DC Circuits

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I. Introduction

Circuits exist in every piece of electronics with the simplest circuit consisting of a resistor connected to a source supplying electromotive force such as a battery. The movement of electrons through the circuit components is known as a current and can be thought of as being pushed through the loop by a characteristic voltage. This relationship is expressed in Ohm's Law: $V = IR$, where R represents the resistance of the component. Circuits can be analyzed using a set of rules called Kirchhoff's Laws and combined with Ohm's Law allow us to predict the voltage, current, and resistance for various parts of a circuit. In the first few following experiments we will use a multimeter to measure voltage and current of several circuits to verify these laws. In the remaining experiments we will use various materials such as metals, alloys, semiconductors, and superconductors to study the effect of temperature on resistance.

II. Simple Circuits

To test our known circuit laws, we will setup a simple circuit as shown in Figure 1.

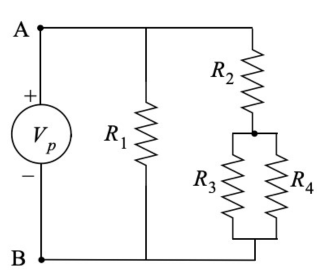

Figure 1: Simple Circuit 1

Here we have a voltage V_p = 5.00V supplied by a Keithley 2231A power supply and a current limit of 20mA to prevent a meltdown in one of the resistors should something go wrong. Safety first. The resistors are connected as depicted in Figure 1 using a breadboard and insulated wires with known resistances R. First, the resistors were measured independently to find how accurate both our probe and the system are, and the values are shown in Table 1.

Table 1: Resistors for Simple Circuit 1

Expected Resistance (kΩ)	*Measured Resistance (kΩ)*
$R_1 = 10.0$	$R_1 = 9.992 \pm 0.001$
$R_2 = 2.10$	$R_2 = 2.093 \pm 0.001$
$R_3 = 5.11$	$R_3 = 5.090 \pm 0.001$
$R_4 = 6.81$	$R_4 = 6.792 \pm 0.001$

Using Ohm's Law and Kirchhoff's Laws, we can find that the expected value for the total resistance of the circuit R_{AB} = 3.34 kΩ. This value was measured using the probe connected to points A and B with the power supply off to be $R_{AB} = 3.333 \pm 0.010$ kΩ. Similarly, we can calculate the expected voltages across each resistor and measure this voltage by connecting the voltage by connecting the probe to the wire on either side of the resistor. This was done for each resistor and the calculated and measured voltages are shown in Table 2.

Table 2: Voltages for Simple Circuit 1

Expected Voltage (V)	*Measured Voltage (V)*
$V_1 = 5.00$	$V_1 = 4.997 \pm 0.001$
$V_2 = 2.09$	$V_2 = 2.092 \pm 0.001$
$V_3 = 2.91$	$V_3 = 2.905 \pm 0.001$
$V_4 = 2.91$	$V_4 = 2.905 \pm 0.001$

Next, we will create a circuit with two power sources as shown in Figure 2.

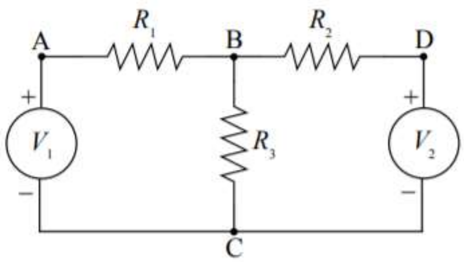

Figure 2: Simple Circuit 2

Here we have a voltage V_1 = 5.00V and V_2 = 1.50V supplied by the same Keithley 2231A power supply from different channels. Again, the current limit is set to 20mA, and the circuit will be constructed using insulated wires and a breadboard with relevant resistors. The resistors were measured independently, and the values are shown in Table 3.

Table 3: Resistors for Simple Circuit 2	
Expected Resistance (kΩ)	*Measured Resistance (kΩ)*
$R_1 = 10.0$	$R_1 = 9.992 \pm 0.001$
$R_2 = 2.10$	$R_2 = 2.092 \pm 0.001$
$R_3 = 1.00$	$R_3 = 0.992 \pm 0.001$

Using Ohm's Law and Kirchhoff's Laws, we can find the expected voltages across each resistor and measure this voltage by connecting the voltage by connecting the probe to the wire on either side of the resistor. This was done for each resistor and the calculated and measured voltages are shown in Table 4.

Table 4: Voltages for Simple Circuit 2	
Expected Voltage (V)	*Measured Voltage (V)*
$V_1 = 4.23$	$V_1 = 4.227 \pm 0.001$
$V_2 = 0.730$	$V_2 = 0.736 \pm 0.001$
$V_3 = 1.0$	$V_3 = 0.770 \pm 0.001$

All the measured voltages for both simple circuits are within a 1% difference of the expected values which provide significant evidence for the use of our circuit laws. However, a source of uncertainty in these measurements is the quality of the connecting wires. Many of these are bent out of shape and slightly rusted on the ends. Care was taken to straighten them and find either pristine ends or trim new ones to reduce the error from the rust. A second source is the wires connecting each resistor to each other and to the power supply. Ideally, the wires would have no inherent resistance; however, there is a small resistance associated with each wire. This was attempted to be solved by using comparatively large resistances in the kΩ range whereas the wires have a resistance of a few Ω.

III. Superposition Theorem

To test the Superposition Theorem, we will use the exact same circuit as Experiment 2 but create a short circuit by removing the red V_2 jack and connecting it to its own black jack connected to the power supply. Measuring the voltage V_{BC1} we can calculate the current

$$I_{31} = V_{BC1} / R_3$$

$$V_{BC1} = 0.317 \pm 0.001 \text{ V}$$

$$I_{31} = V_{BC1} / R_3 = 0.317 \text{ V}/0.992 \text{ k}\Omega =$$

$$0.3196 \pm 0.0034 \text{ mA}$$

The expected value of I_{31} = 0.32 mA, is well within the uncertainty. Similarly, shorting V_1 we will measure V_{BC2} and calculate

$$I_{32} = V_{BC2} / R_3$$

$$V_{BC2} = 0.455 \pm 0.001 \text{ V}$$

$$I_{32} = V_{BC2} / R_3 = 0.455 \text{ V}/0.992 \text{ k}\Omega =$$

$$0.4587 \pm 0.0047 \text{ mA}$$

The expected value of I_{32} = 0.45 mA, within a reasonable range of the measured value. Provided that the Superposition

Theorem holds, as we would expect, $I_{31} + I_{32}$ should give us back the initial current I_3:

$$I_{31} + I_{32} = 0.3196 \text{ mA} + 0.4587 \text{ mA} = 0.7783 \pm 0.0058 \text{ mA}$$

$$I_3 = V_3 / R_3 = 0.770 \text{ V}/0.992 \text{ k}\Omega = 0.7762 \pm 0.0078 \text{ mA}$$

These values are within a reasonable margin of error. Therefore, we have strong supporting evidence for the validity of the Superposition Theorem.

IV. Unknown Circuit

Now we will use the laws and properties of circuits we have verified to predict the voltage and resistance of an unknown circuit. Additionally, we will use the concept of Thevenin equivalencies for voltage and resistance wherein a circuit can be reduced to a simple one containing a characteristic voltage V_T and resistance R_T. The unknown circuit has two terminals to which the hand-held DMM probe is connected. The measured voltage V_{exp} = 1.596 ± 0.001V and current I_{exp} = 0.311 ± 0.001 mA were recorded. Using the known internal resistance of the voltmeter R_i = 10 MΩ and internal resistance of the current meter r_i = 10 Ω, we can calculate the Thevenin equivalencies and values shown below.

$$V_T = \frac{I_{exp}V_{exp}(R_i - r_i)}{I_{exp}R_i - V_{exp}}$$

$$V_T = 1.5968 \ \pm \ 0.0052\, V$$

$$R_T = \frac{R_i(V_{exp} - I_{exp}r_i)}{I_{exp}R_i - V_{exp}}$$

$$R_T = 5.124 \ \pm \ 0.017\, k\Omega$$

$$V = \frac{V_T R_i}{R_i - r_i + \frac{V_T}{I_{exp}}}$$

$$V = 1.5960 \ \pm \ 0.0074\, V$$

$$I = \frac{V_T}{R_T + r_i}$$

$$I = 0.3110 \ \pm \ 0.0014\, mA$$

Comparing the values V and I to the experimental values V_{exp} and I_{exp} we can see that they have a miniscule difference. This even more backing evidence for the validity of our circuit laws as well as the Thevenin equivalencies.

V. Qualitative Analysis of Resistance as a Function of Temperature

Thus far we have worked with resistors assuming their resistance remains constant because for our previous setups this has been adequate. However, interesting things happen to materials as their temperature changes, specifically, as they become very cold. However, this variance in resistance is not true for all types of materials and in the following experiment we will test the resistance as a function of temperature for four materials: a pure metal as copper wire, an alloy as constantan wire, a semiconductor, and a superconductor as YBCO film between two layers of stainless steel. We will measure the first three (metal, alloy, semiconductor) by connecting the sample to our hand-held DMM and measuring the resistance at room temperature. Then we will lower the sample into liquid nitrogen at 77 K and record the corresponding resistance. These values are displayed in Table 5.

Table 5: Resistance at Varying Temperatures

Sample	*Resistance at Room Temperature (Ω)*	*Resistance in Liquid Nitrogen (Ω)*
Copper Wire	88.5 ± 0.1	10.8 ± 0.1
Constantan Wire	98.3 ± 0.1	97.0 ± 0.1
Semiconductor	119.6 ± 0.1	40.00 ± 0.1 MΩ

Note that the prove was unable to read the actual resistance for the semiconductor as it was way too high, so 40.00 MΩ was recorded as it appeared to be the highest value recorded.

The patterns expected for the resistance as temperature decreases are as follows: resistance decrease for a pure metal, very slightly decreases for an alloy, and increase drastically for a semiconductor. As expected, all three of these phenomena were observed in this experiment.

The procedure for the superconductor will be slightly different. This is because as a superconductor becomes superconducting, its resistance drastically diminishes, and the resistance of the copper wires becomes relevant as it will be larger than the resistance of the superconductor. A four wire connection fixes this by having two leads connect to the ends of the sample leading to a power supply to provide current I through the sample. The other two leads connected to the sample lead back to the voltmeter where we will measure the resistance. Additionally, to get a more accurate reading we will ditch the hand-held DMM for a high-resolution bench top DMM set to the 200mV range. With the power supply set to $I = 1.0$ A we can find the resistance R of the superconductor using Ohm's Law $V = IR$ where V is the measured voltage.

The resistance of the superconductor at room temperature was found to be $R = 59.539 \pm 0.001\ \Omega$. After slowly lowering the sample into the liquid nitrogen, the measured resistance leveled out around $R = 0.0040 \pm 0.0010$ mΩ. This sudden drop in resistance was observed as the sample reached its superconducting temperature and effectively became zero resistance. Errors here are that the entire system (wires, etc.) was not down to the sample's superconducting temperature and therefore warmed up the sample slightly as well as the errors associated with the measuring devices. However, the extraordinary property of super-conductance was observed.

VI. Quantitative Analysis of Resistance as a Function of Temperature for a Semiconductor

In the previous experiment we saw that as the semiconductor cooled, its resistance increased. Now, we want to perform a similar experiment to find a precise functional dependence of resistance on temperature. To do this we will have the setup as shown in Figure 3.

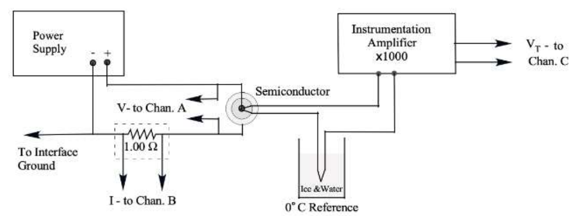

Figure 3: Test for resistance as a function of time for a semiconductor

The power supply will provide constant current measured through the current probe which contains a 1 Ω resistor, effectively converting the current to a voltage without the need for a numerical calculation. The temperature will be measured using a copper-constantan thermocouple whose voltage will be amplified and recorded along with the other quantities as a function of time.

With the power supply set to 10 V and a current control of 10 mA, we slowly lower the sample into the vapor above the liquid nitrogen. The voltage, current, temperature, and resistance are recorded as a function of time and can be seen in Figure 4. Associated graphs are shown in Figure 5 and Figure 6 which will be used for calculations in the proceeding section.

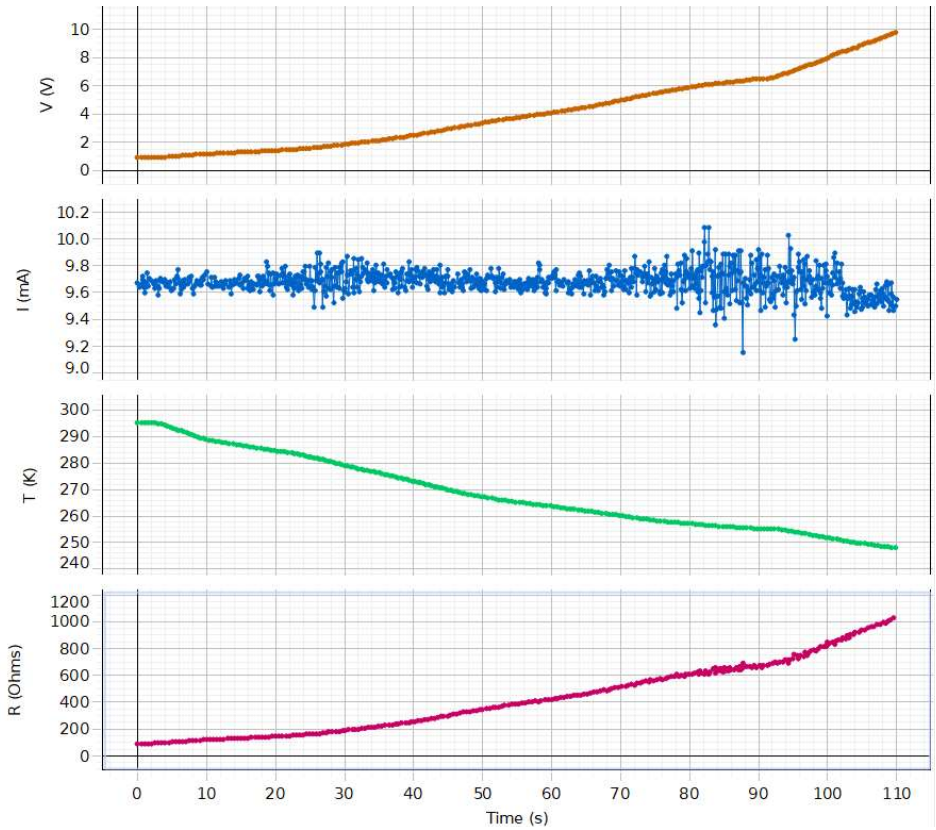

Figure 4: Voltage, current, temperature, and resistance as a semiconductor lowered into liquid nitrogen vapor

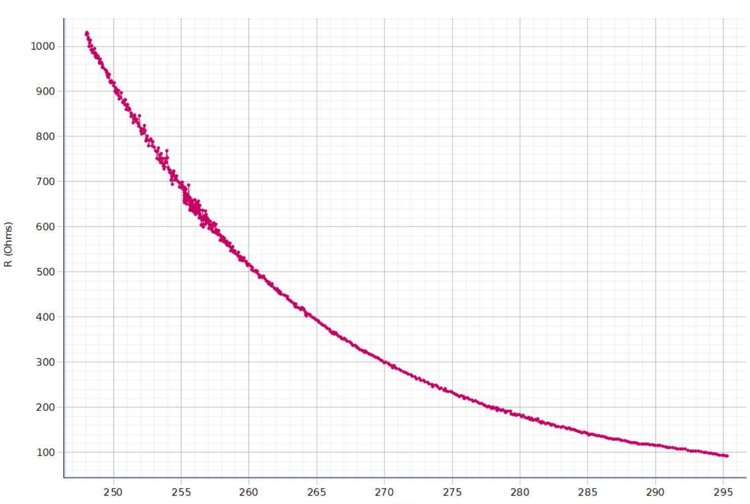

Figure 5: Resistance vs. temperature for a semiconductor lowered into liquid nitrogen vapor

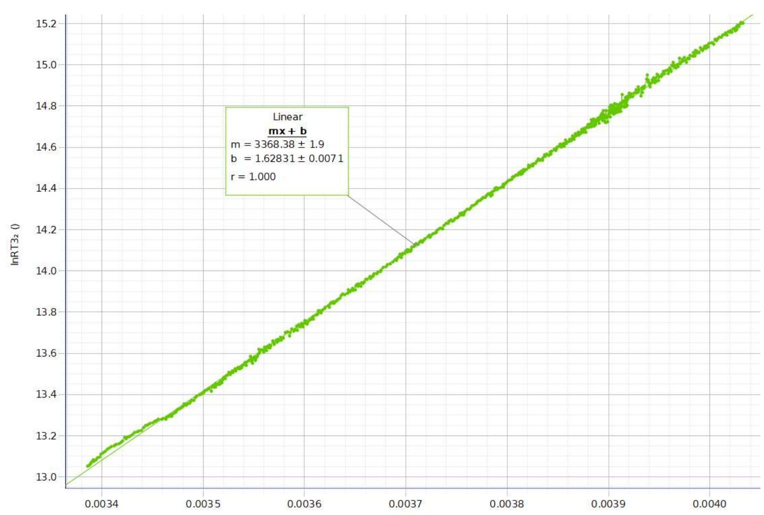

Figure 4: ln(RT^3/2) vs. 1/T for a semiconductor lowered into liquid nitrogen vapor

We know that the theoretical variation involving the resistance R, temperature T, energy gap E_p, and Boltzman constant k_B is given by the following equation.

$$\ln\left(RT^{\frac{3}{2}}\right) = \frac{E_g}{2Tk_B}$$

As shown in Figure 5, plotting resistance as a function of temperature we get a shape resembling an exponential function. Plotting this same relationship as $\ln(RT^{3/2})$ vs. 1/T determined by the theoretical relation above should result in a straight line. This plot can be seen in Figure 6 and as expected, a linear fit to the data produces an equation y = 3368.38x + 1.6283 where our slope is 3368.38 ± 1.9. By the hypothetical relationship, we know that this slope m can be used to calculate the energy gap as follows.

$$m = \frac{E_g}{2k_B}$$

This results in an energy gap E_g = 0.58053 ± 0.00017 eV. As expected, this value falls between the range [0.5, 1.0] eV and provides supporting evidence for the relationship between resistance and temperature for a semiconductor.

VII. Conclusion

In the experiments conducted here we have drawn many important conclusions about resistors in circuits. For experiments one, two, and three we showed how using Ohm's Law, Kirchhoff's Law, and the Superposition Theorem, we can predict the voltage, current, and resistance for given circuits to very precise values. This is important especially for uses in areas such as consumer electronics, household equipment, and laboratory research where we depend on electronics as tools that we want to be predictable and dependable.

Secondly, in the last two experiments we showed that resistance is not a static property of a material. Resistance is highly dependent on temperature for many materials, especially metals, semiconductors, and superconductors. Areas where this dependence is most important include computer processors where semiconductors are highly present for their resistance properties and advanced machinery such as maglev trains, MRI machines, and particle accelerators where superconductors are used for their extraordinary property of negligible resistance. We showed that as the material temperature decreases, resistance in pure metals decreases, resistance in alloys remain nearly intact, resistance in semiconductors increases, and resistance in superconductors tends towards zero. As we took a closer look at semiconductors, we saw that they have a predictable functional dependence on temperature and can be modeled with extreme precision. Understanding the interaction between resistance and temperature is highly important for implementation of theoretical systems as practical circuitry.