

1 **The Sensitivity to Oscillation Parameters**
2 **from a Simultaneous Beam and**
3 **Atmospheric Neutrino Analysis that**
4 **combines the T2K and SK Experiments**



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10 *Doctor of Philosophy*
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Abstract

12

13 A simultaneous beam and atmospheric oscillation analysis that combines the
14 T2K and SK experiments has been presented. The first sensitivities of the joint
15 analysis are reported, with the intention for the two collaborations to publish a
16 data analysis in the near-future. This analysis leverages the different energies
17 and baselines of the two experiments and provides strong sensitivities on δ_{CP} ,
18 $\sin^2(\theta_{23})$ and Δm_{32}^2 . To do this, a Bayesian Markov Chain Monte Carlo technique
19 is utilised to generate parameter estimates and credible intervals. Constraints
20 from the T2K near detector are also used to constrain the uncertainties of both
21 beam and atmospheric predictions.

22 For a known set of oscillation parameters close to the preferred values from
23 a T2K-only data fit, the sensitivity of the joint analysis to $\sin^2(\theta_{23})$ is increased
24 compared to the beam-only analysis. Furthermore, the sensitivity of the joint
25 analysis to select the correct mass hierarchy hypothesis is drastically improved
26 compared to the beam-only analysis, culminating in a substantial preference as
27 classified by Jeffrey's scale. This statement is stronger than the sensitivity of the
28 beam-only analysis, either with or without external constraints on $\sin^2(\theta_{13})$. The
29 sensitivities of the beam-only and joint beam-atmospheric analyses have also been
30 compared for a known set of oscillation parameters which are CP-conserving.
31 The joint analysis displays an improved ability to select the correct phase of δ_{CP}
32 and octant of $\sin^2(\theta_{23})$ compared to the beam-only analysis. This thesis illustrates
33 the benefit of the combined beam and atmospheric analysis, which could also be
34 extended for use in the next-generation Hyper-Kamiokande experiment.

Acknowledgements

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Statement of Originality

77 The work, and associated interpretation, presented within this thesis are
78 my own and were produced by myself. Where applicable, results and figures
79 taken from others have been attributed and referenced. This work has not been
80 submitted for any other qualification, at this university or any other.

81 The background of neutrino physics history and a description of the T2K and
82 SK experiments are provided in Chapters 2 and 3, respectively. The Bayesian
83 fitting techniques and MaCh3 framework used throughout this analysis are de-
84 tailed in Chapter 4. These chapters present entirely background knowledge
85 relevant for understanding the work presented within this thesis. Chapter 5
86 documents the simulation and reconstruction of neutrino events. This includes a
87 section of work done by myself to validate the `fiTQun` reconstruction software
88 for use on SK-V data.

89 Chapter 6 details the event selections and systematics used within this oscilla-
90 tion analysis. The selections were developed by others within the T2K and SK
91 collaborations and have been appropriately referenced. The implementation and
92 validation of the simultaneous support for the beam and atmospheric samples
93 selected by the SK detector, in the MaCh3 fitting framework, has been entirely my
94 own work. The systematic models invoked within this analysis were developed
95 by others, and relevant references have been included. The implementation
96 of these systematics within the MaCh3 fitting framework has been entirely my
97 own work and includes generating the systematic response functions for each
98 systematic from the Super-Kamiokande Monte Carlo. The implementation
99 and validation of the near detector sample and systematics were performed
100 by Clarence Wret on behalf of the MaCh3 working group.

101 Chapter 7 documents a new method of calculating the oscillation probabilities
102 for atmospheric neutrinos at SK. The method and validation of the ‘smearing’
103 technique were entirely my own. The methodology for including effects from
104 production height systematics was developed by others but the implementation
105 and validation were my own. Several performance increases, including the
106 interfacing of alternative oscillation calculation engines, were my own work.
107 The sensitivities, and interpretation, provided within Chapter 8 are produced
108 entirely by myself.

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146

Introduction

147 Current astrophysical measurements show that the universe is matter-dominated,
148 despite current theoretical models suggesting that an equal amount of matter
149 and antimatter were created in the Big Bang. For an imbalance to occur, the
150 Sakharov conditions [1] require the violation of charge parity (CP) symmetries.
151 CP violation has been observed in quark mixing but is insufficient to explain
152 the observed asymmetry. As the Standard Model relates the neutrino and the
153 antineutrino through these symmetries, CP violation could be found in the
154 neutrino sector. This would be observed as a difference between neutrino and
155 antineutrino oscillation. Current neutrino oscillation measurements contain hints
156 of CP violation [2] but no conclusive measurement has been achieved. One of the
157 main goals of neutrino oscillation experiments is to continue the search for CP
158 violation, requiring a precise measurement of all oscillation parameters.

159 The Super-Kamiokande (SK) detector observes atmospheric neutrinos emitted
160 from the subsequent decays of cosmic rays. It is also situated as the far detector
161 of the Tokai-to-Kamioka (T2K) experiment and measures the interactions of
162 neutrinos produced from the J-PARC (anti-)neutrino beam facility. Previous
163 oscillation analyses published by these two experiments have treated the datasets
164 independently. However, due to the different energies, path lengths, and density
165 of matter in which the neutrinos pass through, a combined analysis can leverage

166 the constraints from both datasets and may be able to improve measurements
167 of neutrino oscillation parameters.

168 This thesis introduces a joint beam and atmospheric neutrino analysis using
169 beam samples observed at the near and far detectors of the T2K experiment and
170 atmospheric samples present in SK. It combines the beam analysis presented in
171 [2] and the atmospheric analysis documented in [3]. This corresponds to run1-10
172 of the T2K experiment with approximately equal exposure taken in neutrino
173 and antineutrino beam modes, alongside more than 3000 days of atmospheric
174 events. The results in this thesis are presented as sensitivities to the δ_{CP} , $\sin^2(\theta_{13})$,
175 $\sin^2(\theta_{23})$, and Δm_{32}^2 oscillation parameters. Crucially, the combination of beam
176 and atmospheric neutrinos gives strong sensitivity to the mass hierarchy due to
177 the correlation between the matter resonance and $\sin^2(\theta_{23})$. The sensitivities are
178 generated by the MaCh3 Bayesian Markov Chain Monte Carlo fitting framework.

179 Chapter 2 provides a concise overview of neutrino physics history including
180 the discovery of the neutrino along with the first evidence for neutrino oscillation.
181 It also includes a brief discussion of the theory underpinning the PMNS formal-
182 ism of neutrino oscillation alongside a summary of the current measurements
183 of each oscillation parameter.

184 The T2K and SK experiments are detailed in Chapter 3. This includes the
185 design and calibration of the SK detector along with a brief description of the
186 composition and detection techniques of T2K's two near detectors. The neutrino
187 beamline, and the 'off-axis' technique, are also briefly summarised.

188 This thesis presents a Bayesian neutrino oscillation analysis that uses Markov
189 Chain Monte Carlo techniques. This analysis strategy, along with a summary
190 of the fundamental concepts of Bayesian inference, is described in Chapter 4.
191 This includes a discussion about the conditions that are required to correctly
192 sample the parameter space along with the methods used to calculate parameter
193 estimations and build credible intervals.

194 Chapter 5 details the simulations and reconstruction tools used to build Monte
195 Carlo predictions of each sample used within this analysis. This includes the

196 models used to provide flux predictions of the beam and atmospheric neutrinos
197 as well as the models invoked with this analysis to simulate neutrino interactions.

198 A description of the beam samples used at the near and far detector and the
199 atmospheric samples used at the SK detector is presented in Chapter 6. These
200 include energy and interaction mode comparisons along with documenting the
201 event selection cuts. This chapter also describes the systematic models used
202 to quantify the uncertainty in the flux predictions of both beam and atmo-
203 spheric neutrinos, the interaction models, and the response of the detectors
204 used within this analysis.

205 A novel atmospheric neutrino oscillation probability calculation method
206 is documented in Chapter 7. This is required to ensure reliable Monte Carlo
207 sampling of a rapidly varying region of oscillation parameter space. This chapter
208 also documents the uncertainties related to the Earth’s density as well as the
209 production height of neutrinos in the upper atmosphere.

210 Chapter 8 presents the sensitivities of this joint beam and atmospheric neu-
211 trino oscillation analysis. The results are provided for two different sets of known
212 values. The application of the external constraints on $\sin^2(\theta_{13})$ has also been
213 considered. The sensitivities of the joint analysis are compared to the beam-only
214 analysis and show the benefits of the combined analysis. These results have been
215 presented by the T2K and SK collaborations at the Neutrino 2022 conference [4].

216 A summarised discussion of the sensitivity results and the outlook for the
217 analysis, including the implications of this analysis on the next generation of
218 neutrino experiments, is provided in Chapter 9.

2

219

220

Neutrino Oscillation Physics

221 When first proposed, neutrinos were expected to be approximately massless
222 fermions that only interact through weak and gravitational forces. This meant
223 they were very difficult to detect as they can pass through significant amounts
224 of matter without interacting. Despite this, experimental neutrino physics has
225 developed many different detection techniques and observed neutrinos from
226 both natural and artificial sources. In direct tension with Standard Model physics,
227 neutrinos have been determined to oscillate between different lepton flavours,
228 requiring them to have mass.

229 The observation techniques which led to the discovery of the neutrino are doc-
230 umented in section 2.1. The theory underpinning neutrino oscillation is described
231 in section 2.2 and includes the approximations which can be made to simplify
232 the understanding of neutrino oscillation in the two-flavour approximation. Past,
233 current, and future neutrino experiments are detailed in section 2.3, including the
234 reactor, atmospheric, and long-baseline accelerator neutrino sources that have
235 been used to successfully constrain oscillation parameters. Finally, the current
236 state of oscillation parameter measurements are summarised in section 2.4.

2.1 Discovery of Neutrinos

At the start of the 20th century, the electrons emitted from the β -decay of the nucleus were found to have a continuous energy spectrum [5, 6]. This observation seemingly broke the energy conservation invoked within that period's nuclear models. In 1930, Pauli provided a solution to this problem in the form of a new particle, the neutrino (originally termed "neutron"). It was theorized to be an electrically neutral spin-1/2 fermion with a mass smaller than that of the electron [7]. This neutrino was emitted with the electron in β -decay to alleviate the apparent breaking of energy conservation. As a predecessor of today's weak interaction model, Fermi's theory of β -decay developed the understanding by coupling the four constituent particles: electron, proton, neutron, and neutrino, into a quantitative model [8].

Whilst Pauli was not convinced of the ability to detect neutrinos, the first observations of the particle were made in the mid-1950s when neutrinos from a reactor were observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$ [9, 10]. The detector consisted of two parts: a neutrino interaction medium and a liquid scintillator. The interaction medium was built from two water tanks, loaded with cadmium chloride to allow for increased efficiency in the detection of neutron capture. The positron emitted from IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$, generating a prompt signal and the neutron is captured on the cadmium via $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$, producing a delayed signal. An increase in the coincidence rate was observed when the reactor was operating which was interpreted as interactions from neutrinos generated in the reactor.

After the discovery of the ν_e , the question of how many flavours of neutrino exist was asked. In 1962, a measurement of the ν_μ was conducted at the Brookhaven National Laboratory [11]. A proton beam was directed at a beryllium target, generating pions which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the subsequent interactions of the ν_μ were observed. As the interaction of the neutrino generated muons rather than electrons, it was determined that

the ν_μ was fundamentally different from ν_e . The final observation to be made was that of the ν_τ from the DONUT experiment [12]. Three neutrinos seem the obvious solution as it mirrors the known number of charged leptons (as they form weak isospin doublets) but there could be evidence of more. Several neutrino experiments have found anomalous results [13, 14] which could be attributed to “sterile” neutrinos. These hypothesised particles are not affected by gauge interactions in the Standard Model so their presence can only be inferred through the observation of non-standard oscillation modes. However, cosmological observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [15], as measured from the cosmic microwave background power spectrum. LEP also measured the number of active neutrino flavours to be $N_\nu = 2.9840 \pm 0.0082$ [16] from measurements of the Z-decay width, but this does not strongly constrain the number of sterile neutrinos.

2.2 Theory of Neutrino Oscillation

A neutrino generated with lepton flavour α can change into a different lepton flavour β after propagating some distance. This phenomenon is called neutrino oscillation and requires that neutrinos must have a non-zero mass. This behaviour has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [17–19] mixing matrix which describes how the flavour and mass of neutrinos are associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [20] matrix measured in quark physics.

2.2.1 Three Flavour Oscillations

The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ (indexed ν_α), which are eigenstates of the weak interaction and three mass eigenstates, ν_1 , ν_2 and ν_3 (indexed ν_i). Each mass eigenstate is the superposition

²⁹¹ of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

²⁹² Where U is the 3×3 unitary PMNS matrix and connects the mass and
²⁹³ flavour eigenstates.

²⁹⁴ The weak interaction, when interacting via a W^{\pm} boson, couples to flavour
²⁹⁵ eigenstates so neutrinos interact with leptons of the same flavour. The prop-
²⁹⁶ agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a
²⁹⁷ plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

²⁹⁸ The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
²⁹⁹ three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
³⁰⁰ coordinates). The probability of observing a neutrino of flavour eigenstate β from
³⁰¹ one which originated as flavour α can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)}. \quad (2.3)$$

³⁰² The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

³⁰³ For a relativistic particle, $E_i \gg m_i$, a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

³⁰⁴ Making the approximations that neutrinos are relativistic, the mass eigenstates
³⁰⁵ were created with the same energy, and that $x = L$, where L is the distance
³⁰⁶ travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

307 where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This, combined with further use of unitarity relations
 308 results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \end{aligned} \quad (2.7)$$

309 Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last
 310 term is included for the oscillation probability of antineutrinos. As an important
 311 point to note, the observation of oscillation probability requires a non-zero value
 312 of Δm_{ij}^2 , which in turn requires that neutrinos have differing masses.

313 Typically, the PMNS matrix is parameterised into three mixing angles, a
 314 charge parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

315 Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often
 316 grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many
 317 neutrino experiments aim to measure the PMNS parameters from a wide array
 318 of origins, as is the purpose of this thesis.

319 The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8
 320 is only included for completeness. For an oscillation analysis experiment, any
 321 terms containing this phase disappear due to taking the expectation value of the
 322 PMNS matrix. Measurements of these phases can be performed by experiments
 323 searching for neutrino-less double β -decay [21].

324 A two-flavour approximation can be obtained when one assumes the third
325 mass eigenstate is degenerate with another. This results in the two-flavour
326 approximation being reasonable for understanding the features of the oscillation.
327 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

328 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

329 Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is
330 a sinusoidal function depending upon the distance over which the neutrino
331 propagates. The frequency and amplitude of oscillation are dependent upon
332 $\Delta m^2/4E$ and $\sin^2 2\theta$, respectively. The oscillation probabilities presented thus far
333 assume $c = 1$, where c is the speed of light in a vacuum. In more familiar units, the
334 maximum oscillation probability for a fixed value of θ is given at $L[\text{km}] / E[\text{GeV}] \sim$
335 $1.27/\Delta m^2$. It is this calculation that determines the best L/E value for a given
336 experiment to be designed around for measurements of a specific value of Δm^2 .

337 2.2.2 The MSW Effect

338 The theory of neutrino oscillation in a vacuum has been described in subsec-
339 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating
340 from below the horizon propagate through the matter in the Earth. The coherent
341 scattering of neutrinos from a material target modifies the Hamiltonian of the
342 system. This results in a change of the oscillation probability. This modification
343 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [22–24]. This occurs
344 because charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$, propagated by a W^\pm
345 boson) only affects electron neutrinos whereas the neutral current scattering

³⁴⁶ ($\nu_l + l^- \rightarrow \nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino
³⁴⁷ flavours equally. In the two-flavour approximation, the effective mixing pa-
³⁴⁸ rameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

³⁴⁹ where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F
³⁵⁰ is Fermi's constant. It is clear that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for
³⁵¹ $\Delta m^2 > 0$ which forces $\sin^2(2\theta)$ to be equal to unity. This is colloquially called
³⁵² the matter resonance and regenerates the electron neutrino component of the
³⁵³ neutrino flux [22–24]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

³⁵⁴ At densities lower than this critical value, the oscillation probability will
³⁵⁵ be much closer to that of vacuum oscillation. For antineutrinos, the $N_e \rightarrow$
³⁵⁶ $-N_e$ substitution is made [25].

³⁵⁷ The resonance occurring from the MSW effect depends on the sign of Δm^2 .
³⁵⁸ Therefore, any neutrino oscillation experiment which observes neutrinos and
³⁵⁹ antineutrinos which have propagated through matter can have some sensitivity
³⁶⁰ to the ordering of the neutrino mass eigenstates.

³⁶¹ 2.3 Neutrino Oscillation Measurements

³⁶² As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics
³⁶³ was awarded to the Super-Kamiokande (SK) [26] and Sudbury Neutrino Ob-
³⁶⁴ servatory (SNO) [27] collaborations for the first definitive observation of solar
³⁶⁵ and atmospheric neutrino oscillation [28]. Since then, the field has seen a wide
³⁶⁶ array of oscillation measurements from a variety of neutrino sources. As seen
³⁶⁷ in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio
³⁶⁸ of the propagation baseline, L , to the neutrino energy, E . It is this ratio that
³⁶⁹ determines the type of neutrino oscillation a particular experiment is sensitive to.

As illustrated in Figure 2.1, there are many neutrino sources that span a wide range of energies. The least energetic neutrinos are from reactor and terrestrial sources at $O(1)$ MeV whereas the most energetic neutrinos originate from atmospheric and galactic sources with energies $> O(1)$ TeV.

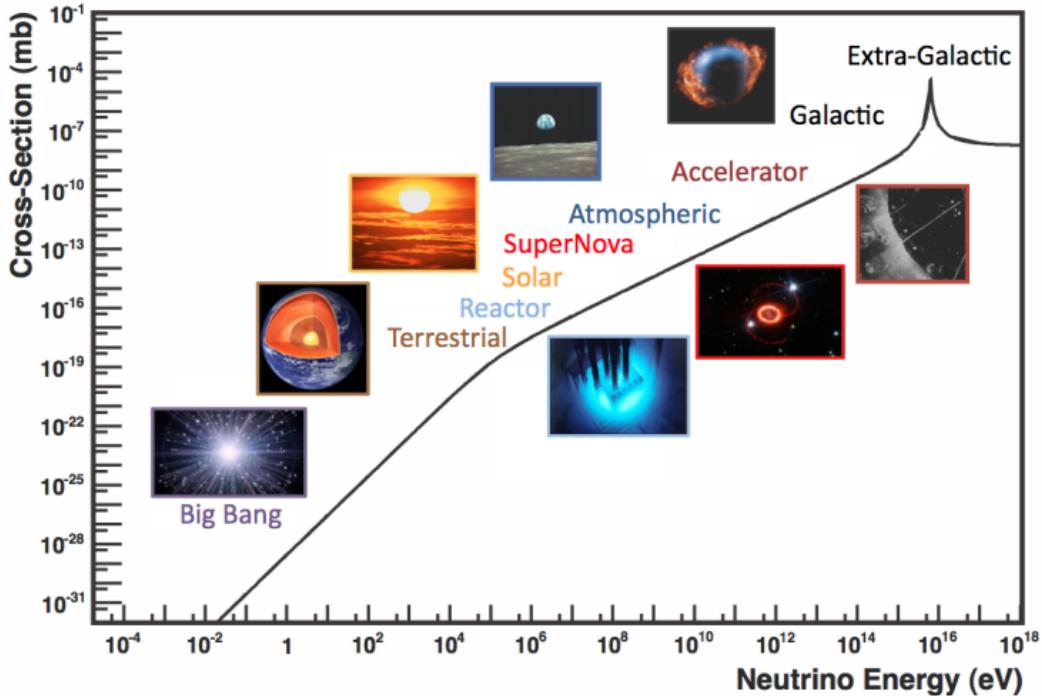


Figure 2.1: The electro-weak cross-section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [29].

2.3.1 Solar Neutrinos

Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun. The solar neutrino flux, given as a function of neutrino energy for different fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion generates the largest flux of neutrinos, the neutrinos are low energy and are difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [30]. Consequently, most experiments focus on the neutrinos from the decay of 8B (via ${}^8B \rightarrow {}^8Be^* + e^+ + \nu_e$), which are higher energy.

The first measurements of solar neutrinos observed a significant reduction in the event rate compared to predictions from the Standard Solar Model [32, 33]. A

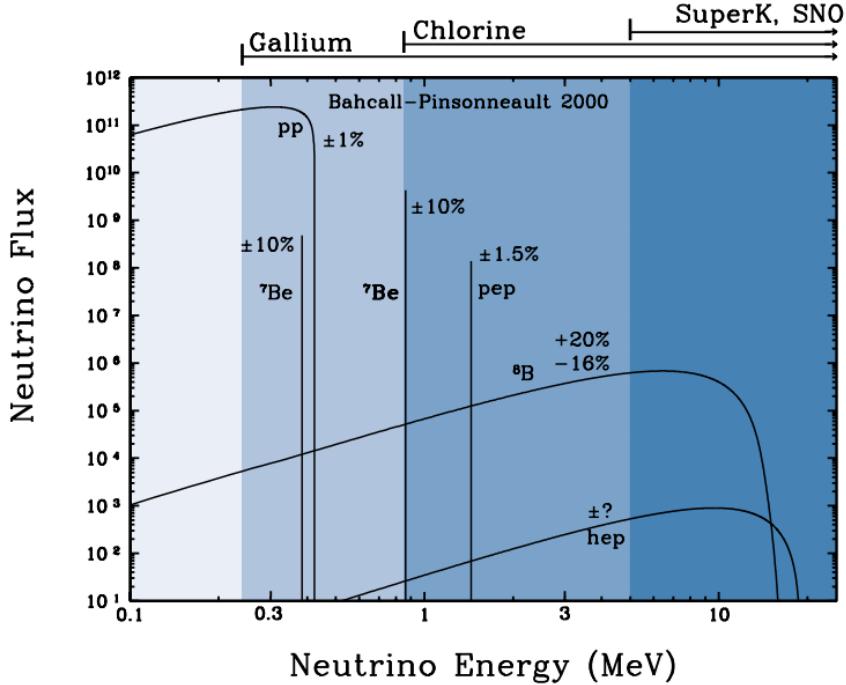
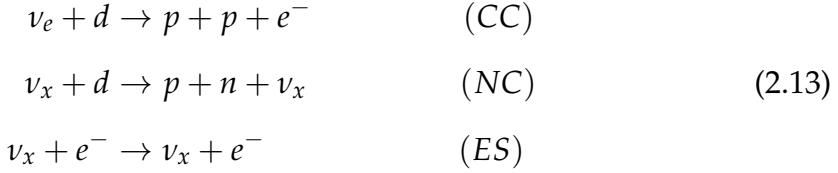


Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [31].

384 proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a
 385 precursory version of the PMNS model [34]. The Kamiokande [35], Gallex [36]
 386 and Sage [37] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

387 The conclusive solution to this problem was determined by the SNO col-
 388 laboration [27]. Using a deuterium water target to observe 8B neutrinos, the
 389 event rate of charged current (CC), neutral current (NC), and elastic scattering
 390 (ES) interactions (given in Equation 2.13) was simultaneously measured. CC
 391 events can only occur for electron neutrinos, whereas the NC channel is agnostic
 392 to neutrino flavour, and the ES channel has a small excess sensitivity for the
 393 detection of electron neutrino interactions. This meant that there were direct
 394 measurements of the ν_e and ν_x neutrino flux. It was concluded that the CC and
 395 ES interaction rates were consistent with the deficit previously observed. Most
 396 importantly, the NC reaction rate was only consistent with the others under the

³⁹⁷ hypothesis of flavour transformation.



³⁹⁸ 2.3.2 Accelerator Neutrinos

³⁹⁹ The concept of using an artificial “neutrino beam” was first realised in 1962 [11].
⁴⁰⁰ Since then, many experiments have adopted the same fundamental concepts.
⁴⁰¹ Typically, a proton beam is aimed at a target producing charged mesons that decay
⁴⁰² to neutrinos. The mesons can be sign-selected by the use of magnetic focusing
⁴⁰³ horns to generate a neutrino or antineutrino beam. Pions are the primary mesons
⁴⁰⁴ that decay and depending on the orientation of the magnetic field, a muon (anti-
⁴⁰⁵)neutrino beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. The decay of
⁴⁰⁶ muons and kaons results in an irreducible intrinsic electron neutrino background.
⁴⁰⁷ In the Tokai-to-Kamioka (T2K) experiment, this background contamination is
⁴⁰⁸ $O(< 1\%)$ [38]. There is also an approximately $\sim 5\%$ “wrong-sign” background
⁴⁰⁹ of $\bar{\nu}_\mu$ generated via the same decays, when operating in neutrino mode. As the
⁴¹⁰ beam is generated by proton interactions (rather than anti-proton interactions),
⁴¹¹ the wrong-sign component in the antineutrino beam is larger when operating
⁴¹² in neutrino mode.

⁴¹³ Tuning the proton energy in the beam and using beam focusing techniques
⁴¹⁴ allows the neutrino energy to be set to a value that maximises the disappear-
⁴¹⁵ ance oscillation probability in the L/E term in Equation 2.10. This means that
⁴¹⁶ accelerator experiments are typically more sensitive to the mixing parameters as
⁴¹⁷ compared to a natural neutrino source. However, the disadvantage compared
⁴¹⁸ to atmospheric neutrino experiments is the cost of building a facility to provide
⁴¹⁹ high-energy neutrinos, with a high flux, which is required for longer baselines.
⁴²⁰ Consequently, there is typically less sensitivity to matter effects and the ordering
⁴²¹ of the neutrino mass eigenstates.

422 A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

423 where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of
 424 neutrinos with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and
 425 $\epsilon(\vec{x})$ is the efficiency and resolution of the detector. In order to leverage the most
 426 out of an accelerator neutrino experiment, the flux and cross-section systematics
 427 need to be constrained. This is typically done via the use of a “near detector”,
 428 situated at a baseline of $O(1)$ km. This detector observes the unoscillated neutrino
 429 flux and constrains the parameters used within the flux and cross-section model.

430 The first accelerator experiments to precisely measure oscillation parameters
 431 were MINOS [39] and K2K [40]. These experiments confirmed the ν_μ disappear-
 432 ance seen in atmospheric neutrino experiments by finding consistent parameter
 433 values for $\sin^2(\theta_{23})$ and Δm_{32}^2 . The current generation of accelerator neutrino
 434 experiments, T2K and NO ν A extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations
 435 and lead the sensitivity to atmospheric mixing parameters as seen in Figure 2.6
 436 [41]. The two experiments differ in their peak neutrino energy, baseline, and
 437 detection technique. The NO ν A experiment is situated at a baseline of 810km
 438 from the NuMI beamline which delivers 2GeV neutrinos. The T2K neutrino beam
 439 is peaked around 0.6GeV and propagates 295km [42]. Additionally, the NO ν A
 440 experiment uses functionally identical detectors (near and far) whereas T2K uses
 441 a plastic scintillator technique at the near detector and a water Cherenkov far
 442 detector. The future generation experiments DUNE [43] and Hyper-Kamiokande
 443 [44] will succeed these experiments as the high-precision era of neutrino oscil-
 444 lation parameter measurements develops.

445 Several anomalous results have been observed in the LSND [13] and Mini-
 446 BooNE [14] detectors which were designed with purposefully short baselines.
 447 Parts of the neutrino community attributed these results to oscillations induced
 448 by a fourth “sterile” neutrino [45] but several searches in other experiments,

⁴⁴⁹ MicroBooNE [46] and KARMEN [47], found no hints of additional neutrino
⁴⁵⁰ species. The solution to these anomalous results is still being determined.

⁴⁵¹ 2.3.3 Atmospheric Neutrinos

⁴⁵² The interactions of primary cosmic ray protons in the Earth's upper atmosphere
⁴⁵³ generate showers of energetic hadrons. These are mostly pions and kaons that
⁴⁵⁴ decay to produce a natural source of neutrinos spanning energies of MeV to
⁴⁵⁵ TeV [48]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu), \end{aligned} \tag{2.15}$$

⁴⁵⁶ such that for a single pion decay, three neutrinos can be produced. The
⁴⁵⁷ atmospheric neutrino flux energy spectra as predicted by the Bartol [49], Honda
⁴⁵⁸ [50–52], and FLUKA [53] models are illustrated in Figure 2.3. The flux distribution
⁴⁵⁹ peaks at an energy of $O(10)$ GeV. The uncertainties associated with these models
⁴⁶⁰ are dominated by the hadronic production of kaon and pions as well as the
⁴⁶¹ primary cosmic flux.

⁴⁶² Unlike long-baseline experiments which have a fixed baseline, the distance
⁴⁶³ atmospheric neutrinos propagate is dependent upon the zenith angle, relative to
⁴⁶⁴ the detector, at which they interact. This is illustrated in Figure 2.4. Neutrinos that
⁴⁶⁵ are generated directly above the detector ($\cos(\theta) = 1.0$) have a baseline equivalent
⁴⁶⁶ to the height of the atmosphere, whereas neutrinos that interact directly below
⁴⁶⁷ the detector ($\cos(\theta) = -1.0$) have to travel a length equal to the diameter of
⁴⁶⁸ the Earth. This means atmospheric neutrinos have a baseline that varies from
⁴⁶⁹ $O(20)$ km to $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will
⁴⁷⁰ be subject to MSW matter resonance as they propagate through the Earth.

⁴⁷¹ Figure 2.5 highlights the atmospheric neutrino flux as a function of the zenith
⁴⁷² angle for different slices of neutrino energy. For medium to high-energy neutrinos
⁴⁷³ (and to a lesser degree for low-energy neutrinos), the flux is approximately
⁴⁷⁴ symmetric around $\cos(\theta) = 0$. To the accuracy of this approximation, the

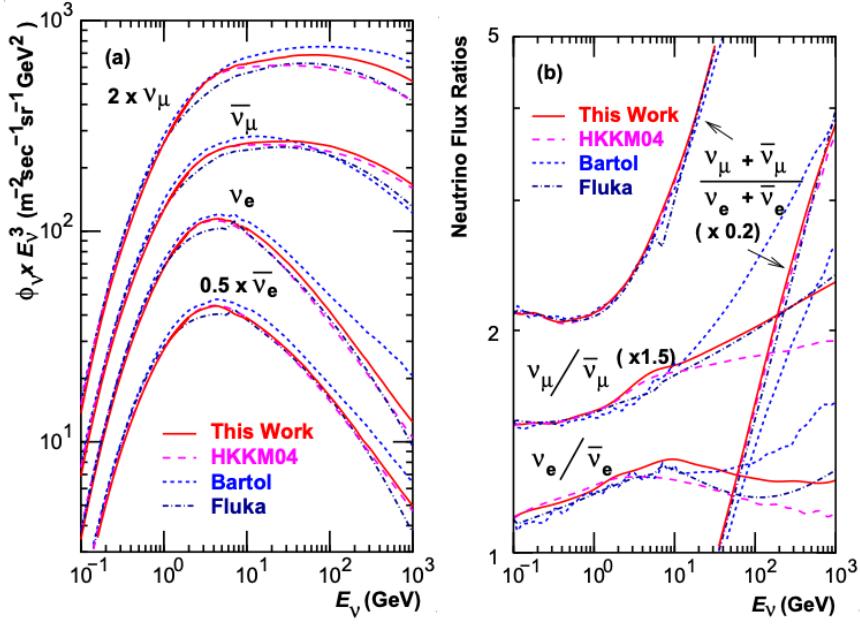


Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [50], the 2004 Honda model (“HKKM04”)[51], the Bartol model [49] and the FLUKA model [53]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [50].

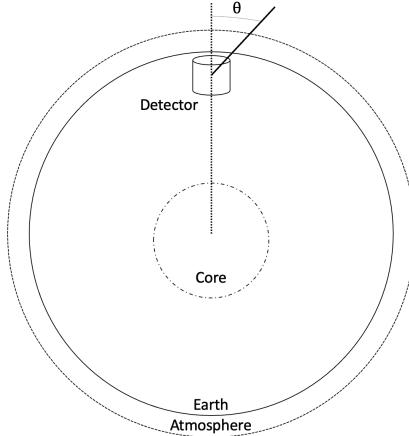


Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [54].

systematic uncertainties associated with atmospheric flux for comparing upward-going and down-going neutrino cancels. This allows the down-going events, which are mostly insensitive to oscillation probabilities, to act as an unoscillated prediction (similar to a near detector in an accelerator neutrino experiment).

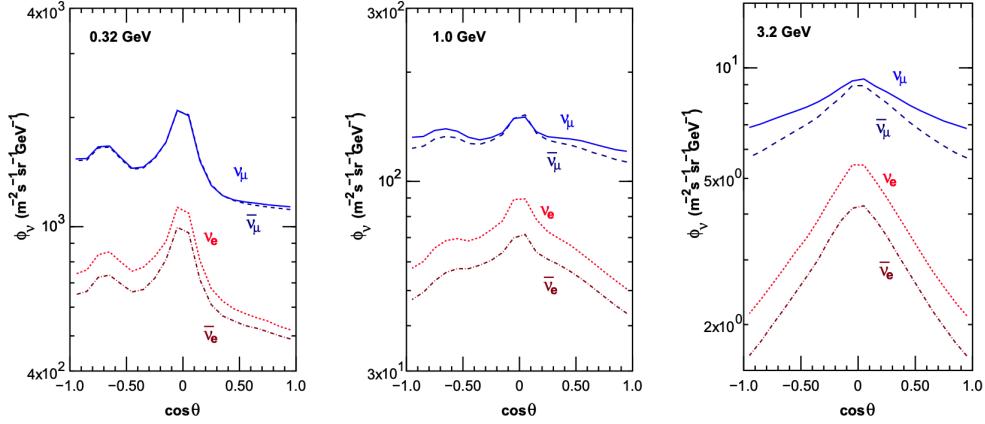


Figure 2.5: Prediction of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [52]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [49], Honda [50] and FLUKA [53] are given in [54].

479 Precursory hints of atmospheric neutrinos were observed in the mid-1960s
 480 searching for $\nu_\mu + X \xrightarrow{(-)} X^* + \mu^\pm$ [55]. This was succeeded by the IMB-3 [56]
 481 and Kamiokande [57] experiments which measured the double ratio of muon
 482 to electron neutrinos in data to Monte Carlo, $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$.
 483 Both experiments were found to have a consistent deficit of muon neutrinos,
 484 with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$, respectively.
 485 Super-Kamiokande (SK) [54] extended this analysis by fitting oscillation pa-
 486 rameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and
 487 $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3} eV^2$.

488 Since then, atmospheric neutrino experiments have been making precision
 489 measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric
 490 neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ
 491 discovery of ν_τ appearance [58]. Figure 2.6 illustrates the current estimates on
 492 the atmospheric mixing parameters, from a wide range of atmospheric and
 493 accelerator neutrino observatories.

494 2.3.4 Reactor Neutrinos

495 As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors
 496 are a very useful artificial source of electron antineutrinos. For reactors that use

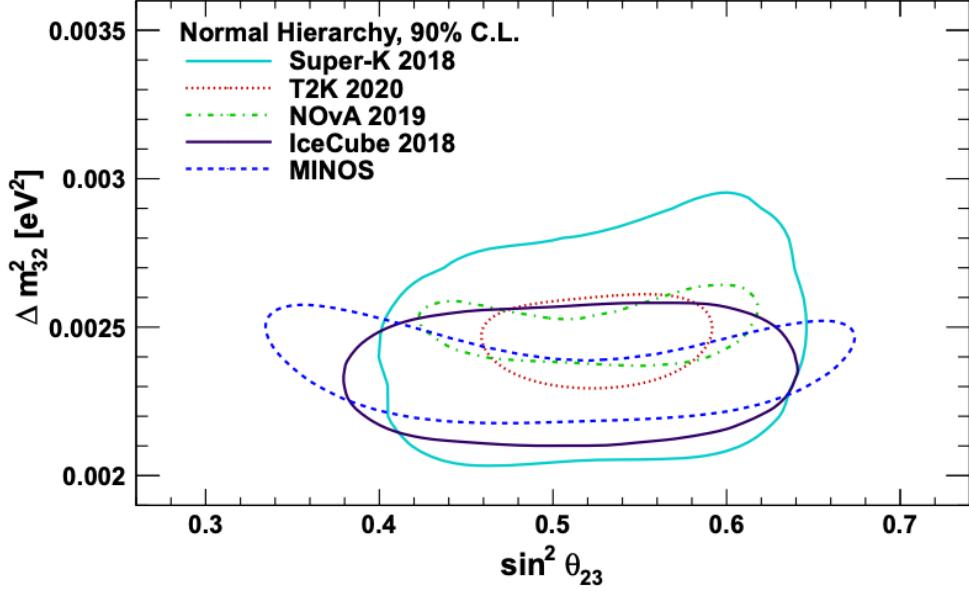


Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long-baseline experiments: SK [59], T2K [60], NOvA [61], IceCube [62] and MINOS [63]. Figure taken from [64].

497 low-enriched uranium ^{235}U as fuel, the antineutrino flux is dominated by the
 498 β -decay fission of ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu [65] as illustrated in Figure 2.7.

499 Due to their low energy, reactor electron antineutrinos predominantly interact
 500 via the inverse β -decay (IBD) interaction. The typical signature contains two
 501 signals delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihila-
 502 tion, and secondly the photon emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after
 503 neutron capture on hydrogen. Searching for both signals improves the detector's
 504 ability to distinguish between background and signal events [66].

505 There are many short baseline experiments ($L \sim O(1)\text{km}$) that have mea-
 506 sured the $\sin^2(\theta_{13})$ and Δm_{32}^2 oscillation parameters. Daya Bay [67], RENO [68]
 507 and Double Chooz [69] have all provided precise measurements, with the first
 508 discovery of a non-zero θ_{13} made by Daya Bay and RENO (and complemented
 509 by T2K [69]). The constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the
 510 field. They are often used as external inputs to accelerator neutrino experiments
 511 to improve their sensitivity to δ_{CP} and mass hierarchy determination.

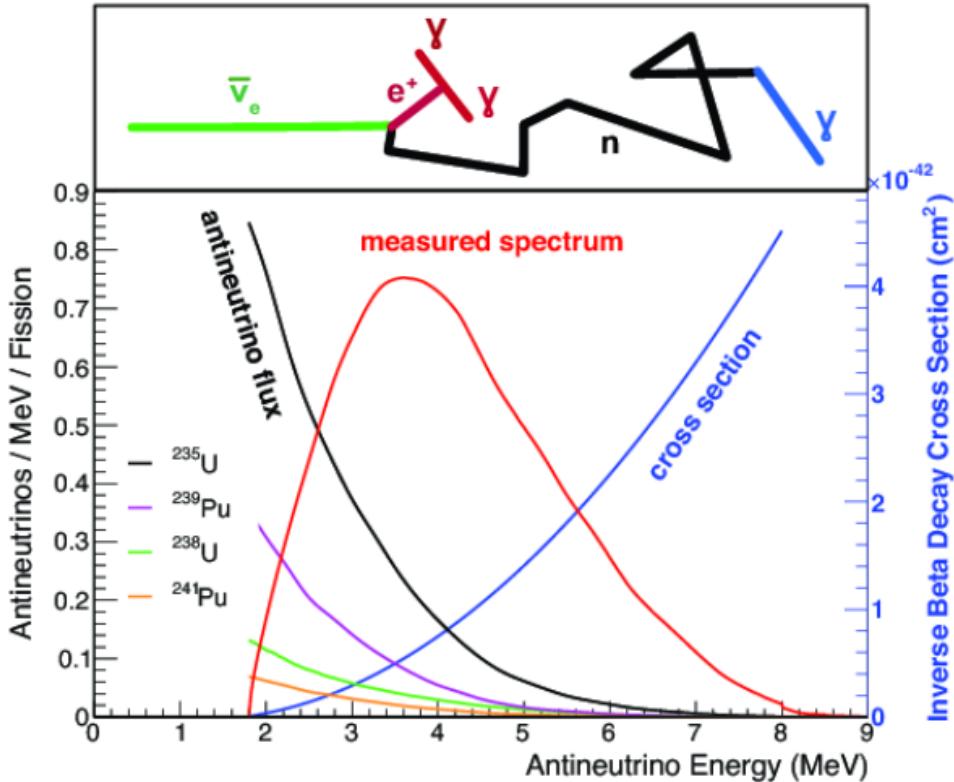


Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [64].

Kamland [70] is the only experiment to have observed reactor neutrinos using a long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have sensitivity to Δm_{21}^2 . Combined with the SK solar neutrino experiment, the combined analysis puts the most stringent constraint on Δm_{21}^2 [71].

2.4 Summary Of Oscillation Parameter Measurements

Since the first evidence of neutrino oscillations, numerous measurements of the mixing parameters have been made. Many experiments use neutrinos as a tool for the discovery of new physics (diffuse supernova background, neutrinoless double beta decay and others) so the PMNS parameters are summarised in the Particle Data Group (PDG) review tables. The analysis presented in this thesis

522 focuses on the 2020 T2K oscillation analysis presented in [2] which uses the 2020
523 PDG constraints [72]. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	0.547 ± 0.021
$\sin^2(\theta_{23})$ (N.H.)	0.545 ± 0.021
Δm_{32}^2 (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2020 Particle Data Group constraints of the oscillation parameters taken from [72]. The value of Δm_{32}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

524 The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance,
525 $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is taken as the average best-fit from the combination of
526 Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within
527 other neutrino oscillation experiments, typically termed the reactor constraint.
528 The $\sin^2(\theta_{12})$ parameter is predominantly measured through electron neutrino
529 disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline
530 reactor neutrino experiment Kamland also has a sensitivity to this parameter
531 and is used in a joint fit to solar data from SNO and SK, using the reactor con-
532 straint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric
533 neutrino experiments. The PDG value is a joint fit of T2K, NO ν A, MINOS and
534 IceCube DeepCore experiments. The latest T2K-only measurement, provided at
535 Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [2].
536 The PDG constraint on Δm_{21}^2 is provided by the KamLAND experiment using
537 solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint
538 from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double
539 Chooz) experiments. Accelerator measurements make some of the most stringent
540 constraints on Δm_{32}^2 although atmospheric experiments have more sensitivity to
541 the mass hierarchy determination. The PDG performs a joint fit of accelerator and

atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [2]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were rejected with $\sim 2\sigma$ intervals, as published in Nature [73]. Since the 2020 PDG publication, there has been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [74], alongside updated Δm_{32}^2 and $\sin^2(\theta_{23})$ measurements.

Throughout this thesis, several sample spectra predictions and contours are presented, which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [75] and “Asimov B” being CP-conserving and further from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

2.5 Overview of Oscillation Effects

The analysis presented within this thesis focuses on the determination of oscillation parameters from a joint atmospheric and beam analysis which combines the SK and T2K experiments. Whilst subject to the same oscillation formalism, the way in which the two samples have sensitivity to the different oscillation parameters differs significantly.

Atmospheric neutrinos have a varying baseline, or “path length” L , such that the distance each neutrino travels before interacting is dependent upon the zenith angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s

surface and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2 (1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (2.16)$$

Where $R_E = 6,371\text{km}$ is the Earth's radius. This assumes a spherically symmetric Earth model. Therefore, the oscillation probability is dependent upon two parameters, $\cos(\theta_Z)$ and E_ν .

The oscillation probability used within this analysis is based on [25]. The neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (2.17)$$

where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino energy, G_F is Fermi's constant, N_e is the electron number density and U is the PMNS matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied for antineutrino propagation. Thus, a model of the Earth's density is required for neutrino propagation. Following the SK methodology [76], this analysis uses the Preliminary Reference Earth Model (PREM) [77] which provides piecewise cubic polynomials as a function of the Earth's radius. This density profile is illustrated in Figure 2.8. As the propagator requires layers of constant density, the SK methodology approximates the PREM model by using four layers of constant density [76], detailed in Table 2.3.

Layer	Outer Radius [km]	Density [g/cm ³]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 2.3: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [77].

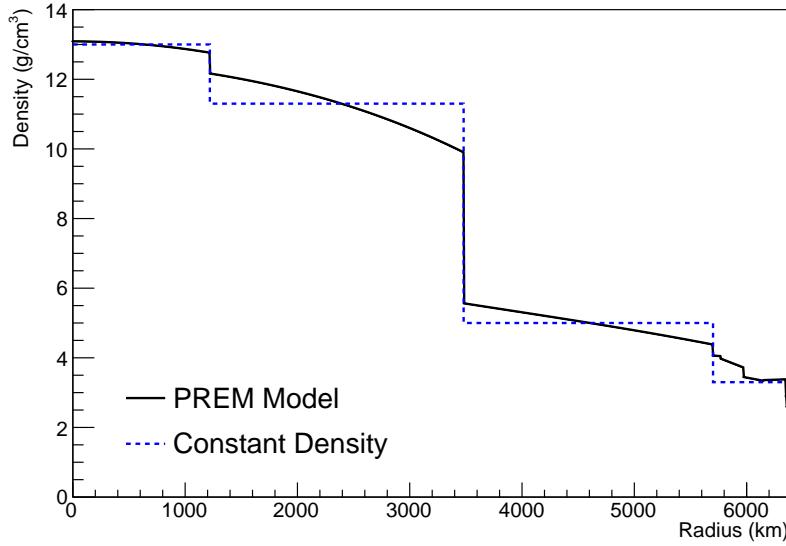


Figure 2.8: The density of the Earth given as a function of the radius, as provided by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

580 The atmospheric neutrino oscillation probabilities can be presented as two di-
 581 mensional “oscillograms” as illustrated in Figure 2.9. The distinct discontinuities,
 582 as a function of $\cos(\theta_Z)$, are due to the discontinuous density in the PREM model.

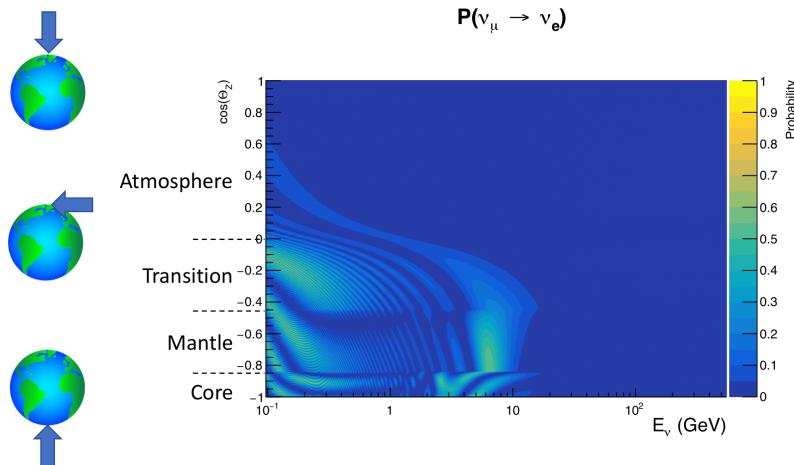


Figure 2.9: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

Atmospheric neutrinos have sensitivity to δ_{CP} through the overall event rate. Figure 2.10 illustrates the difference in oscillation probability between CP-conserving ($\delta_{CP} = 0.$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation parameter set (Table 2.2). The result is a complicated oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric neutrinos comes via the overall normalisation of these events.

The presence of matter means that the effect δ_{CP} has on the oscillation probability is not equal between neutrinos and antineutrinos. Furthermore, the interaction cross-section for neutrinos is larger than for antineutrinos so the two effects have to be disentangled. All of these effects lead to a difference in the number of neutrinos detected compared to antineutrinos. This changes how the δ_{CP} normalisation term is observed, resulting in a very complex sensitivity to δ_{CP} .

The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ are presented in Figure 2.11, where the PREM model has been assumed. The oscillation probability for both neutrinos and antineutrinos is affected in the presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only occur for neutrinos in the normal mass hierarchy and antineutrinos in the inverse mass hierarchy. The exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$, further increasing the atmospheric neutrinos' sensitivity to the parameter.

As the T2K beam flux is centered at the first oscillation maximum ($E_\nu = 0.6\text{GeV}$) [42], the sensitivity to δ_{CP} is predominantly observed as a change in the event-rate of e-like samples in $\nu/\bar{\nu}$ modes. Figure 2.12 illustrates the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a range of δ_{CP} values. A circular modulation of the first oscillation peak (in both magnitude and position) is observed when varying throughout the allowable values of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation maximum than the CP-violating values of $\delta_{CP} = -\pi/2(\delta_{CP} = \pi/2)$. A sub-dominant shift in the energy of the oscillation peak is also present, which aids in separating the two CP-conserving values of δ_{CP} .

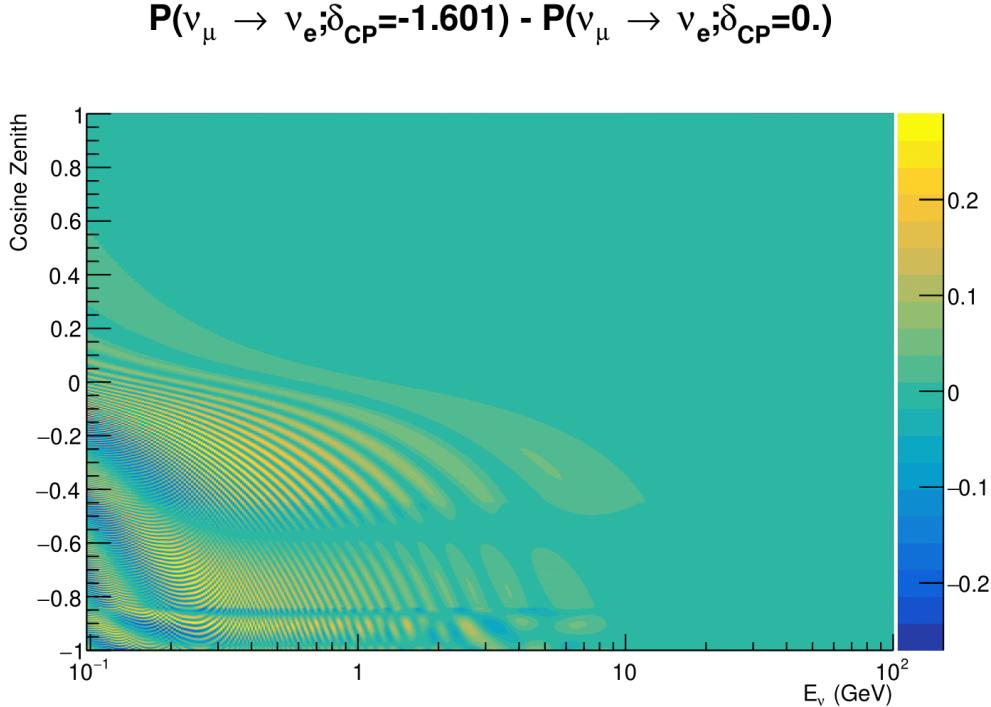


Figure 2.10: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

613 T2K's sensitivity to $\sin^2(\theta_{23})$ and Δm_{32}^2 is observed as a shape-based variation
 614 of the muon-like samples, as illustrated in Figure 2.12. The value of Δm_{32}^2 laterally
 615 shifts the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$.
 616 A variation of $\sin^2(\theta_{23})$ is predominantly observed as a vertical shift of the
 617 oscillation dip with second-order horizontal shifts being due to matter effects.
 618 The beam neutrinos have limited sensitivity to matter effects due to the relatively
 619 shorter baseline as well as the Earth's mantle being a relatively low-density
 620 material (as compared to the Earth's core). For some values of δ_{CP} , the degeneracy
 621 in the number of e-like events allows the mass hierarchy to be broken. This leads
 622 to a δ_{CP} -dependent mass hierarchy sensitivity which can be seen in Figure 2.13.
 623 Whilst all oscillation channels should be included for completeness, the
 624 computational resources required to run a fit are limited and any reasonable
 625 approximations which reduce the number of oscillation probability calculations

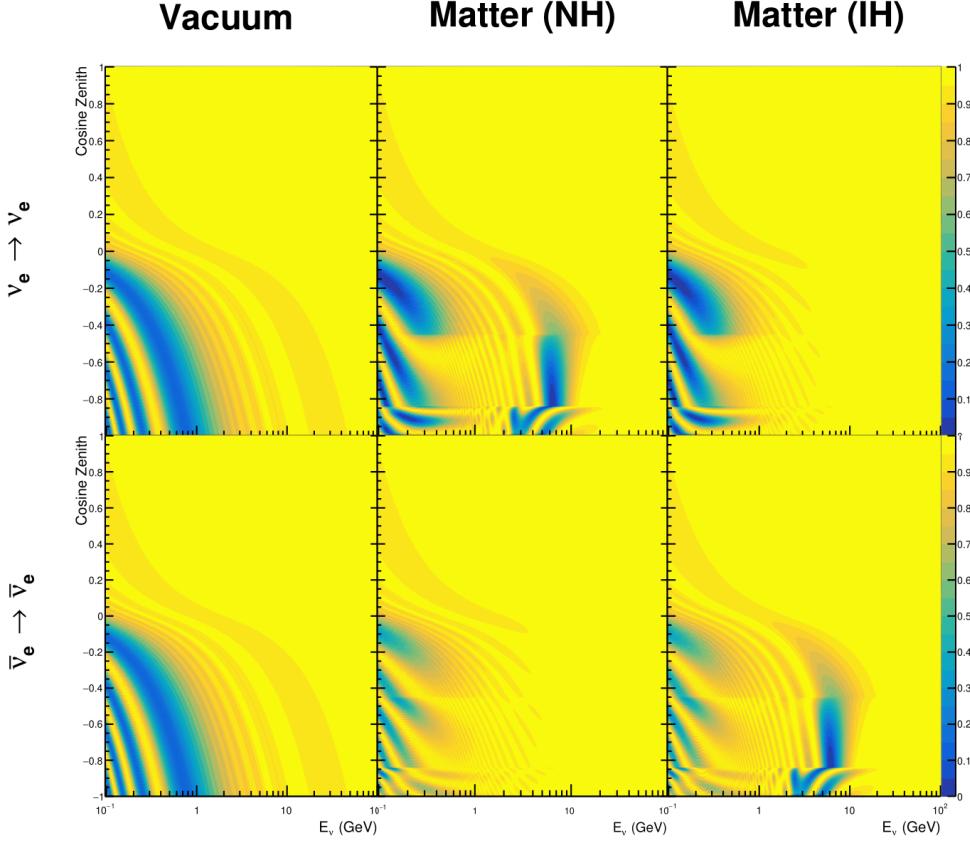


Figure 2.11: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$ flux [78]. Furthermore, as the peak neutrino energy of the beam is well below the threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [58]), only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability

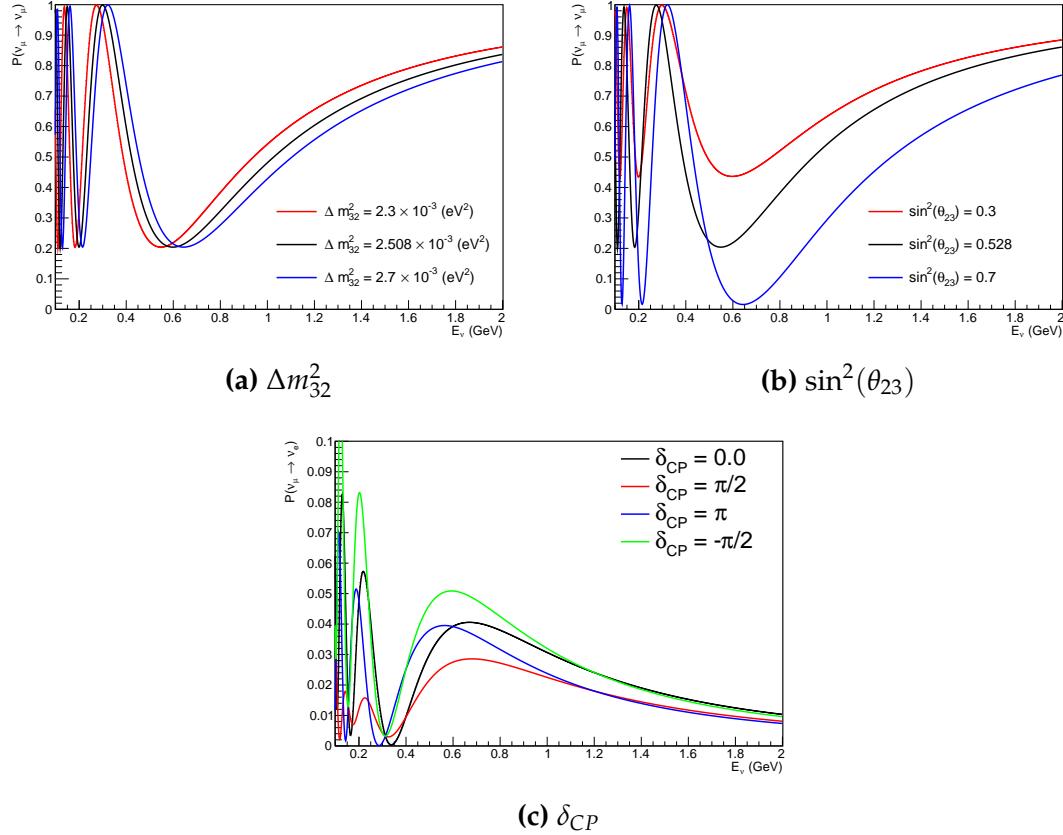


Figure 2.12: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. A path-length of 295km is assumed. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

is very small due to their energy being well above the oscillation maximum (small value of L/E). Whilst these approximations have been made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold. These events can have non-negligible oscillation probabilities due to the further distance they travel.

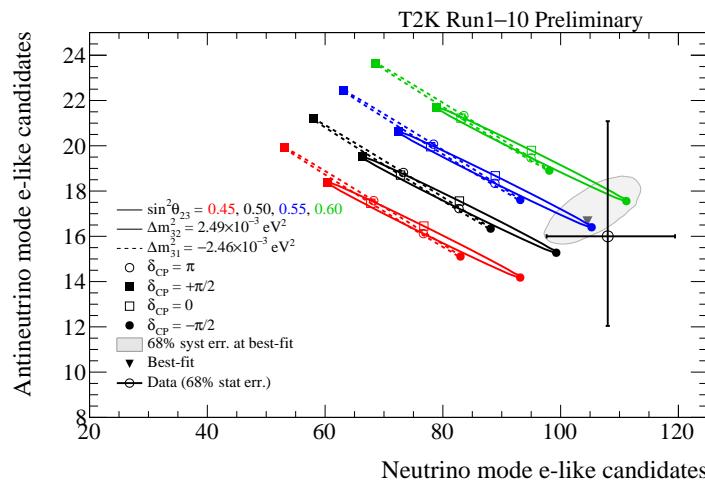


Figure 2.13: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{32}^2 are given.

3

639

640

T2K and SK Experiment Overview

641 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)
642 collaboration has been leading atmospheric neutrino oscillation analyses for
643 over two decades. The detector has provided some of the strongest constraints
644 on proton decay and the first precise measurements of the Δm_{32}^2 and $\sin^2(\theta_{23})$
645 neutrino oscillation parameters. The history, detection technique, and operation
646 of the SK detector is described in section 3.1.

647 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline
648 experiments to use both neutrino and antineutrino beams to precisely measure
649 charge parity violation within the neutrino sector. The T2K experiment observed
650 the first hints of a non-zero $\sin^2(\theta_{13})$ measurement and continues to lead the
651 field with the constraints it provides on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{32}^2 and δ_{CP} . In
652 section 3.2, the techniques that T2K use to generate the neutrino beam and
653 constrain systematic parameter through near detector constraints are described.

654 3.1 The Super-Kamiokande Experiment

655 The SK experiment began taking data in 1996 [79] and has had many modifi-
656 cations throughout its operation. There have been seven defined periods of
657 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were damaged during maintenance [80]. Those that survived were equally distributed throughout the detector in the SK-II era, which resulted in a reduced 19% photo-coverage. From SK-III onwards, repairs to the detector meant the full suite of PMTs was operational recovering the 40% photo-coverage. Before the start of SK-IV, the data acquisition and electronic systems were upgraded. Between SK-IV and SK-V, a significant effort was placed into tank open maintenance and repair/replacement of defective PMTs in preparation for the Gadolinium upgrade; a task for which the author of this thesis was required. Consequently, the detector conditions were significantly changed from this point. SK-VI marked the start of the SK-Gd era, with the detector being doped with gadolinium at a concentration of 0.01% by concentration. SK-VII, which started during the writing of this thesis, has increased the gadolinium concentration to 0.03% for continued operation [81].

The oscillation analysis presented within this thesis focuses on the SK-IV period of running and the data taken within it. This follows from the recent SK analysis presented in [82]. Therefore, the information presented within this section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

Table 3.1: The various SK periods and their respective live-time. The SK-VI live-time is calculated until 1st April 2022. SK-VII started during the writing of this thesis.

3.1.1 The SK Detector

The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank, with diameter 39.3m and height 41.1m, filled with ultrapure water [80]. A diagram of the significant components of the SK detector is given in Figure 3.1.

679 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine is under-
 680 ground with roughly 1km rock overburden (2.7km water equivalent overburden)
 681 [83]. At this depth, the rate of cosmic ray muons is significantly decreased to a
 682 value of $\sim 2\text{Hz}$ (net rate). The top of the tank is covered with stainless steel which
 683 is designed as a working platform for maintenance, calibration, and location for
 684 high voltage and data acquisition electronics.

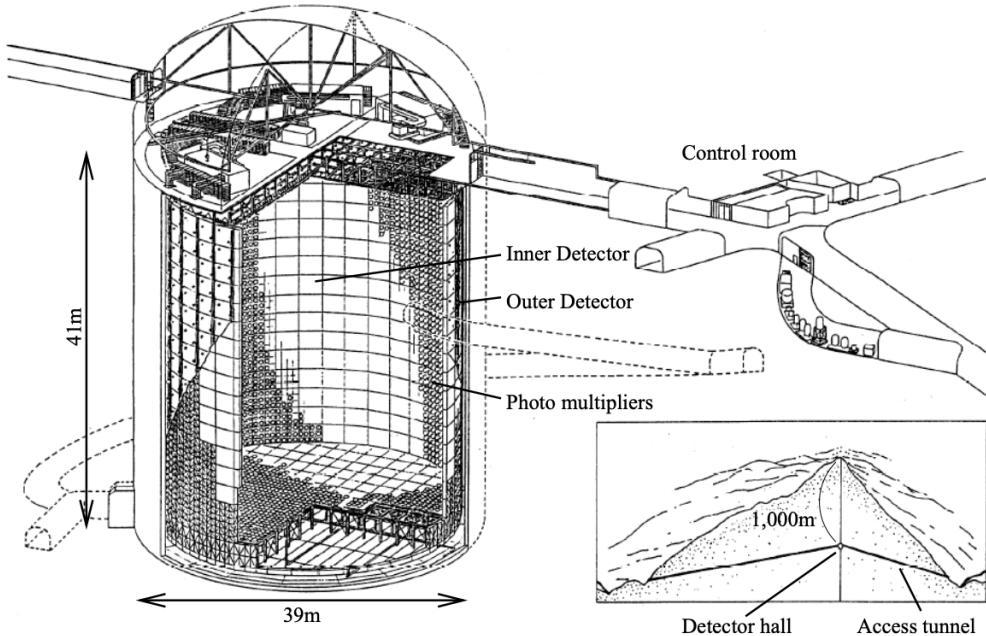


Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [84].

685 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated
 686 inside the tank, with an approximate 2m gap between this structure and the outer
 687 tank wall. The purpose of this structure is to support the photomultiplier tubes
 688 (PMTs). The volume inside and outside the support structure is referred to as the
 689 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the
 690 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs, respectively
 691 [80]. The ID contains a 32kton mass of water. Many analyses performed at SK
 692 use a “fiducial volume” defined by the volume of water inside the ID excluding
 693 some distance to the ID wall. This reduces the volume of the detector which is
 694 sensitive to neutrino events but reduces radioactive backgrounds and allows for

695 better reconstruction performance. The nominal fiducial volume is defined as the
696 area contained inside 2m from the ID wall for a total of 22.5kton water [3].

697 The two regions of the detector (ID and OD) are optically separated with
698 opaque black plastic hung from the support structure. The purpose of this is
699 to determine whether an event entered or exited the ID. This allows cosmic ray
700 muons and partially contained events to be tagged and separated from neutrino
701 events entirely contained within the ID. This black plastic is also used to cover
702 the area between the ID PMTs to reduce photon reflection from the ID walls.
703 Opposite to this, the OD is lined with a reflective material to allow photons to
704 reflect around inside the OD until collected by one of the PMTs. Furthermore,
705 each OD PMT is optically coupled with $50 \times 50\text{cm}$ plates of wavelength shifting
706 acrylic which increases the efficiency of light collection [83].

707 In the SK-IV data-taking period, the photocathode coverage of the detector, or
708 the fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [83]. The PMTs have
709 a quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$
710 for photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$ [85, 86]. The proportion
711 of photoelectrons that produce a signal in the dynode of a PMT, termed the
712 collection efficiency, is $> 70\%$ [83]. The PMTs used within SK are most sensitive
713 to photons with wavelength $300\text{nm} \leq \lambda \leq 600\text{nm}$ [83]. One disadvantage of
714 using PMTs as the detection media is that the Earth's geomagnetic field can
715 modify its response. Therefore, a set of compensation coils is built around the
716 inner surface of the detector to mitigate this effect [83].

717 The SK detector is filled with ultrapure water, which in a perfect world, con-
718 tains no impurities. However, bacteria and organic compounds can significantly
719 degrade the water quality. This decreases the attenuation length, which reduces
720 the total number of photons that hit a PMT. To combat this, a sophisticated water
721 treatment system has been developed [83, 87]. UV lights, mechanical filters, and
722 membrane degasifiers are used to reduce the bacteria, suspended particulates,
723 and radioactive materials from the water. The flow of water within the tank
724 is also critical as it can remove stagnant bacterial growth or build-up of dust

725 on the surfaces within the tank. Gravity drifts impurities in the water towards
726 the bottom of the tank which, if left uncontrolled, can create asymmetric water
727 conditions between the top and bottom of the tank. Typically, the water entering
728 the tank is cooled below the ambient temperature of the tank to control convection
729 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs
730 is sensitive to the PMT temperature [88]. Therefore controlling the temperature
731 gradients within the tank is beneficial for stable measurements.

732 SK-VI is the first phase of the SK experiment to use gadolinium dopants
733 within the ultrapure water [81]. As such, the SK water system had to be replaced
734 to avoid removing the gadolinium concentrate from the ultrapure water [66]. For
735 an inverse β -decay (IBD) interaction on a water target, the emitted neutron is
736 thermally captured on hydrogen. This process releases a 2.2MeV γ ray which is
737 difficult to detect as the resulting Compton scattered electrons are very close to the
738 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons
739 on gadolinium generates γ rays with higher energy (8MeV [66]) meaning they
740 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%
741 gadolinium sulphate by mass) which causes $\approx 50\%$ of neutrons emitted by IBD
742 to be captured on gadolinium[89]. Whilst predominantly useful for low energy
743 analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino
744 event selections [90]. Efforts are currently in place to increase the gadolinium
745 concentrate to 0.03% for $\approx 75\%$ neutron capture efficiency on gadolinium [91].
746 The final stage of loading targets 0.1% concentrate for $\approx 90\%$ neutron capture
747 efficiency on gadolinium.

748 3.1.2 Calibration

749 The calibration of the SK detector is documented in [80] and summarised below.
750 The analysis presented within this thesis is dependent upon ‘high energy events’
751 (Charged particles with $O(> 100)\text{MeV}$ momenta). These are events that are
752 expected to generate a larger number of photons such that each PMT will
753 be hit with multiple photons. The reconstruction of these events depends

upon the charge deposited within each PMT and the timing response of each individual PMT. Therefore, the most relevant calibration techniques to this thesis are outlined.

Before installation, 420 PMTs were calibrated to have identical charge responses and then distributed throughout the tank in a cross-shape pattern (As illustrated by Figure 3.2). These are used as a standardised measure for the rest of the PMTs installed at similar geometric positions within SK to be calibrated against. To perform this calibration, a xenon lamp is located at the center of the SK tank which flashes uniform light at 1Hz. This allows for geometrical effects, water quality variation, and timing effects to be measured in situ throughout normal data-taking periods.

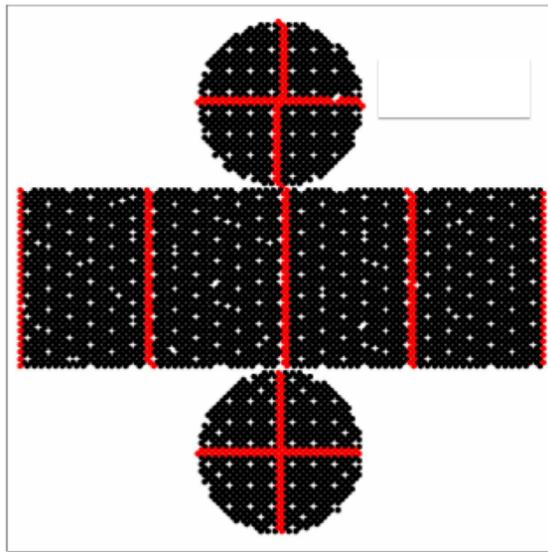


Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [80].

When specifically performing calibration of the detector (in out-of-data taking mode), the water in the tank was circulated to avoid top/bottom asymmetric water quality. Any non-uniformity within the tank significantly affects the PMT hit probability through scattering or absorption. This becomes a dominant effect for very low-intensity light sources that are designed such that only one photon is incident upon a given PMT.

771 The gain of a PMT is defined as the ratio of the total charge of the signal
 772 produced compared to the charge of photoelectrons emitted by the photocathodes
 773 within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”
 774 gain values are measured. The relative gain is the variation of gain among each
 775 of the PMTs whereas the absolute gain is the average gain of all PMTs.

776 The relative gain is calibrated as follows. A laser is used to generate two
 777 measurements: a high-intensity flash that illuminates every PMT with a sufficient
 778 number of photons, and a low-intensity flash in which only a small number
 779 of PMTs collect light. The first measurement creates an average charge, $Q_{obs}(i)$
 780 on PMT i , whereas the second measurement ensures that each hit PMT only
 781 generates a single photoelectron. For the low-intensity measurement, the number
 782 of times each PMT records a charge larger than 1/4 photoelectrons, $N_{obs}(i)$, is
 783 counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i). \end{aligned} \tag{3.1}$$

784 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the
 785 acceptance efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and
 786 collection efficiency of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative
 787 gain for each PMT can be determined by taking the ratio of these quantities.

788 The absolute gain calibration is performed by observing fixed energy γ -rays
 789 of $E_\gamma \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source
 790 situated at the center of the detector. This generates a photon yield of about 0.004
 791 photoelectrons/PMT/event, meaning that $> 99\%$ of PMT signals are generated
 792 from single photoelectrons. A charge distribution is generated by performing
 793 this calibration over all PMTs, and the average value of this distribution is taken
 794 to be the absolute gain value.

795 As mentioned in subsection 3.1.1, the average quantum and collection effi-
 796 ciency for the SK detector PMTs is $\sim 21\%$ and $> 70\%$ respectively. However,
 797 these values do differ between each PMT and need to be calibrated accordingly.

798 Consequently, the NiCf source is also used to calibrate the “quantum \times collection”
799 efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is
800 used as the PMT hit probability is proportional to the QE ($N_{obs}(i) \propto \epsilon(i)$ in
801 Equation 3.1). A Monte Carlo prediction which includes photon absorption,
802 scattering, and reflection is made to estimate the number of photons incident on
803 each PMT and the ratio of the number of predicted to observed hits is calculated.
804 The difference is attributed to the QE efficiency of that PMT. This technique is
805 extended to calculate the relative QE efficiency by normalizing the average of
806 all PMTs which removes the dependence on the light intensity.

807 Due to differing cable lengths and readout electronics, the timing response
808 between a photon hitting the PMT and the signal being captured by the data
809 acquisition can be different between each PMT. Due to threshold triggers (De-
810 scribed in subsection 3.1.3), the time at which a pulse reaches a threshold is
811 dependent upon the size of the pulse. This is known as the ‘time-walk’ effect
812 and also needs to be accounted for in each PMT. To calibrate the timing response,
813 a pulse of light with width 0.2ns is emitted into the detector through a diffuser.
814 Two-dimensional distributions of time and pulse height (or charge) are made
815 for each PMT and are used to calibrate the timing response. This is performed
816 in-situ during data taking with the light source pulsing at 0.03Hz.

817 The top/bottom water quality asymmetry is measured using the NiCf calibra-
818 tion data and cross-referencing these results to the “standard PMTs”. The water
819 attenuation length is continuously measured by the rate of vertically-downgoing
820 cosmic-ray muons which enter via the top of the tank.

821 Dark noise is where a PMT registers a pulse that is consistent with a single
822 photoelectron emitted from photon detection despite the PMT being in complete
823 darkness. This is predominately caused by two processes. Firstly there is
824 intrinsic dark noise which is where photoelectrons gain enough thermal energy
825 to be emitted from the photocathode, and secondly, the radioactive decay of
826 contaminants inside the structure of the PMT. Typical dark noise rate for PMTs
827 used within SK are $O(3)\text{kHz}$ [83]. This is lower than the expected number of

828 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)
829 but instability in this value can cause biases in reconstruction. Dark noise is
830 related to the gain of a PMT and is calibrated using hits inside a time window
831 recorded before an event trigger [92].

832 3.1.3 Data Acquisition and Triggering

833 As the analysis presented in this thesis will only use the SK-IV period of the
834 SK experiment so this subsection focuses on the relevant points of the data
835 acquisition and triggering systems to that SK period. The earlier data acquisition
836 and triggering systems are documented in [93, 94].

837 Before the SK-IV period started, the existing front-end electronics were re-
838 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [95]. When
839 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-
840 time (QTC) converter generates a rectangular pulse. The start of the rectangular
841 pulse indicates the time at which the analog photoelectron signal was received
842 and the width of the pulse indicates the total charge integrated throughout
843 the signal. This is then digitized by time-to-digital converters and sent to the
844 “front-end” PCs.

845 The software trigger (described in [96]) operates by determining the number
846 of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the
847 maximum time that a Cherenkov photon would take to traverse the length of the
848 SK tank [94]. For lower energy events that generate fewer photons, this technique
849 is useful for eliminating background processes like dark noise and radioactive
850 decay which would be expected to be separated in time. When the value of
851 N_{200} exceeds some pre-defined threshold, a software trigger is issued. There are
852 several trigger thresholds used within the SK-IV period which are detailed in
853 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time
854 window are also read out and saved to disk. In the special case of an event that
855 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks
856 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in

857 a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband
858 Intelligent Trigger (WIT)” has been deployed and is described in [97].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

3.1.4 Cherenkov Radiation

860 Cherenkov light is emitted from any highly energetic charged particle traveling
861 with relativistic velocity, β , greater than the local speed of light in a medium [98].
862 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

863 Where n is the refractive index of the medium. Consequently, the Cherenkov
864 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged
865 particle moving through the medium,

$$P_{\text{thres}} = \frac{m}{\sqrt{n^2 - 1}}. \quad (3.3)$$

866 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy
867 for various particles are given in Table 3.3. In contrast, γ -rays are detected
868 indirectly via the combination of photons generated by Compton scattering
869 and pair production. The threshold for detection in the SK detector is typically
870 higher than the threshold for photon production. This is due to the fact that the
871 attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov
872 photons reach the ID PMTs. Then the collection and quantum efficiencies

873 described in subsection 3.1.1 result in the number of detected photons being
874 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

875 The Frank-Tamm equation [99] describes the relationship between the number
876 of Cherenkov photons generated per unit length, dN/dx , the wavelength of the
877 photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dx d\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

878 Where α is the fine structure constant. For a 100MeV momentum electron,
879 approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq$
880 700nm region which the ID PMTs are most sensitive to [83].

881 3.2 The Tokai to Kamioka Experiment

882 The Tokai-to-Kamioka (T2K) experiment is a long-baseline neutrino oscillation
883 experiment located in Japan. Proposed in the early 2000s [84, 100] to replace
884 K2K [101], T2K was designed to observe electron neutrino appearance whilst
885 precisely measuring the oscillation parameters associated with muon neutrino
886 disappearance [102]. The experiment consists of a neutrino beam generated
887 at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near
888 detectors situated 280m from the beam target, and the Super Kamiokande far
889 detector positioned at a 295km baseline. The cross-section view of the T2K
890 experiment is drawn in Figure 3.3.

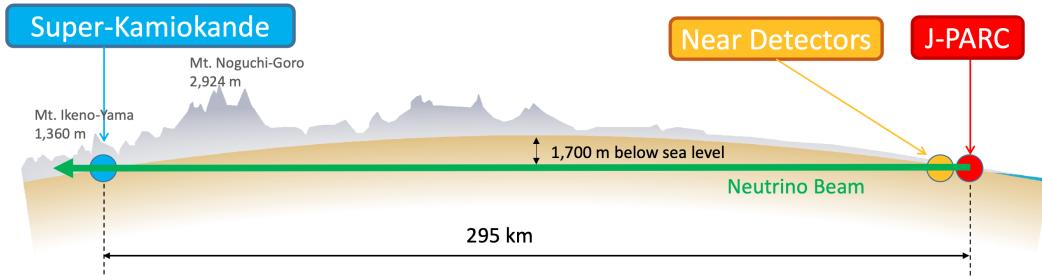


Figure 3.3: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

891 The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$,
 892 Δm_{32}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accu-
 893 racy of parameter estimates are still being made by including new data samples
 894 and developing the models which describe the neutrino interactions and detector
 895 responses [4]. Electron neutrino appearance was first observed at T2K in 2014
 896 [103] with 7.3σ significance.

897 The near detectors provide constraints on the beam flux and cross-section
 898 model parameters used within the oscillation analysis by observing the unoscil-
 899 lated neutrino beam. There are a host of detectors situated in the near detector hall
 900 (as illustrated in Figure 3.4): ND280 (subsection 3.2.3), INGRID (subsection 3.2.4),
 901 NINJA [104], WAGASCI [105], and Baby-MIND [106]. The latter three are not
 902 currently used within the oscillation analysis presented in this thesis.

903 Whilst this thesis presents the ND280 in terms of its purpose for the oscillation
 904 analysis, the detector can also make many cross-section measurements at neutrino
 905 energies of $O(1)\text{GeV}$ for the different targets within the detector [107, 108]. These
 906 measurements are of equal importance as they can lead the way in determining
 907 the model parameters used in the interaction models for the future high-precision
 908 era of neutrino physics.

909 3.2.1 Analysis Overview

910 There are two independent fitters, MaCh3 and BANFF, which perform the near
 911 detector fit. MaCh3 uses a bayesian Markov Chain Monte Carlo fitting technique,

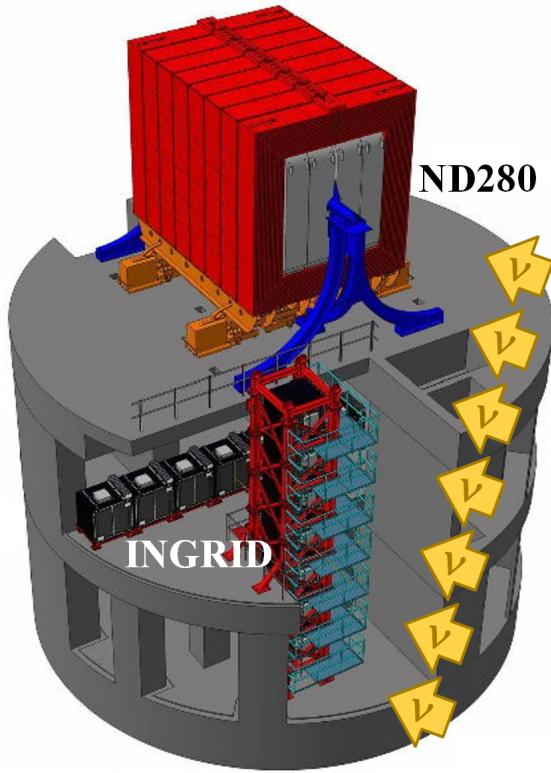


Figure 3.4: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

912 whereas BANFF uses a frequentist gradient descent technique. The output of
 913 each fitter is compared as a method of cross-checking the behaviour of the two
 914 fitters. This is done by comparing: the Monte Carlo predictions using various
 915 tunes, the likelihood that is calculated in each fitter, and the post-fit constraint
 916 associated with every parameter used in the fit. Once validated, the output is
 917 converted into a covariance matrix to describe the error and correlations between
 918 all the flux and cross-section parameters. This is then propagated to the far-
 919 detector oscillation analysis group.

920 The far detector group has three independent fitters: P-Theta, VALOR and
 921 MaCh3. The first two fitters use a hybrid frequentist fitting technique where the
 922 likelihood is minimised with respect to the parameters of interest and marginalised
 923 over all other parameters. These fitters use the covariance provided by the near
 924 detector fitters as a basis for implementing the near detector constraints. The
 925 MaCh3 fitter uses a simultaneous fit of all near and far detector samples. This

removes any Gaussian assumptions when making the covariance matrix from the near detector results. The results for all three fitters are compared using a technique similar to the validation of the near detector fitters.

There are three particular tunes of the T2K flux and cross section model typically considered. Firstly, the “generated” tune which is the set of dial values with which the Monte Carlo was generated. Secondly, the set of dial values which are taken from external data measurements and used as inputs. These are the “pre-fit” dial values. The reason these two sets of dial values are different is that the external data measurements are continually updated but it is very computationally intensive to regenerate a Monte Carlo prediction after each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial values. These are the values taken from the constraints provided by the near detector.

3.2.2 The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [38, 42] and summarised below. The accelerator facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.5 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions

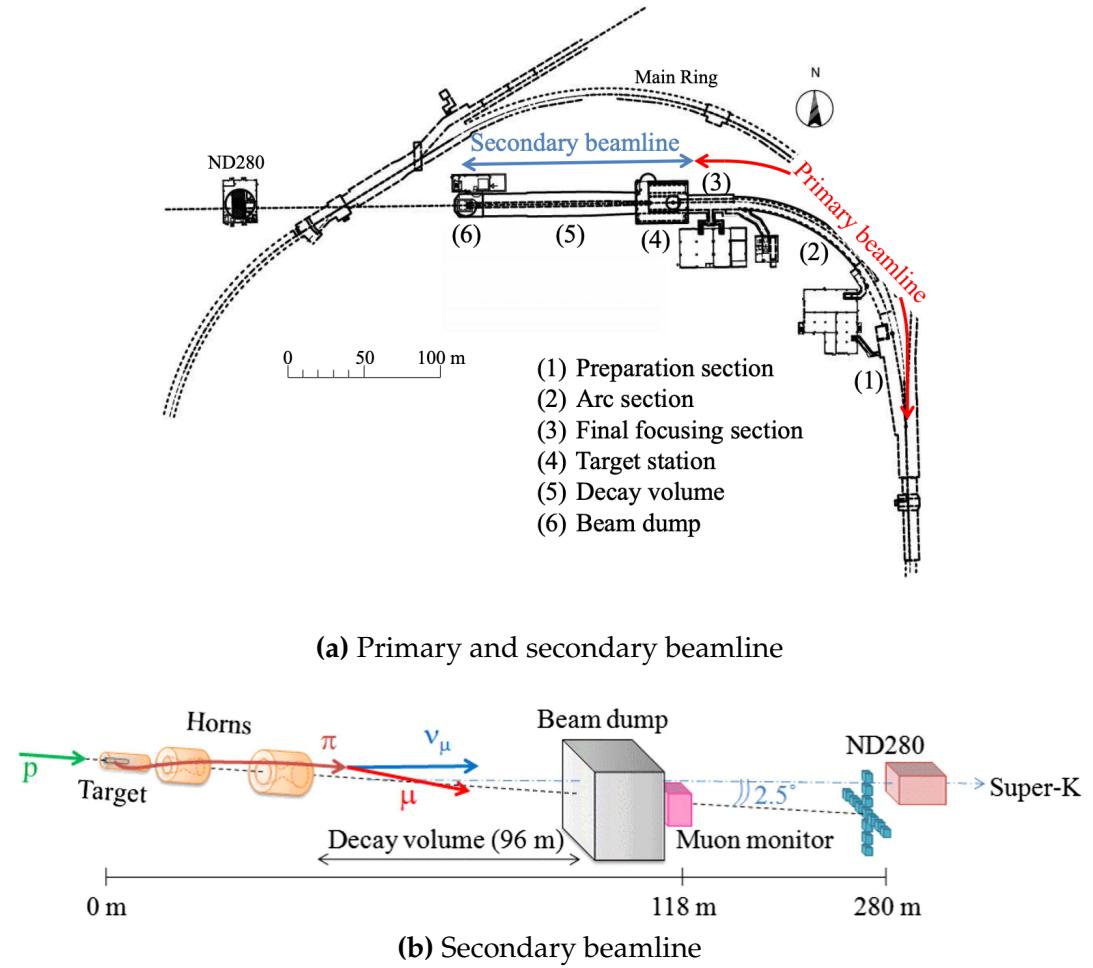


Figure 3.5: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [42]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [109].

and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [38],

$$\begin{array}{ll}
 \pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
 K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
 \rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
 \rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
 K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
 \rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
 \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e.
 \end{array}$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

Figure 3.6 illustrates the different contributions to the FHC and RHC neutrino flux. The low energy flux is dominated by the decay of pions whereas kaon decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic irreducible ν_e background, are predominantly due to muon decay for $E_\nu < 2\text{GeV}$. As the antineutrino production cross-section is smaller than the neutrino cross-section, the wrong-sign component is more dominant in the RHC beam as compared to that in the FHC beam.

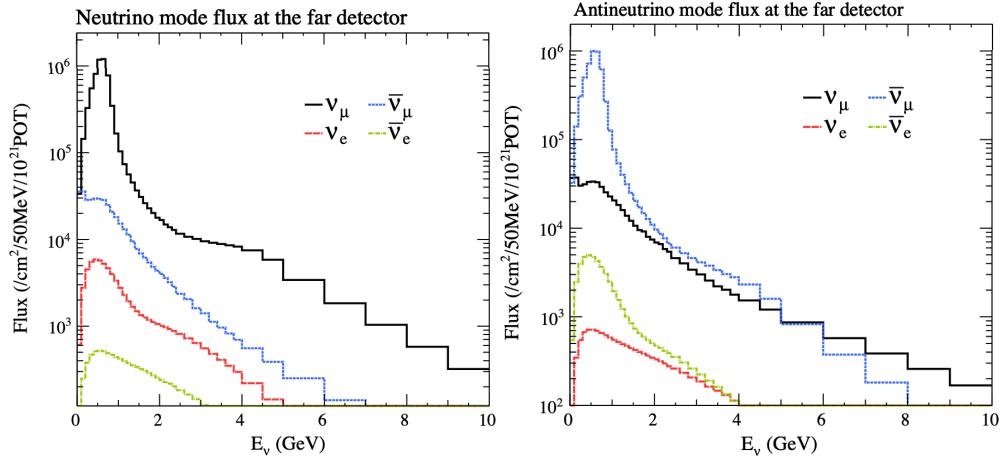


Figure 3.6: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [78].

The beam dump, situated at the end of the decay volume, stops all charged particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector monitors the penetrating muons to determine the beam direction and intensity which is used to constrain some of the beam flux systematics within the analysis [109, 110].

977 The T2K experiment uses an off-axis beam to narrow the neutrino energy
 978 distribution. This was the first implementation of this technique in a long-
 979 baseline neutrino oscillation experiment after its original proposal [111]. Pion
 980 decay, $\pi \rightarrow \mu + \nu_\mu$, is a two-body decay. Consequently, the neutrino energy,
 981 E_ν , can be determined based on the pion energy, E_π , and the angle at which
 982 the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

983 where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
 984 energy pion, the neutrino energy distribution is dependent upon the angle at
 985 which the neutrinos are observed from the initial pion beam direction. For the
 986 295km baseline at T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance
 987 probability, $P(\nu_\mu \rightarrow \nu_e)$, whilst minimising the muon disappearance probability,
 988 $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.7 illustrates the neutrino energy distribution for a range of
 989 off-axis angles, as well as the oscillation probabilities most relevant to T2K.

990 3.2.3 The Near Detector at 280m

991 Whilst all the near detectors are situated in the same “pit” located 280m from
 992 the beam target, the “ND280” detector is the off-axis detector which is situated
 993 at the same off-axis angle as the Super-Kamiokande far detector. It has two
 994 primary functions; firstly it measures the neutrino flux and secondly, it counts the
 995 event rates of different types of neutrino interactions. Both of these constrain the
 996 flux and cross-section systematics invoked within the model for a more accurate
 997 prediction of the expected event rate at the far detector.

998 As illustrated in Figure 3.8, the ND280 detector consists of several sub-
 999 detectors. The most important part of the detector for this analysis is the tracker
 1000 region. This is comprised of two-time projection chambers (TPCs) sandwiched
 1001 between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon
 1002 plastic and water targets for neutrino interactions and provide track recon-
 1003 struction near the interaction vertex. The emitted charged particles can then

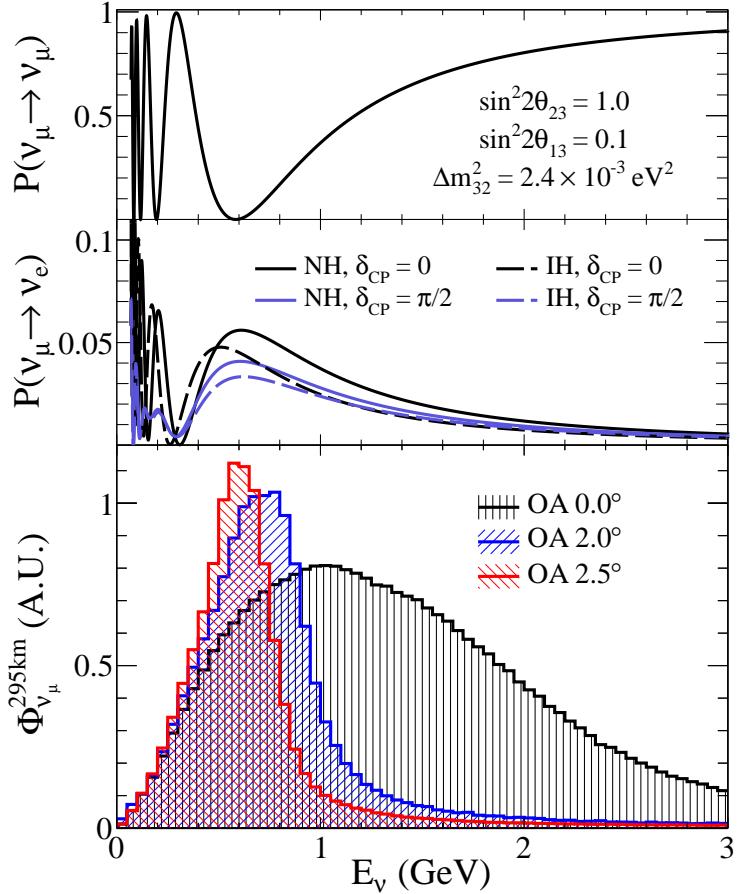


Figure 3.7: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

1004 propagate into the TPCs which provide particle identification and momentum
 1005 reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.3.1
 1006 and subsubsection 3.2.3.2, respectively. The electromagnetic calorimeter (ECAL)
 1007 encapsulates the tracker region alongside the π^0 detector (P0D). The ECAL
 1008 measures the deposited energy from photons emitted from interactions within
 1009 the FGD. The P0D constrains the cross-section of neutral current interactions
 1010 which generate neutral pions, which is one of the largest backgrounds in the
 1011 electron neutrino appearance oscillation channel. The P0D and ECAL detectors
 1012 are detailed in subsubsection 3.2.3.3 and subsubsection 3.2.3.4, respectively.
 1013 The entire detector is located within a large yoke magnet which produces a
 1014 0.2T magnetic field. This design of the magnet also includes a scintillating

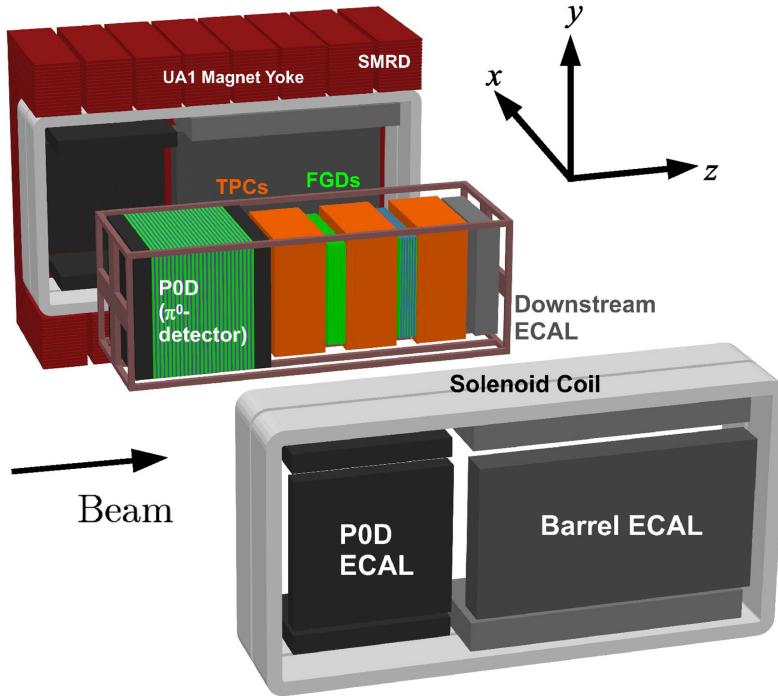


Figure 3.8: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [42].

1015 detector called the side muon range detector (SMRD), which is used to track
 1016 high-angle muons as well as acting as a cosmic veto. The SMRD is described
 1017 in subsubsection 3.2.3.5.

1018 3.2.3.1 Fine Grained Detectors

1019 The T2K tracker region is comprised of two fine-grained detectors (FGD) and
 1020 three Time Projection Chambers (TPC). A detailed description of the FGD design,
 1021 construction, and assembly is found in [112] and summarised below. The FGDs
 1022 are the primary target for neutrino interactions with a mass of 1.1 tonnes per
 1023 FGD. Alongside this, the FGDs are designed to be able to track short-range
 1024 particles which do not exit the sub-detector. Typically, short-range particles are
 1025 low momentum and are observed as tracks that deposit a large amount of energy
 1026 per unit length. This means the FGD needs good granularity to resolve these
 1027 particles. The FGDs have the best timing resolution ($\sim 3\text{ns}$) of any of the sub-
 1028 detectors of the ND280 detector. As such, the FGDs are used for time of flight
 1029 measurements to distinguish forward-going positively charged particles from

1030 backward-going negatively charged particles. Finally, any tracks which pass
1031 through multiple sub-detectors are required to be track matched to the FGD.

1032 Both FGDs are made from square scintillator planes of side length 186cm and
1033 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an
1034 X or Y orientation. A wavelength-shifting fiber is threaded through the center
1035 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is
1036 the most upstream of the two FGDs and contains 15 planes of carbon plastic
1037 scintillator which is a common target in external neutrino scattering data. As
1038 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2
1039 have been replaced with a hybrid water-scintillator target. Due to the complexity
1040 of the nucleus, nuclear effects can not be extrapolated between different nuclei.
1041 Therefore having the ability to take data on one target which is the same as
1042 external data and another target which is the same as the far detector target is
1043 beneficial for reliable model parameter estimates.

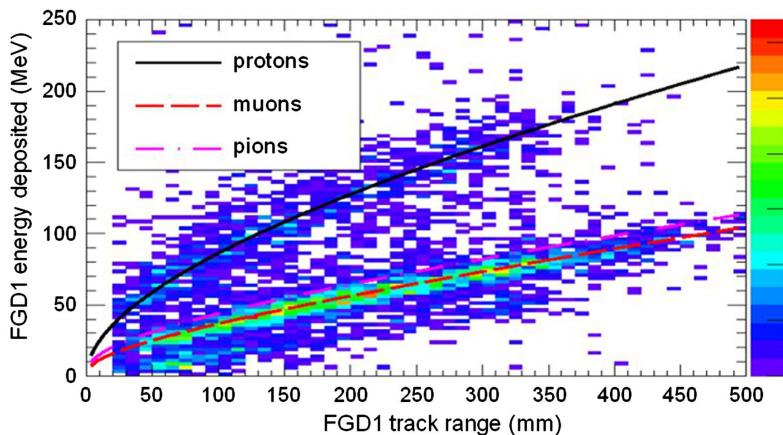


Figure 3.9: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [112].

1044 The integrated deposited energy is used for particle identification in the FGD.
1045 The sub-detector can distinguish protons from other charged particles by com-
1046 paring this quantity from data to Monte Carlo prediction, as seen in Figure 3.9.

1047 3.2.3.2 Time Projection Chambers

1048 The majority of particle identification and momentum measurements within
 1049 ND280 are provided by three Time Projection Chambers (TPCs) [113]. The TPCs
 1050 are located on either side of the FGDs. They are located inside the magnetic
 1051 field meaning the momentum of a charged particle can be determined from
 1052 the bending of the track.

1053 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.10,
 1054 which are made of non-magnetic material. The outer box is filled with CO₂
 1055 which acts as an electrical insulator between the inner box and the ground. The
 1056 inner box forms the field cage which produces a uniform electric drift field of
 1057 $\sim 275\text{V/cm}$ and is filled with an argon gas mixture. Charged particles moving
 1058 through this gas mixture ionize the gas and the subsequent charge is drifted
 1059 towards micromegas detectors. The time and position information in the readout
 1060 allows a three-dimensional image of the neutrino interaction.

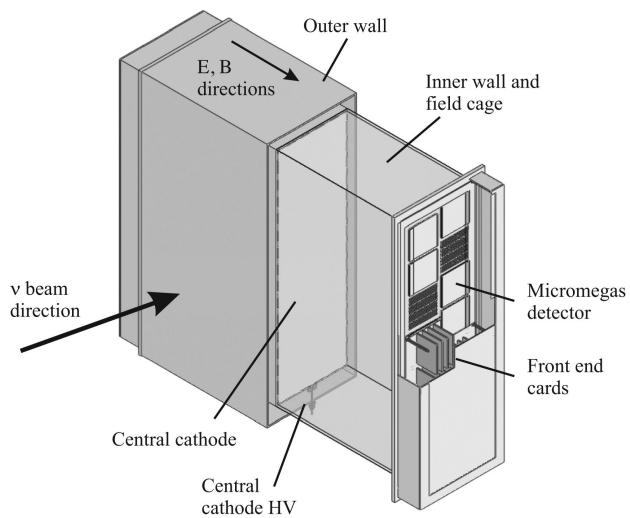


Figure 3.10: Schematic design of a Time Projection Chamber detector. Taken from [113].

1061 The particle identification of tracks that pass through the TPCs is performed
 1062 using dE/dx measurements. Figure 3.11 illustrates the data to Monte Carlo
 1063 distributions of the energy lost by a charged particle passing through the TPC as
 1064 a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$

1065 meaning that electrons and muons can be distinguished. This allows reliable
 1066 measurements of the intrinsic ν_e component of the beam.

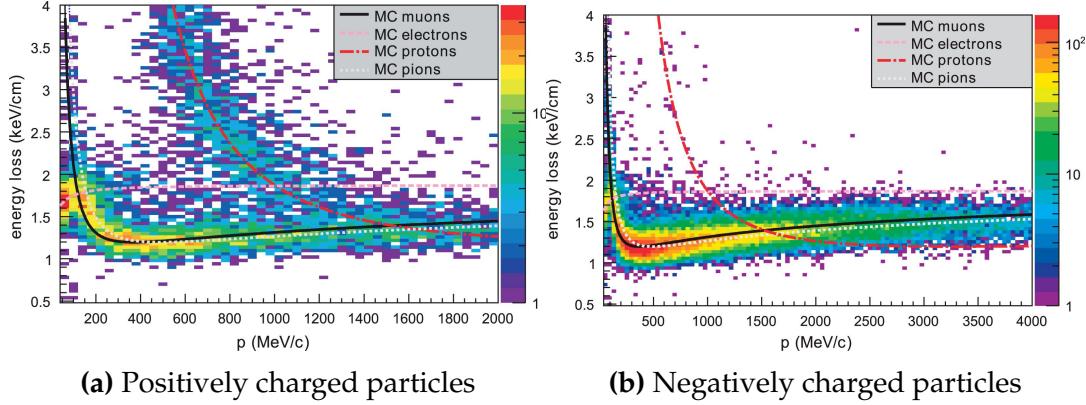


Figure 3.11: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [113].

1067 3.2.3.3 π^0 Detector

1068 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the
 1069 reconstruction algorithm could determine that event to be a charge current ν_e -like
 1070 event. This is one of the main backgrounds hindering the electron neutrino
 1071 appearance searches. The π^0 detector (P0D) measures the cross-section of the
 1072 neutral current induced neutral pion production on a water target to constrain
 1073 this background.

1074 The P0D is a cube of approximately 2.5m length consisting of layers of scin-
 1075 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.12.
 1076 Two electromagnetic calorimeters are positioned at the most upstream and most
 1077 downstream position in the sub-detector and the water target is situated between
 1078 them. The scintillator layers are built from two triangular bars orientated in
 1079 opposite directions to form a rectangular layer. Each triangular scintillator bar is
 1080 threaded with optical fiber which is read out by MPPCs. The high-Z brass and
 1081 lead regions produce electron showers from the photons emitted in π^0 decay.

1082 The sub-detector can generate measurements of NC1 π^0 cross-sections on a
 1083 water target by measuring the event rate both with and without the water target,

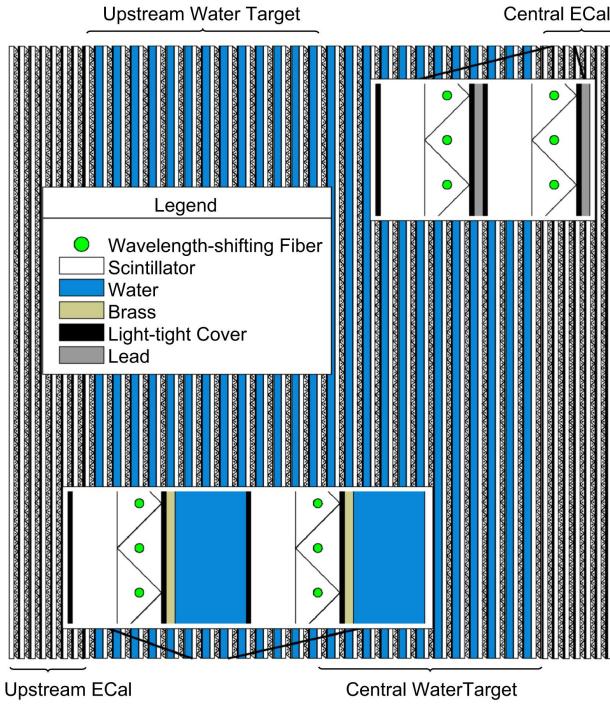


Figure 3.12: A schematic of the P0D side-view. Taken from [114].

1084 with the cross-section on a water target being determined as the difference. The to-
 1085 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

1086 3.2.3.4 Electromagnetic Calorimeter

1087 The electromagnetic calorimeter [115] (ECal) encapsulates the P0D and tracking
 1088 sub-detectors. Its primary purpose is to aid π^0 reconstruction from any interac-
 1089 tion in the tracker. To do this, it measures the energy and direction of photon
 1090 showers from $\pi^0 \rightarrow 2\gamma$ decays. It can also distinguish pion and muon tracks
 1091 depending on the shape of the photon shower deposited.

1092 The ECal is comprised of three sections; the P0D ECal which surrounds the
 1093 P0D, the barrel ECal which encompasses the tracking region, and the downstream
 1094 ECal which is situated downstream of the tracker region. The barrel and down-
 1095 stream ECals are tracking calorimeters that focus on electromagnetic showers
 1096 from high-angle particles emitted from the tracking sub-detectors. Particularly in
 1097 the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis)
 1098 can travel along a single scintillator bar resulting in very few hits. The width of

1099 the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths
1100 to ensure a significant amount of the π^0 energy is contained. As the P0D has
1101 its own calorimetry which reconstructs showers, the P0D ECal determines the
1102 energy which escapes the P0D.

1103 Each ECal is constructed of multiple layers of scintillating bars sandwiched
1104 between lead sheets. The scintillating bars are threaded with optical fiber and read
1105 out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular
1106 to the previous which allows a three-dimensional event reconstruction. The
1107 target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80,
1108 and 6.62 tonnes respectively.

1109 3.2.3.5 Side Muon Range Detector

1110 As illustrated in Figure 3.8, the ECal, FGDs, P0D, and TPCs are enclosed within
1111 the UA1 magnet. Reconditioned after use in the UA1 [116] and NOMAD [117]
1112 experiments, this magnet provides a uniform horizontal magnetic field of 0.2T
1113 with an uncertainty of 2×10^{-4} T.

1114 Built into the UA1 magnet, the side muon range detector (SMRD)[118] moni-
1115 tors high-energy muons which leave the tracking region and permeate through
1116 the ECal. It additionally acts as a cosmic muon veto and trigger.

1117 3.2.4 The Interactive Neutrino GRID

1118 The Interactive Neutrino GRID (INGRID) detector is situated within the same
1119 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”
1120 position and measures the beam direction, spread, and intensity. The detector
1121 was originally designed with 16 identical modules [42] (two modules have since
1122 been decommissioned) and a “proton” module. The design of the detector is 14
1123 modules oriented in a cross with length and height $10\text{m} \times 10\text{m}$, as illustrated
1124 in Figure 3.13.

1125 Each module is composed of iron sheets interlaced with eleven tracking
 1126 scintillator planes for a total target mass of 7.1 tonnes per module. The scin-
 1127 tillator design is an X-Y pattern of 24 bars in both orientations, where each
 1128 bar contains wave-length shifting fibers which are connected to MPPCs. Each
 1129 module is encapsulated inside veto planes to aid the rejection of charged particles
 1130 entering the module.

1131 The proton module is different from the other modules in that it consists
 1132 of entirely scintillator planes with no iron target. The scintillator bars are also
 1133 smaller than those used in the other modules to increase the granularity of
 1134 the detector and improve tracking capabilities. The module sits in the center
 1135 of the beamline and is designed to give precise measurements of quasi-elastic
 1136 charged current interactions to evaluate the performance of the Monte Carlo
 1137 simulation of the beamline.

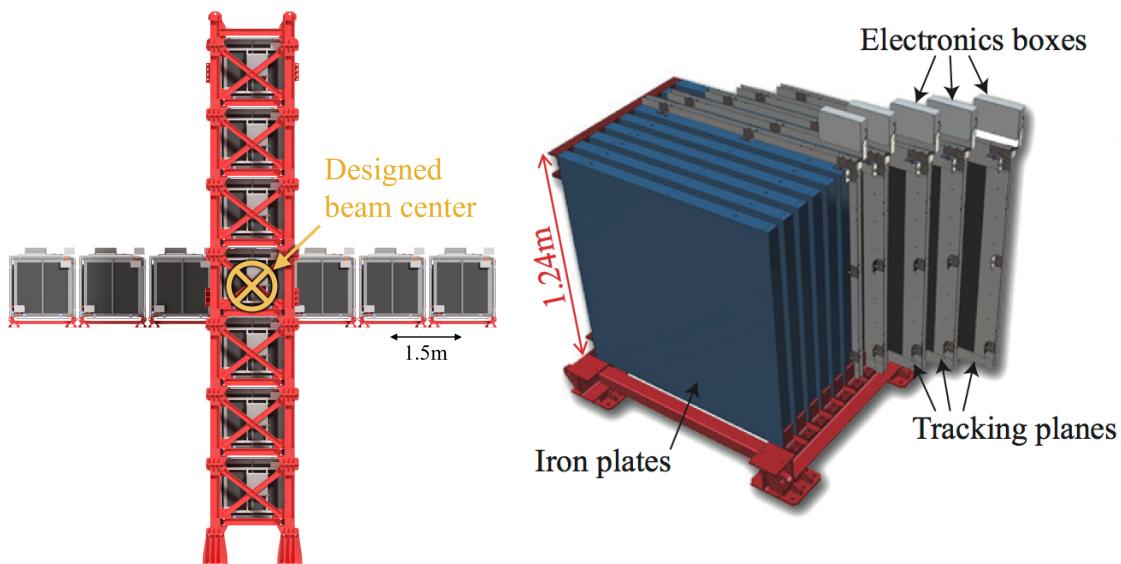


Figure 3.13: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [42].

1138 The INGRID detector can measure the beam direction to an uncertainty of
 1139 0.4mrad and the beam centre within a resolution of 10cm [42]. The beam direction
 1140 in both the vertical and horizontal directions is discussed in [119] and it is found
 1141 to be in good agreement with the MUMON monitor described in subsection 3.2.2.

4

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Bayesian Statistics and Markov Chain Monte Carlo Techniques

1145 This thesis presents a Bayesian oscillation analysis. To extract the oscillation
1146 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter
1147 explains the theory of how parameter estimates can be determined using this
1148 technique and condenses the material found in the literature [120–123].

1149 The oscillation parameter determination presented here is built upon a simultaneous
1150 fit to neutrino beam samples in the near detector, beam samples at SK,
1151 and atmospheric samples at SK. In total, there are four oscillation parameters of
1152 interest ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP}), two oscillation parameters to which
1153 this study will not be sensitive ($\sin^2(\theta_{12})$, Δm_{21}^2) and many nuisance parameters
1154 that control the systematic uncertainty models.

1155 This analysis uses a Monte Carlo technique to generate a multi-dimensional
1156 probability distribution across all of the model parameters used in the fit. To
1157 determine an estimate for each parameter, this multi-dimensional object is in-
1158 tegrated over all other parameters. This process is called Marginalisation and
1159 is described in subsection 4.3.1. Monte Carlo techniques approximate the prob-
1160 ability distribution of each parameter within the limit of generating infinite
1161 samples. As ever, generating a large number of samples is time and resource-

1162 dependent. Therefore, an MCMC technique is utilised within this analysis to
1163 reduce the required number of steps to sufficiently sample the parameter space.
1164 This technique is described in further detail in subsection 4.2.1.

1165 The Bayesian analysis techniques used within this thesis are built within the
1166 MaCh3 framework [124]. This uses a custom MCMC library package exclusively
1167 supported and developed by the MaCh3 collaborators (which includes the author
1168 of this thesis).

1169 4.1 Bayesian Statistics

1170 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
1171 footing such that a probability model of both data and parameters is required.
1172 This is the joint probability distribution $P(D, \vec{\theta})$ and can be described by the
1173 prior distribution for model parameters $P(\vec{\theta})$ and the likelihood of the data given
1174 the model parameters $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

1175 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the
1176 parameters within the model. For example, if the risk of developing health
1177 problems is known to increase with age, the prior distribution would describe the
1178 increase. For the purpose of this analysis, the prior distribution is typically
1179 the best-fit values taken from external data measurements with a Gaussian
1180 uncertainty. The prior distribution can also contain correlations between model
1181 parameters. In an analysis using Monte Carlo techniques, the likelihood of
1182 measuring some data assuming some set of model parameters is calculated
1183 by comparing the Monte Carlo prediction generated at that particular set of
1184 model parameters to the data.

1185 It is parameter estimation that is important for this analysis and as such, Bayes'
1186 theorem [125] is applied to calculate the probability for each parameter to have a

₁₁₈₇ certain value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior
₁₁₈₈ distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

₁₁₈₉ The denominator in Equation 4.2 is the integral of the joint probability distri-
₁₁₉₀ bution over all values of all parameters used within the fit. For brevity, the
₁₁₉₁ posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

₁₁₉₂ For the purposes of this analysis, it is acceptable to neglect the normalisation
₁₁₉₃ term and focus on this proportional relationship.

₁₁₉₄ 4.1.1 Application of Prior Knowledge

₁₁₉₅ The posterior distribution is proportional to the prior uncertainty applied to
₁₁₉₆ each parameter, as illustrated by Equation 4.3. This means that it is possible
₁₁₉₇ to change the prior after the posterior distribution has been determined. The
₁₁₉₈ prior uncertainty of a particular parameter can be ‘divided’ out of the posterior
₁₁₉₉ distribution and the resulting distribution can be reweighted using the new
₁₂₀₀ prior uncertainty that is to be applied. The methodology and implementation
₁₂₀₁ of changing the prior follows that described in [126].

₁₂₀₂ An example implementation that is useful for this analysis is the application
₁₂₀₃ of the “reactor constraint”. As discussed in section 2.4, an external constraint
₁₂₀₄ on $\sin^2(\theta_{13})$ is determined from measurements taken from reactor experiments.
₁₂₀₅ However, the sensitivities from just using the T2K and SK samples is equally
₁₂₀₆ as important. Without this technique, two fits would have to be run, doubling
₁₂₀₇ the required resources. Therefore, the key benefit for this analysis is the fact
₁₂₀₈ that only a single ‘fit’ has to be performed to provide the results both with and
₁₂₀₉ without the reactor constraint applied.

1210 4.2 Monte Carlo Simulation

1211 Monte Carlo techniques are used to numerically solve a complex problem that
1212 does not necessarily have an analytical solution. These techniques rely on
1213 building a large ensemble of samples from an unknown distribution and then
1214 using the ensemble to approximate the properties of the distribution.

1215 An example that uses Monte Carlo techniques is to calculate the area under-
1216 neath a curve. For example, take the problem of calculating the area under a
1217 straight line with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can
1218 calculate the area under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte
1219 Carlo techniques, one can calculate the area under this line by throwing many
1220 random values for the x and y components of each sample and then calculating
1221 whether that point falls below the line. The area can then be calculated by the
1222 ratio of points below the line to the total number of samples thrown multiplied by
1223 the total area in which samples were scattered. The study is shown in Figure 4.1
1224 highlights this technique and finds the area under the curve to be 29.9 compared
1225 to an analytical solution of 30.0. The deviation of the numerical to analytical
1226 solution can be attributed to the number of samples used in the study. The
1227 accuracy of the approximation in which the properties of the Monte Carlo samples
1228 replicate those of the desired distribution is dependent on the number of samples
1229 used. Replicating this study with a differing number of Monte Carlo samples
1230 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo
1231 techniques are only accurate within the limit of a high number of samples.

1232 Whilst the above example has an analytical solution, these techniques are just
1233 as applicable to complex solutions. Clearly, any numerical solution is only as
1234 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is
1235 dependent upon the number of samples generated to approximate the properties
1236 of the distribution. Furthermore, if the positions at which the samples are
1237 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique
1238 significantly drops. Given the example in Figure 4.1, if the region in which the

1239 samples are scattered significantly extends passed the region of interest, many
1240 calculations will be calculated but do not add to the ability of the Monte Carlo
1241 technique to achieve the correct result. For instance, any sample evaluated at
1242 $y \geq 5$ could be removed without affecting the final result. This does bring in
1243 an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,
1244 one needs to know the distribution beforehand.

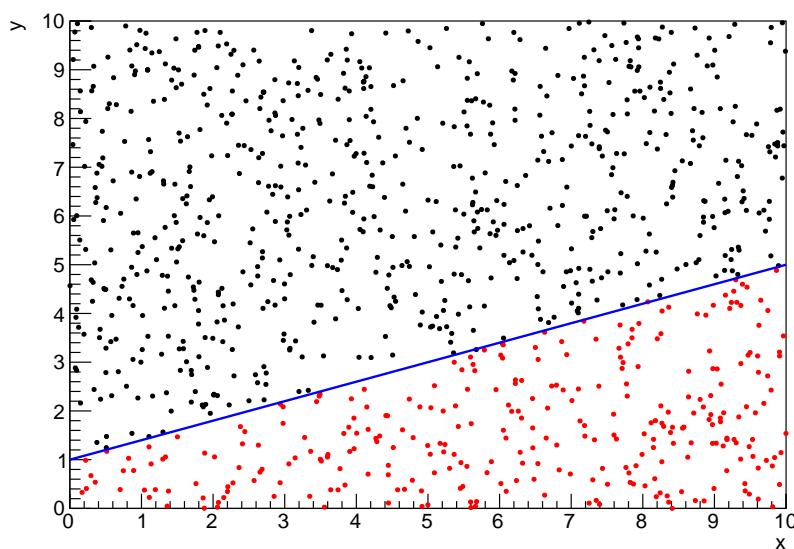


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

1245 4.2.1 Markov Chain Monte Carlo

1246 This analysis utilises a multi-dimensional probability distribution, with some
1247 dimensions being significantly more constrained than others. These constraints
1248 can be from prior knowledge of parameter distributions from external data or
1249 un-physical regions in which parameters can not exist. To maximise the efficiency
1250 of building the posterior distribution, a Markov Chain Monte Carlo (MCMC)
1251 technique is used. This employs a Markov chain to select the points at which
1252 to sample the posterior distribution. It performs a semi-random stochastic walk
1253 through the allowable parameter space. This builds a posterior distribution

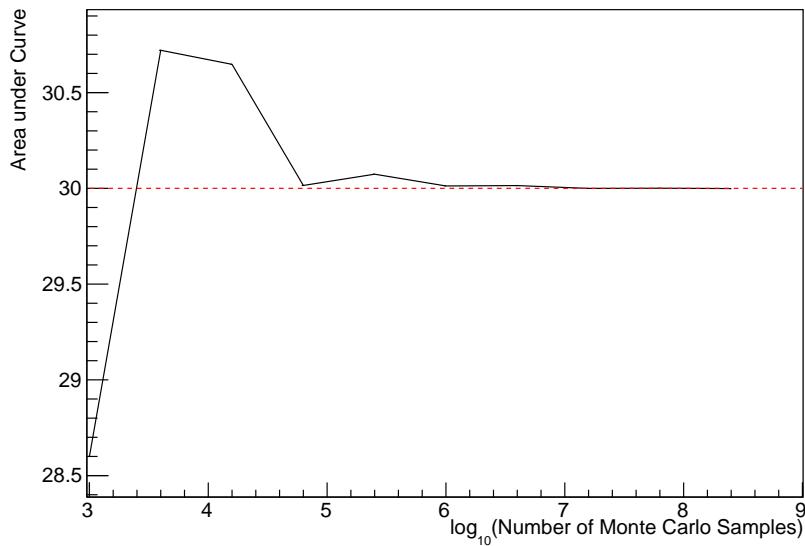


Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1254 which has the property that the density of sampled points is proportional to the
 1255 probability density of that parameter. This means that the samples produced by
 1256 this technique are not statistically independent but they will cover the space
 1257 of the distribution.

1258 A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the
 1259 position of \vec{x}_i . The space in which the Markov chain selects samples is dependent
 1260 upon the total number of parameters utilised within the fit, where a discrete point
 1261 in this space is described by the N-dimensional vector \vec{x} . In a perfectly operating
 1262 Markov chain, the position of the next step depends solely on the previous step
 1263 and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving
 1264 the multi-dimensionality of the fit used within this analysis, each step becomes
 1265 correlated with several of the steps preceding itself. Providing the MCMC chain is
 1266 well optimised, it will begin to converge towards a unique stationary distribution.
 1267 The period between the chain's initial starting point and the convergence to the
 1268 unique stationary distribution is colloquially known as the burn-in period. Once
 1269 the chain reaches the stationary distribution, all points sampled after that point

1270 will look like samples from that distribution.

1271 Further details of the theories underpinning MCMC techniques are discussed
1272 in [121] but can be summarised by the requirement that the chain satisfies the
1273 three ‘regularity conditions’:

- 1274 • Irreducibility: From every position in the parameter space \vec{x} , there must
1275 exist a non-zero probability for every other position in the parameter space
1276 to be reached.
- 1277 • Recurrence: Once the chain arrives at the stationary distribution, every step
1278 following from that position must be samples from the same stationary
1279 distribution.
- 1280 • Aperiodicity: The chain must not repeat the same sequence of steps at any
1281 point throughout the sampling period.

1282 The output of the chain after burn-in (i.e. the sampled points after the chain
1283 has reached the stationary distribution) can be used to approximate the posterior
1284 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1285 stationary distribution found by the chain be the posterior distribution, one
1286 can use the Metropolis-Hastings algorithm. This guides the stochastic process
1287 depending on the likelihood of the current proposed step compared to that
1288 of the previous step.

1289 4.2.2 Metropolis-Hastings Algorithm

1290 As a requirement for MCMCs, the Markov chain implemented in this technique
1291 must have a unique stationary distribution that is equivalent to the posterior
1292 distribution. To ensure this requirement and that the regularity conditions are
1293 met, this analysis utilises the Metropolis-Hastings (MH) algorithm [127, 128].
1294 For the i^{th} step in the chain, the MH algorithm determines the position in the
1295 parameter space to which the chain moves to based on the current step, \vec{x}_i , and
1296 the proposed step, \vec{y}_{i+1} . The proposed step is randomly selected from some

1297 proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely on the current step (ie. not
1298 the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the
1299 current step or the proposed step, determined by whether the proposed step is
1300 accepted or rejected. To decide if the proposed step is selected, the acceptance
1301 probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)}\right). \quad (4.4)$$

1302 Where $P(\vec{y}_{i+1}|D)$ is the posterior probability distribution as introduced in
1303 section 4.1. To simplify this calculation, the proposal function is required to
1304 be symmetric such that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate
1305 Gaussian distribution centered on \vec{x}_i is used to throw parameter proposals. This
1306 reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)}\right). \quad (4.5)$$

1307 After calculating this quantity, a random number, β , is generated uniformly
1308 between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise,
1309 the chain sets the next step equal to the current step. This procedure is repeated
1310 for subsequent steps. This can be interpreted as if the posterior probability
1311 of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq$
1312 $P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true,
1313 ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability
1314 $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped
1315 in any local minima in the potentially non-Gaussian posterior distribution. The
1316 outcome of this technique is that the density of steps taken in a discrete region
1317 is directly proportional to the probability density in that region.

1318 4.2.3 MCMC Optimisation

1319 As discussed in subsection 4.2.2, the proposal function invoked within the MH
1320 algorithm can take any form and the chain will still converge to the stationary
1321 distribution. At each set of proposed parameter values, a prediction of each

sample spectra has to be generated which requires significant computational resources. Therefore, the number of steps taken before the unique stationary distribution is found should be minimised as only steps after convergence add information to the oscillation analysis. Furthermore, the chain should entirely cover the allowable parameter space to ensure that all values have been considered. Tuning the distance that the proposal function jumps between steps on a parameter-by-parameter basis can both minimise the length of the burn-in period and ensure that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently small.

The effect of changing the width of the proposal function is highlighted in Figure 4.3. Three scenarios, each with the same underlying stationary distribution (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between the three scenarios is the width of the proposal function, colloquially known as the ‘step size, σ ’. Each scenario starts at an initial parameter value of 10.0 which would be considered an extreme variation. For the case where $\sigma = 0.1$, it is clear to see that the chain takes a long time to reach the expected region of the parameter. This indicates that this chain would have a large burn-in period and does not converge to the stationary distribution until step ~ 500 . Furthermore, whilst the chain does move towards the expected region, each step is significantly correlated with the previous. Considering the case where $\sigma = 5.0$, the chain approaches the expected parameter region almost instantly meaning that the burn-in period is not significant. However, there are clearly large regions of steps where the chain does not move. This is likely due to the chain proposing steps in the tails of the distribution which have a low probability of being accepted. Consequently, this chain would take a significant number of steps to fully span the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can see a relatively small burn-in period of approximately 100 steps. Once the chain reaches the stationary distribution, it moves throughout the expected region of parameter values many times, sufficiently sampling the full parameter region. This example is a single parameter varying across a continuous distribution and does not fully reflect the difficulties in the many-hundred multi-variate parameter

1352 distribution used within this analysis. However, it does give a conceptual idea of
1353 the importance of selecting the proposal function and associated step size.

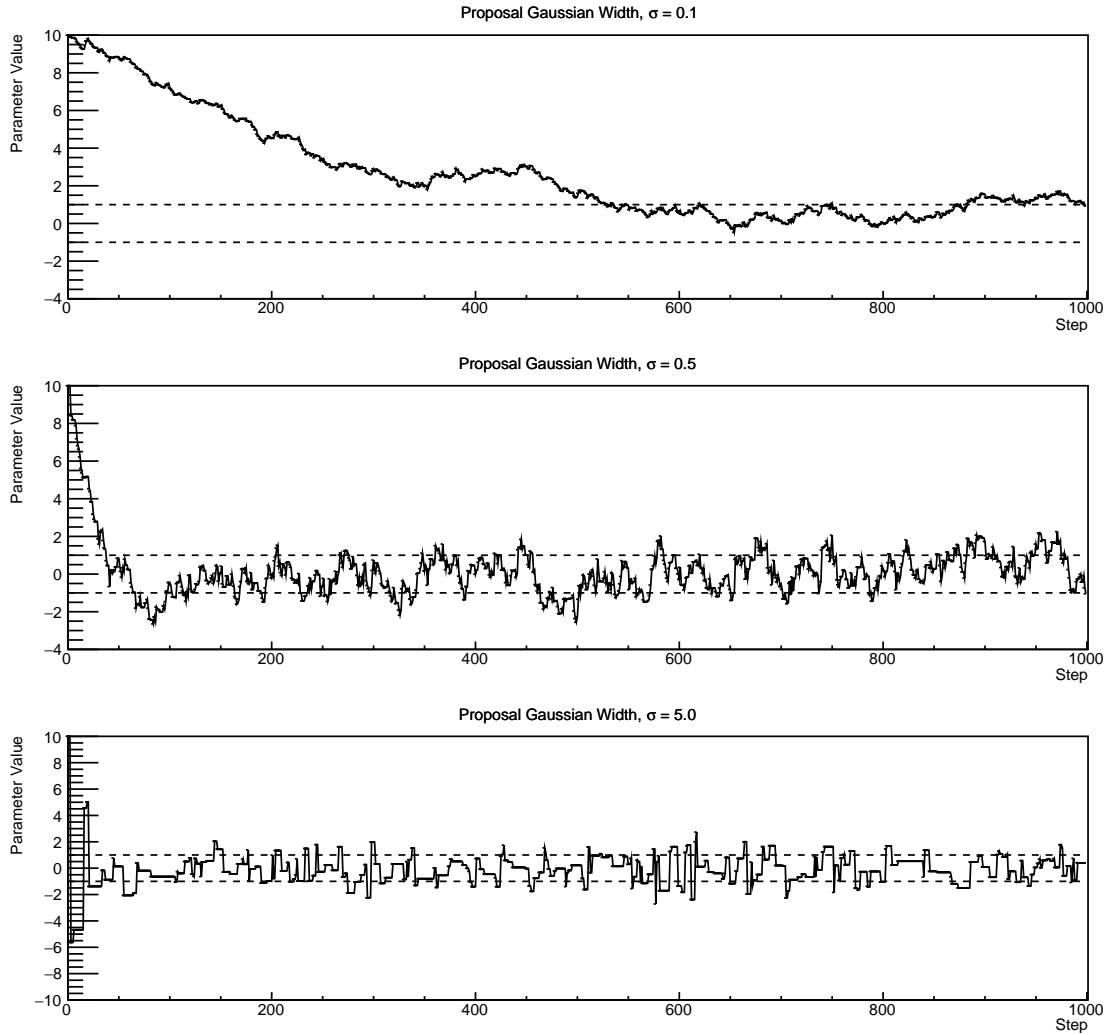


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1354 As discussed, step size tuning directly correlates to the average step accep-
1355 tance rate. If the step size is too small, many steps will be accepted but the
1356 chain moves slowly. If the opposite is true, many steps will be rejected as the
1357 chain proposes steps in the tails of the distribution. Discussion in [129] suggests
1358 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be

¹³⁵⁹ approximately $\sim 25\%$. An “ideal” step size [129] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

¹³⁶⁰ where N_p is the number of parameters included in the MCMC fit. However,
¹³⁶¹ the complex correlations between systematics mean that some parameters have
¹³⁶² to be hand-tuned and many efforts have been taken to select a set of parameter-
¹³⁶³ by-parameter step sizes to approximately reach the ideal acceptance rate.

¹³⁶⁴ Figure 4.4 highlights the likelihood as calculated by the fit in subsection 8.3.4
¹³⁶⁵ as a function of the number of steps in each chain. In practice, many independent
¹³⁶⁶ MCMC chains are run simultaneously to parallelise the task of performing the
¹³⁶⁷ fit. This figure overlays the distribution found in each chain. As seen, the
¹³⁶⁸ likelihood decreases from its initial value and converges towards a stationary
¹³⁶⁹ distribution after $\sim 1 \times 10^5$ steps.

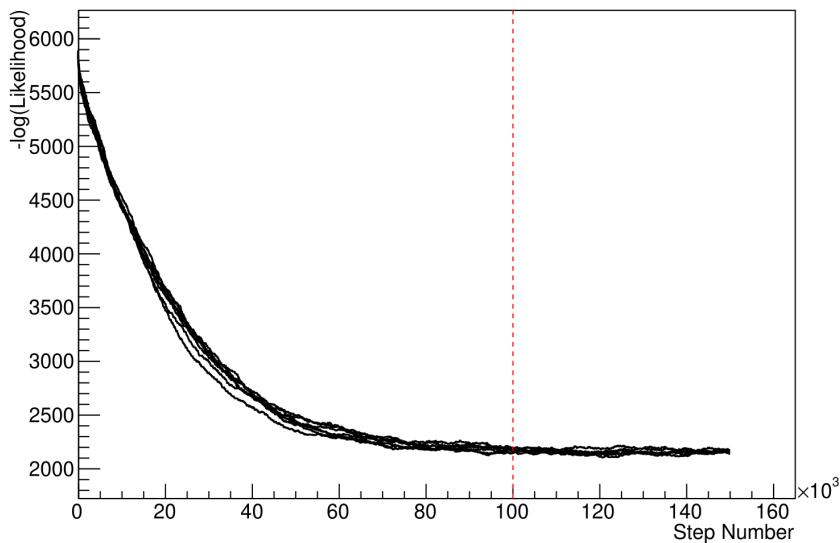


Figure 4.4: The log-likelihood from the fit detailed in subsection 8.3.4 as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

¹³⁷⁰ Multiple configurations of this analysis have been performed throughout this
¹³⁷¹ thesis where different samples or systematics have been used. For all of these
¹³⁷² configurations, it was found that a burnin period of 1×10^5 was sufficient.

1373 4.3 Understanding the MCMC Results

1374 The previous sections have described how to generate the posterior probability
1375 distribution using Bayesian MCMC techniques. However, this analysis focuses
1376 on oscillation parameter determination. The posterior distribution output from
1377 the chain is a high-dimension object, with as many dimensions as there are
1378 parameters included in the oscillation analysis. However, this multi-dimensional
1379 object is difficult to conceptualize so parameter estimations are often presented
1380 in one or two-dimensional projections of this probability distribution. To do
1381 this, marginalisation techniques are invoked.

1382 4.3.1 Marginalisation

1383 The output of the MCMC chain is a highly dimensional probability distribution
1384 which is very difficult to interpret. From the standpoint of an oscillation analysis
1385 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters
1386 of interest are most relevant. Despite this, the best fit values and uncertainties on
1387 the oscillation parameters of interest should correctly encapsulate the correlations
1388 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).
1389 For this joint beam and atmospheric analysis, the oscillation parameters of
1390 interest are $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . All other parameters (includ-
1391 ing the oscillation parameters this fit is insensitive to) are deemed nuisance
1392 parameters. To generate these projections, the posterior distribution is integrated
1393 over all nuisance parameters. This is called marginalisation. This technique
1394 also explains why it is acceptable to neglect the normalisation constant of the
1395 posterior distribution, which was discussed in section 4.1.

1396 A simple example of the marginalisation technique is to imagine the scenario
1397 where two coins are flipped. To determine the probability that the first coin
1398 returned a ‘head’, the exact result of the second coin flip is disregarded and
1399 simply integrated over. For the parameters of interest, $\vec{\theta}_i$, the marginalised

1400 posterior is calculated by integrating over the nuisance parameters, $\vec{\theta}_n$. In this
1401 case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.7)$$

1402 Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and
1403 correlations between the parameters of interest and the nuisance parameters.
1404 In practice, this is simply taking the one or two-dimensional projection of the
1405 multi-dimensional probability distribution.

1406 While in principle an easy solution to a complex problem, correlations be-
1407 tween the interesting and nuisance parameters can bias the marginalised results.
1408 A similar effect is found when the parameters being marginalised over have
1409 non-Gaussian probability distributions. For example, Figure 4.5 highlights
1410 the marginalisation bias in the probability distribution found for a parameter
1411 when requiring a correlated parameter to have a positive value. Due to the
1412 complex nature of the oscillation parameter fit presented in this thesis, there
1413 are correlations occurring between the oscillation parameters of interest and the
1414 other nuisance parameters included in the fit.

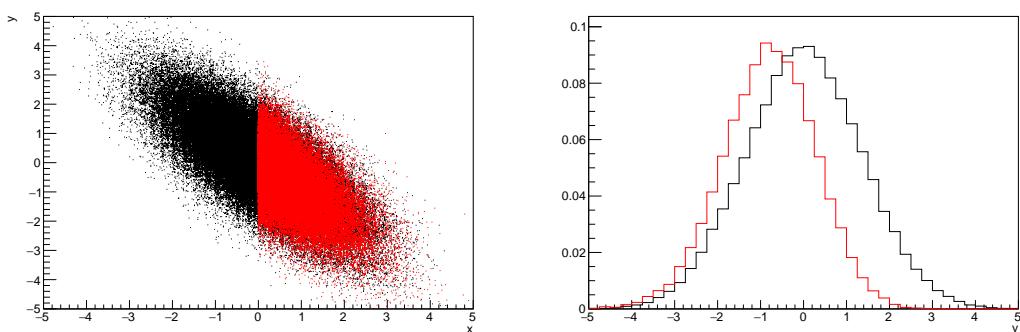


Figure 4.5: Left: The two-dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two-dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

4.3.2 Parameter Estimation and Credible Intervals

The purpose of this analysis is to determine the best fit values for the oscillation parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . The posterior probability density, taken from the output MCMC chain, is binned in these parameters. The parameter best-fit point is then taken to be the value that has the highest posterior probability. This is performed in both one and two-dimensional projections.

However, the single best-fit point in a given parameter is not of much use on its own. The uncertainty on the best-fit point must also be presented using credible intervals. The definition of the 1σ credible interval is that there is 68% belief that the parameter is within those bounds. For a more generalised definition, the credible interval is the region, R , of the posterior distribution that contains a specific fraction of the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha. \quad (4.8)$$

Where θ is the parameter being evaluated. This technique then calculates the $\alpha \times 100\%$ credible interval.

In practice, this analysis uses the highest posterior density (HPD) credible intervals which are calculated through the following method. First, the probability distribution is area-normalised such that it has an integrated area equal to 1.0. The bins of probability are then summed from the highest to lowest until the sum exceeds the 1σ level (0.68 in this example). This process is repeated for a range of credible intervals, notably the 1σ , 2σ and 3σ along with other levels where the critical values for each level can be found in [72]. This process can be repeated for the two-dimensional probability distributions by creating two-dimensional contours rather than one-dimensional credible intervals.

¹⁴³⁹ 4.3.3 Bayesian Model Comparisons

¹⁴⁴⁰ Due to the matter resonance, this analysis has some sensitivity to the mass
¹⁴⁴¹ hierarchy of neutrino states (whether Δm_{32}^2 is positive or negative) and the
¹⁴⁴² octant of $\sin^2(\theta_{23})$. The Bayesian approach utilised within this analysis gives an
¹⁴⁴³ intuitive method of model comparison by determining which hypothesis is most
¹⁴⁴⁴ favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal
¹⁴⁴⁵ hierarchy, NH , and inverted hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

¹⁴⁴⁶ The middle term defines the Bayes factor, $B(NH/IH)$, which is a data-driven
¹⁴⁴⁷ interpretation of how strong the data prefers one hierarchy to the other. For this
¹⁴⁴⁸ analysis, equal priors on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) =$
¹⁴⁴⁹ $P(\vec{\theta}_{IH}) = 0.5$). In practice, the MCMC chain proposes a value of $|\Delta m_{32}^2|$ and
¹⁴⁵⁰ then applies a 50% probability that the value is sign flipped. Consequently,
¹⁴⁵¹ the Bayes factor can be calculated from the ratio of the probability density in
¹⁴⁵² either hypothesis. This equates to counting the number of steps taken in the
¹⁴⁵³ normal and inverted hierarchies and taking the ratio. The same approach can be
¹⁴⁵⁴ taken to compare the upper octant (UO) compared to the lower octant (LO)
¹⁴⁵⁵ hypothesis of $\sin^2(\theta_{23})$.

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [130]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

¹⁴⁵⁶ Whilst the value of the Bayes factor should always be shown, the Jeffreys scale
¹⁴⁵⁷ [130] (highlighted in Table 4.1) gives an indication of the strength of preference

¹⁴⁵⁸ for one model compared to the other. Other interpretations of the strength of
¹⁴⁵⁹ preference of a model exist, e.g. the Kass and Raferty Scale [131].

¹⁴⁶⁰ 4.3.4 Comparison of MCMC Output to Expectation

¹⁴⁶¹ To ensure the fit is performing well, a best-fit spectrum is produced using the
¹⁴⁶² posterior probability distribution and compared with the data, allowing easy
¹⁴⁶³ by-eye comparisons to be made. A simple method of doing this is to perform a
¹⁴⁶⁴ comparison in the fitting parameters (e.g. the reconstructed neutrino energy for
¹⁴⁶⁵ T2K far detector beam samples) of the spectra generated by the MCMC chain to
¹⁴⁶⁶ ‘data’. This ‘data’ could be true data or some variation of Monte Carlo prediction.
¹⁴⁶⁷ This allows easy comparison of the MCMC probability distribution to the data. To
¹⁴⁶⁸ perform this, N steps from the post-burnin MCMC chain are randomly selected.
¹⁴⁶⁹ From these, the Monte Carlo prediction at each step is generated by reweighting
¹⁴⁷⁰ the model parameters to the values specified at that step. Due to the probability
¹⁴⁷¹ density being directly correlated with the density of steps in a certain region,
¹⁴⁷² parameter values close to the best fit value are most likely to be selected.

¹⁴⁷³ In practice, for each bin of the fitting parameters has a probability distribution
¹⁴⁷⁴ of event rates, with one entry per sampled MCMC step. This distribution is
¹⁴⁷⁵ binned where the bin with the highest probability is selected as the mean and an
¹⁴⁷⁶ error on the width of this probability distribution is calculated using the HPD
¹⁴⁷⁷ approach highlighted in subsection 4.3.2. Consequently, the best fit distribution in
¹⁴⁷⁸ the fit parameter is not necessarily that which would be attained by reweighting
¹⁴⁷⁹ the Monte Carlo prediction to the most probable parameter values.

¹⁴⁸⁰ A similar study can be performed to illustrate the freedom of the model
¹⁴⁸¹ parameter space prior to the fit. This can be done by throwing parameter values
¹⁴⁸² from the prior uncertainty of each parameter.

5

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Simulation, Reconstruction, and Event Reduction

1486 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1487 neutrino spectrum at the far detector is required. This includes modeling the
1488 flux generation, neutrino interactions, and detector effects. All of the simulation
1489 packages required to do this are briefly described in section 5.1. The reconstruc-
1490 tion of neutrino events in the far detector, including the `fitQun` algorithm, is
1491 documented in section 5.2. This also includes data quality checks of the SK-V
1492 data which the author performed for the T2K oscillation analysis presented at the
1493 Neutrino 2020 conference [2]. Finally, section 5.3 describes the steps taken in the
1494 SK detector to trigger on events of interest whilst removing the comparatively
1495 large rate of cosmic ray muon events.

1496 5.1 Simulation

1497 In order to generate a Monte Carlo prediction of the expected event rate at
1498 the far detector, all the processes in the beam and atmospheric fluxes, neutrino
1499 interaction, and detector need to be modeled.

1500 5.1.1 Neutrino Flux

1501 The beamline simulation consists of three distinct parts: the initial hadron interaction modeled by FLUKA [132], the target station geometry and particle tracking performed by JNUBEAM, [38, 133] and any hadronic re-interactions simulated by GCALOR [134]. The primary hadronic interactions are $O(10)\text{GeV}$, where FLUKA matches external cross-section data better than GCALOR [135]. However, FLUKA is not very adaptable so a small simulation is built to model the interactions in the target and the output is then passed to JNUBEAM and GCALOR for propagation. The hadronic interactions are tuned to data from the NA61/SHINE [136–138] and HARP [139] experiments. The tuning is done by reweighting the FLUKA and GCALOR predictions to match the external data multiplicity and cross-section measurements, based on final state particle kinematics [135]. The culmination of this simulation package generates the predicted flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.6.

1514 The atmospheric neutrino flux is simulated by the HKKM model [50, 52]. The primary cosmic ray flux is tuned to AMS [140] and BESS [141] data assuming the US-standard atmosphere '76 [142] density profile and includes geomagnetic field effects. The primary cosmic rays interact to generate pions and muons. The interaction of these secondary particles to generate neutrinos is handled by DPMJET-III [143] for energies above 32GeV and JAM [52, 144] for energies below that value [48]. These hadronic interactions are tuned to BESS and L3 data [145, 146] using the same methodology as the tuning of the beamline simulation. The energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions and kaons can travel further than their vertically-going counterparts resulting in a larger probability of decaying to neutrinos. The symmetry is broken in lower-energy neutrinos due to geomagnetic effects, which modify the track of the primary cosmic rays. Updates to the HKKM model are currently ongoing [147].

5.1.2 Neutrino Interaction

Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [148, 149] models the interactions of the neutrinos in the detectors. For the purposes of this analysis, quasi-elastic (QE), meson exchange (MEC), single meson production (PROD), coherent pion production (COH), and deep inelastic scattering (DIS) interactions are simulated. These interaction categories can be further broken down by whether they were propagated via a W^\pm boson in Charged Current (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC interactions have a charged lepton in the final state, which can be flavour-tagged in reconstruction to determine the flavour of the neutrino. In contrast, NC interactions have a neutrino in the final state so no flavour information can be determined from the observables left in the detector after an interaction. This is the reason why neutrinos that interact through NC modes are assumed to not oscillate within this analysis. Both CC and NC interactions are modeled for all the above interaction categories, other than MEC interactions which are only modeled for CC events.

As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of neutrino interactions around $E_\nu \sim 0.5\text{GeV}$. The NEUT implementation adopts the Llewellyn Smith [150] model for neutrino-nucleus interactions, where the nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is approximated by a spectral-function [151] model that simulates the effects of Fermi momentum and Pauli blocking. The cross-section of QE interactions is controlled by vector and axial-vector form factors parameterised by the BBBA05 [152] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data [153], respectively. NEUT implements the Valencia [154] model to simulate MEC events, where two nucleons and two holes in the nuclear target are produced (often called 2p2h interactions).

For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant. These predominantly produce charged and neutral pions although γ , kaon,

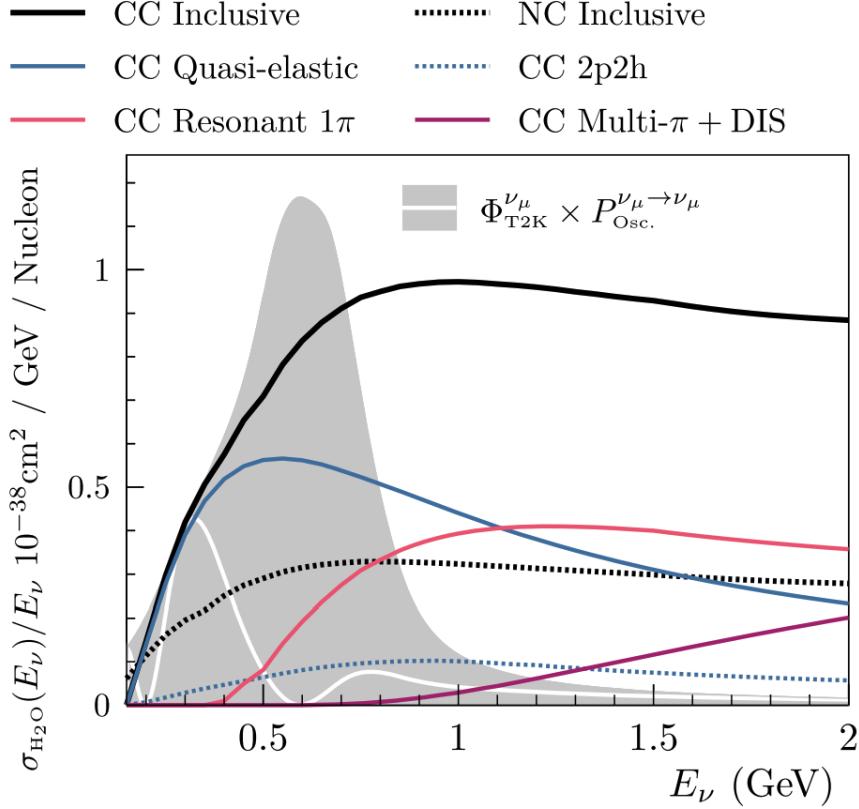


Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [148].

and η production is also considered. To simulate these interactions, the Berger-Sehgal [155] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [156] model to simulate these COH interactions.

DIS and multi- π producing interactions become the most dominant for energies $> O(5)\text{GeV}$. PYTHIA [157] is used to simulate any interaction with invariant mass $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces at least two mesons but has $W < 2\text{GeV}/c^2$, the

₁₅₆₉ Bronner model is used [158]. Both of these models use Parton distribution
₁₅₇₀ functions based on the Bodek-Yang model [159–161].

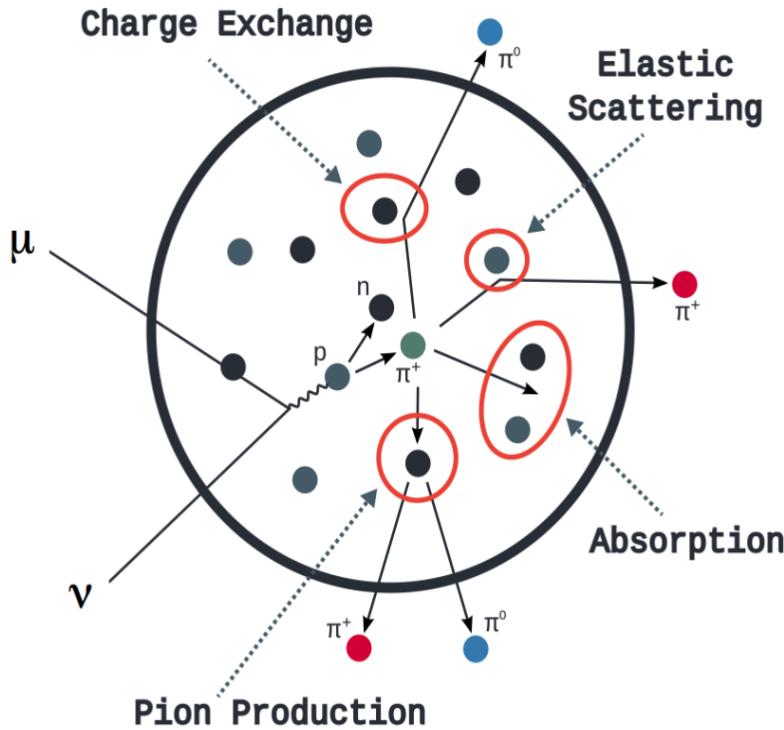


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [162].

₁₅₇₁ Any pion that is produced within the nucleus can re-interact through final
₁₅₇₂ state interactions before it exits, as illustrated by the scattering, absorption,
₁₅₇₃ production, and exchange interactions in Figure 5.2. These re-interactions alter
₁₅₇₄ the observable particles within the detector. For instance, if the charged pion
₁₅₇₅ from a CC PROD interaction is absorbed, the observables would mimic a CC QE
₁₅₇₆ interaction. To simulate these effects, NEUT uses a semi-classical intranuclear
₁₅₇₇ cascade model [148]. This cascade functions by stepping the pion through the
₁₅₇₈ nucleus in fixed-length steps equivalent to $dx = R_N/100$, where R_N is the radius
₁₅₇₉ of the nucleus. At each step, the simulation allows the pion to interact through
₁₅₈₀ scattering, charged exchange, absorption, or production with an interaction-
₁₅₈₁ dependent probability calculated from a fit to external data [163]. This cascade
₁₅₈₂ continues until the pion is absorbed or exits the nucleus.

1583 5.1.3 Detector

1584 Once the final state particle kinematics have been determined by NEUT, they
1585 are passed into the detector simulation. The near detectors, ND280 and INGRID,
1586 are simulated using a GEANT4 package [42, 164] to simulate the detector geom-
1587 etry, particle tracking, and energy deposition. The response of the detectors is
1588 simulated using the elecSim package [42].

1589 The far detector simulation is based upon the original Kamiokande experi-
1590 ment software which uses the GEANT3-based SKDETSIM [42, 165] package. This
1591 simulates the interactions of particles in the water as well as Cherenkov light
1592 production. The water quality and PMT calibration measurements detailed in
1593 subsection 3.1.2 are also used within this simulation to make accurate predictions
1594 of the detector response.

1595 Any event which generates optical photons that occurs in SK will be observed
1596 by the PMT array, where each PMT records the time and accumulated charge.
1597 This recorded information is shown in event displays similar to those illustrated
1598 in Figure 5.3 for simulated Monte Carlo events. To be useful for physics analyses,
1599 this series of PMT hit information needs to be reconstructed to determine the
1600 number and identity of particles and their kinematics (or track parameters): four-
1601 vertex, direction, and momentum. The reconstruction uses the fact that the charge
1602 and timing distribution of photons generated by a particular particle in an event is
1603 dependent upon its initial kinematics. Electron and muon rings are distinguished
1604 by their “fuzziness”. Muons are heavier and less affected by scattering or
1605 showering meaning they typically produce “crisp” rings. Electrons are more
1606 likely to interact via electromagnetic showering or scattering which results in
1607 larger variations of their direction from the initial direction. Consequently,
1608 electrons typically produce “fuzzier” rings compared to muons.

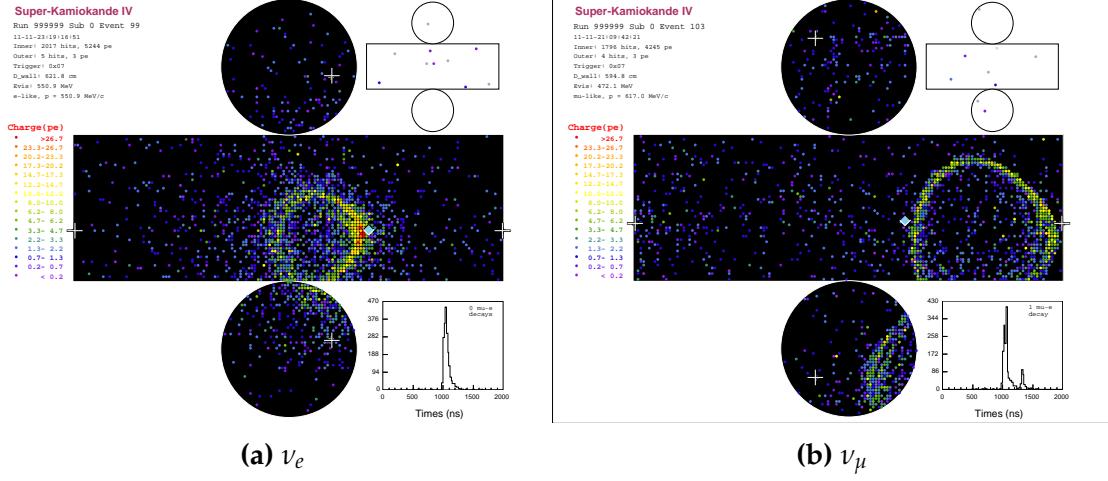


Figure 5.3: Event displays from Monte Carlo simulation at Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [166].

5.2 Event Reconstruction at SK

For the purposes of this analysis, the `fitQun` reconstruction algorithm [167] is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by maximising a likelihood function (or minimising a log-likelihood function) which includes information from PMTs which were hit and those that were not hit. The `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [168].

The `fitQun` algorithm improves upon the previous `APFit` algorithm [169] which has been used for many previous SK analyses. `APFit` fits the vertex from timing information and then fits the direction of the particle from PMT hits within a 43 deg Cherenkov cone (assuming an ultra-relativistic particle) using a fitting estimator. A Hough transformation is used to find the radius of a ring (related to the momentum through Equation 3.2) as well as the number of rings contained within the event. The analysis presented here uses the `fitQun` algorithm as it improves both the accuracy of the fit parameters and the rejection of neutral

1626 current π^0 events as compared to APFit [170, 171].

1627 Any event in SK can consist of prompt (or primary) and decay (or secondary)
1628 particles. For example, a charged current muon neutrino interaction can gen-
1629 erate two particles that have the potential of generating Cherenkov photons
1630 (assuming the proton is below the Cherenkov threshold): the prompt muon,
1631 and the secondary decay-electron from the muon, approximately $2\mu\text{s}$ later. To
1632 reconstruct all particles within an event, it is divided into time clusters which are
1633 called “subevents”. Subevents after the primary subevent are considered to
1634 be decay electrons.

1635 The main steps of the `fitQun` reconstruction algorithm are:

- 1636 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit
1637 metric based on PMT hit times
- 1638 • **Peak finding:** The initial time of each subevent is determined by clustering
1639 events by time residuals
- 1640 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,
1641 a maximum likelihood technique searches for a single particle generating
1642 light. Electron, muon, charged pion, and proton hypotheses are considered
- 1643 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple
1644 light-producing particles are considered using the same maximum likeli-
1645 hood technique. Electron-like or charged pion-like rings are added until
1646 the likelihood stops improving

1647 To find all the subevents in an event, a vertex goodness metric is calculated
1648 for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right), \quad (5.1)$$

1649 where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n, \quad (5.2)$$

1650 is the residual hit time. It is the difference in time between the PMT hit time
 1651 t^i , of the i^{th} PMT, and the expected time of the PMT hit if the photon was at
 1652 the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in water
 1653 and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1654 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends
 1655 to zero resulting in subevents appearing as spikes in the goodness metric. The
 1656 proposed fit vertex and time are grid-scanned, and the values which maximise
 1657 the goodness metric are selected as the “pre-fit vertex”. Whilst this predicts a
 1658 vertex for use in the clustering algorithm, the final vertex is fit using the higher-
 1659 precision maximum likelihood method described below.

1660 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1661 a function of t to determine the number of subevents. A peak-finding algorithm
 1662 is then used on the goodness metric, requiring the goodness metric to exceed
 1663 some threshold and drop below a reduced threshold before any subsequent
 1664 additional peaks are considered. The thresholds are set such that the rate of
 1665 false peak finding is minimised while still attaining good data to Monte Carlo
 1666 agreement. To improve performance, the pre-fit vertex for each delayed subevent
 1667 is re-calculated after PMT hits from the previous subevent are masked. This
 1668 improves the decay-electron tagging performance. Once all subevents have
 1669 been determined, the time window around each subevent is then defined by the
 1670 earliest and latest time which satisfies $-180 < T_{Res}^i < 800\text{ns}$. The subevents and
 1671 associated time windows are then used as seeds for further reconstruction.

1672 For a given subevent, the `fitQun` algorithm constructs a likelihood based on
 1673 the accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}). \quad (5.3)$$

1674 Where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and
 1675 momenta, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the proba-
 1676 bility of the i^{th} tube to not register a hit given the track parameters and particle
 1677 hypothesis. The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$,
 1678 represents the probability density function of observing charge q_i and time t_i on
 1679 the i^{th} PMT given the specified track parameters and particle hypothesis.

1680 The predicted charge is calculated based on contributions from both the
 1681 direct light and the scattered light. The direct light contribution is determined
 1682 based on the integration of the Cherenkov photon profile along the track. PMT
 1683 angular acceptance, water quality, and calibration measurements discussed in
 1684 subsection 3.1.2 are included to accurately predict the charge probability density
 1685 at each PMT. The scattered and reflected light is calculated in a similar way,
 1686 although it includes a scattering function that depends on the vertex of the
 1687 particle and the position of the PMT. The charge likelihood is calculated by
 1688 comparing the prediction to the observed charge in the PMT which is tuned
 1689 to the PMT simulation.

1690 The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} ,
 1691 and time t of the track as well as the particle hypothesis. The expected time
 1692 for PMT hits is calculated by assuming unscattered photons being emitted from
 1693 the midpoint of the track, S_{mid} ,

$$t_{\text{exp}}^i = t + S_{\text{mid}}/c + |R_{\text{PMT}}^i - \vec{x} - S_{\text{mid}}\vec{d}|/c_n, \quad (5.4)$$

1694 where c is the speed of light in a vacuum. The time likelihood is then expressed
 1695 in terms of the residual difference between the PMT hit time and the expected
 1696 hit time, $t_{\text{Res}}^i = t^i - t_{\text{exp}}^i$. The particle hypothesis and momentum also affect the
 1697 Cherenkov photon distribution. These parameters modify the shape of the time
 1698 likelihood density since in reality not all photons are emitted at the midpoint of
 1699 the track. As with the charge likelihood, the contributions from both the direct
 1700 and scattered light to the time likelihood density are calculated separately, which
 1701 are both calculated from particle gun Monte Carlo studies.

1702 The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined
 1703 as the best-fit parameters. In practice MINUIT [172] is used to minimise the value
 1704 of $-\ln L(\Gamma, \vec{\theta})$. The `fiTQun` algorithm considers an electron-like, muon-like, and
 1705 charged pion-like hypothesis for events with a single final state particle, denoted
 1706 “single-ring events”. The particle’s identity is determined by taking the ratio of
 1707 the likelihood of each of the hypotheses. For instance, electrons and muons are
 1708 distinguished by considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison
 1709 to the reconstructed momentum of the electron hypothesis, as illustrated by
 1710 Figure 5.4. The coefficients of the discriminator between electron-like and muon-
 1711 like events are determined from Monte Carlo studies [167]. Similar distributions
 1712 exist for distinguishing electron-like events from π^0 -like events, and muon-like
 1713 events from pion-like events. The cuts are defined as,

$$\begin{aligned} \text{Electron/Muon} : & \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 : & \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} : & \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

1714 as taken from [173], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the
 1715 single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed
 1716 invariant mass of the two photons emitted from π^0 decay. Typically, the distance
 1717 between a particular entry in these two-dimensional distributions and the cut-line
 1718 is termed the PID parameter and is illustrated in Figure 5.5.

1719 The `fiTQun` algorithm also considers a π^0 hypothesis. To do this, it performs
 1720 a fit looking for two standard electron-hypothesis tracks which point to the
 1721 same four-vertex. This assumes the electron tracks are generated from photon-
 1722 conversion so the electron tracks actually appear offset from the proposed π^0
 1723 vertex. For these fits, the conversion length, direction, and momentum of each
 1724 photon are also considered as track parameters which are then fit in the same
 1725 methodology as the standard single-ring hypotheses.

1726 Whilst lower energy events are predominantly single-ring events, higher
 1727 energy neutrino events can generate final states with multiple particles which

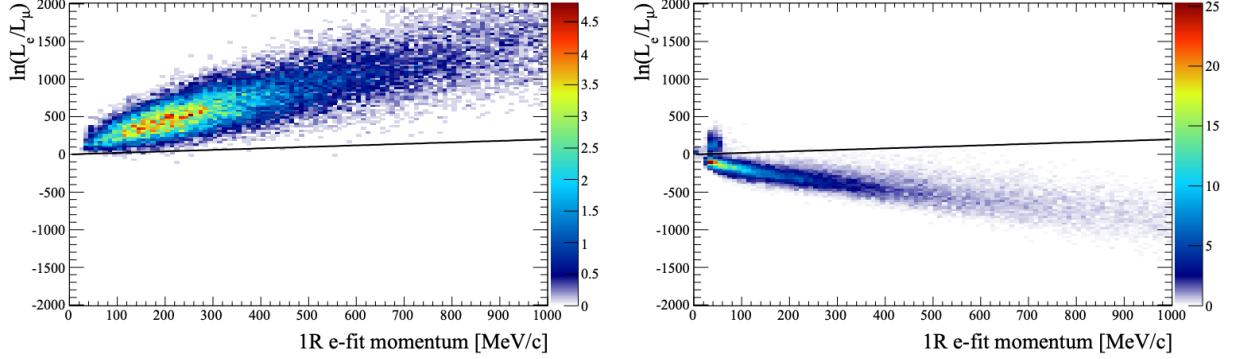


Figure 5.4: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, with coefficients obtained from Monte Carlo studies. Figures from [167].

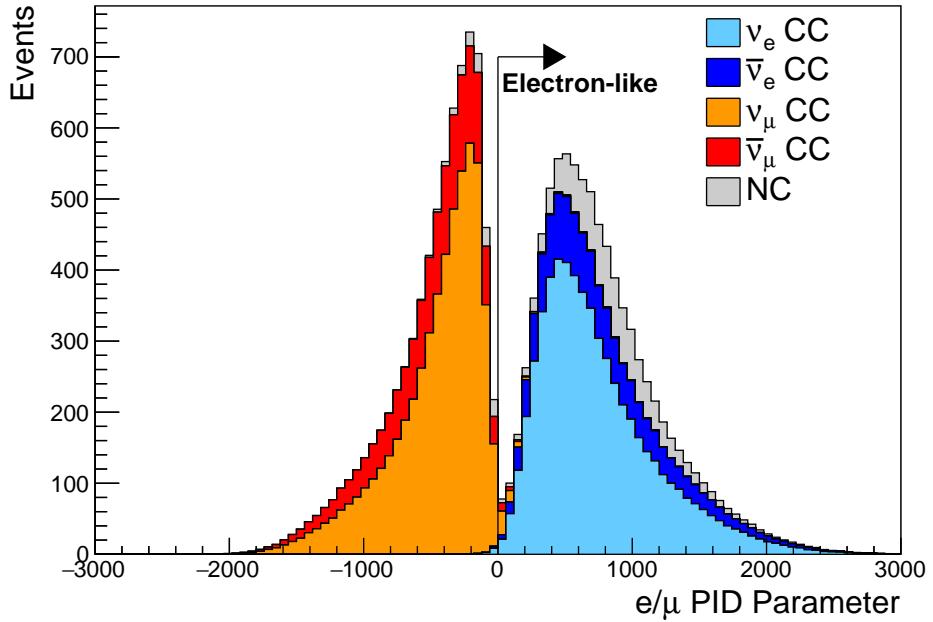


Figure 5.5: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1728 generate Cherenkov photons. These “multi-ring” hypotheses are also considered
 1729 in the `fitQun` algorithm. When calculating the charge likelihood density, the
 1730 predicted charge associated with each ring is calculated separately and then
 1731 summed to calculate the total accumulated charge on each PMT. Similarly, the
 1732 time likelihood for the multi-ring hypothesis is calculated assuming each ring

is independent. Each track is time-ordered based on the time of flight from the center of the track to the PMT and the direct light from any ring incident on the PMT is assumed to arrive before any scattered light. To reduce computational resource usage, the multi-ring fits only consider electron-like and charged pion-like rings as the pion fit can be used as a proxy for a muon fit due to their similar mass. Due to the pions ability to interact through the strong force, they are more likely to hard-scatter. That means a single charged pion can produce multiple rings in different directions. There is an additional freedom, the fraction of kinetic energy lost in a single ring segment, which is added into the `fitQun` pion fit to cover this difference. Pion and muon rings are indistinguishable when this fraction tends to unity.

Multi-ring fits proceed by proposing another ring to the previous fit and then fitting the parameters in the method described above. Typically, multi-ring fits have the largest likelihood because of the additional degrees of freedom introduced. A likelihood value is calculated for the n -ring and $(n + 1)$ -ring hypotheses, where the additional ring is only included if the likelihood value is above 9.35, based on Monte Carlo studies in [174].

5.2.1 Validation of Reconstruction in SK-V

Understanding how the modelling of the detector conditions and stability effects the reconstruction is critical for ensuring accurate measurements. It is important to note that the detector systematics used in the 2020 T2K-only [2] oscillation analysis are determined using data-to-Monte Carlo comparisons of the SK-IV data [175]. Due to tank-open maintenance occurring between SK-IV and SK-V, the dark rate of each PMT was observed to increase in SK-V due to light exposure for a significant time during the repairs. This increase can be seen in Figure 5.6. Run-10 of the T2K experiment was conducted in the SK-V period, so the consistency of SK-IV and SK-V data needs to be studied to determine whether the SK-IV-defined systematics can be applied to the run-10 data. Consequently, the author of this thesis assessed the quality of `fitQun` event reconstruction for SK-V data.

This comparison study was performed using the stopping muon data set for both the SK-IV and SK-V periods. This data sample is used due to the high rate of interactions ($O(200)$ events per hour) as well as having similar energies to muons from CCQE ν_μ interactions from beam interactions. The rate of cosmic muons does depend on the solar activity cycle [176] but has been neglected in this comparison study. This is because the shape of the distributions is most important for the purposes of being compared to the detector systematics. The SK-IV and SK-V data samples consist of 2398.42 and 626.719 hours of data which equates to 686k and 192k events respectively. These samples do not correspond to the full data sets of either period but do contain enough events to be systematics limited rather than statistics limited.

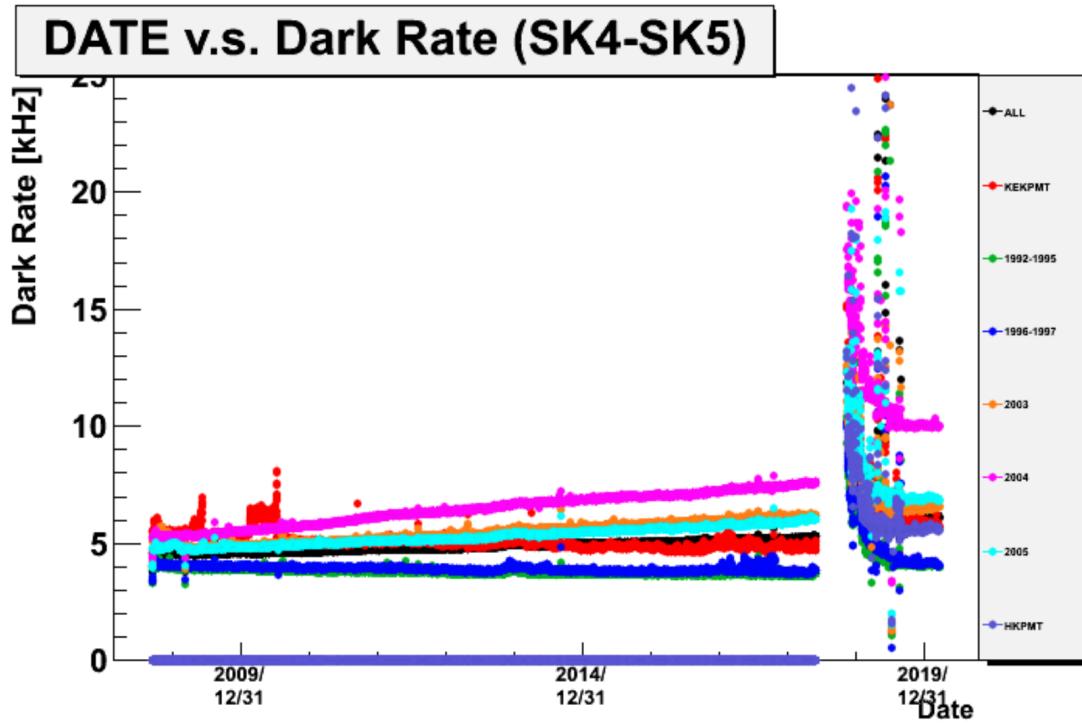


Figure 5.6: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [175].

The predicted charge calculated in the `fitQun` algorithm includes a contribution from the photoelectron emission due to dark noise. Therefore, the increase

1775 in the SK-V dark rate needs to be accounted for. In practice, the average dark rate
 1776 in each SK period is calculated and used as an input in the reconstruction. This is
 1777 calculated by averaging the dark rate per run for each period separately, using
 1778 the calibration measurements detailed in subsection 3.1.2. The average dark rate
 1779 from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The
 1780 charges associated with the muon and decay electron subevents are illustrated in
 1781 Figure 5.7. The photoelectron emission from dark noise is more significant for
 1782 events that have lower energy. This is because this contribution becomes more
 1783 comparable to the number of photoelectrons emitted from incident photons in
 1784 lower-energy events. This behaviour is observed in the data, where the charge
 1785 deposited by the muon subevent is mostly unaffected by the increase in dark rate,
 1786 whilst the charge associated with the decay-electron is clearly affected.

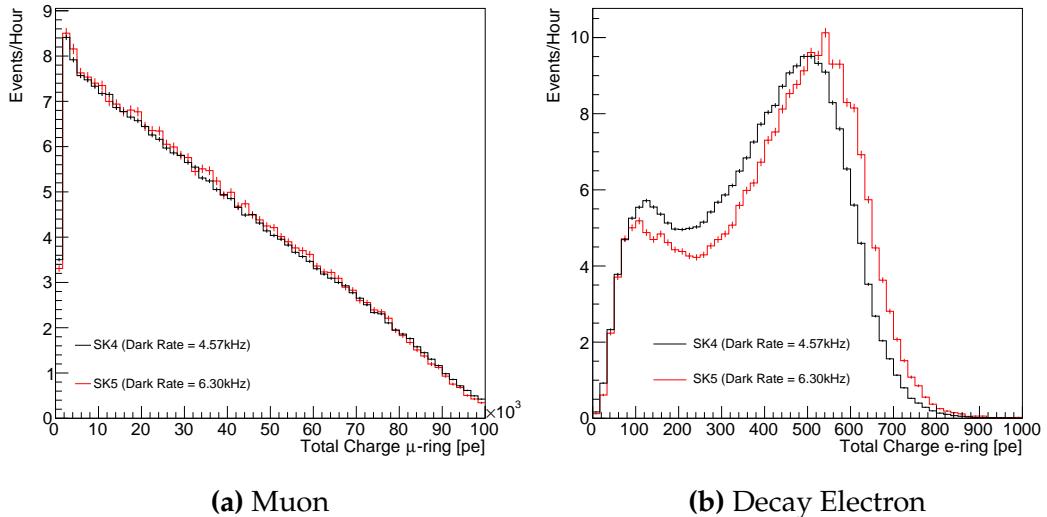


Figure 5.7: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent (left) and the associated decay electron subevent (right).

1787 The energy scale systematic is estimated from data-to-Monte Carlo differences
 1788 in the stopping muon sample in [59] and found to be 2.1%. To determine the
 1789 consistency of SK-IV and SK-V with respect to the energy scale systematic, the
 1790 muon momentum distribution is compared between the two SK periods. As
 1791 the total number of Cherenkov photons is integrated across the track length,

₁₇₉₂ the reconstructed momentum divided by track length (or range) is compared
₁₇₉₃ between SK-IV and SK-V as illustrated in Figure 5.8.

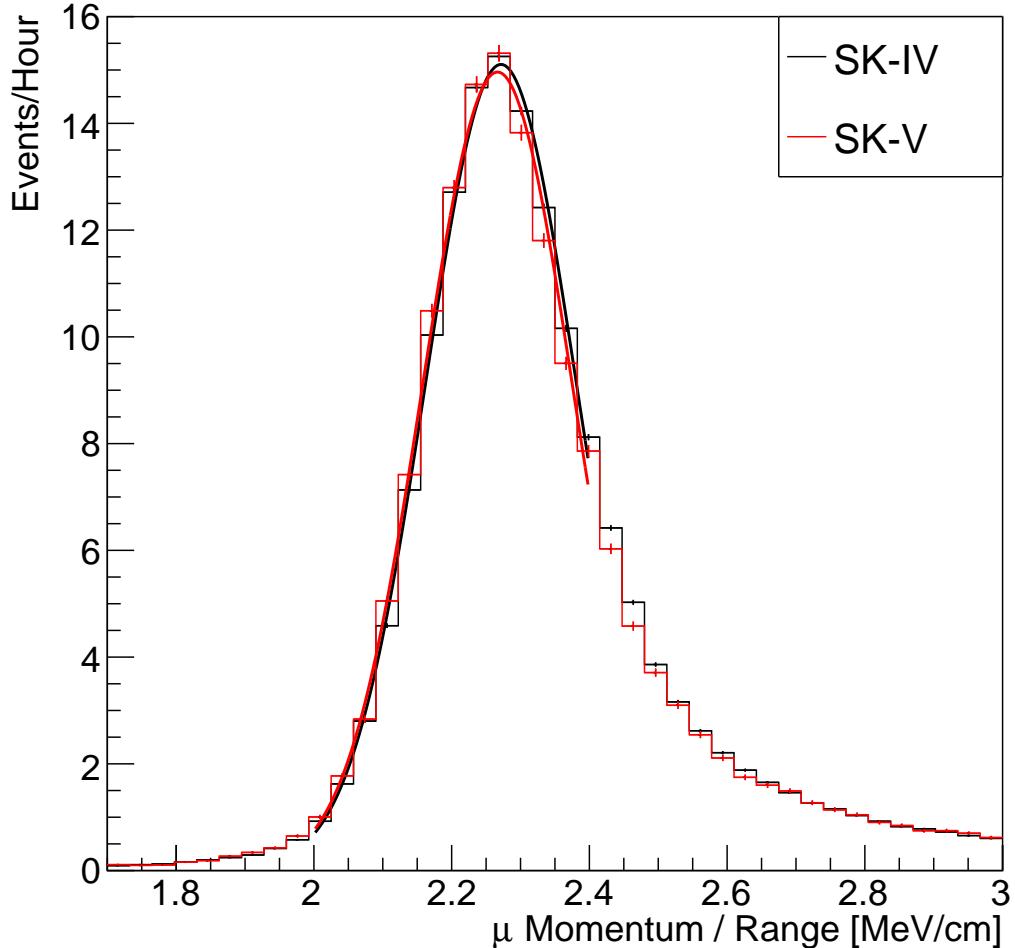


Figure 5.8: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range [2.0, 2.4] MeV/cm and illustrated as the solid curve.

₁₇₉₄ The consistency between these muon distributions has been computed in two
₁₇₉₅ ways. Firstly, a Gaussian is fit to the peak of each distribution separately, whose
₁₇₉₆ mean is found to be (2.272 ± 0.003) MeV/cm and (2.267 ± 0.006) MeV/cm for SK-
₁₇₉₇ IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of
₁₇₉₈ the Gaussian fits are consistent with the expected stopping power of a minimum

1799 ionising muon for a target material (water) with $Z/A \sim 0.5$ [177]. The second
 1800 consistency check is performed by introducing a nuisance parameter, α , which
 1801 modifies the SK-V distribution. The value of α which minimises the χ^2 value
 1802 between the SK-IV and SK-V is determined by scanning across a range of values.
 1803 This is repeated by applying the nuisance parameter as both a multiplicative
 1804 factor and an additive shift. The χ^2 distributions for different values of α is
 1805 illustrated in Figure 5.9. The values which minimise the χ^2 are found to be 0.0052
 1806 and 1.0024 for the additive and multiplicative implementations, respectively. No
 1807 evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic
 1808 has been found in the reconstructed momentum distribution of SK-IV and SK-V.

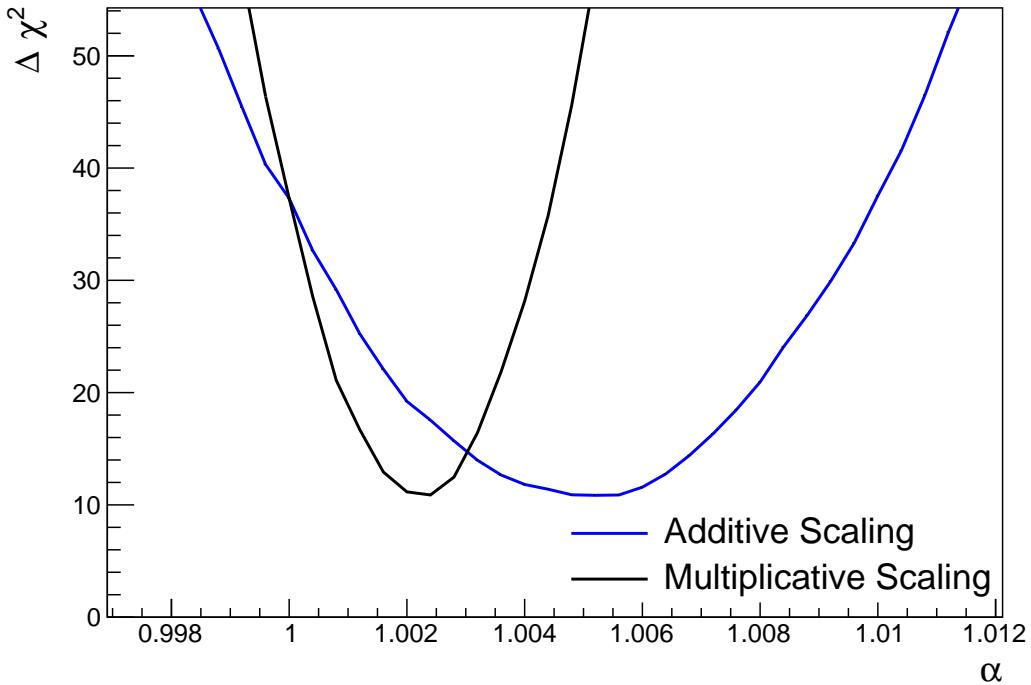


Figure 5.9: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1809 5.3 Event Reduction at SK

1810 In normal data-taking operations, the SK detector observes many background
 1811 events alongside the beam and atmospheric neutrino signal events of physics
 1812 interest for this thesis. Cosmic ray muons and flasher events, which are the spon-
 1813 taneous discharge of a given PMT, contribute the largest amount of background
 1814 events in the energy range relevant to this thesis. Therefore the data recorded
 1815 is reduced with the aim of removing these background events. The reduction
 1816 process is detailed in [3, 54] and briefly summarised below.

1817 Atmospheric neutrino events observed in the SK detector are categorised
 1818 into three different types of samples: fully contained (FC), partially contained
 1819 (PC) and up-going muon (Up- μ), using PMT hit signatures in the inner and
 1820 outer detector (ID and OD, respectively). To identify FC neutrino events, it is
 1821 required that the neutrino interacts inside the fiducial volume of the ID and that
 1822 no significant OD activity is observed. For this analysis, an event is defined to be
 1823 in the fiducial volume provided the event vertex is at least 0.5m away from the
 1824 ID walls. PC events have the same ID requirements but can have a larger signal
 1825 present inside the OD. Typically, only high energy muons from ν_μ interactions can
 1826 penetrate the ID wall. The Up- μ sample contains events where muons are created
 1827 from neutrino interactions in the OD water or rock below the tank. They then
 1828 propagate upwards through the detector. Downward-going muons generated
 1829 from neutrino interactions above the tank are neglected because of the difficulty
 1830 in separating their signature from the cosmic muon shower background. The
 1831 sample categories are visually depicted in Figure 5.10.

1832 Based on the event characteristics, as defined by the `fitQun` event reconstruc-
 1833 tion software, the FC events are categorised by

- 1834 • **Visible Energy:** equal to the sum of the reconstructed kinetic energy of
 1835 particles above the Cerenkov threshold for all rings present in the event.
 1836 The purpose is to separate events into sub-GeV and multi-GeV categories.

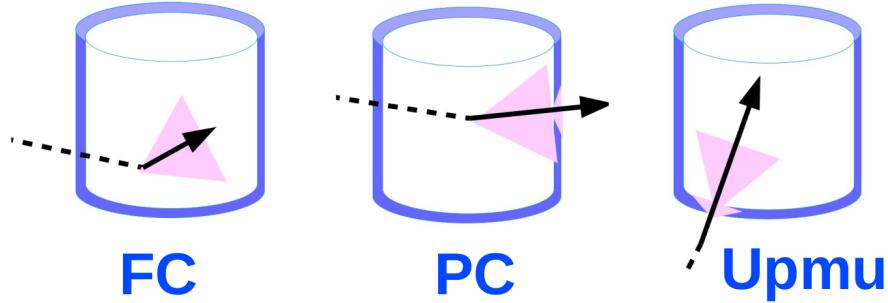


Figure 5.10: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC), and up-going muon ($\text{Up-}\mu$) samples included in this analysis.

- **Number of observed Cerenkov rings.** The purpose is to separate single-ring and multi-ring events, where single-ring events predominantly consist of quasi-elastic interactions and multi-ring events are typically resonant pion production or deep inelastic scattering events.

- **Particle identification parameter of the most energetic ring:** A value determined from the maximum likelihood value based on `fitQun`'s electron, muon, or pion hypothesis. The purpose is to separate electron-like and muon-like events.

- **Number of decay electrons:** The purpose is to separate quasi-elastic events (which have one decay electron emitted from the muon decay) and resonant pion production events (which have two decay electrons emitted from the muon and pion).

The PC and Up- μ categories are broken down into “through-going” and “stopping” samples depending on whether the muon leaves the detector. This is because the PC stopping events deposit the entire energy of the interaction into the detector, resulting in better reconstruction. The energy of events that exit the detector has to be estimated, with a typically worse resolution, which introduces much larger systematic uncertainties. Through-going Up- μ samples are further broken down by whether any hadronic showering was observed in the event which typically indicates DIS interactions. The expected neutrino energy for the different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events

¹⁸⁵⁸ peak around 0.7GeV and 3GeV respectively, with slightly different peak energies
¹⁸⁵⁹ for ν_e and ν_μ oscillation channels. PC and Up- μ are almost entirely comprised
¹⁸⁶⁰ of ν_μ events and peak around 7GeV and 100GeV, respectively.

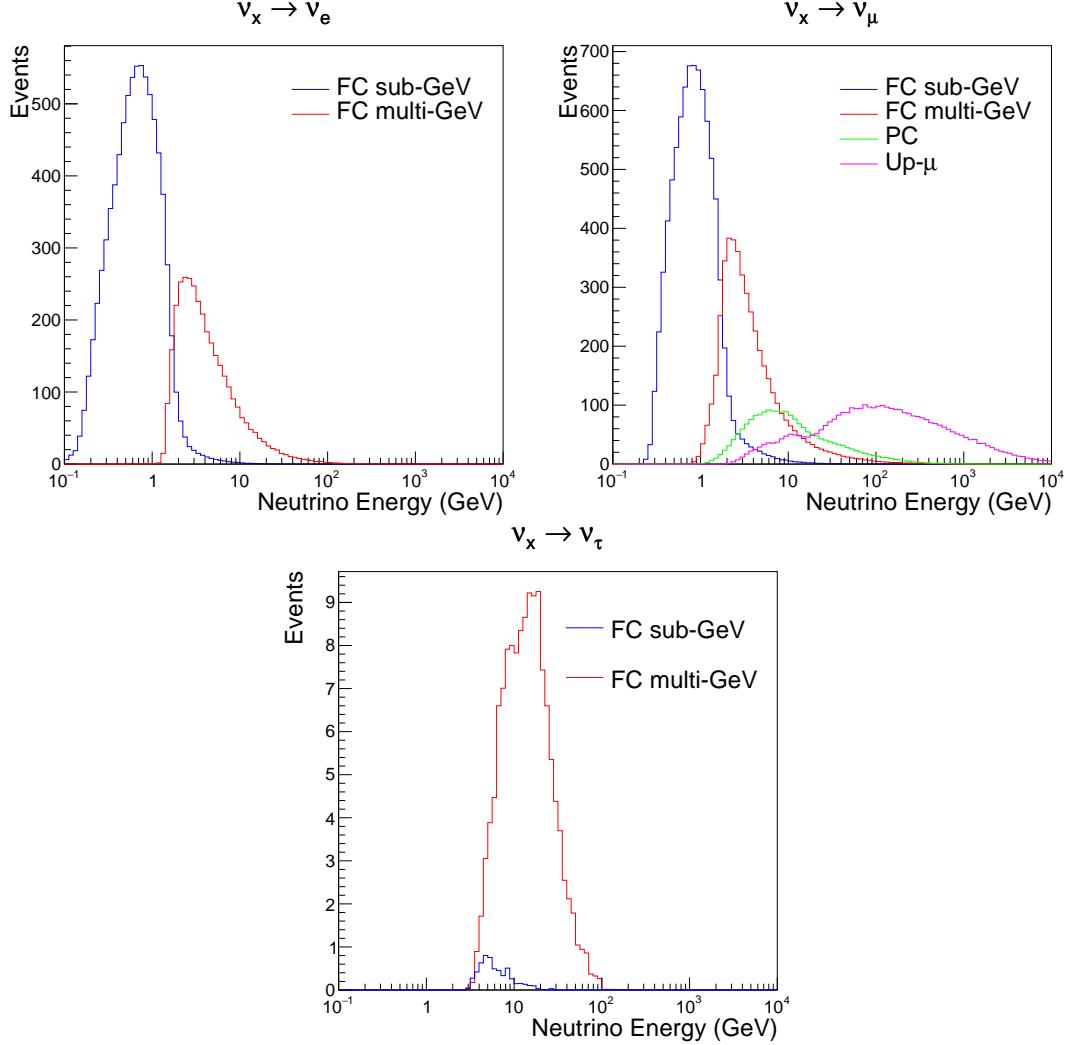


Figure 5.11: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). ν_x represents the flavours of neutrinos produced in the cosmic ray showers (electron and muon). Asimov A oscillation parameters are assumed (given in Table 2.2).

¹⁸⁶¹ The first two steps in the FC reconstruction remove the majority of cosmic
¹⁸⁶² ray muons by requiring a significant amount of ID activity compared to that
¹⁸⁶³ measured in the OD. Events that pass this cut are typically very high momentum
¹⁸⁶⁴ muons or events that leave very little activity in the OD. Consequently, a third

reduction step is then applied to select cosmic-ray muons that pass the initial reduction step. A purpose-built cosmic muon fitter is used to determine the entrance (or exit) position of the muon and a cut is applied to OD activity contained within 8m of this position. Flasher events are removed in the fourth reduction step which is based on the close proximity of PMT hits surrounding the PMT producing the flash. Events that pass all these reduction steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses information from the more precise fitter to repeat the previous two steps with tighter cuts. Muons below the Cherenkov threshold can not generate optical photons in the ID but the associated decay electron can due to its lower mass. These are the types of events targeted in the fifth reduction step. The final cuts require the event vertex to be within the fiducial volume (0.5m from the wall although the nominal distance is 2.0m), visible energy $E_{vis} > 30\text{MeV}$ and fewer than 16 hits within the higher energy OD cluster. The culmination of the fully contained reduction results in 8.09 events/day in the nominal fiducial volume [82]. The uncertainty in the reconstruction is calculated by comparing Monte Carlo prediction to data. The largest discrepancy is found to be 1.3% in the fourth reduction step.

The PC and Up- μ events are processed through their own reduction processes detailed in [54]. Both of these samples are reconstructed with the APFit algorithm rather than `f1TQun`. This is because the efficiency of reconstructing events that leave the detector has not been sufficiently studied for reliable systematic uncertainties with `f1TQun`. The PC and Up- μ samples acquire events at approximately 0.66 and 1.44 events/day.

Beam neutrinos events undergo the same reduction steps as FC events and are then subject to further cuts [178]. The GPS system that links the timing between the beam facility and SK needs to be operating correctly and there should be no activity within the detector in the previous $100\mu\text{s}$ before the trigger. The events then need to triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1894 The beam neutrino samples are not split by visible energy since their energy
 1895 range is smaller than the atmospheric neutrino events. Following the T2K
 1896 analysis in [2], only single-ring beam neutrino events are considered. Similar to
 1897 atmospheric event selection, the number of decay electrons is used as a proxy for
 1898 distinguishing CCQE and CCRES events. The expected neutrino energy, broken
 1899 down by the number of decay electrons, is given in Figure 5.12.

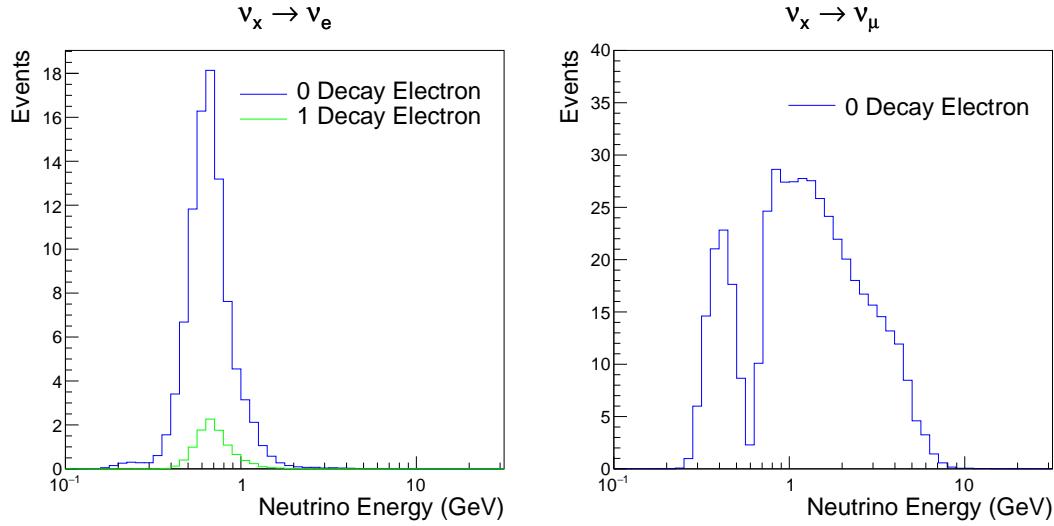


Figure 5.12: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

6

1900

1901

Sample Selections and Systematics

1902 The oscillation analysis presented within this thesis is built upon a simultaneous
1903 fit to atmospheric samples at SK, neutrino beam samples in the near detector,
1904 and beam samples at SK. This is the first simultaneous oscillation analysis of
1905 beam and atmospheric samples supported by the T2K and SK collaborations.
1906 Notably, the author of this thesis has been responsible for the building and
1907 developing the MaCh3 framework to support all sets of samples simultaneously.
1908 The definitions of the samples are documented in section 6.1, section 6.2, and
1909 section 6.3, respectively. The data collected and used within this analysis is
1910 detailed in Table 6.1. The near and far detector data corresponds to T2K runs
1911 2-9 and runs 1-10, respectively. The accumulated POT and beam power for runs
1912 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

1913 The difference in POT recorded at the near and far detector is due to the
 1914 difference in downtime. The SK detector is very stable with almost 100% of
 1915 data recorded during beam operation. Due to various technical and operational
 1916 issues, the downtime of the near detector is significantly higher due to its more
 1917 complex design and operating requirements.

1918 The systematic parameters invoked within the flux, detector, and interaction
 1919 models used within this analysis are documented in section 6.4. The standard
 1920 configuration of the joint beam and atmospheric data fit utilises far detector sys-
 1921 tematics provided in the official inputs from the two experiments. Additionally,
 1922 a correlated detector model which fits the parameters used in sample selections
 1923 to data has been developed and documented in subsection 6.4.5.

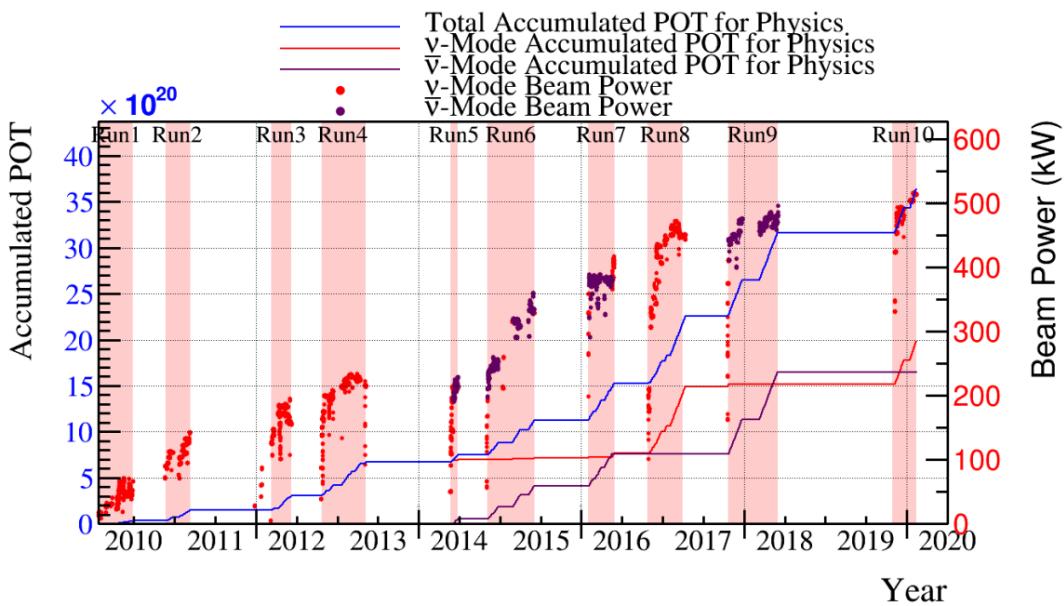


Figure 6.1: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

1924 6.1 Atmospheric Samples

1925 The atmospheric event selection follows the official SK-IV analysis presented
1926 in [3] and is documented below. The Monte Carlo prediction used within this
1927 analysis corresponds to 500 years worth of neutrino events, which is scaled down
1928 to match the SK-IV livetime of 3244.4 days.

1929 The fully contained (FC), partially contained (PC), and upward going muon
1930 events ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further
1931 broken down into different samples based on reconstruction information. This
1932 section details the samples used within this oscillation analysis, alongside the
1933 chosen binning.

1934 FC events are first separated by the visible energy deposited within the
1935 detector. This is calculated as the sum of the reconstructed kinetic energy
1936 above the Cherenkov threshold for all rings present in the event. Events are
1937 separated by whether they were above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates
1938 “subGeV” and “multiGeV” events. Typically, lower energy events consist of
1939 charged current quasi-elastic (CCQE) interactions which are better understood
1940 and simpler to reconstruct resulting in smaller systematic uncertainties. Events
1941 are further separated by the number of rings associated with the event due to
1942 similar reasoning. As the oscillation probability is dependant upon the flavour
1943 of neutrino, electron and muon events are separated using a similar likelihood
1944 method to that discussed in section 5.2. To reduce computational resources
1945 required for the reconstruction, only electron and pion hypotheses are considered
1946 so this separation cut depends on the ratio of the electron to pion likelihoods,
1947 $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1948 Charged current resonant pion production (CCRES) interactions generate a final-
1949 state pion. This can decay, mostly likely through a muon, into a decay electron.
1950 Therefore any electron-like event with one decay electron or muon-like event
1951 with two decay electrons was most likely produced by a CCRES interaction.
1952 Consequently, the number of decay electrons can be used to distinguish CCQE

¹⁹⁵³ and CCRES interaction modes. Ultimately, FC subGeV events are separated
¹⁹⁵⁴ into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

¹⁹⁵⁵ In addition to the cuts discussed above, multiGeV samples also have addi-
¹⁹⁵⁶ tional cuts to separate samples which target neutrino and antineutrino events.
¹⁹⁵⁷ As discussed in section 2.5, the matter resonance only occurs for neutrinos in the
¹⁹⁵⁸ normal hierarchy and antineutrinos in the inverted mass hierarchy. Therefore,
¹⁹⁵⁹ having flavour-enriched samples aids in the determination of the mass hierarchy.
¹⁹⁶⁰ For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

¹⁹⁶¹ The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be captured by an
¹⁹⁶² oxygen nucleus than the π^+ from ν_e interactions [179]. These pions then decay,
¹⁹⁶³ mostly through muons, to electrons. Therefore the number of tagged decay
¹⁹⁶⁴ electrons associated with an event gives an indication of whether the interaction
¹⁹⁶⁵ was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
¹⁹⁶⁶ The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
¹⁹⁶⁷ where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with
¹⁹⁶⁸ only 7% antineutrino background, the rest consisting of NC backgrounds.

1969 The number of decay electrons discriminator works reasonably well for single-
1970 ring events. However, this is not the case for multi-ring events. A multiGeV
1971 multiring electron-like (MME) likelihood cut was introduced in [180, 181]. This
1972 is a two-stage likelihood selection cut. Four observables are used in the first
1973 likelihood cut to distinguish $CC\nu_e$ and $CC\bar{\nu}_e$ events from background:

- 1974 • The number of decay electrons
1975 • The maximum distance between the vertex of the neutrino and the decay
1976 electrons
1977 • The energy deposited by the highest energy ring
1978 • The particle identification of that highest energy ring

1979 Background events consist of $CC\nu_\mu$ and NC interactions. Typically, the
1980 majority of the energy in these background events is carried by the hadronic
1981 system. Additionally, muons tend to travel further than the pions from $CC\nu_e$
1982 before decaying. Thus, the parameters used within the likelihood cut target these
1983 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring e -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring e -like events with one or more decay electrons
MultiGeV-mulike	Single ring μ -like events
MultiRing-elike-nue	Two or more ring events with leading energy e -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy e -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy μ -like ring and only requires $E_{vis} > 0.6\text{GeV}$
MultiRing-Other1	Two or more ring events with leading energy e -like ring and failed the MME likelihood cut

Table 6.3: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

1984 Neutrino and antineutrino events are then separated by a second likelihood
1985 method ($\nu/\bar{\nu}$ separation) detailed in [59]. This uses the number of decay electrons,

1986 the number of reconstructed rings, and the event’s transverse momentum. The
1987 last two parameters are used because higher-energy samples tend to have more
1988 pions produced above the Cherenkov threshold which results in more rings
1989 compared to an antineutrino interaction. Furthermore, the angular distribution
1990 also tends to be more forward peaked in antineutrino interactions as compared
1991 to neutrino interactions [3]. These FC multiGeV sample definitions are de-
1992 tailed in Table 6.3.

1993 The PC and up- μ samples are split by the amount of energy deposited within
1994 the outer detector, into “stopping” and “through-going” samples. If an event
1995 leaves the detector, the energy it takes with it has to be estimated which increases
1996 the systematic uncertainty compared to events entirely contained within the
1997 inner detector. This estimation is particularly poor at high energies, thus the
1998 up- μ through-going events are not binned in reconstructed momentum. The
1999 through-going up- μ are further separated by the presence of any electromagnetic
2000 showering in the event, as the assumption of non-showering muon does not give
2001 reliable reconstruction for these types of events [54]. In total, 13 FC, 2 PC, and
2002 3 up- μ atmospheric samples are included within this analysis.

2003 The atmospheric samples are binned in direct observables: reconstructed
2004 lepton momentum and direction, as given by Table 6.5. The distribution of
2005 the reconstructed lepton momentum (for samples that only have one bin in
2006 reconstructed zenith angle) and reconstructed direction for each atmospheric
2007 sample used within this analysis is illustrated in Figure 6.2.

2008 The reconstructed lepton momemtum, illustrated by interaction mode break-
2009 down, of some representative atmospheric samples is given in Figure 6.3. The
2010 equivalent distributions of all atmospheric samples used within this analy-
2011 sis can be found in [182]. The low energy samples tend to be dominated by
2012 the interaction mode they target (CCQE for SubGeV-elike-0dcy and CC1 π for
2013 SubGeV-elike-1dcy samples). The higher energy samples include much more
2014 CCOther interactions, especially at larger reconstructed lepton momentum.

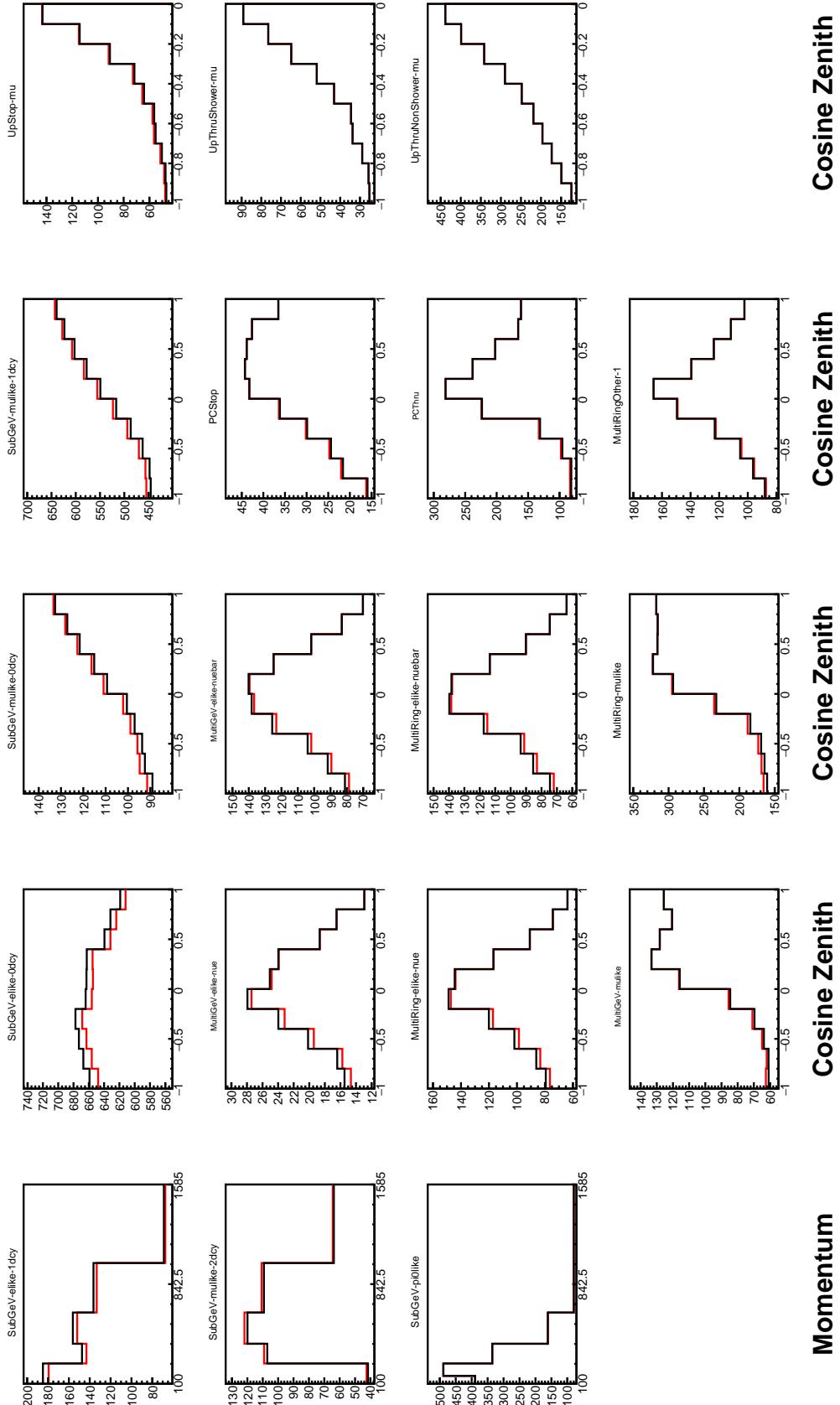


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThru	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Each row sums to 100% by definition. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV- <i>elike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>elike</i> -1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>pi0like</i>	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>mulike</i>	10	3.0, 3.4, 5.0
MultiRing- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>mulike</i>	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing- <i>Other1</i>	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table 6.5: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples.

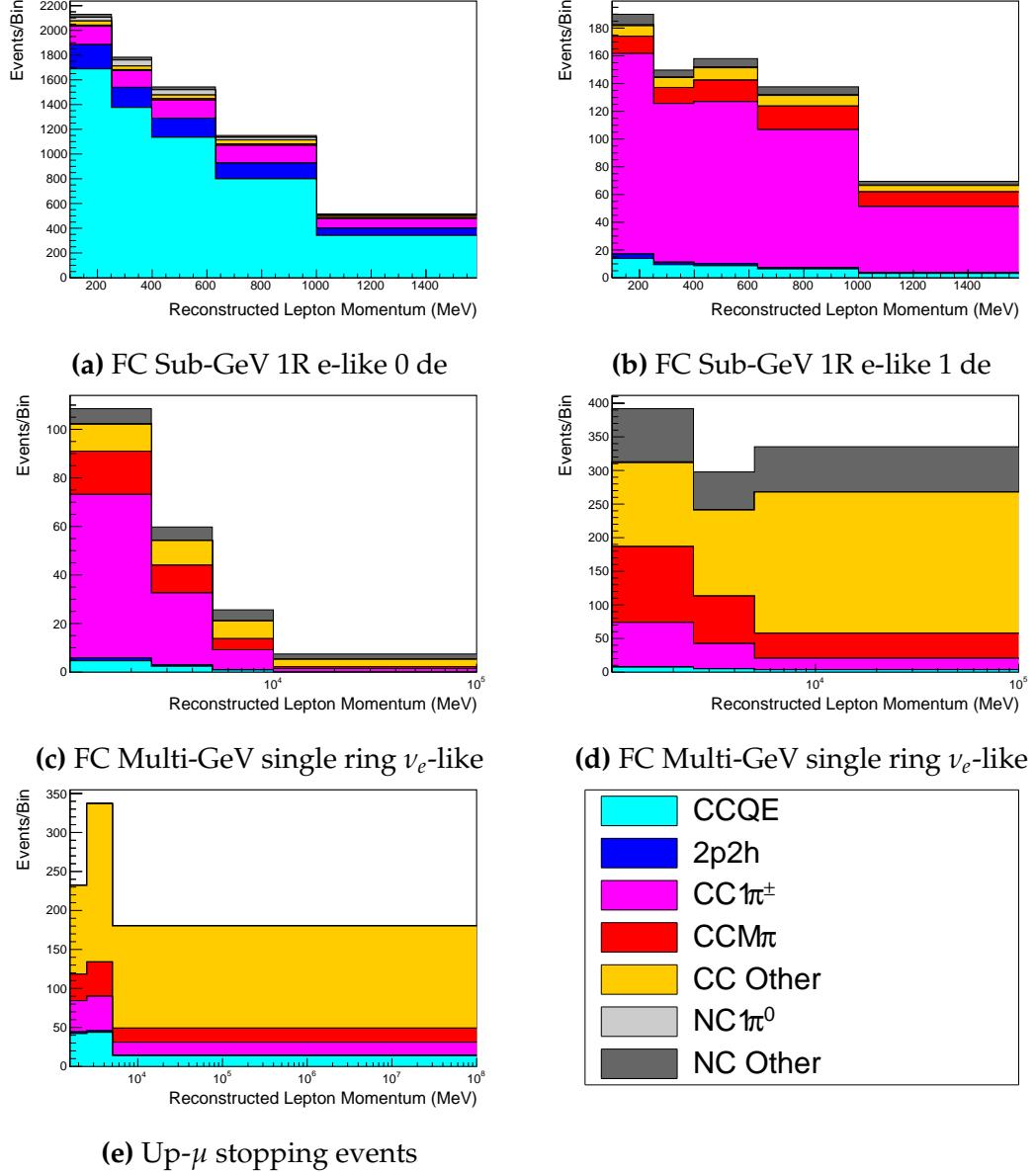


Figure 6.3: Breakdown by interaction mode of some representative atmospheric samples used within this analysis, illustrated as a function of reconstructed lepton momentum. The binning is provided in Table 6.5. Asimov A oscillation parameters are used to generate these plots. The interaction mode breakdown of all atmospheric samples used within this analysis can be found in [182].

2015 6.2 Near Detector Beam Samples

2016 The near detector sample selections are documented in detail within [183] and
2017 summarised below. Samples are selected based upon which of the two Fine
2018 Grained Detector (FGD) the vertex is reconstructed in as well as the operating
2019 mode of the beam: FHC or RHC. Wrong-sign neutrino background samples are
2020 considered in the RHC mode in order to add additional constraints on model
2021 parameters. Samples from the wrong-sign component of the FHC beam mode
2022 are not included as they are statistically insignificant compared to those samples
2023 already listed.

2024 The reconstruction algorithm uses a clustering algorithm to group hits within
2025 the TPC. It then adds information from the upstream FGD to form a track
2026 that passes through both sub-detectors. In FHC(RHC), the highest momentum
2027 negative(positive) curvature track is defined as the muon candidate. Before
2028 being assigned a sample, these candidate muon events must pass CC-inclusive
2029 cuts, as defined in [184]:

- 2030 • Event Timing: The DAQ must be operational and the event must occur
2031 within the expected beam time window consistent with the beam spill
- 2032 • TPC Requirement: The muon-candidate track path must intercept one or
2033 more TPCs
- 2034 • Fiducial volume: The event must originate from within the fiducial volume
2035 defined in [185]
- 2036 • Upstream Background: Remove events that have muon tracks that originate
2037 upstream of the FGDs by requiring no high-momentum tracks within
2038 150mm upstream of the candidate vertex. Additionally, events that occur
2039 within the downstream FGD are vetoed if a secondary track starts within
2040 the upstream FGD

- 2041 • Broken track removal: All candidates where the muon candidate is broken
 2042 in two are removed

- 2043 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-
 2044 like events, from electron-like or proton-like, using a likelihood cut

2045 In addition to these cuts, RHC neutrino events also have to undergo the
 2046 following cuts to aid in the separation of neutrino and antineutrino [186]:

- 2047 • TPC Requirement: The track path must intercept TPC2
 2048 • Positive Track: The highest momentum track must have a positive recon-
 2049 structed charge
 2050 • TPC1 Veto: Remove any events originating upstream of TPC1

2051 Once all CC-inclusive events have been determined, they are further split by
 2052 pion multiplicity: CC0 π , CC1 π , and CCOther. Pions in the TPCs are selected by
 2053 requiring a second track to be observed, which is separate from the muon track
 2054 and is in the same beam spill window and sub-detector. The number of FGD
 2055 pions is equal to the number of Michel electrons which were tagged within the
 2056 same sub-detector and spill window. If this value is equal to zero, the number
 2057 of FGD pions is equivalent to the number of pion-like tracks which have dE/dx
 2058 measurements consistent with the pion hypothesis. The pion tracks from both
 2059 FGD and TPC events are required to have a vertex consistent with that of the
 2060 muon candidate. The Michel electron tagging is preferential as a delayed Michel
 2061 is almost always a pion meaning this cut has a higher purity [184, 187], whereas a
 2062 track in the FGD that is consistent with a pion could be another particle resulting
 2063 in a lower purity. Michel electrons are neglected in the TPC as the pions very
 2064 rarely stop due to the low density.

2065 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

- 2066 • ν_μ CC0 π Selection: No electrons in TPC and no charged pions or decay
 2067 electrons within the TPC or FGD

- 2068 • ν_μ CC1 π **Selection:** Exactly one charged pion in either the TPC or FGD
- 2069 • ν_μ CCOther **Selection:** All events which are not classified into the above
- 2070 two selections

2071 Counting the three selections for each FGD in FHC and RHC running, includ-
2072 ing the wrong-sign background in RHC, 18 near detector samples are used within
2073 this analysis. These samples are binned in reconstructed lepton momentum
2074 (illustrated in Figure 6.4) and direction with respect to the beam. The binning
2075 is chosen such that each event has at least 20 Monte Carlo events in each bin
2076 [185]. This is to ensure that the bins are coarse enough to ensure the reduction
2077 of statistical errors, whilst also being fine enough to sample the high-resolution
2078 peak regions. The exact binning is detailed in [185].

2079 6.3 Far Detector Beam Samples

2080 The beam neutrino events which occur at the SK detector, which pass the
2081 reduction cuts detailed in section 5.3, are separated based on whether the beam
2082 was operating in FHC or RHC mode. The events are then separated into three
2083 samples: electron-like (1Re), muon-like (1R μ), and CC1 π^+ -like (1Re1de) which
2084 are observed as electron-like events with an associated decay electron [175].
2085 As discussed in section 6.1, positively charged pions emitted from neutrino
2086 interactions are more likely to produce decay electrons than negatively charged
2087 pions. Consequently, the CC1 π^+ -like sample is only selected when the beam is
2088 operating in FHC mode. Therefore, five beam samples measured at SK are
2089 used in this analysis.

2090 The fiducial volume definition for beam samples is slightly different from that
2091 used for the atmospheric samples. It uses both the distance to the closest wall
2092 (dWall) and the distance to the wall along the trajectory of the particle (toWall).
2093 This allows events that originate close to the wall but are facing into the tank to be
2094 included within the analysis, which would have otherwise been removed. These
2095 additional events are beneficial for a statistics-limited experiment. The exact

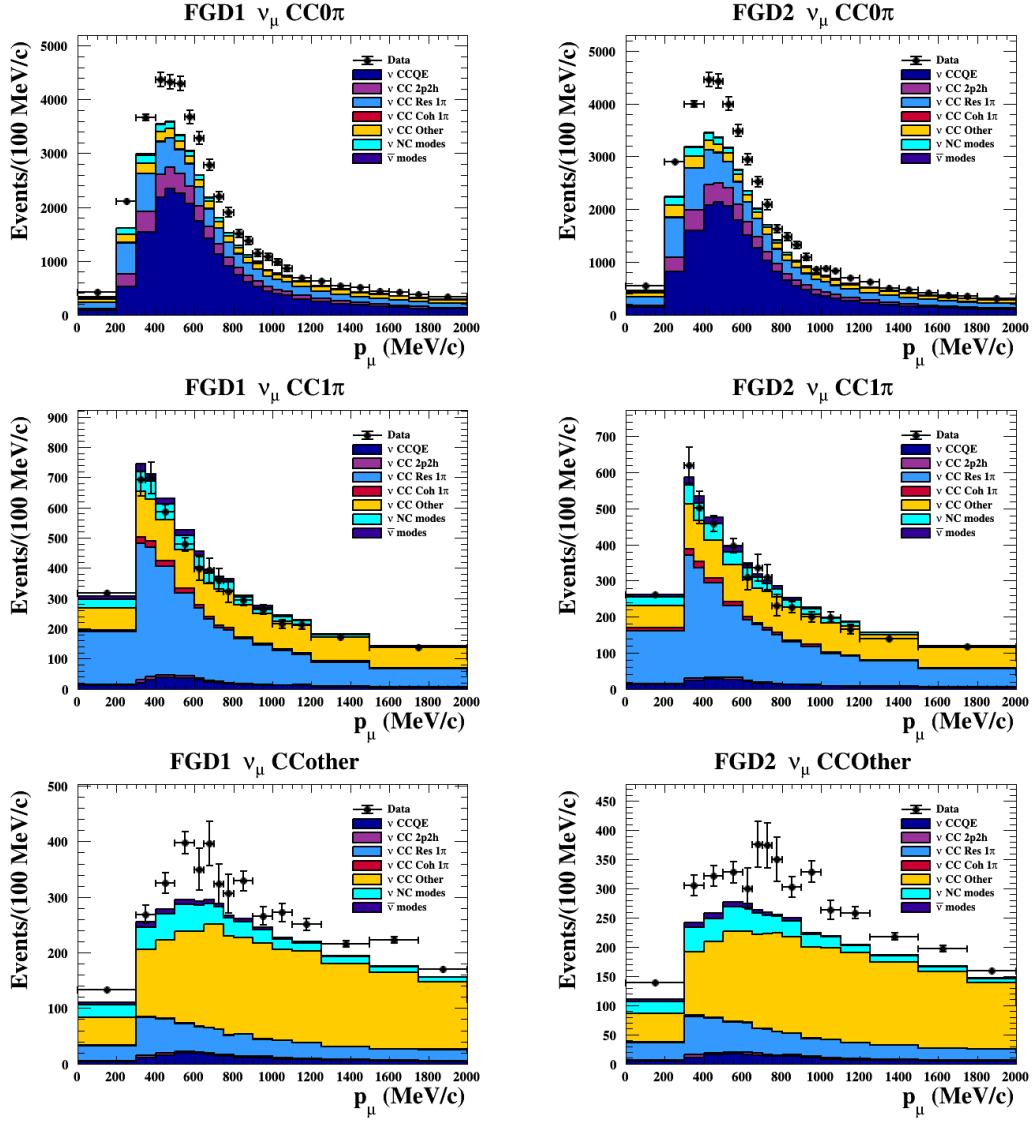


Figure 6.4: The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $CC\nu_\mu 0\pi$, $CC\nu_\mu 1\pi$ and $CC\nu_\mu$ Other categories. Figures taken from [183].

2096 cut values for both `dWall` and `tWall` are different for each of the three types of
 2097 sample and are optimised based on T2K sensitivity to δ_{CP} [173, 188]. They are:

2098 **1Re event selection** For an event to be classified as a 1Re-like, the event must sat-
 2099 isfy:

- 2100
 - Fully-contained and have $dWall > 80\text{cm}$ and $tWall > 170\text{cm}$
 - Total of one ring which is reconstructed as electron-like with reconstructed

2102 momentum $P_e > 100\text{MeV}$

2103 • Zero decay electrons are associated with the event

2104 • Passes π^0 rejection cut discussed in section 5.2

2105 The zero decay electron cut removes non-CCQE interactions and the π^0
 2106 rejection cut is designed to remove neutral current π^0 background events which
 2107 can be easily reconstructed as 1Re-like events.

2108 The zero decay electron cut removes non-CCQE interactions and the π^0
 2109 rejection cut is designed to remove neutral current π^0 background events which
 2110 can be easily reconstructed as 1Re-like events.

2111 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re
 2112 sample. The only differences are that the dWall and toWall criteria are changed
 2113 to $> 50\text{cm}$ and $> 270\text{cm}$, respectively, and exactly one decay electron is required
 2114 from the π^+ decay.

2115 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

2116 • Fully-contained and have $\text{dWall} > 50\text{cm}$ and $\text{toWall} > 250\text{cm}$

2117 • Total of one ring which is reconstructed as muon-like with reconstructed
 2118 momentum $P_\mu > 200\text{MeV}$

2119 • Fewer than two decay electrons are associated with the event

2120 • Passes π^+ rejection cut discussed in section 5.2

2121 All of these samples are binned in reconstructed neutrino energy. This is
 2122 possible under a particular interaction mode assumption, as the direction from
 2123 the source is known extremely well. For the 1Re-like and 1R μ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})}. \quad (6.2)$$

2124 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing
 2125 lepton, respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nucleus
 2126 [175], θ_{beam} is the angle between the beam and the direction of the outgoing
 2127 lepton, and E_l and P_l are the energy and momentum of that outgoing lepton.

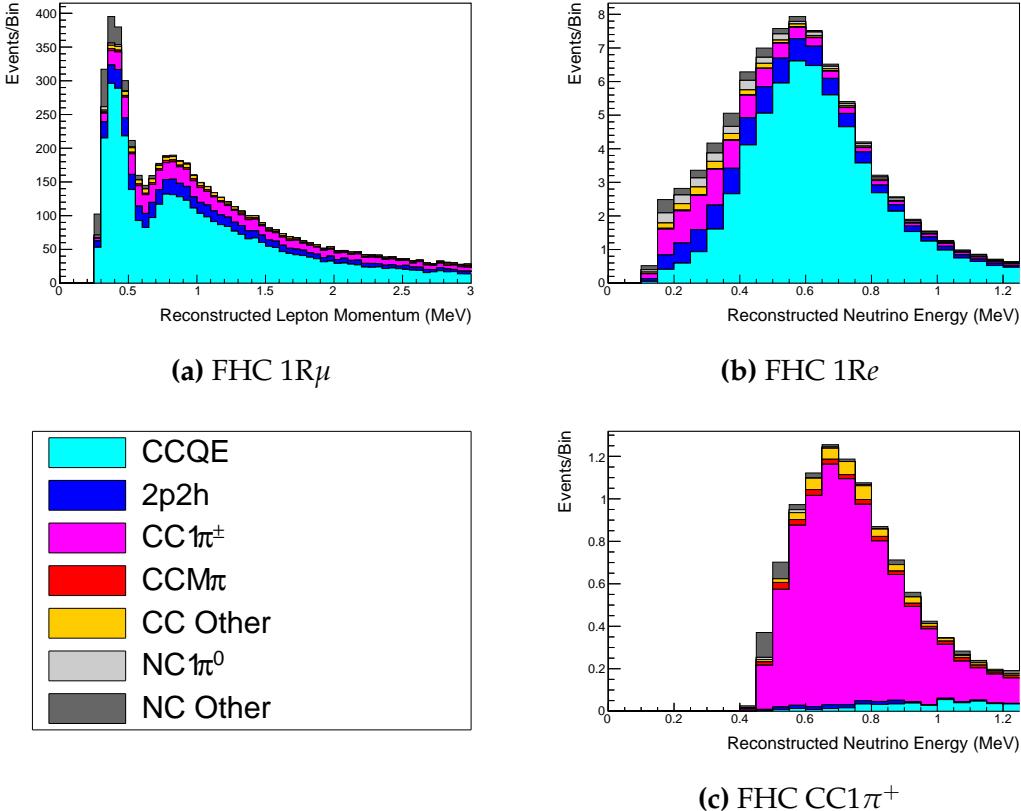


Figure 6.5: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1R e -like, and CC1 π^+ -like samples. The AsimovA oscillation parameters are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0 \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV .

2128 The reconstructed neutrino energy of the CC1 π^+ -like events also accounts
 2129 for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))}. \quad (6.3)$$

2130 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
 2131 considered as a two-body process, with the delta baryon, is assumed. This follows
 2132 the T2K oscillation analysis presented in [2], although recent developments of

2133 the interaction model in the latest T2K oscillation analysis do include effects
2134 from binding energy in this calculation [189].

2135 The reconstructed neutrino energy for the FHC samples is illustrated in
2136 Figure 6.5. As expected, the $1R\mu$ -like and $1Re$ -like samples are heavily dominated
2137 by CCQE interactions, with smaller contributions from $2p2h$ meson exchange and
2138 resonant pion production interactions. The $CC1\pi^+$ -like sample predominantly
2139 consists of charged current resonant pion production interactions. The $1Re$ -like
2140 and $CC1\pi^+$ -like samples are also binned by the angle between the neutrino beam
2141 and the reconstructed lepton momentum. This is to aid in charged current and
2142 neutral current separation, as indicated in Figure 6.6. This is because the neutral
2143 current backgrounds are predominantly due to π^0 -decays, which decay into two
2144 γ rays. The opening angle of which (alongside the different final state kinematics)
2145 can produce a slightly broader angular distribution compared to the final state
2146 particles originating from charged current ν_e interactions.

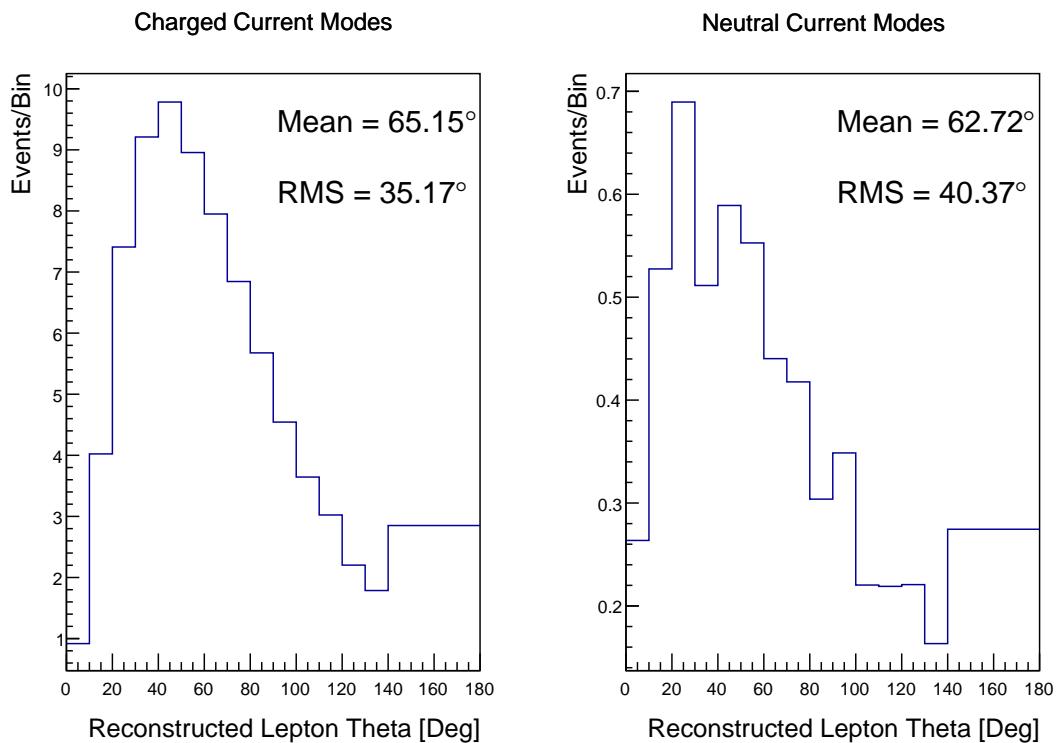


Figure 6.6: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are 35.17° and 40.37° , respectively.

2₁₄₇ 6.4 Systematic Uncertainties

2₁₄₈ The systematic model parameters for this analysis are split into groups, or blocks,
2₁₄₉ depending on their purpose. They consist of flux uncertainties, neutrino-matter
2₁₅₀ interaction systematics, and detector efficiencies. There are also uncertainties on
2₁₅₁ the oscillation parameters to which this analysis is not sensitive, namely Δm_{21}^2
2₁₅₂ and $\sin^2(\theta_{12})$. These oscillation parameter uncertainties are taken from the 2020
2₁₅₃ PDG measurements [72]. As described in chapter 4, each model parameter used
2₁₅₄ within this analysis requires a prior uncertainty. This is provided via separate
2₁₅₅ covariance matrices for each block. The covariance matrices can include prior
2₁₅₆ correlations between parameters within a single block, but the separate treatment
2₁₅₇ means prior correlations can not be included for parameters in different groups.
2₁₅₈ Some parameters in these models have no reasonably motivated uncertainties
2₁₅₉ and are assigned flat priors which do not modify the likelihood penalty. In
2₁₆₀ practice, these flat prior parameters are actually assigned a Gaussian with a
2₁₆₁ very large width to ensure the covariance matrix is positive definite. They are
2₁₆₂ then checked at run time to determine if they contribute to the likelihood. The
2₁₆₃ flux, neutrino interaction, and detector modeling simulations have already been
2₁₆₄ discussed in section 5.1 and section 5.2. The uncertainties invoked within each
2₁₆₅ of these models are described below.

2₁₆₆ 6.4.1 Beam Flux

2₁₆₇ The neutrino beam flux systematics are based upon the uncertainty in the mod-
2₁₆₈eling of the components of the beam simulation. This includes the model of
2₁₆₉ hadron productions and reinteractions, the shape, intensity, and alignment of
2₁₇₀ the beam with respect to the target, and the uniformity of the magnetic field
2₁₇₁ produced by the horn, alongside other effects. The uncertainty, as a function
2₁₇₂ of neutrino energy, is illustrated in Figure 6.7 which includes a depiction of
2₁₇₃ the total uncertainty as well as the contribution from individual components.
2₁₇₄ The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$) is

2175 dominated by uncertainties in the beam profile and alignment. Outside of this
2176 region, uncertainties on hadron production dominate the error.

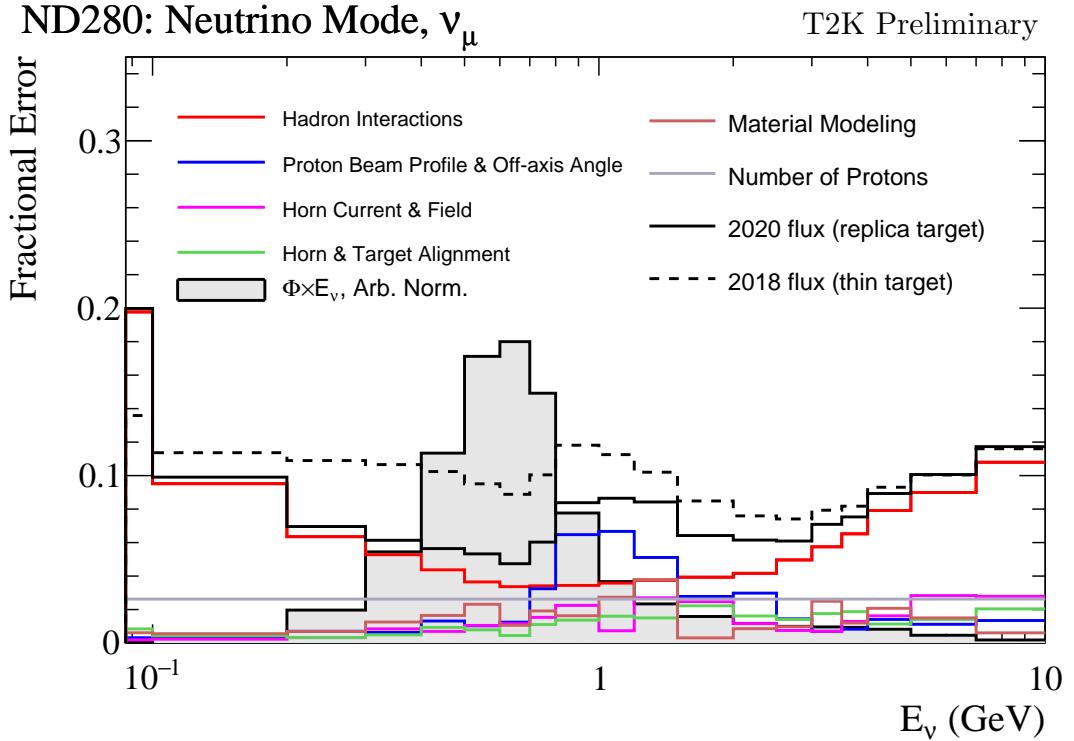


Figure 6.7: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [190]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [191].

2177 The beam flux uncertainties are described by one hundred parameters. They
2178 are split between the ND280 and SK detectors and binned by neutrino flavour:
2179 ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. The response is then broken down as a function of neutrino
2180 energy. The bin density in the neutrino energy is the same for the ν_μ in FHC
2181 and $\bar{\nu}_\mu$ in RHC beams, and narrows for neutrino energies close to the oscillation
2182 maximum of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.6. All of these
2183 systematic uncertainties are applied as normalisation parameters with Gaussian
2184 priors centered at 1.0 and error specified from a covariance matrix provided
2185 by the T2K beam group [191].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.6: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

2186 6.4.2 Atmospheric Flux

2187 The atmospheric neutrino flux is modeled by the HKKM model [50]. 16 systematic
 2188 uncertainties are applied to control the normalisation of each neutrino flavour,
 2189 energy, and direction. They are summarised below:

- 2190 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour
 2191 is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$
 2192 and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction
 2193 uncertainties for the production of pions and kaons [50]. The strength of
 2194 the response is dependent upon the neutrino energy. The uncertainty is
 2195 parameterized following Figure 11 in [50].
- 2196 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$
 2197 are controlled by the difference between the HKKM model [50], FLUKA
 2198 [53] and Bartol models [49]. Three independent parameters are applied in
 2199 the energy ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 2200 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent)
 2201 production uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using
 2202 the same methodology as the relative normalisation parameters.
- 2203 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-
 2204 tematics, the difference between the HKKM, FLUKA, and Bartol model

2205 predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of
 2206 events as a function of zenith angle.

- 2207 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) mostly originate in
 2208 kaon decay. Measurements of the ratio of K/π production [192] are used to
 2209 control the systematic uncertainty of the expected ratio of pion and kaon
 2210 production.
- 2211 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic
 2212 field, the flux of primary cosmic rays varies across the same period. The
 2213 uncertainty is calculated by taking a ± 1 year variation, equating to a 10%
 2214 uncertainty for the SK-IV period.
- 2215 • **Atmospheric Density:** The height of the interaction of the primary cosmic
 2216 rays is dependent upon the atmospheric density. The HKKM assumes the
 2217 US standard 1976 [142] profile. This systematic controls the uncertainty in
 2218 that model.

2219 The total uncertainty is dominated by the absolute and relative normalisation
 2220 parameters. The effect of which is illustrated in Figure 6.8. Generally, the
 2221 uncertainty is large at low energy, reducing to $O(10\%)$ around the peak of the
 2222 flux distribution and then increasing once the neutrino energy exceeds 10GeV.

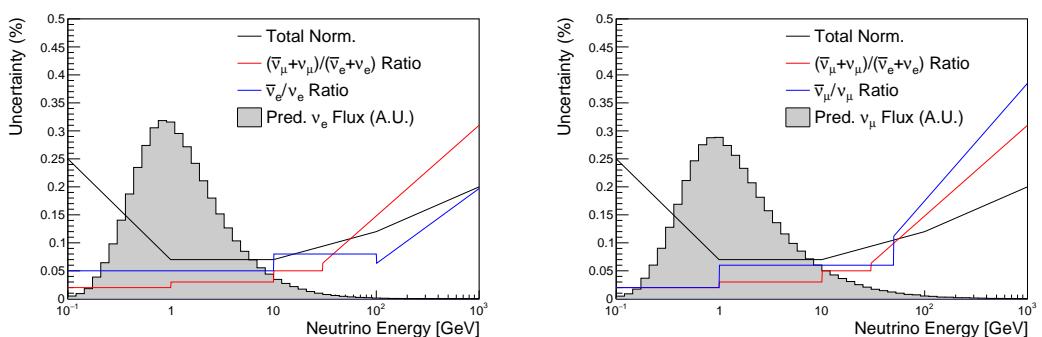


Figure 6.8: The uncertainty evaluated on the atmospheric ν_e (left) and ν_μ (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2223 Updates to the HKKM and Bartol models are underway [147] to use a similar
2224 tuning technique to that used in the beam flux predictions. After those updates,
2225 it may be possible to include correlations in the hadron production uncertainty
2226 systematics for beam and atmospheric flux predictions.

2227 6.4.3 Neutrino Interaction

2228 Neutrino interactions in the detectors are modeled by NEUT. The two indepen-
2229 dent oscillation analyses, T2K-only [193] and the SK-only [59], have developed
2230 separate interaction models. To maximise sensitivity out of this simultaneous
2231 beam and atmospheric analysis, a correlated interaction model has been defined
2232 in [182]. Where applicable, correlations allow the systematic uncertainties applied
2233 to the atmospheric samples to be constrained by near detector neutrino beam
2234 measurements. This can lead to stronger sensitivity to oscillation parameters
2235 as compared to an uncorrelated model.

2236 The low-energy T2K systematic model has a more sophisticated treatment
2237 of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of
2238 this model have been performed to external data [193]. However, the model
2239 is not designed for high-energy atmospheric events, like those illustrated in
2240 Figure 5.11. Therefore the high energy systematic model from the SK-only
2241 analysis is implemented for the relevant multi-GeV, PC, and up- μ samples.
2242 The T2K CCQE model is more sophisticated so it has been implemented for
2243 all samples within this analysis, where separate low-energy and high-energy
2244 dials have been implemented. The low-energy dials are constrained by the near
2245 detector measurements and are uncorrelated to their high-energy counterparts.
2246 The author of this thesis was responsible for implementing and validating the
2247 combined cross-section model as documented in [182, 194].

2248 The high energy systematic model includes parameters developed from
2249 comparisons of Nieves and Rein-Seghal models which affect resonant pion
2250 producing interactions, comparisons of the GRV98 and CKMT models which
2251 control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the ν_τ cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.9 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.10 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

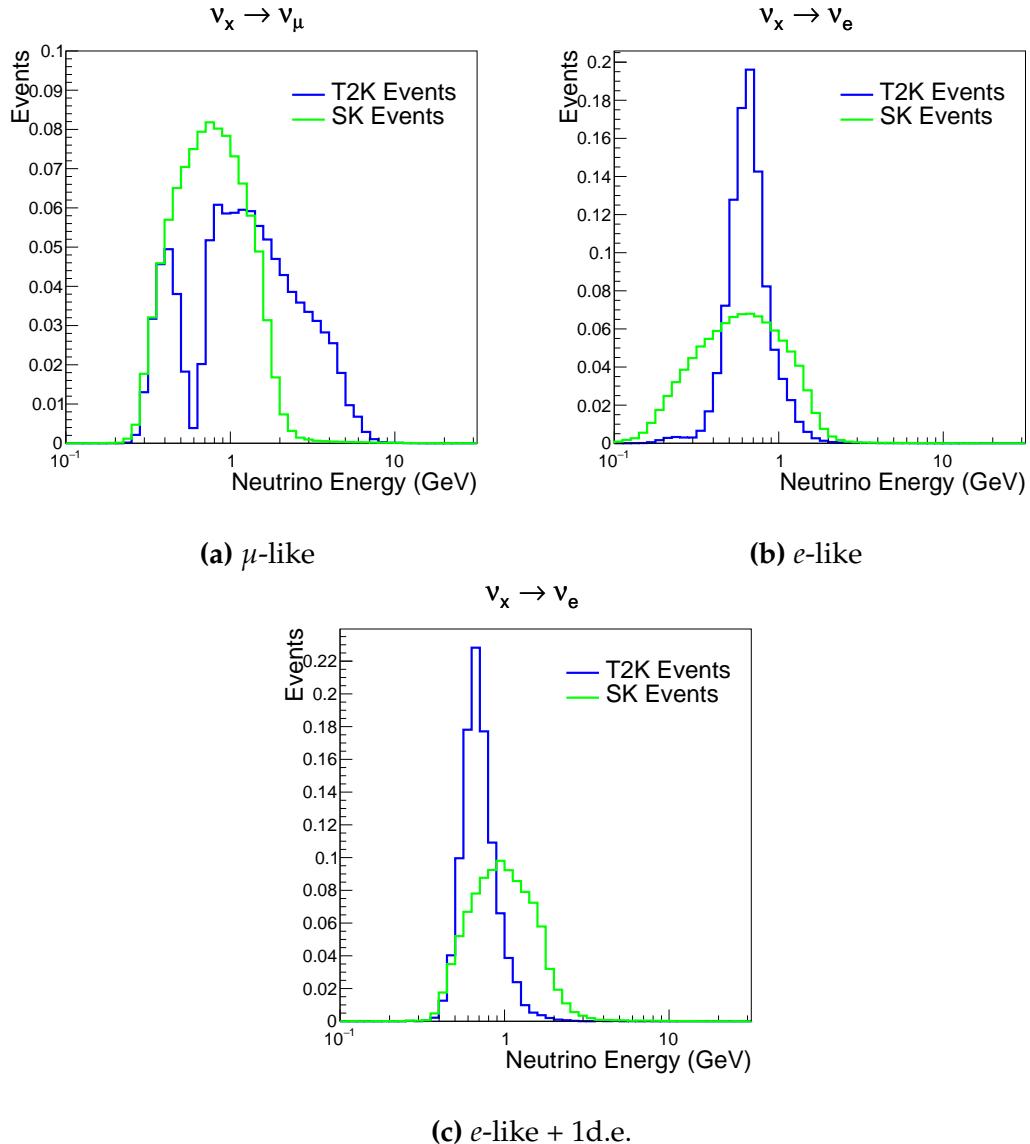


Figure 6.9: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

The T2K systematic model [193] is applied in a similar methodology to the SK model parameters. It consists of 19 shape parameters and 24 normalisation parameters. Four additional parameters, which model the uncertainty in the binding energy, are applied in a way to shift the momentum of the lepton emitted from a nucleus. This controls the uncertainty specified on the 27MeV binding energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.10: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

2288 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated
 2289 uncertainty, or those which have not been fit to external data, are assigned a
 2290 flat prior which does not affect the penalty term.

2291 On top of the combination of the SK and T2K interaction models, several
 2292 other parameters have been specifically developed for the joint oscillation anal-
 2293 ysis. The majority of the atmospheric samples' δ_{CP} sensitivity comes from the
 2294 normalisation of subGeV electron-like events. These are modeled using a spectral
 2295 function to approximate the nuclear ground state. However, the near detector is
 2296 not able to constrain the model so an additional systematic is introduced which
 2297 models an alternative Continuous Random Phase Approximation (CRPA) nuclear
 2298 ground state. This dial approximates the event weights if a CRPA model had

been assumed rather than a spectral function. This dial only applies to ν_e and $\bar{\nu}_e$ as the near detector does not constraint ν_e cross-section measurements. It is applied as a shape parameter.

Further additions to the model have been introduced due to the inclusion of the subGeV π^0 atmospheric sample. This particularly targets charged current and neutral current π^0 producing interactions to help constrain the systematic uncertainties. Therefore, an uncertainty that affects neutral current resonant π^0 production is incorporated into this analysis. Comparisons of NEUT's NC resonant pion production predictions have been made to MiniBooNE [195] data and a consistent 16% to 21% underprediction is observed [182]. Consequently, a conservative 30% normalisation parameter is invoked.

Down-going events are mostly insensitive to oscillation parameters and can act similar to the near detector within an accelerator experiment (Details will be discussed in chapter 7). This region of phase space can act as a sideband and allows the cross-section model and near detector constraint to be studied. The distribution of events in this region is calculated using the technique outlined in subsection 4.3.4. The results are illustrated in Figure 6.11. For CCQE-targeting samples, the application of the near detector constraint is well within the statistical fluctuation of the down-going data. This means there is no significant tension is observed between the data and the Monte Carlo prediction after the near detector constraint is applied. This is not the case for samples with target CCRES interactions. The electron-like data is consistent with the constrained prediction at high reconstructed momenta but diverges at lower momentum, whereas the muon-like sample is under-predicted throughout the range of momenta. To combat this disagreement, an additional cross-section systematic dial, specifically designed to inflate the low pion momentum systematics was developed in [182]. This is a shape parameter implemented through a splined response.

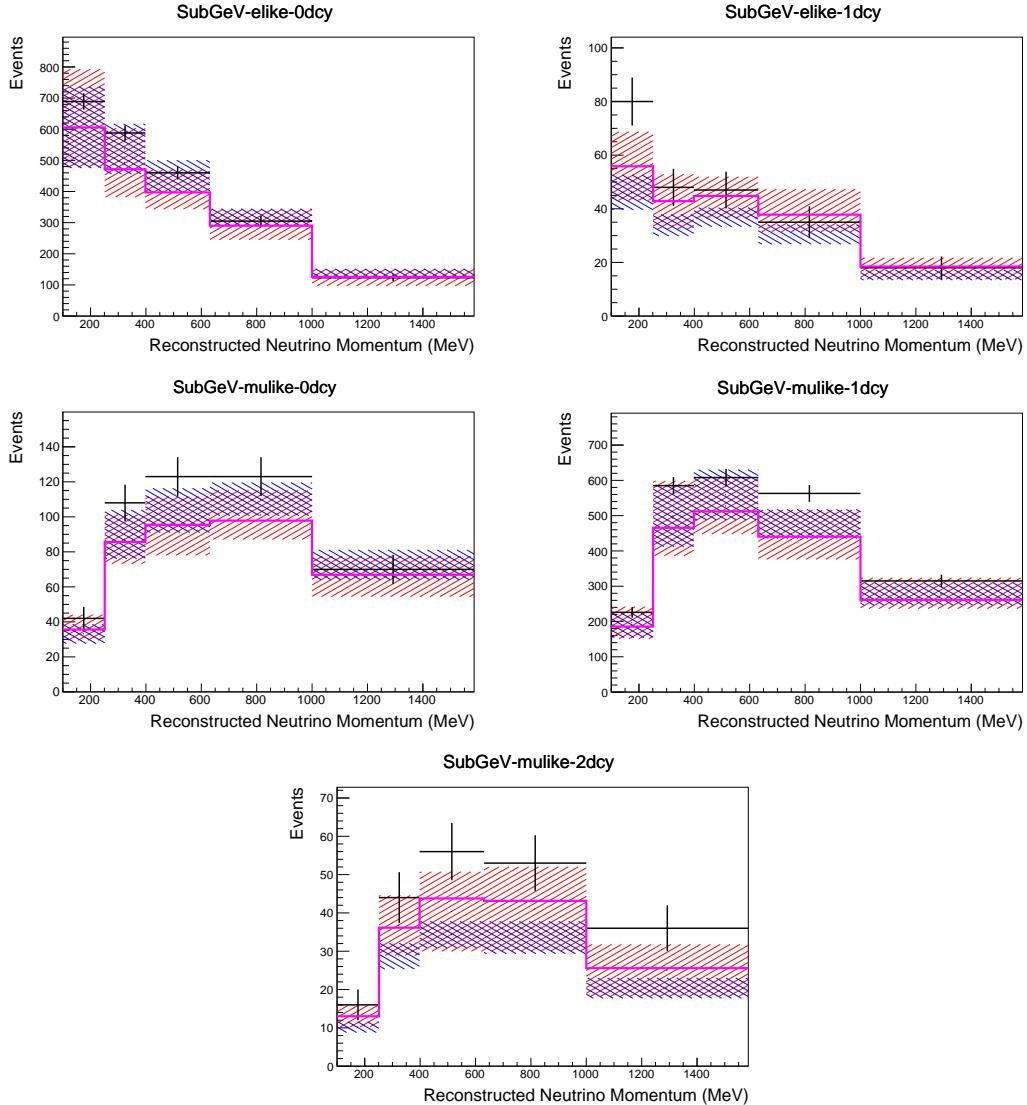


Figure 6.11: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

2326 6.4.4 Near Detector

2327 The systematics applied due to uncertainties arising from the response of the near
 2328 detector is documented in [122]. The response is described by 574 normalisation
 2329 parameters binned in the selected sample as well as momentum and angle,

2330 P_μ and $\cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance
2331 matrix with each parameter being assigned a Gaussian prior from that covariance
2332 matrix. These normalisation parameters are built from underlying systematics,
2333 e.g. pion secondary interaction systematics, which are randomly thrown and
2334 the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. Two thousand throws are
2335 evaluated and a covariance matrix response is created. This allows significant
2336 correlations between FGD1 and FGD2 samples, as well as adjacent $P_\mu \times \cos(\theta_\mu)$
2337 bins. Statistical uncertainties are accounted for by including fluctuations of each
2338 event's weight from a Poisson distribution.

2339 Similar to the cross-section systematics, MaCh3 and BANFF are used to
2340 constrain the uncertainty of these systematics through independent validations.
2341 Each fitter generates a post-fit covariance matrix which is compared and passed
2342 to the far-detector oscillation analysis working group. As the analysis presented
2343 within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all
2344 three sets of samples and their respective systematics is performed.

2345 6.4.5 Far Detector

2346 Two configurations of the far detector systematic model implementation have
2347 been considered. Firstly, the far detector systematic uncertainties for beam and
2348 atmospheric samples are taken from their respective analysis inputs, denoted
2349 “official inputs” analysis, with no correlations assumed between the beam and at-
2350 mospheric samples. The beam- and atmospheric-specific inputs are documented
2351 in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative
2352 detector model has been developed which correlates the response of the SK
2353 detector systematics between the beam and atmospheric samples. Here, the
2354 distribution of parameters used for applying event cuts (e.g. electron-muon
2355 PID separation) is modified within the fit. It follows a similar methodology to
2356 the beam far detector systematics implementation but performs a joint fit of
2357 the beam and atmospheric data. This alternative implementation is detailed
2358 in subsubsection 6.4.5.3.

2359 **6.4.5.1 Beam Samples**

2360 There are 45 systematics which describe the response of the far detector to
 2361 beam events [175], split into 44 normalisation parameters and one energy scale
 2362 systematic. The energy scale systematic is applied as a multiplicative scaling
 2363 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo
 2364 differences in the stopping muon sample in [59] and found to be 2.1%. The
 2365 normalisation parameters are assigned a Gaussian error centered at one with
 2366 width taken from a covariance matrix. A detailed breakdown of the generation
 2367 of the covariance matrix is found in [188]. To build the covariance matrix, a fit
 2368 is performed on atmospheric data which has been selected using beam sample
 2369 selection cuts. These cuts use the variables, L^i , where the index i is detailed in
 2370 Table 6.7. Each L^i is a smear, α , and shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i. \quad (6.4)$$

2371 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in
 2372 Table 6.7. The shift and smear parameters are nuisance parameters with no prior
 2373 constraints. They are binned by final-state topology, j , where the binning is given
 2374 in Table 6.8. The final-state topology binning is because the detector will respond
 2375 differently to events that have one or multiple rings. For example, the detector
 2376 will be able to distinguish single-ring events better than two overlapping ring
 2377 events, resulting in different systematic uncertainty for one-ring events compared
 2378 to two-ring events. This approach is used to allow the cut parameter distributions
 2379 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.7: List of cut variables that are included within the shift/smear fit documented in [188].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 μ	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 μ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 π^0	Only one π^0 in the final state
1 π^\pm or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.8: Reconstructed event topology categories on which the SK detector systematics [188] are based.

2380 The mis-modeling of π^0 events is also considered. If one of the two rings
 2381 from a π^0 event is missed, this will be reconstructed as a CC ν_e -like event. This
 2382 is one of the largest systematics hindering the electron neutrino appearance
 2383 analyses. Consequently, additional systematics have been introduced to con-
 2384 strain the mis-modeling of π^0 events in SK, binned by reconstructed neutrino
 2385 energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples is
 2386 constructed. These events are built by overlaying one electron-like ring from
 2387 the SK atmospheric neutrino samples or decay electron ring from a stopping
 2388 cosmic ray muon with one simulated photon ring. Both rings are chosen so
 2389 that momenta and opening angle follow the decay kinematics of NC π^0 events
 2390 from the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the
 2391 SK Monte Carlo are produced to compare with the hybrid- π^0 data samples and
 2392 the difference in the fraction of events that pass the ν_e selection criteria is used
 2393 to assign the systematic errors. In order to investigate any data to Monte Carlo
 2394 differences that may originate from either the higher energy ring or lower energy
 2395 ring, two samples are built; a sample in which the electron constitutes the higher
 2396 energy ring from the π^0 decay (called the primary sample) and another one in
 2397 which it constitutes the lower energy ring (called the secondary sample). The
 2398 standard T2K ν_e fitQun event selection criteria are used to select events.

2399 Final contributions to the covariance matrix are determined by supplemen-
 2400 tary uncertainties obtained by comparing stopping muon data to Monte Carlo

prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data to Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or fewer decay electrons. Similarly, the rate at which fake decay electrons are reconstructed by `fiTQun` is estimated by comparing the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events is also applied. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [175, 196, 197].

Two additional sources of uncertainty are included: secondary and photoneuclear interactions. These are estimated by varying the underlying parameters are building a distribution of sample event rates. These contributions are then added in quadrature to the above covariance matrix. The final uncertainty on the SK detector systematics are provided in Figure 6.12.

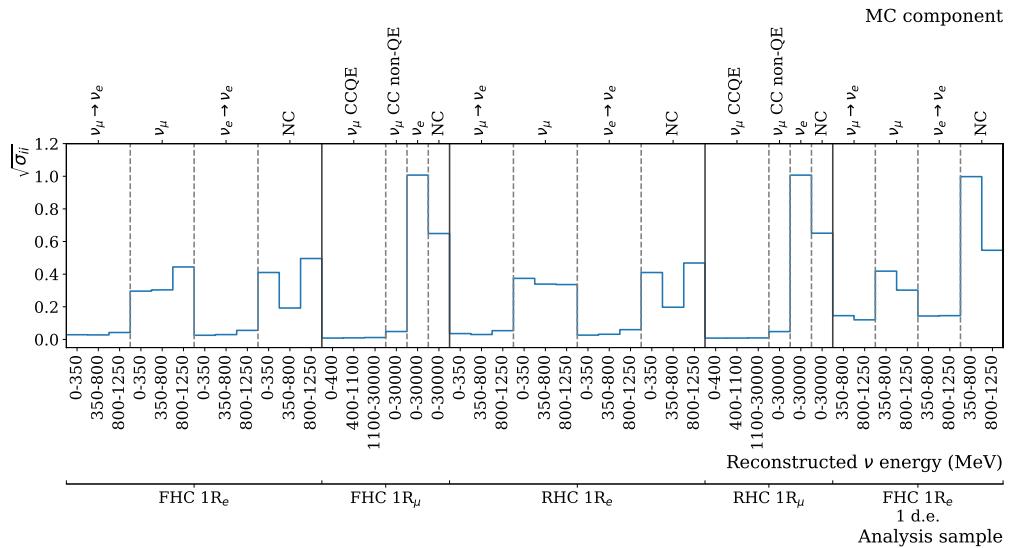


Figure 6.12: The fractional uncertainty on each of the 44 parameters describing the SK detector systematics (The energy scale systematic is neglected). The parameters are split by sample, oscillation channel, interaction mode and reconstructed neutrino energy.

2418 6.4.5.2 Atmospheric Samples

2419 The detector systematics for atmospheric samples, documented in [3], are split
2420 into two sub-groups: those which are related to particle identification and ring
2421 counting systematics, and those which are related to calibration, separation,
2422 and reduction uncertainties.

2423 The particle identification systematics consist of five parameters. The ring sep-
2424 aration systematic enforces an anti-correlated response between the single-ring
2425 and multi-ring samples. This is implemented as a fractional increase/decrease
2426 in the overall normalisation of each sample, depending on the distance to the
2427 nearest wall from an event's vertex. The coefficients of the normalisation are
2428 estimated prior to the fit and depend on the particular atmospheric sample. Two
2429 electron-muon separation systematics are included within this model which
2430 anti-correlates the response of the electron-like and muon-like samples: one for
2431 single-ring events and another for multi-ring events.

2432 The multi-ring electron-like separation likelihood, discussed in section 6.1,
2433 encodes the ability of the detector to separate neutrino from anti-neutrino events.
2434 Two normalisation parameters vary the relative normalisation of multi-ring ν_e
2435 and $\bar{\nu}_e$ samples whilst keeping a consistent overall event rate.

2436 There are 22 systematics related to calibration measurements, including effects
2437 from backgrounds, reduction, and showering effects. They are documented in
2438 [3] and are briefly summarised in Table 6.9. They are applied via normalisation
2439 parameters, with the separation systematics requiring the conservation of event
2440 rate across all samples.

2441 6.4.5.3 Correlated Detector Model

2442 A complete uncertainty model of the SK detector would be able to determine
2443 the systematic shift on the sample spectra for a variation of the underlying
2444 parameters, e.g. PMT angular acceptance. However, this is computationally
2445 intensive, requiring Monte Carlo predictions to be made for each plausible
2446 variation. Consequently, an effective parameter model has been utilised for

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

Table 6.9: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

²⁴⁴⁷ a correlated detector model following from the T2K-only model implementation
²⁴⁴⁸ documented in subsubsection 6.4.5.1. It correlates the detector systematics
²⁴⁴⁹ between the far-detector beam and subGeV atmospheric samples due to their
²⁴⁵⁰ similar energies and interaction types. As there are no equivalent beam samples,
²⁴⁵¹ the multi-GeV, multiring, PC, and Up- μ samples will be subject to the particle
²⁴⁵² identification systematics implementation as described in subsubsection 6.4.5.2
²⁴⁵³ rather than using this correlated detector model. The calibration systematics also
²⁴⁵⁴ described in the aforementioned chapter still apply to all atmospheric samples.
²⁴⁵⁵ The correlated detector model utilises the same smear and shift parameters
²⁴⁵⁶ documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,

the shift and smear parameters are split by visible energy deposited within the detector, with binning specified in Table 6.10. This is because atmospheric events are categorised by subGeV and multi-GeV events based on visible energy, so this splitting is required when correlating the systematic model for beam and atmospheric events. Alongside the technical requirement, higher energy events will be better reconstructed due to fractionally less noise within the detector. As a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

where k is the visible energy bin.

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

Table 6.10: Visible energy binning for which the correlated SK detector systematics are based

The implementation of this systematic model takes the events reconstructed values of the cut parameters, modifies them by the particular shift and smear parameter for that event, and then re-applies event selection. This causes event migration, which is a new feature incorporated into the MaCh3 framework which is only achievable due to the event-by-event reweighting scheme.

Particular care has to be taken when varying the ring counting parameter. This is because the number of rings is a finite value (one-ring, two-ring, etc.) which can not be continuously varied through this shift and smear technique. Consequently a continuous ring counting parameter, RC_i , is calculated for the i^{th} event, following the definition in [174]: the preferred likelihoods from all considered one-ring (L_{1R}) and two-ring (L_{2R}) fits are determined. The difference

2476 is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is
 2477 then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

2478 where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an
 2479 intermediate likelihood value used within the `fitQun` algorithm to decide the
 2480 number of rings associated with a particular event. However, fake-ring merging
 2481 algorithms are applied after this likelihood value is used. Consequently, this
 2482 ring counting parameter does not always exactly correspond to the number of
 2483 reconstructed rings. This can be seen in Figure 6.13.

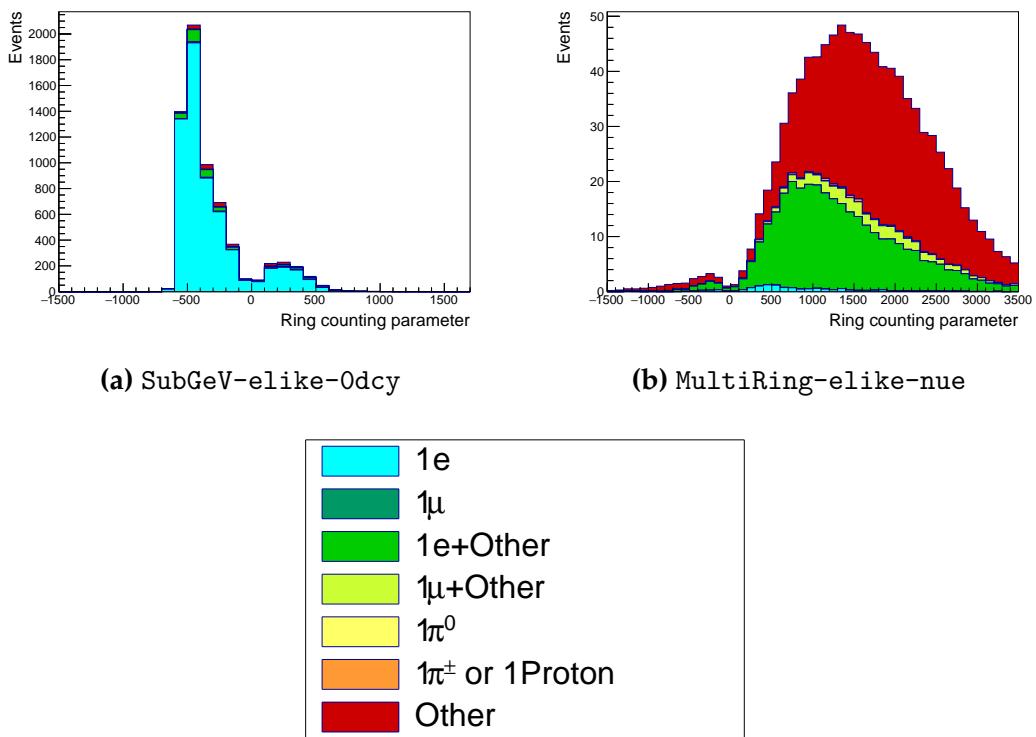


Figure 6.13: The ring counting parameter as defined in Equation 6.6 for the SubGeV-elike-0dcy and MultiRing-elike-nue samples.

2484 As the `fitQun` algorithm does not provide a likelihood value after the fake-
 2485 ring algorithms have been applied, the ring counting parameter distribution is
 2486 correlated to the final number of reconstructed rings through “maps”. These

are two-dimensional distributions of the ring counting parameter and the final number of reconstructed rings. An example is illustrated in Figure 6.14. In principle, the `fitQun` reconstruction algorithm should be re-run after the variation in the ring counting parameter. However, this is not computationally viable. Therefore the “maps” are used as a reweighting template.

The maps are split by final state topology and true neutrino flavour and all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps are row-normalised to represent the probability of X rings for a given RC_i value. Prior to the oscillation fit, an event’s nominal weight is calculated as $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th} event and $W^i(x, y)$ is the bin content in map associated with the i^{th} event, where x number of rings and y is ring counting parameter. Then during the fit, the value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as the event weight for the i^{th} event. This is the only cut variable that uses a reweighting technique rather than event migration.

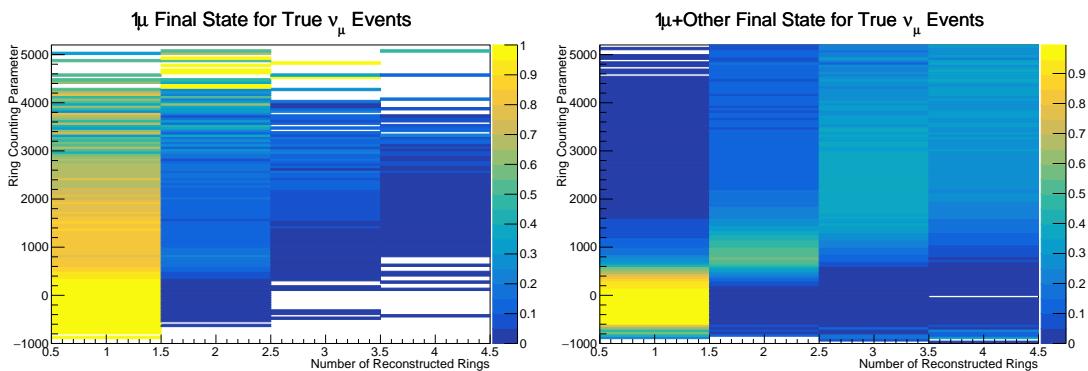


Figure 6.14: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance matrix. This is not possible in the alternative model as no covariance matrix is used. Thus, the implementation of the π^0 systematics has been modified. The inputs from the hybrid π^0 sample are included via the use of “ χ^2 maps”,

which are two-dimensional histograms in α_{jk}^i and β_{jk}^i parameters over some range. Illustrative examples of the χ^2 maps are given in Figure 6.15. Due to their nature, the shift and smear parameters are typically very correlated. A map is produced for each cut parameter given in Table 6.7 and for each visible energy bin given in Table 6.10.

The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo and data in the particle identification parameters documented in Table 6.7. The Monte Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-section and flux nuisance parameters are thrown from their prior uncertainties. The χ^2 between the scaled Monte Carlo and data is calculated and the relevant point in the χ^2 map is filled.

The implementation within this alternative detector model is to add the bin contents of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to the likelihood penalty. Only $1\pi^0$ final state topology shift and smear parameters use this prior uncertainty.

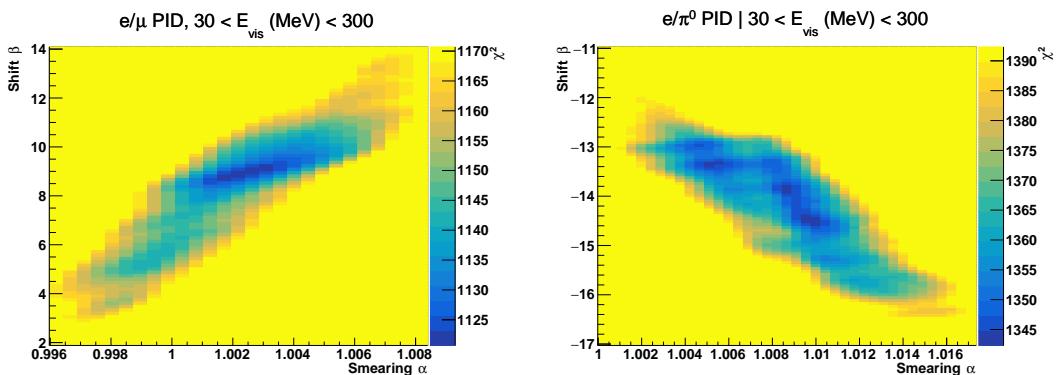


Figure 6.15: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$.

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [198] was built in tandem between the author of this thesis and the T2K-SK working group [175] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular

interaction mode, number of tagged decay electrons, and whether the primary particle generated Cherenkov light. They are assigned Gaussian uncertainties with widths described by a covariance matrix. Furthermore, the secondary interaction and photo-nuclear effects need to be accounted for in this detector model using a different implementation than that in subsubsection 6.4.5.1. This was done by including a shape parameter for each of the secondary interactions and the photo-nuclear systematic parameters.

There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior constraints from the hybrid π^0 samples.

One final complexity of this correlated detector model is that the two sets of samples, beam and subGeV atmospheric, use slightly different parameters to distinguish electron and muon-like events. The T2K samples use the value of $\log(L_e/L_\mu)$ whereas the atmospheric samples use the value of $\log(L_e/L_\pi)$, where L_X is the likelihood for hypothesis X. This is because the T2K fits use single-ring fiTQun fitting techniques, whereas multi-ring fits are applied to the atmospheric samples where only the electron and pion hypothesis are considered. The correlation between the two likelihood ratios is illustrated in Figure 6.16. As discussed in section 5.2, the pion hypothesis is a very good approximation of the muon hypothesis due to their similar mass. Consequently, using the same shift and smear parameters correlated between the beam and subGeV atmospheric samples is deemed a good approximation.

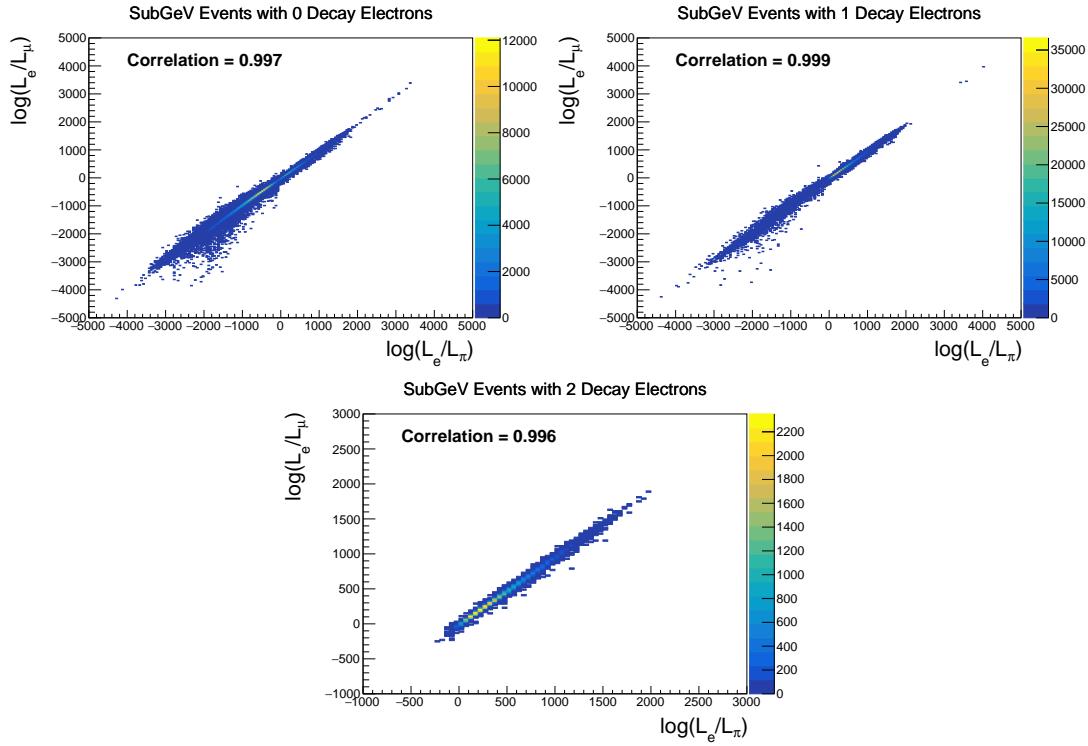


Figure 6.16: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

2547 6.5 Likelihood Calculation

2548 This analysis performs a joint oscillation parameter fit of the ND280 beam
 2549 samples, the T2K far detector beam samples, and the SK atmospheric samples
 2550 introduced in this chapter.

2551 Once the Monte Carlo predictions of each beam and atmospheric sample
 2552 have been built, a likelihood needs to be constructed. This is done by comparing
 2553 the binned Monte Carlo prediction to binned data. The Monte Carlo prediction
 2554 is calculated at a particular point, $\vec{\theta}$, in the model parameter space such that
 2555 $N_i^{MC} = N_i^{MC}(\vec{\theta})$, where N_i represents the bin content of the i^{th} bin. The data
 2556 and Monte Carlo spectra are represented by N_i^D and N_i^{MC} , respectively. The bin
 2557 contents for the beam near detector, beam far detector and atmospheric samples
 2558 are denoted with *ND*, *FD*, and *Atm*, respectively. Taking the FHC1Rmu far detector
 2559 sample as an example, the binning index runs over all the reconstructed neutrino

2560 energy bins. The likelihood calculation between the data and the Monte Carlo
 2561 prediction for a particular bin follows a Poisson distribution, where the data
 2562 is treated as a fluctuation of the simulation.

2563 The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction,
 2564 which is typically used for sensitivity studies and denoted ‘Asimov data’. The
 2565 process for building Asimov data is as follows. The Monte Carlo prediction is
 2566 reweighted using a particular set of oscillation parameters (potentially those
 2567 listed in Table 2.2) and systematic parameter tune. The resulting spectra for each
 2568 sample is then defined to be the Asimov data for that sample. Whilst this results
 2569 in unphysical non-integer data predictions, it eliminates statistical fluctuations
 2570 from the data. Therefore, the results of a fit to Asimov data should not include any
 2571 biases from statistical fluctuations. Furthermore, these results should produce
 2572 posterior probability distributions consistent with the parameters which were
 2573 used to make the data prediction. That is to say, the fit results should return the
 2574 known parameters. Any biases seen would be attributed to correlations between
 2575 each oscillation parameter and correlations between oscillation and systematic
 2576 parameters. Consequently, Asimov fit results present the maximum precision
 2577 at which the oscillation parameters could be measured to.

2578 Following the T2K analysis presented in [2], the likelihood contribution for
 2579 the near detector samples also includes a Monte Carlo statistical uncertainty
 2580 term, derived from the Barlow and Beeston statistical treatment [199, 200]. It
 2581 includes a contribution to the likelihood that treats the generated Monte Carlo
 2582 prediction as a statistical fluctuation of the actual true simulation assuming an
 2583 infinite amount of statistics had been created. The technical implementation of
 2584 this additional likelihood term is documented in [183] and briefly summarised
 2585 as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (6.7)$$

2586 where β_i represents a scaling parameter for the i^{th} bin that relates the bin
 2587 content for the amount of Monte Carlo actually generated N_i^{MC} to the bin content

if an infinite amount of Monte Carlo statistics had been generated $N_{i,true}^{MC}$, such that $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$. In the case where a sufficient amount of Monte Carlo statistics had been generated, $\beta_i = 1$. An analytical solution for β_i is given in [183]. Additionally, $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2 / N_i^{MC}}$ where $\sqrt{\sum_i w_i^2}$ represents the sum of the square of the weights of the Monte Carlo events which fall into bin i .

An additional contribution to the likelihood comes from the variation of the systematic model parameters. For those parameters with well-motivated uncertainty estimates, a covariance matrix, V , describes the prior knowledge of each parameter as well as any correlations between the parameters. Due to a technical implementation, a single covariance matrix describes each “block” of model parameters, e.g. beam flux systematics. The covariance matrix associated with the k^{th} block is denoted V^k . There are seven individual blocks of systematics ($N_b = 7$): oscillation parameters, beam flux systematics, atmospheric flux systematics, neutrino interaction systematics, near detector systematics, beam far detector systematics, and atmospheric far detector systematics. The number of parameters in the k^{th} block is defined as $n(k)$.

The equation for the likelihood \mathcal{L} includes all the terms discussed above,

$$\begin{aligned}
 -\ln(\mathcal{L}) = & \\
 & \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
 & + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
 & + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
 & + \frac{1}{2} \sum_k^{\text{N}_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
 \end{aligned} \tag{6.8}$$

The negative log-likelihood value is determined at each step of the MCMC to build the posterior distribution defined in chapter 4. This value is minimised when the Monte Carlo prediction tends towards the data spectrum.

7

2608

2609

Oscillation Probability Calculation

2610 It is important to understand how and where the sensitivity to the oscillation parameters comes from for both atmospheric and beam samples. An
2611 overview of how these samples respond to changes in δ_{CP} , Δm_{32}^2 , and $\sin^2(\theta_{23})$
2612 is given in section 2.5. This section also explains the additional complexities
2613 involved when performing an atmospheric neutrino analysis as compared to
2614 a beam-only analysis.

2616 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to $\delta_{CP,zaza}$
2617 due to the quickly varying oscillation probability in this region. Therefore, a
2618 “sub-sampling” approach has been developed to reduce these biases ensuring
2619 accurate and reliable sensitivity measurements. This technique ensures that small-
2620 scale unresolvable features of the oscillation probability have been averaged over
2621 whilst the large-scale features in the oscillation probability are unaffected. The
2622 documentation and validation of this technique are found in section 7.1. The
2623 oscillation probability calculation is computationally intensive due to the large
2624 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-
2625 mentation choice made within the fitting framework, as detailed in section 7.2,
2626 ensures that the analysis can be done in a timely manner.

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.3. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrino oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation. This implementation of an external technique is described in section 7.4.

7.1 Treatment of Fast Oscillations

As shown in Figure 7.1, atmospheric neutrino oscillations have a significantly more complex structure for upgoing neutrinos with energy below 1GeV. This is because the L/E dependence of the oscillation probability in this region induces rapid variations for small changes in L or E . As discussed in section 2.5, this is also the region in which atmospheric neutrinos have sensitivity to δ_{CP} . In practice, the direction of the neutrino is inferred from the direction of the final state particles traveling in the detector. The correlation between these two directions can be particularly weak for low-energy neutrino interactions. This creates a distinct difference from the beam neutrinos where the position of the source is very precisely known.

As a consequence of the unresolvable structure, an event rate consistent with the averaged oscillation probability is observed in the subGeV upgoing region. This creates a computational problem: A significantly large amount of Monte Carlo statistics would be required to accurately predict the number of events if Monte Carlo averaging was the only technique used. This section describes

2657 the ‘sub-sampling’ approach developed for this analysis and compares it to the
2658 methodology used within the SK-only analysis.

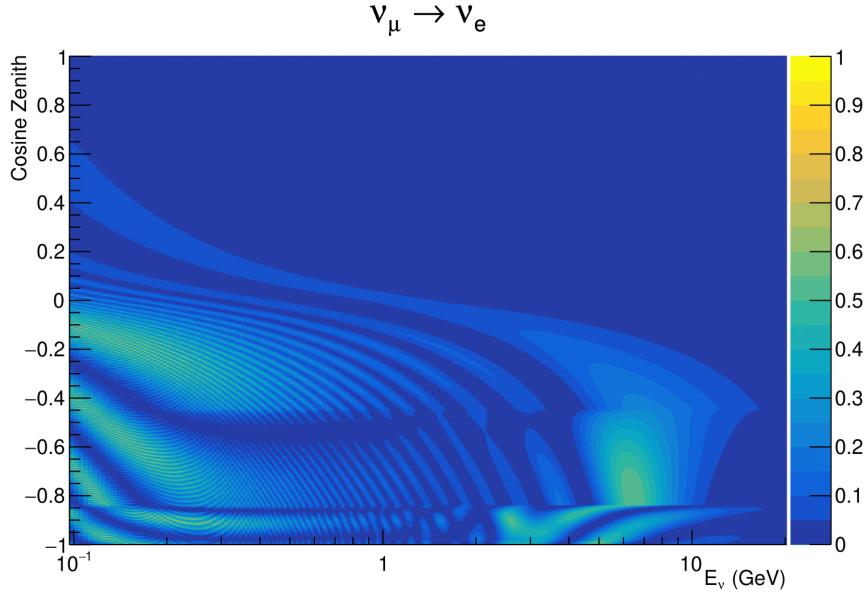


Figure 7.1: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2659 The official SK-only analysis uses the osc3++ oscillation parameter fitter
2660 [76]. To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’
2661 technique. For a given Monte Carlo neutrino event, the nearest twenty Monte
2662 Carlo neighbours in reconstructed lepton momentum and zenith angle are
2663 found and a distribution of their neutrino energies is built. The RMS, σ , of
2664 this distribution is then used to compute an average oscillation probability for
2665 the given neutrino Monte Carlo event.

2666 For the i^{th} event, the oscillation weight is calculated as

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1, -0.5, 0.5, 1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.1)$$

2667 where $P(E, L)$ is the oscillation probability calculation for neutrino energy E
2668 and path length L and the two path lengths, \bar{L}_i and L_β are described below. All
2669 of the oscillation probability calculations are performed with a fixed zenith angle
2670 such that the same density profile is used. The uncertainty in the production

2671 height is controlled by using an “average” production height, \bar{L}_i , which represents
2672 the average path length computed using twenty production heights taken from
2673 the Honda flux model’s prediction [52]. These inputs are provided in 5% intervals
2674 of the cumulative distribution function. The value of \bar{L}_i is calculated as:

$$\bar{L}_i = \frac{1}{20} \sum_{j=1}^{20} \sqrt{(R_E + h_j)^2 - R_E^2 (1 - \cos^2 \theta_i)} - R_E \cos \theta_i. \quad (7.2)$$

2675 Where R_E is the Earth’s radius and θ_i is the zenith angle of the i^{th} event.
2676 The production heights h_j represent the $(j \times 5)^{th}$ percentile of the cumulative
2677 distribution function. L_β values (where the values of β are given in Equa-
2678 tion 7.1) are similarly calculated but instead use different combinations of four
2679 production heights,

$$\begin{aligned} L_{-1.0} &= \frac{1}{4} L(45, 50, 55, 60), \\ L_{-0.5} &= \frac{1}{4} L(35, 40, 65, 70), \\ L_{+0.5} &= \frac{1}{4} L(25, 30, 75, 68), \\ L_{+1.0} &= \frac{1}{4} L(15, 20, 85, 89). \end{aligned} \quad (7.3)$$

2680 Where $L(i, j, k, l)$ represents the sum of the path lengths with fixed zenith
2681 angle and production heights corresponding to the i^{th} , j^{th} , k^{th} and l^{th} percentile
2682 of the cumulative distribution function. The values that are taken as β (and
2683 values for L_β) are chosen to smooth the oscillation contours in Δm_{32}^2 without
2684 incurring loss of sensitivity [201].

2685 This averaging technique works because of the inference between the zenith
2686 angle and the reconstructed direction of final state particles in the detector. For
2687 low-energy neutrinos, where the resolution of the true neutrino direction is poor,
2688 σ_i will be large, resulting in significant averaging effects. Contrary to this, the
2689 inferred direction of high-energy neutrinos will be much closer to the true value,
2690 meaning that σ_i will be smaller, culminating in small averaging effects.

In practice, these calculations are performed prior to the fit as only oscillation parameters at fixed points are considered. The MCMC technique used in this thesis requires oscillation probabilities to be evaluated at arbitrary parameter values, not known *a priori*. Calculating the five oscillation probabilities per event required by the SK technique is computationally infeasible, so a different averaging technique is used. However, the concept of the averaging technique can be taken from it.

To perform a similar averaging as the SK analysis, a sub-sampling approach using binned oscillograms has been devised. A coarsely binned oscillogram is defined in $\cos(\theta_Z)$ and E_ν . For a given set of oscillation parameters, a single oscillation probability will be assigned to each coarse bin. This value will then apply to all Monte Carlo events which fall into that bin. To assign these oscillation probabilities, the probability is calculated at $N \times N$ points on a grid within a particular bin. This ensemble of oscillation probabilities is averaged to define the coarse bin's oscillation probability, assuming a flat prior in E_ν and $\cos(\theta_Z)$ within the bin. Figure 7.2 illustrates the $N = 2$ example where the assigned value to a coarse bin is the average of the four fine bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear on either axis, the sub-division of the fine bins is linear within the range of a coarse bin.

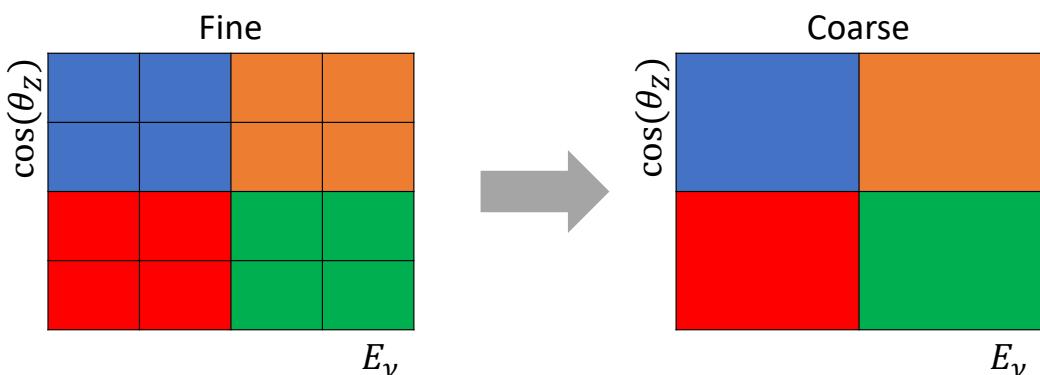


Figure 7.2: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2710 The coarse binning is defined with 67×52 bins in true neutrino energy \times
 2711 cosine zenith. It is picked to be identical to that provided in [201]. In general, the
 2712 binning is logarithmically spaced in neutrino energy but has some hand-picked
 2713 bin edges around the matter resonance to smoothly increased the bin density.
 2714 This is to avoid smearing this region which can be well sampled by the Monte
 2715 Carlo. The cosine zenith binning is approximately linearly spaced across the
 2716 allowable range but the values of layer transitions are hit precisely: -0.8376 (core-
 2717 mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for
 2718 downgoing events as this is a region unaffected by the fast oscillation wavelengths
 2719 and reduces the total number of calculations required to perform the calculation.

2720 The choice of N is justified based on two studies. Firstly, the variation of event
 2721 rates of each sample is studied as a function of N . For a given set of oscillation
 2722 parameters thrown from the PDG prior constraints (detailed in Table 2.1), the
 2723 oscillation probabilities are calculated using a given value of N . Each sample
 2724 is re-weighted and the event rate is stored. The value of N is scanned from
 2725 1, which corresponds to no averaging, to 19, which corresponds to the largest
 2726 computationally viable subdivision binning. The event rate of each sample at
 2727 large N is expected to converge to a stationary value due to the fine binning fully
 2728 sampling the small-scale structure. Figure 7.3 illustrates this behaviour for the
 2729 SubGeV_elike_0dcy sample for 9 different throws of the oscillation parameters.

2730 Denoting the event rate for one sample for a given throw t at each N by λ_t^N ,
 2731 the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The
 2732 variance in the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right) \right]^2. \quad (7.4)$$

2733 In practice, the following procedure is undertaken. For a particular throw,
 2734 the difference between the event rate at a particular choice of N and the mean
 2735 of the distribution is calculated. This is illustrated in Figure 7.4. This value
 2736 is then calculated for all the 2000 throws, generating a distribution of $\lambda_t^N - \bar{\lambda}_t$.

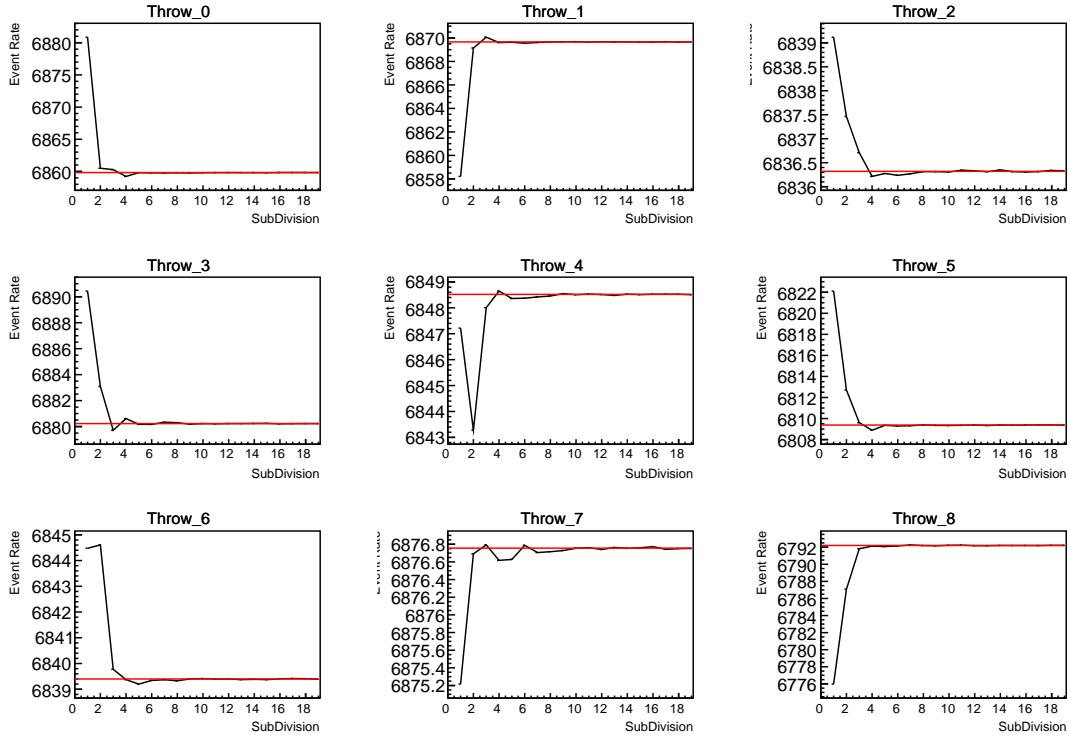


Figure 7.3: Event rate of the SubGeV_elike_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2737 This is repeated for each of the values of N considered within this study. The
 2738 distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.5. As expected,
 2739 the distribution gets narrower and tends towards zero for the higher values of N .

2740 The aim of the study is to find the lowest value of N such that this variance
 2741 is below 0.001. This utilises the width of the distributions given in Figure 7.5.
 2742 This is the typical threshold used by T2K fitters to validate systematic imple-
 2743 mentation so has been set as the same criteria. The results of this study for
 2744 each atmospheric sample used within this thesis are illustrated in Figure 7.6 for
 2745 2000 throws of the oscillation parameters. As can be seen, the variance is below
 2746 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy
 2747 and SubGeV_elike_0dcy samples.

2748 The second study to determine the value of N is as follows. The likelihood
 2749 for each sample is computed against an Asimov data set created with Asimov A

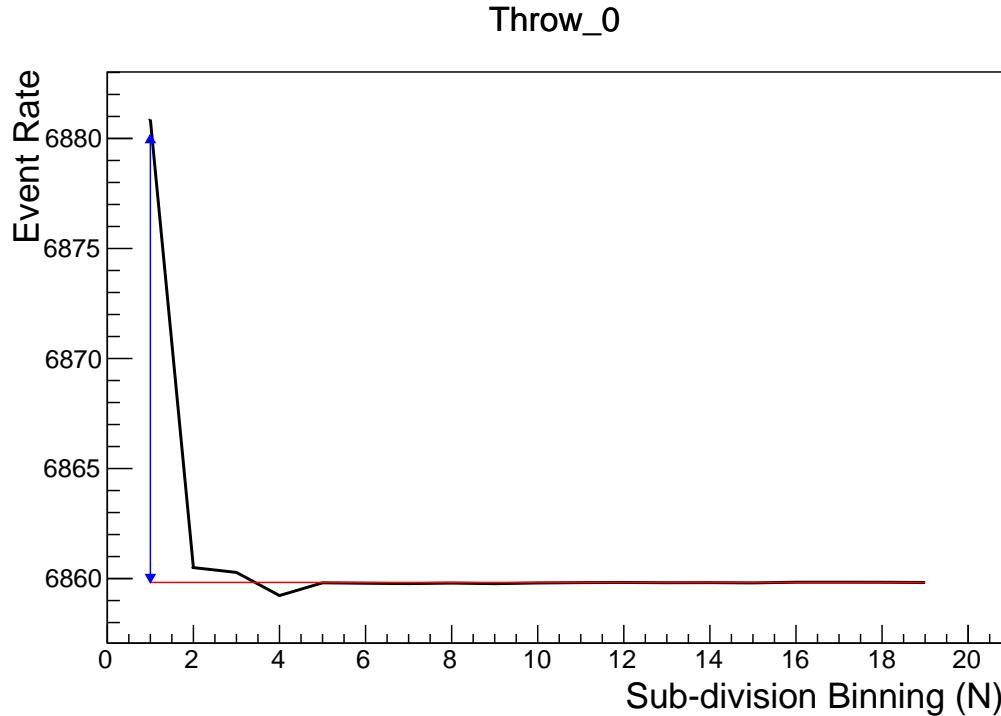


Figure 7.4: Event rate of the SubGeV_elike_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions, N , per coarse bin. The difference between the mean event rate (red), $\bar{\lambda}$, and the event rate at $N = 1$, $\lambda^{N=1}$ is defined as $\lambda^N - \bar{\lambda}$ and illustrated by the blue arrow.

2750 oscillation parameters (Table 2.2). Following Equation 7.4, the variance of the log-
2751 likelihood over all considered N is computed. The results are shown in Figure 7.7.

2752 A choice of $N = 10$ sub-divisions per coarse bin has a variance in both
2753 event rate and log-likelihood residuals less than the required threshold of 0.001.
2754 The largest value of the likelihood variance is of order 10^{-7} , corresponding to
2755 an error on the log-likelihood of about 3×10^{-4} which is small enough to be
2756 negligible for the oscillation analysis.

2757 Figure 7.8 illustrates the effect of the smearing using $N = 10$. The fast oscilla-
2758 tions in the sub-GeV upgoing region have been replaced with a normalisation
2759 effect whilst the large matter resonance structure remains.

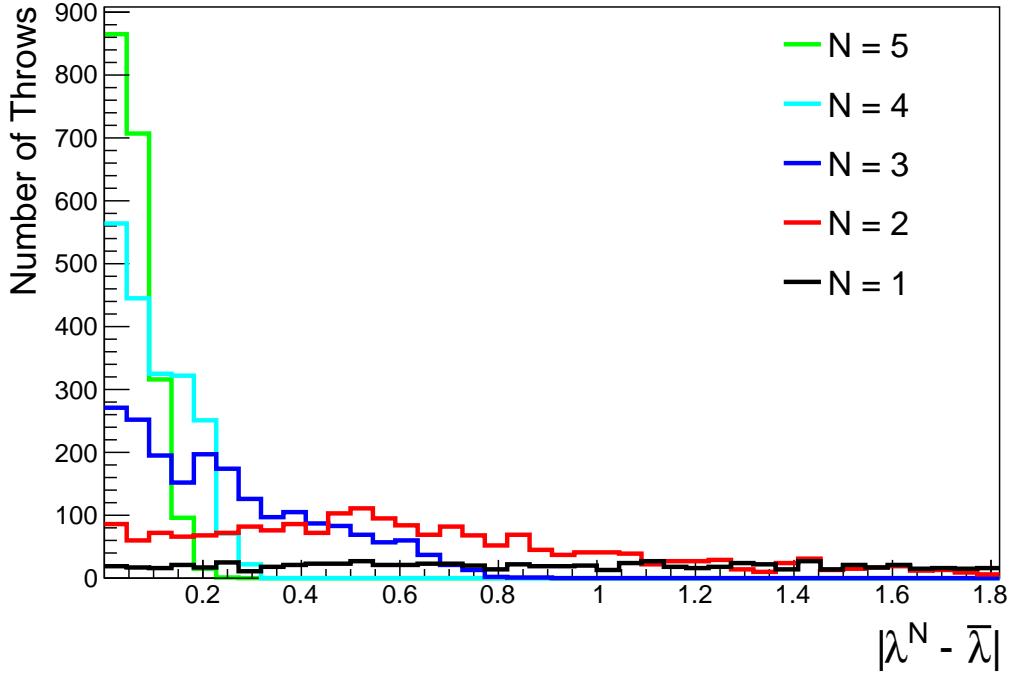


Figure 7.5: The distribution of $\lambda^N - \bar{\lambda}$ for various values of N . As expected, the distribution gets narrower for larger values of N .

2760 7.2 Calculation Engine

2761 As previously discussed in section 7.1, the calculation of oscillation probabilities
 2762 is performed at run-time. Consequently, the time per calculation is crucial for fit
 2763 performance. The initial fitting framework used for this analysis was developed
 2764 with ProbGPU [202]. This is a GPU-only implementation of the prob3 engine
 2765 [203]. It is primarily designed for neutrino propagation in a beam experiment
 2766 (single layer of constant density) with the atmospheric propagation code not
 2767 being used prior to the analysis in this thesis.

2768 Another engine, CUDAProb3 [204], has been interfaced with the fitting frame-
 2769 work used in this analysis. This interfacing was done by the author of this
 2770 thesis. It has been specifically optimised for atmospheric neutrino oscillation
 2771 calculation so does not contain the code to replace the beam oscillation calculation.
 2772 The engine utilises object-orientated techniques as compared to the functional
 2773 implementation of ProbGPU. This allows the energy and cosine zenith arrays to

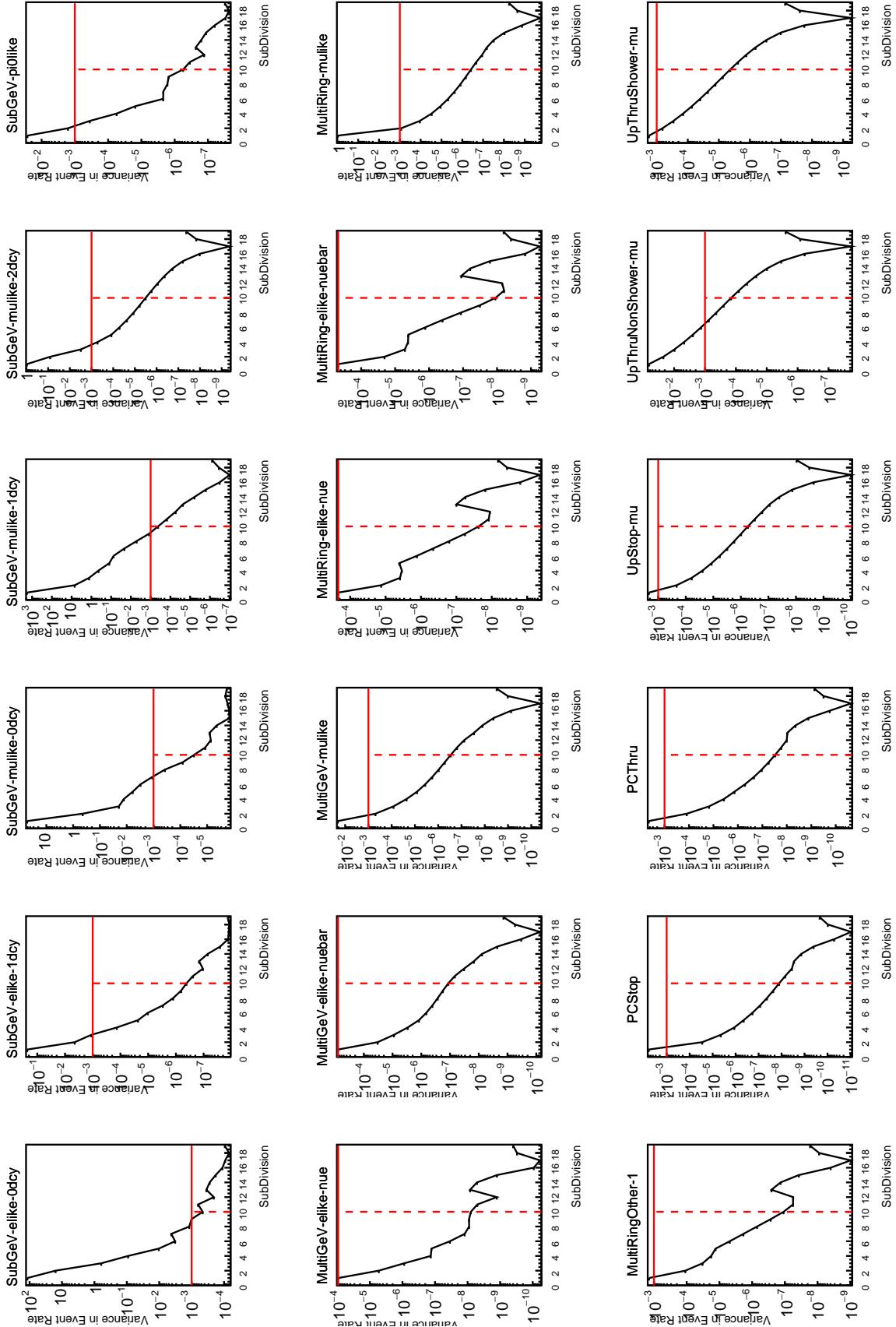


Figure 7.6: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

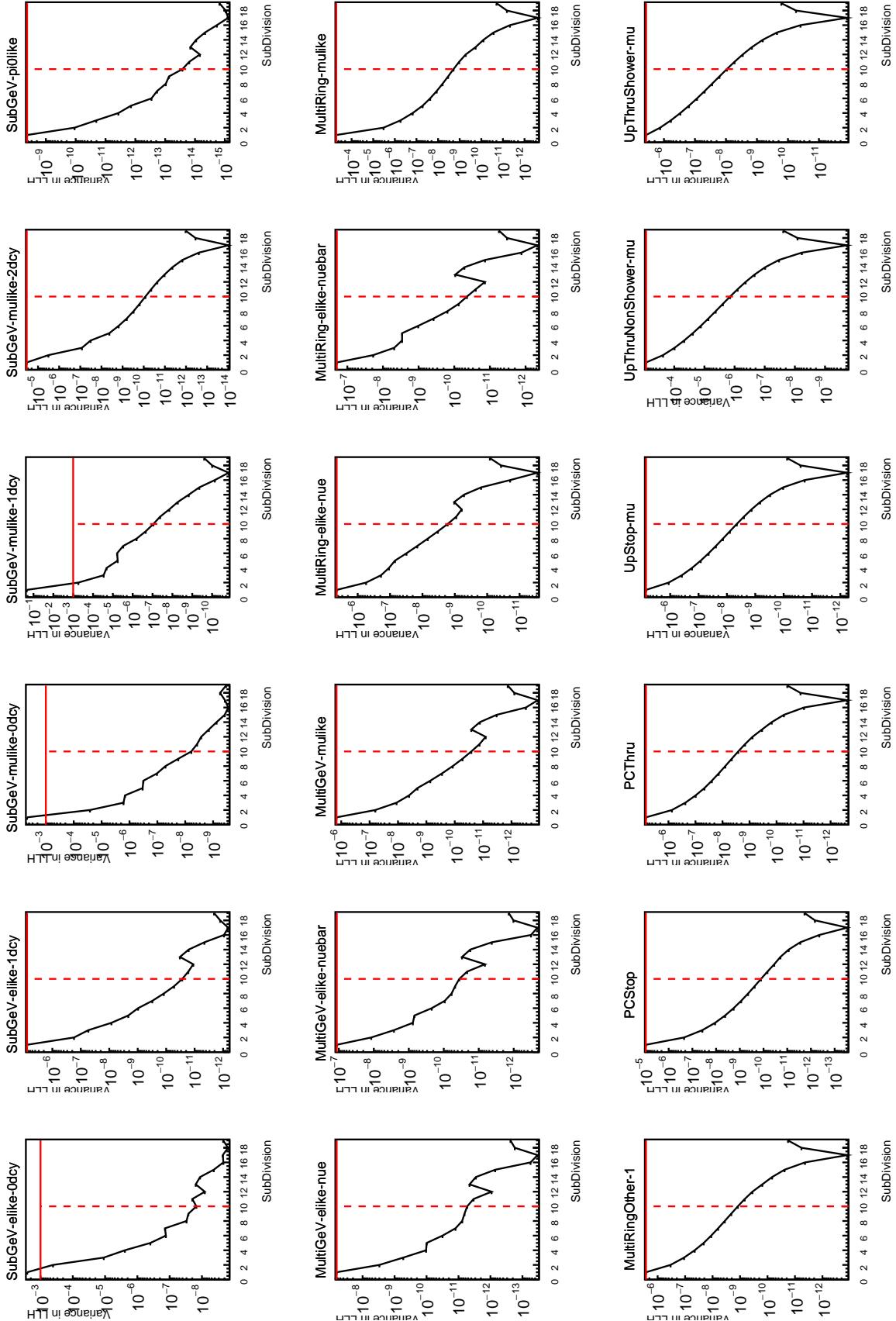


Figure 7.7: Variance of sample likelihood, when compared to 'Asimov data' set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

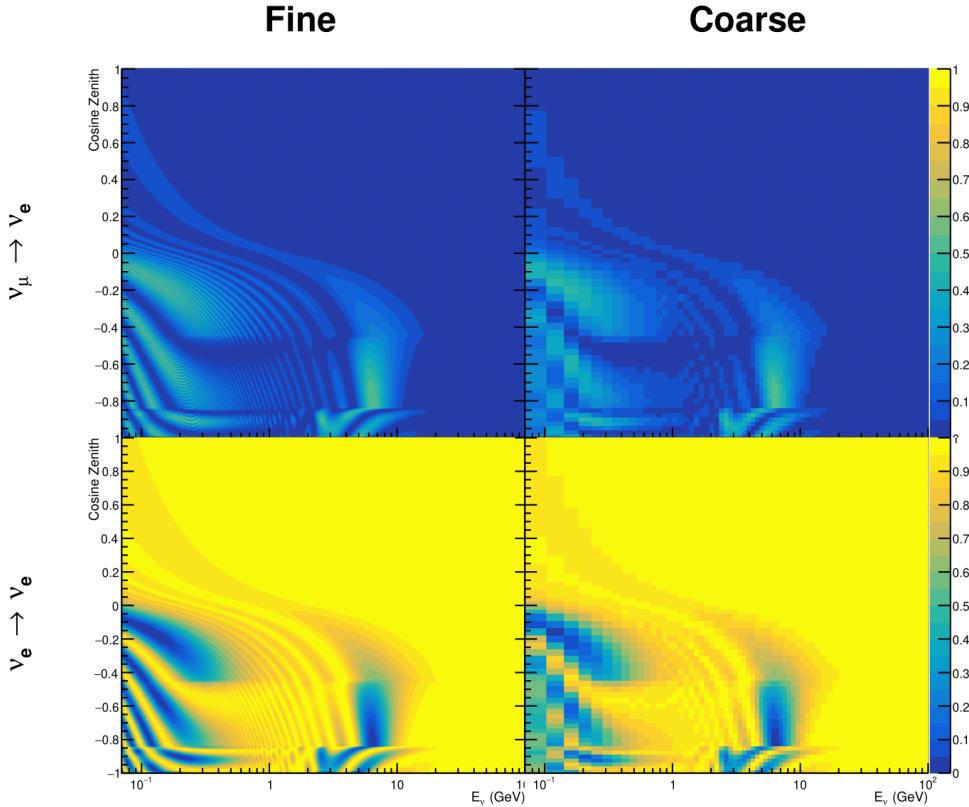


Figure 7.8: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.6 and Figure 7.7.

2774 be kept on GPU memory, rather than having to load these arrays onto GPU
 2775 memory for each calculation. Reducing the memory transfer between CPU and
 2776 GPU significantly reduces the time required for calculation. This can be seen
 2777 in Figure 7.9, where the GPU implementation of CUDAProb3 is approximately
 2778 three times faster than the ProbGPU engine.

2779 Another significant advantage of CUDAProb3 is that it contains a CPU multi-
 2780 threaded implementation which is not possible with the ProbGPU or prob3 engines.
 2781 This eliminates the requirement for GPU resources when submitting jobs to batch
 2782 systems. As illustrated in Figure 7.9, the calculation speed depends on the number

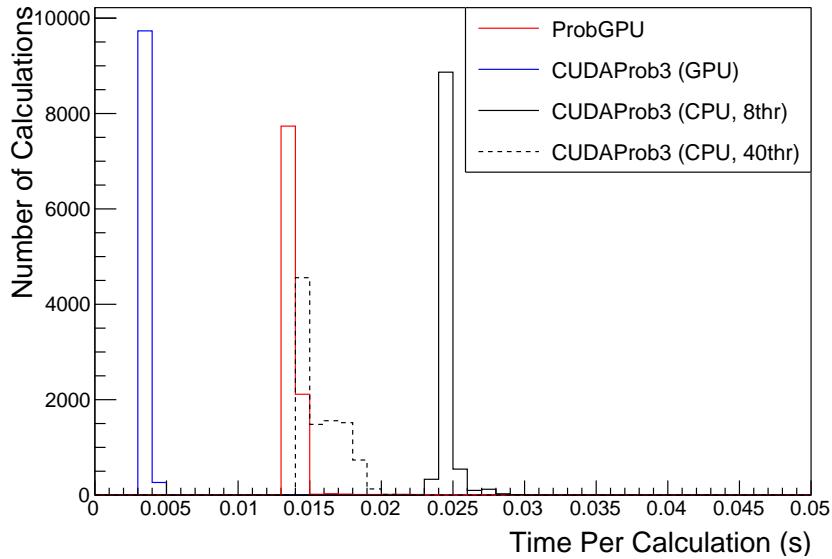


Figure 7.9: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.1, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multi-threaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

of available threads. Using 8 threads (which is typical of the batch systems being used) is approximately twice as slow as the ProbGPU engine implementation, but would allow the fitting framework to be run on many more resources. This fact is utilised for any SK-only fits but GPU resources are required for any fits which include beam samples due to the ProbGPU requirement. Based on the benefits shown by the implementation in this section, efforts are being placed into including linear propagation for beam neutrino propagation into the engine [205].

7.3 Matter Density Profile

For an experiment observing neutrinos propagating through the Earth, a model of the Earth’s density profile is required. The model used within this analysis is based on the Preliminary Reference Earth Model (PREM) [77], as illustrated in Figure 2.8. Table 2.3 documents the density and radii of the layers used within the constant density approximation used by the SK-only analysis [76]. The

density measurements provided in the PREM model are provided in terms of mass density, whereas neutrino oscillations are sensitive to the electron number density. This value can be computed as the product of the chemical composition, or the Z/A value, and the mass density of each layer. Currently, the only way to measure the chemical composition value for layers close to the Earth's core is through neutrino oscillations. The chemical composition of the upper layers of the Earth's Mantle and the Transition zone is well known due to it being predominantly pyrolite which has a chemical composition value of 0.496 [206]. The chemical composition dial for the core layers is set to a value of 0.468, as calculated in [207]. As this value is less well known, it is assigned a Gaussian error with a standard deviation equivalent to the difference in chemical composition in core and mantle layers. Figure 7.10 illustrates the effect of moving from the $Z/A = 0.5$ method which is used in the official SK-only analysis to these more precise values.

The beam oscillation probability in this thesis uses a baseline of 295km, density 2.6g/cm^3 , and chemical composition 0.5 as is done by the official T2K-only analysis [208].

For a neutrino with given $E_\nu, \cos(\theta_Z)$, the oscillation probability calculation engine must be passed a list of the matter regions that the neutrino traversed, with the path length and fixed density in each region. However, a neutrino passing through the earth experiences a range of radii, and thus a range of densities, in each region. In the SK-only analysis, the earth density model used is piecewise-constant, thereby ignoring this effect. For this thesis, the density values for the calculation engine are found by averaging the earth density along the neutrino's path in each layer,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt, \quad (7.5)$$

where t_i are the intersection points between each layer and t is the path length of the trajectory across the layer. This leads to an improved approximation. For this averaging, the simplification of the PREM model developed in [209] is

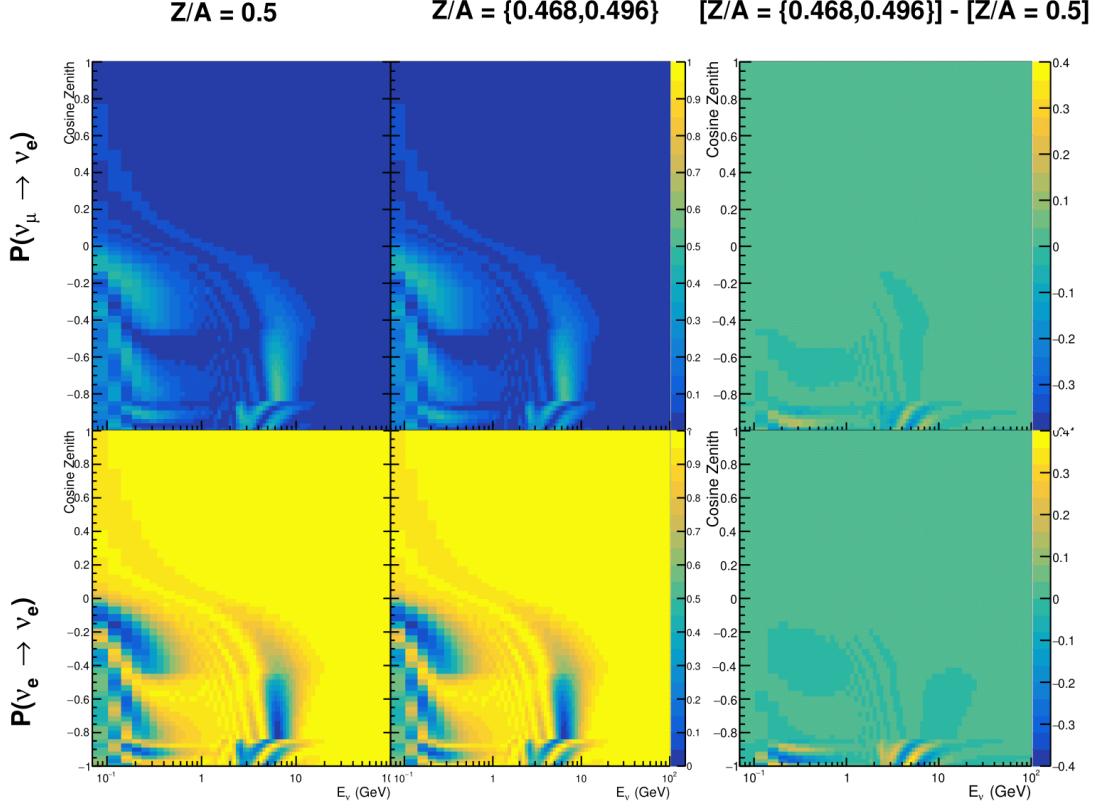


Figure 7.10: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 2.3. The right column illustrates the difference in oscillation probability between the two different techniques.

used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [201]. The coefficients of the quadratic fit to each layer are given in Table 7.1 with the final distribution illustrated in Figure 7.11. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in Figure 7.12. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -0.45$ in the fixed density model, which is due to the transition to

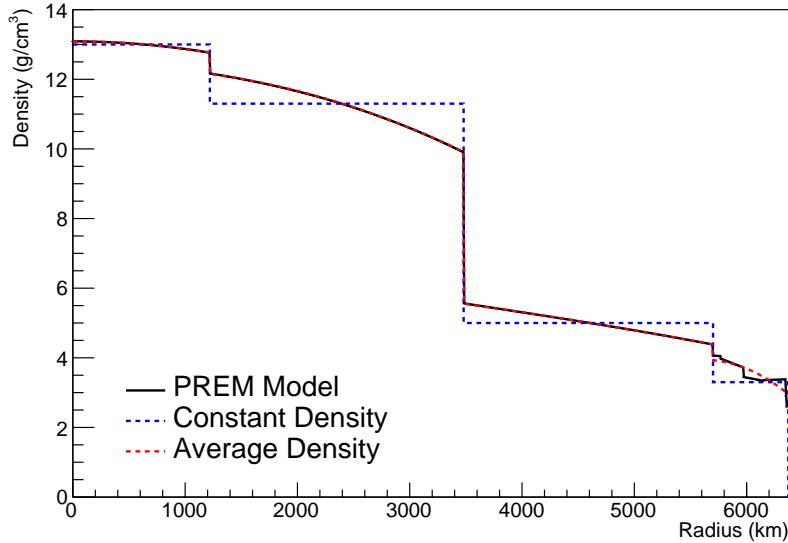


Figure 7.11: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.1: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [201], where $x = R / R_{\text{Earth}}$.

mantle layer boundary, has been reduced. This is expected as the difference in the density across this boundary is significantly smaller in the quadratic density model as compared to the constant density model. Whilst the difference in density across the other layer transitions is reduced, there is still a significant difference. This means the discontinuities in the oscillation probabilities remain but are significantly reduced. However, as the quadratic density approximation matches the PREM model well in this region, these discontinuities are due to the Earth model rather than an artifact of the oscillation calculation.

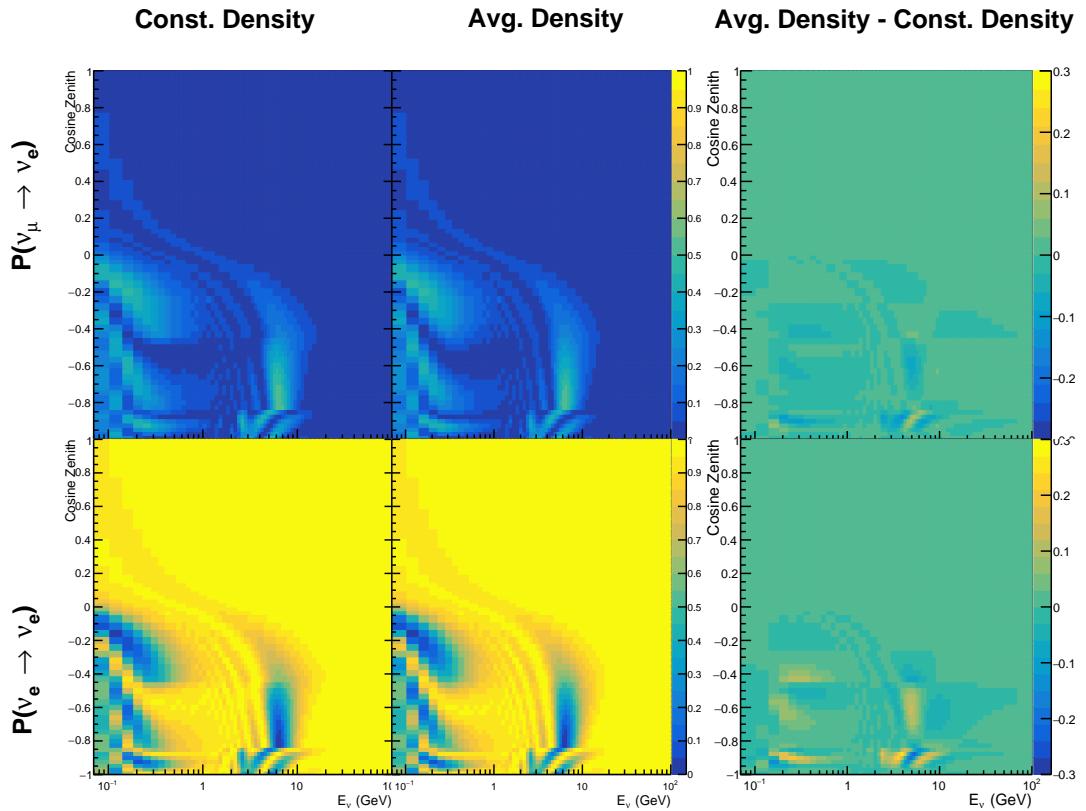


Figure 7.12: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2841 7.4 Production Height Averaging

2842 As discussed in section 2.5, the height at which the cosmic ray flux interacts
 2843 in the atmosphere is not known on an event-by-event basis. The production
 2844 height can vary from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only
 2845 analysis methodology (described in section 7.1) for including the uncertainty
 2846 on the production height is to include variations from the Honda model when
 2847 pre-calculating the oscillation probabilities prior to the fit. This technique is not
 2848 possible for this analysis which uses continuous oscillation parameters that can
 2849 not be known prior to the fit. Consequently, an analytical averaging technique
 2850 was developed in [201]. The author of this thesis was not responsible for the
 2851 derivation of the technique but has performed the implementation and validation
 2852 of the technique for this analysis alone.

2853 Using the 20 production heights per Monte Carlo neutrino event, provided
 2854 as 5% percentiles from the Honda flux model, a production height distribution
 2855 $p_j(h|E_\nu, \cos \theta_Z)$ is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a
 2856 histogram is filled with 20 evenly spaced bins in production height h between
 2857 0 and 50km. The neutrino energy and cosine zenith binning of the histogram
 2858 are the same as that provided in section 7.1. The average production height,
 2859 $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. This assumes a linear average over
 2860 the four flavours of neutrino which are considered to be generated in cosmic
 2861 ray showers. The production height binning of this histogram is then translated
 2862 into $\delta t(h) = t(\bar{h}) - t(h)$, where $t(x)$ is the distance travelled along the trajectory
 2863 in the atmosphere from some production height, x .

2864 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed.
 2865 The time-ordered product of these is then used as the overall transition amplitude
 2866 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.6)$$

2867 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.7)$$

2868 is expressed as a diagonalised time-dependent solution to the Schrodinger
 2869 equation. The 0th layer is the propagation through the atmosphere and is the
 2870 only term that depends on the production height. Using the substitution $t_0 =$
 2871 $t(\bar{h}) - \delta t(h)$, it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.8)$$

2872 Thus Equation 7.6 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.9)$$

2873 The oscillation probability averaged over production height is then calculated
 2874 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)_{ij}^* \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m) \delta t]. \end{aligned} \quad (7.10)$$

2875 It is important to note that the exact value of \bar{h} used does not matter as the
 2876 values of δt would change to compensate for any modification to the value of \bar{h} .

2877 In practice, implementation in CUDAProb3 [204] is relatively straightforward
 2878 as the majority of these terms are already calculated in the standard oscillation
 2879 calculation. Figure 7.13 illustrates the results of the production height averaging.

2880 As expected, the main effect is observed in the low-energy downward-going
 2881 and horizontal-going events. Upward-going events have to travel the radius
 2882 of the Earth, $R_E = 6371\text{km}$, where the production height uncertainty is a small
 2883 fraction of the total path length.

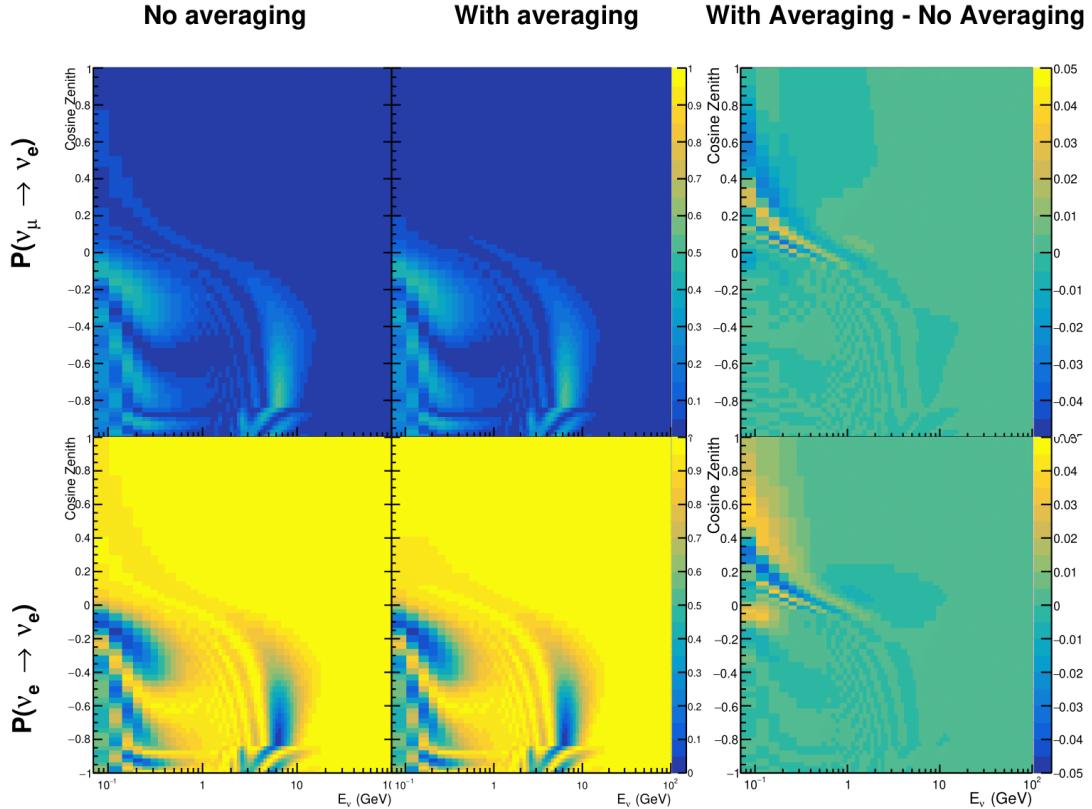


Figure 7.13: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

8

2884

2885

Oscillation Analysis

2886 Using the samples and systematics defined in chapter 6, this chapter documents
2887 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK
2888 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4
2889 is used for all studies performed within this thesis.

2890 The MaCh3 framework has been validated through many tests. The code
2891 that handles the beam far detector samples was developed by the author and
2892 validated by comparison to the 2020 T2K analysis [2]. The sample event rates and
2893 likelihood evaluations of beam samples generated by the framework used within
2894 this thesis were compared to those from the T2K analysis by the author of this
2895 thesis. Variations of the sample predictions were compared at $\pm 1\sigma$ and $\pm 3\sigma$ and
2896 good agreement was found in all cases. A similar study, led by Dr. C. Wret was
2897 used to validate the near detector portion of the code [194]. The implementation
2898 of the atmospheric samples within MaCh3 was completed and cross-checked by
2899 the author of this thesis against the P-Theta framework (introduced in section 3.2).
2900 Both fitters are provided with the same inputs and can therefore cross-validate
2901 each other. These validations compared the event rate and likelihood calculation.
2902 Documentation of all the above validations can be found in [194]. These stringent
2903 validations ensure that the code is doing as intended.

2904 8.1 Monte Carlo Prediction

- 2905 Using the three sets of dial values (generated, pre-fit, and post-fit tunes) defined
 2906 in subsection 6.4.3, the predicted event rates for each sample are given in Table 8.1.
 2907 The oscillated and un-oscillated event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThrus	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1: The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

- 2908 Generally, the samples that target CCQE interaction modes observe a decrease
 2909 in prediction when comparing the generated values with the pre-fit dial values.
 2910 This is in accordance with the Monte Carlo being produced at $M_A^{QE} = 1.21\text{GeV}$
 2911 [153] whilst the pre-fit dial value is set to $M_A^{QE} = 1.03\text{GeV}$ as suggested by [193].

Furthermore, the predicted event rates of samples that target CCRES interaction modes are significantly reduced when considering the post-BANFF fit. This follows the observations in subsection 6.4.3. The strength of the accelerator neutrino experiment can be seen in the remarkable difference between the oscillated and unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very clear decrease in the expected event rate between the oscillated and un-oscillated predictions which is not as obvious as in the atmospheric samples. This is due to the fact that the beam energy is tuned to the maximum disappearance probability, which is not the case for the naturally generated atmospheric neutrinos.

8.2 Likelihood Scans

Using the definition of the likelihood presented in section 6.5, the contribution of each sample to the likelihood from a variation of a particular parameter can be studied. This process identifies which samples drive the determination of the oscillation parameters in the joint fit. Figure 8.1 presents the variation of all the samples (beam and atmospheric) at the far detector to the oscillation parameters of interest: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . These plots are colloquially called ‘likelihood scans’ (or ‘log-likelihood scans’). The process of making these plots is as follows. An Asimov data set is built using the AsimovA oscillation parameters and pre-fit systematic tune. The Monte Carlo is then reweighted using the value of the oscillation parameter at each point on the x-axis of the scan. The likelihood is then calculated between the Asimov data and Monte Carlo prediction and plotted.

Due to the caveat of fixed systematic parameters and the correlations between oscillation parameters being ignored when creating these likelihood scans, the value of $\chi^2 = 1$ (or $-2 \times \ln(\text{Likelihood}) = 1$) does not equate to the typical 1σ sensitivity. However, it does give an indication of which samples respond most strongly to variations in a particular oscillation parameter. The point at

²⁹⁴⁰ which the likelihood tends to zero illustrates the value of the parameter used
²⁹⁴¹ to build the Asimov data prediction.

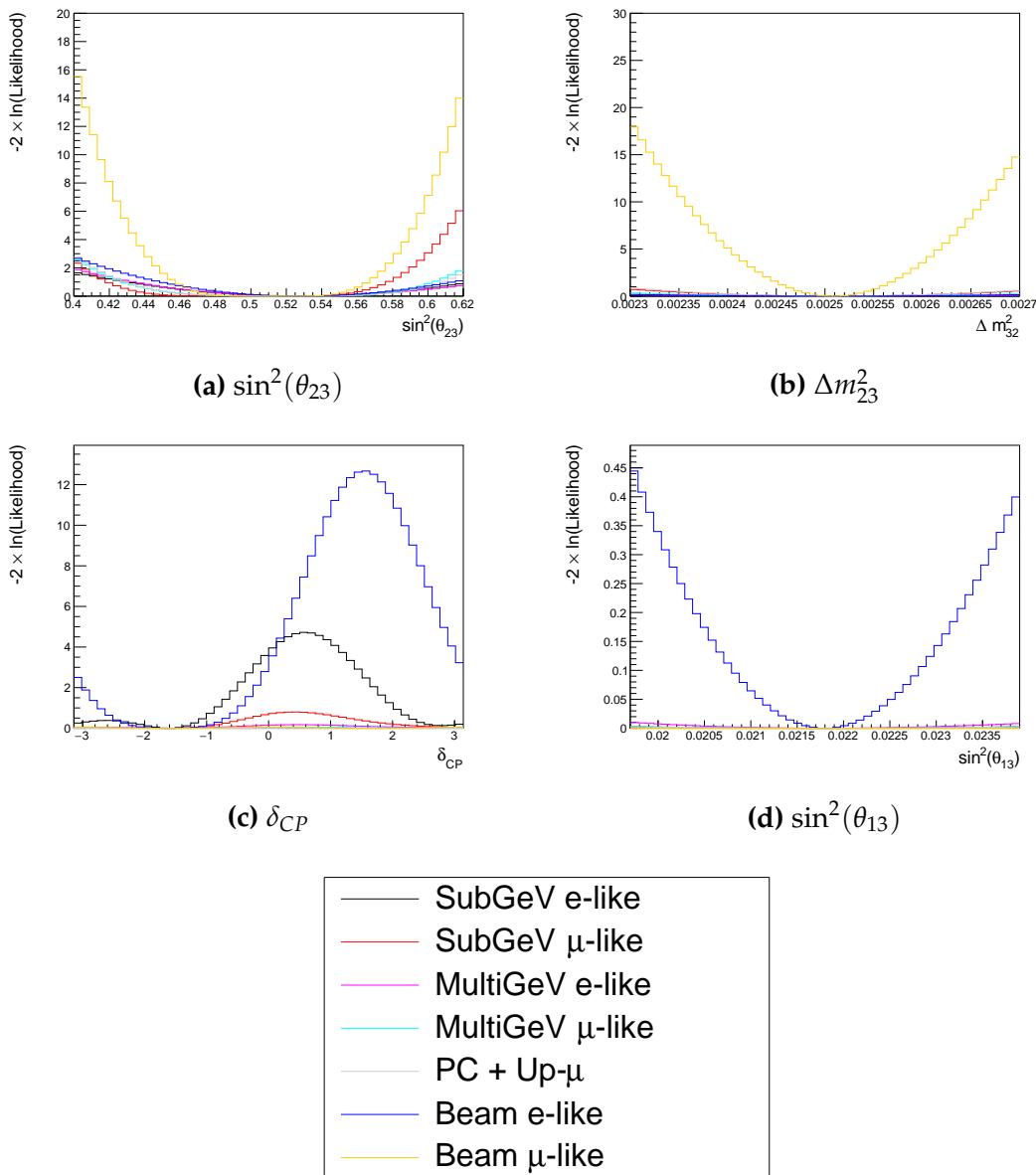


Figure 8.1: The response of the likelihood, as defined in section 6.5, illustrating the response of the samples to a variation of an oscillation parameter.

The sensitivity to $\sin^2(\theta_{23})$ is mostly dominated by the beam muon-like samples. The response of an individual atmospheric sample is small but non-negligible such that the summed response over all atmospheric samples becomes comparable to that of the muon-like beam samples. Consequently, the sensitivity

of the joint fit to $\sin^2(\theta_{23})$ would be expected to be greater than the beam-only analysis. The only sample that responds to the $\sin^2(\theta_{13})$ oscillation parameter is the electron-like beam sample. Consequently, no increase in sensitivity beyond that of the T2K-only analysis would be expected from the joint fit. Regardless, the sensitivity of the beam sample is significantly weaker than the external reactor constraint so prior knowledge will dominate any sensitivity to $\sin^2(\theta_{13})$ which is included within this thesis. The Δm_{21}^2 and $\sin^2(\theta_{12})$ parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to Δm_{32}^2 is completely dominated by the beam muon-like samples. This is because the beam neutrino energy is specifically tuned to match the maximal disappearance probability. Despite this, improvements to the $|\Delta m_{32}^2|$ sensitivity may be expected due to additional mass hierarchy determination added by the atmospheric samples.

Two-dimensional scans of the appearance ($\sin^2(\theta_{13}) - \delta_{CP}$) and disappearance ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) parameters are illustrated in Figure 8.2 and Figure 8.3, respectively. The caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still apply.

The appearance log-likelihood scans show the distinct difference in how the beam and atmospheric samples respond. The beam samples have an approximately constant width of the 2σ and 3σ contours, throughout all ranges of δ_{CP} . Whereas, the response of the atmospheric samples to $\sin^2(\theta_{13})$ is very strongly correlated to the value of δ_{CP} . At higher values of $\sin^2(\theta_{13})$, two lobes appear around $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2.4$. Consequently, this difference allows some of the degeneracy in a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood scans, the 2σ continuous contour in δ_{CP} for beam samples becomes closed when the atmospheric samples are added. This may result in a stronger sensitivity to δ_{CP} . Similarly, the width of the 3σ contours also becomes dependent upon the value of δ_{CP} . Furthermore, atmospheric samples have little sensitivity to $\sin^2(\theta_{13})$ on their own, as evidenced in Figure 8.1, but may improve sensitivity to the parameter when combined within the simultaneous

2976 fit. It is important to remember that these likelihood scans are not sensitivity
2977 measurements as the systematic parameters are fixed and the correlation between
2978 oscillation parameters is neglected. However, they are a very encouraging result
2979 for the joint fit.

2980 The disappearance log-likelihood scans in $\sin^2(\theta_{23}) - \Delta m_{32}^2$ space (Figure 8.3)
2981 show the expected behaviour when considering the one-dimensional scans
2982 already discussed. The uncertainty on the width of $|\Delta m_{32}^2|$ is mostly driven by the
2983 beam samples. However, the width of this contour in the inverted mass region
2984 ($\Delta m_{32}^2 < 0$) is significantly reduced due to the ability of the atmospheric samples
2985 to select the correct (normal) mass hierarchy. The width of the uncertainty
2986 in $\sin^2(\theta_{23})$ is also reduced compared to the beam-only sensitivities, with a
2987 further decrease in the inverted hierarchy region due to the better mass hierarchy
2988 determination.

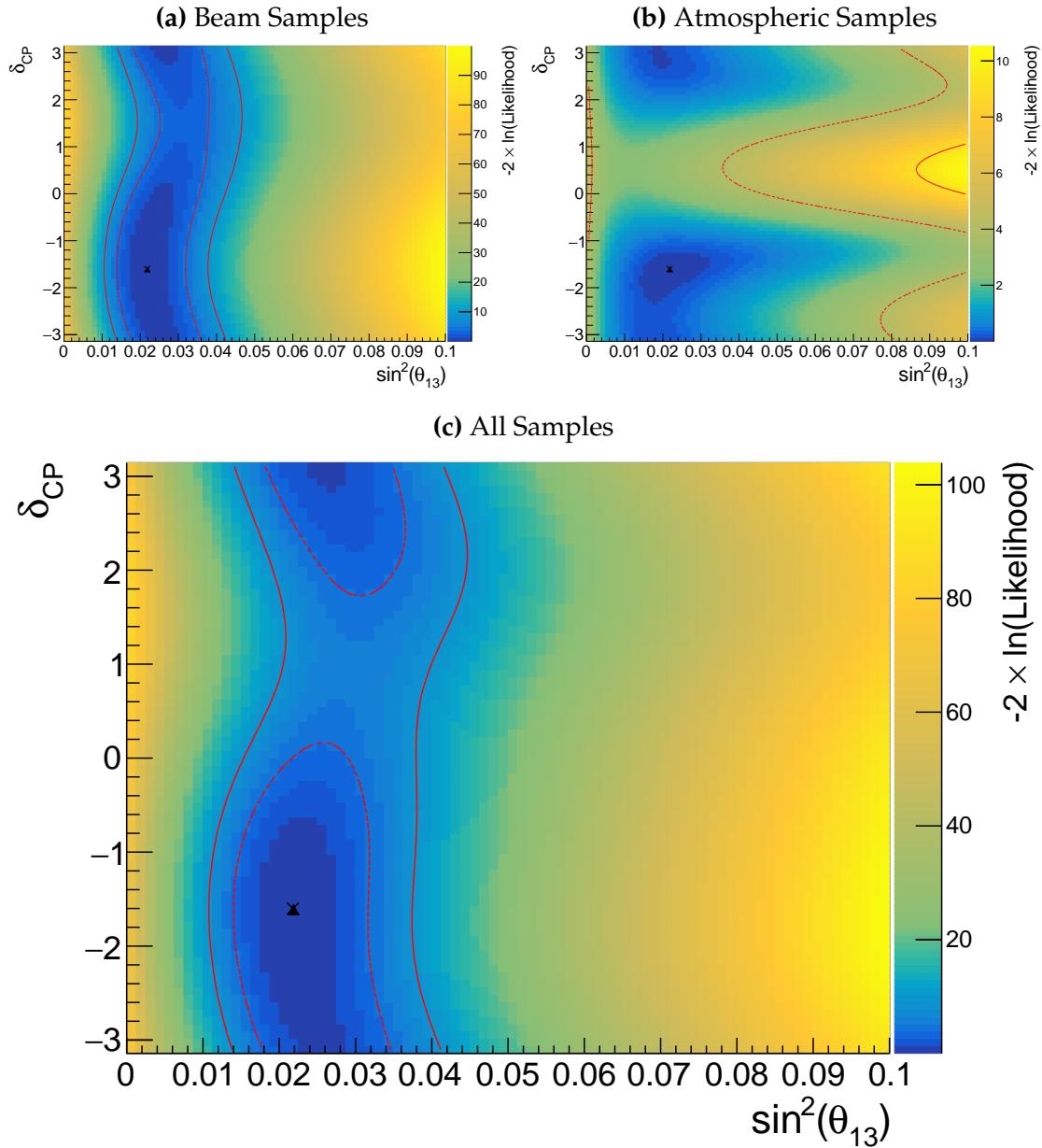


Figure 8.2: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})-\delta_{CP}$) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

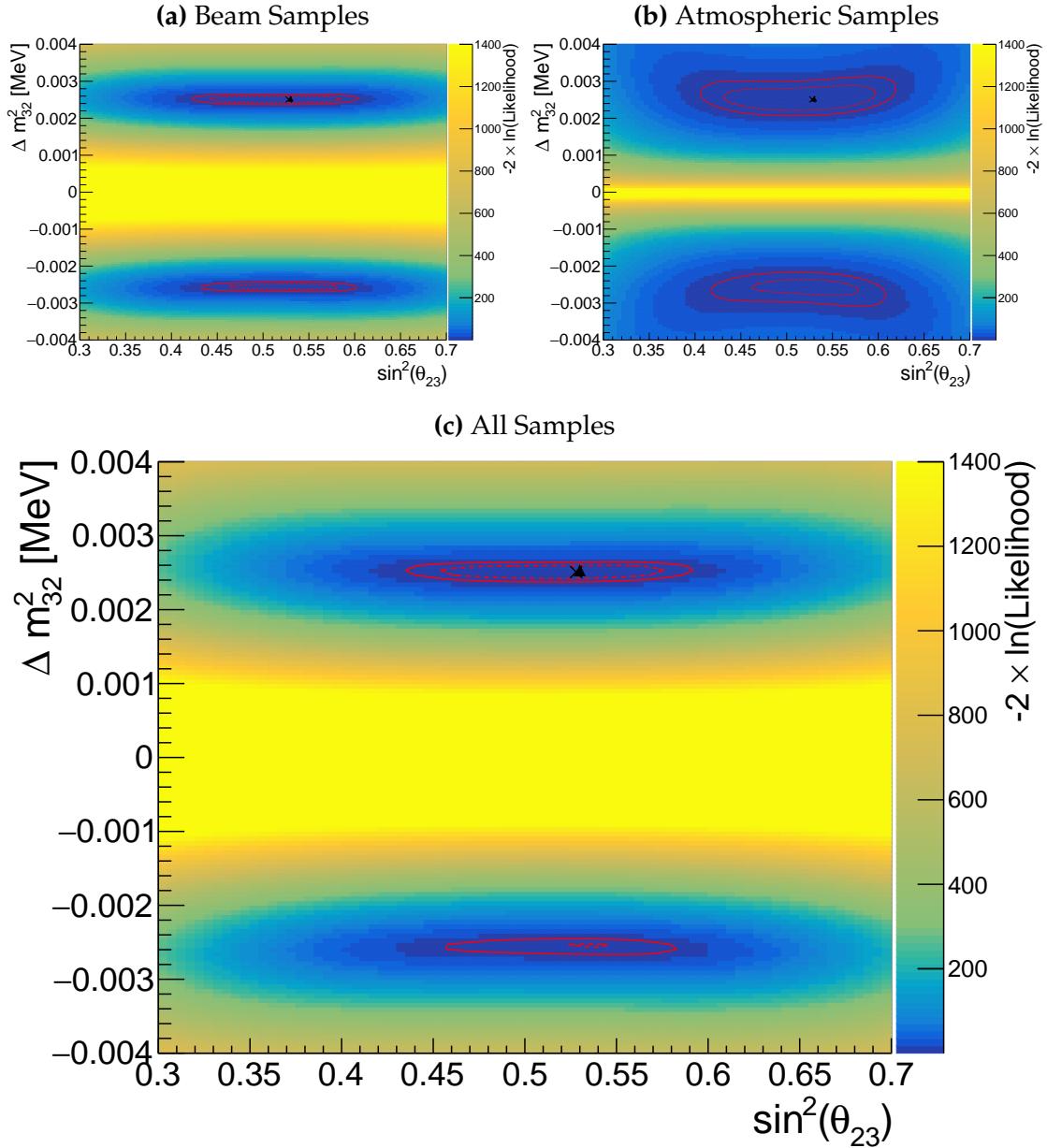


Figure 8.3: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23})$)– Δm_{32}^2) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

The likelihood scans illustrated thus far only consider the sensitivity of this analysis for a fixed set of true oscillation parameters, namely Asimov A defined in Table 2.2. Whilst computationally infeasible to run many fits at different parameter sets, it is possible to calculate the likelihood response to different Asimov data sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing true values of δ_{CP} and $\sin^2(\theta_{23})$, respectively. For both of these plots, the other oscillation parameters are fixed at their Asimov A values. Consequently, the caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still applies.

To explain how these plots are made, consider Figure 8.4. This plot is built by considering multiple one-dimensional log-likelihood scans, each creating an Asimov data set with the value of δ_{CP} taken from the x-axis. The likelihood to this particular Asimov data set is calculated after reweighting the Monte Carlo prediction to each value of δ_{CP} on the y-axis.

Figure 8.4 illustrates the sensitivity to δ_{CP} . To interpret this plot, larger contours result in more parameter space being excluded from the 1σ region. The 1σ intervals contain regions where the beam and atmospheric samples have discontinuous contours. For example, for the x-axis value of $\delta_{CP} = 0$, the beam samples sensitivity would include two discontinuous regions excluded from the 1σ interval: $\delta_{CP} \sim 0$ and $\delta_{CP} \sim \pi$. This behaviour is also seen in atmospheric samples response but at a value of $\delta_{CP} \sim -1$. This difference allows the joint fit to have increased sensitivity to these regions. Consequently, the difference between the beam-only and joint beam-atmospheric fit should be studied using multiple Asimov data sets.

Despite the increased sensitivity at 1σ , the 2σ intervals from the joint fit are more similar to the two independent sensitivities and the off-diagonal degeneracies mostly remain. This indicates that the joint fit has the strength to aid parameter determination but can not entirely break the degeneracies in δ_{CP} at higher confidence levels.

3018 Figure 8.5 illustrates a similar analysis as above, although the value of $\sin^2(\theta_{23})$
3019 is varied and δ_{CP} is fixed to the Asimov A parameter value. Due to the beam
3020 parameters and baseline being tuned to specifically target this oscillation parame-
3021 ter, the average sensitivity of the beam samples is stronger than the atmospheric
3022 samples. However, the degeneracy around maximal mixing ($\sin^2(\theta_{23}) = 0.5$) is
3023 significantly more peaked in the beam samples compared to the atmospheric
3024 samples. This means that a value of $\sin^2(\theta_{23}) \sim 0.56$ would be contained within
3025 the 1σ confidence interval for a true value of $\sin^2(\theta_{23}) \sim 0.46$ if using the beam-
3026 only analysis, whereas it would be excluded in the joint analysis.

3027 This behaviour is strengthened when considering the 2σ intervals, to the
3028 point where two distinct discontinuous regions of the 2σ intervals exist around
3029 the Asimov point $\sin^2(\theta_{23}) \sim 0.41, 0.6$. Given the caveat of only considering
3030 likelihood scans, the joint analysis would mostly eliminate the discontinuous
3031 intervals in these regions. This means that the joint fit could feasibly have an
3032 increased preference for the correct octant hypothesis.

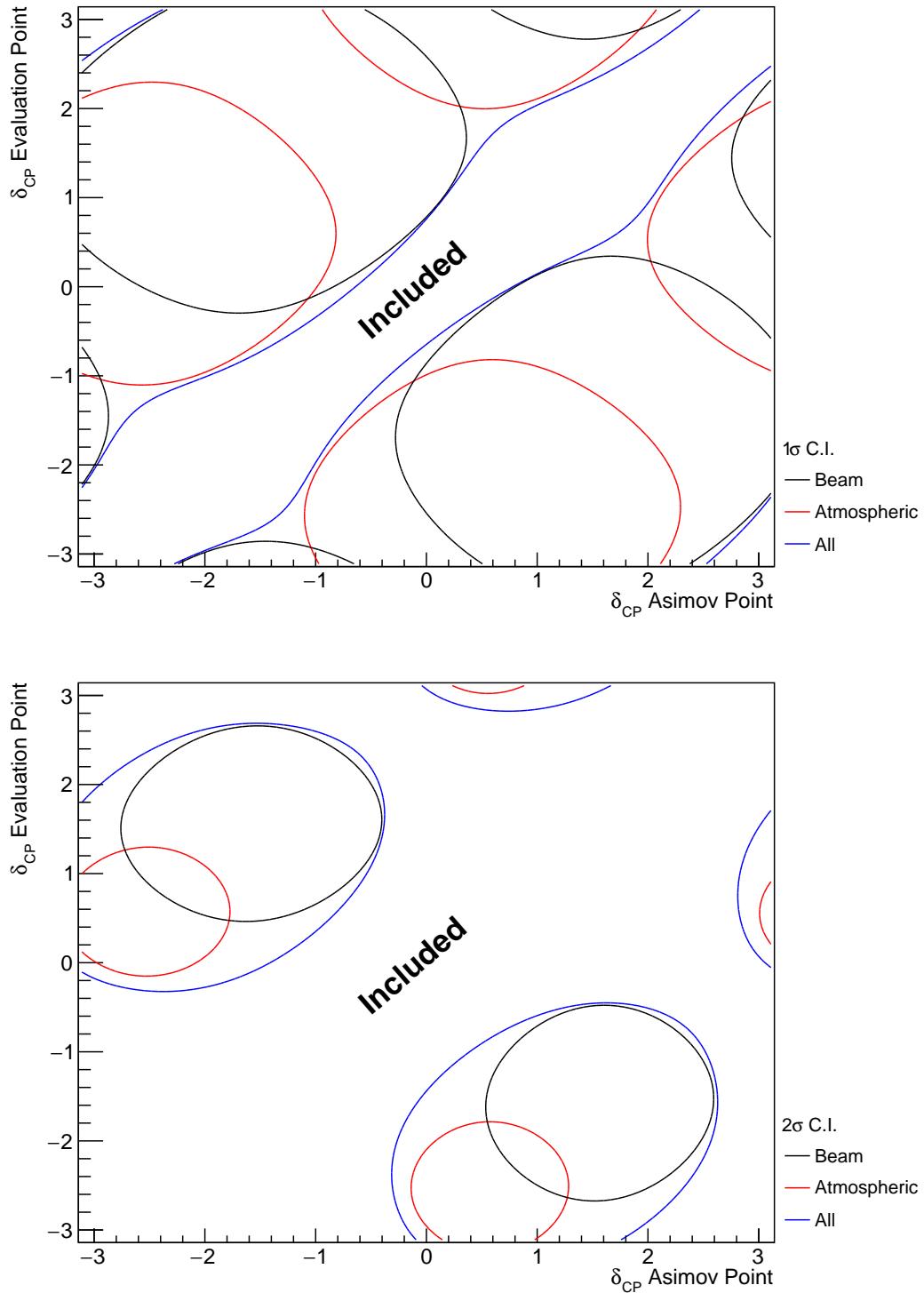


Figure 8.4: A series of one-dimensional likelihood scans over δ_{CP} , where an Asimov data set is built for each value of δ_{CP} on the x-axis and the likelihood is evaluated for each value of δ_{CP} on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

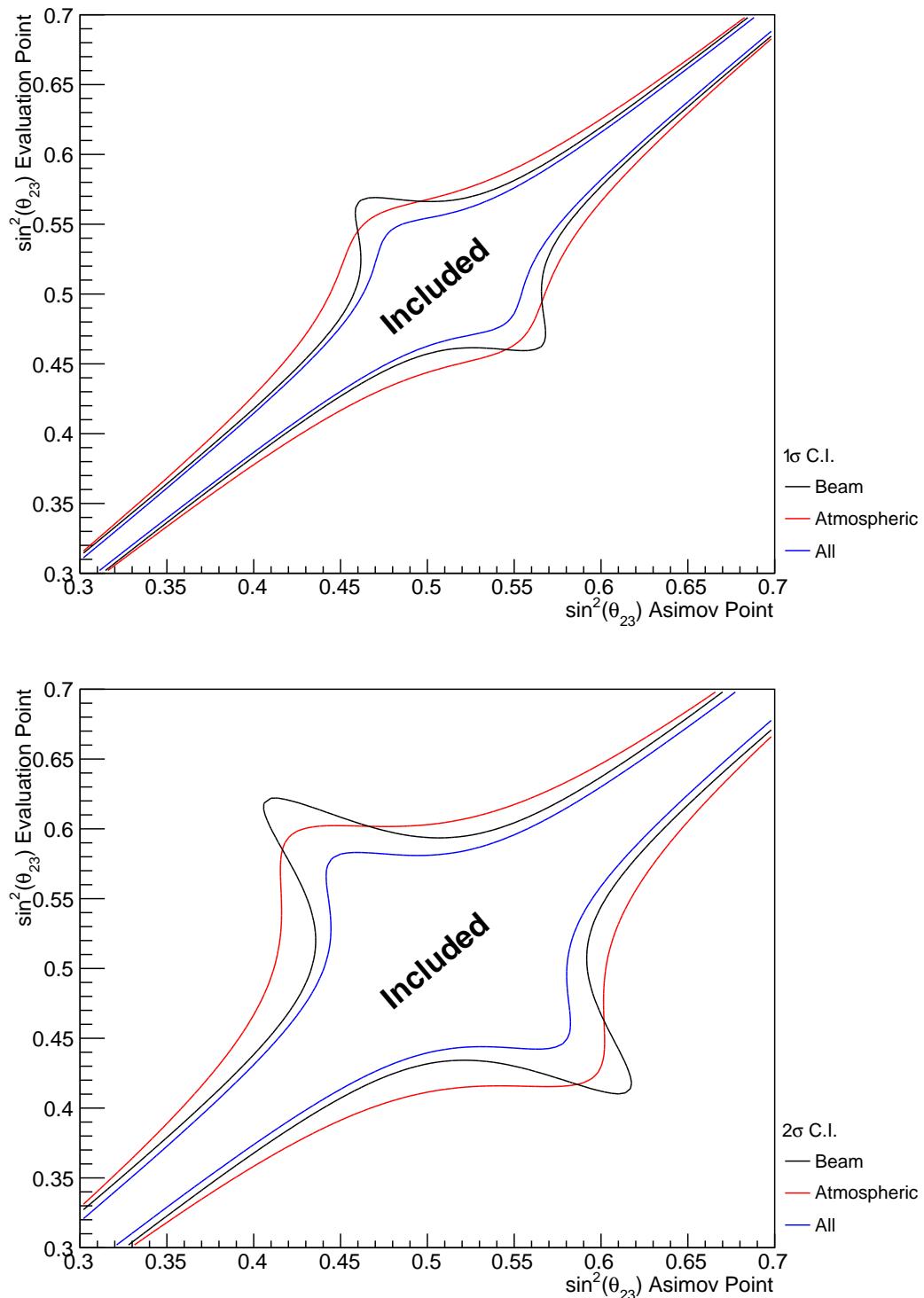


Figure 8.5: A series of one-dimensional likelihood scans over $\sin^2(\theta_{23})$, where an Asimov data set is built for each value of $\sin^2(\theta_{23})$ on the x-axis and the likelihood is evaluated for each value of $\sin^2(\theta_{23})$ on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions, M_A^{QE} , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h CtoO normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics that have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric and beam samples can have similar sensitivity to these systematics due to their similar composition in energy and interaction mode. As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.

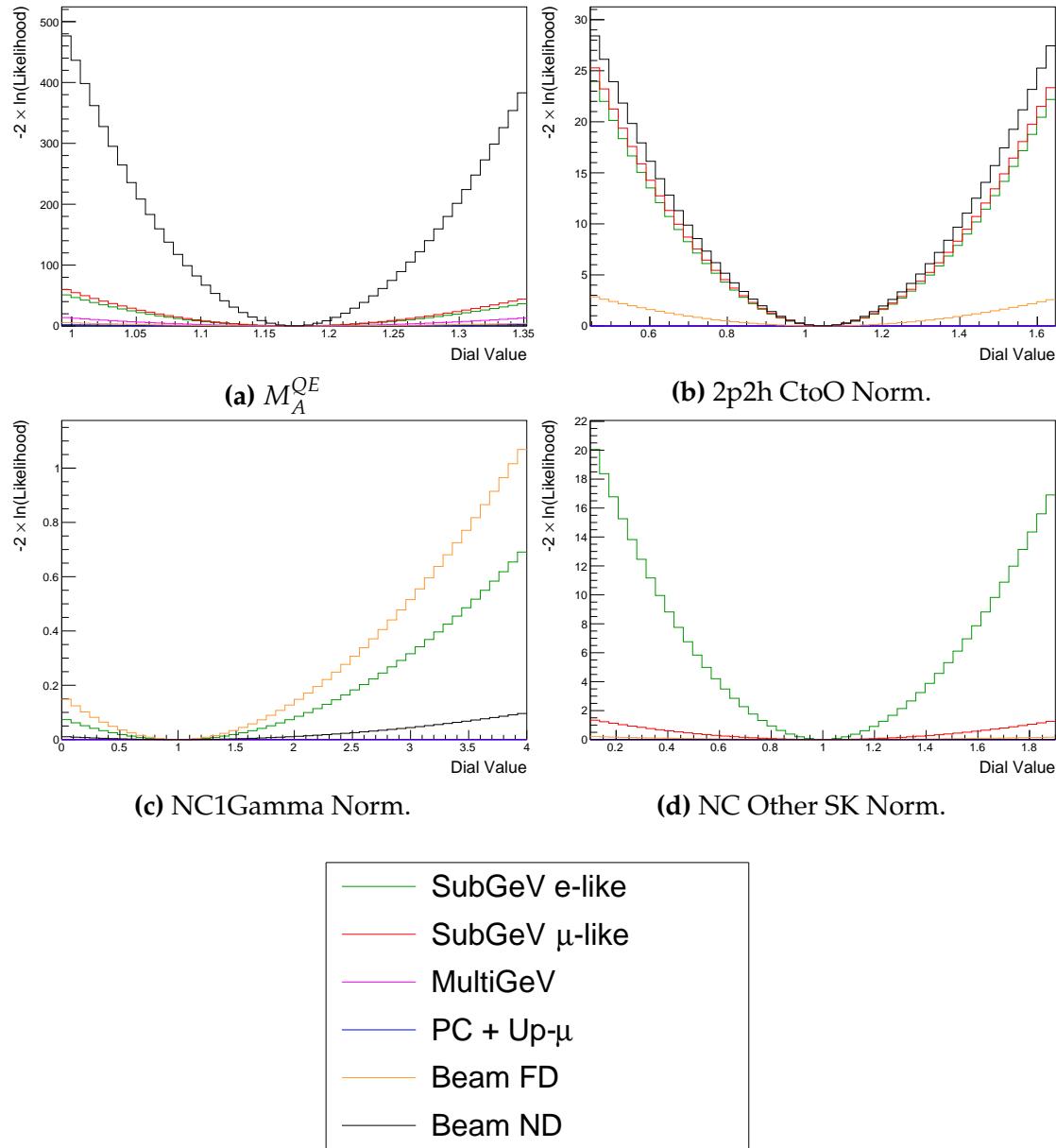


Figure 8.6: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

3058 8.3 Sensitivity Studies

3059 The sensitivities of the joint T2K and SK oscillation analysis are presented in
3060 the form of Asimov fits. These fits consider beam samples from the near and
3061 far detector alongside atmospheric samples at SK. This technique builds an
3062 Asimov data set (following section 6.5) using the AsimovA oscillation parameters
3063 and post-BANFF systematic tune, which is then fit. This technique eliminates
3064 statistical fluctuations from the data, therefore, providing the maximum sen-
3065 sitivity of the analysis.

3066 In practice, the Asimov fits presented within this analysis are modified from
3067 the above definition. An Asimov prediction of both beam and atmospheric far
3068 detector samples is fit whilst the true data is used for near detector samples. The
3069 Asimov predictions at the far detector are built using the post-BANFF tune (as
3070 discussed in section 3.2). These modifications mean that the results are equivalent
3071 to performing a far detector Asimov fit using inputs from the BANFF data fit.
3072 Consequently, this allows the results to be cross-checked with the results from
3073 the P-Theta analysis. The comparison has been performed and is documented in
3074 [210]. No significant discrepancies were found between the fitters.

3075 This section proceeds with the following studies. Firstly, the sensitivity
3076 of the atmospheric samples using the correlated detector model is detailed in
3077 subsection 8.3.1. This includes studying the choice of applying the 2020 PDG
3078 reactor constraint [72] to the atmospheric samples, which is documented in
3079 subsection 8.3.2. Additionally, the effect of applying the near-detector constraints
3080 onto the atmospheric samples is discussed in subsection 8.3.3. The main result is
3081 the sensitivity of the simultaneous beam and atmospheric fit. The sensitivities,
3082 both with and without the application of the reactor constraint, are presented
3083 in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate the benefit
3084 of the joint analysis, the sensitivities are compared to the 2020 T2K beam-only
3085 sensitivities [2, 211] in subsection 8.3.6 and subsection 8.3.7. The T2K analysis
3086 is used as a reference as it uses the same samples and a similar systematic

model. As shown in section 8.2, the response of the beam and atmospheric samples change depending upon the true set of oscillation parameters assumed. Therefore, subsection 8.3.8 documents the sensitivities at an alternative oscillation parameter set. These results have been presented at the Neutrino 2022 conference on behalf of the T2K and SK collaborations [4].

8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

This section presents the results of an Asimov fit using samples from the near detector and only atmospheric samples from the far detector. The results are presented as one-dimensional or two-dimensional histograms which have been marginalised over all other parameters using the technique outlined in subsection 4.3.1. Each histogram displays the posterior probability density and illustrates the credible intervals, calculated using the technique in subsection 4.3.2. For this fit, a flat prior is used for $\sin^2(\theta_{13})$ meaning that the reactor constraint is not applied. The Asimov data is generated assuming the AsimovA oscillation parameter set defined in Table 2.2 and the post-BANFF systematic parameter tune.

Figure 8.7 illustrates the posterior probability density for δ_{CP} , marginalised over both hierarchies. The fit favours the known oscillation parameter ($\delta_{CP} = -1.601$) although the posterior probability is very flat through the range of $-\pi < \delta_{CP} < -1$ and $2 < \delta_{CP} < \pi$. There is also a region around $\delta_{CP} \sim 0.4$ which is disfavoured at 2σ . This indicates that the SK samples can rule out some parts of the CP conserving parameter space reasonably well, near $\delta_{CP} \sim 0.4$, when the true value of $\delta_{CP} \sim -\pi/2$.

The posterior probability density in Δm_{32}^2 is given in Figure 8.8. This distribution includes steps in both the normal hierarchy (NH, $\Delta m_{32}^2 > 0$) and the inverse hierarchy (IH, $\Delta m_{32}^2 < 0$). The highest posterior probability density is found within the NH 1σ credible interval, which agrees with the known oscillation parameter value, $2.509 \times 10^{-3}\text{eV}^2$. However, all of the credible intervals span both of the hierarchy hypotheses.

Without Reactor Constraint, Both Hierarchies

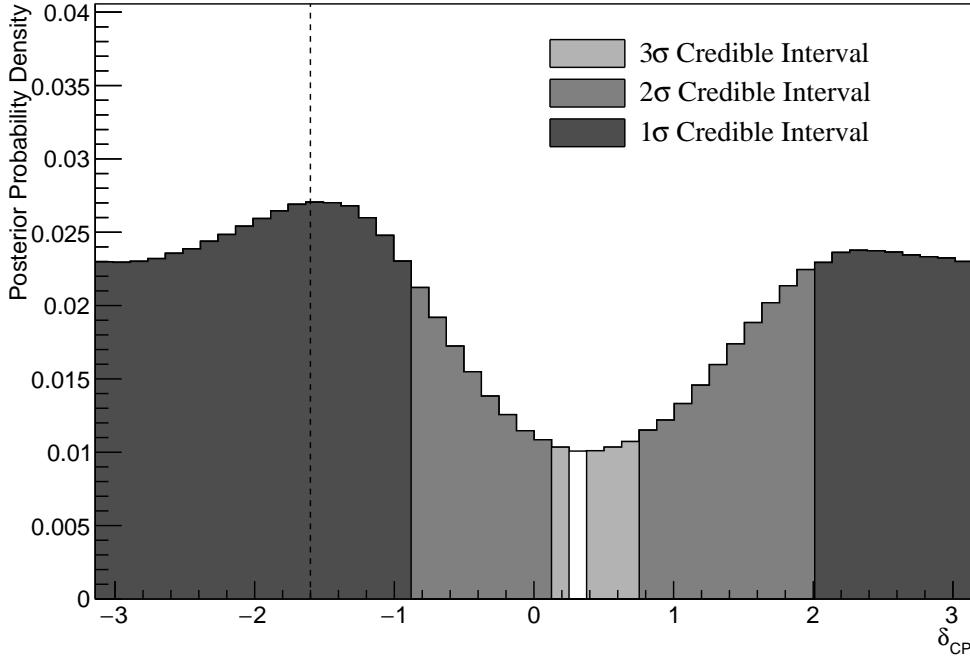


Figure 8.7: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.17	0.40	0.58
IH ($\Delta m_{32}^2 < 0$)	0.13	0.29	0.42
Sum	0.31	0.69	1.00

Table 8.2: The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.37$ and $B(\text{UO}/\text{LO}) = 2.24$.

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO, $\sin^2 \theta_{23} < 0.5$) or upper octant (UO, $\sin^2 \theta_{23} > 0.5$). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy hypothesis is $B(\text{NH}/\text{IH}) = 1.37$. Jeffrey's

Without Reactor Constraint, Both Hierarchies

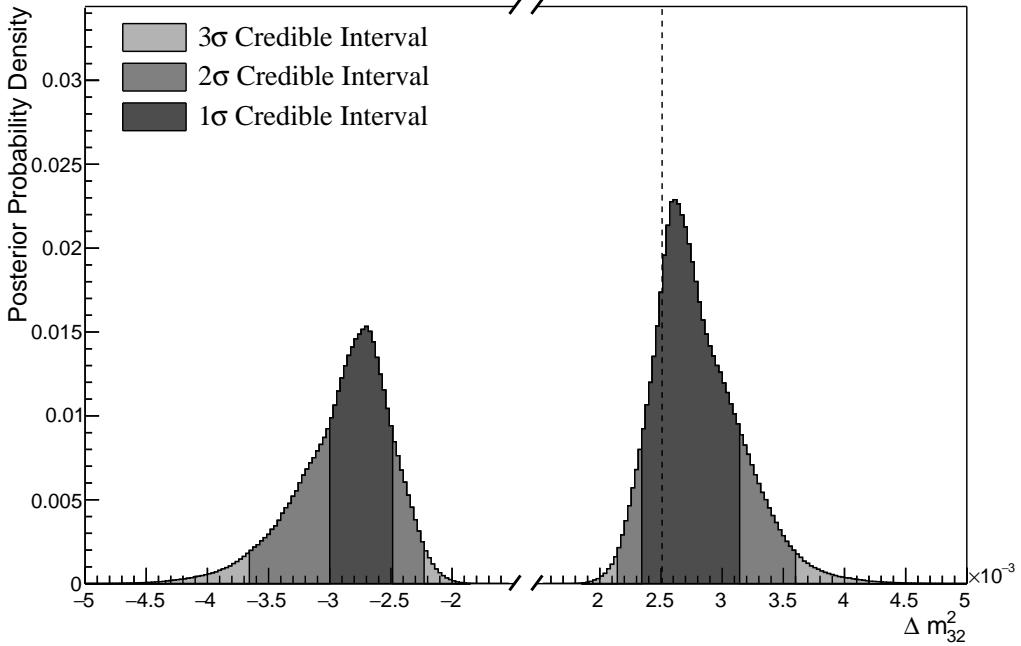


Figure 8.8: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

scale, given in Table 4.1, states this value of the Bayes factor indicates a weak preference for the normal hierarchy hypothesis which is correct given the known oscillation parameters. The Bayes factor for choice of octant is $B(\text{UO}/\text{LO}) = 2.24$. This is also identifying the correct hypothesis (UO) albeit with a strength classified as a weak preference. Both of these show that the fit is returning the correct choice of hypotheses (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The 1 σ credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. As the distribution is binned, the highest posterior density is presented as the center of the bin with the highest posterior density with an error equal to the bin width. For the known Asimov value of $\delta_{CP} = -1.601$, the 1 σ credible interval rules out a region between

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-\pi, -0.88], [1.88, \pi]$	-1.57 ± 0.07
δ_{CP} , (IH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-3.00, -2.49], [2.34, 3.14]$	2.61 ± 0.02
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.41, 3.04]$	2.59 ± 0.03
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-3.11, -2.41]$	-2.73 ± 0.03
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.584]$	0.542 ± 0.006
$\sin^2(\theta_{23})$ (NH)	$[0.488, 0.596]$	0.554 ± 0.006
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.584]$	0.542 ± 0.006

Table 8.3: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3136 $\delta_{CP} = -0.88$ and $\delta_{CP} = 1.96$, when marginalising over both hierarchies. The
3137 position of the highest posterior density is $\delta_{CP} = -1.57 \pm 0.07$ which is clearly
3138 compatible with the known oscillation parameter value.

3139 The sensitivity of the atmospheric samples to $\sin^2(\theta_{13})$ is presented in Fig-
3140 ure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity
3141 to $\sin^2(\theta_{13})$ will be small. This behaviour is also seen in the fit results, where the
3142 width of the 1σ credible intervals span the region of $\sin^2(\theta_{13}) = [0.008, 0.08]$. This
3143 is more than an order of magnitude worse than the constraint from reactor
3144 experiments [72].

3145 As previously discussed, the correlations between oscillation parameters are
3146 also important to understand how the atmospheric samples respond. Figure 8.10
3147 illustrates the two dimensional $\sin^2(\theta_{13})-\delta_{CP}$ sensitivity, marginalised over all
3148 other parameters. The shape of the 1σ credible interval shows that the constrain-
3149 ing power of the fit on δ_{CP} is dependent upon the value of $\sin^2(\theta_{13})$. Furthermore,
3150 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.

Without Reactor Constraint, Both Hierarchies

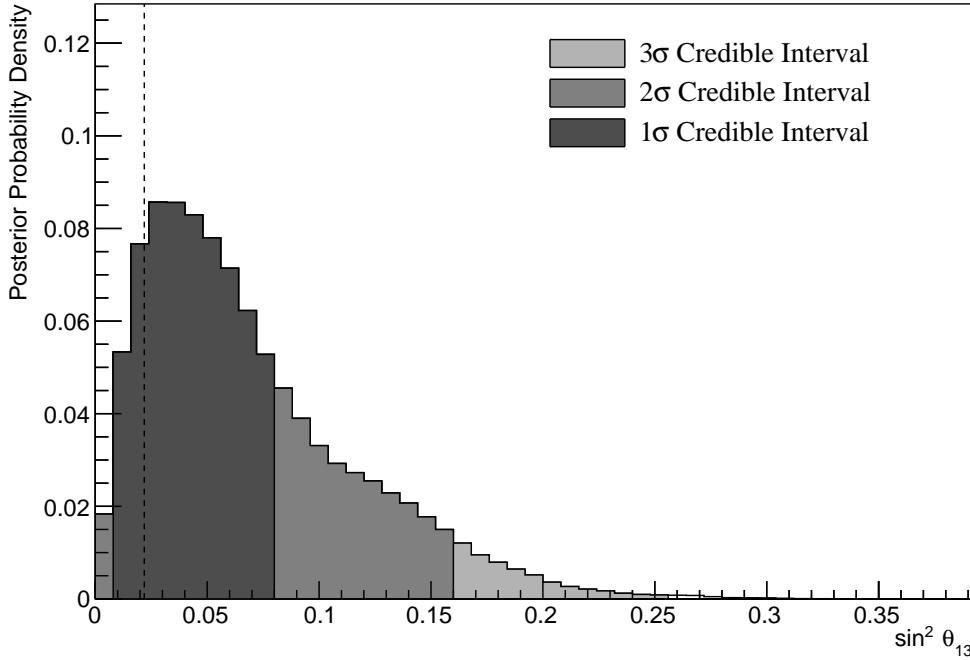


Figure 8.9: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{13})$.

Whilst the atmospheric samples do not strongly constrain the value of $\sin^2(\theta_{13})$, the value of $\sin^2(\theta_{13})$ does impact the atmospheric samples' sensitivity to δ_{CP} .

The $\sin^2(\theta_{23})-\Delta m_{32}^2$ disappearance contours are illustrated in Figure 8.11. As expected, the area contained in the inverted hierarchy 1σ credible interval is slightly smaller than that in the normal hierarchy. This follows from the Bayes factor showing a weak preference for NH meaning that more of the steps will exist in the $\Delta m_{32}^2 > 0$ region. The known oscillation parameters of $\sin^2(\theta_{23}) = 0.528$ and $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ are contained within the 1σ credible interval.

Figure 8.12 illustrates the two-dimensional projections for each permutation of oscillation parameters which this analysis is sensitive to: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . The purpose of this plot is to illustrate the correlations between the oscillation parameters. The contours are calculated whilst marginalising over both hierarchies, however, only the NH is illustrated when plotting the Δm_{32}^2 parameter. As expected the correlations play a significant role in these

Without Reactor Constraint, Both Hierarchies

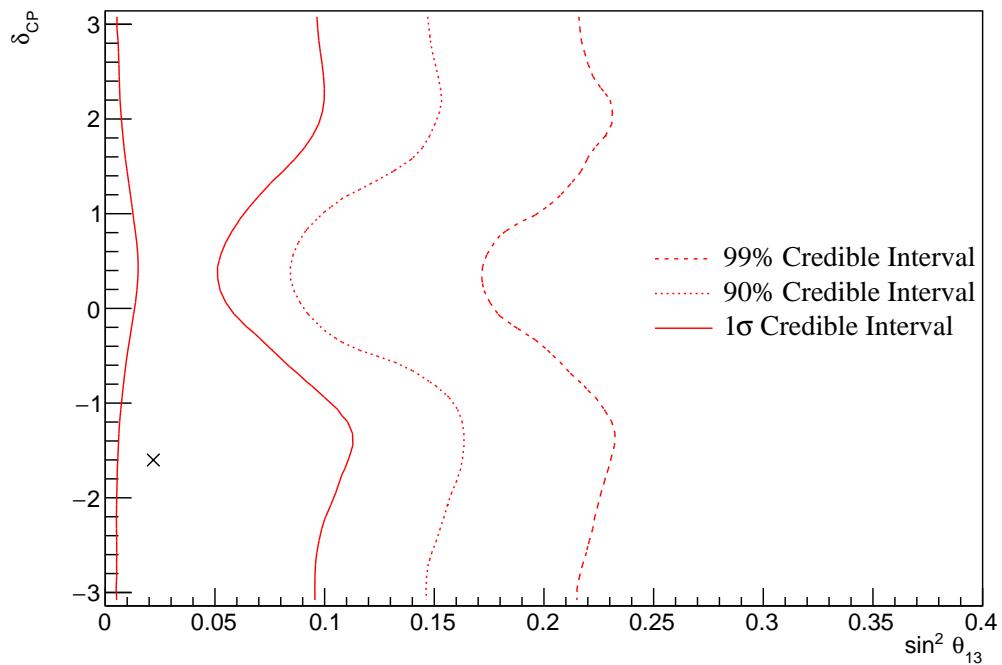


Figure 8.10: The two-dimensional posterior probability density distribution in $\delta_{CP}-\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of $\delta_{CP}-\sin^2(\theta_{13})$.

3165 sensitivity measurements, especially the choice of the $\sin^2(\theta_{13})$ constraint. Most
 3166 notably, the application of reactor constraint would be expected to alter both the
 3167 width and position of the Δm_{32}^2 intervals due to the strong correlation between
 3168 the parameters.

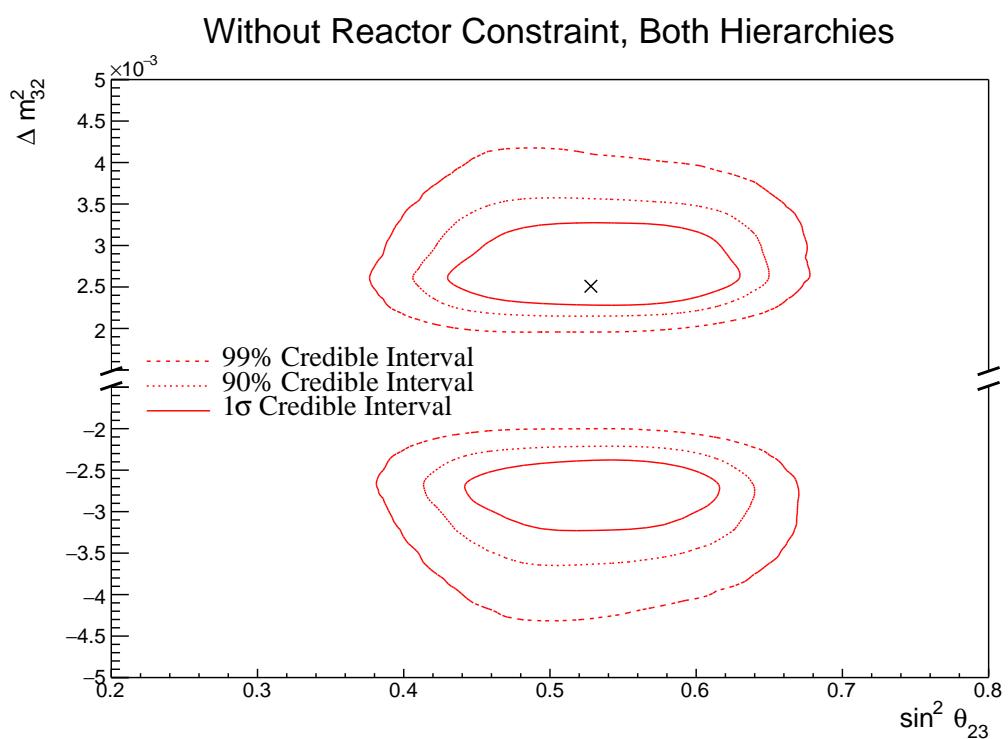


Figure 8.11: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

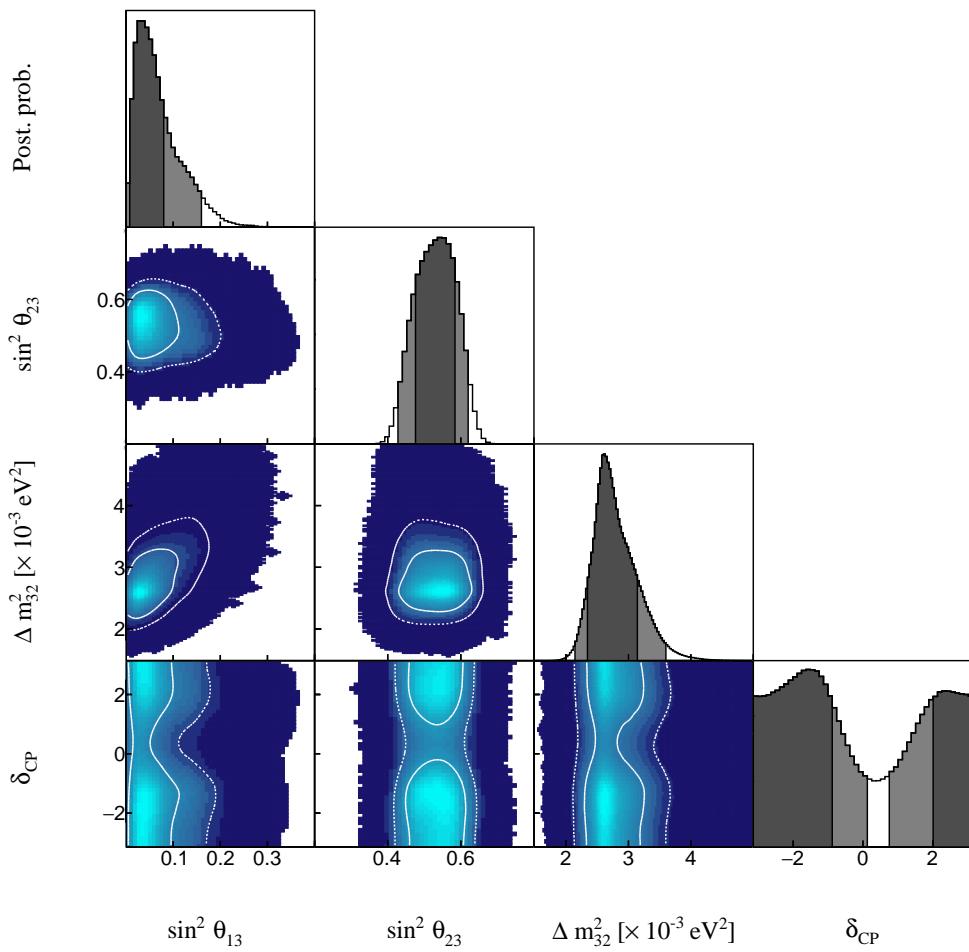


Figure 8.12: The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3169 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

3170 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor
 3171 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that
 3172 the marginalisation effects could contribute to differing sensitivities when the
 3173 external reactor constraint is applied. Using the technique discussed in subsec-
 3174 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted
 3175 to include the reactor constraint of $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ [72].

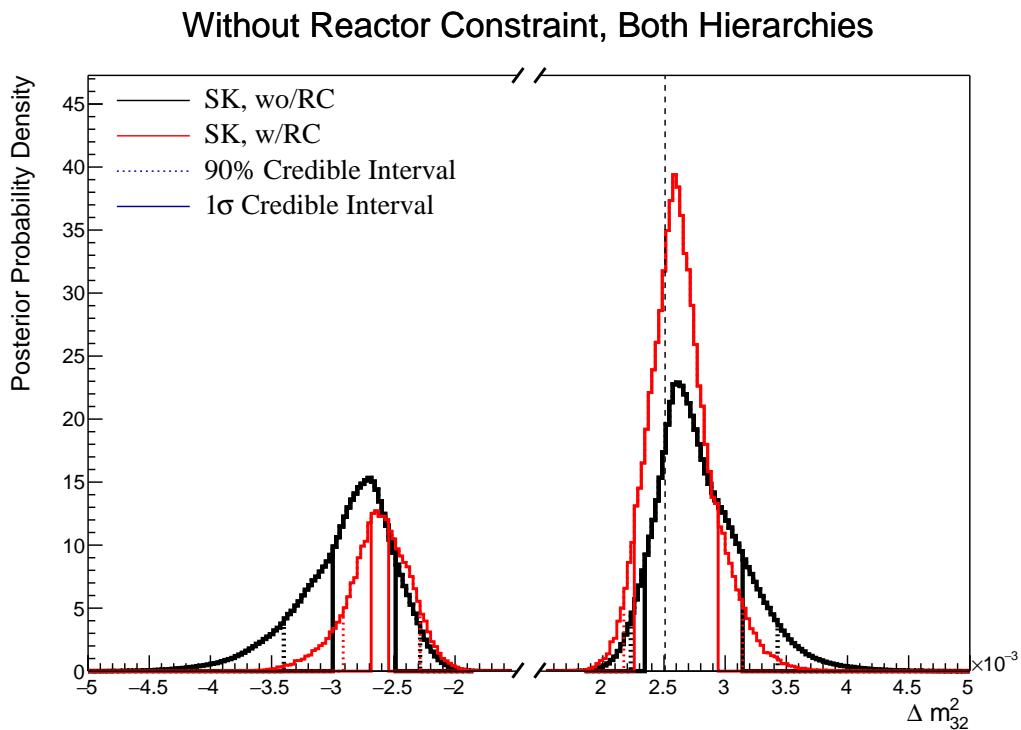


Figure 8.13: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of Δm_{32}^2 .

3176 The reactor constraint increases the sensitivity of the atmospheric samples to
 3177 Δm_{32}^2 as illustrated in Figure 8.13. The 1σ credible interval in Δm_{32}^2 is determined
 3178 to be $[-2.69, -2.54] \times 10^{-3} \text{ eV}^2$ and $[2.25, 2.94] \times 10^{-3} \text{ eV}^2$. The width of the IH
 3179 credible interval is reduced by $\sim 70\%$ when the reactor constraint is applied. Due
 3180 to the marginalisation effects observed in Figure 8.12, the favoured region of Δm_{32}^2
 3181 moves closer to zero for both hierarchies. A clear explanation of this behaviour is

3182 illustrated in Figure 8.14, which shows the posterior distribution in the Δm_{32}^2 –
 3183 $\sin^2(\theta_{13})$ parameters. The correlation between Δm_{32}^2 and $\sin^2(\theta_{13})$ is such that
 3184 lower values of $\sin^2(\theta_{13})$ tend towards lower values of $|\Delta m_{32}^2|$. Therefore the
 3185 application of the reactor constraint moves the posterior distribution towards
 3186 the known oscillation parameter.

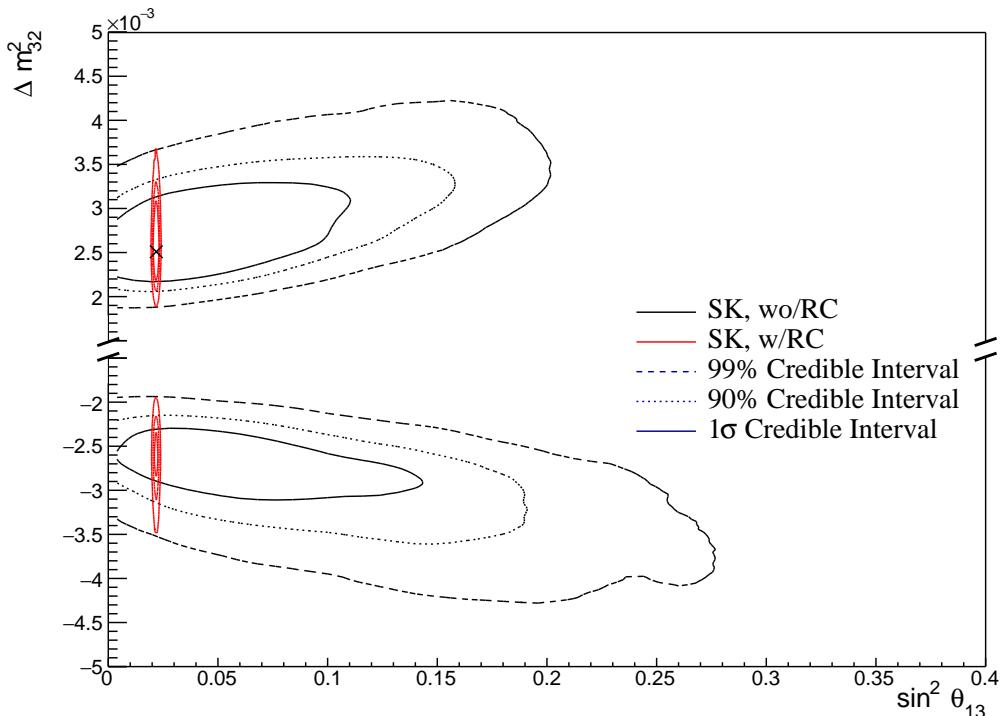


Figure 8.14: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$ compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{13})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.21	0.53	0.74
IH ($\Delta m_{32}^2 < 0$)	0.08	0.18	0.26
Sum	0.29	0.71	1.00

Table 8.4: The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 2.85$ and $B(\text{UO}/\text{LO}) = 2.39$.

3187 Table 8.4 presents the fraction of steps in each hierarchy and octant model
3188 for the fit after the reactor constraint has been applied. The reactor constraint
3189 significantly increases the preference for the correct hierarchy, increasing the
3190 Bayes factor from $B(\text{NH}/\text{IH}) = 1.37$ to $B(\text{NH}/\text{IH}) = 2.85$ when the reactor
3191 constraint is applied. This is still defined as a weak preference for the NH
3192 hypothesis according to Jeffrey's scale, however, it is a stronger preference than
3193 when the constraint is not applied. The preference for the correct octant model is
3194 also slightly increased by the application of the reactor constraint.

3195 8.3.3 Impact of Near Detector Constraints for Atmospheric Sam- 3196 ples

3197 The choice of applying the near detector constraints to the low-energy atmo-
3198 spheric samples was introduced in subsection 6.4.3. This subsection illustrates the
3199 effect of removing the ND constraint on the sensitivity of the atmospheric samples
3200 to the oscillation parameters. To do this, the fit presented in subsection 8.3.1 has
3201 been compared to another fit where the constraints from the near detector have
3202 not been included. This is the only case where the near detector constraints are
3203 neglected throughout this chapter. For both fits, the Asimov data was generated
3204 assuming the ‘AsimovA’ oscillation parameter set defined in Table 2.2 and the
3205 post-BANFF systematic parameter tune.

3206 The change in sensitivity on δ_{CP} is given in Figure 8.15. The reactor constraint
3207 is not applied in either of the fits within this comparison. The fit which includes
3208 the near detector constraint is slightly more peaked at the known oscillation
3209 parameter value. The width of the 1σ credible intervals are approximately the
3210 same (identical to within a bin width) and the same conclusion holds for the
3211 higher credible intervals. The change in sensitivity to other oscillation parameters
3212 has been studied and no significant discrepancies were found. This shows that
3213 the exact choice of constraint does not significantly affect the physics conclusions
3214 one would make from this analysis.

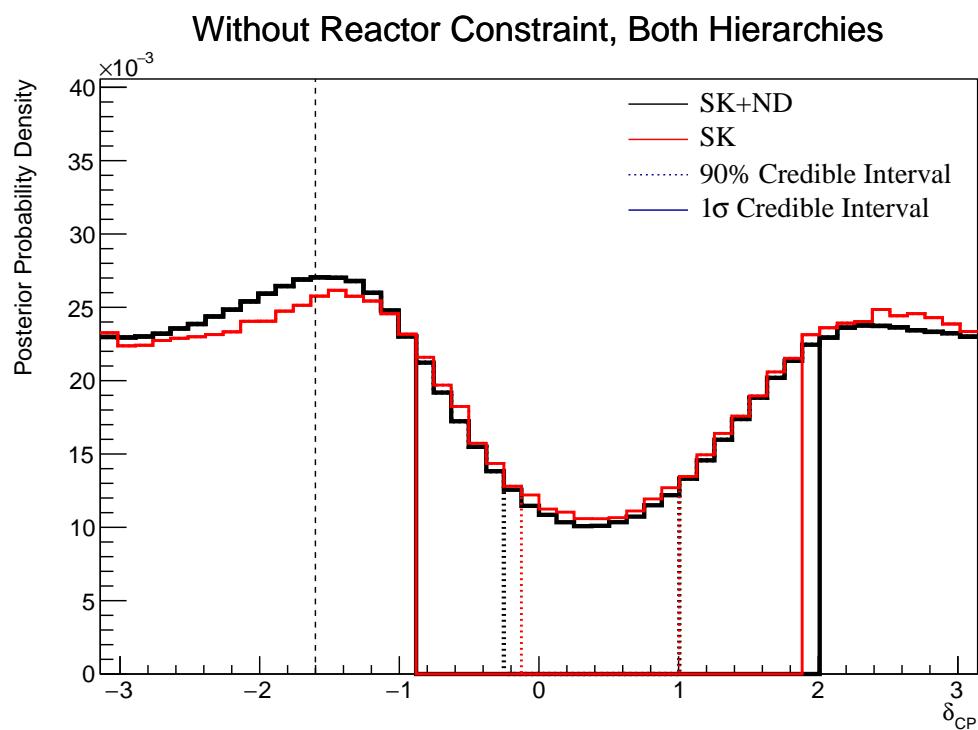


Figure 8.15: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of δ_{CP} .

3215 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3216 straint

3217 This section presents the sensitivities of the simultaneous beam and atmospheric
 3218 analysis where the reactor constraint is not applied. Similar to the previous
 3219 studies, the Asimov data is built assuming the post-BANFF systematic tune and
 3220 Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near
 3221 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.

Without Reactor Constraint, Both Hierarchies

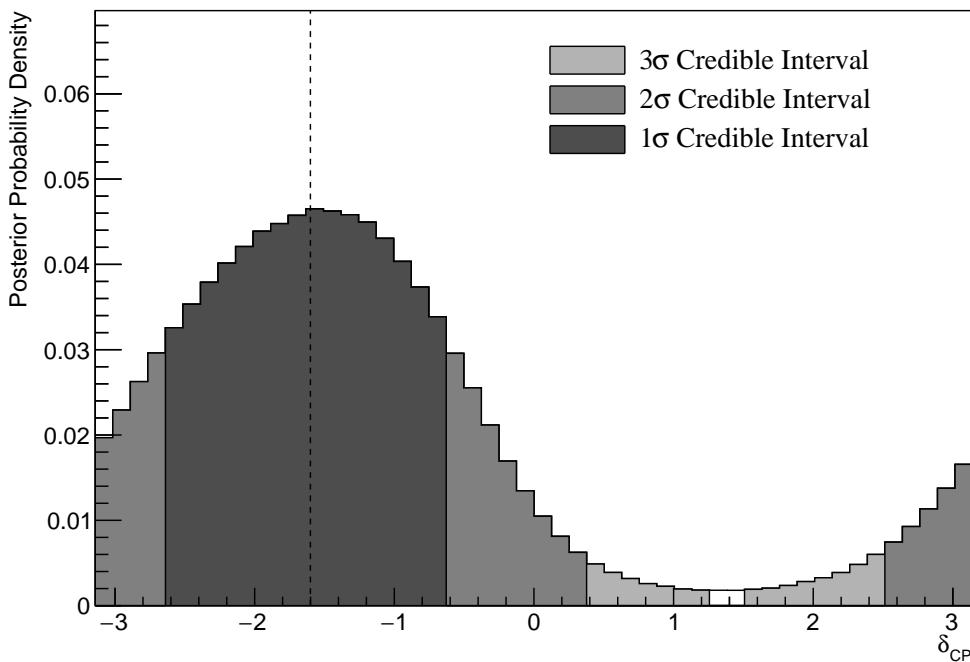


Figure 8.16: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

3222 The sensitivity to δ_{CP} , marginalised over both hierarchies, is given in Fig-
 3223 ure 8.16. The credible intervals and highest posterior distribution for each
 3224 oscillation parameter is given in Table 8.5. The highest posterior probability
 3225 density is $\delta_{CP} = -1.57 \pm 0.07$ and is compatible with the known value of
 3226 $\delta_{CP} = -1.601$. The CP-conserving values of $\delta_{CP} = 0, \pm\pi$ are disfavoured at
 3227 1 σ credible interval. There is also a region around $\delta_{CP} = 1.4$ which is disfavoured
 3228 at more than 3 σ . Whilst these conclusions can only be made at this particular

3229 Asimov point, it does show that if the true value of δ_{CP} were CP-violating,
3230 this joint analysis would be able to disfavour CP conserving values at over 1σ
3231 without any external constraints.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-2.64, -0.63]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-2.76, -0.63]$	-1.45 ± 0.07
δ_{CP} , (IH)	$[-2.39, -0.88]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.58]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.51]$	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.480, 0.545]	0.518 ± 0.003
$\sin^2(\theta_{23})$ (NH)	[0.480, 0.545]	0.508 ± 0.003
$\sin^2(\theta_{23})$ (IH)	[0.480, 0.545]	0.513 ± 0.003

Table 8.5: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3232 The sensitivity to Δm_{32}^2 is illustrated in Figure 8.17. Notably, the 1σ credible
3233 interval is entirely contained within the NH region, as further evidenced by
3234 Table 8.5. This illustrates good sensitivity to the mass hierarchy as it is correctly
3235 selecting the correct hypothesis. This is reflected in the 1σ credible intervals being
3236 approximately the same when they are constructed considering both hierarchies
3237 and when considering only the NH region. The NH distribution favours this
3238 region surrounding the known Asimov point, $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$, where
3239 the highest posterior probability density is at $\Delta m_{32}^2 = (2.51 \pm 0.01) \times 10^{-3}\text{eV}^2$.

3240 The fraction of steps in each of the mass hierarchy regions and octants of
3241 $\sin^2(\theta_{23})$ is given in Table 8.6. The Bayes factors are determined to be $B(\text{NH}/\text{IH}) =$
3242 3.67 and $B(\text{UO}/\text{LO}) = 1.74$. Jeffrey's scale states that this value of the mass
3243 hierarchy Bayes factor illustrates substantial evidence for the NH hypothesis.

³²⁴⁴ This corresponds to the correct hypothesis given the known oscillation parameters
³²⁴⁵ and is a stronger statement than the atmospheric-only analysis can provide. It is
³²⁴⁶ important to note that this substantial preference requires no external constraints.
³²⁴⁷ The Bayes factor for octant determination represents a weak preference for the
³²⁴⁸ upper octant, therefore, selecting the correct octant hypothesis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.29	0.50	0.79
IH ($\Delta m_{32}^2 < 0$)	0.08	0.13	0.21
Sum	0.37	0.63	1.00

Table 8.6: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 3.67$ and $B(\text{UO}/\text{LO}) = 1.74$.

Without Reactor Constraint, Both Hierarchies

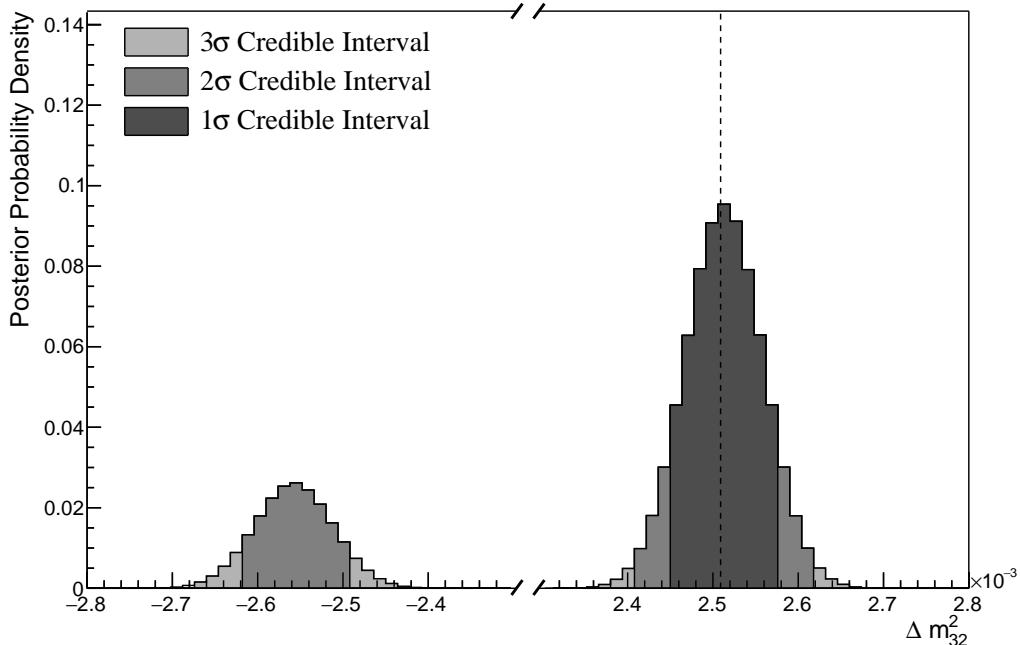


Figure 8.17: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

³²⁴⁹ The sensitivity to $\sin^2(\theta_{23})$ is presented in Figure 8.18. There is a clear

3250 preference for the upper octant but the peak of the distribution is relatively
 3251 flat. It peaks at $\sin^2(\theta_{23}) = 0.509 \pm 0.003$ which is in the region of the known
 3252 value of $\sin^2(\theta_{23}) = 0.528$. The difference in the highest posterior distribution
 3253 and the width of the credible interval is relatively unchanged when consid-
 3254 ering different hierarchy hypotheses showing no strong correlation between
 3255 $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

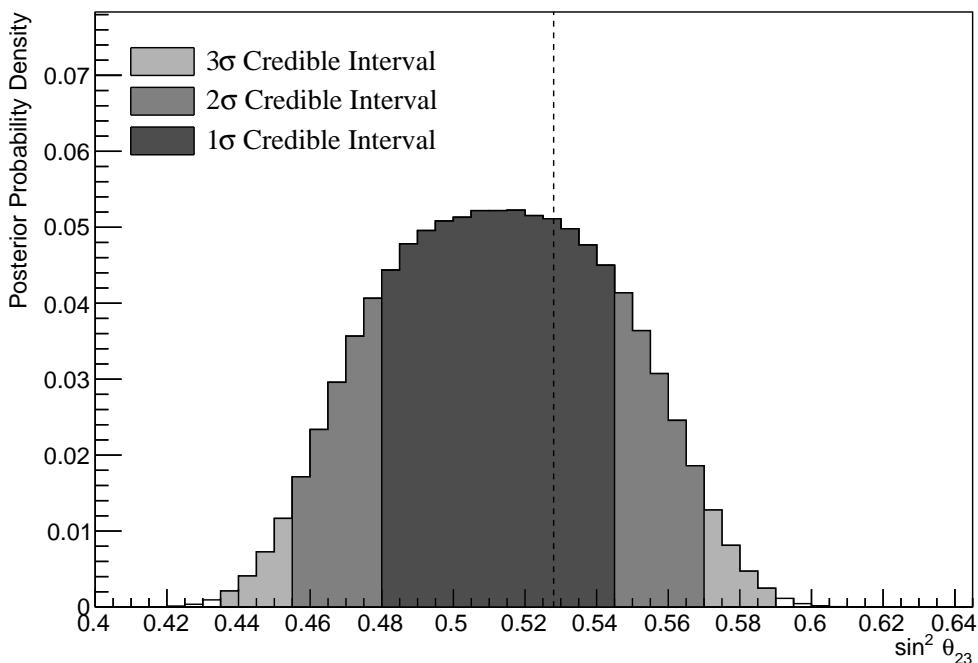


Figure 8.18: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3256 The sensitivity presented as a function of the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$)
 3257 is given in Figure 8.19. As expected, the contours follow the likelihood shape
 3258 given in Figure 8.2, where the 2σ credible intervals have a closed contour exclu-
 3259 sing the region around $\delta_{CP} \sim 1.2$. The width of the 3σ credible interval in $\sin^2(\theta_{13})$
 3260 is dependent upon the value of δ_{CP} . Close to the Asimov point, $\delta_{CP} = -1.601$, the
 3261 width of the 3σ credible interval approximately spans $\sin^2(\theta_{13}) = [0.013, 0.04]$.
 3262 This is reduced to a region of $\sin^2(\theta_{13}) = [0.023, 0.042]$ at the most disfavoured
 3263 value of δ_{CP} . The 1σ credible interval is consistent with the known oscillation

3264 parameter. Application of the reactor constraint would be expected to decrease
3265 the width of the 1σ credible intervals in δ_{CP} due to the triangular shape of
3266 the posterior probability.

3267 The sensitivity in terms of the disappearance parameters, $\sin^2(\theta_{23}) - \Delta m_{32}^2$, is
3268 given in Figure 8.20. The area contained within the IH contours is significantly
3269 smaller than the area within the NH contours. The IH credible intervals are
3270 also notably tighter in the $\sin^2(\theta_{23})$ dimension. No significant correlation is
3271 observed between $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

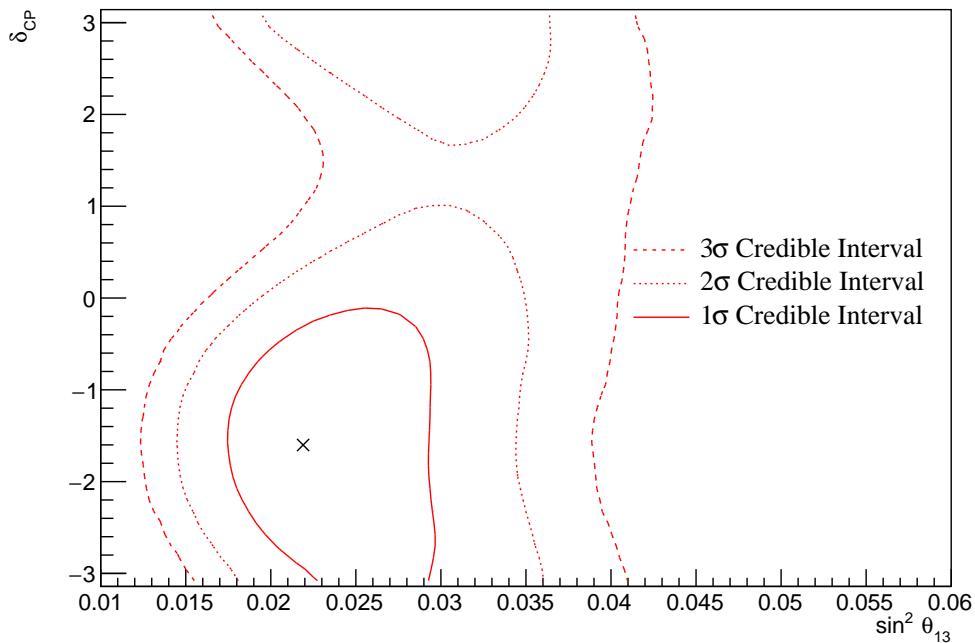


Figure 8.19: The two-dimensional posterior probability density distribution in $\delta_{CP}-\sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\delta_{CP}-\sin^2(\theta_{13})$.

3272 The two-dimensional posterior distribution for each permutation of the oscil-
3273 lation parameters of interest is given in Figure 8.21. The most notable observation
3274 is that the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ are anti-correlated. If the value of $\sin^2(\theta_{13})$ was
3275 constrained closer to the known oscillation parameter value, the preferred value
3276 of $\sin^2(\theta_{23})$ would increase. This would move the highest posterior probability

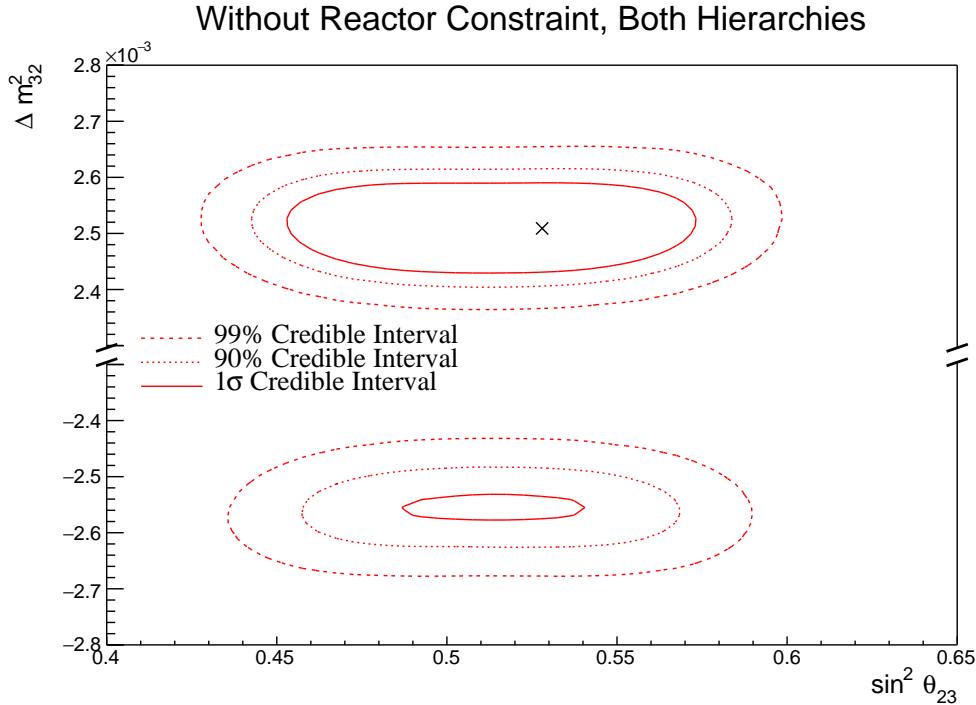


Figure 8.20: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

closer in line with the known value and could lead to an increase in the preference for the correct octant hypothesis (UO).

Furthermore, the δ_{CP} and $|\Delta m_{32}^2|$ oscillation parameters are anti-correlated, such that higher values of $|\Delta m_{32}^2|$ prefer lower values of δ_{CP} . Whilst this is an interesting result on its own, the width of the Δm_{32}^2 contours also depend on $\sin^2(\theta_{13})$. This introduces another correlation effect that could modify the sensitivity to δ_{CP} once the reactor constraint is applied.

The correlation between $\sin^2(\theta_{13})$ and Δm_{32}^2 can be seen in Figure 8.22. A much larger fraction of the posterior distribution is contained in the NH for lower values of $\sin^2(\theta_{13})$. Consequently, the application of the reactor constraint would be expected to significantly increase the preference for NH.

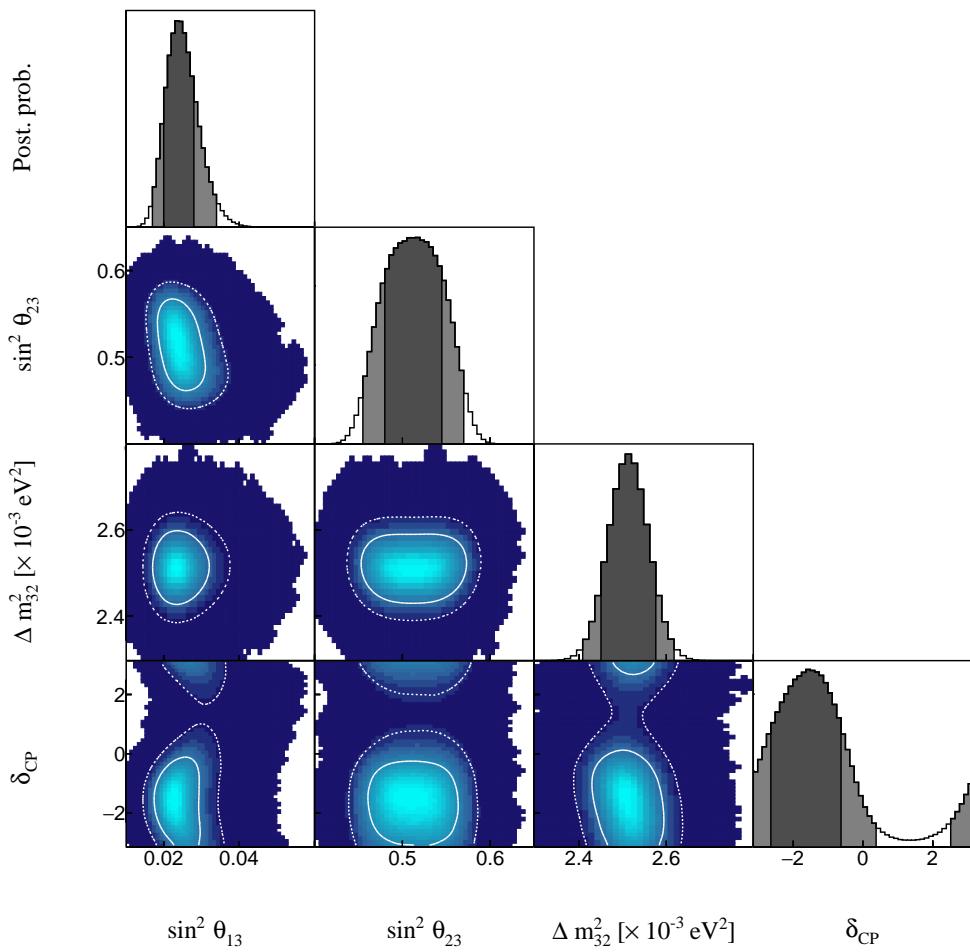


Figure 8.21: The posterior probability density distribution from the joint beam-atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

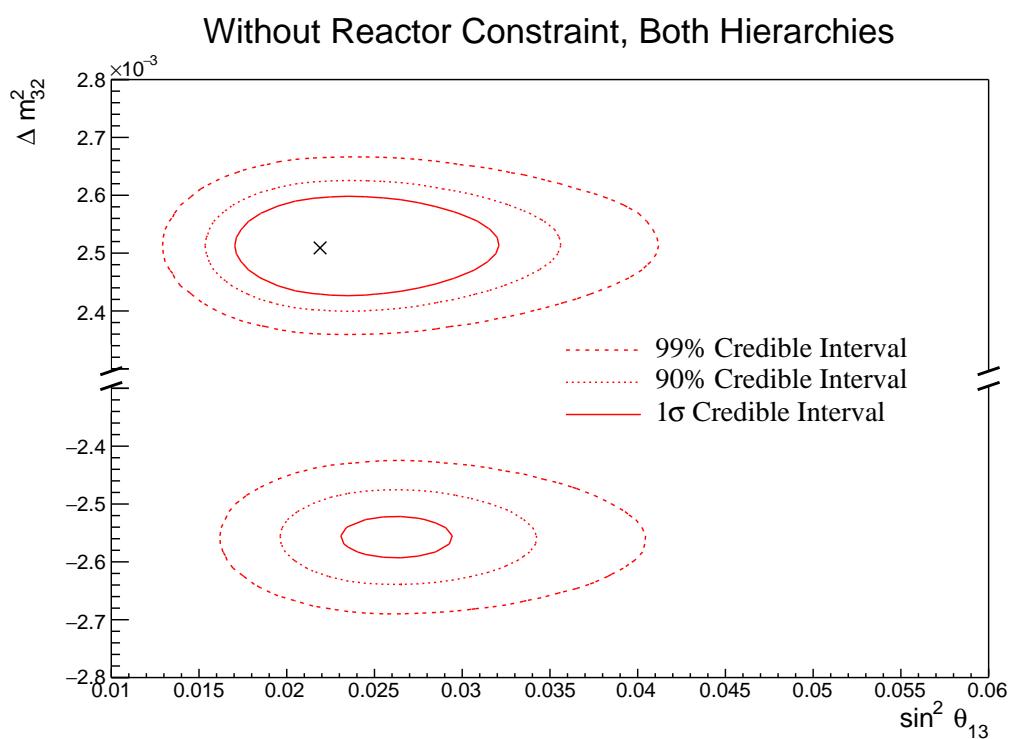


Figure 8.22: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{13})$.

3288 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3289 This section presents the sensitivities of the joint beam-atmospheric fit when
 3290 the reactor constraint is applied to $\sin^2(\theta_{13})$. As with the previous studies, the
 3291 Asimov data is made using the AsimovA oscillation parameter set defined in
 3292 Table 2.2 and the post-BANFF systematic parameter tune.

3293 **With Reactor Constraint, Both Hierarchies**

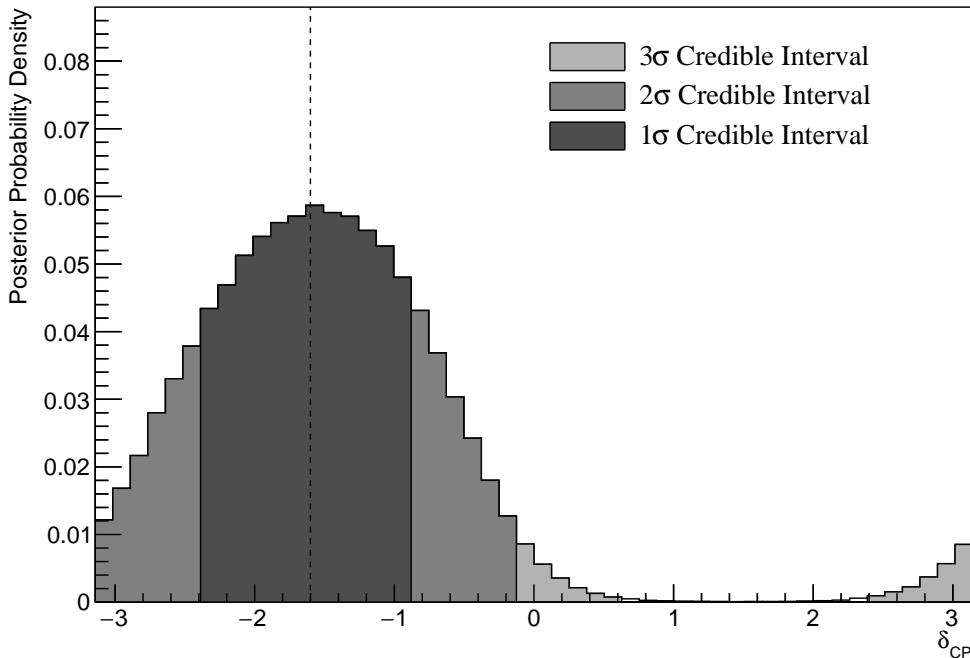


Figure 8.23: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of δ_{CP} .

3293 Figure 8.23 illustrates the sensitivity to δ_{CP} , marginalised over both hierarchies.
 3294 The CP-conserving value of $\delta_{CP} = 0$ is disfavoured at 2σ whilst the value of $\delta_{CP} =$
 3295 $\pm\pi$ is very close to being disfavoured at 2σ . Furthermore, the 3σ credible interval
 3296 excludes the region of $\delta_{CP} = [0.63, 2.39]$, thus clearly disfavouring the region of
 3297 $\delta_{CP} = \pi/2$ at more than 3σ for this particular set of known oscillation parameters.
 3298 The width of the 1σ credible intervals and the position of the highest posterior
 3299 probability density is given in Table 8.7. The highest posterior probability density
 3300 in δ_{CP} is calculated as $\delta_{CP} = -1.57 \pm 0.07$ showing no significant biases in the
 3301 determination of the known oscillation parameters.

The effect of applying the reactor constraint for δ_{CP} in the joint beam-atmospheric fit is presented in Figure 8.24. The reactor constraint significantly improves the ability of the fit to select the known parameter value. This behaviour is evidenced by the tightening of the 1σ and 90% credible intervals and the disfavoured region, centered at $\delta_{CP} \sim \pi/2$, becoming wider when the reactor constraint is applied. This follows from the correlations shown in Figure 8.19, where a lower value of $\sin^2(\theta_{13})$ results in tighter constraints on δ_{CP} .

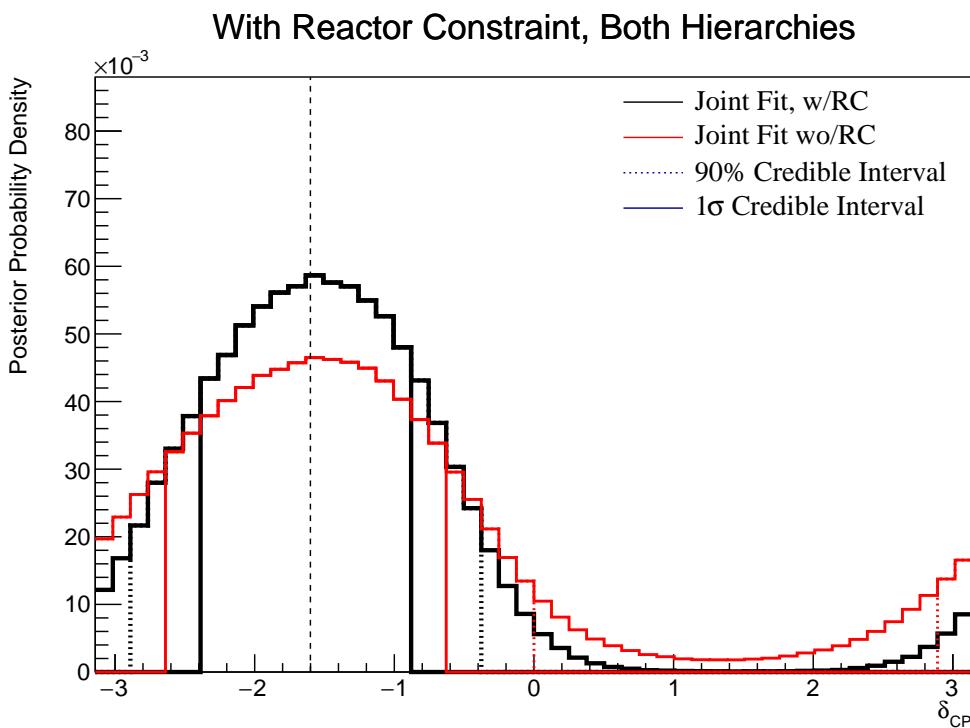


Figure 8.24: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, is given in Figure 8.25. The highest posterior probability density is located at $\sin^2(\theta_{23}) = 0.528 \pm 0.03$ which agrees with the known value of $\sin^2(\theta_{23}) = 0.528$. The distribution clearly favours the UO with almost the entirety of the 1σ credible interval being contained in that region. Figure 8.26 highlights the sensitivity of the joint fit both with and without the reactor constraint. The fit where the

Parameter	Interval	HPD
δ_{CP} , (BH)	[-2.39, -0.88]	-1.57 ± 0.07
δ_{CP} , (NH)	[-2.39, -0.75]	-1.57 ± 0.07
δ_{CP} , (IH)	[-2.14, -1.01]	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.56]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	[-2.60, -2.51]	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (NH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (IH)	[0.500, 0.560]	0.538 ± 0.03

Table 8.7: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

reactor constraint is applied selects the known value much better. This is a result of the marginalisation effects between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as observed in Figure 8.21.

The fraction of steps from the joint fit, after the reactor constraint is applied, is given in Table 8.8 and split by the two hierarchy and two octant hypotheses. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.03, whilst significantly increasing the fraction of steps within the NH-UO region from 0.50 to 0.62. The application of the reactor constraint increases the Bayes factor from $B(\text{NH}/\text{IH}) = 3.67$ to $B(\text{NH}/\text{IH}) = 6.47$. There is a very clear preference for the correct hypothesis, with the Jeffreys scale stating a substantial preference for both fits. The Bayes factor for selecting the correct octant is calculated as $B(\text{UO}/\text{LO}) = 2.64$. Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

The sensitivity of the joint beam-atmospheric fit to Δm_{32}^2 , with the reactor constraint applied, is presented in Figure 8.27. The 1σ credible interval is

With Reactor Constraint, Both Hierarchies

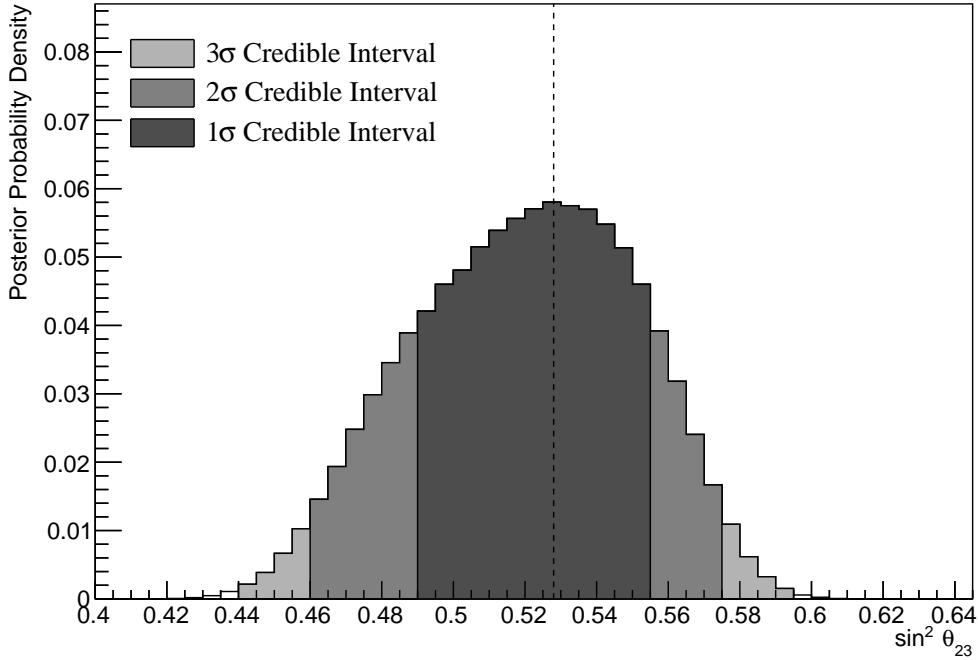


Figure 8.25: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.24	0.62	0.87
IH ($\Delta m_{32}^2 < 0$)	0.03	0.10	0.13
Sum	0.27	0.73	1.00

Table 8.8: The distribution of steps in a joint beam-atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{UO}/\text{LO}) = 2.64$.

3331 entirely contained within the NH region and the position of the highest posterior
 3332 probability density is given as $(2.49 \pm 0.01) \times 10^{-3} \text{ eV}^2$. This illustrates no bias
 3333 between the fit results and the known oscillation parameters. The application
 3334 of the reactor constraint does not significantly move the position or width of
 3335 the credible intervals.

3336 The sensitivity to the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$) is given in Fig-
 3337 ure 8.28. The distribution is mostly uncorrelated between the two parameters and

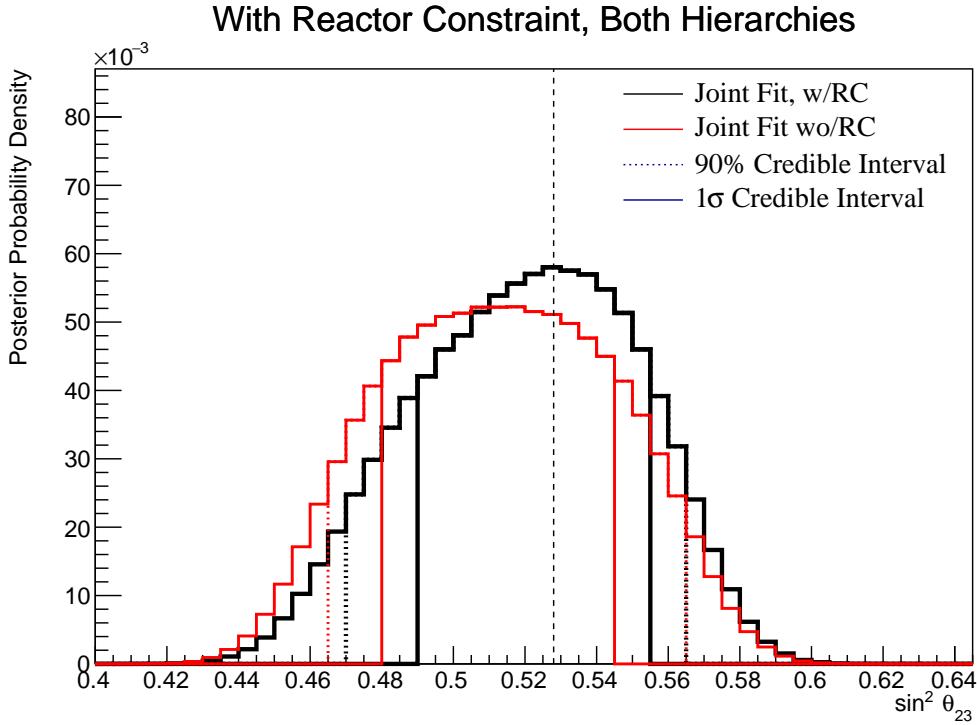


Figure 8.26: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

is centered at the known oscillation parameters. The 1σ credible interval excludes $\delta_{CP} = 0$ and $\delta_{CP} = \pm\pi$. Furthermore, the 3σ credible intervals exclude the region of $\delta_{CP} = \pi/2$.

The sensitivity to the disappearance parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) is illustrated in Figure 8.29. The 1σ credible interval is entirely contained within the NH region reflecting the same results as the one-dimensional marginalised results in Figure 8.27. Both the NH and IH regions favour the UO, with a visually similar preference in both hierarchies. The width of the 1σ contour, in Δm_{32}^2 , does not significantly depend upon the value or octant of $\sin^2(\theta_{23})$. This shows that there are no strong correlations between these two parameters.

Figure 8.30 illustrates the posterior distribution for each permutation of two oscillation parameters of interest. The application of the reactor constraint significantly reduces the correlations previously seen in Figure 8.21.

With Reactor Constraint, Both Hierarchies

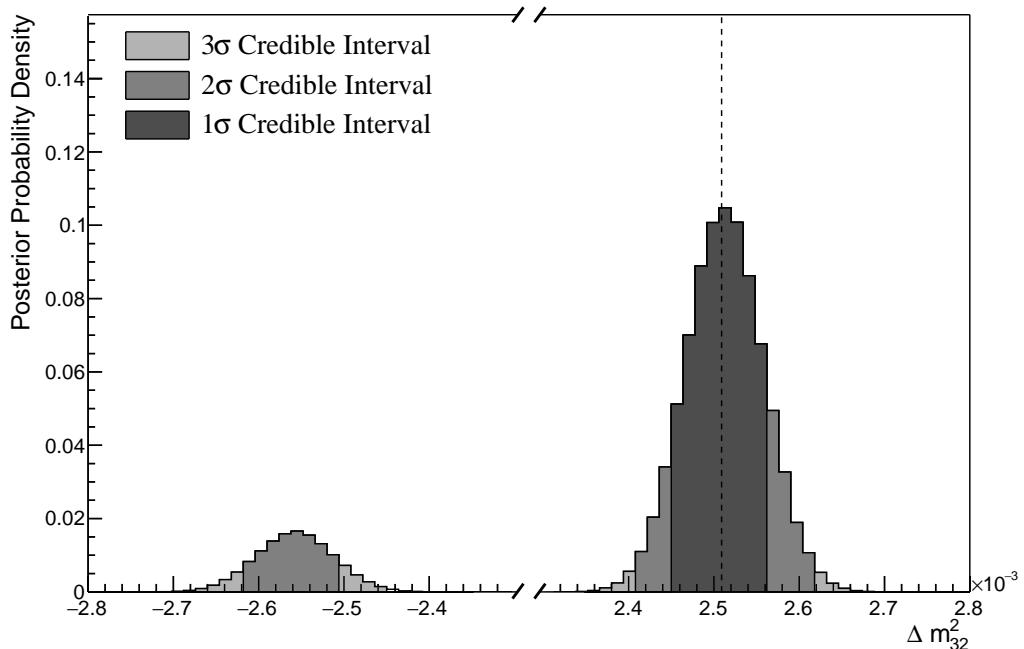


Figure 8.27: The one-dimensional posterior probability density distribution in Δm_{32}^2 from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of Δm_{32}^2 .

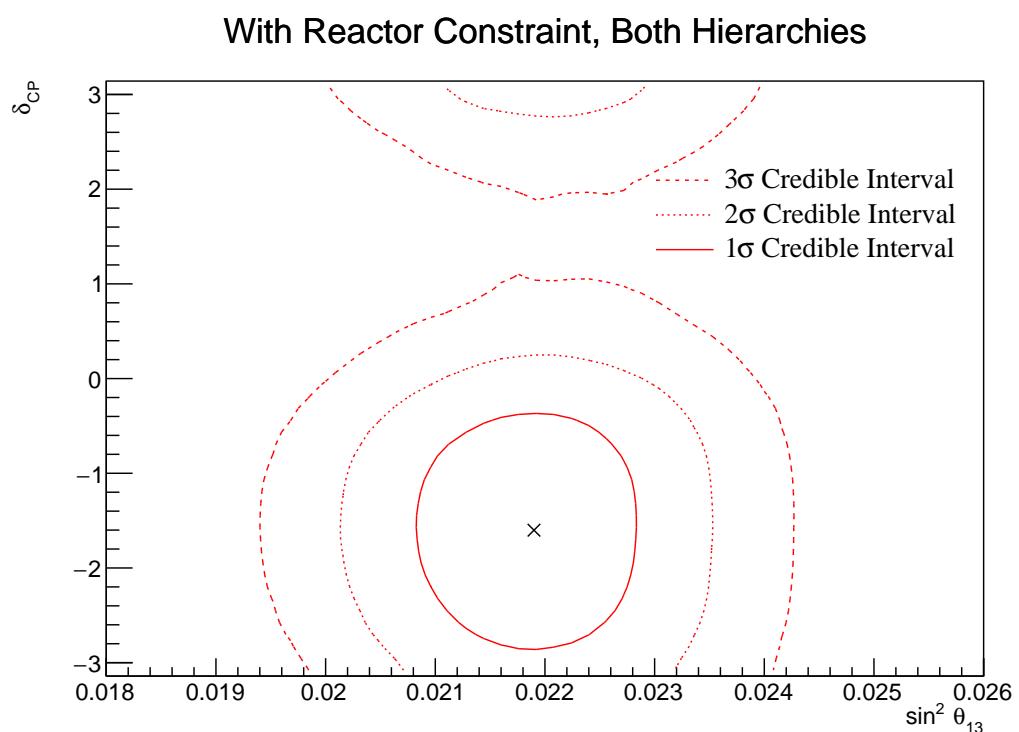


Figure 8.28: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\delta_{CP} - \sin^2(\theta_{13})$.

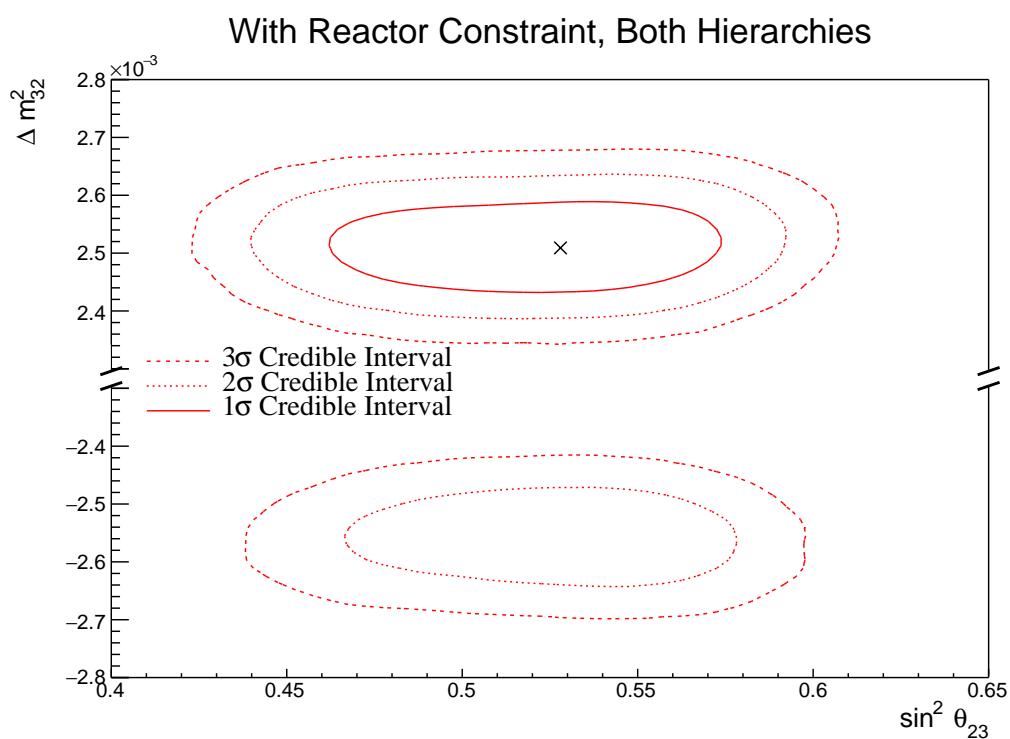


Figure 8.29: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

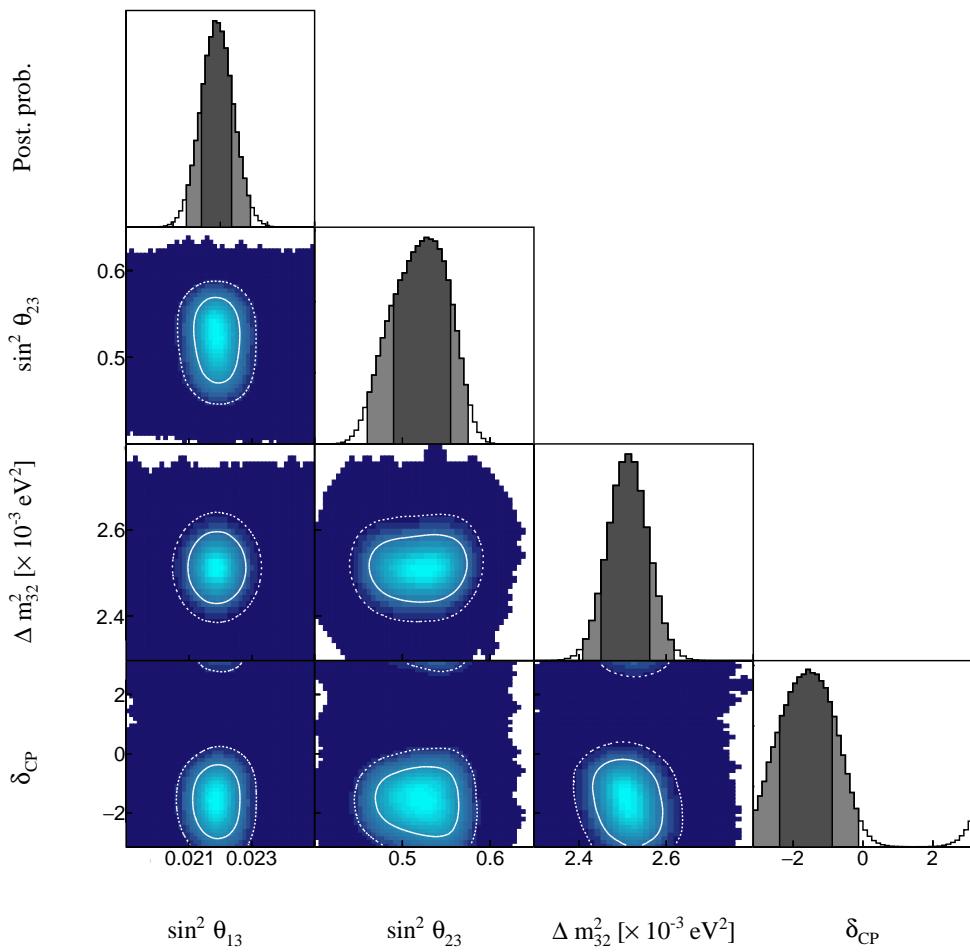


Figure 8.30: The posterior probability density distribution from the joint beam-atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3351 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3352 Constraint

3353 The benefits of the joint beam-atmospheric analysis can be determined by compar-
 3354 ing the sensitivities to the beam-only analysis presented in [2, 211]. This section
 3355 presents those comparisons for sensitivities built using the Asimov A oscillation
 3356 parameters defined in Table 2.2 and the post-BANFF systematic tune. The reactor
 3357 constraint is not applied within either of the fits used in these comparisons.

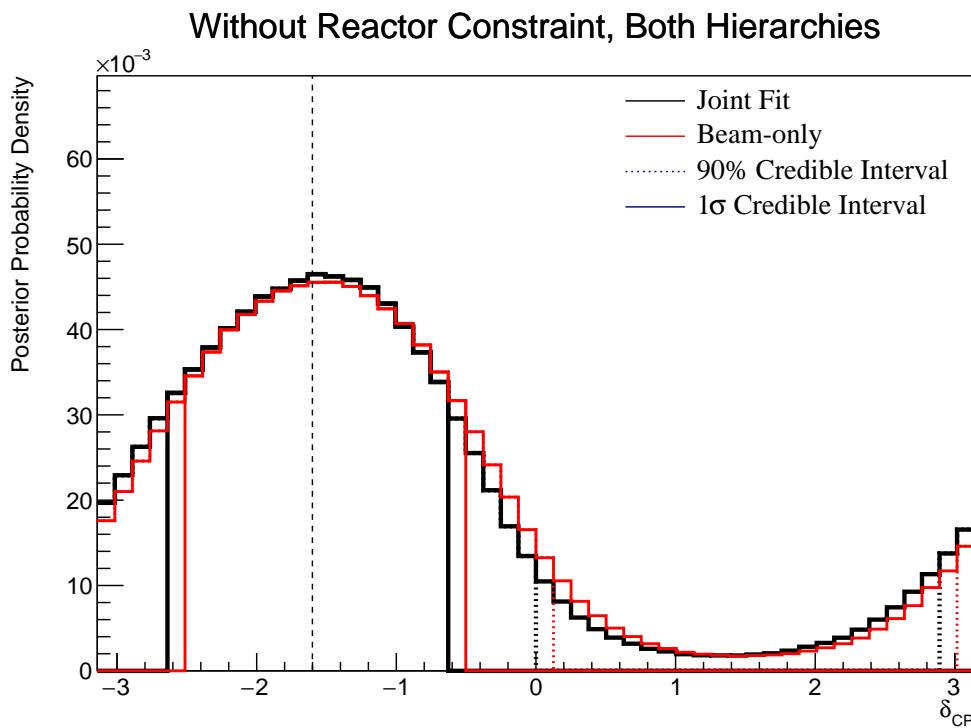


Figure 8.31: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3358 The sensitivity, marginalised over both hierarchies, to δ_{CP} from the joint beam-
 3359 atmospheric and beam-only fits is presented in Figure 8.31. As expected from the
 3360 likelihood scans (Figure 8.4), the sensitivity to δ_{CP} is not significantly increased.
 3361 This is because the known oscillation parameter value lies at the position where
 3362 the beam samples dominate the sensitivity compared to the SK samples.

3363 The sensitivity to Δm_{32}^2 is compared between the joint beam-atmospheric fit
 3364 and beam-only fit in Figure 8.32. The 1σ credible interval of the joint beam-
 3365 atmospheric fit is entirely contained within the NH region. This shows the
 3366 significant increase in the ability of the fit to determine the correct mass hierarchy,
 3367 compared to the beam-only analysis. This is further evidenced by the fact that the
 3368 90% credible intervals from the joint fit are also tighter in the IH region compared
 3369 to the beam-only analysis. The Bayes factor for mass hierarchy determination
 3370 for the beam-only and joint beam-atmospheric fits are $B(\text{NH}/\text{IH}) = 1.91$ and
 3371 $B(\text{NH}/\text{IH}) = 3.67$, respectively. According to Jeffrey's scale, the beam-only
 3372 analysis represents a weak preference for the correct hierarchy whereas the
 3373 joint fit returns a substantial preference for the NH hypothesis. Notably, this
 3374 conclusion does not require any external constraints and clearly illustrates the
 3375 benefit of the joint analysis.

Without Reactor Constraint, Both Hierarchies

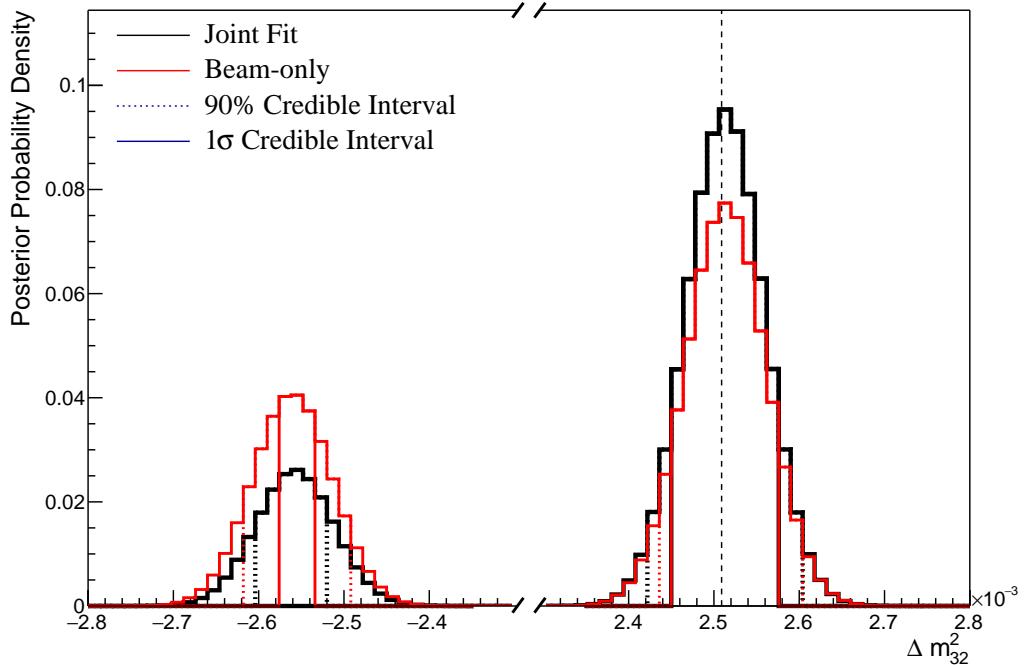


Figure 8.32: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, for both the beam-only and joint beam-atmospheric analysis are presented in Figure 8.33. The peak of the posterior distribution from the joint analysis is more aligned with the known value of $\sin^2(\theta_{23}) = 0.528$ compared to the beam-only analysis. The Bayes factors for the beam-only and joint beam-atmospheric fit are $B(\text{UO}/\text{LO}) = 1.56$ and $B(\text{UO}/\text{LO}) = 1.74$, respectively. Therefore, the joint beam-atmospheric fit does prefer the UO more strongly than the beam-only analysis, albeit slightly.

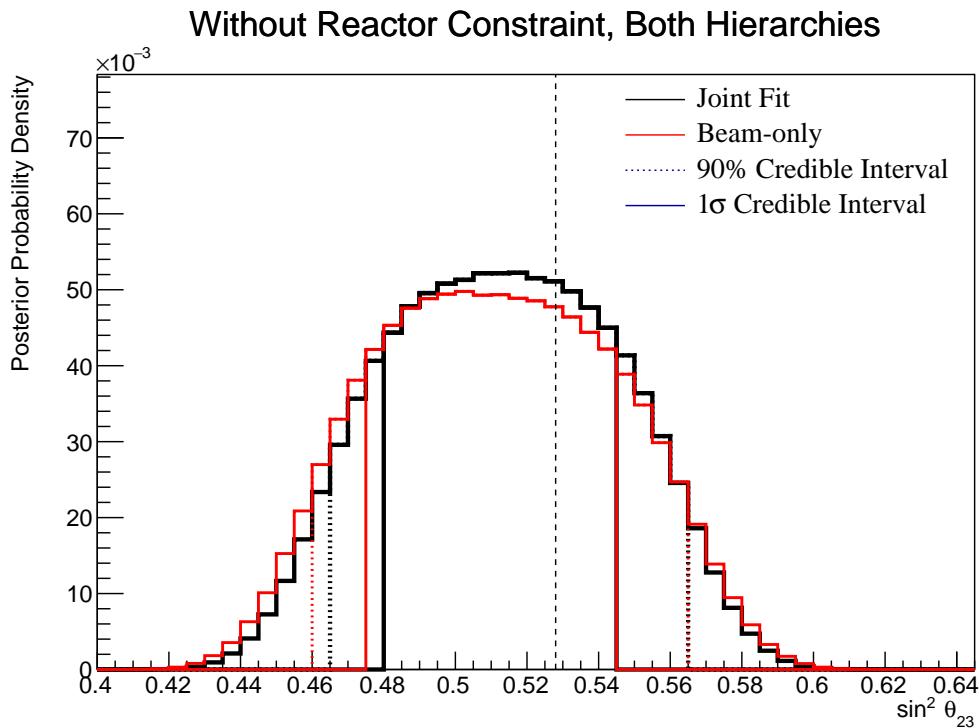


Figure 8.33: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

Whilst the beam-only and joint beam-atmospheric fits have similar sensitivity to δ_{CP} and $\sin^2(\theta_{23})$ when projected in one-dimension, the benefit of the joint analysis becomes more obvious when the sensitivities are presented in two-dimensions. The sensitivity of the two fits to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$) is illustrated in Figure 8.34. The width of the 99% joint fit credible interval in $\sin^2(\theta_{13})$ is squeezed in the region of $\delta_{CP} \sim 0$ compared to the

beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The 1σ and 90% also exhibit slightly tighter constraints on δ_{CP} . This is most prevalent in the region of $\delta_{CP} \sim 0$ and $\sin^2(\theta_{13}) \sim 0.03$. Whilst the atmospheric samples do not have significant sensitivity to $\sin^2(\theta_{13})$ (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters $\sin^2(\theta_{23}) - \Delta m_{32}^2$ is presented in Figure 8.35 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. Even in the NH region, the widths of the credible intervals in $\sin^2(\theta_{23})$ decreases, albeit to a smaller extent.

The comparison in sensitivity to $\delta_{CP} - \Delta m_{32}^2$ is illustrated in Figure 8.36. The contours from the joint beam-atmospheric fit are much smaller in the IH region as compared to the beam-only analysis. This culminates in a region around $\delta_{CP} \sim \pi/2$ in the H region which is excluded at 3σ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to Δm_{32}^2 , as a function of $\sin^2(\theta_{13})$, is presented in Figure 8.37. Similar to previous observations, the Δm_{32}^2 contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH 1σ credible intervals exclude the region around the reactor constraint. This suggests that the application of the reactor constraint would further increase the preference for NH in the joint fit compared to its effect on the beam-only analysis.

The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as illustrated

Without Reactor Constraint, Both Hierarchies

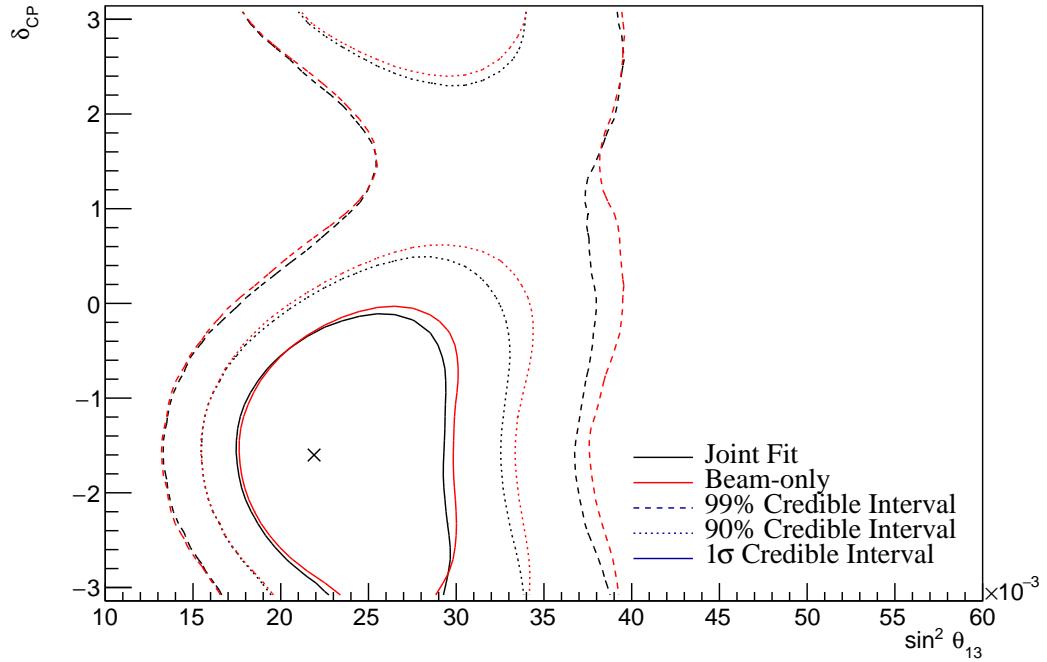


Figure 8.34: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of δ_{CP} – $\sin^2(\theta_{13})$.

³⁴¹⁹ by Figure 8.38. The joint analysis disfavours the wrong octant hypothesis more
³⁴²⁰ strongly in the region of high $\sin^2(\theta_{13})$. This change in correlation means that the
³⁴²¹ application of the reactor constraint could affect the two analyses differently.

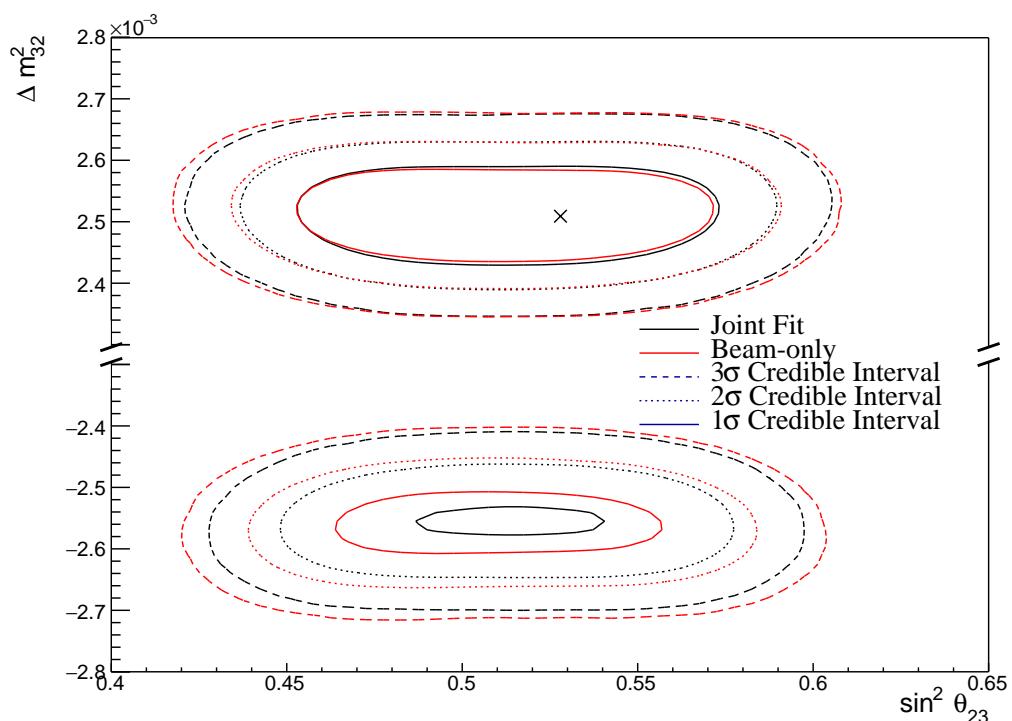


Figure 8.35: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

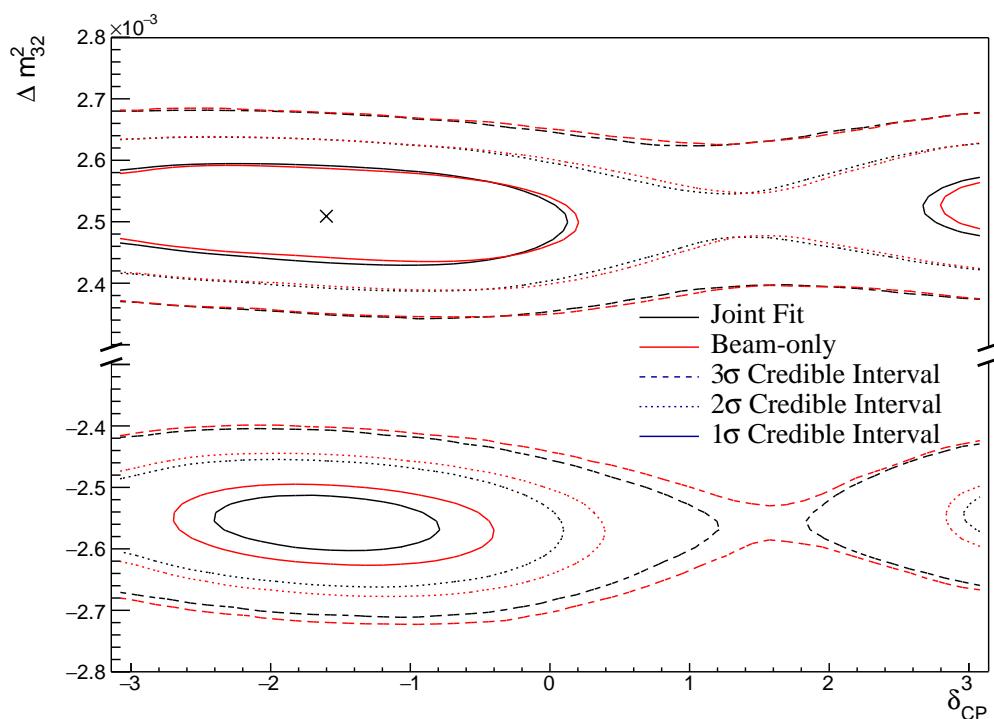


Figure 8.36: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \Delta_{CP}$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \Delta_{CP}$.

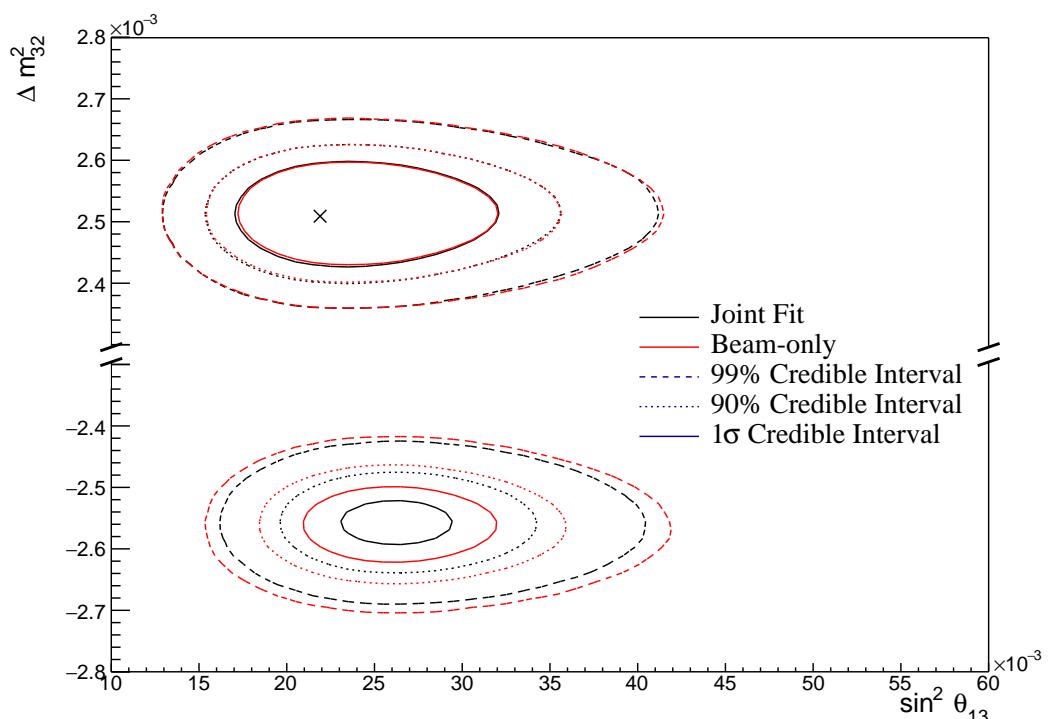


Figure 8.37: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

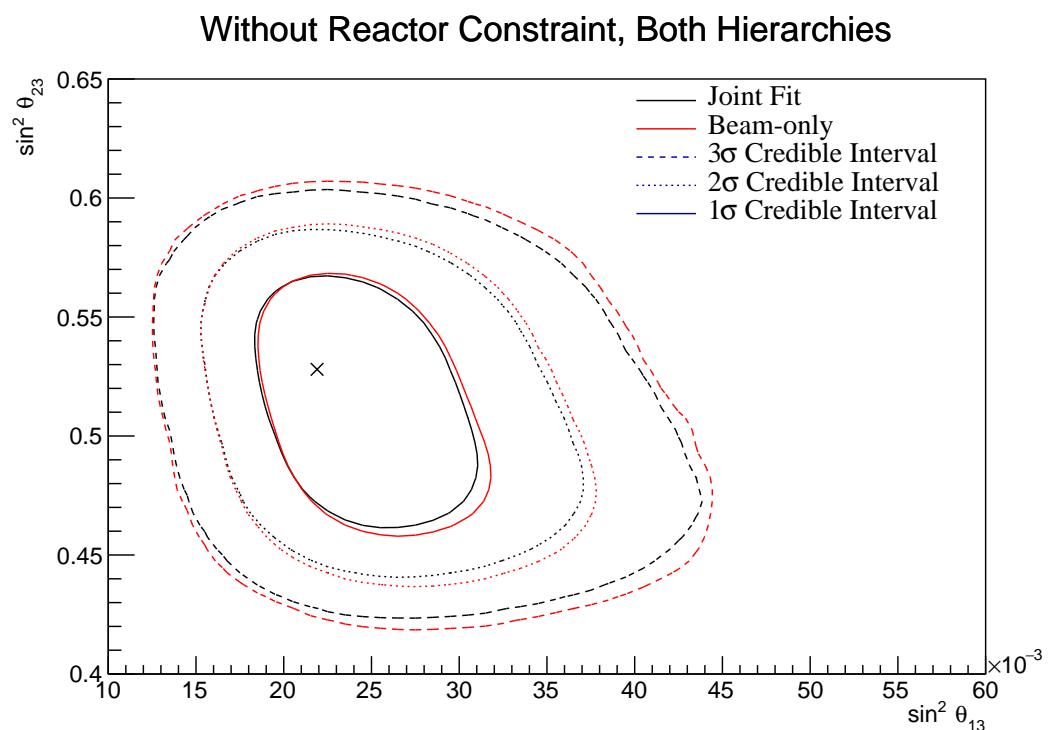


Figure 8.38: The two-dimensional posterior probability density distribution in $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$.

3422 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3423 straint

3424 This section illustrates the comparison between the joint beam-atmospheric and
 3425 beam-only fits when the reactor constraint is applied. As shown in Figure 8.37,
 3426 the application of the reactor constraint is expected to significantly increase
 3427 the joint fit's preference for the NH hypothesis, compared to the beam-only
 3428 analysis. Figure 8.39 illustrates the sensitivities of the two fits to the disappearance
 3429 parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$). This plot further illustrates the benefit of the joint
 3430 beam-atmospheric analysis. The 1σ credible interval in the IH region is entirely
 3431 removed in the joint analysis but not for the beam-only analysis.

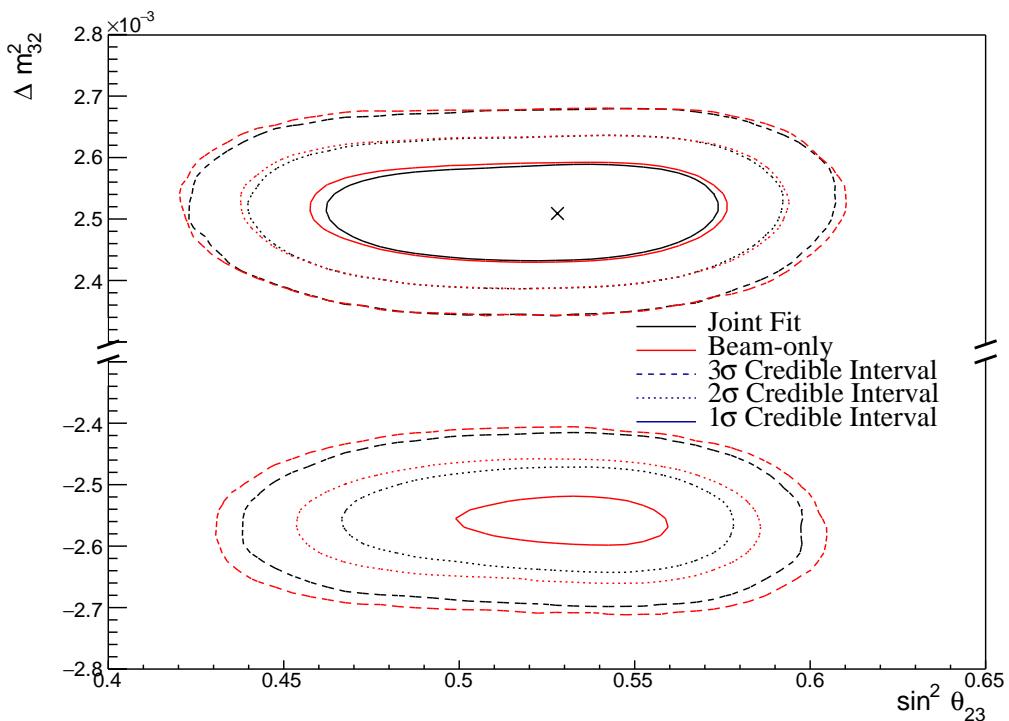


Figure 8.39: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is applied in both fits. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

3432 The credible intervals of the joint fit are also tighter in the $\sin^2(\theta_{23})$ dimension
 3433 than the beam-only analysis in both mass hierarchy regions. This shows that
 3434 beyond the ability of the joint fit to prefer the NH more strongly than the beam-

only analysis, the precision to which it can measure $\sin^2(\theta_{23})$ is also improved. The Bayes factor for NH preference is calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{NH}/\text{IH}) = 3.09$ for the joint beam-atmospheric and beam-only analysis, respectively. This important conclusion illustrates that the joint beam-atmospheric analysis can provide a substantial preference for the correct hypothesis (NH) whilst the beam-only analysis can not.

The Bayes factors for UO preference which are $B(\text{UO}/\text{LO}) = 2.86$ and $B(\text{UO}/\text{LO}) = 2.47$ for the joint beam-atmospheric and beam-only analysis, respectively. Both of these represent a mild preference for the correct octant (UO) but a stronger preference is observed in the joint analysis.

The sensitivity of the beam-only and joint beam-atmospheric analyses, to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$), are compared in Figure 8.40. These results are marginalised over both hierarchies. For this particular set of known oscillation parameters (AsimovA defined in Table 2.2), the beam-only analysis dominates the sensitivity. The joint fit does slightly increase the sensitivity to δ_{CP} but it does not change any conclusions that would be made. As expected, the prior knowledge dominates any sensitivity either fit would have on $\sin^2(\theta_{13})$.

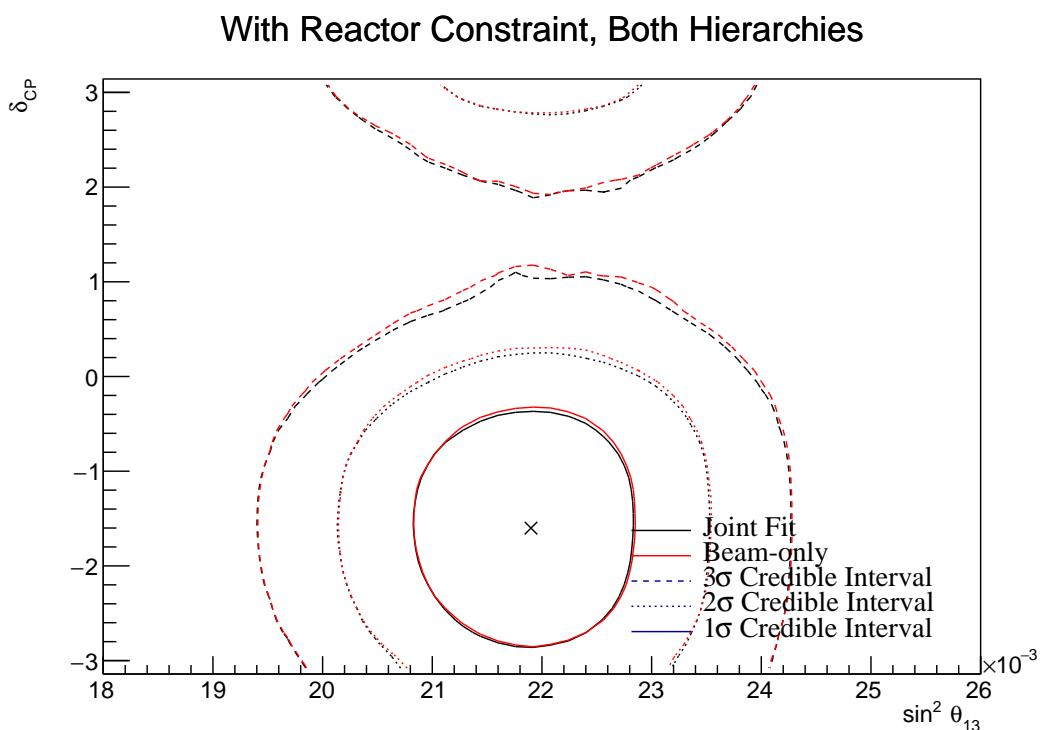


Figure 8.40: The two-dimensional posterior probability density distribution in $\delta_{CP}-\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies. The marker represents the known value of $\delta_{CP}-\sin^2(\theta_{13})$.

3452 8.3.8 Alternate Asimov Parameter Set

3453 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the
 3454 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters
 3455 are defined at a region of δ_{CP} which is preferred by the T2K experiment. This
 3456 explains why the addition of the atmospheric samples does not significantly in-
 3457 crease the sensitivity to δ_{CP} , as illustrated in subsection 8.3.6 and subsection 8.3.7.
 3458 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,
 3459 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when
 3460 building the Asimov data.

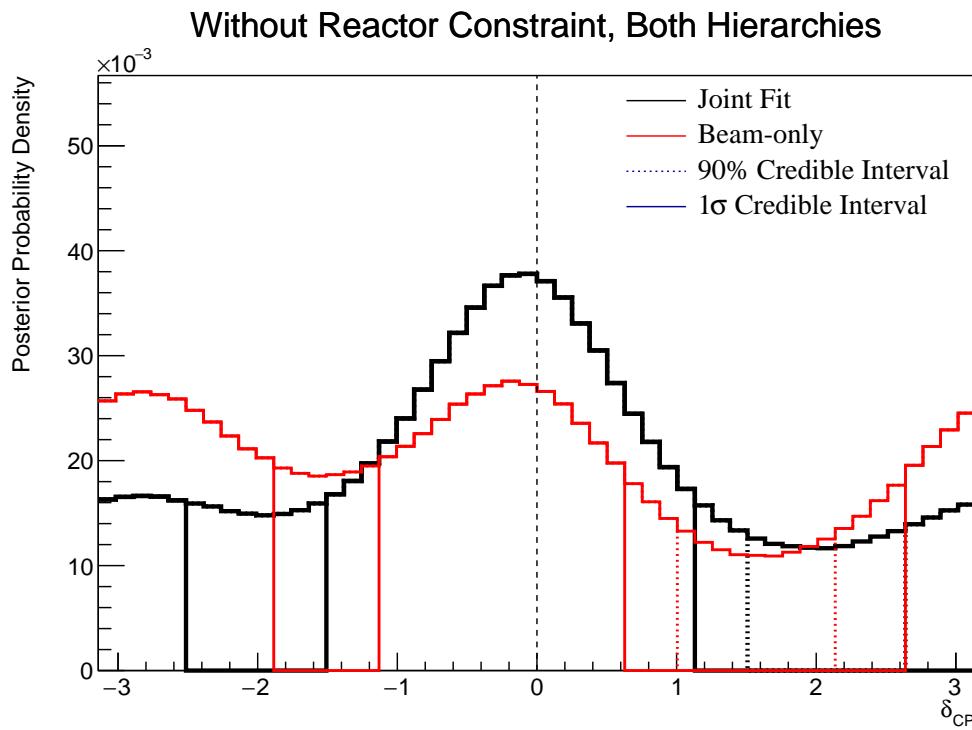


Figure 8.41: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3461 The sensitivity to δ_{CP} for the joint beam-atmospheric fit is presented in
 3462 Figure 8.41. The results are compared to those from the beam-only analysis
 3463 in [2, 211]. The reactor constraint is not applied in either of the fits. The

3464 shape of the posterior distribution from the joint analysis is more peaked at
3465 the known oscillation parameter value compared to the beam-only analysis,
3466 which has approximately the same posterior probability density at $\delta_{CP} = 0$
3467 and $\delta_{CP} = \pm\pi$. This shows the ability of the joint analysis to better determine
3468 the correct phase of δ_{CP} if the true value were CP-conserving. The 1σ credible
3469 intervals and the position of the highest posterior probability density are given in
3470 Table 8.9. The highest posterior density for the joint beam-atmospheric analysis
3471 is $\delta_{CP} = -0.06 \pm 0.06$ which is consistent with the known value.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -2.51], [-1.51, 1.13]$	-0.06 ± 0.06
δ_{CP} , (NH)	$[-1.13, 1.63]$	0.06 ± 0.06
δ_{CP} , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44 ± 0.06
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.52], [2.46, 2.56]$	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.47, 2.56]$	2.52 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.61, -2.52]$	-2.57 ± 0.01
$\sin^2(\theta_{23})$ (BH)	$[0.430, 0.480], [0.545, 0.585]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (NH)	$[0.430, 0.485], [0.550, 0.580]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (IH)	$[0.435, 0.480], [0.540, 0.585]$	0.568 ± 0.003

Table 8.9: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

3472 Naively, if just the 1σ credible interval were considered without observing
3473 the shape of the distribution, it would appear that the joint analysis would
3474 have a worse sensitivity to δ_{CP} due to the larger interval around $\delta_{CP} = 0$. The
3475 1σ credible interval for the beam-only analysis is given as the range $\delta_{CP} =$
3476 $[-\pi, -1.88], [-1.13, 0.63]$ and $[2.64, \pi]$ which contains 56% of all values of δ_{CP} .
3477 Whereas, the joint beam-atmospheric analysis contains 52% of all δ_{CP} values
3478 within the 1σ credible interval. Therefore, if the area within the 1σ credible

3479 interval were to be compared between the two fits, the joint analysis would
3480 be shown to have better precision.

3481 This apparent contradiction stems from the methodology in which the credible
3482 interval is calculated. The technique used in this analysis (documented in
3483 subsection 4.3.2) fills the credible interval by selecting bins in order of probability
3484 density until 68% of the posterior density is contained. If instead, the credible
3485 interval were calculated by expanding around the highest posterior probability,
3486 the benefits of the joint fit would be more obvious. In the case where the shape
3487 of the posterior was uni-modal, these two techniques would be equivalent to
3488 statistical fluctuations.

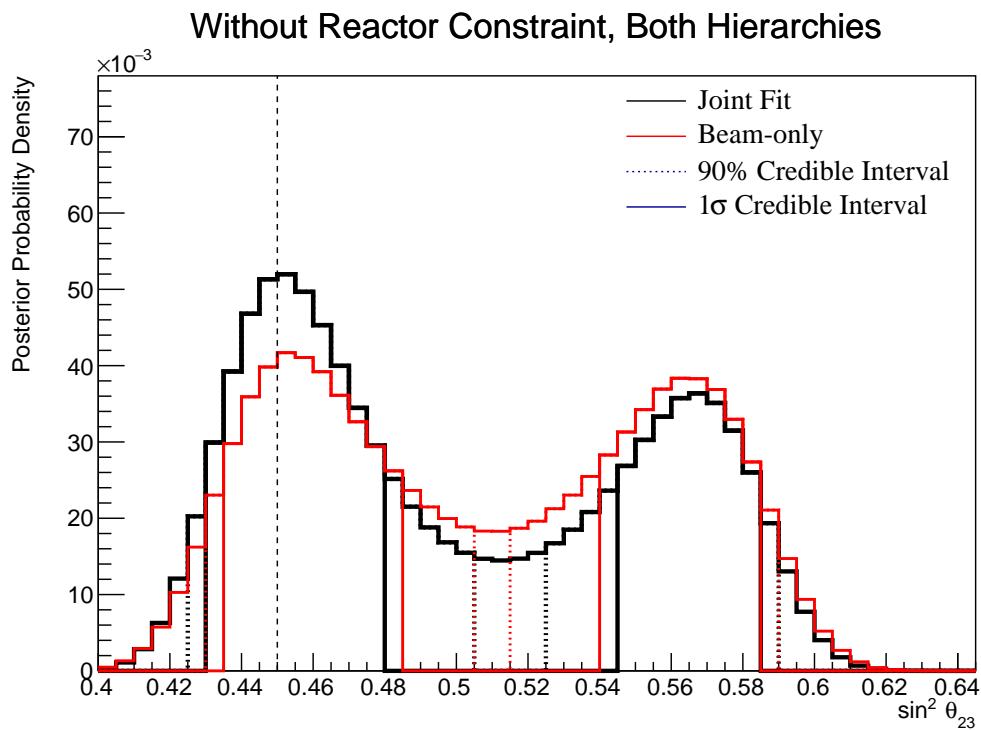


Figure 8.42: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3489 The sensitivity of the joint beam-atmospheric fit to $\sin^2(\theta_{23})$ is presented in
3490 Figure 8.42. The sensitivity is compared to that of the beam-only analysis in [2,
3491 211]. The reactor constraint is not applied in either of the fits being compared.

3492 The joint beam-atmospheric fit has a much larger probability density in the region
 3493 surrounding the known oscillation parameter, $\sin^2(\theta_{23}) = 0.45$. This shows the
 3494 better octant determination of the joint analysis compared to the beam-only fit.
 3495 The ratio of the posterior density at the peak of the lower octant to the peak of
 3496 the upper octant from the joint fit is 1.43 compared to 1.09 from the beam-only
 3497 analysis. The area contained within the 1σ credible interval for the joint analysis
 3498 is $\sin^2(\theta_{23}) = [0.430, 0.480]$ and $\sin^2(\theta_{23}) = [0.545, 0.585]$, whereas the beam-
 3499 only analysis includes $\sin^2(\theta_{23}) = [0.435, 0.485]$ and $\sin^2(\theta_{23}) = [0.540, 0.585]$.
 3500 This corresponds to a $\sim 5\%$ (binning dependent) increase in precision from
 3501 the joint analysis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.35	0.24	0.59
IH ($\Delta m_{32}^2 < 0$)	0.19	0.22	0.41
Sum	0.54	0.46	1.00

Table 8.10: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$.

3502 The distribution of steps, split by hierarchy and octant hypothesis, is presented
 3503 in Table 8.10. The Bayes factor for hierarchy and octant determination are
 3504 $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$, respectively. These values compare
 3505 to $B(\text{NH}/\text{IH}) = 1.08$ and $B(\text{LO}/\text{UO}) = 0.91$ from the beam-only analysis. This
 3506 evidences the joint analysis's ability to select the correct octant and hierarchy
 3507 hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6
 3508 show how the preferences for the correct octant and hierarchy depend on the
 3509 true value of δ_{CP} and $\sin^2(\theta_{23})$.

3510 The sensitivity of the beam-only and joint beam-atmospheric analysis to
 3511 Δm_{32}^2 is given in Figure 8.43. The joint analysis has a stronger preference for the
 3512 correct hierarchy (NH) which is shown by the higher Bayes factor compared
 3513 to the beam-only analysis. This is further evidenced by the width of the 90%

Without Reactor Constraint, Both Hierarchies

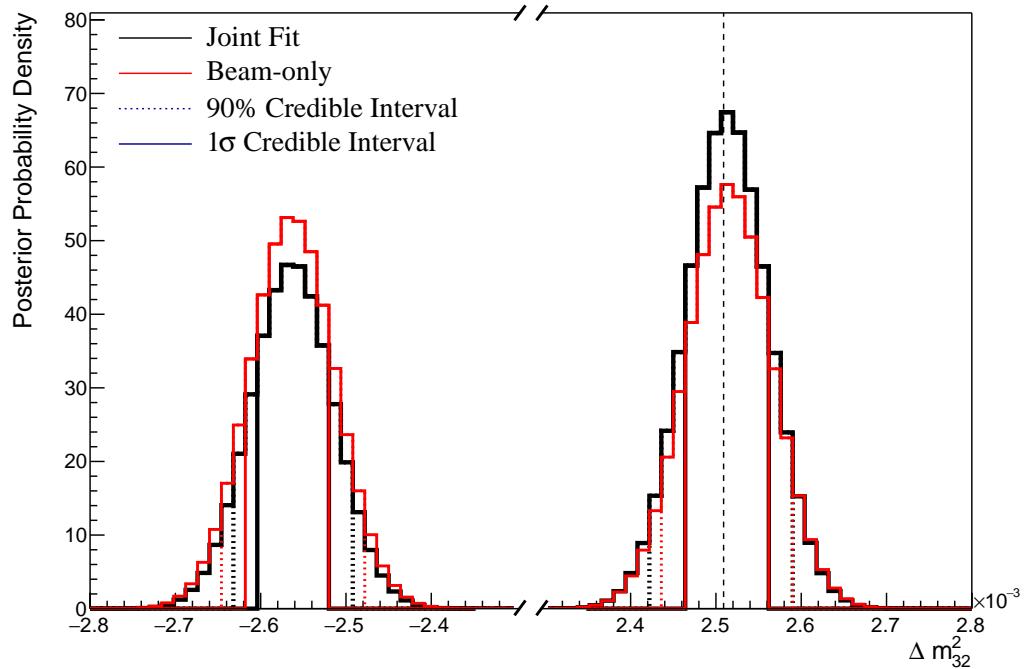


Figure 8.43: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 211]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

credible interval in the IH region being tighter in the joint analysis compared to the beam-only analysis.

³⁵¹⁴ credible interval in the IH region being tighter in the joint analysis compared

³⁵¹⁵ to the beam-only analysis.

3516 8.3.9 Effect of Systematics

3517 The effect of systematics on each sample used in this analysis is calculated using
 3518 the posterior predictive method documented in subsection 4.3.4. The distribution
 3519 of each sample’s spectrum has been generated by sampling 2000 steps from the
 3520 posterior distribution of the joint beam-atmospheric fit. This technique reweights
 3521 the Monte Carlo prediction using the systematic values given by a particular
 3522 step, stores the sample spectra, and repeats for the desired number of steps.
 3523 The oscillation parameters are always fixed at Asimov A values to only study
 3524 the effect of systematic parameters.

3525 Figure 8.44 illustrates the distribution for the SubGeV-*elike*-0dcy atmospheric
 3526 sample. The fit being sampled is detailed in subsection 8.3.4. The distribution
 3527 closely resembles the Asimov data spectrum. This would be expected from
 3528 an Asimov fit where the Monte Carlo is fit to itself but gives more credibility
 3529 to the results of the fit.

3530 The total event rate for each sample from each of the sampled steps is
 3531 calculated and the fractional uncertainty, $\Delta N_i / N_i$ where N_i is the event rate of the
 3532 i^{th} sample, is calculated. These values are presented in Table 8.11. In general, the
 3533 impact of the systematics has a $\sim 3\%$ uncertainty on the event rate of atmospheric
 3534 samples, where CC1 π -targeting samples have slightly larger uncertainties than
 3535 the CCQE-like samples. The fractional uncertainties on the beam samples are
 3536 compared to those from the beam-only analysis presented in [2, 211]. The
 3537 uncertainties on the one-ring muon samples are mostly unchanged, whereas
 3538 the uncertainties on the one-ring electron samples are different. As discussed in
 3539 section 8.2, the atmospheric samples should be able to add constraints on the NC
 3540 background events present in the FHC1Re and RHC1Re samples. The uncertainty
 3541 reduction seen in those samples is due to those additional constraints. The reason
 3542 why the FHC1Re1de has a higher uncertainty in this analysis is due to the addition
 3543 of the ad-hoc systematic introduced for CC1 π interactions (see subsection 6.4.3).

3544 Beyond the impact of the uncertainty on each sample’s event rate, the post-fit
 3545 constraint on each systematic parameter should be checked. Figure 8.45 illustrates

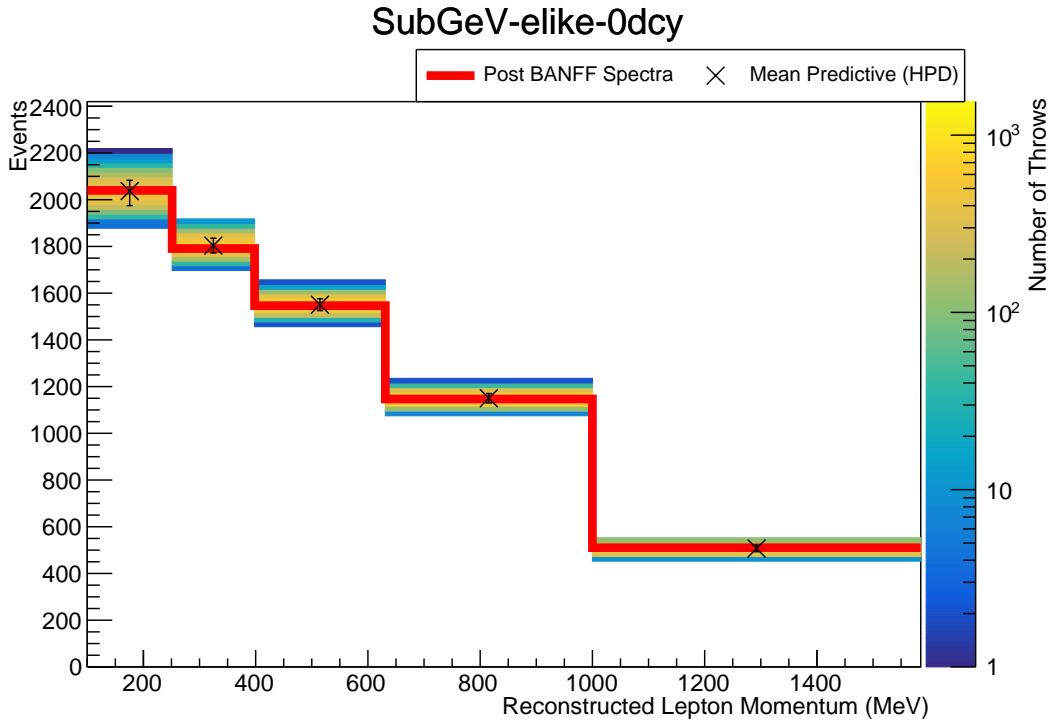


Figure 8.44: Result of the posterior predictive method for the SubGeV_elike_0dcy sample after sampling 2000 steps from the joint beam-atmospheric chain detailed in subsection 8.3.4 (Coloured histogram). The mean and uncertainty is presented for each bin. The Asimov data prediction (denoted ‘Post BANFF Spectra’, Red) assumes the post-BANFF tune and Asimov A oscillation parameters.

the central value and uncertainty on a select group of interaction systematics, for both the joint beam-atmospheric (from subsection 8.3.4) and the beam-only analysis. From the discussion in section 8.2, the uncertainty on systematics which are strongly constrained by the near detector should not significantly change when adding the atmospheric analysis. This behaviour is evidenced by the fact that the ratio of constraints between the two fits are very similar (within a few %) for almost all systematics. The only systematic which is more constrained in the joint beam-atmospheric analysis is the *NCOtherSK* normalisation parameter, which has a $O(10\%)$ tighter constraint. As expected, the atmospheric samples have been able to constrain this systematic which leads to the reduction in uncertainty for the beam electron-like samples.

Sample	Percentage Uncertainty (%)	
	Joint Analysis	Beam-only Analysis
SubGeV-elike-0dcy	2.53	-
SubGeV-elike-1dcy	3.28	-
SubGeV-mulike-0dcy	2.62	-
SubGeV-mulike-1dcy	2.23	-
SubGeV-mulike-2dcy	3.96	-
SubGeV-pi0like	2.84	-
MultiGeV-elike-nue	5.14	-
MultiGeV-elike-nuebar	2.79	-
MultiGeV-mulike	2.99	-
MultiRing-elike-nue	2.94	-
MultiRing-elike-nuebar	2.83	-
MultiRing-mulike	2.89	-
MultiRingOther-1	2.70	-
PCStop	3.22	-
PCThrus	2.99	-
UpStop-mu	2.95	-
UpThruNonShower-mu	2.70	-
UpThruShower-mu	3.19	-
FHC1Rmu	2.49	2.33
RHC1Rmu	2.89	2.93
FHC1Re	4.12	4.57
RHC1Re	5.15	5.65
FHC1Re1de	13.38	11.51

Table 8.11: The percentage uncertainty, $\Delta N / N$, as calculated from sampling 2000 throws from a joint beam-atmospheric chain. The same values for the beam samples are provided from the beam-only analysis [2, 211]. These uncertainties consider all systematic parameters to be sampled from the fit whilst the oscillation parameters are fixed at the Asimov A oscillation set.

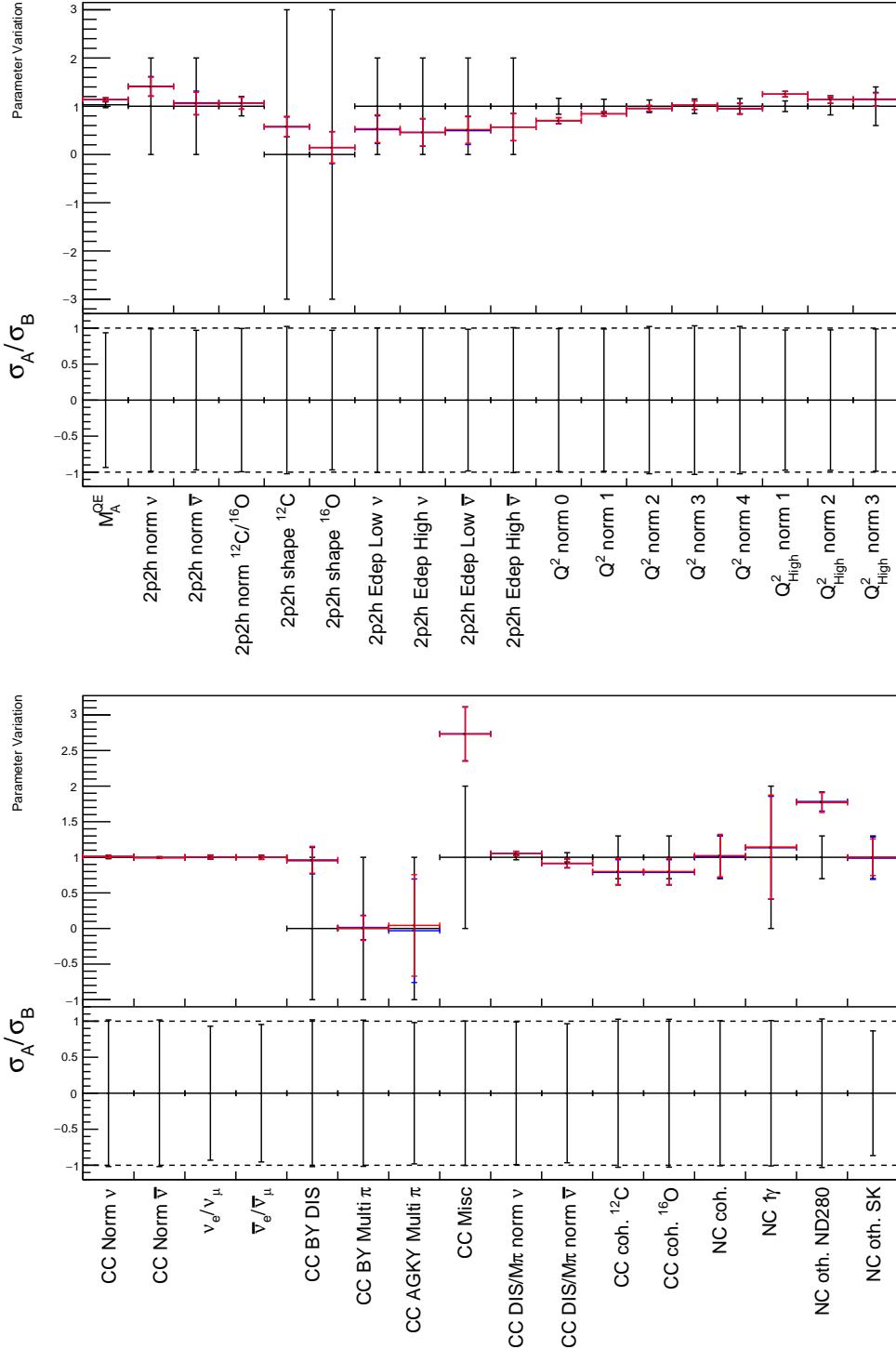


Figure 8.45: Central values and 1σ uncertainties for a select group of interaction systematics. The constraints from the prior uncertainty (Black), joint atmospheric-beam fit given in subsection 8.3.4 (Red) and beam-only analysis [2, 211] (Blue) are presented. The top part of each plot presents the parameter variation and the bottom part represents the ratio of the uncertainty between the joint beam-atmospheric and beam-only fits, where a value below 1.0 means the joint fit has a tighter constraint than the beam-only analysis.

3557 8.3.10 Alternative Detector Model Concept

3558 DB: Complete this section

3559 As a plausible extension to the analysis already presented, this section con-
 3560 siders the joint beam-atmospheric fit results when the correlated detector model
 3561 documented in subsubsection 6.4.5.3 is used instead of the inputs provided by the
 3562 two experiments. The results shown here should be considered preliminary as the
 3563 results need further investigation to ensure their reliability. This technique fits the
 3564 detector systematics simultaneously with the sample spectra so the definition of
 3565 the likelihood, given in section 6.5, needs to be modified. Following the method
 3566 outlined in DB: Xiaoyue's thesis, an additional term needs to be added to the
 3567 likelihood. This additional term is,

$$\frac{1}{2} \sum_s^{SGSamples} \sum_k^{E_k bins} \left(\rho(R_s | E_k, \vec{\theta}) \rho(P_s^{e/\mu} | E_k, \vec{\theta}) \rho(P_s^{e/\pi^0} | E_k, \vec{\theta}) \rho(P_s^{\mu/\pi^+} | E_k, \vec{\theta}) \right). \quad (8.1)$$

3568 This fits the ring counting R_s , electron-muon $P^{e/\mu}$, electron-neutral pion P^{e/π^0}
 3569 and muon-charged pion P^{μ/π^+} particle identification distribution parameters,
 3570 in bins of visible energy E_k , of the SubGeV atmospheric and beam samples
 3571 (SG). In the situation where infinite Monte Carlo statistics were generated, a
 3572 many-dimensional (the reconstructed lepton momentum and direction and the
 3573 four particle identification parameters in the case of an atmospheric sample)
 3574 spectra could be determined. However, this is not the case so the standard
 3575 spectra (reconstructed lepton momentum and direction) is fit along with four one-
 3576 dimensional distribution of each parameter identification parameter. Therefore,
 3577 shape-likelihoods are used for each particle identification parameter,

$$\rho(X | E_k, \vec{\theta}) = \sum_{i=0}^j \left[N_i^d \times \log \left(\frac{N_i^p(\vec{\theta})}{\sum_{i=0}^X N_i^p(\vec{\theta})} \right) \right]. \quad (8.2)$$

3578 Where X refers to a particle identification parameter, $N_i^{p(d)}$ refers to the i^{th} bin
 3579 value from the predicted MC distribution (data distribution) and j represents
 3580 the total bin number of the X distribution.

3581 8.4 Summary of Sensitivity Studies

3582 The sensitivities to each oscillation parameter from the joint beam-atmospheric
 3583 and beam-only fits, which use the Asimov A oscillation parameter set, are
 3584 summarised in Table 8.12. As the posterior distribution to δ_{CP} is cyclical, only
 3585 the position of the highest posterior density (HPD) is given. The uncertainty
 3586 corresponds to the width of the bin in which the HPD is located. Furthermore,
 3587 the Δm_{32}^2 reported values only consider the NH credible interval region as the
 3588 full discussion can be found in the previous section.

Fit	δ_{CP} (HPD)	$\Delta m_{32}^2 [\times 10^{-3} \text{eV}^2]$	$\sin^2(\theta_{23})$	$\sin^2(\theta_{13}) [\times 10^{-2}]$
Asimov A	-1.601	2.509	0.528	2.19
Beam	-1.45 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	$2.45^{+0.45}_{-0.35}$
Beam w/RC	-1.57 ± 0.06	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	$2.19^{+0.06}_{-0.07}$
Joint	-1.57 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	$2.35^{+0.45}_{-0.35}$
Joint w/RC	-1.57 ± 0.06	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	$2.18^{+0.07}_{-0.06}$

Table 8.12: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 211] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead.

3589 The Bayes factors from the beam-only and joint atmospheric-beam analyses
 3590 are presented in Table 8.13. The strength of each preference, from Jeffrey's
 3591 scale (Table 4.1), is also given.

3592 To summarise this information, the joint fit prefers a tighter 1σ credible
 3593 interval in $\sin^2(\theta_{23})$ along with a stronger Bayes factor for preferring the correct
 3594 octant hypothesis. The increase in sensitivity to $|\Delta m_{32}^2|$ between the two fits is
 3595 negligible but the joint analysis substantially prefers the correct mass hierarchy
 3596 hypothesis. It does not require any external constraints on $\sin^2(\theta_{13})$ to make
 3597 this statement. The joint analysis also produces a value of $\sin^2(\theta_{13})$ closer to the
 3598 known value compared to the beam-only analysis. When the reactor constraint

Fit	$B(\text{NH}/\text{IH})$		$B(\text{UO}/\text{LO})$	
	Value	Strength	Value	Strength
Asimov A				
Beam	1.91	Weak	1.56	Weak
Beam w/RC	3.09	Weak	2.47	Weak
Joint	3.67	Substantial	1.74	Weak
Joint w/RC	6.47	Substantial	2.64	Weak
Asimov B				
Beam	1.08	Weak	0.91	Weak
Beam w/RC	0.98	Weak	1.15	Weak
Joint	1.43	Weak	1.19	Weak
Joint w/RC	1.36	Weak	1.52	Weak

Table 8.13: A comparison of the Bayes factors for mass hierarchy and $\sin^2(\theta_{23})$ octant hypotheses, from the beam-only [2, 211] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The strength of the preference for the normal mass hierarchy and upper octants are provided by Jeffrey's scale Table 8.13.

3599 is applied, the preference for the correct hierarchy and octant (NH and UO)
 3600 hypotheses increase but does not change the statement which would be made.

3601 The fits from the Asimov B comparisons (subsection 8.3.9) show the improved
 3602 ability for the joint analysis to more precisely select the true value of δ_{CP} if it were
 3603 CP-conserving, compared to the beam-only analysis. This is evidenced by the
 3604 area contained within the 1σ credible interval decreasing by $\sim 4\%$. Furthermore,
 3605 the joint fit is able to better determine the octant of $\sin^2(\theta_{23})$ when the true
 3606 value is moved further away from the boundary as evidenced by the larger
 3607 Bayes factor. There is also a $\sim 5\%$ reduction of area contained within the 1σ
 3608 credible interval in $\sin^2(\theta_{23})$.

9

3609

3610

Conclusions and Outlook

3611 This thesis has presented a joint beam and atmospheric neutrino oscillation
3612 analysis from the Tokai-to-Kamioka (T2K) and Super-Kamiokande (SK) collab-
3613 orations. It reports the sensitivities of the analysis, with the intention for the
3614 two collaborations to publish a data analysis in the near-future. It combines
3615 the two independent analyses presented by the collaborations [2, 3] and uses
3616 3244.4 days of SK livetime and 1.97×10^{21} (1.63×10^{21}) POT recorded at the far
3617 detector in the neutrino(antineutrino) beam operating mode. The ND280 near
3618 detector is used within this analysis to constrain the beam flux and cross-section
3619 systematics. It uses 1.15×10^{21} POT and 8.34×10^{20} POT in the neutrino and
3620 antineutrino running modes, respectively.

3621 This analysis uses a Bayesian Markov Chain Monte Carlo fitting technique
3622 implemented within the MaCh3 framework. This work has significantly developed
3623 the fitting framework, both in terms of technical features and performance.
3624 This includes supporting new samples, systematics, and oscillation channels.
3625 These developments have become the foundation of the fitter's expansion into
3626 other neutrino oscillation experiments. Beyond these improvements, a novel
3627 technique for calculating the atmospheric neutrino oscillation probabilities has
3628 been developed. This calculation uses a sub-sampling linear-averaging approach
3629 to ensure that the sensitivities being calculated are not biased due to insufficient

3630 Monte Carlo statistics in a region of rapidly varying probability. It illustrates a
3631 computationally feasible method of reliably calculating oscillation probabilities
3632 that can be utilised within any fitting framework.

3633 The sensitivity of the joint beam-atmospheric analysis is presented in Table 9.1,
3634 and compared to the beam-only analysis [2]. The sensitivities are evaluated
3635 using a set of known oscillation parameter values close to the results from a
3636 previous T2K analysis [75] (denoted AsimovA in Table 9.1). The joint analysis
3637 has a stronger sensitivity to $\sin^2(\theta_{23})$, as evidenced by the tighter 1σ credible
3638 intervals when the constraints from reactor experiments are not applied. The
3639 joint fit's sensitivity to δ_{CP} is marginally stronger than beam-only analysis. Whilst
3640 the sensitivity to $|\Delta m_{32}^2|$ is mostly unchanged between the two analyses, the
3641 sensitivity to select the correct hierarchy is significantly improved. This follows
3642 from a substantial preference for the normal hierarchy hypothesis presented
3643 within the joint analysis, as classified by Jeffrey's scale [130]. This is notable as
3644 the beam-only analysis cannot make this statement, either with or without the
3645 application of the reactor constraint. The joint fit's preference for the correct
3646 hierarchy increases once the reactor constraint is applied. The preference for
3647 selecting the correct octant of $\sin^2(\theta_{23})$ is classified as weak by Jeffrey's scale but
3648 is still stronger than the statement made by the beam-only analysis.

3649 The sensitivities of the beam-only and joint atmospheric-beam fit have also
3650 been compared at a set of known oscillation parameters which are CP-conserving
3651 and in the lower octant of $\sin^2(\theta_{23})$. The joint analysis has a $\sim 5\%$ improved abil-
3652 ity to select the known values more precisely compared to the beam-only analysis.

3653 Whilst this analysis provides the first sensitivities of a joint beam and atmo-
3654 spheric analysis, there are more improvements that could be made. Since this
3655 analysis began, the T2K collaboration has released an updated oscillation analysis
3656 with additional near and far detector samples alongside a more sophisticated
3657 interaction model [4]. The overall change in oscillation parameter measurement
3658 observed by T2K is relatively minor but the stronger constraints on the systemat-
3659 ics could impact this joint analysis to a larger extent. Further developments could

Fit	δ_{CP} (HPD)	Δm_{32}^2 [$\times 10^{-3}\text{eV}^2$]	$\sin^2(\theta_{23})$	$B(\text{NH/IH})$	$B(\text{UO/LO})$
Asimov A	-1.601	2.509	0.528	NH	UO
Beam	$-1.45^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	1.91	1.56
Beam w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	3.09	2.47
Joint	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	3.67	1.74
Joint w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	6.47	2.64

Table 9.1: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 211] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead. The Bayes factors are provided for the mass hierarchy preference: normal hierarchy (NH) and inverse hierarchy (IH), and $\sin^2(\theta_{23})$ octant preference: upper octant (UO) and lower octant (LO).

3660 consider the effect of correlating the beam and atmospheric flux uncertainties
 3661 relating to hadron production, where updates of the Bartol and Honda models
 3662 may allow this to be studied [147].

3663 Beyond these model improvements, more data is available than what is
 3664 assumed for this analysis. The T2K experiment has accumulated an additional
 3665 $1.78 \times 10^{20}\text{POT}$ in neutrino mode. Similarly, there are several early SK periods
 3666 that have not been considered within this analysis as the reconstruction software
 3667 used in this analysis has not been validated for those periods. SK will also
 3668 continue to accumulate statistics with Gadolinium doping. Developments in the
 3669 atmospheric sample selections may also benefit from the Gadolinium dopants as
 3670 neutron capture will aid in neutrino/antineutrino separation leading to better
 3671 mass hierarchy sensitivity. This would require including interaction systematics
 3672 for neutron capture of Gadolinium which has already started [212].

3673 This analysis shows the increased sensitivity to oscillation parameters from
 3674 the combination of beam and atmospheric samples. It has developed the MaCh3
 3675 fitting framework and has laid the foundations of the fitter's expansion into other
 3676 neutrino oscillation experiments. The sensitivities presented in this thesis, and

³⁶⁷⁷ the techniques that were used to generate them, are significant to the future of
³⁶⁷⁸ neutrino oscillation physics which will likely perform similar analyses. As such,
³⁶⁷⁹ they have been presented by the T2K and SK collaborations at the Neutrino 2022
³⁶⁸⁰ conference [4]. Moving towards the next generation of neutrino experiments,
³⁶⁸¹ this analysis has the potential to become the basis of the oscillation analysis
³⁶⁸² for the future Hyper-Kamiokande experiment which aims to make conclusive
³⁶⁸³ statements about CP-violation in the neutrino sector.

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