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Simultaneous Measurement of Oscillation Parameters in Beam and Atmospheric Neutrino Data from Tokai-to-Kamioka and Super-Kamiokande Experiments

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13 **Simultaneous Measurement of**

14 **Oscillation Parameters in Beam and**

15 **Atmospheric Neutrino Data from**

16 **Tokai-to-Kamioka and**

17 **Super-Kamiokande Experiments**

18 *Abstract*

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¹⁴⁶ **Chapter 1**

¹⁴⁷ **Introduction**

¹⁴⁸ **Chapter 2**

¹⁴⁹ **Neutrino Oscillation Physics**

¹⁵⁰ When first proposed, neutrinos were expected to be massless fermions that only in-
¹⁵¹ teract through weak and gravitational forces. This meant they were very difficult to
¹⁵² detect as they can pass through significant amounts of matter without interacting. De-
¹⁵³ spite this, experimental neutrino physics has developed with many different detection
¹⁵⁴ techniques and neutrino sources being used today. In direct tension with standard
¹⁵⁵ model physics, neutrinos have been determined to oscillate between different lepton
¹⁵⁶ flavours, requiring them to have mass.

¹⁵⁷ The observation techniques which lead to the discovery of the neutrino are doc-
¹⁵⁸ umented in section 2.1. The theory underpinning neutrino oscillation is described
¹⁵⁹ in section 2.2 and includes the approximations which can be made to simplify the
¹⁶⁰ understanding of neutrino oscillation in the two-flavour approximation. Past, current,
¹⁶¹ and future neutrino experiments are detailed in section 2.3, including the reactor,
¹⁶² atmospheric, and long-baseline accelerator neutrino sources that have been used to
¹⁶³ successfully constrain oscillation parameters. Finally, the current state of oscillation
¹⁶⁴ parameter measurements are summarised in section 2.4.

¹⁶⁵ **2.1. Discovery of Neutrinos**

¹⁶⁶ At the start of the 20th century, the electrons emitted from the β -decay of the nucleus
¹⁶⁷ were found to have a continuous energy spectrum [1,2]. This observation seemingly
¹⁶⁸ broke the energy conservation invoked within that period's nuclear models. Postulated

¹⁶⁹ in 1930 by Pauli as the solution to this problem, the neutrino (originally termed
¹⁷⁰ “neutron”) was theorized to be an electrically neutral spin-1/2 fermion with a mass of
¹⁷¹ the same order of magnitude as the electron [3]. This neutrino was to be emitted with
¹⁷² the electron in β -decay to alleviate the apparent breaking of energy conservation. As a
¹⁷³ predecessor of today’s weak interaction model, Fermi’s theory of β -decay developed
¹⁷⁴ the understanding by coupling the four constituent particles; electron, proton, neutron,
¹⁷⁵ and neutrino, into a consistent model [4].

¹⁷⁶ Whilst Pauli was not convinced of the ability to detect neutrinos, the first observa-
¹⁷⁷ tions of the particle were made in the mid-1950s when neutrinos from a reactor were
¹⁷⁸ observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [5, 6]. The detector
¹⁷⁹ consisted of two parts: a neutrino interaction medium and a liquid scintillator. The
¹⁸⁰ interaction medium was built from two water tanks. These were loaded with cadmium
¹⁸¹ chloride to allow increased efficiency of neutron capture. The positron emitted from
¹⁸² IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured
¹⁸³ on the cadmium via $n + {}^{108}Cd \rightarrow {}^{109*}Cd \rightarrow {}^{109}Cd + \gamma$, producing a delayed signal. An
¹⁸⁴ increase in the coincidence rate was observed when the reactor was operating which
¹⁸⁵ was interpreted as interactions from neutrinos generated in the reactor.

¹⁸⁶ After the discovery of the ν_e , the natural question of how many flavours of neutrino
¹⁸⁷ exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven
¹⁸⁸ National Laboratory [7]. A proton beam was directed at a beryllium target, generating
¹⁸⁹ a π -dominated beam which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the subsequent
¹⁹⁰ interactions of the ν_μ were observed. As the subsequent interaction of the neutrino
¹⁹¹ generates muons rather than electrons, it was determined the ν_μ was fundamentally
¹⁹² different from ν_e . The final observation to be made was that of the ν_τ from the DONUT
¹⁹³ experiment [8]. Three neutrinos seem the obvious solution as it mirrors the known
¹⁹⁴ number of charged lepton (as they form weak isospin doublets) but there could be

195 evidence of more. Several neutrino experiments have found anomalous results [9, 10]
196 which could be attributed to sterile neutrinos. However, cosmological observations
197 indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [11], as measured from
198 the cosmic microwave background power spectrum, and LEP measured the number
199 of active neutrino flavours to be $N_\nu 2.9840 \pm 0.0082$ [12] from measurements of the
200 Z -decay width.

201 2.2. Theory of Neutrino Oscillation

202 As direct evidence of beyond Standard Model physics, a neutrino generated with
203 lepton flavour α can change into a different lepton flavour β after propagating some
204 distance. This phenomenon is called neutrino oscillation and requires that neutrinos
205 must have a non-zero mass (as seen in subsection 2.2.1). This observation is direct
206 evidence of beyond standard model physics. This behaviour has been characterised
207 by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13–15] mixing matrix which
208 describes how the flavour and mass of neutrinos are associated. This is analogous to
209 the Cabibbo-Kobayashi-Maskawa (CKM) [16] matrix measured in quark physics.

210 2.2.1. Three Flavour Oscillations

211 The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ (indexed
212 ν_α), which are eigenstates of the weak interaction and three mass eigenstates, ν_1 , ν_2 and
213 ν_3 (indexed ν_i). Each mass eigenstate is the superposition of all three flavour states,

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle. \quad (2.1)$$

²¹⁴ Where U is the PMNS matrix which is unitary and connects the mass and flavour
²¹⁵ eigenstates.

²¹⁶ The weak interaction couples to flavour eigenstates so neutrinos interact with
²¹⁷ leptons of the same flavour. The propagation of a neutrino flavour eigenstate, in a
²¹⁸ vacuum, can be re-written as a plane-wave solution to the time-dependent Schrödinger
²¹⁹ equation,

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

²²⁰ The probability of observing a neutrino of flavour eigenstate β from one which
²²¹ originated as flavour α can be calculated as,

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-i(\phi_j - \phi_i)} \quad (2.3)$$

²²² The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
²²³ three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
²²⁴ coordinates). Therefore,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

²²⁵ For a relativistic particle, $E_i \gg m_i$,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

²²⁶ Making the approximations that neutrinos are relativistic, the mass eigenstates
²²⁷ were created with the same energy and that $x = L$, where L is the distance traveled by
²²⁸ the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

²²⁹ where $\Delta m_{ij}^2 = m_j^2 - m_i^2$. This, combined with further use of unitarity relations
²³⁰ results in Equation 2.3 becoming

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \quad (2.7)$$

²³¹ Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last term is
²³² included for the oscillation probability of antineutrinos.

²³³ Typically, the PMNS matrix is parameterised into three mixing angles, a charge
²³⁴ parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

²³⁵ Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often
²³⁶ grouped; (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many
²³⁷ neutrino experiments aim to measure the PMNS parameters from a wide array of
²³⁸ origins, as is the purpose of this thesis.

²³⁹ The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8 is only
²⁴⁰ included for completeness. For an oscillation analysis experiment, any terms contain-
²⁴¹ ing this phase disappear due to taking the expectation value of the PMNS matrix.
²⁴² Measurements of these phases are typically performed by experiments searching for
²⁴³ neutrino-less double β -decay [17].

²⁴⁴ A two flavour approximation can be obtained when one assumes the third mass
²⁴⁵ eigenstate is degenerate with another. As discussed in section 2.3, it is found that
²⁴⁶ $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$. This results in the two flavour approximation being reasonable for
²⁴⁷ understanding the features of the oscillation. In this two flavour case, the mixing
²⁴⁸ matrix becomes,

$$U_{2 \text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

²⁴⁹ This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

²⁵⁰ Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is a sinusoidal
²⁵¹ function depending upon the distance over which the neutrino propagates. The
²⁵² frequency and amplitude of oscillation are dependent upon $\Delta m^2/4E$ and $\sin^2 2\theta$,
²⁵³ respectively. The oscillation probabilities presented thus far assume $c = 1$, where
²⁵⁴ c is the speed of light in vacuum. In more familiar units, the maximum oscillation
²⁵⁵ probability for a fixed value of θ is given at $L[km]/E[GeV] \sim 1.27/\Delta m^2$. It is this
²⁵⁶ calculation that determines the best L/E value for a given experiment to be designed
²⁵⁷ around for measurements of a specific value of Δm^2 .

²⁵⁸ 2.2.2. The MSW Effect

²⁵⁹ The theory of neutrino oscillation in a vacuum has been described in subsection 2.2.1.
²⁶⁰ However, the beam neutrinos and atmospheric neutrinos originating from below the
²⁶¹ horizon propagate through matter in the Earth. The coherent scattering of neutrinos
²⁶² from a material target modifies the Hamiltonian of the system. This results in a change
²⁶³ in the oscillation probability. Notably, charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$,
²⁶⁴ propagated by a W boson) only affects electron neutrinos whereas the neutral current
²⁶⁵ scattering ($\nu_l + l^- \rightarrow \nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino
²⁶⁶ flavours equally. In the two-flavour approximation, the effective mixing parameter
²⁶⁷ becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F is Fermi's constant. It is clear to see that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for $\Delta m^2 > 0$ which results in a divergent mixing parameter. This resonance is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect (or more colloquially, the matter resonance) which regenerates the electron neutrino component of the neutrino flux [18–20]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

At densities lower than this critical value, the oscillation probability will be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$ [21]. The resonance occurring from the MSW effect depends on the sign of Δm^2 . Therefore, any neutrino oscillation experiment which observes neutrinos and antineutrinos which have propagated through matter can have some sensitivity to the ordering of the neutrino mass eigenstates.

2.3. Neutrino Oscillation Measurements

As evidence of beyond standard model physics, the 2015 Nobel Prize in Physics was awarded to the Super-Kamiokande (SK) [22] and Sudbury Neutrino Observatory (SNO) [23] collaborations for the first definitive observation of solar and atmospheric

²⁸⁴ neutrino oscillation [24]. Since then, the field has seen a wide array of oscillation
²⁸⁵ measurements from a variety of neutrino sources. As seen in subsection 2.2.1, the
²⁸⁶ neutrino oscillation probability is dependent on the ratio of the propagation baseline, L ,
²⁸⁷ to the neutrino energy, E . It is this ratio that determines the type of neutrino oscillation
²⁸⁸ a particular experiment is sensitive to.

²⁸⁹ As illustrated in Figure 2.1, there are many neutrino sources that span a wide
²⁹⁰ range of energies. The least energetic neutrinos are from diffuse supernovae and
²⁹¹ terrestrial neutrinos at $O(1)$ MeV whereas the most energetic neutrinos originate from
²⁹² atmospheric and galactic neutrinos of $> O(1)$ TeV.

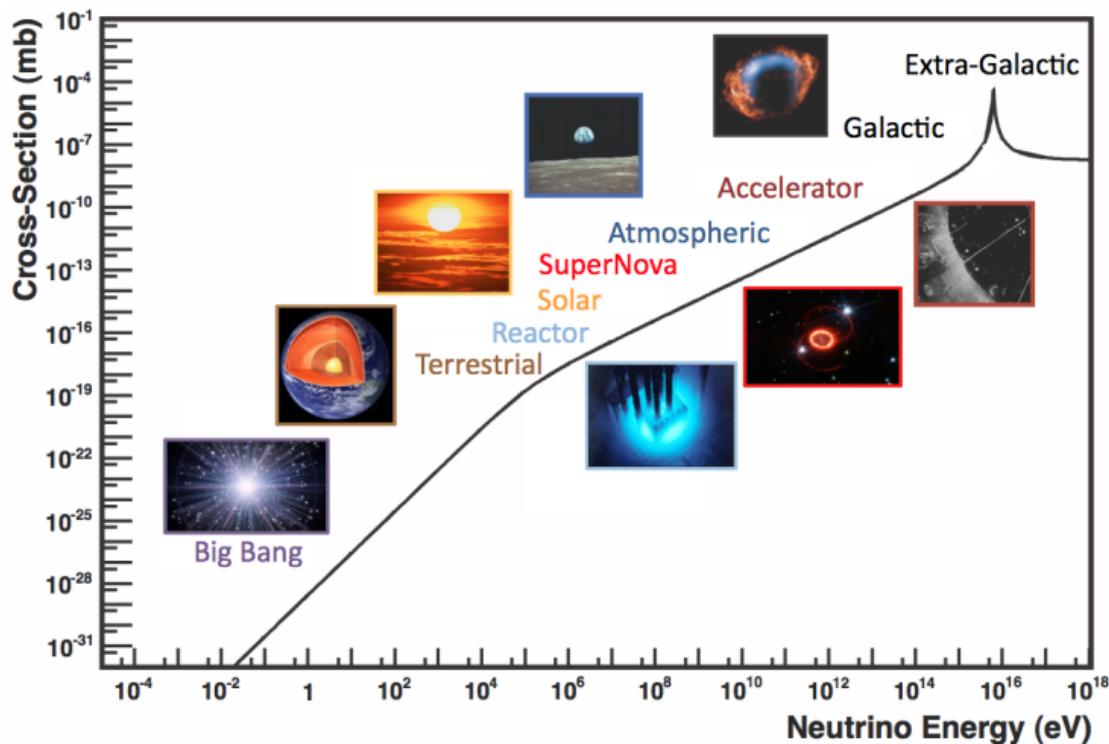


Figure 2.1.: The cross-section of neutrinos from various natural and man-made sources as a function of neutrino energy. Taken from [25]

²⁹³ 2.3.1. Solar Neutrinos

²⁹⁴ Solar neutrinos are emitted from fusion reaction chains at the center of the Sun. The
²⁹⁵ solar neutrino flux, given as a function of neutrino energy for different fusion and
²⁹⁶ decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion generates the
²⁹⁷ largest flux of neutrinos, the neutrinos are of low energy and are difficult to reconstruct
²⁹⁸ due to the IBD interaction threshold of 1.8MeV. Consequently, most experiments focus
²⁹⁹ on the neutrinos from the decay of 8B (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher
³⁰⁰ energy.

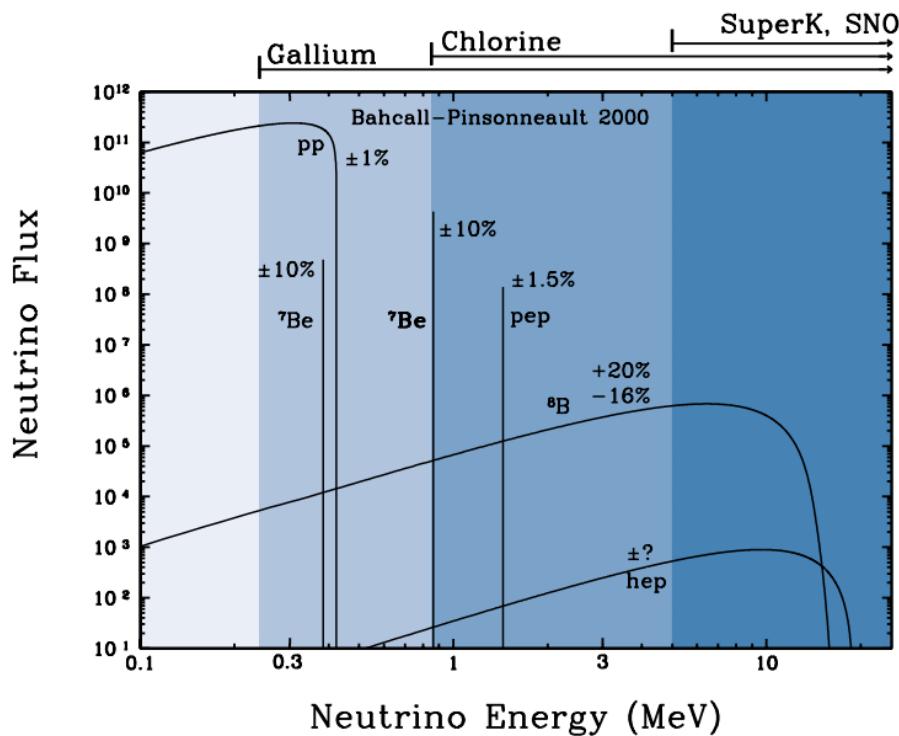
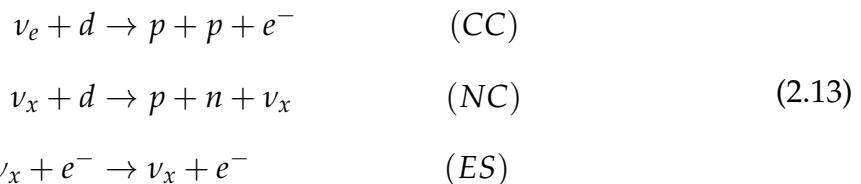


Figure 2.2.: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [26].

³⁰¹ The first measurements of solar neutrinos observed a significant reduction in the
³⁰² event rate compared to predictions from the Standard Solar Model [27, 28]. The
³⁰³ proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a

³⁰⁴ precursory version of the PMNS model [29]. The Kamiokande [30], Gallex [31] and
³⁰⁵ Sage [32] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

³⁰⁶ The conclusive solution to this problem was determined by the SNO collaboration
³⁰⁷ [33]. Using a deuterium water target to observe 8B neutrinos, the event rate of charged
³⁰⁸ current (CC), neutral current (NC), and elastic scattering (ES) interactions (Given in
³⁰⁹ Equation 2.13) was simultaneously measured. CC events can only occur for electron
³¹⁰ neutrinos, whereas the NC channel is agnostic to neutrino flavour, and the ES reaction
³¹¹ has a slight excess sensitivity to electron neutrino interactions. This meant that there
³¹² were direct measurements of the ν_e and ν_x neutrino flux. It was concluded that the
³¹³ CC and ES interaction rates were consistent with the deficit previously observed.
³¹⁴ Most importantly, the NC reaction rate was only consistent with the others under the
³¹⁵ hypothesis of flavour transformation.



³¹⁶ Many experiments have since measured the neutrino flux of different interaction
³¹⁷ chains within the sun [34–36]. The most recent measurement was that of CNO neutrinos
³¹⁸ which were recently observed with 5σ significance by the Borexino collaboration.
³¹⁹ Future neutrino experiments aim to further these spectroscopic measurements of
³²⁰ different fusion chains within the Sun [37–39]. Solar neutrinos act as an irreducible
³²¹ background for dark matter experiments like DARWIN but oscillation parameter
³²² measurements can be made [40].

³²³ 2.3.2. Atmospheric Neutrinos

³²⁴ The interactions of primary cosmic ray protons in Earth's upper atmosphere generate
³²⁵ showers of energetic hadrons. These are mostly pions and kaons which when they
³²⁶ decay produce a natural source of neutrinos spanning energies of MeV to TeV [41].
³²⁷ The main decay is via

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu) \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu) \end{aligned} \tag{2.14}$$

³²⁸ such that for a single pion decay, three neutrinos are typically produced. The
³²⁹ atmospheric neutrino flux energy spectra as predicted by the Bartol [42], Honda
³³⁰ [43–45], and FLUKA [46] models are illustrated in Figure 2.3. The flux distribution
³³¹ peaks at an energy of $O(10)\text{GeV}$. The uncertainties associated with these models
³³² are dominated by the hadronic production of kaon and pions as well as the primary
³³³ cosmic flux.

³³⁴ Unlike long-baseline experiments which have a fixed baseline, the distance at-
³³⁵ mospheric neutrinos propagate is dependent upon the zenith angle at which they
³³⁶ interact. This is illustrated in Figure 2.4. Neutrinos that are generated directly above
³³⁷ the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the height of the atmosphere
³³⁸ whereas neutrinos that interact directly below the detector ($\cos(\theta) = -1.0$) have to
³³⁹ travel a length equal to the diameter of the Earth. This means atmospheric neutrinos
³⁴⁰ have a baseline that varies from $O(20)\text{km}$ to $O(6 \times 10^3)\text{km}$. Any neutrino generated
³⁴¹ at or below the horizon will be subject to matter effects as they propagate through the
³⁴² Earth.

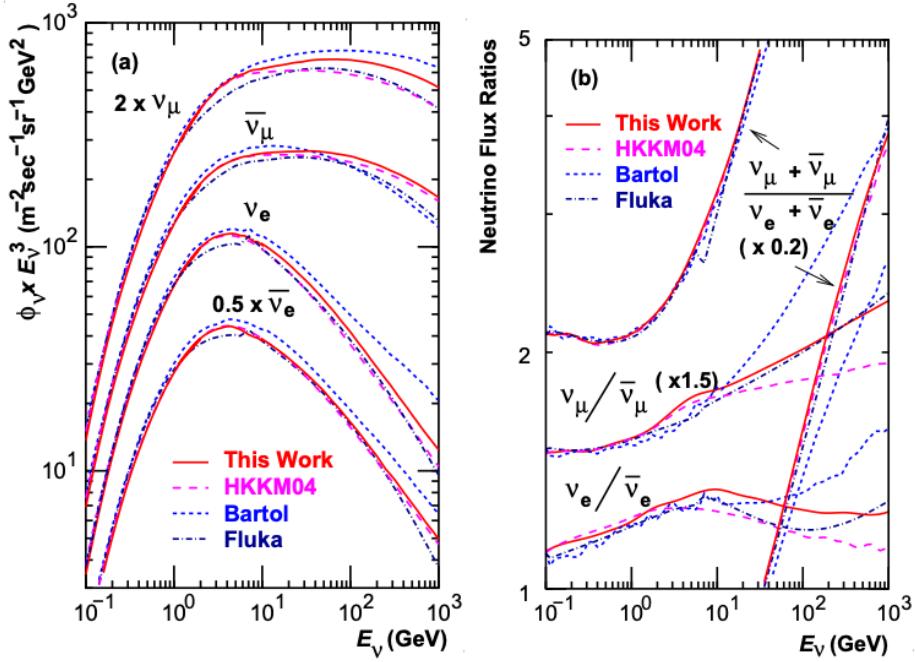


Figure 2.3.: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [43], the 2004 Honda model (“HKKM04”) [44], the Bartol model [42] and the FLUKA model [46]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [43].

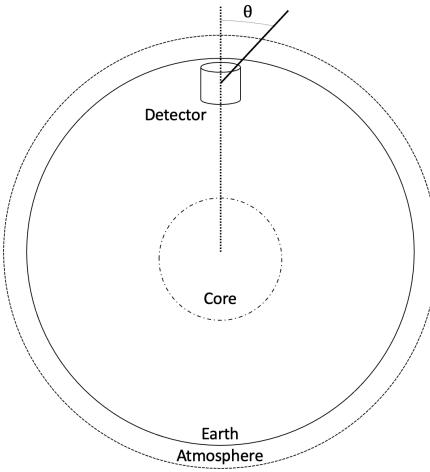


Figure 2.4.: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [47].

Figure 2.5 highlights the neutrino flux as a function of the zenith angle for different

slices of neutrino energy. For medium to high-energy neutrinos (and to a lesser degree

for low-energy neutrinos), the flux is approximately symmetric around $\cos(\theta) = 0$.

³⁴³ Figure 2.5 highlights the neutrino flux as a function of the zenith angle for different
³⁴⁴ slices of neutrino energy. For medium to high-energy neutrinos (and to a lesser degree
³⁴⁵ for low-energy neutrinos), the flux is approximately symmetric around $\cos(\theta) = 0$.

To the accuracy of this approximation, the systematic uncertainties associated with atmospheric flux for comparing upward-going and down-going neutrino cancels. This allows the down-going events, which are mostly insensitive to oscillation probabilities, to act as an unoscillated prediction (similar to a near detector in an accelerator neutrino experiment).

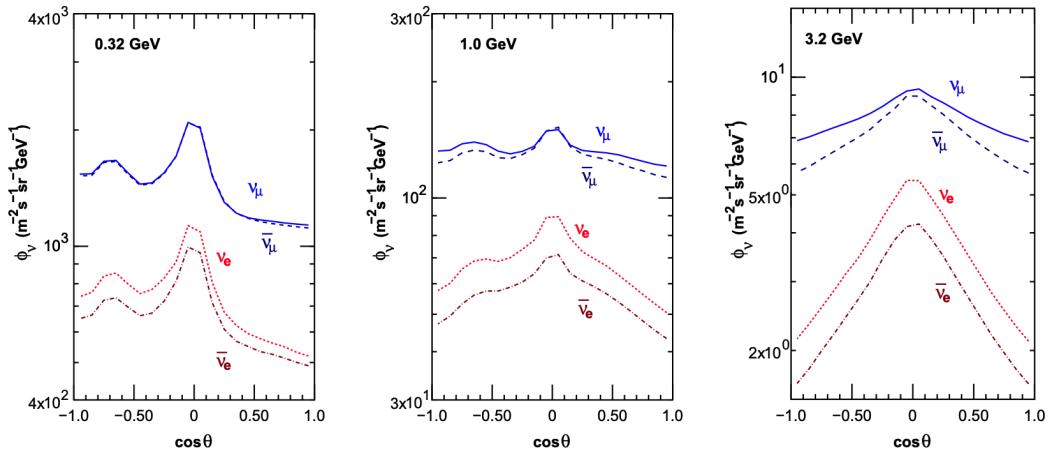


Figure 2.5.: Prediction of ν_e , $\bar{\nu}_e$, ν_μ , $\bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [45]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [42], Honda [43] and FLUKA [46] are given in [47].

Precursory hints of atmospheric neutrinos were observed in the mid-1960s searching for $\nu_\mu + X \xrightarrow{(-)} X^* + \mu^\pm$ [48], although it was called an anomaly at the time of measurement. This was succeeded with the IMB-3 [49] and Kamiokande [50] experiments which measured the ratio of muon neutrinos compared to electron neutrinos $R(\nu_\mu/\nu_e)$. Both experiments were found to have a consistent deficit of muon neutrinos, with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.60^{+0.07}_{-0.06} \pm 0.05$. Super-Kamiokande (SK) [47] extended this analysis by fitting oscillation parameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}$ eV 2 .

Since then, atmospheric neutrino experiments have been making precision measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ discovery of ν_τ

³⁶² appearance [51]. Figure 2.6 illustrates the current estimates on the atmospheric mixing
³⁶³ parameters from a wide range of atmospheric and accelerator neutrino observatories.

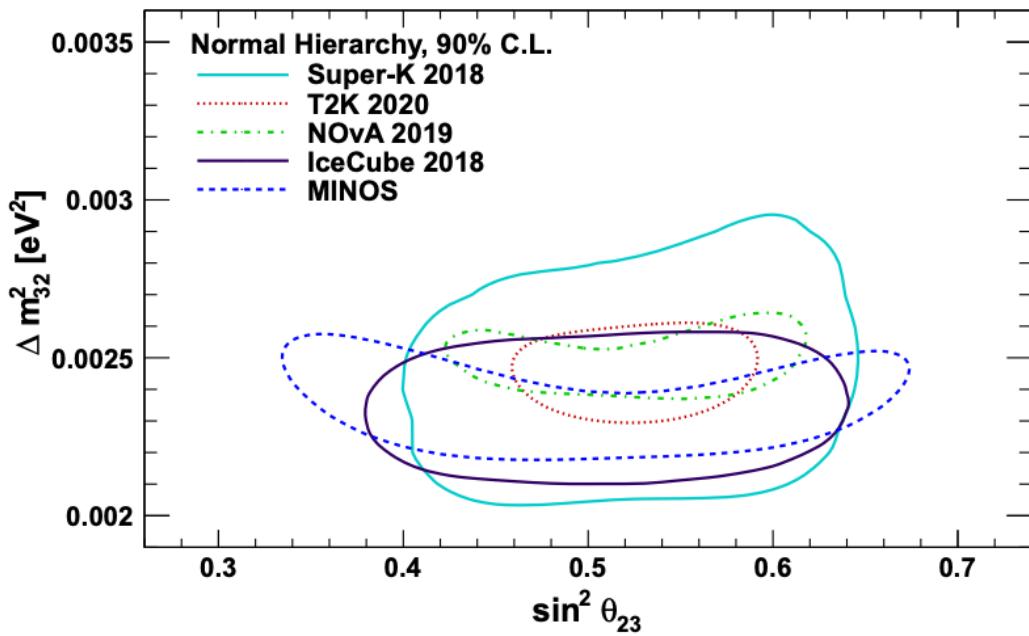


Figure 2.6.: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{23}^2 , from atmospheric and long baseline experiments: SK [52], T2K [53], NOvA [54], IceCube [55] and MINOS [56]. Figure taken from [57].

³⁶⁴ 2.3.3. Accelerator Neutrinos

³⁶⁵ The concept of using a man-made “neutrino beam” was first realised in 1962 [58].
³⁶⁶ Since then, many experiments have followed which all use the same fundamental
³⁶⁷ concepts. Typically, a proton beam is aimed at a target producing charged mesons that
³⁶⁸ decay to neutrinos. The mesons can be sign-selected by the use of magnetic focusing
³⁶⁹ horns to generate a neutrino or antineutrino beam. Pions are the primary meson that
³⁷⁰ decay and depending on the orientation of the magnetic field, a muon (anti-)neutrino
³⁷¹ beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. The decay of muons and
³⁷² kaons does result in an irreducible intrinsic electron neutrino background. In T2K,
³⁷³ this background contamination is $O(< 1\%)$ [59]. There is also an approximately $\sim 5\%$

³⁷⁴ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated via the same decays. As the beam is
³⁷⁵ generated by proton interactions (rather than anti-proton interactions), the wrong-sign
³⁷⁶ component in the antineutrino beam is larger when operating in neutrino mode.

³⁷⁷ Tuning the proton energy in the beam and using beam focusing techniques allows
³⁷⁸ the neutrino energy to be set to a value that maximises the disappearance oscillation
³⁷⁹ probability in the L/E term in Equation 2.10. This means that accelerator experiments
³⁸⁰ are typically more sensitive to the mixing parameters as compared to a natural neutrino
³⁸¹ source. However, the disadvantage compared to atmospheric neutrino experiments is
³⁸² that the baseline has to be shorter due to the lower flux. Consequently, there is typically
³⁸³ less sensitivity to matter effects and the ordering of the neutrino mass eigenstates.

³⁸⁴ A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.15)$$

³⁸⁵ where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of neutrinos
³⁸⁶ with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and $\epsilon(\vec{x})$ is the
³⁸⁷ efficiency and resolution of the detector. In order to leverage the most out of an
³⁸⁸ accelerator neutrino experiment, the flux and cross-section systematics need to be
³⁸⁹ constrained. This is typically done via the use of a “near detector”, situated at a baseline
³⁹⁰ of $O(1)$ km. This detector observes the unoscillated neutrino flux and constrains the
³⁹¹ parameters used within the flux and cross-section model.

³⁹² The first accelerator experiments to precisely measure oscillation parameters were
³⁹³ MINOS [60] and K2K [61]. These experiments confirmed the ν_μ disappearance seen in
³⁹⁴ atmospheric neutrino experiments by finding consistent parameter values for $\sin^2(\theta_{23})$
³⁹⁵ and Δm_{23}^2 . The current generation of accelerator neutrino experiments, T2K and NO ν A

³⁹⁶ extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead the sensitivity to atmospheric mix-
³⁹⁷ ing parameters as seen in Figure 2.6 [62]. The two experiments differ in their peak
³⁹⁸ neutrino energy, baseline, and detection technique. The NO ν A experiment is situated
³⁹⁹ at a baseline of 810km from the NuMI beamline which delivers 2GeV neutrinos. The
⁴⁰⁰ T2K neutrino beam is peaked around 0.6GeV and propagates 295km. The NO ν A
⁴⁰¹ experiment also uses functionally identical detectors (near and far) which allow the
⁴⁰² approximate cancellation of detector systematics whereas T2K uses a plastic scintil-
⁴⁰³ lator technique at the near detector and a water Cherenkov far detector. The future
⁴⁰⁴ generation experiments DUNE [63] and Hyper-Kamiokande [64] will succeed these
⁴⁰⁵ experiments as the high-precision era of neutrino oscillation parameter measurements
⁴⁰⁶ develops.

⁴⁰⁷ Several anomalous results have been observed in the LSND [9] and MiniBooNE [10]
⁴⁰⁸ detectors which were designed with purposefully short baselines. Parts of the neu-
⁴⁰⁹ trino community attributed these results to oscillations induced by a fourth “sterile”
⁴¹⁰ neutrino [65] but several searches in other experiments, MicroBooNE [66] and KAR-
⁴¹¹ MEN [67], found no hints of additional neutrino species. The solution to the anomalous
⁴¹² results is still being determined.

⁴¹³ 2.3.4. Reactor Neutrinos

⁴¹⁴ As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very
⁴¹⁵ useful man-made source of electron antineutrinos. For reactors that use low-enriched
⁴¹⁶ uranium ^{235}U as fuel, the antineutrino flux is dominated by the β -decay fission of ^{235}U ,
⁴¹⁷ ^{238}U , ^{239}Pu and ^{241}Pu [68] as illustrated in Figure 2.7.

⁴¹⁸ Due to their low energy, reactor electron antineutrinos predominantly interact
⁴¹⁹ via the inverse β -decay (IBD) interaction. The typical signature contains two signals

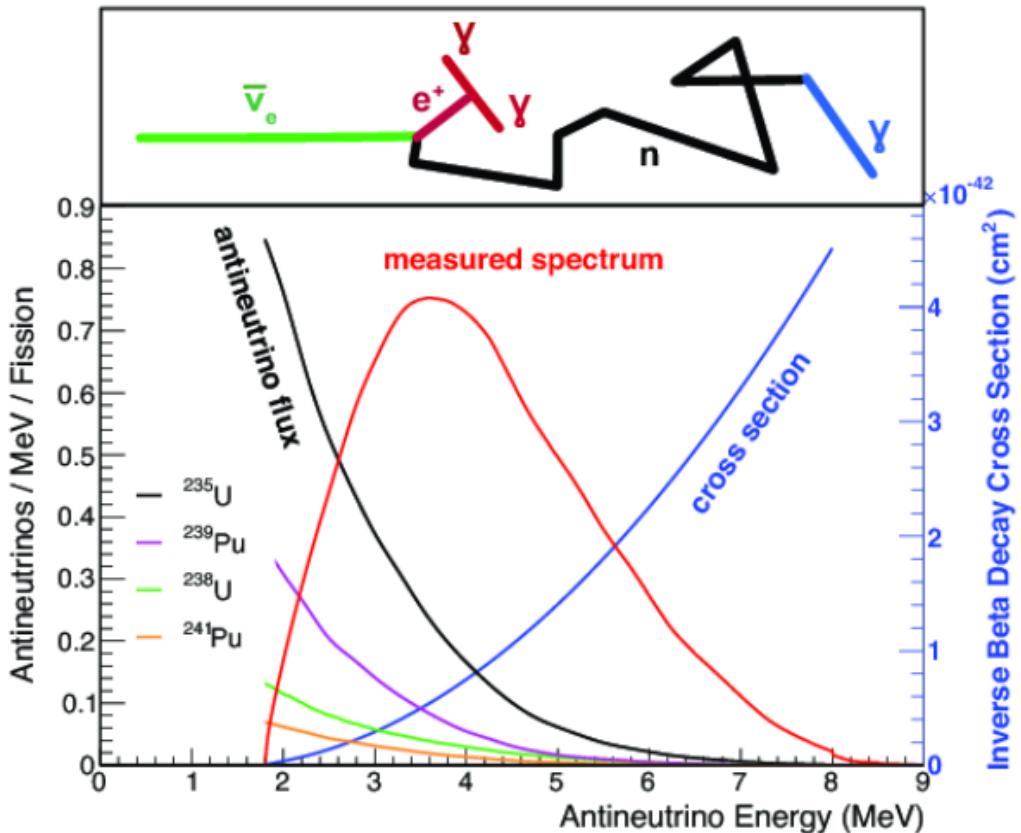


Figure 2.7.: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [69].

delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihilation, and secondly the photons emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [70]. Recently, SK included gadolinium dopants into the ultra-pure water to increase the energy released from the photon cascade to $\sim 8\text{MeV}$ and reduce the time of the delayed signal to $\sim 28\mu\text{s}$.

There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured the $\sin^2(\theta_{13})$ and Δm_{23}^2 oscillation parameters. Daya Bay [71], RENO [72] and Double Chooz [73] have all provided precise measurements, with the first discovery of a

429 non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K [73]). The
430 constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field and are often used as
431 external inputs to accelerator neutrino experiments to improve their sensitivity to δ_{CP}
432 and mass hierarchy determination. JUNO-TAO [74], a small collaboration within the
433 larger JUNO experiment, is a next-generation reactor experiment that aims to precisely
434 measure the isotopic antineutrino yields from the different fission chains. Alongside
435 this, it aims to explain the ‘5MeV excess’ [75–77] by conducting a search for sterile
436 neutrinos with a mass scale of around 1eV.

437 Kamland [78] is the only experiment to have observed reactor neutrinos using a
438 long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have
439 sensitivity to Δm_{12}^2 . Combined with the SK solar neutrino experiment, the combined
440 analysis puts the most stringent constraint on Δm_{12}^2 [79].

441 2.4. Summary

442 Since observing the first evidence of neutrino oscillations in the late 1990’s, numerous
443 measurements of the mixing parameters have been made. Many experiments use
444 neutrinos as a tool for discovery of new physics (diffuse supernova background,
445 neutrinoless double beta decay and others) so the PMNS parameters are summarised
446 in the Particle Data Group (PDG) review tables. The analysis presented in this thesis
447 focuses on the 2020 T2K oscillation analysis presented in [80] where the 2018 PDG
448 constraints [81] were used. These constraints are outlined in Table 2.1.

449 The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance,
450 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is take as the average best-fit from the combination of Daya Bay,
451 Reno and Double Chooz. It is often used as a prior uncertainty within other neu-
452 trino oscillation experiments, typically termed the reactor constraint. The $\sin^2(\theta_{12})$

Parameter	2018 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.12 \pm 0.08) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H., Q1)	$0.421^{+0.033}_{-0.025}$
$\sin^2(\theta_{23})$ (I.H., Q2)	$0.592^{+0.023}_{-0.030}$
$\sin^2(\theta_{23})$ (N.H., Q1)	$0.417^{+0.025}_{-0.028}$
$\sin^2(\theta_{23})$ (N.H., Q2)	$0.597^{+0.024}_{-0.030}$
Δm_{32}^2 (I.H.)	$(-2.56 \pm 0.04) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.51 \pm 0.05) \times 10^{-3} \text{ eV}^2$

Table 2.1.: The 2018 Particle Data Group constraints of the oscillation parameters taken from [81]. The value of Δm_{23}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

parameter is predominantly measured through electron neutrino disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline reactor neutrino experiment Kamland also has sensitivity to this parameter and is used in a joint fit to solar data from SNO and SK, using the reactor constraint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric neutrino experiments. The PDG value is a joint fit of T2K, NOvA, MINOS and IceCube DeepCore experiments. The latest T2K-only measurement, provided at Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [80]. The PDG constraint on Δm_{12}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{23}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchy separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [80]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were

⁴⁶⁹ rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analysis
⁴⁷⁰ have reduced the rejection intervals to 90%. Since the 2018 PDG publication, there has
⁴⁷¹ been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [82], alongside updated
⁴⁷² Δm_{23}^2 and $\sin^2(\theta_{23})$ measurements.

⁴⁷³ Throughout this thesis, several sample spectra predictions and contours are pre-
⁴⁷⁴ sented which require oscillation parameters to be assumed. Table 2.2 defines two sets
⁴⁷⁵ of oscillation parameters, with “Asimov A” set being close to the preferred values
⁴⁷⁶ from a previous T2K-only fit [83] and “Asimov B” being CP-conserving and further
⁴⁷⁷ from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2.: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

⁴⁷⁸ **Chapter 3**

⁴⁷⁹ **T2K and SK Experiment Overview**

⁴⁸⁰ As the successor of the Kamiokande experiment, the Super-Kamiokande (SK) collabora-
⁴⁸¹ ration has been leading atmospheric neutrino oscillation analyses for over two decades.
⁴⁸² The detector has provided some of the strongest constraints on proton decay and the
⁴⁸³ first precise measurements of the Δm_{23}^2 and $\sin^2(\theta_{23})$ neutrino oscillation parameters.
⁴⁸⁴ The ability of the detector to low-energy neutrino events has been significantly im-
⁴⁸⁵ proved with the recent gadolinium doping of the ultra-pure water target. The history,
⁴⁸⁶ detection technique, and operation of the SK detector is described in section 3.1.

⁴⁸⁷ The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline ex-
⁴⁸⁸ periments to use both neutrino and antineutrino beams to precisely measure the
⁴⁸⁹ charge parity violation within the neutrino sector. With the SK detector observing
⁴⁹⁰ the oscillated neutrino flux, the T2K experiment observed the first hints of a non-zero
⁴⁹¹ $\sin^2(\theta_{13})$ measurement and continues to lead the field with the constraints it provides
⁴⁹² on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{23}^2 and δ_{CP} . The techniques which T2K uses in gener-
⁴⁹³ ating its neutrino beam as well as the near-detector used to constrain the flux and
⁴⁹⁴ cross-section parameters used in this analysis are documented in section 3.2.

⁴⁹⁵ **3.1. The Super-Kamiokande Experiment**

⁴⁹⁶ The SK experiment began taking data in 1996 [84] and has had many modifications
⁴⁹⁷ throughout its lifespan. There have been seven defined periods of data taking as
⁴⁹⁸ noted in Table 3.1. Data taking began in SK-I which ran for five years. Between the

499 SK-I and SK-II periods, a significant proportion of the PMTs were damaged during
 500 maintenance. Those that survived were equally distributed throughout the detector
 501 in the SK-II era, which resulted in a reduced photo-coverage. From SK-III onwards,
 502 repairs to the detector meant the full suite of PMTs was operational. Before the
 503 start of SK-IV, the data acquisition and electronic systems were upgraded. Between
 504 SK-IV and SK-V, a significant effort was placed into tank open maintenance and
 505 repair/replacement of defective PMTs, a task for which the author of this thesis was
 506 required. Consequently, the detector conditions were significantly different between
 507 the two operational periods. SK-VI saw the start of the 0.01% gadolinium doped water.
 508 SK-VII, which started during the writing of this thesis, has increased the gadolinium
 509 concentration to 0.03% for continued operation [85].

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

Table 3.1.: The various SK periods and respective live-time. The SK-VI live-time is calculated until 1st April 2022. SK-VII started during the writing of this thesis.

510 3.1.1. The SK Detector

511 The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank with a
 512 diameter 39.3m and height 41.1m filled with ultrapure water [86]. A diagram of the
 513 significant components of the SK detector is given in Figure 3.1. The SK detector is
 514 situated in the Kamioka mine in Gifu, Japan. The mine is underground with roughly
 515 1km rock overburden (2.7km water equivalent overburden) [87]. At this depth, the

⁵¹⁶ rate of cosmic ray muons is significantly decreased to a value of $\sim 2\text{Hz}$. The top of
⁵¹⁷ the tank is covered with stainless steel which is designed as a working platform for
⁵¹⁸ maintenance, calibration, and location for high voltage and data acquisition electronics.

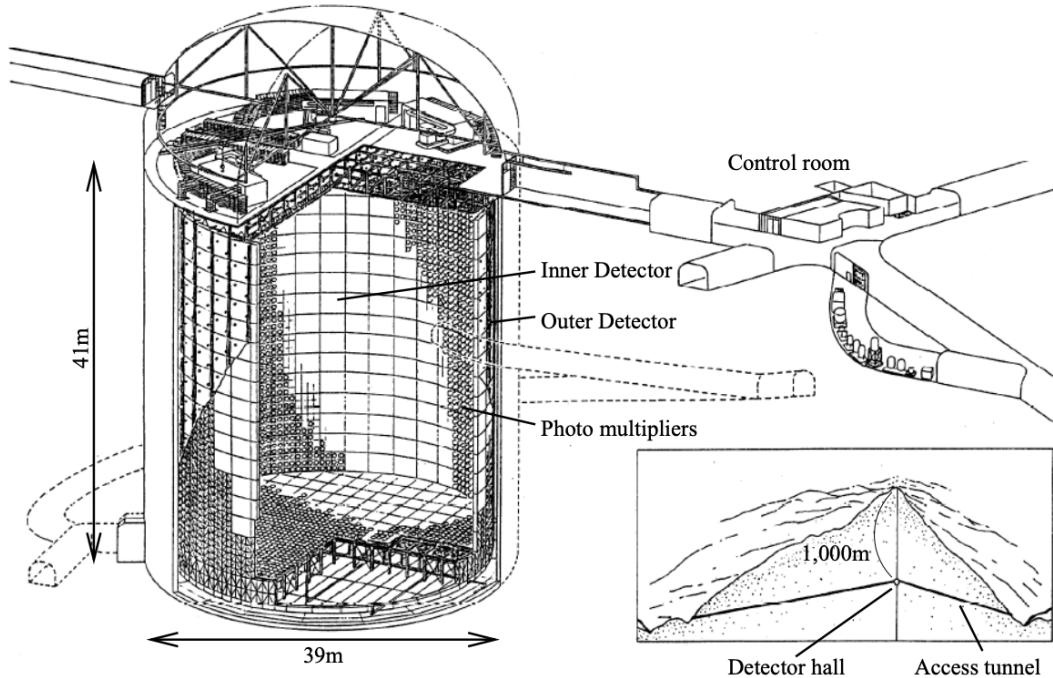


Figure 3.1.: A schematic diagram of the Super-Kamiokande Detector. Taken from [88].

⁵¹⁹ A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated inside the
⁵²⁰ tank, with an approximate 2m gap between this structure and the outer tank wall. The
⁵²¹ purpose of this structure is to support the photomultiplier tubes (PMTs). The volume
⁵²² inside and outside the support structure is referred to as the inner detector (ID) and
⁵²³ outer detector (OD), respectively. In the SK-IV era, the ID and OD are instrumented
⁵²⁴ by 11,129 50cm and 1,885 20cm PMTs respectively [86]. The ID contains a 32kton
⁵²⁵ mass of water. Many analyses performed at SK use a “fiducial volume” defined by the
⁵²⁶ volume of water inside the ID excluding some distance to the ID wall. This reduces the
⁵²⁷ volume of the detector which is sensitive to neutrino events but reduces radioactive
⁵²⁸ backgrounds and allows for better reconstruction performance. The nominal fiducial

529 volume is defined as the area contained inside 2m from the ID wall for a total of
530 22.5kton water [89].

531 The two regions of the detector (ID and OD) are optically separated with opaque
532 black plastic. The purpose of this is to determine whether a track entered or exited
533 the ID. This allows cosmic ray muons and partially contained events to be tagged and
534 separated from neutrino events entirely contained within the ID. This black plastic is
535 also used to cover the area between the ID PMTs to reduce photon reflection from the
536 ID walls. Opposite to this, the OD is lined with a reflective material to allow photons to
537 reflect around inside the OD until collected by one of the PMTs. Furthermore, each OD
538 PMT is backed with $50 \times 50\text{cm}$ plates of wavelength shifting acrylic which increases
539 the efficiency of light collection [87].

540 In the SK-IV data-taking period, the photocathode coverage of the detector, or the
541 fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [87]. The PMTs have a
542 quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$ for
543 photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$. The proportion of photoelectrons
544 that produce a signal in the dynode of a PMT, termed the collection efficiency, is
545 $> 70\%$ [87]. The PMTs used within SK are most sensitive to photons with wavelength
546 $300\text{nm} \leq \lambda \leq 600\text{nm}$ [87]. One disadvantage of using PMTs as the detection media
547 is that the Earth's geomagnetic field can modify its response. Therefore, a set of
548 compensation coils is built around the inner surface of the detector to mitigate this
549 effect [90].

550 As mentioned, the SK detector is filled with ultrapure water, which in a perfect
551 world would contain no impurities. However, bacteria and organic compounds can
552 significantly degrade the water quality. This decreases the attenuation length, which
553 reduces the total number of photons that hit a PMT. To combat this, a sophisticated
554 water treatment system has been developed [87, 91]. UV lights, mechanical filters,

and membrane degasifiers are used to reduce the bacteria, suspended particulates, and radioactive materials from the water. The flow of water within the tank is also critical as it can remove stagnant bacterial growth or build-up of dust on the surfaces within the tank. Gravity drifts impurities in the water towards the bottom of the tank which, if left uncontrolled, can create asymmetric water conditions between the top and bottom of the tank. Typically, the water entering the tank is cooled below the ambient temperature of the tank to control convection and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs is sensitive to the PMT temperature [92] so controlling the temperature gradients within the tank is beneficial for stable measurements.

SK-VI is the first phase of the SK experiment to use gadolinium dopants within the ultrapure water [85]. As such, the SK water system had to be replaced to avoid removing the gadolinium concentrate from the ultrapure water [93]. For an inverse β -decay (IBD) interaction in a water target, the emitted neutron is thermally captured on hydrogen. This process releases 2.2MeV γ rays which are difficult to detect as the resulting Compton scattered electrons are very close to the Cherenkov threshold, limiting the number of photons produced. Thermal capture of neutrons on gadolinium generates γ rays with higher energy (8MeV [70]) meaning they are more easily detected. SK-VI has 0.01% Gd loading (0.02% gadolinium sulphate by mass) which causes \approx 50% of neutrons emitted by IBD to be captured on gadolinium [94, 95]. Whilst predominantly useful for low energy analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino event selections [96]. Efforts are currently in place to increase the gadolinium concentrate to 0.03% for \approx 75% neutron capture efficiency on gadolinium [97]. The final stage of loading targets 0.1% concentrate.

579 3.1.2. Calibration

580 The calibration of the SK detector is documented in [86] and summarised below. The
581 analysis presented within this thesis is dependent upon ‘high energy events’ (Charged
582 particles with $O(> 100)\text{MeV}$ momenta). These are events that are expected to generate
583 a larger number of photons such that each PMT will be hit with multiple photons.
584 The reconstruction of these events depends upon the charge deposited within each
585 PMT and the timing response of each individual PMT. Therefore, the most relevant
586 calibration techniques to this thesis are outlined.

587 Before installation, 420 PMTs were calibrated to have identical charge responses
588 and then distributed throughout the tank in a cross-shape pattern (As illustrated by
589 Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed
590 at similar geometric positions within SK to be calibrated against. To perform this
591 calibration, a xenon lamp is located at the center of the SK tank which flashes uniform
592 light at 1Hz. This allows for geometrical effects, water quality variation, and timing
593 effects to be measured in-situ throughout normal data-taking periods.

594 When specifically performing calibration of the detector (in out-of-data taking
595 mode), the water in the tank was circulated to avoid top/bottom asymmetric water
596 quality. Any non-uniformity within the tank significantly affects the PMT hit proba-
597 bility through scattering or absorption. This becomes a dominant effect for the very
598 low-intensity light sources discussed later which are designed such that only one
599 photon is incident upon a given PMT.

600 The “gain” of a PMT is defined as the ratio of the total charge of the signal produced
601 compared to the charge of photoelectrons emitted by the photocathodes within the
602 PMT. To calibrate the signal of each PMT, the “relative” and “absolute” gain values are

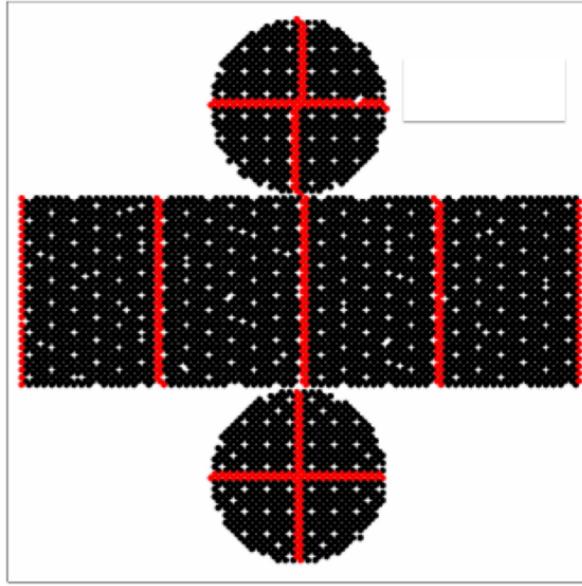


Figure 3.2.: The location of “standard PMTs” (red) inside the SK detector. Taken from [86].

603 measured. The relative gain is the variation of gain among each of the PMTs whereas
 604 the absolute gain is the average gain of all PMTs.

605 The relative gain is calibrated as follows. A laser is used to generate two measure-
 606 ments: a high-intensity flash that illuminates every PMT with a sufficient number of
 607 photons, and a low-intensity flash in which only a small number of PMTs collect light.
 608 The first measurement creates an average charge, $Q_{obs}(i)$ on PMT i , whereas the second
 609 measurement ensures that each hit PMT only generates a single photoelectron. For the
 610 low-intensity measurement, the number of times each PMT records a charge larger
 611 than 1/4 photoelectrons, $N_{obs}(i)$, is counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i), \end{aligned} \tag{3.1}$$

612 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the acceptance
 613 efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and collection efficiency

614 of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative gain for each PMT can
615 determined by taking the ratio of these quantities.

616 The absolute gain calibration is performed by observing fixed energy γ -rays of
617 $E_{\gamma} \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source situated at
618 the center of the detector. This generates a photon yield of about 0.004 photoelec-
619 trons/PMT/event, meaning that $> 99\%$ of PMT signals are generated from single
620 photoelectrons. A charge distribution is generated by performing this calibration over
621 all PMTs, and the average value of this distribution is taken to be the absolute gain
622 value.

623 As mentioned in subsection 3.1.1, the average quantum and collection efficiency
624 for the SK detector is $\sim 21\%$ and $> 70\%$ respectively. However, these values do differ
625 between each PMT and need to be calibrated accordingly. Consequently, the NiCf
626 source is also used to calibrate the “quantum \times collection” efficiency (denoted “QE”)
627 value of each PMT. The NiCf low-intensity source is used as the PMT hit probability
628 is proportional to the QE ($N_{\text{obs}}(i) \propto \epsilon(i)$ in Equation 3.1). A Monte Carlo prediction
629 which includes photon absorption, scattering, and reflection is made to estimate the
630 number of photons incident on each PMT and the ratio of the number of predicted
631 to observed hits is calculated. The difference is attributed to the QE efficiency of that
632 PMT. This technique is extended to calculate the relative QE efficiency by normalizing
633 the average of all PMTs which removes the dependence on the light intensity.

634 Due to differing cable lengths and readout electronics, the timing response between
635 a photon hitting the PMT and the signal being captured by the data acquisition can be
636 different between each PMT. Due to threshold triggers (Described in subsection 3.1.3),
637 the time at which a pulse reaches a threshold is dependent upon the size of the pulse.
638 This is known as the ‘time-walk’ effect and also needs to be accounted for in each PMT.
639 To calibrate the timing response, a pulse of light with width 0.2ns is emitted into the

640 detector through a diffuser. Two-dimensional distributions of time and pulse height
641 (or charge) are made for each PMT and are used to calibrate the timing response. This
642 is performed in-situ during data taking with the light source pulsing at 0.03Hz.

643 The top/bottom water quality asymmetry is measured using the NiCf calibration
644 data and cross-referencing these results to the “standard PMTs”. The water attenuation
645 length is continuously measured by the rate of vertically-downgoing cosmic-ray
646 muons which enter via the top of the tank.

647 Dark noise is the phenomenon where a PMT registers a pulse that is consistent
648 with a single photoelectron emitted from photon detection despite the PMT being in
649 complete darkness. This is predominately caused by two processes. Firstly there is
650 intrinsic dark noise which is where photoelectrons gain enough thermal energy to be
651 emitted from the photocathode, and secondly, the radioactive decay of contaminants
652 inside the structure of the PMT. Typical dark noise rate for PMTs used within SK are
653 $O(3)$ kHz [87]. This is lower than the expected number of photons generated for a ‘high
654 energy event’ (As described in subsection 3.1.4) but instability in this value can cause
655 biases in reconstruction. Dark noise is related to the gain of a PMT and is calibrated
656 using hits inside a time window recorded before an event trigger [98].

657 3.1.3. Data Acquisition and Triggering

658 The analysis presented in this thesis only uses the SK-IV period of the SK experiment
659 so this subsection focuses on the relevant points of the data acquisition and triggering
660 systems to that SK period. The earlier data acquisition and triggering systems are
661 documented in [99, 100].

662 Before the SK-IV period started, the existing front-end electronics were replaced
663 with “QTC-Based Electrons with Ethernet, QBEE” systems [101]. When the QBEE

observes a signal above a 1/4 photoelectron threshold, the charge-to-time (QTC) converter generates a rectangular pulse. The start of the rectangular pulse indicates the time at which the analog photoelectron signal was received and the width of the pulse indicates the total charge integrated throughout the signal. This is then digitized by time-to-digital converters and sent to the “front-end” PCs. The digitized signal from every QBEE is then chronologically ordered and sent to the “merger” PCs. It is the merger PCs that apply the software trigger. Any triggered events are passed to the “organizer” PC. This sorts the data stream of multiple merger PCs into chronologically ordered events which are then saved to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

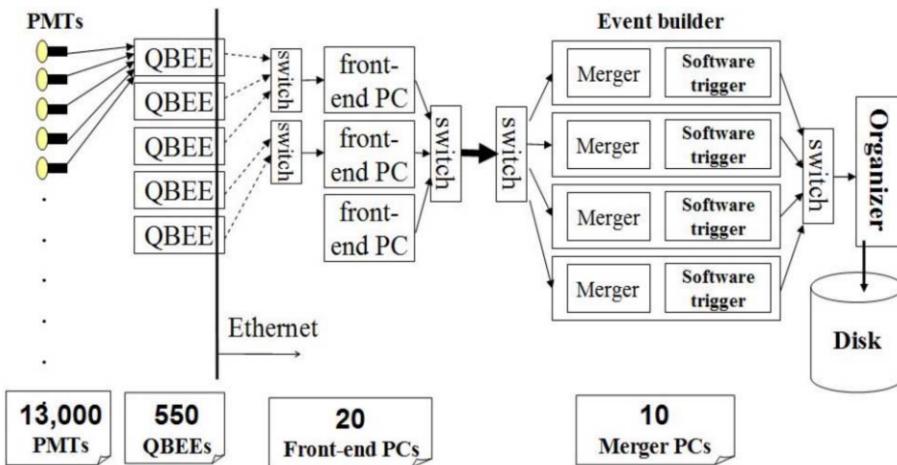


Figure 3.3.: Schematic view of the data flow through the data acquisition and online system. Taken from [102].

The software trigger (described in [103]) operates by determining the number of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the maximum time that a Cherenkov photon would take to traverse the length of the SK tank [100]. For lower energy events that generate fewer photons, this technique is useful for eliminating background processes like dark noise and radioactive decay which would be expected to separate in time. When the value of N_{200} exceeds some threshold, a software trigger is issued. There are several trigger thresholds used within

the SK-IV period which are detailed in Table 3.2. If one of these thresholds is met, the PMT hits within an extended time window are also read out and saved to disk. In the special case of an event that exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband Intelligent Trigger (WIT)” has been deployed and is described in [104].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2.: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

3.1.4. Cherenkov Radiation

Cherenkov light is emitted from any highly energetic charged particle traveling with relativistic velocity, β , greater than the local speed of light in a medium [105]. Cherenkov light is formed at the surface of a cone with characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

where n is the refractive index of the medium. Consequently, the Cherenkov momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}} \quad (3.3)$$

For water, where $n = 1.33$, the Cherenkov threshold momentum and energy for various particles are given in Table 3.3. In contrast, γ -rays are detected indirectly via the combination of photons generated by Compton scattering and pair production. The threshold for detection in the SK detector is typically higher than the threshold for photon production. This is due to the fact that the attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov photons reach the ID PMTs. Then the collection and quantum efficiencies described in subsection 3.1.1 result in the number of detected photons being lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3.: The threshold momentum and energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

The Frank-Tamm equation [106] describes the relationship between the number of Cherenkov photons generated per unit length, dN/dx , the wavelength of the photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

where α is the fine structure constant. For a 100MeV momentum electron, approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq 700\text{nm}$ region which the ID PMTs are most sensitive to [87].

3.2. The Tokai to Kamioka Experiment

The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation experiment located in Japan. Proposed in the early 2000s [107, 108] to replace K2K [109], T2K was designed to observe electron neutrino appearance whilst precisely measuring the oscillation parameters associated with muon neutrino disappearance [110]. The experiment consists of a neutrino beam generated at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near detectors situated 280m from the beam target, and the Super Kamiokande far detector positioned at a 295km baseline. The cross-section view of the T2K experiment is drawn in Figure 3.4.

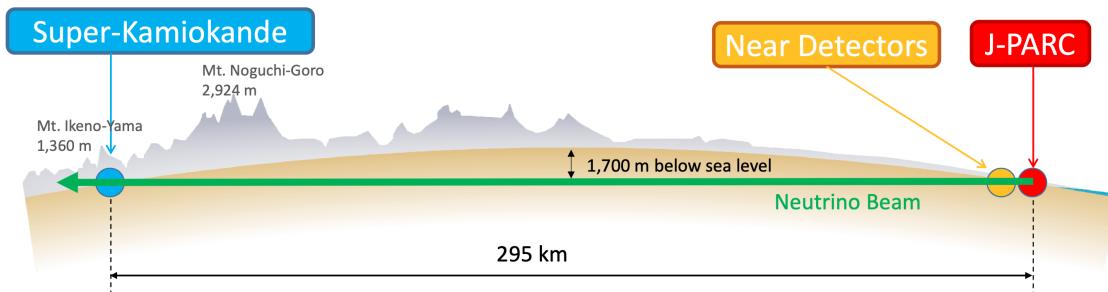


Figure 3.4.: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$, Δm_{23}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accuracy of parameter estimates are still being made by including new data samples and developing the models which describe the neutrino interactions and detector responses [111]. Electron neutrino appearance was first observed at T2K in 2014 [112] with 7.3σ significance.

The near detectors provide constraints on the beam flux and cross-section model parameters used within the oscillation analysis by observing the unoscillated neutrino beam. There are a host of detectors situated in the near detector hall (As illustrated in Figure 3.5): ND280 (subsection 3.2.2), INGRID (subsection 3.2.3), NINJA [113], WAGASCI [114], and Baby-MIND [115]. The latter three are not currently used within the oscillation analysis presented within this thesis.

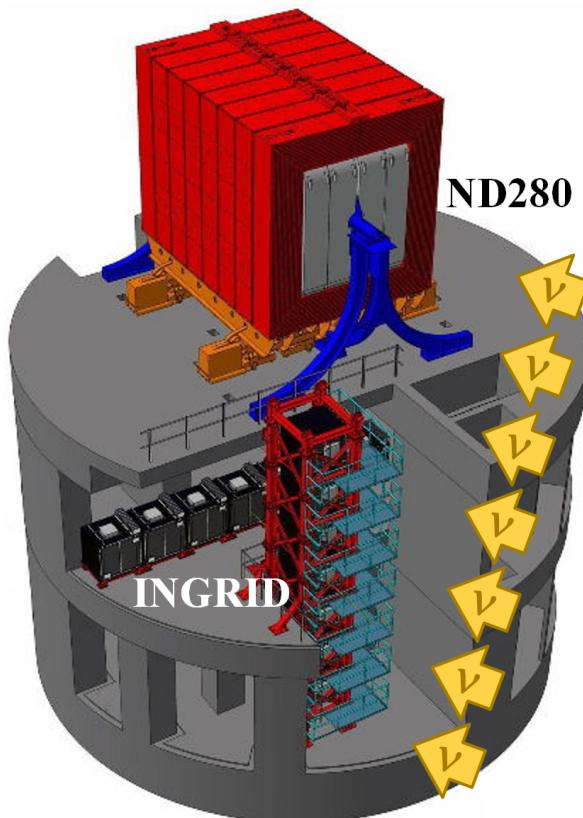


Figure 3.5.: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

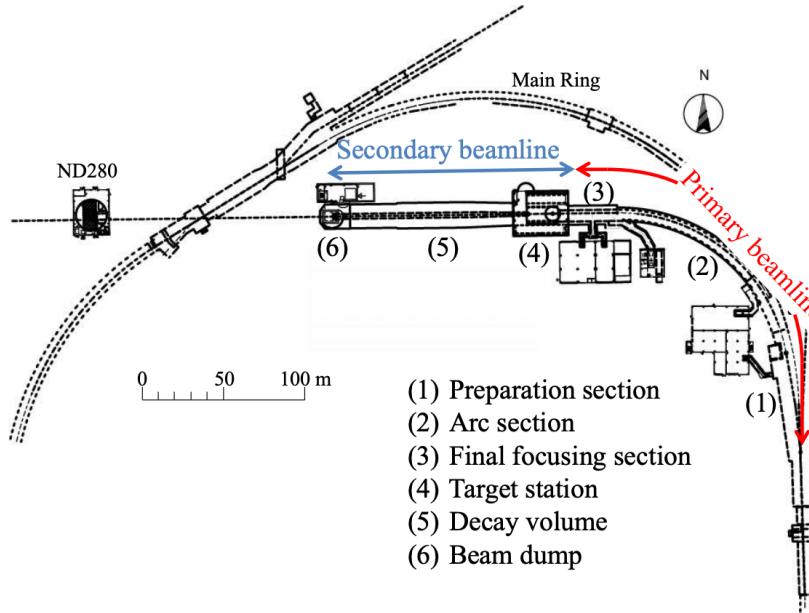
Whilst this thesis presents the ND280 in terms of its purpose for the oscillation analysis, the detector can also make many cross-section measurements at neutrino energies of $O(1)$ GeV for the different targets within the detector [116, 117]. These measurements are of equal importance as they can lead the way in determining the model parameters used in the interaction models for the future high-precision era of neutrino physics.

DB: Discuss BANFF, PTheta, MaCh3 and covariance

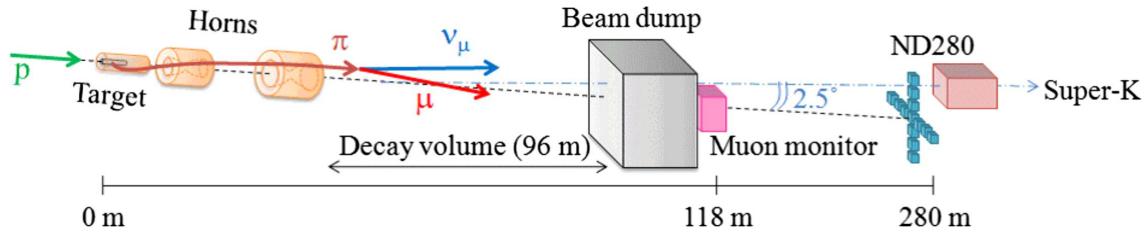
3.2.1. The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [59, 118] and summarised below. The accelerating facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [59],



(a) Primary and secondary beamline



(b) Secondary beamline

Figure 3.6.: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [118]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [119].

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$K^+ \rightarrow \mu^+ + \nu_\mu$$

$$K^- \rightarrow \mu^- + \bar{\nu}_\mu$$

$$\rightarrow \pi^0 + e^+ + \nu_e$$

$$\rightarrow \pi^0 + e^- + \bar{\nu}_e$$

$$\rightarrow \pi^0 + \mu^+ + \nu_\mu$$

$$\rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu$$

$$K_L^0 \rightarrow \pi^- + e^+ + \nu_e$$

$$K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e$$

$$\rightarrow \pi^- + \mu^+ + \nu_\mu$$

$$\rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$$

$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$

$$\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino flux.

The low energy flux is dominated by the decay of pions whereas kaon decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic irreducible ν_e background, are predominantly due to muon decay for $E_\nu < 2\text{GeV}$. As the antineutrino production cross-section is smaller than the neutrino cross-section, the wrong-sign component is more dominant in the RHC beam as compared to that in the FHC beam.

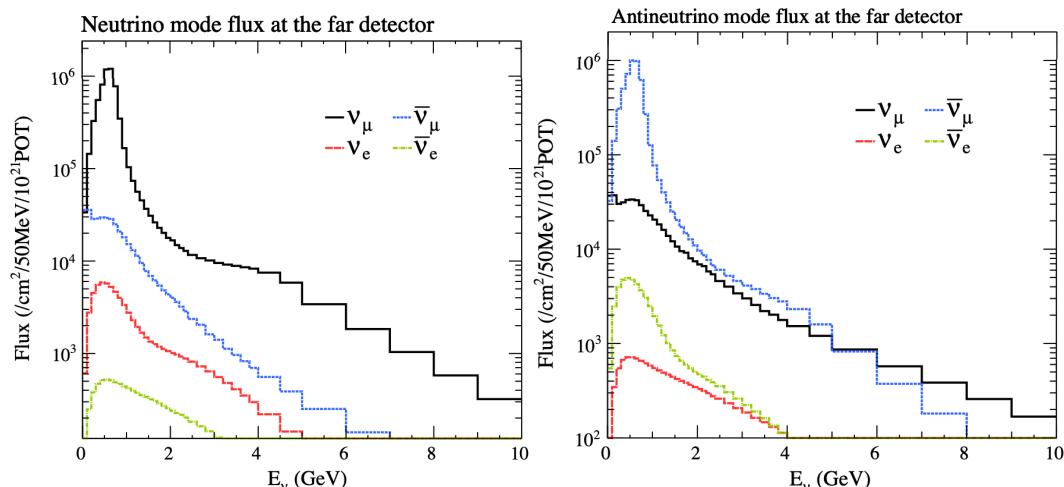


Figure 3.7.: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [120].

The beam dump, situated at the end of the decay volume, stops all charged particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector monitors the

⁷⁶⁹ penetrating muons to determine the beam direction and intensity which is used to
⁷⁷⁰ constrain some of the beam flux systematics within the analysis [119, 121].

⁷⁷¹ The T2K experiment uses an off-axis beam to narrow the neutrino energy distribution.
⁷⁷² This was the first implementation of this technique in a long-baseline neutrino
⁷⁷³ oscillation experiment after its original proposal [122]. Pion decay, $\pi \rightarrow \mu + \nu_\mu$, is a
⁷⁷⁴ two-body decay. Consequently, the neutrino energy, E_ν , can be determined based on
⁷⁷⁵ the pion energy, E_π , and the angle at which the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

⁷⁷⁶ where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
⁷⁷⁷ energy pion, the neutrino energy distribution is dependent upon the angle at which the
⁷⁷⁸ neutrinos are observed from the initial pion beam direction. For the 295km baseline at
⁷⁷⁹ T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance probability, $P(\nu_\mu \rightarrow \nu_e)$,
⁷⁸⁰ whilst minimising the muon disappearance probability, $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8
⁷⁸¹ illustrates the neutrino energy distribution for a range of off-axis angles, as well as the
⁷⁸² oscillation probabilities most relevant to T2K.

⁷⁸³ 3.2.2. The Near Detector at 280m

⁷⁸⁴ Whilst all the near detectors are situated in the same “pit” located at 280m from the
⁷⁸⁵ beamline, the “ND280” detector is the off-axis detector which is situated at the same
⁷⁸⁶ off-axis angle as the Super-Kamiokande far detector. It has two primary functions;
⁷⁸⁷ firstly it measures the neutrino flux and secondly it counts the event rates of different
⁷⁸⁸ types of neutrino interactions. Both of these constrain the flux and cross-section

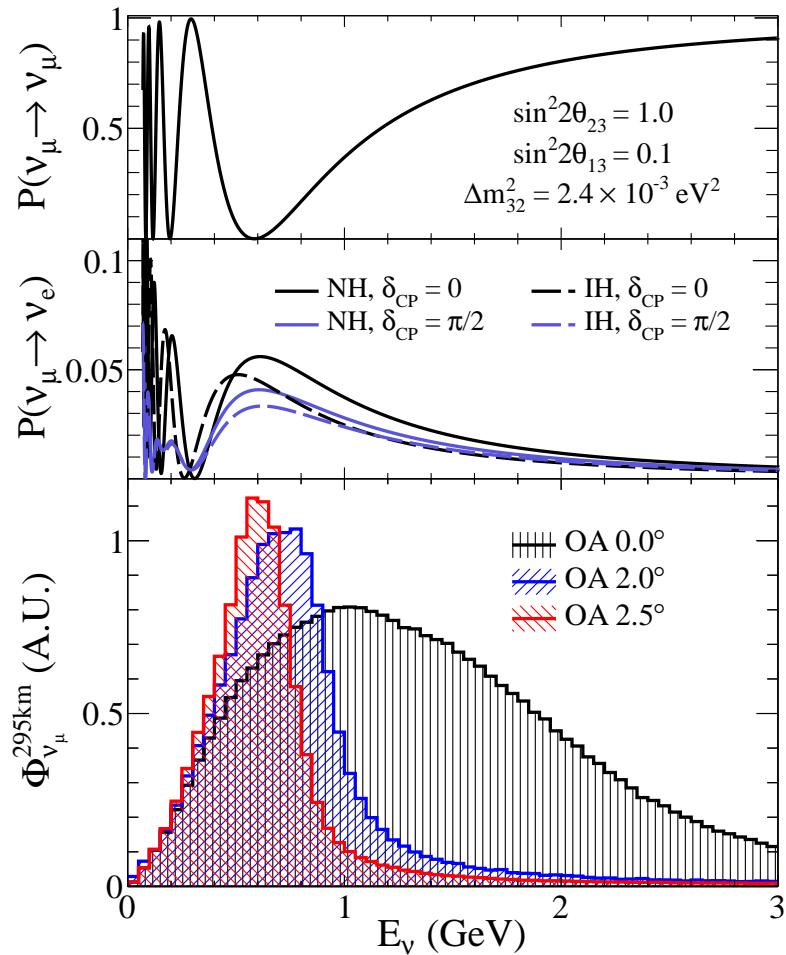


Figure 3.8.: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

789 systematics invoked within the model for a more accurate prediction of the expected
 790 event rate at the far detector.

791 As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors.
 792 The most important part of the detector for this analysis is the tracker region. This is
 793 comprised of two time projection chambers (TPCs) sandwiched between three fine
 794 grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water tar-
 795 gets for neutrino interactions and provide track reconstruction near the interaction
 796 vertex. The emitted charged particles can then propagate into the TPCs which pro-
 797 vide particle identification and momentum reconstruction. The FGDs and TPCs are

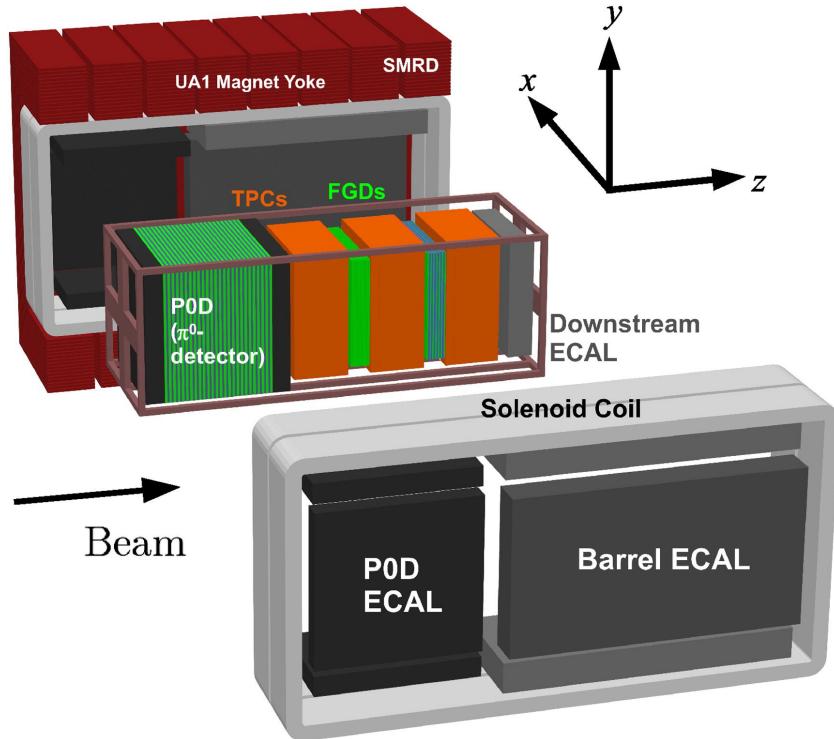


Figure 3.9.: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [118].

798 further described in subsubsection 3.2.2.1 and subsubsection 3.2.2.2 respectively. The
 799 electromagnetic calorimeter (ECAL) encapsulates the tracker region alongside the π^0
 800 detector (P0D). The ECAL measures the deposited energy from photons emitted from
 801 interactions within the FGD. The P0D constrains the cross-section of neutral current
 802 interactions which generate neutral pions, which is one of the largest backgrounds in
 803 the electron neutrino appearance oscillation channel. The P0D and ECAL detectors
 804 are detailed in subsubsection 3.2.2.3 and subsubsection 3.2.2.4 respectively. The entire
 805 detector is located within a large yoke magnet which produces a 0.2T magnetic field.
 806 This design of the magnet also includes a scintillating detector called the side muon
 807 range detector (SMRD) which is used to track high-angle muons as well as acting as a
 808 cosmic veto. The SMRD is described in subsubsection 3.2.2.5.

809 **3.2.2.1. Fine Grained Detectors**

810 The T2K tracker region is comprised of two fine grained detectors (FGD) and three
811 Time Projection Chambers (TPC). A detailed description of the FGD design, construc-
812 tion, and assembly is found in [123] and summarised below. The FGDs are the primary
813 target for neutrino interactions with a mass of 1.1 tonnes per FGD. Alongside this,
814 the FGDs are designed to be able to track short-range particles which do not exit the
815 FGD. Typically, short-range particles are low momentum and are observed as tracks
816 that deposit a large amount of energy per unit length. This means the FGD needs
817 good granularity to resolve these particles. The FGDs have the best timing resolution
818 ($\sim 3\text{ns}$) of any of the sub-detectors of the ND280 detector. As such, the FGDs are
819 used for time of flight measurements to distinguish forward going positively charged
820 particles from backward going negatively charged particles. Finally, any tracks which
821 pass through multiple sub-detectors are required to be track matched to the FGD.

822 Both FGDs are made from square scintillator planes of side length 186cm and
823 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an X or Y
824 orientation. A wavelength shifting fiber is threaded through the center of each bar and
825 is read out by a multi-pixel photon counter (MPPC). FGD1 is the most upstream of
826 the two FGDs and contains 15 planes of carbon plastic scintillator which is a common
827 target in external neutrino scattering data. As the far detector is a pure water target, 7
828 of the 15 scintillator planes in FGD2 have been replaced with a hybrid water-scintillator
829 target. Due to the complexity of the nucleus, nuclear effects can not be extrapolated
830 between different nuclei. Therefore having the ability to take data on one target which
831 is the same as external data and another target which is the same as the far detector
832 target is beneficial for reliable model parameter estimates.

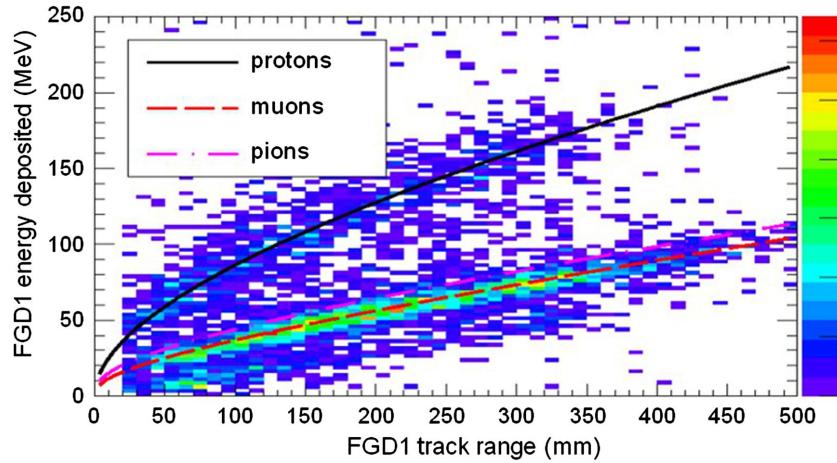


Figure 3.10.: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [123].

The integrated deposited energy is used for particle identification. The FGD can distinguish protons from other charged particles by comparing the integrated deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

3.2.2.2. Time Projection Chambers

The majority of particle identification and momentum measurements within ND280 are provided by three Time Projection Chambers (TPCs) [124]. The TPCs are located on either side of the FGDs. They are located inside of the magnetic field meaning the momentum of a charged particle can be determined from the bending of the track.

Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11, which are made of non-magnetic material. The outer box is filled with CO₂ which acts as an electrical insulator between the inner box and the ground. The inner box forms the field cage which produces a uniform electric drift field of $\sim 275\text{V}/\text{cm}$ and is filled with an argon gas mixture. Charged particles moving through this gas mixture ionize the gas and the ionised charge is drifted towards micromegas detectors which measure the ionization charge. The time and position information in the readout allows a three-dimensional image of the neutrino interaction.

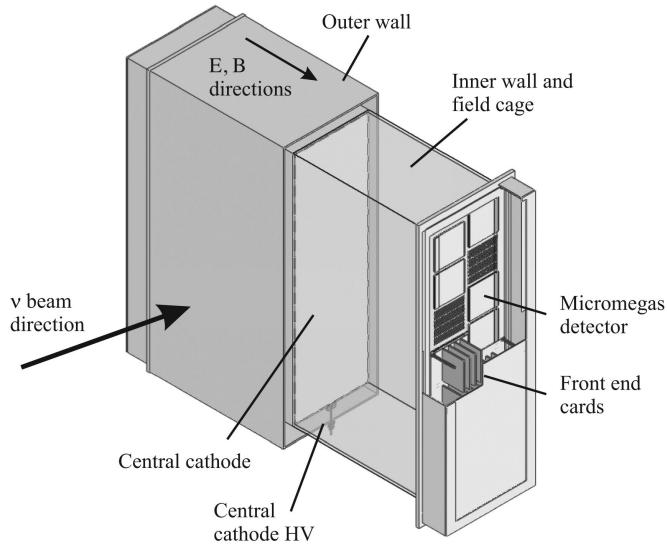


Figure 3.11.: Schematic design of a Time Projection Chamber detector. Taken from [124].

The particle identification of tracks that pass through the TPCs is performed using

dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo distributions of the energy lost by a charged particle passing through the TPC as a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$ meaning that electrons and muons can be distinguished. This allows reliable measurements of the intrinsic ν_e component of the beam.

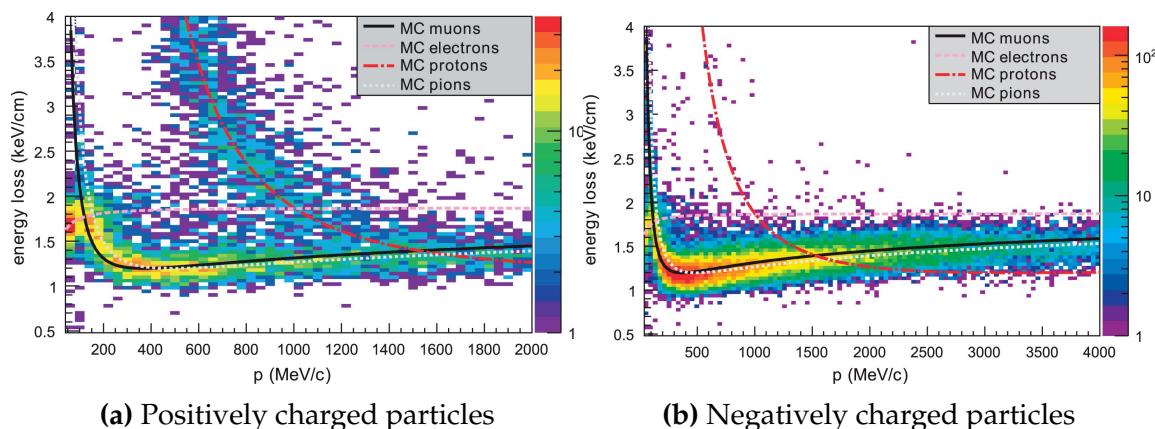


Figure 3.12.: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [124].

⁸⁵⁵ 3.2.2.3. π^0 Detector

⁸⁵⁶ If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the recon-
⁸⁵⁷ struction will determine that event to be a charge current ν_e -like event. This is one of
⁸⁵⁸ the main backgrounds hindering the electron neutrino appearance searches. The π^0
⁸⁵⁹ detector (P0D) measures the cross-section of the neutral current induced neutral pion
⁸⁶⁰ production on a water target to constrain this background.

⁸⁶¹ The P0D is a cube of approximately 2.5m length consisting of layers of scintillating
⁸⁶² bars, brass and lead sheets, and water bags as illustrated in Figure 3.13. Two electro-
⁸⁶³ magnetic calorimeters are positioned at the most upstream and most downstream
⁸⁶⁴ position in the sub-detector and the water target is situated in between them. The
⁸⁶⁵ scintillator layers are built from two triangular bars orientated in opposite directions
⁸⁶⁶ to form a rectangular layer. Each triangular scintillator bar is threaded with optical
⁸⁶⁷ fiber which is read out by MPPCs. The high-Z brass and lead regions produce electron
⁸⁶⁸ showers from the photons emitted in π^0 decay.

⁸⁶⁹ The sub-detector can generate measurements of NC1 π^0 cross-sections on a water
⁸⁷⁰ target by measuring the event rate both with and without the water target, with the
⁸⁷¹ cross-section on a water target being determined as the difference. The total active
⁸⁷² mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

⁸⁷³ 3.2.2.4. Electromagnetic Calorimeter

⁸⁷⁴ The electromagnetic calorimeter [126] (ECal) encapsulates the P0D and tracking sub-
⁸⁷⁵ detectors. Its primary purpose is to aid π^0 reconstruction from any interaction in
⁸⁷⁶ the tracker. To do this, it measures the energy and direction of photon showers from
⁸⁷⁷ $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks depending on the shape
⁸⁷⁸ of the photon shower deposited.

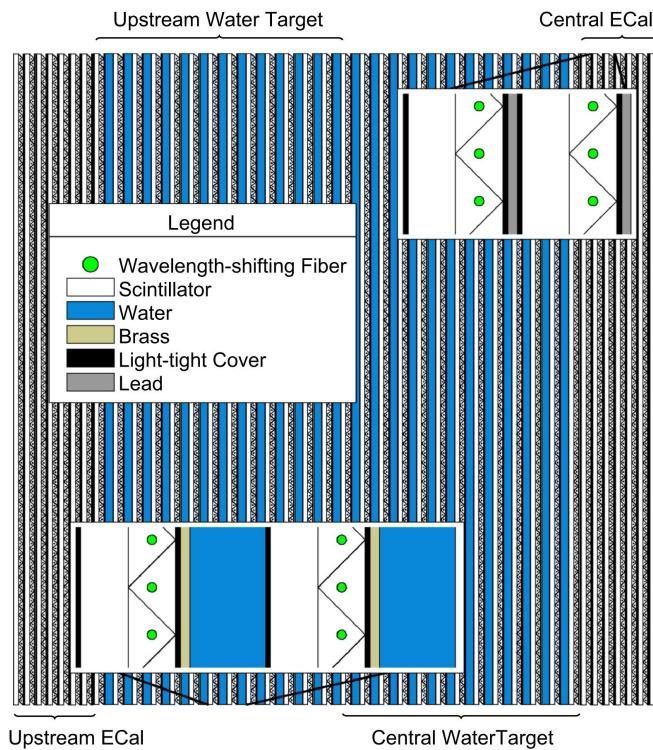


Figure 3.13.: A schematic of the P0D side-view. Taken from [125].

879 The ECal is comprised of three sections; the P0D ECal which surrounds the P0D,
 880 the barrel ECal which encompasses the tracking region, and the downstream ECal
 881 which is situated downstream of the tracker region. The barrel and downstream
 882 ECals are tracking calorimeters that focus on electromagnetic showers from high-angle
 883 particles emitted from the tracking sub-detectors. Particularly in the TPC, high-angle
 884 tracks (those which travel perpendicularly to the beam-axis) can travel along a single
 885 scintillator bar resulting in very few hits. The width of the barrel and downstream
 886 ECal corresponds to ~ 11 electron radiation lengths to ensure a significant amount of
 887 the π^0 energy is contained. As the P0D has its own calorimetry which reconstructs
 888 showers, the P0D ECal determines the energy which escapes the P0D.

889 Each ECal is constructed of multiple layers of scintillating bars sandwiched between
 890 lead sheets. The scintillating bars are threaded with optical fiber and read out by
 891 MPPCs. Each sequential layer of the scintillator is orientated perpendicular to the
 892 previous which allows a three dimensional event reconstruction. The target mass

893 of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80 and 6.62 tonnes
894 respectively.

895 **3.2.2.5. Side Muon Range Detector**

896 As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within the
897 UA1 magnet. Originally designed for the NOMAD [127] experiment and reconditioned
898 for use in the T2K experiment [128], the UA1 magnet provides a uniform horizontal
899 magnetic field of 0.2T with an uncertainty of 2×10^{-4} T.

900 Built into the UA1 magnet, the side muon range detector (SMRD) [129] monitors
901 high-energy muons which leave the tracking region and permeate through the ECal.
902 It additionally acts as a cosmic muon veto and trigger.

903 **3.2.3. The Interactive Neutrino GRID**

904 The Interactive Neutrino GRID (INGRID) detector is situated within the same “pit” as
905 the other near detectors. It is aligned with the beam in the “on-axis” position and mea-
906 sures the beam direction, spread, and intensity. The detector was originally designed
907 with 16 identical modules [118] (two modules have since been decommissioned) and a
908 “proton” module. The design of the detector is cross-shaped with length and height
909 10m × 10m as illustrated in Figure 3.14.

910 Each module is composed of iron sheets interlaced with eleven tracking scintillator
911 planes for a total target mass of 7.1 tonnes per module. The scintillator design is an X-Y
912 pattern of 24 bars in both orientations, where each bar contains wave-length shifting
913 fibers which are connected to multi-pixel photon counters (MPPCs). Each module is
914 encapsulated inside veto planes to aid the rejection of charged particles entering the
915 module.

916 The proton module is different from the other modules in that it consists of entirely
 917 scintillator planes with no iron target. The scintillator bars are also smaller than those
 918 used in the other modules to increase the granularity of the detector and improve
 919 tracking capabilities. The module sits in the center of the beamline and is designed to
 920 give precise measurements of quasi-elastic charged current interactions to evaluate
 921 the performance of the Monte Carlo simulation of the beamline.

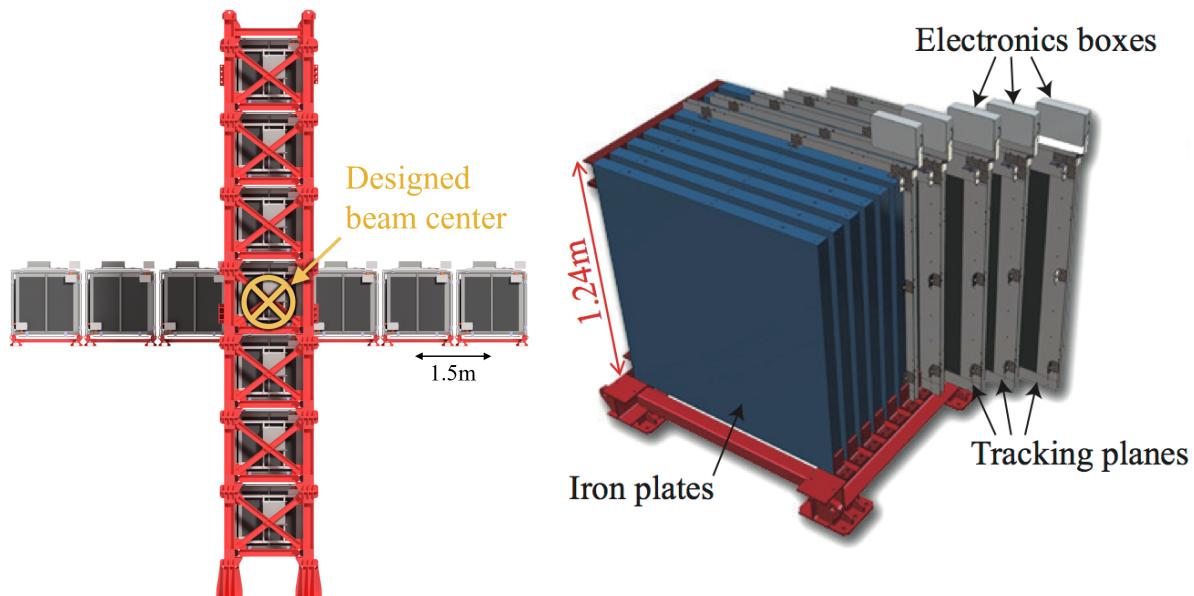


Figure 3.14.: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [118].

922 The INGRID detector can measure the beam direction to an uncertainty of 0.4mrad
 923 and the beam center within a resolution of 10cm [118]. The beam direction in both the
 924 vertical and horizontal directions is discussed in [130] and it is found to be in good
 925 agreement with the MUMON monitor described in subsection 3.2.1.

₉₂₆ **Chapter 4**

₉₂₇ **Bayesian Statistics and Markov Chain**

₉₂₈ **Monte Carlo Techniques**

₉₂₉ This thesis presents a Bayesian oscillation analysis. To extract the oscillation parameters,
₉₃₀ a Markov Chain Monte Carlo (MCMC) method is used. This chapter explains
₉₃₁ the theory of how parameter estimates can be determined using this technique and
₉₃₂ condenses the material found in the literature [131–134].

₉₃₃ The oscillation parameter determination presented within this thesis is built upon
₉₃₄ a simultaneous fit to neutrino beam data in the near detector, beam data at SK and
₉₃₅ atmospheric data at SK. In total, there are four oscillation parameters of interest
₉₃₆ ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP}), two oscillation parameters to which this study
₉₃₇ will not be sensitive ($\sin^2(\theta_{12})$, Δm_{12}^2) and many nuisance parameters that control the
₉₃₈ systematic uncertainty models invoked within this study.

₉₃₉ The MCMC technique generates a multi-dimensional probability distribution across
₉₄₀ all of the model parameters used in the fit. To determine the parameter estimate of a
₉₄₁ single parameter, this multi-dimensional object is integrated over all other parameters.
₉₄₂ This process is called Marginalisation and is further described in subsection 4.3.1.
₉₄₃ Monte Carlo techniques approximate the probability distribution of each parameter
₉₄₄ within the limit of generating infinite samples. As ever, generating a large number of
₉₄₅ samples is time and resource-dependent. Therefore, an MCMC technique is utilised
₉₄₆ within this analysis to reduce the required number of steps to sufficiently sample the
₉₄₇ parameter space. This technique is described in further detail in subsection 4.2.1.

948 4.1. Bayesian Statistics

949 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
950 footing such that a probability model of both data and parameters is required. This is
951 the joint probability distribution $P(D, \vec{\theta})$ and can be described by the prior distribution
952 for model parameters $P(\vec{\theta})$ and the likelihood of the data given the model parameters
953 $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

954 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the parameters
955 within the model. For example, if the risk of developing health problems is known
956 to increase with age, the prior distribution would describe the increase. For the
957 purpose of this analysis, the prior distribution is typically the best-fit values taken
958 from external data measurements with a Gaussian uncertainty. The prior distribution
959 can also contain correlations between model parameters. In an analysis using Monte
960 Carlo techniques, the likelihood of measuring some data assuming some set of model
961 parameters is calculated by comparing the Monte Carlo prediction generated at that
962 particular set of model parameters to the data.

963 It is parameter estimation that is important for this analysis and as such, we apply
964 Bayes' theorem [135] to calculate the probability for each parameter to have a certain
965 value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior distribution
966 (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

967 The denominator in Equation 4.2 is the integral of the joint probability distribution

968 over all values of all parameters used within the fit. For brevity, we say that the
 969 posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

970 In subsection 4.3.1, we see that for the cases used within this analysis, it is reason-
 971 able to know the posterior to some normalisation constant.

972 4.2. Monte Carlo Simulation

973 Monte Carlo techniques are used to numerically solve a complex problem that does
 974 not necessarily have an analytical solution. These techniques rely on building a large
 975 ensemble of samples from an unknown distribution and then using the ensemble to
 976 approximate the properties of the distribution.

977 An example that uses Monte Carlo techniques is to calculate the area underneath
 978 a curve. For example, take the problem of calculating the area under a straight line
 979 with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can calculate the area
 980 under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte Carlo techniques,
 981 one can calculate the area under this line by throwing many random values for the x
 982 and y components of each sample and then calculating whether that point falls below

the line. The area can then be calculated by the ratio of points below the line to the total number of samples thrown multiplied by the total area in which samples were scattered. The study is shown in Figure 4.1 highlights this technique and finds the area under the curve to be 29.9 compared to an analytical solution of 30.0. The deviation of the numerical to analytical solution can be attributed to the number of samples used in the study. The accuracy of the approximation in which the properties of the Monte Carlo samples replicate those of the desired distribution is dependent on the number of samples used. Replicating this study with a differing number of Monte Carlo samples used in each study (As shown in Figure 4.2) highlights how the Monte Carlo techniques are only accurate within the limit of a high number of samples.

Whilst the above example has an analytical solution, these techniques are just as applicable to complex solutions. Clearly, any numerical solution is only as useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is dependent upon the number of samples generated to approximate the properties of the distribution. Furthermore, if the positions at which the samples are evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique significantly drops. Given the example in Figure 4.1, if the region in which the samples are scattered significantly extends passed the region of interest, many calculations will be calculated but do not add to the ability of the Monte Carlo technique to achieve the correct result. For instance, any sample evaluated at a $y \geq 5$ could be removed without affecting the final result. This does bring in an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling, one needs to know the distribution beforehand.

4.2.1. Markov Chain Monte Carlo

This analysis utilises a multi-dimensional probability distribution, with some dimensions being significantly more constrained than others. This could be from prior

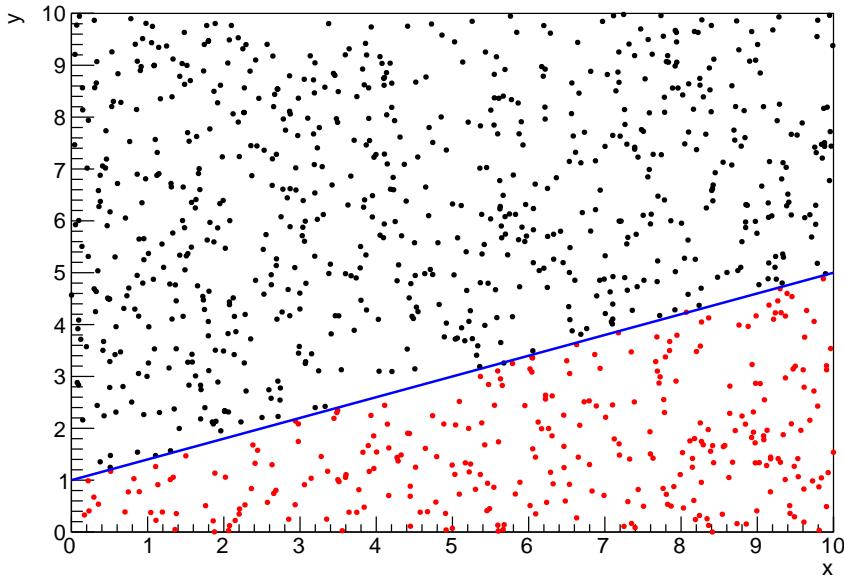


Figure 4.1.: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

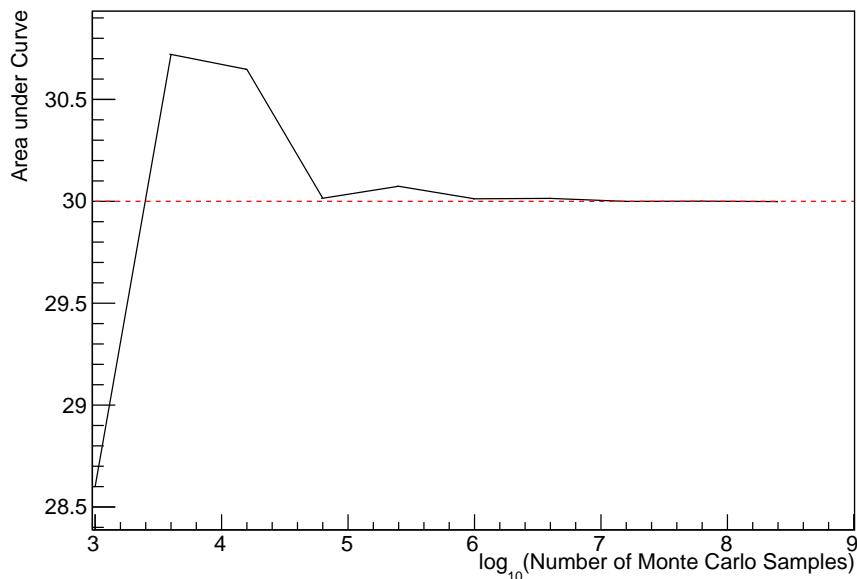


Figure 4.2.: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

knowledge of parameter distributions from external data or un-physical regions in which parameters can not exist. Consequently, the Monte Carlo techniques used need to be as efficient as possible. For this analysis, the Markov Chain Monte Carlo (MCMC) technique is chosen. An MCMC technique is a Monte Carlo technique that uses a Markov chain to select which points at which to sample the parameter distribution. This technique performs a semi-random stochastic walk through the allowable parameter space. This builds a posterior distribution which has the property that the density of sampled points is proportional to the probability density of that parameter. This does mean that the samples produced by this technique are not statistically independent but they will cover the space of the distribution.

A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the position of \vec{x}_i . The space in which the Markov chain selects samples is dependent upon the total number of parameters utilised within the fit, where a discrete point in this space is described by the N-dimensional space \vec{x} . In a perfectly operating Markov chain, the position of the next step depends solely on the previous step and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving the multi-dimensionality of the fit used within this analysis, each step becomes correlated with several of the steps preceding itself. This behaviour is further explained in subsection 4.2.3. Providing the MCMC chain is well optimised, it will begin to converge towards a unique stationary distribution. The period between the chain's initial starting point and the convergence to the unique stationary distribution is colloquially known as the burn-in period. This is discussed further in subsection 4.2.3. Once the chain reaches the stationary distribution, all points sampled after that point will look like samples from that distribution.

1032 Further details of the theories underpinning MCMC techniques are discussed
 1033 in [132] but can be summarised by the requirement that the chain satisfies the three
 1034 ‘regularity conditions’:

- 1035 • Irreducibility: From every position in the parameter space \vec{x} , there must exist a
 non-zero probability for every other position in the parameter space to be reached.
- 1037 • Recurrence: Once the chain arrives at the stationary distribution, every step fol-
 lowing from that position must be samples from the same stationary distribution.
- 1039 • Aperiodicity: The chain must not repeat the same sequence of steps at any point
 throughout the sampling period.

1041 The output of the chain after burn-in (ie. the sampled points after the chain
 1042 has reached the stationary distribution) can be used to approximate the posterior
 1043 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
 1044 stationary distribution found by the chain be the posterior distribution, one can use
 1045 the Metropolis-Hastings algorithm. This guides the stochastic process depending on
 1046 the likelihood of the current proposed step compared to that of the previous step.
 1047 Implementation and other details of this technique are discussed in subsection 4.2.2.

1048 **4.2.2. Metropolis-Hastings Algorithm**

1049 As a requirement for MCMCs, the Markov chain implemented in this technique must
 1050 have a unique stationary distribution that is equivalent to the posterior distribution.
 1051 To ensure this requirement and that the regularity conditions are met, this analysis
 1052 utilises the Metropolis-Hastings (MH) algorithm [136,137]. For the i^{th} step in the chain,
 1053 the MH algorithm determines the position in the parameter space to which the chain
 1054 moves to based on the current step, \vec{x}_i , and the proposed step, \vec{y}_{i+1} . The proposed step
 1055 is randomly selected from some proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely

on the current step (ie. not the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the current step or the proposed step determined by whether the proposed step is accepted or rejected. To decide if the proposed step is selected, the acceptance probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)} \right). \quad (4.4)$$

Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To simplify this calculation, the proposal function is required to be symmetric such that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution is used to throw parameter proposals from. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)} \right). \quad (4.5)$$

After calculating this quantity, a random number, β , is generated uniformly between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise, the chain sets the next step equal to the current step and this procedure is repeated. This can be interpreted as if the posterior probability of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true, ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped in any local minima in the potentially non-Gaussian posterior distribution. The outcome of this technique is that the density of steps taken in a discrete region is directly proportional to the probability density in that region.

1074 4.2.3. MCMC Optimisation

1075 As discussed in subsection 4.2.2, the proposal function invoked within the MH algo-
1076 rithm can take any form and the chain will still converge to the stationary distribution.
1077 At each set of proposed parameter values, a prediction of the same spectra has to be
1078 generated which requires significant computational resources. Therefore, the number
1079 of steps taken before the unique stationary distribution is found should be minimised
1080 as only steps after convergence add information to the oscillation analysis. Further-
1081 more, the chain should entirely cover the allowable parameter space to ensure that all
1082 values have been considered. Tuning the distance that the proposal function jumps
1083 between steps on a parameter-by-parameter basis can both minimise the length of the
1084 burn-in period and ensure that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently
1085 small.

1086 The effect of changing the width of the proposal function is highlighted in Figure 4.3.
1087 Three scenarios, each with the same underlying stationary distribution (A Gaussian of
1088 width 1.0 and mean 0.), are presented. The only difference between the three scenarios
1089 is the width of the proposal function, colloquially known as the ‘step size σ ’. Each
1090 scenario starts at an initial parameter value of 10.0 which would be considered an
1091 extreme variation. For the case where $\sigma = 0.1$, it is clear to see that the chain takes
1092 a long time to reach the expected region of the parameter. This indicates that this
1093 chain would have a large burn-in period and does not converge to the stationary
1094 distribution until step ~ 500 . Furthermore, whilst the chain does move towards the
1095 expected region, each step is significantly correlated with the previous. Considering
1096 the case where $\sigma = 5.0$, the chain approaches the expected parameter region almost
1097 instantly meaning that the burn-in period is not significant. However, there are clearly
1098 large regions of steps where the chain does not move. This is likely due to the chain
1099 proposing steps in the tails of the distribution which have a low probability of being

1100 accepted. Consequently, this chain would take a significant number of steps to fully
 1101 span the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can see
 1102 a relatively small burn-in period of approximately 100 steps. Once the chain reaches
 1103 the stationary distribution, it moves throughout the expected region of parameter
 1104 values many times, sufficiently sampling the full parameter region. This example is a
 1105 single parameter varying across a continuous distribution and does not fully reflect
 1106 the difficulties in the many-hundred multi-variate parameter distribution used within
 1107 this analysis. However, it does give a conceptual idea of the importance of selecting
 1108 the proposal function and associated step size.

1109 As discussed, step size tuning directly correlates to the average step acceptance
 1110 rate. If the step size is too small, many steps will be accepted but the chain moves
 1111 slowly. If the opposite is true, many steps will be rejected as the chain proposes steps
 1112 in the tails of the distribution. Discussion in [138] suggests that the ‘ideal’ acceptance
 1113 rate of a high dimension MCMC chain should be approximately $\sim 25\%$. An “ideal”
 1114 step size [138] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

1115 where N_p is the number of parameters included in the MCMC fit. However, the
 1116 complex correlations between systematics mean that some parameters have to be hand
 1117 tuned and many efforts have been taken to select a set of parameter-by-parameter step
 1118 sizes to approximately reach the ideal acceptance rate.

1119 Figure 4.3 highlights the likelihood as calculated by the fit in [DB: Link to AsimovA](#)
 1120 **Sensitivity Section** as a function of the number of steps in each chain. In practice,
 1121 many independent MCMC chains are run simultaneously to parallelise the task of

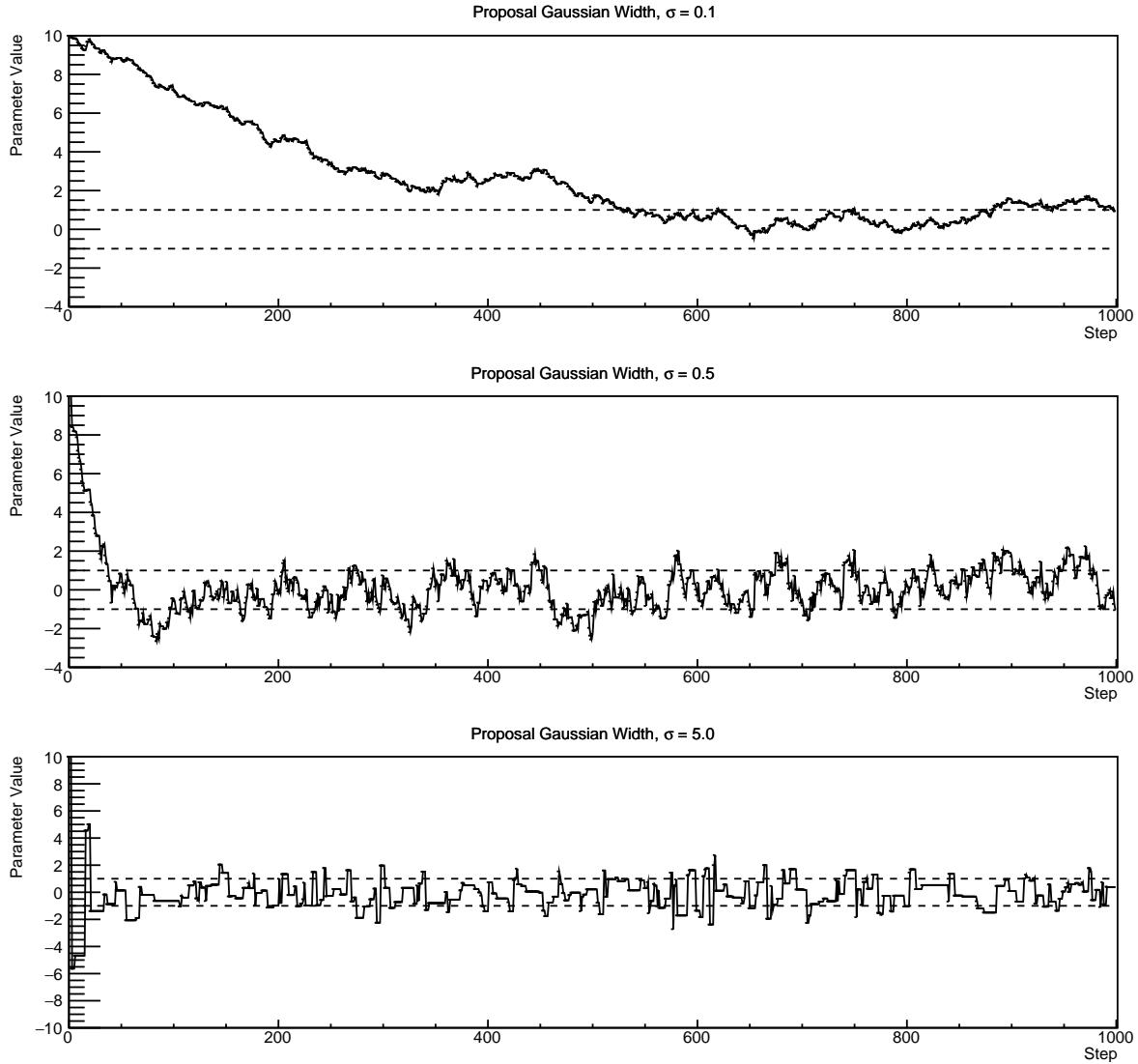


Figure 4.3.: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1122 performing the fit. This figure overlays the distribution found in each chain. As seen,
 1123 the likelihood decreases from its initial value and converges towards a stationary
 1124 distribution after $\sim 1 \times 10^5$ steps.

1125 Multiple configurations of this analysis have been performed throughout this thesis
 1126 where different samples or systematics have been used. For all of these configurations,
 1127 it was found that a burnin period of 1×10^5 was sufficient in all cases.

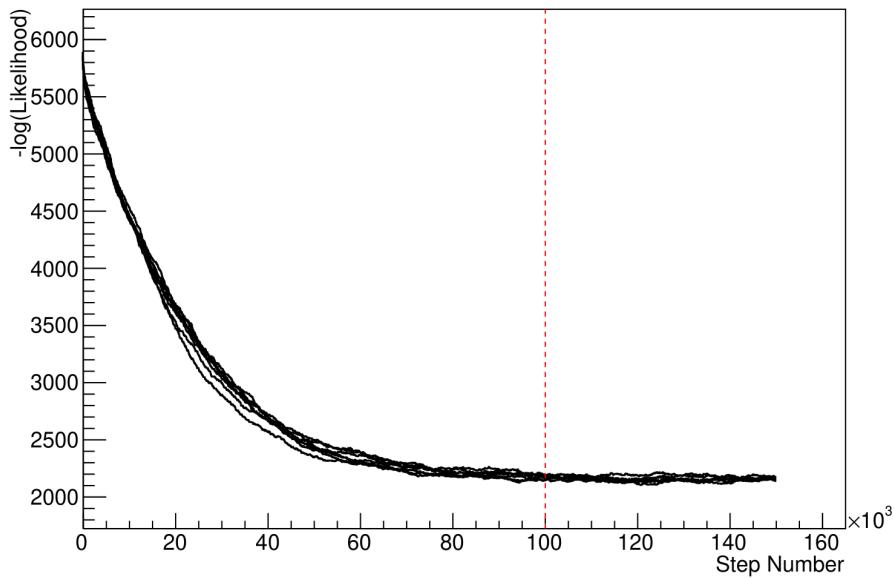


Figure 4.4.: The log-likelihood from the fit detailed in DB: Link to AsimovA Sensitivity Section as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

¹¹²⁸ 4.3. Understanding the MCMC Results

¹¹²⁹ The previous sections have described how to generate the posterior probability distri-
¹¹³⁰ bution using Bayesian MCMC techniques. However, this analysis focuses on oscillation
¹¹³¹ parameter determination. The posterior distribution output from the chain is a high
¹¹³² dimension object, with as many dimensions as there are parameters included in the os-
¹¹³³ cillation analysis. However, this multi-dimensional object is difficult to conceptualize
¹¹³⁴ so parameter estimations are often presented in one or two-dimensional projections
¹¹³⁵ of this probability distribution. To do this, we invoke the marginalisation technique
¹¹³⁶ highlighted in subsection 4.3.1.

¹¹³⁷ 4.3.1. Marginalisation

¹¹³⁸ The output of the MCMC chain is a highly dimensional probability distribution
¹¹³⁹ which is very difficult to interpret. From the standpoint of an oscillation analysis
¹¹⁴⁰ experiment, the one or two-dimensional ‘projections’ of the oscillation parameters of
¹¹⁴¹ interest are most relevant. Despite this, the best fit values and uncertainties on the
¹¹⁴² oscillation parameters of interest should correctly encapsulate the correlations to the
¹¹⁴³ other systematic uncertainties (colloquially called ‘nuisance’ parameters). For this joint
¹¹⁴⁴ beam and atmospheric analysis, the oscillation parameters of interest are $\sin^2(\theta_{23})$,
¹¹⁴⁵ $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP} . All other parameters (Including the oscillation parameter
¹¹⁴⁶ this fit is insensitive to) are deemed nuisance parameters. To generate these projections,
¹¹⁴⁷ we rely upon integrating the posterior distribution over all nuisance parameters. This
¹¹⁴⁸ is called marginalisation. A simple example of this technique is to imagine the scenario
¹¹⁴⁹ where two coins are flipped. To determine the probability that the first coin returned
¹¹⁵⁰ a ‘head’, the exact result of the second coin flip is disregarded and simply integrated
¹¹⁵¹ over. For the parameters of interest, $\vec{\theta}_i$, we can calculate the marginalised posterior by
¹¹⁵² integrating over the nuisance parameters, $\vec{\theta}_n$. In this case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}} \quad (4.7)$$

¹¹⁵³ Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and correlations
¹¹⁵⁴ between the parameters of interest and the nuisance parameters. In practice, this
¹¹⁵⁵ is simply taking the one or two-dimensional projection of the multi-dimensional
¹¹⁵⁶ probability distribution.

Whilst in principle an easy solution to a complex problem, correlations between the interesting and nuisance parameters can bias the marginalised results. A similar effect is found when the parameters being marginalised over have non-Gaussian probability distributions. For example, Figure 4.5 highlights the marginalisation bias in the probability distribution found for a parameter when requiring a correlated parameter to have a positive parameter value. Due to the complex nature of this oscillation parameter fit presented in this thesis, there are correlations occurring between the oscillation parameters of interest and the other nuisance parameters included in the fit.

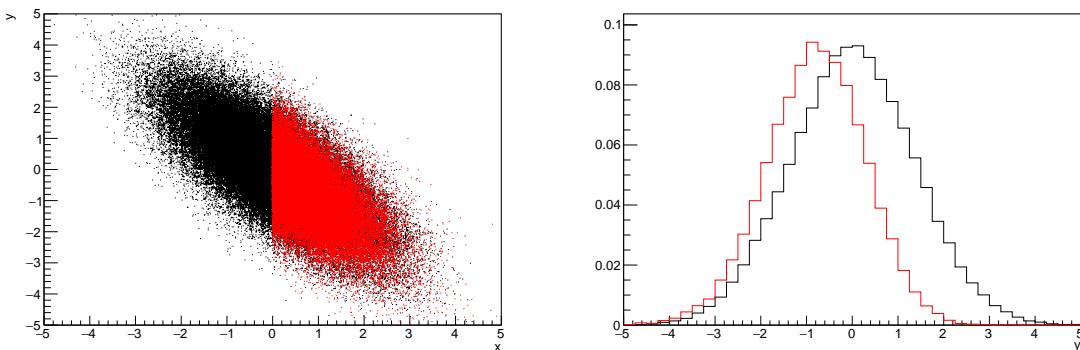


Figure 4.5.: Left: The two dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

4.3.2. Parameter Estimation and Credible Intervals

The purpose of this analysis is to determine the best fit values for the oscillation parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP} . Typically, the results presented take the form of one or two-dimension marginalised probability distributions for the appearance ($\sin^2(\theta_{13})$ and δ_{CP}) and disappearance ($\sin^2(\theta_{23})$ and Δm_{23}^2) parameters. The posterior probability density

₁₁₇₂ taken from the output MCMC chain is binned in these parameters. The parameter
₁₁₇₃ best-fit point is then taken to be the value that has the highest posterior probability.
₁₁₇₄ This is performed in both one and two-dimensional projections.

₁₁₇₅ However, the single best-fit point in a given parameter is not of much use on its
₁₁₇₆ own. We would also like to determine the uncertainty, or credible interval, on that
₁₁₇₇ best-fit point. The definition of the 1σ credible interval is that we have 68% belief that
₁₁₇₈ the parameter is within those bounds. For a more generalised definition, the credible
₁₁₇₉ interval is the region, R , of the posterior distribution that contains a specific fraction of
₁₁₈₀ the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha \quad (4.8)$$

₁₁₈₁ Where θ is the parameter on which we calculate the credible interval. This technique
₁₁₈₂ then calculates the $\alpha \times 100\%$ credible interval.

₁₁₈₃ In practice, this analysis uses the highest posterior density (HPD) credible intervals
₁₁₈₄ which are calculated through the following method. First, the probability distribution
₁₁₈₅ is area-normalised such that it has an integrated area equal to 1.0. The bins of proba-
₁₁₈₆ bility are then summed from the highest to lowest until the sum exceeds the 1σ level
₁₁₈₇ (0.68 in this example). This process is repeated for a range of credible intervals, notably
₁₁₈₈ the 1σ , 2σ and 3σ along with other levels where the critical values for each level can
₁₁₈₉ be found in [139]. This process can be repeated for the two-dimensional probability
₁₁₉₀ distributions by creating two-dimensional contours of credible intervals rather than a
₁₁₉₁ one-dimensional result.

¹¹⁹² 4.3.3. Bayesian Model Comparisons

¹¹⁹³ Due to the matter resonance, this analysis has some sensitivity to the mass hierarchy
¹¹⁹⁴ of neutrino states (whether Δm_{23}^2 is positive or negative) and the octant of $\sin^2(\theta_{23})$
¹¹⁹⁵ . The Bayesian approach utilised within this analysis gives an intuitive method of
¹¹⁹⁶ model comparison by determining which hypothesis is most favourable. Taking the
¹¹⁹⁷ ratio of Equation 4.3 for the two hypotheses of normal hierarchy, NH , and inverted
¹¹⁹⁸ hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

¹¹⁹⁹ The middle term defines the Bayes factor which is a data-driven interpretation of
¹²⁰⁰ how strong the data prefers one hierarchy to the other. For this analysis, equal priors
¹²⁰¹ on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) = P(\vec{\theta}_{IH}) = 0.5$). In practice,
¹²⁰² the MCMC chain proposes a value of $|\Delta m_{23}^2|$ and then applies a 50% probability
¹²⁰³ that the value is sign flipped. Consequently, the Bayes factor can be calculated from
¹²⁰⁴ the ratio of the probability density in either hypothesis. This equates to counting the
¹²⁰⁵ number of steps taken in the normal and inverted hierarchies and taking the ratio. The
¹²⁰⁶ same approach can be taken to compare the upper octant (UO) compared to the lower
¹²⁰⁷ octant (LO) hypothesis of $\sin^2(\theta_{23})$.

¹²⁰⁸ Whilst the value of the Bayes factor should always be shown, the Jeffreys scale [140]
¹²⁰⁹ (highlighted in Table 4.1) gives an indication of the strength of preference for one model
¹²¹⁰ compared to the other. Other interpretations of the strength of preference of a model
¹²¹¹ exist, e.g. the Kass and Raferty Scale [141].

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1.: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [140]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

1212 4.3.4. Comparison of MCMC Output to Expectation

1213 To ensure the fit is performing well, a best-fit spectrum is produced using the pos-
 1214 terior probability distribution and compared with the data, allowing easy by-eye
 1215 comparisons to be made. A simple method of doing this is to perform a comparison
 1216 in the fitting parameters (For instance, the reconstructed neutrino energy and lepton
 1217 direction for T2K far detector beam samples) of the spectra generated by the MCMC
 1218 chain to ‘data’. This ‘data’ could be true data or some variation of Monte Carlo predic-
 1219 tion. This allows easy comparison of the MCMC probability distribution to the data.
 1220 To perform this, N steps from the post burn-in MCMC chain are randomly selected
 1221 (Where for all plots of this style in this thesis, $N = 3000$). From these, the Monte Carlo
 1222 prediction at each step is generated by reweighting the model parameters to the values
 1223 specified at that step. Due to the probability density being directly correlated with
 1224 the density of steps in a certain region, parameter values close to the best fit value are
 1225 most likely to be selected.

1226 In practice, for each bin of the fitting parameters has a probability distribution
 1227 of event rates, with one entry per sampled MCMC step. This distribution is binned
 1228 where the bin with the highest probability is selected as the mean and an error on

1229 the width of this probability distribution is calculated using the approach highlighted
1230 in subsection 4.3.2. Consequently, the best fit distribution in the fit parameter is not
1231 necessarily that which would be attained by reweighting the Monte Carlo prediction
1232 to the most probable parameter values.

1233 A similar study can be performed to illustrate the freedom of the model parameter
1234 space prior to the fit. This can be done by throwing parameter values from the prior
1235 uncertainty of each parameter. This becomes troublesome for parameters with no
1236 prior uncertainty as the range is technically infinite. Where applicable solutions to
1237 remove these have been addressed.

1238 **Chapter 5**

1239 **Simulation, Reconstruction, and Event
Reduction**

1241 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1242 neutrino spectrum at the far detector is required. This includes modeling the flux
1243 generation, neutrino interactions, and detector effects. All of the simulation packages
1244 required to do this are briefly described in section 5.1. The reconstruction of neutrino
1245 events inside the far detector, including the `fitQun` algorithm, is documented in
1246 section 5.2. This also includes data quality checks of the SK-V data which the author
1247 performed for the T2K oscillation analysis presented at Neutrino 2020 [80]. Finally,
1248 section 5.3 describes the steps taken in the SK detector to trigger on events of interest
1249 whilst removing the comparatively large rate of cosmic ray muon events.

1250 **5.1. Simulation**

1251 In order to generate a Monte Carlo prediction of the expected event rate at the far
1252 detector, all the processes in the beam and atmospheric flux, neutrino interaction, and
1253 detector need to be modeled. Each of these parts is individually modeled and each of
1254 them is detailed below.

1255 The beamline simulation consists of three distinct parts: the initial hadron inter-
1256 action modeled by FLUKA [142], the target station geometry and particle tracking

performed by JNUBEAM, [143, 144] and any hadronic re-interactions simulated by GCALOR [145]. The primary hadronic interactions are $O(10)\text{GeV}$, where FLUKA matches external cross-section data better than GCALOR [146]. However, FLUKA is not very adaptable so a small simulation is built to model the interactions in the target and the output is then passed to JNUBEAM and GCALOR for propagation. The hadronic interactions are tuned to data from the NA61/SHINE [147–149] and HARP [150] experiments. The tuning is done by reweighting the FLUKA and GCALOR predictions to match the external data multiplicity and cross-section measurements, based on final state particle kinematics [146]. The culmination of this simulation package generates the predicted flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.7.

The atmospheric neutrino flux **predictions are is** simulated by the HKKM model [43, 45]. The primary cosmic ray flux is tuned to AMS [151] and BESS [152] data assuming the US-standard atmosphere '76 [153] density profile and includes geomagnetic field effects. The primary cosmic rays interact to generate pions and muons. The interaction of these secondary particles to generate neutrinos is handled by DPMJET-III [154] for energies above 32GeV and JAM [45, 155] for energies below that value **DB: Question for Giles: Why different generators for above/below 32GeV?**. These hadronic interactions are tuned to BESS and L3 data [156, 157] using the same methodology as the tuning of the beamline simulation. The energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions and kaons can travel further than their vertically-going counterparts resulting in a larger probability of decaying to neutrinos. The symmetry is broken in lower-energy neutrinos due to geomagnetic effects, which modify the track of the primary cosmic rays. Updates to the HKKM model are currently ongoing [158].

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [159, 160] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a W^\pm boson in Charged Current (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. ~~This is the reason why NC events~~ ~~This is the reason why neutrinos which interact through NC modes~~ are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events. ~~As the SK detector is only sensitive to charged particles above Cherenkov threshold, all charged current interactions are simulated whilst only neutral current processes that can produce charged particles (NCDIS, NCCOH, and NCPROD including π^0 production) are modeled. NC MEC interactions can only produce charged particles through secondary re-interactions which is a low cross-section process.~~

As illustrated in Figure 5.1, CCQE interactions dominate the ~~low-energy~~ cross-section of neutrino interactions ~~around $E_\nu \sim 0.5\text{GeV}$~~ . The NEUT implementation adopts the Llewellyn Smith [161] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [162] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions ~~are is~~ controlled by vector and axial-vector form factors parameterised by the BBBA05 [163] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data [164], respectively. NEUT

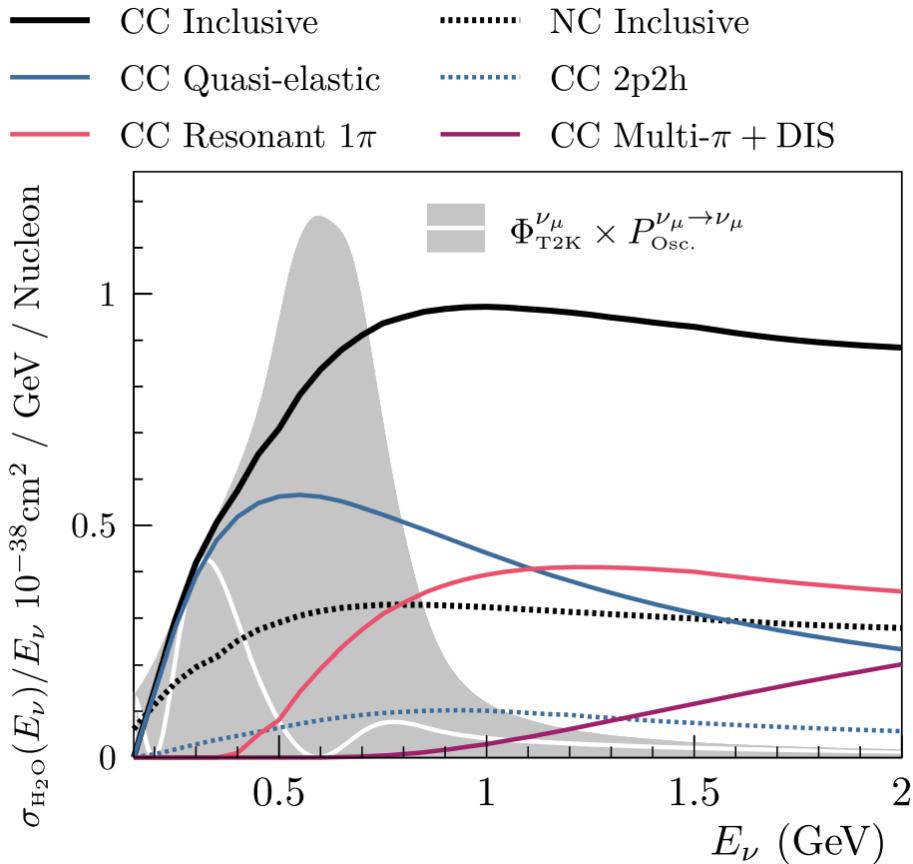


Figure 5.1.: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [159].

1310 implements the Valencia [165] model to simulate MEC events, where two nucleons
 1311 and two holes in the nuclear target are produced (Often called 2p2h interactions).

1312 For neutrinos of energy $O(1)$ GeV, PROD interactions become dominant. These
 1313 predominantly produce charged and neutral pions although γ , kaon, and η production
 1314 is also considered. To simulate these interactions, the Berger-Sehgal [166] model is
 1315 implemented within NEUT. It simulates the excitation of a nucleon from a neutrino
 1316 interaction, production of an intermediate baryon, and the **consequential subsequent**
 1317 decay to a single meson or γ . Pions can also be produced through COH interactions,
 1318 which occur when the incoming neutrino interacts with the entire oxygen **nuclei**

1319 target nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-
1320 Sehgal [167] model to simulate these COH interactions.

1321 DIS and multi- π producing interactions become the most dominant for energies
1322 $> O(5)\text{GeV}$. PYTHIA [168] is used to simulate any interaction with invariant mass
1323 $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces
1324 at least two mesons but has $W < 2\text{GeV}/c^2$, the Bronner model is **invoked used** [169].
1325 Both of these models use parton distribution functions based on the Bodek-Yang
1326 model [170–172].

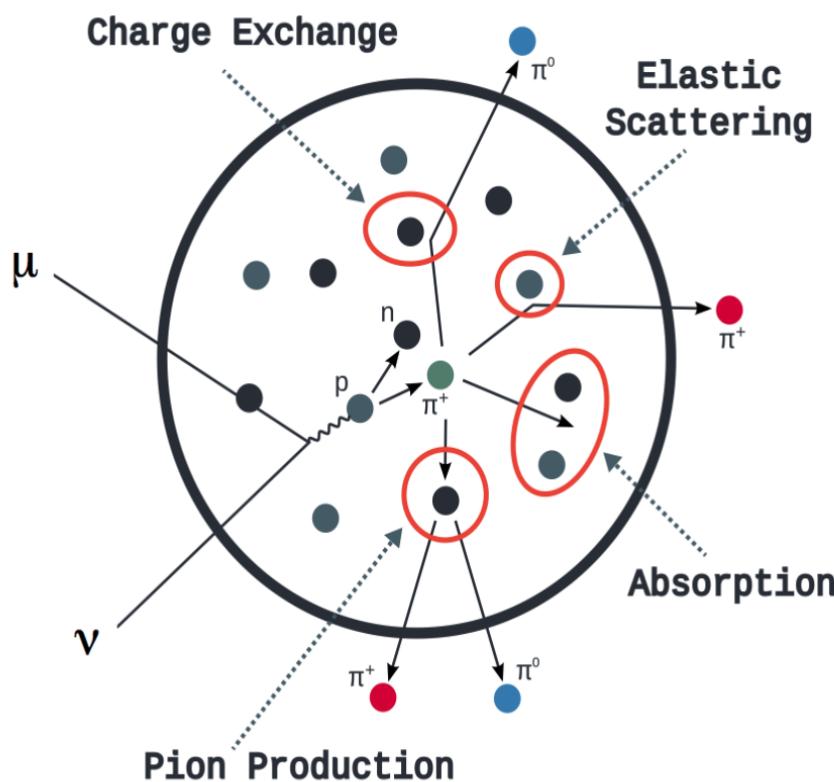


Figure 5.2.: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [173].

1327 Any pion which is produced within the nucleus can re-interact through final state
1328 interactions before it exits, as illustrated by the scattering, absorption, production, and
1329 exchange interactions in Figure 5.2. These re-interactions alter the observable particles

1330 within the detector. For instance, if the charged pion from a CC PROD interaction is
1331 absorbed, the observables would mimic a CC QE interaction. To simulate these effects,
1332 NEUT uses a semi-classical intranuclear cascade model [159]. This cascade functions by
1333 stepping the pion through the nucleus in fixed-length steps equivalent to $dx = R_N/100$,
1334 where R_N is the radius of the nucleus. At each step, the simulation allows the pion
1335 to interact through scattering, charged exchange, absorption, or production with an
1336 interaction-dependent probability calculated from a fit to external data [174]. This
1337 cascade continues until the pion is absorbed or exits the nucleus.

1338 Once the final state particle kinematics have been determined **from by** NEUT, they
1339 are passed into the detector simulation. The near detectors, ND280 and INGRID, are
1340 simulated using a GEANT4 package [118,175] to simulate the detector geometry, particle
1341 tracking, and energy deposition. The response of the detectors is simulated using
1342 the elecSim package [118]. The far detector simulation is based upon the original
1343 Kamiokande experiment software which uses the GEANT3-based SKDETSIM [118,176]
1344 package. This **controls simulates** the interactions of particles in the water as well as
1345 Cherenkov light production. The water quality and PMT calibration measurements
1346 detailed in subsection 3.1.2 are also used within this simulation to make accurate
1347 predictions of the detector response.

1348 5.2. Event Reconstruction at SK

1349 **Any above Cherenkov threshold event which Any event which generates optical**
1350 **photons that** occurs in SK will be **recorded observed** by the PMT array, where each
1351 PMT records the time and accumulated charge. This recorded information is shown
1352 in event displays similar to those illustrated in Figure 5.3. To be useful for physics
1353 analyses, this series of PMT hit information needs to be reconstructed to determine

the particle's identity and kinematics (or track parameters): four-vertex, direction, and momenta. ~~This is because The reconstruction uses the fact that~~ the charge and timing distribution of photons generated by a particular particle in an event is dependent upon its initial kinematics. ~~The concept of distinguishing electron and muon events is from the “fuzziness” of the ring.~~ Electron and muon rings are distinguished by their “fuzziness”. Muons are heavier and less affected by scattering or showering meaning they typically produce “crisp” rings. Electrons are more likely to interact via electromagnetic showering or scattering which results in larger variations of their direction from the initial direction. Consequently, electrons typically produce “fuzzier” rings compared to muons.

1364 The below paragraph has been reordered

For the purposes of this analysis, the `fiTQun` reconstruction algorithm is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by minimising a likelihood function which includes information from PMTs which were hit and those that were not hit. `fiTQun` performs a simultaneous fit of particle kinematics and identity, improving both the accuracy of the fit parameters and the rejection of neutral current π^0 events [177, 178]. The `fiTQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [179] and is described in [180] which is summarised below. The `fiTQun` algorithm improves upon the APFit reconstruction algorithm which has been used for many previous SK analyses. APFit fits the vertex from timing information and then fits the momentum and direction of the particle from PMT hits within a 43 deg Cherenkov cone (which assumes an ultra-relativistic particle). It then fits the particle identity once the track parameters have been fit.

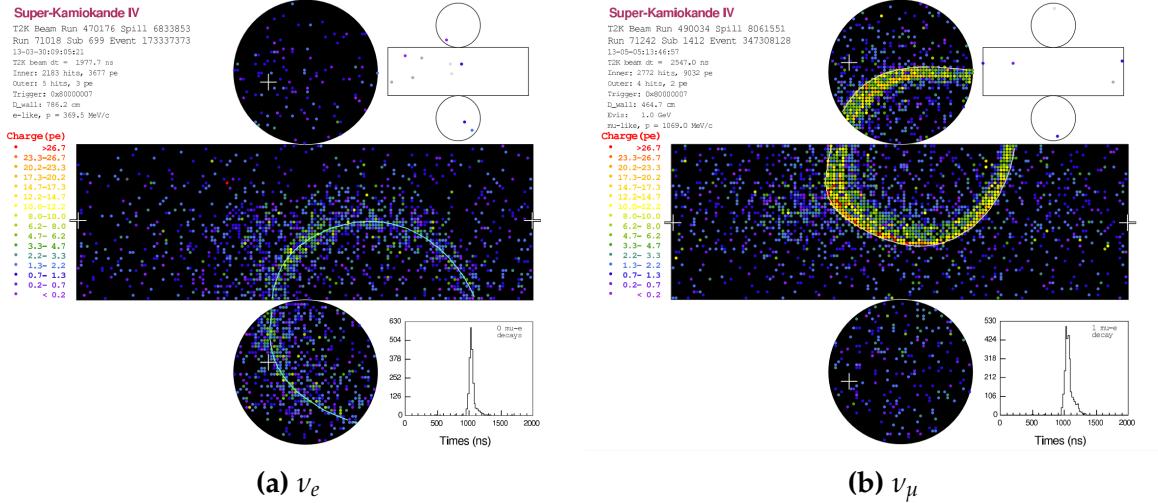


Figure 5.3.: Event displays from Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [181].

1380 The below has been added as an overview of fiTQun

1381 The fiTQun reconstruction algorithm proceeds by:

- 1382 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit metric
- 1384 • **Peak finding:** The initial time of the event is determined by clustering events by time residuals
- 1386 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction, a maximum likelihood technique searches for a single particle generating light
- 1388 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple light-producing particles are considered using the same maximum likelihood technique

1390 An event in SK can consist of a primary and decay particles. An event in SK can

1391 **consist of multiple particles.** For example, a charge current muon neutrino interaction
 1392 can generate two particles that have the potential of generating Cherenkov photons:
 1393 the primary muon, and the secondary decay-electron from the muon. To ensure both

¹³⁹⁴ particle are reconstructed separately, each event is divided into time clusters which
¹³⁹⁵ are called “subevents”. The number of subevents is equal to the number of decay
¹³⁹⁶ electrons plus one (the primary event). To find all the subevents in an event, a vertex
¹³⁹⁷ goodness metric is calculated for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right) \quad (5.1)$$

¹³⁹⁸ where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n \quad (5.2)$$

¹³⁹⁹ is the residual hit time. It is the difference in time between the PMT hit time, t^i ,
¹⁴⁰⁰ of the i^{th} PMT and the expected time of the PMT hit if the photon was emitted at
¹⁴⁰¹ the start of the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in
¹⁴⁰² water and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
¹⁴⁰³ proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends to
¹⁴⁰⁴ zero resulting in subevents appearing as spikes in the goodness metric. The proposed
¹⁴⁰⁵ fit vertex and time are grid-scanned, and the values which maximise the goodness
¹⁴⁰⁶ metric are selected as the “pre-fit vertex”. Whilst this predicts a vertex for use in
¹⁴⁰⁷ the clustering algorithm, the final vertex is fit using the higher-precision maximum
¹⁴⁰⁸ likelihood method described below.

¹⁴⁰⁹ Once the pre-fit vertex has been determined, the goodness metric is scanned as
¹⁴¹⁰ a function of t to determine the number of subevents. A peak-finding algorithm is
¹⁴¹¹ then used on the goodness metric, requiring the goodness metric to exceed some

threshold and drop below a reduced threshold before any subsequent additional
 peaks are considered. The thresholds are set such that the rate of false peak finding
 is minimised while still attaining good data to Monte Carlo agreement. To improve
 performance, the pre-fit vertex for each delayed subevent is re-calculated after PMT
 hits from the previous subevent are masked. This improves the decay-electron tagging
 performance. Once all subevents have been determined, the time window around
 each subevent is then defined by the earliest and latest time which satisfies $-180 <$
 $T_{Res}^i < 800\text{ns}$. The subevents and associated time windows are then used as seeds for
 further reconstruction.

For a given subevent, the `fitQun` algorithm constructs a likelihood based on the
 accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}), \quad (5.3)$$

where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and mo-
 ments, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the probability of the
 i^{th} tube to not register a hit given the track parameters and particle hypothesis. The
 charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$, represents the probability
 density function of observing charge q_i and time t_i on the i^{th} PMT given the specified
 track parameters and particle hypothesis.

I don't think the below paragraph is actually needed?

As the generation and propagation of the optical photons are independent of the
 PMT and electronics response, it is natural to split the calculation into two. This split
 was also performed to . Firstly, the expected number of photoelectrons (or predicted

charge), $\mu_i = \mu_i(\vec{\theta}, \Gamma)$, at the i^{th} PMT is calculated. This value is then substituted into the likelihood function. This allows the charge likelihood density $f_q(q_i|\mu_i)$ and unhit probability $P_i(\text{unhit}|\mu_i)$ to be expressed via quantities that are only dependent on the response of the PMT.

The predicted charge is calculated based on contributions from both the direct light and the scattered light. The direct light contribution is determined based on the integration of the Cherenkov photon profile along the track. PMT angular acceptance, water quality, and calibration measurements discussed in subsection 3.1.2 are included to accurately predict the charge probability density at each PMT. The scattered light is calculated in a similar way, although it includes a scattering function that depends on the vertex of the particle and the position of the PMT. The charge likelihood is calculated by comparing the prediction to the observed charge in the PMT.

The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} , and time t of the track **parameters** as well as the particle hypothesis. The expected time for PMT hits is calculated by assuming unscattered photons being emitted from the midpoint of the track, S_{mid} ,

$$t_{exp}^i = t + S_{mid}/c + |R_{PMT}^i - \vec{x} - S_{mid}\vec{d}|/c_n, \quad (5.4)$$

where c is the speed of light in a vacuum. The time likelihood is then expressed in terms of the residual difference between the PMT hit time and the expected hit time, $t_{Res}^i = t^i - t_{exp}^i$. The particle hypothesis and momentum also affect the Cherenkov photon distribution. These parameters modify the shape of the time likelihood density since in reality not all photons are emitted at the midpoint of the track. As with the charge likelihood, the contributions from both the direct and scattered light to the time

1455 likelihood density are calculated separately, which are both calculated from particle
1456 gun studies.

1457 **Below paragraphs, figures and equations are changed**

1458 The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined as
1459 the best-fit parameters. In practice MINUIT [182] is used to minimise the value of
1460 $-\ln L(\Gamma, \vec{\theta})$. The `fitQun` algorithm considers an electron-like, muon-like, and charged
1461 pion-like hypothesis for events with a single final state particle, denoted “single-ring
1462 events”. The particle’s identity is determined by taking the ratio of the likelihood
1463 of each of the hypotheses. For instance, electrons and muons are distinguished by
1464 considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison to the reconstructed
1465 momentum of the electron hypothesis, as illustrated by Figure 5.4. The coefficients
1466 of the discriminator between electron-like and muon-like events is determined from
1467 Monte Carlo studies [180]. Similar distributions exist for distinguishing electron-like
1468 events from π^0 -like events, and muon-like events from pion-like events. They are
1469 defined as,

$$\begin{aligned} \text{Electron/Muon} &: \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 &: \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} &: \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

1470 as taken from [183], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the
1471 single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed
1472 invariant mass of the two photons emitted from π^0 decay. Typically, the distance
1473 between a particular entry in these two-dimensional distributions and the cut-line is
1474 termed the PID parameter and is illustrated in Figure 5.5.

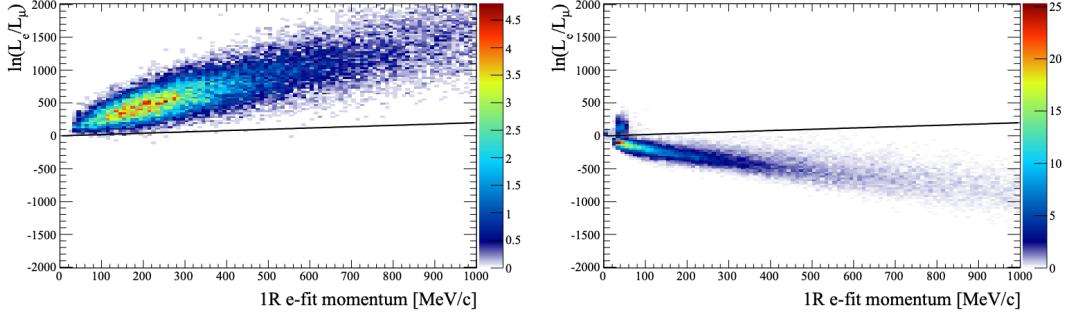


Figure 5.4.: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, which coefficients obtained from Monte Carlo studies. Figures taken from [180].

Review stop

1475

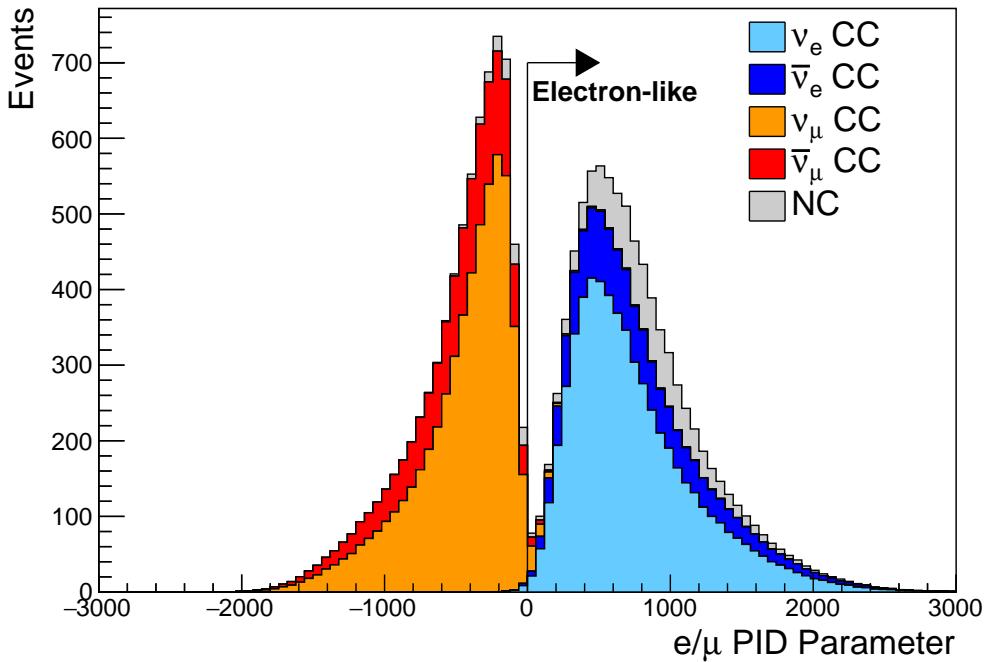


Figure 5.5.: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1476

The `fitQun` algorithm also considers a π^0 hypothesis. To do this, it performs

1477 a fit looking for two standard electron-hypothesis tracks which point to the same

1478 four-vertex. This assumes the electron tracks are generated from photon-conversion so
1479 the electron tracks actually appear offset from the proposed π^0 vertex. For these fits,
1480 the conversion length, direction, and momentum of each photon are also considered
1481 as track parameters which are then fit in the same methodology as the standard
1482 single-ring hypotheses.

1483 Whilst lower energy events are predominately single-ring events, higher energy
1484 neutrino events can generate final states with multiple particles which generate
1485 Cherenkov photons. These “multi-ring” hypotheses are also considered in the `fitQun`
1486 algorithm. When calculating the charge likelihood density, the predicted charge asso-
1487 ciated with each ring is calculated separately and then summed to calculate the total
1488 accumulated charge on each PMT. Similarly, the time likelihood for the multi-ring
1489 hypothesis is calculated assuming each ring is independent. Each track is time-ordered
1490 based on the time of flight from the center of the track to the PMT and the direct light
1491 from any ring incident on the PMT is assumed to arrive before any scattered light. To
1492 reduce computational resource `usage`, the multi-ring fits only consider electron-like
1493 and charged pion-like rings as the pion fit can be used as a proxy for a muon fit due to
1494 their similar mass.

1495 Multi-ring fits proceed by proposing another ring to the previous fit and then
1496 fitting the parameters in the method described above. Typically, multi-ring fits have
1497 the largest likelihood because of the additional degrees of freedom introduced. A
1498 likelihood value is calculated for the n -ring and $(n + 1)$ -ring hypotheses, where
1499 the additional ring is only included if the likelihood value is above 9.35, based
1500 on Monte Carlo studies in [184]. Consequently, the additional ring is only added
1501 if the ratio of likelihoods passes a criterion, which is determined by Monte Carlo
1502 studies.

1503 5.2.1. Validation of Reconstruction in SK-V

1504 As an example of how the reconstruction depends on the detector conditions, the
1505 author of this thesis assessed the quality of event reconstruction for SK-V data. The
1506 detector systematics ~~invoked-within used in~~ the T2K-only oscillation analysis are
1507 determined using data-to-Monte Carlo comparisons of the SK-IV data [185]. Due to
1508 tank-open maintenance occurring between SK-IV and SK-V, the dark rate of each PMT
1509 was observed to increase in SK-V due to light exposure for a significant time during
1510 the repairs. This increase can be seen in Figure 5.6. Run-10 of the T2K experiment was
1511 conducted in the SK-V period, so the consistency of SK-IV and SK-V data needs to
1512 be studied to determine whether the SK-IV-defined systematics can be applied to the
1513 run-10 data. This comparison study was performed using the stopping muon data set
1514 for both the SK-IV and SK-V periods. This data sample is used due to the high rate of
1515 interactions ($O(200)$ events per hour) as well as having similar energies to muons from
1516 CCQE ν_μ interactions from beam interactions. The rate of cosmic muons does depend
1517 on the solar activity cycle [186] but has been neglected in this comparison study. This
1518 is because the shape of the distributions is most important for the purposes of being
1519 compared to the detector systematics. The SK-IV and SK-V data samples consist of
1520 2398.42 and 626.719 hours of data which equates to 686k and 192k events respectively.

1521 The predicted charge calculated in the `fitQun` ~~charge-likelihood prediction al-~~
1522 ~~gorithm~~ includes a contribution from the photoelectron emission due to dark noise.
1523 Therefore, the increase in the SK-V dark rate needs to be accounted for. In practice, the
1524 average dark rate in each SK period is calculated and used as an input in the reconstruc-
1525 tion. This is calculated by averaging the dark rate per run for each period separately,
1526 using the calibration measurements detailed in subsection 3.1.2. The average dark
1527 rate from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The
1528 ~~associated-charge charges associated~~ with the muon and decay electron subevents

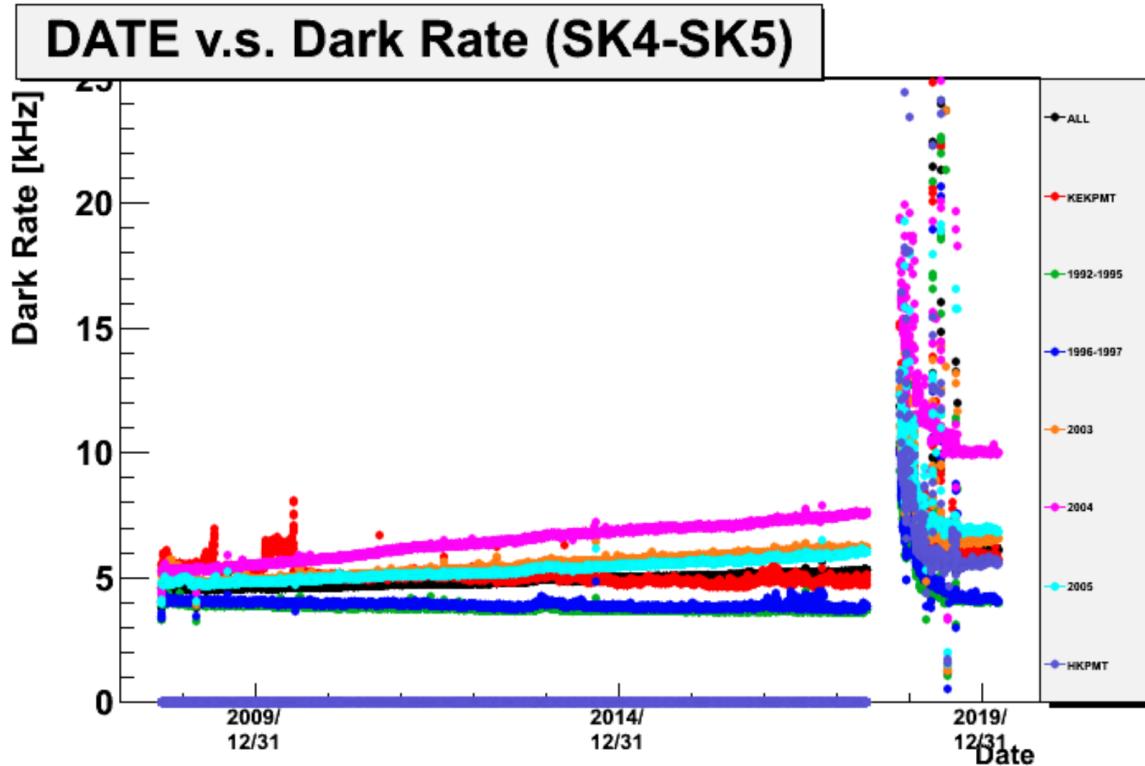


Figure 5.6.: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [185].

are illustrated in Figure 5.7. The photoelectron emission from dark noise ~~will be more noticeable is more significant~~ for events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in lower-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent is mostly unaffected by the increase in dark rate, whilst the charge associated with the decay-electron is clearly affected.

~~The energy scale systematic for the SK-IV period was determined to be 2.1%. It is defined to be equal to the difference between data and Monte Carlo prediction in the stopping muon data sample. The energy scale systematic is estimated from data-to-Monte Carlo differences in the stopping muon sample in [187] and found to be 2.1%.~~ To determine the consistency of SK-IV and SK-V with respect to the energy

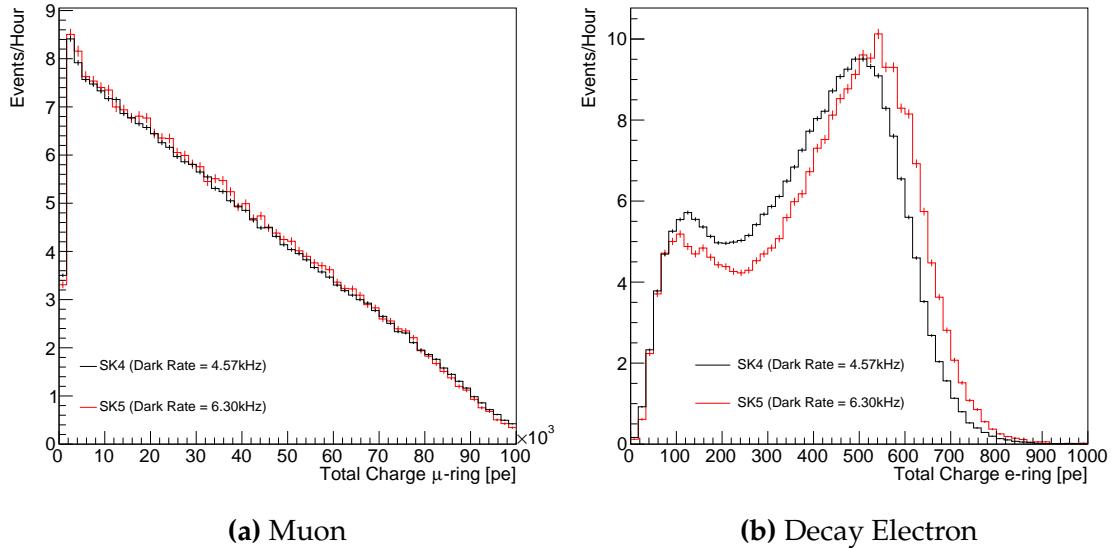


Figure 5.7.: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent and the associated decay electron subevent.

scale systematic, the muon momentum distribution is compared between the two SK periods. As the total number of Cherenkov photons is integrated across the track length, the reconstructed momentum divided by track length (or range) is compared between SK-IV and SK-V as illustrated in Figure 5.8.

The consistency between these distributions has been computed in two ways.

Firstly, a Gaussian is fit to *the peak of* each distribution separately, **whose .The** mean **of** **which** is found to be (2.272 ± 0.003) MeV/cm and (2.267 ± 0.006) MeV/cm for SK-IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of the Gaussian fits are consistent with the expected stopping power of a minimum ionising muon for a target material (water) with $Z/A \sim 0.5$ [188]. The second consistency check is performed by introducing a nuisance parameter, α , which modifies the SK-V distribution. The value of α which minimises the χ^2 value between the SK-IV and SK-V is determined by scanning across a range of values. This is repeated by applying the nuisance parameter as both a multiplicative factor and an additive shift. The χ^2 distributions for different values of α is illustrated in Figure 5.9. The

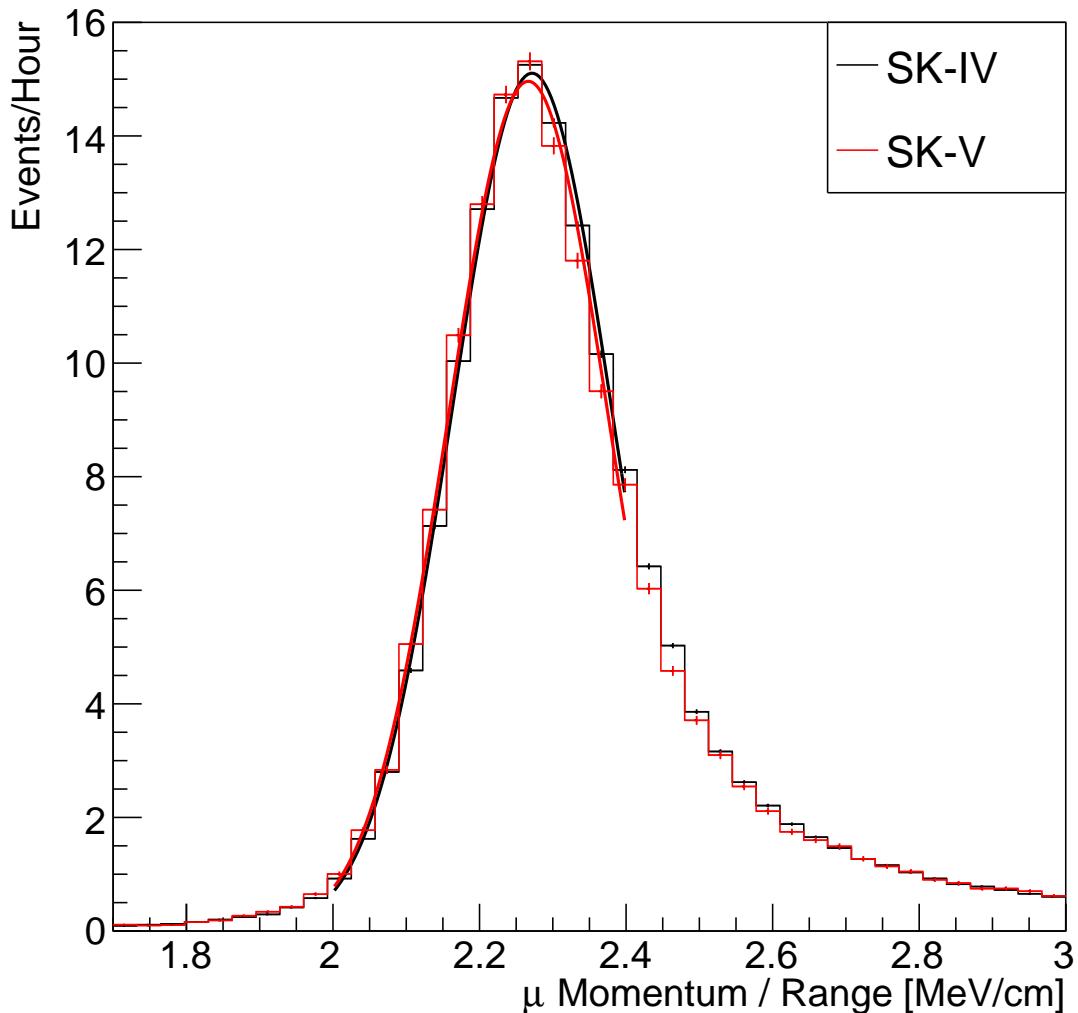


Figure 5.8.: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range [2.0, 2.4] MeV/cm and illustrated as the solid curve.

values which minimise the χ^2 are found to be 0.0052 and 1.0024 for the additive and multiplicative implementations, respectively. No evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic has been found in the reconstructed momentum distribution of SK-IV and SK-V.

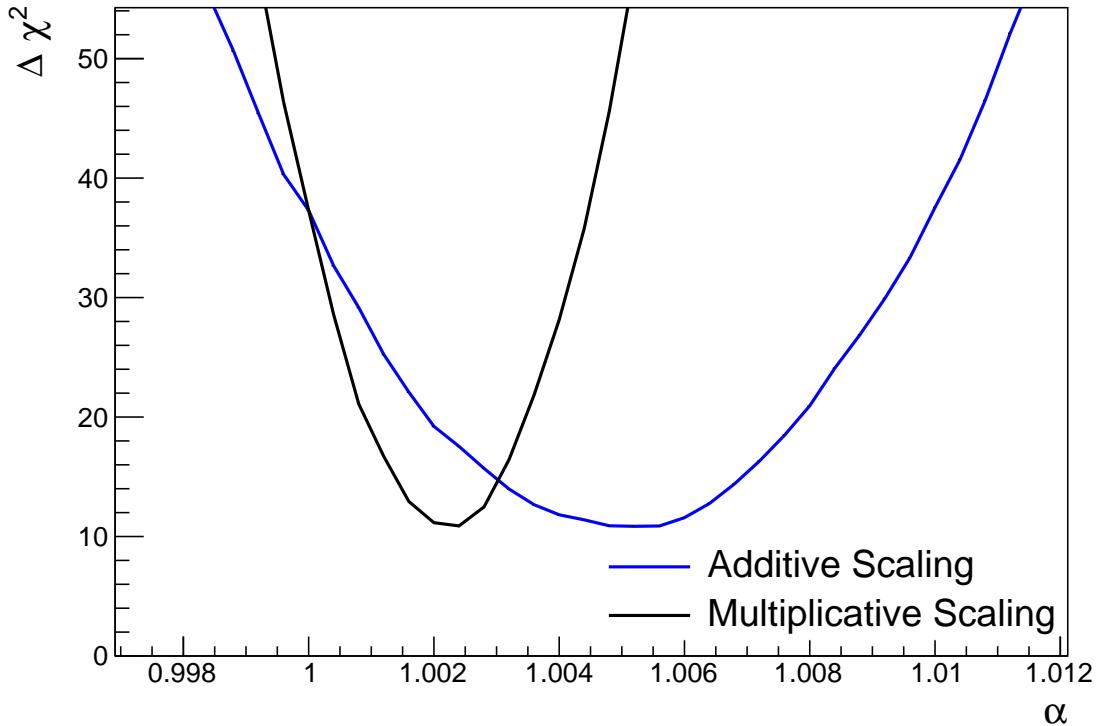


Figure 5.9.: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1559 5.3. Event Reduction at SK

1560 Atmospheric neutrino events observed in the SK detector are categorised into three
 1561 different types of samples: fully contained (FC), partially contained (PC) and up-going
 1562 muon (Up- μ), using PMT hit signatures in the inner and outer detector (ID and OD,
 1563 respectively). To identify FC neutrino events, it is required that the neutrino interacts
 1564 inside the fiducial volume of the ID and that no significant OD activity is observed. For
 1565 this analysis, an event is defined to be in the fiducial volume provided the event vertex
 1566 is at least 0.5m away from the ID walls. PC events have the same ID requirements
 1567 but can have a larger signal present inside the OD. **Typically these events are higher**

1568 **energy muon interactions that penetrate the ID walls** Typically, only high energy
 1569 **muons from ν_μ interactions can penetrate the ID wall.** The Up- μ sample contains
 1570 events where muons are created from neutrino interactions in the OD water or rock
 1571 below the tank. They then propagate upwards through the detector. Downward-going
 1572 muons generated from neutrino interactions above the tank are neglected is because of
 1573 the difficulty in separating their signature from the cosmic muon shower background.
 1574 The sample categories are visually depicted in Figure 5.10.

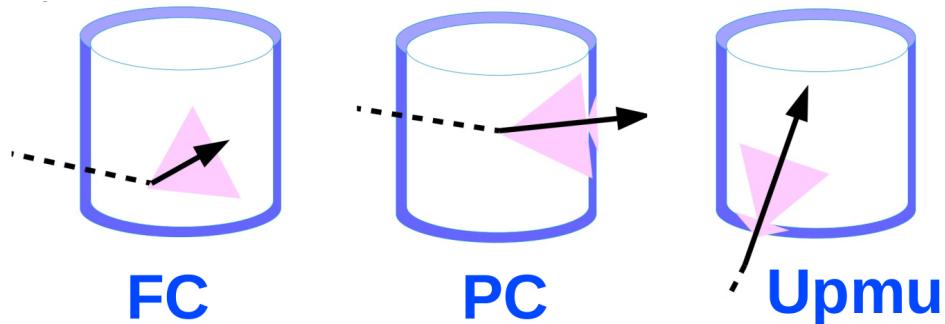


Figure 5.10.: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC) and up-going muon (Up- μ) samples included in this analysis.

1575 Based on the event characteristics, as defined by the `fitQun` event reconstruction
 1576 software, the FC events are categorised by

- 1577 • **Visible Energy:** equal to the sum of the reconstructed kinetic energy of particles
 1578 above the Cerenkov threshold for all rings present in the event. The purpose is to
 1579 separate events into sub-GeV and multi-GeV categories.
- 1580 • **Number of observed Cerenkov rings.** The purpose is to separate single-ring and
 1581 multi-ring events, where single-ring events predominantly consist of quasi-elastic
 1582 interactions and multi-ring events are typically resonant pion production or deep
 1583 inelastic scattering events.

- 1584 • **Particle identification parameter of the most energetic ring:** A value deter-
 1585 mined from the maximum likelihood value based on `fitQun`'s electron, muon, or
 1586 pion hypothesis. The purpose is to separate electron-like and muon-like events.

- 1587 • **Number of decay electrons:** The purpose is to separate quasi-elastic events
 1588 (which have one decay electron emitted from the muon decay) and resonant pion
 1589 production events (which have two decay electrons emitted from the muon and
 1590 pion).

1591 The PC and Up- μ categories are broken down into “through-going” and “stopping”
 1592 samples depending on whether the muon leaves the detector. This is because the **PC**
 1593 stopping events deposit the entire energy of the interaction into the detector, resulting
 1594 in better reconstruction. The energy of events that exit the detector has to be estimated,
 1595 **with typically worse resolution**, which introduces much larger systematic uncertain-
 1596 ties. Through-going Up- μ samples are further broken down by whether any hadronic
 1597 showering was observed in the event which typically indicates DIS interactions. The
 1598 expected neutrino energy for the different categories is given in Figure 5.11. FC sub-
 1599 GeV and multi-GeV events peak around 0.7GeV and 3GeV respectively, with slightly
 1600 different peak energies for ν_e and $n\nu_\mu$ oscillation channels. PC and Up- μ are almost
 1601 entirely comprised of ν_μ events and peak around 7GeV and 100GeV, respectively.

1602 In normal data-taking operations, the SK detector observes many background
 1603 events alongside the beam and atmospheric neutrino signal events of physics inter-
 1604 est **for this thesis**. Cosmic ray muons and flasher events, which are the spontaneous
 1605 discharge of a given PMT, contribute the largest amount of background events in the en-
 1606 ergy range relevant to **any analysis searching for neutrino events this thesis**. Lower
 1607 energy analyses like DSNB searches are also subject to radioactive backgrounds [189].
 1608 Therefore the data recorded is reduced with the aim of removing these background
 1609 events. The reduction process is detailed in [47, 89] and briefly summarised below.

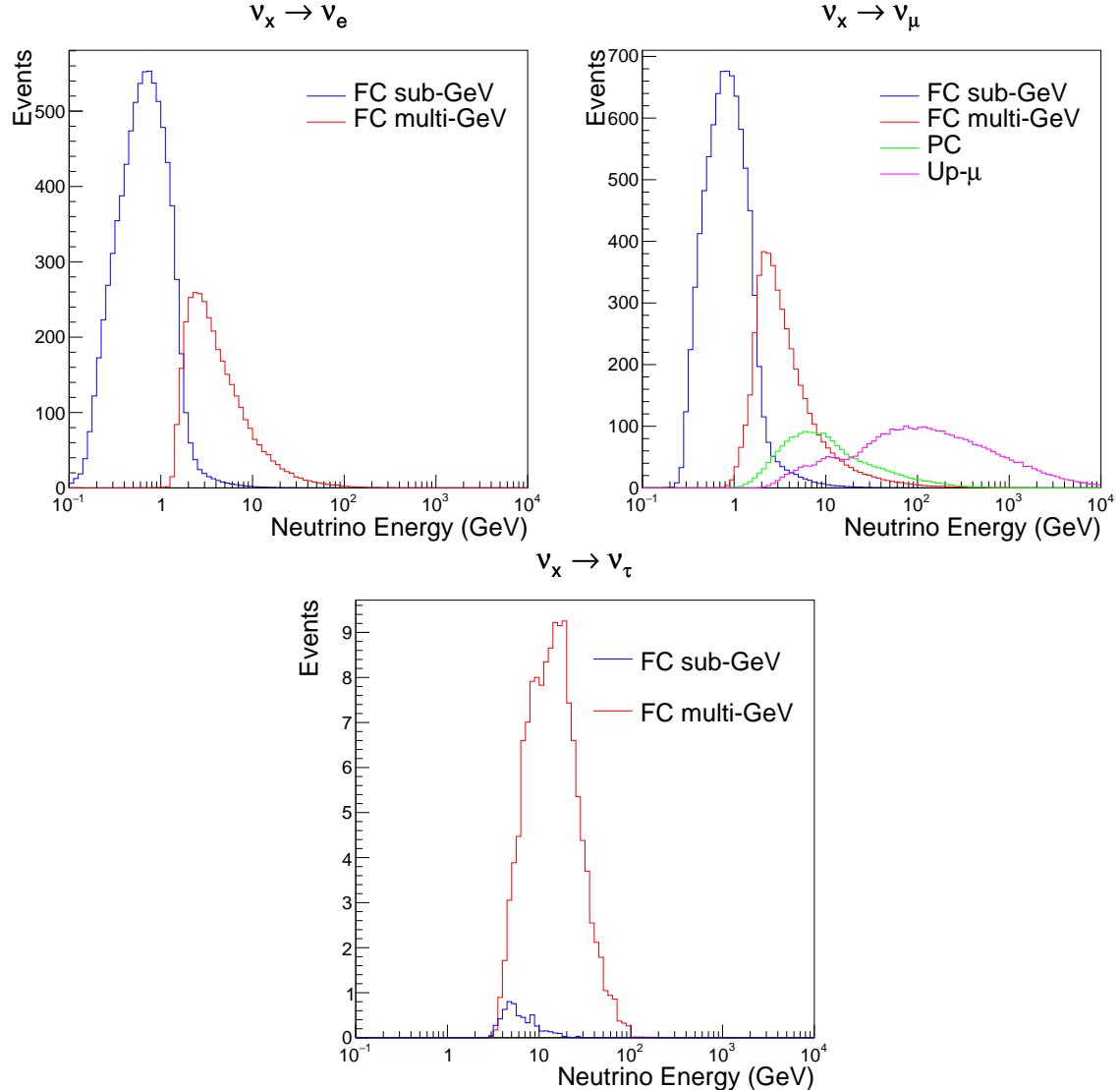


Figure 5.11.: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). Asimov A oscillation parameters are assumed (given in Table 2.2).

1610 The first two steps in the FC reconstruction remove the majority of cosmic ray
 1611 muons by requiring a significant amount of ID activity compared to that measured in
 1612 the OD. Events that pass this cut are typically very high momentum muons or events
 1613 that leave very little activity in the OD. Consequently, a third reduction step is then
 1614 applied to select cosmic-ray muons that pass the initial reduction step. A purpose-built
 1615 cosmic muon fitter is used to determine the entrance (or exit) position of the muon and

1616 a cut is applied to OD activity contained within 8m of this position. Flasher events are
 1617 removed in the fourth reduction step which is based on the close proximity of PMT
 1618 hits surrounding the PMT producing the flash. Events that pass all these reduction
 1619 steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses
 1620 information from the more precise fitter to repeat the previous two steps with tighter
 1621 cuts. Muons below the Cherenkov threshold can not generate optical photons in the
 1622 ID but the associated decay electron can due to its lower mass. These are the types of
 1623 events targeted in the fifth reduction step. The final cuts require the event vertex to be
 1624 within the fiducial volume (0.5m from the wall although the nominal distance is 2.0m),
 1625 visible energy $E_{vis} > 30\text{MeV}$ and fewer than 16 hits within the higher energy OD
 1626 cluster. The culmination of the fully contained reduction results in 8.09 events/day in
 1627 the nominal fiducial volume [190]. The uncertainty in the reconstruction is calculated
 1628 by comparing Monte Carlo prediction to data. The largest discrepancy is found to be
 1629 1.3% in the fourth reduction step.

1630 The PC and Up- μ events are processed through their own reduction processes
 1631 detailed in [47]. Both of these samples are reconstructed with the APFit algorithm
 1632 rather than fitQun. This is because the efficiency of reconstructing events that leave
 1633 the detector has not been sufficiently studied for reliable systematic uncertainties wth
 1634 fitQun. The PC and Up- μ samples acquire events at approximately 0.66 and 1.44
 1635 events/day.

1636 Events due to beam neutrinos undergo the same reduction steps as FC events and
 1637 are then subject to further cuts [191]. The GPS system which links the timing between
 1638 the beam facility and SK needs to be operating correctly and there should be no activity
 1639 within the detector in the previous $100\mu\text{s}$ before the trigger. The events then need to
 1640 triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1641 Due to the lower energy beam neutrinos, the T2K samples are not dependent
 1642 upon the visible energy neutrino as the range of neutrino energies are smaller than
 1643 that found in atmospheric neutrinos. Furthermore, the 2020 T2K-only oscillation
 1644 analysis only considers events which contain a single ring. Similar to atmospheric
 1645 event selection, the number of decay electrons is used as a proxy for distinguishing
 1646 CCQE and CCRES events. The expected neutrino energy, broken down by number
 1647 of decay electrons, is given in ???. The beam neutrinos samples are not split by
 1648 visible energy since their energy range is smaller than the atmospheric neutrino
 1649 events. Following the T2K analysis in [80], only single-ring beam neutrino events
 1650 are considered. Similar to atmospheric event selection, the number of decay electrons
 1651 is used as a proxy for distinguishing CCQE and CCRES events. The expected neutrino
 1652 energy, broken down by number of decay electrons, is given in Figure 5.12.

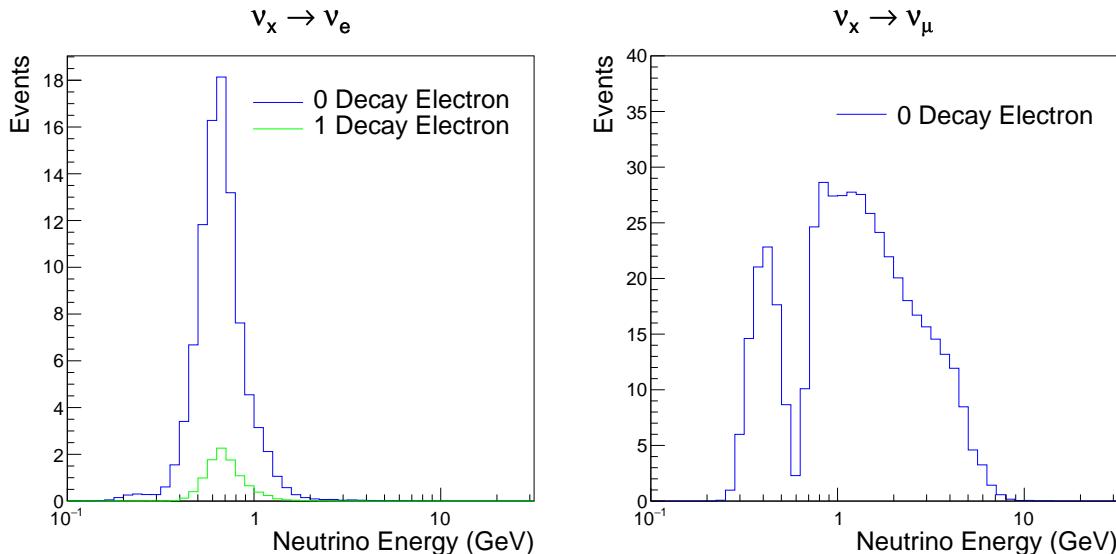


Figure 5.12.: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

₁₆₅₃ **Chapter 6**

₁₆₅₄ **Sample Selections and Systematics**

₁₆₅₅ The oscillation analysis presented within this thesis is built upon a simultaneous
₁₆₅₆ fit to atmospheric data at SK, neutrino beam data in the near detector, and beam
₁₆₅₇ data measured at SK. The definitions of these samples are documented in section 6.1,
₁₆₅₈ section 6.2, and section 6.3, respectively. The data collected and used within this
₁₆₅₉ analysis is detailed in Table 6.1. The near and far detector data corresponds to T2K
₁₆₆₀ runs 2-9 and runs 1-10, respectively. The accumulated POT and beam power for runs
₁₆₆₁ 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1.: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

₁₆₆₂ The difference in POT recorded at the near and far detector is due to the difference
₁₆₆₃ in downtime of the respective detector. The SK detector is very stable with almost 100%
₁₆₆₄ of data recorded during beam operation. Due to various technical and operational
₁₆₆₅ issues, the downtime of the near detector is significantly higher due to its more
₁₆₆₆ complex design and operating requirements.

1667 The systematic parameters invoked within the flux, detector, and interaction models
 1668 used within this analysis are documented in section 6.4. The standard configuration of
 1669 the joint beam and atmospheric data fit utilises far detector systematics provided in
 1670 the official inputs from the two experiments. Additionally, a correlated detector model
 1671 which fits the parameters used in sample selections to data has been developed and
 1672 documented in subsection 6.4.5.

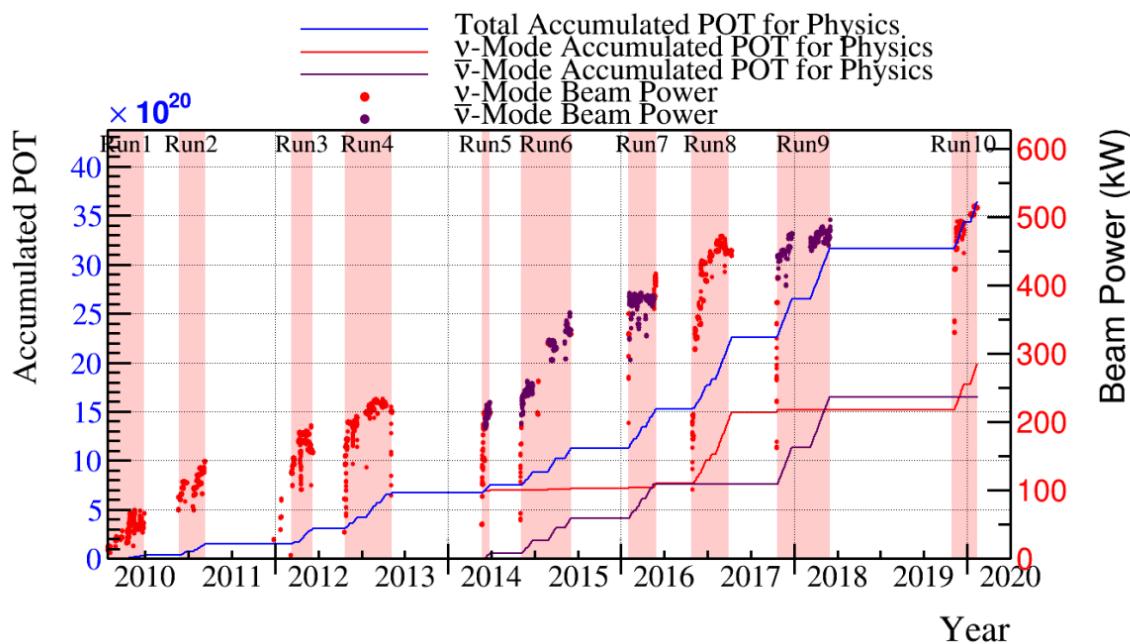


Figure 6.1.: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

1673 6.1. Atmospheric Samples

1674 The atmospheric event selection follows the official SK-IV analysis presented in [89]
1675 and is documented below. The Monte Carlo prediction used within this analysis
1676 corresponds to 500 years worth of neutrino events, which is scaled down to match the
1677 SK-IV livetime of 3244.4 days.

1678 The fully contained (FC), partially contained (PC), and upward going muon events
1679 ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further broken down
1680 into different samples based on reconstruction information. This section details the
1681 samples used within this oscillation analysis, alongside the chosen binning, used
1682 within the fit.

1683 FC events are first separated by the visible energy deposited within the detector.
1684 This is calculated as the sum of the reconstructed kinetic energy above the Cherenkov
1685 threshold for all rings present in the event. Events are separated by whether they were
1686 above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates “subGeV” and “multiGeV” events.
1687 Typically, lower energy events consist of charge current quasi-elastic (CCQE) inter-
1688 actions which are better understood and simpler to reconstruct resulting in smaller
1689 systematic uncertainties. Events are further separated by the number of rings as-
1690 sociated with the event due to similar reasoning. As the oscillation probability is
1691 dependent upon the flavour of neutrino, electron and muon events are separated
1692 using a similar likelihood method to that discussed in section 5.2. To reduce computa-
1693 tional resources required for the reconstruction, only electron and pion hypotheses are
1694 considered so this separation cut depends on the ratio of the electron to pion likeli-
1695 hoods, $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1696 Charged current resonant pion production (CCRES) interactions generate a final-state
1697 pion. This can decay, mostly likely through a muon, into a decay electron. Therefore

- any electron-like event with one decay electron or muon-like event with two decay electrons was most likely produced by a CCRES interaction. Consequently, the number of decay electrons can be used to distinguish CCQE and CCRES interaction modes. Ultimately, FC subGeV events are separated into the samples listed in Table 6.2.

Sample Name	Description
SubGeV-e-like-0dcy	Single ring e -like events with zero decay electrons
SubGeV-e-like-1dcy	Single ring e -like events with one or more decay electrons
SubGeV-mulike-0dcy	Single ring μ -like events with zero decay electrons
SubGeV-mulike-1dcy	Single ring μ -like events with one decay electrons
SubGeV-mulike-2dcy	Single ring μ -like events with two or more decay electrons
SubGeV-pi0like	Two e -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2.: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

- In addition to the cuts discussed above, multiGeV samples also have additional cuts to separate samples which target neutrino and antineutrino separation. As discussed in section 7.1, the matter resonance only occurs for neutrinos in normal hierarchy and antineutrinos in an inverted mass hierarchy. Therefore, having flavour-enriched samples aids in the determination of the mass hierarchy. For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

- The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be absorbed within the oxygen nucleus compared to the π^+ from ν_e interactions [192]. These pions then

1709 decay, mostly through muons, to electrons. Therefore the number of tagged decay
 1710 electrons associated with an event gives an indication of whether the interaction
 1711 was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
 1712 The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
 1713 where the MultiGeV-*e*like-nue has 78% purity of neutrino interactions with only 7%
 1714 antineutrino background in that sample and the rest of the sample comprising of
 1715 neutral current backgrounds.

1716 This relatively simple discriminator works reasonably well for single-ring events.
 1717 However, this is not the case for multi-ring events. A multiGeV multiring separation
 1718 (MME) likelihood cut which specifically targets multiGeV multiRing electron-like
 1719 events was introduced in [193, 194]. This is a two-stage likelihood selection cut. Four
 1720 observables are used within the first likelihood cut to distinguish CC ν_e and CC $\bar{\nu}_e$
 1721 events from background:

- 1722 • The number of decay electrons
- 1723 • The maximum distance between the vertex of the neutrino and the decay electrons
- 1724 • The energy deposited by the leading energy ring
- 1725 • The reconstructed particle identification of that highest energy ring

1726 Background events consist of CC ν_μ and NC interactions. Typically these produce
 1727 events where the majority of the energy is carried by the hadronic system. Additionally,
 1728 muons tend to travel further than the pions from CC ν_e before decaying. Consequently,
 1729 the parameters used within the likelihood cut target the typical background interaction
 1730 kinematics.

1731 Neutrino and antineutrino events are then separated by a second likelihood method
 1732 ($\nu/\bar{\nu}$ separation) detailed in [52]. This uses the number of decay electrons, the number
 1733 of reconstructed rings, and the event's transverse momentum. The last two parameters

Sample Name	Description
MultiGeV- <i>e</i> -like-nue	Single ring <i>e</i> -like events with zero decay electrons
MultiGeV- <i>e</i> -like-nuebar	Single ring <i>e</i> -like events with one or more decay electrons
MultiGeV- <i>μ</i> -like	Single ring <i>μ</i> -like events
MultiRing- <i>e</i> -like-nue	Two or more ring events with leading energy <i>e</i> -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing- <i>e</i> -like-nuebar	Two or more ring events with leading energy <i>e</i> -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing- <i>μ</i> -like	Two or more ring events with leading energy <i>μ</i> -like ring and only requires $E_{vis} > 0.6\text{GeV}$ DB: Why is this not }1.33\text{GeV? N
MultiRing-Other1	Two or more ring events with leading energy <i>e</i> -like ring and failed the MME likelihood cut

Table 6.3.: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1734 are used because higher-energy samples tend to have more pions produced above
1735 the Cherenkov threshold which results in more rings compared to an antineutrino
1736 interaction. Furthermore, the angular distribution also tends to be more forward
1737 peaked in antineutrino interactions as compared to neutrino interactions [89]. These
1738 FC multiGeV sample definitions are detailed in Table 6.3.

1739 The PC and up- μ events are split by the amount of energy deposited within the
1740 outer detector, into “stopping” and “through-going” samples. This is because the
1741 momentum of events leaving the detector has to be approximated, which increases the
1742 systematic uncertainty. This estimate is particularly poor for very high-energy events.
1743 This is why up- μ through-going events are not binned in reconstructed momentum.
1744 If an event leaves the detector, the energy it takes with it has to be estimated which
1745 increases the systematic uncertainty compared to events entirely contained within the
1746 inner detector. The through-going up- μ are further separated by the presence of any
1747 electromagnetic showering in the event, as the assumption of non-showering muon

₁₇₄₈ does not give reliable reconstruction for these types of events [47]. In total, 13 FC, 2
₁₇₄₉ PC, and 3 up- μ atmospheric samples are included within this analysis.

Sample	CC ν_e	CC $\bar{\nu}_e$	CC($\nu_\mu + \bar{\nu}_\mu$)	CC($\nu_\tau + \bar{\nu}_\tau$)	NC
SubGeV- e -like-0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- e -like-1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- μ -like-0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- μ -like-1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- μ -like-2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- π 0-like	6.923	2.368	0.928	0.011	89.77
MultiGeV- e -like-nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- e -like-nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- μ -like	0.024	0.005	99.67	0.245	0.058
MultiRing- e -like-nue	59.32	12.39	4.906	3.385	20
MultiRing- e -like-nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- μ -like	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThrus	0.564	0.207	98.65	0.	0.576
UpStop- μ	0.829	0.370	98.51	0.	0.289
UpThruNonShower- μ	0.206	0.073	99.62	0.	0.103
UpThruShower- μ	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

₁₇₅₀ The atmospheric samples are binned in direct observables: reconstructed lepton
₁₇₅₁ momentum and direction, as given by Table 6.5. The distribution of the reconstructed
₁₇₅₂ lepton momentum (for samples that only have one bin reconstructed zenith angle)
₁₇₅₃ and reconstructed direction for each atmospheric sample used within this analysis is
₁₇₅₄ illustrated in Figure 6.2. The by-mode breakdown of each of the atmospheric samples
₁₇₅₅ is given in Appendix A.

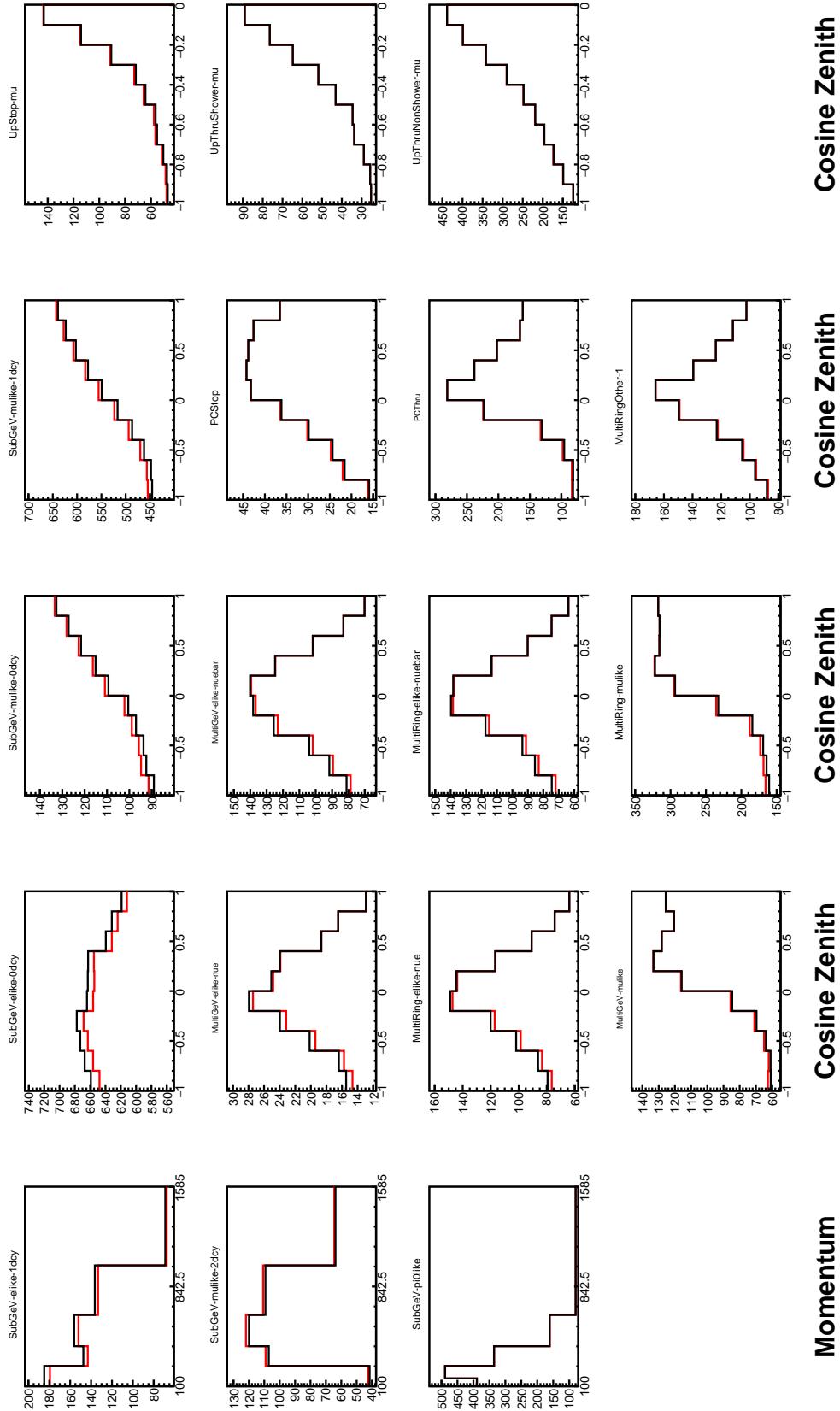


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV-elike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-elike-1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-pi0like	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV-elike-nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-elike-nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-mulike	10	3.0, 3.4, 5.0
MultiRing-elike-nue	10	3.0, 3.4, 3.7, 5.0
MultiRing-elike-nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing-mulike	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing-Other1	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table 6.5.: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples. **DB: Does this belong in the appendix?**

1756 6.2. Near Detector Beam Samples

1757 The near detector sample selections are documented in detail within [195] and sum-
1758 marised below. Samples are selected based upon the which Fine Grained Detector
1759 (FGD) that the vertex is reconstructed in as well as the operating mode of the beam:
1760 FHC or RHC. For additional constraints on model parameters, wrong-sign neutrino
1761 samples are also considered when the beam is operating in RHC mode. Samples
1762 from the wrong-sign component of the FHC beam mode are not included as they are
1763 statistically insignificant compared to those samples already listed.

1764 **The below paragraph has been added** The reconstruction algorithm uses a cluster-
1765 ing algorithm to group hits within the TPC. It then adds information from the upstream
1766 FGD to form a track which passes through both sub-detectors. In FHC(RHC), the
1767 highest momentum negative(positive) curvature track is defined as the muon candi-
1768 date. Before being assigned a sample, these candidate muon events must undergo
1769 CC-inclusive cuts, as defined in [196]:

- 1770 • Event Timing: The DAQ must be operational and the event must occur within
1771 the expected beam time window consistent with the beam spill
- 1772 • TPC Requirement: The muon-candidate track path must intercept one or more
1773 TPCs
- 1774 • Fiducial volume: The event must originate from within the fiducial volume. The
1775 fiducial volumes are defined as a region within each sub-detector [197].
- 1776 • Upstream Background: Remove events that have muons tracks that originate
1777 upstream of the FGDs by requiring no high-momentum tracks within 150mm
1778 upstream of the candidate vertex. Additionally, events that occur within the
1779 downstream FGD are vetoed if a secondary track starts within the upstream FGD

- 1780 • Broken track removal: All candidates where the muon candidate is broken in two
- 1781 are removed
- 1782 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-like
- 1783 events using a likelihood cut

1784 In addition to these cuts, RHC neutrino events also have to undergo the following

1785 cuts to aid in the separation of neutrino and antineutrino [198]:

- 1786 • TPC Requirement: The track path must intercept TPC2
- 1787 • Positive Track: The highest momentum track must be positive
- 1788 • TPC1 Veto: Remove any events originating upstream of TPC1

1789 Once all CC-inclusive events have been determined, they are further separated into

1790 sub-samples that target the constraints on interaction modes most relevant at the far
 1791 detector. They are split by pion multiplicity: CC0 π , CC1 π , and CCOther. These target
 1792 specific interaction modes CCQE, CCRES, and other CC background interactions,
 1793 respectively. Pions in the TPCs and FGDs are selected by requiring a second track to be
 1794 observed, which is separate from the muon track and is in the same beam spill window
 1795 and sub-detector. If the pion originated within a FGD, it must also pass through the
 1796 sequential downstream TPC (TPC2 for FGD1, TPC3 for FGD2).

1797 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

- 1798 • ν_μ **CC0 π Selection:** No electrons in TPC and no charged pions or decay electrons
 1799 within the TPC or FGD
- 1800 • ν_μ **CC1 π Selection:** Exactly one charged pion in either the TPC or FGD, where
 1801 the number of charged pions in the FGD is equal to the number of decay electrons

- 1802 • ν_μ CCOther Selection: All events which are not classified into the above two
1803 selections.

1804 Counting the three selections for each FGD in FHC and RHC running, including
1805 the wrong-sign background in RHC, 18 near detector samples are used within this
1806 analysis. These samples are binned in reconstructed lepton momentum (illustrated in
1807 Figure 6.3) and direction with respect to the beam. The binning is chosen such that
1808 each event has at least 20 Monte Carlo events in each bin [197]. This is to ensure that
1809 the bins are coarse enough to ensure the reduction of statistical errors, whilst also
1810 being fine enough to sample the high-resolution peak regions. The exact binning is
1811 detailed in [197].

1812 6.3. Far Detector Beam Samples

1813 The beam neutrino events which occur at the SK detector, which pass the reduction cuts
1814 detailed in section 5.3, are separated depending on whether the beam was operating
1815 in FH or RHC mode. The events are then separated into three samples: electron-like
1816 ($1Re$), muon-like ($1R\mu$), and CC $1\pi^+$ -like ($1Re1de$) which are observed as electron-like
1817 events with an associated decay electron [185]. As discussed in section 6.1, positively
1818 charged pions emitted from neutrino interactions are more likely to produce decay
1819 electrons than negatively charged pions. Consequently, the CC $1\pi^+$ -like sample is only
1820 selected when the beam is operating in FHC mode. Therefore, five beam samples
1821 measured at SK are used in this analysis.

1822 The fiducial volume definition for beam samples is slightly different from that used
1823 within the atmospheric samples. It uses both the distance to the closest wall (d_{Wall})
1824 and the distance to the wall along the trajectory of the particle (to_{Wall}). This allows
1825 events that originate close to the wall but are facing into the tank to be included within

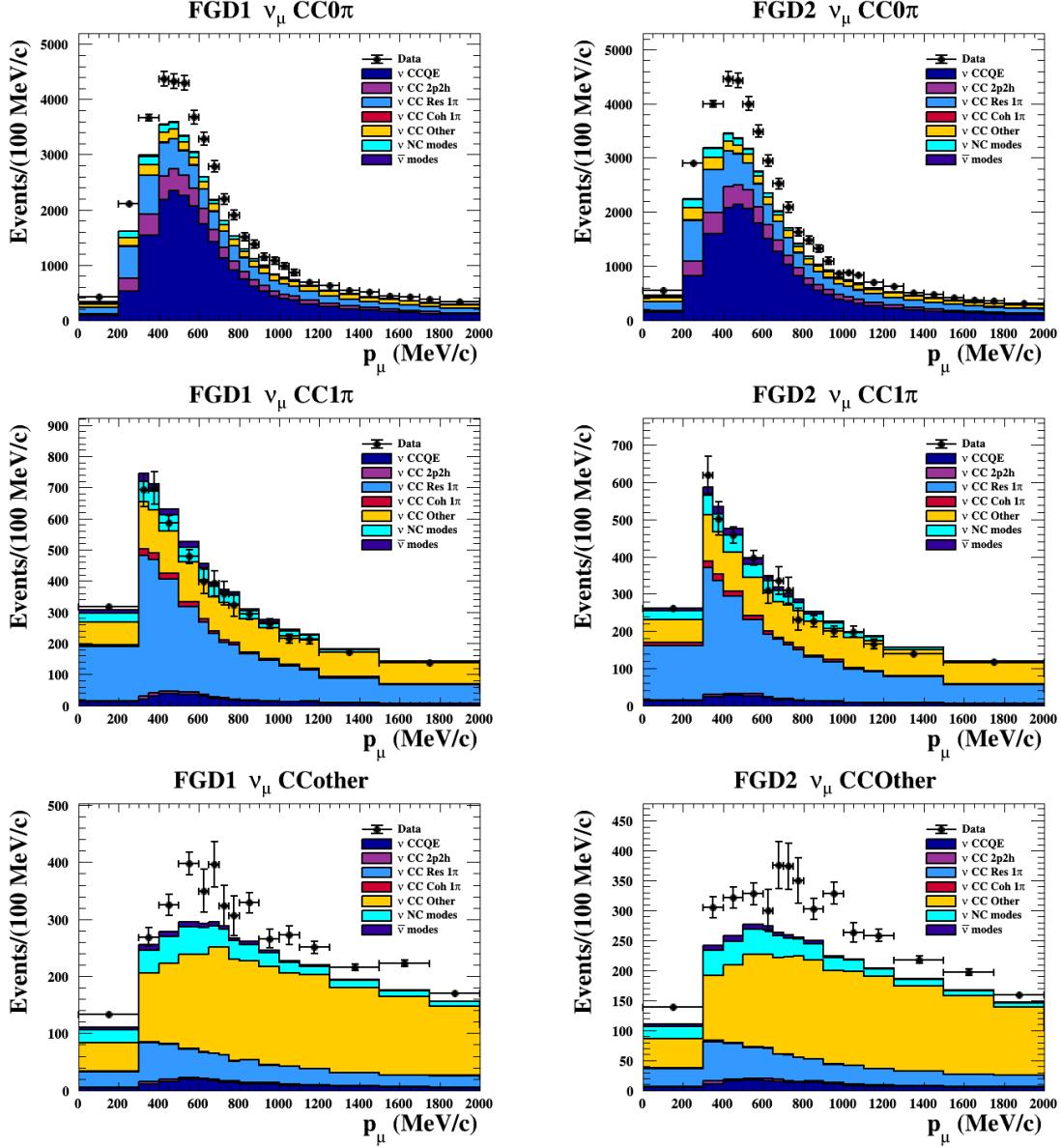


Figure 6.3.: The nominal Monte Carlo predictions for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $\text{CC}\nu_\mu 0\pi$, $\text{CC}\nu_\mu 1\pi$ and $\text{CC}\nu_\mu$ Other categories. Figures taken from [195].

the analysis, which would have otherwise been removed. These additional events are beneficial for a statistics-limited experiment. The exact cut values for both dWall and toWall are different for each of the three types of sample and are optimised based on T2K sensitivity to δ_{CP} [183, 199]. They are:

DB: Diagram of dWall and toWall needed?

1831 **1Re event selection** For an event to be classified as a 1Re-like, the event must follow:

- 1832 • Fully-contained and within $d_{\text{Wall}} > 80\text{cm}$ and $t_{\text{toWall}} > 170\text{cm}$
- 1833 • Total of one ring which is reconstructed as electron-like with reconstructed momentum $P_e > 100\text{MeV}$
- 1834 • Zero decay electrons are associated with the event
- 1835 • Passes π^0 rejection cut discussed in section 5.2

1837 The zero decay electron cut specifically targets CCQE interactions. Whereas, the π^0 1838 rejection cut is designed to remove neutral current π^0 background events which can 1839 be easily reconstructed as 1Re-like events.

1840 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re sample. 1841 The only difference is that the d_{Wall} and t_{toWall} criteria are changed to $> 50\text{cm}$ and 1842 $> 270\text{cm}$, respectively. Furthermore, exactly one decay electron is required from the 1843 π^+ decay.

1844 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

- 1845 • Fully-contained and within $d_{\text{Wall}} > 50\text{cm}$ and $t_{\text{toWall}} > 250\text{cm}$
- 1846 • Total of one ring which is reconstructed as muon-like with reconstructed momentum $P_\mu > 200\text{MeV}$
- 1847 • Fewer than two decay electrons are associated with the event
- 1849 • Passes π^+ rejection cut discussed in section 5.2

1850 As pions and muons have similar masses, the Cherenkov rings they generate have
1851 similar opening angles. To enhance the purity, the events have to pass the π^+ rejection
1852 cut which is specifically optimised to separate the two types of events.

1853 All of these samples are binned in reconstructed neutrino energy. This is possible
1854 as the direction from the source is known extremely well. This value is calculated for
1855 the 1Re-like and 1R μ -like samples assuming CCQE interactions,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})} \quad (6.2)$$

1856 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing lepton,
1857 respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nuclei [185], θ_{beam} is
1858 the angle between the beam and the direction of the outgoing lepton, and E_l and P_l
1859 are the energy and momentum of that outgoing lepton.

1860 The reconstructed neutrino energy of the CC1 π^+ -like events is modified to include
1861 the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))} \quad (6.3)$$

1862 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
1863 considered as a two-body process with the delta baryon is assumed. This follows
1864 the T2K oscillation analysis presented in [80], although recent developments of the
1865 interaction model in the latest T2K oscillation analysis do include effects from binding
1866 energy in this calculation [200].

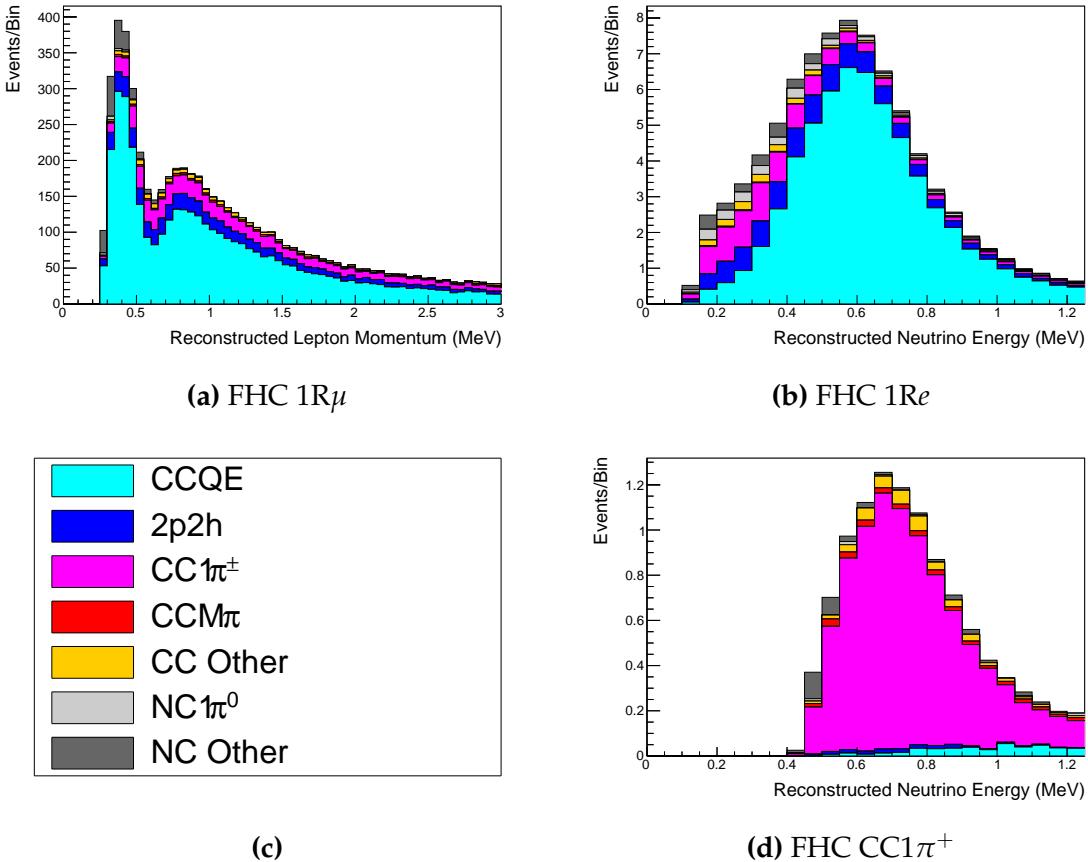


Figure 6.4.: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1Re-like and CC1 π^+ -like samples. Asimov A oscillation parameter sets are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0. \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV.

The reconstructed neutrino energy for the FHC samples is illustrated in Figure 6.4.

As expected, the 1R μ -like and 1Re-like samples are heavily dominated by CCQE interactions, with smaller contributions from 2p2h meson exchange and resonant pion production interactions. The CC1 π^+ -like sample predominantly consists of charged current resonant pion production interactions. The 1Re-like and CC1 π^+ -like samples are also binned by the angle between the neutrino beam and the reconstructed lepton momentum. This is to aid in charged current and neutral current separation, as indicated in Figure 6.5. This is because the neutral current backgrounds are predominantly due to π^0 -decays, where the opening angle of the two gammas alongside the different

- ¹⁸⁷⁶ final state kinematics produces a slightly broader angular distribution compared to
¹⁸⁷⁷ the final state particles originating from charged current ν_e interactions.

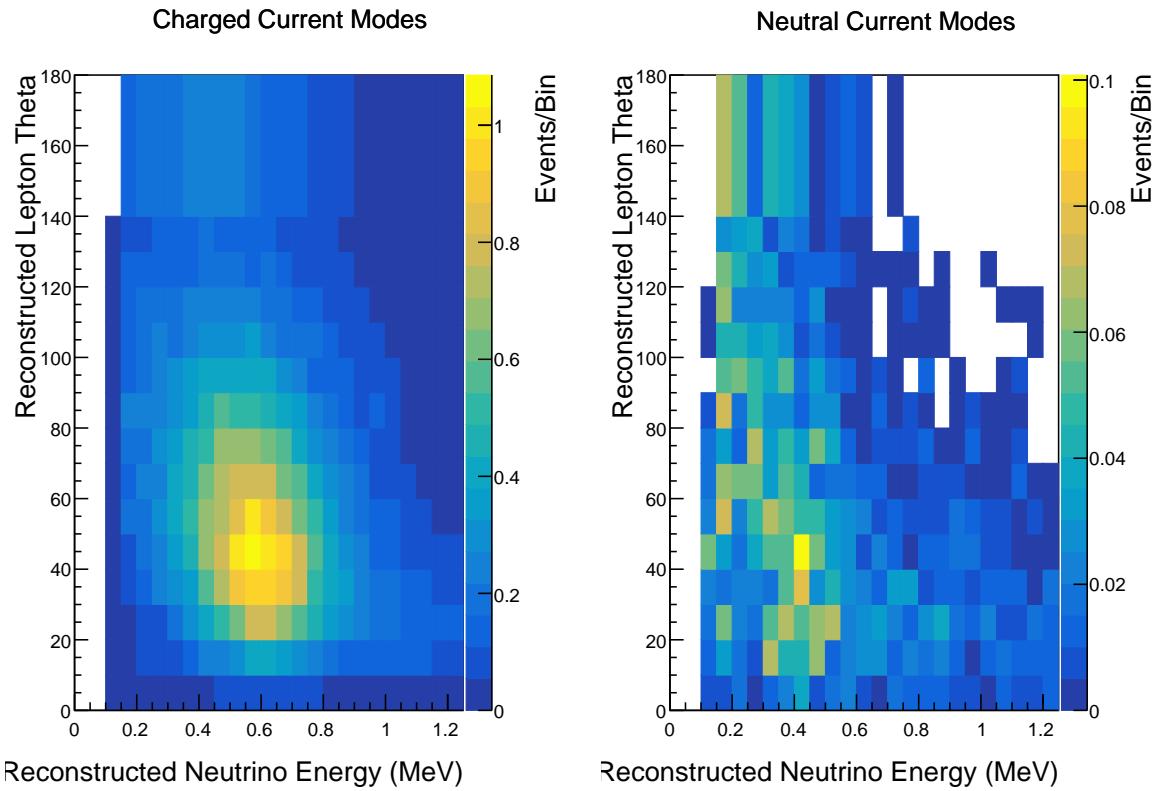


Figure 6.5.: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). **DB: Is this needed or will it just bring up more questions?**

¹⁸⁷⁸ 6.4. Systematic Uncertainties

¹⁸⁷⁹ The systematic model parameters for this analysis are split into groups, or blocks,
¹⁸⁸⁰ depending on their purpose. They consist of flux uncertainties, neutrino-matter
¹⁸⁸¹ interaction systematics, and detector efficiencies. There are also uncertainties on the
¹⁸⁸² oscillation parameters which this analysis will not be sensitive to, Δm_{12}^2 and $\sin^2(\theta_{12})$.
¹⁸⁸³ These uncertainties are taken from the 2018 PDG measurements [81]. As described in
¹⁸⁸⁴ chapter 4, each model parameter used within this analysis requires a prior uncertainty.
¹⁸⁸⁵ This is provided via separate covariance matrices for each block. The covariance
¹⁸⁸⁶ matrices can include prior correlations between parameters within a single block, but
¹⁸⁸⁷ the separate treatment means prior uncertainties can not be included for parameters
¹⁸⁸⁸ in different groups. Some parameters in these models have no reasonably motivated
¹⁸⁸⁹ uncertainties and are assigned flat priors which do not modify the likelihood penalty.
¹⁸⁹⁰ The flux, neutrino interaction, and detector modeling simulations have already been
¹⁸⁹¹ discussed in section 5.1 and section 5.2. The uncertainties invoked within each of these
¹⁸⁹² models are described below.

¹⁸⁹³ 6.4.1. Beam Flux

¹⁸⁹⁴ The neutrino beam flux systematics is based upon the uncertainty in the modeling of
¹⁸⁹⁵ the components of the beam. This includes the hadron production model and their re-
¹⁸⁹⁶ interactions, the shape, intensity, and alignment of the beam with respect to the target,
¹⁸⁹⁷ and the uniformity of the magnetic field produced by the horn, alongside other effects.
¹⁸⁹⁸ The uncertainty, as a function of neutrino energy, is illustrated in Figure 6.6 which
¹⁸⁹⁹ includes a depiction of the total uncertainty as well as the contribution from individual
¹⁹⁰⁰ components. The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$)

¹⁹⁰¹ is dominated by the measurements of the beam profile and alignment. Outside of this
¹⁹⁰² region, the uncertainties within hadron production dominate the uncertainty.

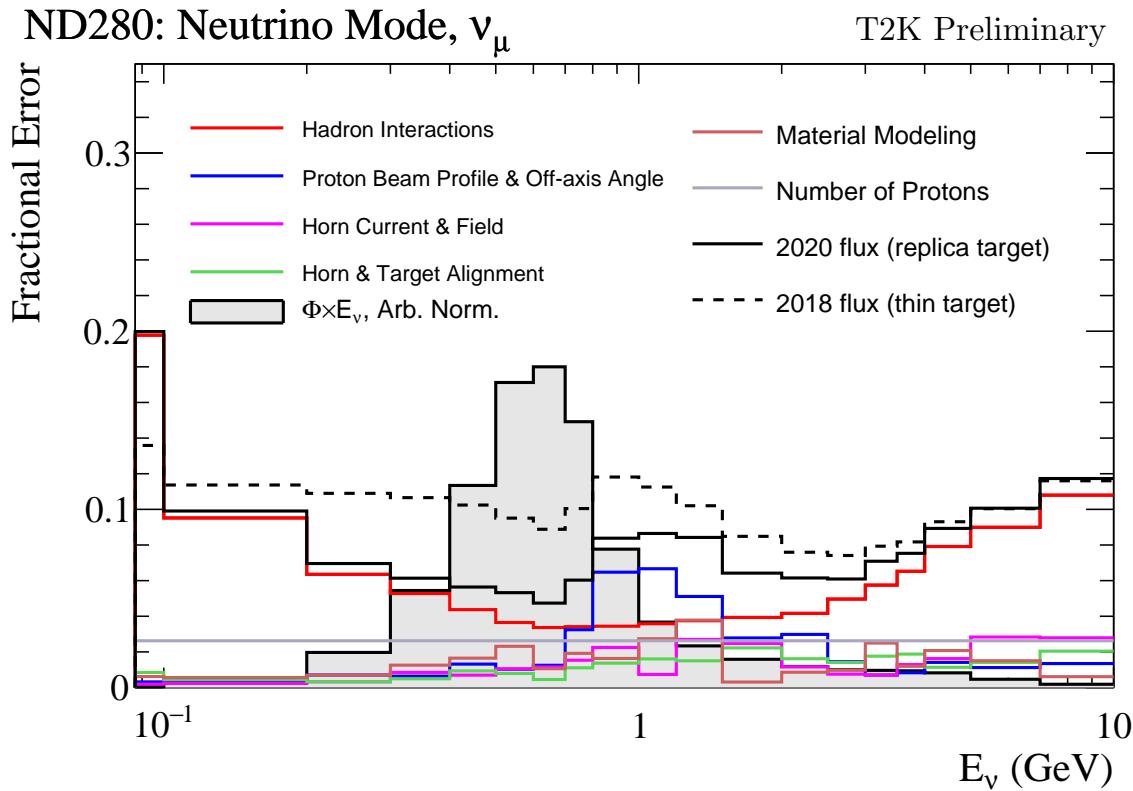


Figure 6.6.: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [201]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [202].

¹⁹⁰³ The beam flux uncertainties are described by one hundred parameters. They are
¹⁹⁰⁴ split between the ND280 and SK detectors and binned by neutrino flavour: ν_μ , $\bar{\nu}_\mu$, ν_e
¹⁹⁰⁵ and $\bar{\nu}_e$. The response is then broken down as a function of neutrino energy. The bin
¹⁹⁰⁶ density in the neutrino energy is the same for the ν_μ in FHC and $\bar{\nu}_\mu$ in RHC beams,
¹⁹⁰⁷ and narrows for neutrino energies close to the oscillation maxima of $E_\nu = 0.6\text{GeV}$.
¹⁹⁰⁸ This binning is specified in Table 6.6. All of these systematic uncertainties are applied
¹⁹⁰⁹ as normalisation parameters with Gaussian priors centered at 1.0 and error specified
¹⁹¹⁰ from a covariance matrix provided by the T2K beam group [202].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.6.: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

1911 6.4.2. Atmospheric Flux

1912 The atmospheric neutrino flux is modeled by the HKKM model [43]. 16 systematic
 1913 uncertainties are applied to control the normalisation of each neutrino flavour, energy,
 1914 and direction. All of the parameters are given Gaussian priors centered at 0 and width
 1915 equal to one. They are summarised below:

- 1916 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour is
 1917 controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$ and $E_\nu >$
 1918 1GeV , respectively. This is driven mostly by hadronic interaction uncertainties for
 1919 the production of pions and kaons [43]. The strength of the response is dependent
 1920 upon the neutrino energy.
- 1921 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$ are
 1922 controlled by the difference between the HKKM model [43], FLUKA [46] and
 1923 Bartol models [42]. Three independent parameters are applied in the energy
 1924 ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 1925 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent) pro-
 1926 duction uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using the same
 1927 methodology as the relative normalisation parameters.

1928 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two systematics,
1929 the difference between the HKKM, FLUKA, and Bartol model predictions, as a
1930 function of $\cos(\theta_Z)$, is used to control the normalisation of events as a function of
1931 zenith angle.

1932 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) become dependent upon
1933 kaon decay as the dominant source of neutrinos. Measurements of the ratio of
1934 K/π [203] are used to control the systematic uncertainty of the expected ratio of
1935 pion and kaon production.

1936 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic field,
1937 the flux of primary cosmic rays varies across the same period. The uncertainty is
1938 calculated by taking a ± 1 year variation, equating to a 10% uncertainty for the
1939 SK-IV period.

1940 • **Atmospheric Density:** The height of the interaction of the primary cosmic rays is
1941 dependent upon the atmospheric density. The HKKM assumes the US standard
1942 1976 [153] profile. This systematic controls the uncertainty in that model.

1943 Updates to the HKKM and Bartol models are underway [158] to use a similar
1944 tuning technique to that used in the beam flux predictions. After those updates, it may
1945 be possible to include correlations in the hadron production uncertainty systematics
1946 for beam and atmospheric flux predictions.

1947 6.4.3. Neutrino Interaction

1948 The neutrino interactions which occur within all the detectors are modeled by NEUT.
1949 The two independent oscillation analyses, T2K-only [204] and the SK-only [52], have
1950 developed separate interaction models. To leverage the most sensitivity out of this
1951 simultaneous beam and atmospheric analysis, a correlated interaction model has been

1952 defined. Where applicable, correlations allow the systematic uncertainties applied to
1953 the atmospheric samples to be constrained by measurements of the near detector in
1954 the beam experiment. This can lead to stronger sensitivity to oscillation parameters as
1955 compared to an uncorrelated model.

1956 The low energy T2K systematic model has a more sophisticated treatment of CCQE,
1957 CCMEC, and CCRES uncertainties which is due to the purpose-made cross-section
1958 measurements made by the near detector. Furthermore, extensive comparisons of this
1959 model have been performed to external data [204]. However, the model is not designed
1960 for high-energy atmospheric events, like those illustrated in Figure 5.11. Therefore
1961 the high energy systematic model from the SK-only analysis is implemented for the
1962 relevant multiGeV, PC, and up- μ samples. The CCQE systematic parameters invoked
1963 within the SK high energy model are actually contained within T2K's CCQE model.
1964 Consequently, the more sophisticated CCQE and CCMEC T2K model parameters
1965 have been incorporated into the high energy model but are uncorrelated from the low
1966 energy counterparts.

1967 The high energy systematic model includes parameters developed from com-
1968 parisons of Nieves and Rein-Seghal models which affect resonant pion producing
1969 interactions, comparisons of the GRV98 and CKMT models which control DIS interac-
1970 tions, and hadron multiplicity measurements which modulate the normalisation of
1971 multi-pion producing events. The uncertainty of the ν_τ cross-section is particularly
1972 large and is controlled by a 25% normalisation uncertainty. These parameters are
1973 applied via normalisation or shape parameters. The former linearly scales the weight
1974 of all affected Monte-Carlo events, whereas the latter can increase or decrease a partic-
1975 ular event's weight depending on its neutrino energy and mode of interaction. The
1976 response of the shape parameters is defined by third-order polynomial splines which
1977 return a weight for a particular neutrino energy. To reduce computational resources

for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.7 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.8 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

The T2K systematic model [204] is applied in a similar methodology to the SK model parameters. It consists of 19 shape parameters and 24 normalisation parameters. Four additional parameters, which model the uncertainty in the binding energy, are applied in a way to shift the momentum of the lepton emitted from a nucleus. This controls the uncertainty specified on the 27MeV binding energy assumed within Equation 6.2.

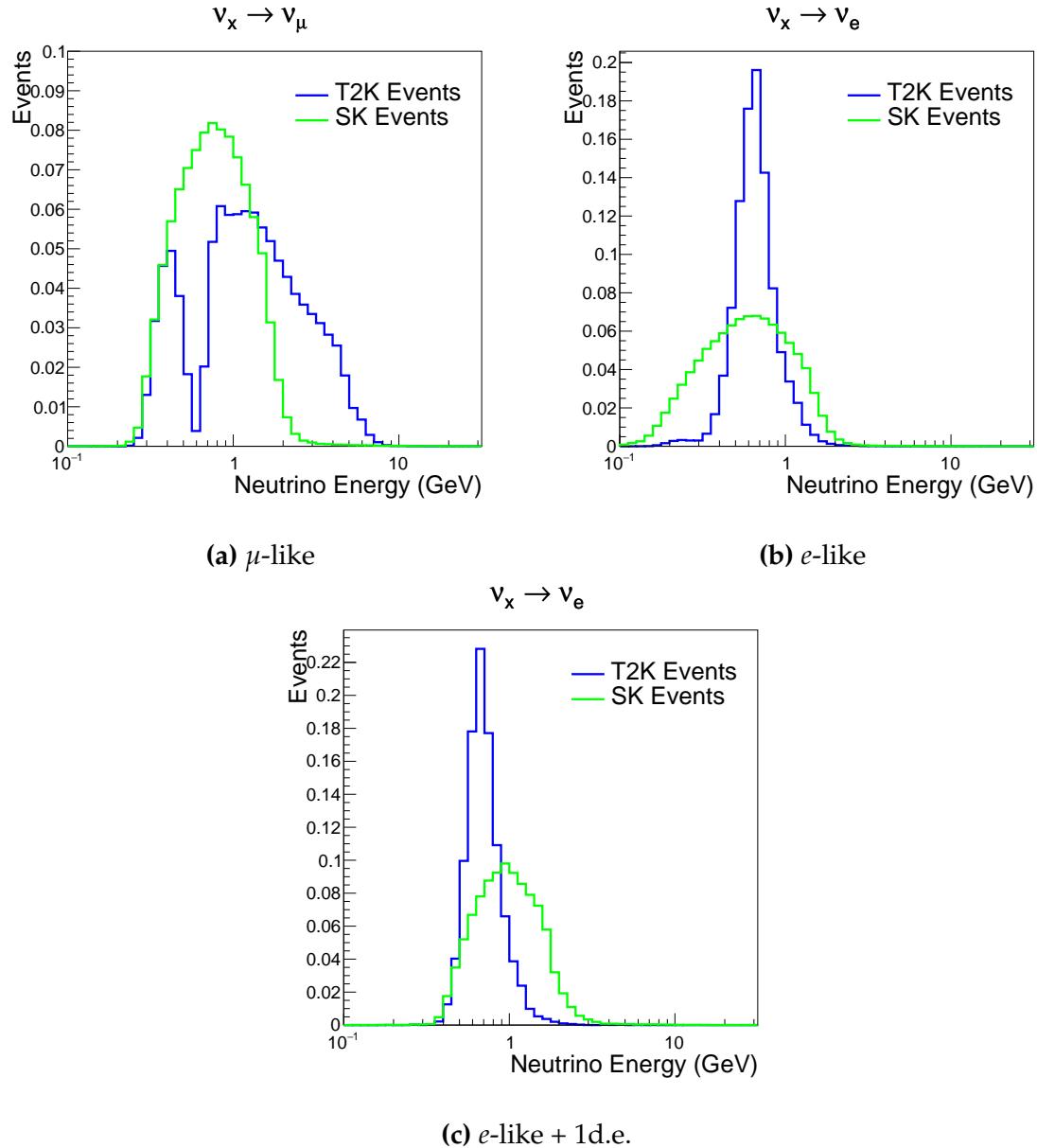


Figure 6.7.: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

²⁰⁰⁴ The majority of these parameters are assigned a Gaussian prior uncertainty. Those
²⁰⁰⁵ that have no reasonably motivated uncertainty, or those which have not been fit to
²⁰⁰⁶ external data, are assigned a flat prior which does not affect the penalty term.

2007 There are three particular tunes of the T2K flux and low energy cross section model
2008 typically considered. Firstly, the “generated” tune which is the set of dial values with

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC π^0	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.8.: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

which the Monte Carlo was generated. Secondly, the set of dial values which are taken from external data measurements and used as inputs. These are the “pre-fit” dial values. The reason these two sets of dial values are different is that the external data measurements are continually updated but it is very computationally intensive to regenerate a Monte Carlo prediction after each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial values. These are the values taken from a fit to the beam near detector data. This fit is performed by two independent fitting frameworks, MaCh3 and BANFF, which ensures reliable measurements. The output of each fitter is converted into a covariance matrix to describe the error and correlations between all the flux and cross-section parameters. This is then propagated to the far-detector

2019 oscillation analysis group for use in the P-Theta fitting framework. As MaCh3 can
2020 perform a near detector fit, it is included within the simultaneous fit of the far-detector
2021 beam and atmospheric oscillation analysis. This is because this technique does not
2022 require any assumption of Gaussian posterior distributions which is required in the
2023 covariance matrix methodology.

2024 On top of the combination of the SK and T2K interaction models, several other
2025 parameters have been specifically developed for the joint oscillation analysis. The
2026 majority of the atmospheric samples' δ_{CP} sensitivity comes from the normalisation
2027 of subGeV electron-like events. These are modeled using a spectral function model
2028 to approximate the nuclear ground state. However, the near detector is not able to
2029 constrain the model. Therefore, an additional systematic is introduced which models
2030 an alternative Continous Random Phase Approximation (CRPA) nuclear ground
2031 state. The reasoning is documented in [205]. As the near detector can not sufficiently
2032 constrain the model, this dial approximates the event weights if a CRPA model had
2033 been assumed rather than a spectral function. This dial only effects ν_e and $\bar{\nu}_e$ and is
2034 applied as a shape parameter.

2035 Further additions to the model have been introduced due to the inclusion of the
2036 subGeV π^0 atmospheric sample. This particularly targets charged current and neutral
2037 current π^0 producing interactions to help constrain the systematic uncertainties. There
2038 is no analogous sample in the T2K beam-only analysis so no significant effort has
2039 been placed into building a sufficient uncertainty model. Therefore, an uncertainty
2040 that affects neutral current resonant π^0 production is incorporated into this analysis.
2041 Comparisons of NEUT's NC resonant pion production predictions have been made to
2042 MiniBooNE [206] data and a consistent 16% to 21% underprediction is observed [205].
2043 Consequently, a conservative 30% normalisation parameter is invoked.

2044 Down-going events are mostly insensitive to oscillation parameters and can act
 2045 similar to the near detector within an accelerator experiment (Details will be dis-
 2046 cussed in chapter 7). This region of phase space can act as a sideband and allows the
 2047 cross-section model and near detector constraint to be studied. The distribution of
 2048 events in this region is calculated using the technique outlined in subsection 4.3.4. For
 2049 CCQE-targeting samples, the application of the near detector constraint is well within
 2050 the statistical fluctuation of the down-going data such that no significant tension is
 2051 observed between the data and the Monte Carlo prediction after the near detector
 2052 constraint is applied. This is not the case for samples with target CCRES interac-
 2053 tions. The electron-like data is consistent with the constrained prediction at high
 2054 reconstructed momenta but diverges at lower momentum, whereas the muon-like
 2055 sample is under-predicted throughout the range of momenta. To combat this disagree-
 2056 ment, an additional cross-section systematic dial, specifically designed to inflate the
 2057 low pion momentum systematics was developed in [205]. This is a shape parameter
 2058 implemented through a splined response.

2059 6.4.4. Near Detector

2060 The systematics applied due to uncertainties arising from the response of the near
 2061 detector is documented in [133]. The response is described by 574 normalisation param-
 2062 eters binned in the selected sample as well as momentum and angle, P_μ and $\cos(\theta_\mu)$,
 2063 of the final-state muon. These are applied via a covariance matrix with each parameter
 2064 being assigned a Gaussian prior from that covariance matrix. These normalisation
 2065 parameters are built from underlying systematics, e.g. pion secondary interaction
 2066 systematics, which are randomly thrown and the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is
 2067 determined. Two thousand throws are evaluated and a covariance matrix response is
 2068 created. This allows significant correlations between FGD1 and FGD2 samples, as well

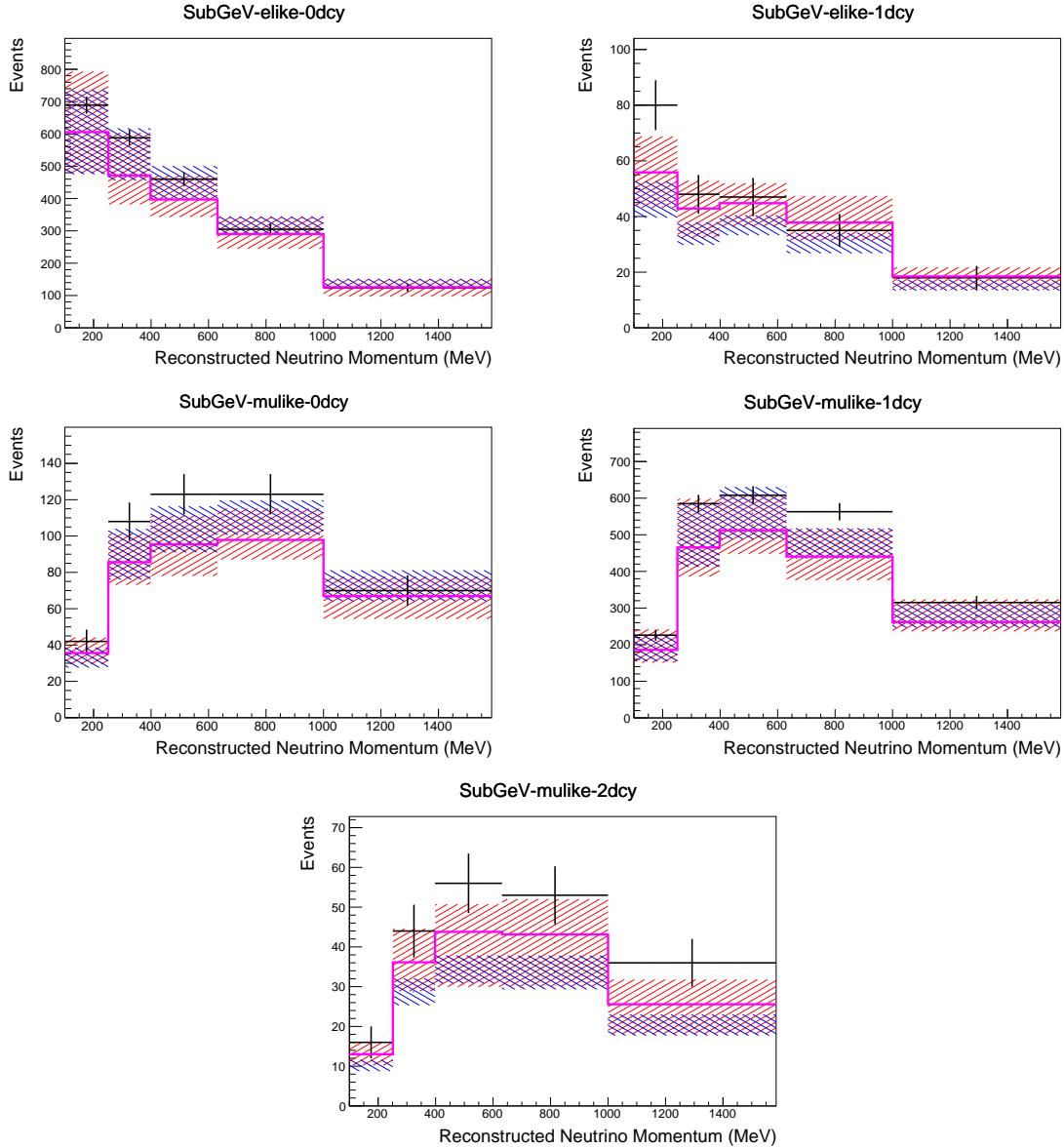


Figure 6.9.: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

as adjacent $P_\mu \times \cos(\theta_\mu)$ bins. Statistical uncertainties are accounted for by including fluctuations of each event's weight from a Poisson distribution.

2071 Similar to the cross-section systematics, MaCh3 and BANFF are used to constrain
2072 the uncertainty of these systematics through independent validations. Each fitter
2073 generates a post-fit covariance matrix which is compared and passed to the far-detector
2074 oscillation analysis working group. As the analysis presented within this thesis uses
2075 the MaCh3 framework, a joint oscillation analysis fit of all three sets of samples and
2076 their respective systematics is performed.

2077 **6.4.5. Far Detector**

2078 Two configurations of the far detector systematic model implementation have been
2079 considered. Firstly, the far detector systematic uncertainties for beam and atmospheric
2080 samples are taken from their respective analysis inputs, denoted “official inputs” anal-
2081 ysis. Consequently, no correlations are assumed between the beam and atmospheric
2082 samples. The generation of the beam- and atmospheric-specific inputs are documented
2083 in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, a correlated detector
2084 model has been developed. Here, the distribution of parameters used for applying
2085 event cuts (e.g. electron-muon PID separation) is modified within the fit. It follows
2086 a similar methodology to the beam far detector systematics implementation but per-
2087 forms a joint fit of the beam and atmospheric data. This alternative implementation is
2088 detailed in subsubsection 6.4.5.3.

2089 **6.4.5.1. Beam Samples**

2090 There are 45 systematics which describe the response of the far detector to beam
2091 events [185], split into 44 normalisation parameters and one energy scale systematic.
2092 The energy scale systematic is applied as a multiplicative scaling of the reconstructed
2093 neutrino energy. It is described by a Gaussian, centered at one with equal to the

2094 difference in Monte Carlo to data comparisons performed in [187]. The normalisation
 2095 parameters are assigned a Gaussian error centralised at one with width taken from a
 2096 covariance matrix. A detailed breakdown of the generation of the covariance matrix
 2097 is found in [199]. To build the covariance matrix, a fit is performed on atmospheric
 2098 data which has been selected using beam sample selection cuts. These cuts use the
 2099 variables, L^i , where the index i is detailed in Table 6.7. Each L^i is a smear, α , and shift,
 2100 β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i \quad (6.4)$$

2101 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in Table 6.7.
 2102 The shift and smear parameters are binned by final-state topology, j , where the binning
 2103 is given in Table 6.8. The final-state topology binning is because the detector will
 2104 respond differently to events that have one or multiple rings. For example, the detector
 2105 will be able to distinguish single-ring events better than two overlapping ring events,
 2106 resulting in smaller systematic uncertainty for one-ring events compared to two-ring
 2107 events. This approach is used to allow the cut parameter distributions to be modified
 2108 within the fit, allowing for better data to Monte Carlo agreement. Only the shape
 2109 of each of the cut variables is used within this fit, such that physics effects are not
 2110 considered.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.7.: List of cut variables that are included within the shift/smear fit documented in [199].

Category	Description
$1e$	Only one electron above Cherenkov threshold in the final state
1μ	Only one muon above Cherenkov threshold in the final state
$1e+other$	One electron and one or more other charged particles above Cherenkov threshold in the final state
$1\mu+other$	One muon and one or more other charged particles above Cherenkov threshold in the final state
$1\pi^0$	Only one π^0 in the final state
$1\pi^\pm$ or $1p$	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.8.: Reconstructed event topology categories on which the SK detector systematics [199] are based.

2111 Beyond the uncertainty on the PID cut criteria, the mis-modeling of π^0 events
 2112 is also considered. If one of the two rings from a π^0 event is missed, this will be
 2113 reconstructed as a $CC\nu_e$ -like event. This is one of the largest systematics hindering the
 2114 electron neutrino appearance analyses. Consequently, additional systematics has been
 2115 introduced to constrain the mis-modeling of π^0 events in SK, binned by reconstructed
 2116 neutrino energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples
 2117 is constructed. These events are built by overlaying one electron-like ring from the
 2118 SK atmospheric neutrino samples or decay electron ring from a stopping cosmic ray
 2119 muon with one simulated photon ring. Both rings are chosen so that momenta and
 2120 opening angle follow the decay kinematics of NC π^0 events from the T2K-MC. Hybrid-
 2121 π^0 Monte Carlo samples with both rings from the SK Monte Carlo are produced
 2122 to compare with the hybrid- π^0 data samples and the difference in the fraction of
 2123 events that pass the ν_e selection criteria is used to assign the systematic errors. In
 2124 order to investigate any data to Monte Carlo differences that may originate from
 2125 either the higher energy ring or lower energy ring, two samples are built; a sample
 2126 in which the electron constitutes the higher energy ring from the π^0 decay called the
 2127 primary sample and another one in which it constitutes the lower energy ring called

2128 the secondary sample. The standard T2K ν_e fiTQun event selection criteria are used to
2129 select events.

2130 Final contributions to the covariance matrix are determined by supplementary
2131 uncertainties attained by comparing stopping muon data to Monte Carlo prediction,
2132 as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated
2133 by the stopping muon data to Monte Carlo differences by comparing the number
2134 of one decay electron events to the number of events with one or fewer decay elec-
2135 trons. Similarly, the rate at which fake decay electrons are reconstructed by fiTQun
2136 is estimated by comparing the number of two decay electron events to the number
2137 of events with one or two reconstructed decay electrons. The two sources of sys-
2138 tematics are added in quadrature weighted by the number of events with one true
2139 decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of
2140 $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events. Additional
2141 normalisation uncertainties based on neutrino flavour and interaction mode are also
2142 defined in [185, 207, 208].

2143 Two additional sources of uncertainty are included: secondary and photo-nuclear
2144 interactions. These are estimated by varying the underlying parameters are building a
2145 distribution of sample event rates. These contributions are then added in quadrature
2146 to the above covariance matrix.

2147 6.4.5.2. Atmospheric Samples

2148 The systematic parameters which control the detector systematics for atmospheric
2149 samples, documented in [89], are split into two sub-groups. Those which are related
2150 to particle identification and ring counting systematics and those which are related to
2151 calibration, separation, and reduction uncertainties.

2152 The particle identification systematics consist of five parameters. The ring separation systematic enforces an anti-correlated response between the single-ring and
 2153 multi-ring samples. This is implemented as a fractional increase/decrease in the overall normalisation of each sample, depending on the distance to the nearest wall
 2154 from an event's vertex. The coefficients of the normalisation are estimated prior to the fit and depend on the particular atmospheric sample. The single-ring and multi-ring
 2155 PID systematics encode the detector's ability to separate electron-like and muon-like
 2156 events and are implemented in a similar way to the ring separation systematic.
 2157

2160 The multi-ring electron-like separation likelihood, discussed in section 6.1, encodes
 2161 the ability of the detector to separate neutrino from anti-neutrino events. As an important
 2162 systematic in the mass hierarchy determination, systematic uncertainties control
 2163 the relative normalisations of the ν_e and $\bar{\nu}_e$ enriched samples. Two normalisation
 2164 parameters are implemented which vary the event rate of each multi-ring sample,
 2165 whilst ensuring the total event rate is conserved.

2166 There are 22 systematics related to calibration measurements, including effects
 2167 from backgrounds, reduction, and showering effects. They are documented in [89] and
 2168 briefly summarised in Table 6.9. They are applied via normalisation parameters, with
 2169 the separation systematics requiring the conservation of event rate across all samples.

2170 6.4.5.3. Correlated Detector Model

2171 A complete uncertainty model of the SK detector would be able to determine the systematic shift on the sample spectra for a variation of the underlying parameters,
 2172 e.g. PMT angular acceptance. However, this is computationally intensive, requiring
 2173 Monte Carlo predictions to be made for each plausible variation. Consequently, an
 2174 effective parameter model has been utilised for a correlated detector model following
 2175 from the T2K-only model implementation documented in subsubsection 6.4.5.1. The
 2176

Table 6.9.: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

²¹⁷⁷ implementation performs a simultaneous fit of detector and oscillation parameters,
²¹⁷⁸ for the detector parameters given in Table 6.7.

²¹⁷⁹ The correlated detector model utilises the same smear and shift parameters docu-
²¹⁸⁰ mented in subsubsection 6.4.5.1, split by final state topology. Beyond this, the shift
²¹⁸¹ and smear parameters are split by visible energy deposited within the detector, with

binning specified in Table 6.10. This is because atmospheric events are categorised by subGeV and multiGeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. This implementation correlates the detector systematics between the far-detector beam and subGeV atmospheric samples due to their similar energies and interaction types. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where k is the visible energy bin. As there are no equivalent beam samples, the multiGeV, multiring, PC, and Up- μ samples will be subject to the ATMPD particle identification systematics implementation as described in subsubsection 6.4.5.2 rather than using this correlated detector model. The calibration systematics also described in the aforementioned chapter still apply to all atmospheric samples.

Index	Range (MeV)
0	$30 \geq x > 300$
1	$300 \geq x > 700$
2	$700 \geq x > 1330$
3	$1330 \geq x$

Table 6.10.: Visible energy binning for which the correlated SK detector systematics are based

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear parameter for that event, and then re-applies event selection. This invokes event migration, which is

2198 a new feature incorporated into the MaCh3 framework which is only achievable due
2199 to the event-by-event reweighting scheme.

2200 Particular care has to be taken when varying the ring counting parameter. This
2201 is because the number of rings is a finite value (one-ring, two-rings, etc.) which can
2202 not be continuously varied through this shift and smear technique. Consequently a
2203 ring counting parameter, RC_i , is calculated for the i^{th} event, following the definition
2204 in [184]. The likelihood from all considered one-ring (L_{1R}) and two-ring (L_{2R}) fits
2205 are compared to determine the preferred hypothesis. The difference is computed as
2206 $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is then defined as,

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

2207 where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an intermedi-
2208 ate likelihood value used within the `fitQun` algorithm to decide the number of rings
2209 associated with a particular event. However, fake-ring merging algorithms are applied
2210 after this likelihood value is used. Consequently, this ring counting parameter does
2211 not always exactly correspond to the number of reconstructed rings. This can be seen
2212 in Figure 6.10.

2213 As the `fitQun` algorithm does not provide a likelihood value after the fake-ring
2214 algorithms have been applied, the ring counting parameter distribution is correlated to
2215 the final number of reconstructed rings through “maps”. These are two-dimensional
2216 distributions of the ring counting parameter and the final number of reconstructed
2217 rings. An example is illustrated in Figure 6.11. In principle, the `fitQun` reconstruction
2218 algorithm should be re-run after the variation in the ring counting parameter. However,

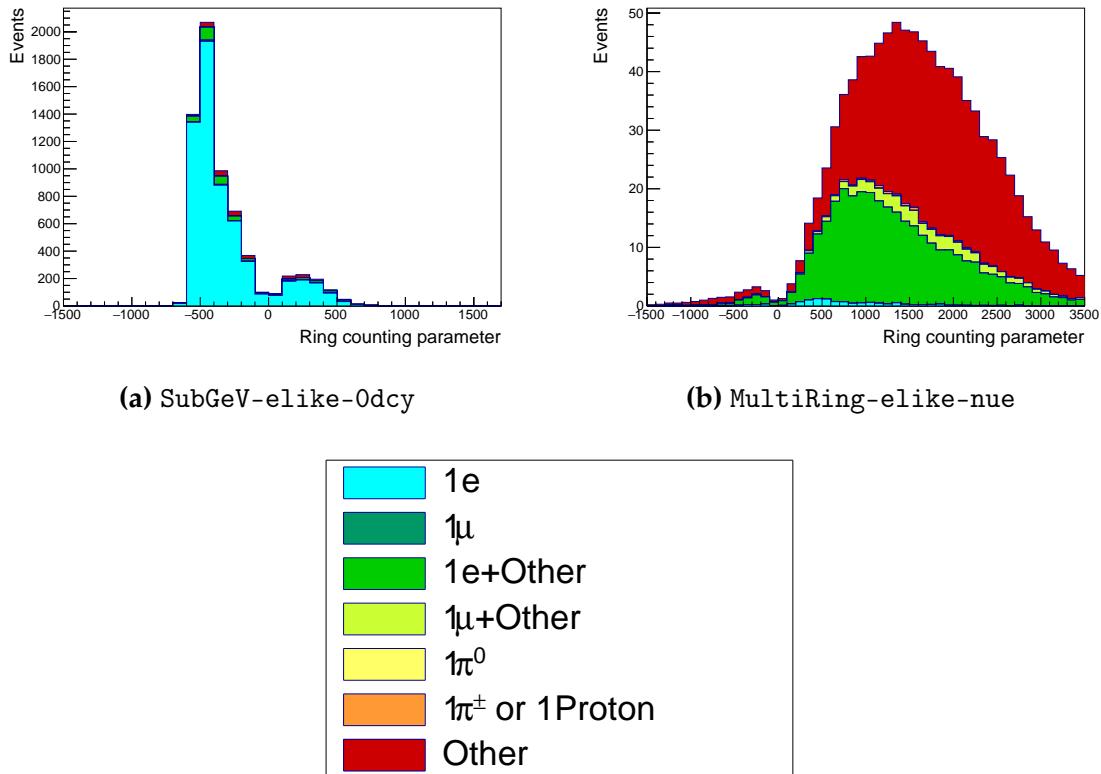


Figure 6.10.: The ring counting parameter as defined in Equation 6.6 for the SubGeV-*elike*-0dcy and MultiRing-*elike*-*nue* samples.

²²¹⁹ this is not computationally viable. Therefore the “maps” are used as a reweighting
²²²⁰ template.

The maps are split by final state topology and true neutrino flavour and all fitQun
 -reconstructed Monte Carlo events are used to fill them. The maps are row-normalised
 to represent the probability of X number of rings for a given RC_i value. Prior to the
 fit, an event's nominal weight is calculated as $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the
 reconstructed number of rings for the i^{th} event and $W^i(x, y)$ is the bin content in map
 associated with the i^{th} event, where x number of rings and y is ring counting parameter.
 Then during the fit, the value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as
 the event weight for the i^{th} event. This is the only cut variable that uses a reweighting
 scheme rather than event migration.

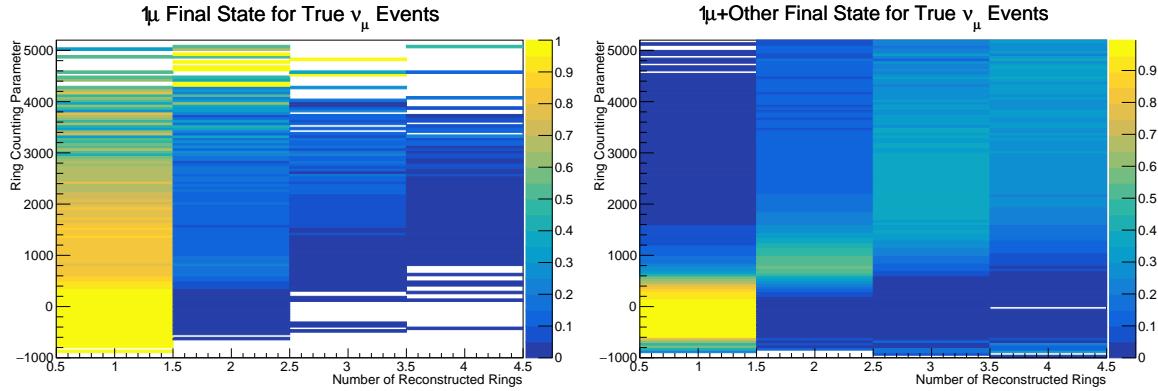


Figure 6.11.: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2230 The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance
 2231 matrix. This is not possible in the alternative model as no covariance matrix is used.
 2232 Thus, the implementation of the π^0 systematics has been modified. The inputs from
 2233 the hybrid π^0 sample is included via the use of “ χ^2 maps”, which are two-dimensional
 2234 histograms in α_{jk}^i and β_{jk}^i parameters over some range. Illustrative examples of the χ^2
 2235 maps are given in Figure 6.12. Due to their nature, the shift and smear parameters are
 2236 typically very correlated. A map is produced for each cut parameter given in Table 6.7
 2237 and for each visible energy bin given in Table 6.10.

2238 The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo and
 2239 data in the particle identification parameters documented in Table 6.7. The Monte
 2240 Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-section and flux
 2241 nuisance parameters are thrown from their prior uncertainties. The χ^2 between the
 2242 scaled Monte Carlo and data is calculated and the relevant point in the χ^2 map is filled.

2243 The implementation within this alternative detector model is to add the bin contents
 2244 of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to the likelihood
 2245 penalty. Only $1\pi^0$ final state topology shift and smear parameters use this prior
 2246 uncertainty.

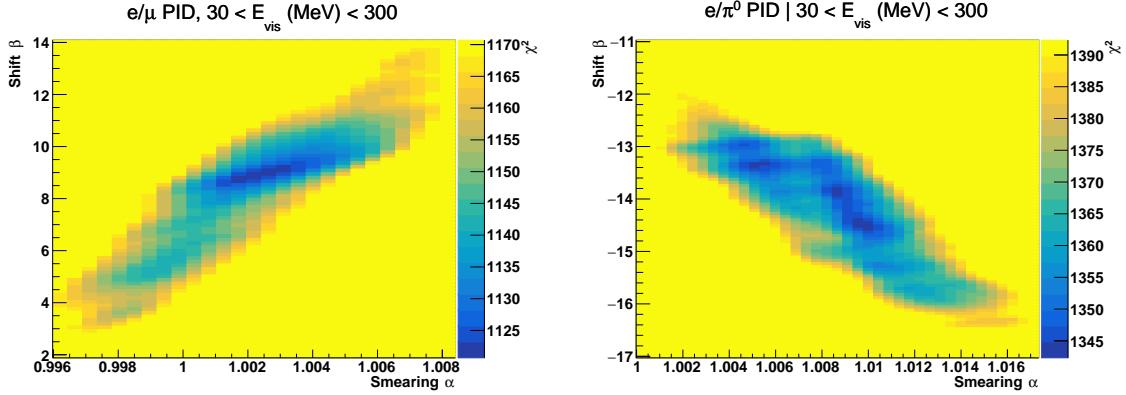


Figure 6.12.: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \geq E_{vis}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \geq E_{vis}(\text{MeV}) < 300$.

2247 Similarly, the implementation of the supplementary systematics documented in
 2248 subsubsection 6.4.5.1 needs to be modified. A new framework [209] was built in
 2249 tandem with the T2K-SK working group [185] so the additional parameters can be
 2250 incorporated into the MaCh3 framework. These are applied as normalisation parame-
 2251 ters, depending on the particular interaction mode, number of tagged decay electrons,
 2252 and whether the primary particle generated Cherenkov light. They are assigned
 2253 Gaussian uncertainties with widths described by a covariance matrix. Furthermore,
 2254 the secondary interaction and photo-nuclear effects need to be accounted for in this
 2255 detector model using a different implementation than that in subsubsection 6.4.5.1.
 2256 This was done by including a shape parameter for each of the secondary interactions
 2257 and the photo-nuclear systematic parameters.

2258 There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior constraints
 2259 from the hybrid π^0 samples.

2260 One final complexity of this correlated detector model is that the two sets of sam-
 2261 ples, beam and subGeV atmospheric, use slightly different parameters to distinguish
 2262 electron and muon-like events. The T2K samples use the value of $\log(L_e/L_\mu)$ whereas
 2263 the atmospheric samples use the value of $\log(L_e/L_\pi)$, where L_X is the likelihood for

2264 hypothesis X. This is because the T2K fits use single-ring `fitQun` fitting techniques,
 2265 whereas multi-ring fits are applied to the atmospheric samples where only the electron
 2266 and pion hypothesis are considered. The correlation between the two likelihood ratios
 2267 is illustrated in Figure 6.13. As discussed in section 5.2, the pion hypothesis is a very
 2268 good approximation of the muon hypothesis due to their similar mass. Consequently,
 2269 using the same shift and smear parameters correlated between the beam and subGeV
 2270 atmospheric samples is deemed a good approximation.

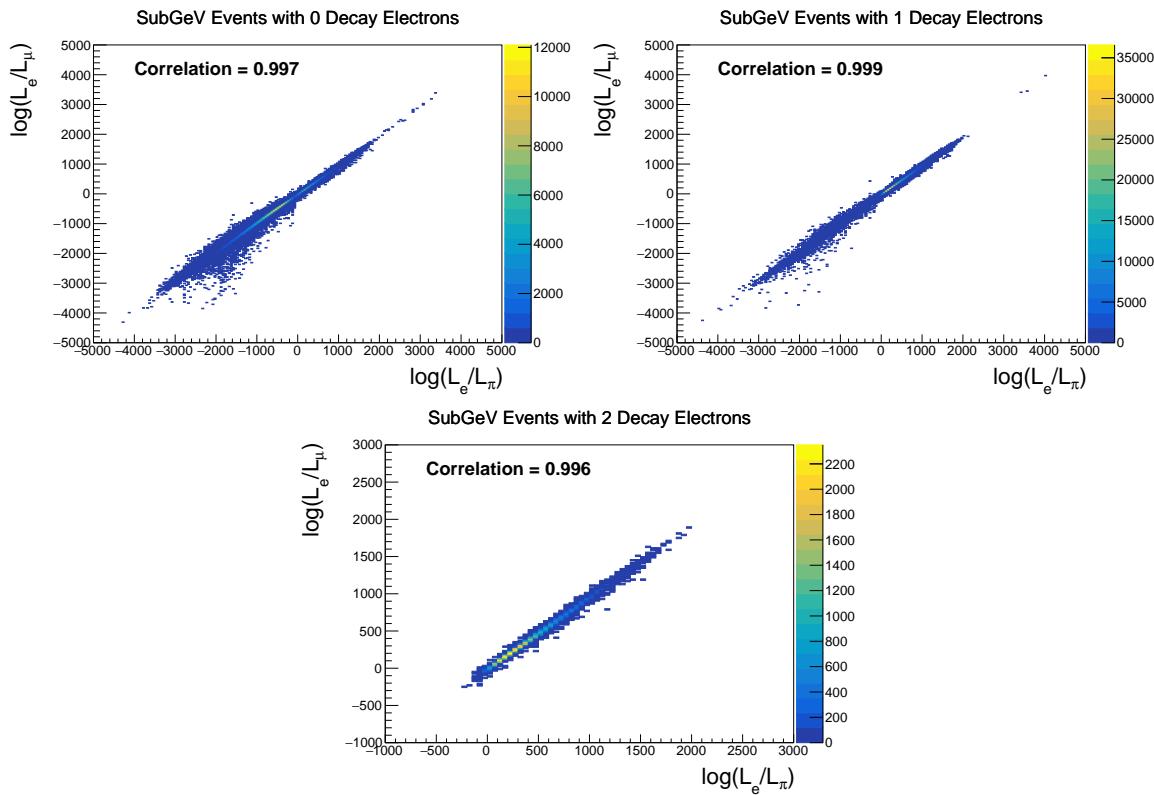


Figure 6.13.: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

2271

Chapter 7

2272

Oscillation Probability Calculation

2273 It is important to understand how and where the sensitivity to the oscillation pa-
2274 rameters comes from for both atmospheric and beam samples. An overview of how
2275 these samples observe changes in δ_{CP} , Δm_{23}^2 , and $\sin^2(\theta_{23})$ is given in section 7.1. It
2276 also explains the additional complexities involved when performing an atmospheric
2277 neutrino analysis as compared to a beam-only analysis.

2278 Without additional techniques, atmospheric sub-GeV upward-going neutrinos
2279 ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to δ_{CP} due to the
2280 quickly varying oscillation probability in this region. Therefore, a “sub-sampling”
2281 approach has been developed to reduce these biases ensuring accurate and reliable
2282 sensitivity measurements. This technique ensures that small-scale unresolvable fea-
2283 tures of the oscillation probability have been averaged over whilst the large-scale
2284 features in the oscillation probability are unaffected. The documentation and valida-
2285 tion of this technique are found in section 7.2. The oscillation probability calculation is
2286 computationally intensive due to the large number of matrix multiplications needed.
2287 Consequently, the CUDAProb3 implementation choice made within the fitting frame-
2288 work, as detailed in section 7.3, ensures that the analysis can be done in a timely
2289 manner.

2290 Whilst the beam neutrinos are assumed to propagate through a constant density
2291 slab of material, the density variations through the Earth result in more complex
2292 oscillation patterns. Furthermore, the uncertainty in the electron density can modify
2293 the oscillation probability for the denser core layers of the Earth. The model of the

2294 Earth used within this analysis is detailed in section 7.4. This includes information
2295 about the official SK-only methodology as well as improvements that can be made
2296 to remove some of the approximations made in that analysis. Another complexity of
2297 atmospheric neutrinos oscillation studies is that the height of production in the atmo-
2298 sphere is not known on an event-by-event basis. An analytical averaging technique
2299 that approximates the uncertainty of the oscillation probability has been followed,
2300 with the author of this thesis being responsible for the implementation and validation.
2301 This implementation of an external technique is illustrated in section 7.5.

2302 7.1. Overview

2303 DB: Should this be moved into an earlier chapter? The selections chapter references
2304 the matter resonance which has not yet been explained at that point

2305 The analysis presented within this thesis focuses on the determination of oscillation
2306 parameters from atmospheric and beam neutrinos. Whilst subject to the same oscil-
2307 lation formalism, the way in which the two samples have sensitivity to the different
2308 oscillation parameters differs quite significantly.

2309 Atmospheric neutrinos have a varying baseline, or “path length”, L , such that
2310 the distance each neutrino travels before interacting is dependent upon the zenith
2311 angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s surface
2312 and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the neutrino was
2313 generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (7.1)$$

2314 Where $R_E = 6,371\text{km}$ is the Earth's radius. Consequently, the oscillation probability
2315 is dependent upon two parameters, $\cos(\theta_Z)$ and E_ν .

2316 The oscillation probability used within this analysis is based on [21]. The neutrino
2317 wavefunction in the vacuum Hamiltonian evolves in each layer of constant matter
2318 density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (7.2)$$

2319 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
2320 energy, G_F is Fermi's constant, N_e is the electron number density and U is the PMNS
2321 matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied for antineutrino
2322 propagation. Thus, a model of the Earth's density is required for atmospheric neutrino
2323 propagation. Following the official SK-only methodology [210], this analysis uses the
2324 Preliminary Reference Earth Model (PREM) [211]. This model provides piecewise cubic
2325 polynomials as a function of the Earth's radius which results in the density profile
2326 illustrated in Figure 7.1. As discussed, the propagator requires layers of constant
2327 density. The SK methodology approximates the PREM model by using four layers of
2328 constant density [210]. The details of these layers are detailed in Table 7.1.

Layer	Outer Radius [km]	Density [g/cm^3]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 7.1.: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [211].

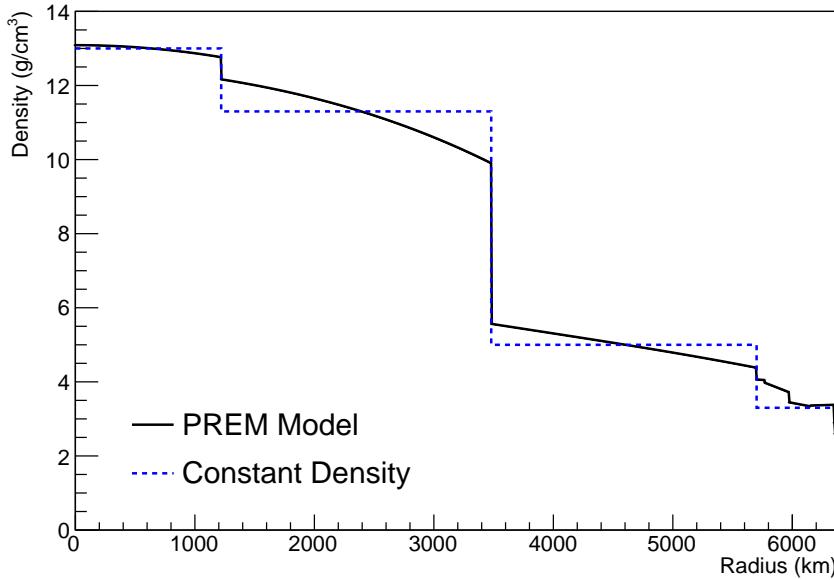


Figure 7.1.: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

2329 The atmospheric neutrino oscillation probabilities can be presented as two dimen-
 2330 sional “oscillograms” as illustrated in Figure 7.2. The distinct discontinuities, as a
 2331 function of $\cos(\theta_Z)$, are due to the discrete change in density invoked within the PREM
 2332 model.

2333 Atmospheric neutrinos do have sensitivity to δ_{CP} through a normalisation term.
 2334 Figure 7.3 illustrates the difference in oscillation probability between CP-conserving
 2335 ($\delta_{CP} = 0$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation
 2336 parameter set (Table 2.2). The result is a complicated oscillation pattern in the appear-
 2337 ance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient
 2338 resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric
 2339 neutrinos comes via the overall normalisation of these events.

2340 The presence of matter means that the effect δ_{CP} has on the oscillation probability
 2341 is not equal between neutrinos and antineutrinos, which would be expected when
 2342 propagating through a vacuum. This is further extenuated by the fact that SK can

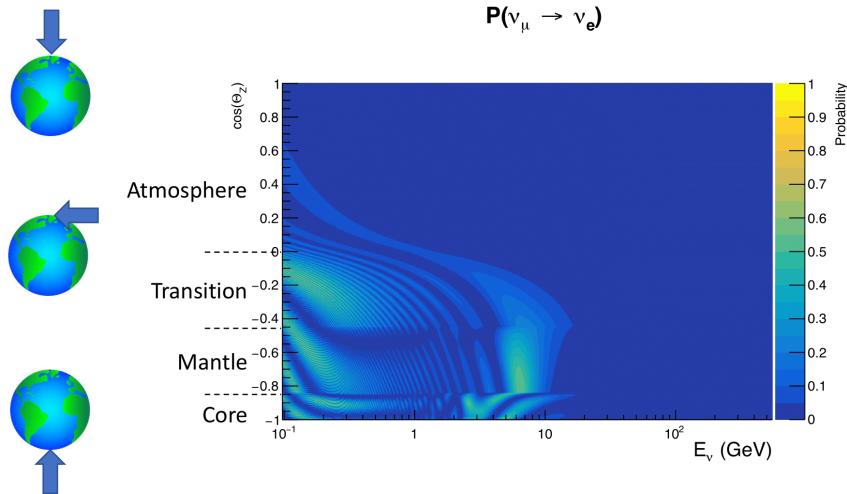


Figure 7.2.: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

not distinguish neutrinos and antineutrinos well and that the cross-section neutrino interaction is larger than that for antineutrinos. Finally, sample selections (discussed in section 6.1) targeting different neutrino interaction modes result in an imbalance in the percentage of neutrinos to anti-neutrinos. This is because negatively charged pions from antineutrino interactions are more likely to be captured by a nucleus compared to a positively charged pion. All of these effects lead to a difference in the number of neutrinos detected compared to antineutrinos. This changes how the δ_{CP} normalisation term is observed, resulting in a very complex sensitivity to δ_{CP} .

Atmospheric neutrinos are subject to matter effects as they travel through the dense matter in the Earth. The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ are presented in Figure 7.4, where the PREM model has been assumed. The oscillation probability for both neutrinos and antineutrinos is affected in the presence of matter. However, the resonance effects around $O(5)$ GeV only occur for neutrinos in normal mass hierarchy and antineutrinos in inverse mass hierarchy. The

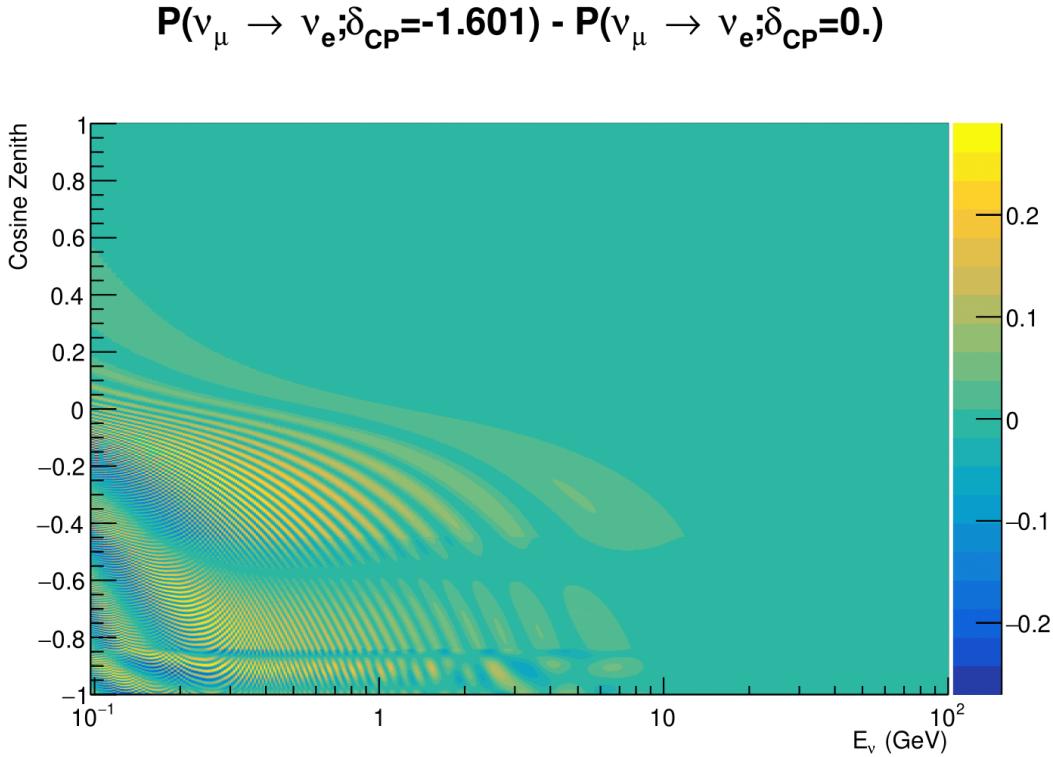


Figure 7.3.: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

2357 exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$ meaning that the
 2358 atmospheric neutrinos have sensitivity to $\sin^2(\theta_{23})$.

2359 As the T2K beam flux is centered at the first oscillation maximum ($E_\nu = 0.6\text{GeV}$),
 2360 the sensitivity to δ_{CP} is predominantly observed as a change in the event-rate of e-like
 2361 samples in $\nu/\bar{\nu}$ modes. Figure 7.5 illustrates the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability
 2362 for a range of δ_{CP} values. A circular modulation of the first oscillation peak (in both
 2363 magnitude and position) is observed when varying throughout the allowable values
 2364 of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation
 2365 maximum than the CP-violating values of $\delta_{CP} = -\pi/2 (\delta_{CP} = \pi/2)$. A sub-dominant

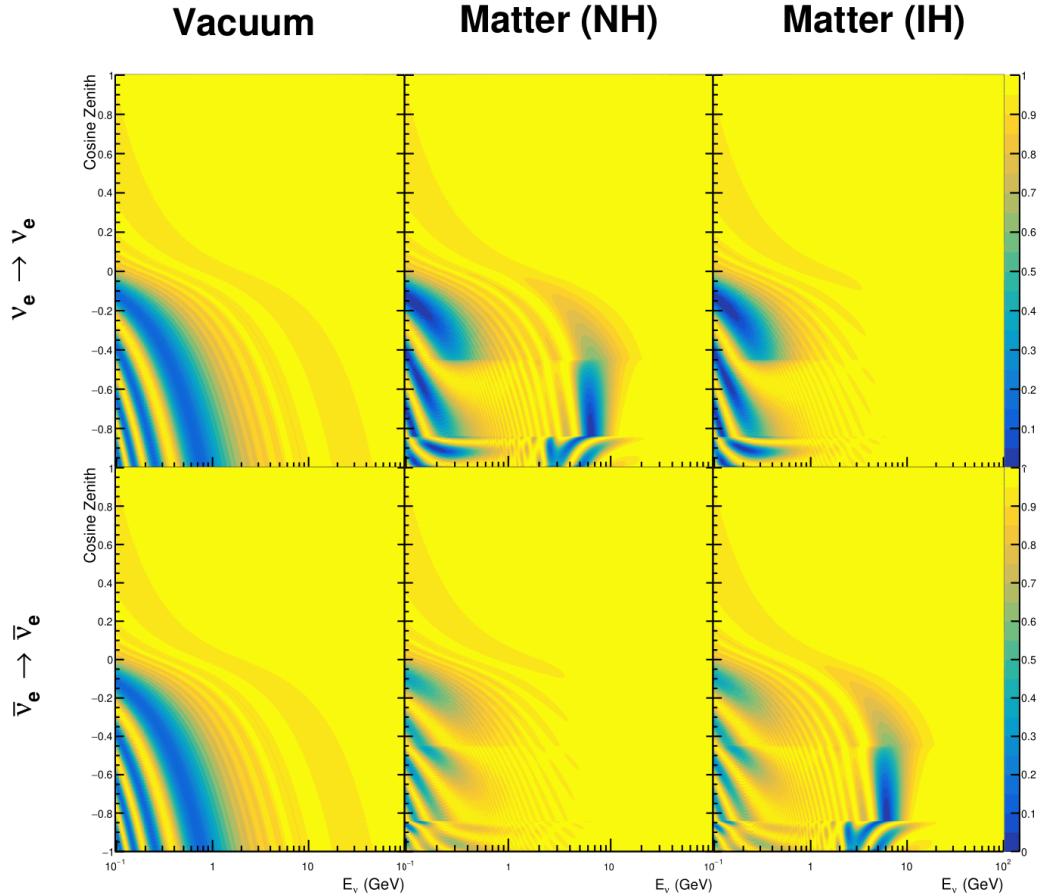


Figure 7.4.: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

shift in the energy of the oscillation peak is also present to aid in separating the two
 CP-conserving values of δ_{CP} .

T2K’s sensitivity to the $\sin^2(\theta_{23})$ and Δm_{23}^2 is observed as a shape-based variation
 of the muon-like samples, as illustrated in Figure 7.5. The value of Δm_{32}^2 laterally shifts
 the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$ oscillation

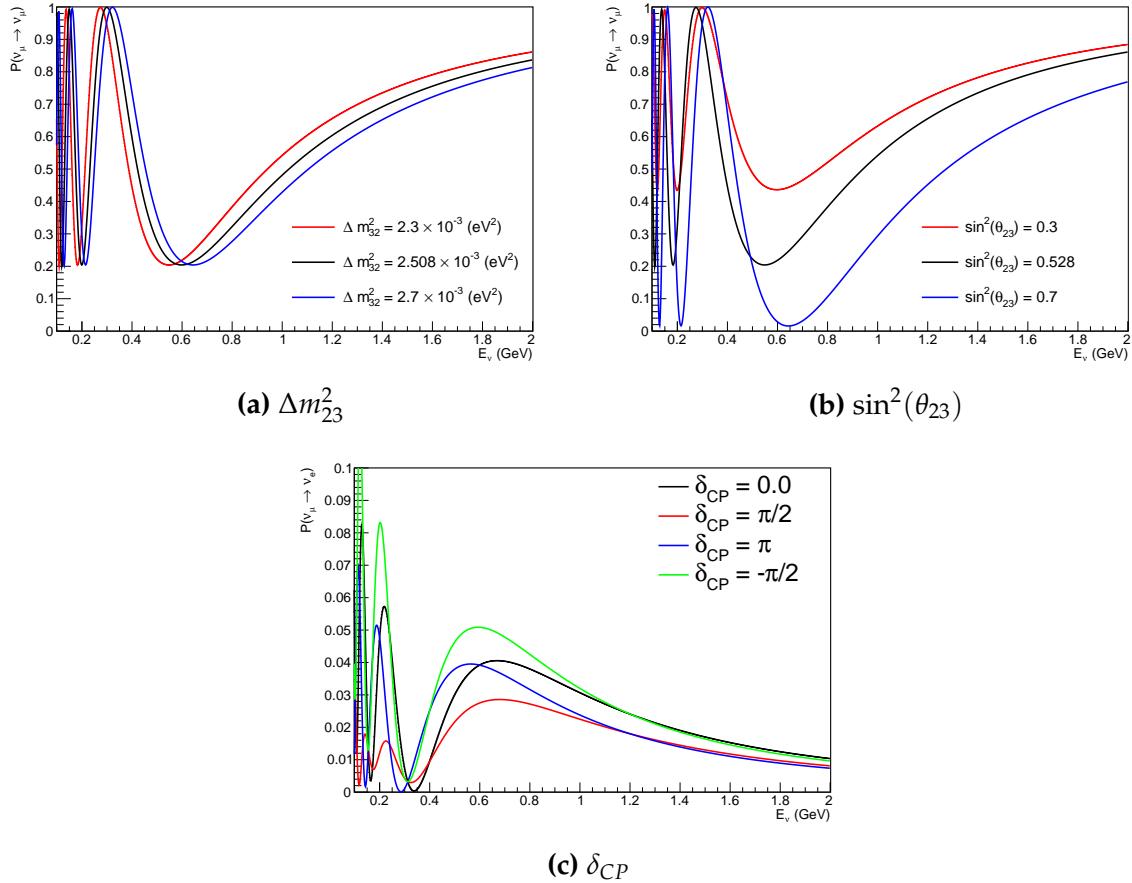


Figure 7.5.: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as a vertical shift of the oscillation dip with second-order horizontal shifts being due to matter effects. The beam neutrinos have limited sensitivity to matter effects due to the relatively shorter baseline as well as the Earth’s mantle being a relatively low-density material (as compared to the Earth’s core). For some values of δ_{CP} , the degeneracy in the number of e-like events allows the mass hierarchy to be resolved. This leads to a δ_{CP} -dependent mass hierarchy sensitivity which can be seen in Figure 7.6.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations

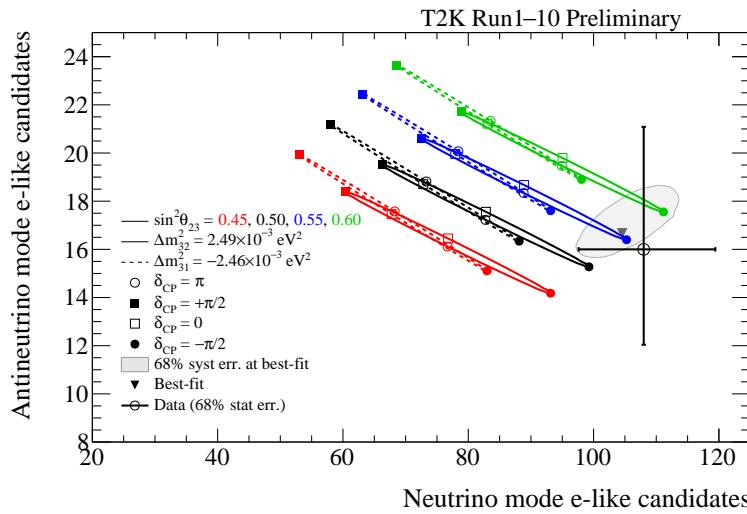


Figure 7.6.: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{23}^2 are given.

which reduce the number of oscillation probability calculations that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux. Furthermore, as the peak neutrino energy of the beam is well below the threshold for charged current tau production ($E_\nu = 3.5 \text{ GeV}$ [51], only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very small due to the short baseline. Whilst these approximations can be made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold.

2391 7.2. Treatment of Fast Oscillations

2392 As shown in Figure 7.7, atmospheric neutrino oscillations have a significantly more
2393 complex structure for upgoing neutrinos with energy below 1GeV. This is because the
2394 L/E dependence of the oscillation probability in this region induces rapid variations
2395 for small changes in L or E . As discussed in section 7.1, this is also the region in which
2396 atmospheric neutrinos have sensitivity to δ_{CP} . In practice, the direction of the neutrino
2397 is inferred from the direction of the final state particles traveling in the detector, which
2398 can be poor for low-energy neutrino interactions. This creates a distinct difference
2399 from the beam neutrinos where the position of the source is very precisely known.

2400 As a consequence of the unresolvable structure, an average oscillation probability
2401 is observed in the subGeV upgoing region. This creates a computational problem; A
2402 significantly large amount of Monte Carlo statistics would be required to accurately
2403 predict the number of events if Monte Carlo averaging was the only technique used.
2404 This section describes the ‘sub-sampling’ approach developed for this analysis and
2405 compares it to the methodology used within the SK-only analysis.

2406 The official SK-only analysis uses the osc3++ oscillation parameter fitter [210].
2407 To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’ technique.
2408 For a given neutrino event, the nearest twenty neighbours in reconstructed lepton
2409 momentum and zenith angle are found and a distribution of their neutrino energies is
2410 built. The RMS, σ , of this distribution is then used to compute an average oscillation
2411 probability for the given neutrino Monte Carlo event.

2412 For the i^{th} event, the oscillation weight is calculated as

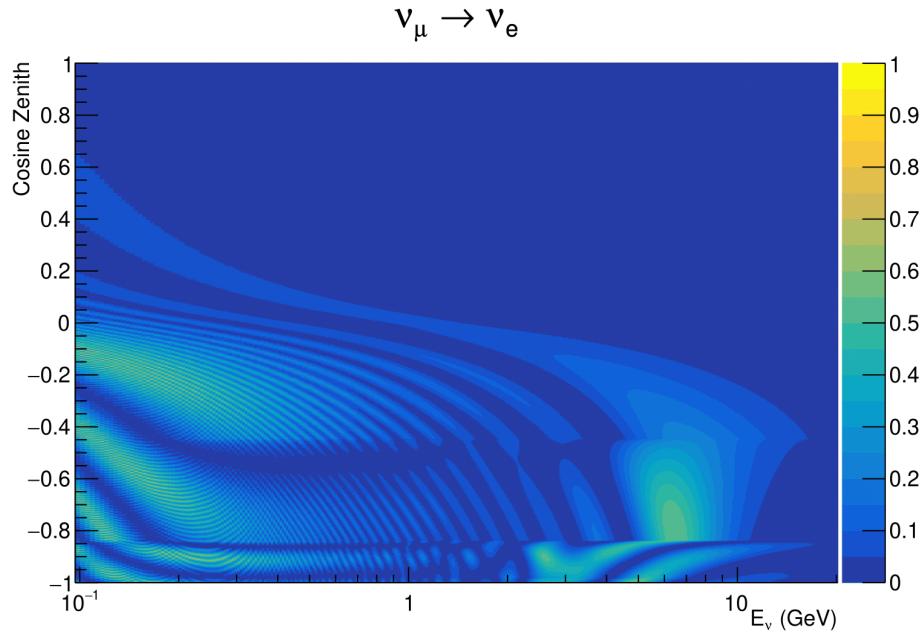


Figure 7.7.: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5} \sum_{\beta=-1,-0.5,0.5,1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.3)$$

where $P(E, L)$ is the oscillation probability calculation for neutrino energy E and path length L and the two path lengths, \bar{L}_i and L_β are discussed below. All of the oscillation probability calculations are performed with a fixed zenith angle such that the same density profile is used.

The uncertainty in the production height is controlled by using an “average” production height, \bar{L}_i , which represents the average path length computed using twenty production heights taken from the Honda flux model’s prediction [45]. For a given event, the production heights are sampled in steps of 5% of their cumulative distribution function. L_β values are similarly calculated but instead use different combinations of four production heights,

$$\begin{aligned}
 L_{-1.0} &= \frac{1}{4}L(45, 50, 55, 60), \\
 L_{-0.5} &= \frac{1}{4}L(35, 40, 65, 70), \\
 L_{+0.5} &= \frac{1}{4}L(25, 30, 75, 68), \\
 L_{+1.0} &= \frac{1}{4}L(15, 20, 85, 89).
 \end{aligned} \tag{7.4}$$

2423 This averaging technique works because of the inference between the zenith angle
 2424 and the reconstructed direction of final state particles in the detector. For low-energy
 2425 neutrinos, where the resolution of the true neutrino direction is poor, σ_i will be large,
 2426 resulting in significant averaging effects. Contrary to this, the inferred direction of
 2427 high-energy neutrinos will be much closer to the true value, meaning that σ_i will be
 2428 smaller, culminating in small averaging effects.

2429 In practice, this technique is performed before the fit in order to deal with the
 2430 computational cost. This is possible as the Osc3++ framework uses binned oscillation
 2431 parameters rather than continuous so the oscillation parameters used in the fit are
 2432 known prior to run-time. The framework used in this analysis uses continuous
 2433 oscillation parameters, and due to the MCMC fitting technique, there is no way to
 2434 know which oscillation parameter values will be selected *a priori*. Therefore, the
 2435 oscillation parameter calculation has to be performed at run-time. Computing five
 2436 oscillation probabilities per event would require far too many computational resources
 2437 to be viable. Therefore SK technique can not be used within this analysis. However,
 2438 the concept of the averaging technique can be taken from it.

2439 To perform a similar averaging as the SK analysis, a sub-sampling approach using
 2440 binned oscillograms has been devised. The technique can be explained by considering
 2441 a “fine” and “coarse” oscillogram. The fine oscillograms are used to define the array of

2442 $\cos(\theta_Z)$ and E_ν used in the oscillation engine. The coarse oscillograms cover the same
2443 phase-space but have fewer bins, where the value of a particular coarse bin is taken
2444 as the linear average (flat prior in E_ν and $\cos(\theta_Z)$) of all fine bins which falls into it.
2445 The coarse oscillogram is then used for determining the oscillation weight for a given
2446 event. The binning which is used to calculate the oscillation probabilities, known as
2447 the ‘fine’ binning, has $N \times N$ subdivisions per coarse bin. Figure 7.8 illustrates the
2448 $N = 2$ example where the assigned value to a coarse bin is the average of the four fine
2449 bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear
2450 on either axis, the sub-division of the fine bins is linear over the range of a coarse bin.

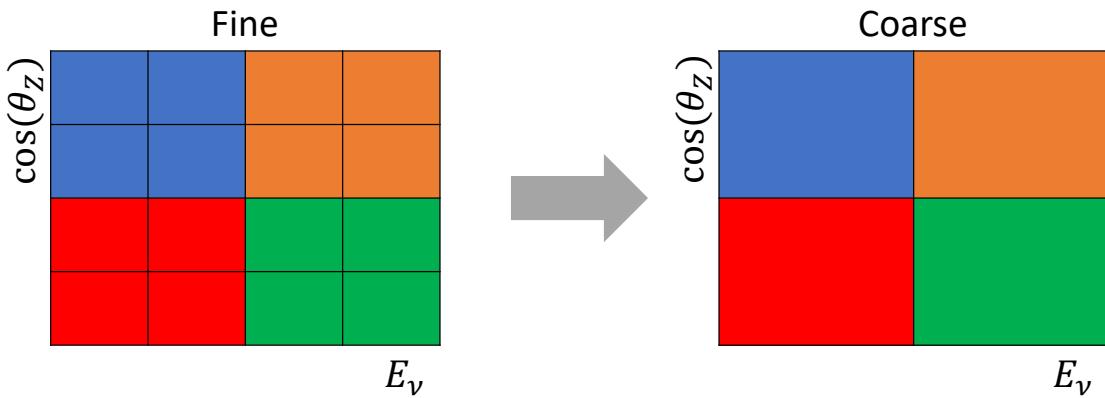


Figure 7.8.: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2451 The coarse binning is defined with 67×52 bins in true neutrino energy \times cosine
2452 zenith. It is picked to be identical to that provided in [212]. In general, the binning is
2453 logarithmically spaced in neutrino energy but has some hand-picked bin edges. Firstly,
2454 the bin density around the matter resonance is smoothly increased around the matter
2455 resonance region. This is to avoid smearing this region which can be well sampled by
2456 the Monte Carlo. Secondly, bin edges are selected to hit $0.4, 0.6, 1, 10, 30, 50, 100\text{GeV}$.
2457 This is to ensure that the Coulomb correction systematic and the atmospheric flux
2458 systematics definitions in neutrino energy can be hit. The cosine zenith binning is

2459 approximately linearly spaced across the allowable range but the values of layer
2460 transitions are hit precisely: -0.8376 (core-mantle) and -0.4464 (mantle/transition
2461 zone). Bins are spread further apart for downgoing events as this is a region unaffected
2462 by the fast oscillation wavelengths and reduces the total number of calculations
2463 required to perform the calculation.

2464 The choice of N is justified based on two studies. Firstly, the variation of event rates
2465 of each sample is studied as a function of N . For a given set of oscillation parameters
2466 thrown from the PDG prior constraints (detailed in Table 2.1), the oscillation probabili-
2467 ties are calculated using a given value of N . Each sample is re-weighted and the event
2468 rate is stored. The value of N is scanned from 1, which corresponds to no averaging, to
2469 19, which corresponds to the largest computationally viable subdivision binning. The
2470 event rate of each sample at large N is expected to converge to a stationary value due
2471 to the fine binning fully sampling the small-scale structure. Figure 7.9 illustrates this
2472 behaviour for the SubGeV_elike_0dcy sample for 9 different throws of the oscillation
2473 parameters.

2474 Denoting the event rate for one sample for a given throw t at each N by λ_t^N , the
2475 average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The variance in
2476 the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right) \right]^2. \quad (7.5)$$

2477 **The Below (and associated figures) has been added** In practice the following
2478 procedure is undertaken. For a particular throw, the difference between the event
2479 rate at a particular choice of N and the mean of the distribution is calculated. This
2480 is illustrated in Figure 7.10. This value is then calculated for all the 2000 throws,

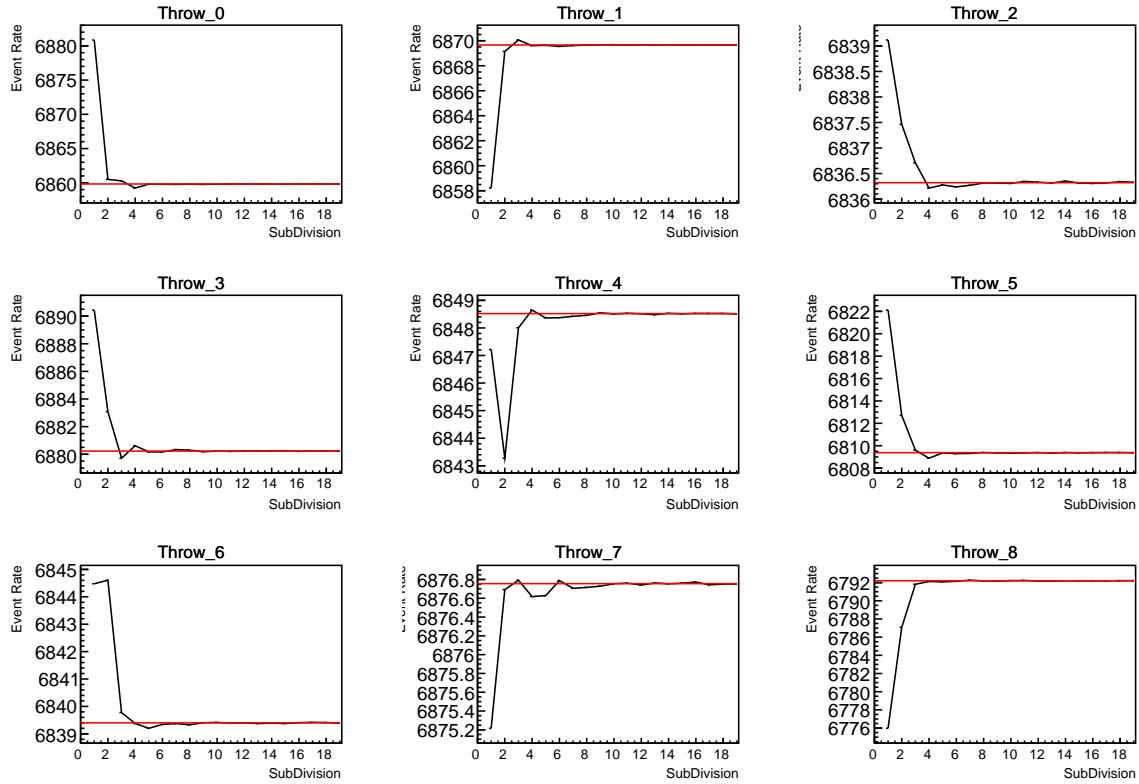


Figure 7.9.: Event rate of the SubGeV_elike_0dcy sample as a function of the number of subdivisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

generating a distribution of $\lambda_t^N - \bar{\lambda}_t$. This is repeated for each of the values of N considered within this study. The distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.11. As expected, the distribution gets narrower and tends towards zero for the higher values of N .

End Review

The aim of the study is to find the lowest value of N such that this variance is below 0.001. This utilises the width of the distributions given in Figure 7.11. This is the typical threshold used by T2K fitters to validate systematic implementation so has been set as the same criteria. The results of this study for each atmospheric sample used within this thesis are illustrated in Figure 7.12 for 2000 throws of the oscillation

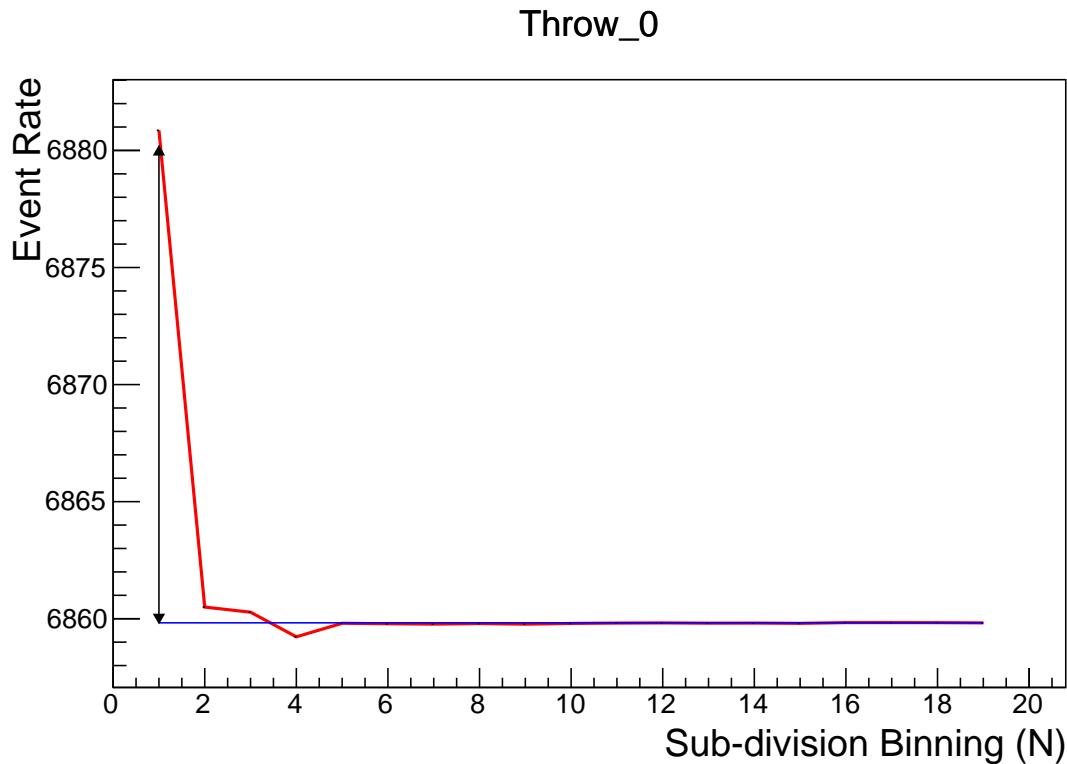


Figure 7.10.

parameters. As can be seen, the variance is below the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy and SubGeV_elike_0dcy samples.

The second study to determine the value of N is as follows. The likelihood for each sample is computed against an Asimov data set created with Asimov A oscillation parameters (Table 2.2). Following Equation 7.5, the variance of the log-likelihood over all considered N is computed. The results are shown in Figure 7.13.

A choice of $N = 10$ sub-divisions per coarse bin has a variance in both event rate and log-likelihood residuals less than the required threshold of 0.001. The largest value of the likelihood variance is of order 10^{-7} , corresponding to an error on the log-likelihood of about 3×10^{-4} which is small enough to be negligible for the oscillation analysis.

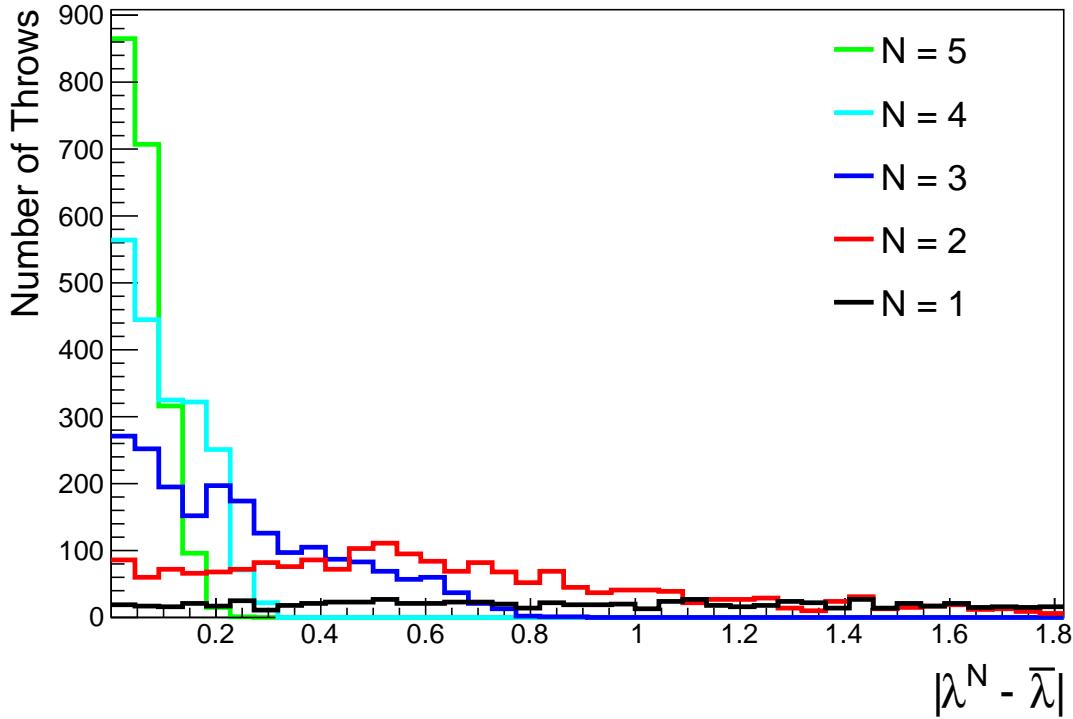


Figure 7.11.

2502 Figure 7.14 illustrates the effect of the smearing using $N = 10$. The fast oscillations
 2503 in the sub-GeV upgoing region have been replaced with a normalisation effect whilst
 2504 the large matter resonance structure remains.

2505 **7.3. Calculation Engine**

2506 As previously discussed in section 7.2, the calculation of oscillation probabilities is per-
 2507 formed at run-time due to utilising continuous oscillation parameters. Consequently,
 2508 the time per calculation is crucial for fit performance. The initial fitting framework
 2509 used for this analysis was developed with ProbGPU [213]. This is a GPU-only implemen-
 2510 tation of the prob3 engine [214]. It is primarily designed for neutrino propagation in a
 2511 beam experiment (single layer of constant density) with the atmospheric propagation
 2512 code not being used prior to the analysis in this thesis.

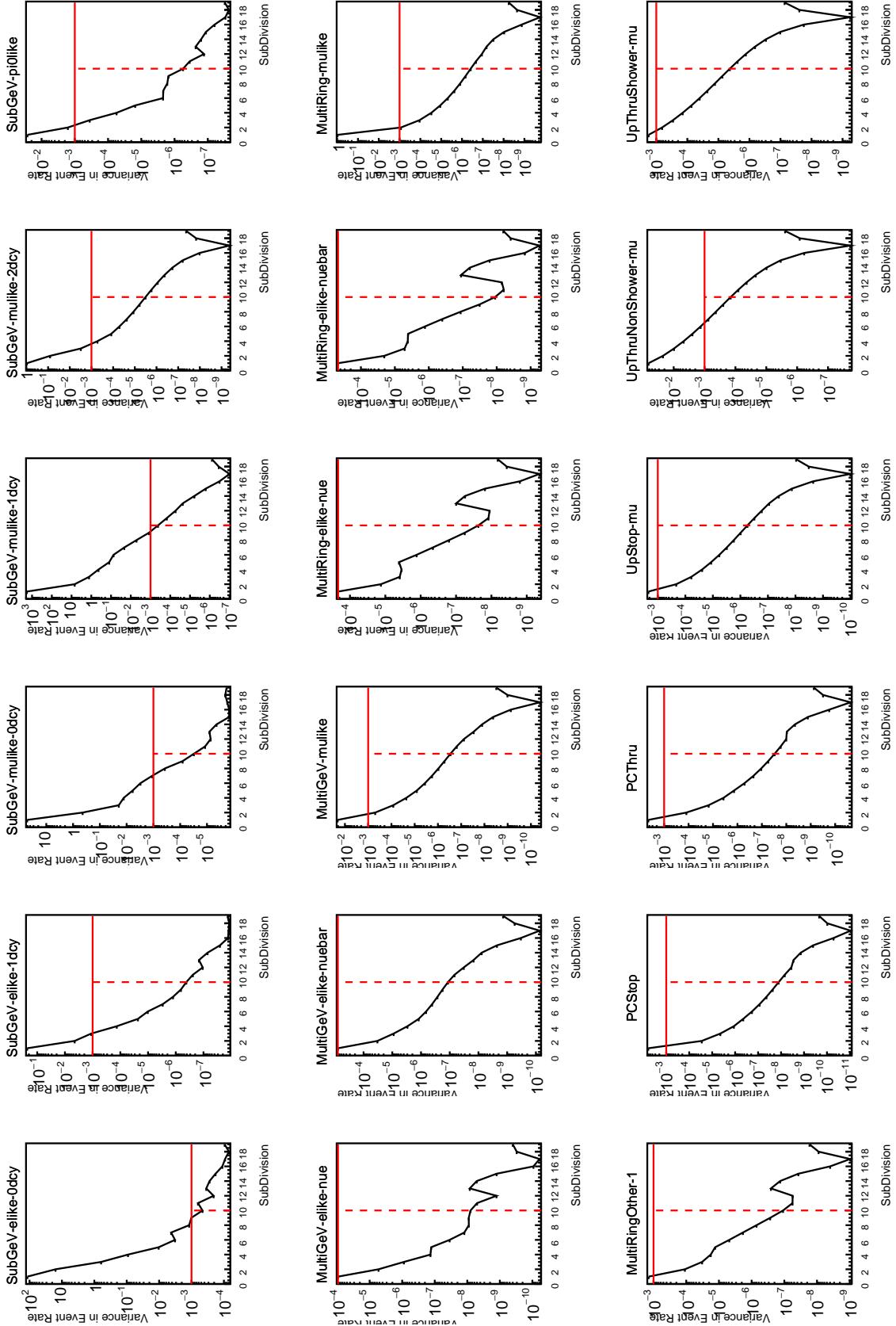


Figure 7.12.: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

Oscillation Probability Calculation

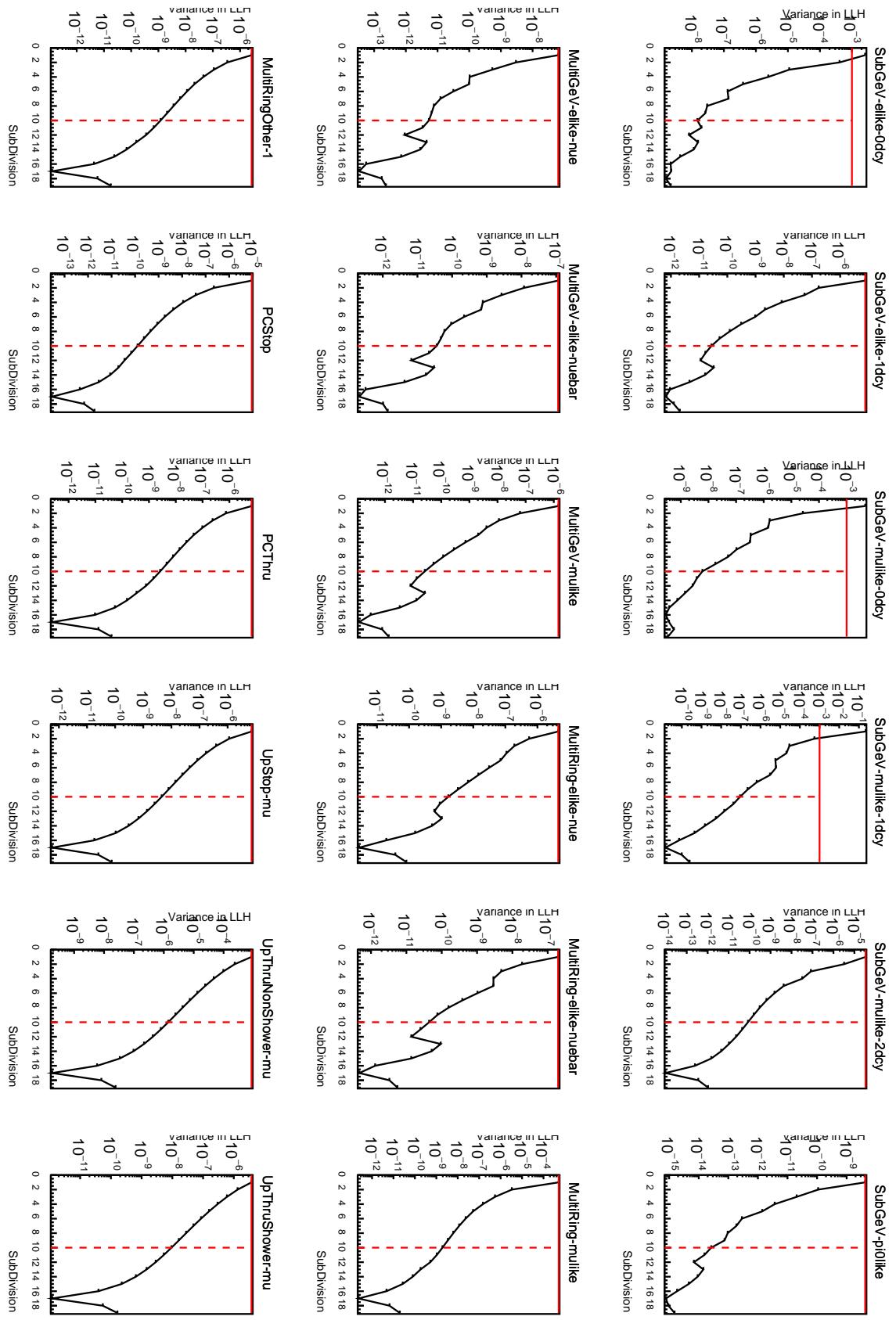


Figure 7.13: Variance of sample likelihood, when compared to 'Asimov data' set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

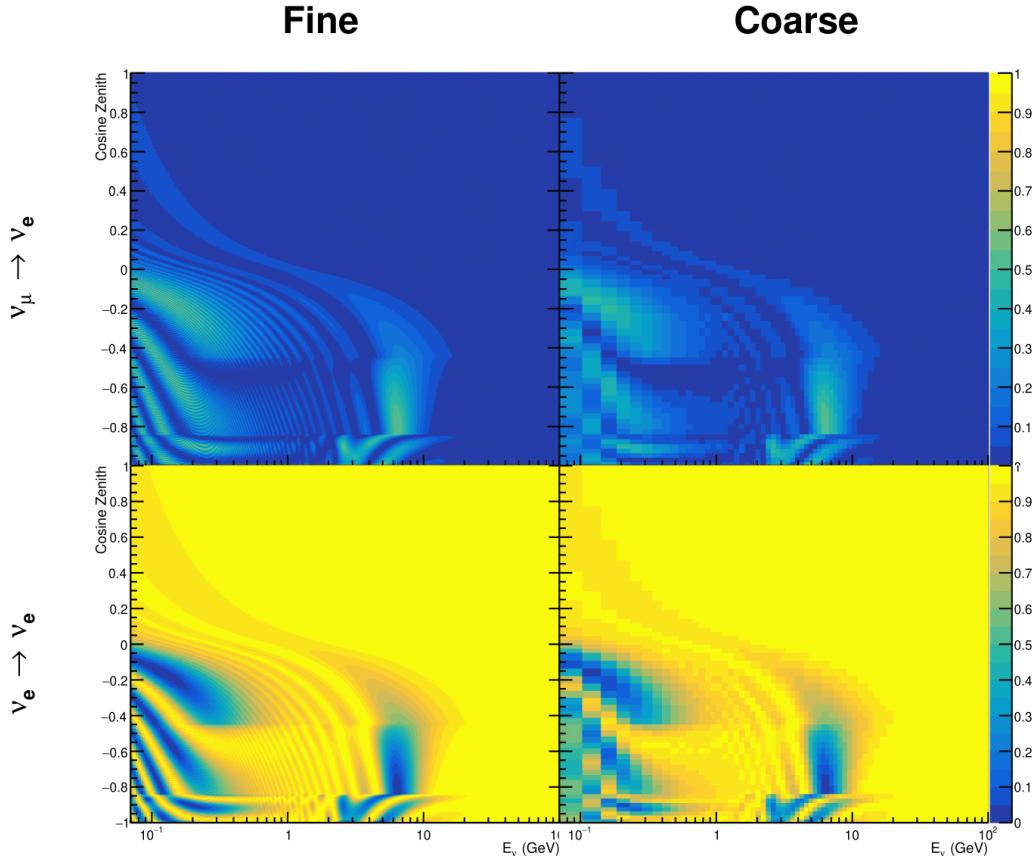


Figure 7.14.: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.12 and Figure 7.13.

2513 Another engine, CUDAProb3 [215], has been implemented within the fitting frame-
 2514 work used in this analysis. It has been specifically optimised for atmospheric neutrino
 2515 oscillation calculation so does not contain the code to replace the beam oscillation
 2516 calculation. The engine utilises object-orientated techniques as compared to the func-
 2517 tional implementation of ProbGPU. This allows the energy and cosine zenith arrays to
 2518 be kept on GPU memory, rather than having to load these arrays onto GPU memory
 2519 for each calculation. General memory interfacing is one of the slowest tasks which

2520 GPUs can do, so being able to eliminate this significantly reduces the time required
 2521 for calculation. This can be seen in Figure 7.15, where the GPU implementation of
 2522 CUDAProb3 is approximately three times faster than the ProbGPU engine.

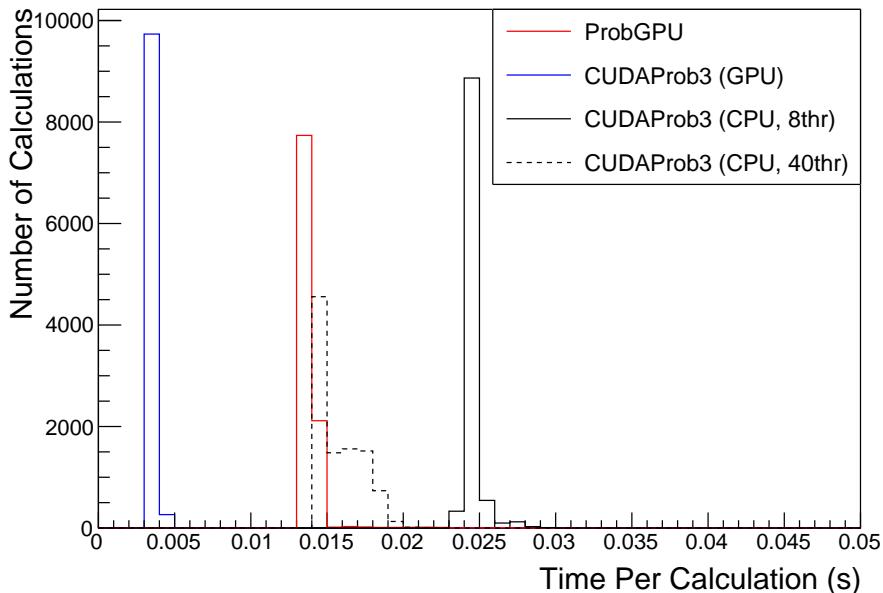


Figure 7.15.: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscilloscopes, following the technique given in section 7.2, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multithreaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

2523 Another significant advantage of CUDAProb3 is that it contains a CPU multithreaded
 2524 implementation which is not possible with the ProbGPU or prob3 engines. This elimi-
 2525 nates the requirement for GPU resources when submitting jobs to batch systems. As
 2526 illustrated in Figure 7.15, the calculation speed depends on the number of available
 2527 threads. Using 8 threads (which is typical of the batch systems being used) is ap-
 2528 proximately twice as slow as the ProbGPU engine implementation, but would allow
 2529 the fitting framework to be run on many more resources. This fact is utilised for any
 2530 SK-only fits but GPU resources are required for any fits which include beam samples
 2531 due to the ProbGPU requirement. Based on the benefits shown by the implementation

2532 in this section, efforts are being placed into including linear propagation for beam
2533 neutrino propagation into the engine [216].

2534 7.4. Matter Density Profile

2535 For an experiment observing atmospheric neutrinos propagating through the Earth, a
2536 model of the Earth's density profile is required. The model used within this analysis is
2537 the Preliminary Reference Earth Model (PREM) [211], as illustrated in Figure 7.1. As
2538 discussed in section 7.1, the propagator used within the calculation engine requires
2539 constant density layers. To follow the official SK-only analysis [210], the average
2540 density of each layer has been taken from the PREM model. Table 7.1 documents
2541 the density and radii of the layers used within this approximation. The density
2542 measurements provided in the PREM model are provided in terms of mass density,
2543 whereas neutrino oscillations are sensitive to the electron number density. This value
2544 can be computed as the product of the chemical composition, or the Z/A value, and
2545 the mass density of each layer. Currently, the only way to calculate the chemical
2546 composition value for layers close to the Earth's core is through neutrino oscillations.
2547 The chemical composition of the upper layers of the Earth's Mantle and the Transition
2548 zone is well known due to it being predominantly pyrolite which has a chemical
2549 composition value of 0.496 [217]. The components of the Earth's core region are less
2550 well known. Consequently, the chemical composition dial for the core layers is set to a
2551 value of 0.468, as calculated in [218]. This value is assigned a Gaussian error with a
2552 standard deviation equivalent to the difference in chemical composition in core and
2553 mantle layers. Figure 7.16 illustrates the effect of moving from the $Z/A = 0.5$ method
2554 which is used in the official SK-only analysis [210] to these more precise values.

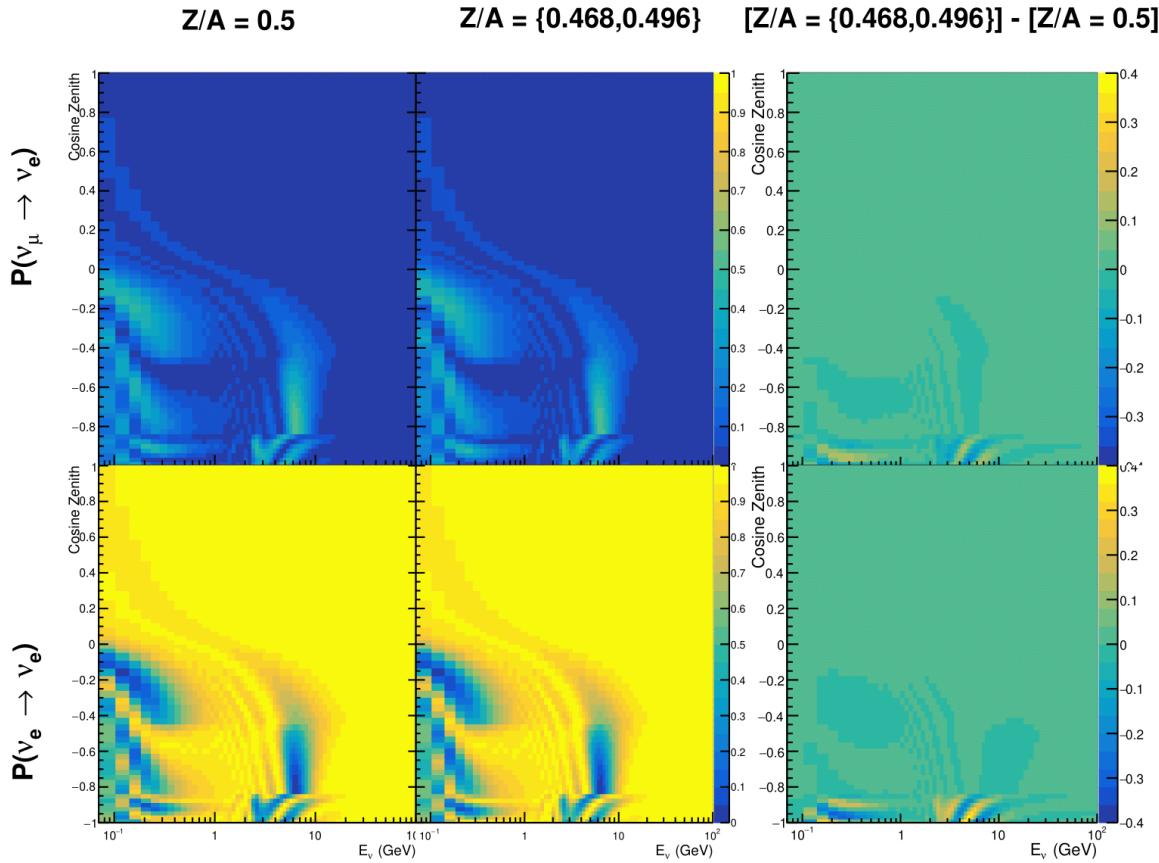


Figure 7.16.: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 7.1. The right column illustrates the difference in oscillation probability between the two different techniques.

2555 The beam oscillation probability in this thesis uses a baseline of 295km, density
 2556 2.6g/cm^3 , and chemical composition 0.5 as is done by the official T2K-only analysis
 2557 [219].

2558 Whilst the propagator requires a fixed density layer model of the Earth, the density
 2559 only has to be fixed for a specific $E_\nu \times \cos(\theta_Z)$ bin in a given layer. As the density is a
 2560 function of radius, which is a function of the direction in which a neutrino propagates,

2561 a better approximation of the PREM model can be made if a $\cos(\theta_Z)$ -specific density is
2562 calculated.

2563 To achieve this, the average density, $\langle \rho \rangle_i$, in the i^{th} layer, is calculated as the density,
2564 $\rho(t)$, integrated over the track a given $\cos(\theta_Z)$,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt \quad (7.6)$$

2565 where t_i are the intersection points between each layer and t is the path length of
2566 the trajectory across the layer.

2567 The oscillation probability calculation speed is approximately linear in the number
2568 of layers invoked within the Earth model. Therefore a four-layer model is still utilized
2569 with the only difference to the official SK-only analysis being that the four-layer model
2570 used for each value of $\cos(\theta_Z)$ is different. Following the method outlined in [220],
2571 a four-layer piecewise quadratic polynomial is fit to the PREM model for the four
2572 layers defined in Table 7.1. This fit was not performed by the author of the thesis
2573 and is documented in [212]. The coefficients of the quadratic fit to each layer are
2574 given in Table 7.2 with the final distribution illustrated in Figure 7.17. The quadratic
2575 approximation is clearly much closer to the PREM model as compared to the constant
2576 density approximation.

2577 The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in
2578 Figure 7.18. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -$
2579 0.45 in the fixed density model, which is due to the transition to mantle layer boundary,
2580 has been reduced. This is expected as the difference in the density across this boundary
2581 is significantly smaller in the quadratic density model as compared to the constant
2582 density model. Whilst the difference in density across the other layer transitions

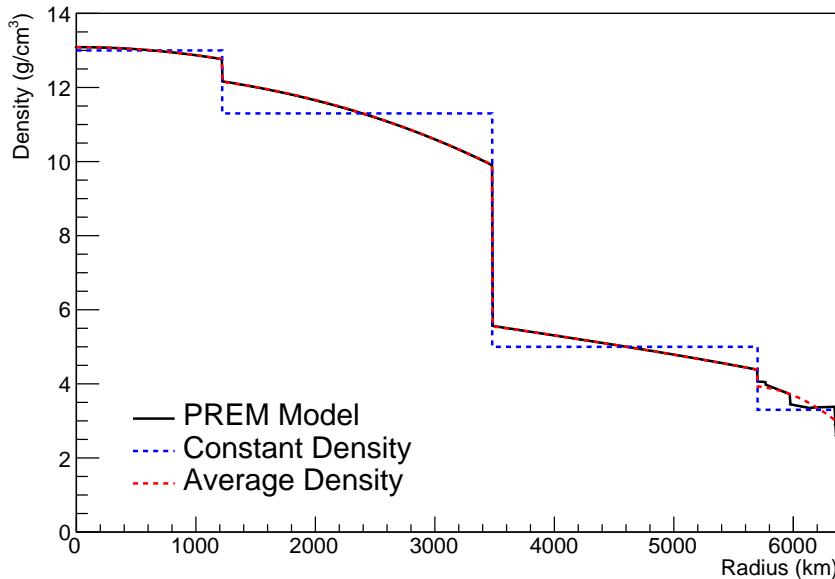


Figure 7.17.: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.2.: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [212], where $x = R/R_{Earth}$.

is reduced, there is still a significant difference. This means the discontinuities in the oscillation probabilities remain but are significantly reduced. However, as the quadratic density approximation matches the PREM model well in this region, these discontinuities are due to the Earth model rather than an artifact of the oscillation calculation.

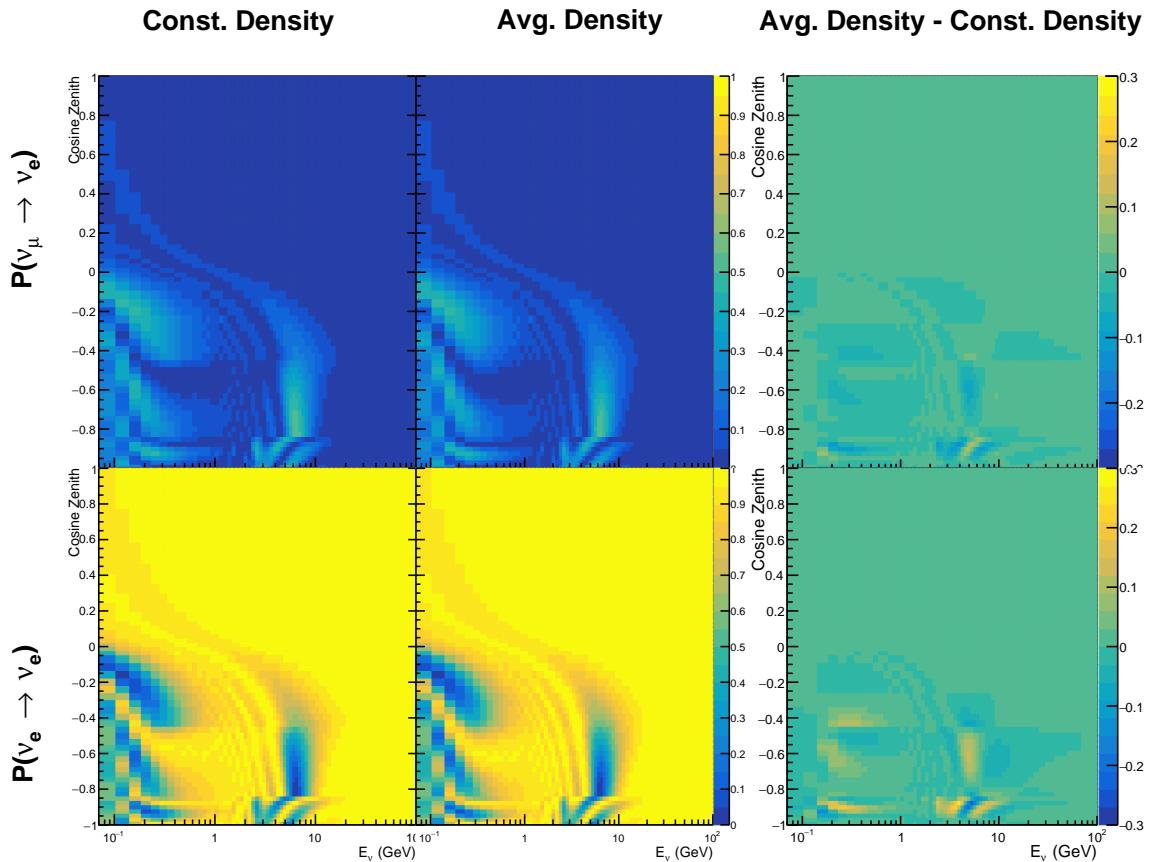


Figure 7.18.: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_\mu)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2588 7.5. Production Height Averaging

2589 As discussed in section 7.1, the height at which the cosmic ray flux interacts in the
2590 atmosphere is not known on an event-by-event basis. The production height can vary
2591 from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only analysis methodology
2592 (described in section 7.2) for including the uncertainty on the production height is
2593 to include variations from the Honda model when pre-calculating the oscillation
2594 probabilities prior to the fit. This technique is not possible for this analysis which
2595 uses continuous oscillation parameters that can not be known prior to the fit. Conse-
2596 quently, an analytical averaging technique was developed in [212]. The author of this
2597 thesis was not responsible for the derivation of the technique but has performed the
2598 implementation and validation of the technique for this analysis alone.

2599 Using the 20 production heights per Monte Carlo neutrino event, provided as 5%
2600 percentiles from the Honda flux model, a production height distribution $p_j(h|E_\nu, \cos\theta_Z)$
2601 is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a histogram is filled with
2602 20 evenly spaced bins in production height h between 0 and 50km. The neutrino energy
2603 and cosine zenith binning of the histogram is the same as that provided in section 7.2.
2604 The average production height, $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. The
2605 production height binning of this histogram is then translated into $\delta t(h) = t(\bar{h}) - t(h)$,
2606 where $t(h)$ is the distance travelled along the trajectory.

2607 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed. The
2608 time-ordered product of these is then used as the overall transition amplitude via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.7)$$

2609

where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.8)$$

2610 is expressed as a diagonalised time-dependent solution to the Schrodinger equation.

2611 The 0^{th} layer is the propagation through the atmosphere and is the only term that
 2612 depends on the production height. Using the substitution $t_0 = t(\bar{h}) - \delta t(h)$, it can be
 2613 shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.9)$$

2614

Thus Equation 7.7 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.10)$$

2615

The oscillation probability averaged over production height is then calculated as

$$\begin{aligned}
 \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\
 &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\
 &= \sum_{km} (B_k)_{ij} (B_m)_{ij}^* \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m)\delta t]
 \end{aligned} \tag{7.11}$$

2616 In practice, implementation in CUDAProb3 [215] is relatively straightforward as
 2617 the majority of these terms are already calculated in the standard oscillation calculation.
 2618 Figure 7.19 illustrates the results of the production height averaging. As expected,
 2619 the main effect is observed in the low-energy downward-going and horizontal-going
 2620 events. Upward-going events have to travel the radius of the Earth, $R_E = 6371\text{km}$,
 2621 where the production height uncertainty is a small fraction of the total path length.

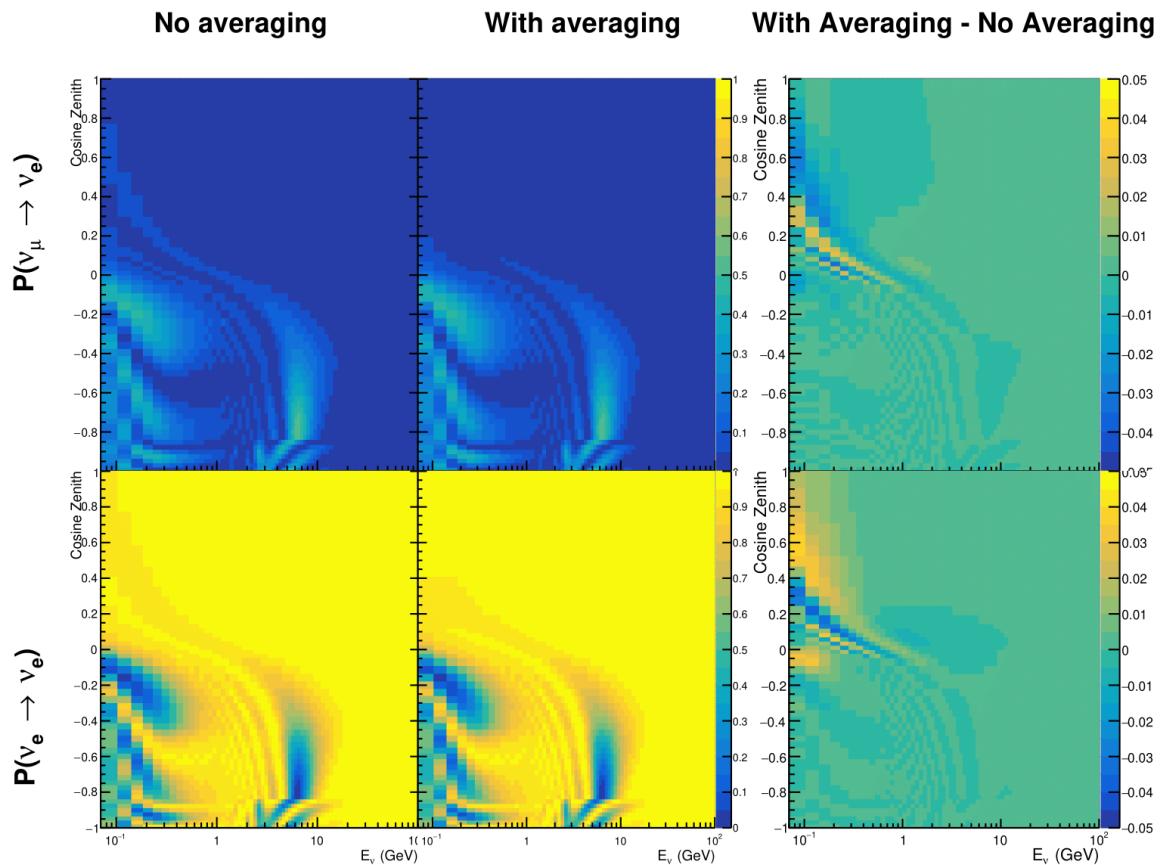


Figure 7.19.: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_\mu)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

2622 **Chapter 8**

2623 **Oscillation Analysis**

2624 **8.1. Likelihood Calculation**

2625 This analysis performs a joint oscillation parameter fit of the ND280, and the SK
2626 atmospheric samples.

2627 Once the Monte Carlo predictions of each beam and atmospheric sample has been
2628 built, following from chapter 6, a likelihood needs to be constructed. This is done
2629 by comparing the Monte Carlo prediction to “data”. The data can consist of either
2630 an Asmiov Monte Carlo prediction, which is typically used for sensitivity studies,
2631 or real data. The Monte Carlo prediction is calculated at a particular point, $\vec{\theta}$, in the
2632 model parameter space, $N_i^{MC} = N_i^{MC}(\vec{\theta})$. Both the data and Monte Carlo spectra are
2633 binned, where the i^{th} bin content is represented by N_i^D and N_i^{MC} , respectively. The bin
2634 contents for the beam near detector, beam far detector and atmospheric samples are
2635 denoted with ND , FD and Atm , respectively. The binning index, i , runs over all the
2636 bins within the sample and all samples with that set. Taking the beam far detector
2637 samples as example, it would run over all the reconstructed neutrino energy bins in all
2638 samples (FHC1R μ , RHC1R μ , etc.). The likelihood calculation between data and Monte
2639 Carlo for a particular bin follows a Poisson distribution, where the data is treated as a
2640 fluctuation of the simulation.

2641 Following the T2K analysis presented in [80], the likelihood contribution from the
2642 near detector also includes a Monte Carlo statistical uncertainty term, derived from

2643 the Barlow and Beeston statistical treatment [221, 222]. In addition to treating the data
 2644 as a fluctuation of the Monte Carlo prediction, it includes a contribution from the
 2645 likelihood that the generated simulation is a statistical fluctuation of the actual true
 2646 simulation assuming infinite statistics. The technical implementation of this additional
 2647 likelihood term is documented in [195]. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (8.1)$$

2648 where β_i represents a scaling parameter for each bin i , which is a value based on
 2649 the amount of Monte Carlo statistics in a bin [195]. $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2 / N_i^{MC}}$, and $\sqrt{\sum_i w_i^2}$
 2650 represents the sum of the square of the weights of the Monte Carlo events which fall
 2651 into bin i .

2652 Additional contributions to the likelihood come from the variation of the systematic
 2653 model parameters. For those parameters with well-motivated uncertainty estimates,
 2654 a covariance matrix, V describes the prior knowledge of each parameter as well as
 2655 any correlations between the parameters. Due to the technical implementation, a
 2656 single covariance matrix describes each “block” of model parameters, e.g. beam flux
 2657 systematics. For simplicity, the covariance matrix associated with the k^{th} block is
 2658 denoted V^k . This substitution results in $\vec{\theta} = \sum_k^{N_b} \vec{\theta}^k$ and $V = \sum_k^{N_b} V^k$, for N_b number
 2659 of blocks describing: oscillation parameters, beam flux, atmospheric flux, neutrino
 2660 interaction, near detector, beam far detector and atmospheric far detector systematics
 2661 detailed in section 6.4. The number of parameters in the k^{th} block is defined as $n(k)$.

2662 The final likelihood term is defined as,

$$\begin{aligned}
& -\ln(\mathcal{L}) = & (8.2) \\
& \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
& + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
& + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
& + \frac{1}{2} \sum_k^{N_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
\end{aligned}$$

2663 This is the value determined at each step of the MCMC to build the posterior

2664 distribution, as discussed in chapter 4.

2665 8.1.1. Likelihood Scans

2666 Using the defintion of the likelihood presented in section 8.1, the response of each
2667 sample to a variation particular parameter can be studied. Figure 8.1 presents the
2668 variation of all the samples (beam and atmospheric) at SK. Each plot represents a
2669 “scan”, where a particular parameter is scanned in some range. The “data” being
2670 used within the definition of the likelihood equation is built using the Asimov A
2671 oscillation parameter values defined in Table 2.2 alongside the pre-fit dial values as
2672 discussed in subsection 6.4.3. Due to the correlations between oscillation parameters,
2673 the value of $\chi^2 \sim 1$ does not equate to the typical 1σ sensitivity. However, it does give
2674 an indication of which samples response the strongest to a variation in the oscillation
2675 parameters. The point at which the likelihood tends to zero illustrates the value of the
2676 parameter used to build the Asimov data prediction. The likelihood scans only include
2677 the sample response and ignore the penalty contribution term from the variation of
2678 the parameter.

2679 The response to Δm_{23}^2 is much larger in beam samples, specifically μ -like samples,
2680 compared to atmospheric samples. This is to be expected as the beam neutrino
2681 energy can be specifically tuned to match the maximal disappearance probability.
2682 As discussed in section 7.1, the determination of the mass hierarchy is significantly
2683 enhanced when using the atmospheric samples due to them transitioning through the
2684 Earth’s core. So whilst the atmospheric samples do not add much information to the
2685 constraint of $|\Delta m_{32}^2|$ beyond that of the beam analysis, they do enhance the ability to
2686 determine the sign of the parameter.

2687 The sensitivity to $\sin^2(\theta_{23})$ is again dominated by the T2K experiment. However,
2688 the difference in the response for atmospheric and beam samples is much smaller. Con-
2689 sequently, one would expect that the joint fit would become more sensivity to $\sin^2(\theta_{23})$

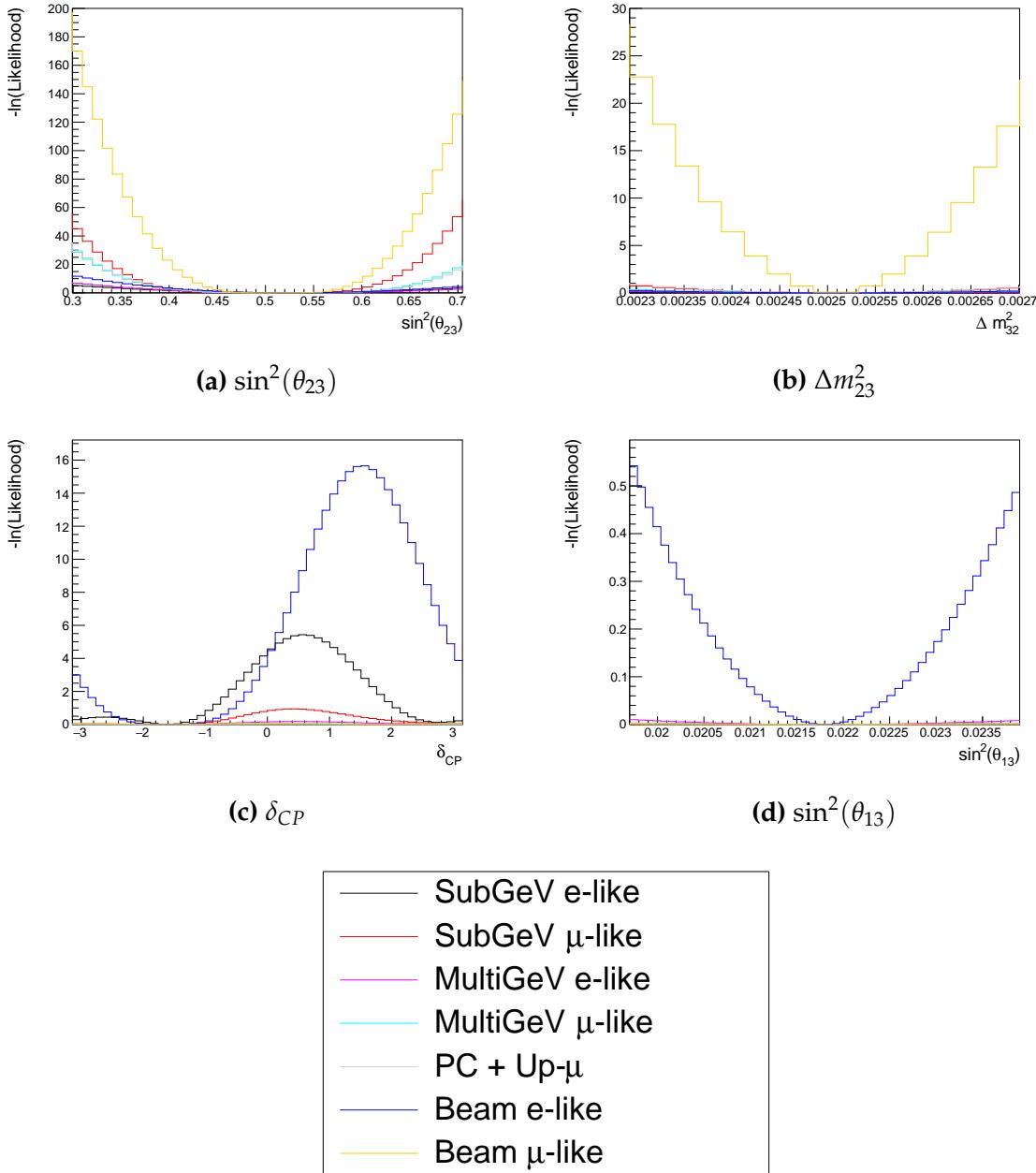


Figure 8.1.: The response of the likelihood, as defined in section 8.1, illustrating the response of the samples to the oscillation parameters. Δm_{12}^2 and $\sin^2(\theta_{12})$ are negated because these samples have no sensitivity to those parameters. The Asimov data set is built using the pre-fit dial values assuming Asimov A oscillation parameters defined in Table 2.2. DB: Need finer binning on delmsq23

than just T2K experiment alone. The summed response over all atmospheric samples becomes comparable to that of the muon-like beam samples. For this particular choice of Asimov point, the only samples which respond to the $\sin^2(\theta_{13})$ parameter are the

2693 electron-like beam samples. Consequently, no increase in sensitivity beyond that of
2694 the T2K-only analysis is expected at that Asimov point. The Δm_{12}^2 and $\sin^2(\theta_{12})$ are
2695 not considered as there is simply no sensitivity in any sample considered within this
2696 analysis.

2697 As discussed, the correlations between oscillation parameters induce marginali-
2698 sation effects within the response of the likelihood. That is to say, the response to
2699 δ_{CP} is affected by the choice of $\sin^2(\theta_{13})$ or $\sin^2(\theta_{23})$. The two-dimensional scans of
2700 the appearance ($\sin^2(\theta_{13}) - \delta_{CP}$) and disappearance ($\sin^2(\theta_{23}) - \Delta m_{23}^2$) parameters are
2701 illustrated in Figure 8.2 and Figure 8.3, respectively.

2702 The appearance log-likelihood scans show the distinct difference in how the beam
2703 and atmospheric samples respond. The beam samples have an approximately constant
2704 width of the 2σ and 3σ contours, throughout all ranges of δ_{CP} . The atmospheric
2705 samples response to δ_{CP} is very strongly correlated to the choice of $\sin^2(\theta_{13})$, with
2706 the strongest constraints around $\delta_{CP} \sim 1$. Consequently, this difference allows some
2707 of the degeneracy in a beam-only fit to be broken. Comparing the beam and joint
2708 fit log-likelihood scans, the 2σ continuous contour in δ_{CP} for beam samples is broken
2709 when the atmospheric samples are added. Furthermore, the width of the 3σ contours
2710 also becomes dependent upon the value of δ_{CP} . Whilst these are encouraging results
2711 for the joint fit, these are not sensitivity measurements as the nuisance parameters are
2712 fixed.

2713 The disappearance log-likelihood scans in $\sin^2(\theta_{23}) - \Delta m_{23}^2$ space show the expected
2714 result when considering the one-dimensional scans already discussed. The uncertainty
2715 on the width of $|\Delta m_{32}^2|$ is mostly driven by the beam-only sensitivities. However, the
2716 width of this contour in the inverted mass region ($\Delta m_{32}^2 < 0$) is significantly reduced
2717 due to the ability of the atmospheric samples to select the correct mass hierarchy (these
2718 log-likelihood scans use the Asimov A oscillation probabilities which assumes true

2719 normal hierarchy). The width of the uncertainty in $\sin^2(\theta_{23})$ is also reduce compared
2720 to a beam-only analysis, with a further decrease in the inverted hierarchy region due
2721 to mass hierarchy determination.

2722 In addition to the oscillation parameters, the response to the systematic model
2723 parameters can also be considered. Due to the correlated cross section model, the most
2724 informative DB: Finish this

2725 8.2. Monte Carlo Prediction

2726 Using the three sets of dial values defined in subsection 6.4.3, the predicted event rates
2727 for each sample are defined in Table 8.1. Both the oscillated event rates assuming
2728 Asimov A oscillation parameters (defined in Table 2.2) and the un-oscillated event
2729 rates are given.

2730 Generally, samples which target CCQE interaction modes observe a decrease in
2731 prediction when using the pre-fit dial values. This is in accordance with the Monte
2732 Carlo being produced assumed $M_A^{QE} = 1.21\text{GeV}$ whilst the pre-fit dial value should
2733 be $M_A^{QE} = 1.03\text{GeV}$, as suggested by [204]. Furthermore, the predicted event rates
2734 of samples which target CCRES interaction modes is significantly reduced when
2735 considering the post-ND fit. This follows the observations in subsection 6.4.3. The
2736 strength of the accelerator neutrino experiment can also be seen in the remarkable
2737 difference between the oscillated and unoscillated predictions in the FHC1Rmu and
2738 RHC1Rmu samples. There is a very obvious decrease in the expected event rate between
2739 the two predictions which is not as clearly observed in the atmospheric samples.
2740 This is due to the fact that the beam energy is tuned to the maximum disappearance
2741 probability, which is not the case for the naturally generated atmospheric neutrinos.

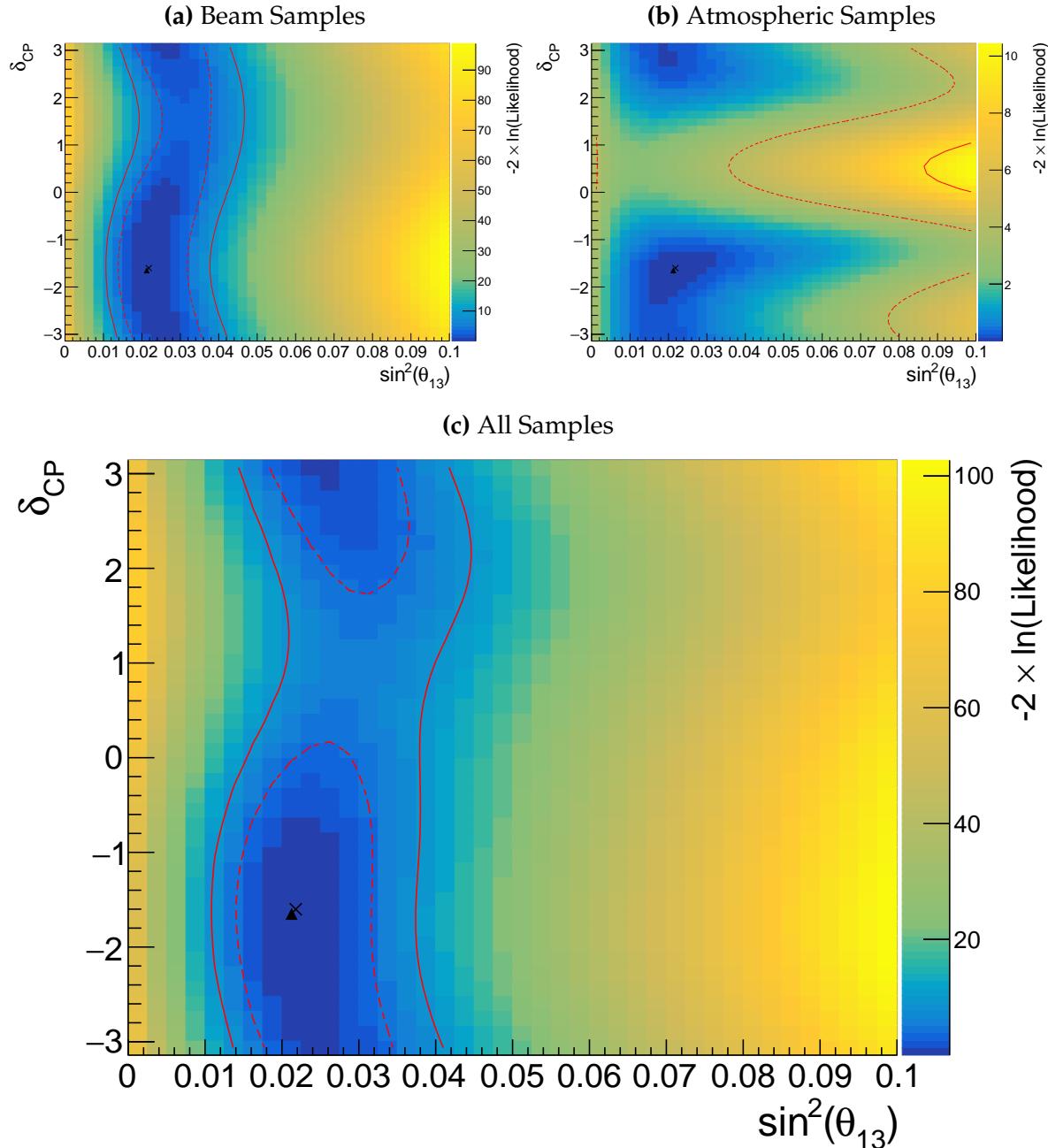


Figure 8.2.: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})$ - δ_{CP}) parameters showing the response of the beam samples (top), atmospheric samples (middle) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are assumed to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

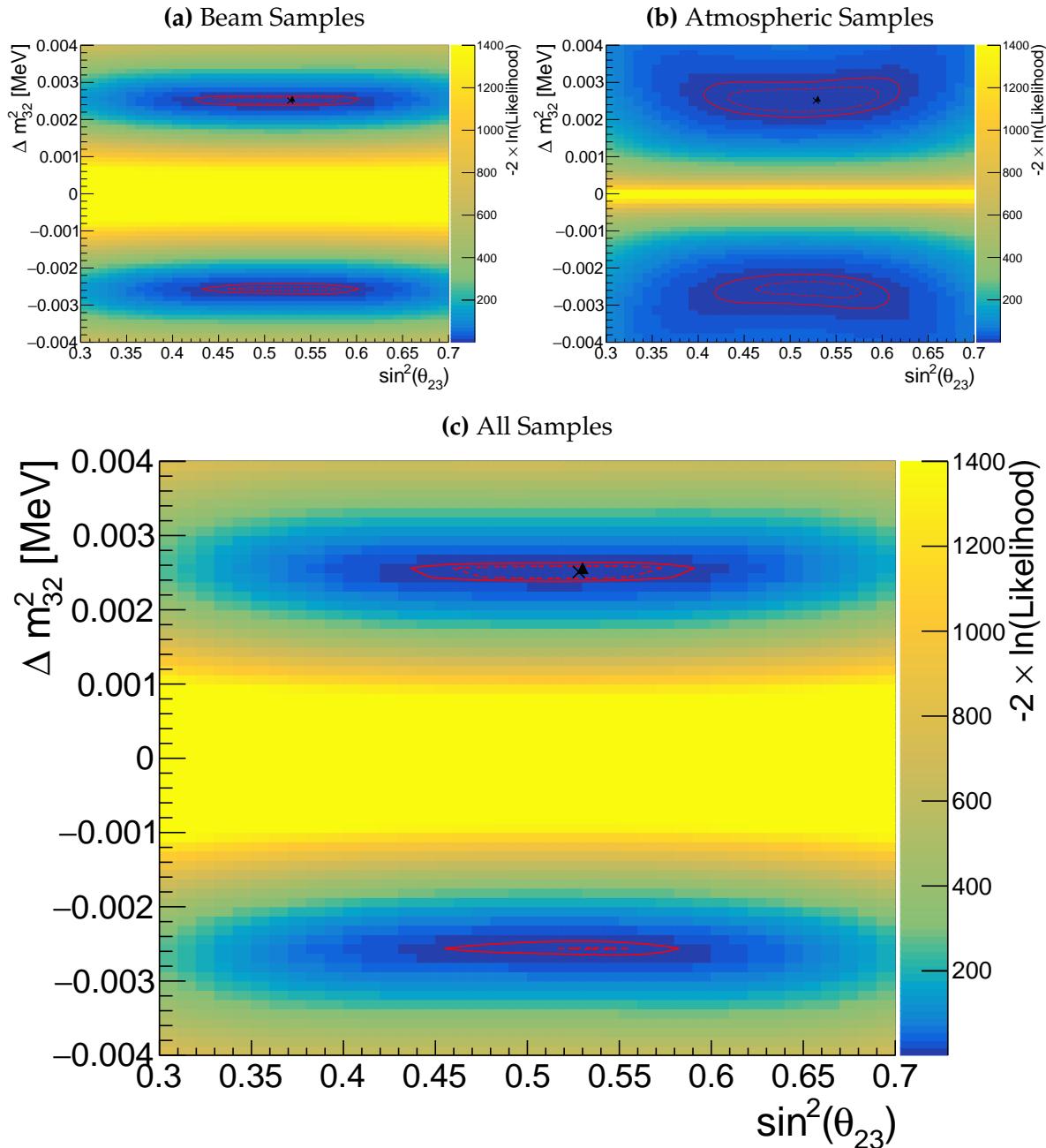


Figure 8.3.: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23})$ - Δm_{23}^2) parameters showing the response of the beam samples (top), atmospheric samples (middle) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are assumed to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThrus	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1.: The Monte Carlo prediction of each sample observed at SK used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. The oscillated predictions assumed Asimov A oscillation parameters provided in Table 2.2.

2743 **Appendix A**

2744 **Atmospheric Sample Spectra**

2745 This appendix documents the interaction mode breakdown of all the atmospheric
2746 samples used within the analysis. The generated tune of the model parameters and the
2747 Asimov A oscillation parameter set (defined in Table 2.2) are assumed. The livetime of
2748 SK-IV is taken to be 3244.4 days.

2749 **A.1. Fully Contained Sub-GeV Samples**

2750 The interaction mode breakdown of the fully contained Sub-GeV samples are shown
2751 in Figure A.1 and Figure A.2, for the samples with enriched CC0 π and CC1 π^\pm
2752 respectively.

2753 The CC0 π sample are dominated by CCQE events ($\sim 70\%$) with smaller contributions of 2p2h ($\sim 12\%$) and CC1 π ($\sim 10\%$) components. The energy peaks around
2754 300 MeV, which is slightly below that of the T2K samples but still has significant
2755 contribution upto 1 GeV which overlaps the T2K sample energy range.
2756

2757 The one-ring CC1 π samples, where the pion is tagged via its decay electron, are
2758 dominated by CC1 π events ($\sim 75\%$) with a small contribution of CCM π ($\sim 10\%$).
2759 The two-ring pion sample is mostly dominated by the NC1 π^0 via resonances, and
2760 has several equally-sized contributions from CCQE, NC1 π^\pm via resonances, and NC
2761 coherent pion production, where the π^0 likely comes from nucleon and π^\pm final state
2762 interactions in the nucleus.

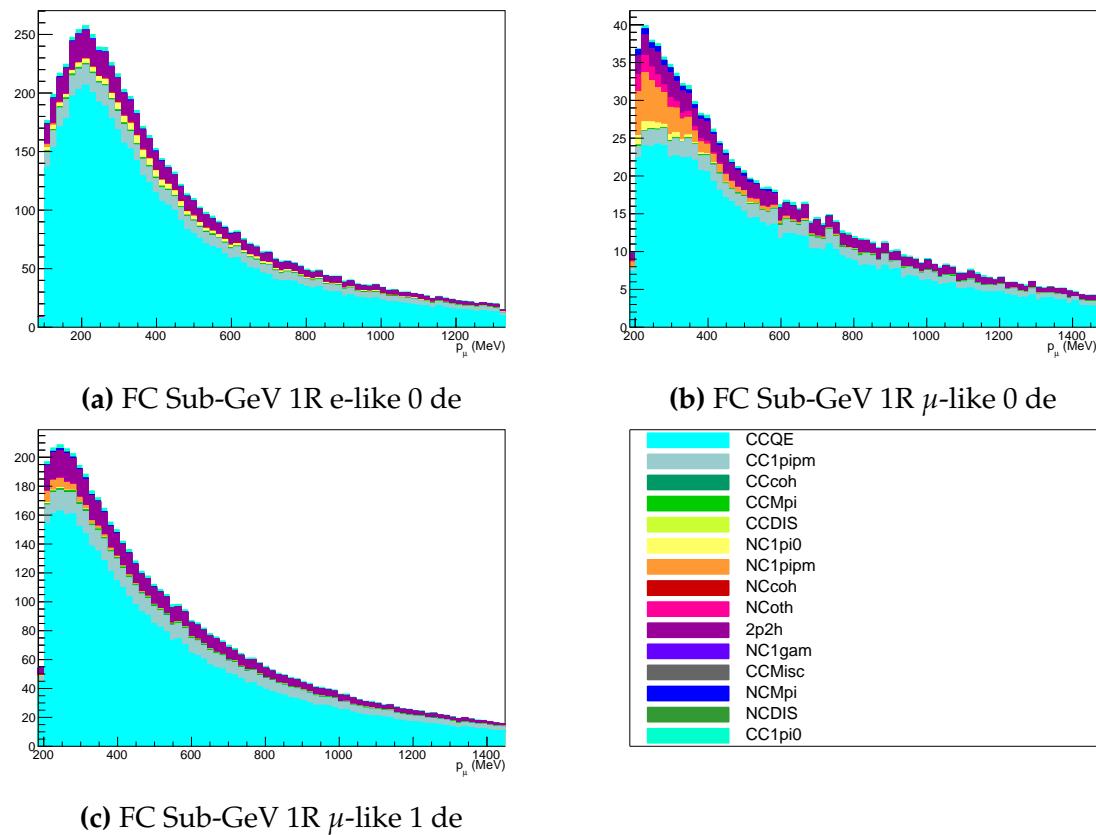


Figure A.1: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting CC 0π events.

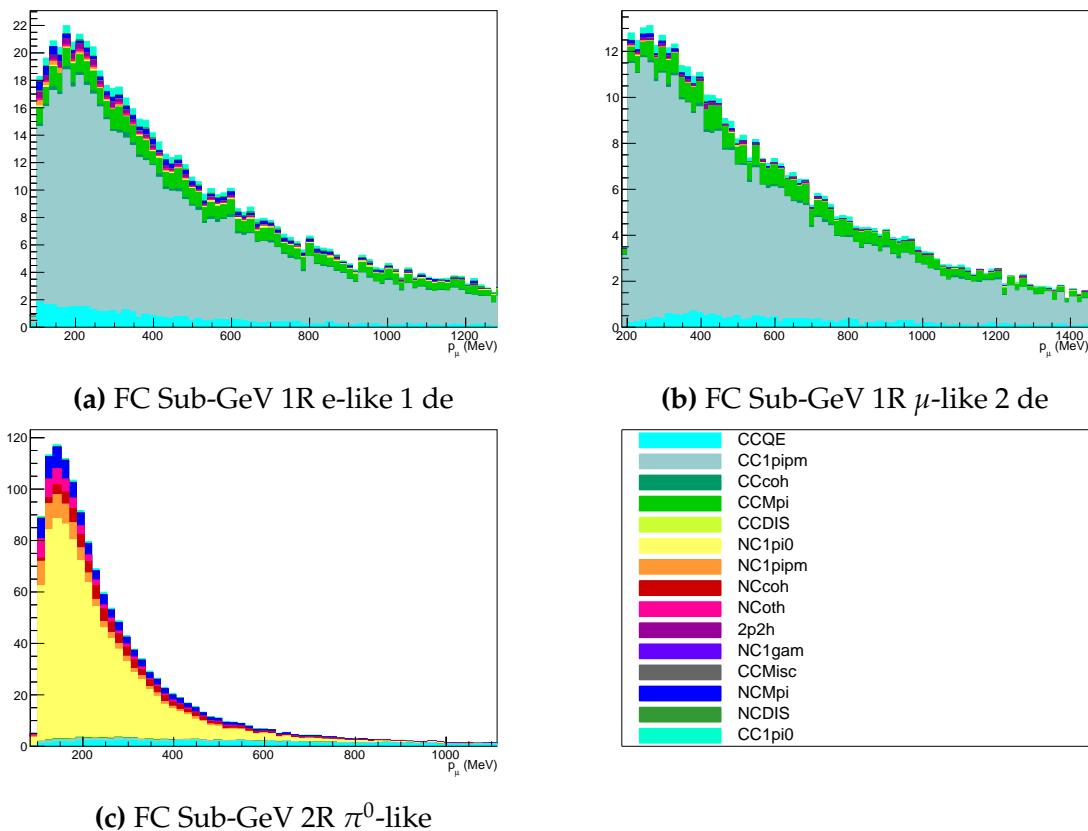


Figure A.2: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting single pion events.

2763 A.2. Fully Contained Multi-GeV Samples

2764 The interaction mode breakdown of fully contained multi-GeV samples is highlighted
 2765 in Figure A.3. Due to the event selection applied in SK which targets π^+ and π^-
 2766 separation, the ν_e sample mainly consists of events with pions (single pion production
 2767 or multi-pion/DIS interactions). The pion separation is explained in Section section 6.1.
 2768 This reasoning also explains the significant CCQE contribution of the $\bar{\nu}_e$ sample. The
 2769 muon-like sample is dominated by CCQE interactions with $\sim 10 - 15\%$ 2p2h and
 2770 CC1 π contribution of events.

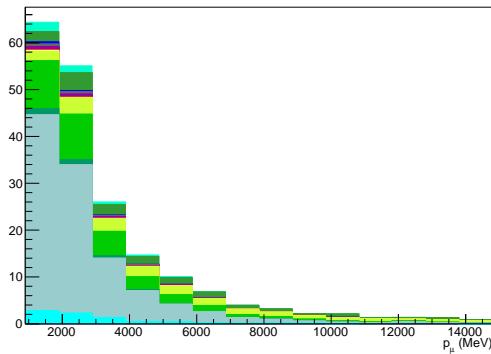
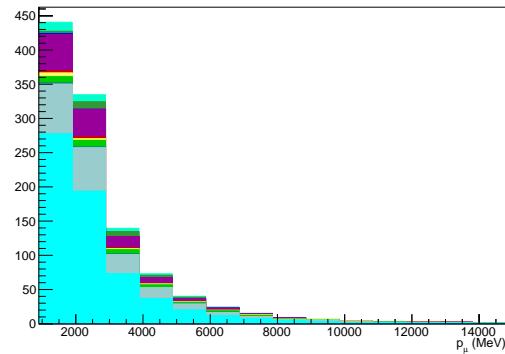
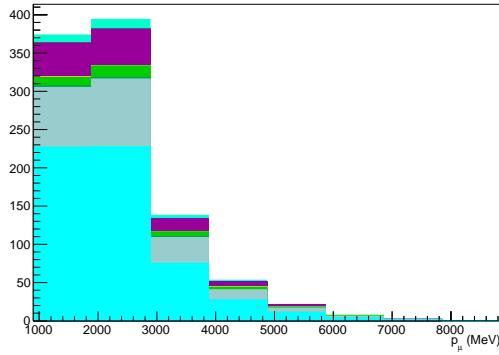
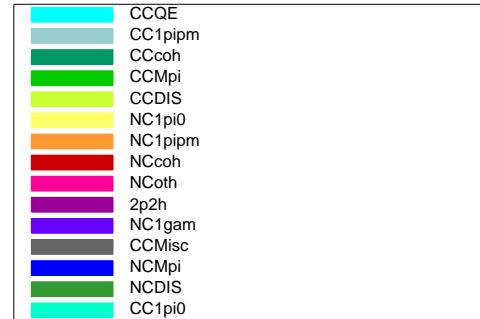
(a) FC Multi-GeV single ring ν_e -like(b) FC Multi-GeV single ring $\bar{\nu}_e$ -like(c) FC Multi-GeV single ring μ -like

Figure A.3: Breakdown by interaction mode of the FC Multi-GeV single ring atmospheric samples.

A.3. Fully Contained Multi-Ring Samples

The interaction mode breakdown of fully contained multi-ring events is shown in Figure A.4. These samples see more interaction modes contributing in general, and there is a much larger contribution from multi-pion and DIS interaction modes, compared to the other samples.

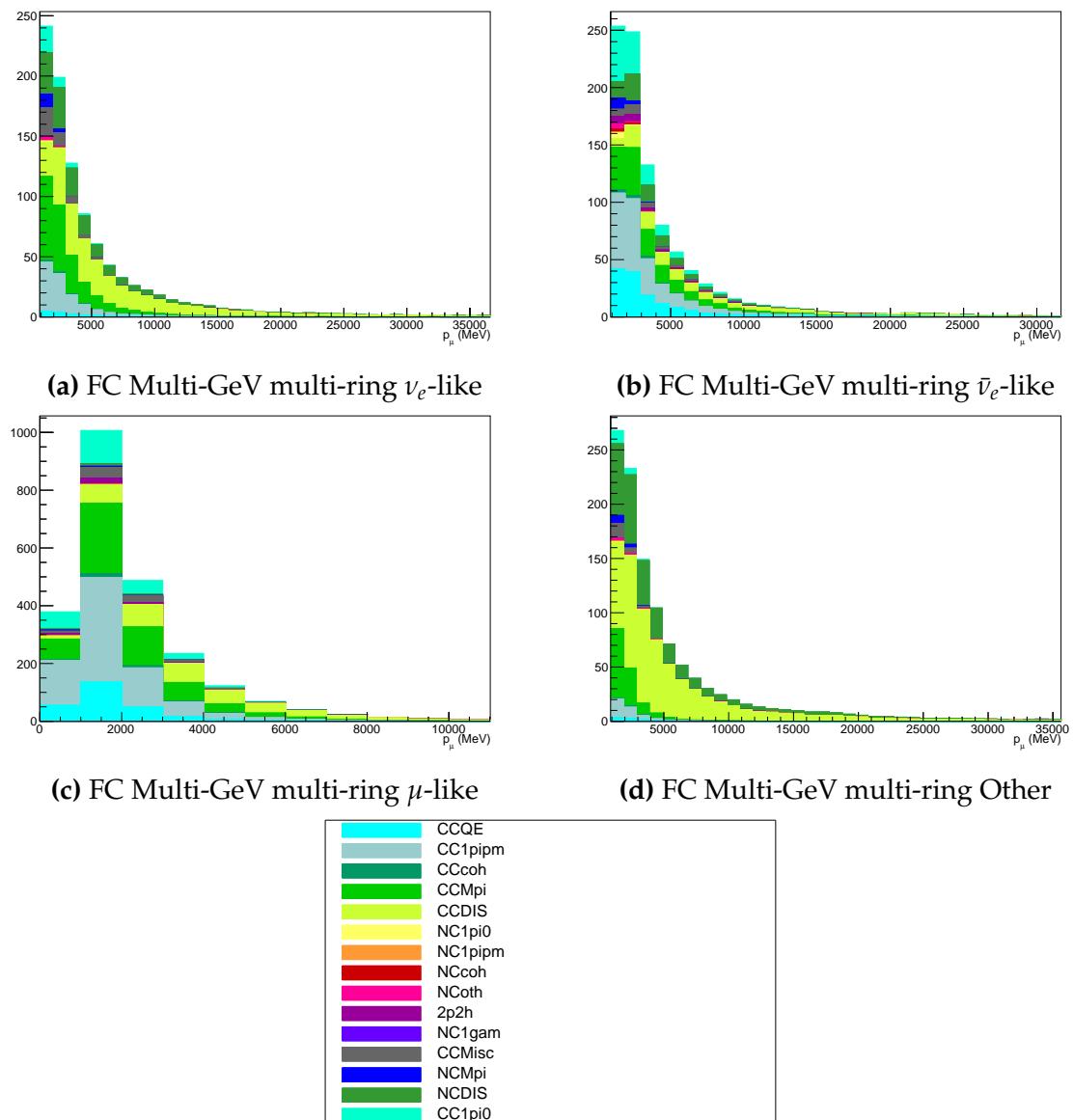


Figure A.4: Breakdown by interaction mode of the FC Multi-GeV multi-ring atmospheric samples.

²⁷⁷⁶ A.4. Partially Contained Samples

²⁷⁷⁷ The breakdown for partially contained samples is highlighted in Figure A.5. As with
²⁷⁷⁸ the multi-ring samples, there is no dominating interaction mode. The neutrino energies
²⁷⁷⁹ of events in this sample extend into the tens of GeV and become dominated by DIS
²⁷⁸⁰ interaction modes in the high energy limit.

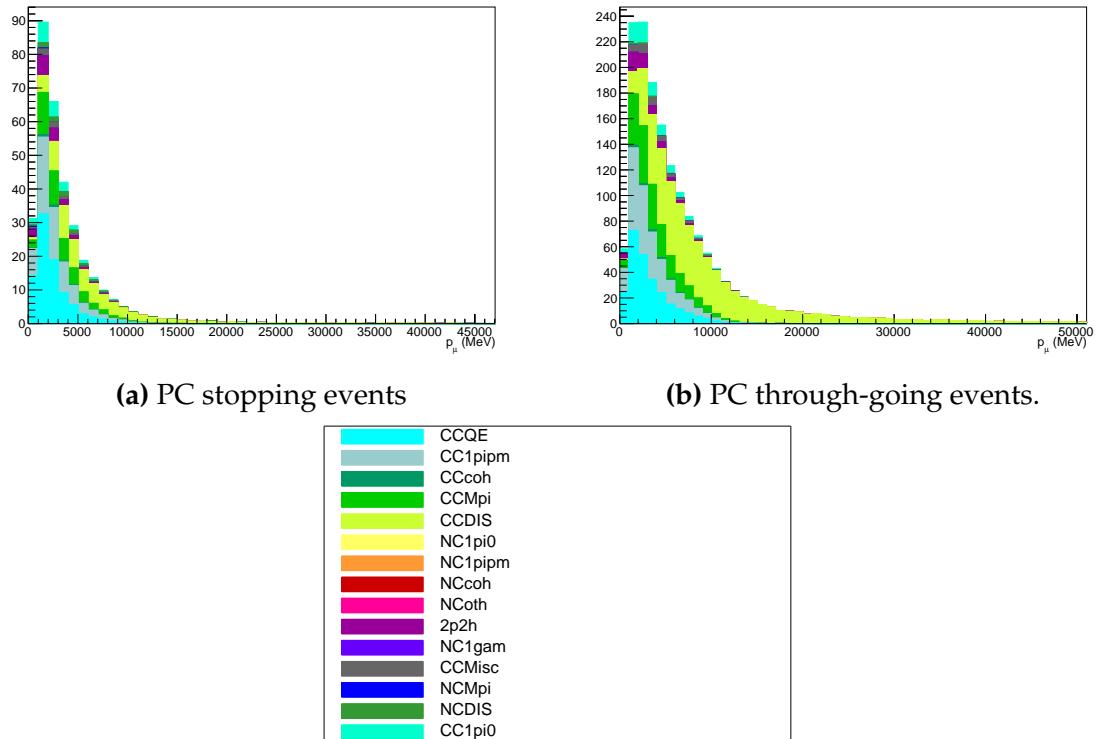
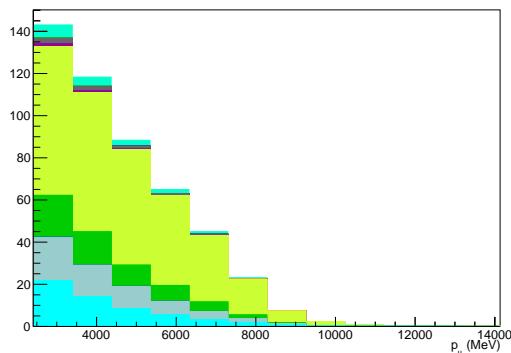
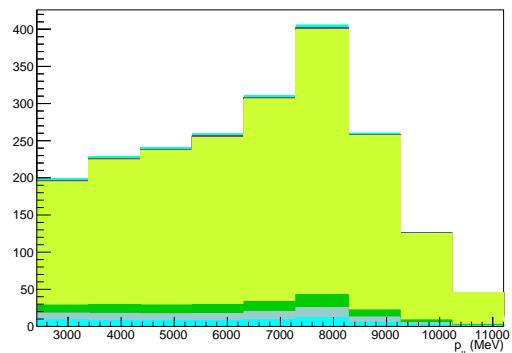
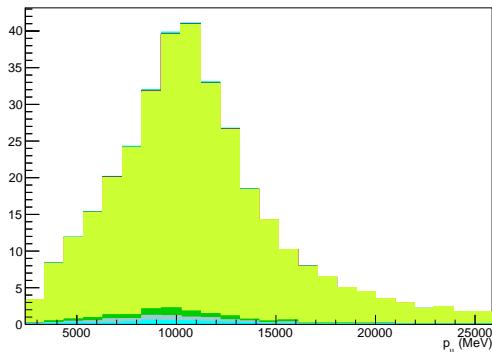
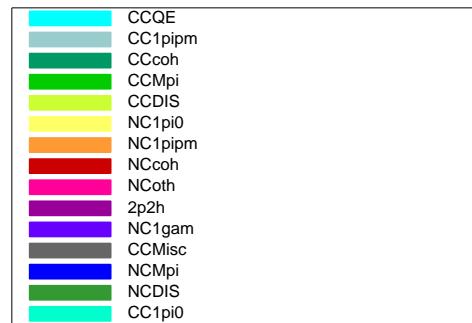


Figure A.5.: Breakdown by interaction mode of the PC atmospheric samples.

2781 A.5. Upward-Going Muon Samples

2782 The breakdown for upward-going muons is illustrated in Figure A.6. These samples
 2783 are significantly dominated by DIS interactions with energies extending up into the
 2784 hundreds of GeV.

(a) Up- μ stopping events(b) Up- μ through going non showering events(c) Up- μ through going showering events**Figure A.6.:** Breakdown by interaction mode of the atmospheric upward going muon samples.

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