The Pendulum

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I. Introduction

A pendulum is defined as any rigid object suspended by a fixed point in such a way that it can oscillate back and forth due to the force of gravity. The period of these oscillations are highly sensitive and are determined by the length of the pendulum and strength of gravity at its location. This allows for pendulums to be used as measuring devices for the specific gravitational acceleration (*g*) at a location which is further explored in this experiment to test the accuracy of current measurements for *g*. Initially we can use bobs connected to a fixed point via nylon string and use small angle formulas to approximate a simple harmonic oscillator.

II. Determining the Local Acceleration of Gravity

For our first experiment we wanted to find the local acceleration *g* using a brass bob suspended by a string by measuring the period of an oscillation. Precise measurements were obtained through use of a photogate. The brass bob was measured to have a mass of 84.8 ± 0.1 g, length of 63.67 ± 0.01 mm, and diameter of 18.96 ± 0.01 mm. The center of mass of our brass bob was 752 ± 1 mm below the fixed point of the string.

The moment of inertia for a simple cylinder is known to be:

$$I_{CM} = m(l^2 + 3d^2/4) / 12$$

Using this and the parallel axis theorem:

$$I_p = I_{CM} + mL_{CM}^2$$

we can determine I_p. Using the relationship:

$$L_p = I_p/(mL_{CM})$$

we find that the effective length for a simple pendulum L_p = 753 ± 1 mm. L_p is important for these calculations because it allows us to relate the frequency and therefore period to the acceleration the pendulum experiences due to local acceleration *g* which we are trying to measure. We can find the period for the simple pendulum in terms of *g* and rearrange the equation to solve for *g*:

$$T = 2\pi\sqrt{g/Lp}$$

$$g = 4\pi^2 L_p/T^2$$

Experimentally, the pendulum period was recorded to be 1.7394 ± 0.00003s and subsequent value for *g* = 9.819 ± 0.008 m/s^2. With the expected value for *g* = 9.795 ± 0.005 m/s^2 this measured *g* is larger by 2.6σ. This is a fairly significant difference in measured *g* value but not enough to raise concerns about the procedure because it is still within the accepted 3σ range.

III. Dependence of Period on Length

Results from the first experiment cannot yet be conclusive because several variables have not yet been tested with this method. In this experiment I will show obtained value of *g* does not change when the period and length are changed. This is because by changing the length of the pendulum, the period must also change and as a result the calculated acceleration *g* remains the same. Rearranging the equation used to calculate *g* using period and effective length, we obtain:

$$T^2 = 4\pi^2/g * L_p$$

This equation is useful for creating a visual representation of the period-length dependence when graphing T^2 vs. L_p. This is shown in Figure 1 where varying pendulum lengths produced corresponding periods that had nearly perfect correlation. Using a least squares line regression finds a slope of 4.0170 ± 0.0032 s^2/m. This slope is equal to $4\pi^2/g$ which yields an experimental g of 9.828 ± 0.008 m/s^2. This is 3.5σ above the expected value of 9.795 ± 0.005 m/s^2 which is a highly unlikely result (0.02%). Combined with the precision of nearly perfect correlation between varying lengths and periods, either the expected value of g_{local} is wrong or (more likely) the experiment setup had a slight variation that carried the whole way through the procedure.

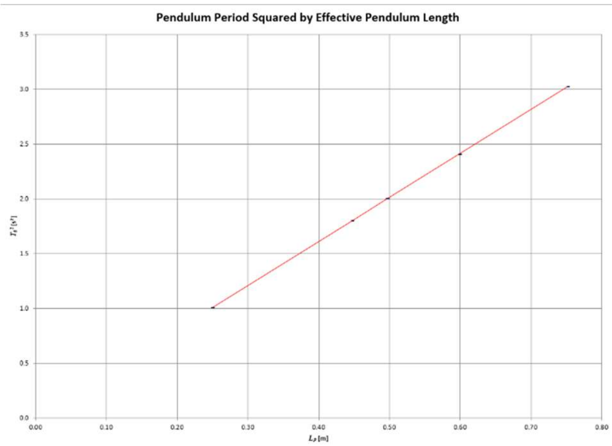

Figure 1. X-axis represents the length of the pendulum and Y-axis represents the square of the oscillation period with a least squares regression line having slope equal to 4π2/g.

IV. Equivalence Principle

Another variable that was not tested in the initial experiment was the mass of the bob. Substituting in values for I_p from the parallel axis theorem into the equation for period we find that mass should cancel out:

$$T = 2\pi\sqrt{\frac{\frac{1}{12}\cancel{m}\left(l^2 + \frac{3}{4}d^2\right) + \cancel{m}L^2}{g\cancel{m}L}}$$

To prove this experimentally, a dimensionally identical bob made of aluminum was tied off at the same spot as the brass bob and their measured periods were compared in order to determine any significant difference due to mass. The same brass bob from previous experiments was used in comparison to an aluminum bob weighing 26.8 ± 0.1 g, length 63.23 ± 0.01 mm, diameter 18.90 ± 0.01 mm, and identical $L_{CM} = 752 \pm 1$mm below the fixed point of the string. The measured period $T_{Al} = 1.73910 \pm 0.00003$ s differed from the period $T_{Br} = 1.74130 \pm 0.00004$ s by 0.034σ from the expected difference of 0 assuming the periods are equal. This highly accurate measurement suggests that the assumption of mass independence holds true for the period of oscillation.

V. Dependence of Pendulum Period on its Amplitude

The last variable which has been ignored up to this point has been the amplitude of the oscillator. This is because for small angles such as those used in the previous experiments, the period is not dependent on amplitude. In order to correct this, a new setup is required in which a simple pendulum and photogate setup will be used in order to find the period of arbitrary angles. This simple pendulum was a brass rod of length 500 ± 1 mm, mass 84.4 ± 0.1 g, and center of mass 245.0 ± 0.5 mm below its fixed point.

The period of such a pendulum is found using the average period (T_0) over several oscillations and change in angle (θ):

$$T = T_0[2/\pi\, K(k)]$$

$$k = \sin^2(\theta/2)$$

The resulting periods of several arbitrary angles were calculated using the experimental data and plotted on a graph of Period vs. Amplitude as shown in Figure 2. The theoretical curve of the relationship between period and amplitude is shown along with error bars representing the experimental values. These values are quite close to the expected theoretical calculations and therefore reinforce the use of the calculations.

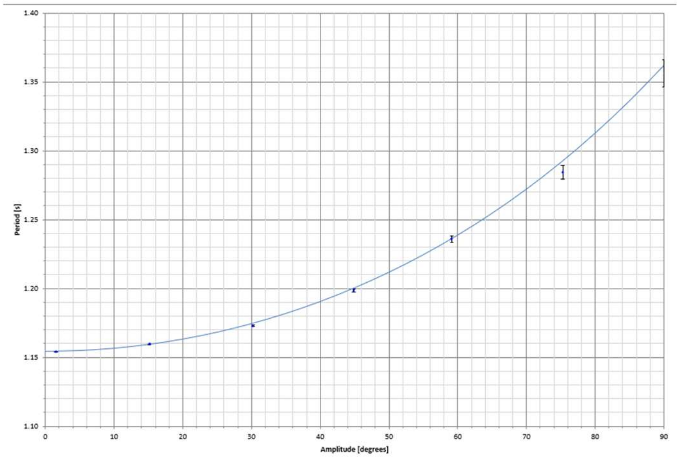

Figure 2. X-axis represents the measured amplitude of the oscillation and the Y-axis represents the calculated period of the brass rod.

VI. Conclusion

The data gathered in this experiment follow closely to the theoretical calculations which they are compared to. This supports the validity of such calculated assumptions about the oscillation of pendulums in the local gravitational field. Additionally, the strength and consistency of such a field is confirmed by the evidence presented in the experiment. While not proven in this somewhat limited experiment, it provides further evidence that the mass of an object is consistent independent of its circumstance in a uniform gravitational field.