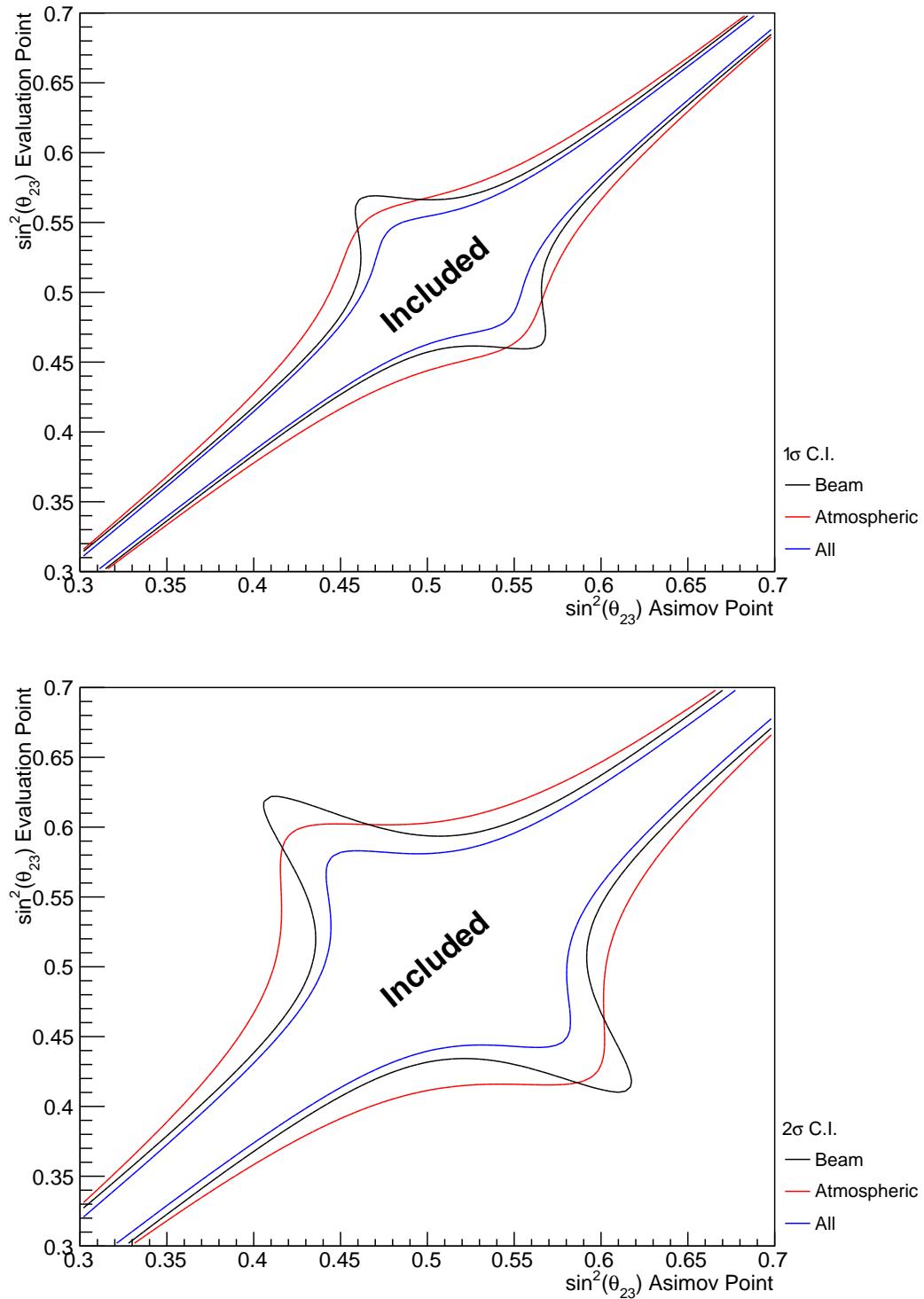
2897 Despite the increased sensitivity at  $1\sigma$ , the  $2\sigma$  intervals from the joint fit  
2898 are more similar to the two independent sensitivities and the off-diagonal de-  
2899 generacies mostly remain. This indicates that the joint fit has the strength to  
2900 aid parameter determination but can not entirely break the degeneracies in  $\delta_{CP}$   
2901 at higher confidence levels.

2902 Figure 8.5 illustrates a similar analysis as above, although the value of  $\sin^2(\theta_{23})$   
2903 is varied and  $\delta_{CP}$  is fixed to the Asimov A parameter value. Due to the beam

parameters and baseline being tuned to specifically target this oscillation parameter, the average sensitivity of the beam samples is stronger than the atmospheric samples. However, the degeneracy around maximal mixing ( $\sin^2(\theta_{23}) = 0.5$ ) is significantly more peaked in the beam samples compared to the atmospheric samples. This behaviour is strengthened when considering the  $2\sigma$  intervals, to the point where two distinct discontinuous regions of the  $2\sigma$  intervals exist around the Asimov point  $\sin^2(\theta_{23}) \sim 0.41, 0.6$ . Given the caveat of only considering likelihood scans, the joint analysis would mostly eliminate the discontinuous intervals in these regions. This means that the joint fit could feasibly have an increased preference for the correct octant hypothesis.

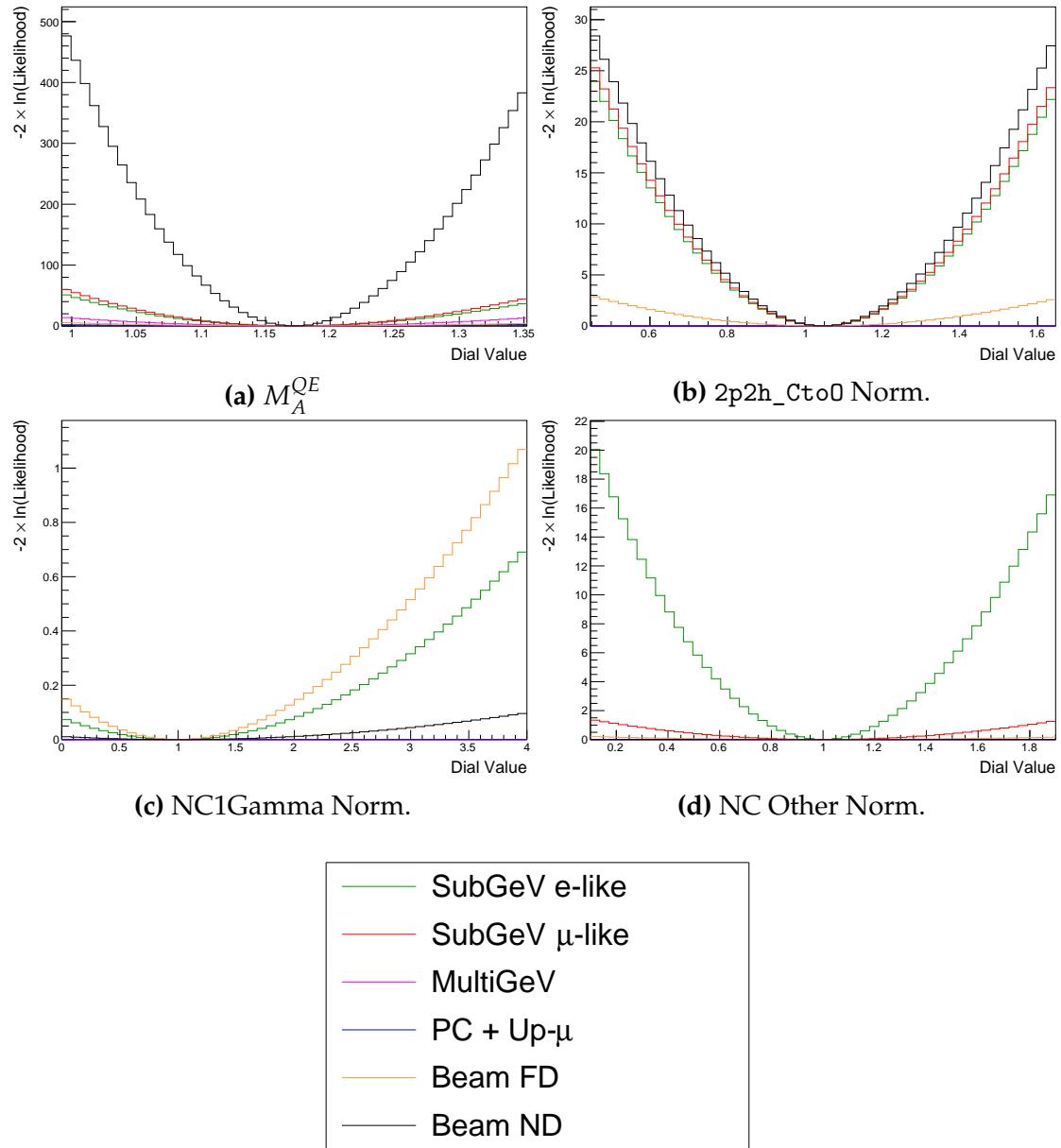


**Figure 8.4:** A series of one-dimensional likelihood scans over  $\delta_{CP}$ , where an Asimov data set is built for each value of  $\delta_{CP}$  on the x-axis and the likelihood is evaluated for each value of  $\delta_{CP}$  on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the  $1\sigma$  (Top) and  $2\sigma$  (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.



**Figure 8.5:** A series of one-dimensional likelihood scans over  $\sin^2(\theta_{23})$ , where an Asimov data set is built for each value of  $\sin^2(\theta_{23})$  on the x-axis and the likelihood is evaluated for each value of  $\sin^2(\theta_{23})$  on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the  $1\sigma$  (Top) and  $2\sigma$  (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than what they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions,  $M_A^{QE}$ , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h\_Cto0 normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics which have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric samples are significantly more sensitive to these systematics than the beam samples due to their similar interaction contributions but relatively higher statistics (Table 8.1). As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be considerably more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.



**Figure 8.6:** The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

### 2939 8.3 Sensitivities

2940 The sensitivities of the joint T2K and SK oscillation analysis are presented in  
2941 the form of Asimov fits. This technique builds an Asimov data set (follow-  
2942 ing section 8.2) using the AsimovA oscillation parameters and post-BANFF  
2943 systematic tune.

2944 In practice, the Asimov fits presented within this analysis are modified from  
2945 the above definition. An Asimov prediction of both beam and atmospheric far  
2946 detector samples is fit whilst the true data is used for near detector samples.  
2947 The Asimov predictions at the far detector are built using the BANFF tuning (as  
2948 discussed in section 3.2). These modifications mean that the results are equivalent  
2949 to performing a far detector Asimov fit using inputs from the BANFF data fit.  
2950 Consequently, this allows the results to be cross-checked with the results from  
2951 the P-Theta analysis. The comparison has been performed and is documented in  
2952 [220]. No significant discrepancies were found between the fitters.

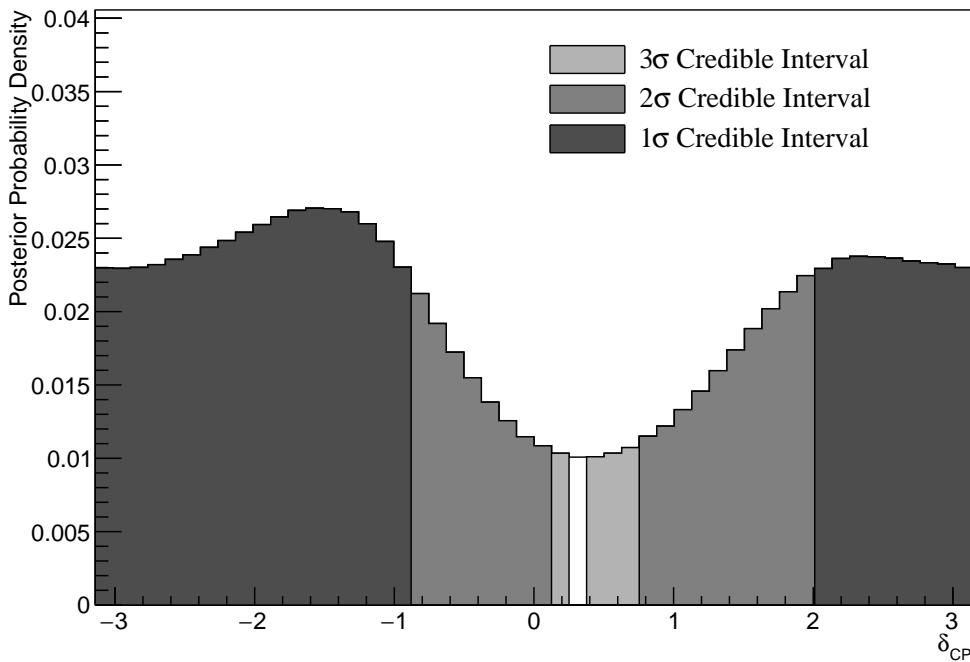
2953 This section proceeds with the following studies. Firstly, the sensitivity of  
2954 the atmospheric samples after the T2K cross-section has been applied to the low-  
2955 energy events is detailed in subsection 8.3.1. This includes studying the choice of  
2956 applying the 2020 PDG reactor constraint [75] to the atmospheric samples, which  
2957 is documented in subsection 8.3.2. Additionally, the effect of applying the near-  
2958 detector constraints onto the atmospheric samples is discussed in subsection 8.3.3.  
2959 The main result is the sensitivity of the simultaneous beam and atmospheric fit.  
2960 The sensitivities, both with and without the application of the reactor constraint,  
2961 are presented in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate  
2962 the benefit of the joint analysis, the sensitivities are compared to the 2020 T2K  
2963 sensitivities [74, 182] in subsection 8.3.6 and subsection 8.3.7. As shown in  
2964 subsection 8.2.1, the response of the beam and atmospheric samples change  
2965 depending upon the true set of oscillation parameters assumed. Therefore,  
2966 subsection 8.3.8 documents the sensitivities at an alternative oscillation parameter

<sup>2967</sup> set. It is important to note that these results have been published at the Neutrino  
<sup>2968</sup> 2022 conference on behalf of the T2K and SK collaborations [108].

### 2969 8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

2970 This section presents the results of an Asimov fit using samples from the near  
 2971 detector and only atmospheric samples from the far detector. The results are  
 2972 presented as one-dimensional or two-dimensional histograms which have been  
 2973 marginalised over all other parameters using the technique outlined in sub-  
 2974 section 4.3.1. Each histogram displays the posterior probability density and  
 2975 illustrates the credible intervals, calculated using the technique in subsection 4.3.2.  
 2976 For these fits in this subsection, a flat penalty term is used for  $\sin^2(\theta_{13})$  such the  
 2977 reactor constraint is not applied. The Asimov data is generated assuming the  
 2978 AsimovA oscillation parameter set defined in Table 2.2 and the post-BANFF  
 2979 systematic parameter tune.

Without Reactor Constraint, Both Hierarchies

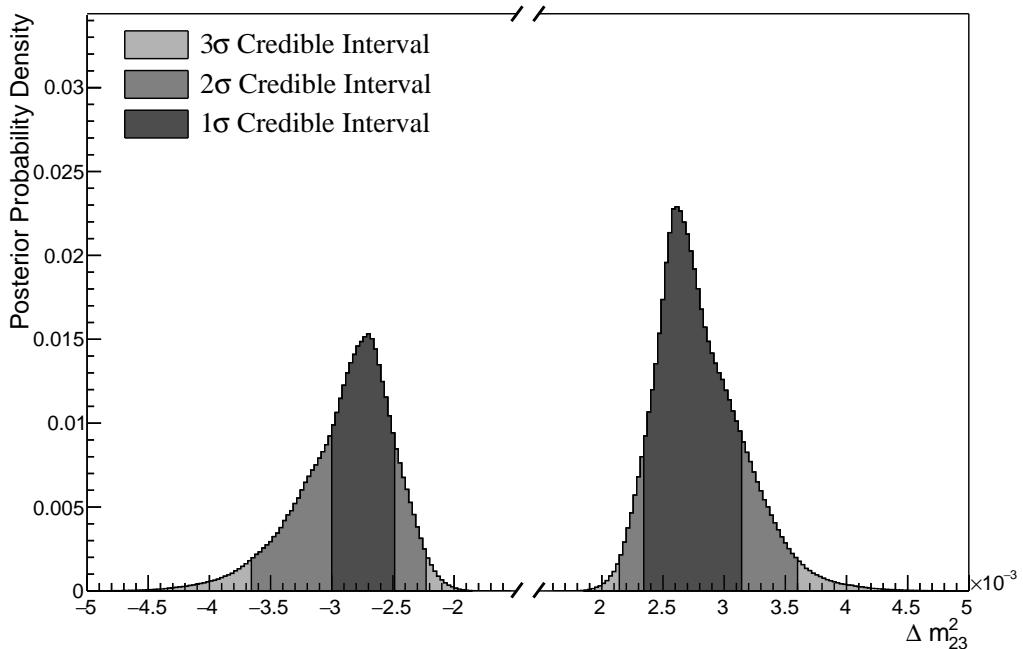


**Figure 8.7:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$ , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

2980 Figure 8.7 illustrates the posterior probability density for  $\delta_{CP}$ , marginalised  
 2981 over both hierarchies. If instead, only steps in the normal hierarchy were  
 2982 considered, the shape of the contours would change. The fit favours the known

oscillation parameter ( $\delta_{CP} = -1.601$ ) although the posterior probability is very flat through the range of  $-\pi < \delta_{CP} < -1$  and  $2 < \delta_{CP} < \pi$ . There is also a region around  $\delta_{CP} \sim 0.4$  which is disfavoured at  $2\sigma$ . This indicates that the SK samples can rule out some parts of the CP conserving parameter space reasonably well, near  $\delta_{CP} \sim 0.4$ , when the true value of  $\delta_{CP} \sim \pi/2$ .

### Without Reactor Constraint, Both Hierarchies



**Figure 8.8:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$ , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

The posterior probability density in  $\Delta m_{32}^2$  is given in Figure 8.8. This distribution includes steps in both the normal hierarchy (NH,  $\Delta m_{32}^2 > 0$ ) and the inverse hierarchy (IH,  $\Delta m_{32}^2 < 0$ ). The highest posterior probability density is found within the NH, which agrees with the known oscillation parameter value. However, all of the credible intervals span both of the hierarchies hypotheses. If instead, only steps in the normal hierarchy were considered, the shape of the contours would change. The known oscillation parameter is  $2.509 \times 10^{-3} \text{ eV}^2$ , which is contained within the  $1\sigma$  credible interval.

	LO ( $\sin^2 \theta_{23} < 0.5$ )	UO ( $\sin^2 \theta_{23} > 0.5$ )	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.17	0.40	0.58
IH ( $\Delta m_{32}^2 < 0$ )	0.13	0.29	0.42
Sum	0.31	0.69	1.00

**Table 8.2:** The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as  $B(\text{NH}/\text{IH}) = 1.37$  and  $B(\text{UO}/\text{LO}) = 2.24$ .

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO,  $\sin^2 \theta_{23} < 0.5$ ) or upper octant (UO,  $\sin^2 \theta_{23} > 0.5$ ). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy model is  $B(\text{NH}/\text{IH}) = 1.37$ . Table 4.1 states this value of the Bayes factor indicates a weak preference for the normal hierarchy model. The Bayes factor for choice of octant is  $B(\text{UO}/\text{LO}) = 2.24$ . This is also classified as a weak preference for the UO. Both of these show that the fit is returning the correct choice of models (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The  $1\sigma$  credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. For the known Asimov value of  $\delta_{CP} = -1.601$ , the  $1\sigma$  credible interval rules out a region between  $\delta_{CP} = -0.86$  and  $\delta_{CP} = 1.96$ , when marginalising over both hierarchies. The position of the highest posterior density is  $\delta_{CP} = -1.57$  which is clearly compatible with the known oscillation parameter value.

The sensitivity of the atmospheric samples to  $\sin^2(\theta_{13})$  is presented in Figure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity

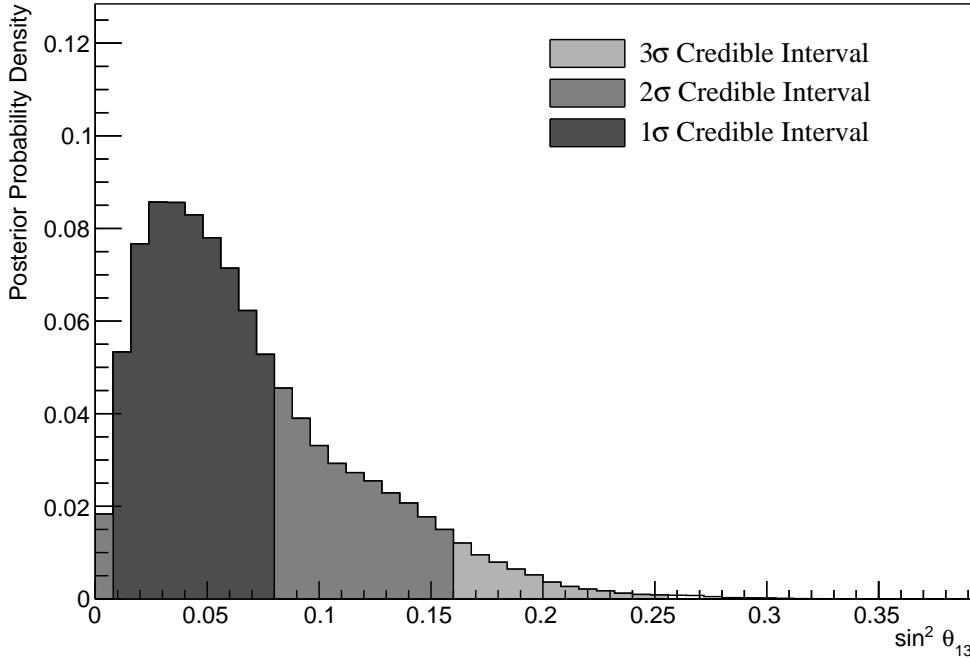
Parameter	Interval	HPD
$\delta_{CP}$ , (BH)	$[-\pi, -0.86], [1.96, \pi]$	-1.57
$\delta_{CP}$ , (NH)	$[-\pi, -0.86], [1.88, \pi]$	-1.57
$\delta_{CP}$ , (IH)	$[-\pi, -0.94], [1.96, \pi]$	-1.57
$\Delta m_{32}^2$ (BH) [ $\times 10^{-3}\text{eV}^2$ ]	$[-3.00, -2.50], [2.35, 3.15]$	2.65
$\Delta m_{32}^2$ (NH) [ $\times 10^{-3}\text{eV}^2$ ]	$[2.39, 3.04]$	2.64
$\Delta m_{32}^2$ (IH) [ $\times 10^{-3}\text{eV}^2$ ]	$[-3.15, -2.45]$	-2.70
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.59]$	0.542
$\sin^2(\theta_{23})$ (NH)	$[0.476, 0.59]$	0.554
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.59]$	0.542

**Table 8.3:** The position of the highest posterior probability density (HPD) and width of the  $1\sigma$  credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3018 to  $\sin^2(\theta_{13})$  will be small. This behaviour is also seen in the fit results, where the  
 3019 width of the  $1\sigma$  credible intervals span the region of  $\sin^2(\theta_{13}) = [0.008, 0.08]$ . This  
 3020 is more than an order of magnitude worse than the constraint from reactor  
 3021 experiments [75].

3022 As previously discussed, the correlations between oscillation parameters are  
 3023 also important to understand how the atmospheric samples respond. Figure 8.10  
 3024 illustrates the two dimensional  $\sin^2(\theta_{13}) - \delta_{CP}$  sensitivity, marginalised over all  
 3025 other parameters. The displayed contours are calculated by marginalising over  
 3026 both hierarchies. The shape of the  $1\sigma$  credible interval shows that the constraining  
 3027 power of the fit on  $\delta_{CP}$  is dependent upon the value of  $\sin^2(\theta_{13})$ . Furthermore,  
 3028 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.  
 3029 Whilst the atmospheric samples do not strongly constrain the value of  $\sin^2(\theta_{13})$ ,  
 3030 the value of  $\sin^2(\theta_{13})$  does impact the atmospheric sensitivity to  $\delta_{CP}$ . A value of  
 3031  $\sin^2(\theta_{13}) \sim 0.02$  would select a continuous contour over all values of  $\delta_{CP}$ . This  
 3032 shows the effect of the marginalisation effect previously described.

### Without Reactor Constraint, Both Hierarchies



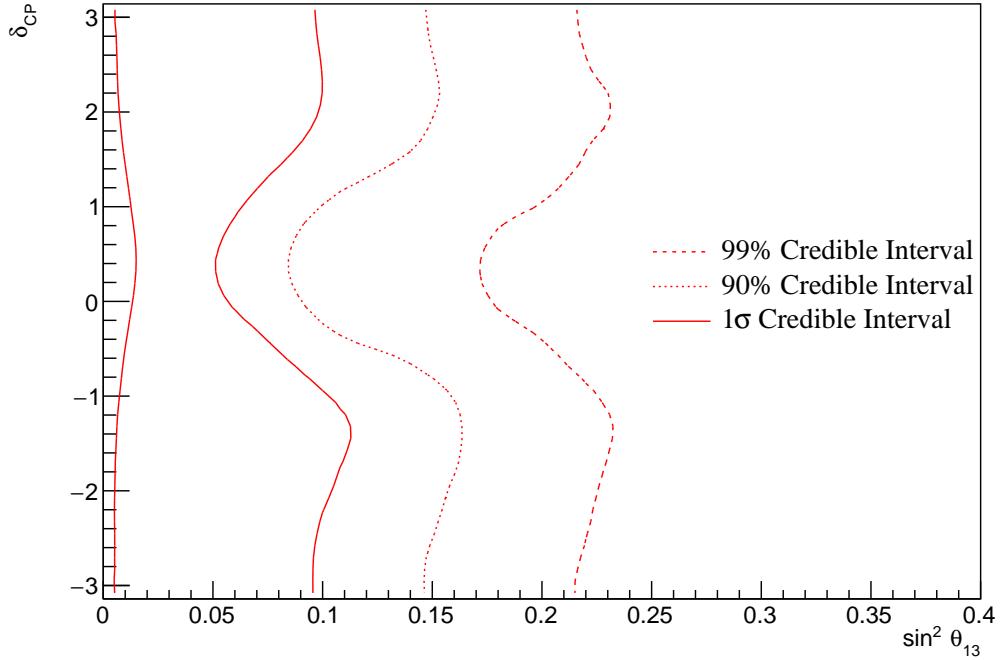
**Figure 8.9:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{13})$ , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3033     The  $\sin^2(\theta_{23}) - \Delta m_{32}^2$  disappearance contours are illustrated in Figure 8.11.

3034     As expected, the area contained in the inverted hierarchy  $1\sigma$  credible interval is  
 3035     slightly smaller than that in the normal hierarchy. This follows from the Bayes  
 3036     factor showing a weak preference for NH meaning that more of the steps will exist  
 3037     in the  $\Delta m_{32}^2 > 0$  region. The known oscillation parameters of  $\sin^2(\theta_{23}) = 0.528$   
 3038     and  $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$  are contained within the  $1\sigma$  credible interval.

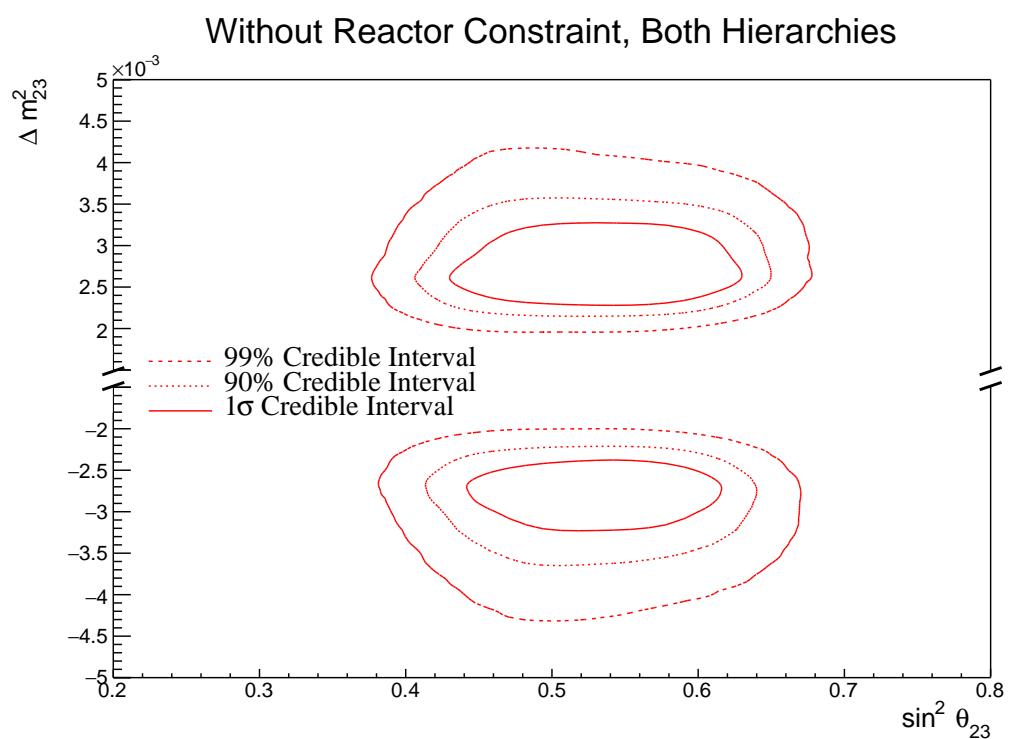
3039     Figure 8.12 illustrates the two-dimensional projections for each permutation of  
 3040     oscillation parameters which this analysis is sensitive to:  $\delta_{CP}$ ,  $\sin^2(\theta_{13})$ ,  $\sin^2(\theta_{23})$ ,  
 3041     and  $\Delta m_{32}^2$ . The purpose of this plot is to illustrate the correlations between the  
 3042     oscillation parameters. The contours are calculated whilst marginalising over  
 3043     both hierarchies, however, only the NH is illustrated when plotting the  $\Delta m_{32}^2$   
 3044     parameter. As expected the correlations play a significant role in these sensitivity  
 3045     measurements, especially the choice of the  $\sin^2(\theta_{13})$  constraint. The application  
 3046     of reactor constraint would be expected to alter both the width and position of the

### Without Reactor Constraint, Both Hierarchies

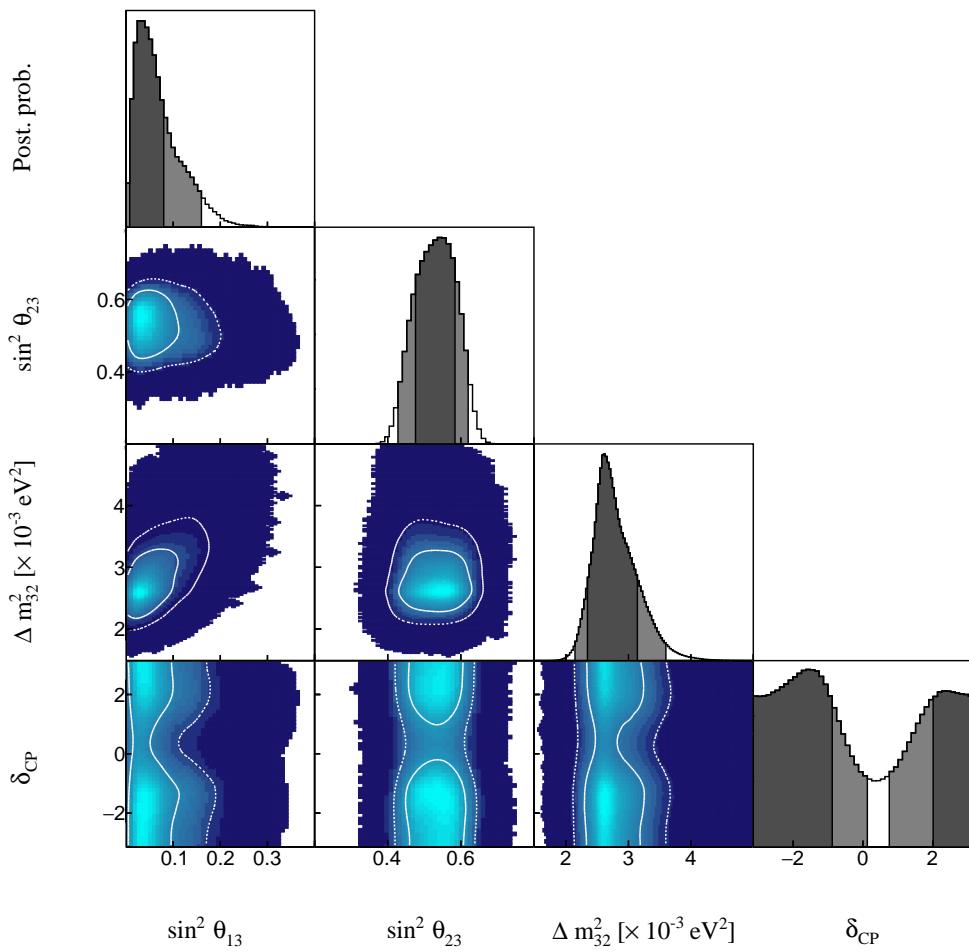


**Figure 8.10:** The two-dimensional posterior probability density distribution in  $\delta_{CP}$  –  $\sin^2(\theta_{13})$ , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3047  $\Delta m_{32}^2$ ,  $\delta_{CP}$ , and  $\sin^2(\theta_{23})$  constraints. The majority of the octant model preference  
 3048 comes from the region of  $\sin^2(\theta_{13}) \sim 0.03$  such that the application of the reactor  
 3049 constraint would not be expected to significantly change the octant preference.  
 3050 The reactor constraint would result in lower values of  $|\Delta m_{32}^2|$ . Interestingly, the  
 3051 distribution of steps in the  $\delta_{CP}$ - $\sin^2(\theta_{13})$  plot is slightly flatter in the region of the  
 3052 reactor constraint. Both the posterior distribution from this fit and the distribution  
 3053 in Figure 8.2 show a region of low negative log-likelihood extending out towards  
 3054 higher values of  $\sin^2(\theta_{13})$  in the  $\delta_{CP} \sim -\pi/2$  and  $\delta_{CP} \sim 2$  region. Consequently,  
 3055 the reactor constraint could feasibly reduce the sensitivity of the atmospheric  
 3056 samples to  $\delta_{CP}$ , due to the previously discussed marginalisation effects.



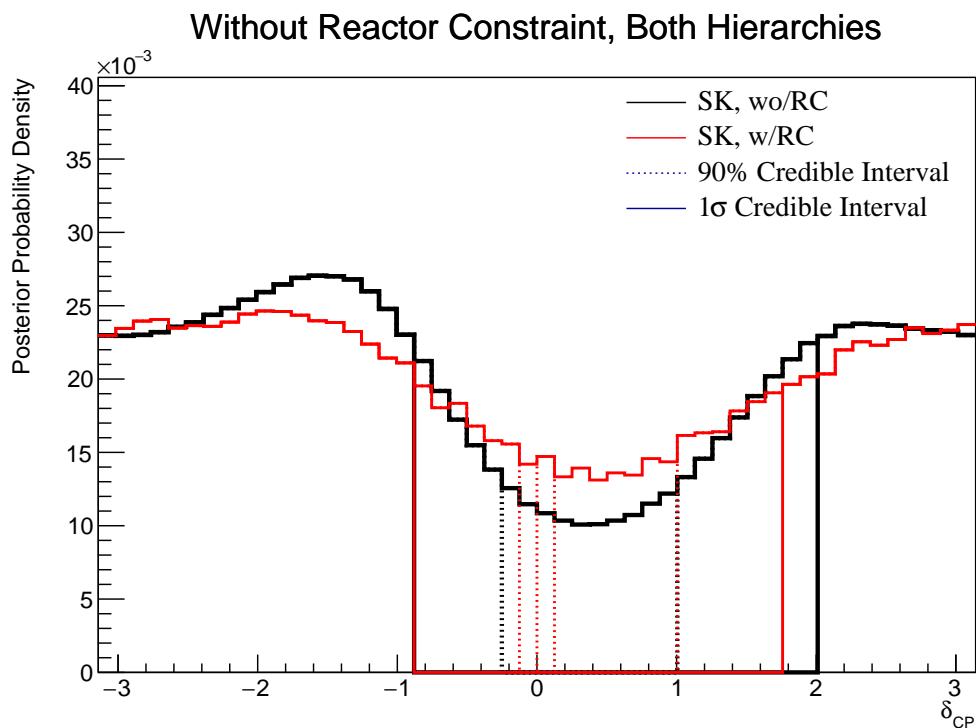
**Figure 8.11:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2 - \sin^2(\theta_{23})$ , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.



**Figure 8.12:** The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

### 3057 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

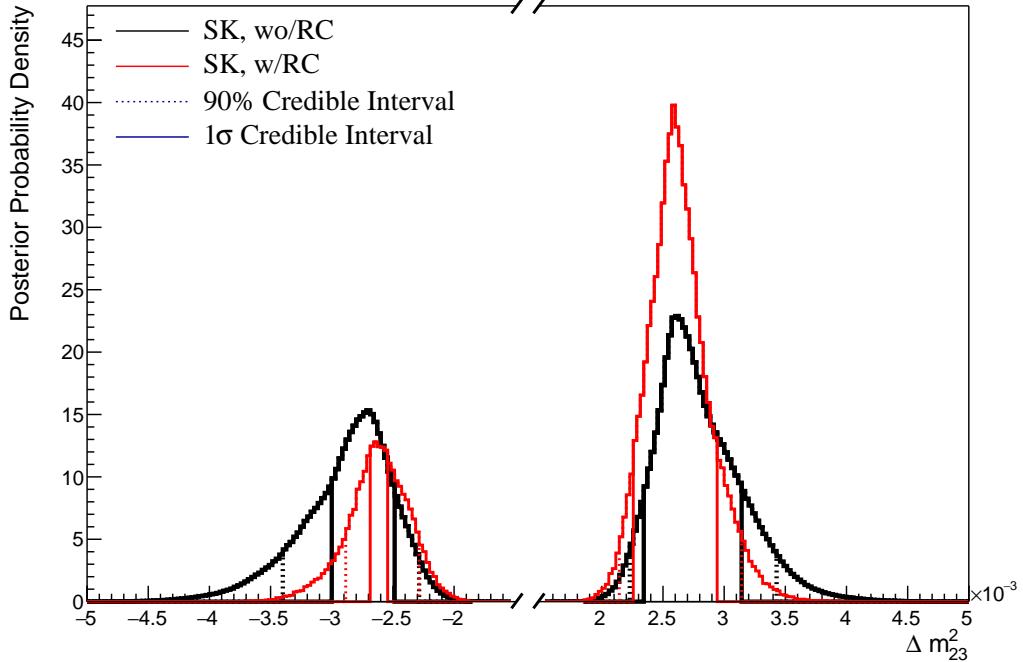
3058 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor  
 3059 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that  
 3060 the marginalisation effects could contribute to differing sensitivities when the  
 3061 external reactor constraint is applied. Using the technique discussed in subsec-  
 3062 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted  
 3063 to include the reactor constraint of  $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$  [75].



**Figure 8.13:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$  compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

3064 Figure 8.13 illustrates the sensitivity to  $\delta_{CP}$  of the atmospheric fit with reactor  
 3065 constraint applied. The distribution is less peaked than the previous results.  
 3066 This is due to the expected marginalisation effect previously discussed. The  
 3067 width of the  $1\sigma$  credible interval is increased when the reactor constraint is  
 3068 applied, indicating less sensitivity to  $\delta_{CP}$  in the region of  $\sin^2(\theta_{13})$  preferred  
 3069 by the reactor constraint.

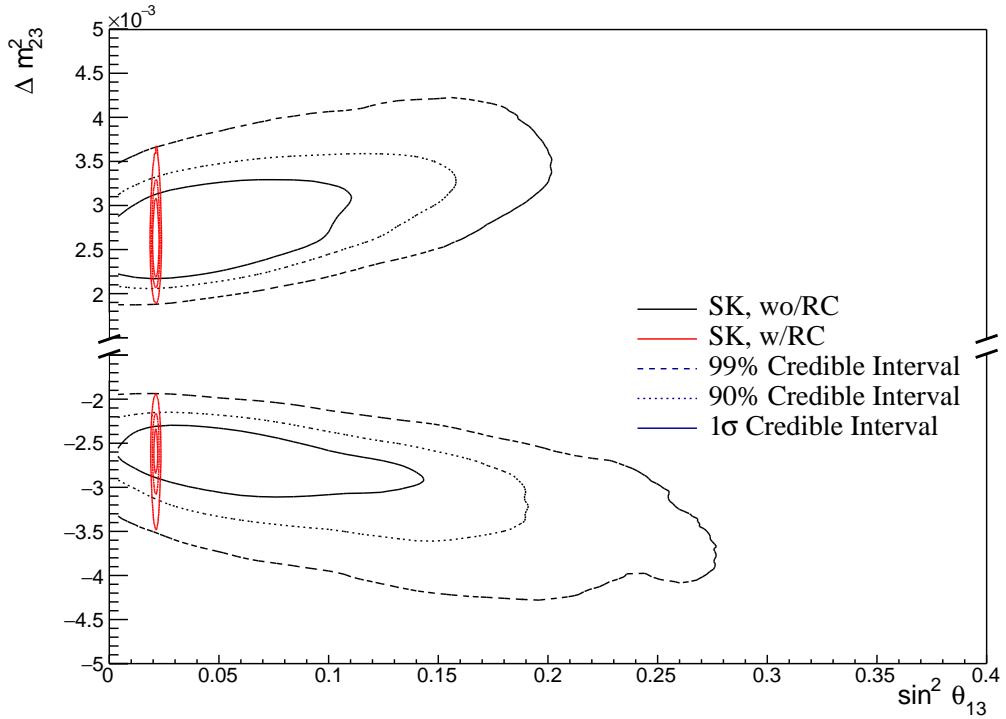
### Without Reactor Constraint, Both Hierarchies



**Figure 8.14:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

The reactor constraint increases the sensitivity of the atmospheric samples to  $\Delta m_{32}^2$  as illustrated in Figure 8.14. The  $1\sigma$  credible interval in  $\Delta m_{32}^2$  is determined to be  $[-2.70, -2.55] \times 10^{-3}\text{eV}^2$  and  $[2.25, 2.95] \times 10^{-3}\text{eV}^2$ . The width of the IH credible interval is reduced by  $\sim 70\%$  when the reactor constraint is applied. Due to the marginalisation effects observed in Figure 8.12, the favoured region of  $\Delta m_{32}^2$  moves closer to zero for both hierarchies. A clear explanation of this behaviour is illustrated in Figure 8.15 which illustrates the posterior distribution in the  $\Delta m_{32}^2 - \sin^2(\theta_{13})$  parameters, marginalised over both hierarchies. The correlation between  $\Delta m_{32}^2$  and  $\sin^2(\theta_{13})$  is such that lower values of  $\sin^2(\theta_{13})$  tend towards lower values of  $|\Delta m_{32}^2|$ . This moves the posterior distribution towards the known oscillation parameter  $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$ .

Table 8.4 presents the fraction of steps in each hierarchy and octant model for the fit after the reactor constraint has been applied. The reactor constraint significantly increases the NH preference, increasing the Bayes factor from



**Figure 8.15:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  –  $\sin^2(\theta_{13})$  compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies.

	LO ( $\sin^2 \theta_{23} < 0.5$ )	UO ( $\sin^2 \theta_{23} > 0.5$ )	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.21	0.53	0.74
IH ( $\Delta m_{32}^2 < 0$ )	0.08	0.18	0.26
Sum	0.29	0.71	1.00

**Table 8.4:** The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as  $B(\text{NH}/\text{IH}) = 2.86$  and  $B(\text{UO}/\text{LO}) = 2.39$ .

3084  $B(\text{NH}/\text{IH}) = 1.37$  to  $B(\text{NH}/\text{IH}) = 2.86$  when the reactor constraint is applied.

3085 This is still defined as a weak preference for NH hypothesis according to Jeffrey's

3086 scale (see Table 4.1), however, it is a stronger preference than when the constraint

3087 is not applied. The preference for the correct octant model is slightly increased

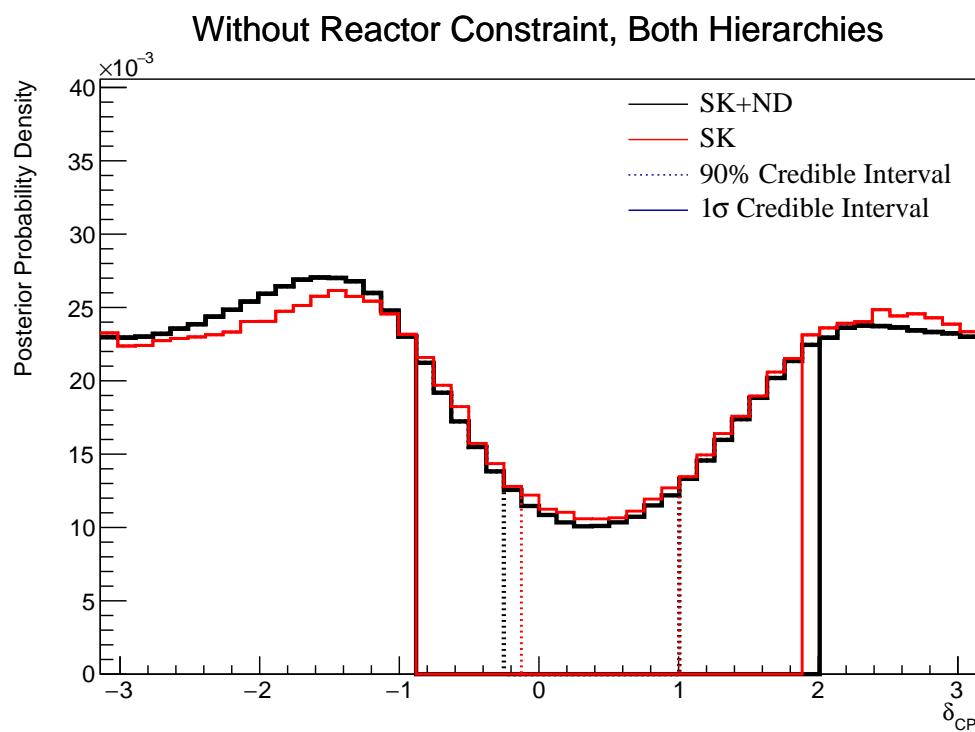
3088 by the application of the reactor constraint which is consistent with expectation.

3089 However, the conclusion that would be made does not significantly change.

### 3090    8.3.3 Application of Near Detector Constraints for Atmospheric 3091    Samples

3092    The choice of applying the near detector constraints to the low-energy atmo-  
3093    spheric samples was introduced in subsection 6.4.3. This subsection illustrates  
3094    the effect of that choice on the sensitivities of the atmospheric samples to the  
3095    oscillation parameters. This Asimov data was generated assuming the ‘AsimovA’  
3096    oscillation parameter set defined in Table 2.2 and the post-BANFF systematic  
3097    parameter tune.

3098    The change in sensitivity on  $\delta_{CP}$  is given in Figure 8.16. The reactor constraint  
3099    is not applied in either of the fits within this comparison. The shape of the  
3100    posterior is similar although less peaked at the Asimov point ( $\delta_{CP} = -1.601$ )  
3101    and more symmetric between the regions of  $\delta_{CP} = -1.601$  and  $\delta_{CP} \sim 2.5$ . The  
3102    width of the  $1\sigma$  credible intervals are approximately the same (identical to within  
3103    a bin width) and the same conclusion holds for the higher credible intervals. The  
3104    change in sensitivity to other oscillation parameters has been studied and no  
3105    significant discrepancies were found. As expected, the sensitivities are statistics  
3106    dominated and the exact choice of systematic model and constraint does not  
3107    significantly affect the physics conclusions one would make from this analysis.

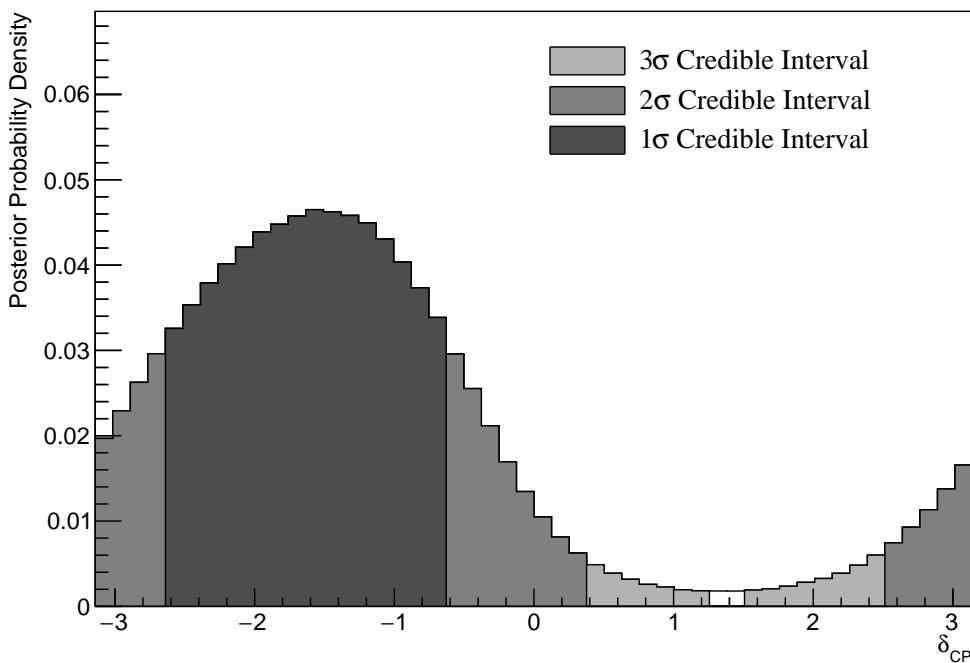


**Figure 8.16:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$  compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies.

### 3108 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3109 straint

3110 This section presents the sensitivities of the simultaneous beam and atmospheric  
 3111 analysis where the reactor constraint is not applied. Similar to the previous  
 3112 studies, the Asimov data is built assuming the post-BANFF cross-section tune  
 3113 and Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near  
 3114 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.  
 3115 The sensitivity to  $\delta_{CP}$ , marginalised over both hierarchies, is given in Figure 8.17.

Without Reactor Constraint, Both Hierarchies



**Figure 8.17:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$ , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

3116 The credible intervals and highest posterior distribution for each oscillation  
 3117 parameter is given in Table 8.5. The highest posterior probability density is  
 3118  $\delta_{CP} = -1.58$  and is compatible with the known Asimov A value of  $\delta_{CP} = -1.601$ .  
 3119 The CP-conserving values of  $\delta_{CP} = 0, \pi, -\pi$  are disfavoured at 1 $\sigma$  credible  
 3120 interval. There is also a region around  $\delta_{CP} = 1.4$  which is disfavoured at more  
 3121 than 3 $\sigma$ . Whilst these conclusions can only be made at this particular Asimov

point, it does show that if the true value of  $\delta_{CP}$  was CP-violating, this joint analysis would be able to disfavour CP conserving values at over  $1\sigma$  without any external constraints. The highest posterior probability density does move further away from the Asimov point when only steps in the NH region are considered. This is due to the correlations between the value of  $\delta_{CP}$  and the mass hierarchy, as will be later discussed.

Parameter	Interval	HPD
$\delta_{CP}$ , (BH)	[−2.64, −0.63]	-1.57
$\delta_{CP}$ , (NH)	[−2.76, −0.63]	-1.45
$\delta_{CP}$ , (IH)	[−2.39, −0.88]	-1.57
$\Delta m_{32}^2$ (BH) [ $\times 10^{-3}\text{eV}^2$ ]	[2.46, 2.58]	2.49
$\Delta m_{32}^2$ (NH) [ $\times 10^{-3}\text{eV}^2$ ]	[2.48, 2.56]	2.51
$\Delta m_{32}^2$ (IH) [ $\times 10^{-3}\text{eV}^2$ ]	[−2.60, −2.52]	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (NH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (IH)	[0.48, 0.55]	0.521

**Table 8.5:** The position of the highest posterior probability density (HPD) and width of the  $1\sigma$  credible interval for the joint beam and atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

The sensitivity to  $\Delta m_{32}^2$  is illustrated in Figure 8.18, marginalised over both hierarchies. Notably, the  $1\sigma$  credible interval is entirely contained within the normal hierarchy region, as illustrated in Table 8.5. This illustrates reasonable sensitivity to the mass hierarchy model. This is also reflected in the  $1\sigma$  credible intervals being approximately the same when they are made considering both hierarchies and when considering only the NH. The known oscillation parameter is  $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$ . The normal hierarchy distribution favours this value with the highest posterior probability density of  $\Delta m_{32}^2 = 2.51 \times 10^{-3}\text{eV}^2$ .

The fraction of steps in each of the mass hierarchy regions and octants of  $\sin^2(\theta_{23})$  is given in Table 8.6. The Bayes factors are determined to be  $B(\text{NH}/\text{IH}) =$

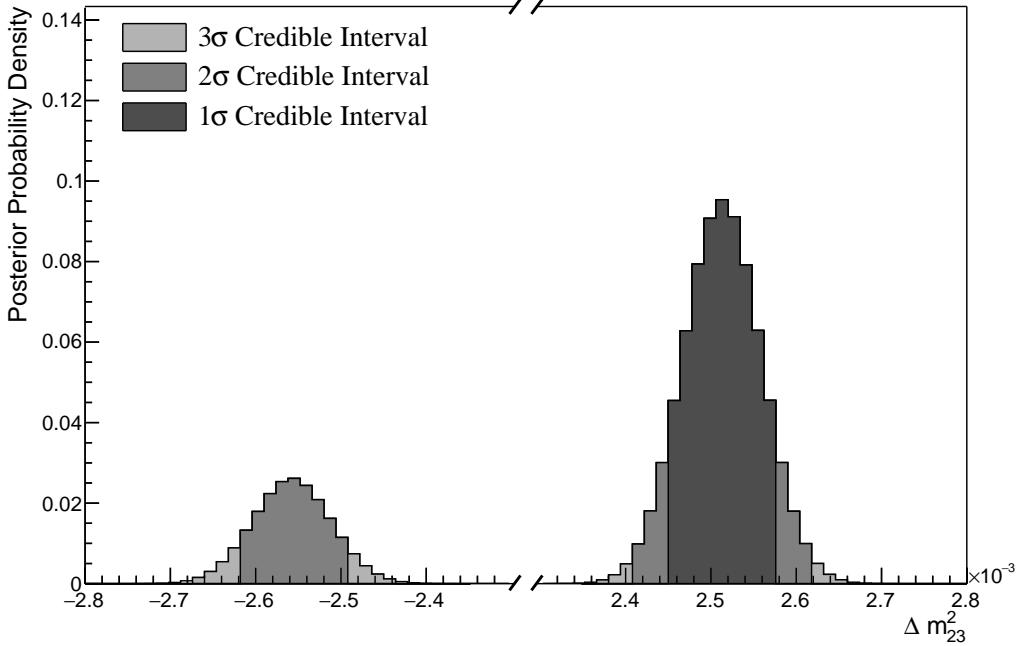
3138 3.67 and  $B(\text{UO}/\text{LO}) = 1.74$ . Jeffrey's scale (presented in Table 4.1) states that  
 3139 this value of the hierarchy Bayes factor illustrates substantial evidence for the  
 3140 normal hierarchy hypothesis. This corresponds to the correct hypothesis given  
 3141 the known oscillation parameters. It is a stronger statement than the atmospheric-  
 3142 only analysis can provide. It is important to note that this is a substantial  
 3143 preference that requires no external constraints required. The Bayes factor for  
 3144 octant determination represents a weak preference for the upper octant but does  
 3145 select the correct octant model.

	LO ( $\sin^2 \theta_{23} < 0.5$ )	UO ( $\sin^2 \theta_{23} > 0.5$ )	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.29	0.50	0.79
IH ( $\Delta m_{32}^2 < 0$ )	0.08	0.13	0.21
Sum	0.37	0.63	1.00

**Table 8.6:** The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as  $B(\text{NH}/\text{IH}) = 3.67$  and  $B(\text{UO}/\text{LO}) = 1.74$ .

3146 The sensitivity to  $\sin^2(\theta_{23})$  is presented in Figure 8.19. There is a clear  
 3147 preference for the upper octant but the peak of the distribution is relatively  
 3148 flat. It peaks at  $\sin^2(\theta_{23}) = 0.509$  which is in the region of the known value of  
 3149  $\sin^2(\theta_{23}) = 0.528$ . The difference in the highest posterior distribution and the  
 3150 width of the credible interval is relatively unchanged when considering different  
 3151 hierarchy models showing no strong correlation between  $\sin^2(\theta_{23})$  and  $|\Delta m_{32}^2|$ .  
 3152 The sensitivity presented as a function of the appearance parameters ( $\sin^2(\theta_{13}) -$   
 3153  $\delta_{CP}$ ) is given in Figure 8.20. As expected, the contours follow that given in  
 3154 Figure 8.2, where the  $2\sigma$  credible intervals have a closed contour excluding the  
 3155 region around  $\delta_{CP} \sim 1.2$ . The width of the  $3\sigma$  credible interval is also clearly  
 3156 dependent upon the value of  $\delta_{CP}$ . Close to the Asimov point,  $\delta_{CP} = -1.601$ , the  
 3157 width of the  $3\sigma$  credible interval approximately spans  $\sin^2(\theta_{13}) = [0.013, 0.04]$ .  
 3158 This is reduced to a region of  $\sin^2(\theta_{13}) = [0.023, 0.042]$  at the most disfavoured  
 3159 value of  $\delta_{CP}$ . This follows the behaviour shown in the likelihood scans. The  $1\sigma$

### Without Reactor Constraint, Both Hierarchies



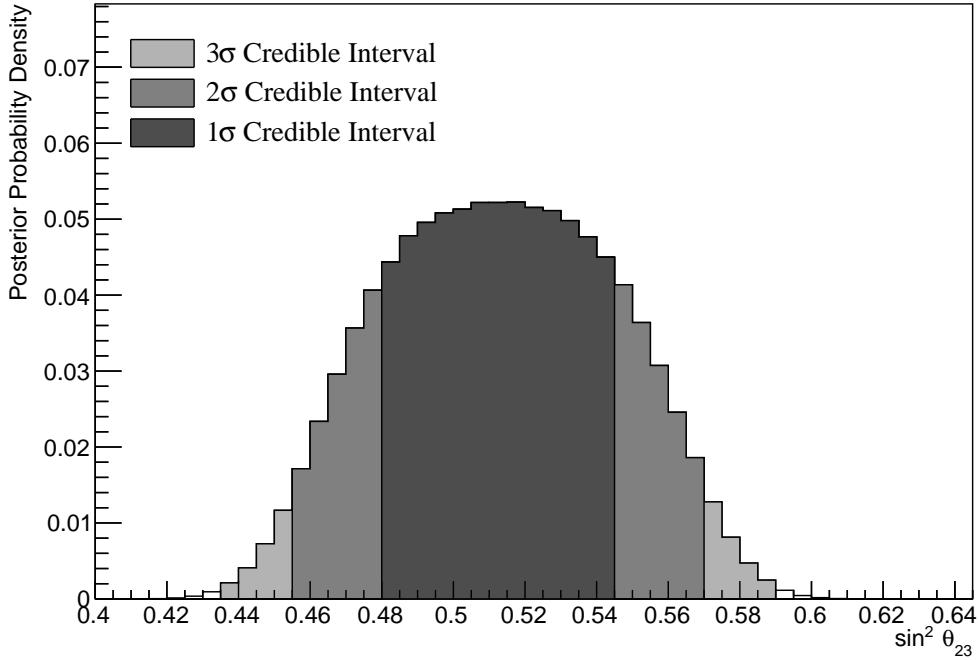
**Figure 8.18:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$ , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

credible interval is consistent with both the known oscillation parameter and the reactor constraint ( $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ ). Application of the reactor constraint would be expected to decrease the width of the 1 $\sigma$  credible intervals of  $\delta_{CP}$  due to the triangular shape of the posterior probability.

The sensitivity in terms of the ‘disappearance’ parameters marginalised over both hierarchies is given in Figure 8.21. The area contained within the IH credible intervals is significantly smaller than those in the NH region. This is reflected in the IH credible intervals being tighter in the  $\sin^2(\theta_{23})$  dimension. No significant correlation is observed between the value of  $\sin^2(\theta_{23})$  and  $|\Delta m_{32}^2|$ .

The two-dimensional posterior distribution for each permutation of the oscillation parameters of interest is given in Figure 8.22. The most notable observation is that the  $\sin^2(\theta_{13})$  and  $\sin^2(\theta_{23})$  are anti-correlated. If the value of  $\sin^2(\theta_{13})$  was known to be closer to the known oscillation parameter value, the preferred value of  $\sin^2(\theta_{23})$  would increase furthering the preference for the UO. That

### Without Reactor Constraint, Both Hierarchies



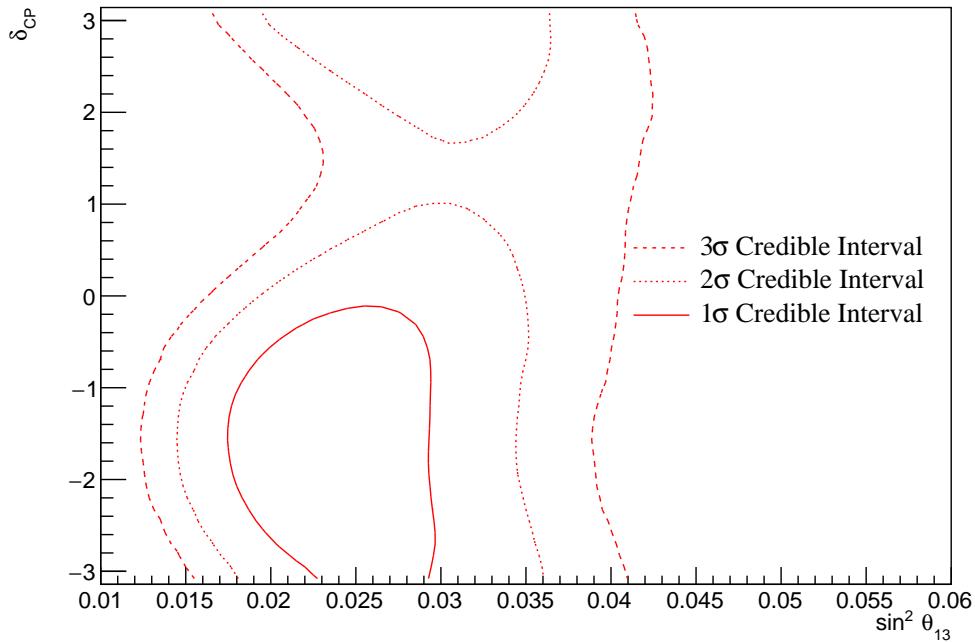
**Figure 8.19:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$ , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied.

would move the highest posterior probability closer in line with the Asimov value. This also means that the preference for the UO would be increased if the reactor constraint was to be applied.

Furthermore, the  $\delta_{CP}$  and  $|\Delta m_{32}^2|$  oscillation parameters are anti-correlated, such that higher values of  $|\Delta m_{32}^2|$  prefer lower values of  $\delta_{CP}$ . Whilst this is an interesting result on its own, the width of the  $\Delta m_{32}^2$  contours also depend on  $\sin^2(\theta_{13})$ . This introduces another correlation effect that could modify the sensitivity to  $\delta_{CP}$  once the reactor constraint is applied.

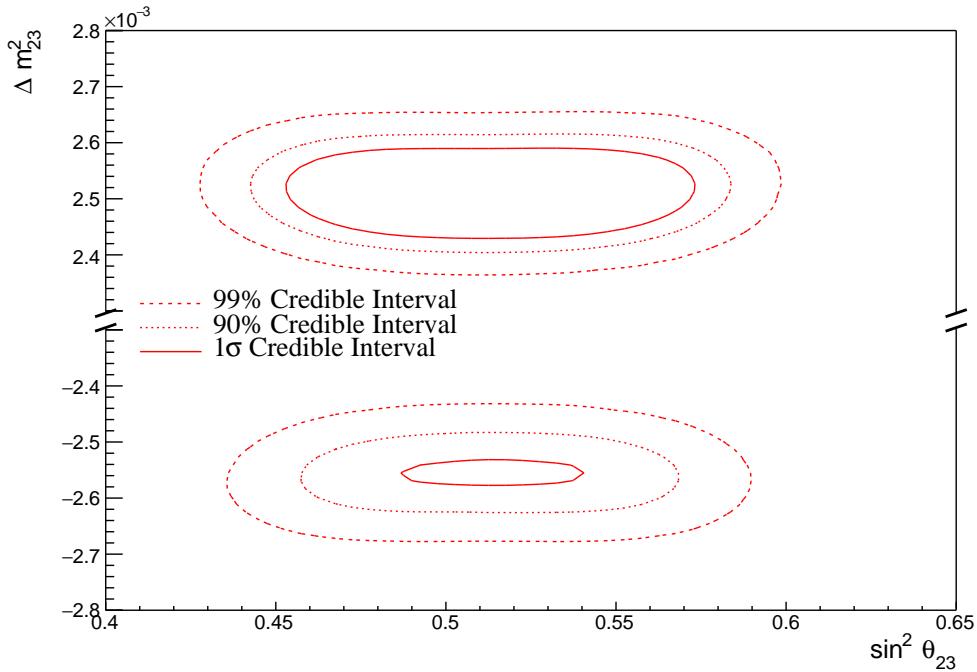
The correlation between  $\sin^2(\theta_{13})$  and  $\Delta m_{32}^2$  can be seen in Figure 8.23. A much larger fraction of the posterior distribution is contained in the NH for lower values of  $\sin^2(\theta_{13})$ . Consequently, the application of the reactor constraint would be expected to significantly increase the preference for NH.

### Without Reactor Constraint, Both Hierarchies

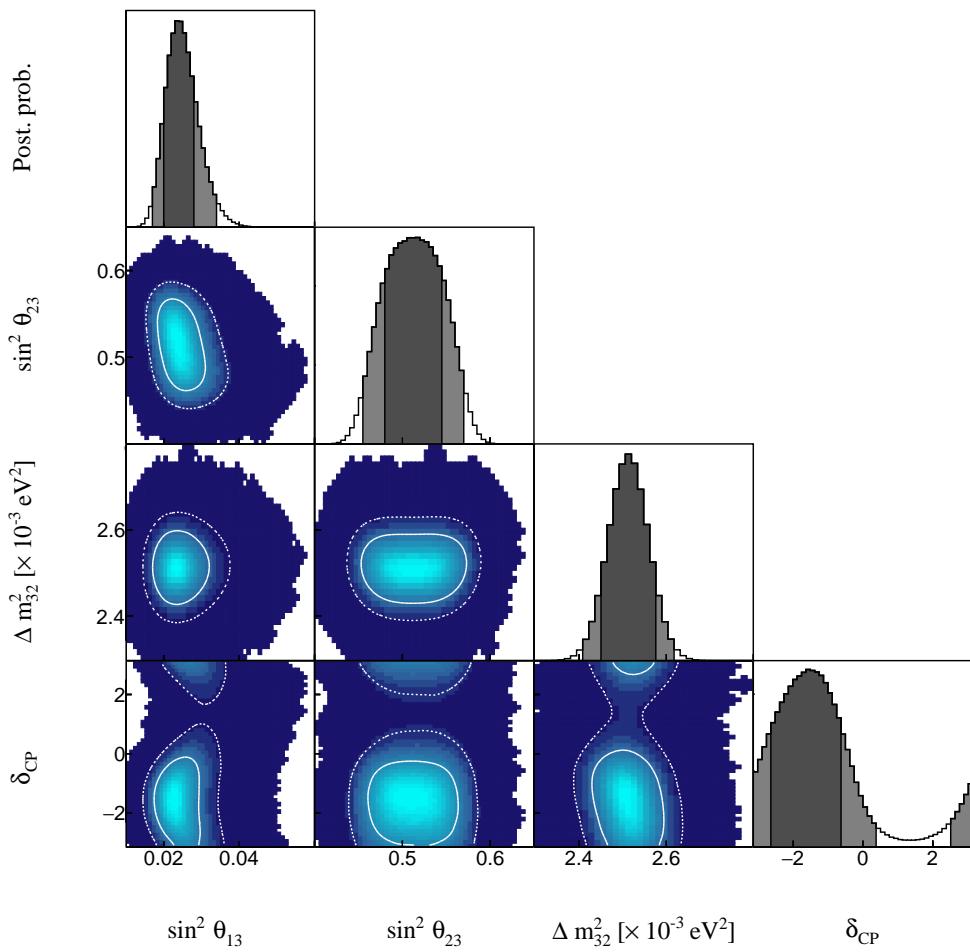


**Figure 8.20:** The two-dimensional posterior probability density distribution in  $\delta_{CP}$  –  $\sin^2(\theta_{13})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

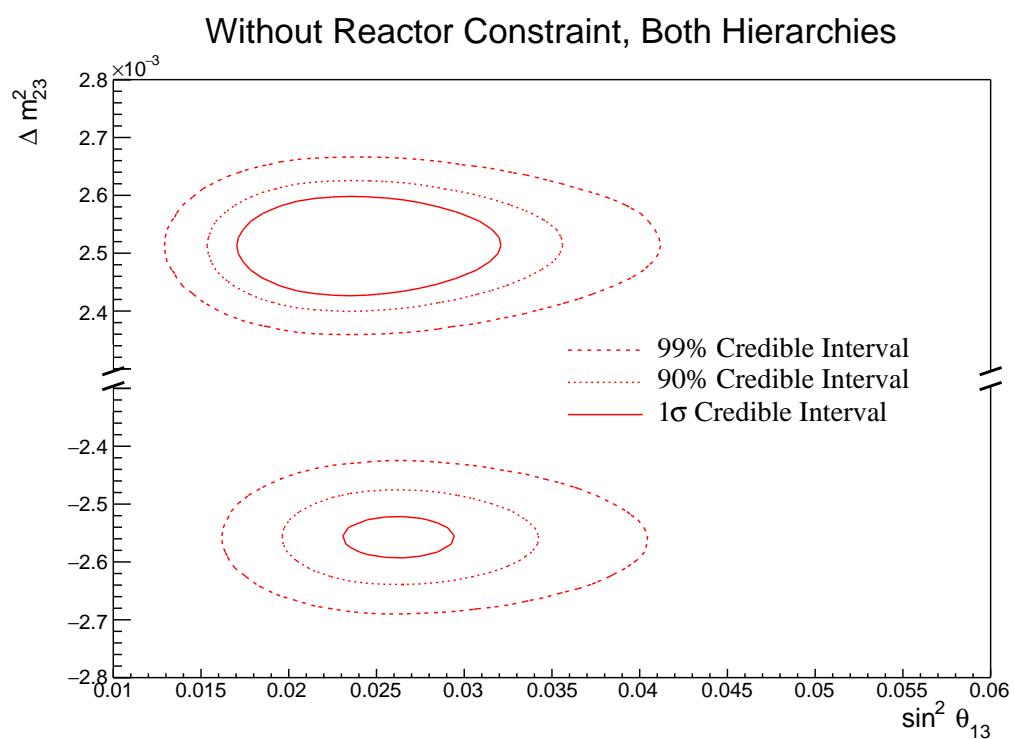
### Without Reactor Constraint, Both Hierarchies



**Figure 8.21:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  –  $\sin^2(\theta_{23})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.



**Figure 8.22:** The posterior probability density distribution from the joint beam and atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

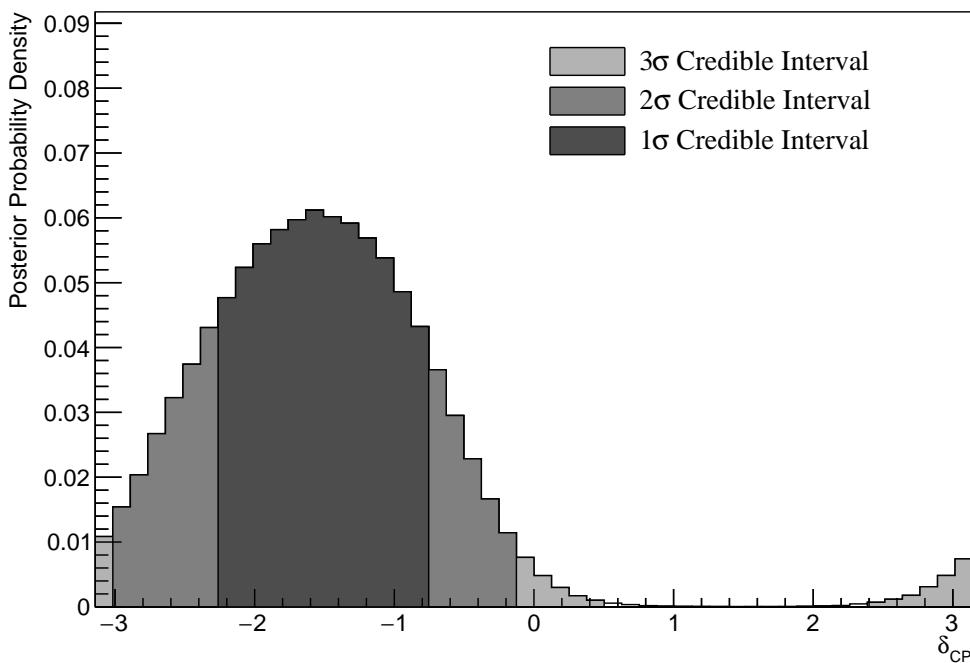


**Figure 8.23:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2 - \sin^2(\theta_{13})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

### 3186 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3187 This section presents the sensitivities of the joint beam and atmospheric fit when  
 3188 the reactor constraint is applied to  $\sin^2(\theta_{13})$ . As with the previous studies, the  
 3189 Asimov data is made using the AsimovA oscillation parameter set defined in  
 3190 Table 2.2 and the post-BANFF systematic parameter tune.

With Reactor Constraint, Both Hierarchies

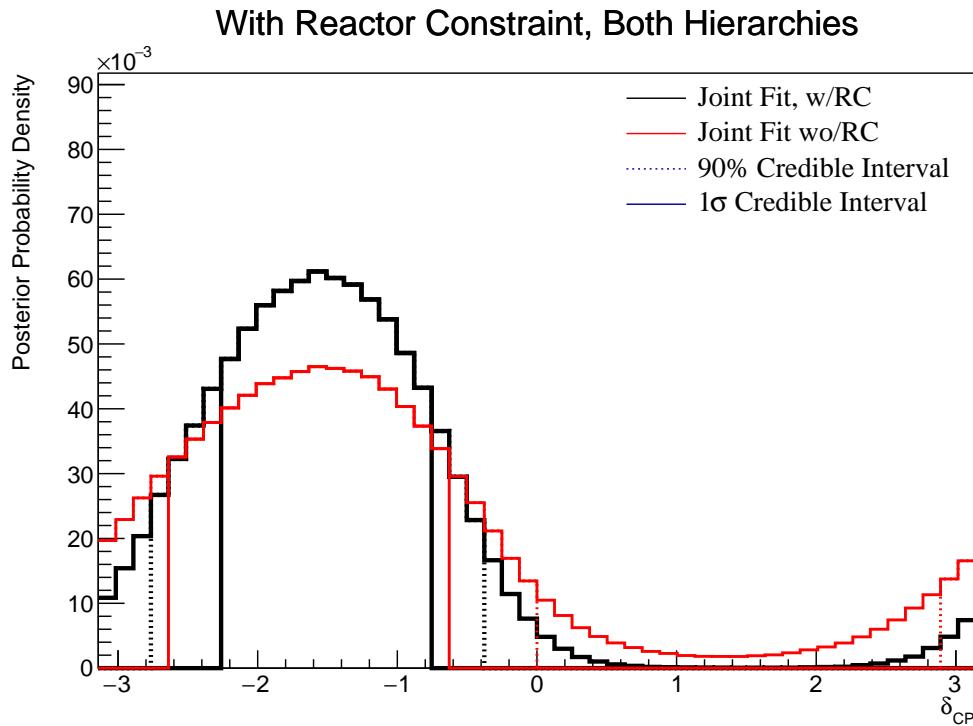


**Figure 8.24:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$ , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3191 Figure 8.24 illustrates the sensitivity to  $\delta_{CP}$ , marginalised over both hierarchies.  
 3192 The CP-conserving values of  $\delta_{CP} = -\pi, 0, \pi$  are disfavoured at  $2\sigma$ . Furthermore,  
 3193 the  $3\sigma$  credible interval excludes the region of  $\delta_{CP} = [0.50, 2.39]$ . Thus clearly  
 3194 disfavouring the region of  $\delta_{CP} = \pi/2$  at more than  $3\sigma$  for this particular set  
 3195 of known oscillation parameters. The width of the  $1\sigma$  credible intervals and  
 3196 the position of the highest posterior probability density is given in Table 8.7.  
 3197 The highest posterior probability density in  $\delta_{CP}$  is calculated as  $\delta_{CP} = -1.57$   
 3198 showing no significant biases in the determination of the known oscillation

parameters. The posterior distribution is more peaked around the known oscillation parameter value of  $\delta_{CP} = -1.601$ , as compared to the sensitivities when the reactor constraint is not applied (subsection 8.3.4). This follows from the correlations shown in Figure 8.20, where a lower value of  $\sin^2(\theta_{13})$  results in tighter constraints on  $\delta_{CP}$ .

The effect of applying the reactor constraint for  $\delta_{CP}$  in the joint beam-atmospheric fit is presented in Figure 8.25. The posterior distribution from the two fits are marginalised over both hierarchies. Clearly, the reactor constraint improves the ability of the fit to select the known oscillation parameter as the shape of the distribution is much more peaked. This is also evidenced by the tightening of the  $1\sigma$  and 90% credible intervals. Additionally, the disfavoured region of  $1 < \delta_{CP} < 2$  is wider when the reactor constraint is applied.



**Figure 8.25:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$  compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

The sensitivity to  $\sin^2(\theta_{23})$ , marginalised over both hierarchies, is given in Figure 8.26. The highest posterior probability density is located at  $\sin^2(\theta_{23}) = 0.527$

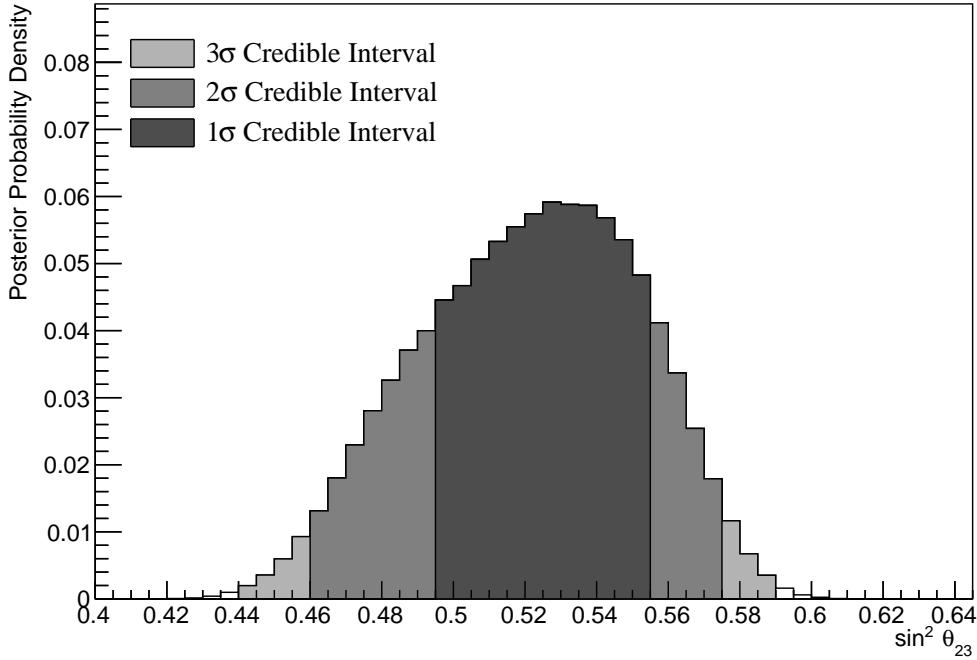
Parameter	Interval	HPD
$\delta_{CP}$ , (BH)	[-2.26, -0.75]	-1.57
$\delta_{CP}$ , (NH)	[-2.26, -0.75]	-1.57
$\delta_{CP}$ , (IH)	[-2.13, -1.00]	-1.57
$\Delta m_{32}^2$ (BH) [ $\times 10^{-3}\text{eV}^2$ ]	[2.46, 2.52]	2.49
$\Delta m_{32}^2$ (NH) [ $\times 10^{-3}\text{eV}^2$ ]	[2.48, 2.56]	2.51
$\Delta m_{32}^2$ (IH) [ $\times 10^{-3}\text{eV}^2$ ]	[-2.60, -2.52]	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (NH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (IH)	[0.50, 0.56]	0.539

**Table 8.7:** The position of the highest posterior probability density (HPD) and width of the  $1\sigma$  credible interval for the joint beam and atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

which agrees with the known value of  $\sin^2(\theta_{23}) = 0.528$ . The distribution clearly favours the UO with almost the entirety of the  $1\sigma$  credible interval contained in the region. Figure 8.27 highlights the sensitivity of the joint fit both with and without the reactor constraint. The fit where the reactor constraint is applied selects the known value much better ( $\sin^2(\theta_{23}) = 0.528$ ). Furthermore, the reactor constraint increases the UO preference which is evidenced by the distribution moving further away from the octant boundary. This indicates that there are marginalisation effects between the two mixing parameters. This follows from the correlation illustrated between  $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$  in Figure 8.22. The posterior distribution of the fit with reactor constraint is more peaked compared to the flatter distribution when the reactor constraint is not applied.

The fraction of steps contained within the two hierarchy and two octant models is given in Table 8.8. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.02, whilst significantly increasing the fraction of steps within the NH-UO region from 0.53 to 0.64. The application of the reactor constraint increases the Bayes factor

### With Reactor Constraint, Both Hierarchies



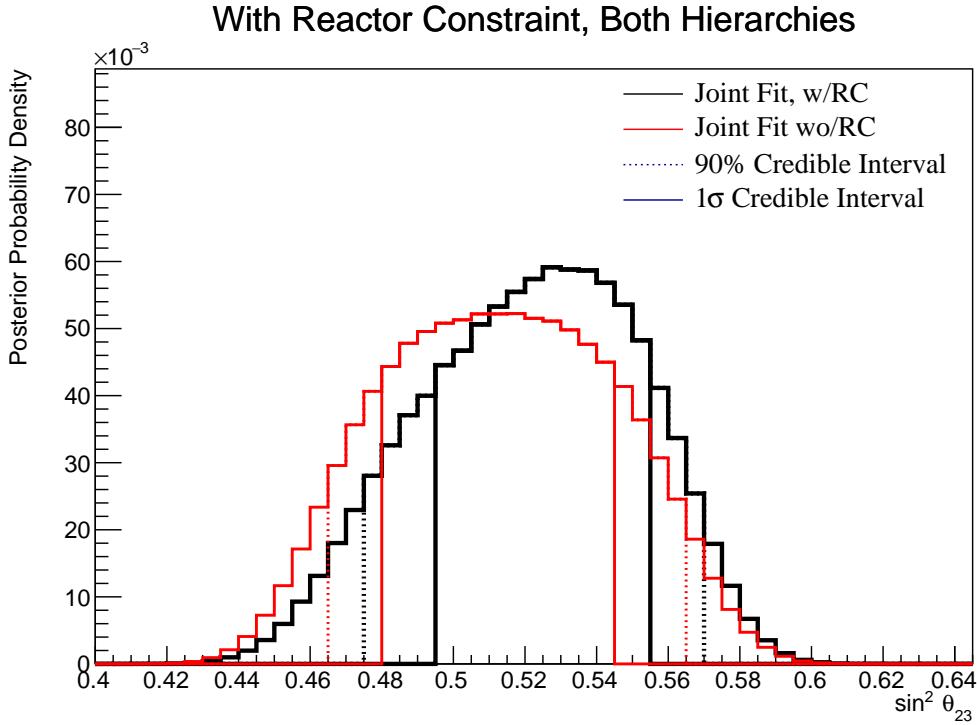
**Figure 8.26:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

from  $B(\text{NH}/\text{IH}) = 3.67$  to  $B(\text{NH}/\text{IH}) = 7.29$ . There is a very clear preference for the NH, with the Jeffreys scale stating a substantial preference for both fits (see subsection 4.3.3). The Bayes factor for UO preference is calculated as  $B(\text{UO}/\text{LO}) = 2.86$ . Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

	LO ( $\sin^2 \theta_{23} < 0.5$ )	UO ( $\sin^2 \theta_{23} > 0.5$ )	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.24	0.64	0.88
IH ( $\Delta m_{32}^2 < 0$ )	0.02	0.10	0.12
Sum	0.26	0.74	1.00

**Table 8.8:** The distribution of steps in a joint beam and atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as  $B(\text{NH}/\text{IH}) = 7.29$  and  $B(\text{UO}/\text{LO}) = 2.86$ .

The sensitivity to  $\Delta m_{32}^2$ , with the reactor constraint applied, is presented in



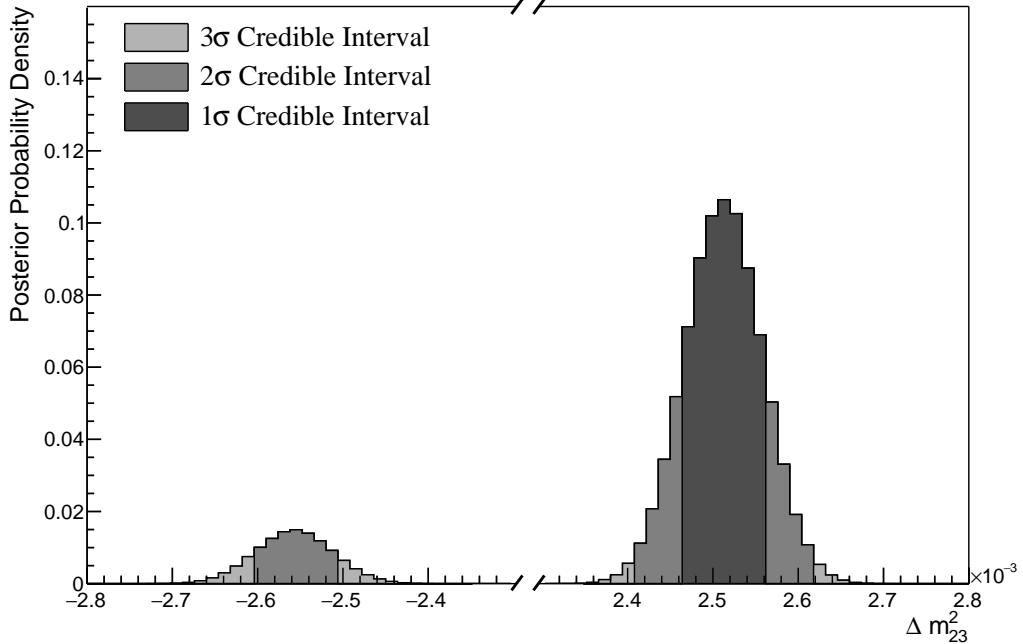
**Figure 8.27:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

3235 Figure 8.28. The posterior distribution is marginalised over both hierarchies. As  
 3236 expected, the  $1\sigma$  credible interval is entirely contained within the NH region. The  
 3237 position of the highest posterior probability density is given as  $2.49 \times 10^{-3} \text{ eV}^2$ ,  
 3238 illustrating no significant bias between the fit results and the known oscillation  
 3239 parameters. The application of the reactor constraint does move significantly the  
 3240 position of the credible intervals but does reduce their width.

3241 The sensitivity to the appearance parameters ( $\sin^2(\theta_{13}) - \delta_{CP}$ ) is given in  
 3242 Figure 8.29. The distribution is mostly uncorrelated between the two parameters  
 3243 and is centered at the known oscillation parameters. The  $1\sigma$  credible interval  
 3244 excludes  $\delta_{CP} = 0$  and  $\delta_{CP} = (-)\pi$ . Furthermore, the  $3\sigma$  credible intervals  
 3245 exclude the region of  $\delta_{CP} = \pi/2$ .

3246 The sensitivity to the disappearance parameters ( $\sin^2(\theta_{23}) - \Delta m_{32}^2$ ) is illus-  
 3247 trated in Figure 8.30. As expected from the one-dimensional distribution, the  $1\sigma$   
 3248 credible interval is entirely contained within the NH region. Both the NH and

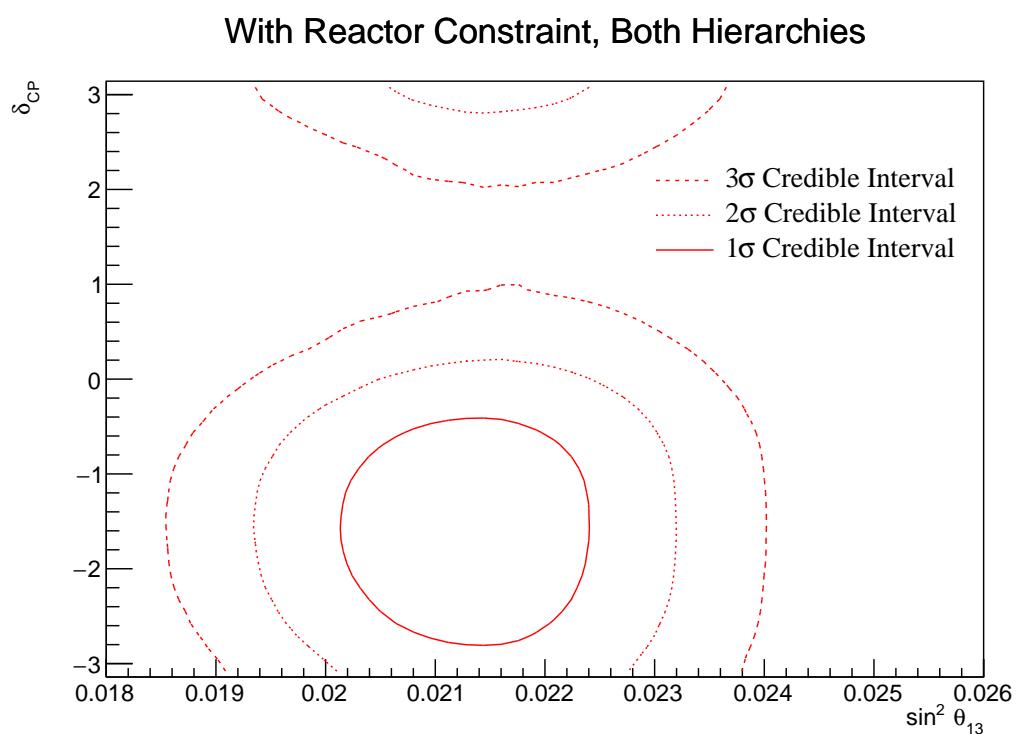
### With Reactor Constraint, Both Hierarchies



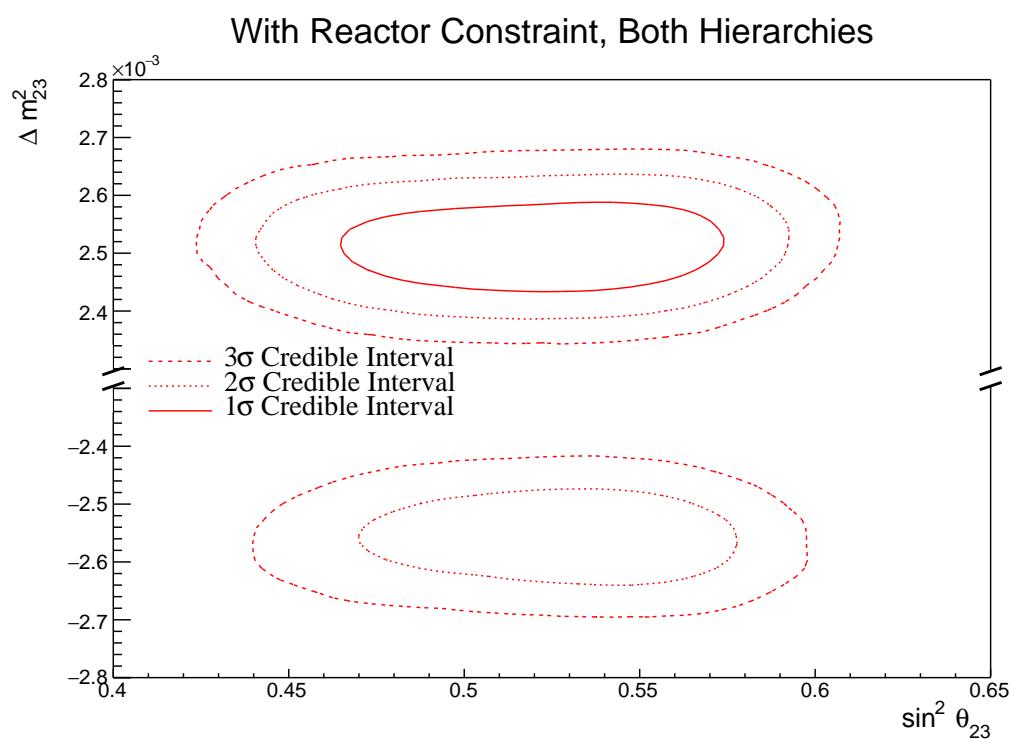
**Figure 8.28:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$ , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3249 IH regions favour the UO, with a visually similar preference in both hierarchies.  
 3250 The width of the  $\Delta m_{32}^2$  1 $\sigma$  credible interval does not significantly depend upon  
 3251 the value or octant of  $\sin^2(\theta_{23})$ . This shows that there are no strong correlations  
 3252 between these two parameters.

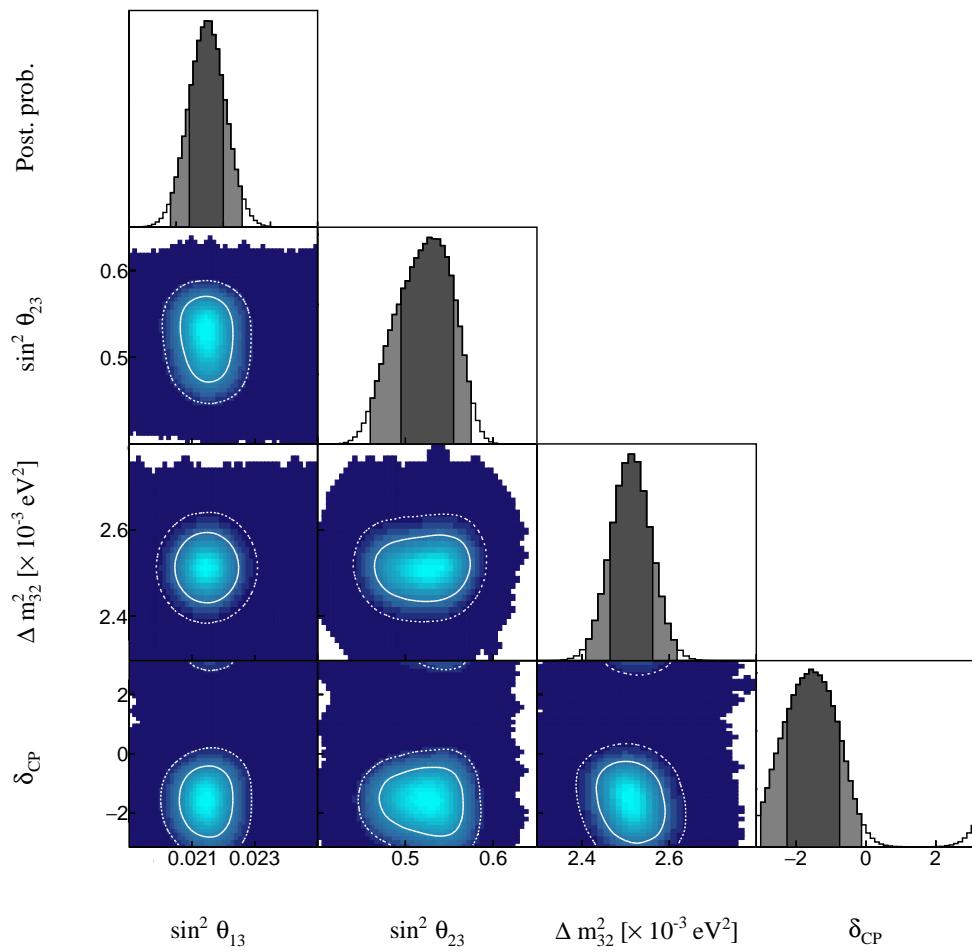
3253 Figure 8.31 illustrates the posterior distribution for each permutation of  
 3254 two oscillation parameters of interest. The application of the reactor constraint  
 3255 significantly reduces the correlations previously seen in Figure 8.22. There is  
 3256 still a small correlation between  $\delta_{CP}$  and  $\Delta m_{32}^2$ . The application of the reactor  
 3257 constraint has not significantly affected this correlation. The width of the 1 $\sigma$   
 3258 credible interval in  $\Delta m_{32}^2$  is wider for a value of  $\delta_{CP} = 0$  as compared to a value  
 3259 of  $\delta_{CP} = \pi$ . Similarly, the width of the 1 $\sigma$  credible interval in  $\delta_{CP}$  is smaller  
 3260 for lower values of  $\sin^2(\theta_{23})$ .



**Figure 8.29:** The two-dimensional posterior probability density distribution in  $\delta_{CP} - \sin^2(\theta_{13})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.



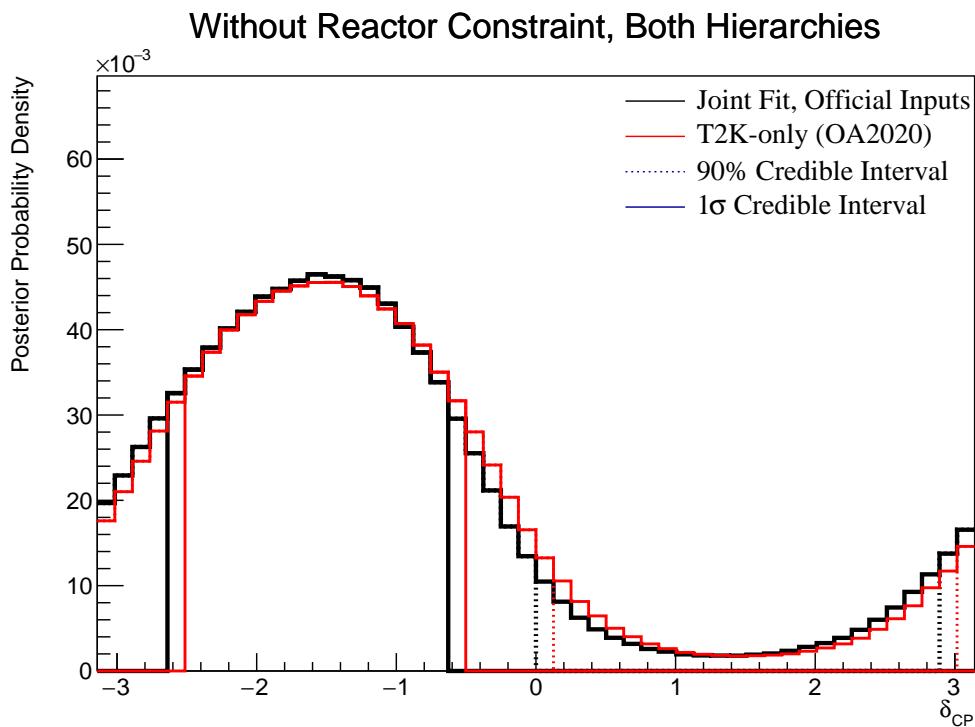
**Figure 8.30:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2 - \sin^2(\theta_{23})$ , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.



**Figure 8.31:** The posterior probability density distribution from the joint beam and atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

### 3261 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3262 Constraint

3263 The benefits of the joint beam and atmospheric analysis can be determined by  
 3264 comparing the sensitivities to the beam-only analysis. This section presents those  
 3265 comparisons for sensitivities built using the Asimov A oscillation parameters  
 3266 defined in Table 2.2 and the post-BANFF systematic tune. The reactor constraint  
 3267 is not applied within either of the fits used in these comparisons.

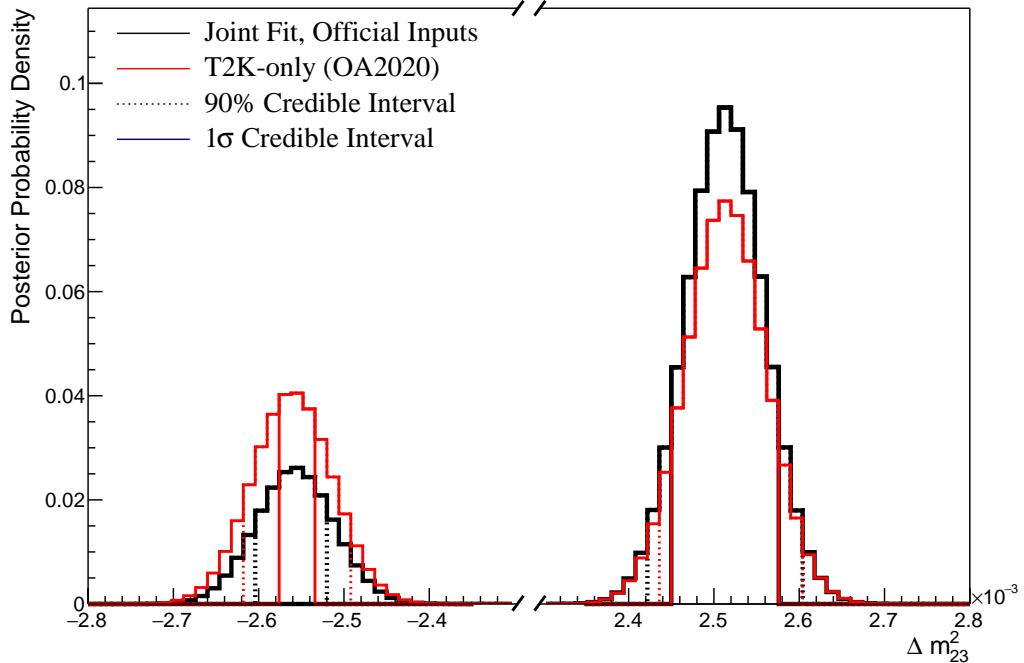


**Figure 8.32:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3268 The sensitivity, marginalised over both hierarchies, to  $\delta_{CP}$  from the joint  
 3269 beam-atmospheric and beam-only fits is presented in Figure 8.32. As expected  
 3270 from the likelihood scans (Figure 8.4), the sensitivity to  $\delta_{CP}$  is not significantly  
 3271 increased. This is because the known oscillation parameter value ( $\delta_{CP} = -1.601$ )  
 3272 lies at the position where the beam samples dominate the sensitivity compared  
 3273 to the SK samples.

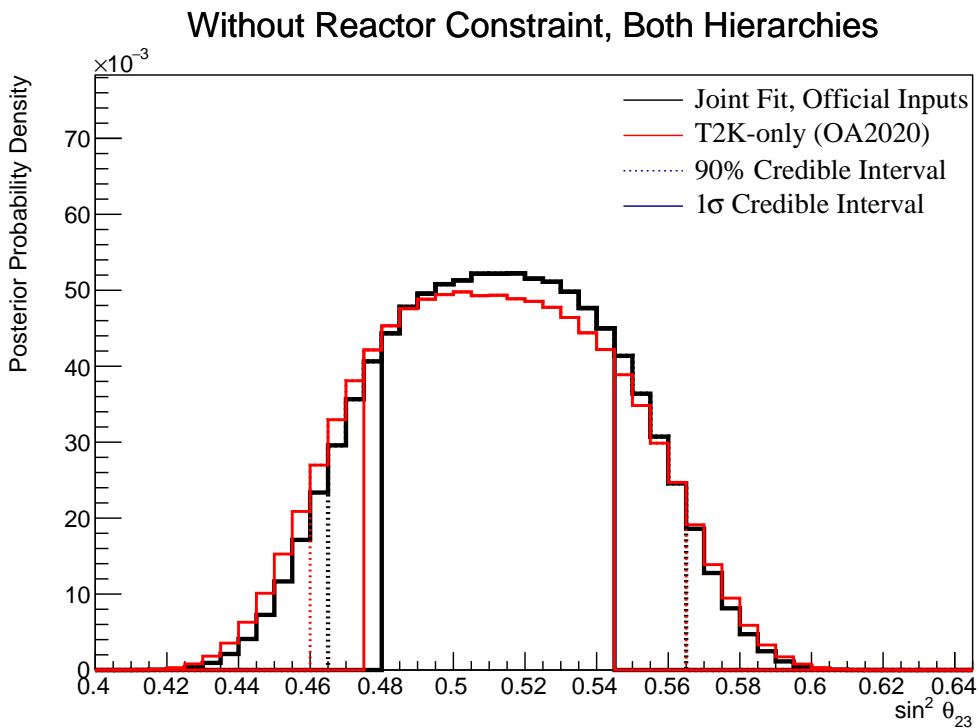
3274 The sensitivity to  $\Delta m_{32}^2$  of the joint beam-atmospheric fit is illustrated in  
 3275 Figure 8.33, where the posterior distribution has been marginalised over both  
 3276 hierarchies. The  $1\sigma$  credible interval of the joint beam and atmospheric fit is  
 3277 entirely contained within the NH region. This shows the significant increase  
 3278 in the ability of the fit to determine the correct mass hierarchy, as compared  
 3279 to the beam-only analysis. This is further evidenced by the fact that the 90%  
 3280 credible intervals from the joint fit are also tighter in the IH region as compared  
 3281 to the beam-only analysis. The Bayes factor for mass hierarchy determination  
 3282 for the beam-only and joint beam and atmospheric are  $B(\text{NH}/\text{IH}) = 1.91$  and  
 3283  $B(\text{NH}/\text{IH}) = 3.67$ , respectively. According to Jeffrey's scale (Table 4.1), the beam-  
 3284 only analysis represents a weak preference for the NH hypothesis whereas the  
 3285 joint fit returns a substantial preference for the NH hypothesis. To summarise, the  
 3286 joint beam-atmospheric fit has a substantial preference for the correct hierarchy  
 3287 without the requirement of external constraints.

### Without Reactor Constraint, Both Hierarchies



**Figure 8.33:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

The sensitivity to  $\sin^2(\theta_{23})$ , marginalised over both hierarchies, for both the beam-only and joint beam and atmospheric analysis are presented in Figure 8.34. The peak of the posterior distribution from the joint analysis is more aligned with the known value of  $\sin^2(\theta_{23}) = 0.528$  as compared to the beam-only analysis. This indicates that the marginalisation effects from other oscillation parameters ( $\sin^2(\theta_{13}) - \sin^2(\theta_{23})$ ) presented in Figure 8.22) are less prevalent in the projection of this parameter. The Bayes factors for the beam-only and joint beam-atmospheric fit are  $B(\text{UO}/\text{LO}) = 1.56$  and  $B(\text{UO}/\text{LO}) = 1.74$ , respectively. Therefore, the joint beam-atmospheric fit does prefer the UO more strongly than the beam-only analysis, albeit slightly.



**Figure 8.34:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

Whilst the beam-only and joint beam-atmospheric fits have similar sensitivity to  $\delta_{CP}$  and  $\sin^2(\theta_{23})$  when projected in one-dimension, the benefit of the joint analysis becomes more obvious when the sensitivities are presented in two-

dimensions. The sensitivity of the two fits to the appearance parameters ( $\delta_{CP} - \sin^2(\theta_{13})$ ) are illustrated in Figure 8.35.

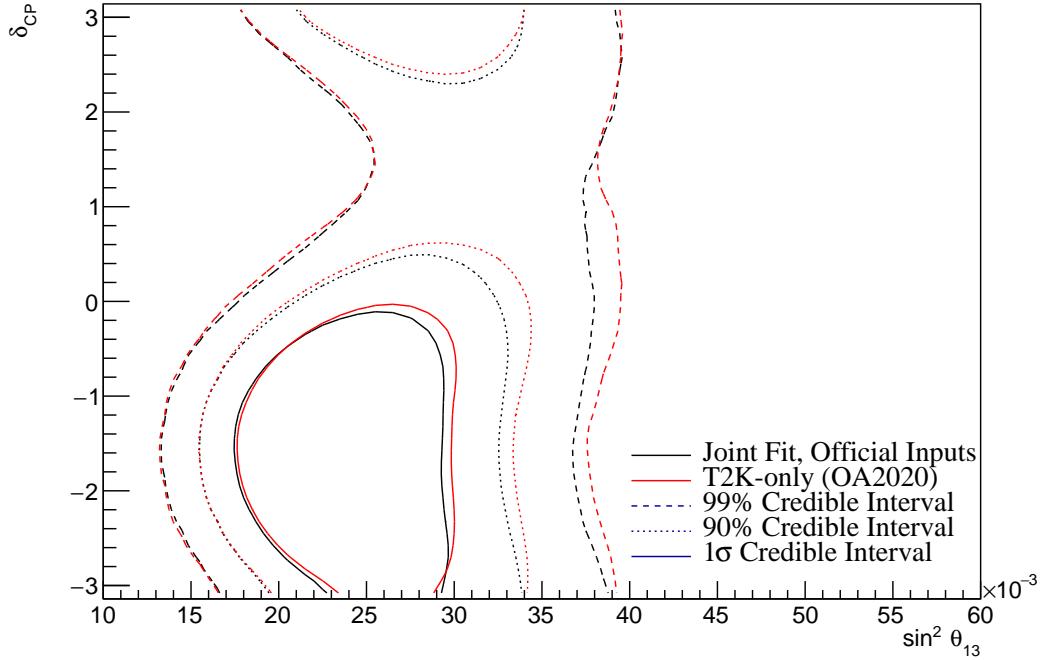
The width of the 99% joint fit credible interval in  $\sin^2(\theta_{13})$  is squeezed in the region of  $\delta_{CP} \sim 0$  compared to the beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The  $1\sigma$  and 90% also exhibit slightly tighter constraints on  $\delta_{CP}$ . This is most prevalent in the region of  $\delta_{CP} \sim 0$  and  $\sin^2(\theta_{13}) \sim 0.03$ . Whilst the atmospheric samples do not have significant sensitivity to  $\sin^2(\theta_{13})$  (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters  $\sin^2(\theta_{23}) - \Delta m_{32}^2$ , marginalised over both hierarchies, is presented in Figure 8.36 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. The  $1\sigma$  joint fit credible interval in the IH region more strongly favours the UO as compared to the beam-only fit. Even in the NH region, the width of the credible intervals in  $\sin^2(\theta_{23})$  decrease, albeit to a smaller extent.

The change in sensitivity to  $\delta_{CP} - \Delta m_{32}^2$  is illustrated in Figure 8.37. As expected, the contours presented within the IH region are much smaller in the joint fit due to the increased sensitivity to mass hierarchy determination. This culminates in a region around  $\delta_{CP} \sim \pi/2$  which is excluded at  $3\sigma$ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to  $\Delta m_{32}^2$  and  $\sin^2(\theta_{23})$ , as a function of  $\sin^2(\theta_{13})$ , is presented in Figure 8.38 and Figure 8.39, respectively. These sensitivities are marginalised

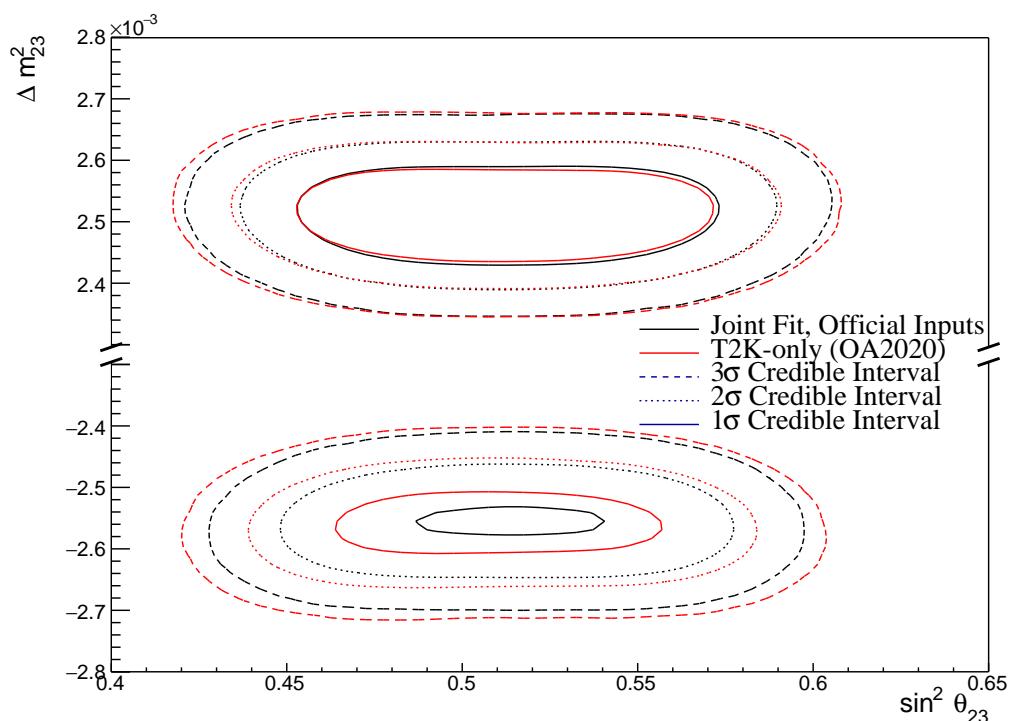
### Without Reactor Constraint, Both Hierarchies



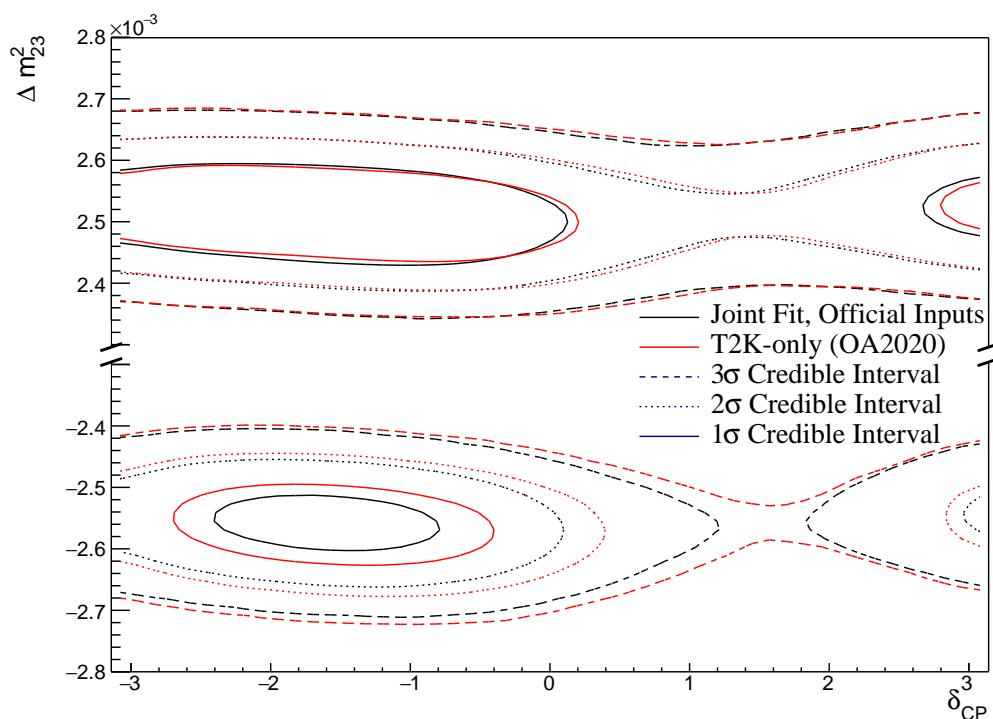
**Figure 8.35:** The two-dimensional posterior probability density distribution in  $\delta_{CP}$  –  $\sin^2(\theta_{13})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

over both hierarchies. As expected from the previous observations, the  $\Delta m_{32}^2$  contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH  $1\sigma$  credible intervals exclude the region around the reactor constraint. This is not a bias from the fit as the known value for  $\Delta m_{32}^2$  is in the NH region. This does suggest that the application of the reactor constraint would further increase the preference for NH in the joint fit as compared to its effect on the beam-only analysis.

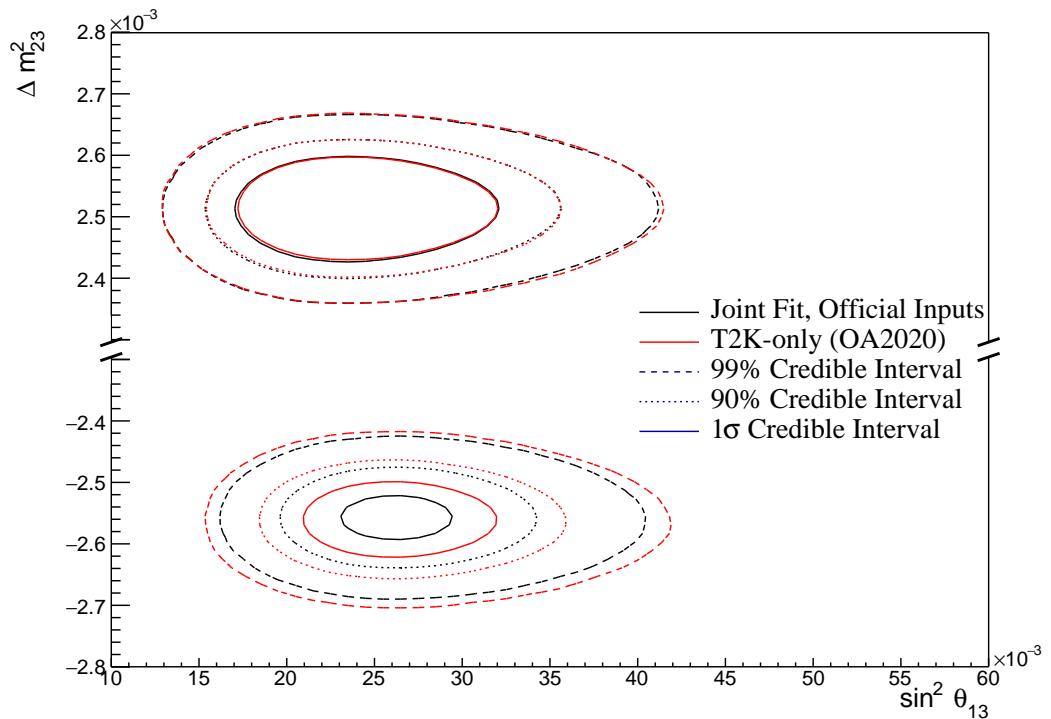
The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the  $\sin^2(\theta_{13})$  and  $\sin^2(\theta_{23})$  parameters, as illustrated by Figure 8.39. The joint analysis disfavours the wrong octant hypothesis more strongly in the region of high  $\sin^2(\theta_{13})$ . This suggests that the application of the reactor constraint will favour the UO more strongly in the joint analysis compared to the beam-only analysis.



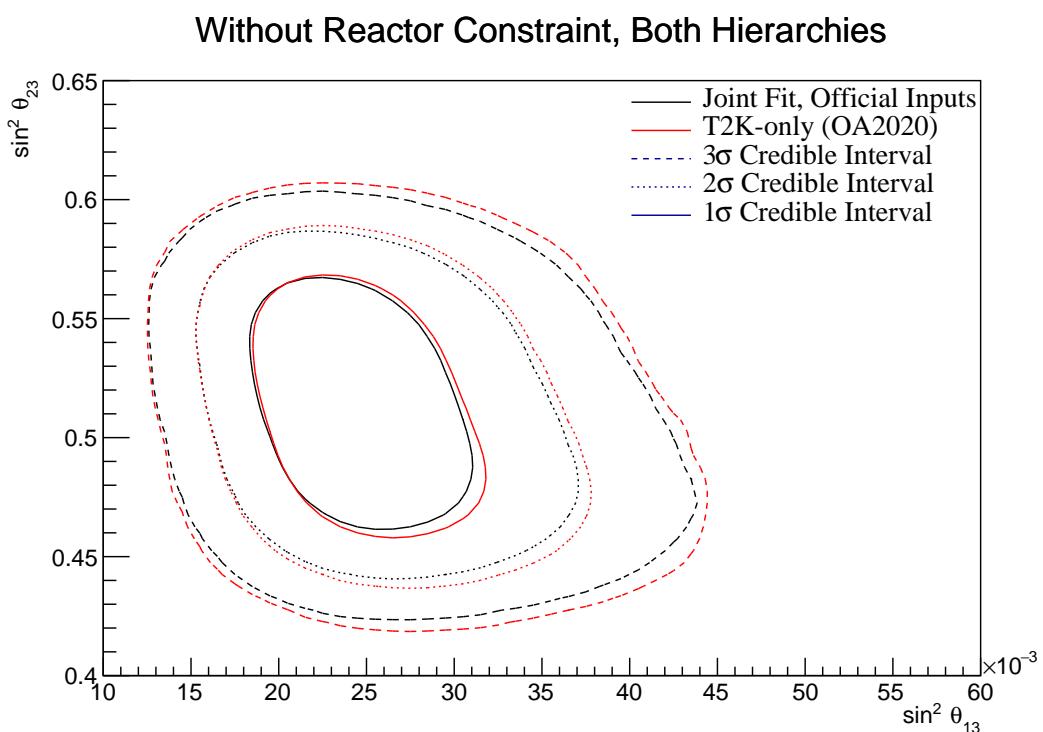
**Figure 8.36:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  –  $\sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.



**Figure 8.37:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2 - \Delta_{CP}$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.



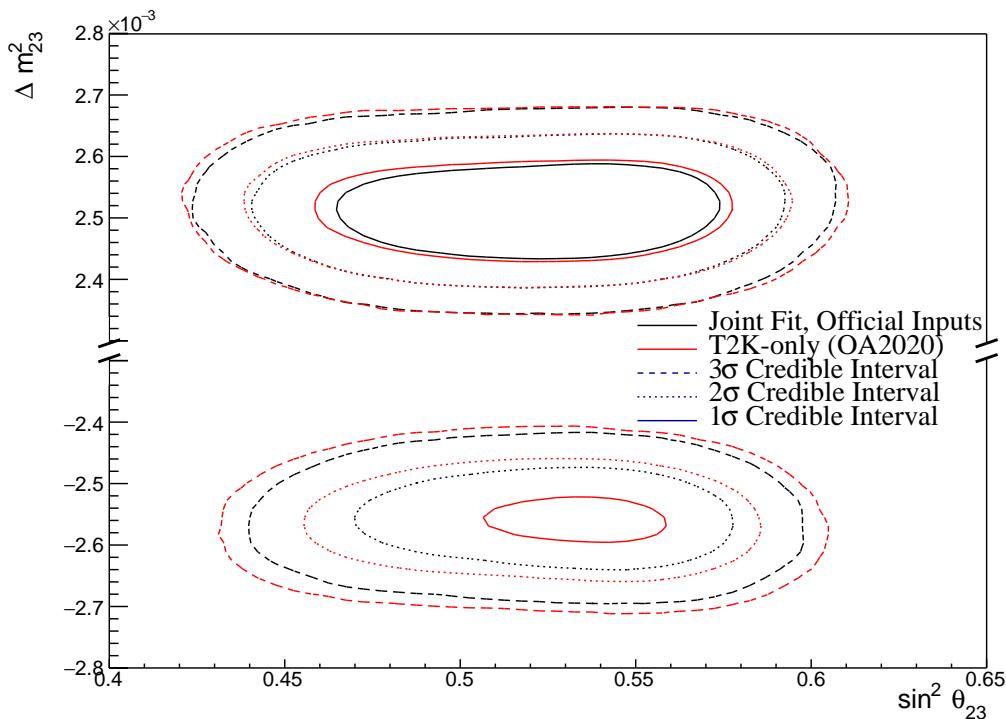
**Figure 8.38:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  –  $\sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.



**Figure 8.39:** The two-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$  –  $\sin^2(\theta_{13})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

### 3344 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3345 straint

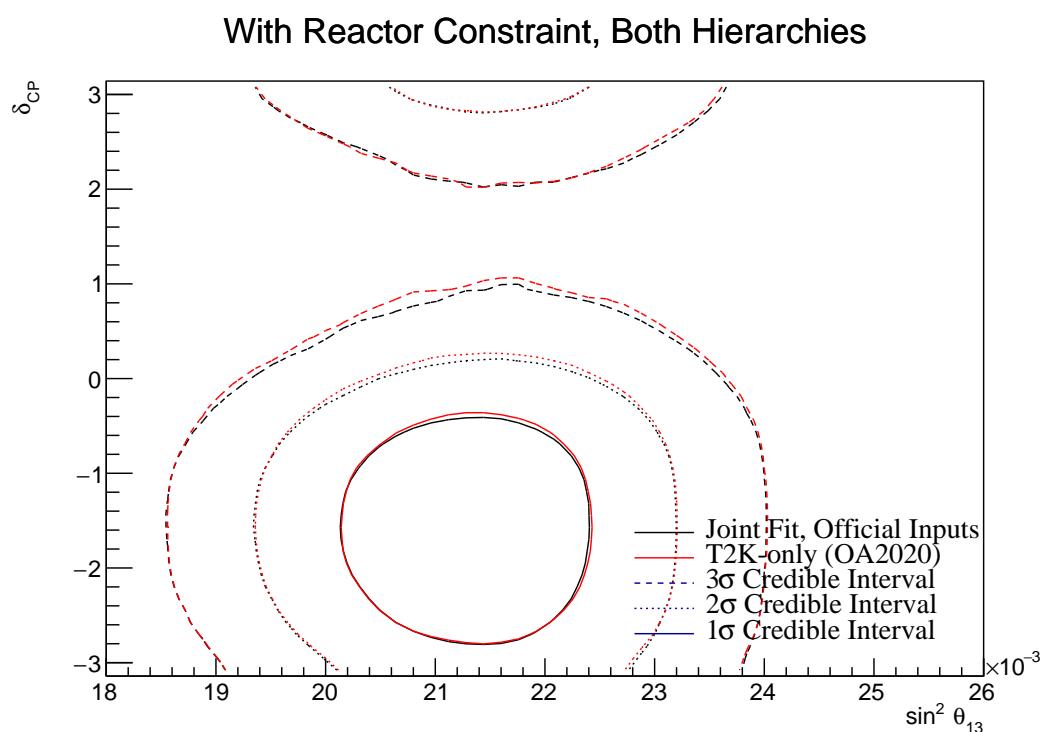
3346 The comparison between the beam-only and joint beam-atmospheric fits are  
 3347 compared in subsection 8.3.6. Those comparisons were made with the reactor  
 3348 constraint not applied to either of the fits. This section illustrates the com-  
 3349 parison when the reactor constraint is applied. As shown in Figure 8.38, the  
 3350 application of the reactor constraint is expected to significantly increase the  
 3351 joint fit's preference for the NH hypothesis, as compared to the beam-only  
 3352 analysis. Figure 8.40 illustrates the sensitivities of the two fits to the disappearance  
 3353 parameters ( $\sin^2(\theta_{23}) - \Delta m_{32}^2$ ) marginalised over both hierarchies and with the  
 3354 reactor constraint applied. This plot clearly illustrates the benefit of the joint  
 3355 beam and atmospheric analysis. The  $1\sigma$  credible interval in the IH region is  
 3356 entirely removed in the joint analysis, illustrating the improved NH preference.



**Figure 8.40:** The two-dimensional posterior probability density distribution in  $\Delta m_{32}^2 - \sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

3357        The credible intervals of the joint fit are also tighter in the  $\sin^2(\theta_{23})$  dimension  
3358        than the beam-only analysis in both mass hierarchy regions. This shows that  
3359        beyond the ability of the joint fit to prefer the NH more strongly than the beam-  
3360        only analysis, the precision to which it can measure  $\sin^2(\theta_{23})$  is also improved.  
3361        The Bayes factor for NH preference is calculated as  $B(\text{NH}/\text{IH}) = 7.29$  and  
3362         $B(\text{NH}/\text{IH}) = 3.41$  for the joint beam-atmospheric and beam-only analysis,  
3363        respectively. Whilst both present a significant preference for the NH hypothesis  
3364        (Table 4.1), the joint fit's preference is much stronger. A similar conclusion can be  
3365        made regarding the Bayes factors for UO preference which are  $B(\text{UO}/\text{LO}) = 2.86$   
3366        and  $B(\text{UO}/\text{LO}) = 2.67$  for the joint beam-atmospheric and beam-only analysis,  
3367        respectively. Both of these represent a mild preference for the UO but there is  
3368        a stronger preference observed in the joint analysis.

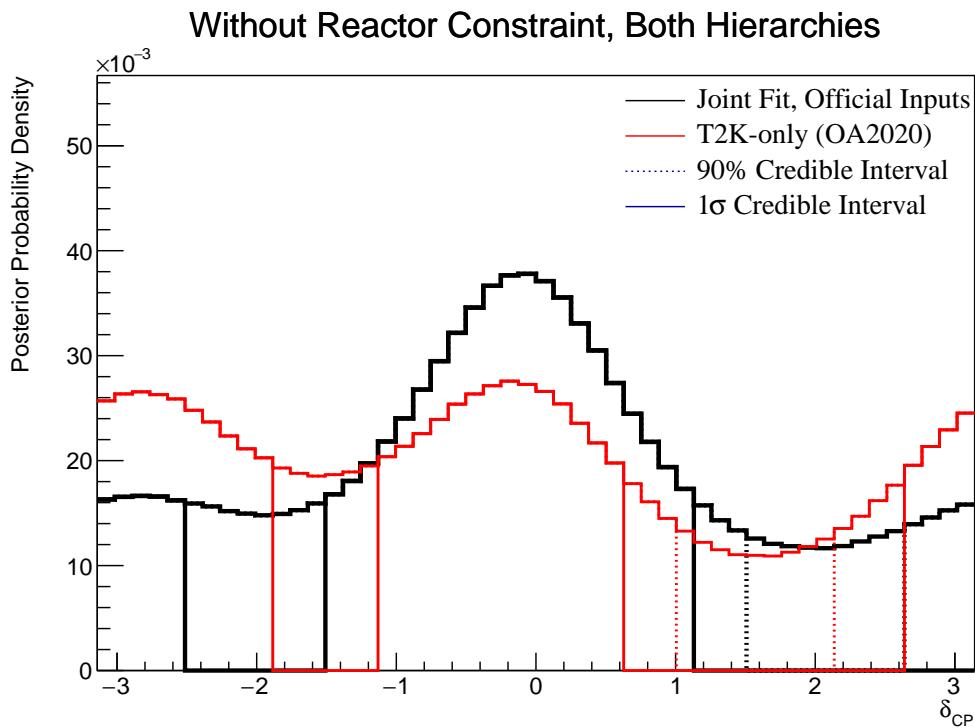
3369        The sensitivity of the beam-only and joint beam-atmospheric analyses, to the  
3370        appearance parameters ( $\delta_{CP} - \sin^2(\theta_{13})$ ), are compared in Figure 8.41. These  
3371        results are marginalised over both hierarchies and include the reactor constraint  
3372        on  $\sin^2(\theta_{13})$ . For this particular set of known oscillation parameters (AsimovA  
3373        defined in Table 2.2), the beam-only analysis dominates the sensitivity. The  
3374        joint fit does slightly increase the sensitivity to  $\delta_{CP}$  but it does not change any  
3375        conclusions that would be made.



**Figure 8.41:** The two-dimensional posterior probability density distribution in  $\delta_{CP}$  –  $\sin^2(\theta_{13})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

### 3376 8.3.8 Effect of Asimov Parameter Set

3377 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the  
 3378 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters  
 3379 are defined at a region of  $\delta_{CP}$  which is dominated by the T2K experiment. This  
 3380 explains why the addition of the atmospheric samples does not significantly in-  
 3381 crease the sensitivity to  $\delta_{CP}$ , as illustrated in subsection 8.3.6 and subsection 8.3.7.  
 3382 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,  
 3383 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when  
 3384 building the Asimov data.



**Figure 8.42:** The one-dimensional posterior probability density distribution in  $\delta_{CP}$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3385 The sensitivity to  $\delta_{CP}$  for the joint beam and atmospheric fit is presented  
 3386 in Figure 8.42. The results are compared to those from the beam-only analysis  
 3387 in [182]. The reactor constraint is not applied in either of the fits. The known  
 3388 oscillation parameter value is  $\delta_{CP} = 0$ . The shape of the posterior distribution

from the joint analysis is more peaked at  $\delta_{CP} = 0$  as compared to the beam-only analysis which has approximately the same posterior probability density at  $\delta_{CP} = 0$  and  $\delta_{CP} = \pi$ . This shows the ability of the joint analysis to better determine the correct phase of  $\delta_{CP}$  if the true value was CP-conserving. The  $1\sigma$  credible intervals and the position of the highest posterior probability density are given in Table 8.9.

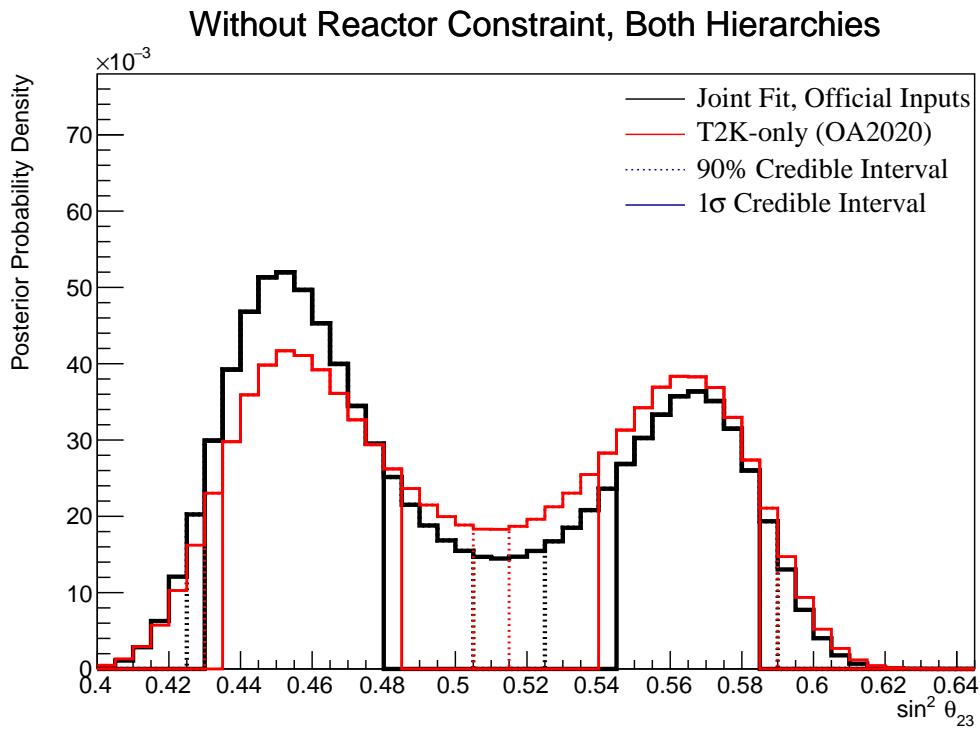
Parameter	Interval	HPD
$\delta_{CP}$ , (BH)	$[-\pi, -2.51], [-1.51, 1.31]$	-0.06
$\delta_{CP}$ , (NH)	$[-1.13, 1.63]$	0.06
$\delta_{CP}$ , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44
$\Delta m_{32}^2$ (BH) [ $\times 10^{-3}$ eV $^2$ ]	$[-2.60, -2.49], [2.46, 2.59]$	2.51
$\Delta m_{32}^2$ (NH) [ $\times 10^{-3}$ eV $^2$ ]	$[2.47, 2.56]$	2.52
$\Delta m_{32}^2$ (IH) [ $\times 10^{-3}$ eV $^2$ ]	$[-2.61, -2.52]$	-2.57
$\sin^2(\theta_{23})$ (BH)	$[0.43, 0.48], [0.55, 0.59]$	0.45
$\sin^2(\theta_{23})$ (NH)	$[0.43, 0.49], [0.55, 0.58]$	0.45
$\sin^2(\theta_{23})$ (IH)	$[0.44, 0.48], [0.54, 0.59]$	0.57

**Table 8.9:** The position of the highest posterior probability density (HPD) and width of the  $1\sigma$  credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

Naively, if just the  $1\sigma$  credible interval were considered without observing the shape of the distribution, it would appear that the joint analysis would have a worse sensitivity to  $\delta_{CP}$  due to the larger interval around  $\delta_{CP}$ . The  $1\sigma$  credible interval for the beam-only analysis is given as the range  $\delta_{CP} = [-\pi, -1.88], [-1.13, 0.63]$  and  $[2.64, \pi]$  which contains 56% of all values of  $\delta_{CP}$ . The joint beam and atmospheric analysis contains 52% of all  $\delta_{CP}$  values within the  $1\sigma$  credible interval. Therefore, if the area within the  $1\sigma$  credible interval were to be compared between the two fits, the joint analysis would be shown to have better precision.

This contradiction stems from the methodology in which the credible interval is calculated. The technique used in this analysis (documented in subsection 4.3.2)

3405 fills the credible interval by selecting bins in order of magnitude until 68% of the  
 3406 posterior density is contained. If instead, the credible interval was calculated  
 3407 by expanding around the highest posterior probability, the benefits of the joint  
 3408 fit would be more obvious. In the case where the shape of the posterior was  
 3409 Gaussian, these two techniques would be equivalent.



**Figure 8.43:** The one-dimensional posterior probability density distribution in  $\sin^2(\theta_{23})$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3410 The sensitivity of the joint beam and atmospheric fit to  $\sin^2(\theta_{23})$  is presented  
 3411 in Figure 8.43. The sensitivity is compared to that of the beam-only analysis  
 3412 in [182]. The reactor constraint is not applied in either of the fits being com-  
 3413 pared. The Asimov parameter value is  $\sin^2(\theta_{23}) = 0.45$  and the sensitivities are  
 3414 marginalised over both hierarchies. Clearly, the joint beam and atmospheric  
 3415 fit has a much larger probability density in the region surrounding the known  
 3416 oscillation parameters. This shows the better octant determination of the joint  
 3417 analysis compared to the beam-only fit. The ratio of the posterior density at

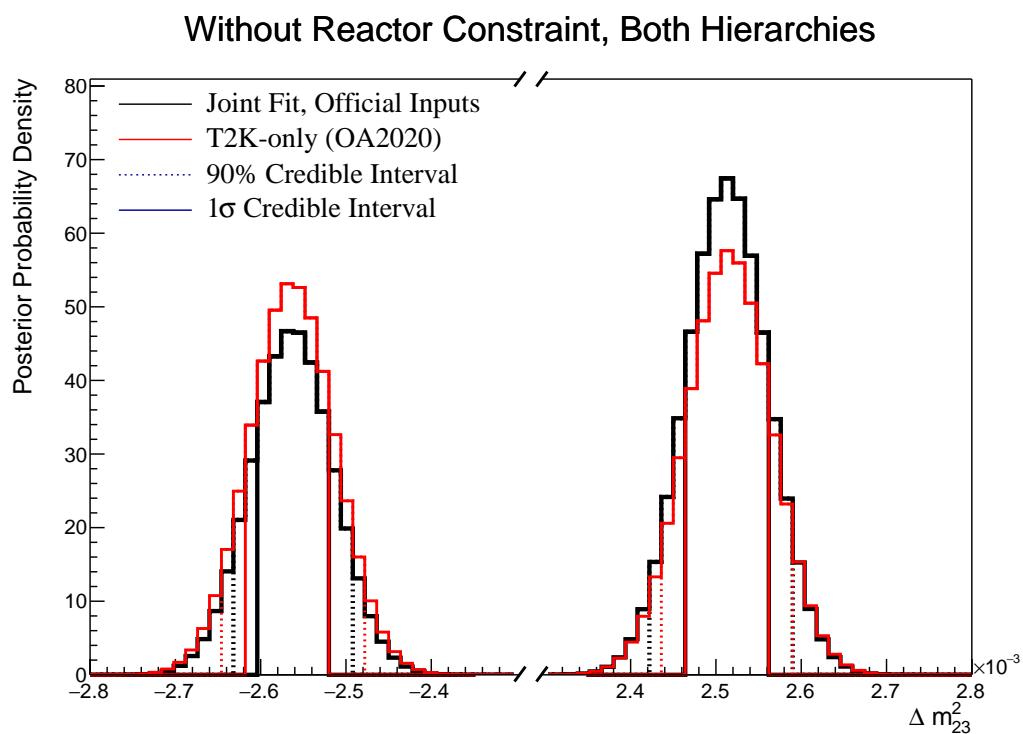
the peak of the lower octant to the peak of the upper octant from the joint fit is 1.43 compared to 1.09 from the beam-only analysis. This shows further support for the joint analysis in correctly selecting the lower octant, which is the correct hypothesis given the known oscillation parameters.

	LO ( $\sin^2 \theta_{23} < 0.5$ )	UO ( $\sin^2 \theta_{23} > 0.5$ )	Sum
NH ( $\Delta m_{32}^2 > 0$ )	0.35	0.24	0.59
IH ( $\Delta m_{32}^2 < 0$ )	0.19	0.22	0.41
Sum	0.54	0.46	1.00

**Table 8.10:** The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as  $B(\text{NH}/\text{IH}) = 1.43$  and  $B(\text{LO}/\text{UO}) = 1.19$ .

The distribution of steps, split by hierarchy and octant hypothesis, is presented in Table 8.10. The Bayes factor for hierarchy and octant determination are  $B(\text{NH}/\text{IH}) = 1.43$  and  $B(\text{LO}/\text{UO}) = 1.19$ , respectively. The octant Bayes factor is now presented as LO/UO as the known oscillation parameter is contained within the lower octant. These values compare to  $B(\text{NH}/\text{IH}) = 1.08$  and  $B(\text{LO}/\text{UO}) = 0.91$  from the beam-only analysis. This shows additional evidence of the joint analysis's preference for selecting the correct octant and hierarchy hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6 show how the preference for the correct octant and hierarchy depend on the true value of  $\delta_{CP}$  and  $\sin^2(\theta_{23})$ .

The sensitivity of the beam-only and joint beam-atmospheric analysis to  $\Delta m_{32}^2$  is given in Figure 8.44. Both of the results are marginalised over both hierarchies and the reactor constraint is not applied in either analysis. The joint analysis has a stronger preference for the correct hierarchy (NH) which is shown by the higher Bayes factor ( $B(\text{NH}/\text{IH}) = 1.43$ ) compared to the beam-only analysis ( $B(\text{NH}/\text{IH}) = 1.08$ ).



**Figure 8.44:** The one-dimensional posterior probability density distribution in  $\Delta m_{32}^2$  compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

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## Conclusions and Outlook

# Appendices

# A

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## Atmospheric Sample Spectra

3443 This appendix documents the interaction mode breakdown of all the atmospheric  
3444 samples used within the analysis. The generated tune of the model parameters  
3445 and the Asimov A oscillation parameter set (defined in Table 2.2) are assumed.  
3446 The livetime of SK-IV is taken to be 3244.4 days.

### 3447 A.1 Binning

3448 The lepton momentum and cosine zenith binning edges for the atmospheric  
3449 samples used within this analysis are defined in Table A.1.

### 3450 A.2 Fully Contained Sub-GeV Samples

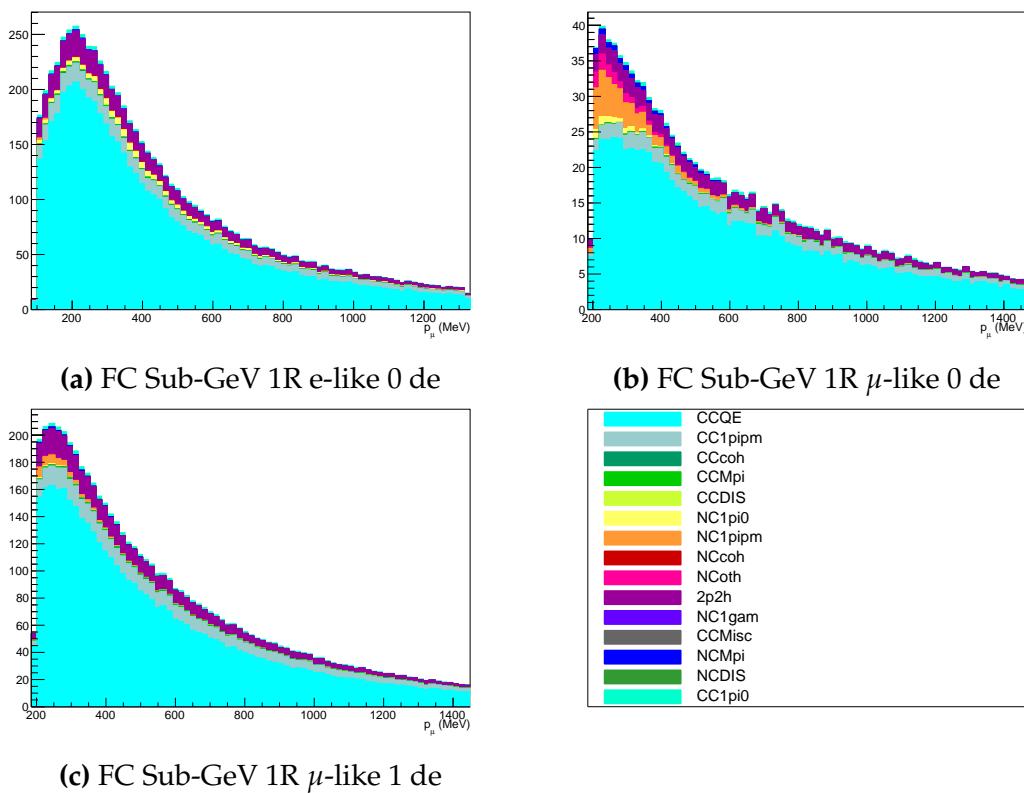
3451 The interaction mode breakdown of the fully contained Sub-GeV samples are  
3452 shown in Figure A.1 and Figure A.2, for the samples with enriched CC0 $\pi$  and  
3453 CC1 $\pi^\pm$  respectively.

3454 The CC0 $\pi$  sample are dominated by CCQE events ( $\sim 70\%$ ) with smaller  
3455 contributions of 2p2h ( $\sim 12\%$ ) and CC1 $\pi$  ( $\sim 10\%$ ) components. The energy peaks  
3456 around 300 MeV, which is slightly below that of the T2K samples but still has  
3457 significant contribution upto 1 GeV which overlaps the T2K sample energy range.

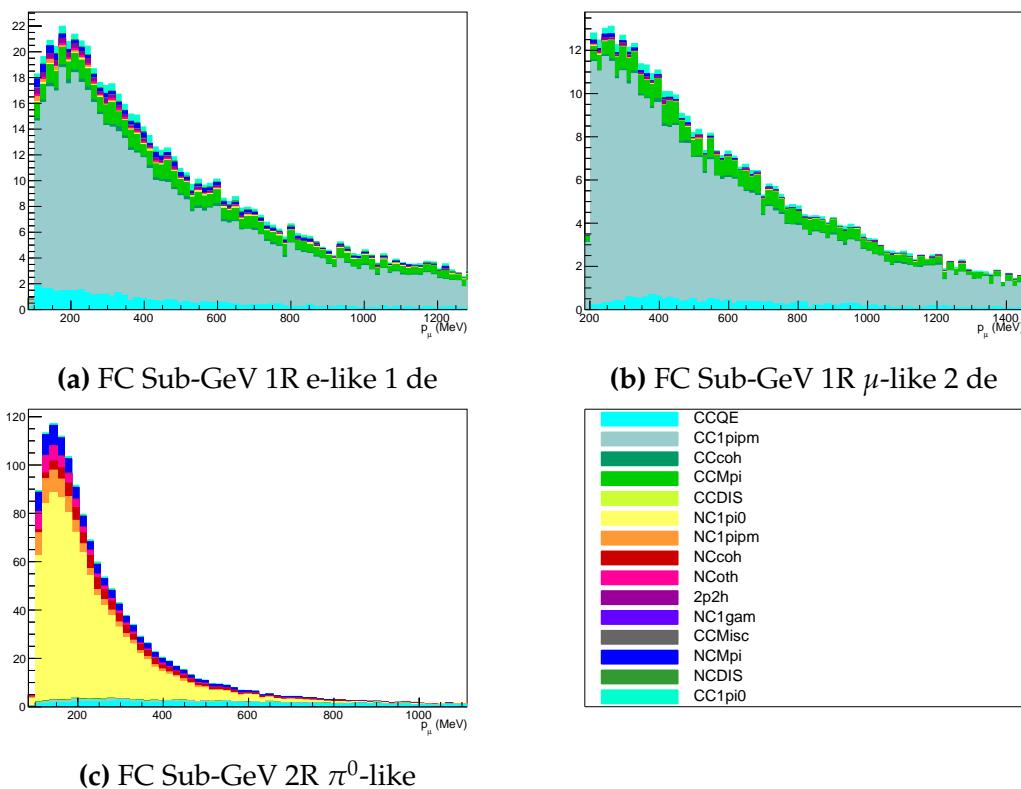
Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ( $\log_{10}(P)$ MeV)
SubGeV-elike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-elike-1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-pi0like	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV-elike-nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-elike-nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-mulike	10	3.0, 3.4, 5.0
MultiRing-elike-nue	10	3.0, 3.4, 3.7, 5.0
MultiRing-elike-nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing-mulike	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing-Other1	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

**Table A.1:** The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$  Bins” column illustrates the number of bins uniformly distributed over the  $-1.0 \leq \cos(\theta_Z) \leq 1.0$  region for fully and partially contained samples and  $-1.0 \leq \cos(\theta_Z) \leq 0.0$  region for up- $\mu$  samples.

3458      The one-ring CC1 $\pi$  samples, where the pion is tagged via its decay electron,  
 3459      are dominated by CC1 $\pi$  events ( $\sim 75\%$ ) with a small contribution of CCM $\pi$   
 3460      ( $\sim 10\%$ ). The two-ring pion sample is mostly dominated by the NC1 $\pi^0$  via  
 3461      resonances, and has several equally-sized contributions from CCQE, NC1 $\pi^\pm$  via  
 3462      resonances, and NC coherent pion production, where the  $\pi^0$  likely comes from  
 3463      nucleon and  $\pi^\pm$  final state interactions in the nucleus.



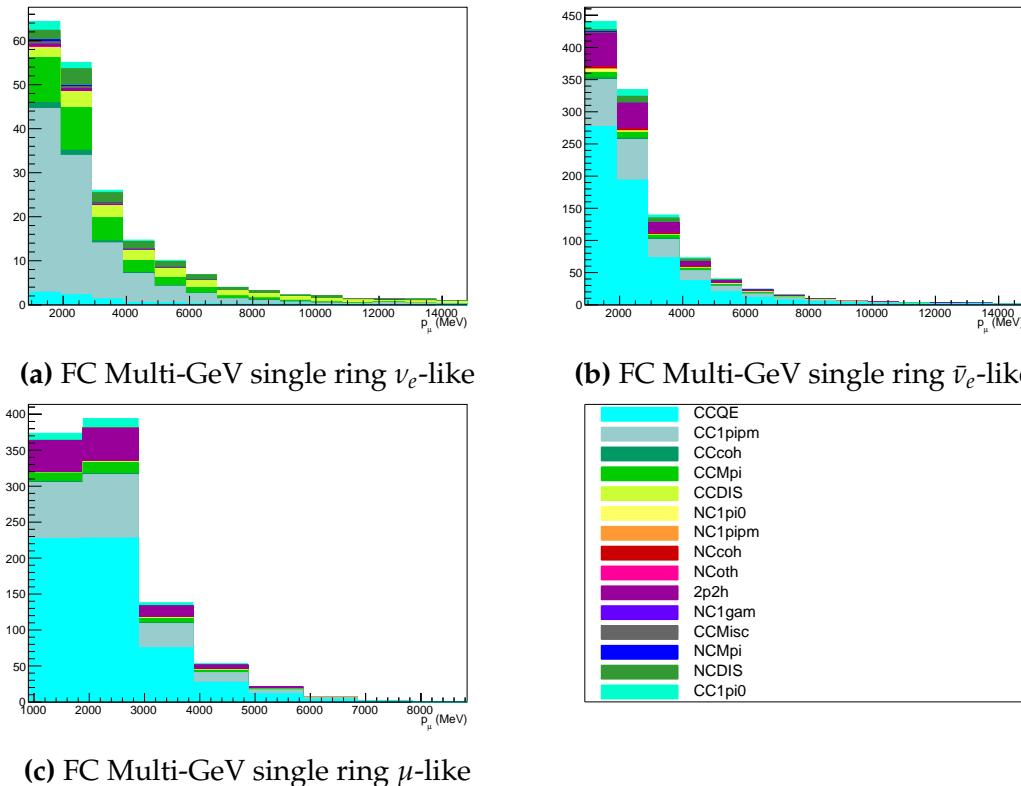
**Figure A.1:** Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting CC $0\pi$  events.



**Figure A.2:** Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting single pion events.

### 3464 A.3 Fully Contained Multi-GeV Samples

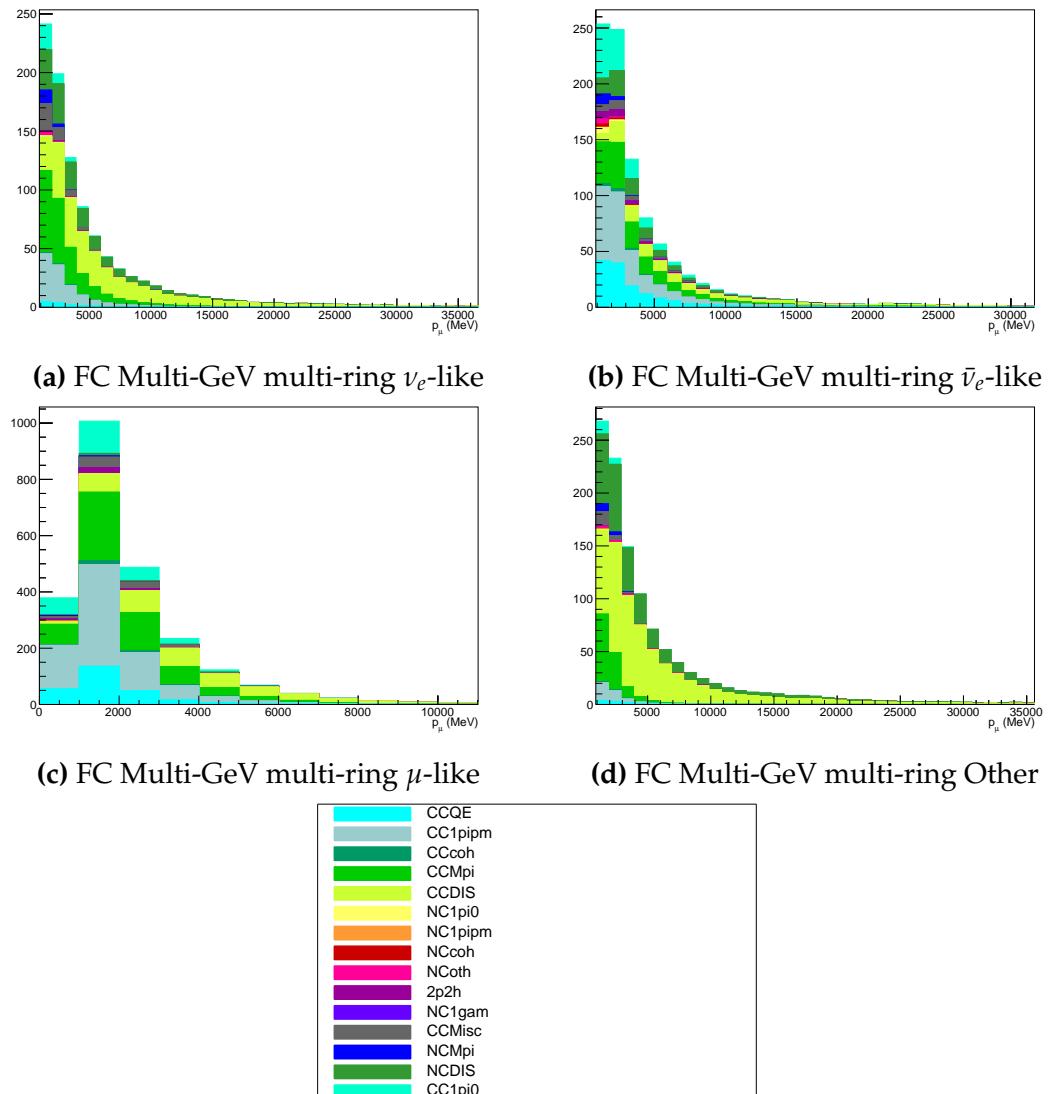
3465 The interaction mode breakdown of fully contained multi-GeV samples is high-  
 3466 lighted in Figure A.3. Due to the event selection applied in SK which targets  $\pi^+$   
 3467 and  $\pi^-$  separation, the  $\nu_e$  sample mainly consists of events with pions (single pion  
 3468 production or multi-pion/DIS interactions). The pion separation is explained in  
 3469 Section section 6.1. This reasoning also explains the significant CCQE contribution  
 3470 of the  $\bar{\nu}_e$  sample. The muon-like sample is dominated by CCQE interactions with  
 3471  $\sim 10 - 15\%$  2p2h and CC1 $\pi$  contribution of events.



**Figure A.3:** Breakdown by interaction mode of the FC Multi-GeV single ring atmospheric samples.

## 3472 A.4 Fully Contained Multi-Ring Samples

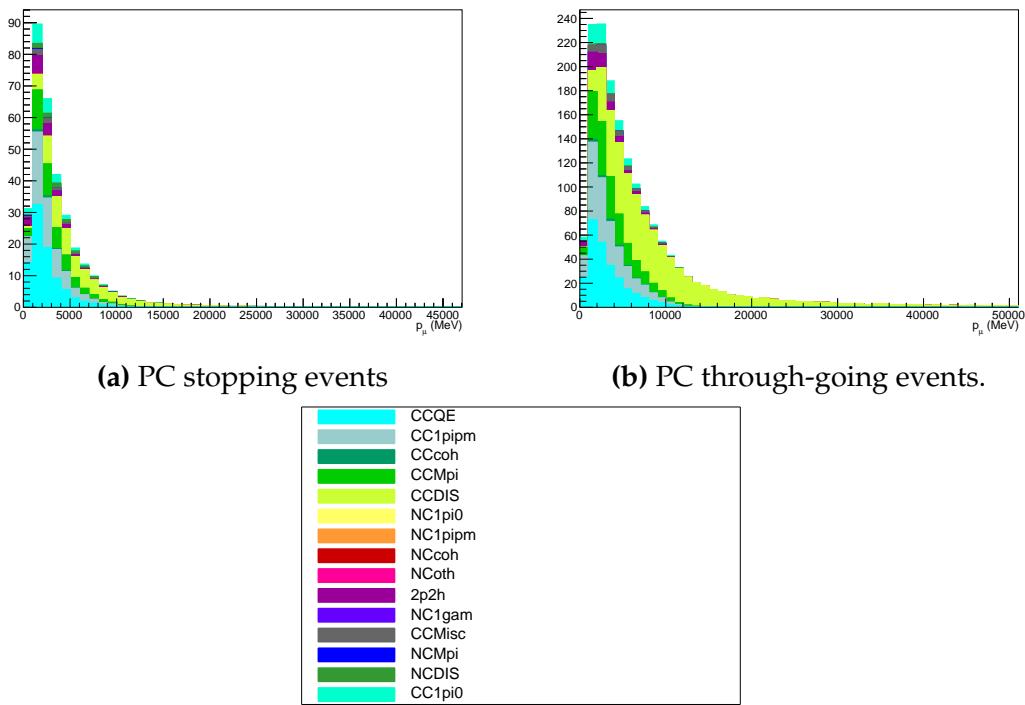
3473 The interaction mode breakdown of fully contained multi-ring events is shown  
 3474 in Figure A.4. These samples see more interaction modes contributing in general,  
 3475 and there is a much larger contribution from multi-pion and DIS interaction  
 3476 modes, compared to the other samples.



**Figure A.4:** Breakdown by interaction mode of the FC Multi-GeV multi-ring atmospheric samples.

## <sup>3477</sup> A.5 Partially Contained Samples

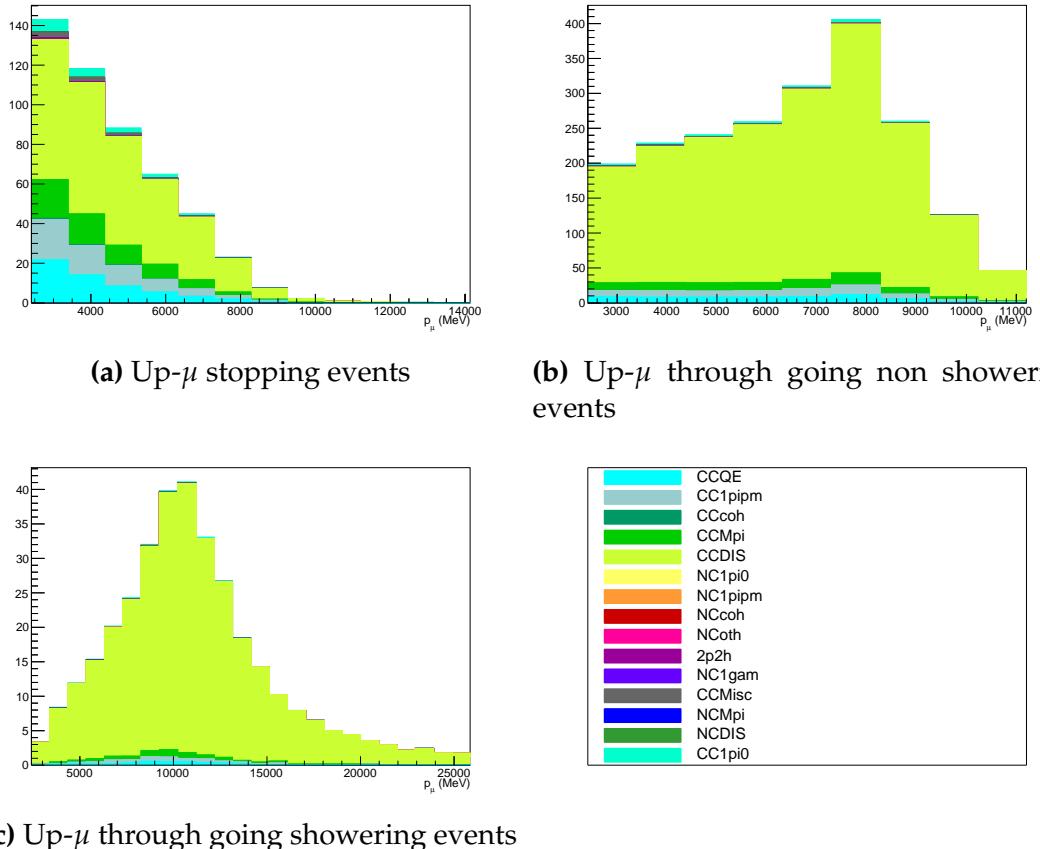
<sup>3478</sup> The breakdown for partially contained samples is highlighted in Figure A.5.  
<sup>3479</sup> As with the multi-ring samples, there is no dominating interaction mode. The  
<sup>3480</sup> neutrino energies of events in this sample extend into the tens of GeV and become  
<sup>3481</sup> dominated by DIS interaction modes in the high energy limit.



**Figure A.5:** Breakdown by interaction mode of the PC atmospheric samples.

## 3482 A.6 Upward-Going Muon Samples

3483 The breakdown for upward-going muons is illustrated in Figure A.6. These  
 3484 samples are significantly dominated by DIS interactions with energies extending  
 3485 up into the hundreds of GeV.



**Figure A.6:** Breakdown by interaction mode of the atmospheric upward going muon samples.

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