

Simultaneous Measurement of Oscillation Parameters in Beam and Atmospheric Neutrino Data from Tokai-to-Kamioka and Super-Kamiokande Experiments

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¹⁰ A Dissertation Submitted to Oxford University

¹¹ for the Degree of Doctor of Philosophy

¹²

13 **Simultaneous Measurement of**

14 **Oscillation Parameters in Beam and**

15 **Atmospheric Neutrino Data from**

16 **Tokai-to-Kamioka and**

17 **Super-Kamiokande Experiments**

18 *Abstract*

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Declaration

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This dissertation is the result of my own work, except where ex-
34 plicit reference is made to the work of others, and has not been sub-
35 mitted for another qualification to this or any other university. This
36 dissertation does not exceed the word limit for the respective Degree
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¹³² Chapter 1

¹³³ Introduction

¹³⁴ **Chapter 2**

¹³⁵ **Neutrino Oscillation Physics**

¹³⁶ When first proposed, neutrinos were expected to be massless fermions that only in-
¹³⁷ teract through weak and gravitational forces. This meant they were very difficult to
¹³⁸ detect as they can pass through significant amounts of matter without interacting. De-
¹³⁹ spite this, experimental neutrino physics has developed with many different detection
¹⁴⁰ techniques and neutrino sources being used today. In direct tension with standard
¹⁴¹ model physics, neutrinos have been determined to oscillate between different lepton
¹⁴² flavours, requiring them to have mass.

¹⁴³ The observation techniques which lead to the discovery of the neutrino are doc-
¹⁴⁴ umented in section 2.1. The theory underpinning neutrino oscillation is described
¹⁴⁵ in section 2.2 and includes the approximations which can be made to simplify the
¹⁴⁶ understanding of neutrino oscillation in the two-flavour approximation. Past, current,
¹⁴⁷ and future neutrino experiments are detailed in section 2.3, including the reactor,
¹⁴⁸ atmospheric, and long-baseline accelerator neutrino sources that have been used to
¹⁴⁹ successfully constrain oscillation parameters. Finally, the current state of oscillation
¹⁵⁰ parameter measurements are summarised in section 2.4.

¹⁵¹ **2.1 Discovery of Neutrinos**

¹⁵² At the start of the 20th century, the electrons emitted from the β -decay of the nucleus
¹⁵³ were found to have a continuous energy spectrum [1,2]. This observation seemingly
¹⁵⁴ broke the energy conservation invoked within that period's nuclear models. Postulated

155 in 1930 by Pauli as the solution to this problem, the neutrino (originally termed
156 “neutron”) was theorized to be an electrically neutral spin-1/2 fermion with a mass of
157 the same order of magnitude as the electron [3]. This neutrino was to be emitted with
158 the electron in β -decay to alleviate the apparent breaking of energy conservation. As a
159 predecessor of today’s weak interaction model, Fermi’s theory of β -decay developed
160 the understanding by coupling the four constituent particles; electron, proton, neutron,
161 and neutrino, into a consistent model [4].

162 Whilst Pauli was not convinced of the ability to detect neutrinos, the first observa-
163 tions of the particle were made in the mid-1950s when neutrinos from a reactor were
164 observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [5, 6]. The detector
165 consisted of two parts: a neutrino interaction medium and a liquid scintillator. The
166 interaction medium was built from two water tanks. These were loaded with cadmium
167 chloride to allow increased efficiency of neutron capture. The positron emitted from
168 IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured
169 on the cadmium via $n + {}^{108}Cd \rightarrow {}^{109*}Cd \rightarrow {}^{109}Cd + \gamma$, producing a delayed signal. An
170 increase in the coincidence rate was observed when the reactor was operating which
171 was interpreted as interactions from neutrinos generated in the reactor.

172 After the discovery of the ν_e , the natural question of how many flavours of neutrino
173 exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven
174 National Laboratory [7]. A proton beam was directed at a beryllium target, gener-
175 ating a π -dominated beam which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the
176 subsequent interactions of the ν_μ were observed. As the subsequent interaction of
177 the neutrino generates muons rather than electrons, it was determined the ν_μ was
178 fundamentally different from ν_e . The final observation to be made was that of the ν_τ
179 from the DONUT experiment [8]. Three neutrinos seem the obvious solution as it
180 mirrors the known number of charged lepton (as they form weak isospin doublets) but

181 there could be evidence of more. Several neutrino experiments have found anomalous
182 results [9, 10] which could be attributed to sterile neutrinos. However, cosmological
183 observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [11], as mea-
184 sured from the cosmic microwave background power spectrum, and Stanford Linear
185 Accelerator found the number of active neutrino flavours to be $N_\nu 2.9840 \pm 0.0082$ [12]
186 from measurements of the Z-decay width.

187 2.2 Theory of Neutrino Oscillation

188 As direct evidence of beyond Standard Model physics, a neutrino generated with
189 lepton flavour α can change into a different lepton flavour β after propagating some
190 distance. This phenomenon is called neutrino oscillation and requires that neutrinos
191 must have a non-zero mass (as seen in subsection 2.2.1). This observation is direct
192 evidence of beyond standard model physics. This behaviour has been characterised
193 by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13–15] mixing matrix which
194 describes how the flavour and mass of neutrinos are associated. This is analogous to
195 the Cabibbo-Kobayashi-Maskawa (CKM) [16] matrix measured in quark physics.

196 2.2.1 Three Flavour Oscillations

197 The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ (indexed
198 ν_α), which are eigenstates of the weak interaction and three mass eigenstates, ν_1 , ν_2 and
199 ν_3 (indexed ν_i). Each mass eigenstate is the superposition of all three flavour states,

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i} |\nu_\alpha\rangle. \quad (2.1)$$

200 Where U is the PMNS matrix which is unitary and connects the mass and flavour

201 eigenstates.

202 The weak interaction couples to flavour eigenstates so neutrinos interact with

203 leptons of the same flavour. The propagation of a neutrino flavour eigenstate, in a

204 vacuum, can be re-written as a plane-wave solution to the time-dependent Schrödinger

205 equation,

$$|\nu_\alpha(t)\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

206 The probability of observing a neutrino of flavour eigenstate β from one which

207 originated as flavour α can be calculated as,

$$P(\nu_\alpha \rightarrow \nu_\beta) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* e^{-i(\phi_j - \phi_i)} \quad (2.3)$$

208 The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the

209 three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position

210 coordinates). Therefore,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

211 For a relativistic particle, $E_i \gg m_i$,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

212 Making the approximations that neutrinos are relativistic, the mass eigenstates
213 were created with the same energy and that $x = L$, where L is the distance traveled by
214 the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

215 where $\Delta m_{ij}^2 = m_j^2 - m_i^2$. This, combined with further use of unitarity relations
216 results in Equation 2.3 becoming

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \quad (2.7)$$

217 Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last term is
218 included for the oscillation probability of antineutrinos.

219 Typically, the PMNS matrix is parameterised into three mixing angles, a charge
220 parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

221 Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often
 222 grouped; (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many
 223 neutrino experiments aim to measure the PMNS parameters from a wide array of
 224 origins, as is the purpose of this thesis.

225 The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8 is only
 226 included for completeness. For an oscillation analysis experiment, any terms contain-
 227 ing this phase disappear due to taking the expectation value of the PMNS matrix.
 228 Measurements of these phases are typically performed by experiments searching for
 229 neutrino-less double β -decay [17].

230 A two flavour approximation can be obtained when one assumes the third mass
 231 eigenstate is degenerate with another. As discussed in section 2.3, it is found that
 232 $\Delta m_{21}^2 \ll |\Delta m_{31}^2|$. This results in the two flavour approximation being reasonable for
 233 understanding the features of the oscillation. In this two flavour case, the mixing
 234 matrix becomes,

$$U_{2 \text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

²³⁵ This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

²³⁶ Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is a sinusoidal
²³⁷ function depending upon the distance over which the neutrino propagates. The
²³⁸ frequency and amplitude of oscillation are dependent upon $\Delta m^2/4E$ and $\sin^2 2\theta$,
²³⁹ respectively. The oscillation probabilities presented thus far assume $c = 1$, where
²⁴⁰ c is the speed of light in vacuum. In more familiar units, the maximum oscillation
²⁴¹ probability for a fixed value of θ is given at $L[km]/E[GeV] \sim 1.27/\Delta m^2$. It is this
²⁴² calculation that determines the best L/E value for a given experiment to be designed
²⁴³ around for measurements of a specific value of Δm^2 .

²⁴⁴ 2.2.2 The MSW Effect

²⁴⁵ The theory of neutrino oscillation in a vacuum has been described in subsection 2.2.1.
²⁴⁶ However, the beam neutrinos and atmospheric neutrinos originating from below the
²⁴⁷ horizon propagate through matter in the Earth. The coherent scattering of neutrinos
²⁴⁸ from a material target modifies the Hamiltonian of the system. This results in a change
²⁴⁹ in the oscillation probability. Notably, charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$,
²⁵⁰ propagated by a W boson) only affects electron neutrinos whereas the neutral current
²⁵¹ scattering ($\nu_l + l^- \rightarrow \nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino
²⁵² flavours equally. In the two-flavour approximation, the effective mixing parameter
²⁵³ becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F is Fermi's constant. It is clear to see that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for $\Delta m^2 > 0$ which results in a divergent mixing parameter. This resonance is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect (or more colloquially, the matter resonance) which regenerates the electron neutrino component of the neutrino flux [18–20]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

At densities lower than this critical value, the oscillation probability will be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$ [21]. The resonance occurring from the MSW effect depends on the sign of Δm^2 . Therefore, any neutrino oscillation experiment which observes neutrinos and antineutrinos which have propagated through matter can have some sensitivity to the ordering of the neutrino mass eigenstates.

2.3 Neutrino Oscillation Measurements

As evidence of beyond standard model physics, the 2015 Nobel Prize in Physics was awarded to the Super-Kamiokande (SK) [22] and Sudbury Neutrino Observatory (SNO) [23] collaborations for the first definitive observation of solar and atmospheric

²⁷⁰ neutrino oscillation [24]. Since then, the field has seen a wide array of oscillation
²⁷¹ measurements from a variety of neutrino sources. As seen in subsection 2.2.1, the
²⁷² neutrino oscillation probability is dependent on the ratio of the propagation baseline, L ,
²⁷³ to the neutrino energy, E . It is this ratio that determines the type of neutrino oscillation
²⁷⁴ a particular experiment is sensitive to.

²⁷⁵ As illustrated in Figure 2.1, there are many neutrino sources that span a wide
²⁷⁶ range of energies. The least energetic neutrinos are from diffuse supernovae and
²⁷⁷ terrestrial neutrinos at $O(1)$ MeV whereas the most energetic neutrinos originate from
²⁷⁸ atmospheric and galactic neutrinos of $> O(1)$ TeV.

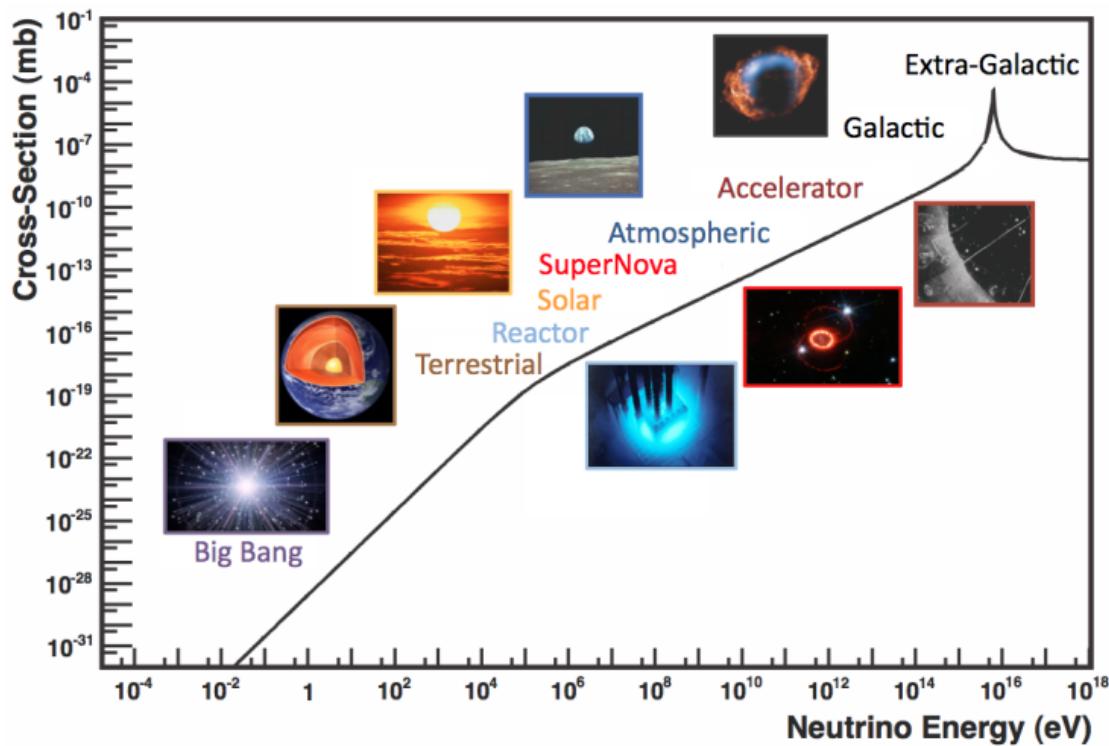


Figure 2.1: The cross-section of neutrinos from various natural and man-made sources as a function of neutrino energy. Taken from [25]

²⁷⁹ 2.3.1 Solar Neutrinos

²⁸⁰ Solar neutrinos are emitted from fusion reaction chains at the center of the Sun. The
²⁸¹ solar neutrino flux, given as a function of neutrino energy for different fusion and
²⁸² decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion generates the
²⁸³ largest flux of neutrinos, the neutrinos are of low energy and are difficult to reconstruct
²⁸⁴ due to the IBD interaction threshold of 1.8MeV. Consequently, most experiments focus
²⁸⁵ on the neutrinos from the decay of 8B (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher
²⁸⁶ energy.

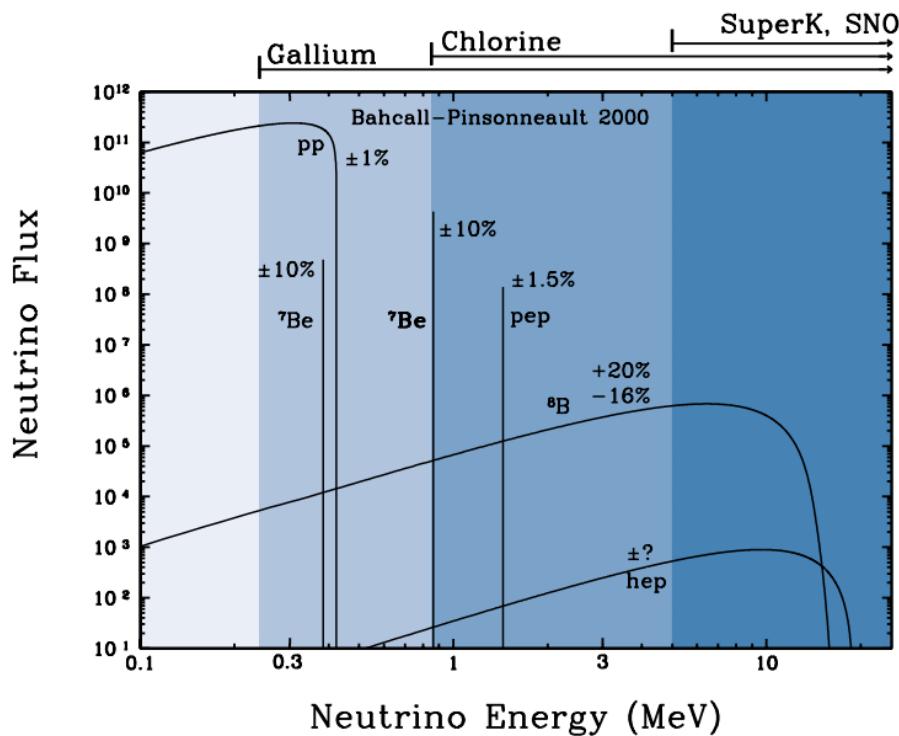


Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [26].

²⁸⁷ The first measurements of solar neutrinos observed a significant reduction in the
²⁸⁸ event rate compared to predictions from the Standard Solar Model [27, 28]. The
²⁸⁹ proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a

²⁹⁰ precursory version of the PMNS model [29]. The Kamiokande [30], Gallex [31] and
²⁹¹ Sage [32] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

²⁹² The conclusive solution to this problem was determined by the SNO collaboration
²⁹³ [33]. Using a deuterium water target to observe 8B neutrinos, the event rate of charged
²⁹⁴ current (CC), neutral current (NC), and elastic scattering (ES) interactions (Given in
²⁹⁵ Equation 2.13) was simultaneously measured. CC events can only occur for electron
²⁹⁶ neutrinos, whereas the NC channel is agnostic to neutrino flavour, and the ES reaction
²⁹⁷ has a slight excess sensitivity to electron neutrino interactions. This meant that there
²⁹⁸ were direct measurements of the ν_e and ν_x neutrino flux. It was concluded that the
²⁹⁹ CC and ES interaction rates were consistent with the deficit previously observed.
³⁰⁰ Most importantly, the NC reaction rate was only consistent with the others under the
³⁰¹ hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (\text{CC}) \\ \nu_x + d &\rightarrow p + n + \nu_x & (\text{NC}) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (\text{ES}) \end{aligned} \tag{2.13}$$

³⁰² Many experiments have since measured the neutrino flux of different interaction
³⁰³ chains within the sun [34–36]. The most recent measurement was that of CNO neutrinos
³⁰⁴ which were recently observed with 5σ significance by the Borexino collaboration.
³⁰⁵ Future neutrino experiments aim to further these spectroscopic measurements of
³⁰⁶ different fusion chains within the Sun [37–39]. Solar neutrinos act as an irreducible
³⁰⁷ background for dark matter experiments like DARWIN but oscillation parameter
³⁰⁸ measurements can be made [40].

³⁰⁹ 2.3.2 Atmospheric Neutrinos

- ³¹⁰ The interactions of primary cosmic ray protons in Earth's upper atmosphere generate
³¹¹ showers of energetic hadrons. These are mostly pions and kaons which when they
³¹² decay produce a natural source of neutrinos spanning energies of MeV to TeV [41].
³¹³ The main decay is via

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu) \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu) \end{aligned} \tag{2.14}$$

³¹⁴ such that for a single pion decay, three neutrinos are typically produced. The
³¹⁵ atmospheric neutrino flux energy spectra as predicted by the Bartol [42], Honda
³¹⁶ [43–45], and FLUKA [46] models are illustrated in Figure 2.3. The flux distribution
³¹⁷ peaks at an energy of $O(10)\text{GeV}$. The uncertainties associated with these models
³¹⁸ are dominated by the hadronic production of kaon and pions as well as the primary
³¹⁹ cosmic flux.

³²⁰ Unlike long-baseline experiments which have a fixed baseline, the distance at-
³²¹ mospheric neutrinos propagate is dependent upon the zenith angle at which they
³²² interact. This is illustrated in Figure 2.4. Neutrinos that are generated directly above
³²³ the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the height of the atmosphere
³²⁴ whereas neutrinos that interact directly below the detector ($\cos(\theta) = -1.0$) have to
³²⁵ travel a length equal to the diameter of the Earth. This means atmospheric neutrinos
³²⁶ have a baseline that varies from $O(20)\text{km}$ to $O(6 \times 10^3)\text{km}$. Any neutrino generated
³²⁷ at or below the horizon will be subject to matter effects as they propagate through the
³²⁸ Earth.

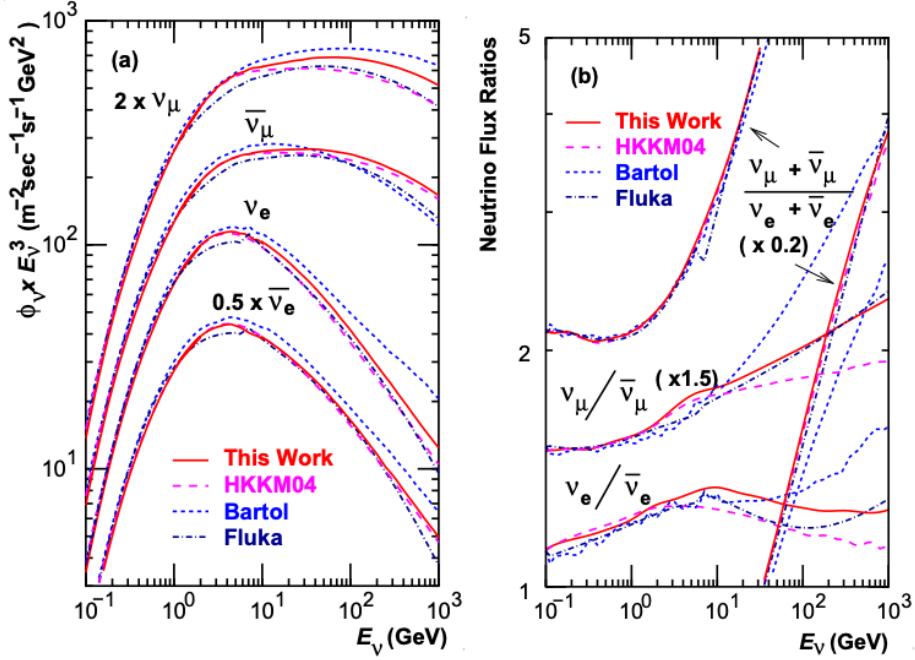


Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [43], the 2004 Honda model (“HKKM04”) [44], the Bartol model [42] and the FLUKA model [46]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [43].

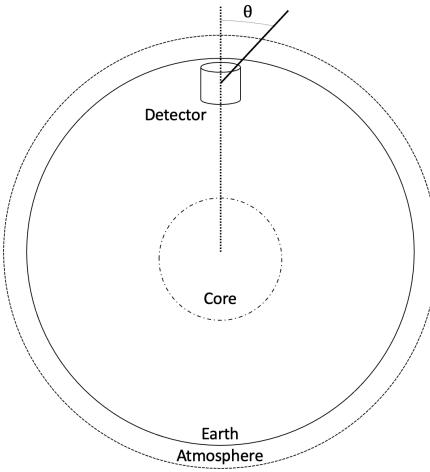


Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [47].

Figure 2.5 highlights the neutrino flux as a function of the zenith angle for different

slices of neutrino energy. For medium to high-energy neutrinos (and to a lesser degree for low-energy neutrinos), the flux is approximately symmetric around $\cos(\theta) = 0$.

To the accuracy of this approximation, the systematic uncertainties associated with atmospheric flux for comparing upward-going and down-going neutrino cancels. This allows the down-going events, which are mostly insensitive to oscillation probabilities, to act as an unoscillated prediction (similar to a near detector in an accelerator neutrino experiment).

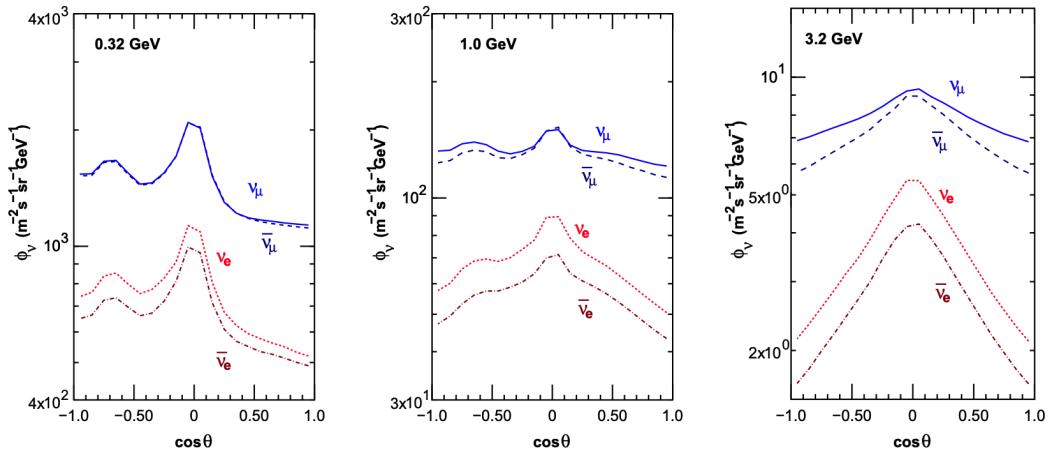


Figure 2.5: Prediction of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [45]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [42], Honda [43] and FLUKA [46] are given in [47].

Precursory hints of atmospheric neutrinos were observed in the mid-1960s searching for $\nu_\mu + X \xrightarrow{(-)} X^* + \mu^\pm$ [48], although it was called an anomaly at the time of measurement. This was succeeded with the IMB-3 [49] and Kamiokande [50] experiments which measured the ratio of muon neutrinos compared to electron neutrinos $R(\nu_\mu/\nu_e)$. Both experiments were found to have a consistent deficit of muon neutrinos, with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.60^{+0.07}_{-0.06} \pm 0.05$. Super-Kamiokande (SK) [47] extended this analysis by fitting oscillation parameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}$ eV 2 .

Since then, atmospheric neutrino experiments have been making precision measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ discovery of ν_τ

³⁴⁸ appearance [51]. Figure 2.6 illustrates the current estimates on the atmospheric mixing
³⁴⁹ parameters from a wide range of atmospheric and accelerator neutrino observatories.

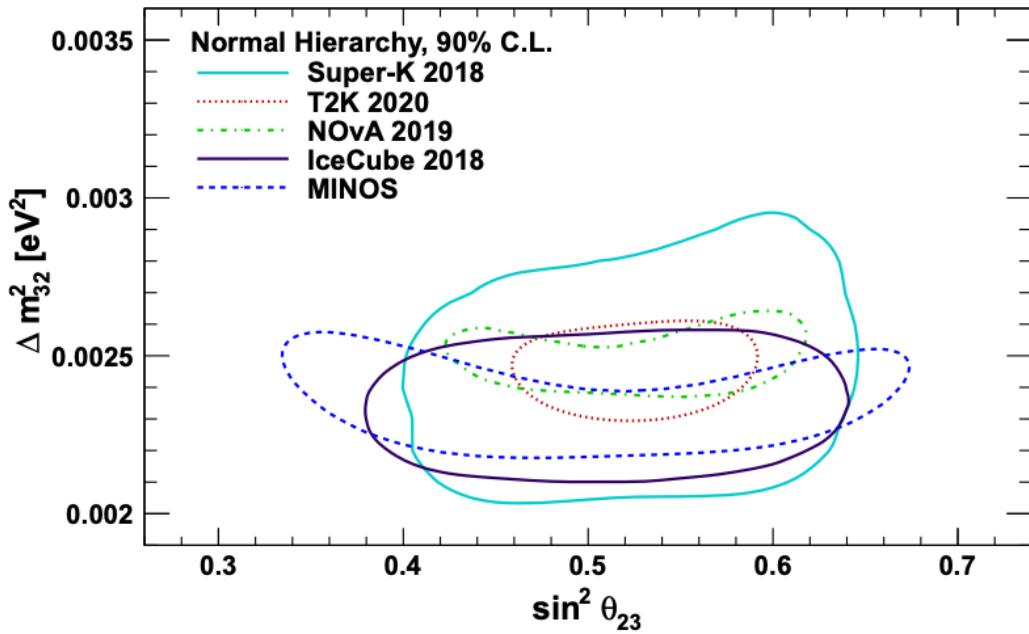


Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long baseline experiments: SK [52], T2K [53], NOvA [54], IceCube [55] and MINOS [56]. Figure taken from [57].

³⁵⁰ 2.3.3 Accelerator Neutrinos

³⁵¹ The concept of using a man-made “neutrino beam” was first realised in 1962 [58].
³⁵² Since then, many experiments have followed which all use the same fundamental
³⁵³ concepts. Typically, a proton beam is aimed at a target producing charged mesons that
³⁵⁴ decay to neutrinos. The mesons can be sign-selected by the use of magnetic focusing
³⁵⁵ horns to generate a neutrino or antineutrino beam. Pions are the primary meson that
³⁵⁶ decay and depending on the orientation of the magnetic field, a muon (anti-)neutrino
³⁵⁷ beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. The decay of muons and
³⁵⁸ kaons does result in an irreducible intrinsic electron neutrino background. In T2K,
³⁵⁹ this background contamination is $O(< 1\%)$ [59]. There is also an approximately $\sim 5\%$

³⁶⁰ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated via the same decays. As the beam is
³⁶¹ generated by proton interactions (rather than anti-proton interactions), the wrong-sign
³⁶² component in the antineutrino beam is larger when operating in neutrino mode.

³⁶³ Tuning the proton energy in the beam and using beam focusing techniques allows
³⁶⁴ the neutrino energy to be set to a value that maximises the disappearance oscillation
³⁶⁵ probability in the L/E term in Equation 2.10. This means that accelerator experiments
³⁶⁶ are typically more sensitive to the mixing parameters as compared to a natural neutrino
³⁶⁷ source. However, the disadvantage compared to atmospheric neutrino experiments is
³⁶⁸ that the baseline has to be shorter due to the lower flux. Consequently, there is typically
³⁶⁹ less sensitivity to matter effects and the ordering of the neutrino mass eigenstates.

³⁷⁰ A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.15)$$

³⁷¹ where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of neutrinos
³⁷² with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and $\epsilon(\vec{x})$ is the
³⁷³ efficiency and resolution of the detector. In order to leverage the most out of an
³⁷⁴ accelerator neutrino experiment, the flux and cross-section systematics need to be
³⁷⁵ constrained. This is typically done via the use of a “near detector”, situated at a baseline
³⁷⁶ of $O(1)$ km. This detector observes the unoscillated neutrino flux and constrains the
³⁷⁷ parameters used within the flux and cross-section model.

³⁷⁸ The first accelerator experiments to precisely measure oscillation parameters were
³⁷⁹ MINOS [60] and K2K [61]. These experiments confirmed the ν_μ disappearance seen in
³⁸⁰ atmospheric neutrino experiments by finding consistent parameter values for $\sin^2(\theta_{23})$
³⁸¹ and Δm_{23}^2 . The current generation of accelerator neutrino experiments, T2K and NO ν A

³⁸² extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead the sensitivity to atmospheric mix-
³⁸³ ing parameters as seen in Figure 2.6 [62]. The two experiments differ in their peak
³⁸⁴ neutrino energy, baseline, and detection technique. The NO ν A experiment is situated
³⁸⁵ at a baseline of 810km from the NuMI beamline which delivers 2GeV neutrinos. The
³⁸⁶ T2K neutrino beam is peaked around 0.6GeV and propagates 295km. The NO ν A
³⁸⁷ experiment also uses functionally identical detectors (near and far) which allow the
³⁸⁸ approximate cancellation of detector systematics whereas T2K uses a plastic scintil-
³⁸⁹ lator technique at the near detector and a water Cherenkov far detector. The future
³⁹⁰ generation experiments DUNE [63] and Hyper-Kamiokande [64] will succeed these
³⁹¹ experiments as the high-precision era of neutrino oscillation parameter measurements
³⁹² develops.

³⁹³ Several anomalous results have been observed in the LSND [9] and MiniBooNE [10]
³⁹⁴ detectors which were designed with purposefully short baselines. Parts of the neu-
³⁹⁵ trino community attributed these results to oscillations induced by a fourth “sterile”
³⁹⁶ neutrino [65] but several searches in other experiments, MicroBooNE [66] and KAR-
³⁹⁷ MEN [67], found no hints of additional neutrino species. The solution to the anomalous
³⁹⁸ results is still being determined.

³⁹⁹ 2.3.4 Reactor Neutrinos

⁴⁰⁰ As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very
⁴⁰¹ useful man-made source of electron antineutrinos. For reactors that use low-enriched
⁴⁰² uranium ^{235}U as fuel, the antineutrino flux is dominated by the β -decay fission of ^{235}U ,
⁴⁰³ ^{238}U , ^{239}Pu and ^{241}Pu [68] as illustrated in Figure 2.7.

⁴⁰⁴ Due to their low energy, reactor electron antineutrinos predominantly interact
⁴⁰⁵ via the inverse β -decay (IBD) interaction. The typical signature contains two signals

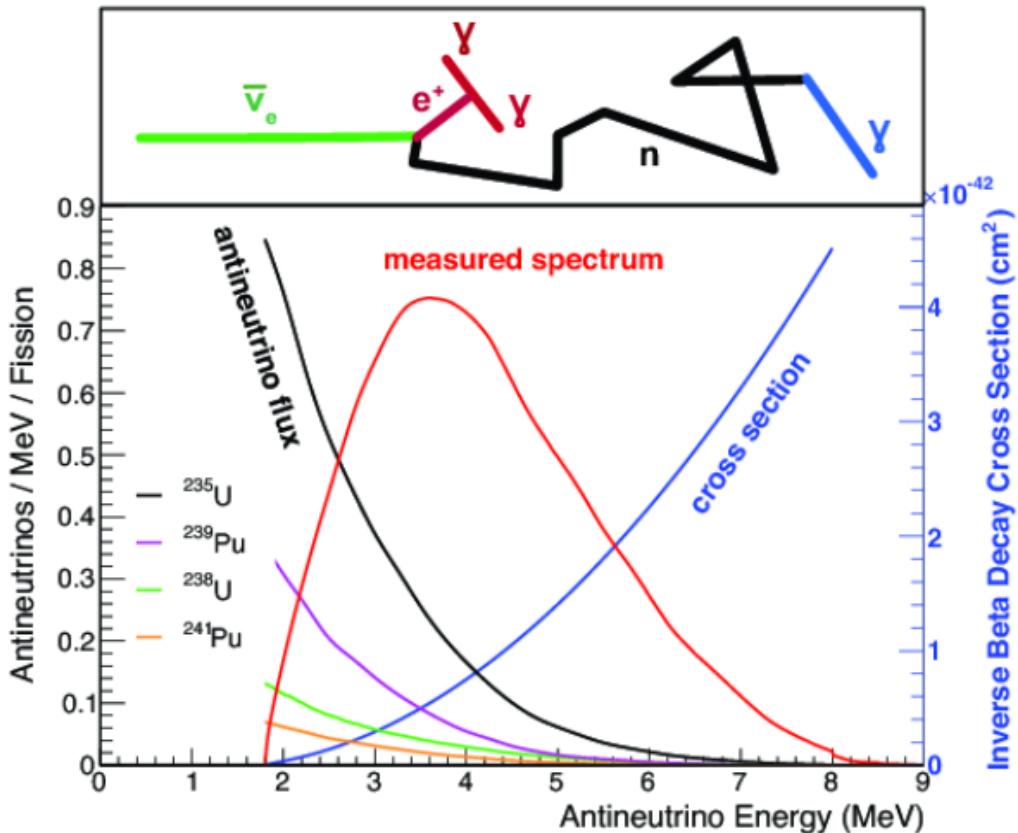


Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [69].

delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihilation, and secondly the photons emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [70]. Recently, SK included gadolinium dopants into the ultra-pure water to increase the energy released from the photon cascade to $\sim 8\text{MeV}$ and reduce the time of the delayed signal to $\sim 28\mu\text{s}$.

There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured the $\sin^2(\theta_{13})$ and Δm_{23}^2 oscillation parameters. Daya Bay [71], RENO [72] and Double Chooz [73] have all provided precise measurements, with the first discovery of a

415 non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K [73]). The
416 constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field and are often used as
417 external inputs to accelerator neutrino experiments to improve their sensitivity to δ_{CP}
418 and mass hierarchy determination. JUNO-TAO [74], a small collaboration within the
419 larger JUNO experiment, is a next-generation reactor experiment that aims to precisely
420 measure the isotopic antineutrino yields from the different fission chains. Alongside
421 this, it aims to explain the ‘5MeV excess’ [75–77] by conducting a search for sterile
422 neutrinos with a mass scale of around 1eV.

423 Kamland [78] is the only experiment to have observed reactor neutrinos using a
424 long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have
425 sensitivity to Δm_{12}^2 . Combined with the SK solar neutrino experiment, the combined
426 analysis puts the most stringent constraint on Δm_{12}^2 [79].

427 2.4 Summary

428 Since observing the first evidence of neutrino oscillations in the late 1990’s, numerous
429 measurements of the mixing parameters have been made. Many experiments use
430 neutrinos as a tool for discovery of new physics (diffuse supernova background,
431 neutrinoless double beta decay and others) so the PMNS parameters are summarised
432 in the Particle Data Group (PDG) review tables. The analysis presented in this thesis
433 focuses on the 2020 T2K oscillation analysis presented in [80] where the 2018 PDG
434 constraints [81] were used. These constraints are outlined in Table 2.1.

435 The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance,
436 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is take as the average best-fit from the combination of Daya Bay,
437 Reno and Double Chooz. It is often used as a prior uncertainty within other neu-
438 trino oscillation experiments, typically termed the reactor constraint. The $\sin^2(\theta_{12})$

Parameter	2018 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.12 \pm 0.08) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H., Q1)	$0.421^{+0.033}_{-0.025}$
$\sin^2(\theta_{23})$ (I.H., Q2)	$0.592^{+0.023}_{-0.030}$
$\sin^2(\theta_{23})$ (N.H., Q1)	$0.417^{+0.025}_{-0.028}$
$\sin^2(\theta_{23})$ (N.H., Q2)	$0.597^{+0.024}_{-0.030}$
Δm_{32}^2 (I.H.)	$(-2.56 \pm 0.04) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.51 \pm 0.05) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2018 Particle Data Group constraints of the oscillation parameters taken from [81]. The value of Δm_{23}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

parameter is predominantly measured through electron neutrino disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline reactor neutrino experiment Kamland also has sensitivity to this parameter and is used in a joint fit to solar data from SNO and SK, using the reactor constraint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric neutrino experiments. The PDG value is a joint fit of T2K, NOvA, MINOS and IceCube DeepCore experiments. The latest T2K-only measurement, provided at Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [80]. The PDG constraint on Δm_{12}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{23}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchy separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [80]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were

455 rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analysis
456 have reduced the rejection intervals to 90%. Since the 2018 PDG publication, there has
457 been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [82], alongside updated
458 Δm_{23}^2 and $\sin^2(\theta_{23})$ measurements.

459 Throughout this thesis, several sample spectra predictions and contours are pre-
460 sented which require oscillation parameters to be assumed. Table 2.2 defines two sets
461 of oscillation parameters, with “Asimov A” set being close to the preferred values
462 from a previous T2K-only fit [83] and “Asimov B” being CP-conserving and further
463 from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

464 **Chapter 3**

465 **T2K and SK Experiment Overview**

466 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK) collabora-
467 ration has been leading atmospheric neutrino oscillation analyses for over two decades.
468 The detector has provided some of the strongest constraints on proton decay and the
469 first precise measurements of the Δm_{23}^2 and $\sin^2(\theta_{23})$ neutrino oscillation parameters.
470 The ability of the detector to low-energy neutrino events has been significantly im-
471 proved with the recent gadolinium doping of the ultra-pure water target. The history,
472 detection technique, and operation of the SK detector is described in section 3.1.

473 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline ex-
474 periments to use both neutrino and antineutrino beams to precisely measure the
475 charge parity violation within the neutrino sector. With the SK detector observing
476 the oscillated neutrino flux, the T2K experiment observed the first hints of a non-zero
477 $\sin^2(\theta_{13})$ measurement and continues to lead the field with the constraints it provides
478 on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{23}^2 and δ_{CP} . The techniques which T2K uses in gener-
479 ating its neutrino beam as well as the near-detector used to constrain the flux and
480 cross-section parameters used in this analysis are documented in section 3.2.

481 **3.1 The Super-Kamiokande Experiment**

482 The SK experiment began taking data in 1996 [84] and has had many modifications
483 throughout its lifespan. There have been seven defined periods of data taking as
484 noted in Table 3.1. Data taking began in SK-I which ran for five years. Between the

485 SK-I and SK-II periods, a significant proportion of the PMTs were damaged during
 486 maintenance. Those that survived were equally distributed throughout the detector
 487 in the SK-II era, which resulted in a reduced photo-coverage. From SK-III onwards,
 488 repairs to the detector meant the full suite of PMTs was operational. Before the
 489 start of SK-IV, the data acquisition and electronic systems were upgraded. Between
 490 SK-IV and SK-V, a significant effort was placed into tank open maintenance and
 491 repair/replacement of defective PMTs, a task for which the author of this thesis was
 492 required. Consequently, the detector conditions were significantly different between
 493 the two operational periods. SK-VI saw the start of the 0.01% gadolinium doped water.
 494 SK-VII, which started during the writing of this thesis, has increased the gadolinium
 495 concentration to 0.03% for continued operation [85].

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

Table 3.1: The various SK periods and respective live-time. The SK-VI live-time is calculated until 1st April 2022. SK-VII started during the writing of this thesis.

496 3.1.1 The SK Detector

497 The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank with a
 498 diameter 39.3m and height 41.1m filled with ultrapure water [86]. A diagram of the
 499 significant components of the SK detector is given in Figure 3.1. The SK detector is
 500 situated in the Kamioka mine in Gifu, Japan. The mine is underground with roughly
 501 1km rock overburden (2.7km water equivalent overburden) [87]. At this depth, the

502 rate of cosmic ray muons is significantly decreased to a value of $\sim 2\text{Hz}$. The top of
 503 the tank is covered with stainless steel which is designed as a working platform for
 504 maintenance, calibration, and location for high voltage and data acquisition electronics.

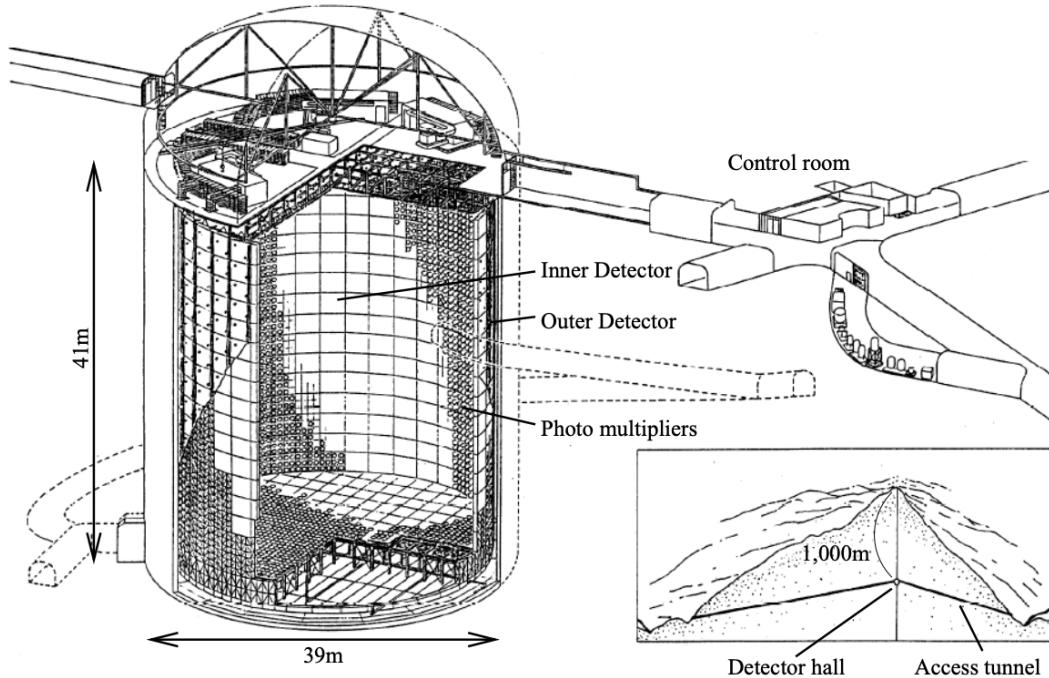


Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [88].

505 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated inside the
 506 tank, with an approximate 2m gap between this structure and the outer tank wall. The
 507 purpose of this structure is to support the photomultiplier tubes (PMTs). The volume
 508 inside and outside the support structure is referred to as the inner detector (ID) and
 509 outer detector (OD), respectively. In the SK-IV era, the ID and OD are instrumented
 510 by 11,129 50cm and 1,885 20cm PMTs respectively [86]. The ID contains a 32kton
 511 mass of water. Many analyses performed at SK use a “fiducial volume” defined by the
 512 volume of water inside the ID excluding some distance to the ID wall. This reduces the
 513 volume of the detector which is sensitive to neutrino events but reduces radioactive
 514 backgrounds and allows for better reconstruction performance. The nominal fiducial

515 volume is defined as the area contained inside 2m from the ID wall for a total of
516 22.5kton water [89].

517 The two regions of the detector (ID and OD) are optically separated with opaque
518 black plastic. The purpose of this is to determine whether a track entered or exited
519 the ID. This allows cosmic ray muons and partially contained events to be tagged and
520 separated from neutrino events entirely contained within the ID. This black plastic is
521 also used to cover the area between the ID PMTs to reduce photon reflection from the
522 ID walls. Opposite to this, the OD is lined with a reflective material to allow photons to
523 reflect around inside the OD until collected by one of the PMTs. Furthermore, each OD
524 PMT is backed with $50 \times 50\text{cm}$ plates of wavelength shifting acrylic which increases
525 the efficiency of light collection [87].

526 In the SK-IV data-taking period, the photocathode coverage of the detector, or the
527 fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [87]. The PMTs have a
528 quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$ for
529 photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$. The proportion of photoelectrons
530 that produce a signal in the dynode of a PMT, termed the collection efficiency, is
531 $> 70\%$ [87]. The PMTs used within SK are most sensitive to photons with wavelength
532 $300\text{nm} \leq \lambda \leq 600\text{nm}$ [87]. One disadvantage of using PMTs as the detection media
533 is that the Earth's geomagnetic field can modify its response. Therefore, a set of
534 compensation coils is built around the inner surface of the detector to mitigate this
535 effect [90].

536 As mentioned, the SK detector is filled with ultrapure water, which in a perfect
537 world would contain no impurities. However, bacteria and organic compounds can
538 significantly degrade the water quality. This decreases the attenuation length, which
539 reduces the total number of photons that hit a PMT. To combat this, a sophisticated
540 water treatment system has been developed [87, 91]. UV lights, mechanical filters,

541 and membrane degasifiers are used to reduce the bacteria, suspended particulates,
542 and radioactive materials from the water. The flow of water within the tank is also
543 critical as it can remove stagnant bacterial growth or build-up of dust on the surfaces
544 within the tank. Gravity drifts impurities in the water towards the bottom of the
545 tank which, if left uncontrolled, can create asymmetric water conditions between
546 the top and bottom of the tank. Typically, the water entering the tank is cooled
547 below the ambient temperature of the tank to control convection and inhibit bacteria
548 growth. Furthermore, the rate of dark noise hits within PMTs is sensitive to the PMT
549 temperature [92] so controlling the temperature gradients within the tank is beneficial
550 for stable measurements.

551 SK-VI is the first phase of the SK experiment to use gadolinium dopants within
552 the ultrapure water [85]. As such, the SK water system had to be replaced to avoid
553 removing the gadolinium concentrate from the ultrapure water [93]. For an inverse
554 β -decay (IBD) interaction in a water target, the emitted neutron is thermally captured
555 on hydrogen. This process releases 2.2MeV γ rays which are difficult to detect as
556 the resulting Compton scattered electrons are very close to the Cherenkov threshold,
557 limiting the number of photons produced. Thermal capture of neutrons on gadolin-
558 ium generates γ rays with higher energy (8MeV [70]) meaning they are more easily
559 detected. SK-VI has 0.01% Gd loading (0.02% gadolinium sulphate by mass) which
560 causes \approx 50% of neutrons emitted by IBD to be captured on gadolinium [94, 95].
561 Whilst predominantly useful for low energy analyses, Gd loading allows better $\nu/\bar{\nu}$
562 separation for atmospheric neutrino event selections [96]. Efforts are currently in place
563 to increase the gadolinium concentrate to 0.03% for \approx 75% neutron capture efficiency
564 on gadolinium [97]. The final stage of loading targets 0.1% concentrate.

565 3.1.2 Calibration

566 The calibration of the SK detector is documented in [86] and summarised below. The
567 analysis presented within this thesis is dependent upon ‘high energy events’ (Charged
568 particles with $O(> 100)\text{MeV}$ momenta). These are events that are expected to generate
569 a larger number of photons such that each PMT will be hit with multiple photons.
570 The reconstruction of these events depends upon the charge deposited within each
571 PMT and the timing response of each individual PMT. Therefore, the most relevant
572 calibration techniques to this thesis are outlined.

573 Before installation, 420 PMTs were calibrated to have identical charge responses
574 and then distributed throughout the tank in a cross-shape pattern (As illustrated by
575 Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed
576 at similar geometric positions within SK to be calibrated against. To perform this
577 calibration, a xenon lamp is located at the center of the SK tank which flashes uniform
578 light at 1Hz. This allows for geometrical effects, water quality variation, and timing
579 effects to be measured in-situ throughout normal data-taking periods.

580 When specifically performing calibration of the detector (in out-of-data taking
581 mode), the water in the tank was circulated to avoid top/bottom asymmetric water
582 quality. Any non-uniformity within the tank significantly affects the PMT hit proba-
583 bility through scattering or absorption. This becomes a dominant effect for the very
584 low-intensity light sources discussed later which are designed such that only one
585 photon is incident upon a given PMT.

586 The “gain” of a PMT is defined as the ratio of the total charge of the signal produced
587 compared to the charge of photoelectrons emitted by the photocathodes within the
588 PMT. To calibrate the signal of each PMT, the “relative” and “absolute” gain values are

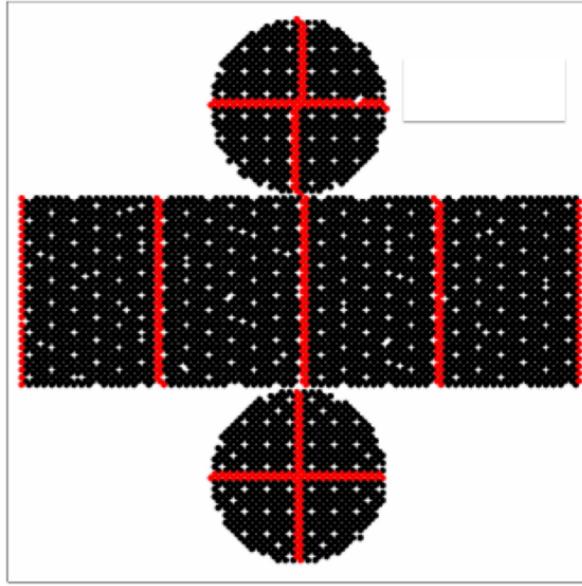


Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [86].

589 measured. The relative gain is the variation of gain among each of the PMTs whereas
 590 the absolute gain is the average gain of all PMTs.

591 The relative gain is calibrated as follows. A laser is used to generate two measure-
 592 ments: a high-intensity flash that illuminates every PMT with a sufficient number of
 593 photons, and a low-intensity flash in which only a small number of PMTs collect light.
 594 The first measurement creates an average charge, $Q_{obs}(i)$ on PMT i , whereas the second
 595 measurement ensures that each hit PMT only generates a single photoelectron. For the
 596 low-intensity measurement, the number of times each PMT records a charge larger
 597 than 1/4 photoelectrons, $N_{obs}(i)$, is counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i), \end{aligned} \tag{3.1}$$

598 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the acceptance
 599 efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and collection efficiency

600 of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative gain for each PMT can
 601 determined by taking the ratio of these quantities.

602 The absolute gain calibration is performed by observing fixed energy γ -rays of
 603 $E_{\gamma} \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source situated at
 604 the center of the detector. This generates a photon yield of about 0.004 photoelec-
 605 trons/PMT/event, meaning that $> 99\%$ of PMT signals are generated from single
 606 photoelectrons. A charge distribution is generated by performing this calibration over
 607 all PMTs, and the average value of this distribution is taken to be the absolute gain
 608 value.

609 As mentioned in subsection 3.1.1, the average quantum and collection efficiency
 610 for the SK detector is $\sim 21\%$ and $> 70\%$ respectively. However, these values do differ
 611 between each PMT and need to be calibrated accordingly. Consequently, the NiCf
 612 source is also used to calibrate the “quantum \times collection” efficiency (denoted “QE”)
 613 value of each PMT. The NiCf low-intensity source is used as the PMT hit probability
 614 is proportional to the QE ($N_{\text{obs}}(i) \propto \epsilon(i)$ in Equation 3.1). A Monte Carlo prediction
 615 which includes photon absorption, scattering, and reflection is made to estimate the
 616 number of photons incident on each PMT and the ratio of the number of predicted
 617 to observed hits is calculated. The difference is attributed to the QE efficiency of that
 618 PMT. This technique is extended to calculate the relative QE efficiency by normalizing
 619 the average of all PMTs which removes the dependence on the light intensity.

620 Due to differing cable lengths and readout electronics, the timing response between
 621 a photon hitting the PMT and the signal being captured by the data acquisition can be
 622 different between each PMT. Due to threshold triggers (Described in subsection 3.1.3),
 623 the time at which a pulse reaches a threshold is dependent upon the size of the pulse.
 624 This is known as the ‘time-walk’ effect and also needs to be accounted for in each PMT.
 625 To calibrate the timing response, a pulse of light with width 0.2ns is emitted into the

626 detector through a diffuser. Two-dimensional distributions of time and pulse height
627 (or charge) are made for each PMT and are used to calibrate the timing response. This
628 is performed in-situ during data taking with the light source pulsing at 0.03Hz.

629 The top/bottom water quality asymmetry is measured using the NiCf calibration
630 data and cross-referencing these results to the “standard PMTs”. The water attenuation
631 length is continuously measured by the rate of vertically-downgoing cosmic-ray
632 muons which enter via the top of the tank.

633 Dark noise is the phenomenon where a PMT registers a pulse that is consistent
634 with a single photoelectron emitted from photon detection despite the PMT being in
635 complete darkness. This is predominately caused by two processes. Firstly there is
636 intrinsic dark noise which is where photoelectrons gain enough thermal energy to be
637 emitted from the photocathode, and secondly, the radioactive decay of contaminants
638 inside the structure of the PMT. Typical dark noise rate for PMTs used within SK are
639 $O(3)$ kHz [87]. This is lower than the expected number of photons generated for a ‘high
640 energy event’ (As described in subsection 3.1.4) but instability in this value can cause
641 biases in reconstruction. Dark noise is related to the gain of a PMT and is calibrated
642 using hits inside a time window recorded before an event trigger [98].

643 3.1.3 Data Acquisition and Triggering

644 The analysis presented in this thesis only uses the SK-IV period of the SK experiment
645 so this subsection focuses on the relevant points of the data acquisition and triggering
646 systems to that SK period. The earlier data acquisition and triggering systems are
647 documented in [99, 100].

648 Before the SK-IV period started, the existing front-end electronics were replaced
649 with “QTC-Based Electrons with Ethernet, QBEE” systems [101]. When the QBEE

observes a signal above a 1/4 photoelectron threshold, the charge-to-time (QTC) converter generates a rectangular pulse. The start of the rectangular pulse indicates the time at which the analog photoelectron signal was received and the width of the pulse indicates the total charge integrated throughout the signal. This is then digitized by time-to-digital converters and sent to the “front-end” PCs. The digitized signal from every QBEE is then chronologically ordered and sent to the “merger” PCs. It is the merger PCs that apply the software trigger. Any triggered events are passed to the “organizer” PC. This sorts the data stream of multiple merger PCs into chronologically ordered events which are then saved to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

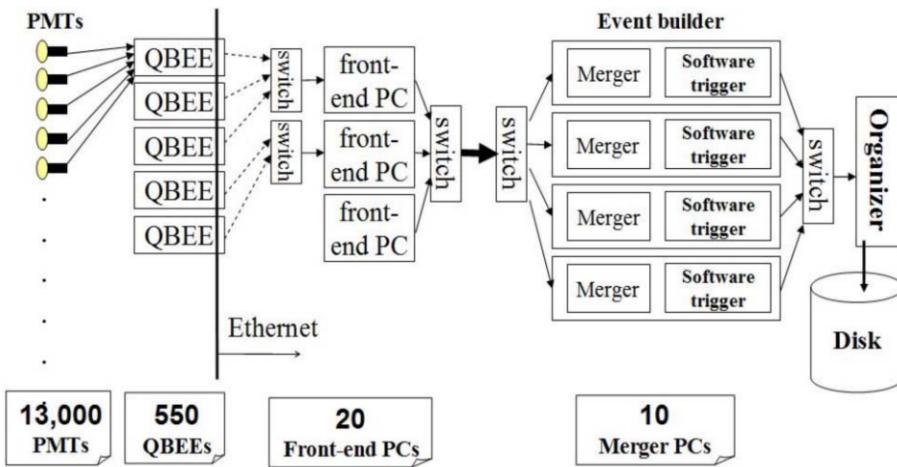


Figure 3.3: Schematic view of the data flow through the data acquisition and online system. Taken from [102].

The software trigger (described in [103]) operates by determining the number of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the maximum time that a Cherenkov photon would take to traverse the length of the SK tank [100]. For lower energy events that generate fewer photons, this technique is useful for eliminating background processes like dark noise and radioactive decay which would be expected to separate in time. When the value of N_{200} exceeds some threshold, a software trigger is issued. There are several trigger thresholds used within

the SK-IV period which are detailed in Table 3.2. If one of these thresholds is met, the PMT hits within an extended time window are also read out and saved to disk. In the special case of an event that exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband Intelligent Trigger (WIT)” has been deployed and is described in [104].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

3.1.4 Cherenkov Radiation

Cherenkov light is emitted from any highly energetic charged particle traveling with relativistic velocity, β , greater than the local speed of light in a medium [105]. Cherenkov light is formed at the surface of a cone with characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

678 where n is the refractive index of the medium. Consequently, the Cherenkov
 679 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged particle
 680 moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}} \quad (3.3)$$

681 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy for
 682 various particles are given in Table 3.3. In contrast, γ -rays are detected indirectly via
 683 the combination of photons generated by Compton scattering and pair production.
 684 The threshold for detection in the SK detector is typically higher than the threshold
 685 for photon production. This is due to the fact that the attenuation of photons in the
 686 water means that typically $\sim 75\%$ of Cherenkov photons reach the ID PMTs. Then the
 687 collection and quantum efficiencies described in subsection 3.1.1 result in the number
 688 of detected photons being lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

689 The Frank-Tamm equation [106] describes the relationship between the number of
 690 Cherenkov photons generated per unit length, dN/dx , the wavelength of the photons
 691 generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

692 where α is the fine structure constant. For a 100MeV momentum electron, approx-
 693 imately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq 700\text{nm}$
 694 region which the ID PMTs are most sensitive to [87].

695 3.2 The Tokai to Kamioka Experiment

696 The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation exper-
 697 iment located in Japan. Proposed in the early 2000s [107, 108] to replace K2K [109],
 698 T2K was designed to observe electron neutrino appearance whilst precisely measuring
 699 the oscillation parameters associated with muon neutrino disappearance [110]. The
 700 experiment consists of a neutrino beam generated at the Japan Proton Accelerator
 701 Research Complex (J-PARC), a suite of near detectors situated 280m from the beam
 702 target, and the Super Kamiokande far detector positioned at a 295km baseline. The
 703 cross-section view of the T2K experiment is drawn in Figure 3.4.

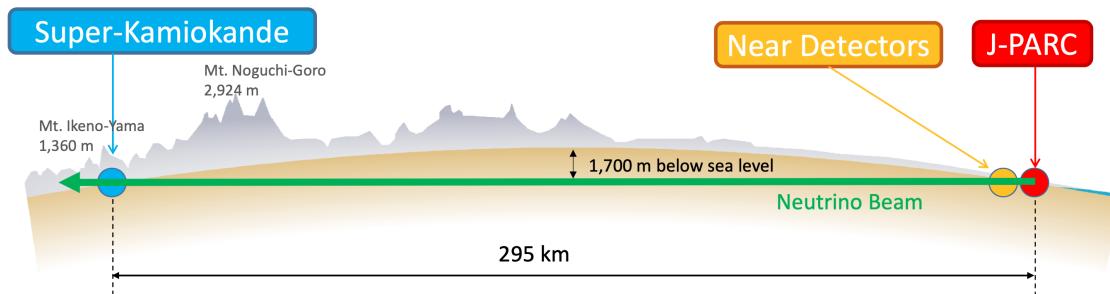


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$, Δm_{23}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accuracy of parameter estimates are still being made by including new data samples and developing the models which describe the neutrino interactions and detector responses [111]. Electron neutrino appearance was first observed at T2K in 2014 [112] with 7.3σ significance.

The near detectors provide constraints on the beam flux and cross-section model parameters used within the oscillation analysis by observing the unoscillated neutrino beam. There are a host of detectors situated in the near detector hall (As illustrated in Figure 3.5): ND280 (subsection 3.2.2), INGRID (subsection 3.2.3), NINJA [113], WAGASCI [114], and Baby-MIND [115]. The latter three are not currently used within the oscillation analysis presented within this thesis.

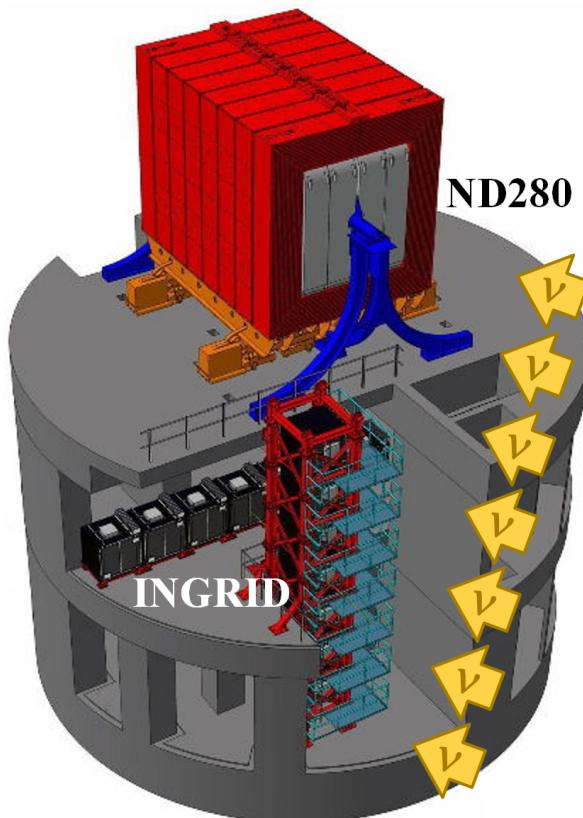


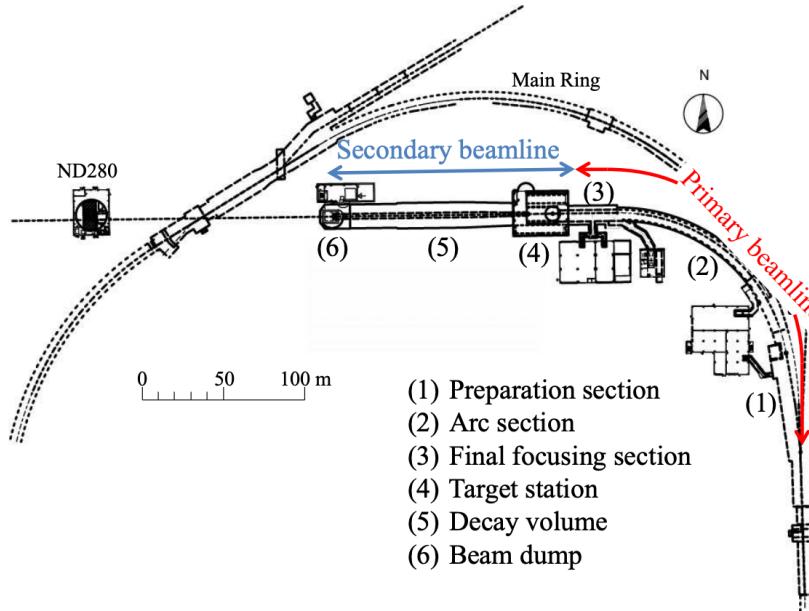
Figure 3.5: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

Whilst this thesis presents the ND280 in terms of its purpose for the oscillation analysis, the detector can also make many cross-section measurements at neutrino energies of $O(1)$ GeV for the different targets within the detector [116, 117]. These measurements are of equal importance as they can lead the way in determining the model parameters used in the interaction models for the future high-precision era of neutrino physics.

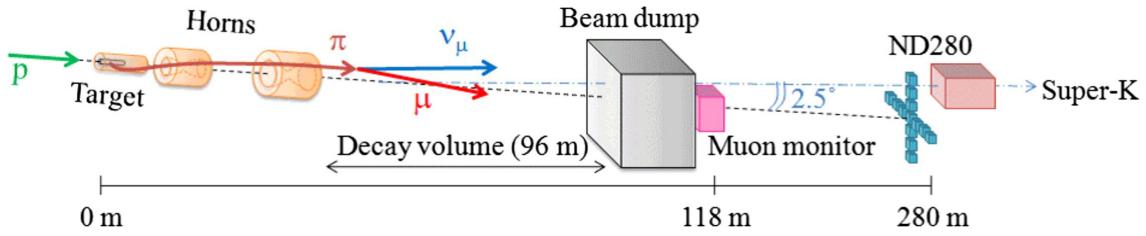
3.2.1 The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [59, 118] and summarised below. The accelerating facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [59],



(a) Primary and secondary beamline



(b) Secondary beamline

Figure 3.6: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [118]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [119].

$$\begin{array}{ll}
 \pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
 K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
 \rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
 \rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
 K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
 \rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
 \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e
 \end{array}$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino flux.

The low energy flux is dominated by the decay of pions whereas kaon decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic irreducible ν_e background, are predominantly due to muon decay for $E_\nu < 2\text{GeV}$. As the antineutrino production cross-section is smaller than the neutrino cross-section, the wrong-sign component is more dominant in the RHC beam as compared to that in the FHC beam.

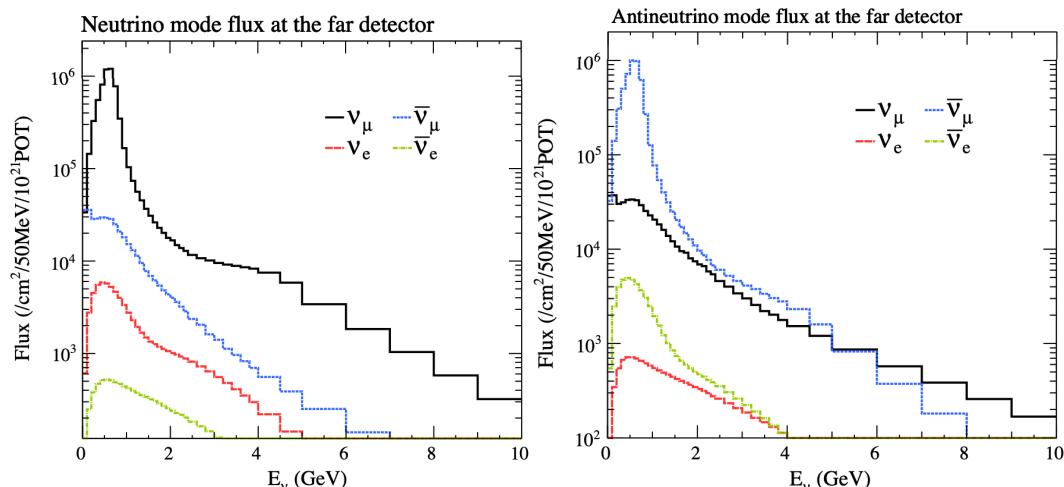


Figure 3.7: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [120].

The beam dump, situated at the end of the decay volume, stops all charged particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector monitors the

⁷⁵⁴ penetrating muons to determine the beam direction and intensity which is used to
⁷⁵⁵ constrain some of the beam flux systematics within the analysis [119, 121].

⁷⁵⁶ The T2K experiment uses an off-axis beam to narrow the neutrino energy distribution.
⁷⁵⁷ This was the first implementation of this technique in a long-baseline neutrino
⁷⁵⁸ oscillation experiment after its original proposal [122]. Pion decay, $\pi \rightarrow \mu + \nu_\mu$, is a
⁷⁵⁹ two-body decay. Consequently, the neutrino energy, E_ν , can be determined based on
⁷⁶⁰ the pion energy, E_π , and the angle at which the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

⁷⁶¹ where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
⁷⁶² energy pion, the neutrino energy distribution is dependent upon the angle at which the
⁷⁶³ neutrinos are observed from the initial pion beam direction. For the 295km baseline at
⁷⁶⁴ T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance probability, $P(\nu_\mu \rightarrow \nu_e)$,
⁷⁶⁵ whilst minimising the muon disappearance probability, $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8
⁷⁶⁶ illustrates the neutrino energy distribution for a range of off-axis angles, as well as the
⁷⁶⁷ oscillation probabilities most relevant to T2K.

⁷⁶⁸ 3.2.2 The Near Detector at 280m

⁷⁶⁹ Whilst all the near detectors are situated in the same “pit” located at 280m from the
⁷⁷⁰ beamline, the “ND280” detector is the off-axis detector which is situated at the same
⁷⁷¹ off-axis angle as the Super-Kamiokande far detector. It has two primary functions;
⁷⁷² firstly it measures the neutrino flux and secondly it counts the event rates of different
⁷⁷³ types of neutrino interactions. Both of these constrain the flux and cross-section

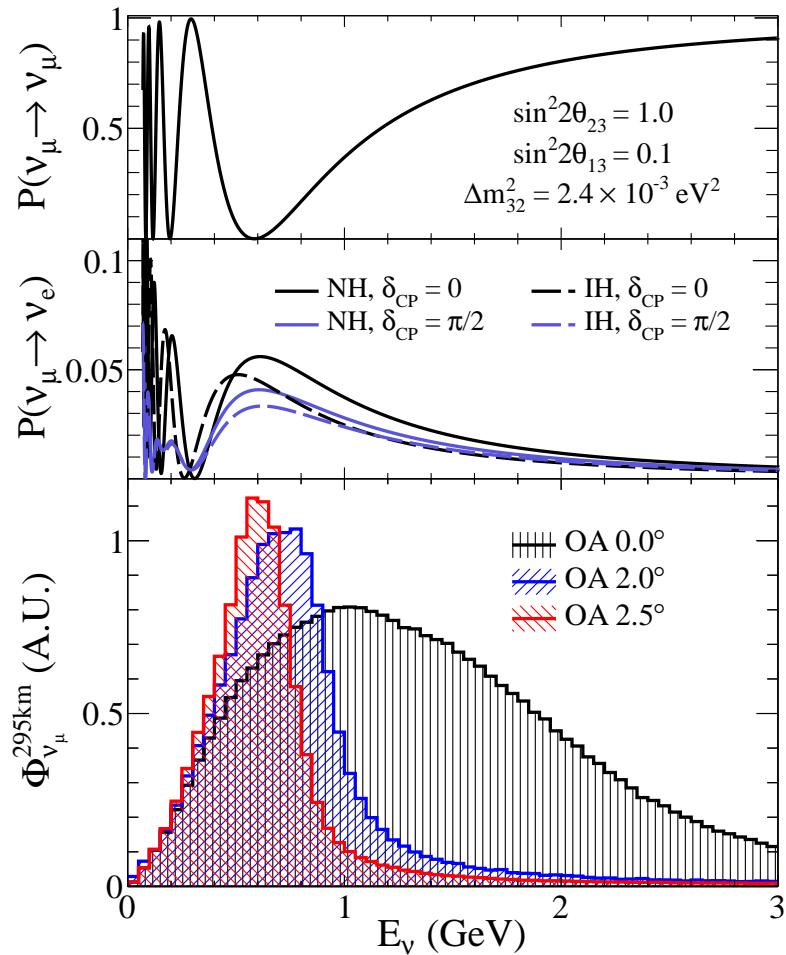


Figure 3.8: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

systematics invoked within the model for a more accurate prediction of the expected event rate at the far detector.

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are

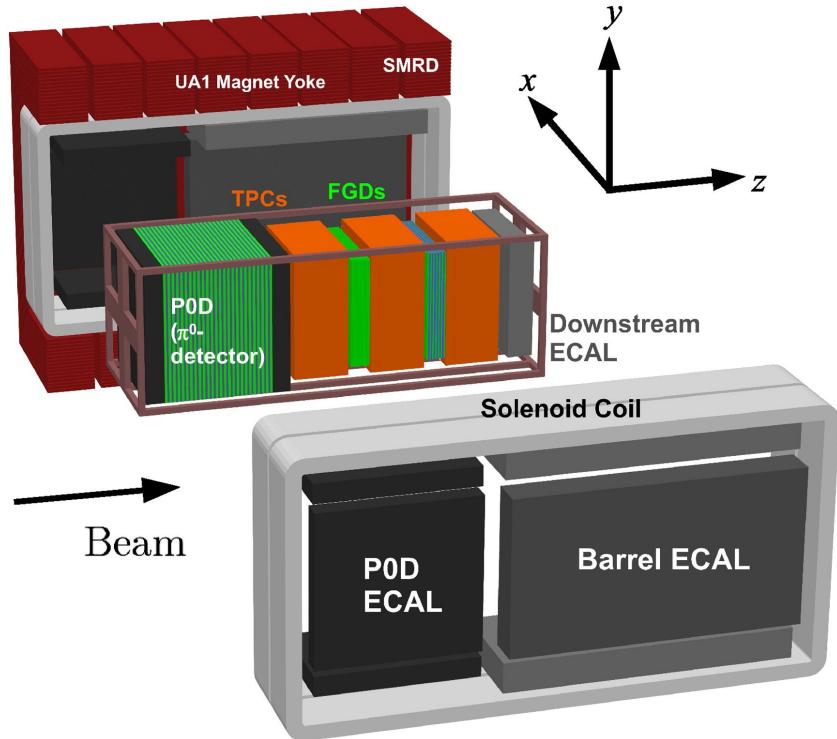


Figure 3.9: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [118].

783 further described in subsubsection 3.2.2.1 and subsubsection 3.2.2.2 respectively. The
 784 electromagnetic calorimeter (ECAL) encapsulates the tracker region alongside the π^0
 785 detector (P0D). The ECAL measures the deposited energy from photons emitted from
 786 interactions within the FGD. The P0D constrains the cross-section of neutral current
 787 interactions which generate neutral pions, which is one of the largest backgrounds in
 788 the electron neutrino appearance oscillation channel. The P0D and ECAL detectors
 789 are detailed in subsubsection 3.2.2.3 and subsubsection 3.2.2.4 respectively. The entire
 790 detector is located within a large yoke magnet which produces a 0.2T magnetic field.
 791 This design of the magnet also includes a scintillating detector called the side muon
 792 range detector (SMRD) which is used to track high-angle muons as well as acting as a
 793 cosmic veto. The SMRD is described in subsubsection 3.2.2.5.

794 3.2.2.1 Fine Grained Detectors

795 The T2K tracker region is comprised of two fine grained detectors (FGD) and three
796 Time Projection Chambers (TPC). A detailed description of the FGD design, construc-
797 tion, and assembly is found in [123] and summarised below. The FGDs are the primary
798 target for neutrino interactions with a mass of 1.1 tonnes per FGD. Alongside this,
799 the FGDs are designed to be able to track short-range particles which do not exit the
800 FGD. Typically, short-range particles are low momentum and are observed as tracks
801 that deposit a large amount of energy per unit length. This means the FGD needs
802 good granularity to resolve these particles. The FGDs have the best timing resolution
803 ($\sim 3\text{ns}$) of any of the sub-detectors of the ND280 detector. As such, the FGDs are
804 used for time of flight measurements to distinguish forward going positively charged
805 particles from backward going negatively charged particles. Finally, any tracks which
806 pass through multiple sub-detectors are required to be track matched to the FGD.

807 Both FGDs are made from square scintillator planes of side length 186cm and
808 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an X or Y
809 orientation. A wavelength shifting fiber is threaded through the center of each bar and
810 is read out by a multi-pixel photon counter (MPPC). FGD1 is the most upstream of
811 the two FGDs and contains 15 planes of carbon plastic scintillator which is a common
812 target in external neutrino scattering data. As the far detector is a pure water target, 7
813 of the 15 scintillator planes in FGD2 have been replaced with a hybrid water-scintillator
814 target. Due to the complexity of the nucleus, nuclear effects can not be extrapolated
815 between different nuclei. Therefore having the ability to take data on one target which
816 is the same as external data and another target which is the same as the far detector
817 target is beneficial for reliable model parameter estimates.

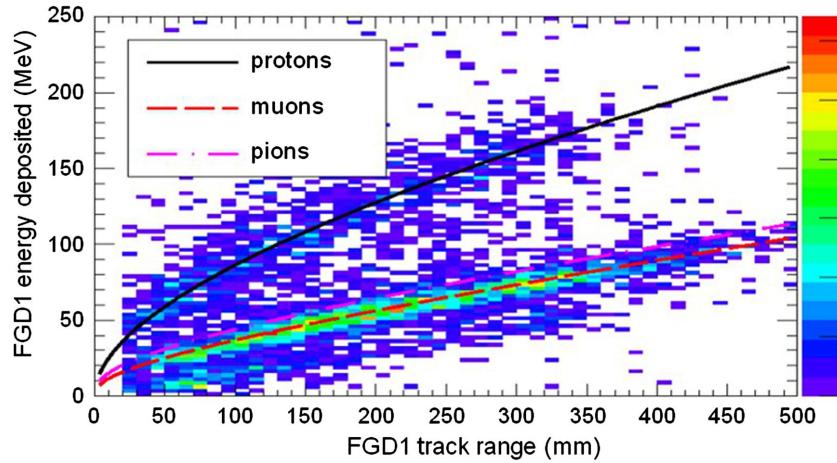


Figure 3.10: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [123].

The integrated deposited energy is used for particle identification. The FGD can distinguish protons from other charged particles by comparing the integrated deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

3.2.2.2 Time Projection Chambers

The majority of particle identification and momentum measurements within ND280 are provided by three Time Projection Chambers (TPCs) [124]. The TPCs are located on either side of the FGDs. They are located inside of the magnetic field meaning the momentum of a charged particle can be determined from the bending of the track.

Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11, which are made of non-magnetic material. The outer box is filled with CO₂ which acts as an electrical insulator between the inner box and the ground. The inner box forms the field cage which produces a uniform electric drift field of $\sim 275\text{V/cm}$ and is filled with an argon gas mixture. Charged particles moving through this gas mixture ionize the gas and the ionised charge is drifted towards micromegas detectors which measure the ionization charge. The time and position information in the readout allows a three-dimensional image of the neutrino interaction.

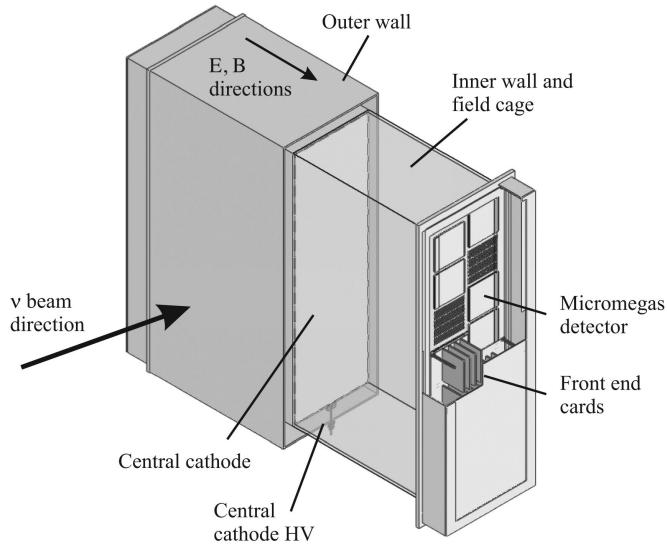


Figure 3.11: Schematic design of a Time Projection Chamber detector. Taken from [124].

The particle identification of tracks that pass through the TPCs is performed using

dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo distributions of the energy lost by a charged particle passing through the TPC as a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$ meaning that electrons and muons can be distinguished. This allows reliable measurements of the intrinsic ν_e component of the beam.

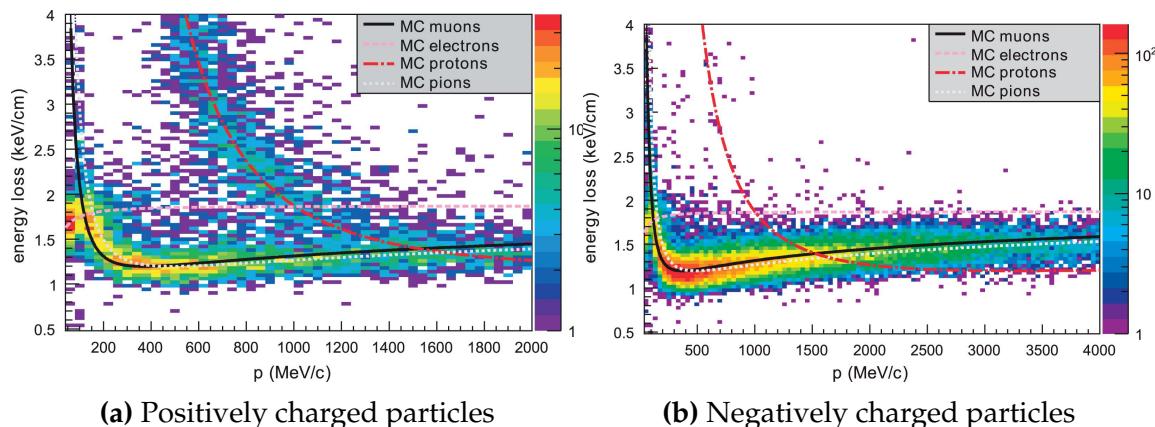


Figure 3.12: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [124].

840 3.2.2.3 π^0 Detector

841 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the recon-
 842 struction will determine that event to be a charge current ν_e -like event. This is one of
 843 the main backgrounds hindering the electron neutrino appearance searches. The π^0
 844 detector (P0D) measures the cross-section of the neutral current induced neutral pion
 845 production on a water target to constrain this background.

846 The P0D is a cube of approximately 2.5m length consisting of layers of scintillating
 847 bars, brass and lead sheets, and water bags as illustrated in Figure 3.13. Two electro-
 848 magnetic calorimeters are positioned at the most upstream and most downstream
 849 position in the sub-detector and the water target is situated in between them. The
 850 scintillator layers are built from two triangular bars orientated in opposite directions
 851 to form a rectangular layer. Each triangular scintillator bar is threaded with optical
 852 fiber which is read out by MPPCs. The high-Z brass and lead regions produce electron
 853 showers from the photons emitted in π^0 decay.

854 The sub-detector can generate measurements of NC1 π^0 cross-sections on a water
 855 target by measuring the event rate both with and without the water target, with the
 856 cross-section on a water target being determined as the difference. The total active
 857 mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

858 3.2.2.4 Electromagnetic Calorimeter

859 The electromagnetic calorimeter [126] (ECal) encapsulates the P0D and tracking sub-
 860 detectors. Its primary purpose is to aid π^0 reconstruction from any interaction in
 861 the tracker. To do this, it measures the energy and direction of photon showers from
 862 $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks depending on the shape
 863 of the photon shower deposited.

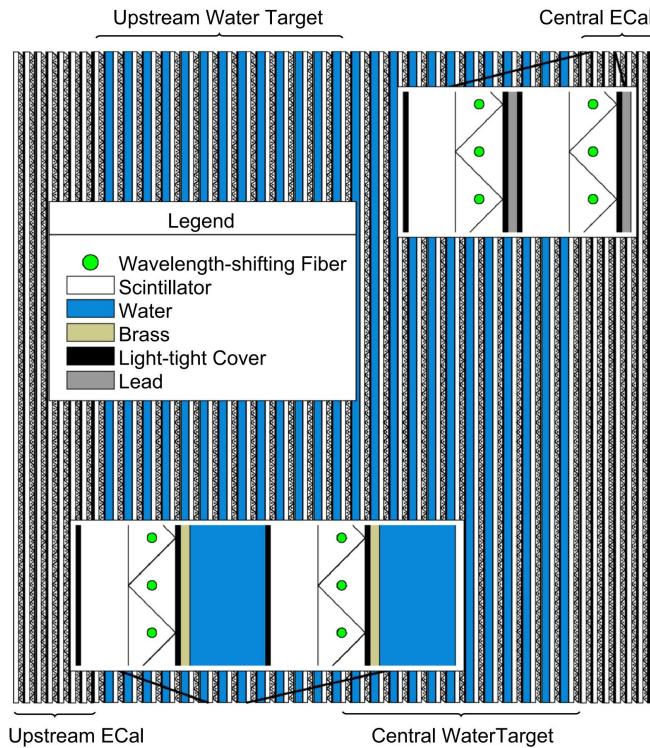


Figure 3.13: A schematic of the P0D side-view. Taken from [125].

The ECal is comprised of three sections; the P0D ECal which surrounds the P0D, the barrel ECal which encompasses the tracking region, and the downstream ECal which is situated downstream of the tracker region. The barrel and downstream ECals are tracking calorimeters that focus on electromagnetic showers from high-angle particles emitted from the tracking sub-detectors. Particularly in the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis) can travel along a single scintillator bar resulting in very few hits. The width of the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths to ensure a significant amount of the π^0 energy is contained. As the P0D has its own calorimetry which reconstructs showers, the P0D ECal determines the energy which escapes the P0D.

Each ECal is constructed of multiple layers of scintillating bars sandwiched between lead sheets. The scintillating bars are threaded with optical fiber and read out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular to the

⁸⁷⁷ previous which allows a three dimensional event displays. The target mass of the P0D
⁸⁷⁸ ECal, barrel ECal, and downstream ECal are 1.50, 4.80 and 6.62 tonnes respectively.

⁸⁷⁹ **3.2.2.5 Side Muon Range Detector**

⁸⁸⁰ As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within the
⁸⁸¹ UA1 magnet. Originally designed for the NOMAD [127] experiment and reconditioned
⁸⁸² for use in the T2K experiment [128], the UA1 magnet provides a uniform horizontal
⁸⁸³ magnetic field of 0.2T with an uncertainty of 2×10^{-4} T.

⁸⁸⁴ Built into the UA1 magnet, the side muon range detector (SMRD) [129] monitors
⁸⁸⁵ high-energy muons which leave the tracking region and permeate through the ECal.
⁸⁸⁶ It additionally acts as a cosmic muon veto and trigger.

⁸⁸⁷ **3.2.3 The Interactive Neutrino GRID**

⁸⁸⁸ The Interactive Neutrino GRID (INGRID) detector is situated within the same “pit” as
⁸⁸⁹ the other near detectors. It is aligned with the beam in the “on-axis” position and mea-
⁸⁹⁰ sures the beam direction, spread, and intensity. The detector was originally designed
⁸⁹¹ with 16 identical modules [118] (two modules have since been decommissioned) and a
⁸⁹² “proton” module. The design of the detector is cross-shaped with length and height
⁸⁹³ 10m × 10m as illustrated in Figure 3.14.

⁸⁹⁴ Each module is composed of iron sheets interlaced with eleven tracking scintillator
⁸⁹⁵ planes for a total target mass of 7.1 tonnes per module. The scintillator design is an X-Y
⁸⁹⁶ pattern of 24 bars in both orientations, where each bar contains wave-length shifting
⁸⁹⁷ fibers which are connected to multi-pixel photon counters (MPPCs). Each module is
⁸⁹⁸ encapsulated inside veto planes to aid the rejection of charged particles entering the
⁸⁹⁹ module.

900 The proton module is different from the other modules in that it consists of entirely
 901 scintillator planes with no iron target. The scintillator bars are also smaller than those
 902 used in the other modules to increase the granularity of the detector and improve
 903 tracking capabilities. The module sits in the center of the beamline and is designed to
 904 give precise measurements of quasi-elastic charged current interactions to evaluate
 905 the performance of the Monte Carlo simulation of the beamline.

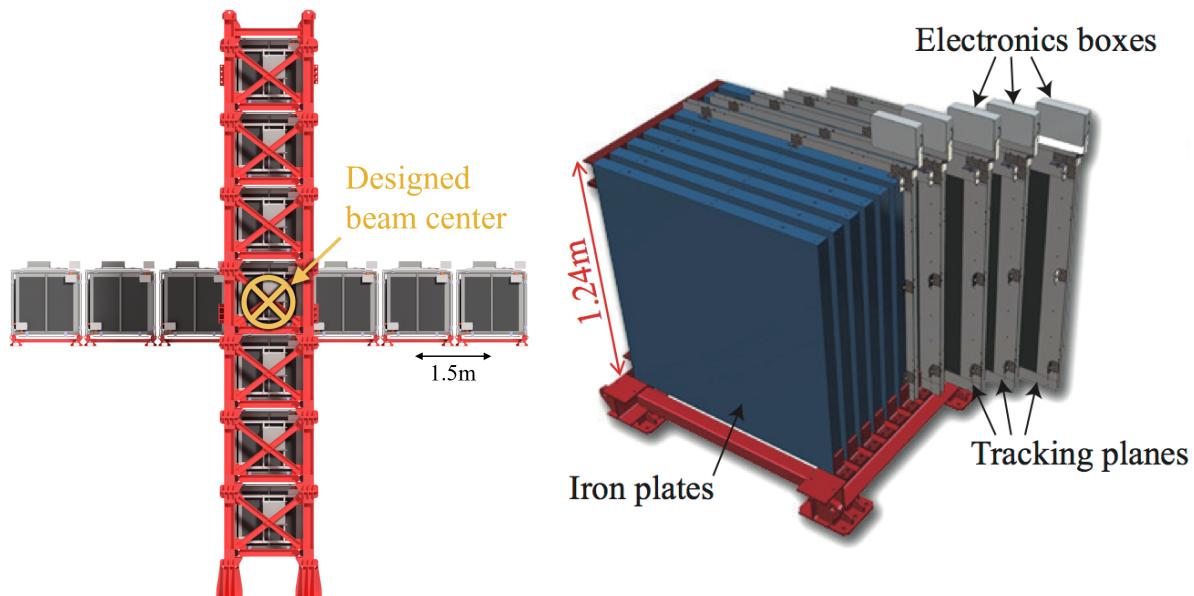


Figure 3.14: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [118].

906 The INGRID detector can measure the beam direction to an uncertainty of 0.4mrad
 907 and the beam center within a resolution of 10cm [118]. The beam direction in both the
 908 vertical and horizontal directions is discussed in [130] and it is found to be in good
 909 agreement with the MUMON monitor described in subsection 3.2.1.

₉₁₀ **Chapter 4**

₉₁₁ **Bayesian Statistics and Markov Chain**

₉₁₂ **Monte Carlo Techniques**

₉₁₃ This thesis presents a Bayesian oscillation analysis. To extract the oscillation parameters,
₉₁₄ a Markov Chain Monte Carlo (MCMC) method is used. This chapter explains
₉₁₅ the theory of how parameter estimates can be determined using this technique and
₉₁₆ condenses the material found in the literature [131–134].

₉₁₇ The oscillation parameter determination presented within this thesis is built upon
₉₁₈ a simultaneous fit to neutrino beam data in the near detector, beam data at SK and
₉₁₉ atmospheric data at SK. In total, there are four oscillation parameters of interest
₉₂₀ ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP}), two oscillation parameters to which this study
₉₂₁ will not be sensitive ($\sin^2(\theta_{12})$, Δm_{12}^2) and many nuisance parameters that control the
₉₂₂ systematic uncertainty models invoked within this study.

₉₂₃ The MCMC technique generates a multi-dimensional probability distribution across
₉₂₄ all of the model parameters used in the fit. To determine the parameter estimate of a
₉₂₅ single parameter, this multi-dimensional object is integrated over all other parameters.
₉₂₆ This process is called Marginalisation and is further described in subsection 4.3.1.
₉₂₇ Monte Carlo techniques approximate the probability distribution of each parameter
₉₂₈ within the limit of generating infinite samples. As ever, generating a large number of
₉₂₉ samples is time and resource-dependent. Therefore, an MCMC technique is utilised
₉₃₀ within this analysis to reduce the required number of steps to sufficiently sample the
₉₃₁ parameter space. This technique is described in further detail in subsection 4.2.1.

932 4.1 Bayesian Statistics

933 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
934 footing such that a probability model of both data and parameters is required. This is
935 the joint probability distribution $P(D, \vec{\theta})$ and can be described by the prior distribution
936 for model parameters $P(\vec{\theta})$ and the likelihood of the data given the model parameters
937 $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

938 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the parameters
939 within the model. For example, if the risk of developing health problems is known
940 to increase with age, the prior distribution would describe the increase. For the
941 purpose of this analysis, the prior distribution is typically the best-fit values taken
942 from external data measurements with a Gaussian uncertainty. The prior distribution
943 can also contain correlations between model parameters. In an analysis using Monte
944 Carlo techniques, the likelihood of measuring some data assuming some set of model
945 parameters is calculated by comparing the Monte Carlo prediction generated at that
946 particular set of model parameters to the data.

947 It is parameter estimation that is important for this analysis and as such, we apply
948 Bayes' theorem [135] to calculate the probability for each parameter to have a certain
949 value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior distribution
950 (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

951 The denominator in Equation 4.2 is the integral of the joint probability distribution

952 over all values of all parameters used within the fit. For brevity, we say that the
 953 posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

954 In subsection 4.3.1, we see that for the cases used within this analysis, it is reason-
 955 able to know the posterior to some normalisation constant.

956 4.2 Monte Carlo Simulation

957 Monte Carlo techniques are used to numerically solve a complex problem that does
 958 not necessarily have an analytical solution. These techniques rely on building a large
 959 ensemble of samples from an unknown distribution and then using the ensemble to
 960 approximate the properties of the distribution.

961 An example that uses Monte Carlo techniques is to calculate the area underneath
 962 a curve. For example, take the problem of calculating the area under a straight line
 963 with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can calculate the area
 964 under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte Carlo techniques,
 965 one can calculate the area under this line by throwing many random values for the x
 966 and y components of each sample and then calculating whether that point falls below

967 the line. The area can then be calculated by the ratio of points below the line to the
968 total number of samples thrown multiplied by the total area in which samples were
969 scattered. The study is shown in Figure 4.1 highlights this technique and finds the area
970 under the curve to be 29.9 compared to an analytical solution of 30.0. The deviation
971 of the numerical to analytical solution can be attributed to the number of samples
972 used in the study. The accuracy of the approximation in which the properties of the
973 Monte Carlo samples replicate those of the desired distribution is dependent on the
974 number of samples used. Replicating this study with a differing number of Monte
975 Carlo samples used in each study (As shown in Figure 4.2) highlights how the Monte
976 Carlo techniques are only accurate within the limit of a high number of samples.

977 Whilst the above example has an analytical solution, these techniques are just as
978 applicable to complex solutions. Clearly, any numerical solution is only as useful as its
979 efficiency. As discussed, the accuracy of the Monte Carlo technique is dependent upon
980 the number of samples generated to approximate the properties of the distribution.
981 Furthermore, if the positions at which the samples are evaluated are not ‘cleverly’
982 picked, the efficiency of the Monte Carlo technique significantly drops. Given the
983 example in Figure 4.1, if the region in which the samples are scattered significantly
984 extends passed the region of interest, many calculations will be calculated but do
985 not add to the ability of the Monte Carlo technique to achieve the correct result. For
986 instance, any sample evaluated at a $y \geq 5$ could be removed without affecting the final
987 result. This does bring in an aspect of the ‘chicken and egg’ problem in that to achieve
988 efficient sampling, one needs to know the distribution beforehand.

989 4.2.1 Markov Chain Monte Carlo

990 This analysis utilises a multi-dimensional probability distribution, with some dimen-
991 sions being significantly more constrained than others. This could be from prior

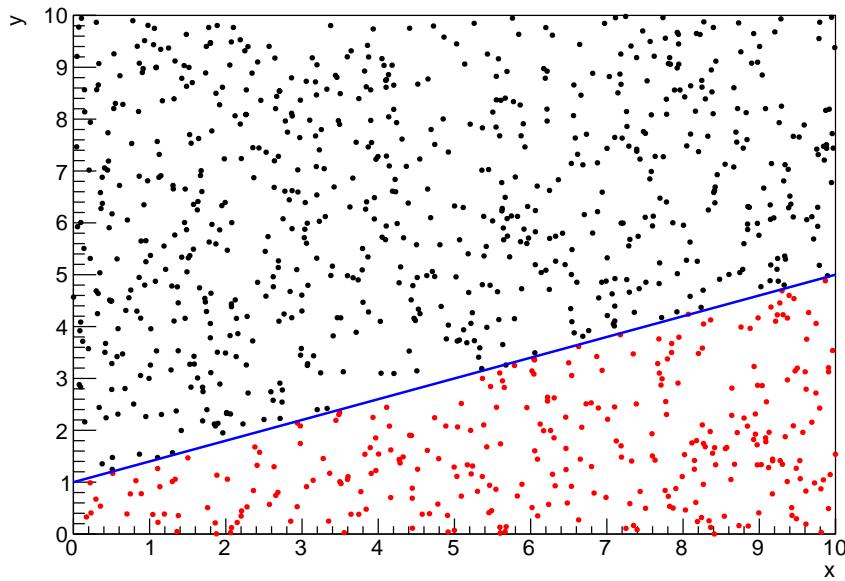


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

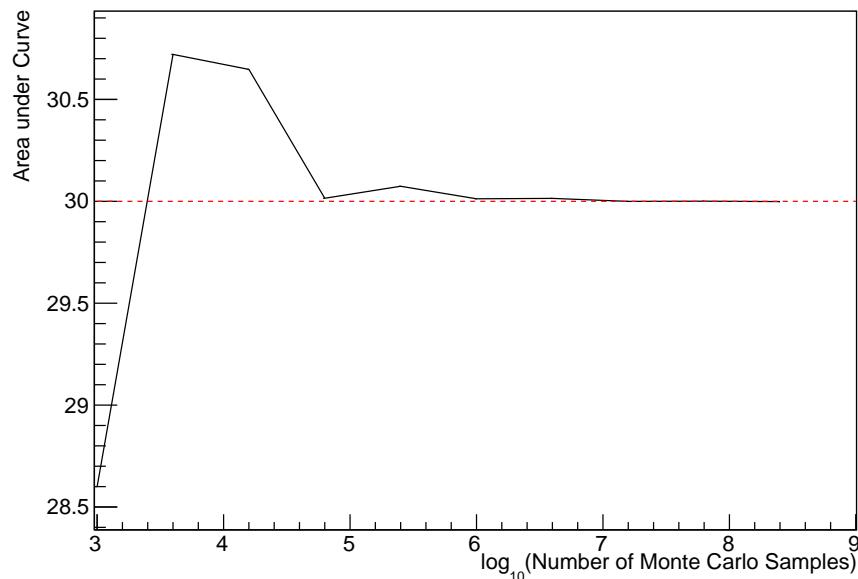


Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

knowledge of parameter distributions from external data or un-physical regions in which parameters can not exist. Consequently, the Monte Carlo techniques used need to be as efficient as possible. For this analysis, the Markov Chain Monte Carlo (MCMC) technique is chosen. An MCMC technique is a Monte Carlo technique that uses a Markov chain to select which points at which to sample the parameter distribution. This technique performs a semi-random stochastic walk through the allowable parameter space. This builds a posterior distribution which has the property that the density of sampled points is proportional to the probability density of that parameter. This does mean that the samples produced by this technique are not statistically independent but they will cover the space of the distribution.

A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the position of \vec{x}_i . The space in which the Markov chain selects samples is dependent upon the total number of parameters utilised within the fit, where a discrete point in this space is described by the N-dimensional space \vec{x} . In a perfectly operating Markov chain, the position of the next step depends solely on the previous step and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving the multi-dimensionality of the fit used within this analysis, each step becomes correlated with several of the steps preceding itself. This behaviour is further explained in subsection 4.2.3. Providing the MCMC chain is well optimised, it will begin to converge towards a unique stationary distribution. The period between the chain's initial starting point and the convergence to the unique stationary distribution is colloquially known as the burn-in period. This is discussed further in subsection 4.2.3. Once the chain reaches the stationary distribution, all points sampled after that point will look like samples from that distribution.

1016 Further details of the theories underpinning MCMC techniques are discussed
1017 in [132] but can be summarised by the requirement that the chain satisfies the three
1018 ‘regularity conditions’:

- 1019 • Irreducibility: From every position in the parameter space \vec{x} , there must exist a
1020 non-zero probability for every other position in the parameter space to be reached.
- 1021 • Recurrence: Once the chain arrives at the stationary distribution, every step fol-
1022 lowing from that position must be samples from the same stationary distribution.
- 1023 • Aperiodicity: The chain must not repeat the same sequence of steps at any point
1024 throughout the sampling period.

1025 The output of the chain after burn-in (ie. the sampled points after the chain
1026 has reached the stationary distribution) can be used to approximate the posterior
1027 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1028 stationary distribution found by the chain be the posterior distribution, one can use
1029 the Metropolis-Hastings algorithm. This guides the stochastic process depending on
1030 the likelihood of the current proposed step compared to that of the previous step.
1031 Implementation and other details of this technique are discussed in subsection 4.2.2.

1032 4.2.2 Metropolis-Hastings Algorithm

1033 As a requirement for MCMCs, the Markov chain implemented in this technique must
1034 have a unique stationary distribution that is equivalent to the posterior distribution.
1035 To ensure this requirement and that the regularity conditions are met, this analysis
1036 utilises the Metropolis-Hastings (MH) algorithm [136,137]. For the i^{th} step in the chain,
1037 the MH algorithm determines the position in the parameter space to which the chain
1038 moves to based on the current step, \vec{x}_i , and the proposed step, \vec{y}_{i+1} . The proposed step
1039 is randomly selected from some proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely

1040 on the current step (ie. not the further history of the chain). The next step in the chain
 1041 \vec{x}_{i+1} can be either the current step or the proposed step determined by whether the
 1042 proposed step is accepted or rejected. To decide if the proposed step is selected, the
 1043 acceptance probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)} \right). \quad (4.4)$$

1044 Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To
 1045 simplify this calculation, the proposal function is required to be symmetric such that
 1046 $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution is used to
 1047 throw parameter proposals from. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)} \right). \quad (4.5)$$

1048 After calculating this quantity, a random number, β , is generated uniformly be-
 1049 tween 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise, the chain
 1050 sets the next step equal to the current step and this procedure is repeated. This can be
 1051 interpreted as if the posterior probability of the proposed step is greater than that of
 1052 the current step, ($P(\vec{y}_{i+1}|D) \geq P(\vec{x}_i|D)$), the proposed step will always be accepted.
 1053 If the opposite is true, ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted
 1054 with probability $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get
 1055 trapped in any local minima in the potentially non-Gaussian posterior distribution.
 1056 The outcome of this technique is that the density of steps taken in a discrete region is
 1057 directly proportional to the probability density in that region.

1058 4.2.3 MCMC Optimisation

1059 As discussed in subsection 4.2.2, the proposal function invoked within the MH algo-
1060 rithm can take any form and the chain will still converge to the stationary distribution.
1061 At each set of proposed parameter values, a prediction of the same spectra has to be
1062 generated which requires significant computational resources. Therefore, the number
1063 of steps taken before the unique stationary distribution is found should be minimised
1064 as only steps after convergence add information to the oscillation analysis. Further-
1065 more, the chain should entirely cover the allowable parameter space to ensure that all
1066 values have been considered. Tuning the distance that the proposal function jumps
1067 between steps on a parameter-by-parameter basis can both minimise the length of the
1068 burn-in period and ensure that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently
1069 small.

1070 The effect of changing the width of the proposal function is highlighted in Figure 4.3.
1071 Three scenarios, each with the same underlying stationary distribution (A Gaussian of
1072 width 1.0 and mean 0.), are presented. The only difference between the three scenarios
1073 is the width of the proposal function, colloquially known as the ‘step size σ ’. Each
1074 scenario starts at an initial parameter value of 10.0 which would be considered an
1075 extreme variation. For the case where $\sigma = 0.1$, it is clear to see that the chain takes
1076 a long time to reach the expected region of the parameter. This indicates that this
1077 chain would have a large burn-in period and does not converge to the stationary
1078 distribution until step ~ 500 . Furthermore, whilst the chain does move towards the
1079 expected region, each step is significantly correlated with the previous. Considering
1080 the case where $\sigma = 5.0$, the chain approaches the expected parameter region almost
1081 instantly meaning that the burn-in period is not significant. However, there are clearly
1082 large regions of steps where the chain does not move. This is likely due to the chain
1083 proposing steps in the tails of the distribution which have a low probability of being

1084 accepted. Consequently, this chain would take a significant number of steps to fully
 1085 span the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can see
 1086 a relatively small burn-in period of approximately 100 steps. Once the chain reaches
 1087 the stationary distribution, it moves throughout the expected region of parameter
 1088 values many times, sufficiently sampling the full parameter region. This example is a
 1089 single parameter varying across a continuous distribution and does not fully reflect
 1090 the difficulties in the many-hundred multi-variate parameter distribution used within
 1091 this analysis. However, it does give a conceptual idea of the importance of selecting
 1092 the proposal function and associated step size.

1093 As discussed, step size tuning directly correlates to the average step acceptance
 1094 rate. If the step size is too small, many steps will be accepted but the chain moves
 1095 slowly. If the opposite is true, many steps will be rejected as the chain proposes steps
 1096 in the tails of the distribution. Discussion in [138] suggests that the ‘ideal’ acceptance
 1097 rate of a high dimension MCMC chain should be approximately $\sim 25\%$. An “ideal”
 1098 step size [138] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

1099 where N_p is the number of parameters included in the MCMC fit. However, the
 1100 complex correlations between systematics mean that some parameters have to be hand
 1101 tuned and many efforts have been taken to select a set of parameter-by-parameter step
 1102 sizes to approximately reach the ideal acceptance rate.

1103 Figure 4.3 highlights the likelihood as calculated by the fit in [DB: Link to AsimovA](#)
 1104 **Sensitivity Section** as a function of the number of steps in each chain. In practice,
 1105 many independent MCMC chains are run simultaneously to parallelise the task of

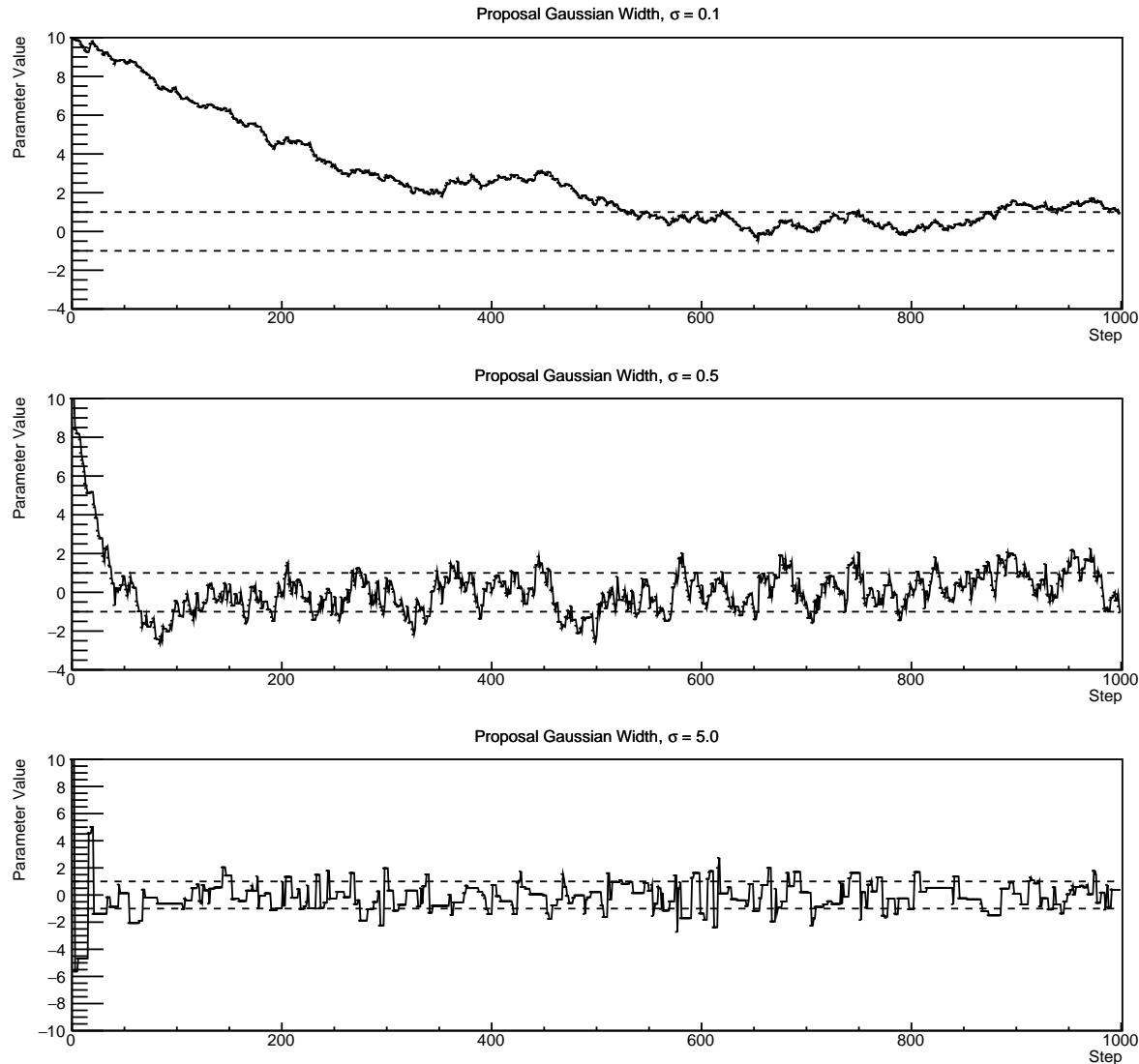


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1106 performing the fit. This figure overlays the distribution found in each chain. As seen,
 1107 the likelihood decreases from its initial value and converges towards a stationary
 1108 distribution after $\sim 1 \times 10^5$ steps.

1109 Multiple configurations of this analysis have been performed throughout this thesis
 1110 where different samples or systematics have been used. For all of these configurations,
 1111 it was found that a burnin period of 1×10^5 was sufficient in all cases.

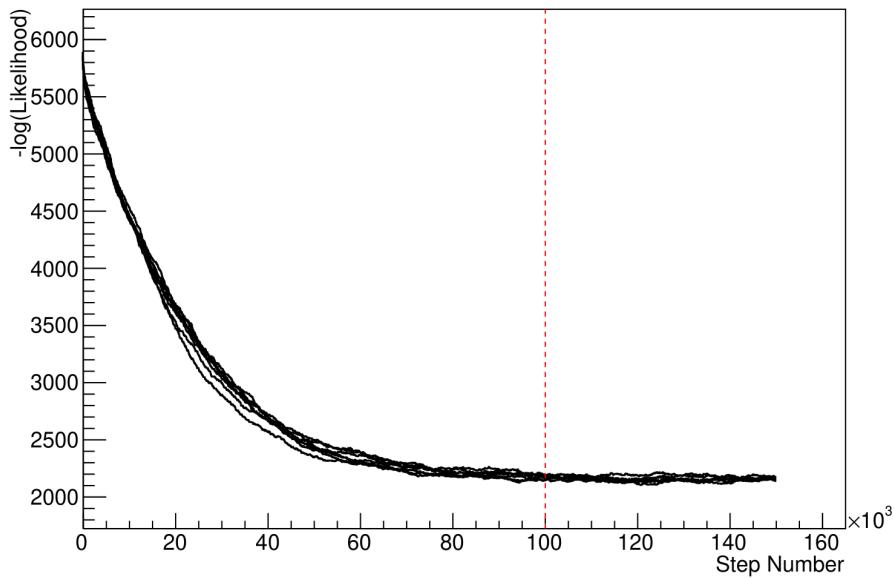


Figure 4.4: The log-likelihood from the fit detailed in DB: [Link to AsimovA Sensitivity Section](#) as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

1112 4.3 Understanding the MCMC Results

1113 The previous sections have described how to generate the posterior probability distri-
 1114 bution using Bayesian MCMC techniques. However, this analysis focuses on oscillation
 1115 parameter determination. The posterior distribution output from the chain is a high
 1116 dimension object, with as many dimensions as there are parameters included in the os-
 1117 cillation analysis. However, this multi-dimensional object is difficult to conceptualize
 1118 so parameter estimations are often presented in one or two-dimensional projections
 1119 of this probability distribution. To do this, we invoke the marginalisation technique
 1120 highlighted in subsection 4.3.1.

1121 4.3.1 Marginalisation

1122 The output of the MCMC chain is a highly dimensional probability distribution
1123 which is very difficult to interpret. From the standpoint of an oscillation analysis
1124 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters of
1125 interest are most relevant. Despite this, the best fit values and uncertainties on the
1126 oscillation parameters of interest should correctly encapsulate the correlations to the
1127 other systematic uncertainties (colloquially called ‘nuisance’ parameters). For this joint
1128 beam and atmospheric analysis, the oscillation parameters of interest are $\sin^2(\theta_{23})$,
1129 $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP} . All other parameters (Including the oscillation parameter
1130 this fit is insensitive to) are deemed nuisance parameters. To generate these projections,
1131 we rely upon integrating the posterior distribution over all nuisance parameters. This
1132 is called marginalisation. A simple example of this technique is to imagine the scenario
1133 where two coins are flipped. To determine the probability that the first coin returned
1134 a ‘head’, the exact result of the second coin flip is disregarded and simply integrated
1135 over. For the parameters of interest, $\vec{\theta}_i$, we can calculate the marginalised posterior by
1136 integrating over the nuisance parameters, $\vec{\theta}_n$. In this case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}} \quad (4.7)$$

1137 Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and correlations
1138 between the parameters of interest and the nuisance parameters. In practice, this
1139 is simply taking the one or two-dimensional projection of the multi-dimensional
1140 probability distribution.

Whilst in principle an easy solution to a complex problem, correlations between the interesting and nuisance parameters can bias the marginalised results. A similar effect is found when the parameters being marginalised over have non-Gaussian probability distributions. For example, Figure 4.5 highlights the marginalisation bias in the probability distribution found for a parameter when requiring a correlated parameter to have a positive parameter value. Due to the complex nature of this oscillation parameter fit presented in this thesis, there are correlations occurring between the oscillation parameters of interest and the other nuisance parameters included in the fit.

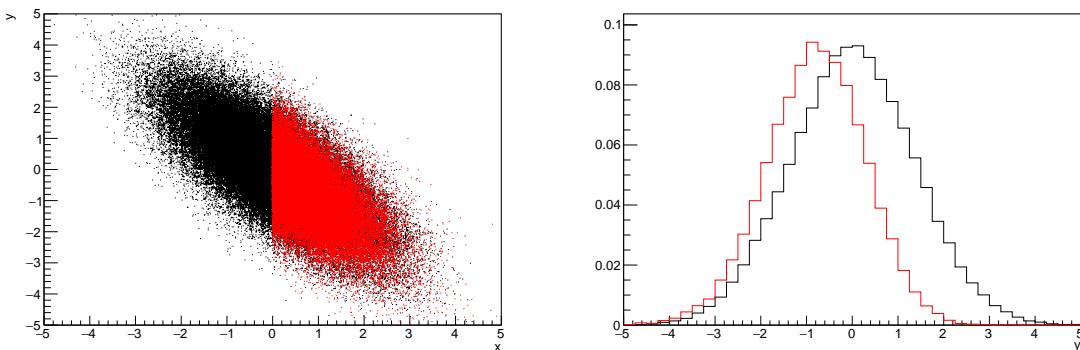


Figure 4.5: Left: The two dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

4.3.2 Parameter Estimation and Credible Intervals

The purpose of this analysis is to determine the best fit values for the oscillation parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{23}^2 , and δ_{CP} . Typically, the results presented take the form of one or two-dimension marginalised probability distributions for the appearance ($\sin^2(\theta_{13})$ and δ_{CP}) and disappearance ($\sin^2(\theta_{23})$ and Δm_{23}^2) parameters. The posterior probability density

₁₁₅₆ taken from the output MCMC chain is binned in these parameters. The parameter
₁₁₅₇ best-fit point is then taken to be the value that has the highest posterior probability.
₁₁₅₈ This is performed in both one and two-dimensional projections.

₁₁₅₉ However, the single best-fit point in a given parameter is not of much use on its
₁₁₆₀ own. We would also like to determine the uncertainty, or credible interval, on that
₁₁₆₁ best-fit point. The definition of the 1σ credible interval is that we have 68% belief that
₁₁₆₂ the parameter is within those bounds. For a more generalised definition, the credible
₁₁₆₃ interval is the region, R , of the posterior distribution that contains a specific fraction of
₁₁₆₄ the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha \quad (4.8)$$

₁₁₆₅ Where θ is the parameter on which we calculate the credible interval. This technique
₁₁₆₆ then calculates the $\alpha \times 100\%$ credible interval.

₁₁₆₇ In practice, this analysis uses the highest posterior density (HPD) credible intervals
₁₁₆₈ which are calculated through the following method. First, the probability distribution
₁₁₆₉ is area-normalised such that it has an integrated area equal to 1.0. The bins of proba-
₁₁₇₀ bility are then summed from the highest to lowest until the sum exceeds the 1σ level
₁₁₇₁ (0.68 in this example). This process is repeated for a range of credible intervals, notably
₁₁₇₂ the 1σ , 2σ and 3σ along with other levels where the critical values for each level can
₁₁₇₃ be found in [139]. This process can be repeated for the two-dimensional probability
₁₁₇₄ distributions by creating two-dimensional contours of credible intervals rather than a
₁₁₇₅ one-dimensional result.

¹¹⁷⁶ 4.3.3 Bayesian Model Comparisons

¹¹⁷⁷ Due to the matter resonance, this analysis has some sensitivity to the mass hierarchy
¹¹⁷⁸ of neutrino states (whether Δm_{23}^2 is positive or negative) and the octant of $\sin^2(\theta_{23})$
¹¹⁷⁹ . The Bayesian approach utilised within this analysis gives an intuitive method of
¹¹⁸⁰ model comparison by determining which hypothesis is most favourable. Taking the
¹¹⁸¹ ratio of Equation 4.3 for the two hypotheses of normal hierarchy, NH , and inverted
¹¹⁸² hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

¹¹⁸³ The middle term defines the Bayes factor which is a data-driven interpretation of
¹¹⁸⁴ how strong the data prefers one hierarchy to the other. For this analysis, equal priors
¹¹⁸⁵ on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) = P(\vec{\theta}_{IH}) = 0.5$). In practice,
¹¹⁸⁶ the MCMC chain proposes a value of $|\Delta m_{23}^2|$ and then applies a 50% probability
¹¹⁸⁷ that the value is sign flipped. Consequently, the Bayes factor can be calculated from
¹¹⁸⁸ the ratio of the probability density in either hypothesis. This equates to counting the
¹¹⁸⁹ number of steps taken in the normal and inverted hierarchies and taking the ratio. The
¹¹⁹⁰ same approach can be taken to compare the upper octant (UO) compared to the lower
¹¹⁹¹ octant (LO) hypothesis of $\sin^2(\theta_{23})$.

¹¹⁹² Whilst the value of the Bayes factor should always be shown, the Jeffreys scale [140]
¹¹⁹³ (highlighted in Table 4.1) gives an indication of the strength of preference for one model
¹¹⁹⁴ compared to the other. Other interpretations of the strength of preference of a model
¹¹⁹⁵ exist, e.g. the Kass and Raferty Scale [141].

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [140]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

1196 4.3.4 Comparison of MCMC Output to Expectation

1197 To ensure the fit is performing well, a best-fit spectra is produced using the posterior
1198 probability distribution and compared with the data, allowing easy by-eye compar-
1199 isons to be made. A simple method of doing this is to perform a comparison in the
1200 fitting parameters (For instance, the reconstructed neutrino energy and lepton direc-
1201 tion for T2K far detector beam samples) of the spectra generated by the MCMC chain to
1202 ‘data’. This ‘data’ could be true data or some variation of Monte Carlo prediction. This
1203 allows easy comparison of the MCMC probability distribution to the data. To perform
1204 this, N steps from the post burn-in MCMC chain are randomly selected (Where for all
1205 plots of this style in this thesis, $N = 3000$). From these, the Monte Carlo prediction at
1206 each step is generated by reweighting the model parameters to the values specified at
1207 that step. Due to the probability density being directly correlated with the density of
1208 steps in a certain region, parameter values close to the best fit value are most likely to
1209 be selected.

1210 In practice, for each bin of the fitting parameters has a probability distribution
1211 of event rates, with one entry per sampled MCMC step. This distribution is binned
1212 where the bin with the highest probability is selected as the mean and an error on

1213 the width of this probability distribution is calculated using the approach highlighted
1214 in subsection 4.3.2. Consequently, the best fit distribution in the fit parameter is not
1215 necessarily that which would be attained by reweighting the Monte Carlo prediction
1216 to the most probable parameter values.

1217 A similar study can be performed to illustrate the freedom of the model parameter
1218 space prior to the fit. This can be done by throwing parameter values from the prior
1219 uncertainty of each parameter. This becomes troublesome for parameters with no
1220 prior uncertainty as the range is technically infinite. Where applicable solutions to
1221 remove these have been addressed.

1222 **Chapter 5**

1223 **Simulation, Reconstruction, and Event
Reduction**

1225 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1226 neutrino spectrum at the far detector is required. This includes modeling the flux
1227 generation, neutrino interactions, and detector effects. All of the simulation packages
1228 required to do this are briefly described in section 5.1. The reconstruction of neutrino
1229 events inside the far detector, including the `fitQun` algorithm, is documented in
1230 section 5.2. This also includes data quality checks of the SK-V data which the author
1231 performed for the T2K oscillation analysis presented at Neutrino 2020 [80]. Finally,
1232 section 5.3 describes the steps taken in the SK detector to trigger on events of interest
1233 whilst removing the comparatively large rate of cosmic ray muon events.

1234 **5.1 Simulation**

1235 In order to generate a Monte Carlo prediction of the expected event rate at the far
1236 detector, all the processes in the beam and atmospheric flux, neutrino interaction, and
1237 detector need to be modeled. Each of these parts is individually modeled and each of
1238 them is detailed below.

1239 The beamline simulation consists of three distinct parts: the initial hadron inter-
1240 action modeled by FLUKA [142], the target station geometry and particle tracking

1241 performed by JNUBEAM, [143, 144] and any hadronic re-interactions simulated by
1242 GCALOR [145]. The primary hadronic interactions are $O(10)\text{GeV}$, where FLUKA
1243 matches external cross-section data better than GCALOR [146]. However, FLUKA
1244 is not very adaptable so a small simulation is built to model the interactions in the
1245 target and the output is then passed to JNUBEAM and GCALOR for propagation.
1246 The hadronic interactions are tuned to data from the NA61/SHINE [147–149] and
1247 HARP [150] experiments. The tuning is done by reweighting the FLUKA and GCALOR
1248 predictions to match the external data multiplicity and cross-section measurements,
1249 based on final state particle kinematics [146]. The culmination of this simulation
1250 package generates the predicted flux for neutrino and antineutrino beam modes which
1251 are illustrated in Figure 3.7.

1252 The atmospheric neutrino flux predictions are simulated by the HKKM model
1253 [43, 45]. The primary cosmic ray flux is tuned to AMS [151] and BESS [152] data assum-
1254 ing the US-standard atmosphere '76 [153] density profile and includes geomagnetic
1255 field effects. The primary cosmic rays interact to generate pions and muons. The
1256 interaction of these secondary particles to generate neutrinos is handled by DPMJET-
1257 III [154] for energies above 32GeV and JAM [45, 155] for energies below that value **DB:**
1258 **Question for Giles: Why different generators for above/below 32GeV?** These hadronic
1259 interactions are tuned to BESS and L3 data [156, 157] using the same methodology
1260 as the tuning of the beamline simulation. The energy and cosine zenith predictions
1261 of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3 and Figure 2.5, respectively. The flux is
1262 approximately symmetrical and peaked around the horizon ($\cos(\theta_Z) = 0.0$). This is be-
1263 cause horizontally-going pions and kaons can travel further than their vertically-going
1264 counterparts resulting in a larger probability of decaying to neutrinos. The symmetry
1265 is broken in low-energy neutrinos due to geomagnetic effects, which modify the track
1266 of the primary cosmic rays. Updates to the HKKM model are currently ongoing [158].

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [159, 160] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a W^\pm boson in Charged Current (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. This is the reason why NC events are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events. The SK detector is only sensitive to charged particles, so all charged current interactions are simulated whilst only neutral current processes that produce charged mesons (NCDIS, NCCOH, and NCProd) are modeled. NC MEC interactions can only produce charged particles through secondary re-interactions which is a low cross-section process.

As illustrated in Figure 5.1, CC QE interactions dominate the low-energy cross-section of neutrino interactions. The NEUT implementation adopts the Llewellyn Smith [161] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [162] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions are controlled by vector and axial-vector form factors parameterised by the BBBA05 [163] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data [164], respectively. NEUT implements the Valencia [165] model to simulate MEC events, where two nucleons and two holes in the nuclear target are produced (Often called 2p2h interactions).

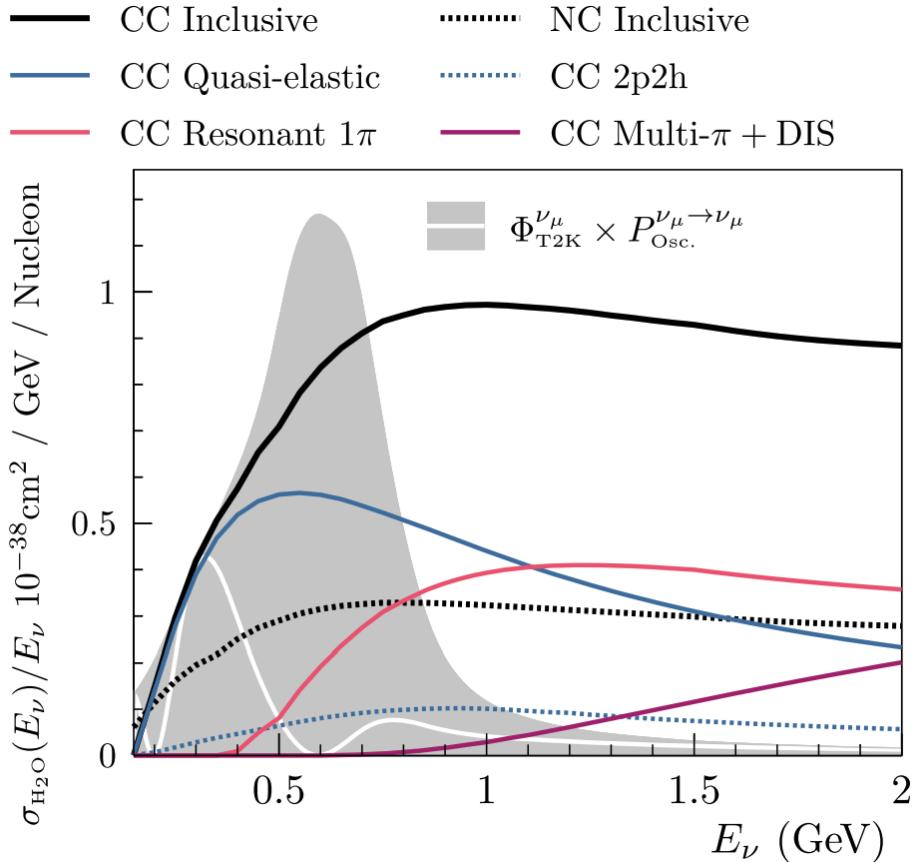


Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [159].

For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant. These predominantly produce charged and neutral pions although γ , kaon, and η production is also considered. To simulate these interactions, the Berger-Sehgal [166] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the consequential decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nuclei target leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [167] model to simulate these COH interactions.

1303 DIS and multi- π producing interactions become the most dominant for energies
1304 $> O(5)\text{GeV}$. PYTHIA [168] is used to simulate any interaction with invariant mass,
1305 $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces
1306 at least two mesons but has $W < 2\text{GeV}/c^2$, the Bronner model is invoked [169].
1307 Both of these models use Parton distribution functions based on the Bodek-Yang
1308 model [170–172].

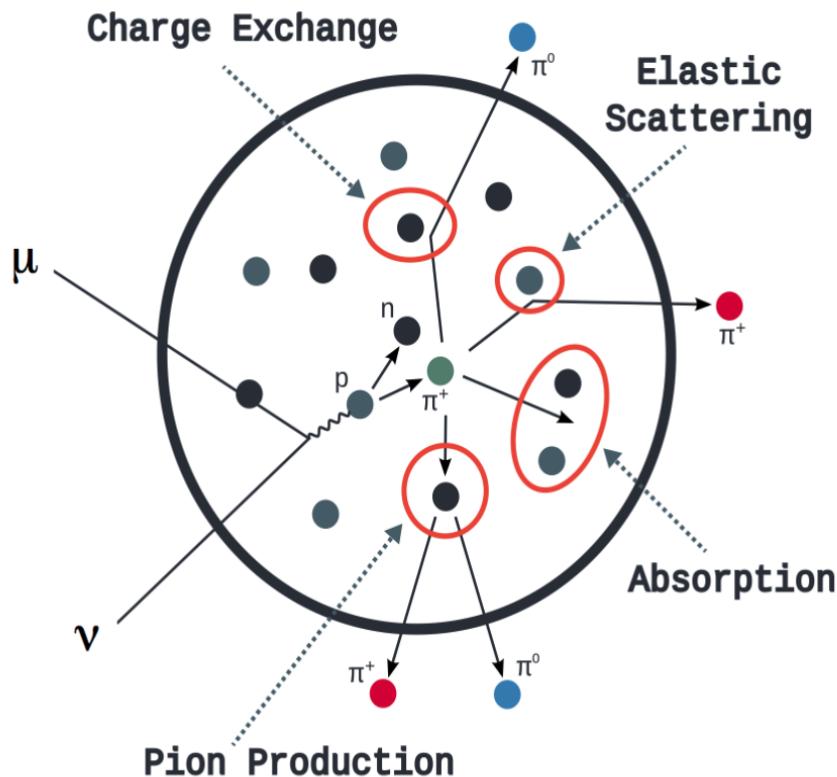


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [173].

1309 Any pion which is produced within the nucleus can re-interact through final state
1310 interactions before it exits, as illustrated by the scattering, absorption, production, and
1311 exchange interactions in Figure 5.2. These re-interactions alter the observable particles
1312 within the detector. For instance, if the charged pion from a CC PROD interaction is
1313 absorbed, the observables would mimic a CC QE interaction. To simulate these effects,

1314 NEUT uses a semi-classical intranuclear cascade model [159]. This cascade functions by
1315 stepping the pion through the nucleus in fixed-length steps equivalent to $dx = R_N/100$,
1316 where R_N is the radius of the nucleus. At each step, the simulation allows the pion
1317 to interact through scattering, charged exchange, absorption, or production with an
1318 interaction-dependent probability calculated from a fit to external data [174]. This
1319 cascade continues until the pion is absorbed or exits the nucleus.

1320 Once the final state particle kinematics have been determined from NEUT, they
1321 are passed into the detector simulation. The near detectors, ND280 and INGRID, are
1322 simulated using a GEANT4 package [118,175] to simulate the detector geometry, particle
1323 tracking, and energy deposition. The response of the detectors is simulated using
1324 the elecSim package [118]. The far detector simulation is based upon the original
1325 Kamiokande experiment software which uses the GEANT3-based SKDETSIM [118,176]
1326 package. This controls the interactions of particles in the water as well as Cherenkov
1327 light production. The water quality and PMT calibration measurements detailed in
1328 subsection 3.1.2 are also used within this simulation to make accurate predictions of
1329 the detector response.

1330 5.2 Event Reconstruction at SK

1331 Any above Cherenkov threshold event which occurs in SK will be recorded by the
1332 PMT array, where each PMT records the time and accumulated charge. This recorded
1333 information is shown in event displays similar to those illustrated in Figure 5.3. To
1334 be useful for physics analyses, this series of PMT hit information needs to be recon-
1335 structed to determine the particle's identity and kinematics (or track parameters):
1336 four-vertex, direction, and momenta. This is because the charge and timing distribu-
1337 tion of photons generated by a particular particle in an event is dependent upon its

1338 initial kinematics. The concept of distinguishing electron and muon events is from the
 1339 “fuzziness” of the ring. Muons are heavier and less affected by scattering or showering
 1340 meaning they typically produce “crisp” rings. Electrons are more likely to interact
 1341 via electromagnetic showering or scattering which results in larger variations of their
 1342 direction from the initial direction. Consequently, electrons typically produce “fuzzier”
 1343 rings compared to muons.

1344 For the purposes of this analysis, the `fitQun` reconstruction algorithm is utilised.
 1345 Its core function is to compare a prediction of the accumulated charge and timing
 1346 distribution from each PMT, generated for a particular particle identity and track
 1347 parameters, to that observed in the neutrino event. It determines the preferred values
 1348 by minimising a likelihood function which includes information from PMTs which
 1349 were hit and those that were not hit. The `fitQun` algorithm improves upon the APFit
 1350 reconstruction algorithm which has been used for many previous SK analyses. APFit
 1351 fits the vertex from timing information and then fits the momentum and direction
 1352 of the particle from PMT hits within a 43 deg Cherenkov cone (which assumes an
 1353 ultra-relativistic particle). It then fits the particle identity once the track parameters
 1354 have been fit. Conversely, `fitQun` performs a simultaneous fit of particle kinematics
 1355 and identity, improving both the accuracy of the fit parameters and the rejection of
 1356 neutral current π^0 events [177,178]. The `fitQun` algorithm is based on the key concepts
 1357 of the MiniBooNE reconstruction algorithm [179] and is described in [180] which is
 1358 summarised below.

1359 An event in SK can consist of multiple particles. For example, a charge current
 1360 muon neutrino interaction can generate two particles that have the potential of gen-
 1361 erating Cherenkov photons: the primary muon, and the secondary decay-electron
 1362 from the muon. To ensure both subevents are reconstructed separately, each event is
 1363 divided into time clusters which are called “subevents”. The number of subevents is

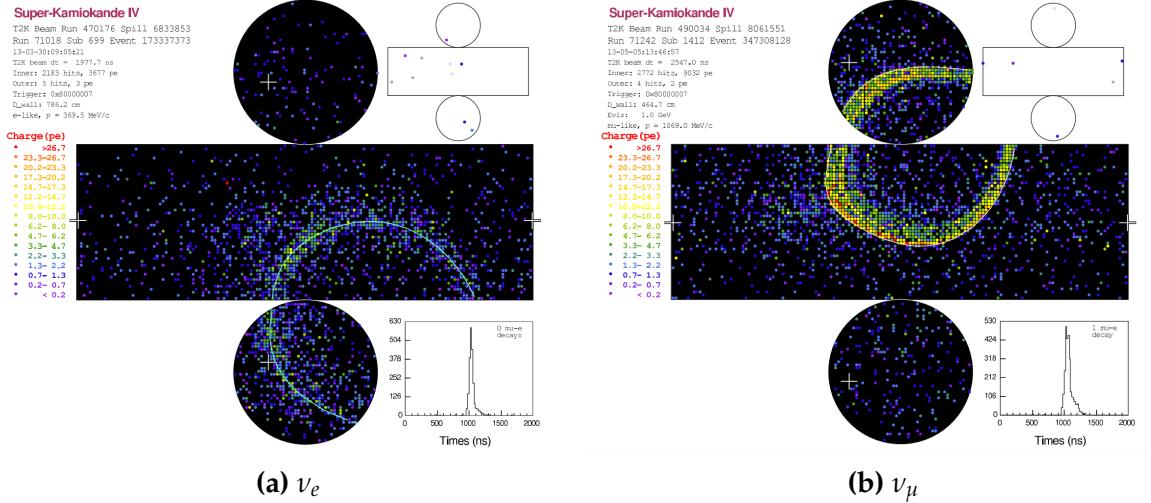


Figure 5.3: Event displays from Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [181].

equal to the number of decay electrons minus one (the primary event). To find all the subevents in an event, a vertex goodness metric is calculated for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right) \quad (5.1)$$

where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n \quad (5.2)$$

is the residual hit time. It is the difference in time between the PMT hit time, t^i , of the i^{th} PMT and the expected time of the PMT hit if the photon was emitted at the start of the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in

1371 water and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1372 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends to
 1373 zero resulting in subevents appearing as spikes in the goodness metric. The proposed
 1374 fit vertex and time are grid-scanned, and the values which maximise the goodness
 1375 metric are selected as the “pre-fit vertex”. Whilst this predicts a vertex for use in
 1376 the clustering algorithm, the final vertex is fit using the higher-precision maximum
 1377 likelihood method described below.

1378 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1379 a function of t to determine the number of subevents. A peak-finding algorithm is
 1380 then used on the goodness metric, requiring the goodness metric to exceed some
 1381 threshold and drop below a reduced threshold before any subsequent additional
 1382 peaks are considered. The thresholds are set such that the rate of false peak finding
 1383 is minimised while still attaining good data to Monte Carlo agreement. To improve
 1384 performance, the pre-fit vertex for each delayed subevent is re-calculated after PMT
 1385 hits from the previous subevent are masked. This improves the decay-electron tagging
 1386 performance. Once all subevents have been determined, the time window around
 1387 each subevent is then defined by the earliest and latest time which satisfies $-180 <$
 1388 $T_{Res}^i < 800\text{ns}$. The subevents and associated time windows are then used as seeds for
 1389 further reconstruction.

1390 For a given subevent, the `fiTQun` algorithm constructs a likelihood based on the
 1391 accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}), \quad (5.3)$$

1392 where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and mo-
 1393 ments, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ defines the probability
 1394 of the i^{th} tube to not register a hit given the track parameters and particle hypothesis.
 1395 The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$, represent the prob-
 1396 ability density function of observing charge q_i and time t_i on the i^{th} PMT given the
 1397 specified track parameters and particle hypothesis.

1398 As the generation and propagation of the optical photons are independent of the
 1399 PMT and electronics response, it is natural to split the calculation into two. Firstly,
 1400 the expected number of photoelectrons (or predicted charge), $\mu_i = \mu_i(\vec{\theta}, \Gamma)$, at the i^{th}
 1401 PMT is calculated. This value is then substituted into the likelihood function. This
 1402 allows the charge likelihood density $f_q(q_i|\mu_i)$ and unhit probability $P_i(\text{unhit}|\mu_i)$ to be
 1403 expressed via quantities that are only dependent on the response of the PMT.

1404 The predicted charge is calculated based on contributions from both the direct
 1405 light and the scattered light. The direct light contribution is determined based on the
 1406 integration of the Cherenkov photon profile along the track. PMT angular acceptance,
 1407 water quality, and calibration measurements discussed in subsection 3.1.2 are included
 1408 to accurately predict the charge probability density at each PMT. The scattered light
 1409 is calculated in a similar way, although it includes a scattering function that depends
 1410 on the vertex of the particle and the position of the PMT. The charge likelihood is
 1411 calculated by comparing the prediction to the observed charge in the PMT.

1412 The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} , and
 1413 time t of the track parameters as well as the particle hypothesis. The expected time
 1414 for PMT hits is calculated by assuming unscattered photons being emitted from the
 1415 midpoint of the track, S_{mid} ,

$$t_{exp}^i = t + S_{mid}/c + |R_{PMT}^i - \vec{x} - S_{mid}\vec{d}|/c_n, \quad (5.4)$$

where c is the speed of light in a vacuum. The time likelihood is then expressed in terms of the residual difference between the PMT hit time and the expected hit time, $t_{Res}^i = t^i - t_{exp}^i$. The particle hypothesis and momentum also affect the Cherenkov photon distribution. These parameters modify the shape of the time likelihood density since in reality not all photons are emitted at the midpoint of the track. As with the charge likelihood, the contributions from both the direct and scattered light to the time likelihood density are calculated separately, which are both calculated from particle gun studies.

The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined as the best-fit parameters. In practice MINUIT [182] is used to minimise the value of $-\ln L(\Gamma, \vec{\theta})$. The `fitQun` algorithm considers an electron-like, muon-like, and charged pion-like hypothesis for events with a single final state particle, denoted “single-ring events”. The particle’s identity is determined by taking the ratio of the likelihood of each of the hypotheses. For instance, electrons and muons are distinguished by considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison to the reconstructed momentum of the electron hypothesis [180]. This distance from this criteria is termed the PID parameter and is illustrated in Figure 5.4.

The `fitQun` algorithm also considers a π^0 hypothesis. To do this, it performs a fit looking for two standard electron-hypothesis tracks which point to the same four-vertex. This assumes the electron tracks are generated from photon-conversion so the electron tracks actually appear offset from the proposed π^0 vertex. For these fits, the conversion length, direction, and momenta of each photon are also considered as track

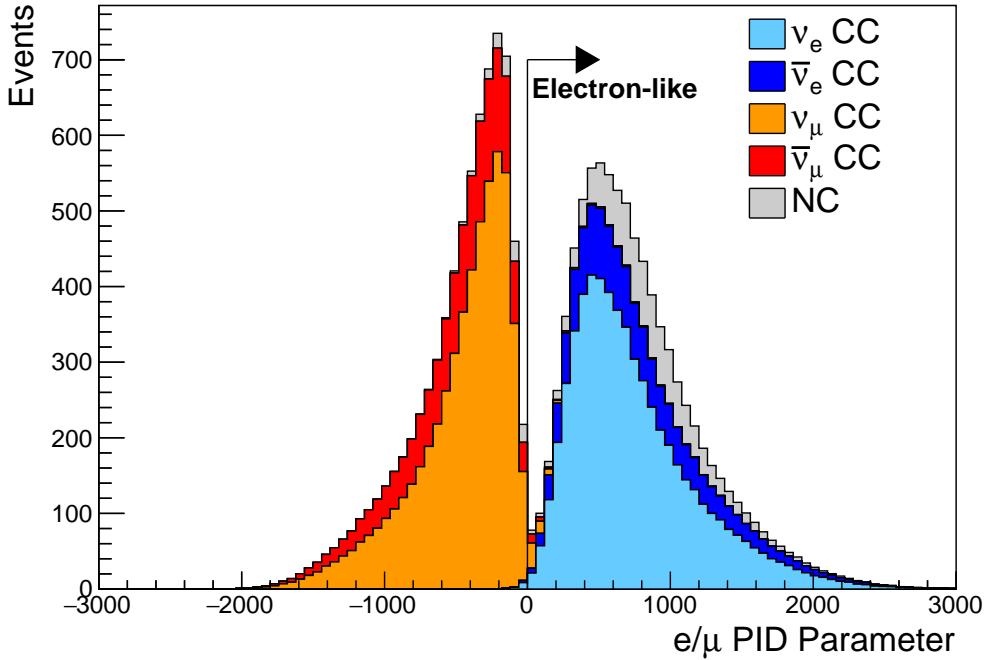


Figure 5.4: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

¹⁴³⁸ parameters which are then fit in the same methodology as the standard single-ring
¹⁴³⁹ hypotheses.

¹⁴⁴⁰ Whilst low energy events are predominately single-ring events, higher energy
¹⁴⁴¹ neutrino events can generate final states with multiple particles which generate
¹⁴⁴² Cherenkov photons. These “multi-ring” hypotheses are also considered in the `fitQun`
¹⁴⁴³ algorithm. When calculating the charge likelihood density, the predicted charge
¹⁴⁴⁴ associated with each ring is calculated separately and then merged to calculate the
¹⁴⁴⁵ total accumulated charge on each PMT. Similarly, the time likelihood for the multi-ring
¹⁴⁴⁶ hypothesis is calculated assuming each ring is independent. Each track is time-ordered
¹⁴⁴⁷ based on the time of flight from the center of the track to the PMT and the direct light
¹⁴⁴⁸ from any ring incident on the PMT is assumed to arrive before any scattered light. To
¹⁴⁴⁹ reduce computational resources, the multi-ring fits only consider electron-like and

1450 charged pion-like rings as the pion fit can be used as a proxy for a muon fit due to
1451 their similar mass.

1452 Multi-ring fits proceed by proposing another ring to the previous fit and then
1453 fitting the parameters in the method described above. Typically, multi-ring fits have
1454 the largest likelihood because of the additional degrees of freedom introduced. Conse-
1455 quently, the additional ring is only added if the ratio of likelihoods passes a criterion,
1456 which is determined by Monte Carlo studies.

1457 As an example of how the reconstruction depends on the detector conditions, the
1458 author of this thesis assessed the quality of event reconstruction for SK-V data. The
1459 detector systematics invoked within the T2K-only oscillation analysis are determined
1460 using data to Monte Carlo comparisons using the SK-IV data [183]. Due to tank-open
1461 maintenance occurring between SK-IV and SK-V, the dark rate of each PMT was
1462 observed to increase in SK-V due to light exposure for a significant time during the
1463 repairs. This increase can be seen in Figure 5.5. Run-10 of the T2K experiment was
1464 conducted in the SK-V period, so the consistency of SK-IV and SK-V data needs to
1465 be studied to determine whether the SK-IV-defined systematics can be applied to the
1466 run-10 data. This comparison study was performed using the stopping muon data set
1467 for both the SK-IV and SK-V periods. This data sample is used due to the high rate of
1468 interactions ($O(200)$ events per hour) as well as having similar energies to muons from
1469 CCQE ν_μ interactions from beam interactions. The rate of cosmic muons does depend
1470 on the solar activity cycle [184] but has been neglected in this comparison study. This
1471 is because the shape of the distributions is most important for the purposes of being
1472 compared to the detector systematics. The SK-IV and SK-V data samples consist of
1473 2398.42 and 626.719 hours of data which equates to 686k and 192k events respectively.

1474 The predicted charge calculated in the `fitQun` charge likelihood prediction includes
1475 a contribution from the photoelectron emission due to dark noise. Therefore, the

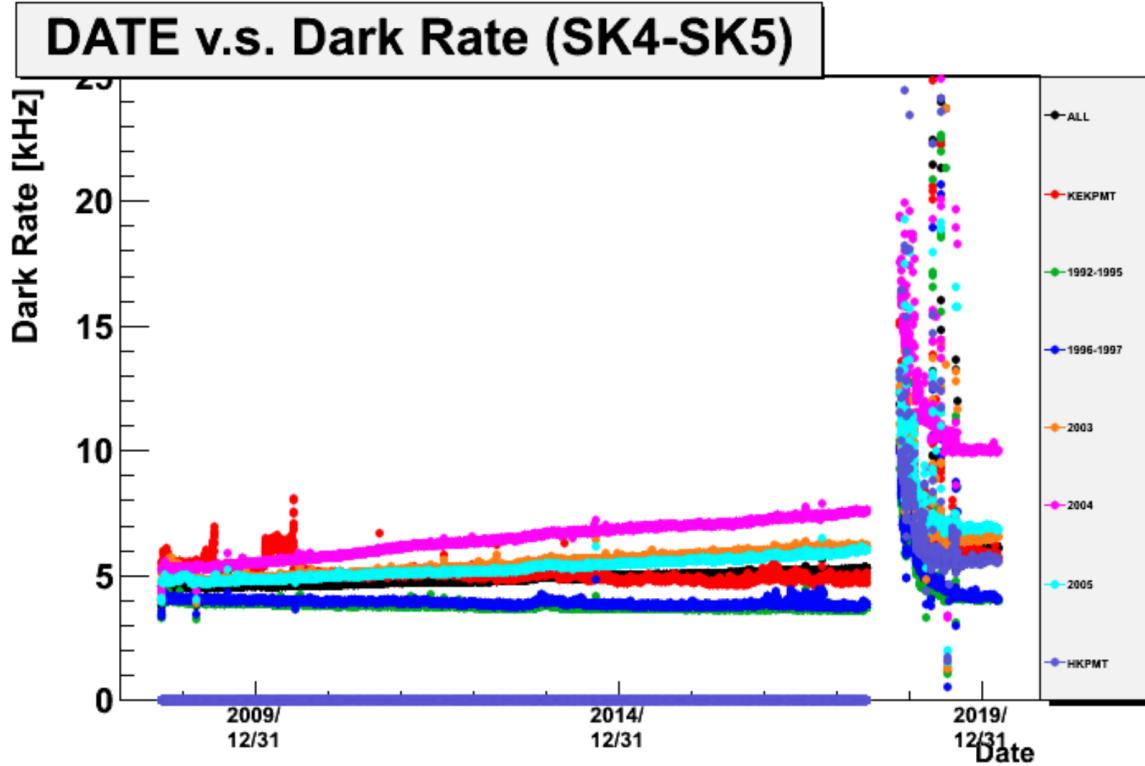


Figure 5.5: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [183].

increase in the SK-V dark rate needs to be accounted for. In practice, the average dark rate in each SK period is calculated and used as an input in the reconstruction. This is calculated by averaging the dark rate per run for each period separately, using the calibration measurements detailed in subsection 3.1.2. The average dark rate from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The associated charge with the muon and decay electron subevents are illustrated in Figure 5.6. The photoelectron emission from dark noise will be more noticeable for events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in low-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent

¹⁴⁸⁶ is mostly unaffected by the increase in dark rate, whilst the charge associated with the
¹⁴⁸⁷ decay-electron is clearly affected.

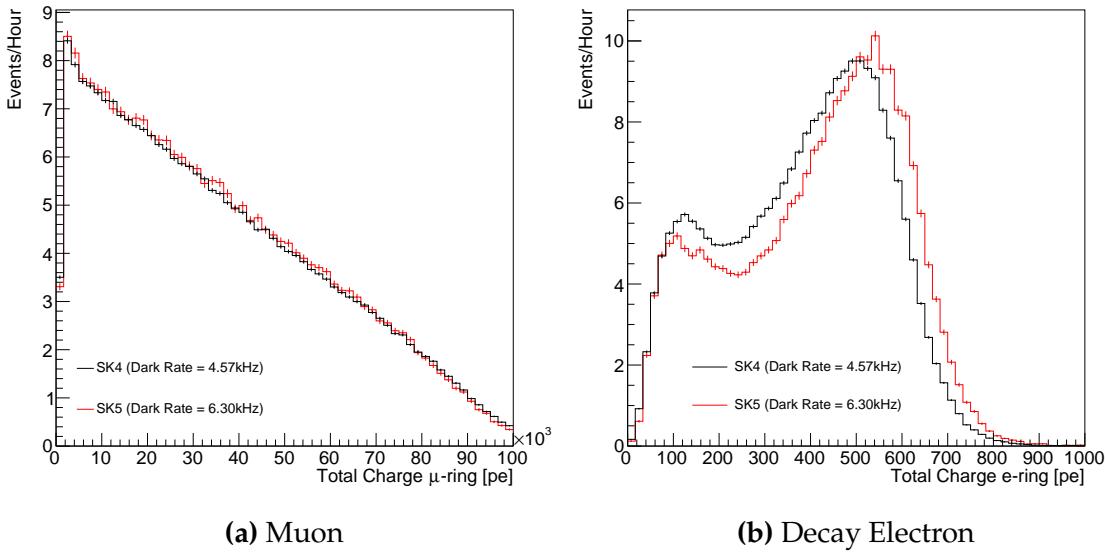


Figure 5.6: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent and the associated decay electron subevent.

¹⁴⁸⁸ The energy scale systematic for the SK-IV period was determined to be 2.1% [185].
¹⁴⁸⁹ It is defined to be equal to the difference between data and Monte Carlo prediction
¹⁴⁹⁰ in the stopping muon data sample. To determine the consistency of the SK-IV and
¹⁴⁹¹ SK-V with respect to the energy scale systematic, the muon momentum distribution is
¹⁴⁹² compared between the two SK periods. As the total number of Cherenkov photons
¹⁴⁹³ is integrated across the track length, the reconstructed momentum divided by track
¹⁴⁹⁴ length (or range) is compared between SK-IV and SK-V as illustrated in Figure 5.7.

¹⁴⁹⁵ The consistency between these distributions has been computed in two ways.
¹⁴⁹⁶ Firstly, a Gaussian is fit to each distribution separately. The mean of which is found to
¹⁴⁹⁷ be $(2.272 \pm 0.003)\text{MeV/cm}$ and $(2.267 \pm 0.006)\text{MeV/cm}$ for SK-IV and SK-V respec-
¹⁴⁹⁸ tively. The ratio of these is equal to 1.002 ± 0.003 . The mean of the Gaussian fits are
¹⁴⁹⁹ consistent with the expected stopping power of a minimum ionising muon for a target

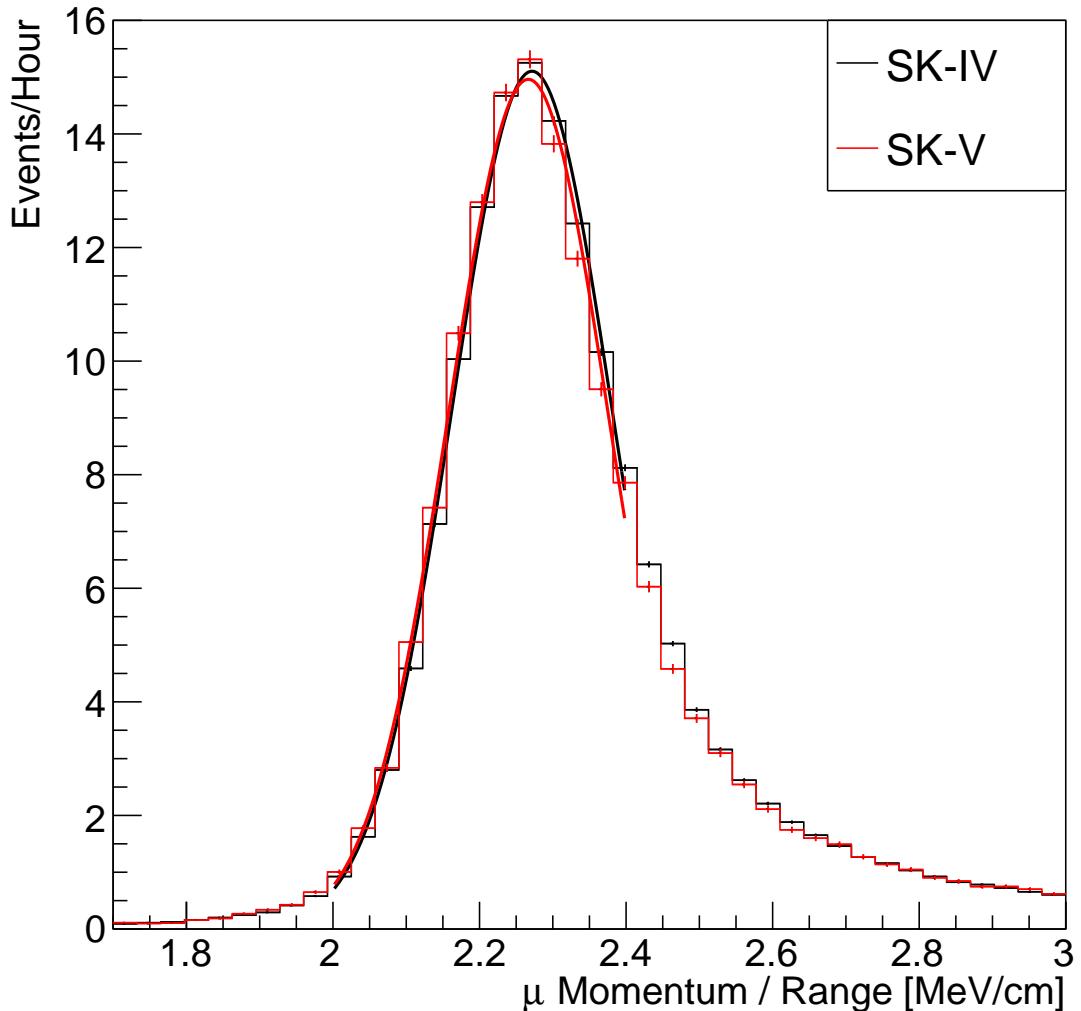


Figure 5.7: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range [2.0, 2.4] MeV/cm and illustrated as the solid curve.

material (water) with $Z/A \sim 0.5$ [186]. The second consistency check is performed by introducing a nuisance parameter, α , which modifies the SK-V distribution. The value of α which minimises the χ^2 value between the SK-IV and SK-V is determined by scanning across a range of values. This is repeated by applying the nuisance parameter as both a multiplicative factor and an additive shift. The χ^2 distributions for different values of α is illustrated in Figure 5.8. The values which minimise the χ^2 are found to

- 1506 be 0.0052 and 1.0024 for the additive and multiplicative implementations, respectively.
1507 No evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic
1508 has been found in the reconstructed momentum distribution of SK-IV and SK-V.

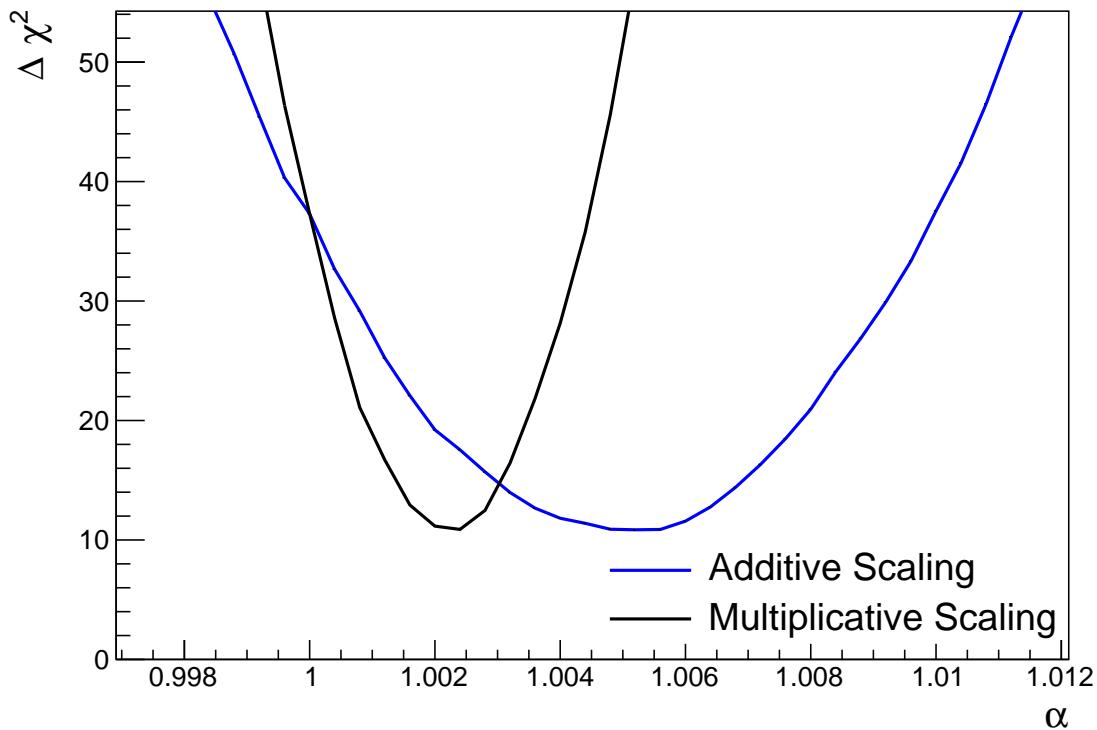


Figure 5.8: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1509 5.3 Event Reduction at SK

- 1510 Atmospheric neutrino events observed in the SK detector are categorised into three
1511 different types of samples: fully contained (FC), partially contained (PC) and up-
1512 going muon (Up- μ), using PMT hit signatures in the inner and outer detector (ID
1513 and OD, respectively). To identify FC neutrino events, it is required that the neutrino

1514 interacts inside the fiducial volume of the ID such that no significant OD activity is
 1515 observed. For this analysis, an event is defined to be in the fiducial volume providing
 1516 the event vertex is at least 0.5m away from the ID walls. PC events have the same
 1517 ID requirements but can have a larger signal present inside the OD. Typically these
 1518 events are higher energy muon interactions that penetrate the ID walls. The Up- μ
 1519 sample contains events where muons are created from neutrino interactions in the
 1520 OD water or rock below the tank. They then propagate upwards through the detector.
 1521 The reason downward-going muons generated from neutrino interactions above the
 1522 tank are neglected is due to the difficulty in separating their signature from the cosmic
 1523 muon shower background. The sample categories are visually depicted in Figure 5.9.

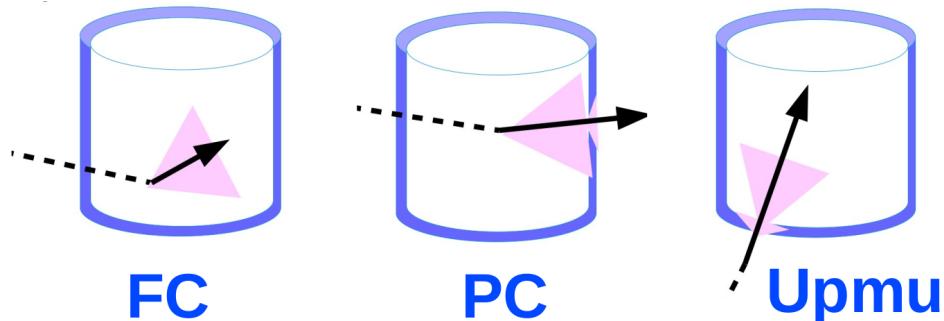


Figure 5.9: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC) and up-going muon (Up- μ) samples included in this analysis.

1524 Based on the event characteristics, as defined by the `fitQun` event reconstruction
 1525 software, the FC events are categorised by

- 1526 • **Visible Energy:** equal to the sum of the reconstructed kinetic energy above the
 1527 Cerenkov threshold for all rings present in the event. The purpose is to separate
 1528 events into sub-GeV and multi-GeV categories.
- 1529 • **Number of observed Cerenkov rings.** The purpose is to separate single-ring and
 1530 multi-ring events, where single-ring events predominantly consist of quasi-elastic
 1531 interactions and multi-ring events are typically resonant pion production or deep
 1532 inelastic scattering events.

- **Particle identification parameter of the most energetic ring:** A value determined from the maximum likelihood value based on `fitQun`'s electron, muon, or pion hypothesis. The purpose is to separate electron-like and muon-like events.
- **Number of decay electrons:** The purpose is to separate quasi-elastic events (which have one decay electron emitted from the muon decay) and resonant pion production events (which have two decay electrons emitted from the muon and pion).

The PC and Up- μ categories are broken down into “through-going” and “stopping” samples depending on whether the muon left the detector. This is because the stopping events deposit the entire energy of the interaction into the detector, resulting in better reconstruction. The energy of events that exit the detector has to be estimated which introduces much larger systematic uncertainties. Through-going Up- μ samples are further broken down by whether any hadronic showering was observed in the event which typically indicates DIS interactions. The expected neutrino energy for the different categories is given in Figure 5.10. FC sub-GeV and multi-GeV events peak around 0.7GeV and 3GeV respectively, with slightly different peak energies for $\nu_x \rightarrow \nu_e$ and $\nu_x \rightarrow \nu_\mu$ oscillation channels. PC and Up- μ are almost entirely comprised of $\nu_x \rightarrow \nu_\mu$ events and peak around 7GeV and 100GeV, respectively.

In normal data-taking operations, the SK detector observes many background events alongside the beam and atmospheric neutrino signal events of physics interest. Cosmic ray muons and flasher events, which are the spontaneous discharge of a given PMT, contribute the largest amount of background events in the energy range relevant to any analysis searching for neutrino events. Lower energy analyses like DSNB searches are also subject to radioactive backgrounds [187]. Therefore the data recorded is reduced with the aim of removing these background events. The reduction process is detailed in [47, 89] and briefly summarised below.

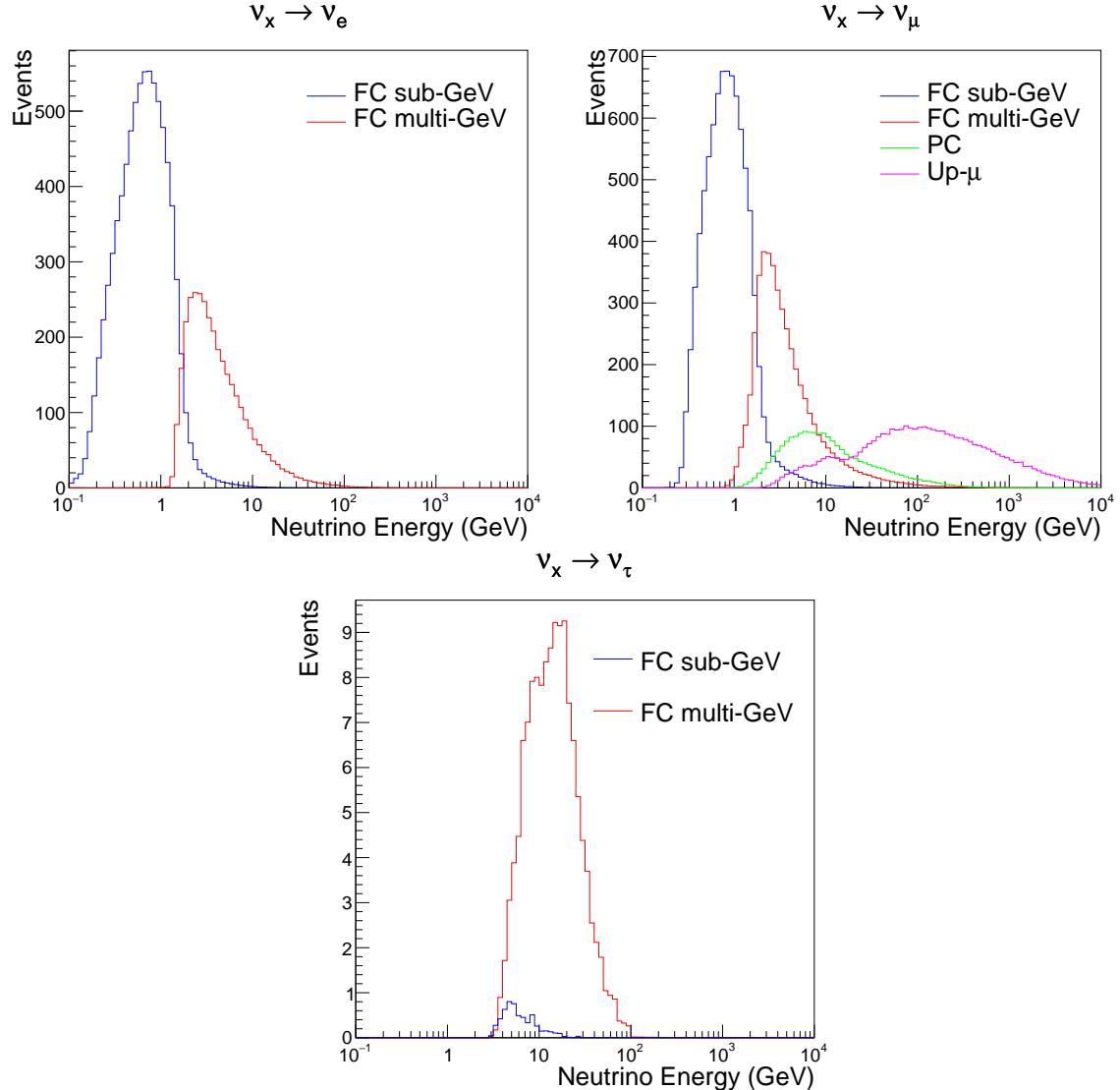


Figure 5.10: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). All systematic dials are set to their nominal values and the Asimov A oscillation parameters are assumed.

1559 The first two steps in the FC reconstruction remove the majority of cosmic ray
 1560 muons by requiring a significant amount of ID activity compared to that measured in
 1561 the OD. Events that pass this cut are typically very high momentum muons or events
 1562 that leave very little activity in the OD. Consequently, a third reduction step is then
 1563 applied to select cosmic-ray muons that pass the initial reduction step. A purpose-built
 1564 cosmic muon fitter is used to determine the entrance (or exit) position of the muon and

1565 a cut is applied to OD activity contained within 8m of this position. Flasher events are
 1566 removed in the fourth reduction step which is based on the close proximity of PMT
 1567 hits surrounding the PMT producing the flash. Events that pass all these reduction
 1568 steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses
 1569 information from the more precise fitter to repeat the previous two steps with tighter
 1570 cuts. Muons below the Cherenkov threshold can not generate optical photons in the
 1571 ID but the associated decay electron can due to its lower mass. These are the types of
 1572 events targeted in the fifth reduction step. The final cuts require the event vertex to be
 1573 within the fiducial volume (0.5m from the wall although the nominal distance is 2.0m),
 1574 visible energy $E_{vis} > 30\text{MeV}$ and fewer than 16 hits within the higher energy OD
 1575 cluster. The culmination of the fully contained reduction results in 8.09 events/day in
 1576 the nominal fiducial volume [188]. The uncertainty in the reconstruction is calculated
 1577 by comparing Monte Carlo prediction to data. The largest discrepancy is found to be
 1578 1.3% in the fourth reduction step.

1579 The PC and Up- μ events are processed through their own reduction processes
 1580 detailed in [47]. Both of these samples are reconstructed with the APFit algorithm
 1581 rather than fiTQun. This is because the efficiency of reconstructing events that leave
 1582 the detector has not been sufficiently studied for reliable systematic uncertainties. The
 1583 PC and Up- μ samples attain events at approximately 0.66 and 1.44 events/day.

1584 Events due to beam neutrinos undergo the same reduction steps as FC events and
 1585 are then subject to further cuts [189]. The GPS system which links the timing between
 1586 the beam facility and SK needs to be operating correctly and there should be no activity
 1587 within the detector in the previous $100\mu\text{s}$ before the trigger. The events then need to
 1588 triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1589 Due to the lower energy beam neutrinos, the T2K samples are not dependent
 1590 upon the visible energy neutrino as the range of neutrino energies are smaller than

that found in atmospheric neutrinos. Furthermore, the 2020 T2K-only oscillation analysis only considers events which contain a single ring. Similar to atmospheric event selection, the number of decay electrons is used as a proxy for distinguishing CCQE and CCRES events. The expected neutrino energy, broken down by number of decay electrons, is given in Figure 5.11.

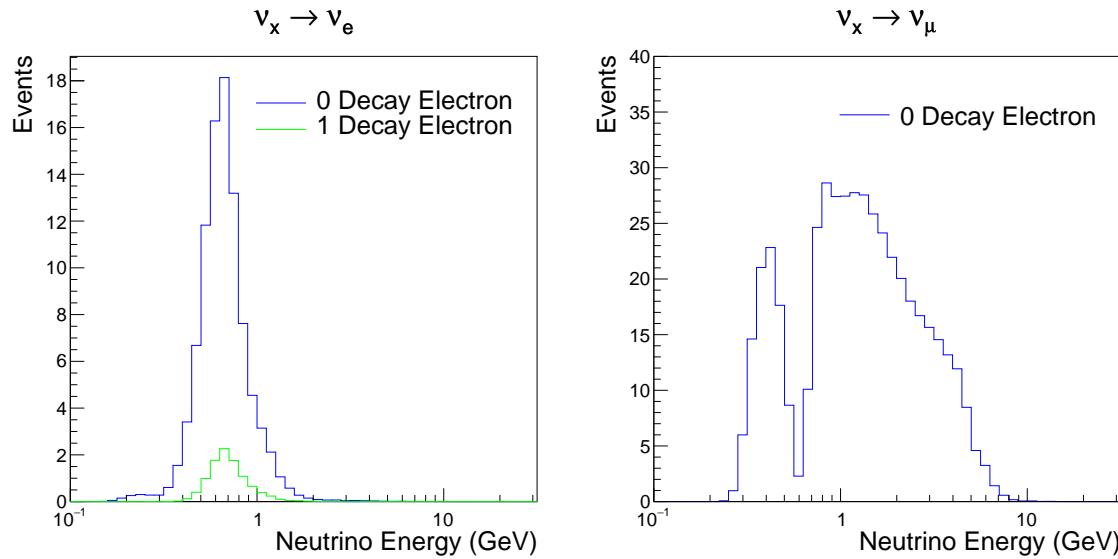


Figure 5.11: The predicted neutrino flux of the beam neutrinos, illustrated as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. All systematic dials are set to their nominal values and the Asimov A oscillation parameters are assumed.

1596 **Chapter 6**

1597 **Sample Selections and Systematics**

1598 **6.1 Systematic Uncertainties**

1599 The systematics for this uncertainty are split into the groups, or blocks, depending
1600 on their purpose. They consist of flux uncertainties, neutrino-matter interaction
1601 systematics and detector efficiencies. There are also uncertainties on the oscillation
1602 parameters which this analysis will not be sensitive to, Δm_{12}^2 and $\sin^2(\theta_{12})$. As
1603 described in chapter 4, each model parameter used within this analysis requires a
1604 prior uncertainty. This is provided via separate covariance matrices for each block.
1605 The covariance matrices can include prior correlations between parameters within a
1606 single block, but the separate treatment means prior uncertainties can not be included
1607 for parameters in different groups. Alternatively, some parameters have no reasonably
1608 motivated uncertainties. These parameters are assigned flat priors which do not
1609 change the likelihood penalty. The flux, neutrino interaction and detector modelling
1610 has already been discussed in section 5.1. The uncertainties invoked within these
1611 models are described below.

1612 **6.1.1 Beam Flux**

1613 The neutrino beam flux systematics are based upon our uncertainty in the modelling of
1614 the components of the beam. This includes: the hadron production model and their re-
1615 interactions, the shape, intensity and alignment of the beam with respect to the target,

and the uniformity of the magnetic field produced by the horn, alongside other effects. The uncertainty, as a function of neutrino energy, is illustrated in Figure 6.1 which includes the total uncertainty as well as the individual components. The uncertainty for events below, and much higher than, the peak neutrino energy is dominated by hadron production and re-interaction systematics. The beam profile and alignment of the proton beam dominates the systematic uncertainty for events with $E_\nu \sim 1\text{GeV}$.

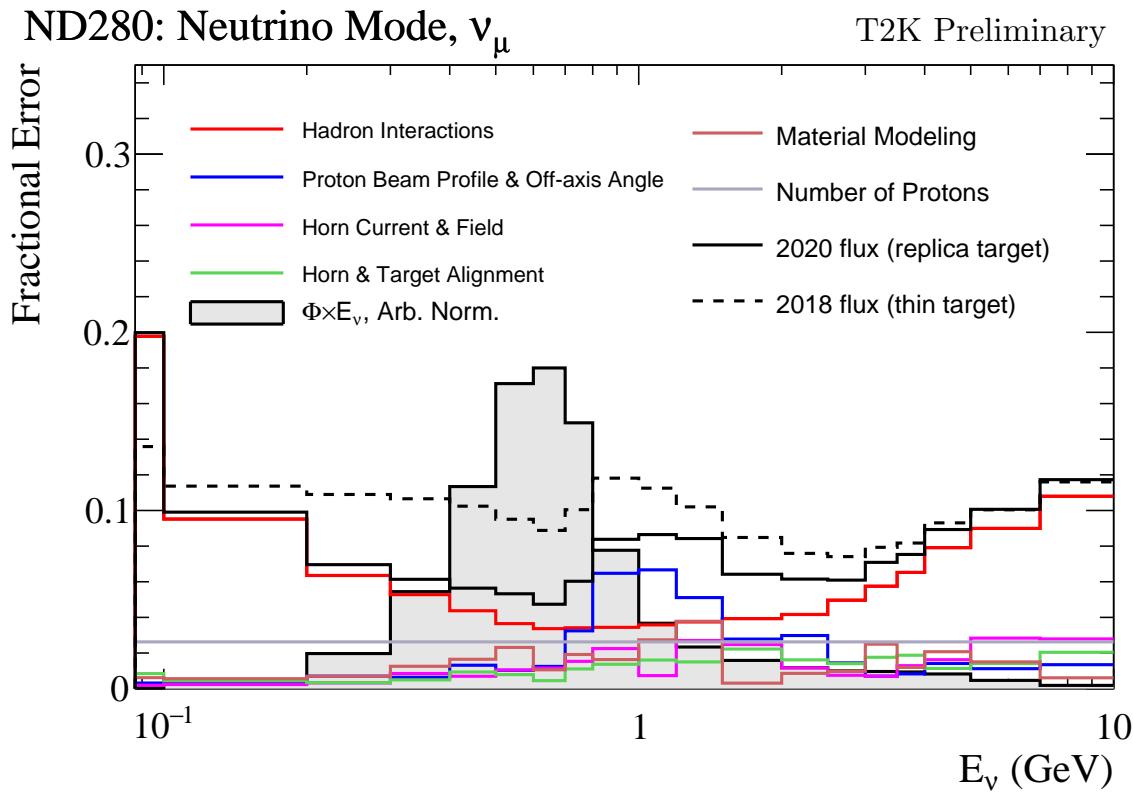


Figure 6.1: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [190].

The beam flux uncertainties are described by one hundred parameters. They are split between both ND280 and SK detectors and binned by neutrino flavour: $\nu_\mu, \bar{\nu}_\mu, \nu_e$ and $\bar{\nu}_e$. The response is then broken down as a function of neutrino energy. The bin density in the neutrino energy is the same for the FHC- ν_μ and RHC- $\bar{\nu}_\mu$, and narrows

for neutrino energies close to the oscillation maxima of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.1. All of these systematic uncertainties are applied as normalisation parameters with Gaussian priors centered at 1.0 and error specified from a covariance matrix provided by the T2K beam group.

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.1: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam mode. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam mode. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

6.1.2 Atmospheric Flux

The atmospheric neutrino flux is modelled by the HKKM model, however 16 systematic uncertainties are applied to control the normalisation of each neutrino flavour, energy and direction. All of the parameters are given Gaussian priors centered at 0 and width 1.. They are summarised below:

- **Absolute Normalisation:** The overall normalisation of each neutrino flavour is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$ and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction uncertainties for the production of pions and kaons [43]. The strength of the response is dependent upon the neutrino energy.

- **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$ are controlled by the difference between the HKKM model [43], FLUKA [46] and

1642 Bartol models [42]. Three independent parameters are applied in the energy
1643 ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.

- 1644 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent) produce uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied in the same way as the relative normalisation parameters.
- 1647 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two systematics, the difference between the HKKM, FLUKA and Bartol model predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of events as a function of zenith angle.
- 1651 • **K/π Ratio:** Higher energy neutrinos ($E_\nu < 10\text{GeV}$) become dependent upon kaon decay as the dominant source of neutrinos. Measurements of the ratio of K/π [191] are used to control the systematic uncertainty of the expected ratio of pion and kaon production.
- 1655 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic field, the flux of primary cosmic rays is modulated across the same period. The uncertainty is calculated by taking a ± 1 year variation, equating to a 10% uncertainty for the SK-IV period.
- 1659 • **Atmospheric Density:** The height of the interaction of the primary cosmic rays is dependent upon the atmospheric density. The HKKM assumes the US standard 1976 [153] profile. This systematic controls the uncertainty in that model.

1662 Updates to the HKKM and Bartol models are underway to use a similar tuning
1663 technique to that used in the beam flux predictions. After those updates, it may be
1664 possible to include correlations in the hadron production uncertainty systematics for
1665 beam and atmospheric flux predictions.

1666 6.1.3 Neutrino Interaction

1667 The neutrino interactions which occur within all the detectors are modelled by NEUT.
1668 The two independent oscillation analyses, T2K beam only and the SK atmospheric
1669 only, have developed separate interaction models. The T2K-only analysis uses the
1670 systematics model defined in [192] and the SK-only analysis uses the uncertainties
1671 detailed in [52]. To leverage the most sensitivity out of this joint beam and atmospheric
1672 analysis, a correlated interaction model has been defined. Where applicable, these
1673 correlations allow the systematic uncertainties applied to the atmospheric samples to
1674 be constrained by measurements of the near detector in the beam experiment leading
1675 to stronger sensitivity to oscillation parameters as compared to an uncorrelated model.
1676 An in-depth discussion of the reasoning and validity of enforcing correlations is
1677 documented in [193] and briefly summarised below.

1678 The low energy T2K systematic model has a more sophisticated treatment of CCQE,
1679 CCMEC and CCRES uncertainties which is due to the purpose made cross-section
1680 measurements made by the near detector. Furthermore, extensive testing of this model
1681 has been performed by the working group responsible for this model [192]. However,
1682 it is not designed for the high energy atmospheric events illustrated in Figure 5.10.
1683 Therefore the high energy systematic model from the SK-only analysis is implemented
1684 for the relevant multiGeV samples. The CCQE systematic parameters invoked within
1685 the SK high energy model are actually contained within T2K's CCQE model. Conse-
1686 quently, the more sophisticated CCQE and CCMEC T2K model parameters have been
1687 incorporated into the high energy model but are uncorrelated from the low energy
1688 counterparts. This results in a more complete model but without any constraint from
1689 the near detector measurements.

1690 The high energy systematic model includes parameters developed from compar-
1691 isons of Nieves and Rein-Seghal models which affect CCRES interactions, comparisons
1692 of the GRV98 and CKMT models which control DIS interactions, and hadron multiplic-
1693 ity measurements which modulate the normalisation of $CCN\pi$ events. The uncertainty
1694 of the ν_τ cross-section is particularly large and is controlled by a 25% normalisation
1695 uncertainty. These parameters are applied via normalisation or shape parameters. The
1696 former linearly scales the weight of all effected Monte-Carlo events, whereas the latter
1697 can increase or decrease a particular events weight depending on its neutrino energy
1698 and mode of interaction. The response of the shape parameters are defined by third
1699 order polynominal splines which return a weight for a particular neutrino energy. In
1700 total, 17 normalisation and 15 shape parameters are included in the more sophisticated
1701 high energy model.

1702 Figure 6.2 indicates the predicted neutrino energy distibution for both beam and
1703 subGeV atmospheric samples, and Figure 6.3 illustrates the fractional contribution
1704 of the different interaction modes per sample. There is clearly significant overlap in
1705 neutrino energy between the subGeV atmospheric and beam samples, allowing similar
1706 kinematics in the final state particles. Comparing beam samples with zero decay
1707 electrons and atmospheric electron-like(muon-like) samples with zero(zero or one)
1708 decay electrons, there is a very similar contribution of CCQE, CC 2p2h and $CC1\pi^\pm$
1709 interactions. The samples which target $CC1\pi^\pm$ interactions, FHC 1Re1de beam sample
1710 and atmospheric electron-like(muon-like) samples with one(two) decay electrons, also
1711 consist of very similar mode interactions. As a consequence of the similarity in energy
1712 and mode contributions, correlating the systematic model between the beam and
1713 subGeV atmospheric samples ensures that this analysis attains the largest sensitivity
1714 to oscillation parameters while still ensuring neutrino interaction systematics are
1715 correctly accounted for. Due to its sophisticated CCQE model, the T2K systematic
1716 model was chosen as the basis of the correlated model.

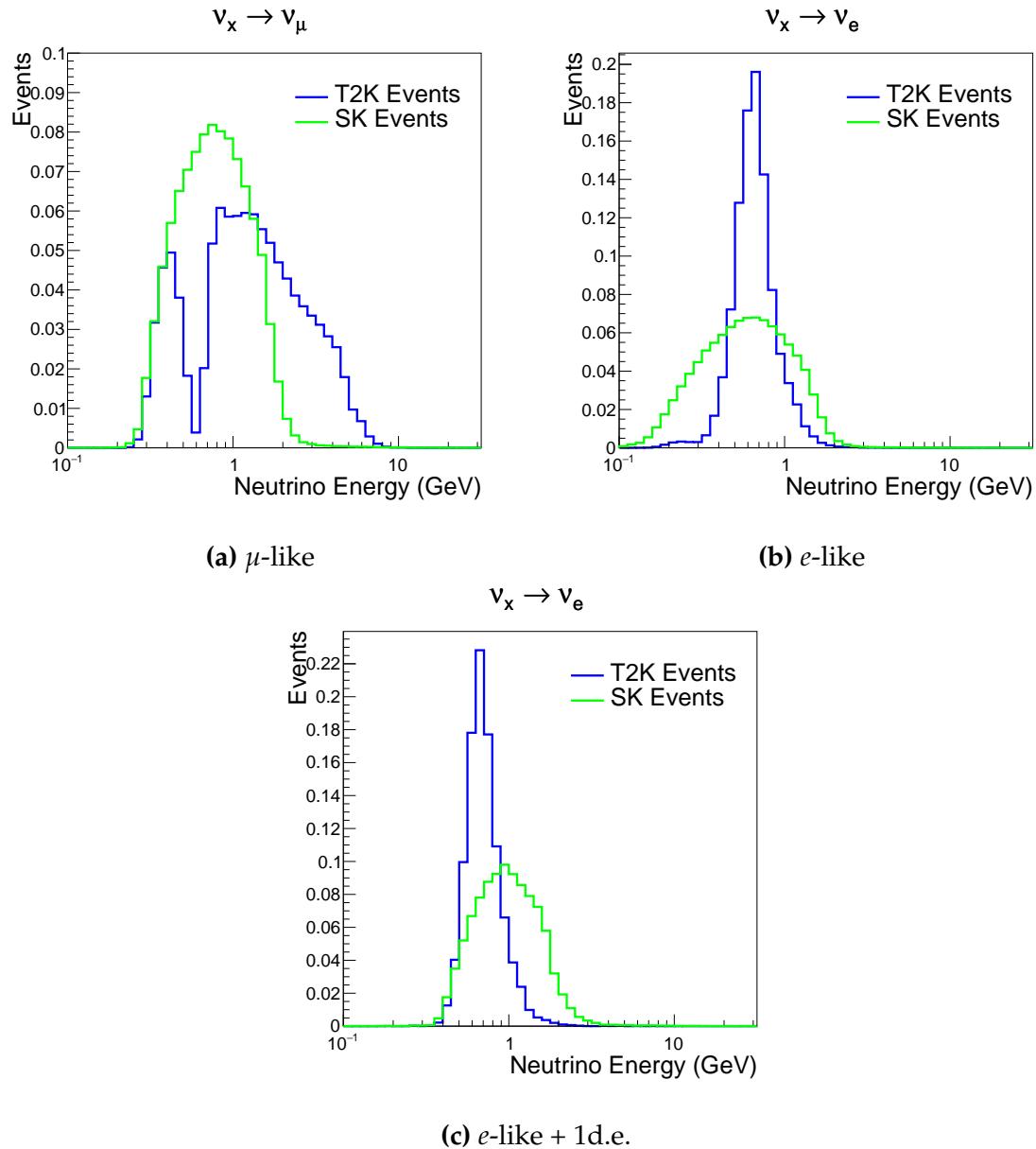


Figure 6.2: The prediction neutrino energy distribution for subGeV atmospheric and beam samples, given for muon-like samples FHC+RHC 1R μ beam samples compared to the subGeV μ -like 0+1 decay electrons (d.e.) atmospheric samples, electron-like 0d.e. samples FHC+RHC 1Re beam samples compared to the subGeV e -like 1d.e. sample, and electron-like 1d.e. sample FHC1Re1de beam sample compared to the subGeV e -like 1d.e. atmospheric sample.

1717 The T2K uncertainty model is applied in a similar methodology to the SK model
1718 parameters. It consists of 19 shape parameters applied via third order polynomial
1719 splines and 24 normalisation parameters. Four additional parameters, which model
1720 the uncertainty in the binning energy, are applied in a way to shift the momentum

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC π^0	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.3: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. All systematic dials are set to their nominal values and the Asimov A oscillation parameters are assumed. The Charged Current (CC) modes are broken into quasi-elastic (QE), meson exchange (2p2h), resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction category. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production and other.

of lepton emitted from a nucleus. The majority of these parameters are assigned a Gaussian prior uncertainty. Those that have no theoretical reasoning, or those which have not been fit to external data, are assigned a flat prior which does not affect the penalty term. The CCQE model parameters were tuned to MiniBooNE [194] and MINERνA [195] measurements and CCRES model parameters are tuned to ANL and BNL experiments [196].

On top of the combination of the SK and T2K interaction models, several other parameters have been specifically developed for the joint oscillation analysis. As the majority of the atmospheric samples' δ_{CP} sensitivity comes from the normali-

1730 sation of subGeV electron-like events, additional dial which models an alternative
1731 Continous Random Phase Approximation (CRPA) nuclear ground state has been im-
1732 plemented [193]. As the near detector can not sufficiently constrain the model, this
1733 dial approximates the event weights if a CRPA model had been assumed rather than a
1734 spectral function. This dial only effects ν_e and $\bar{\nu}_e$ and is applied as a shape parameter.

1735 Further additions to the model have been included due to the the subGeV π^0
1736 atmospheric sample. This particulary targets charged current and neutral current
1737 π^0 producing interactions to constrain the systematic uncertainties. However, there
1738 is no analogous sample in the T2K beam-only analysis so no significant effort has
1739 been placed into building a sufficient uncertainty model. Therefore, an uncertainty
1740 which effects neutral current resonant π^0 production is incorporated in this analysis.
1741 Comparisons of NEUT's NC resonant pion production predictions have been made
1742 to MiniBooNE [197] data and a consistent 16% to 21% underprediction is observed.
1743 Consequently, a conservative 30% normalisation is invoked.

1744 **DB: Adler Angle Dial**

1745 **DB: ND Constraint/BANFF**

1746 **6.1.4 Near Detector**

1747 The systematics applied due to uncertainties arising from the response of the near
1748 detector is contained within 574 normalisation parameters binned in momentum and
1749 angle, $P_\mu \text{ ad } \cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance
1750 matrix with each parameter been assigned a Gaussian prior from that covariance
1751 matrix. These normalisation parameters are built from underlying systematics, e.g.
1752 pion secondary interaction systematics, which are randomly thrown and the variation
1753 in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. This is performed 2000 times and a covariance

1754 matrix response is created. This allows significant correlations between FGD1 and
1755 FGD2 samples, as well as adjacent bins. Statistical uncertainties are accounted for by
1756 including fluctuations of each event's weight from a Poisson distribution.

1757 Similar to the cross section systematics, MaCh3 and BANFF are used to constrain
1758 the uncertainty of these systematics through independent validations. Each fitter
1759 generates a post-fit covariance matrix which is compared and passed to the far-detector
1760 oscillation analysis working group. However, as the analysis presented within this
1761 thesis uses the MaCh3 framework, a joint oscillation analysis fit of all three sets of
1762 samples is performed. From the T2K-only perspective, this joint analysis including
1763 atmospheric samples allows additional constraints on the systematic uncertainties
1764 where correlations have been invoked.

1765 6.1.5 Far Detector

1766 Two configurations of the far detector systematic model implementation have been
1767 considered. Firstly, the far detector systematic uncertainties for beam and atmo-
1768 spheric samples are taken from their respective analysis inputs, denoted “official
1769 inputs” analysis. Consequently, no correlations are assumed between the beam and
1770 atmospheric samples. The generation of the beam- and atmospheric- specific inputs
1771 are documented in subsubsection 6.1.5.1 and subsubsection 6.1.5.2 for the beam and
1772 atmospheric samples, respectively. Secondly, a correlated detector model has been
1773 considered. Here, the distribution of parameters used for applying event cuts (e.g.
1774 electron-muon separation) are modified within the fit, following similar methodology
1775 to the beam far detector systematics model implementation. However, it has been
1776 designed to ensure that the atmospheric data is not double-counted, which would be
1777 the case for the official inputs analysis. This alternative implementation is detailed in
1778 subsubsection 6.1.5.3.

1779 6.1.5.1 Beam Samples

1780 There are 45 systematics which describe the response of the far detector, specifically for
 1781 beam sample neutrino events. 44 of these parameters are normalisation parameters and
 1782 are split by the interaction mode, true neutrino flavour, reconstructed neutrino energy
 1783 and sample which they effect. The final parameter is the energy scale uncertainty. It is
 1784 applied as a multiplicative factor to the reconstructed neutrino energy. The value of
 1785 the systematic is taken from Monte Carlo to data differences illustrated in [185]. The
 1786 normalisation parameters are assigned a Gaussian error centrelised at 1.0 and error
 1787 taken from a covariance matrix. A detailed breakdown of the following procedure is
 1788 found in [198]. To build the covariance matrix, first a fit is performed to atmospheric
 1789 data which has been selected using beam sample selection cuts. The variable which
 1790 defines each cut, L (e.g. the electron-muon pid parameter) is assigned a smear, α , and
 1791 shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i \quad (6.1)$$

1792 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) pid cut parameters given in Table 6.2.
 1793 The shift and smear parameters are binned by final-state topology, j , where the binning
 1794 is given in Table 6.3. This approach is used to allow the cut parameter distributions to
 1795 be modified within the fit which allows better data to Monte Carlo agreement.

1796 Beyond the uncertainty on the pid cut criteria, the mis-modelling of π^0 events is also
 1797 considered. If one of the two rings from a π^0 event is missed, this will be reconstructed
 1798 as a $CC\nu_e$ event. This is one of the largest systematics hindering the electron neutrino
 1799 appearance analyses. Consequently, a systematic has been introduced to constrain
 1800 the mis-modelling of π^0 events in SK. To evaluate this systematic uncertainty, a set of

Cut Variable	Parameter Name
0	<code>fitQun e/μ PID</code>
1	<code>fitQun e/π⁰ PID</code>
2	<code>fitQun μ/π PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.2: List of cut variables which are included within the shift/smear fit documented in [198].

Index	Category	Description
0	$1e$	Only one electron above Cerenkov threshold in the final state
1	1μ	Only one muon above Cerenkov threshold in the final state
2	$1e+other$	One electron and one or more other charged particles above Cerenkov threshold in the final state
3	$1\mu+other$	One muon and one or more other charged particles above Cerenkov threshold in the final state
4	$1\pi^0$	Only one π^0 in the final state
5	$1\pi^\pm$ or $1p$	Only one hadron (typically charged pion or proton) in the final state
6	Other	Any other final state

Table 6.3: Reconstructed event topology categories on which the SK detector systematics [198] are based.

1801 “hybrid- π^0 samples is constructed. These events are built by overlaying one electron-
 1802 like ring from the SK atmospheric neutrino samples or decay electron ring from a
 1803 stopping cosmic ray muon with one simulated photon ring. Both rings are chosen so
 1804 that momenta and opening angle follow the decay kinematics of NC π^0 events from
 1805 the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the SK Monte
 1806 Carlo are produced to compare with the hybrid- π^0 data samples and the difference in
 1807 the fraction of events that pass the ν_e selection criteria is used to assign the systematic
 1808 error. In order to investigate any data to Monte Carlo differences which may originate
 1809 from either the higher energy ring or lower energy ring, two samples are built; a
 1810 sample in which the electron constitutes the higher energy ring from the π^0 decay
 1811 called the primary sample, and another one in which it constitutes the lower energy
 1812 ring called the secondary sample. The standard T2K ν_e fitQun event selection criteria
 1813 are used to select events.

Final contributions to the covariance matrix are determined by supplementary uncertainties attained by comparing stopping muon data to Monte Carlo prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data/Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or less decay electrons. The rate at which fake decay electrons are reconstructed by `f1TQun` is estimated in a similar way with the only difference being the ratio compares the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. The muon mis-identification rate is estimated by comparing the number of electron-like events which have one decay electron to the total number of events with one decay electron. This systematic is estimated as a 30% effect in the rate of muon mis-identification. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [183,199,200].

This covariance matrix is then added in quadrature with two other covariance matrices. These are matrices which describe the uncertainties due to secondary interactions which modify the final state kinematics and the photo-nucleon interactions. These are generated by studying the effect of each samples event rates when considering variations of the underlying parameters.

1835 **6.1.5.2 Atmospheric Samples**

1836 **6.1.5.3 Correlated Detector Model**

1837 A complete uncertainty model of the SK detector would be able to determine the
1838 systematic shift on the sample spectra for a variation of the underlying parameters, e.g.
1839 PMT angular acceptance. However, this is particularly resource intensive, requiring
1840 Monte Carlo predictions to be made for each plausible variation. Consequently an
1841 effective parameter model has been utilised for a correlated detector model. This
1842 follows from the T2K-only model implementation documented in subsubsection 6.1.5.1.
1843 The T2K-only implementation can not be used for atmospheric sample systematics
1844 because it is built upon a fit to atmospheric data. Consequently, an implementation
1845 where the cut distributions (given in Table 6.2) from both beam and atmospheric
1846 samples are fit, whilst simultaneously fitting for oscillation parameters. The fit to the
1847 cut variables performs a shape-only fit to ensure that no double-counting occurs.

1848 The correlated detector model utilises the same smear and shift parameters doc-
1849 umented in subsubsection 6.1.5.1, split by final state topology. This splitting is done
1850 because the detector will respond differently for events which have one or multiple
1851 rings. For example, the detector will be able to distinguish single-ring events better
1852 than two overlapping ring events, resulting in smaller systematic uncertainty for one
1853 ring events compared to two ring events. Furthermore, the shift and smear param-
1854 eters are split by visible energy deposited within the tank has been included, with
1855 binning specified in Table 6.4. This is because atmospheric events are categorised by
1856 subGeV and multiGeV events based on visible energy, so this splitting is required
1857 when correlating the systematic model for beam and atmospheric events. Alongside
1858 the technical requirement, higher energy events will be better reconstructed due to
1859 fractionally less noise within the detector. Consequently, this analysis correlates the

1860 detector systematics between the far-detector beam and subGeV atmospheric samples
1861 due to their similar energies and interaction types. As a result of the inclusion of visible
1862 energy binning, Equation 6.1 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.2)$$

1863 where k is the visible energy bin. The multi-GeV, multi-ring, PC and Up- μ samples
1864 will be subject to the ATMPD detector systematics implementation as described in
1865 subsubsection 6.1.5.2.

Index	Range (MeV)
0	$30 \geq x > 300$
1	$300 \geq x > 700$
2	$700 \geq x > 1330$
3	$1330 \geq x$

Table 6.4: Reconstructed event topology categories on which the SK detector systematics are based

1866 The implementation of this systematic model takes the events reconstructed values
1867 of the cut parameters, modifies them by the particular shift and smear parameter for
1868 that event, and then re-applies event selection. This invokes event migration, which is
1869 a new feature incorporated into the MaCh3 framework which is only achievable due
1870 to the event-by-event reweighting scheme.

1871 Particular care has to be taken when varying the ring counting parameter. This is
1872 because the number of rings is a finite value (one-ring, two-rings, etc.) which can not be
1873 continuously varied. Consequently a ring counting parameter, RC_i , is calculated for the
1874 i^{th} event, following the definition in [201]. The likelihood from all considered one-ring
1875 ($1R$) and two-ring ($2R$) fits are compared to determine the preferred hypothesis. This

1876 is done by searching for the minimum log likelihoods, $\log(L_{1R})$ and $\log(L_{2R})$. The
1877 difference is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter
1878 is then defined as,

$$RC_i = \text{sgn}(\Delta_{LLH} - C_{Thres}) \times \sqrt{|\Delta_{LLH} - C_{Thres}|}, \quad (6.3)$$

1879 where $C_{Thres} = 150.0 - 0.6 \times P_{2R}$, and P_{2R} is the momentum of the preferred two-
1880 ring hypothesis, and $\text{sgn}(x) = x/|x|$. The co-efficients used within the definition
1881 of C_{Thres} are calculated based Monte Carlo studies. This ring counting parameter
1882 corresponds to a intermediate likelihood value used within the `fitQun` algorithm to
1883 decide the number of rings associated with a particular event. However, fake-ring
1884 merging algorithms are applied after this likelihood value is used to determine the
1885 number of rings associated with an event. Consequently, this ring counting parameter
1886 does not always exactly correspond to the number of reconstructed rings. This can be
1887 seen in [DB: Plot of RC Parameter](#).

1888 [DB: Plot of RC Parameter](#)

1889 As the `fitQun` algorithm does not provide a likelihood value after the fake-ring
1890 algorithms have been applied, the ring counting parameter distribution is connected
1891 to the final number of reconstructed rings through “maps”. These are two dimen-
1892 sional distributions linking the ring counting parameter and the final number of
1893 reconstructed rings. An example is illustrated in [DB: Plot of RC Map](#). In principle,
1894 the `fitQun` reconstruction algorithm should be re-ran after the variation in the ring
1895 counting parameter. However, this is not computationally viable. Therefore the “maps”
1896 are used as a reweighting template.

1897 The maps are split by final state topology and true neutrino flavour and all `f1TQun`-
 1898 reconstructed Monte Carlo events are used to fill them. To ensure conservation of event
 1899 rate, the maps are normalised such that the total event rate across all number of recon-
 1900 structed rings is equal to one. Prior to the fit, an event's nominal weight is calculated as
 1901 $W(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th} event and
 1902 $W(x, y)$ is the bin content in the associated map for x number of rings and ring count-
 1903 ing parameter L . Then during the fit, the value of $R = W(N_{Rings}^i, \bar{L}_{jk}^i) / W(N_{Rings}^i, L_{jk}^i)$
 1904 is calculated as the ring-counting weight for the i^{th} event. This is the only cut variable
 1905 which uses a reweighting scheme rather than event migration.

1906 **DB: Plot of RC Map**

1907 The π^0 systematics introduced in subsection 6.1.4 were expected to be applied
 1908 via a covariance matrix. As this alternative technique performs a simultaneous fit
 1909 between detector distributions and oscillation parameters, the implementation of the
 1910 π^0 systematics has been modified. In practice, the inputs from the hybrid π^0 sample
 1911 is included via the use of “ χ^2 maps”, which are two dimensional histograms in α
 1912 and β parameters over some range. Illustrative examples of the χ^2 maps are given
 1913 in Figure 6.4. Due to their nature, the shift and smear parameter are typically very
 1914 correlated.

1915 The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo and
 1916 data in the particle identification parameters documented in Table 6.2. The Monte
 1917 Carlo distribution is modified with the α and β scaling, whilst cross-section and flux
 1918 nuisance parameters are thrown from there prior uncertainties, and the χ^2 between
 1919 the scaled Monte Carlo and data is calculated and the relevant point in the χ^2 map is
 1920 filled. Then in the fit, the likelihood penalty term is found for the particular particle
 1921 identification parameter by using the value of the relevant χ^2 map for the α and β

parameter at that step in the MCMC fit. For this fit, only $1\pi^0$ final state topology shift and smear parameters use the hybrid $\pi^0 \chi^2$ prior uncertainty.

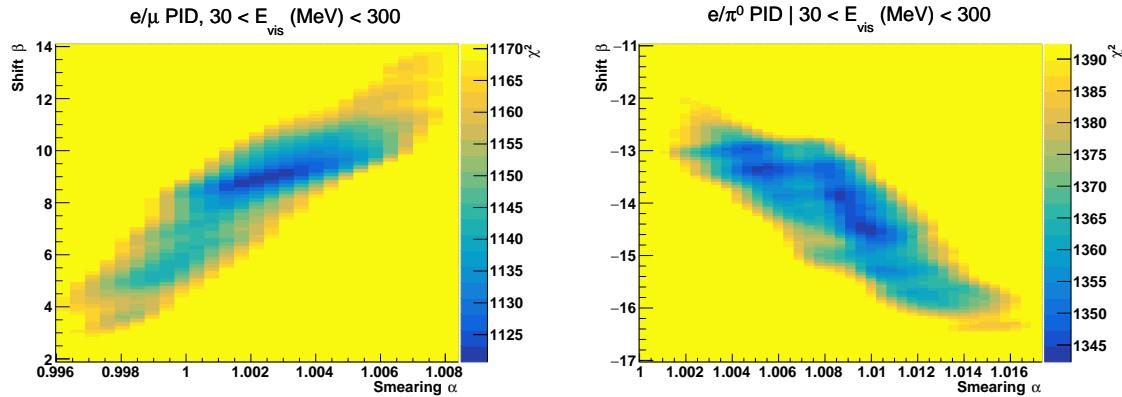


Figure 6.4: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \geq E_{vis}$ (MeV) < 300 . Right: Electron- π^0 separation PID parameter for events with $30 \geq E_{vis}$ (MeV) < 300 .

Similarly, the supplementary systematics which are added into the covariance from stopping muon and decay electron studies need to be included. A new framework [202] was built in tandem with the T2K-SK working group [183] so the additional parameters can be incorporated in the MaCh3 framework. These are applied as normalisation parameters, depending on the particular interaction mode, number of tagged decay electrons and whether the primary particle generated Cherenkov light. They are assigned Gaussian uncertainties with widths described by a covariance matrix.

Finally, the secondary interaction and photo-nuclear effects need to be accounted for in this detector model. In the T2K-only analysis, a covariance matrix was built to describe the response of the samples to variations of these parameters which was then added in quadrature to the detector covariance matrix. However, this technique can not be applied in the correlated detector model. Consequently, a binned response of each of the secondary interaction systematic parameters and the photo-nuclear

₁₉₃₈ response was generated and included through splined shape parameters, similar to
₁₉₃₉ the application of shape parameters in the cross-section model (see subsection 6.1.3).

₁₉₄₀ There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior constraints
₁₉₄₁ from the hybrid π^0 samples.

₁₉₄₂ **DB: Correlations of e/mu and e/pi**

1943 **Chapter 7**

1944 **Oscillation Probability Calculation**

1945 It is important to understand how and where the sensitivity to the oscillation pa-
1946 rameters comes from for both atmospheric and beam samples. An overview of how
1947 these samples observe changes in δ_{CP} , Δm_{23}^2 , and $\sin^2(\theta_{23})$ is given in section 7.1. It
1948 also explains the additional complexities involved when performing an atmospheric
1949 neutrino analysis as compared to a beam-only analysis.

1950 Without additional techniques, atmospheric sub-GeV upward-going neutrinos
1951 ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to δ_{CP} due to the
1952 quickly varying oscillation probability in this region. Therefore, a “sub-sampling”
1953 approach has been developed to reduce these biases ensuring accurate and reliable
1954 sensitivity measurements. This technique ensures that small-scale unresolvable fea-
1955 tures of the oscillation probability have been averaged over whilst the large-scale
1956 features in the oscillation probability are unaffected. The documentation and valida-
1957 tion of this technique are found in section 7.2. The oscillation probability calculation is
1958 computationally intensive due to the large number of matrix multiplications needed.
1959 Consequently, the CUDAProb3 implementation choice made within the fitting frame-
1960 work, as detailed in section 7.3, ensures that the analysis can be done in a timely
1961 manner.

1962 Whilst the beam neutrinos are assumed to propagate through a constant density
1963 slab of material, the density variations through the Earth result in more complex
1964 oscillation patterns. Furthermore, the uncertainty in the electron density can modify
1965 the oscillation probability for the denser core layers of the Earth. The model of the

1966 Earth used within this analysis is detailed in section 7.4. This includes information
 1967 about the official SK-only methodology as well as improvements that can be made
 1968 to remove some of the approximations made in that analysis. Another complexity of
 1969 atmospheric neutrinos oscillation studies is that the height of production in the atmo-
 1970 sphere is not known on an event-by-event basis. An analytical averaging technique
 1971 that approximates the uncertainty of the oscillation probability has been followed,
 1972 with the author of this thesis being responsible for the implementation and validation.
 1973 This implementation of an external technique is illustrated in section 7.5.

1974 7.1 Overview

1975 The analysis presented within this thesis focuses on the determination of oscillation
 1976 parameters from atmospheric and beam neutrinos. Whilst subject to the same oscil-
 1977 lation formalism, the way in which the two samples have sensitivity to the different
 1978 oscillation parameters differs quite significantly.

1979 Atmospheric neutrinos have a varying baseline, or “path length”, L , such that
 1980 the distance each neutrino travels before interacting is dependent upon the zenith
 1981 angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s surface
 1982 and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the neutrino was
 1983 generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (7.1)$$

1984 Where $R_E = 6,371\text{km}$ is the Earth’s radius. Consequently, the oscillation probabil-
 1985 ity is dependent upon two parameters, $\cos(\theta_Z)$ and E_ν .

1986 The oscillation probability used within this analysis is based on [21]. The neutrino
1987 wavefunction in the vacuum Hamiltonian evolves in each layer of constant matter
1988 density via

$$i\frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu}\psi_j(t) - \sum_k \sqrt{2}G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (7.2)$$

1989 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
1990 energy, G_F is Fermi's constant, N_e is the electron number density and U is the PMNS
1991 matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied for antineutrino
1992 propagation. Thus, a model of the Earth's density is required for atmospheric neutrino
1993 propagation. Following the official SK-only methodology [203], this analysis uses the
1994 Preliminary Reference Earth Model (PREM) [204]. This model provides piecewise cubic
1995 polynomials as a function of the Earth's radius which results in the density profile
1996 illustrated in Figure 7.1. As discussed, the propagator requires layers of constant
1997 density. The SK methodology approximates the PREM model by using four layers of
1998 constant density [203]. The details of these layers are detailed in Table 7.1.

Layer	Outer Radius [km]	Density [g/cm ³]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 7.1: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [204].

1999 The atmospheric neutrino oscillation probabilities can be presented as two dimensional
2000 “oscillograms” as illustrated in Figure 7.2. The distinct discontinuities, as a

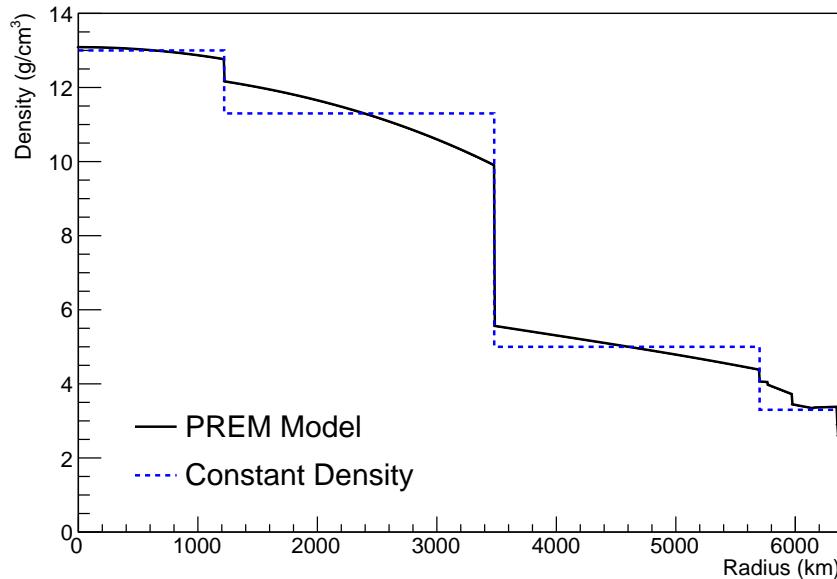


Figure 7.1: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

function of $\cos(\theta_Z)$, are due to the discrete change in density invoked within the PREM model.

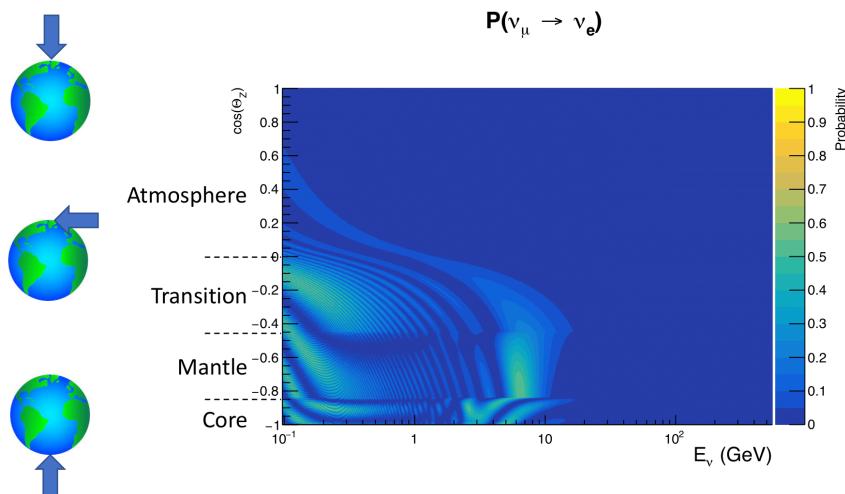


Figure 7.2: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

2003 Atmospheric neutrinos do have sensitivity to δ_{CP} through a normalisation term.
2004 Figure 7.3 illustrates the difference in oscillation probability between CP-conserving
2005 ($\delta_{CP} = 0.$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation
2006 parameter set (Table 2.2). The result is a complicated oscillation pattern in the appear-
2007 ance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient
2008 resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric
2009 neutrinos comes via the overall normalisation of these events.

2010 The presence of matter means that the effect δ_{CP} has on the oscillation probability
2011 is not equal between neutrinos and antineutrinos, which would be expected when
2012 propagating through a vacuum. This is further extenuated by the fact that SK can
2013 not distinguish neutrinos and antineutrinos well and that the cross-section neutrino
2014 interaction is larger than that for antineutrinos. Finally, sample selections (discussed
2015 in DB: [Link to selection chapter](#)) targeting different neutrino interaction modes result
2016 in an imbalance in the percentage of neutrinos to anti-neutrinos. This is because
2017 negatively charged pions from antineutrino interactions are more likely to be captured
2018 by a nucleus compared to a positively charged pion. All of these effects lead to
2019 a difference in the number of neutrinos detected compared to antineutrinos. This
2020 changes how the δ_{CP} normalisation term is observed, resulting in a very complex
2021 sensitivity to δ_{CP} .

2022 Atmospheric neutrinos are subject to matter effects as they travel through the dense
2023 matter in the Earth. The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$
2024 and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ are presented in Figure 7.4, where the PREM model has been assumed.
2025 The oscillation probability for both neutrinos and antineutrinos is affected in the
2026 presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only occur for
2027 neutrinos in normal mass hierarchy and antineutrinos in inverse mass hierarchy. The

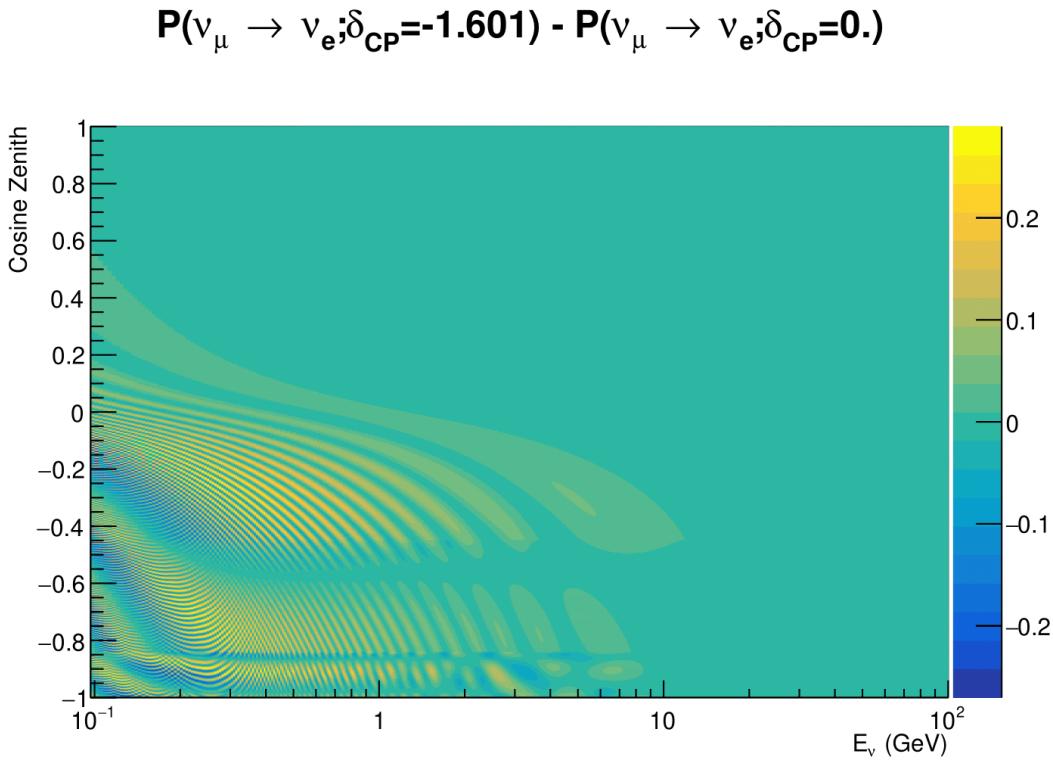


Figure 7.3: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

2028 exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$ meaning that the
 2029 atmospheric neutrinos have sensitivity to $\sin^2(\theta_{23})$.

2030 As the T2K beam flux is centered at the first oscillation maximum ($E_\nu = 0.6\text{GeV}$),
 2031 the sensitivity to δ_{CP} is predominantly observed as a change in the event-rate of e-like
 2032 samples in $\nu/\bar{\nu}$ modes. Figure 7.5 illustrates the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability
 2033 for a range of δ_{CP} values. A circular modulation of the first oscillation peak (in both
 2034 magnitude and position) is observed when varying throughout the allowable values
 2035 of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation
 2036 maximum than the CP-violating values of $\delta_{CP} = -\pi/2 (\delta_{CP} = \pi/2)$. A sub-dominant

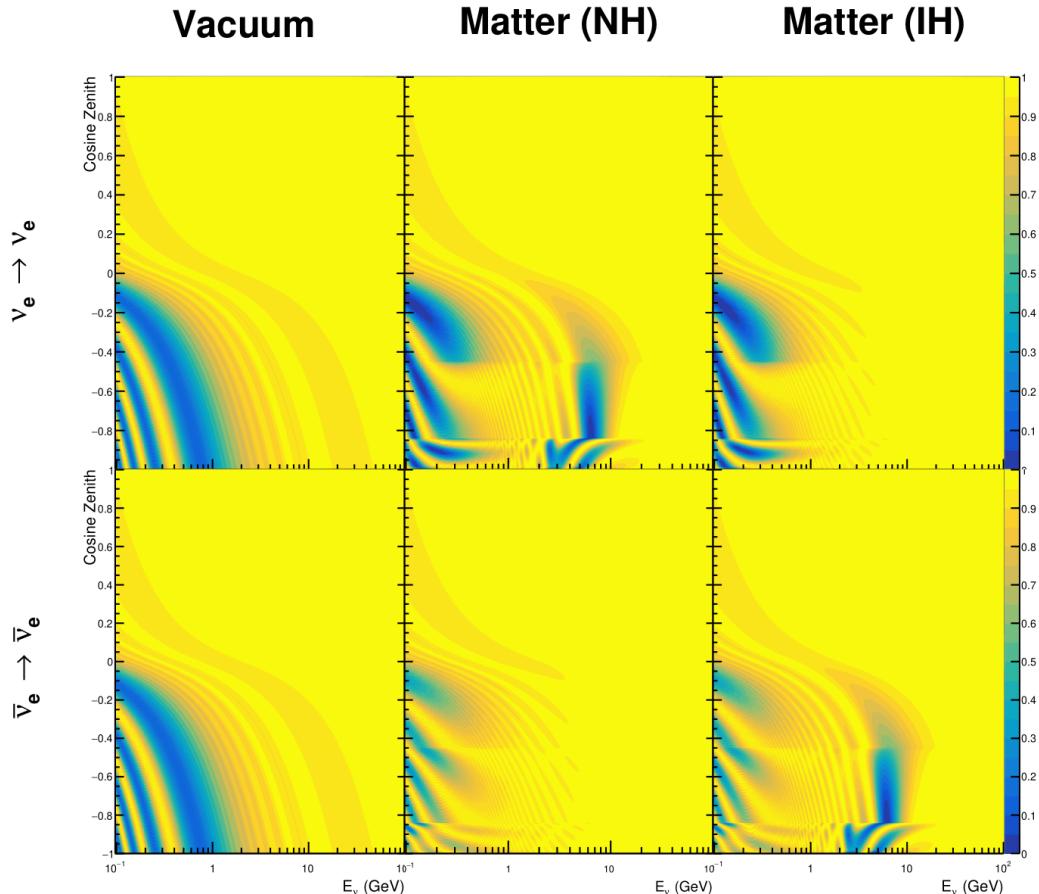


Figure 7.4: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

shift in the energy of the oscillation peak is also present to aid in separating the two
CP-conserving values of δ_{CP} .

T2K’s sensitivity to the $\sin^2(\theta_{23})$ and Δm_{23}^2 is observed as a shape-based variation
of the muon-like samples, as illustrated in Figure 7.5. The value of Δm_{32}^2 laterally shifts
the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$ oscillation

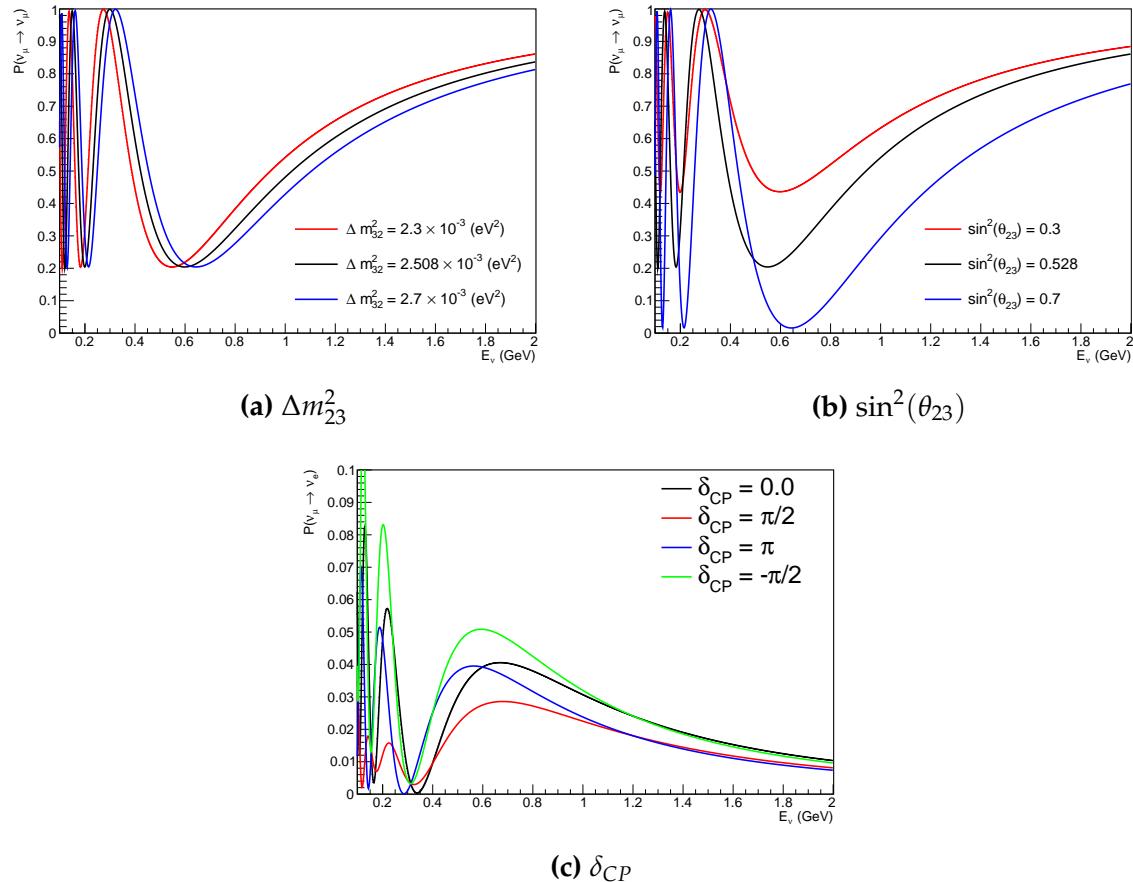


Figure 7.5: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as a vertical shift of the oscillation dip with second-order horizontal shifts being due to matter effects. The beam neutrinos have limited sensitivity to matter effects due to the relatively shorter baseline as well as the Earth’s mantle being a relatively low-density material (as compared to the Earth’s core). For some values of δ_{CP} , the degeneracy in the number of e-like events allows the mass hierarchy to be resolved. This leads to a δ_{CP} -dependent mass hierarchy sensitivity which can be seen in Figure 7.6.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations

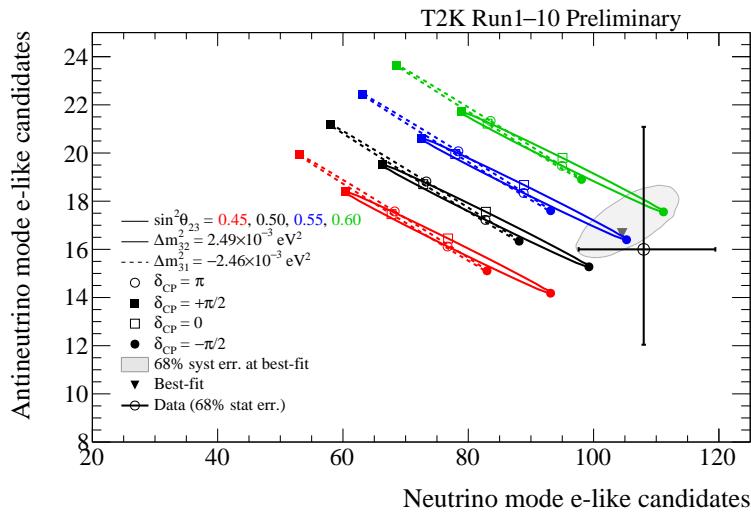


Figure 7.6: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{23}^2 are given.

which reduce the number of oscillation probability calculations that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux. Furthermore, as the peak neutrino energy of the beam is well below the threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [51], only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very small due to the short baseline. Whilst these approximations can be made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold.

2062 7.2 Treatment of Fast Oscillations

2063 As shown in Figure 7.7, atmospheric neutrino oscillations have a significantly more
2064 complex structure for upgoing neutrinos with energy below 1GeV. This is because the
2065 L/E dependence of the oscillation probability in this region induces rapid variations
2066 for small changes in L or E . As discussed in section 7.1, this is also the region in which
2067 atmospheric neutrinos have sensitivity to δ_{CP} . In practice, the direction of the neutrino
2068 is inferred from the direction of the final state particles traveling in the detector, which
2069 can be poor for low-energy neutrino interactions. This creates a distinct difference
2070 from the beam neutrinos where the position of the source is very precisely known.

2071 As a consequence of the unresolvable structure, an average oscillation probability
2072 is observed in the subGeV upgoing region. This creates a computational problem; A
2073 significantly large amount of Monte Carlo statistics would be required to accurately
2074 predict the number of events if Monte Carlo averaging was the only technique used.
2075 This section describes the ‘sub-sampling’ approach developed for this analysis and
2076 compares it to the methodology used within the SK-only analysis.

2077 The official SK-only analysis uses the osc3++ oscillation parameter fitter [203].
2078 To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’ technique.
2079 For a given neutrino event, the nearest twenty neighbours in reconstructed lepton
2080 momentum and zenith angle are found and a distribution of their neutrino energies is
2081 built. The RMS, σ , of this distribution is then used to compute an average oscillation
2082 probability for the given neutrino Monte Carlo event.

2083 For the i^{th} event, the oscillation weight is calculated as

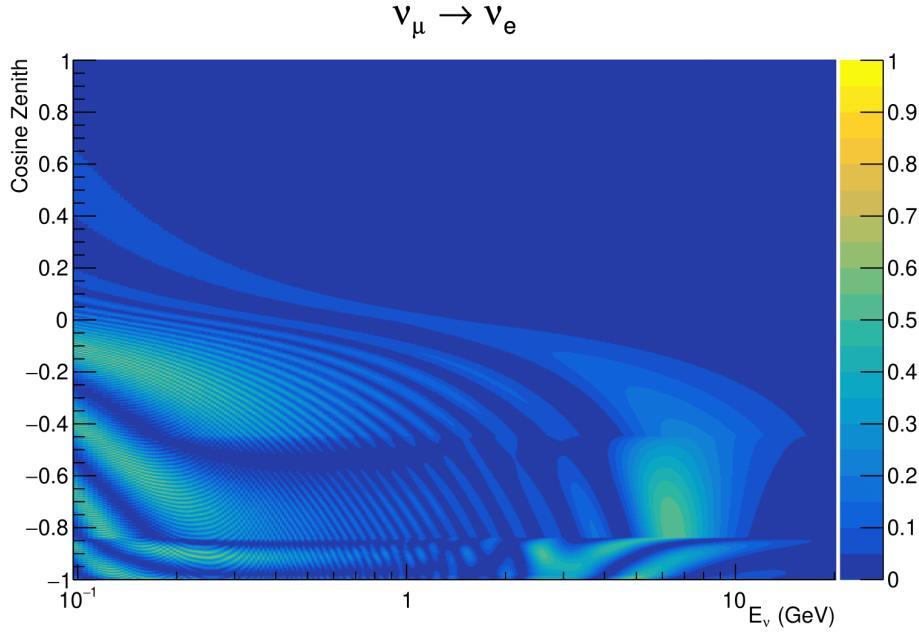


Figure 7.7: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5} \sum_{\beta=-1,-0.5,0.5,1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.3)$$

2084 where $P(E, L)$ is the oscillation probability calculation for neutrino energy E and
 2085 path length L and the two path lengths, \bar{L}_i and L_β are discussed below. All of the
 2086 oscillation probability calculations are performed with a fixed zenith angle such that
 2087 the same density profile is used.

2088 The uncertainty in the production height is controlled by using an “average” pro-
 2089 duction height, \bar{L}_i , which represents the average path length computed using twenty
 2090 production heights taken from the Honda flux model’s prediction [45]. For a given
 2091 event, the production heights are sampled in steps of 5% of their cumulative distribu-
 2092 tion function. L_β values are similarly calculated but instead use different combinations
 2093 of four production heights,

$$\begin{aligned}
 L_{-1.0} &= \frac{1}{4}L(45, 50, 55, 60), \\
 L_{-0.5} &= \frac{1}{4}L(35, 40, 65, 70), \\
 L_{+0.5} &= \frac{1}{4}L(25, 30, 75, 68), \\
 L_{+1.0} &= \frac{1}{4}L(15, 20, 85, 89).
 \end{aligned} \tag{7.4}$$

2094 This averaging technique works because of the inference between the zenith angle
 2095 and the reconstructed direction of final state particles in the detector. For low-energy
 2096 neutrinos, where the resolution of the true neutrino direction is poor, σ_i will be large,
 2097 resulting in significant averaging effects. Contrary to this, the inferred direction of
 2098 high-energy neutrinos will be much closer to the true value, meaning that σ_i will be
 2099 smaller, culminating in small averaging effects.

2100 In practice, this technique is performed before the fit in order to deal with the
 2101 computational cost. This is possible as the Osc3++ framework uses binned oscillation
 2102 parameters rather than continuous so the oscillation parameters used in the fit are
 2103 known prior to run-time. The framework used in this analysis uses continuous
 2104 oscillation parameters, and due to the MCMC fitting technique, there is no way to
 2105 know which oscillation parameter values will be selected *a priori*. Therefore, the
 2106 oscillation parameter calculation has to be performed at run-time. Computing five
 2107 oscillation probabilities per event would require far too many computational resources
 2108 to be viable. Therefore SK technique can not be used within this analysis. However,
 2109 the concept of the averaging technique can be taken from it.

2110 To perform a similar averaging as the SK analysis, a sub-sampling approach using
 2111 binned oscillograms has been devised. The technique can be explained by considering
 2112 a “fine” and “coarse” oscillogram. The fine oscillograms are used to define the array of

2113 $\cos(\theta_Z)$ and E_ν used in the oscillation engine. The coarse oscillograms cover the same
2114 phase-space but have fewer bins, where the value of a particular coarse bin is taken
2115 as the linear average (flat prior in E_ν and $\cos(\theta_Z)$) of all fine bins which falls into it.
2116 The coarse oscillogram is then used for determining the oscillation weight for a given
2117 event. The binning which is used to calculate the oscillation probabilities, known as
2118 the ‘fine’ binning, has $N \times N$ subdivisions per coarse bin. Figure 7.8 illustrates the
2119 $N = 2$ example where the assigned value to a coarse bin is the average of the four fine
2120 bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear
2121 on either axis, the sub-division of the fine bins is linear over the range of a coarse bin.

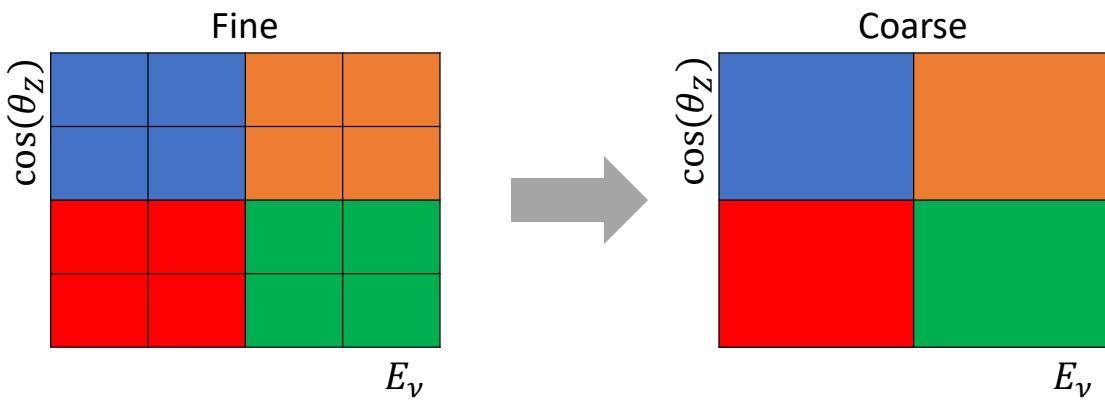


Figure 7.8: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2122 The coarse binning is defined with 67×52 bins in true neutrino energy \times cosine
2123 zenith. It is picked to be identical to that provided in [205]. In general, the binning is
2124 logarithmically spaced in neutrino energy but has some hand-picked bin edges. Firstly,
2125 the bin density around the matter resonance is smoothly increased around the matter
2126 resonance region. This is to avoid smearing this region which can be well sampled by
2127 the Monte Carlo. Secondly, bin edges are selected to hit $0.4, 0.6, 1, 10, 30, 50, 100\text{GeV}$.
2128 This is to ensure that the Coulomb correction systematic and the atmospheric flux
2129 systematics definitions in neutrino energy can be hit. The cosine zenith binning is

approximately linearly spaced across the allowable range but the values of layer transitions are hit precisely: -0.8376 (core-mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for downgoing events as this is a region unaffected by the fast oscillation wavelengths and reduces the total number of calculations required to perform the calculation.

The choice of N is justified based on two studies. Firstly, the variation of event rates of each sample is studied as a function of N . For a given set of oscillation parameters thrown from the PDG prior constraints (detailed in Table 2.1), the oscillation probabilities are calculated using a given value of N . Each sample is re-weighted and the event rate is stored. The value of N is scanned from 1, which corresponds to no averaging, to 24, which corresponds to the largest computationally viable subdivision binning. The event rate of each sample at large N is expected to converge to a stationary value due to the fine binning fully sampling the small-scale structure. Figure 7.9 illustrates this behaviour for the SubGeV_elike_0dcy sample for 30 different throws of the oscillation parameters.

Denoting the event rate for one sample for a given throw t at each N by λ_t^N , the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The variance in the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t) \right]^2. \quad (7.5)$$

The aim of the study is to find the lowest value of N such that this variance is below 0.001. This is the typical threshold used by T2K fitters to validate systematic implementation so has been set as the same criteria. The results of this study for each atmospheric sample used within this thesis are illustrated in Figure 7.10 for

SubGeV-elike-0dcy

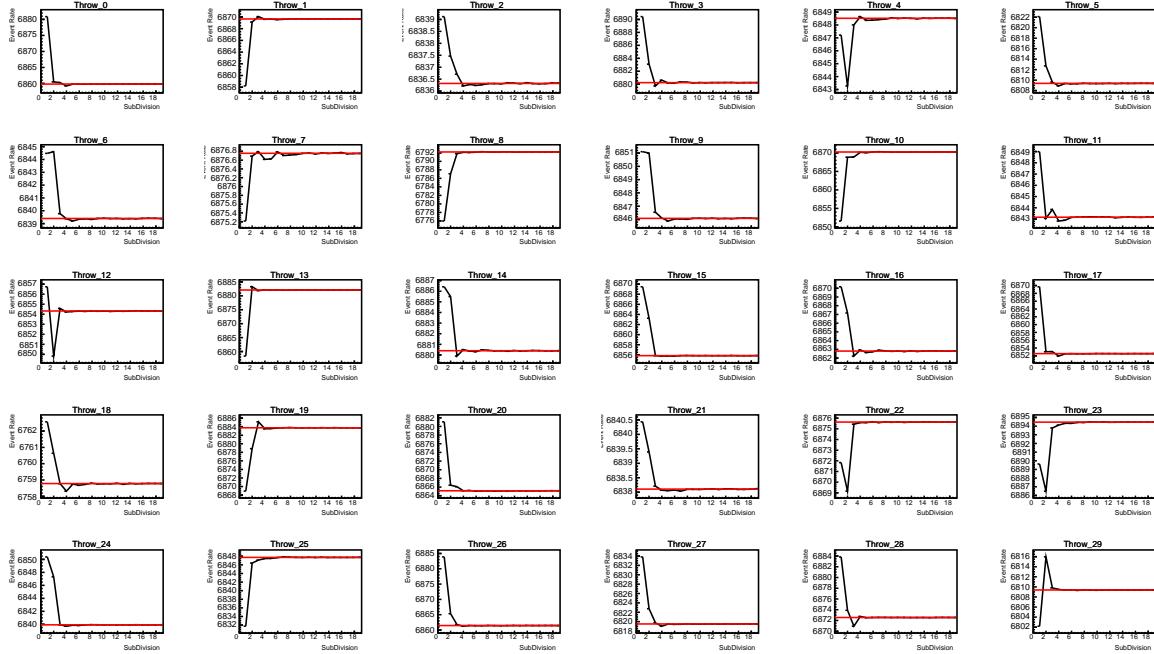


Figure 7.9: Event rate of the SubGeV_elike_0dcy sample as a function of the number of subdivisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2152 2000 throws of the oscillation parameters. As can be seen, the variance is below
 2153 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy and
 2154 SubGeV_elike_0dcy samples.

2155 The second study to determine the value of N is as follows. The likelihood for each
 2156 sample is computed against an Asimov data set created with Asimov A oscillation
 2157 parameters (Table 2.2). Following Equation 7.5, the variance of the log-likelihood over
 2158 all considered N is computed. The results are shown in Figure 7.11.

2159 A choice of $N = 10$ sub-divisions per coarse bin has a variance in both event rate
 2160 and log-likelihood residuals less than the required threshold of 0.001. The largest
 2161 value of the likelihood variance is of order 10^{-7} , corresponding to an error on the log-

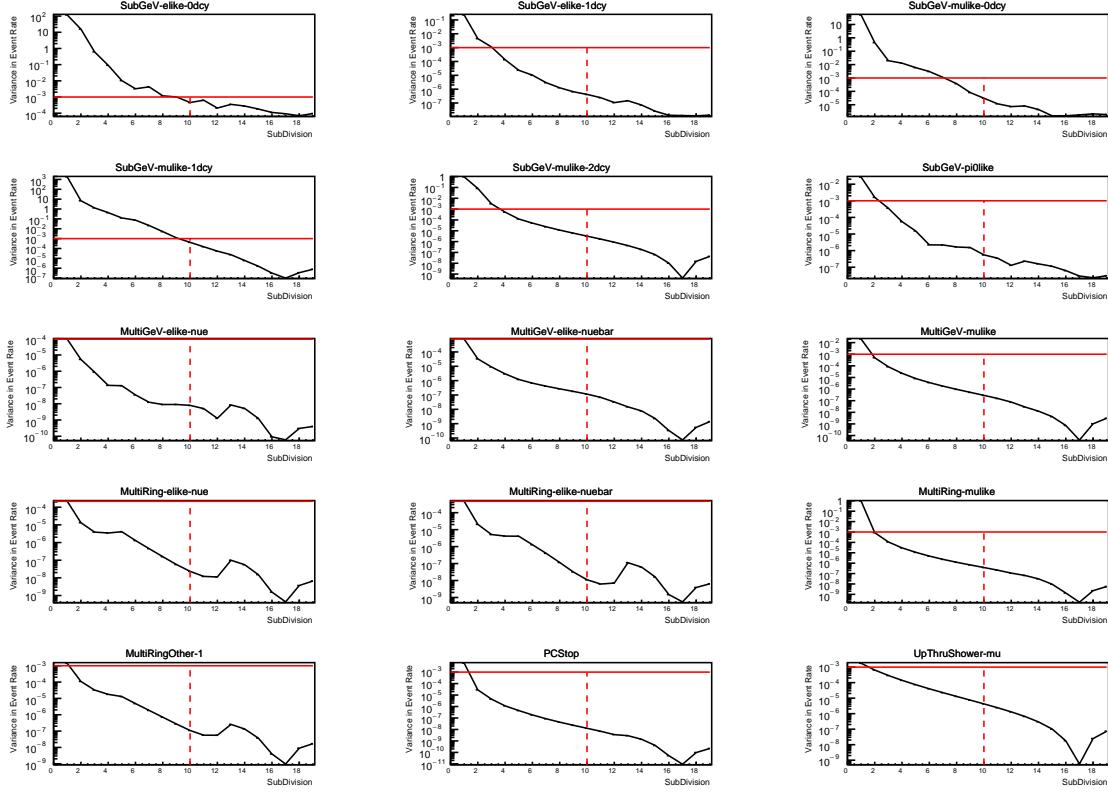


Figure 7.10: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

2162 likelihood of about 3×10^{-4} which is small enough to be negligible for the oscillation
 2163 analysis.

2164 Figure 7.12 illustrates the effect of the smearing using $N = 10$. The fast oscillations
 2165 in the sub-GeV upgoing region have been replaced with a normalisation effect whilst
 2166 the large matter resonance structure remains.

2167 7.3 Calculation Engine

2168 As previously discussed in section 7.2, the calculation of oscillation probabilities is per-
 2169 formed at run-time due to utilising continuous oscillation parameters. Consequently,
 2170 the time per calculation is crucial for fit performance. The initial fitting framework

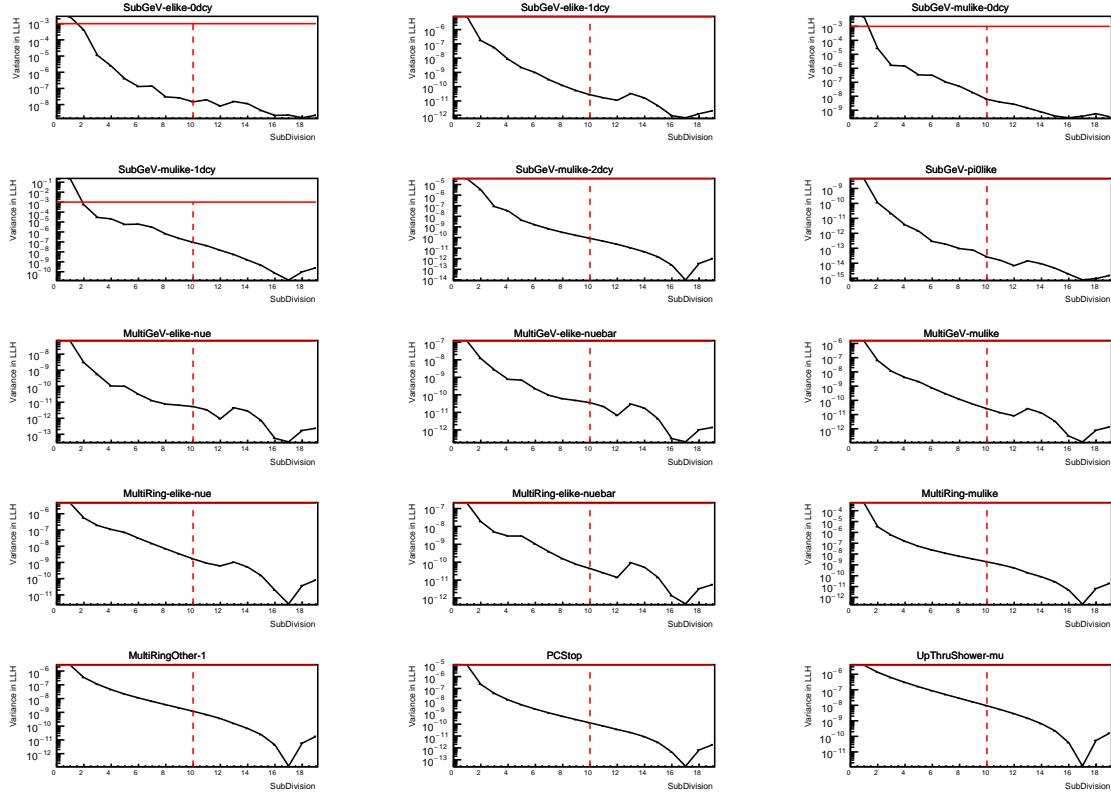


Figure 7.11: Variance of sample likelihood, when compared to ‘Asimov data’ set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

used for this analysis was developed with ProbGPU [206]. This is a GPU-only implementation of the prob3 engine [207]. It is primarily designed for neutrino propagation in a beam experiment (single layer of constant density) with the atmospheric propagation code not being used prior to the analysis in this thesis.

Another engine, CUDAProb3 [208], has been implemented within the fitting framework used in this analysis. It has been specifically optimised for atmospheric neutrino oscillation calculation so does not contain the code to replace the beam oscillation calculation. The engine utilises object-orientated techniques as compared to the functional implementation of ProbGPU. This allows the energy and cosine zenith arrays to be kept on GPU memory, rather than having to load these arrays onto GPU memory for each calculation. General memory interfacing is one of the slowest tasks which

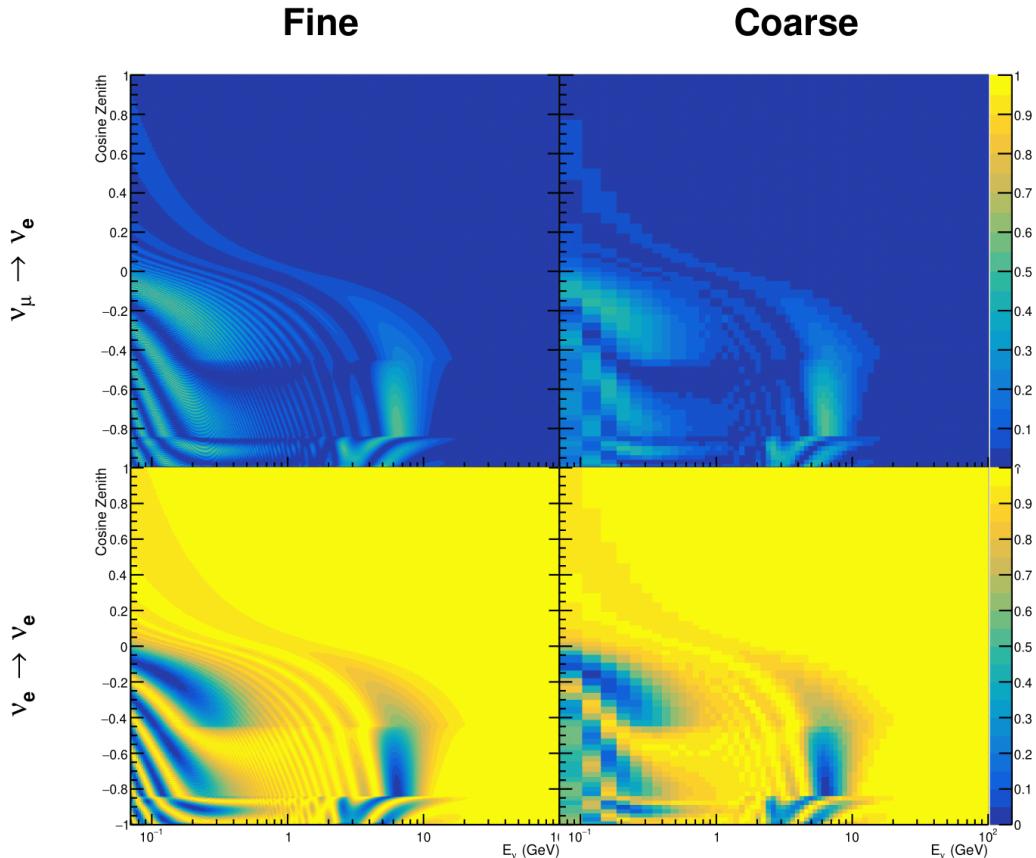


Figure 7.12: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.10 and Figure 7.11.

2182 GPUs can do, so being able to eliminate this significantly reduces the time required
 2183 for calculation. This can be seen in Figure 7.13, where the GPU implementation of
 2184 CUDAProb3 is approximately three times faster than the ProbGPU engine.

2185 Another significant advantage of CUDAProb3 is that it contains a CPU multithreaded
 2186 implementation which is not possible with the ProbGPU or prob3 engines. This elimi-
 2187 nates the requirement for GPU resources when submitting jobs to batch systems. As
 2188 illustrated in Figure 7.13, the calculation speed depends on the number of available

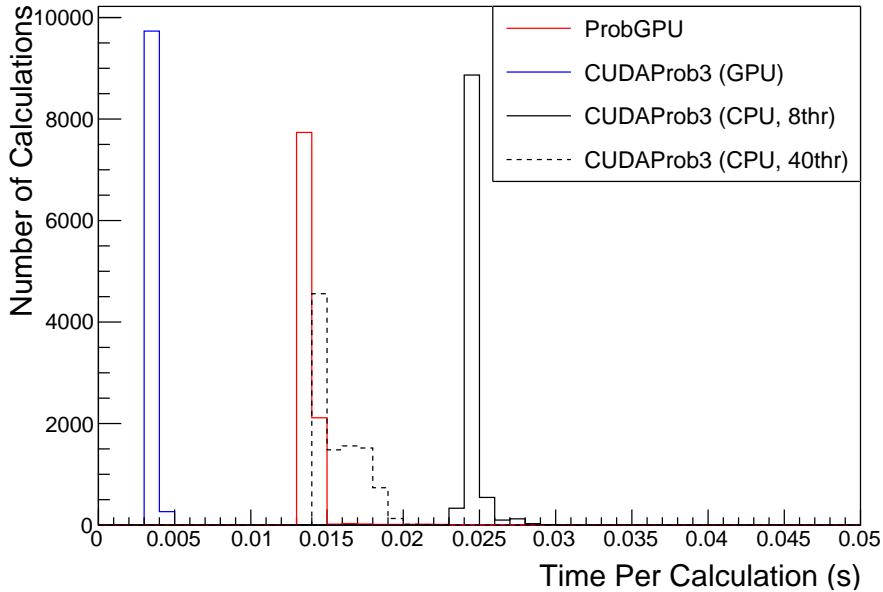


Figure 7.13: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscilloscopes, following the technique given in section 7.2, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multithreaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

2189 threads. Using 8 threads (which is typical of the batch systems being used) is ap-
 2190 proximately twice as slow as the ProbGPU engine implementation, but would allow
 2191 the fitting framework to be run on many more resources. This fact is utilised for any
 2192 SK-only fits but GPU resources are required for any fits which include beam samples
 2193 due to the ProbGPU requirement. Based on the benefits shown by the implementation
 2194 in this section, efforts are being placed into including linear propagation for beam
 2195 neutrino propagation into the engine [209].

2196 7.4 Matter Density Profile

2197 For an experiment observing atmospheric neutrinos propagating through the Earth, a
 2198 model of the Earth’s density profile is required. The model used within this analysis is

the Preliminary Reference Earth Model (PREM) [204], as illustrated in Figure 7.1. As discussed in section 7.1, the propagator used within the calculation engine requires constant density layers. To follow the official SK-only analysis [203], the average density of each layer has been taken from the PREM model. Table 7.1 documents the density and radii of the layers used within this approximation. The density measurements provided in the PREM model are provided in terms of mass density, whereas neutrino oscillations are sensitive to the electron number density. This value can be computed as the product of the chemical composition, or the Z/A value, and the mass density of each layer. Currently, the only way to calculate the chemical composition value for layers close to the Earth’s core is through neutrino oscillations. The chemical composition of the upper layers of the Earth’s Mantle and the Transition zone is well known due to it being predominantly pyrolite which has a chemical composition value of 0.496 [210]. The components of the Earth’s core region are less well known. Consequently, the chemical composition dial for the core layers is set to a value of 0.468, as calculated in [211]. This value is assigned a Gaussian error with a standard deviation equivalent to the difference in chemical composition in core and mantle layers. Figure 7.14 illustrates the effect of moving from the $Z/A = 0.5$ method which is used in the official SK-only analysis [203] to these more precise values.

The beam oscillation probability in this thesis uses a baseline of 295km, density $2.6\text{g}/\text{cm}^3$, and chemical composition 0.5 as is done by the official T2K-only analysis [212].

Whilst the propagator requires a fixed density layer model of the Earth, the density only has to be fixed for a specific $E_\nu \times \cos(\theta_Z)$ bin in a given layer. As the density is a function of radius, which is a function of the direction in which a neutrino propagates, a better approximation of the PREM model can be made if a $\cos(\theta_Z)$ -specific density is calculated.

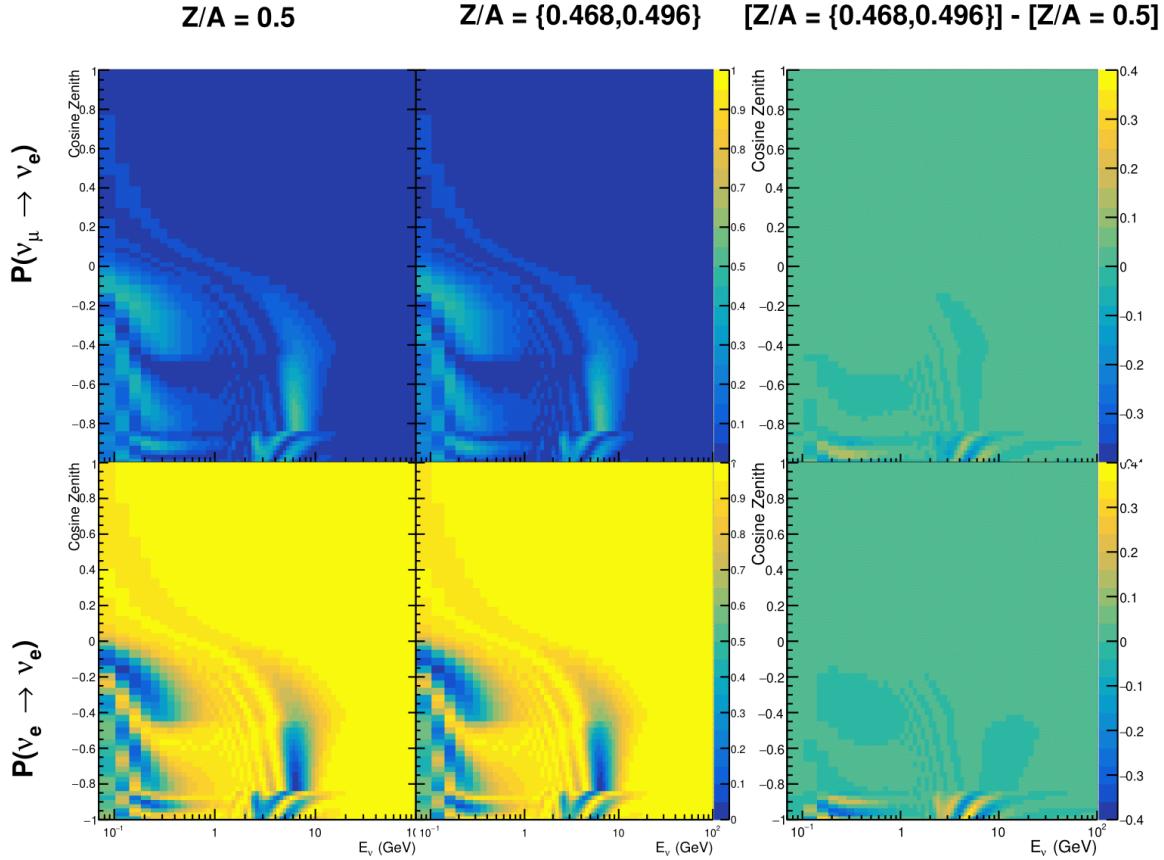


Figure 7.14: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 7.1. The right column illustrates the difference in oscillation probability between the two different techniques.

2225 To achieve this, the average density, $\langle \rho \rangle_i$, in the i^{th} layer, is calculated as the density,
 2226 $\rho(t)$, integrated over the track a given $\cos(\theta_Z)$,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt \quad (7.6)$$

2227 where t_i are the intersection points between each layer and t is the path length of
 2228 the trajectory across the layer.

2229 The oscillation probability calculation speed is approximately linear in the number
 2230 of layers invoked within the Earth model. Therefore a four-layer model is still utilized
 2231 with the only difference to the official SK-only analysis being that the four-layer model
 2232 used for each value of $\cos(\theta_Z)$ is different. Following the method outlined in [213],
 2233 a four-layer piecewise quadratic polynomial is fit to the PREM model for the four
 2234 layers defined in Table 7.1. This fit was not performed by the author of the thesis
 2235 and is documented in [205]. The coefficients of the quadratic fit to each layer are
 2236 given in Table 7.2 with the final distribution illustrated in Figure 7.15. The quadratic
 2237 approximation is clearly much closer to the PREM model as compared to the constant
 2238 density approximation.

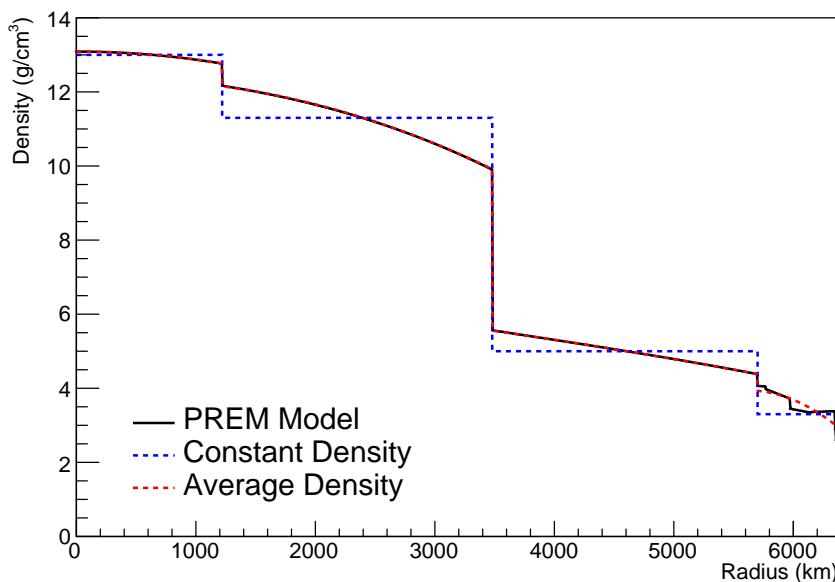


Figure 7.15: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.2: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [205], where $x = R/R_{Earth}$.

2239 The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in
 2240 Figure 7.16. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -$
 2241 0.45 in the fixed density model, which is due to the transition to mantle layer boundary,
 2242 has been reduced. This is expected as the difference in the density across this boundary
 2243 is significantly smaller in the quadratic density model as compared to the constant
 2244 density model. Whilst the difference in density across the other layer transitions
 2245 is reduced, there is still a significant difference. This means the discontinuities in
 2246 the oscillation probabilities remain but are significantly reduced. However, as the
 2247 quadratic density approximation matches the PREM model well in this region, these
 2248 discontinuities are due to the Earth model rather than an artifact of the oscillation
 2249 calculation.

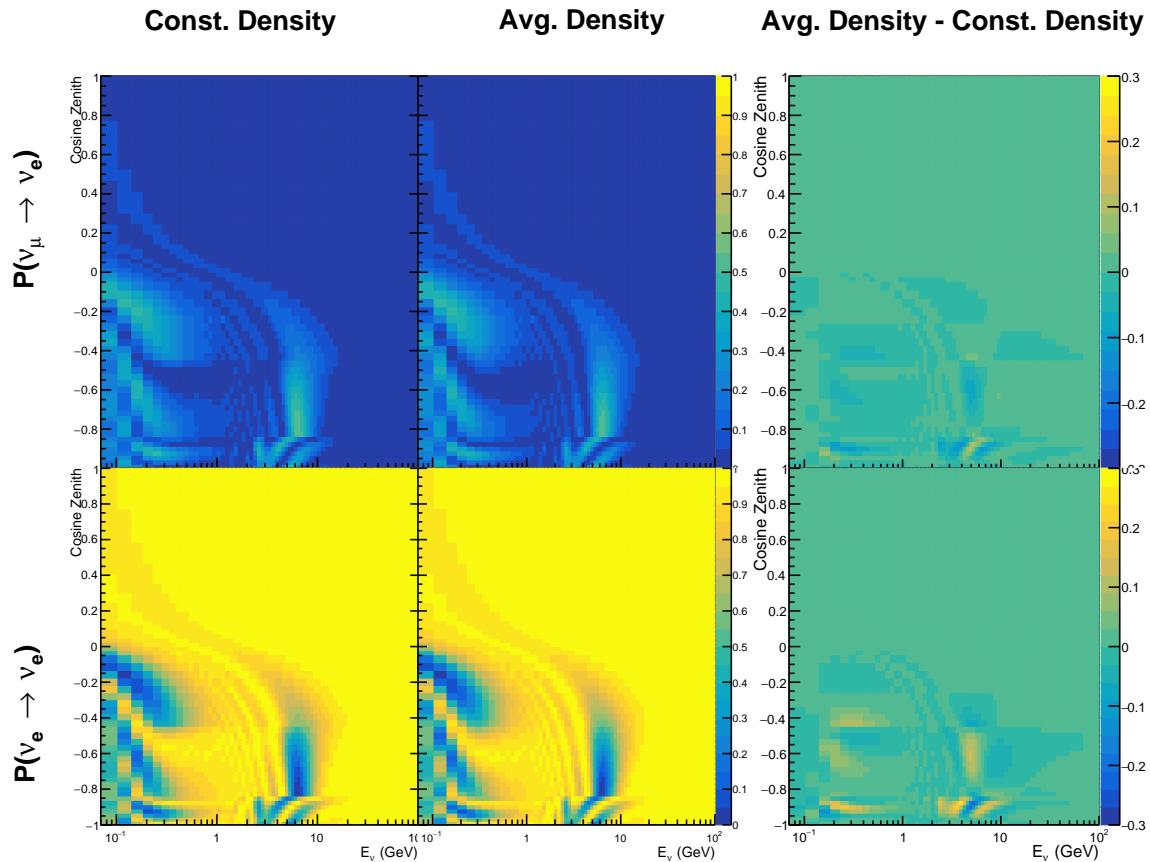


Figure 7.16: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_\mu)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2250 7.5 Production Height Averaging

2251 As discussed in section 7.1, the height at which the cosmic ray flux interacts in the
2252 atmosphere is not known on an event-by-event basis. The production height can vary
2253 from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only analysis methodology
2254 (described in section 7.2) for including the uncertainty on the production height is
2255 to include variations from the Honda model when pre-calculating the oscillation
2256 probabilities prior to the fit. This technique is not possible for this analysis which
2257 uses continuous oscillation parameters that can not be known prior to the fit. Conse-
2258 quently, an analytical averaging technique was developed in [205]. The author of this
2259 thesis was not responsible for the derivation of the technique but has performed the
2260 implementation and validation of the technique for this analysis alone.

2261 Using the 20 production heights per Monte Carlo neutrino event, provided as 5%
2262 percentiles from the Honda flux model, a production height distribution $p_j(h|E_\nu, \cos \theta_Z)$
2263 is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a histogram is filled with
2264 20 evenly spaced bins in production height h between 0 and 50km. The neutrino energy
2265 and cosine zenith binning of the histogram is the same as that provided in section 7.2.
2266 The average production height, $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. The
2267 production height binning of this histogram is then translated into $\delta t(h) = t(\bar{h}) - t(h)$,
2268 where $t(h)$ is the distance travelled along the trajectory.

2269 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed. The
2270 time-ordered product of these is then used as the overall transition amplitude via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.7)$$

²²⁷¹ where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.8)$$

²²⁷² is expressed as a diagonalised time-dependent solution to the Schrodinger equation.
²²⁷³ The 0^{th} layer is the propagation through the atmosphere and is the only term that
²²⁷⁴ depends on the production height. Using the substitution $t_0 = t(\bar{h}) - \delta t(h)$, it can be
²²⁷⁵ shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.9)$$

²²⁷⁶ Thus Equation 7.7 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.10)$$

²²⁷⁷ The oscillation probability averaged over production height is then calculated as

$$\begin{aligned}
 \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\
 &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\
 &= \sum_{km} (B_k)_{ij} (B_m)_{ij}^* \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m)\delta t]
 \end{aligned} \tag{7.11}$$

2278 In practice, implementation in CUDAProb3 [208] is relatively straightforward as
 2279 the majority of these terms are already calculated in the standard oscillation calculation.
 2280 Figure 7.17 illustrates the results of the production height averaging. As expected,
 2281 the main effect is observed in the low-energy downward-going and horizontal-going
 2282 events. Upward-going events have to travel the radius of the Earth, $R_E = 6371\text{km}$,
 2283 where the production height uncertainty is a small fraction of the total path length.

2284

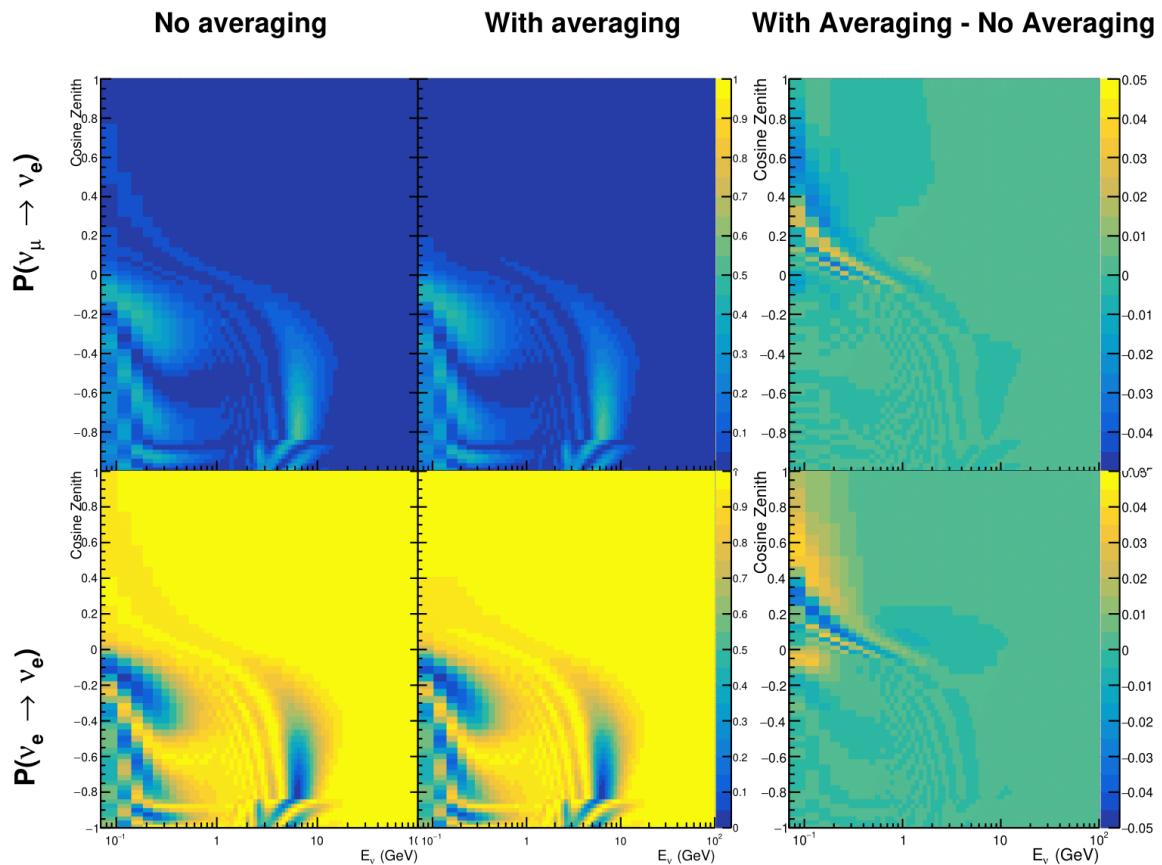


Figure 7.17: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_\mu)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

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