How to calculate and combine effect-si

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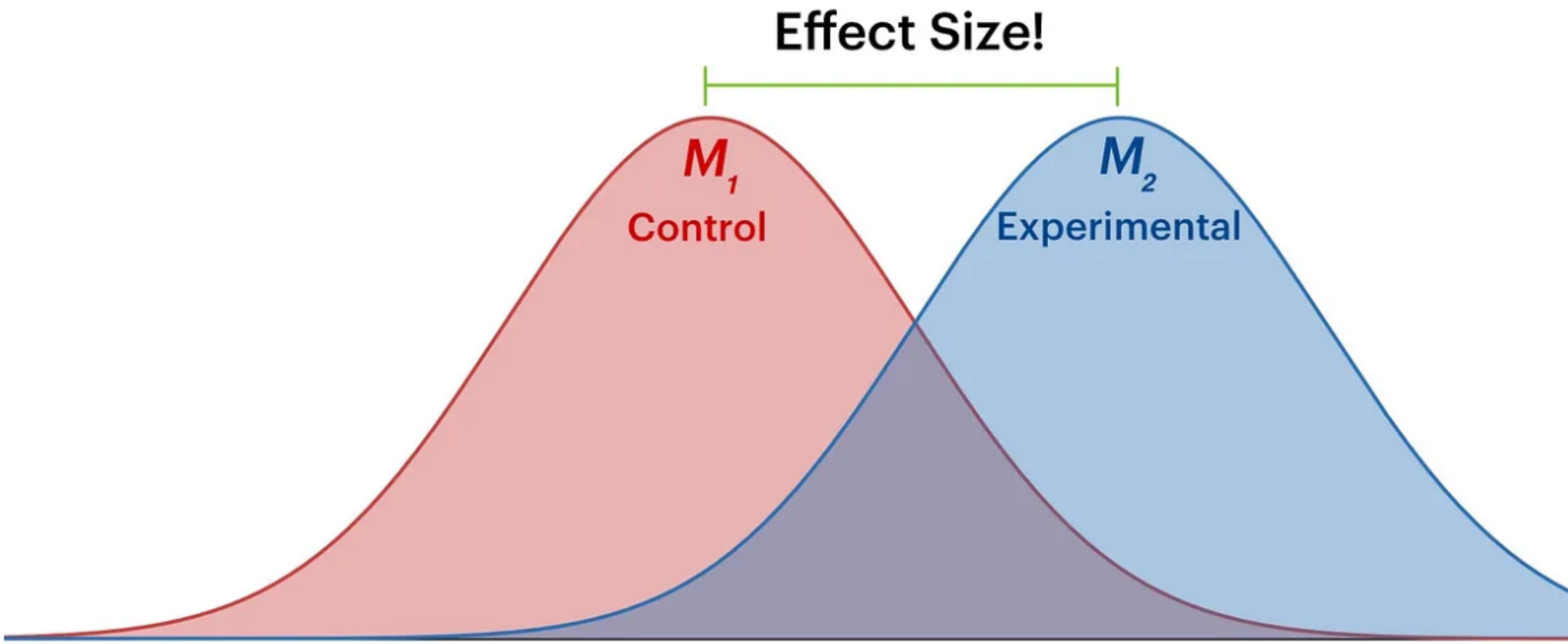
What is an effect-size?

A metric that quantifies the **direction** and **magnitude** of an experimental/observational effect:

- **Extracted directly** from publications or **calculated** from re
  data
- **Standardized** to be comparable across multiple primary st
- **Reliable**: effectively represent the underlying data (e.g., be
  of ratios with low denominators)

What is an effect-size (Visually)?



Effect-size and p-value

- **Statistical vs. Practical Significance**
  Small effect sizes can produce significant p-values with lar
  samples but have little real-world impact.
- **Impact of Sample Size**
  Large datasets make even tiny effects statistically significa
  risking misinterpretation.
- **Criticisms of p-values**
  see e.g., Hasley, 2019; Chen et al., 2023). Complement with
  confidence intervals or Bayesian approaches.

Effect-size and p-value

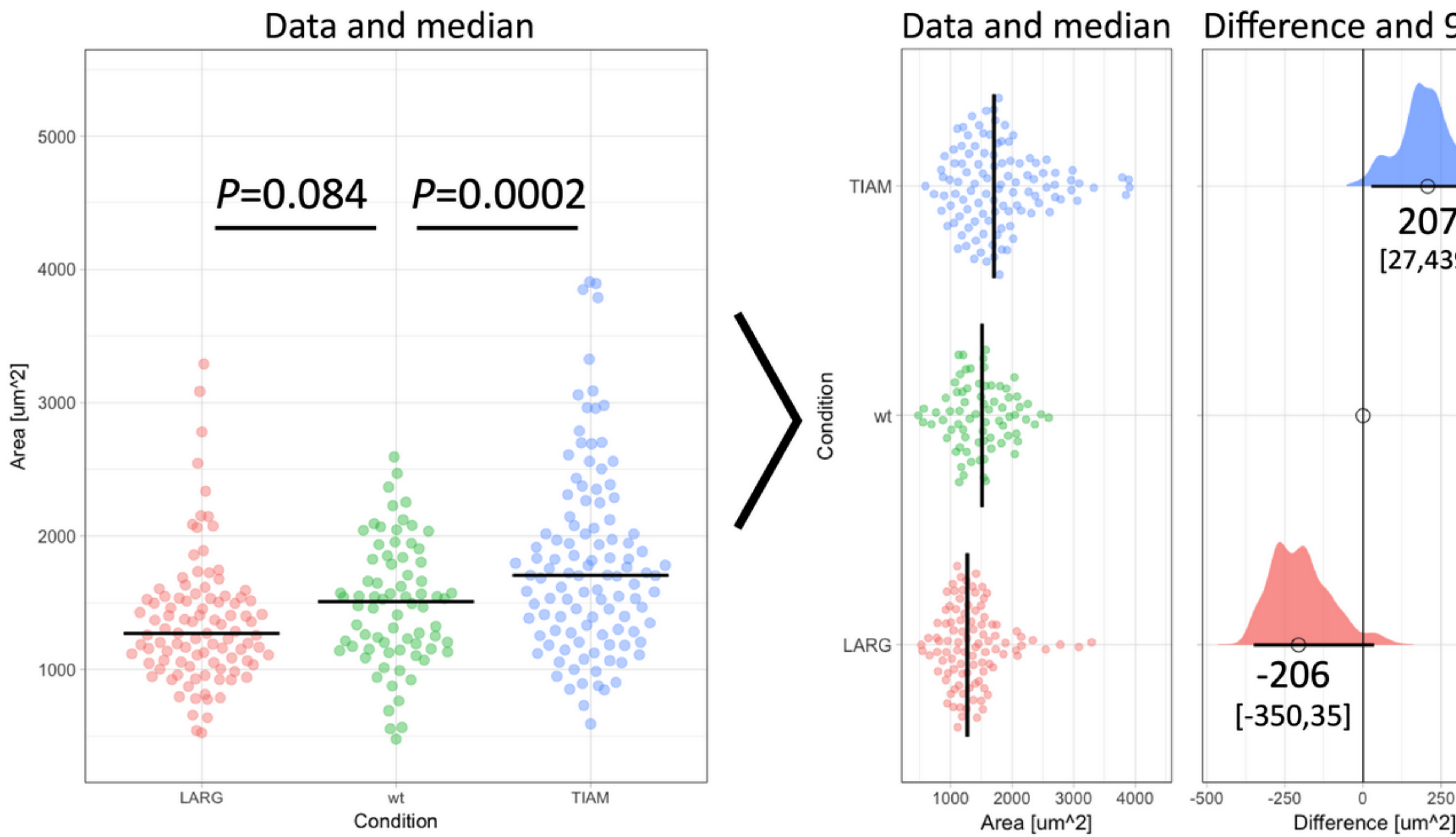


Figure 1: Transformation of an ordinary graph with p-values into a visualization of the d

True vs. Estimated Effect Size

Effect sizes are typically represented by the Greek letter theta

- θ_k: the ‘true’ (unknown) effect size of study *k*
- $\hat{\theta}_k$: the observed (estimated) effect size from data in study

The observed effect size deviates from the true effect size
sampling error:

- $\theta_k \neq \hat{\theta}_k$ because $\theta_k = \hat{\theta}_k + \varepsilon_k$ (where ε_k is the sampling error)

Goal of meta-analysis

Reduce sampling error to produce accurate estimates that ar
close as possible to the **true effect size**.

We do not know much

Unknown parameters:

- θ_k, $\hat{\theta}_k$, ε_k,

What we do:

==> $\hat{\theta}_k$ˆ: Estimated through the mean value (of a sampling distribution)

==> ε_k : Estimated through the standard error (SE).

(standard error of the mean : $SE = s/\sqrt{n}$; with n: sample size, s standard dev)

In a perfect world

Imagine we know the **true mean** (μ) and **standard deviation** distribution perfectly.

e.g.

- mu <- 100 # True mean
- sigma <- 15 # True standard deviation

True or estimated effect-size?

- **What happens if we sample this distribution?**

How to calculate and combine effect-sizes?

An Effect Size for All Your Needs

Depending on your experimental design, research question, a
type of outcome you are interested in, various effect sizes car
applicable.

An Effect Size for All Your Needs

- Continuous outcomes

Effect Type	Description
Cohen's d	Effect size between two means.
Hedges' g	Corrected Cohen's d for small samples.
Correlation (r)	Strength and direction of a linear relations
Eta-squared ((^2))	Proportion of variance explained in ANOVA
Partial Eta-squared	Proportion of variance explained by an effe controlling others.

An Effect Size for All Your Needs

- Continuous outcomes

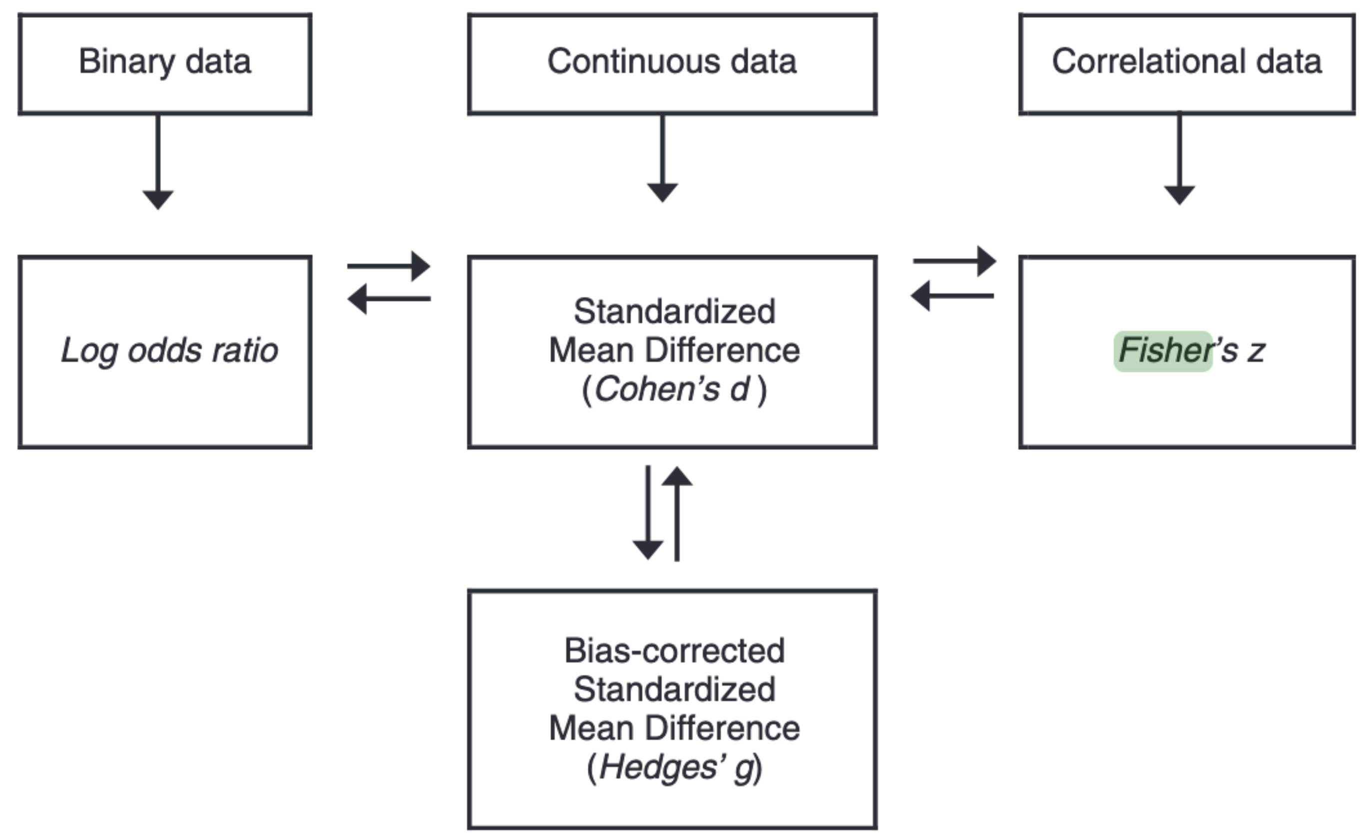


Figure 7.1 Converting among effect sizes.

An Effect Size for All Your Needs

- Discrete outcomes

Effect Type	Description
Odds Ratio (OR)	Measures the strength of association between events.
Risk Ratio (RR)	Compares the risk of an event between two gr
Phi Coefficient	Measures the association between two binary

Mean Difference (MD) Calculation

1. Mean Difference (MD)

$$MD = M_1 - M_2$$

with M1: Mean of group 1 - M2: Mean of group 2

2. Standard Error of the Mean Difference (SE MD)

$$SE_{MD} = SD_{pooled}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

With- SD1 : Standard deviation of group 1 - SD2 : Standard deviation of group 2 - n1 , n_
sizes

3. Pooled Standard Deviation (spooled)

$$SD_{pooled} = \sqrt{\frac{(n_1 - 1) \cdot SD_1^2 + (n_2 - 1) \cdot SD_2^2}{n_1 + n_2 - 2}}$$

Plant Growth in Different Soil Types

Variable	Soil A	Soil B
Mean Height ((M))	25 cm	20 cm
Standard Deviation ((SD))	4 cm	5 cm
Sample Size ((n))	30	30

Calculations

1. **Mean Difference**:

$$MD = 25 - 20 = 5 \text{ cm}$$

2. **Pooled Standard Deviation**:

$$SE_{MD} = \sqrt{\frac{(30-1)\cdot 4^2 + (30-1)\cdot 5^2}{30+30-2}} \cdot \sqrt{\frac{1}{30} + \frac{1}{30}} = 1.18$$

Variables to Retrieve from Primary Studies

1. **Mean ((M))**
   - For each compared group.
2. **Standard Deviation ((SD))**
   - Standard deviation for each group.
3. **Sample Size ((n))**
   - Number of observations in each group.

Advantages and Disadvantages of Me
Difference (MD)

Advantages:

- **Simplicity**: Easy to understand and interpret.
- **Direct Measurement**: Represents a direct measure of the effect between two groups.
- **General Applicability**: Useful across various fields (biology, psychology, etc.).

Disadvantages:

- **Sensitivity to Samples**: Can be influenced by sample variability.
- **Data Distribution**: Requires a normal distribution for valid interpretations.

Standardized Mean Difference (Coher

$$d = \frac{M_1 - M_2}{S_{within}}$$

$$S_{within} = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

Where: - M_1 and M_2 represent the sample means of the two groups. S_{within}, is the within-groups standard deviation: - n_1 and n_2 are the sample sizes of the two groups S_2 are the standard deviations of the two groups.

$$SE_d = \sqrt{\frac{n_1 + n_2}{n_1 n_2} + \frac{d^2}{2(n_1 + n_2)}}$$

Standardized Mean Difference (Coher

- Interpretation : SMD= 2 -> a difference of 2 standard deviat
- BUT bias when the sample size of a study is small, especial
  n≤ 20 (L. V. Hedges 1981).

Hedges' (g)

$$g = d \times \left(1 - \frac{3}{4N - 1}\right)$$

- (N): Total sample size ((N = n_1 + n_2))

$$SE_g = \sqrt{J^2 \times SE_d^2}$$

Plant growth in different soil

Variable	Soil A	Soil B
Mean Height ((M))	25 cm	20 cm
Standard Deviation ((SD))	4 cm	5 cm
Sample Size ((n))	30	30

Calculations

2. **Cohen's (d)**:

$$d = \frac{5}{4.58} \approx 1.09$$

3. **Hedges' (g)**:

$$g = 1.09 \times \left(1 - \frac{3}{4(60) - 1}\right) \approx 1.08$$

Advantages and Disadvantages of Cohen's (d) and Hedges' (g)

Advantages:

- **Standardized Measure**: Allows for comparison across studies.
- **Applicability**: Useful in various fields (ecology, psychology, etc.).

Disadvantages:

- **Sample Size Sensitivity**: May be affected by small sample sizes.
- **Assumption of Normality**: Requires normality for accurate interpretation.
- **Interpretability**: Not so clear understanding of effect sizes.

Ratio Calculation

1. Ratio (R)

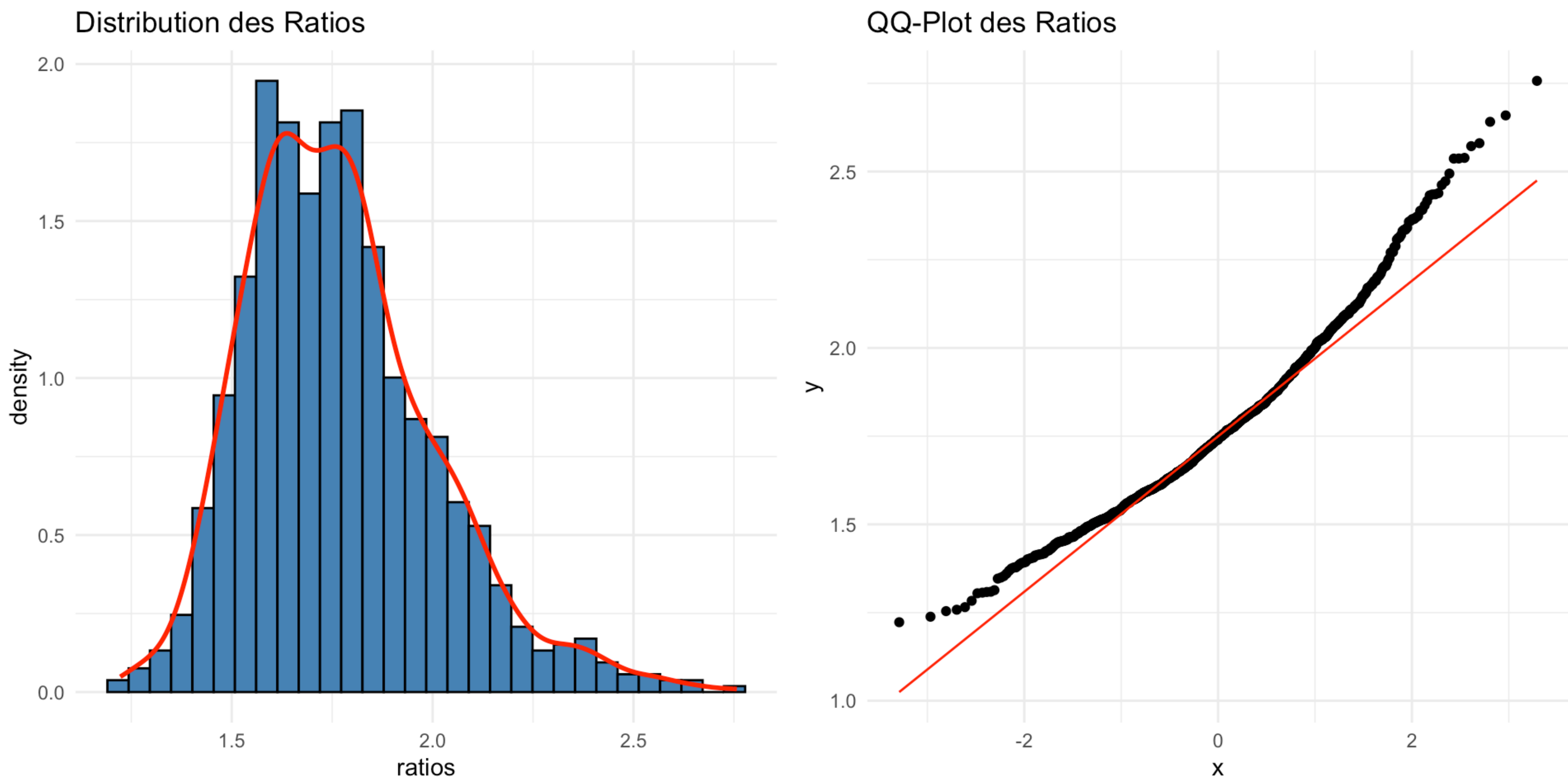
$$R = \frac{X_1}{X_2}$$

- X_1: Mean value or proportion in group 1
- X_2: Mean value or proportion in group 2

where: - SD_{pooled}: Pooled standard deviation - X_1: Mean of group 1 - X_2: Mean of group 2
Sample sizes of the two groups

Ratio :non-normal distribution

- Lead to non normal distribution



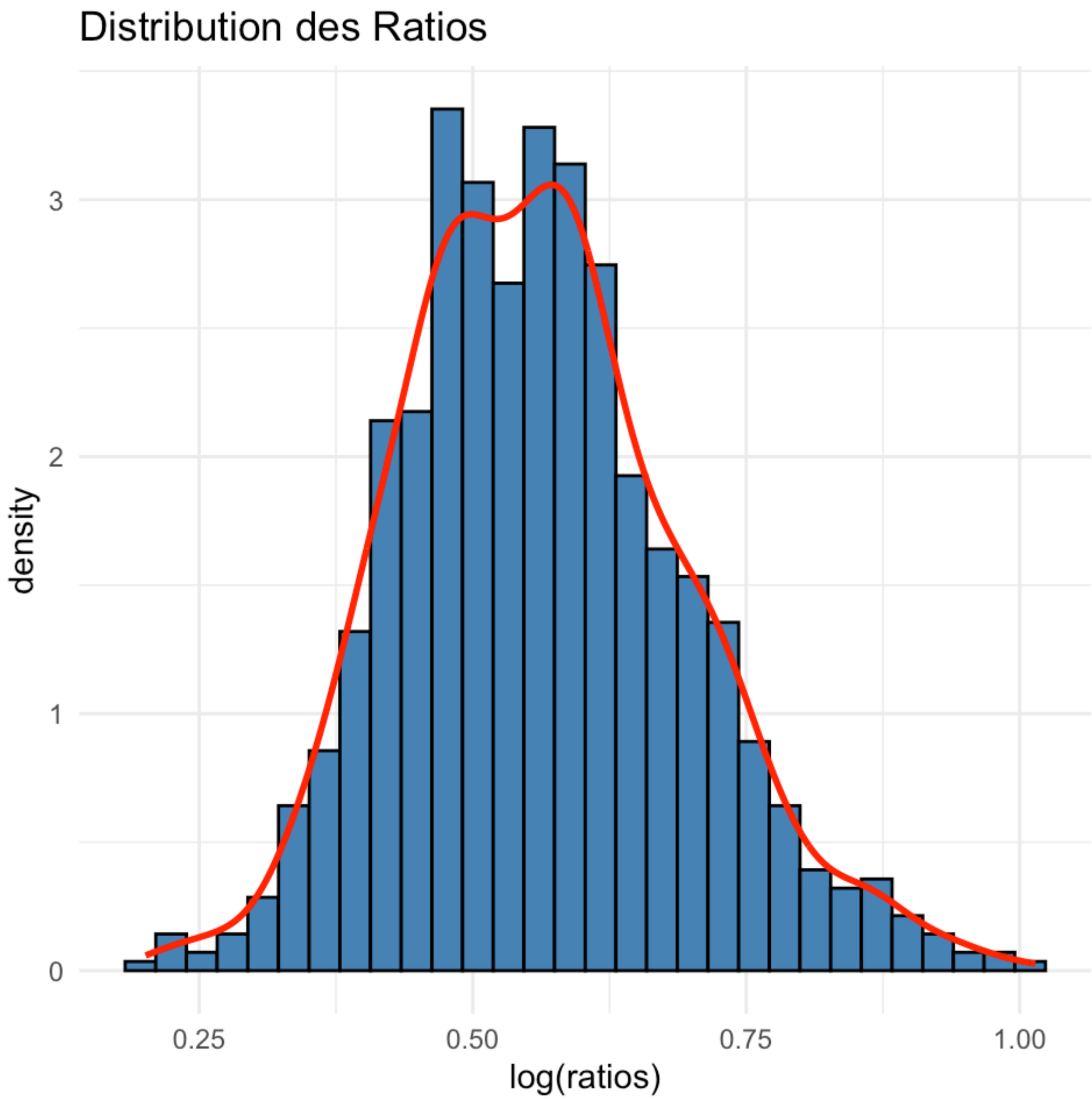
Logarithmic Transformation for Ratio

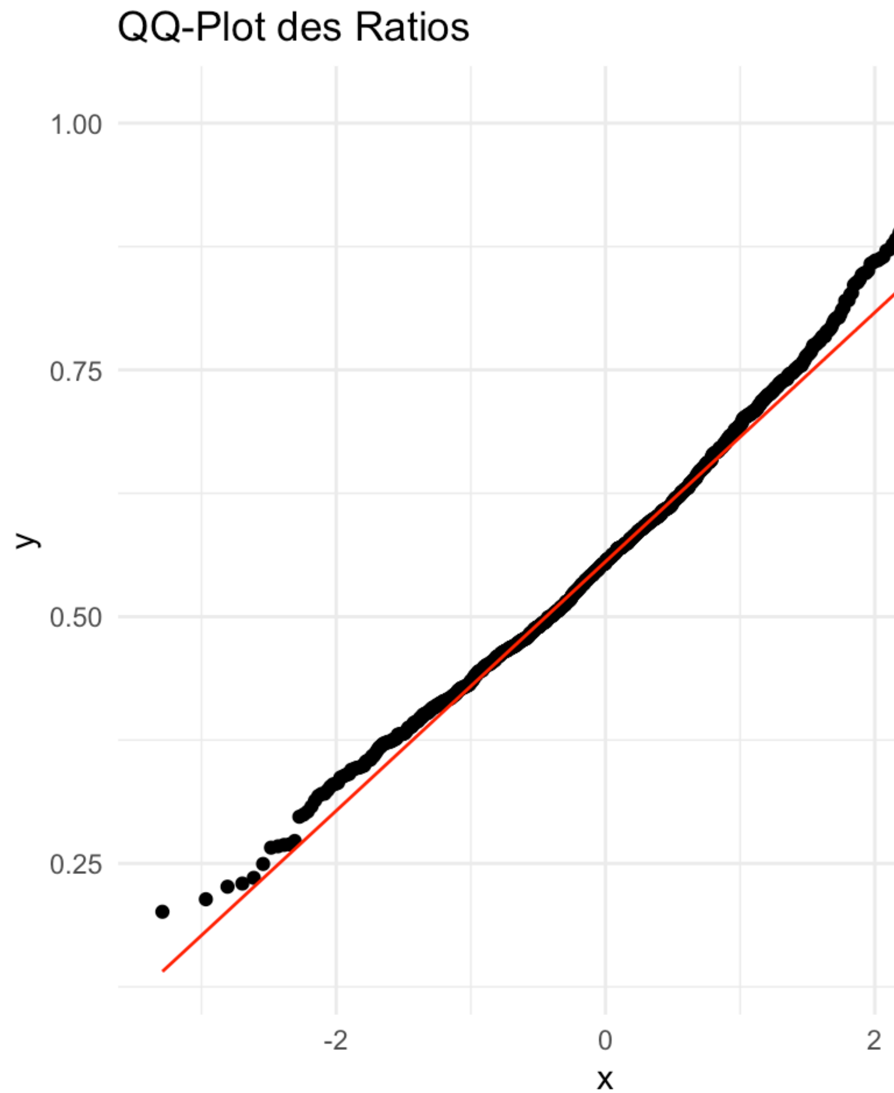
$$\log R = \ln\left(\frac{X_1}{X_2}\right)$$

Log transformation is often used to stabilize variance and nor
the distribution.

$$SE_{\ln R} = SD_{\text{pooled}}\sqrt{\left(\frac{1}{n_1 \cdot (X_1)^2}\right) + \left(\frac{1}{n_2 \cdot (X_2)^2}\right)}$$

Logarithmic Transformation for Ratio





Adjustments for Ratio-Based Effect S

1. **Correction for Small Sample Size**:
   - **Log Response Ratio (LnR)** tends to be biased when sample sizes are small.
   - Apply bias corrections to reduce overestimation or underestimation.
   - Variance estimations need adjustment, especially when the response ratio is high

Corrected Log Response Ratio (LnR_corr)

$$\mathrm{LnR}_{corr} = \ln\left(\frac{X_1}{X_2}\right) - \frac{1}{2} \cdot \left(\frac{SD_{X1}^2}{n_1 X_1^2} + \frac{SD_{X2}^2}{n_2 X_2^2}\right)$$

Where:
- (X_1, X_2): Group means.
- (SD_{X1}, SD_{X2}): Standard deviations.
- (n_1, n_2): Sample sizes.

References: Lajeunesse, M. J. (2011)

Comparing Plant Growth

Variable	Soil A	Soil B
Mean Plant Height (X)	25 cm	20 cm
Standard Deviation (SD)	4 cm	5 cm
Sample Size (n)	30	30

Ratio and LogRatio

$$R = \frac{25}{20} = 1.25$$

and

$$\log R = \ln(1.25) \approx 0.223$$

Standard Error of Log Ratio (SE Log Ratio):

$$SE_{\ln R} = 4.53 \cdot \sqrt{\left(\frac{1}{30 \cdot 25^2}\right) + \left(\frac{1}{30 \cdot 20^2}\right)} \approx 0.053$$

Correlation

Correlation Coefficient (r)

$$r = \frac{Cov(X, Y)}{SD_X \cdot SD_Y}$$

Where: - (Cov(X, Y)): Covariance between variables (X) and (Y) - (SD_X): Standard dev
variable (X) - (SD_Y): Standard deviation of variable (Y)

Standard Error of the Correlation Coefficient (SE)

$$SE_r = \frac{1 - r^2}{\sqrt{n - 2}}$$

Where: - n: Sample size

Correlation

- Non normal distribution

How to calculate and combine effect-sizes?

Fisher’s Transformation

Fisher’s transformation is used to stabilize the variance and make the sampling distribu
correlation coefficient more normal:

$$z_r = \frac{1}{2} \cdot \ln\left(\frac{1+r}{1-r}\right)$$

Where:
- r: Observed correlation coefficient
- z_r: Fisher-transformed value

Standard Error of (z_r)

$$SE_{z_r} = \frac{1}{\sqrt{n-3}}$$

Where:
- n : Sample size

Key Takeaways: Soil Experiment Exar

- **Comparison of Plant Growth in Two Soil Types**:

Variable	Soil A	Soil B
Mean Height	25 cm	20 cm
Standard Deviation	4 cm	5 cm
Sample Size (n)	30	30

- **Effect Size Calculations**:
  - **Mean Difference (MD)**: 5 cm
  - **Cohen's d**: 1.09
  - **Hedges' g**: 1.08
  - **Ratio**: 1.25
  - **log(Ratio)**: 0.0223

Important Considerations for Effect S

1. **Data Characteristics**:
    - Ensure assumptions of normality and homogeneity of variance are met.
    - Consider small sample size biases, especially with Cohen's d.
2. **Interpretation**:
    - Choose the appropriate effect size based on study design (e.g., continuous vs. cat data).
    - Understand practical significance beyond just statistical significance.
3. **Use of Correct Formulas**:
    - When sample sizes differ, use **pooled standard deviations** for accurate mean diffe
    - Apply corrections like Hedges' g for small sample sizes to minimize bias.

Key References for Effect Size Calculations

1. **Borenstein et al. (2009)**:
   - Comprehensive guide on effect size calculations and interpretations.
   - Essential for meta-analyses and evidence synthesis.
2. **The `esc` and `metafor` Packages**:
   - R tools for calculating various effect sizes.
   - Use `metafor` for meta-analytic models and conversions between effect types.
3. **Practical Meta-Analysis (Lipsey & Wilson)**:
   - Resource for practical guidance on interpreting and reporting effect sizes.
   - Includes guidance on which effect size to use based on research context.