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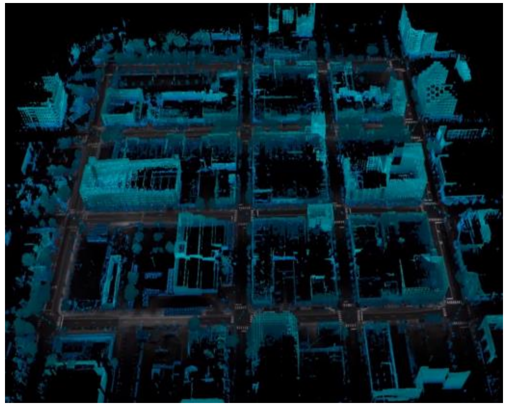
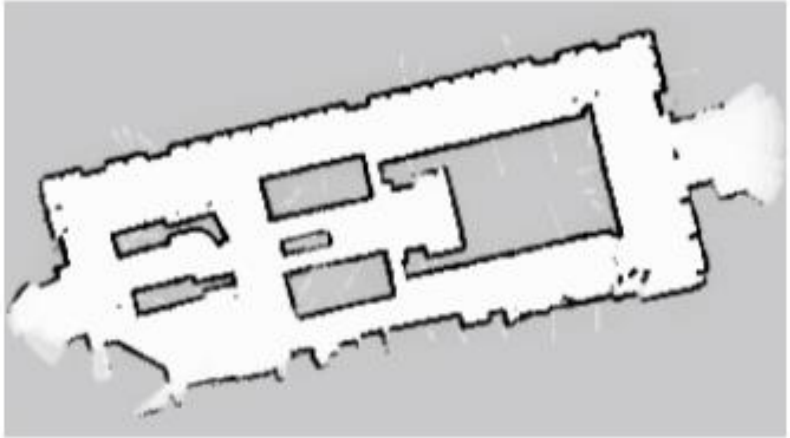
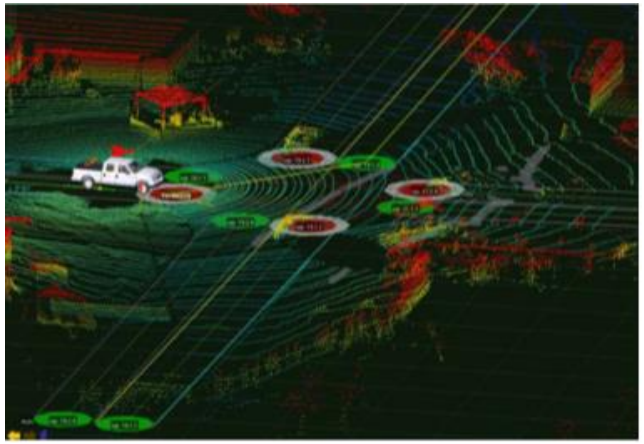






# BAYES FILTERS

## ECEN 633: Robotic Localization and Mapping

Slides Based on probabilistic-robotics.org and a slide deck by Ryan Eustice.

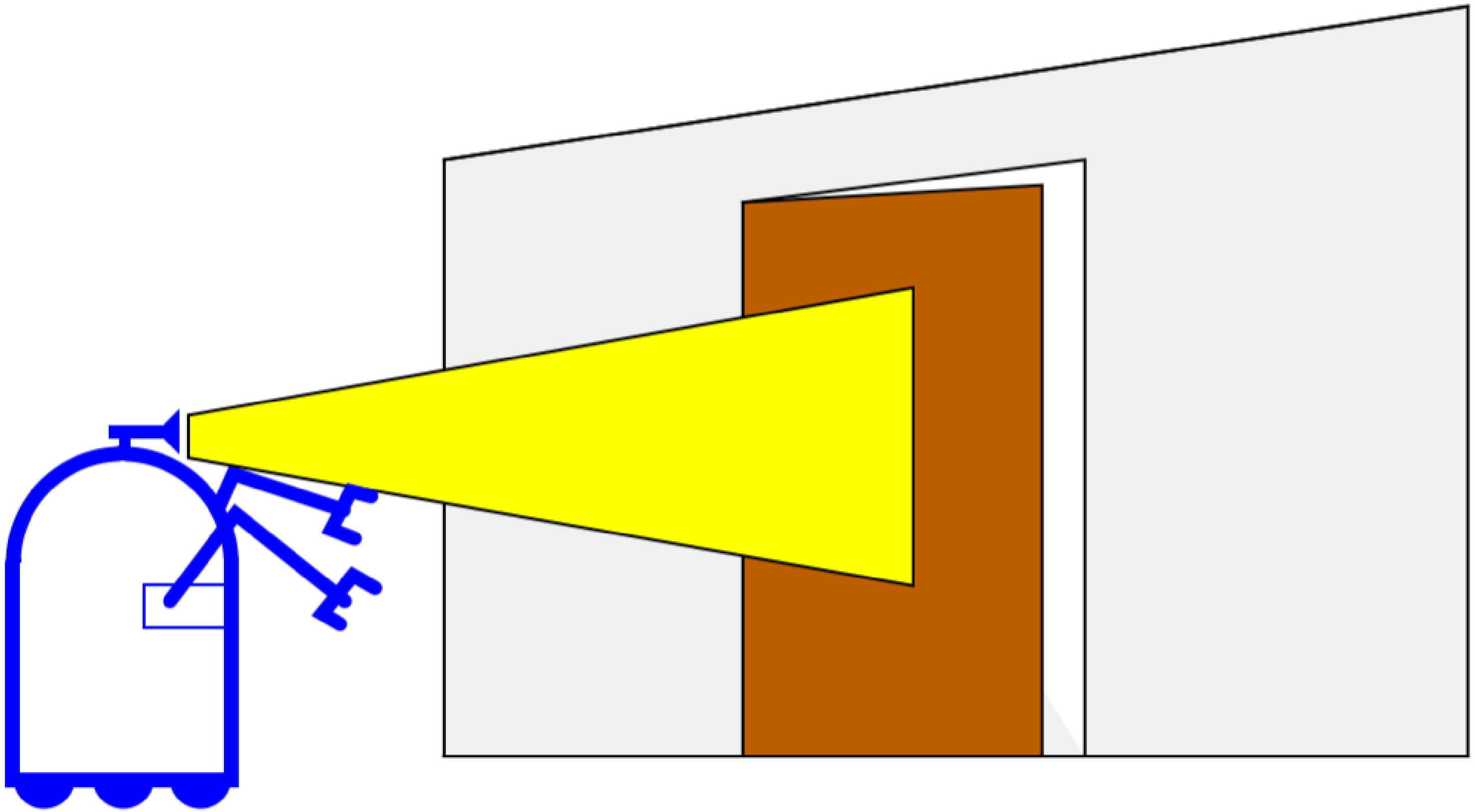
# Agenda

- Estimation via **Bayes Filters!**

# Estimation using Bayes Filters

# Simple State Estimation Example

- Suppose a robot obtains measurement z
  - E.g. robot estimates state of door using its camera
- What is P(open | z)



# Causal vs. Diagnostic Reasoning

- P( open | z ) is ***diagnostic***
- P( z | open ) is ***causal***
- Often causal knowledge is easier to obtain.
- Bayes rule allows us to swap them out

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z)}$$

**Bayes Rule**

# Example

- $z = sense\_open$
- $P(z = sense\_open \mid open) = 0.6$ $\quad P(z = sense\_open \mid \neg open) = 0.3$
- $P(open) = P(\neg open) = 0.5$

$$P(open \mid z) = \frac{P(z \mid open)P(open)}{P(z \mid open)P(open) + P(z \mid \neg open)P(\neg open)}$$

$$P(open \mid z = sense\_open) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

- z raises the probability that the door is open

# Combining Evidence

- Suppose our robot obtains another observation $z_2$.
- How can we incorporate this information?
- More generally, how can we estimate $P(x \mid z_1, \ldots, z_n)$ ?

# Recursive Bayesian Updating

$$P(x \mid z_1, \ldots, z_n) = \frac{P(z_n \mid x, z_1, \ldots, z_{n-1})\, P(x \mid z_1, \ldots, z_{n-1})}{P(z_n \mid z_1, \ldots, z_{n-1})}$$

**Markov assumption:** $z_n$ is independent of $z_1, \ldots, z_{n-1}$ if we know $x$

$$P(x \mid z_1, \ldots, z_n) = \frac{P(z_n \mid x)\, P(x \mid z_1, \ldots, z_{n-1})}{P(z_n \mid z_1, \ldots, z_{n-1})}$$

$$= \eta\, P(z_n \mid x)\, P(x \mid z_1, \ldots, z_{n-1})$$

$$= \eta_{1\ldots n} \prod_{i=1\ldots n} P(z_i \mid x)\, P(x)$$

# Example: Second (Poorer) Measurement

- $P(z_2 = sense\_open \mid open) = 0.5 \qquad P(z_2 = sense\_open \mid \neg open) = 0.6$
- $P(open \mid z_1 = sense\_open) = 2/3$

$$P(open \mid z_2, z_1) = \frac{P(z_2 \mid open)\, P(open \mid z_1)}{P(z_2 \mid open)\, P(open \mid z_1) + P(z_2 \mid \neg open)\, P(\neg open \mid z_1)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

- $z_2$ decreases the probability that the door is open

$$P(open \mid z) = \frac{P(z \mid open) P(open)}{P(z \mid open) P(open) + P(z \mid \neg open) P(\neg open)}$$

# Actions

- Often the world is ***dynamic*** due to:
  - Actions carried out **by the robot**
  - Actions carried out **by other agents**
  - **Time** passing

- How can we **incorporate** such **actions**?

# Typical Actions

- The robot **turns its wheels** to move
- The robot **uses its manipulator** to grasp an object
- Plants grow over **time…**
- If you don't know what is happening **than more time can increase uncertainty…**

- Actions are **never carried out with absolute certainty.**
- In contrast to measurements, **actions generally increase uncertainty.**

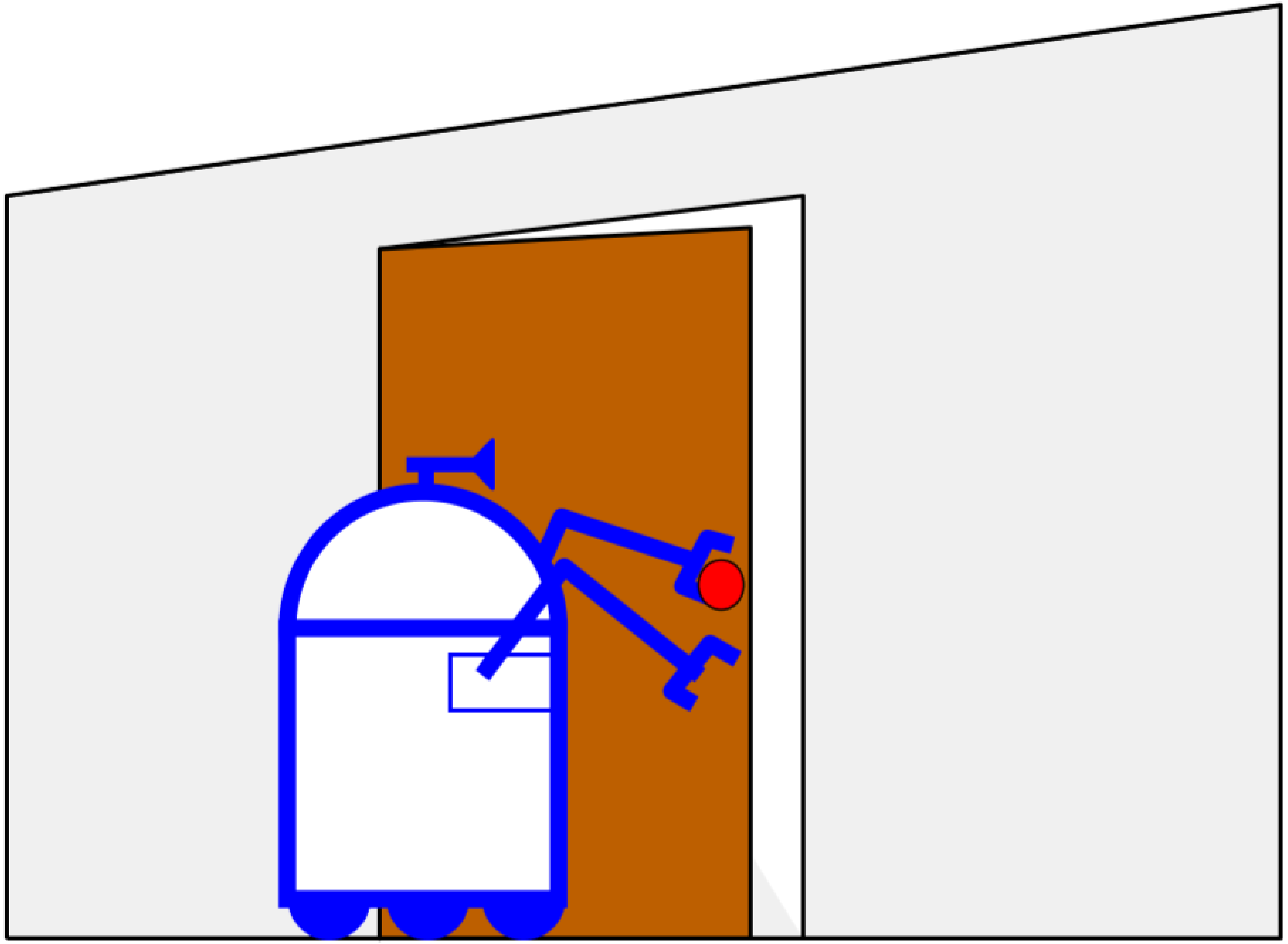
# Modeling Actions

- To incorporate the outcome of an action $u$ into the current "belief", we use the conditional pdf

$$P(x \mid u, x')$$

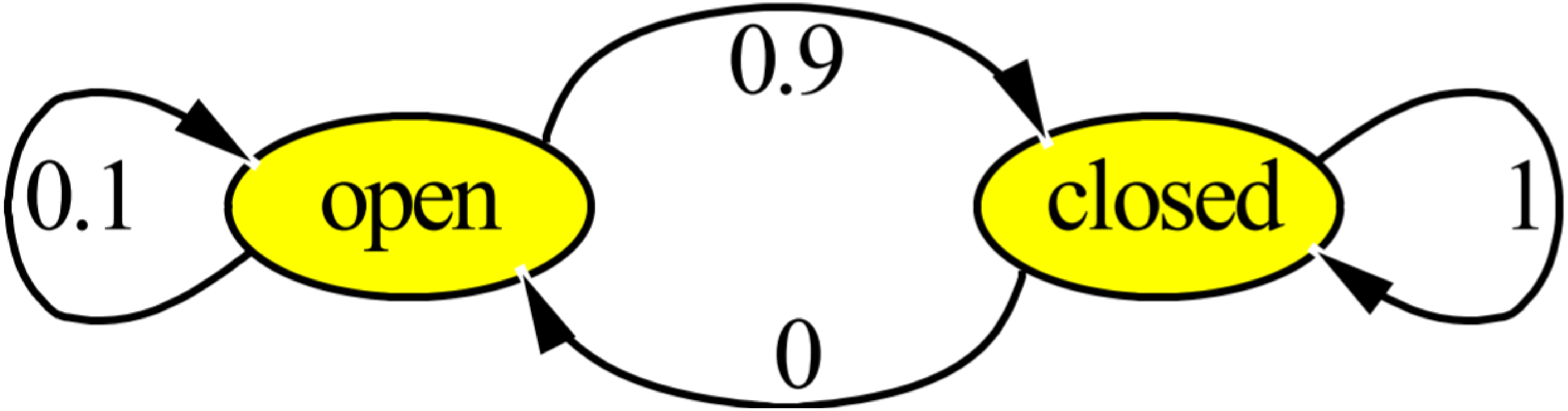
- This term specifies the pdf that describes how likely **executing $u$ changes the state from $x'$ to $x$.**

# Example: Closing the door



# State Transitions

$P(x \mid u, x')$ for $u =$ "close door":




If the door is open, the action "close door" succeeds in 90% of all cases.

# Integrating the Outcome of Actions

Continuous case:

$$p(x \mid u) = \int p(x \mid u, x')p(x')\,dx'$$

Discrete case:

$$P(x \mid u) = \sum P(x \mid u, x')P(x')$$

# Example: The Resulting Belief

$$
\begin{aligned}
P(closed \mid u, z_1, z_2) &= \sum P(closed \mid u, x')P(x' \mid z_1, z_2) \\
&= P(closed \mid u, open)P(open \mid z_1, z_2) \\
&\quad + P(closed \mid u, closed)P(closed \mid z_1, z_2) \\
&= \frac{9}{10} * \frac{5}{8} + \frac{1}{1} * \frac{3}{8} = \frac{15}{16}
\end{aligned}
$$

$$
\begin{aligned}
P(open \mid u, z_1, z_2) &= \sum P(open \mid u, x')P(x' \mid z_1, z_2) \\
&= P(open \mid u, open)P(open \mid z_1, z_2) \\
&\quad + P(open \mid u, closed)P(closed \mid z_1, z_2) \\
&= \frac{1}{10} * \frac{5}{8} + \frac{0}{1} * \frac{3}{8} = \frac{1}{16} \\
&= 1 - P(closed \mid u, z_1, z_2)
\end{aligned}
$$

# Bayes Filters: Framework

- Given:
  - Stream of observations $z$ and action data $u$ :

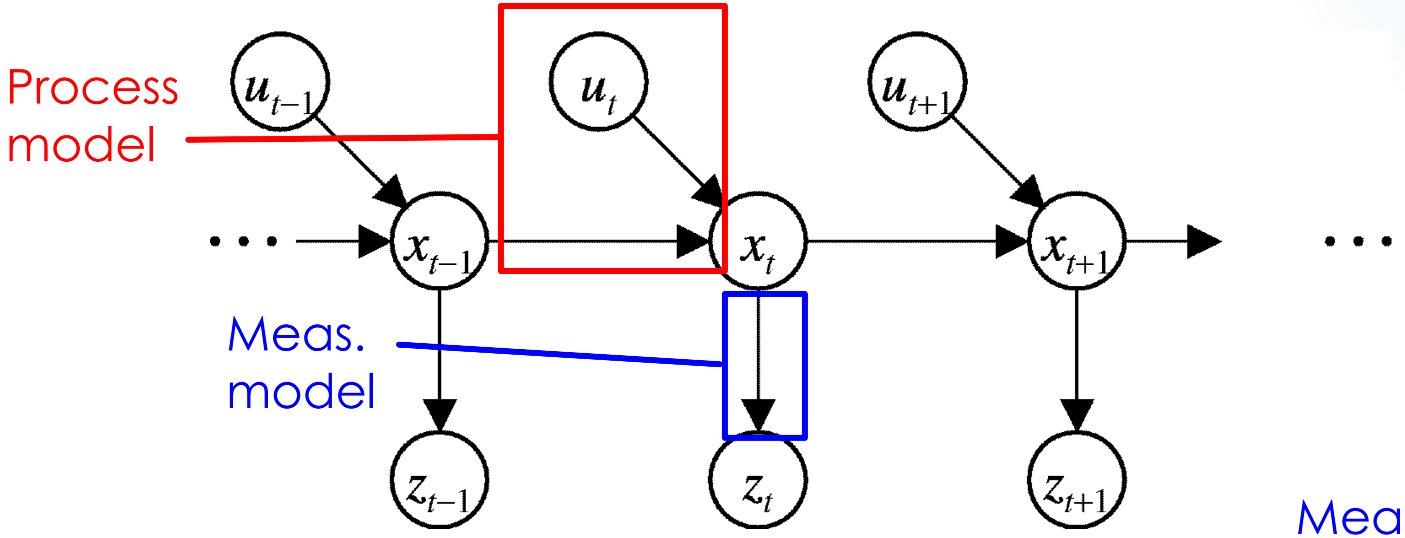
$$d_t = \{u_1, z_1 \ldots, u_t, z_t\}$$

  - Measurement (Sensor) model $P(z \mid x)$.
  - Process (Action) model $P(x \mid u, x')$.
  - Prior probability of the system state $P(x)$. .

- Wanted:
  - Estimate the state $X$ of a dynamical system.
  - The posterior of the state is also called the **Belief**:

$$Bel(x_t) = P(x_t \mid u_1, z_1 \ldots, u_t, z_t)$$

# Markov Assumption




$$p(z_t \mid x_{0:t}, z_{1:t}, u_{1:t}) = p(z_t \mid x_t)$$ Measurement model

$$p(x_t \mid x_{1:t-1}, z_{1:t-1}, u_{1:t}) = p(x_t \mid x_{t-1}, u_t)$$ Process model

- Underlying Assumptions
  - Perfect model structure, no approximation errors
  - Independent measurement noise
  - Random controls

# Bayes Filters

$z$ = observation
$u$ = action
$x$ = state

$$Bel(x_t) = p(x_t \mid u_1, z_1, \ldots, u_t, z_t)$$

**Bayes** $$= \eta\, p(z_t \mid x_t, u_1, z_1, \ldots, u_t)\, p(x_t \mid u_1, z_1, \ldots, u_t)$$

**Markov** $$= \eta\, p(z_t \mid x_t)\, p(x_t \mid u_1, z_1, \ldots, u_t)$$

**Total prob.** $$= \eta\, p(z_t \mid x_t) \int p(x_t \mid u_1, z_1, \ldots, u_t, x_{t-1})\, p(x_{t-1} \mid u_1, z_1, \ldots, u_t)\, dx_{t-1}$$

**Markov** $$= \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1} \mid u_1, z_1, \ldots, u_t)\, dx_{t-1}$$

**Markov** $$= \eta\, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1})\, p(x_{t-1} \mid u_1, z_1, \ldots, z_{t-1})\, dx_{t-1}$$

$$= \underbrace{\eta\, p(z_t \mid x_t)}_{\text{Correction Step}} \underbrace{\int p(x_t \mid u_t, x_{t-1})\, \underbrace{Bel(x_{t-1})}_{\text{Prior Belief}}\, dx_{t-1}}_{\text{Update Step}}$$

$$Bel(x_t) = \eta \, p(z_t \mid x_t) \int p(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

Algorithm **Bayes_filter**( *Bel(x), d* ):

$\eta$=0

If $d$ is a perceptual data item $z$ then

For all $x$ do

$$\overline{Bel}(x) = p(z \mid x) Bel(x)$$

$$h = h + \overline{Bel}(x)$$

For all $x$ do

$$\overline{Bel}(x) = h^{-1} \overline{Bel}(x)$$

Else if $d$ is an action data item $u$ then

For all $x$ do

$$\overline{Bel}(x) = \sum p(x \mid u, x') Bel(x') dx'$$

Return $\overline{Bel}(x)$

# Bayes Algorithm: Predictor / Corrector Structure

**Algorithm Bayes_filter**($bel(x_{t-1}), u_t, z_t$):
  *for all* $x_t$ *do*
    $\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1})\, bel(x_{t-1})\, dx_{t-1}$
    $bel(x_t) = \eta\, p(z_t \mid x_t)\, \overline{bel}(x_t)$
  *endfor*
  *return* $bel(x_t)$

**Table 2.1** The general algorithm for Bayes filtering.

# Many Families of Bayes Filters

$$Bel(x_t) = \eta P(z_t \mid x_t) \int P(x_t \mid u_t, x_{t-1}) Bel(x_{t-1}) dx_{t-1}$$

- Kalman Filters
- Particle Filters
- Hidden Markov models
- Dynamic Bayesian Networks
- Partially observable Markov decision processes (POMDPs)

# Bayes Filters - Summary

- Bayes rule allows us to compute probabilities that are hard to assess otherwise.

- Under the Markov assumption, recursive Bayesian updating can be used to efficiently combine evidence.

- Bayes filters are a probabilistic tool for estimating the state of dynamic systems.