A Comparative Study of Techniques for Estimation and Inference of Nonlinear Stochastic Time Series

---

Dexter Barrows, B.Sc.

Masters Thesis Defence
McMaster University
April 2016

1. Framing
2. Stochastic SIR Model
3. Hamiltonian HMC
4. Iterated Filtering 2
5. Fitting Results
6. Forecasting Frameworks
7. S-maps & Seasonal Outbreaks
8. Spatiotemporal Epidemics
9. Parallelism & Future Directions

Framing


SIResampling
ParticleMCMC
ParticleFilter
DewdropRegression
SIS
EnsembleKF
SeasonalARIMA
GARMA
ASIR
IF
RPF
MIF2
HMM
RHF
GLM
ABC
S-map
GEnKF
SimplexProjection
KalmanFilter
MCMCFilter
HamiltonianMCMC
ARIMA

2

Stochastic SIR Model

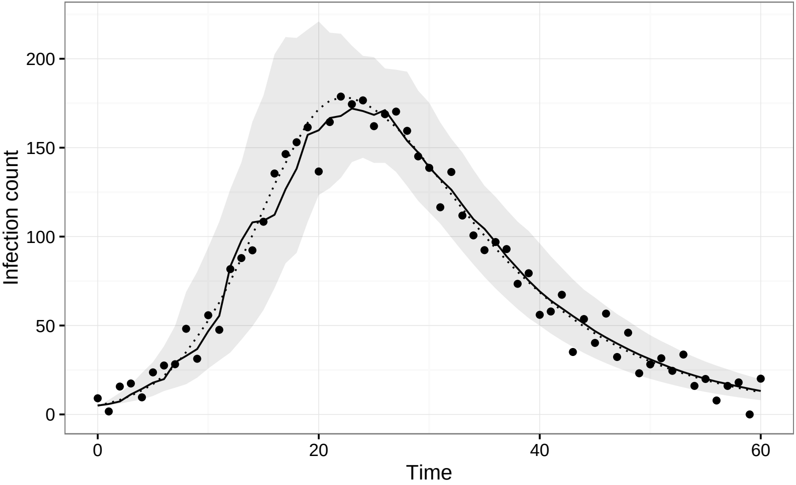
Stochastic SIR model

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

$$+$$

$$\beta_{t+1} = \exp\left[\beta_t + \eta\left(\bar{\beta} - \beta_t\right) + \mathcal{N}(0, \sigma_{proc})\right]$$

Model simulations



Solid – sample trajectory · dots – sample data · dotted – average · ribbon – overall

3

Hamiltonian MCMC

MCMC

Iteratively construct Markov chain to approximate posterior

1. Choose starting parameter set
2. Generate N samples by
   - 2.1 Propose new sample
   - 2.2 Compute acceptance ratio
   - 2.3 Accept/reject sample

HMC

Proposal via Hamiltonian dynamics

1. Choose starting parameter set
2. Generate N samples by
   - 2.1 Resample moments
   - 2.2 Simulate Hamiltonian dynamics using Leapfrog integration
   - 2.3 Compute acceptance ratio
   - 2.4 Accept/reject sample

Hamiltonian Dynamics

Energy

Kinetic $$K(r) = \frac{1}{2} r^T M^{-1} r$$

Potential $$U(\theta) = -\log(\mathcal{L}(\theta) p(\theta))$$

Hamiltonian $$H(\theta, r) = U(\theta) + K(r)$$

Dynamics simulation
$$\frac{d\theta}{dt} = M^{-1} r$$
$$\frac{dr}{dt} = -\nabla U(\theta)$$

Iterated Filtering 2

4

Basic particle filter

Iterative prediction-update cycle prunes particle cohort of poor parameter estimates

1. Initialize particles with parameter sets
2. For each data point
   - 2.1 Evolve particle states
   - 2.2 Weight via likelihood
   - 2.3 Resample proportional to weights

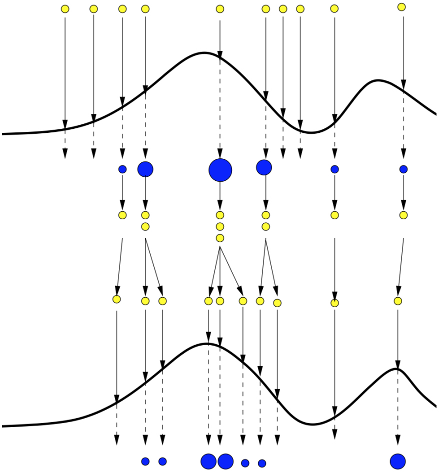


Image source [2]

Iterated Filtering 2 (IF2)

Evolution of MIF (IFI)

Multiple passes through data

Treat parameter estimates as stochastic processes

- Alleviates risk of particle collapse
- Process noise decreases with passes

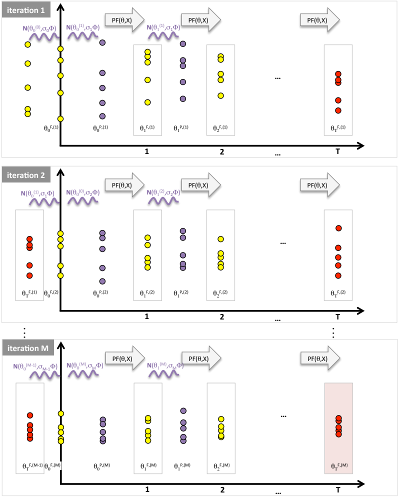
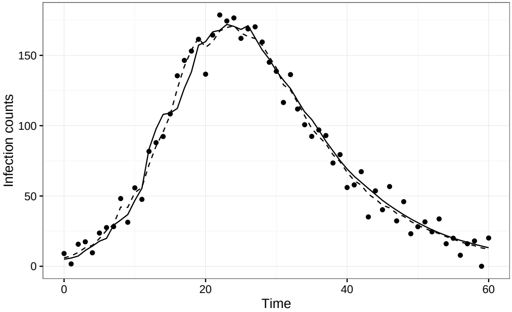


Image source [2]

5 Fitting Results

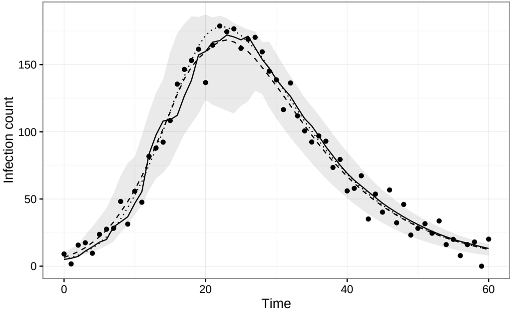
IF2

state estimates



HMC

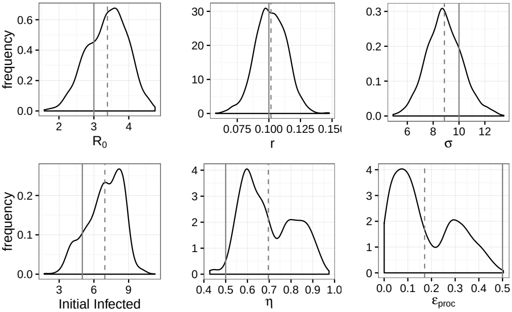
state estimates



Dashed – estimates · solid – true · dots – data · ribbon – trajectories

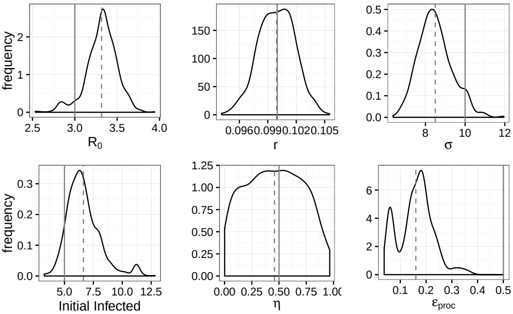
IF2

final particle swarm samples



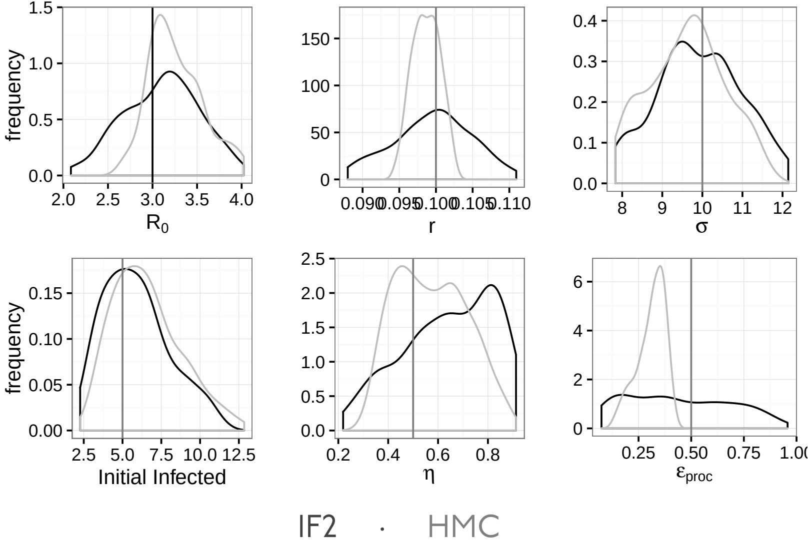
HMC

posterior distribution estimates

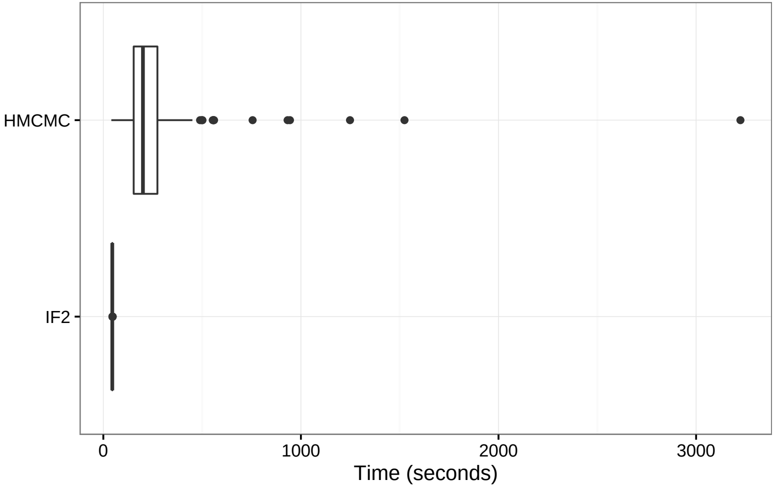


Dashed – medians · solid – true

Mean parameter estimate distributions



Running times



IF2: 5.7x faster than HMC

6

Forecasting Frameworks

IF2

Parametric bootstrapping
+ forward simulation

- Provides additional samples from posterior distribution
- Forward simulation using states point estimate, posterior samples

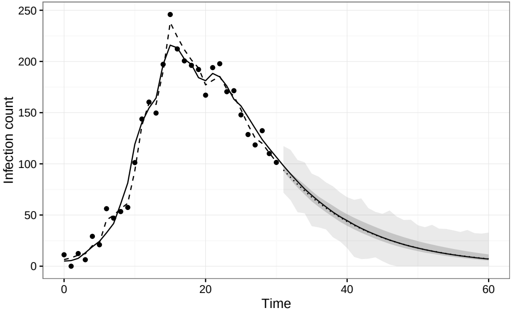
HMC

State reconstructions
+ forward simulation

- Reconstruct states using latent process noise samples
- Simulate forward

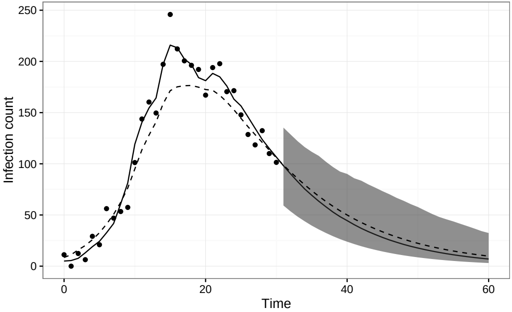
IF2

forecast



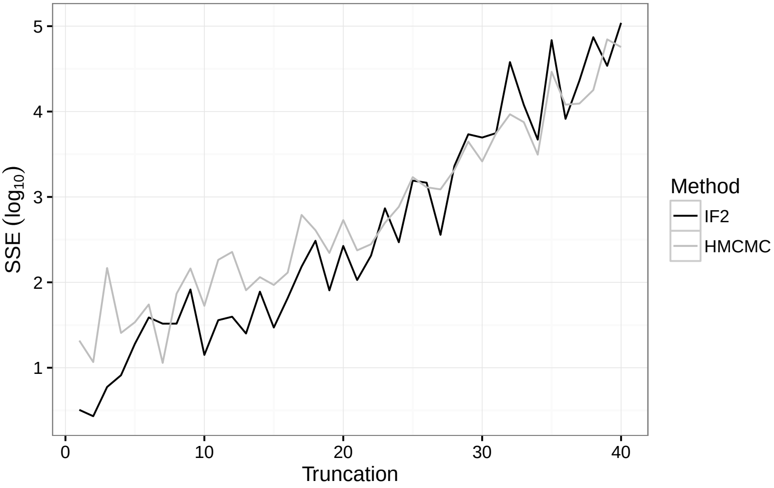
HMC

forecast



Dotted/dashed – mean · Dark ribbon - state · Light ribbon - observation

Forecast accuracy comparison



7

S-maps & Seasonal Outbreaks

Stochastic SIRS model

$$\frac{dS}{dt} = -\Gamma(t)\beta SI + \alpha R$$

$$\frac{dI}{dt} = \Gamma(t)\beta SI - \gamma I$$

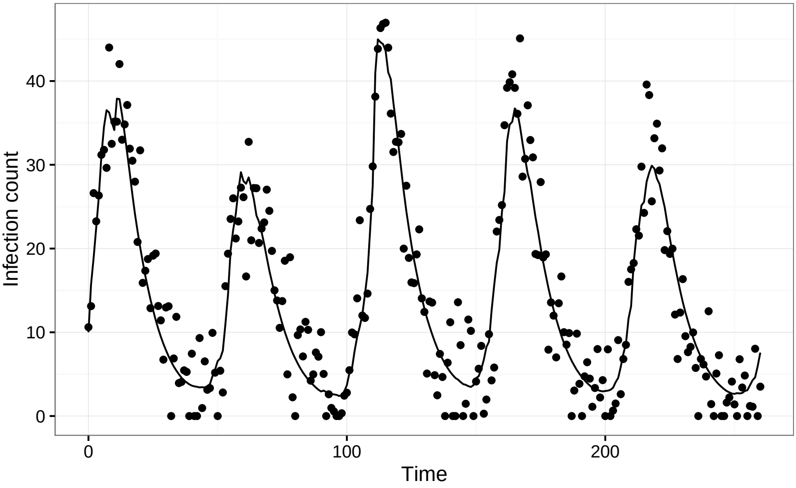
$$\frac{dR}{dt} = \gamma I - \alpha R$$

$$+$$

$$\Gamma(t) = \exp\left[2\left(\cos\left(\frac{2\pi}{365}t\right)\right)\right]$$

$$\beta_{t+1} = \exp\left[\beta_t + \eta\left(\bar{\beta} - \beta_t\right) + \mathcal{N}(0, \sigma_{proc})\right]$$

SIRS model simulation

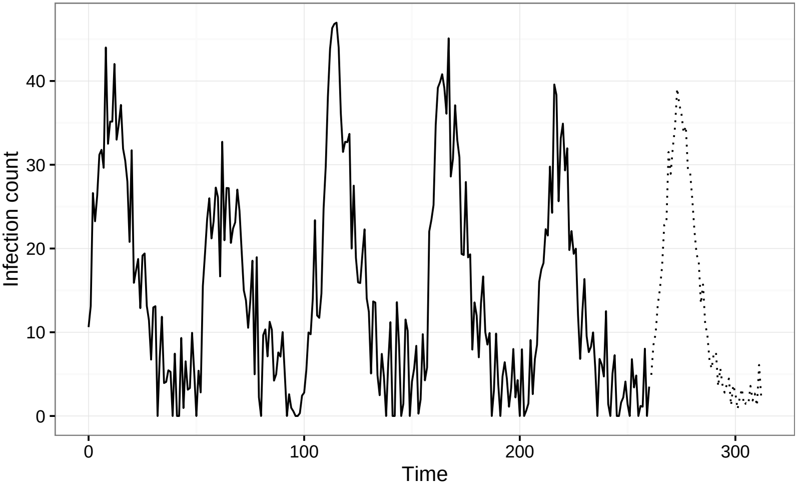


S-map

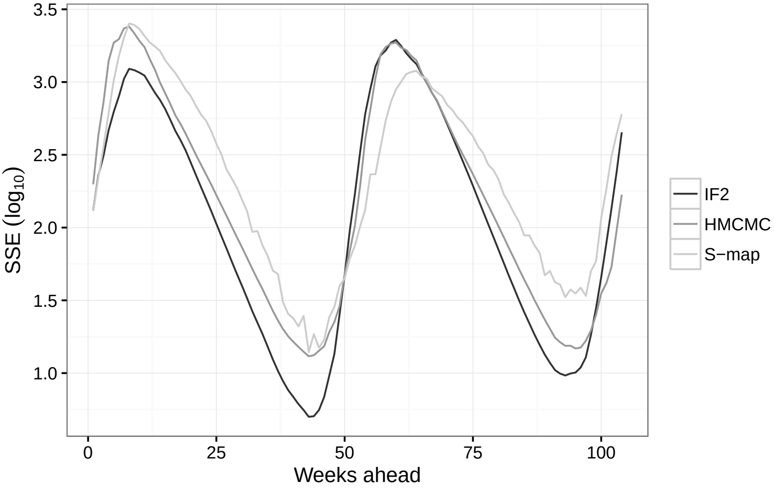
A little bit of history repeating

- Construct global weighted mapping of time-lagged vectors (the library) to future states
- Weightings are used to penalize poorly matching library vectors

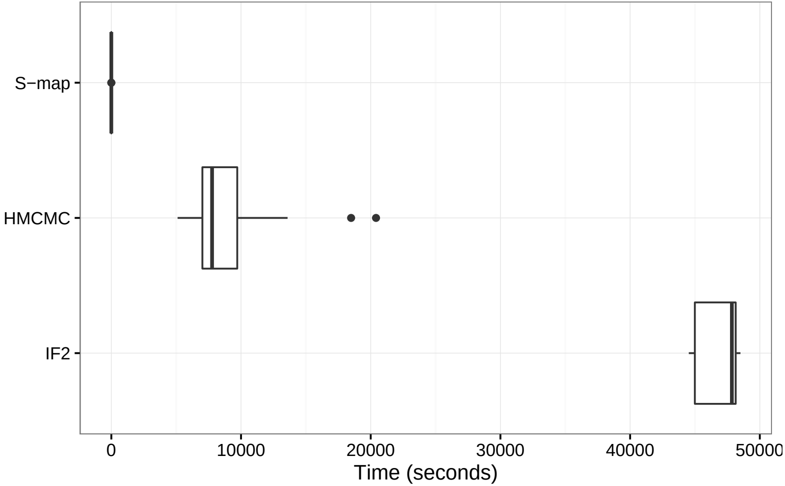
S-map forecast



SIRS model forecasting error



SIRS model forecasting runtimes



S-map: 316,000x faster than IF2, 61,800x faster than HMC

8

Spatiotemporal Epidemics

Stochastic Spatial SIR model

$$\frac{dS_i}{dt} = -\left(1 - \phi\frac{M}{M+1}\right)\beta_i S_i I_i - \frac{\phi}{M+1} S_i \sum_{j=1}^{M} \beta_{ij} I_j$$

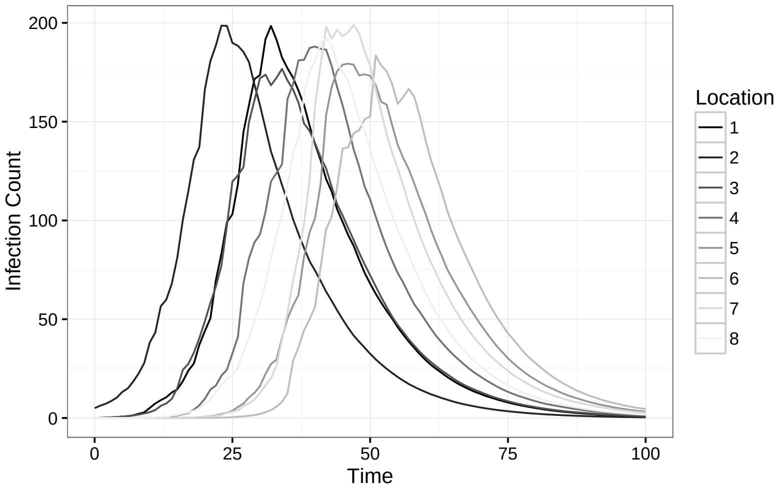
$$\frac{dI_i}{dt} = \left(1 - \phi\frac{M}{M+1}\right)\beta_i S_i I_i + \frac{\phi}{M+1} S_i \sum_{j=1}^{M} \beta_{ij} I_j - \gamma I_i$$

$$\frac{dR_i}{dt} = \gamma I_i$$

$$+$$

$$\beta_{i,t+1} = \exp\left[\beta_{i,t} + \eta\left(\bar{\beta} - \beta_{i,t}\right) + \mathcal{N}(0, \sigma_{proc})\right]$$

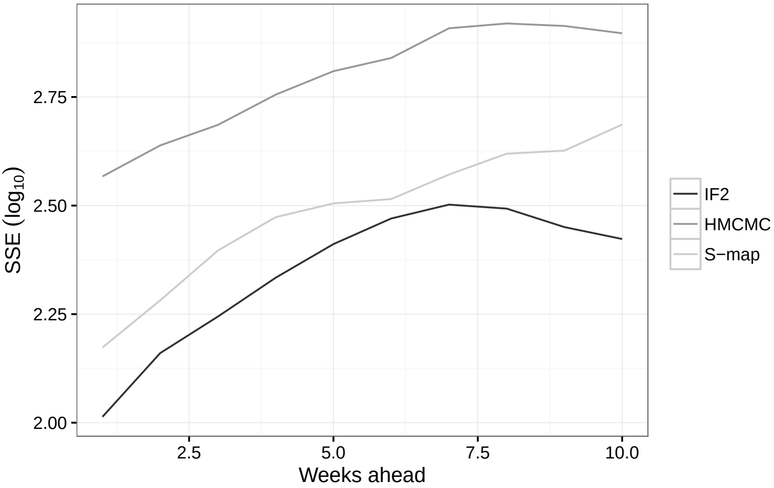
Stochastic spatial SIR model simulation (ring topology)



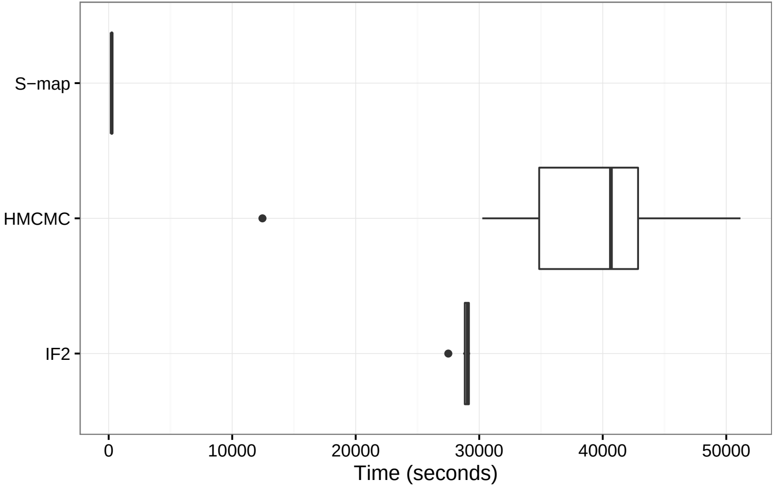
Dewdrop Regression

- “Stitching” together multiple short time series into single library
- Requires scaling

Spatial SIR model forecasting error



Spatial SIR model forecasting runtimes



S-map: 116x faster than IF2, 156x faster than HMC

9

Parallelism & Future Directions

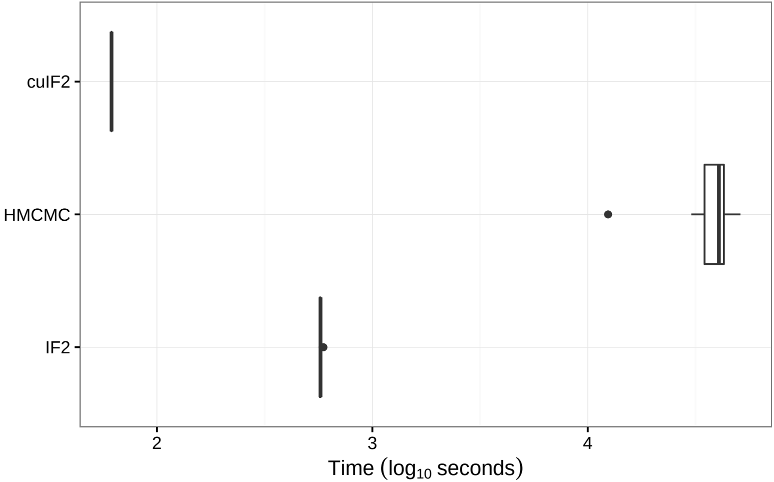
Conclusions

- IF2 produces superior forecasts in all scenarios
- S-mapping runs orders of magnitude faster than other methods

More than Moore

- Moore's Law is ceasing to hold
- Focus now on distributed computing
- MCMC-based methods are resistant to parallelization
  - Chain construction requires iterative dependence
- IF2 exhibits high parallel potential
  - Preliminary CUDA (GPU-accelerated) implementation - `cuIF2`

Spatial SIR model fitting times



`cuIF2`: 9.33x faster than IF2, 617x faster than HMC

Questions

[1] D. Barrows.
A Comparative Study of Techniques for Estimation and Inference of Nonlinear Stochastic Time Series.
Master's thesis, McMaster University, Hamilton, Canada, 2016.

[2] A. Doucet, N. de Freitas, and N. Gordon.
An Introduction to Sequential Monte Carlo Methods.
Sequential Monte Carlo Methods in Practice, pages 3–14, 2001.

THE BEST THESIS DEFENSE IS A GOOD THESIS OFFENSE.