Duffing Oscillator and Diffusion Limited Aggregation

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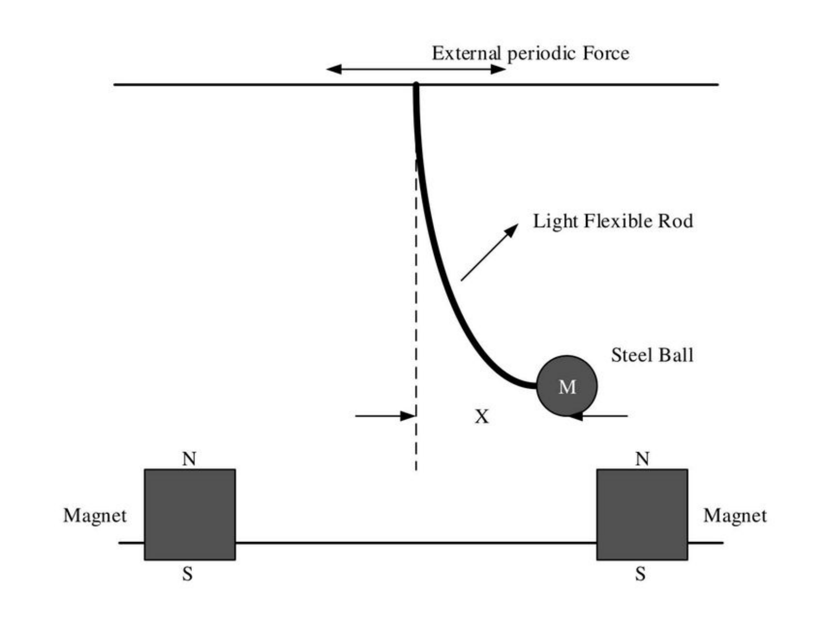
Duffing Oscillator

A pendulum under the influence of magnets on either side exhibits chaotic behavior for certain values of its parameters.

$$F[x, \dot{x}, t] = -\gamma \dot{x} + 2ax - 4bx^3 + F_0 \cos(\omega t).$$

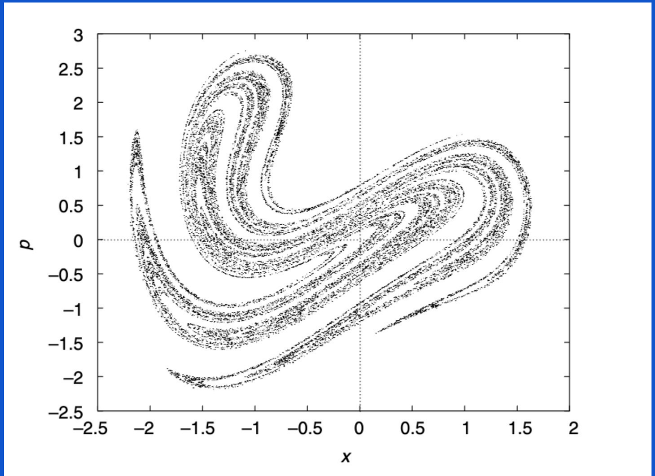


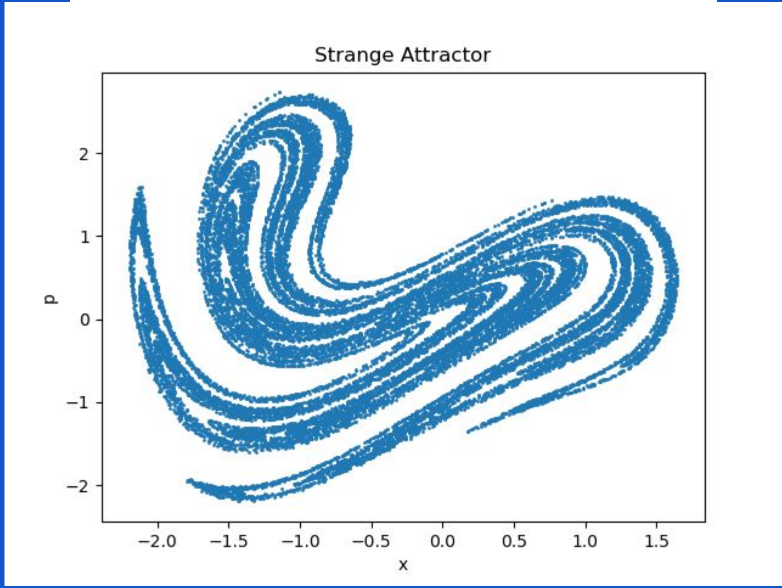
https://www.researchgate.net/figure/Duffing-oscillator-model_fig1_337322516

Strange Attractor

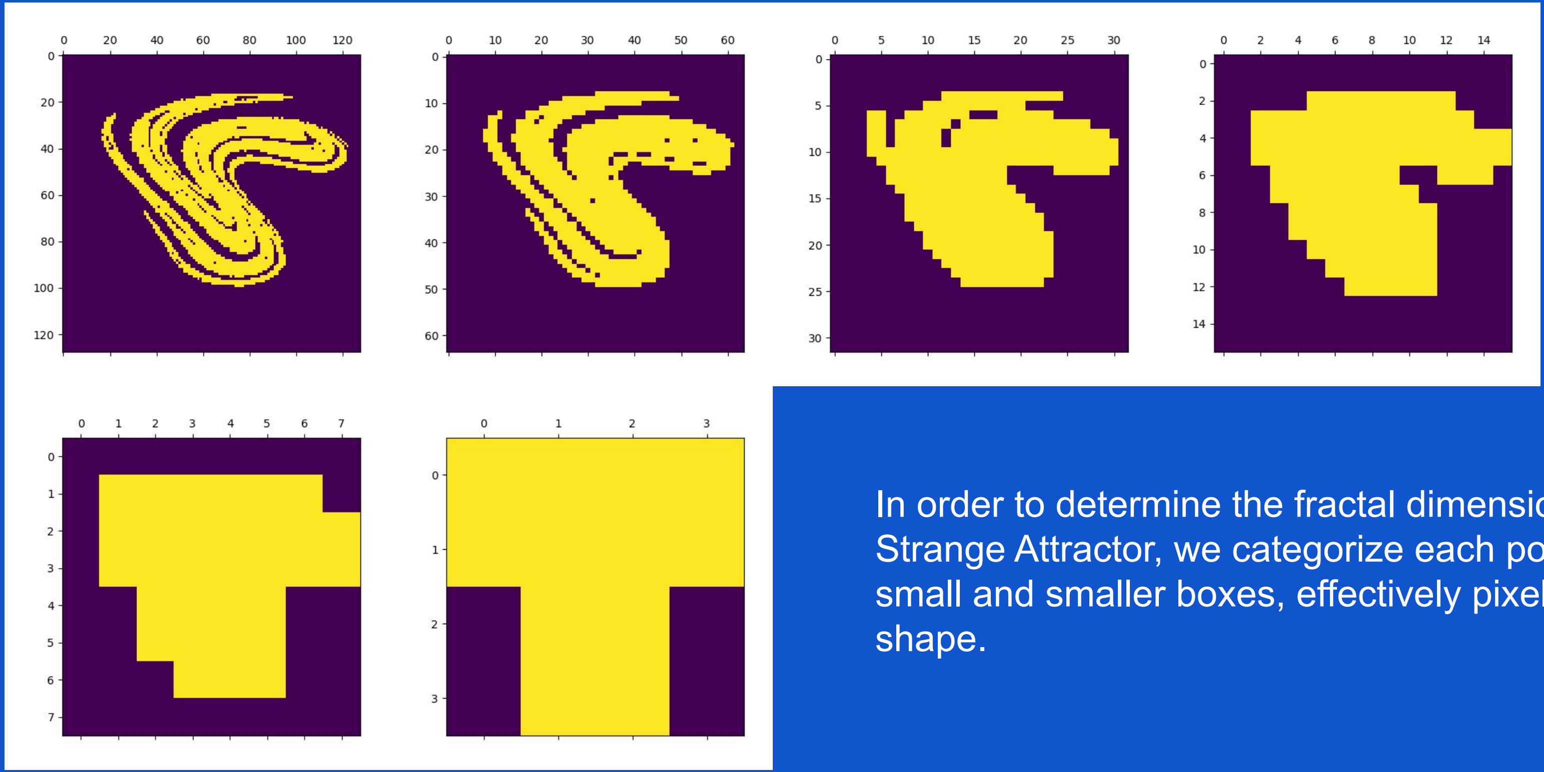
If we measure the position and velocity of these chaotic oscillations every period, then the resulting figure is a very predictable Strange Attractor.

The period is calculated using the frequency parameter ($2\pi/\omega$).





Determining the Fractal Dimension



In order to determine the fractal dimension of the Strange Attractor, we categorize each point using small and smaller boxes, effectively pixelating the shape.

Fractal Dimension of Strange Attractor

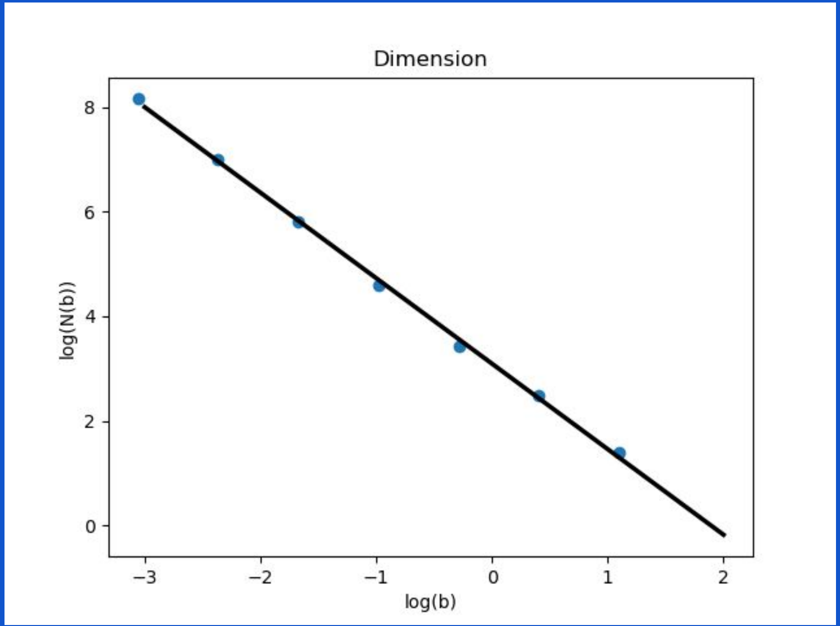
The dimension D_f of a line would be 1 because the number of boxes N(b) should scale linearly with the size of the boxes b.

The dimension D_f of a surface would be 2 because the number of boxes N(b) should scale quadratically with the size of the boxes b.

For Strange Attractor of 25,000 points,

Expected D_f= 1.68

Measured D_f= 1.63

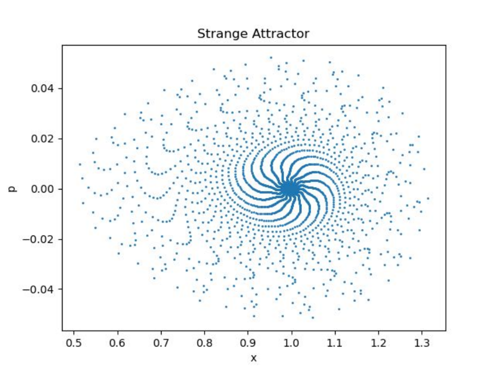


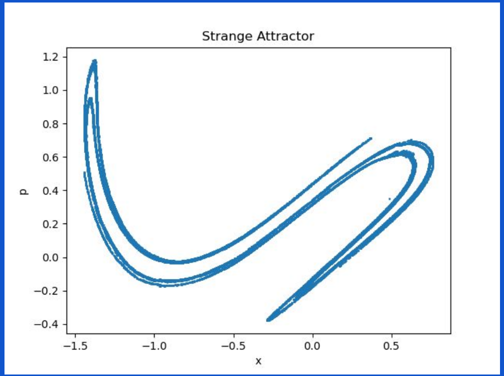
$$N(b) \propto b^{-D_{\mathrm{f}}}, \text{ small } b,$$

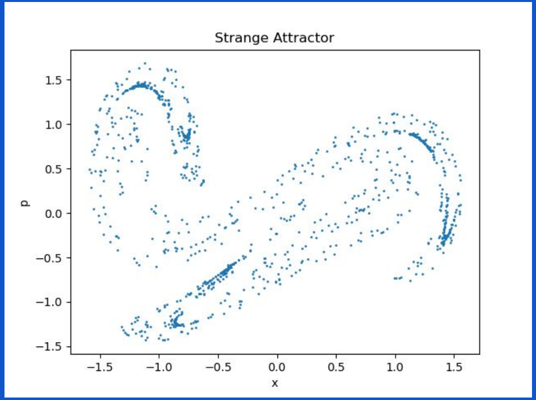
Variation of Parameters

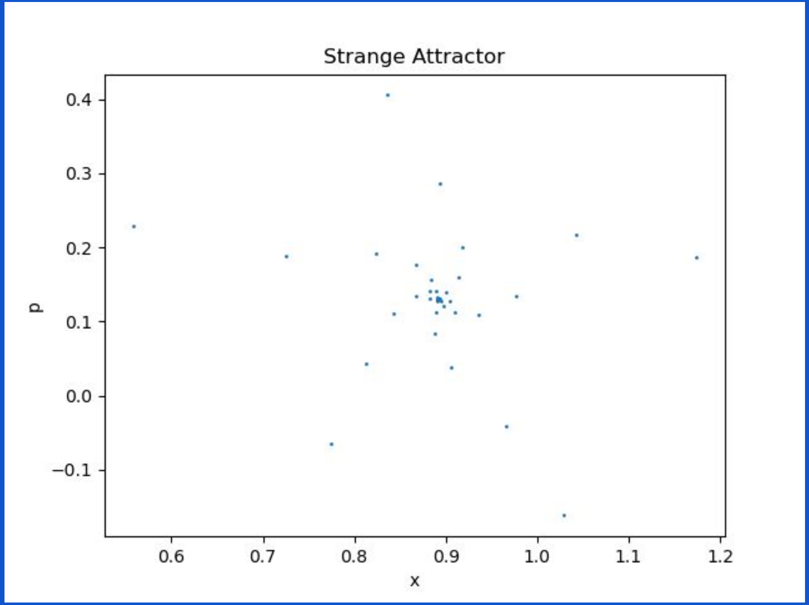
Parameter	Starting value	Effect of Change
Steps per Period	200	When changed to 10, the steps per period exhibited no noticeable change in either the Strange Attractor shape or the D_f value. At 1, the attractor becomes a point
m	1	New value: 100 - Spiral Egg Pattern - D_f=0.77
a	1/2	New value: 0.1 - no pattern
b	1/4	New value: 2.5 - no pattern
F_0	2.0	New value: 1 - Three Wing Pattern - D_f=1.25
ω	2.4	New value: 5 - Five Spiral Dot Pattern - D_f=0.63
γ	0.1	New value: 0.5 - Thick Double Wing Pattern - D_f=1.11

Interesting Strange Attractors









Europa Mission

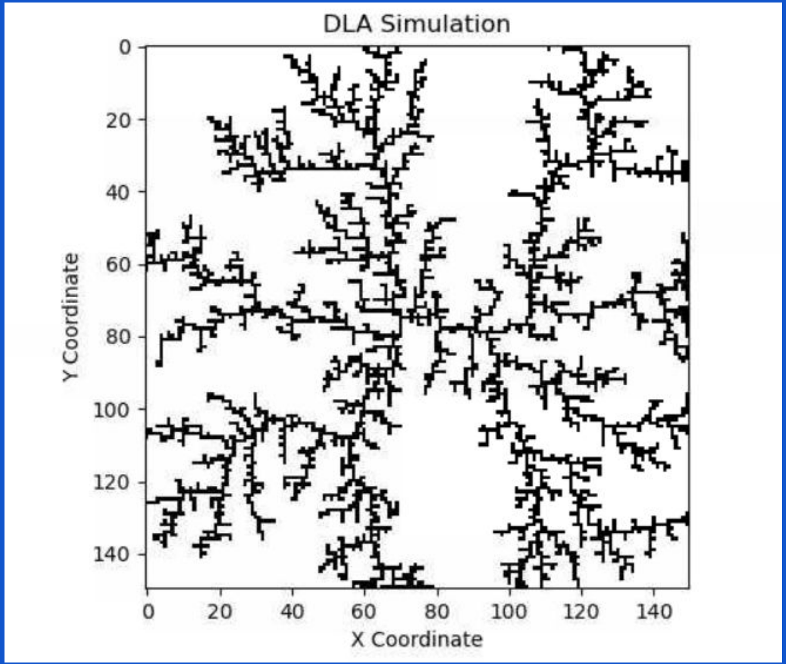
We used the ScyPy ODE algorithm to solve this ODE. Considering that at a step per period of 10, the results were still accurate, I would rate this algorithm very highly. If it were used for a mission to Europa, and the computers were strong enough for a higher step size (maybe on the order of 1000), I would certainly trust the resulting flight path. That being said, the calculations would likely be much faster in C++.

Diffusion Limited Aggregation (DLA) Algorithm

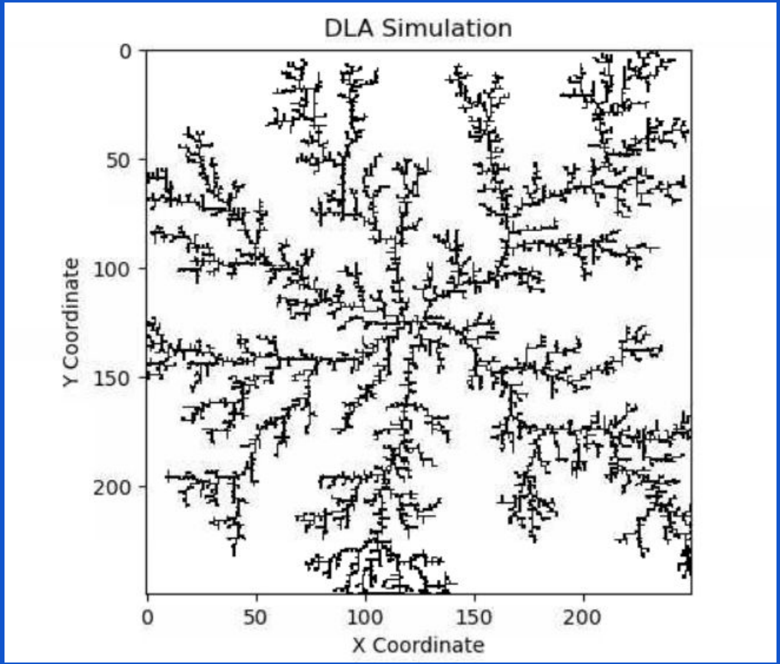
DLA is an algorithm designed to simulate crystal aggregate growth

1. Initialize a "seed" by setting an occupied site at the center of the lattice
2. Start a random walker randomly on the border of the lattice
3. Draw an occupied site when the random walker becomes adjacent to an occupied site in the cluster
4. Repeat starting random walkers until the set number of occupied sites is reached

Dendritic Clusters

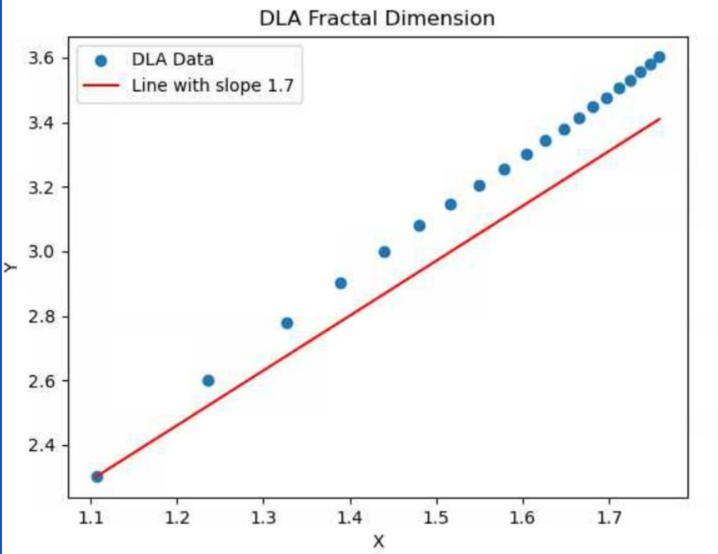


150x150 lattice with 4000 sites



250x250 lattice with 9000 sites

Fractal Dimension Graphs



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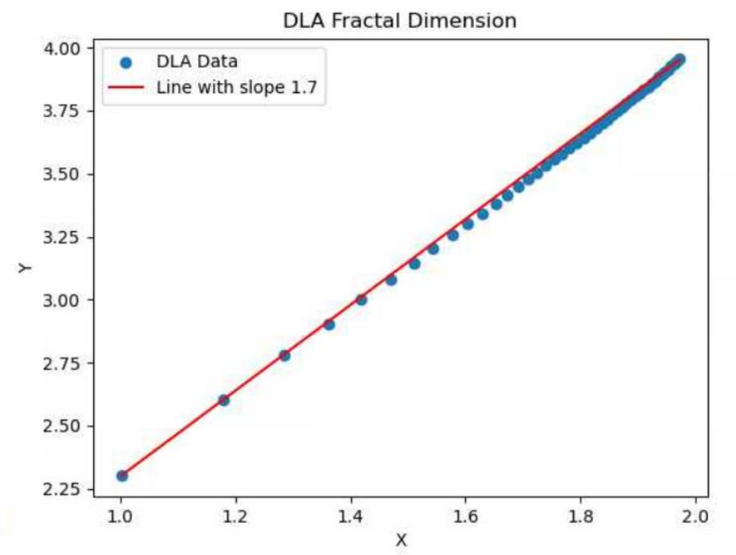

250x250 lattice with 9000 sites

Larger lattice converges closer to expectation for fractal dimension of 1.7

DLA Results

- The expectation for the radius of gyration derivation according to theory should yield a slope of 1.7 for the log(R_g) vs. log(N) graph. Our results show that increasing the size of the cluster converge this slope to 1.7.
- Small changes in this simulation drastically affect the outcome due to the stochastic nature of the algorithm. Initializing a seed somewhere other than the center or initializing multiple seeds creates more unique cluster outcomes. Increasing the step size (step size taken by the random walker) does not appear to affect the outcome of the algorithm.