

1 **The Sensitivity to Oscillation Parameters**
2 **from a Simultaneous Beam and**
3 **Atmospheric Neutrino Analysis that**
4 **combines the T2K and SK Experiments**



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10 *Doctor of Philosophy*
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Abstract

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Acknowledgements

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⁷¹ I don't think there's any rule against it. Of course, you could also just remove
⁷² the subsections and do one big traditional acknowledgement section.

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Introduction

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Neutrino Oscillation Physics

129 When first proposed, neutrinos were expected to be approximately massless
130 fermions that only interact through weak and gravitational forces. This meant
131 they were very difficult to detect as they can pass through significant amounts
132 of matter without interacting. Despite this, experimental neutrino physics has
133 developed many different detection techniques and observed neutrinos from
134 both natural and artificial sources. In direct tension with Standard Model physics,
135 neutrinos have been determined to oscillate between different lepton flavours,
136 requiring them to have mass.

137 The observation techniques which led to the discovery of the neutrino are doc-
138 umented in section 2.1. The theory underpinning neutrino oscillation is described
139 in section 2.2 and includes the approximations which can be made to simplify
140 the understanding of neutrino oscillation in the two-flavour approximation. Past,
141 current, and future neutrino experiments are detailed in section 2.3, including the
142 reactor, atmospheric, and long-baseline accelerator neutrino sources that have
143 been used to successfully constrain oscillation parameters. Finally, the current
144 state of oscillation parameter measurements are summarised in section 2.4.

2.1 Discovery of Neutrinos

At the start of the 20th century, the electrons emitted from the β -decay of the nucleus were found to have a continuous energy spectrum [1, 2]. This observation seemingly broke the energy conservation invoked within that period's nuclear models. In 1930, Pauli provided a solution to this problem in the form of a new particle, the neutrino (originally termed "neutron"). It was theorized to be an electrically neutral spin-1/2 fermion with a mass smaller than that of the electron [3]. This neutrino was emitted with the electron in β -decay to alleviate the apparent breaking of energy conservation. As a predecessor of today's weak interaction model, Fermi's theory of β -decay developed the understanding by coupling the four constituent particles: electron, proton, neutron, and neutrino, into a quantitative model [4].

Whilst Pauli was not convinced of the ability to detect neutrinos, the first observations of the particle were made in the mid-1950s when neutrinos from a reactor were observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [5, 6]. The detector consisted of two parts: a neutrino interaction medium and a liquid scintillator. The interaction medium was built from two water tanks, loaded with cadmium chloride to allow for increased efficiency in the detection of neutron capture. The positron emitted from IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured on the cadmium via $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$, producing a delayed signal. An increase in the coincidence rate was observed when the reactor was operating which was interpreted as interactions from neutrinos generated in the reactor.

After the discovery of the ν_e , the question of how many flavours of neutrino exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven National Laboratory [7]. A proton beam was directed at a beryllium target, generating pions which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the subsequent interactions of the ν_μ were observed. As the subsequent interaction of the neutrino generated muons rather than electrons, it was determined that

¹⁷⁴ the ν_μ was fundamentally different from ν_e . The final observation to be made
¹⁷⁵ was that of the ν_τ from the DONUT experiment [8]. Three neutrinos seem the
¹⁷⁶ obvious solution as it mirrors the known number of charged leptons (as they form
¹⁷⁷ weak isospin doublets) but there could be evidence of more. Several neutrino
¹⁷⁸ experiments have found anomalous results [9, 10] which could be attributed
¹⁷⁹ to “sterile” neutrinos. These hypothesised particles are not affected by gauge
¹⁸⁰ interactions in the Standard Model so their presence can only be inferred through
¹⁸¹ the observation of non-standard oscillation modes. However, cosmological
¹⁸² observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [11], as
¹⁸³ measured from the cosmic microwave background power spectrum. LEP also
¹⁸⁴ measured the number of active neutrino flavours to be $N_\nu = 2.9840 \pm 0.0082$ [12]
¹⁸⁵ from measurements of the Z-decay width, but this does not strongly constrain
¹⁸⁶ the number of sterile neutrinos.

¹⁸⁷ 2.2 Theory of Neutrino Oscillation

¹⁸⁸ A neutrino generated with lepton flavour α can change into a different lepton
¹⁸⁹ flavour β after propagating some distance. This phenomenon is called neutrino
¹⁹⁰ oscillation and requires that neutrinos must have a non-zero mass. This behaviour
¹⁹¹ has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [13–
¹⁹² 15] mixing matrix which describes how the flavour and mass of neutrinos are
¹⁹³ associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [16]
¹⁹⁴ matrix measured in quark physics.

¹⁹⁵ 2.2.1 Three Flavour Oscillations

¹⁹⁶ The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ
¹⁹⁷ (indexed ν_α), which are eigenstates of the weak interaction and three mass
¹⁹⁸ eigenstates, ν_1 , ν_2 and ν_3 (indexed ν_i). Each mass eigenstate is the superposition

¹⁹⁹ of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

²⁰⁰ Where U is the 3×3 PMNS matrix which is unitary and connects the mass
²⁰¹ and flavour eigenstates.

²⁰² The weak interaction, when interacting via a W^{\pm} boson, couples to flavour
²⁰³ eigenstates so neutrinos interact with leptons of the same flavour. The prop-
²⁰⁴ agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a
²⁰⁵ plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

²⁰⁶ The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
²⁰⁷ three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
²⁰⁸ coordinates). The probability of observing a neutrino of flavour eigenstate β from
²⁰⁹ one which originated as flavour α can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)} \quad (2.3)$$

²¹⁰ The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

²¹¹ For a relativistic particle, $E_i \gg m_i$, a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

²¹² Making the approximations that neutrinos are relativistic, the mass eigenstates
²¹³ were created with the same energy and that $x = L$, where L is the distance
²¹⁴ travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This, combined with further use of unitarity relations results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right) \end{aligned} \quad (2.7)$$

Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last term is included for the oscillation probability of antineutrinos. As an important point to note, the observation of oscillation probability requires a non-zero value of Δm_{ij}^2 , which in turn requires that neutrinos have differing masses.

Typically, the PMNS matrix is parameterised into three mixing angles, a charge parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many neutrino experiments aim to measure the PMNS parameters from a wide array of origins, as is the purpose of this thesis.

The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8 is only included for completeness. For an oscillation analysis experiment, any terms containing this phase disappear due to taking the expectation value of the PMNS matrix. Measurements of these phases can be performed by experiments searching for neutrino-less double β -decay [17].

232 A two-flavour approximation can be obtained when one assumes the third
233 mass eigenstate is degenerate with another. This results in the two-flavour
234 approximation being reasonable for understanding the features of the oscillation.
235 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

236 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

237 Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is
238 a sinusoidal function depending upon the distance over which the neutrino
239 propagates. The frequency and amplitude of oscillation are dependent upon
240 $\Delta m^2/4E$ and $\sin^2 2\theta$, respectively. The oscillation probabilities presented thus far
241 assume $c = 1$, where c is the speed of light in a vacuum. In more familiar units, the
242 maximum oscillation probability for a fixed value of θ is given at $L[\text{km}] / E[\text{GeV}] \sim$
243 $1.27/\Delta m^2$. It is this calculation that determines the best L/E value for a given
244 experiment to be designed around for measurements of a specific value of Δm^2 .

245 2.2.2 The MSW Effect

246 The theory of neutrino oscillation in a vacuum has been described in subsec-
247 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating
248 from below the horizon propagate through the matter in the Earth. The coherent
249 scattering of neutrinos from a material target modifies the Hamiltonian of the
250 system which results in a change in the oscillation probability. This modification
251 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [18–20]. This occurs
252 because charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$, propagated by a W boson)
253 only affects electron neutrinos whereas the neutral current scattering ($\nu_l + l^- \rightarrow$

²⁵⁴ $\nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino flavours equally.
²⁵⁵ In the two-flavour approximation, the effective mixing parameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

²⁵⁶ where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F
²⁵⁷ is Fermi's constant. It is clear that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for
²⁵⁸ $\Delta m^2 > 0$, which results in a divergent mixing parameter, colloquially called the
²⁵⁹ matter resonance. This resonance regenerates the electron neutrino component of
²⁶⁰ the neutrino flux [18–20]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

²⁶¹ At densities lower than this critical value, the oscillation probability will
²⁶² be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$
²⁶³ [21]. The resonance occurring from the MSW effect depends on the sign of Δm^2 .
²⁶⁴ Therefore, any neutrino oscillation experiment which observes neutrinos and
²⁶⁵ antineutrinos which have propagated through matter can have some sensitivity
²⁶⁶ to the ordering of the neutrino mass eigenstates.

²⁶⁷ 2.3 Neutrino Oscillation Measurements

²⁶⁸ As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics
²⁶⁹ was awarded to the Super-Kamiokande (SK) [22] and Sudbury Neutrino Ob-
²⁷⁰ servatory (SNO) [23] collaborations for the first definitive observation of solar
²⁷¹ and atmospheric neutrino oscillation [24]. Since then, the field has seen a wide
²⁷² array of oscillation measurements from a variety of neutrino sources. As seen
²⁷³ in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio
²⁷⁴ of the propagation baseline, L , to the neutrino energy, E . It is this ratio that
²⁷⁵ determines the type of neutrino oscillation a particular experiment is sensitive to.

²⁷⁶ As illustrated in Figure 2.1, there are many neutrino sources that span a
²⁷⁷ wide range of energies. The least energetic neutrinos are from reactor and

²⁷⁸ terrestrial sources at $O(1)$ MeV whereas the most energetic neutrinos originate
²⁷⁹ from atmospheric and galactic neutrinos of $> O(1)$ TeV.

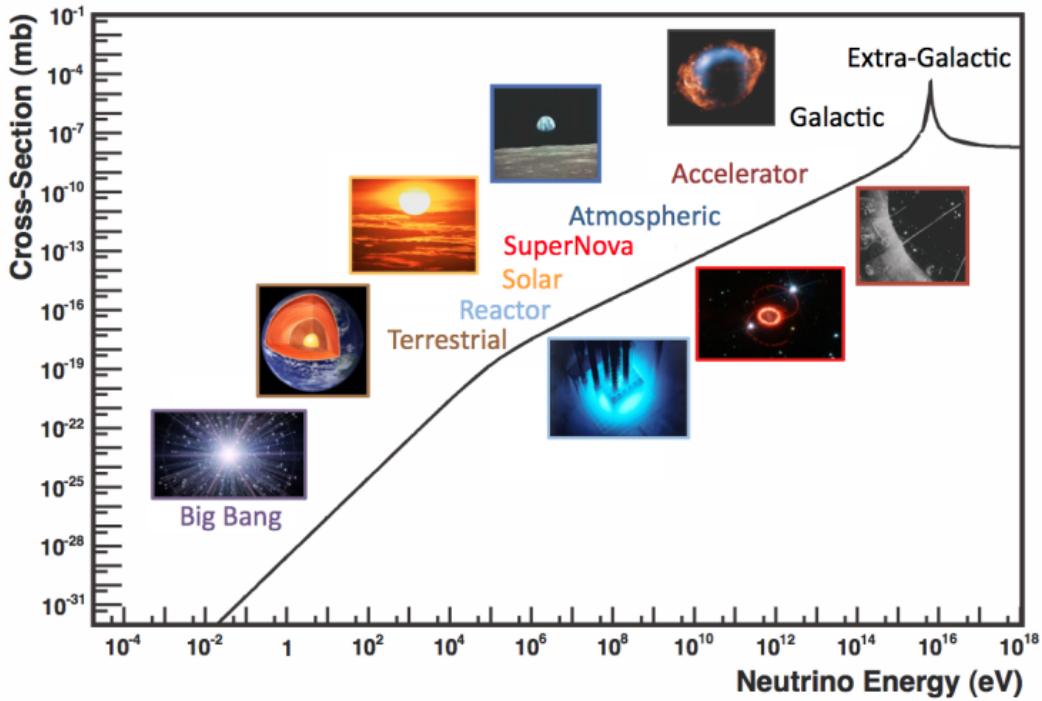


Figure 2.1: The electro-weak cross-section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [25]

²⁸⁰ 2.3.1 Solar Neutrinos

²⁸¹ Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun.
²⁸² The solar neutrino flux, given as a function of neutrino energy for different
²⁸³ fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion
²⁸⁴ generates the largest flux of neutrinos, the neutrinos are low energy and are
²⁸⁵ difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [26].
²⁸⁶ Consequently, most experiments focus on the neutrinos from the decay of 8B
²⁸⁷ (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher energy.

²⁸⁸ The first measurements of solar neutrinos observed a significant reduction in
²⁸⁹ the event rate compared to predictions from the Standard Solar Model [28, 29]. A
²⁹⁰ proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a

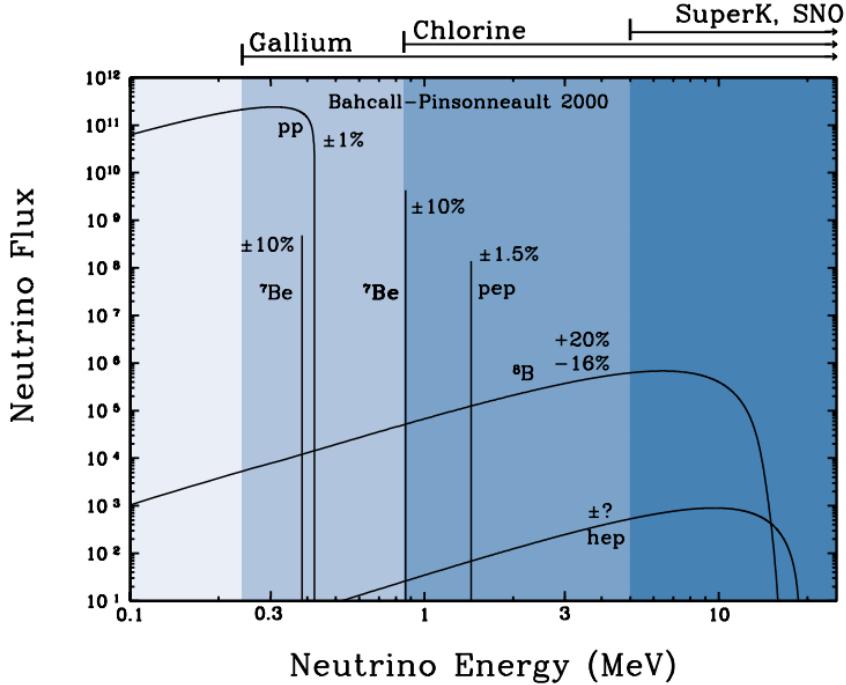


Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [27].

291 precursory version of the PMNS model [30]. The Kamiokande [31], Gallex [32]
 292 and Sage [33] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

293 The conclusive solution to this problem was determined by the SNO col-
 294 laboration [34]. Using a deuterium water target to observe 8B neutrinos, the
 295 event rate of charged current (CC), neutral current (NC), and elastic scattering
 296 (ES) interactions (Given in Equation 2.13) was simultaneously measured. CC
 297 events can only occur for electron neutrinos, whereas the NC channel is agnostic
 298 to neutrino flavour, and the ES reaction has a small excess sensitivity for the
 299 detection of electron neutrino interactions. This meant that there were direct
 300 measurements of the ν_e and ν_x neutrino flux. It was concluded that the CC and
 301 ES interaction rates were consistent with the deficit previously observed. Most
 302 importantly, the NC reaction rate was only consistent with the others under the

303 hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (CC) \\ \nu_x + d &\rightarrow p + n + \nu_x & (NC) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (ES) \end{aligned} \quad (2.13)$$

304 Since the SNO measurement, many experiments have since measured the
 305 neutrino flux of different interaction chains within the sun [35–37]. The most
 306 recent measurement was that of CNO-cycle neutrinos which were recently
 307 observed with 5σ significance by the Borexino collaboration [35].

308 2.3.2 Accelerator Neutrinos

309 The concept of using an artificial “neutrino beam” was first realised in 1962 [38].
 310 Since then, many experiments have adopted the same fundamental concepts.
 311 Typically, a proton beam is aimed at a target producing charged mesons that
 312 decay to neutrinos. The mesons can be sign-selected by the use of magnetic
 313 focusing horns to generate a neutrino or antineutrino beam. Pions are the primary
 314 mesons that decay and depending on the orientation of the magnetic field, a
 315 muon (anti-)neutrino beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$.
 316 The decay of muons and kaons results in an irreducible intrinsic electron neutrino
 317 background. In T2K, this background contamination is $O(< 1\%)$ [39]. There is
 318 also an approximately $\sim 5\%$ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated
 319 via the same decays. As the beam is generated by proton interactions (rather
 320 than anti-proton interactions), the wrong-sign component in the antineutrino
 321 beam is larger when operating in neutrino mode.

322 Tuning the proton energy in the beam and using beam focusing techniques
 323 allows the neutrino energy to be set to a value that maximises the disappear-
 324 ance oscillation probability in the L/E term in Equation 2.10. This means that
 325 accelerator experiments are typically more sensitive to the mixing parameters as
 326 compared to a natural neutrino source. However, the disadvantage compared
 327 to atmospheric neutrino experiments is the cost of building a facility to provide

328 high-energy neutrinos, with a high flux, which is required for longer baselines.
 329 Consequently, there is typically less sensitivity to matter effects and the ordering
 330 of the neutrino mass eigenstates.

331 A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

332 where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of
 333 neutrinos with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and
 334 $\epsilon(\vec{x})$ is the efficiency and resolution of the detector. In order to leverage the most
 335 out of an accelerator neutrino experiment, the flux and cross-section systematics
 336 need to be constrained. This is typically done via the use of a “near detector”,
 337 situated at a baseline of $O(1)$ km. This detector observes the unoscillated neutrino
 338 flux and constrains the parameters used within the flux and cross-section model.

339 The first accelerator experiments to precisely measure oscillation parameters
 340 were MINOS [40] and K2K [41]. These experiments confirmed the ν_μ disappear-
 341 ance seen in atmospheric neutrino experiments by finding consistent parameter
 342 values for $\sin^2(\theta_{23})$ and Δm_{32}^2 . The current generation of accelerator neutrino
 343 experiments, T2K and NO ν A extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead
 344 the sensitivity to atmospheric mixing parameters as seen in Figure 2.6 [42]. The
 345 two experiments differ in their peak neutrino energy, baseline, and detection tech-
 346 nique. The NO ν A experiment is situated at a baseline of 810km from the NuMI
 347 beamline which delivers 2GeV neutrinos. The T2K neutrino beam is peaked
 348 around 0.6GeV and propagates 295km. Additionally, the NO ν A experiment
 349 uses functionally identical detectors (near and far) whereas T2K uses a plastic
 350 scintillator technique at the near detector and a water Cherenkov far detector.
 351 The future generation experiments DUNE [43] and Hyper-Kamiokande [44]
 352 will succeed these experiments as the high-precision era of neutrino oscillation
 353 parameter measurements develops.

354 Several anomalous results have been observed in the LSND [9] and Mini-
 355 BooNE [10] detectors which were designed with purposefully short baselines.

356 Parts of the neutrino community attributed these results to oscillations induced
357 by a fourth “sterile” neutrino [45] but several searches in other experiments,
358 MicroBooNE [46] and KARMEN [47], found no hints of additional neutrino
359 species. The solution to the anomalous results is still being determined.

360 2.3.3 Atmospheric Neutrinos

361 The interactions of primary cosmic ray protons in the Earth’s upper atmosphere
362 generate showers of energetic hadrons. These are mostly pions and kaons that
363 decay to produce a natural source of neutrinos spanning energies of MeV to
364 TeV [48]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu) \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu) \end{aligned} \tag{2.15}$$

365 such that for a single pion decay, three neutrinos can be produced. The
366 atmospheric neutrino flux energy spectra as predicted by the Bartol [49], Honda
367 [50–52], and FLUKA [53] models are illustrated in Figure 2.3. The flux distribution
368 peaks at an energy of $O(10)$ GeV. The uncertainties associated with these models
369 are dominated by the hadronic production of kaon and pions as well as the
370 primary cosmic flux.

371 Unlike long-baseline experiments which have a fixed baseline, the distance
372 atmospheric neutrinos propagate is dependent upon the zenith angle at which
373 they interact. This is illustrated in Figure 2.4. Neutrinos that are generated
374 directly above the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the
375 height of the atmosphere whereas neutrinos that interact directly below the
376 detector ($\cos(\theta) = -1.0$) have to travel a length equal to the diameter of the Earth.
377 This means atmospheric neutrinos have a baseline that varies from $O(20)$ km to
378 $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will be subject
379 to MSW matter resonance as they propagate through the Earth.

380 Figure 2.5 highlights the neutrino flux as a function of the zenith angle for
381 different slices of neutrino energy. For medium to high-energy neutrinos (and to

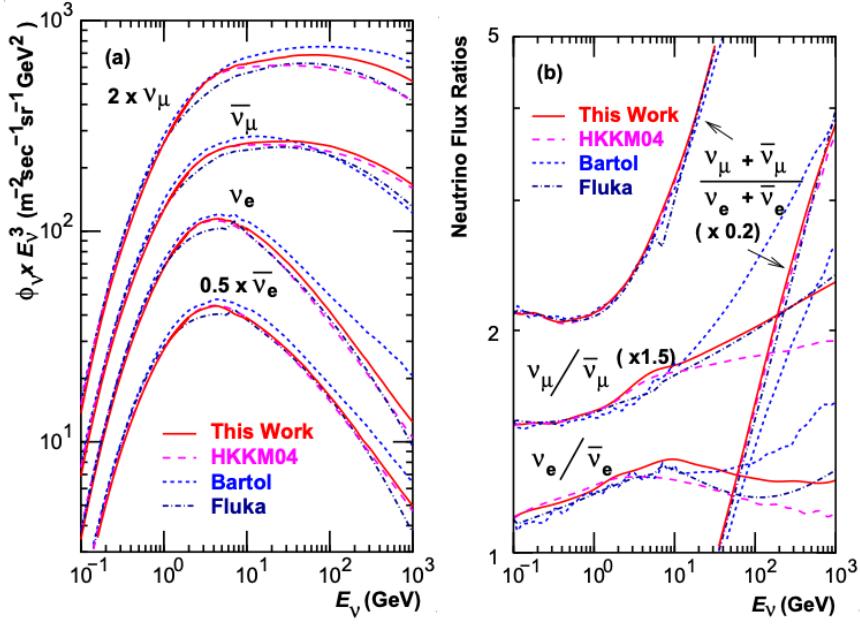


Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [50], the 2004 Honda model (“HKKM04”)[51], the Bartol model [49] and the FLUKA model [53]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [50].

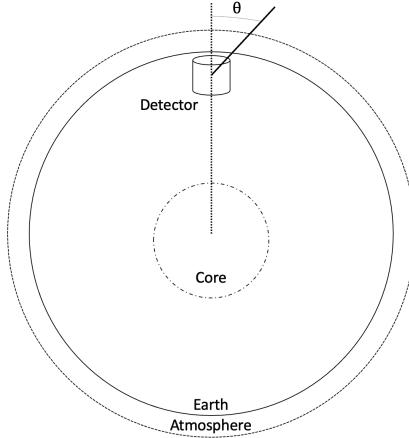


Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [54].

382 a lesser degree for low-energy neutrinos), the flux is approximately symmetric
 383 around $\cos(\theta) = 0$. To the accuracy of this approximation, the systematic
 384 uncertainties associated with atmospheric flux for comparing upward-going
 385 and down-going neutrino cancels. This allows the down-going events, which are

- 386 mostly insensitive to oscillation probabilities, to act as an unoscillated prediction
 387 (similar to a near detector in an accelerator neutrino experiment).

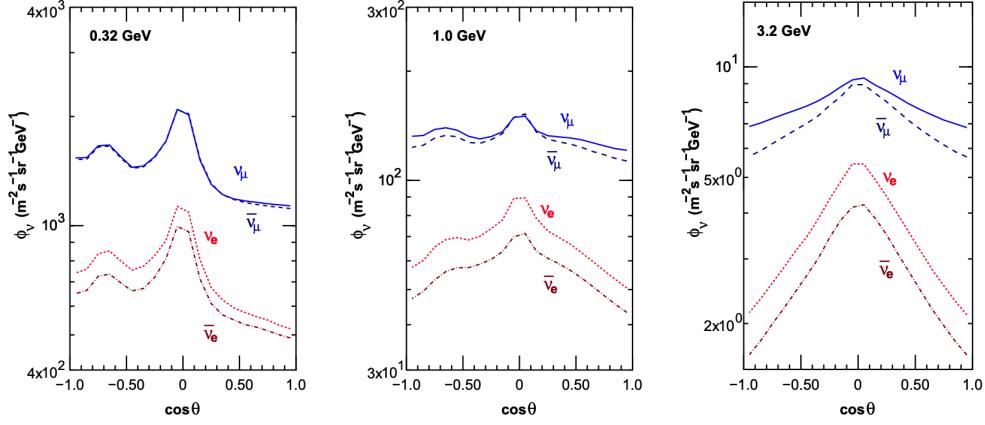


Figure 2.5: Prediction of ν_e , $\bar{\nu}_e$, ν_μ , $\bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [52]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [49], Honda [50] and FLUKA [53] are given in [54].

388 Precursory hints of atmospheric neutrinos were observed in the mid-1960s
 389 searching for $\nu_\mu + X \rightarrow X^* + \mu^\pm$ [55]. This was succeeded by the IMB-3 [56]
 390 and Kamiokande [57] experiments which measured the double ratio of muon
 391 to electron neutrinos in data to Monte Carlo, $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$.
 392 Both experiments were found to have a consistent deficit of muon neutrinos,
 393 with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$, respectively.
 394 Super-Kamiokande (SK) [54] extended this analysis by fitting oscillation pa-
 395 rameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and
 396 $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}\text{eV}^2$.

397 Since then, atmospheric neutrino experiments have been making precision
 398 measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric
 399 neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ
 400 discovery of ν_τ appearance [58]. Figure 2.6 illustrates the current estimates
 401 on the atmospheric mixing parameters from a wide range of atmospheric and
 402 accelerator neutrino observatories.

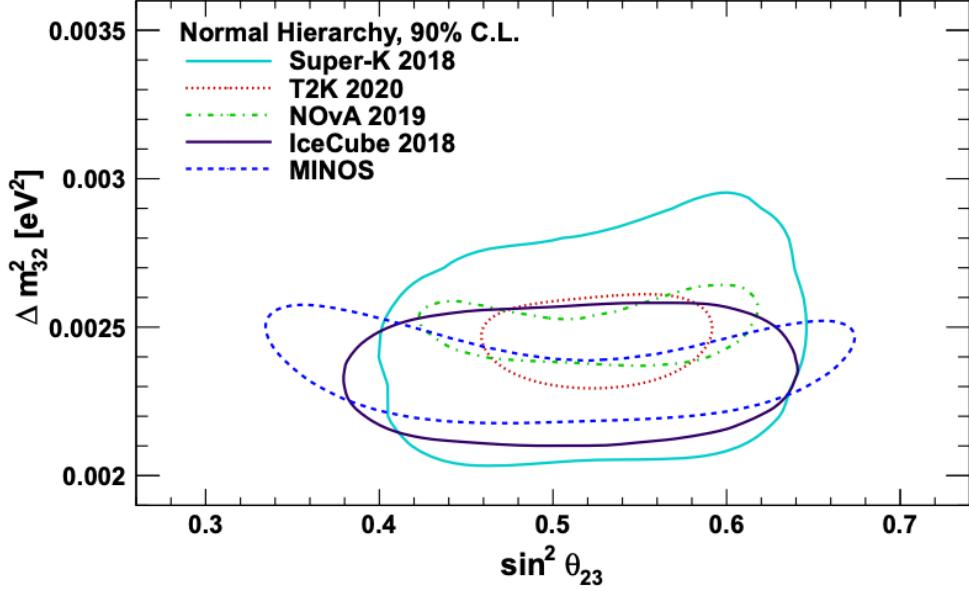


Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long-baseline experiments: SK [59], T2K [60], NOvA [61], IceCube [62] and MINOS [63]. Figure taken from [64].

2.3.4 Reactor Neutrinos

As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very useful artificial source of electron antineutrinos. For reactors that use low-enriched uranium ^{235}U as fuel, the antineutrino flux is dominated by the β -decay fission of ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu [65] as illustrated in Figure 2.7.

Due to their low energy, reactor electron antineutrinos predominantly interact via the inverse β -decay (IBD) interaction. The typical signature contains two signals delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihilation, and secondly the photon emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [67].

There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured the $\sin^2(\theta_{13})$ and Δm_{32}^2 oscillation parameters. Daya Bay [68], RENO [69] and Double Chooz [70] have all provided precise measurements, with the first discovery of a non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K [70]). The constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field and

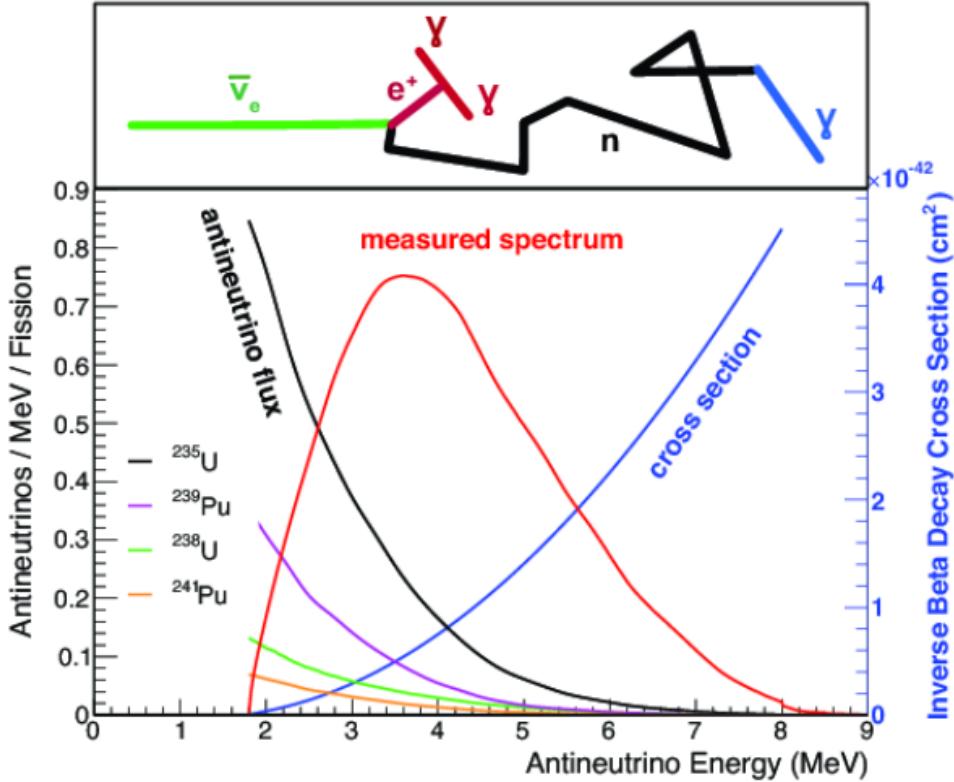


Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [66].

are often used as external inputs to accelerator neutrino experiments to improve their sensitivity to δ_{CP} and mass hierarchy determination. JUNO-TAO [71], a small collaboration within the larger JUNO experiment, is a next-generation reactor experiment that aims to precisely measure the isotopic antineutrino yields from the different fission chains.

Kamland [72] is the only experiment to have observed reactor neutrinos using a long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have sensitivity to Δm_{21}^2 . Combined with the SK solar neutrino experiment, the combined analysis puts the most stringent constraint on Δm_{21}^2 [73].

2.4 Summary Of Oscillation Parameter Measurements

Since the first evidence of neutrino oscillations, numerous measurements of the mixing parameters have been made. Many experiments use neutrinos as a tool for the discovery of new physics (diffuse supernova background, neutrinoless double beta decay and others) so the PMNS parameters are summarised in the Particle Data Group (PDG) review tables. The analysis presented in this thesis focuses on the 2020 T2K oscillation analysis presented in [74] which the 2020 PDG constraints [75] were used. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	0.547 ± 0.021
$\sin^2(\theta_{23})$ (N.H.)	0.545 ± 0.021
Δm_{32}^2 (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2020 Particle Data Group constraints of the oscillation parameters taken from [75]. The value of Δm_{32}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is take as the average best-fit from the combination of Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within other neutrino oscillation experiments, typically termed the reactor constraint. The $\sin^2(\theta_{12})$ parameter is predominantly measured through electron neutrino disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline reactor neutrino experiment Kamland also has a sensitivity to this parameter and is used in a joint fit to solar data from SNO and SK, using the reactor constraint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric neutrino experiments. The PDG value is a joint fit of T2K, NO ν A , MINOS and IceCube DeepCore experiments. The latest T2K-only measurement, provided at Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [74].

The PDG constraint on Δm_{21}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{32}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [74]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analyses have reduced the credible intervals to 90%. Since the 2020 PDG publication, there has been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [76], alongside updated Δm_{32}^2 and $\sin^2(\theta_{23})$ measurements.

Throughout this thesis, several sample spectra predictions and contours are presented which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [77] and “Asimov B” being CP-conserving and further from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

3

466

467

T2K and SK Experiment Overview

468 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)
469 collaboration has been leading atmospheric neutrino oscillation analyses for
470 over two decades. The detector has provided some of the strongest constraints
471 on proton decay and the first precise measurements of the Δm_{23}^2 and $\sin^2(\theta_{23})$
472 neutrino oscillation parameters. The history, detection technique, and operation
473 of the SK detector is described in section 3.1.

474 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline
475 experiments to use both neutrino and antineutrino beams to precisely measure
476 charge parity violation within the neutrino sector. The T2K experiment observed
477 the first hints of a non-zero $\sin^2(\theta_{13})$ measurement and continues to lead the
478 field with the constraints it provides on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{23}^2 and δ_{CP} .
479 In section 3.2, the techniques that T2K use to generate the neutrino beam and
480 constrain systematic parameter through near detector constraints are described.

481 3.1 The Super-Kamiokande Experiment

482 The SK experiment began taking data in 1996 [78] and has had many modifi-
483 cations throughout its operation. There have been seven defined periods of
484 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

485 years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were
 486 damaged during maintenance [79]. Those that survived were equally distributed
 487 throughout the detector in the SK-II era, which resulted in a reduced 19% photo-
 488 coverage. From SK-III onwards, repairs to the detector meant the full suite of
 489 PMTs was operational recovering the 40% photo-coverage. Before the start of
 490 SK-IV, the data acquisition and electronic systems were upgraded. Between
 491 SK-IV and SK-V, a significant effort was placed into tank open maintenance and
 492 repair/replacement of defective PMTs in preparation for the Gadolinium upgrade,
 493 a task for which the author of this thesis was required. Consequently, the detector
 494 conditions were significantly changed from this point. SK-VI marked the start of
 495 the SK-Gd era, with the detector being doped with gadolinium at a concentration
 496 of 0.01% by concentration. SK-VII, which started during the writing of this thesis,
 497 has increased the gadolinium concentration to 0.03% for continued operation [80].

498 The oscillation analysis presented within this thesis focuses on the SK-IV
 499 period of running and the data taken within it. This follows from the recent
 500 SK analysis presented in [81]. Therefore, the information presented within this
 501 section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

502 **Table 3.1:** The various SK periods and their respective live-time. The SK-VI live-time is
 503 calculated until 1st April 2022. SK-VII started during the writing of this thesis.

502 3.1.1 The SK Detector

503 The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank
 504 with a diameter 39.3m and height 41.1m filled with ultrapure water [79]. A
 505 diagram of the significant components of the SK detector is given in Figure 3.1.

506 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine
 507 is underground with roughly 1km rock overburden (2.7km water equivalent
 508 overburden) [82]. At this depth, the rate of cosmic ray muons is significantly
 509 decreased to a value of $\sim 2\text{Hz}$ (net rate) [83]. The top of the tank is covered
 510 with stainless steel which is designed as a working platform for maintenance,
 511 calibration, and location for high voltage and data acquisition electronics.

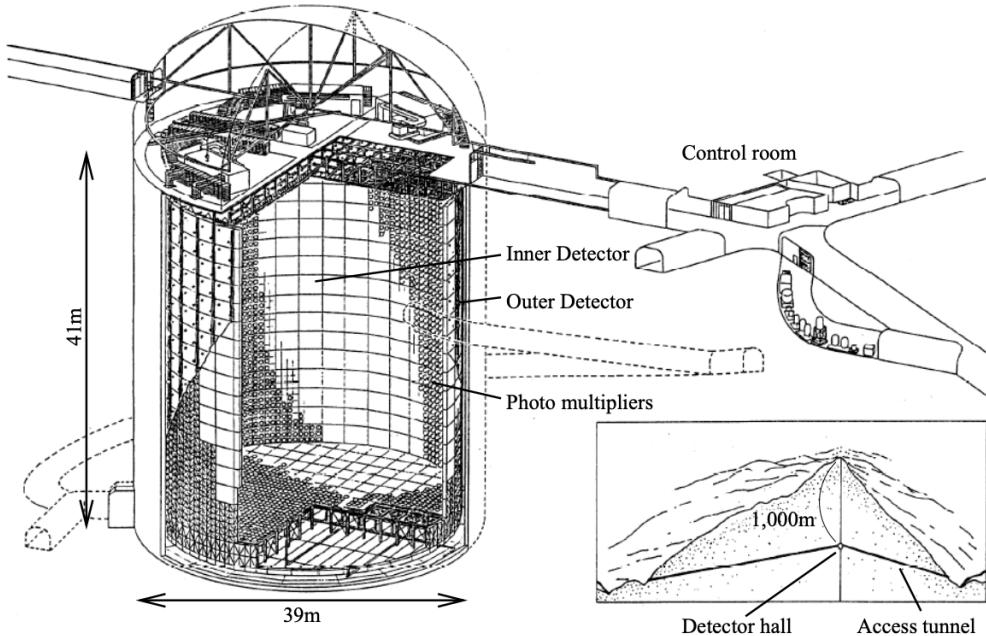


Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [84].

512 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated
 513 inside the tank, with an approximate 2m gap between this structure and the outer
 514 tank wall. The purpose of this structure is to support the photomultiplier tubes
 515 (PMTs). The volume inside and outside the support structure is referred to as the
 516 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the
 517 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs respectively
 518 [79]. The ID contains a 32kton mass of water. Many analyses performed at SK
 519 use a “fiducial volume” defined by the volume of water inside the ID excluding
 520 some distance to the ID wall. This reduces the volume of the detector which is
 521 sensitive to neutrino events but reduces radioactive backgrounds and allows for

522 better reconstruction performance. The nominal fiducial volume is defined as the
523 area contained inside 2m from the ID wall for a total of 22.5kton water [85].

524 The two regions of the detector (ID and OD) are optically separated with
525 opaque black plastic hung from the support structure. The purpose of this is
526 to determine whether an event entered or exited the ID. This allows cosmic ray
527 muons and partially contained events to be tagged and separated from neutrino
528 events entirely contained within the ID. This black plastic is also used to cover
529 the area between the ID PMTs to reduce photon reflection from the ID walls.
530 Opposite to this, the OD is lined with a reflective material to allow photons to
531 reflect around inside the OD until collected by one of the PMTs. Furthermore,
532 each OD PMT is optically coupled with $50 \times 50\text{cm}$ plates of wavelength shifting
533 acrylic which increases the efficiency of light collection [82].

534 In the SK-IV data-taking period, the photocathode coverage of the detector, or
535 the fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [82]. The PMTs have
536 a quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$
537 for photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$ [86, 87]. The proportion
538 of photoelectrons that produce a signal in the dynode of a PMT, termed the
539 collection efficiency, is $> 70\%$ [82]. The PMTs used within SK are most sensitive
540 to photons with wavelength $300\text{nm} \leq \lambda \leq 600\text{nm}$ [82]. One disadvantage of
541 using PMTs as the detection media is that the Earth's geomagnetic field can
542 modify its response. Therefore, a set of compensation coils is built around the
543 inner surface of the detector to mitigate this effect [83].

544 The SK detector is filled with ultrapure water, which in a perfect world, con-
545 tains no impurities. However, bacteria and organic compounds can significantly
546 degrade the water quality. This decreases the attenuation length, which reduces
547 the total number of photons that hit a PMT. To combat this, a sophisticated water
548 treatment system has been developed [82, 88]. UV lights, mechanical filters, and
549 membrane degasifiers are used to reduce the bacteria, suspended particulates,
550 and radioactive materials from the water. The flow of water within the tank
551 is also critical as it can remove stagnant bacterial growth or build-up of dust

552 on the surfaces within the tank. Gravity drifts impurities in the water towards
553 the bottom of the tank which, if left uncontrolled, can create asymmetric water
554 conditions between the top and bottom of the tank. Typically, the water entering
555 the tank is cooled below the ambient temperature of the tank to control convection
556 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs
557 is sensitive to the PMT temperature [89] so controlling the temperature gradients
558 within the tank is beneficial for stable measurements.

559 SK-VI is the first phase of the SK experiment to use gadolinium dopants
560 within the ultrapure water [80]. As such, the SK water system had to be replaced
561 to avoid removing the gadolinium concentrate from the ultrapure water [90]. For
562 an inverse β -decay (IBD) interaction on a water target, the emitted neutron is
563 thermally captured on hydrogen. This process releases a 2.2MeV γ ray which is
564 difficult to detect as the resulting Compton scattered electrons are very close to the
565 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons
566 on gadolinium generates γ rays with higher energy (8MeV [67]) meaning they
567 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%
568 gadolinium sulphate by mass) which causes \approx 50% of neutrons emitted by IBD to
569 be captured on gadolinium[91, 92] . Whilst predominantly useful for low energy
570 analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino
571 event selections [93]. Efforts are currently in place to increase the gadolinium
572 concentrate to 0.03% for \approx 75% neutron capture efficiency on gadolinium [94].
573 The final stage of loading targets 0.1% concentrate for \approx 90% neutron capture
574 efficiency on gadolinium.

575 3.1.2 Calibration

576 The calibration of the SK detector is documented in [79] and summarised below.
577 The analysis presented within this thesis is dependent upon ‘high energy events’
578 (Charged particles with $O(> 100)$ MeV momenta). These are events that are
579 expected to generate a larger number of photons such that each PMT will
580 be hit with multiple photons. The reconstruction of these events depends

upon the charge deposited within each PMT and the timing response of each individual PMT. Therefore, the most relevant calibration techniques to this thesis are outlined.

Before installation, 420 PMTs were calibrated to have identical charge responses and then distributed throughout the tank in a cross-shape pattern (As illustrated by Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed at similar geometric positions within SK to be calibrated against. To perform this calibration, a xenon lamp is located at the center of the SK tank which flashes uniform light at 1Hz. This allows for geometrical effects, water quality variation, and timing effects to be measured in situ throughout normal data-taking periods.

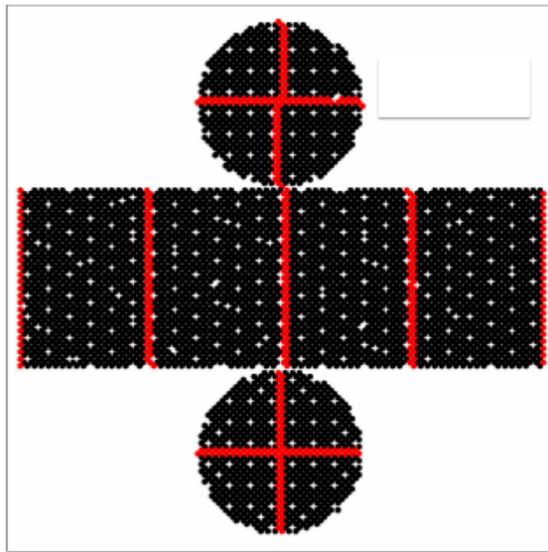


Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [79].

When specifically performing calibration of the detector (in out-of-data taking mode), the water in the tank was circulated to avoid top/bottom asymmetric water quality. Any non-uniformity within the tank significantly affects the PMT hit probability through scattering or absorption. This becomes a dominant effect for the very low-intensity light sources discussed later which are designed such that only one photon is incident upon a given PMT.

598 The gain of a PMT is defined as the ratio of the total charge of the signal
 599 produced compared to the charge of photoelectrons emitted by the photocathodes
 600 within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”
 601 gain values are measured. The relative gain is the variation of gain among each
 602 of the PMTs whereas the absolute gain is the average gain of all PMTs.

603 The relative gain is calibrated as follows. A laser is used to generate two
 604 measurements: a high-intensity flash that illuminates every PMT with a sufficient
 605 number of photons, and a low-intensity flash in which only a small number
 606 of PMTs collect light. The first measurement creates an average charge, $Q_{obs}(i)$
 607 on PMT i , whereas the second measurement ensures that each hit PMT only
 608 generates a single photoelectron. For the low-intensity measurement, the number
 609 of times each PMT records a charge larger than 1/4 photoelectrons, $N_{obs}(i)$, is
 610 counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i), \end{aligned} \tag{3.1}$$

611 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the
 612 acceptance efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and
 613 collection efficiency of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative
 614 gain for each PMT can be determined by taking the ratio of these quantities.

615 The absolute gain calibration is performed by observing fixed energy γ -rays
 616 of $E_\gamma \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source
 617 situated at the center of the detector. This generates a photon yield of about 0.004
 618 photoelectrons/PMT/event, meaning that $> 99\%$ of PMT signals are generated
 619 from single photoelectrons. A charge distribution is generated by performing
 620 this calibration over all PMTs, and the average value of this distribution is taken
 621 to be the absolute gain value.

622 As mentioned in subsection 3.1.1, the average quantum and collection effi-
 623 ciency for the SK detector PMTs is $\sim 21\%$ and $> 70\%$ respectively. However,
 624 these values do differ between each PMT and need to be calibrated accordingly.

625 Consequently, the NiCf source is also used to calibrate the “quantum \times collection”
626 efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is
627 used as the PMT hit probability is proportional to the QE ($N_{obs}(i) \propto \epsilon(i)$ in
628 Equation 3.1). A Monte Carlo prediction which includes photon absorption,
629 scattering, and reflection is made to estimate the number of photons incident on
630 each PMT and the ratio of the number of predicted to observed hits is calculated.
631 The difference is attributed to the QE efficiency of that PMT. This technique is
632 extended to calculate the relative QE efficiency by normalizing the average of
633 all PMTs which removes the dependence on the light intensity.

634 Due to differing cable lengths and readout electronics, the timing response
635 between a photon hitting the PMT and the signal being captured by the data
636 acquisition can be different between each PMT. Due to threshold triggers (De-
637 scribed in subsection 3.1.3), the time at which a pulse reaches a threshold is
638 dependent upon the size of the pulse. This is known as the ‘time-walk’ effect
639 and also needs to be accounted for in each PMT. To calibrate the timing response,
640 a pulse of light with width 0.2ns is emitted into the detector through a diffuser.
641 Two-dimensional distributions of time and pulse height (or charge) are made
642 for each PMT and are used to calibrate the timing response. This is performed
643 in-situ during data taking with the light source pulsing at 0.03Hz.

644 The top/bottom water quality asymmetry is measured using the NiCf calibra-
645 tion data and cross-referencing these results to the “standard PMTs”. The water
646 attenuation length is continuously measured by the rate of vertically-downgoing
647 cosmic-ray muons which enter via the top of the tank.

648 Dark noise is where a PMT registers a pulse that is consistent with a single
649 photoelectron emitted from photon detection despite the PMT being in complete
650 darkness. This is predominately caused by two processes. Firstly there is
651 intrinsic dark noise which is where photoelectrons gain enough thermal energy
652 to be emitted from the photocathode, and secondly, the radioactive decay of
653 contaminants inside the structure of the PMT. Typical dark noise rate for PMTs
654 used within SK are $O(3)\text{kHz}$ [82]. This is lower than the expected number of

655 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)
656 but instability in this value can cause biases in reconstruction. Dark noise is
657 related to the gain of a PMT and is calibrated using hits inside a time window
658 recorded before an event trigger [95].

659 3.1.3 Data Acquisition and Triggering

660 As the analysis presented in this thesis will only use the SK-IV period of the
661 SK experiment so this subsection focuses on the relevant points of the data
662 acquisition and triggering systems to that SK period. The earlier data acquisition
663 and triggering systems are documented in [96, 97].

664 Before the SK-IV period started, the existing front-end electronics were re-
665 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [98]. When
666 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-
667 time (QTC) converter generates a rectangular pulse. The start of the rectangular
668 pulse indicates the time at which the analog photoelectron signal was received
669 and the width of the pulse indicates the total charge integrated throughout the
670 signal. This is then digitized by time-to-digital converters and sent to the “front-
671 end” PCs. The digitized signal from every QBEE is then chronologically ordered
672 and sent to the “merger” PCs. It is the merger PCs that apply the software trigger.
673 Any triggered events are passed to the “organizer” PC. This sorts the data stream
674 of multiple merger PCs into chronologically ordered events which are then saved
675 to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

676 The software trigger (described in [100]) operates by determining the number
677 of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the
678 maximum time that a Cherenkov photon would take to traverse the length of the
679 SK tank [97]. For lower energy events that generate fewer photons, this technique
680 is useful for eliminating background processes like dark noise and radioactive
681 decay which would be expected to be separated in time. When the value of
682 N_{200} exceeds some pre-defined threshold, a software trigger is issued. There are
683 several trigger thresholds used within the SK-IV period which are detailed in

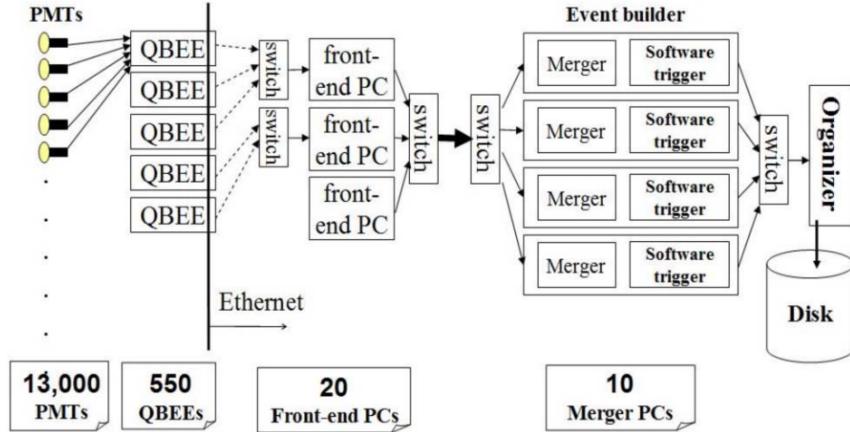


Figure 3.3: Schematic view of the data flow through the data acquisition and online system. Taken from [99].

684 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time
 685 window are also read out and saved to disk. In the special case of an event that
 686 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks
 687 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in
 688 a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband
 689 Intelligent Trigger (WIT)” has been deployed and is described in [101].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

690 3.1.4 Cherenkov Radiation

691 Cherenkov light is emitted from any highly energetic charged particle traveling
 692 with relativistic velocity, β , greater than the local speed of light in a medium [102].

693 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

694 where n is the refractive index of the medium. Consequently, the Cherenkov
 695 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged
 696 particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}} \quad (3.3)$$

697 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy
 698 for various particles are given in Table 3.3. In contrast, γ -rays are detected
 699 indirectly via the combination of photons generated by Compton scattering
 700 and pair production. The threshold for detection in the SK detector is typically
 701 higher than the threshold for photon production. This is due to the fact that the
 702 attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov
 703 photons reach the ID PMTs. Then the collection and quantum efficiencies
 704 described in subsection 3.1.1 result in the number of detected photons being
 705 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

706 The Frank-Tamm equation [103] describes the relationship between the num-
 707 ber of Cherenkov photons generated per unit length, dN/dx , the wavelength of
 708 the photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

709 where α is the fine structure constant. For a 100MeV momentum electron,
 710 approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq$
 711 700nm region which the ID PMTs are most sensitive to [82].

712 3.2 The Tokai to Kamioka Experiment

713 The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation
 714 experiment located in Japan. Proposed in the early 2000s [104, 105] to replace
 715 K2K [106], T2K was designed to observe electron neutrino appearance whilst
 716 precisely measuring the oscillation parameters associated with muon neutrino
 717 disappearance [107]. The experiment consists of a neutrino beam generated
 718 at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near
 719 detectors situated 280m from the beam target, and the Super Kamiokande far
 720 detector positioned at a 295km baseline. The cross-section view of the T2K
 721 experiment is drawn in Figure 3.4.

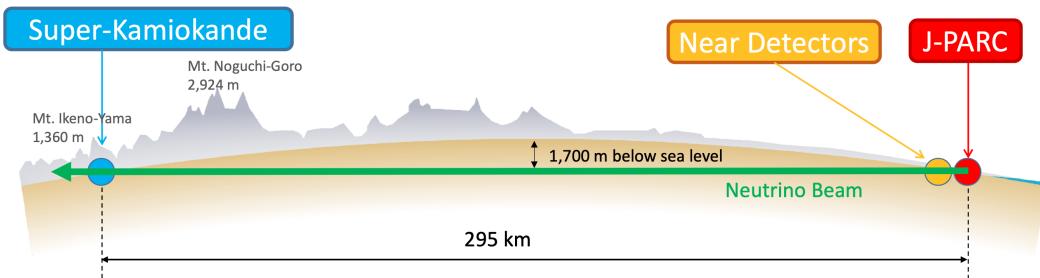


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

722 The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$
 723 , Δm_{23}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and
 724 accuracy of parameter estimates are still being made by including new data
 725 samples and developing the models which describe the neutrino interactions
 726 and detector responses [108]. Electron neutrino appearance was first observed
 727 at T2K in 2014 [109] with 7.3σ significance.

The near detectors provide constraints on the beam flux and cross-section model parameters used within the oscillation analysis by observing the unoscillated neutrino beam. There are a host of detectors situated in the near detector hall (As illustrated in Figure 3.5): ND280 (subsection 3.2.2), INGRID (subsection 3.2.3), NINJA [110], WAGASCI [111], and Baby-MIND [112]. The latter three are not currently used within the oscillation analysis presented in this thesis.

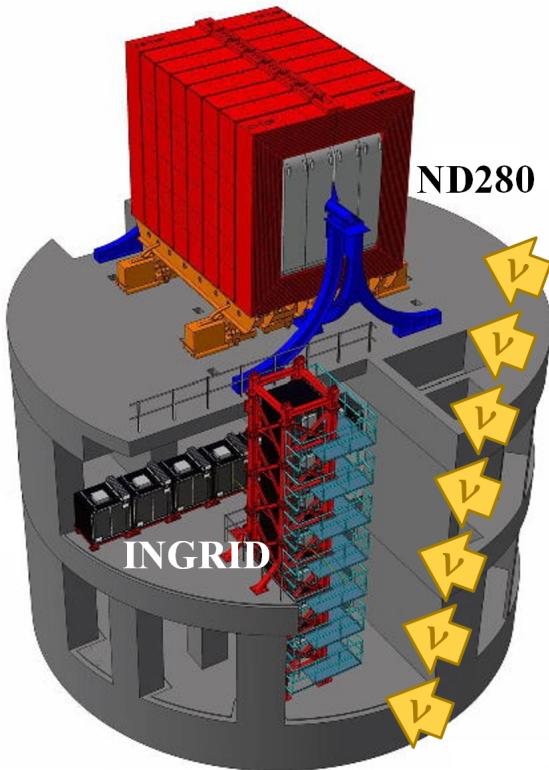


Figure 3.5: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

Whilst this thesis presents the ND280 in terms of its purpose for the oscillation analysis, the detector can also make many cross-section measurements at neutrino energies of $O(1)\text{GeV}$ for the different targets within the detector [113, 114]. These measurements are of equal importance as they can lead the way in determining the model parameters used in the interaction models for the future high-precision era of neutrino physics.

There are two independent fitters, MaCh3 and BANFF, which perform the near detector fit. MaCh3 is the basis of this analysis and uses a bayesian Markov

742 Chain Monte Carlo fitting technique, whereas BANFF uses a frequentist gradient
743 descent technique. The output of each fitter is converted into a covariance matrix
744 to describe the error and correlations between all the flux and cross-section
745 parameters. This is then propagated to the far-detector oscillation analysis group
746 for use in the P-Theta and VALOR fitting framework. As MaCh3 can handle both
747 near and far detector samples, it does not use this covariance matrix and instead
748 opts for a simultaneous fit of the two detector measurements. This is an analysis
749 choice that removes the assumption of Gaussian posterior distributions required
750 when building the post-fit covariance matrix.

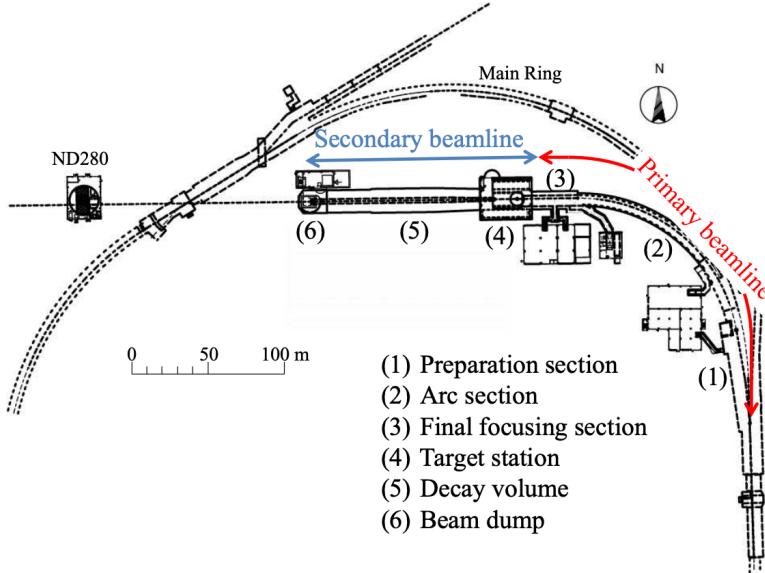
751 **DB: MaCh3 vs PTheta and Valor**

752 There are three particular tunes of the T2K flux and low energy cross section
753 model typically considered. Firstly, the “generated” tune which is the set of
754 dial values with which the Monte Carlo was generated. Secondly, the set of dial
755 values which are taken from external data measurements and used as inputs.
756 These are the “pre-fit” dial values. The reason these two sets of dial values are
757 different is that the external data measurements are continually updated but it
758 is very computationally intensive to regenerate a Monte Carlo prediction after
759 each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial
760 values. These are the values taken from the fit to the beam near detector data.

761 **3.2.1 The Neutrino Beam**

762 The neutrino beam used within the T2K experiment is described in [39, 115]
763 and summarised below. The accelerator facility at J-PARC is composed of two
764 sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic
765 of the beamline, focusing mostly on the components of the secondary beamline.
766 The primary beamline has three accelerators that progressively accelerate pro-
767 tons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR)
768 synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy
769 of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted
770 per “spill” from the MR and directed towards a graphite target (a rod of length

91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.



(a) Primary and secondary beamline

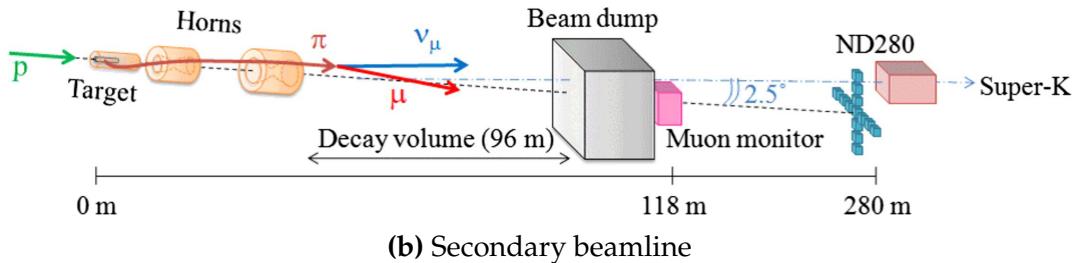


Figure 3.6: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [115]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [116].

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [39],

$$\begin{aligned}
& \pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
& K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
& \rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
& \rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
& K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
& \rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
& \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e
\end{aligned}$$

779 The electrically charged component of the secondary beam is focused towards
 780 the far detector by the three magnetic horns. These horns direct charged particles
 781 of a particular polarity towards SK whilst defocusing the oppositely charged
 782 particles. This allows a mostly neutrino or mostly antineutrino beam to be used
 783 within the experiment, denoted as “forward horn current (FHC)” or “reverse
 784 horn current (RHC)” respectively.
 785

786 Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino
 787 flux. The low energy flux is dominated by the decay of pions whereas kaon
 788 decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-
 789 sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic
 790 irreducible ν_e background, are predominantly due to muon decay for $E_\nu <$
 791 2GeV . As the antineutrino production cross-section is smaller than the neutrino
 792 cross-section, the wrong-sign component is more dominant in the RHC beam
 793 as compared to that in the FHC beam.
 794

794 The beam dump, situated at the end of the decay volume, stops all charged
 795 particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector
 796 monitors the penetrating muons to determine the beam direction and inten-
 797 sity which is used to constrain some of the beam flux systematics within the
 798 analysis [116, 118].
 799

799 The T2K experiment uses an off-axis beam to narrow the neutrino energy
 800 distribution. This was the first implementation of this technique in a long-
 801 baseline neutrino oscillation experiment after its original proposal [119]. Pion
 802 decay, $\pi \rightarrow \mu + \nu_\mu$, is a two-body decay. Consequently, the neutrino energy,
 803

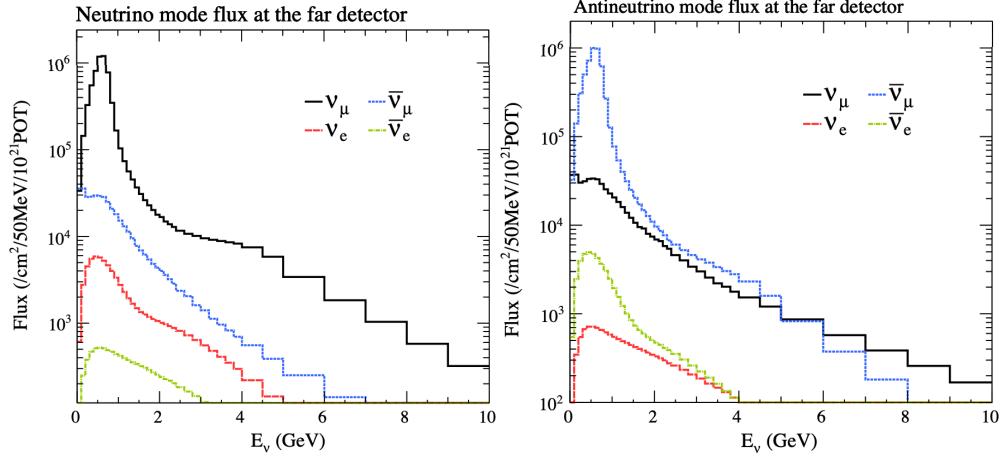


Figure 3.7: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [117].

803 E_ν , can be determined based on the pion energy, E_π , and the angle at which
 804 the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

805 where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
 806 energy pion, the neutrino energy distribution is dependent upon the angle at
 807 which the neutrinos are observed from the initial pion beam direction. For the
 808 295km baseline at T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance
 809 probability, $P(\nu_\mu \rightarrow \nu_e)$, whilst minimising the muon disappearance probability,
 810 $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8 illustrates the neutrino energy distribution for a range of
 811 off-axis angles, as well as the oscillation probabilities most relevant to T2K.

812 3.2.2 The Near Detector at 280m

813 Whilst all the near detectors are situated in the same “pit” located at 280m from
 814 the beamline, the “ND280” detector is the off-axis detector which is situated at
 815 the same off-axis angle as the Super-Kamiokande far detector. It has two primary
 816 functions; firstly it measures the neutrino flux and secondly, it counts the event
 817 rates of different types of neutrino interactions. Both of these constrain the flux

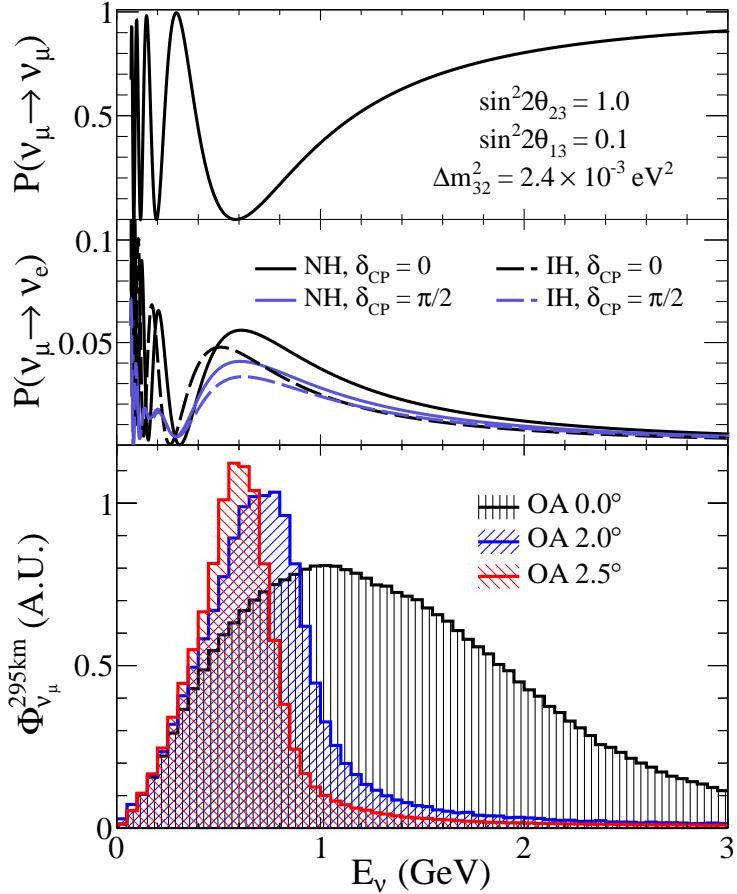


Figure 3.8: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

and cross-section systematics invoked within the model for a more accurate prediction of the expected event rate at the far detector.

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two-time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.2.1 and subsubsection 3.2.2.2 respectively. The electromagnetic calorimeter (ECAL)

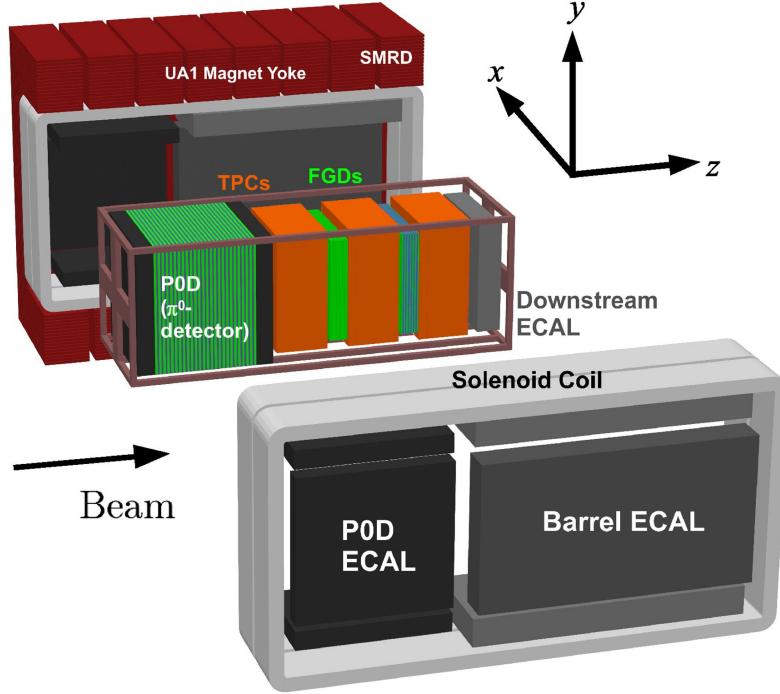


Figure 3.9: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [115].

encapsulates the tracker region alongside the π^0 detector (P0D). The ECAL measures the deposited energy from photons emitted from interactions within the FGD. The P0D constrains the cross-section of neutral current interactions which generate neutral pions, which is one of the largest backgrounds in the electron neutrino appearance oscillation channel. The P0D and ECAL detectors are detailed in subsubsection 3.2.2.3 and subsubsection 3.2.2.4 respectively. The entire detector is located within a large yoke magnet which produces a 0.2T magnetic field. This design of the magnet also includes a scintillating detector called the side muon range detector (SMRD) which is used to track high-angle muons as well as acting as a cosmic veto. The SMRD is described in subsubsection 3.2.2.5.

3.2.2.1 Fine Grained Detectors

The T2K tracker region is comprised of two fine-grained detectors (FGD) and three Time Projection Chambers (TPC). A detailed description of the FGD design, construction, and assembly is found in [120] and summarised below. The FGDs are the primary target for neutrino interactions with a mass of 1.1 tonnes per FGD.

844 Alongside this, the FGDs are designed to be able to track short-range particles
845 which do not exit the FGD. Typically, short-range particles are low momentum
846 and are observed as tracks that deposit a large amount of energy per unit length.
847 This means the FGD needs good granularity to resolve these particles. The
848 FGDs have the best timing resolution ($\sim 3\text{ns}$) of any of the sub-detectors of the
849 ND280 detector. As such, the FGDs are used for time of flight measurements
850 to distinguish forward-going positively charged particles from backward-going
851 negatively charged particles. Finally, any tracks which pass through multiple
852 sub-detectors are required to be track matched to the FGD.

853 Both FGDs are made from square scintillator planes of side length 186cm and
854 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an
855 X or Y orientation. A wavelength-shifting fiber is threaded through the center
856 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is
857 the most upstream of the two FGDs and contains 15 planes of carbon plastic
858 scintillator which is a common target in external neutrino scattering data. As
859 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2
860 have been replaced with a hybrid water-scintillator target. Due to the complexity
861 of the nucleus, nuclear effects can not be extrapolated between different nuclei.
862 Therefore having the ability to take data on one target which is the same as
863 external data and another target which is the same as the far detector target is
864 beneficial for reliable model parameter estimates.

865 The integrated deposited energy is used for particle identification. The FGD
866 can distinguish protons from other charged particles by comparing the integrated
867 deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

868 3.2.2.2 Time Projection Chambers

869 The majority of particle identification and momentum measurements within
870 ND280 are provided by three Time Projection Chambers (TPCs) [121]. The
871 TPCs are located on either side of the FGDs. They are located inside of the

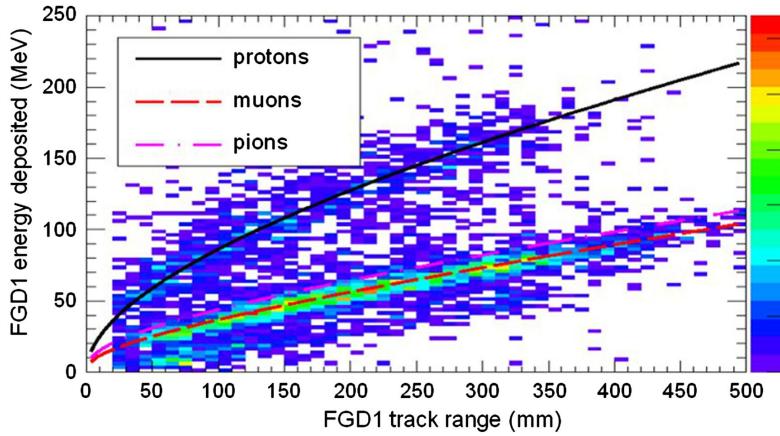


Figure 3.10: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [120].

872 magnetic field meaning the momentum of a charged particle can be determined
873 from the bending of the track.

874 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11,
875 which are made of non-magnetic material. The outer box is filled with CO₂ which
876 acts as an electrical insulator between the inner box and the ground. The inner box
877 forms the field cage which produces a uniform electric drift field of $\sim 275\text{V/cm}$
878 and is filled with an argon gas mixture. Charged particles moving through this
879 gas mixture ionize the gas and the ionised charge is drifted towards micromegas
880 detectors which measure the ionization charge. The time and position information
881 in the readout allows a three-dimensional image of the neutrino interaction.

882 The particle identification of tracks that pass through the TPCs is performed
883 using dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo
884 distributions of the energy lost by a charged particle passing through the TPC as
885 a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$
886 meaning that electrons and muons can be distinguished. This allows reliable
887 measurements of the intrinsic ν_e component of the beam.

888 3.2.2.3 π^0 Detector

889 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the
890 reconstruction will determine that event to be a charge current ν_e -like event.

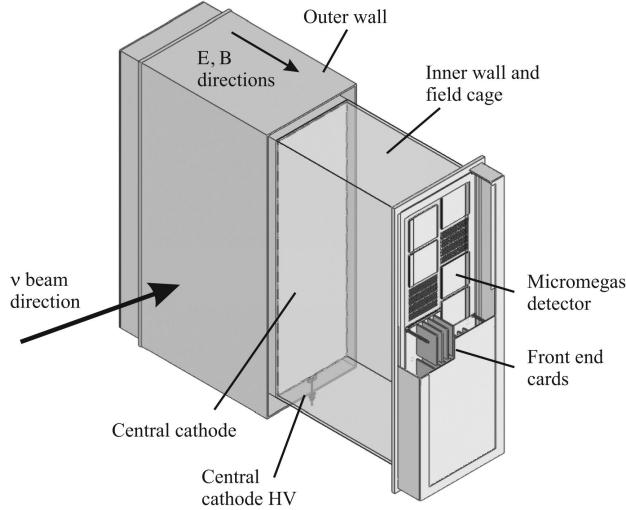


Figure 3.11: Schematic design of a Time Projection Chamber detector. Taken from [121].

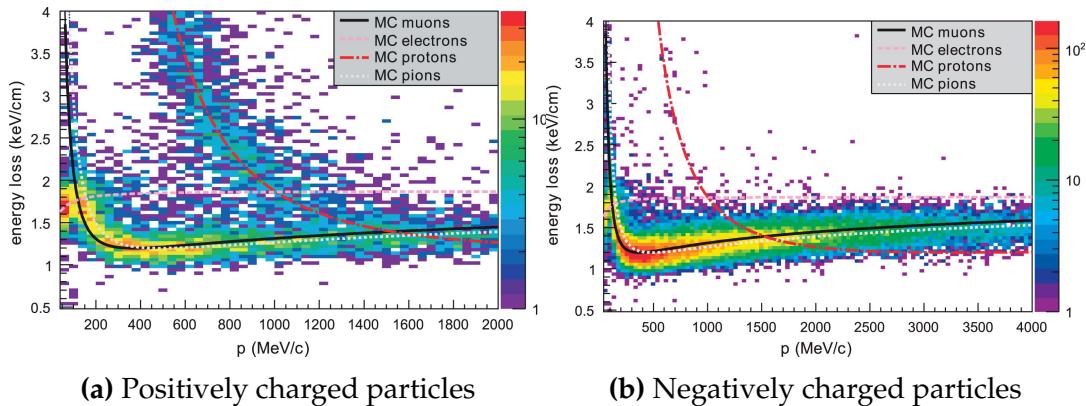


Figure 3.12: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [121].

891 This is one of the main backgrounds hindering the electron neutrino appearance
 892 searches. The π^0 detector (P0D) measures the cross-section of the neutral current
 893 induced neutral pion production on a water target to constrain this background.

894 The P0D is a cube of approximately 2.5m length consisting of layers of scin-
 895 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.13.

896 Two electromagnetic calorimeters are positioned at the most upstream and most
 897 downstream position in the sub-detector and the water target is situated in
 898 between them. The scintillator layers are built from two triangular bars orientated
 899 in opposite directions to form a rectangular layer. Each triangular scintillator bar
 900 is threaded with optical fiber which is read out by MPPCs. The high-Z brass and

901 lead regions produce electron showers from the photons emitted in π^0 decay.

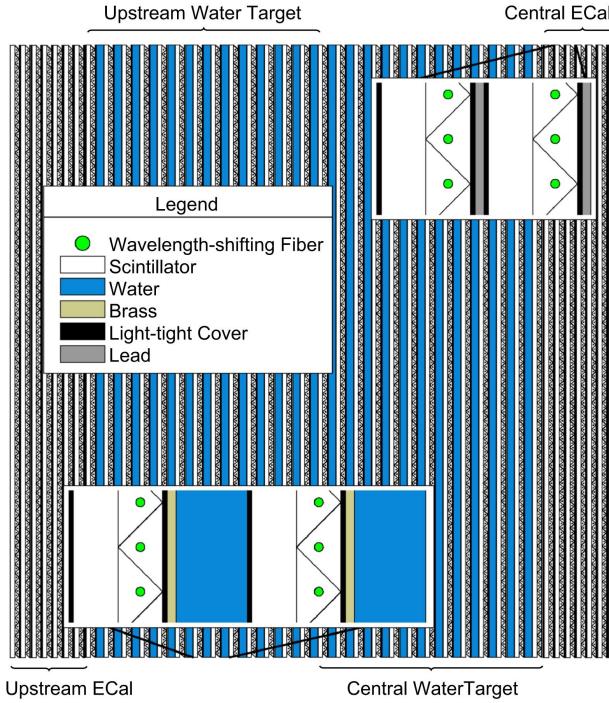


Figure 3.13: A schematic of the P0D side-view. Taken from [122].

902 The sub-detector can generate measurements of NC1 π^0 cross-sections on a
 903 water target by measuring the event rate both with and without the water target,
 904 with the cross-section on a water target being determined as the difference. The to-
 905 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

906 3.2.2.4 Electromagnetic Calorimeter

907 The electromagnetic calorimeter [123] (ECal) encapsulates the P0D and tracking
 908 sub-detectors. Its primary purpose is to aid π^0 reconstruction from any interac-
 909 tion in the tracker. To do this, it measures the energy and direction of photon
 910 showers from $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks
 911 depending on the shape of the photon shower deposited.

912 The ECal is comprised of three sections; the P0D ECal which surrounds the
 913 P0D, the barrel ECal which encompasses the tracking region, and the downstream
 914 ECal which is situated downstream of the tracker region. The barrel and down-
 915 stream ECals are tracking calorimeters that focus on electromagnetic showers

916 from high-angle particles emitted from the tracking sub-detectors. Particularly in
917 the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis)
918 can travel along a single scintillator bar resulting in very few hits. The width of
919 the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths
920 to ensure a significant amount of the π^0 energy is contained. As the P0D has
921 its own calorimetry which reconstructs showers, the P0D ECal determines the
922 energy which escapes the P0D.

923 Each ECal is constructed of multiple layers of scintillating bars sandwiched
924 between lead sheets. The scintillating bars are threaded with optical fiber and read
925 out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular
926 to the previous which allows a three-dimensional event reconstruction. The
927 target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80,
928 and 6.62 tonnes respectively.

929 3.2.2.5 Side Muon Range Detector

930 As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within
931 the UA1 magnet. Reconditioned after use in the UA1 [124] and NOMAD [125]
932 experiments, this magnet provides a uniform horizontal magnetic field of 0.2T
933 with an uncertainty of 2×10^{-4} T.

934 Built into the UA1 magnet, the side muon range detector (SMRD)[126] moni-
935 tors high-energy muons which leave the tracking region and permeate through
936 the ECal. It additionally acts as a cosmic muon veto and trigger.

937 3.2.3 The Interactive Neutrino GRID

938 The Interactive Neutrino GRID (INGRID) detector is situated within the same
939 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”
940 position and measures the beam direction, spread, and intensity. The detector
941 was originally designed with 16 identical modules [115] (two modules have since
942 been decommissioned) and a “proton” module. The design of the detector is 14

modules oriented in a cross with length and height $10\text{m} \times 10\text{m}$, as illustrated in Figure 3.14.

Each module is composed of iron sheets interlaced with eleven tracking scintillator planes for a total target mass of 7.1 tonnes per module. The scintillator design is an X-Y pattern of 24 bars in both orientations, where each bar contains wave-length shifting fibers which are connected to multi-pixel photon counters (MPPCs). Each module is encapsulated inside veto planes to aid the rejection of charged particles entering the module.

The proton module is different from the other modules in that it consists of entirely scintillator planes with no iron target. The scintillator bars are also smaller than those used in the other modules to increase the granularity of the detector and improve tracking capabilities. The module sits in the center of the beamline and is designed to give precise measurements of quasi-elastic charged current interactions to evaluate the performance of the Monte Carlo simulation of the beamline.

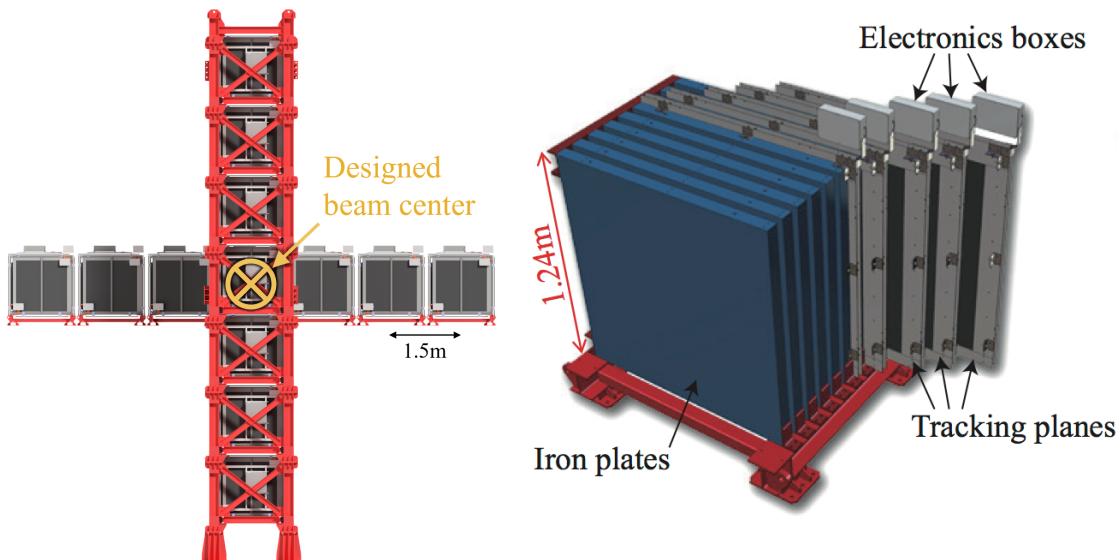


Figure 3.14: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [115].

The INGRID detector can measure the beam direction to an uncertainty of 0.4mrad and the beam centre within a resolution of 10cm [115]. The beam

⁹⁶⁰ direction in both the vertical and horizontal directions is discussed in [127]
⁹⁶¹ and it is found to be in good agreement with the MUMON monitor described
⁹⁶² in subsection 3.2.1.

4

963

964

Bayesian Statistics and Markov Chain Monte Carlo Techniques

965

966 This thesis presents a Bayesian oscillation analysis. To extract the oscillation
967 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter
968 explains the theory of how parameter estimates can be determined using this
969 technique and condenses the material found in the literature [128–131].

970 The oscillation parameter determination presented here is built upon a si-
971 multaneous fit to neutrino beam data in the near detector, beam data at SK, and
972 atmospheric data at SK. In total, there are four oscillation parameters of interest
973 ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP}), two oscillation parameters to which this
974 study will not be sensitive ($\sin^2(\theta_{12})$, Δm_{21}^2) and many nuisance parameters that
975 control the systematic uncertainty models.

976 This analysis uses a Monte Carlo technique to generate a multi-dimensional
977 probability distribution across all of the model parameters used in the fit. To
978 determine an estimate for each parameter, this multi-dimensional object is in-
979 tegrated over all other parameters. This process is called Marginalisation and
980 is described in subsection 4.3.1. Monte Carlo techniques approximate the prob-
981 ability distribution of each parameter within the limit of generating infinite
982 samples. As ever, generating a large number of samples is time and resource-

dependent. Therefore, an MCMC technique is utilised within this analysis to reduce the required number of steps to sufficiently sample the parameter space. This technique is described in further detail in subsection 4.2.1.

DB: Introduce MaCh3 and say what I did on it

4.1 Bayesian Statistics

Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal footing such that a probability model of both data and parameters is required. This is the joint probability distribution $P(D, \vec{\theta})$ and can be described by the prior distribution for model parameters $P(\vec{\theta})$ and the likelihood of the data given the model parameters $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the parameters within the model. For example, if the risk of developing health problems is known to increase with age, the prior distribution would describe the increase. For the purpose of this analysis, the prior distribution is typically the best-fit values taken from external data measurements with a Gaussian uncertainty. The prior distribution can also contain correlations between model parameters. In an analysis using Monte Carlo techniques, the likelihood of measuring some data assuming some set of model parameters is calculated by comparing the Monte Carlo prediction generated at that particular set of model parameters to the data.

It is parameter estimation that is important for this analysis and as such, we apply Bayes' theorem [132] to calculate the probability for each parameter to have a certain value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

1007 The denominator in Equation 4.2 is the integral of the joint probability dis-
1008 tribution over all values of all parameters used within the fit. For brevity, we
1009 say that the posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

1010 For the purposes of this analysis, it is acceptable to neglect the normalisation
1011 term and focus on this proportional relationship.

1012 4.1.1 Application of Prior Knowledge

1013 The posterior distribution is proportional to the prior uncertainty applied to
1014 each parameter, as illustrated by Equation 4.3. This means that it is possible
1015 to change the prior after the posterior distribution has been determined. The
1016 prior uncertainty of a particular parameter can be ‘divided’ out of the posterior
1017 distribution and the resulting distribution can be reweighted using the new
1018 prior uncertainty that is to be applied. The methodology and implementation
1019 of changing the prior follows that described in [133].

1020 An example implementation that is useful for this analysis is the application
1021 of the “reactor constraint”. As discussed in section 2.4, an external constraint
1022 on $\sin^2(\theta_{13})$ is determined from measurements taken from reactor experiments.
1023 However, the sensitivities from just using the T2K and SK samples is equally
1024 as important. Without this technique, two fits would have to be run, doubling
1025 the required resources. Therefore, the key benefit for this analysis is the fact that
1026 only a single ‘fit’ has to be performed and can be used to build the two posterior
1027 distributions of the with and without reactor constraint applied.

1028 4.2 Monte Carlo Simulation

1029 Monte Carlo techniques are used to numerically solve a complex problem that
1030 does not necessarily have an analytical solution. These techniques rely on

1031 building a large ensemble of samples from an unknown distribution and then
1032 using the ensemble to approximate the properties of the distribution.

1033 An example that uses Monte Carlo techniques is to calculate the area under-
1034 neath a curve. For example, take the problem of calculating the area under a
1035 straight line with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can
1036 calculate the area under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte
1037 Carlo techniques, one can calculate the area under this line by throwing many
1038 random values for the x and y components of each sample and then calculating
1039 whether that point falls below the line. The area can then be calculated by the
1040 ratio of points below the line to the total number of samples thrown multiplied by
1041 the total area in which samples were scattered. The study is shown in Figure 4.1
1042 highlights this technique and finds the area under the curve to be 29.9 compared
1043 to an analytical solution of 30.0. The deviation of the numerical to analytical
1044 solution can be attributed to the number of samples used in the study. The
1045 accuracy of the approximation in which the properties of the Monte Carlo samples
1046 replicate those of the desired distribution is dependent on the number of samples
1047 used. Replicating this study with a differing number of Monte Carlo samples
1048 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo
1049 techniques are only accurate within the limit of a high number of samples.

1050 Whilst the above example has an analytical solution, these techniques are just
1051 as applicable to complex solutions. Clearly, any numerical solution is only as
1052 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is
1053 dependent upon the number of samples generated to approximate the properties
1054 of the distribution. Furthermore, if the positions at which the samples are
1055 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique
1056 significantly drops. Given the example in Figure 4.1, if the region in which the
1057 samples are scattered significantly extends passed the region of interest, many
1058 calculations will be calculated but do not add to the ability of the Monte Carlo
1059 technique to achieve the correct result. For instance, any sample evaluated at
1060 a $y \geq 5$ could be removed without affecting the final result. This does bring in

- ¹⁰⁶¹ an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,
¹⁰⁶² one needs to know the distribution beforehand.

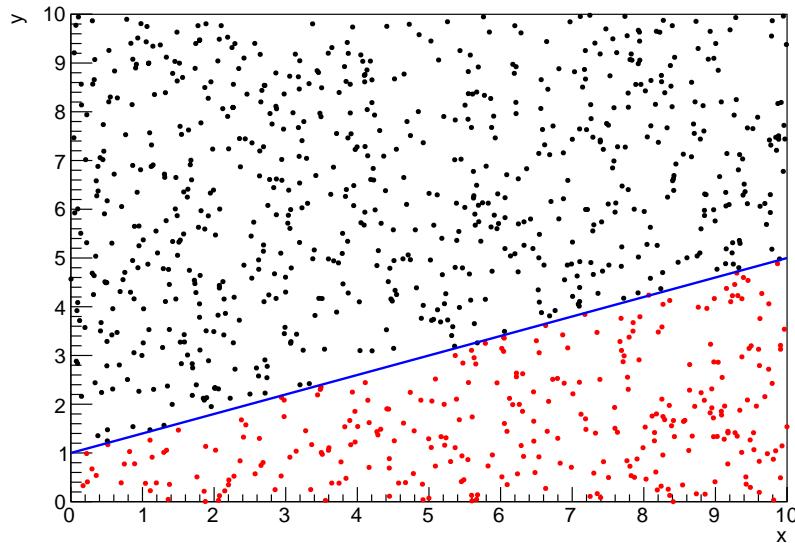


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

¹⁰⁶³ 4.2.1 Markov Chain Monte Carlo

- ¹⁰⁶⁴ This analysis utilises a multi-dimensional probability distribution, with some
¹⁰⁶⁵ dimensions being significantly more constrained than others. These constraints
¹⁰⁶⁶ can be from prior knowledge of parameter distributions from external data or
¹⁰⁶⁷ un-physical regions in which parameters can not exist. To maximise the efficiency
¹⁰⁶⁸ of building the posterior distribution, a Markov Chain Monte Carlo (MCMC)
¹⁰⁶⁹ technique is used. This employs a Markov chain to select the points at which
¹⁰⁷⁰ to sample the posterior distribution. It performs a semi-random stochastic walk
¹⁰⁷¹ through the allowable parameter space. This builds a posterior distribution
¹⁰⁷² which has the property that the density of sampled points is proportional to the
¹⁰⁷³ probability density of that parameter. This means that the samples produced by
¹⁰⁷⁴ this technique are not statistically independent but they will cover the space
¹⁰⁷⁵ of the distribution.

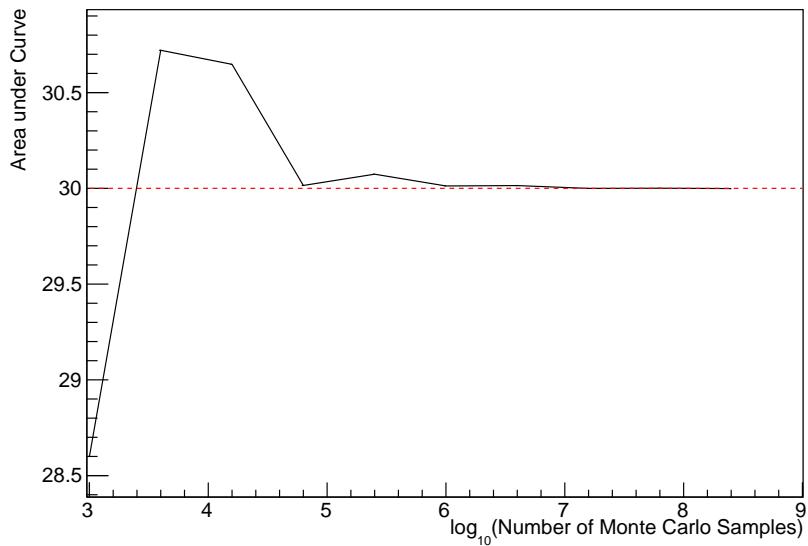


Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1076 A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the
 1077 position of \vec{x}_i . The space in which the Markov chain selects samples is dependent
 1078 upon the total number of parameters utilised within the fit, where a discrete point
 1079 in this space is described by the N-dimensional space \vec{x} . In a perfectly operating
 1080 Markov chain, the position of the next step depends solely on the previous step
 1081 and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving
 1082 the multi-dimensionality of the fit used within this analysis, each step becomes
 1083 correlated with several of the steps preceding itself. Providing the MCMC chain is
 1084 well optimised, it will begin to converge towards a unique stationary distribution.
 1085 The period between the chain's initial starting point and the convergence to the
 1086 unique stationary distribution is colloquially known as the burn-in period. Once
 1087 the chain reaches the stationary distribution, all points sampled after that point
 1088 will look like samples from that distribution.

1089 Further details of the theories underpinning MCMC techniques are discussed
 1090 in [129] but can be summarised by the requirement that the chain satisfies the
 1091 three 'regularity conditions':

- 1092 • Irreducibility: From every position in the parameter space \vec{x} , there must
1093 exist a non-zero probability for every other position in the parameter space
1094 to be reached.
- 1095 • Recurrence: Once the chain arrives at the stationary distribution, every step
1096 following from that position must be samples from the same stationary
1097 distribution.
- 1098 • Aperiodicity: The chain must not repeat the same sequence of steps at any
1099 point throughout the sampling period.

1100 The output of the chain after burn-in (i.e. the sampled points after the chain
1101 has reached the stationary distribution) can be used to approximate the posterior
1102 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1103 stationary distribution found by the chain be the posterior distribution, one
1104 can use the Metropolis-Hastings algorithm. This guides the stochastic process
1105 depending on the likelihood of the current proposed step compared to that
1106 of the previous step.

1107 **4.2.2 Metropolis-Hastings Algorithm**

1108 As a requirement for MCMCs, the Markov chain implemented in this technique
1109 must have a unique stationary distribution that is equivalent to the posterior
1110 distribution. To ensure this requirement and that the regularity conditions are
1111 met, this analysis utilises the Metropolis-Hastings (MH) algorithm [134, 135].
1112 For the i^{th} step in the chain, the MH algorithm determines the position in the
1113 parameter space to which the chain moves to based on the current step, \vec{x}_i , and
1114 the proposed step, \vec{y}_{i+1} . The proposed step is randomly selected from some
1115 proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely on the current step (ie. not
1116 the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the
1117 current step or the proposed step determined by whether the proposed step is

₁₁₁₈ accepted or rejected. To decide if the proposed step is selected, the acceptance
₁₁₁₉ probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_i) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)} \right). \quad (4.4)$$

₁₁₂₀ Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To
₁₁₂₁ simplify this calculation, the proposal function is required to be symmetric such
₁₁₂₂ that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution is
₁₁₂₃ used to throw parameter proposals. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_i) = \min \left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)} \right). \quad (4.5)$$

₁₁₂₄ **DB: Figure out what Giles means**

₁₁₂₅ After calculating this quantity, a random number, β , is generated uniformly
₁₁₂₆ between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_i)$, the proposed step is accepted. Otherwise,
₁₁₂₇ the chain sets the next step equal to the current step. This procedure is repeated
₁₁₂₈ for subsequent steps. This can be interpreted as if the posterior probability
₁₁₂₉ of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq$
₁₁₃₀ $P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true,
₁₁₃₁ ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability
₁₁₃₂ $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped
₁₁₃₃ in any local minima in the potentially non-Gaussian posterior distribution. The
₁₁₃₄ outcome of this technique is that the density of steps taken in a discrete region
₁₁₃₅ is directly proportional to the probability density in that region.

₁₁₃₆ 4.2.3 MCMC Optimisation

₁₁₃₇ As discussed in subsection 4.2.2, the proposal function invoked within the MH
₁₁₃₈ algorithm can take any form and the chain will still converge to the stationary
₁₁₃₉ distribution. At each set of proposed parameter values, a prediction of the same
₁₁₄₀ spectra has to be generated which requires significant computational resources.
₁₁₄₁ Therefore, the number of steps taken before the unique stationary distribution
₁₁₄₂ is found should be minimised as only steps after convergence add information

to the oscillation analysis. Furthermore, the chain should entirely cover the allowable parameter space to ensure that all values have been considered. Tuning the distance that the proposal function jumps between steps on a parameter-by-parameter basis can both minimise the length of the burn-in period and ensure that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently small.

The effect of changing the width of the proposal function is highlighted in Figure 4.3. Three scenarios, each with the same underlying stationary distribution (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between the three scenarios is the width of the proposal function, colloquially known as the ‘step size σ ’. Each scenario starts at an initial parameter value of 10.0 which would be considered an extreme variation. For the case where $\sigma = 0.1$, it is clear to see that the chain takes a long time to reach the expected region of the parameter. This indicates that this chain would have a large burn-in period and does not converge to the stationary distribution until step ~ 500 . Furthermore, whilst the chain does move towards the expected region, each step is significantly correlated with the previous. Considering the case where $\sigma = 5.0$, the chain approaches the expected parameter region almost instantly meaning that the burn-in period is not significant. However, there are clearly large regions of steps where the chain does not move. This is likely due to the chain proposing steps in the tails of the distribution which have a low probability of being accepted. Consequently, this chain would take a significant number of steps to fully span the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can see a relatively small burn-in period of approximately 100 steps. Once the chain reaches the stationary distribution, it moves throughout the expected region of parameter values many times, sufficiently sampling the full parameter region. This example is a single parameter varying across a continuous distribution and does not fully reflect the difficulties in the many-hundred multi-variate parameter distribution used within this analysis. However, it does give a conceptual idea of the importance of selecting the proposal function and associated step size.

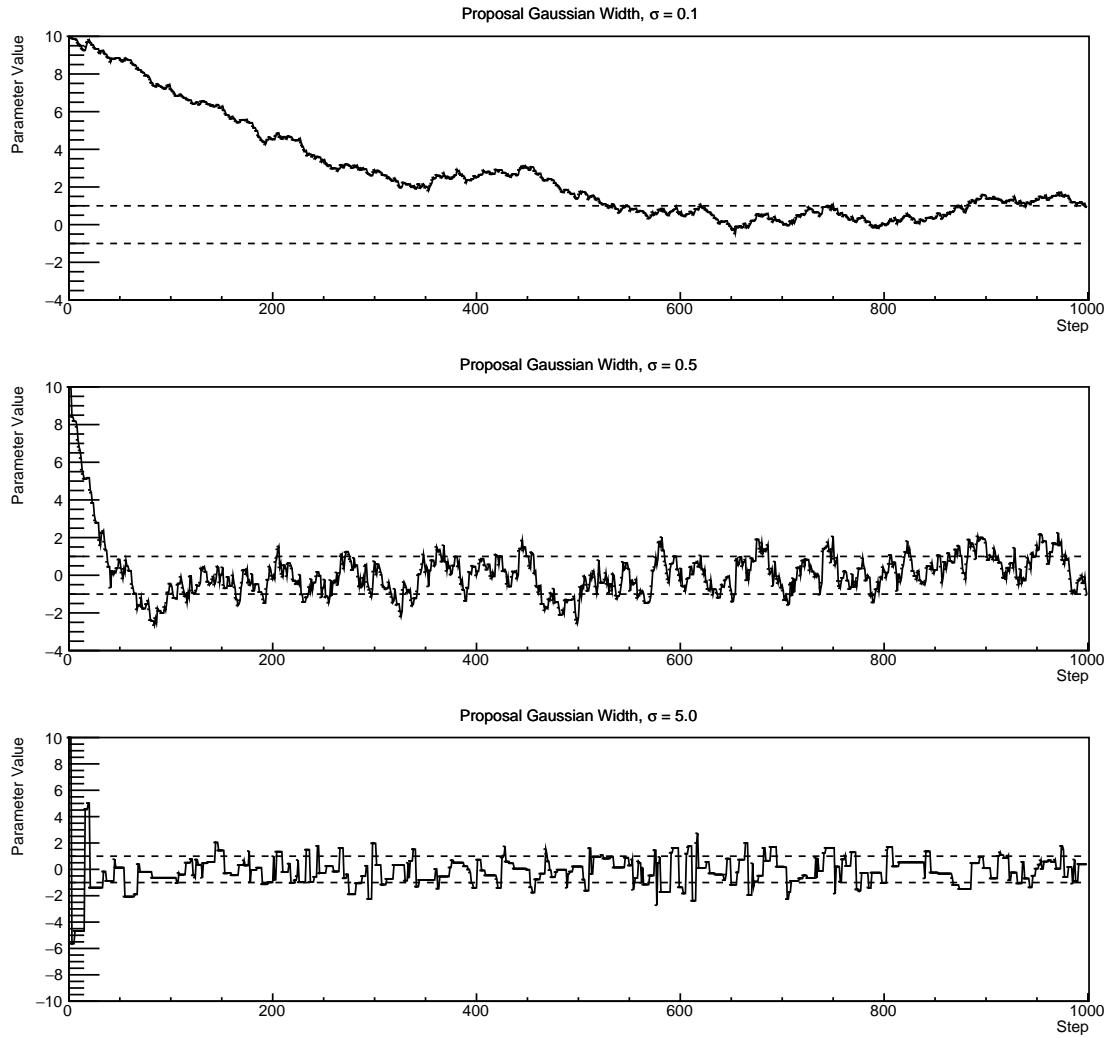


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1172 As discussed, step size tuning directly correlates to the average step accep-
 1173 tance rate. If the step size is too small, many steps will be accepted but the
 1174 chain moves slowly. If the opposite is true, many steps will be rejected as the
 1175 chain proposes steps in the tails of the distribution. Discussion in [136] suggests
 1176 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be
 1177 approximately $\sim 25\%$. An “ideal” step size [136] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

1178 where N_p is the number of parameters included in the MCMC fit. However,
 1179 the complex correlations between systematics mean that some parameters have
 1180 to be hand-tuned and many efforts have been taken to select a set of parameter-
 1181 by-parameter step sizes to approximately reach the ideal acceptance rate.

1182 Figure 4.4 highlights the likelihood as calculated by the fit in DB: [Link to](#)
 1183 [AsimovA Sensitivity Section](#) as a function of the number of steps in each chain. In
 1184 practice, many independent MCMC chains are run simultaneously to parallelise
 1185 the task of performing the fit. This figure overlays the distribution found in each
 1186 chain. As seen, the likelihood decreases from its initial value and converges
 1187 towards a stationary distribution after $\sim 1 \times 10^5$ steps.

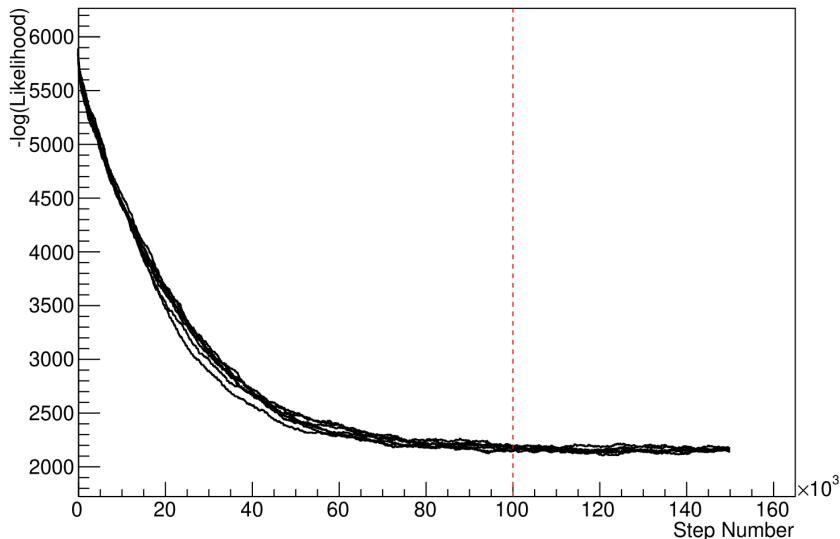


Figure 4.4: The log-likelihood from the fit detailed in DB: [Link to](#) [AsimovA Sensitivity Section](#) as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

1188 Multiple configurations of this analysis have been performed throughout this
 1189 thesis where different samples or systematics have been used. For all of these con-
 1190 figurations, it was found that a burnin period of 1×10^5 was sufficient in all cases.

1191 4.3 Understanding the MCMC Results

1192 The previous sections have described how to generate the posterior probability
1193 distribution using Bayesian MCMC techniques. However, this analysis focuses
1194 on oscillation parameter determination. The posterior distribution output from
1195 the chain is a high-dimension object, with as many dimensions as there are
1196 parameters included in the oscillation analysis. However, this multi-dimensional
1197 object is difficult to conceptualize so parameter estimations are often presented
1198 in one or two-dimensional projections of this probability distribution. To do this,
1199 we invoke the marginalisation technique highlighted in subsection 4.3.1.

1200 4.3.1 Marginalisation

1201 The output of the MCMC chain is a highly dimensional probability distribution
1202 which is very difficult to interpret. From the standpoint of an oscillation analysis
1203 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters
1204 of interest are most relevant. Despite this, the best fit values and uncertainties on
1205 the oscillation parameters of interest should correctly encapsulate the correlations
1206 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).
1207 For this joint beam and atmospheric analysis, the oscillation parameters of
1208 interest are $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . All other parameters (including the
1209 oscillation parameters this fit is insensitive to) are deemed nuisance parameters.
1210 To generate these projections, we rely upon integrating the posterior distribution
1211 over all nuisance parameters. This is called marginalisation. This technique
1212 also explains why it is acceptable to neglect the normalisation constant of the
1213 posterior distribution, which was discussed in section 4.1.

1214 A simple example of the marginalisation technique is to imagine the scenario
1215 where two coins are flipped. To determine the probability that the first coin
1216 returned a ‘head’, the exact result of the second coin flip is disregarded and
1217 simply integrated over. For the parameters of interest, $\vec{\theta}_i$, we can calculate the

₁₂₁₈ marginalised posterior by integrating over the nuisance parameters, $\vec{\theta}_n$. In this
₁₂₁₉ case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}} \quad (4.7)$$

₁₂₂₀ Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and
₁₂₂₁ correlations between the parameters of interest and the nuisance parameters.
₁₂₂₂ In practice, this is simply taking the one or two-dimensional projection of the
₁₂₂₃ multi-dimensional probability distribution.

₁₂₂₄ While in principle an easy solution to a complex problem, correlations be-
₁₂₂₅ tween the interesting and nuisance parameters can bias the marginalised results.
₁₂₂₆ A similar effect is found when the parameters being marginalised over have
₁₂₂₇ non-Gaussian probability distributions. For example, Figure 4.5 highlights the
₁₂₂₈ marginalisation bias in the probability distribution found for a parameter when
₁₂₂₉ requiring a correlated parameter to have a positive parameter value. Due to
₁₂₃₀ the complex nature of the oscillation parameter fit presented in this thesis, there
₁₂₃₁ are correlations occurring between the oscillation parameters of interest and the
₁₂₃₂ other nuisance parameters included in the fit.

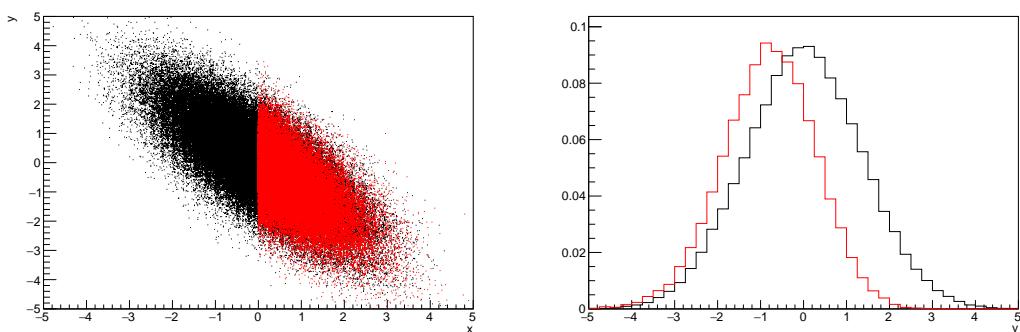


Figure 4.5: Left: The two-dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two-dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

1233 4.3.2 Parameter Estimation and Credible Intervals

1234 The purpose of this analysis is to determine the best fit values for the oscillation
1235 parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$,
1236 $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . The posterior probability density, taken from the output
1237 MCMC chain, is binned in these parameters. The parameter best-fit point is then
1238 taken to be the value that has the highest posterior probability. This is performed
1239 in both one and two-dimensional projections.

1240 However, the single best-fit point in a given parameter is not of much use on
1241 its own. We would also like to determine the uncertainty, or credible interval,
1242 on that best-fit point. The definition of the 1σ credible interval is that we have
1243 68% belief that the parameter is within those bounds. For a more generalised
1244 definition, the credible interval is the region, R , of the posterior distribution that
1245 contains a specific fraction of the total probability, such that

$$\int_R P(\theta|D) d\theta = \alpha \quad (4.8)$$

1246 Where θ is the parameter on which we calculate the credible interval. This
1247 technique then calculates the $\alpha \times 100\%$ credible interval.

1248 In practice, this analysis uses the highest posterior density (HPD) credible in-
1249 tervals which are calculated through the following method. First, the probability
1250 distribution is area-normalised such that it has an integrated area equal to 1.0.
1251 The bins of probability are then summed from the highest to lowest until the sum
1252 exceeds the 1σ level (0.68 in this example). This process is repeated for a range of
1253 credible intervals, notably the 1σ , 2σ and 3σ along with other levels where the
1254 critical values for each level can be found in [75]. This process can be repeated
1255 for the two-dimensional probability distributions by creating two-dimensional
1256 contours of credible intervals rather than a one-dimensional result.

1257 4.3.3 Bayesian Model Comparisons

1258 Due to the matter resonance, this analysis has some sensitivity to the mass
 1259 hierarchy of neutrino states (whether Δm_{32}^2 is positive or negative) and the
 1260 octant of $\sin^2(\theta_{23})$. The Bayesian approach utilised within this analysis gives an
 1261 intuitive method of model comparison by determining which hypothesis is most
 1262 favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal
 1263 hierarchy, NH , and inverted hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

1264 The middle term defines the Bayes factor, $B(NH/IH)$, which is a data-driven
 1265 interpretation of how strong the data prefers one hierarchy to the other. For this
 1266 analysis, equal priors on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) =$
 1267 $P(\vec{\theta}_{IH}) = 0.5$). In practice, the MCMC chain proposes a value of $|\Delta m_{32}^2|$ and
 1268 then applies a 50% probability that the value is sign flipped. Consequently,
 1269 the Bayes factor can be calculated from the ratio of the probability density in
 1270 either hypothesis. This equates to counting the number of steps taken in the
 1271 normal and inverted hierarchies and taking the ratio. The same approach can be
 1272 taken to compare the upper octant (UO) compared to the lower octant (LO)
 1273 hypothesis of $\sin^2(\theta_{23})$.

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [137]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

1274 Whilst the value of the Bayes factor should always be shown, the Jeffreys scale
 1275 [137] (highlighted in Table 4.1) gives an indication of the strength of preference

1276 for one model compared to the other. Other interpretations of the strength of
1277 preference of a model exist, e.g. the Kass and Raferty Scale [138].

1278 4.3.4 Comparison of MCMC Output to Expectation

1279 To ensure the fit is performing well, a best-fit spectrum is produced using the
1280 posterior probability distribution and compared with the data, allowing easy
1281 by-eye comparisons to be made. A simple method of doing this is to perform a
1282 comparison in the fitting parameters (For instance, the reconstructed neutrino
1283 energy and lepton direction for T2K far detector beam samples) of the spectra
1284 generated by the MCMC chain to ‘data’. This ‘data’ could be true data or some
1285 variation of Monte Carlo prediction. This allows easy comparison of the MCMC
1286 probability distribution to the data. To perform this, N steps from the post-burnin
1287 MCMC chain are randomly selected. From these, the Monte Carlo prediction
1288 at each step is generated by reweighting the model parameters to the values
1289 specified at that step. Due to the probability density being directly correlated
1290 with the density of steps in a certain region, parameter values close to the best
1291 fit value are most likely to be selected.

1292 In practice, for each bin of the fitting parameters has a probability distribution
1293 of event rates, with one entry per sampled MCMC step. This distribution is
1294 binned where the bin with the highest probability is selected as the mean and an
1295 error on the width of this probability distribution is calculated using the approach
1296 highlighted in subsection 4.3.2. Consequently, the best fit distribution in the fit
1297 parameter is not necessarily that which would be attained by reweighting the
1298 Monte Carlo prediction to the most probable parameter values.

1299 A similar study can be performed to illustrate the freedom of the model
1300 parameter space prior to the fit. This can be done by throwing parameter values
1301 from the prior uncertainty of each parameter.

5

1302

1303

1304

Simulation, Reconstruction, and Event Reduction

1305 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1306 neutrino spectrum at the far detector is required. This includes modeling the
1307 flux generation, neutrino interactions, and detector effects. All of the simulation
1308 packages required to do this are briefly described in section 5.1. The reconstruc-
1309 tion of neutrino events in the far detector, including the `fitQun` algorithm, is
1310 documented in section 5.2. This also includes data quality checks of the SK-V
1311 data which the author performed for the T2K oscillation analysis presented at the
1312 Neutrino 2020 conference [74]. Finally, section 5.3 describes the steps taken in the
1313 SK detector to trigger on events of interest whilst removing the comparatively
1314 large rate of cosmic ray muon events.

1315 5.1 Simulation

1316 In order to generate a Monte Carlo prediction of the expected event rate at
1317 the far detector, all the processes in the beam and atmospheric fluxes, neutrino
1318 interaction, and detector need to be modeled.

1319 5.1.1 Neutrino Flux

1320 The beamline simulation consists of three distinct parts: the initial hadron
1321 interaction modeled by FLUKA [139], the target station geometry and particle
1322 tracking performed by JNUBEAM, [140, 141] and any hadronic re-interactions
1323 simulated by GCALOR [142]. The primary hadronic interactions are $O(10)\text{GeV}$,
1324 where FLUKA matches external cross-section data better than GCALOR [143].
1325 However, FLUKA is not very adaptable so a small simulation is built to model
1326 the interactions in the target and the output is then passed to JNUBEAM and
1327 GCALOR for propagation. The hadronic interactions are tuned to data from the
1328 NA61/SHINE [144–146] and HARP [147] experiments. The tuning is done by
1329 reweighting the FLUKA and GCALOR predictions to match the external data
1330 multiplicity and cross-section measurements, based on final state particle kine-
1331 matics [143]. The culmination of this simulation package generates the predicted
1332 flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.7.

1333 The atmospheric neutrino flux is simulated by the HKKM model [50, 52]. The
1334 primary cosmic ray flux is tuned to AMS [148] and BESS [149] data assuming
1335 the US-standard atmosphere '76 [150] density profile and includes geomagnetic
1336 field effects. The primary cosmic rays interact to generate pions and muons.
1337 The interaction of these secondary particles to generate neutrinos is handled by
1338 DPMJET-III [151] for energies above 32GeV and JAM [52, 152] for energies below
1339 that value **DB: Question for Giles: Why different generators for above/below**
1340 **32GeV?**. These hadronic interactions are tuned to BESS and L3 data [153, 154]
1341 using the same methodology as the tuning of the beamline simulation. The
1342 energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3
1343 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked
1344 around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions
1345 and kaons can travel further than their vertically-going counterparts resulting
1346 in a larger probability of decaying to neutrinos. The symmetry is broken in
1347 lower-energy neutrinos due to geomagnetic effects, which modify the track of the
1348 primary cosmic rays. Updates to the HKKM model are currently ongoing [155].

1349 5.1.2 Neutrino Interaction

1350 Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [156, 157]
1351 models the interactions of the neutrinos in the detectors. For the purposes of this
1352 analysis, quasi-elastic (QE), meson exchange (MEC), single meson production
1353 (PROD), coherent pion production (COH), and deep inelastic scattering (DIS)
1354 interactions are simulated. These interaction categories can be further broken
1355 down by whether they were propagated via a W^\pm boson in Charged Current
1356 (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC
1357 interactions have a charged lepton in the final state, which can be flavour-tagged
1358 in reconstruction to determine the flavour of the neutrino. In contrast, NC
1359 interactions have a neutrino in the final state so no flavour information can be
1360 determined from the observables left in the detector after an interaction. This
1361 is the reason why neutrinos that interact through NC modes are assumed to
1362 not oscillate within this analysis. Both CC and NC interactions are modeled
1363 for all the above interaction categories, other than MEC interactions which are
1364 only modeled for CC events.

1365 As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of
1366 neutrino interactions around $E_\nu \sim 0.5\text{GeV}$. The NEUT implementation adopts
1367 the Llewellyn Smith [158] model for neutrino-nucleus interactions, where the
1368 nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is
1369 approximated by a spectral-function [159] model that simulates the effects of
1370 Fermi momentum and Pauli blocking. The cross-section of QE interactions is
1371 controlled by vector and axial-vector form factors parameterised by the BBBA05
1372 [160] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data
1373 [161], respectively. NEUT implements the Valencia [162] model to simulate MEC
1374 events, where two nucleons and two holes in the nuclear target are produced
1375 (often called 2p2h interactions).

1376 For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant.
1377 These predominantly produce charged and neutral pions although γ , kaon,

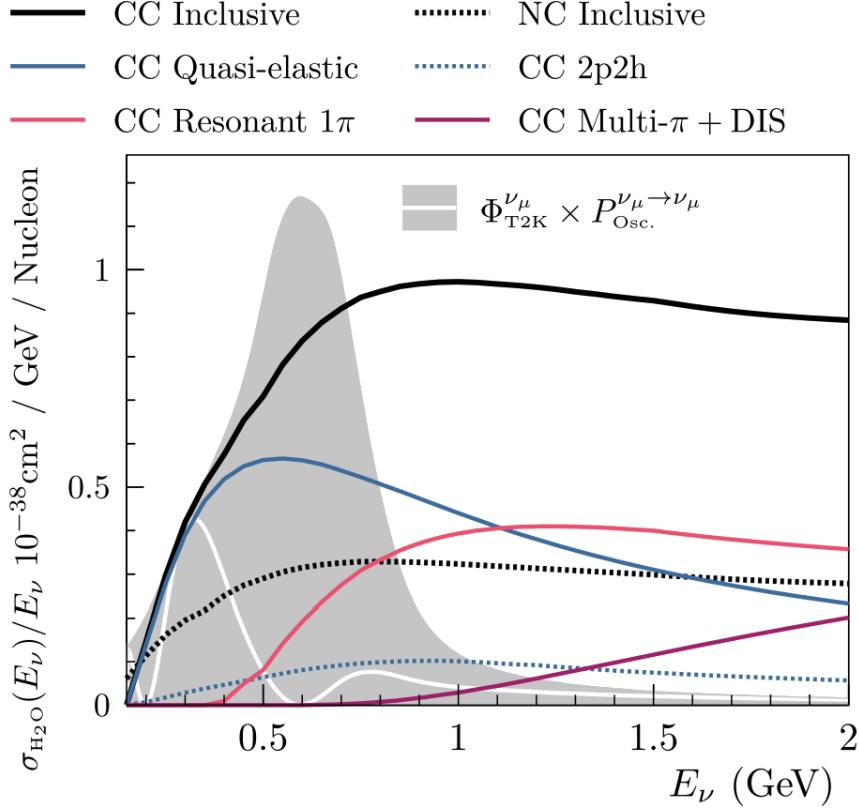


Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [156].

and η production is also considered. To simulate these interactions, the Berger-Sehgal [163] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [164] model to simulate these COH interactions.

DIS and multi- π producing interactions become the most dominant for energies $> O(5)$ GeV. PYTHIA [165] is used to simulate any interaction with invariant mass $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces at least two mesons but has $W < 2\text{GeV}/c^2$, the

¹³⁸⁹ Bronner model is used [166]. Both of these models use Parton distribution
¹³⁹⁰ functions based on the Bodek-Yang model [167–169].

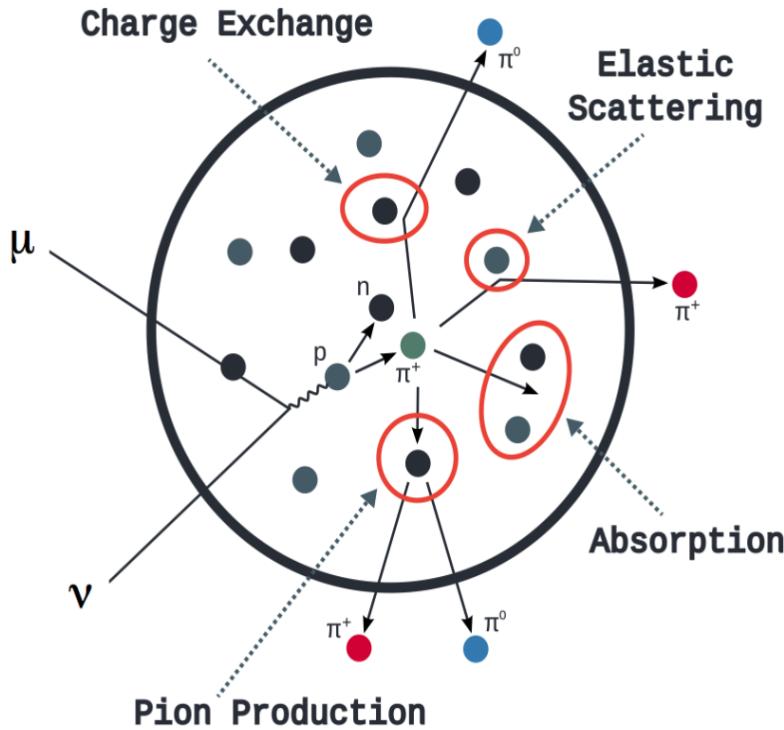


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [170].

¹³⁹¹ Any pion that is produced within the nucleus can re-interact through final
¹³⁹² state interactions before it exits, as illustrated by the scattering, absorption,
¹³⁹³ production, and exchange interactions in Figure 5.2. These re-interactions alter
¹³⁹⁴ the observable particles within the detector. For instance, if the charged pion
¹³⁹⁵ from a CC PROD interaction is absorbed, the observables would mimic a CC QE
¹³⁹⁶ interaction. To simulate these effects, NEUT uses a semi-classical intranuclear
¹³⁹⁷ cascade model [156]. This cascade functions by stepping the pion through the
¹³⁹⁸ nucleus in fixed-length steps equivalent to $dx = R_N/100$, where R_N is the radius
¹³⁹⁹ of the nucleus. At each step, the simulation allows the pion to interact through
¹⁴⁰⁰ scattering, charged exchange, absorption, or production with an interaction-
¹⁴⁰¹ dependent probability calculated from a fit to external data [171]. This cascade
¹⁴⁰² continues until the pion is absorbed or exits the nucleus.

1403 5.1.3 Detector

1404 Once the final state particle kinematics have been determined by NEUT, they
1405 are passed into the detector simulation. The near detectors, ND280 and INGRID,
1406 are simulated using a GEANT4 package [115, 172] to simulate the detector geom-
1407 etry, particle tracking, and energy deposition. The response of the detectors is
1408 simulated using the elecSim package [115]. The far detector simulation is based
1409 upon the original Kamiokande experiment software which uses the GEANT3-based
1410 SKDETSIM [115, 173] package. This simulates the interactions of particles in
1411 the water as well as Cherenkov light production. The water quality and PMT
1412 calibration measurements detailed in subsection 3.1.2 are also used within this
1413 simulation to make accurate predictions of the detector response.

1414 **DB: Make sure the rest of this section flows and is relevant to simulation**

1415 Any event which generates optical photons that occurs in SK will be ob-
1416 served by the PMT array, where each PMT records the time and accumulated
1417 charge. This recorded information is shown in event displays similar to those
1418 illustrated in Figure 5.3. To be useful for physics analyses, this series of PMT
1419 hit information needs to be reconstructed to determine the number and identity
1420 of particles and their kinematics (or track parameters): four-vertex, direction,
1421 and momentum. The reconstruction uses the fact that the charge and timing
1422 distribution of photons generated by a particular particle in an event is dependent
1423 upon its initial kinematics. Electron and muon rings are distinguished by their
1424 “fuzziness”. Muons are heavier and less affected by scattering or showering
1425 meaning they typically produce “crisp” rings. Electrons are more likely to interact
1426 via electromagnetic showering or scattering which results in larger variations
1427 of their direction from the initial direction. Consequently, electrons typically
1428 produce “fuzzier” rings compared to muons.

1429 5.2 Event Reconstruction at SK

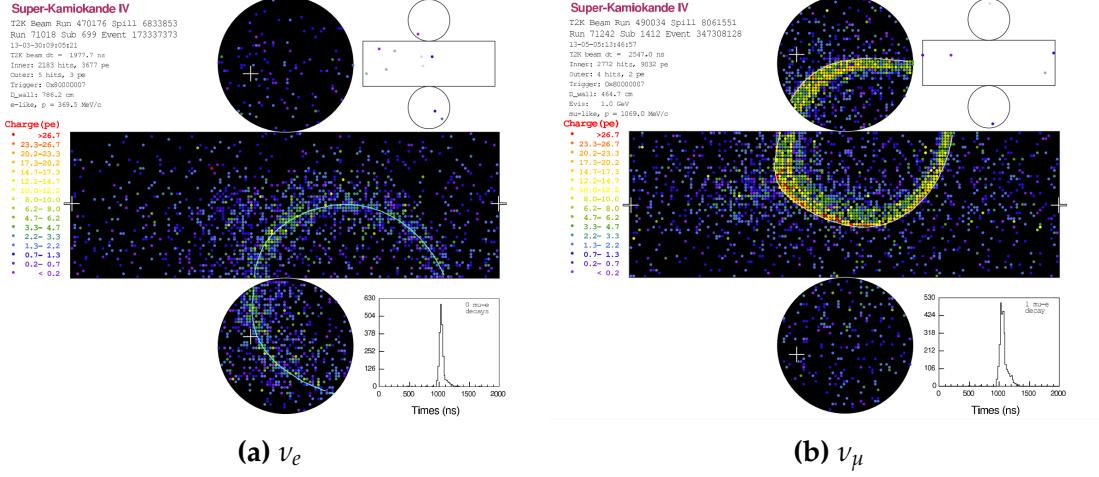


Figure 5.3: Event displays from Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzier” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [174].

DB: Better separate FiTQun and APFit from the below paragraph, and add a little more information about APFit - Hough transform For the purposes of this analysis, the `fitQun` reconstruction algorithm [175] is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by maximising a likelihood function (or minimising a log-likelihood function) which includes information from PMTs which were hit and those that were not hit. The `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [176]. It improves upon the previous `APFit` reconstruction algorithm which has been used for many previous SK analyses. `APFit` fits the vertex from timing information and then fits the momentum and direction of the particle from PMT hits within a 43 deg Cherenkov cone (assuming an ultra-relativistic particle). Due to the additional information being used in the fit, `fitQun` improves both the accuracy of the fit parameters and the rejection of neutral current π^0 events as compared to `APFit` [177, 178].

An event in SK can consist of prompt (or primary) and decay (or secondary) particles. For example, a charged current muon neutrino interaction can gen-

1448 erate two particles that have the potential of generating Cherenkov photons
1449 (assuming the proton is below the Cherenkov threshold): the prompt muon,
1450 and the secondary decay-electron from the muon, approximately $2\mu\text{s}$ later. To
1451 reconstruct all particles within an event, it is divided into time clusters which are
1452 called “subevents”. Subevents after the primary subevent are considered to
1453 be decay electrons.

1454 The main steps of the `fitQun` reconstruction algorithm are:

- 1455 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit
1456 metric based on PMT hit times
- 1457 • **Peak finding:** The initial time of each subevent is determined by clustering
1458 events by time residuals
- 1459 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,
1460 a maximum likelihood technique searches for a single particle generating
1461 light. Electron, muon, charged pion, and proton hypotheses are considered
- 1462 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple
1463 light-producing particles are considered using the same maximum likeli-
1464 hood technique. Electron-like or charged pion-like rings are added until
1465 the likelihood stops improving

1466 To find all the subevents in an event, a vertex goodness metric is calculated
1467 for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{\text{Res}}^i(\vec{x}, t)}{\sigma} \right)^2 \right) \quad (5.1)$$

1468 where

$$T_{\text{Res}}^i(\vec{x}, t) = t^i - t - |R_{\text{PMT}}^i - \vec{x}| / c_n \quad (5.2)$$

1469 is the residual hit time. It is the difference in time between the PMT hit time
1470 t^i , of the i^{th} PMT, and the expected time of the PMT hit if the photon was at

1471 the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in water
 1472 and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1473 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends
 1474 to zero resulting in subevents appearing as spikes in the goodness metric. The
 1475 proposed fit vertex and time are grid-scanned, and the values which maximise
 1476 the goodness metric are selected as the “pre-fit vertex”. Whilst this predicts a
 1477 vertex for use in the clustering algorithm, the final vertex is fit using the higher-
 1478 precision maximum likelihood method described below.

1479 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1480 a function of t to determine the number of subevents. A peak-finding algorithm
 1481 is then used on the goodness metric, requiring the goodness metric to exceed
 1482 some threshold and drop below a reduced threshold before any subsequent
 1483 additional peaks are considered. The thresholds are set such that the rate of
 1484 false peak finding is minimised while still attaining good data to Monte Carlo
 1485 agreement. To improve performance, the pre-fit vertex for each delayed subevent
 1486 is re-calculated after PMT hits from the previous subevent are masked. This
 1487 improves the decay-electron tagging performance. Once all subevents have
 1488 been determined, the time window around each subevent is then defined by the
 1489 earliest and latest time which satisfies $-180 < T_{Res}^i < 800\text{ns}$. The subevents and
 1490 associated time windows are then used as seeds for further reconstruction.

1491 For a given subevent, the `fitQun` algorithm constructs a likelihood based on
 1492 the accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}), \quad (5.3)$$

1493 where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and
 1494 momenta, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the proba-
 1495 bility of the i^{th} tube to not register a hit given the track parameters and particle
 1496 hypothesis. The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$,

1497 represents the probability density function of observing charge q_i and time t_i on
1498 the i^{th} PMT given the specified track parameters and particle hypothesis.

1499 The predicted charge is calculated based on contributions from both the
1500 direct light and the scattered light. The direct light contribution is determined
1501 based on the integration of the Cherenkov photon profile along the track. PMT
1502 angular acceptance, water quality, and calibration measurements discussed in
1503 subsection 3.1.2 are included to accurately predict the charge probability density
1504 at each PMT. The scattered and reflected light is calculated in a similar way,
1505 although it includes a scattering function that depends on the vertex of the
1506 particle and the position of the PMT. The charge likelihood is calculated by
1507 comparing the prediction to the observed charge in the PMT which is tuned
1508 to the PMT simulation.

1509 The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} ,
1510 and time t of the track as well as the particle hypothesis. The expected time
1511 for PMT hits is calculated by assuming unscattered photons being emitted from
1512 the midpoint of the track, S_{mid} ,

$$t_{exp}^i = t + S_{mid}/c + |R_{PMT}^i - \vec{x} - S_{mid}\vec{d}|/c_n, \quad (5.4)$$

1513 where c is the speed of light in a vacuum. The time likelihood is then expressed
1514 in terms of the residual difference between the PMT hit time and the expected
1515 hit time, $t_{Res}^i = t^i - t_{exp}^i$. The particle hypothesis and momentum also affect the
1516 Cherenkov photon distribution. These parameters modify the shape of the time
1517 likelihood density since in reality not all photons are emitted at the midpoint of
1518 the track. As with the charge likelihood, the contributions from both the direct
1519 and scattered light to the time likelihood density are calculated separately, which
1520 are both calculated from particle gun Monte Carlo studies.

1521 The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined
1522 as the best-fit parameters. In practice MINUIT [179] is used to minimise the value
1523 of $-\ln L(\Gamma, \vec{\theta})$. The `fitQun` algorithm considers an electron-like, muon-like, and
1524 charged pion-like hypothesis for events with a single final state particle, denoted

“single-ring events”. The particle’s identity is determined by taking the ratio of the likelihood of each of the hypotheses. For instance, electrons and muons are distinguished by considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison to the reconstructed momentum of the electron hypothesis, as illustrated by Figure 5.4. The coefficients of the discriminator between electron-like and muon-like events are determined from Monte Carlo studies [175]. Similar distributions exist for distinguishing electron-like events from π^0 -like events, and muon-like events from pion-like events. The cuts are defined as,

$$\begin{aligned} \text{Electron/Muon} : & \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 : & \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} : & \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

as taken from [180], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed invariant mass of the two photons emitted from π^0 decay. Typically, the distance between a particular entry in these two-dimensional distributions and the cut-line is termed the PID parameter and is illustrated in Figure 5.5.

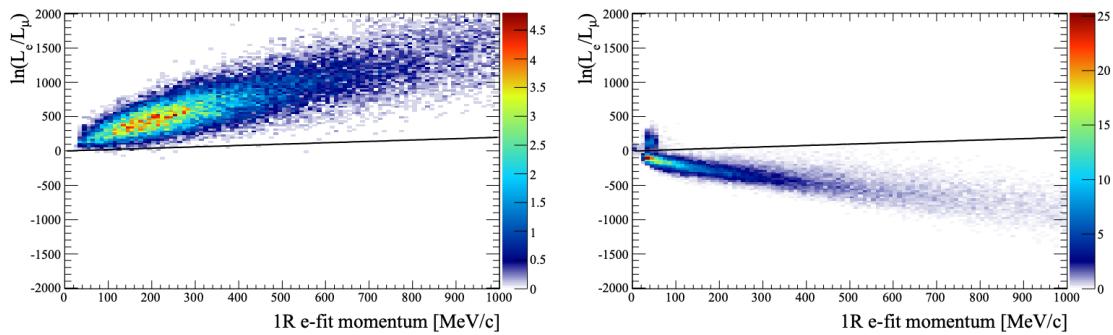


Figure 5.4: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, with coefficients obtained from Monte Carlo studies. Figures from [175].

The `fitQun` algorithm also considers a π^0 hypothesis. To do this, it performs a fit looking for two standard electron-hypothesis tracks which point to the

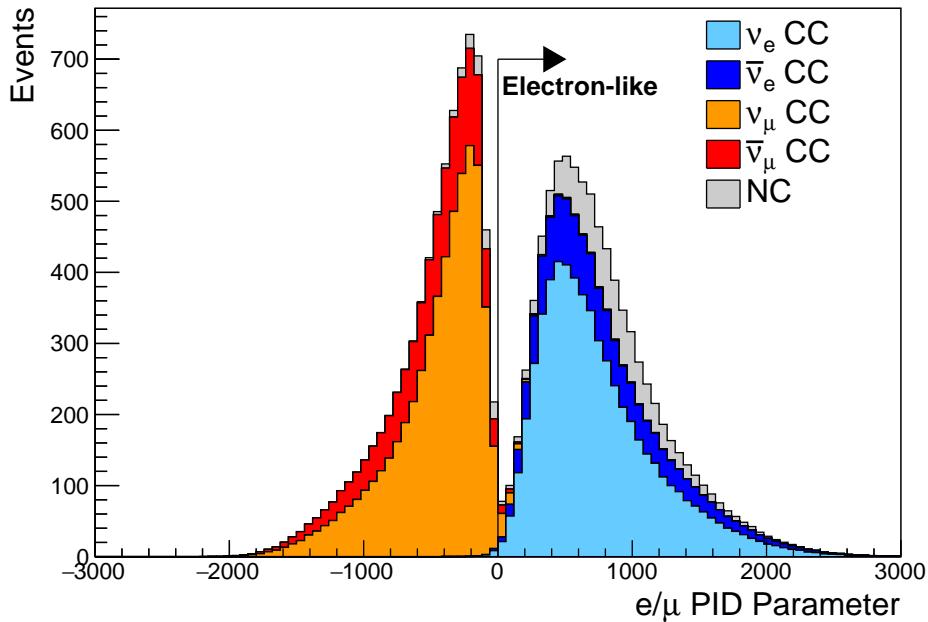


Figure 5.5: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1540 same four-vertex. This assumes the electron tracks are generated from photon-
 1541 conversion so the electron tracks actually appear offset from the proposed π^0
 1542 vertex. For these fits, the conversion length, direction, and momentum of each
 1543 photon are also considered as track parameters which are then fit in the same
 1544 methodology as the standard single-ring hypotheses.

1545 Whilst lower energy events are predominantly single-ring events, higher
 1546 energy neutrino events can generate final states with multiple particles which
 1547 generate Cherenkov photons. These “multi-ring” hypotheses are also considered
 1548 in the `fitQun` algorithm. When calculating the charge likelihood density, the
 1549 predicted charge associated with each ring is calculated separately and then
 1550 summed to calculate the total accumulated charge on each PMT. Similarly,
 1551 the time likelihood for the multi-ring hypothesis is calculated assuming each
 1552 ring is independent. Each track is time-ordered based on the time of flight
 1553 from the center of the track to the PMT and the direct light from any ring

1554 incident on the PMT is assumed to arrive before any scattered light. To reduce
1555 computational resource usage, the multi-ring fits only consider electron-like
1556 and charged pion-like rings as the pion fit can be used as a proxy for a muon
1557 fit due to their similar mass.

1558 **DB: Figure out how pion-like rings are different from the muon hypothesis**
1559 Multi-ring fits proceed by proposing another ring to the previous fit and
1560 then fitting the parameters in the method described above. Typically, multi-ring
1561 fits have the largest likelihood because of the additional degrees of freedom
1562 introduced. A likelihood value is calculated for the n -ring and $(n + 1)$ -ring
1563 hypotheses, where the additional ring is only included if the likelihood value
1564 is above 9.35, based on Monte Carlo studies in [181].

1565 5.2.1 Validation of Reconstruction in SK-V

1566 **DB: Link this into the previous section**

1567 As an example of how the reconstruction depends on the detector conditions,
1568 the author of this thesis assessed the quality of `fitQun` event reconstruction for
1569 SK-V data. The detector systematics used in the T2K-only oscillation analysis
1570 are determined using data-to-Monte Carlo comparisons of the SK-IV data [182].
1571 Due to tank-open maintenance occurring between SK-IV and SK-V, the dark
1572 rate of each PMT was observed to increase in SK-V due to light exposure for a
1573 significant time during the repairs. This increase can be seen in Figure 5.6. Run-10
1574 of the T2K experiment was conducted in the SK-V period, so the consistency
1575 of SK-IV and SK-V data needs to be studied to determine whether the SK-IV-
1576 defined systematics can be applied to the run-10 data. This comparison study was
1577 performed using the stopping muon data set for both the SK-IV and SK-V periods.
1578 This data sample is used due to the high rate of interactions ($O(200)$ events per
1579 hour) as well as having similar energies to muons from CCQE ν_μ interactions from
1580 beam interactions. The rate of cosmic muons does depend on the solar activity
1581 cycle [183] but has been neglected in this comparison study. This is because the
1582 shape of the distributions is most important for the purposes of being compared

1583 to the detector systematics. The SK-IV and SK-V data samples consist of 2398.42
1584 and 626.719 hours of data which equates to 686k and 192k events respectively.
1585 These samples do not correspond to the full data sets of either period but do
1586 contain enough events to be systematics limited rather than statistics limited.

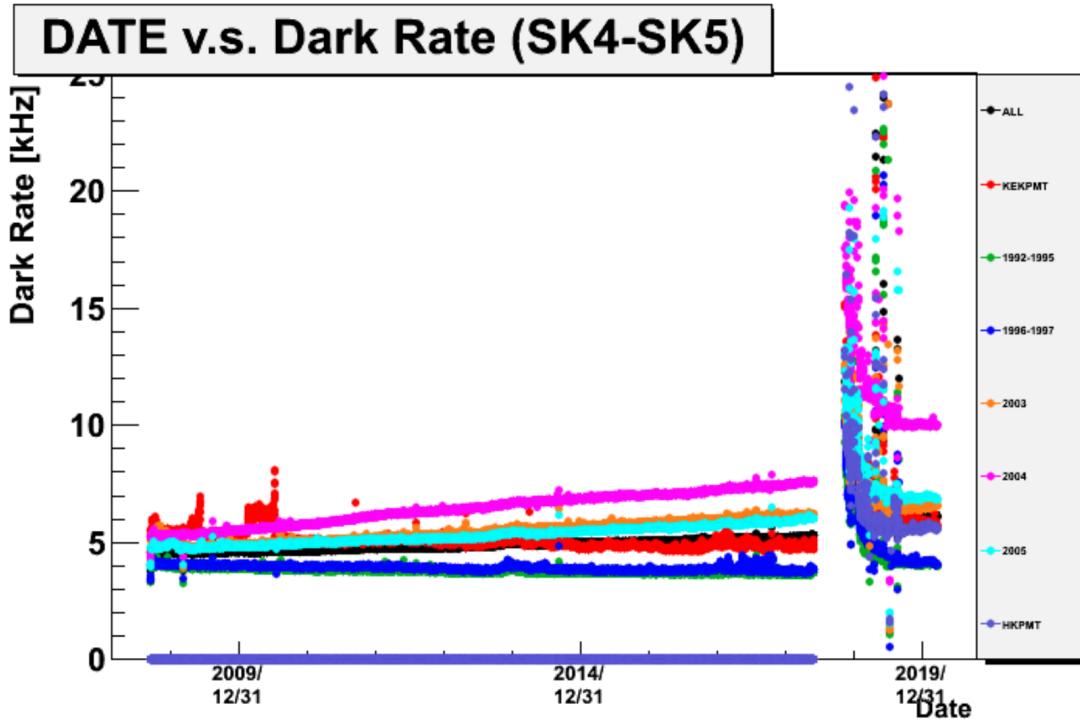


Figure 5.6: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [182].

1587 The predicted charge calculated in the `fitQun` algorithm includes a contribu-
1588 tion from the photoelectron emission due to dark noise. Therefore, the increase
1589 in the SK-V dark rate needs to be accounted for. In practice, the average dark rate
1590 in each SK period is calculated and used as an input in the reconstruction. This is
1591 calculated by averaging the dark rate per run for each period separately, using
1592 the calibration measurements detailed in subsection 3.1.2. The average dark rate
1593 from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The
1594 charges associated with the muon and decay electron subevents are illustrated in
1595 Figure 5.7. The photoelectron emission from dark noise is more significant for

events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in lower-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent is mostly unaffected by the increase in dark rate, whilst the charge associated with the decay-electron is clearly affected.

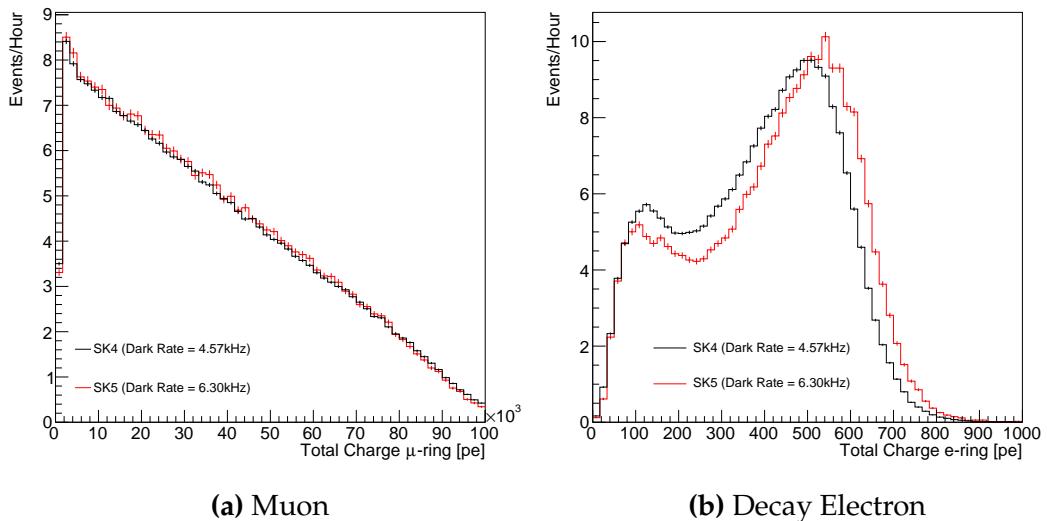


Figure 5.7: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent (left) and the associated decay electron subevent (right).

The energy scale systematic is estimated from data-to-Monte Carlo differences in the stopping muon sample in [184] and found to be 2.1%. To determine the consistency of SK-IV and SK-V with respect to the energy scale systematic, the muon momentum distribution is compared between the two SK periods. As the total number of Cherenkov photons is integrated across the track length, the reconstructed momentum divided by track length (or range) is compared between SK-IV and SK-V as illustrated in Figure 5.8.

The consistency between these muon distributions has been computed in two ways. Firstly, a Gaussian is fit to the peak of each distribution separately, whose mean is found to be $(2.272 \pm 0.003)\text{MeV/cm}$ and $(2.267 \pm 0.006)\text{MeV/cm}$ for SK-IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of the Gaussian fits are consistent with the expected stopping power of a minimum

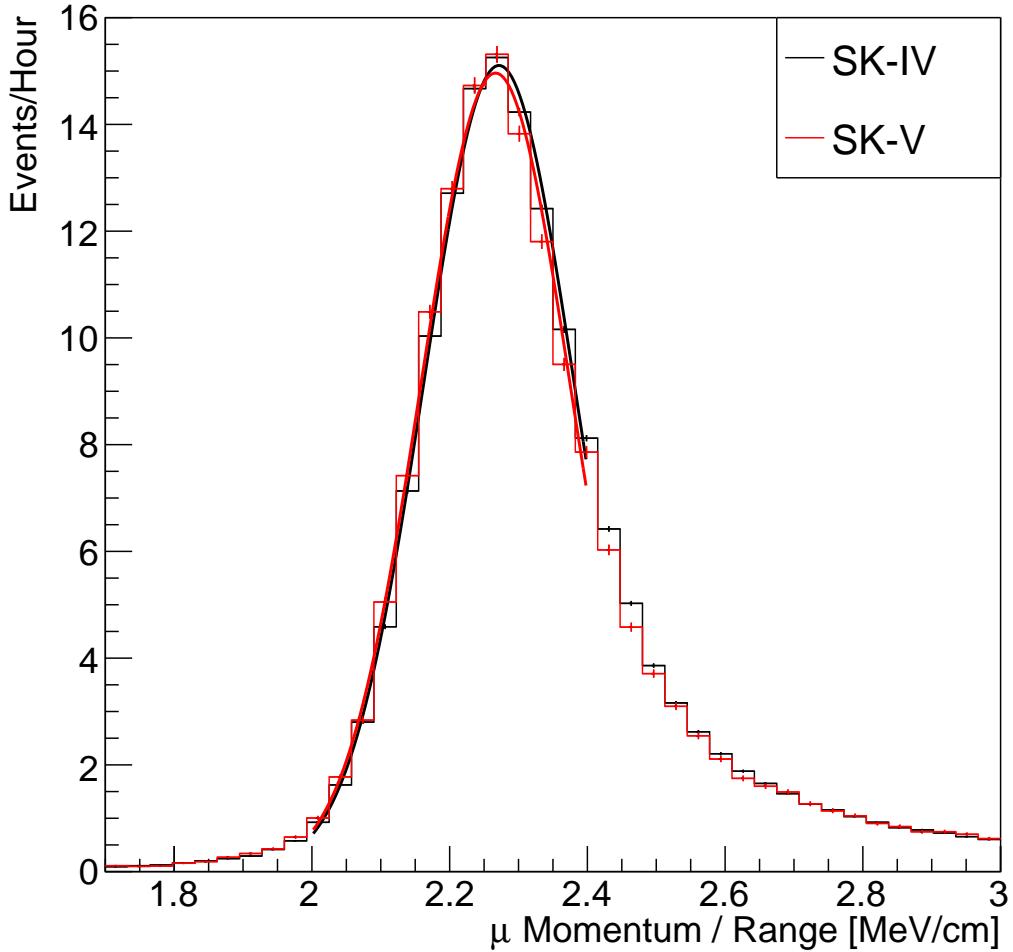


Figure 5.8: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-V (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range $[2.0, 2.4]$ MeV/cm and illustrated as the solid curve.

¹⁶¹³ ionising muon for a target material (water) with $Z/A \sim 0.5$ [185]. The second
¹⁶¹⁴ consistency check is performed by introducing a nuisance parameter, α , which
¹⁶¹⁵ modifies the SK-V distribution. The value of α which minimises the χ^2 value
¹⁶¹⁶ between the SK-IV and SK-V is determined by scanning across a range of values.
¹⁶¹⁷ This is repeated by applying the nuisance parameter as both a multiplicative
¹⁶¹⁸ factor and an additive shift. The χ^2 distributions for different values of α is
¹⁶¹⁹ illustrated in Figure 5.9. The values which minimise the χ^2 are found to be 0.0052

1620 and 1.0024 for the additive and multiplicative implementations, respectively. No
1621 evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic
1622 has been found in the reconstructed momentum distribution of SK-IV and SK-V.

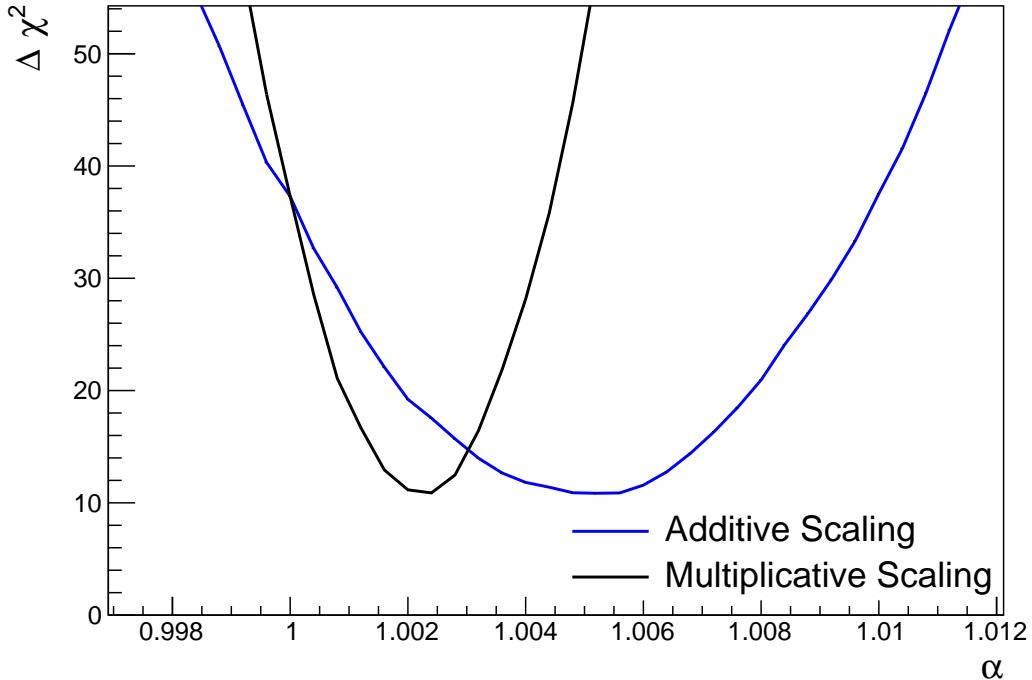


Figure 5.9: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1623 5.3 Event Reduction at SK

1624 In normal data-taking operations, the SK detector observes many background
1625 events alongside the beam and atmospheric neutrino signal events of physics
1626 interest for this thesis. Cosmic ray muons and flasher events, which are the spon-
1627 taneous discharge of a given PMT, contribute the largest amount of background
1628 events in the energy range relevant to this thesis. Therefore the data recorded
1629 is reduced with the aim of removing these background events. The reduction
1630 process is detailed in [54, 85] and briefly summarised below.

Atmospheric neutrino events observed in the SK detector are categorised into three different types of samples: fully contained (FC), partially contained (PC) and up-going muon (Up- μ), using PMT hit signatures in the inner and outer detector (ID and OD, respectively). To identify FC neutrino events, it is required that the neutrino interacts inside the fiducial volume of the ID and that no significant OD activity is observed. For this analysis, an event is defined to be in the fiducial volume provided the event vertex is at least 0.5m away from the ID walls. PC events have the same ID requirements but can have a larger signal present inside the OD. Typically, only high energy muons from ν_μ interactions can penetrate the ID wall. The Up- μ sample contains events where muons are created from neutrino interactions in the OD water or rock below the tank. They then propagate upwards through the detector. Downward-going muons generated from neutrino interactions above the tank are neglected because of the difficulty in separating their signature from the cosmic muon shower background. The sample categories are visually depicted in Figure 5.10.

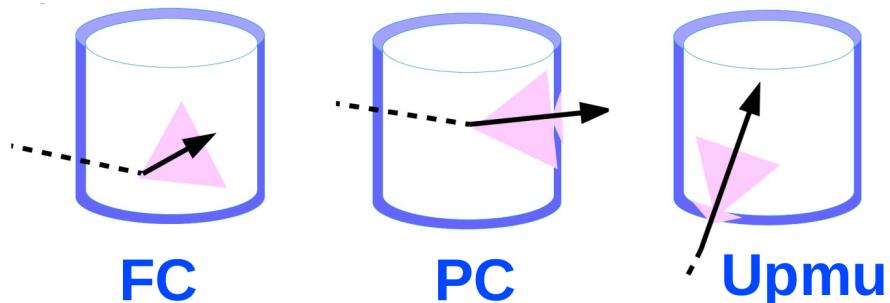


Figure 5.10: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC), and up-going muon (Up- μ) samples included in this analysis.

Based on the event characteristics, as defined by the `fitQun` event reconstruction software, the FC events are categorised by

- **Visible Energy:** equal to the sum of the reconstructed kinetic energy of particles above the Cerenkov threshold for all rings present in the event.
- The purpose is to separate events into sub-GeV and multi-GeV categories.

- **Number of observed Cerenkov rings.** The purpose is to separate single-ring and multi-ring events, where single-ring events predominantly consist of quasi-elastic interactions and multi-ring events are typically resonant pion production or deep inelastic scattering events.
- **Particle identification parameter of the most energetic ring:** A value determined from the maximum likelihood value based on `fitQun`'s electron, muon, or pion hypothesis. The purpose is to separate electron-like and muon-like events.
- **Number of decay electrons:** The purpose is to separate quasi-elastic events (which have one decay electron emitted from the muon decay) and resonant pion production events (which have two decay electrons emitted from the muon and pion).

The PC and Up- μ categories are broken down into “through-going” and “stopping” samples depending on whether the muon leaves the detector. This is because the PC stopping events deposit the entire energy of the interaction into the detector, resulting in better reconstruction. The energy of events that exit the detector has to be estimated, with a typically worse resolution, which introduces much larger systematic uncertainties. Through-going Up- μ samples are further broken down by whether any hadronic showering was observed in the event which typically indicates DIS interactions. The expected neutrino energy for the different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events peak around 0.7GeV and 3GeV respectively, with slightly different peak energies for ν_e and ν_μ oscillation channels. PC and Up- μ are almost entirely comprised of ν_μ events and peak around 7GeV and 100GeV, respectively.

The first two steps in the FC reconstruction remove the majority of cosmic ray muons by requiring a significant amount of ID activity compared to that measured in the OD. Events that pass this cut are typically very high momentum muons or events that leave very little activity in the OD. Consequently, a third reduction step is then applied to select cosmic-ray muons that pass the initial

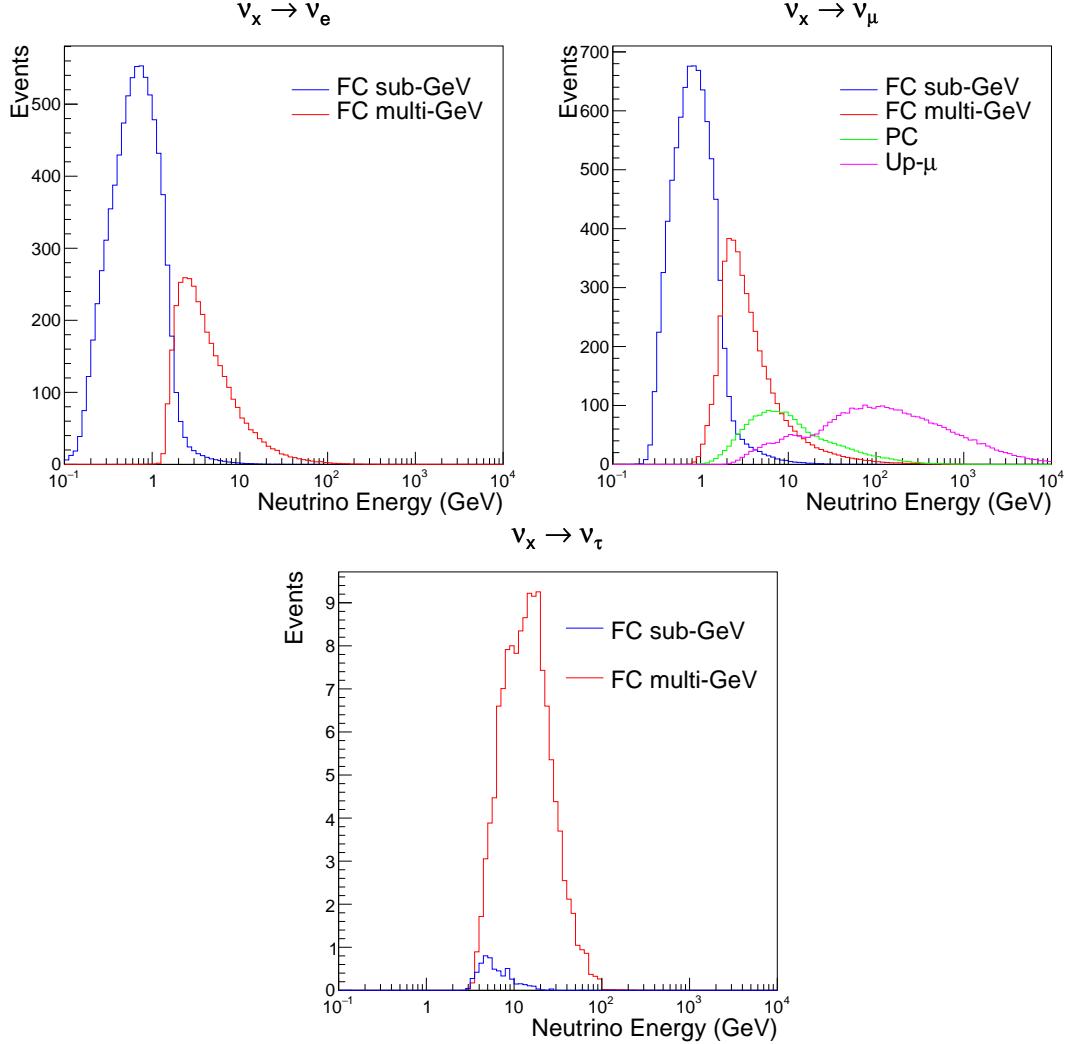


Figure 5.11: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). ν_x represents the flavours of neutrinos produced in the cosmic ray showers (electron and muon). Asimov A oscillation parameters are assumed (given in Table 2.2).

reduction step. A purpose-built cosmic muon fitter is used to determine the entrance (or exit) position of the muon and a cut is applied to OD activity contained within 8m of this position. Flasher events are removed in the fourth reduction step which is based on the close proximity of PMT hits surrounding the PMT producing the flash. Events that pass all these reduction steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses information from the more precise fitter to repeat the previous two steps with

1687 tighter cuts. Muons below the Cherenkov threshold can not generate optical
1688 photons in the ID but the associated decay electron can due to its lower mass.
1689 These are the types of events targeted in the fifth reduction step. The final
1690 cuts require the event vertex to be within the fiducial volume (0.5m from the
1691 wall although the nominal distance is 2.0m), visible energy $E_{vis} > 30\text{MeV}$ and
1692 fewer than 16 hits within the higher energy OD cluster. The culmination of
1693 the fully contained reduction results in 8.09 events/day in the nominal fiducial
1694 volume [81]. The uncertainty in the reconstruction is calculated by comparing
1695 Monte Carlo prediction to data. The largest discrepancy is found to be 1.3%
1696 in the fourth reduction step.

1697 The PC and Up- μ events are processed through their own reduction processes
1698 detailed in [54]. Both of these samples are reconstructed with the APFit algorithm
1699 rather than fiTQun. This is because the efficiency of reconstructing events that
1700 leave the detector has not been sufficiently studied for reliable systematic uncer-
1701 tainties with fiTQun. The PC and Up- μ samples acquire events at approximately
1702 0.66 and 1.44 events/day.

1703 Beam neutrinos events undergo the same reduction steps as FC events and are
1704 then subject to further cuts [186]. The GPS system that links the timing between
1705 the beam facility and SK needs to be operating correctly and there should be no
1706 activity within the detector in the previous $100\mu\text{s}$ before the trigger. The events
1707 then need to triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1708 The beam neutrino samples are not split by visible energy since their energy
1709 range is smaller than the atmospheric neutrino events. Following the T2K
1710 analysis in [74], only single-ring beam neutrino events are considered. Similar to
1711 atmospheric event selection, the number of decay electrons is used as a proxy for
1712 distinguishing CCQE and CCRES events. The expected neutrino energy, broken
1713 down by the number of decay electrons, is given in Figure 5.12.

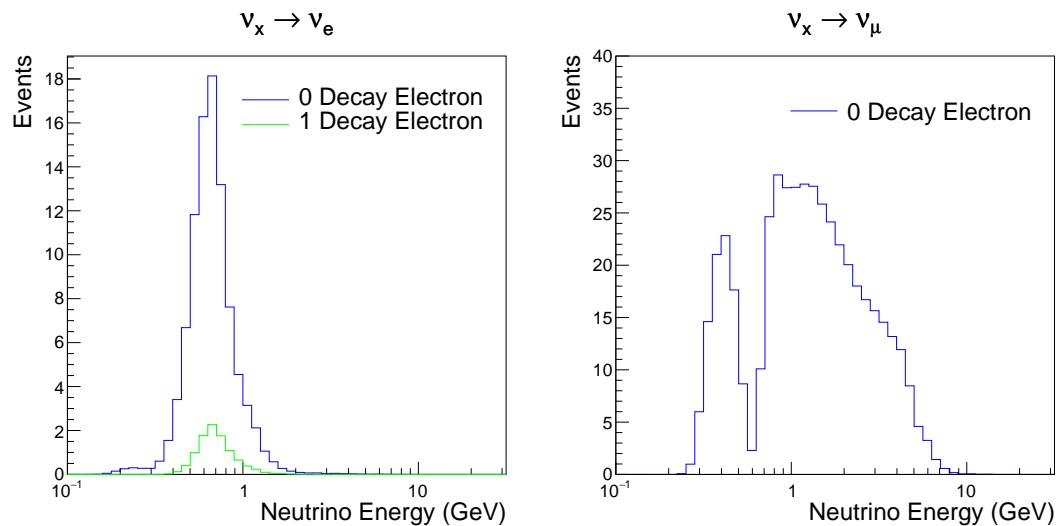


Figure 5.12: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

6

1714

1715

Sample Selections and Systematics

1716 The oscillation analysis presented within this thesis is built upon a simultaneous
1717 fit to atmospheric data at SK, neutrino beam data in the near detector, and
1718 beam data measured at SK. The definitions of these samples are documented
1719 in section 6.1, section 6.2, and section 6.3, respectively. The data collected and
1720 used within this analysis is detailed in Table 6.1. The near and far detector data
1721 corresponds to T2K runs 2-9 and runs 1-10, respectively. The accumulated POT
1722 and beam power for runs 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

1723 The difference in POT recorded at the near and far detector is due to the
1724 difference in downtime. The SK detector is very stable with almost 100% of
1725 data recorded during beam operation. Due to various technical and operational

¹⁷²⁶ issues, the downtime of the near detector is significantly higher due to its more
¹⁷²⁷ complex design and operating requirements.

¹⁷²⁸ The systematic parameters invoked within the flux, detector, and interaction
¹⁷²⁹ models used within this analysis are documented in section 6.4. The standard
¹⁷³⁰ configuration of the joint beam and atmospheric data fit utilises far detector sys-
¹⁷³¹ tematics provided in the official inputs from the two experiments. Additionally,
¹⁷³² a correlated detector model which fits the parameters used in sample selections
¹⁷³³ to data has been developed and documented in subsection 6.4.5.

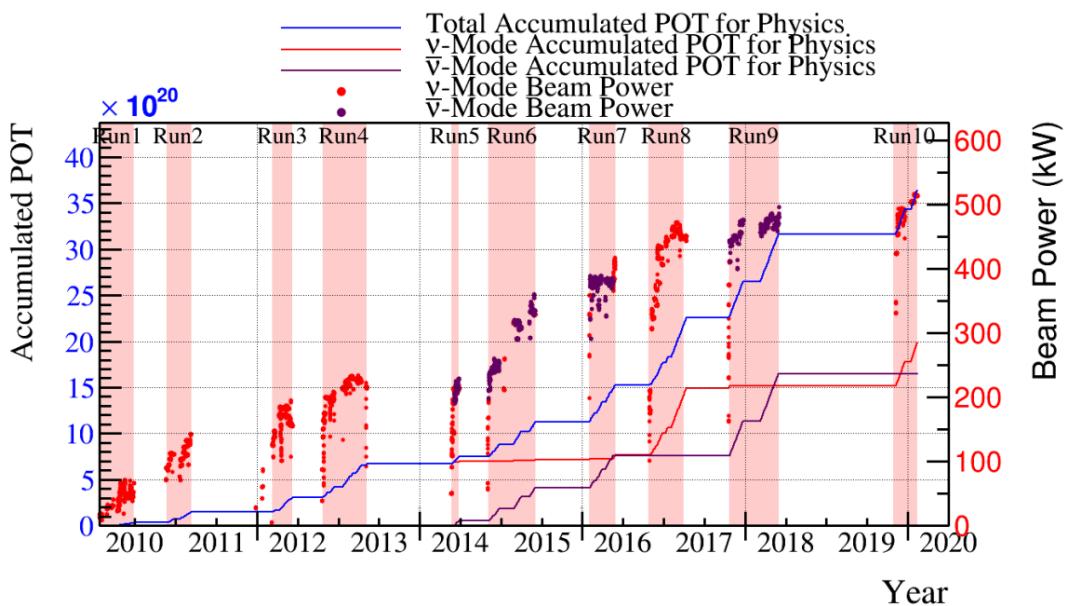


Figure 6.1: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

¹⁷³⁴ DB: Say that I have merged T2K and SK which hadn't previously been done

1735 6.1 Atmospheric Samples

1736 The atmospheric event selection follows the official SK-IV analysis presented
1737 in [85] and is documented below. The Monte Carlo prediction used within this
1738 analysis corresponds to 500 years worth of neutrino events, which is scaled down
1739 to match the SK-IV livetime of 3244.4 days.

1740 The fully contained (FC), partially contained (PC), and upward going muon
1741 events ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further
1742 broken down into different samples based on reconstruction information. This
1743 section details the samples used within this oscillation analysis, alongside the
1744 chosen binning.

1745 FC events are first separated by the visible energy deposited within the
1746 detector. This is calculated as the sum of the reconstructed kinetic energy
1747 above the Cherenkov threshold for all rings present in the event. Events are
1748 separated by whether they were above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates
1749 “subGeV” and “multiGeV” events. Typically, lower energy events consist of
1750 charged current quasi-elastic (CCQE) interactions which are better understood
1751 and simpler to reconstruct resulting in smaller systematic uncertainties. Events
1752 are further separated by the number of rings associated with the event due to
1753 similar reasoning. As the oscillation probability is dependant upon the flavour
1754 of neutrino, electron and muon events are separated using a similar likelihood
1755 method to that discussed in section 5.2. To reduce computational resources
1756 required for the reconstruction, only electron and pion hypotheses are considered
1757 so this separation cut depends on the ratio of the electron to pion likelihoods,
1758 $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1759 Charged current resonant pion production (CCRES) interactions generate a final-
1760 state pion. This can decay, mostly likely through a muon, into a decay electron.
1761 Therefore any electron-like event with one decay electron or muon-like event
1762 with two decay electrons was most likely produced by a CCRES interaction.
1763 Consequently, the number of decay electrons can be used to distinguish CCQE

¹⁷⁶⁴ and CCRES interaction modes. Ultimately, FC subGeV events are separated
¹⁷⁶⁵ into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

¹⁷⁶⁶ In addition to the cuts discussed above, multiGeV samples also have addi-
¹⁷⁶⁷ tional cuts to separate samples which target neutrino and antineutrino events.
¹⁷⁶⁸ As discussed in section 7.1, the matter resonance only occurs for neutrinos in the
¹⁷⁶⁹ normal hierarchy and antineutrinos in the inverted mass hierarchy. Therefore,
¹⁷⁷⁰ having flavour-enriched samples aids in the determination of the mass hierarchy.
¹⁷⁷¹ For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

¹⁷⁷² The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be captured by an
¹⁷⁷³ oxygen nucleus than the π^+ from ν_e interactions [187]. These pions then decay,
¹⁷⁷⁴ mostly through muons, to electrons. Therefore the number of tagged decay
¹⁷⁷⁵ electrons associated with an event gives an indication of whether the interaction
¹⁷⁷⁶ was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
¹⁷⁷⁷ The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
¹⁷⁷⁸ where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with
¹⁷⁷⁹ only 7% antineutrino background, the rest consisting of NC backgrounds.

1780 The number of decay electrons discriminator works reasonably well for single-
 1781 ring events. However, this is not the case for multi-ring events. A multiGeV
 1782 multiring electron-like (MME) likelihood cut was introduced in [188, 189]. This
 1783 is a two-stage likelihood selection cut. Four observables are used in the first
 1784 likelihood cut to distinguish $CC\nu_e$ and $CC\bar{\nu}_e$ events from background:

- 1785 • The number of decay electrons
 1786 • The maximum distance between the vertex of the neutrino and the decay
 1787 electrons
 1788 • The energy deposited by the highest energy ring
 1789 • The particle identification of that highest energy ring

1790 Background events consist of $CC\nu_\mu$ and NC interactions. Typically, the
 1791 majority of the energy in these background events is carried by the hadronic
 1792 system. Additionally, muons tend to travel further than the pions from $CC\nu_e$
 1793 before decaying. Thus, the parameters used within the likelihood cut target these
 1794 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring e -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring e -like events with one or more decay electrons
MultiGeV-mulike	Single ring μ -like events
MultiRing-elike-nue	Two or more ring events with leading energy e -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy e -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy μ -like ring and only requires $E_{vis} > 0.6\text{GeV}$ DB: Why is this not }1.33\text{GeV
MultiRing-Other1	Two or more ring events with leading energy e -like ring and failed the MME likelihood cut

Table 6.3: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1795 Neutrino and antineutrino events are then separated by a second likelihood
 1796 method ($\nu/\bar{\nu}$ separation) detailed in [59]. This uses the number of decay electrons,

1797 the number of reconstructed rings, and the event’s transverse momentum. The
1798 last two parameters are used because higher-energy samples tend to have more
1799 pions produced above the Cherenkov threshold which results in more rings
1800 compared to an antineutrino interaction. Furthermore, the angular distribution
1801 also tends to be more forward peaked in antineutrino interactions as compared
1802 to neutrino interactions [85]. These FC multiGeV sample definitions are de-
1803 tailed in Table 6.3.

1804 The PC and up- μ samples are split by the amount of energy deposited within
1805 the outer detector, into “stopping” and “through-going” samples. If an event
1806 leaves the detector, the energy it takes with it has to be estimated which increases
1807 the systematic uncertainty compared to events entirely contained within the
1808 inner detector. This estimation is particularly poor at high energies, thus the
1809 up- μ through-going events are not binned in reconstructed momentum. The
1810 through-going up- μ are further separated by the presence of any electromagnetic
1811 showering in the event, as the assumption of non-showering muon does not give
1812 reliable reconstruction for these types of events [54]. In total, 13 FC, 2 PC, and
1813 3 up- μ atmospheric samples are included within this analysis.

1814 The atmospheric samples are binned in direct observables: reconstructed
1815 lepton momentum and direction, as given by Table A.1. The distribution of
1816 the reconstructed lepton momentum (for samples that only have one bin in
1817 reconstructed zenith angle) and reconstructed direction for each atmospheric
1818 sample used within this analysis is illustrated in Figure 6.2. The by-mode
1819 breakdown of each of the atmospheric samples is given in Appendix A.

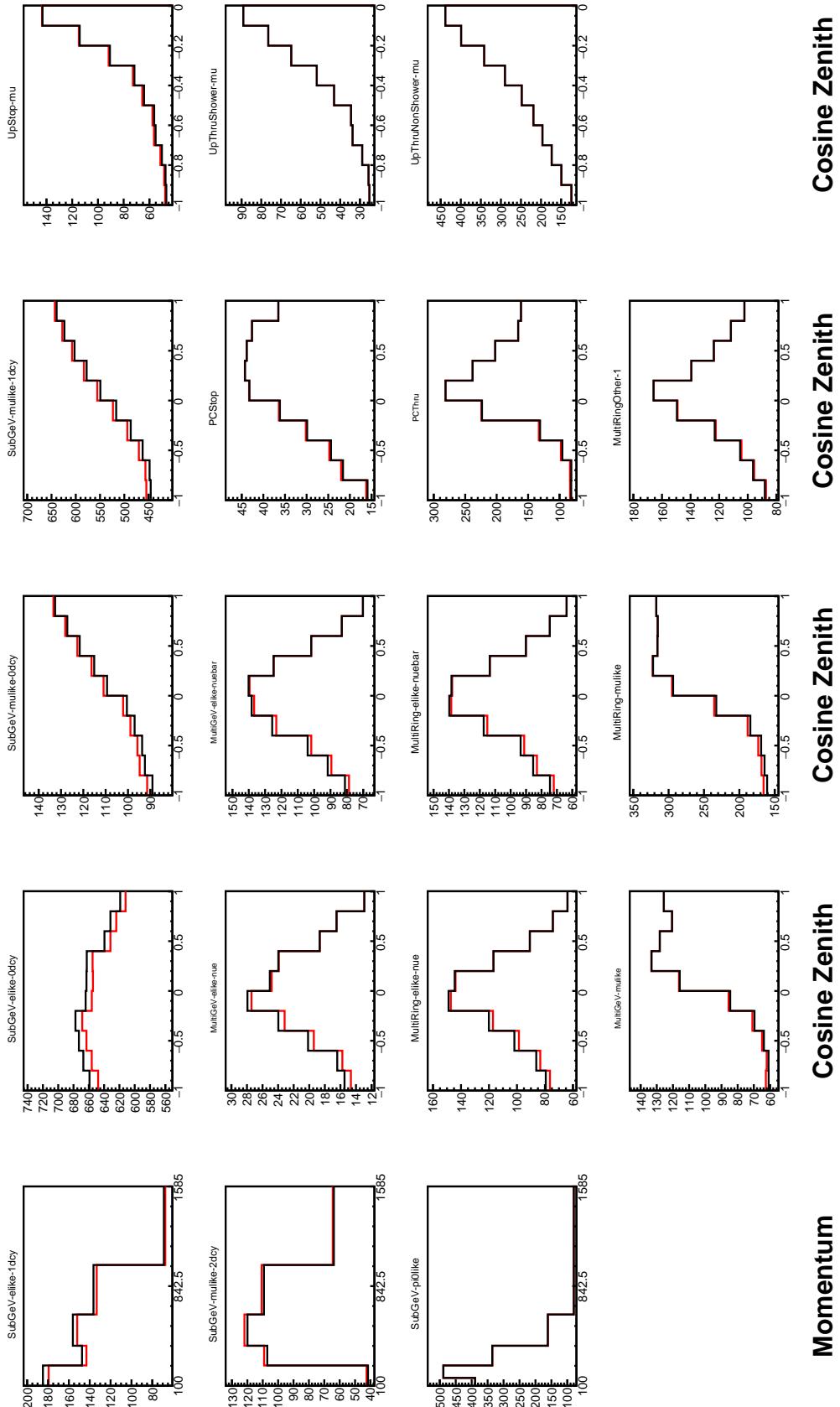


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThru	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Each row sums to 100% by definition. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

1820 6.2 Near Detector Beam Samples

1821 The near detector sample selections are documented in detail within [190] and
1822 summarised below. Samples are selected based upon which of the two Fine
1823 Grained Detector (FGD) the vertex is reconstructed in as well as the operating
1824 mode of the beam: FHC or RHC. Wrong-sign neutrino background samples are
1825 considered in the RHC mode in order to add additional constraints on model
1826 parameters. Samples from the wrong-sign component of the FHC beam mode
1827 are not included as they are statistically insignificant compared to those samples
1828 already listed.

1829 The reconstruction algorithm uses a clustering algorithm to group hits within
1830 the TPC. It then adds information from the upstream FGD to form a track
1831 that passes through both sub-detectors. In FHC(RHC), the highest momentum
1832 negative(positive) curvature track is defined as the muon candidate. Before
1833 being assigned a sample, these candidate muon events must pass CC-inclusive
1834 cuts, as defined in [191]:

- 1835 • Event Timing: The DAQ must be operational and the event must occur
1836 within the expected beam time window consistent with the beam spill
- 1837 • TPC Requirement: The muon-candidate track path must intercept one or
1838 more TPCs
- 1839 • Fiducial volume: The event must originate from within the fiducial volume
1840 defined in [192].
- 1841 • Upstream Background: Remove events that have muon tracks that originate
1842 upstream of the FGDs by requiring no high-momentum tracks within
1843 150mm upstream of the candidate vertex. Additionally, events that occur
1844 within the downstream FGD are vetoed if a secondary track starts within
1845 the upstream FGD

- 1846 • Broken track removal: All candidates where the muon candidate is broken
1847 in two are removed

- 1848 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-
1849 like events, from electron-like or proton-like, using a likelihood cut

1850 In addition to these cuts, RHC neutrino events also have to undergo the
1851 following cuts to aid in the separation of neutrino and antineutrino [193]:

- 1852 • TPC Requirement: The track path must intercept TPC2

- 1853 • Positive Track: The highest momentum track must have a positive recon-
1854 structed charge

- 1855 • TPC1 Veto: Remove any events originating upstream of TPC1

1856 Once all CC-inclusive events have been determined, they are further split
1857 by pion multiplicity: CC0 π , CC1 π , and CCOther. This breakdown targets the
1858 specific interaction modes CCQE, CCRES, and other CC background interactions,
1859 respectively. Pions in the TPCs and FGDs are selected by requiring a second track
1860 to be observed, which is separate from the muon track and is in the same beam
1861 spill window and sub-detector. If the pion originated within a FGD, it must also
1862 pass through the sequential downstream TPC (TPC2 for FGD1, TPC3 for FGD2).

1863 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

1864 **DB: Understand pion cuts at ND**

- 1865 • ν_μ CC0 π **Selection:** No electrons in TPC and no charged pions or decay
1866 electrons within the TPC or FGD

- 1867 • ν_μ CC1 π **Selection:** Exactly one charged pion in either the TPC or FGD,
1868 where the number of charged pions in the FGD is equal to the number of
1869 decay electrons

- 1870 • ν_μ CCOther **Selection:** All events which are not classified into the above
1871 two selections.

Counting the three selections for each FGD in FHC and RHC running, including the wrong-sign background in RHC, 18 near detector samples are used within this analysis. These samples are binned in reconstructed lepton momentum (illustrated in Figure 6.3) and direction with respect to the beam. The binning is chosen such that each event has at least 20 Monte Carlo events in each bin [192]. This is to ensure that the bins are coarse enough to ensure the reduction of statistical errors, whilst also being fine enough to sample the high-resolution peak regions. The exact binning is detailed in [192].

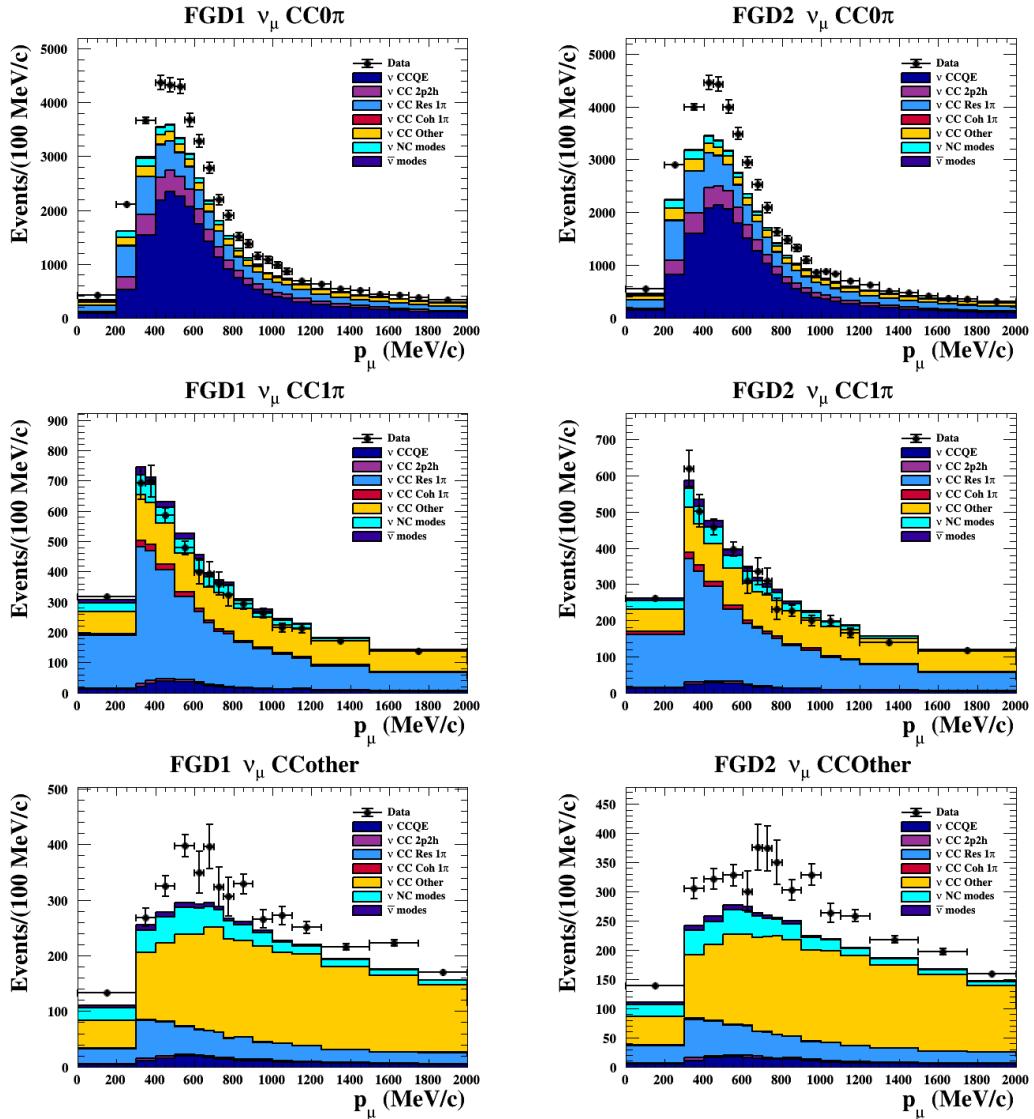


Figure 6.3: The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $CC\nu_\mu 0\pi$, $CC\nu_\mu 1\pi$ and $CC\nu_\mu$ Other categories. Figures taken from [190].

1880 6.3 Far Detector Beam Samples

1881 The beam neutrino events which occur at the SK detector, which pass the
 1882 reduction cuts detailed in section 5.3, are separated based on whether the beam
 1883 was operating in FHC or RHC mode. The events are then separated into three
 1884 samples: electron-like ($1Re$), muon-like ($1R\mu$), and $CC1\pi^+$ -like ($1Re1de$) which
 1885 are observed as electron-like events with an associated decay electron [182].
 1886 As discussed in section 6.1, positively charged pions emitted from neutrino
 1887 interactions are more likely to produce decay electrons than negatively charged
 1888 pions. Consequently, the $CC1\pi^+$ -like sample is only selected when the beam is
 1889 operating in FHC mode. Therefore, five beam samples measured at SK are
 1890 used in this analysis.

1891 The fiducial volume definition for beam samples is slightly different from that
 1892 used for the atmospheric samples. It uses both the distance to the closest wall
 1893 (`dWall`) and the distance to the wall along the trajectory of the particle (`toWall`).
 1894 This allows events that originate close to the wall but are facing into the tank to be
 1895 included within the analysis, which would have otherwise been removed. These
 1896 additional events are beneficial for a statistics-limited experiment. The exact
 1897 cut values for both `dWall` and `toWall` are different for each of the three types of
 1898 sample and are optimised based on T2K sensitivity to δ_{CP} [180, 194]. They are:

1899 **1Re event selection** For an event to be classified as a $1Re$ -like, the event must sat-
 1900 isfy:

- 1901 • Fully-contained and have $dWall > 80\text{cm}$ and $toWall > 170\text{cm}$
- 1902 • Total of one ring which is reconstructed as electron-like with reconstructed
 1903 momentum $P_e > 100\text{MeV}$
- 1904 • Zero decay electrons are associated with the event
- 1905 • Passes π^0 rejection cut discussed in section 5.2

1906 The zero decay electron cut removes non-CCQE interactions and the π^0
 1907 rejection cut is designed to remove neutral current π^0 background events which
 1908 can be easily reconstructed as 1Re-like events.

1909 The zero decay electron cut removes non-CCQE interactions and the π^0
 1910 rejection cut is designed to remove neutral current π^0 background events which
 1911 can be easily reconstructed as 1Re-like events.

1912 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re
 1913 sample. The only differences are that the `dWall` and `toWall` criteria are changed
 1914 to $> 50\text{cm}$ and $> 270\text{cm}$, respectively, and exactly one decay electron is required
 1915 from the π^+ decay.

1916 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

- 1917 • Fully-contained and have `dWall` $> 50\text{cm}$ and `toWall` $> 250\text{cm}$
- 1918 • Total of one ring which is reconstructed as muon-like with reconstructed
 1919 momentum $P_\mu > 200\text{MeV}$
- 1920 • Fewer than two decay electrons are associated with the event
- 1921 • Passes π^+ rejection cut discussed in section 5.2

1922 All of these samples are binned in reconstructed neutrino energy. This is
 1923 possible under a particular interaction mode assumption, as the direction from
 1924 the source is known extremely well. For the 1Re-like and 1R μ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})} \quad (6.2)$$

1925 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing
 1926 lepton, respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nucleus
 1927 [182], θ_{beam} is the angle between the beam and the direction of the outgoing
 1928 lepton, and E_l and P_l are the energy and momentum of that outgoing lepton.

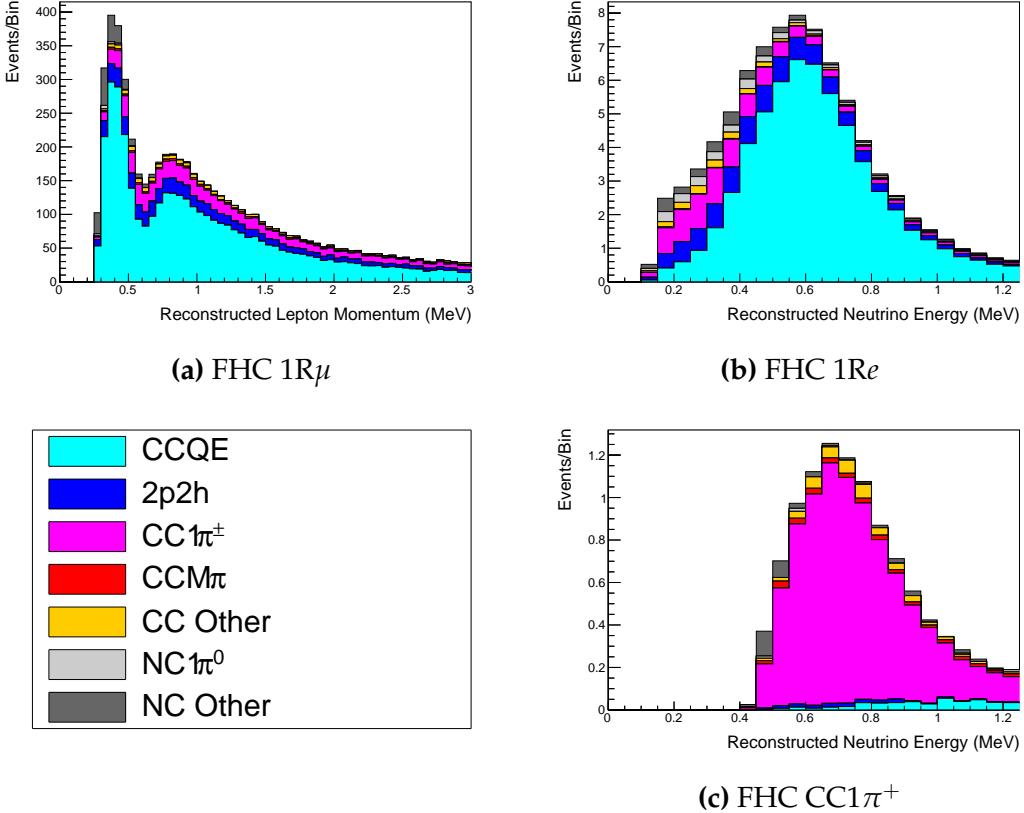


Figure 6.4: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1Re-like, and CC1 π^+ -like samples. The AsimovA oscillation parameters are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0 \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV.

1929 The reconstructed neutrino energy of the CC1 π^+ -like events also accounts
 1930 for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))} \quad (6.3)$$

1931 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
 1932 considered as a two-body process, with the delta baryon, is assumed. This follows
 1933 the T2K oscillation analysis presented in [74], although recent developments of
 1934 the interaction model in the latest T2K oscillation analysis do include effects
 1935 from binding energy in this calculation [195].

1936 The reconstructed neutrino energy for the FHC samples is illustrated in
 1937 Figure 6.4. As expected, the 1R μ -like and 1Re-like samples are heavily dominated

by CCQE interactions, with smaller contributions from 2p2h meson exchange and resonant pion production interactions. The CC1 π^+ -like sample predominantly consists of charged current resonant pion production interactions. The 1Re-like and CC1 π^+ -like samples are also binned by the angle between the neutrino beam and the reconstructed lepton momentum. This is to aid in charged current and neutral current separation, as indicated in Figure 6.5. This is because the neutral current backgrounds are predominantly due to π^0 -decays, where the opening angle of the two gammas alongside the different final state kinematics produces a slightly broader angular distribution compared to the final state particles originating from charged current ν_e interactions.

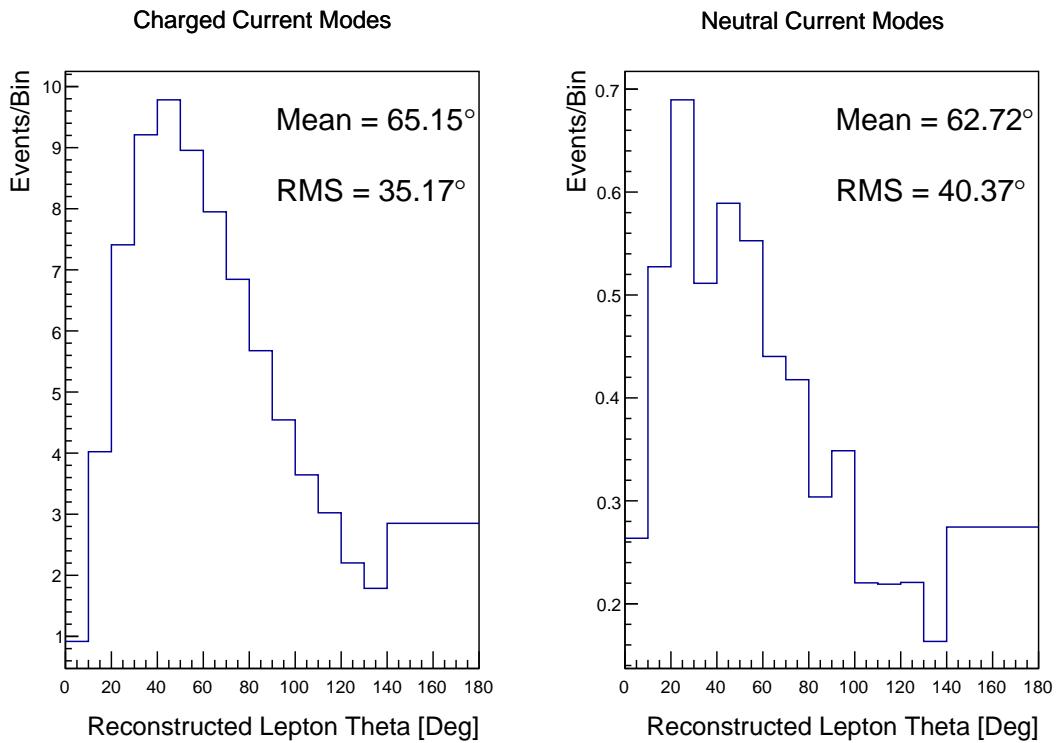


Figure 6.5: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are 35.17° and 40.37° , respectively.

1948 6.4 Systematic Uncertainties

1949 The systematic model parameters for this analysis are split into groups, or blocks,
 1950 depending on their purpose. They consist of flux uncertainties, neutrino-matter
 1951 interaction systematics, and detector efficiencies. There are also uncertainties on
 1952 the oscillation parameters to which this analysis is not sensitive, namely Δm_{21}^2
 1953 and $\sin^2(\theta_{12})$. These oscillation parameter uncertainties are taken from the 2020
 1954 PDG measurements [75]. As described in chapter 4, each model parameter used
 1955 within this analysis requires a prior uncertainty. This is provided via separate
 1956 covariance matrices for each block. The covariance matrices can include prior
 1957 correlations between parameters within a single block, but the separate treatment
 1958 means prior correlations can not be included for parameters in different groups.
 1959 Some parameters in these models have no reasonably motivated uncertainties
 1960 and are assigned flat priors which do not modify the likelihood penalty. In
 1961 practice, these flat prior parameters are actually assigned a Gaussian with a
 1962 very large width to ensure the covariance matrix is positive definite. They are
 1963 then checked at run time to determine if they contribute to the likelihood. The
 1964 flux, neutrino interaction, and detector modeling simulations have already been
 1965 discussed in section 5.1 and section 5.2. The uncertainties invoked within each
 1966 of these models are described below.

1967 6.4.1 Beam Flux

1968 The neutrino beam flux systematics are based upon the uncertainty in the mod-
 1969 eling of the components of the beam simulation. This includes the model of
 1970 hadron productions and reinteractions, the shape, intensity, and alignment of
 1971 the beam with respect to the target, and the uniformity of the magnetic field
 1972 produced by the horn, alongside other effects. The uncertainty, as a function
 1973 of neutrino energy, is illustrated in Figure 6.6 which includes a depiction of
 1974 the total uncertainty as well as the contribution from individual components.
 1975 The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$) is

1976 dominated by uncertainties in the beam profile and alignment. Outside of this
1977 region, uncertainties on hadron production dominate the error.

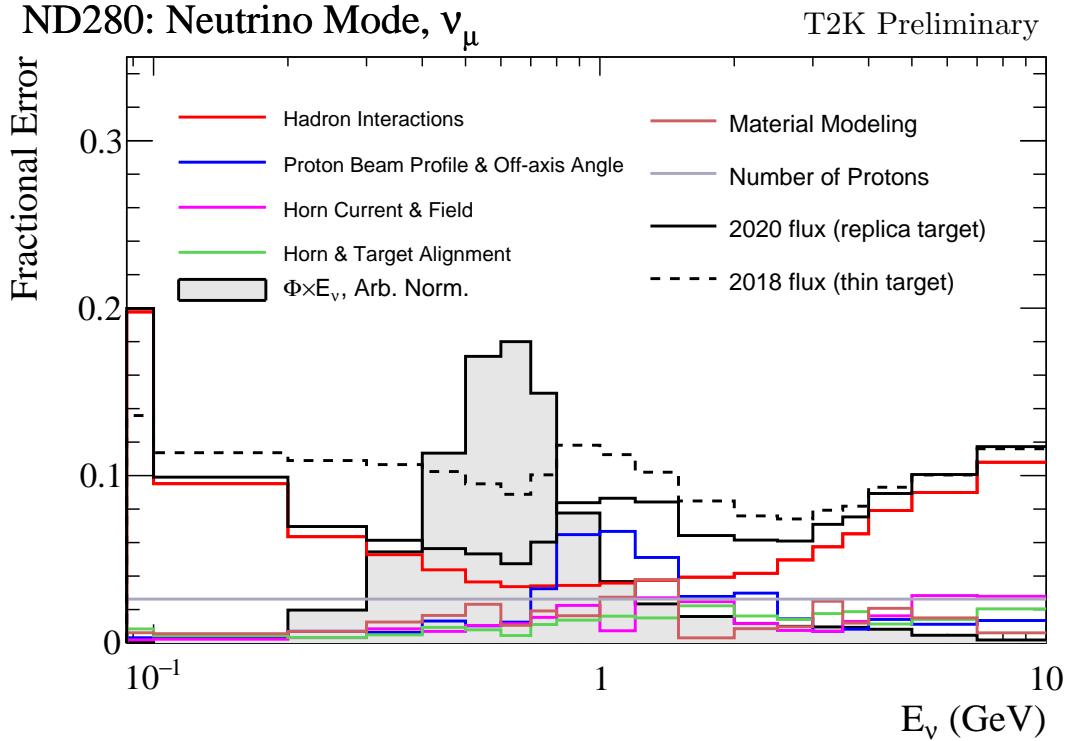


Figure 6.6: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [196]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [197].

1978 The beam flux uncertainties are described by one hundred parameters. They
1979 are split between the ND280 and SK detectors and binned by neutrino flavour:
1980 ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. The response is then broken down as a function of neutrino
1981 energy. The bin density in the neutrino energy is the same for the ν_μ in FHC
1982 and $\bar{\nu}_\mu$ in RHC beams, and narrows for neutrino energies close to the oscillation
1983 maximum of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.5. All of these
1984 systematic uncertainties are applied as normalisation parameters with Gaussian
1985 priors centered at 1.0 and error specified from a covariance matrix provided
1986 by the T2K beam group [197].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.5: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

1987 6.4.2 Atmospheric Flux

1988 The atmospheric neutrino flux is modeled by the HKKM model [50]. 16 systematic
 1989 uncertainties are applied to control the normalisation of each neutrino flavour,
 1990 energy, and direction. They are summarised below:

- 1991 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour
 1992 is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$
 1993 and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction
 1994 uncertainties for the production of pions and kaons [50]. The strength of
 1995 the response is dependent upon the neutrino energy. The uncertainty is
 1996 parameterized following Figure 11 in [50].
- 1997 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$
 1998 are controlled by the difference between the HKKM model [50], FLUKA
 1999 [53] and Bartol models [49]. Three independent parameters are applied in
 2000 the energy ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 2001 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent)
 2002 production uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using
 2003 the same methodology as the relative normalisation parameters.
- 2004 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-
 2005 tematics, the difference between the HKKM, FLUKA, and Bartol model

2006 predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of
 2007 events as a function of zenith angle.

- 2008 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) mostly originate in
 2009 kaon decay. Measurements of the ratio of K/π production [198] are used to
 2010 control the systematic uncertainty of the expected ratio of pion and kaon
 2011 production.
- 2012 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic
 2013 field, the flux of primary cosmic rays varies across the same period. The
 2014 uncertainty is calculated by taking a ± 1 year variation, equating to a 10%
 2015 uncertainty for the SK-IV period.

- 2016 • **Atmospheric Density:** The height of the interaction of the primary cosmic
 2017 rays is dependent upon the atmospheric density. The HKKM assumes the
 2018 US standard 1976 [150] profile. This systematic controls the uncertainty in
 2019 that model.

2020 The total uncertainty is dominated by the absolute and relative normalisation
 2021 parameters. The effect of which is illustrated in Figure 6.7. Generally, the
 2022 uncertainty is large at low energy, reducing to $O(10\%)$ around the peak of the
 2023 flux distribution and then increasing once the neutrino energy exceeds 10GeV.

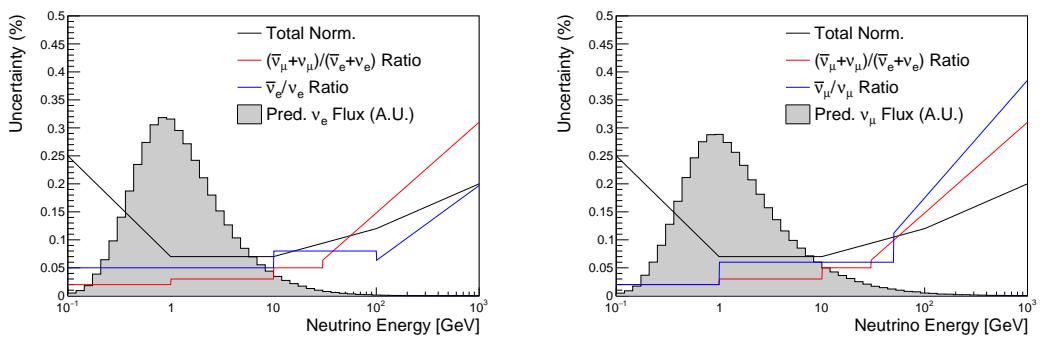


Figure 6.7: The uncertainty evaluated on the atmospheric ν_e (left) and ν_μ (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2024 Updates to the HKKM and Bartol models are underway [155] to use a similar
2025 tuning technique to that used in the beam flux predictions. After those updates,
2026 it may be possible to include correlations in the hadron production uncertainty
2027 systematics for beam and atmospheric flux predictions.

2028 6.4.3 Neutrino Interaction

2029 Neutrino interactions in the detectors are modeled by NEUT. The two indepen-
2030 dent oscillation analyses, T2K-only [199] and the SK-only [59], have developed
2031 separate interaction models. To maximise sensitivity out of this simultaneous
2032 beam and atmospheric analysis, a correlated interaction model has been defined
2033 in [200]. Where applicable, correlations allow the systematic uncertainties applied
2034 to the atmospheric samples to be constrained by near detector neutrino beam
2035 measurements. This can lead to stronger sensitivity to oscillation parameters
2036 as compared to an uncorrelated model.

2037 The low-energy T2K systematic model has a more sophisticated treatment
2038 of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of
2039 this model have been performed to external data [199]. However, the model
2040 is not designed for high-energy atmospheric events, like those illustrated in
2041 Figure 5.11. Therefore the high energy systematic model from the SK-only
2042 analysis is implemented for the relevant multi-GeV, PC, and up- μ samples.
2043 The T2K CCQE model is more sophisticated so it has been implemented for
2044 all samples within this analysis, where separate low-energy and high-energy
2045 dials have been implemented. The low-energy dials are constrained by the near
2046 detector measurements and are uncorrelated to their high-energy counterparts.
2047 The author of this thesis was responsible for implementing and validating the
2048 combined cross-section model as documented in [200, 201].

2049 The high energy systematic model includes parameters developed from
2050 comparisons of Nieves and Rein-Seghal models which affect resonant pion
2051 producing interactions, comparisons of the GRV98 and CKMT models which
2052 control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the ν_τ cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.8 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.9 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

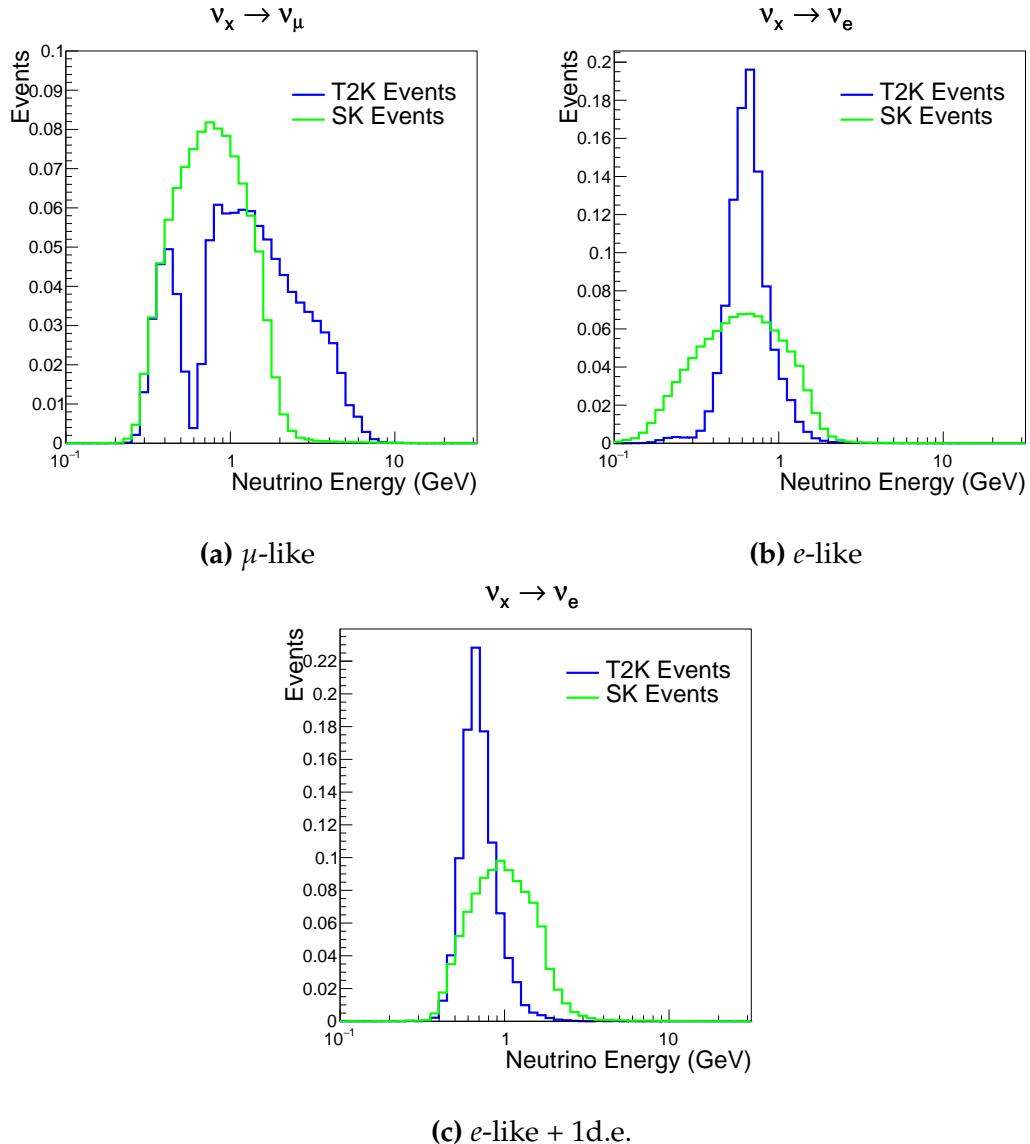


Figure 6.8: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

The T2K systematic model [199] is applied in a similar methodology to the SK model parameters. It consists of 19 shape parameters and 24 normalisation parameters. Four additional parameters, which model the uncertainty in the binding energy, are applied in a way to shift the momentum of the lepton emitted from a nucleus. This controls the uncertainty specified on the 27MeV binding energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.9: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

2089 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated
 2090 uncertainty, or those which have not been fit to external data, are assigned a
 2091 flat prior which does not affect the penalty term.

2092 On top of the combination of the SK and T2K interaction models, several
 2093 other parameters have been specifically developed for the joint oscillation anal-
 2094 ysis. The majority of the atmospheric samples' δ_{CP} sensitivity comes from the
 2095 normalisation of subGeV electron-like events. These are modeled using a spectral
 2096 function to approximate the nuclear ground state. However, the near detector is
 2097 not able to constrain the model so an additional systematic is introduced which
 2098 models an alternative Continuous Random Phase Approximation (CRPA) nuclear
 2099 ground state. This dial approximates the event weights if a CRPA model had

2100 been assumed rather than a spectral function. This dial only applies to ν_e and
2101 $\bar{\nu}_e$ as the near detector does not constraint ν_e cross-section measurements. It
2102 is applied as a shape parameter.

2103 Further additions to the model have been introduced due to the inclusion of
2104 the subGeV π^0 atmospheric sample. This particularly targets charged current
2105 and neutral current π^0 producing interactions to help constrain the systematic
2106 uncertainties. Therefore, an uncertainty that affects neutral current resonant
2107 π^0 production is incorporated into this analysis. Comparisons of NEUT's NC
2108 resonant pion production predictions have been made to MiniBooNE [202] data
2109 and a consistent 16% to 21% underprediction is observed [200]. Consequently,
2110 a conservative 30% normalisation parameter is invoked.

2111 Down-going events are mostly insensitive to oscillation parameters and can
2112 act similar to the near detector within an accelerator experiment (Details will
2113 be discussed in chapter 7). This region of phase space can act as a sideband
2114 and allows the cross-section model and near detector constraint to be studied.
2115 The distribution of events in this region is calculated using the technique out-
2116 lined in subsection 4.3.4. The results are illustrated in Figure 6.10. For CCQE-
2117 targeting samples, the application of the near detector constraint is well within
2118 the statistical fluctuation of the down-going data such that no significant tension
2119 is observed between the data and the Monte Carlo prediction after the near
2120 detector constraint is applied. This is not the case for samples with target CCRES
2121 interactions. The electron-like data is consistent with the constrained prediction
2122 at high reconstructed momenta but diverges at lower momentum, whereas the
2123 muon-like sample is under-predicted throughout the range of momenta. To
2124 combat this disagreement, an additional cross-section systematic dial, specifically
2125 designed to inflate the low pion momentum systematics was developed in [200].
2126 This is a shape parameter implemented through a splined response.

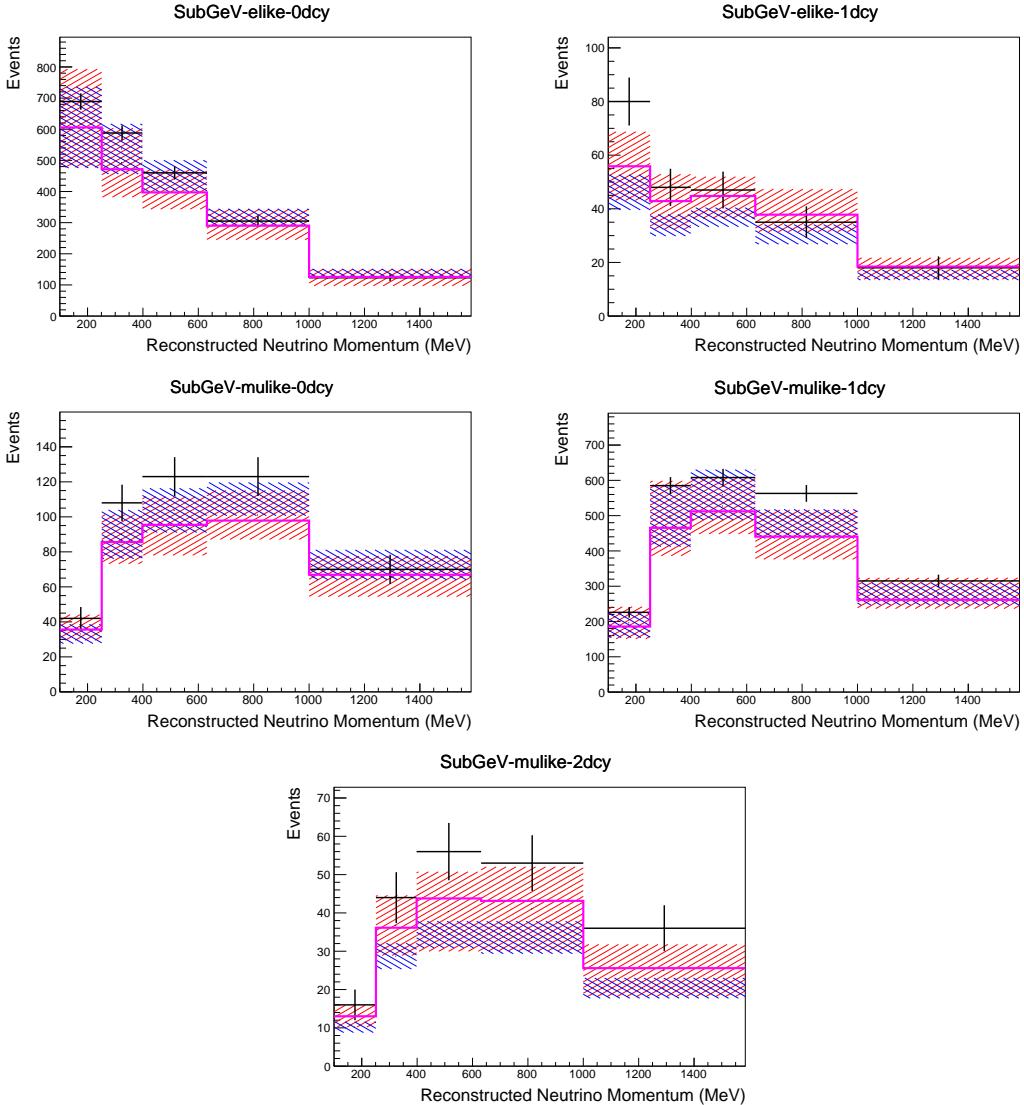


Figure 6.10: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

2127 6.4.4 Near Detector

2128 The systematics applied due to uncertainties arising from the response of the near
 2129 detector is documented in [130]. The response is described by 574 normalisation
 2130 parameters binned in the selected sample as well as momentum and angle,

2131 P_μ and $\cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance
2132 matrix with each parameter being assigned a Gaussian prior from that covariance
2133 matrix. These normalisation parameters are built from underlying systematics,
2134 e.g. pion secondary interaction systematics, which are randomly thrown and
2135 the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. Two thousand throws are
2136 evaluated and a covariance matrix response is created. This allows significant
2137 correlations between FGD1 and FGD2 samples, as well as adjacent $P_\mu \times \cos(\theta_\mu)$
2138 bins. Statistical uncertainties are accounted for by including fluctuations of each
2139 event's weight from a Poisson distribution.

2140 Similar to the cross-section systematics, MaCh3 and BANFF are used to
2141 constrain the uncertainty of these systematics through independent validations.
2142 Each fitter generates a post-fit covariance matrix which is compared and passed
2143 to the far-detector oscillation analysis working group. As the analysis presented
2144 within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all
2145 three sets of samples and their respective systematics is performed.

2146 6.4.5 Far Detector

2147 Two configurations of the far detector systematic model implementation have
2148 been considered. Firstly, the far detector systematic uncertainties for beam and
2149 atmospheric samples are taken from their respective analysis inputs, denoted
2150 “official inputs” analysis, with no correlations assumed between the beam and at-
2151 mospheric samples. The beam- and atmospheric-specific inputs are documented
2152 in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative
2153 detector model has been developed which correlates the response of the SK
2154 detector systematics between the beam and atmospheric samples. Here, the
2155 distribution of parameters used for applying event cuts (e.g. electron-muon
2156 PID separation) is modified within the fit. It follows a similar methodology to
2157 the beam far detector systematics implementation but performs a joint fit of
2158 the beam and atmospheric data. This alternative implementation is detailed
2159 in subsubsection 6.4.5.3.

2160 **6.4.5.1 Beam Samples**

2161 There are 45 systematics which describe the response of the far detector to
2162 beam events [182], split into 44 normalisation parameters and one energy scale
2163 systematic. The energy scale systematic is applied as a multiplicative scaling
2164 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo
2165 differences in the stopping muon sample in [184] and found to be 2.1%. The
2166 normalisation parameters are assigned a Gaussian error centered at one with
2167 width taken from a covariance matrix. A detailed breakdown of the generation
2168 of the covariance matrix is found in [194]. To build the covariance matrix, a fit
2169 is performed on atmospheric data which has been selected using beam sample
2170 selection cuts. These cuts use the variables, L^i , where the index i is detailed in
2171 Table 6.6. Each L^i is a smear, α , and shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i \quad (6.4)$$

2172 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in
2173 Table 6.6. The shift and smear parameters are nuisance parameters with no prior
2174 constraints. They are binned by final-state topology, j , where the binning is given
2175 in Table 6.7. The final-state topology binning is because the detector will respond
2176 differently to events that have one or multiple rings. For example, the detector
2177 will be able to distinguish single-ring events better than two overlapping ring
2178 events, resulting in different systematic uncertainty for one-ring events compared
2179 to two-ring events. This approach is used to allow the cut parameter distributions
2180 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.6: List of cut variables that are included within the shift/smear fit documented in [194].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 μ	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 μ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 π^0	Only one π^0 in the final state
1 π^\pm or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.7: Reconstructed event topology categories on which the SK detector systematics [194] are based.

2181 The mis-modeling of π^0 events is also considered. If one of the two rings
 2182 from a π^0 event is missed, this will be reconstructed as a CC ν_e -like event. This
 2183 is one of the largest systematics hindering the electron neutrino appearance
 2184 analyses. Consequently, additional systematics have been introduced to con-
 2185 strain the mis-modeling of π^0 events in SK, binned by reconstructed neutrino
 2186 energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples is
 2187 constructed. These events are built by overlaying one electron-like ring from
 2188 the SK atmospheric neutrino samples or decay electron ring from a stopping
 2189 cosmic ray muon with one simulated photon ring. Both rings are chosen so
 2190 that momenta and opening angle follow the decay kinematics of NC π^0 events
 2191 from the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the
 2192 SK Monte Carlo are produced to compare with the hybrid- π^0 data samples and
 2193 the difference in the fraction of events that pass the ν_e selection criteria is used
 2194 to assign the systematic errors. In order to investigate any data to Monte Carlo
 2195 differences that may originate from either the higher energy ring or lower energy
 2196 ring, two samples are built; a sample in which the electron constitutes the higher
 2197 energy ring from the π^0 decay (called the primary sample) and another one in
 2198 which it constitutes the lower energy ring (called the secondary sample). The
 2199 standard T2K ν_e fitQun event selection criteria are used to select events.

2200 Final contributions to the covariance matrix are determined by supplemen-
 2201 tary uncertainties obtained by comparing stopping muon data to Monte Carlo

prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data to Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or fewer decay electrons. Similarly, the rate at which fake decay electrons are reconstructed by `f iTQun` is estimated by comparing the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events is also applied. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [182, 203, 204].

Two additional sources of uncertainty are included: secondary and photoneuclear interactions. These are estimated by varying the underlying parameters are building a distribution of sample event rates. These contributions are then added in quadrature to the above covariance matrix.

DB: Matrix Diagonals from SK Det Matrix

6.4.5.2 Atmospheric Samples

The detector systematics for atmospheric samples, documented in [85], are split into two sub-groups: those which are related to particle identification and ring counting systematics, and those which are related to calibration, separation, and reduction uncertainties.

The particle identification systematics consist of five parameters. The ring separation systematic enforces an anti-correlated response between the single-ring and multi-ring samples. This is implemented as a fractional increase/decrease in the overall normalisation of each sample, depending on the distance to the nearest wall from an event's vertex. The coefficients of the normalisation are estimated prior to the fit and depend on the particular atmospheric sample. Two electron-muon separation systematics are included within this model which

2231 anti-correlates the response of the electron-like and muon-like samples: one for
2232 single-ring events and another for multi-ring events.

2233 The multi-ring electron-like separation likelihood, discussed in section 6.1,
2234 encodes the ability of the detector to separate neutrino from anti-neutrino events.
2235 Two normalisation parameters vary the relative normalisation of multi-ring ν_e
2236 and $\bar{\nu}_e$ samples whilst keeping a consistent overall event rate.

2237 There are 22 systematics related to calibration measurements, including effects
2238 from backgrounds, reduction, and showering effects. They are documented in
2239 [85] and are briefly summarised in Table 6.8. They are applied via normalisation
2240 parameters, with the separation systematics requiring the conservation of event
2241 rate across all samples.

2242 6.4.5.3 Correlated Detector Model

2243 A complete uncertainty model of the SK detector would be able to determine
2244 the systematic shift on the sample spectra for a variation of the underlying
2245 parameters, e.g. PMT angular acceptance. However, this is computationally
2246 intensive, requiring Monte Carlo predictions to be made for each plausible
2247 variation. Consequently, an effective parameter model has been utilised for
2248 a correlated detector model following from the T2K-only model implementation
2249 documented in subsubsection 6.4.5.1. It correlates the detector systematics
2250 between the far-detector beam and subGeV atmospheric samples due to their
2251 similar energies and interaction types. As there are no equivalent beam samples,
2252 the multi-GeV, multiring, PC, and Up- μ samples will be subject to the particle
2253 identification systematics implementation as described in subsubsection 6.4.5.2
2254 rather than using this correlated detector model. The calibration systematics also
2255 described in the aforementioned chapter still apply to all atmospheric samples.

2256 The correlated detector model utilises the same smear and shift parameters
2257 documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,
2258 the shift and smear parameters are split by visible energy deposited within the
2259 detector, with binning specified in Table 6.9. This is because atmospheric events

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

Table 6.8: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

are categorised by subGeV and multi-GeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where k is the visible energy bin.

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

Table 6.9: Visible energy binning for which the correlated SK detector systematics are based

parameter for that event, and then re-applies event selection. This causes event migration, which is a new feature incorporated into the MaCh3 framework which is only achievable due to the event-by-event reweighting scheme.

Particular care has to be taken when varying the ring counting parameter. This is because the number of rings is a finite value (one-ring, two-ring, etc.) which can not be continuously varied through this shift and smear technique. Consequently a continuous ring counting parameter, RC_i , is calculated for the i^{th} event, following the definition in [181]: the preferred likelihoods from all considered one-ring (L_{1R}) and two-ring (L_{2R}) fits are determined. The difference is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an intermediate likelihood value used within the `fitQun` algorithm to decide the number of rings associated with a particular event. However, fake-ring merging algorithms are applied after this likelihood value is used. Consequently, this ring counting parameter does not always exactly correspond to the number of reconstructed rings. This can be seen in Figure 6.11.

As the `fitQun` algorithm does not provide a likelihood value after the fake-ring algorithms have been applied, the ring counting parameter distribution is correlated to the final number of reconstructed rings through “maps”. These are two-dimensional distributions of the ring counting parameter and the final number of reconstructed rings. An example is illustrated in Figure 6.12. In principle,

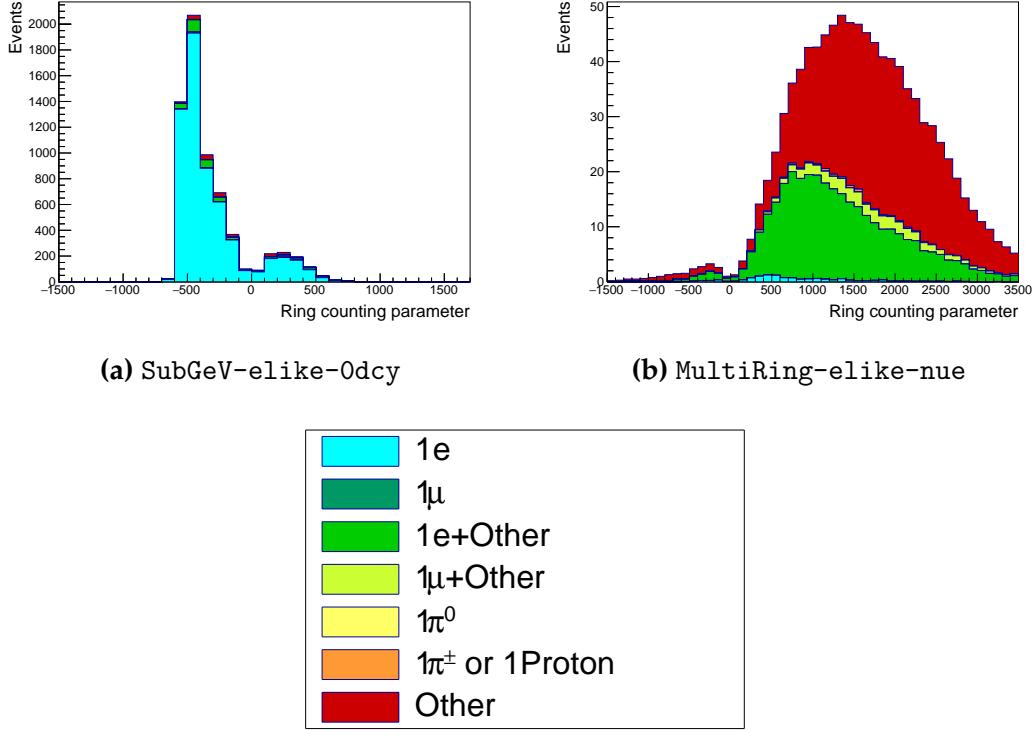


Figure 6.11: The ring counting parameter as defined in Equation 6.6 for the SubGeV-elike-0dcy and MultiRing-elike-nue samples.

the `fitQun` reconstruction algorithm should be re-run after the variation in the ring counting parameter. However, this is not computationally viable. Therefore the “maps” are used as a reweighting template.

The maps are split by final state topology and true neutrino flavour and all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps are row-normalised to represent the probability of X rings for a given RC_i value. Prior to the oscillation fit, an event’s nominal weight is calculated as $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th} event and $W^i(x, y)$ is the bin content in map associated with the i^{th} event, where x is the number of rings and y is the ring counting parameter. Then during the fit, the value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i)/W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as the event weight for the i^{th} event. This is the only cut variable that uses a reweighting technique rather than event migration.

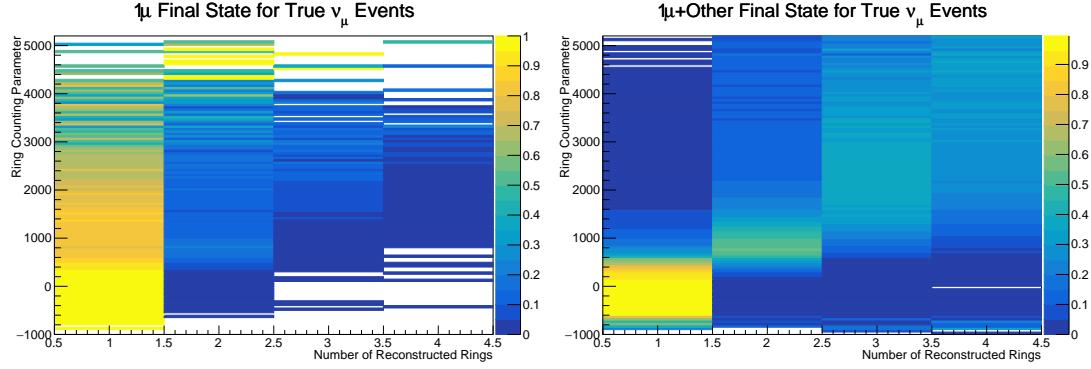


Figure 6.12: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2303 The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance
 2304 matrix. This is not possible in the alternative model as no covariance matrix
 2305 is used. Thus, the implementation of the π^0 systematics has been modified.
 2306 The inputs from the hybrid π^0 sample are included via the use of “ χ^2 maps”,
 2307 which are two-dimensional histograms in α_{jk}^i and β_{jk}^i parameters over some
 2308 range. Illustrative examples of the χ^2 maps are given in Figure 6.13. Due to
 2309 their nature, the shift and smear parameters are typically very correlated. A
 2310 map is produced for each cut parameter given in Table 6.6 and for each visible
 2311 energy bin given in Table 6.9.

2312 The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo
 2313 and data in the particle identification parameters documented in Table 6.6. The
 2314 Monte Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-
 2315 section and flux nuisance parameters are thrown from their prior uncertainties.
 2316 The χ^2 between the scaled Monte Carlo and data is calculated and the relevant
 2317 point in the χ^2 map is filled.

2318 The implementation within this alternative detector model is to add the bin
 2319 contents of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to
 2320 the likelihood penalty. Only 1 π^0 final state topology shift and smear parameters
 2321 use this prior uncertainty.

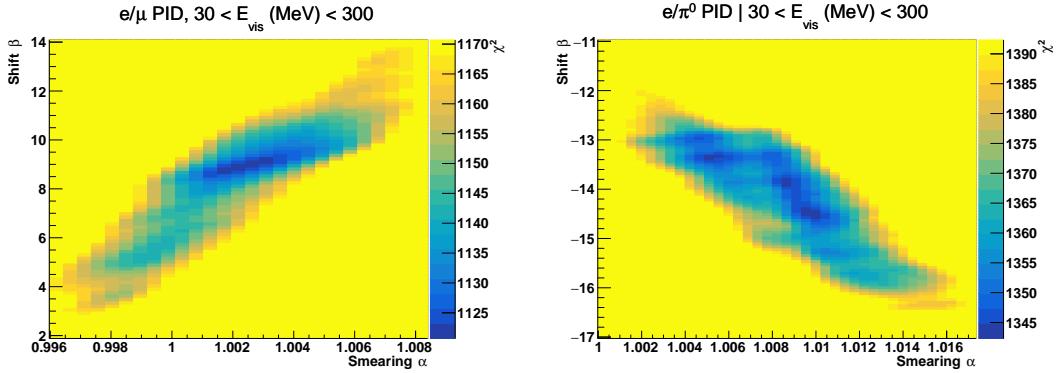


Figure 6.13: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$.

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [205] was built in tandem with the T2K-SK working group [182] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular interaction mode, number of tagged decay electrons, and whether the primary particle generated Cherenkov light. They are assigned Gaussian uncertainties with widths described by a covariance matrix. Furthermore, the secondary interaction and photo-nuclear effects need to be accounted for in this detector model using a different implementation than that in subsubsection 6.4.5.1. This was done by including a shape parameter for each of the secondary interactions and the photo-nuclear systematic parameters.

There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior constraints from the hybrid π^0 samples.

One final complexity of this correlated detector model is that the two sets of samples, beam and subGeV atmospheric, use slightly different parameters to distinguish electron and muon-like events. The T2K samples use the value of $\log(L_e/L_\mu)$ whereas the atmospheric samples use the value of $\log(L_e/L_\pi)$, where L_X is the likelihood for hypothesis X. This is because the T2K fits use single-ring fiTQun fitting techniques, whereas multi-ring fits are applied to the atmospheric samples where only the electron and pion hypothesis are considered.

2342 The correlation between the two likelihood ratios is illustrated in Figure 6.14. As
 2343 discussed in section 5.2, the pion hypothesis is a very good approximation of the
 2344 muon hypothesis due to their similar mass. Consequently, using the same shift
 2345 and smear parameters correlated between the beam and subGeV atmospheric
 2346 samples is deemed a good approximation.

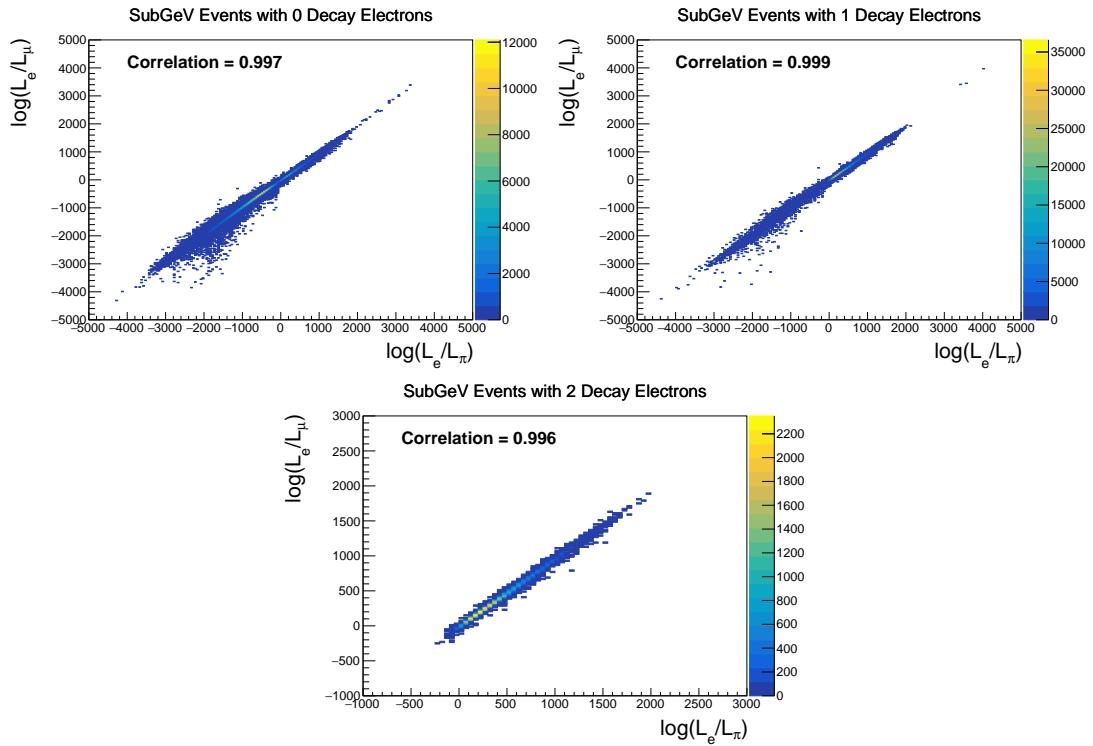


Figure 6.14: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

7

2347

2348

Oscillation Probability Calculation

2349 It is important to understand how and where the sensitivity to the oscillation
2350 parameters comes from for both atmospheric and beam samples. An overview
2351 of how these samples observe changes in δ_{CP} , Δm^2_{23} , and $\sin^2(\theta_{23})$ affect
2352 these samples is given in section 7.1. This section also explains the additional
2353 complexities involved when performing an atmospheric neutrino analysis as
2354 compared to a beam-only analysis.

2355 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to δ_{CP}
2356 due to the quickly varying oscillation probability in this region. Therefore, a
2357 “sub-sampling” approach has been developed to reduce these biases ensuring
2358 accurate and reliable sensitivity measurements. This technique ensures that small-
2359 scale unresolvable features of the oscillation probability have been averaged over
2360 whilst the large-scale features in the oscillation probability are unaffected. The
2361 documentation and validation of this technique are found in section 7.2. The
2362 oscillation probability calculation is computationally intensive due to the large
2363 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-
2364 mentation choice made within the fitting framework, as detailed in section 7.3,
2365 ensures that the analysis can be done in a timely manner.

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.4. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrinos oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation.

This implementation of an external technique is illustrated in section 7.5.

7.1 Overview

DB: Should this be moved into an earlier chapter? The selections chapter references the matter resonance which has not yet been explained at that point

The analysis presented within this thesis focuses on the determination of oscillation parameters from atmospheric and beam neutrinos. Whilst subject to the same oscillation formalism, the way in which the two samples have sensitivity to the different oscillation parameters differs significantly.

Atmospheric neutrinos have a varying baseline, or “path length”, L , such that the distance each neutrino travels before interacting is dependent upon the zenith angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s surface and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (7.1)$$

2392 Where $R_E = 6,371\text{km}$ is the Earth's radius. Consequently, the oscillation
 2393 probability is dependent upon two parameters, $\cos(\theta_Z)$ and E_ν .

2394 The oscillation probability used within this analysis is based on [21]. The
 2395 neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of
 2396 constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (7.2)$$

2397 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
 2398 energy, G_F is Fermi's constant, N_e is the electron number density and U is the
 2399 PMNS matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied
 2400 for antineutrino propagation. Thus, a model of the Earth's density is required
 2401 for neutrino propagation. Following the official SK-only methodology [206],
 2402 this analysis uses the Preliminary Reference Earth Model (PREM) [207] which
 2403 provides piecewise cubic polynomials as a function of the Earth's radius. This
 2404 density profile is illustrated in Figure 7.1. As the propagator requires layers of
 2405 constant density, the SK methodology approximates the PREM model by using
 2406 four layers of constant density [206], detailed in Table 7.1.

Layer	Outer Radius [km]	Density [g/cm ³]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 7.1: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [207].

2407 The atmospheric neutrino oscillation probabilities can be presented as two di-
 2408 mensional “oscillograms” as illustrated in Figure 7.2. The distinct discontinuities,
 2409 as a function of $\cos(\theta_Z)$, are due to the discontinuous density in the PREM model.

2410 Atmospheric neutrinos have sensitivity to δ_{CP} through the overall event
 2411 rate. Figure 7.3 illustrates the difference in oscillation probability between CP-
 2412 conserving ($\delta_{CP} = 0$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from

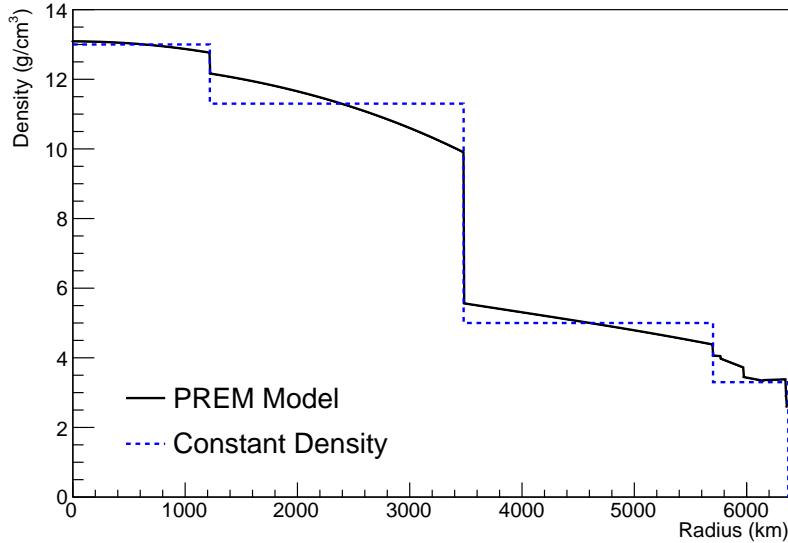


Figure 7.1: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

2413 Asimov A oscillation parameter set (Table 2.2). The result is a complicated
 2414 oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos.
 2415 The detector does not have sufficient resolution to resolve these individual
 2416 patterns so the sensitivity to δ_{CP} for atmospheric neutrinos comes via the overall
 2417 normalisation of these events.

2418 The presence of matter means that the effect δ_{CP} has on the oscillation
 2419 probability is not equal between neutrinos and antineutrinos. Furthermore,
 2420 the interaction cross-section for neutrinos is larger than antineutrinos so the
 2421 two effects have to be disentangled. These effects are further convoluted by
 2422 detector efficiencies as SK cannot distinguish neutrinos and antineutrinos well.
 2423 Furthermore, the sample selections discussed in section 6.1 have difference
 2424 efficiencies for neutrino-antineutrino selections. All of these effects lead to
 2425 a difference in the number of neutrinos detected compared to antineutrinos.
 2426 This changes how the δ_{CP} normalisation term is observed, resulting in a very
 2427 complex sensitivity to δ_{CP} .

2428 The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow$

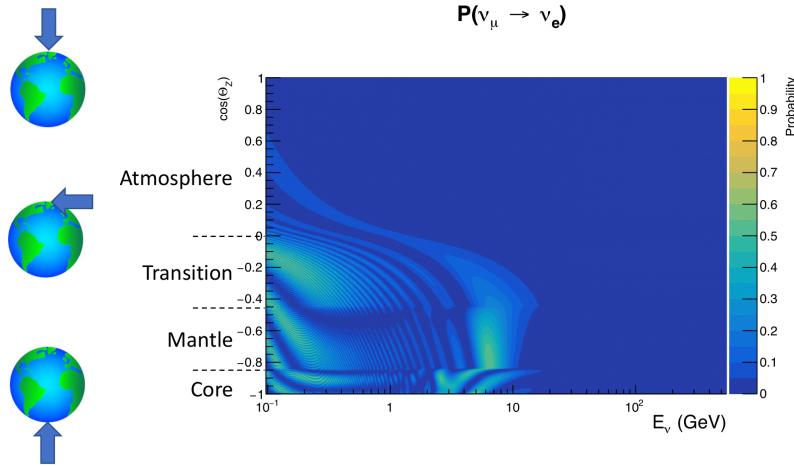


Figure 7.2: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

2429 $\bar{\nu}_e$) are presented in Figure 7.4, where the PREM model has been assumed.
 2430 The oscillation probability for both neutrinos and antineutrinos is affected in
 2431 the presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only
 2432 occur for neutrinos in normal mass hierarchy and antineutrinos in inverse mass
 2433 hierarchy. The exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$
 2434 , further increasing the atmospheric neutrinos’ sensitivity to the parameter.

2435 As the T2K beam flux is centered at the first oscillation maximum ($E_\nu =$
 2436 0.6GeV), the sensitivity to δ_{CP} is predominantly observed as a change in the
 2437 event-rate of e-like samples in $\nu/\bar{\nu}$ modes. Figure 7.5 illustrates the $P(\nu_\mu \rightarrow \nu_e)$
 2438 oscillation probability for a range of δ_{CP} values. A circular modulation of the
 2439 first oscillation peak (in both magnitude and position) is observed when varying
 2440 throughout the allowable values of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$
 2441 have a lower(higher) oscillation maximum than the CP-violating values of $\delta_{CP} =$
 2442 $-\pi/2(\delta_{CP} = \pi/2)$. A sub-dominant shift in the energy of the oscillation peak is
 2443 also present, which aids in separating the two CP-conserving values of δ_{CP} .

2444 T2K’s sensitivity to $\sin^2(\theta_{23})$ and Δm_{23}^2 is observed as a shape-based variation

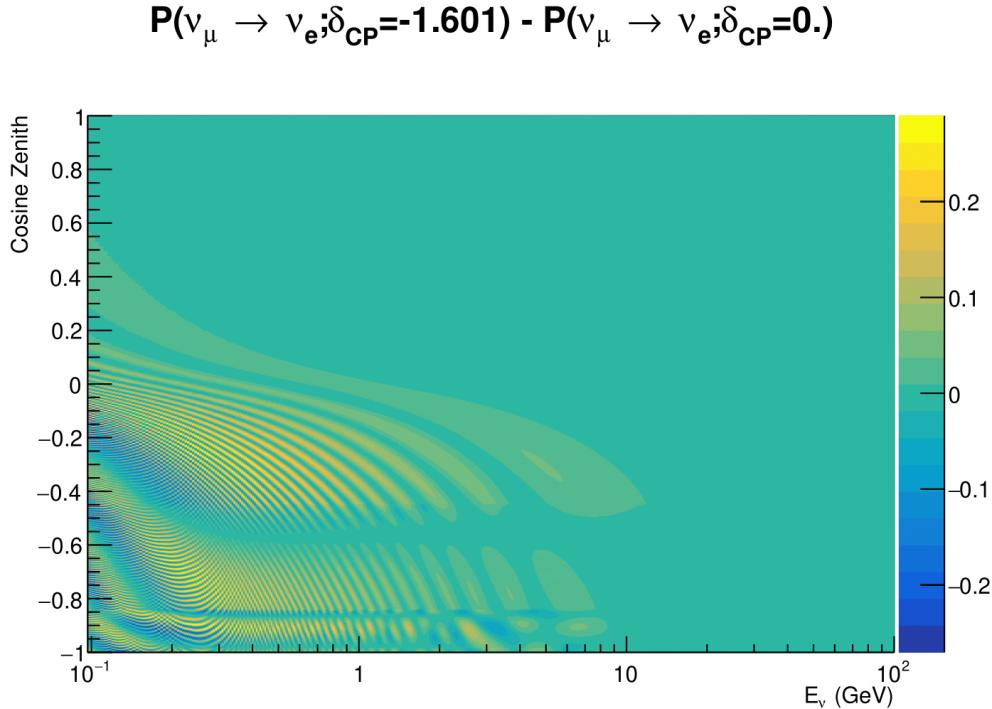


Figure 7.3: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

of the muon-like samples, as illustrated in Figure 7.5. The value of Δm_{32}^2 laterally shifts the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$ oscillation probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as a vertical shift of the oscillation dip with second-order horizontal shifts being due to matter effects. The beam neutrinos have limited sensitivity to matter effects due to the relatively shorter baseline as well as the Earth's mantle being a relatively low-density material (as compared to the Earth's core). For some values of δ_{CP} , the degeneracy in the number of e-like events allows the mass hierarchy to be broken. This leads to a δ_{CP} -dependent mass hierarchy sensitivity which can be seen in Figure 7.6.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations which reduce the number of oscillation probability calculations

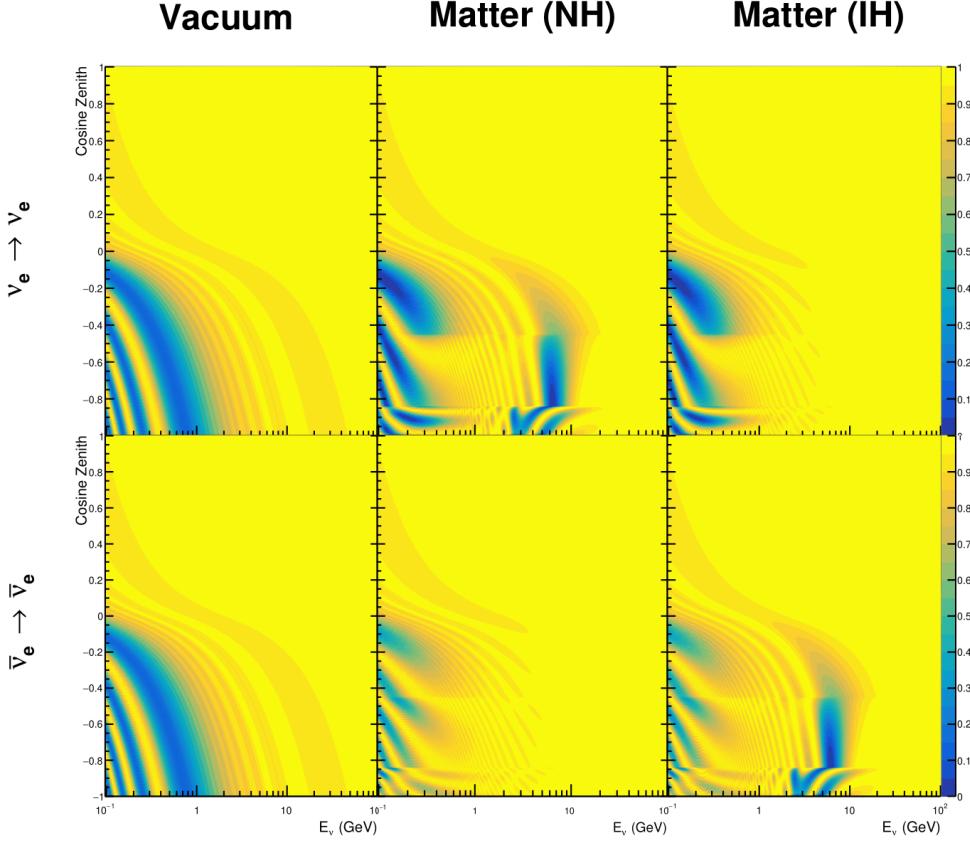


Figure 7.4: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux. Furthermore, as the peak neutrino energy of the beam is well below the threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [58], only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very

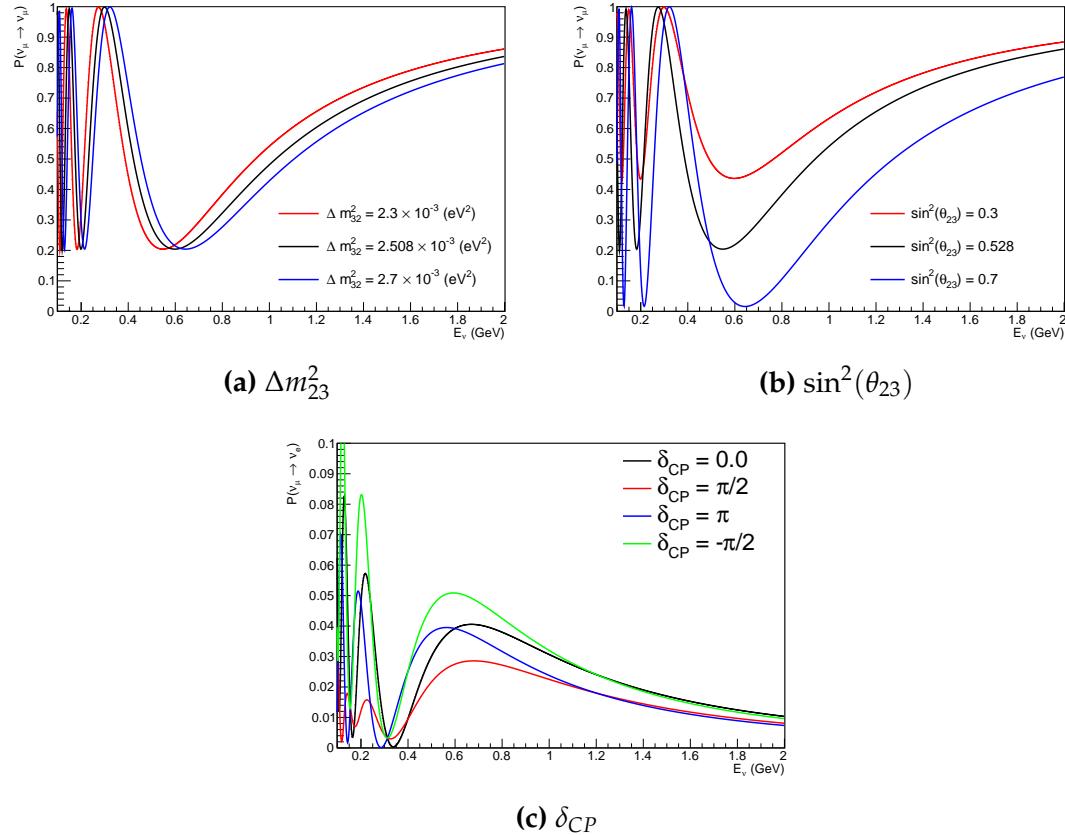


Figure 7.5: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

2465 small due to the short baseline. Whilst these approximations have been made for
 2466 the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude
 2467 as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well
 2468 above the tau production threshold.

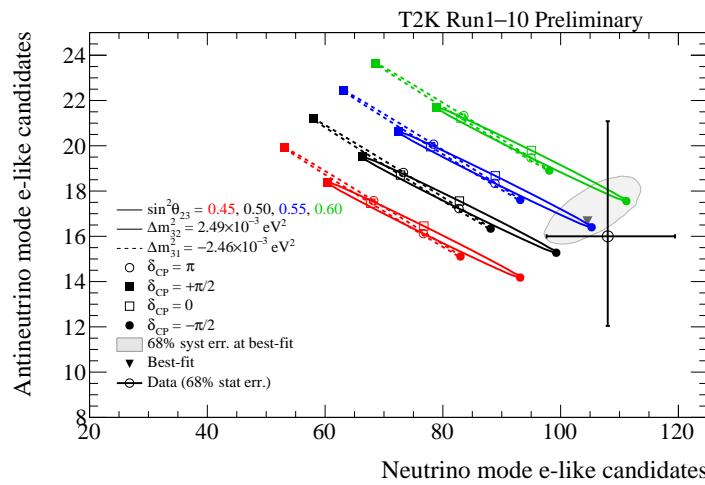


Figure 7.6: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{23}^2 are given.

2469 7.2 Treatment of Fast Oscillations

2470 As shown in Figure 7.7, atmospheric neutrino oscillations have a significantly
 2471 more complex structure for upgoing neutrinos with energy below 1GeV. This is
 2472 because the L/E dependence of the oscillation probability in this region induces
 2473 rapid variations for small changes in L or E . As discussed in section 7.1, this
 2474 is also the region in which atmospheric neutrinos have sensitivity to δ_{CP} . In
 2475 practice, the direction of the neutrino is inferred from the direction of the final
 2476 state particles traveling in the detector, which can be poor for low-energy neutrino
 2477 interactions. This creates a distinct difference from the beam neutrinos where
 2478 the position of the source is very precisely known.

2479 As a consequence of the unresolvable structure, an event rate consistent with
 2480 the averaged oscillation probability is observed in the subGeV upgoing region.
 2481 This creates a computational problem: A significantly large amount of Monte
 2482 Carlo statistics would be required to accurately predict the number of events
 2483 if Monte Carlo averaging was the only technique used. This section describes
 2484 the ‘sub-sampling’ approach developed for this analysis and compares it to the
 2485 methodology used within the SK-only analysis.

2486 The official SK-only analysis uses the osc3++ oscillation parameter fitter
 2487 [206]. To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’
 2488 technique. For a given Monte Carlo neutrino event, the nearest twenty Monte
 2489 Carlo neighbours in reconstructed lepton momentum and zenith angle are
 2490 found and a distribution of their neutrino energies is built. The RMS, σ , of
 2491 this distribution is then used to compute an average oscillation probability for
 2492 the given neutrino Monte Carlo event.

2493 For the i^{th} event, the oscillation weight is calculated as

$$2494 W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1, -0.5, 0.5, 1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.3)$$

2495 where $P(E, L)$ is the oscillation probability calculation for neutrino energy
 2496 E and path length L and the two path lengths, \bar{L}_i and L_β are discussed below.

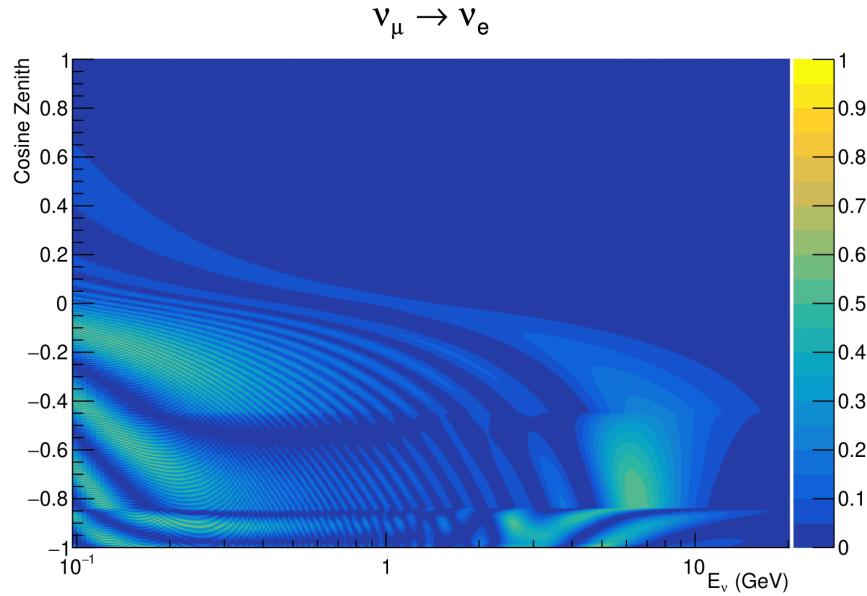


Figure 7.7: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2496 All of the oscillation probability calculations are performed with a fixed zenith
2497 angle such that the same density profile is used.

2498 The uncertainty in the production height is controlled by using an “average”
2499 production height, \bar{L}_i , which represents the average path length computed using
2500 twenty production heights taken from the Honda flux model’s prediction [52].
2501 For a given event, the production heights are sampled in steps of 5% of their
2502 cumulative distribution function. L_β values are similarly calculated but instead
2503 use different combinations of four production heights,

$$\begin{aligned}
 L_{-1.0} &= \frac{1}{4}L(45, 50, 55, 60), \\
 L_{-0.5} &= \frac{1}{4}L(35, 40, 65, 70), \\
 L_{+0.5} &= \frac{1}{4}L(25, 30, 75, 68), \\
 L_{+1.0} &= \frac{1}{4}L(15, 20, 85, 89).
 \end{aligned} \tag{7.4}$$

2504 This averaging technique works because of the inference between the zenith
2505 angle and the reconstructed direction of final state particles in the detector. For

2506 low-energy neutrinos, where the resolution of the true neutrino direction is poor,
2507 σ_i will be large, resulting in significant averaging effects. Contrary to this, the
2508 inferred direction of high-energy neutrinos will be much closer to the true value,
2509 meaning that σ_i will be smaller, culminating in small averaging effects.

2510 In practice, these calculations are performed prior to the fit as only oscillation
2511 parameters at fixed points are considered. The MCMC technique used in this
2512 thesis requires oscillation probabilities to be evaluated at arbitrary parameter
2513 values, not known *a priori*. Calculating the five oscillation probabilities per
2514 event required by the SK technique is computationally infeasible, so a differencct
2515 averaging technique is used. However, the concept of the averaging technique
2516 can be taken from it.

2517 To perform a similar averaging as the SK analysis, a sub-sampling approach
2518 using binned oscillograms has been devised. A coarsly binned oscillogram is
2519 defined in $\cos(\theta_Z)$ and E_ν . For a given set of oscillation parameters, a single
2520 oscillation probability will be assigned to each coarse bin. This value will then
2521 apply to all Monte Carlo events which fall into that bin. To assign these oscillation
2522 probabilities, the probability is calculated at $N \times N$ points on a grid within a
2523 particular bin. This ensemble of oscillation probabilities is averaged to define
2524 the coarse bin's oscillation probability, assuming a flat prior in E_ν and $\cos(\theta_Z)$.
2525 Figure 7.8 illustrates the $N = 2$ example where the assigned value to a coarse
2526 bin is the average of the four fine bins which fall in that coarse bin. Whilst the
2527 coarse bin edges do not have to be linear on either axis, the sub-division of the
2528 fine bins is linear over the range of a coarse bin.

2529 The coarse binning is defined with 67×52 bins in true neutrino energy \times
2530 cosine zenith. It is picked to be identical to that provided in [208]. In general, the
2531 binning is logarithmically spaced in neutrino energy but has some hand-picked
2532 bin edges around the matter resonance to smoothly increased the bin density.
2533 This is to avoid smearing this region which can be well sampled by the Monte
2534 Carlo. The cosine zenith binning is approximately linearly spaced across the

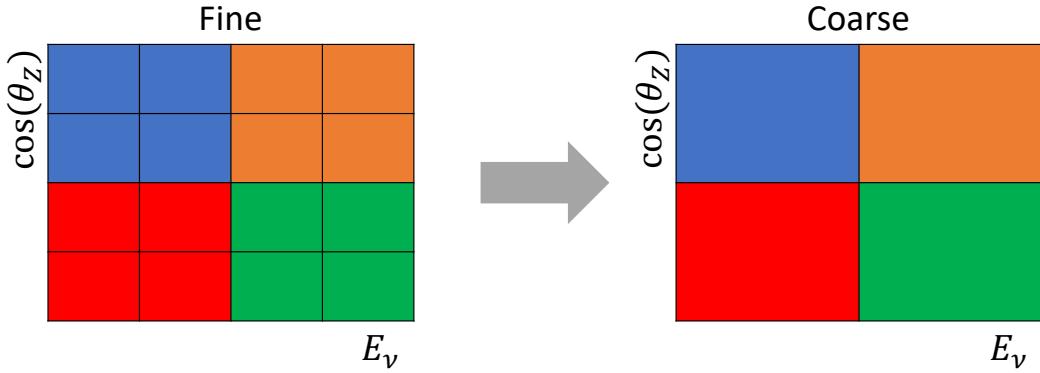


Figure 7.8: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

allowable range but the values of layer transitions are hit precisely: -0.8376 (core-mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for downgoing events as this is a region unaffected by the fast oscillation wavelengths and reduces the total number of calculations required to perform the calculation.

The choice of N is justified based on two studies. Firstly, the variation of event rates of each sample is studied as a function of N . For a given set of oscillation parameters thrown from the PDG prior constraints (detailed in Table 2.1), the oscillation probabilities are calculated using a given value of N . Each sample is re-weighted and the event rate is stored. The value of N is scanned from 1, which corresponds to no averaging, to 19, which corresponds to the largest computationally viable subdivision binning. The event rate of each sample at large N is expected to converge to a stationary value due to the fine binning fully sampling the small-scale structure. Figure 7.9 illustrates this behaviour for the SubGeV_elike_0dcy sample for 9 different throws of the oscillation parameters.

Denoting the event rate for one sample for a given throw t at each N by λ_t^N , the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The variance in the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} (\lambda_t^N - \bar{\lambda}_t) \right]^2. \quad (7.5)$$

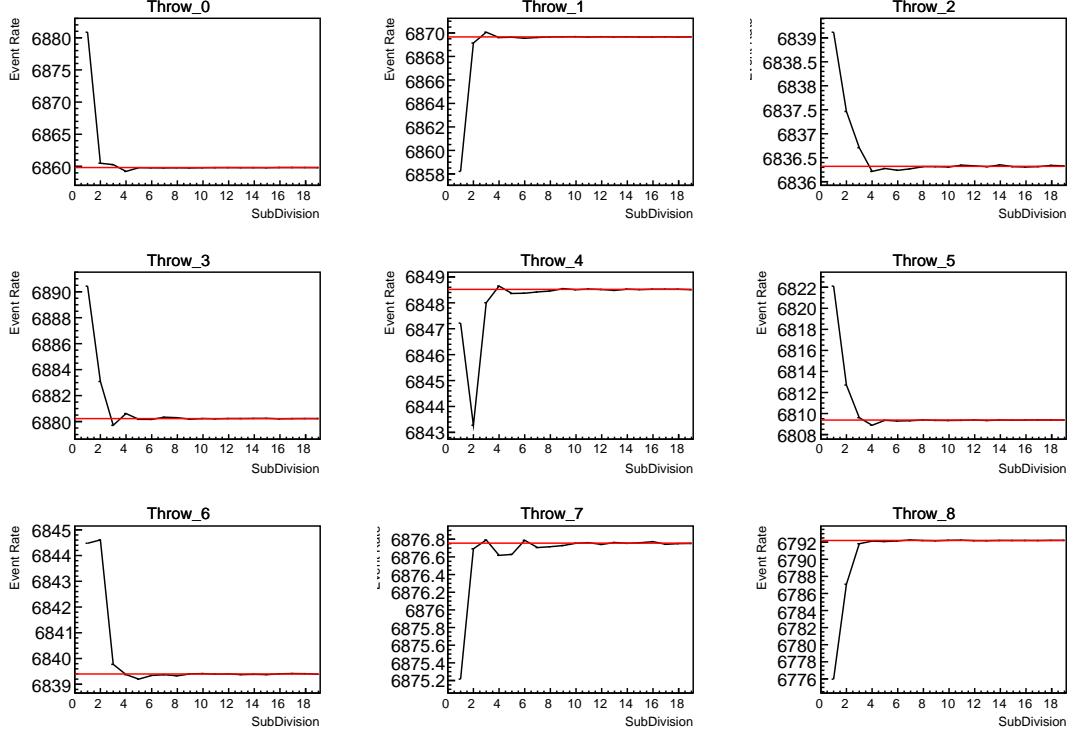


Figure 7.9: Event rate of the SubGeV_elike_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2552 In practice the following procedure is undertaken. For a particular throw,
 2553 the difference between the event rate at a particular choice of N and the mean
 2554 of the distribution is calculated. This is illustrated in Figure 7.10. This value
 2555 is then calculated for all the 2000 throws, generating a distribution of $\lambda_t^N - \bar{\lambda}_t$.
 2556 This is repeated for each of the values of N considered within this study. The
 2557 distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.11. As expected,
 2558 the distribution gets narrower and tends towards zero for the higher values of N .

2559 The aim of the study is to find the lowest value of N such that this variance
 2560 is below 0.001. This utilises the width of the distributions given in Figure 7.11.
 2561 This is the typical threshold used by T2K fitters to validate systematic imple-
 2562 mentation so has been set as the same criteria. The results of this study for
 2563 each atmospheric sample used within this thesis are illustrated in Figure 7.12 for
 2564 2000 throws of the oscillation parameters. As can be seen, the variance is below

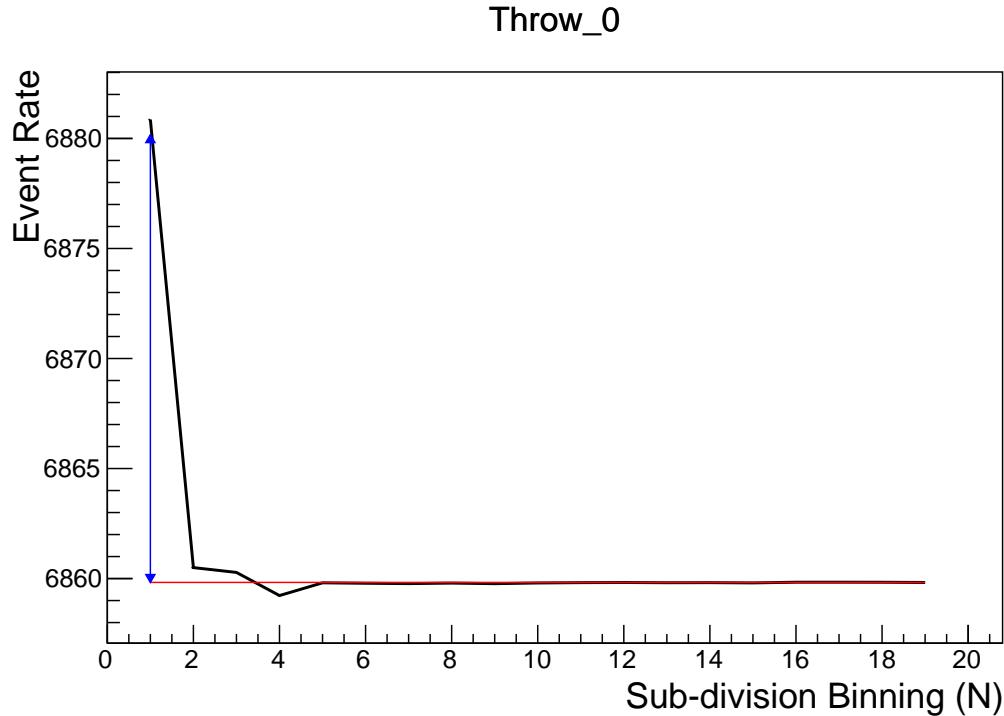


Figure 7.10: Event rate of the SubGeV_elike_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions, N , per coarse bin. The difference between the mean event rate (red), $\bar{\lambda}$, and the event rate at $N = 1$, $\lambda^{N=1}$ is defined as $\lambda^N - \bar{\lambda}$ and illustrated by the blue arrow.

2565 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy
2566 and SubGeV_elike_0dcy samples.

2567 The second study to determine the value of N is as follows. The likelihood
2568 for each sample is computed against an Asimov data set created with Asimov
2569 A oscillation parameters (Table 2.2). Following Equation 7.5, the variance of
2570 the log-likelihood over all considered N is computed. The results are shown
2571 in Figure 7.13.

2572 A choice of $N = 10$ sub-divisions per coarse bin has a variance in both
2573 event rate and log-likelihood residuals less than the required threshold of 0.001.
2574 The largest value of the likelihood variance is of order 10^{-7} , corresponding to
2575 an error on the log-likelihood of about 3×10^{-4} which is small enough to be
2576 negligible for the oscillation analysis.

2577 Figure 7.14 illustrates the effect of the smearing using $N = 10$. The fast oscil-

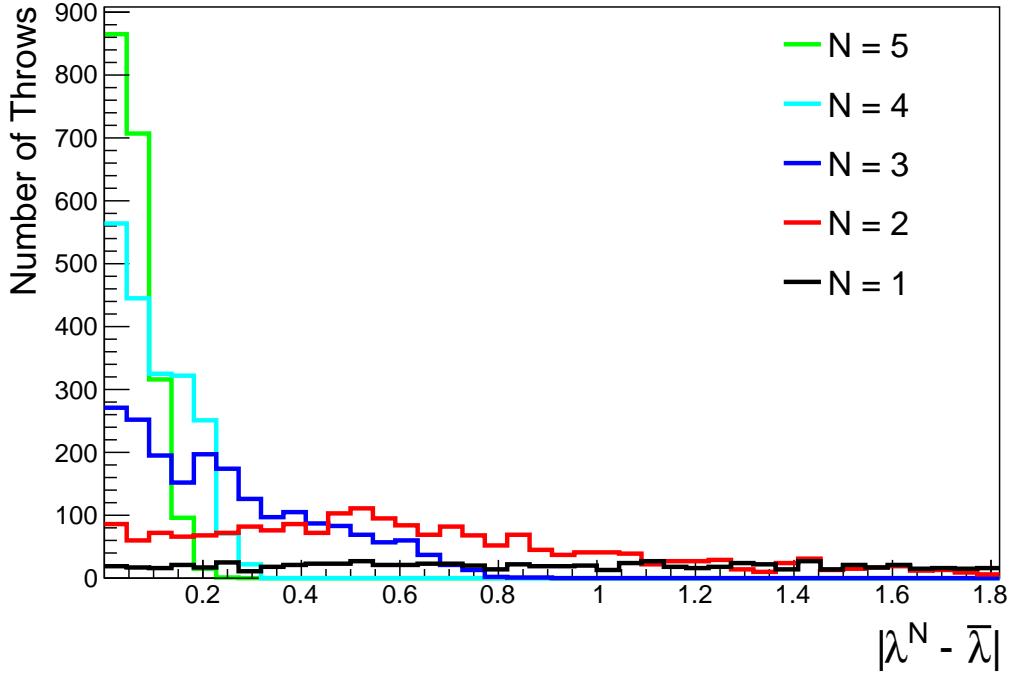


Figure 7.11: The distribution of $\lambda^N - \bar{\lambda}$ for various values of N . As expected, the distribution gets narrower for larger values of N .

2578 lations in the sub-GeV upgoing region have been replaced with a normalisation
 2579 effect whilst the large matter resonance structure remains.

2580 7.3 Calculation Engine

2581 As previously discussed in section 7.2, the calculation of oscillation probabilities
 2582 is performed at run-time. Consequently, the time per calculation is crucial for fit
 2583 performance. The initial fitting framework used for this analysis was developed
 2584 with ProbGPU [209]. This is a GPU-only implementation of the prob3 engine
 2585 [210]. It is primarily designed for neutrino propagation in a beam experiment
 2586 (single layer of constant density) with the atmospheric propagation code not
 2587 being used prior to the analysis in this thesis.

2588 Another engine, CUDAProb3 [211], has been interfaced with the fitting frame-
 2589 work used in this analysis. It has been specifically optimised for atmospheric
 2590 neutrino oscillation calculation so does not contain the code to replace the

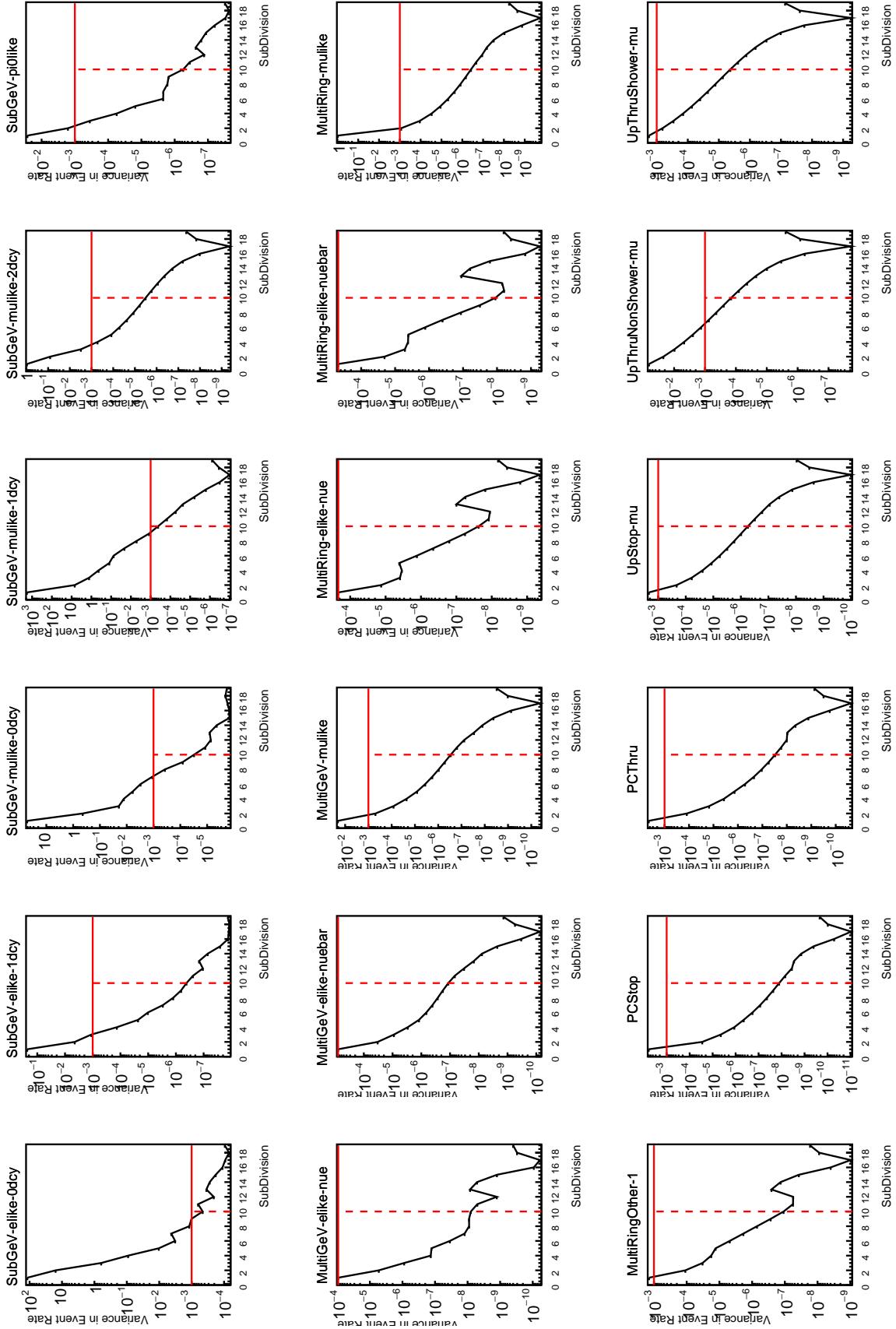


Figure 7.12: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

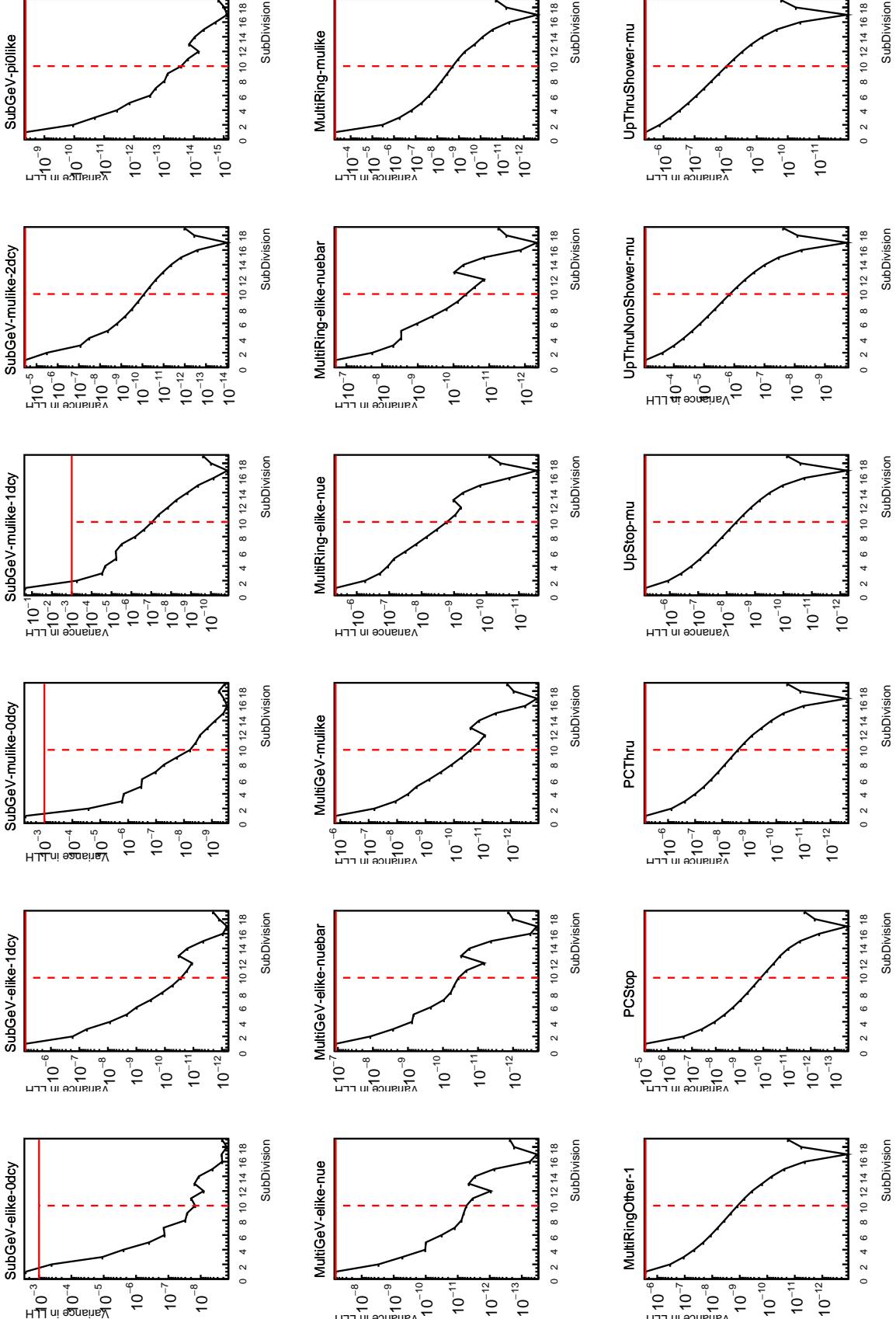


Figure 7.13: Variance of sample likelihood, when compared to ‘Asimov data’ set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

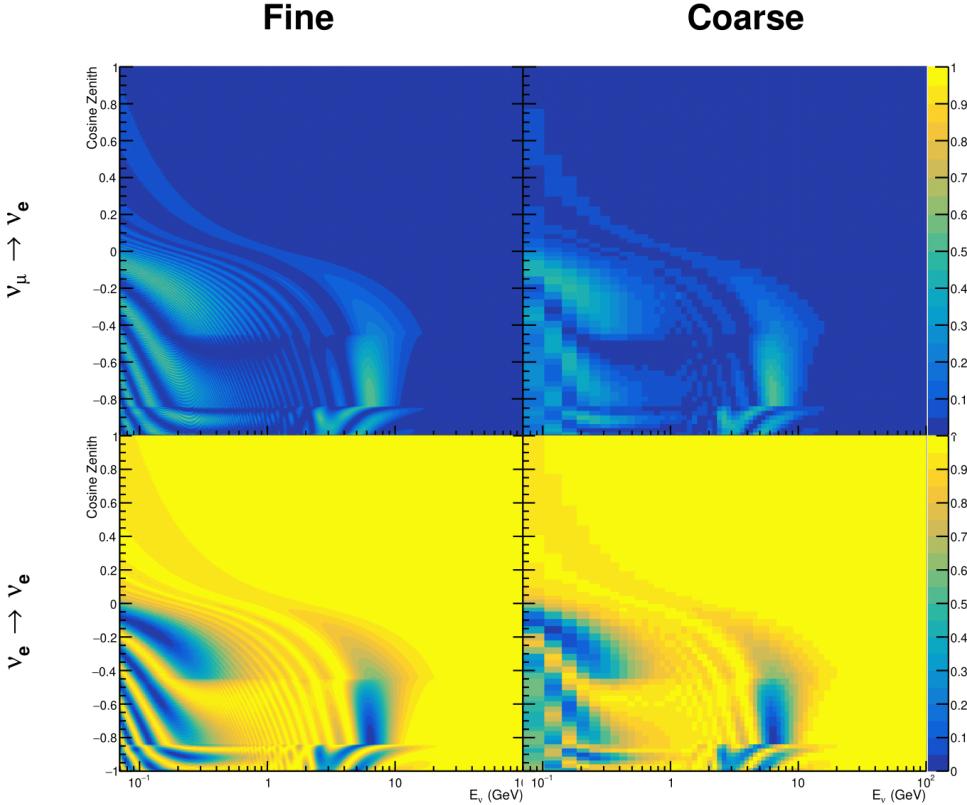


Figure 7.14: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.12 and Figure 7.13.

beam oscillation calculation. The engine utilises object-orientated techniques as compared to the functional implementation of ProbGPU. This allows the energy and cosine zenith arrays to be kept on GPU memory, rather than having to load these arrays onto GPU memory for each calculation. Reducing the memory transfer between CPU and GPU significantly reduces the time required for calculation. This can be seen in Figure 7.15, where the GPU implementation of CUDAProb3 is approximately three times faster than the ProbGPU engine.

Another significant advantage of CUDAProb3 is that it contains a CPU multithreaded implementation which is not possible with the ProbGPU or prob3

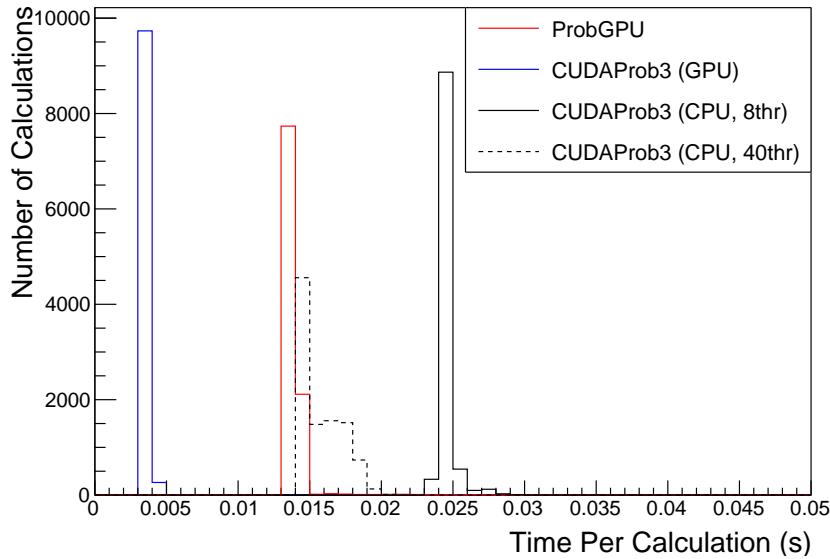


Figure 7.15: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.2, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multithreaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

2600 engines. This eliminates the requirement for GPU resources when submitting
 2601 jobs to batch systems. As illustrated in Figure 7.15, the calculation speed depends
 2602 on the number of available threads. Using 8 threads (which is typical of the
 2603 batch systems being used) is approximately twice as slow as the ProbGPU engine
 2604 implementation, but would allow the fitting framework to be run on many more
 2605 resources. This fact is utilised for any SK-only fits but GPU resources are required
 2606 for any fits which include beam samples due to the ProbGPU requirement. Based
 2607 on the benefits shown by the implementation in this section, efforts are being
 2608 placed into including linear propagation for beam neutrino propagation into
 2609 the engine [212].

2610 7.4 Matter Density Profile

2611 For an experiment observing neutrinos propagating through the Earth, a model
 2612 of the Earth’s density profile is required. The model used within this analysis is

based on the Preliminary Reference Earth Model (PREM) [207], as illustrated in Figure 7.1. Table 7.1 documents the density and radii of the layers used within the constant density approximaton used by the SK-only analysis [206]. The density measurements provided in the PREM model are provided in terms of mass density, whereas neutrino oscillations are sensitive to the electron number density. This value can be computed as the product of the chemical composition, or the Z/A value, and the mass density of each layer. Currently, the only way to measure the chemical composition value for layers close to the Earth's core is through neutrino oscillations. The chemical composition of the upper layers of the Earth's Mantle and the Transition zone is well known due to it being predominantly pyrolite which has a chemical composition value of 0.496 [213]. The chemical composition dial for the core layers is set to a value of 0.468, as calculated in [214]. As this value is lesss well known, it is assigned a Gaussian error with a standard deviation equivalent to the difference in chemical composition in core and mantle layers. Figure 7.16 illustrates the effect of moving from the $Z/A = 0.5$ method which is used in the official SK-only analysis to these more precise values.

The beam oscillation probability in this thesis uses a baseline of 295km, density $2.6\text{g}/\text{cm}^3$, and chemical composition 0.5 as is done by the official T2K-only analysis [215].

For a neutrino with given E_ν , $\cos(\theta_Z)$, the oscillation probability calculation engine must be passed a list of the matter regions that the neutrino traversed, with the path length and fixed density in each region. However, a neutrino passing through the earth experiences a range of radii, and thus a range of densities, in each region. In the SK-only analysis, the earth density model used is piecewise-constant, thereby ignoring this effect. For this thesis, the density values for the calculation engine are found by averaging the earth density along the neutrino's path,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt \quad (7.6)$$

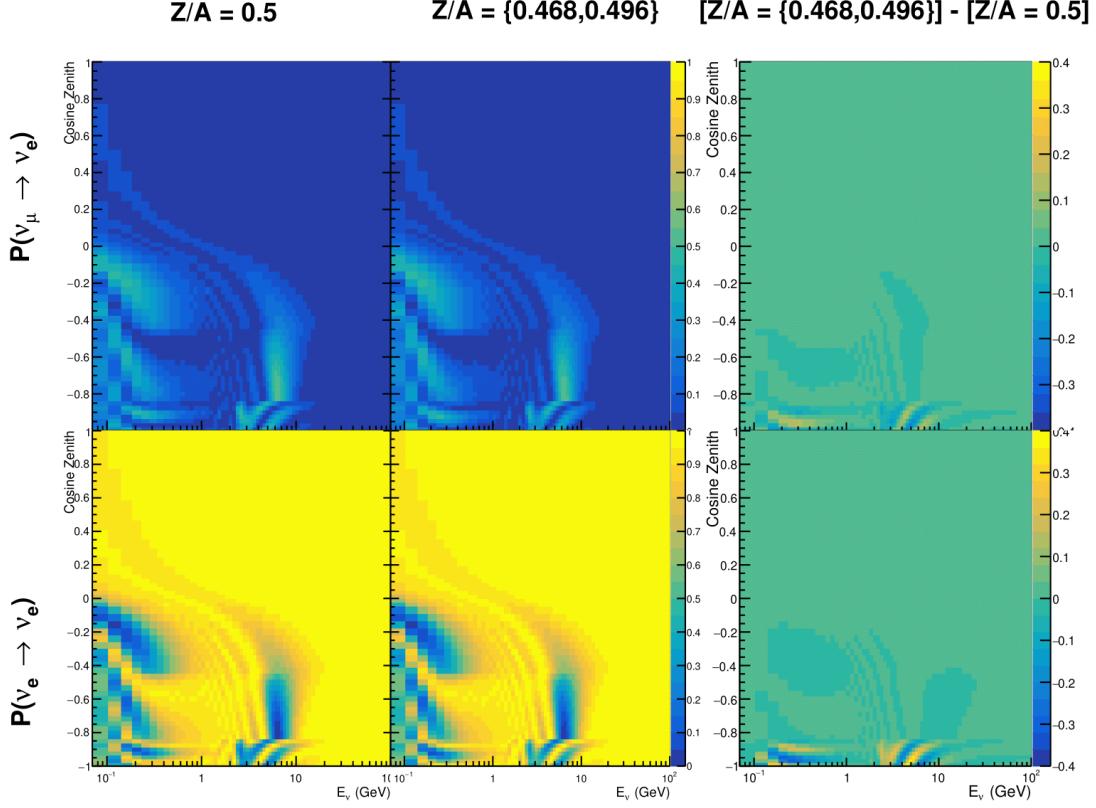


Figure 7.16: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 7.1. The right column illustrates the difference in oscillation probability between the two different techniques.

where t_i are the intersection points between each layer and t is the path length of the trajectory across the layer. This leads to an improved approximation. For this averaging, the simplification of the PREM model developed in [216] is used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [208]. The coefficients of the quadratic fit to each layer are given in Table 7.2 with the final distribution illustrated in Figure 7.17. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

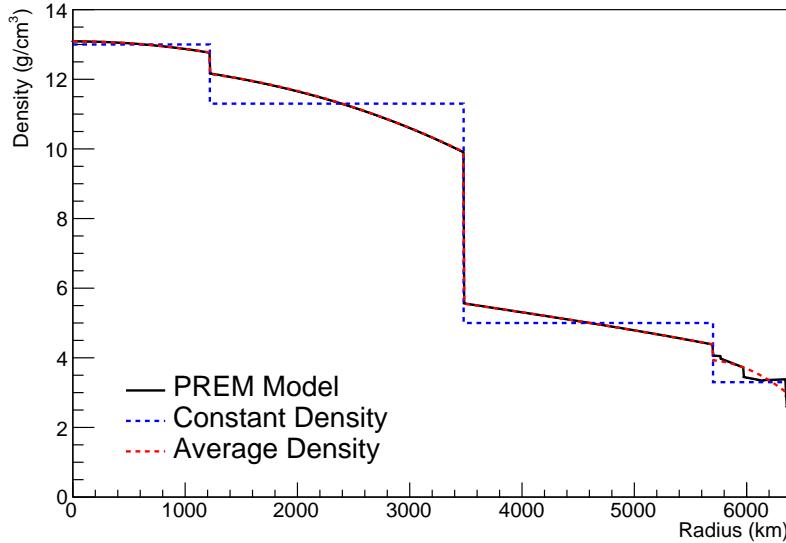


Figure 7.17: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.2: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [208], where $x = R / R_{\text{Earth}}$.

2650 The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted
 2651 in Figure 7.18. The slight discontinuity in the oscillation probability around
 2652 $\cos(\theta_Z) \sim -0.45$ in the fixed density model, which is due to the transition to
 2653 mantle layer boundary, has been reduced. This is expected as the difference in
 2654 the density across this boundary is significantly smaller in the quadratic density
 2655 model as compared to the constant density model. Whilst the difference in
 2656 density across the other layer transitions is reduced, there is still a significant
 2657 difference. This means the discontinuities in the oscillation probabilities remain
 2658 but are significantly reduced. However, as the quadratic density approximation
 2659 matches the PREM model well in this region, these discontinuities are due to the
 2660 Earth model rather than an artifact of the oscillation calculation.

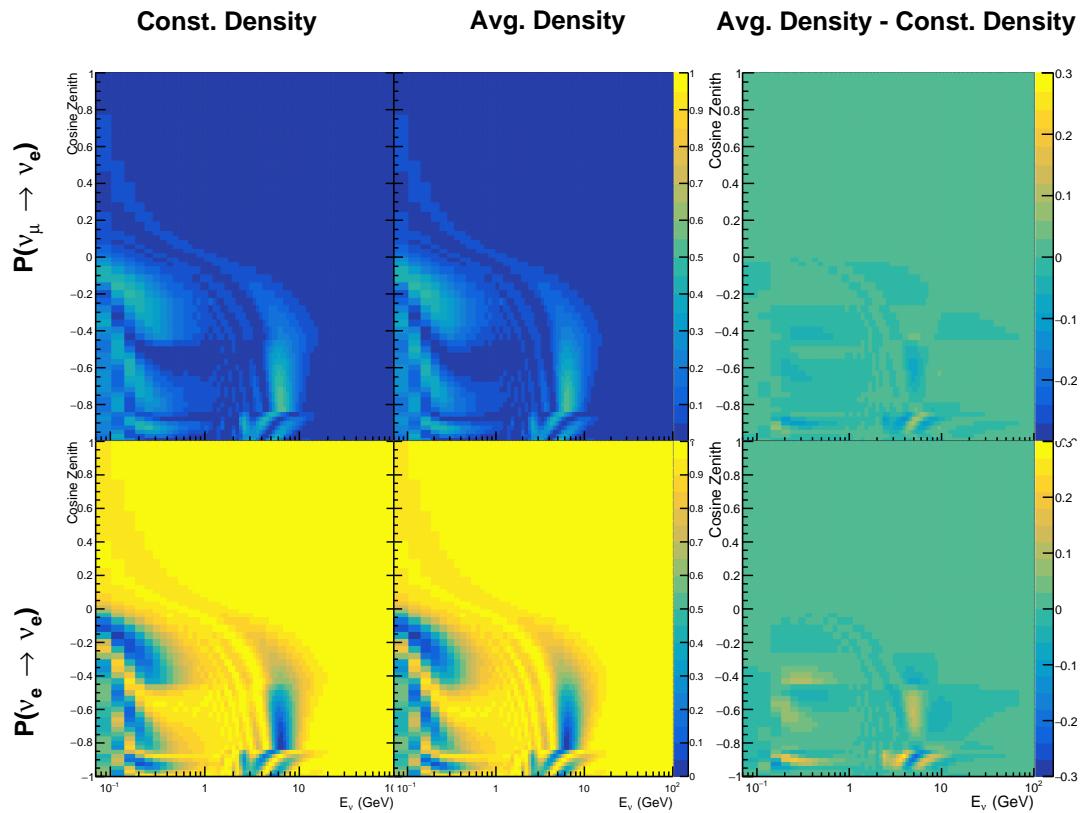


Figure 7.18: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2661 7.5 Production Height Averaging

2662 As discussed in section 7.1, the height at which the cosmic ray flux interacts
 2663 in the atmosphere is not known on an event-by-event basis. The production
 2664 height can vary from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only
 2665 analysis methodology (described in section 7.2) for including the uncertainty
 2666 on the production height is to include variations from the Honda model when
 2667 pre-calculating the oscillation probabilities prior to the fit. This technique is not
 2668 possible for this analysis which uses continuous oscillation parameters that can
 2669 not be known prior to the fit. Consequently, an analytical averaging technique
 2670 was developed in [208]. The author of this thesis was not responsible for the
 2671 derivation of the technique but has performed the implementation and validation
 2672 of the technique for this analysis alone.

2673 Using the 20 production heights per Monte Carlo neutrino event, provided
 2674 as 5% percentiles from the Honda flux model, a production height distribution
 2675 $p_j(h|E_\nu, \cos \theta_Z)$ is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a
 2676 histogram is filled with 20 evenly spaced bins in production height h between
 2677 0 and 50km. The neutrino energy and cosine zenith binning of the histogram
 2678 is the same as that provided in section 7.2. The average production height,
 2679 $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. The production height binning
 2680 of this histogram is then translated into $\delta t(h) = t(\bar{h}) - t(h)$, where $t(h)$ is the
 2681 distance travelled along the trajectory.

2682 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed.
 2683 The time-ordered product of these is then used as the overall transition amplitude
 2684 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.7)$$

2685 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[i a_k (t_{n+1} - t_n)] \end{aligned} \quad (7.8)$$

2686 is expressed as a diagonalised time-dependent solution to the Schrodinger
 2687 equation. The 0th layer is the propagation through the atmosphere and is the
 2688 only term that depends on the production height. Using the substitution $t_0 =$
 2689 $t(\bar{h}) - \delta t(h)$, it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h}) D_0(\delta t). \quad (7.9)$$

2690 Thus Equation 7.7 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, \bar{h}) D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[i a_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[i a_k \delta t]. \end{aligned} \quad (7.10)$$

2691 The oscillation probability averaged over production height is then calculated
 2692 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)^*_{ij} \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m)\delta t] \end{aligned} \quad (7.11)$$

2693 In practice, implementation in CUDAProb3 [211] is relatively straightforward
 2694 as the majority of these terms are already calculated in the standard oscillation
 2695 calculation. Figure 7.19 illustrates the results of the production height averaging.
 2696 As expected, the main effect is observed in the low-energy downward-going
 2697 and horizontal-going events. Upward-going events have to travel the radius
 2698 of the Earth, $R_E = 6371\text{km}$, where the production height uncertainty is a small
 2699 fraction of the total path length.

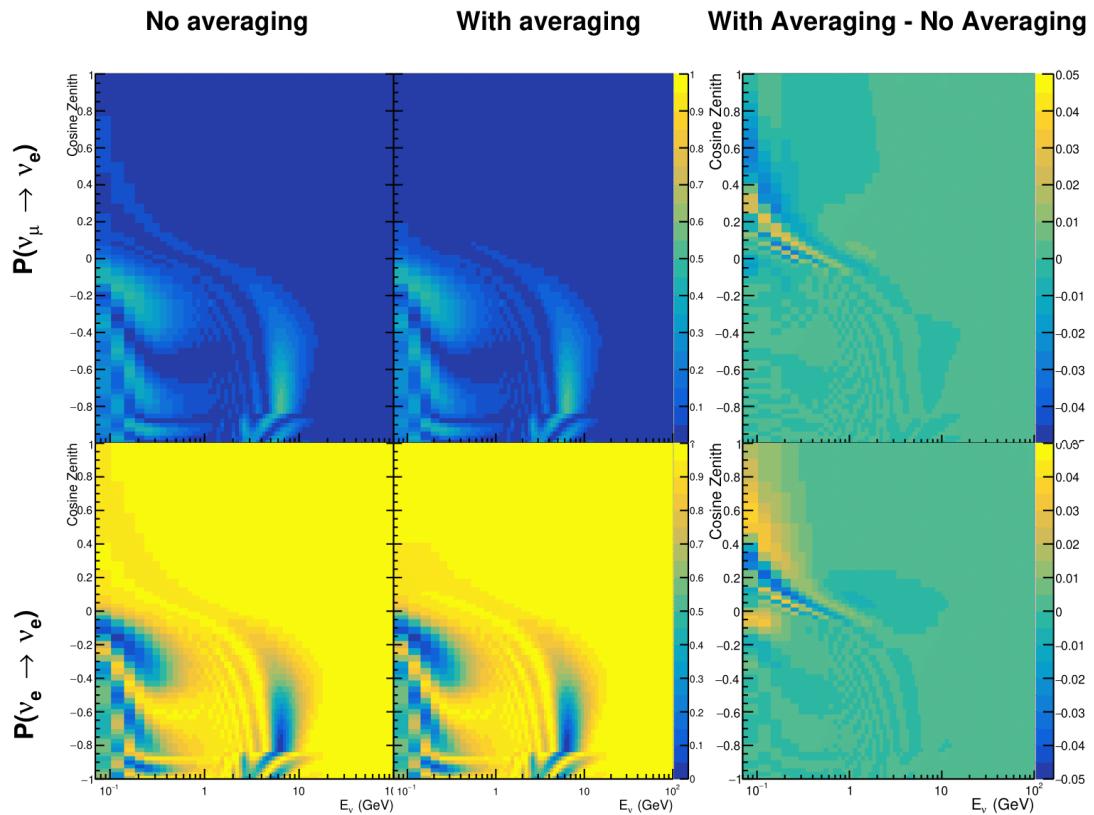


Figure 7.19: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

8

2700

2701

Oscillation Analysis

2702 Using the samples and systematics defined in chapter 6, this chapter documents
2703 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK
2704 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4
2705 is used for all studies performed within this thesis.

2706 The MaCh3 framework used throughout this thesis has been validated through
2707 many tests. The code which handles the beam far detector samples was developed
2708 by the author and validated by comparison to the 2020 T2K analysis [74]. The
2709 sample event rates and likelihood evaluations of beam samples generated by
2710 the framework used within this thesis were compared to those from the T2K
2711 analysis by the author of this thesis. Variations of the sample predictions were
2712 compared at $\pm 1\sigma$ and $\pm 3\sigma$ and good agreement was found in all cases. A similar
2713 study, led by Dr. C. Wret was used to validate the near detector portion of
2714 the code [201]. The implementation of the atmospheric samples within MaCh3
2715 was completed and cross-checked by the author of this thesis against the P-
2716 Theta framework (Introduced in section 3.2). Both fitters are provided with
2717 the same inputs and can therefore cross-validate each other. These validations
2718 compared the event rate and likelihood calculation. Documentation of all the
2719 above validations can be found in [201].

2720 8.1 Monte Carlo Prediction

2721 Using the three sets of dial values (generated, pre-BANFF, and post-BANFF tunes)
 2722 defined in subsection 6.4.3, the predicted event rates for each sample are defined
 2723 in Table 8.1. The oscillated (AsimovA defined in Table 2.2) and un-oscillated
 2724 event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThru	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1: The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

2725 Generally, the samples which target CCQE interaction modes observe a
 2726 decrease in prediction when using the pre-fit dial values. This is in accordance
 2727 with the Monte Carlo being produced assumed $M_A^{QE} = 1.21\text{GeV}$ [161] whilst

the pre-fit dial value is set to $M_A^{QE} = 1.03\text{GeV}$ as suggested by [199]. Furthermore, the predicted event rates of samples that target CCRES interaction modes are significantly reduced when considering the post-BANFF fit. This follows the observations in subsection 6.4.3. The strength of the accelerator neutrino experiment can be seen in the remarkable difference between the oscillated and unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very clear decrease in the expected event rate between the oscillated and un-oscillated predictions which is not as obvious as in the atmospheric samples. This is due to the fact that the beam energy is tuned to the maximum disappearance probability which is not the case for the naturally generated atmospheric neutrinos.

8.2 Likelihood Calculation

This analysis performs a joint oscillation parameter fit of the ND280 beam samples, the T2K far detector beam samples, and the SK atmospheric samples.

Once the Monte Carlo predictions of each beam and atmospheric sample have been built (see chapter 6) a likelihood needs to be constructed. This is done by comparing the binned Monte Carlo prediction to binned data. The Monte Carlo prediction is calculated at a particular point, $\vec{\theta}$, in the model parameter space such that $N_i^{MC} = N_i^{MC}(\vec{\theta})$, where N_i represents the bin content of the i^{th} bin. The data and Monte Carlo spectra are represented by N_i^D and N_i^{MC} , respectively. The bin contents for the beam near detector, beam far detector and atmospheric samples are denoted with *ND*, *FD*, and *Atm*, respectively. The binning index, i , runs over all the bins within a sample. Taking the FHC1Rmu far detector sample as an example, the binning index runs over all the reconstructed neutrino energy bins. The likelihood calculation between the data and the Monte Carlo prediction for a particular bin follows a Poisson distribution, where the data is treated as a fluctuation of the simulation.

The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction, which is typically used for sensitivity studies and denoted ‘Asimov data’. The process for building Asimov data is as follows. The Monte Carlo prediction is

2757 reweighted using a particular set of oscillation parameters (potentially those
 2758 listed in Table 2.2) and systematic parameter tune. The resulting spectra for each
 2759 sample is then defined to be the Asimov data for that sample. Whilst this results
 2760 in unphysical non-integer data predictions, it eliminates statistical fluctuations
 2761 from the data. Therefore, the results of a fit to Asimov data should not include any
 2762 biases from statistical fluctuations. Furthermore, these results should produce
 2763 posterior probability distributions consistent with the parameters which were
 2764 used to make the data prediction. That is to say, the fit results should return the
 2765 known parameters. Any biases seen would be attributed to correlations between
 2766 each oscillation parameter and correlations between oscillation and systematic
 2767 parameters. Consequently, Asimov fit results present the maximum precision
 2768 at which the oscillation parameters could be measured to.

2769 Following the T2K analysis presented in [74], the likelihood contribution for
 2770 the near detector samples also includes a Monte Carlo statistical uncertainty term,
 2771 derived from the Barlow and Beeston statistical treatment [217, 218]. In addition
 2772 to treating the data as a Poisson fluctuation of the Monte Carlo prediction, it
 2773 includes a contribution to the likelihood that which treats the generated Monte
 2774 Carlo prediction as a statistical fluctuation of the actual true simulation assuming
 2775 an infinite amount of statistics had been created. The technical implementation
 2776 of this additional likelihood term is documented in [190] and briefly summarised
 2777 as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (8.1)$$

2778 where β_i represents a scaling parameter for the i^{th} bin that relates the bin
 2779 content for the amount of Monte Carlo actually generated N_i^{MC} to the bin content
 2780 if an infinite amount of Monte Carlo statistics had been generated $N_{i,true}^{MC}$, such
 2781 that $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$. In the case where a sufficient amount of Monte Carlo
 2782 statistics had been generated, $\beta_i = 1$. An analytical solution for β_i is given in
 2783 [190]. Additionally, $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2 / N_i^{MC}}$ where $\sqrt{\sum_i w_i^2}$ represents the sum of
 2784 the square of the weights of the Monte Carlo events which fall into bin i .

An additional contribution to the likelihood comes from the variation of the systematic model parameters. For those parameters with well-motivated uncertainty estimates, a covariance matrix, V , describes the prior knowledge of each parameter as well as any correlations between the parameters. Due to a technical implementation, a single covariance matrix describes each “block” of model parameters, e.g. beam flux systematics. The covariance matrix associated with the k^{th} block is denoted V^k . This substitution results in $\vec{\theta} = \sum_k^{N_b} \vec{\theta}^k$ and $V = \sum_k^{N_b} V^k$ where N_b denotes the number of blocks. A single covariance matrix is provided for: the oscillation parameters, the beam flux parameters, the atmospheric flux parameters, the neutrino interaction systematics, the near detector parameters, the beam far detector systematics, and the atmospheric far detector systematics. All of these are detailed in section 6.4. The number of parameters in the k^{th} block is defined as $n(k)$.

The equation for the likelihood \mathcal{L} includes all the terms discussed above. It is defined as,

$$\begin{aligned}
-\ln(\mathcal{L}) = & \\
& \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
& + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
& + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
& + \frac{1}{2} \sum_k^{N_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
\end{aligned} \tag{8.2}$$

The negative log-likelihood value is determined at each step of the MCMC to build the posterior distribution defined in chapter 4. This value is minimised when the Monte Carlo prediction tends towards the data spectrum.

2803 8.2.1 Likelihood Scans

2804 Using the definition of the likelihood presented in section 8.2, the response of each
2805 sample to a variation of a particular parameter can be studied. Figure 8.1 presents
2806 the variation of all the samples (beam and atmospheric) at the far detector to the
2807 oscillation parameters of interest: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . These plots
2808 are colloquially called ‘likelihood scan’ (or ‘log-likelihood scans’). The process
2809 of making these plots is as follows. An Asimov data set (following technique
2810 detailed in section 8.2) is built using the AsimovA oscillation parameters and
2811 pre-fit systematic tune. The Monte Carlo is then reweighted using the value of
2812 the oscillation parameter at each point on the x-axis of the scan. The likelihood
2813 is then calculated between the Asimov data and Monte Carlo prediction and
2814 plotted. This process identifies which samples drive the determination of the
2815 oscillation parameters in the joint fit.

2816 Due to the caveat of fixed systematic parameters and the correlations between
2817 oscillation parameters being ignored when creating these likelihood scans, the
2818 value of $\chi^2 = 1$ (or $-2 \times \ln(\text{Likelihood}) = 1$) does not equate to the typical
2819 1σ sensitivity. However, it does give an indication of which samples respond
2820 the strongest to a variation in a particular oscillation parameter. The point at
2821 which the likelihood tends to zero illustrates the value of the parameter used
2822 to build the Asimov data prediction.

2823 The sensitivity to $\sin^2(\theta_{23})$ is mostly dominated by the beam muon-like
2824 samples. The response of an individual atmospheric sample is small but non-
2825 negligible such that the summed response over all atmospheric samples becomes
2826 comparable to that of the muon-like beam samples. Consequently, the sensitivity
2827 of the joint fit to $\sin^2(\theta_{23})$ would be expected to be greater than the beam-only
2828 analysis. The only sample which respond to the $\sin^2(\theta_{13})$ oscillation parameter is
2829 the electron-like beam samples. Consequently, no increase in sensitivity beyond
2830 that of the T2K-only analysis would be expected from the joint fit. Regardless, the
2831 sensitivity of the beam sample is significantly weaker than the external reactor
2832 constraint so prior knowledge will dominate any measurement that is included

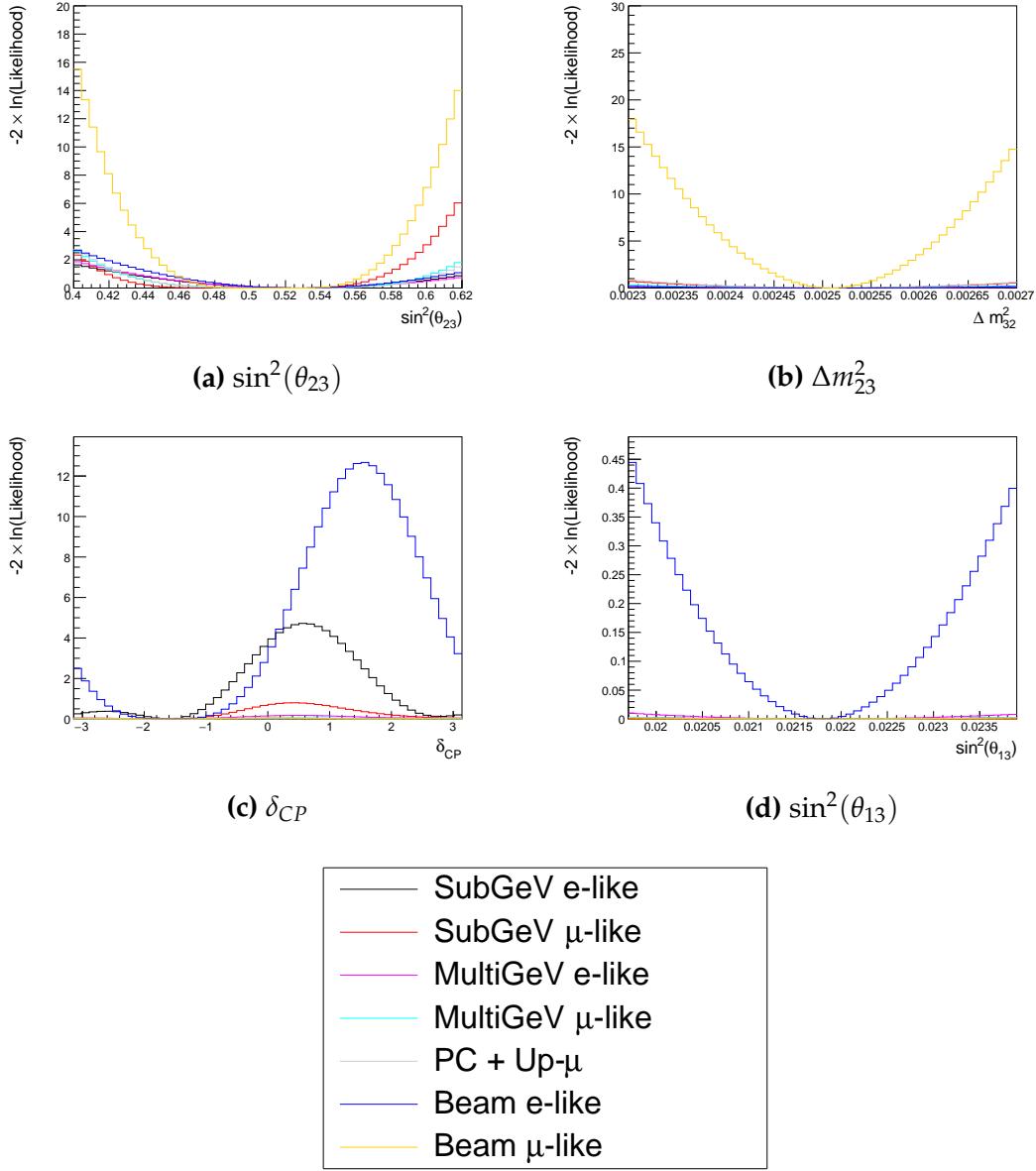


Figure 8.1: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to a variation of an oscillation parameter.

within this thesis. The Δm_{21}^2 and $\sin^2(\theta_{12})$ parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to Δm_{32}^2 is completely dominated by the beam muon-like samples. This is because the beam neutrino energy can be specifically tuned to match the maximal disappearance probability. Despite this, improvements to the $|\Delta m_{32}^2|$ sensitivity may be expected due to additional mass hierarchy determination

2839 added by the atmospheric samples.

2840 Two-dimensional scans of the appearance ($\sin^2(\theta_{13})-\delta_{CP}$) and disappearance
2841 ($\sin^2(\theta_{23})-\Delta m_{32}^2$) parameters are illustrated in Figure 8.2 and Figure 8.3, respec-
2842 tively. The caveat of fixed systematic parameters and correlations between other
2843 oscillation parameters being neglected still apply.

2844 The appearance log-likelihood scans show the distinct difference in how
2845 the beam and atmospheric samples respond. The beam samples have an ap-
2846 proximately constant width of the 2σ and 3σ contours, throughout all ranges
2847 of δ_{CP} . The response of the atmospheric samples to $\sin^2(\theta_{13})$ is very strongly
2848 correlated to the value of δ_{CP} being evaluated, with the strongest constraints
2849 around $\delta_{CP} \sim 1$. Consequently, this difference allows some of the degeneracy in
2850 a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood
2851 scans, the 2σ continuous contour in δ_{CP} for beam samples is broken when the
2852 atmospheric samples are added. This may result in a stronger sensitivity to
2853 δ_{CP} . Similarly, the width of the 3σ contours also becomes dependent upon the
2854 value of δ_{CP} . Whilst these are encouraging results for the joint fit, these are
2855 not sensitivity measurements as the systematic parameters are fixed and the
2856 correlation between oscillation parameters is neglected. An interesting point to
2857 note is that the atmospheric samples have little sensitivity to $\sin^2(\theta_{13})$ on their
2858 own, as evidenced in Figure 8.1, but can improve sensitivity to the parameter
2859 when combined within the simultaneous fit.

2860 The response of the atmospheric samples in Figure 8.2 shows an interesting
2861 behaviour when considering the application of the reactor constraint. At higher
2862 values of $\sin^2(\theta_{13})$, two lobes appear around $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2.4$. If this
2863 distribution was projected onto the δ_{CP} axis, these lobes would mean the posterior
2864 distribution would have a significant dip between these values. However, the
2865 region of $\sin^2(\theta_{13})$ near the reactor constraint ($\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$)
2866 is flatter across the range of δ_{CP} . Therefore, if we were to project only this
2867 region onto the δ_{CP} axis, the dip between the peaks would not be as significant.
2868 If this behaviour was to be seen in the results of a fit, these marginalisation

2869 effects would actually conspire to reduce the sensitivity to δ_{CP} if the reactor
2870 constraint was to be applied.

2871 The disappearance log-likelihood scans in $\sin^2(\theta_{23})$ - Δm_{32}^2 space (Figure 8.3)
2872 show the expected behaviour when considering the one-dimensional scans
2873 already discussed. The uncertainty on the width of $|\Delta m_{32}^2|$ is mostly driven
2874 by the beam-only sensitivities. However, the width of this contour in the
2875 inverted mass region ($\Delta m_{32}^2 < 0$) is significantly reduced due to the ability
2876 of the atmospheric samples to select the correct (normal) mass hierarchy. The
2877 width of the uncertainty in $\sin^2(\theta_{23})$ is also reduced compared to the beam-only
2878 sensitivities, with a further decrease in the inverted hierarchy region due to the
2879 better mass hierarchy determination.

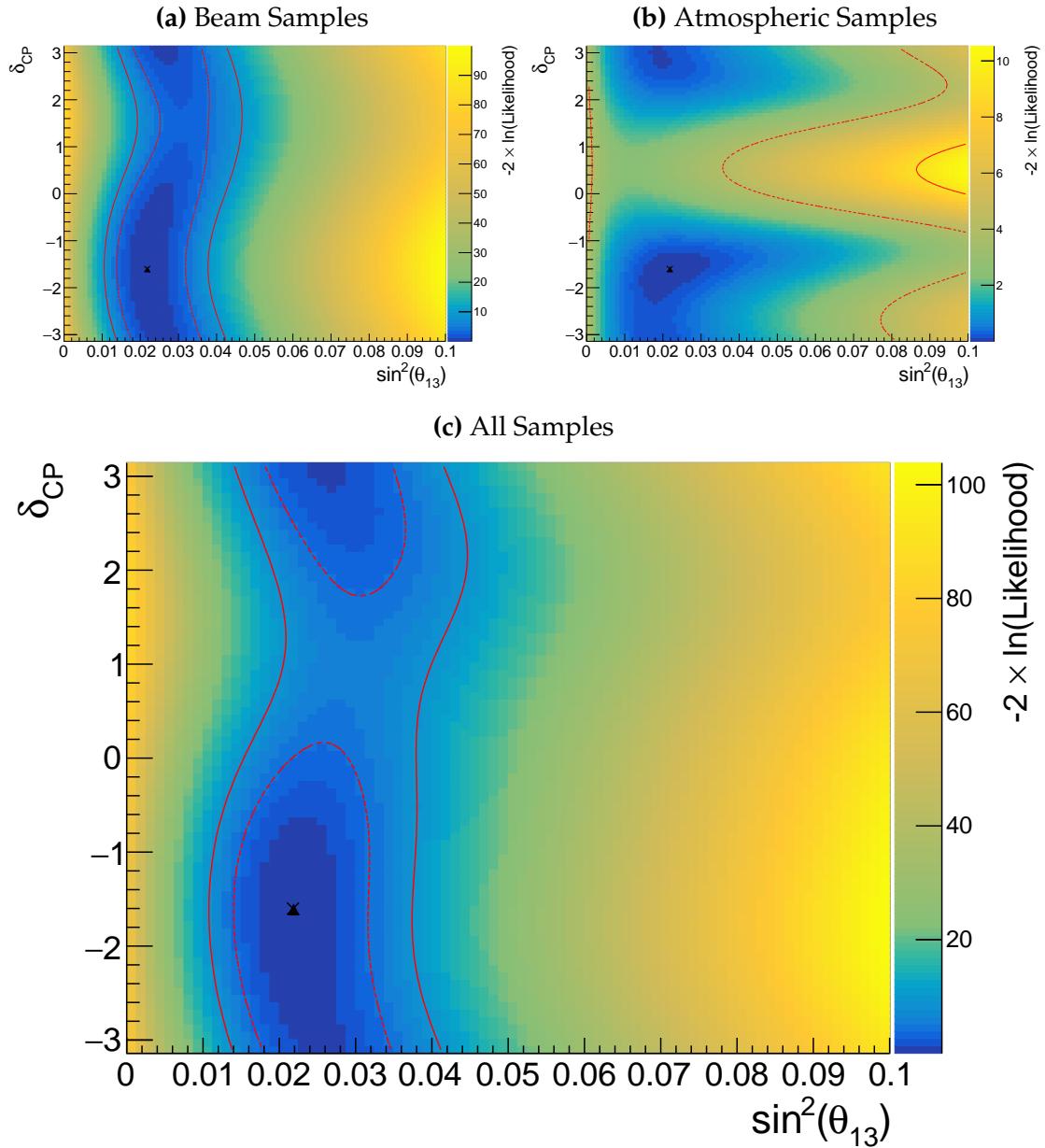


Figure 8.2: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})$ - δ_{CP}) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

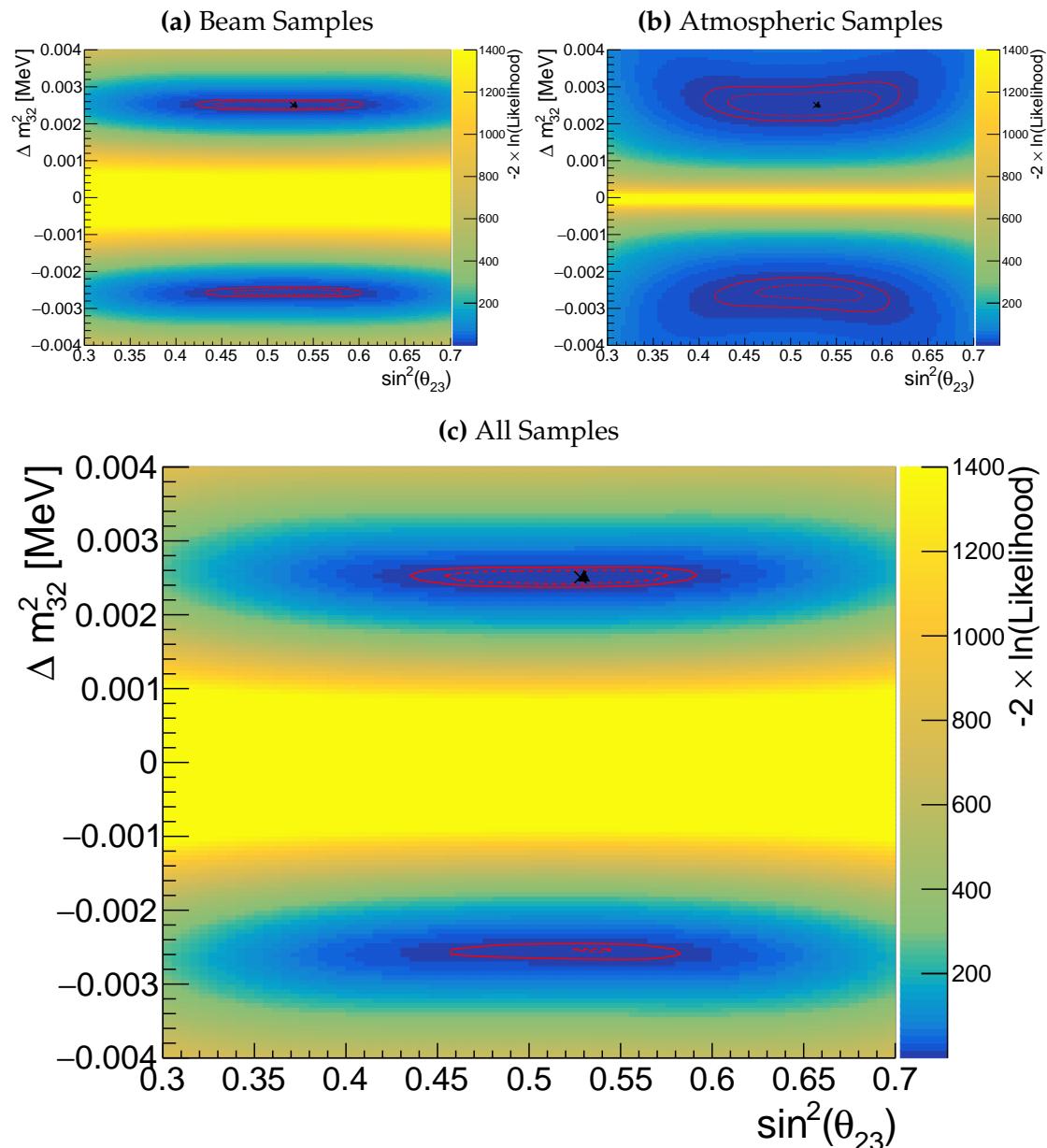


Figure 8.3: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23})$)- Δm_{32}^2) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

2880 The likelihood scans illustrated thus far only consider the sensitivity of this
2881 analysis for a fixed set of true oscillation parameters, namely Asimov A defined in
2882 Table 2.2. Whilst computational infeasible to run many fits at different parameter
2883 sets, it is possible to calculate the likelihood response to different Asimov data
2884 sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing
2885 true values of δ_{CP} and $\sin^2(\theta_{23})$, respectively. For both of these plots, the other
2886 oscillation parameters are fixed at their Asimov A values. Consequently, the
2887 caveat of fixed systematic parameters and correlations between other oscillation
2888 parameters being neglected still applies.

2889 To explain how these plots are made, consider Figure 8.4. This plot is built
2890 by considering multiple one-dimensional log-likelihood scans, each creating
2891 an Asimov data with the value of δ_{CP} taken from the x-axis. The likelihood to
2892 a particular Asimov data set is calculated after reweighting the Monte Carlo
2893 prediction to each value of δ_{CP} on the y-axis.

2894 Figure 8.4 illustrates the sensitivity to δ_{CP} . Notably, the 1σ intervals contain
2895 regions in the off-diagonal for which the beam and atmospheric samples have
2896 broken and discontinuous contours. This indicates that there are regions of δ_{CP}
2897 which are degenerate. For example, for the x-axis value of $\delta_{CP} = 0$, the beam
2898 samples sensitivity would include two discontinuous regions excluded from
2899 the 1σ interval: $\delta_{CP} \sim 0$ and $\delta_{CP} \sim \pi$. The offset in δ_{CP} between the beam
2900 and atmospheric samples allows the joint fit to have increased sensitivity in
2901 these regions. Consequently, the difference between the beam-only and joint
2902 beam-atmospheric fit should be studied using multiple Asimov data sets.

2903 Despite the increased sensitivity at 1σ , the 2σ intervals from the joint fit
2904 are more similar to the two independent sensitivities and the off-diagonal de-
2905 generacies mostly remain. This indicates that the joint fit has the strength to
2906 aid parameter determination but can not entirely break the degeneracies in δ_{CP}
2907 at higher confidence levels.

2908 Figure 8.5 illustrates a similar analysis as above, although the value of $\sin^2(\theta_{23})$
2909 is varied and δ_{CP} is fixed to the Asimov A parameter value. Due to the beam

parameters and baseline being tuned to specifically target this oscillation parameter, the average sensitivity of the beam samples is stronger than the atmospheric samples. However, the degeneracy around maximal mixing ($\sin^2(\theta_{23}) = 0.5$) is significantly more peaked in the beam samples compared to the atmospheric samples. This behaviour is strengthened when considering the 2σ intervals, to the point where two distinct discontinuous regions of the 2σ intervals exist around the Asimov point $\sin^2(\theta_{23}) \sim 0.41, 0.6$. Given the caveat of only considering likelihood scans, the joint analysis would mostly eliminate the discontinuous intervals in these regions. This means that the joint fit could feasibly have an increased preference for the correct octant hypothesis.

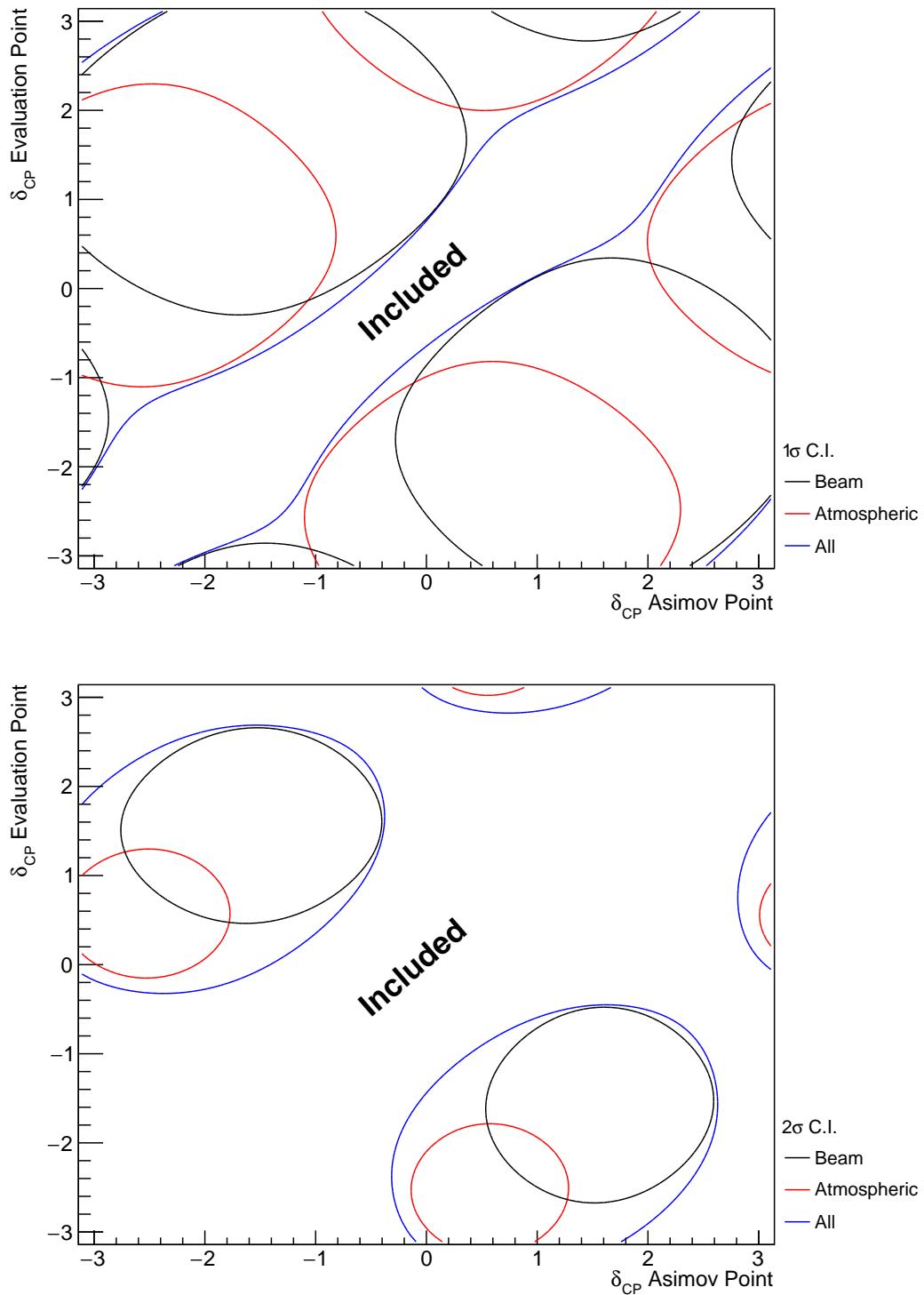


Figure 8.4: A series of one-dimensional likelihood scans over δ_{CP} , where an Asimov data set is built for each value of δ_{CP} on the x-axis and the likelihood is evaluated for each value of δ_{CP} on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

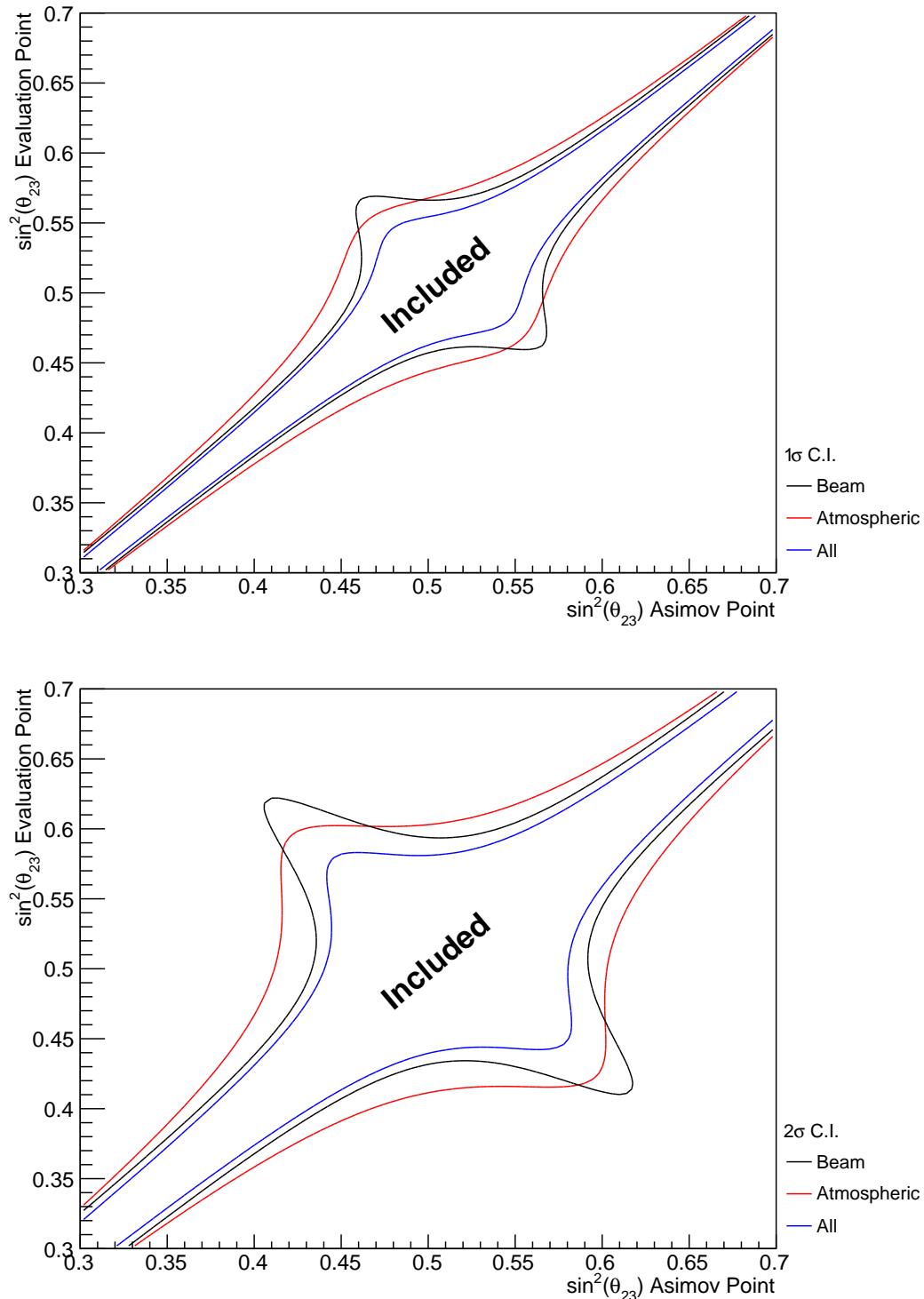


Figure 8.5: A series of one-dimensional likelihood scans over $\sin^2(\theta_{23})$, where an Asimov data set is built for each value of $\sin^2(\theta_{23})$ on the x-axis and the likelihood is evaluated for each value of $\sin^2(\theta_{23})$ on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than what they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions, M_A^{QE} , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h_Cto0 normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics which have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric samples are significantly more sensitive to these systematics than the beam samples due to their similar interaction contributions but relatively higher statistics (Table 8.1). As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be considerably more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.

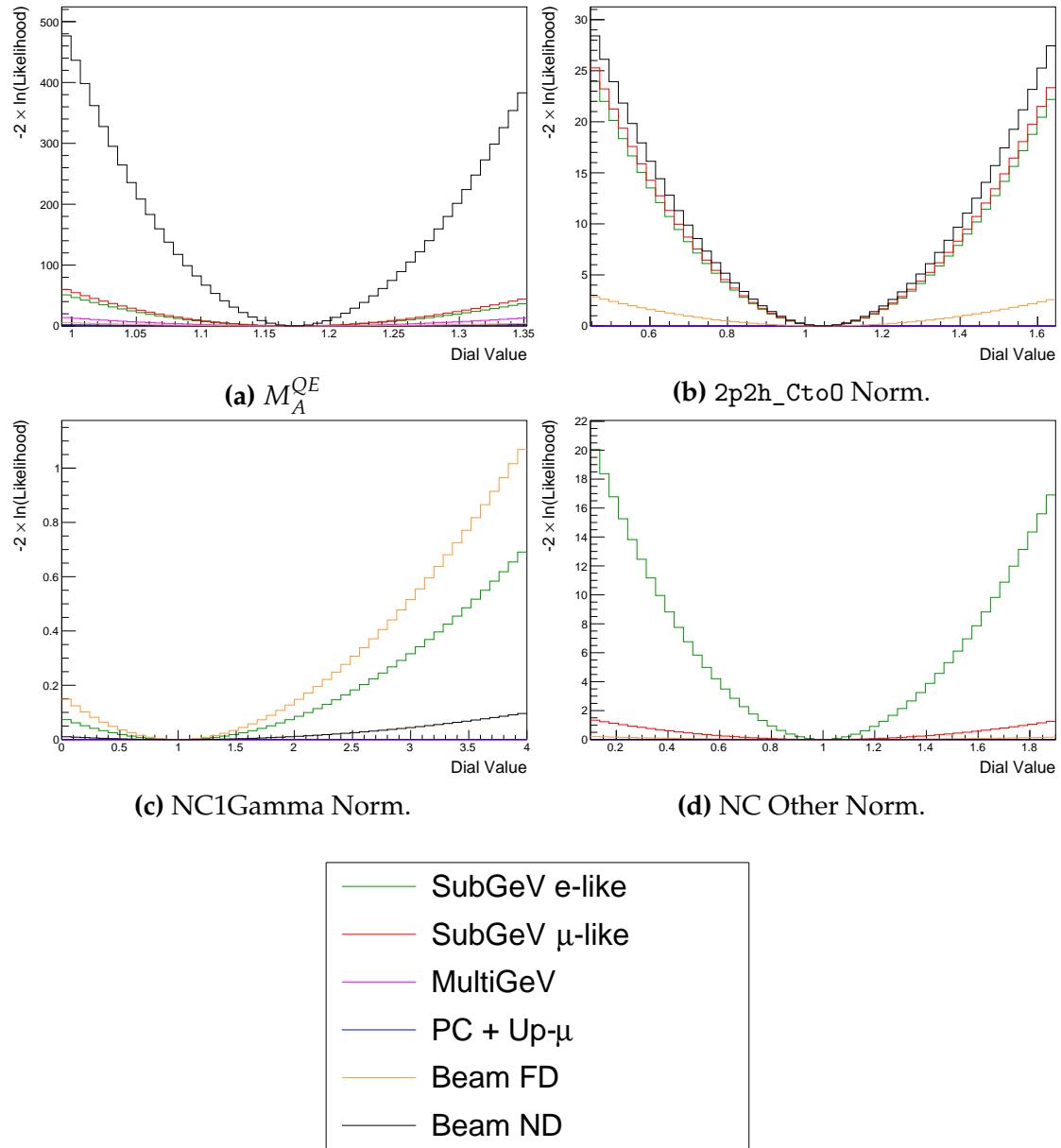


Figure 8.6: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

2945 8.3 Sensitivities

2946 The sensitivities of the joint T2K and SK oscillation analysis are presented in
2947 the form of Asimov fits. This technique builds an Asimov data set (follow-
2948 ing section 8.2) using the AsimovA oscillation parameters and post-BANFF
2949 systematic tune.

2950 In practice, the Asimov fits presented within this analysis are modified from
2951 the above definition. An Asimov prediction of both beam and atmospheric far
2952 detector samples is fit whilst the true data is used for near detector samples.
2953 The Asimov predictions at the far detector are built using the BANFF tuning (as
2954 discussed in section 3.2). These modifications mean that the results are equivalent
2955 to performing a far detector Asimov fit using inputs from the BANFF data fit.
2956 Consequently, this allows the results to be cross-checked with the results from
2957 the P-Theta analysis. The comparison has been performed and is documented in
2958 [219]. No significant discrepancies were found between the fitters.

2959 This section proceeds with the following studies. Firstly, the sensitivity of
2960 the atmospheric samples after the T2K cross-section has been applied to the low-
2961 energy events is detailed in subsection 8.3.1. This includes studying the choice of
2962 applying the 2020 PDG reactor constraint [75] to the atmospheric samples, which
2963 is documented in subsection 8.3.2. Additionally, the effect of applying the near-
2964 detector constraints onto the atmospheric samples is discussed in subsection 8.3.3.
2965 The main result is the sensitivity of the simultaneous beam and atmospheric fit.
2966 The sensitivities, both with and without the application of the reactor constraint,
2967 are presented in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate
2968 the benefit of the joint analysis, the sensitivities are compared to the 2020 T2K
2969 sensitivities [74, 182] in subsection 8.3.6 and subsection 8.3.7. As shown in
2970 subsection 8.2.1, the response of the beam and atmospheric samples change
2971 depending upon the true set of oscillation parameters assumed. Therefore,
2972 subsection 8.3.8 documents the sensitivities at an alternative oscillation parameter

²⁹⁷³ set. It is important to note that these results have been published at the Neutrino
²⁹⁷⁴ 2022 conference on behalf of the T2K and SK collaborations [108].

2975 8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

2976 This section presents the results of an Asimov fit using samples from the near
 2977 detector and only atmospheric samples from the far detector. The results are
 2978 presented as one-dimensional or two-dimensional histograms which have been
 2979 marginalised over all other parameters using the technique outlined in sub-
 2980 section 4.3.1. Each histogram displays the posterior probability density and
 2981 illustrates the credible intervals, calculated using the technique in subsection 4.3.2.
 2982 For these fits in this subsection, a flat penalty term is used for $\sin^2(\theta_{13})$ such the
 2983 reactor constraint is not applied. The Asimov data is generated assuming the
 2984 AsimovA oscillation parameter set defined in Table 2.2 and the post-BANFF
 2985 systematic parameter tune.

Without Reactor Constraint, Both Hierarchies

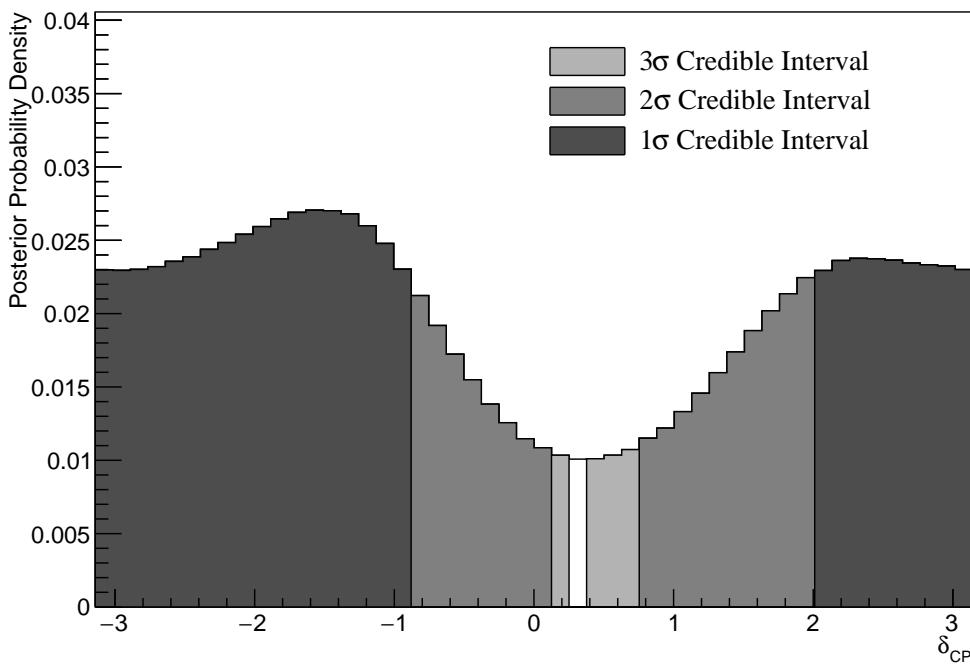


Figure 8.7: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

2986 Figure 8.7 illustrates the posterior probability density for δ_{CP} , marginalised
 2987 over both hierarchies. If instead, only steps in the normal hierarchy were
 2988 considered, the shape of the contours would change. The fit favours the known

oscillation parameter ($\delta_{CP} = -1.601$) although the posterior probability is very flat through the range of $-\pi < \delta_{CP} < -1$ and $2 < \delta_{CP} < \pi$. There is also a region around $\delta_{CP} \sim 0.4$ which is disfavoured at 2σ . This indicates that the SK samples can rule out some parts of the CP conserving parameter space reasonably well, near $\delta_{CP} \sim 0.4$, when the true value of $\delta_{CP} \sim \pi/2$.

Without Reactor Constraint, Both Hierarchies

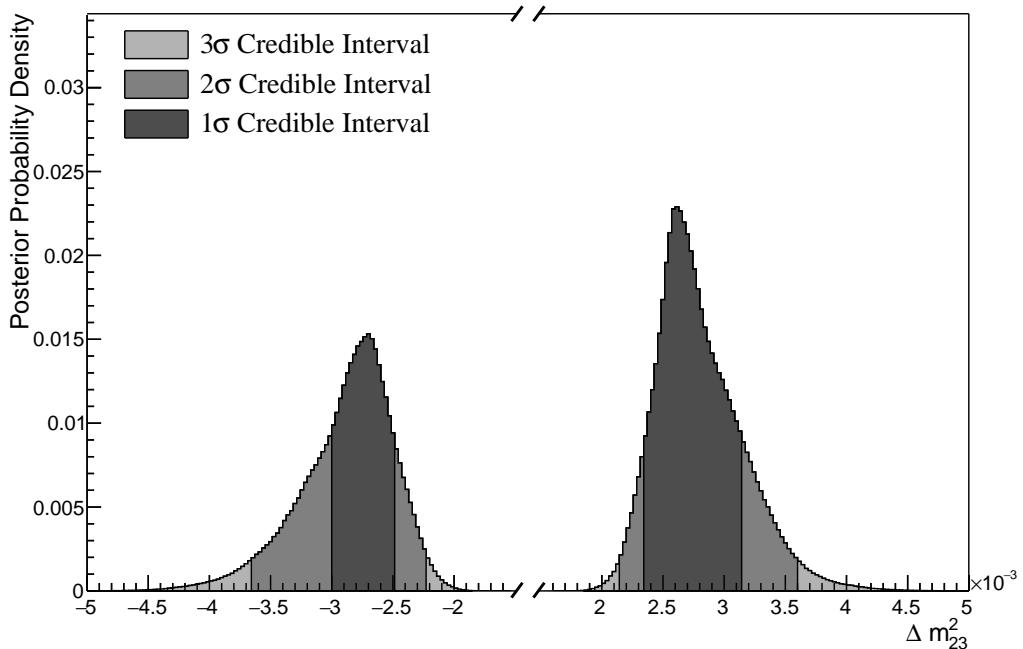


Figure 8.8: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

The posterior probability density in Δm_{32}^2 is given in Figure 8.8. This distribution includes steps in both the normal hierarchy (NH, $\Delta m_{32}^2 > 0$) and the inverse hierarchy (IH, $\Delta m_{32}^2 < 0$). The highest posterior probability density is found within the NH, which agrees with the known oscillation parameter value. However, all of the credible intervals span both of the hierarchies hypotheses. If instead, only steps in the normal hierarchy were considered, the shape of the contours would change. The known oscillation parameter is $2.509 \times 10^{-3} \text{ eV}^2$, which is contained within the 1σ credible interval.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.17	0.40	0.58
IH ($\Delta m_{32}^2 < 0$)	0.13	0.29	0.42
Sum	0.31	0.69	1.00

Table 8.2: The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.37$ and $B(\text{UO}/\text{LO}) = 2.24$.

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO, $\sin^2 \theta_{23} < 0.5$) or upper octant (UO, $\sin^2 \theta_{23} > 0.5$). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy model is $B(\text{NH}/\text{IH}) = 1.37$. Table 4.1 states this value of the Bayes factor indicates a weak preference for the normal hierarchy model. The Bayes factor for choice of octant is $B(\text{UO}/\text{LO}) = 2.24$. This is also classified as a weak preference for the UO. Both of these show that the fit is returning the correct choice of models (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The 1σ credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. For the known Asimov value of $\delta_{CP} = -1.601$, the 1σ credible interval rules out a region between $\delta_{CP} = -0.86$ and $\delta_{CP} = 1.96$, when marginalising over both hierarchies. The position of the highest posterior density is $\delta_{CP} = -1.57$ which is clearly compatible with the known oscillation parameter value.

The sensitivity of the atmospheric samples to $\sin^2(\theta_{13})$ is presented in Figure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -0.86], [1.96, \pi]$	-1.57
δ_{CP} , (NH)	$[-\pi, -0.86], [1.88, \pi]$	-1.57
δ_{CP} , (IH)	$[-\pi, -0.94], [1.96, \pi]$	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-3.00, -2.50], [2.35, 3.15]$	2.65
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.39, 3.04]$	2.64
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-3.15, -2.45]$	-2.70
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.59]$	0.542
$\sin^2(\theta_{23})$ (NH)	$[0.476, 0.59]$	0.554
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.59]$	0.542

Table 8.3: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3024 to $\sin^2(\theta_{13})$ will be small. This behaviour is also seen in the fit results, where the
 3025 width of the 1σ credible intervals span the region of $\sin^2(\theta_{13}) = [0.008, 0.08]$. This
 3026 is more than an order of magnitude worse than the constraint from reactor
 3027 experiments [75].

3028 As previously discussed, the correlations between oscillation parameters are
 3029 also important to understand how the atmospheric samples respond. Figure 8.10
 3030 illustrates the two dimensional $\sin^2(\theta_{13}) - \delta_{CP}$ sensitivity, marginalised over all
 3031 other parameters. The displayed contours are calculated by marginalising over
 3032 both hierarchies. The shape of the 1σ credible interval shows that the constraining
 3033 power of the fit on δ_{CP} is dependent upon the value of $\sin^2(\theta_{13})$. Furthermore,
 3034 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.
 3035 Whilst the atmospheric samples do not strongly constrain the value of $\sin^2(\theta_{13})$,
 3036 the value of $\sin^2(\theta_{13})$ does impact the atmospheric sensitivity to δ_{CP} . A value of
 3037 $\sin^2(\theta_{13}) \sim 0.02$ would select a continuous contour over all values of δ_{CP} . This
 3038 shows the effect of the marginalisation effect previously described.

Without Reactor Constraint, Both Hierarchies

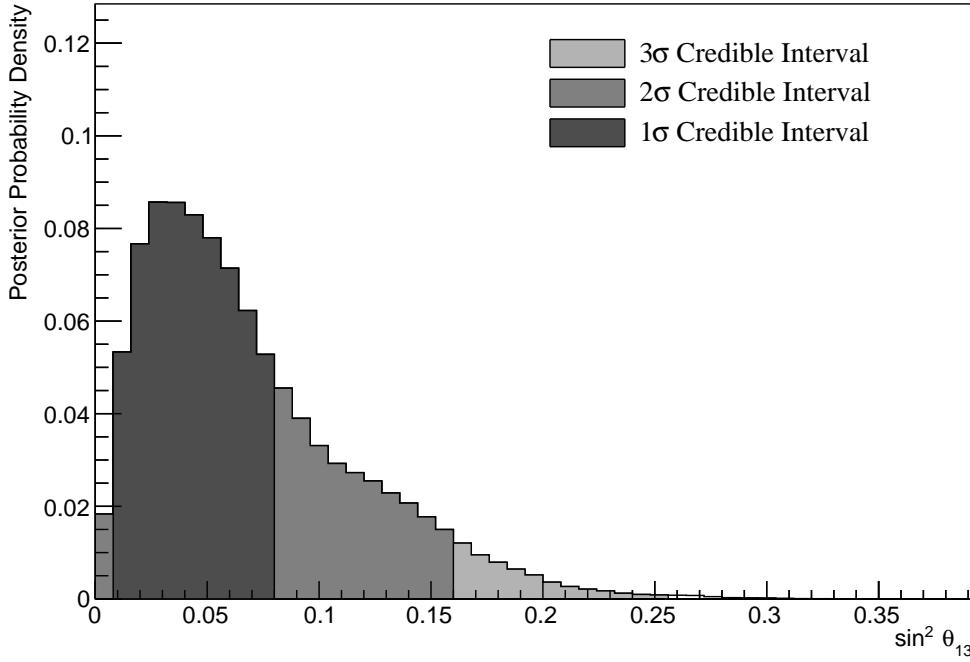


Figure 8.9: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3039 The $\sin^2(\theta_{23}) - \Delta m_{32}^2$ disappearance contours are illustrated in Figure 8.11.

3040 As expected, the area contained in the inverted hierarchy 1σ credible interval is
 3041 slightly smaller than that in the normal hierarchy. This follows from the Bayes
 3042 factor showing a weak preference for NH meaning that more of the steps will exist
 3043 in the $\Delta m_{32}^2 > 0$ region. The known oscillation parameters of $\sin^2(\theta_{23}) = 0.528$
 3044 and $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ are contained within the 1σ credible interval.

3045 Figure 8.12 illustrates the two-dimensional projections for each permutation of
 3046 oscillation parameters which this analysis is sensitive to: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$,
 3047 and Δm_{32}^2 . The purpose of this plot is to illustrate the correlations between the
 3048 oscillation parameters. The contours are calculated whilst marginalising over
 3049 both hierarchies, however, only the NH is illustrated when plotting the Δm_{32}^2
 3050 parameter. As expected the correlations play a significant role in these sensitivity
 3051 measurements, especially the choice of the $\sin^2(\theta_{13})$ constraint. The application
 3052 of reactor constraint would be expected to alter both the width and position of the

Without Reactor Constraint, Both Hierarchies

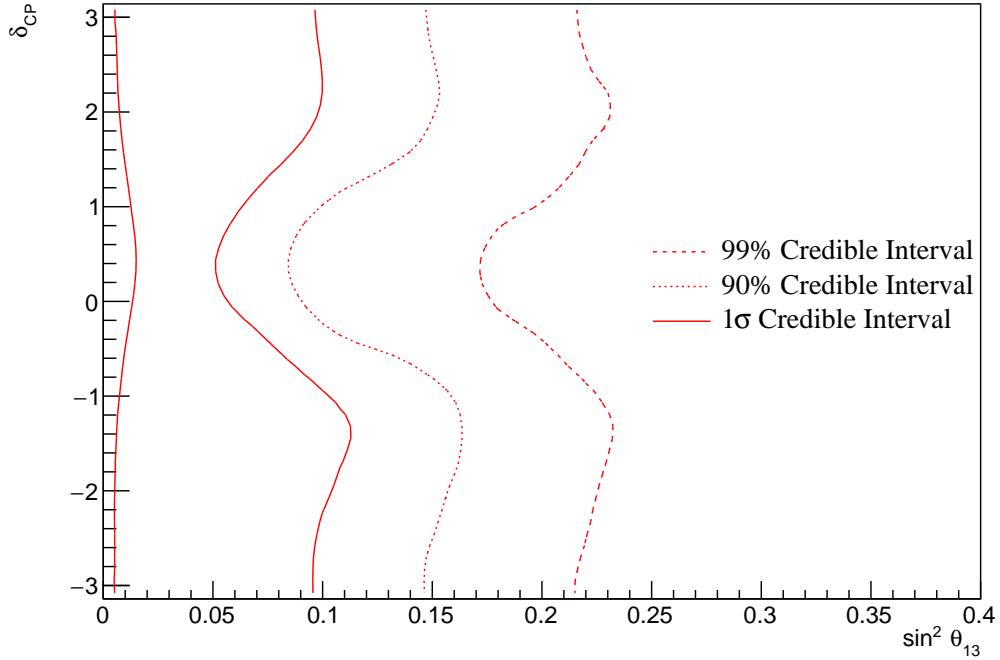


Figure 8.10: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

3053 Δm_{32}^2 , δ_{CP} , and $\sin^2(\theta_{23})$ constraints. The majority of the octant model preference
 3054 comes from the region of $\sin^2(\theta_{13}) \sim 0.03$ such that the application of the reactor
 3055 constraint would not be expected to significantly change the octant preference.
 3056 The reactor constraint would result in lower values of $|\Delta m_{32}^2|$. Interestingly, the
 3057 distribution of steps in the δ_{CP} - $\sin^2(\theta_{13})$ plot is slightly flatter in the region of the
 3058 reactor constraint. Both the posterior distribution from this fit and the distribution
 3059 in Figure 8.2 show a region of low negative log-likelihood extending out towards
 3060 higher values of $\sin^2(\theta_{13})$ in the $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2$ region. Consequently,
 3061 the reactor constraint could feasibly reduce the sensitivity of the atmospheric
 3062 samples to δ_{CP} , due to the previously discussed marginalisation effects.

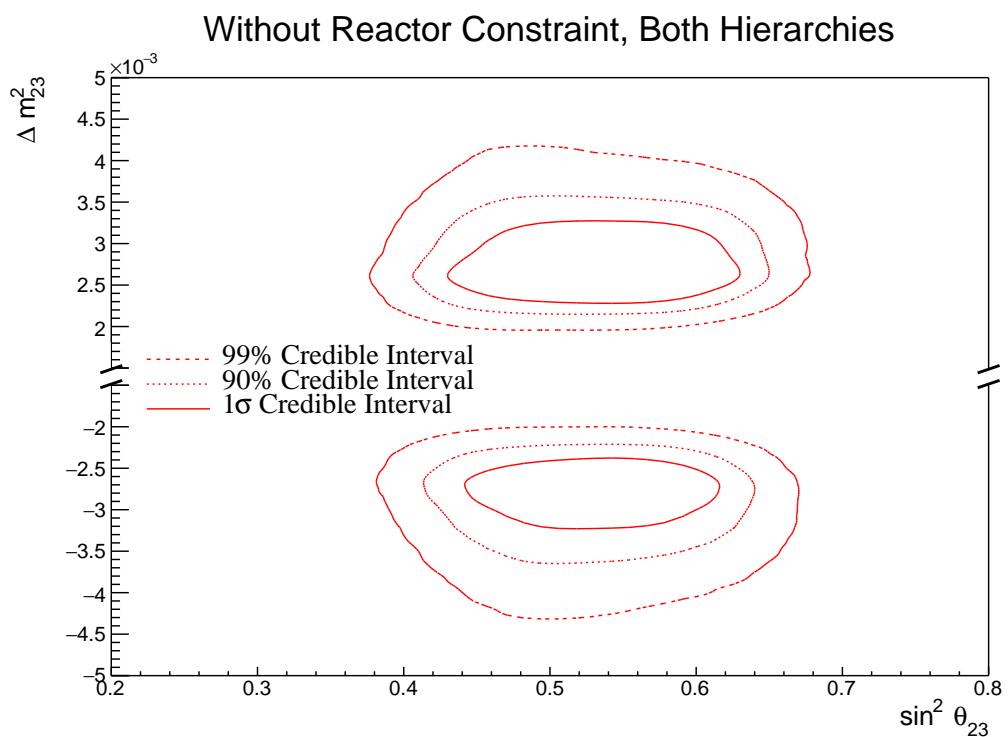


Figure 8.11: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied.

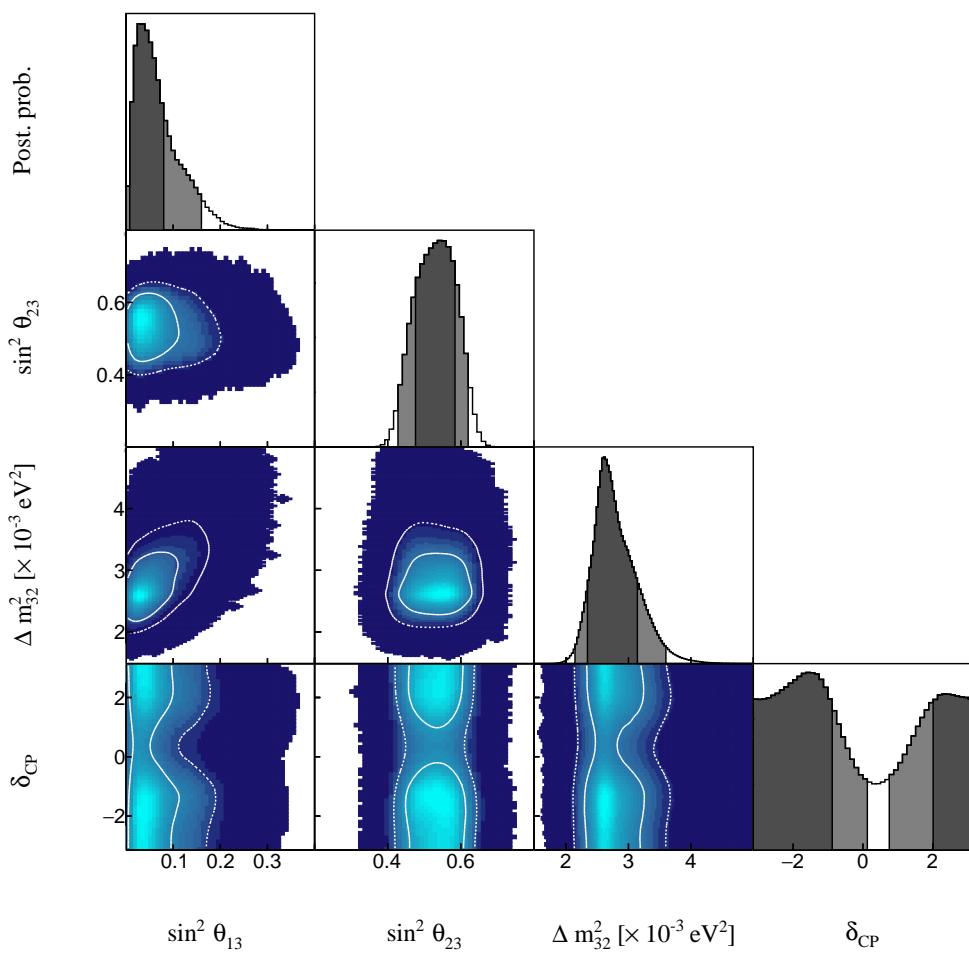


Figure 8.12: The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3063 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

3064 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor
 3065 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that
 3066 the marginalisation effects could contribute to differing sensitivities when the
 3067 external reactor constraint is applied. Using the technique discussed in subsec-
 3068 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted
 3069 to include the reactor constraint of $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ [75].

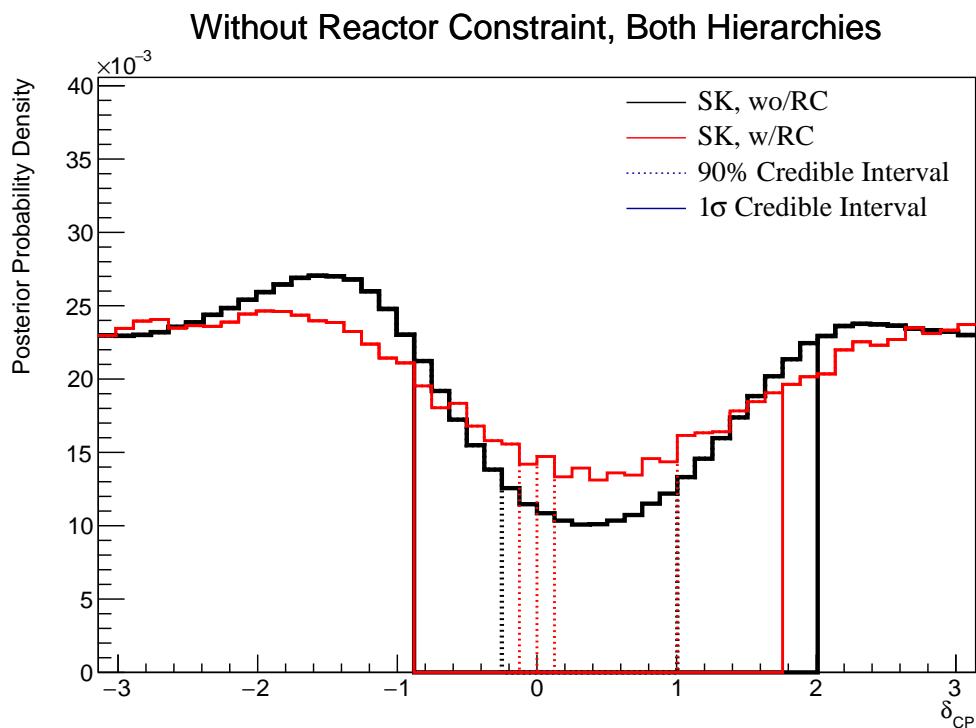


Figure 8.13: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

3070 Figure 8.13 illustrates the sensitivity to δ_{CP} of the atmospheric fit with reactor
 3071 constraint applied. The distribution is less peaked than the previous results.
 3072 This is due to the expected marginalisation effect previously discussed. The
 3073 width of the 1σ credible interval is increased when the reactor constraint is
 3074 applied, indicating less sensitivity to δ_{CP} in the region of $\sin^2(\theta_{13})$ preferred
 3075 by the reactor constraint.

Without Reactor Constraint, Both Hierarchies

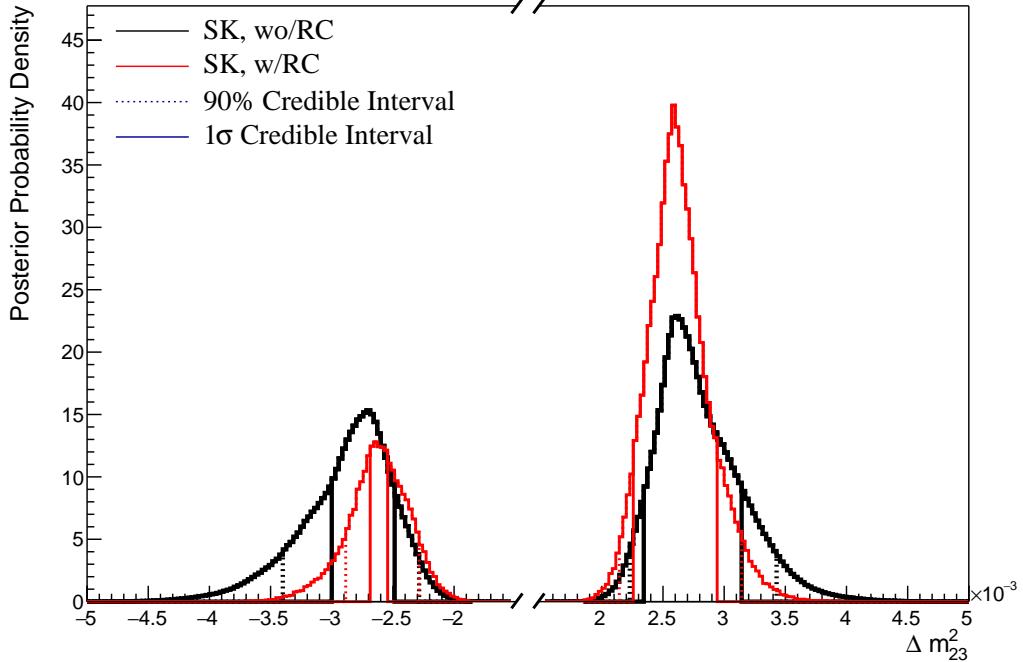


Figure 8.14: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies.

3076 The reactor constraint increases the sensitivity of the atmospheric samples to
 3077 Δm_{32}^2 as illustrated in Figure 8.14. The 1σ credible interval in Δm_{32}^2 is determined
 3078 to be $[-2.70, -2.55] \times 10^{-3}\text{eV}^2$ and $[2.25, 2.95] \times 10^{-3}\text{eV}^2$. The width of the IH
 3079 credible interval is reduced by $\sim 70\%$ when the reactor constraint is applied. Due
 3080 to the marginalisation effects observed in Figure 8.12, the favoured region of Δm_{32}^2
 3081 moves closer to zero for both hierarchies. A clear explanation of this behaviour
 3082 is illustrated in Figure 8.15 which illustrates the posterior distribution in the
 3083 $\Delta m_{32}^2 - \sin^2(\theta_{13})$ parameters, marginalised over both hierarchies. The correlation
 3084 between Δm_{32}^2 and $\sin^2(\theta_{13})$ is such that lower values of $\sin^2(\theta_{13})$ tend towards
 3085 lower values of $|\Delta m_{32}^2|$. This moves the posterior distribution towards the known
 3086 oscillation parameter $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$.

3087 Table 8.4 presents the fraction of steps in each hierarchy and octant model
 3088 for the fit after the reactor constraint has been applied. The reactor constraint
 3089 significantly increases the NH preference, increasing the Bayes factor from

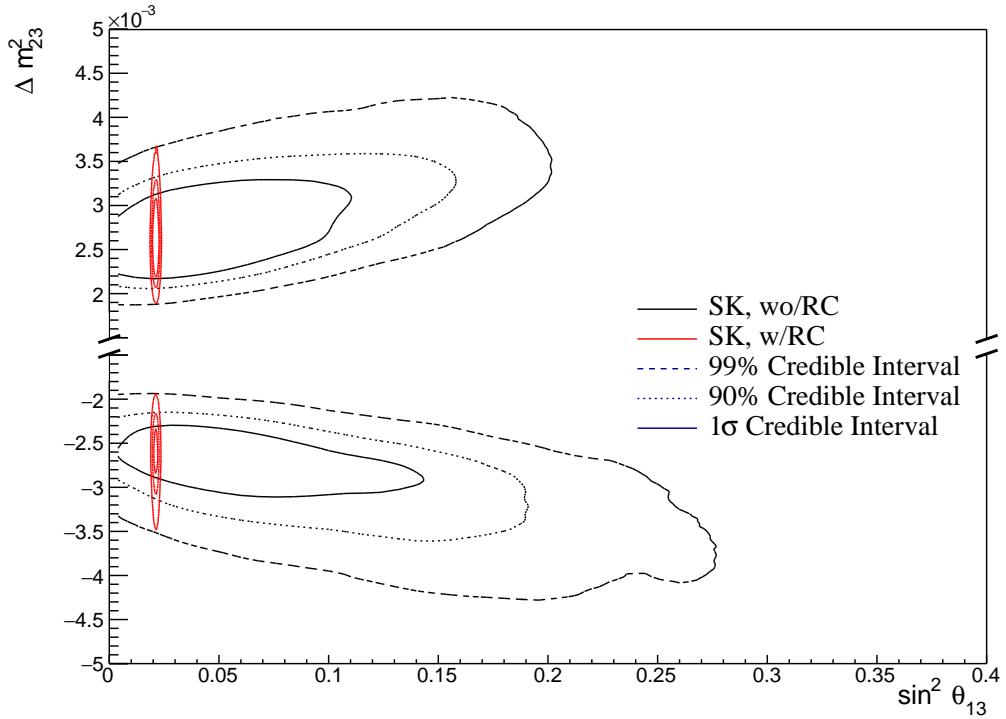


Figure 8.15: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{13})$ compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.21	0.53	0.74
IH ($\Delta m_{32}^2 < 0$)	0.08	0.18	0.26
Sum	0.29	0.71	1.00

Table 8.4: The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 2.86$ and $B(\text{UO}/\text{LO}) = 2.39$.

3090 $B(\text{NH}/\text{IH}) = 1.37$ to $B(\text{NH}/\text{IH}) = 2.86$ when the reactor constraint is applied.

3091 This is still defined as a weak preference for NH hypothesis according to Jeffrey's

3092 scale (see Table 4.1), however, it is a stronger preference than when the constraint

3093 is not applied. The preference for the correct octant model is slightly increased

3094 by the application of the reactor constraint which is consistent with expectation.

3095 However, the conclusion that would be made does not significantly change.

3096 8.3.3 Application of Near Detector Constraints for Atmospheric 3097 Samples

3098 The choice of applying the near detector constraints to the low-energy atmo-
3099 spheric samples was introduced in subsection 6.4.3. This subsection illustrates
3100 the effect of that choice on the sensitivities of the atmospheric samples to the
3101 oscillation parameters. This Asimov data was generated assuming the ‘AsimovA’
3102 oscillation parameter set defined in Table 2.2 and the post-BANFF systematic
3103 parameter tune.

3104 The change in sensitivity on δ_{CP} is given in Figure 8.16. The reactor constraint
3105 is not applied in either of the fits within this comparison. The shape of the
3106 posterior is similar although less peaked at the Asimov point ($\delta_{CP} = -1.601$)
3107 and more symmetric between the regions of $\delta_{CP} = -1.601$ and $\delta_{CP} \sim 2.5$. The
3108 width of the 1σ credible intervals are approximately the same (identical to within
3109 a bin width) and the same conclusion holds for the higher credible intervals. The
3110 change in sensitivity to other oscillation parameters has been studied and no
3111 significant discrepancies were found. As expected, the sensitivities are statistics
3112 dominated and the exact choice of systematic model and constraint does not
3113 significantly affect the physics conclusions one would make from this analysis.

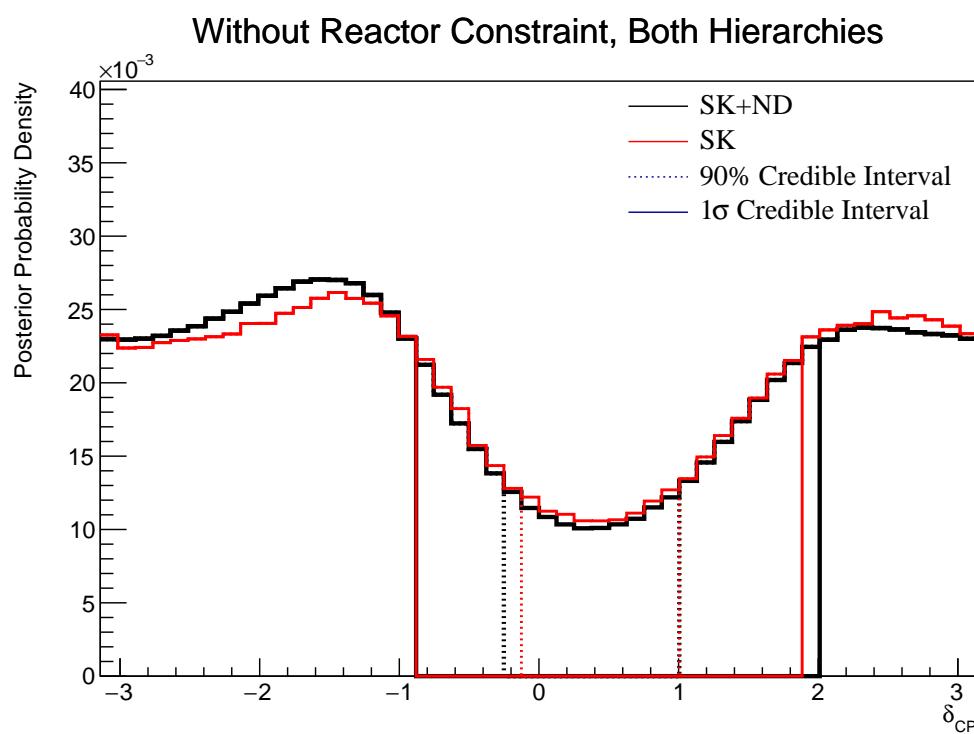


Figure 8.16: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies.

3114 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3115 straint

3116 This section presents the sensitivities of the simultaneous beam and atmospheric
 3117 analysis where the reactor constraint is not applied. Similar to the previous
 3118 studies, the Asimov data is built assuming the post-BANFF cross-section tune
 3119 and Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near
 3120 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.
 3121 The sensitivity to δ_{CP} , marginalised over both hierarchies, is given in Figure 8.17.

Without Reactor Constraint, Both Hierarchies

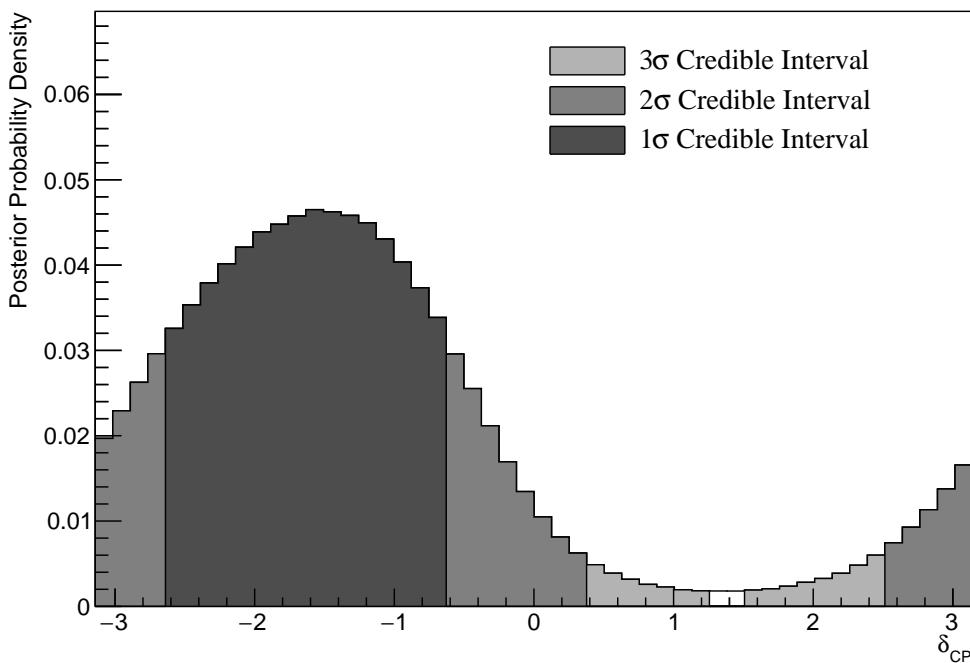


Figure 8.17: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

3122 The credible intervals and highest posterior distribution for each oscillation
 3123 parameter is given in Table 8.5. The highest posterior probability density is
 3124 $\delta_{CP} = -1.58$ and is compatible with the known Asimov A value of $\delta_{CP} = -1.601$.
 3125 The CP-conserving values of $\delta_{CP} = 0, \pi, -\pi$ are disfavoured at 1 σ credible
 3126 interval. There is also a region around $\delta_{CP} = 1.4$ which is disfavoured at more
 3127 than 3 σ . Whilst these conclusions can only be made at this particular Asimov

point, it does show that if the true value of δ_{CP} was CP-violating, this joint analysis would be able to disfavour CP conserving values at over 1σ without any external constraints. The highest posterior probability density does move further away from the Asimov point when only steps in the NH region are considered. This is due to the correlations between the value of δ_{CP} and the mass hierarchy, as will be later discussed.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-2.64, -0.63]$	-1.57
δ_{CP} , (NH)	$[-2.76, -0.63]$	-1.45
δ_{CP} , (IH)	$[-2.39, -0.88]$	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.46, 2.58]	2.49
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.48, 2.56]	2.51
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.52]$	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (NH)	[0.48, 0.55]	0.509
$\sin^2(\theta_{23})$ (IH)	[0.48, 0.55]	0.521

Table 8.5: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam and atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

The sensitivity to Δm_{32}^2 is illustrated in Figure 8.18, marginalised over both hierarchies. Notably, the 1σ credible interval is entirely contained within the normal hierarchy region, as illustrated in Table 8.5. This illustrates reasonable sensitivity to the mass hierarchy model. This is also reflected in the 1σ credible intervals being approximately the same when they are made considering both hierarchies and when considering only the NH. The known oscillation parameter is $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$. The normal hierarchy distribution favours this value with the highest posterior probability density of $\Delta m_{32}^2 = 2.51 \times 10^{-3}\text{eV}^2$.

The fraction of steps in each of the mass hierarchy regions and octants of $\sin^2(\theta_{23})$ is given in Table 8.6. The Bayes factors are determined to be $B(\text{NH}/\text{IH}) =$

3144 3.67 and $B(\text{UO}/\text{LO}) = 1.74$. Jeffrey's scale (presented in Table 4.1) states that
3145 this value of the hierarchy Bayes factor illustrates substantial evidence for the
3146 normal hierarchy hypothesis. This corresponds to the correct hypothesis given
3147 the known oscillation parameters. It is a stronger statement than the atmospheric-
3148 only analysis can provide. It is important to note that this is a substantial
3149 preference that requires no external constraints required. The Bayes factor for
3150 octant determination represents a weak preference for the upper octant but does
3151 select the correct octant model.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.29	0.50	0.79
IH ($\Delta m_{32}^2 < 0$)	0.08	0.13	0.21
Sum	0.37	0.63	1.00

Table 8.6: The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 3.67$ and $B(\text{UO}/\text{LO}) = 1.74$.

3152 The sensitivity to $\sin^2(\theta_{23})$ is presented in Figure 8.19. There is a clear
3153 preference for the upper octant but the peak of the distribution is relatively
3154 flat. It peaks at $\sin^2(\theta_{23}) = 0.509$ which is in the region of the known value of
3155 $\sin^2(\theta_{23}) = 0.528$. The difference in the highest posterior distribution and the
3156 width of the credible interval is relatively unchanged when considering different
3157 hierarchy models showing no strong correlation between $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.
3158 The sensitivity presented as a function of the appearance parameters ($\sin^2(\theta_{13}) -$
3159 δ_{CP}) is given in Figure 8.20. As expected, the contours follow that given in
3160 Figure 8.2, where the 2σ credible intervals have a closed contour excluding the
3161 region around $\delta_{CP} \sim 1.2$. The width of the 3σ credible interval is also clearly
3162 dependent upon the value of δ_{CP} . Close to the Asimov point, $\delta_{CP} = -1.601$, the
3163 width of the 3σ credible interval approximately spans $\sin^2(\theta_{13}) = [0.013, 0.04]$.
3164 This is reduced to a region of $\sin^2(\theta_{13}) = [0.023, 0.042]$ at the most disfavoured
3165 value of δ_{CP} . This follows the behaviour shown in the likelihood scans. The 1σ

Without Reactor Constraint, Both Hierarchies

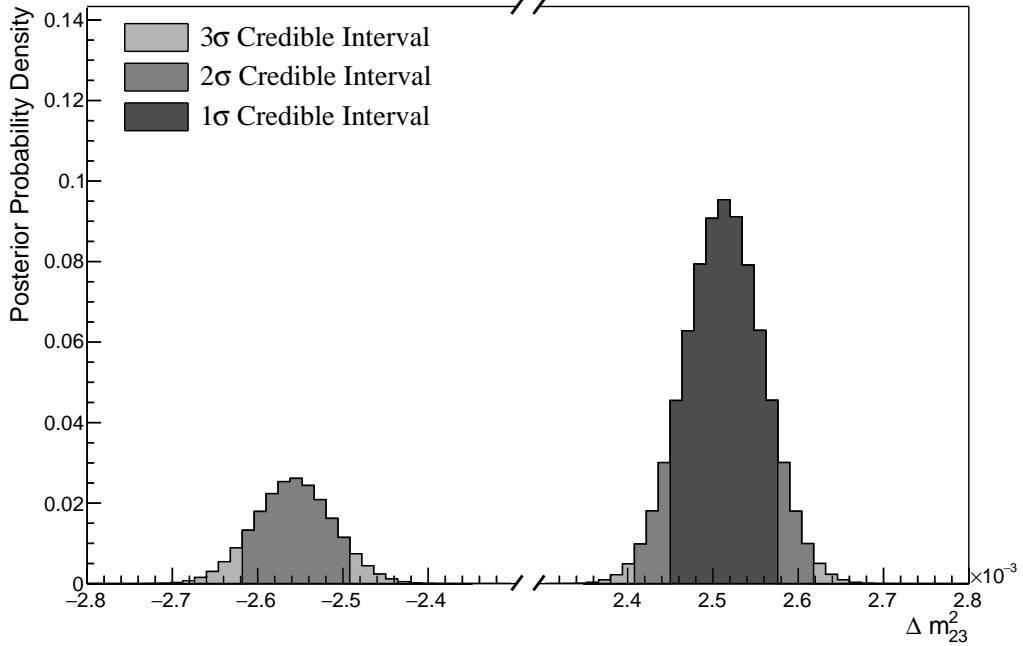


Figure 8.18: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

credible interval is consistent with both the known oscillation parameter and the reactor constraint ($\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$). Application of the reactor constraint would be expected to decrease the width of the 1 σ credible intervals of δ_{CP} due to the triangular shape of the posterior probability.

The sensitivity in terms of the ‘disappearance’ parameters marginalised over both hierarchies is given in Figure 8.21. The area contained within the IH credible intervals is significantly smaller than those in the NH region. This is reflected in the IH credible intervals being tighter in the $\sin^2(\theta_{23})$ dimension. No significant correlation is observed between the value of $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

The two-dimensional posterior distribution for each permutation of the oscillation parameters of interest is given in Figure 8.22. The most notable observation is that the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ are anti-correlated. If the value of $\sin^2(\theta_{13})$ was known to be closer to the known oscillation parameter value, the preferred value of $\sin^2(\theta_{23})$ would increase furthering the preference for the UO. That

Without Reactor Constraint, Both Hierarchies

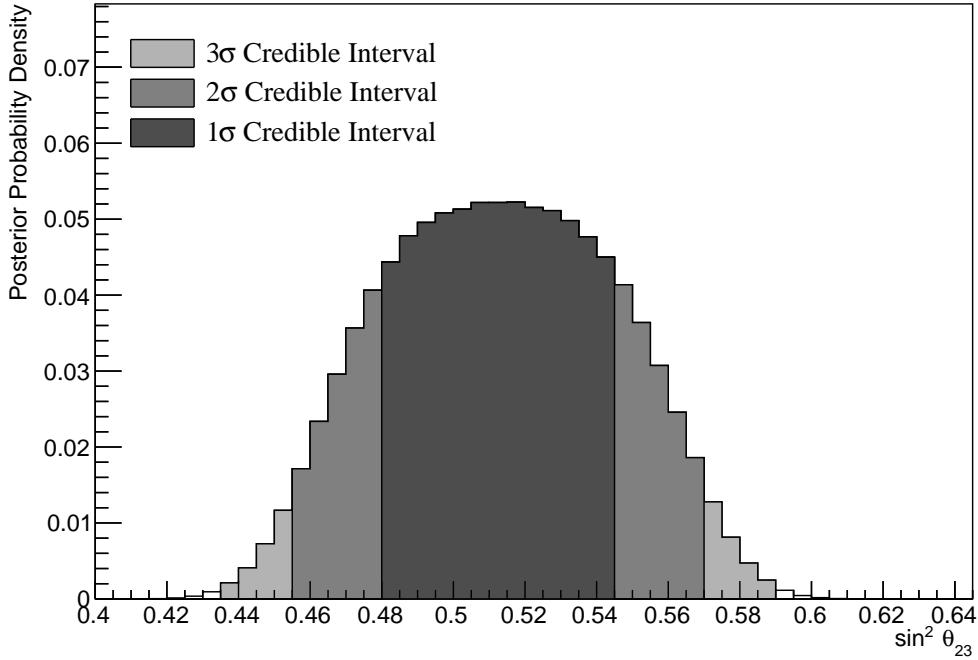


Figure 8.19: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied.

would move the highest posterior probability closer in line with the Asimov value. This also means that the preference for the UO would be increased if the reactor constraint was to be applied.

Furthermore, the δ_{CP} and $|\Delta m_{32}^2|$ oscillation parameters are anti-correlated, such that higher values of $|\Delta m_{32}^2|$ prefer lower values of δ_{CP} . Whilst this is an interesting result on its own, the width of the Δm_{32}^2 contours also depend on $\sin^2(\theta_{13})$. This introduces another correlation effect that could modify the sensitivity to δ_{CP} once the reactor constraint is applied.

The correlation between $\sin^2(\theta_{13})$ and Δm_{32}^2 can be seen in Figure 8.23. A much larger fraction of the posterior distribution is contained in the NH for lower values of $\sin^2(\theta_{13})$. Consequently, the application of the reactor constraint would be expected to significantly increase the preference for NH.

Without Reactor Constraint, Both Hierarchies

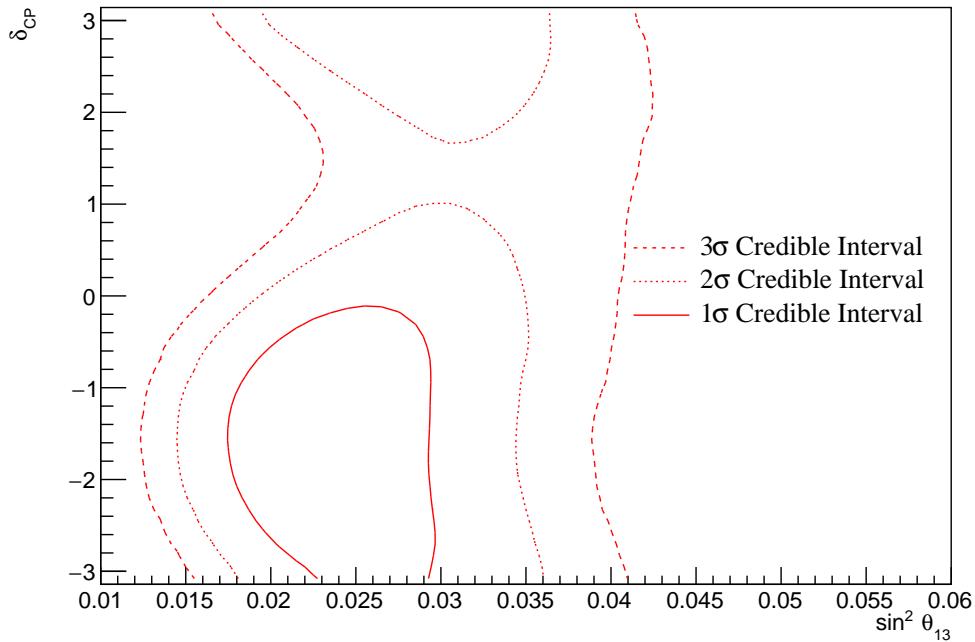


Figure 8.20: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

Without Reactor Constraint, Both Hierarchies

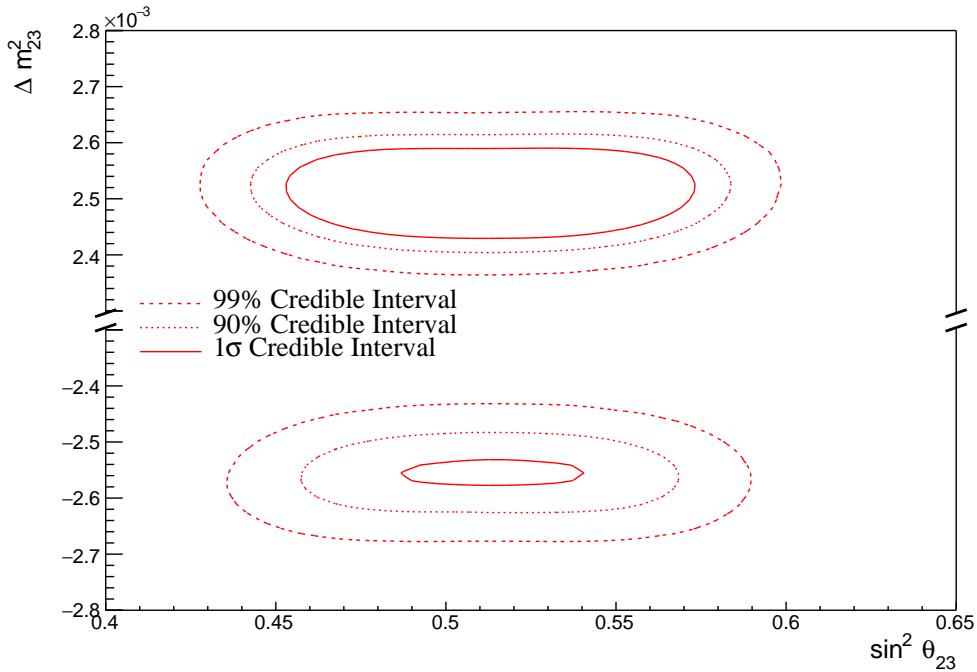


Figure 8.21: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

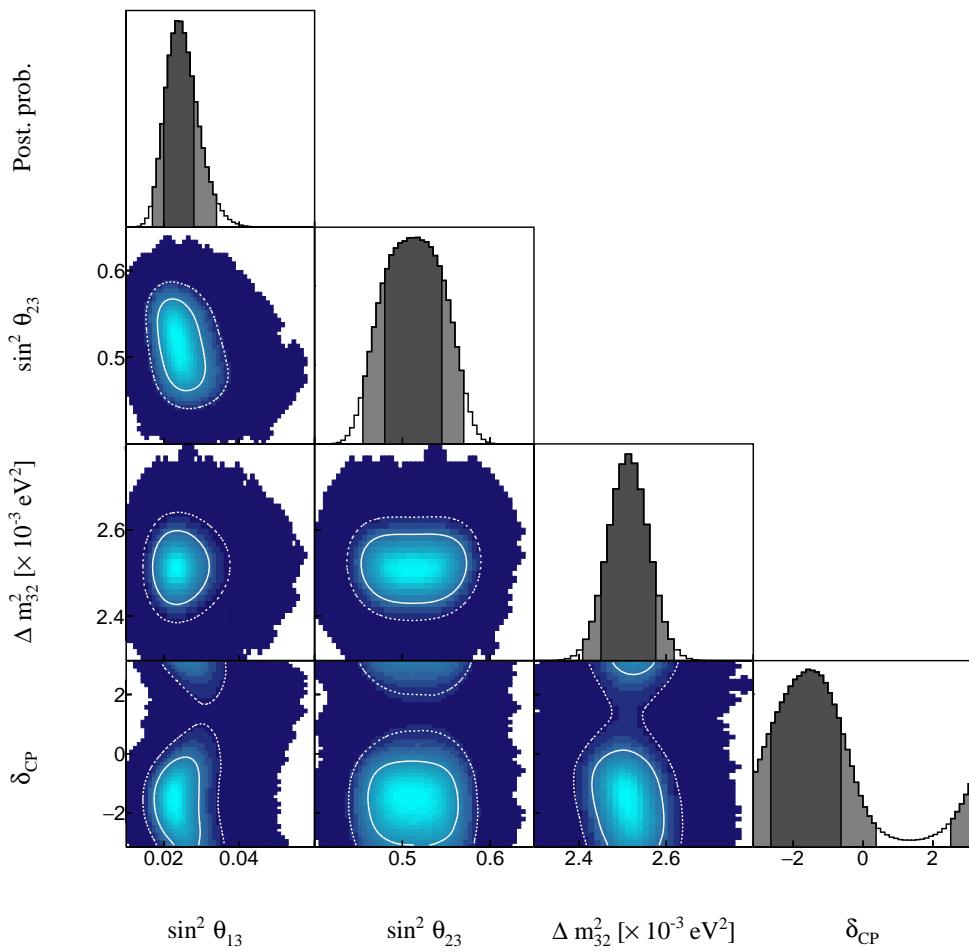


Figure 8.22: The posterior probability density distribution from the joint beam and atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

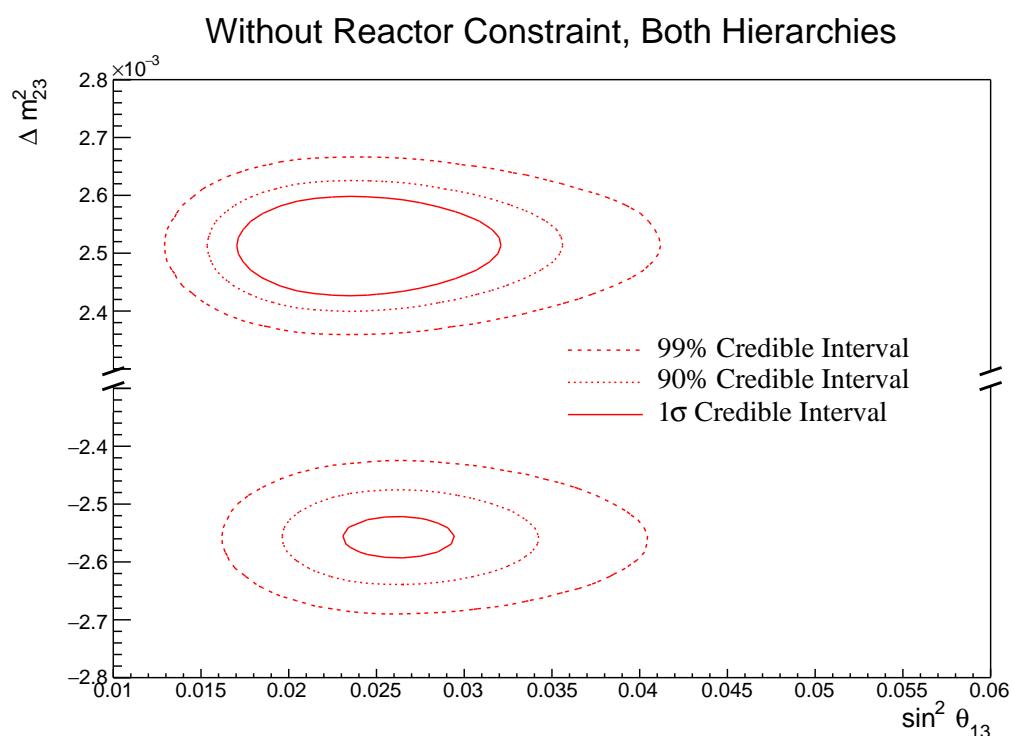


Figure 8.23: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit. The reactor constraint is not applied.

3192 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3193 This section presents the sensitivities of the joint beam and atmospheric fit when
 3194 the reactor constraint is applied to $\sin^2(\theta_{13})$. As with the previous studies, the
 3195 Asimov data is made using the AsimovA oscillation parameter set defined in
 3196 Table 2.2 and the post-BANFF systematic parameter tune.

With Reactor Constraint, Both Hierarchies

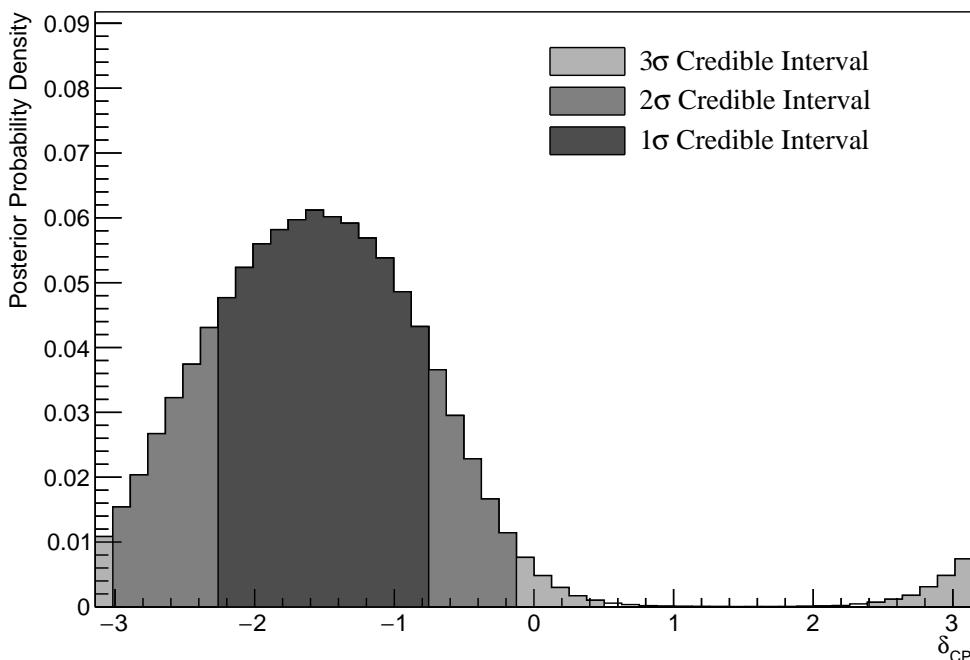


Figure 8.24: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3197 Figure 8.24 illustrates the sensitivity to δ_{CP} , marginalised over both hierarchies.
 3198 The CP-conserving values of $\delta_{CP} = -\pi, 0, \pi$ are disfavoured at 2σ . Furthermore,
 3199 the 3σ credible interval excludes the region of $\delta_{CP} = [0.50, 2.39]$. Thus clearly
 3200 disfavouring the region of $\delta_{CP} = \pi/2$ at more than 3σ for this particular set
 3201 of known oscillation parameters. The width of the 1σ credible intervals and
 3202 the position of the highest posterior probability density is given in Table 8.7.
 3203 The highest posterior probability density in δ_{CP} is calculated as $\delta_{CP} = -1.57$
 3204 showing no significant biases in the determination of the known oscillation

parameters. The posterior distribution is more peaked around the known oscillation parameter value of $\delta_{CP} = -1.601$, as compared to the sensitivities when the reactor constraint is not applied (subsection 8.3.4). This follows from the correlations shown in Figure 8.20, where a lower value of $\sin^2(\theta_{13})$ results in tighter constraints on δ_{CP} .

The effect of applying the reactor constraint for δ_{CP} in the joint beam-atmospheric fit is presented in Figure 8.25. The posterior distribution from the two fits are marginalised over both hierarchies. Clearly, the reactor constraint improves the ability of the fit to select the known oscillation parameter as the shape of the distribution is much more peaked. This is also evidenced by the tightening of the 1σ and 90% credible intervals. Additionally, the disfavoured region of $1 < \delta_{CP} < 2$ is wider when the reactor constraint is applied.

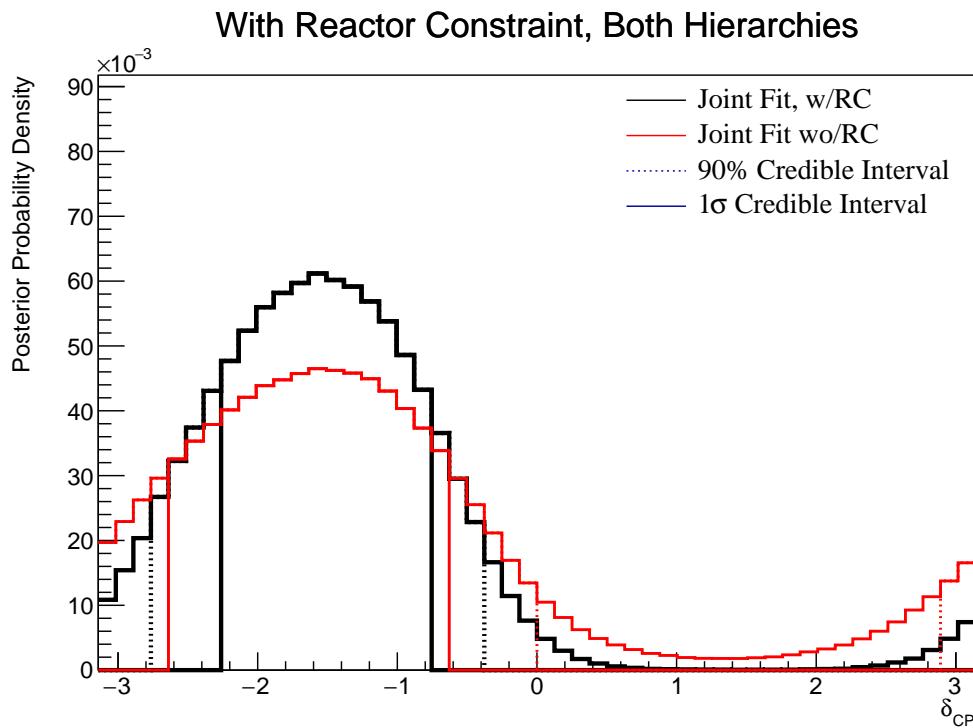


Figure 8.25: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, is given in Figure 8.26. The highest posterior probability density is located at $\sin^2(\theta_{23}) = 0.527$

Parameter	Interval	HPD
δ_{CP} , (BH)	[-2.26, -0.75]	-1.57
δ_{CP} , (NH)	[-2.26, -0.75]	-1.57
δ_{CP} , (IH)	[-2.13, -1.00]	-1.57
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.46, 2.52]	2.49
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.48, 2.56]	2.51
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	[-2.60, -2.52]	-2.55
$\sin^2(\theta_{23})$ (BH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (NH)	[0.49, 0.55]	0.527
$\sin^2(\theta_{23})$ (IH)	[0.50, 0.56]	0.539

Table 8.7: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam and atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

which agrees with the known value of $\sin^2(\theta_{23}) = 0.528$. The distribution clearly favours the UO with almost the entirety of the 1σ credible interval contained in the region. Figure 8.27 highlights the sensitivity of the joint fit both with and without the reactor constraint. The fit where the reactor constraint is applied selects the known value much better ($\sin^2(\theta_{23}) = 0.528$). Furthermore, the reactor constraint increases the UO preference which is evidenced by the distribution moving further away from the octant boundary. This indicates that there are marginalisation effects between the two mixing parameters. This follows from the correlation illustrated between $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$ in Figure 8.22. The posterior distribution of the fit with reactor constraint is more peaked compared to the flatter distribution when the reactor constraint is not applied.

The fraction of steps contained within the two hierarchy and two octant models is given in Table 8.8. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.02, whilst significantly increasing the fraction of steps within the NH-UO region from 0.53 to 0.64. The application of the reactor constraint increases the Bayes factor

With Reactor Constraint, Both Hierarchies

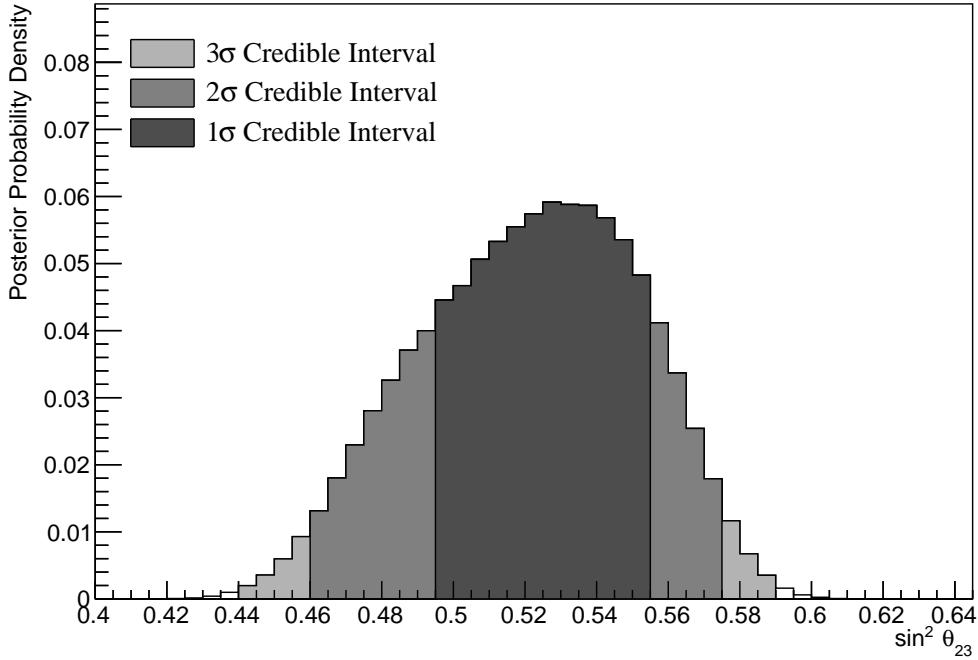


Figure 8.26: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

from $B(\text{NH}/\text{IH}) = 3.67$ to $B(\text{NH}/\text{IH}) = 7.29$. There is a very clear preference for the NH, with the Jeffreys scale stating a substantial preference for both fits (see subsection 4.3.3). The Bayes factor for UO preference is calculated as $B(\text{UO}/\text{LO}) = 2.86$. Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.24	0.64	0.88
IH ($\Delta m_{32}^2 < 0$)	0.02	0.10	0.12
Sum	0.26	0.74	1.00

Table 8.8: The distribution of steps in a joint beam and atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 7.29$ and $B(\text{UO}/\text{LO}) = 2.86$.

The sensitivity to Δm_{32}^2 , with the reactor constraint applied, is presented in

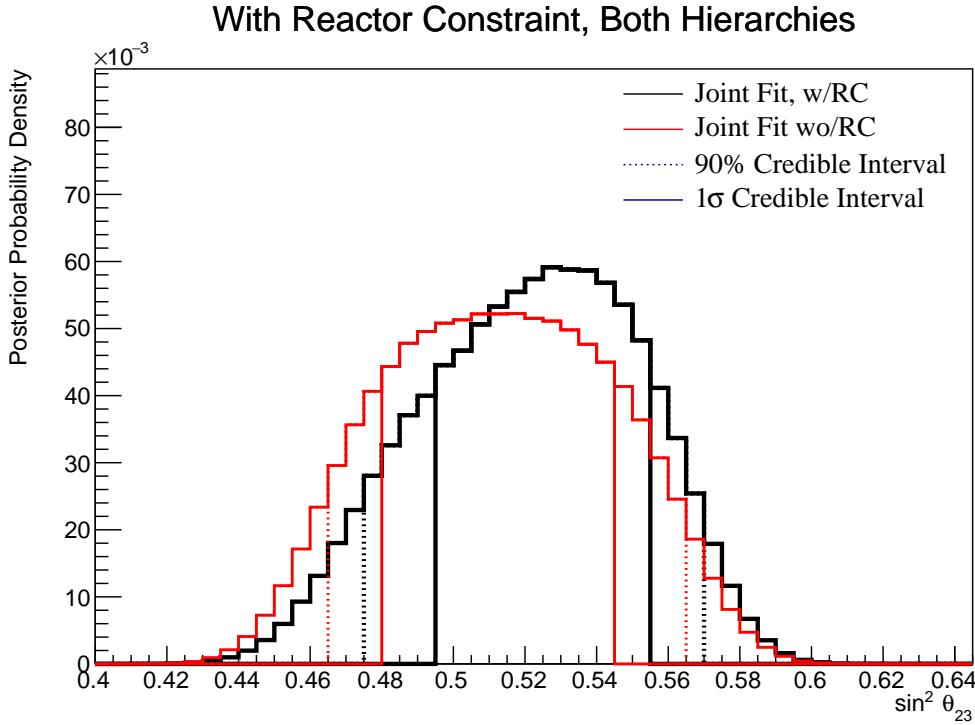


Figure 8.27: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Red) and the joint beam and atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies.

3241 Figure 8.28. The posterior distribution is marginalised over both hierarchies. As
 3242 expected, the 1σ credible interval is entirely contained within the NH region. The
 3243 position of the highest posterior probability density is given as $2.49 \times 10^{-3} \text{ eV}^2$,
 3244 illustrating no significant bias between the fit results and the known oscillation
 3245 parameters. The application of the reactor constraint does move significantly the
 3246 position of the credible intervals but does reduce their width.

3247 The sensitivity to the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$) is given in
 3248 Figure 8.29. The distribution is mostly uncorrelated between the two parameters
 3249 and is centered at the known oscillation parameters. The 1σ credible interval
 3250 excludes $\delta_{CP} = 0$ and $\delta_{CP} = (-)\pi$. Furthermore, the 3σ credible intervals
 3251 exclude the region of $\delta_{CP} = \pi/2$.

3252 The sensitivity to the disappearance parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) is illus-
 3253 trated in Figure 8.30. As expected from the one-dimensional distribution, the 1σ
 3254 credible interval is entirely contained within the NH region. Both the NH and

With Reactor Constraint, Both Hierarchies

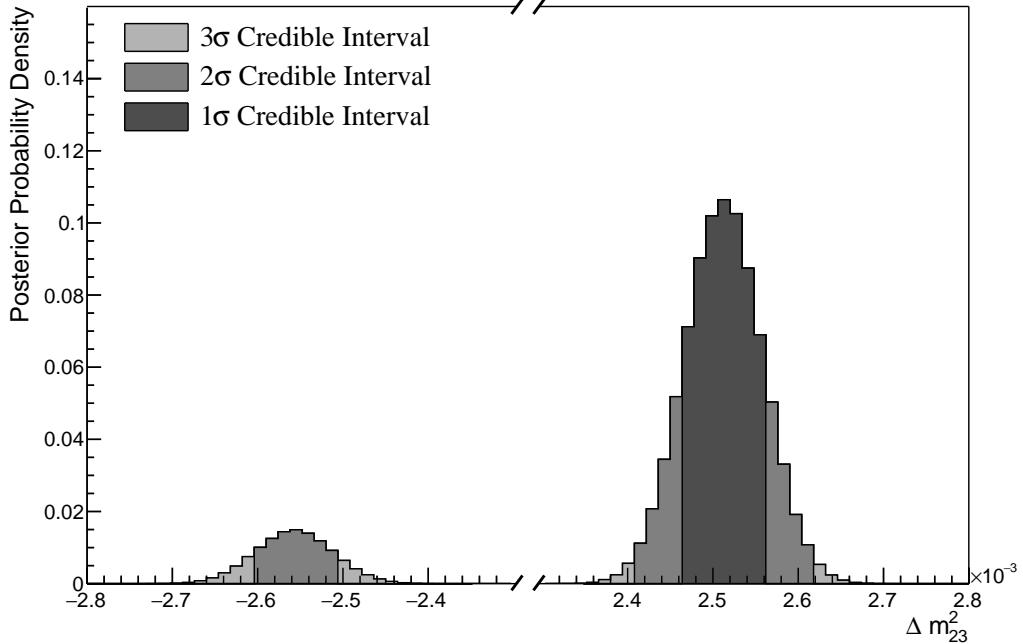


Figure 8.28: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

3255 IH regions favour the UO, with a visually similar preference in both hierarchies.
 3256 The width of the Δm_{32}^2 1 σ credible interval does not significantly depend upon
 3257 the value or octant of $\sin^2(\theta_{23})$. This shows that there are no strong correlations
 3258 between these two parameters.

3259 Figure 8.31 illustrates the posterior distribution for each permutation of
 3260 two oscillation parameters of interest. The application of the reactor constraint
 3261 significantly reduces the correlations previously seen in Figure 8.22. There is
 3262 still a small correlation between δ_{CP} and Δm_{32}^2 . The application of the reactor
 3263 constraint has not significantly affected this correlation. The width of the 1 σ
 3264 credible interval in Δm_{32}^2 is wider for a value of $\delta_{CP} = 0$ as compared to a value
 3265 of $\delta_{CP} = \pi$. Similarly, the width of the 1 σ credible interval in δ_{CP} is smaller
 3266 for lower values of $\sin^2(\theta_{23})$.

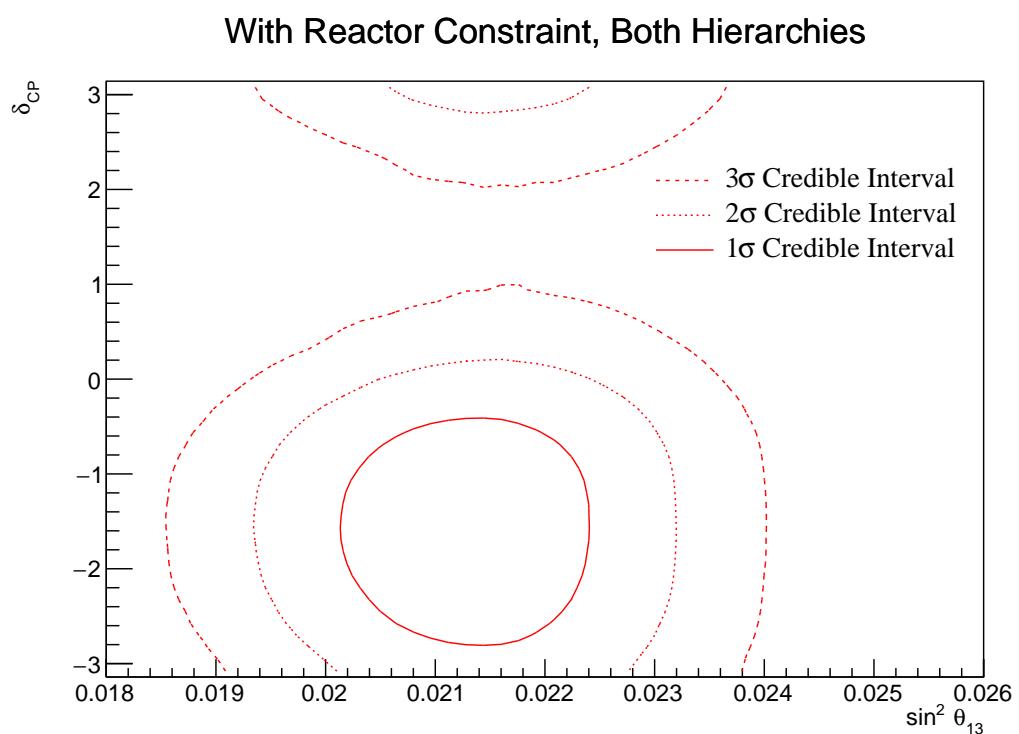


Figure 8.29: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

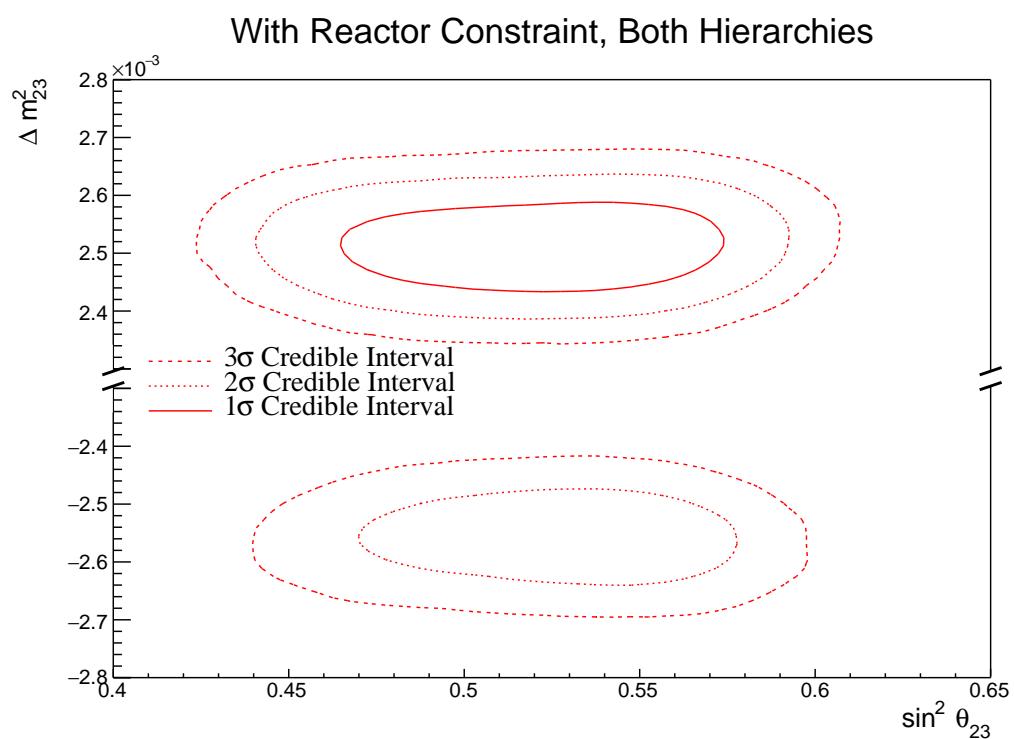


Figure 8.30: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam and atmospheric fit where the reactor constraint is applied.

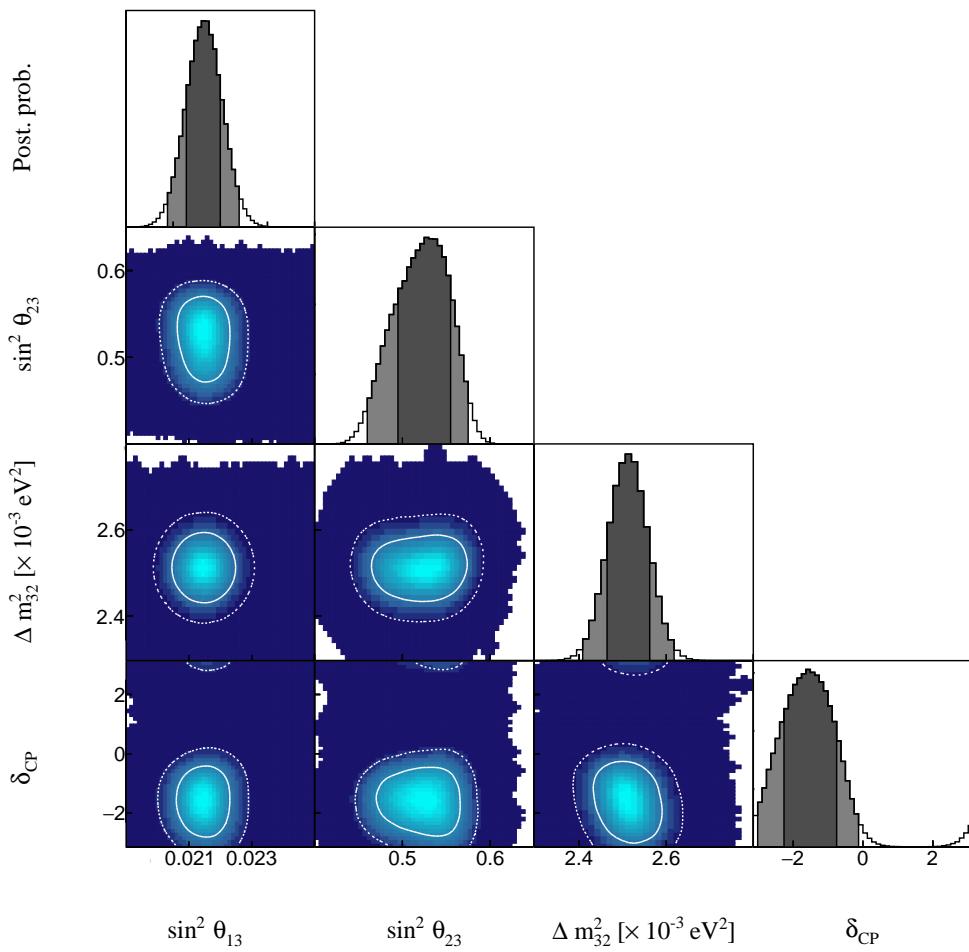


Figure 8.31: The posterior probability density distribution from the joint beam and atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3267 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3268 Constraint

3269 The benefits of the joint beam and atmospheric analysis can be determined by
 3270 comparing the sensitivities to the beam-only analysis. This section presents those
 3271 comparisons for sensitivities built using the Asimov A oscillation parameters
 3272 defined in Table 2.2 and the post-BANFF systematic tune. The reactor constraint
 3273 is not applied within either of the fits used in these comparisons.

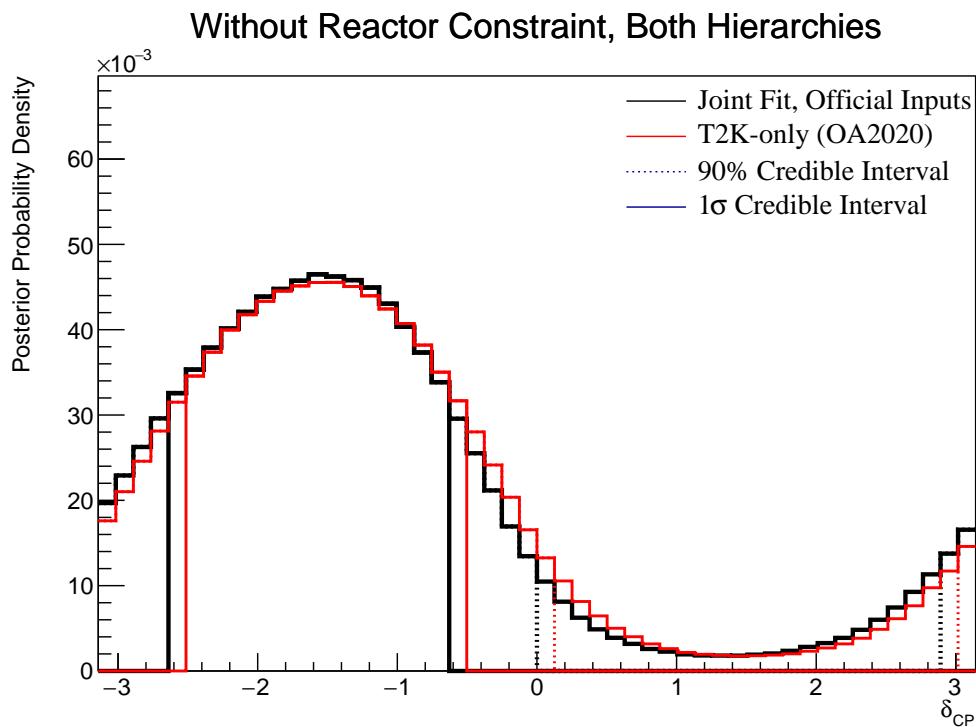


Figure 8.32: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3274 The sensitivity, marginalised over both hierarchies, to δ_{CP} from the joint
 3275 beam-atmospheric and beam-only fits is presented in Figure 8.32. As expected
 3276 from the likelihood scans (Figure 8.4), the sensitivity to δ_{CP} is not significantly
 3277 increased. This is because the known oscillation parameter value ($\delta_{CP} = -1.601$)
 3278 lies at the position where the beam samples dominate the sensitivity compared
 3279 to the SK samples.

The sensitivity to Δm_{32}^2 of the joint beam-atmospheric fit is illustrated in Figure 8.33, where the posterior distribution has been marginalised over both hierarchies. The 1σ credible interval of the joint beam and atmospheric fit is entirely contained within the NH region. This shows the significant increase in the ability of the fit to determine the correct mass hierarchy, as compared to the beam-only analysis. This is further evidenced by the fact that the 90% credible intervals from the joint fit are also tighter in the IH region as compared to the beam-only analysis. The Bayes factor for mass hierarchy determination for the beam-only and joint beam and atmospheric are $B(\text{NH}/\text{IH}) = 1.91$ and $B(\text{NH}/\text{IH}) = 3.67$, respectively. According to Jeffrey's scale (Table 4.1), the beam-only analysis represents a weak preference for the NH hypothesis whereas the joint fit returns a substantial preference for the NH hypothesis. To summarise, the joint beam-atmospheric fit has a substantial preference for the correct hierarchy without the requirement of external constraints.

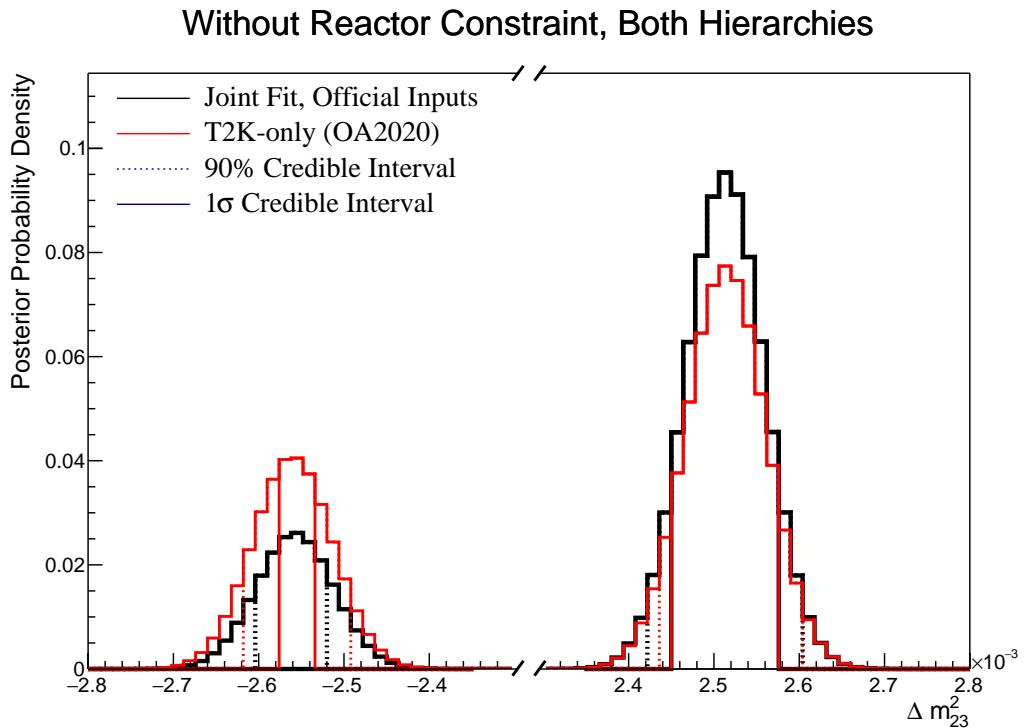


Figure 8.33: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, for both the beam-only and joint beam and atmospheric analysis are presented in Figure 8.34. The peak of the posterior distribution from the joint analysis is more aligned with the known value of $\sin^2(\theta_{23}) = 0.528$ as compared to the beam-only analysis. This indicates that the marginalisation effects from other oscillation parameters ($\sin^2(\theta_{13}) - \sin^2(\theta_{23})$) presented in Figure 8.22) are less prevalent in the projection of this parameter. The Bayes factors for the beam-only and joint beam-atmospheric fit are $B(\text{UO}/\text{LO}) = 1.56$ and $B(\text{UO}/\text{LO}) = 1.74$, respectively. Therefore, the joint beam-atmospheric fit does prefer the UO more strongly than the beam-only analysis, albeit slightly.

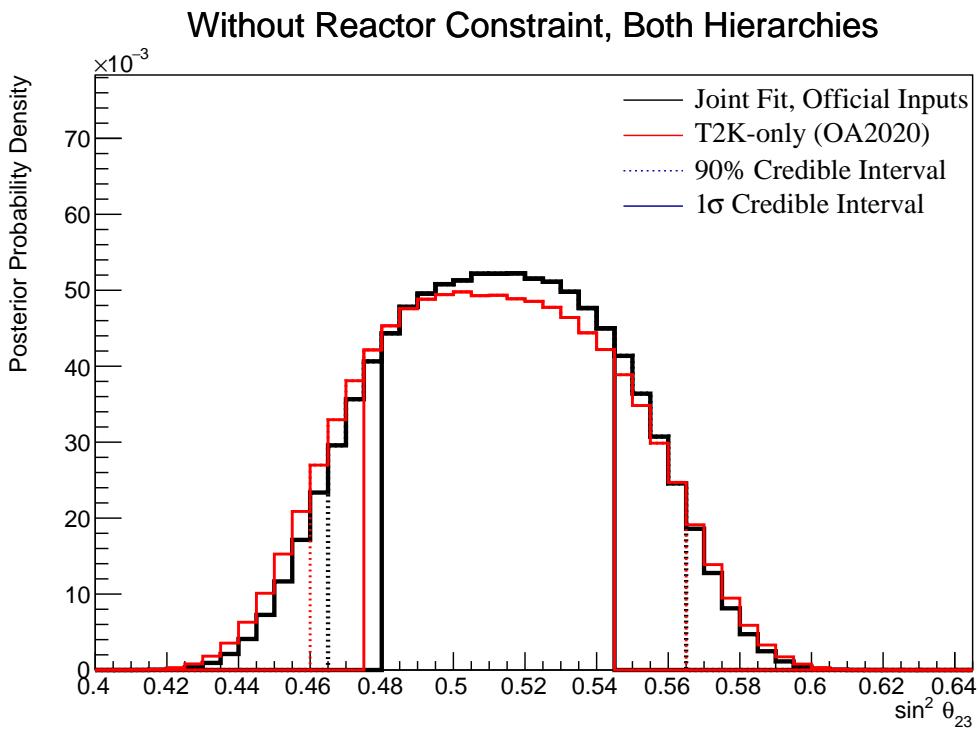


Figure 8.34: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

Whilst the beam-only and joint beam-atmospheric fits have similar sensitivity to δ_{CP} and $\sin^2(\theta_{23})$ when projected in one-dimension, the benefit of the joint analysis becomes more obvious when the sensitivities are presented in two-

dimensions. The sensitivity of the two fits to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$) are illustrated in Figure 8.35.

The width of the 99% joint fit credible interval in $\sin^2(\theta_{13})$ is squeezed in the region of $\delta_{CP} \sim 0$ compared to the beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The 1σ and 90% also exhibit slightly tighter constraints on δ_{CP} . This is most prevalent in the region of $\delta_{CP} \sim 0$ and $\sin^2(\theta_{13}) \sim 0.03$. Whilst the atmospheric samples do not have significant sensitivity to $\sin^2(\theta_{13})$ (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters $\sin^2(\theta_{23}) - \Delta m_{32}^2$, marginalised over both hierarchies, is presented in Figure 8.36 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. The 1σ joint fit credible interval in the IH region more strongly favours the UO as compared to the beam-only fit. Even in the NH region, the width of the credible intervals in $\sin^2(\theta_{23})$ decrease, albeit to a smaller extent.

The change in sensitivity to $\delta_{CP} - \Delta m_{32}^2$ is illustrated in Figure 8.37. As expected, the contours presented within the IH region are much smaller in the joint fit due to the increased sensitivity to mass hierarchy determination. This culminates in a region around $\delta_{CP} \sim \pi/2$ which is excluded at 3σ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to Δm_{32}^2 and $\sin^2(\theta_{23})$, as a function of $\sin^2(\theta_{13})$, is presented in Figure 8.38 and Figure 8.39, respectively. These sensitivities are marginalised

Without Reactor Constraint, Both Hierarchies

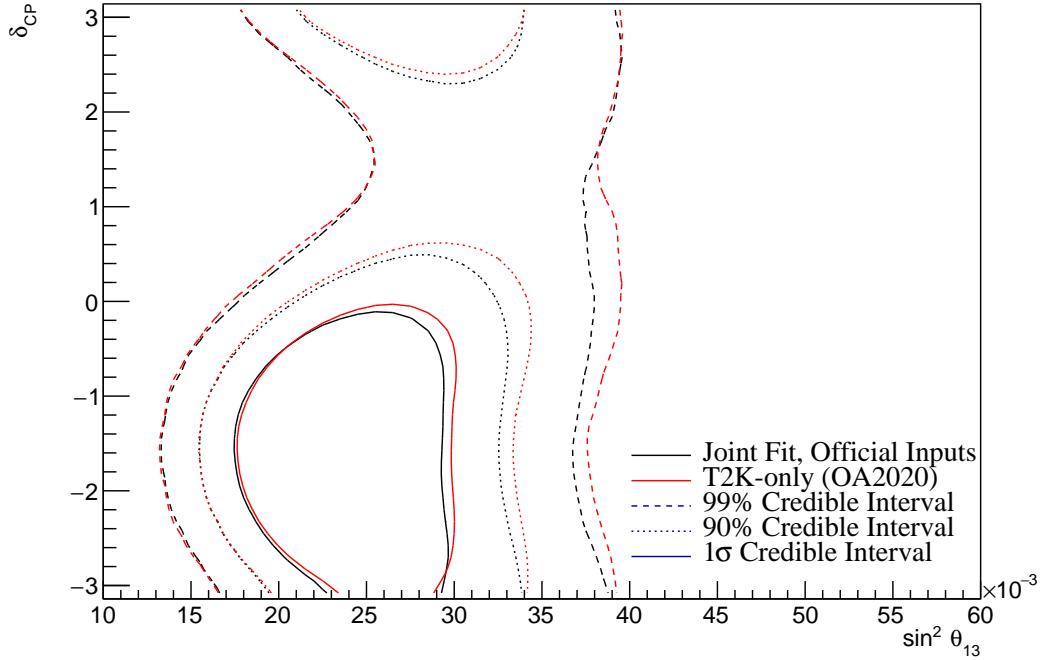


Figure 8.35: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

over both hierarchies. As expected from the previous observations, the Δm_{32}^2 contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH 1σ credible intervals exclude the region around the reactor constraint. This is not a bias from the fit as the known value for Δm_{32}^2 is in the NH region. This does suggest that the application of the reactor constraint would further increase the preference for NH in the joint fit as compared to its effect on the beam-only analysis.

The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as illustrated by Figure 8.39. The joint analysis disfavours the wrong octant hypothesis more strongly in the region of high $\sin^2(\theta_{13})$. This suggests that the application of the reactor constraint will favour the UO more strongly in the joint analysis compared to the beam-only analysis.

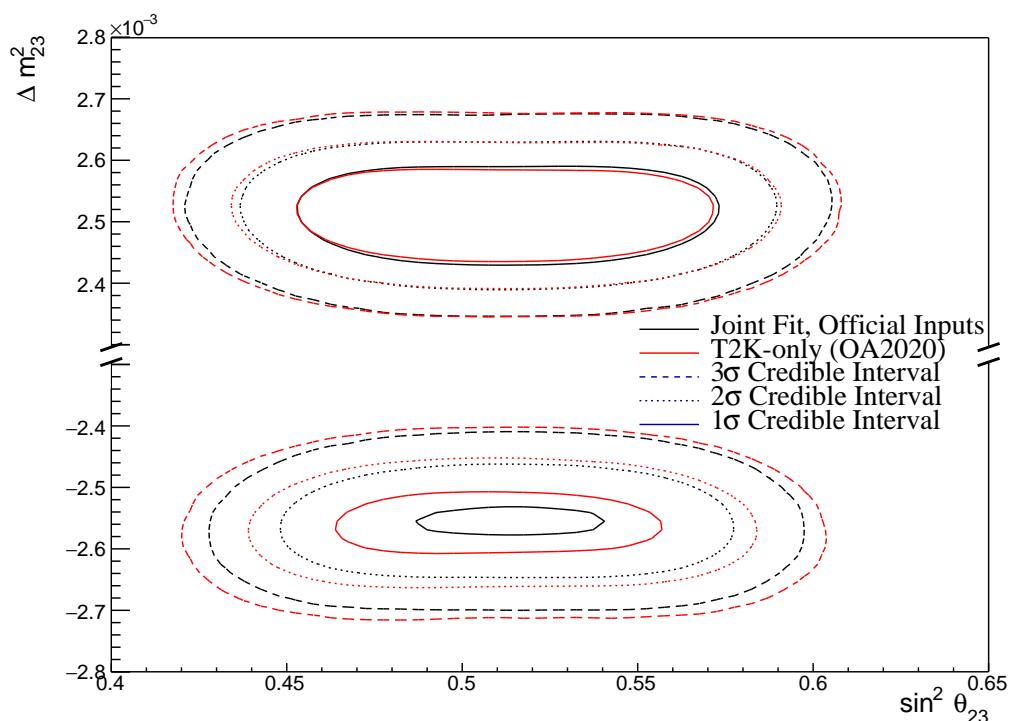


Figure 8.36: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

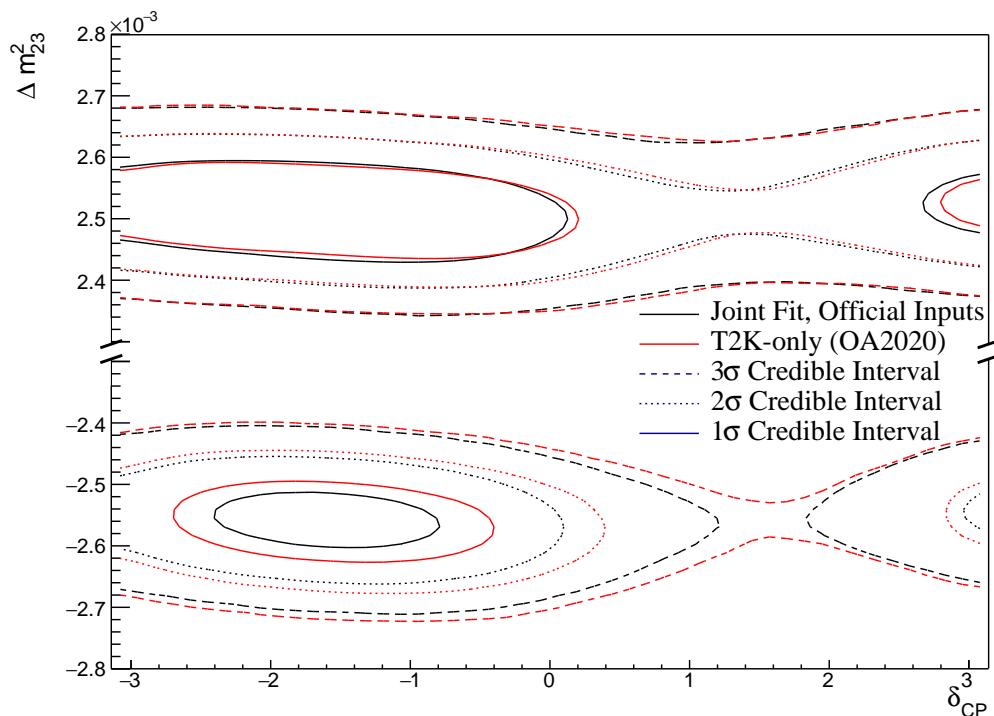


Figure 8.37: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \Delta_{CP}$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

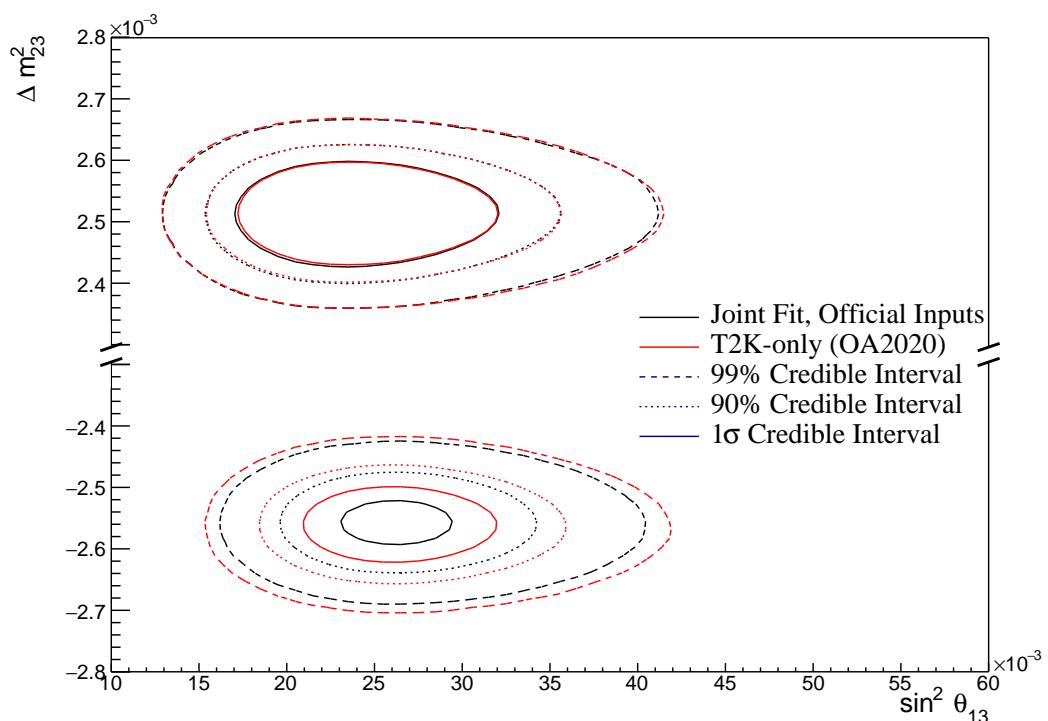


Figure 8.38: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

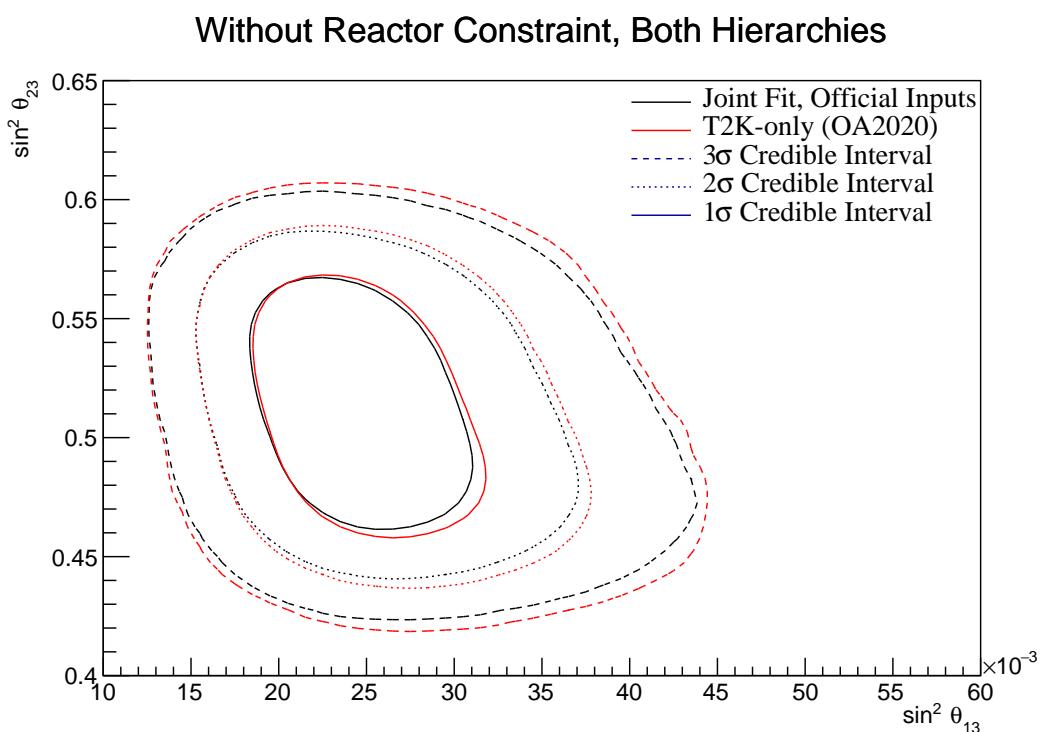


Figure 8.39: The two-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3350 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3351 straint

3352 The comparison between the beam-only and joint beam-atmospheric fits are
 3353 compared in subsection 8.3.6. Those comparisons were made with the reactor
 3354 constraint not applied to either of the fits. This section illustrates the com-
 3355 parison when the reactor constraint is applied. As shown in Figure 8.38, the
 3356 application of the reactor constraint is expected to significantly increase the
 3357 joint fit's preference for the NH hypothesis, as compared to the beam-only
 3358 analysis. Figure 8.40 illustrates the sensitivities of the two fits to the disappearance
 3359 parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) marginalised over both hierarchies and with the
 3360 reactor constraint applied. This plot clearly illustrates the benefit of the joint
 3361 beam and atmospheric analysis. The 1σ credible interval in the IH region is
 3362 entirely removed in the joint analysis, illustrating the improved NH preference.

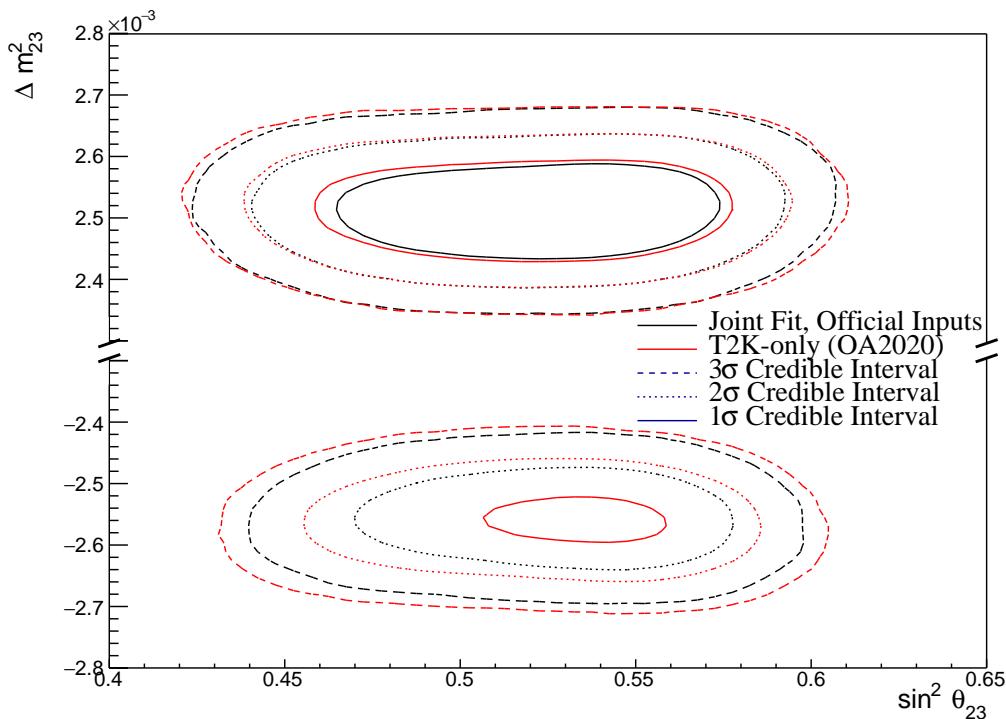


Figure 8.40: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

3363 The credible intervals of the joint fit are also tighter in the $\sin^2(\theta_{23})$ dimension
3364 than the beam-only analysis in both mass hierarchy regions. This shows that
3365 beyond the ability of the joint fit to prefer the NH more strongly than the beam-
3366 only analysis, the precision to which it can measure $\sin^2(\theta_{23})$ is also improved.
3367 The Bayes factor for NH preference is calculated as $B(\text{NH}/\text{IH}) = 7.29$ and
3368 $B(\text{NH}/\text{IH}) = 3.41$ for the joint beam-atmospheric and beam-only analysis,
3369 respectively. Whilst both present a significant preference for the NH hypothesis
3370 (Table 4.1), the joint fit's preference is much stronger. A similar conclusion can be
3371 made regarding the Bayes factors for UO preference which are $B(\text{UO}/\text{LO}) = 2.86$
3372 and $B(\text{UO}/\text{LO}) = 2.67$ for the joint beam-atmospheric and beam-only analysis,
3373 respectively. Both of these represent a mild preference for the UO but there is
3374 a stronger preference observed in the joint analysis.

3375 The sensitivity of the beam-only and joint beam-atmospheric analyses, to the
3376 appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$), are compared in Figure 8.41. These
3377 results are marginalised over both hierarchies and include the reactor constraint
3378 on $\sin^2(\theta_{13})$. For this particular set of known oscillation parameters (AsimovA
3379 defined in Table 2.2), the beam-only analysis dominates the sensitivity. The
3380 joint fit does slightly increase the sensitivity to δ_{CP} but it does not change any
3381 conclusions that would be made.

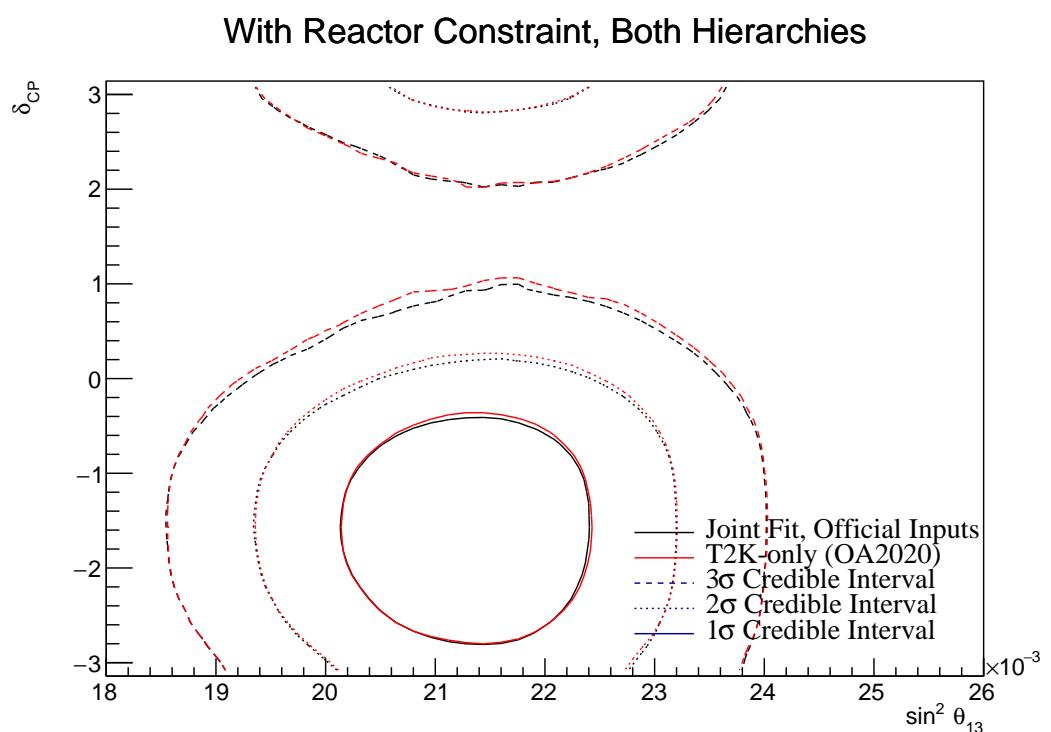


Figure 8.41: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies.

3382 8.3.8 Effect of Asimov Parameter Set

3383 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the
 3384 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters
 3385 are defined at a region of δ_{CP} which is dominated by the T2K experiment. This
 3386 explains why the addition of the atmospheric samples does not significantly in-
 3387 crease the sensitivity to δ_{CP} , as illustrated in subsection 8.3.6 and subsection 8.3.7.
 3388 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,
 3389 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when
 3390 building the Asimov data.

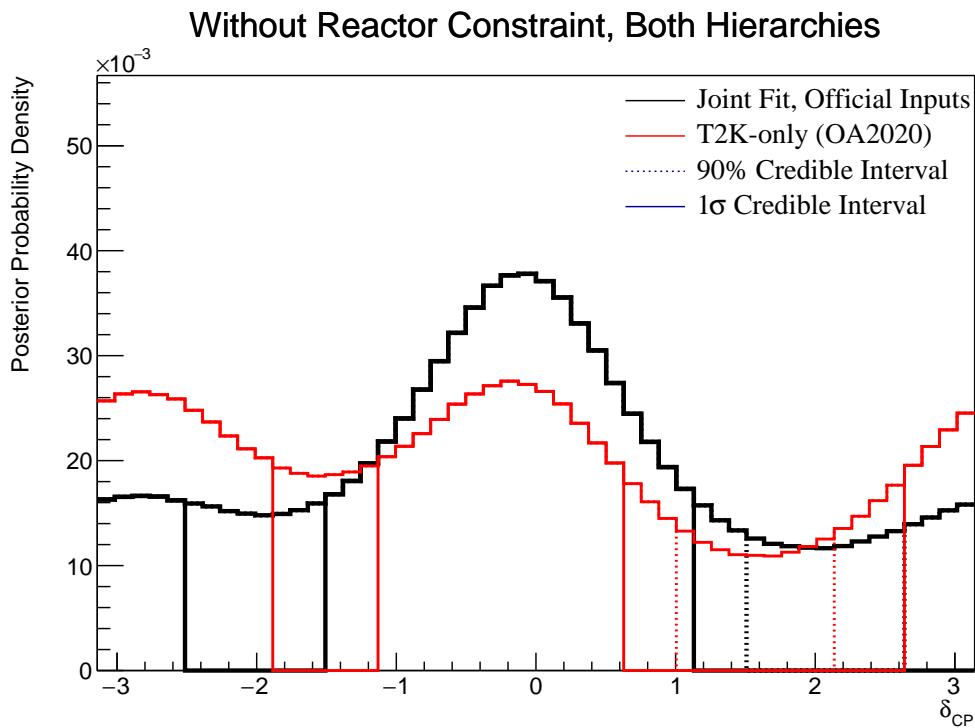


Figure 8.42: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3391 The sensitivity to δ_{CP} for the joint beam and atmospheric fit is presented
 3392 in Figure 8.42. The results are compared to those from the beam-only analysis
 3393 in [182]. The reactor constraint is not applied in either of the fits. The known
 3394 oscillation parameter value is $\delta_{CP} = 0$. The shape of the posterior distribution

from the joint analysis is more peaked at $\delta_{CP} = 0$ as compared to the beam-only analysis which has approximately the same posterior probability density at $\delta_{CP} = 0$ and $\delta_{CP} = \pi$. This shows the ability of the joint analysis to better determine the correct phase of δ_{CP} if the true value was CP-conserving. The 1σ credible intervals and the position of the highest posterior probability density are given in Table 8.9.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -2.51], [-1.51, 1.31]$	-0.06
δ_{CP} , (NH)	$[-1.13, 1.63]$	0.06
δ_{CP} , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44
Δm_{32}^2 (BH) [$\times 10^{-3}$ eV 2]	$[-2.60, -2.49], [2.46, 2.59]$	2.51
Δm_{32}^2 (NH) [$\times 10^{-3}$ eV 2]	$[2.47, 2.56]$	2.52
Δm_{32}^2 (IH) [$\times 10^{-3}$ eV 2]	$[-2.61, -2.52]$	-2.57
$\sin^2(\theta_{23})$ (BH)	$[0.43, 0.48], [0.55, 0.59]$	0.45
$\sin^2(\theta_{23})$ (NH)	$[0.43, 0.49], [0.55, 0.58]$	0.45
$\sin^2(\theta_{23})$ (IH)	$[0.44, 0.48], [0.54, 0.59]$	0.57

Table 8.9: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

Naively, if just the 1σ credible interval were considered without observing the shape of the distribution, it would appear that the joint analysis would have a worse sensitivity to δ_{CP} due to the larger interval around δ_{CP} . The 1σ credible interval for the beam-only analysis is given as the range $\delta_{CP} = [-\pi, -1.88], [-1.13, 0.63]$ and $[2.64, \pi]$ which contains 56% of all values of δ_{CP} . The joint beam and atmospheric analysis contains 52% of all δ_{CP} values within the 1σ credible interval. Therefore, if the area within the 1σ credible interval were to be compared between the two fits, the joint analysis would be shown to have better precision.

This contradiction stems from the methodology in which the credible interval is calculated. The technique used in this analysis (documented in subsection 4.3.2)

3411 fills the credible interval by selecting bins in order of magnitude until 68% of the
3412 posterior density is contained. If instead, the credible interval was calculated
3413 by expanding around the highest posterior probability, the benefits of the joint
3414 fit would be more obvious. In the case where the shape of the posterior was
3415 Gaussian, these two techniques would be equivalent.

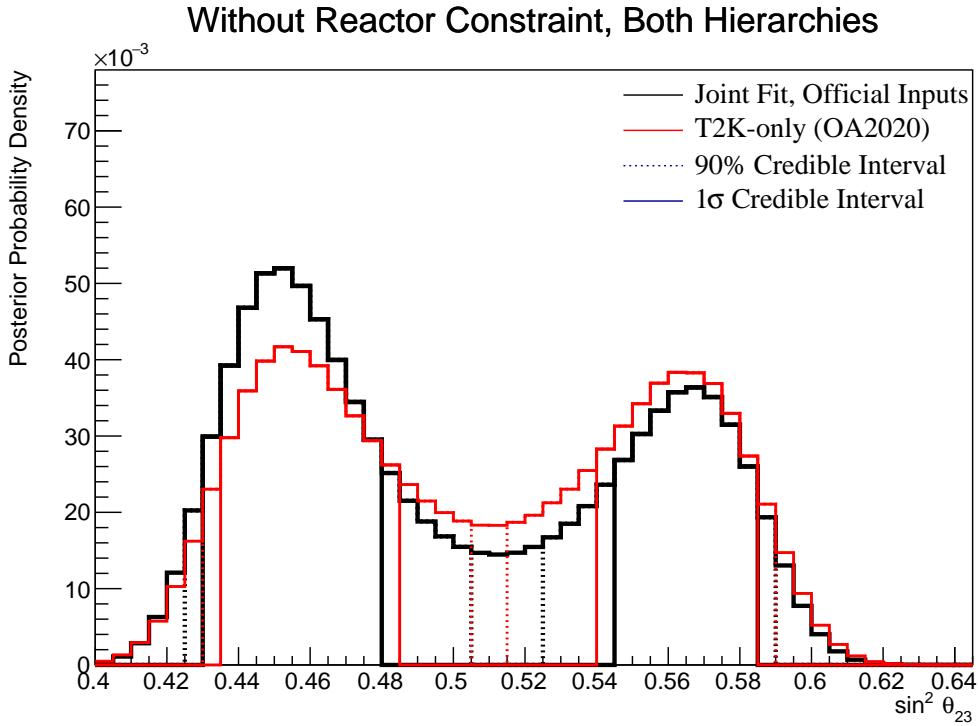


Figure 8.43: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

3416 The sensitivity of the joint beam and atmospheric fit to $\sin^2(\theta_{23})$ is presented
3417 in Figure 8.43. The sensitivity is compared to that of the beam-only analysis
3418 in [182]. The reactor constraint is not applied in either of the fits being com-
3419 pared. The Asimov parameter value is $\sin^2(\theta_{23}) = 0.45$ and the sensitivities are
3420 marginalised over both hierarchies. Clearly, the joint beam and atmospheric
3421 fit has a much larger probability density in the region surrounding the known
3422 oscillation parameters. This shows the better octant determination of the joint
3423 analysis compared to the beam-only fit. The ratio of the posterior density at

3424 the peak of the lower octant to the peak of the upper octant from the joint fit is
3425 1.43 compared to 1.09 from the beam-only analysis. This shows further support
3426 for the joint analysis in correctly selecting the lower octant, which is the correct
3427 hypothesis given the known oscillation parameters.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.35	0.24	0.59
IH ($\Delta m_{32}^2 < 0$)	0.19	0.22	0.41
Sum	0.54	0.46	1.00

Table 8.10: The distribution of steps in a joint beam and atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$.

3428 The distribution of steps, split by hierarchy and octant hypothesis, is presented
3429 in Table 8.10. The Bayes factor for hierarchy and octant determination are
3430 $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$, respectively. The octant Bayes factor
3431 is now presented as LO/UO as the known oscillation parameter is contained
3432 within the lower octant. These values compare to $B(\text{NH}/\text{IH}) = 1.08$ and
3433 $B(\text{LO}/\text{UO}) = 0.91$ from the beam-only analysis. This shows additional evidence
3434 of the joint analysis's preference for selecting the correct octant and hierarchy
3435 hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6
3436 show how the preference for the correct octant and hierarchy depend on the
3437 true value of δ_{CP} and $\sin^2(\theta_{23})$.

3438 The sensitivity of the beam-only and joint beam-atmospheric analysis to
3439 Δm_{32}^2 is given in Figure 8.44. Both of the results are marginalised over both
3440 hierarchies and the reactor constraint is not applied in either analysis. The joint
3441 analysis has a stronger preference for the correct hierarchy (NH) which is shown
3442 by the higher Bayes factor ($B(\text{NH}/\text{IH}) = 1.43$) compared to the beam-only
3443 analysis ($B(\text{NH}/\text{IH}) = 1.08$).

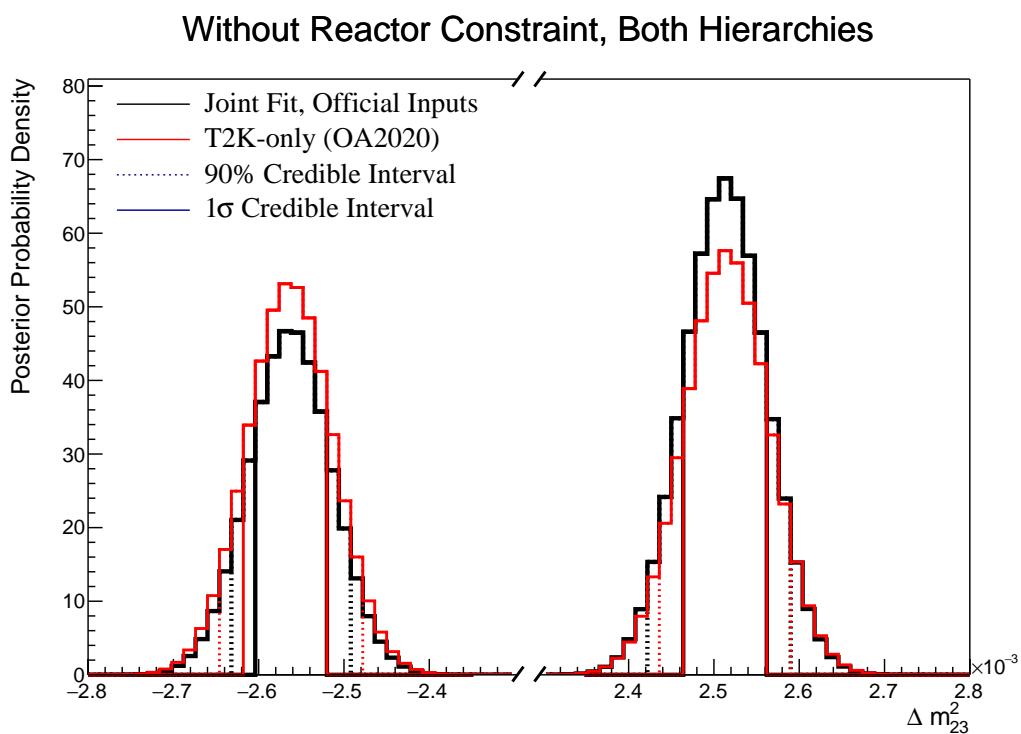


Figure 8.44: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [182]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies.

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Conclusions and Outlook

Appendices

A

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Atmospheric Sample Spectra

3449 This appendix documents the interaction mode breakdown of all the atmospheric
3450 samples used within the analysis. The generated tune of the model parameters
3451 and the Asimov A oscillation parameter set (defined in Table 2.2) are assumed.
3452 The livetime of SK-IV is taken to be 3244.4 days.

3453 A.1 Binning

3454 The lepton momentum and cosine zenith binning edges for the atmospheric
3455 samples used within this analysis are defined in Table A.1.

3456 A.2 Fully Contained Sub-GeV Samples

3457 The interaction mode breakdown of the fully contained Sub-GeV samples are
3458 shown in Figure A.1 and Figure A.2, for the samples with enriched CC0 π and
3459 CC1 π^\pm respectively.

3460 The CC0 π sample are dominated by CCQE events ($\sim 70\%$) with smaller
3461 contributions of 2p2h ($\sim 12\%$) and CC1 π ($\sim 10\%$) components. The energy peaks
3462 around 300 MeV, which is slightly below that of the T2K samples but still has
3463 significant contribution upto 1 GeV which overlaps the T2K sample energy range.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV-elike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-elike-1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-mulike-2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV-pi0like	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV-elike-nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-elike-nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV-mulike	10	3.0, 3.4, 5.0
MultiRing-elike-nue	10	3.0, 3.4, 3.7, 5.0
MultiRing-elike-nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing-mulike	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing-Other1	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table A.1: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples.

3464 The one-ring CC1 π samples, where the pion is tagged via its decay electron,
 3465 are dominated by CC1 π events ($\sim 75\%$) with a small contribution of CCM π
 3466 ($\sim 10\%$). The two-ring pion sample is mostly dominated by the NC1 π^0 via
 3467 resonances, and has several equally-sized contributions from CCQE, NC1 π^\pm via
 3468 resonances, and NC coherent pion production, where the π^0 likely comes from
 3469 nucleon and π^\pm final state interactions in the nucleus.

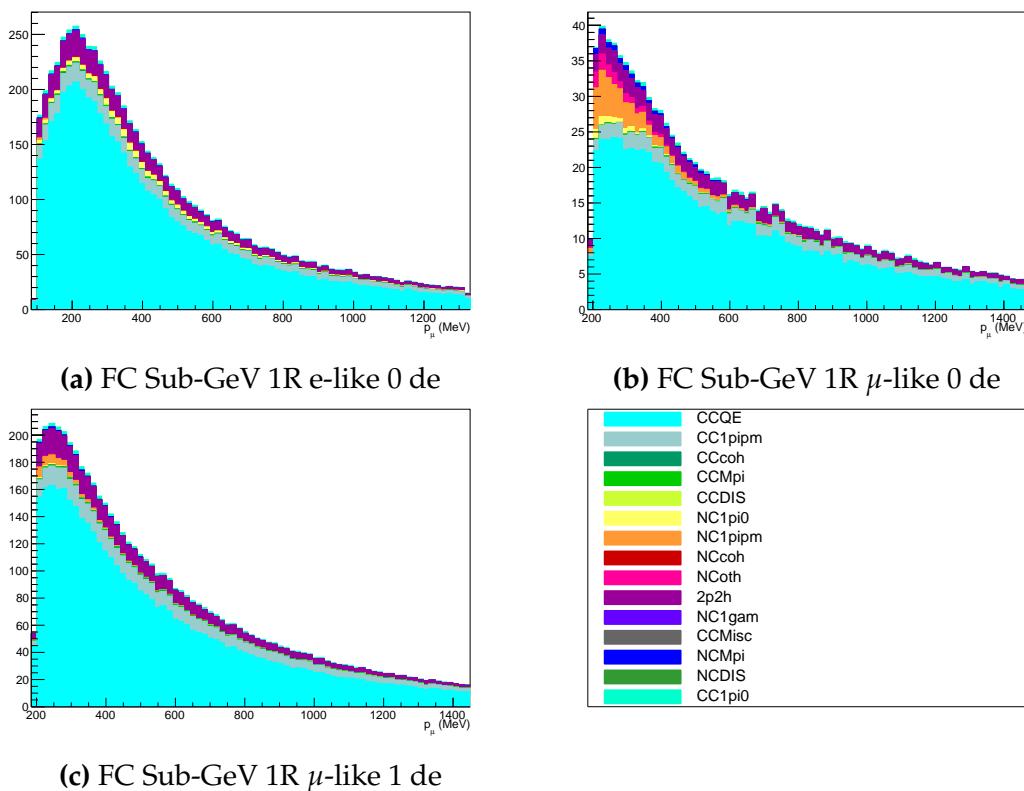


Figure A.1: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting CC 0π events.

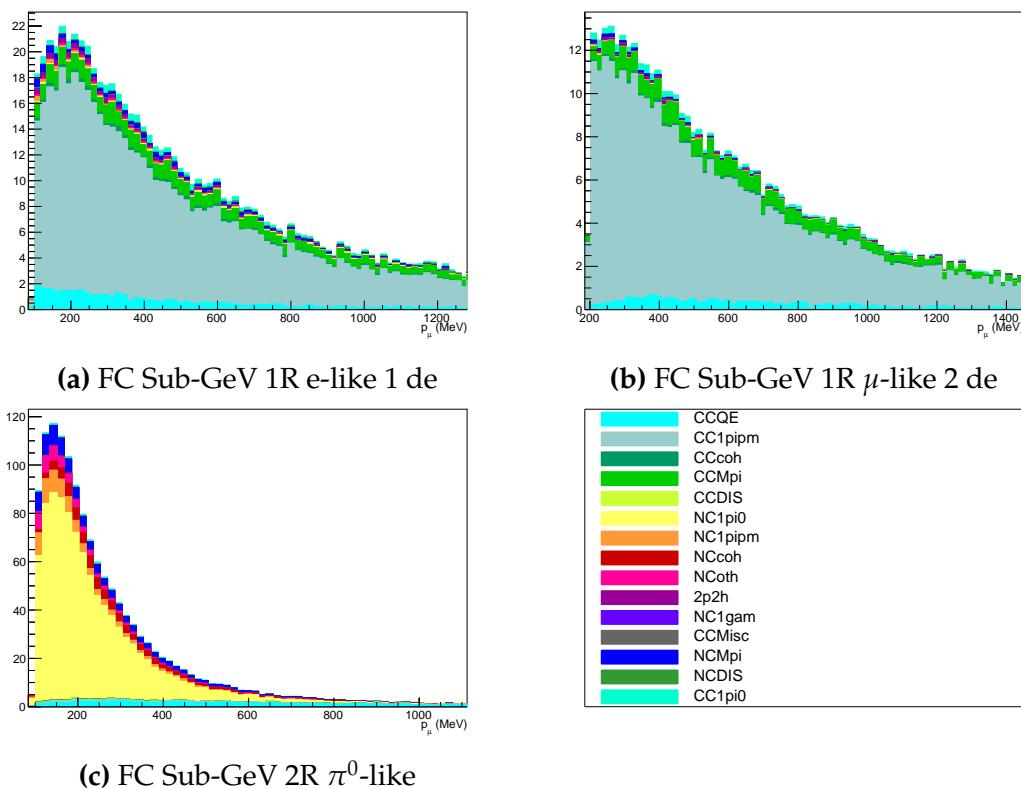


Figure A.2: Breakdown by interaction mode of the FC Sub-GeV atmospheric samples targeting single pion events.

3470 A.3 Fully Contained Multi-GeV Samples

3471 The interaction mode breakdown of fully contained multi-GeV samples is high-
 3472 lighted in Figure A.3. Due to the event selection applied in SK which targets π^+
 3473 and π^- separation, the ν_e sample mainly consists of events with pions (single pion
 3474 production or multi-pion/DIS interactions). The pion separation is explained in
 3475 Section section 6.1. This reasoning also explains the significant CCQE contribution
 3476 of the $\bar{\nu}_e$ sample. The muon-like sample is dominated by CCQE interactions with
 3477 $\sim 10 - 15\%$ 2p2h and CC1 π contribution of events.

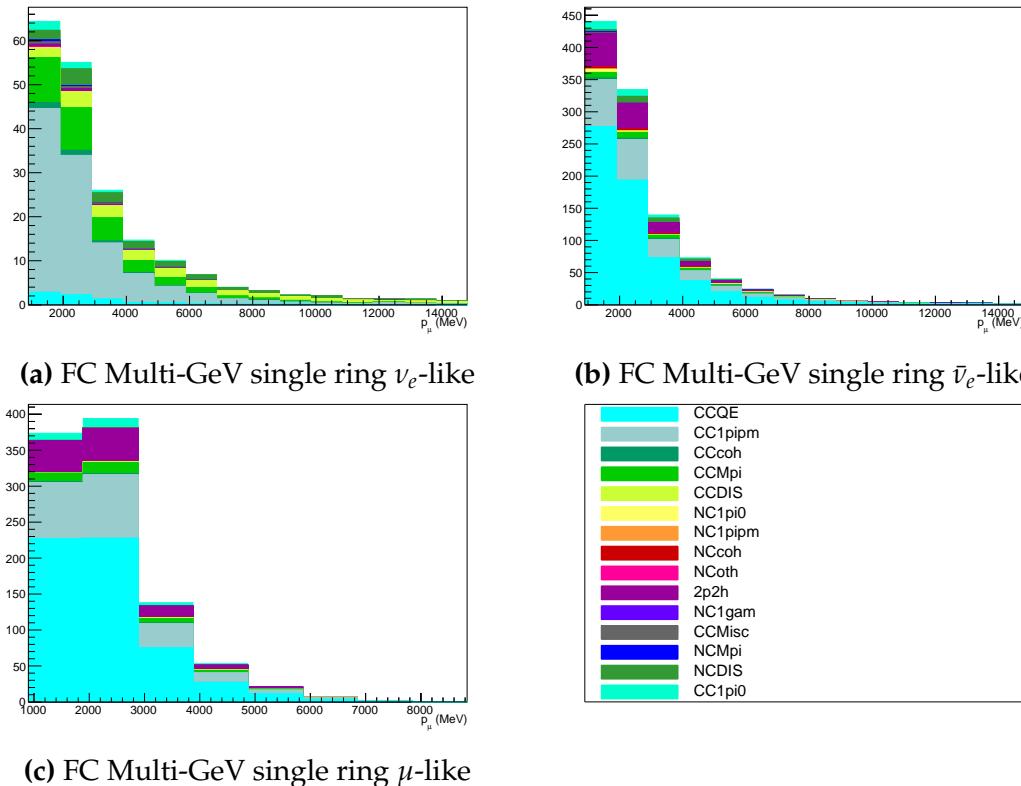


Figure A.3: Breakdown by interaction mode of the FC Multi-GeV single ring atmospheric samples.

3478 A.4 Fully Contained Multi-Ring Samples

3479 The interaction mode breakdown of fully contained multi-ring events is shown
 3480 in Figure A.4. These samples see more interaction modes contributing in general,
 3481 and there is a much larger contribution from multi-pion and DIS interaction
 3482 modes, compared to the other samples.

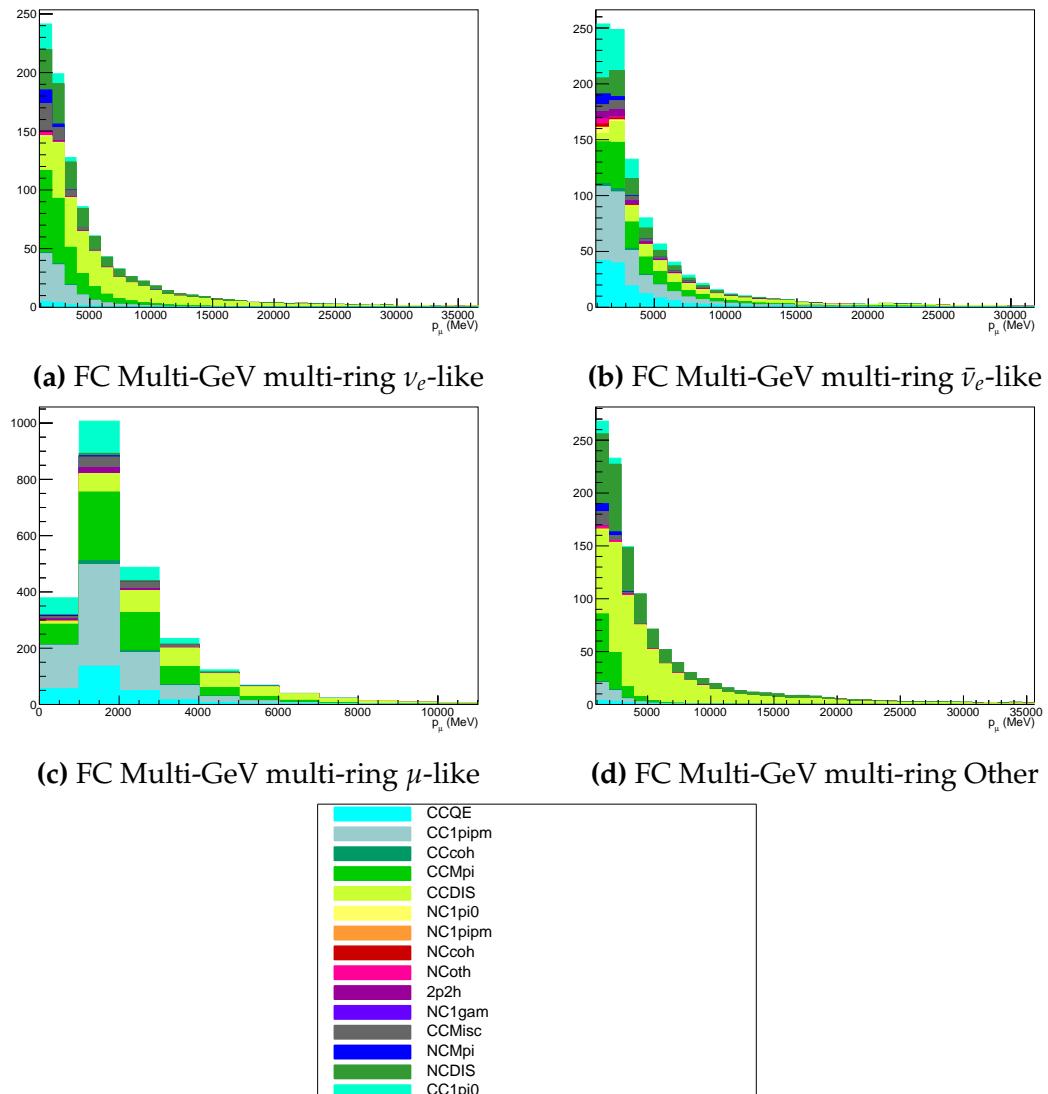


Figure A.4: Breakdown by interaction mode of the FC Multi-GeV multi-ring atmospheric samples.

3483 A.5 Partially Contained Samples

3484 The breakdown for partially contained samples is highlighted in Figure A.5.
 3485 As with the multi-ring samples, there is no dominating interaction mode. The
 3486 neutrino energies of events in this sample extend into the tens of GeV and become
 3487 dominated by DIS interaction modes in the high energy limit.

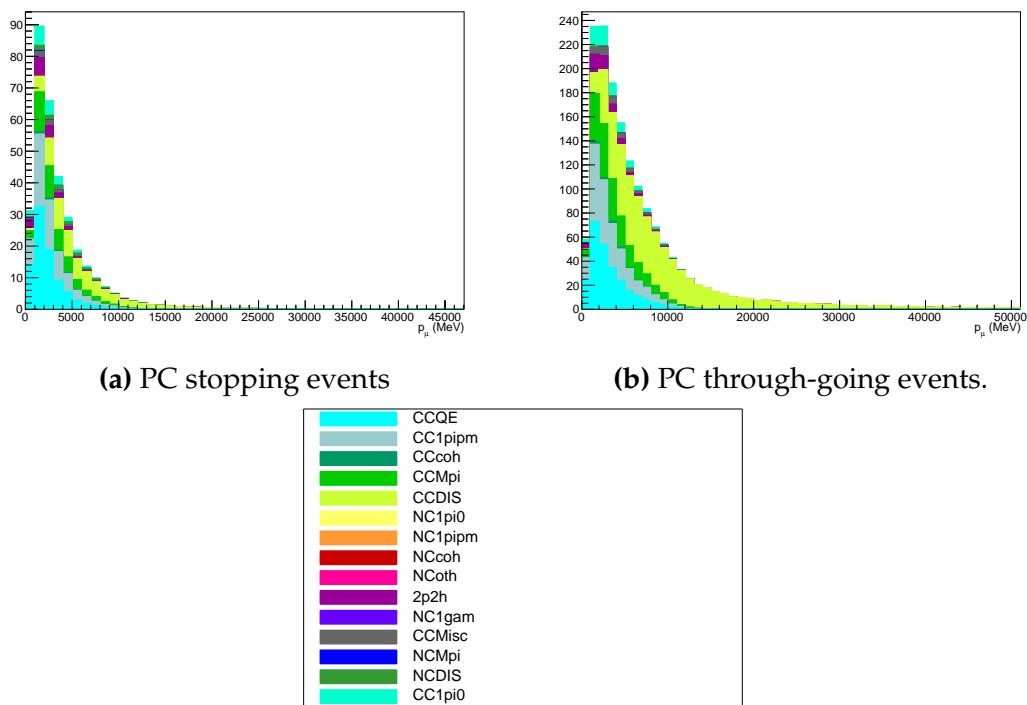


Figure A.5: Breakdown by interaction mode of the PC atmospheric samples.

3488 A.6 Upward-Going Muon Samples

3489 The breakdown for upward-going muons is illustrated in Figure A.6. These
 3490 samples are significantly dominated by DIS interactions with energies extending
 3491 up into the hundreds of GeV.

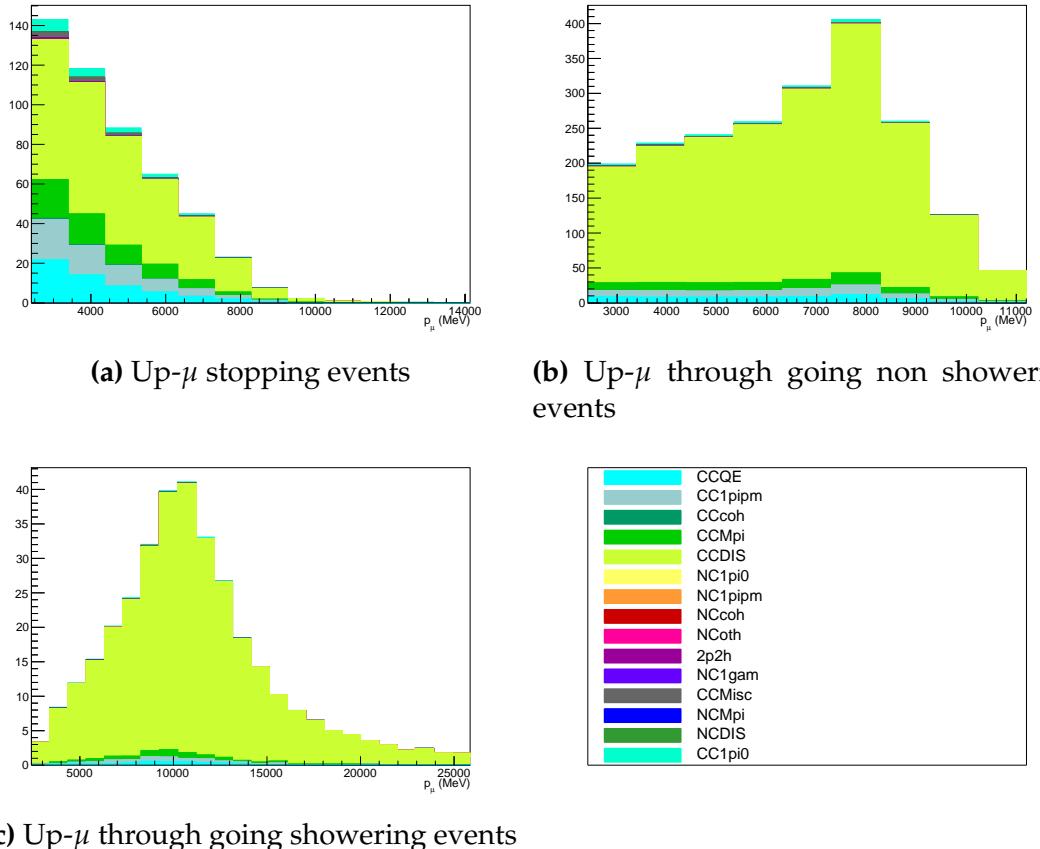


Figure A.6: Breakdown by interaction mode of the atmospheric upward going muon samples.

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