Beta **beta(α_1, α_2)**

Possible applications: Used as a rough model in the absence of data (see Sec. 6.9); distribution of a random proportion, such as the proportion of defective items in a shipment; time to complete a task, e.g., in a PERT network

Density (see Fig. 6.7)

$$f(x) = \begin{cases} \dfrac{x^{\alpha_1 - 1}(1-x)^{\alpha_2 - 1}}{B(\alpha_1, \alpha_2)} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $B(\alpha_1, \alpha_2)$ is the *beta function*, defined by

$$B(z_1, z_2) = \int_0^1 t^{z_1 - 1}(1-t)^{z_2 - 1}\, dt$$

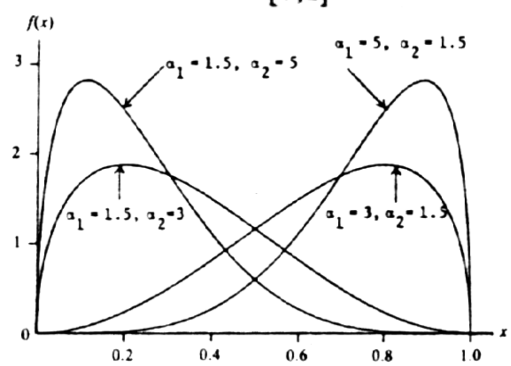
for any real numbers $z_1 > 0$ and $z_2 > 0$. Some properties of the beta function:

$$B(z_1, z_2) = B(z_2, z_1), \qquad B(z_1, z_2) = \frac{\Gamma(z_1)\Gamma(z_2)}{\Gamma(z_1 + z_2)}$$

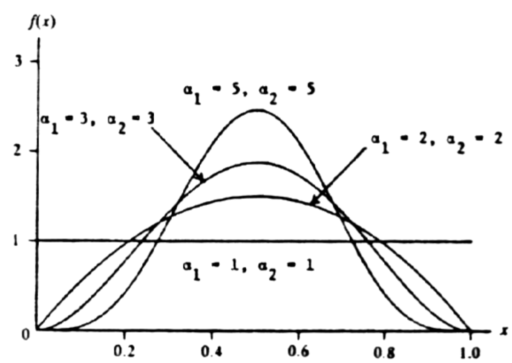
Distribution: No closed form, in general. If either α_1 or α_2 is a positive integer, a binomial expansion can be used to obtain $F(x)$, which will be a polynomial in x, and the powers of x will be, in general, positive real numbers ranging from 0 through $\alpha_1 + \alpha_2 - 1$

Parameters: Shape parameters $\alpha_1 > 0$ and $\alpha_2 > 0$

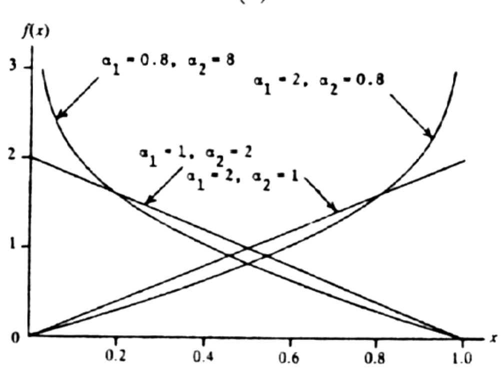
Range: [0,1]



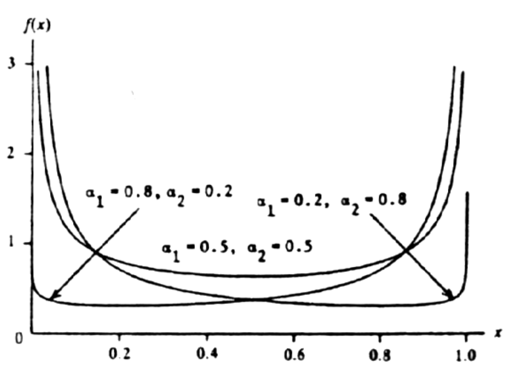
(a)



(b)



(c)



(d)

Pearson type V	**PT5(α, β)**
Possible applications	Time to perform some task (density takes on shapes similar to lognormal, but can have a larger "spike" close to $x = 0$)
Density (see Fig. 6.8)	$f(x) = \begin{cases} \dfrac{x^{-(\alpha+1)} e^{-\beta/x}}{\beta^{-\alpha}\Gamma(\alpha)} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$
Distribution	$F(x) = \begin{cases} 1 - F_G\left(\dfrac{1}{x}\right) & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}$ where $F_G(x)$ is the distribution function of a gamma($\alpha, 1/\beta$) random variable
Parameters	Shape parameter $\alpha > 0$, scale parameter $\beta > 0$
Range	$[0, \infty)$
Mean	$\dfrac{\beta}{\alpha - 1}$ for $\alpha > 1$

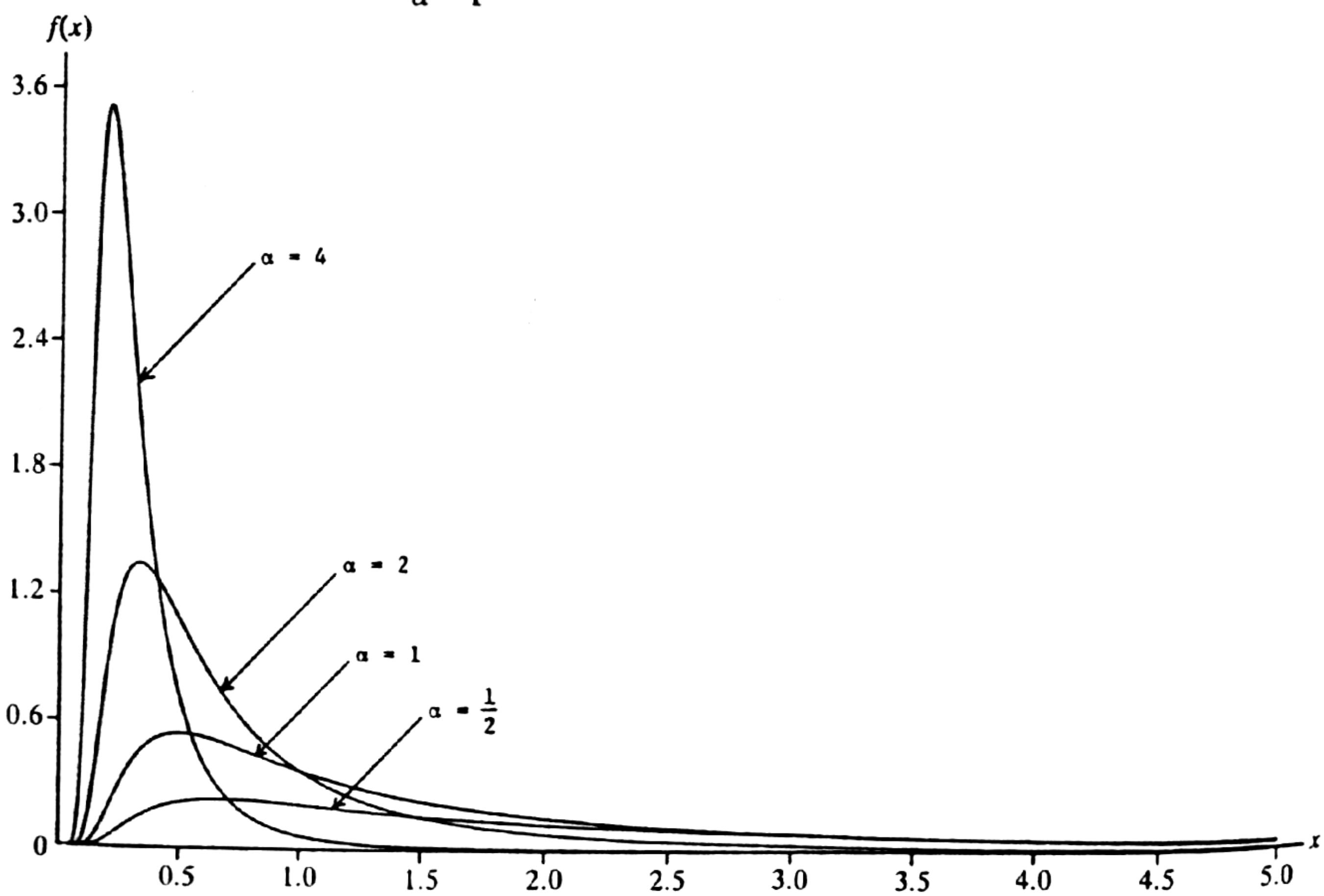


FIGURE 6.8
PT5(α,1) density functions.