Interference and Polarization

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I. Introduction

Interference is a phenomenon that occurs in waves where the intersection of two or more waves results in addition or subtraction of the wave amplitude called constructive and destructive interference respectively. Interference occurs in all types of waves provided that they are not polarized perpendicular to each other. Polarization refers to the orientation of a transverse wave as it propagates through space with types including linear and circular polarization. The direction of polarization in electromagnetic waves denotes the direction of the variation in the electric field. In the following experiments we will use these concepts to study the wavelength of electromagnetic waves, the effects of polarizing filters, and refraction of light.

II. Measurement of the Wavelength with a Michelson Interferometer

As we have done before in previous experiments, we will be working with a Michelson Interferometer. However, this interferometer will have several modifications so that we can study some interesting interactions between microwaves and wire grid polarizers. The setup for such an instrument is shown in Figure 1 where we have a microwave transmitter at point D, microwave receiver at point E, adjustable reflective plates at points A and B, and a wire grid polarizer at point C. The wire grid works as a polarizer for microwaves because of its unique size and layout. Waves which pass the grid having an electric field oriented along the length of the wires are blocked entirely while waves whose electric field is oriented perpendicular to the wires are free to pass. This results in the transmitted wave being polarized in the direction perpendicular to the wire grid. For our setup we will emit microwaves at 10.5 GHz and a wire grid of aluminum rods spaced 1.5 cm apart. This corresponds to roughly half of the wavelength of the microwaves and thus can interact with each other. The now polarized transmitted is reflected off the plate at point A, returning to the wire grid at C, where it is reflected into the receiver at E.

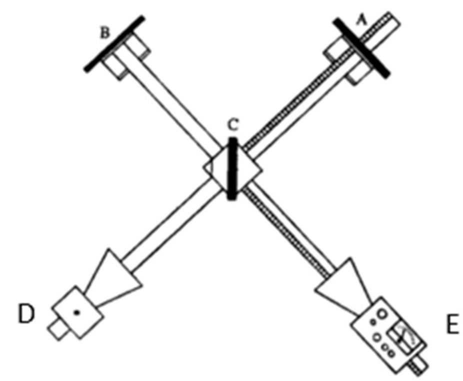

Figure 1: Microwave Interferometer setup

To find the frequency, we measure the wavelength of the emitted microwaves and record the distance *d* traveled from C to A and maxima index *m* when a maximum intensity of the received waves at E is found. The wavelength can then be calculated as follows:

$$\lambda = 2\frac{\Delta d}{\Delta m}$$

Recording these values for several maxima and finding their slope will give the best average value of one-half wavelength $\frac{\Delta d}{\Delta m}$. Ten of these values were recorded and distances d are plotted as a function of index m in Figure 2 and corresponding least squares regression fit in Table 1.

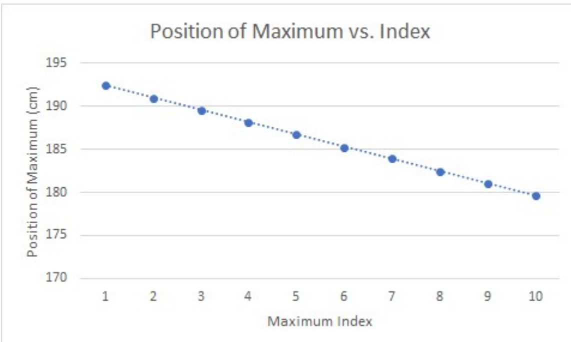

Figure 2: Position of Maximum vs. Index for microwave-wire grid interferometer

	Slope	Intercept
Value	-1.42485	193.8667
Error	0.006749	0.041875
Correlation	0.999821	0.061299

Table 1: Least-squares regression fit for maxima position vs. index for microwave-wire grid interferometer

As the reflector was moved towards the center of the interferometer, the distances were decreasing. Therefore, the slope of the line is negative. The slope corresponding to the regression fit is y = -1.42485x + 193.8667. The slope $\frac{\Delta d}{\Delta m}$ was found to be -1.4249 ± 0.0067 cm and wavelength λ = 2.8498 ± 0.0095 cm. Converting this to frequency f:

$$f = \frac{c}{\lambda}$$

We find our calculated frequency f = 10.527 ± 0.035 GHz with a correlation of 0.9998 is off by 0.26% from the expected frequency of f = 10.5 GHz. A source of uncertainty for the measurements in this experiment is that the detector was very sensitive. Any sort of movement around the sensor affected the recorded intensity of the beam for several reasons. The wire grid polarizer used has inconsistencies: varying diameter in the wires and imperfect orientation of the wires. Additionally, the setup does not actually measure the intensity of the microwave. The emitter is an oscillating dipole that creates an electromagnetic wave. The detector is also a dipole that is excited to oscillate at the frequency of the wave incident on it. The detector measures the field's strength not the intensity of the wave so it will observe approximately a cos(x) pattern rather than the theoretical $\cos^2(x)$ pattern. This however does not have a significant impact on our measurements because both cos(x) and $\cos^2(x)$ have the same positions of maxima which are the only values we care about.

III. Polarization

Now we want to test the polarization of the transmitter with respect to the receiver. To do this, we setup the transmitter-receiver pair as shown in Figure 3. Here, the microwave transmitter and receiver are spaced roughly 35cm apart.

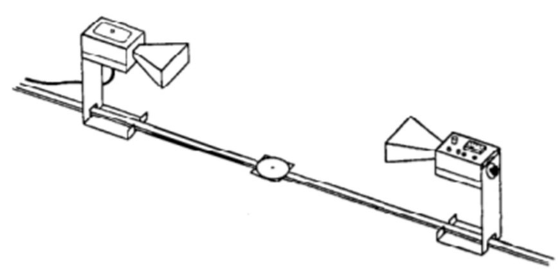

Figure 3: Polarization setup with rotating transmitter

The transmitter can be rotated to produce microwaves polarized in the direction of the cone. As the transmitter is rotated, we expect to see changes in the intensity of the received waves as a function of the angle θ modeled by $I_{max}\cos^2(\theta)$ where I_{max} is the maximum intensity reading. To measure this experimentally we start with the transmitter rotated to θ = -90º and rotate it to θ = 90º in increments of 10º. The output of the meter was recorded for each angle value. The meter reading was plotted as a function of the angle of rotation of the receiver and can be seen in Figure 4 and can be compared to the shape of a $\cos^2(\theta)$ function shown in Figure 5.

Figure 4: Microwave receiver reading as a function of transmitter rotation

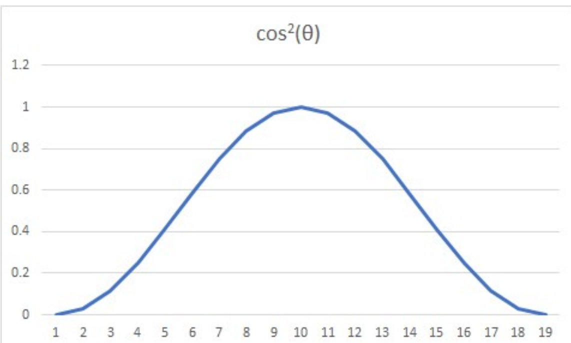

Figure 5

The estimated uncertainty for the receiver measurements are ± 0.050 mA. This uncertainty is caused mostly by the needle moving back and forth while the setup was stationary. This was most likely caused by sources of movement and other light sources in the room. The shapes are clearly very similar with a few edits. The tails of Figure 4 indicate that the receiver has a minimum threshold before a reading is made. Thus, the intensity does not begin to rise until ± 70º. The reading for θ = -20º appears to be a faulty measurement which can be attributed to the techniques used for measuring the intensity because as stated before, this is a measure of the strength of the electric field, that although is close, it is not the true intensity of the wave.

Now we will introduce our wire grid polarizer back into the setup as shown in Figure 6. The polarizing direction is perpendicular to the direction of the incident microwaves. The intensity reading on the receiver is I = 0.810 ± 0.050 mA to begin. Rotating both transmitter and receiver by 90º yields a maximum intensity of I = 23.700 ± 0.050 mA for the transmitted wave. This indicates that the direction of polarization was as expected, perpendicular to the wire grid polarizer.

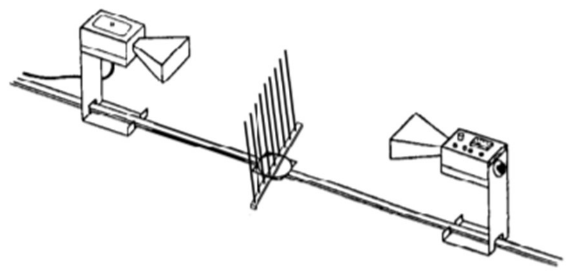

Figure 6: Microwave polarizing wire grid setup

Now we will show that it is simply the relative orientation of the grid and microwaves that determine polarization. The transmitter and receiver are reset to 0º and the wire grid rotated 90º until the wires were parallel to the table. The intensity increased to a maximum value equivalent to the one above. We'd expect the functional dependence for the intensity as a function of angle of the rods with respect to the vertical to be sinusoidal as it reaches a maximum when the grid polarizer is horizontal.

The receiver was rotated to 90º so its polarization axis is perpendicular to the axis of the transmitter. As in the procedure above, the polarizer was rotated from 0º to 90º with respect to the vertical. The peak intensity was found at ~ 45º. Therefore, the functional dependence should be similar to $\sin^2(x)$.

Now we will show that the wire grid polarizer also acts as a mirror for waves polarized parallel to the wires. The goniometer was rotated from 180º to 90º as shown in Figure 7 to find the maximum intensity reading.

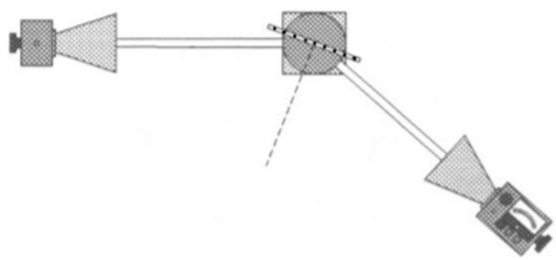

Figure 7: Microwave polarizer wire grid as a mirror setup

The angle of incidence and the angle of reflection were measured to be about $40.0º \pm 1.0º$ as expected. Therefore, it can be concluded that the wire grid behaves like a mirror for waves polarized along the length of the wires as they are reflected into the receiver. However, this is not a perfect reflection because as stated before the wire grid is an imperfect device due to inconsistencies in the wires' orientations.

IV. Linear Polarization

For this experiment we will use a polarized laser. However, its polarization direction is not precisely known. To correct this, we place a Polaroid filter in front of the laser with its polarization angle set to zero degrees so that the electric field oscillates in the vertical direction. A wire–grid polarizer, similar in theory to the one used in previous experiments, is coupled to a rotary motion sensor, and mounted near the middle of the optical track. This wire-grid polarizer is built for waves of this wavelength and is a much more precise piece of equipment. There is a light sensor past the wire grid that measures the intensity of the light beam. The wire grid is at a slight angle so that light reflected from it can be detected with a second sensor placed between it and the laser. The gains on the transmitted beam detector and the reflected beam detector were adjusted so that the maximum voltage did not exceed readable values of about 4.4 V. The voltage recorded by the sensors will be proportional to the intensity of the light beam. The setup can be seen in Figure 8.

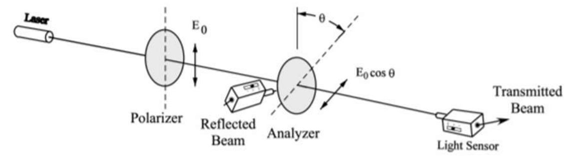

Figure 8: Linear polarizer setup

The wire grid was set to its most negative angle and rotated through its entire range of angles. The intensity as a function of angle was recorded in for both the transmitted and reflected beams. The transmitted beam had a maximum intensity at 0º whereas the reflected beam had a minimum. This follows what we would expect from Malus' Law:

$$I = I_{max} cos^2(\theta)$$

The beams were 180º out of phase, and the reflected beam can be represented with:

$$I = I_{max}\left(1 - cos^2(\theta)\right) = I_{max} sin^2(\theta)$$

A graph of the intensities was made where the dark green plot represents the intensity read at the transmitted sensor and the lighter colored plot is the intensity read at the reflected sensor. The transmitted beam was fit to a $\cos^2(\theta)$ curve and relevant statistics can been seen in Figure 9.

Adjustments had to be made to the reflected beam sensor as the beam moved off the surface of the detector and there were

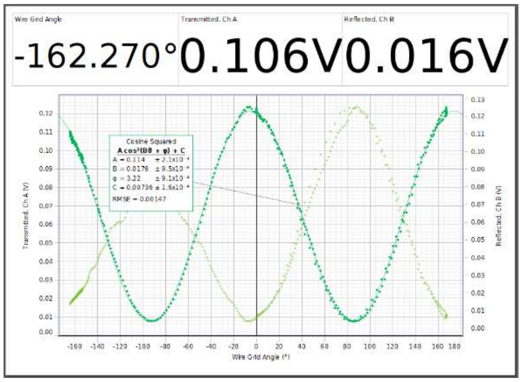

Figure 9: Measure of intensity vs. wire-grid angle for transmitted and reflected laser beams

problems with the recorded intensity. This is one of the possible sources of error as the full intensity of the light was not captured. Another source of uncertainty for this experiment is that the lights in the room were on during the experiment which interfered with the intensity measured by the sensors. This could be reduced by running the experiment in a dark room, however, we care about the qualitative observations in this experiment, so the exact values of the intensity do not matter as much.

A second Polaroid filter was placed between the wire grid and the transmitted beam detector. This polarizer was rotated to 90º with respect to the first filter. The wire grid was then moved through its entire range of angles and the intensity of the beam was recorded which can be seen in Figure 10.

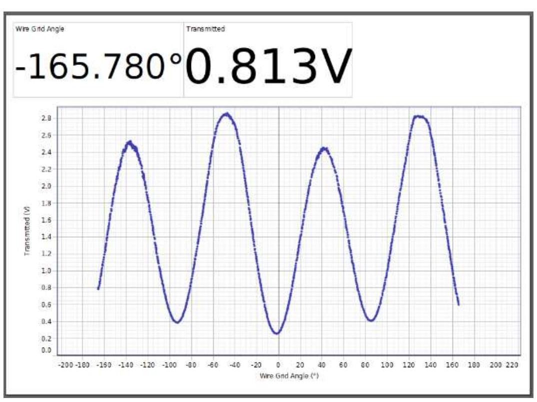

Figure 10: Measure of intensity of the transmitted wave as a function of wire grid angle for a three-polarizer setup

The beam had an intensity minimum at 0º and maximums at 90º which is consistent with the expectation of a $\sin^2(2x)$ function. This however cannot be fit to a sine curve properly because of inconsistencies in the measurements. As you can see, the maximum peaks vary by about 0.5V. This does not matter for our conclusion because the shape of the graph is the most important factor. However, this shows an error in the setup similar to the one seen before for a single polarizer. The wire-grid used for both setups would ideally be a flat plane which perfectly transmits the polarized light. However, the disc is not entirely flat and as it is rotated it causes the transmitted and reflected beams to wobble and on occasion land outside the sensor's measurement radius which causes the variance in maxima intensity.

V. Brewster's Law

Using the same laser, we will now test Brewster's law. The setup will consist of a polarized light beam incident on a plastic "D" shape placed on a rotatable optical bench. The setup is shown in Figure 11.

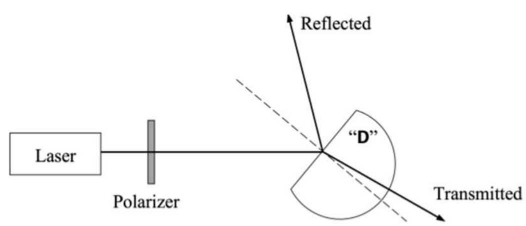

Figure 11: Brewster's law setup

Brewster's Law tells us that there is a configuration of polarizing angle and reflection angle at which the reflected beam disappears. This is the Brewster condition for external reflection. As the table is rotated, a white screen is placed next to the table in order to view the reflected beam. At $\theta = 56.1º \pm 1.0º$, the reflected beam vanished. This is the Brewster condition for external reflection. The refractive index n_2 for the plastic can be calculated using Brewster's Law:

$$\tan(\theta) = \frac{n_2}{n_1}$$

where n_2 is the index of refraction for the plastic and n_1 is the index of refraction for air ($n_{air} = 1$).

$$n_2 = \tan(56.1º) = 1.488 \pm 0.056$$

Since the Brewster angle is only possible when the electromagnetic waves are parallel to the crystal plane of the material (in this case, the horizontal), the angle on the Polarizer was 90º with respect to the vertical. The error in this calculation is simply from the measure of the angle θ on the optical bench.

We can also determine that the Brewster angle is independent of order of the mediums. The polarizer was moved from its place between the laser and the plastic D to be between the D and the screen. The polarizer has the same effect on the reflected beam in this order and at the Brewster angle the intensity goes to zero. However, the same polarizer had no effect on the beam transmitted through the plastic. This is consistent with expectations.

Similarly, we can measure the condition for internal reflection. This is done by rotating the plastic "D" piece 180º so that the light enters the curved side first. The observed Brewster angle for this curved side was observed to be $\theta = 35.2º \pm 1.0º$. The Polarizer angle was 90º yet again, so the electric field was parallel to the horizontal surface of the plastic D. Brewster's Law still applies; however, in this case, n_2 is the refractive index of air and n_1 is the refractive index of the plastic.

$$n_1 = \frac{n_2}{\tan(\theta)} = \frac{1}{\tan(35.2º)}$$

$$n = 1.1418 \pm 0.026$$

Another estimate for the refractive index of the plastic can be found by slightly rotating the plane away from total reflection. This occurred at an angle $\theta = 37.0º \pm 1.0º$. The index of refraction from the angle for total internal reflection can be calculated with:

$$n_1 = \frac{n_2}{\sin(\theta)} = \frac{1}{\sin(37º)}$$

$$n = 1.662 \pm 0.014$$

These three values for the indices of refraction are very similar, but clearly the best estimates are achieved through the first two procedures as they are more precise.

VI. Other Polarizing Experiments

Calcite is known as a birefringent material, that is, its refractive index depends on the polarization of the light passing through it. A piece of calcite is placed on a sheet of paper with words written on it. A double image of the words could be seen through the calcite as the light passing through the material takes multiple paths through the material according to its polarization. As the piece of calcite is rotated, the images rotate around each other. However, when viewed through a polarizer, the multiple images disappear. This is because the light which is seen through the polarizer is polarized in the same direction and only one image is seen.

Light from a computer screen is also polarized and can be used to demonstrate depolarization. Viewing the computer monitor's screen through the polarizer and rotating it shows that the computer screen is polarized light because the intensity of the light through the polarizer decreases for various angles as we'd expect for polarized light. At particular angles, the light from the computer disappears completely, and we can only see darkness through the polarizer. However, when we put a piece of wax paper in front of the screen and viewed it through a polarizer, the computer screen turned dark as before, but light still shines through the wax paper as though no polarizer is present. Therefore, the light transmitted through the paper must not be linearly polarized and it is depolarized.

Observing a Helium Neon laser shining through a tank of a colloidal solution shows that the degree of polarization of scattered light in this case is a function of incident angle. While the laser itself is not polarized, when viewing the beam of the laser in the tank through a Polarizer, the intensity of the beam changes as the polarizer is rotated.

VII. Conclusion

Throughout these experiments we have confirmed several concepts and laws about electromagnetic radiation. First, in all the above experiments we showed how light can be polarized and that by using specialized polarizers we can filter out specific orientations of the electromagnetic spectrum according to their electric field variation. In experiments one, two, and three, we used wire-grid polarizers which are specially crafted for a certain wavelength of light and proved that they can also act as mirrors for light not polarized in the proper orientation.

We showed that these, along with polaroid filters, filter light dependent on a given angle offset. In experiment two we showed that this offset is a relative angle, that either the emitted light or the filter can be oriented, and the same polarizing result is produced. In experiment three we confirmed Malus' Law which predicts the intensity of light transmitted and reflected from a polarizing filter as a function of the relative angle and that this holds for multiple polarizing filters.

In experiment four we confirmed Brewster's Laws about the internal and external reflection angles. Using these angles, we showed that it is possible to calculate the refractive index of a material through several different strategies. Furthermore, we showed that the order in which the light passes through the given mediums does not change the outcome for a Brewster angle reflection, but it does for the transmitted beam.

Finally, we demonstrated a few applications of the principles studied in these experiments. Using a birefringent material such as calcite, we can understand how light passing through an object can be distorted based on the atomic structure of the material. In the case of a birefringent material, we showed that it has a varying index of refraction based on the polarization of incident light.

An important principle was also shown with the polarized light emitted from a computer monitor. We showed that the light was polarized and that by placing a wax paper between the screen and the filter, the light passing through the paper was depolarized. We saw a similar effect with the HeNe laser through the colloidal solution where a material can change the polarization of light. In the case of the HeNe laser, we showed that light was polarized in differing amounts through the fluid.