Comparing heuristics for graph edit distance computation

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https://github.com/dbblumenthal/gedlib

Graph Edit Distance (Definition)

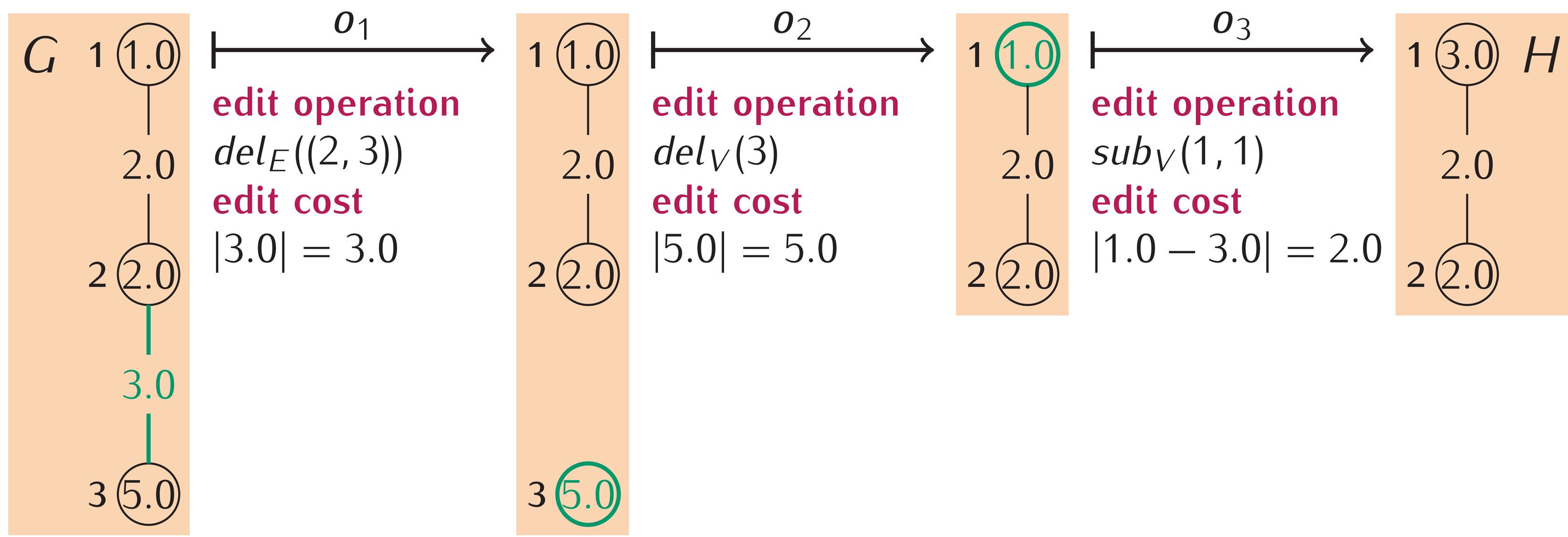
- $\mathrm{GED}(G,H) := \min\{c(P) \mid P \text{ edit path between } G \text{ and } H\}$
- labeled graphs: $G = (V^G, E^G, \ell_V^G, \ell_E^G)$ and $H = (V^H, E^H, \ell_V^H, \ell_E^H)$
- labeling functions: $\ell_V^G : V^G \to \Sigma_V$ and $\ell_E^G : E^G \to \Sigma_E$
- node and edge label spaces: Σ_V and Σ_E
- edit path: edit operation sequence $P = (o_1, \ldots, o_r)$ transforms G into H
- edit operations and edit costs ($i \in V^G$, $k \in V^H$, $(i,j) \in V^G$, $(k,l) \in E^H$):

node edit op.	edit cost	edge edit op.	edit cost
$sub_V(i,k)$	$c_V(\ell_V^G(i), \ell_V^H(k))$	$sub_E((i,j),(k,l))$	$c_E(\ell_E^G(i,j), \ell_E^H(k,l))$
$del_V(i)$	$c_V(\ell_V^G(i), \epsilon)$	$del_E((i,j))$	$c_E(\ell_E^G(i,j), \epsilon)$
$ins_V(k)$	$c_V(\epsilon, \ell_V^H(k))$	$ins_E((k,l))$	$c_E(\epsilon, \ell_E^H(k,l))$

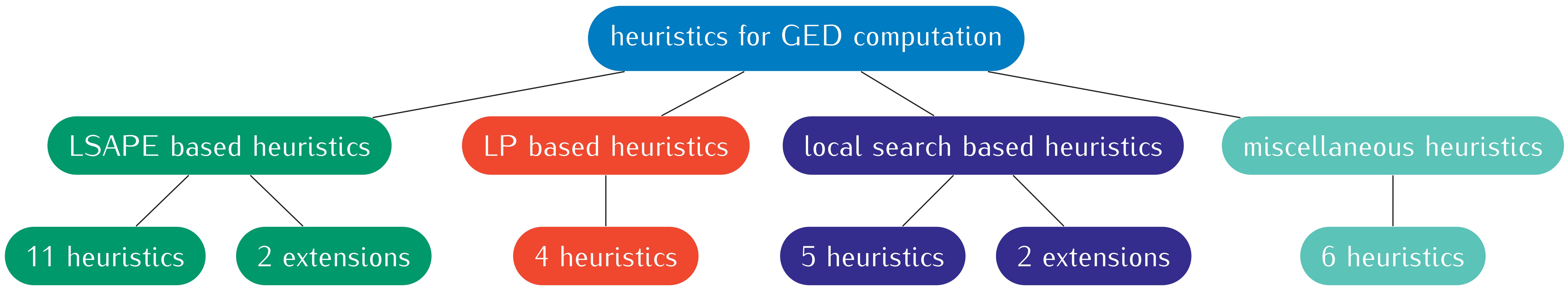
- edit path cost: $c(P) = \sum_{i=1}^{r} c(o_i)$

Graph Edit Distance (Example)

- node and edge label spaces: $\Sigma_V = \Sigma_E = \mathbb{R}_{\geq 0}$
- sub. costs: $c_V(\alpha,\beta) = c_E(\alpha,\beta) = |\alpha - \beta|$
- del. and ins. costs: $c_V(\alpha,\epsilon) = c_V(\epsilon,\alpha) = c_E(\alpha,\epsilon) = c_E(\epsilon,\alpha) = |\alpha|$
- edit path cost: $c(P) = c(o_1) + c(o_2) + c(o_3) = 3.0 + 5.0 + 2.0 = 10.0$



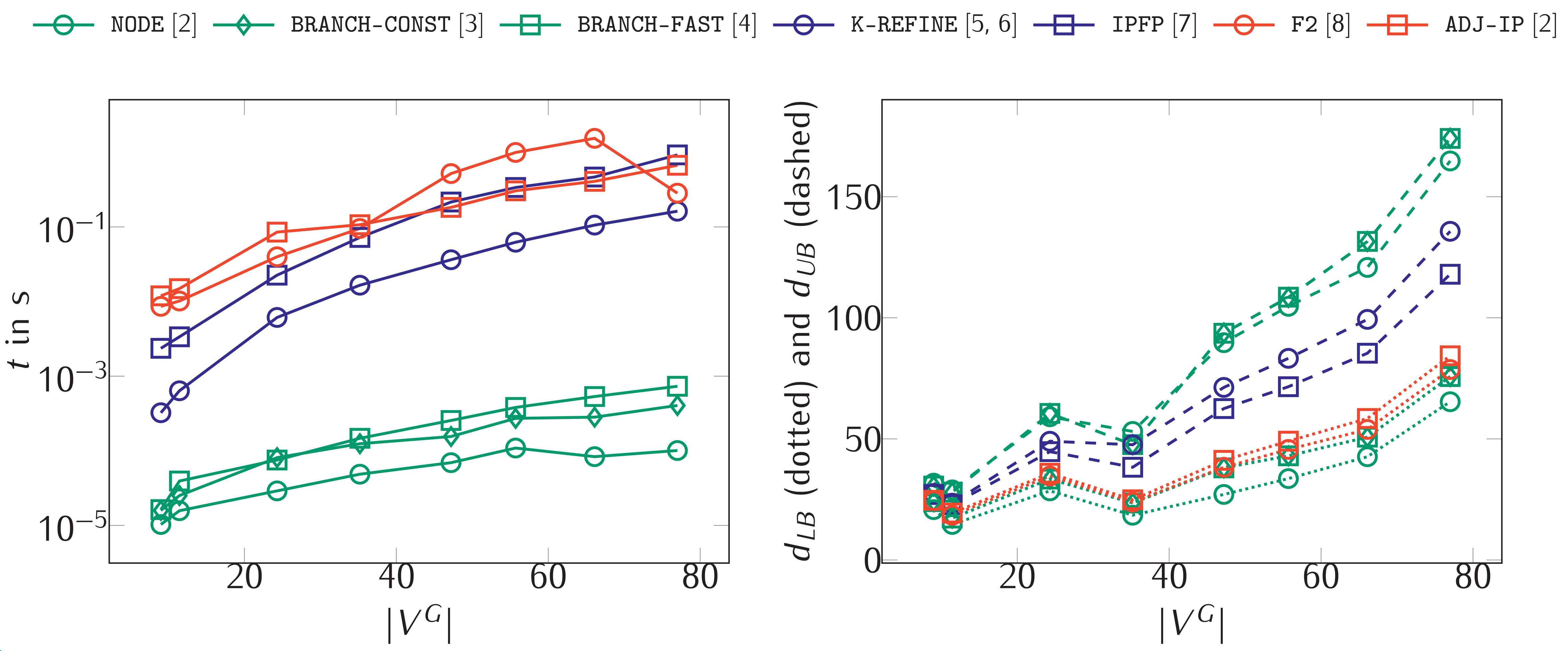
Suggested Taxonomy and Compared Heuristics



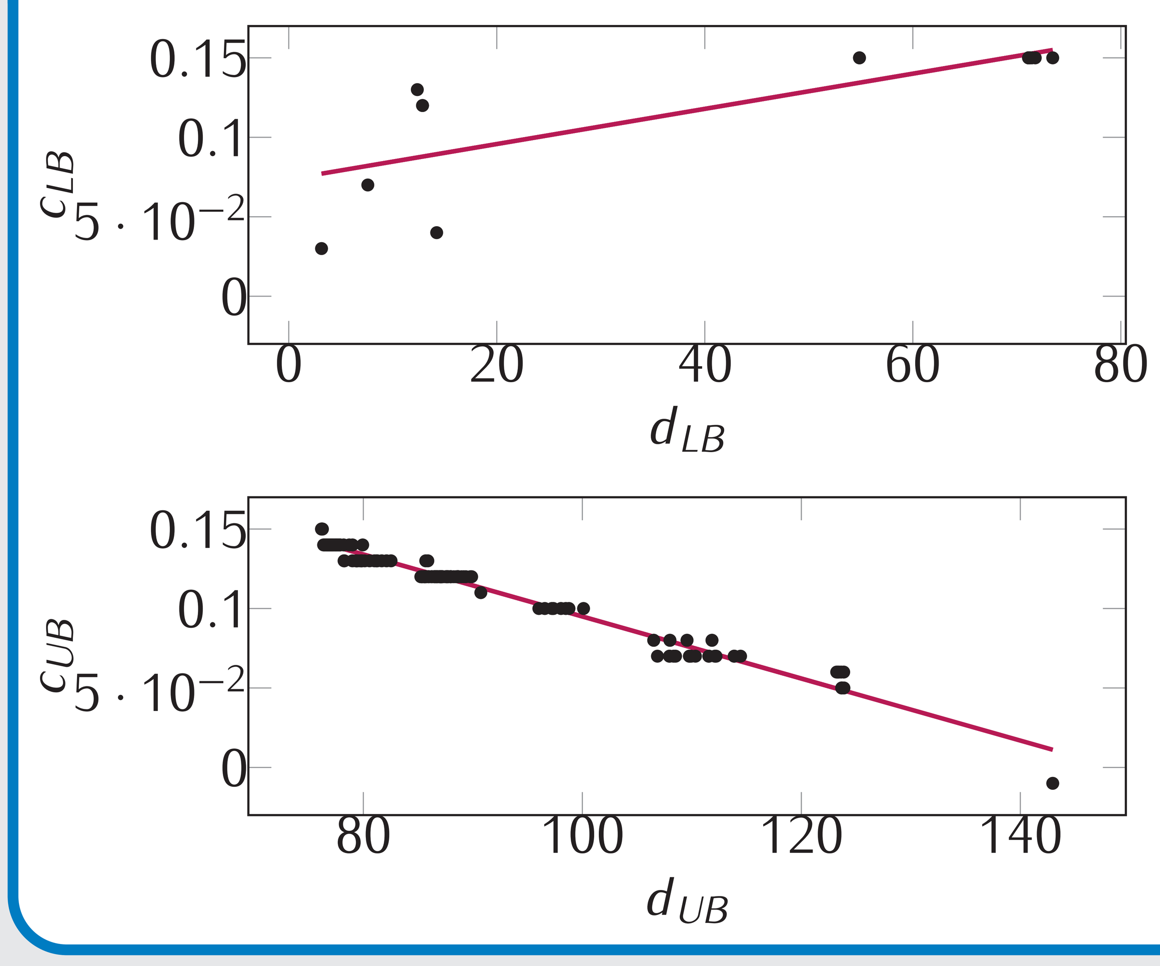
Experimental Setup

- datasets: 6 widely used benchmark datasets from IAM Graph Database Repository [1]
- implementation: to ensure comparability, all heuristics were re-implemented in C++
- metrics: runtime t, lower and upper bounds d_{LB} and d_{UB}, and classification coefficients c_{LB} and c_{UB}

Runtimes and Bounds of Globally Best Heuristics



Bounds vs. Classif. Coeff.



Main Insights

- tightness of lower bound critical: use LP based heuristic F2 [8] or ADJ-IP [2]
- tightness of upper bound critical: use local search based heuristic IPFP [7] or K-REFINE [5, 6]
- runtime critical: use LSAPE based heuristic NODE [2], BRANCH-CONST [3], or BRANCH-FAST [4]
- stability w. r. t. graph size: ordering of heuristics w. r. t. tightness is independent of graph size
- confirmed hypothesis: tight bounds exhibit better classification coefficients than loose bounds
- gap between best lower and upper bounds: only grows moderately with increasing graph size

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