

# S-map and SIRS

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March 11, 2016

## 1 S-maps

A family of forecasting methods that shy away from the mechanistic model-based approaches outlined in the previous sections have been developed by Sugihara (references) over the last several decades. As these methods do not include a mechanistic model in their forecasting process, they also do not attempt to perform parameter inference. Instead they attempt to reconstruct the underlying dynamical process as a weighted linear model from a time series.

maybe clarify that "weighted linear model" is not as simple/stupid as it might initially sound?

One such method, the sequential locally weighted global linear maps (S-map), builds a global linear map model and uses it to produce forecasts directly. Despite relying on a linear mapping, the S-map does not assume the time series on which it is operating is the product of linear system dynamics, and in fact was developed to accommodate non-linear dynamics.

explain more what this means?

good.

REFERENCES???

doesn't one also have to consider embedding lag?

The S-map works by first constructing a time series embedding of length  $E$ , known as the library and denoted  $\{\mathbf{x}_i\}$ . Consider a time series of length  $T$  denoted  $x_1, x_2, \dots, x_T$ . Each element in the time series with indices in the range  $E, E+1, \dots, T$  will have a corresponding entry in the library such that a given element  $x_t$  will correspond to a library vector of the form  $\mathbf{x}_i = (x_t, x_{t-1}, \dots, x_{t-E+1})$ . Next, given a forecast length  $L$  (representing  $L$  time steps into the future), each library vector  $\mathbf{x}_i$  is assigned a prediction from the time series  $y_i = x_{t+L}$ , where  $x_t$  is the first entry in  $\mathbf{x}_i$ . Finally, a forecast  $\hat{y}_t$  for specified predictor vector  $\mathbf{x}_t$  (usually from the library itself), is generated using an exponentially weighted function of the library  $\{\mathbf{x}_i\}$ , predictions  $\{y_i\}$ , and predictor vector  $\mathbf{x}_t$ .

This function is defined as follows:

First construct a matrix  $A$  and vector  $b$  defined as

define the norm (double-bars) here?

$$\begin{aligned} A(i, j) &= w(\|\mathbf{x}_i - \mathbf{x}_t\|) \mathbf{x}_i(j) \\ b(i) &= w(\|\mathbf{x}_i - \mathbf{x}_t\|) y_i \end{aligned} \tag{1}$$

where  $i$  ranges over 1 to the length of the library, and  $j$  ranges over  $[0, E]$ . It should be noted that in the above equations and the ones that follow,  $x_t(0) = 1$  to account for the

is set to

linear term in the map.

The weighting function  $w$  is defined as

$$w(d) = \exp\left(\frac{-\theta d}{\bar{d}}\right), \quad (2)$$

where  $d$  is the euclidean distance between the predictor vector and library vectors in Equation (1) and  $\bar{d}$  is the average of these distances. We can then see that  $\theta$  serves as a way to specify the appropriate level of penalization applied to poorly-matching library vectors – if  $\theta$  is 0 all weights are the same (no penalization), and increasing  $\theta$  increases the level of penalization.

Now we solve the system  $Ac = b$  to obtain the linear weightings used in to generate the forecast according to

$$\hat{y}_t = \sum_{j=0}^E c_t(j) \mathbf{x}_t(j). \quad (3)$$

In this way we have produced a forecast value for a single time. This process can be repeated for a sequence of times  $T + 1, T + 2, \dots$  to project a time series into the future.

can you give a slightly more heuristic description too?

## 2 S-map Algorithm

The above description can be summarized in Algorithm 1.

---

### Algorithm 1: S-map

---

```

/* Select a starting point */
Input : Time series  $x_1, x_2, \dots, x_T$ , embedding dimension  $E$ , distance penalization  $\theta$ ,
        forecast length  $L$ , predictor vector  $\mathbf{x}_t$ 

/* Construct library  $\{\mathbf{x}_i\}$  */
1 for  $i = E : T$  do
2    $\mathbf{x}_i = (x_i, x_{i-1}, \dots, x_{i-E+1})$ 

/* Construct mapping from library vectors to predictions */
3 for  $i = 1 : (T_E + 1)$  do
4   for  $j = 1 : E$  do
5      $A(i, j) = w(\|\mathbf{x}_i - \mathbf{x}_t\|)\mathbf{x}_i(j)$ 
6 for  $i = 1 : (T_E + 1)$  do
7    $b(i) = w(\|\mathbf{x}_i - \mathbf{x}_t\|)y_i$ 

/* Use SVD to solve the mapping system,  $Ac = b$  */
8  $SVD(Ac = b)$ 

/* Compute forecast */
9  $\hat{y}_t = \sum_{j=0}^E c_t(j)\mathbf{x}_t(j)$ 

/* Forecasted value in time series */
Output: Forecast  $\hat{y}_t$ 

```

---

## 3 SIRS Model

In an epidemic or infectious disease context, the S-map algorithm will only really work on time series that appear cyclic. While there is nothing mechanically that prevents it from operating on a time series that do not appear cyclic, S-mapping requires a long time series in order to build a quality library. Without one the forecasting process would produce unreliable data.

results

With that in mind, the only fair way to compare the efficacy of s-mapping to IF2 or Hamiltonian MCMC is to generate data from a SIRS model with a seasonal component, and have all methods operate on the resulting time series.

"fair" seems too strong? perhaps say something more that admits that we need to use S-mapping on a problem for which it is suitable (i.e., this is one disadvantage of S-mapping)

The basic skeleton of the SIRS model is similar to the stochastic SIR model described

previously. The deterministic ODE component of the model is as follows.

$$\begin{aligned}\frac{dS}{dt} &= -\Gamma(t)\beta SI + \eta R \\ \frac{dI}{dt} &= \Gamma(t)\beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I - \eta R,\end{aligned}\tag{4}$$

There are two new features here. We have a <sup>rate of waning immunity</sup> re-susceptibility rate  $\eta$  through which people become able to be reinfected, and a seasonality factor  $\Gamma$  defined as

$$\Gamma(t) = \exp\left(2\cos\left(\frac{2\pi}{365}t\right) - 2\right).$$

(5)

This function oscillates between 1 and  $e^{-4}$  (close to 0) and is meant to represent transmission damping during the off-season, for example summer for influenza. Further, it displays flatter troughs and sharper peaks to exaggerate its effect in peak season.

As before,  $\beta$  is allowed to walk restricted by a geometric mean, described by

$$\beta_{t+1} = \exp\left(\log(\beta_t) + \eta(\log(\bar{\beta}) - \log(\beta_t)) + \epsilon_t\right).$$

(6)

When simulated for the equivalent of 5 years (260 weeks), and adding noise drawn from  $\mathcal{N}(0, \sigma)$  we obtain Figure 1.

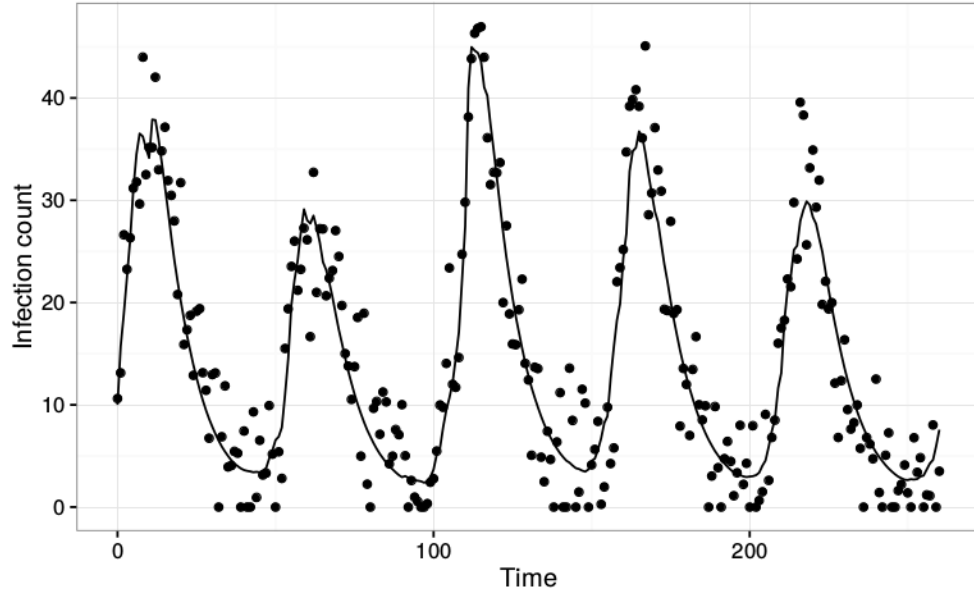


Figure 1: Five cycles generated by the SIRS function. The solid line the the true number of cases, dots show case counts with added observation noise. The Parameter values were  $R_0 = 3.0$ ,  $\gamma = 0.1$ ,  $\eta = 1$ ,  $\sigma = 5$ , and 10 initial cases.

We can see how the S-map can reconstruct the next cycle in the time series in Figure 2.

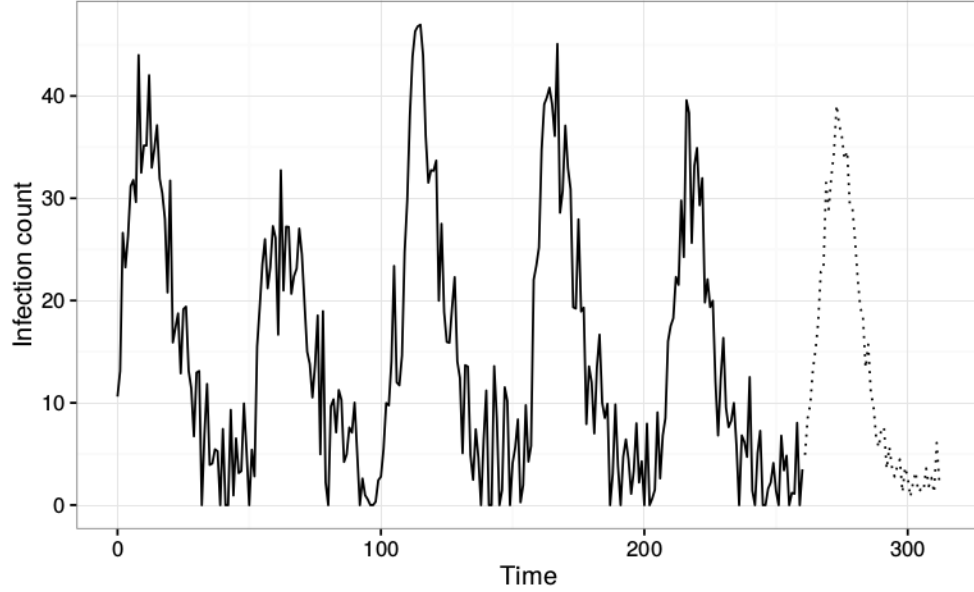


Figure 2: S-map applied to the data from the previous figure. The solid line shows the infection counts with observation noise [from](#) the previous plot, and the dotted line is the S-map forecast. Parameters chosen were  $E = 14$  and  $\theta = 3$ .

The parameters used in the S-map algorithm to obtain the forecast used in Figure [2](#) were obtained using a grid search of potential parameters outlined in [\(Sugihara ref\)](#). The script is included in the appendices.

## 4 SIRS Model Forecasting

Naturally we wish to compare the efficacy of this comparatively simple technique against the more complex and more computationally taxing frameworks we have established to perform forecasting using IF2 and HMC MC.

To do this we generated a series of artificial time series of length 260 meant to represent 5 years of weekly incidence counts and used each method to forecast up to 2 years into the future. Our goal here was to determine how forecast error changed with forecast length.

The results of the simulation are shown in Figure [3](#).

same again (you don't *\*have\** to change this style if you don't want to, just a suggestion)

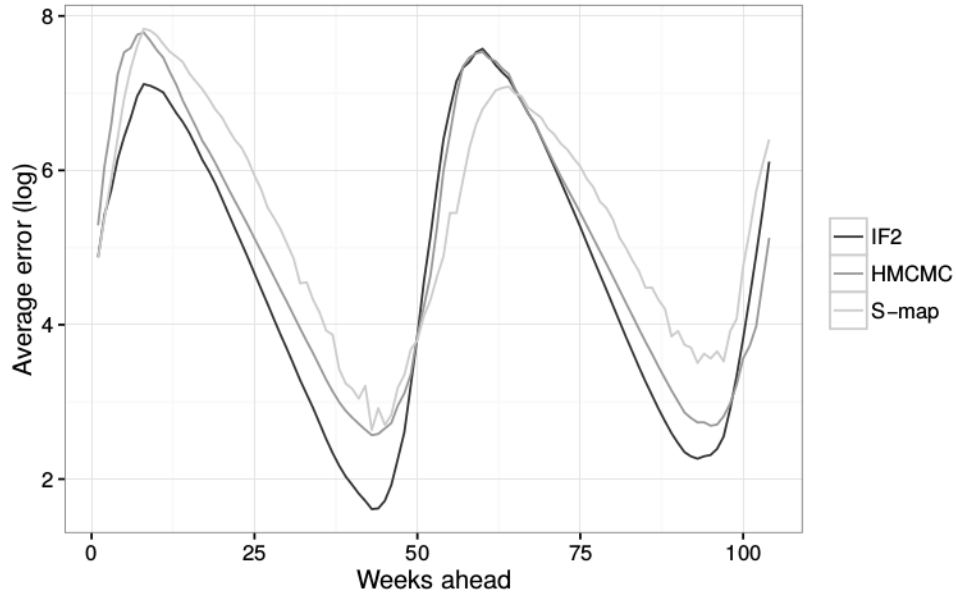


Figure 3: Error as a function of forecast length.

we may have talked about this, but confidence intervals on error bounds (might make the plot too busy though)?

Interestingly, all methods produce roughly the same result, which is to say the spike in each outbreak cycle are difficult to accurately predict. IF2 produces better results than either HMC MC and the S-map for the majority of forecast lengths, with the S-map producing the poorest results with the exception of the second rise in infection rates in which it outperforms the other methods.

While the S-map may not provide the same fidelity or forecast as IF2 or HMC MC, it shines when it comes to running time. Figure 4 shows the running times over 20 simulations.

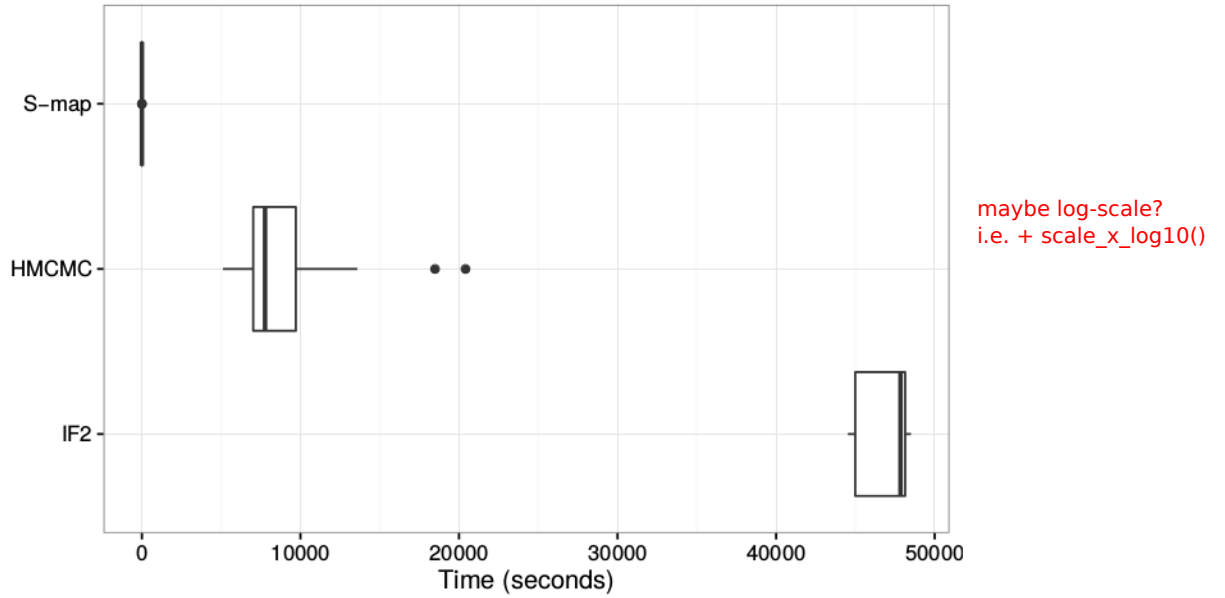


Figure 4: Runtimes for producing SIRS forecasts. The box shows the middle 50th quantile, the bold line is the median, and the dots are outliers.

careful about 'outliers' - dots in box-whisker plots are simply those beyond the whiskers (approx 1.5 times interquartile range), not \*statistically defined\* outliers

It is clear from the above figure that the S-map running times are minute compared to the other methods, but to emphasize the degree: The average running time for the S-map is about  $1.49 \times 10^{-1}$  seconds, for IF2 it is about  $4.70 \times 10^4$ , and for HMCMC it is about  $9.20 \times 10^3$ . This is a speed-up of over 316,000x compared to IF2 and over 61,800x compared to HMCMC.

format running times \*not\* in scientific notation? (0.15, 47000, 9200)



# Appendices

## A SIRS R Function Code

R code to simulate the [outlines](#) SIRS function.

```
1 StocSIRS <- function(y, pars, T, steps) {
2
3   out <- matrix(NA, nrow = (T+1), ncol = 4)
4     do you know the with(as.list(c(y,pars)), { ... }) trick?
5   R0 <- pars[['R0']]
6   r <- pars[['r']]
7   N <- pars[['N']]
8   eta <- pars[['eta']]
9   berr <- pars[['berr']]
10    re <- pars[['re']]
11
12   S <- y[['S']]
13   I <- y[['I']]
14   R <- y[['R']]
15
16   B0 <- R0 * r / N
17   B <- B0
18
19   out[1,] <- c(S,I,R,B)
20
21   h <- 1 / steps
22
23   for ( i in 1:(T*steps) ) {
24
25     #Bfac <- 1/2 - cos((2*pi/365)*i)/2
26     Bfac <- exp(2*cos((2*pi/365)*i) - 2)
27
28     B <- exp( log(B) + eta*(log(B0) - log(B)) + rnorm(1, 0, berr) )
29
30     BSI <- Bfac*B*S*I
31     rI <- r*I
32     reR <- re*R
33
34     dS <- -BSI + reR
35     dI <- BSI - rI
36     dR <- rI - reR
37
38     S <- S + h*dS #newInf
39     I <- I + h*dI #newInf - h*dR
40     R <- R + h*dR #h*dR
41
42     if (i %% steps == 0)
43       out[i/steps+1,] <- c(S,I,R,B)
44   }
```

```

45 }
46
47 colnames(out) ← c("S", "I", "R", "B")
48 return(out)
49
50 }
51
52 ### suggested parameters
53 #
54 # T      ← 200
55 # i_infec ← 10
56 # steps  ← 7
57 # N      ← 500
58 # sigma  ← 5
59 #
60 # pars ← c(R0 = 3.0, # new infected people per infected person
61 #          r = 0.1, # recovery rate
62 #          N = 500, # population size
63 #          eta = 0.5, # geometric random walk
64 #          berr = 0.5, # Beta geometric walk noise
65 #          re = 1) # resuceptibility rate

```

try to be consistent about SMAP, S-map, s-map, ...

## B SMAP Code

This code implements an SMAP function on a user-provided time series.

```

1 library(pracma)
2
3 smap ← function(data, E, theta, stepsAhead) {
4
5   # construct library
6   tseries ← as.vector(data)
7   liblen ← length(tseries) - E + 1 - stepsAhead
8   lib ← matrix(NA, liblen, E)
9
10  for (i in 1:E) {
11    lib[,i] ← tseries[(E-i+1):(liblen+E-i)]
12  }
13
14  # predict from the last index
15  tslen ← length(tseries)
16  predictee ← rev(t(as.matrix(tseries[(tslen-E+1):tslen])))
17  predictions ← numeric(stepsAhead)
18
19  #allPredictees ← matrix(NA, stepsAhead, E)
20
21  # for each prediction index (number of steps ahead)
22  for(i in 1:stepsAhead) {
23
24    # set up weight calculation

```

```

25     predmat ← repmat(predictee, liblen, 1)
26     distances ← sqrt( rowSums( abs(lib - predmat)^2 ) )
27     meanDist ← mean(distances)
28
29     # calculate weights
30     weights ← exp( - (theta * distances) / meanDist )
31
32     # construct A, B
33
34     preds ← tseries[(E+i):(liblen+E+i-1)]
35
36     A ← cbind( rep(1.0, liblen), lib ) * repmat(as.matrix(weights), 1,
37           E+1)
38     B ← as.matrix(preds * weights)
39
40     # solve system for C
41
42     Asvd ← svd(A)
43     C ← Asvd$v %*% diag(1/Asvd$d) %*% t(Asvd$u) %*% B
44
45     # get prediction
46
47     predsum ← sum(C * c(1,predictee))
48
49     # save
50
51     predictions[i] ← predsum
52
53     # next predictee
54
55     #predictee ← c( predsum, predictee[-E] )
56     #allPredictees[i,] ← predictee
57 }
58
59 return(predictions)
60
61 }

```

## C SMAP Parameter Optimization Code

This code determines the optimal parameter values to be used by the S-map algorithm.

```

1 library(deSolve)
2 library(ggplot2)
3 library(RColorBrewer)
4 library(pracma)    ## when using less-common packages I like to leave a comment about
5                    ## what functions from the package are used ...
6 set.seed(1010)
7

```

```

8 ## external files
9 ##
10 stoc_sirs_file ← paste(getwd(), "../sir-functions", "StocSIRS.r", sep = "/"
11                        ")
12 smap_file      ← paste(getwd(), "smap.r", sep = "/")
13 source(stoc_sirs_file)
14 source(smap_file)
15
16
17 ## parameters
18 ##
19 T      ← 6*52
20 Tlim   ← T - 52
21 i_infec ← 10
22 steps  ← 7
23 N      ← 500
24 sigma  ← 5
25
26 true_pars ← c( R0 = 3.0, # new infected people per infected person
27               r = 0.1, # recovery rate
28               N = 500, # population size
29               eta = 0.5, # geometric random walk
30               berr = 0.5, # Beta geometric walk noise
31               re = 1) # resuseptibility rate
32
33 true_init_cond ← c(S = N - i_infec,
34                   I = i_infec,
35                   R = 0)
36
37 ## trial parameter values to check.options
38 ##
39 Elist ← 1:20
40 thetalist ← 10*exp(-(seq(0,9.5,0.5)))
41 nTrials ← 100
42
43 ssemat ← matrix(NA, 20, 20)
44
45 for (i in 1:length(Elist)) {
46   for (j in 1:length(thetalist)) {
47
48     ssemean ← 0
49
50     for (k in 1:nTrials) {
51
52       E ← Elist[i]
53       theta ← thetalist[j]
54
55       ## get true trajectory
56       ##
57       sdeout ← StocSIRS(true_init_cond, true_pars, T, steps)
58
59       ## perturb to get data
60       ##

```

```

61   infec_counts_raw ← sdeout[1:(Tlim+1), 'I'] + rnorm(Tlim+1, 0, sigma)
62   infec_counts     ← ifelse(infec_counts_raw < 0, 0, infec_counts_raw)
63
64   predictions ← smap(infec_counts, E, theta, 52)
65
66   err ← sdeout[(Tlim+2):dim(sdeout)[1], 'I'] - predictions
67   sse ← sum(err^2)
68
69   ssemean ← ssemean + (sse / nTrials)
70
71 }
72
73 ssemat[i, j] ← ssemean
74
75
76 }
77 }
78
79 quartz()      ## this is platform-specific ... use dev.new() ?
80 image(-ssemat)
81 quartz()
82 filled.contour(-ssemat)
83
84 #print(ssemat)
85 #cms ← colMeans(ssemat)
86 #rms ← rowMeans(ssemat)
87
88 #Emin ← Elist[which.min(rms)]
89 #thetamin ← thetalist[which.min(cms)]
90 #print(Emin)
91 #print(thetamin)
92
93 mininds ← which(ssemat==min(ssemat), arr.ind=TRUE)
94
95 Emin ← Elist[mininds[, 'row']]
96 thetamin ← thetalist[mininds[, 'col']]
97
98 print(Emin)
99 print(thetamin)

```

## D RStan SIRS Code

This code implements a periodic SIRS model in Rstan.

```

1 data {
2
3   int      <lower=1>  T;      // total integration steps
4   real     y[T];      // observed number of cases
5   int      <lower=1>  N;      // population size
6   real     h;         // step size

```

```

7
8 }
9
10 parameters {
11
12     real <lower=0, upper=10>      R0;      // R0
13     real <lower=0, upper=10>      r;        // recovery rate
14     real <lower=0, upper=10>      re;       // resusceptibility rate
15     real <lower=0, upper=20>      sigma;    // observation error
16     real <lower=0, upper=30>      Iinit;    // initial infected
17     real <lower=0, upper=1>       eta;      // geometric walk attraction
18         strength
19     real <lower=0, upper=1>       berr;     // beta walk noise
20     real <lower=-1.5, upper=1.5>   Bnoise[T]; // Beta vector
21 }
22
23 //transformed parameters {
24 //     real B0 ← R0 * r / N;
25 //}
26
27 model {
28
29     real S[T];
30     real I[T];
31     real R[T];
32     real B[T];
33     real B0;
34
35     real pi;
36     real Bfac;
37
38     pi ← 3.1415926535;
39
40     B0 ← R0 * r / N;
41
42     B[1] ← B0;
43
44     S[1] ← N - Iinit;
45     I[1] ← Iinit;
46     R[1] ← 0.0;
47
48     for (t in 2:T) {
49
50         Bnoise[t] ~ normal(0, berr);
51         Bfac ← exp(2*cos((2*pi/365)*t) - 2);
52         B[t] ← exp( log(B0) + eta * ( log(B[t-1]) - log(B0) ) + Bnoise[t] )
53         ;
54
55         S[t] ← S[t-1] + h*( - Bfac*B[t]*S[t-1]*I[t-1] + re*R[t-1] );
56         I[t] ← I[t-1] + h*( Bfac*B[t]*S[t-1]*I[t-1] - I[t-1]*r );
57         R[t] ← R[t-1] + h*( I[t-1]*r - re*R[t-1] );
58
59         if (y[t] > 0) {

```

```

59         y[t] ~ normal( I[t], sigma );
60     }
61
62 }
63
64 R0      ~ lognormal(1,1);
65 r       ~ lognormal(1,1);
66 sigma   ~ lognormal(1,1);
67 re      ~ lognormal(1,1);
68 Iinit   ~ normal(y[1], sigma);
69
70 }

```

## E IF2 SIRS Code

This code implements a periodic SIRS model using IF2 in C++.

```

1  /* Author: Dexter Barrows
2     Github: dbarrows.github.io
3
4     */
5
6  #include <stdio.h>
7  #include <math.h>
8  #include <sys/time.h>
9  #include <time.h>
10 #include <stdlib.h>
11 #include <string>
12 #include <cmath>
13 #include <cstdlib>
14 #include <fstream>
15
16 // #include "rand.h"
17 // #include "timer.h"
18
19 #define Treal      100          // time to simulate over
20 #define R0true     3.0          // infectiousness
21 #define rtrue      0.1          // recovery rate
22 #define retrue     0.05         // resusceptibility rate
23 #define Nreal      500.0        // population size
24 #define etatrue    0.5          // real drift attraction strength
25 #define berrtrue   0.5          // real beta drift noise
26 #define merr       5.0          // expected measurement error
27 #define I0         5.0          // Initial infected individuals
28
29 #define PSC        0.5          // scale factor for more sensitive
30     parameters
31
32 #include <Rcpp.h>
33 using namespace Rcpp;

```

```

33
34 struct State {
35     double S;
36     double I;
37     double R;
38 };
39
40 struct Particle {
41     double R0;
42     double r;
43     double re;
44     double sigma;
45     double eta;
46     double berr;
47     double B;
48     double S;
49     double I;
50     double R;
51     double Sinit;
52     double Iinit;
53     double Rinit;
54 };
55
56 struct ParticleInfo {
57     double R0mean;    double R0sd;
58     double rmean;    double rsd;
59     double remean;    double resd;
60     double sigmamean; double sigmasd;
61     double etamean;   double etasd;
62     double berrmean;  double berrsd;
63     double Sinitmean; double Sinitsd;
64     double Iinitmean; double Iinitsd;
65     double Rinitmean; double Rinitsd;
66 };
67
68
69 int timeval_subtract (double *result, struct timeval *x, struct timeval *y)
70 ;
71 int check_double(double x, double y);
72 void exp_euler_SIRS(double h, double t0, double tn, int N, Particle *
73     particle);
74 void copyParticle(Particle * dst, Particle * src);
75 void perturbParticles(Particle * particles, int N, int NP, int passnum,
76     double coolrate);
77 void particleDiagnostics(ParticleInfo * partInfo, Particle * particles, int
78     NP);
79 void getStateMeans(State * state, Particle* particles, int NP);
80 NumericMatrix if2(NumericVector * data, int T, int N);
81 double randu();
82 double randn();
83
84 // [[Rcpp::export]]
85 Rcpp::List if2_sirs(NumericVector data, int T, int N, int NP, int nPasses,
86     double coolrate) {

```



```

82
83 int npar = 9;
84
85 NumericMatrix paramdata(NP, npar);
86 NumericMatrix means(nPasses, npar);
87 NumericMatrix sds(nPasses, npar);
88 NumericMatrix statemeans(T, 3);
89 NumericMatrix statedata(NP, 4);
90
91 srand(time(NULL)); // Seed PRNG with system time
92
93 double w[NP]; // particle weights
94
95 Particle particles[NP]; // particle estimates for current step
96 Particle particles_old[NP]; // intermediate particle states for
    resampling
97
98 printf("Initializing particle states\n");
99
100 // initialize particle parameter states (seeding)
101 for (int n = 0; n < NP; n++) {
102
103     double R0can, rcan, recan, sigmacan, linitcan, etacan, berrcan;
104
105     do {
106         R0can = R0true + R0true*randn();
107     } while (R0can < 0);
108     particles[n].R0 = R0can;
109
110     do {
111         rcan = rtrue + rtrue*randn();
112     } while (rcan < 0);
113     particles[n].r = rcan;
114
115     do {
116         recan = retrue + retrue*randn();
117     } while (recan < 0);
118     particles[n].re = recan;
119
120     particles[n].B = (double) R0can * rcan / N;
121
122     do {
123         sigmacan = merr + merr*randn();
124     } while (sigmacan < 0);
125     particles[n].sigma = sigmacan;
126
127     do {
128         etacan = etatrue + PSC*etatrue*randn();
129     } while (etacan < 0 || etacan > 1);
130     particles[n].eta = etacan;
131
132     do {
133         berrcan = berrtrue + PSC*berrtrue*randn();
134     } while (berrcan < 0);

```

```

135     particles[n].berr = berrcan;
136
137     do {
138         Iinitcan = I0 + I0*randn();
139     } while (Iinitcan < 0 || N < Iinitcan);
140     particles[n].Sinit = N - Iinitcan;
141     particles[n].Iinit = Iinitcan;
142     particles[n].Rinit = 0.0;
143
144 }
145
146 // START PASSES THROUGH DATA
147
148 printf("Starting filter\n");
149 printf("-----\n");
150 printf("Pass\n");
151
152
153 for (int pass = 0; pass < nPasses; pass++) {
154
155     printf("...%d / %d\n", pass, nPasses);
156
157     // reset particle system evolution states
158     for (int n = 0; n < NP; n++) {
159
160         particles[n].S = particles[n].Sinit;
161         particles[n].I = particles[n].Iinit;
162         particles[n].R = particles[n].Rinit;
163         particles[n].B = (double) particles[n].R0 * particles[n].r / N;
164
165     }
166
167     if (pass == (nPasses-1)) {
168         State sMeans;
169         getStateMeans(&sMeans, particles, NP);
170         statemeans(0,0) = sMeans.S;
171         statemeans(0,1) = sMeans.I;
172         statemeans(0,2) = sMeans.R;
173     }
174
175     for (int t = 1; t < T; t++) {
176
177         // generate individual predictions and weight
178         for (int n = 0; n < NP; n++) {
179
180             exp_euler_SIRS(1.0/7.0, (double) t-1, (double) t, N, &
                particles[n]);
181
182             double merr_par = particles[n].sigma;
183             double y_diff = data[t] - particles[n].I;
184
185             w[n] = 1.0/(merr_par*sqrt(2.0*M_PI)) * exp( - y_diff*y_diff
                / (2.0*merr_par*merr_par) );
186

```

```

187     }
188
189     // cumulative sum
190     for (int n = 1; n < NP; n++) {
191         w[n] += w[n-1];
192     }
193
194     // save particle states to resample from
195     for (int n = 0; n < NP; n++){
196         copyParticle(&particles_old[n], &particles[n]);
197     }
198
199     // resampling
200     for (int n = 0; n < NP; n++) {
201
202         double w_r = randu() * w[NP-1];
203         int i = 0;
204         while (w_r > w[i]) {
205             i++;
206         }
207
208         // i is now the index to copy state from
209         copyParticle(&particles[n], &particles_old[i]);
210
211     }
212
213     // between-iteration perturbations, not after last time step
214     if (t < (T-1))
215         perturbParticles(particles, N, NP, pass, coolrate);
216
217     if (pass == (nPasses-1)) {
218         State sMeans;
219         getStateMeans(&sMeans, particles, NP);
220         statemeans(t,0) = sMeans.S;
221         statemeans(t,1) = sMeans.I;
222         statemeans(t,2) = sMeans.R;
223     }
224
225 }
226
227 ParticleInfo pInfo;
228 particleDiagnostics(&pInfo, particles, NP);
229
230 means(pass, 0) = pInfo.R0mean;
231 means(pass, 1) = pInfo.rmean;
232 means(pass, 2) = pInfo.remean;
233 means(pass, 3) = pInfo.sigamean;
234 means(pass, 4) = pInfo.etamean;
235 means(pass, 5) = pInfo.berrmean;
236 means(pass, 6) = pInfo.Sinitmean;
237 means(pass, 7) = pInfo.Iinitmean;
238 means(pass, 8) = pInfo.Rinitmean;
239
240 sds(pass, 0) = pInfo.R0sd;

```

```

241     sds(pass, 1) = pInfo.rsd;
242     sds(pass, 2) = pInfo.resd;
243     sds(pass, 3) = pInfo.sigmasd;
244     sds(pass, 4) = pInfo.etasd;
245     sds(pass, 5) = pInfo.berrsd;
246     sds(pass, 6) = pInfo.Sinit;
247     sds(pass, 7) = pInfo.Iinit;
248     sds(pass, 8) = pInfo.Rinit;
249
250     // between-pass perturbations, not after last pass
251     if (pass < (nPasses + 1))
252         perturbParticles(particles, N, NP, pass, coolrate);
253
254 }
255
256 ParticleInfo pInfo;
257 particleDiagnostics(&pInfo, particles, NP);
258
259 printf("Parameter results (mean | sd)\n");
260 printf("-----\n");
261 printf("R0      %f %f\n", pInfo.R0mean, pInfo.R0sd);
262 printf("r      %f %f\n", pInfo.rmean, pInfo.rsd);
263 printf("re      %f %f\n", pInfo.remean, pInfo.resd);
264 printf("sigma    %f %f\n", pInfo.sigamean, pInfo.sigmasd);
265 printf("eta      %f %f\n", pInfo.etamean, pInfo.etasd);
266 printf("berr     %f %f\n", pInfo.berrmean, pInfo.berrsd);
267 printf("S_init   %f %f\n", pInfo.Sinitmean, pInfo.Sinit);
268 printf("I_init   %f %f\n", pInfo.Iinitmean, pInfo.Iinit);
269 printf("R_init   %f %f\n", pInfo.Rinitmean, pInfo.Rinit);
270
271 printf("\n");
272
273
274
275 // Get particle results to pass back to R
276
277 for (int n = 0; n < NP; n++) {
278
279     paramdata(n, 0) = particles[n].R0;
280     paramdata(n, 1) = particles[n].r;
281     paramdata(n, 2) = particles[n].re;
282     paramdata(n, 3) = particles[n].sigma;
283     paramdata(n, 4) = particles[n].eta;
284     paramdata(n, 5) = particles[n].berr;
285     paramdata(n, 6) = particles[n].Sinit;
286     paramdata(n, 7) = particles[n].Iinit;
287     paramdata(n, 8) = particles[n].Rinit;
288
289 }
290
291 for (int n = 0; n < NP; n++) {
292
293     statedata(n, 0) = particles[n].S;
294     statedata(n, 1) = particles[n].I;

```

```

295     statedata(n, 2) = particles[n].R;
296     statedata(n, 3) = particles[n].B;
297
298 }
299
300
301
302     return Rcpp::List::create( Rcpp::Named("paramdata") = paramdata,
303                               Rcpp::Named("means") = means,
304                               Rcpp::Named("statemeans") = statemeans,
305                               Rcpp::Named("statedata") = statedata,
306                               Rcpp::Named("sds") = sds);
307
308 }
309
310
311 /* Use the Explicit Euler integration scheme to integrate SIR model
312    forward in time
313    double h      - time step size
314    double t0     - start time
315    double tn     - stop time
316    double * y    - current system state; a three-component vector
317                   representing [S I R], susceptible-infected-recovered
318
319 */
318 void exp_euler_SIRS(double h, double t0, double tn, int N, Particle *
319   particle) {
320
321     int num_steps = floor( (tn-t0) / h );
322
323     double S = particle->S;
324     double I = particle->I;
325     double R = particle->R;
326
327     double R0 = particle->R0;
328     double r = particle->r;
329     double re = particle->re;
330     double B0 = R0 * r / N;
331     double eta = particle->eta;
332     double berr = particle->berr;
333
334     double B = particle->B;
335
336     for(int i = 0; i < num_steps; i++) {
337         //double Bfac = 0.5 - 0.95*cos( (2.0*M_PI/365)*(t0*num_steps+i) )
338         //                /2.0;
339         double Bfac = exp(2*cos((2*M_PI/365)*(t0*num_steps+i)) - 2);
340         B = exp( log(B) + eta*(log(B0) - log(B)) + berr*randn() );
341
342         double BSI = Bfac*B*S*I;
343         double rI = r*I;
344         double reR = re*R;

```

```

345         // get derivatives
346         double dS = - BSI + reR;
347         double dI = BSI - rI;
348         double dR = rI - reR;
349
350         // step forward by h
351         S += h*dS;
352         I += h*dI;
353         R += h*dR;
354
355     }
356
357     particle->S = S;
358     particle->I = I;
359     particle->R = R;
360     particle->B = B;
361 }
362 }
363
364
365 /* Particle pertubation function to be run between iterations and passes
366
367 */
368 void perturbParticles(Particle * particles, int N, int NP, int passnum,
369     double coolrate) {
370
371     //double coolcoef = exp( - (double) passnum / coolrate );
372     double coolcoef = pow(coolrate, passnum);
373
374     double spreadR0      = coolcoef * R0true / 10.0;
375     double spreadr       = coolcoef * rtrue / 10.0;
376     double spreadre      = coolcoef * retrue / 10.0;
377     double spreadsigma   = coolcoef * merr / 10.0;
378     double spreadIinit   = coolcoef * I0 / 10.0;
379     double spreadeta     = coolcoef * etatrue / 10.0;
380     double spreadberr    = coolcoef * berrtrue / 10.0;
381
382
383     double R0can, rcan, recan, sigmacan, Iinitcan, etacan, berrcan;
384
385     for (int n = 0; n < NP; n++) {
386
387         do {
388             R0can = particles[n].R0 + spreadR0*randn();
389         } while (R0can < 0);
390         particles[n].R0 = R0can;
391
392         do {
393             rcan = particles[n].r + spreadr*randn();
394         } while (rcan < 0);
395         particles[n].r = rcan;
396
397         do {

```

```

398         recan = particles[n].re + spreadre*randn();
399     } while (recan < 0);
400     particles[n].re = recan;
401
402     do {
403         sigmacan = particles[n].sigma + spreadsigma*randn();
404     } while (sigmacan < 0);
405     particles[n].sigma = sigmacan;
406
407     do {
408         etacan = particles[n].eta + PSC*spreadeta*randn();
409     } while (etacan < 0 || etacan > 1);
410     particles[n].eta = etacan;
411
412     do {
413         berrcan = particles[n].berr + PSC*spreadberr*randn();
414     } while (berrcan < 0);
415     particles[n].berr = berrcan;
416
417     do {
418         Iinitcan = particles[n].Iinit + spreadIinit*randn();
419     } while (Iinitcan < 0 || Iinitcan > 500);
420     particles[n].Iinit = Iinitcan;
421     particles[n].Sinit = N - Iinitcan;
422
423 }
424
425 }
426
427
428 /* Convenience function for particle resampling process
429
430 */
431 void copyParticle(Particle * dst, Particle * src) {
432
433     dst->R0      = src->R0;
434     dst->r        = src->r;
435     dst->re       = src->re;
436     dst->sigma    = src->sigma;
437     dst->eta      = src->eta;
438     dst->berr     = src->berr;
439     dst->B        = src->B;
440     dst->S        = src->S;
441     dst->I        = src->I;
442     dst->R        = src->R;
443     dst->Sinit    = src->Sinit;
444     dst->Iinit    = src->Iinit;
445     dst->Rinit    = src->Rinit;
446
447 }
448
449 void particleDiagnostics(ParticleInfo * partInfo, Particle * particles, int
450     NP) {

```

```

451 double R0mean      = 0.0,
452       rmean        = 0.0,
453       remean        = 0.0,
454       sigmamean     = 0.0,
455       etamean       = 0.0,
456       berrmean      = 0.0,
457       Sinitmean     = 0.0,
458       Iinitmean     = 0.0,
459       Rinitmean     = 0.0;
460
461 // means
462
463 for (int n = 0; n < NP; n++) {
464
465     R0mean      += particles[n].R0;
466     rmean       += particles[n].r;
467     remean      += particles[n].re;
468     etamean     += particles[n].eta,
469     berrmean    += particles[n].berr,
470     sigmamean   += particles[n].sigma;
471     Sinitmean   += particles[n].Sinit;
472     Iinitmean   += particles[n].Iinit;
473     Rinitmean   += particles[n].Rinit;
474
475 }
476
477 R0mean      /= NP;
478 rmean       /= NP;
479 remean      /= NP;
480 sigmamean   /= NP;
481 etamean     /= NP;
482 berrmean    /= NP;
483 Sinitmean   /= NP;
484 Iinitmean   /= NP;
485 Rinitmean   /= NP;
486
487 // standard deviations
488
489 double R0sd     = 0.0,
490       rsd       = 0.0,
491       resd      = 0.0,
492       sigmasd   = 0.0,
493       etasd     = 0.0,
494       berrsd    = 0.0,
495       Sinitsd   = 0.0,
496       Iinitsd   = 0.0,
497       Rinitsd   = 0.0;
498
499 for (int n = 0; n < NP; n++) {
500
501     R0sd      += ( particles[n].R0 - R0mean ) * ( particles[n].R0 -
502               R0mean );
503     rsd       += ( particles[n].r - rmean ) * ( particles[n].r - rmean );
504     resd      += ( particles[n].re - remean ) * ( particles[n].re - remean );

```



```

    );
504     sigmasd += ( particles[n].sigma - sigmamean ) * ( particles[n].
        sigma - sigmamean );
505     etasd += ( particles[n].eta - etamean ) * ( particles[n].eta -
        etamean );
506     berrsd += ( particles[n].berr - berrmean ) * ( particles[n].berr -
        berrmean );
507     Sinitd += ( particles[n].Sinit - Sinitmean ) * ( particles[n].
        Sinit - Sinitmean );
508     Iinitd += ( particles[n].Iinit - Iinitmean ) * ( particles[n].
        Iinit - Iinitmean );
509     Rinitd += ( particles[n].Rinit - Rinitmean ) * ( particles[n].
        Rinit - Rinitmean );
510
511 }
512
513 R0sd      /= NP;
514 rsd       /= NP;
515 resd      /= NP;
516 sigmasd   /= NP;
517 etasd     /= NP;
518 berrsd    /= NP;
519 Sinitd    /= NP;
520 Iinitd    /= NP;
521 Rinitd    /= NP;
522
523 partInfo->R0mean    = R0mean;
524 partInfo->R0sd      = R0sd;
525 partInfo->rmean     = rmean;
526 partInfo->rsd       = rsd;
527 partInfo->remean    = remean;
528 partInfo->resd      = resd;
529 partInfo->sigmamean = sigmamean;
530 partInfo->sigmasd   = sigmasd;
531 partInfo->etamean   = etamean;
532 partInfo->etasd     = etasd;
533 partInfo->berrmean   = berrmean;
534 partInfo->berrsd    = berrsd;
535 partInfo->Sinitmean = Sinitmean;
536 partInfo->Sinitd    = Sinitd;
537 partInfo->Iinitmean = Iinitmean;
538 partInfo->Iinitd    = Iinitd;
539 partInfo->Rinitmean = Rinitmean;
540 partInfo->Rinitd    = Rinitd;
541
542 }
543
544 double randu() {
545
546     return (double) rand() / (double) RAND_MAX;
547
548 }
549
550 void getStateMeans(State * state, Particle* particles, int NP) {

```

```

551
552     double Smean = 0, Imean = 0, Rmean = 0;
553
554     for (int n = 0; n < NP; n++) {
555         Smean += particles[n].S;
556         Imean += particles[n].I;
557         Rmean += particles[n].R;
558     }
559
560     state->S = (double) Smean / NP;
561     state->I = (double) Imean / NP;
562     state->R = (double) Rmean / NP;
563
564 }
565
566
567 /*  Return a normally distributed random number with mean 0 and standard
568     deviation 1
569     Uses the polar form of the Box-Muller transformation
570     From http://www.design.caltech.edu/erik/Misc/Gaussian.html
571 */
572 double randn() {
573     double x1, x2, w, y1;
574
575     do {
576         x1 = 2.0 * randu() - 1.0;
577         x2 = 2.0 * randu() - 1.0;
578         w = x1 * x1 + x2 * x2;
579     } while ( w >= 1.0 );
580
581     w = sqrt( (-2.0 * log( w ) ) / w );
582     y1 = x1 * w;
583
584     return y1;
585
586 }

```