

1 **The Sensitivity to Oscillation Parameters**
2 **from a Simultaneous Beam and**
3 **Atmospheric Neutrino Analysis that**
4 **combines the T2K and SK Experiments**



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10 *Doctor of Philosophy*
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Abstract

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Acknowledgements

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⁷² the subsections and do one big traditional acknowledgement section.

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Statement of Originality

84 The work, and associated interpretation, presented within this thesis are
85 my own and were produced by myself. Where applicable, results and figures
86 taken from others have been attributed and referenced. This work has not been
87 submitted for any other qualification, at this university or any other.

88 The background of neutrino physics history and a description of the T2K and
89 SK experiments are provided in Chapters 2 and 3, respectively. The Bayesian
90 fitting techniques and MaCh3 framework used throughout this analysis are de-
91 tailed in Chapter 4. These chapters present entirely background knowledge
92 relevant for understanding the work presented within this thesis. Chapter 5
93 documents the simulation and reconstruction of neutrino events. This includes a
94 section of work done by myself to validate the `fiTQun` reconstruction software
95 for use on SK-V data.

96 Chapter 6 details the event selections and systematics used within this oscilla-
97 tion analysis. The selections were developed by others within the T2K and SK
98 collaborations and have been appropriately referenced. The implementation and
99 validation of the simultaneous support for the beam and atmospheric samples
100 selected by the SK detector, in the MaCh3 fitting framework, has been entirely my
101 own work. The systematic models invoked within this analysis were developed
102 by others, and relevant references have been included. The implementation
103 of these systematics within the MaCh3 fitting framework has been entirely my
104 own work and includes generating the systematic response functions for each
105 systematic from the Super-Kamiokande Monte Carlo. The implementation
106 and validation of the near detector sample and systematics were performed
107 by Clarence Wret on behalf of the MaCh3 working group.

108 Chapter 7 documents a new method of calculating the oscillation probabilities
109 for atmospheric neutrinos at SK. The method and validation of the ‘smearing’
110 technique were entirely my own. The methodology for including effects from
111 production height systematics was developed by others but the implementation
112 and validation were my own. Several performance increases, including the
113 interfacing of alternative oscillation calculation engines, were my own work.
114 The sensitivities, and interpretation, provided within Chapter 8 are produced
115 entirely by myself.

Contents

116

117	1 Introduction	1
118	2 Neutrino Oscillation Physics	4
119	2.1 Discovery of Neutrinos	5
120	2.2 Theory of Neutrino Oscillation	6
121	2.3 Neutrino Oscillation Measurements	10
122	2.4 Summary Of Oscillation Parameter Measurements	20
123	2.5 Overview of Oscillation Effects	21
124	3 T2K and SK Experiment Overview	29
125	3.1 The Super-Kamiokande Experiment	29
126	3.2 The Tokai to Kamioka Experiment	40
127	4 Bayesian Statistics and Markov Chain Monte Carlo Techniques	55
128	4.1 Bayesian Statistics	56
129	4.2 Monte Carlo Simulation	58
130	4.3 Understanding the MCMC Results	66
131	5 Simulation, Reconstruction, and Event Reduction	71
132	5.1 Simulation	71
133	5.2 Event Reconstruction at SK	77
134	5.3 Event Reduction at SK	88
135	6 Sample Selections and Systematics	93
136	6.1 Atmospheric Samples	95
137	6.2 Near Detector Beam Samples	103
138	6.3 Far Detector Beam Samples	105
139	6.4 Systematic Uncertainties	111
140	6.5 Likelihood Calculation	132
141	7 Oscillation Probability Calculation	135
142	7.1 Treatment of Fast Oscillations	136
143	7.2 Calculation Engine	143
144	7.3 Matter Density Profile	147
145	7.4 Production Height Averaging	152

¹⁴⁶	8 Oscillation Analysis	155
¹⁴⁷	8.1 Monte Carlo Prediction	156
¹⁴⁸	8.2 Likelihood Scans	157
¹⁴⁹	8.3 Sensitivity Studies	169
¹⁵⁰	8.4 Summary of Sensitivity Studies	221
¹⁵¹	9 Conclusions and Outlook	223

1

152

153

Introduction

154 Current measurements show that the universe is matter-dominated, despite
155 current theoretical knowledge suggesting that an equal amount of matter and
156 antimatter were created in the Big Bang. One explanation of this behaviour is
157 through the violation of charge parity (CP) symmetries, one requirement of the
158 Sakharov conditions [1]. The Standard Model relates a particle and its antiparticle
159 through these symmetries, which if violated, could result in the observed matter-
160 antimatter imbalance. CP-violation has been observed in quark mixing but is
161 insufficient to explain the observed asymmetry. Neutrino oscillation physics
162 has the potential to include CP-violating terms through the δ_{CP} phase contained
163 within the PMNS mechanism. Current measurements hint of a non-zero value [2]
164 but no conclusive measurement has been achieved. Therefore, one of the main
165 goals of neutrino oscillation experiments is to precisely measure this parameter.
166 This requires a precise measurement of all oscillation parameters including the
167 currently undetermined neutrino mass hierarchy.

168 The Super-Kamiokande (SK) detector is situated as the far detector of the
169 Tokai-to-Kamioka (T2K) experiment and observes neutrinos from the beam origi-
170 nating in J-PARC alongside the flux of atmospheric neutrinos emitted from the
171 primary and secondary interactions of cosmic rays. Previous oscillation analyses
172 published by these two experiments have treated the datasets independently.

173 However, due to the different energies, path lengths, and density of matter in
174 which the neutrinos pass through, a combined analysis will be able to leverage the
175 constraints from both experiments and be able to break some of the degeneracies
176 in oscillation parameter space.

177 This thesis details the sensitivities of a joint beam and atmospheric neutrino
178 analysis using beam samples observed at the near and far detectors of the
179 T2K experiment and atmospheric samples present in SK. It combines the beam
180 analysis presented in [2] and the atmospheric analysis documented in [3]. This
181 corresponds to run1-10 of the T2K experiment with approximately equal exposure
182 taken in neutrino and antineutrino beam modes, alongside more than 3000 days
183 of atmospheric events. This analysis has sensitivity to the δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$,
184 and Δm_{32}^2 oscillation parameters. Crucially, the combination of beam and atmo-
185 spheric neutrinos should give strong sensitivity to the mass hierarchy due to
186 the correlation between the matter resonance and $\sin^2(\theta_{23})$. The sensitivities are
187 generated by the MaCh3 Bayesian Markov Chain Monte Carlo fitting framework.
188 This analysis lays the foundation of a joint analysis in Hyper-Kamiokande, which
189 is one of the next-generation neutrino oscillation experiments.

190 Chapter 2 provides a concise overview of neutrino physics history includ-
191 ing the discovery of the neutrino along with the first evidence for neutrino
192 oscillation. It also includes a brief discussion of the theory underpinning the
193 PMNS formalism alongside a summary of the current measurements of each
194 oscillaton parameter.

195 The T2K and SK experiments are detailed in Chapter 3. This includes the
196 design and calibration of the SK detector along with a brief insight into the
197 composition and detection techniques of T2K's two near detectors. The neutrino
198 beamline, and the 'off-axis' trick, are also briefly summarised.

199 This thesis presents a Bayesian neutrino oscillation analysis that uses Markov
200 Chain Monte Carlo techniques. This analysis strategy, along with a summary
201 of the fundamental concepts of Bayesian inference, is described in Chapter 4.
202 This includes a discussion about the conditions that are required to correctly

203 sample the parameter space along with the methods used to calculate parameter
204 estimations and build credible intervals.

205 Chapter 5 details the simulations and reconstruction tools used to build Monte
206 Carlo predictions of each sample used within this analysis. This includes the
207 models used to provide a flux prediction of the beam and atmospheric neutrinos
208 as well as the models invoked with this analysis to simulate neutrino interactions.
209 Validation of the far detector’s reconstruction tools has been documented which
210 compares the change in detector response between two different periods of SK.

211 A description of the beam samples used at the near and far detector and the
212 atmospheric samples used at the SK detector is presented in Chapter 6. These
213 include energy and interaction mode comparisons along with documenting the
214 similarities between the event selection cuts. This chapter also includes the
215 systematic models used to control the uncertainty within the flux predictions of
216 both beam and atmospheric neutrinos, the interaction models, and the response
217 of the detectors used within this analysis.

218 A novel atmospheric neutrino oscillation calculation method is documented in
219 Chapter 7. This is required to ensure reliable Monte Carlo sampling of a rapidly
220 varying region of oscillation parameter space. This chapter also documents
221 systematics used to control the uncertainties related to the Earth’s density as well
222 as the production height of neutrinos in the upper atmosphere.

223 Chapter 8 presents the sensitivities of this joint beam and atmospheric neu-
224 trino oscillation analysis. The results are provided for two different sets of
225 known values. The application of the reactor constraint on $\sin^2(\theta_{23})$ has also been
226 considered. The sensitivities of the joint analysis are compared to the beam-only
227 analysis and show the benefits of the combined analysis. These results have been
228 presented by the T2K and SK collaborations at the Neutrino 2022 conference [4].

229 A summarised discussion of the sensitivity results and the outlook for the
230 analysis, including the implications of analysis on the next generation of neutrino
231 experiments, is provided in Chapter 9.

2

232

233

Neutrino Oscillation Physics

234 When first proposed, neutrinos were expected to be approximately massless
235 fermions that only interact through weak and gravitational forces. This meant
236 they were very difficult to detect as they can pass through significant amounts
237 of matter without interacting. Despite this, experimental neutrino physics has
238 developed many different detection techniques and observed neutrinos from
239 both natural and artificial sources. In direct tension with Standard Model physics,
240 neutrinos have been determined to oscillate between different lepton flavours,
241 requiring them to have mass.

242 The observation techniques which led to the discovery of the neutrino are doc-
243 umented in section 2.1. The theory underpinning neutrino oscillation is described
244 in section 2.2 and includes the approximations which can be made to simplify
245 the understanding of neutrino oscillation in the two-flavour approximation. Past,
246 current, and future neutrino experiments are detailed in section 2.3, including the
247 reactor, atmospheric, and long-baseline accelerator neutrino sources that have
248 been used to successfully constrain oscillation parameters. Finally, the current
249 state of oscillation parameter measurements are summarised in section 2.4.

250 2.1 Discovery of Neutrinos

251 At the start of the 20th century, the electrons emitted from the β -decay of the
252 nucleus were found to have a continuous energy spectrum [5, 6]. This observation
253 seemingly broke the energy conservation invoked within that period’s nuclear
254 models. In 1930, Pauli provided a solution to this problem in the form of a
255 new particle, the neutrino (originally termed “neutron”). It was theorized to
256 be an electrically neutral spin-1/2 fermion with a mass smaller than that of the
257 electron [7]. This neutrino was emitted with the electron in β -decay to alleviate
258 the apparent breaking of energy conservation. As a predecessor of today’s weak
259 interaction model, Fermi’s theory of β -decay developed the understanding by
260 coupling the four constituent particles: electron, proton, neutron, and neutrino,
261 into a quantitative model [8].

262 Whilst Pauli was not convinced of the ability to detect neutrinos, the first
263 observations of the particle were made in the mid-1950s when neutrinos from
264 a reactor were observed via the inverse β -decay (IBD) process, $\bar{\nu}_e + p \rightarrow n + e^+$
265 [9, 10]. The detector consisted of two parts: a neutrino interaction medium and
266 a liquid scintillator. The interaction medium was built from two water tanks,
267 loaded with cadmium chloride to allow for increased efficiency in the detection
268 of neutron capture. The positron emitted from IBD annihilates, $e^+ + e^- \rightarrow 2\gamma$,
269 generating a prompt signal and the neutron is captured on the cadmium via
270 $n + ^{108}Cd \rightarrow ^{109*}Cd \rightarrow ^{109}Cd + \gamma$, producing a delayed signal. An increase in
271 the coincidence rate was observed when the reactor was operating which was
272 interpreted as interactions from neutrinos generated in the reactor.

273 After the discovery of the ν_e , the question of how many flavours of neu-
274 trino exist was asked. In 1962, a measurement of the ν_μ was conducted at the
275 Brookhaven National Laboratory [11]. A proton beam was directed at a beryllium
276 target, generating pions which then decayed via $\pi^\pm \rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu)$, and the
277 subsequent interactions of the ν_μ were observed. As the subsequent interaction
278 of the neutrino generated muons rather than electrons, it was determined that

the ν_μ was fundamentally different from ν_e . The final observation to be made was that of the ν_τ from the DONUT experiment [12]. Three neutrinos seem the obvious solution as it mirrors the known number of charged leptons (as they form weak isospin doublets) but there could be evidence of more. Several neutrino experiments have found anomalous results [13, 14] which could be attributed to “sterile” neutrinos. These hypothesised particles are not affected by gauge interactions in the Standard Model so their presence can only be inferred through the observation of non-standard oscillation modes. However, cosmological observations indicate the number of neutrino species $N_{eff} = 2.99 \pm 0.17$ [15], as measured from the cosmic microwave background power spectrum. LEP also measured the number of active neutrino flavours to be $N_\nu = 2.9840 \pm 0.0082$ [16] from measurements of the Z-decay width, but this does not strongly constrain the number of sterile neutrinos.

2.2 Theory of Neutrino Oscillation

A neutrino generated with lepton flavour α can change into a different lepton flavour β after propagating some distance. This phenomenon is called neutrino oscillation and requires that neutrinos must have a non-zero mass. This behaviour has been characterised by the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [17–19] mixing matrix which describes how the flavour and mass of neutrinos are associated. This is analogous to the Cabibbo-Kobayashi-Maskawa (CKM) [20] matrix measured in quark physics.

2.2.1 Three Flavour Oscillations

The PMNS parameterisation defines three flavour eigenstates, ν_e , ν_μ and ν_τ (indexed ν_α), which are eigenstates of the weak interaction and three mass eigenstates, ν_1 , ν_2 and ν_3 (indexed ν_i). Each mass eigenstate is the superposition

³⁰⁴ of all three flavour states,

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle. \quad (2.1)$$

³⁰⁵ Where U is the 3×3 PMNS matrix which is unitary and connects the mass
³⁰⁶ and flavour eigenstates.

³⁰⁷ The weak interaction, when interacting via a W^{\pm} boson, couples to flavour
³⁰⁸ eigenstates so neutrinos interact with leptons of the same flavour. The prop-
³⁰⁹ agation of a neutrino flavour eigenstate, in a vacuum, can be re-written as a
³¹⁰ plane-wave solution to the time-dependent Schrödinger equation,

$$|\nu_{\alpha}(t)\rangle = \sum_i U_{\alpha i}^{*} |\nu_i\rangle e^{-i\phi_i}. \quad (2.2)$$

³¹¹ The ϕ_i term can be expressed in terms of the energy, E_i , and magnitude of the
³¹² three momenta, p_i , of the neutrino, $\phi_i = E_i t - p_i x$ (t and x being time and position
³¹³ coordinates). The probability of observing a neutrino of flavour eigenstate β from
³¹⁴ one which originated as flavour α can be calculated as,

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\langle \nu_{\beta} | \nu_{\alpha}(t) \rangle|^2 = \sum_{i,j} U_{\alpha i}^{*} U_{\beta i} U_{\alpha j} U_{\beta j}^{*} e^{-i(\phi_j - \phi_i)}. \quad (2.3)$$

³¹⁵ The term within the exponential can be represented as,

$$\phi_j - \phi_i = E_j t - E_i t - p_j x + p_i x. \quad (2.4)$$

³¹⁶ For a relativistic particle, $E_i \gg m_i$, a Taylor series expansion means,

$$p_i = \sqrt{E_i^2 - m_i^2} \approx E_i - \frac{m_i^2}{2E_i}. \quad (2.5)$$

³¹⁷ Making the approximations that neutrinos are relativistic, the mass eigenstates
³¹⁸ were created with the same energy and that $x = L$, where L is the distance
³¹⁹ travelled by the neutrino, Equation 2.4 then becomes

$$\phi_j - \phi_i = \frac{\Delta m_{ij}^2 L}{2E}, \quad (2.6)$$

320 where $\Delta m_{ij}^2 = m_i^2 - m_j^2$. This, combined with further use of unitarity relations
 321 results in Equation 2.3 becoming

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\ &\quad + (-) 2 \sum_{i>j} \Im \left(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \right) \sin \left(\frac{\Delta m_{ij}^2 L}{2E} \right). \end{aligned} \quad (2.7)$$

322 Where $\delta_{\alpha\beta}$ is the Kronecker delta function and the negative sign on the last
 323 term is included for the oscillation probability of antineutrinos. As an important
 324 point to note, the observation of oscillation probability requires a non-zero value
 325 of Δm_{ij}^2 , which in turn requires that neutrinos have differing masses.

326 Typically, the PMNS matrix is parameterised into three mixing angles, a
 327 charge parity (CP) violating phase δ_{CP} , and two Majorana phases $\alpha_{1,2}$,

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{Atmospheric, Accelerator}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix}}_{\text{Reactor, Accelerator}} \times \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Reactor, Solar}} \underbrace{\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{Majorana}}. \quad (2.8)$$

328 Where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. The oscillation parameters are often
 329 grouped: (1,2) as “solar”, (2,3) as “atmospheric” and (1,3) as “reactor”. Many
 330 neutrino experiments aim to measure the PMNS parameters from a wide array
 331 of origins, as is the purpose of this thesis.

332 The Majorana phase, $\alpha_{1,2}$, included within the fourth matrix in Equation 2.8
 333 is only included for completeness. For an oscillation analysis experiment, any
 334 terms containing this phase disappear due to taking the expectation value of the
 335 PMNS matrix. Measurements of these phases can be performed by experiments
 336 searching for neutrino-less double β -decay [21].

337 A two-flavour approximation can be obtained when one assumes the third
338 mass eigenstate is degenerate with another. This results in the two-flavour
339 approximation being reasonable for understanding the features of the oscillation.
340 In this two-flavour case, the mixing matrix becomes,

$$U_{2\text{ Flav.}} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \quad (2.9)$$

341 This culminates in the oscillation probability,

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right), \\ P(\nu_\alpha \rightarrow \nu_\beta) &= \sin^2(2\theta) \sin^2\left(\frac{\Delta m^2 L}{4E}\right). \end{aligned} \quad (2.10)$$

342 Where $\alpha \neq \beta$. For a fixed neutrino energy, the oscillation probability is
343 a sinusoidal function depending upon the distance over which the neutrino
344 propagates. The frequency and amplitude of oscillation are dependent upon
345 $\Delta m^2/4E$ and $\sin^2 2\theta$, respectively. The oscillation probabilities presented thus far
346 assume $c = 1$, where c is the speed of light in a vacuum. In more familiar units, the
347 maximum oscillation probability for a fixed value of θ is given at $L[\text{km}] / E[\text{GeV}] \sim$
348 $1.27/\Delta m^2$. It is this calculation that determines the best L/E value for a given
349 experiment to be designed around for measurements of a specific value of Δm^2 .

350 2.2.2 The MSW Effect

351 The theory of neutrino oscillation in a vacuum has been described in subsec-
352 tion 2.2.1. However, the beam neutrinos and atmospheric neutrinos originating
353 from below the horizon propagate through the matter in the Earth. The coherent
354 scattering of neutrinos from a material target modifies the Hamiltonian of the
355 system which results in a change in the oscillation probability. This modification
356 is termed the Mikheyev-Smirnov-Wolfenstein (MSW) effect [22–24]. This occurs
357 because charged current scattering ($\nu_e + e^- \rightarrow \nu_e + e^-$, propagated by a W boson)
358 only affects electron neutrinos whereas the neutral current scattering ($\nu_l + l^- \rightarrow$

³⁵⁹ $\nu_l + l^-$, propagated by a Z^0 boson) interacts through all neutrino flavours equally.
³⁶⁰ In the two-flavour approximation, the effective mixing parameter becomes

$$\sin^2(2\theta) \rightarrow \sin^2(2\theta_m) = \frac{\sin^2(2\theta)}{(A/\Delta m^2 - \cos(2\theta))^2 + \sin^2(2\theta)}, \quad (2.11)$$

³⁶¹ where $A = 2\sqrt{2}G_F N_e E$, N_e is the electron density of the medium and G_F
³⁶² is Fermi's constant. It is clear that there exists a value of $A = \Delta m^2 \cos(2\theta)$ for
³⁶³ $\Delta m^2 > 0$, which results in a divergent mixing parameter, colloquially called the
³⁶⁴ matter resonance. This resonance regenerates the electron neutrino component of
³⁶⁵ the neutrino flux [22–24]. The density at which the resonance occurs is given by

$$N_e = \frac{\Delta m^2 \cos(2\theta)}{2\sqrt{2}G_F E}. \quad (2.12)$$

³⁶⁶ At densities lower than this critical value, the oscillation probability will
³⁶⁷ be much closer to that of vacuum oscillation. For antineutrinos, $N_e \rightarrow -N_e$
³⁶⁸ [25]. The resonance occurring from the MSW effect depends on the sign of Δm^2 .
³⁶⁹ Therefore, any neutrino oscillation experiment which observes neutrinos and
³⁷⁰ antineutrinos which have propagated through matter can have some sensitivity
³⁷¹ to the ordering of the neutrino mass eigenstates.

³⁷² 2.3 Neutrino Oscillation Measurements

³⁷³ As evidence of beyond Standard Model physics, the 2015 Nobel Prize in Physics
³⁷⁴ was awarded to the Super-Kamiokande (SK) [26] and Sudbury Neutrino Ob-
³⁷⁵ servatory (SNO) [27] collaborations for the first definitive observation of solar
³⁷⁶ and atmospheric neutrino oscillation [28]. Since then, the field has seen a wide
³⁷⁷ array of oscillation measurements from a variety of neutrino sources. As seen
³⁷⁸ in subsection 2.2.1, the neutrino oscillation probability is dependent on the ratio
³⁷⁹ of the propagation baseline, L , to the neutrino energy, E . It is this ratio that
³⁸⁰ determines the type of neutrino oscillation a particular experiment is sensitive to.

³⁸¹ As illustrated in Figure 2.1, there are many neutrino sources that span a
³⁸² wide range of energies. The least energetic neutrinos are from reactor and

³⁸³ terrestrial sources at $O(1)$ MeV whereas the most energetic neutrinos originate
³⁸⁴ from atmospheric and galactic neutrinos of $> O(1)$ TeV.

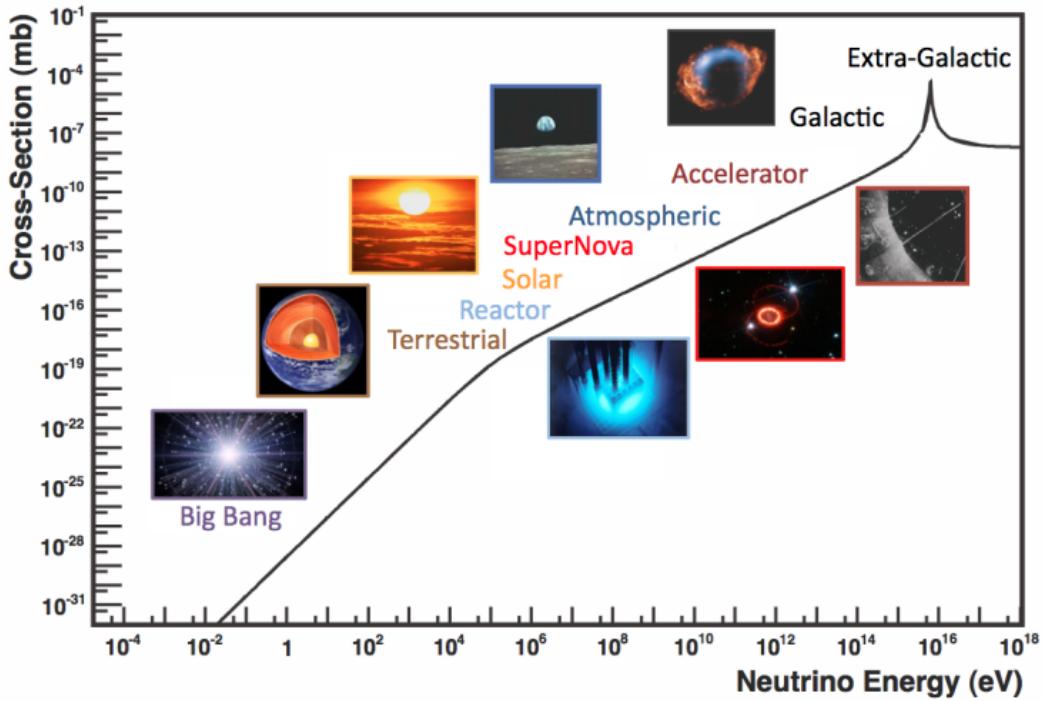


Figure 2.1: The electro-weak cross-section for $\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$ scattering on free electrons from various natural and man-made neutrino sources, as a function of neutrino energy. Taken from [29]

³⁸⁵ 2.3.1 Solar Neutrinos

³⁸⁶ Solar neutrinos are emitted from fusion reaction chains at the centre of the Sun.
³⁸⁷ The solar neutrino flux, given as a function of neutrino energy for different
³⁸⁸ fusion and decay chains, is illustrated in Figure 2.2. Whilst proton-proton fusion
³⁸⁹ generates the largest flux of neutrinos, the neutrinos are low energy and are
³⁹⁰ difficult to reconstruct due to the IBD interaction threshold of 1.8MeV [30].
³⁹¹ Consequently, most experiments focus on the neutrinos from the decay of 8B
³⁹² (via $^8B \rightarrow ^8Be^* + e^+ + \nu_e$), which are higher energy.

³⁹³ The first measurements of solar neutrinos observed a significant reduction in
³⁹⁴ the event rate compared to predictions from the Standard Solar Model [32, 33]. A
³⁹⁵ proposed solution to this “solar neutrino problem” was $\nu_e \leftrightarrow \nu_\mu$ oscillations in a

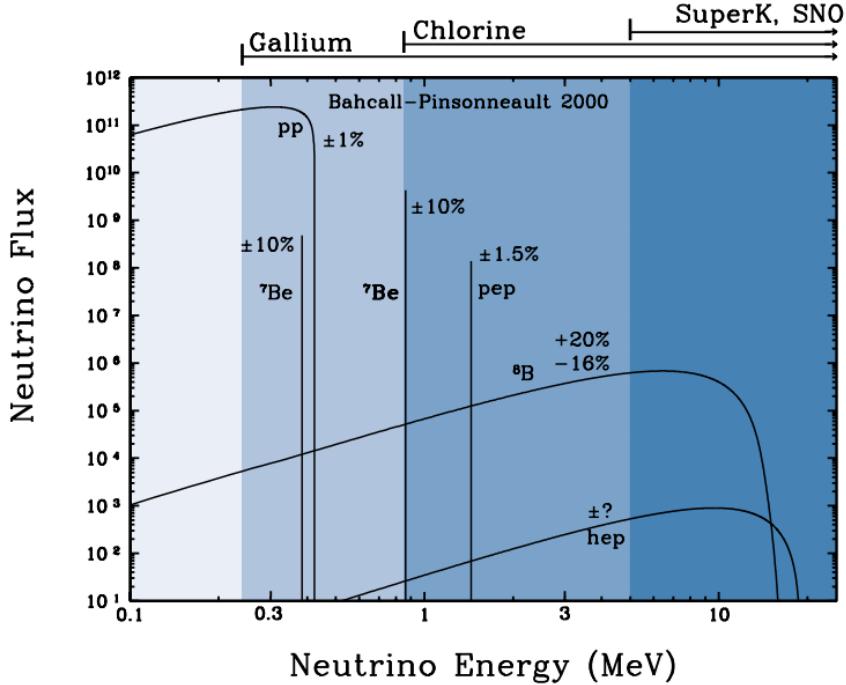


Figure 2.2: The solar neutrino flux as a function of neutrino energy for various fusion reactions and decay chains as predicted by the Standard Solar Model. Taken from [31].

³⁹⁶ precursory version of the PMNS model [34]. The Kamiokande [35], Gallex [36]
³⁹⁷ and Sage [37] experiments confirmed the ~ 0.5 factor deficit of solar neutrinos.

³⁹⁸ The conclusive solution to this problem was determined by the SNO col-
³⁹⁹ laboration [27]. Using a deuterium water target to observe 8B neutrinos, the
⁴⁰⁰ event rate of charged current (CC), neutral current (NC), and elastic scattering
⁴⁰¹ (ES) interactions (Given in Equation 2.13) was simultaneously measured. CC
⁴⁰² events can only occur for electron neutrinos, whereas the NC channel is agnostic
⁴⁰³ to neutrino flavour, and the ES reaction has a small excess sensitivity for the
⁴⁰⁴ detection of electron neutrino interactions. This meant that there were direct
⁴⁰⁵ measurements of the ν_e and ν_x neutrino flux. It was concluded that the CC and
⁴⁰⁶ ES interaction rates were consistent with the deficit previously observed. Most
⁴⁰⁷ importantly, the NC reaction rate was only consistent with the others under the

408 hypothesis of flavour transformation.

$$\begin{aligned} \nu_e + d &\rightarrow p + p + e^- & (CC) \\ \nu_x + d &\rightarrow p + n + \nu_x & (NC) \\ \nu_x + e^- &\rightarrow \nu_x + e^- & (ES) \end{aligned} \quad (2.13)$$

409 Since the SNO measurement, many experiments have since measured the
410 neutrino flux of different interaction chains within the sun [38–40]. The most
411 recent measurement was that of CNO-cycle neutrinos which were recently
412 observed with 5σ significance by the Borexino collaboration [38].

413 2.3.2 Accelerator Neutrinos

414 The concept of using an artificial “neutrino beam” was first realised in 1962 [11].
415 Since then, many experiments have adopted the same fundamental concepts.
416 Typically, a proton beam is aimed at a target producing charged mesons that
417 decay to neutrinos. The mesons can be sign-selected by the use of magnetic
418 focusing horns to generate a neutrino or antineutrino beam. Pions are the primary
419 mesons that decay and depending on the orientation of the magnetic field, a
420 muon (anti-)neutrino beam is generated via $\pi^+ \rightarrow \mu^+ + \nu_\mu$ or $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$.
421 The decay of muons and kaons results in an irreducible intrinsic electron neutrino
422 background. In T2K, this background contamination is $O(< 1\%)$ [41]. There is
423 also an approximately $\sim 5\%$ “wrong-sign” neutrino background of $\bar{\nu}_\mu$ generated
424 via the same decays. As the beam is generated by proton interactions (rather
425 than anti-proton interactions), the wrong-sign component in the antineutrino
426 beam is larger when operating in neutrino mode.

427 Tuning the proton energy in the beam and using beam focusing techniques
428 allows the neutrino energy to be set to a value that maximises the disappear-
429 ance oscillation probability in the L/E term in Equation 2.10. This means that
430 accelerator experiments are typically more sensitive to the mixing parameters as
431 compared to a natural neutrino source. However, the disadvantage compared
432 to atmospheric neutrino experiments is the cost of building a facility to provide

⁴³³ high-energy neutrinos, with a high flux, which is required for longer baselines.

⁴³⁴ Consequently, there is typically less sensitivity to matter effects and the ordering

⁴³⁵ of the neutrino mass eigenstates.

⁴³⁶ A neutrino experiment measures

$$R(\vec{x}) = \Phi(E_\nu) \times \sigma(E_\nu) \times \epsilon(\vec{x}) \times P(\nu_\alpha \rightarrow \nu_\beta), \quad (2.14)$$

⁴³⁷ where $R(\vec{x})$ is the event rate of neutrinos at position \vec{x} , $\Phi(E_\nu)$ is the flux of
⁴³⁸ neutrinos with energy E_ν , $\sigma(E_\nu)$ is the cross-section of the neutrino interaction and
⁴³⁹ $\epsilon(\vec{x})$ is the efficiency and resolution of the detector. In order to leverage the most
⁴⁴⁰ out of an accelerator neutrino experiment, the flux and cross-section systematics
⁴⁴¹ need to be constrained. This is typically done via the use of a “near detector”,
⁴⁴² situated at a baseline of $O(1)$ km. This detector observes the unoscillated neutrino
⁴⁴³ flux and constrains the parameters used within the flux and cross-section model.

⁴⁴⁴ The first accelerator experiments to precisely measure oscillation parameters
⁴⁴⁵ were MINOS [42] and K2K [43]. These experiments confirmed the ν_μ disappear-
⁴⁴⁶ ance seen in atmospheric neutrino experiments by finding consistent parameter
⁴⁴⁷ values for $\sin^2(\theta_{23})$ and Δm_{32}^2 . The current generation of accelerator neutrino
⁴⁴⁸ experiments, T2K and NO ν A extended this field by observing $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ and lead
⁴⁴⁹ the sensitivity to atmospheric mixing parameters as seen in Figure 2.6 [44]. The
⁴⁵⁰ two experiments differ in their peak neutrino energy, baseline, and detection tech-
⁴⁵¹ nique. The NO ν A experiment is situated at a baseline of 810km from the NuMI
⁴⁵² beamline which delivers 2GeV neutrinos. The T2K neutrino beam is peaked
⁴⁵³ around 0.6GeV and propagates 295km [45]. Additionally, the NO ν A experiment
⁴⁵⁴ uses functionally identical detectors (near and far) whereas T2K uses a plastic
⁴⁵⁵ scintillator technique at the near detector and a water Cherenkov far detector.
⁴⁵⁶ The future generation experiments DUNE [46] and Hyper-Kamiokande [47]
⁴⁵⁷ will succeed these experiments as the high-precision era of neutrino oscillation
⁴⁵⁸ parameter measurements develops.

⁴⁵⁹ Several anomalous results have been observed in the LSND [13] and Mini-
⁴⁶⁰ BooNE [14] detectors which were designed with purposefully short baselines.

461 Parts of the neutrino community attributed these results to oscillations induced
462 by a fourth “sterile” neutrino [48] but several searches in other experiments,
463 MicroBooNE [49] and KARMEN [50], found no hints of additional neutrino
464 species. The solution to the anomalous results is still being determined.

465 2.3.3 Atmospheric Neutrinos

466 The interactions of primary cosmic ray protons in the Earth’s upper atmosphere
467 generate showers of energetic hadrons. These are mostly pions and kaons that
468 decay to produce a natural source of neutrinos spanning energies of MeV to
469 TeV [51]. The main decay is via,

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm + (\nu_\mu, \bar{\nu}_\mu), \\ \mu^\pm &\rightarrow e^\pm + (\nu_e, \bar{\nu}_e) + (\nu_\mu, \bar{\nu}_\mu), \end{aligned} \tag{2.15}$$

470 such that for a single pion decay, three neutrinos can be produced. The
471 atmospheric neutrino flux energy spectra as predicted by the Bartol [52], Honda
472 [53–55], and FLUKA [56] models are illustrated in Figure 2.3. The flux distribution
473 peaks at an energy of $O(10)$ GeV. The uncertainties associated with these models
474 are dominated by the hadronic production of kaon and pions as well as the
475 primary cosmic flux.

476 Unlike long-baseline experiments which have a fixed baseline, the distance
477 atmospheric neutrinos propagate is dependent upon the zenith angle at which
478 they interact. This is illustrated in Figure 2.4. Neutrinos that are generated
479 directly above the detector ($\cos(\theta) = 1.0$) have a baseline equivalent to the
480 height of the atmosphere, whereas neutrinos that interact directly below the
481 detector ($\cos(\theta) = -1.0$) have to travel a length equal to the diameter of the Earth.
482 This means atmospheric neutrinos have a baseline that varies from $O(20)$ km to
483 $O(6 \times 10^3)$ km. Any neutrino generated at or below the horizon will be subject
484 to MSW matter resonance as they propagate through the Earth.

485 Figure 2.5 highlights the neutrino flux as a function of the zenith angle for
486 different slices of neutrino energy. For medium to high-energy neutrinos (and to

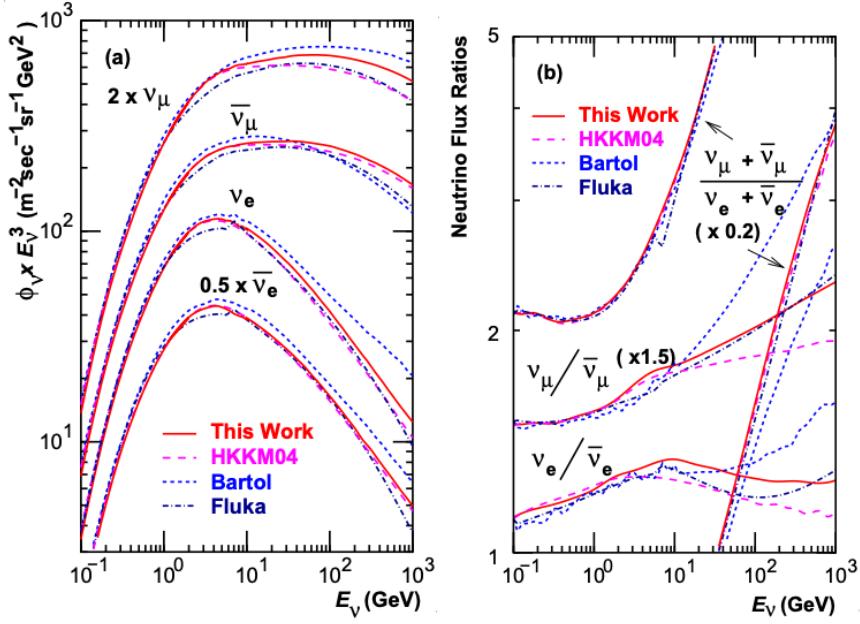


Figure 2.3: Left panel: The atmospheric neutrino flux for different neutrino flavours as a function of neutrino energy as predicted by the 2007 Honda model (“This work”) [53], the 2004 Honda model (“HKKM04”) [54], the Bartol model [52] and the FLUKA model [56]. Right panel: The ratio of the muon to electron neutrino flux as predicted by all the quoted models. Both figures taken from [53].

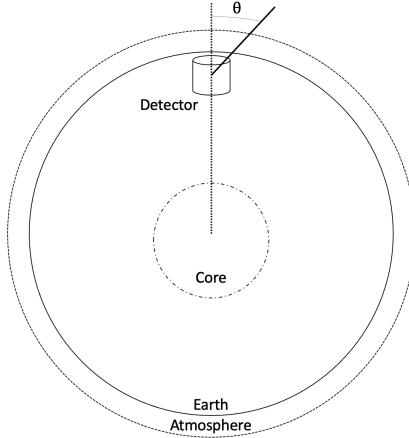


Figure 2.4: A diagram illustrating the definition of zenith angle as used in the Super Kamiokande experiment [57].

487 a lesser degree for low-energy neutrinos), the flux is approximately symmetric
 488 around $\cos(\theta) = 0$. To the accuracy of this approximation, the systematic
 489 uncertainties associated with atmospheric flux for comparing upward-going
 490 and down-going neutrino cancels. This allows the down-going events, which are

- 491 mostly insensitive to oscillation probabilities, to act as an unoscillated prediction
 492 (similar to a near detector in an accelerator neutrino experiment).

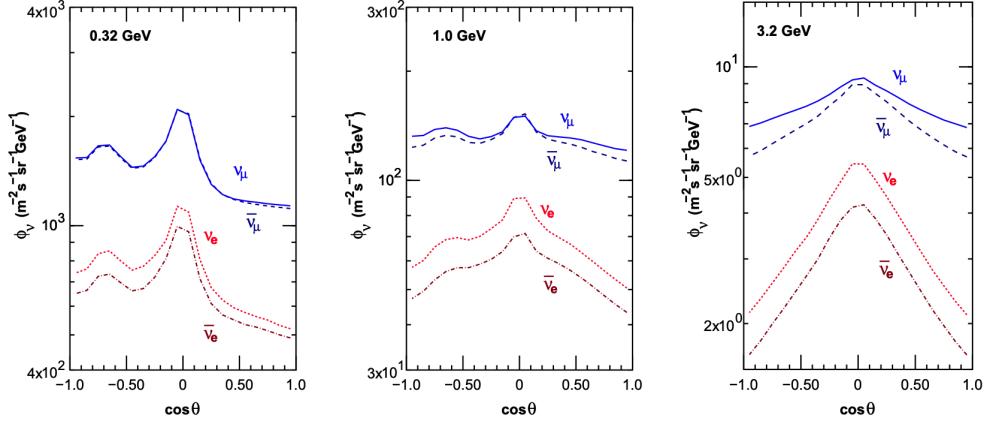


Figure 2.5: Prediction of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ fluxes as a function of zenith angle as calculated by the HKKM model [55]. The left, middle and right panels represent three values of neutrino energy, 0.32GeV, 1.0GeV and 3.2GeV respectively. Predictions for other models including Bartol [52], Honda [53] and FLUKA [56] are given in [57].

493 Precursory hints of atmospheric neutrinos were observed in the mid-1960s
 494 searching for $\nu_\mu + X \rightarrow X^* + \mu^\pm$ [58]. This was succeeded by the IMB-3 [59]
 495 and Kamiokande [60] experiments which measured the double ratio of muon
 496 to electron neutrinos in data to Monte Carlo, $R(\nu_\mu/\nu_e) = (\mu/e)_{Data}/(\mu/e)_{MC}$.
 497 Both experiments were found to have a consistent deficit of muon neutrinos,
 498 with $R(\nu_\mu/\nu_e) = 0.67 \pm 0.17$ and $R(\nu_\mu/\nu_e) = 0.658 \pm 0.016 \pm 0.035$, respectively.
 499 Super-Kamiokande (SK) [57] extended this analysis by fitting oscillation pa-
 500 rameters in $P(\nu_\mu \rightarrow \nu_\tau)$ which found best fit parameters $\sin^2(2\theta) > 0.92$ and
 501 $1.5 \times 10^{-3} < \Delta m^2 < 3.4 \times 10^{-3} \text{ eV}^2$.

502 Since then, atmospheric neutrino experiments have been making precision
 503 measurements of the $\sin^2(\theta_{23})$ and Δm^2_{32} oscillation parameters. Atmospheric
 504 neutrino oscillation is dominated by $P(\nu_\mu \rightarrow \nu_\tau)$, where SK observed a 4.6σ
 505 discovery of ν_τ appearance [61]. Figure 2.6 illustrates the current estimates on
 506 the atmospheric mixing parameters, from a wide range of atmospheric and
 507 accelerator neutrino observatories.

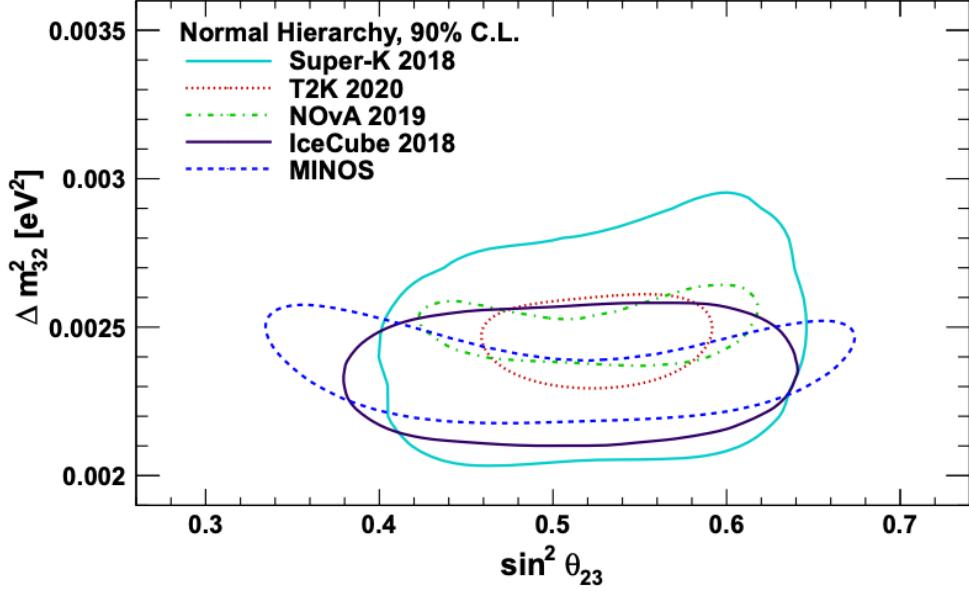


Figure 2.6: Constraints on the atmospheric oscillation parameters, $\sin^2(\theta_{23})$ and Δm_{32}^2 , from atmospheric and long-baseline experiments: SK [62], T2K [63], NOvA [64], IceCube [65] and MINOS [66]. Figure taken from [67].

508 2.3.4 Reactor Neutrinos

509 As illustrated in the first discovery of neutrinos (section 2.1), nuclear reactors
 510 are a very useful artificial source of electron antineutrinos. For reactors that use
 511 low-enriched uranium ^{235}U as fuel, the antineutrino flux is dominated by the
 512 β -decay fission of ^{235}U , ^{238}U , ^{239}Pu and ^{241}Pu [68] as illustrated in Figure 2.7.

513 Due to their low energy, reactor electron antineutrinos predominantly interact
 514 via the inverse β -decay (IBD) interaction. The typical signature contains two
 515 signals delayed by $O(200)\mu\text{s}$; firstly the prompt photons from positron annihila-
 516 tion, and secondly the photon emitted ($E_{tot}^\gamma = 2.2\text{MeV}$) from de-excitation after
 517 neutron capture on hydrogen. Searching for both signals improves the detector's
 518 ability to distinguish between background and signal events [69].

519 There are many short baseline experiments ($L \sim O(1)\text{km}$) that have measured
 520 the $\sin^2(\theta_{13})$ and Δm_{32}^2 oscillation parameters. Daya Bay [70], RENO [71] and
 521 Double Chooz [72] have all provided precise measurements, with the first discov-
 522 ery of a non-zero θ_{13} made by Daya Bay and RENO (and complemented by T2K
 523 [72]). The constraints on $\sin^2(\theta_{13})$ by the reactor experiments lead the field. They

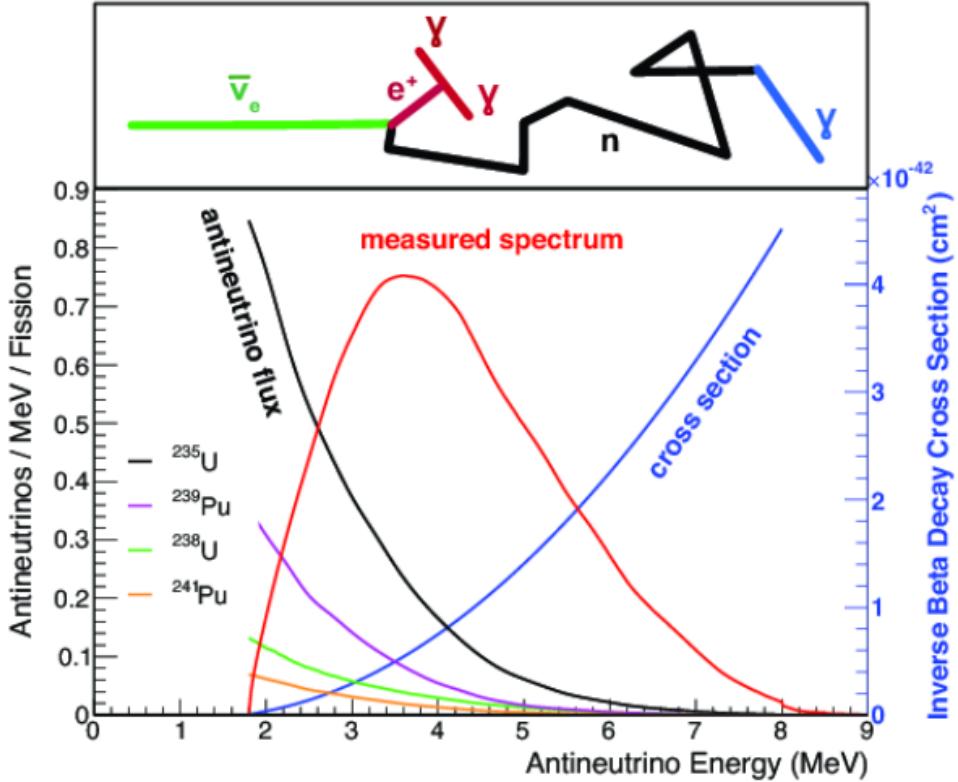


Figure 2.7: Reactor electron antineutrino fluxes for ^{235}U (Black), ^{238}U (Green), ^{239}Pu (Purple), and ^{241}Pu (Orange) isotopes. The inverse β -decay cross-section (Blue) and corresponding measurable neutrino spectrum (Red) are also given. Top panel: Schematic of Inverse β -decay interaction including the eventual capture of the emitted neutron. This capture emits a γ -ray which provides a second signal of the event. Taken from [67].

are often used as external inputs to accelerator neutrino experiments to improve their sensitivity to δ_{CP} and mass hierarchy determination. JUNO-TAO [73], a small collaboration within the larger JUNO experiment, is a next-generation reactor experiment that aims to precisely measure the isotopic antineutrino yields from the different fission chains.

Kamland [74] is the only experiment to have observed reactor neutrinos using a long baseline (flux weighted averaged baseline of $L \sim 180\text{km}$) which allows it to have sensitivity to Δm_{21}^2 . Combined with the SK solar neutrino experiment, the combined analysis puts the most stringent constraint on Δm_{21}^2 [75].

2.4 Summary Of Oscillation Parameter Measurements

Since the first evidence of neutrino oscillations, numerous measurements of the mixing parameters have been made. Many experiments use neutrinos as a tool for the discovery of new physics (diffuse supernova background, neutrinoless double beta decay and others) so the PMNS parameters are summarised in the Particle Data Group (PDG) review tables. The analysis presented in this thesis focuses on the 2020 T2K oscillation analysis presented in [2] which the 2020 PDG constraints [76] were used. These constraints are outlined in Table 2.1.

Parameter	2020 Constraint
$\sin^2(\theta_{12})$	0.307 ± 0.013
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.18 \pm 0.07) \times 10^{-2}$
$\sin^2(\theta_{23})$ (I.H.)	0.547 ± 0.021
$\sin^2(\theta_{23})$ (N.H.)	0.545 ± 0.021
Δm_{32}^2 (I.H.)	$(-2.546^{+0.034}_{-0.040}) \times 10^{-3} \text{ eV}^2$
Δm_{32}^2 (N.H.)	$(2.453 \pm 0.034) \times 10^{-3} \text{ eV}^2$

Table 2.1: The 2020 Particle Data Group constraints of the oscillation parameters taken from [76]. The value of Δm_{32}^2 is given for both normal hierarchy (N.H.) and inverted hierarchy (I.H.) and $\sin^2(\theta_{23})$ is broken down by whether its value is below (Q1) or above (Q2) 0.5.

The $\sin^2(\theta_{13})$ measurement stems from the electron antineutrino disappearance, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, and is taken as the average best-fit from the combination of Daya Bay, Reno and Double Chooz. It is often used as a prior uncertainty within other neutrino oscillation experiments, typically termed the reactor constraint. The $\sin^2(\theta_{12})$ parameter is predominantly measured through electron neutrino disappearance, $P(\nu_e \rightarrow \nu_{\mu,\tau})$, in solar neutrino experiments. The long-baseline reactor neutrino experiment Kamland also has a sensitivity to this parameter and is used in a joint fit to solar data from SNO and SK, using the reactor constraint. Measurements of $\sin^2(\theta_{23})$ are made by long-baseline and atmospheric neutrino experiments. The PDG value is a joint fit of T2K, NO ν A, MINOS and IceCube DeepCore experiments. The latest T2K-only measurement, provided at Neutrino2020 and is the basis of this thesis, is given as $\sin^2(\theta_{23}) = 0.546^{+0.024}_{-0.046}$ [2].

The PDG constraint on Δm_{21}^2 is provided by the KamLAND experiment using solar and geoneutrino data. This measurement utilised a $\sin^2(\theta_{13})$ constraint from accelerator (T2K, MINOS) and reactor neutrino (Daya Bay, RENO, Double Chooz) experiments. Accelerator measurements make some of the most stringent constraints on Δm_{32}^2 although atmospheric experiments have more sensitivity to the mass hierarchy determination. The PDG performs a joint fit of accelerator and atmospheric data, in both normal and inverted hierarchies separately. The latest T2K-only result is $\Delta m_{32}^2 = 2.49^{+0.058}_{-0.082} \times 10^{-3} \text{ eV}^2$ favouring normal hierarchy [2]. The value of δ_{CP} is largely undetermined. CP-conserving values of 0 and π were rejected with $\sim 2\sigma$ intervals, as published in Nature, although more recent analyses have reduced the credible intervals to 90%. Since the 2020 PDG publication, there has been a new measurement of $\sin^2(\theta_{13}) = (2.20 \pm 0.07) \times 10^{-2}$ [77], alongside updated Δm_{32}^2 and $\sin^2(\theta_{23})$ measurements.

Throughout this thesis, several sample spectra predictions and contours are presented, which require oscillation parameters to be assumed. Table 2.2 defines two sets of oscillation parameters, with “Asimov A” set being close to the preferred values from a previous T2K-only fit [78] and “Asimov B” being CP-conserving and further from maximal θ_{23} mixing.

Parameter	Asimov A	Asimov B
Δm_{12}^2	$7.53 \times 10^{-5} \text{ eV}^2$	
Δm_{32}^2	$2.509 \times 10^{-3} \text{ eV}^2$	
$\sin^2(\theta_{12})$	0.304	
$\sin^2(\theta_{13})$	0.0219	
$\sin^2(\theta_{23})$	0.528	0.45
δ_{CP}	-1.601	0.0

Table 2.2: Reference values of the neutrino oscillation parameters for two different oscillation parameter sets.

2.5 Overview of Oscillation Effects

The analysis presented within this thesis focuses on the determination of oscillation parameters from atmospheric and beam neutrinos. Whilst subject to the

574 same oscillation formalism, the way in which the two samples have sensitivity
575 to the different oscillation parameters differs significantly.

576 Atmospheric neutrinos have a varying baseline, or “path length” L , such that
577 the distance each neutrino travels before interacting is dependent upon the zenith
578 angle, θ_Z . As primary cosmic rays can interact anywhere between the Earth’s
579 surface and $\sim 50\text{km}$ above that, the height, h , in the atmosphere at which the
580 neutrino was generated also affects the path length,

$$L = \sqrt{(R_E + h)^2 - R_E^2(1 - \cos^2(\theta_Z))} - R_E \cos(\theta_Z). \quad (2.16)$$

581 Where $R_E = 6,371\text{km}$ is the Earth’s radius. This assumes a spherically
582 symmetric Earth model. Therefore, the oscillation probability is dependent upon
583 two parameters, $\cos(\theta_Z)$ and E_ν .

584 The oscillation probability used within this analysis is based on [25]. The
585 neutrino wavefunction in the vacuum Hamiltonian evolves in each layer of
586 constant matter density via

$$i \frac{d\psi_j(t)}{dt} = \frac{m_j^2}{2E_\nu} \psi_j(t) - \sum_k \sqrt{2} G_F N_e U_{ej} U_{ke}^\dagger \psi_k(t), \quad (2.17)$$

587 where m_j^2 is the square of the j^{th} vacuum eigenstate mass, E_ν is the neutrino
588 energy, G_F is Fermi’s constant, N_e is the electron number density and U is the
589 PMNS matrix. The transformation $N_e \rightarrow -N_e$ and $\delta_{CP} \rightarrow -\delta_{CP}$ is applied for
590 antineutrino propagation. Thus, a model of the Earth’s density is required for
591 neutrino propagation. Following the official SK-only methodology [79], this
592 analysis uses the Preliminary Reference Earth Model (PREM) [80] which provides
593 piecewise cubic polynomials as a function of the Earth’s radius. This density
594 profile is illustrated in Figure 2.8. As the propagator requires layers of constant
595 density, the SK methodology approximates the PREM model by using four layers
596 of constant density [79], detailed in Table 2.3.

597 The atmospheric neutrino oscillation probabilities can be presented as two di-
598 mensional “oscillograms” as illustrated in Figure 2.9. The distinct discontinuities,
599 as a function of $\cos(\theta_Z)$, are due to the discontinuous density in the PREM model.

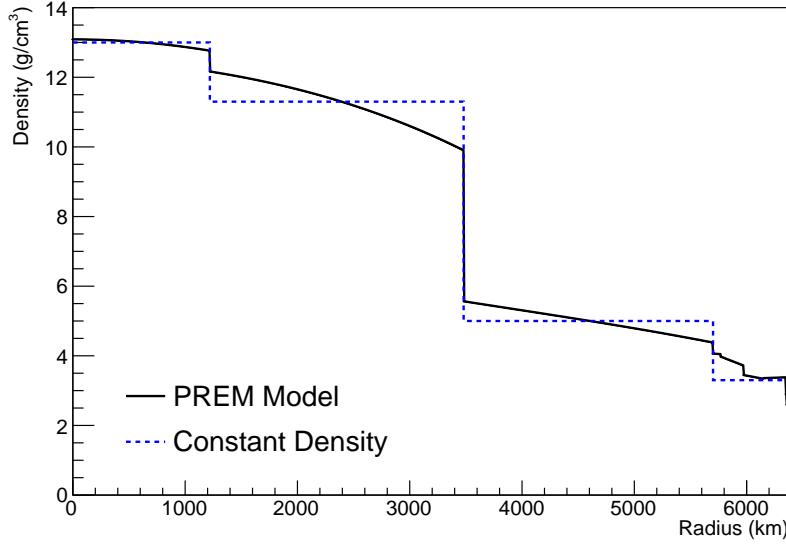


Figure 2.8: The density of the Earth given as a function of the radius, as given by the PREM model (Black), and the constant density four-layer approximation (Blue), as used in the official SK-only analysis.

Layer	Outer Radius [km]	Density [g/cm ³]	Chemical composition (Z/A)
Inner Core	1220	13	0.468 ± 0.029
Outer Core	3480	11.3	0.468 ± 0.029
Lower Mantle	5701	5.0	0.496
Transition Zone	6371	3.3	0.496

Table 2.3: Description of the four layers of the Earth invoked within the constant density approximation of the PREM model [80].

Atmospheric neutrinos have sensitivity to δ_{CP} through the overall event rate. Figure 2.10 illustrates the difference in oscillation probability between CP-conserving ($\delta_{CP} = 0.$) and a CP-violating ($\delta_{CP} = -1.601$) value taken from Asimov A oscillation parameter set (Table 2.2). The result is a complicated oscillation pattern in the appearance probability for sub-GeV upgoing neutrinos. The detector does not have sufficient resolution to resolve these individual patterns so the sensitivity to δ_{CP} for atmospheric neutrinos comes via the overall normalisation of these events.

The presence of matter means that the effect δ_{CP} has on the oscillation probability is not equal between neutrinos and antineutrinos. Furthermore, the interaction cross-section for neutrinos is larger than for antineutrinos so the two

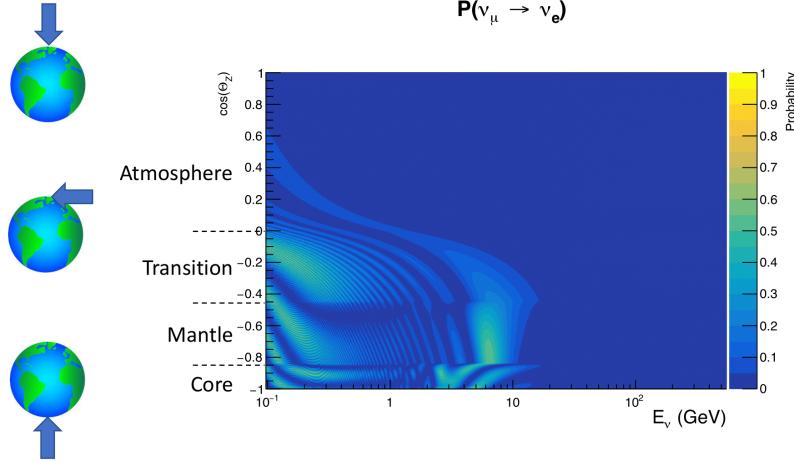


Figure 2.9: An “oscillogram” that depicts the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability as a function of neutrino energy and cosine of the zenith angle. The zenith angle is defined such that $\cos(\theta_Z) = 1.0$ represents neutrinos that travel from directly above the detector. The four-layer constant density PREM model approximation is used and Asimov A oscillation parameters are assumed (Table 2.2).

611 effects have to be disentangled. These effects are further convoluted by detector
 612 efficiencies as SK cannot distinguish neutrinos and antineutrinos well. All of
 613 these effects lead to a difference in the number of neutrinos detected compared
 614 to antineutrinos. This changes how the δ_{CP} normalisation term is observed,
 615 resulting in a very complex sensitivity to δ_{CP} .

616 The vacuum and matter oscillation probabilities for $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow$
 617 $\bar{\nu}_e)$ are presented in Figure 2.11, where the PREM model has been assumed. The
 618 oscillation probability for both neutrinos and antineutrinos is affected in the
 619 presence of matter. However, the resonance effects around $O(5)\text{GeV}$ only occur
 620 for neutrinos in the normal mass hierarchy and antineutrinos in the inverse mass
 621 hierarchy. The exact position and amplitude of the resonance depend on $\sin^2(\theta_{23})$,
 622 further increasing the atmospheric neutrinos’ sensitivity to the parameter.

623 As the T2K beam flux is centered at the first oscillation maximum ($E_\nu =$
 624 0.6GeV) [45], the sensitivity to δ_{CP} is predominantly observed as a change in the
 625 event-rate of e-like samples in $\nu/\bar{\nu}$ modes. Figure 2.12 illustrates the $P(\nu_\mu \rightarrow \nu_e)$
 626 oscillation probability for a range of δ_{CP} values. A circular modulation of the

$$\mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = -1.601) - \mathbf{P}(\nu_\mu \rightarrow \nu_e; \delta_{CP} = 0.)$$

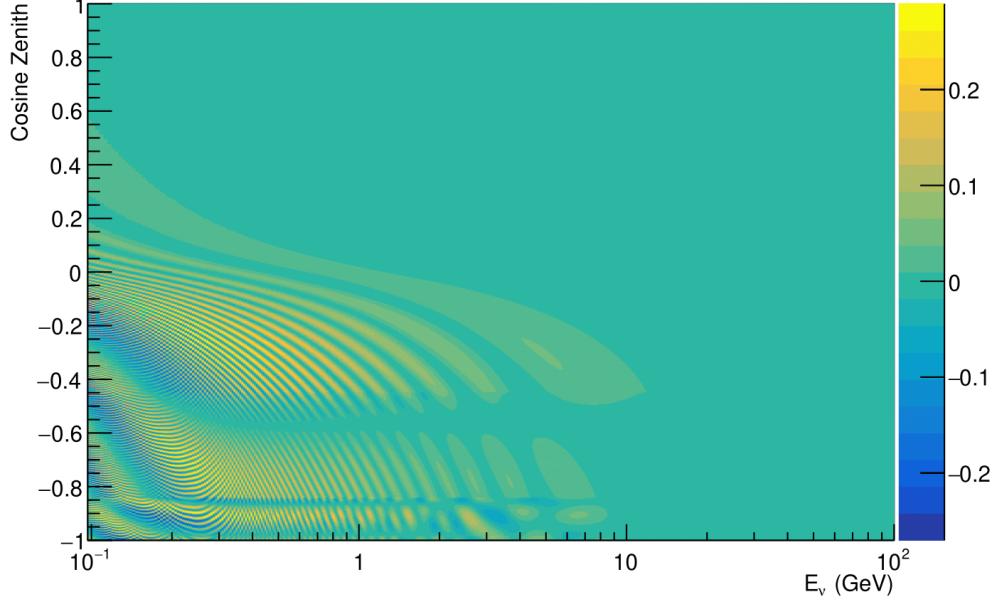


Figure 2.10: The effect of δ_{CP} for atmospheric neutrinos given in terms of the neutrino energy and zenith angle. This oscillogram compares the $P(\nu_\mu \rightarrow \nu_e)$ oscillation probability for a CP conserving ($\delta_{CP} = 0.0$) and a CP violating ($\delta_{CP} = -1.601$) value taken from the Asimov A parameter set. The other oscillation parameters assume the Asimov A oscillation parameter set given in Table 2.2.

first oscillation peak (in both magnitude and position) is observed when varying throughout the allowable values of δ_{CP} . The CP-conserving values of $\delta_{CP} = 0, \pi$ have a lower(higher) oscillation maximum than the CP-violating values of $\delta_{CP} = -\pi/2 (\delta_{CP} = \pi/2)$. A sub-dominant shift in the energy of the oscillation peak is also present, which aids in separating the two CP-conserving values of δ_{CP} .

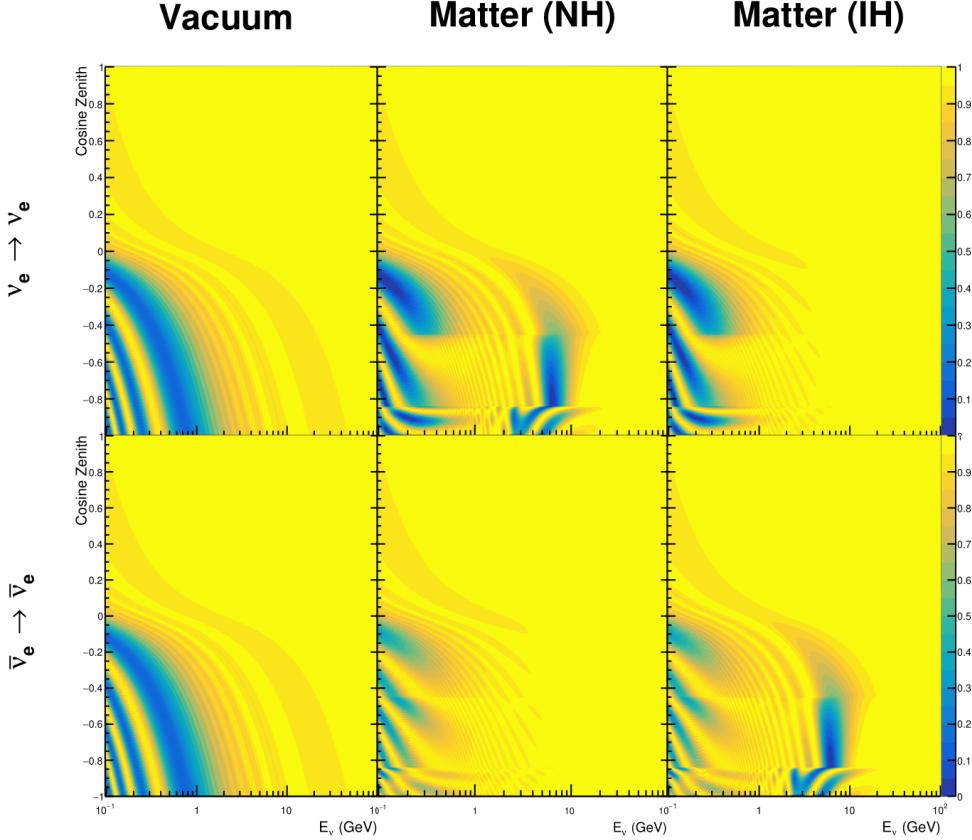


Figure 2.11: An illustration of the matter-induced effects on the oscillation probability, given as a function of neutrino energy and zenith angle. The top row of panels gives the $P(\nu_e \rightarrow \nu_e)$ oscillation probability and the bottom row illustrates the $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ oscillation probability. The left column highlights the oscillation probability in a vacuum, whereas the middle and right column represents the oscillation probabilities when the four-layer fixed density PREM model is assumed. All oscillation probabilities assume the “Asimov A” set given in Table 2.2, but importantly, the right column sets an inverted mass hierarchy. The “matter resonance” effects at $E_\nu \sim 5\text{GeV}$ can be seen in the $P(\nu_e \rightarrow \nu_e)$ for normal mass hierarchy and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ for inverted hierarchy.

632 T2K’s sensitivity to $\sin^2(\theta_{23})$ and Δm_{32}^2 is observed as a shape-based variation
 633 of the muon-like samples, as illustrated in Figure 2.12. The value of Δm_{32}^2 laterally
 634 shifts the position of the oscillation dip (around $E_\nu \sim 0.6\text{GeV}$) in the $P(\nu_\mu \rightarrow \nu_\mu)$
 635 oscillation probability. A variation of $\sin^2(\theta_{23})$ is predominantly observed as
 636 a vertical shift of the oscillation dip with second-order horizontal shifts being
 637 due to matter effects. The beam neutrinos have limited sensitivity to matter
 638 effects due to the relatively shorter baseline as well as the Earth’s mantle being

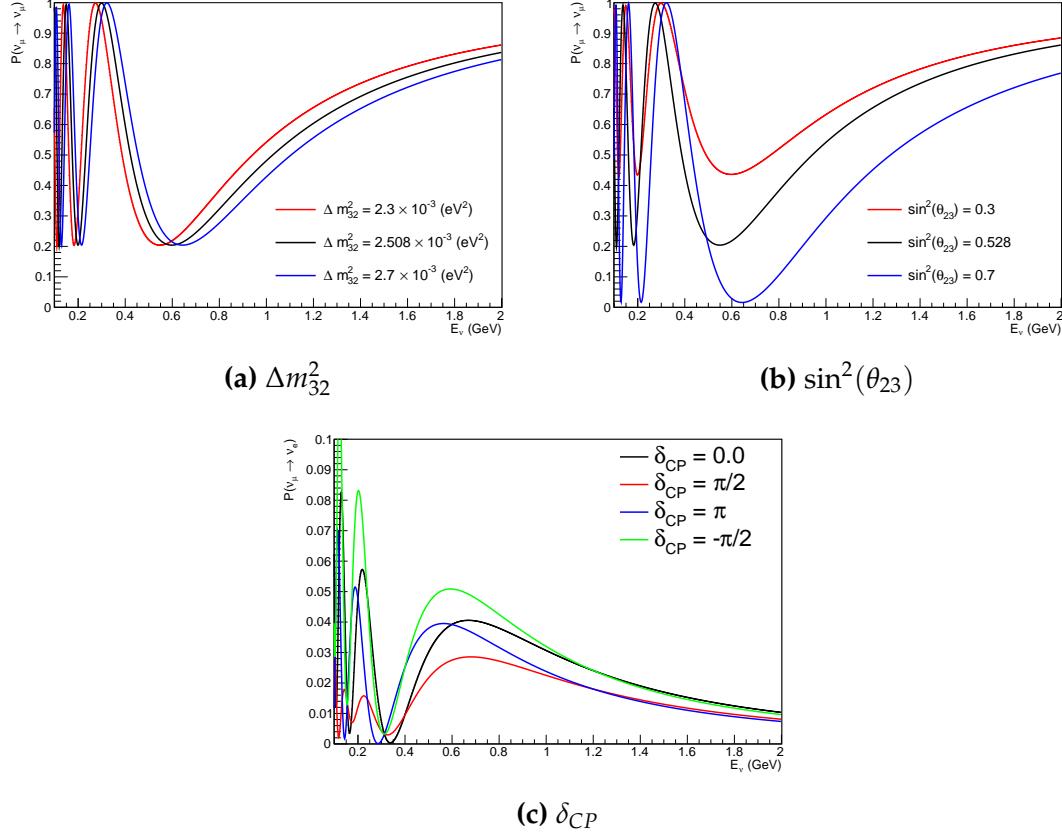


Figure 2.12: The oscillation probability for beam neutrino events given as a function of neutrino energy. All oscillation parameters assume the “Asimov A” set given in Table 2.2 unless otherwise stated. A path-length of 295km is assumed. Each panel represents a change in one of the oscillation parameters whilst keeping the remaining parameters fixed.

639 a relatively low-density material (as compared to the Earth’s core). For some
 640 values of δ_{CP} , the degeneracy in the number of e-like events allows the mass
 641 hierarchy to be broken. This leads to a δ_{CP} -dependent mass hierarchy sensitivity
 642 which can be seen in Figure 2.13.

643 Whilst all oscillation channels should be included for completeness, the
 644 computational resources required to run a fit are limited and any reasonable
 645 approximations which reduce the number of oscillation probability calculations
 646 that need to be made should be applied. The $\nu_e \rightarrow \nu_{e,\mu,\tau}$ (and antineutrino
 647 equivalent) oscillations can be ignored for beam neutrinos as the $\nu_e/\bar{\nu}_e$ fluxes are
 648 approximately two orders of magnitude smaller than the corresponding $\nu_\mu/\bar{\nu}_\mu$
 649 flux. Furthermore, as the peak neutrino energy of the beam is well below the

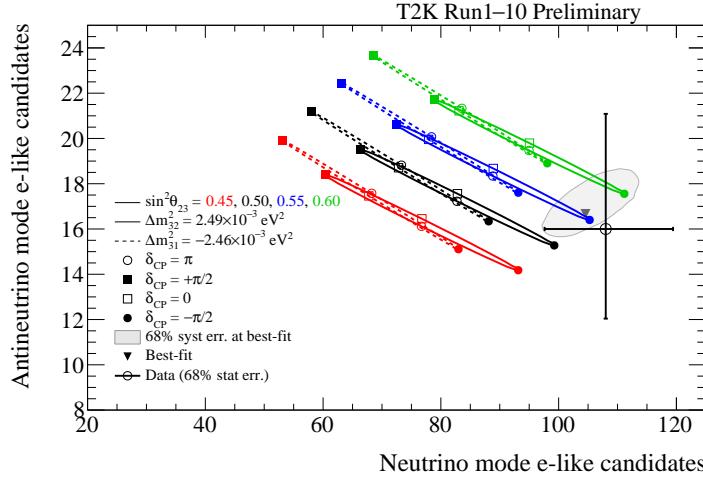


Figure 2.13: The number of electron-like events in the FHC and RHC operating mode of the beam, as a function of the oscillation probabilities. Both normal hierarchy (Solid) and inverse hierarchy (Dashed) values of Δm_{32}^2 are given.

threshold for charged current tau production ($E_\nu = 3.5\text{GeV}$ [61]), only a small proportion of the neutrinos produced in the beam have the required energy. For the few neutrinos that have sufficient energy, the oscillation probability is very small due to their energy being well above the oscillation maximum (small value of L/E). Whilst these approximations have been made for the beam neutrinos, the atmospheric flux of ν_e is of the same order of magnitude as the ν_μ flux and the energy distribution of atmospheric neutrinos extends well above the tau production threshold. These events can have non-negligible oscillation probabilities due to the further distance they travel.

3

659

660

T2K and SK Experiment Overview

661 As the successor of the Kamiokande experiment, the Super-Kamiokande (SK)
662 collaboration has been leading atmospheric neutrino oscillation analyses for
663 over two decades. The detector has provided some of the strongest constraints
664 on proton decay and the first precise measurements of the Δm_{32}^2 and $\sin^2(\theta_{23})$
665 neutrino oscillation parameters. The history, detection technique, and operation
666 of the SK detector is described in section 3.1.

667 The Tokai-to-Kamioka (T2K) experiment was one of the first long-baseline
668 experiments to use both neutrino and antineutrino beams to precisely measure
669 charge parity violation within the neutrino sector. The T2K experiment observed
670 the first hints of a non-zero $\sin^2(\theta_{13})$ measurement and continues to lead the
671 field with the constraints it provides on $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, Δm_{32}^2 and δ_{CP} . In
672 section 3.2, the techniques that T2K use to generate the neutrino beam and
673 constrain systematic parameter through near detector constraints are described.

674 3.1 The Super-Kamiokande Experiment

675 The SK experiment began taking data in 1996 [81] and has had many modifi-
676 cations throughout its operation. There have been seven defined periods of
677 data taking as noted in Table 3.1. Data taking began in SK-I which ran for five

years. Between the SK-I and SK-II periods, approximately 55% of the PMTs were damaged during maintenance [82]. Those that survived were equally distributed throughout the detector in the SK-II era, which resulted in a reduced 19% photo-coverage. From SK-III onwards, repairs to the detector meant the full suite of PMTs was operational recovering the 40% photo-coverage. Before the start of SK-IV, the data acquisition and electronic systems were upgraded. Between SK-IV and SK-V, a significant effort was placed into tank open maintenance and repair/replacement of defective PMTs in preparation for the Gadolinium upgrade; a task for which the author of this thesis was required. Consequently, the detector conditions were significantly changed from this point. SK-VI marked the start of the SK-Gd era, with the detector being doped with gadolinium at a concentration of 0.01% by concentration. SK-VII, which started during the writing of this thesis, has increased the gadolinium concentration to 0.03% for continued operation [83].

The oscillation analysis presented within this thesis focuses on the SK-IV period of running and the data taken within it. This follows from the recent SK analysis presented in [84]. Therefore, the information presented within this section focuses on that period.

Period	Start Date	End Date	Live-time (days)
I	April 1996	July 2001	1489.19
II	October 2002	October 2005	798.59
III	July 2006	September 2008	518.08
IV	September 2008	May 2018	3244.4
V	January 2019	July 2020	461.02
VI	July 2020	May 2022	583.3
VII	May 2022	Ongoing	N/A

Table 3.1: The various SK periods and their respective live-time. The SK-VI live-time is calculated until 1st April 2022. SK-VII started during the writing of this thesis.

3.1.1 The SK Detector

The basic structure of the Super-Kamiokande (SK) detector is a cylindrical tank with a diameter 39.3m and height 41.1m filled with ultrapure water [82]. A diagram of the significant components of the SK detector is given in Figure 3.1.

699 The SK detector is situated in the Kamioka mine in Gifu, Japan. The mine is under-
 700 ground with roughly 1km rock overburden (2.7km water equivalent overburden)
 701 [85]. At this depth, the rate of cosmic ray muons is significantly decreased to a
 702 value of $\sim 2\text{Hz}$ (net rate). The top of the tank is covered with stainless steel which
 703 is designed as a working platform for maintenance, calibration, and location for
 704 high voltage and data acquisition electronics.

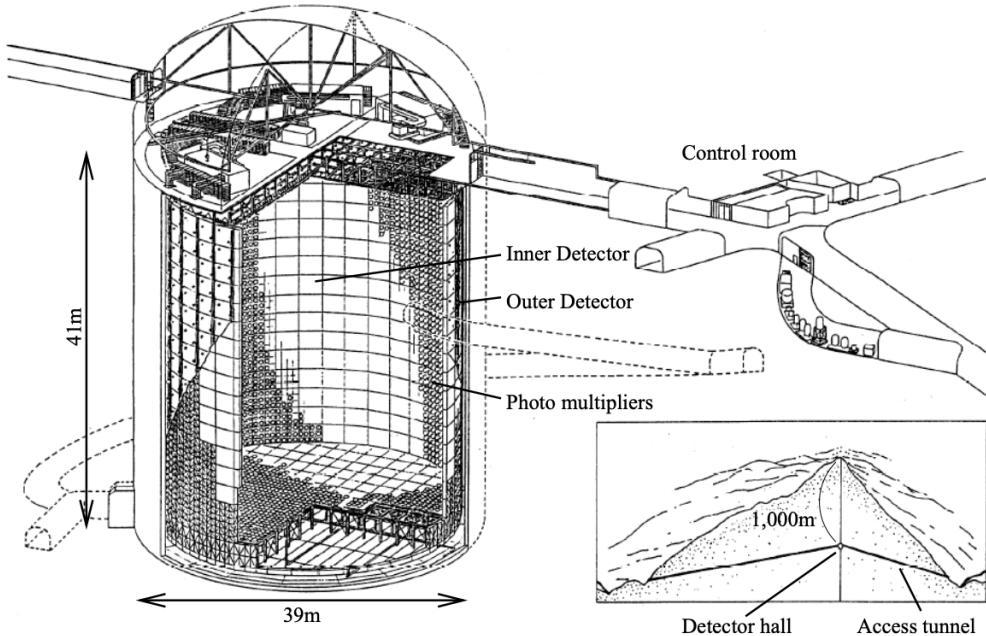


Figure 3.1: A schematic diagram of the Super-Kamiokande Detector. Taken from [86].

705 A smaller cylindrical structure (36.2m diameter, 33.8m height) is situated
 706 inside the tank, with an approximate 2m gap between this structure and the outer
 707 tank wall. The purpose of this structure is to support the photomultiplier tubes
 708 (PMTs). The volume inside and outside the support structure is referred to as the
 709 inner detector (ID) and outer detector (OD), respectively. In the SK-IV era, the
 710 ID and OD are instrumented by 11,129 50cm and 1,885 20cm PMTs respectively
 711 [82]. The ID contains a 32kton mass of water. Many analyses performed at SK
 712 use a “fiducial volume” defined by the volume of water inside the ID excluding
 713 some distance to the ID wall. This reduces the volume of the detector which is
 714 sensitive to neutrino events but reduces radioactive backgrounds and allows for

715 better reconstruction performance. The nominal fiducial volume is defined as the
716 area contained inside 2m from the ID wall for a total of 22.5kton water [3].

717 The two regions of the detector (ID and OD) are optically separated with
718 opaque black plastic hung from the support structure. The purpose of this is
719 to determine whether an event entered or exited the ID. This allows cosmic ray
720 muons and partially contained events to be tagged and separated from neutrino
721 events entirely contained within the ID. This black plastic is also used to cover
722 the area between the ID PMTs to reduce photon reflection from the ID walls.
723 Opposite to this, the OD is lined with a reflective material to allow photons to
724 reflect around inside the OD until collected by one of the PMTs. Furthermore,
725 each OD PMT is optically coupled with $50 \times 50\text{cm}$ plates of wavelength shifting
726 acrylic which increases the efficiency of light collection [85].

727 In the SK-IV data-taking period, the photocathode coverage of the detector, or
728 the fraction of the ID wall instrumented with PMTs, is $\sim 40\%$ [85]. The PMTs have
729 a quantum efficiency (the ratio of detected electrons to incident photons) of $\sim 21\%$
730 for photons with wavelengths of $360\text{nm} < \lambda < 390\text{nm}$ [87, 88]. The proportion
731 of photoelectrons that produce a signal in the dynode of a PMT, termed the
732 collection efficiency, is $> 70\%$ [85]. The PMTs used within SK are most sensitive
733 to photons with wavelength $300\text{nm} \leq \lambda \leq 600\text{nm}$ [85]. One disadvantage of
734 using PMTs as the detection media is that the Earth's geomagnetic field can
735 modify its response. Therefore, a set of compensation coils is built around the
736 inner surface of the detector to mitigate this effect [85].

737 The SK detector is filled with ultrapure water, which in a perfect world, con-
738 tains no impurities. However, bacteria and organic compounds can significantly
739 degrade the water quality. This decreases the attenuation length, which reduces
740 the total number of photons that hit a PMT. To combat this, a sophisticated water
741 treatment system has been developed [85, 89]. UV lights, mechanical filters, and
742 membrane degasifiers are used to reduce the bacteria, suspended particulates,
743 and radioactive materials from the water. The flow of water within the tank
744 is also critical as it can remove stagnant bacterial growth or build-up of dust

745 on the surfaces within the tank. Gravity drifts impurities in the water towards
746 the bottom of the tank which, if left uncontrolled, can create asymmetric water
747 conditions between the top and bottom of the tank. Typically, the water entering
748 the tank is cooled below the ambient temperature of the tank to control convection
749 and inhibit bacteria growth. Furthermore, the rate of dark noise hits within PMTs
750 is sensitive to the PMT temperature [90]. Therefore controlling the temperature
751 gradients within the tank is beneficial for stable measurements.

752 SK-VI is the first phase of the SK experiment to use gadolinium dopants
753 within the ultrapure water [83]. As such, the SK water system had to be replaced
754 to avoid removing the gadolinium concentrate from the ultrapure water [69]. For
755 an inverse β -decay (IBD) interaction on a water target, the emitted neutron is
756 thermally captured on hydrogen. This process releases a 2.2MeV γ ray which is
757 difficult to detect as the resulting Compton scattered electrons are very close to the
758 Cherenkov threshold, limiting detection capability. Thermal capture of neutrons
759 on gadolinium generates γ rays with higher energy (8MeV [69]) meaning they
760 are more easily detected and reconstructed. SK-VI has 0.01% Gd loading (0.02%
761 gadolinium sulphate by mass) which causes \approx 50% of neutrons emitted by IBD
762 to be captured on gadolinium[91]. Whilst predominantly useful for low energy
763 analyses, Gd loading allows better $\nu/\bar{\nu}$ separation for atmospheric neutrino
764 event selections [92]. Efforts are currently in place to increase the gadolinium
765 concentrate to 0.03% for \approx 75% neutron capture efficiency on gadolinium [93].
766 The final stage of loading targets 0.1% concentrate for \approx 90% neutron capture
767 efficiency on gadolinium.

768 3.1.2 Calibration

769 The calibration of the SK detector is documented in [82] and summarised below.
770 The analysis presented within this thesis is dependent upon ‘high energy events’
771 (Charged particles with $O(> 100)$ MeV momenta). These are events that are
772 expected to generate a larger number of photons such that each PMT will
773 be hit with multiple photons. The reconstruction of these events depends

774 upon the charge deposited within each PMT and the timing response of each
775 individual PMT. Therefore, the most relevant calibration techniques to this
776 thesis are outlined.

777 Before installation, 420 PMTs were calibrated to have identical charge re-
778 sponds and then distributed throughout the tank in a cross-shape pattern (As
779 illustrated by Figure 3.2). These are used as a standardised measure for the rest
780 of the PMTs installed at similar geometric positions within SK to be calibrated
781 against. To perform this calibration, a xenon lamp is located at the center of the
782 SK tank which flashes uniform light at 1Hz. This allows for geometrical effects,
783 water quality variation, and timing effects to be measured in situ throughout
784 normal data-taking periods.

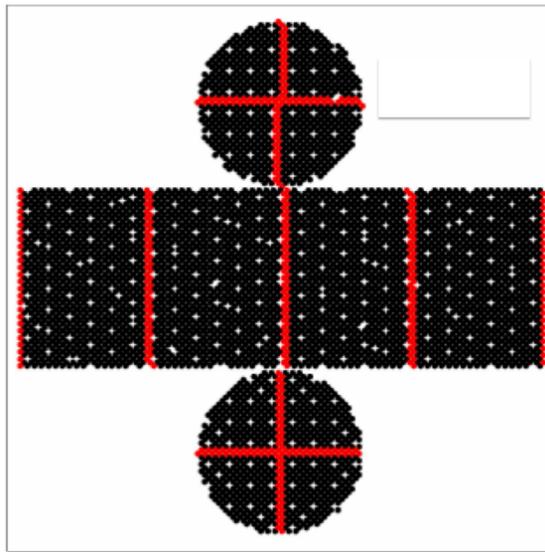


Figure 3.2: The location of “standard PMTs” (red) inside the SK detector. Taken from [82].

785 When specifically performing calibration of the detector (in out-of-data taking
786 mode), the water in the tank was circulated to avoid top/bottom asymmetric
787 water quality. Any non-uniformity within the tank significantly affects the PMT
788 hit probability through scattering or absorption. This becomes a dominant effect
789 for very low-intensity light sources that are designed such that only one photon
790 is incident upon a given PMT.

791 The gain of a PMT is defined as the ratio of the total charge of the signal
 792 produced compared to the charge of photoelectrons emitted by the photocathodes
 793 within the PMT. To calibrate the signal of each PMT, the “relative” and “absolute”
 794 gain values are measured. The relative gain is the variation of gain among each
 795 of the PMTs whereas the absolute gain is the average gain of all PMTs.

796 The relative gain is calibrated as follows. A laser is used to generate two
 797 measurements: a high-intensity flash that illuminates every PMT with a sufficient
 798 number of photons, and a low-intensity flash in which only a small number
 799 of PMTs collect light. The first measurement creates an average charge, $Q_{obs}(i)$
 800 on PMT i , whereas the second measurement ensures that each hit PMT only
 801 generates a single photoelectron. For the low-intensity measurement, the number
 802 of times each PMT records a charge larger than 1/4 photoelectrons, $N_{obs}(i)$, is
 803 counted. The values measured can be expressed as

$$\begin{aligned} Q_{obs}(i) &\propto I_H \times f(i) \times \epsilon(i) \times G(i), \\ N_{obs}(i) &\propto I_L \times f(i) \times \epsilon(i). \end{aligned} \tag{3.1}$$

804 Where I_H and I_L is the intensity of the high and low flashes, $f(i)$ is the
 805 acceptance efficiency of the i^{th} PMT, $\epsilon(i)$ is the product of the quantum and
 806 collection efficiency of the i^{th} PMT and $G(i)$ is the gain of the i^{th} PMT. The relative
 807 gain for each PMT can be determined by taking the ratio of these quantities.

808 The absolute gain calibration is performed by observing fixed energy γ -rays
 809 of $E_\gamma \sim 9\text{MeV}$ emitted isotropically from neutron capture on a NiCf source
 810 situated at the center of the detector. This generates a photon yield of about 0.004
 811 photoelectrons/PMT/event, meaning that $> 99\%$ of PMT signals are generated
 812 from single photoelectrons. A charge distribution is generated by performing
 813 this calibration over all PMTs, and the average value of this distribution is taken
 814 to be the absolute gain value.

815 As mentioned in subsection 3.1.1, the average quantum and collection effi-
 816 ciency for the SK detector PMTs is $\sim 21\%$ and $> 70\%$ respectively. However,
 817 these values do differ between each PMT and need to be calibrated accordingly.

Consequently, the NiCf source is also used to calibrate the “quantum \times collection” efficiency (denoted “QE”) value of each PMT. The NiCf low-intensity source is used as the PMT hit probability is proportional to the QE ($N_{obs}(i) \propto \epsilon(i)$ in Equation 3.1). A Monte Carlo prediction which includes photon absorption, scattering, and reflection is made to estimate the number of photons incident on each PMT and the ratio of the number of predicted to observed hits is calculated. The difference is attributed to the QE efficiency of that PMT. This technique is extended to calculate the relative QE efficiency by normalizing the average of all PMTs which removes the dependence on the light intensity.

Due to differing cable lengths and readout electronics, the timing response between a photon hitting the PMT and the signal being captured by the data acquisition can be different between each PMT. Due to threshold triggers (Described in subsection 3.1.3), the time at which a pulse reaches a threshold is dependent upon the size of the pulse. This is known as the ‘time-walk’ effect and also needs to be accounted for in each PMT. To calibrate the timing response, a pulse of light with width 0.2ns is emitted into the detector through a diffuser. Two-dimensional distributions of time and pulse height (or charge) are made for each PMT and are used to calibrate the timing response. This is performed in-situ during data taking with the light source pulsing at 0.03Hz.

The top/bottom water quality asymmetry is measured using the NiCf calibration data and cross-referencing these results to the “standard PMTs”. The water attenuation length is continuously measured by the rate of vertically-downgoing cosmic-ray muons which enter via the top of the tank.

Dark noise is where a PMT registers a pulse that is consistent with a single photoelectron emitted from photon detection despite the PMT being in complete darkness. This is predominately caused by two processes. Firstly there is intrinsic dark noise which is where photoelectrons gain enough thermal energy to be emitted from the photocathode, and secondly, the radioactive decay of contaminants inside the structure of the PMT. Typical dark noise rate for PMTs used within SK are $O(3)$ kHz [85]. This is lower than the expected number of

848 photons generated for a ‘high energy event’ (As described in subsection 3.1.4)
849 but instability in this value can cause biases in reconstruction. Dark noise is
850 related to the gain of a PMT and is calibrated using hits inside a time window
851 recorded before an event trigger [94].

852 3.1.3 Data Acquisition and Triggering

853 As the analysis presented in this thesis will only use the SK-IV period of the
854 SK experiment so this subsection focuses on the relevant points of the data
855 acquisition and triggering systems to that SK period. The earlier data acquisition
856 and triggering systems are documented in [95, 96].

857 Before the SK-IV period started, the existing front-end electronics were re-
858 placed with “QTC-Based Electronics with Ethernet, QBEE” systems [97]. When
859 the QBEE observes a signal above a 1/4 photoelectron threshold, the charge-to-
860 time (QTC) converter generates a rectangular pulse. The start of the rectangular
861 pulse indicates the time at which the analog photoelectron signal was received
862 and the width of the pulse indicates the total charge integrated throughout the
863 signal. This is then digitized by time-to-digital converters and sent to the “front-
864 end” PCs. The digitized signal from every QBEE is then chronologically ordered
865 and sent to the “merger” PCs. It is the merger PCs that apply the software trigger.
866 Any triggered events are passed to the “organizer” PC. This sorts the data stream
867 of multiple merger PCs into chronologically ordered events, which are then saved
868 to disk. The schematic of data flow from PMTs to disk is illustrated in Figure 3.3.

869 The software trigger (described in [99]) operates by determining the number
870 of PMT hits within a 200ns sliding window, N_{200} . This window coincides with the
871 maximum time that a Cherenkov photon would take to traverse the length of the
872 SK tank [96]. For lower energy events that generate fewer photons, this technique
873 is useful for eliminating background processes like dark noise and radioactive
874 decay which would be expected to be separated in time. When the value of
875 N_{200} exceeds some pre-defined threshold, a software trigger is issued. There are
876 several trigger thresholds used within the SK-IV period which are detailed in

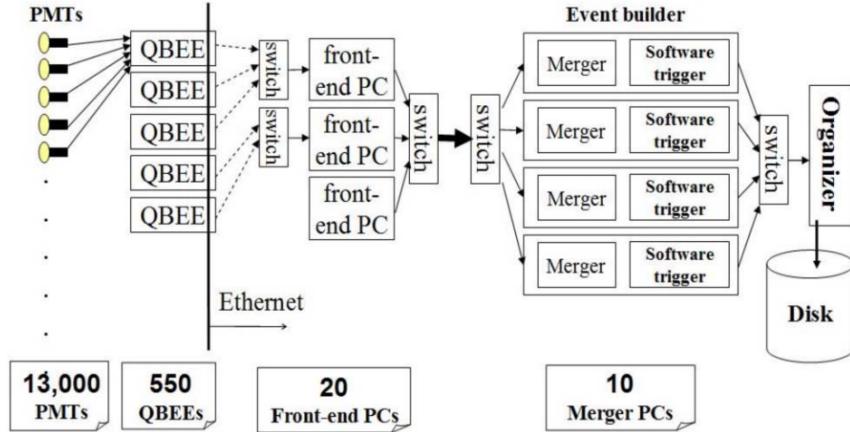


Figure 3.3: Schematic view of the data flow through the data acquisition and online system. Taken from [98].

877 Table 3.2. If one of these thresholds is met, the PMT hits within an extended time
 878 window are also read out and saved to disk. In the special case of an event that
 879 exceeds the SHE trigger but does not exceed the OD trigger, the AFT trigger looks
 880 for delayed coincidences of 2.2MeV gamma rays emitted from neutron capture in
 881 a $535\mu\text{s}$ window after the SHE trigger. A similar but more complex “Wideband
 882 Intelligent Trigger (WIT)” has been deployed and is described in [100].

Trigger	Acronym	Condition	Extended time window (μs)
Super Low Energy	SLE	>34/31 hits	1.3
Low Energy	LE	>47 hits	40
High Energy	HE	>50 hits	40
Super High Energy	SHE	>70/58 hits	40
Outer Detector	OD	>22 hits in OD	N/A

Table 3.2: The trigger thresholds and extended time windows saved around an event which were utilised throughout the SK-IV period. The exact thresholds can change and the values listed here represent the thresholds at the start and end of the SK-IV period.

3.1.4 Cherenkov Radiation

883 Cherenkov light is emitted from any highly energetic charged particle traveling
 884 with relativistic velocity, β , greater than the local speed of light in a medium [101].

886 Cherenkov light is formed at the surface of a cone with a characteristic pitch angle,

$$\cos(\theta) = \frac{1}{\beta n}. \quad (3.2)$$

887 Where n is the refractive index of the medium. Consequently, the Cherenkov
 888 momentum threshold, P_{thres} , is dependent upon the mass, m , of the charged
 889 particle moving through the medium,

$$P_{thres} = \frac{m}{\sqrt{n^2 - 1}}. \quad (3.3)$$

890 For water, where $n = 1.33$, the Cherenkov threshold momentum and energy
 891 for various particles are given in Table 3.3. In contrast, γ -rays are detected
 892 indirectly via the combination of photons generated by Compton scattering
 893 and pair production. The threshold for detection in the SK detector is typically
 894 higher than the threshold for photon production. This is due to the fact that the
 895 attenuation of photons in the water means that typically $\sim 75\%$ of Cherenkov
 896 photons reach the ID PMTs. Then the collection and quantum efficiencies
 897 described in subsection 3.1.1 result in the number of detected photons being
 898 lower than the number of photons which reach the PMTs.

Particle	Threshold Momentum (MeV)	Threshold Energy (MeV)
Electron	0.5828	0.7751
Muon	120.5	160.3
Pion	159.2	211.7
Proton	1070.0	1423.1

Table 3.3: The threshold momentum and total energy for a particle to generate Cherenkov light in ultrapure water, as calculated in Equation 3.2 in ultrapure water which has refractive index $n = 1.33$.

899 The Frank-Tamm equation [102] describes the relationship between the num-
 900 ber of Cherenkov photons generated per unit length, dN/dx , the wavelength of
 901 the photons generated, λ , and the relativistic velocity of the charged particle,

$$\frac{d^2N}{dxd\lambda} = 2\pi\alpha \left(1 - \frac{1}{n^2\beta^2}\right) \frac{1}{\lambda^2}. \quad (3.4)$$

where α is the fine structure constant. For a 100MeV momentum electron, approximately 330 photons will be produced per centimeter in the $300\text{nm} \leq \lambda \leq 700\text{nm}$ region which the ID PMTs are most sensitive to [85].

3.2 The Tokai to Kamioka Experiment

The Tokai to Kamioka (T2K) experiment is a long-baseline neutrino oscillation experiment located in Japan. Proposed in the early 2000s [86, 103] to replace K2K [104], T2K was designed to observe electron neutrino appearance whilst precisely measuring the oscillation parameters associated with muon neutrino disappearance [105]. The experiment consists of a neutrino beam generated at the Japan Proton Accelerator Research Complex (J-PARC), a suite of near detectors situated 280m from the beam target, and the Super Kamiokande far detector positioned at a 295km baseline. The cross-section view of the T2K experiment is drawn in Figure 3.4.

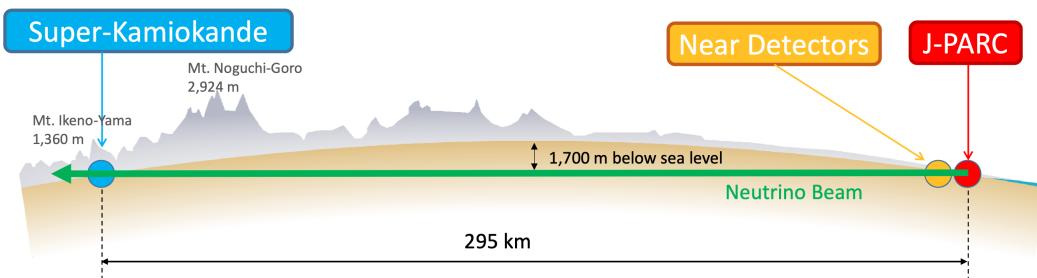


Figure 3.4: The cross-section view of the Tokai to Kamioka experiment illustrating the beam generation facility at J-PARC, the near detector situated at a baseline of 280m and the Super Kamiokande far detector situated 295km from the beam target.

The T2K collaboration makes world-leading measurements of the $\sin^2(\theta_{23})$, Δm_{32}^2 , and δ_{CP} oscillation parameters. Improvements in the precision and accuracy of parameter estimates are still being made by including new data samples and developing the models which describe the neutrino interactions and detector responses [4]. Electron neutrino appearance was first observed at T2K in 2014 [106] with 7.3σ significance.

921 The near detectors provide constraints on the beam flux and cross-section
 922 model parameters used within the oscillation analysis by observing the unoscil-
 923 lated neutrino beam. There are a host of detectors situated in the near detector hall
 924 (As illustrated in Figure 3.5): ND280 (subsection 3.2.3), INGRID (subsection 3.2.4),
 925 NINJA [107], WAGASCI [108], and Baby-MIND [109]. The latter three are not
 926 currently used within the oscillation analysis presented in this thesis.

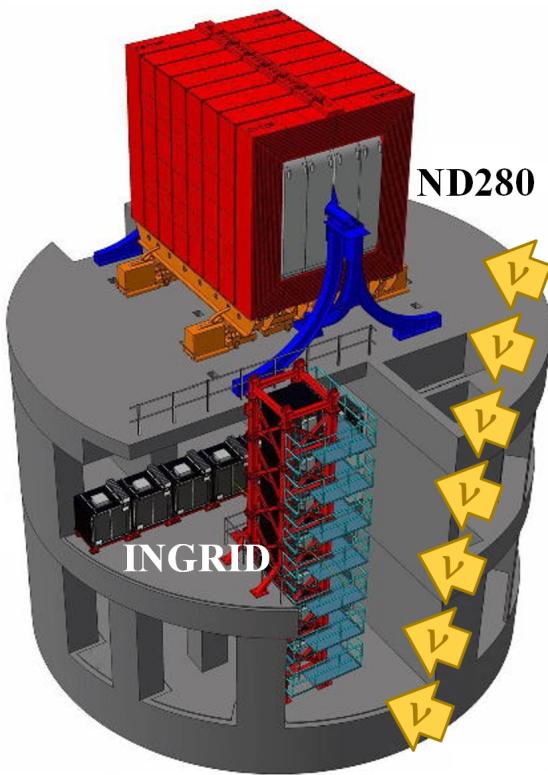


Figure 3.5: The near detector suite for the T2K experiment showing the ND280 and INGRID detectors. The distance between the detectors and the beam target is 280m.

927 Whilst this thesis presents the ND280 in terms of its purpose for the oscillation
 928 analysis, the detector can also make many cross-section measurements at neutrino
 929 energies of $O(1)$ GeV for the different targets within the detector [110, 111]. These
 930 measurements are of equal importance as they can lead the way in determining
 931 the model parameters used in the interaction models for the future high-precision
 932 era of neutrino physics.

3.2.1 Analysis Overview

There are two independent fitters, MaCh3 and BANFF, which perform the near detector fit. MaCh3 uses a bayesian Markov Chain Monte Carlo fitting technique, whereas BANFF uses a frequentist gradient descent technique. The output of each fitter is compared as a method of cross-checking the behaviour of the two fitters. This is done by comparing: the Monte Carlo predictions using various tunes, the likelihood that is calculated in each fitter and the post-fit constraint associated with every parameter used in the fit. Once validated, the output converted into a covariance matrix to describe the error and correlations between all the flux and cross-section parameters. This is then propagated to the far-detector oscillation analysis group.

The far detector group has three independent fitters: P-Theta, VALOR and MaCh3. The first two fitters use a hybrid frequentist fitting technique where the likelihood is minimised with respect to the parameters of interest and marginalised over all other parameters. These fitters use the covariance provided by the near detector fitters as a basis for implementing the near detector constraints. The MaCh3 fitter uses a simultaneous fit of all near and far detector samples. This removes any Gaussian assumptions when making the covariance matrix from the near detector results. The results for all three fitters are compares using a technique similar to the validation of the near detector fitters.

There are three particular tunes of the T2K flux and low energy cross section model typically considered. Firstly, the “generated” tune which is the set of dial values with which the Monte Carlo was generated. Secondly, the set of dial values which are taken from external data measurements and used as inputs. These are the “pre-fit” dial values. The reason these two sets of dial values are different is that the external data measurements are continually updated but it is very computationally intensive to regenerate a Monte Carlo prediction after each update. The final tune is the “post-fit”, “post-ND fit” or “post-BANFF” dial values. These are the values taken from the constraints provided by the near detector.

3.2.2 The Neutrino Beam

The neutrino beam used within the T2K experiment is described in [41, 45] and summarised below. The accelerator facility at J-PARC is composed of two sections; the primary and secondary beamlines. Figure 3.6 illustrates a schematic of the beamline, focusing mostly on the components of the secondary beamline. The primary beamline has three accelerators that progressively accelerate protons; a linear accelerator, a rapid-cycling synchrotron, and the main-ring (MR) synchrotron. Once fully accelerated by the MR, the protons have a kinetic energy of 30GeV. Eight bunches of these protons, separated by 500ns, are extracted per “spill” from the MR and directed towards a graphite target (a rod of length 91.4cm and diameter 2.6cm). Spills are extracted at 0.5Hz with $\sim 3 \times 10^{14}$ protons contained per spill.

The secondary beamline consists of three main components: the target station, the decay volume, and the beam dump. The target station is comprised of the target, beam monitors, and three magnetic focusing horns. The proton beam interacts with the graphite target to form a secondary beam of mostly pions and kaons. The secondary beam travels through a 96m long decay volume, generating neutrinos through the following decays [41],

$$\begin{array}{ll}
\pi^+ \rightarrow \mu^+ + \nu_\mu & \pi^- \rightarrow \mu^- + \bar{\nu}_\mu \\
K^+ \rightarrow \mu^+ + \nu_\mu & K^- \rightarrow \mu^- + \bar{\nu}_\mu \\
\rightarrow \pi^0 + e^+ + \nu_e & \rightarrow \pi^0 + e^- + \bar{\nu}_e \\
\rightarrow \pi^0 + \mu^+ + \nu_\mu & \rightarrow \pi^0 + \mu^- + \bar{\nu}_\mu \\
K_L^0 \rightarrow \pi^- + e^+ + \nu_e & K_L^0 \rightarrow \pi^+ + e^- + \bar{\nu}_e \\
\rightarrow \pi^- + \mu^+ + \nu_\mu & \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu \\
\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e & \mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e.
\end{array}$$

The electrically charged component of the secondary beam is focused towards the far detector by the three magnetic horns. These horns direct charged particles of a particular polarity towards SK whilst defocusing the oppositely charged particles. This allows a mostly neutrino or mostly antineutrino beam to be used within the experiment, denoted as “forward horn current (FHC)” or “reverse horn current (RHC)” respectively.

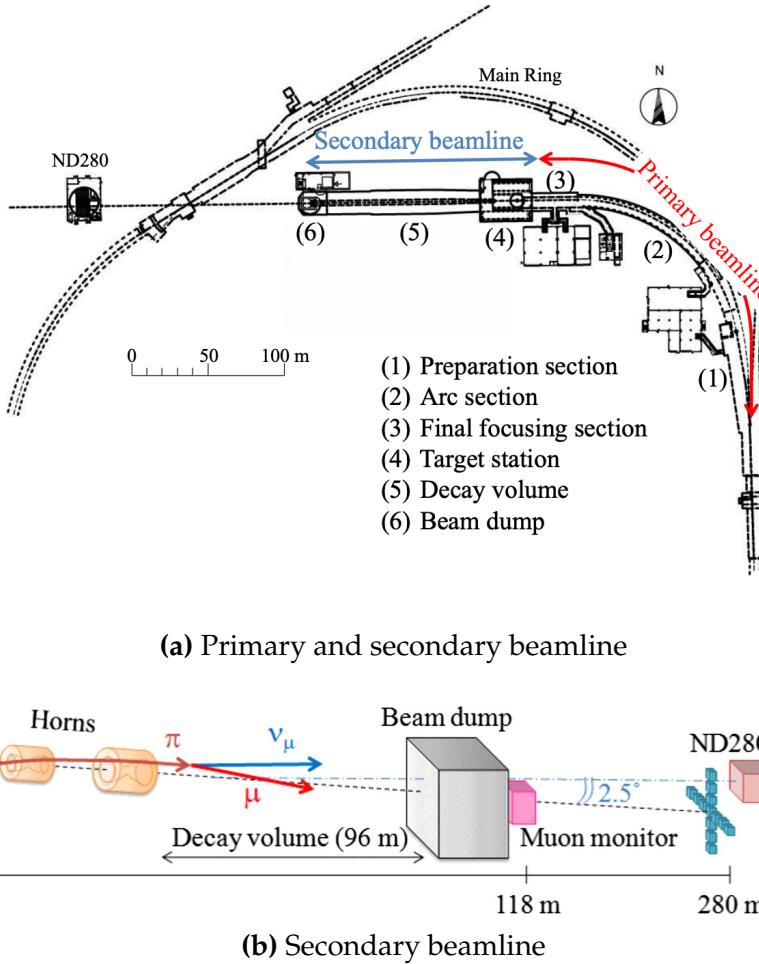


Figure 3.6: Top panel: Bird's eye view of the most relevant part of primary and secondary beamline used within the T2K experiment. The primary beamline is the main-ring proton synchrotron, kicker magnet, and graphite target. The secondary beamline consists of the three focusing horns, decay volume, and beam dump. Figure taken from [45]. Bottom panel: The side-view of the secondary beamline including the focusing horns, beam dump and neutrino detectors. Figure taken from [112].

988 Figure 3.7 illustrates the different contributions to the FHC and RHC neutrino
 989 flux. The low energy flux is dominated by the decay of pions whereas kaon
 990 decay becomes the dominant source of neutrinos for $E_\nu > 3\text{GeV}$. The “wrong-
 991 sign” component, which is the $\bar{\nu}_\mu$ background in a ν_μ beam, and the intrinsic
 992 irreducible ν_e background, are predominantly due to muon decay for $E_\nu <$
 993 2GeV . As the antineutrino production cross-section is smaller than the neutrino
 994 cross-section, the wrong-sign component is more dominant in the RHC beam
 995 as compared to that in the FHC beam.

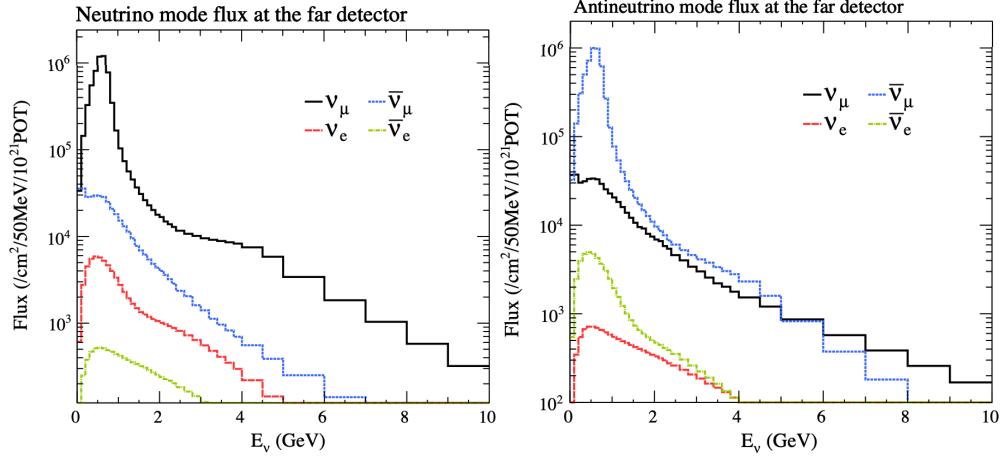


Figure 3.7: The Monte Carlo prediction of the energy spectrum for each flavour of neutrino (ν_e , $\bar{\nu}_e$, ν_μ and $\bar{\nu}_\mu$) in the neutrino dominated beam FHC mode (Left) and antineutrino dominated beam RHC mode (Right) expected at SK. Taken from [113].

996 The beam dump, situated at the end of the decay volume, stops all charged
 997 particles other than highly energetic muons ($p_\mu > 5\text{GeV}$). The MuMon detector
 998 monitors the penetrating muons to determine the beam direction and inten-
 999 sity which is used to constrain some of the beam flux systematics within the
 1000 analysis [112, 114].

1001 The T2K experiment uses an off-axis beam to narrow the neutrino energy
 1002 distribution. This was the first implementation of this technique in a long-
 1003 baseline neutrino oscillation experiment after its original proposal [115]. Pion
 1004 decay, $\pi \rightarrow \mu + \nu_\mu$, is a two-body decay. Consequently, the neutrino energy,
 1005 E_ν , can be determined based on the pion energy, E_π , and the angle at which
 1006 the neutrino is emitted, θ ,

$$E_\nu = \frac{m_\pi^2 - m_\mu^2}{2(E_\pi - p_\pi \cos(\theta))}, \quad (3.5)$$

1007 where m_π and m_μ are the mass of the pion and muon respectively. For a fixed
 1008 energy pion, the neutrino energy distribution is dependent upon the angle at
 1009 which the neutrinos are observed from the initial pion beam direction. For the
 1010 295km baseline at T2K, $E_\nu = 0.6\text{GeV}$ maximises the electron neutrino appearance
 1011 probability, $P(\nu_\mu \rightarrow \nu_e)$, whilst minimising the muon disappearance probability,

¹⁰¹² $P(\nu_\mu \rightarrow \nu_\mu)$. Figure 3.8 illustrates the neutrino energy distribution for a range of
¹⁰¹³ off-axis angles, as well as the oscillation probabilities most relevant to T2K.

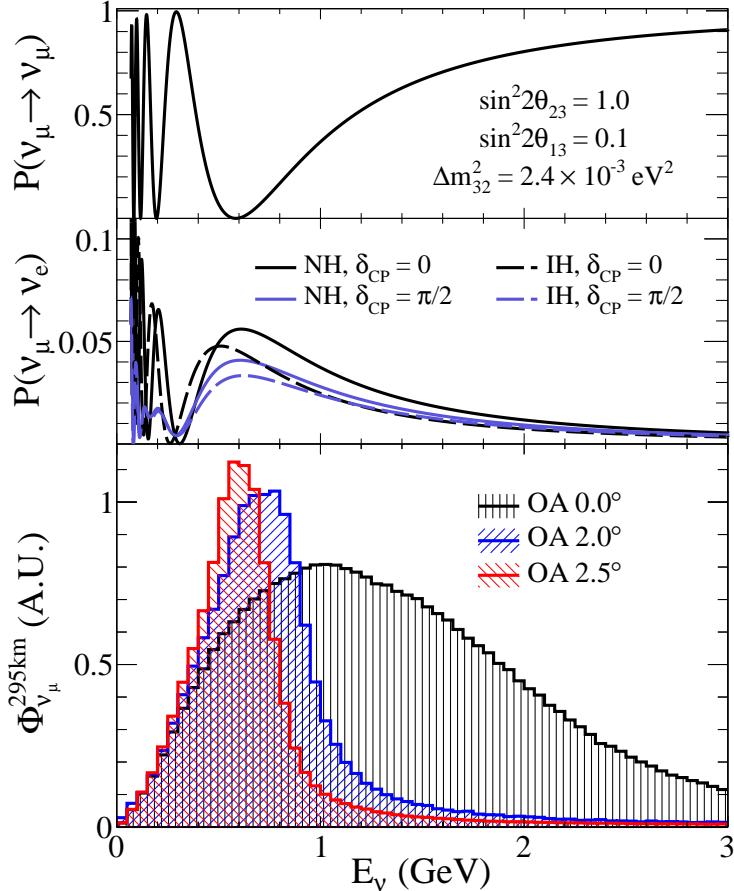


Figure 3.8: Top panel: T2K muon neutrino disappearance probability as a function of neutrino energy. Middle panel: T2K electron neutrino appearance probability as a function of neutrino energy. Bottom panel: The neutrino flux distribution for three different off-axis angles (Arbitrary units) as a function of neutrino energy.

3.2.3 The Near Detector at 280m

¹⁰¹⁴ ¹⁰¹⁵ Whilst all the near detectors are situated in the same “pit” located at 280m from
¹⁰¹⁶ the beamline, the “ND280” detector is the off-axis detector which is situated at
¹⁰¹⁷ the same off-axis angle as the Super-Kamiokande far detector. It has two primary
¹⁰¹⁸ functions; firstly it measures the neutrino flux and secondly, it counts the event
¹⁰¹⁹ rates of different types of neutrino interactions. Both of these constrain the flux
¹⁰²⁰ and cross-section systematics invoked within the model for a more accurate
¹⁰²¹ prediction of the expected event rate at the far detector.

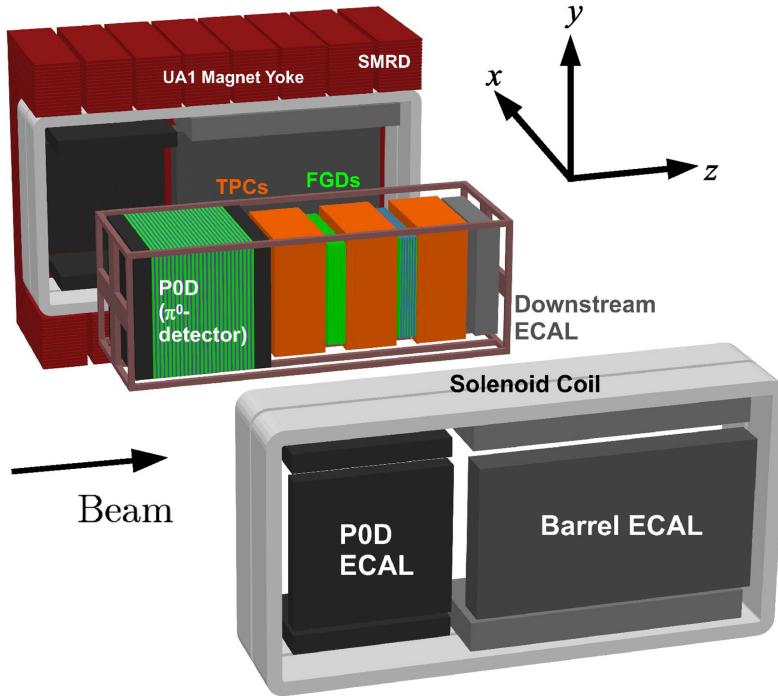


Figure 3.9: The components of the ND280 detector. The neutrino beam travels from left to right. Taken from [45].

As illustrated in Figure 3.9, the ND280 detector consists of several sub-detectors. The most important part of the detector for this analysis is the tracker region. This is comprised of two-time projection chambers (TPCs) sandwiched between three fine grain detectors (FGDs). The FGDs contain both hydrocarbon plastics and water targets for neutrino interactions and provide track reconstruction near the interaction vertex. The emitted charged particles can then propagate into the TPCs which provide particle identification and momentum reconstruction. The FGDs and TPCs are further described in subsubsection 3.2.3.1 and subsubsection 3.2.3.2 respectively. The electromagnetic calorimeter (ECAL) encapsulates the tracker region alongside the π^0 detector (P0D). The ECAL measures the deposited energy from photons emitted from interactions within the FGD. The P0D constrains the cross-section of neutral current interactions which generate neutral pions, which is one of the largest backgrounds in the electron neutrino appearance oscillation channel. The P0D and ECAL detectors are detailed in subsubsection 3.2.3.3 and subsubsection 3.2.3.4 respectively. The entire detector is located within a large yoke magnet which produces a 0.2T magnetic field.

1038 field. This design of the magnet also includes a scintillating detector called the
1039 side muon range detector (SMRD), which is used to track high-angle muons as
1040 well as acting as a cosmic veto. The SMRD is described in subsubsection 3.2.3.5.

1041 3.2.3.1 Fine Grained Detectors

1042 The T2K tracker region is comprised of two fine-grained detectors (FGD) and
1043 three Time Projection Chambers (TPC). A detailed description of the FGD design,
1044 construction, and assembly is found in [116] and summarised below. The FGDS
1045 are the primary target for neutrino interactions with a mass of 1.1 tonnes per FGD.
1046 Alongside this, the FGDS are designed to be able to track short-range particles
1047 which do not exit the FGD. Typically, short-range particles are low momentum
1048 and are observed as tracks that deposit a large amount of energy per unit length.
1049 This means the FGD needs good granularity to resolve these particles. The
1050 FGDS have the best timing resolution ($\sim 3\text{ns}$) of any of the sub-detectors of the
1051 ND280 detector. As such, the FGDS are used for time of flight measurements
1052 to distinguish forward-going positively charged particles from backward-going
1053 negatively charged particles. Finally, any tracks which pass through multiple
1054 sub-detectors are required to be track matched to the FGD.

1055 Both FGDS are made from square scintillator planes of side length 186cm and
1056 width 2.02cm. Each plane consists of two layers of 192 scintillator bars in an
1057 X or Y orientation. A wavelength-shifting fiber is threaded through the center
1058 of each bar and is read out by a multi-pixel photon counter (MPPC). FGD1 is
1059 the most upstream of the two FGDS and contains 15 planes of carbon plastic
1060 scintillator which is a common target in external neutrino scattering data. As
1061 the far detector is a pure water target, 7 of the 15 scintillator planes in FGD2
1062 have been replaced with a hybrid water-scintillator target. Due to the complexity
1063 of the nucleus, nuclear effects can not be extrapolated between different nuclei.
1064 Therefore having the ability to take data on one target which is the same as
1065 external data and another target which is the same as the far detector target is
1066 beneficial for reliable model parameter estimates.

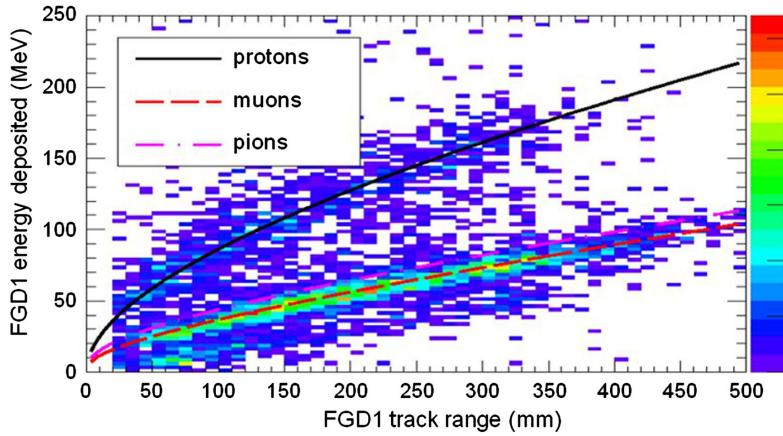


Figure 3.10: Comparison of data to Monte Carlo prediction of integrated deposited energy as a function of track length for particles that stopped in FGD1. Taken from [116].

1067 The integrated deposited energy is used for particle identification. The FGD
 1068 can distinguish protons from other charged particles by comparing the integrated
 1069 deposited energy from data to Monte Carlo prediction as seen in Figure 3.10.

1070 3.2.3.2 Time Projection Chambers

1071 The majority of particle identification and momentum measurements within
 1072 ND280 are provided by three Time Projection Chambers (TPCs) [117]. The
 1073 TPCs are located on either side of the FGDs. They are located inside of the
 1074 magnetic field meaning the momentum of a charged particle can be determined
 1075 from the bending of the track.

1076 Each TPC module consists of two gas-tight boxes, as shown in Figure 3.11,
 1077 which are made of non-magnetic material. The outer box is filled with CO₂ which
 1078 acts as an electrical insulator between the inner box and the ground. The inner box
 1079 forms the field cage which produces a uniform electric drift field of $\sim 275\text{V/cm}$
 1080 and is filled with an argon gas mixture. Charged particles moving through this
 1081 gas mixture ionize the gas and the ionised charge is drifted towards micromegas
 1082 detectors which measure the ionization charge. The time and position information
 1083 in the readout allows a three-dimensional image of the neutrino interaction.

1084 The particle identification of tracks that pass through the TPCs is performed
 1085 using dE/dx measurements. Figure 3.12 illustrates the data to Monte Carlo

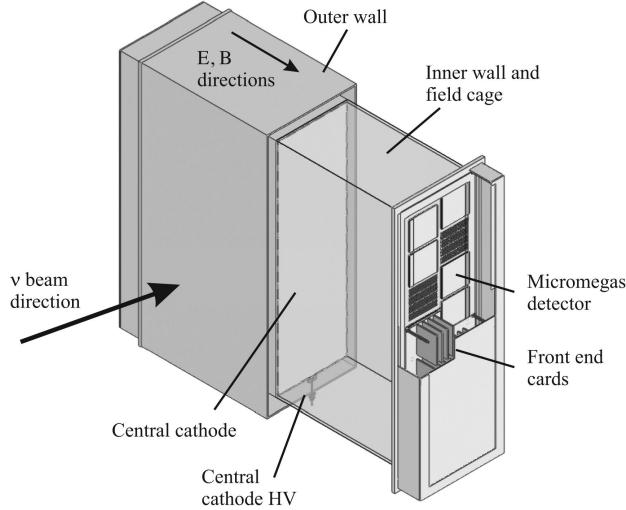


Figure 3.11: Schematic design of a Time Projection Chamber detector. Taken from [117].

1086 distributions of the energy lost by a charged particle passing through the TPC as
 1087 a function of the reconstructed particle momentum. The resolution is $7.8 \pm 0.2\%$
 1088 meaning that electrons and muons can be distinguished. This allows reliable
 1089 measurements of the intrinsic ν_e component of the beam.

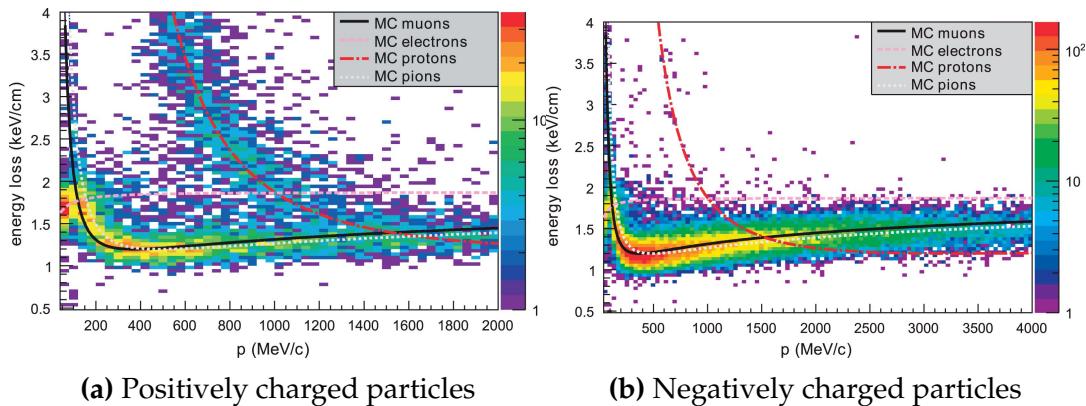


Figure 3.12: The distribution of energy loss as a function of reconstructed momentum for charged particles passing through the TPC, comparing data to Monte Carlo prediction. Taken from [117].

1090 3.2.3.3 π^0 Detector

1091 If one of the γ -rays from a $\pi^0 \rightarrow 2\gamma$ decay is missed at the far detector, the
 1092 reconstruction will determine that event to be a charge current ν_e -like event.
 1093 This is one of the main backgrounds hindering the electron neutrino appearance

1094 searches. The π^0 detector (P0D) measures the cross-section of the neutral current
1095 induced neutral pion production on a water target to constrain this background.

1096 The P0D is a cube of approximately 2.5m length consisting of layers of scin-
1097 tillating bars, brass and lead sheets, and water bags as illustrated in Figure 3.13.
1098 Two electromagnetic calorimeters are positioned at the most upstream and most
1099 downstream position in the sub-detector and the water target is situated in
1100 between them. The scintillator layers are built from two triangular bars orientated
1101 in opposite directions to form a rectangular layer. Each triangular scintillator bar
1102 is threaded with optical fiber which is read out by MPPCs. The high-Z brass and
1103 lead regions produce electron showers from the photons emitted in π^0 decay.

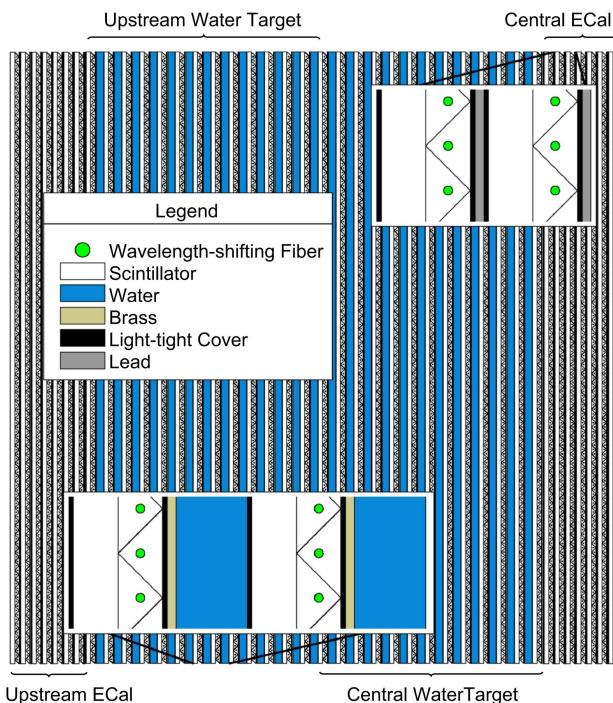


Figure 3.13: A schematic of the P0D side-view. Taken from [118].

1104 The sub-detector can generate measurements of NC1 π^0 cross-sections on a
1105 water target by measuring the event rate both with and without the water target,
1106 with the cross-section on a water target being determined as the difference. The to-
1107 tal active mass is 16.1 tonnes when filled with water and 13.3 tonnes when empty.

3.2.3.4 Electromagnetic Calorimeter

The electromagnetic calorimeter [119] (ECal) encapsulates the P0D and tracking sub-detectors. Its primary purpose is to aid π^0 reconstruction from any interaction in the tracker. To do this, it measures the energy and direction of photon showers from $\pi^0 \rightarrow 2\gamma$ decay. It can also distinguish pion and muon tracks depending on the shape of the photon shower deposited.

The ECal is comprised of three sections; the P0D ECal which surrounds the P0D, the barrel ECal which encompasses the tracking region, and the downstream ECal which is situated downstream of the tracker region. The barrel and downstream ECals are tracking calorimeters that focus on electromagnetic showers from high-angle particles emitted from the tracking sub-detectors. Particularly in the TPC, high-angle tracks (those which travel perpendicularly to the beam-axis) can travel along a single scintillator bar resulting in very few hits. The width of the barrel and downstream ECal corresponds to ~ 11 electron radiation lengths to ensure a significant amount of the π^0 energy is contained. As the P0D has its own calorimetry which reconstructs showers, the P0D ECal determines the energy which escapes the P0D.

Each ECal is constructed of multiple layers of scintillating bars sandwiched between lead sheets. The scintillating bars are threaded with optical fiber and read out by MPPCs. Each sequential layer of the scintillator is orientated perpendicular to the previous which allows a three-dimensional event reconstruction. The target mass of the P0D ECal, barrel ECal, and downstream ECal are 1.50, 4.80, and 6.62 tonnes respectively.

3.2.3.5 Side Muon Range Detector

As illustrated in Figure 3.9, the ECal, FGDs, P0D, and TPCs are enclosed within the UA1 magnet. Reconditioned after use in the UA1 [120] and NOMAD [121] experiments, this magnet provides a uniform horizontal magnetic field of 0.2T with an uncertainty of 2×10^{-4} T.

1136 Built into the UA1 magnet, the side muon range detector (SMRD)[122] monitors
1137 high-energy muons which leave the tracking region and permeate through
1138 the ECal. It additionally acts as a cosmic muon veto and trigger.

1139 **3.2.4 The Interactive Neutrino GRID**

1140 The Interactive Neutrino GRID (INGRID) detector is situated within the same
1141 “pit” as the other near detectors. It is aligned with the beam in the “on-axis”
1142 position and measures the beam direction, spread, and intensity. The detector
1143 was originally designed with 16 identical modules [45] (two modules have since
1144 been decommissioned) and a “proton” module. The design of the detector is 14
1145 modules oriented in a cross with length and height $10\text{m} \times 10\text{m}$, as illustrated
1146 in Figure 3.14.

1147 Each module is composed of iron sheets interlaced with eleven tracking
1148 scintillator planes for a total target mass of 7.1 tonnes per module. The scintillator
1149 design is an X-Y pattern of 24 bars in both orientations, where each bar contains
1150 wave-length shifting fibers which are connected to multi-pixel photon counters
1151 (MPPCs). Each module is encapsulated inside veto planes to aid the rejection
1152 of charged particles entering the module.

1153 The proton module is different from the other modules in that it consists
1154 of entirely scintillator planes with no iron target. The scintillator bars are also
1155 smaller than those used in the other modules to increase the granularity of
1156 the detector and improve tracking capabilities. The module sits in the center
1157 of the beamline and is designed to give precise measurements of quasi-elastic
1158 charged current interactions to evaluate the performance of the Monte Carlo
1159 simulation of the beamline.

1160 The INGRID detector can measure the beam direction to an uncertainty of
1161 0.4mrad and the beam centre within a resolution of 10cm [45]. The beam direction
1162 in both the vertical and horizontal directions is discussed in [123] and it is found
1163 to be in good agreement with the MUMON monitor described in subsection 3.2.2.

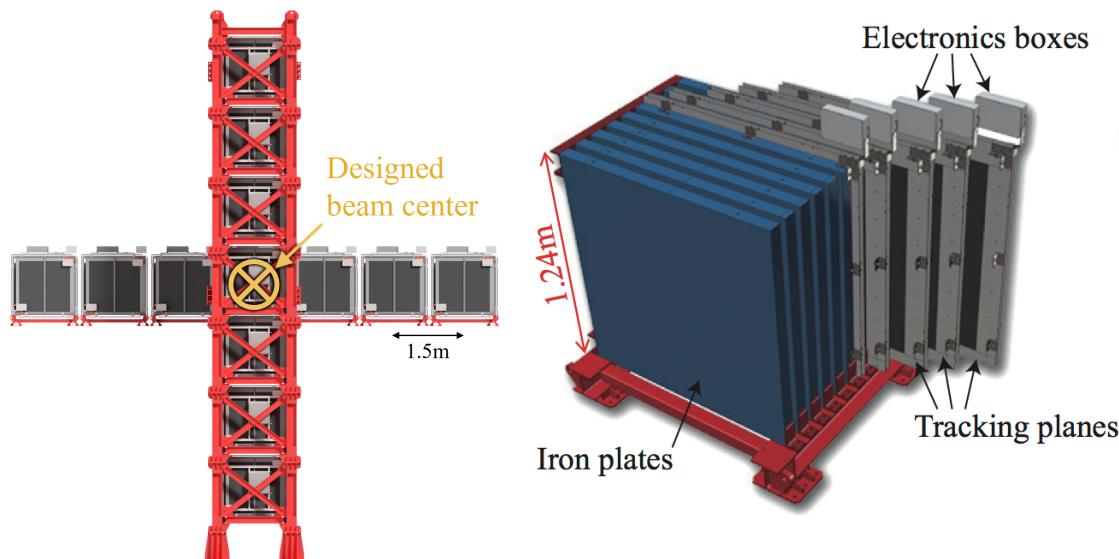


Figure 3.14: Left panel: The Interactive Neutrino GRID on-axis Detector. 14 modules are arranged in a cross-shape configuration, with the center modules being directly aligned with the on-axis beam. Right panel: The layout of a single module of the INGRID detector. Both figures are recreated from [45].

4

1164

1165

1166

Bayesian Statistics and Markov Chain Monte Carlo Techniques

1167 This thesis presents a Bayesian oscillation analysis. To extract the oscillation
1168 parameters, a Markov Chain Monte Carlo (MCMC) method is used. This chapter
1169 explains the theory of how parameter estimates can be determined using this
1170 technique and condenses the material found in the literature [124–127].

1171 The oscillation parameter determination presented here is built upon a si-
1172 multaneous fit to neutrino beam data in the near detector, beam data at SK, and
1173 atmospheric data at SK. In total, there are four oscillation parameters of interest
1174 ($\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP}), two oscillation parameters to which this
1175 study will not be sensitive ($\sin^2(\theta_{12})$, Δm_{21}^2) and many nuisance parameters that
1176 control the systematic uncertainty models.

1177 This analysis uses a Monte Carlo technique to generate a multi-dimensional
1178 probability distribution across all of the model parameters used in the fit. To
1179 determine an estimate for each parameter, this multi-dimensional object is in-
1180 tegrated over all other parameters. This process is called Marginalisation and
1181 is described in subsection 4.3.1. Monte Carlo techniques approximate the prob-
1182 ability distribution of each parameter within the limit of generating infinite
1183 samples. As ever, generating a large number of samples is time and resource-

1184 dependent. Therefore, an MCMC technique is utilised within this analysis to
1185 reduce the required number of steps to sufficiently sample the parameter space.
1186 This technique is described in further detail in subsection 4.2.1.

1187 The Bayesian analysis techniques used within this thesis are built within the
1188 MaCh3 framework [128]. This uses a custom MCMC library package exclusively
1189 supported and developed by the MaCh3 collaborators (which includes the author
1190 of this thesis).

1191 4.1 Bayesian Statistics

1192 Bayesian inference treats observable data, D , and model parameters, $\vec{\theta}$, on equal
1193 footing such that a probability model of both data and parameters is required.
1194 This is the joint probability distribution $P(D, \vec{\theta})$ and can be described by the
1195 prior distribution for model parameters $P(\vec{\theta})$ and the likelihood of the data given
1196 the model parameters $P(D|\vec{\theta})$,

$$P(D, \vec{\theta}) = P(D|\vec{\theta})P(\vec{\theta}). \quad (4.1)$$

1197 The prior distribution, $P(\vec{\theta})$, describes all previous knowledge about the
1198 parameters within the model. For example, if the risk of developing health
1199 problems is known to increase with age, the prior distribution would describe the
1200 increase. For the purpose of this analysis, the prior distribution is typically
1201 the best-fit values taken from external data measurements with a Gaussian
1202 uncertainty. The prior distribution can also contain correlations between model
1203 parameters. In an analysis using Monte Carlo techniques, the likelihood of
1204 measuring some data assuming some set of model parameters is calculated
1205 by comparing the Monte Carlo prediction generated at that particular set of
1206 model parameters to the data.

1207 It is parameter estimation that is important for this analysis and as such, Bayes'
1208 theorem [129] is applied to calculate the probability for each parameter to have a

₁₂₀₉ certain value given the observed data, $P(\vec{\theta}|D)$, which is known as the posterior
₁₂₁₀ distribution (often termed the posterior). This can be expressed as

$$P(\vec{\theta}|D) = \frac{P(D|\vec{\theta})P(\vec{\theta})}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.2)$$

₁₂₁₁ The denominator in Equation 4.2 is the integral of the joint probability distri-
₁₂₁₂ bution over all values of all parameters used within the fit. For brevity, the
₁₂₁₃ posterior distribution is

$$P(\vec{\theta}|D) \propto P(D|\vec{\theta})P(\vec{\theta}). \quad (4.3)$$

₁₂₁₄ For the purposes of this analysis, it is acceptable to neglect the normalisation
₁₂₁₅ term and focus on this proportional relationship.

₁₂₁₆ 4.1.1 Application of Prior Knowledge

₁₂₁₇ The posterior distribution is proportional to the prior uncertainty applied to
₁₂₁₈ each parameter, as illustrated by Equation 4.3. This means that it is possible
₁₂₁₉ to change the prior after the posterior distribution has been determined. The
₁₂₂₀ prior uncertainty of a particular parameter can be ‘divided’ out of the posterior
₁₂₂₁ distribution and the resulting distribution can be reweighted using the new
₁₂₂₂ prior uncertainty that is to be applied. The methodology and implementation
₁₂₂₃ of changing the prior follows that described in [130].

₁₂₂₄ An example implementation that is useful for this analysis is the application
₁₂₂₅ of the “reactor constraint”. As discussed in section 2.4, an external constraint
₁₂₂₆ on $\sin^2(\theta_{13})$ is determined from measurements taken from reactor experiments.
₁₂₂₇ However, the sensitivities from just using the T2K and SK samples is equally
₁₂₂₈ as important. Without this technique, two fits would have to be run, doubling
₁₂₂₉ the required resources. Therefore, the key benefit for this analysis is the fact that
₁₂₃₀ only a single ‘fit’ has to be performed and can be used to build the two posterior
₁₂₃₁ distributions of the with and without reactor constraint applied.

1232 4.2 Monte Carlo Simulation

1233 Monte Carlo techniques are used to numerically solve a complex problem that
1234 does not necessarily have an analytical solution. These techniques rely on
1235 building a large ensemble of samples from an unknown distribution and then
1236 using the ensemble to approximate the properties of the distribution.

1237 An example that uses Monte Carlo techniques is to calculate the area under-
1238 neath a curve. For example, take the problem of calculating the area under a
1239 straight line with gradient $M = 0.4$ and intercept $C = 1.0$. Analytically, one can
1240 calculate the area under the line is equal to 30 units for $0 \leq x \leq 10$. Using Monte
1241 Carlo techniques, one can calculate the area under this line by throwing many
1242 random values for the x and y components of each sample and then calculating
1243 whether that point falls below the line. The area can then be calculated by the
1244 ratio of points below the line to the total number of samples thrown multiplied by
1245 the total area in which samples were scattered. The study is shown in Figure 4.1
1246 highlights this technique and finds the area under the curve to be 29.9 compared
1247 to an analytical solution of 30.0. The deviation of the numerical to analytical
1248 solution can be attributed to the number of samples used in the study. The
1249 accuracy of the approximation in which the properties of the Monte Carlo samples
1250 replicate those of the desired distribution is dependent on the number of samples
1251 used. Replicating this study with a differing number of Monte Carlo samples
1252 used in each study (As shown in Figure 4.2) highlights how the Monte Carlo
1253 techniques are only accurate within the limit of a high number of samples.

1254 Whilst the above example has an analytical solution, these techniques are just
1255 as applicable to complex solutions. Clearly, any numerical solution is only as
1256 useful as its efficiency. As discussed, the accuracy of the Monte Carlo technique is
1257 dependent upon the number of samples generated to approximate the properties
1258 of the distribution. Furthermore, if the positions at which the samples are
1259 evaluated are not ‘cleverly’ picked, the efficiency of the Monte Carlo technique
1260 significantly drops. Given the example in Figure 4.1, if the region in which the

1261 samples are scattered significantly extends passed the region of interest, many
1262 calculations will be calculated but do not add to the ability of the Monte Carlo
1263 technique to achieve the correct result. For instance, any sample evaluated at
1264 a $y \geq 5$ could be removed without affecting the final result. This does bring in
1265 an aspect of the ‘chicken and egg’ problem in that to achieve efficient sampling,
1266 one needs to know the distribution beforehand.

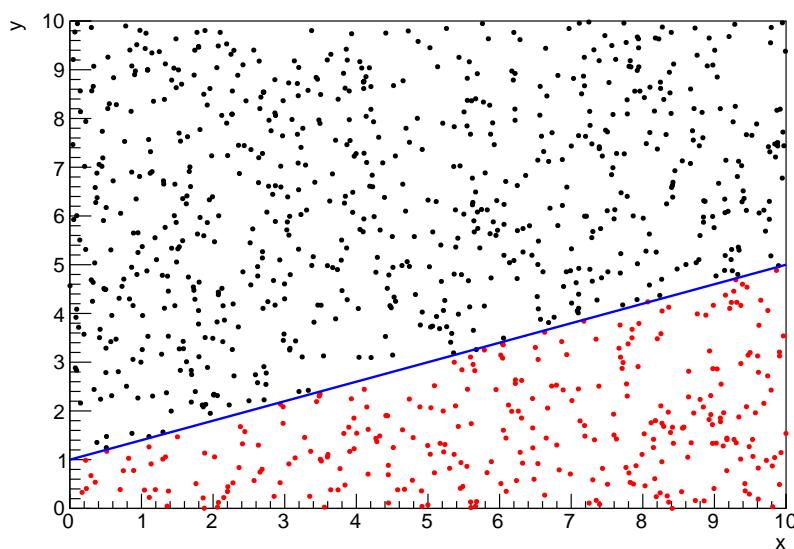


Figure 4.1: Example of using Monte Carlo techniques to find the area under the blue line. The gradient and intercept of the line are 0.4 and 1.0 respectively. The area found to be under the curve using one thousand samples is 29.9 units.

1267 4.2.1 Markov Chain Monte Carlo

1268 This analysis utilises a multi-dimensional probability distribution, with some
1269 dimensions being significantly more constrained than others. These constraints
1270 can be from prior knowledge of parameter distributions from external data or
1271 un-physical regions in which parameters can not exist. To maximise the efficiency
1272 of building the posterior distribution, a Markov Chain Monte Carlo (MCMC)
1273 technique is used. This employs a Markov chain to select the points at which
1274 to sample the posterior distribution. It performs a semi-random stochastic walk
1275 through the allowable parameter space. This builds a posterior distribution

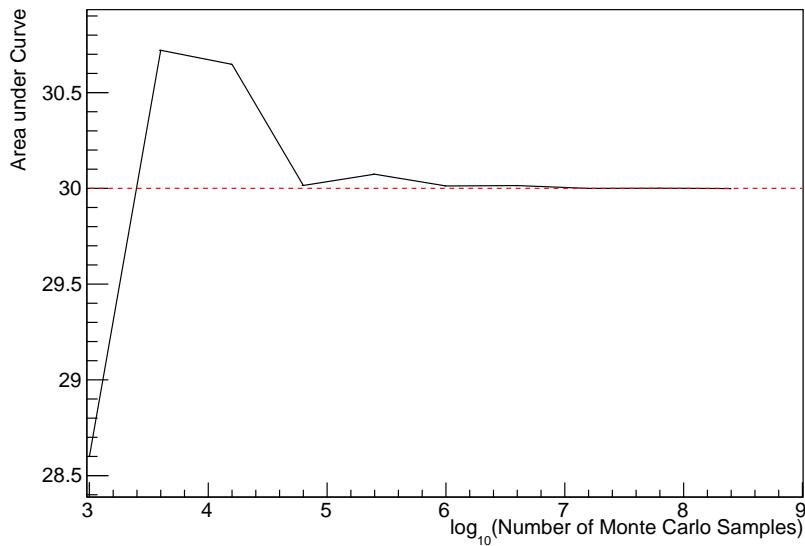


Figure 4.2: The area under a line of gradient 0.4 and intercept 1.0 for the range $0 \leq x \leq 10$ as calculated using Monte Carlo techniques as a function of the number of samples used in each repetition. The analytical solution to the area is 30 units as given by the red line.

1276 which has the property that the density of sampled points is proportional to the
 1277 probability density of that parameter. This means that the samples produced by
 1278 this technique are not statistically independent but they will cover the space
 1279 of the distribution.

1280 A Markov chain functions by selecting the position of step \vec{x}_{i+1} based on the
 1281 position of \vec{x}_i . The space in which the Markov chain selects samples is dependent
 1282 upon the total number of parameters utilised within the fit, where a discrete point
 1283 in this space is described by the N-dimensional space \vec{x} . In a perfectly operating
 1284 Markov chain, the position of the next step depends solely on the previous step
 1285 and not on the further history of the chain (\vec{x}_0, \vec{x}_1 , etc.). However, in solving
 1286 the multi-dimensionality of the fit used within this analysis, each step becomes
 1287 correlated with several of the steps preceding itself. Providing the MCMC chain is
 1288 well optimised, it will begin to converge towards a unique stationary distribution.
 1289 The period between the chain's initial starting point and the convergence to the
 1290 unique stationary distribution is colloquially known as the burn-in period. Once
 1291 the chain reaches the stationary distribution, all points sampled after that point

1292 will look like samples from that distribution.

1293 Further details of the theories underpinning MCMC techniques are discussed
1294 in [125] but can be summarised by the requirement that the chain satisfies the
1295 three ‘regularity conditions’:

- 1296 • Irreducibility: From every position in the parameter space \vec{x} , there must
1297 exist a non-zero probability for every other position in the parameter space
1298 to be reached.
- 1299 • Recurrence: Once the chain arrives at the stationary distribution, every step
1300 following from that position must be samples from the same stationary
1301 distribution.
- 1302 • Aperiodicity: The chain must not repeat the same sequence of steps at any
1303 point throughout the sampling period.

1304 The output of the chain after burn-in (i.e. the sampled points after the chain
1305 has reached the stationary distribution) can be used to approximate the posterior
1306 distribution and model parameters $\vec{\theta}$. To achieve the requirement that the unique
1307 stationary distribution found by the chain be the posterior distribution, one
1308 can use the Metropolis-Hastings algorithm. This guides the stochastic process
1309 depending on the likelihood of the current proposed step compared to that
1310 of the previous step.

1311 4.2.2 Metropolis-Hastings Algorithm

1312 As a requirement for MCMCs, the Markov chain implemented in this technique
1313 must have a unique stationary distribution that is equivalent to the posterior
1314 distribution. To ensure this requirement and that the regularity conditions are
1315 met, this analysis utilises the Metropolis-Hastings (MH) algorithm [131, 132].
1316 For the i^{th} step in the chain, the MH algorithm determines the position in the
1317 parameter space to which the chain moves to based on the current step, \vec{x}_i , and
1318 the proposed step, \vec{y}_{i+1} . The proposed step is randomly selected from some

1319 proposal function $f(\vec{x}_{i+1}|\vec{x}_i)$, which depends solely on the current step (ie. not
1320 the further history of the chain). The next step in the chain \vec{x}_{i+1} can be either the
1321 current step or the proposed step determined by whether the proposed step is
1322 accepted or rejected. To decide if the proposed step is selected, the acceptance
1323 probability, $\alpha(\vec{x}_i, \vec{y}_i)$, is calculated as

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)f(\vec{x}_i|\vec{y}_{i+1})}{P(\vec{x}_i|D)f(\vec{y}_{i+1}|\vec{x}_i)}\right). \quad (4.4)$$

1324 Where $P(\vec{y}_{i+1}|D)$ is the posterior distribution as introduced in section 4.1. To
1325 simplify this calculation, the proposal function is required to be symmetric such
1326 that $f(\vec{x}_i|\vec{y}_{i+1}) = f(\vec{y}_{i+1}|\vec{x}_i)$. In practice, a multi-variate Gaussian distribution
1327 centered on \vec{x}_i is used to throw parameter proposals. This reduces Equation 4.4 to

$$\alpha(\vec{x}_i, \vec{y}_{i+1}) = \min\left(1, \frac{P(\vec{y}_{i+1}|D)}{P(\vec{x}_i|D)}\right). \quad (4.5)$$

1328 After calculating this quantity, a random number, β , is generated uniformly
1329 between 0 and 1. If $\beta \leq \alpha(\vec{x}_i, \vec{y}_{i+1})$, the proposed step is accepted. Otherwise,
1330 the chain sets the next step equal to the current step. This procedure is repeated
1331 for subsequent steps. This can be interpreted as if the posterior probability
1332 of the proposed step is greater than that of the current step, ($P(\vec{y}_{i+1}|D) \geq$
1333 $P(\vec{x}_i|D)$), the proposed step will always be accepted. If the opposite is true,
1334 ($P(\vec{y}_{i+1}|D) \leq P(\vec{x}_i|D)$), the proposed step will be accepted with probability
1335 $P(\vec{x}_i|D)/P(\vec{y}_{i+1}|D)$. This ensures that the Markov chain does not get trapped
1336 in any local minima in the potentially non-Gaussian posterior distribution. The
1337 outcome of this technique is that the density of steps taken in a discrete region
1338 is directly proportional to the probability density in that region.

1339 4.2.3 MCMC Optimisation

1340 As discussed in subsection 4.2.2, the proposal function invoked within the MH
1341 algorithm can take any form and the chain will still converge to the stationary
1342 distribution. At each set of proposed parameter values, a prediction of the same
1343 spectra has to be generated which requires significant computational resources.

1344 Therefore, the number of steps taken before the unique stationary distribution
1345 is found should be minimised as only steps after convergence add information
1346 to the oscillation analysis. Furthermore, the chain should entirely cover the
1347 allowable parameter space to ensure that all values have been considered. Tuning
1348 the distance that the proposal function jumps between steps on a parameter-by-
1349 parameter basis can both minimise the length of the burn-in period and ensure
1350 that the correlation between step \vec{x}_i and \vec{x}_j is sufficiently small.

1351 The effect of changing the width of the proposal function is highlighted in
1352 Figure 4.3. Three scenarios, each with the same underlying stationary distribution
1353 (A Gaussian of width 1.0 and mean 0.), are presented. The only difference between
1354 the three scenarios is the width of the proposal function, colloquially known as
1355 the ‘step size σ ’. Each scenario starts at an initial parameter value of 10.0 which
1356 would be considered an extreme variation. For the case where $\sigma = 0.1$, it is
1357 clear to see that the chain takes a long time to reach the expected region of the
1358 parameter. This indicates that this chain would have a large burn-in period and
1359 does not converge to the stationary distribution until step ~ 500 . Furthermore,
1360 whilst the chain does move towards the expected region, each step is significantly
1361 correlated with the previous. Considering the case where $\sigma = 5.0$, the chain
1362 approaches the expected parameter region almost instantly meaning that the
1363 burn-in period is not significant. However, there are clearly large regions of steps
1364 where the chain does not move. This is likely due to the chain proposing steps
1365 in the tails of the distribution which have a low probability of being accepted.
1366 Consequently, this chain would take a significant number of steps to fully span
1367 the allowable parameter region. For the final scenario, where $\sigma = 0.5$, you can
1368 see a relatively small burn-in period of approximately 100 steps. Once the chain
1369 reaches the stationary distribution, it moves throughout the expected region of
1370 parameter values many times, sufficiently sampling the full parameter region.
1371 This example is a single parameter varying across a continuous distribution and
1372 does not fully reflect the difficulties in the many-hundred multi-variate parameter

1373 distribution used within this analysis. However, it does give a conceptual idea of
1374 the importance of selecting the proposal function and associated step size.

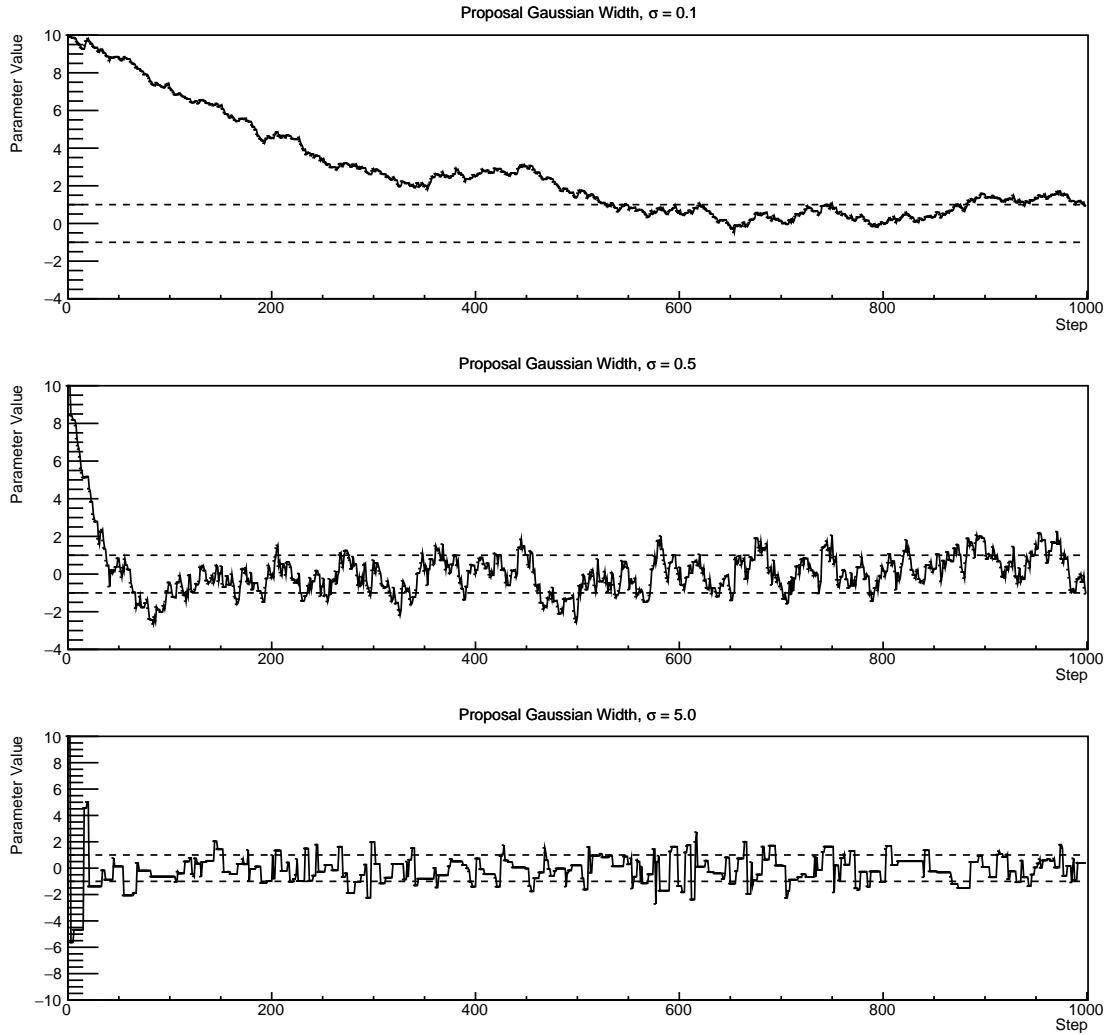


Figure 4.3: Three MCMC chains, each with a stationary distribution equal to a Gaussian centered at 0 and width 1 (As indicated by the black dotted lines). All of the chains use a Gaussian proposal function but have different widths (or ‘step size σ ’). The top panel has $\sigma = 0.1$, middle panel has $\sigma = 0.5$ and the bottom panel has $\sigma = 5.0$.

1375 As discussed, step size tuning directly correlates to the average step accep-
1376 tance rate. If the step size is too small, many steps will be accepted but the
1377 chain moves slowly. If the opposite is true, many steps will be rejected as the
1378 chain proposes steps in the tails of the distribution. Discussion in [133] suggests
1379 that the ‘ideal’ acceptance rate of a high dimension MCMC chain should be

1380 approximately $\sim 25\%$. An “ideal” step size [133] of

$$\sigma = \frac{2.4}{N_p}, \quad (4.6)$$

1381 where N_p is the number of parameters included in the MCMC fit. However,
1382 the complex correlations between systematics mean that some parameters have
1383 to be hand-tuned and many efforts have been taken to select a set of parameter-
1384 by-parameter step sizes to approximately reach the ideal acceptance rate.

1385 Figure 4.4 highlights the likelihood as calculated by the fit in subsection 8.3.4
1386 as a function of the number of steps in each chain. In practice, many independent
1387 MCMC chains are run simultaneously to parallelise the task of performing the
1388 fit. This figure overlays the distribution found in each chain. As seen, the
1389 likelihood decreases from its initial value and converges towards a stationary
1390 distribution after $\sim 1 \times 10^5$ steps.

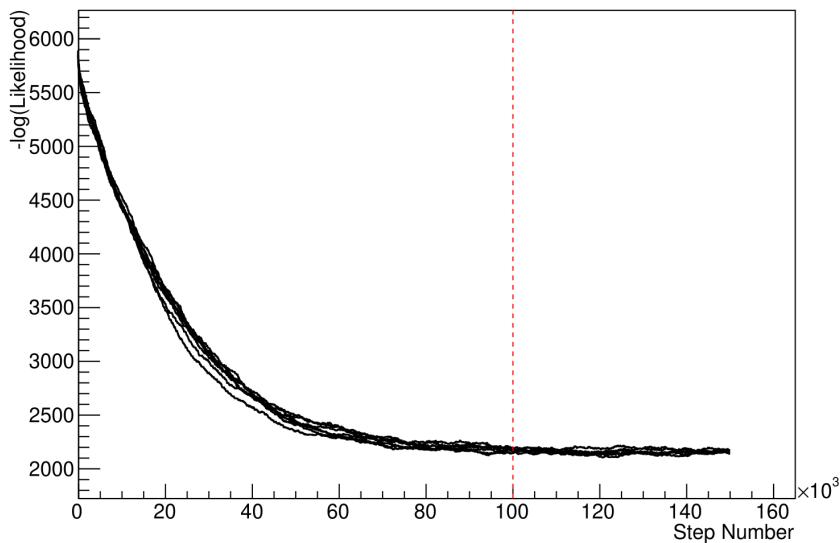


Figure 4.4: The log-likelihood from the fit detailed in subsection 8.3.4 as a function of the number of steps accumulated in each fit. Many independent MCMC chains were run in parallel and overlaid on this plot. The red line indicates the 1×10^5 step burn-in period after which the log-likelihood becomes stable.

1391 Multiple configurations of this analysis have been performed throughout this
1392 thesis where different samples or systematics have been used. For all of these con-
1393 figurations, it was found that a burnin period of 1×10^5 was sufficient in all cases.

1394 4.3 Understanding the MCMC Results

1395 The previous sections have described how to generate the posterior probability
1396 distribution using Bayesian MCMC techniques. However, this analysis focuses
1397 on oscillation parameter determination. The posterior distribution output from
1398 the chain is a high-dimension object, with as many dimensions as there are
1399 parameters included in the oscillation analysis. However, this multi-dimensional
1400 object is difficult to conceptualize so parameter estimations are often presented
1401 in one or two-dimensional projections of this probability distribution. To do
1402 this, marginalisation techniques are invoked.

1403 4.3.1 Marginalisation

1404 The output of the MCMC chain is a highly dimensional probability distribution
1405 which is very difficult to interpret. From the standpoint of an oscillation analysis
1406 experiment, the one or two-dimensional ‘projections’ of the oscillation parameters
1407 of interest are most relevant. Despite this, the best fit values and uncertainties on
1408 the oscillation parameters of interest should correctly encapsulate the correlations
1409 to the other systematic uncertainties (colloquially called ‘nuisance’ parameters).
1410 For this joint beam and atmospheric analysis, the oscillation parameters of
1411 interest are $\sin^2(\theta_{23})$, $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . All other parameters (includ-
1412 ing the oscillation parameters this fit is insensitive to) are deemed nuisance
1413 parameters. To generate these projections, the posterior distribution is integrated
1414 over all nuisance parameters. This is called marginalisation. This technique
1415 also explains why it is acceptable to neglect the normalisation constant of the
1416 posterior distribution, which was discussed in section 4.1.

1417 A simple example of the marginalisation technique is to imagine the scenario
1418 where two coins are flipped. To determine the probability that the first coin
1419 returned a ‘head’, the exact result of the second coin flip is disregarded and
1420 simply integrated over. For the parameters of interest, $\vec{\theta}_i$, the marginalised

¹⁴²¹ posterior is calculated by integrating over the nuisance parameters, $\vec{\theta}_n$. In this
¹⁴²² case, Equation 4.2 becomes

$$P(\vec{\theta}_i|D) = \frac{\int P(D|\vec{\theta}_i, \vec{\theta}_n)P(\vec{\theta}_i, \vec{\theta}_n)d\vec{\theta}_n}{\int P(D|\vec{\theta})P(\vec{\theta})d\vec{\theta}}. \quad (4.7)$$

¹⁴²³ Where $P(\vec{\theta}_i, \vec{\theta}_n)$ encodes the prior knowledge about the uncertainty and
¹⁴²⁴ correlations between the parameters of interest and the nuisance parameters.
¹⁴²⁵ In practice, this is simply taking the one or two-dimensional projection of the
¹⁴²⁶ multi-dimensional probability distribution.

¹⁴²⁷ While in principle an easy solution to a complex problem, correlations be-
¹⁴²⁸ tween the interesting and nuisance parameters can bias the marginalised results.
¹⁴²⁹ A similar effect is found when the parameters being marginalised over have
¹⁴³⁰ non-Gaussian probability distributions. For example, Figure 4.5 highlights the
¹⁴³¹ marginalisation bias in the probability distribution found for a parameter when
¹⁴³² requiring a correlated parameter to have a positive parameter value. Due to
¹⁴³³ the complex nature of the oscillation parameter fit presented in this thesis, there
¹⁴³⁴ are correlations occurring between the oscillation parameters of interest and the
¹⁴³⁵ other nuisance parameters included in the fit.

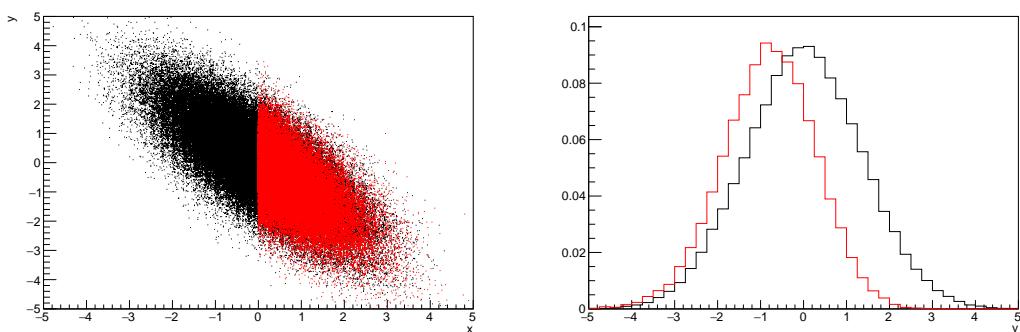


Figure 4.5: Left: The two-dimensional probability distribution for two correlated parameters x and y . The red distribution shows the two-dimensional probability distribution when $0 \leq x \leq 5$. Right: The marginalised probability distribution for the y parameter found when requiring the x to be bound between $-5 \leq x \leq 5$ and $0 \leq x \leq 5$ for the black and red distribution, respectively.

1436 4.3.2 Parameter Estimation and Credible Intervals

1437 The purpose of this analysis is to determine the best fit values for the oscillation
 1438 parameters that the beam and atmospheric samples are sensitive to: $\sin^2(\theta_{23})$,
 1439 $\sin^2(\theta_{13})$, Δm_{32}^2 , and δ_{CP} . The posterior probability density, taken from the output
 1440 MCMC chain, is binned in these parameters. The parameter best-fit point is then
 1441 taken to be the value that has the highest posterior probability. This is performed
 1442 in both one and two-dimensional projections.

1443 However, the single best-fit point in a given parameter is not of much use on its
 1444 own. The uncertainty on the best-fit point must also be presented using credible
 1445 intervals. The definition of the 1σ credible interval is that there is 68% belief
 1446 that the parameter is within those bounds. For a more generalised definition,
 1447 the credible interval is the region, R , of the posterior distribution that contains
 1448 a specific fraction of the total probability, such that

$$\int_R P(\theta|D)d\theta = \alpha. \quad (4.8)$$

1449 Where θ is the parameter being evaluated. This technique then calculates
 1450 the $\alpha \times 100\%$ credible interval.

1451 In practice, this analysis uses the highest posterior density (HPD) credible in-
 1452 tervals which are calculated through the following method. First, the probability
 1453 distribution is area-normalised such that it has an integrated area equal to 1.0.
 1454 The bins of probability are then summed from the highest to lowest until the sum
 1455 exceeds the 1σ level (0.68 in this example). This process is repeated for a range of
 1456 credible intervals, notably the 1σ , 2σ and 3σ along with other levels where the
 1457 critical values for each level can be found in [76]. This process can be repeated
 1458 for the two-dimensional probability distributions by creating two-dimensional
 1459 contours of credible intervals rather than a one-dimensional result.

4.3.3 Bayesian Model Comparisons

Due to the matter resonance, this analysis has some sensitivity to the mass hierarchy of neutrino states (whether Δm_{32}^2 is positive or negative) and the octant of $\sin^2(\theta_{23})$. The Bayesian approach utilised within this analysis gives an intuitive method of model comparison by determining which hypothesis is most favourable. Taking the ratio of Equation 4.3 for the two hypotheses of normal hierarchy, NH , and inverted hierarchy, IH , gives

$$\frac{P(\vec{\theta}_{NH}|D)}{P(\vec{\theta}_{IH}|D)} = \frac{P(D|\vec{\theta}_{NH})}{P(D|\vec{\theta}_{IH})} \times \frac{P(\vec{\theta}_{NH})}{P(\vec{\theta}_{IH})}. \quad (4.9)$$

The middle term defines the Bayes factor, $B(NH/IH)$, which is a data-driven interpretation of how strong the data prefers one hierarchy to the other. For this analysis, equal priors on both mass hierarchy hypotheses are chosen ($P(\vec{\theta}_{NH}) = P(\vec{\theta}_{IH}) = 0.5$). In practice, the MCMC chain proposes a value of $|\Delta m_{32}^2|$ and then applies a 50% probability that the value is sign flipped. Consequently, the Bayes factor can be calculated from the ratio of the probability density in either hypothesis. This equates to counting the number of steps taken in the normal and inverted hierarchies and taking the ratio. The same approach can be taken to compare the upper octant (UO) compared to the lower octant (LO) hypothesis of $\sin^2(\theta_{23})$.

$\log_{10}(B_{AB})$	B_{AB}	Strength of Preference
< 0.0	< 1	No preference for hypothesis A (Supports hypothesis B)
0.0 – 0.5	1.0 – 3.16	Preference for hypothesis A is weak
0.5 – 1.0	3.16 – 10.0	Preference for hypothesis A is substantial
1.0 – 1.5	10.0 – 31.6	Preference for hypothesis A is strong
1.5 – 2.0	31.6 – 100.0	Preference for hypothesis A is very strong
> 2.0	> 100.0	Decisive preference for hypothesis A

Table 4.1: Jeffreys scale for strength of preference for two models A and B as a function of the calculated Bayes factor ($B_{AB} = B(A/B)$) between the two models [134]. The original scale is given in terms of $\log_{10}(B(A/B))$ but converted to linear scale for easy comparison throughout this thesis.

Whilst the value of the Bayes factor should always be shown, the Jeffreys scale [134] (highlighted in Table 4.1) gives an indication of the strength of preference

1479 for one model compared to the other. Other interpretations of the strength of
1480 preference of a model exist, e.g. the Kass and Raferty Scale [135].

1481 4.3.4 Comparison of MCMC Output to Expectation

1482 To ensure the fit is performing well, a best-fit spectrum is produced using the
1483 posterior probability distribution and compared with the data, allowing easy
1484 by-eye comparisons to be made. A simple method of doing this is to perform a
1485 comparison in the fitting parameters (e.g. the reconstructed neutrino energy for
1486 T2K far detector beam samples) of the spectra generated by the MCMC chain to
1487 ‘data’. This ‘data’ could be true data or some variation of Monte Carlo prediction.
1488 This allows easy comparison of the MCMC probability distribution to the data. To
1489 perform this, N steps from the post-burnin MCMC chain are randomly selected.
1490 From these, the Monte Carlo prediction at each step is generated by reweighting
1491 the model parameters to the values specified at that step. Due to the probability
1492 density being directly correlated with the density of steps in a certain region,
1493 parameter values close to the best fit value are most likely to be selected.

1494 In practice, for each bin of the fitting parameters has a probability distribution
1495 of event rates, with one entry per sampled MCMC step. This distribution is
1496 binned where the bin with the highest probability is selected as the mean and an
1497 error on the width of this probability distribution is calculated using the approach
1498 highlighted in subsection 4.3.2. Consequently, the best fit distribution in the fit
1499 parameter is not necessarily that which would be attained by reweighting the
1500 Monte Carlo prediction to the most probable parameter values.

1501 A similar study can be performed to illustrate the freedom of the model
1502 parameter space prior to the fit. This can be done by throwing parameter values
1503 from the prior uncertainty of each parameter.

5

1504

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1506

Simulation, Reconstruction, and Event Reduction

1507 As a crucial part of the oscillation analysis, an accurate prediction of the expected
1508 neutrino spectrum at the far detector is required. This includes modeling the
1509 flux generation, neutrino interactions, and detector effects. All of the simulation
1510 packages required to do this are briefly described in section 5.1. The reconstruc-
1511 tion of neutrino events in the far detector, including the `fitQun` algorithm, is
1512 documented in section 5.2. This also includes data quality checks of the SK-V
1513 data which the author performed for the T2K oscillation analysis presented at the
1514 Neutrino 2020 conference [2]. Finally, section 5.3 describes the steps taken in the
1515 SK detector to trigger on events of interest whilst removing the comparatively
1516 large rate of cosmic ray muon events.

1517 5.1 Simulation

1518 In order to generate a Monte Carlo prediction of the expected event rate at
1519 the far detector, all the processes in the beam and atmospheric fluxes, neutrino
1520 interaction, and detector need to be modeled.

5.1.1 Neutrino Flux

The beamline simulation consists of three distinct parts: the initial hadron interaction modeled by FLUKA [136], the target station geometry and particle tracking performed by JNUBEAM, [41, 137] and any hadronic re-interactions simulated by GCALOR [138]. The primary hadronic interactions are $O(10)\text{GeV}$, where FLUKA matches external cross-section data better than GCALOR [139]. However, FLUKA is not very adaptable so a small simulation is built to model the interactions in the target and the output is then passed to JNUBEAM and GCALOR for propagation. The hadronic interactions are tuned to data from the NA61/SHINE [140–142] and HARP [143] experiments. The tuning is done by reweighting the FLUKA and GCALOR predictions to match the external data multiplicity and cross-section measurements, based on final state particle kinematics [139]. The culmination of this simulation package generates the predicted flux for neutrino and antineutrino beam modes which are illustrated in Figure 3.7.

The atmospheric neutrino flux is simulated by the HKKM model [53, 55]. The primary cosmic ray flux is tuned to AMS [144] and BESS [145] data assuming the US-standard atmosphere '76 [146] density profile and includes geomagnetic field effects. The primary cosmic rays interact to generate pions and muons. The interaction of these secondary particles to generate neutrinos is handled by DPMJET-III [147] for energies above 32GeV and JAM [55, 148] for energies below that value [51]. These hadronic interactions are tuned to BESS and L3 data [149, 150] using the same methodology as the tuning of the beamline simulation. The energy and cosine zenith predictions of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ flux are given in Figure 2.3 and Figure 2.5, respectively. The flux is approximately symmetrical and peaked around the horizon ($\cos(\theta_Z) = 0.0$). This is because horizontally-going pions and kaons can travel further than their vertically-going counterparts resulting in a larger probability of decaying to neutrinos. The symmetry is broken in lower-energy neutrinos due to geomagnetic effects, which modify the track of the primary cosmic rays. Updates to the HKKM model are currently ongoing [151].

1550 5.1.2 Neutrino Interaction

1551 Once a flux prediction has been made for all three detectors, NEUT 5.4.0 [152, 153]
 1552 models the interactions of the neutrinos in the detectors. For the purposes of this
 1553 analysis, quasi-elastic (QE), meson exchange (MEC), single meson production
 1554 (PROD), coherent pion production (COH), and deep inelastic scattering (DIS)
 1555 interactions are simulated. These interaction categories can be further broken
 1556 down by whether they were propagated via a W^\pm boson in Charged Current
 1557 (CC) interactions or via a Z^0 boson in Neutral Current (NC) interactions. CC
 1558 interactions have a charged lepton in the final state, which can be flavour-tagged
 1559 in reconstruction to determine the flavour of the neutrino. In contrast, NC
 1560 interactions have a neutrino in the final state so no flavour information can be
 1561 determined from the observables left in the detector after an interaction. This
 1562 is the reason why neutrinos that interact through NC modes are assumed to
 1563 not oscillate within this analysis. Both CC and NC interactions are modeled
 1564 for all the above interaction categories, other than MEC interactions which are
 1565 only modeled for CC events.

1566 As illustrated in Figure 5.1, CCQE interactions dominate the cross-section of
 1567 neutrino interactions around $E_\nu \sim 0.5\text{GeV}$. The NEUT implementation adopts
 1568 the Llewellyn Smith [154] model for neutrino-nucleus interactions, where the
 1569 nuclear ground state of any bound nucleons (neutrino-oxygen interactions) is
 1570 approximated by a spectral-function [155] model that simulates the effects of
 1571 Fermi momentum and Pauli blocking. The cross-section of QE interactions is
 1572 controlled by vector and axial-vector form factors parameterised by the BBBA05
 1573 [156] model and a dipole form factor with $M_A^{QE} = 1.21\text{GeV}$ fit to external data
 1574 [157], respectively. NEUT implements the Valencia [158] model to simulate MEC
 1575 events, where two nucleons and two holes in the nuclear target are produced
 1576 (often called 2p2h interactions).

1577 For neutrinos of energy $O(1)\text{GeV}$, PROD interactions become dominant.
 1578 These predominantly produce charged and neutral pions although γ , kaon,

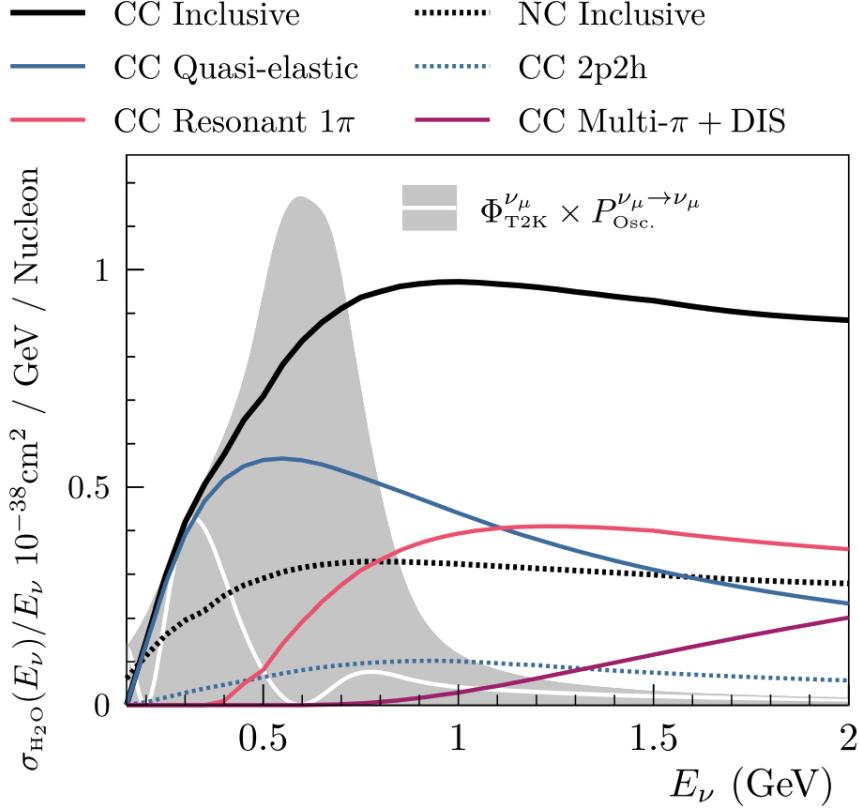


Figure 5.1: The NEUT prediction of the ν_μ -H₂O cross-section overlaid on the T2K ν_μ flux. The charged current (black, solid) and neutral current (black, dashed) inclusive, charged current quasi-elastic (blue, solid), charged current 2p2h (blue, dashed), charged current single pion production (pink), and charged current multi- π and DIS (Purple) cross-sections are illustrated. Figure taken from [152].

and η production is also considered. To simulate these interactions, the Berger-Sehgal [159] model is implemented within NEUT. It simulates the excitation of a nucleon from a neutrino interaction, production of an intermediate baryon, and the subsequent decay to a single meson or γ . Pions can also be produced through COH interactions, which occur when the incoming neutrino interacts with the entire oxygen nucleus leaving a single pion outside of the nucleus. NEUT utilises the Berger-Sehgal [160] model to simulate these COH interactions.

DIS and multi- π producing interactions become the most dominant for energies $> O(5)\text{GeV}$. PYTHIA [161] is used to simulate any interaction with invariant mass $W > 2\text{GeV}/c^2$, which produces at least one meson. For any interaction which produces at least two mesons but has $W < 2\text{GeV}/c^2$, the

₁₅₉₀ Bronner model is used [162]. Both of these models use Parton distribution
₁₅₉₁ functions based on the Bodek-Yang model [163–165].

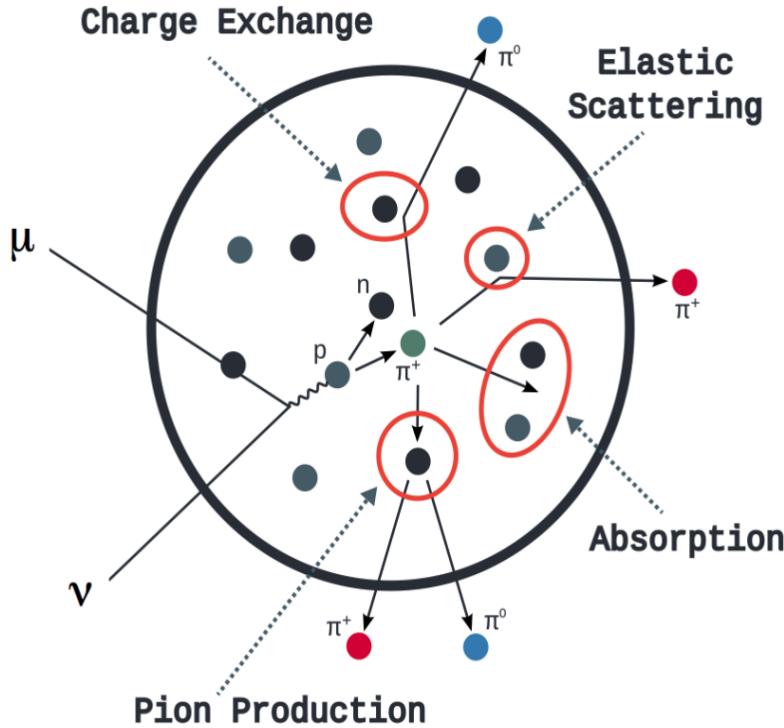


Figure 5.2: Illustration of the various processes which a pion can undergo before exiting the nucleus. Taken from [166].

₁₅₉₂ Any pion that is produced within the nucleus can re-interact through final
₁₅₉₃ state interactions before it exits, as illustrated by the scattering, absorption,
₁₅₉₄ production, and exchange interactions in Figure 5.2. These re-interactions alter
₁₅₉₅ the observable particles within the detector. For instance, if the charged pion
₁₅₉₆ from a CC PROD interaction is absorbed, the observables would mimic a CC QE
₁₅₉₇ interaction. To simulate these effects, NEUT uses a semi-classical intranuclear
₁₅₉₈ cascade model [152]. This cascade functions by stepping the pion through the
₁₅₉₉ nucleus in fixed-length steps equivalent to $dx = R_N/100$, where R_N is the radius
₁₆₀₀ of the nucleus. At each step, the simulation allows the pion to interact through
₁₆₀₁ scattering, charged exchange, absorption, or production with an interaction-
₁₆₀₂ dependent probability calculated from a fit to external data [167]. This cascade
₁₆₀₃ continues until the pion is absorbed or exits the nucleus.

1604 5.1.3 Detector

1605 Once the final state particle kinematics have been determined by NEUT, they
1606 are passed into the detector simulation. The near detectors, ND280 and INGRID,
1607 are simulated using a GEANT4 package [45, 168] to simulate the detector geom-
1608 etry, particle tracking, and energy deposition. The response of the detectors is
1609 simulated using the elecSim package [45].

1610 The far detector simulation is based upon the original Kamiokande experi-
1611 ment software which uses the GEANT3-based SKDETSIM [45, 169] package. This
1612 simulates the interactions of particles in the water as well as Cherenkov light
1613 production. The water quality and PMT calibration measurements detailed in
1614 subsection 3.1.2 are also used within this simulation to make accurate predictions
1615 of the detector response.

1616 Any event which generates optical photons that occurs in SK will be observed
1617 by the PMT array, where each PMT records the time and accumulated charge.
1618 This recorded information is shown in event displays similar to those illustrated
1619 in Figure 5.3 for simulated Monte Carlo events. To be useful for physics analyses,
1620 this series of PMT hit information needs to be reconstructed to determine the
1621 number and identity of particles and their kinematics (or track parameters): four-
1622 vertex, direction, and momentum. The reconstruction uses the fact that the charge
1623 and timing distribution of photons generated by a particular particle in an event is
1624 dependent upon its initial kinematics. Electron and muon rings are distinguished
1625 by their “fuzziness”. Muons are heavier and less affected by scattering or
1626 showering meaning they typically produce “crisp” rings. Electrons are more
1627 likely to interact via electromagnetic showering or scattering which results in
1628 larger variations of their direction from the initial direction. Consequently,
1629 electrons typically produce “fuzzier” rings compared to muons.

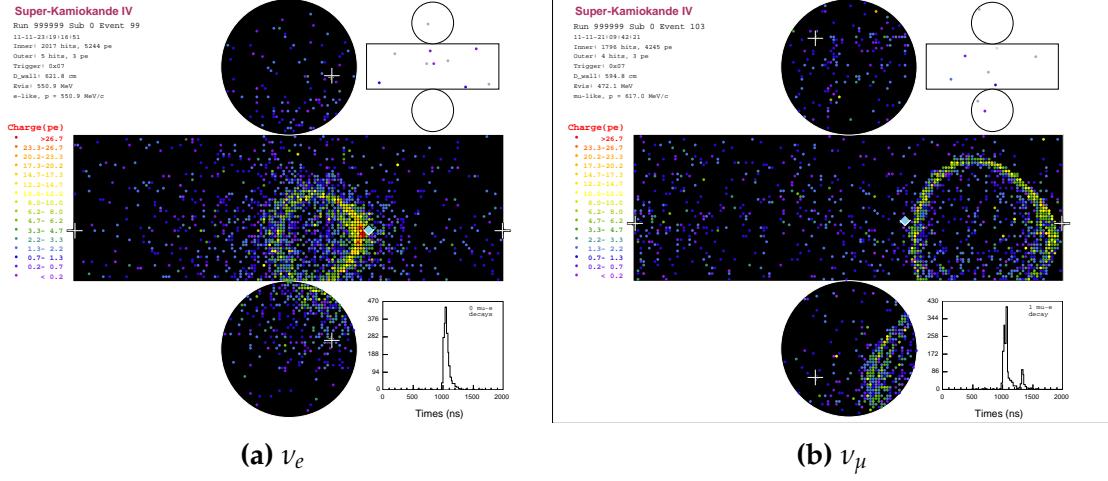


Figure 5.3: Event displays from Monte Carlo simulation at Super Kamiokande illustrating the “crisp” ring from a muon and the typically “fuzzy” electron ring. Each pixel represents a PMT and the color scheme denotes the accumulated charge deposited on that PMT. Figures taken from [170].

5.2 Event Reconstruction at SK

For the purposes of this analysis, the `fitQun` reconstruction algorithm [171] is utilised. Its core function is to compare a prediction of the accumulated charged and timing distribution from each PMT, generated for a particular particle identity and track parameters, to that observed in the neutrino event. It determines the preferred values by maximising a likelihood function (or minimising a log-likelihood function) which includes information from PMTs which were hit and those that were not hit. The `fitQun` algorithm is based on the key concepts of the MiniBooNE reconstruction algorithm [172].

The `fitQun` algorithm improves upon the previous `APFit` algorithm [173] which has been used for many previous SK analyses. `APFit` fits the vertex from timing information and then fits the direction of the particle from PMT hits within a 43 deg Cherenkov cone (assuming an ultra-relativistic particle) using a fitting estimator. A Hough transformation is used to find the radius of a ring (related to the momentum through Equation 3.2) as well as the number of rings contained within the event. The analysis presented here uses the `fitQun` algorithm as it improves both the accuracy of the fit parameters and the rejection of neutral

1647 current π^0 events as compared to APFit [174, 175].

1648 Any event in SK can consist of prompt (or primary) and decay (or secondary)
1649 particles. For example, a charged current muon neutrino interaction can gen-
1650 erate two particles that have the potential of generating Cherenkov photons
1651 (assuming the proton is below the Cherenkov threshold): the prompt muon,
1652 and the secondary decay-electron from the muon, approximately $2\mu\text{s}$ later. To
1653 reconstruct all particles within an event, it is divided into time clusters which are
1654 called “subevents”. Subevents after the primary subevent are considered to
1655 be decay electrons.

1656 The main steps of the `fitQun` reconstruction algorithm are:

- 1657 • **Vertex pre-fitting:** An estimate of the vertex is made using a goodness-of-fit
1658 metric based on PMT hit times
- 1659 • **Peak finding:** The initial time of each subevent is determined by clustering
1660 events by time residuals
- 1661 • **Single-ring fits:** Given the pre-fit vertex and estimated time of interaction,
1662 a maximum likelihood technique searches for a single particle generating
1663 light. Electron, muon, charged pion, and proton hypotheses are considered
- 1664 • **Multi-ring fits:** Seeded from the single-ring fits, hypotheses with multiple
1665 light-producing particles are considered using the same maximum likeli-
1666 hood technique. Electron-like or charged pion-like rings are added until
1667 the likelihood stops improving

1668 To find all the subevents in an event, a vertex goodness metric is calculated
1669 for some vertex position \vec{x} and time t ,

$$G(\vec{x}, t) = \sum_i^{\text{hit PMTs}} \exp \left(-\frac{1}{2} \left(\frac{T_{Res}^i(\vec{x}, t)}{\sigma} \right)^2 \right), \quad (5.1)$$

1670 where

$$T_{Res}^i(\vec{x}, t) = t^i - t - |R_{PMT}^i - \vec{x}| / c_n, \quad (5.2)$$

1671 is the residual hit time. It is the difference in time between the PMT hit time
 1672 t^i , of the i^{th} PMT, and the expected time of the PMT hit if the photon was at
 1673 the vertex. R_{PMT}^i is the position of the i^{th} PMT, c_n is the speed of light in water
 1674 and $\sigma = 4\text{ns}$ which is comparable to the time resolution of the PMT. When the
 1675 proposed fit values of time and vertex are close to the true values, $T_{Res}^i(\vec{x}, t)$ tends
 1676 to zero resulting in subevents appearing as spikes in the goodness metric. The
 1677 proposed fit vertex and time are grid-scanned, and the values which maximise
 1678 the goodness metric are selected as the “pre-fit vertex”. Whilst this predicts a
 1679 vertex for use in the clustering algorithm, the final vertex is fit using the higher-
 1680 precision maximum likelihood method described below.

1681 Once the pre-fit vertex has been determined, the goodness metric is scanned as
 1682 a function of t to determine the number of subevents. A peak-finding algorithm
 1683 is then used on the goodness metric, requiring the goodness metric to exceed
 1684 some threshold and drop below a reduced threshold before any subsequent
 1685 additional peaks are considered. The thresholds are set such that the rate of
 1686 false peak finding is minimised while still attaining good data to Monte Carlo
 1687 agreement. To improve performance, the pre-fit vertex for each delayed subevent
 1688 is re-calculated after PMT hits from the previous subevent are masked. This
 1689 improves the decay-electron tagging performance. Once all subevents have
 1690 been determined, the time window around each subevent is then defined by the
 1691 earliest and latest time which satisfies $-180 < T_{Res}^i < 800\text{ns}$. The subevents and
 1692 associated time windows are then used as seeds for further reconstruction.

1693 For a given subevent, the `fitQun` algorithm constructs a likelihood based on
 1694 the accumulated charge q_i and time information t_i from the i^{th} PMT,

$$L(\Gamma, \vec{\theta}) = \prod_j^{\text{unhit}} P_j(\text{unhit}|\Gamma, \vec{\theta}) \prod_i^{\text{hit}} \{1 - P_i(\text{unhit}|\Gamma, \vec{\theta})\} f_q(q_i|\Gamma, \vec{\theta}) f_t(t_i|\Gamma, \vec{\theta}). \quad (5.3)$$

1695 Where $\vec{\theta}$ defines the track parameters; vertex position, direction vector and
 1696 momenta, and Γ represents the particle hypothesis. $P_i(\text{unhit}|\Gamma, \vec{\theta})$ is the proba-
 1697 bility of the i^{th} tube to not register a hit given the track parameters and particle
 1698 hypothesis. The charge likelihood, $f_q(q_i|\Gamma, \vec{\theta})$, and time likelihood, $f_t(t_i|\Gamma, \vec{\theta})$,
 1699 represents the probability density function of observing charge q_i and time t_i on
 1700 the i^{th} PMT given the specified track parameters and particle hypothesis.

1701 The predicted charge is calculated based on contributions from both the
 1702 direct light and the scattered light. The direct light contribution is determined
 1703 based on the integration of the Cherenkov photon profile along the track. PMT
 1704 angular acceptance, water quality, and calibration measurements discussed in
 1705 subsection 3.1.2 are included to accurately predict the charge probability density
 1706 at each PMT. The scattered and reflected light is calculated in a similar way,
 1707 although it includes a scattering function that depends on the vertex of the
 1708 particle and the position of the PMT. The charge likelihood is calculated by
 1709 comparing the prediction to the observed charge in the PMT which is tuned
 1710 to the PMT simulation.

1711 The time likelihood is approximated to depend on the vertex \vec{x} , direction \vec{d} ,
 1712 and time t of the track as well as the particle hypothesis. The expected time
 1713 for PMT hits is calculated by assuming unscattered photons being emitted from
 1714 the midpoint of the track, S_{mid} ,

$$t_{\text{exp}}^i = t + S_{\text{mid}}/c + |R_{\text{PMT}}^i - \vec{x} - S_{\text{mid}}\vec{d}|/c_n, \quad (5.4)$$

1715 where c is the speed of light in a vacuum. The time likelihood is then expressed
 1716 in terms of the residual difference between the PMT hit time and the expected
 1717 hit time, $t_{\text{Res}}^i = t^i - t_{\text{exp}}^i$. The particle hypothesis and momentum also affect the
 1718 Cherenkov photon distribution. These parameters modify the shape of the time
 1719 likelihood density since in reality not all photons are emitted at the midpoint of
 1720 the track. As with the charge likelihood, the contributions from both the direct
 1721 and scattered light to the time likelihood density are calculated separately, which
 1722 are both calculated from particle gun Monte Carlo studies.

The track parameters and particle identity which maximise $L(\Gamma, \vec{\theta})$ are defined as the best-fit parameters. In practice MINUIT [176] is used to minimise the value of $-\ln L(\Gamma, \vec{\theta})$. The `fiTQun` algorithm considers an electron-like, muon-like, and charged pion-like hypothesis for events with a single final state particle, denoted “single-ring events”. The particle’s identity is determined by taking the ratio of the likelihood of each of the hypotheses. For instance, electrons and muons are distinguished by considering the value of $\ln(L(e, \vec{\theta}_e)/L(\mu, \vec{\theta}_\mu))$ in comparison to the reconstructed momentum of the electron hypothesis, as illustrated by Figure 5.4. The coefficients of the discriminator between electron-like and muon-like events are determined from Monte Carlo studies [171]. Similar distributions exist for distinguishing electron-like events from π^0 -like events, and muon-like events from pion-like events. The cuts are defined as,

$$\begin{aligned} \text{Electron/Muon} : & \ln(L_e/L_\mu) > 0.2 \times p_e^{rec} [\text{MeV}], \\ \text{Electron}/\pi^0 : & \ln(L_e/L_{\pi^0}) < 175 - 0.875 \times m_{\gamma\gamma} [\text{MeV}], \\ \text{Muon/Pion} : & \ln(L_\mu/L_{\pi^\pm}) < 0.15 \times p_\mu^{rec} [\text{MeV}], \end{aligned} \quad (5.5)$$

as taken from [177], where p_e^{rec} and p_μ^{rec} are the reconstructed momentum of the single-ring electron and muon fits, respectively. $m_{\gamma\gamma}$ represents the reconstructed invariant mass of the two photons emitted from π^0 decay. Typically, the distance between a particular entry in these two-dimensional distributions and the cut-line is termed the PID parameter and is illustrated in Figure 5.5.

The `fiTQun` algorithm also considers a π^0 hypothesis. To do this, it performs a fit looking for two standard electron-hypothesis tracks which point to the same four-vertex. This assumes the electron tracks are generated from photon-conversion so the electron tracks actually appear offset from the proposed π^0 vertex. For these fits, the conversion length, direction, and momentum of each photon are also considered as track parameters which are then fit in the same methodology as the standard single-ring hypotheses.

Whilst lower energy events are predominantly single-ring events, higher energy neutrino events can generate final states with multiple particles which

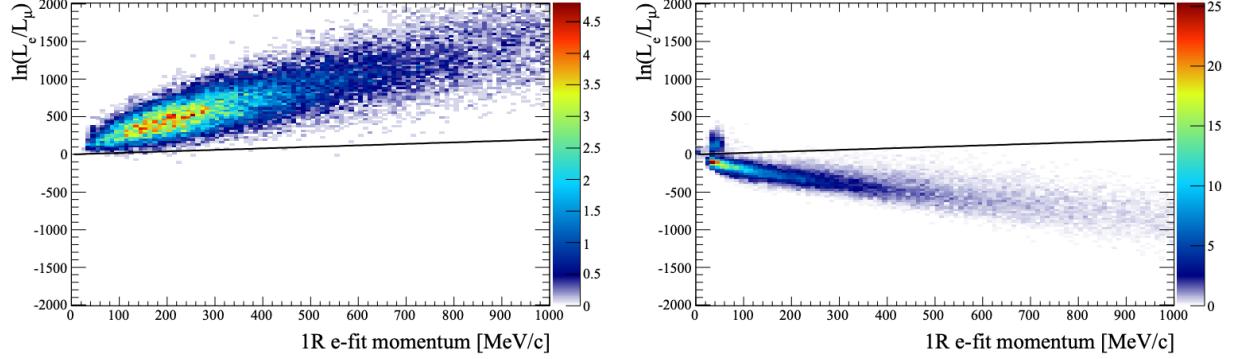


Figure 5.4: The difference of the electron-like and muon-like log-likelihood compared to the reconstructed single-ring fit momentum for atmospheric ν_e (left) and ν_μ (right) samples. The black line represents the cut used to discriminate electron-like and muon-like events, with coefficients obtained from Monte Carlo studies. Figures from [171].

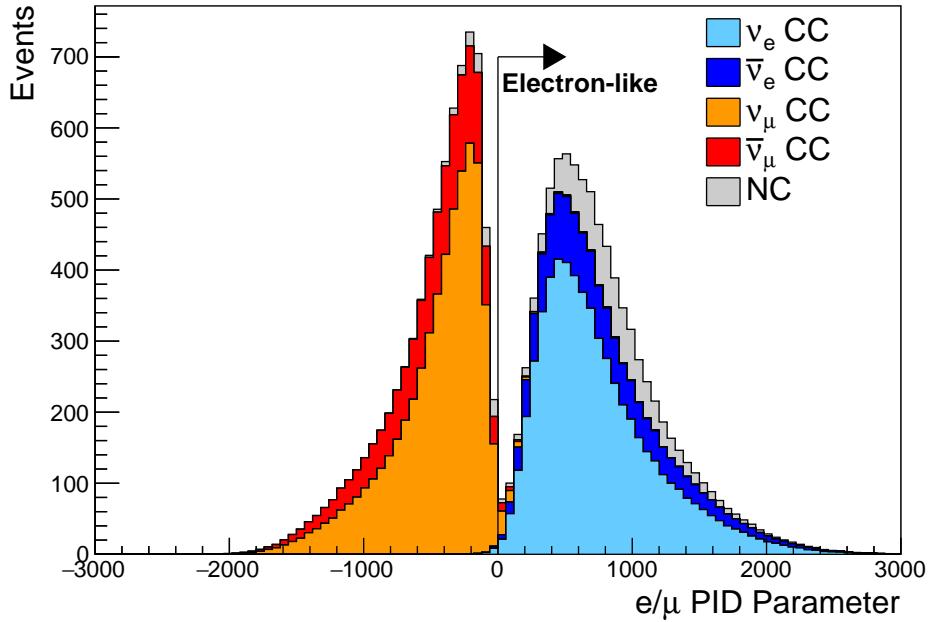


Figure 5.5: The electron/muon PID separation parameter for all sub-GeV single-ring events in SK-IV. The charged current (CC) component is broken down in four flavours of neutrino (ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$). Events with positive values of the parameter are determined to be electron-like.

1749 generate Cherenkov photons. These “multi-ring” hypotheses are also considered
 1750 in the `fitQun` algorithm. When calculating the charge likelihood density, the
 1751 predicted charge associated with each ring is calculated separately and then
 1752 summed to calculate the total accumulated charge on each PMT. Similarly, the
 1753 time likelihood for the multi-ring hypothesis is calculated assuming each ring

is independent. Each track is time-ordered based on the time of flight from the center of the track to the PMT and the direct light from any ring incident on the PMT is assumed to arrive before any scattered light. To reduce computational resource usage, the multi-ring fits only consider electron-like and charged pion-like rings as the pion fit can be used as a proxy for a muon fit due to their similar mass. Due to the pions ability to interact through the strong force, they are more likely to hard-scatter. That means a single charged pion can produce multiple rings in different directions. There is an additional freedom, the fraction of kinetic energy lost in a single ring segment, which is added into the `fiTQun` pion fit to cover this difference. Pion and muon rings are indistinguishable when this fraction tends to unity.

Multi-ring fits proceed by proposing another ring to the previous fit and then fitting the parameters in the method described above. Typically, multi-ring fits have the largest likelihood because of the additional degrees of freedom introduced. A likelihood value is calculated for the n -ring and $(n + 1)$ -ring hypotheses, where the additional ring is only included if the likelihood value is above 9.35, based on Monte Carlo studies in [178].

5.2.1 Validation of Reconstruction in SK-V

Understanding how the modelling of the detector conditions and stability effects the reconstruction is critical for ensuring accurate measurements. It is important to note that the detector systematics used in the 2020 T2K-only [2] oscillation analysis are determined using data-to-Monte Carlo comparisons of the SK-IV data [179]. Due to tank-open maintenance occurring between SK-IV and SK-V, the dark rate of each PMT was observed to increase in SK-V due to light exposure for a significant time during the repairs. This increase can be seen in Figure 5.6. Run-10 of the T2K experiment was conducted in the SK-V period, so the consistency of SK-IV and SK-V data needs to be studied to determine whether the SK-IV-defined systematics can be applied to the run-10 data. Consequently, the author of this thesis assessed the quality of `fiTQun` event reconstruction for SK-V data.

This comparison study was performed using the stopping muon data set for both the SK-IV and SK-V periods. This data sample is used due to the high rate of interactions ($O(200)$ events per hour) as well as having similar energies to muons from CCQE ν_μ interactions from beam interactions. The rate of cosmic muons does depend on the solar activity cycle [180] but has been neglected in this comparison study. This is because the shape of the distributions is most important for the purposes of being compared to the detector systematics. The SK-IV and SK-V data samples consist of 2398.42 and 626.719 hours of data which equates to 686k and 192k events respectively. These samples do not correspond to the full data sets of either period but do contain enough events to be systematics limited rather than statistics limited.

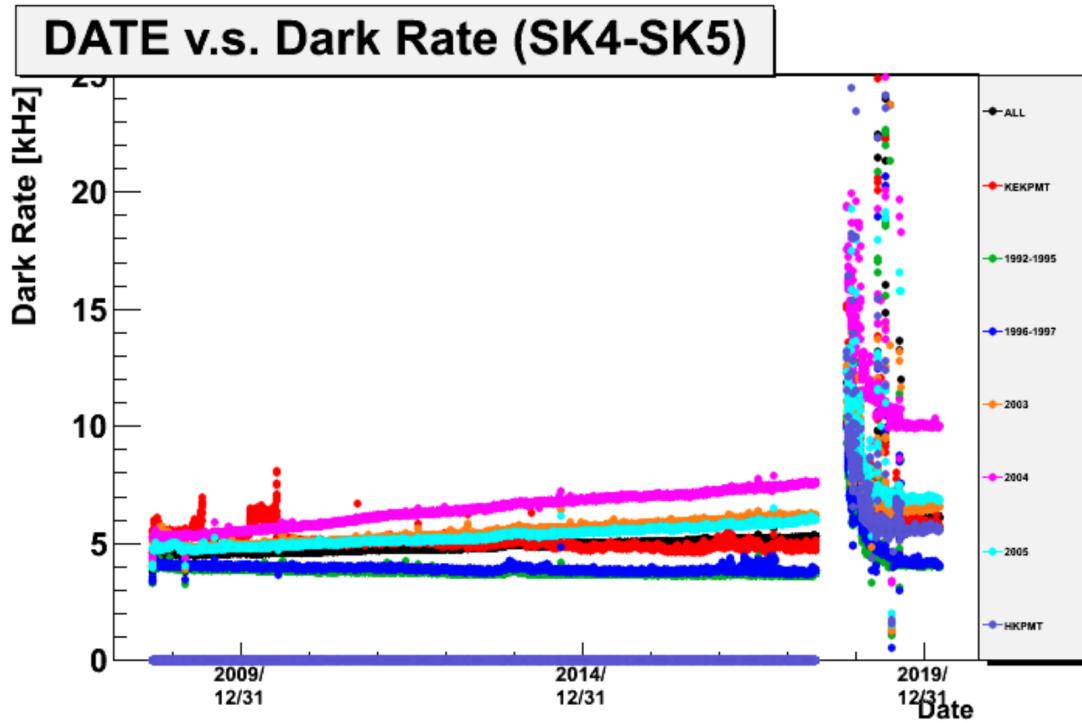


Figure 5.6: The variation of the measured dark rate as a function of date, broken down by PMT type. The SK-IV and SK-V periods span September 2008 to May 2018 and January 2019 to July 2020, respectively. The break in measurement in 2018 corresponds to the period of tank repair and refurbishment. Figure adapted from [179].

The predicted charge calculated in the `fitQun` algorithm includes a contribution from the photoelectron emission due to dark noise. Therefore, the increase

in the SK-V dark rate needs to be accounted for. In practice, the average dark rate in each SK period is calculated and used as an input in the reconstruction. This is calculated by averaging the dark rate per run for each period separately, using the calibration measurements detailed in subsection 3.1.2. The average dark rate from SK-IV and SK-V were found to be 4.57kHz and 6.30kHz, respectively. The charges associated with the muon and decay electron subevents are illustrated in Figure 5.7. The photoelectron emission from dark noise is more significant for events that have lower energy. This is because this contribution becomes more comparable to the number of photoelectrons emitted from incident photons in lower-energy events. This behaviour is observed in the data, where the charge deposited by the muon subevent is mostly unaffected by the increase in dark rate, whilst the charge associated with the decay-electron is clearly affected.

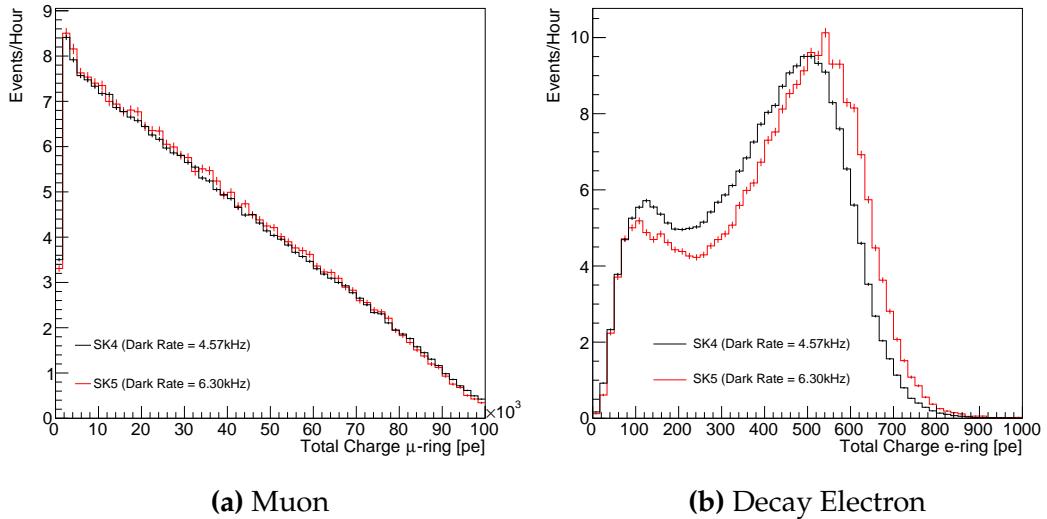


Figure 5.7: Comparison of the measured raw charge deposited per event from the stopping muon data samples between SK-IV (Blue) and SK-V (Red), split by the primary muon subevent (left) and the associated decay electron subevent (right).

The energy scale systematic is estimated from data-to-Monte Carlo differences in the stopping muon sample in [62] and found to be 2.1%. To determine the consistency of SK-IV and SK-V with respect to the energy scale systematic, the muon momentum distribution is compared between the two SK periods. As the total number of Cherenkov photons is integrated across the track length,

₁₈₁₃ the reconstructed momentum divided by track length (or range) is compared
₁₈₁₄ between SK-IV and SK-V as illustrated in Figure 5.8.

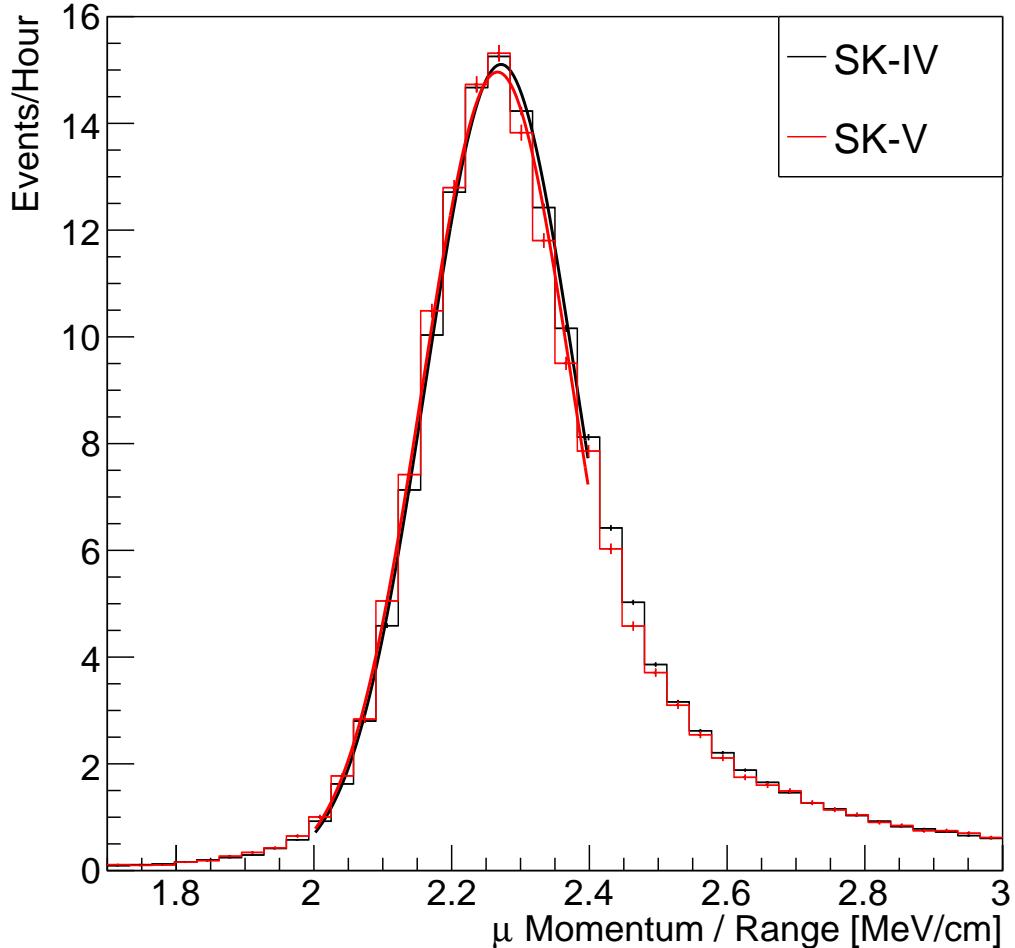


Figure 5.8: The distribution of the reconstructed momentum from the muon ring divided by the distance between the reconstructed muon and decay electron vertices as found in the stopping muon data sets of SK-IV (Black) and SK-IV (Red). Only events with one tagged decay electron are considered. A Gaussian fit is considered in the range [2.0, 2.4] MeV/cm and illustrated as the solid curve.

₁₈₁₅ The consistency between these muon distributions has been computed in two
₁₈₁₆ ways. Firstly, a Gaussian is fit to the peak of each distribution separately, whose
₁₈₁₇ mean is found to be (2.272 ± 0.003) MeV/cm and (2.267 ± 0.006) MeV/cm for SK-
₁₈₁₈ IV and SK-V respectively. The ratio of these is equal to 1.002 ± 0.003 . The means of
₁₈₁₉ the Gaussian fits are consistent with the expected stopping power of a minimum

ionising muon for a target material (water) with $Z/A \sim 0.5$ [181]. The second consistency check is performed by introducing a nuisance parameter, α , which modifies the SK-V distribution. The value of α which minimises the χ^2 value between the SK-IV and SK-V is determined by scanning across a range of values. This is repeated by applying the nuisance parameter as both a multiplicative factor and an additive shift. The χ^2 distributions for different values of α is illustrated in Figure 5.9. The values which minimise the χ^2 are found to be 0.0052 and 1.0024 for the additive and multiplicative implementations, respectively. No evidence of shifts larger than the 2.1% uncertainty on the energy scale systematic has been found in the reconstructed momentum distribution of SK-IV and SK-V.

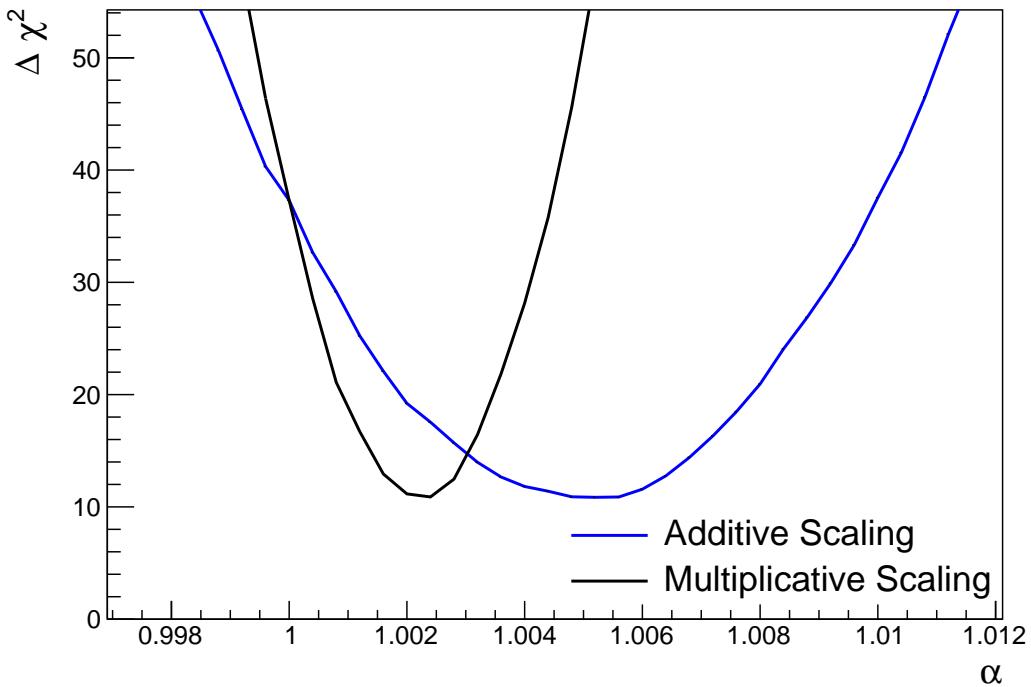


Figure 5.9: The χ^2 difference between the SK-IV and SK-V reconstructed muon momentum divided by range when the SK-V is modified by the scaling parameter α . Both additive (Blue) and multiplicative (Black) scaling factors have been considered. In practice, the additive scaling factor actually uses the value of $(\alpha - 1.0)$ but is illustrated like this so the results can be shown on the same axis range.

1830 5.3 Event Reduction at SK

1831 In normal data-taking operations, the SK detector observes many background
1832 events alongside the beam and atmospheric neutrino signal events of physics
1833 interest for this thesis. Cosmic ray muons and flasher events, which are the spon-
1834 taneous discharge of a given PMT, contribute the largest amount of background
1835 events in the energy range relevant to this thesis. Therefore the data recorded
1836 is reduced with the aim of removing these background events. The reduction
1837 process is detailed in [3, 57] and briefly summarised below.

1838 Atmospheric neutrino events observed in the SK detector are categorised
1839 into three different types of samples: fully contained (FC), partially contained
1840 (PC) and up-going muon (Up- μ), using PMT hit signatures in the inner and
1841 outer detector (ID and OD, respectively). To identify FC neutrino events, it is
1842 required that the neutrino interacts inside the fiducial volume of the ID and that
1843 no significant OD activity is observed. For this analysis, an event is defined to be
1844 in the fiducial volume provided the event vertex is at least 0.5m away from the
1845 ID walls. PC events have the same ID requirements but can have a larger signal
1846 present inside the OD. Typically, only high energy muons from ν_μ interactions can
1847 penetrate the ID wall. The Up- μ sample contains events where muons are created
1848 from neutrino interactions in the OD water or rock below the tank. They then
1849 propagate upwards through the detector. Downward-going muons generated
1850 from neutrino interactions above the tank are neglected because of the difficulty
1851 in separating their signature from the cosmic muon shower background. The
1852 sample categories are visually depicted in Figure 5.10.

1853 Based on the event characteristics, as defined by the `fitQun` event reconstruc-
1854 tion software, the FC events are categorised by

- 1855 • **Visible Energy:** equal to the sum of the reconstructed kinetic energy of
1856 particles above the Cerenkov threshold for all rings present in the event.
1857 The purpose is to separate events into sub-GeV and multi-GeV categories.

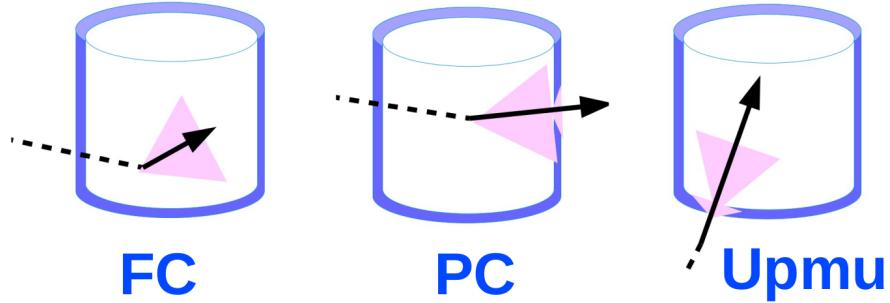


Figure 5.10: A depiction of the topology patterns for fully-contained (FC), partially-contained (PC), and up-going muon ($\text{Up-}\mu$) samples included in this analysis.

- **Number of observed Cerenkov rings.** The purpose is to separate single-ring and multi-ring events, where single-ring events predominantly consist of quasi-elastic interactions and multi-ring events are typically resonant pion production or deep inelastic scattering events.
- **Particle identification parameter of the most energetic ring:** A value determined from the maximum likelihood value based on `fitQun`'s electron, muon, or pion hypothesis. The purpose is to separate electron-like and muon-like events.
- **Number of decay electrons:** The purpose is to separate quasi-elastic events (which have one decay electron emitted from the muon decay) and resonant pion production events (which have two decay electrons emitted from the muon and pion).

The PC and Up- μ categories are broken down into “through-going” and “stopping” samples depending on whether the muon leaves the detector. This is because the PC stopping events deposit the entire energy of the interaction into the detector, resulting in better reconstruction. The energy of events that exit the detector has to be estimated, with a typically worse resolution, which introduces much larger systematic uncertainties. Through-going Up- μ samples are further broken down by whether any hadronic showering was observed in the event which typically indicates DIS interactions. The expected neutrino energy for the different categories is given in Figure 5.11. FC sub-GeV and multi-GeV events

¹⁸⁷⁹ peak around 0.7GeV and 3GeV respectively, with slightly different peak energies
¹⁸⁸⁰ for ν_e and ν_μ oscillation channels. PC and Up- μ are almost entirely comprised
¹⁸⁸¹ of ν_μ events and peak around 7GeV and 100GeV, respectively.

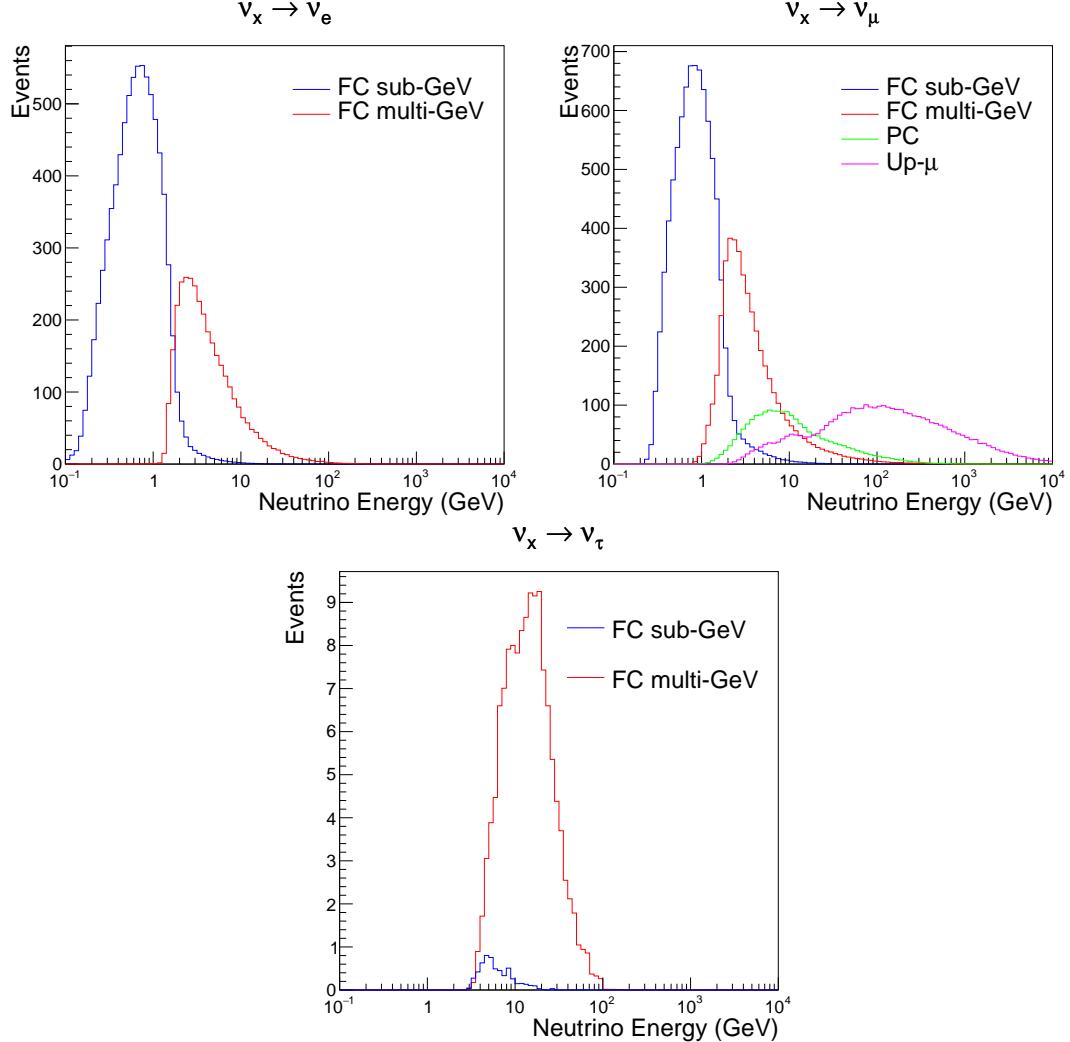


Figure 5.11: The predicted neutrino flux of the fully contained (FC) sub-GeV and multi-GeV, partially contained (PC), and upward-going muon (Up- μ) events. The prediction is broken down by the $\nu_x \rightarrow \nu_e$ prediction (top left), $\nu_x \rightarrow \nu_\mu$ prediction (top right) and $\nu_x \rightarrow \nu_\tau$ prediction (bottom). ν_x represents the flavours of neutrinos produced in the cosmic ray showers (electron and muon). Asimov A oscillation parameters are assumed (given in Table 2.2).

¹⁸⁸² The first two steps in the FC reconstruction remove the majority of cosmic
¹⁸⁸³ ray muons by requiring a significant amount of ID activity compared to that
¹⁸⁸⁴ measured in the OD. Events that pass this cut are typically very high momentum
¹⁸⁸⁵ muons or events that leave very little activity in the OD. Consequently, a third

reduction step is then applied to select cosmic-ray muons that pass the initial reduction step. A purpose-built cosmic muon fitter is used to determine the entrance (or exit) position of the muon and a cut is applied to OD activity contained within 8m of this position. Flasher events are removed in the fourth reduction step which is based on the close proximity of PMT hits surrounding the PMT producing the flash. Events that pass all these reduction steps are reconstructed with the APFit algorithm. The fifth step of the reduction uses information from the more precise fitter to repeat the previous two steps with tighter cuts. Muons below the Cherenkov threshold can not generate optical photons in the ID but the associated decay electron can due to its lower mass. These are the types of events targeted in the fifth reduction step. The final cuts require the event vertex to be within the fiducial volume (0.5m from the wall although the nominal distance is 2.0m), visible energy $E_{vis} > 30\text{MeV}$ and fewer than 16 hits within the higher energy OD cluster. The culmination of the fully contained reduction results in 8.09 events/day in the nominal fiducial volume [84]. The uncertainty in the reconstruction is calculated by comparing Monte Carlo prediction to data. The largest discrepancy is found to be 1.3% in the fourth reduction step.

The PC and Up- μ events are processed through their own reduction processes detailed in [57]. Both of these samples are reconstructed with the APFit algorithm rather than `f1TQun`. This is because the efficiency of reconstructing events that leave the detector has not been sufficiently studied for reliable systematic uncertainties with `f1TQun`. The PC and Up- μ samples acquire events at approximately 0.66 and 1.44 events/day.

Beam neutrinos events undergo the same reduction steps as FC events and are then subject to further cuts [182]. The GPS system that links the timing between the beam facility and SK needs to be operating correctly and there should be no activity within the detector in the previous $100\mu\text{s}$ before the trigger. The events then need to triggered between $-2\mu\text{s}$ and $10\mu\text{s}$ of the expected spill timing.

1915 The beam neutrino samples are not split by visible energy since their energy
 1916 range is smaller than the atmospheric neutrino events. Following the T2K
 1917 analysis in [2], only single-ring beam neutrino events are considered. Similar to
 1918 atmospheric event selection, the number of decay electrons is used as a proxy for
 1919 distinguishing CCQE and CCRES events. The expected neutrino energy, broken
 1920 down by the number of decay electrons, is given in Figure 5.12.

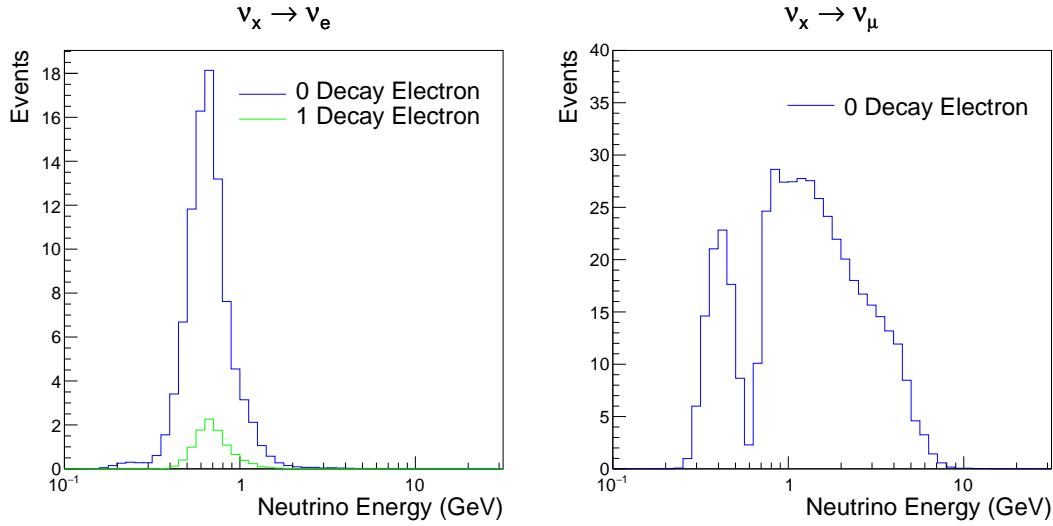


Figure 5.12: The predicted flux of beam neutrinos, as a function of neutrino energy. The predictions are broken down by the number of decay electrons associated with the particular events. Asimov A oscillation parameters are assumed (given in Table 2.2).

6

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1922

Sample Selections and Systematics

1923 The oscillation analysis presented within this thesis is built upon a simultaneous
1924 fit to atmospheric samples at SK, neutrino beam samples in the near detector,
1925 and beam samples at SK. This is the first simultaneous oscillation analysis of
1926 beam and atmospheric samples supported by the T2K and SK collaborations.
1927 Notably, the author of this thesis has been responsible for the building and
1928 developing the MaCh3 framework to support all sets of samples simultaneously.
1929 The definitions of the samples are documented in section 6.1, section 6.2, and
1930 section 6.3, respectively. The data collected and used within this analysis is
1931 detailed in Table 6.1. The near and far detector data corresponds to T2K runs
1932 2-9 and runs 1-10, respectively. The accumulated POT and beam power for runs
1933 1 – 10 are illustrated in Figure 6.1.

Data Type	Total
Near Detector FHC	1.15×10^{21} POT
Near Detector RHC	8.34×10^{20} POT
Far Detector FHC	1.97×10^{21} POT
Far Detector RHC	1.63×10^{21} POT
Atmospheric SK-IV	3244.4 days

Table 6.1: The amount of data collected in each detector used within this analysis. The data collected at the near and far detector, for both neutrino beam (FHC) and antineutrino beam (RHC), is measured as the number of protons on target (POT).

The difference in POT recorded at the near and far detector is due to the difference in downtime. The SK detector is very stable with almost 100% of data recorded during beam operation. Due to various technical and operational issues, the downtime of the near detector is significantly higher due to its more complex design and operating requirements.

The systematic parameters invoked within the flux, detector, and interaction models used within this analysis are documented in section 6.4. The standard configuration of the joint beam and atmospheric data fit utilises far detector systematics provided in the official inputs from the two experiments. Additionally, a correlated detector model which fits the parameters used in sample selections to data has been developed and documented in subsection 6.4.5.

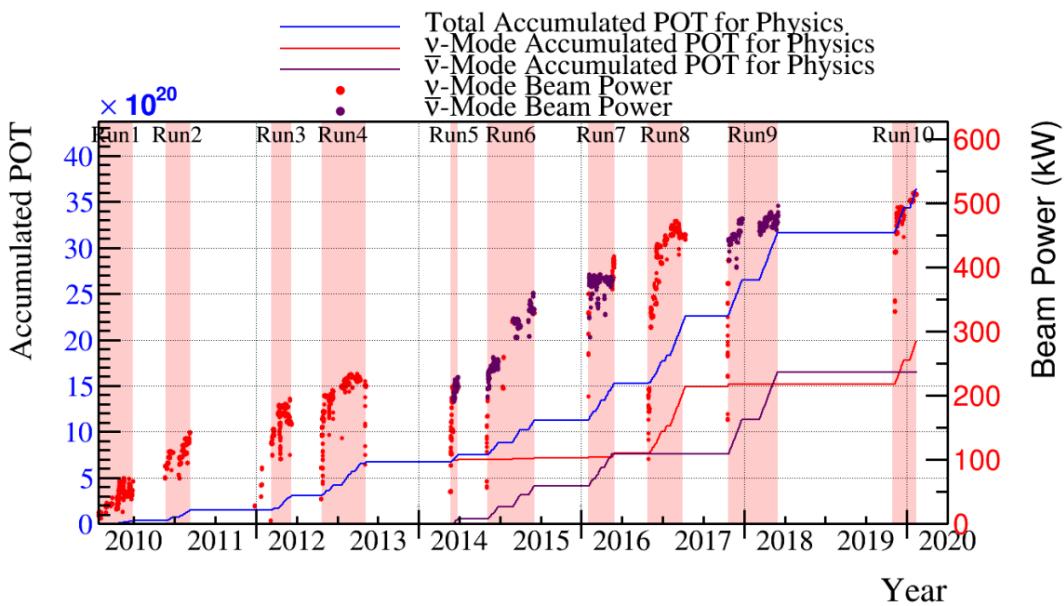


Figure 6.1: The accumulated beam data, measured as the number of protons on target (POT). The total data (blue) is given which comprises of the neutrino beam (red) and antineutrino (purple) components. The beam power for neutrino and antineutrino beams is given as the markers using the same colour scheme. The timescale runs from Run 1 which started in January 2010 until Run 10 which ended in February 2020. The ratio of accumulated data in neutrino and antineutrino beam is 54.7% : 45.3%.

1945 6.1 Atmospheric Samples

1946 The atmospheric event selection follows the official SK-IV analysis presented
1947 in [3] and is documented below. The Monte Carlo prediction used within this
1948 analysis corresponds to 500 years worth of neutrino events, which is scaled down
1949 to match the SK-IV livetime of 3244.4 days.

1950 The fully contained (FC), partially contained (PC), and upward going muon
1951 events ($\text{up-}\mu$) which pass the reduction cuts discussed in section 5.3 are further
1952 broken down into different samples based on reconstruction information. This
1953 section details the samples used within this oscillation analysis, alongside the
1954 chosen binning.

1955 FC events are first separated by the visible energy deposited within the
1956 detector. This is calculated as the sum of the reconstructed kinetic energy
1957 above the Cherenkov threshold for all rings present in the event. Events are
1958 separated by whether they were above or below $E_{\text{vis}} = 1.33\text{GeV}$. This separates
1959 “subGeV” and “multiGeV” events. Typically, lower energy events consist of
1960 charged current quasi-elastic (CCQE) interactions which are better understood
1961 and simpler to reconstruct resulting in smaller systematic uncertainties. Events
1962 are further separated by the number of rings associated with the event due to
1963 similar reasoning. As the oscillation probability is dependant upon the flavour
1964 of neutrino, electron and muon events are separated using a similar likelihood
1965 method to that discussed in section 5.2. To reduce computational resources
1966 required for the reconstruction, only electron and pion hypotheses are considered
1967 so this separation cut depends on the ratio of the electron to pion likelihoods,
1968 $\log(L_e/L_\pi)$. Finally, the number of decay electrons is used to classify events.
1969 Charged current resonant pion production (CCRES) interactions generate a final-
1970 state pion. This can decay, mostly likely through a muon, into a decay electron.
1971 Therefore any electron-like event with one decay electron or muon-like event
1972 with two decay electrons was most likely produced by a CCRES interaction.
1973 Consequently, the number of decay electrons can be used to distinguish CCQE

¹⁹⁷⁴ and CCRES interaction modes. Ultimately, FC subGeV events are separated
¹⁹⁷⁵ into the samples listed in Table 6.2.

Sample Name	Description
SubGeV- <i>e</i> like-0dcy	Single ring <i>e</i> -like events with zero decay electrons
SubGeV- <i>e</i> like-1dcy	Single ring <i>e</i> -like events with one or more decay electrons
SubGeV- <i>μ</i> like-0dcy	Single ring <i>μ</i> -like events with zero decay electrons
SubGeV- <i>μ</i> like-1dcy	Single ring <i>μ</i> -like events with one decay electrons
SubGeV- <i>μ</i> like-2dcy	Single ring <i>μ</i> -like events with two or more decay electrons
SubGeV- <i>π</i> 0like	Two <i>e</i> -like ring events with zero decay electrons and reconstructed π^0 mass $85 \leq m_{\pi^0} < 215$ MeV

Table 6.2: The fully contained subGeV samples, defined as events with visible energy $E_{vis} < 1.33$ GeV, used within this oscillation analysis.

¹⁹⁷⁶ In addition to the cuts discussed above, multiGeV samples also have addi-
¹⁹⁷⁷ tional cuts to separate samples which target neutrino and antineutrino events.
¹⁹⁷⁸ As discussed in section 2.5, the matter resonance only occurs for neutrinos in the
¹⁹⁷⁹ normal hierarchy and antineutrinos in the inverted mass hierarchy. Therefore,
¹⁹⁸⁰ having flavour-enriched samples aids in the determination of the mass hierarchy.
¹⁹⁸¹ For a CCRES interaction,

$$\begin{aligned}
 \bar{\nu}_e + N &\rightarrow e^+ + N' + \pi^-, \\
 \nu_e + N &\rightarrow e^- + N' + \pi^+ \\
 &\quad \downarrow \mu^+ + \nu_\mu \\
 &\quad \downarrow e^+ + \nu_e + \bar{\nu}_\mu.
 \end{aligned} \tag{6.1}$$

¹⁹⁸² The π^- emitted from a $\bar{\nu}_e$ interaction is more likely to be captured by an
¹⁹⁸³ oxygen nucleus than the π^+ from ν_e interactions [183]. These pions then decay,
¹⁹⁸⁴ mostly through muons, to electrons. Therefore the number of tagged decay
¹⁹⁸⁵ electrons associated with an event gives an indication of whether the interaction
¹⁹⁸⁶ was due to a neutrino or antineutrino: zero for $\bar{\nu}_e$ events, and one for ν_e events.
¹⁹⁸⁷ The ability to separate neutrino from antineutrino events is illustrated in Table 6.4,
¹⁹⁸⁸ where the MultiGeV-*e*like-nue has 78% purity of CC neutrino interactions with
¹⁹⁸⁹ only 7% antineutrino background, the rest consisting of NC backgrounds.

1990 The number of decay electrons discriminator works reasonably well for single-
 1991 ring events. However, this is not the case for multi-ring events. A multiGeV
 1992 multiring electron-like (MME) likelihood cut was introduced in [184, 185]. This
 1993 is a two-stage likelihood selection cut. Four observables are used in the first
 1994 likelihood cut to distinguish $CC\nu_e$ and $CC\bar{\nu}_e$ events from background:

- 1995 • The number of decay electrons
 1996 • The maximum distance between the vertex of the neutrino and the decay
 1997 electrons
 1998 • The energy deposited by the highest energy ring
 1999 • The particle identification of that highest energy ring

2000 Background events consist of $CC\nu_\mu$ and NC interactions. Typically, the
 2001 majority of the energy in these background events is carried by the hadronic
 2002 system. Additionally, muons tend to travel further than the pions from $CC\nu_e$
 2003 before decaying. Thus, the parameters used within the likelihood cut target these
 2004 typical background interaction kinematics.

Sample Name	Description
MultiGeV-elike-nue	Single ring e -like events with zero decay electrons
MultiGeV-elike-nuebar	Single ring e -like events with one or more decay electrons
MultiGeV-mulike	Single ring μ -like events
MultiRing-elike-nue	Two or more ring events with leading energy e -like ring and passed both MME and $\nu/\bar{\nu}$ separation cuts
MultiRing-elike-nuebar	Two or more ring events with leading energy e -like ring and passed MME and failed $\nu/\bar{\nu}$ separation cuts
MultiRing-mulike	Two or more ring events with leading energy μ -like ring and only requires $E_{vis} > 0.6\text{GeV}$
MultiRing-Other1	Two or more ring events with leading energy e -like ring and failed the MME likelihood cut

Table 6.3: The fully contained multiGeV samples used within this oscillation analysis. Both the sample name and description are given.

2005 Neutrino and antineutrino events are then separated by a second likelihood
 2006 method ($\nu/\bar{\nu}$ separation) detailed in [62]. This uses the number of decay electrons,

2007 the number of reconstructed rings, and the event’s transverse momentum. The
2008 last two parameters are used because higher-energy samples tend to have more
2009 pions produced above the Cherenkov threshold which results in more rings
2010 compared to an antineutrino interaction. Furthermore, the angular distribution
2011 also tends to be more forward peaked in antineutrino interactions as compared
2012 to neutrino interactions [3]. These FC multiGeV sample definitions are de-
2013 tailed in Table 6.3.

2014 The PC and up- μ samples are split by the amount of energy deposited within
2015 the outer detector, into “stopping” and “through-going” samples. If an event
2016 leaves the detector, the energy it takes with it has to be estimated which increases
2017 the systematic uncertainty compared to events entirely contained within the
2018 inner detector. This estimation is particularly poor at high energies, thus the
2019 up- μ through-going events are not binned in reconstructed momentum. The
2020 through-going up- μ are further separated by the presence of any electromagnetic
2021 showering in the event, as the assumption of non-showering muon does not give
2022 reliable reconstruction for these types of events [57]. In total, 13 FC, 2 PC, and
2023 3 up- μ atmospheric samples are included within this analysis.

2024 The atmospheric samples are binned in direct observables: reconstructed
2025 lepton momentum and direction, as given by Table 6.5. The distribution of
2026 the reconstructed lepton momentum (for samples that only have one bin in
2027 reconstructed zenith angle) and reconstructed direction for each atmospheric
2028 sample used within this analysis is illustrated in Figure 6.2.

2029 The reconstructed lepton momemtum, illustrated by interaction mode break-
2030 down, of some representative atmospheric samples is given in Figure 6.3. The
2031 equivalent distributions of all atmospheric samples used within this analy-
2032 sis can be found in [186]. The low energy samples tend to be dominated by
2033 the interaction mode they target (CCQE for SubGeV-elike-0dcy and CC1 π for
2034 SubGeV-elike-1dcy samples). The higher energy samples include much more
2035 CCOther interactions, especially at larger reconstructed lepton momentum.

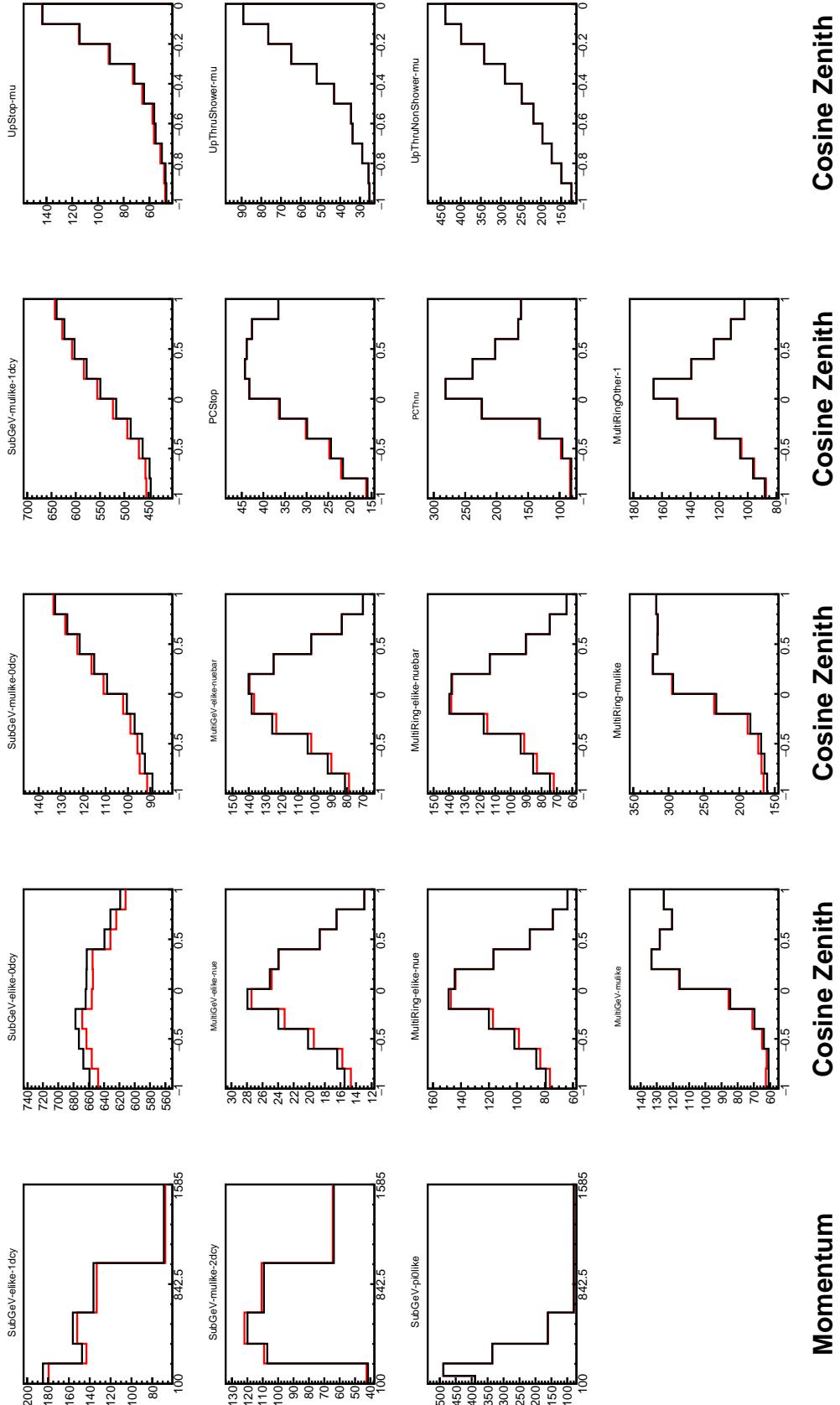


Figure 6.2: Comparison of the SK-IV atmospheric samples between predictions made with the CP-violating Asimov A (Black) and CP-conserving Asimov B (Red) oscillation parameter sets (given in Table 2.2). The subGeV samples CCRES and π^0 -like samples are given in their reconstructed lepton momentum. All other samples are presented in their reconstructed zenith angle projection.

Sample	$CC\nu_e$	$CC\bar{\nu}_e$	$CC(\nu_\mu + \bar{\nu}_\mu)$	$CC(\nu_\tau + \bar{\nu}_\tau)$	NC
SubGeV- <i>elike</i> -0dcy	72.17	23.3	0.724	0.033	3.77
SubGeV- <i>elike</i> -1dcy	86.81	1.773	7.002	0.062	4.351
SubGeV- <i>mulike</i> -0dcy	1.003	0.380	90.07	0.036	8.511
SubGeV- <i>mulike</i> -1dcy	0.023	0.	98.46	0.029	1.484
SubGeV- <i>mulike</i> -2dcy	0.012	0.	99.25	0.030	0.711
SubGeV- <i>pi0like</i>	6.923	2.368	0.928	0.011	89.77
MultiGeV- <i>elike</i> -nue	78.18	7.041	3.439	1.886	9.451
MultiGeV- <i>elike</i> -nuebar	56.68	37.81	0.174	0.614	4.718
MultiGeV- <i>mulike</i>	0.024	0.005	99.67	0.245	0.058
MultiRing- <i>elike</i> -nue	59.32	12.39	4.906	3.385	20
MultiRing- <i>elike</i> -nuebar	52.39	31.03	1.854	1.585	13.14
MultiRing- <i>mulike</i>	0.673	0.080	97.33	0.342	1.578
MultiRingOther-1	27.98	2.366	34.93	4.946	29.78
PCStop	8.216	3.118	84.45	0.	4.214
PCThrus	0.564	0.207	98.65	0.	0.576
UpStop-mu	0.829	0.370	98.51	0.	0.289
UpThruNonShower-mu	0.206	0.073	99.62	0.	0.103
UpThruShower-mu	0.128	0.054	99.69	0.	0.132

Table 6.4: The purity of each atmospheric sample used within this analysis, broken down by charged current (CC) and neutral current (NC) interactions and which neutrino flavour interacted within the detector. Each row sums to 100% by definition. Asimov A oscillation parameter sets are assumed (given in Table 2.2). Electron neutrino and antineutrino events are separated to illustrate the ability of the separation likelihood cuts used within the multiGeV and multiring sample selections.

Sample	$\cos(\theta_Z)$ Bins	Momentum Bin Edges ($\log_{10}(P)$ MeV)
SubGeV- <i>elike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>elike</i> -1dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -0dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -1dcy	10	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>mulike</i> -2dcy	1	2.0, 2.4, 2.6, 2.8, 3.0, 3.2
SubGeV- <i>pi0like</i>	1	2.0, 2.2, 2.4, 2.6, 2.8, 3.2
MultiGeV- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 4.0, 5.0
MultiGeV- <i>mulike</i>	10	3.0, 3.4, 5.0
MultiRing- <i>elike</i> -nue	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>elike</i> -nuebar	10	3.0, 3.4, 3.7, 5.0
MultiRing- <i>mulike</i>	10	2.0, 3.124, 3.4, 3.7, 5.0
MultiRing- <i>Other1</i>	10	3.0, 3.4, 3.7, 4.0, 5.0
PC-Stop	10	2.0, 3.4, 5.0
PC-Through	10	2.0, 3.124, 3.4, 3.7, 5.0
Upmu-Stop	10	3.2, 3.4, 3.7, 8.0
Upmu-Through-Showering	10	2.0, 8.0
Upmu-Through-NonShowering	10	2.0, 8.0

Table 6.5: The reconstructed cosine zenith and lepton momentum binning assigned to the atmospheric samples. The “ $\cos(\theta_Z)$ Bins” column illustrates the number of bins uniformly distributed over the $-1.0 \leq \cos(\theta_Z) \leq 1.0$ region for fully and partially contained samples and $-1.0 \leq \cos(\theta_Z) \leq 0.0$ region for up- μ samples.

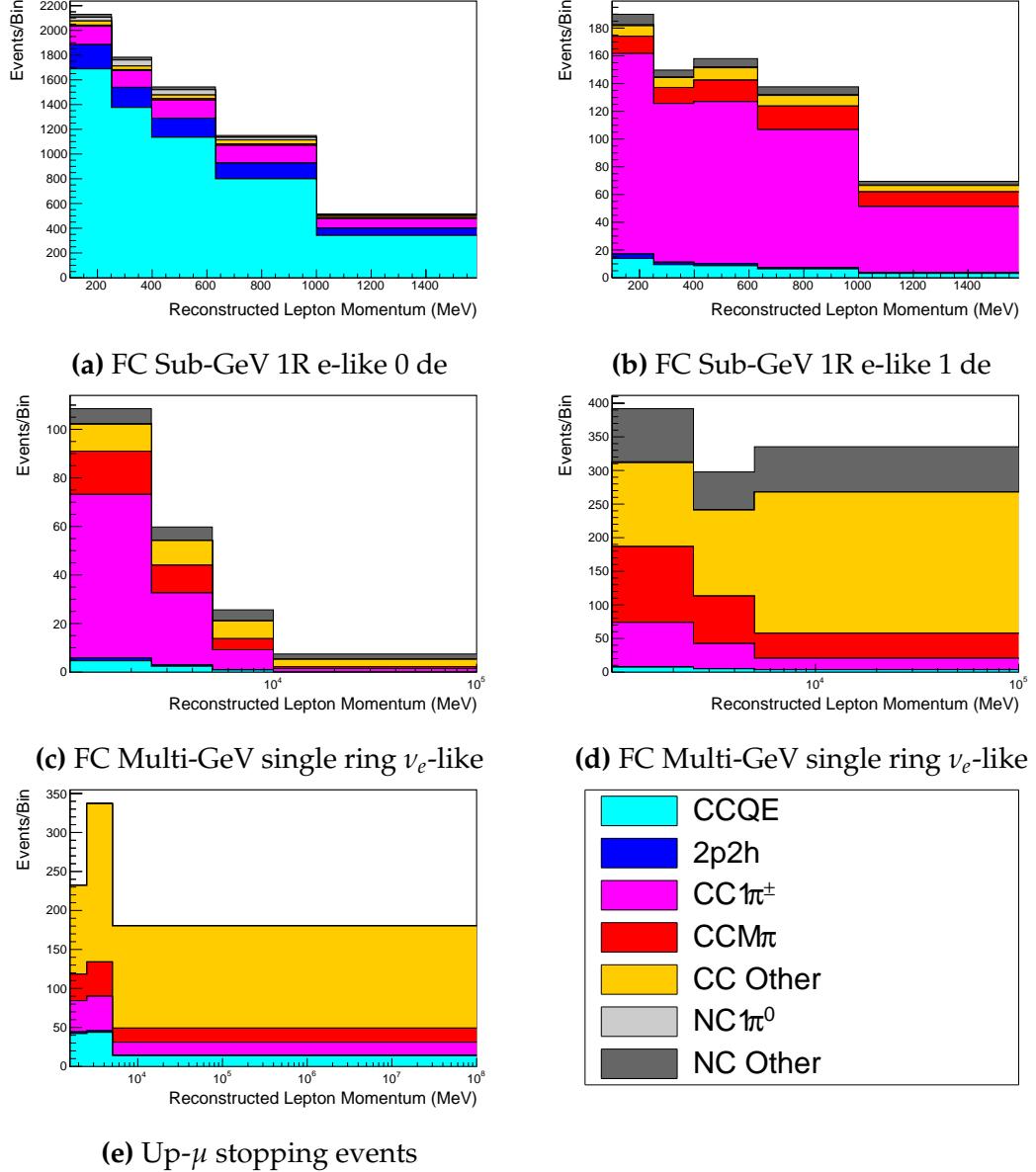


Figure 6.3: Breakdown by interaction mode of some representative atmospheric samples used within this analysis, illustrated as a function of reconstructed lepton momentum. The binning is provided in Table 6.5. Asimov A oscillation parameters are used to generate these plots. The interaction mode breakdown of all atmospheric samples used within this analysis can be found in [186].

2036 6.2 Near Detector Beam Samples

2037 The near detector sample selections are documented in detail within [187] and
2038 summarised below. Samples are selected based upon which of the two Fine
2039 Grained Detector (FGD) the vertex is reconstructed in as well as the operating
2040 mode of the beam: FHC or RHC. Wrong-sign neutrino background samples are
2041 considered in the RHC mode in order to add additional constraints on model
2042 parameters. Samples from the wrong-sign component of the FHC beam mode
2043 are not included as they are statistically insignificant compared to those samples
2044 already listed.

2045 The reconstruction algorithm uses a clustering algorithm to group hits within
2046 the TPC. It then adds information from the upstream FGD to form a track
2047 that passes through both sub-detectors. In FHC(RHC), the highest momentum
2048 negative(positive) curvature track is defined as the muon candidate. Before
2049 being assigned a sample, these candidate muon events must pass CC-inclusive
2050 cuts, as defined in [188]:

- 2051 • Event Timing: The DAQ must be operational and the event must occur
2052 within the expected beam time window consistent with the beam spill
- 2053 • TPC Requirement: The muon-candidate track path must intercept one or
2054 more TPCs
- 2055 • Fiducial volume: The event must originate from within the fiducial volume
2056 defined in [189]
- 2057 • Upstream Background: Remove events that have muon tracks that originate
2058 upstream of the FGDs by requiring no high-momentum tracks within
2059 150mm upstream of the candidate vertex. Additionally, events that occur
2060 within the downstream FGD are vetoed if a secondary track starts within
2061 the upstream FGD

- 2062 • Broken track removal: All candidates where the muon candidate is broken
 2063 in two are removed

- 2064 • Muon PID: Measurements of dE/dx in a TPC are used to distinguish muon-
 2065 like events, from electron-like or proton-like, using a likelihood cut

2066 In addition to these cuts, RHC neutrino events also have to undergo the
 2067 following cuts to aid in the separation of neutrino and antineutrino [190]:

- 2068 • TPC Requirement: The track path must intercept TPC2
 2069 • Positive Track: The highest momentum track must have a positive recon-
 2070 structed charge
 2071 • TPC1 Veto: Remove any events originating upstream of TPC1

2072 Once all CC-inclusive events have been determined, they are further split by
 2073 pion multiplicity: CC0 π , CC1 π , and CCOther. Pions in the TPCs are selected by
 2074 requiring a second track to be observed, which is separate from the muon track
 2075 and is in the same beam spill window and sub-detector. The number of FGD
 2076 pions is equal to the number of Michel electrons which were tagged within the
 2077 same sub-detector and spill window. If this value is equal to zero, the number
 2078 of FGD pions is equivalent to the number of pion-like tracks which have dE/dx
 2079 measurements consistent with the pion hypothesis. The pion tracks from both
 2080 FGD and TPC events are required to have a vertex consistent with that of the
 2081 muon candidate. The Michel electron tagging is preferential as a delayed Michel
 2082 is almost always a pion meaning this cut has a higher purity [188, 191], whereas a
 2083 track in the FGD that is consistent with a pion could be another particle resulting
 2084 in a lower purity. Michel electrons are neglected in the TPC as the pions very
 2085 rarely stop due to the low density.

2086 CC0 π , CC1 π , and CCOther samples are defined with the following cuts:

- 2087 • ν_μ CC0 π Selection: No electrons in TPC and no charged pions or decay
 2088 electrons within the TPC or FGD

- 2089 • ν_μ **CC1 π Selection:** Exactly one charged pion in either the TPC or FGD
 2090 • ν_μ **CCOther Selection:** All events which are not classified into the above
 2091 two selections

2092 Counting the three selections for each FGD in FHC and RHC running, includ-
 2093 ing the wrong-sign background in RHC, 18 near detector samples are used within
 2094 this analysis. These samples are binned in reconstructed lepton momentum
 2095 (illustrated in Figure 6.4) and direction with respect to the beam. The binning
 2096 is chosen such that each event has at least 20 Monte Carlo events in each bin
 2097 [189]. This is to ensure that the bins are coarse enough to ensure the reduction
 2098 of statistical errors, whilst also being fine enough to sample the high-resolution
 2099 peak regions. The exact binning is detailed in [189].

2100 6.3 Far Detector Beam Samples

2101 The beam neutrino events which occur at the SK detector, which pass the
 2102 reduction cuts detailed in section 5.3, are separated based on whether the beam
 2103 was operating in FHC or RHC mode. The events are then separated into three
 2104 samples: electron-like (1Re), muon-like (1R μ), and CC1 π^+ -like (1Re1de) which
 2105 are observed as electron-like events with an associated decay electron [179].
 2106 As discussed in section 6.1, positively charged pions emitted from neutrino
 2107 interactions are more likely to produce decay electrons than negatively charged
 2108 pions. Consequently, the CC1 π^+ -like sample is only selected when the beam is
 2109 operating in FHC mode. Therefore, five beam samples measured at SK are
 2110 used in this analysis.

2111 The fiducial volume definition for beam samples is slightly different from that
 2112 used for the atmospheric samples. It uses both the distance to the closest wall
 2113 (dWall) and the distance to the wall along the trajectory of the particle (toWall).
 2114 This allows events that originate close to the wall but are facing into the tank to be
 2115 included within the analysis, which would have otherwise been removed. These
 2116 additional events are beneficial for a statistics-limited experiment. The exact

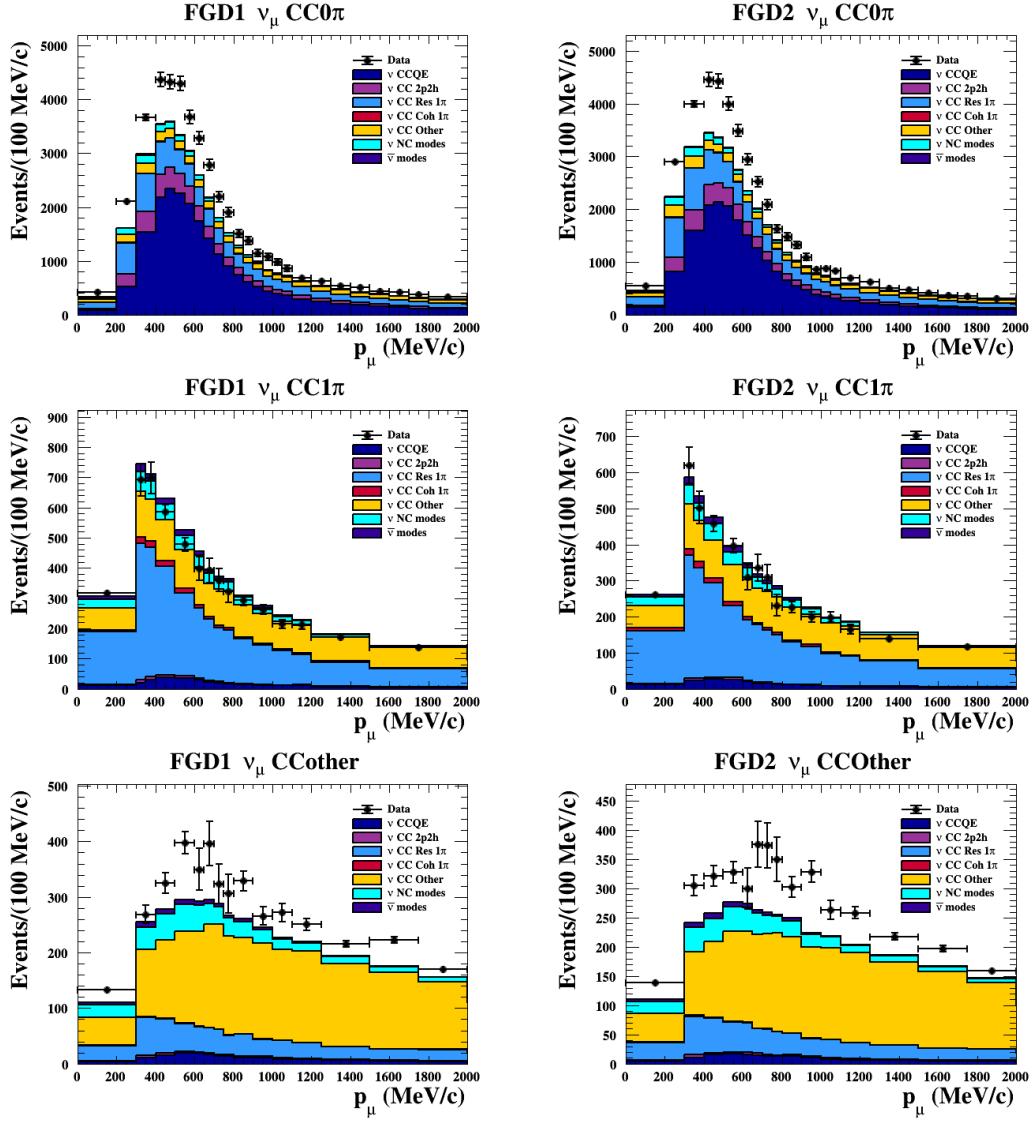


Figure 6.4: The nominal Monte Carlo predictions compared to data for the FGD1 and FGD2 samples in neutrino beam mode, broken down into the $CC\nu_\mu 0\pi$, $CC\nu_\mu 1\pi$ and $CC\nu_\mu$ Other categories. Figures taken from [187].

2117 cut values for both `dWall` and `tWall` are different for each of the three types of
 2118 sample and are optimised based on T2K sensitivity to δ_{CP} [177, 192]. They are:

2119 **1Re event selection** For an event to be classified as a 1Re-like, the event must sat-
 2120 isfy:

- 2121 • Fully-contained and have $dWall > 80\text{cm}$ and $tWall > 170\text{cm}$
- 2122 • Total of one ring which is reconstructed as electron-like with reconstructed

2123 momentum $P_e > 100\text{MeV}$

2124 • Zero decay electrons are associated with the event

2125 • Passes π^0 rejection cut discussed in section 5.2

2126 The zero decay electron cut removes non-CCQE interactions and the π^0
2127 rejection cut is designed to remove neutral current π^0 background events which
2128 can be easily reconstructed as 1Re-like events.

2129 The zero decay electron cut removes non-CCQE interactions and the π^0
2130 rejection cut is designed to remove neutral current π^0 background events which
2131 can be easily reconstructed as 1Re-like events.

2132 **CC1 π^+ event selection** This event selection is very similar to that of the 1Re
2133 sample. The only differences are that the dWall and toWall criteria are changed
2134 to $> 50\text{cm}$ and $> 270\text{cm}$, respectively, and exactly one decay electron is required
2135 from the π^+ decay.

2136 **1R μ event selection** A 1R μ -like event is determined by the following cuts:

2137 • Fully-contained and have $\text{dWall} > 50\text{cm}$ and $\text{toWall} > 250\text{cm}$

2138 • Total of one ring which is reconstructed as muon-like with reconstructed
2139 momentum $P_\mu > 200\text{MeV}$

2140 • Fewer than two decay electrons are associated with the event

2141 • Passes π^+ rejection cut discussed in section 5.2

2142 All of these samples are binned in reconstructed neutrino energy. This is
2143 possible under a particular interaction mode assumption, as the direction from
2144 the source is known extremely well. For the 1Re-like and 1R μ -like samples,

$$E_\nu^{rec} = \frac{(M_N - V_{nuc})E_l - m_l^2/2 + M_N V_{nuc} - V_{nuc}^2/2 + (M_P^2 + M_N^2)/2}{M_N - V_{nuc} - E_l + P_l \cos(\theta_{beam})}. \quad (6.2)$$

2145 Where M_N , M_P and m_l are the masses of the neutron, proton and outgoing
 2146 lepton, respectively. $V_{nuc} = 27\text{MeV}$ is the binding energy of the oxygen nucleus
 2147 [179], θ_{beam} is the angle between the beam and the direction of the outgoing
 2148 lepton, and E_l and P_l are the energy and momentum of that outgoing lepton.

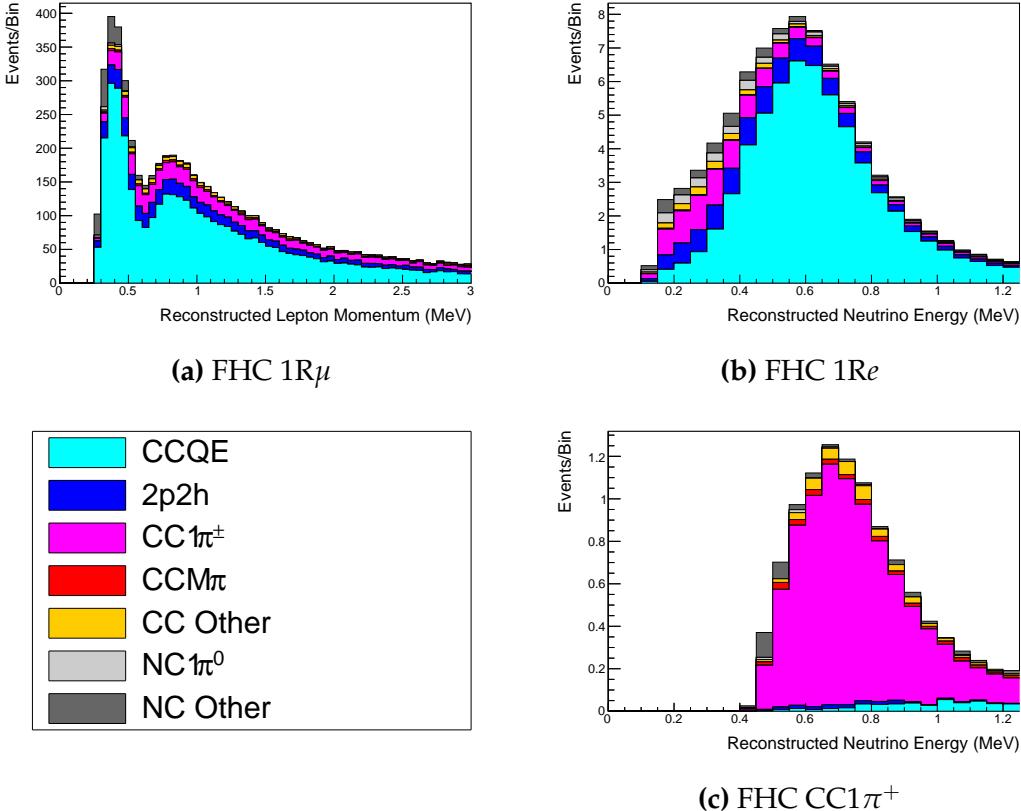


Figure 6.5: The reconstructed neutrino energy, as defined by Equation 6.2 and Equation 6.3, for the 1R μ -like, 1R e -like, and CC1 π^+ -like samples. The AsimovA oscillation parameters are assumed (given in Table 2.2). These samples are the FHC mode samples. For ease of viewing, the 1R μ sample only shows the $0 \leq E_\nu^{rec} < 3.0\text{GeV}$ but the binning extends to 30.0GeV .

2149 The reconstructed neutrino energy of the CC1 π^+ -like events also accounts
 2150 for the delta resonance produced within the interaction,

$$E_\nu^{rec} = \frac{2M_N E_l + M_{\Delta^{++}}^2 - M_N^2 - m_l^2}{2(M_N - E_l + P_l \cos(\theta_{beam}))}. \quad (6.3)$$

2151 Where $M_{\Delta^{++}}$ is the mass of the delta baryon. Binding energy effects are not
 2152 considered as a two-body process, with the delta baryon, is assumed. This follows
 2153 the T2K oscillation analysis presented in [2], although recent developments of

2154 the interaction model in the latest T2K oscillation analysis do include effects
2155 from binding energy in this calculation [193].

2156 The reconstructed neutrino energy for the FHC samples is illustrated in
2157 Figure 6.5. As expected, the $1R\mu$ -like and $1Re$ -like samples are heavily dominated
2158 by CCQE interactions, with smaller contributions from $2p2h$ meson exchange and
2159 resonant pion production interactions. The $CC1\pi^+$ -like sample predominantly
2160 consists of charged current resonant pion production interactions. The $1Re$ -like
2161 and $CC1\pi^+$ -like samples are also binned by the angle between the neutrino beam
2162 and the reconstructed lepton momentum. This is to aid in charged current and
2163 neutral current separation, as indicated in Figure 6.6. This is because the neutral
2164 current backgrounds are predominantly due to π^0 -decays, which decay into two
2165 γ rays. The opening angle of which (alongside the different final state kinematics)
2166 can produce a slightly broader angular distribution compared to the final state
2167 particles originating from charged current ν_e interactions.

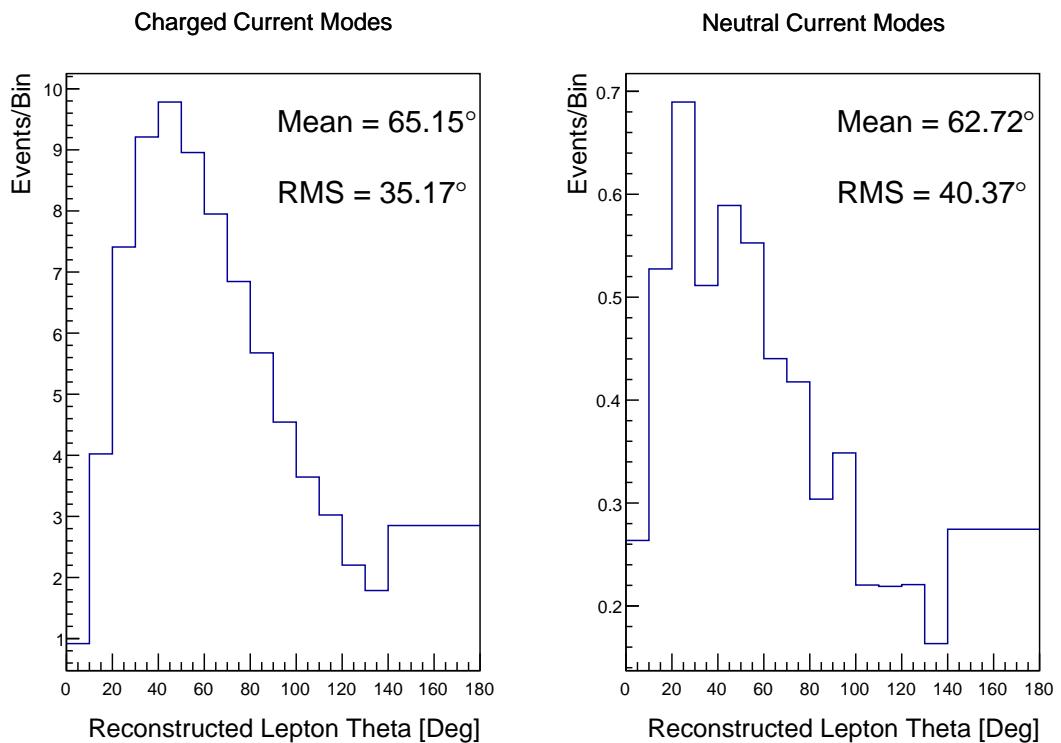


Figure 6.6: The distribution of the angle between the neutrino beam direction and the reconstructed final state lepton, for the FHC 1Re-like sample. The distribution is broken down by neutrino interaction mode into charged current (left) and neutral current (right) components. Asimov A oscillation parameter sets are assumed (given in Table 2.2). The RMS of the charged and neutral current plots are 35.17° and 40.37° , respectively.

2168 6.4 Systematic Uncertainties

2169 The systematic model parameters for this analysis are split into groups, or blocks,
2170 depending on their purpose. They consist of flux uncertainties, neutrino-matter
2171 interaction systematics, and detector efficiencies. There are also uncertainties on
2172 the oscillation parameters to which this analysis is not sensitive, namely Δm_{21}^2
2173 and $\sin^2(\theta_{12})$. These oscillation parameter uncertainties are taken from the 2020
2174 PDG measurements [76]. As described in chapter 4, each model parameter used
2175 within this analysis requires a prior uncertainty. This is provided via separate
2176 covariance matrices for each block. The covariance matrices can include prior
2177 correlations between parameters within a single block, but the separate treatment
2178 means prior correlations can not be included for parameters in different groups.
2179 Some parameters in these models have no reasonably motivated uncertainties
2180 and are assigned flat priors which do not modify the likelihood penalty. In
2181 practice, these flat prior parameters are actually assigned a Gaussian with a
2182 very large width to ensure the covariance matrix is positive definite. They are
2183 then checked at run time to determine if they contribute to the likelihood. The
2184 flux, neutrino interaction, and detector modeling simulations have already been
2185 discussed in section 5.1 and section 5.2. The uncertainties invoked within each
2186 of these models are described below.

2187 6.4.1 Beam Flux

2188 The neutrino beam flux systematics are based upon the uncertainty in the mod-
2189eling of the components of the beam simulation. This includes the model of
2190 hadron productions and reinteractions, the shape, intensity, and alignment of
2191 the beam with respect to the target, and the uniformity of the magnetic field
2192 produced by the horn, alongside other effects. The uncertainty, as a function
2193 of neutrino energy, is illustrated in Figure 6.7 which includes a depiction of
2194 the total uncertainty as well as the contribution from individual components.
2195 The uncertainty around the peak of the energy distribution ($E_\nu \sim 0.6\text{GeV}$) is

2196 dominated by uncertainties in the beam profile and alignment. Outside of this
2197 region, uncertainties on hadron production dominate the error.

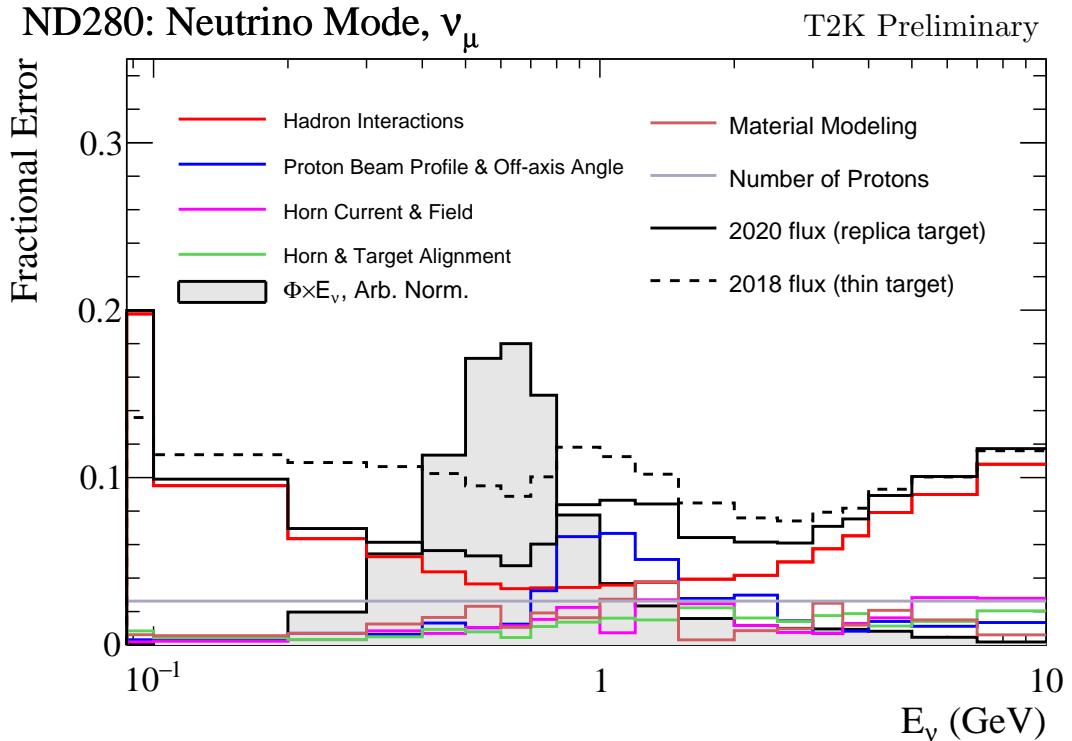


Figure 6.7: The total uncertainty evaluated on the near detector ν_μ flux prediction constrained by the replica-target data, illustrated as a function of neutrino energy. The solid(dashed) line indicates the uncertainty used within this analysis(the T2K 2018 analysis [194]). The solid histogram indicates the neutrino flux as a function of energy. Figure taken from [195].

2198 The beam flux uncertainties are described by one hundred parameters. They
2199 are split between the ND280 and SK detectors and binned by neutrino flavour:
2200 ν_μ , $\bar{\nu}_\mu$, ν_e and $\bar{\nu}_e$. The response is then broken down as a function of neutrino
2201 energy. The bin density in the neutrino energy is the same for the ν_μ in FHC
2202 and $\bar{\nu}_\mu$ in RHC beams, and narrows for neutrino energies close to the oscillation
2203 maximum of $E_\nu = 0.6\text{GeV}$. This binning is specified in Table 6.6. All of these
2204 systematic uncertainties are applied as normalisation parameters with Gaussian
2205 priors centered at 1.0 and error specified from a covariance matrix provided
2206 by the T2K beam group [195].

Neutrino Flavour	Sign	Neutrino Energy Bin Edges (GeV)
μ	Right	0., 0.4, 0.5, 0.6, 0.7, 1., 1.5, 2.5, 3.5, 5., 7., 30.
μ	Wrong	0., 0.7, 1., 1.5, 2.5, 30.
e	Right	0., 0.5, 0.7, 0.8, 1.5, 2.5, 4., 30.
e	Wrong	0., 2.5, 30.

Table 6.6: The neutrino energy binning for the different neutrino flavours. “Right” sign indicates neutrinos in the FHC beam and antineutrinos in the RHC beam. “Wrong” sign indicates antineutrinos in the FHC beam and neutrinos in the RHC beam. The binning of the detector response is identical for the FHC and RHC modes as well as at ND280 and SK.

2207 6.4.2 Atmospheric Flux

2208 The atmospheric neutrino flux is modeled by the HKKM model [53]. 16 systematic
 2209 uncertainties are applied to control the normalisation of each neutrino flavour,
 2210 energy, and direction. They are summarised below:

- 2211 • **Absolute Normalisation:** The overall normalisation of each neutrino flavour
 2212 is controlled by two independent systematic uncertainties, for $E_\nu < 1\text{GeV}$
 2213 and $E_\nu > 1\text{GeV}$, respectively. This is driven mostly by hadronic interaction
 2214 uncertainties for the production of pions and kaons [53]. The strength of
 2215 the response is dependent upon the neutrino energy. The uncertainty is
 2216 parameterized following Figure 11 in [53].
- 2217 • **Relative Normalisation:** Uncertainties on the ratio of $(\nu_\mu + \bar{\nu}_\mu) / (\nu_e + \bar{\nu}_e)$
 2218 are controlled by the difference between the HKKM model [53], FLUKA
 2219 [56] and Bartol models [52]. Three independent parameters are applied in
 2220 the energy ranges: $E_\nu < 1\text{GeV}$, $1\text{GeV} < E_\nu < 10\text{GeV}$, and $E_\nu > 10\text{GeV}$.
- 2221 • **$\nu/\bar{\nu}$ Normalisation:** The uncertainties in the π^+/π^- (and kaon equivalent)
 2222 production uncertainties in the flux of $\nu/\bar{\nu}$. The response is applied using
 2223 the same methodology as the relative normalisation parameters.
- 2224 • **Up/Down and Vertical/Horizontal Ratio:** Similar to the above two sys-
 2225 tematics, the difference between the HKKM, FLUKA, and Bartol model

2226 predictions, as a function of $\cos(\theta_Z)$, is used to control the normalisation of
 2227 events as a function of zenith angle.

- 2228 • **K/π Ratio:** Higher energy neutrinos ($E_\nu > 10\text{GeV}$) mostly originate in
 2229 kaon decay. Measurements of the ratio of K/π production [196] are used to
 2230 control the systematic uncertainty of the expected ratio of pion and kaon
 2231 production.
- 2232 • **Solar Activity:** As the 11-year solar cycle can affect the Earth's magnetic
 2233 field, the flux of primary cosmic rays varies across the same period. The
 2234 uncertainty is calculated by taking a ± 1 year variation, equating to a 10%
 2235 uncertainty for the SK-IV period.
- 2236 • **Atmospheric Density:** The height of the interaction of the primary cosmic
 2237 rays is dependent upon the atmospheric density. The HKKM assumes the
 2238 US standard 1976 [146] profile. This systematic controls the uncertainty in
 2239 that model.

2240 The total uncertainty is dominated by the absolute and relative normalisation
 2241 parameters. The effect of which is illustrated in Figure 6.8. Generally, the
 2242 uncertainty is large at low energy, reducing to $O(10\%)$ around the peak of the
 2243 flux distribution and then increasing once the neutrino energy exceeds 10GeV.

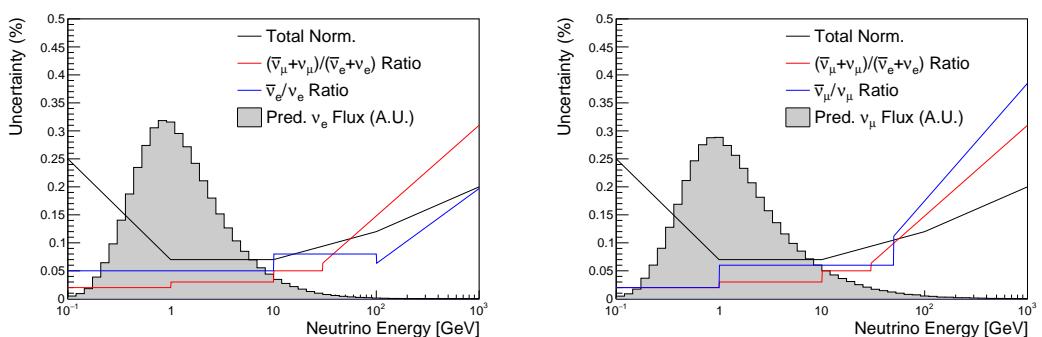


Figure 6.8: The uncertainty evaluated on the atmospheric ν_e (left) and ν_μ (right) flux predictions. The absolute normalisation and flavour ratio uncertainties are given. The solid histogram indicates the neutrino flux as a function of energy.

2244 Updates to the HKKM and Bartol models are underway [151] to use a similar
2245 tuning technique to that used in the beam flux predictions. After those updates,
2246 it may be possible to include correlations in the hadron production uncertainty
2247 systematics for beam and atmospheric flux predictions.

2248 6.4.3 Neutrino Interaction

2249 Neutrino interactions in the detectors are modeled by NEUT. The two indepen-
2250 dent oscillation analyses, T2K-only [197] and the SK-only [62], have developed
2251 separate interaction models. To maximise sensitivity out of this simultaneous
2252 beam and atmospheric analysis, a correlated interaction model has been defined
2253 in [186]. Where applicable, correlations allow the systematic uncertainties applied
2254 to the atmospheric samples to be constrained by near detector neutrino beam
2255 measurements. This can lead to stronger sensitivity to oscillation parameters
2256 as compared to an uncorrelated model.

2257 The low-energy T2K systematic model has a more sophisticated treatment
2258 of CCQE, 2p2h, and CCRES uncertainties, where extensive comparisons of
2259 this model have been performed to external data [197]. However, the model
2260 is not designed for high-energy atmospheric events, like those illustrated in
2261 Figure 5.11. Therefore the high energy systematic model from the SK-only
2262 analysis is implemented for the relevant multi-GeV, PC, and up- μ samples.
2263 The T2K CCQE model is more sophisticated so it has been implemented for
2264 all samples within this analysis, where separate low-energy and high-energy
2265 dials have been implemented. The low-energy dials are constrained by the near
2266 detector measurements and are uncorrelated to their high-energy counterparts.
2267 The author of this thesis was responsible for implementing and validating the
2268 combined cross-section model as documented in [186, 198].

2269 The high energy systematic model includes parameters developed from
2270 comparisons of Nieves and Rein-Seghal models which affect resonant pion
2271 producing interactions, comparisons of the GRV98 and CKMT models which
2272 control DIS interactions, and hadron multiplicity measurements which modulate

the normalisation of multi-pion producing events. The uncertainty on the ν_τ cross-section is particularly large and is controlled by a 25% normalisation uncertainty. These uncertainties are applied via normalisation or shape parameters. The former linearly scales the weight of all affected Monte-Carlo events, whereas the latter can increase or decrease a particular event's weight depending on its neutrino energy and mode of interaction. The response of the shape parameters is defined by third-order polynomial splines which return a weight for a particular neutrino energy. To reduce computational resources for the far detector fit, the response is binned by neutrino energy and sample binning: lepton momentum and cosine zenith binning for atmospheric splined responses and reconstructed neutrino energy and direction binning for beam samples. In total, 17 normalisation and 15 shape parameters are included in the high-energy model within this analysis.

Figure 6.9 indicates the predicted neutrino energy distribution for both beam and subGeV atmospheric samples. There is clearly significant overlap in neutrino energy between the subGeV atmospheric and beam samples, allowing similar kinematics in the final state particles. Figure 6.10 illustrates the fractional contribution of the different interaction modes per sample.

Comparing beam and atmospheric samples which target CCQE interactions (S.G. e-like 0de, S.G. μ -like [0,1]de, [FHC,RHC] 1R μ -like and [FHC,RHC] 1R e-like samples), there is a very similar contribution of CCQE, CC 2p2h, and CC1 π^\pm interactions. The samples which target CC1 π^\pm interactions, (S.G. e-like 0de, S.G. μ -like 2de and FHC 1R+1d.e e-like) also consist of very similar mode interactions.

As a consequence of the similarity in energy and mode contributions, correlating the systematic model between the beam and subGeV atmospheric samples ensures that this analysis attains the largest sensitivity to oscillation parameters while still ensuring neutrino interaction systematics are correctly accounted for. Due to its more sophisticated CCQE and 2p2h model, the T2K systematic model was chosen as the basis of the correlated model.

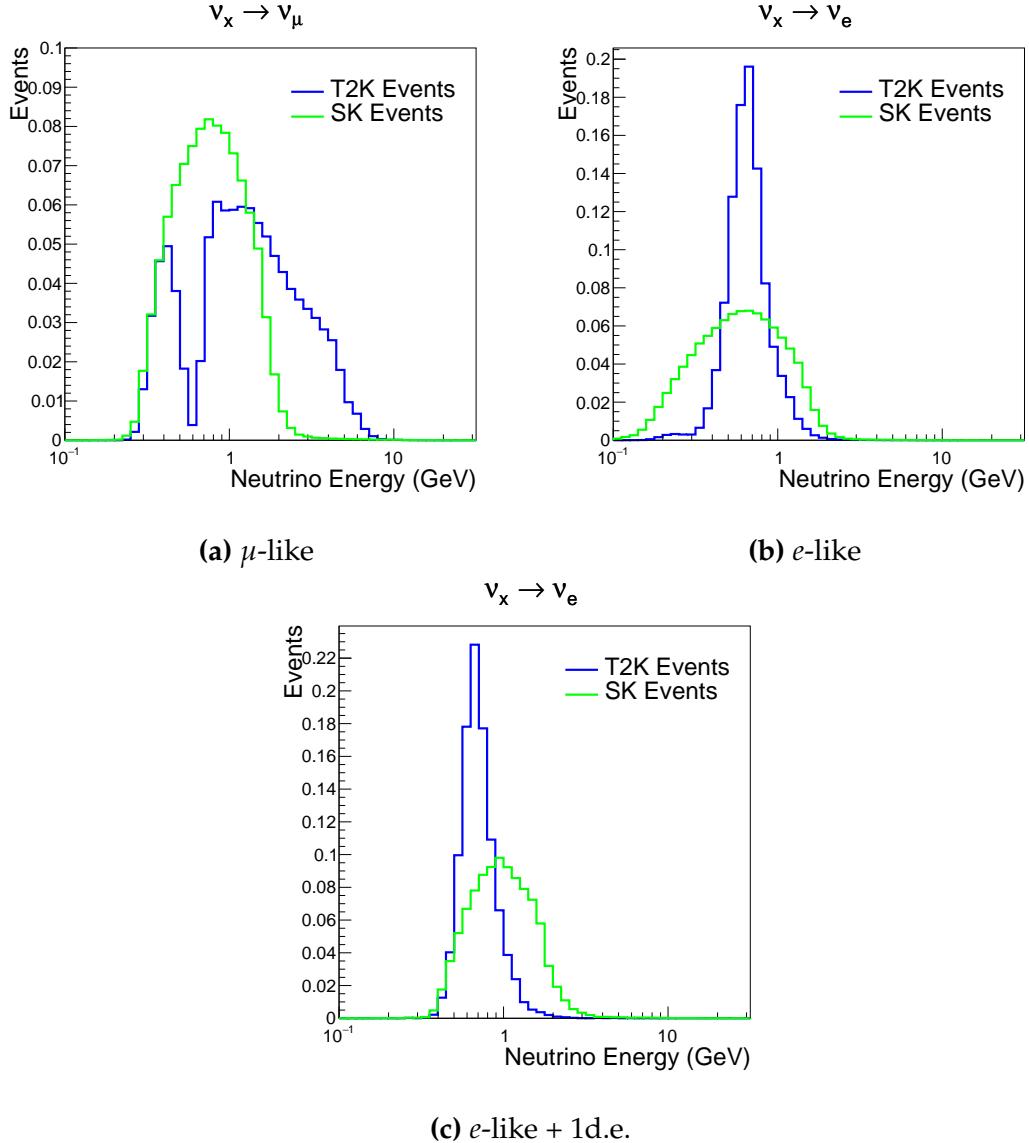


Figure 6.9: The predicted neutrino energy distribution for subGeV atmospheric and beam samples. FHC and RHC beam samples are summed together Asimov A oscillation parameters are assumed (given in Table 2.2). Beam and atmospheric samples with similar cuts are compared against one another.

2303 The T2K systematic model [197] is applied in a similar methodology to the
 2304 SK model parameters. It consists of 19 shape parameters and 24 normalisation
 2305 parameters. Four additional parameters, which model the uncertainty in the
 2306 binding energy, are applied in a way to shift the momentum of the lepton emitted
 2307 from a nucleus. This controls the uncertainty specified on the 27MeV binding
 2308 energy assumed within Equation 6.2. The majority of these parameters are

	CC QE	CC 2p2h	CC $1\pi^\pm$	CC $M\pi$	CC Other	NC $1\pi^0$	NC $1\pi^\pm$	NC $M\pi$	NC Coh.	NC Other
FHC 1R+1d.e. e-like	0.04	0.02	0.83	0.03	0.04	0.01	0.01	0.01	0.00	0.01
RHC 1R e-like	0.62	0.12	0.11	0.01	0.02	0.06	0.01	0.01	0.01	0.04
FHC 1R e-like	0.68	0.12	0.10	0.00	0.02	0.04	0.01	0.00	0.00	0.02
RHC 1R μ -like	0.62	0.13	0.17	0.02	0.03	0.00	0.02	0.00	0.00	0.00
FHC 1R μ -like	0.62	0.12	0.16	0.02	0.03	0.00	0.03	0.00	0.00	0.00
S.G. π^0 -like	0.05	0.01	0.02	0.00	0.01	0.68	0.06	0.07	0.06	0.04
S.G. μ -like 2de	0.04	0.01	0.80	0.10	0.04	0.00	0.00	0.00	0.00	0.00
S.G. μ -like 1de	0.72	0.11	0.12	0.01	0.02	0.00	0.01	0.00	0.00	0.00
S.G. μ -like 0de	0.68	0.11	0.10	0.01	0.02	0.01	0.05	0.01	0.00	0.02
S.G. e-like 1de	0.05	0.01	0.75	0.10	0.05	0.00	0.01	0.02	0.00	0.01
S.G. e-like 0de	0.73	0.11	0.10	0.01	0.02	0.02	0.00	0.00	0.00	0.00

Figure 6.10: The interaction mode contribution of each sample given as a fraction of the total event rate in that sample. Asimov A oscillation parameters are assumed (given in Table 2.2). The Charged Current (CC) modes are broken into quasi-elastic (QE), 2p2h, resonant charged pion production ($1\pi^\pm$), multi-pion production ($M\pi$), and other interaction categories. Neutral Current (NC) interaction modes are given in interaction mode categories: π^0 production, resonant charged pion production, multi-pion production, and others.

2309 assigned a Gaussian prior uncertainty. Those that have no reasonably motivated
 2310 uncertainty, or those which have not been fit to external data, are assigned a
 2311 flat prior which does not affect the penalty term.

2312 On top of the combination of the SK and T2K interaction models, several
 2313 other parameters have been specifically developed for the joint oscillation anal-
 2314 ysis. The majority of the atmospheric samples' δ_{CP} sensitivity comes from the
 2315 normalisation of subGeV electron-like events. These are modeled using a spectral
 2316 function to approximate the nuclear ground state. However, the near detector is
 2317 not able to constrain the model so an additional systematic is introduced which
 2318 models an alternative Continuous Random Phase Approximation (CRPA) nuclear
 2319 ground state. This dial approximates the event weights if a CRPA model had

been assumed rather than a spectral function. This dial only applies to ν_e and $\bar{\nu}_e$ as the near detector does not constraint ν_e cross-section measurements. It is applied as a shape parameter.

Further additions to the model have been introduced due to the inclusion of the subGeV π^0 atmospheric sample. This particularly targets charged current and neutral current π^0 producing interactions to help constrain the systematic uncertainties. Therefore, an uncertainty that affects neutral current resonant π^0 production is incorporated into this analysis. Comparisons of NEUT's NC resonant pion production predictions have been made to MiniBooNE [199] data and a consistent 16% to 21% underprediction is observed [186]. Consequently, a conservative 30% normalisation parameter is invoked.

Down-going events are mostly insensitive to oscillation parameters and can act similar to the near detector within an accelerator experiment (Details will be discussed in chapter 7). This region of phase space can act as a sideband and allows the cross-section model and near detector constraint to be studied. The distribution of events in this region is calculated using the technique outlined in subsection 4.3.4. The results are illustrated in Figure 6.11. For CCQE-targeting samples, the application of the near detector constraint is well within the statistical fluctuation of the down-going data. This means there is no significant tension is observed between the data and the Monte Carlo prediction after the near detector constraint is applied. This is not the case for samples with target CCRES interactions. The electron-like data is consistent with the constrained prediction at high reconstructed momenta but diverges at lower momentum, whereas the muon-like sample is under-predicted throughout the range of momenta. To combat this disagreement, an additional cross-section systematic dial, specifically designed to inflate the low pion momentum systematics was developed in [186]. This is a shape parameter implemented through a splined response.

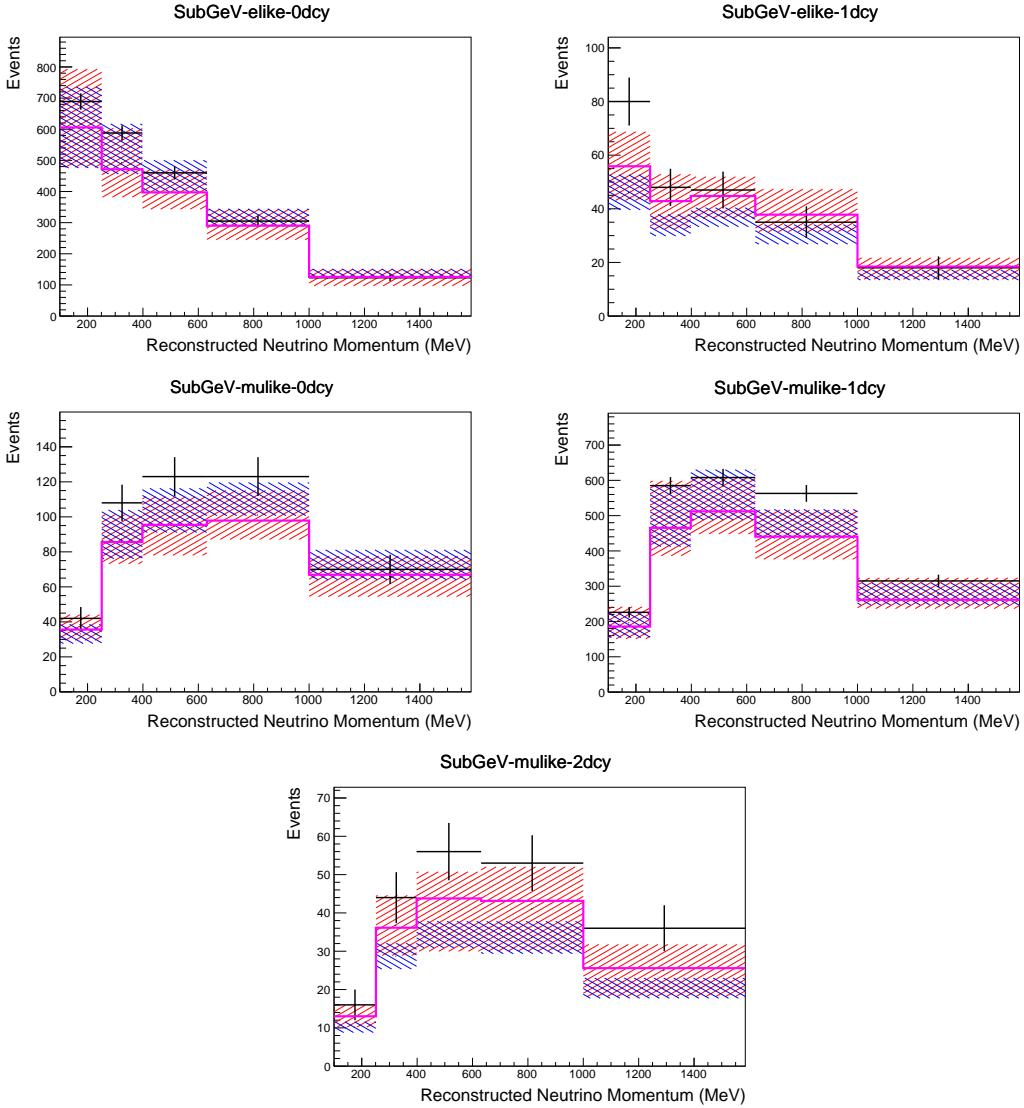


Figure 6.11: Down-going atmospheric subGeV single-ring samples comparing the mean and error of the pre-fit and post-fit Monte Carlo predictions in red and blue, respectively. The magenta histogram illustrates the Monte Carlo prediction using the generated dial values. The black points illustrate the down-going data with statistical errors given. The mean and errors of the Monte Carlo predictions are calculated by the techniques documented in subsection 4.3.4. The pre-fit spectrum is calculated by throwing the cross-section and atmospheric flux dial values from the pre-fit covariance matrix. The post-fit spectrum is calculated by sampling the cross-section dial values from an ND fit MCMC chain, whilst still throwing the atmospheric flux dials from the pre-fit covariance.

6.4.4 Near Detector

2348 The systematics applied due to uncertainties arising from the response of the near
 2349 detector is documented in [126]. The response is described by 574 normalisation
 2350 parameters binned in the selected sample as well as momentum and angle,

P_μ and $\cos(\theta_\mu)$, of the final-state muon. These are applied via a covariance matrix with each parameter being assigned a Gaussian prior from that covariance matrix. These normalisation parameters are built from underlying systematics, e.g. pion secondary interaction systematics, which are randomly thrown and the variation in each $P_\mu \times \cos(\theta_\mu)$ bin is determined. Two thousand throws are evaluated and a covariance matrix response is created. This allows significant correlations between FGD1 and FGD2 samples, as well as adjacent $P_\mu \times \cos(\theta_\mu)$ bins. Statistical uncertainties are accounted for by including fluctuations of each event's weight from a Poisson distribution.

Similar to the cross-section systematics, MaCh3 and BANFF are used to constrain the uncertainty of these systematics through independent validations. Each fitter generates a post-fit covariance matrix which is compared and passed to the far-detector oscillation analysis working group. As the analysis presented within this thesis uses the MaCh3 framework, a joint oscillation analysis fit of all three sets of samples and their respective systematics is performed.

6.4.5 Far Detector

Two configurations of the far detector systematic model implementation have been considered. Firstly, the far detector systematic uncertainties for beam and atmospheric samples are taken from their respective analysis inputs, denoted “official inputs” analysis, with no correlations assumed between the beam and atmospheric samples. The beam- and atmospheric-specific inputs are documented in subsubsection 6.4.5.1 and subsubsection 6.4.5.2. Secondly, an alternative detector model has been developed which correlates the response of the SK detector systematics between the beam and atmospheric samples. Here, the distribution of parameters used for applying event cuts (e.g. electron-muon PID separation) is modified within the fit. It follows a similar methodology to the beam far detector systematics implementation but performs a joint fit of the beam and atmospheric data. This alternative implementation is detailed in subsubsection 6.4.5.3.

2380 **6.4.5.1 Beam Samples**

2381 There are 45 systematics which describe the response of the far detector to
 2382 beam events [179], split into 44 normalisation parameters and one energy scale
 2383 systematic. The energy scale systematic is applied as a multiplicative scaling
 2384 of the reconstructed neutrino energy. It is estimated from data-to-Monte Carlo
 2385 differences in the stopping muon sample in [62] and found to be 2.1%. The
 2386 normalisation parameters are assigned a Gaussian error centered at one with
 2387 width taken from a covariance matrix. A detailed breakdown of the generation
 2388 of the covariance matrix is found in [192]. To build the covariance matrix, a fit
 2389 is performed on atmospheric data which has been selected using beam sample
 2390 selection cuts. These cuts use the variables, L^i , where the index i is detailed in
 2391 Table 6.7. Each L^i is a smear, α , and shift, β parameter such that,

$$L_j^i \rightarrow \bar{L}_j^i = \alpha_j^i L + \beta_j^i. \quad (6.4)$$

2392 Where L_j^i (\bar{L}_j^i) correspond to nominal(varied) PID cut parameters given in
 2393 Table 6.7. The shift and smear parameters are nuisance parameters with no prior
 2394 constraints. They are binned by final-state topology, j , where the binning is given
 2395 in Table 6.8. The final-state topology binning is because the detector will respond
 2396 differently to events that have one or multiple rings. For example, the detector
 2397 will be able to distinguish single-ring events better than two overlapping ring
 2398 events, resulting in different systematic uncertainty for one-ring events compared
 2399 to two-ring events. This approach is used to allow the cut parameter distributions
 2400 to be modified within the fit, allowing for better data to Monte Carlo agreement.

Cut Variable	Parameter Name
0	<code>fitQun e/mu PID</code>
1	<code>fitQun e/pi0 PID</code>
2	<code>fitQun mu/pi PID</code>
3	<code>fitQun Ring-Counting Parameter</code>

Table 6.7: List of cut variables that are included within the shift/smear fit documented in [192].

Category	Description
1e	Only one electron above Cherenkov threshold in the final state
1 μ	Only one muon above Cherenkov threshold in the final state
1e+other	One electron and one or more other charged particles above Cherenkov threshold in the final state
1 μ +other	One muon and one or more other charged particles above Cherenkov threshold in the final state
1 π^0	Only one π^0 in the final state
1 π^\pm or 1p	Only one hadron (typically charged pion or proton) in the final state
Other	Any other final state

Table 6.8: Reconstructed event topology categories on which the SK detector systematics [192] are based.

2401 The mis-modeling of π^0 events is also considered. If one of the two rings
 2402 from a π^0 event is missed, this will be reconstructed as a CC ν_e -like event. This
 2403 is one of the largest systematics hindering the electron neutrino appearance
 2404 analyses. Consequently, additional systematics have been introduced to con-
 2405 strain the mis-modeling of π^0 events in SK, binned by reconstructed neutrino
 2406 energy. To evaluate this systematic uncertainty, a set of “hybrid- π^0 ” samples is
 2407 constructed. These events are built by overlaying one electron-like ring from
 2408 the SK atmospheric neutrino samples or decay electron ring from a stopping
 2409 cosmic ray muon with one simulated photon ring. Both rings are chosen so
 2410 that momenta and opening angle follow the decay kinematics of NC π^0 events
 2411 from the T2K-MC. Hybrid- π^0 Monte Carlo samples with both rings from the
 2412 SK Monte Carlo are produced to compare with the hybrid- π^0 data samples and
 2413 the difference in the fraction of events that pass the ν_e selection criteria is used
 2414 to assign the systematic errors. In order to investigate any data to Monte Carlo
 2415 differences that may originate from either the higher energy ring or lower energy
 2416 ring, two samples are built; a sample in which the electron constitutes the higher
 2417 energy ring from the π^0 decay (called the primary sample) and another one in
 2418 which it constitutes the lower energy ring (called the secondary sample). The
 2419 standard T2K ν_e fitQun event selection criteria are used to select events.

2420 Final contributions to the covariance matrix are determined by supplemen-
 2421 tary uncertainties obtained by comparing stopping muon data to Monte Carlo

prediction, as first introduced in section 5.2. The efficiency of tagging decay electrons is estimated by the stopping muon data to Monte Carlo differences by comparing the number of one decay electron events to the number of events with one or fewer decay electrons. Similarly, the rate at which fake decay electrons are reconstructed by `fiTQun` is estimated by comparing the number of two decay electron events to the number of events with one or two reconstructed decay electrons. The two sources of systematics are added in quadrature weighted by the number of events with one true decay electron yielding a 0.2% systematic uncertainty. A fiducial volume systematic of $\pm 2.5\text{cm}$ which corresponds to a 0.5% shift in the normalisation of events is also applied. Additional normalisation uncertainties based on neutrino flavour and interaction mode are also defined in [179, 200, 201].

Two additional sources of uncertainty are included: secondary and photoneuclear interactions. These are estimated by varying the underlying parameters are building a distribution of sample event rates. These contributions are then added in quadrature to the above covariance matrix. The final uncertainty on the SK detector systematics are provided in Figure 6.12.

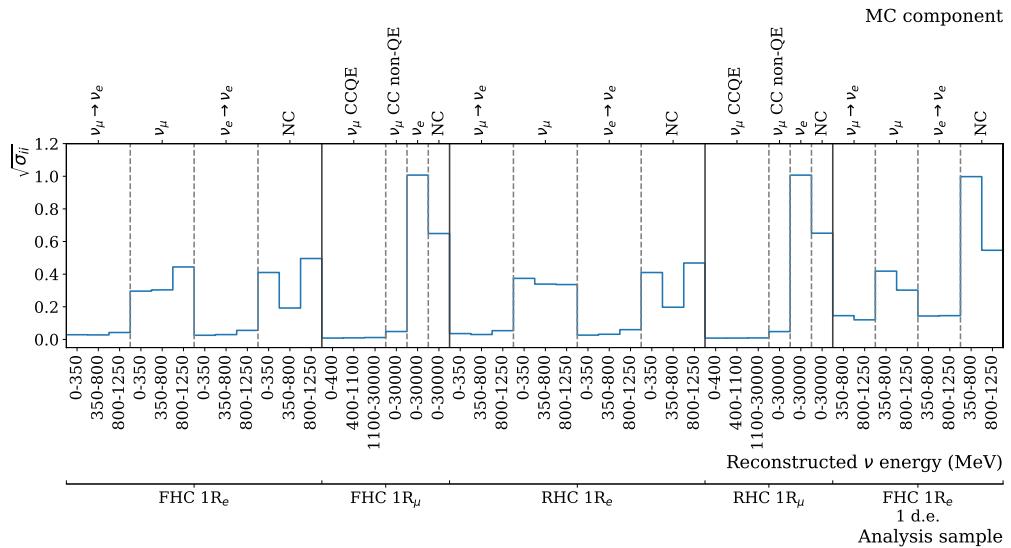


Figure 6.12: The fractional uncertainty on each of the 44 parameters describing the SK detector systematics (The energy scale systematic is neglected). The parameters are split by sample, oscillation channel, interaction mode and reconstructed neutrino energy.

2439 6.4.5.2 Atmospheric Samples

2440 The detector systematics for atmospheric samples, documented in [3], are split
2441 into two sub-groups: those which are related to particle identification and ring
2442 counting systematics, and those which are related to calibration, separation,
2443 and reduction uncertainties.

2444 The particle identification systematics consist of five parameters. The ring sep-
2445 aration systematic enforces an anti-correlated response between the single-ring
2446 and multi-ring samples. This is implemented as a fractional increase/decrease
2447 in the overall normalisation of each sample, depending on the distance to the
2448 nearest wall from an event's vertex. The coefficients of the normalisation are
2449 estimated prior to the fit and depend on the particular atmospheric sample. Two
2450 electron-muon separation systematics are included within this model which
2451 anti-correlates the response of the electron-like and muon-like samples: one for
2452 single-ring events and another for multi-ring events.

2453 The multi-ring electron-like separation likelihood, discussed in section 6.1,
2454 encodes the ability of the detector to separate neutrino from anti-neutrino events.
2455 Two normalisation parameters vary the relative normalisation of multi-ring ν_e
2456 and $\bar{\nu}_e$ samples whilst keeping a consistent overall event rate.

2457 There are 22 systematics related to calibration measurements, including effects
2458 from backgrounds, reduction, and showering effects. They are documented in
2459 [3] and are briefly summarised in Table 6.9. They are applied via normalisation
2460 parameters, with the separation systematics requiring the conservation of event
2461 rate across all samples.

2462 6.4.5.3 Correlated Detector Model

2463 A complete uncertainty model of the SK detector would be able to determine
2464 the systematic shift on the sample spectra for a variation of the underlying
2465 parameters, e.g. PMT angular acceptance. However, this is computationally
2466 intensive, requiring Monte Carlo predictions to be made for each plausible
2467 variation. Consequently, an effective parameter model has been utilised for

Index	Description
0	Partially contained reduction
1	Fully contained reduction
2	Separation of fully contained and partially contained events
3	Separation of stopping and through-going partially contained events in top of detector
4	Separation of stopping and through-going partially contained events in barrel of detector
5	Separation of stopping and through-going partially contained events in bottom of detector
6	Background due to cosmic rays
7	Background due to flasher events
8	Vertex systematic moving events into and out of fiducial volume
9	Upward going muon event reduction
10	Separation of stopping and through-going in upward going muon events
11	Energy systematic in upward going muon events
12	Reconstruction of the path length of upward going muon events
13	Separation of showering and non-showering upward going muon events
14	Background of stopping upward going muon events
15	Background of non-showering through-going upward going muon events
16	Background of showering through-going upward going muon events
17	Efficiency of tagging two rings from π^0 decay
18	Efficiency of decay electron tagging
19	Background from downgoing cosmic muons
20	Asymmetry of energy deposition in tank
21	Energy scale deposition

Table 6.9: Sources of systematic errors specified within the grouped into the “calibration” systematics model.

²⁴⁶⁸ a correlated detector model following from the T2K-only model implementation
²⁴⁶⁹ documented in subsubsection 6.4.5.1. It correlates the detector systematics
²⁴⁷⁰ between the far-detector beam and subGeV atmospheric samples due to their
²⁴⁷¹ similar energies and interaction types. As there are no equivalent beam samples,
²⁴⁷² the multi-GeV, multiring, PC, and Up- μ samples will be subject to the particle
²⁴⁷³ identification systematics implementation as described in subsubsection 6.4.5.2
²⁴⁷⁴ rather than using this correlated detector model. The calibration systematics also
²⁴⁷⁵ described in the aforementioned chapter still apply to all atmospheric samples.
²⁴⁷⁶ The correlated detector model utilises the same smear and shift parameters
²⁴⁷⁷ documented in subsubsection 6.4.5.1, split by final state topology. Beyond this,

2478 the shift and smear parameters are split by visible energy deposited within the
 2479 detector, with binning specified in Table 6.10. This is because atmospheric events
 2480 are categorised by subGeV and multi-GeV events based on visible energy, so
 2481 this splitting is required when correlating the systematic model for beam and
 2482 atmospheric events. Alongside the technical requirement, higher energy events
 2483 will be better reconstructed due to fractionally less noise within the detector. As
 2484 a result of the inclusion of visible energy binning, Equation 6.4 becomes

$$L_{jk}^i \rightarrow \bar{L}_{jk}^i = \alpha_{jk}^i L + \beta_{jk}^i, \quad (6.5)$$

2485 where k is the visible energy bin.

Index	Range (MeV)
0	$30 \geq E_{vis} > 300$
1	$300 \geq E_{vis} > 700$
2	$700 \geq E_{vis} > 1330$
3	$E_{vis} \geq 1330$

Table 6.10: Visible energy binning for which the correlated SK detector systematics are based

2486 The implementation of this systematic model takes the events reconstructed
 2487 values of the cut parameters, modifies them by the particular shift and smear
 2488 parameter for that event, and then re-applies event selection. This causes event
 2489 migration, which is a new feature incorporated into the MaCh3 framework which
 2490 is only achievable due to the event-by-event reweighting scheme.

2491 Particular care has to be taken when varying the ring counting parameter.
 2492 This is because the number of rings is a finite value (one-ring, two-ring, etc.)
 2493 which can not be continuously varied through this shift and smear technique.
 2494 Consequently a continuous ring counting parameter, RC_i , is calculated for the
 2495 i^{th} event, following the definition in [178]: the preferred likelihoods from all
 2496 considered one-ring (L_{1R}) and two-ring (L_{2R}) fits are determined. The difference

2497 is computed as $\Delta_{LLH} = \log(L_{1R}) - \log(L_{2R})$. The ring counting parameter is
 2498 then defined as

$$RC_i = \text{sgn}(\Delta_{LLH}) \times \sqrt{|\Delta_{LLH}|}, \quad (6.6)$$

2499 where $\text{sgn}(x) = x/|x|$. This ring counting parameter corresponds to an
 2500 intermediate likelihood value used within the `fitQun` algorithm to decide the
 2501 number of rings associated with a particular event. However, fake-ring merging
 2502 algorithms are applied after this likelihood value is used. Consequently, this
 2503 ring counting parameter does not always exactly correspond to the number of
 2504 reconstructed rings. This can be seen in Figure 6.13.

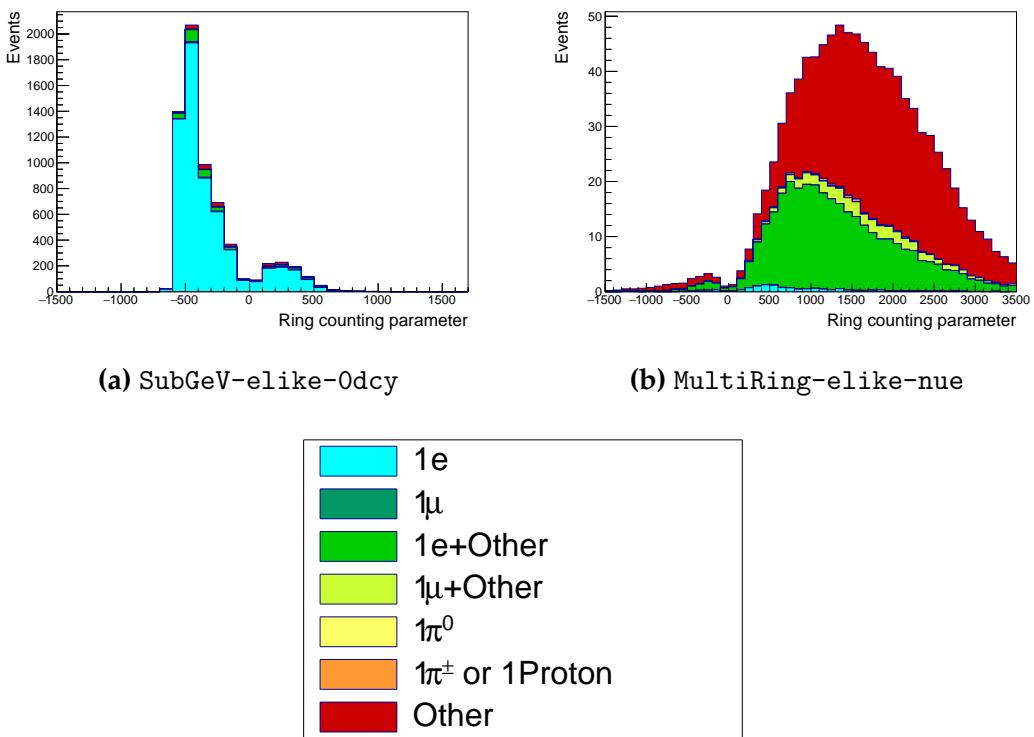


Figure 6.13: The ring counting parameter as defined in Equation 6.6 for the SubGeV-elike-0dcy and MultiRing-elike-nue samples.

2505 As the `fitQun` algorithm does not provide a likelihood value after the fake-
 2506 ring algorithms have been applied, the ring counting parameter distribution is
 2507 correlated to the final number of reconstructed rings through “maps”. These

2508 are two-dimensional distributions of the ring counting parameter and the final
 2509 number of reconstructed rings. An example is illustrated in Figure 6.14. In
 2510 principle, the `fitQun` reconstruction algorithm should be re-run after the variation
 2511 in the ring counting parameter. However, this is not computationally viable.
 2512 Therefore the “maps” are used as a reweighting template.

2513 The maps are split by final state topology and true neutrino flavour and
 2514 all `fitQun`-reconstructed Monte Carlo events are used to fill them. The maps
 2515 are row-normalised to represent the probability of X rings for a given RC_i
 2516 value. Prior to the oscillation fit, an event’s nominal weight is calculated as
 2517 $W^i(N_{Rings}^i, L_{jk}^i)$, where N_{Rings}^i is the reconstructed number of rings for the i^{th}
 2518 event and $W^i(x, y)$ is the bin content in map associated with the i^{th} event, where
 2519 x number of rings and y is ring counting parameter. Then during the fit, the
 2520 value of $R = W^i(N_{Rings}^i, \bar{L}_{jk}^i) / W^i(N_{Rings}^i, L_{jk}^i)$ is calculated as the event weight
 2521 for the i^{th} event. This is the only cut variable that uses a reweighting technique
 2522 rather than event migration.

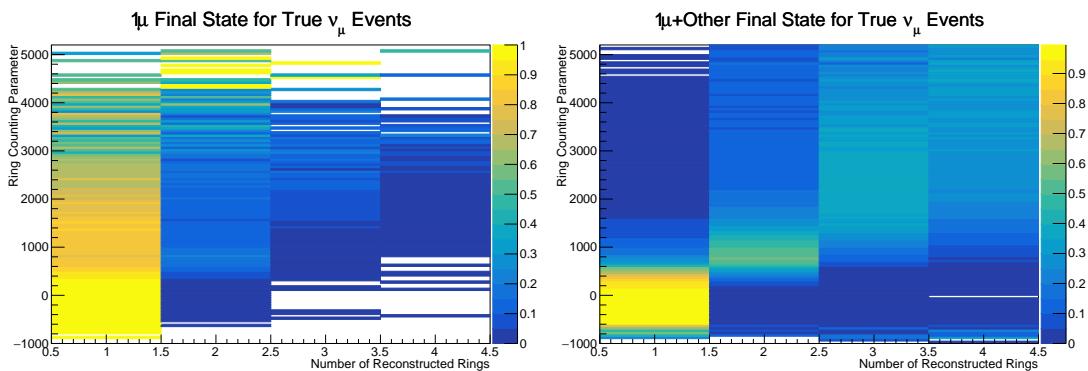


Figure 6.14: The ring counting parameter, defined in Equation 6.6, as a function of the number of reconstructed rings as found by the `fitQun` reconstruction algorithm. Left: true ν_μ events with only one muon above the Cherenkov threshold in the final state. Right: true ν_μ events with one muon and at least one other charged particle above the Cherenkov threshold in the final state.

2523 The π^0 systematics introduced in subsection 6.4.4 are applied via a covariance
 2524 matrix. This is not possible in the alternative model as no covariance matrix
 2525 is used. Thus, the implementation of the π^0 systematics has been modified.
 2526 The inputs from the hybrid π^0 sample are included via the use of “ χ^2 maps”,

which are two-dimensional histograms in α_{jk}^i and β_{jk}^i parameters over some range. Illustrative examples of the χ^2 maps are given in Figure 6.15. Due to their nature, the shift and smear parameters are typically very correlated. A map is produced for each cut parameter given in Table 6.7 and for each visible energy bin given in Table 6.10.

The maps are filled through the χ^2 comparison of the hybrid π^0 Monte Carlo and data in the particle identification parameters documented in Table 6.7. The Monte Carlo distribution is modified by the α_{jk}^i and β_{jk}^i scaling, whilst cross-section and flux nuisance parameters are thrown from their prior uncertainties. The χ^2 between the scaled Monte Carlo and data is calculated and the relevant point in the χ^2 map is filled.

The implementation within this alternative detector model is to add the bin contents of the maps, for the relevant values of the α_{jk}^i and β_{jk}^i parameters, to the likelihood penalty. Only $1\pi^0$ final state topology shift and smear parameters use this prior uncertainty.

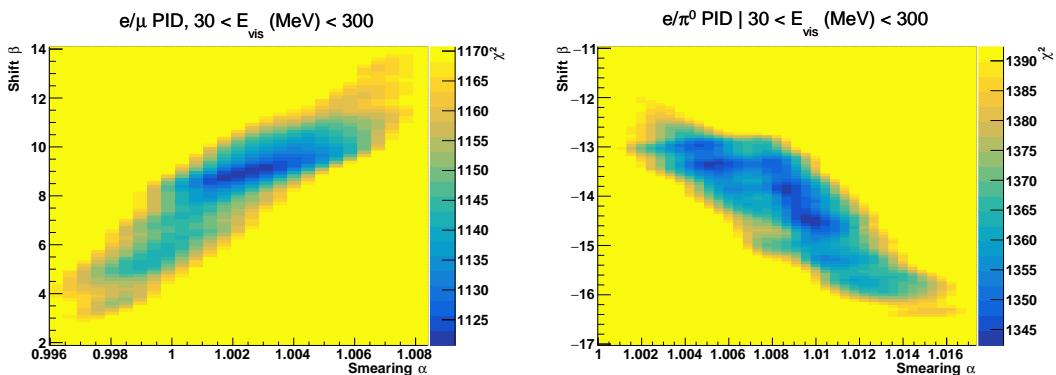


Figure 6.15: The χ^2 between the hybrid- π^0 Monte Carlo and data samples, as a function of smear (α) and shift (β) parameters, for events which have $1\pi^0$ final state topology. Left: Electron-muon separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$. Right: Electron- π^0 separation PID parameter for events with $30 \leq E_{\text{vis}}(\text{MeV}) < 300$.

Similarly, the implementation of the supplementary systematics documented in subsubsection 6.4.5.1 needs to be modified. A new framework [202] was built in tandem between the author of this thesis and the T2K-SK working group [179] so the additional parameters can be incorporated into the MaCh3 framework. These are applied as normalisation parameters, depending on the particular

2547 interaction mode, number of tagged decay electrons, and whether the primary
2548 particle generated Cherenkov light. They are assigned Gaussian uncertainties
2549 with widths described by a covariance matrix. Furthermore, the secondary
2550 interaction and photo-nuclear effects need to be accounted for in this detector
2551 model using a different implementation than that in subsubsection 6.4.5.1. This
2552 was done by including a shape parameter for each of the secondary interactions
2553 and the photo-nuclear systematic parameters.

2554 There are a total of 224 α_{jk}^i and β_{jk}^i parameters, of which 32 have prior
2555 constraints from the hybrid π^0 samples.

2556 One final complexity of this correlated detector model is that the two sets
2557 of samples, beam and subGeV atmospheric, use slightly different parameters
2558 to distinguish electron and muon-like events. The T2K samples use the value
2559 of $\log(L_e/L_\mu)$ whereas the atmospheric samples use the value of $\log(L_e/L_\pi)$,
2560 where L_X is the likelihood for hypothesis X. This is because the T2K fits use
2561 single-ring fiTQun fitting techniques, whereas multi-ring fits are applied to the
2562 atmospheric samples where only the electron and pion hypothesis are considered.
2563 The correlation between the two likelihood ratios is illustrated in Figure 6.16. As
2564 discussed in section 5.2, the pion hypothesis is a very good approximation of the
2565 muon hypothesis due to their similar mass. Consequently, using the same shift
2566 and smear parameters correlated between the beam and subGeV atmospheric
2567 samples is deemed a good approximation.

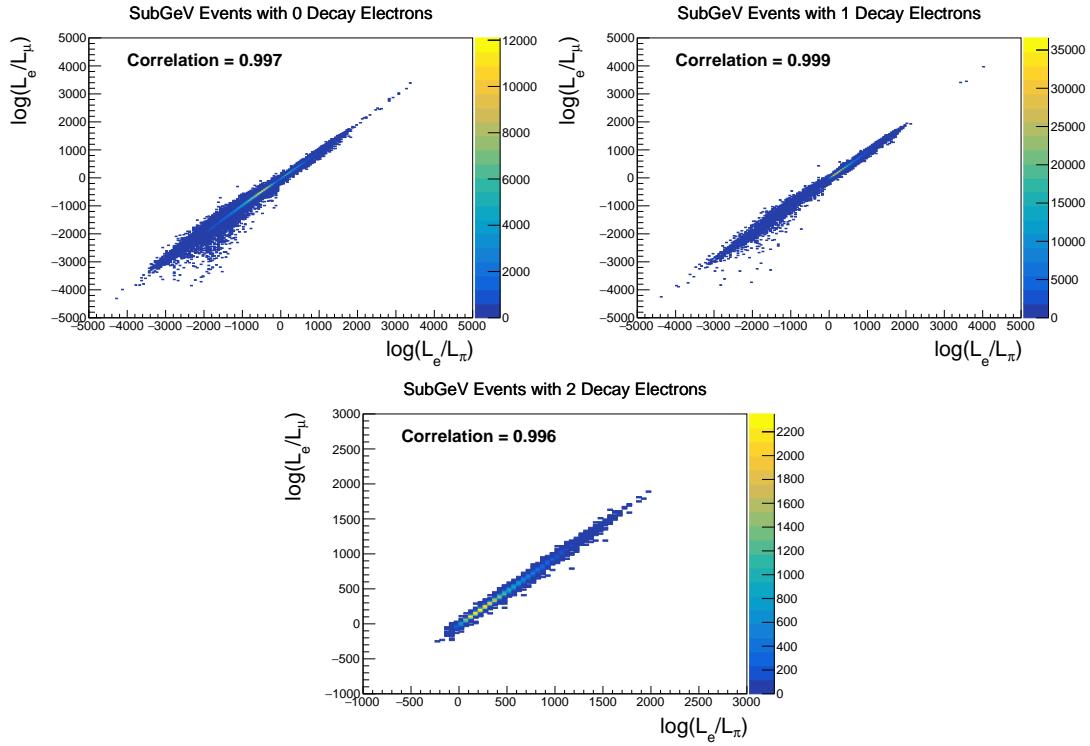


Figure 6.16: The distribution of $\log(L_e/L_\mu)$ compared to $\log(L_e/L_\pi)$ for subGeV events with zero (top left), one (top right) or two (bottom) decay electrons. The correlation in the distribution is calculated as 0.997, 0.999 and 0.996, respectively.

6.5 Likelihood Calculation

This analysis performs a joint oscillation parameter fit of the ND280 beam samples, the T2K far detector beam samples, and the SK atmospheric samples introduced in this chapter.

Once the Monte Carlo predictions of each beam and atmospheric sample have been built, a likelihood needs to be constructed. This is done by comparing the binned Monte Carlo prediction to binned data. The Monte Carlo prediction is calculated at a particular point, $\vec{\theta}$, in the model parameter space such that $N_i^{MC} = N_i^{MC}(\vec{\theta})$, where N_i represents the bin content of the i^{th} bin. The data and Monte Carlo spectra are represented by N_i^D and N_i^{MC} , respectively. The bin contents for the beam near detector, beam far detector and atmospheric samples are denoted with *ND*, *FD*, and *Atm*, respectively. Taking the FHC1Rmu far detector sample as an example, the binning index runs over all the reconstructed neutrino

2581 energy bins. The likelihood calculation between the data and the Monte Carlo
 2582 prediction for a particular bin follows a Poisson distribution, where the data
 2583 is treated as a fluctuation of the simulation.

2584 The data can consist of either real data or an ‘Asimov’ Monte Carlo prediction,
 2585 which is typically used for sensitivity studies and denoted ‘Asimov data’. The
 2586 process for building Asimov data is as follows. The Monte Carlo prediction is
 2587 reweighted using a particular set of oscillation parameters (potentially those
 2588 listed in Table 2.2) and systematic parameter tune. The resulting spectra for each
 2589 sample is then defined to be the Asimov data for that sample. Whilst this results
 2590 in unphysical non-integer data predictions, it eliminates statistical fluctuations
 2591 from the data. Therefore, the results of a fit to Asimov data should not include any
 2592 biases from statistical fluctuations. Furthermore, these results should produce
 2593 posterior probability distributions consistent with the parameters which were
 2594 used to make the data prediction. That is to say, the fit results should return the
 2595 known parameters. Any biases seen would be attributed to correlations between
 2596 each oscillation parameter and correlations between oscillation and systematic
 2597 parameters. Consequently, Asimov fit results present the maximum precision
 2598 at which the oscillation parameters could be measured to.

2599 Following the T2K analysis presented in [2], the likelihood contribution for
 2600 the near detector samples also includes a Monte Carlo statistical uncertainty
 2601 term, derived from the Barlow and Beeston statistical treatment [203, 204]. It
 2602 includes a contribution to the likelihood that treats the generated Monte Carlo
 2603 prediction as a statistical fluctuation of the actual true simulation assuming an
 2604 infinite amount of statistics had been created. The technical implementation of
 2605 this additional likelihood term is documented in [187] and briefly summarised
 2606 as follows. The term is defined as,

$$\frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2}, \quad (6.7)$$

2607 where β_i represents a scaling parameter for the i^{th} bin that relates the bin
 2608 content for the amount of Monte Carlo actually generated N_i^{MC} to the bin content

2609 if an infinite amount of Monte Carlo statistics had been generated $N_{i,true}^{MC}$, such
 2610 that $N_{i,true}^{MC} = \beta_i \times N_i^{MC}$. In the case where a sufficient amount of Monte Carlo
 2611 statistics had been generated, $\beta_i = 1$. An analytical solution for β_i is given in
 2612 [187]. Additionally, $\sigma_{\beta_i} = \sqrt{\sum_i w_i^2 / N_i^{MC}}$ where $\sqrt{\sum_i w_i^2}$ represents the sum of
 2613 the square of the weights of the Monte Carlo events which fall into bin i .

2614 An additional contribution to the likelihood comes from the variation of
 2615 the systematic model parameters. For those parameters with well-motivated
 2616 uncertainty estimates, a covariance matrix, V , describes the prior knowledge of
 2617 each parameter as well as any correlations between the parameters. Due to a
 2618 technical implementation, a single covariance matrix describes each “block” of
 2619 model parameters, e.g. beam flux systematics. The covariance matrix associated
 2620 with the k^{th} block is denoted V^k . There are seven individual blocks of system-
 2621 atics ($N_b = 7$): oscillation parameters, beam flux systematics, atmospheric flux
 2622 systematics, neutrino interaction systematics, near detector systematics, beam
 2623 far detector systematics, and atmospheric far detector systematics. The number
 2624 of parameters in the k^{th} block is defined as $n(k)$.

2625 The equation for the likelihood \mathcal{L} includes all the terms discussed above,

$$\begin{aligned}
 -\ln(\mathcal{L}) = & \\
 & \sum_i^{\text{NDbins}} N_i^{\text{ND},MC}(\vec{\theta}) - N_i^{\text{ND},D} + N_i^{\text{ND},D} \times \ln \left[N_i^{\text{ND},D} / N_i^{\text{ND},MC}(\vec{\theta}) \right] + \frac{(\beta_i - 1)^2}{2\sigma_{\beta_i}^2} \\
 & + \sum_i^{\text{FDbins}} N_i^{\text{FD},MC}(\vec{\theta}) - N_i^{\text{FD},D} + N_i^{\text{FD},D} \times \ln \left[N_i^{\text{FD},D} / N_i^{\text{FD},MC}(\vec{\theta}) \right] \\
 & + \sum_i^{\text{Atmbins}} N_i^{\text{Atm},MC}(\vec{\theta}) - N_i^{\text{Atm},D} + N_i^{\text{Atm},D} \times \ln \left[N_i^{\text{Atm},D} / N_i^{\text{Atm},MC}(\vec{\theta}) \right] \\
 & + \frac{1}{2} \sum_k^{\text{N}_b} \sum_i^{n(k)} \sum_j^{n(k)} (\vec{\theta}^k)_i (V^k)_{ij}^{-1} (\vec{\theta}^k)_j.
 \end{aligned} \tag{6.8}$$

2626 The negative log-likelihood value is determined at each step of the MCMC
 2627 to build the posterior distribution defined in chapter 4. This value is minimised
 2628 when the Monte Carlo prediction tends towards the data spectrum.

7

2629

2630

Oscillation Probability Calculation

2631 It is important to understand how and where the sensitivity to the oscillation parameters comes from for both atmospheric and beam samples. An
2632 overview of how these samples respond to changes in δ_{CP} , Δm_{32}^2 , and $\sin^2(\theta_{23})$
2633 is given in section 2.5. This section also explains the additional complexities
2634 involved when performing an atmospheric neutrino analysis as compared to
2635 a beam-only analysis.

2636 Without additional techniques, atmospheric sub-GeV upward-going neutrinos ($E_\nu < 1.33\text{GeV}, \cos(\theta_Z) < 0.$) can artificially inflate the sensitivity to $\delta_{CP,zaza}$
2637 due to the quickly varying oscillation probability in this region. Therefore, a
2638 “sub-sampling” approach has been developed to reduce these biases ensuring
2639 accurate and reliable sensitivity measurements. This technique ensures that small-
2640 scale unresolvable features of the oscillation probability have been averaged over
2641 whilst the large-scale features in the oscillation probability are unaffected. The
2642 documentation and validation of this technique are found in section 7.1. The
2643 oscillation probability calculation is computationally intensive due to the large
2644 number of matrix multiplications needed. Consequently, the CUDAProb3 imple-
2645 mentation choice made within the fitting framework, as detailed in section 7.2,
2646 ensures that the analysis can be done in a timely manner.

Whilst the beam neutrinos are assumed to propagate through a constant density slab of material, the density variations through the Earth result in more complex oscillation patterns for atmospheric neutrinos. Furthermore, the uncertainty in the electron density can modify the oscillation probability for the denser core layers of the Earth. The model of the Earth used within this analysis is detailed in section 7.3. This includes information about the official SK-only methodology as well as improvements that have been made to remove some of the approximations used in that analysis. Another complexity of atmospheric neutrino oscillation studies is that the height of production in the atmosphere is not known on an event-by-event basis. An analytical averaging technique that approximates the uncertainty of the oscillation probability has been followed, with the author of this thesis being responsible for the implementation and validation. This implementation of an external technique is described in section 7.4.

7.1 Treatment of Fast Oscillations

As shown in Figure 7.1, atmospheric neutrino oscillations have a significantly more complex structure for upgoing neutrinos with energy below 1GeV. This is because the L/E dependence of the oscillation probability in this region induces rapid variations for small changes in L or E . As discussed in section 2.5, this is also the region in which atmospheric neutrinos have sensitivity to δ_{CP} . In practice, the direction of the neutrino is inferred from the direction of the final state particles traveling in the detector. The correlation between these two directions can be particularly weak for low-energy neutrino interactions. This creates a distinct difference from the beam neutrinos where the position of the source is very precisely known.

As a consequence of the unresolvable structure, an event rate consistent with the averaged oscillation probability is observed in the subGeV upgoing region. This creates a computational problem: A significantly large amount of Monte Carlo statistics would be required to accurately predict the number of events if Monte Carlo averaging was the only technique used. This section describes

2678 the ‘sub-sampling’ approach developed for this analysis and compares it to the
2679 methodology used within the SK-only analysis.

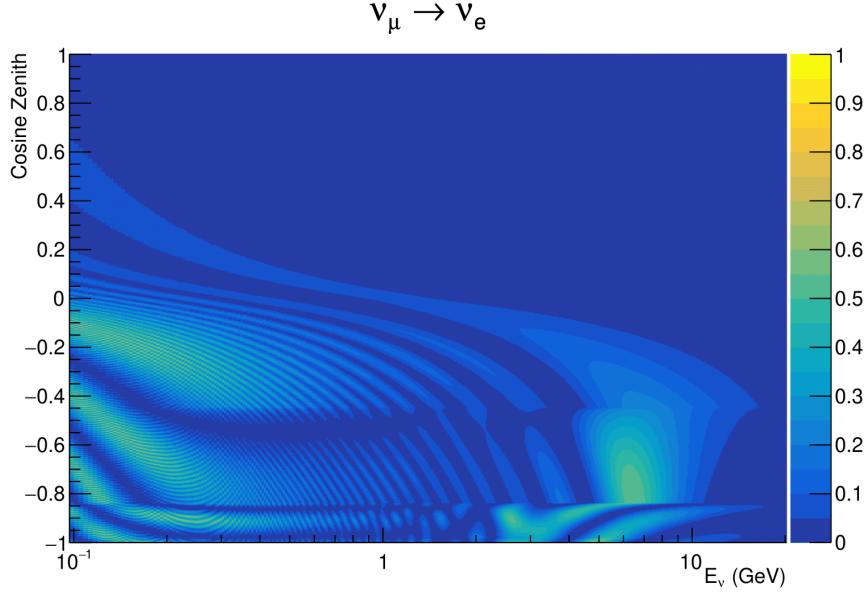


Figure 7.1: The oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, given as a function of neutrino energy and zenith angle, which highlights an example of the “fast” oscillations in the sub-GeV upgoing region.

2680 The official SK-only analysis uses the osc3++ oscillation parameter fitter
2681 [79]. To perform the fast oscillation averaging, it uses a ‘nearest-neighbour’
2682 technique. For a given Monte Carlo neutrino event, the nearest twenty Monte
2683 Carlo neighbours in reconstructed lepton momentum and zenith angle are
2684 found and a distribution of their neutrino energies is built. The RMS, σ , of
2685 this distribution is then used to compute an average oscillation probability for
2686 the given neutrino Monte Carlo event.

2687 For the i^{th} event, the oscillation weight is calculated as

$$W_i = \frac{1}{5}P(E_i, \bar{L}_i) + \frac{1}{5}\sum_{\beta=-1, -0.5, 0.5, 1} P(E_i + \beta\sigma_i, L_\beta), \quad (7.1)$$

2688 where $P(E, L)$ is the oscillation probability calculation for neutrino energy E
2689 and path length L and the two path lengths, \bar{L}_i and L_β are described below. All
2690 of the oscillation probability calculations are performed with a fixed zenith angle
2691 such that the same density profile is used. The uncertainty in the production

height is controlled by using an “average” production height, \bar{L}_i , which represents the average path length computed using twenty production heights taken from the Honda flux model’s prediction [55]. These inputs are provided in 5% intervals of the cumulative distribution function. The value of \bar{L}_i is calculated as:

$$\bar{L}_i = \frac{1}{20} \sum_{j=1}^{20} \sqrt{(R_E + h_j)^2 - R_E^2 (1 - \cos^2 \theta_i)} - R_E \cos \theta_i. \quad (7.2)$$

Where R_E is the Earth’s radius and θ_i is the zenith angle of the i^{th} event. The production heights h_j represent the $(j \times 5)^{th}$ percentile of the cumulative distribution function. L_β values (where the values of β are given in Equation 7.1) are similarly calculated but instead use different combinations of four production heights,

$$\begin{aligned} L_{-1.0} &= \frac{1}{4} L(45, 50, 55, 60), \\ L_{-0.5} &= \frac{1}{4} L(35, 40, 65, 70), \\ L_{+0.5} &= \frac{1}{4} L(25, 30, 75, 68), \\ L_{+1.0} &= \frac{1}{4} L(15, 20, 85, 89). \end{aligned} \quad (7.3)$$

Where $L(i, j, k, l)$ represents the sum of the path lengths with fixed zenith angle and production heights corresponding to the i^{th} , j^{th} , k^{th} and l^{th} percentile of the cumulative distribution function. The values that are taken as β (and values for L_β) are chosen to smooth the oscillation contours in Δm_{32}^2 without incurring loss of sensitivity [205].

This averaging technique works because of the inference between the zenith angle and the reconstructed direction of final state particles in the detector. For low-energy neutrinos, where the resolution of the true neutrino direction is poor, σ_i will be large, resulting in significant averaging effects. Contrary to this, the inferred direction of high-energy neutrinos will be much closer to the true value, meaning that σ_i will be smaller, culminating in small averaging effects.

In practice, these calculations are performed prior to the fit as only oscillation parameters at fixed points are considered. The MCMC technique used in this thesis requires oscillation probabilities to be evaluated at arbitrary parameter values, not known *a priori*. Calculating the five oscillation probabilities per event required by the SK technique is computationally infeasible, so a different averaging technique is used. However, the concept of the averaging technique can be taken from it.

To perform a similar averaging as the SK analysis, a sub-sampling approach using binned oscillograms has been devised. A coarsely binned oscillogram is defined in $\cos(\theta_Z)$ and E_ν . For a given set of oscillation parameters, a single oscillation probability will be assigned to each coarse bin. This value will then apply to all Monte Carlo events which fall into that bin. To assign these oscillation probabilities, the probability is calculated at $N \times N$ points on a grid within a particular bin. This ensemble of oscillation probabilities is averaged to define the coarse bin's oscillation probability, assuming a flat prior in E_ν and $\cos(\theta_Z)$ within the bin. Figure 7.2 illustrates the $N = 2$ example where the assigned value to a coarse bin is the average of the four fine bins which fall in that coarse bin. Whilst the coarse bin edges do not have to be linear on either axis, the sub-division of the fine bins is linear within the range of a coarse bin.

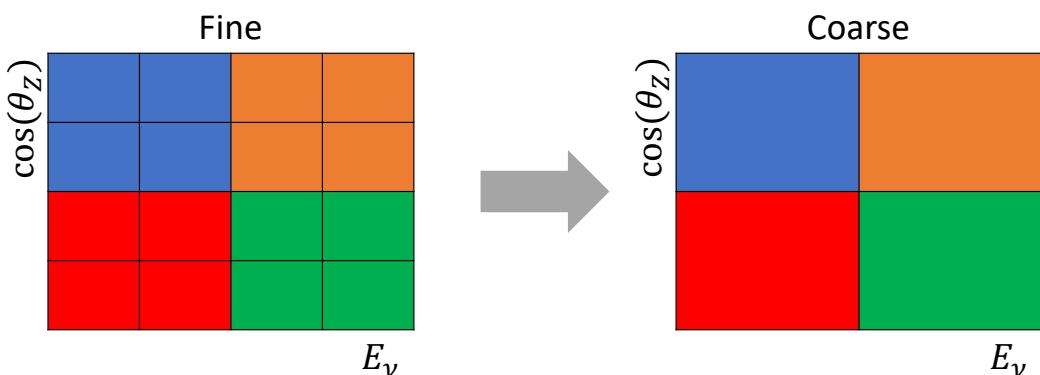


Figure 7.2: Illustration of the averaging procedure for $N = 2$. The oscillation probabilities calculated on the finer left binning are averaged to obtain the oscillation probabilities in the coarser right binning. These averaged oscillation probabilities with the coarser binning are then applied to each event during the fit.

2731 The coarse binning is defined with 67×52 bins in true neutrino energy \times
 2732 cosine zenith. It is picked to be identical to that provided in [205]. In general, the
 2733 binning is logarithmically spaced in neutrino energy but has some hand-picked
 2734 bin edges around the matter resonance to smoothly increased the bin density.
 2735 This is to avoid smearing this region which can be well sampled by the Monte
 2736 Carlo. The cosine zenith binning is approximately linearly spaced across the
 2737 allowable range but the values of layer transitions are hit precisely: -0.8376 (core-
 2738 mantle) and -0.4464 (mantle/transition zone). Bins are spread further apart for
 2739 downgoing events as this is a region unaffected by the fast oscillation wavelengths
 2740 and reduces the total number of calculations required to perform the calculation.

2741 The choice of N is justified based on two studies. Firstly, the variation of event
 2742 rates of each sample is studied as a function of N . For a given set of oscillation
 2743 parameters thrown from the PDG prior constraints (detailed in Table 2.1), the
 2744 oscillation probabilities are calculated using a given value of N . Each sample
 2745 is re-weighted and the event rate is stored. The value of N is scanned from
 2746 1, which corresponds to no averaging, to 19, which corresponds to the largest
 2747 computationally viable subdivision binning. The event rate of each sample at
 2748 large N is expected to converge to a stationary value due to the fine binning fully
 2749 sampling the small-scale structure. Figure 7.3 illustrates this behaviour for the
 2750 SubGeV_elike_0dcy sample for 9 different throws of the oscillation parameters.

2751 Denoting the event rate for one sample for a given throw t at each N by λ_t^N ,
 2752 the average over all considered N values ($\bar{\lambda}_t = \frac{1}{24} \sum_{N=1}^{24} \lambda_t^N$) is computed. The
 2753 variance in the event rate at each N is then calculated as

$$\text{Var}[\lambda^N] = \frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right)^2 - \left[\frac{1}{N_{\text{throws}}} \sum_{t=1}^{N_{\text{throws}}} \left(\lambda_t^N - \bar{\lambda}_t \right) \right]^2. \quad (7.4)$$

2754 In practice, the following procedure is undertaken. For a particular throw,
 2755 the difference between the event rate at a particular choice of N and the mean
 2756 of the distribution is calculated. This is illustrated in Figure 7.4. This value
 2757 is then calculated for all the 2000 throws, generating a distribution of $\lambda_t^N - \bar{\lambda}_t$.

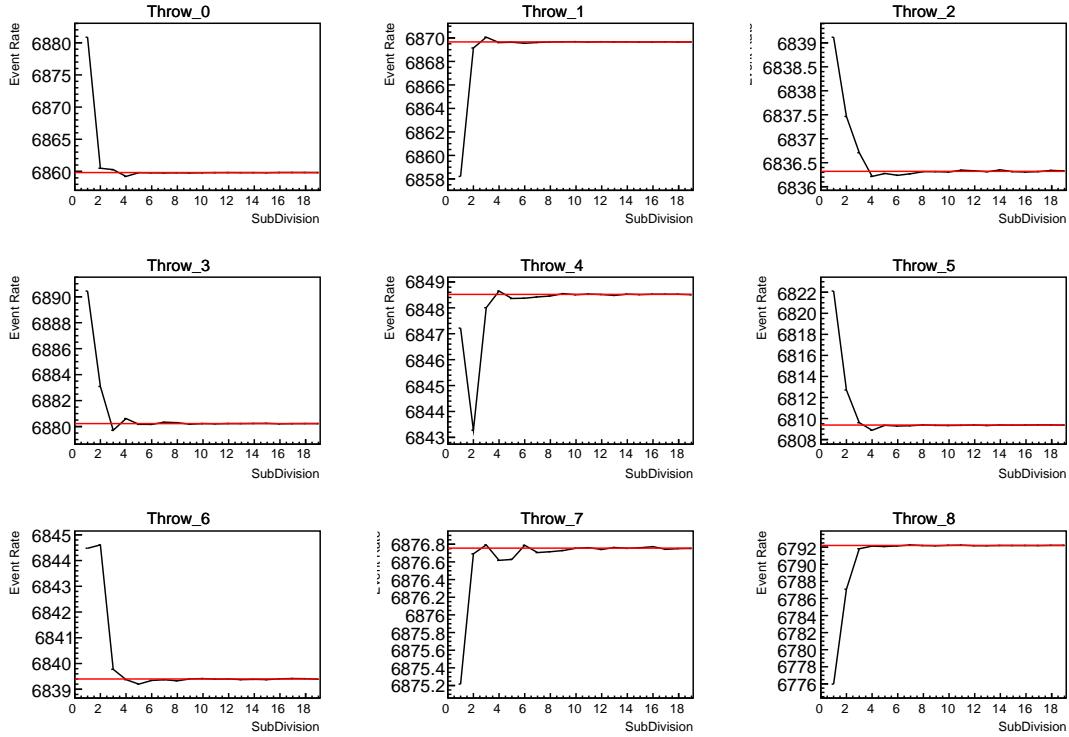


Figure 7.3: Event rate of the SubGeV_elike_0dcy sample as a function of the number of sub-divisions per coarse bin. Each subplot represents the event rate of the sample at a different oscillation parameter set thrown from the PDG priors detailed in Table 2.1. The red line in each subplot represents the mean of the event rate over the different values of sub-divisions for that particular oscillation parameter throw.

2758 This is repeated for each of the values of N considered within this study. The
 2759 distributions of this value, for $N = \{1, 5\}$, are given in Figure 7.5. As expected,
 2760 the distribution gets narrower and tends towards zero for the higher values of N .

2761 The aim of the study is to find the lowest value of N such that this variance
 2762 is below 0.001. This utilises the width of the distributions given in Figure 7.5.
 2763 This is the typical threshold used by T2K fitters to validate systematic imple-
 2764 mentation so has been set as the same criteria. The results of this study for
 2765 each atmospheric sample used within this thesis are illustrated in Figure 7.6 for
 2766 2000 throws of the oscillation parameters. As can be seen, the variance is below
 2767 the threshold at $N = 10$, and is driven primarily by the SubGeV_mulike_1dcy
 2768 and SubGeV_elike_0dcy samples.

2769 The second study to determine the value of N is as follows. The likelihood
 2770 for each sample is computed against an Asimov data set created with Asimov A

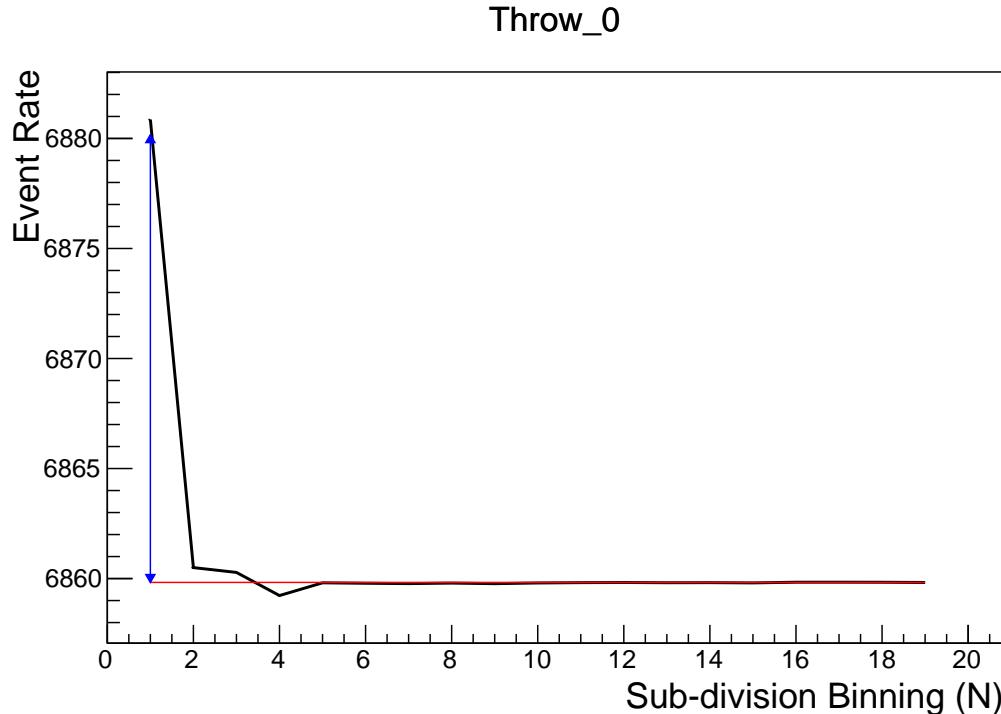


Figure 7.4: Event rate of the SubGeV_elike_0dcy sample, for a particular oscillation parameter throw, as a function of the number of sub-divisions, N , per coarse bin. The difference between the mean event rate (red), $\bar{\lambda}$, and the event rate at $N = 1$, $\lambda^{N=1}$ is defined as $\lambda^N - \bar{\lambda}$ and illustrated by the blue arrow.

oscillation parameters (Table 2.2). Following Equation 7.4, the variance of the log-likelihood over all considered N is computed. The results are shown in Figure 7.7.

A choice of $N = 10$ sub-divisions per coarse bin has a variance in both event rate and log-likelihood residuals less than the required threshold of 0.001. The largest value of the likelihood variance is of order 10^{-7} , corresponding to an error on the log-likelihood of about 3×10^{-4} which is small enough to be negligible for the oscillation analysis.

Figure 7.8 illustrates the effect of the smearing using $N = 10$. The fast oscillations in the sub-GeV upgoing region have been replaced with a normalisation effect whilst the large matter resonance structure remains.

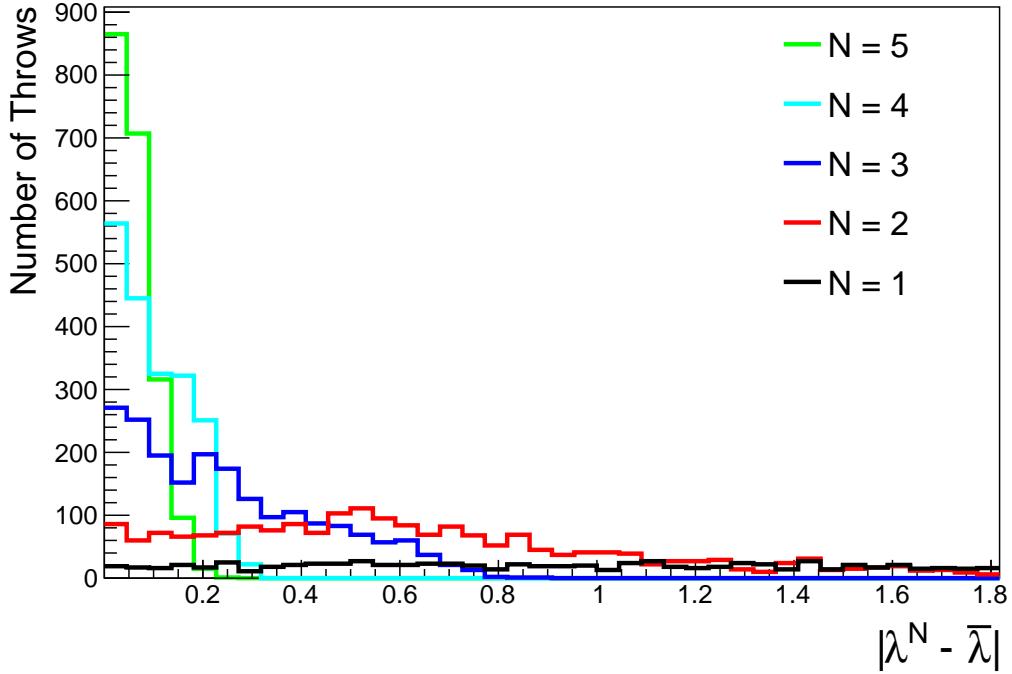


Figure 7.5: The distribution of $\lambda^N - \bar{\lambda}$ for various values of N . As expected, the distribution gets narrower for larger values of N .

2781 7.2 Calculation Engine

2782 As previously discussed in section 7.1, the calculation of oscillation probabilities
 2783 is performed at run-time. Consequently, the time per calculation is crucial for fit
 2784 performance. The initial fitting framework used for this analysis was developed
 2785 with ProbGPU [206]. This is a GPU-only implementation of the prob3 engine
 2786 [207]. It is primarily designed for neutrino propagation in a beam experiment
 2787 (single layer of constant density) with the atmospheric propagation code not
 2788 being used prior to the analysis in this thesis.

2789 Another engine, CUDAProb3 [208], has been interfaced with the fitting frame-
 2790 work used in this analysis. This interfacing was done by the author of this
 2791 thesis. It has been specifically optimised for atmospheric neutrino oscillation
 2792 calculation so does not contain the code to replace the beam oscillation calculation.
 2793 The engine utilises object-orientated techniques as compared to the functional
 2794 implementation of ProbGPU. This allows the energy and cosine zenith arrays to

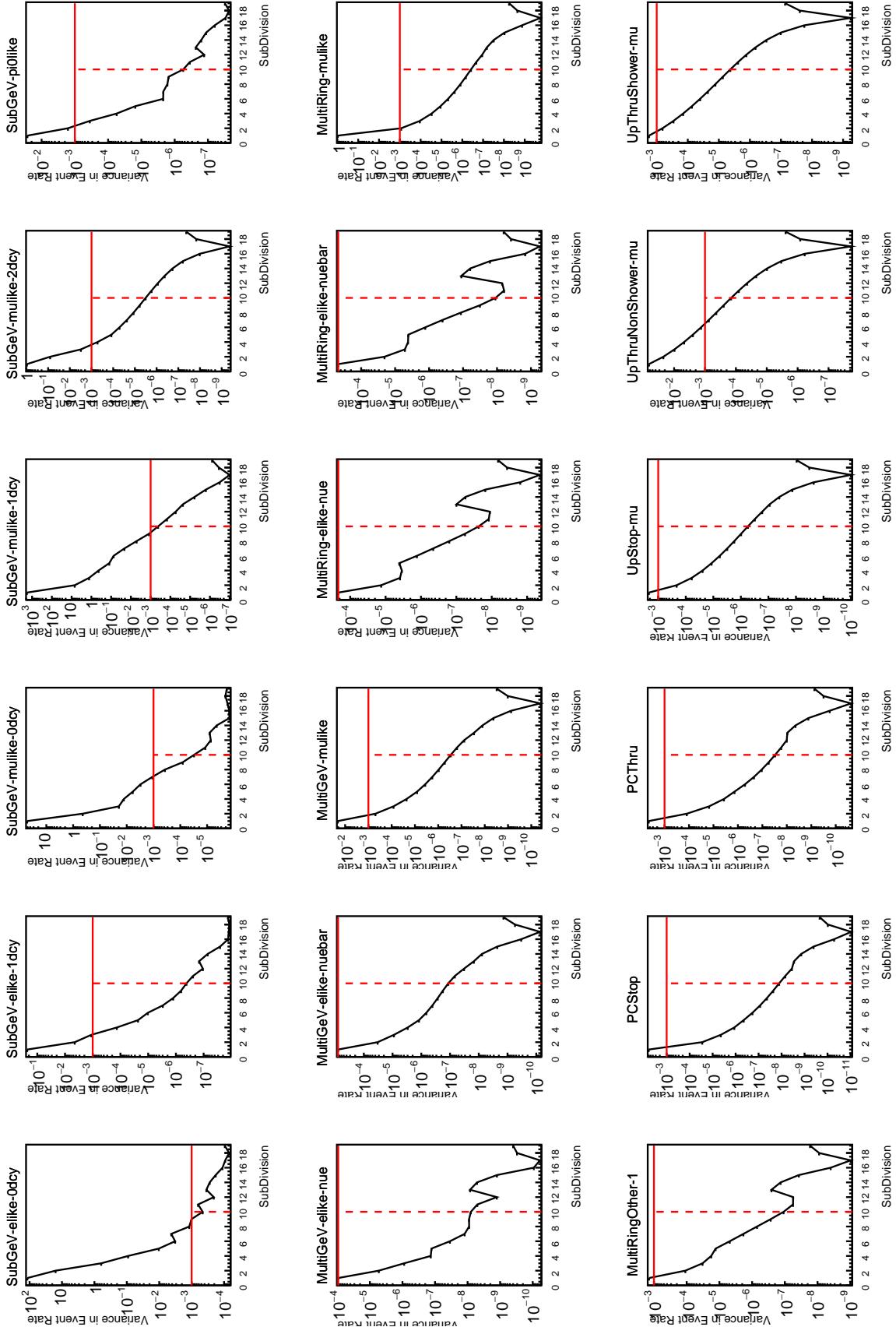


Figure 7.6: Variance of event rate for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line indicates the variance at a sub-division $N = 10$.

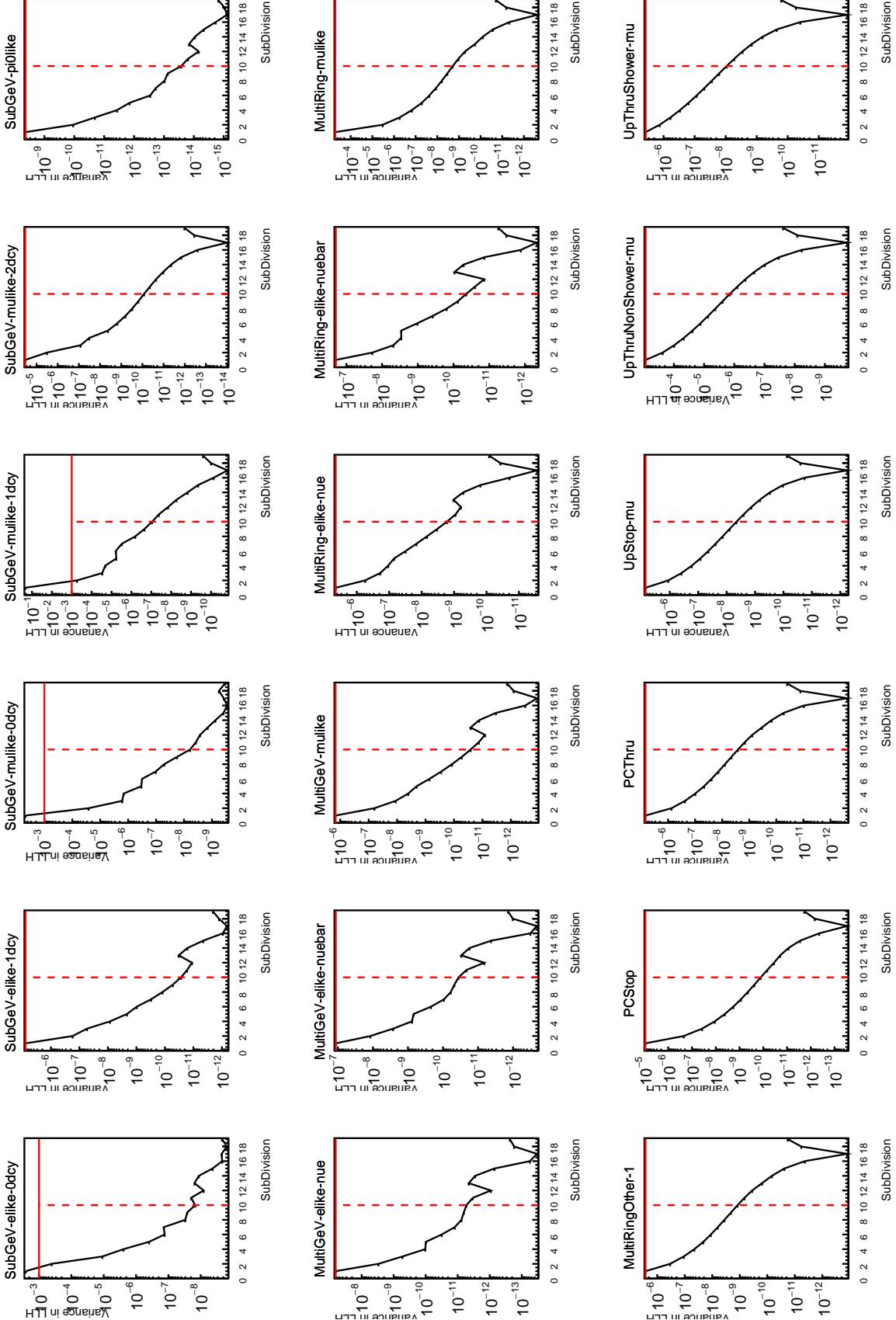


Figure 7.7: Variance of sample likelihood, when compared to 'Asimov data' set at Asimov A, for each atmospheric sample as a function of the number of sub-divisions per coarse bin. The solid red line indicates the 0.1% threshold and the dashed red line is a graphical indication of the variance at a sub-division $N = 10$.

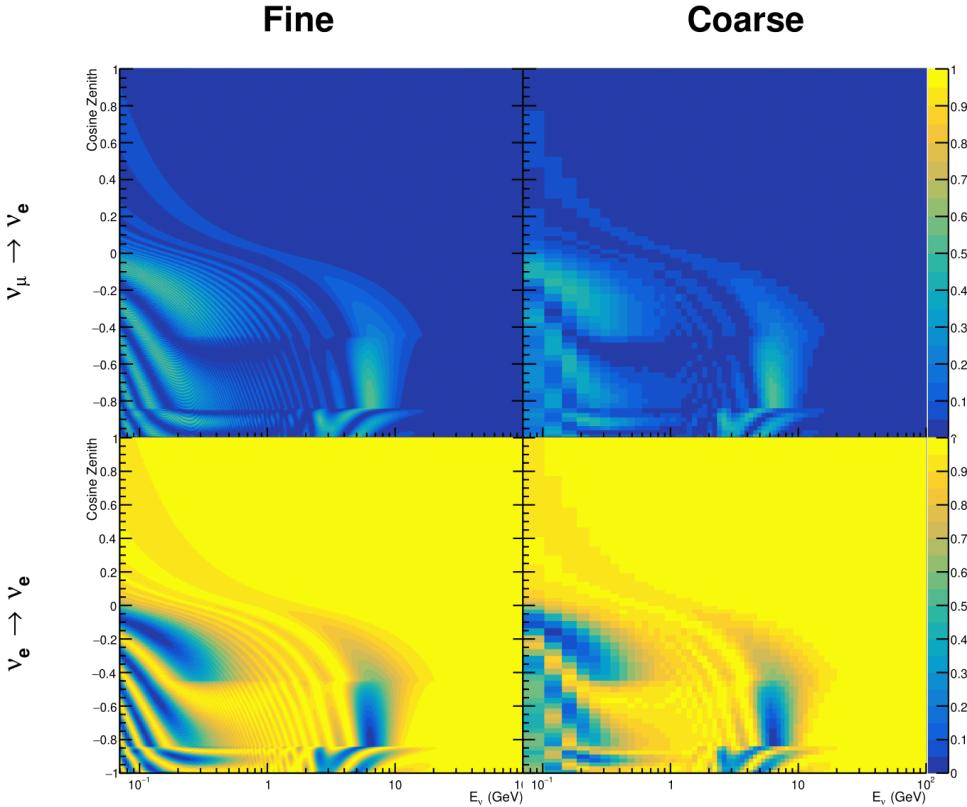


Figure 7.8: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives the “fine” binning used to calculate the oscillation probabilities and the right column illustrates the “coarse” binning used to reweight the Monte Carlo events. The fine binning choice is given with $N = 10$, which was determined to be below the threshold from Figure 7.6 and Figure 7.7.

2795 be kept on GPU memory, rather than having to load these arrays onto GPU
 2796 memory for each calculation. Reducing the memory transfer between CPU and
 2797 GPU significantly reduces the time required for calculation. This can be seen
 2798 in Figure 7.9, where the GPU implementation of CUDAProb3 is approximately
 2799 three times faster than the ProbGPU engine.

2800 Another significant advantage of CUDAProb3 is that it contains a CPU multi-
 2801 threaded implementation which is not possible with the ProbGPU or prob3 engines.
 2802 This eliminates the requirement for GPU resources when submitting jobs to batch
 2803 systems. As illustrated in Figure 7.9, the calculation speed depends on the number

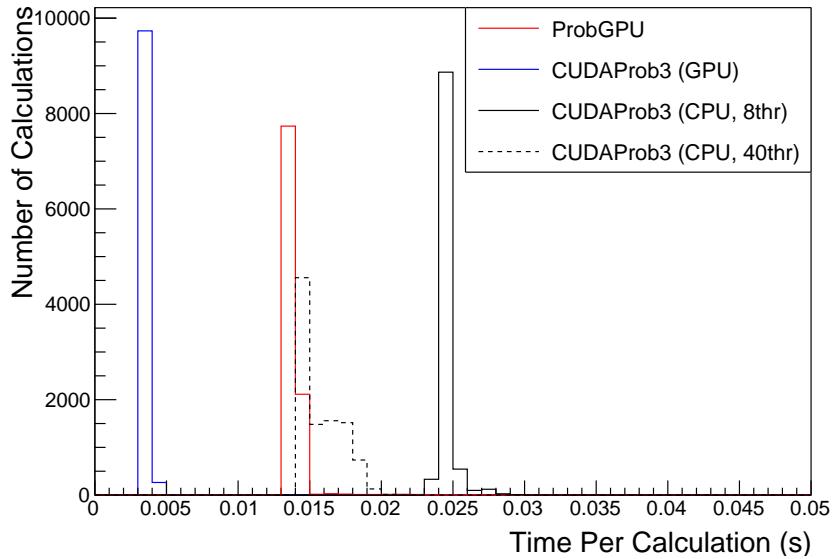


Figure 7.9: The calculation time taken to both calculate the oscillation probabilities and fill the “coarse” oscillograms, following the technique given in section 7.1, for the CUDAProb3 and ProbGPU (Red) calculation engines. CUDAProb3 has both a GPU (Blue) and CPU (Black) implementation, where the CPU implementation is multi-threaded. Therefore, 8-threads (solid) and 40-threads (dashed) configurations have been tested. Prob3, which is a CPU single-thread implementation has a mean step time of 1.142s.

of available threads. Using 8 threads (which is typical of the batch systems being used) is approximately twice as slow as the ProbGPU engine implementation, but would allow the fitting framework to be run on many more resources. This fact is utilised for any SK-only fits but GPU resources are required for any fits which include beam samples due to the ProbGPU requirement. Based on the benefits shown by the implementation in this section, efforts are being placed into including linear propagation for beam neutrino propagation into the engine [209].

7.3 Matter Density Profile

For an experiment observing neutrinos propagating through the Earth, a model of the Earth’s density profile is required. The model used within this analysis is based on the Preliminary Reference Earth Model (PREM) [80], as illustrated in Figure 2.8. Table 2.3 documents the density and radii of the layers used within the constant density approximation used by the SK-only analysis [79]. The

density measurements provided in the PREM model are provided in terms of mass density, whereas neutrino oscillations are sensitive to the electron number density. This value can be computed as the product of the chemical composition, or the Z/A value, and the mass density of each layer. Currently, the only way to measure the chemical composition value for layers close to the Earth's core is through neutrino oscillations. The chemical composition of the upper layers of the Earth's Mantle and the Transition zone is well known due to it being predominantly pyrolite which has a chemical composition value of 0.496 [210]. The chemical composition dial for the core layers is set to a value of 0.468, as calculated in [211]. As this value is less well known, it is assigned a Gaussian error with a standard deviation equivalent to the difference in chemical composition in core and mantle layers. Figure 7.10 illustrates the effect of moving from the $Z/A = 0.5$ method which is used in the official SK-only analysis to these more precise values.

The beam oscillation probability in this thesis uses a baseline of 295km, density 2.6g/cm^3 , and chemical composition 0.5 as is done by the official T2K-only analysis [212].

For a neutrino with given $E_\nu, \cos(\theta_Z)$, the oscillation probability calculation engine must be passed a list of the matter regions that the neutrino traversed, with the path length and fixed density in each region. However, a neutrino passing through the earth experiences a range of radii, and thus a range of densities, in each region. In the SK-only analysis, the earth density model used is piecewise-constant, thereby ignoring this effect. For this thesis, the density values for the calculation engine are found by averaging the earth density along the neutrino's path in each layer,

$$\langle \rho \rangle_i = \frac{1}{t_{i+1} - t_i} \int_{t_i}^{t_{i+1}} \rho(t) dt, \quad (7.5)$$

where t_i are the intersection points between each layer and t is the path length of the trajectory across the layer. This leads to an improved approximation. For this averaging, the simplification of the PREM model developed in [213] is

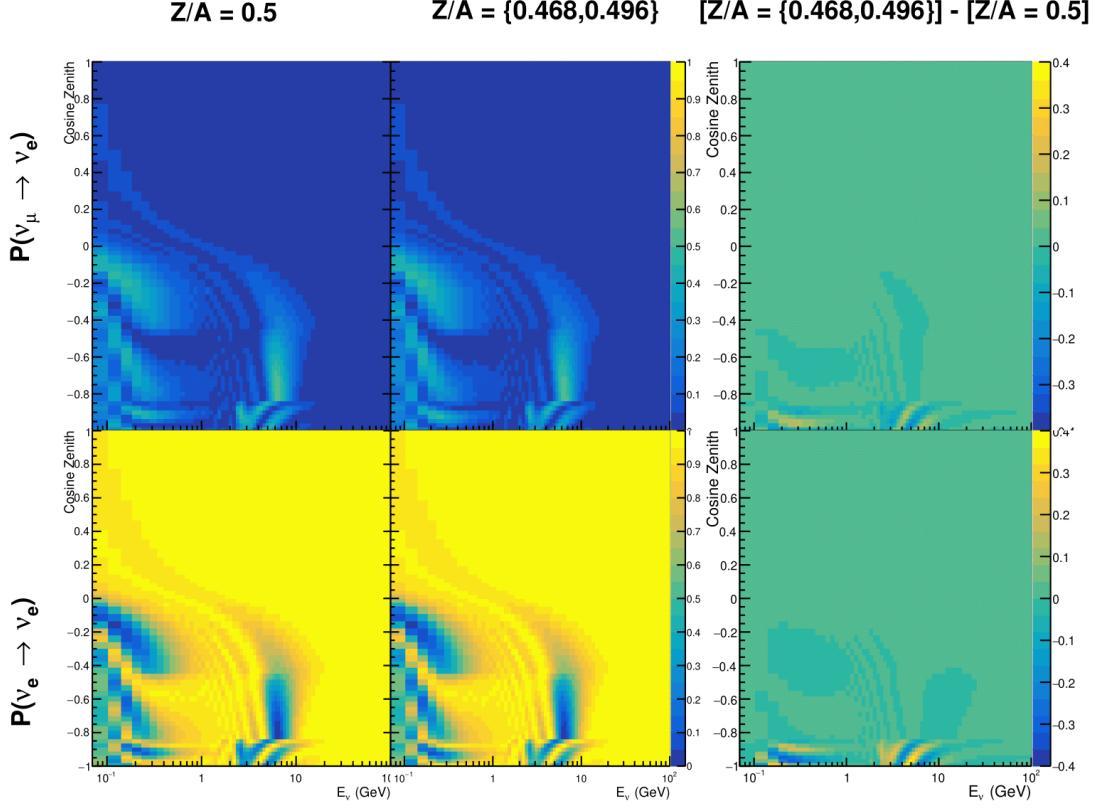


Figure 7.10: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the constant $Z/A = 0.5$ approximation which is used in the official SK-only analysis. The middle column gives the probabilities where $Z/A = [0.468, 0.498]$ values are used, as given in Table 2.3. The right column illustrates the difference in oscillation probability between the two different techniques.

used. The layers of the prem model are combined into four to reduce calculation time, with a quadratic fit to each section. This fit was not performed by the author of the thesis and is documented in [205]. The coefficients of the quadratic fit to each layer are given in Table 7.1 with the final distribution illustrated in Figure 7.11. The quadratic approximation is clearly much closer to the PREM model as compared to the constant density approximation.

The effect of using the quadratic density per $\cos(\theta_Z)$ model is highlighted in Figure 7.12. The slight discontinuity in the oscillation probability around $\cos(\theta_Z) \sim -0.45$ in the fixed density model, which is due to the transition to

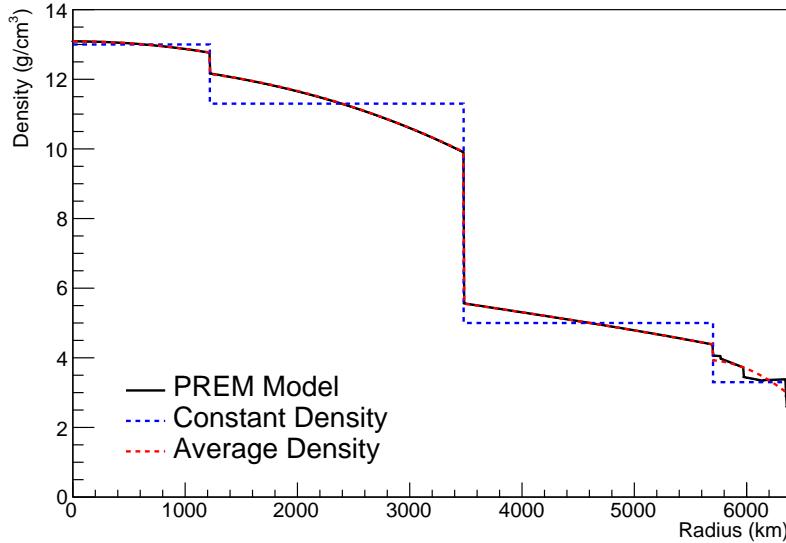


Figure 7.11: The density of the Earth given as a function of the radius, as given by the PREM model (Black), the constant density four-layer approximation (Blue), as used in the official SK-only analysis, and the quadratic approximation of the PREM model (Red).

Layer	Outer Radius [km]	Density [g/cm ³]
Inner Core	1220	$13.09 - 8.84x^2$
Outer Core	3480	$12.31 + 1.09x - 10.02x^2$
Lower Mantle	5701	$6.78 - 1.56x - 1.25x^2$
Transition Zone	6371	$-50.42 + 123.33x - 69.95x^2$

Table 7.1: The quadratic polynomial fits to the PREM model for four assumed layers of the PREM model. The fit to calculate the coefficients is given in [205], where $x = R / R_{\text{Earth}}$.

mantle layer boundary, has been reduced. This is expected as the difference in the density across this boundary is significantly smaller in the quadratic density model as compared to the constant density model. Whilst the difference in density across the other layer transitions is reduced, there is still a significant difference. This means the discontinuities in the oscillation probabilities remain but are significantly reduced. However, as the quadratic density approximation matches the PREM model well in this region, these discontinuities are due to the Earth model rather than an artifact of the oscillation calculation.

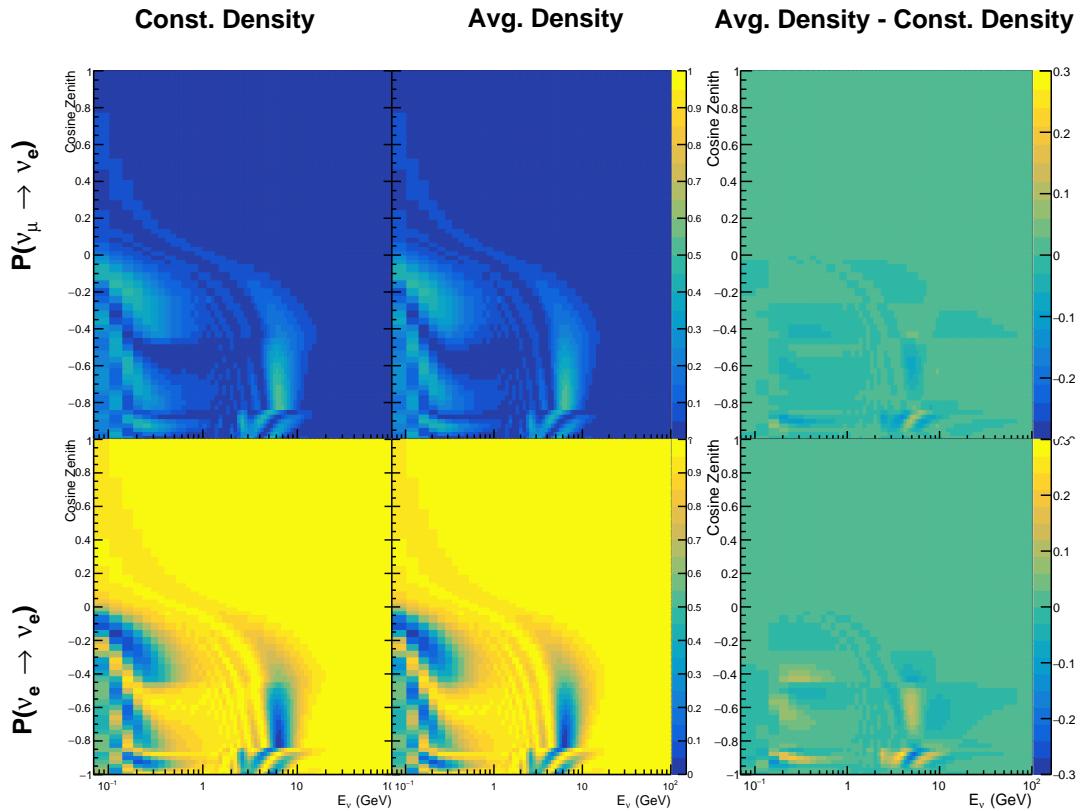


Figure 7.12: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where the four-layer constant density approximation is used. The middle column gives the probabilities where the density is integrated over the trajectory, using the quadratic PREM approximation, for each $\cos(\theta_Z)$ is used. The right column illustrates the difference in oscillation probability between the two different techniques.

2862 7.4 Production Height Averaging

2863 As discussed in section 2.5, the height at which the cosmic ray flux interacts
 2864 in the atmosphere is not known on an event-by-event basis. The production
 2865 height can vary from the Earth’s surface to $\sim 50\text{km}$ above that. The SK-only
 2866 analysis methodology (described in section 7.1) for including the uncertainty
 2867 on the production height is to include variations from the Honda model when
 2868 pre-calculating the oscillation probabilities prior to the fit. This technique is not
 2869 possible for this analysis which uses continuous oscillation parameters that can
 2870 not be known prior to the fit. Consequently, an analytical averaging technique
 2871 was developed in [205]. The author of this thesis was not responsible for the
 2872 derivation of the technique but has performed the implementation and validation
 2873 of the technique for this analysis alone.

2874 Using the 20 production heights per Monte Carlo neutrino event, provided
 2875 as 5% percentiles from the Honda flux model, a production height distribution
 2876 $p_j(h|E_\nu, \cos \theta_Z)$ is built for each neutrino flavour $j = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$. In practice, a
 2877 histogram is filled with 20 evenly spaced bins in production height h between
 2878 0 and 50km. The neutrino energy and cosine zenith binning of the histogram
 2879 are the same as that provided in section 7.1. The average production height,
 2880 $\bar{h} = \int dh \frac{1}{4} \sum_j p_j(h|E_\nu, \cos(\theta_Z))$, is calculated. This assumes a linear average over
 2881 the four flavours of neutrino which are considered to be generated in cosmic
 2882 ray showers. The production height binning of this histogram is then translated
 2883 into $\delta t(h) = t(\bar{h}) - t(h)$, where $t(x)$ is the distance travelled along the trajectory
 2884 in the atmosphere from some production height, x .

2885 For the i^{th} traversed layer, the transition amplitude, $D_i(t_{i+1}, t_i)$, is computed.
 2886 The time-ordered product of these is then used as the overall transition amplitude
 2887 via

$$A(t_{n+1}, t_0) = D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1) D_0(t_1, t_0), \quad (7.6)$$

2888 where,

$$\begin{aligned} D_n(t_{n+1}, t_n) &= \exp[-iH_n(t_{n+1} - t_n)] \\ &= \sum_{k=1}^3 C_k \exp[ia_k(t_{n+1} - t_n)] \end{aligned} \quad (7.7)$$

2889 is expressed as a diagonalised time-dependent solution to the Schrodinger
 2890 equation. The 0th layer is the propagation through the atmosphere and is the
 2891 only term that depends on the production height. Using the substitution $t_0 =$
 2892 $t(\bar{h}) - \delta t(h)$, it can be shown that

$$D_0(t_1, t_0) = D_0(t_1, \bar{h})D_0(\delta t). \quad (7.8)$$

2893 Thus Equation 7.6 becomes

$$\begin{aligned} A(t_{n+1}, t_0) &= D_n(t_{n+1}, t_n) \dots D_1(t_2, t_1)D_0(t_1, \bar{h})D(\delta t) \\ &= A(t_{n+1}, \bar{h}) \sum_{k=1}^3 C_k \exp[ia_k \delta t], \\ &= \sum_{k=1}^3 B_k \exp[ia_k \delta t]. \end{aligned} \quad (7.9)$$

2894 The oscillation probability averaged over production height is then calculated
 2895 as

$$\begin{aligned} \bar{P}(\nu_j \rightarrow \nu_i) &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) P(\nu_j \rightarrow \nu_i) \\ &= \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) A(t_{n+1}, t_0) A^*(t_{n+1}, t_0) \\ &= \sum_{km} (B_k)_{ij} (B_m)_{ij}^* \int d(\delta t) p_j(\delta t | E_\nu, \cos \theta_Z) \exp[i(a_k - a_m) \delta t]. \end{aligned} \quad (7.10)$$

2896 It is important to note that the exact value of \bar{h} used does not matter as the
 2897 values of δt would change to compensate for any modification to the value of \bar{h} .

2898 In practice, implementation in CUDAProb3 [208] is relatively straightforward
 2899 as the majority of these terms are already calculated in the standard oscillation
 2900 calculation. Figure 7.13 illustrates the results of the production height averaging.

2901 As expected, the main effect is observed in the low-energy downward-going
 2902 and horizontal-going events. Upward-going events have to travel the radius
 2903 of the Earth, $R_E = 6371\text{km}$, where the production height uncertainty is a small
 2904 fraction of the total path length.

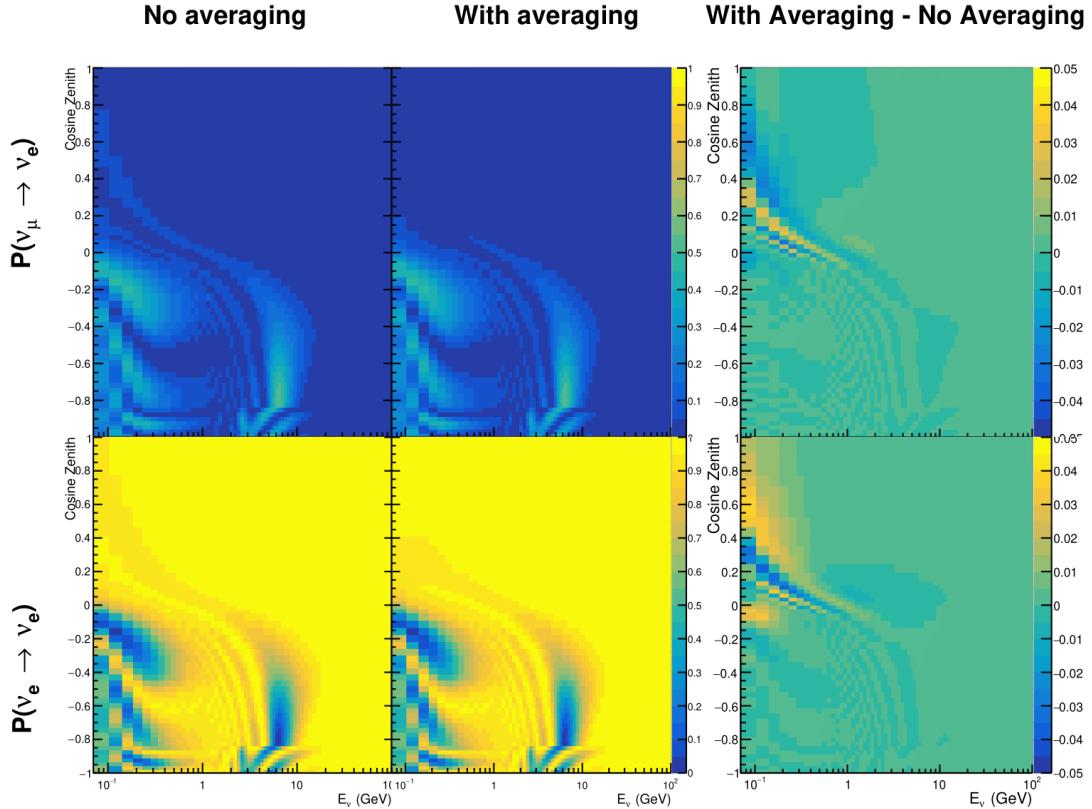


Figure 7.13: The oscillation probability, $P(\nu_\mu \rightarrow \nu_e)$ (top row) and $P(\nu_e \rightarrow \nu_e)$ (bottom row), given as a function of neutrino energy and zenith angle. The left column gives probabilities where a fixed production height of 25km is used. The middle column gives the probabilities where the production height is analytically averaged. The right column illustrates the difference in oscillation probability between the two different techniques.

8

2905

2906

Oscillation Analysis

2907 Using the samples and systematics defined in chapter 6, this chapter documents
2908 a simultaneous beam and atmospheric oscillation analysis from the T2K and SK
2909 experiments. The MaCh3 Bayesian MCMC framework introduced in chapter 4
2910 is used for all studies performed within this thesis.

2911 The MaCh3 framework has been validated through many tests. The code
2912 that handles the beam far detector samples was developed by the author and
2913 validated by comparison to the 2020 T2K analysis [2]. The sample event rates and
2914 likelihood evaluations of beam samples generated by the framework used within
2915 this thesis were compared to those from the T2K analysis by the author of this
2916 thesis. Variations of the sample predictions were compared at $\pm 1\sigma$ and $\pm 3\sigma$ and
2917 good agreement was found in all cases. A similar study, led by Dr. C. Wret was
2918 used to validate the near detector portion of the code [198]. The implementation
2919 of the atmospheric samples within MaCh3 was completed and cross-checked by
2920 the author of this thesis against the P-Theta framework (introduced in section 3.2).
2921 Both fitters are provided with the same inputs and can therefore cross-validate
2922 each other. These validations compared the event rate and likelihood calculation.
2923 Documentation of all the above validations can be found in [198]. These stringent
2924 validations ensure that the code is doing as intended.

2925 8.1 Monte Carlo Prediction

- 2926 Using the three sets of dial values (generated, pre-fit, and post-fit tunes) defined
 2927 in subsection 6.4.3, the predicted event rates for each sample are given in Table 8.1.
 2928 The oscillated and un-oscillated event rates are calculated for each tune.

Sample	Total Predicted Events					
	Generated		Pre-fit		Post-fit	
	Osc	UnOsc	Osc	UnOsc	Osc	UnOsc
SubGeV- <i>elike</i> -0dcy	7121.0	7102.6	6556.8	6540.0	7035.2	7015.7
SubGeV- <i>elike</i> -1dcy	704.8	725.5	693.8	712.8	565.7	586.0
SubGeV- <i>mulike</i> -0dcy	1176.5	1737.2	1078.6	1588.1	1182.7	1757.1
SubGeV- <i>mulike</i> -1dcy	5850.7	8978.1	5351.7	8205.1	5867.0	9009.9
SubGeV- <i>mulike</i> -2dcy	446.9	655.2	441.6	647.7	345.9	505.6
SubGeV- <i>pi0like</i>	1438.8	1445.4	1454.9	1461.1	1131.1	1136.2
MultiGeV- <i>elike</i> -nue	201.4	195.6	201.1	195.3	202.6	196.7
MultiGeV- <i>elike</i> -nuebar	1141.5	1118.3	1060.7	1039.5	1118.5	1095.7
MultiGeV- <i>mulike</i>	1036.7	1435.8	963.1	1334.1	1015.2	1405.9
MultiRing- <i>elike</i> -nue	1025.1	982.2	1026.8	984.3	1029.8	986.4
MultiRing- <i>elike</i> -nuebar	1014.8	984.5	991.0	962.0	1008.9	978.5
MultiRing- <i>mulike</i>	2510.0	3474.4	2475.6	3425.8	2514.6	3480.4
MultiRingOther-1	1204.5	1279.1	1205.8	1280.3	1207.4	1281.0
PCStop	349.2	459.2	338.4	444.7	346.8	456.1
PCThrus	1692.8	2192.5	1661.5	2149.8	1689.2	2187.8
UpStop-mu	751.2	1295.0	739.7	1271.6	750.4	1293.0
UpThruNonShower-mu	2584.4	3031.6	2577.9	3019.4	2586.8	3034.0
UpThruShower-mu	473.0	488.6	473.2	488.7	473.8	489.4
FHC1Rmu	328.0	1409.2	301.1	1274.7	345.1	1568.0
RHC1Rmu	133.0	432.3	122.7	396.2	135.0	443.9
FHC1Re	84.6	19.2	77.4	18.2	93.7	19.7
RHC1Re	15.7	6.4	14.6	6.1	15.9	6.3
FHC1Re1de	10.5	3.2	10.3	3.1	8.8	2.9

Table 8.1: The Monte Carlo predicted event rate of each far detector sample used within this analysis. Three model parameter tunes are considered, as defined in subsection 6.4.3. Un-oscillated and oscillated predictions are given, where the oscillated predictions assume Asimov A oscillation parameters provided in Table 2.2.

- 2929 Generally, the samples that target CCQE interaction modes observe a decrease
 2930 in prediction when comparing the generated values with the pre-fit dial values.
 2931 This is in accordance with the Monte Carlo being produced at $M_A^{QE} = 1.21\text{GeV}$
 2932 [157] whilst the pre-fit dial value is set to $M_A^{QE} = 1.03\text{GeV}$ as suggested by [197].

Furthermore, the predicted event rates of samples that target CCRES interaction modes are significantly reduced when considering the post-BANFF fit. This follows the observations in subsection 6.4.3. The strength of the accelerator neutrino experiment can be seen in the remarkable difference between the oscillated and unoscillated predictions in the FHC1Rmu and RHC1Rmu samples. There is a very clear decrease in the expected event rate between the oscillated and un-oscillated predictions which is not as obvious as in the atmospheric samples. This is due to the fact that the beam energy is tuned to the maximum disappearance probability, which is not the case for the naturally generated atmospheric neutrinos.

8.2 Likelihood Scans

Using the definition of the likelihood presented in section 6.5, the contribution of each sample to the likelihood from a variation of a particular parameter can be studied. This process identifies which samples drive the determination of the oscillation parameters in the joint fit. Figure 8.1 presents the variation of all the samples (beam and atmospheric) at the far detector to the oscillation parameters of interest: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$, and Δm_{32}^2 . These plots are colloquially called ‘likelihood scans’ (or ‘log-likelihood scans’). The process of making these plots is as follows. An Asimov data set is built using the AsimovA oscillation parameters and pre-fit systematic tune. The Monte Carlo is then reweighted using the value of the oscillation parameter at each point on the x-axis of the scan. The likelihood is then calculated between the Asimov data and Monte Carlo prediction and plotted.

Due to the caveat of fixed systematic parameters and the correlations between oscillation parameters being ignored when creating these likelihood scans, the value of $\chi^2 = 1$ (or $-2 \times \ln(\text{Likelihood}) = 1$) does not equate to the typical 1σ sensitivity. However, it does give an indication of which samples respond most strongly to variations in a particular oscillation parameter. The point at

which the likelihood tends to zero illustrates the value of the parameter used to build the Asimov data prediction.

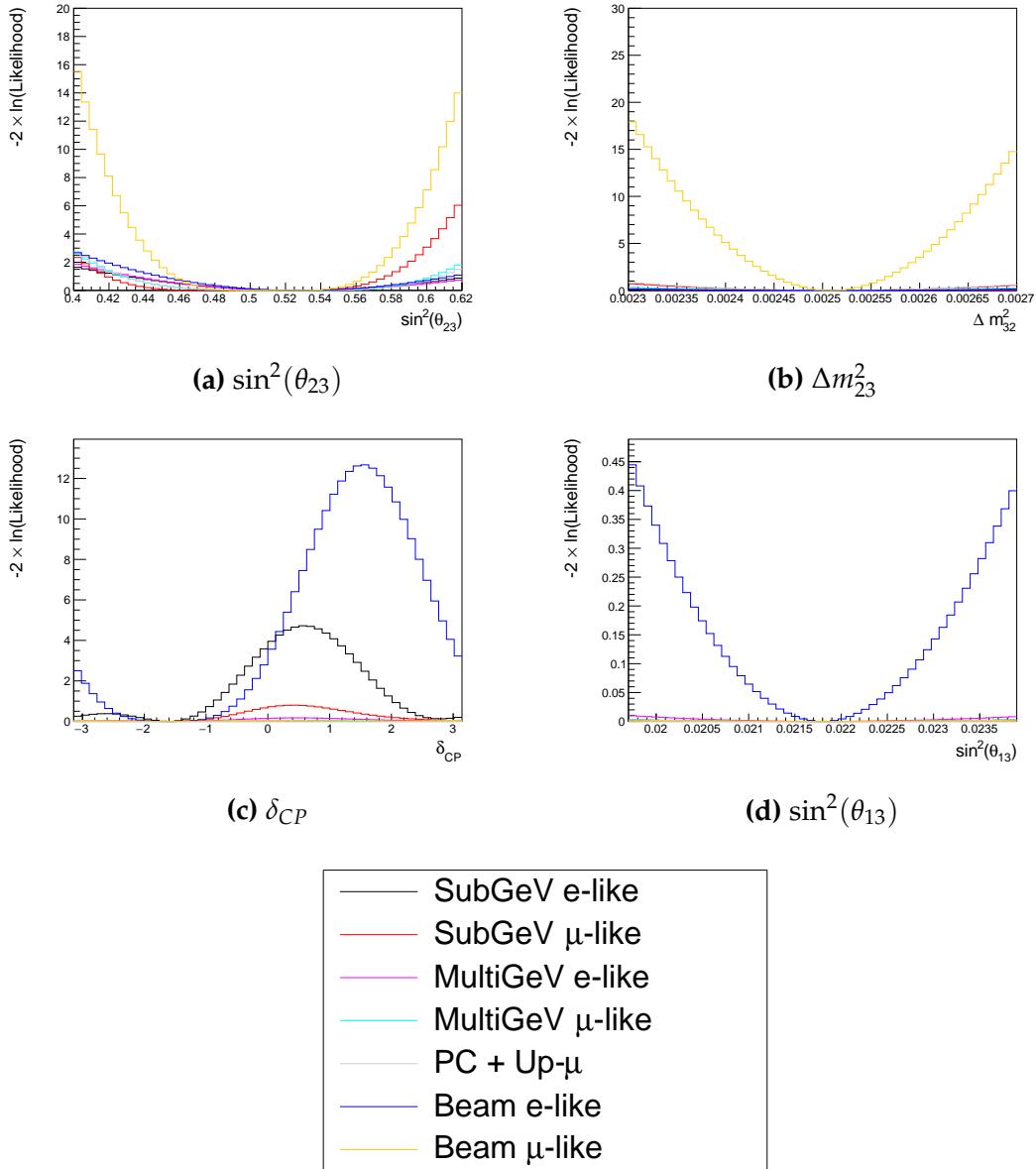


Figure 8.1: The response of the likelihood, as defined in section 6.5, illustrating the response of the samples to a variation of an oscillation parameter.

The sensitivity to $\sin^2(\theta_{23})$ is mostly dominated by the beam muon-like samples. The response of an individual atmospheric sample is small but non-negligible such that the summed response over all atmospheric samples becomes comparable to that of the muon-like beam samples. Consequently, the sensitivity

of the joint fit to $\sin^2(\theta_{23})$ would be expected to be greater than the beam-only analysis. The only sample that responds to the $\sin^2(\theta_{13})$ oscillation parameter is the electron-like beam sample. Consequently, no increase in sensitivity beyond that of the T2K-only analysis would be expected from the joint fit. Regardless, the sensitivity of the beam sample is significantly weaker than the external reactor constraint so prior knowledge will dominate any sensitivity to $\sin^2(\theta_{13})$ which is included within this thesis. The Δm_{21}^2 and $\sin^2(\theta_{12})$ parameters are not considered as there is simply no sensitivity in any sample considered within this analysis. The response to Δm_{32}^2 is completely dominated by the beam muon-like samples. This is because the beam neutrino energy is specifically tuned to match the maximal disappearance probability. Despite this, improvements to the $|\Delta m_{32}^2|$ sensitivity may be expected due to additional mass hierarchy determination added by the atmospheric samples.

Two-dimensional scans of the appearance ($\sin^2(\theta_{13}) - \delta_{CP}$) and disappearance ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) parameters are illustrated in Figure 8.2 and Figure 8.3, respectively. The caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still apply.

The appearance log-likelihood scans show the distinct difference in how the beam and atmospheric samples respond. The beam samples have an approximately constant width of the 2σ and 3σ contours, throughout all ranges of δ_{CP} . Whereas, the response of the atmospheric samples to $\sin^2(\theta_{13})$ is very strongly correlated to the value of δ_{CP} . At higher values of $\sin^2(\theta_{13})$, two lobes appear around $\delta_{CP} \sim -\pi/2$ and $\delta_{CP} \sim 2.4$. Consequently, this difference allows some of the degeneracy in a beam-only fit to be broken. Comparing the beam-only and joint fit likelihood scans, the 2σ continuous contour in δ_{CP} for beam samples becomes closed when the atmospheric samples are added. This may result in a stronger sensitivity to δ_{CP} . Similarly, the width of the 3σ contours also becomes dependent upon the value of δ_{CP} . Furthermore, atmospheric samples have little sensitivity to $\sin^2(\theta_{13})$ on their own, as evidenced in Figure 8.1, but may improve sensitivity to the parameter when combined within the simultaneous

2997 fit. It is important to remember that these likelihood scans are not sensitivity
2998 measurements as the systematic parameters are fixed and the correlation between
2999 oscillation parameters is neglected. However, they are a very encouraging result
3000 for the joint fit.

3001 The disappearance log-likelihood scans in $\sin^2(\theta_{23}) - \Delta m_{32}^2$ space (Figure 8.3)
3002 show the expected behaviour when considering the one-dimensional scans
3003 already discussed. The uncertainty on the width of $|\Delta m_{32}^2|$ is mostly driven by the
3004 beam samples. However, the width of this contour in the inverted mass region
3005 ($\Delta m_{32}^2 < 0$) is significantly reduced due to the ability of the atmospheric samples
3006 to select the correct (normal) mass hierarchy. The width of the uncertainty
3007 in $\sin^2(\theta_{23})$ is also reduced compared to the beam-only sensitivities, with a
3008 further decrease in the inverted hierarchy region due to the better mass hierarchy
3009 determination.

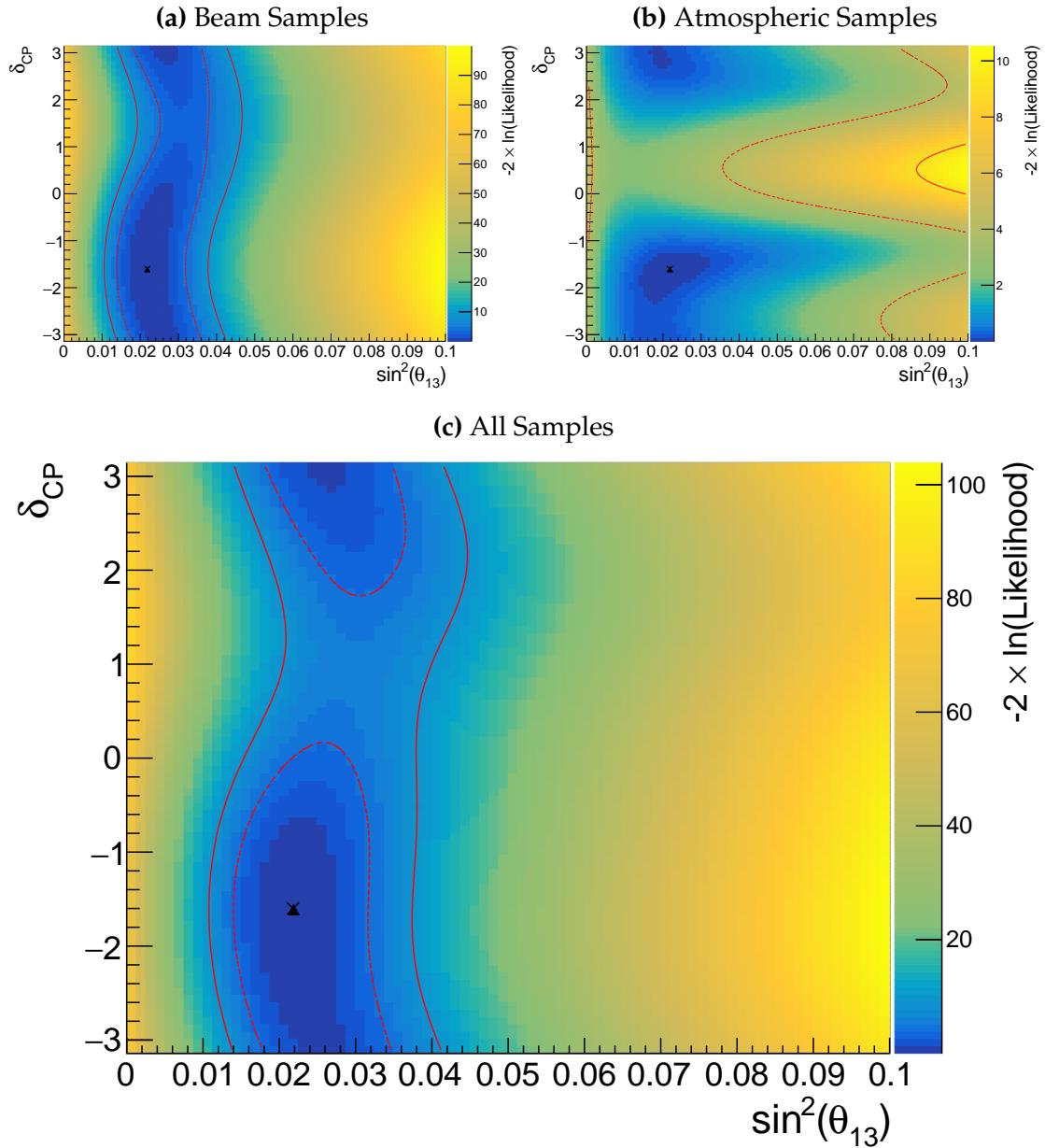


Figure 8.2: Two-dimensional log-likelihood scan of the appearance ($\sin^2(\theta_{13})$)– δ_{CP}) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

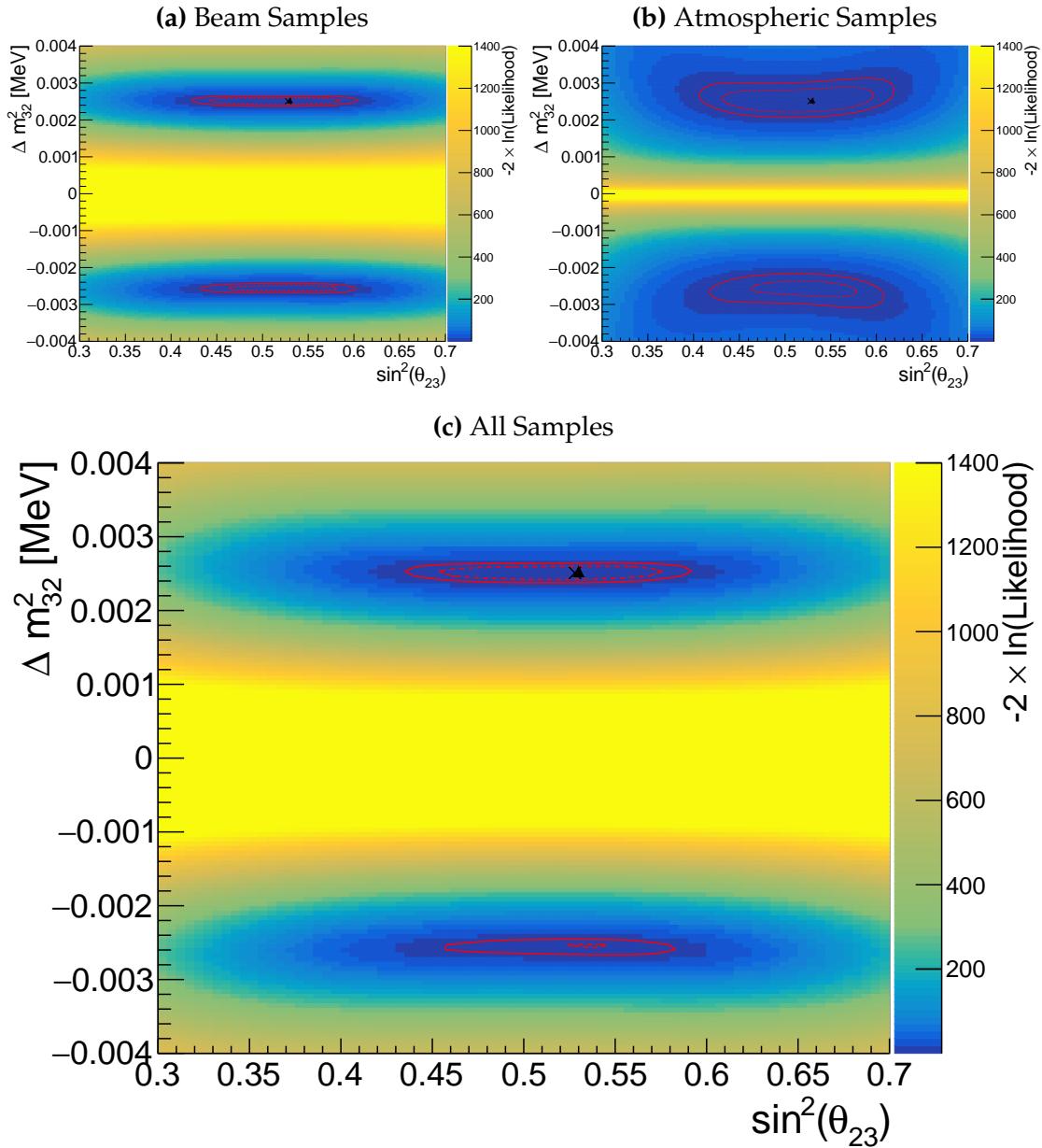


Figure 8.3: Two-dimensional log-likelihood scan of the disappearance ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) parameters showing the response of the beam samples (top left), atmospheric samples (top right) and the summed response (bottom). The Asimov A oscillation parameters, defined in Table 2.2, are known to be the true point (Black Cross). The position of the smallest log-likelihood is highlighted with the triangle. Prior uncertainty terms of the oscillation parameters are neglected. The two(three) sigma contour levels are illustrated with the dashed(solid) red line.

The likelihood scans illustrated thus far only consider the sensitivity of this analysis for a fixed set of true oscillation parameters, namely Asimov A defined in Table 2.2. Whilst computationally infeasible to run many fits at different parameter sets, it is possible to calculate the likelihood response to different Asimov data sets. Figure 8.4 and Figure 8.5 illustrate how the sensitivity changes for differing true values of δ_{CP} and $\sin^2(\theta_{23})$, respectively. For both of these plots, the other oscillation parameters are fixed at their Asimov A values. Consequently, the caveat of fixed systematic parameters and correlations between other oscillation parameters being neglected still applies.

To explain how these plots are made, consider Figure 8.4. This plot is built by considering multiple one-dimensional log-likelihood scans, each creating an Asimov data set with the value of δ_{CP} taken from the x-axis. The likelihood to this particular Asimov data set is calculated after reweighting the Monte Carlo prediction to each value of δ_{CP} on the y-axis.

Figure 8.4 illustrates the sensitivity to δ_{CP} . To interpret this plot, larger contours result in more phase space being excluded from the 1σ region. The 1σ intervals contain regions where the beam and atmospheric samples have discontinuous contours. For example, for the x-axis value of $\delta_{CP} = 0$, the beam samples sensitivity would include two discontinuous regions excluded from the 1σ interval: $\delta_{CP} \sim 0$ and $\delta_{CP} \sim \pi$. This behaviour is also seen in atmospheric samples response but at a value of $\delta_{CP} \sim -1$. This difference allows the joint fit to have increased sensitivity to these regions. Consequently, the difference between the beam-only and joint beam-atmospheric fit should be studied using multiple Asimov data sets.

Despite the increased sensitivity at 1σ , the 2σ intervals from the joint fit are more similar to the two independent sensitivities and the off-diagonal degeneracies mostly remain. This indicates that the joint fit has the strength to aid parameter determination but can not entirely break the degeneracies in δ_{CP} at higher confidence levels.

3039 Figure 8.5 illustrates a similar analysis as above, although the value of $\sin^2(\theta_{23})$
3040 is varied and δ_{CP} is fixed to the Asimov A parameter value. Due to the beam
3041 parameters and baseline being tuned to specifically target this oscillation parame-
3042 ter, the average sensitivity of the beam samples is stronger than the atmospheric
3043 samples. However, the degeneracy around maximal mixing ($\sin^2(\theta_{23}) = 0.5$) is
3044 significantly more peaked in the beam samples compared to the atmospheric
3045 samples. This means that a value of $\sin^2(\theta_{23}) \sim 0.56$ would be contained within
3046 the 1σ confidence interval for a true value of $\sin^2(\theta_{23}) \sim 0.46$ if using the beam-
3047 only analysis, whereas it would be excluded in the joint analysis.

3048 This behaviour is strengthened when considering the 2σ intervals, to the
3049 point where two distinct discontinuous regions of the 2σ intervals exist around
3050 the Asimov point $\sin^2(\theta_{23}) \sim 0.41, 0.6$. Given the caveat of only considering
3051 likelihood scans, the joint analysis would mostly eliminate the discontinuous
3052 intervals in these regions. This means that the joint fit could feasibly have an
3053 increased preference for the correct octant hypothesis.

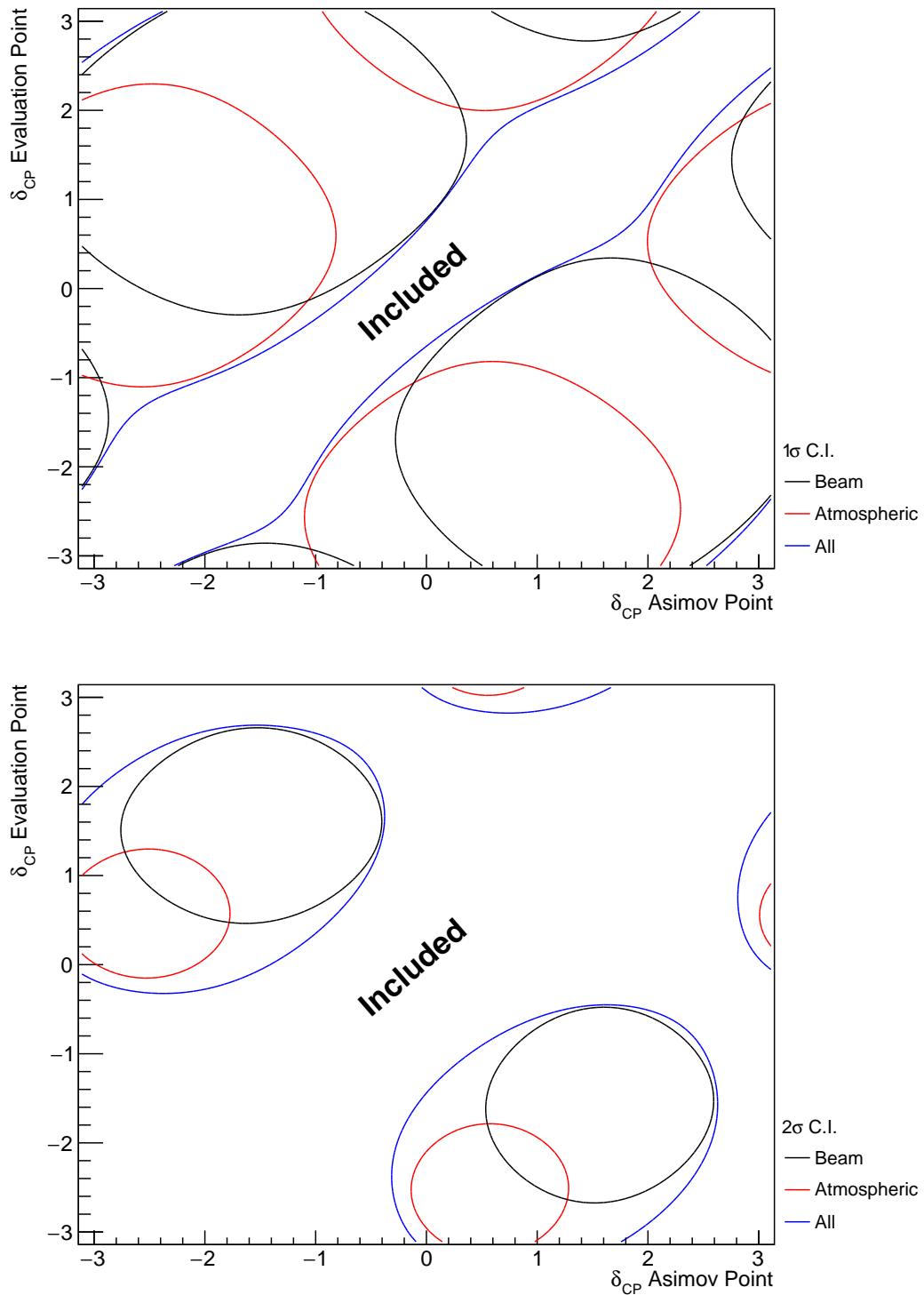


Figure 8.4: A series of one-dimensional likelihood scans over δ_{CP} , where an Asimov data set is built for each value of δ_{CP} on the x-axis and the likelihood is evaluated for each value of δ_{CP} on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

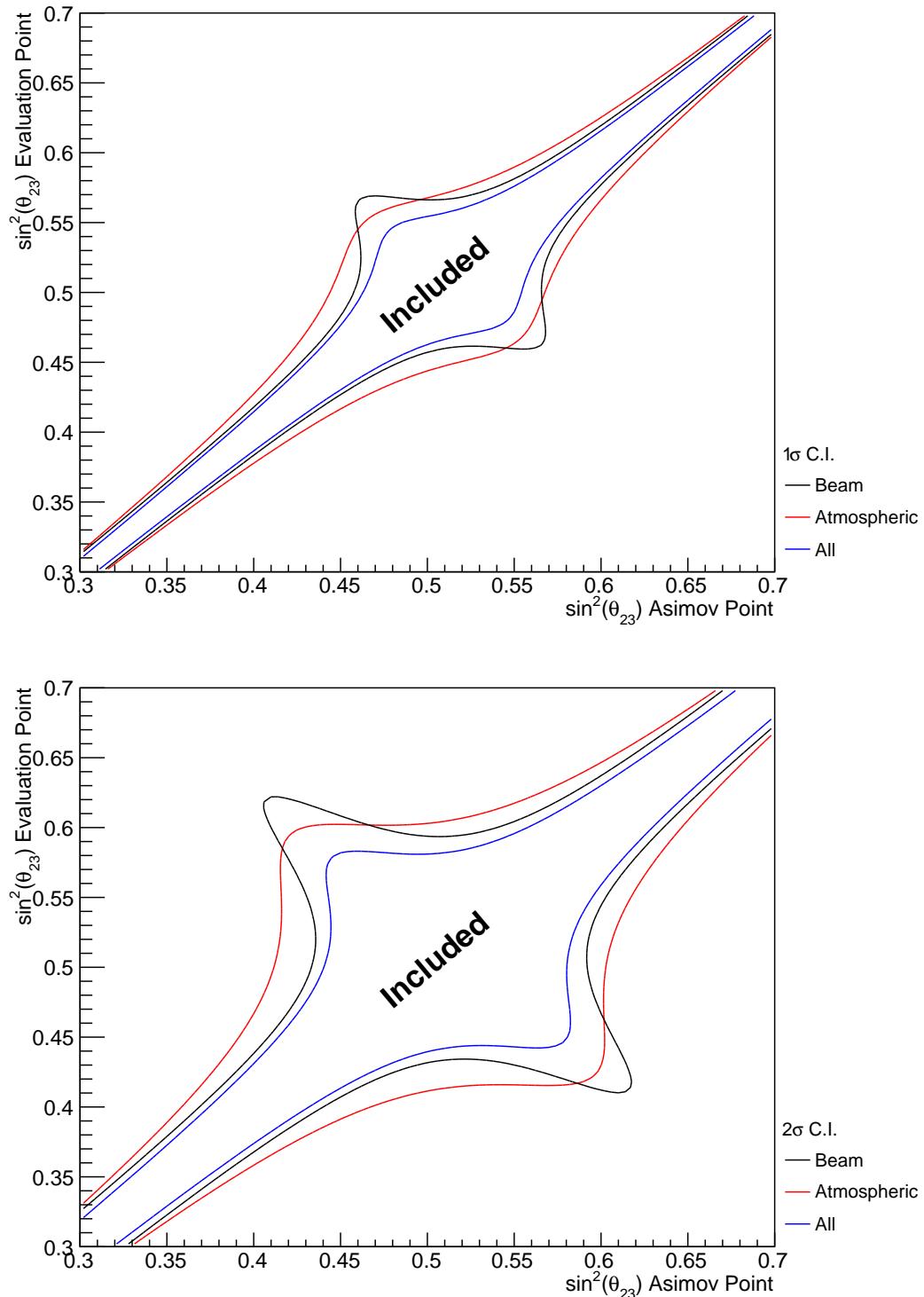


Figure 8.5: A series of one-dimensional likelihood scans over $\sin^2(\theta_{23})$, where an Asimov data set is built for each value of $\sin^2(\theta_{23})$ on the x-axis and the likelihood is evaluated for each value of $\sin^2(\theta_{23})$ on the y-axis. The diagonal represents the minimum log-likelihood and defines the region included within the 1σ (Top) and 2σ (Bottom) confidence intervals. The beam (black) and atmospheric (red) samples are individually plotted and the joint fit (blue) is the sum of the two.

Alongside oscillation parameters (Figure 8.1), the sensitivity to systematic parameters can also be studied for the joint fit. As some of these parameters are correlated between the beam and atmospheric events, the response of the atmospheric samples can modify the constraint. This means the systematics can have additional constraints than they would from a beam-only analysis. Therefore, the response from the beam and the atmospheric samples to various systematic parameters has been compared in Figure 8.6. The Asimov data set has been created using the AsimovA oscillation parameter and the pre-fit systematic tune. For example, the systematic parameter controlling the effective axial mass coupling in CCQE interactions, M_A^{QE} , is clearly dominated by the ND constraint. An example where the response of the atmospheric sample is approximately similar to the near detector constraint is the 2p2h CtoO normalisation systematic. This systematic models the scaling of the 2p2h interaction cross-section on a carbon target to an oxygen target. There are also systematics that have no near detector constraint. For example, the systematic parameters which describe the normalisation of the NC1Gamma and NCOther interaction modes. The atmospheric and beam samples can have similar sensitivity to these systematics due to their similar composition in energy and interaction mode. As an example of how the atmospheric samples can help constrain systematic parameters used within the T2K-only analysis, these NC background events in beam electron-like samples will be more constrained with the additional sensitivity of atmospheric samples. This would be expected to reduce the overall uncertainty of the beam electron-like event rates in the joint analysis compared to the beam-only studies. This could modify the sensitivity of the beam samples due to the more constrained background events.

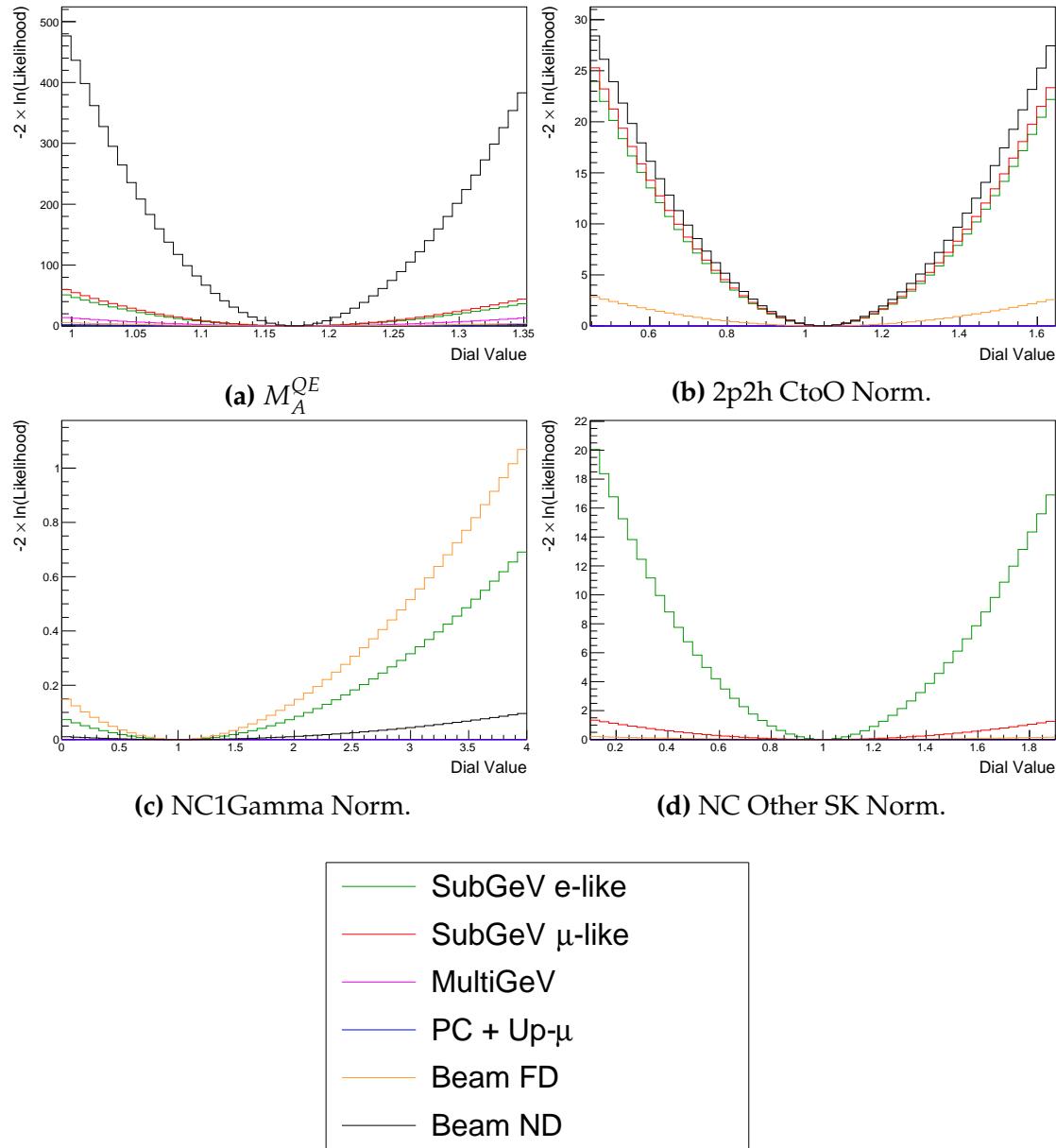


Figure 8.6: The response of the likelihood, as defined in section 8.2, illustrating the response of the samples to the various cross-section systematic parameters.

3079 8.3 Sensitivity Studies

3080 The sensitivities of the joint T2K and SK oscillation analysis are presented in
3081 the form of Asimov fits. These fits consider beam samples from the near and
3082 far detector alongside atmospheric samples at SK. This technique builds an
3083 Asimov data set (following section 6.5) using the AsimovA oscillation parameters
3084 and post-BANFF systematic tune, which is then fit. This technique eliminates
3085 statistical fluctuations from the data, therefore, providing the maximum sen-
3086 sitivity of the analysis.

3087 In practice, the Asimov fits presented within this analysis are modified from
3088 the above definition. An Asimov prediction of both beam and atmospheric far
3089 detector samples is fit whilst the true data is used for near detector samples. The
3090 Asimov predictions at the far detector are built using the post-BANFF tune (as
3091 discussed in section 3.2). These modifications mean that the results are equivalent
3092 to performing a far detector Asimov fit using inputs from the BANFF data fit.
3093 Consequently, this allows the results to be cross-checked with the results from
3094 the P-Theta analysis. The comparison has been performed and is documented in
3095 [214]. No significant discrepancies were found between the fitters.

3096 This section proceeds with the following studies. Firstly, the sensitivity
3097 of the atmospheric samples using the correlated detector model is detailed in
3098 subsection 8.3.1. This includes studying the choice of applying the 2020 PDG
3099 reactor constraint [76] to the atmospheric samples, which is documented in
3100 subsection 8.3.2. Additionally, the effect of applying the near-detector constraints
3101 onto the atmospheric samples is discussed in subsection 8.3.3. The main result is
3102 the sensitivity of the simultaneous beam and atmospheric fit. The sensitivities,
3103 both with and without the application of the reactor constraint, are presented
3104 in subsection 8.3.4 and subsection 8.3.5, respectively. To indicate the benefit
3105 of the joint analysis, the sensitivities are compared to the 2020 T2K beam-only
3106 sensitivities [2, 215] in subsection 8.3.6 and subsection 8.3.7. The T2K analysis
3107 is used as a reference as it uses the same samples and a similar systematic

model. As shown in section 8.2, the response of the beam and atmospheric samples change depending upon the true set of oscillation parameters assumed. Therefore, subsection 8.3.8 documents the sensitivities at an alternative oscillation parameter set. These results have been presented at the Neutrino 2022 conference on behalf of the T2K and SK collaborations [4].

8.3.1 Atmospheric-Only Sensitivity Without Reactor Constraint

This section presents the results of an Asimov fit using samples from the near detector and only atmospheric samples from the far detector. The results are presented as one-dimensional or two-dimensional histograms which have been marginalised over all other parameters using the technique outlined in subsection 4.3.1. Each histogram displays the posterior probability density and illustrates the credible intervals, calculated using the technique in subsection 4.3.2. For this fit, a flat prior is used for $\sin^2(\theta_{13})$ meaning that the reactor constraint is not applied. The Asimov data is generated assuming the AsimovA oscillation parameter set defined in Table 2.2 and the post-BANFF systematic parameter tune.

Figure 8.7 illustrates the posterior probability density for δ_{CP} , marginalised over both hierarchies. The fit favours the known oscillation parameter ($\delta_{CP} = -1.601$) although the posterior probability is very flat through the range of $-\pi < \delta_{CP} < -1$ and $2 < \delta_{CP} < \pi$. There is also a region around $\delta_{CP} \sim 0.4$ which is disfavoured at 2σ . This indicates that the SK samples can rule out some parts of the CP conserving parameter space reasonably well, near $\delta_{CP} \sim 0.4$, when the true value of $\delta_{CP} \sim -\pi/2$.

The posterior probability density in Δm_{32}^2 is given in Figure 8.8. This distribution includes steps in both the normal hierarchy (NH, $\Delta m_{32}^2 > 0$) and the inverse hierarchy (IH, $\Delta m_{32}^2 < 0$). The highest posterior probability density is found within the NH 1σ credible interval, which agrees with the known oscillation parameter value, $2.509 \times 10^{-3}\text{eV}^2$. However, all of the credible intervals span both of the hierarchy hypotheses.

Without Reactor Constraint, Both Hierarchies

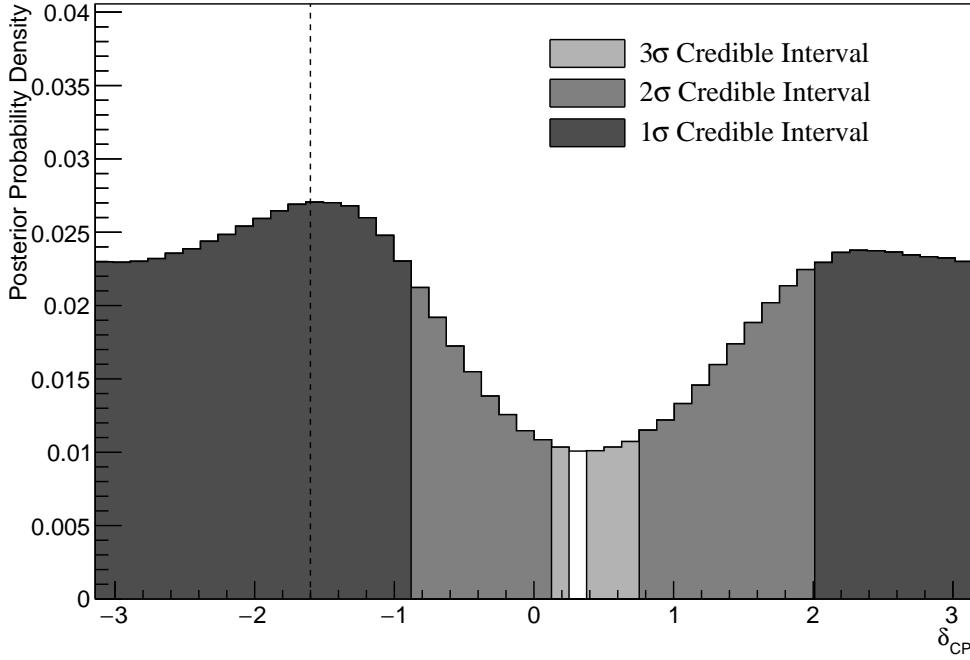


Figure 8.7: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.17	0.40	0.58
IH ($\Delta m_{32}^2 < 0$)	0.13	0.29	0.42
Sum	0.31	0.69	1.00

Table 8.2: The distribution of steps in an SK atmospheric-only fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.37$ and $B(\text{UO}/\text{LO}) = 2.24$.

Following the discussion in subsection 4.3.3, the Bayes factor for hierarchy preference can be calculated by determining the fraction of steps that fall into the NH and the IH regions, as an equal prior is placed on both hypotheses. A similar calculation can be performed by calculating the fraction of steps which fall in the lower octant (LO, $\sin^2 \theta_{23} < 0.5$) or upper octant (UO, $\sin^2 \theta_{23} > 0.5$). The fraction of steps, broken down by hierarchy and octant, are given in Table 8.2. The Bayes factor for preferred hierarchy hypothesis is $B(\text{NH}/\text{IH}) = 1.37$. Jeffrey's

Without Reactor Constraint, Both Hierarchies

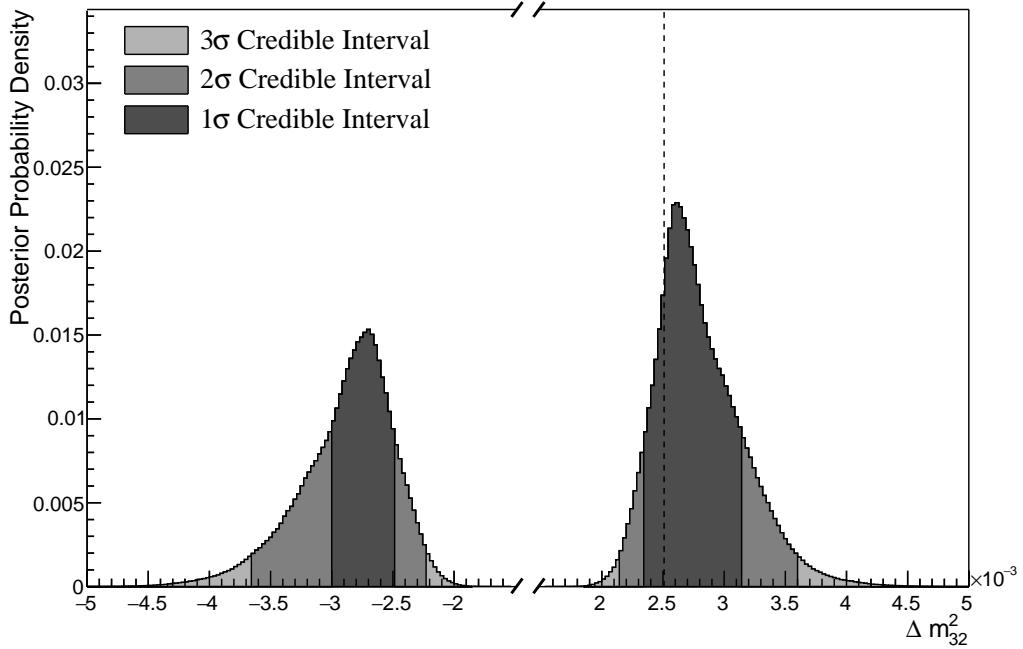


Figure 8.8: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

scale, given in Table 4.1, states this value of the Bayes factor indicates a weak preference for the normal hierarchy hypothesis. The Bayes factor for choice of octant is $B(\text{UO}/\text{LO}) = 2.24$. This is also classified as a weak preference for the UO. Both of these show that the fit is returning the correct choice of hypotheses (NH and UO) for the known Asimov A oscillation parameters defined in Table 2.2.

The 1 σ credible intervals, broken down by hierarchy, and position in parameter space of the highest posterior probability density is given in Table 8.3. These are taken from the one-dimensional projections of the oscillation parameters, marginalised over all other parameters within the fit. As the distribution is binned, the highest posterior density is presented as the center of the bin with the highest posterior density with an error equal to the bin width. For the known Asimov value of $\delta_{CP} = -1.601$, the 1 σ credible interval rules out a region between $\delta_{CP} = -0.88$ and $\delta_{CP} = 1.96$, when marginalising over both hierarchies. The position of the highest posterior density is $\delta_{CP} = -1.57 \pm 0.07$ which is clearly

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-\pi, -0.88], [1.88, \pi]$	-1.57 ± 0.07
δ_{CP} , (IH)	$[-\pi, -0.88], [2.01, \pi]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-3.00, -2.49], [2.34, 3.14]$	2.61 ± 0.02
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.41, 3.04]$	2.59 ± 0.03
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-3.11, -2.41]$	-2.73 ± 0.03
$\sin^2(\theta_{23})$ (BH)	$[0.476, 0.584]$	0.542 ± 0.006
$\sin^2(\theta_{23})$ (NH)	$[0.488, 0.596]$	0.554 ± 0.006
$\sin^2(\theta_{23})$ (IH)	$[0.476, 0.584]$	0.542 ± 0.006

Table 8.3: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the SK atmospheric-only fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3157 compatible with the known oscillation parameter value.

3158 The sensitivity of the atmospheric samples to $\sin^2(\theta_{13})$ is presented in Fig-
 3159 ure 8.9. The likelihood scans presented in Figure 8.1 suggest that the sensitivity
 3160 to $\sin^2(\theta_{13})$ will be small. This behaviour is also seen in the fit results, where the
 3161 width of the 1σ credible intervals span the region of $\sin^2(\theta_{13}) = [0.008, 0.08]$. This
 3162 is more than an order of magnitude worse than the constraint from reactor
 3163 experiments [76].

3164 As previously discussed, the correlations between oscillation parameters are
 3165 also important to understand how the atmospheric samples respond. Figure 8.10
 3166 illustrates the two dimensional $\sin^2(\theta_{13})-\delta_{CP}$ sensitivity, marginalised over all
 3167 other parameters. The shape of the 1σ credible interval shows that the constrain-
 3168 ing power of the fit on δ_{CP} is dependent upon the value of $\sin^2(\theta_{13})$. Furthermore,
 3169 they show a strong resemblance to the likelihood scans illustrated in Figure 8.2.
 3170 Whilst the atmospheric samples do not strongly constrain the value of $\sin^2(\theta_{13})$,
 3171 the value of $\sin^2(\theta_{13})$ does impact the atmospheric samples' sensitivity to δ_{CP} .

Without Reactor Constraint, Both Hierarchies

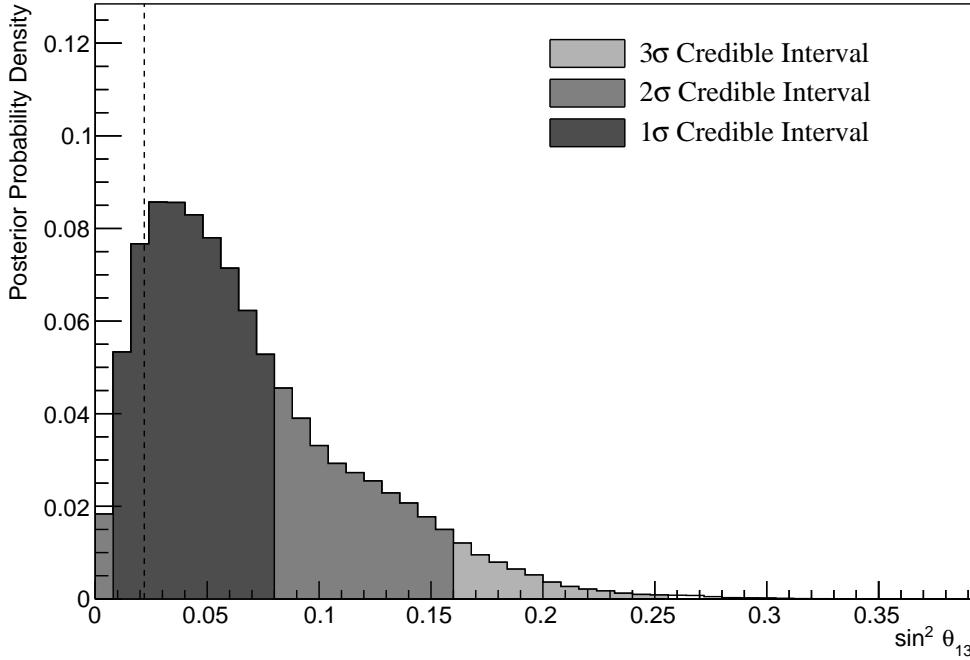


Figure 8.9: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{13})$.

3172 The $\sin^2(\theta_{23}) - \Delta m_{32}^2$ disappearance contours are illustrated in Figure 8.11. As
 3173 expected, the area contained in the inverted hierarchy 1σ credible interval is
 3174 slightly smaller than that in the normal hierarchy. This follows from the Bayes
 3175 factor showing a weak preference for NH meaning that more of the steps will exist
 3176 in the $\Delta m_{32}^2 > 0$ region. The known oscillation parameters of $\sin^2(\theta_{23}) = 0.528$
 3177 and $\Delta m_{32}^2 = 2.509 \times 10^{-3} \text{ eV}^2$ are contained within the 1σ credible interval.

3178 Figure 8.12 illustrates the two-dimensional projections for each permutation of
 3179 oscillation parameters which this analysis is sensitive to: δ_{CP} , $\sin^2(\theta_{13})$, $\sin^2(\theta_{23})$,
 3180 and Δm_{32}^2 . The purpose of this plot is to illustrate the correlations between
 3181 the oscillation parameters. The contours are calculated whilst marginalising
 3182 over both hierarchies, however, only the NH is illustrated when plotting the
 3183 Δm_{32}^2 parameter. As expected the correlations play a significant role in these
 3184 sensitivity measurements, especially the choice of the $\sin^2(\theta_{13})$ constraint. Most
 3185 notably, the application of reactor constraint would be expected to alter both the

Without Reactor Constraint, Both Hierarchies

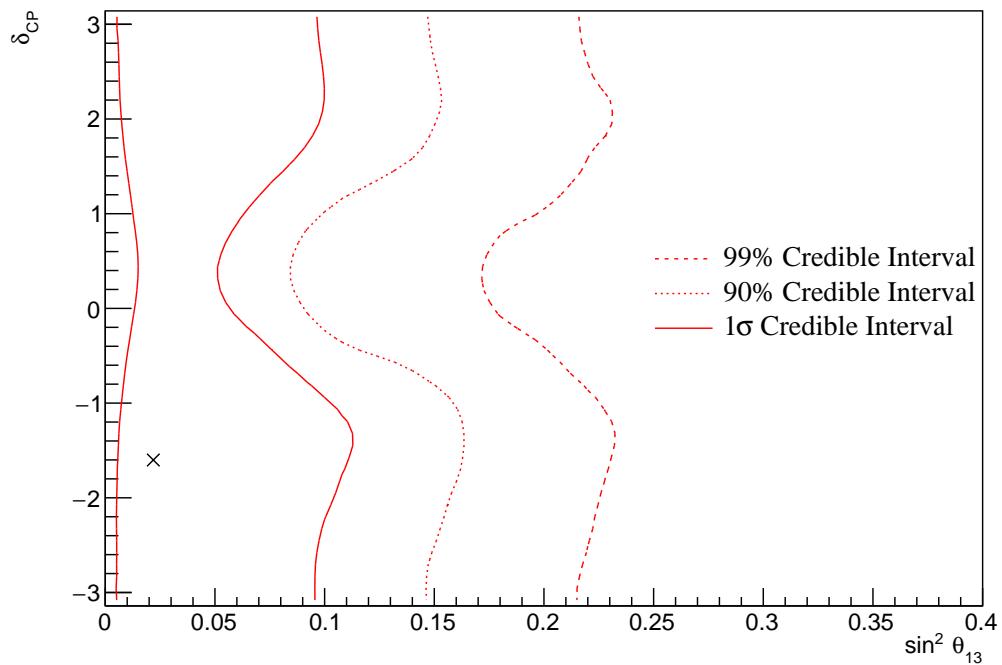


Figure 8.10: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of δ_{CP} – $\sin^2(\theta_{13})$.

³¹⁸⁶ width and position of the Δm_{32}^2 intervals due to the strong correlation between
³¹⁸⁷ the parameters.

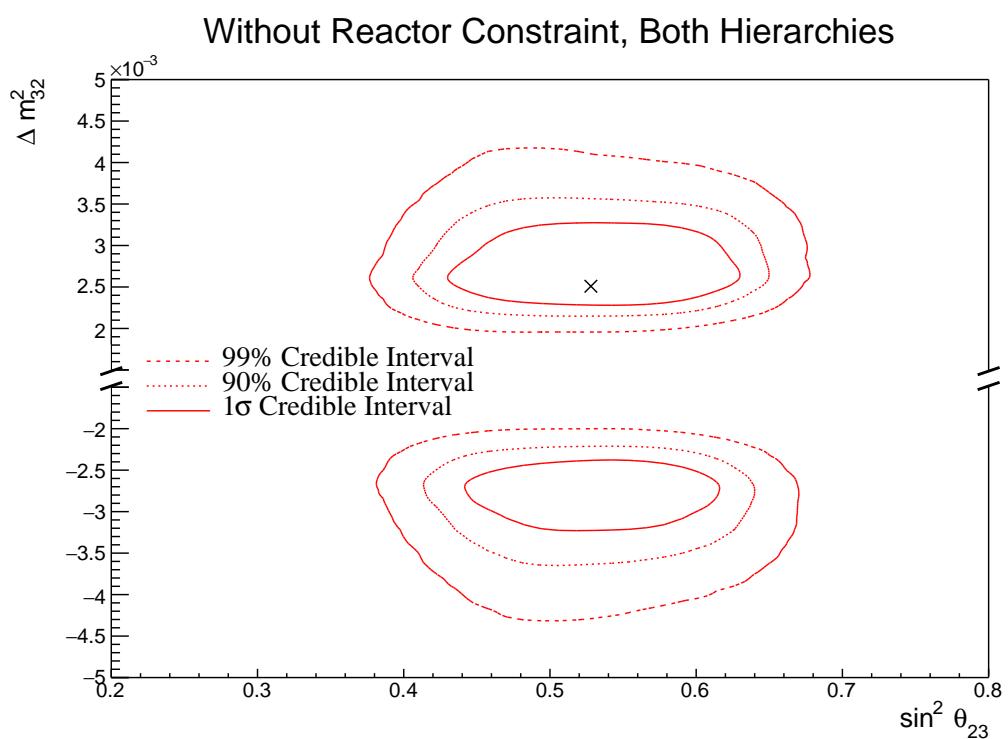


Figure 8.11: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the SK atmospheric-only fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

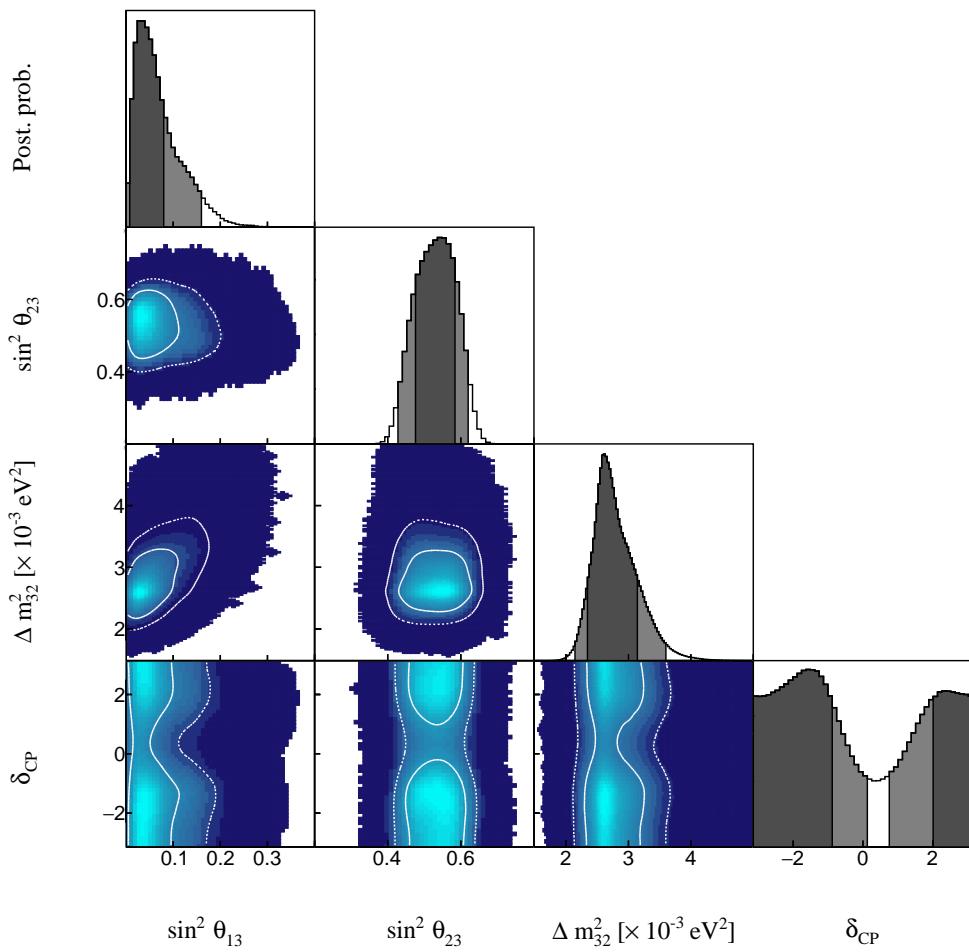


Figure 8.12: The posterior probability density distribution from the SK atmospheric-only fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3188 8.3.2 Atmospheric-Only Sensitivity With Reactor Constraint

3189 The results in subsection 8.3.1 discuss the atmospheric sensitivity when the reactor
 3190 constraint is not applied. The correlations illustrated in Figure 8.12 indicate that
 3191 the marginalisation effects could contribute to differing sensitivities when the
 3192 external reactor constraint is applied. Using the technique discussed in subsec-
 3193 tion 4.1.1, the posterior distribution of the fit in subsection 8.3.1 can be reweighted
 3194 to include the reactor constraint of $\sin^2(\theta_{13}) = (2.18 \pm 0.08) \times 10^{-2}$ [76].

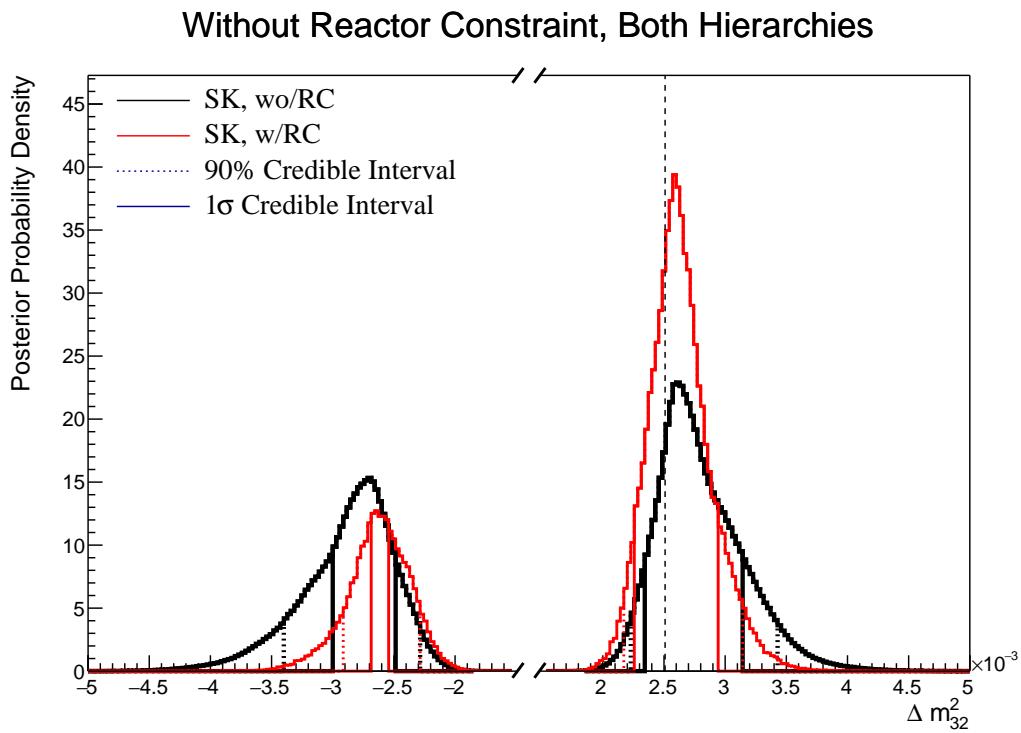


Figure 8.13: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint applied (Red). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of Δm_{32}^2 .

3195 The reactor constraint increases the sensitivity of the atmospheric samples to
 3196 Δm_{32}^2 as illustrated in Figure 8.13. The 1σ credible interval in Δm_{32}^2 is determined
 3197 to be $[-2.69, -2.54] \times 10^{-3} \text{ eV}^2$ and $[2.25, 2.94] \times 10^{-3} \text{ eV}^2$. The width of the IH
 3198 credible interval is reduced by $\sim 70\%$ when the reactor constraint is applied. Due
 3199 to the marginalisation effects observed in Figure 8.12, the favoured region of Δm_{32}^2
 3200 moves closer to zero for both hierarchies. A clear explanation of this behaviour is

3201 illustrated in Figure 8.14, which shows the posterior distribution in the Δm_{32}^2 –
 3202 $\sin^2(\theta_{13})$ parameters. The correlation between Δm_{32}^2 and $\sin^2(\theta_{13})$ is such that
 3203 lower values of $\sin^2(\theta_{13})$ tend towards lower values of $|\Delta m_{32}^2|$. Therefore the
 3204 application of the reactor constraint moves the posterior distribution towards
 3205 the known oscillation parameter.

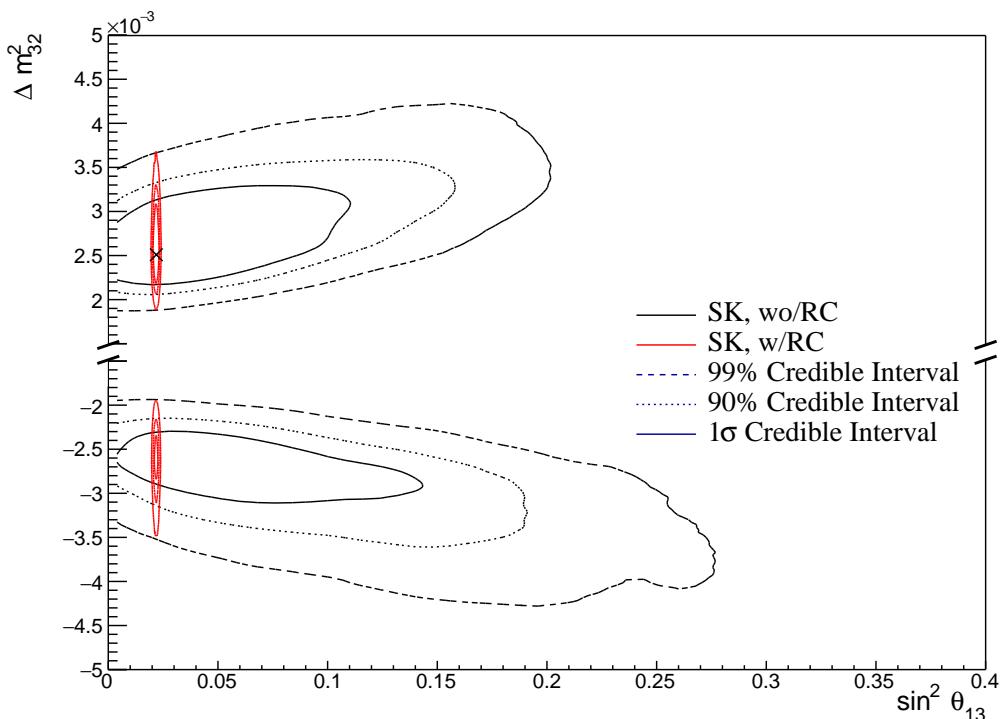


Figure 8.14: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$ compared between the SK atmospheric-only fit (Black) and the SK atmospheric fit with the reactor constraint (Red). The distributions are marginalised over both hierarchies. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{13})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.21	0.53	0.74
IH ($\Delta m_{32}^2 < 0$)	0.08	0.18	0.26
Sum	0.29	0.71	1.00

Table 8.4: The distribution of steps in an SK atmospheric with reactor constraint fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 2.85$ and $B(\text{UO}/\text{LO}) = 2.39$.

3206 Table 8.4 presents the fraction of steps in each hierarchy and octant model
3207 for the fit after the reactor constraint has been applied. The reactor constraint
3208 significantly increases the NH preference, increasing the Bayes factor from
3209 $B(\text{NH}/\text{IH}) = 1.37$ to $B(\text{NH}/\text{IH}) = 2.85$ when the reactor constraint is applied.
3210 This is still defined as a weak preference for the NH hypothesis according to
3211 Jeffrey's scale, however, it is a stronger preference than when the constraint is
3212 not applied. The preference for the correct octant model is also slightly increased
3213 by the application of the reactor constraint.

3214 8.3.3 Impact of Near Detector Constraints for Atmospheric Sam- 3215 ples

3216 The choice of applying the near detector constraints to the low-energy atmo-
3217 spheric samples was introduced in subsection 6.4.3. This subsection illustrates the
3218 effect of removing the ND constraint on the sensitivity of the atmospheric samples
3219 to the oscillation parameters. To do this, the fit presented in subsection 8.3.1 has
3220 been compared to another fit where the constraints from the near detector have
3221 not been included. This is the only case where the near detector constraints are
3222 neglected throughout this chapter. For both fits, the Asimov data was generated
3223 assuming the ‘AsimovA’ oscillation parameter set defined in Table 2.2 and the
3224 post-BANFF systematic parameter tune.

3225 The change in sensitivity on δ_{CP} is given in Figure 8.15. The reactor constraint
3226 is not applied in either of the fits within this comparison. The fit which includes
3227 the near detector constraint is slightly more peaked at the known oscillation
3228 parameter value. The width of the 1σ credible intervals are approximately the
3229 same (identical to within a bin width) and the same conclusion holds for the
3230 higher credible intervals. The change in sensitivity to other oscillation parameters
3231 has been studied and no significant discrepancies were found. This shows that
3232 the exact choice of constraint does not significantly affect the physics conclusions
3233 one would make from this analysis.

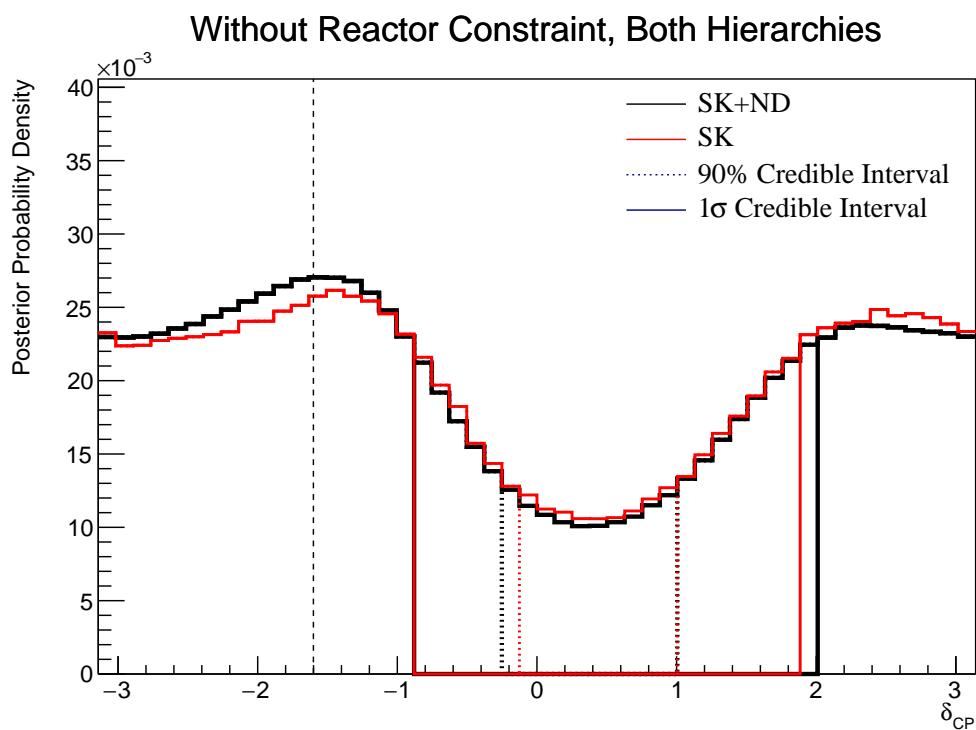


Figure 8.15: The one-dimensional posterior probability density distribution in δ_{CP} compared between the SK atmospheric-only fit where the near detector constraint is (Black) and is not (Red) applied. The distributions are marginalised over both hierarchies. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of δ_{CP} .

3234 8.3.4 Atmospheric and Beam Sensitivity without Reactor Con- 3235 straint

3236 This section presents the sensitivities of the simultaneous beam and atmospheric
 3237 analysis where the reactor constraint is not applied. Similar to the previous
 3238 studies, the Asimov data is built assuming the post-BANFF systematic tune and
 3239 Asimov A oscillation parameters defined in Table 2.2. This fit uses all 18 near
 3240 detector beam samples, 5 far detector beam samples, and 18 atmospheric samples.

Without Reactor Constraint, Both Hierarchies

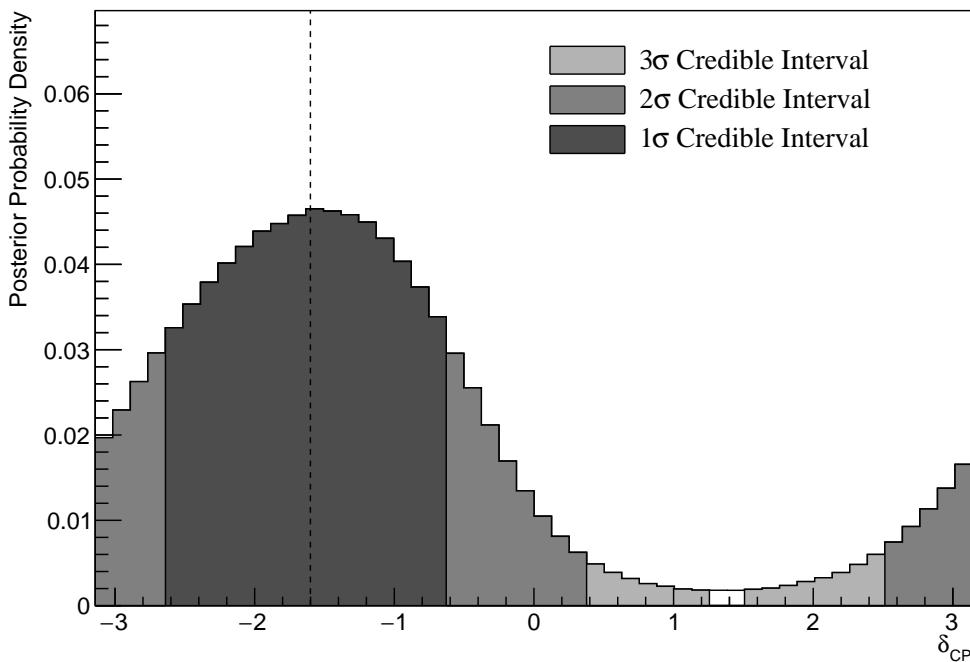


Figure 8.16: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of δ_{CP} .

3241 The sensitivity to δ_{CP} , marginalised over both hierarchies, is given in Fig-
 3242 ure 8.16. The credible intervals and highest posterior distribution for each
 3243 oscillation parameter is given in Table 8.5. The highest posterior probability
 3244 density is $\delta_{CP} = -1.57 \pm 0.07$ and is compatible with the known value of
 3245 $\delta_{CP} = -1.601$. The CP-conserving values of $\delta_{CP} = 0, \pm\pi$ are disfavoured at
 3246 1σ credible interval. There is also a region around $\delta_{CP} = 1.4$ which is disfavoured
 3247 at more than 3σ . Whilst these conclusions can only be made at this particular

3248 Asimov point, it does show that if the true value of δ_{CP} were CP-violating,
3249 this joint analysis would be able to disfavour CP conserving values at over 1σ
3250 without any external constraints.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-2.64, -0.63]$	-1.57 ± 0.07
δ_{CP} , (NH)	$[-2.76, -0.63]$	-1.45 ± 0.07
δ_{CP} , (IH)	$[-2.39, -0.88]$	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.58]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.51]$	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.480, 0.545]	0.518 ± 0.003
$\sin^2(\theta_{23})$ (NH)	[0.480, 0.545]	0.508 ± 0.003
$\sin^2(\theta_{23})$ (IH)	[0.480, 0.545]	0.513 ± 0.003

Table 8.5: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

3251 The sensitivity to Δm_{32}^2 is illustrated in Figure 8.17. Notably, the 1σ credible
3252 interval is entirely contained within the NH region, as further evidenced by
3253 Table 8.5. This illustrates good sensitivity to the mass hierarchy as it is correctly
3254 selecting the correct hypothesis. This is reflected in the 1σ credible intervals being
3255 approximately the same when they are constructed considering both hierarchies
3256 and when considering only the NH region. The NH distribution favours this
3257 region surrounding the known Asimov point, $\Delta m_{32}^2 = 2.509 \times 10^{-3}\text{eV}^2$, where
3258 the highest posterior probability density is at $\Delta m_{32}^2 = (2.51 \pm 0.01) \times 10^{-3}\text{eV}^2$.

3259 The fraction of steps in each of the mass hierarchy regions and octants of
3260 $\sin^2(\theta_{23})$ is given in Table 8.6. The Bayes factors are determined to be $B(\text{NH}/\text{IH}) =$
3261 3.67 and $B(\text{UO}/\text{LO}) = 1.74$. Jeffrey's scale states that this value of the mass
3262 hierarchy Bayes factor illustrates substantial evidence for the NH hypothesis.

This corresponds to the correct hypothesis given the known oscillation parameters and is a stronger statement than the atmospheric-only analysis can provide. It is important to note that this substantial preference requires no external constraints. The Bayes factor for octant determination represents a weak preference for the upper octant, therefore, selecting the correct octant hypothesis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.29	0.50	0.79
IH ($\Delta m_{32}^2 < 0$)	0.08	0.13	0.21
Sum	0.37	0.63	1.00

Table 8.6: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 3.67$ and $B(\text{UO}/\text{LO}) = 1.74$.

Without Reactor Constraint, Both Hierarchies

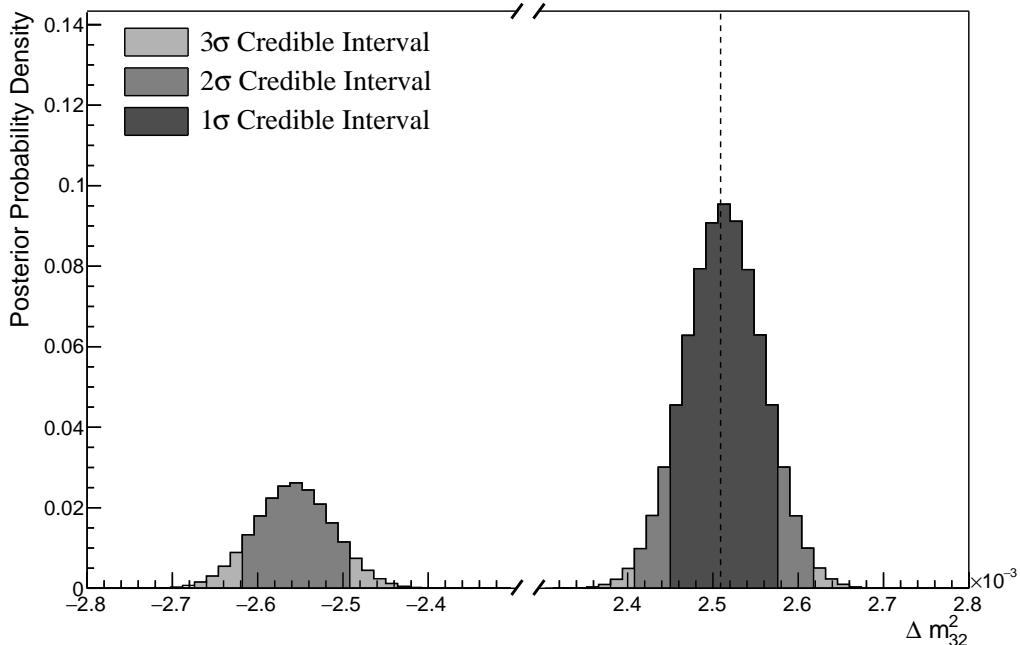


Figure 8.17: The one-dimensional posterior probability density distribution in Δm_{32}^2 , marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of Δm_{32}^2 .

The sensitivity to $\sin^2(\theta_{23})$ is presented in Figure 8.18. There is a clear

3269 preference for the upper octant but the peak of the distribution is relatively
 3270 flat. It peaks at $\sin^2(\theta_{23}) = 0.509 \pm 0.003$ which is in the region of the known
 3271 value of $\sin^2(\theta_{23}) = 0.528$. The difference in the highest posterior distribution
 3272 and the width of the credible interval is relatively unchanged when consid-
 3273 ering different hierarchy hypotheses showing no strong correlation between
 3274 $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

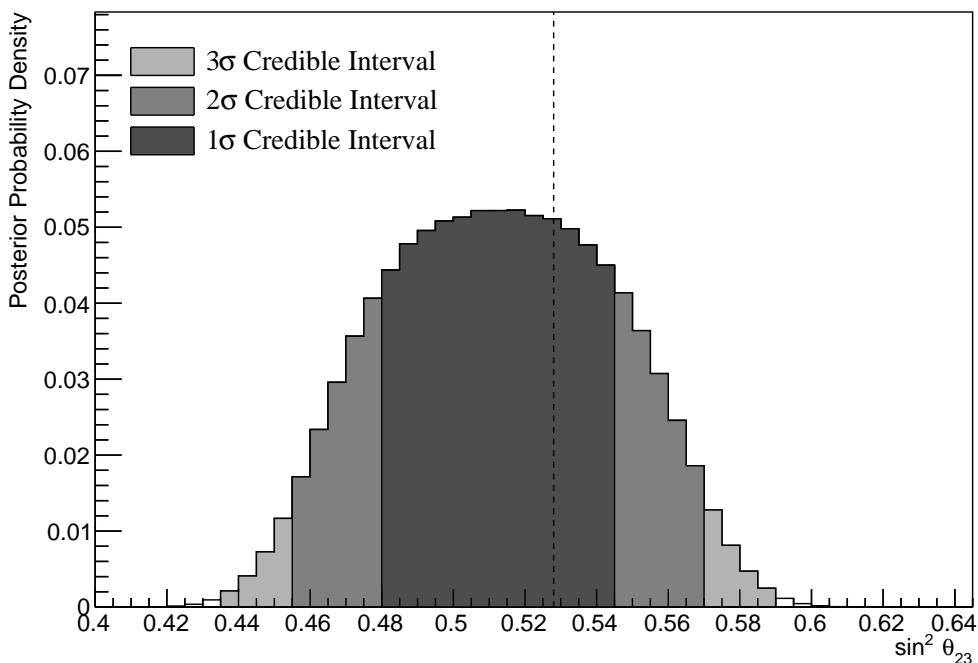


Figure 8.18: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3275 The sensitivity presented as a function of the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$)
 3276 is given in Figure 8.19. As expected, the contours follow the likelihood shape
 3277 given in Figure 8.2, where the 2σ credible intervals have a closed contour exclud-
 3278 ing the region around $\delta_{CP} \sim 1.2$. The width of the 3σ credible interval in $\sin^2(\theta_{13})$
 3279 is dependent upon the value of δ_{CP} . Close to the Asimov point, $\delta_{CP} = -1.601$, the
 3280 width of the 3σ credible interval approximately spans $\sin^2(\theta_{13}) = [0.013, 0.04]$.
 3281 This is reduced to a region of $\sin^2(\theta_{13}) = [0.023, 0.042]$ at the most disfavoured
 3282 value of δ_{CP} . The 1σ credible interval is consistent with the known oscillation

parameter. Application of the reactor constraint would be expected to decrease the width of the 1σ credible intervals in δ_{CP} due to the triangular shape of the posterior probability.

The sensitivity in terms of the disappearance parameters, $\sin^2(\theta_{23}) - \Delta m_{32}^2$, is given in Figure 8.20. The area contained within the IH contours is significantly smaller than the area within the NH contours. The IH credible intervals are also notably tighter in the $\sin^2(\theta_{23})$ dimension. No significant correlation is observed between $\sin^2(\theta_{23})$ and $|\Delta m_{32}^2|$.

Without Reactor Constraint, Both Hierarchies

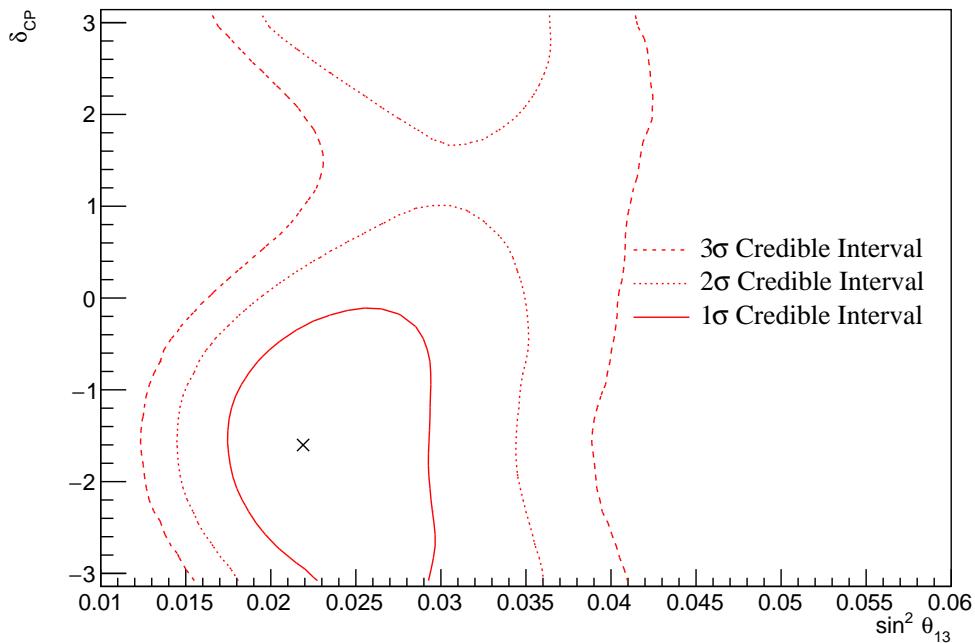


Figure 8.19: The two-dimensional posterior probability density distribution in $\delta_{CP}-\sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\delta_{CP}-\sin^2(\theta_{13})$.

The two-dimensional posterior distribution for each permutation of the oscillation parameters of interest is given in Figure 8.21. The most notable observation is that the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ are anti-correlated. If the value of $\sin^2(\theta_{13})$ was constrained closer to the known oscillation parameter value, the preferred value of $\sin^2(\theta_{23})$ would increase. This would move the highest posterior probability

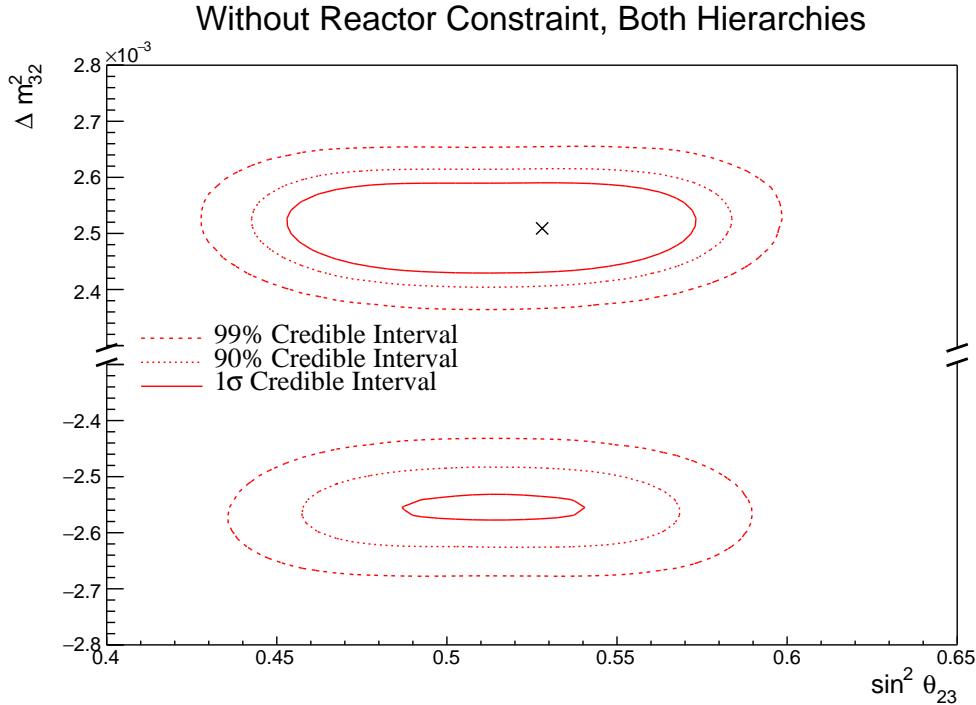


Figure 8.20: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

3296 closer in line with the known value and could lead to an increase in the preference
 3297 for the UO hypothesis.

3298 Furthermore, the δ_{CP} and $|\Delta m_{32}^2|$ oscillation parameters are anti-correlated,
 3299 such that higher values of $|\Delta m_{32}^2|$ prefer lower values of δ_{CP} . Whilst this is
 3300 an interesting result on its own, the width of the Δm_{32}^2 contours also depend
 3301 on $\sin^2(\theta_{13})$. This introduces another correlation effect that could modify the
 3302 sensitivity to δ_{CP} once the reactor constraint is applied.

3303 The correlation between $\sin^2(\theta_{13})$ and Δm_{32}^2 can be seen in Figure 8.22. A
 3304 much larger fraction of the posterior distribution is contained in the NH for lower
 3305 values of $\sin^2(\theta_{13})$. Consequently, the application of the reactor constraint would
 3306 be expected to significantly increase the preference for NH.

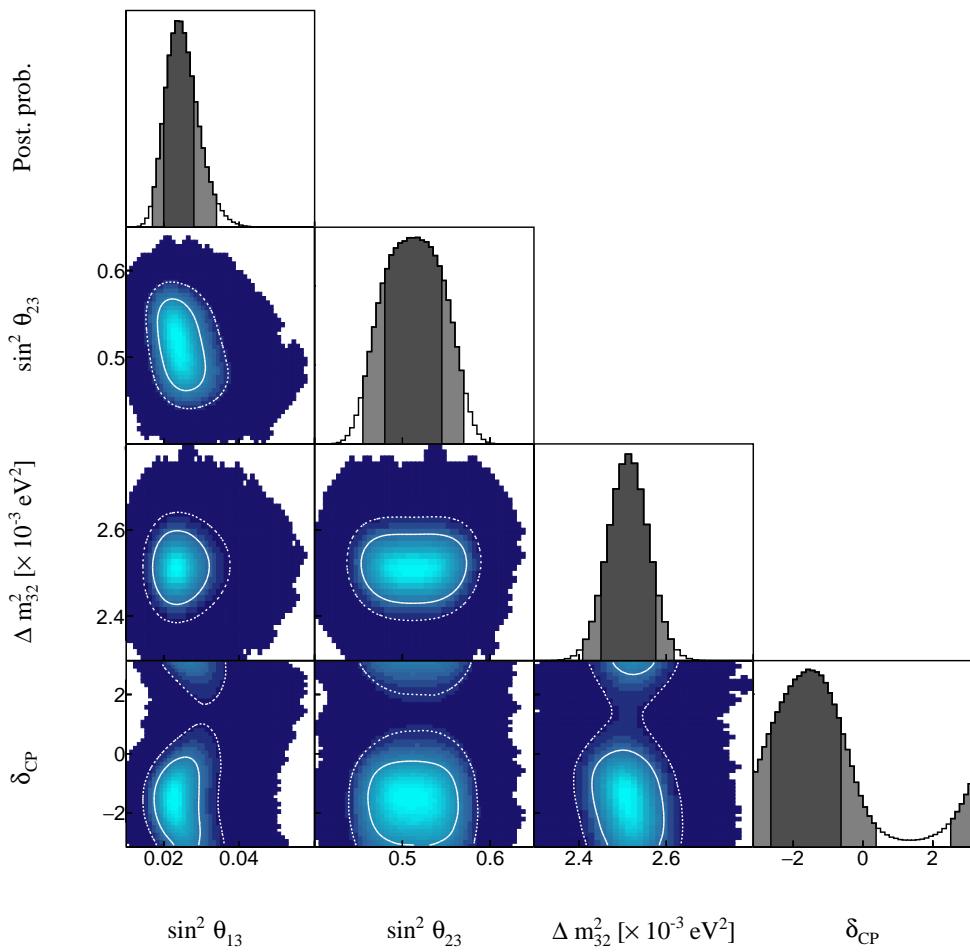


Figure 8.21: The posterior probability density distribution from the joint beam-atmospheric fit. The reactor constraint is not applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

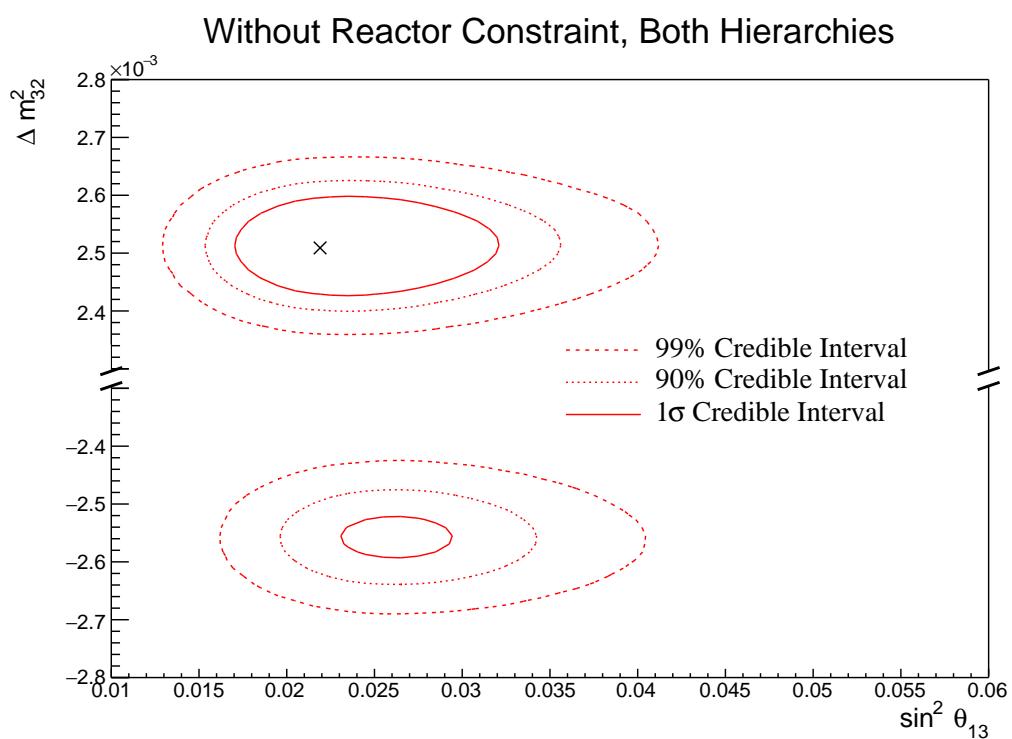


Figure 8.22: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit. The reactor constraint is not applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{13})$.

3307 8.3.5 Atmospheric and Beam Sensitivity with Reactor Constraint

3308 This section presents the sensitivities of the joint beam-atmospheric fit when
 3309 the reactor constraint is applied to $\sin^2(\theta_{13})$. As with the previous studies, the
 3310 Asimov data is made using the AsimovA oscillation parameter set defined in
 3311 Table 2.2 and the post-BANFF systematic parameter tune.

With Reactor Constraint, Both Hierarchies

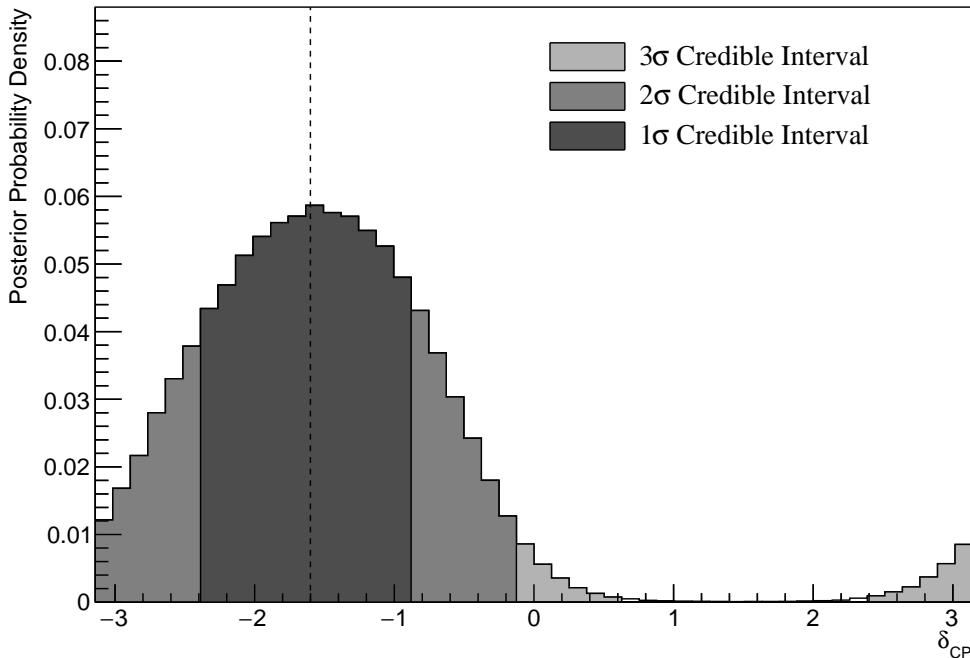


Figure 8.23: The one-dimensional posterior probability density distribution in δ_{CP} , marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of δ_{CP} .

3312 Figure 8.23 illustrates the sensitivity to δ_{CP} , marginalised over both hierarchies.
 3313 The CP-conserving value of $\delta_{CP} = 0$ is disfavoured at 2σ whilst the value of $\delta_{CP} =$
 3314 $\pm\pi$ is very close to being disfavoured at 2σ . Furthermore, the 3σ credible interval
 3315 excludes the region of $\delta_{CP} = [0.63, 2.39]$, thus clearly disfavouring the region of
 3316 $\delta_{CP} = \pi/2$ at more than 3σ for this particular set of known oscillation parameters.
 3317 The width of the 1σ credible intervals and the position of the highest posterior
 3318 probability density is given in Table 8.7. The highest posterior probability density
 3319 in δ_{CP} is calculated as $\delta_{CP} = -1.57 \pm 0.07$ showing no significant biases in the
 3320 determination of the known oscillation parameters.

The effect of applying the reactor constraint for δ_{CP} in the joint beam-atmospheric fit is presented in Figure 8.24. The reactor constraint significantly improves the ability of the fit to select the known parameter value. This behaviour is evidenced by the tightening of the 1σ and 90% credible intervals and the disfavoured region, centered at $\delta_{CP} \sim \pi/2$, becoming wider when the reactor constraint is applied. This follows from the correlations shown in Figure 8.19, where a lower value of $\sin^2(\theta_{13})$ results in tighter constraints on δ_{CP} .

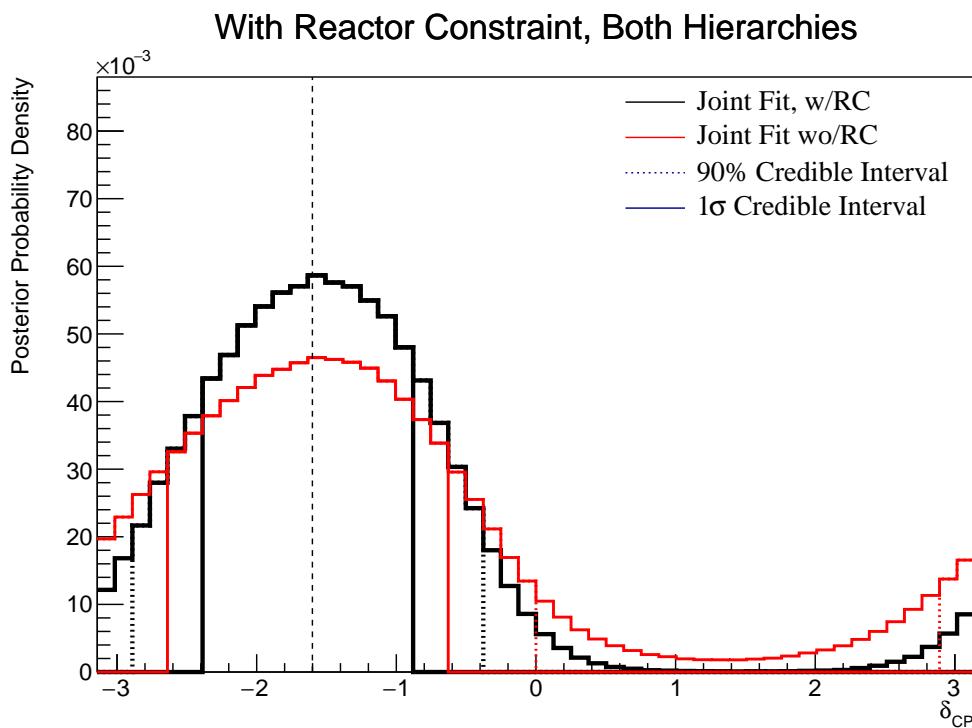


Figure 8.24: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, is given in Figure 8.25. The highest posterior probability density is located at $\sin^2(\theta_{23}) = 0.528 \pm 0.03$ which agrees with the known value of $\sin^2(\theta_{23}) = 0.528$. The distribution clearly favours the UO with almost the entirety of the 1σ credible interval being contained in that region. Figure 8.26 highlights the sensitivity of the joint fit both with and without the reactor constraint. The fit where the

Parameter	Interval	HPD
δ_{CP} , (BH)	[-2.39, -0.88]	-1.57 ± 0.07
δ_{CP} , (NH)	[-2.39, -0.75]	-1.57 ± 0.07
δ_{CP} , (IH)	[-2.14, -1.01]	-1.57 ± 0.07
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	[2.45, 2.56]	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	[2.47, 2.56]	2.51 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	[-2.60, -2.51]	-2.55 ± 0.01
$\sin^2(\theta_{23})$ (BH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (NH)	[0.490, 0.555]	0.528 ± 0.03
$\sin^2(\theta_{23})$ (IH)	[0.500, 0.560]	0.538 ± 0.03

Table 8.7: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit where the reactor constraint is applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH), and inverted hierarchy only (IH).

reactor constraint is applied selects the known value much better. This is a result of the marginalisation effects between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as observed in Figure 8.21.

The fraction of steps from the joint fit, after the reactor constraint is applied, is given in Table 8.8 and split by the two hierarchy and two octant hypotheses. The reactor constraint significantly reduces the fraction of steps that are contained within the IH-LO region from 0.08 to 0.03, whilst significantly increasing the fraction of steps within the NH-UO region from 0.50 to 0.62. The application of the reactor constraint increases the Bayes factor from $B(\text{NH}/\text{IH}) = 3.67$ to $B(\text{NH}/\text{IH}) = 6.47$. There is a very clear preference for the NH, with the Jeffreys scale stating a substantial preference for both fits. The Bayes factor for UO preference is calculated as $B(\text{UO}/\text{LO}) = 2.64$. Whilst still a weak preference, this is certainly a stronger statement than the sensitivity when the reactor constraint is not applied.

The sensitivity of the joint beam-atmospheric fit to Δm_{32}^2 , with the reactor constraint applied, is presented in Figure 8.27. The 1σ credible interval is

With Reactor Constraint, Both Hierarchies

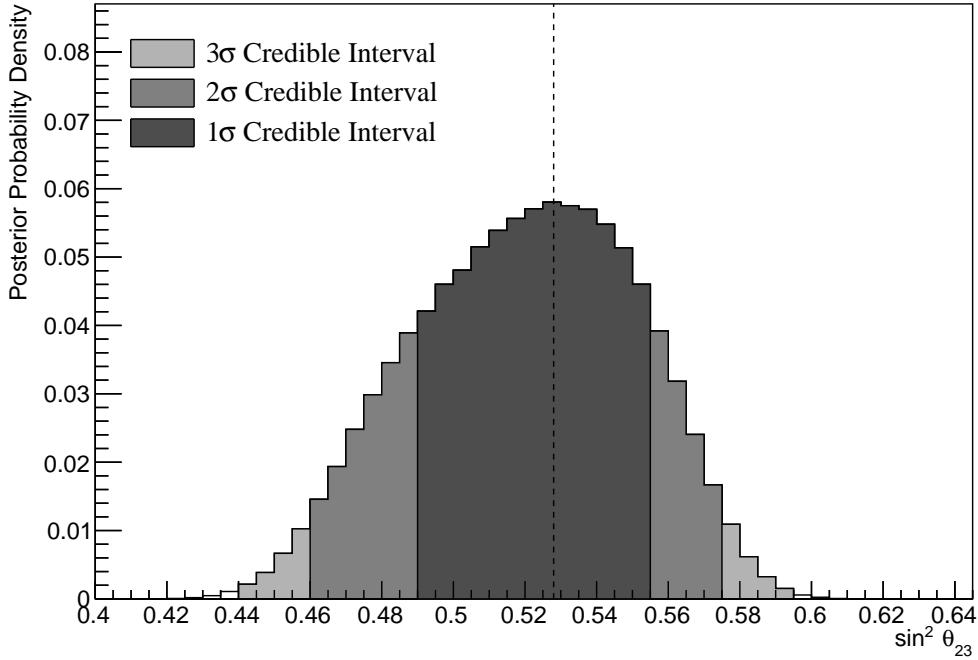


Figure 8.25: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.24	0.62	0.87
IH ($\Delta m_{32}^2 < 0$)	0.03	0.10	0.13
Sum	0.27	0.73	1.00

Table 8.8: The distribution of steps in a joint beam-atmospheric with the reactor constraint fit applied, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{UO}/\text{LO}) = 2.64$.

3350 entirely contained within the NH region and the position of the highest posterior
 3351 probability density is given as $(2.49 \pm 0.01) \times 10^{-3} \text{ eV}^2$. This illustrates no bias
 3352 between the fit results and the known oscillation parameters. The application
 3353 of the reactor constraint does not significantly move the position or width of
 3354 the credible intervals.

3355 The sensitivity to the appearance parameters ($\sin^2(\theta_{13}) - \delta_{CP}$) is given in Fig-
 3356 ure 8.28. The distribution is mostly uncorrelated between the two parameters and

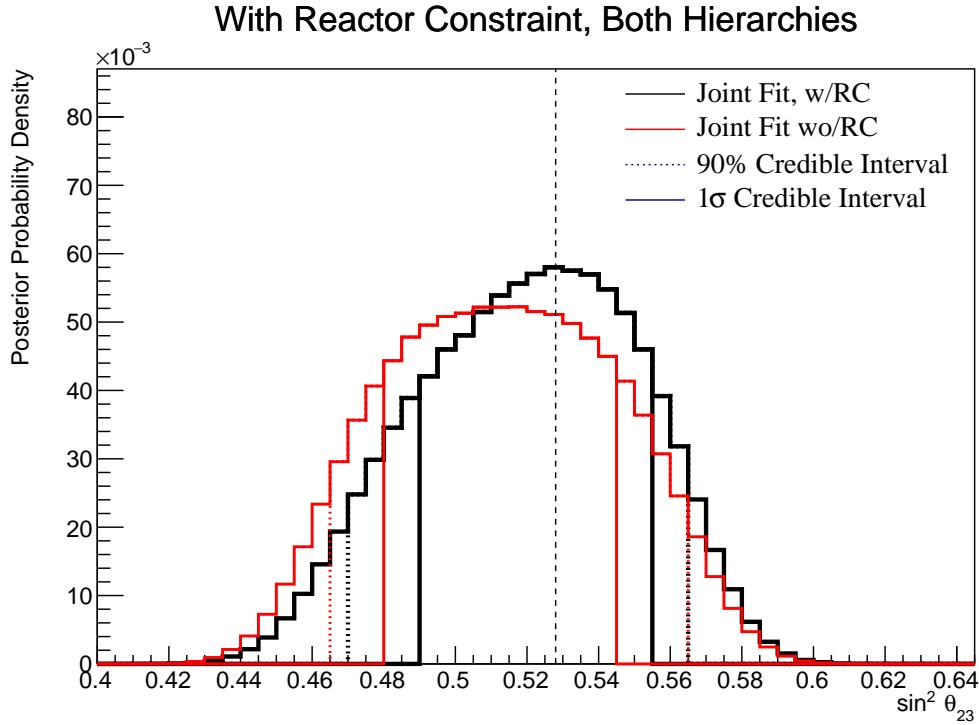


Figure 8.26: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Red) and the joint beam-atmospheric fit with the reactor constraint (Black). The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3357 is centered at the known oscillation parameters. The 1σ credible interval excludes
 3358 $\delta_{CP} = 0$ and $\delta_{CP} = \pm\pi$. Furthermore, the 3σ credible intervals exclude the
 3359 region of $\delta_{CP} = \pi/2$.

3360 The sensitivity to the disappearance parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$) is illustrated
 3361 in Figure 8.29. The 1σ credible interval is entirely contained within the NH
 3362 region reflecting the same results as the one-dimensional marginalised results in
 3363 Figure 8.27. Both the NH and IH regions favour the UO, with a visually similar
 3364 preference in both hierarchies. The width of the 1σ contour, in Δm_{32}^2 , does not
 3365 significantly depend upon the value or octant of $\sin^2(\theta_{23})$. This shows that there
 3366 are no strong correlations between these two parameters.

3367 Figure 8.30 illustrates the posterior distribution for each permutation of
 3368 two oscillation parameters of interest. The application of the reactor constraint
 3369 significantly reduces the correlations previously seen in Figure 8.21.

With Reactor Constraint, Both Hierarchies

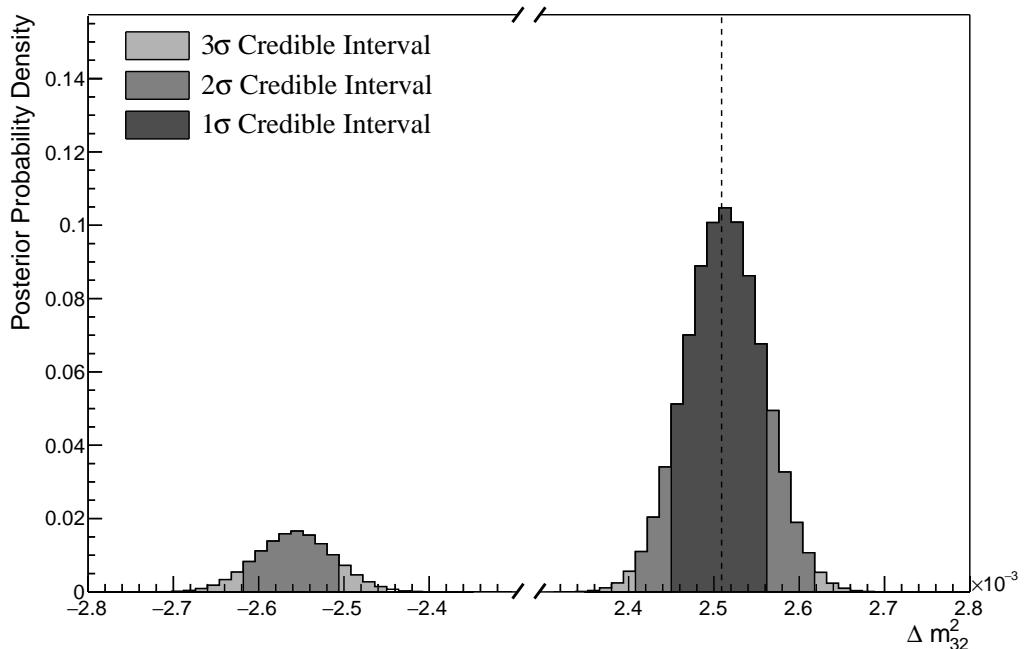


Figure 8.27: The one-dimensional posterior probability density distribution in Δm_{32}^2 from the joint beam-atmospheric fit where the reactor constraint is applied. The vertical dashed line represents the known value of Δm_{32}^2 .

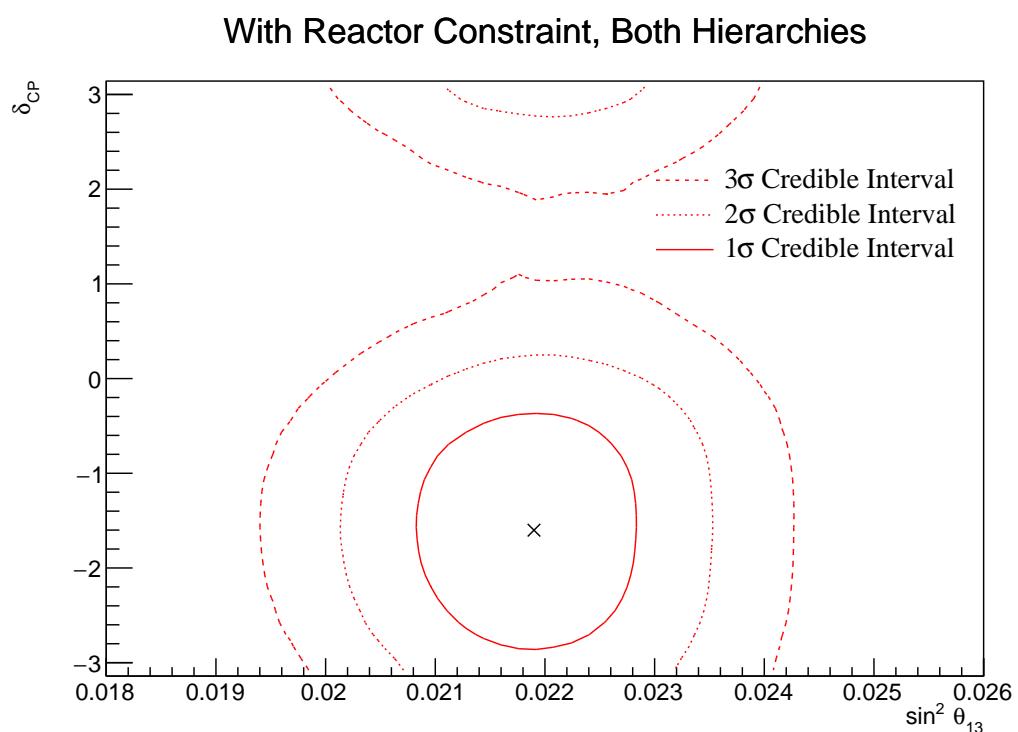


Figure 8.28: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\delta_{CP} - \sin^2(\theta_{13})$.

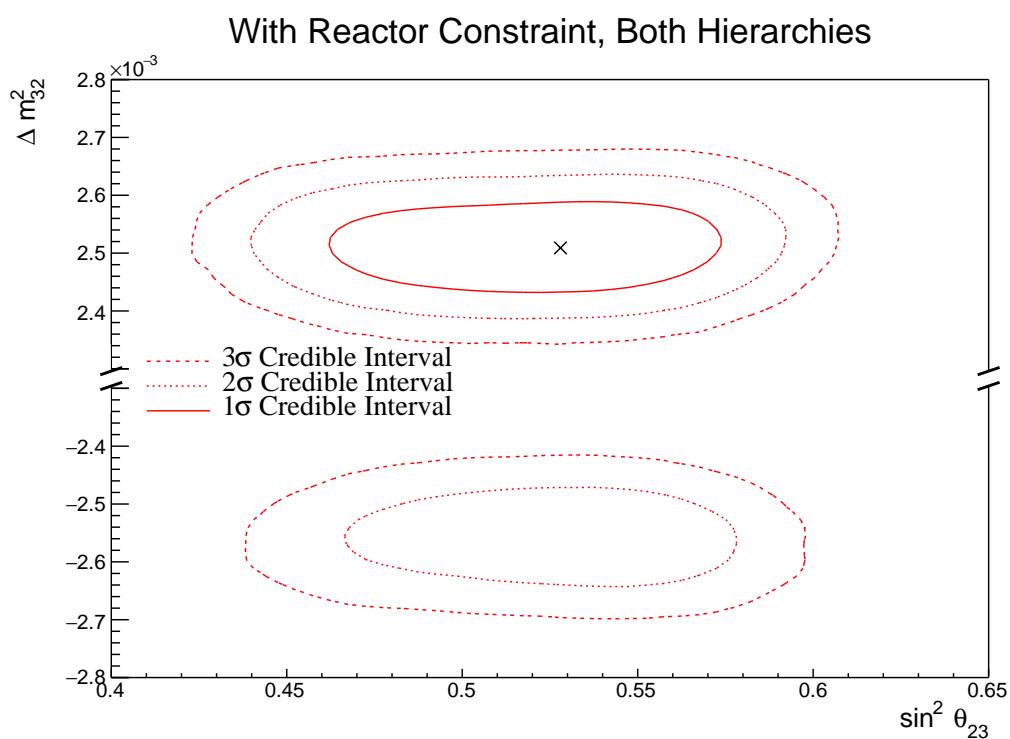


Figure 8.29: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$, marginalised over both hierarchies, from the joint beam-atmospheric fit where the reactor constraint is applied. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

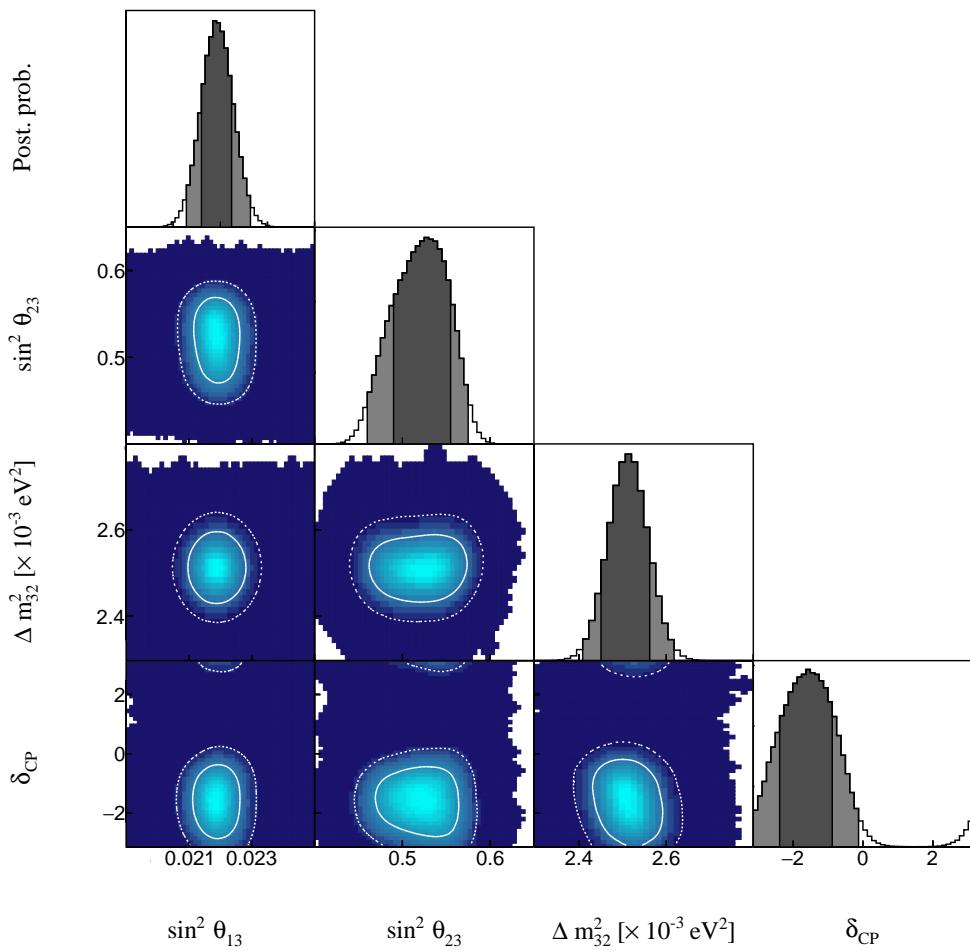


Figure 8.30: The posterior probability density distribution from the joint beam-atmospheric fit where the reactor constraint is applied. The distribution is given for each two-dimensional permutation of the oscillation parameters of interest. The one-dimensional distribution of each parameter is also given.

3370 8.3.6 Comparison to Latest T2K Sensitivities without Reactor 3371 Constraint

3372 The benefits of the joint beam-atmospheric analysis can be determined by compar-
 3373 ing the sensitivities to the beam-only analysis presented in [2, 215]. This section
 3374 presents those comparisons for sensitivities built using the Asimov A oscillation
 3375 parameters defined in Table 2.2 and the post-BANFF systematic tune. The reactor
 3376 constraint is not applied within either of the fits used in these comparisons.

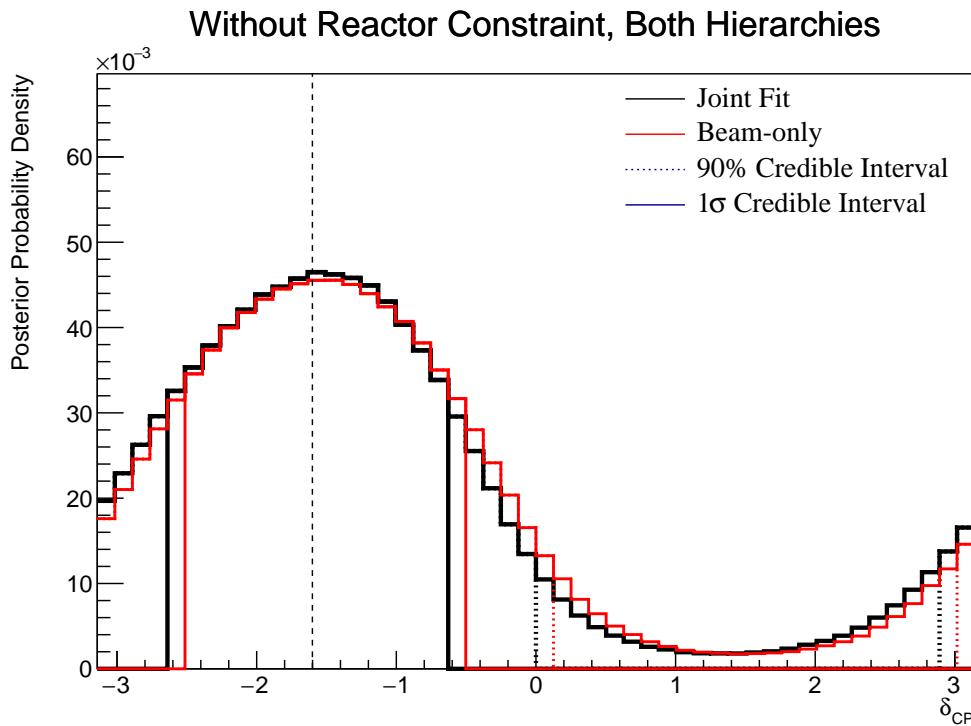


Figure 8.31: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3377 The sensitivity, marginalised over both hierarchies, to δ_{CP} from the joint beam-
 3378 atmospheric and beam-only fits is presented in Figure 8.31. As expected from the
 3379 likelihood scans (Figure 8.4), the sensitivity to δ_{CP} is not significantly increased.
 3380 This is because the known oscillation parameter value lies at the position where
 3381 the beam samples dominate the sensitivity compared to the SK samples.

The sensitivity to Δm_{32}^2 is compared between the joint beam-atmospheric fit and beam-only fit in Figure 8.32. The 1σ credible interval of the joint beam-atmospheric fit is entirely contained within the NH region. This shows the significant increase in the ability of the fit to determine the correct mass hierarchy, compared to the beam-only analysis. This is further evidenced by the fact that the 90% credible intervals from the joint fit are also tighter in the IH region compared to the beam-only analysis. The Bayes factor for mass hierarchy determination for the beam-only and joint beam-atmospheric fits are $B(\text{NH}/\text{IH}) = 1.91$ and $B(\text{NH}/\text{IH}) = 3.67$, respectively. According to Jeffrey's scale, the beam-only analysis represents a weak preference for the NH hypothesis whereas the joint fit returns a substantial preference for the NH hypothesis. Notably, this conclusion does not require any external constraints and clearly illustrates the benefit of the joint analysis.

Without Reactor Constraint, Both Hierarchies

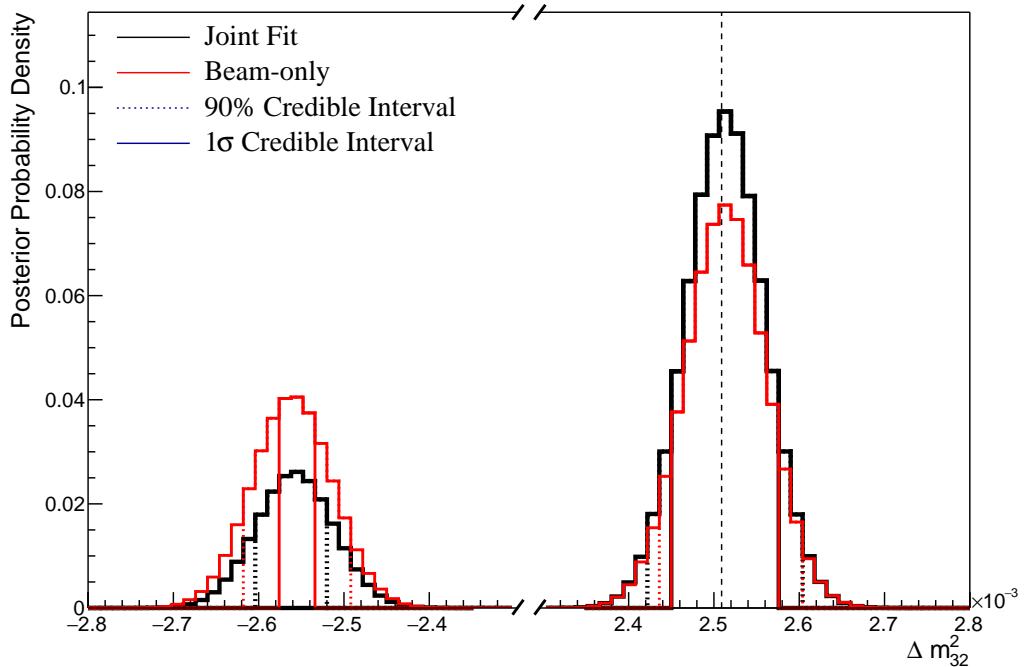


Figure 8.32: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

3395 The sensitivity to $\sin^2(\theta_{23})$, marginalised over both hierarchies, for both the
 3396 beam-only and joint beam-atmospheric analysis are presented in Figure 8.33. The
 3397 peak of the posterior distribution from the joint analysis is more aligned with the
 3398 known value of $\sin^2(\theta_{23}) = 0.528$ compared to the beam-only analysis. The Bayes
 3399 factors for the beam-only and joint beam-atmospheric fit are $B(\text{UO}/\text{LO}) = 1.56$
 3400 and $B(\text{UO}/\text{LO}) = 1.74$, respectively. Therefore, the joint beam-atmospheric fit
 3401 does prefer the UO more strongly than the beam-only analysis, albeit slightly.

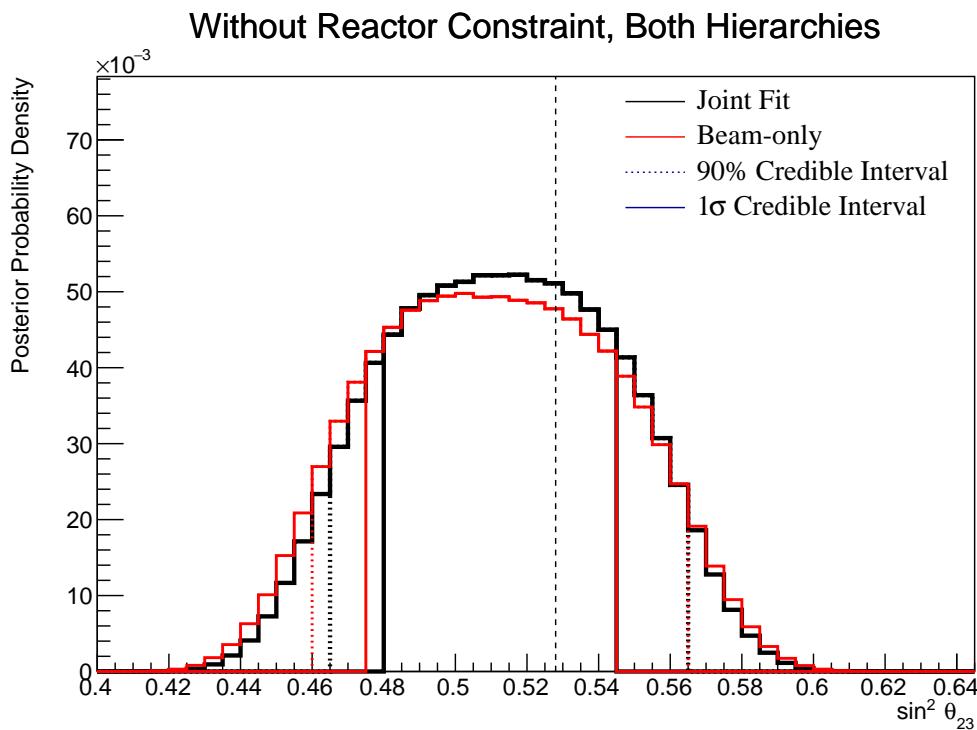


Figure 8.33: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

3402 Whilst the beam-only and joint beam-atmospheric fits have similar sensi-
 3403 tivity to δ_{CP} and $\sin^2(\theta_{23})$ when projected in one-dimension, the benefit of the
 3404 joint analysis becomes more obvious when the sensitivities are presented in
 3405 two-dimensions. The sensitivity of the two fits to the appearance parameters
 3406 ($\delta_{CP} - \sin^2(\theta_{13})$) is illustrated in Figure 8.34. The width of the 99% joint fit credible
 3407 interval in $\sin^2(\theta_{13})$ is squeezed in the region of $\delta_{CP} \sim 0$ compared to the

beam-only analysis. This is the same behaviour that is seen in the appearance likelihood scans presented in Figure 8.2. The 1σ and 90% also exhibit slightly tighter constraints on δ_{CP} . This is most prevalent in the region of $\delta_{CP} \sim 0$ and $\sin^2(\theta_{13}) \sim 0.03$. Whilst the atmospheric samples do not have significant sensitivity to $\sin^2(\theta_{13})$ (as shown in Figure 8.1), they aid in breaking the degeneracy between the oscillation parameters allowing for tighter constraints.

The sensitivity to the disappearance parameters $\sin^2(\theta_{23}) - \Delta m_{32}^2$ is presented in Figure 8.35 for both the beam-only and joint beam-atmospheric fits. Whilst the one-dimensional sensitivity comparisons considered so far show the improvements of the joint fit, the two-dimensional projection really shows the benefit of adding the atmospheric samples to the beam samples. The area contained within the IH credible intervals is drastically reduced in the joint fit. This follows from the better determination of the mass hierarchy seen in the Bayes factor comparisons. Even in the NH region, the widths of the credible intervals in $\sin^2(\theta_{23})$ decreases, albeit to a smaller extent.

The comparison in sensitivity to $\delta_{CP} - \Delta m_{32}^2$ is illustrated in Figure 8.36. The contours from the joint beam-atmospheric fit are much smaller in the IH region as compared to the beam-only analysis. This culminates in a region around $\delta_{CP} \sim \pi/2$ in the H region which is excluded at 3σ . This behaviour is not present within the beam-only analysis. Consistent with the previous observations, the area contained within the IH credible intervals is significantly reduced in comparison to the beam-only analysis.

The sensitivity to Δm_{32}^2 , as a function of $\sin^2(\theta_{13})$, is presented in Figure 8.37. Similar to previous observations, the Δm_{32}^2 contours within IH region of the joint fit are much smaller than the beam-only analysis. Notably, the joint fit IH 1σ credible intervals exclude the region around the reactor constraint. This suggests that the application of the reactor constraint would further increase the preference for NH in the joint fit compared to its effect on the beam-only analysis.

The beam-only and joint beam-atmospheric fits have a slightly different contour shape between the $\sin^2(\theta_{13})$ and $\sin^2(\theta_{23})$ parameters, as illustrated

Without Reactor Constraint, Both Hierarchies

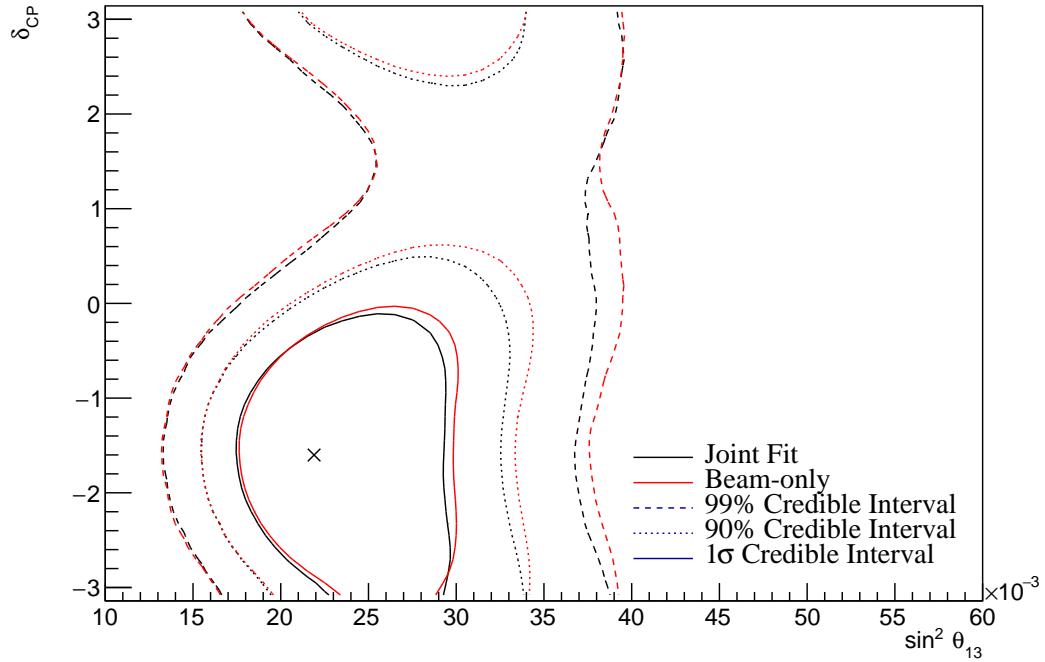


Figure 8.34: The two-dimensional posterior probability density distribution in $\delta_{CP} - \sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of $\delta_{CP} - \sin^2(\theta_{13})$.

³⁴³⁸ by Figure 8.38. The joint analysis disfavours the wrong octant hypothesis more
³⁴³⁹ strongly in the region of high $\sin^2(\theta_{13})$. This change in correlation means that the
³⁴⁴⁰ application of the reactor constraint could affect the two analyses differently.

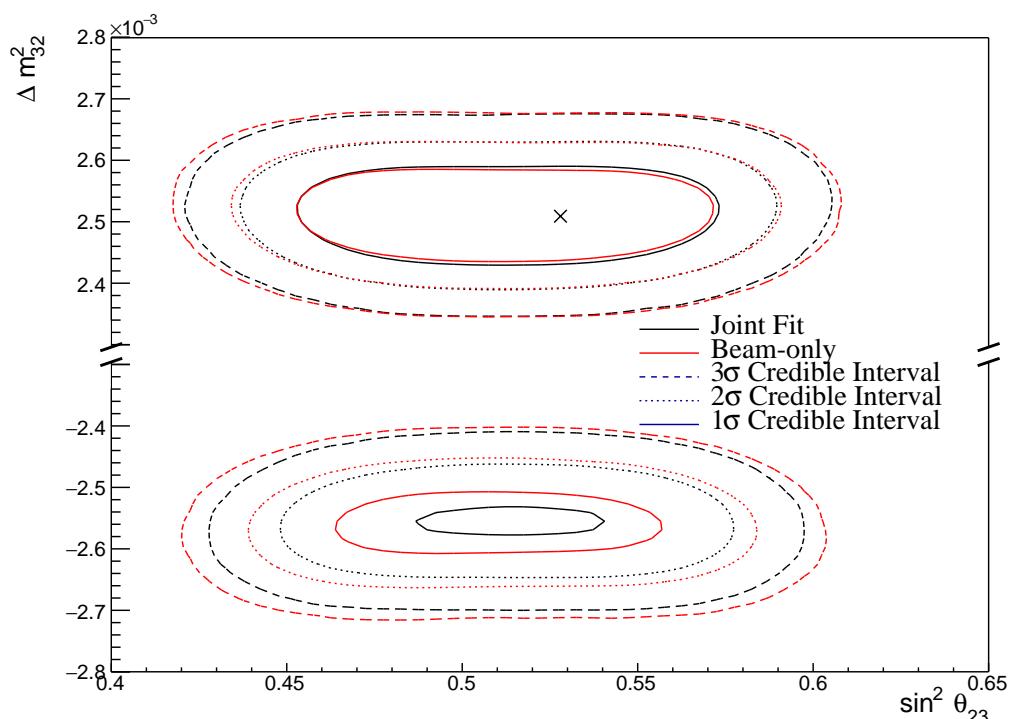


Figure 8.35: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

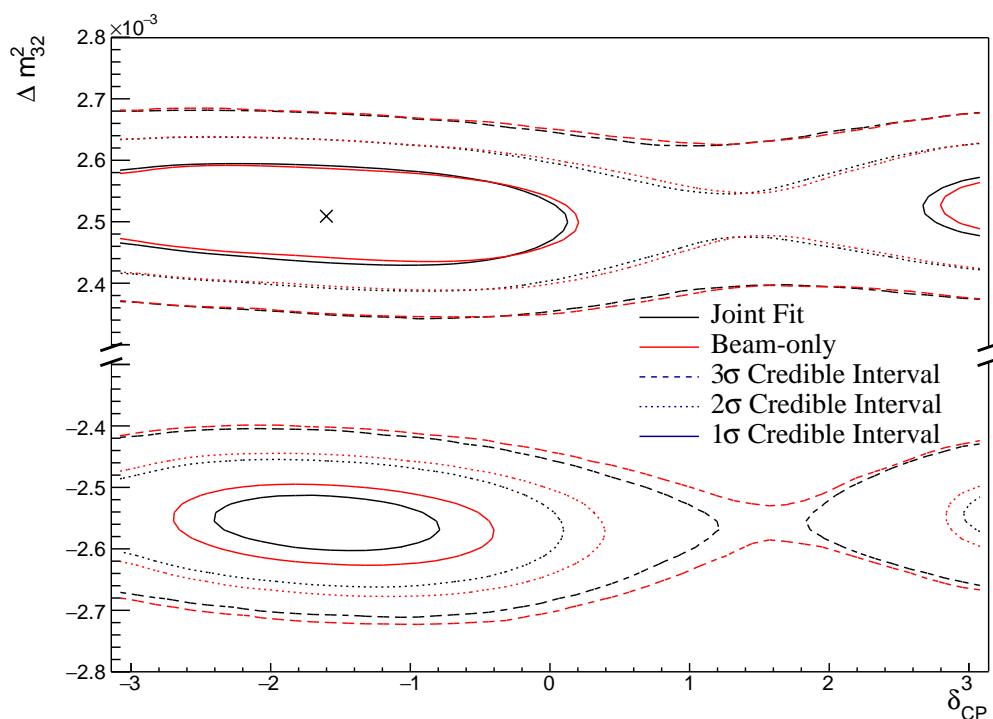


Figure 8.36: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \Delta_{CP}$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \Delta_{CP}$.

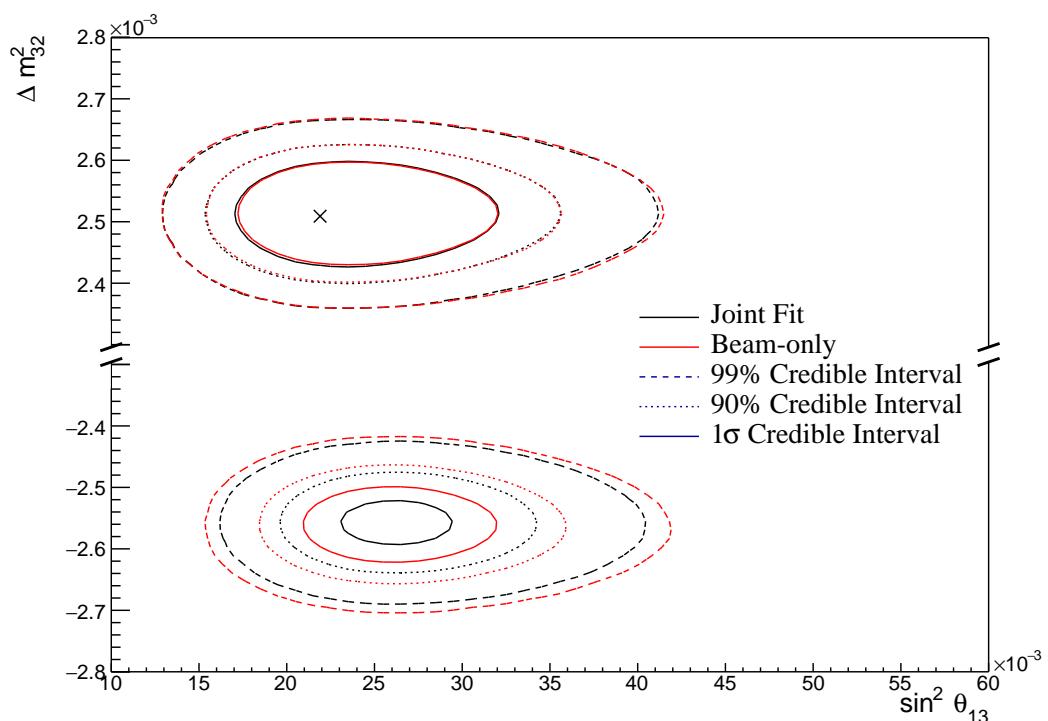


Figure 8.37: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

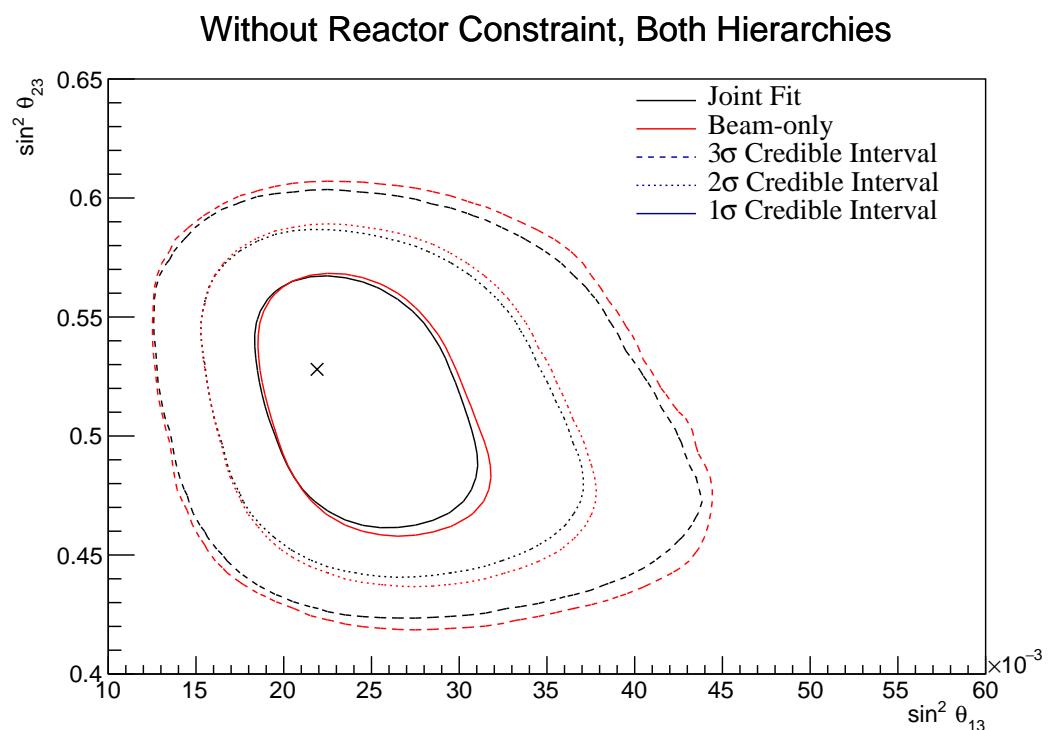


Figure 8.38: The two-dimensional posterior probability density distribution in $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The marker represents the known value of $\sin^2(\theta_{23}) - \sin^2(\theta_{13})$.

3441 8.3.7 Comparison to Latest T2K Sensitivities with Reactor Con- 3442 straint

3443 This section illustrates the comparison between the joint beam-atmospheric and
 3444 beam-only fits when the reactor constraint is applied. As shown in Figure 8.37,
 3445 the application of the reactor constraint is expected to significantly increase
 3446 the joint fit's preference for the NH hypothesis, compared to the beam-only
 3447 analysis. Figure 8.39 illustrates the sensitivities of the two fits to the disappearance
 3448 parameters ($\sin^2(\theta_{23}) - \Delta m_{32}^2$). This plot further illustrates the benefit of the joint
 3449 beam-atmospheric analysis. The 1σ credible interval in the IH region is entirely
 3450 removed in the joint analysis but not for the beam-only analysis.

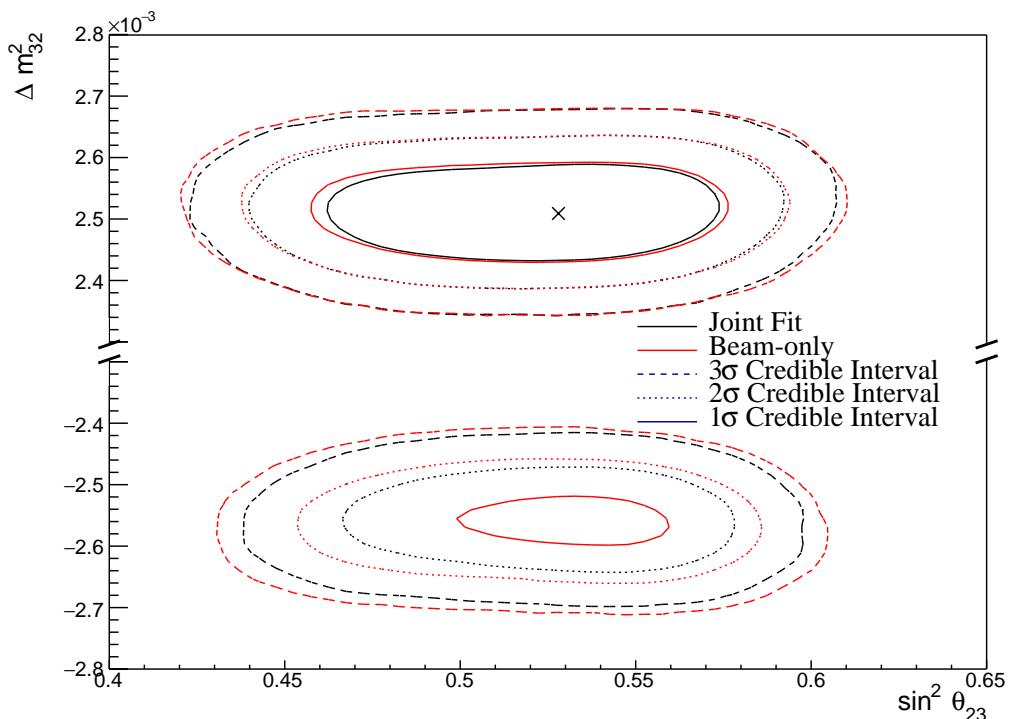


Figure 8.39: The two-dimensional posterior probability density distribution in $\Delta m_{32}^2 - \sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is applied in both fits. The marker represents the known value of $\Delta m_{32}^2 - \sin^2(\theta_{23})$.

3451 The credible intervals of the joint fit are also tighter in the $\sin^2(\theta_{23})$ dimension
 3452 than the beam-only analysis in both mass hierarchy regions. This shows that
 3453 beyond the ability of the joint fit to prefer the NH more strongly than the beam-

only analysis, the precision to which it can measure $\sin^2(\theta_{23})$ is also improved. The Bayes factor for NH preference is calculated as $B(\text{NH}/\text{IH}) = 6.47$ and $B(\text{NH}/\text{IH}) = 3.09$ for the joint beam-atmospheric and beam-only analysis, respectively. This important conclusion illustrates that the joint beam-atmospheric analysis can provide a substantial preference for the NH hypothesis whilst the beam-only analysis can not.

The Bayes factors for UO preference which are $B(\text{UO}/\text{LO}) = 2.86$ and $B(\text{UO}/\text{LO}) = 2.47$ for the joint beam-atmospheric and beam-only analysis, respectively. Both of these represent a mild preference for the UO but a stronger preference is observed in the joint analysis.

The sensitivity of the beam-only and joint beam-atmospheric analyses, to the appearance parameters ($\delta_{CP} - \sin^2(\theta_{13})$), are compared in Figure 8.40. These results are marginalised over both hierarchies. For this particular set of known oscillation parameters (AsimovA defined in Table 2.2), the beam-only analysis dominates the sensitivity. The joint fit does slightly increase the sensitivity to δ_{CP} but it does not change any conclusions that would be made. As expected, the prior knowledge dominates any sensitivity either fit would have on $\sin^2(\theta_{13})$.

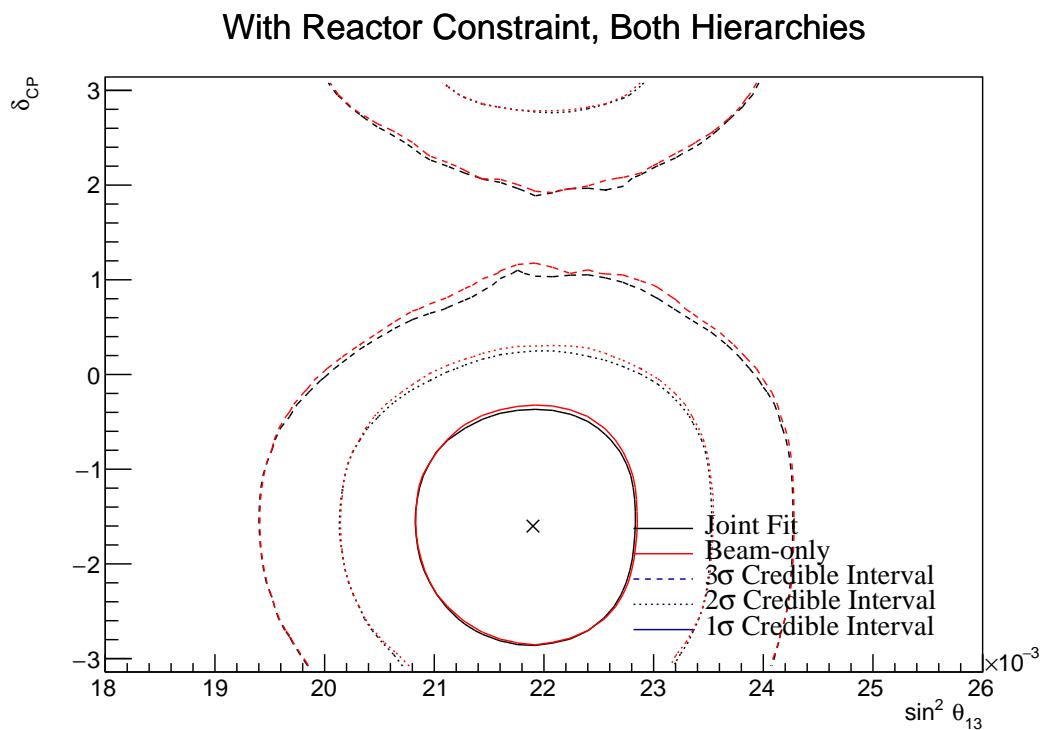


Figure 8.40: The two-dimensional posterior probability density distribution in δ_{CP} – $\sin^2(\theta_{13})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is applied in both fits. The distributions are marginalised over both hierarchies. The marker represents the known value of δ_{CP} – $\sin^2(\theta_{13})$.

3471 8.3.8 Alternate Asimov Parameter Set

3472 Figure 8.4 and Figure 8.5 show that the choice of the parameter set at which the
 3473 Asimov data is made can affect the conclusion. ‘AsimovA’ oscillation parameters
 3474 are defined at a region of δ_{CP} which is preferred by the T2K experiment. This
 3475 explains why the addition of the atmospheric samples does not significantly in-
 3476 crease the sensitivity to δ_{CP} , as illustrated in subsection 8.3.6 and subsection 8.3.7.
 3477 This section presents the sensitivities when ‘AsimovB’ oscillation parameters,
 3478 as defined in Table 2.2, are assumed (alongside the post-BANFF tune) when
 3479 building the Asimov data.

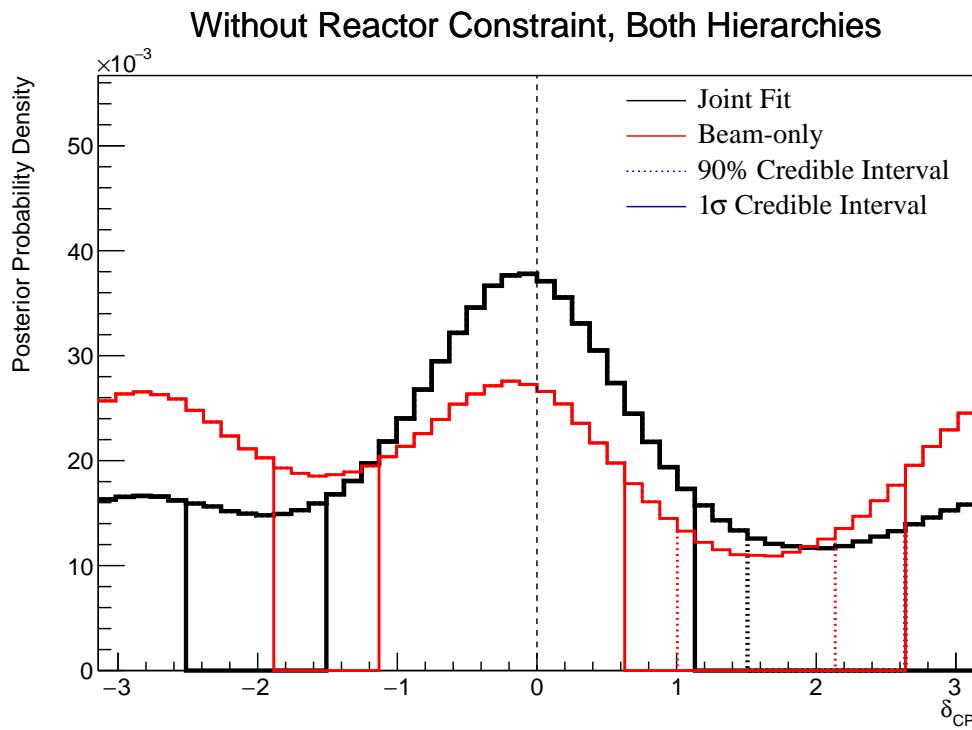


Figure 8.41: The one-dimensional posterior probability density distribution in δ_{CP} compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of δ_{CP} .

3480 The sensitivity to δ_{CP} for the joint beam-atmospheric fit is presented in
 3481 Figure 8.41. The results are compared to those from the beam-only analysis
 3482 in [2, 215]. The reactor constraint is not applied in either of the fits. The

shape of the posterior distribution from the joint analysis is more peaked at the known oscillation parameter value compared to the beam-only analysis, which has approximately the same posterior probability density at $\delta_{CP} = 0$ and $\delta_{CP} = \pm\pi$. This shows the ability of the joint analysis to better determine the correct phase of δ_{CP} if the true value were CP-conserving. The 1σ credible intervals and the position of the highest posterior probability density are given in Table 8.9. The highest posterior density for the joint beam-atmospheric analysis is $\delta_{CP} = -0.06 \pm 0.06$ which is consistent with the known value.

Parameter	Interval	HPD
δ_{CP} , (BH)	$[-\pi, -2.51], [-1.51, 1.13]$	-0.06 ± 0.06
δ_{CP} , (NH)	$[-1.13, 1.63]$	0.06 ± 0.06
δ_{CP} , (IH)	$[-3.02, -1.88], [-1.76, 0.13]$	-0.44 ± 0.06
Δm_{32}^2 (BH) [$\times 10^{-3}\text{eV}^2$]	$[-2.60, -2.52], [2.46, 2.56]$	2.51 ± 0.01
Δm_{32}^2 (NH) [$\times 10^{-3}\text{eV}^2$]	$[2.47, 2.56]$	2.52 ± 0.01
Δm_{32}^2 (IH) [$\times 10^{-3}\text{eV}^2$]	$[-2.61, -2.52]$	-2.57 ± 0.01
$\sin^2(\theta_{23})$ (BH)	$[0.430, 0.480], [0.545, 0.585]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (NH)	$[0.430, 0.485], [0.550, 0.580]$	0.453 ± 0.003
$\sin^2(\theta_{23})$ (IH)	$[0.435, 0.480], [0.540, 0.585]$	0.568 ± 0.003

Table 8.9: The position of the highest posterior probability density (HPD) and width of the 1σ credible interval for the joint beam-atmospheric fit. The reactor constraint is not applied. The values are presented by which hierarchy hypothesis is assumed: marginalised over both hierarchies (BH), normal hierarchy only (NH) and inverted hierarchy only (IH).

Naively, if just the 1σ credible interval were considered without observing the shape of the distribution, it would appear that the joint analysis would have a worse sensitivity to δ_{CP} due to the larger interval around $\delta_{CP} = 0$. The 1σ credible interval for the beam-only analysis is given as the range $\delta_{CP} = [-\pi, -1.88], [-1.13, 0.63]$ and $[2.64, \pi]$ which contains 56% of all values of δ_{CP} . Whereas, the joint beam-atmospheric analysis contains 52% of all δ_{CP} values within the 1σ credible interval. Therefore, if the area within the 1σ credible

interval were to be compared between the two fits, the joint analysis would be shown to have better precision.

This apparent contradiction stems from the methodology in which the credible interval is calculated. The technique used in this analysis (documented in subsection 4.3.2) fills the credible interval by selecting bins in order of probability density until 68% of the posterior density is contained. If instead, the credible interval were calculated by expanding around the highest posterior probability, the benefits of the joint fit would be more obvious. In the case where the shape of the posterior was uni-modal, these two techniques would be equivalent to statistical fluctuations.

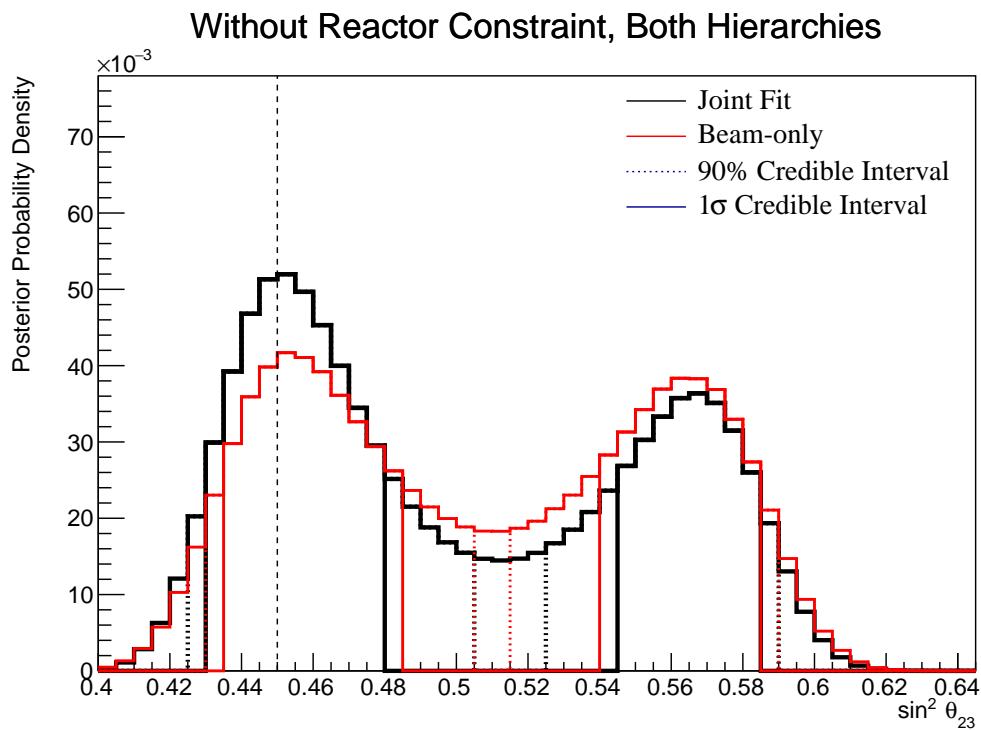


Figure 8.42: The one-dimensional posterior probability density distribution in $\sin^2(\theta_{23})$ compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The distributions are marginalised over both hierarchies. The vertical dashed line represents the known value of $\sin^2(\theta_{23})$.

The sensitivity of the joint beam-atmospheric fit to $\sin^2(\theta_{23})$ is presented in Figure 8.42. The sensitivity is compared to that of the beam-only analysis in [2, 215]. The reactor constraint is not applied in either of the fits being compared.

3511 The joint beam-atmospheric fit has a much larger probability density in the region
 3512 surrounding the known oscillation parameter, $\sin^2(\theta_{23}) = 0.45$. This shows the
 3513 better octant determination of the joint analysis compared to the beam-only fit.
 3514 The ratio of the posterior density at the peak of the lower octant to the peak of
 3515 the upper octant from the joint fit is 1.43 compared to 1.09 from the beam-only
 3516 analysis. The area contained within the 1σ credible interval for the joint analysis
 3517 is $\sin^2(\theta_{23}) = [0.430, 0.480]$ and $\sin^2(\theta_{23}) = [0.545, 0.585]$, whereas the beam-
 3518 only analysis includes $\sin^2(\theta_{23}) = [0.435, 0.485]$ and $\sin^2(\theta_{23}) = [0.540, 0.585]$.
 3519 This corresponds to a $\sim 5\%$ (binning dependent) increase in precision from
 3520 the joint analysis.

	LO ($\sin^2 \theta_{23} < 0.5$)	UO ($\sin^2 \theta_{23} > 0.5$)	Sum
NH ($\Delta m_{32}^2 > 0$)	0.35	0.24	0.59
IH ($\Delta m_{32}^2 < 0$)	0.19	0.22	0.41
Sum	0.54	0.46	1.00

Table 8.10: The distribution of steps in a joint beam-atmospheric fit, presented as the fraction of steps in the upper (UO) and lower (LO) octants and the normal (NH) and inverted (IH) hierarchies. The reactor constraint is not applied. The Bayes factors are calculated as $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$.

3521 The distribution of steps, split by hierarchy and octant hypothesis, is presented
 3522 in Table 8.10. The Bayes factor for hierarchy and octant determination are
 3523 $B(\text{NH}/\text{IH}) = 1.43$ and $B(\text{LO}/\text{UO}) = 1.19$, respectively. These values compare
 3524 to $B(\text{NH}/\text{IH}) = 1.08$ and $B(\text{LO}/\text{UO}) = 0.91$ from the beam-only analysis. This
 3525 evidences the joint analysis's ability to select the correct octant and hierarchy
 3526 hypothesis. Comparisons to the AsimovA Bayes factors presented in Table 8.6
 3527 show how the preferences for the correct octant and hierarchy depend on the
 3528 true value of δ_{CP} and $\sin^2(\theta_{23})$.

3529 The sensitivity of the beam-only and joint beam-atmospheric analysis to
 3530 Δm_{32}^2 is given in Figure 8.43. The joint analysis has a stronger preference for the
 3531 correct hierarchy (NH) which is shown by the higher Bayes factor compared
 3532 to the beam-only analysis. This is further evidenced by the width of the 90%

Without Reactor Constraint, Both Hierarchies

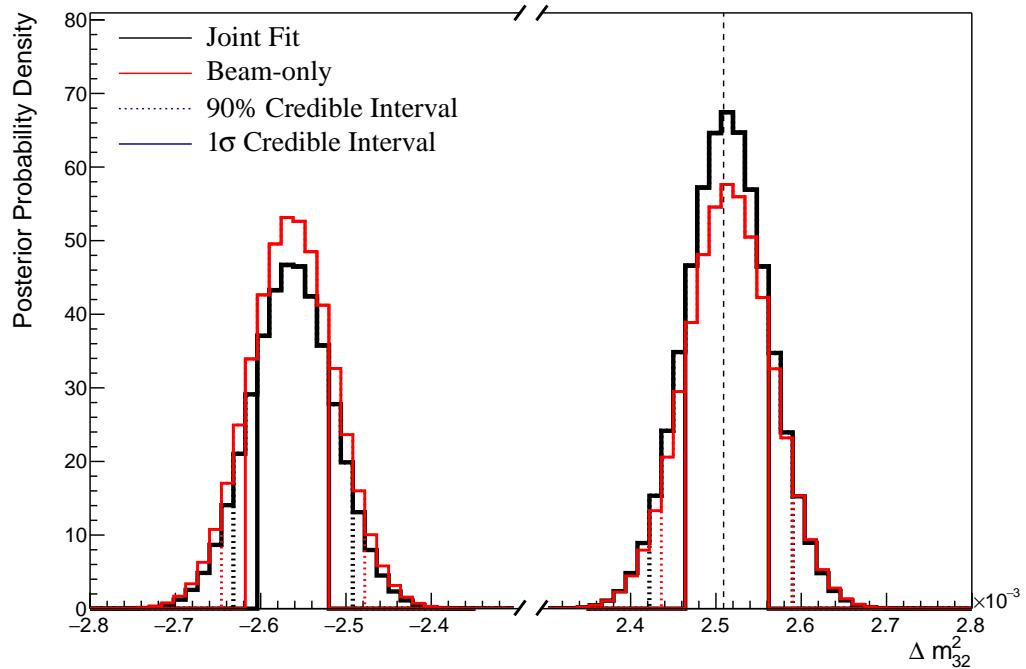


Figure 8.43: The one-dimensional posterior probability density distribution in Δm_{32}^2 compared between the joint beam-atmospheric fit (Black) and the latest T2K sensitivities (Red) [2, 215]. The reactor constraint is not applied in either fit. The vertical dashed line represents the known value of Δm_{32}^2 .

credible interval in the IH region being tighter in the joint analysis compared
³⁵³³ to the beam-only analysis.
³⁵³⁴

3535 8.3.9 Effect of Systematics

3536 The effect of systematics on each sample used in this analysis is calculated using
 3537 the posterior predictive method documented in subsection 4.3.4. The distribution
 3538 of each sample’s spectrum has been generated by sampling 2000 steps from the
 3539 posterior distribution of the joint beam-atmospheric fit. This technique reweights
 3540 the Monte Carlo prediction using the systematic values given by a particular
 3541 step, stores the sample spectra, and repeats for the desired number of steps.
 3542 The oscillation parameters are always fixed at Asimov A values to only study
 3543 the effect of systematic parameters.

3544 Figure 8.44 illustrates the distribution for the SubGeV-*elike*-0dcy atmospheric
 3545 sample. The fit being sampled is detailed in subsection 8.3.4. The distribution
 3546 closely resembles the Asimov data spectrum. This would be expected from
 3547 an Asimov fit where the Monte Carlo is fit to itself but gives more credibility
 3548 to the results of the fit.

3549 The total event rate for each sample from each of the sampled steps is
 3550 calculated and the fractional uncertainty, $\Delta N_i / N_i$ where N_i is the event rate of the
 3551 i^{th} sample, is calculated. These values are presented in Table 8.11. In general, the
 3552 impact of the systematics has a $\sim 3\%$ uncertainty on the event rate of atmospheric
 3553 samples, where CC1 π -targeting samples have slightly larger uncertainties than
 3554 the CCQE-like samples. The fractional uncertainties on the beam samples are
 3555 compared to those from the beam-only analysis presented in [2, 215]. The
 3556 uncertainties on the one-ring muon samples are mostly unchanged, whereas
 3557 the uncertainties on the one-ring electron samples are different. As discussed in
 3558 section 8.2, the atmospheric samples should be able to add constraints on the NC
 3559 background events present in the FHC1Re and RHC1Re samples. The uncertainty
 3560 reduction seen in those samples is due to those additional constraints. The reason
 3561 why the FHC1Re1de has a higher uncertainty in this analysis is due to the addition
 3562 of the ad-hoc systematic introduced for CC1 π interactions (see subsection 6.4.3).

3563 Beyond the impact of the uncertainty on each sample’s event rate, the post-fit
 3564 constraint on each systematic parameter should be checked. Figure 8.45 illustrates

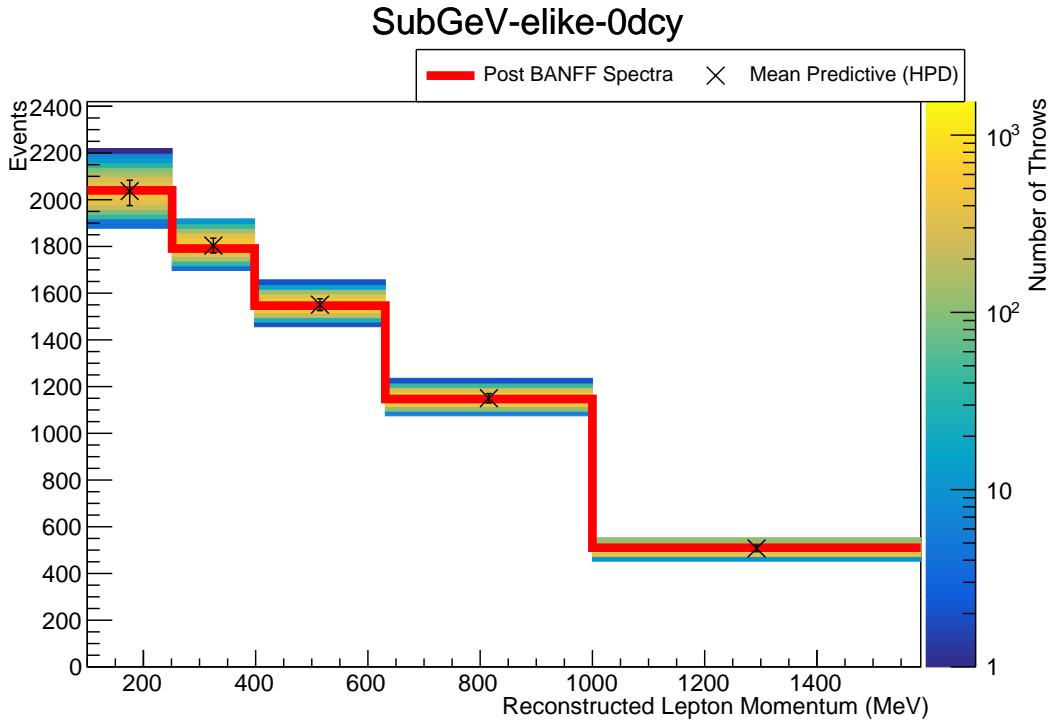


Figure 8.44: Result of the posterior predictive method for the SubGeV_elike_0dcy sample after sampling 2000 steps from the joint beam-atmospheric chain detailed in subsection 8.3.4 (Coloured histogram). The mean and uncertainty is presented for each bin. The Asimov data prediction (denoted ‘Post BANFF Spectra’, Red) assumes the post-BANFF tune and Asimov A oscillation parameters.

the central value and uncertainty on a select group of interaction systematics, for both the joint beam-atmospheric (from subsection 8.3.4) and the beam-only analysis. From the discussion in section 8.2, the uncertainty on systematics which are strongly constrained by the near detector should not significantly change when adding the atmospheric analysis. This behaviour is evidenced by the fact that the ratio of constraints between the two fits are very similar (within a few %) for almost all systematics. The only systematic which is more constrained in the joint beam-atmospheric analysis is the *NCOtherSK* normalisation parameter, which has a $O(10\%)$ tighter constraint. As expected, the atmospheric samples have been able to constrain this systematic which leads to the reduction in uncertainty for the beam electron-like samples.

Sample	Percentage Uncertainty (%)	
	Joint Analysis	Beam-only Analysis
SubGeV-elike-0dcy	2.53	-
SubGeV-elike-1dcy	3.28	-
SubGeV-mulike-0dcy	2.62	-
SubGeV-mulike-1dcy	2.23	-
SubGeV-mulike-2dcy	3.96	-
SubGeV-pi0like	2.84	-
MultiGeV-elike-nue	5.14	-
MultiGeV-elike-nuebar	2.79	-
MultiGeV-mulike	2.99	-
MultiRing-elike-nue	2.94	-
MultiRing-elike-nuebar	2.83	-
MultiRing-mulike	2.89	-
MultiRingOther-1	2.70	-
PCStop	3.22	-
PCThrus	2.99	-
UpStop-mu	2.95	-
UpThruNonShower-mu	2.70	-
UpThruShower-mu	3.19	-
FHC1Rmu	2.49	2.33
RHC1Rmu	2.89	2.93
FHC1Re	4.12	4.57
RHC1Re	5.15	5.65
FHC1Re1de	13.38	11.51

Table 8.11: The percentage uncertainty, $\Delta N / N$, as calculated from sampling 2000 throws from a joint beam-atmospheric chain. The same values for the beam samples are provided from the beam-only analysis [2, 215]. These uncertainties consider all systematic parameters to be sampled from the fit whilst the oscillation parameters are fixed at the Asimov A oscillation set.

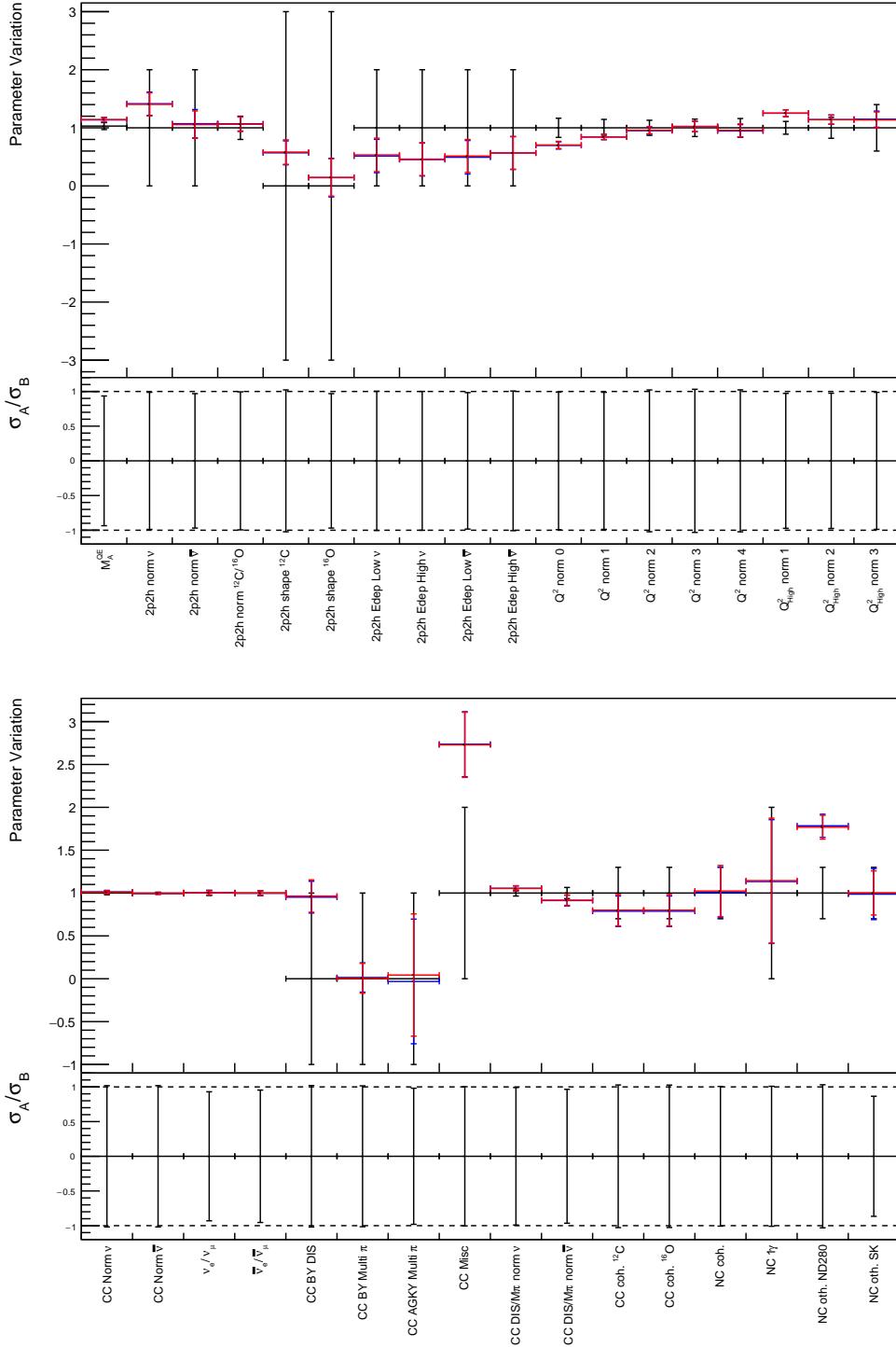


Figure 8.45: Central values and 1σ uncertainties for a select group of interaction systematics. The constraints from the prior uncertainty (Black), joint atmospheric-beam fit given in subsection 8.3.4 (Red) and beam-only analysis [2, 215] (Blue) are presented. The top part of each plot presents the parameter variation and the bottom part represents the ratio of the uncertainty between the joint beam-atmospheric and beam-only fits, where a value below 1.0 means the joint fit has a tighter constraint than the beam-only analysis.

3576 8.4 Summary of Sensitivity Studies

3577 The sensitivities to each oscillation parameter from the joint beam-atmospheric
 3578 and beam-only fits, which use the Asimov A oscillation parameter set, are
 3579 summarised in Table 8.12. As the posterior distribution to δ_{CP} is cyclical, only
 3580 the position of the highest posterior density (HPD) is given. The uncertainty
 3581 corresponds to the width of the bin in which the HPD is located. Furthermore,
 3582 the Δm_{32}^2 reported values only consider the NH credible interval region as the
 3583 full discussion can be found in the previous section.

Fit	δ_{CP} (HPD)	$\Delta m_{32}^2 [\times 10^{-3} \text{eV}^2]$	$\sin^2(\theta_{23})$	$\sin^2(\theta_{13}) [\times 10^{-2}]$
Asimov A	-1.601	2.509	0.528	2.19
Beam	-1.45 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	$2.45^{+0.45}_{-0.35}$
Beam w/RC	-1.57 ± 0.06	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	$2.19^{+0.06}_{-0.07}$
Joint	-1.57 ± 0.06	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	$2.35^{+0.45}_{-0.35}$
Joint w/RC	-1.57 ± 0.06	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	$2.18^{+0.07}_{-0.06}$

Table 8.12: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead.

3584 The Bayes factors from the beam-only and joint atmospheric-beam analyses
 3585 are presented in Table 8.13. The strength of each preference, from Jeffrey's
 3586 scale (Table 4.1), is also given.

3587 To summarise this information, the joint fit prefers a tighter 1σ credible
 3588 interval in $\sin^2(\theta_{23})$ along with a stronger Bayes factor for preferring the correct
 3589 octant hypothesis. The increase in sensitivity to $|\Delta m_{32}^2|$ between the two fits is
 3590 negligible but the joint analysis substantially prefers the correct mass hierarchy
 3591 hypothesis. It does not require any external constraints on $\sin^2(\theta_{13})$ to make
 3592 this statement. The joint analysis also produces a value of $\sin^2(\theta_{13})$ closer to the
 3593 known value compared to the beam-only analysis. When the reactor constraint is

Fit	$B(\text{NH}/\text{IH})$		$B(\text{UO}/\text{LO})$	
	Value	Strength	Value	Strength
Asimov A				
Beam	1.91	Weak	1.56	Weak
Beam w/RC	3.09	Weak	2.47	Weak
Joint	3.67	Substantial	1.74	Weak
Joint w/RC	6.47	Substantial	2.64	Weak
Asimov B				
Beam	1.08	Weak	0.91	Weak
Beam w/RC	0.98	Weak	1.15	Weak
Joint	1.43	Weak	1.19	Weak
Joint w/RC	1.36	Weak	1.52	Weak

Table 8.13: A comparison of the Bayes factors for mass hierarchy and $\sin^2(\theta_{23})$ octant hypotheses, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The strength of the preference for the normal mass hierarchy and upper octants are provided by Jeffrey's scale Table 8.13.

3594 applied, the preference for both the NH and UO hypotheses increases but does
 3595 not change the statement which would be made.

3596 The fits from the Asimov B comparisons (subsection 8.3.9) show the improved
 3597 ability for the joint analysis to more precisely select the true value of δ_{CP} if it were
 3598 CP-conserving, compared to the beam-only analysis. This is evidenced by the
 3599 area contained within the 1σ credible interval decreasing by $\sim 4\%$. Furthermore,
 3600 the joint fit is able to better determine the octant of $\sin^2(\theta_{23})$ when the true
 3601 value is moved further away from the boundary as evidenced by the larger
 3602 Bayes factor. There is also a $\sim 5\%$ reduction of area contained within the 1σ
 3603 credible interval in $\sin^2(\theta_{23})$.

9

3604

3605

Conclusions and Outlook

3606 This thesis has presented the sensitivities of a joint beam and atmospheric neutrino oscillation analysis from the Tokai-to-Kamioka (T2K) and Super-Kamiokande
3607 (SK) experiments combining the two independent analyses presented by the
3608 collaborations [2, 3]. This study uses 3244.4 days of SK livetime and $1.97 \times$
3609 $10^{21}(1.63 \times 10^{21})$ POT recorded at the far detector in the neutrino(antineutrino)
3610 beam operating mode. The ND280 near detector is used within this analysis to
3611 constrain the beam flux and cross-section systematics. It uses 1.15×10^{21} POT and
3612 8.34×10^{20} POT in the neutrino and antineutrino running modes, respectively.
3613

3614 This analysis uses a Bayesian Markov Chain Monte Carlo fitting technique
3615 implemented within the MaCh3 framework. This analysis has significantly developed
3616 the fitting framework, both in terms of technical features and performance.
3617 This includes supporting new samples, systematics, and oscillation channels.
3618 These developments have become the foundation of the fitter's expansion into
3619 other neutrino oscillation experiments. Beyond these improvements, a novel
3620 technique for calculating the atmospheric neutrino oscillation probabilities has
3621 been developed. This calculation uses a sub-sampling linear-averaging approach
3622 to ensure that the sensitivities being calculated are not biased due to insufficient
3623 Monte Carlo statistics in a region of rapidly varying probability. It illustrates a

3624 computationally feasible method of reliably calculating oscillation probabilities
3625 that can be utilised within any fitting framework.

3626 The sensitivity of the joint beam-atmospheric analysis is presented in Table 9.1,
3627 and compared to the beam-only analysis [2]. The sensitivities are evaluated
3628 using a set of known oscillation parameter values close to the results from a
3629 previous T2K analysis [78] (denoted AsimovA in Table 9.1). The joint analysis
3630 has a stronger sensitivity to $\sin^2(\theta_{23})$, as evidenced by the tighter 1σ credible
3631 intervals when the constraints from reactor experiments are not applied. The
3632 joint fit's sensitivity to δ_{CP} is marginally stronger than beam-only analysis. Whilst
3633 the sensitivity to $|\Delta m_{32}^2|$ is mostly unchanged between the two analyses, the
3634 sensitivity to select the correct hierarchy is significantly improved. This follows
3635 from a substantial preference for the normal hierarchy hypothesis presented
3636 within the joint analysis, as classified by Jeffrey's scale [134]. This is notable as
3637 the beam-only analysis cannot make this statement, either with or without the
3638 application of the reactor constraint. The joint fit's preference for the correct
3639 hierarchy increases once the reactor constraint is applied. The preference for
3640 selecting the correct octant of $\sin^2(\theta_{23})$ is classified as weak by Jeffrey's scale but
3641 is still stronger than the statement made by the beam-only analysis.

3642 The sensitivities of the beam-only and joint atmospheric-beam fit have also
3643 been compared at a set of known oscillation parameters which are CP-conserving
3644 and in the lower octant of $\sin^2(\theta_{23})$. The joint analysis has a $\sim 5\%$ improved abil-
3645 ity to select the known values more precisely compared to the beam-only analysis.

3646 Whilst this analysis provides the first sensitivities of a joint beam and atmo-
3647 spheric analysis, there are more improvements that could be made. Since this
3648 analysis began, the T2K collaboration has released an updated oscillation analysis
3649 with additional near and far detector samples alongside a more sophisticated
3650 interaction model [4]. The overall change in oscillation parameter measurement
3651 observed by T2K is relatively minor but the stronger constraints on the systemat-
3652 ics could impact this joint analysis to a larger extent. Further developments could
3653 consider the effect of correlating the beam and atmospheric flux uncertainties

Fit	δ_{CP} (HPD)	Δm_{32}^2 [$\times 10^{-3}\text{eV}^2$]	$\sin^2(\theta_{23})$	$B(\text{NH/IH})$	$B(\text{UO/LO})$
Asimov A	-1.601	2.509	0.528	NH	UO
Beam	$-1.45^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.501^{+0.044}_{-0.026}$	1.91	1.56
Beam w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.08}_{-0.06}$	$0.533^{+0.022}_{-0.043}$	3.09	2.47
Joint	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.07}_{-0.06}$	$0.518^{+0.027}_{-0.038}$	3.67	1.74
Joint w/RC	$-1.57^{+0.06}_{-0.06}$	$2.51^{+0.05}_{-0.06}$	$0.528^{+0.027}_{-0.038}$	6.47	2.64

Table 9.1: A comparison of the sensitivity to each oscillation parameter of interest, from the beam-only [2, 215] and joint atmospheric-beam fits discussed within this thesis. The results are illustrated both with and without the reactor constraint (RC). The best-fit values are taken from the highest posterior density (HPD) and the error comes from the width of the one-dimensional 1σ credible intervals. As the posterior distribution in δ_{CP} is cyclical, the highest posterior distribution is given instead. The Bayes factors are provided for the mass hierarchy preference: normal hierarchy (NH) and inverse hierarchy (IH), and $\sin^2(\theta_{23})$ octant preference: upper octant (UO) and lower octant (LO).

3654 relating to hadron production, where updates of the Bartol and Honda models
 3655 may allow this to be studied [151].

3656 Beyond these model improvements, more data is available than what is
 3657 assumed for this analysis. The T2K experiment has accumulated an additional
 3658 $1.78 \times 10^{20}\text{POT}$ in neutrino mode. Similarly, there are several early SK periods
 3659 that have not been considered within this analysis as the reconstruction software
 3660 used in this analysis has not been validated for those periods. SK will also
 3661 continue to accumulate statistics with Gadolinium doping. Developments in the
 3662 atmospheric sample selections may also benefit from the Gadolinium dopants as
 3663 neutron capture will aid in neutrino/antineutrino separation leading to better
 3664 mass hierarchy sensitivity. This would require including interaction systematics
 3665 for neutron capture of Gadolinium which has already started [216].

3666 This analysis shows the increased sensitivity to oscillation parameters from
 3667 the combination of beam and atmospheric samples. It has developed the MaCh3
 3668 fitting framework and has laid the foundations of the fitter's expansion into other
 3669 neutrino oscillation experiments. The sensitivities presented in this thesis, and
 3670 the techniques that were used to generate them, are significant to the future of

³⁶⁷¹ neutrino oscillation physics which will likely perform similar analyses. As such,
³⁶⁷² they have been presented by the T2K and SK collaborations at the Neutrino 2022
³⁶⁷³ conference [4]. Moving towards the next generation of neutrino experiments,
³⁶⁷⁴ this analysis has the potential to become the basis of the oscillation analysis
³⁶⁷⁵ for the future Hyper-Kamiokande experiment which aims to make conclusive
³⁶⁷⁶ statements about CP-violation in the neutrino sector.

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