Speed of Light

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Introduction

The speed of light is one of the most important physical constants of the universe because it not only represents the velocity of light in a vacuum. As Maxwell's theory of electromagnetism shows us, electric and magnetic fields are two sides of the same coin with the coin in this case being light. The formal equation for this relationship,

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

relates the speed of light c to the magnetic constant known as the permeability of free space μ_0 and electric constant known as the permittivity of free space ε_0. These constants provide a baseline to ground ourselves with respect to a vacuum.

Using the electromagnetic relationship, we can extract a value for the speed of light from an AC circuit at resonance with measurable component specifications. This technique will be done to calculate c in experiment 1 with an LC circuit.

Using the idea that the speed of light is a vacuum baseline, we can find determine c by measuring the speed of voltage signals through a transmission cable. This speed is equivalent to the speed of light through the material in the cable which depends on its index of refraction. We can then extract the speed of light provided we have an accurate measure of the material's refractive index. This procedure will be performed in experiments 2 and 3.

We can also measure the speed of light by finding the time it takes for light to travel a known distance. While this is the most direct measurement technique, it is most likely to introduce errors because with the previous techniques we have essentially slowed down light in a more manageable system. This technique will be performed in experiment 4.

I. Aluminum Capacitor

For this experiment we will use the setup shown in Figure 1. The oscillator is a function generator configured to output sinusoidal AC voltage with adjustable frequencies. The capacitor consists of three identical aluminum disks with diameters of $d_C = 0.456 \pm 0.001$ m separated by glass microscope slides as spacers of thickness $t = 0.87 \pm 0.01$ mm. The disks are stacked together separated by the glass spacers and have a dielectric constant $\kappa_C = 1.0171 \pm 0.0023$, capacitance of $C = 2.894 \pm 0.001$ nF, and area $A_C = 0.32663 \pm 0.00033$ m^2. The inductor is a copper solenoid with diameter $d_L = 5.0 \pm 0.1$ cm, length $l = 25.9 \pm 0.1$ cm, turns $N = 445$, dielectric constant $K_{coil} = 0.923$, inductance $L = 1.839 \pm 0.001$ mH, and resistance $R_L = 5.8 \pm 0.1$ Ω.

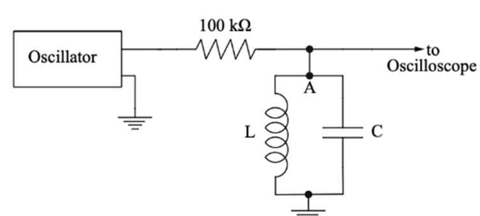

Figure 1: LC circuit with copper solenoid and aluminum capacitor

Using these measured values of the components we can calculate our expected resonant frequency f_0:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 68.989 \pm 1.9\ kHz$$

This will give us a general idea of the range at which we will measure various quantities of the circuit needed to give our best estimate of the resonant frequency. The first quantity we need is the parallel resonance frequency f_{pr} where the phase difference between the supply voltage and voltage measured across the LC circuit. This was found at f_{pr} = 69.24 ± 0.01 kHz. Similarly, we can find the upper and lower half-power frequencies where the phase difference is ±45°. These were found at f_+ = 70.05 ± 0.01 kHz and f_- = 68.61 ± 0.01 kHz. We can calculate our quality factor Q and natural resonant frequency f_{nat} as:

$$Q = \frac{f_{pr}}{f_+ - f_-} = 48.083 \pm 0.012$$

$$f_{nat} = \frac{f_{pr}}{\sqrt{1 - \frac{1}{Q^2}}} = 69.255 \pm 0.020\ kHz$$

The reason we must find all these related frequencies is because we want to exclude any source of error in our final calculation of c. This requires that we have a measure for the resonant frequency of the circuit in an ideal setup. However, with our physical components this presents a problem specifically in the case of the capacitor which has additional extraneous capacitance from the material it sits on, the air above it, and the leads connecting to the rest of the circuit. We can exclude this from our final calculation by finding the resonant frequency f_e of the circuit created by extraneous capacitance. This is done by disconnecting the LC circuit from the oscillator and oscilloscope at point A in the diagram. The resonant frequency of this disconnected circuit was recorded to be f_e = 722.9 ± 0.1 kHz.

Now we can calculate our best estimate of the resonant frequency of the whole circuit and finally our measure of the speed of light:

$$f_0 = \frac{f_{nat}}{\sqrt{1 - (\frac{f_{nat}}{f_e})^2}} = 69.575 \pm 0.022\ kHz$$

$$c = 2\pi f_0 N \sqrt{K_{coil} K_C \frac{A_L A_C}{lt}} = 3.18 \pm 0.075\ x\ 10^8 m/s$$

II. Transmission Line Pulses

In the following sets of experiments, we will use a function generator to send a pulse voltage down a coaxial cable that will have either a 50 Ω terminator attached, an open end, or a shorted end. These will be done for various pulse timescales, and we can find the speed of the signal by finding the time t for the pulse to travel across the cable length l. We will use the setup shown in Figure 2 with cable length l = 30.48 m.

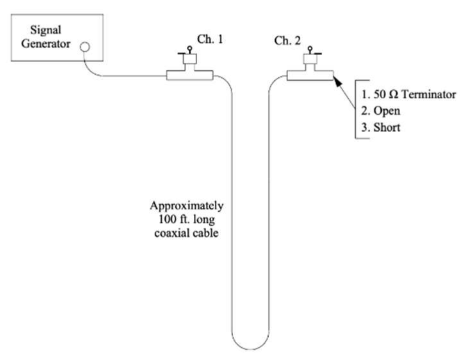

Figure 2: Signal pulse setup with 30m coaxial cable

First, we will analyze the waveforms for short pulses of 4 V and width of 40 nsec. Figures 3 through 8 show the waveforms produced by these experiments where the yellow plot is the input signal of Ch. 1 and blue plot is the received signal of Ch. 2 in the diagram. This setup is analogous to a standing wave on a fixed string where the cable creates nodes and antinodes in the signal voltage.

When terminated with its characteristic impedance through the 50 Ω terminator, there is no reflected signal because the end of the cable acts like an open-ended antinode. The time-delay between these pulses is $t_1 = 152 \pm 1$ ns which results in a velocity $v_1 = 2.005 \pm 10^8$ m/s. The plot can be seen below in Figure 3.

Figure 3: Short pulse across a coaxial cable terminated with its characteristic impedance

Removing the terminator, we now have an open line. The pulses should be an antinode with a reflection, so the waveform recorded at the end should be twice the amplitude of the original pulse. The time-delay between the pulses is $t_2 = 306 \pm 1$ ns which results in a velocity $v_2 = 1.992 \pm 10^8$ m/s as it traveled the distance of the cable twice. The waveform can be seen below in Figure 4.

Figure 4: Short pulse across a coaxial cable with an open end

Adding a short to the end of the coaxial cable creates an analogous node for our voltage pulse. The two recorded pulses therefore must sum to zero which is why the reflected wave is exactly opposite the incident pulse. The time-delay between pulses is $t_3 = 310 \pm 1$ ns which results in a velocity $v_3 = 1.966 \pm 10^8$ m/s as it traveled the distance of the cable twice. The waveform can be seen below in Figure 5.

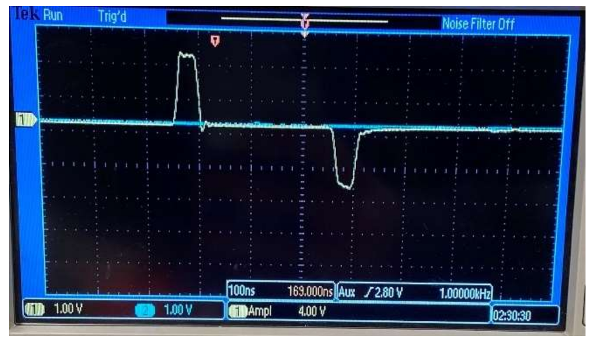

Figure 5: Short pulse across a coaxial cable with a shorted end

Now using the same setup as before we will plot the three different cable ends with longer pulses of 4 V but width 1.5 times the cable delay (228 ns). The waveforms for the 50 Ω terminator, open line, and shorted line can be seen below in Figures 6, 7, and 8 respectively.

Figure 6: Long pulse across a coaxial cable terminated with its characteristic impedance

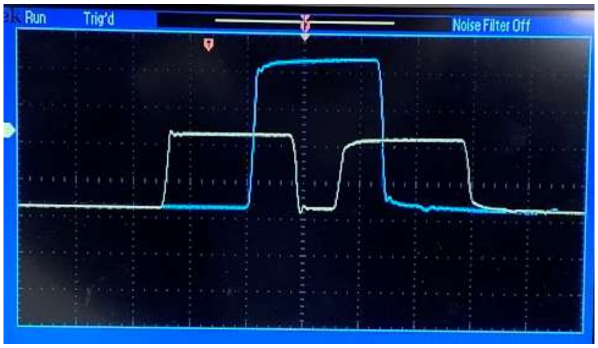

Figure 7: Long pulse across a coaxial cable with an open end

Figure 8: Long pulse across a coaxial cable with a shorted end

III. Transmission Line Sinusoid

Now we will record the resonant frequencies of the same setup as the previous experiment with sinusoidal waves instead of pulses. The input wave will be 4 V produced by a function generator at 10 kHz. We will use the same three cable endings: a 50 Ω terminator, shorted end, and open end.

First, we will analyze the setup with the 50 Ω terminator. Since the signal in the cable with the terminator acts as a standing wave, we know that the resonant frequency will be reached at:

$$f_0 = \frac{v}{\lambda} = \frac{1}{4t_l} = \ 1.645 \text{ MHz}$$

Where the cable length l is one-fourth of the wavelength λ and time-delay $t_l = 152 \pm 1$ ns as calculated from the previous experiment. We can measure the resonant frequency as the frequencies for which phase difference between the input signal and received signal differ by increments of 90°. This was found to be $f_0 = 1.62 \pm 0.01$MHz.

Now, replacing the terminator with a short we will find similarly find frequencies which are multiples of the resonant frequency where the voltage at the input has some extrema. The first five maxima and five minima were recorded for this setup and can be seen plotted below in Figure 9.

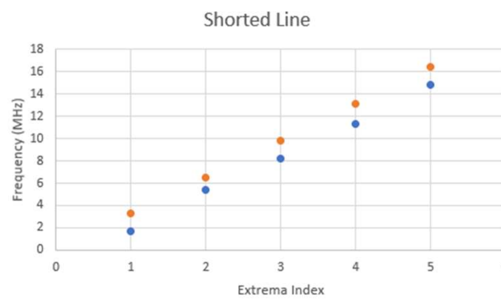

Figure 9: First ten extrema for shorted coaxial cable using sinusoidal input. Maxima labeled in blue, and minima labeled in orange

Using a least-squares regression fit we obtain a value for the resonant frequency at maxima $f_{max} = 1.610 \pm 0.076$ MHz and corresponding time $t_{max} = 155.3$ ns. We also have the resonant frequency at minima $f_{min} = 1.646 \pm 0.0028$ MHz and corresponding time $t_{min} = 151.9$ ns.

Lastly, we will remove the short to create an open line. Exactly like the procedure for the shorted line we will find frequencies which correspond to extrema at the input voltage. The first five maxima and five minima were recorded for this setup and can be seen plotted below in Figure 10.

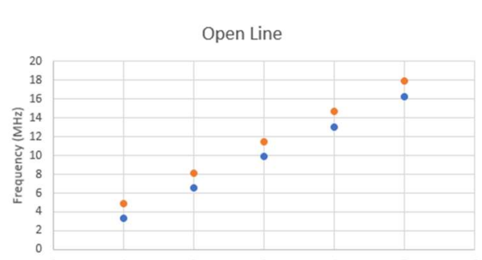

Figure 10: First ten extrema for an open coaxial cable using sinusoidal input. Maxima labeled in blue, and minima labeled in orange

Using a least-squares regression fit we obtain a value for the resonant frequency at maxima f_{max} = 1.617 ± 0.015 MHz and corresponding time t_{max} = 154.6 ns. We also have the resonant frequency at minima f_{min} = 1.64 ± 0.002 MHz and corresponding time t_{min} = 152.4 ns.

IV. Pulsed Laser

In this experiment we will directly measure the time it takes a beam of light to travel a distance *d*. To do so we will use a setup as shown in Figure 11, where a photodetector is placed at position 1 at ~10–20 cm from the laser. We then mark the arrival of the pulse on the oscilloscope as the time when the pulse reached ~ 50% of its maximum height. We then move the detector to position 2 and record the second arrival of the pulse at ~ 50% of its maximum height.

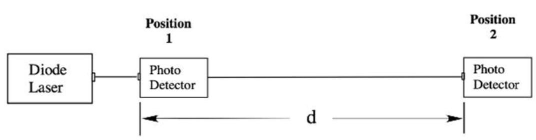

Figure 11: Setup to directly measure the speed of light

The two positions of the detectors were marked on the table using a plumb bob and the distance between the two points measured with a tape. The distance was recorded as d = 5.44 ± 0.005 m and time-delay between the distance Δt = 17.6 ± 0.1 ns. Thus, we can calculate the speed of light c as $\Delta d/\Delta t$ = 3.091 ± 0.018 x 10^8 m/s.

Conclusion

Throughout these experiments we have measured to roughly 5% or less of the generally accepted value for the speed of light c = 2.998 x 10^8 m/s. In experiment I, we proved part of Maxwell's theory of electromagnetism that relates the speed of light to the magnetic and electric constants. This result for the speed of light was further than the other techniques used in later experiments and error can be most attributed to the measured component values of the circuit related to the quality factor which was calculated to be a third of the theoretical value for the same circuit.

In experiments II and III we showed how voltage signals traverse through coaxial transmission cables and that their speed is equivalent to the speed of light divided by their refractive index. For coaxial cables this happens to be roughly 2/3 the speed of light which we measured in various different ways to very high precision.

Finally, in experiment IV we performed one method for directly measuring the speed of light using a diode laser and photo detector. This was performed rather simply and required little calculation but needed to be performed with high precision.

Light although seeming to be an intangible object can be measured through highly physical means as we've shown in these sets of experiments. The speed of light represents one of the most important concepts in modern physics as an integral part of all electromagnetic interactions and especially in relativistic calculations. Greater accuracy in our measures of this universal constant helps physicists produce more accurate predictions in other areas which shows how crucial it is to understand this fundamental property of nature.