Lorentz Force & Faraday's Law

Duncan Beauch with Charlotte Hoelzl

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I. Introduction

The electromagnetic force is one of the strongest of the fundamental forces. The most basic interaction of the electromagnetic force is the repulsion of particles with like charges and attraction of particles with opposite charges. Another electromagnetic interaction is the Lorentz Force, which describes the force on a charged particle in an electric or magnetic field. Faraday's Law of Induction is a consequence of the Lorentz Force. Faraday's Law tells us that a changing magnetic field will induce a voltage in a wire. This is because the charged particles in the wire (electrons) experience a Lorentz Force from the changing magnetic field and are forced along the wire, generating an electromotive force (EMF). In the following experiments we will observe these properties on the macroscopic scale and verify them by calculating the strength of known magnetic fields.

II. Force on a Current Carrying Wire

In this experiment we will calculate the force experienced by a current wire from a magnetic field. We will use this to find the strength of the magnetic field and compare it to the expected value. The setup for this experiment is shown in Figure 1.

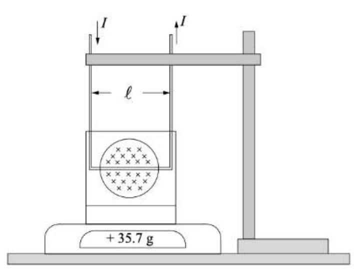

Figure 1: Setup for measuring the force on a current carrying wire

The wire of length $\ell = 45$ mm is placed between the pole faces of a U-shaped magnet. The magnet sits atop an electronic balance which is zeroed when there is no current in the wire and therefore the weight of the magnet is accounted for. As a current is passed through the wire, the wire and magnet experience a repulsive force from each other and as the wire is fixed in place, the magnet is forced downwards into the scale. This force is measured by the scale and the recorded values for varying currents are shown below in Table 1. The force in Newtons exerted on the magnet can be calculated by converting to kilograms and multiplying by the local acceleration of gravity $g_{local} = 9.792$ m/s^2.

We can plot these values as the force exerted vs. current to find the relationship between force and current. This plot is shown in Figure 2. The slope of which will give us a precise measure for $\frac{F}{I}$. The least-squares regression fit is shown in Table 2.

Table 1: Force on Current Carrying Wire

Current (A)	*Scale Reading (g)*	*Force Exerted (N)*
-25	-52.3 ± 0.1	-0.5121216
-20	-41.7 ± 0.1	-0.4083264
-15	-31.3 ± 0.1	-0.3064896
-10	-20.8 ± 0.1	-0.2036736
-5	-10.4 ± 0.1	-0.1018368
0	0.0 ± 0.1	0.0
5	10.1 ± 0.1	0.0988992
10	20.5 ± 0.1	0.200736
15	31.0 ± 0.1	0.303552
20	41.5 ± 0.1	0.406368
25	52.0 ± 0.1	0.509184

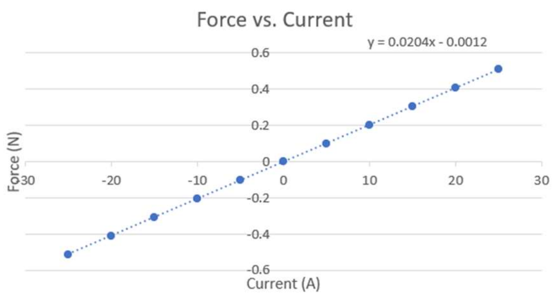

Figure 2: Force exerted vs. current for the current carrying wire in a magnetic field

	Slope	Intercept
Value	0.020373	-0.00125
Error	2.5E-05	0.000395
Correlation	0.999986	0.00131

Table 2: Least-squares regression fit for force exerted vs. current for the current carrying wire in a magnetic field

We know that the strength of the magnetic field B can be found using the following equation:

$$\langle B \rangle \ wire \ = \frac{F}{I\ell}$$

Where ℓ represents the length of the wire, I the current through the wire, and F the measured force. Using our slope value for the quantity $\frac{F}{I}$, we find that $B = 0.4527 \pm 0.0092$ T. Sources of error in this procedure come from the calibration of the electronic scale and consistency in the current from the power source.

III. Magnetic Field Variation

For the previous experiment we were able to find the average strength of the magnetic field around the U-shaped magnet. However, the magnetic field strength is not uniform across the magnet and varies as a function of position. Our goal in this experiment will be to plot $B(x)$, the magnetic field strength as a function of position along the U-shaped magnet and integrate over the length to find the average magnetic field strength felt by the wire.

We will be using a Hall Effect sensor to measure the magnetic field. The sensor is attached to an adjustable pulley which records linear position. A diagram of the setup is shown in Figure 3.

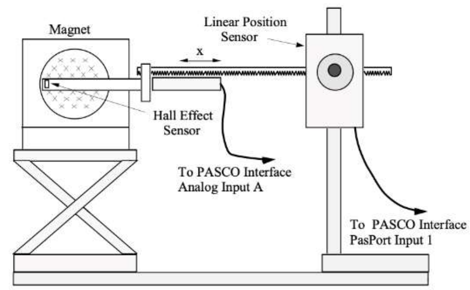

Figure 3: Hall Effect and linear position sensor setup for measuring magnetic field variation

The magnetic field probe is positioned so that as the pulley is rotated on the rotary motion sensor, the tip of the probe moves from a position a few millimeters beyond the U–magnet frame on one side to a few millimeters beyond the frame on the other side and centered between the pole faces and along the centerline of the pole faces. The sensor is zeroed with the tip of the probe well outside of the magnet to account for any technical offset. The pulley is slowly rotated to move the probe tip and plot values of $B(x)$, taking roughly 30 seconds to scan the Hall Effect probe across the magnet to obtain an accurate reading. The graph obtained is the Hall voltage V_H in V vs. position x in mm and is shown in Figure 4 below.

Figure 4: Hall Effect voltage vs. position for the Hall Effect probe scanned across the magnet

The center of the peak in Figure 4 was found to be at x_0 = -58.7 mm. Since we know the length of the wire ℓ = 45 mm we can integrate from x_1 to x_2:

$$x_1 = x_0 + \frac{\ell}{2} = -36.2\,mm$$

$$x_2 = x_0 - \frac{\ell}{2} = -81.2\,mm$$

This guarantees that we only take into account the magnetic field experienced by the wire when it is centered on the magnet. We can then calculate the average strength of the magnetic field experienced by the wire as follows:

$$\langle B_{wire} \rangle = \frac{1}{\ell} \int_{x_1}^{x_2} B(x)dx$$

$$\int_{x_1}^{x_2} B(x)dx = \frac{1}{C} \int_{x_1}^{x_2} V_H(x)dx$$

Where the calibration constant C = 0.312 V/T is provided by the instrument and $\int_{x_1}^{x_2} V_H(x)dx$ is calculated through the Capstone integration tool as shown in Figure 4. However, there is still an offset in the graph which we can take into account by subtracting by the offset multiplied by our length ℓ.

$$\langle B_{wire} \rangle = \frac{1}{C\ell} \int_{x_1}^{x_2} B(x)dx$$

$$\langle B_{wire} \rangle = \frac{1.95 \times 10^{-4} - (0.003 \times 0.045)}{(0.312)(0.045)}$$

$$\langle B_{wire} \rangle = 0.4274 \ \pm 0.0001 \text{ T}$$

Comparing this value to the one obtained in Experiment 1, B = 0.4527 T, we can see that they are very close and only differ by roughly 5%. Some errors in this experiment come from the measurement devices as well as the calculated offset for the Hall Effect integral. The both the offset and integral have an associated error due to the variance in the Hall Effect sensor which can be seen as the jaggedness of the graph in Figure 4.

IV. Magnetic Field Measurement with a Pick-up Loop

In this experiment we will demonstrate Faraday's Law of Induction. Similar to the previous two experiments, we will measure the field strength of the same U-shaped magnet and compare the obtained values. To do this, we will use a rectangular plastic form 45 mm long and 4 mm wide so that it fits perfectly into the bend of the magnet. Inside the plastic is a copper coil with N = 25 turns whose ends are connected to a voltmeter input that will plot electromotive force (EMF) as a function of time. Faraday's Law tells us that as we move the coil through the magnetic field, an EMF will be produced and using the associated equations we can calculate the strength of the magnetic field B. The coil was swiped through the magnet at varying speeds and plots can be seen below ordered from fastest to slowest speeds. This can be seen by the width of the EMF spike, where a quicker withdrawal speed corresponds to a thinner EMF spike. For the following calculations we will use the plot in Figure 5 as it is the smoothest of the generated graphs.

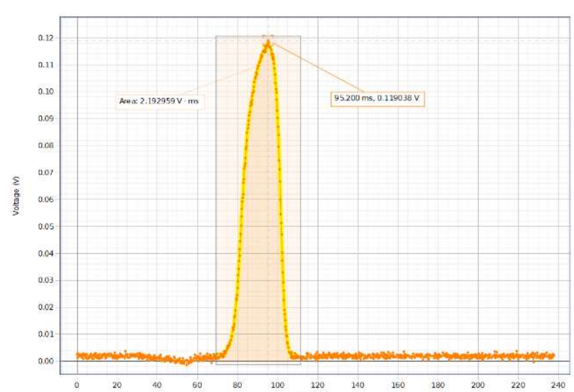

Figure 5: EMF vs. time for a copper coil swiped quickly through U-shaped magnet

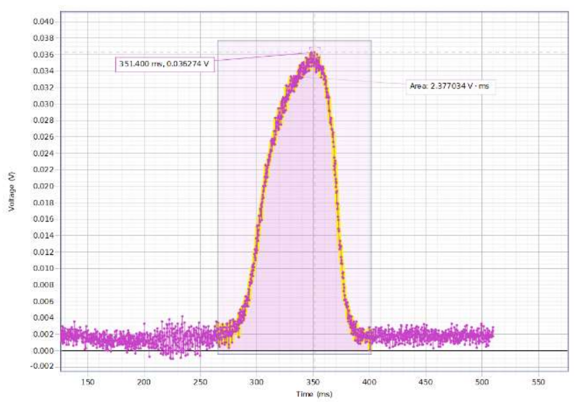

Figure 6: EMF vs. time for a copper coil swiped through U-shaped magnet

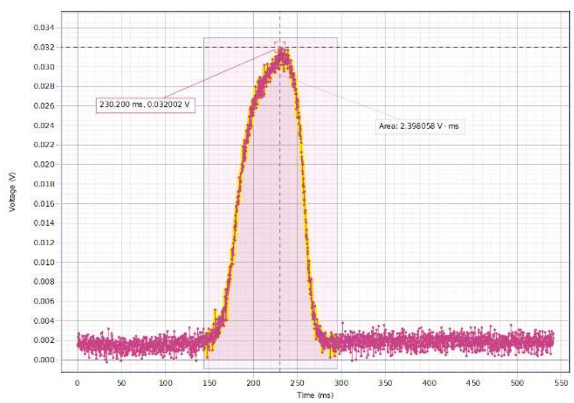

Figure 7: EMF vs. time for a copper coil swiped slowly through U-shaped magnet

$$B_{avg} = \frac{\Phi}{A} = -\frac{1}{NA}\int \varepsilon\, dt$$

$$B_{avg} = \frac{0.002193}{(25)(0.045)(0.004)} = 0.4873 \pm 0.0011\, T$$

The integral of EMF, $\int \varepsilon\, dt$, was taken from the plot in Figure 5 using the integral tool within the Capstone program. This result is also very similar to the calculated values in the previous two experiments. Errors in the procedure can be attributed to the voltage input which recorded the induced EMF and similar to the previous experiment, the jaggedness in the graphs.

V. EMF Induced in a Coil by a Magnet Moving Through It

For this experiment, we will do the opposite procedure of the previous experiment. Rather than force a coil through a magnetic field, we will force a magnet through a coil which will induce an EMF in the coil just the same according to Faraday's Law. A bar magnet with a strength roughly 0.45 T is held 20 cm above an $N = 10$ turn circular coil of wire with the plane of the coil oriented horizontally. The magnet is first released with its north pole downward and then repeated again with the south pole facing downwards. As the magnet falls through the coil the induced emf in the coil is recorded as a function of time, just as before, and can be seen in Figures 8 and 9 respectively.

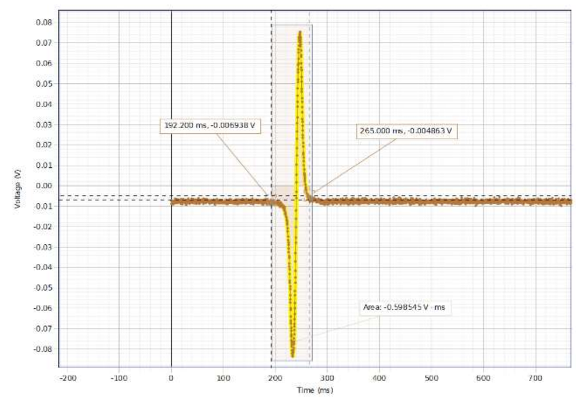

Figure 8: Magnet dropped through the coil with north pole facing downwards

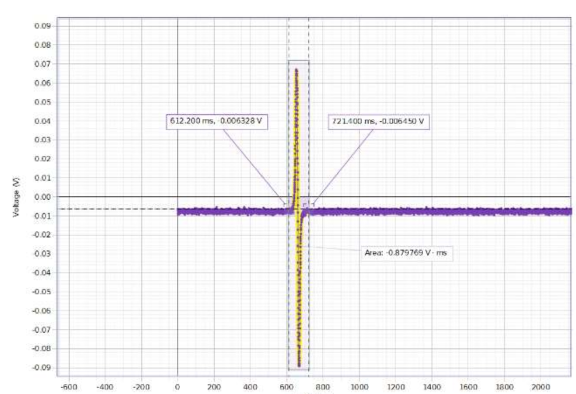

Figure 9: Magnet dropped through the coil with south pole facing downwards

The plots show two voltage pulses of opposite signs which qualitatively agree with Lenz's law. The shapes of the graph were exactly the same except their signs are flipped, but the areas are identical.

To quantitatively verify Lenz's Law we can take the integral over the spikes in the EMF plot which should ideally be zero as the positive and negative EMFs should be equivalent in magnitude. However, the voltage sensor used cannot be zeroed before the experiment which explains the negative offset in the graph. To fix this, an offset A_{off} is calculated and subtracted from the integral obtained using the integral tool in Capstone:

$$A_{off} = -0.058\, Vms$$

$$\int \varepsilon\, dt - A_{off} = -0.0019 \pm 0.002\, Vms$$

This number is effectively zero as expected theoretically and qualitatively. Errors in this procedure come from the estimate for the offset and the EMF measurement device.

VI. The Daedalon Apparatus

In the following experiment, we will use a Daedalon Apparatus, a device used to study the interaction between magnetic fields and beams of electrons. A diagram can be seen in Figure 10.

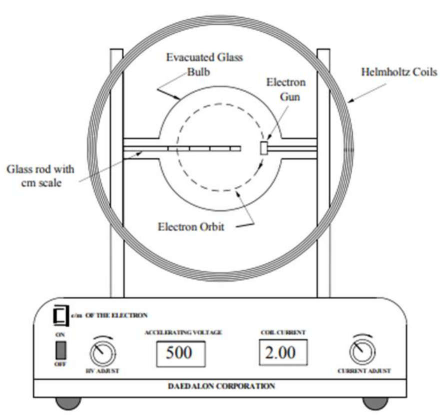

Figure 10: Daedalon Apparatus

Helmholtz coils surround the outside of the apparatus to provide a uniform magnetic field in the horizontal direction. An electron gun is placed inside a glass bulb that has been pumped to near vacuum and filled with a low pressure of helium gas. This allows us to see the beam of electrons as they pass through the bulb by colliding with helium atoms to produce visible light. A ruler spans the length of the bulb which will allow us to accurately measure the beam of electrons. Ideally, this experiment would be oriented to line up with Earth's magnetic field lines in order to reduce its effect on the result. However, the magnetic fields created in these experiments are many magnitudes larger than the strength of the magnetic field of the earth, so its effect is negligible.

By adjusting the voltage of the Helmholtz coils, we can bend the beam of electrons, and at specific settings we can achieve a circular electron beam. We will first perform some qualitative measurements to understand how the apparatus functions. The current is set to 1.410 A, voltage is set to 300 V, and voltage limit of the Kiethley power supply set to 30 V. The current is increased until it produces a circle of diameter 10 cm measured by the internal ruler. A bar magnet with its north pole facing inwards is then aligned with the axis of the coils and the circle is moved away from the bar magnet. Similarly, with the south pole facing inwards, the circle moved closer to the magnet.

We know that the radius R of the circular electron beam can be predicted by the following equation where V represents the input voltage, e represents the charge of an electron, and m_e represents the mass of an electron. We can then rearrange to obtain a relationship which will be helpful in the following calculations. We also know that the magnetic field strength B can be calculated using the input current and calibration constant provided by the apparatus:

$$R = \frac{1}{B}\sqrt{\frac{2V}{\frac{e}{m_e}}}$$

$$V = \frac{1}{2}\left(\frac{e}{m_e}\right)R^2B^2$$

$$B = (0.73)(I)$$

We can use the second equation here to calculate the ratio $\frac{e}{m_e}$ by recording various values of V and R, plotting V vs. R^2 where $\frac{e}{m_e}$ will be a multiple of the slope of the graph. Producing circles of diameters from 6.5 to 11 mm in increments of 0.5 mm. The table of values can be seen in Table 3, corresponding graph in Figure 11, and least-square regression fit in Table 4.

Table 3: Recorded Values of the Daedalon Apparatus

Diameter (m)	Voltage (V)	Radius Squared (m)	Current (A)
0.065 ± 0.001	230 ± 1	0.00106	1.73
0.070 ± 0.001	244 ± 1	0.00123	1.73
0.075 ± 0.001	264 ± 1	0.00141	1.73
0.080 ± 0.001	286 ± 1	0.00160	1.73
0.085 ± 0.001	314 ± 1	0.00181	1.73
0.090 ± 0.001	346 ± 1	0.00203	1.73
0.095 ± 0.001	380 ± 1	0.00226	1.73
0.100 ± 0.001	414 ± 1	0.00250	1.73
0.105 ± 0.001	458 ± 1	0.00276	1.73
0.110 ± 0.001	500 ± 1	0.00303	1.73

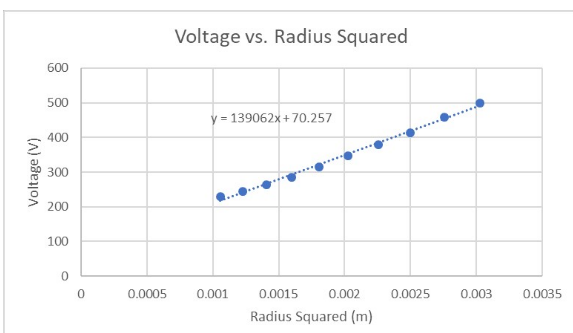

Figure 11: Voltage vs. Radius squared for the Daedalon Apparatus

	Slope	Intercept
Value	139061.82	70.25661
Error	3761.2851	7.763678
Correlation	0.9941815	7.492777

Table 4: Least-squares regression fit for voltage vs. radius squared

The slope of the graph can be converted to the ratio $\frac{e}{m_e}$ by the following:

$$\frac{e}{m_e} = \frac{2(slope)}{B^2} = 1.7435714246(23) \times 10^{11}$$

Comparing this to the expected value of $1.758820024(11) \times 10^{11}$ C/kg, our experimental value is off by 1%. Errors in this procedure come from the recorded values for R and V as the radius of the electron beam orbit was measured via the internal ruler and the voltage was supplied by the power supply.

We can perform a very similar experiment to calculate the ratio $\frac{e}{m_e}$ by keeping the diameter constant and varying the voltage and current supplied. Using the same equations as before we can calculate the magnetic field for each current and by plotting V vs. B^2, we will end up with a slope that is a multiple of $\frac{e}{m_e}$. The recorded values are shown in Table 5, plot in Figure 12, and least-squares regression in Table 6.

	Slope	Intercept
Value	228486436	56.91181
Error	1328591.9	2.028576
Correlation	0.9996958	1.219527

Table 5: Least-squares regression fit for voltage vs. magnetic field squared for the Daedalon Apparatus

Dia...			... Field ...d (T)
			10^{-6}
			10^{-6}
0.1	340	1.55	1.25×10^{-6}
			10^{-6}
			10^{-6}
			10^{-6}
0.1	420	1.75	1.59×10^{-6}
0.1	440	1.8	1.68×10^{-6}
0.1	460	1.85	1.77×10^{-6}
0.1	480	1.89	1.85×10^{-6}
0.1	500	1.93	1.93×10^{-6}

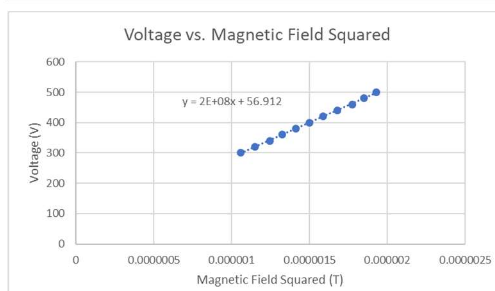

Figure 12: Voltage vs. Magnetic Field Squared for the Daedalon Apparatus

The slope of the graph can be converted to the ratio $\frac{e}{m_e}$ by the following:

$$\frac{e}{m_e} = \frac{2(slope)}{R^2} = 1.82789148800(16) \times 10^{11}$$

Comparing this to the expected value of $1.758820024(11) \times 10^{11}$ C/kg, our experimental value is off by 4%. Errors in this procedure come from the recorded values for I and V as the current and voltage are produced by the power supply which has an internal error.

VII. Conclusion

Throughout these experiments we have tested many principles of electromagnetism. In experiment 1 we observed and predicted the Lorentz Force on a current carrying wire. In experiment 2 we showed that the field strength of a magnet is dependent on the position relative to its poles. In experiment 3 we observed and verified Faraday's Law of Induction. With these first several experiments, we have shown their relationship with one another by connecting them to the average magnetic field strength. For all three experiments, we obtained an accurate value for the B_{avg} of the same magnet.

For experiments 3 and 4, we observed Faraday's Law of Induction and how the north/south poles of a magnetic field affect the sign of the induced EMF. Between these two experiments we showed that the principle of relativity holds for Faraday's Law in that as long as the magnetic field is changing with respect to a coil, an EMF is produced. This is important for generalizing the principle of relativity as we know that it holds for the electromagnetic force.

In experiment 5, we demonstrated several principles of electromagnetism. We provided a visible example of how electrons are affected by magnetic fields in the Daedalon Apparatus where the beam of electrons was deflected by a magnetic field generated by the Helmholtz coils. Moreover, we were able to measure with high precision the ratio $\frac{e}{m_e}$, relating the charge of an electron to its mass. This was done in two different ways using the same apparatus using equations in basic mechanics and electromagnetism.

Understanding electricity and magnetism is extremely important for experiments in modern physics because of the vast use of electronics. We need to be able to predict how our instruments will interact with each other when producing electric and magnetic fields. Many engineering feats in history such as the first intercontinental telegraph cable have had catastrophic failures because of a failure to understand electricity and magnetism.