# Report of experiments

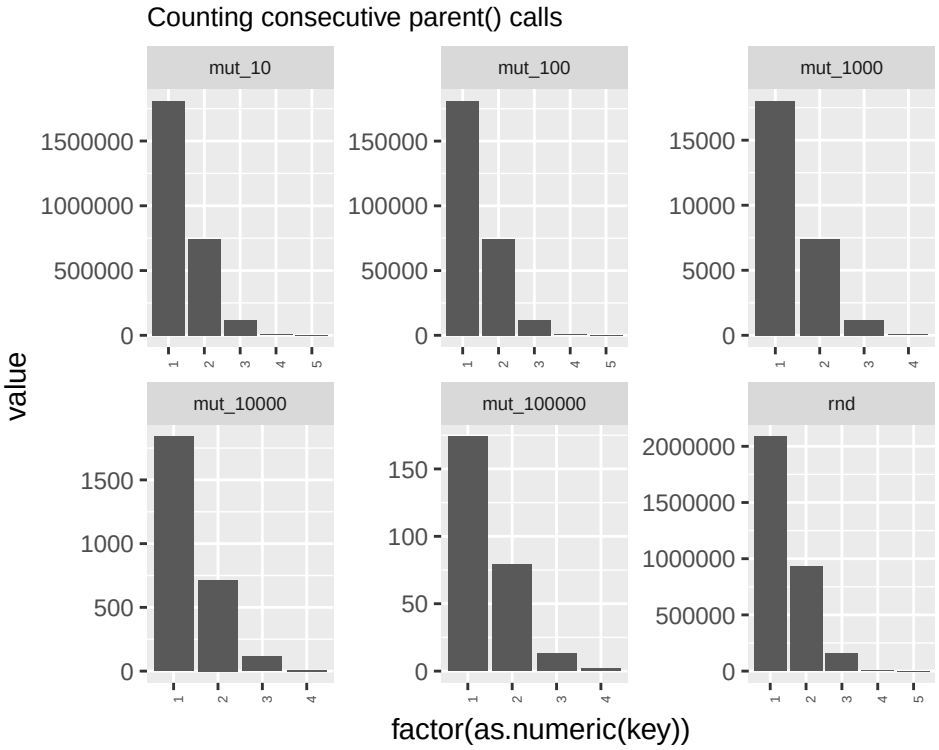
*O. Denas*

*12/28/2016*

## Contents

# 1 Input properties

For various types ("mut_XMs_YMt_Z" means `s` and `t` are random identical strings of length X, and Y million respectively with mutations inserted every Z characters. "rnd_XMs_YMt" means `s` and `t` are random strings of length X, and Y million respectively) of inputs run the MS algorithm and count the number of consecutive `wl()` or `parent()` calls during the `runs` or `ms` construction phase.



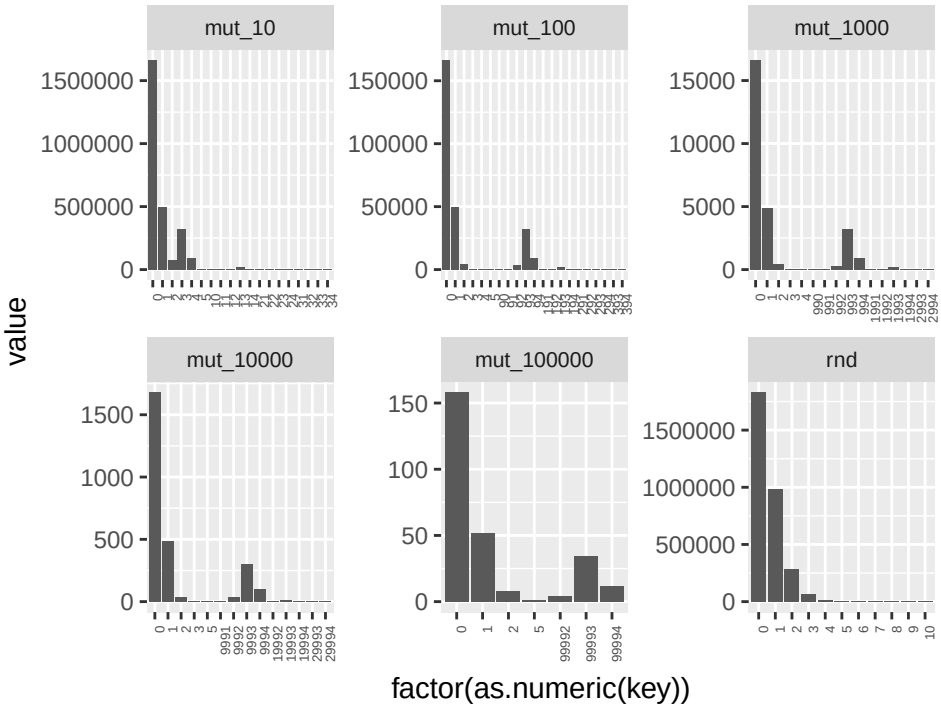



Table 1: Double rank calls that fail for various input types.

| b_path | where | total_calls | fail_calls | perc |
|---|---|---|---|---|
| mut_10 | ms | 2323907 | 1494544 | 64.31 |
| mut_10 | runs | 4999999 | 1525775 | 30.52 |
| mut_100 | ms | 4732340 | 4649474 | 98.25 |
| mut_100 | runs | 4999999 | 4652823 | 93.06 |
| mut_1000 | ms | 4973326 | 4965082 | 99.83 |
| mut_1000 | runs | 4999999 | 4965446 | 99.31 |
| mut_10000 | ms | 4997320 | 4996520 | 99.98 |
| mut_10000 | runs | 4999999 | 4996531 | 99.93 |
| mut_100000 | ms | 4999731 | 4999634 | 100.00 |
| mut_100000 | runs | 4999999 | 4999625 | 99.99 |
| rnd | ms | 1818974 | 690911 | 37.98 |
| rnd | runs | 4999999 | 767488 | 15.35 |

Table 2: Double rank calls that fail for various input types.

| b_path | total_calls | fail_calls | perc |
|---|---|---|---|
| mut_10 | 7323906 | 3020319 | 41.24 |
| mut_100 | 9732339 | 9302297 | 95.58 |
| mut_1000 | 9973325 | 9930528 | 99.57 |
| mut_10000 | 9997319 | 9993051 | 99.96 |
| mut_100000 | 9999730 | 9999259 | 100.00 |
| rnd | 6818973 | 1458399 | 21.39 |

Table 3: Composition of the `runs` vector for various input types.

| vector_value | mut_10 | mut_100 | mut_1000 | mut_10000 | mut_100000 | rnd |
|---|---|---|---|---|---|---|
| 0 | 2676093 | 267660 | 26674 | 2680 | 269 | 3181026 |
| 1 | 2323907 | 4732340 | 4973326 | 4997320 | 4999731 | 1818974 |

# 2 Double vs. single rank

## 2.1 Code

The single rank and double rank implementations in sdsl: rank_support_v.hpp link

```
// RANK(idx)
const uint64_t* p = m_basic_block.data() + ((idx>>8)&0xFFFFFFFFFFFFFFFEULL);
return *p + ((*(p+1)>>(63 - 9*((idx&0x1FF)>>6)))&0x1FF) +
      (idx&0x3F ? trait_type::word_rank(m_v->data(), idx) : 0);

// DOUBLE RANK OD(i, j)
if((i>>8) == (j>>8)){
  const uint64_t* p = m_basic_block.data() + ((i>>8)&0xFFFFFFFFFFFFFFFEULL);
  res.first  = *p + ((*(p+1)>>(63 - 9*((i&0x1FF)>>6)))&0x1FF) +
            (i&0x3F ? trait_type::word_rank(m_v->data(), i) : 0);
  res.second = *p + ((*(p+1)>>(63 - 9*((j&0x1FF)>>6)))&0x1FF) +
            (j&0x3F ? trait_type::word_rank(m_v->data(), j) : 0);
} else {
  const uint64_t* p = m_basic_block.data() + ((i>>8)&0xFFFFFFFFFFFFFFFEULL);
  res.first  = *p + ((*(p+1)>>(63 - 9*((i&0x1FF)>>6)))&0x1FF) +
            (i&0x3F ? trait_type::word_rank(m_v->data(), i) : 0);
  p -= (((i>>8)&0xFFFFFFFFFFFFFFFEULL) - ((j>>8)&0xFFFFFFFFFFFFFFFEULL));
  res.second = *p + ((*(p+1)>>(63 - 9*((j&0x1FF)>>6)))&0x1FF) +
            (j&0x3F ? trait_type::word_rank(m_v->data(), j) : 0);
}
return res

// DOUBLE RANK FC(i, j)
const uint64_t* b = m_basic_block.data();
const uint64_t* pi = b + ((i>>8)&0xFFFFFFFFFFFFFFFEULL);
const uint64_t* pj = b + ((j>>8)&0xFFFFFFFFFFFFFFFEULL);

return (*pi + ((*(pi+1)>>(63 - 9*((i&0x1FF)>>6)))&0x1FF) +
              (i&0x3F ? trait_type::word_rank(m_v->data(), i) : 0),
      *pj + ((*(pj+1)>>(63 - 9*((j&0x1FF)>>6)))&0x1FF) +
              (j&0x3F ? trait_type::word_rank(m_v->data(), j) : 0));
```

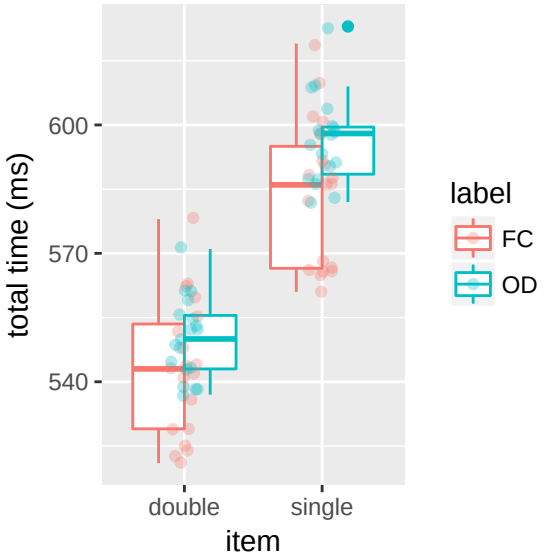
## 2.2 Performance

Table 4: Time (in ms) of 500K calls to `wl()` based on `single_rank()` or `double_rank()` methods on 100MB random DNA input; Mean/sd over 20 repetitions.

| item | label | avg_time | sd_time |
|---|---|---|---|
| double | FC | 543.11 | 15.88 |
| double | OD | 550.00 | 9.27 |
| single | FC | 584.32 | 17.11 |
| single | OD | 596.37 | 10.20 |

Table 5: FC vs. OD implementations. Absolute (FC / OD) and relative (100 * |FC - OD| / OD) ratios of average times

| item | FC | OD | abs_ratio | rel_ratio |
|---|---|---|---|---|
| double | 543.11 | 550.00 | 0.99 | 1.25 |
| single | 584.32 | 596.37 | 0.98 | 2.02 |



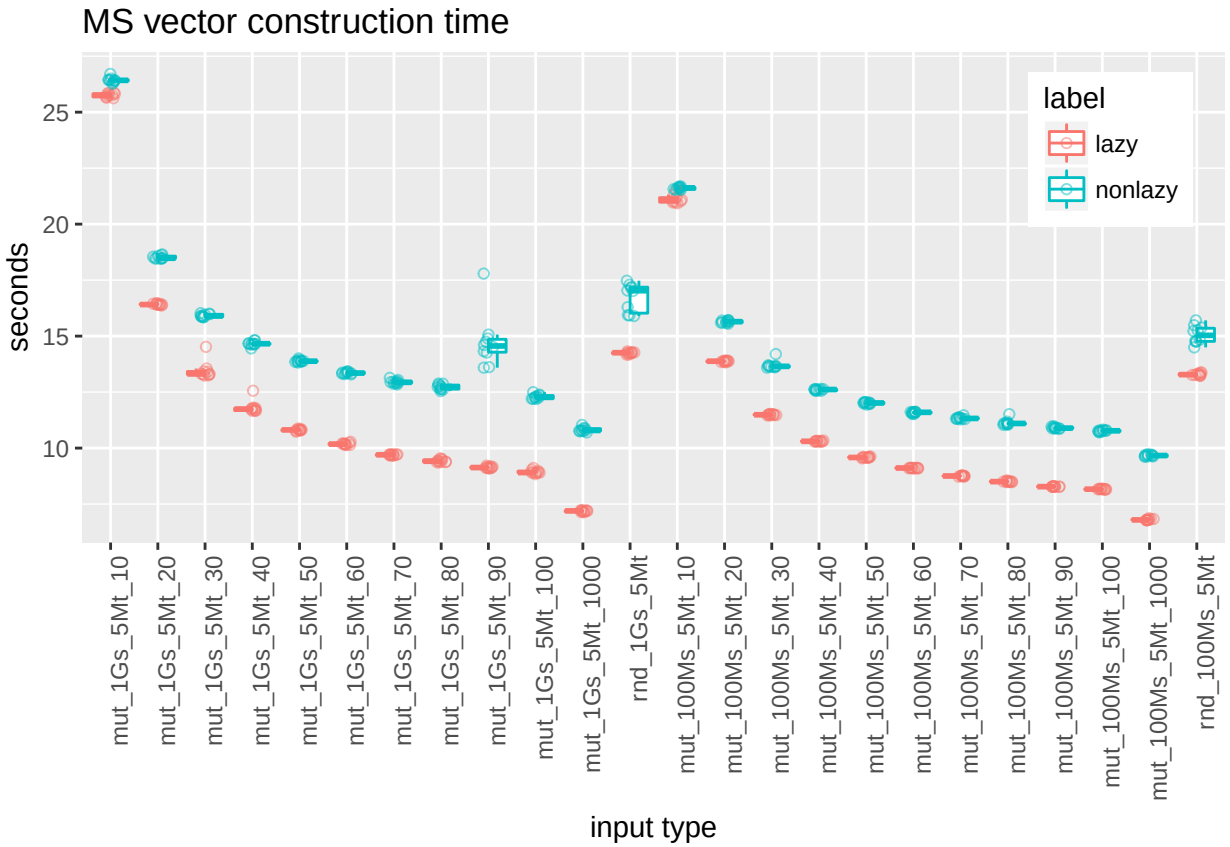
# 3 Lazy vs non-lazy

## 3.1 Code

The lazy and non-lazy versions differ in a couple of lines of code as follows

```
if(flags.lazy){
    for(; I.first <= I.second && h_star < ms_size; ){
        c = t[h_star];
        I = bstep_interval(st, I, c); //I.bstep(c);
        if(I.first <= I.second){
            v = st.lazy_wl(v, c);
            h_star++;
        }
    }
    if(h_star > h_star_prev) // // we must have called lazy_wl(). complete the node
        st.lazy_wl_followup(v);
} else { // non-lazy weiner links
    for(; I.first <= I.second && h_star < ms_size; ){
        c = t[h_star];
        I = bstep_interval(st, I, c); //I.bstep(c);
        if(I.first <= I.second){
            v = st.wl(v, c);
            h_star++;
        }
    }
}
```

### 3.2 Performance



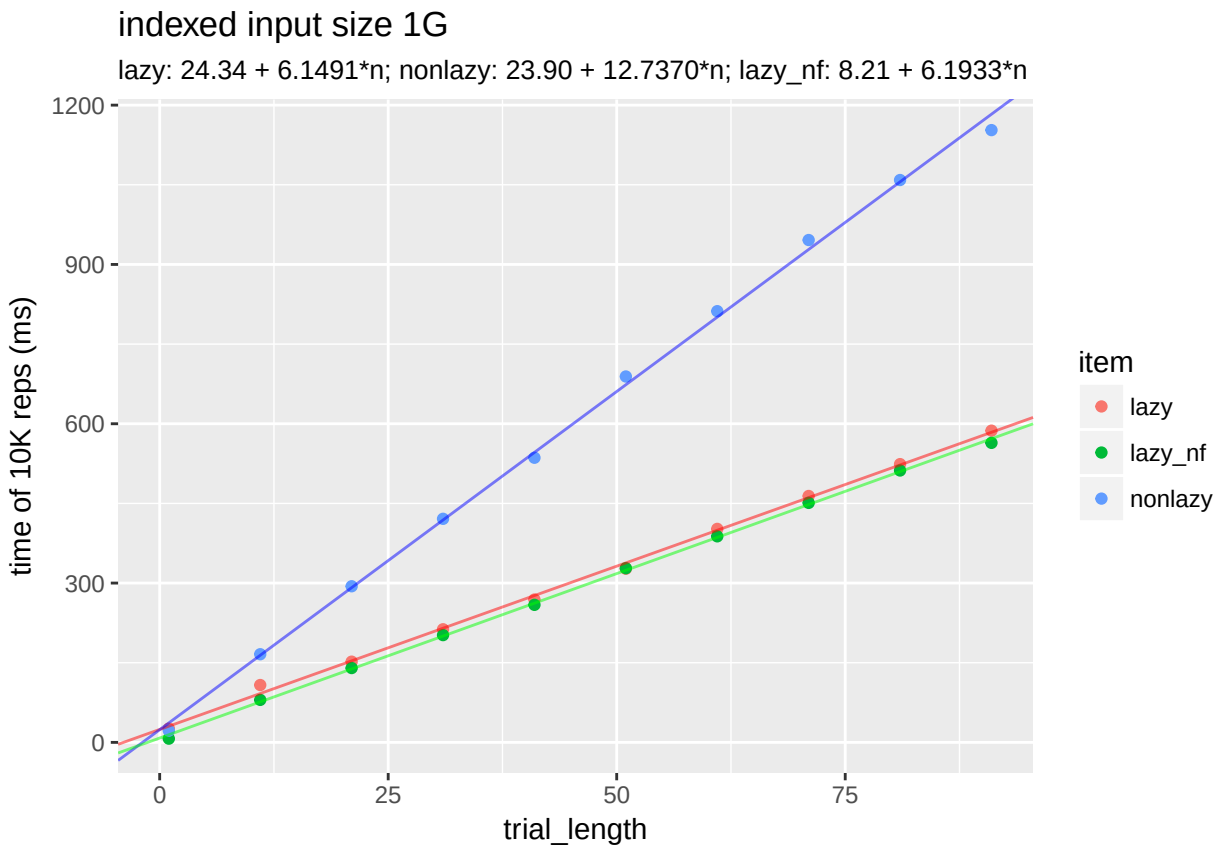

The right panel shows the time to construct the `runs` vector. This stage is the same for both versions and is shown as a control. On the left panel it can be seen that speedup correlates positively with both the size of the indexed string and the mutation period.

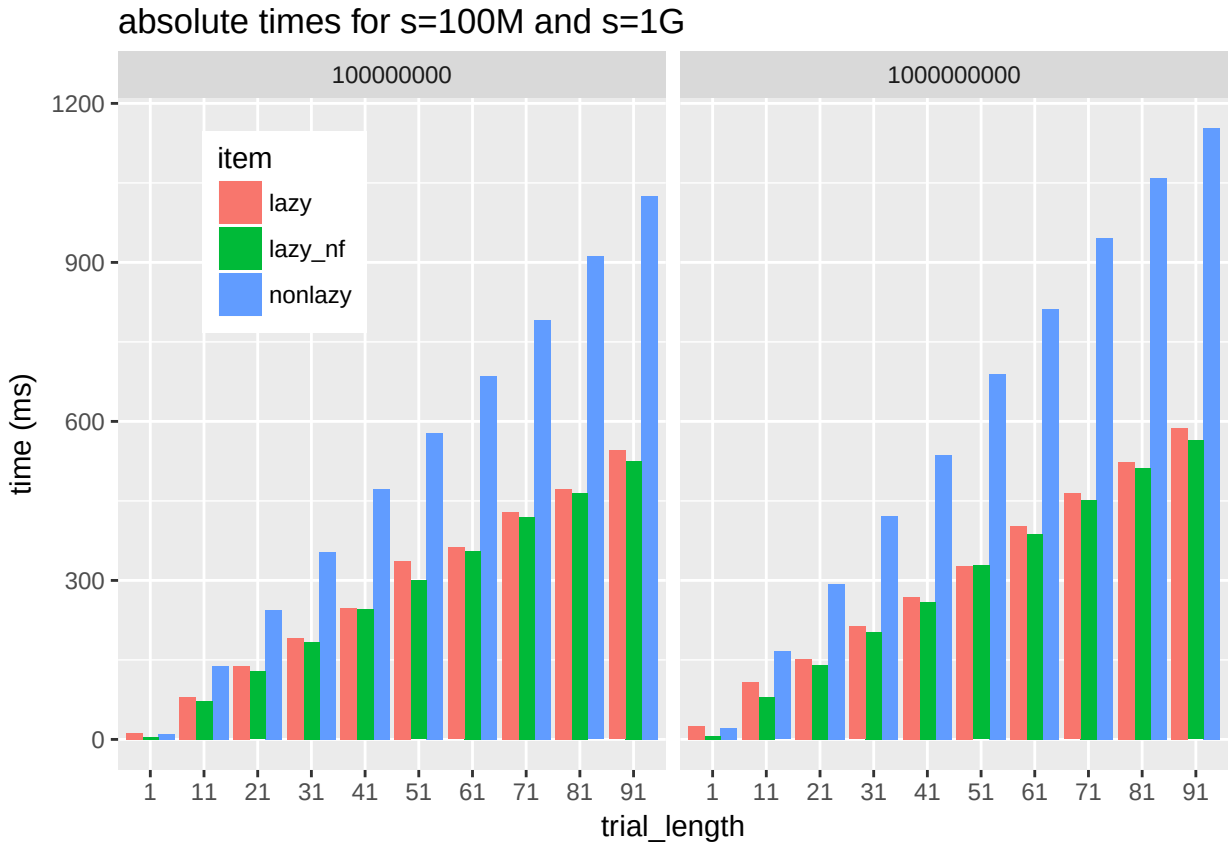
### 3.3 Sandbox timing

Measure the time of 10k repetitions of

- (lazy) $n$ consecutive `lazy_wl()` calls followed by a `lazy_wl_followup()`
- (nonlazy) $n$ consecutive `wl()` calls
- (lazy_nf) $n$ consecutive `lazy_wl()` calls

```
// lazy
for(size_type i = 0; i < trial_length; i++)
    v = st.lazy_wl(v, s_rev[k--]);
if(h_star > h_star_prev) // // we must have called lazy_wl(). complete the node
    st.lazy_wl_followup(v);
...
// non-lazy
for(size_type i = 0; i < trial_length; i++)
    v = st.wl(v, s_rev[k--]);
...
// lazy_nf
for(size_type i = 0; i < trial_length; i++)
    v = st.lazy_wl(v, s_rev[k--]);
```






### 3.4 Check

In the experiments above we ran the program with the "lazy" or "non-lazy" flag and measured. The total time of each experiment can be written as $t_l = l_l + a$ and $t_n = l_n + a$ for the two versions respectively; only the $t$s being known. Furthermore, we have $\hat{l}_l$ and $\hat{l}_n$ estimations – computed by combining the time / wl call with the number of with the count of wl calls in each input (Section "Input Properties"). Hence we should expect

$$\delta t = t_l - t_n = l_l + a - l_n - a = l_l - l_n \approx \delta\hat{l} = \hat{l}_l - \hat{l}_n$$

| b_path | t_l | t_n | l_l | l_n | delta_t | delta_l_hat |
|---|---|---|---|---|---|---|
| mut_100Ms_5Mt_10 | 21.12 | 21.61 | 8.56 | 6.16 | -0.49 | 2.39 |
| mut_100Ms_5Mt_100 | 8.16 | 10.77 | 3.36 | 4.33 | -2.60 | -0.97 |
| mut_100Ms_5Mt_1000 | 6.80 | 9.67 | 2.84 | 4.15 | -2.86 | -1.31 |
| mut_100Ms_5Mt_20 | 13.87 | 15.64 | 5.66 | 5.14 | -1.77 | 0.52 |
| mut_100Ms_5Mt_30 | 11.49 | 13.70 | 4.71 | 4.81 | -2.21 | -0.10 |
| mut_100Ms_5Mt_40 | 10.31 | 12.60 | 4.22 | 4.64 | -2.30 | -0.41 |
| mut_100Ms_5Mt_50 | 9.58 | 12.01 | 3.93 | 4.53 | -2.43 | -0.60 |
| mut_100Ms_5Mt_60 | 9.11 | 11.58 | 3.74 | 4.47 | -2.48 | -0.72 |
| mut_100Ms_5Mt_70 | 8.75 | 11.34 | 3.60 | 4.42 | -2.59 | -0.81 |
| mut_100Ms_5Mt_80 | 8.51 | 11.13 | 3.50 | 4.38 | -2.63 | -0.88 |
| mut_100Ms_5Mt_90 | 8.28 | 10.90 | 3.42 | 4.35 | -2.62 | -0.93 |
| mut_1Gs_5Mt_10 | 25.75 | 26.43 | 7.57 | 6.65 | -0.68 | 0.92 |
| mut_1Gs_5Mt_100 | 8.94 | 12.29 | 3.49 | 4.90 | -3.35 | -1.41 |

| b_path | t_l | t_n | l_l | l_n | delta_t | delta_l_hat |
|---|---|---|---|---|---|---|
| mut_1Gs_5Mt_1000 | 7.19 | 10.82 | 3.08 | 4.72 | -3.63 | -1.64 |
| mut_1Gs_5Mt_20 | 16.42 | 18.52 | 5.30 | 5.68 | -2.10 | -0.37 |
| mut_1Gs_5Mt_30 | 13.46 | 15.92 | 4.55 | 5.36 | -2.46 | -0.81 |
| mut_1Gs_5Mt_40 | 11.81 | 14.66 | 4.17 | 5.20 | -2.85 | -1.02 |
| mut_1Gs_5Mt_50 | 10.81 | 13.89 | 3.95 | 5.10 | -3.08 | -1.15 |
| mut_1Gs_5Mt_60 | 10.19 | 13.36 | 3.80 | 5.03 | -3.17 | -1.24 |
| mut_1Gs_5Mt_70 | 9.70 | 12.95 | 3.69 | 4.99 | -3.26 | -1.30 |
| mut_1Gs_5Mt_80 | 9.43 | 12.72 | 3.61 | 4.95 | -3.29 | -1.35 |
| mut_1Gs_5Mt_90 | 9.14 | 14.74 | 3.55 | 4.93 | -5.60 | -1.38 |
| rnd_100Ms_5Mt | 13.29 | 15.07 | 9.65 | 6.55 | -1.78 | 3.10 |
| rnd_1Gs_5Mt | 14.25 | 16.72 | 8.20 | 6.92 | -2.48 | 1.28 |

The numbers are not identical (process dependent factors might influence the running time of function calls), but they are correlated ($corr(\delta t, \delta\hat{l}) = 0.71$).

# 4 Double rank and fail

## 4.1 Code

```
// Given subtree_double_rank(v, i, j) -> (a.first, a.second) -- to simplify code

// DOUBLE RANK: int i, int j, char c
p = bit_path(c)
result_i, result_j = i, j;
node_type v = m_tree.root();
for (l = 0; l < path_len; ++l, p >>= 1) {
  a = subtree_double_rank(v, m_tree.bv_pos(v) + result_i, m_tree.bv_pos(v) + result_j);

  if(p&1){ // left child
      if(result_i > 0) result_i = a.first;
      if(result_j > 0) result_j = a.second;
  } else { // right child
      if(result_i > 0) result_i -= a.first;
      if(result_j > 0) result_j -= a.second;
  }
  v = m_tree.child(v, p&1); // goto child
}
return(result_i, result_j)

// DOUBLE RANK AND FAIL
p = bit_path(c)
result_i, result_j = i, j;
node_type v = m_tree.root();
for (l = 0; l < path_len; ++l, p >>= 1) {
  a = subtree_double_rank(v, m_tree.bv_pos(v) + result_i, m_tree.bv_pos(v) + result_j);

  if(p&1){ // left child
      if(result_i > 0) result_i = a.first;
      if(result_j > 0) result_j = a.second;
  } else { // right child
      if(result_i > 0) result_i -= a.first;
      if(result_j > 0) result_j -= a.second;
  }
  if(result_i == result_j) // Weiner Link call will fail
    return(0, 0)
  v = m_tree.child(v, p&1); // goto child
}
return(result_i, result_j)
```

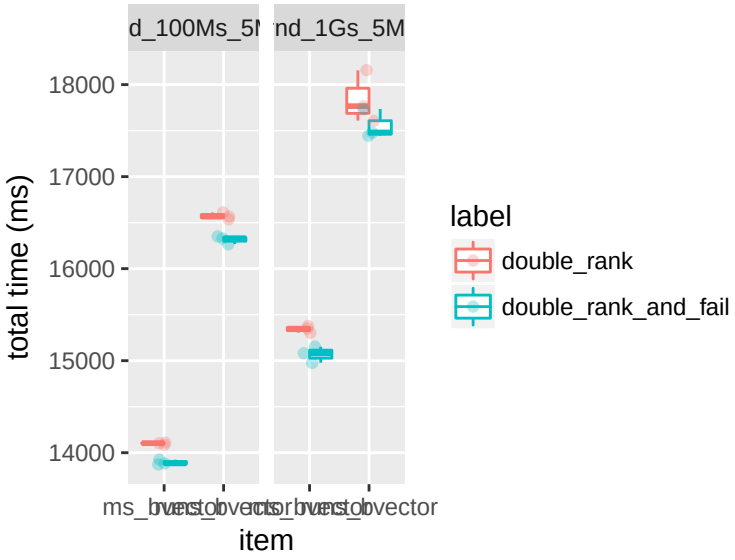
## 4.2 Performance

Table 7: Time (in ms) of 500K calls to `wl()` based on `single_rank()` or `double_rank()` methods on 100MB random DNA input; Mean/sd over 20 repetitions.

| item | label | b_path | avg_time | sd_time |
|---|---|---|---|---|
| ms_bvector | double_rank | rnd_100Ms_5Mt | 14101.33 | 12.22 |
| ms_bvector | double_rank | rnd_1Gs_5Mt | 15342.67 | 35.30 |
| ms_bvector | double_rank_and_fail | rnd_100Ms_5Mt | 13893.33 | 27.23 |
| ms_bvector | double_rank_and_fail | rnd_1Gs_5Mt | 15068.67 | 87.37 |
| runs_bvector | double_rank | rnd_100Ms_5Mt | 16572.33 | 36.30 |
| runs_bvector | double_rank | rnd_1Gs_5Mt | 17843.33 | 281.09 |
| runs_bvector | double_rank_and_fail | rnd_100Ms_5Mt | 16315.00 | 42.33 |
| runs_bvector | double_rank_and_fail | rnd_1Gs_5Mt | 17553.00 | 158.64 |

Table 8: Single vs. double rank. Absolute (double / single) and relative (100 * |double - single| / single) ratios of average times.

| item | b_path | double_rank | double_rank_and_fail | abs_ratio | rel_ratio |
|---|---|---|---|---|---|
| ms_bvector | rnd_100Ms_5Mt | 14101.33 | 13893.33 | 0.99 | 1.48 |
| ms_bvector | rnd_1Gs_5Mt | 15342.67 | 15068.67 | 0.98 | 1.79 |
| runs_bvector | rnd_100Ms_5Mt | 16572.33 | 16315.00 | 0.98 | 1.55 |
| runs_bvector | rnd_1Gs_5Mt | 17843.33 | 17553.00 | 0.98 | 1.63 |



# 5 Parallelization

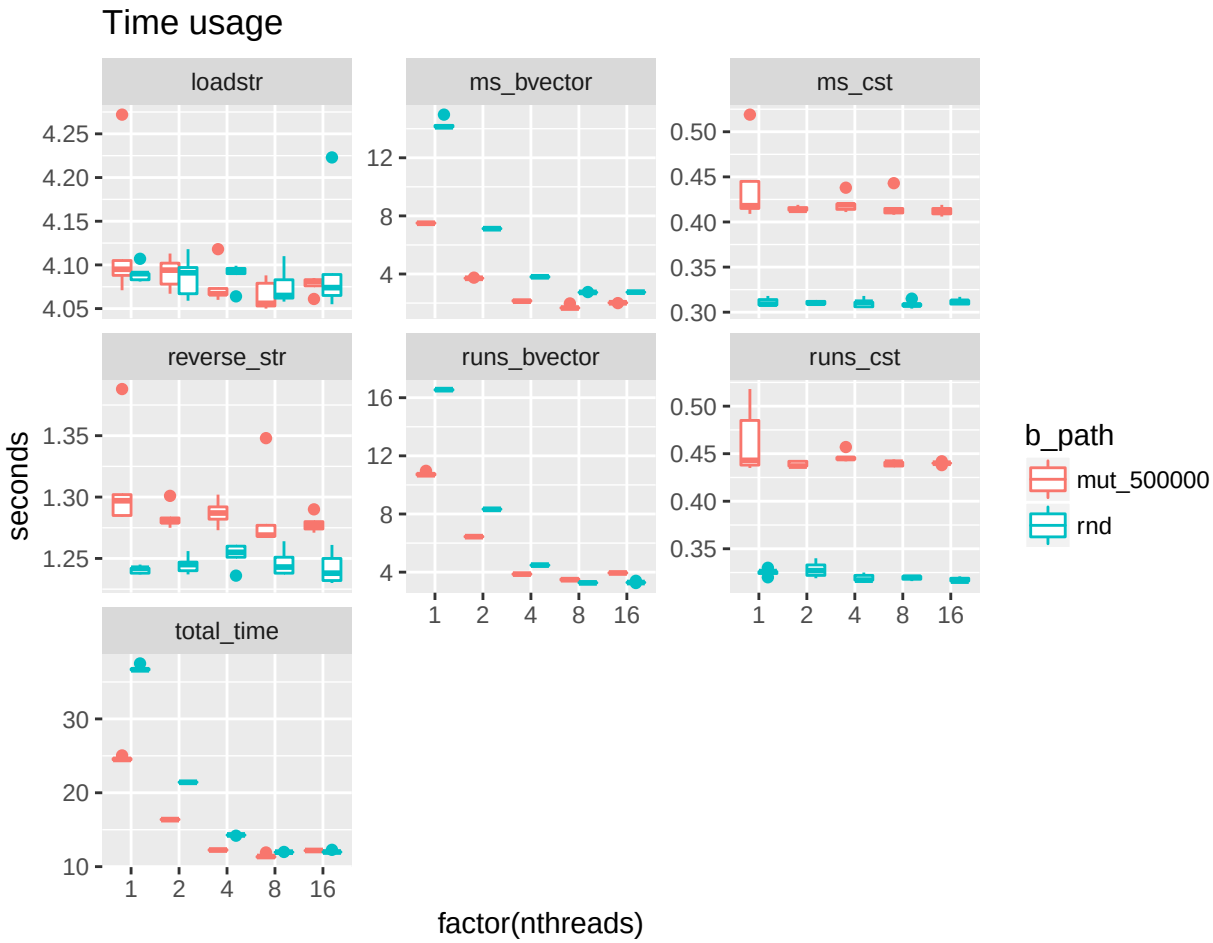
## 5.1 Code

See the pseudo-code in the repo (link)

## 5.2 Performance

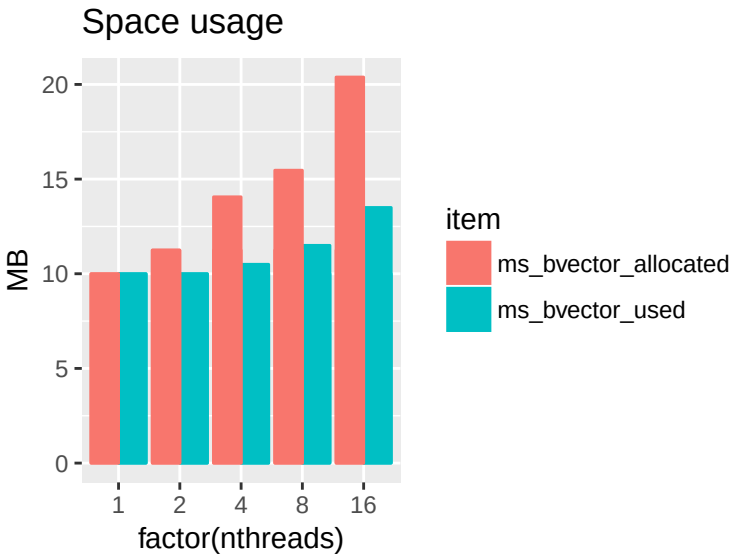
Run the MS construction program on the same input (random strings $s$ of length 100M and $t$ of length 5M) with varying parallelzation degree (nthreads = number of threads).

The time is reported over 5 runs for each fixed number of threads.




Space in MB for the same settings as above.

Each thread allocates its own $ms$ vector with initial size $|t|/nthreads$ then it resizes by a factor of 1.5 each time it needs to. Resizing will always result in a vector smaller than $2|t|$ elements.


Space usage
20
15
10
5
0
MB
1
2
4
8
16
factor(nthreads)
item
ms_bvector_allocated
ms_bvector_used