INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Introducing an AR Model

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Mathematical Decription of AR(1) Model

today value = mean + phi * yesterday's value + noise

$$R_t = \mu + \phi \, R_{t-1} + \epsilon_t$$

- Since only one lagged value on right hand side, this is called:
  - AR model of order 1, or
  - AR(1) model
- AR parameter is ϕ =1 (random walk), =0(white noise)
- For stationarity, $-1 < \phi < 1$ (to be stable and stationary)

Interpretation of AR(1) Parameter

R_t is a time series of stock returns

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$

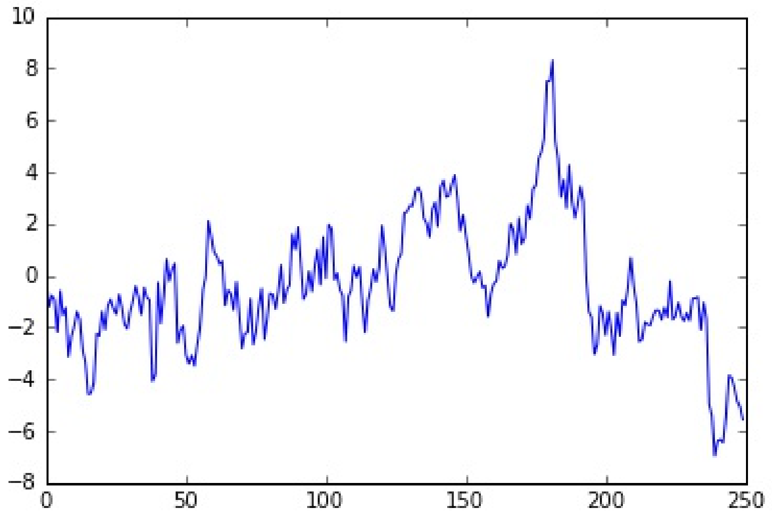
phi is negative, a positive return last period, at time t-1 implies that this period's return is more likely to be negative

- Negative ϕ: Mean Reversion
- Positive ϕ: Momentum

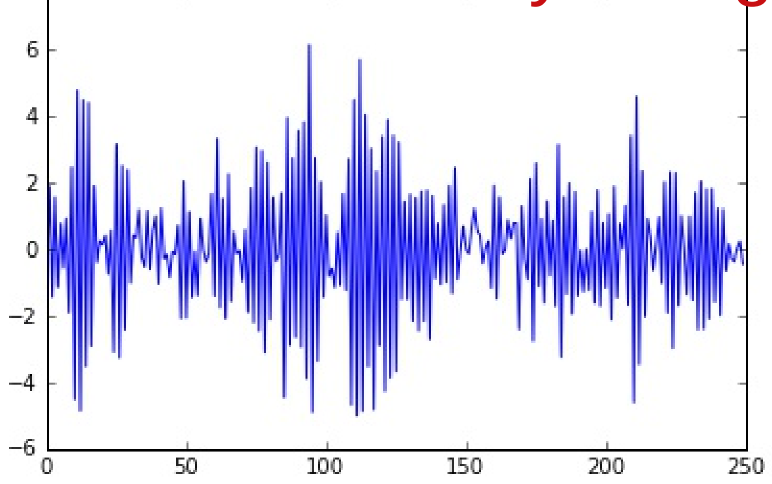
a positive return last period implies that this period's return is expected to be positive

Comparison of AR(1) Time Series

- $\phi = 0.9$ close to random walk

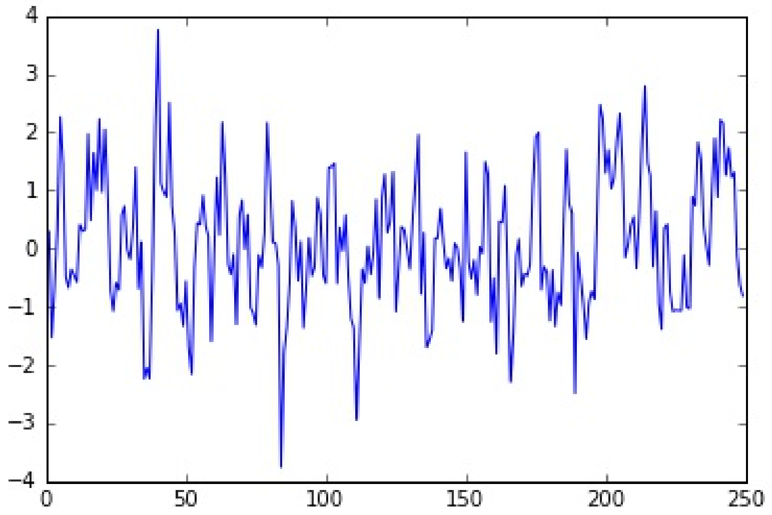


- $\phi = -0.9$ more erratic (a large + value is usually followed by a large - value)

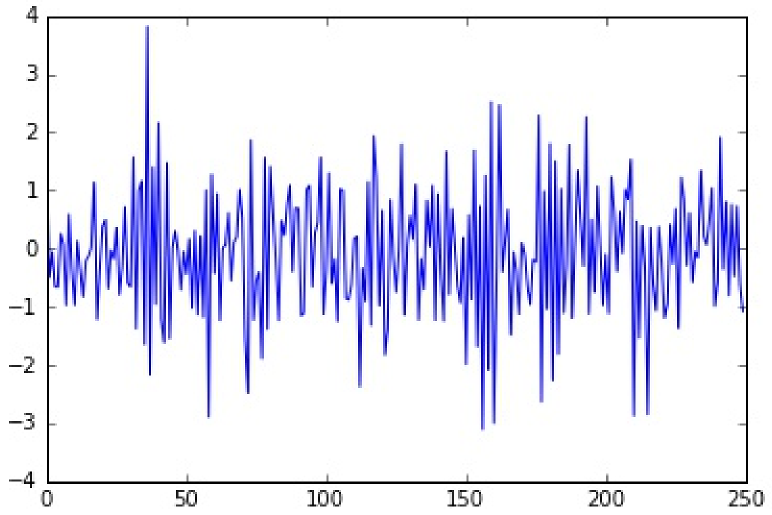


bottom two are similar, but are less exaggerated and closer to white noise

- $\phi = 0.5$



- $\phi = -0.5$

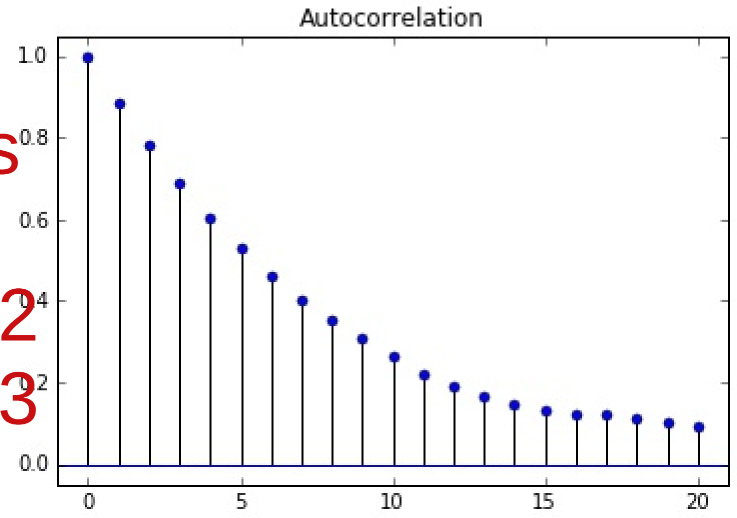


Comparison of AR(1) Autocorrelation Functions

autocorrelation decays exponentially at a rate of phi

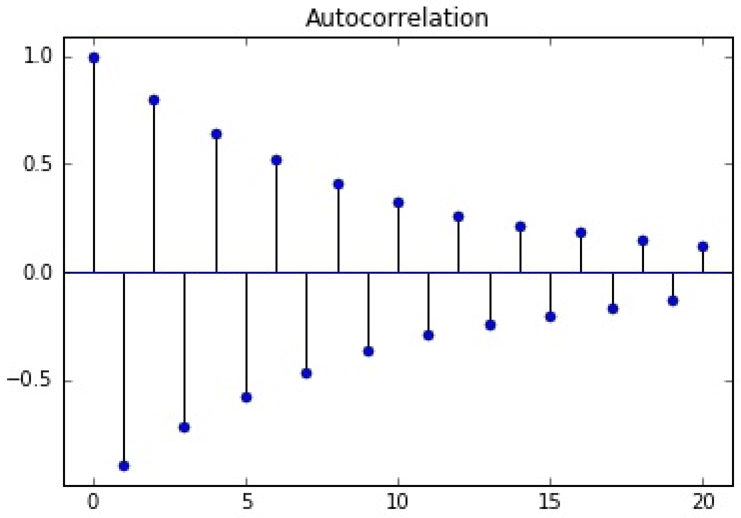
- $\phi = 0.9$

lag-1 auto correlation is 0.9
lag-2 is 0.9^2
lag-3 is 0.9^3

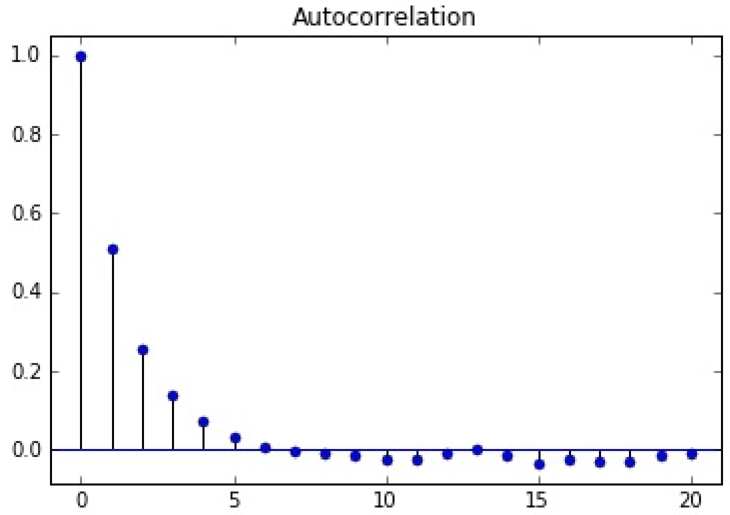


autocorrelation function is still decays exponentialy, but the signs of acf reverse at each lag

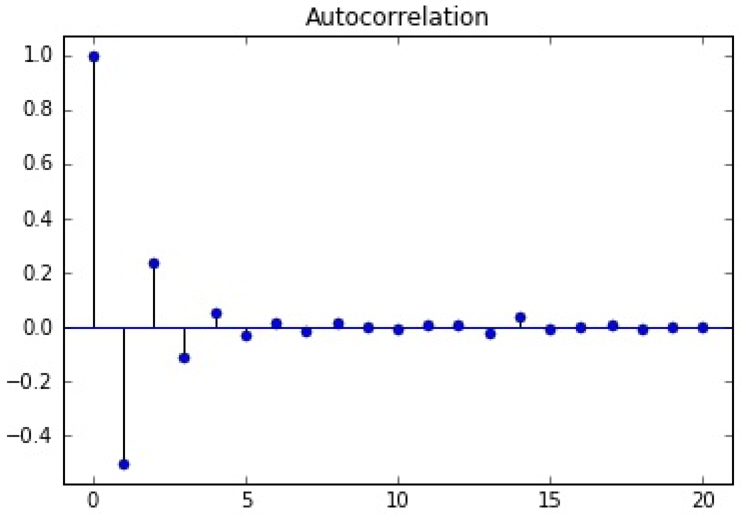
- $\phi = -0.9$



- $\phi = 0.5$



- $\phi = -0.5$



Higher Order AR Models

can be extended to include more lagged values and more phi parameters.

- AR(1)

$$R_t = \mu + \phi_1 R_{t-1} + \epsilon_t$$

- AR(2)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \epsilon_t$$

- AR(3)

$$R_t = \mu + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \phi_3 R_{t-3} + \epsilon_t$$

- ...

Simulating an AR Process

want to study and understand a pure AR process, it is useful to work with simulated data

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1, -0.9])
ma = np.array([1])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

define order and parameters

the convention is a little counterintuitive: must include the zero-lag coeff of 1, and sign of the other coeff is the opposite of what we have been using.

EX: AR(1) with phi=0.9 --> 2nd element of 'ar' array should be -0.9
this is consistent with the time series literature in the field of signal processing

also have to input MA parameters

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Let's practice!

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Estimating and Forecasting an AR Model

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Estimating an AR Model

- To estimate parameters from data (simulated)

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

create an instance of class called mod (data, order of the model)
(1,0) mean --> fit the data to AR(1)
(2,0) --> AR(2)
2nd part of order is MA part

.fit() to estimate model

Estimating an AR Model

- Full output (true $\mu = 0$ and $\phi = 0.9$)

estimated parameters are very close to true parameters

```
print(result.summary())
```

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 5000
Model:                     ARMA(1, 0)   Log Likelihood               -7178.386
Method:                       css-mle   S.D. of innovations              1.017
Date:                Fri, 01 Dec 2017   AIC                          14362.772
Time:                        15:34:50   BIC                          14382.324
Sample:                             0   HQIC                         14369.625

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const         -0.0361      0.152     -0.238      0.812      -0.333       0.261
ar.L1.y        0.9054      0.006    151.020      0.000       0.894       0.917
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1           1.1045           +0.0000j            1.1045            0.0000
-----------------------------------------------------------------------------
```

mean mu

AR(1) parameter

Estimating an AR Model

- Only the estimates of μ and ϕ (true $\mu = 0$ and $\phi = 0.9$)

```
print(result.params)
array([-0.03605989,  0.90535667])
```

see only coeff

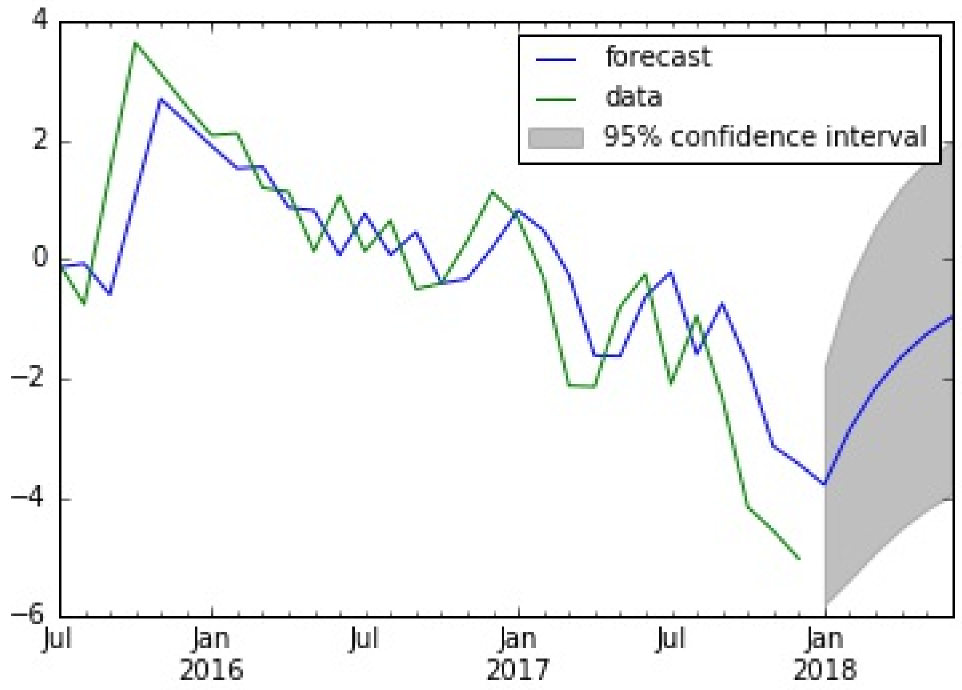
mu

phi

Forecasting an AR Model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```

to do forecast



plot also gives confidence intervals around the out of sample forecasts.

notice how the confidence interval gets wider, the farther out the forecast is

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Choosing the Right Model

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in practice, you will ordinarily not be told the order of the model that you are trying to estimate

Identifying the Order of an AR Model

- The order of an AR(p) model will usually be unknown
- Two techniques to determine order
  - Partial Autocorrelation Function
  - Information criteria

Partial Autocorrelation Funcion (PACF)

measures the incremental benefit of adding another lag.

imagine running several regressions, where u regress returns on more and more lagged values.

$$R_t = \phi_{0,1} + \boxed{\phi_{1,1}} R_{t-1} + \epsilon_{1t}$$

$$R_t = \phi_{0,2} + \phi_{1,2} R_{t-1} + \boxed{\phi_{2,2}} R_{t-2} + \epsilon_{2t}$$

$$R_t = \phi_{0,3} + \phi_{1,3} R_{t-1} + \phi_{2,3} R_{t-2} + \boxed{\phi_{3,3}} R_{t-3} + \epsilon_{3t}$$

$$R_t = \phi_{0,4} + \phi_{1,4} R_{t-1} + \phi_{2,4} R_{t-2} + \phi_{3,4} R_{t-3} + \boxed{\phi_{4,4}} R_{t-4} + \epsilon_{4t}$$

$$\vdots$$

is the lag-4 value of the partial acf, it represent how significant adding a fourth lag is when u already have 3 lags

coefficients in red boxes represent values of the partial autocorrelation function for different lags.

Plot PACF in Python

- Same as ACF, but use plot_pacf instead of plt_acf

- Import module

```
from statsmodels.graphics.tsaplots import plot_pacf
```

- Plot the PACF

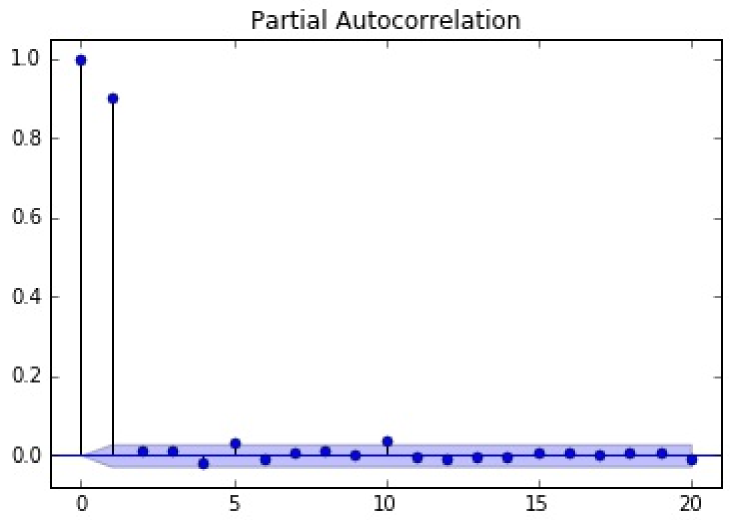
```
plot_pacf(x, lags= 20, alpha=0.05)
```

how many lags of pacf will be plotted
alpha: set the width of the confidence interval

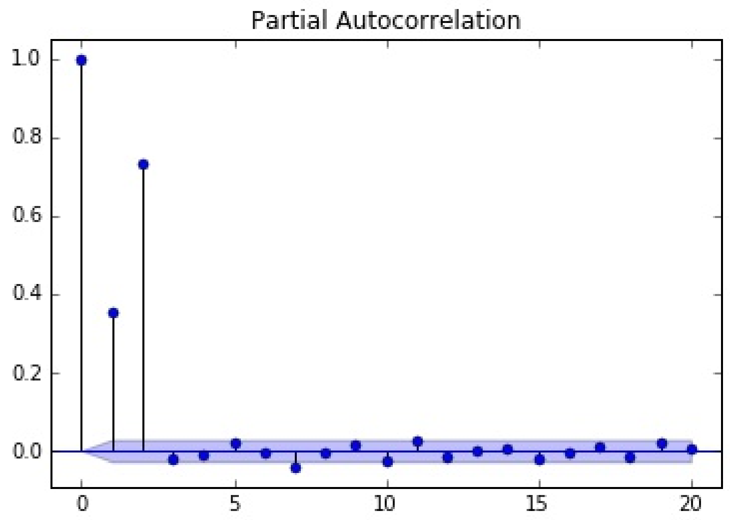
Comparison of PACF for Different AR Models

- AR(1)

only lag-1 pacf is significantly different from zero



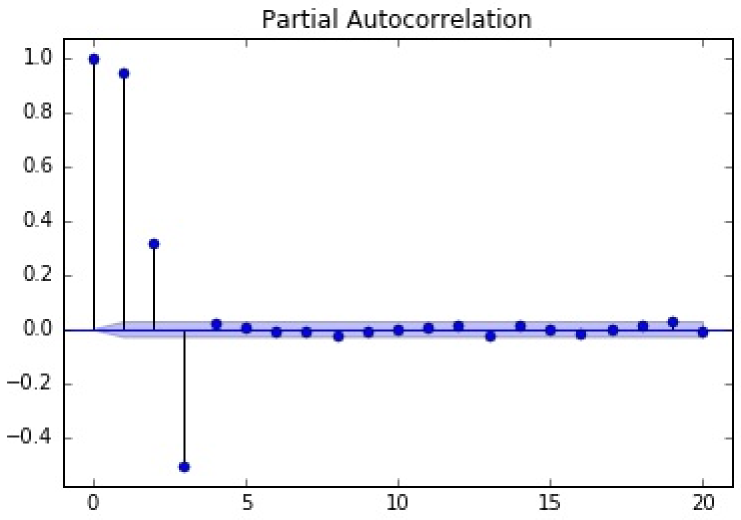
- AR(2)



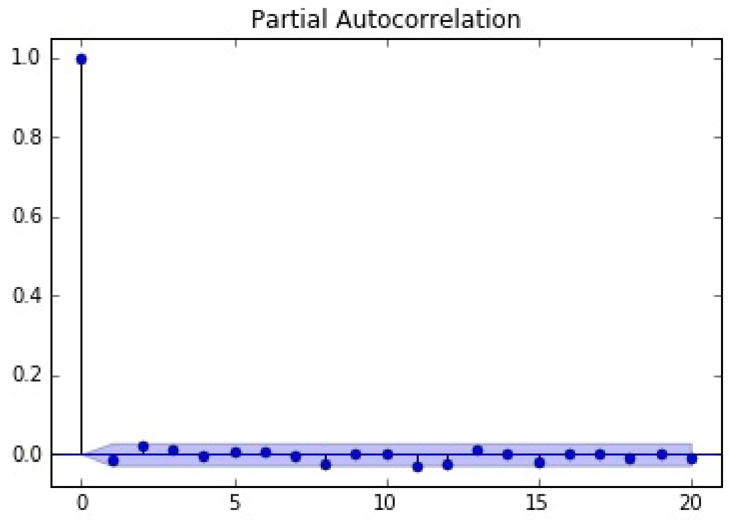
2 lags are different from zero

- AR(3)

3 lags are different from zero



- White Noise



no lags are significantly different from zero

Information Criteria

more parameters in a model, better the model will fit the data.
but this can lead to overfitting of the data

- Information criteria: adjusts goodness-of-fit for number of parameters
- Two popular adjusted goodness-of-fit meaures
  - AIC (Akaike Information Criterion)
  - BIC (Bayesian Information Criterion)

by imposing a penalty based on the no of parameters used.

Information Criteria

- Estimation output

```
                              ARMA Model Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 2500
Model:                     ARMA(2, 0)   Log Likelihood               -3536.481
Method:                       css-mle   S.D. of innovations              0.996
Date:                Fri, 29 Dec 2017   AIC                           7080.963
Time:                        22:53:24   BIC                           7104.259
Sample:                             0   HQIC                          7089.420

==============================================================================
                 coef    std err          z      P>|z|      [95.0% Conf. Int.]
------------------------------------------------------------------------------
const          0.0054      0.010      0.517      0.605        -0.015     0.026
ar.L1.y       -0.6130      0.019    -32.243      0.000        -0.650    -0.576
ar.L2.y       -0.3109      0.019    -16.351      0.000        -0.348    -0.274
                                    Roots
=============================================================================
                 Real           Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1          -0.9859             -1.4982j           1.7935           -0.3426
AR.2          -0.9859             +1.4982j           1.7935            0.3426
-----------------------------------------------------------------------------
```

Getting Information Criteria From statsmodels

- You learned earlier how to fit an AR model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(1,0))
result = mod.fit()
```

- And to get full output

```
result.summary()
```

- Or just the parameters

```
result.params
```

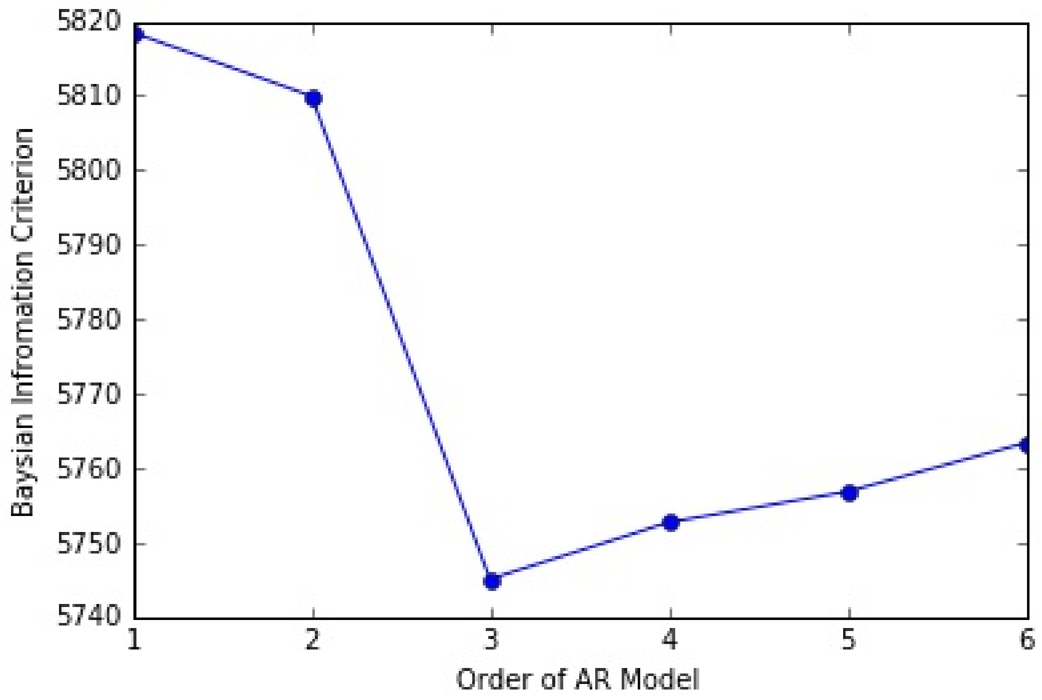
- To get the AIC and BIC

```
result.aic
result.bic
```

Information Criteria

in practice, the way to use the info crieteria is to fit several models, each with a different no of parameters, and choose the one with the lowest BIC

- Fit a simulated AR(3) to different AR(p) models
- Choose p with the lowest BIC



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