



Robotics 2

Adaptive Trajectory Control

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Motivation and approach

- need of adaptation in robot motion control laws
 - large uncertainty on the robot dynamic parameters
 - poor knowledge of the inertial payload
- characteristics of **direct** adaptive control
 - direct aim is to bring to zero the state trajectory error, i.e., position and velocity errors
 - no need to estimate on line the true values of the dynamic coefficients of the robot (as opposed to **indirect** adaptive control)
- main tool and methodology
 - **linear parametrization** of robot dynamics
 - **nonlinear** control law of the **dynamic** type (the controller has its own 'states')



Summary of robot parameters

- parameters assumed to be **known**
 - kinematic description based, e.g., on Denavit-Hartenberg parameters ($\{\alpha_i, d_i, a_i, i = 1, \dots, N\}$ in case of all revolute joints), including link lengths (**kinematic calibration**)
- **uncertain** parameters that can be **identified** off line
 - masses m_i , positions r_{ci} of CoMs, and inertia matrices I_i of each link, appearing in combinations (**dynamic coefficients**) $\Rightarrow p \ll 10 \times N$
- parameters that are **(slowly) varying** during operation
 - viscous F_{Vi} , dry F_{Si} , and stiction F_{Di} friction at each joint $\Rightarrow 1 \div 3 \times N$
- **unknown** and abruptly changing parameters
 - mass, CoM, inertia matrix of the payload w.r.t. the tool center point



when a payload is firmly **attached** to the robot E-E, only the 10 parameters of the last link are modified, influencing however most part of the robot dynamics



Goal of adaptive control

- given a twice-differentiable desired joint trajectory $q_d(t)$
 - with known desired velocity $\dot{q}_d(t)$ and acceleration $\ddot{q}_d(t)$
 - possibly obtained by kinematic inversion + joint interpolation
- execute this trajectory under large dynamic uncertainties
 - with a trajectory tracking error vanishing **asymptotically**

$$e = q_d - q \longrightarrow 0 \quad \dot{e} = \dot{q}_d - \dot{q} \longrightarrow 0$$

- guaranteeing **global stability**, no matter how far are the initial estimates of the unknown/uncertain parameters from their true values and how large is the initial trajectory error
- identification is **not** of particular concern: in general, the estimates of dynamic coefficients will not converge to the true ones!
- if this convergence is a specific extra requirement, then one should use (more complex) **indirect adaptive** schemes



Linear parameterization

$$M(q)\ddot{q} + S(q, \dot{q})\dot{q} + g(q) + F_V\dot{q} = u$$

- there exists always a (p -dimensional) **vector a** of **dynamic coefficients**, so that the robot model takes the **linear** form

$$Y(q, \dot{q}, \ddot{q}) a = u$$

- vector **a** contains only unknown or uncertain coefficients
- each component of **a** is in general a **combination** of the robot physical parameters (not necessarily all of them)
- the model **regression matrix Y** depends linearly on \ddot{q} , quadratically on \dot{q} (for the terms related to kinetic energy), and nonlinearly (trigonometrically) on q



Trajectory controllers based on model estimates

- inverse dynamics feedforward (**FFW**) + PD (**linear**) control

$$u = \underbrace{\hat{M}(q_d)\ddot{q}_d + \hat{S}(q_d, \dot{q}_d)\dot{q}_d + \hat{g}(q_d) + \hat{F}_V\dot{q}_d}_{\hat{u}_d} + K_P e + K_D \dot{e}$$

- (**nonlinear**) control based on feedback linearization (**FBL**)

$$u = \hat{M}(q)(\ddot{q}_d + K_P e + K_D \dot{e}) + \hat{S}(q, \dot{q})\dot{q} + \hat{g}(q) + \hat{F}_V\dot{q}$$

$$\boxed{\hat{M}, \hat{S}, \hat{g}, \hat{F}_V \quad \Longleftrightarrow \quad \text{estimate } \hat{a}}$$

- approximate estimates of dynamic coefficients may lead to **instability** with **FBL** due to temporary 'non-positive' PD gains (e.g., $\hat{M}(q)K_P < 0$!)
- **not easy** to turn these laws in **adaptive** schemes: inertia inversion/use of acceleration (FBL); bounds on PD gains (FFW)



A control law easily made 'adaptive'

- nonlinear trajectory tracking control (without cancellations) having global asymptotic stabilization properties

$$u = \hat{M}(q)\ddot{q}_d + \hat{S}(q, \dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_V\dot{q}_d + K_P e + K_D \dot{e}$$

- a natural **adaptive version** would require ...

$$\dot{\hat{a}} = \text{designing a suitable **update law** (in continuous time)}$$

- without extra assumptions, it can be shown only that joint velocities become eventually "clamped" to those of the **desired** trajectory (zero **velocity** error), but a permanent residual **position** error is left

- idea: **on-line modification** with a **reference velocity**

$$\dot{q}_d \rightarrow \boxed{\dot{q}_r = \dot{q}_d + \Lambda(q_d - q)} \quad \Lambda > 0$$

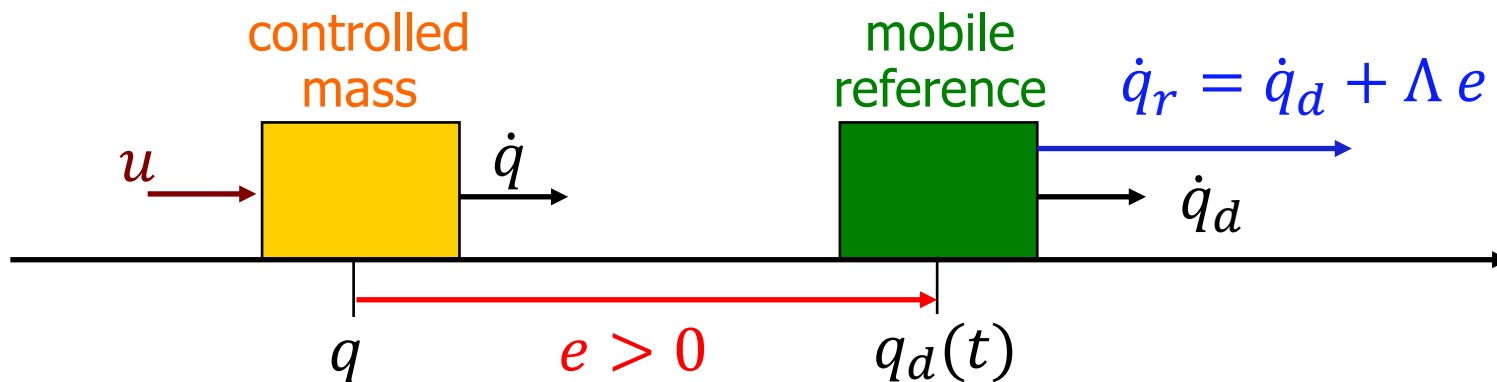
typically $\Lambda = K_D^{-1}K_P$ (all matrices will be chosen **diagonal**)



Intuitive interpretation of \dot{q}_r

- elementary case

- a mass 'lagging behind' its mobile reference ($e > 0$) on a linear rail



➡ 'enhanced' velocity error $s = \dot{q}_r - \dot{q} > \dot{q}_d - \dot{q} = \dot{e}$

$$u = K_D s = K_D (\dot{q}_r - \dot{q}) = K_D (\dot{q}_d + \Lambda e - \dot{q}) = K_D \dot{e} + \underbrace{K_D \Lambda}_{K_P} e$$

- a mass 'leading in front' of its mobile reference ($e < 0$)

➡ in a symmetric way, a 'reduced' velocity error will appear ($s < \dot{e}$)



Adaptive control law design

- substituting $\dot{q}_r = \dot{q}_d + \Lambda e$, $\ddot{q}_r = \ddot{q}_d + \Lambda \dot{e}$ in the previous nonlinear controller for trajectory tracking

$$\begin{aligned} u &= \hat{M}(q)\ddot{q}_r + \hat{S}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_V\dot{q}_r + K_P e + K_D \dot{e} \\ &= Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{a} + K_P e + K_D \dot{e} \end{aligned}$$

dynamic parameterization of
the control law using current estimates (diagonal matrices, >0)
(note here the 4 arguments in $Y(\cdot)$!)

PD stabilization

- update law for the estimates of the dynamic coefficients (\hat{a} becomes the p -dimensional state of the dynamic controller)

$$\dot{\hat{a}} = \Gamma Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\dot{q}_r - \dot{q})$$

$\Gamma > 0$ (diagonal)

estimation gains
(variation rate of estimates)

'modified' velocity error



Asymptotic stability of trajectory error

Theorem

The introduced adaptive controller makes the **tracking error** along the desired trajectory **globally asymptotically stable**

$$e = q_d - q \rightarrow 0, \dot{e} = \dot{q}_d - \dot{q} \rightarrow 0$$

Proof

- a **Lyapunov candidate** for the closed-loop system (robot + dynamic controller) is given by

$$V = \frac{1}{2} s^T M(q) s + \frac{1}{2} e^T R e + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \geq 0$$

$$s = \dot{q}_r - \dot{q} (= \dot{e} + \Lambda e)$$

modified velocity error

$$R > 0$$

constant matrix
(to be specified later)

$$\tilde{a} = a - \hat{a}$$

error in parametric
estimation

$$V = 0 \iff \hat{a} = a, \quad q = q_d, \quad s = 0 \quad (\Rightarrow \dot{q} = \dot{q}_d)$$



Proof (cont)

- the **time derivative** of V is

$$\dot{V} = \frac{1}{2} s^T \dot{M}(q) s + s^T M(q) \dot{s} + e^T R \dot{e} - \tilde{a}^T \Gamma^{-1} \dot{\hat{a}}$$

since $\dot{\tilde{a}} = -\dot{\hat{a}}$ ($\dot{a} = 0$)

- the **closed-loop** dynamics is given by

$$\begin{aligned} M(q)\ddot{q} + S(q, \dot{q})\dot{q} + g(q) + F_V\dot{q} &= \\ &= \hat{M}(q)\ddot{q}_r + \hat{S}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_V\dot{q}_r + K_P e + K_D \dot{e} \end{aligned}$$

subtracting the two sides **from** $M(q)\ddot{q}_r + S(q, \dot{q})\dot{q}_r + g(q) + F_V\dot{q}_r$ leads to

$$\begin{aligned} M(q)\dot{s} + (S(q, \dot{q}) + F_V)s &= \\ &= \tilde{M}(q)\ddot{q}_r + \tilde{S}(q, \dot{q})\dot{q}_r + \tilde{g}(q) + \tilde{F}_V\dot{q}_r - K_P e - K_D \dot{e} \end{aligned}$$

with $\tilde{M} = M - \hat{M}$, $\tilde{S} = S - \hat{S}$, $\tilde{g} = g - \hat{g}$, $\tilde{F}_V = F_V - \hat{F}_V$



Proof (cont)

- from the property of **linearity in the dynamic coefficients**, it follows

$$\boxed{M(q)\dot{s}} + (S(q, \dot{q}) + F_V)s = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\tilde{a} - K_P e - K_D \dot{e}$$

- substituting in \dot{V}** , together with $\hat{a} = \Gamma Y^T s$, and using the skew-symmetry of matrix $\dot{M} - 2S$ we obtain

$$\begin{aligned}\dot{V} &= \frac{1}{2} s^T [\dot{M}(q) - 2S(q, \dot{q})] s - s^T F_V s + s^T Y \tilde{a} \\ &\quad - s^T (K_P e + K_D \dot{e}) + e^T R \dot{e} - \tilde{a}^T Y^T s \\ &= -s^T F_V s - s^T (K_P e + K_D \dot{e}) + e^T R \dot{e}\end{aligned}$$

- replacing** $s = \dot{e} + \Lambda e$ and being $F_V = F_V^T$ (diagonal)

$$\dot{V} = -e^T (\Lambda^T F_V \Lambda + \Lambda^T K_P) e$$

a complete
quadratic form
in e, \dot{e} !

$$\rightarrow -e^T (2\Lambda^T F_V + \Lambda^T K_D + K_P - \boxed{R}) \dot{e} - \dot{e}^T (F_V + K_D) \dot{e}$$



Proof (end)

- defining now (all matrices are **diagonal!**)

$$\Lambda = K_D^{-1} K_P > 0 \quad \textcircled{R} = 2K_P (I + K_D^{-1} F_V) > 0$$

leads to

$$\begin{aligned} \dot{V} &= -e^T \Lambda^T (F_V + K_D) \Lambda e - \dot{e}^T (F_V + K_D) \dot{e} \\ &= -e^T K_P K_D^{-1} (F_V + K_D) K_D^{-1} K_P e - \dot{e}^T (F_V + K_D) \dot{e} \leq 0 \end{aligned}$$

and thus

$$\dot{V} = 0 \iff e = \dot{e} = 0$$

the thesis follows from Barbalat lemma + LaSalle theorem



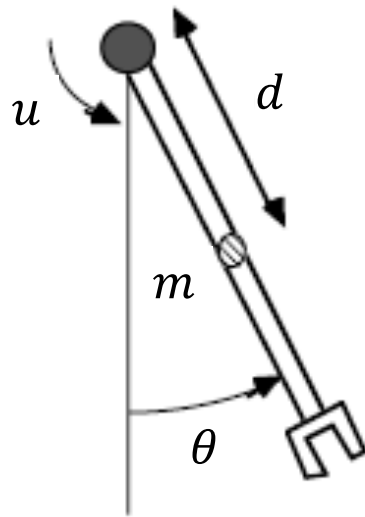
the set of states of convergence has **zero trajectory error** and **a constant value** for \hat{a} , not necessarily the true one ($\tilde{a} \neq 0$)



Remarks

- if the desired trajectory $q_d(t)$ is **persistently exciting**, then also the estimates of the dynamic coefficients converge to their true values
- **condition** of persistent excitation
 - for **linear** systems: # of frequency components in the desired trajectory should be at least **twice as large** as # of unknown coefficients
 - for **nonlinear** systems: the condition can be checked only **a posteriori** (a certain motion integral should be permanently lower bounded)
- in case of known absence of (viscous) friction ($F_V \equiv 0$), the same proof applies (a bit easier in the final part)
- the adaptive controller **does not require** the inverse of the inertia matrix (true or estimated), nor the actual robot acceleration (only the desired acceleration), nor further lower bounds on $K_P > 0, K_D > 0$
- adaptation can be also used **only for a subset** of dynamic coefficients, the remaining being known ($Y a = Y_{adapt} \hat{a}_{adapt} + Y_{known} a_{known}$)
- the **non-adaptive version** (using accurate estimates) is a static tracking controller based on the **passivity** property of robot dynamics

Case study: Single-link under gravity



model $I\ddot{\theta} + mgd \sin \theta + f_V \dot{\theta} = u$ (with friction)

linear parameterization

$$Y(\theta, \dot{\theta}, \ddot{\theta})a = [\ddot{\theta} \quad \sin \theta \quad \dot{\theta}] \begin{bmatrix} I \\ mgd \\ f_V \end{bmatrix} = u$$

adaptive controller

$$\begin{aligned} e &= \theta_d - \theta \\ \dot{\theta}_r &= \dot{\theta}_d + \frac{k_P}{k_D} e \\ \gamma_i &> 0, i = 1, 2, 3 \end{aligned}$$

$$\begin{aligned} u &= \hat{I} \ddot{\theta} + \widehat{mgd} \sin \theta + \hat{f}_V \dot{\theta} + k_P e + k_D \dot{e} \\ \hat{a} &= \begin{pmatrix} \hat{I} \\ \widehat{mgd} \\ \hat{f}_V \end{pmatrix} = \begin{pmatrix} \gamma_1 \ddot{\theta}_r \\ \gamma_2 \sin \theta \\ \gamma_3 \dot{\theta}_r \end{pmatrix} (\dot{\theta}_r - \dot{\theta}) \end{aligned}$$



Simulation data

- **real** dynamic coefficients

$$I = 7.5, \quad mgd = 6, \quad f_V = 1$$

- **initial** estimates

$$\hat{I} = 5, \quad \widehat{mgd} = 5, \quad \hat{f}_V = 2$$

- control parameters

$$k_P = 25, \quad k_D = 10, \quad \gamma_i = 5, \quad i = 1, 2, 3$$

- **test trajectories** (starting with $\theta(0) = 0, \dot{\theta}(0) = 0$)

- **first**

$$\theta_d(t) = -\sin t$$

- **second**

Note: same test trajectories
used also for robust control

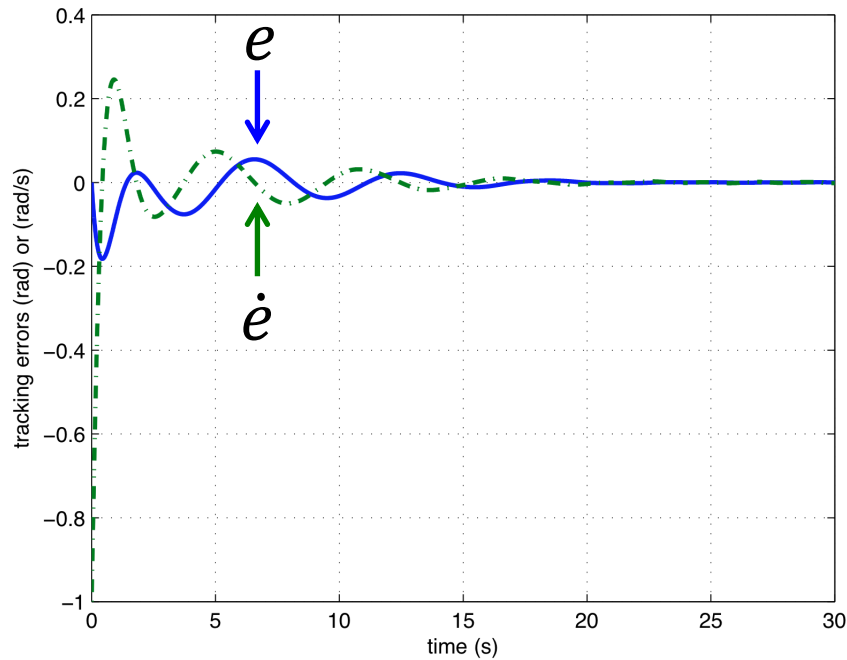
$$\ddot{\theta}_d(t) = \text{(periodic) bang-bang acceleration profile with} \\ A = 1 \text{ rad/s}^2, \omega = 1 \text{ rad/s}$$

Results

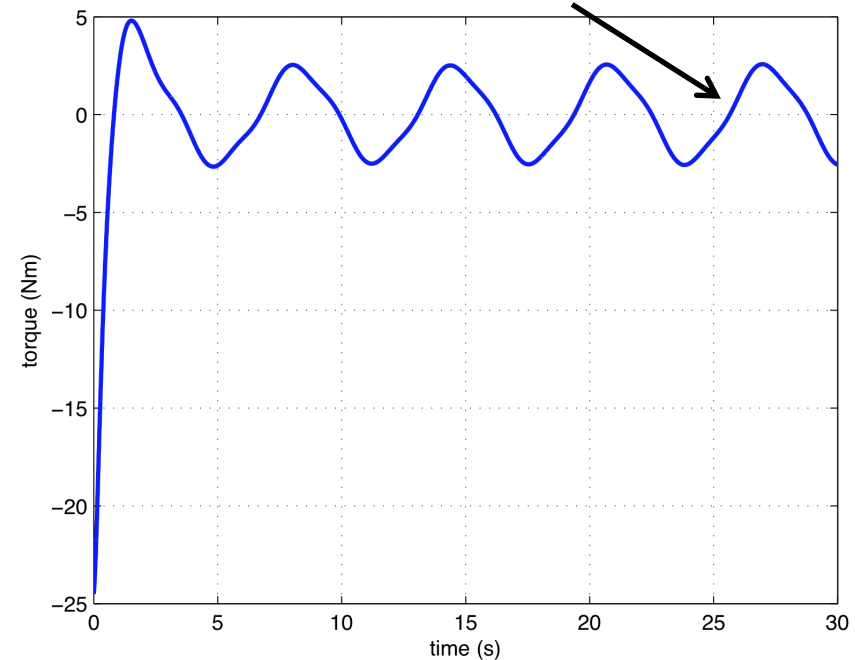
first trajectory



note the nonlinear system dynamics
(no sinusoidal regime at steady state!)



position and velocity errors

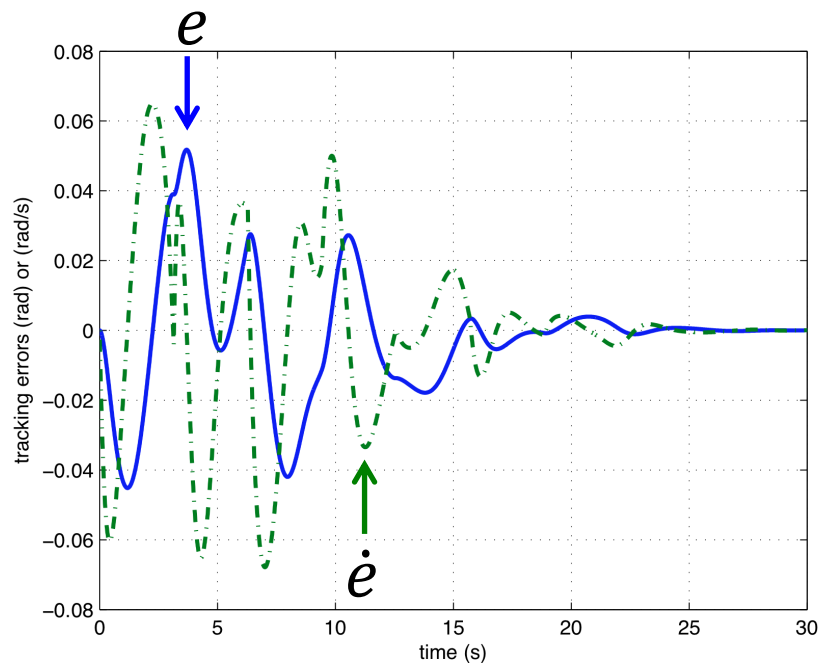


control torque

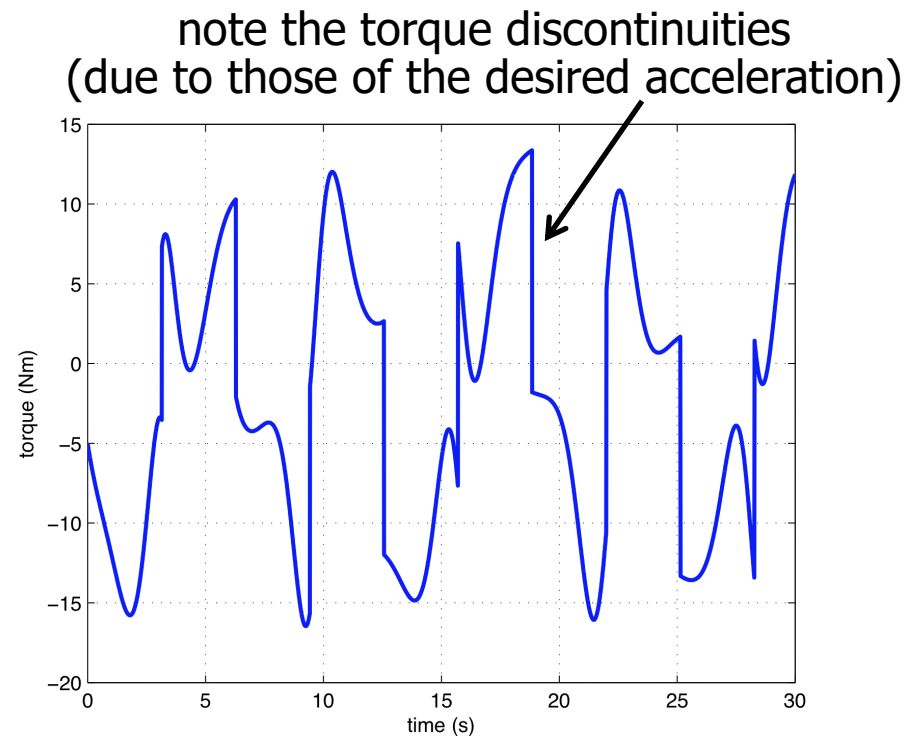
$$\theta_d(t) = -\sin t$$

Results

second trajectory



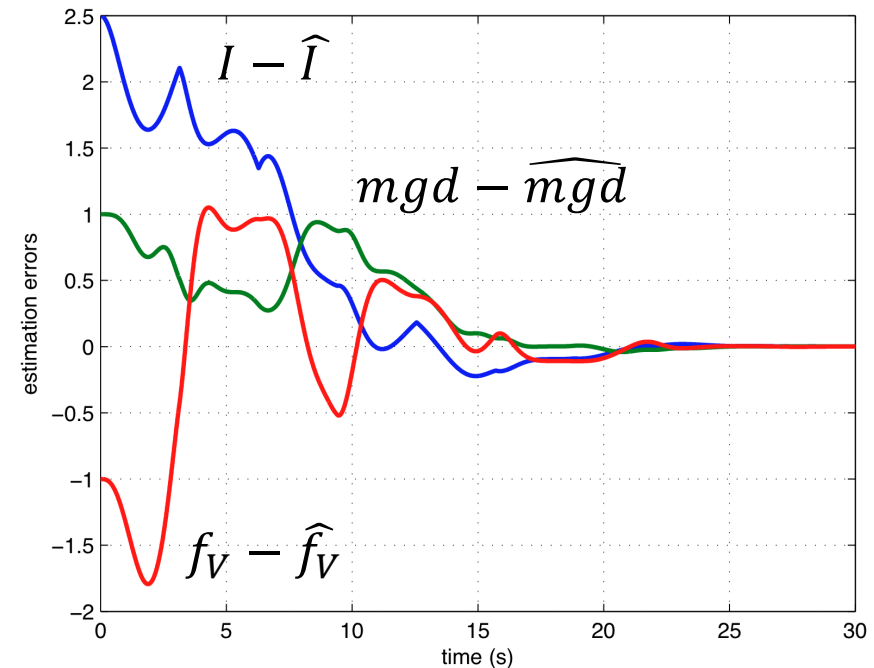
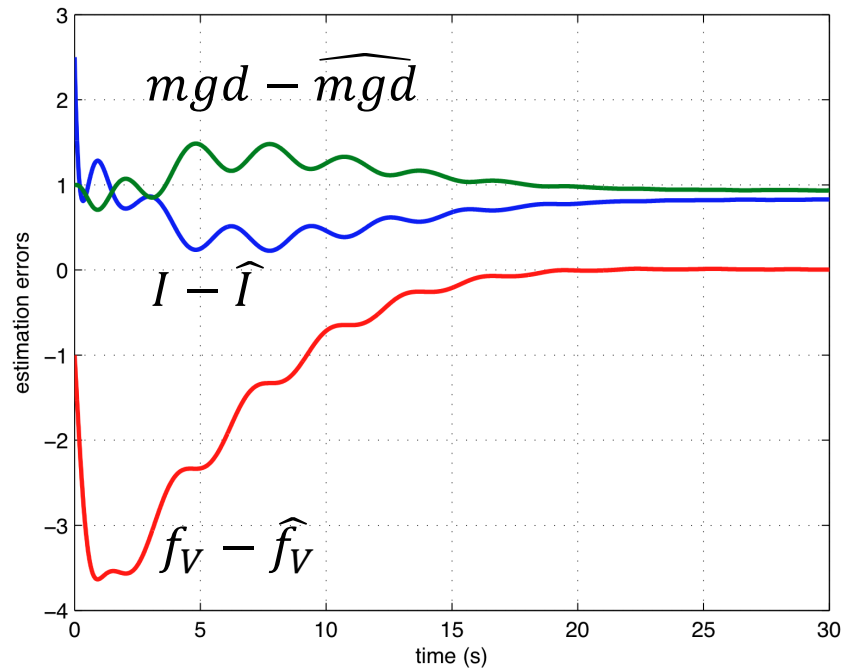
position and velocity errors



control torque

$$\ddot{\theta}_d(t) = \text{(periodic) bang-bang acceleration profile}$$

Estimates of dynamic coefficients



errors $\tilde{a} = a - \hat{a}$

first trajectory

only the estimate of the viscous
friction coefficient converges
to the true value

second trajectory

all three estimates of
dynamic coefficients converge
to their true values



A special case: Adaptive regulation

- adaptation in case q_d is **constant**
- **no special simplifications** for the presented adaptive control law (designed for the general tracking case...)

$$u = \hat{M}(q)\ddot{q}_r + \hat{S}(q, \dot{q})\dot{q}_r + \hat{g}(q) + \hat{F}_v\dot{q}_r + K_P e + K_D \dot{e}$$

$$\dot{\hat{a}} = \Gamma Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\dot{q}_r - \dot{q})$$

since $\dot{q}_r = \Lambda(q_d - q)$ and $\ddot{q}_r = -\Lambda\dot{q}$ **do not** vanish!

- a **different** case would be the availability of an adaptive version of the trajectory tracking controller

$$u = \hat{M}(q)\ddot{q}_d + \hat{S}(q, \dot{q})\dot{q}_d + \hat{g}(q) + \hat{F}_v\dot{q}_d + K_P e + K_D \dot{e}$$

since, when q_d collapses to a constant, **only the adaptation of the gravity term** would be left over (which is what one would naturally expect...)



An efficient adaptive regulator

- use a linear parameterization of the **gravity term** only

$$g(q) = G(q)a_g$$

with a **p_g -dimensional** vector **a_g**

- an adaptive regulator yielding **global asymptotic stability** of the equilibrium state $(q_d, 0)$ is provided by

$$u = G(q)\hat{a}_g + K_P(q_d - q) - K_D\dot{q}$$

$$\dot{\hat{a}}_g = \gamma G^T(q) \left(\frac{2e}{1 + 2\|e\|^2} - \beta\dot{q} \right), \quad \gamma > 0$$

where $e = q_d - q$, $K_P > 0$, $K_D > 0$ (symmetric), and $\beta > 0$ is chosen sufficiently **large**

(see paper by P. Tomei, IEEE TRA, 1991; available as extra material on the course web)



An adaptive regulator

Sketch of asymptotic stability analysis

- use the function

$$V = \frac{\beta}{2} (\dot{q}^T M(q) \dot{q} + e^T K_P e) - \frac{2\dot{q}^T M(q) e}{1 + 2\|e\|^2} + \frac{1}{2} (\hat{a}_g - a_g)^T (\hat{a}_g - a_g)$$

- a sufficient condition for V to be a **Lyapunov candidate** is that

$$\beta > \frac{2M_M}{\sqrt{M_m K_{P,m}}}$$

- a sufficient condition which guarantees **also** that

$$\dot{V} = \dots \leq -a\|e\|^2 - b\|\dot{q}\|^2 \leq 0, \quad a > 0, b > 0$$

is

$$\beta > \max \left\{ \frac{2M_M}{\sqrt{M_m K_{P,m}}}, \frac{1}{K_{D,m}} \left(\frac{K_{D,m}^2}{2K_{P,m}} + 4M_M + \frac{\alpha_S}{\sqrt{2}} \right) \right\}$$

$\|S(q, \dot{q})\| \leq \alpha_S \dot{q}$

Note: for any **symmetric, positive definite** matrix A

$$A_M = \lambda_{\max}(A) = \sqrt{\lambda_{\max}(A^T A)} = \|A\| \quad \text{and thus, e.g., } \frac{1}{2} \dot{q}^T M(q) \dot{q} \geq \frac{1}{2} M_m \|\dot{q}\|^2$$
$$A_m = \lambda_{\min}(A)$$