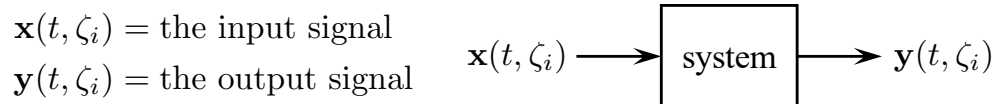


9-2 Systems With Stochastic Inputs

Definitions

- A stochastic process $\mathbf{x}(t, \zeta)$ is a map from \mathcal{S} to a real-valued function of time.
 - For each $\zeta \in \mathcal{S}$, $\mathbf{x}(t, \zeta)$ is a *signal*.
- System: input is a signal $x(t)$. Output is another signal $y(t)$.
 - If the input to a system is a random process, then the input/relationship applies on a sample-by-sample basis.
 - For $\zeta_i \in \mathcal{S}$,

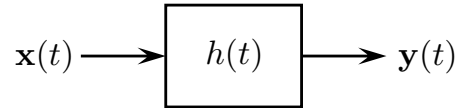


- Kinds of Systems
 - The system is *deterministic* if it operates only the variable t , treating ζ as a parameter.
 - The system is called *stochastic* if it operates on both variables t and ζ .
 - If the system is specified in terms of physical elements or by an equation,
 - * the system is deterministic if the elements or coefficients of the defining equations are deterministic
 - * the system is stochastic if the elements or coefficients of the defining equations are random
 - Memoryless Systems: a system is called *memoryless* if its input/output relationship is given by $\mathbf{y}(t) = g(\mathbf{x}(t))$.
 - LTI Systems: a linear time-invariant system is described in the recorded lectures. The input/output relationship is given by the *convolution* of the input signal with the *impulse response* $h(t)$ of the system:

$$\mathbf{y}(t, \zeta) = \int_{-\infty}^{\infty} \mathbf{x}(u, \zeta) h(t - u) du = \int_{-\infty}^{\infty} \mathbf{x}(t - u, \zeta) h(u) du$$

- In this class, all LTI systems will be described by linear constant-coefficient differential equations *with all zero initial conditions*

LTI System with WSS Input



$$\mu_{\mathbf{y}} = \mu_{\mathbf{x}} \int_{-\infty}^{\infty} h(u) du$$

$$R_{\mathbf{xy}}(\tau) = \int_{-\infty}^{\infty} R_{\mathbf{xx}}(\tau + u) h^*(u) du$$

$$R_{\mathbf{yy}}(\tau) = \int_{-\infty}^{\infty} R_{\mathbf{xy}}(\tau - u) h(u) du$$

Special Case: $\mathbf{x}(t)$ is WSS white random process:

$$\mu_{\mathbf{x}} = 0 \quad R_{\mathbf{xx}}(\tau) = q\delta(\tau)$$

$$\mu_{\mathbf{y}} = 0$$

$$R_{\mathbf{xy}}(\tau) = qh^*(-\tau)$$

$$R_{\mathbf{yy}}(\tau) = q \underbrace{\int_{-\infty}^{\infty} h^*(u - \tau) h(u) du}_{\rho(\tau)}$$