Homework 1

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Homework Set One

ECE 271A

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The purpose of this assignment is to give you experience with Bayesian decision theory. The first four problems are of the pen-and-paper type, while problem 5 is a computer problem. Some of the problems simply require the straightforward application of the principles covered in class. These are intended to give you some practice on the Bayesian manipulations that are usually involved in decision theory. Others involve somewhat deeper thinking and extend in some way what we have seen in class. The problems are not ordered by difficulty.

1. In this problem we will consider the traditional probability scenario of coin tossing. However, we will consider two variations. First, the coin is not fair. Denoting by S the outcome of the coin toss we have

$$P_S(heads) = \alpha,\ \alpha \in [0, 1].$$

Second, you do not observe the coin directly but have to rely on a friend that reports the outcome of the toss. Unfortunately your friend is unreliable, he will sometimes report heads when the outcome was tails and vice-versa. Denoting the report by R we have

$$P_{R|S}(tails|heads) = \theta_1 \quad (1)$$
$$P_{R|S}(heads|tails) = \theta_2 \quad (2)$$

where $\theta_1, \theta_2 \in [0, 1]$. Your job is to, given the report from your friend, guess the outcome of the toss.

a) Given that your friend reports heads, what is the optimal decision function in the minimum probability of error sense. That is, when should you guess heads, and when should you guess tails?

b) Consider the case $\theta_1 = \theta_2$. Can you give an intuitive interpretation to the rule derived in **a)**?

c) You figured out that if you ask your friend to report the outcome of the toss various times, he will produce reports that are statistically independent. You then decide to ask him to report the outcome n times, in the hope that this will reduce the uncertainty. (Note: there is still only one coin toss, but the outcome gets reported n times). What is the new minimum probability of error decision rule?

d) Consider the case $\theta_1 = \theta_2$ and assume that the report sequence is *all heads*. Can you give an intuitive interpretation to the rule derived in **c)**?

2. Problem 2.2.2 in DHS.

3. Problem 2.5.23 in DHS (feel free to use MATLAB here to compute eigenvalues, matrix vector products, and so forth. The goal is not evaluate your mastery of the mechanics of linear algebra, but to give you an intuitive understanding of what whitening is).

4. Problem 2.9.43 in DHS

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1) $P_S(\text{heads}) = \alpha$

$P_{R|S}(\text{tails}|\text{heads}) = \theta_1$, $P_{R|S}(\text{heads}|\text{tails}) = \theta_2$

a) $P_{S|R}(\text{heads}|\text{heads}) = \dfrac{P_{R|S}(\text{heads}|\text{heads})\, P_S(\text{heads})}{P_R(\text{heads})}$

$P_{S|R}(\text{tails}|\text{heads}) = \dfrac{P_{R|S}(\text{heads}|\text{tails})\, P_S(\text{tails})}{P_R(\text{heads})}$

$\Rightarrow P_R(\text{heads})$ cancels

$\Rightarrow (1-\theta_1)\alpha > \theta_2(1-\alpha)$

b) $(1-\theta)\alpha > \theta(1-\alpha)$

$\Rightarrow \frac{1}{\theta} - 1 > \frac{1}{\alpha} - 1$

$\Rightarrow \theta > \alpha$

$\Rightarrow \frac{1}{2} > \alpha$, you go off of the prior

c) maximize the probability of the posterior

$P_{S|R_1,R_2,\dots,R_n}(s|R_1,R_2,\dots,R_n) = \left(\prod_{i=1}^{n} P_{R_i|S}(R_i|s)\right) \cdot P(s)$