An explicit approach for time-optimal trajectory planning for kinematically redundant serial robots

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Kinematically redundant serial robots have become industrially important due their increased workspace and their inherent capability of null space motion resulting in remarkable adaptiveness to specific tasks compared to conventional, non-redundant manipulators. Attempting to increase the cost-effectiveness of industrial processes, introducing minimum-time trajectories may yield economical advantages due to reduced motion cycle times. This contribution presents a method that uses joint space decomposition and analytic inverse kinematics as well as standard optimization techniques to obtain minimum-time B-spline joint trajectories along prescribed task space paths for kinematically redundant serial robots. It is shown that the present method was successfully applied to a planar manipulator.

1 Introduction

Kinematic redundancy refers to a manipulator's kinematic property of featuring more joints $\mathbf{q} \in \mathcal{Q} \subseteq \mathbf{R}^n$ than necessary to assume any task space configuration $\mathbf{z}_\mathrm{E}^\top = \begin{pmatrix} \mathbf{r}_\mathrm{E}^\top & \boldsymbol{\varphi}_\mathrm{E}^\top \end{pmatrix} \in \mathcal{T} \subseteq \mathbf{R}^m$, i.e. $m < n$. In many cases the task of finding minimum-time trajectories is formulated as an optimal control problem of the joint inputs incorporating constraints for tracking a prescribed path. Other approaches apply inverse kinematics methods to the prescribed geometric path and solve an optimal control problem for the evolution along the path eliminating the requirement for an additional path-tracking constraint. While for kinematically non-redundant serial robots often straight-forward analytical solutions for the inverse kinematics problem can be found, for redundant manipulators conventionally Jacobian-based numerical methods are employed. These approaches, however, are often missing appropriate secondary objectives for exploiting the redundancy in order to achieve an optimal solution. The main contribution of this paper is a method to obtain minimum-time joint trajectories along prescribed end-effector paths that employs joint space decomposition enabling analytical inverse kinematics. Describing the resulting trajectories as B-spline curves yields a problem that can be solved using standard optimization techniques. Using the simulation example of a planar manipulator the relevance of the presented method will be shown.

2 Inverse kinematics

While the inverse kinematics problem can often be solved analytically for kinematically non-redundant serial manipulators, this is not possible for redundant robots without further considerations. Most methods are Jacobian-based numeric approaches that generate solutions of the inverse kinematics problem on velocity or acceleration level. These methods require time integration to obtain position information and are thus prone to numerical drift. In order to exploit a manipulator's kinematic redundancy, internal motion in the Jacobian null space [2] is added according to certain performance criteria such as kinematic [3] or dynamic [4] manipulability. In [1] a method for obtaining minimum-time trajectories along predefined, parameterized task space paths of kinematically redundant serial robots is presented. The joint space is decomposed by separating m arbitrarily chosen joint coordinates yielding a *non-redundant* set of m joints and a *redundant* set of $(n-m)$ coordinates. The result of a subsequent optimal control problem for the path parameter and for the time evolution of the *redundant* joints are time-optimal trajectories. The resulting trajectories are only $\mathcal{C}^1$ continuous, which limits the method's range of possible applications. The present contribution adapts the joint space decomposition approach from [1]. A closed-form solution for the non-redundant joint coordinates $\mathbf{q}_\mathrm{nr}$ can be computed from the current position of the redundant joints $\mathbf{q}_\mathrm{r}$ and the geometric end-effector position along the prescribed path $\mathbf{z}_\mathrm{E}(s)$ where s denotes a scalar path coordinate and $s(t) \in [0,1]$, $s(0) = 0$, $s(t_\mathrm{E}) = 1$, i.e. $\mathbf{q}_\mathrm{nr} = \mathbf{q}_\mathrm{nr}(\mathbf{z}_\mathrm{E}(s), \mathbf{q}_\mathrm{r})$.

3 Trajectory description using B-spline curves

For the inverse kinematics approach from the previous section, $(1+r)$ trajectories are required to describe the robot joints' time evolution during the prescribed task. Multi-interval uniform B-spline curves of degree d for the path parameter $s = s(t)$ and the *redundant* joints $\mathbf{q}_\mathrm{r} = \mathbf{q}_\mathrm{r}(t)$ are assumed whose parameters are the control points $\mathbf{c}_\mathrm{s}$ and $\mathbf{c}_\mathrm{r}$ and the common trajectory end time t_E. The degree of the curves are chosen such that the required degree of continuous differentiability is met.

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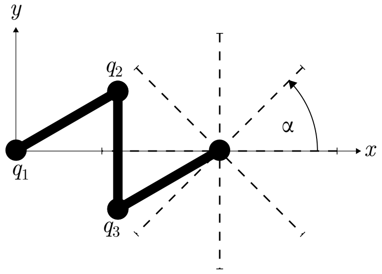

Fig. 1: Three-link SCARA with straight line paths

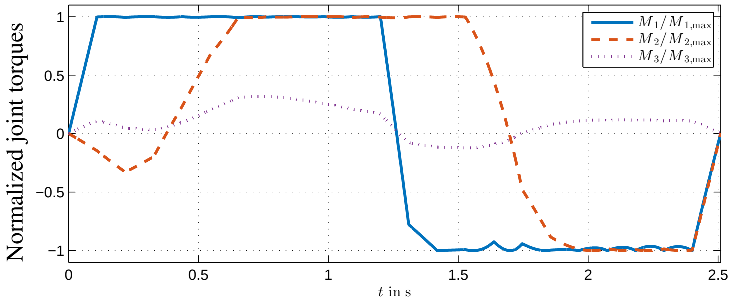

Fig. 2: Resulting normalized joint torques, end-time $t_{\mathrm{E}} = 2.51$ s

4 Optimization problem

The above mentioned B-spline trajectories are the result of the optimization problem $\mathbf{x}^* = \arg\min_{\mathbf{x}} f(\mathbf{x})$ where $f(\mathbf{x}) = t_{\mathrm{E}}$ and $\mathbf{x}^\top = \begin{pmatrix} t_{\mathrm{E}} & \mathbf{c}_s^\top & \mathbf{c}_{\mathrm{r}}^\top \end{pmatrix}$ and are subjected to the dynamics of the robot, expressed with its equations of motion $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{g}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{Q}$, as well as to physical and technological constraints such as limitations in the joint velocities $\dot{\mathbf{q}}$, the joint accelerations $\ddot{\mathbf{q}}$, or the motor torques $\mathbf{Q}$. The path tracking requirement is implicitly fulfilled by employing above mentioned path parameterization and inverse kinematics. The initial guess for $\mathbf{x}$ can be obtained by assuming a conservative value for t_{E} and a generic time evolution for s (e.g. linear from 0 to 1) and applying a standard numerical inverse kinematics procedure yielding a geometrically valid solution for $\mathbf{q}_{\mathrm{r}}(t)$.

5 Application to planar manipulator

The presented approach was applied to the planar three-link SCARA robot with link lengths of $l_i = 1$ m and link masses of $m_i = 10$ kg depicted in Fig. 1. Kinematic redundancy is introduced by only considering the Cartesian position of the end-effector $\mathbf{r}_{\mathrm{E}}$ in the plane but not its orientation as task space coordinates, i.e. $\mathbf{z}_{\mathrm{E}}^\top = \mathbf{r}_{\mathrm{E}}^\top = \begin{pmatrix} x & y \end{pmatrix} \in \mathcal{T} \subset \mathbf{R}^2$. The joint space decomposition is performed such that $\dim \mathbf{q}_{\mathrm{nr}} = 2$ and $\dim \mathbf{q}_{\mathrm{r}} = 1$ for all possible decompositions. In this example the joint torques are bounded at $M_{i,\max} = \pm 10$ Nm and are required to be continuous, requiring $d \geq 3$. The end-effector paths starting from the depicted initial configuration $\mathbf{q}_{\mathrm{init}}^\top = \begin{pmatrix} \frac{1}{6}\pi & -\frac{2}{3}\pi & \frac{2}{3}\pi \end{pmatrix}$ are straight lines of length 1 m at angle α.

6 Results

The results for $\alpha = k\frac{\pi}{4}$, $k = 0, 1, \ldots, 7$ and all choices for q_{r} were obtained using the active-set optimization method of MATLAB's function `fmincon`. Fig. 2 depicts the normalized joint torques of the resulting trajectory for $\alpha = \frac{\pi}{2}$ and $q_{\mathrm{r}} = q_3$. It shows min-max behavior indicating time-optimality.

7 Conclusion

In this contribution it was shown that joint space decomposition allows for an efficient solution of the closed-form inverse kinematics of kinematically redundant serial robots. The redundancy property of such a manipulator is exploited by using separate trajectories for the redundant and non-redundant joints. The resulting trajectories are available on position level, their degree of continuous differentiability depends on the selection of the degree of the B-spline curves. The current method can be further improved by using trajectory splines with non-uniformly distributed knots which are additional optimization variables that allow for improved time resolution where fast dynamic changes occur at the cost of a higher complexity of the optimization problem. In future work, the method presented in this contribution will be applied to a STÄUBLI TX90L 6 DOF industrial robot mounted on a 1 DOF linear axis.

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