AC Circuits

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I. Introduction

Circuits that combine resistors (R), capacitors (C), and inductors (L) are crucial to our understanding and usage of electricity. RC and RL circuits are especially helpful for creating low-pass and high-pass frequency filters. These special circuits allow you to attenuate desired frequencies as they pass through the circuit for specific values of R, C, and L. In the following experiments we will explore the relationship between frequency attenuation and component values as well as the response time of the circuits to square and sinusoidal waves. The circuit will be supplied frequencies by a function generator and evaluated using an oscilloscope set to trigger on the function generator's signal. The circuit will be setup using a breadboard with 22-AWG wires and nominal component values which will be verified before use and calculations with a bench meter. We will be working with RC and LCR circuits and find the relevant Bode plots and Q values.

II. Series RC Circuit: Time Domain

Here we will analyze a simple RC circuit supplied with an AC square wave produced by the function generator. This square wave will have the form shown in Figure 1.

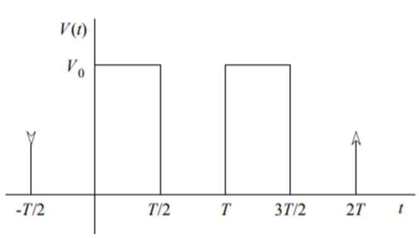

Figure 1: Square wave with frequency 1/T

The function generator produces a 6 V_{PP} square wave with a 3 V_{DC} offset at 1kHz. To build the RC circuit, we will use a resistor R and capacitor C assembled on the breadboard as shown in Figure 2:

$R = 0.992 \pm 0.001$ kΩ

$C = 27.54 \pm 0.01$ nF

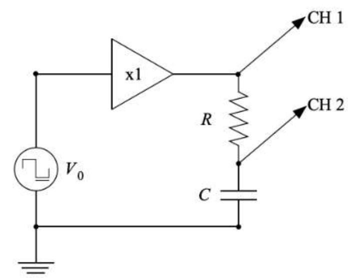

Figure 2: RC circuit setup

The CH1 10X probe measures the voltage produced by the function generator with respect to ground and the CH2 10X probe measures the charging and discharging of the capacitor with respect to ground. We want to determine the time constant τ for this circuit which will be done in two ways. The time constant represents the time it takes for the capacitor to charge to $V_0(1 - \frac{1}{e})$ or the time it takes for the capacitor, once charged, to discharge to $V_0(\frac{1}{e})$. These will be denoted by the terms τ_+ and τ_- respectively.

$$\tau_+ = 25.4 \ \pm 0.2\ \mu s$$

$$\tau_- = 25.0 \ \pm 0.2\ \mu s$$

$$RC = (992\ \Omega)(27.54\ nF) = 27.32 \pm\ 0.029\ \mu\text{s}$$

The time constant can also be calculated as shown above by the product RC which roughly agrees with the experimental values.

III. Series RC Circuit: Frequency Domain

Now we will measure the response of the RC circuit to a sinusoidal AC input from the function generator. This is set to produce a 12 V_{PP} sine wave, with 0 DC offset, at the expected cutoff frequency f_c. Past this frequency, attenuation becomes effective. The cutoff frequency is calculated below using the experimental value of τ:

$$f_c = \frac{1}{2\pi\tau} = 5779\ Hz$$

The cutoff frequency can also be experimentally determined by finding the frequency at which the ratio $\frac{V_{in}}{V_{out}} = \frac{1}{\sqrt{2}}$ or the phase shift ϕ is -45°. This however was difficult to precisely measure due to fluctuations in the system. The best measure of the cutoff frequency was found to be f_C = 6.00 kHz at ϕ = -44 ± 1°.

Now we will create Bode plots for the amplitude and phase shift as functions of frequency by plotting selected points ranging from $\frac{f_c}{100}$ to $100f_c$. These measurements were recorded and compared with the theoretical model (green line) for the amplitude and phase shift plots as shown in Figures 3 and 4 respectively.

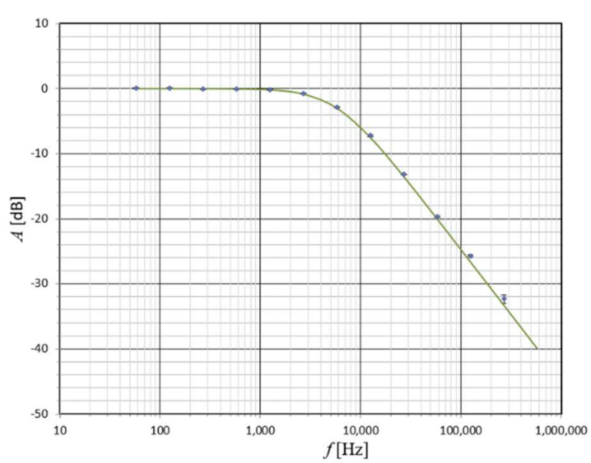

Figure 3: Amplitude vs. Frequency Bode plot on a logarithmic scale for RC circuit

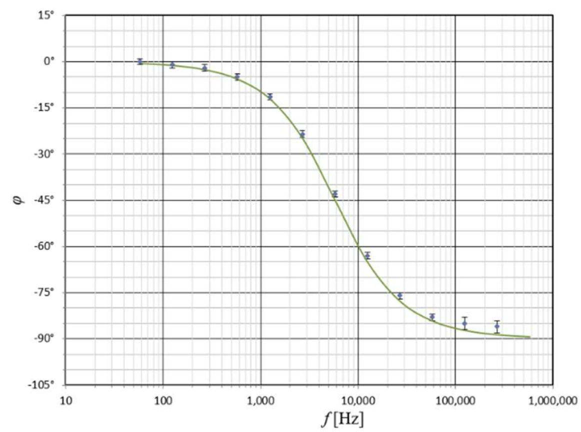

Figure 4: Phase Shift vs. Frequency Bode plot with a logarithmic scale for RC circuit

These experimental datapoints line up with the theoretical model very closely as all of the points and their error bars lay on the expected curve. There were a couple outliers as the frequency reached more extreme values towards the range of 500,000 Hz. This outlier was excluded from the plot as it is likely a technical fault.

IV. Series LCR Circuit: Time Domain

This experiment will analyze attributes of an LCR series circuit similar to that of the RC series circuit. The circuit setup is shown in Figure 5 and components used to create the circuit were tested using the bench meter and are shown below.

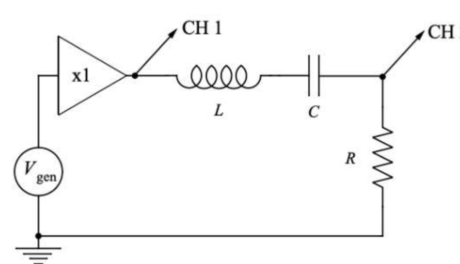

Figure 5: LCR series circuit setup

$R = 100.3 \pm 0.1\ \Omega$

$C = 10.3 \pm 0.1$ nF

$L = 24.76 \pm 0.01$ mH

$R_L = 65.5 \pm 0.1\ \Omega$

Using these verified values we can predict the values for total resistance, natural frequency, decay time, and quality factor as shown in Table 1.

Table 1: ***LCR Circuit Preditions***		
Quantity	*Value*	*Error*
R_{tot} [Ω]	165.800	0.141
ω_0 [s-1]	62,772	307
f_0 [Hz]	9,990	49
τ [us]	298.76	0.28
Q	9.37	0.41

To begin the experiment, we will set the function generator to output a 20 V_{PP} square wave with 0 offset at 100 Hz. Qualitatively, this appears as a dampened sine wave at the point CH2 where the measured quantity is the voltage across the capacitor. Recording the values for the time and voltage of each maximum amplitude we can plot the time *t* vs. maxima index *i* to find the frequency f_d and plot ln|V_i| vs. *i* to find the time constant τ_d. The statistics for least-squares regression fit are shown below in Tables 2 and 3.

Table 2: Least-Squares Regression for t_i vs. i		
	Slope	Intercept
Value	49.05	-25.4
Error	0.16	0.92
Correlation	0.99992	1.3

Table 3: Least-Squares Regression for ln|V| vs. t		
	Slope	Intercept
Value	-0.003765	-7.146
Error	0.000054	0.014
Correlation	0.999	0.020

The slope in Table 2 represents one-half the period f_d and the slope in Table 3 represents the negative inverse of the time constant τ_d. Using these measured quantities, we can calculate the rest of the predicted quantities from earlier:

$$f_d = 10.211 \pm 0.034 \text{ kHz}$$

$$\tau_d = 265.6 \pm 3.8 \text{ s}$$

$$\omega_0 = 64{,}155 \pm 213\, s^{-1}$$

$$f_0 = 10{,}211 \pm 34 \text{ Hz}$$

$$Q_d = 8.52 \pm 0.12$$

$$L_d = 23.70 \pm 0.08 \text{ mH}$$

$$R_{eff} = 178.5 \pm 2.6\, \Omega$$

V. Series LCR Circuit: Frequency Domain

With the function generator now on a sinusoidal output, we will find the resonant frequency of the circuit. This can be done by finding when the phase shift between the current and driving voltage is zero. To do this precisely we will display the two voltages as an X-Y plot where at zero phase shift the plot will converge to a straight line. This was found at f_{res} = 10,126 ± 1 Hz. Using this value, we can calculate *L*:

$$L_{res} \frac{1}{4\pi^2 {f_{res}}^2 C} = 24.70 \pm 0.25\, mH$$

Using a similar technique as before we can find the half-power frequencies f_+ and f_- as the frequency when the phase is shifted by ±45°. These can be used to calculate our quality factor *Q*:

$$f_+ = 10{,}750 \pm 1 \text{ Hz}$$

$$f_- = 9{,}541 \pm 1 \text{ Hz}$$

$$Q = \frac{f_0}{f_+ - f_+} = 8.3755 \pm 0.0098$$

This *Q* value agrees with the experimental *Q* value from the last experiment, both of which differ from the predicted value. To obtain a complete description of the circuit frequency domain we will record the values V_{in}, V_{out}, and ϕ. This allows us to make Bode plots of the amplitude and phase difference as functions of frequency and overlay them with the theoretical models as shown in Figures 6 and 7.

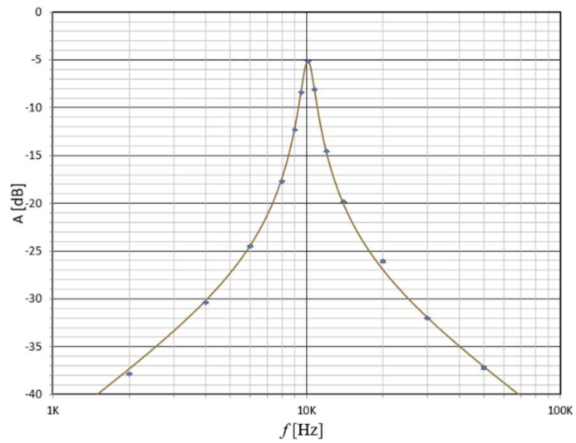

Figure 6: Amplitude vs. Frequency Bode plot on a logarithmic scale for LCR circuit

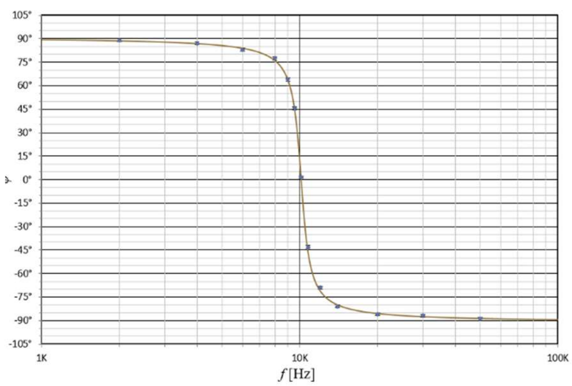

Figure 7: Phase Difference vs. Frequency Bode plot on a logarithmic scale for LCR circuit

VI. Series LCR Circuit: Resonant Amplification

We now set the function generator to produce resonant frequency f_c = 10,126 Hz. At resonance, we expect to read a voltage across the resistor:

$$V_R = V_0 \frac{R}{R_{tot}} = 10.97 \pm 0.21 \text{ V}$$

Measuring this voltage with the oscilloscope we obtain $V_R = V_R = 10.64 \pm 0.01$ V which is very close to the theoretical value.

Switching the resistor and the capacitor in the circuit, we can now measure the voltage across the capacitor. We expect to find the voltage across the capacitor to be:

$$V_c = QV_0 = 170.6 \pm 2.7\, V$$

Measuring the voltage with the oscilloscope we find $V_c = 167.7 \pm 0.1$ V which qualitatively agrees. We can then reverse the process and use our measured value V_c to find a Q value:

$$Q = \frac{V_C}{V_0} = 8.385 \pm 0.042$$

We can perform a similar calculation by switching the inductor and the capacitor, so the oscilloscope measures the voltage across the inductor. We expect to find the voltage across the inductor to be:

$$V_L = QV_0 = 170.6 \pm 2.7\, V$$

Measuring the voltage with the oscilloscope we find that $V_L = 166.6 \pm 0.1$ V which qualitatively agrees, however is less than expected likely due to an internal resistance of the inductor that is not properly accounted for.

VII. Conclusion

Throughout these experiments we have seen how resistors, capacitors, and inductors interact with AC currents. We can create low-pass AC filters with basic RC circuits and even more complicated behavior in combination with inductors. We have provided supporting evidence for the equations that relate resistance, capacitance, inductance, frequency, voltage and Q factors for frequency and time domain. We have proven ways to calculate resonance of circuits and verified the theoretical models shown in the Bode plots. Circuits are integral to all electronics and understanding their predictable behavior allows us to manipulate electricity for our benefit.