
Algorithm 1: `fill_runs_slice(T, t, from, to, v, runs)`

Input: Cst, T, of index string `s` supporting `WeinerLink` and `Parent`;
Query string `t`; Interval over `t`, `[from, to]`; Starting node `v`;
Vector `runs`.

Output: Tripple `(first, last, v)`: first index of the first failing
`WeinerLink` call; last index last failing `WeinerLink` call; `v` the
node corresponding to `last`;

```
1 v ← T.WeinerLink(T.root(), t[to - 1]);
2 idx_set ← FALSE;
3 k ← to - 2;
4 repeat
5   c ← t[k - 1];
6   if T.IsRoot(T.WeinerLink(v, t[to - 1])) then
7     if idx_set = FALSE then
8       first ← k;
9       idx_set ← TRUE;
10    end
11    runs[k] ← 0;
12    repeat
13      v ← T.parent(v);
14      until T.IsRoot(T.WeinerLink(v, c));
15      v ← T.WeinerLink(v, c);
16      last ← k;
17    else
18      runs[k] ← 1
19    end
20    v ← T.WeinerLink(v, c);
21    k ← k - 1;
22 until k > from;
```

Algorithm 2: Build runs in parallel.

Input: Cst, T, of index string `s` supporting `WeinerLink` and `Parent`;
Query string `t`; Number of threads `nthreads` dividing `|t|`.

```
1 S ← |t|/nthreads;
2 R ← [1..nthreads];
3 runs ← [1..t];
4 for i ← 1 to nthreads do // run asynchronously
5   Ri ← fill_runs_slice(T, t, (i - 1) × S, i × S);
6 end
7 for i ← 1 to nthreads - 1 do // run asynchronously
8   fill_runs_slice(T, t, Ri-1[0], Ri[1], Ri[2]);
9 end
```

Algorithm 3: fill_ms_slice($T, t, ms, runs, from, to$)

Input: Cst of index string \bar{s} , T , supporting `WeinerLink` and `Parent`;
Query string t ; Vector ms initialized at 0; Vector $runs$; Interval
over t , $[from, to)$

Output: Last index j of ms

```
1  $k \leftarrow from, j \leftarrow 0, h^* \leftarrow k + 1;$ 
2  $c \leftarrow t[k];$ 
3  $v \leftarrow T.root();$ 
4 repeat //extend current match
5    $h \leftarrow h^*;$ 
6    $u \leftarrow v;$ 
7   repeat
8      $c \leftarrow t[h^*];$ 
9      $u \leftarrow T.WeinerLink(v, c);$ 
10    if ! $T.IsRoot(u)$  then
11       $v \leftarrow u;$ 
12       $h^* \leftarrow h^* + 1;$ 
13    end
14  until ! $T.IsRoot(u)$  and  $h^* < |t|;$ 
15  // resize  $ms$  if needed;
16   $j \leftarrow j + h^* - h + 1$  // set  $ms[j..j + h^* - h + 1]$  to 0;
17   $ms[j + +] \leftarrow 1;$ 
18  if  $h^* < |t|$  then
19    repeat // clip prefixes of  $t[k..h^*]$  until can extend by  $c$ 
20       $v \leftarrow T.Parent(v);$ 
21       $u \leftarrow T.WeinerLink(v, c);$ 
22      until ! $T.IsRoot(u);$ 
23       $h^* \leftarrow h^* + 1;$ 
24  end
25   $k' \leftarrow k + 1$  // index of first 1 in  $runs[k + 1..];$ 
26  repeat
27     $k' \leftarrow k' + 1;$ 
28    until  $k' < |t|$  and  $runs[k'] \neq 0;$ 
29    // resize  $ms$  if needed;
30     $ms[k + 1..k' - 1] \leftarrow 1;$ 
31     $j \leftarrow \max(k' - 1, to)$  // set  $ms[j.. \max(k' - 1, to)]$  to 0;
32 until  $k < to;$ 
```

Algorithm 4: Build \mathbf{ms} in parallel.

Input: Cst of index string \bar{s} , T , supporting `WeinerLink` and `Parent`;
Query string t ; Number of threads `nthreads` dividing $|t|$.

```
1  $S \leftarrow |t|/\text{nthreads}$ ;  
2 for  $i \leftarrow 1$  to nthreads do  
3    $\mathbf{ms}_i \leftarrow [1..2|t|/\text{nthreads}]$ ;  
4 end  
5 for  $i \leftarrow 1$  to nthreads do // run asynchronously  
6    $J_i \leftarrow \text{fill\_ms\_slice}(T, t, \mathbf{ms}_i, \text{runs}, (i - 1) \times S, i \times S)$ ;  
7 end  
8 for  $i \leftarrow 1$  to nthreads - 1 do  
9    $\mathbf{ms}_i \leftarrow \text{resize\_to}(J_i)$ ;  
10 end
```
