9-2 Systems With Stochastic Inputs

Definitions

- A stochastic process $\mathbf{x}(t,\zeta)$ is a map from $\mathcal{S}$ to a real-valued function of time.
  - For each $\zeta \in \mathcal{S}$, $\mathbf{x}(t,\zeta)$ is a *signal.*
- System: input is a signal $x(t)$. Output is another signal $y(t)$.
  - If the input to a system is a random process, then the input/relationship applies on a sample-by-sample basis.
  - For $\zeta_i \in \mathcal{S}$,

$$\begin{aligned} \mathbf{x}(t,\zeta_i) &= \text{the input signal} \\ \mathbf{y}(t,\zeta_i) &= \text{the output signal} \end{aligned}$$

$$\mathbf{x}(t,\zeta_i) \longrightarrow \boxed{\text{system}} \longrightarrow \mathbf{y}(t,\zeta_i)$$

- Kinds of Systems
  - The system is *deterministic* if it operates only the variable t, treating ζ as a parameter.
  - The system is called *stochastic* if it operates on both variales t and ζ.
  - If the system is specified in terms of physical elements or by an equation,
    * the system is deterministic if the elements or coefficients of the defining equations are deterministic
    * the system is stochastic of the elements or coefficients of the defining equations are random
  - Memoryless Systems: a system is called *memoryless* if its input/output relationship is given by $\mathbf{y}(t) = g(\mathbf{x}(t))$.
  - LTI Systems: a linear time-invariant system is described in the recorded lectures. The input/output relationship is given by the *convolution* of the input signal with the *impulse response* $h(t)$ of the system:

$$\mathbf{y}(t,\zeta) = \int_{-\infty}^{\infty} \mathbf{x}(u,\zeta)h(t-u)\,du = \int_{-\infty}^{\infty} \mathbf{x}(t-u,\zeta)h(u)\,du$$

  - In this class, all LTI systems will be described by linear constant-coefficient differential equations *with all zero initial conditions*

LTI System with WSS Input

$$\mathbf{x}(t) \longrightarrow \boxed{h(t)} \longrightarrow \mathbf{y}(t)$$

$$\mu_{\mathbf{y}} = \mu_{\mathbf{x}} \int_{-\infty}^{\infty} h(u)du$$

$$R_{\mathbf{xy}}(\tau) = \int_{-\infty}^{\infty} R_{\mathbf{xx}}(\tau + u)h^*(u)du$$

$$R_{\mathbf{yy}}(\tau) = \int_{-\infty}^{\infty} R_{\mathbf{xy}}(\tau - u)h(u)du$$

Special Case: $\mathbf{x}(t)$ is WSS white random process:

$$\mu_{\mathbf{x}} = 0 \qquad R_{\mathbf{xx}}(\tau) = q\delta(\tau)$$

$$\mu_{\mathbf{y}} = 0$$

$$R_{\mathbf{xy}}(\tau) = qh^*(-\tau)$$

$$R_{\mathbf{yy}}(\tau) = q \underbrace{\int_{-\infty}^{\infty} h^*(u - \tau)h(u)du}_{\rho(\tau)}$$