

1 **The Sensitivity to Oscillation Parameters**
2 **from a Simultaneous Beam and**
3 **Atmospheric Neutrino Analysis that**
4 **combines the T2K and SK Experiments**



6 Daniel Robert Clement Barrow
7 Magdalen College
8 University of Oxford

9 A thesis submitted for the degree of
10 *Doctor of Philosophy*
11 Michaelmas 2022

Abstract

12

13 Lorem ipsum dolor sit amet, consectetur adipiscing elit. Pellentesque sit amet
14 nibh volutpat, scelerisque nibh a, vehicula neque. Integer placerat nulla massa,
15 et vestibulum velit dignissim id. Ut eget nisi elementum, consectetur nibh in,
16 condimentum velit. Quisque sodales dui ut tempus mattis. Duis malesuada arcu
17 at ligula egestas egestas. Phasellus interdum odio at sapien fringilla scelerisque.
18 Mauris sagittis eleifend sapien, sit amet laoreet felis mollis quis. Pellentesque
19 dui ante, finibus eget blandit sit amet, tincidunt eu neque. Vivamus rutrum
20 dapibus ligula, ut imperdiet lectus tincidunt ac. Pellentesque ac lorem sed
21 diam egestas lobortis.

22 Suspendisse leo purus, efficitur mattis urna a, maximus molestie nisl. Aenean
23 porta semper tortor a vestibulum. Suspendisse viverra facilisis lorem, non
24 pretium erat lacinia a. Vestibulum tempus, quam vitae placerat porta, magna
25 risus euismod purus, in viverra lorem dui at metus. Sed ac sollicitudin nunc.
26 In maximus ipsum nunc, placerat maximus tortor gravida varius. Suspendisse
27 pretium, lorem at porttitor rhoncus, nulla urna condimentum tortor, sed suscipit
28 nisi metus ac risus.

29 Aenean sit amet enim quis lorem tristique commodo vitae ut lorem. Duis
30 vel tincidunt lacus. Sed massa velit, lacinia sed posuere vitae, malesuada vel
31 ante. Praesent a rhoncus leo. Etiam sed rutrum enim. Pellentesque lobortis
32 elementum augue, at suscipit justo malesuada at. Lorem ipsum dolor sit amet,
33 consectetur adipiscing elit. Praesent rhoncus convallis ex. Etiam commodo nunc
34 ex, non consequat diam consectetur ut. Pellentesque vitae est nec enim interdum
35 dapibus. Donec dapibus purus ipsum, eget tincidunt ex gravida eget. Donec
36 luctus nisi eu fringilla mollis. Donec eget lobortis diam.

37 Suspendisse finibus placerat dolor. Etiam ornare elementum ex ut vehicula.
38 Donec accumsan mattis erat. Quisque cursus fringilla diam, eget placerat neque
39 bibendum eu. Ut faucibus dui vitae dolor porta, at elementum ipsum semper.
40 Sed ultrices dui non arcu pellentesque placerat. Etiam posuere malesuada turpis,
41 nec malesuada tellus malesuada.

42

Statement of Originality

43 Lorem ipsum dolor sit amet, consectetur adipiscing elit. Pellentesque sit amet
44 nibh volutpat, scelerisque nibh a, vehicula neque. Integer placerat nulla massa,
45 et vestibulum velit dignissim id. Ut eget nisi elementum, consectetur nibh in,
46 condimentum velit. Quisque sodales dui ut tempus mattis. Duis malesuada arcu
47 at ligula egestas egestas. Phasellus interdum odio at sapien fringilla scelerisque.
48 Mauris sagittis eleifend sapien, sit amet laoreet felis mollis quis. Pellentesque
49 dui ante, finibus eget blandit sit amet, tincidunt eu neque. Vivamus rutrum
50 dapibus ligula, ut imperdiet lectus tincidunt ac. Pellentesque ac lorem sed
51 diam egestas lobortis.

52 Suspendisse leo purus, efficitur mattis urna a, maximus molestie nisl. Aenean
53 porta semper tortor a vestibulum. Suspendisse viverra facilisis lorem, non
54 pretium erat lacinia a. Vestibulum tempus, quam vitae placerat porta, magna
55 risus euismod purus, in viverra lorem dui at metus. Sed ac sollicitudin nunc.
56 In maximus ipsum nunc, placerat maximus tortor gravida varius. Suspendisse
57 pretium, lorem at porttitor rhoncus, nulla urna condimentum tortor, sed suscipit
58 nisi metus ac risus.

Acknowledgements

60 Personal

61 This is where you thank your advisor, colleagues, and family and friends.

62 Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum feugiat
63 et est at accumsan. Praesent sed elit mattis, congue mi sed, porta ipsum. In
64 non ullamcorper lacus. Quisque volutpat tempus ligula ac ultricies. Nam sed
65 erat feugiat, elementum dolor sed, elementum neque. Aliquam eu iaculis est,
66 a sollicitudin augue. Cras id lorem vel purus posuere tempor. Proin tincidunt,
67 sapien non dictum aliquam, ex odio ornare mauris, ultrices viverra nisi magna
68 in lacus. Fusce aliquet molestie massa, ut fringilla purus rutrum consectetur.
69 Nam non nunc tincidunt, rutrum dui sit amet, ornare nunc. Donec cursus
70 tortor vel odio molestie dignissim. Vivamus id mi erat. Duis porttitor diam
71 tempor rutrum porttitor. Lorem ipsum dolor sit amet, consectetur adipiscing
72 elit. Sed condimentum venenatis consectetur. Lorem ipsum dolor sit amet,
73 consectetur adipiscing elit.

74 Aenean sit amet lectus nec tellus viverra ultrices vitae commodo nunc. Mauris
75 at maximus arcu. Aliquam varius congue orci et ultrices. In non ipsum vel
76 est scelerisque efficitur in at augue. Nullam rhoncus orci velit. Duis ultricies
77 accumsan feugiat. Etiam consectetur ornare velit et eleifend.

78 Suspendisse sed enim lacinia, pharetra neque ac, ultricies urna. Phasellus sit
79 amet cursus purus. Quisque non odio libero. Etiam iaculis odio a ex volutpat, eget
80 pulvinar augue mollis. Mauris nibh lorem, mollis quis semper quis, consequat
81 nec metus. Etiam dolor mi, cursus a ipsum aliquam, eleifend venenatis ipsum.
82 Maecenas tempus, nibh eget scelerisque feugiat, leo nibh lobortis diam, id laoreet
83 purus dolor eu mauris. Pellentesque habitant morbi tristique senectus et netus
84 et malesuada fames ac turpis egestas. Nulla eget tortor eu arcu sagittis euismod
85 fermentum id neque. In sit amet justo ligula. Donec rutrum ex a aliquet egestas.

86 Institutional

87 If you want to separate out your thanks for funding and institutional support,
88 I don't think there's any rule against it. Of course, you could also just remove
89 the subsections and do one big traditional acknowledgement section.

90 Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut luctus tempor ex at
91 pretium. Sed varius, mauris at dapibus lobortis, elit purus tempor neque, facilisis
92 sollicitudin felis nunc a urna. Morbi mattis ante non augue blandit pulvinar.
93 Quisque nec euismod mauris. Nulla et tellus eu nibh auctor malesuada quis
94 imperdiet quam. Sed eget tincidunt velit. Cras molestie sem ipsum, at faucibus
95 quam mattis vel. Quisque vel placerat orci, id tempor urna. Vivamus mollis,
96 neque in aliquam consequat, dui sem volutpat lorem, sit amet tempor ipsum felis
97 eget ante. Integer lacinia nulla vitae felis vulputate, at tincidunt ligula maximus.
98 Aenean venenatis dolor ante, euismod ultrices nibh mollis ac. Ut malesuada
99 aliquam urna, ac interdum magna malesuada posuere.

Contents

¹⁰¹	1 Introduction	1
¹⁰²	2 Neutrino Oscillation Physics	4
¹⁰³	2.1 Discovery of Neutrinos	5
¹⁰⁴	2.2 Theory of Neutrino Oscillation	6
¹⁰⁵	2.3 Neutrino Oscillation Measurements	10
¹⁰⁶	2.4 Summary Of Oscillation Parameter Measurements	20
¹⁰⁷	2.5 Overview of Oscillation Effects	21
¹⁰⁸	3 T2K and SK Experiment Overview	29
¹⁰⁹	3.1 The Super-Kamiokande Experiment	29
¹¹⁰	3.2 The Tokai to Kamioka Experiment	40
¹¹¹	4 Bayesian Statistics and Markov Chain Monte Carlo Techniques	55
¹¹²	4.1 Bayesian Statistics	56
¹¹³	4.2 Monte Carlo Simulation	58
¹¹⁴	4.3 Understanding the MCMC Results	66
¹¹⁵	5 Simulation, Reconstruction, and Event Reduction	71
¹¹⁶	5.1 Simulation	71
¹¹⁷	5.2 Event Reconstruction at SK	77
¹¹⁸	5.3 Event Reduction at SK	88
¹¹⁹	6 Sample Selections and Systematics	93
¹²⁰	6.1 Atmospheric Samples	95
¹²¹	6.2 Near Detector Beam Samples	103
¹²²	6.3 Far Detector Beam Samples	105
¹²³	6.4 Systematic Uncertainties	111
¹²⁴	6.5 Likelihood Calculation	131
¹²⁵	7 Oscillation Probability Calculation	135
¹²⁶	7.1 Treatment of Fast Oscillations	136
¹²⁷	7.2 Calculation Engine	143
¹²⁸	7.3 Matter Density Profile	147
¹²⁹	7.4 Production Height Averaging	152

¹³⁰	8 Oscillation Analysis	155
¹³¹	8.1 Monte Carlo Prediction	156
¹³²	8.2 Likelihood Scans	157
¹³³	8.3 Sensitivity Studies	169
¹³⁴	8.4 Summary of Sensitivity Studies	221
¹³⁵	9 Conclusions and Outlook	223

1

136

137

Introduction

138 Current measurements illustrate that the universe is matter-dominated, despite
139 the belief that an equal amount of matter and antimatter were created in the
140 Big Bang. One explanation of this behaviour is through the violation of CP-
141 symmetries, one requirement of the Sakharov conditions [1]. The Standard
142 Model relates a particle and its antiparticle through these symmetries, which if
143 violated, could result in the observed matter-antimatter imbalance. CP-violation
144 has been observed in quark mixing but is insufficient to explain the current
145 measurements. Neutrino oscillation physics has the potential to include CP-
146 violating terms through the δ_{CP} phase contained within the PMNS mechanism.
147 Therefore, one of the main goals of neutrino oscillation experiments is to precisely
148 measure this parameter. To allow this, a precise measurement of all oscillation
149 parameters, including the currently undetermined mass hierarchy, is required.

150 The Super-Kamiokande (SK) detector is situated as the far detector of the
151 Tokai-to-Kamioka (T2K) experiment and observes neutrinos from the beam
152 originating in J-PARC alongside the flux of atmospheric neutrinos emitted from
153 the primary and secondary interactions of cosmic rays. Previous oscillation
154 analyses published by the two experiments have been independent of one another.
155 However, due to the different energies, path lengths, and density of matter in
156 which the neutrinos pass through, a combined analysis will be able to leverage the

157 constraints from both experiments and be able to break some of the degeneracies
158 in oscillation parameter space.

159 This thesis details the sensitivities of a joint beam and atmospheric neutrino
160 analysis using beam samples observed at the near and far detectors of the
161 T2K experiment and atmospheric samples present in SK. It combines the beam
162 analysis presented in [2] and the atmospheric analysis documented in [3]. This
163 corresponds to run1-10 of the T2K experiment with approximately equal exposure
164 taken in neutrino and antineutrino beam modes, alongside more than 3000 days
165 of atmospheric events. This analysis will have sensitivity to the δ_{CP} , $\sin^2(\theta_{13})$,
166 $\sin^2(\theta_{23})$, and Δm_{32}^2 oscillation parameters. Crucially, the combination of beam
167 and atmospheric neutrinos should give strong sensitivity to the mass hierarchy
168 due to the correlation between the matter resonance and $\sin^2(\theta_{23})$. This analysis
169 also lays the foundation of a joint analysis in the Hyper-Kamiokande, which is
170 one of the next-generation neutrino oscillation experiments.

171 Chapter 2 provides a concise overview of neutrino physics history includ-
172 ing the discovery of the neutrino along with the first evidence for neutrino
173 oscillation. It also includes a brief discussion of the theory underpinning the
174 PMNS formalism alongside a summary of the current measurements of each
175 oscillaton parameter.

176 The T2K and SK experiments are detailed in Chapter 3. This includes the
177 design and calibration of the SK detector along with a brief insight into the
178 composition and detection techniques of T2K’s two near detectors. The neutrino
179 beamline, and the ‘off-axis’ trick, are also briefly summarised.

180 This thesis presents a Bayesian neutrino oscillation analysis that uses Markov
181 Chain Monte Carlo techniques. This analysis strategy, along with a summary
182 of the fundamental concepts of Bayes’ theorem, is described in Chapter 4. This
183 includes a discussion about the conditions that are required to correctly sam-
184 ple the parameter space along with the methods used to calculate parameter
185 estimations and build credible intervals.

186 Chapter 5 details the simulations and reconstruction tools used to build Monte
187 Carlo predictions of each sample used within this analysis. This includes the
188 models used to provide a flux prediction of the beam and atmospheric neutrinos
189 as well as the models invoked with this analysis to simulate neutrino interactions.
190 Validation of the far detector’s reconstruction tools has been documented which
191 compares the change in detector response between two distinct periods of
192 the SK detector.

193 A description of the beam samples used at the near and far detector and the
194 atmospheric samples used at the SK detector is presented in Chapter 6. These
195 include energy and interaction mode comparisons along with documenting the
196 similarities between the event selection cuts. This chapter also includes the
197 systematic models used to control the uncertainty within the flux predictions of
198 both beam and atmospheric neutrinos, the interaction models, and the response
199 of the detectors used within this analysis.

200 A novel atmospheric neutrino oscillation calculation method is documented in
201 Chapter 7. This is required to ensure reliable Monte Carlo sampling of a rapidly
202 varying region of oscillation parameter space. This chapter also documents
203 the implementation of systematics used to control the uncertainties related
204 to the Earth’s density as well as the production height of neutrinos in the
205 upper atmosphere.

206 Chapter 8 presents the sensitivities of this joint beam and atmospheric neu-
207 trino oscillation analysis. The results are provided for two different sets of
208 known values. The application of the reactor constraint on $\sin^2(\theta_{23})$ has also been
209 considered. The sensitivities of the joint analysis are compared to the beam-only
210 analysis and show the benefits of the combined analysis.

211 A summarised discussion of the sensitivity results and the outlook for the
212 analysis, including the implications of analysis on the next generation of neutrino
213 experiments, is provided in Chapter 9.

2

214

215

Neutrino Oscillation Physics

216 When first proposed, neutrinos were expected to be approximately massless
217 fermions that only interact through weak and gravitational forces. This meant
218 they were very difficult to detect as they can pass through significant amounts
219 of matter without interacting. Despite this, experimental neutrino physics has
220 developed many different detection techniques and observed neutrinos from
221 both natural and artificial sources. In direct tension with Standard Model physics,
222 neutrinos have been determined to oscillate between different lepton flavours,
223 requiring them to have mass.

224 The observation techniques which led to the discovery of the neutrino are doc-
225 umented in section 2.1. The theory underpinning neutrino oscillation is described
226 in section 2.2 and includes the approximations which can be made to simplify
227 the understanding of neutrino oscillation in the two-flavour approximation. Past,
228 current, and future neutrino experiments are detailed in section 2.3, including the
229 reactor, atmospheric, and long-baseline accelerator neutrino sources that have
230 been used to successfully constrain oscillation parameters. Finally, the current
231 state of oscillation parameter measurements are summarised in section 2.4.

2.1 Discovery of Neutrinos

At the start of the 20th century, the electrons emitted from the β -decay of the nucleus were found to have a continuous energy spectrum [4, 5]. This observation seemingly broke the energy conservation invoked within that period's nuclear models. In 1930, Pauli provided a solution to this problem in the form of a new particle, the neutrino (originally termed "neutron"). It was theorized to be an electrically neutral spin-1/2 fermion with a mass smaller than that of the electron [6]. This neutrino was emitted with the electron in β -decay to alleviate the apparent breaking of energy conservation. As a predecessor of today's weak interaction model, Fermi's theory of β -decay developed the understanding by coupling the four constituent particles: electron, proton, neutron, and neutrino, into a quantitative model [7].

Whilst Pauli was not convinced of the ability to detect neutrinos, the first observations of the particle were made in the mid-1950s when neutrinos from a reactor were observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [8, 9]. The detector consisted of two parts: a neutrino interaction medium and a liquid scintillator. The interaction medium was built from two water tanks, loaded with cadmium chloride to allow for increased efficiency in the detection of neutron capture. The positron emitted from IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured on the cadmium via $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$, producing a delayed signal. An increase in the coincidence rate was observed when the reactor was operating which was interpreted as interactions from neutrinos generated in the reactor.

After the discovery of the ν_e , the question of how many flavours of neutrino exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven National Laboratory [10]. A proton beam was directed at a beryllium target, generating pions which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the subsequent interactions of the ν_μ were observed. As the subsequent interaction of the neutrino generated muons rather than electrons, it was determined that

the ν_μ was fundamentally different from ν_e . The final observation to be made was that of the ν_τ from the DONUT experiment [11]. Three neutrinos seem the obvious solution as it mirrors the known number of charged leptons (as they form weak isospin doublets) but there could be evidence of more. Several neutrino experiments have found anomalous results [12, 13] which could be attributed to “sterile” neutrinos. These hypothesised particles are not affected by gauge interactions in the Standard Model so their presence can only be inferred through the observation of non-standard oscillation modes. However, cosmological observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [14], as measured from the cosmic microwave background power spectrum. LEP also measured the number of active neutrino flavours to be $N_\nu = 2.9840 \pm 0.0082$ [15] from measurements of the Z-decay width, but this does not strongly constrain the number of sterile neutrinos.

2.2 Theory of Neutrino Oscillation

A neutrino generated with lepton flavour α can change into a different lepton flavour β after propagating some distance. This phenomenon is called neutrino oscillation and requires that neutrinos must have a non-zero mass. This behaviour has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [16–18] mixing matrix which describes how the flavour and mass of neutrinos are associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [19] matrix measured in quark physics.

2.2.1 Three Flavour Oscillations

The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ (indexed ν_α), which are eigenstates of the weak interaction and three mass eigenstates, ν_1 , ν_2 and ν_3 (indexed ν_i). Each mass eigenstate is the superposition

²⁸⁶ of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

²⁸⁷ Where U is the 3×3 PMNS matrix which is unitary and connects the mass
²⁸⁸ and flavour eigenstates.

²⁸⁹ The weak interaction, when interacting via a W^{\pm} boson, couples to flavour
²⁹⁰ eigenstates so neutrinos interact with leptons of the same flavour. The prop-
²⁹¹ agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a
²⁹² plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

²⁹³ The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
²⁹⁴ three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
²⁹⁵ coordinates). The probability of observing a neutrino of flavour eigenstate β from
²⁹⁶ one which originated as flavour α can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)}. \quad (2.3)$$

²⁹⁷ The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

²⁹⁸ For a relativistic particle, $E_i \gg m_i$, a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

²⁹⁹ Making the approximations that neutrinos are relativistic, the mass eigenstates
³⁰⁰ were created with the same energy and that $x = L$, where L is the distance
³⁰¹ travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

302 where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This, combined with further use of unitarity relations
 303 results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \end{aligned} \quad (2.7)$$

304 Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last
 305 term is included for the oscillation probability of antineutrinos. As an important
 306 point to note, the observation of oscillation probability requires a non-zero value
 307 of Δm_{ij}^2 , which in turn requires that neutrinos have differing masses.

308 Typically, the PMNS matrix is parameterised into three mixing angles, a
 309 charge parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

310 Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often
 311 grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many
 312 neutrino experiments aim to measure the PMNS parameters from a wide array
 313 of origins, as is the purpose of this thesis.

314 The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8
 315 is only included for completeness. For an oscillation analysis experiment, any
 316 terms containing this phase disappear due to taking the expectation value of the
 317 PMNS matrix. Measurements of these phases can be performed by experiments
 318 searching for neutrino-less double β -decay [20].

319 A two-flavour approximation can be obtained when one assumes the third
320 mass eigenstate is degenerate with another. This results in the two-flavour
321 approximation being reasonable for understanding the features of the oscillation.
322 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

323 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

324 Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is
325 a sinusoidal function depending upon the distance over which the neutrino
326 propagates. The frequency and amplitude of oscillation are dependent upon
327 $\Delta m^2/4E$ and $\sin^2 2\theta$, respectively. The oscillation probabilities presented thus far
328 assume $c = 1$, where c is the speed of light in a vacuum. In more familiar units, the
329 maximum oscillation probability for a fixed value of θ is given at $L[\text{km}] / E[\text{GeV}] \sim$
330 $1.27/\Delta m^2$. It is this calculation that determines the best L/E value for a given
331 experiment to be designed around for measurements of a specific value of Δm^2 .

332 2.2.2 The MSW Effect

333 The theory of neutrino oscillation in a vacuum has been described in subsec-
334 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating
335 from below the horizon propagate through the matter in the Earth. The coherent
336 scattering of neutrinos from a material target modifies the Hamiltonian of the
337 system which results in a change in the oscillation probability. This modification
338 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [21–23]. This occurs
339 because charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$, propagated by a W boson)
340 only affects electron neutrinos whereas the neutral current scattering ($\nu_l + l^- \rightarrow$

- ³⁴¹ $\nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino flavours equally.
³⁴² In the two-flavour approximation, the effective mixing parameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

³⁴³ where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F
³⁴⁴ is Fermi's constant. It is clear that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for
³⁴⁵ $\Delta m^2 > 0$, which results in a divergent mixing parameter, colloquially called the
³⁴⁶ matter resonance. This resonance regenerates the electron neutrino component of
³⁴⁷ the neutrino flux [21–23]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

³⁴⁸ At densities lower than this critical value, the oscillation probability will
³⁴⁹ be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$
³⁵⁰ [24]. The resonance occurring from the MSW effect depends on the sign of Δm^2 .
³⁵¹ Therefore, any neutrino oscillation experiment which observes neutrinos and
³⁵² antineutrinos which have propagated through matter can have some sensitivity
³⁵³ to the ordering of the neutrino mass eigenstates.

³⁵⁴ 2.3 Neutrino Oscillation Measurements

³⁵⁵ As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics
³⁵⁶ was awarded to the Super-Kamiokande (SK) [25] and Sudbury Neutrino Ob-
³⁵⁷ servatory (SNO) [26] collaborations for the first definitive observation of solar
³⁵⁸ and atmospheric neutrino oscillation [27]. Since then, the field has seen a wide
³⁵⁹ array of oscillation measurements from a variety of neutrino sources. As seen
³⁶⁰ in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio
³⁶¹ of the propagation baseline, L , to the neutrino energy, E . It is this ratio that
³⁶² determines the type of neutrino oscillation a particular experiment is sensitive to.

³⁶³ As illustrated in Figure 2.1, there are many neutrino sources that span a
³⁶⁴ wide range of energies. The least energetic neutrinos are from reactor and

365 terrestrial sources at $O(1)$ MeV whereas the most energetic neutrinos originate
366 from atmospheric and galactic neutrinos of $> O(1)$ TeV.

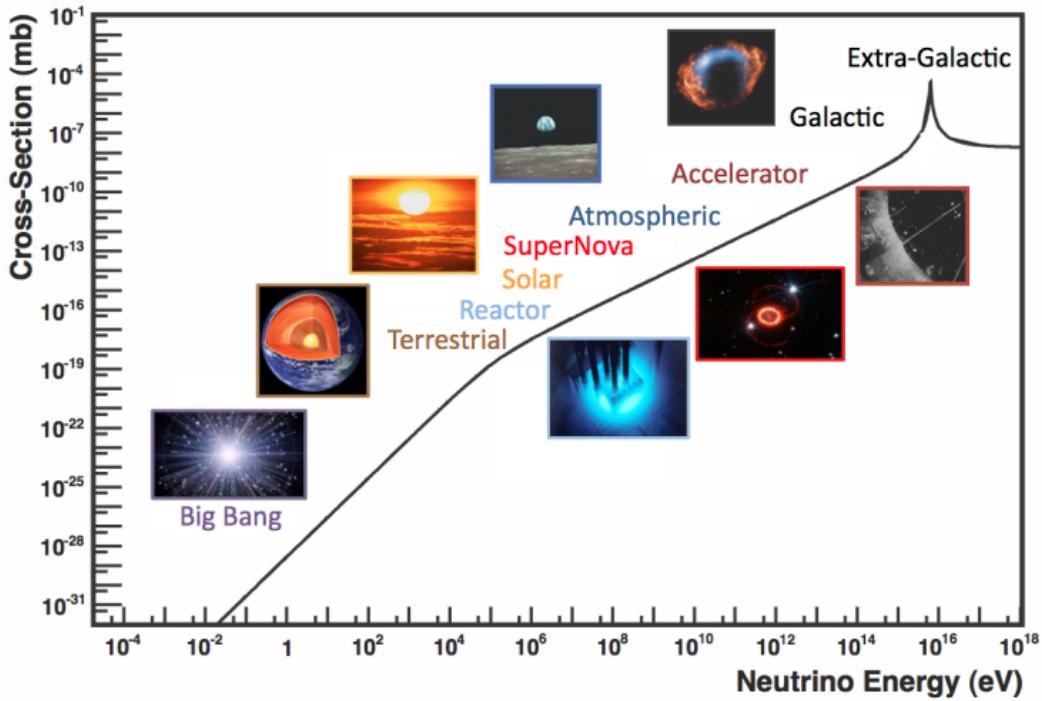


Figure 2.1: The electro-weak cross-section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [28]

367 2.3.1 Solar Neutrinos

368 Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun.
369 The solar neutrino flux, given as a function of neutrino energy for different
370 fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion
371 generates the largest flux of neutrinos, the neutrinos are low energy and are
372 difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [29].
373 Consequently, most experiments focus on the neutrinos from the decay of 8B
374 (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher energy.

375 The first measurements of solar neutrinos observed a significant reduction in
376 the event rate compared to predictions from the Standard Solar Model [31, 32]. A
377 proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a

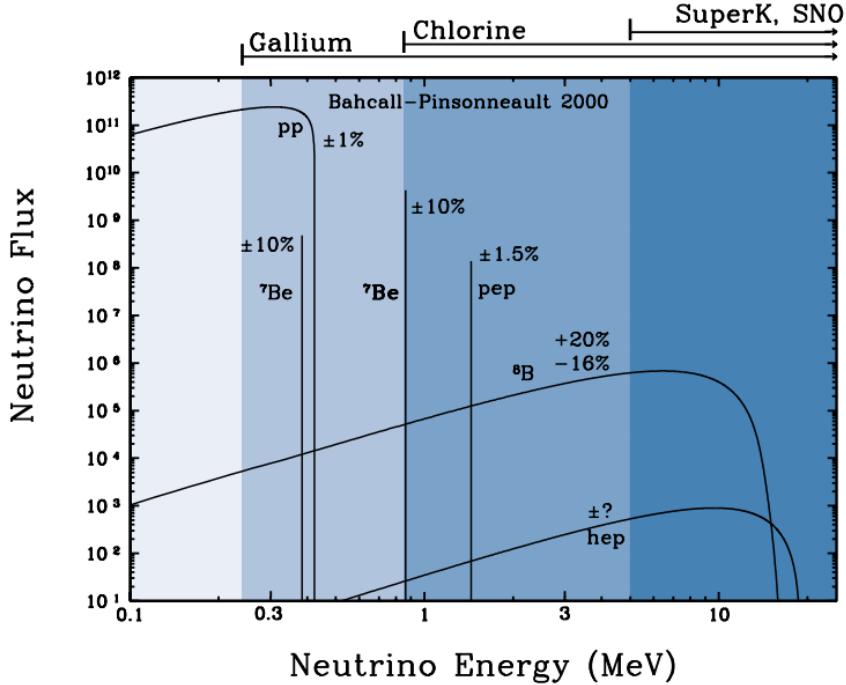


Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [30].

378 precursory version of the PMNS model [33]. The Kamiokande [34], Gallex [35]
 379 and Sage [36] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

380 The conclusive solution to this problem was determined by the SNO col-
 381 laboration [26]. Using a deuterium water target to observe 8B neutrinos, the
 382 event rate of charged current (CC), neutral current (NC), and elastic scattering
 383 (ES) interactions (Given in Equation 2.13) was simultaneously measured. CC
 384 events can only occur for electron neutrinos, whereas the NC channel is agnostic
 385 to neutrino flavour, and the ES reaction has a small excess sensitivity for the
 386 detection of electron neutrino interactions. This meant that there were direct
 387 measurements of the ν_e and ν_x neutrino flux. It was concluded that the CC and
 388 ES interaction rates were consistent with the deficit previously observed. Most
 389 importantly, the NC reaction rate was only consistent with the others under the

³⁹⁰ hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (CC) \\ \nu_x + d &\rightarrow p + n + \nu_x & (NC) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (ES) \end{aligned} \quad (2.13)$$

³⁹¹ Since the SNO measurement, many experiments have since measured the
³⁹² neutrino flux of different interaction chains within the sun [37–39]. The most
³⁹³ recent measurement was that of CNO-cycle neutrinos which were recently
³⁹⁴ observed with 5σ significance by the Borexino collaboration [37].

³⁹⁵ 2.3.2 Accelerator Neutrinos

³⁹⁶ The concept of using an artificial “neutrino beam” was first realised in 1962 [10].
³⁹⁷ Since then, many experiments have adopted the same fundamental concepts.
³⁹⁸ Typically, a proton beam is aimed at a target producing charged mesons that
³⁹⁹ decay to neutrinos. The mesons can be sign-selected by the use of magnetic
⁴⁰⁰ focusing horns to generate a neutrino or antineutrino beam. Pions are the primary
⁴⁰¹ mesons that decay and depending on the orientation of the magnetic field, a
⁴⁰² muon (anti-)neutrino beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$.
⁴⁰³ The decay of muons and kaons results in an irreducible intrinsic electron neutrino
⁴⁰⁴ background. In T2K, this background contamination is $O(< 1\%)$ [40]. There is
⁴⁰⁵ also an approximately $\sim 5\%$ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated
⁴⁰⁶ via the same decays. As the beam is generated by proton interactions (rather
⁴⁰⁷ than anti-proton interactions), the wrong-sign component in the antineutrino
⁴⁰⁸ beam is larger when operating in neutrino mode.

⁴⁰⁹ Tuning the proton energy in the beam and using beam focusing techniques
⁴¹⁰ allows the neutrino energy to be set to a value that maximises the disappear-
⁴¹¹ ance oscillation probability in the L/E term in Equation 2.10. This means that
⁴¹² accelerator experiments are typically more sensitive to the mixing parameters as
⁴¹³ compared to a natural neutrino source. However, the disadvantage compared
⁴¹⁴ to atmospheric neutrino experiments is the cost of building a facility to provide

415 high-energy neutrinos, with a high flux, which is required for longer baselines.
416 Consequently, there is typically less sensitivity to matter effects and the ordering
417 of the neutrino mass eigenstates.

418 A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

419 where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of
420 neutrinos with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and
421 $\epsilon(\vec{x})$ is the efficiency and resolution of the detector. In order to leverage the most
422 out of an accelerator neutrino experiment, the flux and cross-section systematics
423 need to be constrained. This is typically done via the use of a “near detector”,
424 situated at a baseline of $O(1)$ km. This detector observes the unoscillated neutrino
425 flux and constrains the parameters used within the flux and cross-section model.

426 The first accelerator experiments to precisely measure oscillation parameters
427 were MINOS [41] and K2K [42]. These experiments confirmed the ν_μ disappear-
428 ance seen in atmospheric neutrino experiments by finding consistent parameter
429 values for $\sin^2(\theta_{23})$ and Δm_{32}^2 . The current generation of accelerator neutrino
430 experiments, T2K and NO ν A extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead
431 the sensitivity to atmospheric mixing parameters as seen in Figure 2.6 [43]. The
432 two experiments differ in their peak neutrino energy, baseline, and detection tech-
433 nique. The NO ν A experiment is situated at a baseline of 810km from the NuMI
434 beamline which delivers 2GeV neutrinos. The T2K neutrino beam is peaked
435 around 0.6GeV and propagates 295km [44]. Additionally, the NO ν A experiment
436 uses functionally identical detectors (near and far) whereas T2K uses a plastic
437 scintillator technique at the near detector and a water Cherenkov far detector.
438 The future generation experiments DUNE [45] and Hyper-Kamiokande [46]
439 will succeed these experiments as the high-precision era of neutrino oscillation
440 parameter measurements develops.

441 Several anomalous results have been observed in the LSND [12] and Mini-
442 BooNE [13] detectors which were designed with purposefully short baselines.

⁴⁴³ Parts of the neutrino community attributed these results to oscillations induced
⁴⁴⁴ by a fourth “sterile” neutrino [47] but several searches in other experiments,
⁴⁴⁵ MicroBooNE [48] and KARMEN [49], found no hints of additional neutrino
⁴⁴⁶ species. The solution to the anomalous results is still being determined.

⁴⁴⁷ 2.3.3 Atmospheric Neutrinos

⁴⁴⁸ The interactions of primary cosmic ray protons in the Earth’s upper atmosphere
⁴⁴⁹ generate showers of energetic hadrons. These are mostly pions and kaons that
⁴⁵⁰ decay to produce a natural source of neutrinos spanning energies of MeV to
⁴⁵¹ TeV [50]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu), \end{aligned} \tag{2.15}$$

⁴⁵² such that for a single pion decay, three neutrinos can be produced. The
⁴⁵³ atmospheric neutrino flux energy spectra as predicted by the Bartol [51], Honda
⁴⁵⁴ [52–54], and FLUKA [55] models are illustrated in Figure 2.3. The flux distribution
⁴⁵⁵ peaks at an energy of $O(10)$ GeV. The uncertainties associated with these models
⁴⁵⁶ are dominated by the hadronic production of kaon and pions as well as the
⁴⁵⁷ primary cosmic flux.

⁴⁵⁸ Unlike long-baseline experiments which have a fixed baseline, the distance
⁴⁵⁹ atmospheric neutrinos propagate is dependent upon the zenith angle at which
⁴⁶⁰ they interact. This is illustrated in Figure 2.4. Neutrinos that are generated
⁴⁶¹ directly above the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the
⁴⁶² height of the atmosphere, whereas neutrinos that interact directly below the
⁴⁶³ detector ($\cos(\theta) = -1.0$) have to travel a length equal to the diameter of the Earth.
⁴⁶⁴ This means atmospheric neutrinos have a baseline that varies from $O(20)$ km to
⁴⁶⁵ $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will be subject
⁴⁶⁶ to MSW matter resonance as they propagate through the Earth.

⁴⁶⁷ Figure 2.5 highlights the neutrino flux as a function of the zenith angle for
⁴⁶⁸ different slices of neutrino energy. For medium to high-energy neutrinos (and to

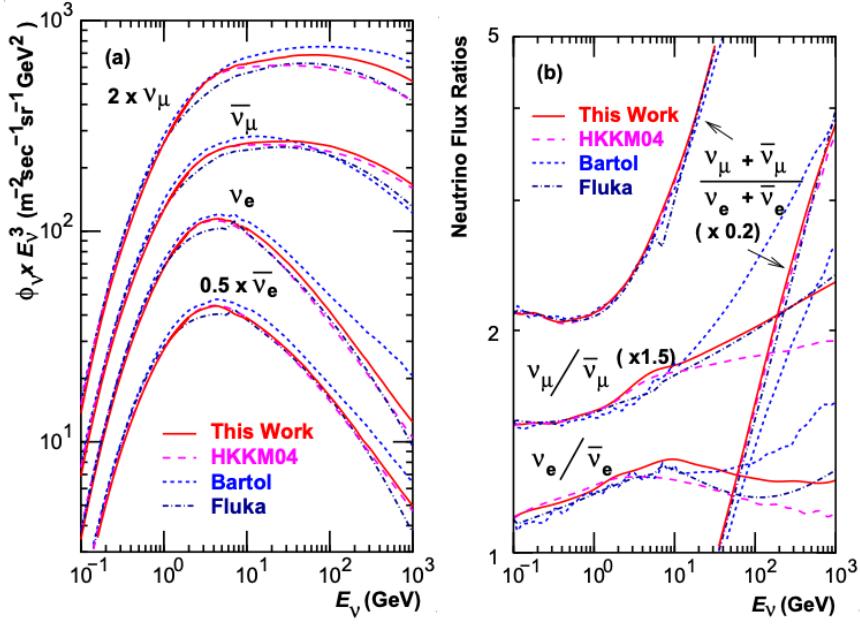


Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [52], the 2004 Honda model (“HKKM04”)[53], the Bartol model [51] and the FLUKA model [55]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [52].

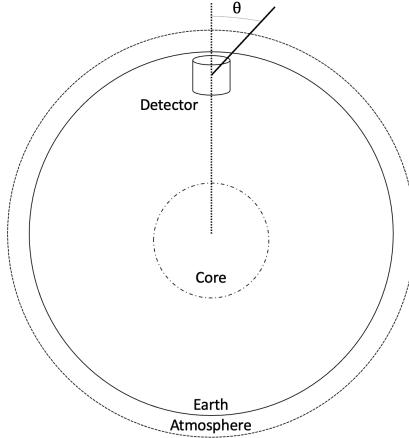


Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [56].

469 a lesser degree for low-energy neutrinos), the flux is approximately symmetric
 470 around $\cos(\theta) = 0$. To the accuracy of this approximation, the systematic
 471 uncertainties associated with atmospheric flux for comparing upward-going
 472 and down-going neutrino cancels. This allows the down-going events, which are

⁴⁷³ mostly insensitive to oscillation probabilities, to act as an unoscillated prediction
⁴⁷⁴ (similar to a near detector in an accelerator neutrino experiment).

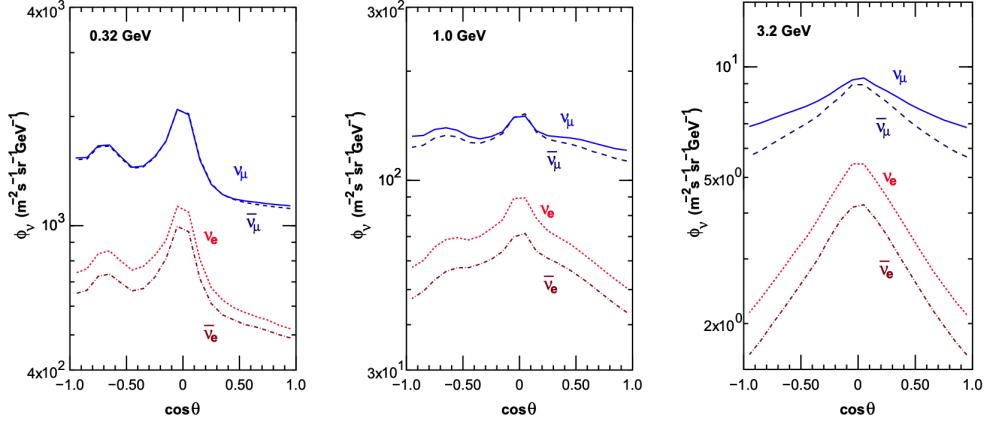


Figure 2.5: Prediction of ν_e , $\bar{\nu}_e$, ν_μ , $\bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [54]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [51], Honda [52] and FLUKA [55] are given in [56].

⁴⁷⁵ Precursory hints of atmospheric neutrinos were observed in the mid-1960s
⁴⁷⁶ searching for $\nu_\mu + X \rightarrow X^* + \mu^\pm$ [57]. This was succeeded by the IMB-3 [58]
⁴⁷⁷ and Kamiokande [59] experiments which measured the double ratio of muon
⁴⁷⁸ to electron neutrinos in data to Monte Carlo, $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$.
⁴⁷⁹ Both experiments were found to have a consistent deficit of muon neutrinos,
⁴⁸⁰ with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$, respectively.
⁴⁸¹ Super-Kamiokande (SK) [56] extended this analysis by fitting oscillation pa-
⁴⁸² rameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and
⁴⁸³ $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3}\text{eV}^2$.

⁴⁸⁴ Since then, atmospheric neutrino experiments have been making precision
⁴⁸⁵ measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric
⁴⁸⁶ neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ
⁴⁸⁷ discovery of ν_τ appearance [60]. Figure 2.6 illustrates the current estimates on
⁴⁸⁸ the atmospheric mixing parameters, from a wide range of atmospheric and
⁴⁸⁹ accelerator neutrino observatories.

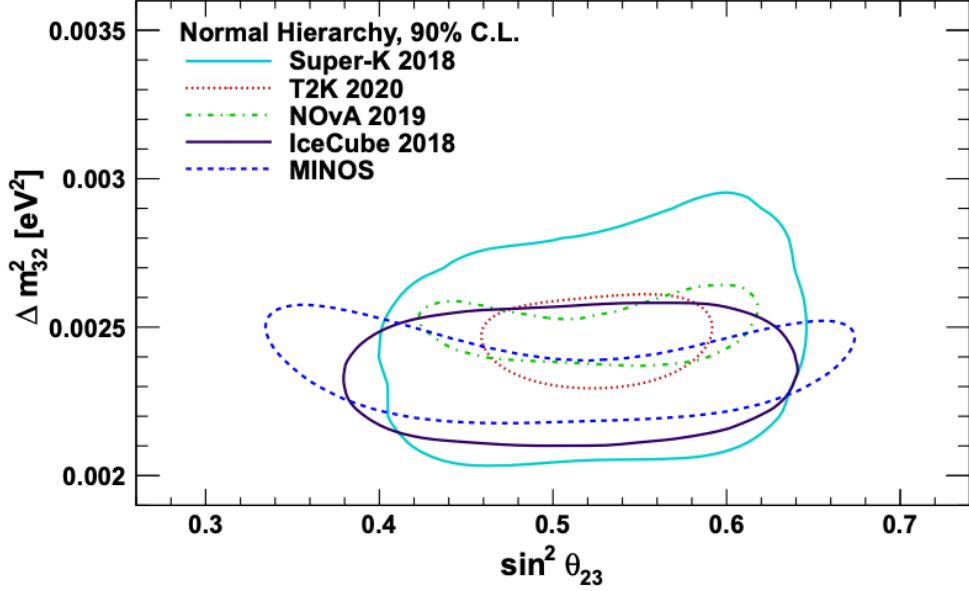


Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long-baseline experiments: SK [61], T2K [62], NO ν A [63], IceCube [64] and MINOS [65]. Figure taken from [66].

2.3.4 Reactor Neutrinos

As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors are a very useful artificial source of electron antineutrinos. For reactors that use low-enriched uranium ^{235}U as fuel, the antineutrino flux is dominated by the β -decay fission of ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu [67] as illustrated in Figure 2.7.

Due to their low energy, reactor electron antineutrinos predominantly interact via the inverse β -decay (IBD) interaction. The typical signature contains two signals delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihilation, and secondly the photon emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after neutron capture on hydrogen. Searching for both signals improves the detector's ability to distinguish between background and signal events [68].

There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured the $\sin^2(\theta_{13})$ and Δm_{32}^2 oscillation parameters. Daya Bay [69], RENO [70] and Double Chooz [71] have all provided precise measurements, with the first discovery of a non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K [71]). The constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field. They

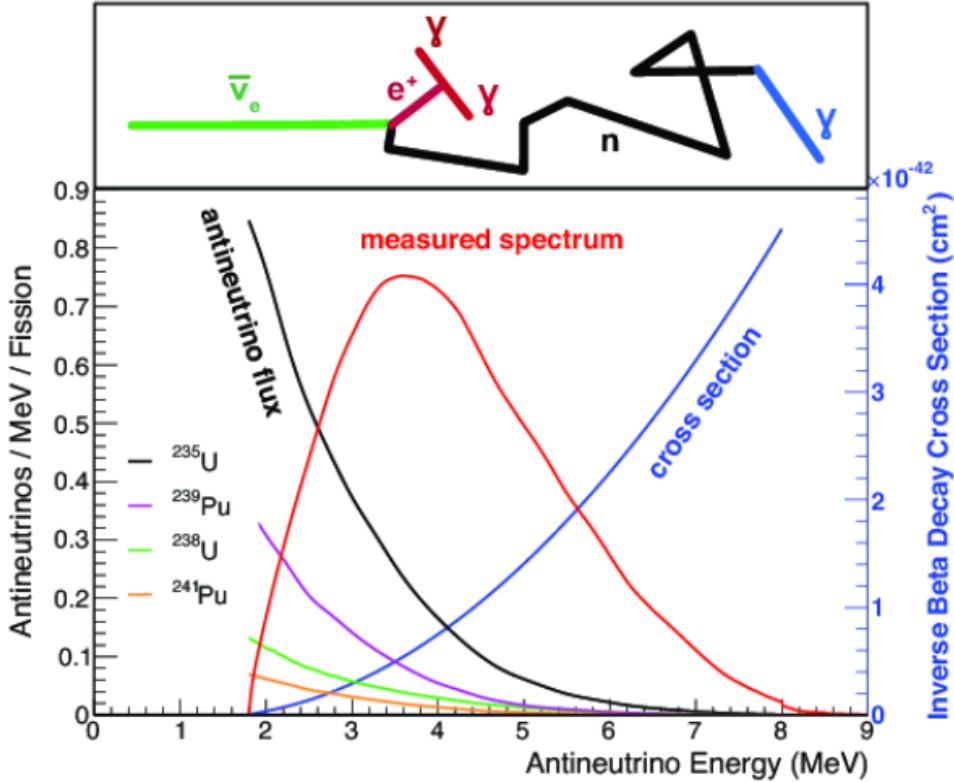


Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [66].

506 are often used as external inputs to accelerator neutrino experiments to improve
 507 their sensitivity to δ_{CP} and mass hierarchy determination. JUNO-TAO [72], a
 508 small collaboration within the larger JUNO experiment, is a next-generation
 509 reactor experiment that aims to precisely measure the isotopic antineutrino yields
 510 from the different fission chains.

511 Kamland [73] is the only experiment to have observed reactor neutrinos using
 512 a long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it
 513 to have sensitivity to Δm_{21}^2 . Combined with the SK solar neutrino experiment,
 514 the combined analysis puts the most stringent constraint on Δm_{21}^2 [74].

515 2.4 Summary Of Oscillation Parameter Measurements

516 Since the first evidence of neutrino oscillations, numerous measurements of the
 517 mixing parameters have been made. Many experiments use neutrinos as a tool
 518 for the discovery of new physics (diffuse supernova background, neutrinoless
 519 double beta decay and others) so the PMNS parameters are summarised in the
 520 Particle Data Group (PDG) review tables. The analysis presented in this thesis
 521 focuses on the 2020 T2K oscillation analysis presented in [2] which the 2020 PDG
 522 constraints [75] were used. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	0.547 ± 0.021
$\sin^2(\theta_{23})$ (N.H.)	0.545 ± 0.021
Δm_{32}^2 (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2020 Particle Data Group constraints of the oscillation parameters taken from [75]. The value of Δm_{32}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

523 The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance,
 524 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is taken as the average best-fit from the combination of
 525 Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within
 526 other neutrino oscillation experiments, typically termed the reactor constraint.
 527 The $\sin^2(\theta_{12})$ parameter is predominantly measured through electron neutrino
 528 disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline
 529 reactor neutrino experiment Kamland also has a sensitivity to this parameter
 530 and is used in a joint fit to solar data from SNO and SK, using the reactor con-
 531 straint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric
 532 neutrino experiments. The PDG value is a joint fit of T2K, NO ν A, MINOS and
 533 IceCube DeepCore experiments. The latest T2K-only measurement, provided at
 534 Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [2].

The PDG constraint on Δm_{21}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{32}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [2]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analyses have reduced the credible intervals to 90%. Since the 2020 PDG publication, there has been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [76], alongside updated Δm_{32}^2 and $\sin^2(\theta_{23})$ measurements.

Throughout this thesis, several sample spectra predictions and contours are presented, which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [77] and “Asimov B” being CP-conserving and further from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

2.5 Overview of Oscillation Effects

The analysis presented within this thesis focuses on the determination of oscillation parameters from atmospheric and beam neutrinos. Whilst subject to the

556 same oscillation formalism, the way in which the two samples have sensitivity
 557 to the different oscillation parameters differs significantly.

558 Atmospheric neutrinos have a varying baseline, or “path length” L , such that
 559 the distance each neutrino travels before interacting is dependent upon the zenith
 560 angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s
 561 surface and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the
 562 neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2(1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (2.16)$$

563 Where $R_E = 6,371\text{km}$ is the Earth’s radius. This assumes a spherically
 564 symmetric Earth model. Therefore, the oscillation probability is dependent upon
 565 two parameters, $\cos(\theta_Z)$ and E_ν .

566 The oscillation probability used within this analysis is based on [24]. The
 567 neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of
 568 constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (2.17)$$

569 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
 570 energy, G_F is Fermi’s constant, N_e is the electron number density and U is the
 571 PMNS matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied for
 572 antineutrino propagation. Thus, a model of the Earth’s density is required for
 573 neutrino propagation. Following the official SK-only methodology [78], this
 574 analysis uses the Preliminary Reference Earth Model (PREM) [79] which provides
 575 piecewise cubic polynomials as a function of the Earth’s radius. This density
 576 profile is illustrated in Figure 2.8. As the propagator requires layers of constant
 577 density, the SK methodology approximates the PREM model by using four layers
 578 of constant density [78], detailed in Table 2.3.

579 The atmospheric neutrino oscillation probabilities can be presented as two di-
 580 mensional “oscillograms” as illustrated in Figure 2.9. The distinct discontinuities,
 581 as a function of $\cos(\theta_Z)$, are due to the discontinuous density in the PREM model.

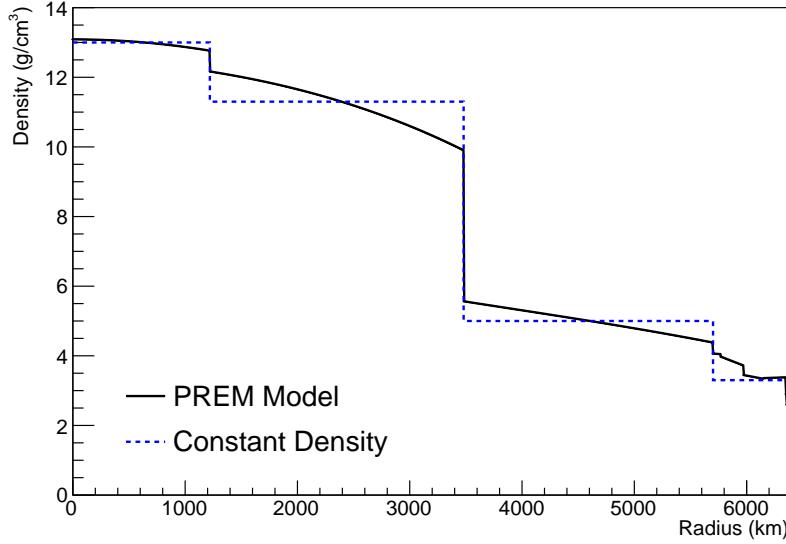


Figure 2.8: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

Layer	Outer Radius [km]	Density [g/cm ³]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 2.3: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [79].

Atmospheric neutrinos have sensitivity to δ_{CP} through the overall event rate. Figure 2.10 illustrates the difference in oscillation probability between CP-conserving ($\delta_{CP} = 0.$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation parameter set (Table 2.2). The result is a complicated oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric neutrinos comes via the overall normalisation of these events.

The presence of matter means that the effect δ_{CP} has on the oscillation probability is not equal between neutrinos and antineutrinos. Furthermore, the interaction cross-section for neutrinos is larger than for antineutrinos so the two

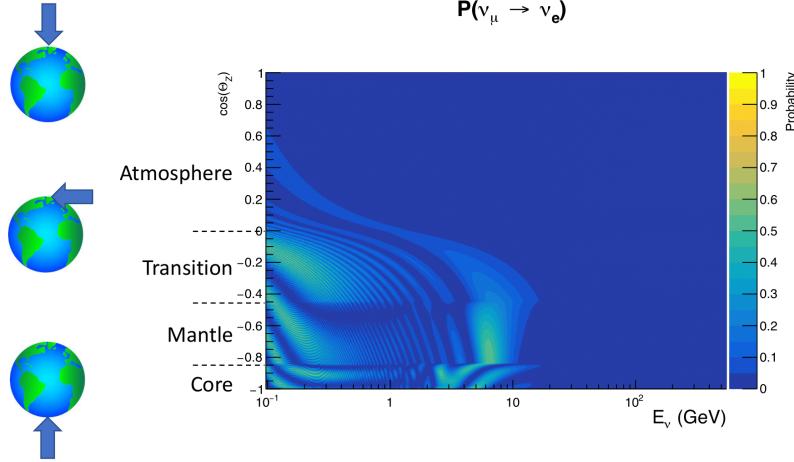


Figure 2.9: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

593 effects have to be disentangled. These effects are further convoluted by detector
 594 efficiencies as SK cannot distinguish neutrinos and antineutrinos well. All of
 595 these effects lead to a difference in the number of neutrinos detected compared
 596 to antineutrinos. This changes how the δ_{CP} normalisation term is observed,
 597 resulting in a very complex sensitivity to δ_{CP} .

598 The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow$
 599 $\bar{\nu}_e)$ are presented in Figure 2.11, where the PREM model has been assumed. The
 600 oscillation probability for both neutrinos and antineutrinos is affected in the
 601 presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only occur
 602 for neutrinos in the normal mass hierarchy and antineutrinos in the inverse mass
 603 hierarchy. The exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$,
 604 further increasing the atmospheric neutrinos’ sensitivity to the parameter.

605 As the T2K beam flux is centered at the first oscillation maximum ($E_\nu =$
 606 0.6GeV) [44], the sensitivity to δ_{CP} is predominantly observed as a change in the
 607 event-rate of e-like samples in $\nu/\bar{\nu}$ modes. Figure 2.12 illustrates the $P(\nu_\mu \rightarrow \nu_e)$
 608 oscillation probability for a range of δ_{CP} values. A circular modulation of the

$$\mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = -1.601) - \mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = 0.)$$

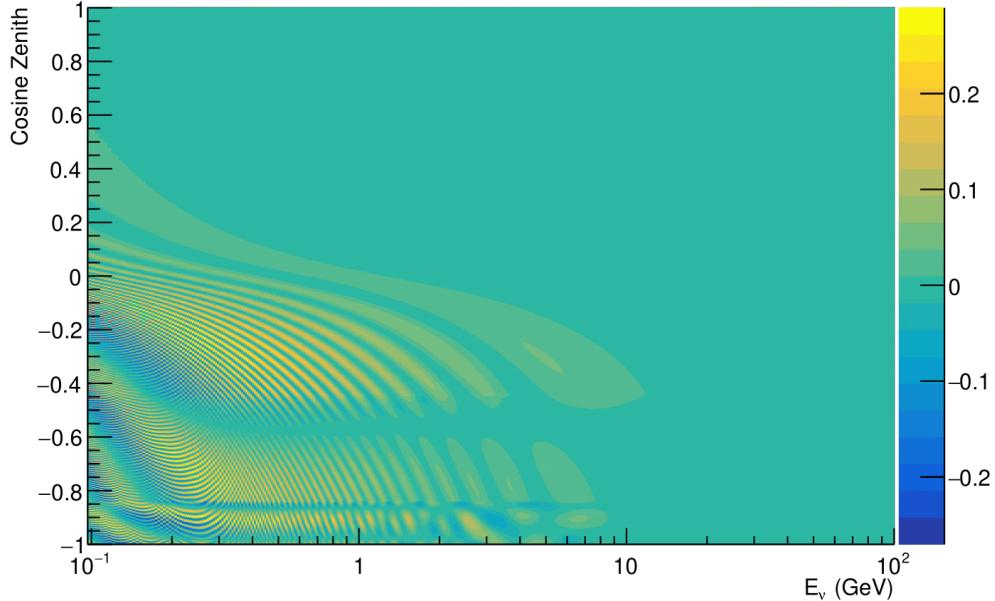


Figure 2.10: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

first oscillation peak (in both magnitude and position) is observed when varying throughout the allowable values of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation maximum than the CP-violating values of $\delta_{CP} = -\pi/2 (\delta_{CP} = \pi/2)$. A sub-dominant shift in the energy of the oscillation peak is also present, which aids in separating the two CP-conserving values of δ_{CP} .

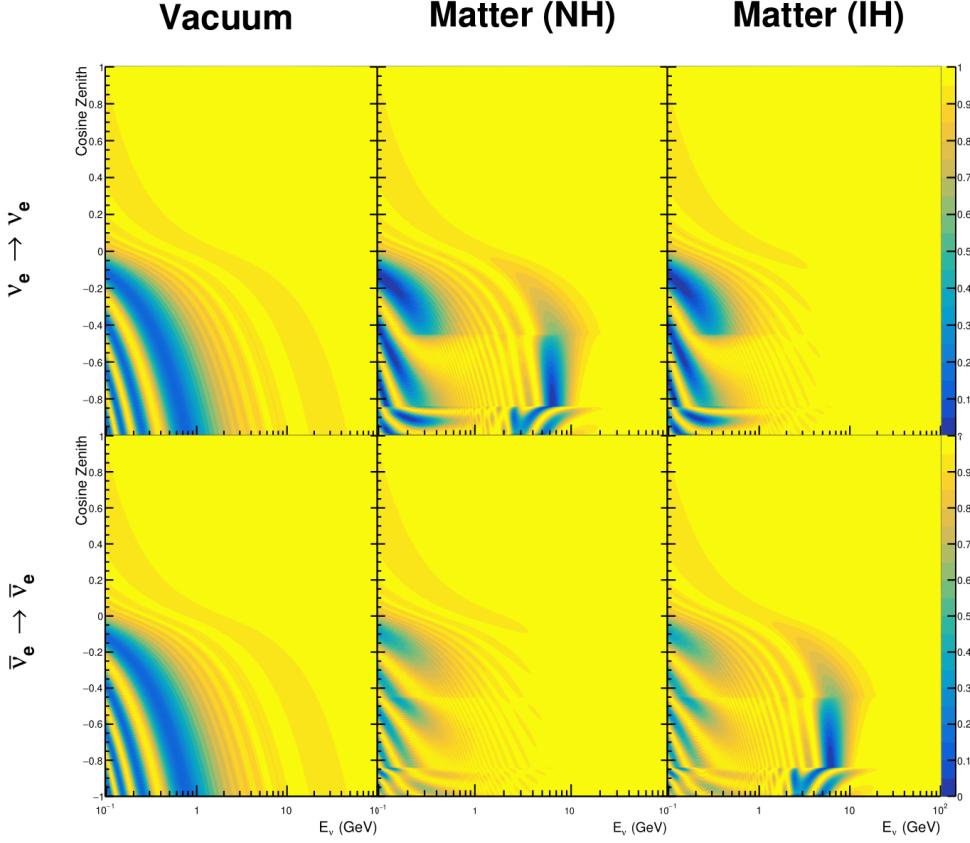


Figure 2.11: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

614 T2K’s sensitivity to $\sin^2(\theta_{23})$ and Δm_{32}^2 is observed as a shape-based variation
 615 of the muon-like samples, as illustrated in Figure 2.12. The value of Δm_{32}^2 laterally
 616 shifts the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$
 617 oscillation probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as
 618 a vertical shift of the oscillation dip with second-order horizontal shifts being
 619 due to matter effects. The beam neutrinos have limited sensitivity to matter
 620 effects due to the relatively shorter baseline as well as the Earth’s mantle being

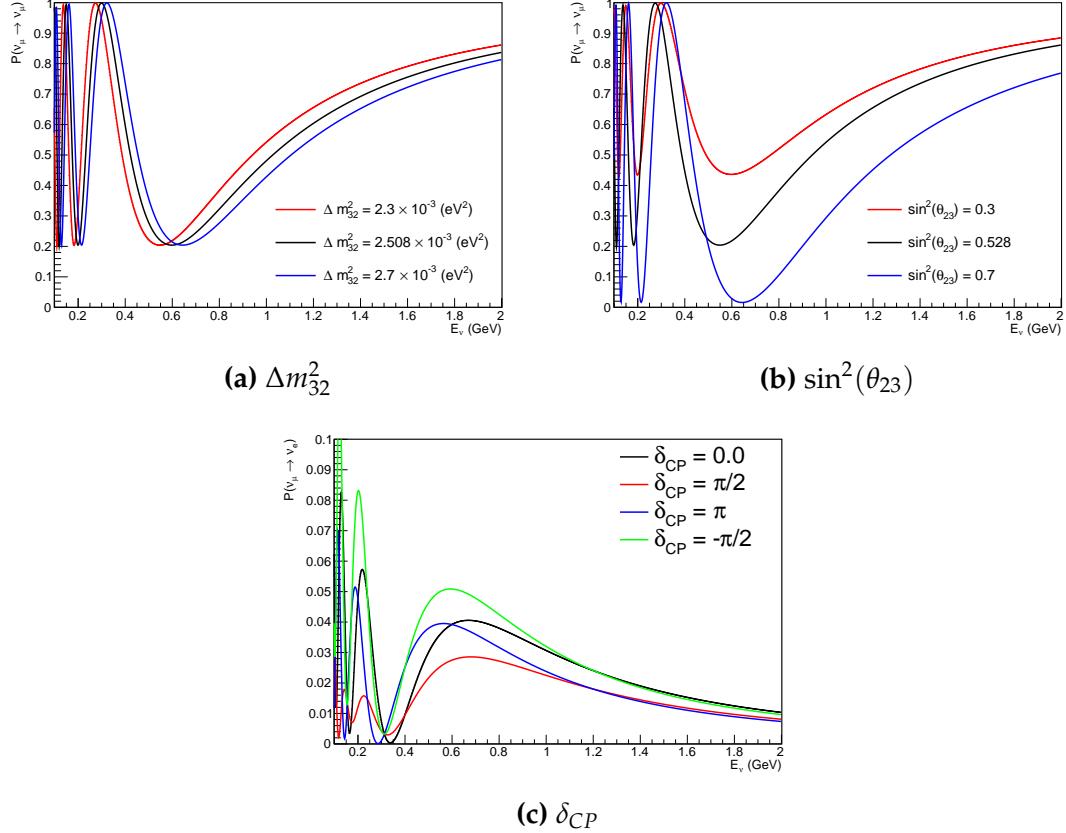


Figure 2.12: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. A path-length of 295km is assumed. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

a relatively low-density material (as compared to the Earth’s core). For some values of δ_{CP} , the degeneracy in the number of e-like events allows the mass hierarchy to be broken. This leads to a δ_{CP} -dependent mass hierarchy sensitivity which can be seen in Figure 2.13.

Whilst all oscillation channels should be included for completeness, the computational resources required to run a fit are limited and any reasonable approximations which reduce the number of oscillation probability calculations that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux. Furthermore, as the peak neutrino energy of the beam is well below the

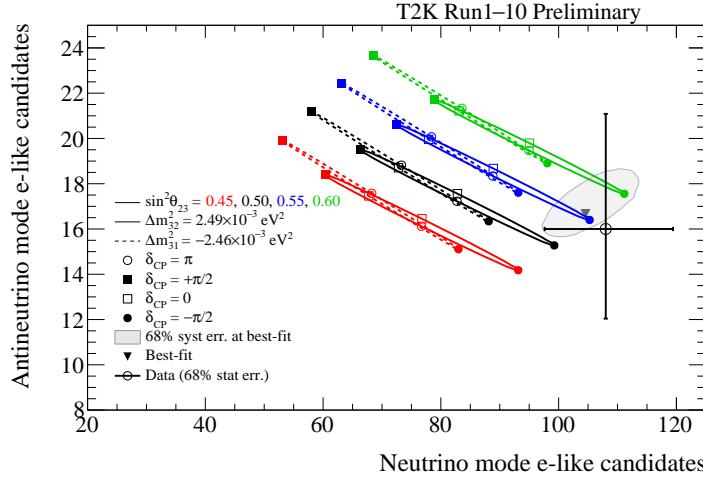


Figure 2.13: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{32}^2 are given.

threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [60]), only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very small due to their energy being well above the oscillation maximum (small value of L/E). Whilst these approximations have been made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold. These events can have non-negligible oscillation probabilities due to the further distance they travel.

3

641

642

T2K and SK Experiment Overview

643 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)
644 collaboration has been leading atmospheric neutrino oscillation analyses for
645 over two decades. The detector has provided some of the strongest constraints
646 on proton decay and the first precise measurements of the Δm_{32}^2 and $\sin^2(\theta_{23})$
647 neutrino oscillation parameters. The history, detection technique, and operation
648 of the SK detector is described in section 3.1.

649 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline
650 experiments to use both neutrino and antineutrino beams to precisely measure
651 charge parity violation within the neutrino sector. The T2K experiment observed
652 the first hints of a non-zero $\sin^2(\theta_{13})$ measurement and continues to lead the
653 field with the constraints it provides on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{32}^2 and δ_{CP} . In
654 section 3.2, the techniques that T2K use to generate the neutrino beam and
655 constrain systematic parameter through near detector constraints are described.

656 3.1 The Super-Kamiokande Experiment

657 The SK experiment began taking data in 1996 [80] and has had many modifi-
658 cations throughout its operation. There have been seven defined periods of
659 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were damaged during maintenance [81]. Those that survived were equally distributed throughout the detector in the SK-II era, which resulted in a reduced 19% photo-coverage. From SK-III onwards, repairs to the detector meant the full suite of PMTs was operational recovering the 40% photo-coverage. Before the start of SK-IV, the data acquisition and electronic systems were upgraded. Between SK-IV and SK-V, a significant effort was placed into tank open maintenance and repair/replacement of defective PMTs in preparation for the Gadolinium upgrade; a task for which the author of this thesis was required. Consequently, the detector conditions were significantly changed from this point. SK-VI marked the start of the SK-Gd era, with the detector being doped with gadolinium at a concentration of 0.01% by concentration. SK-VII, which started during the writing of this thesis, has increased the gadolinium concentration to 0.03% for continued operation [82].

The oscillation analysis presented within this thesis focuses on the SK-IV period of running and the data taken within it. This follows from the recent SK analysis presented in [83]. Therefore, the information presented within this section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

Table 3.1: The various SK periods and their respective live-time. The SK-VI live-time is calculated until 1st April 2022. SK-VII started during the writing of this thesis.

3.1.1 The SK Detector

The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank with a diameter 39.3m and height 41.1m filled with ultrapure water [81]. A diagram of the significant components of the SK detector is given in Figure 3.1.

681 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine is under-
 682 ground with roughly 1km rock overburden (2.7km water equivalent overburden)
 683 [84]. At this depth, the rate of cosmic ray muons is significantly decreased to a
 684 value of $\sim 2\text{Hz}$ (net rate). The top of the tank is covered with stainless steel which
 685 is designed as a working platform for maintenance, calibration, and location for
 686 high voltage and data acquisition electronics.

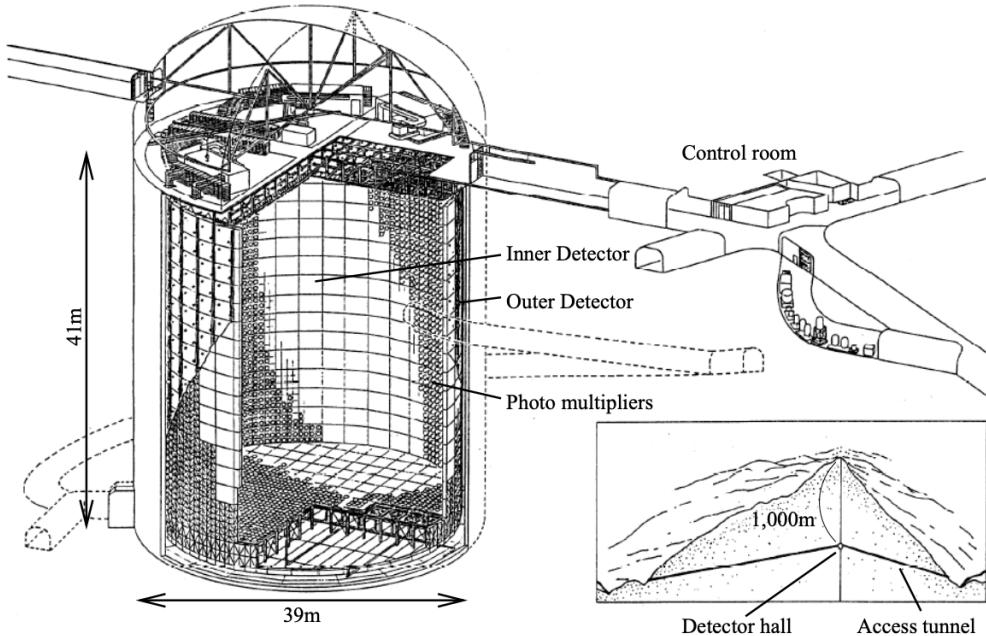


Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [85].

687 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated
 688 inside the tank, with an approximate 2m gap between this structure and the outer
 689 tank wall. The purpose of this structure is to support the photomultiplier tubes
 690 (PMTs). The volume inside and outside the support structure is referred to as the
 691 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the
 692 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs respectively
 693 [81]. The ID contains a 32kton mass of water. Many analyses performed at SK
 694 use a “fiducial volume” defined by the volume of water inside the ID excluding
 695 some distance to the ID wall. This reduces the volume of the detector which is
 696 sensitive to neutrino events but reduces radioactive backgrounds and allows for

697 better reconstruction performance. The nominal fiducial volume is defined as the
698 area contained inside 2m from the ID wall for a total of 22.5kton water [3].

699 The two regions of the detector (ID and OD) are optically separated with
700 opaque black plastic hung from the support structure. The purpose of this is
701 to determine whether an event entered or exited the ID. This allows cosmic ray
702 muons and partially contained events to be tagged and separated from neutrino
703 events entirely contained within the ID. This black plastic is also used to cover
704 the area between the ID PMTs to reduce photon reflection from the ID walls.
705 Opposite to this, the OD is lined with a reflective material to allow photons to
706 reflect around inside the OD until collected by one of the PMTs. Furthermore,
707 each OD PMT is optically coupled with $50 \times 50\text{cm}$ plates of wavelength shifting
708 acrylic which increases the efficiency of light collection [84].

709 In the SK-IV data-taking period, the photocathode coverage of the detector, or
710 the fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [84]. The PMTs have
711 a quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$
712 for photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$ [86, 87]. The proportion
713 of photoelectrons that produce a signal in the dynode of a PMT, termed the
714 collection efficiency, is $> 70\%$ [84]. The PMTs used within SK are most sensitive
715 to photons with wavelength $300\text{nm} \leq \lambda \leq 600\text{nm}$ [84]. One disadvantage of
716 using PMTs as the detection media is that the Earth's geomagnetic field can
717 modify its response. Therefore, a set of compensation coils is built around the
718 inner surface of the detector to mitigate this effect [84].

719 The SK detector is filled with ultrapure water, which in a perfect world, con-
720 tains no impurities. However, bacteria and organic compounds can significantly
721 degrade the water quality. This decreases the attenuation length, which reduces
722 the total number of photons that hit a PMT. To combat this, a sophisticated water
723 treatment system has been developed [84, 88]. UV lights, mechanical filters, and
724 membrane degasifiers are used to reduce the bacteria, suspended particulates,
725 and radioactive materials from the water. The flow of water within the tank
726 is also critical as it can remove stagnant bacterial growth or build-up of dust

727 on the surfaces within the tank. Gravity drifts impurities in the water towards
728 the bottom of the tank which, if left uncontrolled, can create asymmetric water
729 conditions between the top and bottom of the tank. Typically, the water entering
730 the tank is cooled below the ambient temperature of the tank to control convection
731 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs
732 is sensitive to the PMT temperature [89]. Therefore controlling the temperature
733 gradients within the tank is beneficial for stable measurements.

734 SK-VI is the first phase of the SK experiment to use gadolinium dopants
735 within the ultrapure water [82]. As such, the SK water system had to be replaced
736 to avoid removing the gadolinium concentrate from the ultrapure water [68]. For
737 an inverse β -decay (IBD) interaction on a water target, the emitted neutron is
738 thermally captured on hydrogen. This process releases a 2.2MeV γ ray which is
739 difficult to detect as the resulting Compton scattered electrons are very close to the
740 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons
741 on gadolinium generates γ rays with higher energy (8MeV [68]) meaning they
742 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%
743 gadolinium sulphate by mass) which causes \approx 50% of neutrons emitted by IBD
744 to be captured on gadolinium[90]. Whilst predominantly useful for low energy
745 analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino
746 event selections [91]. Efforts are currently in place to increase the gadolinium
747 concentrate to 0.03% for \approx 75% neutron capture efficiency on gadolinium [92].
748 The final stage of loading targets 0.1% concentrate for \approx 90% neutron capture
749 efficiency on gadolinium.

750 3.1.2 Calibration

751 The calibration of the SK detector is documented in [81] and summarised below.
752 The analysis presented within this thesis is dependent upon ‘high energy events’
753 (Charged particles with $O(> 100)$ MeV momenta). These are events that are
754 expected to generate a larger number of photons such that each PMT will
755 be hit with multiple photons. The reconstruction of these events depends

upon the charge deposited within each PMT and the timing response of each individual PMT. Therefore, the most relevant calibration techniques to this thesis are outlined.

Before installation, 420 PMTs were calibrated to have identical charge responses and then distributed throughout the tank in a cross-shape pattern (As illustrated by Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed at similar geometric positions within SK to be calibrated against. To perform this calibration, a xenon lamp is located at the center of the SK tank which flashes uniform light at 1Hz. This allows for geometrical effects, water quality variation, and timing effects to be measured in situ throughout normal data-taking periods.

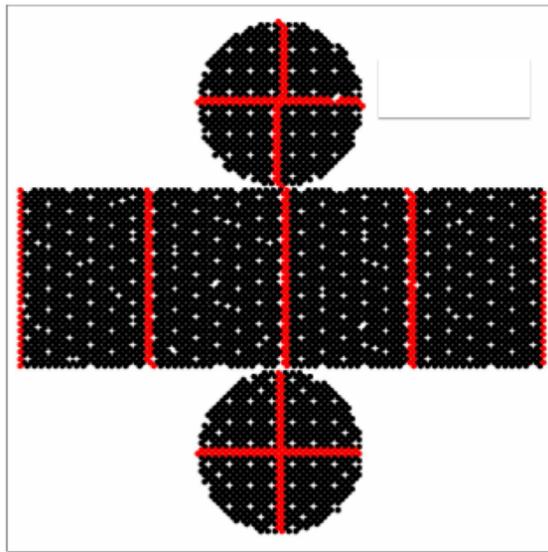


Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [81].

When specifically performing calibration of the detector (in out-of-data taking mode), the water in the tank was circulated to avoid top/bottom asymmetric water quality. Any non-uniformity within the tank significantly affects the PMT hit probability through scattering or absorption. This becomes a dominant effect for very low-intensity light sources that are designed such that only one photon is incident upon a given PMT.

773 The gain of a PMT is defined as the ratio of the total charge of the signal
 774 produced compared to the charge of photoelectrons emitted by the photocathodes
 775 within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”
 776 gain values are measured. The relative gain is the variation of gain among each
 777 of the PMTs whereas the absolute gain is the average gain of all PMTs.

778 The relative gain is calibrated as follows. A laser is used to generate two
 779 measurements: a high-intensity flash that illuminates every PMT with a sufficient
 780 number of photons, and a low-intensity flash in which only a small number
 781 of PMTs collect light. The first measurement creates an average charge, $Q_{obs}(i)$
 782 on PMT i , whereas the second measurement ensures that each hit PMT only
 783 generates a single photoelectron. For the low-intensity measurement, the number
 784 of times each PMT records a charge larger than 1/4 photoelectrons, $N_{obs}(i)$, is
 785 counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i). \end{aligned} \tag{3.1}$$

786 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the
 787 acceptance efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and
 788 collection efficiency of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative
 789 gain for each PMT can be determined by taking the ratio of these quantities.

790 The absolute gain calibration is performed by observing fixed energy γ -rays
 791 of $E_\gamma \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source
 792 situated at the center of the detector. This generates a photon yield of about 0.004
 793 photoelectrons/PMT/event, meaning that $> 99\%$ of PMT signals are generated
 794 from single photoelectrons. A charge distribution is generated by performing
 795 this calibration over all PMTs, and the average value of this distribution is taken
 796 to be the absolute gain value.

797 As mentioned in subsection 3.1.1, the average quantum and collection effi-
 798 ciency for the SK detector PMTs is $\sim 21\%$ and $> 70\%$ respectively. However,
 799 these values do differ between each PMT and need to be calibrated accordingly.

800 Consequently, the NiCf source is also used to calibrate the “quantum \times collection”
801 efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is
802 used as the PMT hit probability is proportional to the QE ($N_{obs}(i) \propto \epsilon(i)$ in
803 Equation 3.1). A Monte Carlo prediction which includes photon absorption,
804 scattering, and reflection is made to estimate the number of photons incident on
805 each PMT and the ratio of the number of predicted to observed hits is calculated.
806 The difference is attributed to the QE efficiency of that PMT. This technique is
807 extended to calculate the relative QE efficiency by normalizing the average of
808 all PMTs which removes the dependence on the light intensity.

809 Due to differing cable lengths and readout electronics, the timing response
810 between a photon hitting the PMT and the signal being captured by the data
811 acquisition can be different between each PMT. Due to threshold triggers (De-
812 scribed in subsection 3.1.3), the time at which a pulse reaches a threshold is
813 dependent upon the size of the pulse. This is known as the ‘time-walk’ effect
814 and also needs to be accounted for in each PMT. To calibrate the timing response,
815 a pulse of light with width 0.2ns is emitted into the detector through a diffuser.
816 Two-dimensional distributions of time and pulse height (or charge) are made
817 for each PMT and are used to calibrate the timing response. This is performed
818 in-situ during data taking with the light source pulsing at 0.03Hz.

819 The top/bottom water quality asymmetry is measured using the NiCf calibra-
820 tion data and cross-referencing these results to the “standard PMTs”. The water
821 attenuation length is continuously measured by the rate of vertically-downgoing
822 cosmic-ray muons which enter via the top of the tank.

823 Dark noise is where a PMT registers a pulse that is consistent with a single
824 photoelectron emitted from photon detection despite the PMT being in complete
825 darkness. This is predominately caused by two processes. Firstly there is
826 intrinsic dark noise which is where photoelectrons gain enough thermal energy
827 to be emitted from the photocathode, and secondly, the radioactive decay of
828 contaminants inside the structure of the PMT. Typical dark noise rate for PMTs
829 used within SK are $O(3)\text{kHz}$ [84]. This is lower than the expected number of

830 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)
831 but instability in this value can cause biases in reconstruction. Dark noise is
832 related to the gain of a PMT and is calibrated using hits inside a time window
833 recorded before an event trigger [93].

834 3.1.3 Data Acquisition and Triggering

835 As the analysis presented in this thesis will only use the SK-IV period of the
836 SK experiment so this subsection focuses on the relevant points of the data
837 acquisition and triggering systems to that SK period. The earlier data acquisition
838 and triggering systems are documented in [94, 95].

839 Before the SK-IV period started, the existing front-end electronics were re-
840 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [96]. When
841 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-
842 time (QTC) converter generates a rectangular pulse. The start of the rectangular
843 pulse indicates the time at which the analog photoelectron signal was received
844 and the width of the pulse indicates the total charge integrated throughout the
845 signal. This is then digitized by time-to-digital converters and sent to the “front-
846 end” PCs. The digitized signal from every QBEE is then chronologically ordered
847 and sent to the “merger” PCs. It is the merger PCs that apply the software trigger.
848 Any triggered events are passed to the “organizer” PC. This sorts the data stream
849 of multiple merger PCs into chronologically ordered events, which are then saved
850 to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

851 The software trigger (described in [98]) operates by determining the number
852 of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the
853 maximum time that a Cherenkov photon would take to traverse the length of the
854 SK tank [95]. For lower energy events that generate fewer photons, this technique
855 is useful for eliminating background processes like dark noise and radioactive
856 decay which would be expected to be separated in time. When the value of
857 N_{200} exceeds some pre-defined threshold, a software trigger is issued. There are
858 several trigger thresholds used within the SK-IV period which are detailed in

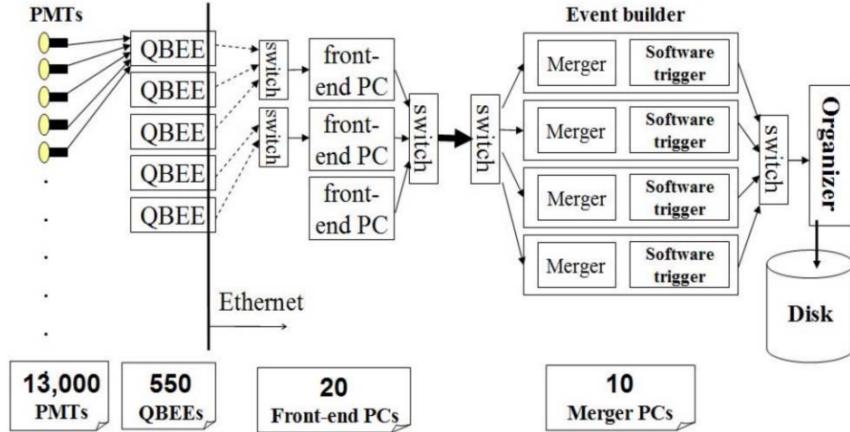


Figure 3.3: Schematic view of the data flow through the data acquisition and online system. Taken from [97].

859 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time
 860 window are also read out and saved to disk. In the special case of an event that
 861 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks
 862 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in
 863 a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband
 864 Intelligent Trigger (WIT)” has been deployed and is described in [99].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

3.1.4 Cherenkov Radiation

866 Cherenkov light is emitted from any highly energetic charged particle traveling
 867 with relativistic velocity, β , greater than the local speed of light in a medium [100].

868 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

869 Where n is the refractive index of the medium. Consequently, the Cherenkov
 870 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged
 871 particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}}. \quad (3.3)$$

872 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy
 873 for various particles are given in Table 3.3. In contrast, γ -rays are detected
 874 indirectly via the combination of photons generated by Compton scattering
 875 and pair production. The threshold for detection in the SK detector is typically
 876 higher than the threshold for photon production. This is due to the fact that the
 877 attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov
 878 photons reach the ID PMTs. Then the collection and quantum efficiencies
 879 described in subsection 3.1.1 result in the number of detected photons being
 880 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

881 The Frank-Tamm equation [101] describes the relationship between the num-
 882 ber of Cherenkov photons generated per unit length, dN/dx , the wavelength of
 883 the photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

884 where α is the fine structure constant. For a 100MeV momentum electron,
 885 approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq$
 886 700nm region which the ID PMTs are most sensitive to [84].

887 3.2 The Tokai to Kamioka Experiment

888 The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation
 889 experiment located in Japan. Proposed in the early 2000s [85, 102] to replace
 890 K2K [103], T2K was designed to observe electron neutrino appearance whilst
 891 precisely measuring the oscillation parameters associated with muon neutrino
 892 disappearance [104]. The experiment consists of a neutrino beam generated
 893 at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near
 894 detectors situated 280m from the beam target, and the Super Kamiokande far
 895 detector positioned at a 295km baseline. The cross-section view of the T2K
 896 experiment is drawn in Figure 3.4.

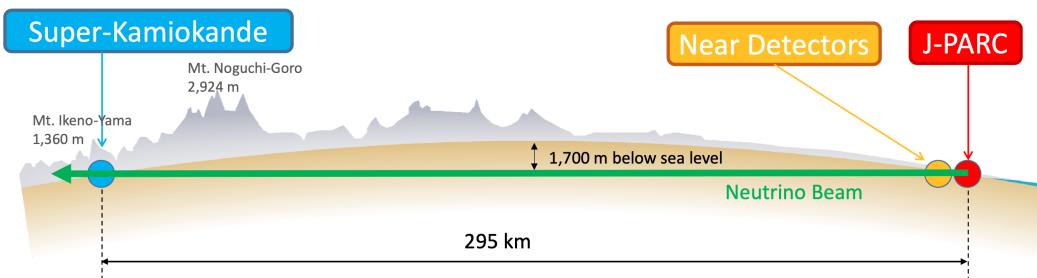


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

897 The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$,
 898 Δm_{32}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accu-
 899 racy of parameter estimates are still being made by including new data samples
 900 and developing the models which describe the neutrino interactions and detector
 901 responses [105]. Electron neutrino appearance was first observed at T2K in 2014
 902 [106] with 7.3σ significance.

903 The near detectors provide constraints on the beam flux and cross-section
 904 model parameters used within the oscillation analysis by observing the unoscil-
 905 lated neutrino beam. There are a host of detectors situated in the near detector hall
 906 (As illustrated in Figure 3.5): ND280 (subsection 3.2.3), INGRID (subsection 3.2.4),
 907 NINJA [107], WAGASCI [108], and Baby-MIND [109]. The latter three are not
 908 currently used within the oscillation analysis presented in this thesis.

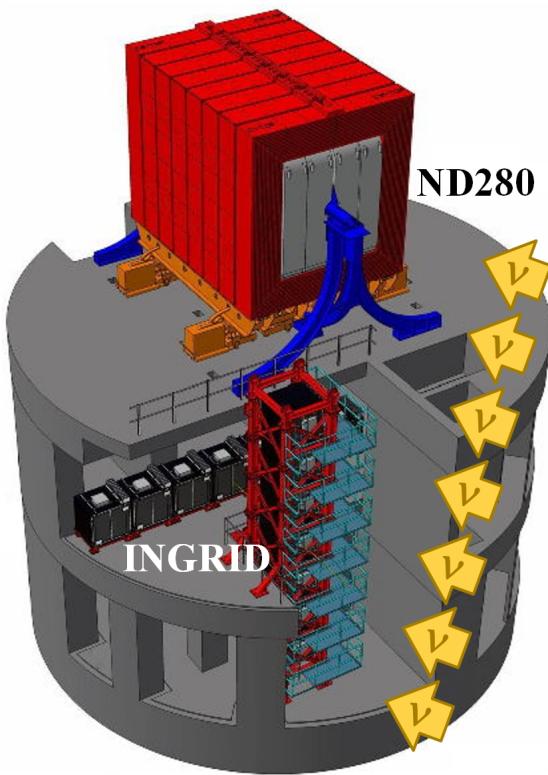


Figure 3.5: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

909 Whilst this thesis presents the ND280 in terms of its purpose for the oscillation
 910 analysis, the detector can also make many cross-section measurements at neutrino
 911 energies of $O(1)$ GeV for the different targets within the detector [110, 111]. These
 912 measurements are of equal importance as they can lead the way in determining
 913 the model parameters used in the interaction models for the future high-precision
 914 era of neutrino physics.

3.2.1 Analysis Overview

There are two independent fitters, MaCh3 and BANFF, which perform the near detector fit. MaCh3 uses a bayesian Markov Chain Monte Carlo fitting technique, whereas BANFF uses a frequentist gradient descent technique. The output of each fitter is compared as a method of cross-checking the behaviour of the two fitters. This is done by comparing: the Monte Carlo predictions using various tunes, the likelihood that is calculated in each fitter and the post-fit constraint associated with every parameter used in the fit. Once validated, the output converted into a covariance matrix to describe the error and correlations between all the flux and cross-section parameters. This is then propagated to the far-detector oscillation analysis group.

The far detector group has three independent fitters: P-Theta, VALOR and MaCh3. The first two fitters use a hybrid frequentist fitting technique where the likelihood is minimised with respect to the parameters of interest and marginalised over all other parameters. These fitters use the covariance provided by the near detector fitters as a basis for implementing the near detector constraints. The MaCh3 fitter uses a simultaneous fit of all near and far detector samples. This removes any Gaussian assumptions when making the covariance matrix from the near detector results. The results for all three fitters are compares using a technique similar to the validation of the near detector fitters.

There are three particular tunes of the T2K flux and low energy cross section model typically considered. Firstly, the “generated” tune which is the set of dial values with which the Monte Carlo was generated. Secondly, the set of dial values which are taken from external data measurements and used as inputs. These are the “pre-fit” dial values. The reason these two sets of dial values are different is that the external data measurements are continually updated but it is very computationally intensive to regenerate a Monte Carlo prediction after each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial values. These are the values taken from the constraints provided by the near detector.

3.2.2 The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [40, 44] and summarised below. The accelerator facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [40],

$$\begin{array}{ll}
\pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
\rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
\rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
\rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e.
\end{array}$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

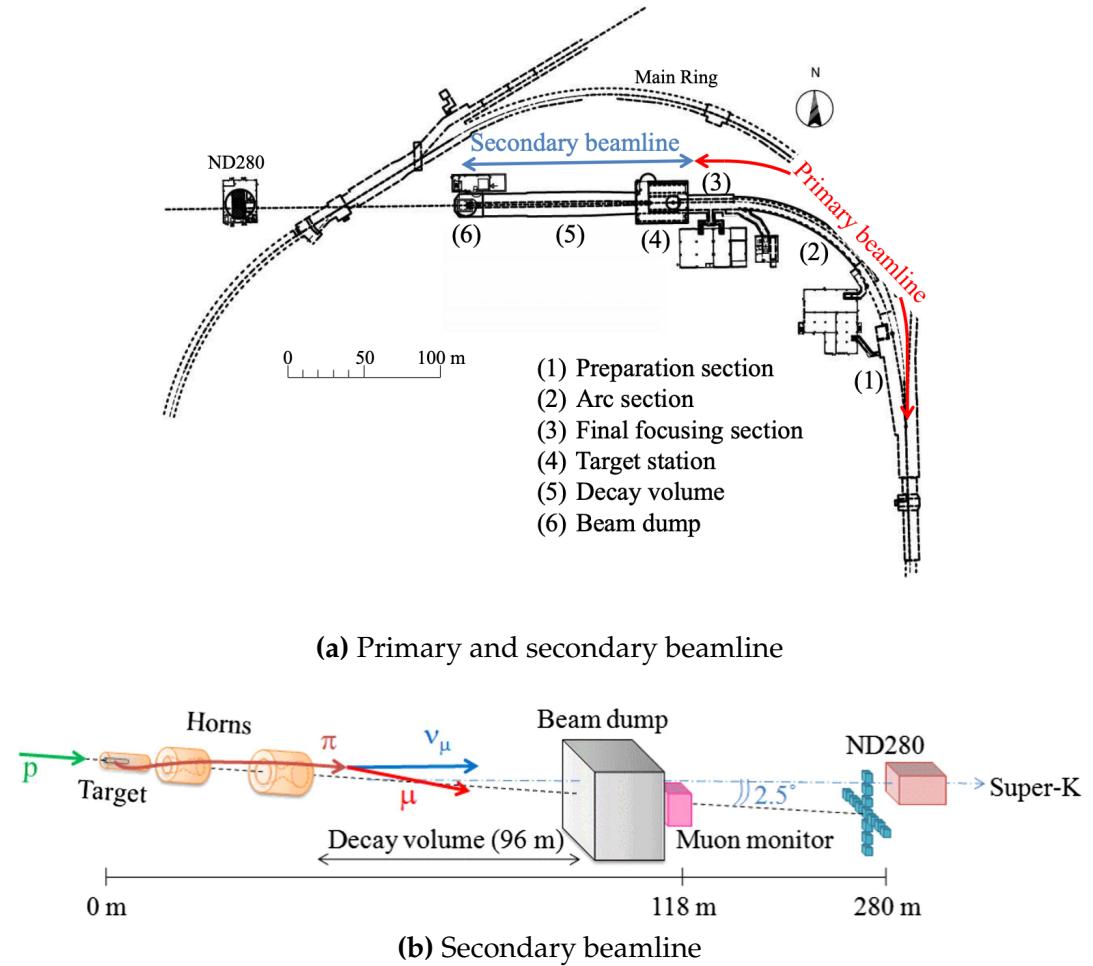


Figure 3.6: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [44]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [112].

Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino flux. The low energy flux is dominated by the decay of pions whereas kaon decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic irreducible ν_e background, are predominantly due to muon decay for $E_\nu < 2\text{GeV}$. As the antineutrino production cross-section is smaller than the neutrino cross-section, the wrong-sign component is more dominant in the RHC beam as compared to that in the FHC beam.

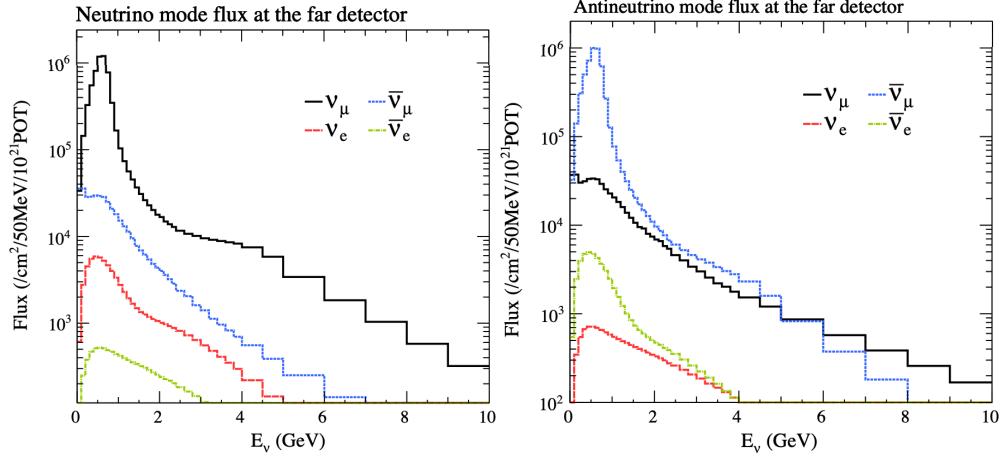


Figure 3.7: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [113].

The beam dump, situated at the end of the decay volume, stops all charged particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector monitors the penetrating muons to determine the beam direction and intensity which is used to constrain some of the beam flux systematics within the analysis [112, 114].

The T2K experiment uses an off-axis beam to narrow the neutrino energy distribution. This was the first implementation of this technique in a long-baseline neutrino oscillation experiment after its original proposal [115]. Pion decay, $\pi \rightarrow \mu + \nu_\mu$, is a two-body decay. Consequently, the neutrino energy, E_ν , can be determined based on the pion energy, E_π , and the angle at which the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

where m_π and m_μ are the mass of the pion and muon respectively. For a fixed energy pion, the neutrino energy distribution is dependent upon the angle at which the neutrinos are observed from the initial pion beam direction. For the 295km baseline at T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance probability, $P(\nu_\mu \rightarrow \nu_e)$, whilst minimising the muon disappearance probability,

⁹⁹⁴ $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8 illustrates the neutrino energy distribution for a range of
⁹⁹⁵ off-axis angles, as well as the oscillation probabilities most relevant to T2K.

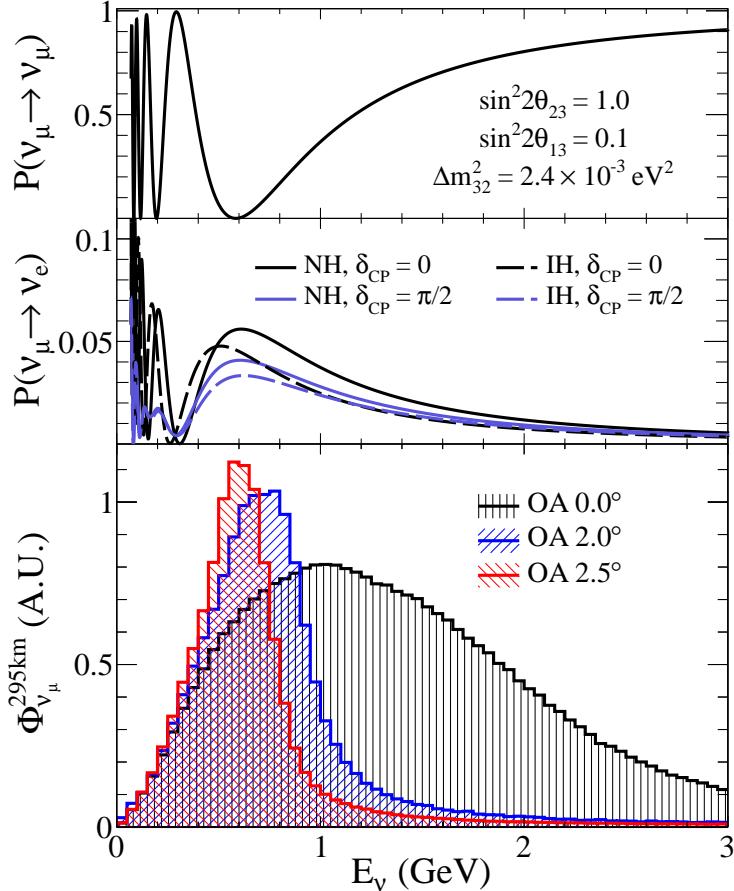


Figure 3.8: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

3.2.3 The Near Detector at 280m

⁹⁹⁷ Whilst all the near detectors are situated in the same “pit” located at 280m from
⁹⁹⁸ the beamline, the “ND280” detector is the off-axis detector which is situated at
⁹⁹⁹ the same off-axis angle as the Super-Kamiokande far detector. It has two primary
¹⁰⁰⁰ functions; firstly it measures the neutrino flux and secondly, it counts the event
¹⁰⁰¹ rates of different types of neutrino interactions. Both of these constrain the flux
¹⁰⁰² and cross-section systematics invoked within the model for a more accurate
¹⁰⁰³ prediction of the expected event rate at the far detector.

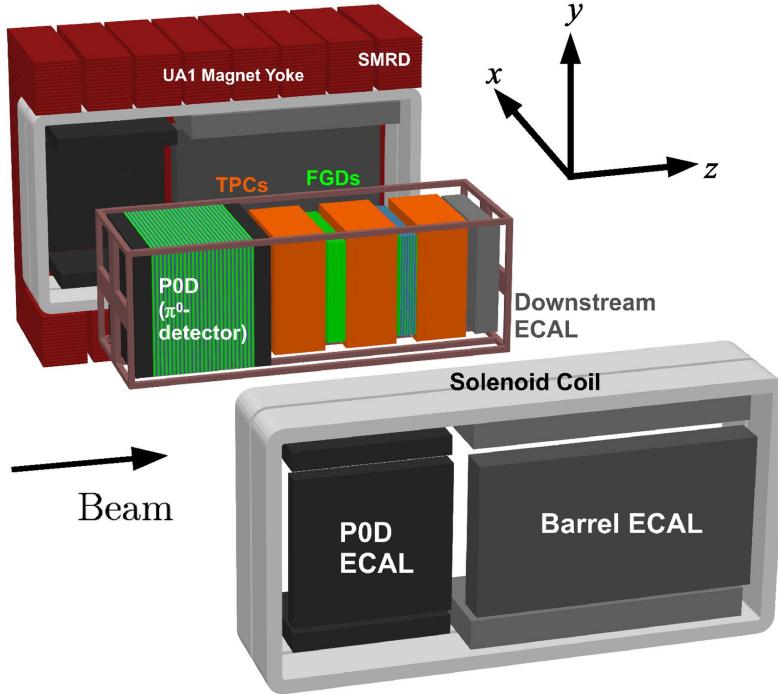


Figure 3.9: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [44].

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two-time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.3.1 and subsubsection 3.2.3.2 respectively. The electromagnetic calorimeter (ECAL) encapsulates the tracker region alongside the π^0 detector (P0D). The ECAL measures the deposited energy from photons emitted from interactions within the FGD. The P0D constrains the cross-section of neutral current interactions which generate neutral pions, which is one of the largest backgrounds in the electron neutrino appearance oscillation channel. The P0D and ECAL detectors are detailed in subsubsection 3.2.3.3 and subsubsection 3.2.3.4 respectively. The entire detector is located within a large yoke magnet which produces a 0.2T magnetic field.

1020 field. This design of the magnet also includes a scintillating detector called the
1021 side muon range detector (SMRD), which is used to track high-angle muons as
1022 well as acting as a cosmic veto. The SMRD is described in subsubsection 3.2.3.5.

1023 **3.2.3.1 Fine Grained Detectors**

1024 The T2K tracker region is comprised of two fine-grained detectors (FGD) and
1025 three Time Projection Chambers (TPC). A detailed description of the FGD design,
1026 construction, and assembly is found in [116] and summarised below. The FGDS
1027 are the primary target for neutrino interactions with a mass of 1.1 tonnes per FGD.
1028 Alongside this, the FGDS are designed to be able to track short-range particles
1029 which do not exit the FGD. Typically, short-range particles are low momentum
1030 and are observed as tracks that deposit a large amount of energy per unit length.
1031 This means the FGD needs good granularity to resolve these particles. The
1032 FGDS have the best timing resolution ($\sim 3\text{ns}$) of any of the sub-detectors of the
1033 ND280 detector. As such, the FGDS are used for time of flight measurements
1034 to distinguish forward-going positively charged particles from backward-going
1035 negatively charged particles. Finally, any tracks which pass through multiple
1036 sub-detectors are required to be track matched to the FGD.

1037 Both FGDS are made from square scintillator planes of side length 186cm and
1038 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an
1039 X or Y orientation. A wavelength-shifting fiber is threaded through the center
1040 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is
1041 the most upstream of the two FGDS and contains 15 planes of carbon plastic
1042 scintillator which is a common target in external neutrino scattering data. As
1043 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2
1044 have been replaced with a hybrid water-scintillator target. Due to the complexity
1045 of the nucleus, nuclear effects can not be extrapolated between different nuclei.
1046 Therefore having the ability to take data on one target which is the same as
1047 external data and another target which is the same as the far detector target is
1048 beneficial for reliable model parameter estimates.

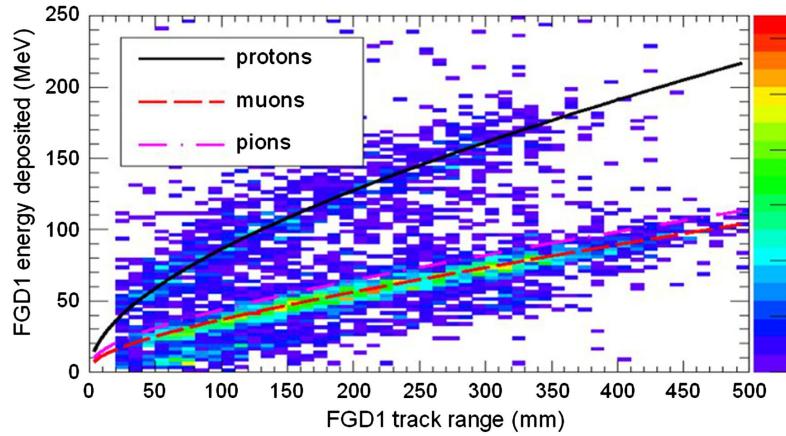


Figure 3.10: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [116].

1049 The integrated deposited energy is used for particle identification. The FGD
 1050 can distinguish protons from other charged particles by comparing the integrated
 1051 deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

1052 3.2.3.2 Time Projection Chambers

1053 The majority of particle identification and momentum measurements within
 1054 ND280 are provided by three Time Projection Chambers (TPCs) [117]. The
 1055 TPCs are located on either side of the FGDs. They are located inside of the
 1056 magnetic field meaning the momentum of a charged particle can be determined
 1057 from the bending of the track.

1058 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11,
 1059 which are made of non-magnetic material. The outer box is filled with CO₂ which
 1060 acts as an electrical insulator between the inner box and the ground. The inner box
 1061 forms the field cage which produces a uniform electric drift field of $\sim 275\text{V/cm}$
 1062 and is filled with an argon gas mixture. Charged particles moving through this
 1063 gas mixture ionize the gas and the ionised charge is drifted towards micromegas
 1064 detectors which measure the ionization charge. The time and position information
 1065 in the readout allows a three-dimensional image of the neutrino interaction.

1066 The particle identification of tracks that pass through the TPCs is performed
 1067 using dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo

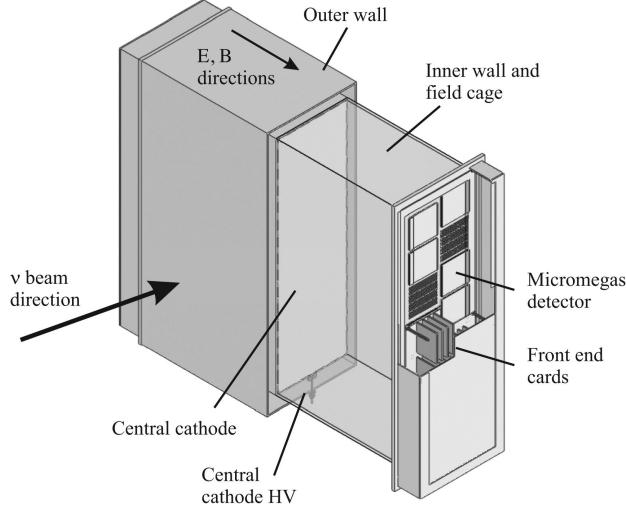


Figure 3.11: Schematic design of a Time Projection Chamber detector. Taken from [117].

1068 distributions of the energy lost by a charged particle passing through the TPC as
 1069 a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$
 1070 meaning that electrons and muons can be distinguished. This allows reliable
 1071 measurements of the intrinsic ν_e component of the beam.

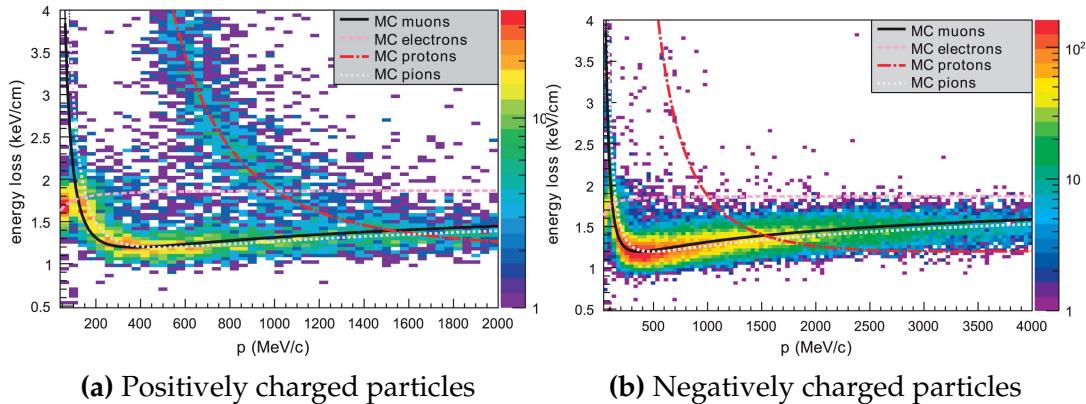


Figure 3.12: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [117].

1072 3.2.3.3 π^0 Detector

1073 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the
 1074 reconstruction will determine that event to be a charge current ν_e -like event.
 1075 This is one of the main backgrounds hindering the electron neutrino appearance

1076 searches. The π^0 detector (P0D) measures the cross-section of the neutral current
1077 induced neutral pion production on a water target to constrain this background.

1078 The P0D is a cube of approximately 2.5m length consisting of layers of scin-
1079 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.13.
1080 Two electromagnetic calorimeters are positioned at the most upstream and most
1081 downstream position in the sub-detector and the water target is situated in
1082 between them. The scintillator layers are built from two triangular bars orientated
1083 in opposite directions to form a rectangular layer. Each triangular scintillator bar
1084 is threaded with optical fiber which is read out by MPPCs. The high-Z brass and
1085 lead regions produce electron showers from the photons emitted in π^0 decay.

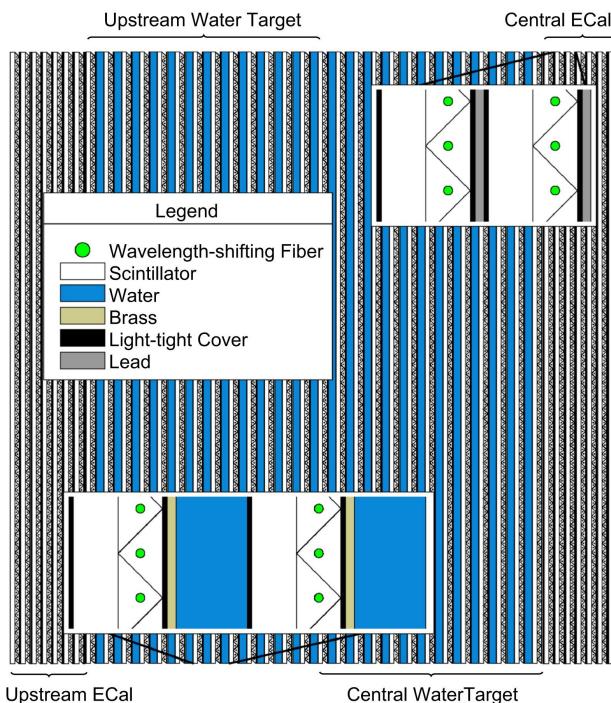


Figure 3.13: A schematic of the P0D side-view. Taken from [118].

1086 The sub-detector can generate measurements of NC1 π^0 cross-sections on a
1087 water target by measuring the event rate both with and without the water target,
1088 with the cross-section on a water target being determined as the difference. The to-
1089 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

1090 3.2.3.4 Electromagnetic Calorimeter

1091 The electromagnetic calorimeter [119] (ECal) encapsulates the P0D and tracking
1092 sub-detectors. Its primary purpose is to aid π^0 reconstruction from any interac-
1093 tion in the tracker. To do this, it measures the energy and direction of photon
1094 showers from $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks
1095 depending on the shape of the photon shower deposited.

1096 The ECal is comprised of three sections; the P0D ECal which surrounds the
1097 P0D, the barrel ECal which encompasses the tracking region, and the downstream
1098 ECal which is situated downstream of the tracker region. The barrel and down-
1099 stream ECals are tracking calorimeters that focus on electromagnetic showers
1100 from high-angle particles emitted from the tracking sub-detectors. Particularly in
1101 the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis)
1102 can travel along a single scintillator bar resulting in very few hits. The width of
1103 the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths
1104 to ensure a significant amount of the π^0 energy is contained. As the P0D has
1105 its own calorimetry which reconstructs showers, the P0D ECal determines the
1106 energy which escapes the P0D.

1107 Each ECal is constructed of multiple layers of scintillating bars sandwiched
1108 between lead sheets. The scintillating bars are threaded with optical fiber and read
1109 out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular
1110 to the previous which allows a three-dimensional event reconstruction. The
1111 target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80,
1112 and 6.62 tonnes respectively.

1113 3.2.3.5 Side Muon Range Detector

1114 As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within
1115 the UA1 magnet. Reconditioned after use in the UA1 [120] and NOMAD [121]
1116 experiments, this magnet provides a uniform horizontal magnetic field of 0.2T
1117 with an uncertainty of 2×10^{-4} T.

1118 Built into the UA1 magnet, the side muon range detector (SMRD)[122] monitors
1119 high-energy muons which leave the tracking region and permeate through
1120 the ECal. It additionally acts as a cosmic muon veto and trigger.

1121 3.2.4 The Interactive Neutrino GRID

1122 The Interactive Neutrino GRID (INGRID) detector is situated within the same
1123 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”
1124 position and measures the beam direction, spread, and intensity. The detector
1125 was originally designed with 16 identical modules [44] (two modules have since
1126 been decommissioned) and a “proton” module. The design of the detector is 14
1127 modules oriented in a cross with length and height $10\text{m} \times 10\text{m}$, as illustrated
1128 in Figure 3.14.

1129 Each module is composed of iron sheets interlaced with eleven tracking
1130 scintillator planes for a total target mass of 7.1 tonnes per module. The scintillator
1131 design is an X-Y pattern of 24 bars in both orientations, where each bar contains
1132 wave-length shifting fibers which are connected to multi-pixel photon counters
1133 (MPPCs). Each module is encapsulated inside veto planes to aid the rejection
1134 of charged particles entering the module.

1135 The proton module is different from the other modules in that it consists
1136 of entirely scintillator planes with no iron target. The scintillator bars are also
1137 smaller than those used in the other modules to increase the granularity of
1138 the detector and improve tracking capabilities. The module sits in the center
1139 of the beamline and is designed to give precise measurements of quasi-elastic
1140 charged current interactions to evaluate the performance of the Monte Carlo
1141 simulation of the beamline.

1142 The INGRID detector can measure the beam direction to an uncertainty of
1143 0.4mrad and the beam centre within a resolution of 10cm [44]. The beam direction
1144 in both the vertical and horizontal directions is discussed in [123] and it is found
1145 to be in good agreement with the MUMON monitor described in subsection 3.2.2.

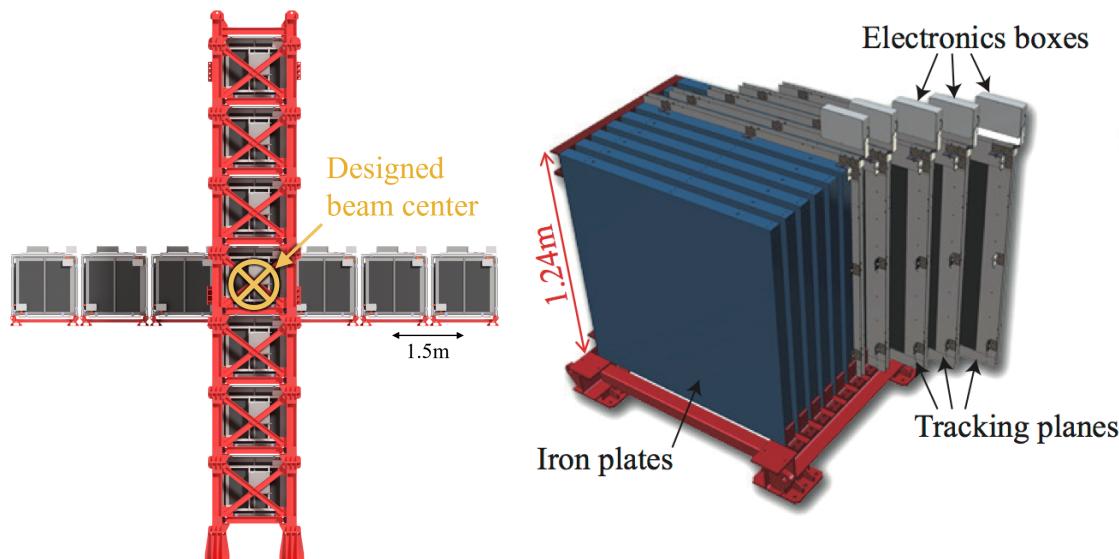


Figure 3.14: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [44].

4

1146

1147

1148

Bayesian Statistics and Markov Chain Monte Carlo Techniques

1149 This thesis presents a Bayesian oscillation analysis. To extract the oscillation
1150 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter
1151 explains the theory of how parameter estimates can be determined using this
1152 technique and condenses the material found in the literature [124–127].

1153 The oscillation parameter determination presented here is built upon a si-
1154 multaneous fit to neutrino beam data in the near detector, beam data at SK, and
1155 atmospheric data at SK. In total, there are four oscillation parameters of interest
1156 ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP}), two oscillation parameters to which this
1157 study will not be sensitive ($\sin^2(\theta_{12})$, Δm_{21}^2) and many nuisance parameters that
1158 control the systematic uncertainty models.

1159 This analysis uses a Monte Carlo technique to generate a multi-dimensional
1160 probability distribution across all of the model parameters used in the fit. To
1161 determine an estimate for each parameter, this multi-dimensional object is in-
1162 tegrated over all other parameters. This process is called Marginalisation and
1163 is described in subsection 4.3.1. Monte Carlo techniques approximate the prob-
1164 ability distribution of each parameter within the limit of generating infinite
1165 samples. As ever, generating a large number of samples is time and resource-

1166 dependent. Therefore, an MCMC technique is utilised within this analysis to
1167 reduce the required number of steps to sufficiently sample the parameter space.
1168 This technique is described in further detail in subsection 4.2.1.

1169 The Bayesian analysis techniques used within this thesis are built within the
1170 MaCh3 framework [128]. This uses a custom MCMC library package exclusively
1171 supported and developed by the MaCh3 collaborators (which includes the author
1172 of this thesis).

1173 4.1 Bayesian Statistics

1174 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
1175 footing such that a probability model of both data and parameters is required.
1176 This is the joint probability distribution $P(D, \vec{\theta})$ and can be described by the
1177 prior distribution for model parameters $P(\vec{\theta})$ and the likelihood of the data given
1178 the model parameters $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

1179 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the
1180 parameters within the model. For example, if the risk of developing health
1181 problems is known to increase with age, the prior distribution would describe the
1182 increase. For the purpose of this analysis, the prior distribution is typically
1183 the best-fit values taken from external data measurements with a Gaussian
1184 uncertainty. The prior distribution can also contain correlations between model
1185 parameters. In an analysis using Monte Carlo techniques, the likelihood of
1186 measuring some data assuming some set of model parameters is calculated
1187 by comparing the Monte Carlo prediction generated at that particular set of
1188 model parameters to the data.

1189 It is parameter estimation that is important for this analysis and as such, Bayes'
1190 theorem [129] is applied to calculate the probability for each parameter to have a

1191 certain value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior
1192 distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

1193 The denominator in Equation 4.2 is the integral of the joint probability distri-
1194 bution over all values of all parameters used within the fit. For brevity, the
1195 posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

1196 For the purposes of this analysis, it is acceptable to neglect the normalisation
1197 term and focus on this proportional relationship.

1198 4.1.1 Application of Prior Knowledge

1199 The posterior distribution is proportional to the prior uncertainty applied to
1200 each parameter, as illustrated by Equation 4.3. This means that it is possible
1201 to change the prior after the posterior distribution has been determined. The
1202 prior uncertainty of a particular parameter can be ‘divided’ out of the posterior
1203 distribution and the resulting distribution can be reweighted using the new
1204 prior uncertainty that is to be applied. The methodology and implementation
1205 of changing the prior follows that described in [130].

1206 An example implementation that is useful for this analysis is the application
1207 of the “reactor constraint”. As discussed in section 2.4, an external constraint
1208 on $\sin^2(\theta_{13})$ is determined from measurements taken from reactor experiments.
1209 However, the sensitivities from just using the T2K and SK samples is equally
1210 as important. Without this technique, two fits would have to be run, doubling
1211 the required resources. Therefore, the key benefit for this analysis is the fact that
1212 only a single ‘fit’ has to be performed and can be used to build the two posterior
1213 distributions of the with and without reactor constraint applied.

1214 4.2 Monte Carlo Simulation

1215 Monte Carlo techniques are used to numerically solve a complex problem that
1216 does not necessarily have an analytical solution. These techniques rely on
1217 building a large ensemble of samples from an unknown distribution and then
1218 using the ensemble to approximate the properties of the distribution.

1219 An example that uses Monte Carlo techniques is to calculate the area under-
1220 neath a curve. For example, take the problem of calculating the area under a
1221 straight line with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can
1222 calculate the area under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte
1223 Carlo techniques, one can calculate the area under this line by throwing many
1224 random values for the x and y components of each sample and then calculating
1225 whether that point falls below the line. The area can then be calculated by the
1226 ratio of points below the line to the total number of samples thrown multiplied by
1227 the total area in which samples were scattered. The study is shown in Figure 4.1
1228 highlights this technique and finds the area under the curve to be 29.9 compared
1229 to an analytical solution of 30.0. The deviation of the numerical to analytical
1230 solution can be attributed to the number of samples used in the study. The
1231 accuracy of the approximation in which the properties of the Monte Carlo samples
1232 replicate those of the desired distribution is dependent on the number of samples
1233 used. Replicating this study with a differing number of Monte Carlo samples
1234 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo
1235 techniques are only accurate within the limit of a high number of samples.

1236 Whilst the above example has an analytical solution, these techniques are just
1237 as applicable to complex solutions. Clearly, any numerical solution is only as
1238 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is
1239 dependent upon the number of samples generated to approximate the properties
1240 of the distribution. Furthermore, if the positions at which the samples are
1241 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique
1242 significantly drops. Given the example in Figure 4.1, if the region in which the

1243 samples are scattered significantly extends passed the region of interest, many
1244 calculations will be calculated but do not add to the ability of the Monte Carlo
1245 technique to achieve the correct result. For instance, any sample evaluated at
1246 a $y \geq 5$ could be removed without affecting the final result. This does bring in
1247 an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,
1248 one needs to know the distribution beforehand.

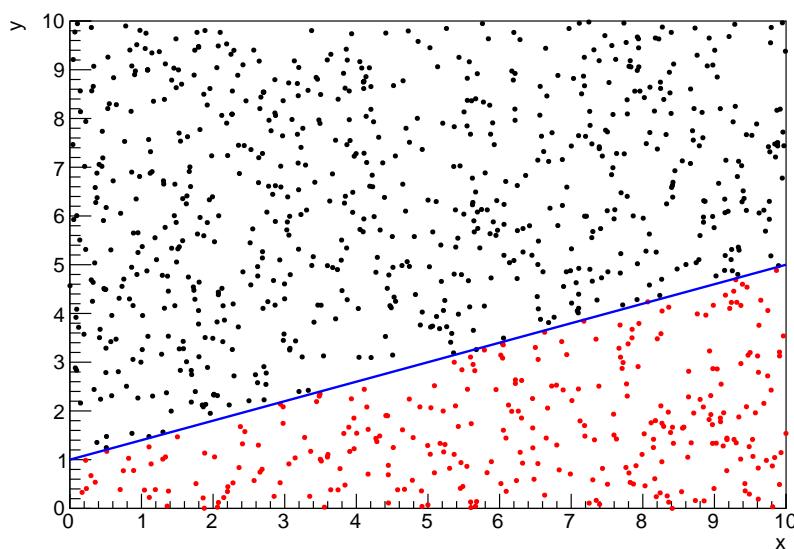


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

1249 4.2.1 Markov Chain Monte Carlo

1250 This analysis utilises a multi-dimensional probability distribution, with some
1251 dimensions being significantly more constrained than others. These constraints
1252 can be from prior knowledge of parameter distributions from external data or
1253 un-physical regions in which parameters can not exist. To maximise the efficiency
1254 of building the posterior distribution, a Markov Chain Monte Carlo (MCMC)
1255 technique is used. This employs a Markov chain to select the points at which
1256 to sample the posterior distribution. It performs a semi-random stochastic walk
1257 through the allowable parameter space. This builds a posterior distribution

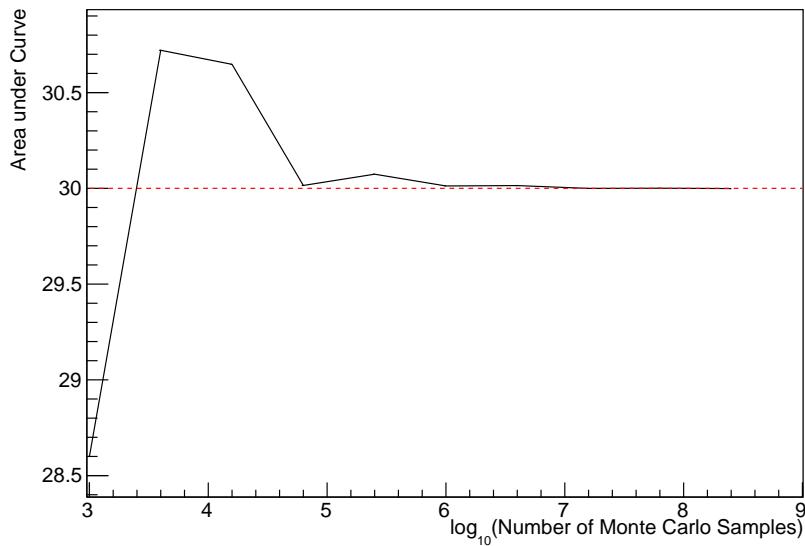


Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1258 which has the property that the density of sampled points is proportional to the
 1259 probability density of that parameter. This means that the samples produced by
 1260 this technique are not statistically independent but they will cover the space
 1261 of the distribution.

1262 A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the
 1263 position of \vec{x}_i . The space in which the Markov chain selects samples is dependent
 1264 upon the total number of parameters utilised within the fit, where a discrete point
 1265 in this space is described by the N-dimensional space \vec{x} . In a perfectly operating
 1266 Markov chain, the position of the next step depends solely on the previous step
 1267 and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving
 1268 the multi-dimensionality of the fit used within this analysis, each step becomes
 1269 correlated with several of the steps preceding itself. Providing the MCMC chain is
 1270 well optimised, it will begin to converge towards a unique stationary distribution.
 1271 The period between the chain's initial starting point and the convergence to the
 1272 unique stationary distribution is colloquially known as the burn-in period. Once
 1273 the chain reaches the stationary distribution, all points sampled after that point

1274 will look like samples from that distribution.

1275 Further details of the theories underpinning MCMC techniques are discussed
1276 in [125] but can be summarised by the requirement that the chain satisfies the
1277 three ‘regularity conditions’:

- 1278 • Irreducibility: From every position in the parameter space \vec{x} , there must
1279 exist a non-zero probability for every other position in the parameter space
1280 to be reached.
- 1281 • Recurrence: Once the chain arrives at the stationary distribution, every step
1282 following from that position must be samples from the same stationary
1283 distribution.
- 1284 • Aperiodicity: The chain must not repeat the same sequence of steps at any
1285 point throughout the sampling period.

1286 The output of the chain after burn-in (i.e. the sampled points after the chain
1287 has reached the stationary distribution) can be used to approximate the posterior
1288 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1289 stationary distribution found by the chain be the posterior distribution, one
1290 can use the Metropolis-Hastings algorithm. This guides the stochastic process
1291 depending on the likelihood of the current proposed step compared to that
1292 of the previous step.

1293 4.2.2 Metropolis-Hastings Algorithm

1294 As a requirement for MCMCs, the Markov chain implemented in this technique
1295 must have a unique stationary distribution that is equivalent to the posterior
1296 distribution. To ensure this requirement and that the regularity conditions are
1297 met, this analysis utilises the Metropolis-Hastings (MH) algorithm [131, 132].
1298 For the i^{th} step in the chain, the MH algorithm determines the position in the
1299 parameter space to which the chain moves to based on the current step, \vec{x}_i , and
1300 the proposed step, \vec{y}_{i+1} . The proposed step is randomly selected from some

proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely on the current step (ie. not the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the current step or the proposed step determined by whether the proposed step is accepted or rejected. To decide if the proposed step is selected, the acceptance probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)}\right). \quad (4.4)$$

Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To simplify this calculation, the proposal function is required to be symmetric such that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution centered on \vec{x}_i is used to throw parameter proposals. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)}\right). \quad (4.5)$$

After calculating this quantity, a random number, β , is generated uniformly between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise, the chain sets the next step equal to the current step. This procedure is repeated for subsequent steps. This can be interpreted as if the posterior probability of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true, ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped in any local minima in the potentially non-Gaussian posterior distribution. The outcome of this technique is that the density of steps taken in a discrete region is directly proportional to the probability density in that region.

4.2.3 MCMC Optimisation

As discussed in subsection 4.2.2, the proposal function invoked within the MH algorithm can take any form and the chain will still converge to the stationary distribution. At each set of proposed parameter values, a prediction of the same spectra has to be generated which requires significant computational resources.

1326 Therefore, the number of steps taken before the unique stationary distribution
1327 is found should be minimised as only steps after convergence add information
1328 to the oscillation analysis. Furthermore, the chain should entirely cover the
1329 allowable parameter space to ensure that all values have been considered. Tuning
1330 the distance that the proposal function jumps between steps on a parameter-by-
1331 parameter basis can both minimise the length of the burn-in period and ensure
1332 that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently small.

1333 The effect of changing the width of the proposal function is highlighted in
1334 Figure 4.3. Three scenarios, each with the same underlying stationary distribution
1335 (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between
1336 the three scenarios is the width of the proposal function, colloquially known as
1337 the ‘step size σ ’. Each scenario starts at an initial parameter value of 10.0 which
1338 would be considered an extreme variation. For the case where $\sigma = 0.1$, it is
1339 clear to see that the chain takes a long time to reach the expected region of the
1340 parameter. This indicates that this chain would have a large burn-in period and
1341 does not converge to the stationary distribution until step ~ 500 . Furthermore,
1342 whilst the chain does move towards the expected region, each step is significantly
1343 correlated with the previous. Considering the case where $\sigma = 5.0$, the chain
1344 approaches the expected parameter region almost instantly meaning that the
1345 burn-in period is not significant. However, there are clearly large regions of steps
1346 where the chain does not move. This is likely due to the chain proposing steps
1347 in the tails of the distribution which have a low probability of being accepted.
1348 Consequently, this chain would take a significant number of steps to fully span
1349 the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can
1350 see a relatively small burn-in period of approximately 100 steps. Once the chain
1351 reaches the stationary distribution, it moves throughout the expected region of
1352 parameter values many times, sufficiently sampling the full parameter region.
1353 This example is a single parameter varying across a continuous distribution and
1354 does not fully reflect the difficulties in the many-hundred multi-variate parameter

1355 distribution used within this analysis. However, it does give a conceptual idea of
 1356 the importance of selecting the proposal function and associated step size.

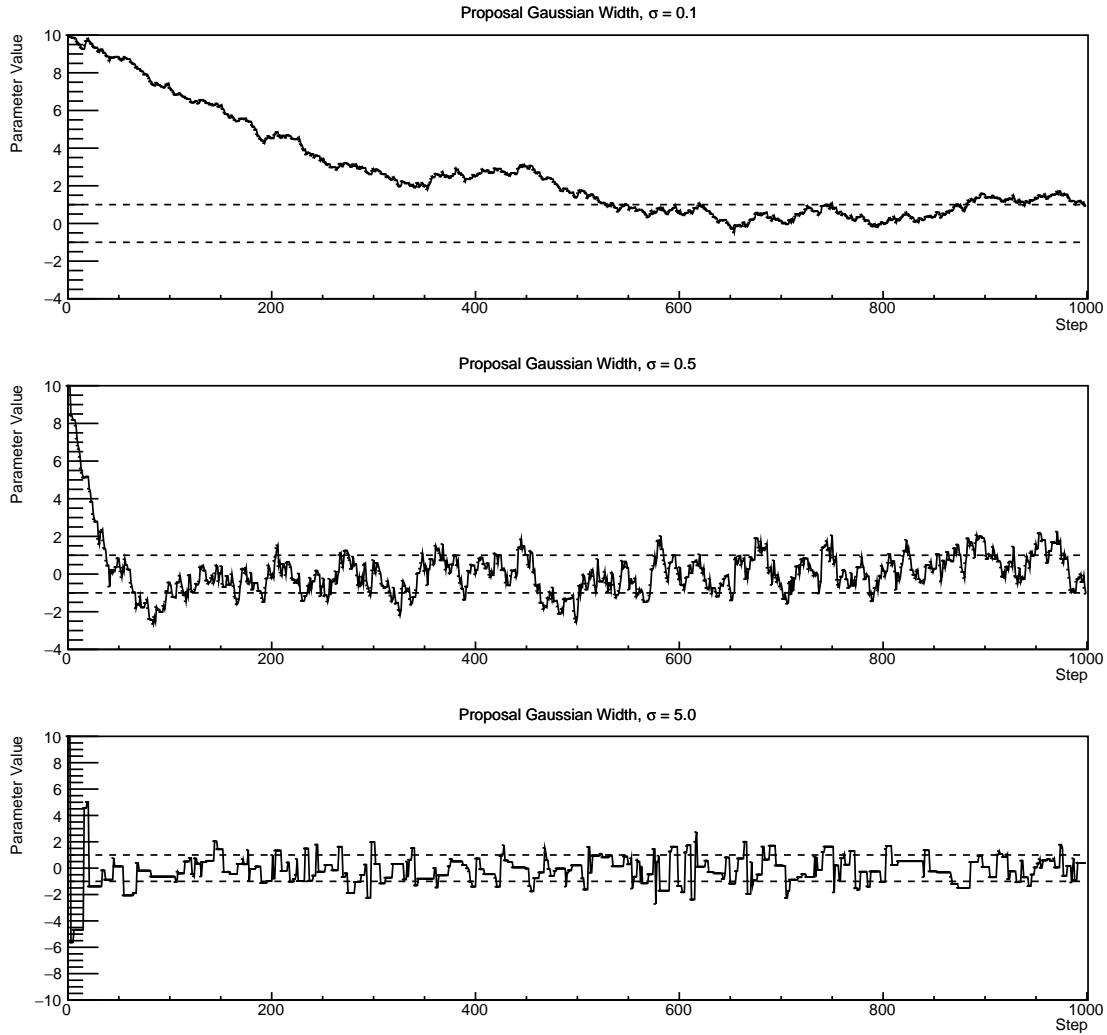


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1357 As discussed, step size tuning directly correlates to the average step accep-
 1358 tance rate. If the step size is too small, many steps will be accepted but the
 1359 chain moves slowly. If the opposite is true, many steps will be rejected as the
 1360 chain proposes steps in the tails of the distribution. Discussion in [133] suggests
 1361 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be

₁₃₆₂ approximately $\sim 25\%$. An “ideal” step size [133] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

₁₃₆₃ where N_p is the number of parameters included in the MCMC fit. However,
₁₃₆₄ the complex correlations between systematics mean that some parameters have
₁₃₆₅ to be hand-tuned and many efforts have been taken to select a set of parameter-
₁₃₆₆ by-parameter step sizes to approximately reach the ideal acceptance rate.

₁₃₆₇ Figure 4.4 highlights the likelihood as calculated by the fit in subsection 8.3.4
₁₃₆₈ as a function of the number of steps in each chain. In practice, many independent
₁₃₆₉ MCMC chains are run simultaneously to parallelise the task of performing the
₁₃₇₀ fit. This figure overlays the distribution found in each chain. As seen, the
₁₃₇₁ likelihood decreases from its initial value and converges towards a stationary
₁₃₇₂ distribution after $\sim 1 \times 10^5$ steps.

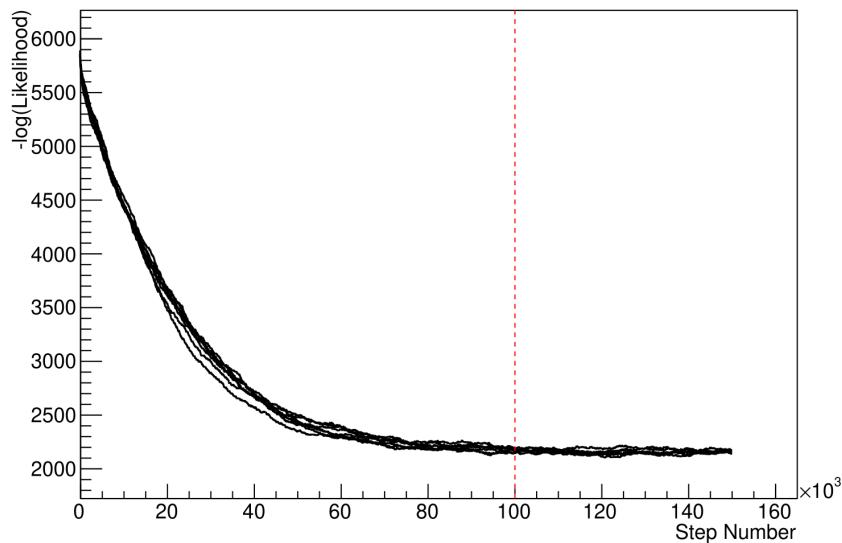


Figure 4.4: The log-likelihood from the fit detailed in subsection 8.3.4 as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

₁₃₇₃ Multiple configurations of this analysis have been performed throughout this
₁₃₇₄ thesis where different samples or systematics have been used. For all of these con-
₁₃₇₅ figurations, it was found that a burnin period of 1×10^5 was sufficient in all cases.

1376 4.3 Understanding the MCMC Results

1377 The previous sections have described how to generate the posterior probability
1378 distribution using Bayesian MCMC techniques. However, this analysis focuses
1379 on oscillation parameter determination. The posterior distribution output from
1380 the chain is a high-dimension object, with as many dimensions as there are
1381 parameters included in the oscillation analysis. However, this multi-dimensional
1382 object is difficult to conceptualize so parameter estimations are often presented
1383 in one or two-dimensional projections of this probability distribution. To do
1384 this, marginalisation techniques are invoked.

1385 4.3.1 Marginalisation

1386 The output of the MCMC chain is a highly dimensional probability distribution
1387 which is very difficult to interpret. From the standpoint of an oscillation analysis
1388 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters
1389 of interest are most relevant. Despite this, the best fit values and uncertainties on
1390 the oscillation parameters of interest should correctly encapsulate the correlations
1391 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).
1392 For this joint beam and atmospheric analysis, the oscillation parameters of
1393 interest are $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . All other parameters (includ-
1394 ing the oscillation parameters this fit is insensitive to) are deemed nuisance
1395 parameters. To generate these projections, the posterior distribution is integrated
1396 over all nuisance parameters. This is called marginalisation. This technique
1397 also explains why it is acceptable to neglect the normalisation constant of the
1398 posterior distribution, which was discussed in section 4.1.

1399 A simple example of the marginalisation technique is to imagine the scenario
1400 where two coins are flipped. To determine the probability that the first coin
1401 returned a ‘head’, the exact result of the second coin flip is disregarded and
1402 simply integrated over. For the parameters of interest, $\vec{\theta}_i$, the marginalised

¹⁴⁰³ posterior is calculated by integrating over the nuisance parameters, $\vec{\theta}_n$. In this
¹⁴⁰⁴ case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.7)$$

¹⁴⁰⁵ Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and
¹⁴⁰⁶ correlations between the parameters of interest and the nuisance parameters.
¹⁴⁰⁷ In practice, this is simply taking the one or two-dimensional projection of the
¹⁴⁰⁸ multi-dimensional probability distribution.

¹⁴⁰⁹ While in principle an easy solution to a complex problem, correlations be-
¹⁴¹⁰ tween the interesting and nuisance parameters can bias the marginalised results.
¹⁴¹¹ A similar effect is found when the parameters being marginalised over have
¹⁴¹² non-Gaussian probability distributions. For example, Figure 4.5 highlights the
¹⁴¹³ marginalisation bias in the probability distribution found for a parameter when
¹⁴¹⁴ requiring a correlated parameter to have a positive parameter value. Due to
¹⁴¹⁵ the complex nature of the oscillation parameter fit presented in this thesis, there
¹⁴¹⁶ are correlations occurring between the oscillation parameters of interest and the
¹⁴¹⁷ other nuisance parameters included in the fit.

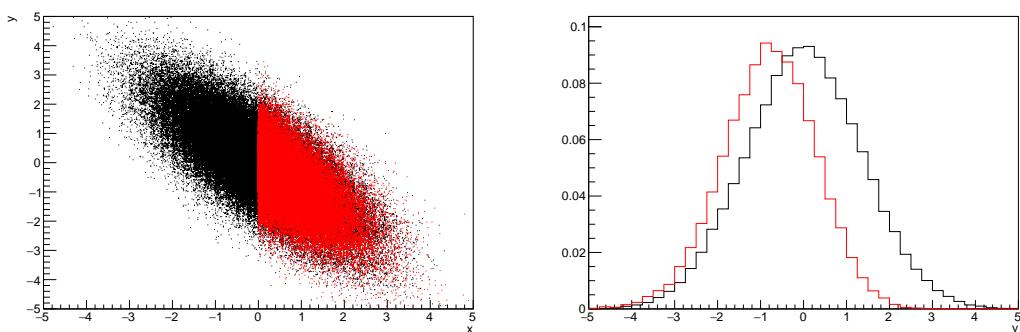


Figure 4.5: Left: The two-dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two-dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

4.3.2 Parameter Estimation and Credible Intervals

The purpose of this analysis is to determine the best fit values for the oscillation parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . The posterior probability density, taken from the output MCMC chain, is binned in these parameters. The parameter best-fit point is then taken to be the value that has the highest posterior probability. This is performed in both one and two-dimensional projections.

However, the single best-fit point in a given parameter is not of much use on its own. The uncertainty on the best-fit point must also be presented using credible intervals. The definition of the 1σ credible interval is that there is 68% belief that the parameter is within those bounds. For a more generalised definition, the credible interval is the region, R , of the posterior distribution that contains a specific fraction of the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha. \quad (4.8)$$

Where θ is the parameter being evaluated. This technique then calculates the $\alpha \times 100\%$ credible interval.

In practice, this analysis uses the highest posterior density (HPD) credible intervals which are calculated through the following method. First, the probability distribution is area-normalised such that it has an integrated area equal to 1.0. The bins of probability are then summed from the highest to lowest until the sum exceeds the 1σ level (0.68 in this example). This process is repeated for a range of credible intervals, notably the 1σ , 2σ and 3σ along with other levels where the critical values for each level can be found in [75]. This process can be repeated for the two-dimensional probability distributions by creating two-dimensional contours of credible intervals rather than a one-dimensional result.

1442 4.3.3 Bayesian Model Comparisons

1443 Due to the matter resonance, this analysis has some sensitivity to the mass
 1444 hierarchy of neutrino states (whether Δm_{32}^2 is positive or negative) and the
 1445 octant of $\sin^2(\theta_{23})$. The Bayesian approach utilised within this analysis gives an
 1446 intuitive method of model comparison by determining which hypothesis is most
 1447 favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal
 1448 hierarchy, NH , and inverted hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

1449 The middle term defines the Bayes factor, $B(NH/IH)$, which is a data-driven
 1450 interpretation of how strong the data prefers one hierarchy to the other. For this
 1451 analysis, equal priors on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) =$
 1452 $P(\vec{\theta}_{IH}) = 0.5$). In practice, the MCMC chain proposes a value of $|\Delta m_{32}^2|$ and
 1453 then applies a 50% probability that the value is sign flipped. Consequently,
 1454 the Bayes factor can be calculated from the ratio of the probability density in
 1455 either hypothesis. This equates to counting the number of steps taken in the
 1456 normal and inverted hierarchies and taking the ratio. The same approach can be
 1457 taken to compare the upper octant (UO) compared to the lower octant (LO)
 1458 hypothesis of $\sin^2(\theta_{23})$.

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [134]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

1459 Whilst the value of the Bayes factor should always be shown, the Jeffreys scale
 1460 [134] (highlighted in Table 4.1) gives an indication of the strength of preference

¹⁴⁶¹ for one model compared to the other. Other interpretations of the strength of
¹⁴⁶² preference of a model exist, e.g. the Kass and Raferty Scale [135].

¹⁴⁶³ 4.3.4 Comparison of MCMC Output to Expectation

¹⁴⁶⁴ To ensure the fit is performing well, a best-fit spectrum is produced using the
¹⁴⁶⁵ posterior probability distribution and compared with the data, allowing easy
¹⁴⁶⁶ by-eye comparisons to be made. A simple method of doing this is to perform a
¹⁴⁶⁷ comparison in the fitting parameters (e.g. the reconstructed neutrino energy for
¹⁴⁶⁸ T2K far detector beam samples) of the spectra generated by the MCMC chain to
¹⁴⁶⁹ ‘data’. This ‘data’ could be true data or some variation of Monte Carlo prediction.
¹⁴⁷⁰ This allows easy comparison of the MCMC probability distribution to the data. To
¹⁴⁷¹ perform this, N steps from the post-burnin MCMC chain are randomly selected.
¹⁴⁷² From these, the Monte Carlo prediction at each step is generated by reweighting
¹⁴⁷³ the model parameters to the values specified at that step. Due to the probability
¹⁴⁷⁴ density being directly correlated with the density of steps in a certain region,
¹⁴⁷⁵ parameter values close to the best fit value are most likely to be selected.

¹⁴⁷⁶ In practice, for each bin of the fitting parameters has a probability distribution
¹⁴⁷⁷ of event rates, with one entry per sampled MCMC step. This distribution is
¹⁴⁷⁸ binned where the bin with the highest probability is selected as the mean and an
¹⁴⁷⁹ error on the width of this probability distribution is calculated using the approach
¹⁴⁸⁰ highlighted in subsection 4.3.2. Consequently, the best fit distribution in the fit
¹⁴⁸¹ parameter is not necessarily that which would be attained by reweighting the
¹⁴⁸² Monte Carlo prediction to the most probable parameter values.

¹⁴⁸³ A similar study can be performed to illustrate the freedom of the model
¹⁴⁸⁴ parameter space prior to the fit. This can be done by throwing parameter values
¹⁴⁸⁵ from the prior uncertainty of each parameter.

5

1486

1487

1488

Simulation, Reconstruction, and Event Reduction

1489 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1490 neutrino spectrum at the far detector is required. This includes modeling the
1491 flux generation, neutrino interactions, and detector effects. All of the simulation
1492 packages required to do this are briefly described in section 5.1. The reconstruc-
1493 tion of neutrino events in the far detector, including the `fitQun` algorithm, is
1494 documented in section 5.2. This also includes data quality checks of the SK-V
1495 data which the author performed for the T2K oscillation analysis presented at the
1496 Neutrino 2020 conference [2]. Finally, section 5.3 describes the steps taken in the
1497 SK detector to trigger on events of interest whilst removing the comparatively
1498 large rate of cosmic ray muon events.

1499 5.1 Simulation

1500 In order to generate a Monte Carlo prediction of the expected event rate at
1501 the far detector, all the processes in the beam and atmospheric fluxes, neutrino
1502 interaction, and detector need to be modeled.

1503 5.1.1 Neutrino Flux

1504 The beamline simulation consists of three distinct parts: the initial hadron interaction
 1505 modeled by FLUKA [136], the target station geometry and particle tracking
 1506 performed by JNUBEAM, [40, 137] and any hadronic re-interactions simulated by
 1507 GCALOR [138]. The primary hadronic interactions are $O(10)$ GeV, where FLUKA
 1508 matches external cross-section data better than GCALOR [139]. However, FLUKA
 1509 is not very adaptable so a small simulation is built to model the interactions
 1510 in the target and the output is then passed to JNUBEAM and GCALOR for
 1511 propagation. The hadronic interactions are tuned to data from the NA61/SHINE
 1512 [140–142] and HARP [143] experiments. The tuning is done by reweighting the
 1513 FLUKA and GCALOR predictions to match the external data multiplicity and
 1514 cross-section measurements, based on final state particle kinematics [139]. The
 1515 culmination of this simulation package generates the predicted flux for neutrino
 1516 and antineutrino beam modes which are illustrated in Figure 3.7.

1517 The atmospheric neutrino flux is simulated by the HKKM model [52, 54]. The
 1518 primary cosmic ray flux is tuned to AMS [144] and BESS [145] data assuming
 1519 the US-standard atmosphere '76 [146] density profile and includes geomagnetic
 1520 field effects. The primary cosmic rays interact to generate pions and muons.
 1521 The interaction of these secondary particles to generate neutrinos is handled by
 1522 DPMJET-III [147] for energies above 32GeV and JAM [54, 148] for energies below
 1523 that value [50]. These hadronic interactions are tuned to BESS and L3 data [149,
 1524 150] using the same methodology as the tuning of the beamline simulation. The
 1525 energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3
 1526 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked
 1527 around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions
 1528 and kaons can travel further than their vertically-going counterparts resulting
 1529 in a larger probability of decaying to neutrinos. The symmetry is broken in
 1530 lower-energy neutrinos due to geomagnetic effects, which modify the track of the
 1531 primary cosmic rays. Updates to the HKKM model are currently ongoing [151].

5.1.2 Neutrino Interaction

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [152, 153] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a W^\pm boson in Charged Current (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. This is the reason why neutrinos that interact through NC modes are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events.

As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of neutrino interactions around $E_\nu \sim 0.5\text{GeV}$. The NEUT implementation adopts the Llewellyn Smith [154] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [155] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions is controlled by vector and axial-vector form factors parameterised by the BBBA05 [156] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data [157], respectively. NEUT implements the Valencia [158] model to simulate MEC events, where two nucleons and two holes in the nuclear target are produced (often called 2p2h interactions).

For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant. These predominantly produce charged and neutral pions although γ , kaon,

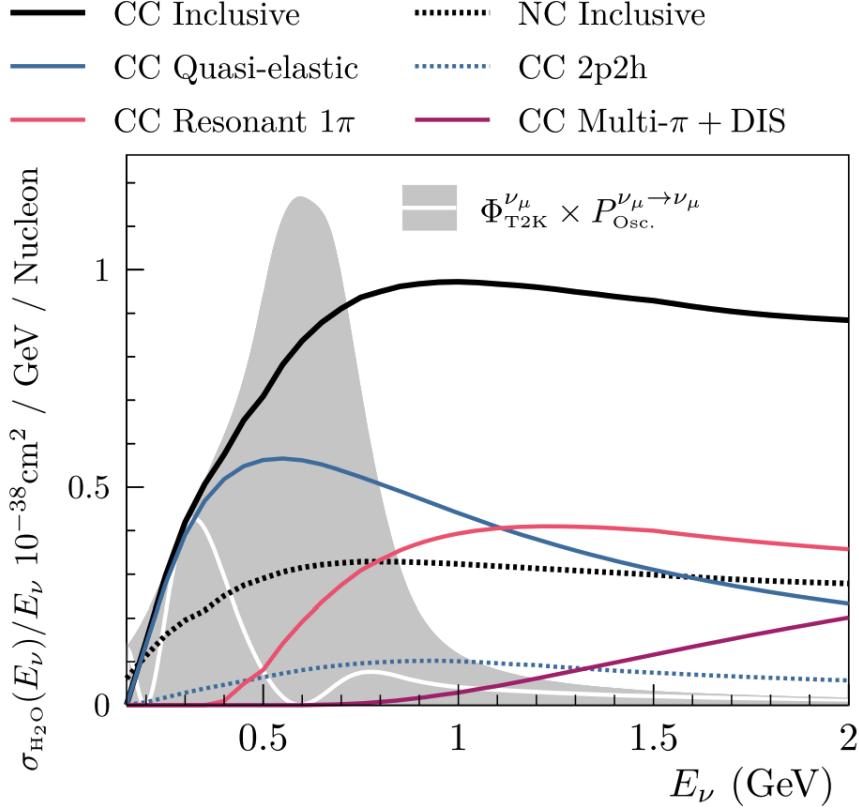


Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [152].

and η production is also considered. To simulate these interactions, the Berger-Sehgal [159] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [160] model to simulate these COH interactions.

DIS and multi- π producing interactions become the most dominant for energies $> O(5)\text{GeV}$. PYTHIA [161] is used to simulate any interaction with invariant mass $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces at least two mesons but has $W < 2\text{GeV}/c^2$, the

₁₅₇₂ Bronner model is used [162]. Both of these models use Parton distribution
₁₅₇₃ functions based on the Bodek-Yang model [163–165].

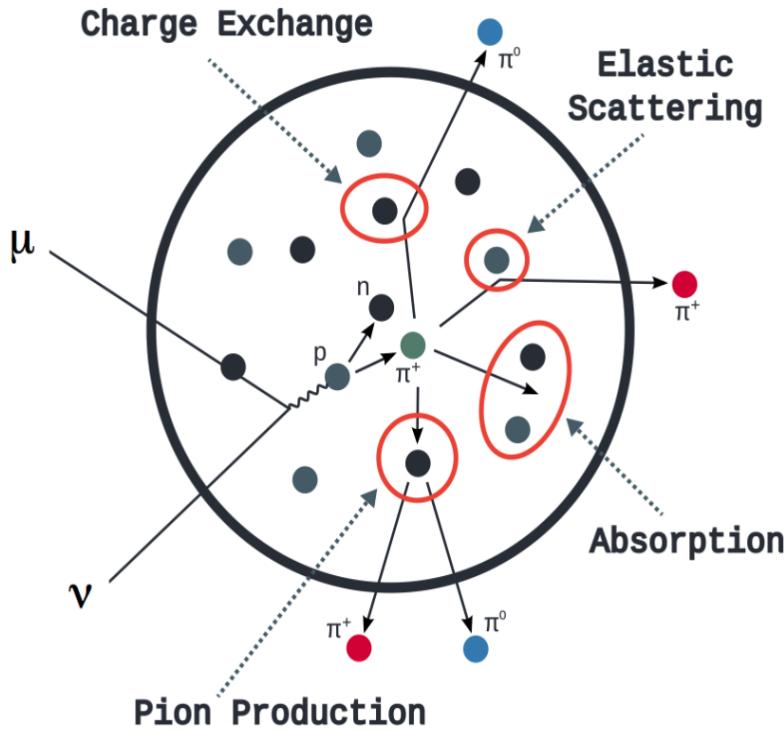


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [166].

₁₅₇₄ Any pion that is produced within the nucleus can re-interact through final
₁₅₇₅ state interactions before it exits, as illustrated by the scattering, absorption,
₁₅₇₆ production, and exchange interactions in Figure 5.2. These re-interactions alter
₁₅₇₇ the observable particles within the detector. For instance, if the charged pion
₁₅₇₈ from a CC PROD interaction is absorbed, the observables would mimic a CC QE
₁₅₇₉ interaction. To simulate these effects, NEUT uses a semi-classical intranuclear
₁₅₈₀ cascade model [152]. This cascade functions by stepping the pion through the
₁₅₈₁ nucleus in fixed-length steps equivalent to $dx = R_N/100$, where R_N is the radius
₁₅₈₂ of the nucleus. At each step, the simulation allows the pion to interact through
₁₅₈₃ scattering, charged exchange, absorption, or production with an interaction-
₁₅₈₄ dependent probability calculated from a fit to external data [167]. This cascade
₁₅₈₅ continues until the pion is absorbed or exits the nucleus.

5.1.3 Detector

Once the final state particle kinematics have been determined by NEUT, they are passed into the detector simulation. The near detectors, ND280 and INGRID, are simulated using a GEANT4 package [44, 168] to simulate the detector geometry, particle tracking, and energy deposition. The response of the detectors is simulated using the elecSim package [44].

The far detector simulation is based upon the original Kamiokande experiment software which uses the GEANT3-based SKDETSIM [44, 169] package. This simulates the interactions of particles in the water as well as Cherenkov light production. The water quality and PMT calibration measurements detailed in subsection 3.1.2 are also used within this simulation to make accurate predictions of the detector response.

Any event which generates optical photons that occurs in SK will be observed by the PMT array, where each PMT records the time and accumulated charge. This recorded information is shown in event displays similar to those illustrated in Figure 5.3 for simulated Monte Carlo events. To be useful for physics analyses, this series of PMT hit information needs to be reconstructed to determine the number and identity of particles and their kinematics (or track parameters): four-vertex, direction, and momentum. The reconstruction uses the fact that the charge and timing distribution of photons generated by a particular particle in an event is dependent upon its initial kinematics. Electron and muon rings are distinguished by their “fuzziness”. Muons are heavier and less affected by scattering or showering meaning they typically produce “crisp” rings. Electrons are more likely to interact via electromagnetic showering or scattering which results in larger variations of their direction from the initial direction. Consequently, electrons typically produce “fuzzier” rings compared to muons.

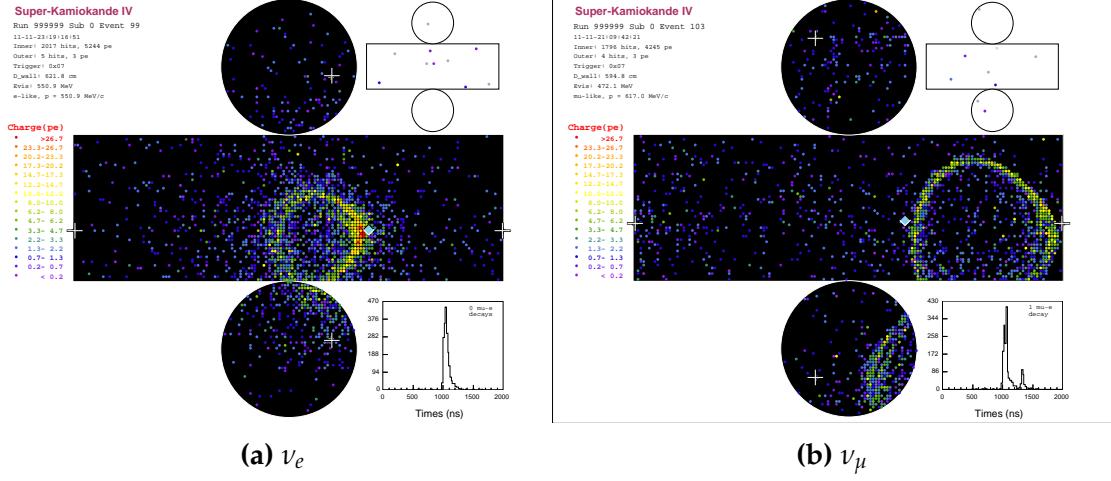


Figure 5.3: Event displays from Monte Carlo simulation at Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [170].

5.2 Event Reconstruction at SK

For the purposes of this analysis, the `fitQun` reconstruction algorithm [171] is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by maximising a likelihood function (or minimising a log-likelihood function) which includes information from PMTs which were hit and those that were not hit. The `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [172].

The `fitQun` algorithm improves upon the previous `APFit` algorithm [173] which has been used for many previous SK analyses. `APFit` fits the vertex from timing information and then fits the direction of the particle from PMT hits within a 43 deg Cherenkov cone (assuming an ultra-relativistic particle) using a fitting estimator. A Hough transformation is used to find the radius of a ring (related to the momentum through Equation 3.2) as well as the number of rings contained within the event. The analysis presented here uses the `fitQun` algorithm as it improves both the accuracy of the fit parameters and the rejection of neutral

1629 current π^0 events as compared to APFit [174, 175].

1630 Any event in SK can consist of prompt (or primary) and decay (or secondary)
1631 particles. For example, a charged current muon neutrino interaction can gen-
1632 erate two particles that have the potential of generating Cherenkov photons
1633 (assuming the proton is below the Cherenkov threshold): the prompt muon,
1634 and the secondary decay-electron from the muon, approximately $2\mu\text{s}$ later. To
1635 reconstruct all particles within an event, it is divided into time clusters which are
1636 called “subevents”. Subevents after the primary subevent are considered to
1637 be decay electrons.

1638 The main steps of the `fitQun` reconstruction algorithm are:

- 1639 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit
1640 metric based on PMT hit times
- 1641 • **Peak finding:** The initial time of each subevent is determined by clustering
1642 events by time residuals
- 1643 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,
1644 a maximum likelihood technique searches for a single particle generating
1645 light. Electron, muon, charged pion, and proton hypotheses are considered
- 1646 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple
1647 light-producing particles are considered using the same maximum likeli-
1648 hood technique. Electron-like or charged pion-like rings are added until
1649 the likelihood stops improving

1650 To find all the subevents in an event, a vertex goodness metric is calculated
1651 for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right), \quad (5.1)$$

1652 where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n, \quad (5.2)$$

1653 is the residual hit time. It is the difference in time between the PMT hit time
 1654 t^i , of the i^{th} PMT, and the expected time of the PMT hit if the photon was at
 1655 the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in water
 1656 and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1657 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends
 1658 to zero resulting in subevents appearing as spikes in the goodness metric. The
 1659 proposed fit vertex and time are grid-scanned, and the values which maximise
 1660 the goodness metric are selected as the “pre-fit vertex”. Whilst this predicts a
 1661 vertex for use in the clustering algorithm, the final vertex is fit using the higher-
 1662 precision maximum likelihood method described below.

1663 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1664 a function of t to determine the number of subevents. A peak-finding algorithm
 1665 is then used on the goodness metric, requiring the goodness metric to exceed
 1666 some threshold and drop below a reduced threshold before any subsequent
 1667 additional peaks are considered. The thresholds are set such that the rate of
 1668 false peak finding is minimised while still attaining good data to Monte Carlo
 1669 agreement. To improve performance, the pre-fit vertex for each delayed subevent
 1670 is re-calculated after PMT hits from the previous subevent are masked. This
 1671 improves the decay-electron tagging performance. Once all subevents have
 1672 been determined, the time window around each subevent is then defined by the
 1673 earliest and latest time which satisfies $-180 < T_{Res}^i < 800\text{ns}$. The subevents and
 1674 associated time windows are then used as seeds for further reconstruction.

1675 For a given subevent, the `fitQun` algorithm constructs a likelihood based on
 1676 the accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}). \quad (5.3)$$

1677 Where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and
 1678 momenta, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the proba-
 1679 bility of the i^{th} tube to not register a hit given the track parameters and particle
 1680 hypothesis. The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$,
 1681 represents the probability density function of observing charge q_i and time t_i on
 1682 the i^{th} PMT given the specified track parameters and particle hypothesis.

1683 The predicted charge is calculated based on contributions from both the
 1684 direct light and the scattered light. The direct light contribution is determined
 1685 based on the integration of the Cherenkov photon profile along the track. PMT
 1686 angular acceptance, water quality, and calibration measurements discussed in
 1687 subsection 3.1.2 are included to accurately predict the charge probability density
 1688 at each PMT. The scattered and reflected light is calculated in a similar way,
 1689 although it includes a scattering function that depends on the vertex of the
 1690 particle and the position of the PMT. The charge likelihood is calculated by
 1691 comparing the prediction to the observed charge in the PMT which is tuned
 1692 to the PMT simulation.

1693 The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} ,
 1694 and time t of the track as well as the particle hypothesis. The expected time
 1695 for PMT hits is calculated by assuming unscattered photons being emitted from
 1696 the midpoint of the track, S_{mid} ,

$$t_{\text{exp}}^i = t + S_{\text{mid}}/c + |R_{\text{PMT}}^i - \vec{x} - S_{\text{mid}}\vec{d}|/c_n, \quad (5.4)$$

1697 where c is the speed of light in a vacuum. The time likelihood is then expressed
 1698 in terms of the residual difference between the PMT hit time and the expected
 1699 hit time, $t_{\text{Res}}^i = t^i - t_{\text{exp}}^i$. The particle hypothesis and momentum also affect the
 1700 Cherenkov photon distribution. These parameters modify the shape of the time
 1701 likelihood density since in reality not all photons are emitted at the midpoint of
 1702 the track. As with the charge likelihood, the contributions from both the direct
 1703 and scattered light to the time likelihood density are calculated separately, which
 1704 are both calculated from particle gun Monte Carlo studies.

1705 The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined
 1706 as the best-fit parameters. In practice MINUIT [176] is used to minimise the value
 1707 of $-\ln L(\Gamma, \vec{\theta})$. The `fitQun` algorithm considers an electron-like, muon-like, and
 1708 charged pion-like hypothesis for events with a single final state particle, denoted
 1709 “single-ring events”. The particle’s identity is determined by taking the ratio of
 1710 the likelihood of each of the hypotheses. For instance, electrons and muons are
 1711 distinguished by considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison
 1712 to the reconstructed momentum of the electron hypothesis, as illustrated by
 1713 Figure 5.4. The coefficients of the discriminator between electron-like and muon-
 1714 like events are determined from Monte Carlo studies [171]. Similar distributions
 1715 exist for distinguishing electron-like events from π^0 -like events, and muon-like
 1716 events from pion-like events. The cuts are defined as,

$$\begin{aligned} \text{Electron/Muon} &: \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 &: \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} &: \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

1717 as taken from [177], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the
 1718 single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed
 1719 invariant mass of the two photons emitted from π^0 decay. Typically, the distance
 1720 between a particular entry in these two-dimensional distributions and the cut-line
 1721 is termed the PID parameter and is illustrated in Figure 5.5.

1722 The `fitQun` algorithm also considers a π^0 hypothesis. To do this, it performs
 1723 a fit looking for two standard electron-hypothesis tracks which point to the
 1724 same four-vertex. This assumes the electron tracks are generated from photon-
 1725 conversion so the electron tracks actually appear offset from the proposed π^0
 1726 vertex. For these fits, the conversion length, direction, and momentum of each
 1727 photon are also considered as track parameters which are then fit in the same
 1728 methodology as the standard single-ring hypotheses.

1729 Whilst lower energy events are predominantly single-ring events, higher
 1730 energy neutrino events can generate final states with multiple particles which

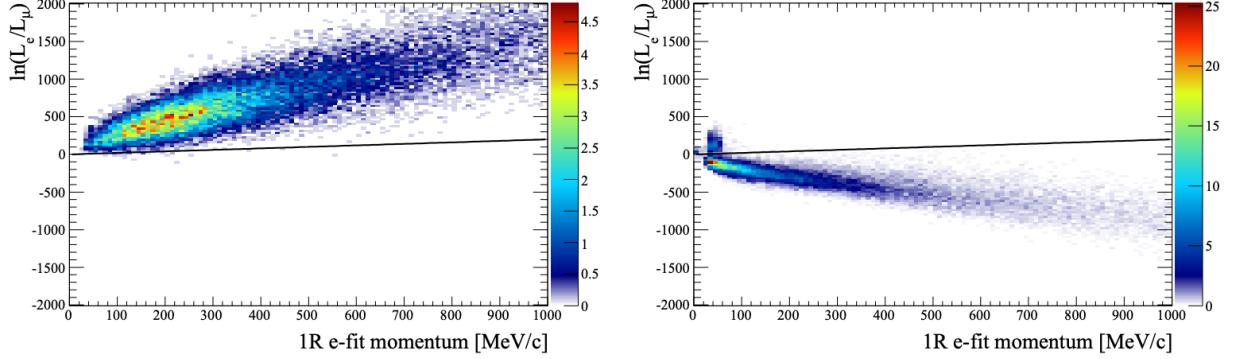


Figure 5.4: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, with coefficients obtained from Monte Carlo studies. Figures from [171].

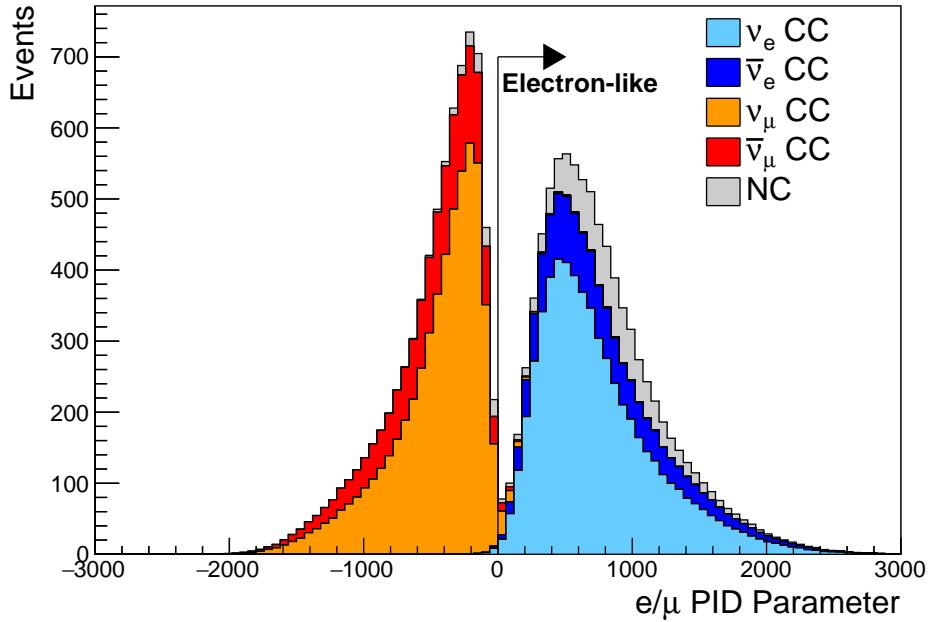


Figure 5.5: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1731 generate Cherenkov photons. These “multi-ring” hypotheses are also considered
 1732 in the `fitQun` algorithm. When calculating the charge likelihood density, the
 1733 predicted charge associated with each ring is calculated separately and then
 1734 summed to calculate the total accumulated charge on each PMT. Similarly, the
 1735 time likelihood for the multi-ring hypothesis is calculated assuming each ring

is independent. Each track is time-ordered based on the time of flight from the center of the track to the PMT and the direct light from any ring incident on the PMT is assumed to arrive before any scattered light. To reduce computational resource usage, the multi-ring fits only consider electron-like and charged pion-like rings as the pion fit can be used as a proxy for a muon fit due to their similar mass. Due to the pions ability to interact through the strong force, they are more likely to hard-scatter. That means a single charged pion can produce multiple rings in different directions. There is an additional freedom, the fraction of kinetic energy lost in a single ring segment, which is added into the `fitQun` pion fit to cover this difference. Pion and muon rings are indistinguishable when this fraction tends to unity.

Multi-ring fits proceed by proposing another ring to the previous fit and then fitting the parameters in the method described above. Typically, multi-ring fits have the largest likelihood because of the additional degrees of freedom introduced. A likelihood value is calculated for the n -ring and $(n + 1)$ -ring hypotheses, where the additional ring is only included if the likelihood value is above 9.35, based on Monte Carlo studies in [178].

5.2.1 Validation of Reconstruction in SK-V

Understanding how the modelling of the detector conditions and stability effects the reconstruction is critical for ensuring accurate measurements. It is important to note that the detector systematics used in the 2020 T2K-only [2] oscillation analysis are determined using data-to-Monte Carlo comparisons of the SK-IV data [179]. Due to tank-open maintenance occurring between SK-IV and SK-V, the dark rate of each PMT was observed to increase in SK-V due to light exposure for a significant time during the repairs. This increase can be seen in Figure 5.6. Run-10 of the T2K experiment was conducted in the SK-V period, so the consistency of SK-IV and SK-V data needs to be studied to determine whether the SK-IV-defined systematics can be applied to the run-10 data. Consequently, the author of this thesis assessed the quality of `fitQun` event reconstruction for SK-V data.

This comparison study was performed using the stopping muon data set for both the SK-IV and SK-V periods. This data sample is used due to the high rate of interactions ($O(200)$ events per hour) as well as having similar energies to muons from CCQE ν_μ interactions from beam interactions. The rate of cosmic muons does depend on the solar activity cycle [180] but has been neglected in this comparison study. This is because the shape of the distributions is most important for the purposes of being compared to the detector systematics. The SK-IV and SK-V data samples consist of 2398.42 and 626.719 hours of data which equates to 686k and 192k events respectively. These samples do not correspond to the full data sets of either period but do contain enough events to be systematics limited rather than statistics limited.

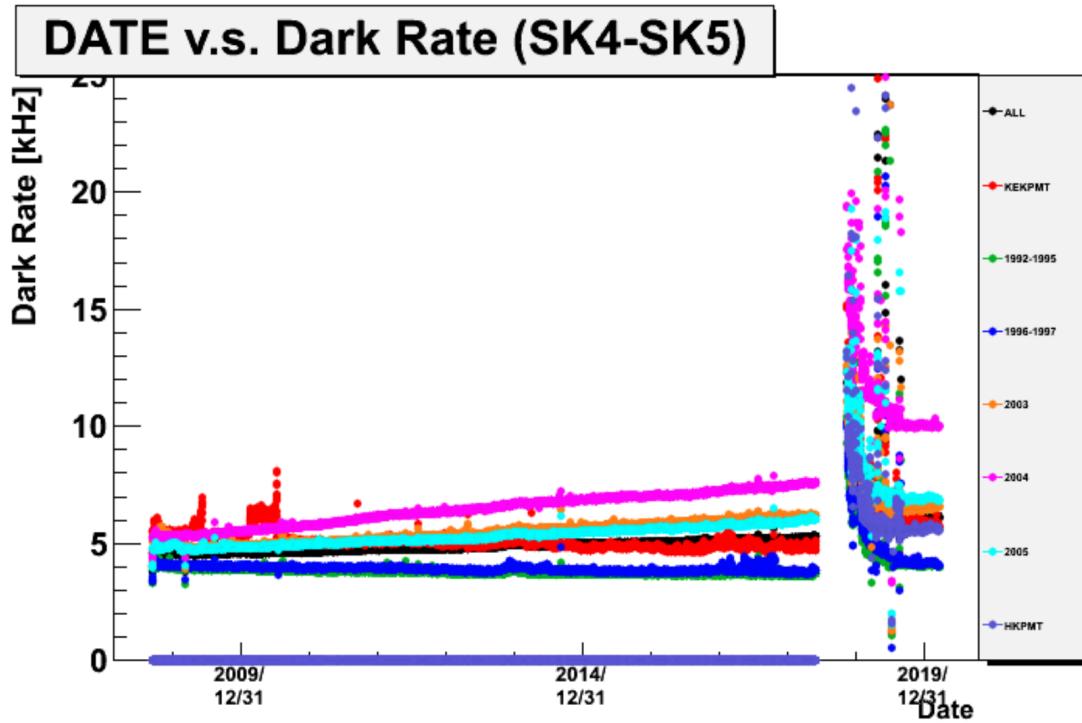


Figure 5.6: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [179].

The predicted charge calculated in the `fitQun` algorithm includes a contribution from the photoelectron emission due to dark noise. Therefore, the increase

in the SK-V dark rate needs to be accounted for. In practice, the average dark rate in each SK period is calculated and used as an input in the reconstruction. This is calculated by averaging the dark rate per run for each period separately, using the calibration measurements detailed in subsection 3.1.2. The average dark rate from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The charges associated with the muon and decay electron subevents are illustrated in Figure 5.7. The photoelectron emission from dark noise is more significant for events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in lower-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent is mostly unaffected by the increase in dark rate, whilst the charge associated with the decay-electron is clearly affected.

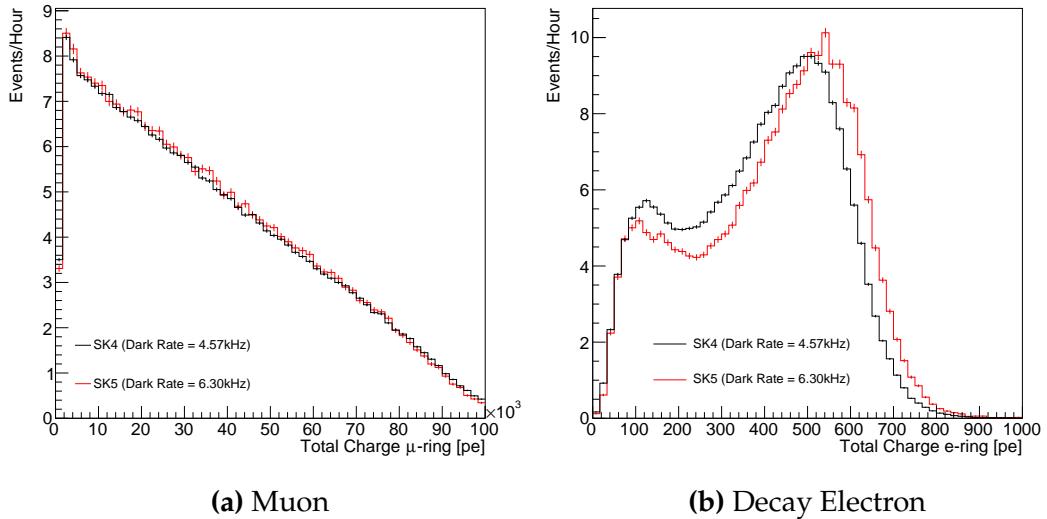


Figure 5.7: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent (left) and the associated decay electron subevent (right).

The energy scale systematic is estimated from data-to-Monte Carlo differences in the stopping muon sample in [61] and found to be 2.1%. To determine the consistency of SK-IV and SK-V with respect to the energy scale systematic, the muon momentum distribution is compared between the two SK periods. As the total number of Cherenkov photons is integrated across the track length,

₁₇₉₅ the reconstructed momentum divided by track length (or range) is compared
₁₇₉₆ between SK-IV and SK-V as illustrated in Figure 5.8.

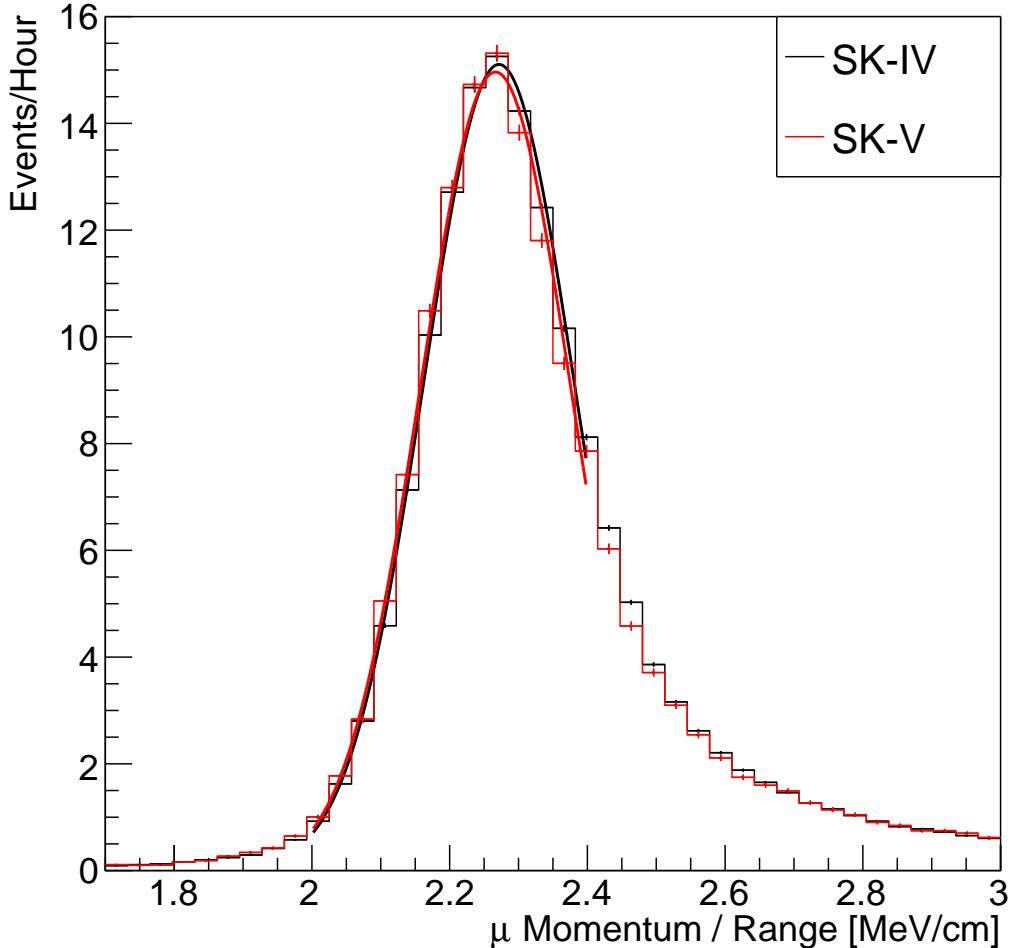


Figure 5.8: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range $[2.0, 2.4]\text{MeV}/\text{cm}$ and illustrated as the solid curve.

₁₇₉₇ The consistency between these muon distributions has been computed in two
₁₇₉₈ ways. Firstly, a Gaussian is fit to the peak of each distribution separately, whose
₁₇₉₉ mean is found to be $(2.272 \pm 0.003)\text{MeV}/\text{cm}$ and $(2.267 \pm 0.006)\text{MeV}/\text{cm}$ for SK-
₁₈₀₀ IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of
₁₈₀₁ the Gaussian fits are consistent with the expected stopping power of a minimum

ionising muon for a target material (water) with $Z/A \sim 0.5$ [181]. The second consistency check is performed by introducing a nuisance parameter, α , which modifies the SK-V distribution. The value of α which minimises the χ^2 value between the SK-IV and SK-V is determined by scanning across a range of values. This is repeated by applying the nuisance parameter as both a multiplicative factor and an additive shift. The χ^2 distributions for different values of α is illustrated in Figure 5.9. The values which minimise the χ^2 are found to be 0.0052 and 1.0024 for the additive and multiplicative implementations, respectively. No evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic has been found in the reconstructed momentum distribution of SK-IV and SK-V.

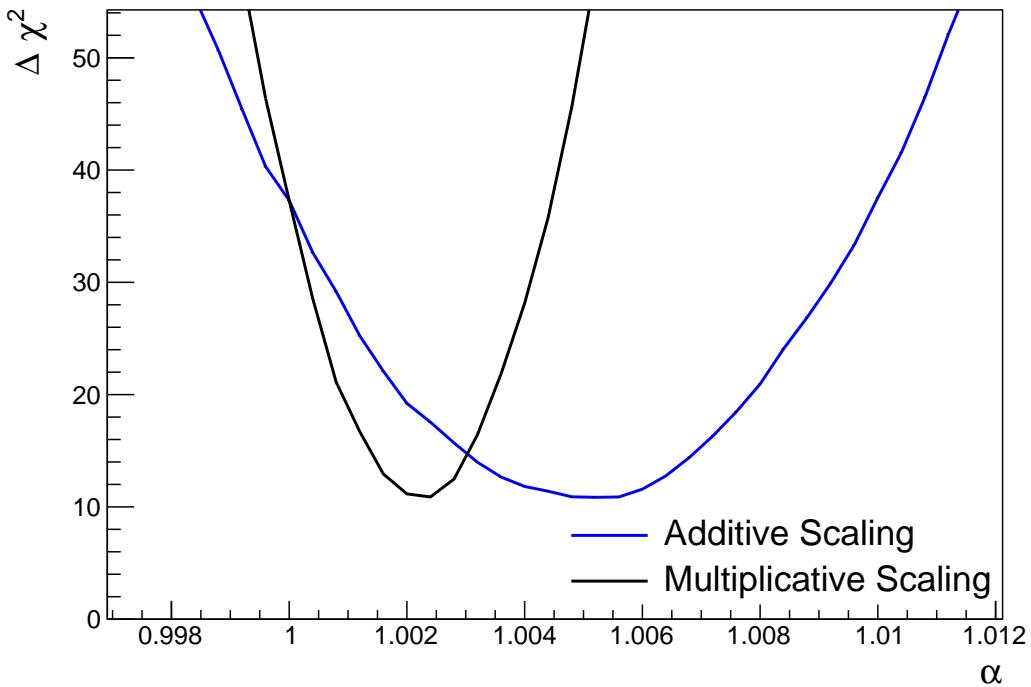


Figure 5.9: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1812 5.3 Event Reduction at SK

1813 In normal data-taking operations, the SK detector observes many background
 1814 events alongside the beam and atmospheric neutrino signal events of physics
 1815 interest for this thesis. Cosmic ray muons and flasher events, which are the spon-
 1816 taneous discharge of a given PMT, contribute the largest amount of background
 1817 events in the energy range relevant to this thesis. Therefore the data recorded
 1818 is reduced with the aim of removing these background events. The reduction
 1819 process is detailed in [3, 56] and briefly summarised below.

1820 Atmospheric neutrino events observed in the SK detector are categorised
 1821 into three different types of samples: fully contained (FC), partially contained
 1822 (PC) and up-going muon (Up- μ), using PMT hit signatures in the inner and
 1823 outer detector (ID and OD, respectively). To identify FC neutrino events, it is
 1824 required that the neutrino interacts inside the fiducial volume of the ID and that
 1825 no significant OD activity is observed. For this analysis, an event is defined to be
 1826 in the fiducial volume provided the event vertex is at least 0.5m away from the
 1827 ID walls. PC events have the same ID requirements but can have a larger signal
 1828 present inside the OD. Typically, only high energy muons from ν_μ interactions can
 1829 penetrate the ID wall. The Up- μ sample contains events where muons are created
 1830 from neutrino interactions in the OD water or rock below the tank. They then
 1831 propagate upwards through the detector. Downward-going muons generated
 1832 from neutrino interactions above the tank are neglected because of the difficulty
 1833 in separating their signature from the cosmic muon shower background. The
 1834 sample categories are visually depicted in Figure 5.10.

1835 Based on the event characteristics, as defined by the `fitQun` event reconstruc-
 1836 tion software, the FC events are categorised by

- 1837 • **Visible Energy:** equal to the sum of the reconstructed kinetic energy of
 1838 particles above the Cerenkov threshold for all rings present in the event.
 1839 The purpose is to separate events into sub-GeV and multi-GeV categories.

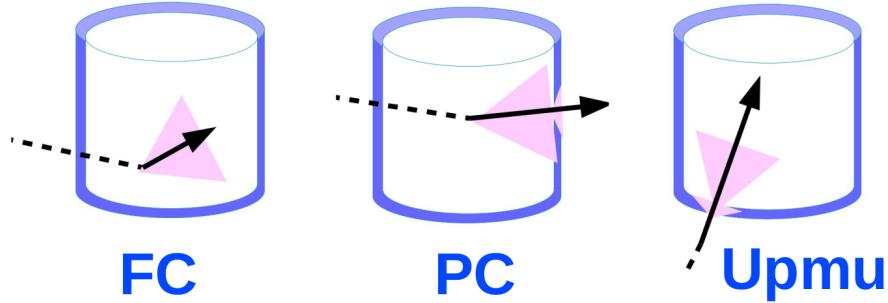


Figure 5.10: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC), and up-going muon ($\text{Up-}\mu$) samples included in this analysis.

1840 • **Number of observed Cerenkov rings.** The purpose is to separate single-
 1841 ring and multi-ring events, where single-ring events predominantly consist
 1842 of quasi-elastic interactions and multi-ring events are typically resonant
 1843 pion production or deep inelastic scattering events.

1844 • **Particle identification parameter of the most energetic ring:** A value de-
 1845 termined from the maximum likelihood value based on `fitQun`'s electron,
 1846 muon, or pion hypothesis. The purpose is to separate electron-like and
 1847 muon-like events.

1848 • **Number of decay electrons:** The purpose is to separate quasi-elastic events
 1849 (which have one decay electron emitted from the muon decay) and resonant
 1850 pion production events (which have two decay electrons emitted from the
 1851 muon and pion).

1852 The PC and Up- μ categories are broken down into “through-going” and
 1853 “stopping” samples depending on whether the muon leaves the detector. This is
 1854 because the PC stopping events deposit the entire energy of the interaction into
 1855 the detector, resulting in better reconstruction. The energy of events that exit the
 1856 detector has to be estimated, with a typically worse resolution, which introduces
 1857 much larger systematic uncertainties. Through-going Up- μ samples are further
 1858 broken down by whether any hadronic showering was observed in the event
 1859 which typically indicates DIS interactions. The expected neutrino energy for the
 1860 different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events

¹⁸⁶¹ peak around 0.7GeV and 3GeV respectively, with slightly different peak energies
¹⁸⁶² for ν_e and ν_μ oscillation channels. PC and Up- μ are almost entirely comprised
¹⁸⁶³ of ν_μ events and peak around 7GeV and 100GeV, respectively.

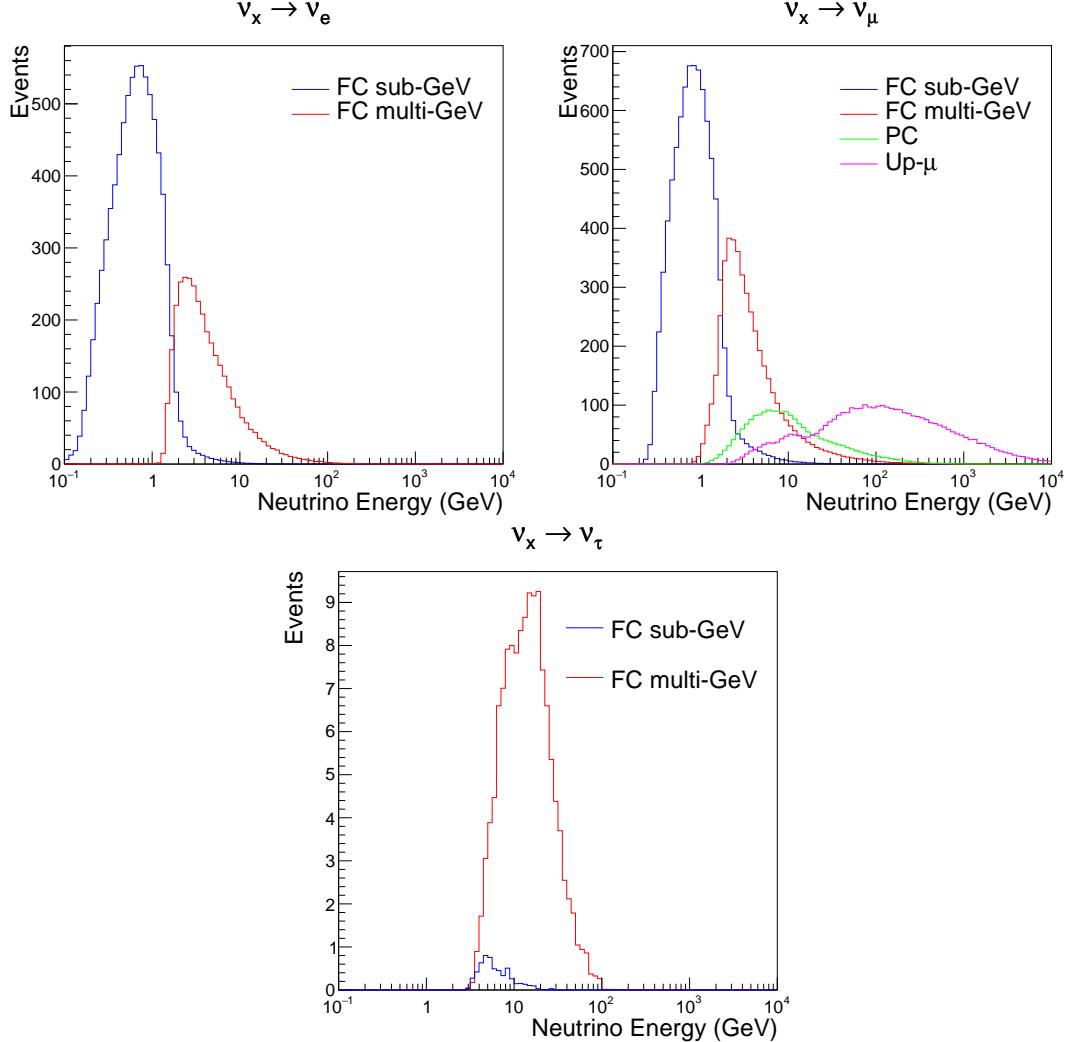


Figure 5.11: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). ν_x represents the flavours of neutrinos produced in the cosmic ray showers (electron and muon). Asimov A oscillation parameters are assumed (given in Table 2.2).

¹⁸⁶⁴ The first two steps in the FC reconstruction remove the majority of cosmic
¹⁸⁶⁵ ray muons by requiring a significant amount of ID activity compared to that
¹⁸⁶⁶ measured in the OD. Events that pass this cut are typically very high momentum
¹⁸⁶⁷ muons or events that leave very little activity in the OD. Consequently, a third

reduction step is then applied to select cosmic-ray muons that pass the initial reduction step. A purpose-built cosmic muon fitter is used to determine the entrance (or exit) position of the muon and a cut is applied to OD activity contained within 8m of this position. Flasher events are removed in the fourth reduction step which is based on the close proximity of PMT hits surrounding the PMT producing the flash. Events that pass all these reduction steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses information from the more precise fitter to repeat the previous two steps with tighter cuts. Muons below the Cherenkov threshold can not generate optical photons in the ID but the associated decay electron can due to its lower mass. These are the types of events targeted in the fifth reduction step. The final cuts require the event vertex to be within the fiducial volume (0.5m from the wall although the nominal distance is 2.0m), visible energy $E_{vis} > 30\text{MeV}$ and fewer than 16 hits within the higher energy OD cluster. The culmination of the fully contained reduction results in 8.09 events/day in the nominal fiducial volume [83]. The uncertainty in the reconstruction is calculated by comparing Monte Carlo prediction to data. The largest discrepancy is found to be 1.3% in the fourth reduction step.

The PC and Up- μ events are processed through their own reduction processes detailed in [56]. Both of these samples are reconstructed with the APFit algorithm rather than `f1TQun`. This is because the efficiency of reconstructing events that leave the detector has not been sufficiently studied for reliable systematic uncertainties with `f1TQun`. The PC and Up- μ samples acquire events at approximately 0.66 and 1.44 events/day.

Beam neutrinos events undergo the same reduction steps as FC events and are then subject to further cuts [182]. The GPS system that links the timing between the beam facility and SK needs to be operating correctly and there should be no activity within the detector in the previous $100\mu\text{s}$ before the trigger. The events then need to triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1897 The beam neutrino samples are not split by visible energy since their energy
 1898 range is smaller than the atmospheric neutrino events. Following the T2K
 1899 analysis in [2], only single-ring beam neutrino events are considered. Similar to
 1900 atmospheric event selection, the number of decay electrons is used as a proxy for
 1901 distinguishing CCQE and CCRES events. The expected neutrino energy, broken
 1902 down by the number of decay electrons, is given in Figure 5.12.

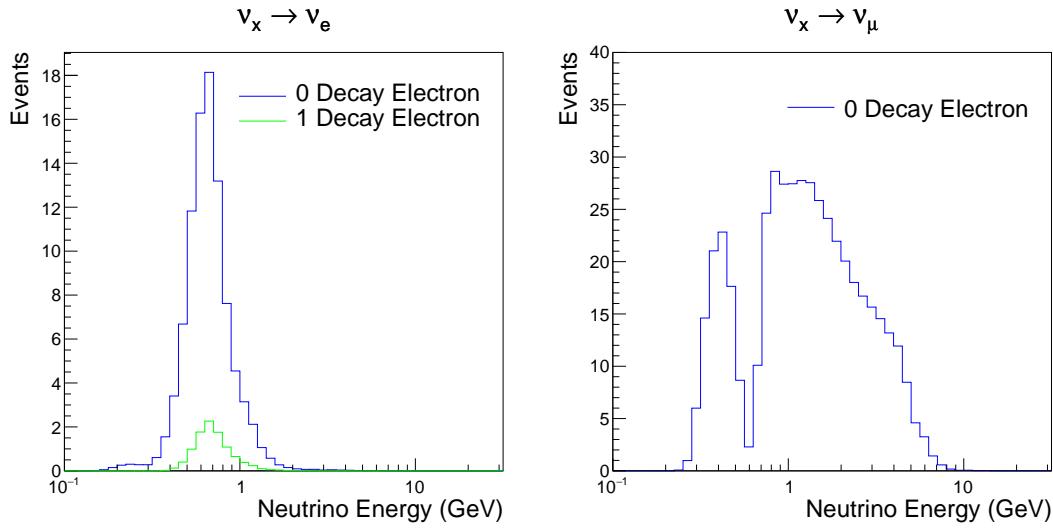


Figure 5.12: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

6

1903

1904

Sample Selections and Systematics

1905 The oscillation analysis presented within this thesis is built upon a simultaneous
1906 fit to atmospheric samples at SK, neutrino beam samples in the near detector,
1907 and beam samples at SK. This is the first simultaneous oscillation analysis of
1908 beam and atmospheric samples supported by the T2K and SK collaborations.
1909 Notably, the author of this thesis has been responsible for the building and
1910 developing the MaCh3 framework to support all sets of samples simultaneously.
1911 The definitions of the samples are documented in section 6.1, section 6.2, and
1912 section 6.3, respectively. The data collected and used within this analysis is
1913 detailed in Table 6.1. The near and far detector data corresponds to T2K runs
1914 2-9 and runs 1-10, respectively. The accumulated POT and beam power for runs
1915 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

1916 The difference in POT recorded at the near and far detector is due to the
 1917 difference in downtime. The SK detector is very stable with almost 100% of
 1918 data recorded during beam operation. Due to various technical and operational
 1919 issues, the downtime of the near detector is significantly higher due to its more
 1920 complex design and operating requirements.

1921 The systematic parameters invoked within the flux, detector, and interaction
 1922 models used within this analysis are documented in section 6.4. The standard
 1923 configuration of the joint beam and atmospheric data fit utilises far detector sys-
 1924 tematics provided in the official inputs from the two experiments. Additionally,
 1925 a correlated detector model which fits the parameters used in sample selections
 1926 to data has been developed and documented in subsection 6.4.5.

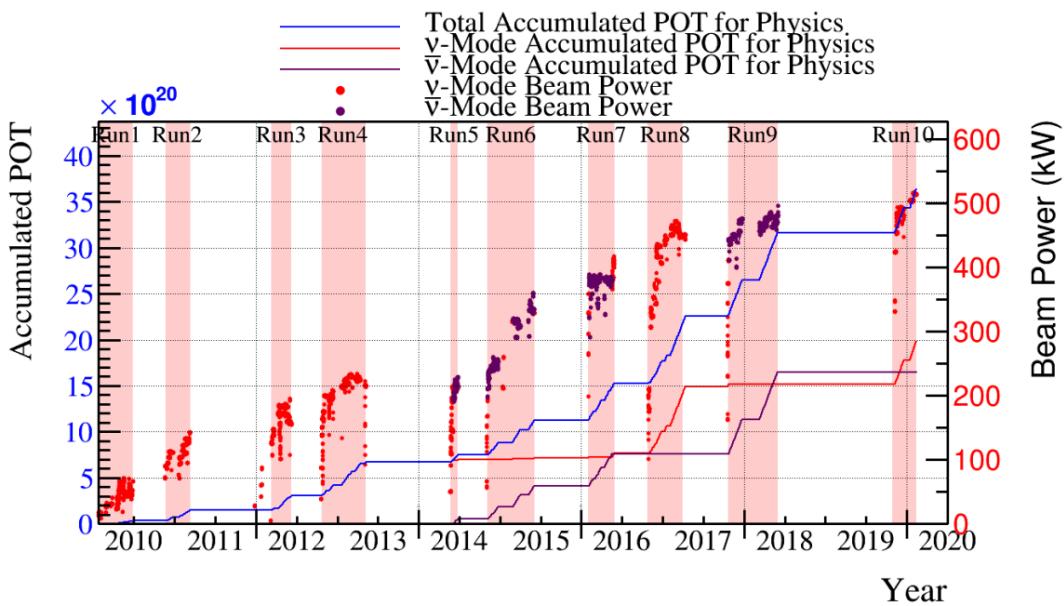


Figure 6.1: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

1927 6.1 Atmospheric Samples

1928 The atmospheric event selection follows the official SK-IV analysis presented
1929 in [3] and is documented below. The Monte Carlo prediction used within this
1930 analysis corresponds to 500 years worth of neutrino events, which is scaled down
1931 to match the SK-IV livetime of 3244.4 days.

1932 The fully contained (FC), partially contained (PC), and upward going muon
1933 events ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further
1934 broken down into different samples based on reconstruction information. This
1935 section details the samples used within this oscillation analysis, alongside the
1936 chosen binning.

1937 FC events are first separated by the visible energy deposited within the
1938 detector. This is calculated as the sum of the reconstructed kinetic energy
1939 above the Cherenkov threshold for all rings present in the event. Events are
1940 separated by whether they were above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates
1941 “subGeV” and “multiGeV” events. Typically, lower energy events consist of
1942 charged current quasi-elastic (CCQE) interactions which are better understood
1943 and simpler to reconstruct resulting in smaller systematic uncertainties. Events
1944 are further separated by the number of rings associated with the event due to
1945 similar reasoning. As the oscillation probability is dependant upon the flavour
1946 of neutrino, electron and muon events are separated using a similar likelihood
1947 method to that discussed in section 5.2. To reduce computational resources
1948 required for the reconstruction, only electron and pion hypotheses are considered
1949 so this separation cut depends on the ratio of the electron to pion likelihoods,
1950 $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1951 Charged current resonant pion production (CCRES) interactions generate a final-
1952 state pion. This can decay, mostly likely through a muon, into a decay electron.
1953 Therefore any electron-like event with one decay electron or muon-like event
1954 with two decay electrons was most likely produced by a CCRES interaction.
1955 Consequently, the number of decay electrons can be used to distinguish CCQE

¹⁹⁵⁶ and CCRES interaction modes. Ultimately, FC subGeV events are separated
¹⁹⁵⁷ into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

¹⁹⁵⁸ In addition to the cuts discussed above, multiGeV samples also have addi-
¹⁹⁵⁹ tional cuts to separate samples which target neutrino and antineutrino events.
¹⁹⁶⁰ As discussed in section 2.5, the matter resonance only occurs for neutrinos in the
¹⁹⁶¹ normal hierarchy and antineutrinos in the inverted mass hierarchy. Therefore,
¹⁹⁶² having flavour-enriched samples aids in the determination of the mass hierarchy.
¹⁹⁶³ For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

¹⁹⁶⁴ The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be captured by an
¹⁹⁶⁵ oxygen nucleus than the π^+ from ν_e interactions [183]. These pions then decay,
¹⁹⁶⁶ mostly through muons, to electrons. Therefore the number of tagged decay
¹⁹⁶⁷ electrons associated with an event gives an indication of whether the interaction
¹⁹⁶⁸ was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
¹⁹⁶⁹ The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
¹⁹⁷⁰ where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with
¹⁹⁷¹ only 7% antineutrino background, the rest consisting of NC backgrounds.

1972 The number of decay electrons discriminator works reasonably well for single-
 1973 ring events. However, this is not the case for multi-ring events. A multiGeV
 1974 multiring electron-like (MME) likelihood cut was introduced in [184, 185]. This
 1975 is a two-stage likelihood selection cut. Four observables are used in the first
 1976 likelihood cut to distinguish $CC\nu_e$ and $CC\bar{\nu}_e$ events from background:

- 1977 • The number of decay electrons
 1978 • The maximum distance between the vertex of the neutrino and the decay
 1979 electrons
 1980 • The energy deposited by the highest energy ring
 1981 • The particle identification of that highest energy ring

1982 Background events consist of $CC\nu_\mu$ and NC interactions. Typically, the
 1983 majority of the energy in these background events is carried by the hadronic
 1984 system. Additionally, muons tend to travel further than the pions from $CC\nu_e$
 1985 before decaying. Thus, the parameters used within the likelihood cut target these
 1986 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring e -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring e -like events with one or more decay electrons
MultiGeV-mulike	Single ring μ -like events
MultiRing-elike-nue	Two or more ring events with leading energy e -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy e -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy μ -like ring and only requires $E_{vis} > 0.6\text{GeV}$
MultiRing-Other1	Two or more ring events with leading energy e -like ring and failed the MME likelihood cut

Table 6.3: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1987 Neutrino and antineutrino events are then separated by a second likelihood
 1988 method ($\nu/\bar{\nu}$ separation) detailed in [61]. This uses the number of decay electrons,

1989 the number of reconstructed rings, and the event’s transverse momentum. The
1990 last two parameters are used because higher-energy samples tend to have more
1991 pions produced above the Cherenkov threshold which results in more rings
1992 compared to an antineutrino interaction. Furthermore, the angular distribution
1993 also tends to be more forward peaked in antineutrino interactions as compared
1994 to neutrino interactions [3]. These FC multiGeV sample definitions are de-
1995 tailed in Table 6.3.

1996 The PC and up- μ samples are split by the amount of energy deposited within
1997 the outer detector, into “stopping” and “through-going” samples. If an event
1998 leaves the detector, the energy it takes with it has to be estimated which increases
1999 the systematic uncertainty compared to events entirely contained within the
2000 inner detector. This estimation is particularly poor at high energies, thus the
2001 up- μ through-going events are not binned in reconstructed momentum. The
2002 through-going up- μ are further separated by the presence of any electromagnetic
2003 showering in the event, as the assumption of non-showering muon does not give
2004 reliable reconstruction for these types of events [56]. In total, 13 FC, 2 PC, and
2005 3 up- μ atmospheric samples are included within this analysis.

2006 The atmospheric samples are binned in direct observables: reconstructed
2007 lepton momentum and direction, as given by Table 6.5. The distribution of
2008 the reconstructed lepton momentum (for samples that only have one bin in
2009 reconstructed zenith angle) and reconstructed direction for each atmospheric
2010 sample used within this analysis is illustrated in Figure 6.2.

2011 The reconstructed lepton momemtum, illustrated by interaction mode break-
2012 down, of some representative atmospheric samples is given in Figure 6.3. The
2013 equivalent distributions of all atmospheric samples used within this analy-
2014 sis can be found in [186]. The low energy samples tend to be dominated by
2015 the interaction mode they target (CCQE for SubGeV-elike-0dcy and CC1 π for
2016 SubGeV-elike-1dcy samples). The higher energy samples include much more
2017 CCOther interactions, especially at larger reconstructed lepton momentum.

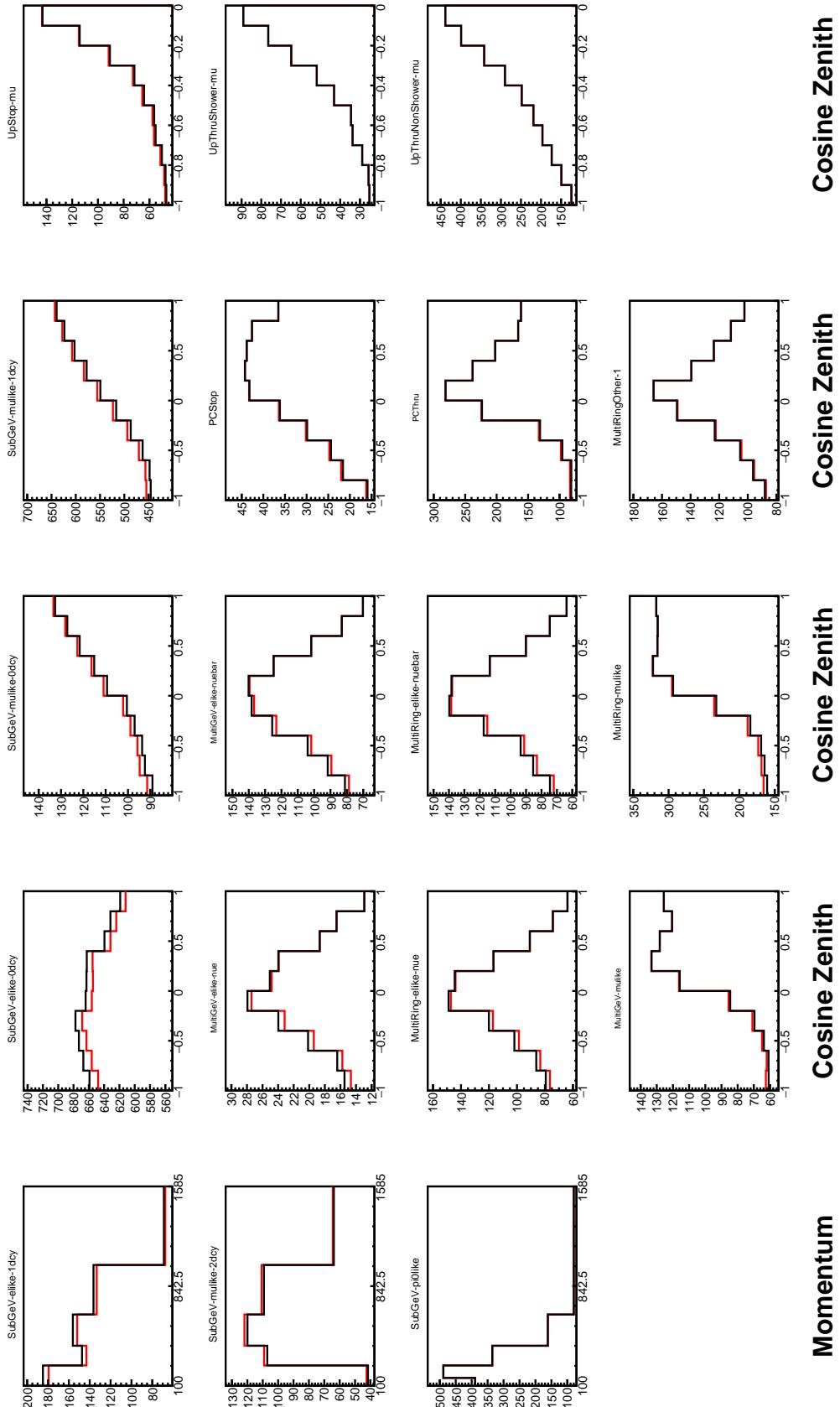


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThrus	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Each row sums to 100% by definition. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV- <i>elike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>elike</i> -1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>pi0like</i>	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>mulike</i>	10	3.0, 3.4, 5.0
MultiRing- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>mulike</i>	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing- <i>Other1</i>	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table 6.5: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples.

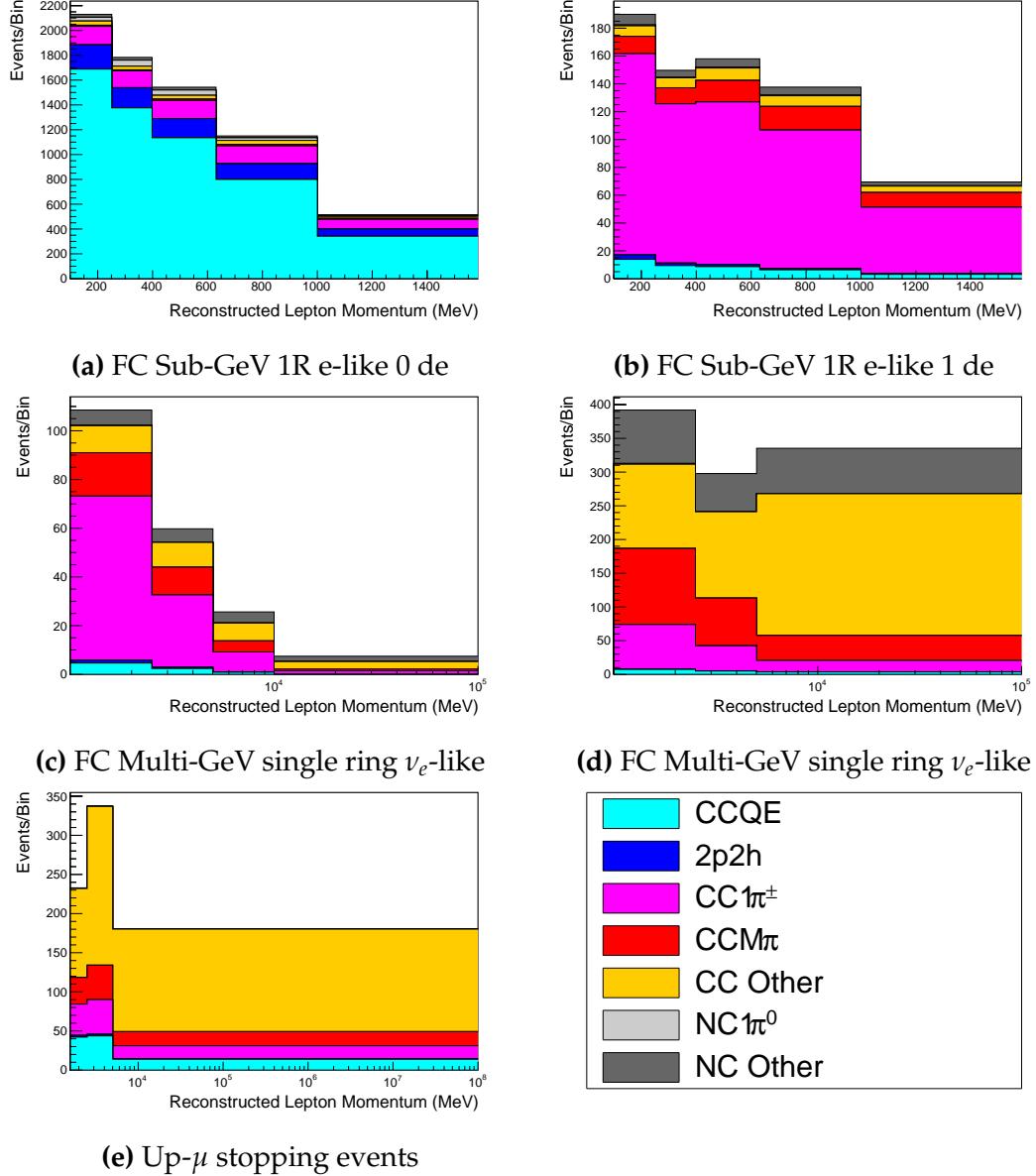


Figure 6.3: Breakdown by interaction mode of some representative atmospheric samples used within this analysis, illustrated as a function of reconstructed lepton momentum. The binning is provided in Table 6.5. Asimov A oscillation parameters are used to generate these plots. The interaction mode breakdown of all atmospheric samples used within this analysis can be found in [186].

2018 6.2 Near Detector Beam Samples

2019 The near detector sample selections are documented in detail within [187] and
2020 summarised below. Samples are selected based upon which of the two Fine
2021 Grained Detector (FGD) the vertex is reconstructed in as well as the operating
2022 mode of the beam: FHC or RHC. Wrong-sign neutrino background samples are
2023 considered in the RHC mode in order to add additional constraints on model
2024 parameters. Samples from the wrong-sign component of the FHC beam mode
2025 are not included as they are statistically insignificant compared to those samples
2026 already listed.

2027 The reconstruction algorithm uses a clustering algorithm to group hits within
2028 the TPC. It then adds information from the upstream FGD to form a track
2029 that passes through both sub-detectors. In FHC(RHC), the highest momentum
2030 negative(positive) curvature track is defined as the muon candidate. Before
2031 being assigned a sample, these candidate muon events must pass CC-inclusive
2032 cuts, as defined in [188]:

- 2033 • Event Timing: The DAQ must be operational and the event must occur
2034 within the expected beam time window consistent with the beam spill
- 2035 • TPC Requirement: The muon-candidate track path must intercept one or
2036 more TPCs
- 2037 • Fiducial volume: The event must originate from within the fiducial volume
2038 defined in [189]
- 2039 • Upstream Background: Remove events that have muon tracks that originate
2040 upstream of the FGDs by requiring no high-momentum tracks within
2041 150mm upstream of the candidate vertex. Additionally, events that occur
2042 within the downstream FGD are vetoed if a secondary track starts within
2043 the upstream FGD

- 2044 • Broken track removal: All candidates where the muon candidate is broken
2045 in two are removed

- 2046 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-
2047 like events, from electron-like or proton-like, using a likelihood cut

2048 In addition to these cuts, RHC neutrino events also have to undergo the
2049 following cuts to aid in the separation of neutrino and antineutrino [190]:

- 2050 • TPC Requirement: The track path must intercept TPC2
2051 • Positive Track: The highest momentum track must have a positive recon-
2052 structed charge
2053 • TPC1 Veto: Remove any events originating upstream of TPC1

2054 Once all CC-inclusive events have been determined, they are further split by
2055 pion multiplicity: CC0 π , CC1 π , and CCOther. Pions in the TPCs are selected by
2056 requiring a second track to be observed, which is separate from the muon track
2057 and is in the same beam spill window and sub-detector. The number of FGD
2058 pions is equal to the number of Michel electrons which were tagged within the
2059 same sub-detector and spill window. If this value is equal to zero, the number
2060 of FGD pions is equivalent to the number of pion-like tracks which have dE/dx
2061 measurements consistent with the pion hypothesis. The pion tracks from both
2062 FGD and TPC events are required to have a vertex consistent with that of the
2063 muon candidate. The Michel electron tagging is preferential as a delayed Michel
2064 is almost always a pion meaning this cut has a higher purity [188, 191], whereas a
2065 track in the FGD that is consistent with a pion could be another particle resulting
2066 in a lower purity. Michel electrons are neglected in the TPC as the pions very
2067 rarely stop due to the low density.

2068 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

- 2069 • ν_μ CC0 π Selection: No electrons in TPC and no charged pions or decay
2070 electrons within the TPC or FGD

- 2071 • ν_μ CC1 π **Selection:** Exactly one charged pion in either the TPC or FGD
- 2072 • ν_μ CCOther **Selection:** All events which are not classified into the above
2073 two selections

2074 Counting the three selections for each FGD in FHC and RHC running, includ-
2075 ing the wrong-sign background in RHC, 18 near detector samples are used within
2076 this analysis. These samples are binned in reconstructed lepton momentum
2077 (illustrated in Figure 6.4) and direction with respect to the beam. The binning
2078 is chosen such that each event has at least 20 Monte Carlo events in each bin
2079 [189]. This is to ensure that the bins are coarse enough to ensure the reduction
2080 of statistical errors, whilst also being fine enough to sample the high-resolution
2081 peak regions. The exact binning is detailed in [189].

2082 6.3 Far Detector Beam Samples

2083 The beam neutrino events which occur at the SK detector, which pass the
2084 reduction cuts detailed in section 5.3, are separated based on whether the beam
2085 was operating in FHC or RHC mode. The events are then separated into three
2086 samples: electron-like (1Re), muon-like (1R μ), and CC1 π^+ -like (1Re1de) which
2087 are observed as electron-like events with an associated decay electron [179].
2088 As discussed in section 6.1, positively charged pions emitted from neutrino
2089 interactions are more likely to produce decay electrons than negatively charged
2090 pions. Consequently, the CC1 π^+ -like sample is only selected when the beam is
2091 operating in FHC mode. Therefore, five beam samples measured at SK are
2092 used in this analysis.

2093 The fiducial volume definition for beam samples is slightly different from that
2094 used for the atmospheric samples. It uses both the distance to the closest wall
2095 (dWall) and the distance to the wall along the trajectory of the particle (toWall).
2096 This allows events that originate close to the wall but are facing into the tank to be
2097 included within the analysis, which would have otherwise been removed. These
2098 additional events are beneficial for a statistics-limited experiment. The exact

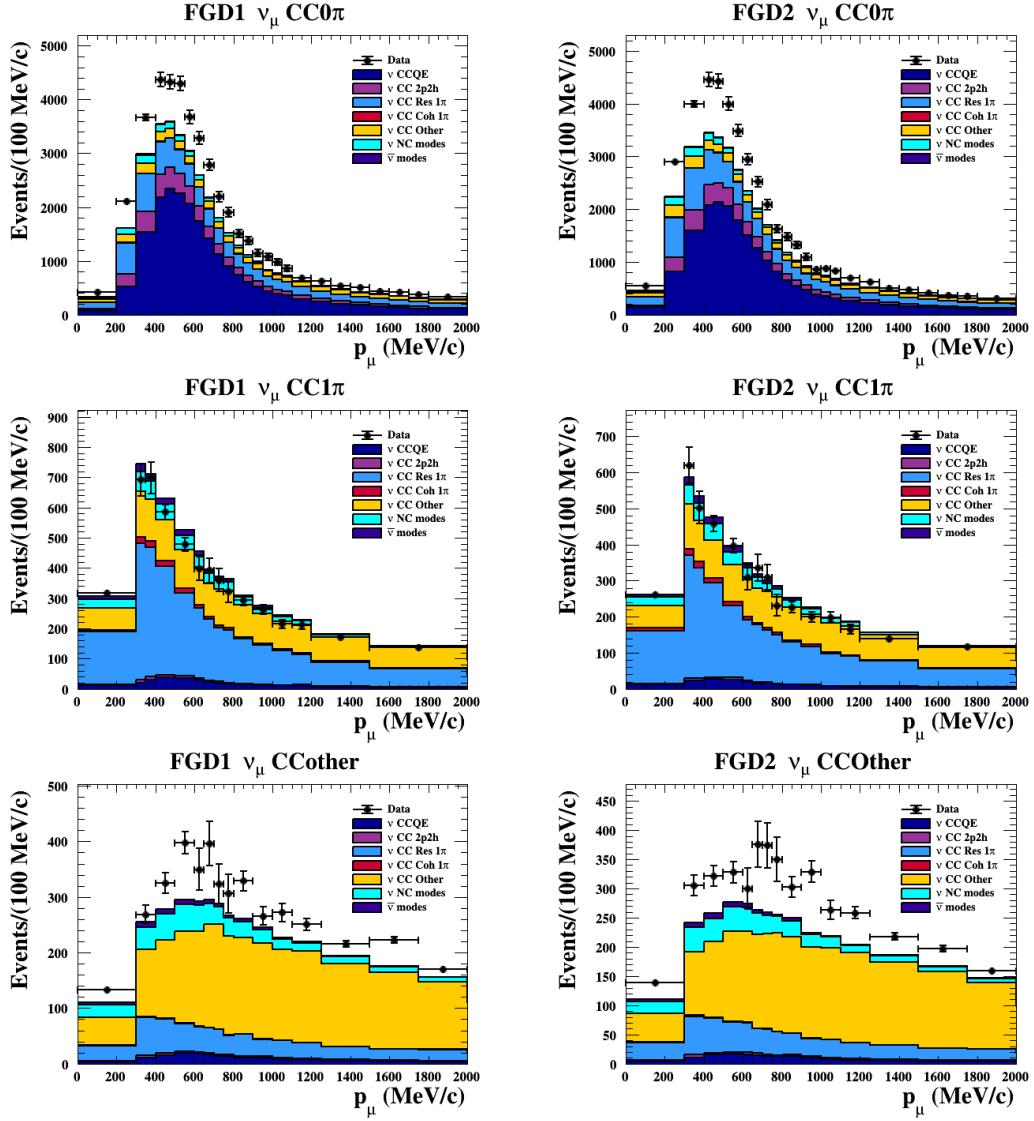


Figure 6.4: The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $CC\nu_\mu 0\pi$, $CC\nu_\mu 1\pi$ and $CC\nu_\mu$ Other categories. Figures taken from [187].

2099 cut values for both `dWall` and `tWall` are different for each of the three types of
 2100 sample and are optimised based on T2K sensitivity to δ_{CP} [177, 192]. They are:

2101 **1Re event selection** For an event to be classified as a 1Re-like, the event must sat-
 2102 isfy:

- 2103
 - Fully-contained and have $dWall > 80\text{cm}$ and $tWall > 170\text{cm}$
 - Total of one ring which is reconstructed as electron-like with reconstructed

2105 momentum $P_e > 100\text{MeV}$

2106 • Zero decay electrons are associated with the event

2107 • Passes π^0 rejection cut discussed in section 5.2

2108 The zero decay electron cut removes non-CCQE interactions and the π^0
 2109 rejection cut is designed to remove neutral current π^0 background events which
 2110 can be easily reconstructed as 1Re-like events.

2111 The zero decay electron cut removes non-CCQE interactions and the π^0
 2112 rejection cut is designed to remove neutral current π^0 background events which
 2113 can be easily reconstructed as 1Re-like events.

2114 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re
 2115 sample. The only differences are that the dWall and toWall criteria are changed
 2116 to $> 50\text{cm}$ and $> 270\text{cm}$, respectively, and exactly one decay electron is required
 2117 from the π^+ decay.

2118 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

2119 • Fully-contained and have $\text{dWall} > 50\text{cm}$ and $\text{toWall} > 250\text{cm}$

2120 • Total of one ring which is reconstructed as muon-like with reconstructed
 2121 momentum $P_\mu > 200\text{MeV}$

2122 • Fewer than two decay electrons are associated with the event

2123 • Passes π^+ rejection cut discussed in section 5.2

2124 All of these samples are binned in reconstructed neutrino energy. This is
 2125 possible under a particular interaction mode assumption, as the direction from
 2126 the source is known extremely well. For the 1Re-like and 1R μ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})}. \quad (6.2)$$

2127 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing
 2128 lepton, respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nucleus
 2129 [179], θ_{beam} is the angle between the beam and the direction of the outgoing
 2130 lepton, and E_l and P_l are the energy and momentum of that outgoing lepton.

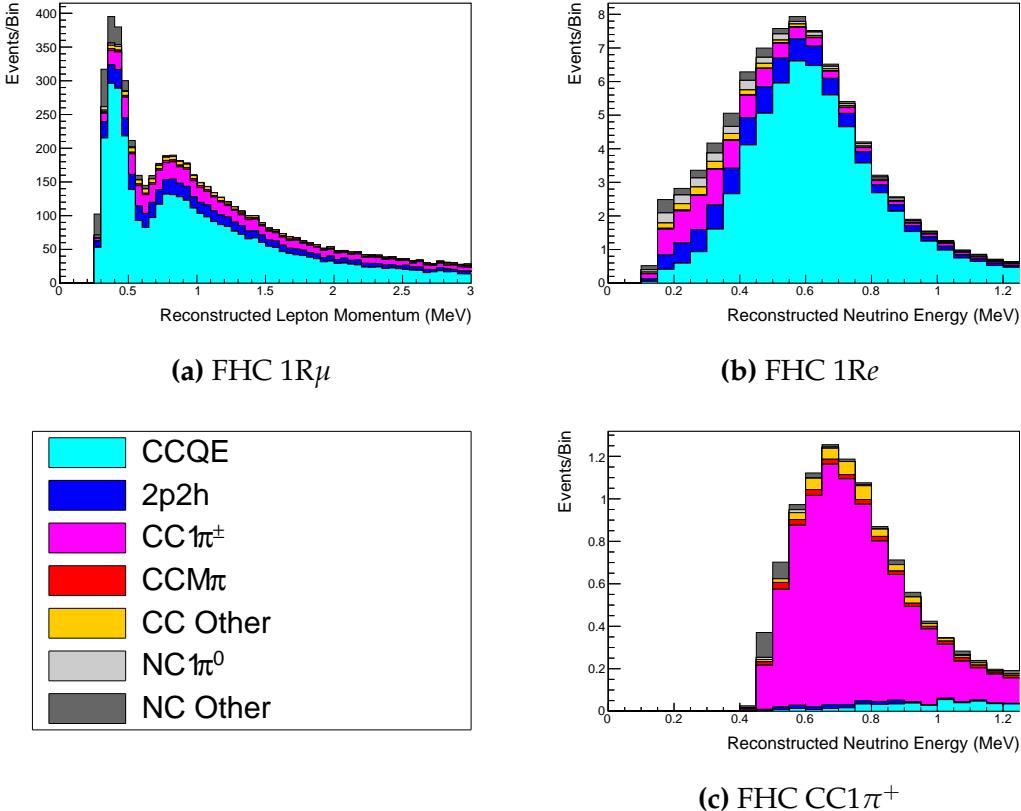


Figure 6.5: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1Re-like, and CC1 π^+ -like samples. The AsimovA oscillation parameters are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0 \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV .

2131 The reconstructed neutrino energy of the CC1 π^+ -like events also accounts
 2132 for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))}. \quad (6.3)$$

2133 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
 2134 considered as a two-body process, with the delta baryon, is assumed. This follows
 2135 the T2K oscillation analysis presented in [2], although recent developments of

2136 the interaction model in the latest T2K oscillation analysis do include effects
2137 from binding energy in this calculation [193].

2138 The reconstructed neutrino energy for the FHC samples is illustrated in
2139 Figure 6.5. As expected, the $1R\mu$ -like and $1Re$ -like samples are heavily dominated
2140 by CCQE interactions, with smaller contributions from $2p2h$ meson exchange and
2141 resonant pion production interactions. The $CC1\pi^+$ -like sample predominantly
2142 consists of charged current resonant pion production interactions. The $1Re$ -like
2143 and $CC1\pi^+$ -like samples are also binned by the angle between the neutrino beam
2144 and the reconstructed lepton momentum. This is to aid in charged current and
2145 neutral current separation, as indicated in Figure 6.6. This is because the neutral
2146 current backgrounds are predominantly due to π^0 -decays, which decay into two
2147 γ rays. The opening angle of which (alongside the different final state kinematics)
2148 can produce a slightly broader angular distribution compared to the final state
2149 particles originating from charged current ν_e interactions.

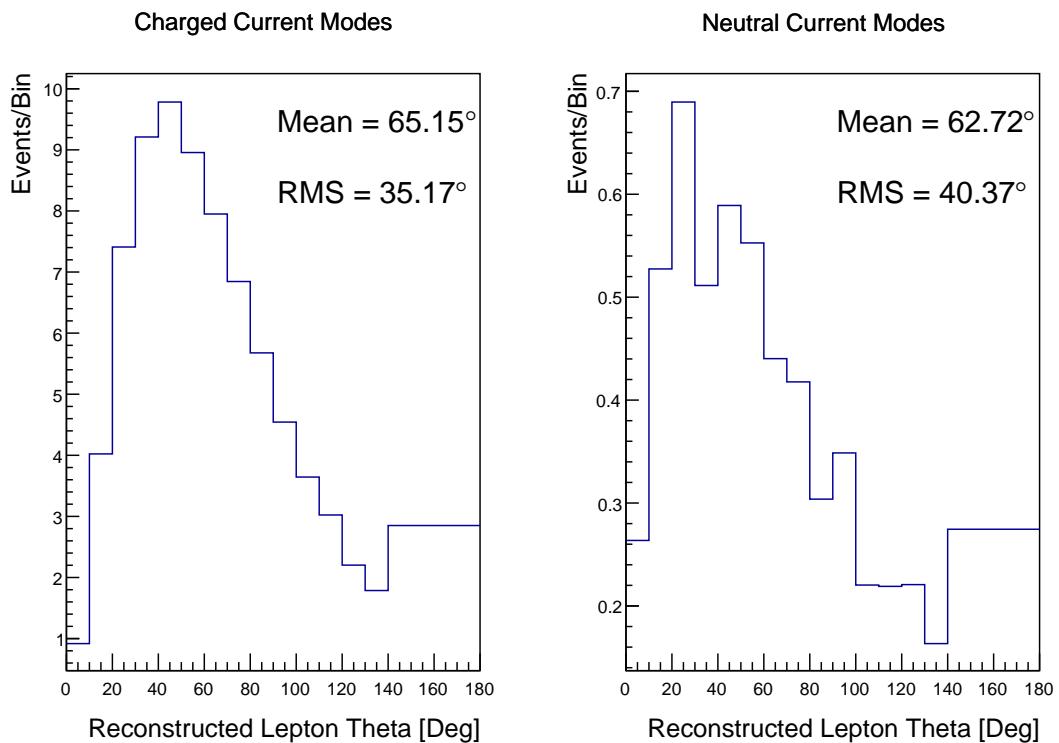


Figure 6.6: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are 35.17° and 40.37° , respectively.

2150 6.4 Systematic Uncertainties

2151 The systematic model parameters for this analysis are split into groups, or blocks,
2152 depending on their purpose. They consist of flux uncertainties, neutrino-matter
2153 interaction systematics, and detector efficiencies. There are also uncertainties on
2154 the oscillation parameters to which this analysis is not sensitive, namely Δm_{21}^2
2155 and $\sin^2(\theta_{12})$. These oscillation parameter uncertainties are taken from the 2020
2156 PDG measurements [75]. As described in chapter 4, each model parameter used
2157 within this analysis requires a prior uncertainty. This is provided via separate
2158 covariance matrices for each block. The covariance matrices can include prior
2159 correlations between parameters within a single block, but the separate treatment
2160 means prior correlations can not be included for parameters in different groups.
2161 Some parameters in these models have no reasonably motivated uncertainties
2162 and are assigned flat priors which do not modify the likelihood penalty. In
2163 practice, these flat prior parameters are actually assigned a Gaussian with a
2164 very large width to ensure the covariance matrix is positive definite. They are
2165 then checked at run time to determine if they contribute to the likelihood. The
2166 flux, neutrino interaction, and detector modeling simulations have already been
2167 discussed in section 5.1 and section 5.2. The uncertainties invoked within each
2168 of these models are described below.

2169 6.4.1 Beam Flux

2170 The neutrino beam flux systematics are based upon the uncertainty in the mod-
2171 eling of the components of the beam simulation. This includes the model of
2172 hadron productions and reinteractions, the shape, intensity, and alignment of
2173 the beam with respect to the target, and the uniformity of the magnetic field
2174 produced by the horn, alongside other effects. The uncertainty, as a function
2175 of neutrino energy, is illustrated in Figure 6.7 which includes a depiction of
2176 the total uncertainty as well as the contribution from individual components.
2177 The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$) is

2178 dominated by uncertainties in the beam profile and alignment. Outside of this
2179 region, uncertainties on hadron production dominate the error.

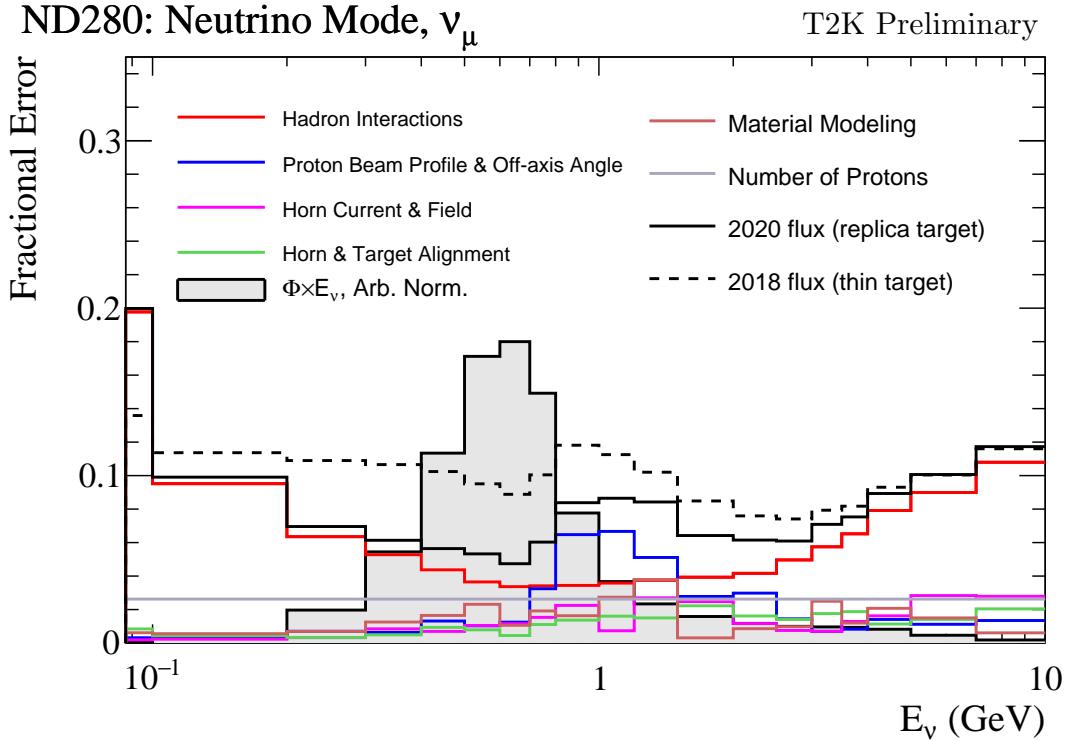


Figure 6.7: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [194]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [195].

2180 The beam flux uncertainties are described by one hundred parameters. They
2181 are split between the ND280 and SK detectors and binned by neutrino flavour:
2182 ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. The response is then broken down as a function of neutrino
2183 energy. The bin density in the neutrino energy is the same for the ν_μ in FHC
2184 and $\bar{\nu}_\mu$ in RHC beams, and narrows for neutrino energies close to the oscillation
2185 maximum of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.6. All of these
2186 systematic uncertainties are applied as normalisation parameters with Gaussian
2187 priors centered at 1.0 and error specified from a covariance matrix provided
2188 by the T2K beam group [195].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.6: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

2189 6.4.2 Atmospheric Flux

2190 The atmospheric neutrino flux is modeled by the HKKM model [52]. 16 systematic
 2191 uncertainties are applied to control the normalisation of each neutrino flavour,
 2192 energy, and direction. They are summarised below:

- 2193 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour
 2194 is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$
 2195 and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction
 2196 uncertainties for the production of pions and kaons [52]. The strength of
 2197 the response is dependent upon the neutrino energy. The uncertainty is
 2198 parameterized following Figure 11 in [52].
- 2199 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$
 2200 are controlled by the difference between the HKKM model [52], FLUKA
 2201 [55] and Bartol models [51]. Three independent parameters are applied in
 2202 the energy ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 2203 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent)
 2204 production uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using
 2205 the same methodology as the relative normalisation parameters.
- 2206 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-
 2207 tematics, the difference between the HKKM, FLUKA, and Bartol model

2208 predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of
 2209 events as a function of zenith angle.

- 2210 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) mostly originate in
 2211 kaon decay. Measurements of the ratio of K/π production [196] are used to
 2212 control the systematic uncertainty of the expected ratio of pion and kaon
 2213 production.
- 2214 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic
 2215 field, the flux of primary cosmic rays varies across the same period. The
 2216 uncertainty is calculated by taking a ± 1 year variation, equating to a 10%
 2217 uncertainty for the SK-IV period.
- 2218 • **Atmospheric Density:** The height of the interaction of the primary cosmic
 2219 rays is dependent upon the atmospheric density. The HKKM assumes the
 2220 US standard 1976 [146] profile. This systematic controls the uncertainty in
 2221 that model.

2222 The total uncertainty is dominated by the absolute and relative normalisation
 2223 parameters. The effect of which is illustrated in Figure 6.8. Generally, the
 2224 uncertainty is large at low energy, reducing to $O(10\%)$ around the peak of the
 2225 flux distribution and then increasing once the neutrino energy exceeds 10GeV.

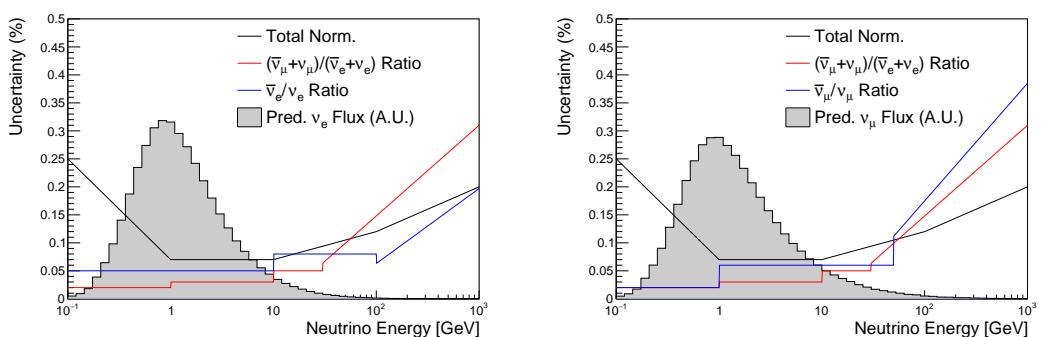


Figure 6.8: The uncertainty evaluated on the atmospheric ν_e (left) and ν_μ (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2226 Updates to the HKKM and Bartol models are underway [151] to use a similar
2227 tuning technique to that used in the beam flux predictions. After those updates,
2228 it may be possible to include correlations in the hadron production uncertainty
2229 systematics for beam and atmospheric flux predictions.

2230 6.4.3 Neutrino Interaction

2231 Neutrino interactions in the detectors are modeled by NEUT. The two indepen-
2232 dent oscillation analyses, T2K-only [197] and the SK-only [61], have developed
2233 separate interaction models. To maximise sensitivity out of this simultaneous
2234 beam and atmospheric analysis, a correlated interaction model has been defined
2235 in [186]. Where applicable, correlations allow the systematic uncertainties applied
2236 to the atmospheric samples to be constrained by near detector neutrino beam
2237 measurements. This can lead to stronger sensitivity to oscillation parameters
2238 as compared to an uncorrelated model.

2239 The low-energy T2K systematic model has a more sophisticated treatment
2240 of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of
2241 this model have been performed to external data [197]. However, the model
2242 is not designed for high-energy atmospheric events, like those illustrated in
2243 Figure 5.11. Therefore the high energy systematic model from the SK-only
2244 analysis is implemented for the relevant multi-GeV, PC, and up- μ samples.
2245 The T2K CCQE model is more sophisticated so it has been implemented for
2246 all samples within this analysis, where separate low-energy and high-energy
2247 dials have been implemented. The low-energy dials are constrained by the near
2248 detector measurements and are uncorrelated to their high-energy counterparts.
2249 The author of this thesis was responsible for implementing and validating the
2250 combined cross-section model as documented in [186, 198].

2251 The high energy systematic model includes parameters developed from
2252 comparisons of Nieves and Rein-Seghal models which affect resonant pion
2253 producing interactions, comparisons of the GRV98 and CKMT models which
2254 control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the ν_τ cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.9 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.10 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

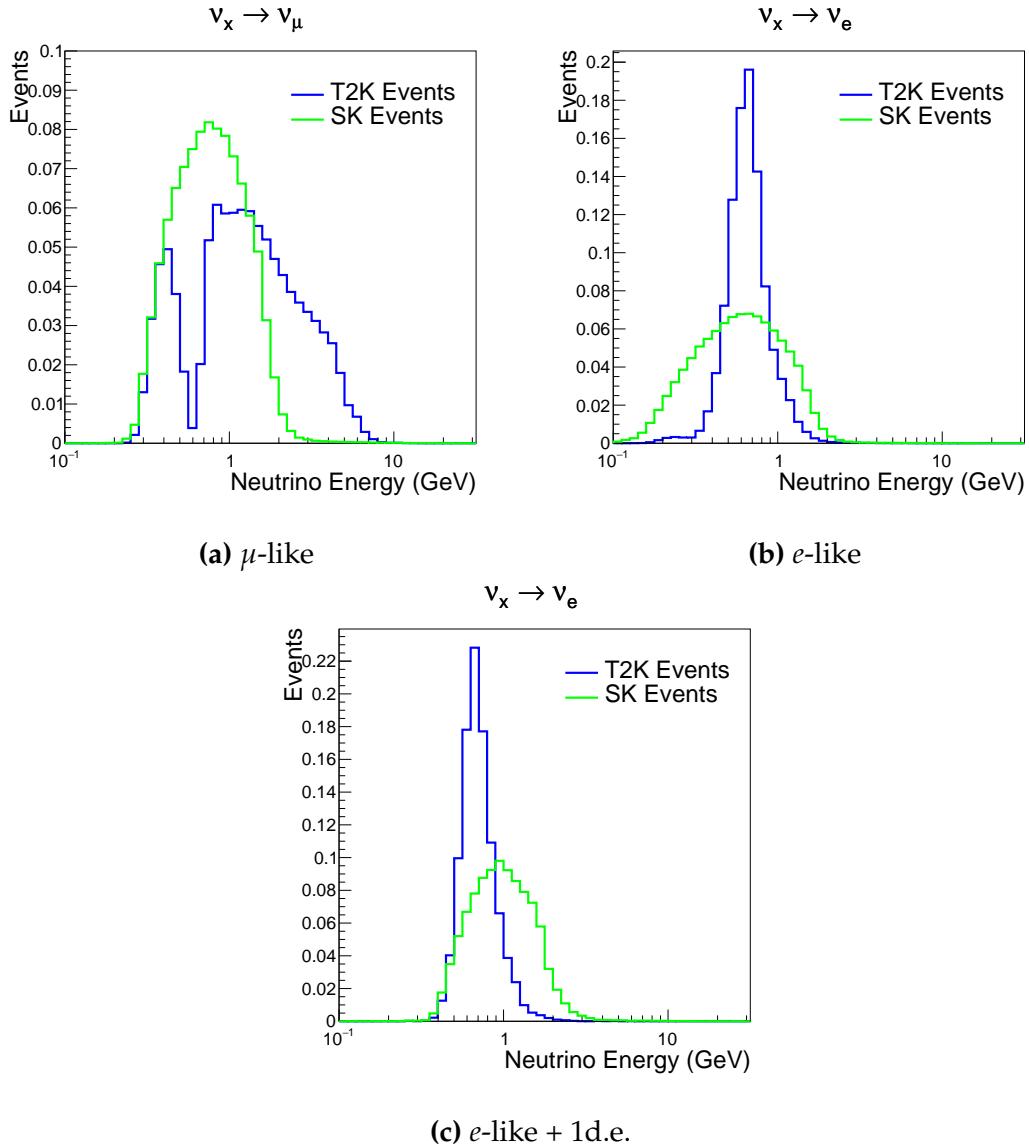


Figure 6.9: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

2285 The T2K systematic model [197] is applied in a similar methodology to the
 2286 SK model parameters. It consists of 19 shape parameters and 24 normalisation
 2287 parameters. Four additional parameters, which model the uncertainty in the
 2288 binding energy, are applied in a way to shift the momentum of the lepton emitted
 2289 from a nucleus. This controls the uncertainty specified on the 27MeV binding
 2290 energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.10: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

2291 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated
 2292 uncertainty, or those which have not been fit to external data, are assigned a
 2293 flat prior which does not affect the penalty term.

2294 On top of the combination of the SK and T2K interaction models, several
 2295 other parameters have been specifically developed for the joint oscillation anal-
 2296 ysis. The majority of the atmospheric samples' δ_{CP} sensitivity comes from the
 2297 normalisation of subGeV electron-like events. These are modeled using a spectral
 2298 function to approximate the nuclear ground state. However, the near detector is
 2299 not able to constrain the model so an additional systematic is introduced which
 2300 models an alternative Continuous Random Phase Approximation (CRPA) nuclear
 2301 ground state. This dial approximates the event weights if a CRPA model had

been assumed rather than a spectral function. This dial only applies to ν_e and $\bar{\nu}_e$ as the near detector does not constraint ν_e cross-section measurements. It is applied as a shape parameter.

Further additions to the model have been introduced due to the inclusion of the subGeV π^0 atmospheric sample. This particularly targets charged current and neutral current π^0 producing interactions to help constrain the systematic uncertainties. Therefore, an uncertainty that affects neutral current resonant π^0 production is incorporated into this analysis. Comparisons of NEUT's NC resonant pion production predictions have been made to MiniBooNE [199] data and a consistent 16% to 21% underprediction is observed [186]. Consequently, a conservative 30% normalisation parameter is invoked.

Down-going events are mostly insensitive to oscillation parameters and can act similar to the near detector within an accelerator experiment (Details will be discussed in chapter 7). This region of phase space can act as a sideband and allows the cross-section model and near detector constraint to be studied. The distribution of events in this region is calculated using the technique outlined in subsection 4.3.4. The results are illustrated in Figure 6.11. For CCQE-targeting samples, the application of the near detector constraint is well within the statistical fluctuation of the down-going data. This means there is no significant tension is observed between the data and the Monte Carlo prediction after the near detector constraint is applied. This is not the case for samples with target CCRES interactions. The electron-like data is consistent with the constrained prediction at high reconstructed momenta but diverges at lower momentum, whereas the muon-like sample is under-predicted throughout the range of momenta. To combat this disagreement, an additional cross-section systematic dial, specifically designed to inflate the low pion momentum systematics was developed in [186]. This is a shape parameter implemented through a splined response.

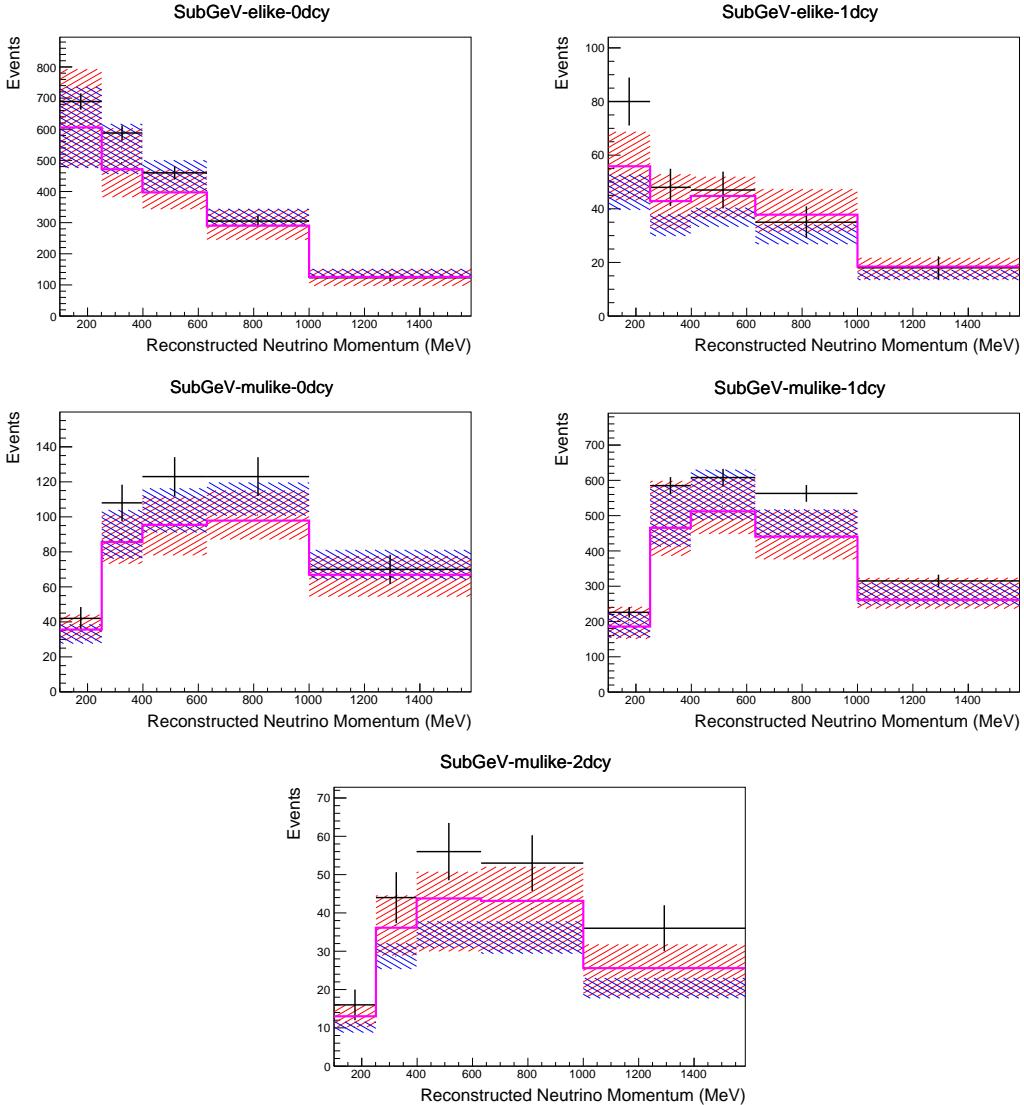


Figure 6.11: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

2329 6.4.4 Near Detector

2330 The systematics applied due to uncertainties arising from the response of the near
 2331 detector is documented in [126]. The response is described by 574 normalisation
 2332 parameters binned in the selected sample as well as momentum and angle,

P_μ and $\cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance matrix with each parameter being assigned a Gaussian prior from that covariance matrix. These normalisation parameters are built from underlying systematics, e.g. pion secondary interaction systematics, which are randomly thrown and the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. Two thousand throws are evaluated and a covariance matrix response is created. This allows significant correlations between FGD1 and FGD2 samples, as well as adjacent $P_\mu \times \cos(\theta_\mu)$ bins. Statistical uncertainties are accounted for by including fluctuations of each event's weight from a Poisson distribution.

Similar to the cross-section systematics, MaCh3 and BANFF are used to constrain the uncertainty of these systematics through independent validations. Each fitter generates a post-fit covariance matrix which is compared and passed to the far-detector oscillation analysis working group. As the analysis presented within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all three sets of samples and their respective systematics is performed.

6.4.5 Far Detector

Two configurations of the far detector systematic model implementation have been considered. Firstly, the far detector systematic uncertainties for beam and atmospheric samples are taken from their respective analysis inputs, denoted “official inputs” analysis, with no correlations assumed between the beam and atmospheric samples. The beam- and atmospheric-specific inputs are documented in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative detector model has been developed which correlates the response of the SK detector systematics between the beam and atmospheric samples. Here, the distribution of parameters used for applying event cuts (e.g. electron-muon PID separation) is modified within the fit. It follows a similar methodology to the beam far detector systematics implementation but performs a joint fit of the beam and atmospheric data. This alternative implementation is detailed in subsubsection 6.4.5.3.

2362 **6.4.5.1 Beam Samples**

2363 There are 45 systematics which describe the response of the far detector to
 2364 beam events [179], split into 44 normalisation parameters and one energy scale
 2365 systematic. The energy scale systematic is applied as a multiplicative scaling
 2366 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo
 2367 differences in the stopping muon sample in [61] and found to be 2.1%. The
 2368 normalisation parameters are assigned a Gaussian error centered at one with
 2369 width taken from a covariance matrix. A detailed breakdown of the generation
 2370 of the covariance matrix is found in [192]. To build the covariance matrix, a fit
 2371 is performed on atmospheric data which has been selected using beam sample
 2372 selection cuts. These cuts use the variables, L^i , where the index i is detailed in
 2373 Table 6.7. Each L^i is a smear, α , and shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i. \quad (6.4)$$

2374 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in
 2375 Table 6.7. The shift and smear parameters are nuisance parameters with no prior
 2376 constraints. They are binned by final-state topology, j , where the binning is given
 2377 in Table 6.8. The final-state topology binning is because the detector will respond
 2378 differently to events that have one or multiple rings. For example, the detector
 2379 will be able to distinguish single-ring events better than two overlapping ring
 2380 events, resulting in different systematic uncertainty for one-ring events compared
 2381 to two-ring events. This approach is used to allow the cut parameter distributions
 2382 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.7: List of cut variables that are included within the shift/smear fit documented in [192].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 μ	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 μ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 π^0	Only one π^0 in the final state
1 π^\pm or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.8: Reconstructed event topology categories on which the SK detector systematics [192] are based.

2383 The mis-modeling of π^0 events is also considered. If one of the two rings
 2384 from a π^0 event is missed, this will be reconstructed as a CC ν_e -like event. This
 2385 is one of the largest systematics hindering the electron neutrino appearance
 2386 analyses. Consequently, additional systematics have been introduced to con-
 2387 strain the mis-modeling of π^0 events in SK, binned by reconstructed neutrino
 2388 energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples is
 2389 constructed. These events are built by overlaying one electron-like ring from
 2390 the SK atmospheric neutrino samples or decay electron ring from a stopping
 2391 cosmic ray muon with one simulated photon ring. Both rings are chosen so
 2392 that momenta and opening angle follow the decay kinematics of NC π^0 events
 2393 from the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the
 2394 SK Monte Carlo are produced to compare with the hybrid- π^0 data samples and
 2395 the difference in the fraction of events that pass the ν_e selection criteria is used
 2396 to assign the systematic errors. In order to investigate any data to Monte Carlo
 2397 differences that may originate from either the higher energy ring or lower energy
 2398 ring, two samples are built; a sample in which the electron constitutes the higher
 2399 energy ring from the π^0 decay (called the primary sample) and another one in
 2400 which it constitutes the lower energy ring (called the secondary sample). The
 2401 standard T2K ν_e fitQun event selection criteria are used to select events.

2402 Final contributions to the covariance matrix are determined by supplemen-
 2403 tary uncertainties obtained by comparing stopping muon data to Monte Carlo

prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data to Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or fewer decay electrons. Similarly, the rate at which fake decay electrons are reconstructed by `fiTQun` is estimated by comparing the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events is also applied. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [179, 200, 201].

Two additional sources of uncertainty are included: secondary and photoneuclear interactions. These are estimated by varying the underlying parameters are building a distribution of sample event rates. These contributions are then added in quadrature to the above covariance matrix. The final uncertainty on the SK detector systematics are provided in Figure 6.12.

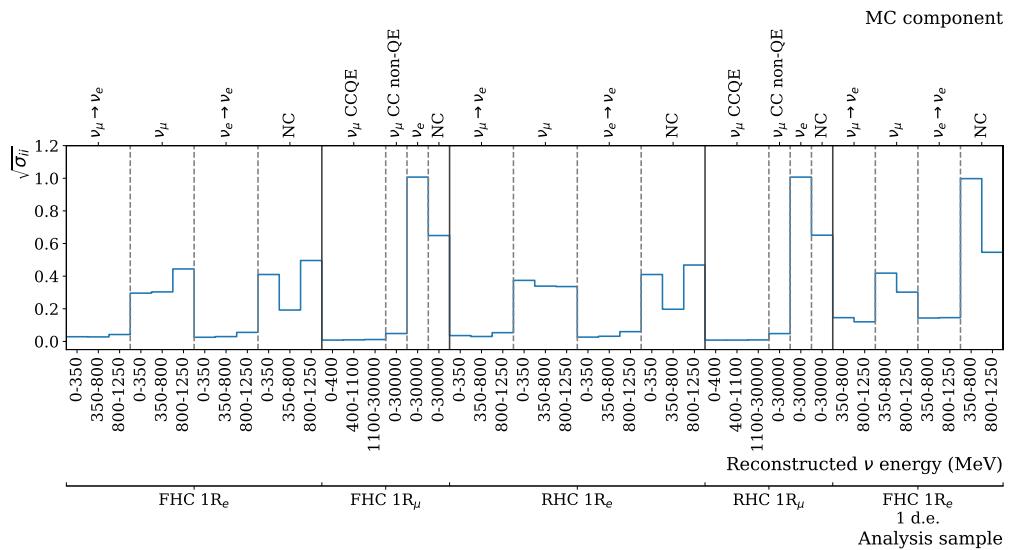


Figure 6.12: The fractional uncertainty on each of the 44 parameters describing the SK detector systematics (The energy scale systematic is neglected). The parameters are split by sample, oscillation channel, interaction mode and reconstructed neutrino energy.

2421 6.4.5.2 Atmospheric Samples

2422 The detector systematics for atmospheric samples, documented in [3], are split
2423 into two sub-groups: those which are related to particle identification and ring
2424 counting systematics, and those which are related to calibration, separation,
2425 and reduction uncertainties.

2426 The particle identification systematics consist of five parameters. The ring sep-
2427 aration systematic enforces an anti-correlated response between the single-ring
2428 and multi-ring samples. This is implemented as a fractional increase/decrease
2429 in the overall normalisation of each sample, depending on the distance to the
2430 nearest wall from an event’s vertex. The coefficients of the normalisation are
2431 estimated prior to the fit and depend on the particular atmospheric sample. Two
2432 electron-muon separation systematics are included within this model which
2433 anti-correlates the response of the electron-like and muon-like samples: one for
2434 single-ring events and another for multi-ring events.

2435 The multi-ring electron-like separation likelihood, discussed in section 6.1,
2436 encodes the ability of the detector to separate neutrino from anti-neutrino events.
2437 Two normalisation parameters vary the relative normalisation of multi-ring ν_e
2438 and $\bar{\nu}_e$ samples whilst keeping a consistent overall event rate.

2439 There are 22 systematics related to calibration measurements, including effects
2440 from backgrounds, reduction, and showering effects. They are documented in
2441 [3] and are briefly summarised in Table 6.9. They are applied via normalisation
2442 parameters, with the separation systematics requiring the conservation of event
2443 rate across all samples.

2444 6.4.5.3 Correlated Detector Model

2445 A complete uncertainty model of the SK detector would be able to determine
2446 the systematic shift on the sample spectra for a variation of the underlying
2447 parameters, e.g. PMT angular acceptance. However, this is computationally
2448 intensive, requiring Monte Carlo predictions to be made for each plausible
2449 variation. Consequently, an effective parameter model has been utilised for

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

Table 6.9: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

2450 a correlated detector model following from the T2K-only model implementation
 2451 documented in subsubsection 6.4.5.1. It correlates the detector systematics
 2452 between the far-detector beam and subGeV atmospheric samples due to their
 2453 similar energies and interaction types. As there are no equivalent beam samples,
 2454 the multi-GeV, multiring, PC, and Up- μ samples will be subject to the particle
 2455 identification systematics implementation as described in subsubsection 6.4.5.2
 2456 rather than using this correlated detector model. The calibration systematics also
 2457 described in the aforementioned chapter still apply to all atmospheric samples.
 2458 The correlated detector model utilises the same smear and shift parameters
 2459 documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,

the shift and smear parameters are split by visible energy deposited within the detector, with binning specified in Table 6.10. This is because atmospheric events are categorised by subGeV and multi-GeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where k is the visible energy bin.

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

Table 6.10: Visible energy binning for which the correlated SK detector systematics are based

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear parameter for that event, and then re-applies event selection. This causes event migration, which is a new feature incorporated into the MaCh3 framework which is only achievable due to the event-by-event reweighting scheme.

Particular care has to be taken when varying the ring counting parameter. This is because the number of rings is a finite value (one-ring, two-ring, etc.) which can not be continuously varied through this shift and smear technique. Consequently a continuous ring counting parameter, RC_i , is calculated for the i^{th} event, following the definition in [178]: the preferred likelihoods from all considered one-ring (L_{1R}) and two-ring (L_{2R}) fits are determined. The difference

2479 is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is
 2480 then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

2481 where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an
 2482 intermediate likelihood value used within the `fitQun` algorithm to decide the
 2483 number of rings associated with a particular event. However, fake-ring merging
 2484 algorithms are applied after this likelihood value is used. Consequently, this
 2485 ring counting parameter does not always exactly correspond to the number of
 2486 reconstructed rings. This can be seen in Figure 6.13.

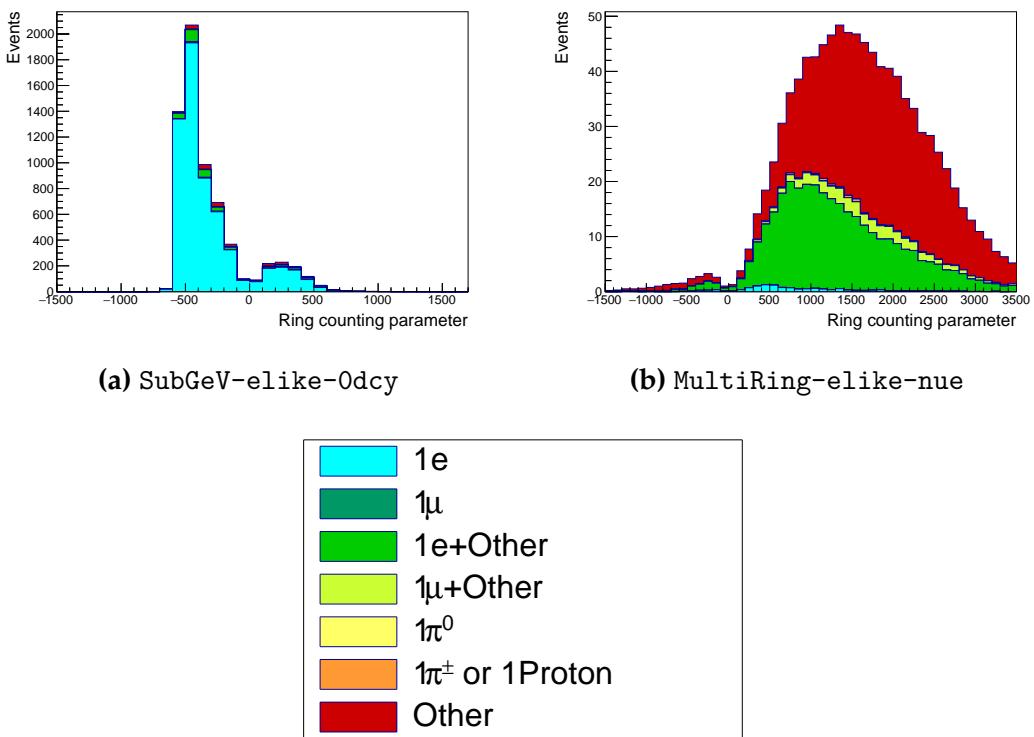


Figure 6.13: The ring counting parameter as defined in Equation 6.6 for the SubGeV-elike-0dcy and MultiRing-elike-nue samples.

2487 As the `fitQun` algorithm does not provide a likelihood value after the fake-
 2488 ring algorithms have been applied, the ring counting parameter distribution is
 2489 correlated to the final number of reconstructed rings through “maps”. These

2490 are two-dimensional distributions of the ring counting parameter and the final
 2491 number of reconstructed rings. An example is illustrated in Figure 6.14. In
 2492 principle, the `fitQun` reconstruction algorithm should be re-run after the variation
 2493 in the ring counting parameter. However, this is not computationally viable.
 2494 Therefore the “maps” are used as a reweighting template.

2495 The maps are split by final state topology and true neutrino flavour and
 2496 all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps
 2497 are row-normalised to represent the probability of X rings for a given RC_i
 2498 value. Prior to the oscillation fit, an event’s nominal weight is calculated as
 2499 $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th}
 2500 event and $W^i(x, y)$ is the bin content in map associated with the i^{th} event, where
 2501 x number of rings and y is ring counting parameter. Then during the fit, the
 2502 value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as the event weight
 2503 for the i^{th} event. This is the only cut variable that uses a reweighting technique
 2504 rather than event migration.

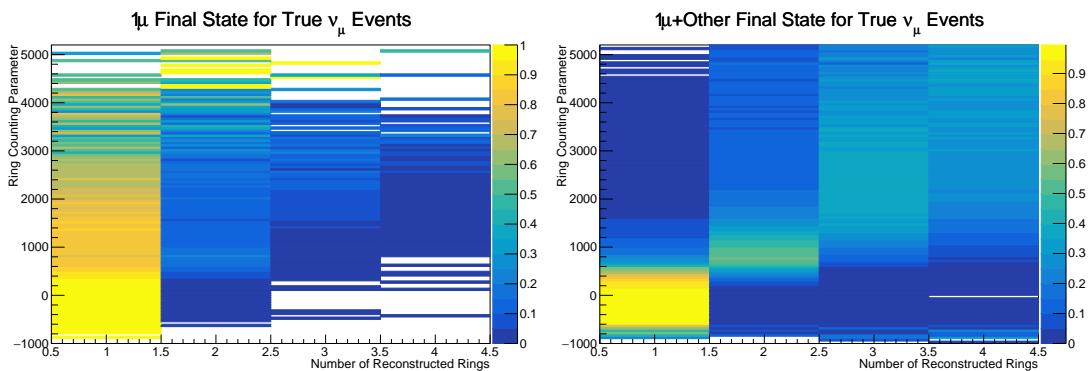


Figure 6.14: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2505 The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance
 2506 matrix. This is not possible in the alternative model as no covariance matrix
 2507 is used. Thus, the implementation of the π^0 systematics has been modified.
 2508 The inputs from the hybrid π^0 sample are included via the use of “ χ^2 maps”,

which are two-dimensional histograms in α_{jk}^i and β_{jk}^i parameters over some range. Illustrative examples of the χ^2 maps are given in Figure 6.15. Due to their nature, the shift and smear parameters are typically very correlated. A map is produced for each cut parameter given in Table 6.7 and for each visible energy bin given in Table 6.10.

The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo and data in the particle identification parameters documented in Table 6.7. The Monte Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-section and flux nuisance parameters are thrown from their prior uncertainties. The χ^2 between the scaled Monte Carlo and data is calculated and the relevant point in the χ^2 map is filled.

The implementation within this alternative detector model is to add the bin contents of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to the likelihood penalty. Only $1\pi^0$ final state topology shift and smear parameters use this prior uncertainty.

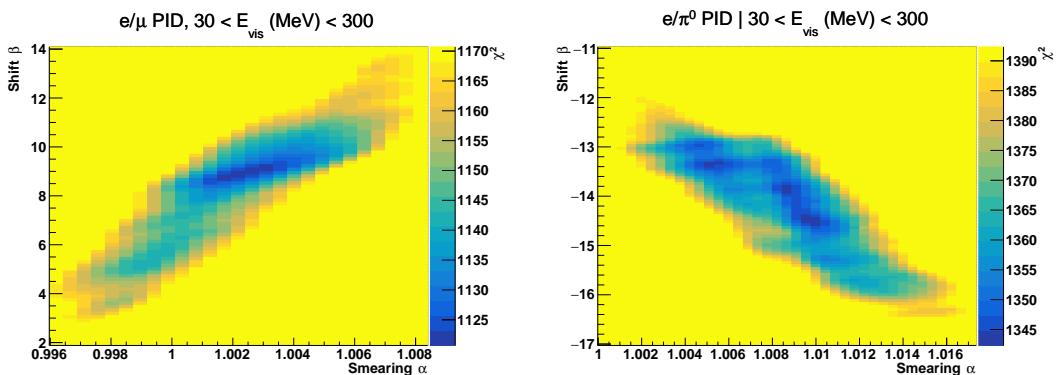


Figure 6.15: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$.

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [202] was built in tandem between the author of this thesis and the T2K-SK working group [179] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular

2529 interaction mode, number of tagged decay electrons, and whether the primary
2530 particle generated Cherenkov light. They are assigned Gaussian uncertainties
2531 with widths described by a covariance matrix. Furthermore, the secondary
2532 interaction and photo-nuclear effects need to be accounted for in this detector
2533 model using a different implementation than that in subsubsection 6.4.5.1. This
2534 was done by including a shape parameter for each of the secondary interactions
2535 and the photo-nuclear systematic parameters.

2536 There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior
2537 constraints from the hybrid π^0 samples.

2538 One final complexity of this correlated detector model is that the two sets
2539 of samples, beam and subGeV atmospheric, use slightly different parameters
2540 to distinguish electron and muon-like events. The T2K samples use the value
2541 of $\log(L_e/L_\mu)$ whereas the atmospheric samples use the value of $\log(L_e/L_\pi)$,
2542 where L_X is the likelihood for hypothesis X. This is because the T2K fits use
2543 single-ring f iTQun fitting techniques, whereas multi-ring fits are applied to the
2544 atmospheric samples where only the electron and pion hypothesis are considered.
2545 The correlation between the two likelihood ratios is illustrated in Figure 6.16. As
2546 discussed in section 5.2, the pion hypothesis is a very good approximation of the
2547 muon hypothesis due to their similar mass. Consequently, using the same shift
2548 and smear parameters correlated between the beam and subGeV atmospheric
2549 samples is deemed a good approximation.

2550 6.5 Likelihood Calculation

2551 This analysis performs a joint oscillation parameter fit of the ND280 beam
2552 samples, the T2K far detector beam samples, and the SK atmospheric samples
2553 introduced in this chapter.

2554 Once the Monte Carlo predictions of each beam and atmospheric sample
2555 have been built, a likelihood needs to be constructed. This is done by comparing
2556 the binned Monte Carlo prediction to binned data. The Monte Carlo prediction
2557 is calculated at a particular point, $\vec{\theta}$, in the model parameter space such that

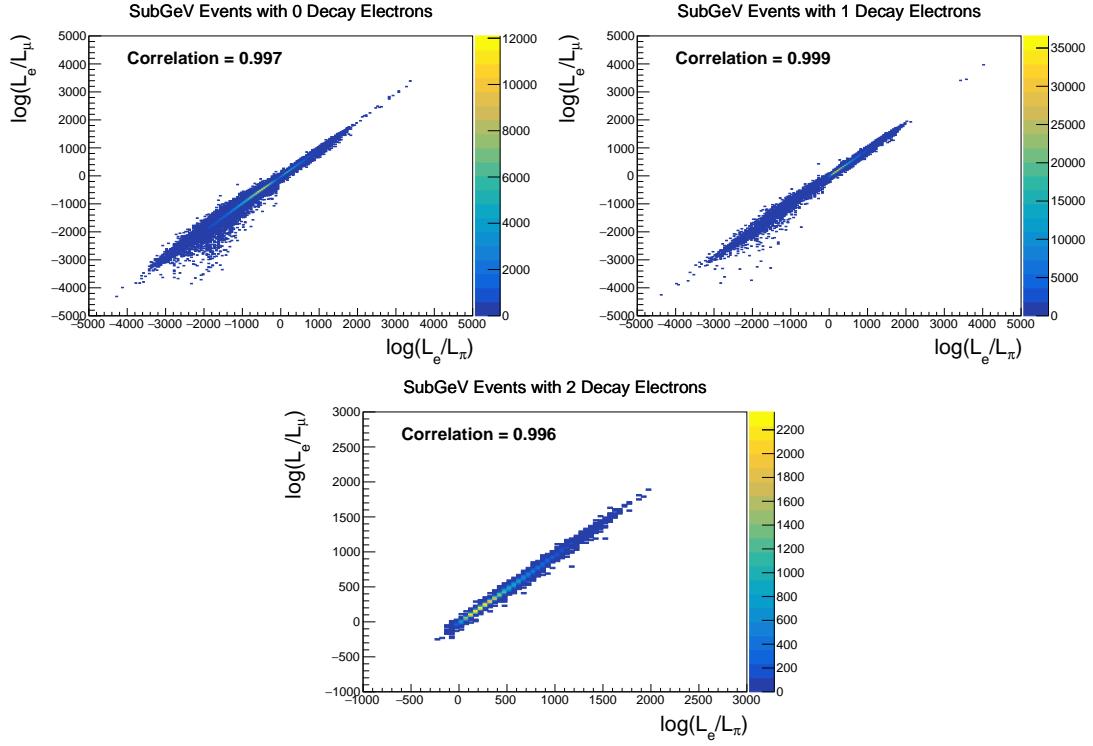


Figure 6.16: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

2558 $N_i^{MC} = N_i^{MC}(\vec{\theta})$, where N_i represents the bin content of the i^{th} bin. The data
 2559 and Monte Carlo spectra are represented by N_i^D and N_i^{MC} , respectively. The bin
 2560 contents for the beam near detector, beam far detector and atmospheric samples
 2561 are denoted with ND , FD , and Atm , respectively. Taking the FHC1Rmu far detector
 2562 sample as an example, the binning index runs over all the reconstructed neutrino
 2563 energy bins. The likelihood calculation between the data and the Monte Carlo
 2564 prediction for a particular bin follows a Poisson distribution, where the data
 2565 is treated as a fluctuation of the simulation.

2566 The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction,
 2567 which is typically used for sensitivity studies and denoted ‘Asimov data’. The
 2568 process for building Asimov data is as follows. The Monte Carlo prediction is
 2569 reweighted using a particular set of oscillation parameters (potentially those
 2570 listed in Table 2.2) and systematic parameter tune. The resulting spectra for each
 2571 sample is then defined to be the Asimov data for that sample. Whilst this results

in unphysical non-integer data predictions, it eliminates statistical fluctuations from the data. Therefore, the results of a fit to Asimov data should not include any biases from statistical fluctuations. Furthermore, these results should produce posterior probability distributions consistent with the parameters which were used to make the data prediction. That is to say, the fit results should return the known parameters. Any biases seen would be attributed to correlations between each oscillation parameter and correlations between oscillation and systematic parameters. Consequently, Asimov fit results present the maximum precision at which the oscillation parameters could be measured to.

Following the T2K analysis presented in [2], the likelihood contribution for the near detector samples also includes a Monte Carlo statistical uncertainty term, derived from the Barlow and Beeston statistical treatment [203, 204]. It includes a contribution to the likelihood that treats the generated Monte Carlo prediction as a statistical fluctuation of the actual true simulation assuming an infinite amount of statistics had been created. The technical implementation of this additional likelihood term is documented in [187] and briefly summarised as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (6.7)$$

where β_i represents a scaling parameter for the i^{th} bin that relates the bin content for the amount of Monte Carlo actually generated N_i^{MC} to the bin content if an infinite amount of Monte Carlo statistics had been generated $N_{i,true}^{MC}$, such that $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$. In the case where a sufficient amount of Monte Carlo statistics had been generated, $\beta_i = 1$. An analytical solution for β_i is given in [187]. Additionally, $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2} / N_i^{MC}$ where $\sqrt{\sum_i w_i^2}$ represents the sum of the square of the weights of the Monte Carlo events which fall into bin i .

DB: Giles did not understand this - Address An additional contribution to the likelihood comes from the variation of the systematic model parameters. For those parameters with well-motivated uncertainty estimates, a covariance matrix, V , describes the prior knowledge of each parameter as well as any

correlations between the parameters. Due to a technical implementation, a single covariance matrix describes each “block” of model parameters, e.g. beam flux systematics. The covariance matrix associated with the k^{th} block is denoted V^k . This substitution results in $\vec{\theta} = \sum_k^{N_b} \vec{\theta}^k$ and $V = \sum_k^{N_b} V^k$ where N_b denotes the number of blocks. A single covariance matrix is provided for: the oscillation parameters, the beam flux parameters, the atmospheric flux parameters, the neutrino interaction systematics, the near detector parameters, the beam far detector systematics, and the atmospheric far detector systematics. The number of parameters in the k^{th} block is defined as $n(k)$.

The equation for the likelihood \mathcal{L} includes all the terms discussed above. It is defined as,

$$\begin{aligned}
-\ln(\mathcal{L}) = & \\
& \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
& + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
& + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
& + \frac{1}{2} \sum_k^{N_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
\end{aligned} \tag{6.8}$$

The negative log-likelihood value is determined at each step of the MCMC to build the posterior distribution defined in chapter 4. This value is minimised when the Monte Carlo prediction tends towards the data spectrum.

7

2614

2615

Oscillation Probability Calculation

2616 It is important to understand how and where the sensitivity to the oscillation parameters comes from for both atmospheric and beam samples. An
2617 overview of how these samples respond to changes in δ_{CP} , Δm_{32}^2 , and $\sin^2(\theta_{23})$
2618 is given in section 2.5. This section also explains the additional complexities
2619 involved when performing an atmospheric neutrino analysis as compared to
2620 a beam-only analysis.

2622 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to $\delta_{CP,zaza}$
2623 due to the quickly varying oscillation probability in this region. Therefore, a
2624 “sub-sampling” approach has been developed to reduce these biases ensuring
2625 accurate and reliable sensitivity measurements. This technique ensures that small-
2626 scale unresolvable features of the oscillation probability have been averaged over
2627 whilst the large-scale features in the oscillation probability are unaffected. The
2628 documentation and validation of this technique are found in section 7.1. The
2629 oscillation probability calculation is computationally intensive due to the large
2630 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-
2631 mentation choice made within the fitting framework, as detailed in section 7.2,
2632 ensures that the analysis can be done in a timely manner.

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.3. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrino oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation. This implementation of an external technique is described in section 7.4.

7.1 Treatment of Fast Oscillations

As shown in Figure 7.1, atmospheric neutrino oscillations have a significantly more complex structure for upgoing neutrinos with energy below 1GeV. This is because the L/E dependence of the oscillation probability in this region induces rapid variations for small changes in L or E . As discussed in section 2.5, this is also the region in which atmospheric neutrinos have sensitivity to δ_{CP} . In practice, the direction of the neutrino is inferred from the direction of the final state particles traveling in the detector. The correlation between these two directions can be particularly weak for low-energy neutrino interactions. This creates a distinct difference from the beam neutrinos where the position of the source is very precisely known.

As a consequence of the unresolvable structure, an event rate consistent with the averaged oscillation probability is observed in the subGeV upgoing region. This creates a computational problem: A significantly large amount of Monte Carlo statistics would be required to accurately predict the number of events if Monte Carlo averaging was the only technique used. This section describes

2663 the ‘sub-sampling’ approach developed for this analysis and compares it to the
2664 methodology used within the SK-only analysis.

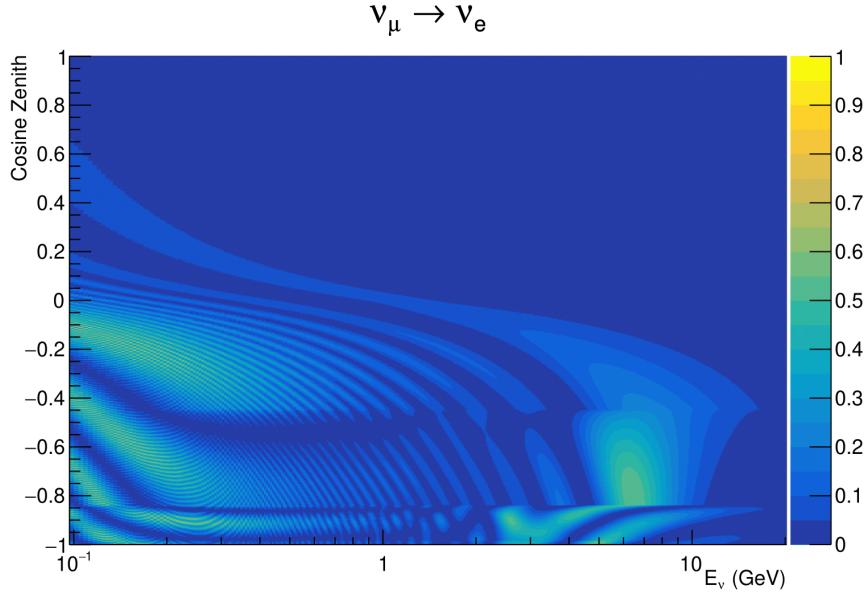


Figure 7.1: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2665 The official SK-only analysis uses the osc3++ oscillation parameter fitter
2666 [78]. To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’
2667 technique. For a given Monte Carlo neutrino event, the nearest twenty Monte
2668 Carlo neighbours in reconstructed lepton momentum and zenith angle are
2669 found and a distribution of their neutrino energies is built. The RMS, σ , of
2670 this distribution is then used to compute an average oscillation probability for
2671 the given neutrino Monte Carlo event.

2672 For the i^{th} event, the oscillation weight is calculated as

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1, -0.5, 0.5, 1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.1)$$

2673 where $P(E, L)$ is the oscillation probability calculation for neutrino energy E
2674 and path length L and the two path lengths, \bar{L}_i and L_β are described below. All
2675 of the oscillation probability calculations are performed with a fixed zenith angle
2676 such that the same density profile is used. The uncertainty in the production

2677 height is controlled by using an “average” production height, \bar{L}_i , which represents
2678 the average path length computed using twenty production heights taken from
2679 the Honda flux model’s prediction [54]. These inputs are provided in 5% intervals
2680 of the cumulative distribution function. The value of \bar{L}_i is calculated as:

$$\bar{L}_i = \frac{1}{20} \sum_{j=1}^{20} \sqrt{(R_E + h_j)^2 - R_E^2 (1 - \cos^2 \theta_i)} - R_E \cos \theta_i. \quad (7.2)$$

2681 Where R_E is the Earth’s radius and θ_i is the zenith angle of the i^{th} event.
2682 The production heights h_j represent the $(j \times 5)^{th}$ percentile of the cumulative
2683 distribution function. L_β values (where the values of β are given in Equa-
2684 tion 7.1) are similarly calculated but instead use different combinations of four
2685 production heights,

$$\begin{aligned} L_{-1.0} &= \frac{1}{4} L(45, 50, 55, 60), \\ L_{-0.5} &= \frac{1}{4} L(35, 40, 65, 70), \\ L_{+0.5} &= \frac{1}{4} L(25, 30, 75, 68), \\ L_{+1.0} &= \frac{1}{4} L(15, 20, 85, 89). \end{aligned} \quad (7.3)$$

2686 Where $L(i, j, k, l)$ represents the sum of the path lengths with fixed zenith
2687 angle and production heights corresponding to the i^{th} , j^{th} , k^{th} and l^{th} percentile
2688 of the cumulative distribution function. The values that are taken as β (and
2689 values for L_β) are chosen to smooth the oscillation contours in Δm_{32}^2 without
2690 incurring loss of sensitivity [205].

2691 This averaging technique works because of the inference between the zenith
2692 angle and the reconstructed direction of final state particles in the detector. For
2693 low-energy neutrinos, where the resolution of the true neutrino direction is poor,
2694 σ_i will be large, resulting in significant averaging effects. Contrary to this, the
2695 inferred direction of high-energy neutrinos will be much closer to the true value,
2696 meaning that σ_i will be smaller, culminating in small averaging effects.

In practice, these calculations are performed prior to the fit as only oscillation parameters at fixed points are considered. The MCMC technique used in this thesis requires oscillation probabilities to be evaluated at arbitrary parameter values, not known *a priori*. Calculating the five oscillation probabilities per event required by the SK technique is computationally infeasible, so a different averaging technique is used. However, the concept of the averaging technique can be taken from it.

To perform a similar averaging as the SK analysis, a sub-sampling approach using binned oscillograms has been devised. A coarsely binned oscillogram is defined in $\cos(\theta_Z)$ and E_ν . For a given set of oscillation parameters, a single oscillation probability will be assigned to each coarse bin. This value will then apply to all Monte Carlo events which fall into that bin. To assign these oscillation probabilities, the probability is calculated at $N \times N$ points on a grid within a particular bin. This ensemble of oscillation probabilities is averaged to define the coarse bin's oscillation probability, assuming a flat prior in E_ν and $\cos(\theta_Z)$ within the bin. Figure 7.2 illustrates the $N = 2$ example where the assigned value to a coarse bin is the average of the four fine bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear on either axis, the sub-division of the fine bins is linear within the range of a coarse bin.

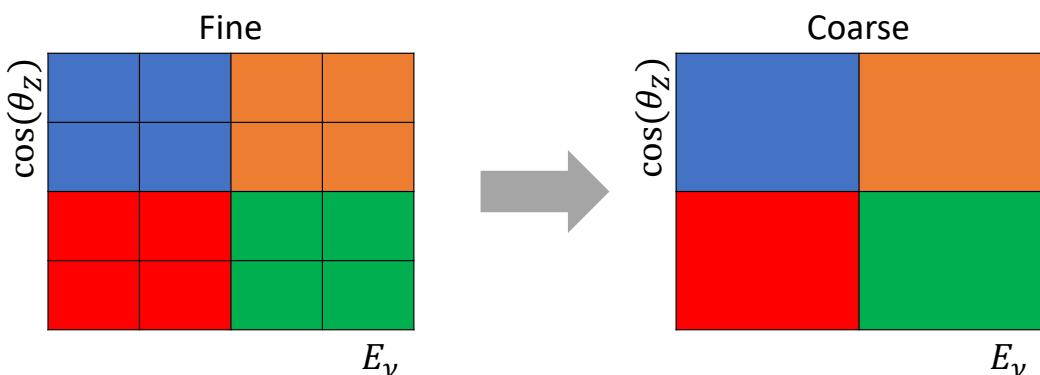


Figure 7.2: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2716 The coarse binning is defined with 67×52 bins in true neutrino energy \times
 2717 cosine zenith. It is picked to be identical to that provided in [205]. In general, the
 2718 binning is logarithmically spaced in neutrino energy but has some hand-picked
 2719 bin edges around the matter resonance to smoothly increased the bin density.
 2720 This is to avoid smearing this region which can be well sampled by the Monte
 2721 Carlo. The cosine zenith binning is approximately linearly spaced across the
 2722 allowable range but the values of layer transitions are hit precisely: -0.8376 (core-
 2723 mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for
 2724 downgoing events as this is a region unaffected by the fast oscillation wavelengths
 2725 and reduces the total number of calculations required to perform the calculation.

2726 The choice of N is justified based on two studies. Firstly, the variation of event
 2727 rates of each sample is studied as a function of N . For a given set of oscillation
 2728 parameters thrown from the PDG prior constraints (detailed in Table 2.1), the
 2729 oscillation probabilities are calculated using a given value of N . Each sample
 2730 is re-weighted and the event rate is stored. The value of N is scanned from
 2731 1, which corresponds to no averaging, to 19, which corresponds to the largest
 2732 computationally viable subdivision binning. The event rate of each sample at
 2733 large N is expected to converge to a stationary value due to the fine binning fully
 2734 sampling the small-scale structure. Figure 7.3 illustrates this behaviour for the
 2735 SubGeV_elike_0dcy sample for 9 different throws of the oscillation parameters.

2736 Denoting the event rate for one sample for a given throw t at each N by λ_t^N ,
 2737 the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The
 2738 variance in the event rate at each N is then calculated as

$$2739 \text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right) \right]^2. \quad (7.4)$$

2740 In practice, the following procedure is undertaken. For a particular throw,
 2741 the difference between the event rate at a particular choice of N and the mean
 2742 of the distribution is calculated. This is illustrated in Figure 7.4. This value
 2743 is then calculated for all the 2000 throws, generating a distribution of $\lambda_t^N - \bar{\lambda}_t$.

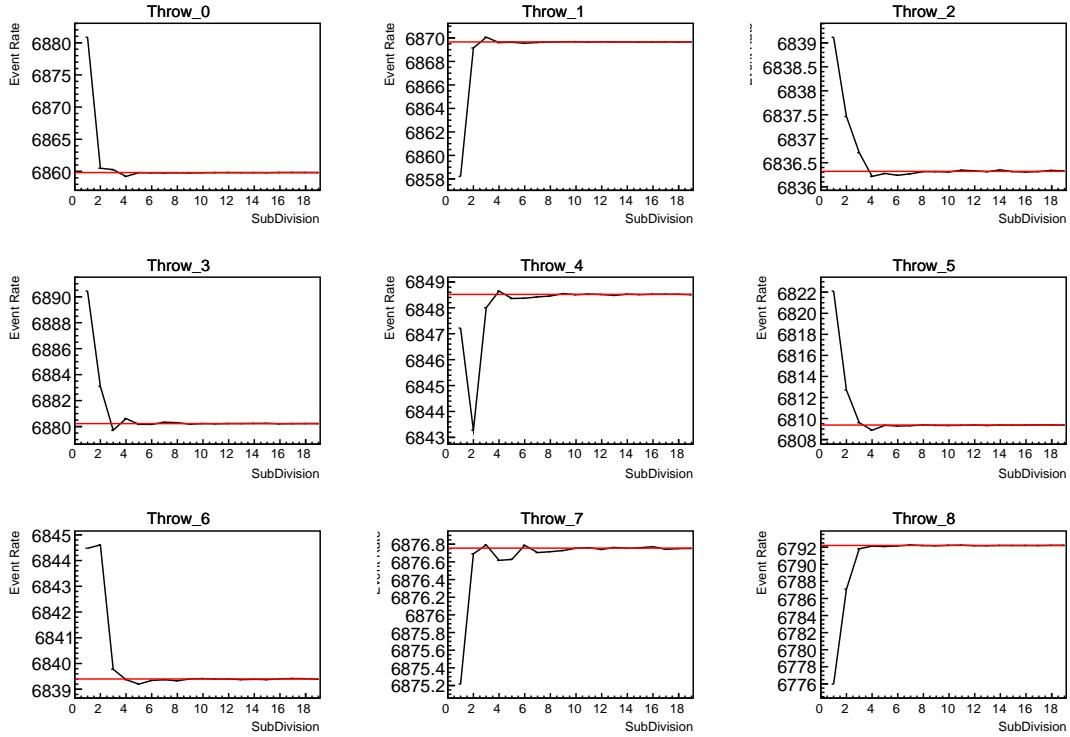


Figure 7.3: Event rate of the SubGeV_elike_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2743 This is repeated for each of the values of N considered within this study. The
 2744 distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.5. As expected,
 2745 the distribution gets narrower and tends towards zero for the higher values of N .

2746 The aim of the study is to find the lowest value of N such that this variance
 2747 is below 0.001. This utilises the width of the distributions given in Figure 7.5.
 2748 This is the typical threshold used by T2K fitters to validate systematic imple-
 2749 mentation so has been set as the same criteria. The results of this study for
 2750 each atmospheric sample used within this thesis are illustrated in Figure 7.6 for
 2751 2000 throws of the oscillation parameters. As can be seen, the variance is below
 2752 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy
 2753 and SubGeV_elike_0dcy samples.

2754 The second study to determine the value of N is as follows. The likelihood
 2755 for each sample is computed against an Asimov data set created with Asimov A

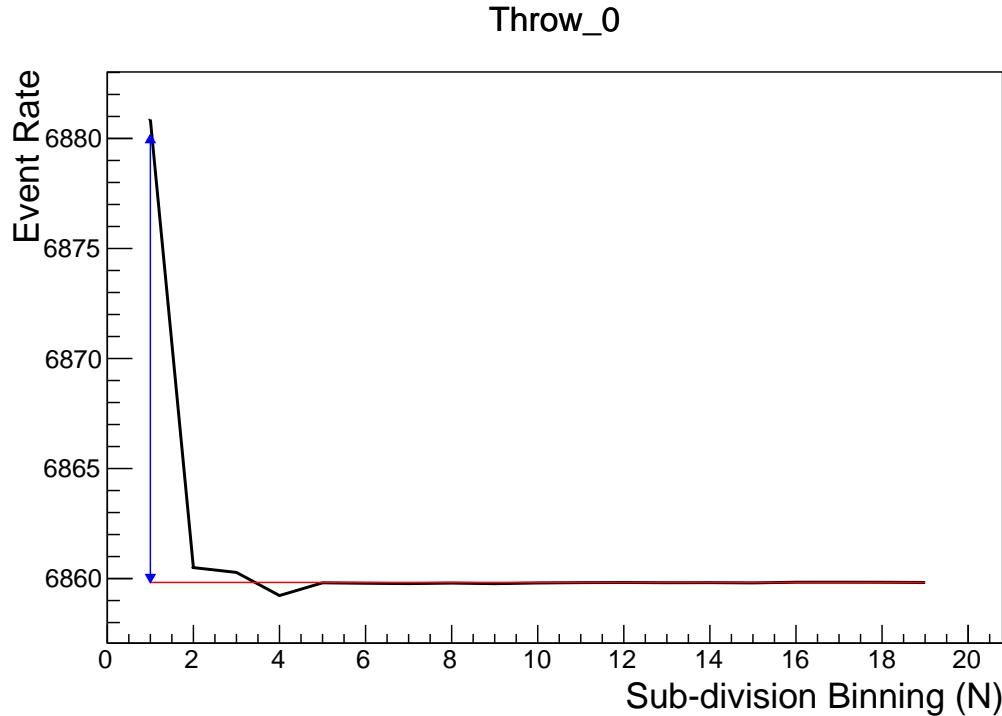


Figure 7.4: Event rate of the SubGeV_elike_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions, N , per coarse bin. The difference between the mean event rate (red), $\bar{\lambda}$, and the event rate at $N = 1$, $\lambda^{N=1}$ is defined as $\lambda^N - \bar{\lambda}$ and illustrated by the blue arrow.

2756 oscillation parameters (Table 2.2). Following Equation 7.4, the variance of the log-
 2757 likelihood over all considered N is computed. The results are shown in Figure 7.7.

2758 A choice of $N = 10$ sub-divisions per coarse bin has a variance in both
 2759 event rate and log-likelihood residuals less than the required threshold of 0.001.
 2760 The largest value of the likelihood variance is of order 10^{-7} , corresponding to
 2761 an error on the log-likelihood of about 3×10^{-4} which is small enough to be
 2762 negligible for the oscillation analysis.

2763 Figure 7.8 illustrates the effect of the smearing using $N = 10$. The fast oscilla-
 2764 tions in the sub-GeV upgoing region have been replaced with a normalisation
 2765 effect whilst the large matter resonance structure remains.

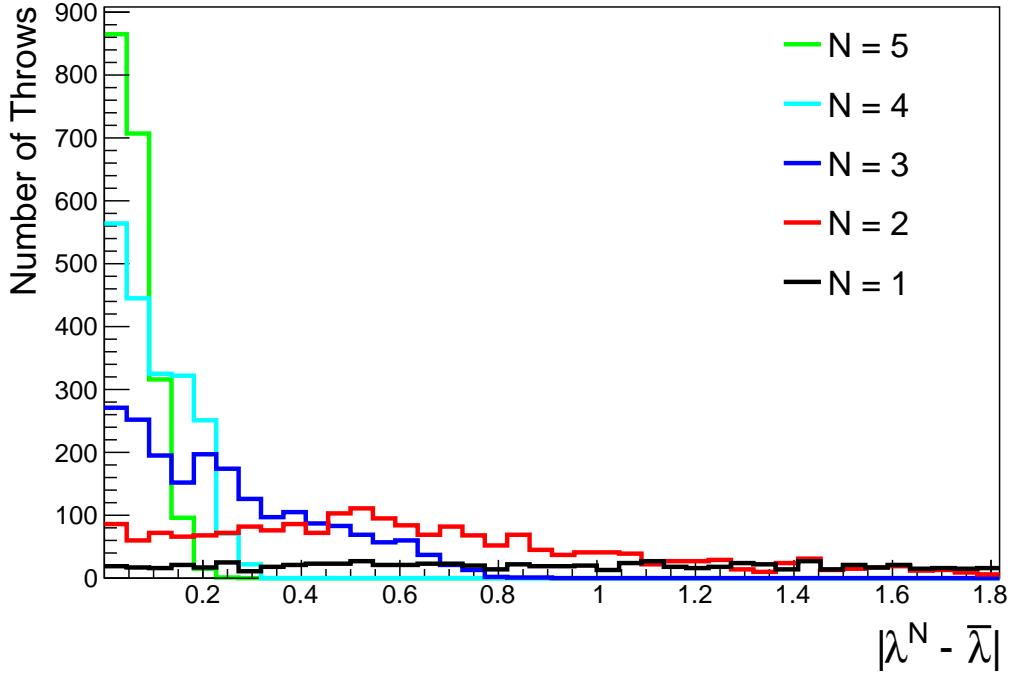


Figure 7.5: The distribution of $\lambda^N - \bar{\lambda}$ for various values of N . As expected, the distribution gets narrower for larger values of N .

2766 7.2 Calculation Engine

2767 As previously discussed in section 7.1, the calculation of oscillation probabilities
 2768 is performed at run-time. Consequently, the time per calculation is crucial for fit
 2769 performance. The initial fitting framework used for this analysis was developed
 2770 with ProbGPU [206]. This is a GPU-only implementation of the prob3 engine
 2771 [207]. It is primarily designed for neutrino propagation in a beam experiment
 2772 (single layer of constant density) with the atmospheric propagation code not
 2773 being used prior to the analysis in this thesis.

2774 Another engine, CUDAProb3 [208], has been interfaced with the fitting frame-
 2775 work used in this analysis. This interfacing was done by the author of this
 2776 thesis. It has been specifically optimised for atmospheric neutrino oscillation
 2777 calculation so does not contain the code to replace the beam oscillation calculation.
 2778 The engine utilises object-orientated techniques as compared to the functional
 2779 implementation of ProbGPU. This allows the energy and cosine zenith arrays to

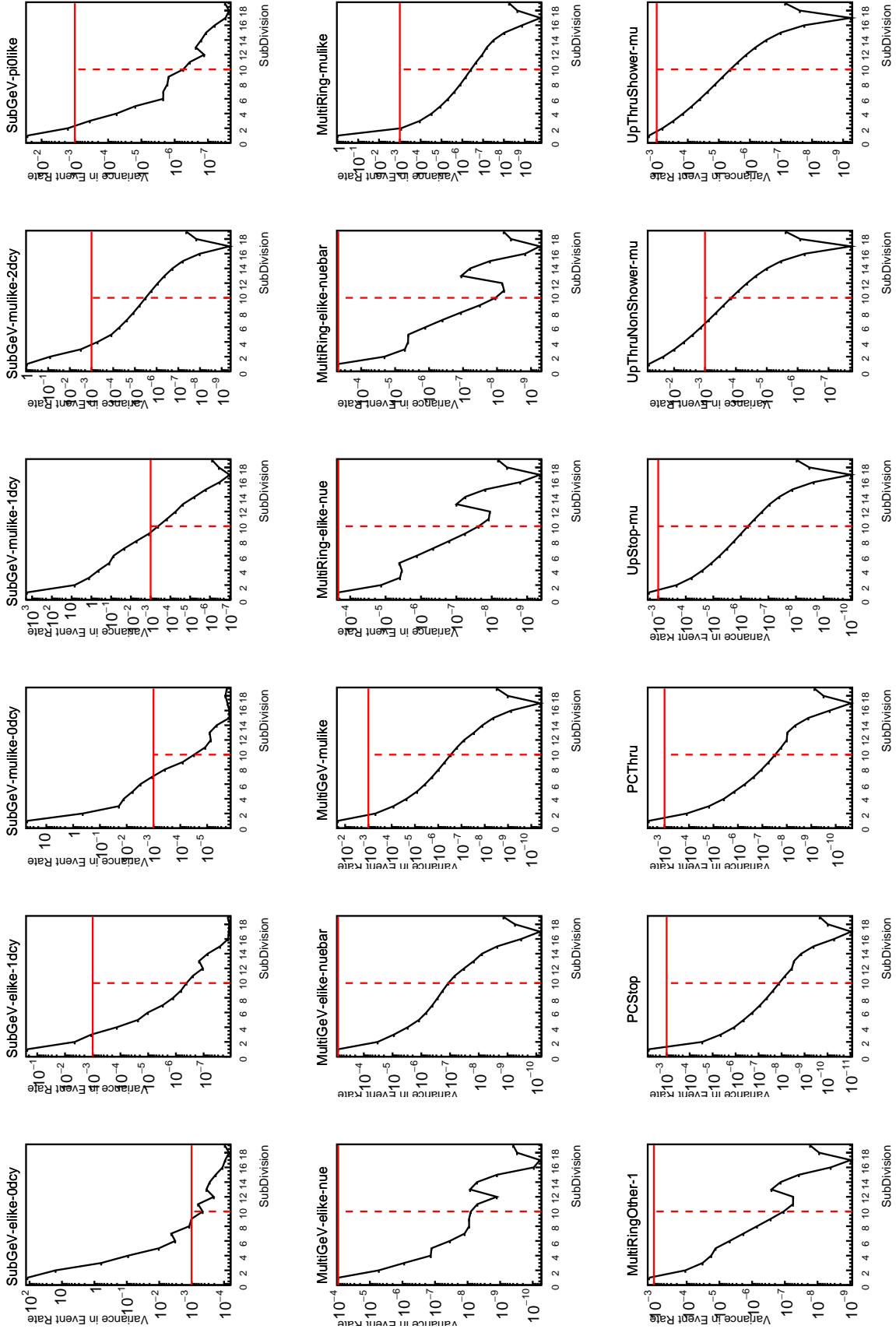


Figure 7.6: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

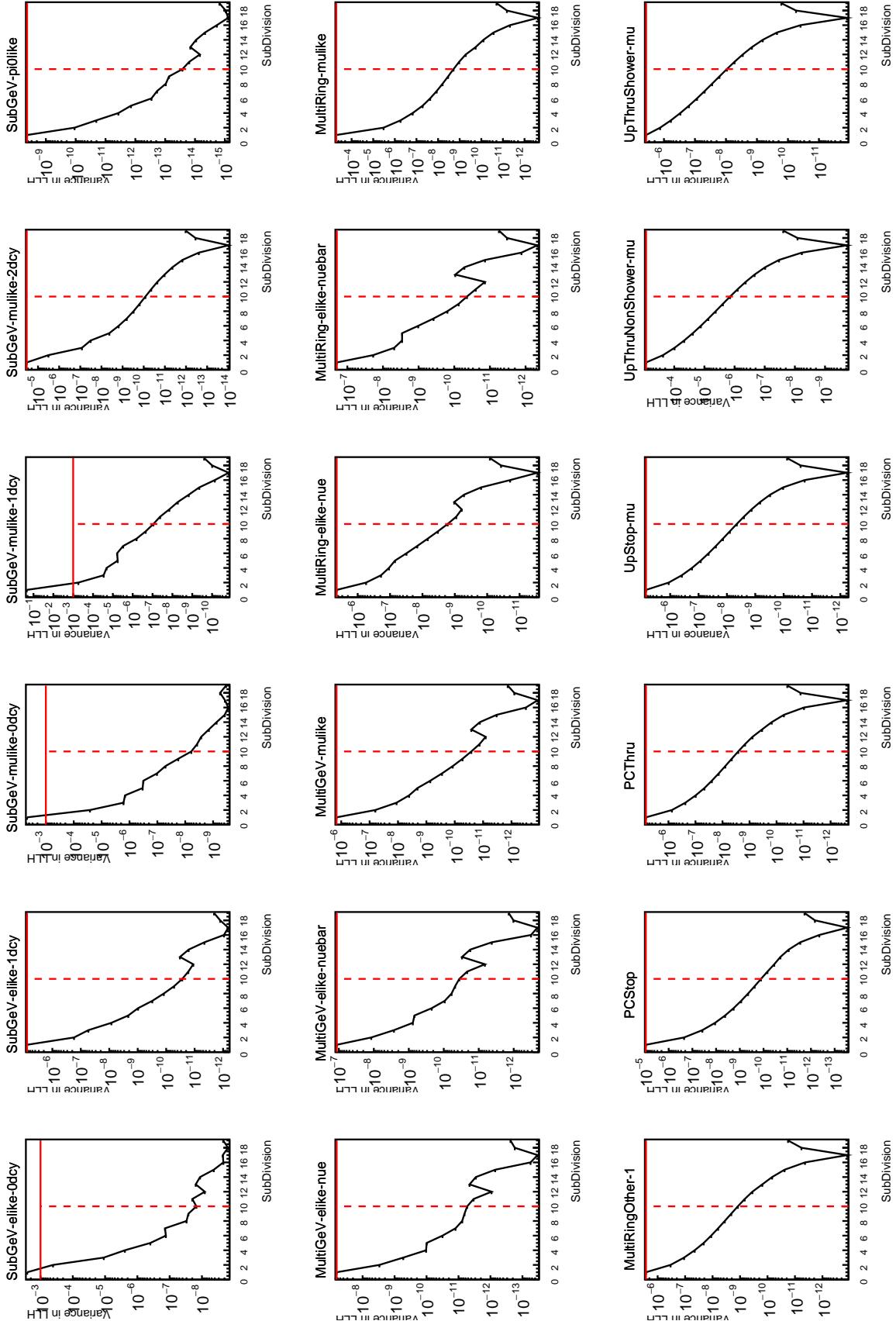


Figure 7.7: Variance of sample likelihood, when compared to 'Asimov data' set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

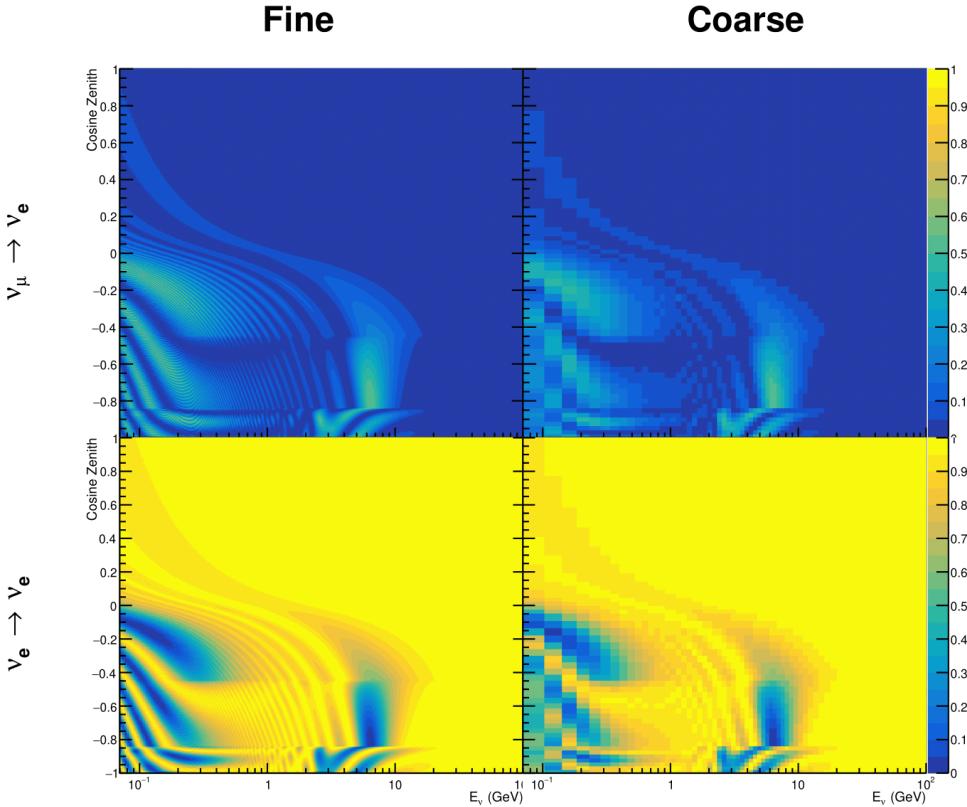


Figure 7.8: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.6 and Figure 7.7.

2780 be kept on GPU memory, rather than having to load these arrays onto GPU
 2781 memory for each calculation. Reducing the memory transfer between CPU and
 2782 GPU significantly reduces the time required for calculation. This can be seen
 2783 in Figure 7.9, where the GPU implementation of CUDAProb3 is approximately
 2784 three times faster than the ProbGPU engine.

2785 Another significant advantage of CUDAProb3 is that it contains a CPU multi-
 2786 threaded implementation which is not possible with the ProbGPU or prob3 engines.
 2787 This eliminates the requirement for GPU resources when submitting jobs to batch
 2788 systems. As illustrated in Figure 7.9, the calculation speed depends on the number

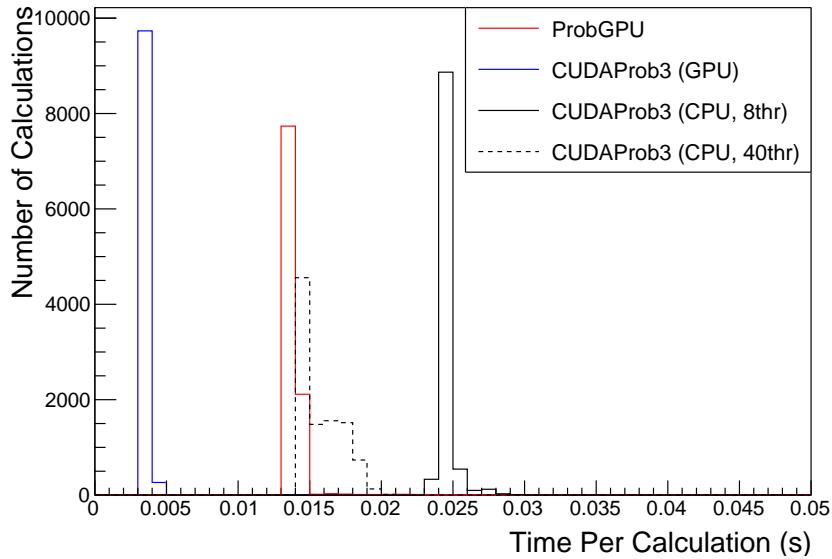


Figure 7.9: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.1, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multi-threaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

of available threads. Using 8 threads (which is typical of the batch systems being used) is approximately twice as slow as the ProbGPU engine implementation, but would allow the fitting framework to be run on many more resources. This fact is utilised for any SK-only fits but GPU resources are required for any fits which include beam samples due to the ProbGPU requirement. Based on the benefits shown by the implementation in this section, efforts are being placed into including linear propagation for beam neutrino propagation into the engine [209].

7.3 Matter Density Profile

For an experiment observing neutrinos propagating through the Earth, a model of the Earth’s density profile is required. The model used within this analysis is based on the Preliminary Reference Earth Model (PREM) [79], as illustrated in Figure 2.8. Table 2.3 documents the density and radii of the layers used within the constant density approximation used by the SK-only analysis [78]. The

density measurements provided in the PREM model are provided in terms of mass density, whereas neutrino oscillations are sensitive to the electron number density. This value can be computed as the product of the chemical composition, or the Z/A value, and the mass density of each layer. Currently, the only way to measure the chemical composition value for layers close to the Earth's core is through neutrino oscillations. The chemical composition of the upper layers of the Earth's Mantle and the Transition zone is well known due to it being predominantly pyrolite which has a chemical composition value of 0.496 [210]. The chemical composition dial for the core layers is set to a value of 0.468, as calculated in [211]. As this value is less well known, it is assigned a Gaussian error with a standard deviation equivalent to the difference in chemical composition in core and mantle layers. Figure 7.10 illustrates the effect of moving from the $Z/A = 0.5$ method which is used in the official SK-only analysis to these more precise values.

The beam oscillation probability in this thesis uses a baseline of 295km, density $2.6\text{g}/\text{cm}^3$, and chemical composition 0.5 as is done by the official T2K-only analysis [212].

For a neutrino with given $E_\nu, \cos(\theta_Z)$, the oscillation probability calculation engine must be passed a list of the matter regions that the neutrino traversed, with the path length and fixed density in each region. However, a neutrino passing through the earth experiences a range of radii, and thus a range of densities, in each region. In the SK-only analysis, the earth density model used is piecewise-constant, thereby ignoring this effect. For this thesis, the density values for the calculation engine are found by averaging the earth density along the neutrino's path in each layer,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt, \quad (7.5)$$

where t_i are the intersection points between each layer and t is the path length of the trajectory across the layer. This leads to an improved approximation. For this averaging, the simplification of the PREM model developed in [213] is

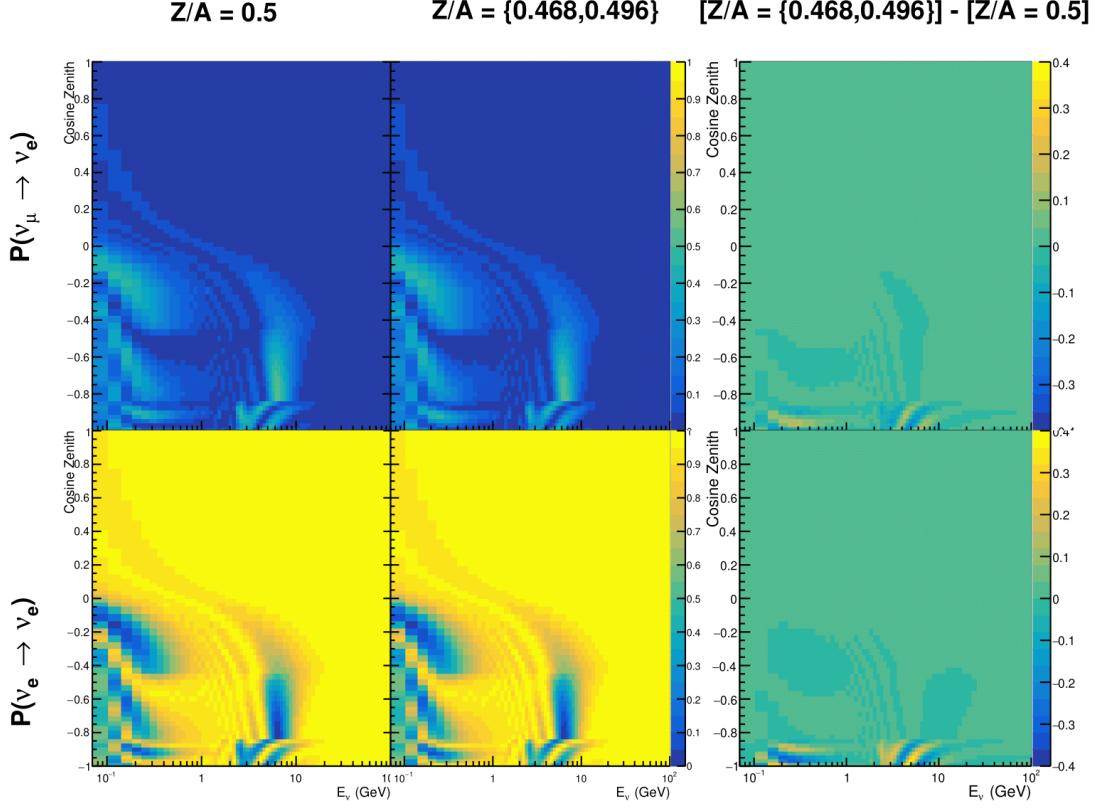


Figure 7.10: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 2.3. The right column illustrates the difference in oscillation probability between the two different techniques.

used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [205]. The coefficients of the quadratic fit to each layer are given in Table 7.1 with the final distribution illustrated in Figure 7.11. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in Figure 7.12. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -0.45$ in the fixed density model, which is due to the transition to

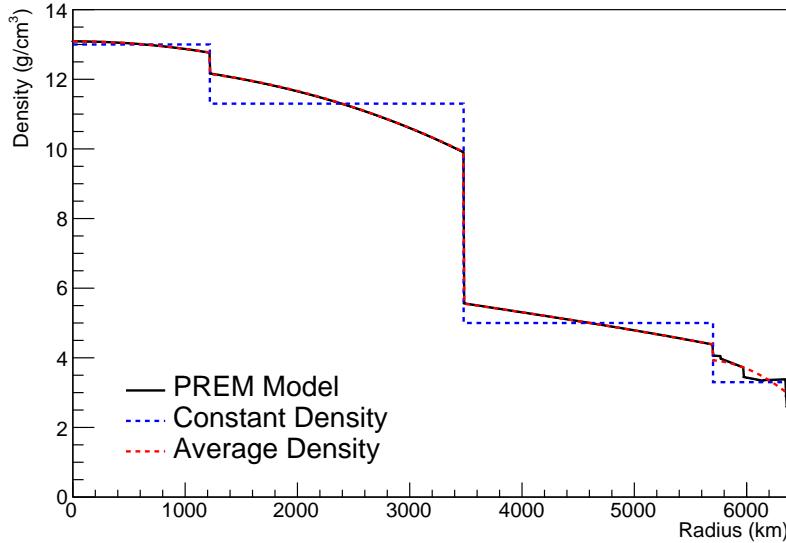


Figure 7.11: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.1: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [205], where $x = R / R_{\text{Earth}}$.

mantle layer boundary, has been reduced. This is expected as the difference in the density across this boundary is significantly smaller in the quadratic density model as compared to the constant density model. Whilst the difference in density across the other layer transitions is reduced, there is still a significant difference. This means the discontinuities in the oscillation probabilities remain but are significantly reduced. However, as the quadratic density approximation matches the PREM model well in this region, these discontinuities are due to the Earth model rather than an artifact of the oscillation calculation.

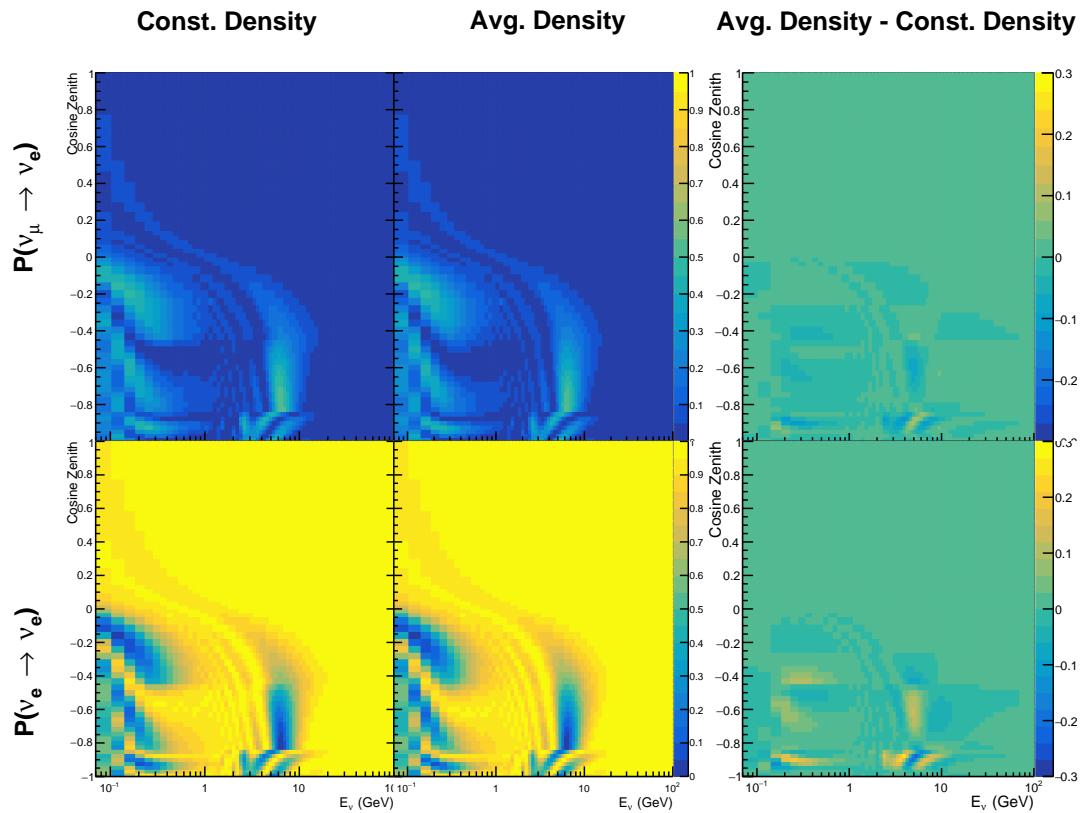


Figure 7.12: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2847 7.4 Production Height Averaging

2848 As discussed in section 2.5, the height at which the cosmic ray flux interacts
2849 in the atmosphere is not known on an event-by-event basis. The production
2850 height can vary from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only
2851 analysis methodology (described in section 7.1) for including the uncertainty
2852 on the production height is to include variations from the Honda model when
2853 pre-calculating the oscillation probabilities prior to the fit. This technique is not
2854 possible for this analysis which uses continuous oscillation parameters that can
2855 not be known prior to the fit. Consequently, an analytical averaging technique
2856 was developed in [205]. The author of this thesis was not responsible for the
2857 derivation of the technique but has performed the implementation and validation
2858 of the technique for this analysis alone.

2859 Using the 20 production heights per Monte Carlo neutrino event, provided
2860 as 5% percentiles from the Honda flux model, a production height distribution
2861 $p_j(h|E_\nu, \cos \theta_Z)$ is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a
2862 histogram is filled with 20 evenly spaced bins in production height h between
2863 0 and 50km. The neutrino energy and cosine zenith binning of the histogram
2864 are the same as that provided in section 7.1. The average production height,
2865 $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. This assumes a linear average over
2866 the four flavours of neutrino which are considered to be generated in cosmic
2867 ray showers. The production height binning of this histogram is then translated
2868 into $\delta t(h) = t(\bar{h}) - t(h)$, where $t(x)$ is the distance travelled along the trajectory
2869 in the atmosphere from some production height, x .

2870 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed.
2871 The time-ordered product of these is then used as the overall transition amplitude
2872 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.6)$$

2873 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.7)$$

2874 is expressed as a diagonalised time-dependent solution to the Schrodinger
 2875 equation. The 0th layer is the propagation through the atmosphere and is the
 2876 only term that depends on the production height. Using the substitution $t_0 =$
 2877 $t(\bar{h}) - \delta t(h)$, it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.8)$$

2878 Thus Equation 7.6 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.9)$$

2879 The oscillation probability averaged over production height is then calculated
 2880 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)_{ij}^* \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m) \delta t]. \end{aligned} \quad (7.10)$$

2881 It is important to note that the exact value of \bar{h} used does not matter as the
 2882 values of δt would change to compensate for any modification to the value of \bar{h} .

2883 In practice, implementation in CUDAProb3 [208] is relatively straightforward
 2884 as the majority of these terms are already calculated in the standard oscillation
 2885 calculation. Figure 7.13 illustrates the results of the production height averaging.

2886 As expected, the main effect is observed in the low-energy downward-going
 2887 and horizontal-going events. Upward-going events have to travel the radius
 2888 of the Earth, $R_E = 6371\text{km}$, where the production height uncertainty is a small
 2889 fraction of the total path length.

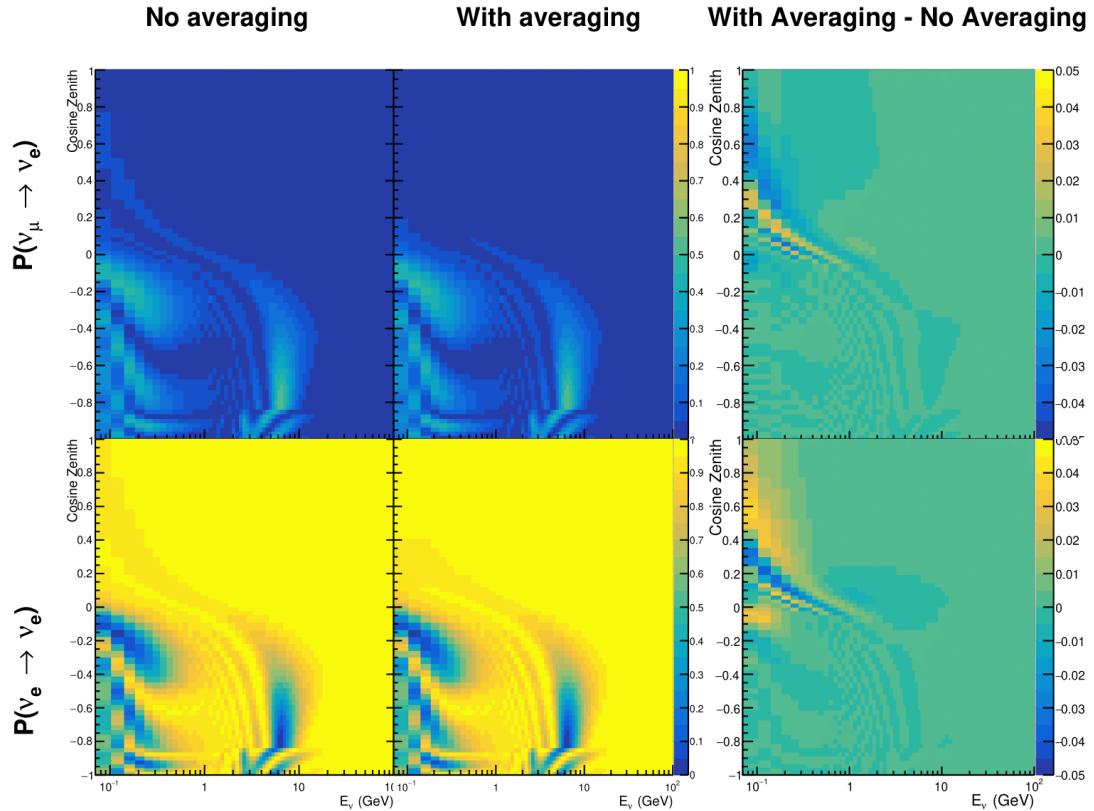


Figure 7.13: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

8

2890

2891

Oscillation Analysis

2892 Using the samples and systematics defined in chapter 6, this chapter documents
2893 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK
2894 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4
2895 is used for all studies performed within this thesis.

2896 The MaCh3 framework has been validated through many tests. The code
2897 that handles the beam far detector samples was developed by the author and
2898 validated by comparison to the 2020 T2K analysis [2]. The sample event rates and
2899 likelihood evaluations of beam samples generated by the framework used within
2900 this thesis were compared to those from the T2K analysis by the author of this
2901 thesis. Variations of the sample predictions were compared at $\pm 1\sigma$ and $\pm 3\sigma$ and
2902 good agreement was found in all cases. A similar study, led by Dr. C. Wret was
2903 used to validate the near detector portion of the code [198]. The implementation
2904 of the atmospheric samples within MaCh3 was completed and cross-checked by
2905 the author of this thesis against the P-Theta framework (introduced in section 3.2).
2906 Both fitters are provided with the same inputs and can therefore cross-validate
2907 each other. These validations compared the event rate and likelihood calculation.
2908 Documentation of all the above validations can be found in [198]. These stringent
2909 validations ensure that the code is doing as intended.

2910 8.1 Monte Carlo Prediction

- 2911 Using the three sets of dial values (generated, pre-fit, and post-fit tunes) defined
 2912 in subsection 6.4.3, the predicted event rates for each sample are given in Table 8.1.
 2913 The oscillated and un-oscillated event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThrus	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1: The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

- 2914 Generally, the samples that target CCQE interaction modes observe a decrease
 2915 in prediction when comparing the generated values with the pre-fit dial values.
 2916 This is in accordance with the Monte Carlo being produced at $M_A^{QE} = 1.21\text{GeV}$
 2917 [157] whilst the pre-fit dial value is set to $M_A^{QE} = 1.03\text{GeV}$ as suggested by [197].

Furthermore, the predicted event rates of samples that target CCRES interaction modes are significantly reduced when considering the post-BANFF fit. This follows the observations in subsection 6.4.3. The strength of the accelerator neutrino experiment can be seen in the remarkable difference between the oscillated and unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very clear decrease in the expected event rate between the oscillated and un-oscillated predictions which is not as obvious as in the atmospheric samples. This is due to the fact that the beam energy is tuned to the maximum disappearance probability, which is not the case for the naturally generated atmospheric neutrinos.

8.2 Likelihood Scans

Using the definition of the likelihood presented in section 6.5, the contribution of each sample to the likelihood from a variation of a particular parameter can be studied. This process identifies which samples drive the determination of the oscillation parameters in the joint fit. Figure 8.1 presents the variation of all the samples (beam and atmospheric) at the far detector to the oscillation parameters of interest: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . These plots are colloquially called ‘likelihood scans’ (or ‘log-likelihood scans’). The process of making these plots is as follows. An Asimov data set is built using the AsimovA oscillation parameters and pre-fit systematic tune. The Monte Carlo is then reweighted using the value of the oscillation parameter at each point on the x-axis of the scan. The likelihood is then calculated between the Asimov data and Monte Carlo prediction and plotted.

Due to the caveat of fixed systematic parameters and the correlations between oscillation parameters being ignored when creating these likelihood scans, the value of $\chi^2 = 1$ (or $-2 \times \ln(\text{Likelihood}) = 1$) does not equate to the typical 1σ sensitivity. However, it does give an indication of which samples respond most strongly to variations in a particular oscillation parameter. The point at

2946 which the likelihood tends to zero illustrates the value of the parameter used
 2947 to build the Asimov data prediction.

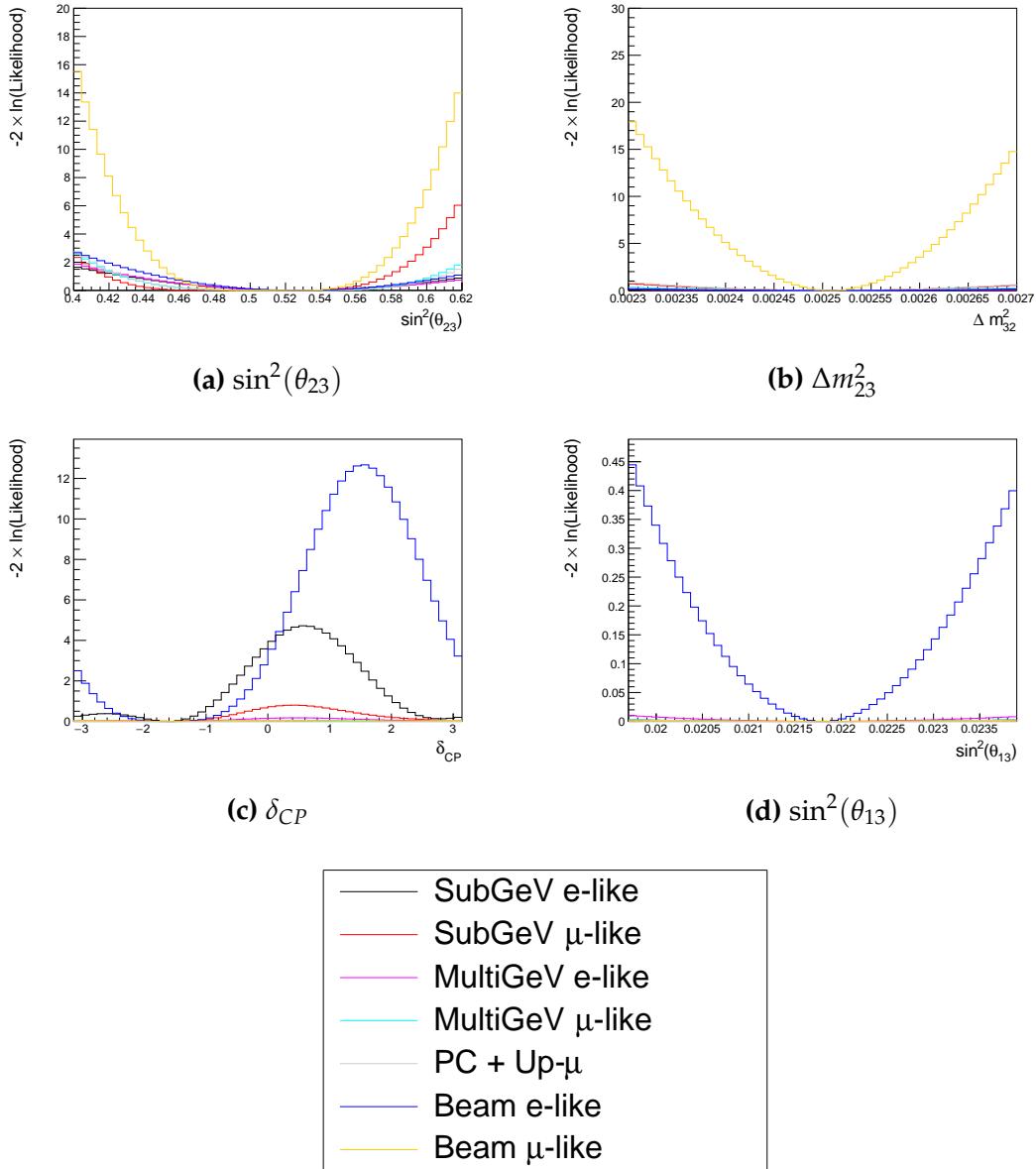


Figure 8.1: The response of the likelihood, as defined in section 6.5, illustrating the response of the samples to a variation of an oscillation parameter.

2948 The sensitivity to $\sin^2(\theta_{23})$ is mostly dominated by the beam muon-like
 2949 samples. The response of an individual atmospheric sample is small but non-
 2950 negligible such that the summed response over all atmospheric samples becomes
 2951 comparable to that of the muon-like beam samples. Consequently, the sensitivity

of the joint fit to $\sin^2(\theta_{23})$ would be expected to be greater than the beam-only analysis. The only sample that responds to the $\sin^2(\theta_{13})$ oscillation parameter is the electron-like beam sample. Consequently, no increase in sensitivity beyond that of the T2K-only analysis would be expected from the joint fit. Regardless, the sensitivity of the beam sample is significantly weaker than the external reactor constraint so prior knowledge will dominate any sensitivity to $\sin^2(\theta_{13})$ which is included within this thesis. The Δm_{21}^2 and $\sin^2(\theta_{12})$ parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to Δm_{32}^2 is completely dominated by the beam muon-like samples. This is because the beam neutrino energy is specifically tuned to match the maximal disappearance probability. Despite this, improvements to the $|\Delta m_{32}^2|$ sensitivity may be expected due to additional mass hierarchy determination added by the atmospheric samples.

Two-dimensional scans of the appearance ($\sin^2(\theta_{13}) - \delta_{CP}$) and disappearance ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) parameters are illustrated in Figure 8.2 and Figure 8.3, respectively. The caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still apply.

The appearance log-likelihood scans show the distinct difference in how the beam and atmospheric samples respond. The beam samples have an approximately constant width of the 2σ and 3σ contours, throughout all ranges of δ_{CP} . Whereas, the response of the atmospheric samples to $\sin^2(\theta_{13})$ is very strongly correlated to the value of δ_{CP} . At higher values of $\sin^2(\theta_{13})$, two lobes appear around $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2.4$. Consequently, this difference allows some of the degeneracy in a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood scans, the 2σ continuous contour in δ_{CP} for beam samples becomes closed when the atmospheric samples are added. This may result in a stronger sensitivity to δ_{CP} . Similarly, the width of the 3σ contours also becomes dependent upon the value of δ_{CP} . Furthermore, atmospheric samples have little sensitivity to $\sin^2(\theta_{13})$ on their own, as evidenced in Figure 8.1, but may improve sensitivity to the parameter when combined within the simultaneous

2982 fit. It is important to remember that these likelihood scans are not sensitivity
2983 measurements as the systematic parameters are fixed and the correlation between
2984 oscillation parameters is neglected. However, they are a very encouraging result
2985 for the joint fit.

2986 The disappearance log-likelihood scans in $\sin^2(\theta_{23}) - \Delta m_{32}^2$ space (Figure 8.3)
2987 show the expected behaviour when considering the one-dimensional scans
2988 already discussed. The uncertainty on the width of $|\Delta m_{32}^2|$ is mostly driven by the
2989 beam samples. However, the width of this contour in the inverted mass region
2990 ($\Delta m_{32}^2 < 0$) is significantly reduced due to the ability of the atmospheric samples
2991 to select the correct (normal) mass hierarchy. The width of the uncertainty
2992 in $\sin^2(\theta_{23})$ is also reduced compared to the beam-only sensitivities, with a
2993 further decrease in the inverted hierarchy region due to the better mass hierarchy
2994 determination.

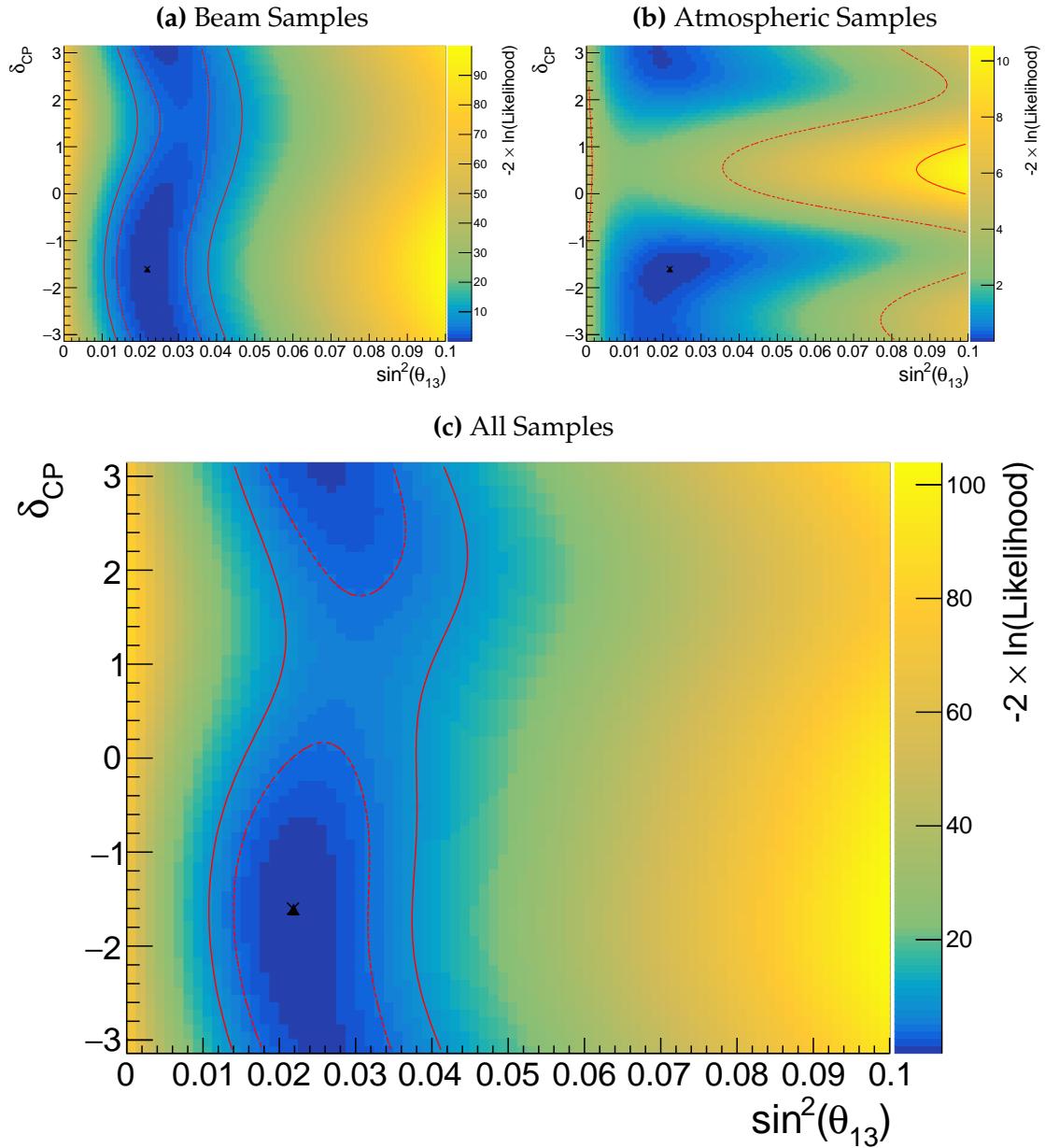


Figure 8.2: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})$)– δ_{CP}) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

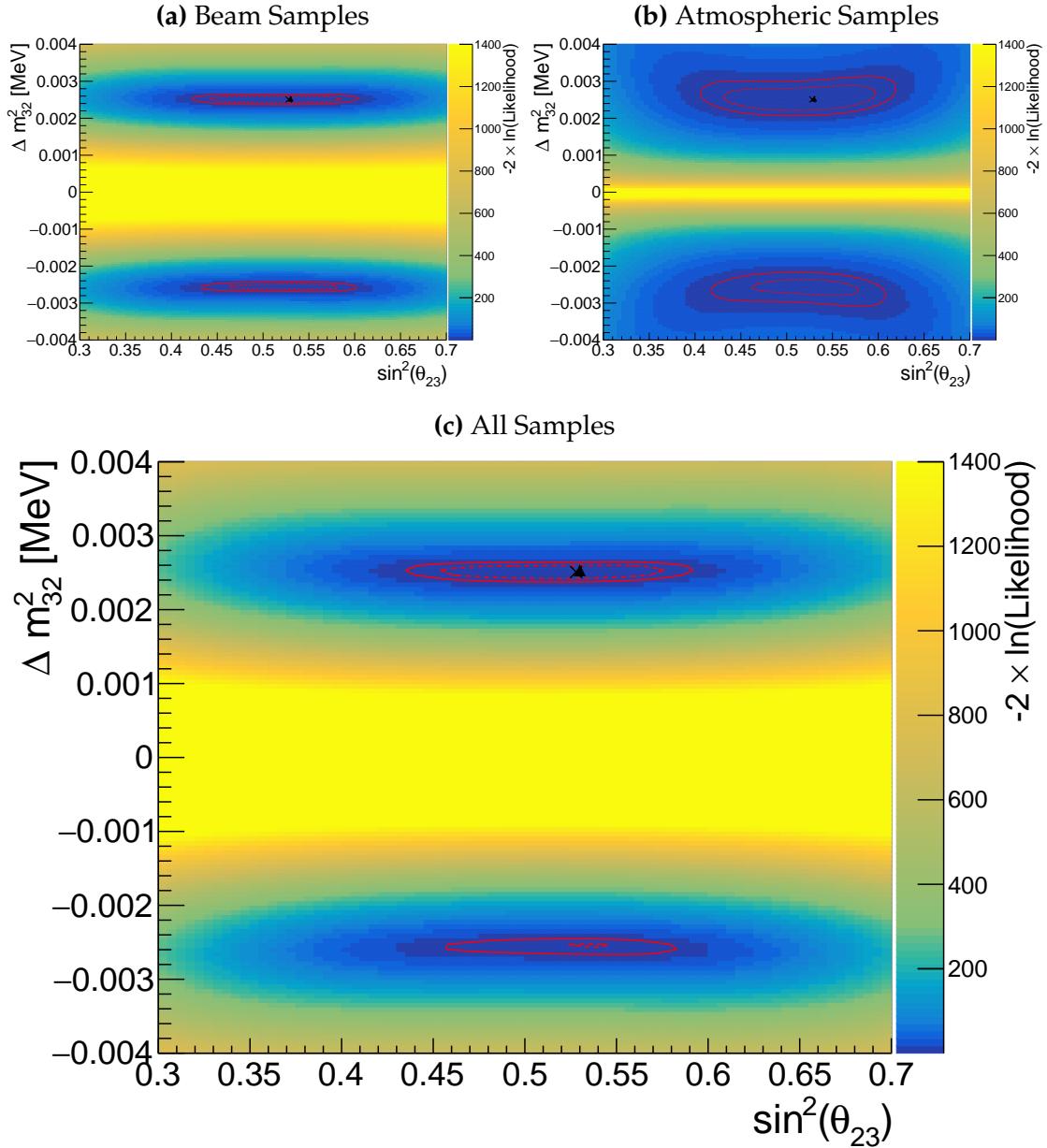


Figure 8.3: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23})$)– Δm_{32}^2) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

The likelihood scans illustrated thus far only consider the sensitivity of this analysis for a fixed set of true oscillation parameters, namely Asimov A defined in Table 2.2. Whilst computationally infeasible to run many fits at different parameter sets, it is possible to calculate the likelihood response to different Asimov data sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing true values of δ_{CP} and $\sin^2(\theta_{23})$, respectively. For both of these plots, the other oscillation parameters are fixed at their Asimov A values. Consequently, the caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still applies.

To explain how these plots are made, consider Figure 8.4. This plot is built by considering multiple one-dimensional log-likelihood scans, each creating an Asimov data set with the value of δ_{CP} taken from the x-axis. The likelihood to this particular Asimov data set is calculated after reweighting the Monte Carlo prediction to each value of δ_{CP} on the y-axis.

Figure 8.4 illustrates the sensitivity to δ_{CP} . To interpret this plot, larger contours result in more phase space being excluded from the 1σ region. The 1σ intervals contain regions where the beam and atmospheric samples have discontinuous contours. For example, for the x-axis value of $\delta_{CP} = 0$, the beam samples sensitivity would include two discontinuous regions excluded from the 1σ interval: $\delta_{CP} \sim 0$ and $\delta_{CP} \sim \pi$. This behaviour is also seen in atmospheric samples response but at a value of $\delta_{CP} \sim -1$. This difference allows the joint fit to have increased sensitivity to these regions. Consequently, the difference between the beam-only and joint beam-atmospheric fit should be studied using multiple Asimov data sets.

Despite the increased sensitivity at 1σ , the 2σ intervals from the joint fit are more similar to the two independent sensitivities and the off-diagonal degeneracies mostly remain. This indicates that the joint fit has the strength to aid parameter determination but can not entirely break the degeneracies in δ_{CP} at higher confidence levels.

Figure 8.5 illustrates a similar analysis as above, although the value of $\sin^2(\theta_{23})$ is varied and δ_{CP} is fixed to the Asimov A parameter value. Due to the beam parameters and baseline being tuned to specifically target this oscillation parameter, the average sensitivity of the beam samples is stronger than the atmospheric samples. However, the degeneracy around maximal mixing ($\sin^2(\theta_{23}) = 0.5$) is significantly more peaked in the beam samples compared to the atmospheric samples. This means that a value of $\sin^2(\theta_{23}) \sim 0.56$ would be contained within the 1σ confidence interval for a true value of $\sin^2(\theta_{23}) \sim 0.46$ if using the beam-only analysis, whereas it would be excluded in the joint analysis.

This behaviour is strengthened when considering the 2σ intervals, to the point where two distinct discontinuous regions of the 2σ intervals exist around the Asimov point $\sin^2(\theta_{23}) \sim 0.41, 0.6$. Given the caveat of only considering likelihood scans, the joint analysis would mostly eliminate the discontinuous intervals in these regions. This means that the joint fit could feasibly have an increased preference for the correct octant hypothesis.

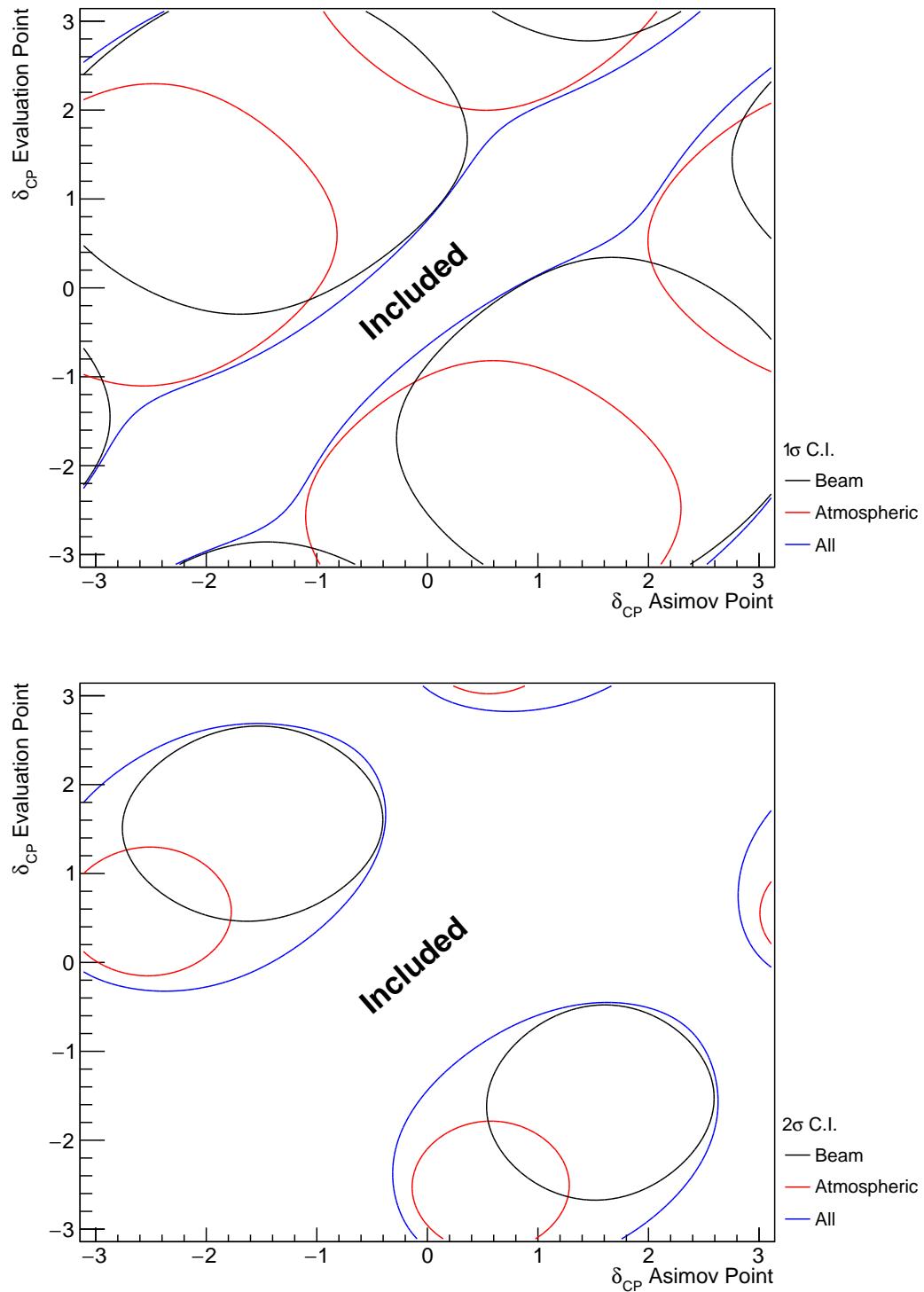


Figure 8.4: A series of one-dimensional likelihood scans over δ_{CP} , where an Asimov data set is built for each value of δ_{CP} on the x-axis and the likelihood is evaluated for each value of δ_{CP} on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

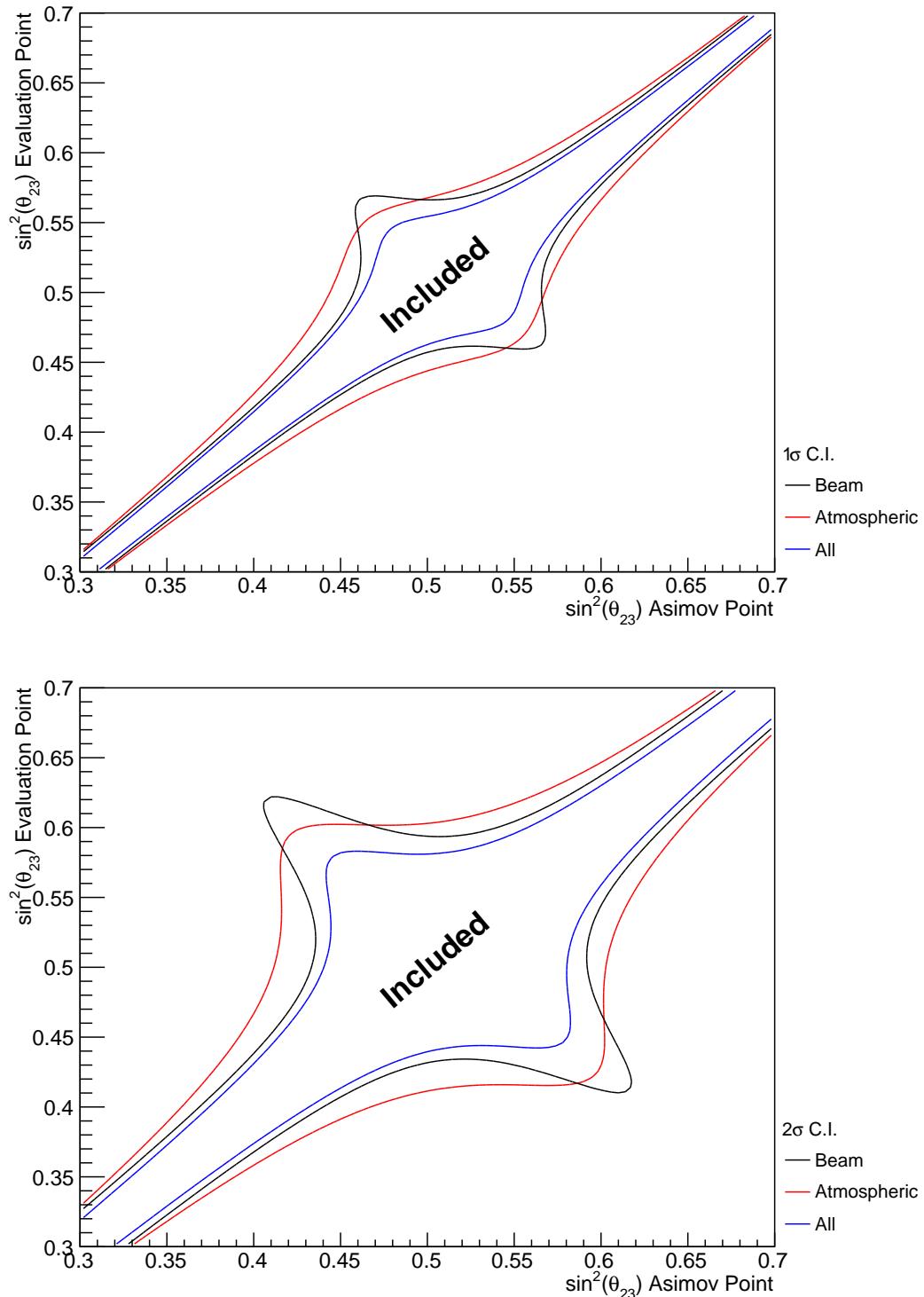


Figure 8.5: A series of one-dimensional likelihood scans over $\sin^2(\theta_{23})$, where an Asimov data set is built for each value of $\sin^2(\theta_{23})$ on the x-axis and the likelihood is evaluated for each value of $\sin^2(\theta_{23})$ on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions, M_A^{QE} , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h CtoO normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics that have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric and beam samples can have similar sensitivity to these systematics due to their similar composition in energy and interaction mode. As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.

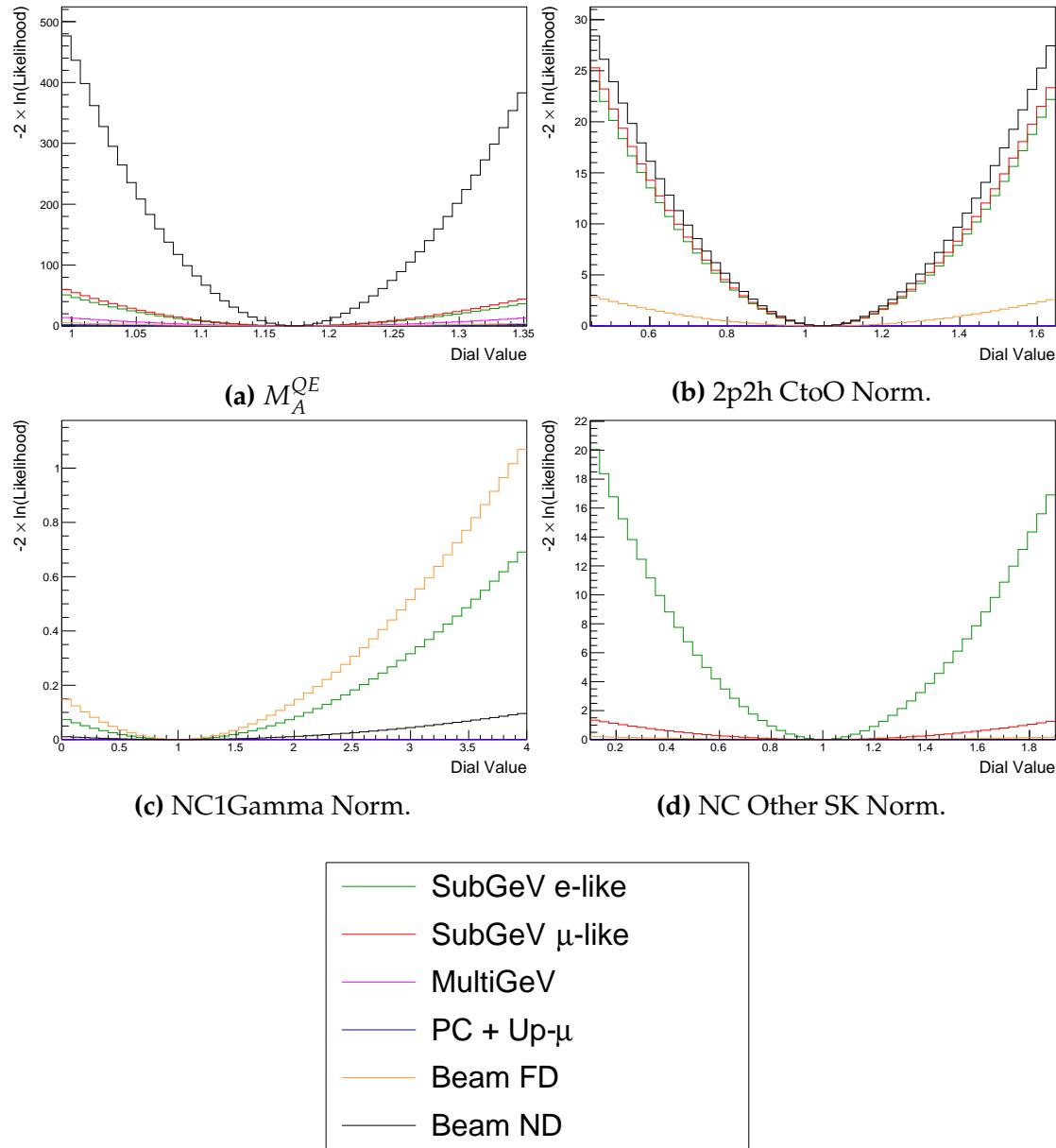


Figure 8.6: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

3064 8.3 Sensitivity Studies

3065 DB: Statistics vs Systematics dominated

3066 The sensitivities of the joint T2K and SK oscillation analysis are presented
3067 in the form of Asimov fits. These fits consider beam samples from the near
3068 and far detector alongside atmospheric samples at SK. This technique builds an
3069 Asimov data set (following section 6.5) using the AsimovA oscillation parameters
3070 and post-BANFF systematic tune, which is then fit. This technique eliminates
3071 statistical fluctuations from the data, therefore, providing the maximum sen-
3072 sitivity of the analysis.

3073 In practice, the Asimov fits presented within this analysis are modified from
3074 the above definition. An Asimov prediction of both beam and atmospheric far
3075 detector samples is fit whilst the true data is used for near detector samples. The
3076 Asimov predictions at the far detector are built using the post-BANFF tune (as
3077 discussed in section 3.2). These modifications mean that the results are equivalent
3078 to performing a far detector Asimov fit using inputs from the BANFF data fit.
3079 Consequently, this allows the results to be cross-checked with the results from
3080 the P-Theta analysis. The comparison has been performed and is documented in
3081 [214]. No significant discrepancies were found between the fitters.

3082 This section proceeds with the following studies. Firstly, the sensitivity
3083 of the atmospheric samples using the correlated detector model is detailed in
3084 subsection 8.3.1. This includes studying the choice of applying the 2020 PDG
3085 reactor constraint [75] to the atmospheric samples, which is documented in
3086 subsection 8.3.2. Additionally, the effect of applying the near-detector constraints
3087 onto the atmospheric samples is discussed in subsection 8.3.3. The main result is
3088 the sensitivity of the simultaneous beam and atmospheric fit. The sensitivities,
3089 both with and without the application of the reactor constraint, are presented
3090 in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate the benefit
3091 of the joint analysis, the sensitivities are compared to the 2020 T2K beam-only
3092 sensitivities [2, 215] in subsection 8.3.6 and subsection 8.3.7. The T2K analysis

3093 is used as a reference as it uses the same samples and a similar systematic
 3094 model. As shown in section 8.2, the response of the beam and atmospheric
 3095 samples change depending upon the true set of oscillation parameters assumed.
 3096 Therefore, subsection 8.3.8 documents the sensitivities at an alternative oscillation
 3097 parameter set. These results have been presented at the Neutrino 2022 conference
 3098 on behalf of the T2K and SK collaborations [105].

3099 8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

3100 This section presents the results of an Asimov fit using samples from the near
 3101 detector and only atmospheric samples from the far detector. The results are
 3102 presented as one-dimensional or two-dimensional histograms which have been
 3103 marginalised over all other parameters using the technique outlined in sub-
 3104 section 4.3.1. Each histogram displays the posterior probability density and
 3105 illustrates the credible intervals, calculated using the technique in subsection 4.3.2.
 3106 For this fit, a flat prior is used for $\sin^2(\theta_{13})$ meaning that the reactor constraint is
 3107 not applied. The Asimov data is generated assuming the AsimovA oscillation pa-
 3108 rameter set defined in Table 2.2 and the post-BANFF systematic parameter tune.

3109 Figure 8.7 illustrates the posterior probability density for δ_{CP} , marginalised
 3110 over both hierarchies. The fit favours the known oscillation parameter ($\delta_{CP} =$
 3111 -1.601) although the posterior probability is very flat through the range of
 3112 $-\pi < \delta_{CP} < -1$ and $2 < \delta_{CP} < \pi$. There is also a region around $\delta_{CP} \sim 0.4$
 3113 which is disfavoured at 2σ . This indicates that the SK samples can rule out some
 3114 parts of the CP conserving parameter space reasonably well, near $\delta_{CP} \sim 0.4$,
 3115 when the true value of $\delta_{CP} \sim -\pi/2$.

3116 The posterior probability density in Δm_{32}^2 is given in Figure 8.8. This distribu-
 3117 tion includes steps in both the normal hierarchy (NH, $\Delta m_{32}^2 > 0$) and the inverse
 3118 hierarchy (IH, $\Delta m_{32}^2 < 0$). The highest posterior probability density is found
 3119 within the NH 1σ credible interval, which agrees with the known oscillation
 3120 parameter value, $2.509 \times 10^{-3}\text{eV}^2$. However, all of the credible intervals span
 3121 both of the hierarchy hypotheses.

Without Reactor Constraint, Both Hierarchies

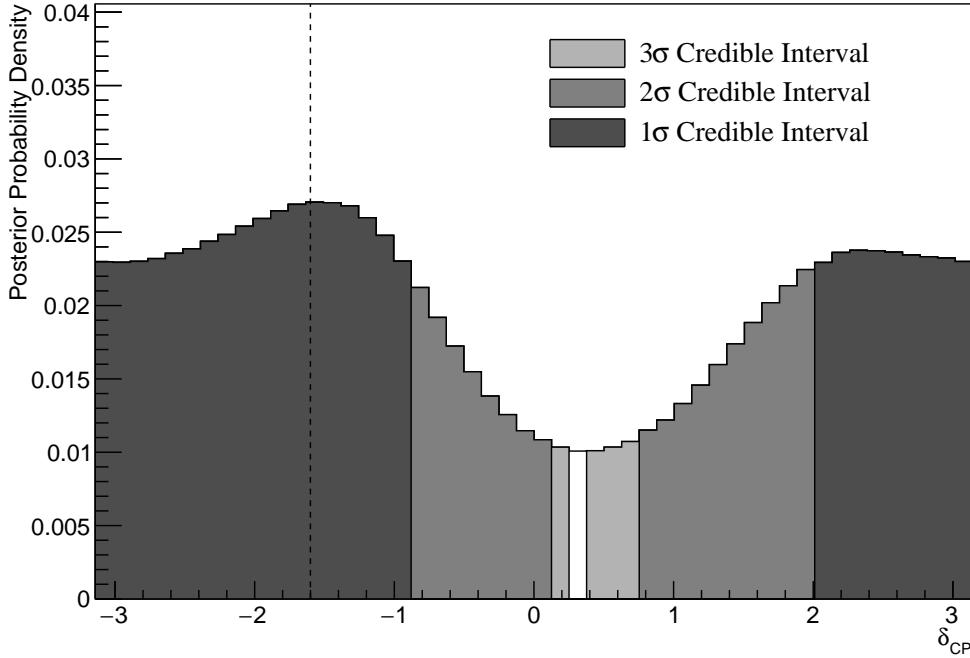


Figure 8.7: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.17	0.40	0.58
IH ($\Delta m_{32}^2 < 0$)	0.13	0.29	0.42
Sum	0.31	0.69	1.00

Table 8.2: The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.37$ and $B(\text{UO}/\text{LO}) = 2.24$.

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO, $\sin^2 \theta_{23} < 0.5$) or upper octant (UO, $\sin^2 \theta_{23} > 0.5$). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy hypothesis is $B(\text{NH}/\text{IH}) = 1.37$. Jeffrey's

Without Reactor Constraint, Both Hierarchies

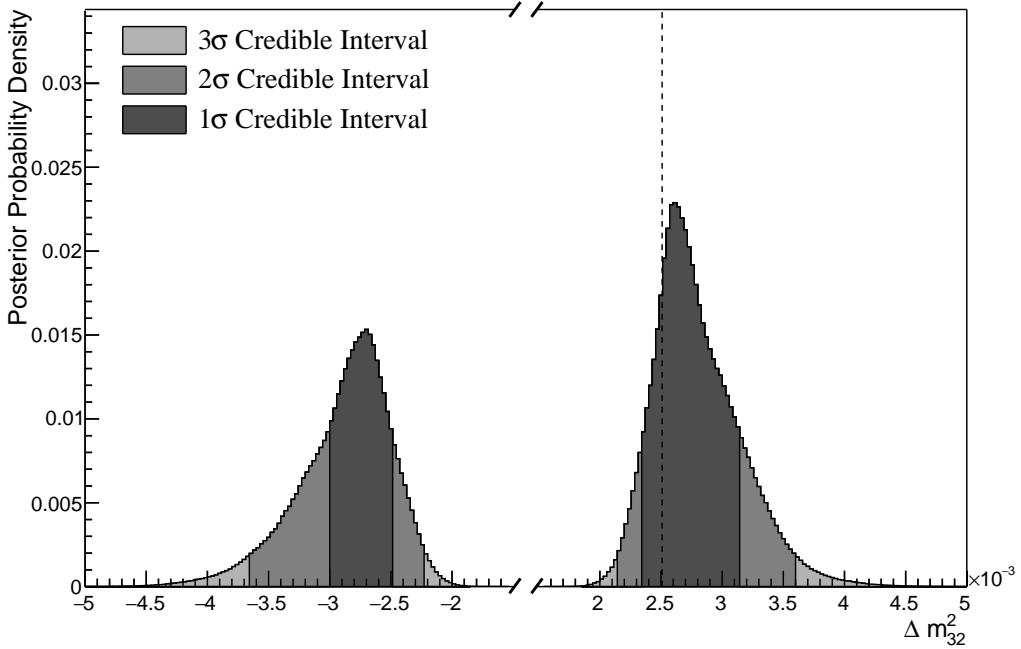


Figure 8.8: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

scale, given in Table 4.1, states this value of the Bayes factor indicates a weak preference for the normal hierarchy hypothesis. The Bayes factor for choice of octant is $B(\text{UO}/\text{LO}) = 2.24$. This is also classified as a weak preference for the UO. Both of these show that the fit is returning the correct choice of hypotheses (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The 1σ credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. As the distribution is binned, the highest posterior density is presented as the center of the bin with the highest posterior density with an error equal to the bin width. For the known Asimov value of $\delta_{CP} = -1.601$, the 1σ credible interval rules out a region between $\delta_{CP} = -0.88$ and $\delta_{CP} = 1.96$, when marginalising over both hierarchies. The position of the highest posterior density is $\delta_{CP} = -1.57 \pm 0.07$ which is clearly

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-\pi, -0.88], [1.88, \pi]$	-1.57 ± 0.07
δ_{CP} , (IH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-3.00, -2.49], [2.34, 3.14]$	2.61 ± 0.02
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.41, 3.04]$	2.59 ± 0.03
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-3.11, -2.41]$	-2.73 ± 0.03
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.584]$	0.542 ± 0.006
$\sin^2(\theta_{23})$ (NH)	$[0.488, 0.596]$	0.554 ± 0.006
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.584]$	0.542 ± 0.006

Table 8.3: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3143 compatible with the known oscillation parameter value.

3144 The sensitivity of the atmospheric samples to $\sin^2(\theta_{13})$ is presented in Fig-
3145 ure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity
3146 to $\sin^2(\theta_{13})$ will be small. This behaviour is also seen in the fit results, where the
3147 width of the 1σ credible intervals span the region of $\sin^2(\theta_{13}) = [0.008, 0.08]$. This
3148 is more than an order of magnitude worse than the constraint from reactor
3149 experiments [75].

3150 As previously discussed, the correlations between oscillation parameters are
3151 also important to understand how the atmospheric samples respond. Figure 8.10
3152 illustrates the two dimensional $\sin^2(\theta_{13})-\delta_{CP}$ sensitivity, marginalised over all
3153 other parameters. The shape of the 1σ credible interval shows that the constrain-
3154 ing power of the fit on δ_{CP} is dependent upon the value of $\sin^2(\theta_{13})$. Furthermore,
3155 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.
3156 Whilst the atmospheric samples do not strongly constrain the value of $\sin^2(\theta_{13})$,
3157 the value of $\sin^2(\theta_{13})$ does impact the atmospheric samples' sensitivity to δ_{CP} .

Without Reactor Constraint, Both Hierarchies

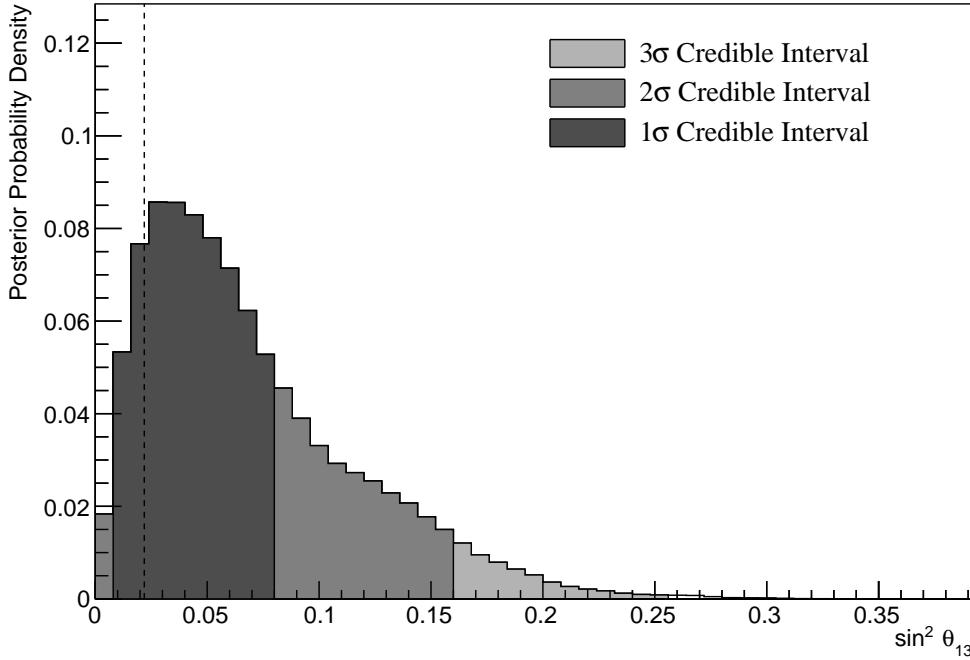


Figure 8.9: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{13})$.

3158 The $\sin^2(\theta_{23}) - \Delta m_{32}^2$ disappearance contours are illustrated in Figure 8.11. As
 3159 expected, the area contained in the inverted hierarchy 1σ credible interval is
 3160 slightly smaller than that in the normal hierarchy. This follows from the Bayes
 3161 factor showing a weak preference for NH meaning that more of the steps will exist
 3162 in the $\Delta m_{32}^2 > 0$ region. The known oscillation parameters of $\sin^2(\theta_{23}) = 0.528$
 3163 and $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ are contained within the 1σ credible interval.

3164 Figure 8.12 illustrates the two-dimensional projections for each permutation of
 3165 oscillation parameters which this analysis is sensitive to: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$,
 3166 and Δm_{32}^2 . The purpose of this plot is to illustrate the correlations between
 3167 the oscillation parameters. The contours are calculated whilst marginalising
 3168 over both hierarchies, however, only the NH is illustrated when plotting the
 3169 Δm_{32}^2 parameter. As expected the correlations play a significant role in these
 3170 sensitivity measurements, especially the choice of the $\sin^2(\theta_{13})$ constraint. Most
 3171 notably, the application of reactor constraint would be expected to alter both the

Without Reactor Constraint, Both Hierarchies

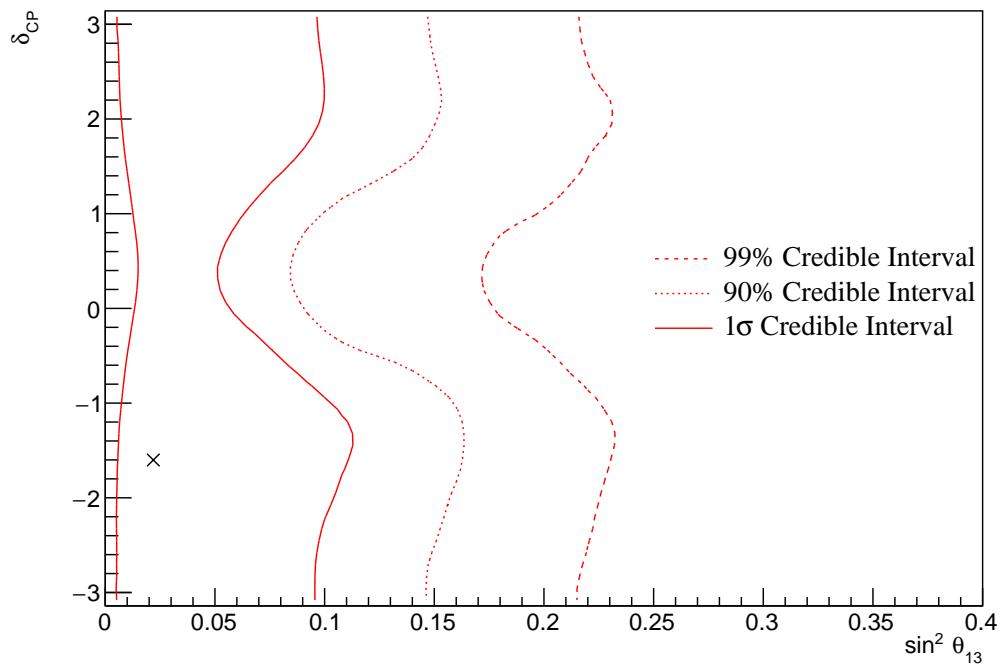


Figure 8.10: The two-dimensional posterior probability density distribution in δ_{CP} - $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of δ_{CP} - $\sin^2(\theta_{13})$.

³¹⁷² width and position of the Δm_{32}^2 intervals due to the strong correlation between
³¹⁷³ the parameters.

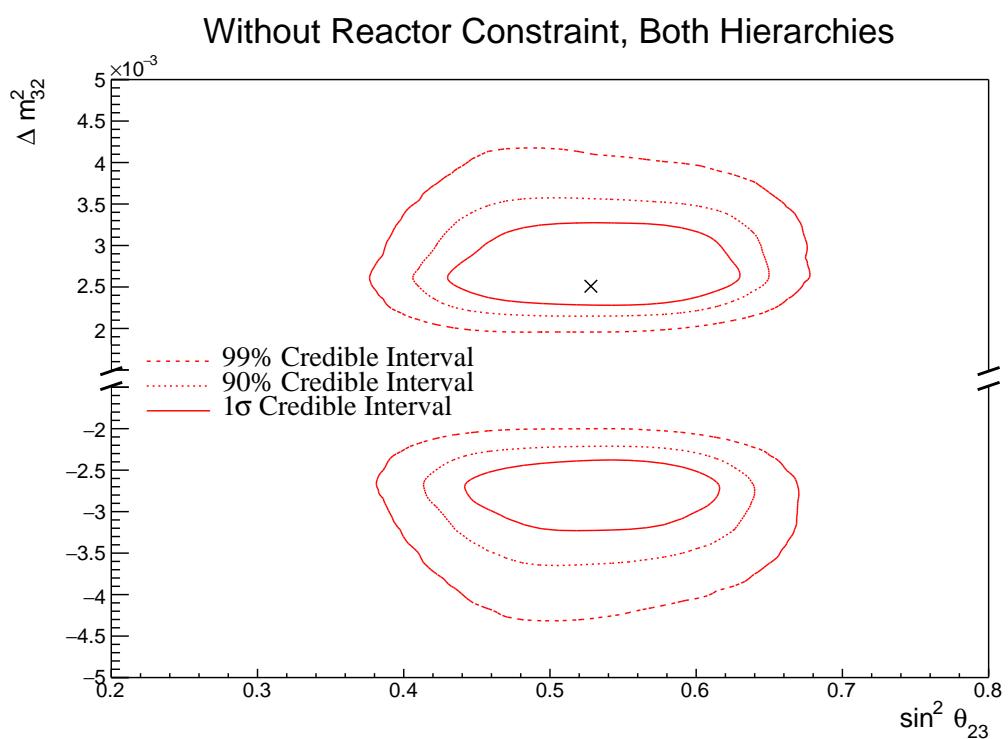


Figure 8.11: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

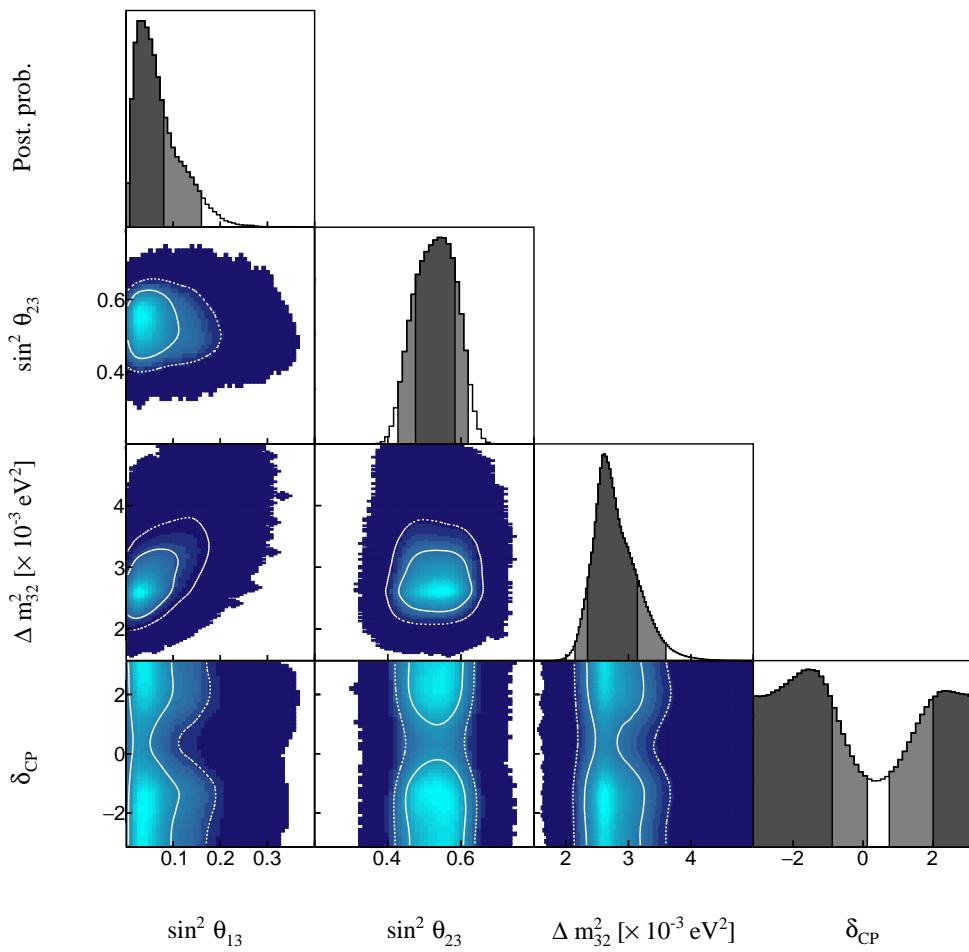


Figure 8.12: The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3174 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

3175 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor
 3176 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that
 3177 the marginalisation effects could contribute to differing sensitivities when the
 3178 external reactor constraint is applied. Using the technique discussed in subsec-
 3179 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted
 3180 to include the reactor constraint of $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ [75].

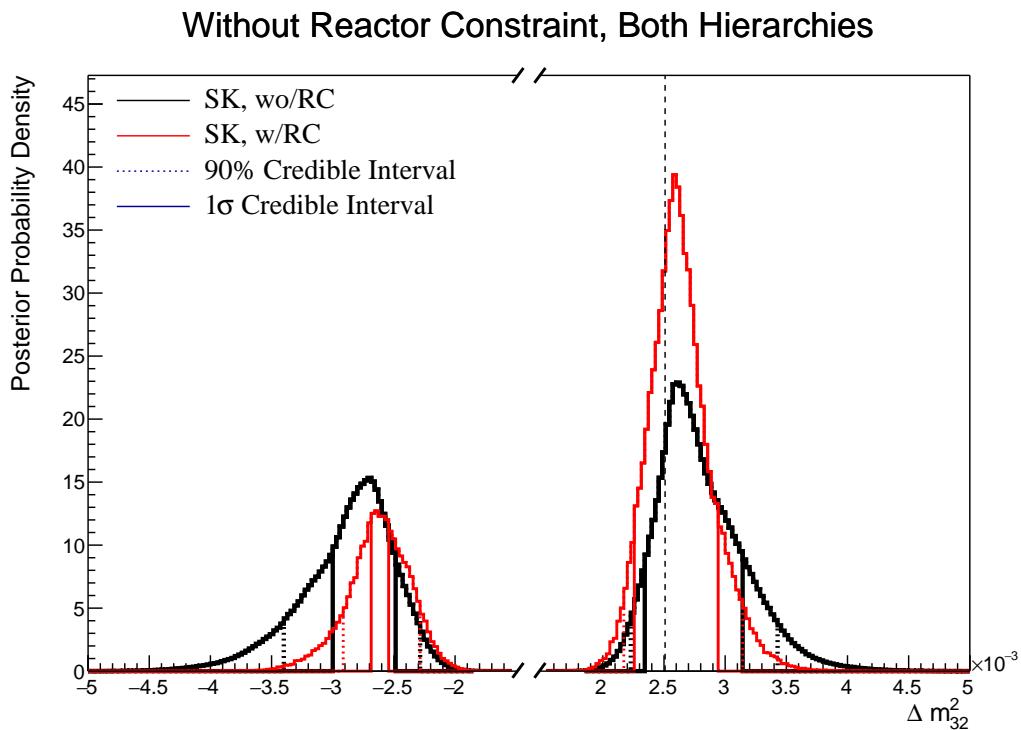


Figure 8.13: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of Δm_{32}^2 .

3181 The reactor constraint increases the sensitivity of the atmospheric samples to
 3182 Δm_{32}^2 as illustrated in Figure 8.13. The 1σ credible interval in Δm_{32}^2 is determined
 3183 to be $[-2.69, -2.54] \times 10^{-3} \text{ eV}^2$ and $[2.25, 2.94] \times 10^{-3} \text{ eV}^2$. The width of the IH
 3184 credible interval is reduced by $\sim 70\%$ when the reactor constraint is applied. Due
 3185 to the marginalisation effects observed in Figure 8.12, the favoured region of Δm_{32}^2
 3186 moves closer to zero for both hierarchies. A clear explanation of this behaviour is

3187 illustrated in Figure 8.14, which shows the posterior distribution in the Δm_{32}^2 –
 3188 $\sin^2(\theta_{13})$ parameters. The correlation between Δm_{32}^2 and $\sin^2(\theta_{13})$ is such that
 3189 lower values of $\sin^2(\theta_{13})$ tend towards lower values of $|\Delta m_{32}^2|$. Therefore the
 3190 application of the reactor constraint moves the posterior distribution towards
 3191 the known oscillation parameter.

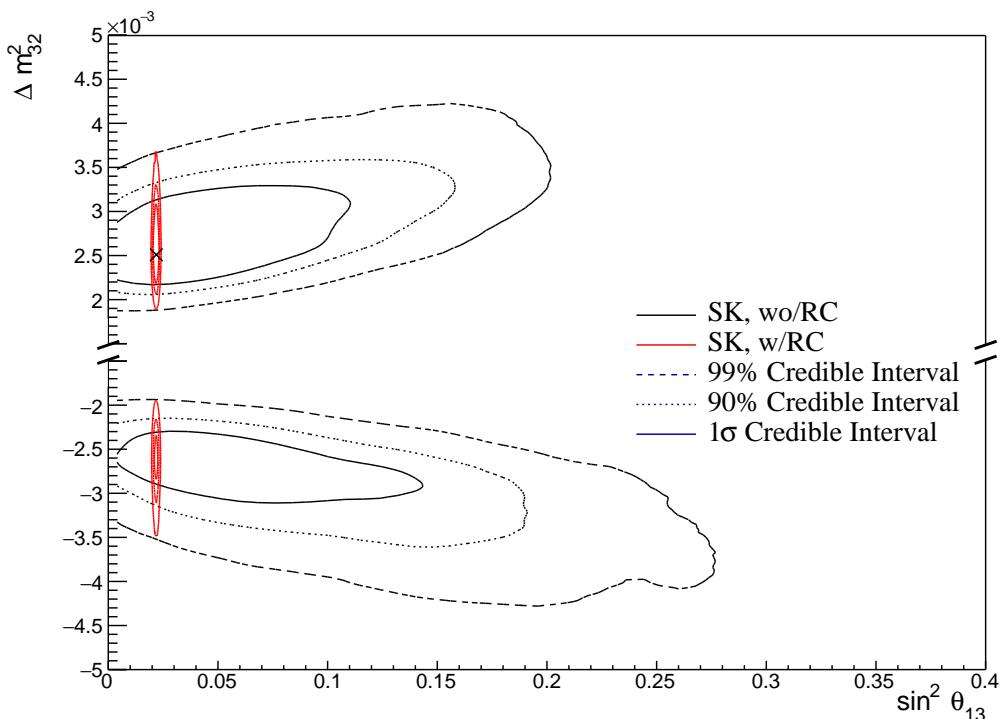


Figure 8.14: The two-dimensional posterior probability density distribution in Δm_{32}^2 – $\sin^2(\theta_{13})$ compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies. The marker represents the known value of Δm_{32}^2 – $\sin^2(\theta_{13})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.21	0.53	0.74
IH ($\Delta m_{32}^2 < 0$)	0.08	0.18	0.26
Sum	0.29	0.71	1.00

Table 8.4: The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 2.85$ and $B(\text{UO}/\text{LO}) = 2.39$.

3192 Table 8.4 presents the fraction of steps in each hierarchy and octant model
3193 for the fit after the reactor constraint has been applied. The reactor constraint
3194 significantly increases the NH preference, increasing the Bayes factor from
3195 $B(\text{NH}/\text{IH}) = 1.37$ to $B(\text{NH}/\text{IH}) = 2.85$ when the reactor constraint is applied.
3196 This is still defined as a weak preference for the NH hypothesis according to
3197 Jeffrey's scale, however, it is a stronger preference than when the constraint is
3198 not applied. The preference for the correct octant model is also slightly increased
3199 by the application of the reactor constraint.

3200 8.3.3 Impact of Near Detector Constraints for Atmospheric Sam- 3201 ples

3202 The choice of applying the near detector constraints to the low-energy atmo-
3203 spheric samples was introduced in subsection 6.4.3. This subsection illustrates the
3204 effect of removing the ND constraint on the sensitivity of the atmospheric samples
3205 to the oscillation parameters. To do this, the fit presented in subsection 8.3.1 has
3206 been compared to another fit where the constraints from the near detector have
3207 not been included. This is the only case where the near detector constraints are
3208 neglected throughout this chapter. For both fits, the Asimov data was generated
3209 assuming the ‘AsimovA’ oscillation parameter set defined in Table 2.2 and the
3210 post-BANFF systematic parameter tune.

3211 The change in sensitivity on δ_{CP} is given in Figure 8.15. The reactor constraint
3212 is not applied in either of the fits within this comparison. The fit which includes
3213 the near detector constraint is slightly more peaked at the known oscillation
3214 parameter value. The width of the 1σ credible intervals are approximately the
3215 same (identical to within a bin width) and the same conclusion holds for the
3216 higher credible intervals. The change in sensitivity to other oscillation parameters
3217 has been studied and no significant discrepancies were found. As expected, the
3218 sensitivities are statistics dominated such that the exact choice of constraint does
3219 not significantly affect the physics conclusions one would make from this analysis.

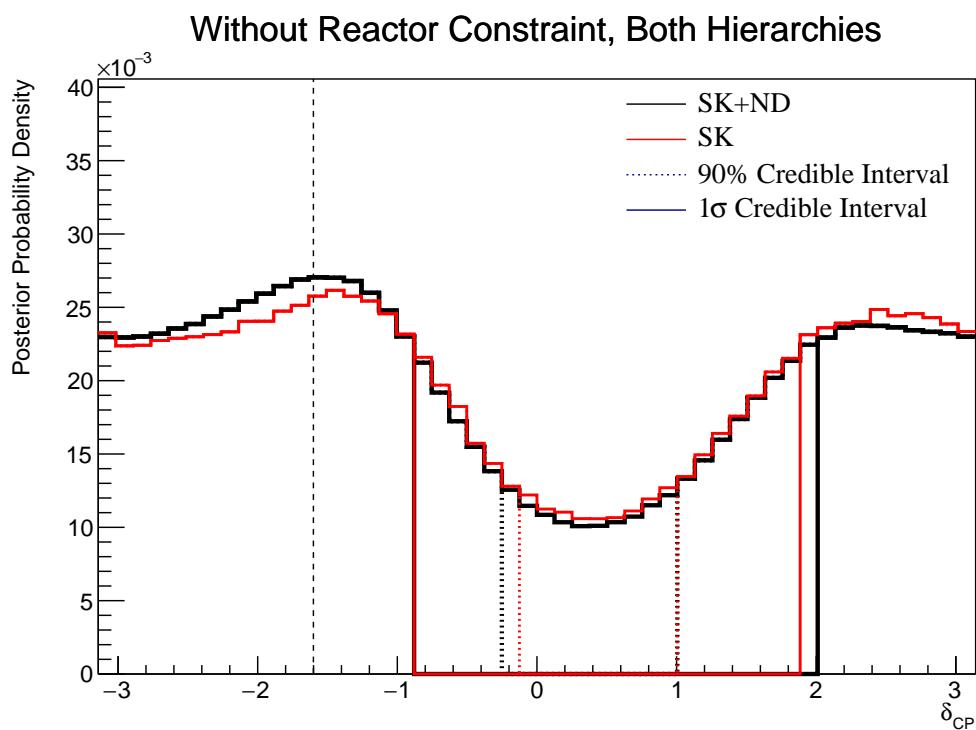


Figure 8.15: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of δ_{CP} .

3220 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3221 straint

3222 This section presents the sensitivities of the simultaneous beam and atmospheric
 3223 analysis where the reactor constraint is not applied. Similar to the previous
 3224 studies, the Asimov data is built assuming the post-BANFF systematic tune and
 3225 Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near
 3226 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.

Without Reactor Constraint, Both Hierarchies

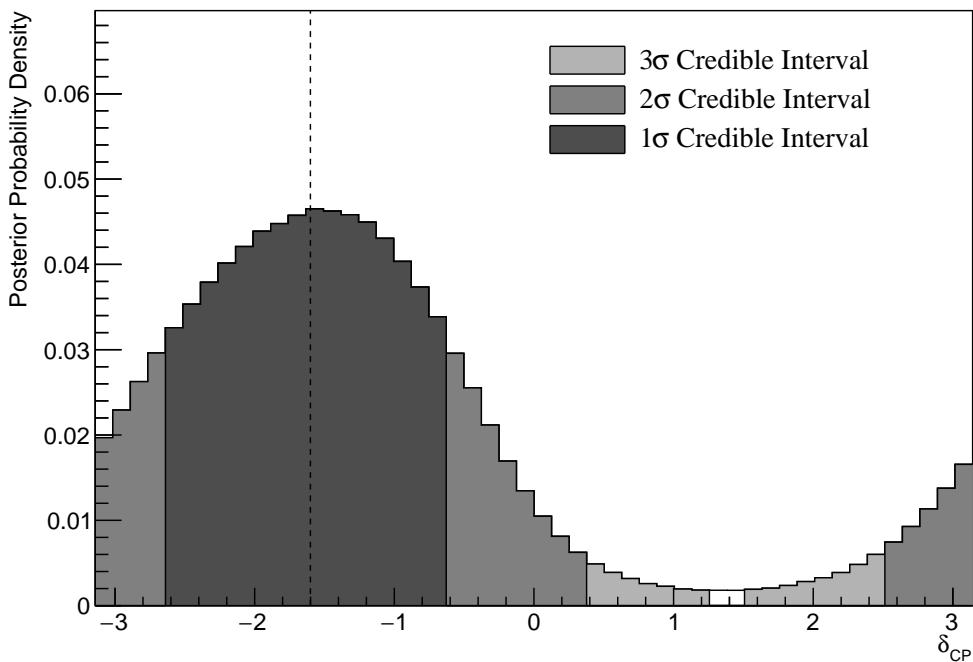


Figure 8.16: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

3227 The sensitivity to δ_{CP} , marginalised over both hierarchies, is given in Fig-
 3228 ure 8.16. The credible intervals and highest posterior distribution for each
 3229 oscillation parameter is given in Table 8.5. The highest posterior probability
 3230 density is $\delta_{CP} = -1.57 \pm 0.07$ and is compatible with the known value of
 3231 $\delta_{CP} = -1.601$. The CP-conserving values of $\delta_{CP} = 0, \pm\pi$ are disfavoured at
 3232 1σ credible interval. There is also a region around $\delta_{CP} = 1.4$ which is disfavoured
 3233 at more than 3σ . Whilst these conclusions can only be made at this particular

³²³⁴ Asimov point, it does show that if the true value of δ_{CP} were CP-violating,
³²³⁵ this joint analysis would be able to disfavour CP conserving values at over 1σ
³²³⁶ without any external constraints.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-2.64, -0.63]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-2.76, -0.63]$	-1.45 ± 0.07
δ_{CP} , (IH)	$[-2.39, -0.88]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.58]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.51]$	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.480, 0.545]	0.518 ± 0.003
$\sin^2(\theta_{23})$ (NH)	[0.480, 0.545]	0.508 ± 0.003
$\sin^2(\theta_{23})$ (IH)	[0.480, 0.545]	0.513 ± 0.003

Table 8.5: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

³²³⁷ The sensitivity to Δm_{32}^2 is illustrated in Figure 8.17. Notably, the 1σ credible
³²³⁸ interval is entirely contained within the NH region, as further evidenced by
³²³⁹ Table 8.5. This illustrates good sensitivity to the mass hierarchy as it is correctly
³²⁴⁰ selecting the correct hypothesis. This is reflected in the 1σ credible intervals being
³²⁴¹ approximately the same when they are constructed considering both hierarchies
³²⁴² and when considering only the NH region. The NH distribution favours this
³²⁴³ region surrounding the known Asimov point, $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$, where
³²⁴⁴ the highest posterior probability density is at $\Delta m_{32}^2 = (2.51 \pm 0.01) \times 10^{-3}\text{eV}^2$.

³²⁴⁵ The fraction of steps in each of the mass hierarchy regions and octants of
³²⁴⁶ $\sin^2(\theta_{23})$ is given in Table 8.6. The Bayes factors are determined to be $B(\text{NH}/\text{IH}) =$
³²⁴⁷ 3.67 and $B(\text{UO}/\text{LO}) = 1.74$. Jeffrey's scale states that this value of the mass
³²⁴⁸ hierarchy Bayes factor illustrates substantial evidence for the NH hypothesis.

3249 This corresponds to the correct hypothesis given the known oscillation parameters
 3250 and is a stronger statement than the atmospheric-only analysis can provide. It is
 3251 important to note that this substantial preference requires no external constraints.
 3252 The Bayes factor for octant determination represents a weak preference for the
 3253 upper octant, therefore, selecting the correct octant hypothesis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.29	0.50	0.79
IH ($\Delta m_{32}^2 < 0$)	0.08	0.13	0.21
Sum	0.37	0.63	1.00

Table 8.6: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 3.67$ and $B(\text{UO}/\text{LO}) = 1.74$.

Without Reactor Constraint, Both Hierarchies

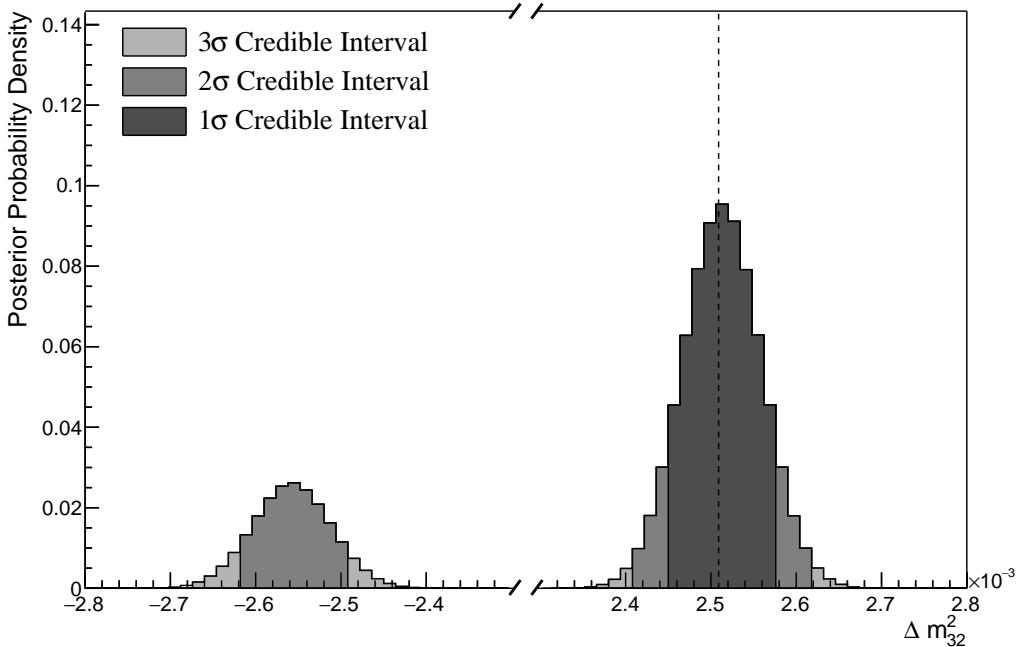


Figure 8.17: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

3254 The sensitivity to $\sin^2(\theta_{23})$ is presented in Figure 8.18. There is a clear

3255 preference for the upper octant but the peak of the distribution is relatively
 3256 flat. It peaks at $\sin^2(\theta_{23}) = 0.509 \pm 0.003$ which is in the region of the known
 3257 value of $\sin^2(\theta_{23}) = 0.528$. The difference in the highest posterior distribution
 3258 and the width of the credible interval is relatively unchanged when consid-
 3259 ering different hierarchy hypotheses showing no strong correlation between
 3260 $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

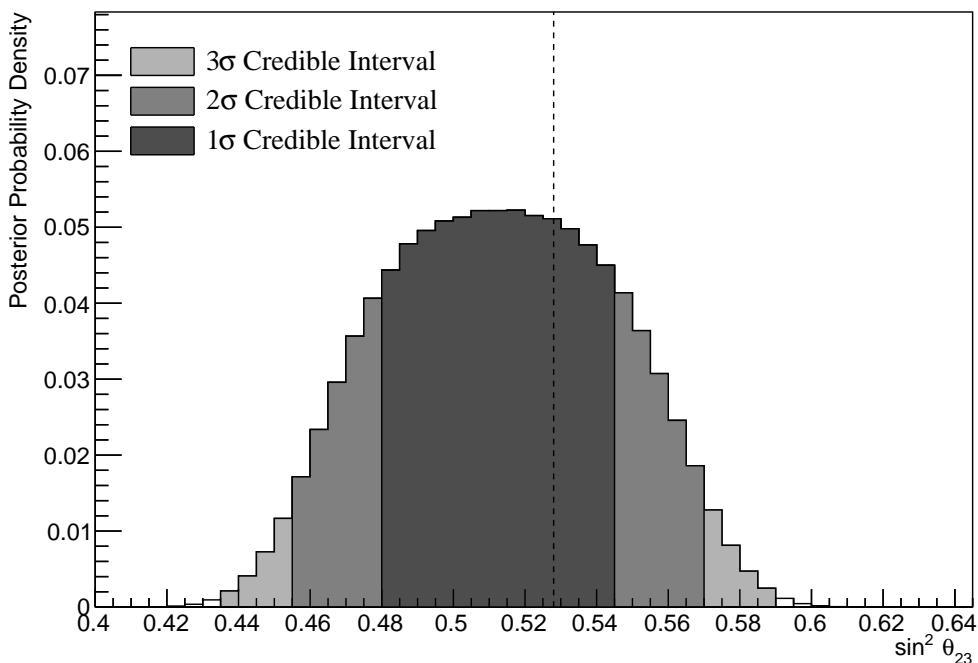


Figure 8.18: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3261 The sensitivity presented as a function of the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$)
 3262 is given in Figure 8.19. As expected, the contours follow the likelihood shape
 3263 given in Figure 8.2, where the 2σ credible intervals have a closed contour exclu-
 3264 ding the region around $\delta_{CP} \sim 1.2$. The width of the 3σ credible interval in $\sin^2(\theta_{13})$
 3265 is dependent upon the value of δ_{CP} . Close to the Asimov point, $\delta_{CP} = -1.601$, the
 3266 width of the 3σ credible interval approximately spans $\sin^2(\theta_{13}) = [0.013, 0.04]$.
 3267 This is reduced to a region of $\sin^2(\theta_{13}) = [0.023, 0.042]$ at the most disfavoured
 3268 value of δ_{CP} . The 1σ credible interval is consistent with the known oscillation

parameter. Application of the reactor constraint would be expected to decrease the width of the 1σ credible intervals in δ_{CP} due to the triangular shape of the posterior probability.

The sensitivity in terms of the disappearance parameters, $\sin^2(\theta_{23}) - \Delta m_{32}^2$, is given in Figure 8.20. The area contained within the IH contours is significantly smaller than the area within the NH contours. The IH credible intervals are also notably tighter in the $\sin^2(\theta_{23})$ dimension. No significant correlation is observed between $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

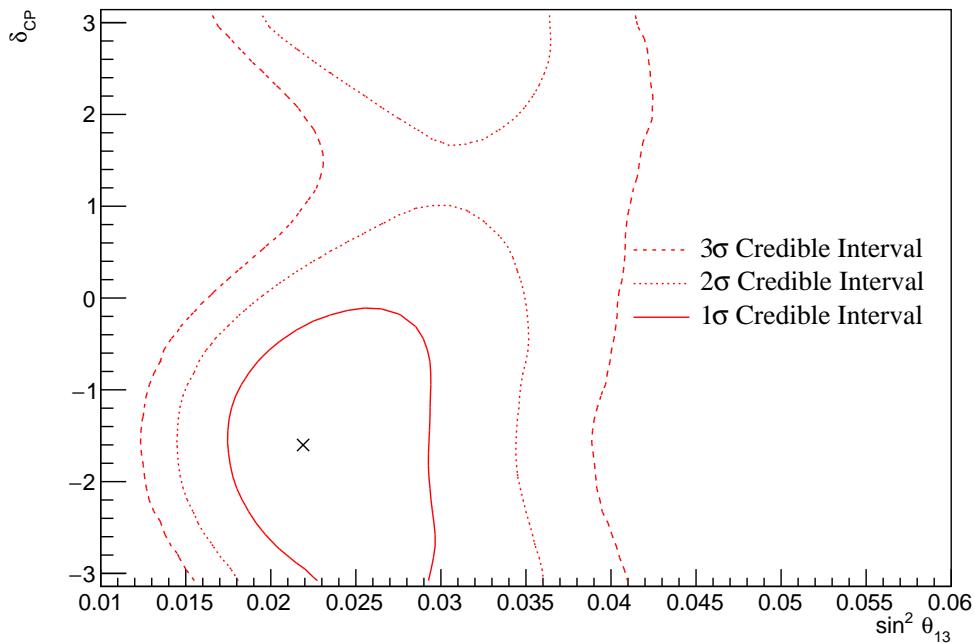


Figure 8.19: The two-dimensional posterior probability density distribution in $\delta_{CP}-\sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\delta_{CP}-\sin^2(\theta_{13})$.

The two-dimensional posterior distribution for each permutation of the oscillation parameters of interest is given in Figure 8.21. The most notable observation is that the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ are anti-correlated. If the value of $\sin^2(\theta_{13})$ was constrained closer to the known oscillation parameter value, the preferred value of $\sin^2(\theta_{23})$ would increase. This would move the highest posterior probability

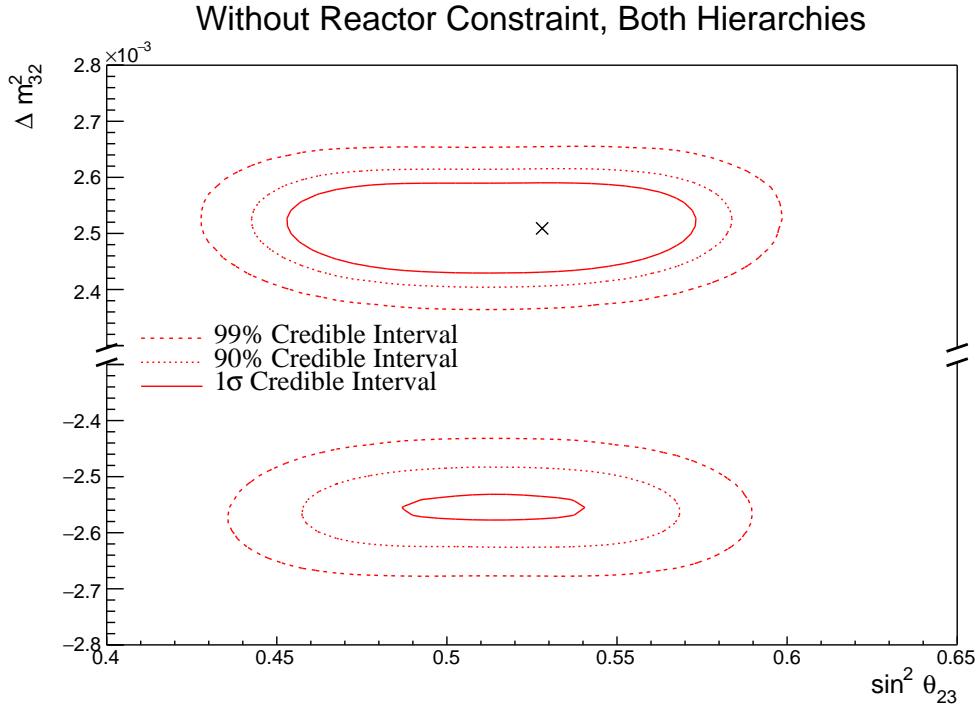


Figure 8.20: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

closer in line with the known value and could lead to an increase in the preference for the UO hypothesis.

Furthermore, the δ_{CP} and $|\Delta m_{32}^2|$ oscillation parameters are anti-correlated, such that higher values of $|\Delta m_{32}^2|$ prefer lower values of δ_{CP} . Whilst this is an interesting result on its own, the width of the Δm_{32}^2 contours also depend on $\sin^2(\theta_{13})$. This introduces another correlation effect that could modify the sensitivity to δ_{CP} once the reactor constraint is applied.

The correlation between $\sin^2(\theta_{13})$ and Δm_{32}^2 can be seen in Figure 8.22. A much larger fraction of the posterior distribution is contained in the NH for lower values of $\sin^2(\theta_{13})$. Consequently, the application of the reactor constraint would be expected to significantly increase the preference for NH.

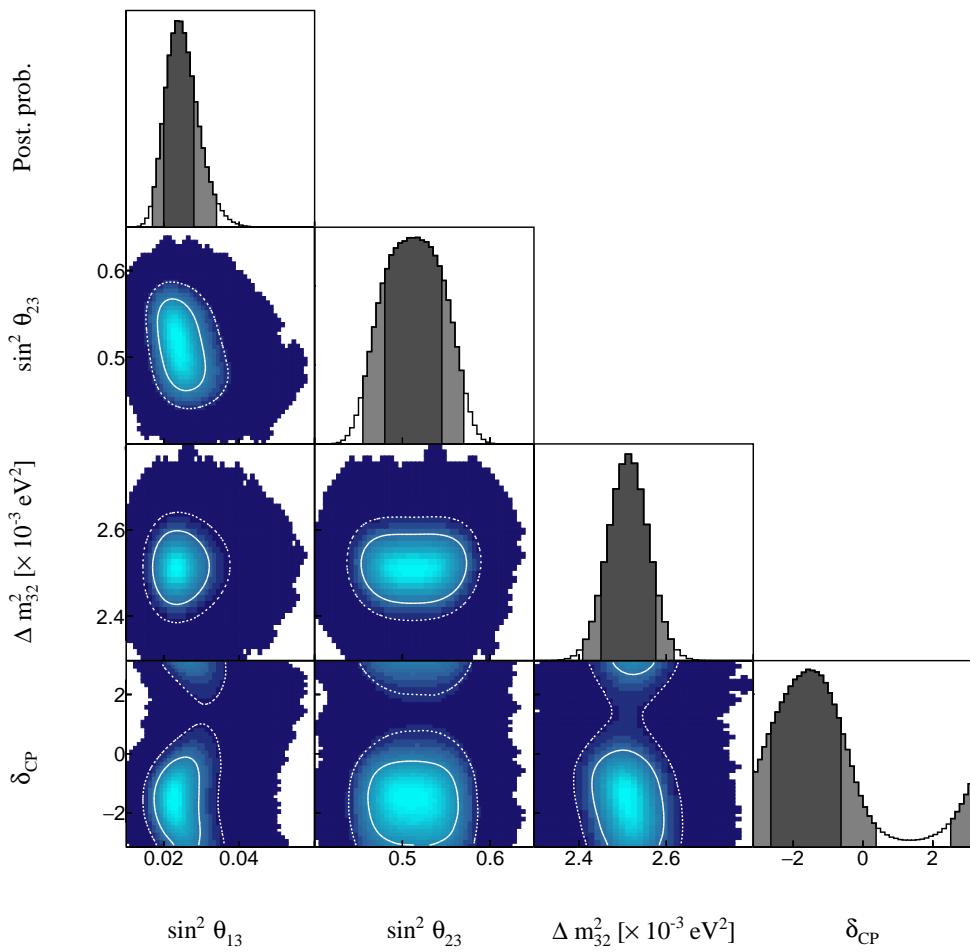


Figure 8.21: The posterior probability density distribution from the joint beam-atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

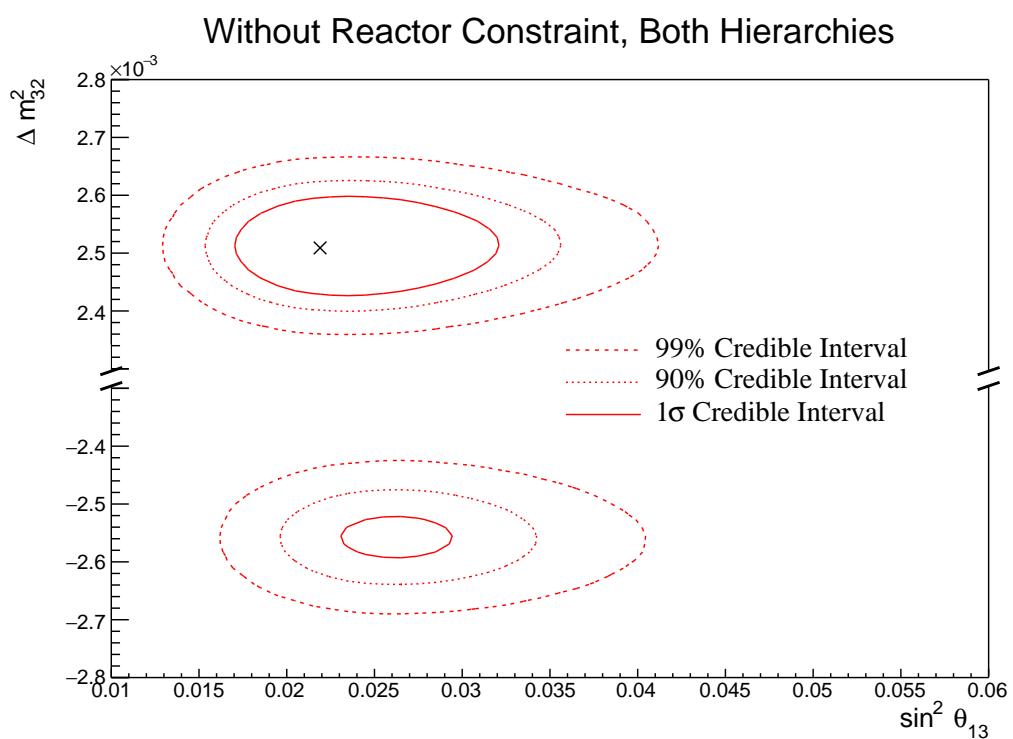


Figure 8.22: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{13})$.

3293 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3294 This section presents the sensitivities of the joint beam-atmospheric fit when
 3295 the reactor constraint is applied to $\sin^2(\theta_{13})$. As with the previous studies, the
 3296 Asimov data is made using the AsimovA oscillation parameter set defined in
 3297 Table 2.2 and the post-BANFF systematic parameter tune.

With Reactor Constraint, Both Hierarchies

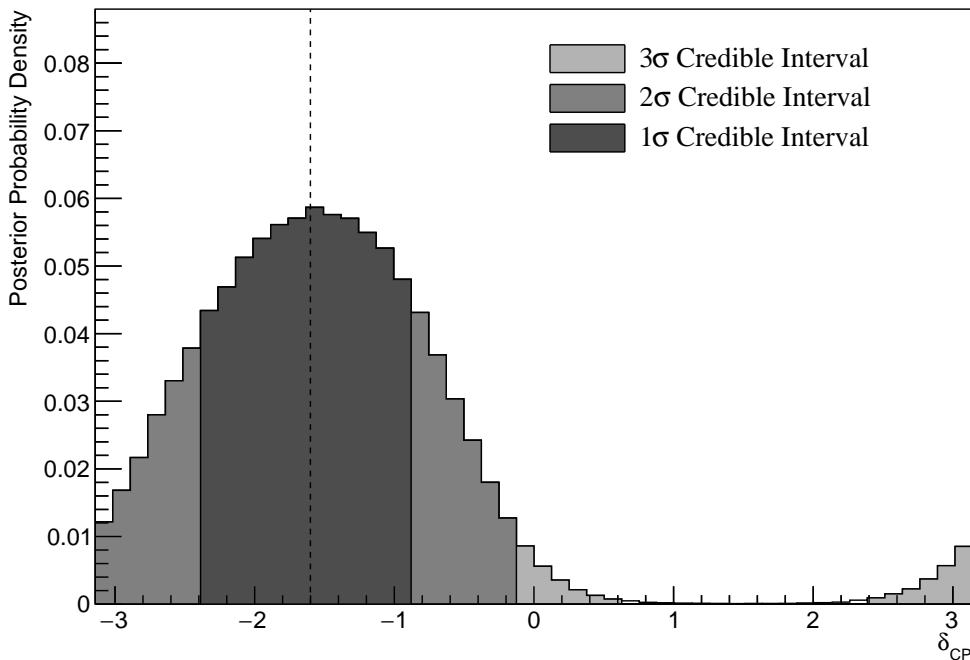


Figure 8.23: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of δ_{CP} .

3298 Figure 8.23 illustrates the sensitivity to δ_{CP} , marginalised over both hierarchies.
 3299 The CP-conserving value of $\delta_{CP} = 0$ is disfavoured at 2σ whilst the value of $\delta_{CP} =$
 3300 $\pm\pi$ is very close to being disfavoured at 2σ . Furthermore, the 3σ credible interval
 3301 excludes the region of $\delta_{CP} = [0.63, 2.39]$, thus clearly disfavouring the region of
 3302 $\delta_{CP} = \pi/2$ at more than 3σ for this particular set of known oscillation parameters.
 3303 The width of the 1σ credible intervals and the position of the highest posterior
 3304 probability density is given in Table 8.7. The highest posterior probability density
 3305 in δ_{CP} is calculated as $\delta_{CP} = -1.57 \pm 0.07$ showing no significant biases in the
 3306 determination of the known oscillation parameters.

3307 The effect of applying the reactor constraint for δ_{CP} in the joint beam-atmospheric
 3308 fit is presented in Figure 8.24. The reactor constraint significantly improves the
 3309 ability of the fit to select the known parameter value. This behaviour is evidenced
 3310 by the tightening of the 1σ and 90% credible intervals and the disfavoured region,
 3311 centered at $\delta_{CP} \sim \pi/2$, becoming wider when the reactor constraint is applied.
 3312 This follows from the correlations shown in Figure 8.19, where a lower value
 3313 of $\sin^2(\theta_{13})$ results in tighter constraints on δ_{CP} .

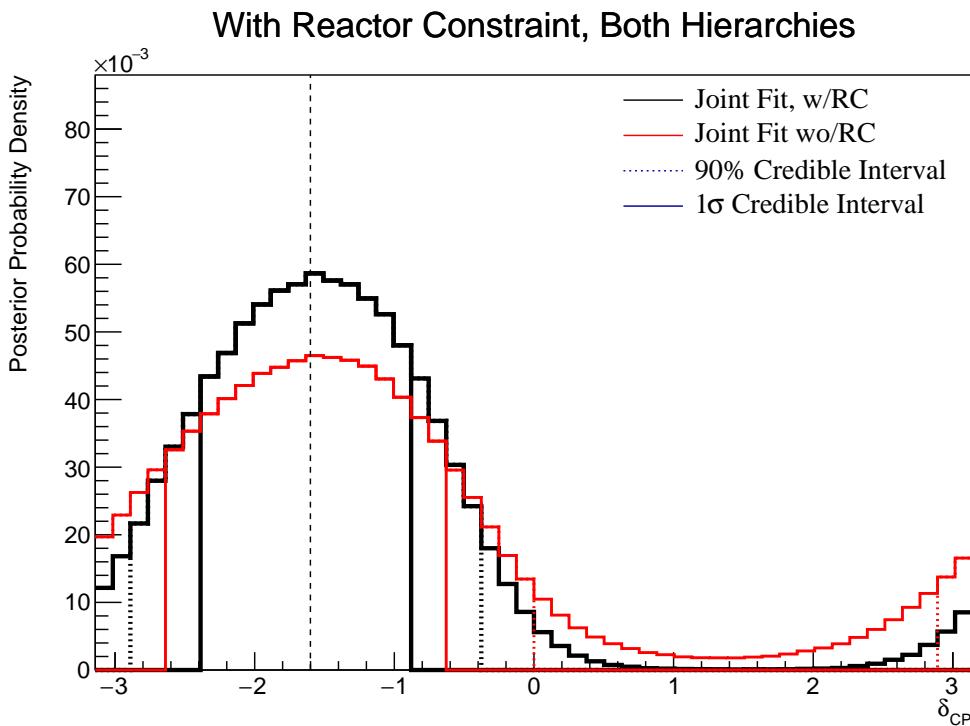


Figure 8.24: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3314 The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, is given in
 3315 Figure 8.25. The highest posterior probability density is located at $\sin^2(\theta_{23}) =$
 3316 0.528 ± 0.03 which agrees with the known value of $\sin^2(\theta_{23}) = 0.528$. The
 3317 distribution clearly favours the UO with almost the entirety of the 1σ credible
 3318 interval being contained in that region. Figure 8.26 highlights the sensitivity
 3319 of the joint fit both with and without the reactor constraint. The fit where the

Parameter	Interval	HPD
δ_{CP} , (BH)	[-2.39, -0.88]	-1.57 ± 0.07
δ_{CP} , (NH)	[-2.39, -0.75]	-1.57 ± 0.07
δ_{CP} , (IH)	[-2.14, -1.01]	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.56]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	[-2.60, -2.51]	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (NH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (IH)	[0.500, 0.560]	0.538 ± 0.03

Table 8.7: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

reactor constraint is applied selects the known value much better. This is a result of the marginalisation effects between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as observed in Figure 8.21.

The fraction of steps from the joint fit, after the reactor constraint is applied, is given in Table 8.8 and split by the two hierarchy and two octant hypotheses. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.03, whilst significantly increasing the fraction of steps within the NH-UO region from 0.50 to 0.62. The application of the reactor constraint increases the Bayes factor from $B(\text{NH}/\text{IH}) = 3.67$ to $B(\text{NH}/\text{IH}) = 6.47$. There is a very clear preference for the NH, with the Jeffreys scale stating a substantial preference for both fits. The Bayes factor for UO preference is calculated as $B(\text{UO}/\text{LO}) = 2.64$. Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

The sensitivity of the joint beam-atmospheric fit to Δm_{32}^2 , with the reactor constraint applied, is presented in Figure 8.27. The 1σ credible interval is

With Reactor Constraint, Both Hierarchies

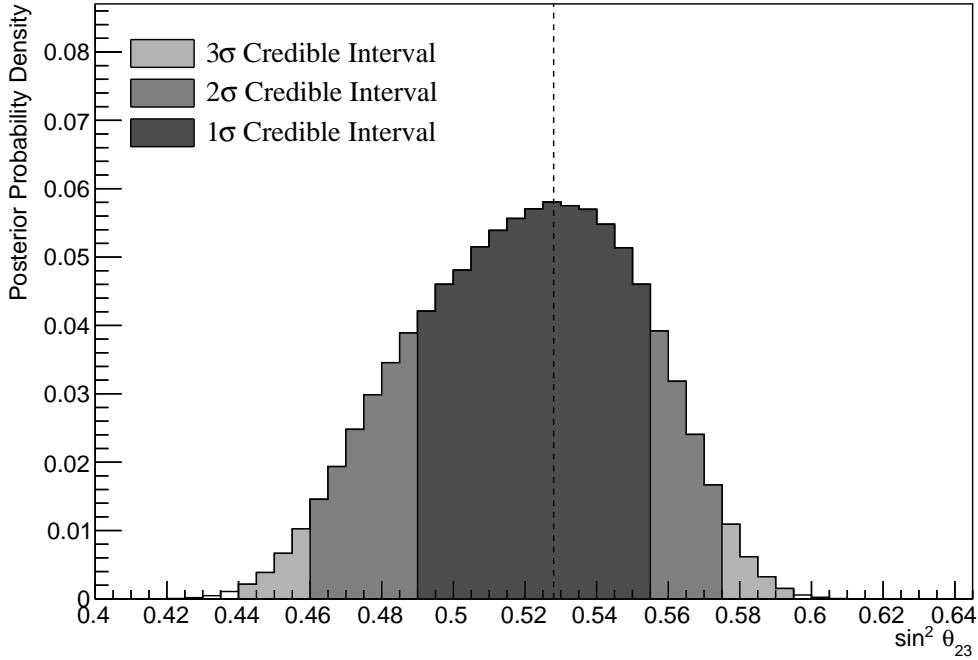


Figure 8.25: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.24	0.62	0.87
IH ($\Delta m_{32}^2 < 0$)	0.03	0.10	0.13
Sum	0.27	0.73	1.00

Table 8.8: The distribution of steps in a joint beam-atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{UO}/\text{LO}) = 2.64$.

3336 entirely contained within the NH region and the position of the highest posterior
 3337 probability density is given as $(2.49 \pm 0.01) \times 10^{-3} \text{ eV}^2$. This illustrates no bias
 3338 between the fit results and the known oscillation parameters. The application
 3339 of the reactor constraint does not significantly move the position or width of
 3340 the credible intervals.

3341 The sensitivity to the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$) is given in Fig-
 3342 ure 8.28. The distribution is mostly uncorrelated between the two parameters and

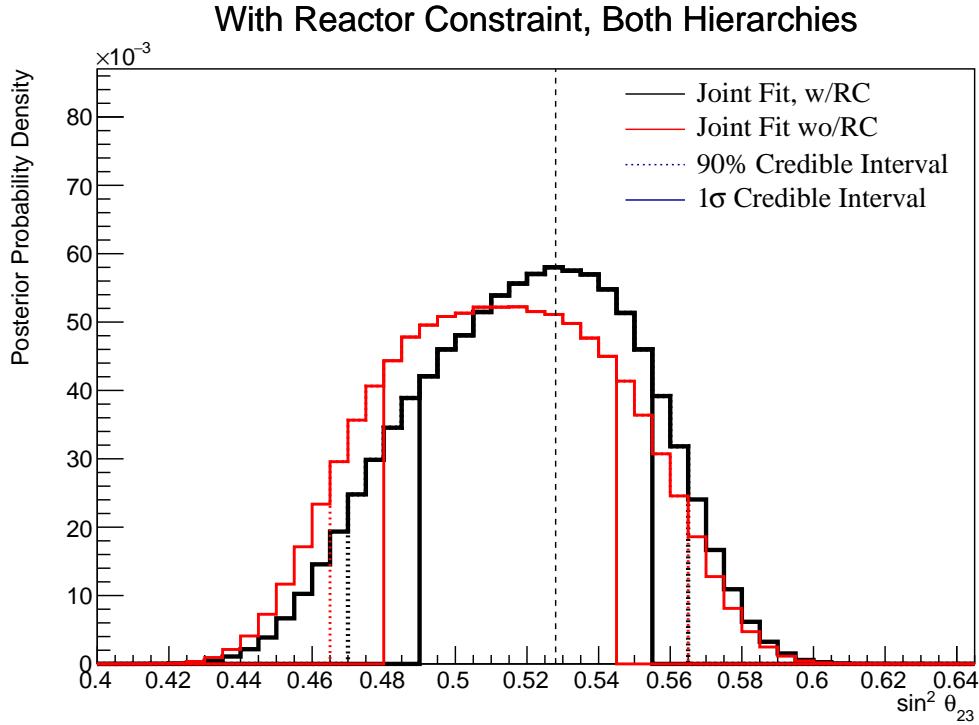


Figure 8.26: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3343 is centered at the known oscillation parameters. The 1σ credible interval excludes
 3344 $\delta_{CP} = 0$ and $\delta_{CP} = \pm\pi$. Furthermore, the 3σ credible intervals exclude the
 3345 region of $\delta_{CP} = \pi/2$.

3346 The sensitivity to the disappearance parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) is illustrated
 3347 in Figure 8.29. The 1σ credible interval is entirely contained within the NH
 3348 region reflecting the same results as the one-dimensional marginalised results in
 3349 Figure 8.27. Both the NH and IH regions favour the UO, with a visually similar
 3350 preference in both hierarchies. The width of the 1σ contour, in Δm_{32}^2 , does not
 3351 significantly depend upon the value or octant of $\sin^2(\theta_{23})$. This shows that there
 3352 are no strong correlations between these two parameters.

3353 Figure 8.30 illustrates the posterior distribution for each permutation of
 3354 two oscillation parameters of interest. The application of the reactor constraint
 3355 significantly reduces the correlations previously seen in Figure 8.21.

With Reactor Constraint, Both Hierarchies

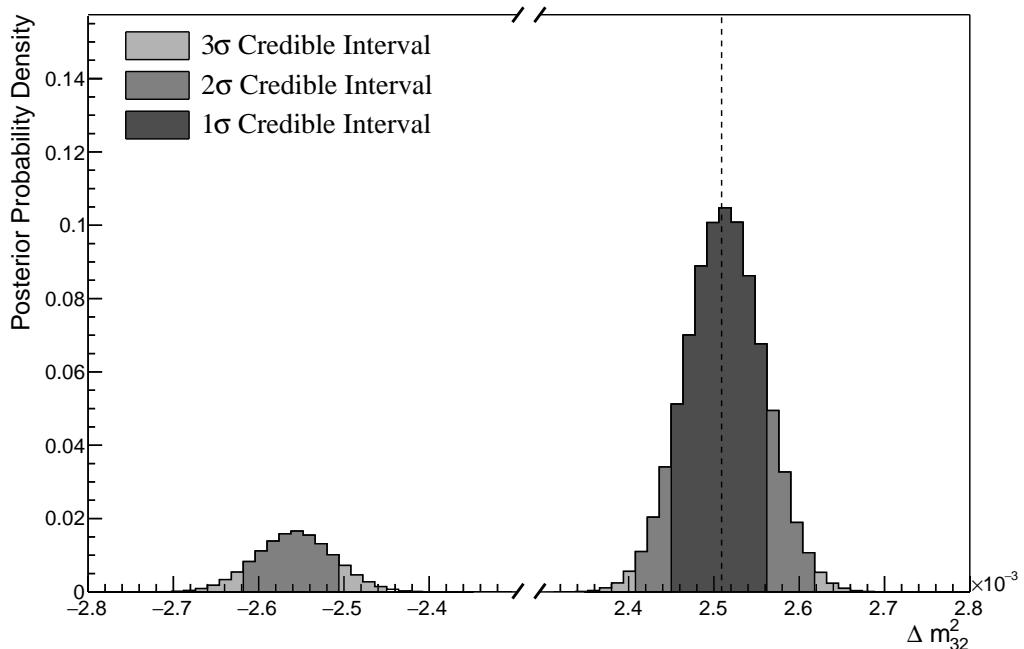


Figure 8.27: The one-dimensional posterior probability density distribution in Δm_{32}^2 from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of Δm_{32}^2 .

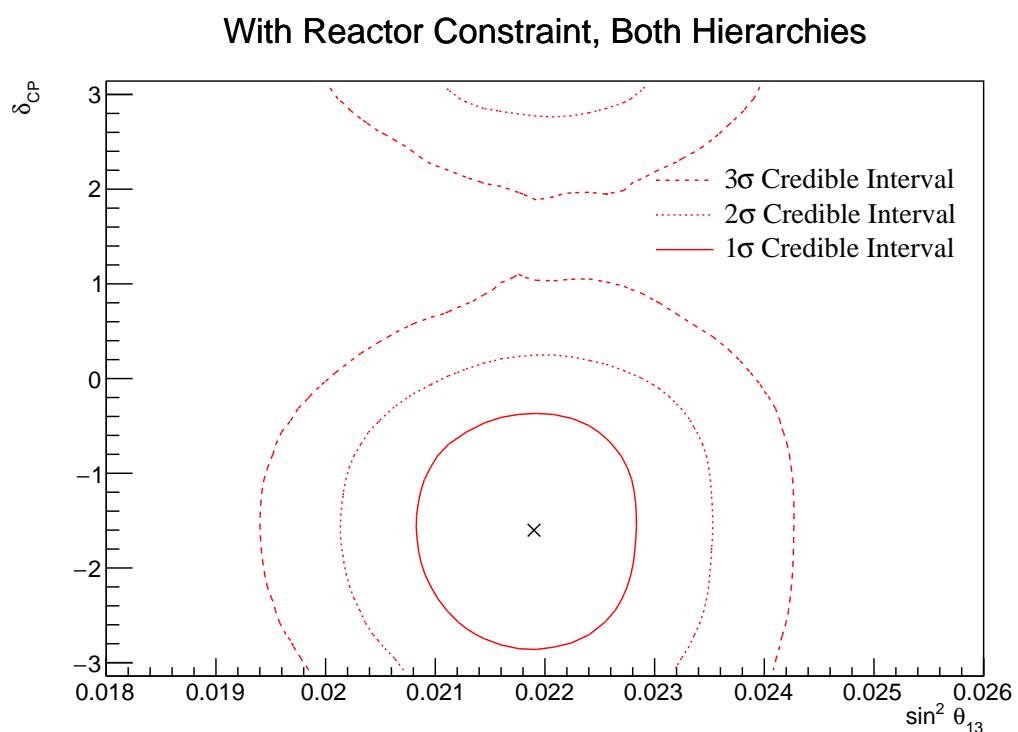


Figure 8.28: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\delta_{CP} - \sin^2(\theta_{13})$.

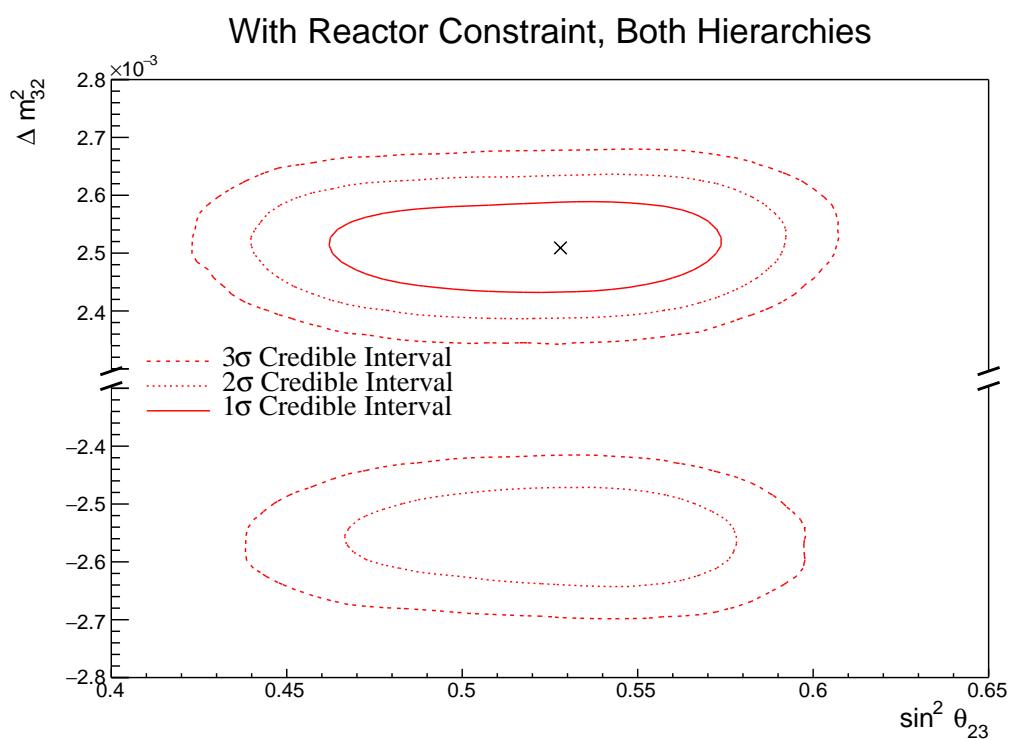


Figure 8.29: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

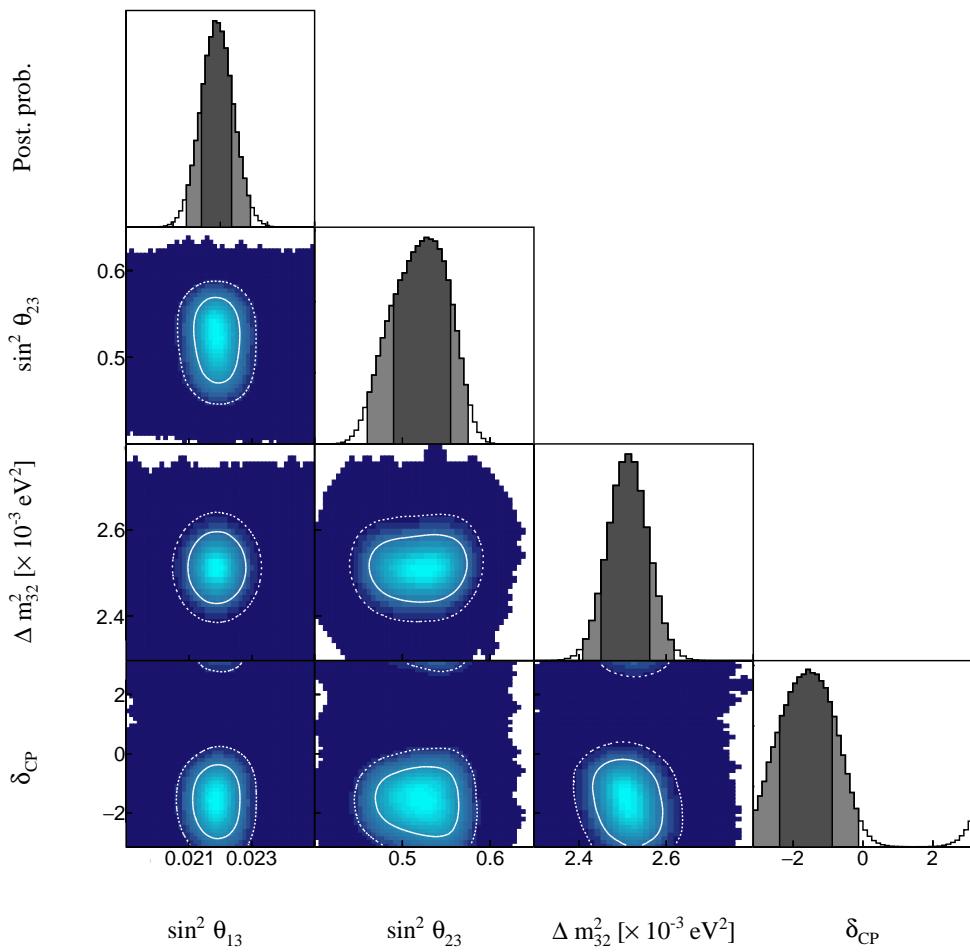


Figure 8.30: The posterior probability density distribution from the joint beam-atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3356 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3357 Constraint

3358 The benefits of the joint beam-atmospheric analysis can be determined by compar-
 3359 ing the sensitivities to the beam-only analysis presented in [2, 215]. This section
 3360 presents those comparisons for sensitivities built using the Asimov A oscillation
 3361 parameters defined in Table 2.2 and the post-BANFF systematic tune. The reactor
 3362 constraint is not applied within either of the fits used in these comparisons.

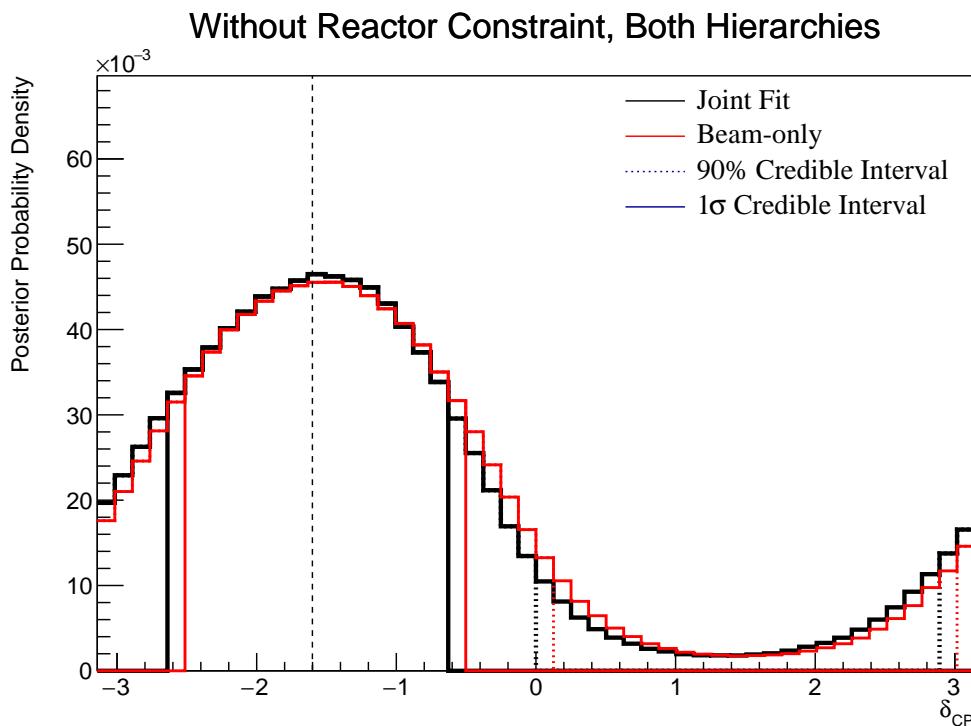


Figure 8.31: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3363 The sensitivity, marginalised over both hierarchies, to δ_{CP} from the joint beam-
 3364 atmospheric and beam-only fits is presented in Figure 8.31. As expected from the
 3365 likelihood scans (Figure 8.4), the sensitivity to δ_{CP} is not significantly increased.
 3366 This is because the known oscillation parameter value lies at the position where
 3367 the beam samples dominate the sensitivity compared to the SK samples.

3368 The sensitivity to Δm_{32}^2 is compared between the joint beam-atmospheric fit
 3369 and beam-only fit in Figure 8.32. The 1σ credible interval of the joint beam-
 3370 atmospheric fit is entirely contained within the NH region. This shows the
 3371 significant increase in the ability of the fit to determine the correct mass hierarchy,
 3372 compared to the beam-only analysis. This is further evidenced by the fact that the
 3373 90% credible intervals from the joint fit are also tighter in the IH region compared
 3374 to the beam-only analysis. The Bayes factor for mass hierarchy determination
 3375 for the beam-only and joint beam-atmospheric fits are $B(\text{NH}/\text{IH}) = 1.91$ and
 3376 $B(\text{NH}/\text{IH}) = 3.67$, respectively. According to Jeffrey's scale, the beam-only
 3377 analysis represents a weak preference for the NH hypothesis whereas the joint fit
 3378 returns a substantial preference for the NH hypothesis. Notably, this conclusion
 3379 does not require any external constraints and clearly illustrates the benefit of
 3380 the joint analysis.

Without Reactor Constraint, Both Hierarchies

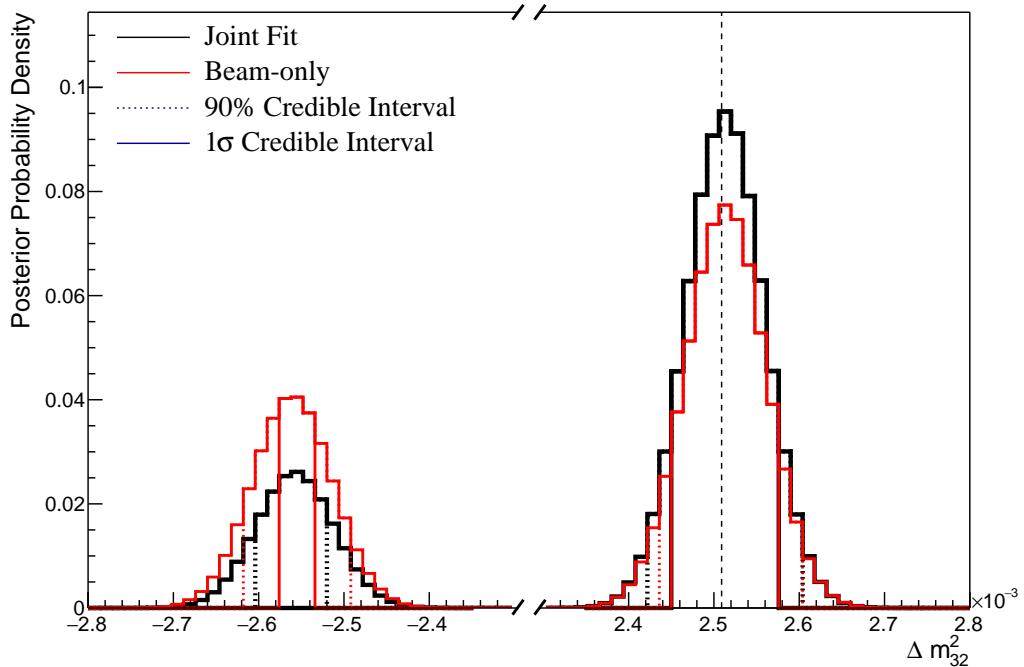


Figure 8.32: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, for both the beam-only and joint beam-atmospheric analysis are presented in Figure 8.33. The peak of the posterior distribution from the joint analysis is more aligned with the known value of $\sin^2(\theta_{23}) = 0.528$ compared to the beam-only analysis. The Bayes factors for the beam-only and joint beam-atmospheric fit are $B(\text{UO}/\text{LO}) = 1.56$ and $B(\text{UO}/\text{LO}) = 1.74$, respectively. Therefore, the joint beam-atmospheric fit does prefer the UO more strongly than the beam-only analysis, albeit slightly.

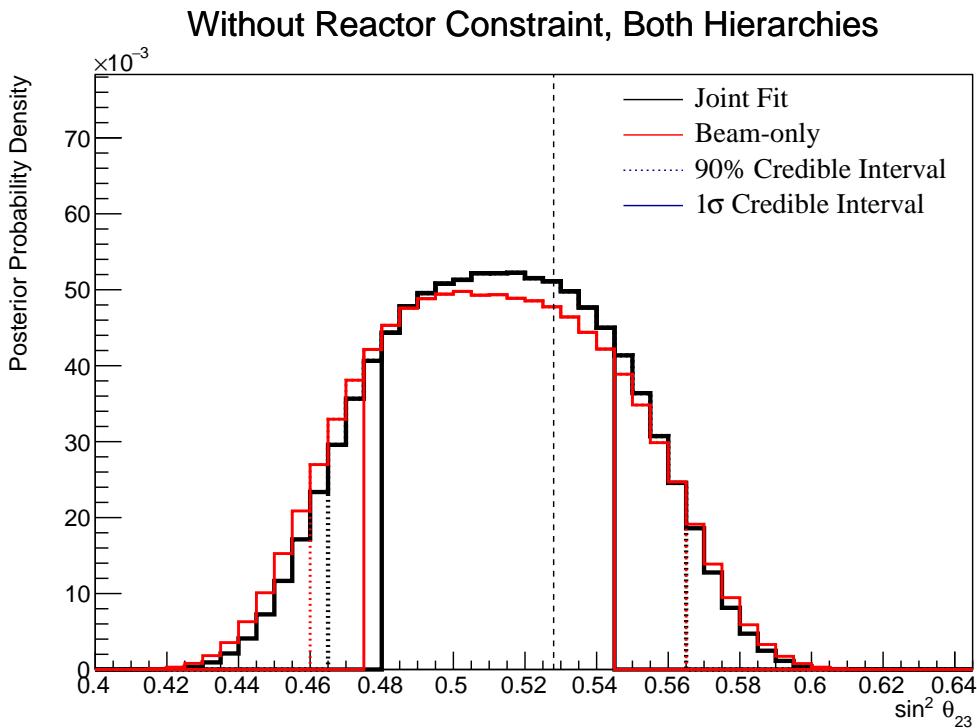


Figure 8.33: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

Whilst the beam-only and joint beam-atmospheric fits have similar sensitivity to δ_{CP} and $\sin^2(\theta_{23})$ when projected in one-dimension, the benefit of the joint analysis becomes more obvious when the sensitivities are presented in two-dimensions. The sensitivity of the two fits to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$) is illustrated in Figure 8.34. The width of the 99% joint fit credible interval in $\sin^2(\theta_{13})$ is squeezed in the region of $\delta_{CP} \sim 0$ compared to the

beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The 1σ and 90% also exhibit slightly tighter constraints on δ_{CP} . This is most prevalent in the region of $\delta_{CP} \sim 0$ and $\sin^2(\theta_{13}) \sim 0.03$. Whilst the atmospheric samples do not have significant sensitivity to $\sin^2(\theta_{13})$ (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters $\sin^2(\theta_{23}) - \Delta m_{32}^2$ is presented in Figure 8.35 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. Even in the NH region, the widths of the credible intervals in $\sin^2(\theta_{23})$ decreases, albeit to a smaller extent.

The comparison in sensitivity to $\delta_{CP} - \Delta m_{32}^2$ is illustrated in Figure 8.36. The contours from the joint beam-atmospheric fit are much smaller in the IH region as compared to the beam-only analysis. This culminates in a region around $\delta_{CP} \sim \pi/2$ in the H region which is excluded at 3σ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to Δm_{32}^2 , as a function of $\sin^2(\theta_{13})$, is presented in Figure 8.37. Similar to previous observations, the Δm_{32}^2 contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH 1σ credible intervals exclude the region around the reactor constraint. This suggests that the application of the reactor constraint would further increase the preference for NH in the joint fit compared to its effect on the beam-only analysis.

The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as illustrated

Without Reactor Constraint, Both Hierarchies

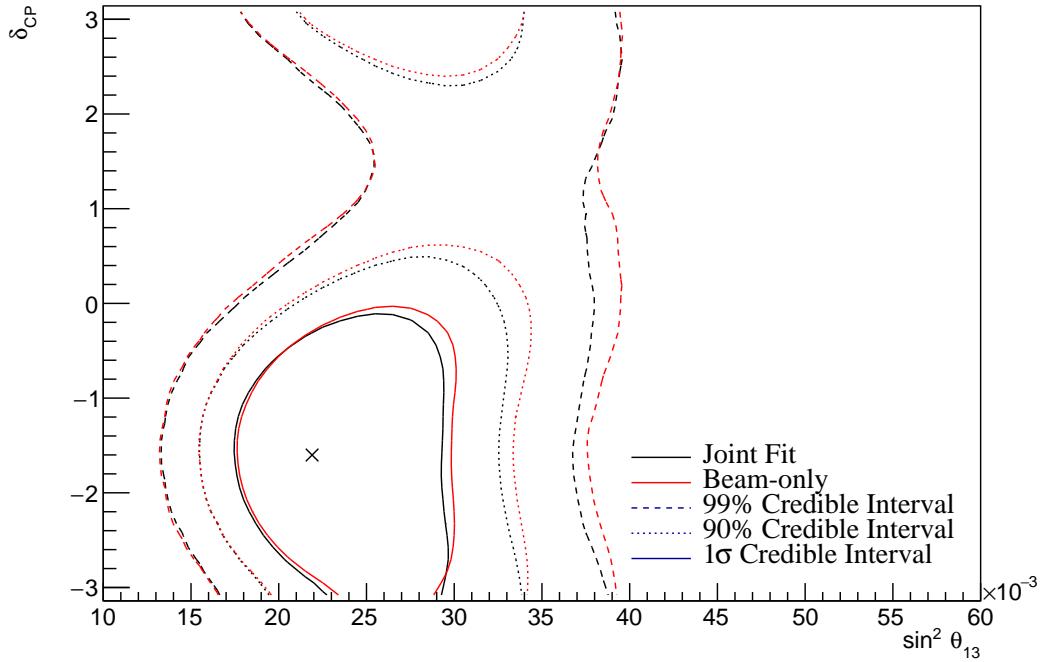


Figure 8.34: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of δ_{CP} – $\sin^2(\theta_{13})$.

³⁴²⁴ by Figure 8.38. The joint analysis disfavours the wrong octant hypothesis more
³⁴²⁵ strongly in the region of high $\sin^2(\theta_{13})$. This change in correlation means that the
³⁴²⁶ application of the reactor constraint could affect the two analyses differently.

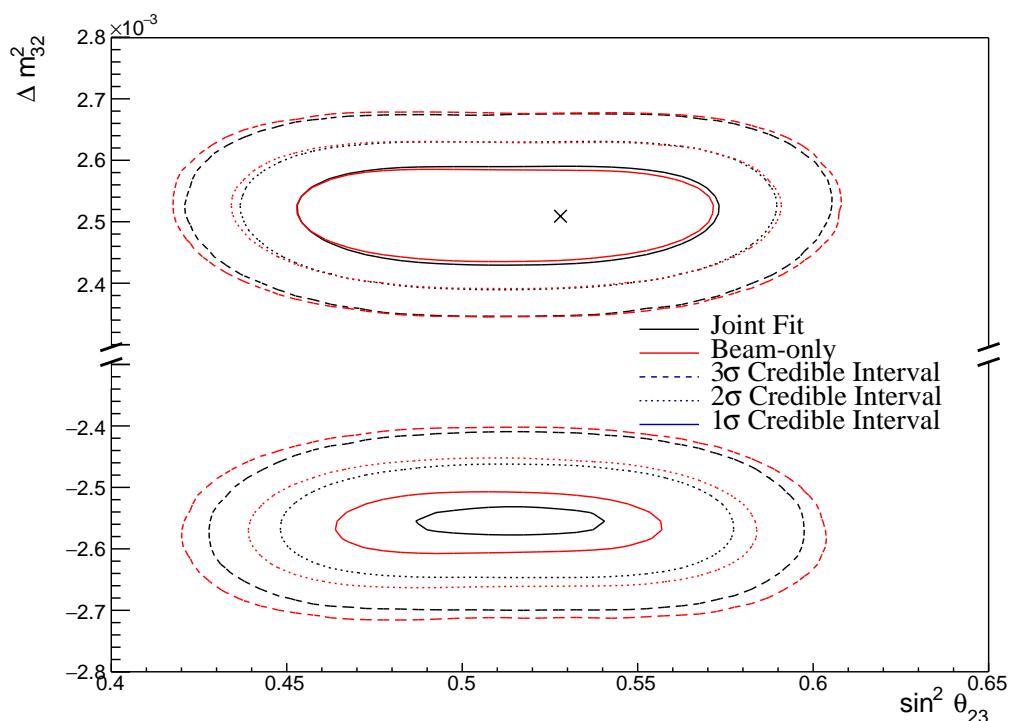


Figure 8.35: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

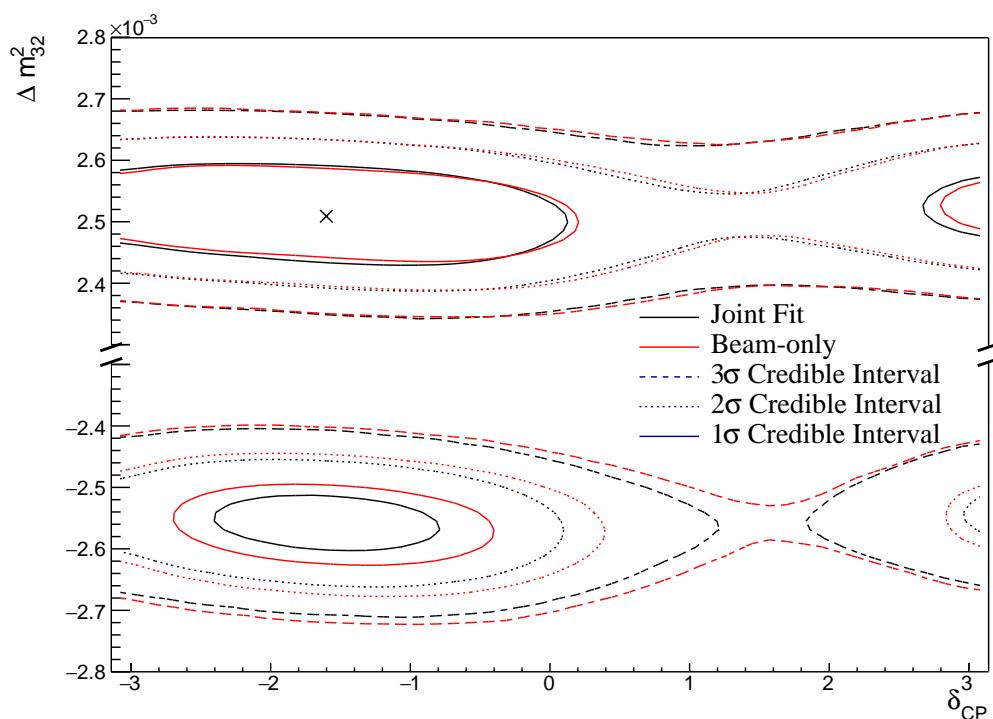


Figure 8.36: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \Delta_{CP}$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \Delta_{CP}$.

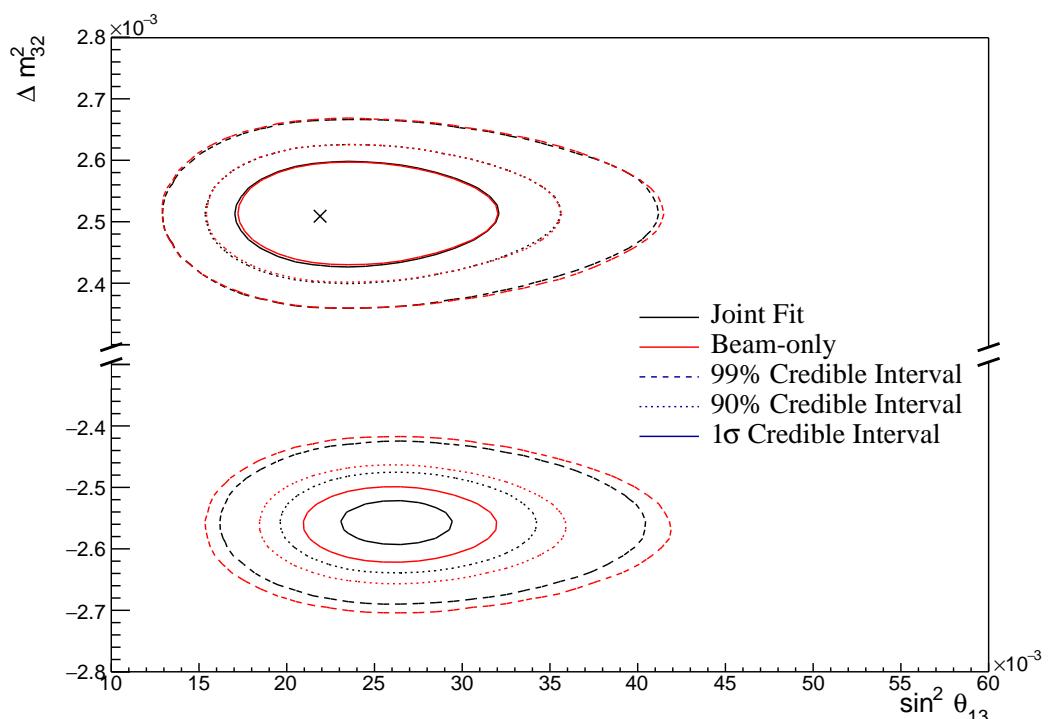


Figure 8.37: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

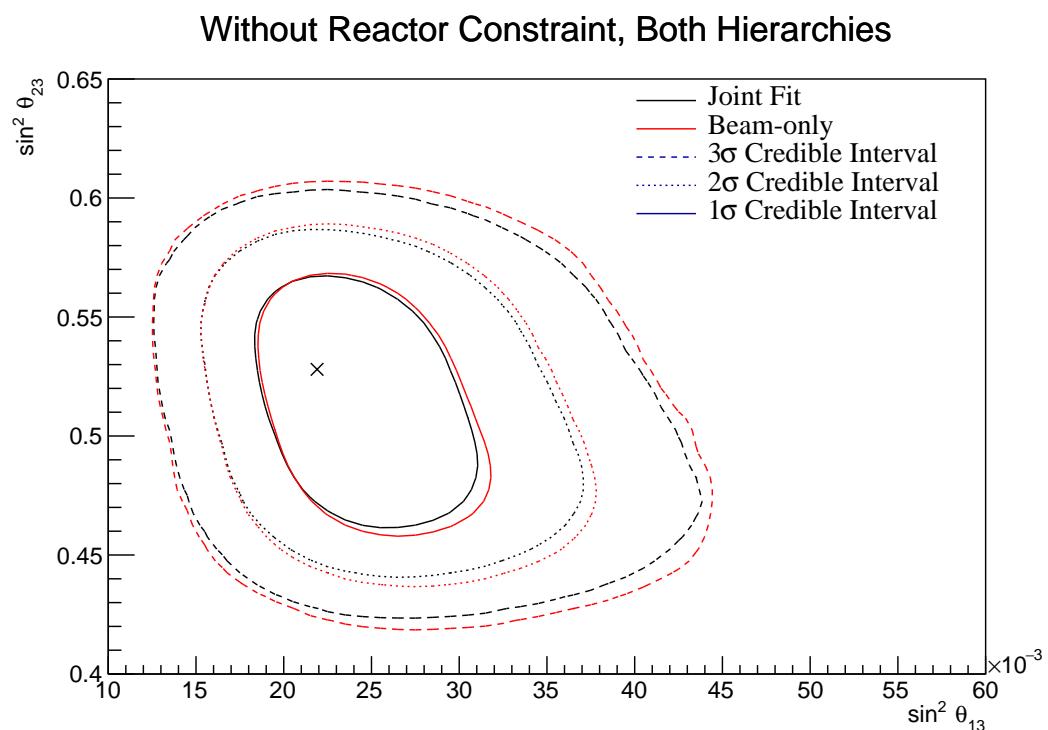


Figure 8.38: The two-dimensional posterior probability density distribution in $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$.

3427 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3428 straint

3429 This section illustrates the comparison between the joint beam-atmospheric and
 3430 beam-only fits when the reactor constraint is applied. As shown in Figure 8.37,
 3431 the application of the reactor constraint is expected to significantly increase
 3432 the joint fit's preference for the NH hypothesis, compared to the beam-only
 3433 analysis. Figure 8.39 illustrates the sensitivities of the two fits to the disappearance
 3434 parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$). This plot further illustrates the benefit of the joint
 3435 beam-atmospheric analysis. The 1σ credible interval in the IH region is entirely
 3436 removed in the joint analysis but not for the beam-only analysis.

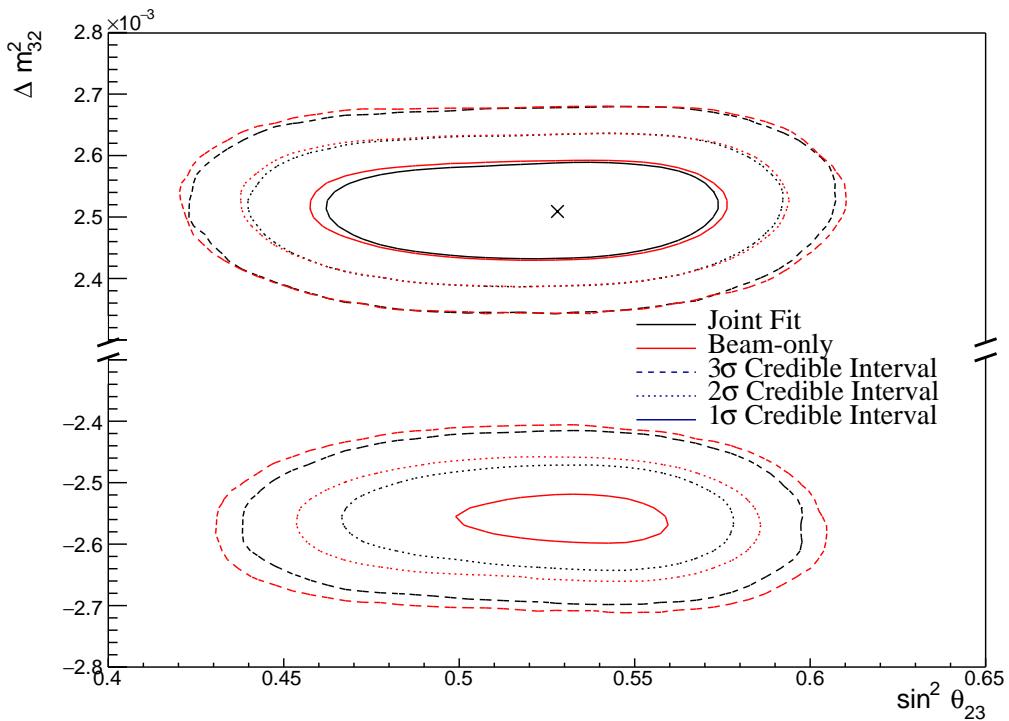


Figure 8.39: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is applied in both fits. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

3437 The credible intervals of the joint fit are also tighter in the $\sin^2(\theta_{23})$ dimension
 3438 than the beam-only analysis in both mass hierarchy regions. This shows that
 3439 beyond the ability of the joint fit to prefer the NH more strongly than the beam-

only analysis, the precision to which it can measure $\sin^2(\theta_{23})$ is also improved. The Bayes factor for NH preference is calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{NH}/\text{IH}) = 3.09$ for the joint beam-atmospheric and beam-only analysis, respectively. This important conclusion illustrates that the joint beam-atmospheric analysis can provide a substantial preference for the NH hypothesis whilst the beam-only analysis can not.

The Bayes factors for UO preference which are $B(\text{UO}/\text{LO}) = 2.86$ and $B(\text{UO}/\text{LO}) = 2.47$ for the joint beam-atmospheric and beam-only analysis, respectively. Both of these represent a mild preference for the UO but a stronger preference is observed in the joint analysis.

The sensitivity of the beam-only and joint beam-atmospheric analyses, to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$), are compared in Figure 8.40. These results are marginalised over both hierarchies. For this particular set of known oscillation parameters (AsimovA defined in Table 2.2), the beam-only analysis dominates the sensitivity. The joint fit does slightly increase the sensitivity to δ_{CP} but it does not change any conclusions that would be made. As expected, the prior knowledge dominates any sensitivity either fit would have on $\sin^2(\theta_{13})$.

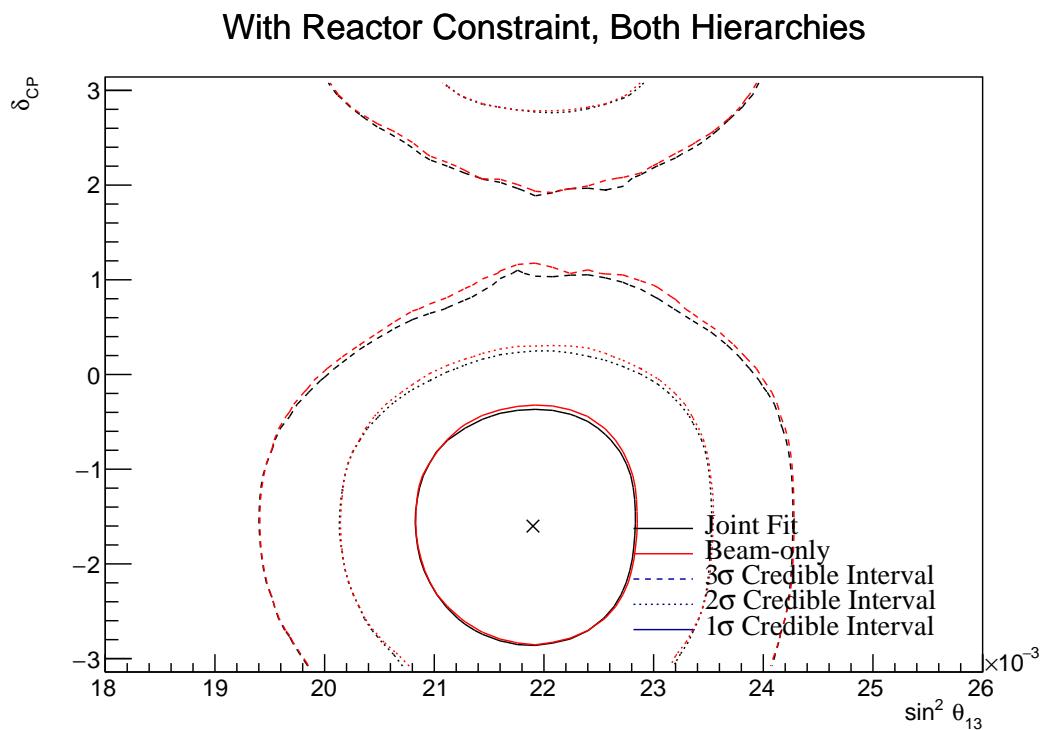


Figure 8.40: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies. The marker represents the known value of δ_{CP} – $\sin^2(\theta_{13})$.

3457 8.3.8 Alternate Asimov Parameter Set

3458 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the
 3459 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters
 3460 are defined at a region of δ_{CP} which is preferred by the T2K experiment. This
 3461 explains why the addition of the atmospheric samples does not significantly in-
 3462 crease the sensitivity to δ_{CP} , as illustrated in subsection 8.3.6 and subsection 8.3.7.
 3463 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,
 3464 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when
 3465 building the Asimov data.

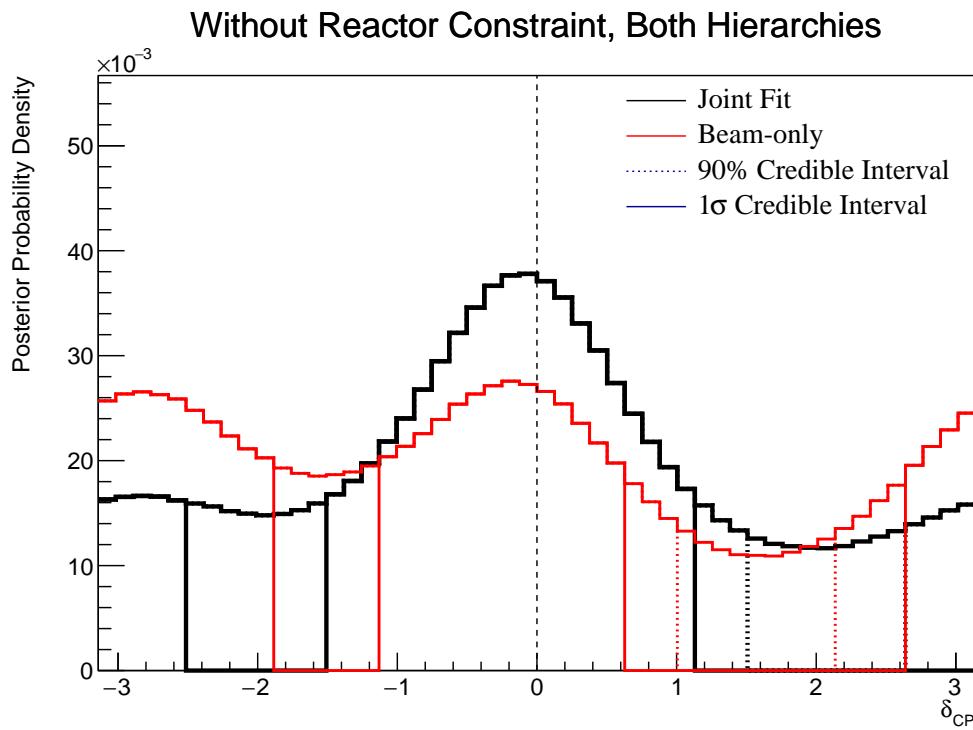


Figure 8.41: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3466 The sensitivity to δ_{CP} for the joint beam-atmospheric fit is presented in
 3467 Figure 8.41. The results are compared to those from the beam-only analysis
 3468 in [2, 215]. The reactor constraint is not applied in either of the fits. The

3469 shape of the posterior distribution from the joint analysis is more peaked at
3470 the known oscillation parameter value compared to the beam-only analysis,
3471 which has approximately the same posterior probability density at $\delta_{CP} = 0$
3472 and $\delta_{CP} = \pm\pi$. This shows the ability of the joint analysis to better determine
3473 the correct phase of δ_{CP} if the true value were CP-conserving. The 1σ credible
3474 intervals and the position of the highest posterior probability density are given in
3475 Table 8.9. The highest posterior density for the joint beam-atmospheric analysis
3476 is $\delta_{CP} = -0.06 \pm 0.06$ which is consistent with the known value.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -2.51], [-1.51, 1.13]$	-0.06 ± 0.06
δ_{CP} , (NH)	$[-1.13, 1.63]$	0.06 ± 0.06
δ_{CP} , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44 ± 0.06
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.52], [2.46, 2.56]$	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.47, 2.56]$	2.52 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.61, -2.52]$	-2.57 ± 0.01
$\sin^2(\theta_{23})$ (BH)	$[0.430, 0.480], [0.545, 0.585]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (NH)	$[0.430, 0.485], [0.550, 0.580]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (IH)	$[0.435, 0.480], [0.540, 0.585]$	0.568 ± 0.003

Table 8.9: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

3477 Naively, if just the 1σ credible interval were considered without observing
3478 the shape of the distribution, it would appear that the joint analysis would
3479 have a worse sensitivity to δ_{CP} due to the larger interval around $\delta_{CP} = 0$. The
3480 1σ credible interval for the beam-only analysis is given as the range $\delta_{CP} =$
3481 $[-\pi, -1.88], [-1.13, 0.63]$ and $[2.64, \pi]$ which contains 56% of all values of δ_{CP} .
3482 Whereas, the joint beam-atmospheric analysis contains 52% of all δ_{CP} values
3483 within the 1σ credible interval. Therefore, if the area within the 1σ credible

3484 interval were to be compared between the two fits, the joint analysis would
3485 be shown to have better precision.

3486 This apparent contradiction stems from the methodology in which the credible
3487 interval is calculated. The technique used in this analysis (documented in
3488 subsection 4.3.2) fills the credible interval by selecting bins in order of probability
3489 density until 68% of the posterior density is contained. If instead, the credible
3490 interval were calculated by expanding around the highest posterior probability,
3491 the benefits of the joint fit would be more obvious. In the case where the shape
3492 of the posterior was uni-modal, these two techniques would be equivalent to
3493 statistical fluctuations.

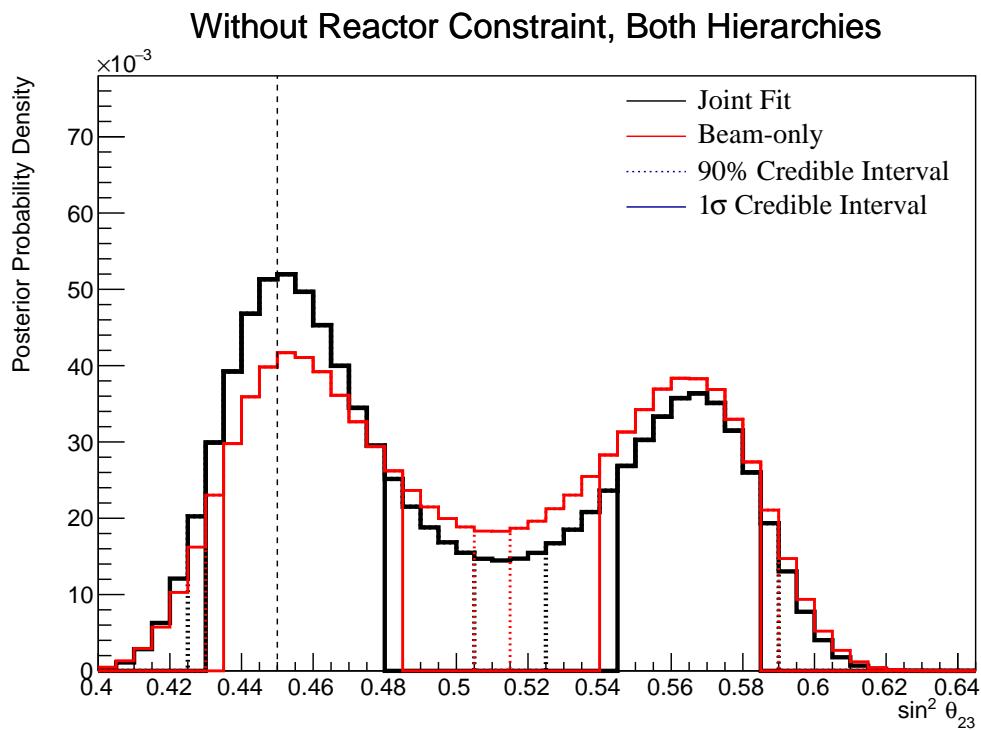


Figure 8.42: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3494 The sensitivity of the joint beam-atmospheric fit to $\sin^2(\theta_{23})$ is presented in
3495 Figure 8.42. The sensitivity is compared to that of the beam-only analysis in [2,
3496 215]. The reactor constraint is not applied in either of the fits being compared.

3497 The joint beam-atmospheric fit has a much larger probability density in the region
 3498 surrounding the known oscillation parameter, $\sin^2(\theta_{23}) = 0.45$. This shows the
 3499 better octant determination of the joint analysis compared to the beam-only fit.
 3500 The ratio of the posterior density at the peak of the lower octant to the peak of
 3501 the upper octant from the joint fit is 1.43 compared to 1.09 from the beam-only
 3502 analysis. The area contained within the 1σ credible interval for the joint analysis
 3503 is $\sin^2(\theta_{23}) = [0.430, 0.480]$ and $\sin^2(\theta_{23}) = [0.545, 0.585]$, whereas the beam-
 3504 only analysis includes $\sin^2(\theta_{23}) = [0.435, 0.485]$ and $\sin^2(\theta_{23}) = [0.540, 0.585]$.
 3505 This corresponds to a $\sim 5\%$ (binning dependent) increase in precision from
 3506 the joint analysis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.35	0.24	0.59
IH ($\Delta m_{32}^2 < 0$)	0.19	0.22	0.41
Sum	0.54	0.46	1.00

Table 8.10: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$.

3507 The distribution of steps, split by hierarchy and octant hypothesis, is presented
 3508 in Table 8.10. The Bayes factor for hierarchy and octant determination are
 3509 $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$, respectively. These values compare
 3510 to $B(\text{NH}/\text{IH}) = 1.08$ and $B(\text{LO}/\text{UO}) = 0.91$ from the beam-only analysis. This
 3511 evidences the joint analysis's ability to select the correct octant and hierarchy
 3512 hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6
 3513 show how the preferences for the correct octant and hierarchy depend on the
 3514 true value of δ_{CP} and $\sin^2(\theta_{23})$.

3515 The sensitivity of the beam-only and joint beam-atmospheric analysis to
 3516 Δm_{32}^2 is given in Figure 8.43. The joint analysis has a stronger preference for the
 3517 correct hierarchy (NH) which is shown by the higher Bayes factor compared
 3518 to the beam-only analysis. This is further evidenced by the width of the 90%

Without Reactor Constraint, Both Hierarchies

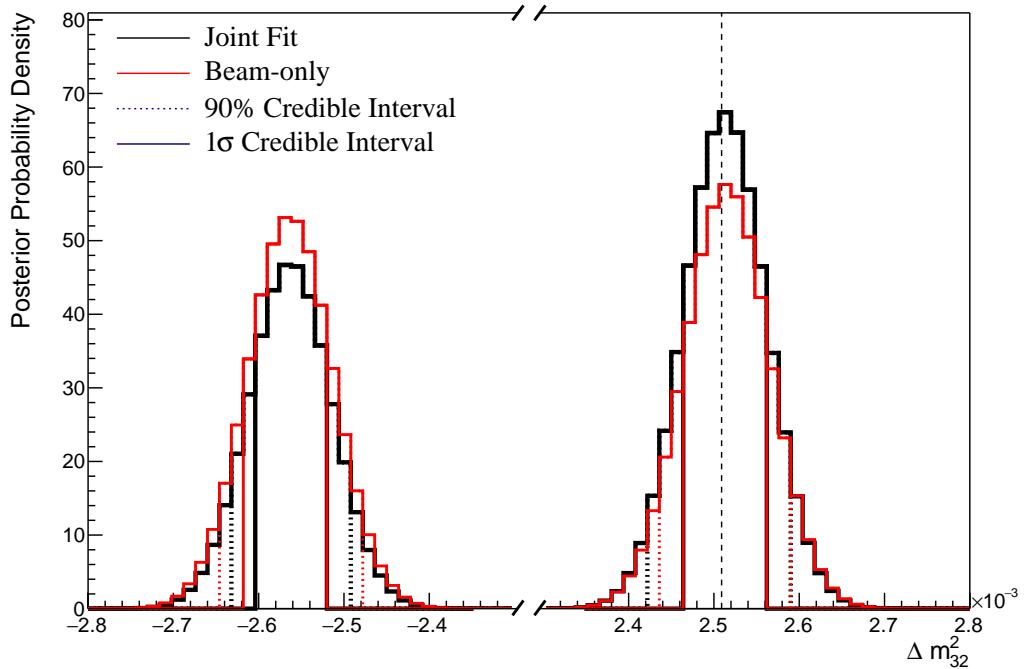


Figure 8.43: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

credible interval in the IH region being tighter in the joint analysis compared to the beam-only analysis.

3519

3520

3521 8.3.9 Effect of Systematics

3522 The effect of systematics on each sample used in this analysis is calculated using
 3523 the posterior predictive method documented in subsection 4.3.4. The distribution
 3524 of each sample’s spectrum has been generated by sampling 2000 steps from the
 3525 posterior distribution of the joint beam-atmospheric fit. This technique reweights
 3526 the Monte Carlo prediction using the systematic values given by a particular
 3527 step, stores the sample spectra, and repeats for the desired number of steps.
 3528 The oscillation parameters are always fixed at Asimov A values to only study
 3529 the effect of systematic parameters.

3530 Figure 8.44 illustrates the distribution for the SubGeV-*elike*-0dcy atmospheric
 3531 sample. The fit being sampled is detailed in subsection 8.3.4. The distribution
 3532 closely resembles the Asimov data spectrum. This would be expected from
 3533 an Asimov fit where the Monte Carlo is fit to itself but gives more credibility
 3534 to the results of the fit.

3535 The total event rate for each sample from each of the sampled steps is
 3536 calculated and the fractional uncertainty, $\Delta N_i / N_i$ where N_i is the event rate of the
 3537 i^{th} sample, is calculated. These values are presented in Table 8.11. In general, the
 3538 impact of the systematics has a $\sim 3\%$ uncertainty on the event rate of atmospheric
 3539 samples, where CC1 π -targeting samples have slightly larger uncertainties than
 3540 the CCQE-like samples. The fractional uncertainties on the beam samples are
 3541 compared to those from the beam-only analysis presented in [2, 215]. The
 3542 uncertainties on the one-ring muon samples are mostly unchanged, whereas
 3543 the uncertainties on the one-ring electron samples are different. As discussed in
 3544 section 8.2, the atmospheric samples should be able to add constraints on the NC
 3545 background events present in the FHC1Re and RHC1Re samples. The uncertainty
 3546 reduction seen in those samples is due to those additional constraints. The reason
 3547 why the FHC1Re1de has a higher uncertainty in this analysis is due to the addition
 3548 of the ad-hoc systematic introduced for CC1 π interactions (see subsection 6.4.3).

3549 Beyond the impact of the uncertainty on each sample’s event rate, the post-fit
 3550 constraint on each systematic parameter should be checked. Figure 8.45 illustrates

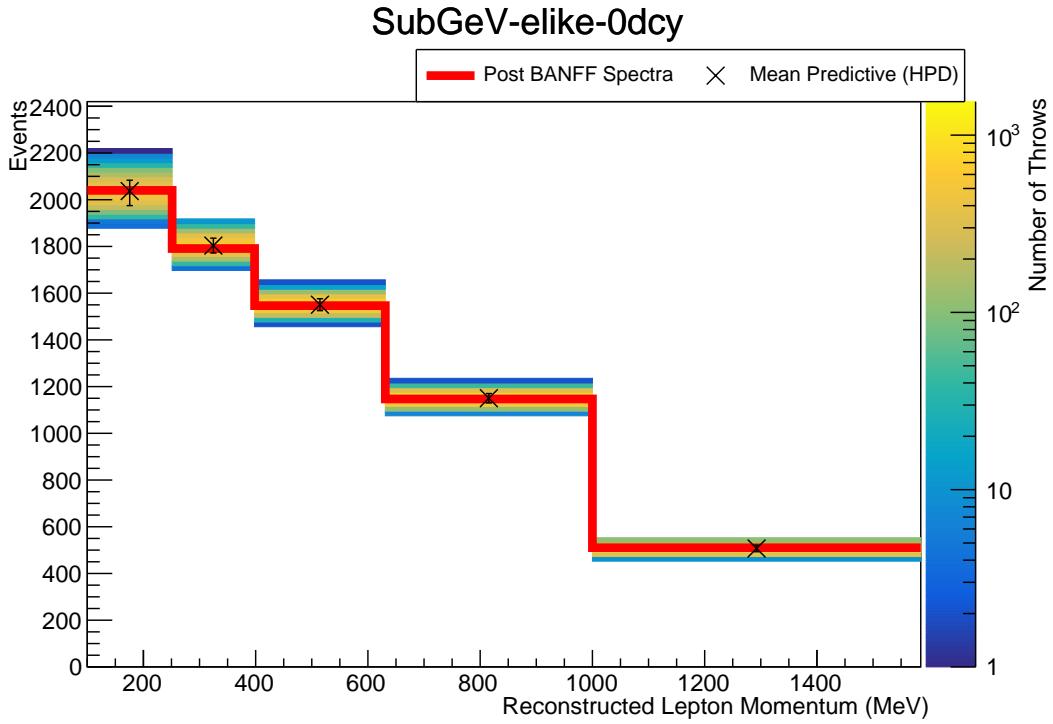


Figure 8.44: Result of the posterior predictive method for the SubGeV_elike_0dcy sample after sampling 2000 steps from the joint beam-atmospheric chain detailed in subsection 8.3.4 (Coloured histogram). The mean and uncertainty is presented for each bin. The Asimov data prediction (denoted ‘Post BANFF Spectra’, Red) assumes the post-BANFF tune and Asimov A oscillation parameters.

the central value and uncertainty on a select group of interaction systematics, for both the joint beam-atmospheric (from subsection 8.3.4) and the beam-only analysis. From the discussion in section 8.2, the uncertainty on systematics which are strongly constrained by the near detector should not significantly change when adding the atmospheric analysis. This behaviour is evidenced by the fact that the ratio of constraints between the two fits are very similar (within a few %) for almost all systematics. The only systematic which is more constrained in the joint beam-atmospheric analysis is the *NCOtherSK* normalisation parameter, which has a $O(10\%)$ tighter constraint. As expected, the atmospheric samples have been able to constrain this systematic which leads to the reduction in uncertainty for the beam electron-like samples.

Sample	Percentage Uncertainty (%)	
	Joint Analysis	Beam-only Analysis
SubGeV-elike-0dcy	2.53	-
SubGeV-elike-1dcy	3.28	-
SubGeV-mulike-0dcy	2.62	-
SubGeV-mulike-1dcy	2.23	-
SubGeV-mulike-2dcy	3.96	-
SubGeV-pi0like	2.84	-
MultiGeV-elike-nue	5.14	-
MultiGeV-elike-nuebar	2.79	-
MultiGeV-mulike	2.99	-
MultiRing-elike-nue	2.94	-
MultiRing-elike-nuebar	2.83	-
MultiRing-mulike	2.89	-
MultiRingOther-1	2.70	-
PCStop	3.22	-
PCThru	2.99	-
UpStop-mu	2.95	-
UpThruNonShower-mu	2.70	-
UpThruShower-mu	3.19	-
FHC1Rmu	2.49	2.33
RHC1Rmu	2.89	2.93
FHC1Re	4.12	4.57
RHC1Re	5.15	5.65
FHC1Re1de	13.38	11.51

Table 8.11: The percentage uncertainty, $\Delta N / N$, as calculated from sampling 2000 throws from a joint beam-atmospheric chain. The same values for the beam samples are provided from the beam-only analysis [2, 215]. These uncertainties consider all systematic parameters to be sampled from the fit whilst the oscillation parameters are fixed at the Asimov A oscillation set.

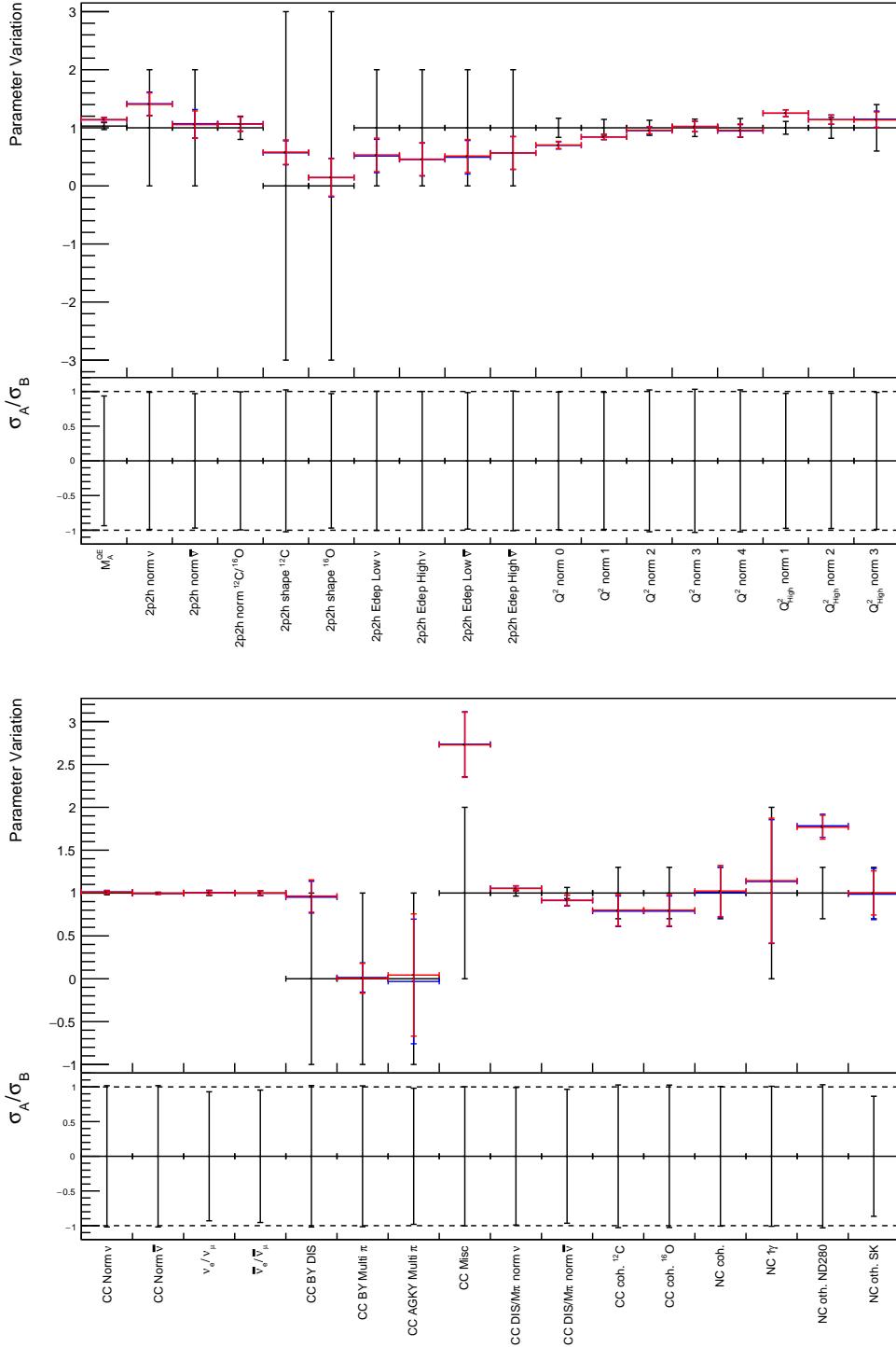


Figure 8.45: Central values and 1σ uncertainties for a select group of interaction systematics. The constraints from the prior uncertainty (Black), joint atmospheric-beam fit given in subsection 8.3.4 (Red) and beam-only analysis [2, 215] (Blue) are presented. The top part of each plot presents the parameter variation and the bottom part represents the ratio of the uncertainty between the joint beam-atmospheric and beam-only fits, where a value below 1.0 means the joint fit has a tighter constraint than the beam-only analysis.

3562 8.4 Summary of Sensitivity Studies

3563 The sensitivities to each oscillation parameter from the joint beam-atmospheric
 3564 and beam-only fits, which use the Asimov A oscillation parameter set, are
 3565 summarised in Table 8.12. As the posterior distribution to δ_{CP} is cyclical, only
 3566 the position of the highest posterior density (HPD) is given. The uncertainty
 3567 corresponds to the width of the bin in which the HPD is located. Furthermore,
 3568 the Δm_{32}^2 reported values only consider the NH credible interval region as the
 3569 full discussion can be found in the previous section.

Fit	δ_{CP} (HPD)	$\Delta m_{32}^2 [\times 10^{-3} \text{eV}^2]$	$\sin^2(\theta_{23})$	$\sin^2(\theta_{13}) [\times 10^{-2}]$
Asimov A	-1.601	2.509	0.528	2.19
Beam	-1.45 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	$2.45^{+0.45}_{-0.35}$
Beam w/RC	-1.57 ± 0.06	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	$2.19^{+0.06}_{-0.07}$
Joint	-1.57 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	$2.35^{+0.45}_{-0.35}$
Joint w/RC	-1.57 ± 0.06	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	$2.18^{+0.07}_{-0.06}$

Table 8.12: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead.

3570 The Bayes factors from the beam-only and joint atmospheric-beam analyses
 3571 are presented in Table 8.13. The strength of each preference, from Jeffrey's
 3572 scale (Table 4.1), is also given.

3573 To summarise this information, the joint fit prefers a tighter 1σ credible
 3574 interval in $\sin^2(\theta_{23})$ along with a stronger Bayes factor for preferring the correct
 3575 octant hypothesis. The increase in sensitivity to $|\Delta m_{32}^2|$ between the two fits is
 3576 negligible but the joint analysis substantially prefers the correct mass hierarchy
 3577 hypothesis. It does not require any external constraints on $\sin^2(\theta_{13})$ to make
 3578 this statement. The joint analysis also produces a value of $\sin^2(\theta_{13})$ closer to the
 3579 known value compared to the beam-only analysis. When the reactor constraint is

Fit	$B(\text{NH}/\text{IH})$		$B(\text{UO}/\text{LO})$	
	Value	Strength	Value	Strength
Asimov A				
Beam	1.91	Weak	1.56	Weak
Beam w/RC	3.09	Weak	2.47	Weak
Joint	3.67	Substantial	1.74	Weak
Joint w/RC	6.47	Substantial	2.64	Weak
Asimov B				
Beam	1.08	Weak	0.91	Weak
Beam w/RC	0.98	Weak	1.15	Weak
Joint	1.43	Weak	1.19	Weak
Joint w/RC	1.36	Weak	1.52	Weak

Table 8.13: A comparison of the Bayes factors for mass hierarchy and $\sin^2(\theta_{23})$ octant hypotheses, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The strength of the preference for the normal mass hierarchy and upper octants are provided by Jeffrey's scale Table 8.13.

3580 applied, the preference for both the NH and UO hypotheses increases but does
 3581 not change the statement which would be made.

3582 The fits from the Asimov B comparisons (subsection 8.3.9) show the improved
 3583 ability for the joint analysis to more precisely select the true value of δ_{CP} if it were
 3584 CP-conserving, compared to the beam-only analysis. This is evidenced by the
 3585 area contained within the 1σ credible interval decreasing by $\sim 4\%$. Furthermore,
 3586 the joint fit is able to better determine the octant of $\sin^2(\theta_{23})$ when the true
 3587 value is moved further away from the boundary as evidenced by the larger
 3588 Bayes factor. There is also a $\sim 5\%$ reduction of area contained within the 1σ
 3589 credible interval in $\sin^2(\theta_{23})$.

9

3590

3591

Conclusions and Outlook

3592 This thesis has presented the sensitivities of a joint beam and atmospheric neutrino oscillation analysis from the Tokai-to-Kamioka (T2K) and Super-Kamiokande
3593 (SK) experiments combining the two independent analyses presented by the
3594 collaborations [2, 3]. This study uses 3244.4 days of SK livetime and $1.97 \times$
3595 $10^{21}(1.63 \times 10^{21})$ POT recorded at the far detector in the neutrino(antineutrino)
3596 beam operating mode. The ND280 near detector is used within this analysis to
3597 constrain the beam flux and cross-section systematics. It uses 1.15×10^{21} POT and
3598 8.34×10^{20} POT in the neutrino and antineutrino running modes, respectively.
3599

3600 This analysis uses a Bayesian Markov Chain Monte Carlo fitting technique
3601 implemented within the MaCh3 framework. This analysis has significantly developed
3602 the fitting framework, both in terms of technical features and performance.
3603 This includes supporting new samples, systematics, and oscillation channels.
3604 These developments have become the foundation of the fitter's expansion into
3605 other neutrino oscillation experiments. Beyond these improvements, a novel
3606 technique for calculating the atmospheric neutrino oscillation probabilities has
3607 been developed. This calculation uses a sub-sampling linear-averaging approach
3608 to ensure that the sensitivities being calculated are not biased due to insufficient
3609 Monte Carlo statistics in a region of rapidly varying probability. It illustrates a

3610 computationally feasible method of reliably calculating oscillation probabilities
3611 that can be utilised within any fitting framework.

3612 The sensitivity of the joint beam-atmospheric analysis is presented in Table 9.1,
3613 and compared to the beam-only analysis [2]. The sensitivities are evaluated
3614 using a set of known oscillation parameter values close to the results from a
3615 previous T2K analysis [77] (denoted AsimovA in Table 9.1). The joint analysis
3616 has a stronger sensitivity to $\sin^2(\theta_{23})$, as evidenced by the tighter 1σ credible
3617 intervals when the constraints from reactor experiments are not applied. The
3618 joint fit's sensitivity to δ_{CP} is marginally stronger than beam-only analysis. Whilst
3619 the sensitivity to $|\Delta m_{32}^2|$ is mostly unchanged between the two analyses, the
3620 sensitivity to select the correct hierarchy is significantly improved. This follows
3621 from a substantial preference for the normal hierarchy hypothesis presented
3622 within the joint analysis, as classified by Jeffrey's scale [134]. This is notable as
3623 the beam-only analysis cannot make this statement, either with or without the
3624 application of the reactor constraint. The joint fit's preference for the correct
3625 hierarchy increases once the reactor constraint is applied. The preference for
3626 selecting the correct octant of $\sin^2(\theta_{23})$ is classified as weak by Jeffrey's scale but
3627 is still stronger than the statement made by the beam-only analysis.

3628 The sensitivities of the beam-only and joint atmospheric-beam fit have also
3629 been compared at a set of known oscillation parameters which are CP-conserving
3630 and in the lower octant of $\sin^2(\theta_{23})$. The joint analysis has a $\sim 5\%$ improved abil-
3631 ity to select the known values more precisely compared to the beam-only analysis.

3632 Whilst this analysis provides the first sensitivities of a joint beam and at-
3633 mospheric analysis, there are more improvements that could be made. Since
3634 this analysis began, the T2K collaboration has released an updated oscillation
3635 analysis with additional near and far detector samples alongside a more so-
3636 phisticated interaction model [105]. The overall change in oscillation parameter
3637 measurement observed by T2K is relatively minor but the stronger constraints
3638 on the systematics could impact this joint analysis to a larger extent. Further
3639 developments could consider the effect of correlating the beam and atmospheric

Fit	δ_{CP} (HPD)	Δm_{32}^2 [$\times 10^{-3}$ eV 2]	$\sin^2(\theta_{23})$	$B(\text{NH/IH})$	$B(\text{UO/LO})$
Asimov A	-1.601	2.509	0.528	NH	UO
Beam	$-1.45^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	1.91	1.56
Beam w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	3.09	2.47
Joint	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	3.67	1.74
Joint w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	6.47	2.64

Table 9.1: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead. The Bayes factors are provided for the mass hierarchy preference: normal hierarchy (NH) and inverse hierarchy (IH), and $\sin^2(\theta_{23})$ octant preference: upper octant (UO) and lower octant (LO).

3640 flux uncertainties relating to hadron production, where updates of the Bartol
3641 and Honda models may allow this to be studied [151].

3642 Beyond these model improvements, more data is available than what is
3643 assumed for this analysis. The T2K experiment has accumulated an additional
3644 1.78×10^{20} POT in neutrino mode. Similarly, there are several early SK periods
3645 that have not been considered within this analysis as the reconstruction software
3646 used in this analysis has not been validated for those periods. SK will also
3647 continue to accumulate statistics with Gadolinium doping. Developments in the
3648 atmospheric sample selections may also benefit from the Gadolinium dopants as
3649 neutron capture will aid in neutrino/antineutrino separation leading to better
3650 mass hierarchy sensitivity. This would require including interaction systematics
3651 for neutron capture of Gadolinium which has already started [216].

3652 This analysis shows the increased sensitivity to oscillation parameters from
3653 the combination of beam and atmospheric samples. It has developed the MaCh3
3654 fitting framework and has laid the foundations of the fitter's expansion into other
3655 neutrino oscillation experiments. The sensitivities presented in this thesis, and
3656 the techniques which were used to generate them, are significant to the future of

³⁶⁵⁷ neutrino oscillation physics which will likely perform similar analyses. As such,
³⁶⁵⁸ they have been presented by the T2K and SK collaborations at the Neutrino 2022
³⁶⁵⁹ conference [105]. Moving towards the next generation of neutrino experiments,
³⁶⁶⁰ this analysis has the potential to become the basis of the oscillation analysis
³⁶⁶¹ for the future Hyper-Kamiokande experiment which aims to make conclusive
³⁶⁶² statements about CP-violation in the neutrino sector.

Bibliography

- [1] Andrei D Sakharov. "Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe". In: *Soviet Physics Uspekhi* 34.5 (May 1991), pp. 392–393. URL: <https://doi.org/10.1070/pu1991v034n05abeh002497>.
- [2] P. Dunne. "Latest Neutrino Oscillation Results from T2K". In: *Neutrino 2020* (2020). URL: <https://zenodo.org/record/3959557>.
- [3] M. Jiang et al. "Atmospheric neutrino oscillation analysis with improved event reconstruction in Super-Kamiokande IV". en. In: *Prog. Theor. Exp. Phys.* 2019.5 (May 2019).
- [4] J. Chadwick. "Intensitätsverteilung im magnetischen Spectrum der β -Strahlen von radium B + C". In: *Verhandl. Dtsc. Phys. Ges.* 16 (1914), p. 383. URL: <http://cds.cern.ch/record/262756>.
- [5] C. D. Ellis and W. A. Wooster. "The average energy of disintegration of radium E". en. In: *Proc. R. Soc. Lond. A Math. Phys. Sci.* 117.776 (Dec. 1927), pp. 109–123.
- [6] W. Pauli. "Dear radioactive ladies and gentlemen". In: *Phys. Today* 31N9 (1978), p. 27.
- [7] E. Fermi. "An attempt of a theory of beta radiation. 1." In: *Z. Phys.* 88 (1934), pp. 161–177.
- [8] F. Reines and C. L. Cowan. "Detection of the Free Neutrino". In: *Phys. Rev.* 92 (3 1953), pp. 830–831. URL: <https://link.aps.org/doi/10.1103/PhysRev.92.830>.
- [9] C. L. Cowan et al. "Detection of the Free Neutrino: a Confirmation". In: *Science* 124.3212 (1956), pp. 103–104. URL: <http://science.sciencemag.org/content/124/3212/103>.
- [10] G. Danby et al. "Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos". In: *Phys. Rev. Lett.* 9 (1 1962), pp. 36–44. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.9.36>.
- [11] K. Kodama et al. "Observation of tau neutrino interactions". In: *Physics Letters B* 504.3 (2001), pp. 218 –224. URL: <http://www.sciencedirect.com/science/article/pii/S0370269301003070>.
- [12] A. Aguilar-Arevalo et al. "Evidence for neutrino oscillations from the observation of anti-neutrino(electron) appearance in a anti-neutrino(muon) beam". In: *Phys. Rev.* D64 (2001), p. 112007. arXiv: [hep-ex/0104049](https://arxiv.org/abs/hep-ex/0104049) [hep-ex].
- [13] A. A. Aguilar-Arevalo et al. "Improved Search for $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ Oscillations in the MiniBooNE Experiment". In: *Phys. Rev. Lett.* 110 (16 2013), p. 161801. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.110.161801>.

- 3699 [14] Planck Collaboration. "Planck 2018 results. VI. Cosmological parameters". In: *aap*
3700 641 (Sept. 2020).
- 3701 [15] J. A. Bagger et al. "Precision electroweak measurements on the Z resonance". In:
3702 *Physics Reports* 427.5 (2006), pp. 257–454. URL:
3703 <http://www.sciencedirect.com/science/article/pii/S0370157305005119>.
- 3704 [16] B. Pontecorvo. "Neutrino Experiments and the Problem of Conservation of
3705 Leptonic Charge". In: *Sov. Phys. JETP* 26 (1968). [Zh. Eksp. Teor.
3706 Fiz.53,1717(1967)], pp. 984–988.
- 3707 [17] B. Pontecorvo. "Inverse beta processes and nonconservation of lepton charge". In:
3708 *Sov. Phys. JETP* 7 (1958). [Zh. Eksp. Teor. Fiz.34,247(1957)], pp. 172–173.
- 3709 [18] M. Kobayashi and T. Maskawa. "CP-Violation in the Renormalizable Theory of
3710 Weak Interaction". In: *Progress of Theoretical Physics* 49.2 (1973), pp. 652–657. URL:
3711 <http://dx.doi.org/10.1143/PTP.49.652>.
- 3712 [19] N. Cabibbo. "Unitary Symmetry and Leptonic Decays". In: *Phys. Rev. Lett.* 10 (12
3713 1963), pp. 531–533. URL:
3714 <https://link.aps.org/doi/10.1103/PhysRevLett.10.531>.
- 3715 [20] A. Maio et al. "Search for Majorana neutrinos with the SNO+ detector at
3716 SNOLAB". In: *Journal of Physics: Conference Series* 587 (2015), p. 012030. URL:
3717 <https://doi.org/10.1088/1742-6596/587/1/012030>.
- 3718 [21] A. Y. Smirnov. "The MSW effect and Solar Neutrinos". In: *ArXiv* (2003). URL:
3719 <https://arxiv.org/abs/hep-ph/0305106>.
- 3720 [22] S. P. Mikheyev and A. Y. Smirnov. "Resonance enhancement of oscillations in
3721 matter and solar neutrino spectroscopy". In: *Soviet Journal of Nuclear Physics* 42 (6
3722 1985), pp. 913–917.
- 3723 [23] L. Wolfenstein. "Neutrino oscillations in matter". In: *Phys. Rev. D* 17 (9 1978),
3724 pp. 2369–2374. URL: <https://link.aps.org/doi/10.1103/PhysRevD.17.2369>.
- 3725 [24] V. D. Barger et al. "Matter Effects on Three-Neutrino Oscillations". In: *Phys. Rev.*
3726 *D* 22 (1980), p. 2718.
- 3727 [25] Y. Ashie et al. "Evidence for an Oscillatory Signature in Atmospheric Neutrino
3728 Oscillations". In: *Phys. Rev. Lett.* 93 (10 2004), p. 101801. URL:
3729 <https://link.aps.org/doi/10.1103/PhysRevLett.93.101801>.
- 3730 [26] Q. R. Ahmad et al. "Direct Evidence for Neutrino Flavor Transformation from
3731 Neutral-Current Interactions in the Sudbury Neutrino Observatory". In: *Phys.*
3732 *Rev. Lett.* 89 (1 2002), p. 011301. URL:
3733 <https://link.aps.org/doi/10.1103/PhysRevLett.89.011301>.
- 3734 [27] Nobel Prize. 2015 Nobel prize in Physics as listed by Nobelprize.org.
3735 https://www.nobelprize.org/nobel_prizes/physics/laureates/2015/.
3736 Accessed: 22-06-2022.
- 3737 [28] J. A. Formaggio and G. P. Zeller. "From eV to EeV: Neutrino Cross Sections
3738 Across Energy Scales". In: *Rev. Mod. Phys.* 84 (2012), pp. 1307–1341. arXiv:
3739 1305.7513 [hep-ex].
- 3740 [29] A. Oralbaev, M. Skorokhvatov, and O. Titov. "The inverse beta decay: a study of
3741 cross section". In: *Journal of Physics: Conference Series* 675.1 (2016), p. 012003. URL:
3742 <https://doi.org/10.1088/1742-6596/675/1/012003>.

- 3743 [30] A. Bellerive. "Review of solar neutrino experiments". en. In: *Int. J. Mod. Phys. A*
3744 19.08 (Mar. 2004), pp. 1167–1179.
- 3745 [31] R. Davis, D. S. Harmer, and K. C. Hoffman. "Search for Neutrinos from the Sun".
3746 In: *Phys. Rev. Lett.* 20 (21 1968), pp. 1205–1209. URL:
3747 <https://link.aps.org/doi/10.1103/PhysRevLett.20.1205>.
- 3748 [32] N. Vinyoles et al. "A new generation of standard solar models". In: *Astrophys. J.*
3749 835.2 (Jan. 2017), p. 202.
- 3750 [33] V. Gribov and B. Pontecorvo. "Neutrino astronomy and lepton charge". en. In: *Phys. Lett. B*
3751 28.7 (Jan. 1969), pp. 493–496.
- 3752 [34] K. S. Hirata et al. "Observation of ${}^8\text{B}$ solar neutrinos in the Kamiokande-II
3753 detector". In: *Phys. Rev. Lett.* 63 (1 1989), pp. 16–19. URL:
3754 <https://link.aps.org/doi/10.1103/PhysRevLett.63.16>.
- 3755 [35] W. Hampel et al. "GALLEX solar neutrino observations: results for GALLEX IV".
3756 en. In: *Phys. Lett. B* 447.1-2 (Feb. 1999), pp. 127–133.
- 3757 [36] J. N. Abdurashitov et al. "Measurement of the solar neutrino capture rate with
3758 gallium metal". In: *Phys. Rev. C* 60 (5 1999), p. 055801. URL:
3759 <https://link.aps.org/doi/10.1103/PhysRevC.60.055801>.
- 3760 [37] Borexino Collaboration. "Comprehensive measurement of pp-chain solar
3761 neutrinos". en. In: *Nature* 562.7728 (Oct. 2018), pp. 505–510.
- 3762 [38] B. Aharmim et al. "A search for neutrinos from the SolarhepReaction and the
3763 diffuse supernova neutrino background with the Sudbury neutrino observatory".
3764 en. In: *Astrophys. J.* 653.2 (Dec. 2006), pp. 1545–1551.
- 3765 [39] Borexino Collaboration. "Experimental evidence of neutrinos produced in the
3766 CNO fusion cycle in the Sun". In: *Nature* 587.7835 (2020), pp. 577–582. URL:
3767 <https://doi.org/10.1038%2Fs41586-020-2934-0>.
- 3768 [40] K. Abe et al. "T2K neutrino flux prediction". In: *Phys. Rev. D* 87 (1 2013),
3769 p. 012001. URL: <https://link.aps.org/doi/10.1103/PhysRevD.87.012001>.
- 3770 [41] D. G. Michael et al. "Observation of Muon Neutrino Disappearance with the
3771 MINOS Detectors in the NuMI Neutrino Beam". In: *Phys. Rev. Lett.* 97 (19 2006),
3772 p. 191801. URL:
3773 <https://link.aps.org/doi/10.1103/PhysRevLett.97.191801>.
- 3774 [42] M. H. Ahn et al. "Measurement of neutrino oscillation by the K2K experiment".
3775 In: *Phys. Rev. D* 74 (7 2006), p. 072003. eprint:
3776 <https://link.aps.org/doi/10.1103/PhysRevD.74.072003>.
- 3777 [43] M. A. Acero et al. "First measurement of neutrino oscillation parameters using
3778 neutrinos and antineutrinos by NOvA". In: *Phys. Rev. Lett.* 123 (15 2019),
3779 p. 151803. URL:
3780 <https://link.aps.org/doi/10.1103/PhysRevLett.123.151803>.
- 3781 [44] K. Abe et al. "The T2K experiment". In: *Nuclear Instruments and Methods in Physics*
3782 *Research Section A* 659.1 (2011), pp. 106 –135. eprint:
3783 <http://www.sciencedirect.com/science/article/pii/S0168900211011910>.
- 3784 [45] B. Abi et al. "Long-baseline neutrino oscillation physics potential of the DUNE
3785 experiment". en. In: *Eur. Phys. J. C Part. Fields* 80.10 (Oct. 2020).

- 3786 [46] Hyper-Kamiokande Proto-Collaboration. "Physics potential of a long-baseline
3787 neutrino oscillation experiment using a J-PARC neutrino beam and
3788 Hyper-Kamiokande". In: *Prog. Theor. Exp. Phys.* 2015.5 (May 2015), pp. 53C02–0.
- 3789 [47] C. Blanco, D. Hooper, and P. Machado. "Constraining sterile neutrino
3790 interpretations of the LSND and MiniBooNE anomalies with coherent neutrino
3791 scattering experiments". In: *Phys. Rev. D* 101.7 (2020). URL:
3792 <https://doi.org/10.1103%2Fphysrevd.101.075051>.
- 3793 [48] MicroBooNE Collaboration. "Search for an Excess of Electron Neutrino
3794 Interactions in MicroBooNE Using Multiple Final State Topologies". In: *arXiv*
3795 (2021). URL: <https://arxiv.org/abs/2110.14054>.
- 3796 [49] B. Armbruster et al. "Upper limits for neutrino oscillations $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ from muon
3797 decay at rest". In: *Phys. Rev. D* 65 (11 2002), p. 112001. URL:
3798 <https://link.aps.org/doi/10.1103/PhysRevD.65.112001>.
- 3799 [50] T. K. Gaisser and M. Honda. "Flux of Atmospheric Neutrinos". In: *Annual Review
3800 of Nuclear and Particle Science* 52.1 (2002), pp. 153–199. URL:
3801 <https://doi.org/10.1146%2Fannurev.nucl.52.050102.090645>.
- 3802 [51] G. D. Barr et al. "Three-dimensional calculation of atmospheric neutrinos". In:
3803 *Phys. Rev. D* 70.2 (2004). URL: <https://doi.org/10.1103/physrevd.70.023006>.
- 3804 [52] M. Honda et al. "Calculation of atmospheric neutrino flux using the interaction
3805 model calibrated with atmospheric muon data". In: *Phys. Rev. D* 75.4 (2007). URL:
3806 <https://doi.org/10.1103/physrevd.75.043006>.
- 3807 [53] M. Honda et al. "New calculation of the atmospheric neutrino flux in a
3808 three-dimensional scheme". In: *Phys. Rev. D* 70 (4 2004), p. 043008. URL:
3809 <https://link.aps.org/doi/10.1103/PhysRevD.70.043008>.
- 3810 [54] M. Honda et al. "Improvement of low energy atmospheric neutrino flux
3811 calculation using the JAM nuclear interaction model". In: *Phys. Rev. D* 83 (12
3812 2011), p. 123001. URL:
3813 <https://link.aps.org/doi/10.1103/PhysRevD.83.123001>.
- 3814 [55] A. Fassò et al. *FLUKA: Status and Prospects for Hadronic Applications*. Ed. by
3815 Andreas Kling et al. Berlin, Heidelberg: Springer Berlin Heidelberg, 2001,
3816 pp. 955–960.
- 3817 [56] Y. Ashie et al. "Measurement of atmospheric neutrino oscillation parameters by
3818 Super-Kamiokande I". In: *Phys. Rev. D* 71.11 (2005). URL:
3819 <https://doi.org/10.1103/physrevd.71.112005>.
- 3820 [57] F. Reines et al. "Evidence for high-energy cosmic-ray neutrino interactions". In:
3821 *Phys. Rev. Lett.* 15.9 (Aug. 1965), pp. 429–433.
- 3822 [58] D. Casper et al. "Measurement of atmospheric neutrino composition with the
3823 IMB-3 detector". In: *Phys. Rev. Lett.* 66 (20 1991), pp. 2561–2564. URL:
3824 <https://link.aps.org/doi/10.1103/PhysRevLett.66.2561>.
- 3825 [59] K. S. Hirata et al. "Observation of a small atmospheric ν_μ/ν_e ratio in
3826 Kamiokande". en. In: *Phys. Lett. B* 280.1-2 (Apr. 1992), pp. 146–152.
- 3827 [60] Z. Li et al. "Measurement of the tau neutrino cross section in atmospheric
3828 neutrino oscillations with Super-Kamiokande". In: *Phys. Rev. D* 98.5 (2018). URL:
3829 <https://doi.org/10.1103/physrevd.98.052006>.

- 3830 [61] K. Abe et al. "Atmospheric neutrino oscillation analysis with external constraints
3831 in Super-Kamiokande I-IV". In: *Phys. Rev. D* 97 (7 2018), p. 072001. URL:
3832 <https://link.aps.org/doi/10.1103/PhysRevD.97.072001>.
- 3833 [62] T2K Collaboration. "Constraint on the matter-antimatter symmetry-violating
3834 phase in neutrino oscillations". en. In: *Nature* 580.7803 (Apr. 2020), pp. 339–344.
- 3835 [63] M. A. Acero et al. "First measurement of neutrino oscillation parameters using
3836 neutrinos and antineutrinos by NOvA". en. In: *Phys. Rev. Lett.* 123.15 (Oct. 2019),
3837 p. 151803.
- 3838 [64] M. G. Aartsen et al. "Measurement of atmospheric neutrino oscillations at 6–56
3839 GeV with IceCube DeepCore". In: *Phys. Rev. Lett.* 120.7 (Feb. 2018).
- 3840 [65] P. Adamson et al. "Combined analysis of $\nu\mu$ Disappearance
3841 and $\nu\mu \rightarrow \nu e$ Appearance in MINOS using accelerator and atmospheric neutrinos".
3842 In: *Phys. Rev. Lett.* 112.19 (May 2014).
- 3843 [66] M. Sajjad Athar et al. "Status and perspectives of neutrino physics". In: *Progress
3844 in Particle and Nuclear Physics* 124 (2022), p. 103947. URL:
3845 <https://doi.org/10.1016/Fj.pnnp.2022.103947>.
- 3846 [67] S. B. Kim, T. Lasserre, and Y. Wang. "Reactor Neutrinos". In: *Adv. High Energy
3847 Phys.* 2013 (2013), pp. 1–34.
- 3848 [68] K Abe et al. "First gadolinium loading to Super-Kamiokande". en. In: *Nucl.
3849 Instrum. Methods Phys. Res. A* 1027.166248 (Mar. 2022), p. 166248.
- 3850 [69] F. P. An et al. "Observation of Electron-Antineutrino Disappearance at Daya Bay".
3851 In: *Phys. Rev. Lett.* 108 (17 2012), p. 171803. URL:
3852 <https://link.aps.org/doi/10.1103/PhysRevLett.108.171803>.
- 3853 [70] J. K. Ahn et al. "Observation of Reactor Electron Antineutrinos Disappearance in
3854 the RENO Experiment". In: *Phys. Rev. Lett.* 108 (19 2012), p. 191802. URL:
3855 <https://link.aps.org/doi/10.1103/PhysRevLett.108.191802>.
- 3856 [71] Y. Abe et al. "Indication of Reactor $\bar{\nu}_e$ Disappearance in the Double Chooz
3857 Experiment". In: *Phys. Rev. Lett.* 108 (13 2012), p. 131801. URL:
3858 <https://link.aps.org/doi/10.1103/PhysRevLett.108.131801>.
- 3859 [72] JUNO Collaboration. "TAO Conceptual Design Report: A Precision
3860 Measurement of the Reactor Antineutrino Spectrum with Sub-percent Energy
3861 Resolution". In: *arXiv* (2020). arXiv: 2005.08745 [physics.ins-det].
- 3862 [73] M. P. Decowski. "KamLAND's precision neutrino oscillation measurements". en.
3863 In: *Nucl. Phys. B*. 908 (July 2016), pp. 52–61.
- 3864 [74] A. Gando et al. "Constraints on θ_{13} from a three-flavor oscillation analysis of
3865 reactor antineutrinos at KamLAND". In: *Phys. Rev. D* 83 (5 2011), p. 052002. URL:
3866 <https://link.aps.org/doi/10.1103/PhysRevD.83.052002>.
- 3867 [75] Particle Data Group. "Review of particle physics". en. In: *Prog. Theor. Exp. Phys.*
3868 2020.8 (Aug. 2020).
- 3869 [76] R. L. Workman et al. "Review of Particle Physics". In: *PTEP* 2022 (2022),
3870 p. 083C01.

- 3871 [77] K. Abe et al. "Precise Measurement of the Neutrino Mixing Parameter θ_{23} from
3872 Muon Neutrino Disappearance in an Off-Axis Beam". In: *Phys. Rev. Lett.* 112 (18
3873 2014), p. 181801. URL:
3874 <https://link.aps.org/doi/10.1103/PhysRevLett.112.181801>.
- 3875 [78] R. Wendell. "Three Flavor Oscillation Analysis of Atmospheric Neutrinos in
3876 Super-Kamiokande". PhD thesis. University of North Carolina, 2008.
- 3877 [79] A. M. Dziewonski and D. L. Anderson. "Preliminary reference Earth model". en.
3878 In: *Phys. Earth Planet. Inter.* 25.4 (June 1981), pp. 297–356.
- 3879 [80] Y. Fukuda et al. "Evidence for oscillation of atmospheric neutrinos". In: *Phys. Rev.*
3880 *Lett.* 81.8 (Aug. 1998), pp. 1562–1567.
- 3881 [81] K. Abe et al. "Calibration of the Super-Kamiokande detector". In: *Nuclear*
3882 *Instruments and Methods in Physics Research Section A* 737 (2014), pp. 253–272. URL:
3883 <https://doi.org/10.1016/j.nima.2013.11.081>.
- 3884 [82] L. Wan. "Atmospheric Neutrino Super-K". In: *Neutrino 2022* (2022). URL:
3885 <https://zenodo.org/record/6694761>.
- 3886 [83] M. Jiang. "Study of the neutrino mass hierarchy with the atmospheric neutrino
3887 data collected in Super-Kamiokande IV". PhD thesis. Kyoto University, 2019.
- 3888 [84] S. Fukuda et al. "The super-kamiokande detector". en. In: *Nucl. Instrum. Methods*
3889 *Phys. Res. A* 501.2-3 (Apr. 2003), pp. 418–462. eprint:
3890 <http://www.sciencedirect.com/science/article/pii/S016890020300425X>.
- 3891 [85] Y. Itow et al. "The JHF-Kamioka neutrino project". In: *arXiv* (2001).
- 3892 [86] H. Kume et al. "20 inch diameter photomultiplier". In: *Nuclear Instruments and*
3893 *Methods in Physics Research* 205.3 (1983), pp. 443–449. URL:
3894 <https://www.sciencedirect.com/science/article/pii/0167508783900078>.
- 3895 [87] A. Suzuki et al. "Improvement of 20 in. diameter photomultiplier tubes". In:
3896 *Nuclear Instruments and Methods in Physics Research Section A* 329.1-2 (May 1993),
3897 pp. 299–313. URL: [https://doi.org/10.1016/0168-9002\(93\)90949-i](https://doi.org/10.1016/0168-9002(93)90949-i).
- 3898 [88] Y. Nakano et al. "Measurement of the radon concentration in purified water in
3899 the Super-Kamiokande IV detector". en. In: *Nucl. Instrum. Methods Phys. Res. A*
3900 977.164297 (Oct. 2020), p. 164297.
- 3901 [89] Hamamatsu. *Hamamatsu Photonics Photomultiplier Tubes Handbook*. URL:
3902 https://www.hamamatsu.com/content/dam/hamamatsu-photonics/sites/documents/99_SALES_LIBRARY/etd/PMT_handbook_v4E.pdf.
- 3904 [90] J. F. Beacom and M. R. Vagins. "Antineutrino Spectroscopy with Large Water
3905 Čerenkov Detectors". In: *Phys. Rev. Lett.* 93 (17 2004), p. 171101. URL:
3906 <https://link.aps.org/doi/10.1103/PhysRevLett.93.171101>.
- 3907 [91] L. Marti et al. "Evaluation of gadolinium's action on water Cherenkov detector
3908 systems with EGADS". en. In: *Nucl. Instrum. Methods Phys. Res. A* 959.163549
3909 (Apr. 2020), p. 163549.
- 3910 [92] M. Vagins. "Solar/DSNB Neutrino SK-Gd". In: *Neutrino 2022* (2022). URL:
3911 <https://zenodo.org/record/6696210>.
- 3912 [93] J. Focht. "Dark Noise Calibration of the Super-Kamiokande Outer Detector".
3913 PhD thesis. Massachusetts Institute of Technology, 2004.

- 3914 [94] T. Tanimori et al. "Design and performance of semi-custom analog IC including
3915 two TACs and two current integrators for 'Super-Kamiokande'". In: *IEEE*
3916 *Transactions on Nuclear Science* 36.1 (1989), pp. 497–501.
- 3917 [95] J. Hosaka et al. "Solar neutrino measurements in Super-Kamiokande-I". In: *Phys.*
3918 *Rev. D* 73 (11 2006), p. 112001.
- 3919 [96] H. Nishino et al. "High-speed charge-to-time converter ASIC for the
3920 Super-Kamiokande detector". en. In: *Nucl. Instrum. Methods Phys. Res. A* 610.3
3921 (Nov. 2009), pp. 710–717.
- 3922 [97] S. Yamada et al. "Commissioning of the New Electronics and Online System for
3923 the Super-Kamiokande Experiment". In: *IEEE Transactions on Nuclear Science* 57.2
3924 (2010), pp. 428–432.
- 3925 [98] S. Yamada et al. "New online system without hardware trigger for the
3926 Super-Kamiokande experiment". In: *2007 IEEE Nuclear Science Symposium*
3927 *Conference Record*. Honolulu, HI, USA: IEEE, Oct. 2007.
- 3928 [99] G. Carminati. "The new wide-band solar neutrino trigger for super-kamiokande".
3929 In: *Phys. Procedia* 61 (2015), pp. 666–672.
- 3930 [100] P. A. Čerenkov. "Visible radiation produced by electrons moving in a medium
3931 with velocities exceeding that of light". In: *Phys. Rev.* 52.4 (Aug. 1937),
3932 pp. 378–379.
- 3933 [101] I. Frank and I. Tamm. "Coherent visible radiation of fast electrons passing
3934 through matter". In: *Selected Papers*. Berlin, Heidelberg: Springer Berlin
3935 Heidelberg, 1991, pp. 29–35.
- 3936 [102] The T2K Collaboration. "Letter of Intent: Neutrino Oscillation Experiment at
3937 JHF". In: *KEK Proposal* (2001). eprint:
3938 <http://neutrino.kek.jp/jhfnu/loi/loi.v2.030528.pdf>.
- 3939 [103] The K2K Collaboration and S. H. Ahn. "Detection of Accelerator-Produced
3940 Neutrinos at a Distance of 250 km". In: (Feb. 2001). arXiv: [hep-ex/0103001](https://arxiv.org/abs/hep-ex/0103001)
3941 [hep-ex].
- 3942 [104] The T2K Collaboration. "Tokai-to-Kamioka (T2K) Long Baseline Neutrino
3943 Oscillation Experiment Proposal". In: *KEK Proposal* (2006). eprint: [http://j-](http://j-parc.jp/researcher/Hadron/en/pac\0606/pdf/p11-Nishikawa.pdf)
3944 [parc.jp/researcher/Hadron/en/pac\0606/pdf/p11-Nishikawa.pdf](http://j-parc.jp/researcher/Hadron/en/pac\0606/pdf/p11-Nishikawa.pdf).
- 3945 [105] C. Bronner. "Accelerator Neutrino I Recent results from T2K". In: *Neutrino 2022*
3946 (2022). URL: <https://zenodo.org/record/6683821>.
- 3947 [106] K. Abe et al. "Observation of Electron Neutrino Appearance in a Muon Neutrino
3948 Beam". In: *Phys. Rev. Lett.* 112 (6 2014), p. 061802. eprint:
3949 <https://link.aps.org/doi/10.1103/PhysRevLett.112.061802>.
- 3950 [107] T. Fukuda et al. "Proposal for precise measurement of neutrino-water
3951 cross-section in NINJA physics run". Proposal for J-PARC and KEK. 2017.
- 3952 [108] T. Ovsiannikova et al. "New experiment WAGASCI to measure cross sections of
3953 neutrino interactions in water and hydrocarbon using J-PARC beam". In: *Physics*
3954 *of Particles and Nuclei* 48.6 (2017), pp. 1014–1017. eprint:
3955 <https://doi.org/10.1134/S1063779617060478>.

- 3956 [109] M. Antonova et al. "Baby MIND: a magnetized segmented neutrino detector for
3957 the WAGASCI experiment". In: *Journal of Instrumentation* 12.07 (2017), p. C07028.
3958 eprint: <http://stacks.iop.org/1748-0221/12/i=07/a=C07028>.
- 3959 [110] K. Abe et al. "First measurement of the charged current $\bar{\nu}_\mu$ double differential
3960 cross section on a water target without pions in the final state". In: *Phys. Rev. D*
3961 102 (1 2020), p. 012007. URL:
3962 <https://link.aps.org/doi/10.1103/PhysRevD.102.012007>.
- 3963 [111] K. Abe et al. "Measurements of $\bar{\nu}_\mu$ and $\bar{\nu}_\mu + \nu_\mu$ charged-current cross-sections
3964 without detected pions or protons on water and hydrocarbon at a mean
3965 anti-neutrino energy of 0.86 GeV". In: *Progress of Theoretical and Experimental
3966 Physics* 2021.4 (Mar. 2021). URL: <https://doi.org/10.1093/ptep/ptab014>.
- 3967 [112] K. Matsuoka et al. "Design and performance of the muon monitor for the T2K
3968 neutrino oscillation experiment". In: *Nuclear Instruments and Methods in Physics
3969 Research Section A* 624.3 (2010), pp. 591–600. eprint:
3970 <http://www.sciencedirect.com/science/article/pii/S016890021002098X>.
- 3971 [113] K. Abe et al. "Improved constraints on neutrino mixing from the T2K experiment
3972 with 3.13×10^{21} protons on target". en. In: *Phys. Rev. D.* 103.11 (June 2021).
- 3973 [114] T. Vladisavljevic. *Predicting the T2K neutrino flux and measuring oscillation
3974 parameters*. 1st ed. Springer theses. Cham, Switzerland: Springer Nature, Sept.
3975 2020.
- 3976 [115] D. Beavis, A. Carroll, and I. Chiang. "Long baseline neutrino oscillation
3977 experiment at the AGS. Physics design report". In: *Office of Scientific and Technical
3978 Information (OSTI)* (Apr. 1995).
- 3979 [116] P.-A. Amaudruz et al. "The T2K fine-grained detectors". In: *Nuclear Instruments
3980 and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and
3981 Associated Equipment* 696 (Dec. 2012), pp. 1–31. URL:
3982 <https://doi.org/10.1016/j.nima.2012.08.020>.
- 3983 [117] N. Abgrall et al. "Time projection chambers for the T2K near detectors". In:
3984 *Nuclear Instruments and Methods in Physics Research Section A: Accelerators,
3985 Spectrometers, Detectors and Associated Equipment* 637.1 (May 2011), pp. 25–46. URL:
3986 <https://doi.org/10.1016/j.nima.2011.02.036>.
- 3987 [118] S. Assylbekov et al. "The T2K ND280 off-axis pi-zero detector". In: *Nuclear
3988 Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers,
3989 Detectors and Associated Equipment* 686 (Sept. 2012), pp. 48–63. URL:
3990 <https://doi.org/10.1016/j.nima.2012.05.028>.
- 3991 [119] D. Allan et al. "The electromagnetic calorimeter for the T2K near detector
3992 ND280". In: *Journal of Instrumentation* 8.10 (2013), P10019–P10019. URL:
3993 <https://doi.org/10.1088%2F1748-0221%2F8%2F10%2Fp10019>.
- 3994 [120] CERN Courier. *UA1 magnet sets off for a second new life*. 2008. URL: <https://cerncourier.com/a/ua1-magnet-sets-off-for-a-second-new-life/>.
- 3995 [121] F. Vannucci. "The NOMAD Experiment at CERN". In: *Advances in High Energy
3996 Physics* 2014 (2014), pp. 1–20. URL: <https://doi.org/10.1155/2014/129694>.

- 3998 [122] S. Aoki et al. "The T2K Side Muon Range Detector (SMRD)". In: *Nuclear*
3999 *Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers,*
4000 *Detectors and Associated Equipment* 698 (Jan. 2013), pp. 135–146. URL:
4001 <https://doi.org/10.1016/j.nima.2012.10.001>.
- 4002 [123] K. Suzuki et al. "Measurement of the muon beam direction and muon flux for the
4003 T2K neutrino experiment". In: *Progress of Theoretical and Experimental Physics*
4004 2015.5 (2015), pp. 53C01–0. URL: <https://doi.org/10.1093%2Fptep%2Fptv054>.
- 4005 [124] S. Brooks et al. *Handbook of Markov Chain Monte Carlo*. CRC Press, 2011.
- 4006 [125] W. R. Gilks, S. Richardson, and D. J. Spiegelhalter. *Markov Chain Monte Carlo in*
4007 *Practice*. Chapman & Hall/CRC Interdisciplinary Statistics, 1995.
- 4008 [126] C. Wret. "Minimising systematic uncertainties in the T2K experiment using
4009 near-detector and external data". PhD thesis. Imperial College London, 2018.
- 4010 [127] K. E. Duffy. "Measurement of the Neutrino Oscillation Parameters $\sin^2 \theta_{23}$, Δm_{32}^2 ,
4011 $\sin^2 \theta_{13}$, and δ_{CP} in Neutrino and Antineutrino Oscillation at T2K". PhD thesis.
4012 Oriel College, University of Oxford, 2016.
- 4013 [128] C. Bojeckho and A. Kabooth. "Muon neutrino disappearance simultaneous fit of
4014 ND280 and SK with Run 1+2+3 data using Markov Chain Monte Carlo Analysis".
4015 In: *T2K Technical Note* 140 (2013).
- 4016 [129] T. Bayes. "An essay toward solving a problem in the doctrine of chances". In: *Phil.*
4017 *Trans. Roy. Soc. Lond.* 53 (1764), pp. 370–418.
- 4018 [130] A. Sztuc. "Standard and Non-Standard Neutrino-Antineutrino Oscillation
4019 Analyses and Event Reconstruction Studies using Markov Chain Monte Carlo
4020 Methods at T2K". PhD thesis. Imperial College London, 2021.
- 4021 [131] N. Metropolis et al. "Equation of State Calculations by Fast Computing
4022 Machines". In: *Journal of Chemical Physics* 21.6 (1970).
- 4023 [132] W. K. Hastings. "Monte Carlo Sampling Methods Using Markov Chains and
4024 Their Applications". In: *Biometrika* 57.1 (1970).
- 4025 [133] J. Dunkley et al. "Fast and reliable Markov chain Monte Carlo technique for
4026 cosmological parameter estimation". en. In: *Mon. Not. R. Astron. Soc.* 356.3 (Jan.
4027 2005), pp. 925–936.
- 4028 [134] H. Jeffreys. *The Theory of Probability*. Oxford Classic Texts in the Physical Sciences.
4029 1939.
- 4030 [135] R. E. Kass and A. E. Raftery. "Bayes factors". en. In: *J. Am. Stat. Assoc.* 90.430
4031 (June 1995), pp. 773–795.
- 4032 [136] T.T. Böhlen et al. "The FLUKA Code: Developments and Challenges for High
4033 Energy and Medical Applications". In: *Nuclear Data Sheets* 120 (2014), pp. 211
4034 –214. eprint:
4035 <http://www.sciencedirect.com/science/article/pii/S0090375214005018>.
- 4036 [137] R. Brun et al. *GEANT: Detector Description and Simulation Tool; Oct 1994*. CERN
4037 Program Library. Long Writeup W5013. Geneva: CERN, 1993. eprint:
4038 <http://cds.cern.ch/record/1082634>.

- 4039 [138] C. Zeitnitz and T.A. Gabriel. "The GEANT-CALOR interface and benchmark
4040 calculations of ZEUS test calorimeters". In: *Nuclear Instruments and Methods in*
4041 *Physics Research Section A* 349.1 (1994), pp. 106–111. eprint:
4042 <http://www.sciencedirect.com/science/article/pii/0168900294906130>.
- 4043 [139] A. Fiorentini et al. "Flux Prediction and Uncertainty Updates with NA61 2009
4044 Thin Target Data and Negative Focussing Mode Predictions". In: *T2K Technical*
4045 *Note* 217 (2017).
- 4046 [140] N. Abgrall et al. "Measurements of cross sections and charged pion spectra in
4047 proton-carbon interactions at 31 GeV". In: *Phys. Rev. C* 84.3 (2011). URL:
4048 <https://doi.org/10.1103/2Fphysrevc.84.034604>.
- 4049 [141] N. Abgrall et al. "Measurement of production properties of positively charged
4050 kaons in proton-carbon interactions at 31 GeV". In: *Phys. Rev. C* 85.3 (2012). URL:
4051 <https://doi.org/10.1103%2Fphysrevc.85.035210>.
- 4052 [142] N. Abgrall et al. "Pion emission from the T2K replica target: Method, results and
4053 application". In: *Nuclear Instruments and Methods in Physics Research Section A* 701
4054 (2013), pp. 99–114. eprint:
4055 <http://www.sciencedirect.com/science/article/pii/S016890021201234X>.
- 4056 [143] M. Apollonio et al. "Forward production of charged pions with incident protons
4057 on nuclear targets at the CERN Proton Synchrotron". In: *Phys. Rev. C* 80 (3 2009),
4058 p. 035208. eprint: <https://link.aps.org/doi/10.1103/PhysRevC.80.035208>.
- 4059 [144] B. Blau et al. "The superconducting magnet of AMS-02". In: *Nuclear Physics B -*
4060 *Proceedings Supplements* 113.1-3 (Dec. 2002), pp. 125–132. URL:
4061 [https://doi.org/10.1016/s0920-5632\(02\)01831-5](https://doi.org/10.1016/s0920-5632(02)01831-5).
- 4062 [145] S. Haino et al. "Measurements of primary and atmospheric cosmic-ray spectra
4063 with the BESS-TeV spectrometer". In: *Physics Letters B* 594.1-2 (July 2004),
4064 pp. 35–46. URL: <https://doi.org/10.1016/j.physletb.2004.05.019>.
- 4065 [146] NASA. *U.S. Standard Atmosphere, 1976.* 1976. URL: <https://ntrs.nasa.gov/api/citations/19770009539/downloads/19770009539.pdf>.
- 4066 [147] S. Roesler, R. Engel, and J. Ranft. "The Monte Carlo Event Generator
4067 DPMJET-III". In: *Advanced Monte Carlo for Radiation Physics, Particle Transport*
4068 *Simulation and Applications*. Springer Berlin Heidelberg, 2001, pp. 1033–1038. URL:
4069 https://doi.org/10.1007/978-3-642-18211-2_166.
- 4070 [148] K. Niita et al. "PHITS—a particle and heavy ion transport code system". In:
4071 *Radiation Measurements* 41.9-10 (Oct. 2006), pp. 1080–1090. URL:
4072 <https://doi.org/10.1016/j.radmeas.2006.07.013>.
- 4073 [149] T. Sanuki et al. "Measurements of atmospheric muon spectra at mountain
4074 altitude". In: *Physics Letters B* 541.3-4 (2002), pp. 234–242. URL:
4075 <https://doi.org/10.1016%2Fs0370-2693%2802%2902265-7>.
- 4076 [150] P. Achard et al. "Measurement of the atmospheric muon spectrum from 20 to
4077 3000 GeV". In: *Physics Letters B* 598.1-2 (2004), pp. 15–32. URL:
4078 <https://doi.org/10.1016%2Fj.physletb.2004.08.003>.
- 4079 [151] K. Sato. "Atmospheric Neutrino Reviews on neutrino fluxes (low E atm nu)". In:
4080 *Neutrino 2022* (2022). URL: <https://zenodo.org/record/6695711>.

- 4082 [152] Y. Hayato and L. Pickering. "The NEUT neutrino interaction simulation program
4083 library". In: *The European Physical Journal Special Topics* 230.24 (Oct. 2021),
4084 pp. 4469–4481. URL: <https://doi.org/10.1140/epjs/s11734-021-00287-7>.
- 4085 [153] Y. Hayato. "A Neutrino Interaction Simulation Program Library NEUT". In: *Acta
4086 Physica Polonica B* 40.9 (2009).
- 4087 [154] C. H. Llewellyn Smith. "Neutrino reactions at accelerator energies". In: *Physics
4088 Reports* 3.5 (1972), pp. 261 –379. eprint:
4089 <http://www.sciencedirect.com/science/article/pii/0370157372900105>.
- 4090 [155] O. Benhar, A. Fabrocini, and S. Fantoni. "The nucleon spectral function in infinite
4091 nuclear matter". In: *Nuclear Physics A* 497 (June 1989), pp. 423–430. URL:
4092 [https://doi.org/10.1016/0375-9474\(89\)90484-3](https://doi.org/10.1016/0375-9474(89)90484-3).
- 4093 [156] R. Bradford et al. "A New Parameterization of the Nucleon Elastic Form Factors".
4094 In: *Nuclear Physics B - Proceedings Supplements* 159 (2006). Proceedings of the 4th
4095 International Workshop on Neutrino-Nucleus Interactions in the Few-GeV
4096 Region, pp. 127 –132. eprint:
4097 <http://www.sciencedirect.com/science/article/pii/S0920563206005184>.
- 4098 [157] A. A. Aguilar-Arevalo et al. "First measurement of the muon neutrino charged
4099 current quasielastic double differential cross section". In: *Phys. Rev. D* 81.9 (2010).
4100 URL: <https://doi.org/10.1103/2Fphysrevd.81.092005>.
- 4101 [158] R. Gran et al. "Neutrino-nucleus quasi-elastic and 2p2h interactions up to 10
4102 GeV". In: *Phys. Rev. D* 88 (11 2013), p. 113007. eprint:
4103 <https://link.aps.org/doi/10.1103/PhysRevD.88.113007>.
- 4104 [159] C. Berger and L. M. Sehgal. "Lepton mass effects in single pion production by
4105 neutrinos". In: *Phys. Rev. D* 76 (11 2007), p. 113004. URL:
4106 <https://link.aps.org/doi/10.1103/PhysRevD.76.113004>.
- 4107 [160] C. Berger and L. M. Sehgal. "Partially conserved axial vector current and
4108 coherent pion production by low energy neutrinos". In: *Phys. Rev. D* 79 (5 2009),
4109 p. 053003. eprint: <https://link.aps.org/doi/10.1103/PhysRevD.79.053003>.
- 4110 [161] T. Sjöstrand. "High-energy-physics event generation with PYTHIA 5.7 and
4111 JETSET 7.4". In: *Computer Physics Communications* 82.1 (Aug. 1994), pp. 74–89.
4112 URL: [https://doi.org/10.1016/0010-4655\(94\)90132-5](https://doi.org/10.1016/0010-4655(94)90132-5).
- 4113 [162] C. Bronner and M. Hartz. "Tuning of the Charged Hadrons Multiplicities for
4114 Deep Inelastic Interactions in NEUT". In: *Proceedings of the 10th International
4115 Workshop on Neutrino-Nucleus Interactions in Few-GeV Region (NuInt15)*. Journal of
4116 the Physical Society of Japan, Dec. 2016. URL:
4117 <https://doi.org/10.7566/jpscp.12.010041>.
- 4118 [163] M. Glück, E. Reya, and A. Vogt. "Dynamical parton distributions revisited". In:
4119 *The European Physical Journal C* 5.3 (1998), pp. 461–470. URL:
4120 <https://doi.org/10.1007%2Fs100529800978>.
- 4121 [164] A. Bodek and U. Yang. "Axial and Vector Structure Functions for Electron- and
4122 Neutrino- Nucleon Scattering Cross Sections at all Q^2 using Effective Leading
4123 order Parton Distribution Functions". In: *arXiv* (2010). URL:
4124 <https://arxiv.org/abs/1011.6592>.

- 4125 [165] A. Bodek and U. Yang. "Update to the Bodek-Yang Unified Model for Electron-
4126 and Neutrino- Nucleon Scattering Cross Sections". In: *arXiv* (2010). URL:
4127 <https://arxiv.org/abs/1012.0261>.
- 4128 [166] S. Gollapinni. "Neutrino Cross section Future". In: *arXiv* (2016). URL:
4129 <https://arxiv.org/abs/1602.05299>.
- 4130 [167] E. S. Pinzon Guerra et al. "Using world π^\pm -nucleus scattering data to constrain
4131 an intranuclear cascade model". In: *Phys. Rev. D* 99 (5 2019), p. 052007.
- 4132 [168] S. Agostinelli et al. "GEANT4: A Simulation toolkit". In: *Nucl. Instrum. Meth.*
4133 A506 (2003), pp. 250–303.
- 4134 [169] R. Brun et al. "GEANT3". In: *Cern Document Server* (Sept. 1987). URL:
4135 <https://cds.cern.ch/record/1119728/>.
- 4136 [170] A. Himmel et al. "Super-Kamiokande events and data quality studies for T2K
4137 Runs 5 and 6". In: *T2K Technical Note* 219 (2015).
- 4138 [171] S. Berkman et al. "fiTQun: A New Reconstruction Algorithm for Super-K". In:
4139 *T2K Technical Note* 146 (2013).
- 4140 [172] R.B. Patterson et al. "The extended-track event reconstruction for MiniBooNE".
4141 In: *Nuclear Instruments and Methods in Physics Research Section A* 608.1 (2009),
4142 pp. 206–224. URL: <https://doi.org/10.1016/j.nima.2009.06.064>.
- 4143 [173] M. Shiozawa. "Reconstruction algorithms in the Super-Kamiokande large water
4144 Cherenkov detector". In: *Nuclear Instruments and Methods in Physics Research*
4145 *Section A: Accelerators, Spectrometers, Detectors and Associated Equipment* 433.1-2
4146 (Aug. 1999), pp. 240–246.
- 4147 [174] K. Abe et al. "Search for CP Violation in Neutrino and Antineutrino Oscillations
4148 by the T2K Experiment with 2.2×10^{21} Protons on Target". In: *Phys. Rev. Lett.*
4149 121.17 (Oct. 2018). URL: <https://doi.org/10.1103/physrevlett.121.171802>.
- 4150 [175] K. Abe et al. "Measurements of neutrino oscillation in appearance and
4151 disappearance channels by the T2K experiment with 6.6×10^{20} protons on
4152 target". In: *Phys. Rev. D* 91.7 (Apr. 2015).
- 4153 [176] F. James. "MINUIT Function Minimization and Error Analysis". In: *Reference*
4154 *Manual, Version 94.1, CERN Program Library Long Writeup D506 D.506* (1994).
4155 eprint:
4156 <https://root.cern.ch/sites/d35c7d8c.web.cern.ch/files/minuit.pdf>.
- 4157 [177] X. Li and M. Wilking. "FiTQun Event Selection Optimization". In: *T2K Technical*
4158 *Note* 319 (2017).
- 4159 [178] S. Tobayama. "An Analysis of the Oscillation of Atmospheric Neutrinos".
4160 PhD thesis. British Columbia U., 2016.
- 4161 [179] D. Barrow et al. "Super-Kamiokande Data Quality, MC, and Systematics in Run
4162 10". In: *T2K Technical Note* 399 (2020).
- 4163 [180] A. Maghrabi, A. Aldosari, and M. Almutairi. "Correlation analyses between solar
4164 activity parameters and cosmic ray muons between 2002 and 2012 at high cutoff
4165 rigidity". In: *Advances in Space Research* 68.7 (Oct. 2021), pp. 2941–2952.
- 4166 [181] J. Beringer et al. "Review of Particle Physics". In: *Phys. Rev. D* 86 (1 2012),
4167 p. 010001. URL: <https://link.aps.org/doi/10.1103/PhysRevD.86.010001>.

- 4168 [182] S. Nakayama K. Iyogi and Y. Obayashi. "T2K data acquisition and FC event
4169 selection at Super-Kamiokande". In: *T2K Technical Note* 027 (2011).
- 4170 [183] L. K. Pik. "Study of the neutrino mass hierarchy with the atmospheric neutrino
4171 data observed in Super-Kamiokande". PhD thesis. Tokyo University, 2012.
- 4172 [184] R. Wendell et al. "Atmospheric neutrino oscillation analysis with subleading
4173 effects in Super-Kamiokande I, II, and III". In: *Phys. Rev. D* 81 (9 2010), p. 092004.
- 4174 [185] J. Hosaka et al. "Three flavor neutrino oscillation analysis of atmospheric
4175 neutrinos in Super-Kamiokande". In: *Phys. Rev. D* 74 (3 2006), p. 032002.
- 4176 [186] D. Barrow et al. "Flux and interaction models for the initial T2K-SK atmospheric
4177 joint fit studies". In: *T2K Technical Note* 422 (2022).
- 4178 [187] L. Munteanu et al. "Constraining the Flux and Cross Section Models with Data
4179 from ND280 using FGD1 and FGD2 for the 2020 Oscillation Analysis". In: *T2K*
4180 *Technical Note* 395 (2020).
- 4181 [188] P. Bartet et al. " ν_μ CC event selections in the ND280 tracker using Run 2+3+4
4182 data". In: *T2K Technical Note* 212 (2015).
- 4183 [189] W. Parker. "Constraining Systematic Uncertainties at T2K using Near Detector
4184 Data". PhD thesis. Royal Holloway University of London, 2020.
- 4185 [190] V. Berardi et al. " $\bar{\nu}_\mu$ event selection in the ND280 tracker using Run 5c and Run 6
4186 anti-neutrino beam data". In: *T2K Technical Note* 246 (2015).
- 4187 [191] B. Kirby J. Kim and M. Wilking. "Michel Electron Tagging in FGD1". In: *T2K*
4188 *Technical Note* 104 (2012).
- 4189 [192] J. Misset. "TN-318: Fit to Super-K Atmospheric Neutrino Data for Optimization
4190 of the fiTQun Fiducial Volume Cuts and Estimation of Detector Uncertainties".
4191 In: *T2K Technical Note* 318 (2017).
- 4192 [193] J. Chakrani et al. "NIWG model and uncertainties for 2021 oscillation analysis".
4193 In: *T2K Technical Note* 414 (2022).
- 4194 [194] M. Wascko. "T2K Status, Results, And Plans". In: *Neutrino 2018* (2018). URL:
4195 <https://zenodo.org/record/1286752>.
- 4196 [195] T. Vladisavljevic et al. "Flux Prediction and Uncertainty with NA61/SHINE 2009
4197 Replica-Target Data (TN354 version 3.3)". In: *T2K Technical Note* 354 (2020).
- 4198 [196] G. Ambrosini et al. "K/ π production ratios from 450 GeV/c protons on
4199 beryllium". en. In: *Phys. Lett. B* 420.1-2 (Feb. 1998), pp. 225–232.
- 4200 [197] E. Atkin et al. "NIWG model and uncertainties for 2019-2020 oscillation analysis".
4201 In: *T2K Technical Note* 344 (2019).
- 4202 [198] D. Barrow et al. "SK atmospheric T2K beam joint fit technical note, MaCh3
4203 details". In: *T2K Technical Note* 426 (2022).
- 4204 [199] A. A. Aguilar-Arevalo et al. "Measurement of ν_μ and $\bar{\nu}_\mu$ induced neutral current
4205 single π^0 production cross sections on mineral oil at $E_\nu \sim \mathcal{O} (1 \text{ GeV})$ ". In: *Phys.*
4206 *Rev. D* 81 (1 2010), p. 013005.
- 4207 [200] P. Perio and J. Imber. "Super-K Systematic Uncertainties for RUN1-4 Joint ν_e and
4208 ν_μ Analyses". In: *T2K Technical Note* 186 (2014).

- 4209 [201] P. Perio and J. Imber. "Update of SK ν_e systematic error for 2012a oscillation
4210 analysis". In: *T2K Technical Note* 107 (2012).
- 4211 [202] D. Barrow and C. Vilela. *T2K-SK Detector Matrix Uncertainties - MaCh3 Integration*.
4212 <https://git.t2k.org/t2k-sk/t2ksk-detcovmat/-/tree/feature/MaCh3Integration>. Accessed: 22-06-2022.
- 4214 [203] R. Barlow and C. Beeston. "Fitting using finite Monte Carlo samples". en. In:
4215 *Comput. Phys. Commun.* 77.2 (Oct. 1993), pp. 219–228.
- 4216 [204] J. S. Conway. "Incorporating Nuisance Parameters in Likelihoods for Multisource
4217 Spectra". In: *arXiv* (2011). URL: <https://arxiv.org/abs/1103.0354>.
- 4218 [205] D. Barrow et al. "Oscillation probability calculation for the T2K+SK atmospheric
4219 joint fit". In: *T2K Technical Note* 425 (2022).
- 4220 [206] R. G. Calland, A. C. Kaboth, and D. Payne. "Accelerated event-by-event neutrino
4221 oscillation reweighting with matter effects on a GPU". In: *IOP Journal of
4222 Instrumentation* 9.04 (Apr. 2014), P04016–P04016.
- 4223 [207] R. Wendell. *Prob3++ Oscillation Probability Calculation*. URL:
4224 <http://www.phy.duke.edu/~raw22/public/Prob3++/>.
- 4225 [208] F. Kallenborn et al. "Massively parallel computation of atmospheric neutrino
4226 oscillations on CUDA-enabled accelerators". In: *Computer Physics Communications*
4227 234 (2019), pp. 235–244.
- 4228 [209] L. Warsame. *MaCh3 Analysis Progress*. Implementation of CUDAProb3 in MaCh3
4229 for DUNE. URL: https://indico.fnal.gov/event/50217/contributions/241232/attachments/155318/202209/MaCh3ProgressforDUNELBL_May17%20%282%29.pdf.
- 4232 [210] S. Bourret et al. "Neutrino oscillation tomography of the Earth with
4233 KM3NeT-ORCA". In: *Journal of Physics: Conference Series* 888 (2017), p. 012114.
- 4234 [211] C. Rott, A. Taketa, and D. Bose. "Spectrometry of the Earth using Neutrino
4235 Oscillations". In: *Scientific Reports* 5.1 (Oct. 2015). URL:
4236 <https://doi.org/10.1038/srep15225>.
- 4237 [212] K. Hagiwara, N. Okamura, and K. Senda. "The earth matter effects in neutrino
4238 oscillation experiments from Tokai to Kamioka and Korea". In: *Journal of High
4239 Energy Physics* 2011.9 (Sept. 2011).
- 4240 [213] D. Typinski. *Earth Gravity*.
4241 <http://www.typnet.net/Essays/EarthGravGraphics/EarthGrav.pdf>.
4242 Accessed: 24-06-2022.
- 4243 [214] D. Barrow. *T2K Beam + SK Atmospheric Joint Fit*. Plenary Slides for May 2022 T2K
4244 Collaboration Meeting. 2022. URL:
4245 https://t2k.org/meet/collab/CM_May_2022/Talks/Friday/T2KSKPlenary.
- 4246 [215] E. Atkin et al. "Measuring PMNS parameters in a joint ND280-SK analysis using
4247 MCMC". In: *T2K Technical Note* 393 (2020).
- 4248 [216] K. Abe et al. "Neutron Tagging following Atmospheric Neutrino Events in a
4249 Water Cherenkov Detector". In: *arXiv* (2022). URL:
4250 <https://arxiv.org/abs/2209.08609>.