Creating Graphs in Mathematica

The purpose of this lab is to learn how to use Mathematica to create two and three dimensional graphs.

Mathematica Commands

1. `Plot[f[x], {x,a,b}]` plots the function $f(x)$ on the interval $a \leq x \leq b$.

   (a) `AxesLabel->{"x", "y"}` option labels the x and y-axes.

   (b) `PlotLabel->"title"` option labels the graph.

   (c) `AspectRatio->r` option specifies the ratio of the height of the plot to the width of the plot. The default is the inverse of the Golden Ratio.

   (d) `PlotRange->{ymin,ymax}` option specifies the y range of the plot.

2. `ParametricPlot[{xt,yt}, {t,a,b}]` graphs the parametric equations $x = x(t)$, $y = y(t)$ on the interval $a \leq t \leq b$.

   (a) `AspectRatio->r` option sets the height-to-width ratio for the plot. The default is the inverse of the Golden Ratio.

   (b) `PlotRange->{{xmin,xmax}, {ymin,ymax}}` option specifies the x and y ranges of the plot.

3. `Plot3D[ f[x,y], {x, a, b}, {y, a, b}]` graphs the surface $z = f(x,y)$ over the region $a \leq x \leq b$ and $c \leq y \leq d$.

   (a) `AxesLabel->{"x", "y", "z"}` option labels the x, y, and z-axes.

   (b) `BoxRatios->{xr, yr, zr}` option specifies the ratios of the bounding box, i.e., the relative lengths of the x, y, and z-axes.

   (c) `PlotPoints->n` option directs Mathematica to evaluate the function on a n by n grid of equally spaced points. The default is a 15 by 15 grid of points.

4. `ContourPlot[ f[x,y], {x, a, b}, {y, a, b}]` graphs level curves of $f(x,y)$ over the region $a \leq x \leq b$ and $c \leq y \leq d$.

   (a) `FrameLabel->{"x", "y"}` option labels the x and y-axes.

   (b) `ContourShading->False` option turns off the shading.

   (c) `Contours->n` option directs Mathematica to plot n level curves. The default is 10.

   (d) `PlotPoints->n` option directs Mathematica to evaluate the function on a n by n grid of equally spaced points. The default is a 15 by 15 grid of points.

5. `ParametricPlot3D[{xt,yt,zt}, {t,a,b}]` graphs the 3-dimensional curve that is represented by the parametric equations $x = x(t)$, $y = y(t)$, $z = z(t)$, on the interval $a \leq t \leq b$.

Two Dimensional Graphs

The graph of the function $y = f(x)$ can be created in Mathematica by using the `Plot` command. The command `Plot[ f[x], {x, a, b}]` graphs the surface $y = f(x)$ over the region $a \leq x \leq b$.

Example 1

Graph the surface given by $z = f(x) = \sin(x)$ over the region $0 \leq x \leq 2\pi$.

```
Clear[f, x];
f[x_] := Sin[x];
Plot[f[x], {x, 0, 2Pi}]
```

It is helpful to label the x and y-axes by including the option `AxesLabel->{"x", "y"}`.

```
Plot[f[x], {x, 0, 2Pi}, AxesLabel->{''x'', ''y''}]
```

It is also helpful to use the `PlotLabel->"title"` option to label the graph.

```
Plot[f[x], {x, 0, 2Pi}, AxesLabel->{''x'', ''y''}, PlotLabel->''sin(x)'']
```

You can control the ratio of the height of the graph to the width of the graph by using the `AspectRatio->r` option. The following command forces the height-to-width ratio to be one.

```
Plot[f[x], {x, 0, 2Pi}, AxesLabel -> {''x'', ''y''}, PlotLabel -> ''sin(x)'',
  AspectRatio -> 1]
```

Notice that the scale on the x-axis is still different than the scale on the y-axis. If it is important to use the same scale on both axes, then the `PlotRange->{ymin,ymax}` option, which specifies the plotting range for the y-axis, can be used in conjunction with the `AspectRatio->r` option. The follow command uses the same scale on both axes.

```
Plot[f[x], {x, 0, 2Pi}, AxesLabel -> {''x'', ''y''}, PlotLabel -> ''sin(x)'',
  AspectRatio -> 1, PlotRange -> {-Pi, Pi}]
```

Two Dimensional Parametric Plots

The command `ParametricPlot[{xt,yt}, {t,a,b}]` graphs the 2-dimensional curve that is represented by the parametric equations $x = x(t)$, $y = y(t)$, on the interval $a \leq t \leq b$.

Example 1

Graph the curve given by $x = \cos t$, $y = \sin t$, $0 \leq t \leq 2\pi$.

```
ParametricPlot[{Cos[t],Sin[t]},{t,0,2Pi},AspectRatio->1,AxesLabel->{"x","y"}]
```

Mathematica should create a graph of a circle with radius 1. Note that if you omit the `AspectRatio->1` option, then the graph will "look" like an ellipse rather than a circle.

Three Dimensional Surfaces

The graph of a surface $z = f(x,y)$ can be created in Mathematica by using the `Plot3D` command. The command `Plot3D[ f[x,y], {x, a, b}, {y, a, b}]` graphs the surface $z = f(x,y)$ over the region $a \leq x \leq b$ and $c \leq y \leq d$.

Example 1

Graph the surface given by $z = f(x,y) = \sin(x)\cos(y)$ over the region $0 \leq x \leq 3\pi$ and $0 \leq y \leq 3\pi$.

```
Clear[f, x, y];
f[x_, y_] := Sin[x] Cos[y];
Plot3D[f[x,y], {x, 0, 3Pi}, {y, 0, 3Pi}]
```

It is helpful to label the x, y, and z-axes by including the option `AxesLabel->{"x", "y", "z"}`.

```
Plot3D[f[x,y], {x, 0, 3Pi}, {y, 0, 3Pi}, AxesLabel->{''x'', ''y'', ''z''}]
```

The graph produced by Mathematica appears "choppy." In order to obtain a smoother graph, you must direct Mathematica to evaluate the function at more points by using the option `PlotPoints->30`. This causes Mathematica to evaluate the function on a 30 by 30 grid of equally spaced points. The default is a 15 by 15 grid of points.

```
Plot3D[f[x,y], {x, 0, 2Pi}, {y, 0, 2Pi}, AxesLabel->{''x'', ''y'', ''z''},
       PlotPoints->30]
```

Example 2

Graph the surface given by $z = f(x,y) = x^2 - 4x + y^2 - 2y + 5$ over the region $0 \leq x \leq 4$ and $-1 \leq y \leq 3$.

```
Clear[f, x, y];
f[x_, y_] := x^2 - 4 x + y^2 - 2y + 5;
Plot3D[f[x,y], {x, 0, 4}, {y, -1, 3}, AxesLabel->{''x'', ''y'', ''z''}]
```

Notice that Mathematica has chosen the length of the z-axis to be shorter than the x and y-axes. This causes the graph to be distorted because the range for the x and y-axes are both four units, whereas the range for the z-axis is eight units. You can control the ratios of the bounding box by using the `BoxRatios->{xr, yr, zr}` option. In this case we want the z-axis to be twice as long as the x and y-axes, so we will use `BoxRatios->{1, 1, 2}`.

```
Plot3D[f[x,y], {x, 0, 4}, {y, -1, 3}, AxesLabel->{''x'', ''y'', ''z''},
       BoxRatios->{1, 1, 2}]
```

Contour Plots

A contour plot of the function $f(x,y)$ can be created in Mathematica by using the `ContourPlot` command. The command `ContourPlot[ f[x,y], {x, a, b}, {y, a, b}]` graphs level curves of $f(x,y)$ over the region $a \leq x \leq b$ and $c \leq y \leq d$.

Example 1

Create a contour plot of $f(x,y) = x^2 + y^2$ over the region $-2 \le x \le 2$ and $-2 \le y \le 2$.

```
Clear[f, x, y];
f[x_, y_] := x^2 + y^2;
ContourPlot[ f[x,y], {x, -2, 2}, {y, -2, 2}]
```

Since the surface $z = f(x,y) = x^2 + y^2$ is an elliptic paraboloid with equal coefficients for x^2 and y^2, it is easy to see that the level curves should be circles. It is helpful to label the x and y-axes by including the option `FrameLabel->{"x", "y"}`. Furthermore, sometimes it is helpful to turn off the shading by using the `ContourShading` option.

```
ContourPlot[ f[x,y], {x, -2, 2}, {y, -2, 2}, FrameLabel->{''x'', ''y''},
             ContourShading->False]
```

Use the `Contours` options to increase the number of contour lines from the default of 10.

```
ContourPlot[ f[x,y], {x, -2, 2}, {y, -2, 2}, FrameLabel->{''x'', ''y''},
             ContourShading->False, Contours->20]
```

Example 2

Create a contour plot of $f(x,y) = \sin(x)\cos(y)$ over the region $0 \le x \le 3\pi$ and $0 \le y \le 3\pi$.

```
Clear[f, x, y];
f[x_, y_] := Sin[x] Cos[y];
ContourPlot[ f[x,y], {x, 0, 3Pi}, {y, 0, 3Pi}, FrameLabel->{''x'', ''y''}]
```

Notice that for this example the contour shading is useful because it allows us to distinguish between relative maximums (light shading) and relative minimums (dark shading). The contour lines appear to be "choppy." In order to obtain smoother level curves, you must direct Mathematica to evaluate the function at more points by using the option `PlotPoints->30`. This causes Mathematica to evaluate the function on a 30 by 30 grid of equally spaced points. The default is a 15 by 15 grid of points.

```
ContourPlot[ f[x,y], {x, 0, 3Pi}, {y, 0, 3Pi}, FrameLabel->{''x'', ''y''},
             PlotPoints->30]
```

Three Dimensional Parametric Plots

`ParametricPlot3D[{xt,yt,zt}, {t,a,b}]` graphs the 3-dimensional curve that is represented by the parametric equations $x = x(t)$, $y = y(t)$, $z = z(t)$, on the interval $a \le t \le b$.

Example 1

Graph the curve given by $x = \cos t$, $y = \sin t$, $z = t$, $0 \le t \le 4\pi$.

```
ParametricPlot3D[{Cos[t],Sin[t],t},{t,0,4Pi},BoxRatios->{1,1,1},
   AxesLabel->{''x'',''y'',''z''}]
```

Mathematica should create a graph that looks like a corkscrew or a ramp in a parking garage.