Waves

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I. Introduction

Waves are disturbances in some medium that behave in an oscillatory manner. Waves can be seen in continuous materials like strings, collections of particles like water, or fields like light. Standing waves such as the vibration of a guitar string oscillate in time in fixed positions whereas traveling waves oscillate over space in a specific direction. Traveling waves transport energy from one place to another and are the most common type of wave. Waves have distinctive characteristics: amplitude and frequency that describe its motion and can be driven or dampened which is often the case in the real world. These characteristics can be described mathematically by wave functions and added together using the concept of superposition. In this experiment we will explore how frequency affects the position and number of nodes in standing waves, measure the speed of sound through air, and obtain resonant frequencies for standing pressure waves.

II. Experiment 1: Standing Waves on an Elastic Cord

For this experiment, we will determine the speed of traveling waves on the cord and resonant frequencies of the resulting standing wave as well as the positions of its nodes. The setup consists of an elastic cord (linear density μ = 4.1 g/m) fixed on one end with the opposite end fixed to a free-hanging weight (m=200g) via a pulley system in order to give a precise tension force in the cord. The pulley is also attached to a motion transducer which can be set to drive the cord at a given sinusoidal frequency. A diagram can be seen in Figure 1.

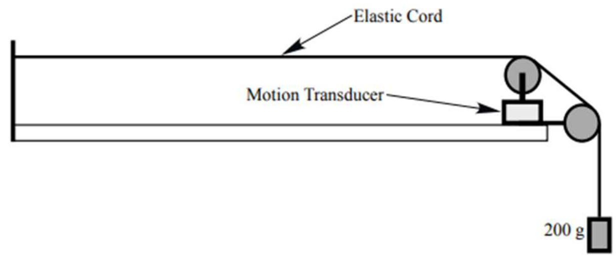

Figure 1: Setup for experiment 1 as described

Using this, we can determine the speed of a traveling wave across the cord as follows:

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{mg}{\mu}} = 22m/s$$

The length from the fixed end of the cord to the pulley attached to the motion transducer was found using a measuring tape to be L = 130 ± 0.5 cm. Using v and L we can predict the fundamental frequencies of the cord as follows:

$$f_n = \frac{v}{2L}$$

The predicted values for the first seven harmonic frequencies are shown in Table 1. The motion transducer frequency was then adjusted until the proper number of antinodes appeared in the standing wave corresponding to the harmonic number n and further tweaked until what appeared to be the resonant frequency was found, characterized by the largest amplitude. The specified frequency was recorded, and corresponding wavelength λ was measured by measuring the distance between the node fixed to one and the next node on the cord using a measuring tape. These values can be seen in Table 1, as well as the measured velocity of the wave. Much uncertainty can be introduced during this step for several reasons: the inaccuracy of measuring with a tape due and difficulty of finding the true position of second node in the middle of the cord.

Table 1: Quantities for Elastic Cord				
N	$f_{predict}$ (Hz)	$f_{measure}$ (Hz)	$\lambda/2$ (m)	$V=\lambda f_n$ (m/s)
1	8.46	8.710	1.3	22.6
2	16.92	17.420	0.661	23.0
3	25.38	26.38	0.441	23.3
4	33.85	35.95	0.328	23.6
5	42.31	44.60	0.265	23.6
6	50.77	53.93	0.220	23.7
7	59.23	62.56	0.188	23.5

Another possible source of error is due to the setup of the experiment. At certain frequencies, the part of the cord between the two pulley wheels was at its resonant frequency. Because this was connected to the segment of cord we were measuring, some disturbance could have been introduced to the system and true resonant frequency not observed.

In order to see how well our measurements agree with the theory of harmonic frequencies we can graph resonant frequency f_n vs. harmonic number n. This should be a linear relationship as we'd expect $f_n=n*f_0$. Using the least squares function in excel with these values we can find a line of best fit whose slope represents the fundamental frequency f_0. The graph is shown in Figure 2 and least squares data in Table 2.

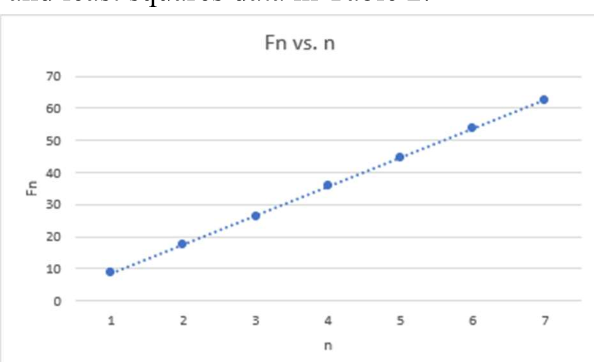

Figure 2: Harmonic number vs. observed resonant frequency

	Slope	Intercept
Value	9.027929	-0.46257
Error	0.045085	0.201625
r	0.999875	0.238566

Table 2: Least-squares regression statistics

Using the value of the slope for line of best fit we can find a more robust measure of the velocity of a traveling wave through the cord:

$$v = \lambda f_0 = (2.6)(9.028) = 23.47\ m/s$$

III. Experiment 2: Standing Waves on a Steel Wire

Similar to experiment 1, we will be using the relationship between velocity of a wave, frequency, and wavelength to determine properties of a system. However, this is a much more precise system using a sonometer with less room for both human and mechanical error. The setup consists of a steel wire above a cm-marked platform fixed on both ends to rigid supports at 10cm and 70cm. The wire is under tension due to a 1kg mass hanging from a lever in such a way that its mechanical advantage is 5. Therefore, the tension force $F_T = 49N$. The mass per length of the steel wire is $\mu = 1.84$ g/m. Using this we can predict the velocity of a wave through the medium will be:

$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{49N}{1.84g/m}} = 163.2m/s$$

Attached to the platform below the steel wire is a magnetic driver which uses an electronic oscillator to pass AC current through the driver coil at a frequency f_{osc}. This produces a magnetic field which drives the wire at twice the frequency ($f_{wire}= 2f_{osc}$) because the wire is attracted by the magnetic field at the maximum magnitude of amplitude of each oscillation which occurs twice per cycle. The frequency and amplitude of the driving oscillation can be adjusted to achieve any desired motion. Also attached to the platform below the wire is an electromagnetic sensor which measures the frequency of the wire f_{wire} by creating a magnetic field through which the wire induces an oscillating voltage at the same rate as f_{wire}. This information is transmitted to an oscilloscope which plots the voltage as a function of time for us to view and taking the period from this oscillation reproduces the frequency of the wire f_{wire}. This sensor must be placed below an antinode on the wire and can be adjusted to any position along the wire in order to do so.

With the magnetic driver off and the oscilloscope reading, the wire was plucked and after transient motion had died down, four complete cycles were recorded. The oscilloscope reading can be seen in Figure 3. The frequency of one cycle in this case is the natural frequency and the average over four cycles was found to be f_0 = 134.7 Hz.

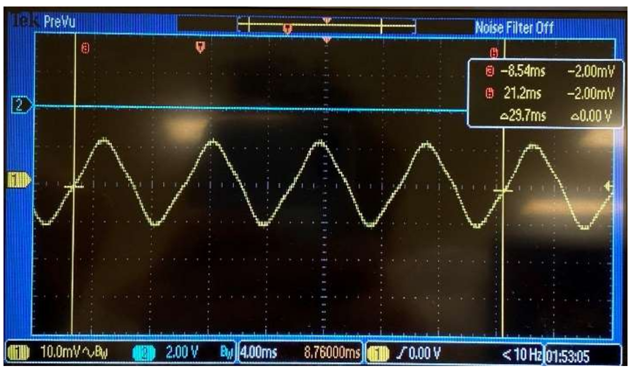

Figure 4: Natural frequency of steel wire

Next, we want to find the decay time τ of the wire to estimate the Q factor of the system. To do this, the wire was plucked again and using the build in clock on the oscilloscope, measured the time for all motion in the wire to subside. After performing this three times, the average decay time was found to be τ = 12 s. Using this, we can estimate our Q factor:

$$Q = \pi \tau f_0 = \pi(12)(134.7) = 5078$$

To test our theoretical prediction, we want to find where the wire reaches resonance with maximum amplitude. Because our Q factor is so high, very subtle changes in the frequency will dramatically change the amplitude and adjusting to find resonance is difficult. After homing in on the highest achievable amplitude, the driving resonant frequency was found to be f_0 = 67.074 Hz. This is extremely close to the expected driving frequency of 67.34 Hz. A graph of the oscillation is shown in Figure 4. The yellow line represents the oscillation of the wire and the blue line represents the driving oscillation. Notice how the wire has twice the frequency of the driver. This is because of the reasons described above and shown graphically.

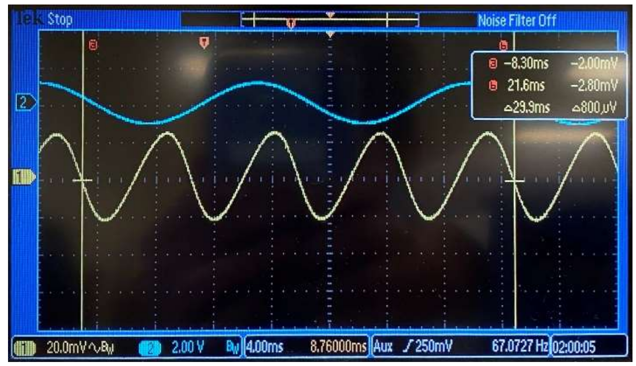

Figure 3: Wire driven to resonance

Now we will repeat this process to find resonant frequencies for higher harmonics. Similar to experiment 1, we want to graph these frequencies to find a linear relationship between frequency and harmonic number, the slope of which will give use a precise measurement of the resonant frequency to compare to the expected value. The procedure was done for the first five harmonics and the resulting graph and least-squares regression statistics in Figure 5 and Table 3 respectively. The data corresponding to these can be found in Table 4.

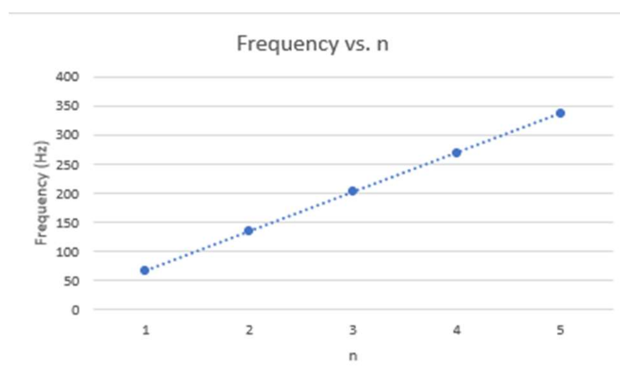

Figure 5: Graph of harmonic number n vs. measured resonant frequency

Slope	Intercept
67.475	-0.515
0.054206396	0.179782
0.999998064	0.171416

Table 3: Least-squares regression statistics for Figure 5

Table 4: Resonant Frequencies by Harmonic Number				
n	f_{osc} (Hz)	f_{wire} (Hz)	λ (m)	Nodes (cm)
1	66.9	133.8	1.3	10, 70
2	134.52	269.04	0.60	10, 40, 70
3	202.02	404.04	0.40	10, 30, 50, 70
4	269.15	538.30	0.30	10, 25, 40, 55, 70
5	336.96	673.92	0.24	10, 22, 34, 46, 58, 70

There are a couple sources of error in this procedure. The detector placement may not be ideal because it is difficult to place the sensor exactly under an antinode for the wire and any slight variation could introduce some uncertainty. Additionally, peak resonance may not be achieved in these cases because the maximum amplitude is merely measured by the human eye and may be off by a few hundredths of a hertz. However, according to the graph and data, the slope of the regression line indicates that the fundamental frequency of the wire f_0 = 67.475 has very high correlation in our data with a coefficient of 0.999998. This measured frequency differs from the theoretical frequency by 0.2%, less than one standard deviation from the expected value.

IV. Experiment 3: Speed of Sound

Now we wish to measure the speed of sound in room-temperature air. To do this, there is a signal generator placed on one end of a plastic tube, with the other closed off by an adjustable piston. The signal generator produces sound waves at 20 Hz and reflected sound can be picked up by an embedded microphone which are translated to the oscilloscope for data acquisition. The speed of sound can be measured this way because a pulse of sound is let out by the generator and reflected wave received in the same place. Using the oscilloscope, we can measure this very precisely using the time elapsed t and length of the tube L. The velocity will be given by:

$$v = \frac{2L}{t}$$

Placing the piston at L = 0.5 m we find that t = 2.93 ms and plugging into our equation above we find v_{sound} = 341.2 m/s. This observed value differs from the expected value of 340 m/s by 0.35%, an accurate measurement. It is also noteworthy that when retracting the piston entirely to the full length of the tube, polarity of the sound wave is flipped. This is due to the fact that by retracting the piston fully we have created a tube of fixed pressure for sound to travel through.

V. Experiment 4: Standing Waves in a Tube

Continuing with the same setup as in experiment 3, we now want to find resonant frequencies of sound as a function of tube length L. Rather than before where we varied the frequencies, we will use the piston to vary the length of the tube while maintaining constant frequency of 840 Hz. This will achieve different resonant lengths for the sound wave produced in the tube where the open end with the signal generator acts as a pressure node and the piston becomes a pressure antinode. This diagram is shown in Figure 6.

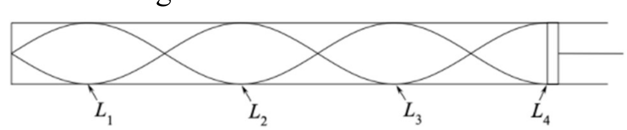

Figure 6: Diagram of sound wave tube with pressure wave and resonant lengths shown

These resonant lengths will be achieved when the largest amplitude is seen in the oscilloscope, coupled with a sharp cry from the signal generator indicating resonance. The resonant lengths were recorded and plotted against their harmonic number in Figure 7 with corresponding data in Table 5.

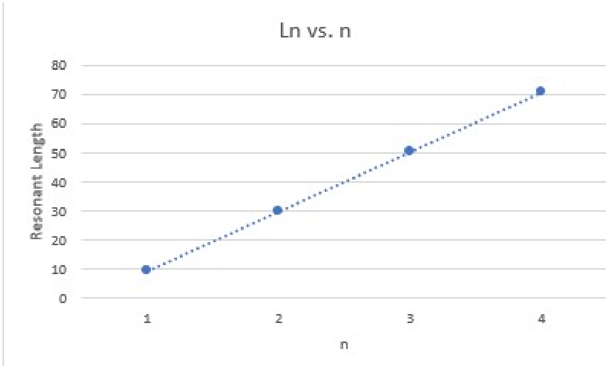

Figure 7: Graph of harmonic number vs. resonant length (cm)

	Slope	Intercept
Value	20.47	-11
Error	0.079372539	0.217371
r	0.999969931	0.177482

Table 5: Least-squares regression statistics

The slope of the regression line represents half the wavelength of the sound pressure wave traveling through the tube. According to our data, the wavelength of sound traveling through the tube is $\lambda = 40.94$ cm. Using this we can find another estimate for the speed of sound through the tube:

$$v = f\lambda = (840)(0.4094) = 343.9 \text{ m/s}$$

This measured velocity differs from the known value for the speed of sound (340 m/s) by 1.1%. This is further off than in experiment 3 but still less than one standard deviation away from the expected value.

VI. Conclusion

Through these four experiments we have provided support for several conclusions. First, we have seen how the velocity of a wave through strings or wires can be modeled by $v_{wave} = \sqrt{\frac{F_T}{\mu}}$ where F_T represents the force of tension in the medium and μ represents the linear density of the material. Wave velocity can also be predicted by the relationship $v = \lambda f_0$ and in experiments 1 and 2 we showed that these two methods produce the same result. Velocity of a fluid wave such as sound cannot be found by the first equation but was instead substituted for $v = \frac{2L}{t}$ where we found the time t it took for the wave to travel a distance L and back. This also proved to be a valid measure of the wave velocity, agreeing with $v = \lambda f_0$ for the known value of the speed of sound.

We have also proven the quantization of resonances. Each point of resonance of a wave can be represented as a multiple of the fundamental resonances. In experiments 1 and 2 we saw this through varying the frequency of the driver to uncover multiple resonant frequencies for a given length of string/wire where each frequency was an integer multiple of the natural frequency f_0. In experiment 3 and 4 we saw this slightly differently in the form of resonant lengths. Using the same frequency, we found resonant lengths by changing the length of the piston and found that each length was an integer multiple of the fundamental length L_0.

Knowledge about the interaction of wave velocities and resonances is important for many applications in science and technology. Construction is the most well known where buildings and bridges can even have their own resonant frequencies that drive them to destruction. Waves are part of everyday life because as we have shown in these experiments, anything from a solid to a fluid can be modeled by a wave and treated as such.