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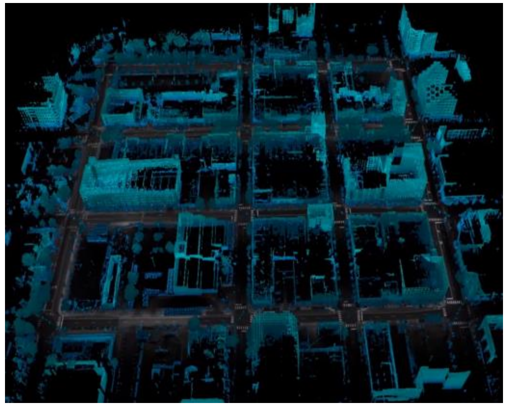
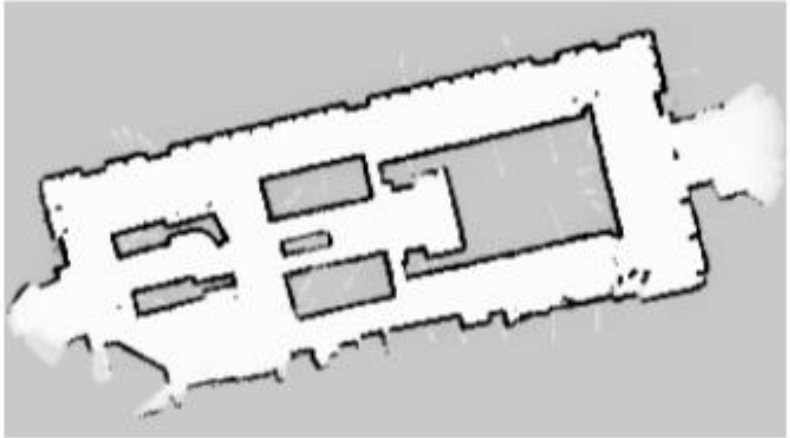
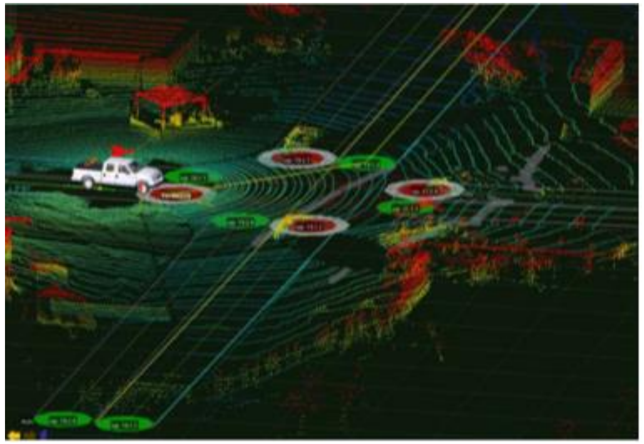


UNSCENTED TRANSFORM AND THE UKF

ECEN 633: Robotic Localization and Mapping

Slides courtesy of Ryan Eustice.

KF, EKF and UKF

- Kalman filter requires linear models
- EKF linearizes via Taylor expansion

Is there a better way to linearize?

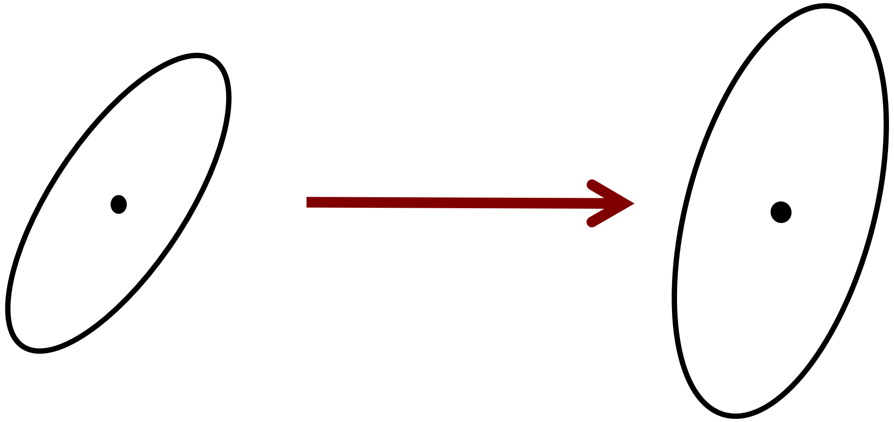
Unscented Transform

↓

Unscented Kalman Filter (UKF)

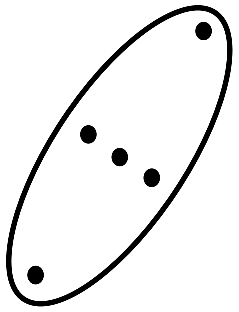
Unscented Transform

Taylor Approximation (EKF)



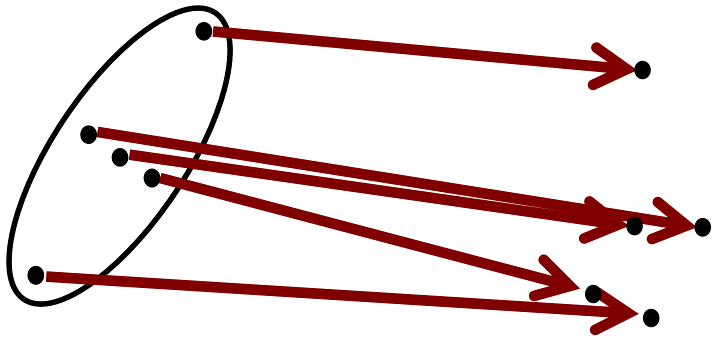
Linearization of the non-linear
function through Taylor expansion

Unscented Transform



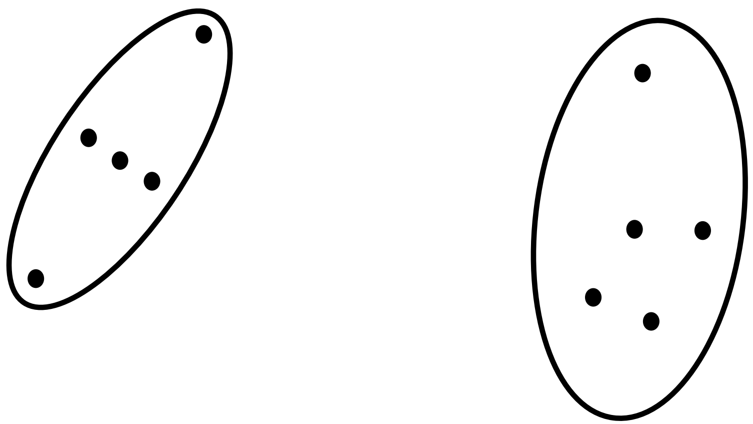
Compute a set of (so-called) sigma points

Unscented Transform



Transform each sigma point
through the non-linear function

Unscented Transform

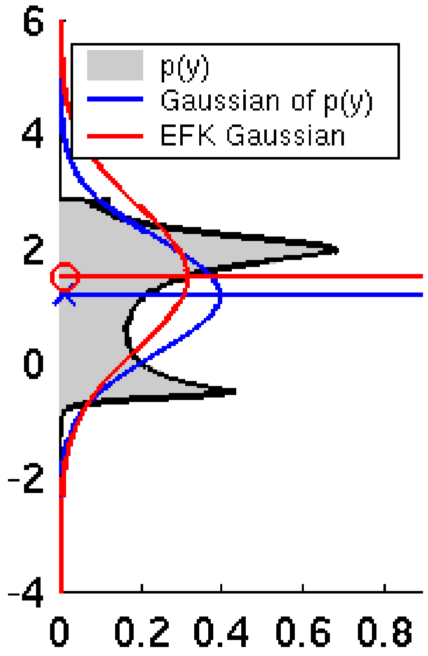


Compute Gaussian from the transformed and weighted sigma points

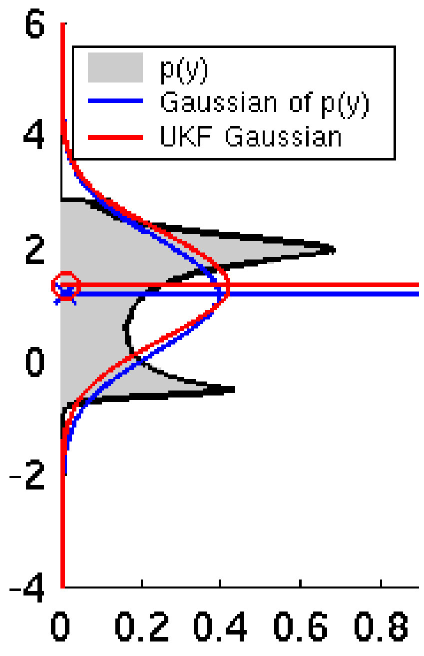
Nonlinear Gaussian Filters

- Approach 2: Unscented Kalman Filter
  - **Approximate the PDF!**
  - Use the full nonlinear plant and observation models and recompute 1st and 2nd order statistics.

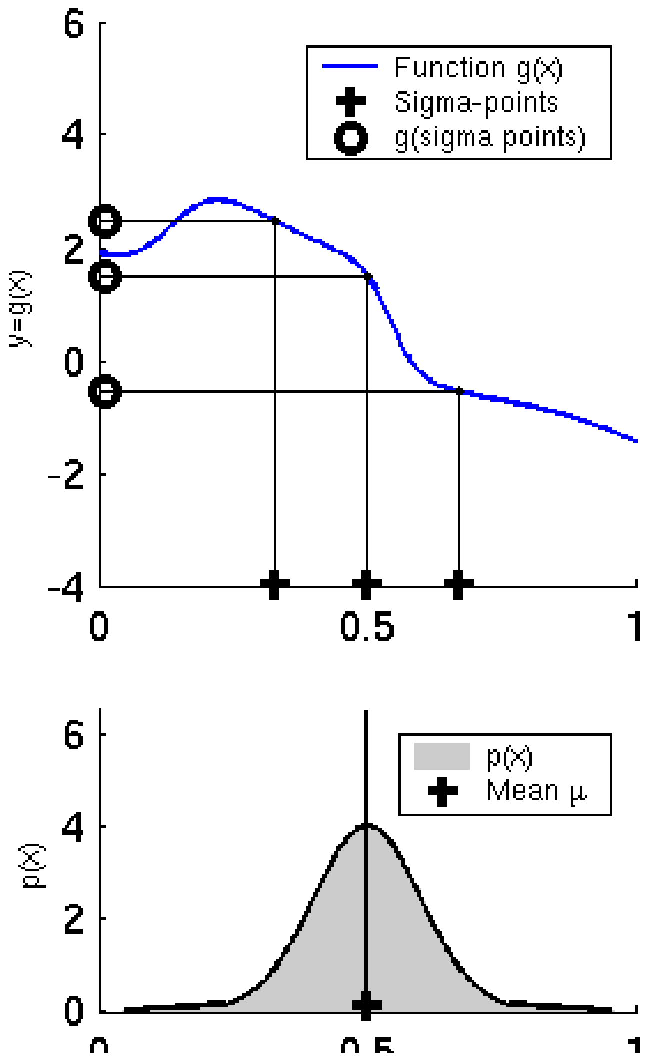
UKF Linearization via Unscented Transform



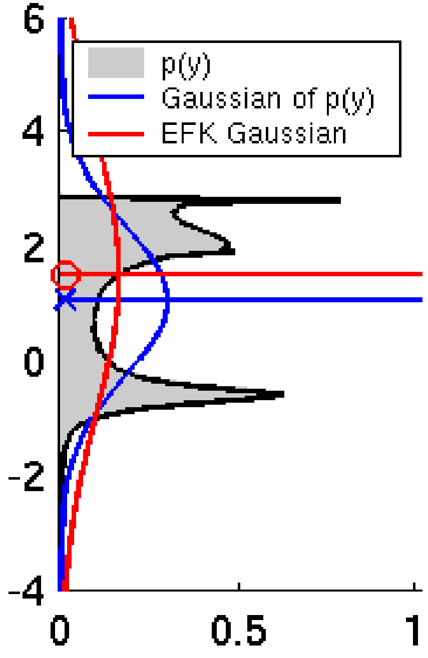
EKF



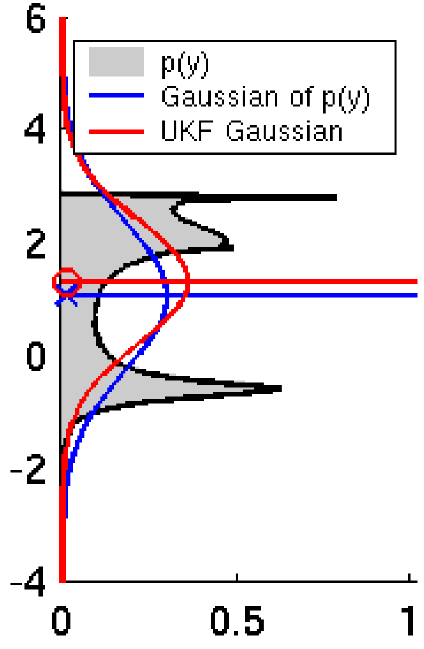
UKF



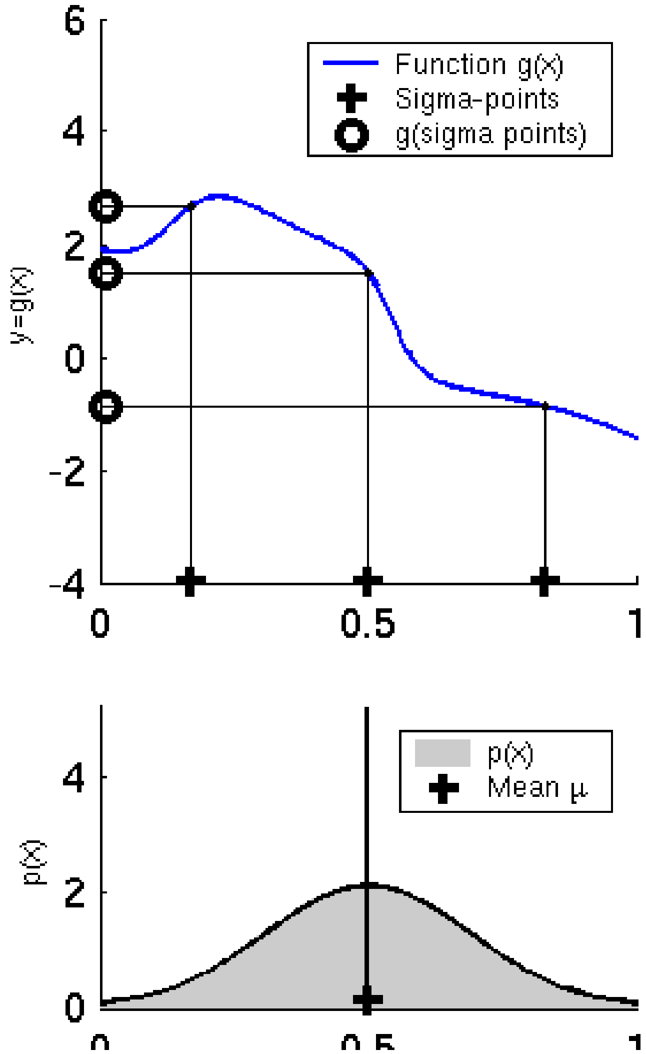
UKF Sigma-Point Estimate: Large Variance



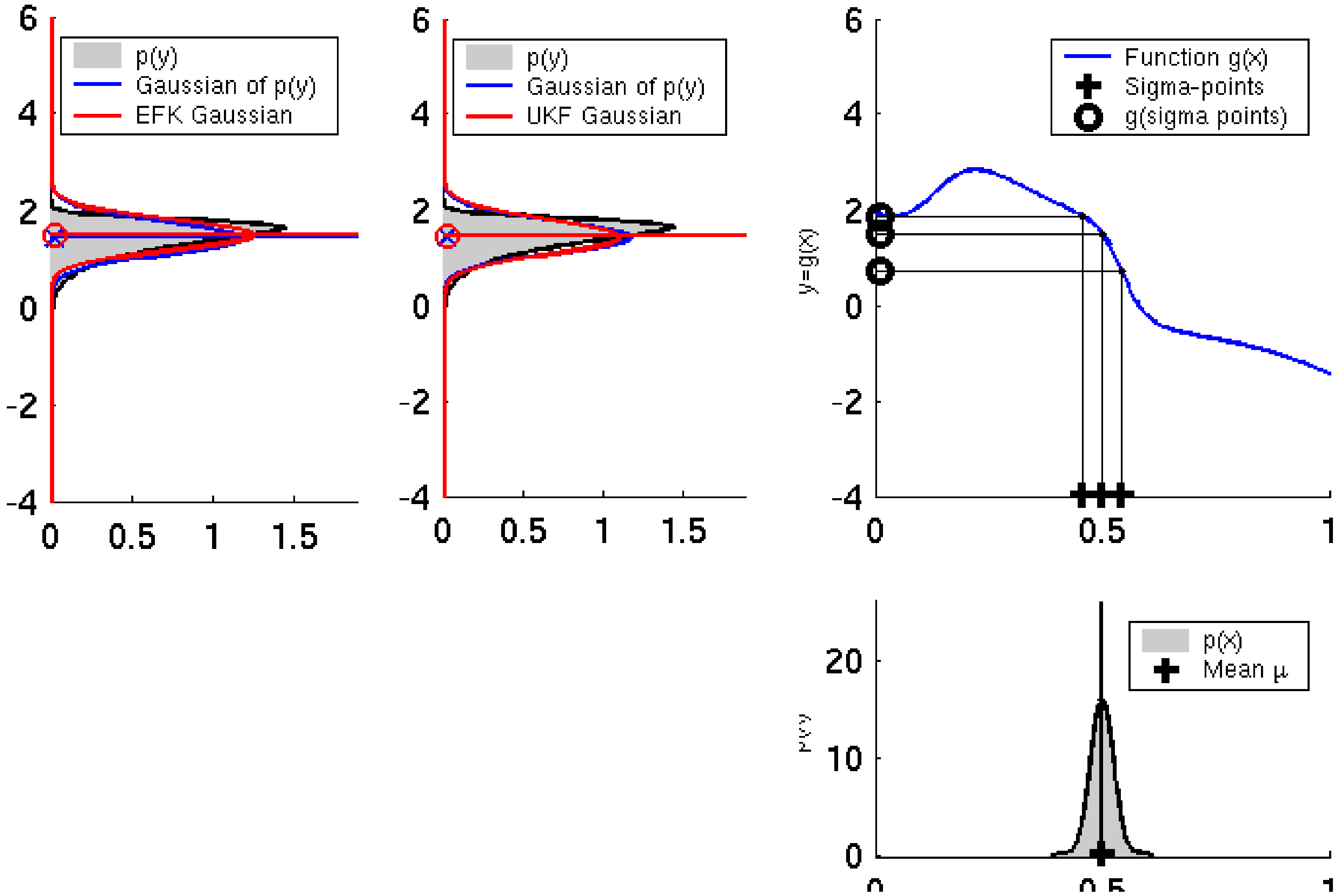
EKF



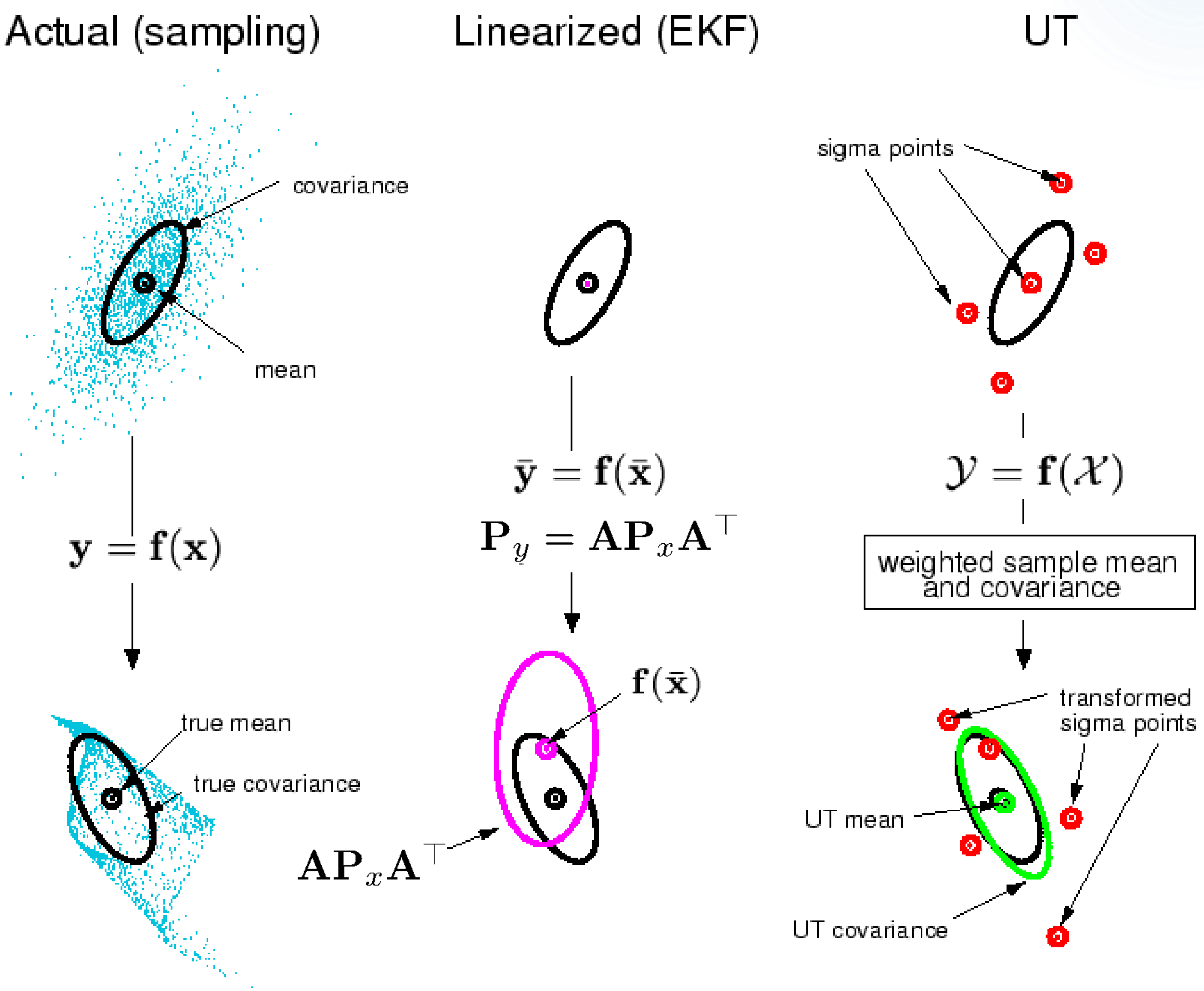
UKF



UKF Sigma-Point Estimate: Narrow Variance



UKF vs. EKF



Courtesy: E.A. Wan and R. van der Merwe

Unscented Transform Overview

- Compute a set of sigma points
- Each sigma point has a weight
- Transform the point through the non-linear function
- Compute a Gaussian from weighted points

- Avoids need to linearize **around the mean** as Taylor expansion (and EKF) does

Courtesy: Cyrill Stachniss

Sigma Points

- How to choose the sigma points?
- How to set the weights?

Courtesy: Cyrill Stachniss

Sigma Points Properties

- How to choose the sigma points?
- How to set the weights?
- Select $\boldsymbol{\mathcal{X}}^{[i]}, w^{[i]}$ so that:

$$\sum_i w^{[i]} = 1$$

$$\boldsymbol{\mu} = \sum_i w^{[i]} \boldsymbol{\mathcal{X}}^{[i]}$$

$$\Sigma = \sum_i w^{[i]} (\boldsymbol{\mathcal{X}}^{[i]} - \boldsymbol{\mu})(\boldsymbol{\mathcal{X}}^{[i]} - \boldsymbol{\mu})^\top$$

- There is no unique solution for $\boldsymbol{\mathcal{X}}^{[i]}, w^{[i]}$

Courtesy: Cyrill Stachniss

Sigma Points

- Choosing the sigma points

$$\mathcal{X}^{[0]} = \mu$$

First sigma point is the mean

Courtesy: Cyrill Stachniss

Sigma Points

- Choosing the sigma points

$$
\begin{aligned}
\boldsymbol{\mathcal{X}}^{[0]} &= \boldsymbol{\mu} \\
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} + \left(\sqrt{(n+\lambda)\,\Sigma}\right)_i \qquad \text{for } i = 1, \ldots, n \\
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} - \left(\sqrt{(n+\lambda)\,\Sigma}\right)_{i-n} \qquad \text{for } i = n+1, \ldots, 2n
\end{aligned}
$$

matrix square root

column vector

dimensionality

scaling parameter

Courtesy: Cyrill Stachniss

Real Symmetric Matrix Square Root

- Defined as S with $\Sigma = SS^{\top}$
- Computed via diagonalization

$$\begin{aligned}
\Sigma &= VDV^{-1} \\
&= V \begin{pmatrix} d_{11} & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & \dots & d_{nn} \end{pmatrix} V^{-1} \\
&= V \begin{pmatrix} \sqrt{d_{11}} & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & \dots & \sqrt{d_{nn}} \end{pmatrix} \begin{pmatrix} \sqrt{d_{11}} & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & \dots & \sqrt{d_{nn}} \end{pmatrix} V^{-1}
\end{aligned}$$

Courtesy: Cyrill Stachniss

Real Symmetric Matrix Square Root

- Thus, we can define

$$S \quad = \underbrace{V \begin{pmatrix} \sqrt{d_{11}} & \dots & 0 \\ 0 & \ddots & 0 \\ 0 & \dots & \sqrt{d_{nn}} \end{pmatrix}}_{\mathbf{P}_y = \mathbf{A}\mathbf{P}_x\mathbf{A}^\top} V^{-1}$$

- so that

$$SS = (VD^{1/2}V^{-1})(VD^{1/2}V^{-1}) = VDV^{-1} = \Sigma$$

- S and $\sum$ have the same Eigenvectors

Cholesky Matrix Square Root

- Alternative definition of the matrix square root

$$L \text{ with } \Sigma = LL^\top$$

- Result of the Cholesky decomposition
- Numerically stable solution
- Often used in UKF implementations

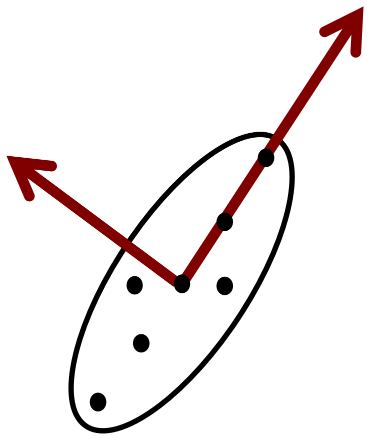
- Actually, any such square root factorization is ok, e.g., could use factorization

$$\Sigma = AA^\top \qquad \text{where } A = VD^{\frac{1}{2}}$$

Sigma Points and Eigenvectors

- Sigma points **can** but **do not have to** lie on the main axes of Σ

$$
\begin{aligned}
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} + \left(\sqrt{(n+\lambda)\,\Sigma}\right)_i \qquad \text{for } i = 1, \ldots, n \\
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} - \left(\sqrt{(n+\lambda)\,\Sigma}\right)_{i-n} \qquad \text{for } i = n+1, \ldots, 2n
\end{aligned}
$$



Sigma Points Example

Sigma =

1.1335	1.9544
1.9544	5.5336

SigmaL =

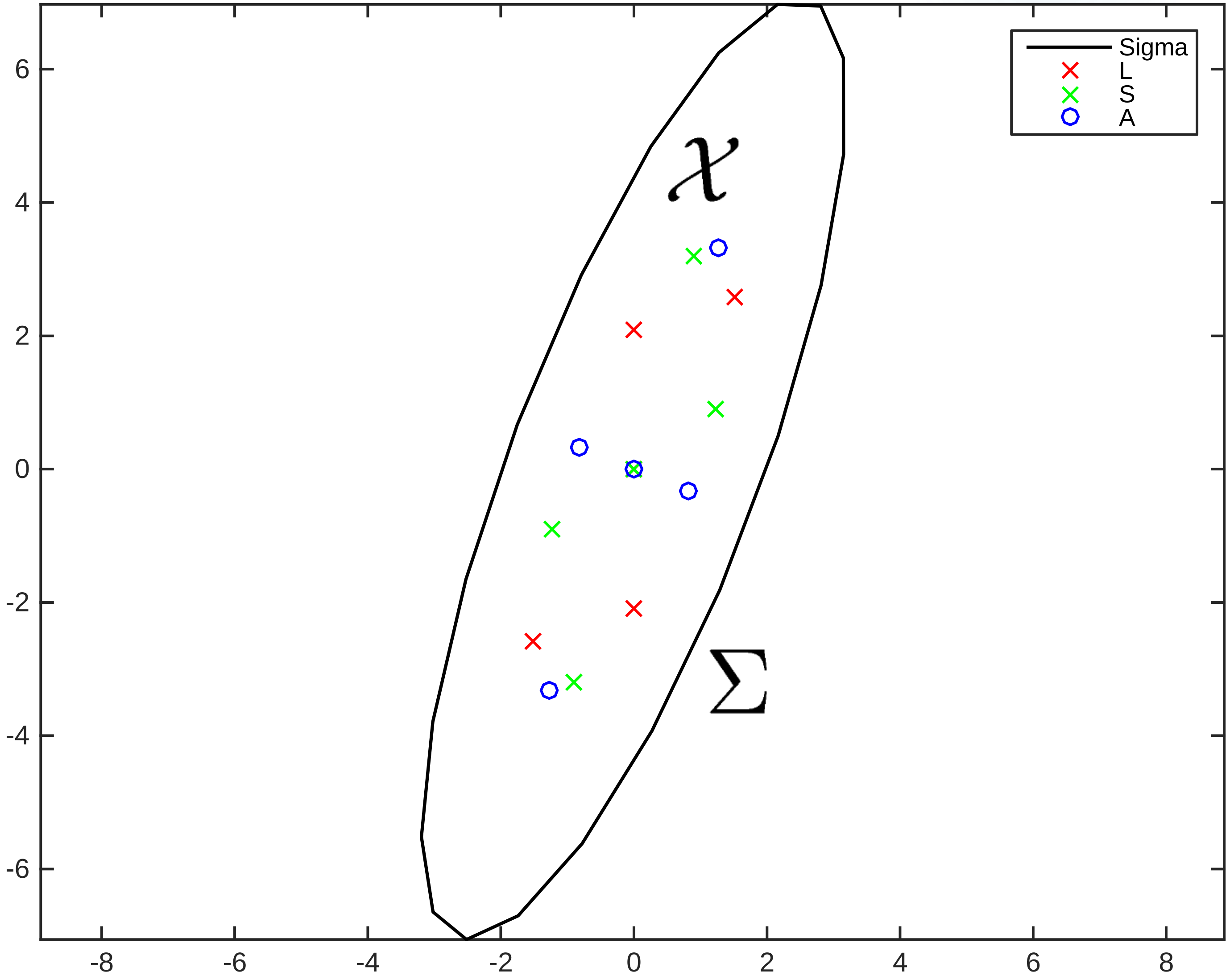
1.1335	1.9544
1.9544	5.5336

SigmaS =

1.1335	1.9544
1.9544	5.5336

SigmaA =

1.1335	1.9544
1.9544	5.5336



Sigma Point Weights

- Weight sigma points

for computing the mean

parameters

$$
\begin{aligned}
w_m^{[0]} &= \frac{\lambda}{n+\lambda} \\
w_c^{[0]} &= w_m^{[0]} + (1 - \alpha^2 + \beta) \\
w_m^{[i]} = w_c^{[i]} &= \frac{1}{2(n+\lambda)} \quad \text{for } i = 1, \ldots, 2n
\end{aligned}
$$

for computing the covariance

Recover the Gaussian

- Compute Gaussian from weighted and transformed points

$$\boldsymbol{\mu}' = \sum_{i=0}^{2n} w_m^{[i]}\, g(\boldsymbol{\mathcal{X}}^{[i]})$$

$$\Sigma' = \sum_{i=0}^{2n} w_c^{[i]}\, (g(\boldsymbol{\mathcal{X}}^{[i]}) - \boldsymbol{\mu}')(g(\boldsymbol{\mathcal{X}}^{[i]}) - \boldsymbol{\mu}')^\top$$

(Scaled) Unscented Transform

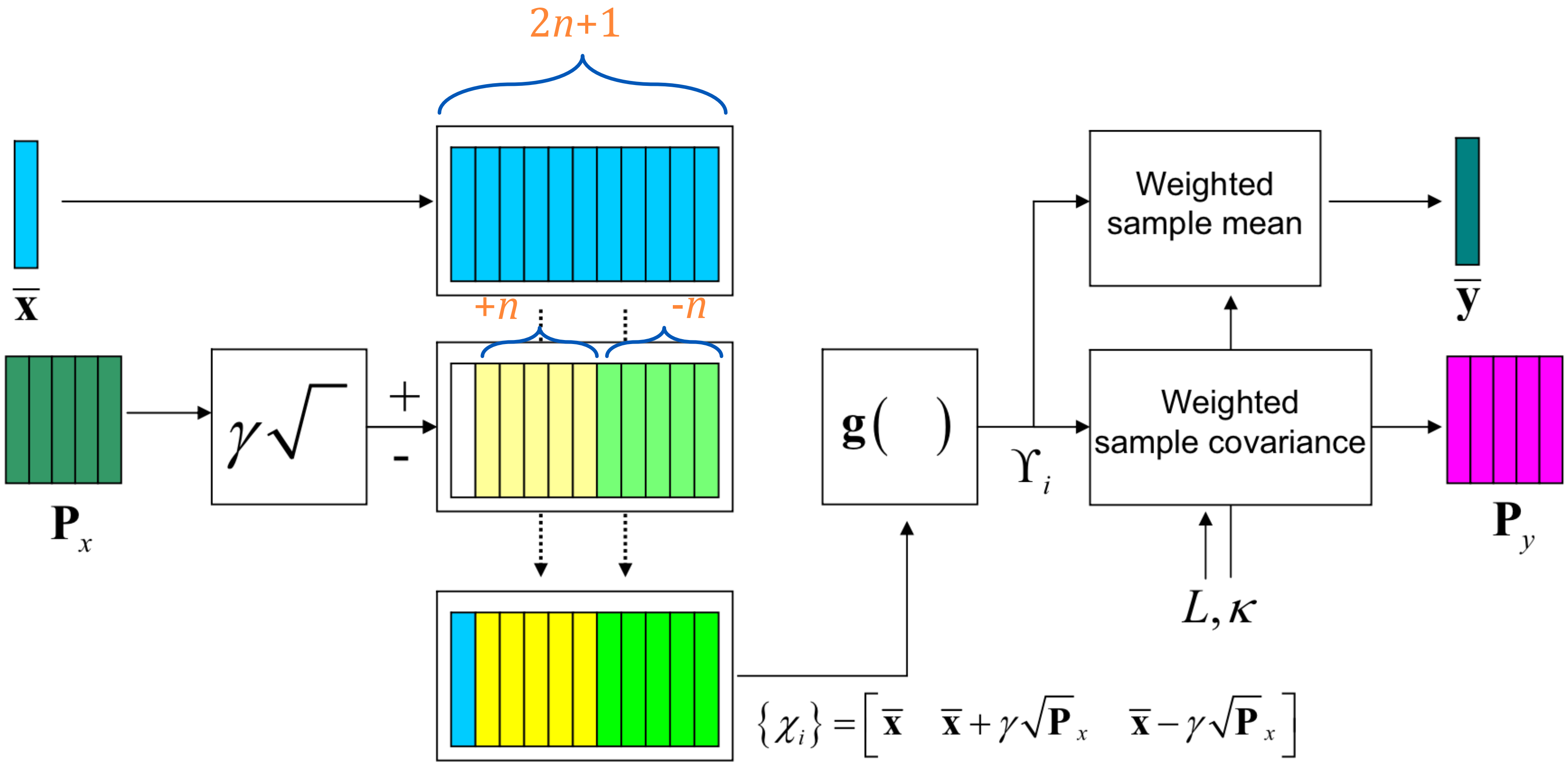


Figure 3.2: Schematic diagram of the unscented transformation.

Source: Van Der Merwe, Thesis

Unscented Transform Summary

- Sigma points

$$\begin{aligned}
\boldsymbol{\mathcal{X}}^{[0]} &= \boldsymbol{\mu} \\
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} + \left(\sqrt{(n+\lambda)\,\Sigma}\right)_i \qquad \text{for } i = 1, \ldots, n \\
\boldsymbol{\mathcal{X}}^{[i]} &= \boldsymbol{\mu} - \left(\sqrt{(n+\lambda)\,\Sigma}\right)_{i-n} \qquad \text{for } i = n+1, \ldots, 2n
\end{aligned}$$

- Weights

$$\begin{aligned}
w_m^{[0]} &= \frac{\lambda}{n+\lambda} \\
w_c^{[0]} &= w_m^{[0]} + (1 - \alpha^2 + \beta) \\
w_m^{[i]} = w_c^{[i]} &= \frac{1}{2(n+\lambda)} \qquad \text{for } i = 1, \ldots, 2n
\end{aligned}$$

Courtesy: Cyrill Stachniss

SUT Parameters

- Free parameters as there is no unique solution
- Scaled Unscented Transform suggests

$$\begin{aligned} \kappa &\geq 0 \\ \alpha &\in (0, 1] \end{aligned}$$

Influence how far the sigma points are away from the mean

$$\lambda = \alpha^2(n+\kappa) - n$$

$$\beta = 2$$

Optimal choice for Gaussians

Courtesy: Cyrill Stachniss

SUT Parameters

- Choose $\kappa \geq 0$
  - to guarantee positive semi-definiteness of the covariance matrix. The specific value of κ is not critical though, so a good default choice is $\kappa = 0$.

- Choose $0 < \alpha \leq 1$
  - to control the "size" of the sigma-point distribution and should be chosen to avoid sampling non-local effects when the nonlinearities are strong; a default choice is $\alpha = 1$.

- Choose $\beta \geq 0$
  - to incorporate knowledge of the higher-order moments of the distribution. For example, for a Gaussian prior the optimal choice is $\beta = 2$.

- The original (un-scaled) UT transform is equivalent to:
  - SUT with $\alpha = 1, \beta = 0$

(Scaled) Unscented Transform

Sigma points

$$\chi^0 = \mu$$

$$\chi^i = \mu \pm \left(\sqrt{(n+\lambda)\Sigma}\right)_i$$

Weights

$$w_m^0 = \frac{\lambda}{n+\lambda} \qquad w_c^0 = \frac{\lambda}{n+\lambda} + (1-\alpha^2+\beta)$$

$$w_m^i = w_c^i = \frac{1}{2(n+\lambda)} \qquad \text{for } i = 1,\ldots,2n$$

Pass sigma points through nonlinear function

$$\psi^i = g(\chi^i)$$

Recover mean and covariance

$$\mu' = \sum_{i=0}^{2n} w_m^i \psi^i$$

$$\Sigma' = \sum_{i=0}^{2n} w_c^i (\psi^i - \mu')(\psi^i - \mu')^T$$

$$\lambda = \alpha^2(n+\kappa) - n$$

$0 < \alpha \le 1$ Sigma point scaling

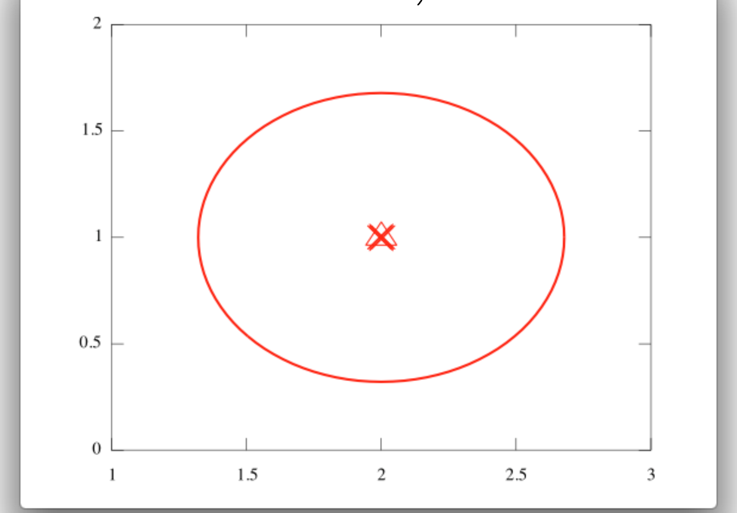
$\beta \ge 0$ Higher-order moment matching

$\kappa \ge 0$ Scalar tuning parameter

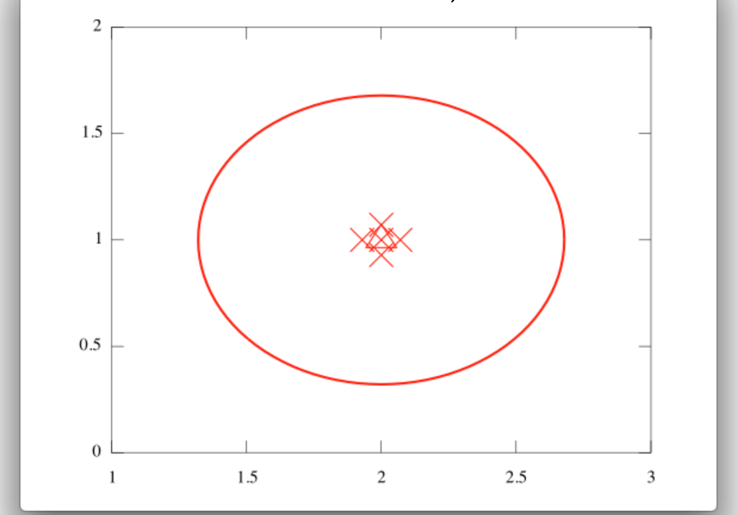
Examples

Courtesy: Cyrill Stachniss

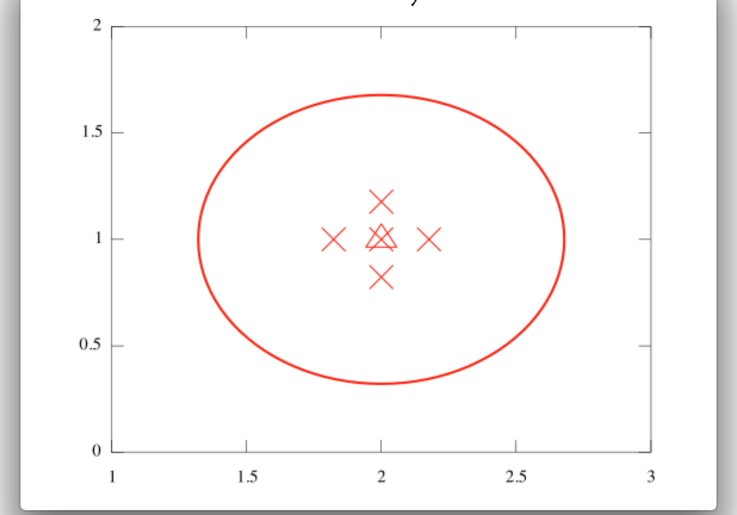
$\kappa = 3, \alpha = 0.01$



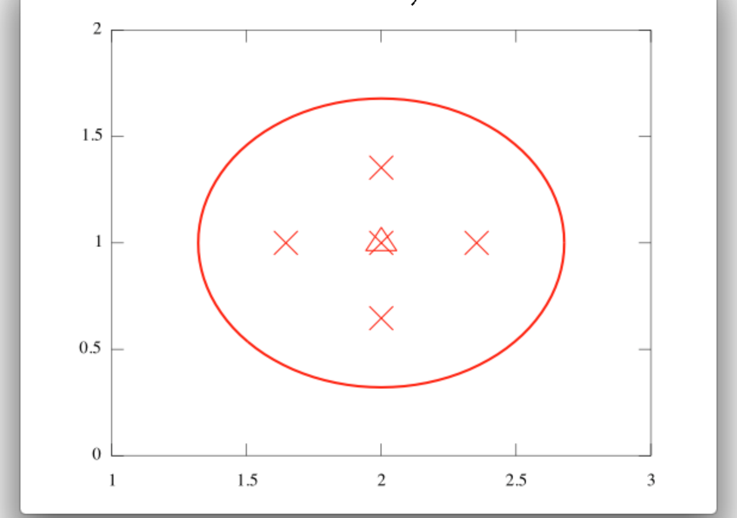
$\kappa = 3, \alpha = 0.1$



$\kappa = 3, \alpha = 0.25$

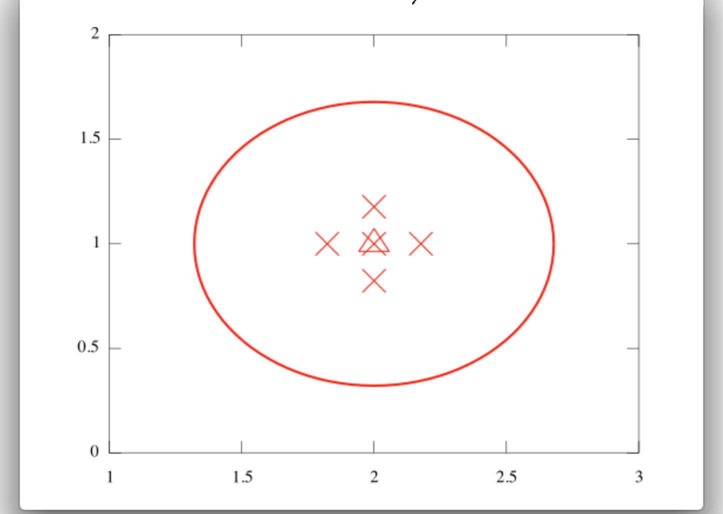


$\kappa = 3, \alpha = 0.75$

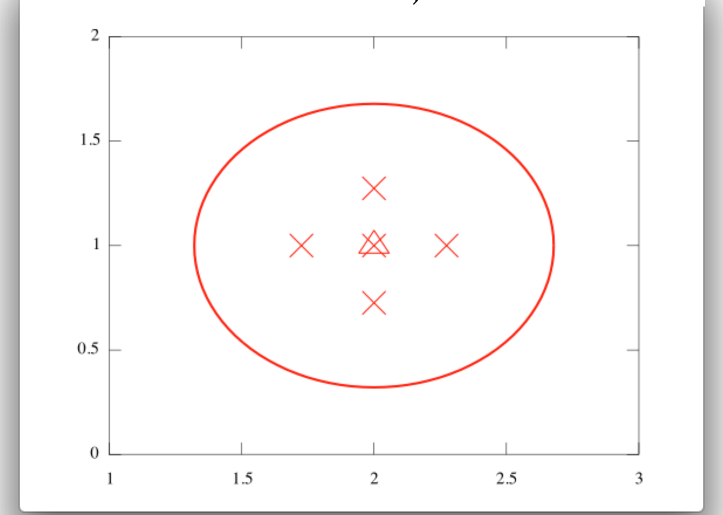


Examples

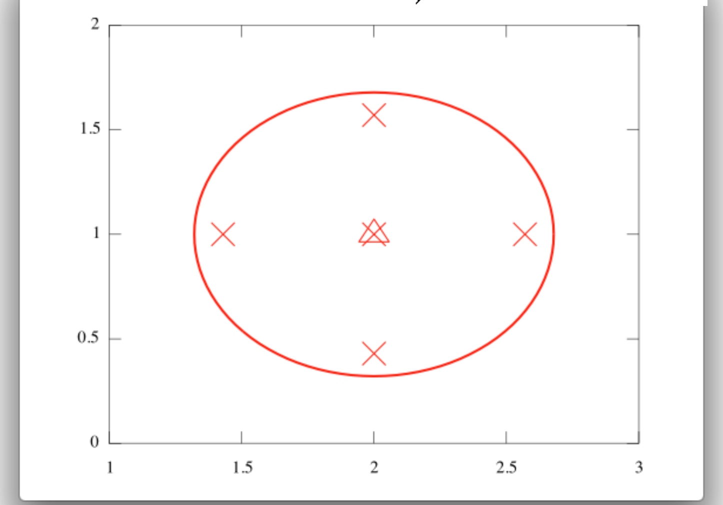
$\kappa = 3, \alpha = 0.25$



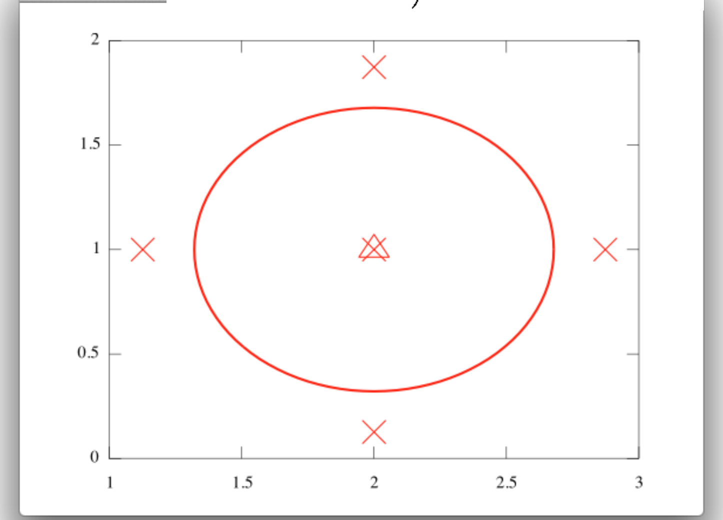
$\kappa = 10, \alpha = 0.25$



$\kappa = 50, \alpha = 0.25$



$\kappa = 120, \alpha = 0.25$



- How to apply UT to estimation??

UKF (Unscented Kalman Filter)

Unscented Kalman Filter

UKF uses the Kalman Update

- KF is the Best Linear Unbiased Estimator (BLUE)
  - i.e., if we restrict our estimator to the class of linear estimators, then the KF is the best *linear* MMSE estimator*

* Note: a nonlinear estimator could do ***better!!***

EKF Algorithm*

1: $\textbf{Extended\_Kalman\_filter}(\boldsymbol{\mu}_{t-1}, \Sigma_{t-1}, \mathbf{u}_t, \mathbf{z}_t)$**:**

2: $\bar{\boldsymbol{\mu}}_t = g(\mathbf{u}_t, \boldsymbol{\mu}_{t-1})$

3: $\bar{\Sigma}_t = G_t\, \Sigma_{t-1}\, G_t^\top + R_t$

4: $K_t = \bar{\Sigma}_t\, H_t^\top (H_t\, \bar{\Sigma}_t\, H_t^\top + Q_t)^{-1}$

5: $\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + K_t(\mathbf{z}_t - h(\bar{\boldsymbol{\mu}}_t))$

6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$

7: $return\ \boldsymbol{\mu}_t, \Sigma_t$

* The form shown assumes additive process and observation model noise

EKF to UKF – Prediction

Unscented

1: $\cancel{\textbf{Extended}}$**_Kalman_filter**$(\boldsymbol{\mu}_{t-1}, \Sigma_{t-1}, \mathbf{u}_t, \mathbf{z}_t)$**:**

2: $\bar{\boldsymbol{\mu}}_t =$ replace this by sigma point
3: $\bar{\Sigma}_t =$ propagation of the motion

4: $K_t = \bar{\Sigma}_t\, H_t^\top (H_t\, \bar{\Sigma}_t\, H_t^\top + Q_t)^{-1}$
5: $\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + K_t(\mathbf{z}_t - h(\bar{\boldsymbol{\mu}}_t))$
6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$
7: $return\ \boldsymbol{\mu}_t, \Sigma_t$

UKF Algorithm – Prediction*

1: $\textbf{Unscented\_Kalman\_filter}(\boldsymbol{\mu}_{t-1}, \Sigma_{t-1}, \mathbf{u}_t, \mathbf{z}_t)$:

2: $\quad \boldsymbol{\mathcal{X}}_{t-1} = (\boldsymbol{\mu}_{t-1} \quad \boldsymbol{\mu}_{t-1} + \sqrt{(n+\lambda)\Sigma_{t-1}} \quad \boldsymbol{\mu}_{t-1} - \sqrt{(n+\lambda)\Sigma_{t-1}})$

3: $\quad \bar{\boldsymbol{\mathcal{X}}}_t^* = g(\mathbf{u}_t, \boldsymbol{\mathcal{X}}_{t-1})$

4: $\quad \bar{\boldsymbol{\mu}}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\boldsymbol{\mathcal{X}}}_t^{*[i]}$

5: $\quad \bar{\Sigma}_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\boldsymbol{\mathcal{X}}}_t^{*[i]} - \bar{\mu}_t)(\bar{\boldsymbol{\mathcal{X}}}_t^{*[i]} - \bar{\mu}_t)^\top + R_t$

* The form shown assumes additive process and observation model noise

EKF to UKF – Correction

Unscented

1: $\cancel{\textbf{Extended}}\textbf{\_Kalman\_filter}(\boldsymbol{\mu}_{t-1}, \Sigma_{t-1}, \mathbf{u}_t, \mathbf{z}_t)$:

2: $\bar{\boldsymbol{\mu}}_t =$ replace this by sigma point

3: $\bar{\Sigma}_t =$ propagation of the motion

use sigma point propagation for the expected observation and Kalman gain

5: $\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + K_t(\mathbf{z}_t - h(\bar{\boldsymbol{\mu}}_t))$

6: $\Sigma_t = (I - K_t\, H_t)\, \bar{\Sigma}_t$

7: $return\ \boldsymbol{\mu}_t, \Sigma_t$

UKF Algorithm – Correction (1)*

$$
\begin{aligned}
&6: \quad \bar{\mathcal{X}}_t = (\bar{\boldsymbol{\mu}}_t \quad \bar{\boldsymbol{\mu}}_t + \sqrt{(n+\lambda)\bar{\Sigma}_t} \quad \bar{\boldsymbol{\mu}}_t - \sqrt{(n+\lambda)\bar{\Sigma}_t})\\
&7: \quad \bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t)\\
&8: \quad \hat{\mathbf{z}}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{Z}}_t^{[i]}\\
&9: \quad S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top + Q_t\\
&10: \quad \bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\boldsymbol{\mu}}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top
\end{aligned}
$$

* The form shown assumes additive process and observation model noise

UKF Algorithm – Correction (1)*

6: $\bar{\mathcal{X}}_t = (\bar{\mu}_t \quad \bar{\mu}_t + \sqrt{(n+\lambda)\bar{\Sigma}_t} \quad \bar{\mu}_t - \sqrt{(n+\lambda)\bar{\Sigma}_t})$

7: $\bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t)$

8: $\hat{\mathbf{z}}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{Z}}_t^{[i]}$

9: $S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top + Q_t \longrightarrow \Sigma_t^{z,z}$

10: $\bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\mu}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top$

11: $K_t = \bar{\Sigma}_t^{x,z} \; S_t^{-1}$

(from BLUE)

* The form shown assumes additive process and observation model noise

UKF Algorithm – Correction (2)

$$\bar{\mathcal{X}}_t = (\bar{\mu}_t \quad \bar{\mu}_t + \sqrt{(n+\lambda)\bar{\Sigma}_t} \quad \bar{\mu}_t - \sqrt{(n+\lambda)\bar{\Sigma}_t})$$

$$\bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t)$$

$$\hat{\mathbf{z}}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{Z}}_t^{[i]}$$

$$S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top + Q_t$$

$$\bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\mu}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{\mathbf{z}}_t)^\top$$

$$K_t = \bar{\Sigma}_t^{x,z} \, S_t^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(\mathbf{z}_t - \hat{\mathbf{z}}_t)$$

$$\Sigma_t = \bar{\Sigma}_t - K_t \, S_t \, K_t^\top$$

return μ_t, Σ_t

UKF

This version of the algorithm implicitly assumes ***additive*** zero-mean process and observation noise

Algorithm Unscented_Kalman_filter$(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$:

$$\mathcal{X}_{t-1} = (\mu_{t-1} \quad \mu_{t-1} + \gamma\sqrt{\Sigma_{t-1}} \quad \mu_{t-1} - \gamma\sqrt{\Sigma_{t-1}})$$

$$\bar{\mathcal{X}}_t^* = g(u_t, \mathcal{X}_{t-1})$$

$$\bar{\mu}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{X}}_t^{*[i]}$$

Take care with means of circular quantities

$$\bar{\Sigma}_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t)(\bar{\mathcal{X}}_t^{*[i]} - \bar{\mu}_t)^T + R_t$$

$$\bar{\mathcal{X}}_t = (\bar{\mu}_t \quad \bar{\mu}_t + \gamma\sqrt{\bar{\Sigma}_t} \quad \bar{\mu}_t - \gamma\sqrt{\bar{\Sigma}_t})$$

$$\bar{\mathcal{Z}}_t = h(\bar{\mathcal{X}}_t)$$

$$\hat{z}_t = \sum_{i=0}^{2n} w_m^{[i]} \bar{\mathcal{Z}}_t^{[i]}$$

$$S_t = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t)^T + Q_t$$

$$\bar{\Sigma}_t^{x,z} = \sum_{i=0}^{2n} w_c^{[i]} (\bar{\mathcal{X}}_t^{[i]} - \bar{\mu}_t)(\bar{\mathcal{Z}}_t^{[i]} - \hat{z}_t)^T$$

$$K_t = \bar{\Sigma}_t^{x,z} S_t^{-1}$$

$$\mu_t = \bar{\mu}_t + K_t(z_t - \hat{z}_t)$$

$$\Sigma_t = \bar{\Sigma}_t - K_t S_t K_t^T$$

return μ_t, Σ_t

Means of Circular Quantities

- Trick is to map angles θ_i to the unit circle

- Take arithmetic mean of Cartesian quantities

$$\overline{\cos} = \sum_{i=0}^{2N} \cos(\theta_i) w_m^{[i]} \quad \overline{\sin} = \sum_{i=0}^{2N} \sin(\theta_i) w_m^{[i]}$$

- Map back to corresponding "average" angle*

$$\bar{\theta} = \text{atan2}(\overline{\sin}, \overline{\cos})$$

Similarly

- Map angular differences, such as

$$(\mathcal{X}^{[i]} - \mu) \quad \text{to} \quad [-\pi, \pi]$$

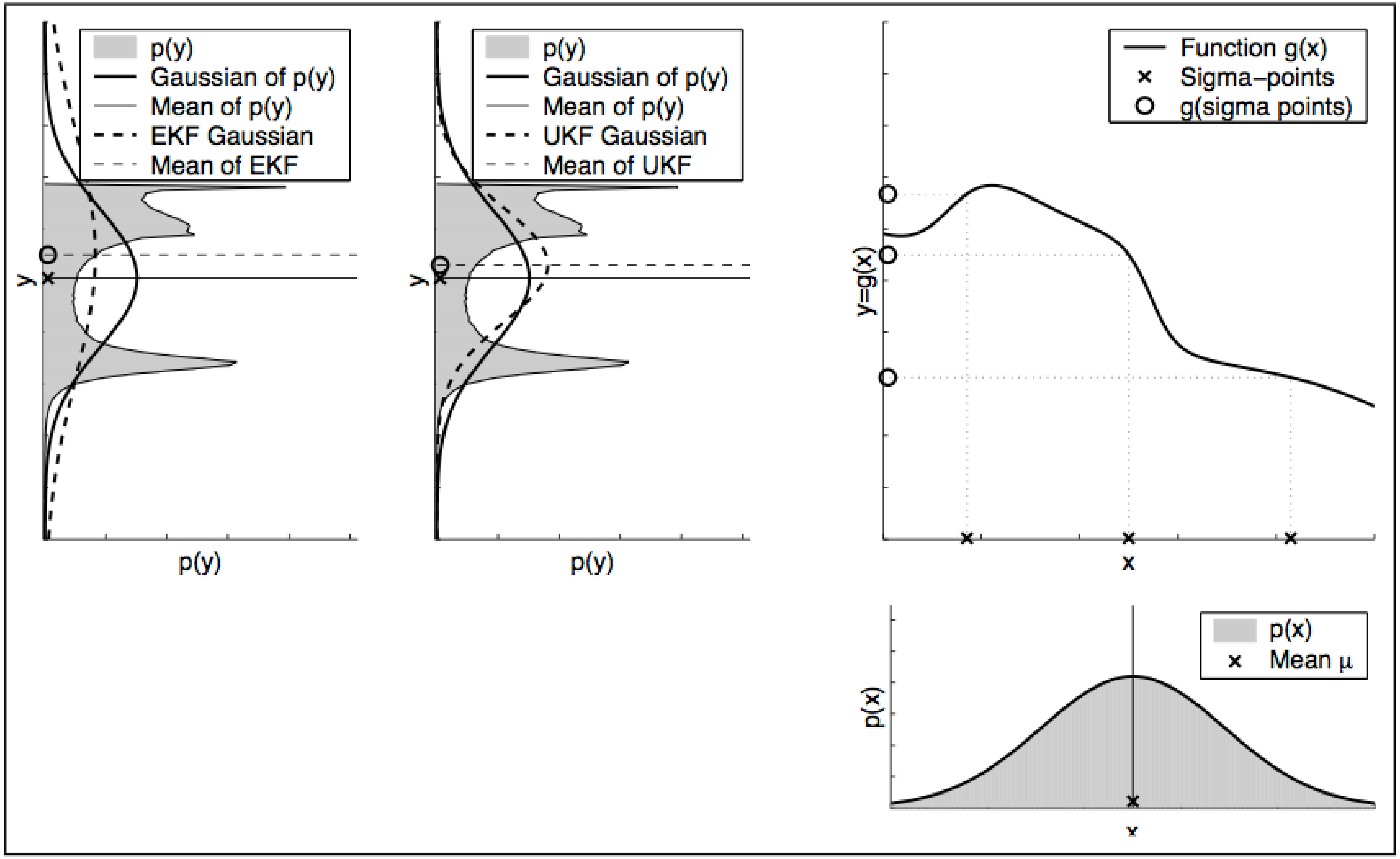
when computing innovation and covariance expressions, e.g.:

$$\Sigma_{xx} = \sum_{i=0}^{2N} w_c^{[i]} (\boldsymbol{\mathcal{X}}^{[i]} - \boldsymbol{\mu}_x)(\boldsymbol{\mathcal{X}}^{[i]} - \boldsymbol{\mu}_x)^\top$$

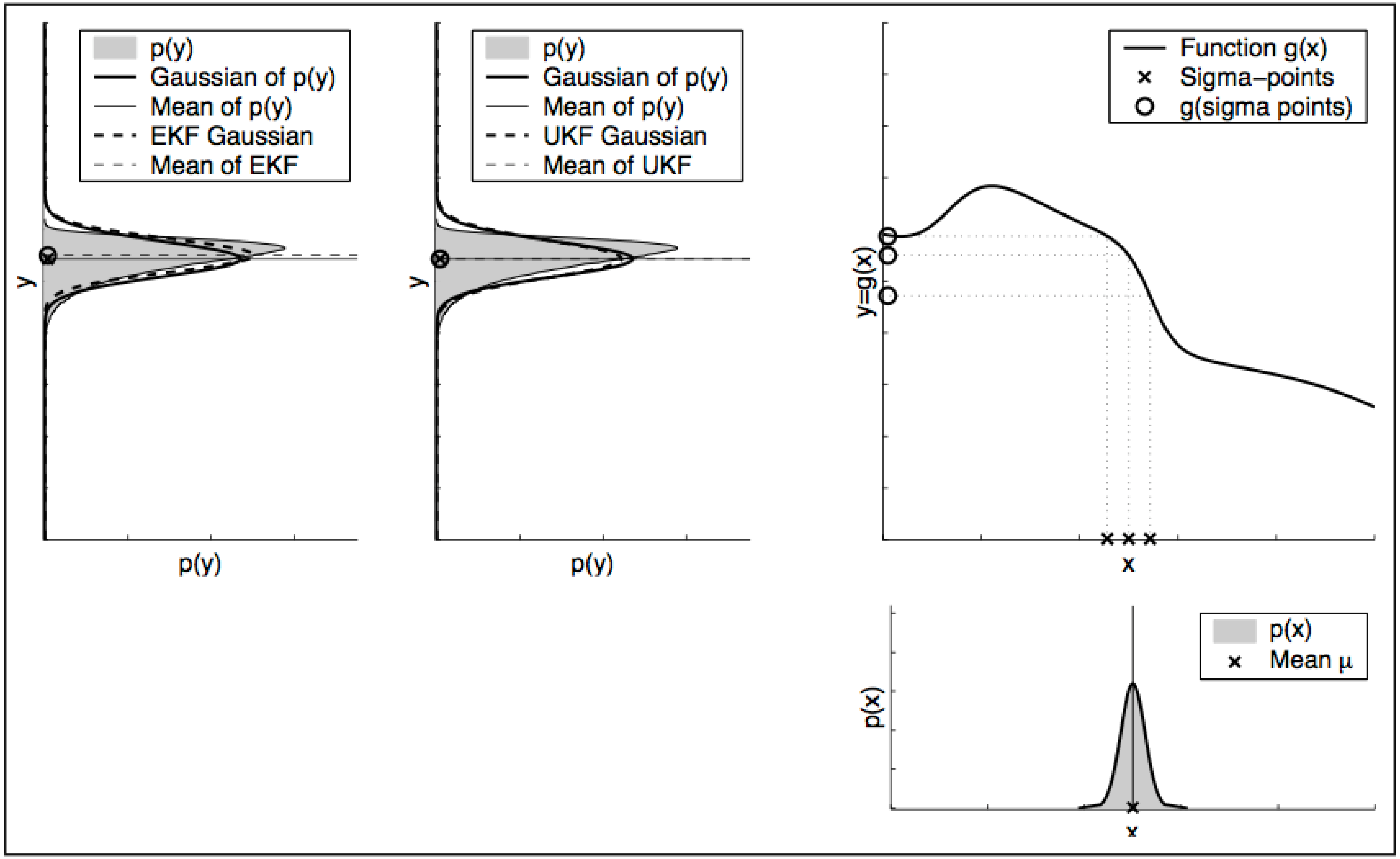
$$\Sigma_{xz} = \sum_{i=0}^{2N} w_c^{[i]} (\boldsymbol{\mathcal{X}}^{[i]} - \boldsymbol{\mu}_x)(\boldsymbol{\mathcal{Z}}^{[i]} - \boldsymbol{\mu}_z)^\top$$

i.e. 2π-0 = 0 !!!

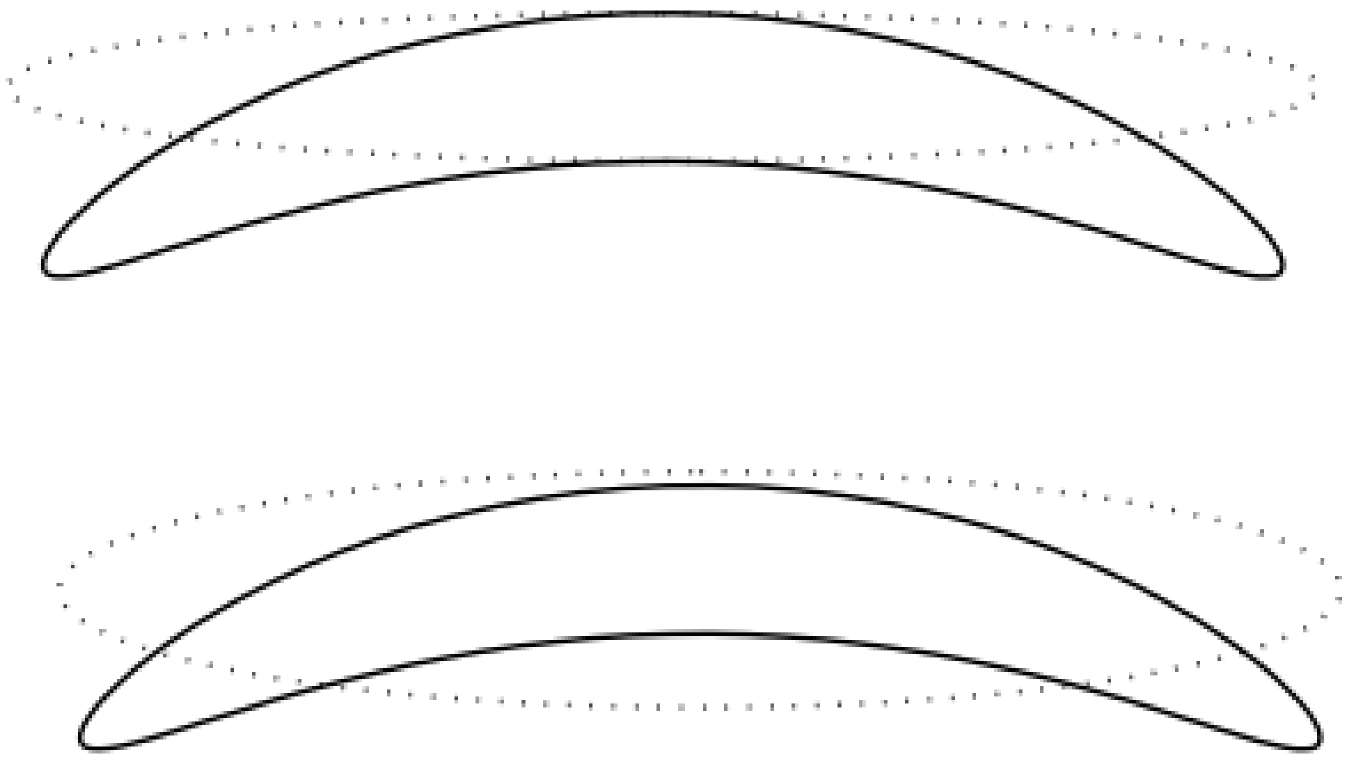
UKF vs. EKF


p(y)
Gaussian of p(y)
Mean of p(y)
EKF Gaussian
Mean of EKF
y
p(y)
p(y)
Gaussian of p(y)
Mean of p(y)
UKF Gaussian
Mean of UKF
y
p(y)
Function g(x)
Sigma–points
g(sigma points)
y=g(x)
x
p(x)
Mean μ
p(x)
x

UKF vs. EKF (Small Covariance)



UKF vs. EKF – Banana Shape



UKF Summary

- **Highly efficient**: Same complexity as EKF, with a constant factor slower (sometimes significant) in typical practical applications
- **Better linearization than EKF**: Accurate in first two derivatives* of Taylor expansion (EKF only first term)
- **Derivative-free**: No Jacobians needed
- **Still not optimal**!

UKF vs. EKF

- Same results as EKF for linear models
- Better approximation than EKF for non-linear models
- Differences often “somewhat small”
- No Jacobians needed for the UKF
- Same complexity class
- Slightly slower than the EKF

Literature

Unscented Transform and UKF

- Thrun et al.: “Probabilistic Robotics”, Chapter 3.4
- “A New Extension of the Kalman Filter to Nonlinear Systems” by Julier and Uhlmann, 1995
- “Sigma-Point Kalman Filters for Probabilistic Inference in Dynamic State-Space Models”, PhD Thesis, Rudolph van der Merwe, 2004