INTRODUCTION TO TIME SERIES ANALYSIS IN PYTHON

Describe Model

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Mathematical Decription of MA(1) Model

moving average

noise yesterday's noise

$$R_t = \mu + \epsilon_t 1 + \theta\, \epsilon_{t-1}$$

- Since only one lagged error on right hand side, this is called:
  - MA model of order 1, or
  - MA(1) model
- MA parameter is θ =0, --> white noise
- Stationary for all values of θ

Interpretation of MA(1) Parameter

$$R_t = \mu + \epsilon_t + \theta\, \epsilon_{t-1}$$

theta is -, a + shock last period, represented by epsilon t-1 would have caused last period's return to be +, but this periods return is more likely to be -.

- Negative θ: One-Period Mean Reversion
- Positive θ: One-Period Momentum
- Note: One-period autocorrelation is $\theta/(1+\theta^2)$, not θ

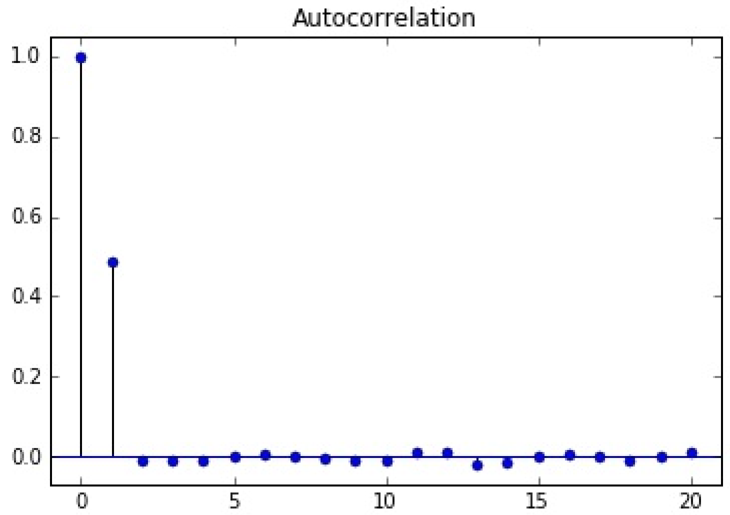
a shock 2 periods ago would have no effect on today's return - only the shock now and last period.

each case, there is zero autocorrelation for MA(1) beyond lag-1

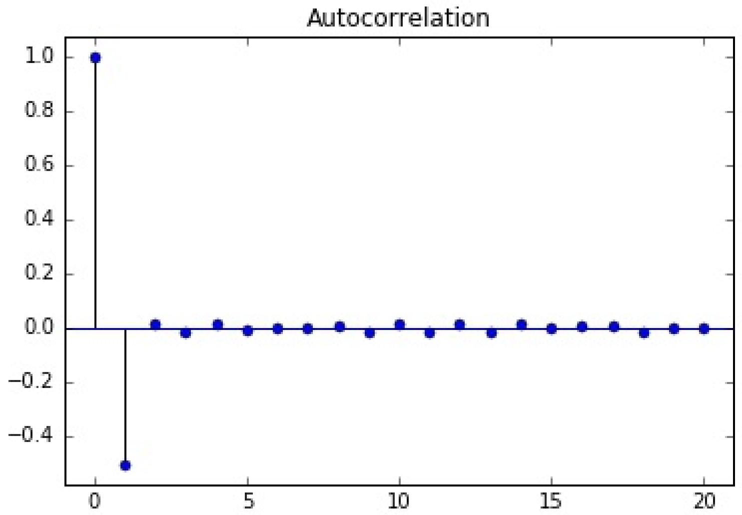
Comparison of MA(1) Autocorrelation Functions

theta > 0 , lag-1 autocorrelation is >0. vice versa

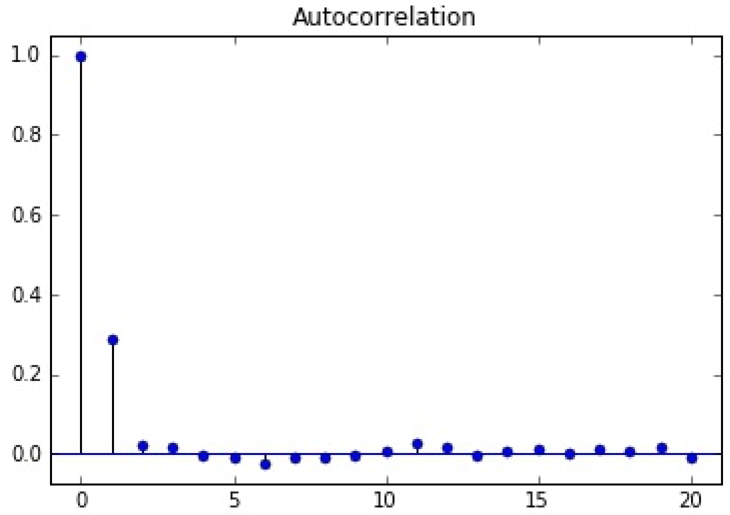
- $\theta = 0.9$



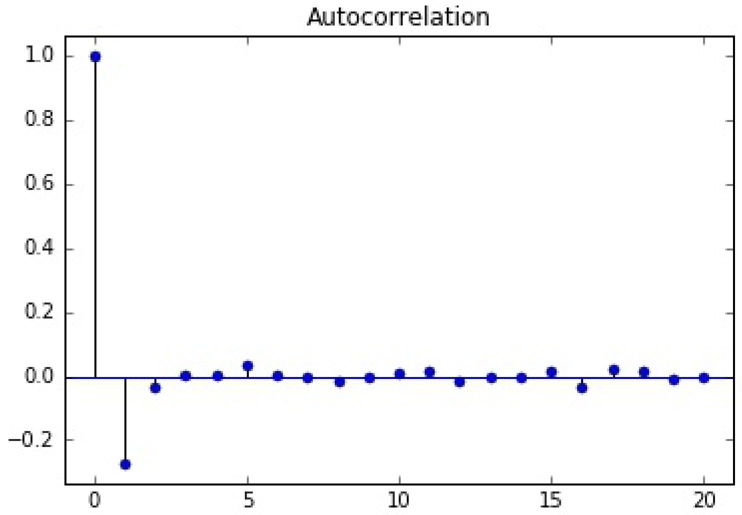
- $\phi = -0.9$



- $\phi = 0.5$

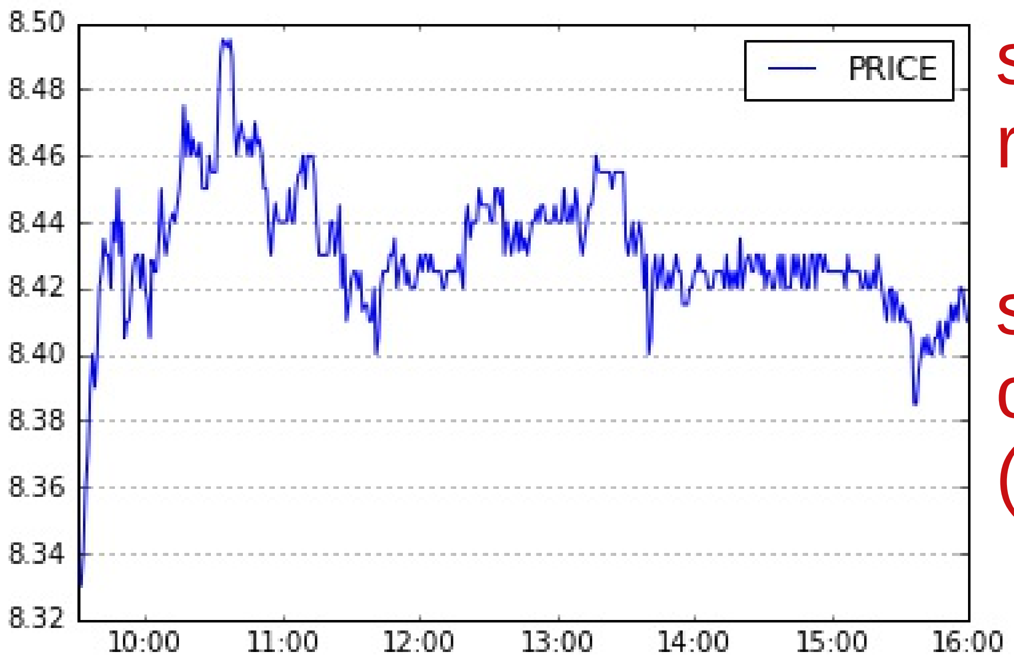


- $\phi = -0.5$



Example of MA(1) Process: Intraday Stock Returns

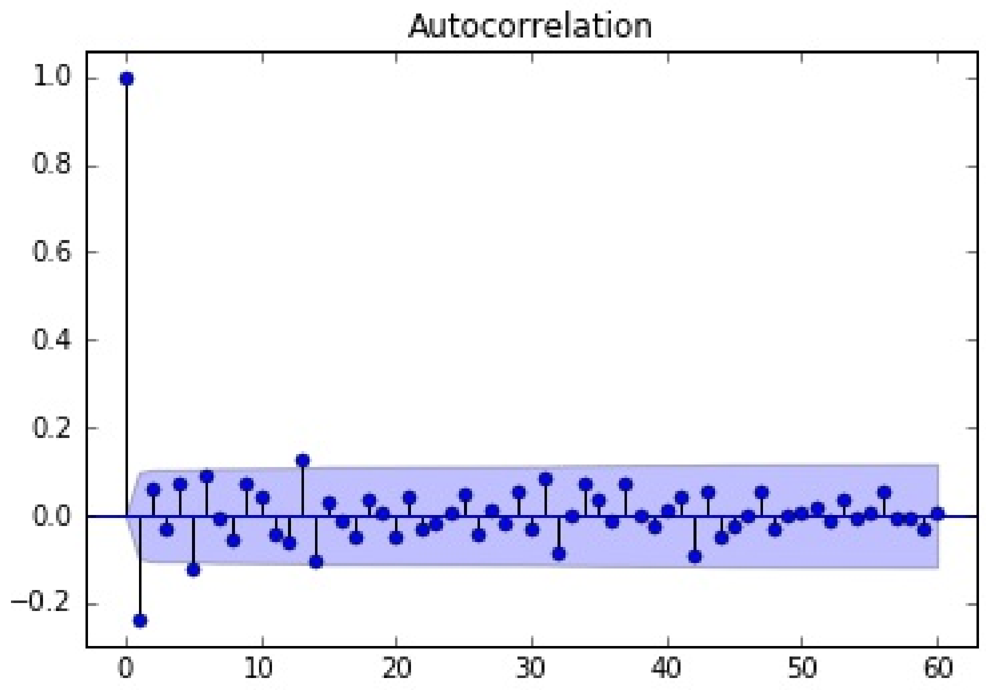
stock price for one day



stocks trade at discrete one-cent increments rather than at continuous prices

stock can bounce back and forth over a one cent range for long periods of time (sometime referred to as "bid/ask bounce"

Autocorrelation Function of Intraday Stock Returns



bid/ask bounce induces a negative lag-1 autocorrelation, but no autocorrelation beyond lag-1

Higher Order MA Models

- MA(1)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1}$$

- MA(2)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1} - \theta_2 \, \epsilon_{t-2}$$

- MA(3)

$$R_t = \mu + \epsilon_t - \theta_1 \, \epsilon_{t-1} - \theta_2 \, \epsilon_{t-2} - \theta_3 \, \epsilon_{t-3}$$

- ...

simulate a pure MA process

Simulating an MA Process

```
from statsmodels.tsa.arima_process import ArmaProcess
ar = np.array([1])
ma = np.array([1, 0.5])
AR_object = ArmaProcess(ar, ma)
simulated_data = AR_object.generate_sample(nsample=1000)
plt.plot(simulated_data)
```

ar order is just an array containing 1
ma order is an array containing 1 and MA(1) parameter theta
No need to reverse sign of theta

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Let's practice!

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Estimation and Forecasting an MA Model

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Estimating an MA Model

same module that u used to estimate the parameters of AR model can be used to estimate the parameters of an MA model

- Same as estimating an AR model (except order=(0,1))

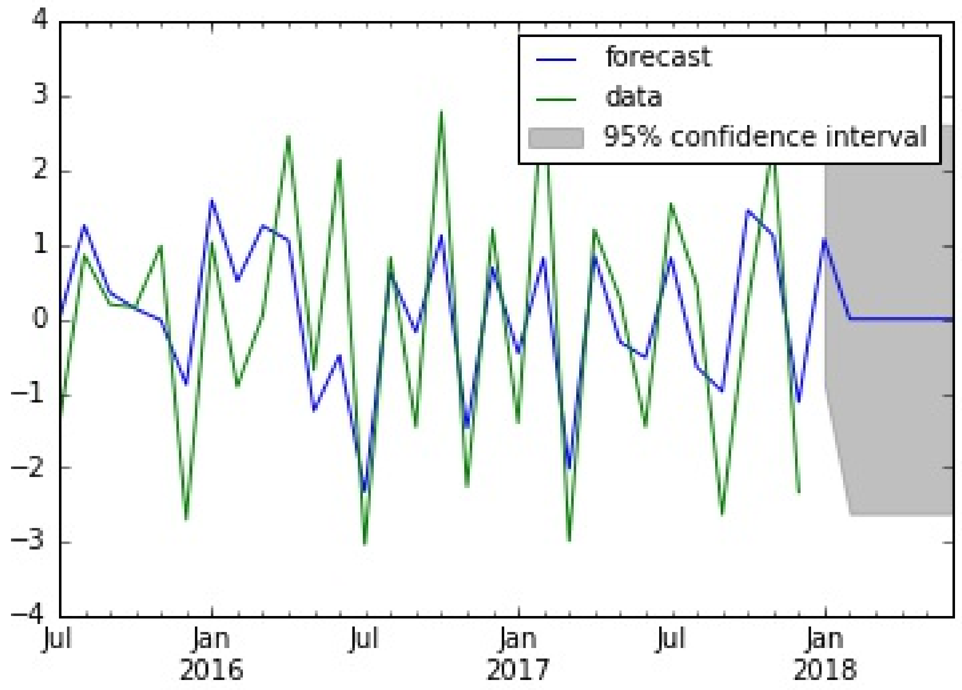
```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(0,1))
result = mod.fit()
```

order is (0,1) for MA(1)
is (1,0) for AR(1)

Forecasting an MA Model

is the same as for an AR model

```
from statsmodels.tsa.arima_model import ARMA
mod = ARMA(simulated_data, order=(0,1))
res = mod.fit()
res.plot_predict(start='2016-07-01', end='2017-06-01')
plt.show()
```



note that with an MA(1) model, unlike an AR model, all forecasts beyond the one-step ahead forecast will be the same.

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ARMA models

an ARMA model is combination of an AR and MA model

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ARMA Model

- ARMA(1,1) model:

$$R_t = \mu + \phi\, R_{t-1} + \epsilon_t + \theta\, \epsilon_{t-1}$$

Converting Between ARMA, AR, and MA Models

- Converting AR(1) into an MA(infinity)

ARMA models can be converted to pure AR or pure MA models

$$R_t = \mu + \phi R_{t-1} + \epsilon_t$$ AR(1) model

$$R_t = \mu + \phi(\mu + R_{t-2} + \epsilon_{t-1}) + \epsilon_t$$ R_t-1 replace by AR(1)

$$R_t = \mu + \phi(\mu + \phi(\mu + R_{t-3} + \epsilon_{t-2}) + \epsilon_{t-1}) + \epsilon_t$$

$$\vdots$$

$$R_t = \mu + \epsilon_t + \phi\,\epsilon_{t-1} + \phi^2\,\epsilon_{t-2} + \phi^3\,\epsilon_{t-3} + \phi^4\,\epsilon_{t-4} + \cdots$$

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