Solutions Manual

for

Microwave Engineering
4^{th} edition

David Pozar

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Chapter 1

1.1 This is an open-ended question where the focus of the answer may be largely chosen by the student or the instructor. Some of the relevant historical developments related to the early days of radio are listed here (as cited from T. S. Sarkar, R. J. Mailloux, A. A. Oliner, M. Salazar-Palma, and D. Sengupta, *History of Wireless*, Wiley, N.J., 2006):

1865: James Clerk Maxwell published his work on the unification of electric and magnetic phenomenon, including the introduction of the displacement current and the theoretical prediction of EM wave propagation.

1872: Mahlon Loomis, a dentist, was issued US Patent 129,971 for "aerial telegraphy by employing an 'aerial' used to radiate or receive pulsations caused by producing a disturbance in the electrical equilibrium of the atmosphere". This sounds a lot like radio, but in fact Loomis was not using an RF source, instead relying on static electricity in the atmosphere. Strictly speaking this method does not involve a propagating EM wave. It was not a practical system.

1887-1888: Heinrich Hertz studied Maxwell's equations and experimentally verified EM wave propagation using spark gap sources with dipole and loop antennas.

1893: Nikola Tesla demonstrated a wireless system with tuned circuits in the transmitter and receiver, with a spark gap source.

1895: Marconi transmitted and received a coded message over a distance of 1.75 miles in Italy.

1894: Oliver Lodge demonstrated wireless transmission of Morse code over a distance of 60 m, using coupled induction coils. This method relied on the inductive coupling between the two coils, and did not involve a propagating EM wave.

1897: Marconi was issued a British Patent 12,039 for wireless telegraphy.

1901: Marconi achieved the first trans-Atlantic wireless transmission.

1943: The US Supreme Court invalidated Marconi's 1904 US patent on tuning using resonant circuits as being superseded by prior art of Tesla, Lodge, and Braun.

So it is clear that many workers contributed to the development of wireless technology during this time period, and that Marconi was not the first to develop a wireless system that relied on the propagation of electromagnetic waves. On the other hand, Marconi was very successful at making radio practical and commercially viable, for both shipping and land-based services.

1.2 $E_y = E_0 \cos(\omega t - kx)$, $E_0 = 5$ V/m, $f = 2.4$ GHz.
$\epsilon_r = 2.54$, $x_1 = 0.1$, $x_2 = 0.15$

a) $\eta = \eta_0/\sqrt{\epsilon_r} = 236.6\,\Omega$

$H_z = E_y/\eta = 0.0211 \cos(\omega t - kz)$

b) $v_p = c/\sqrt{\epsilon_r} = 1.88 \times 10^8$ m/sec

c) $\lambda = v_p/f = 0.0784$ m , $k = 2\pi/\lambda = 80.11\ \text{m}^{-1}$

d) $\Delta\phi = k(x_2 - x_1) = 80.11(.15-.1) = 4.00$ rad $= 229.5°$

1.3 $\bar{E} = E_0(a\hat{x} + b\hat{y})\,e^{-jk_0 z}$; a, b real

Let $\bar{E} = A(\hat{x} - j\hat{y})e^{-jk_0 z} + B(\hat{x} + j\hat{y})e^{-jk_0 z}$

where A, B are the amplitudes of the RHCP and LHCP components. Equating vector components gives

$$\hat{x}: \quad A + B = aE_0$$
$$\hat{y}: \quad -jA + jB = bE_0 , \quad \text{or} \quad A - B = jbE_0$$

so

$$A = E_0(a + jb)/2$$
$$B = E_0(a - jb)/2$$

check: if $a = 1$, $b = 2$ then $A = (\frac{1}{2} + j)E_0$, $B = (\frac{1}{2} - j)E_0$
(agrees with Problem 1.5 from 3rd ed.)

1.4 From eq. (1.76),

$$\bar{H} = \frac{1}{\eta_0}\hat{n} \times \bar{E} \quad , \quad \bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$$

$$\bar{S} = \bar{E} \times \bar{H}^* = \frac{1}{\eta_0}\bar{E} \times \hat{n} \times \bar{E}^*$$

$$= \frac{1}{\eta_0}\left[(\bar{E}\cdot\bar{E}^*)\hat{n} - (\bar{E}\cdot\hat{n})\bar{E}^*\right] \qquad \text{(from B.5)}$$

Since $\bar{k}\cdot\bar{E}_0 = k_0\hat{n}\cdot\bar{E}_0 = 0$ from (1.69) and (1.74), we have

$$\bar{S} = \frac{\hat{n}}{\eta_0}\bar{E}\cdot\bar{E}^* = \frac{\hat{n}}{\eta_0}|E_0|^2 \quad W/m^2 \quad \checkmark$$

1.5 Writing general plane wave fields in each region:

$$\bar{E}^i = \hat{x}\, e^{-jk_0 z} \qquad \bar{H}^i = \frac{\hat{y}}{\eta_0} e^{-jk_0 z} \qquad \text{for } z<0$$

$$\bar{E}^r = \hat{x}\,\Gamma e^{jk_0 z} \qquad \bar{H}^r = \frac{-\hat{y}}{\eta_0}\Gamma e^{jk_0 z} \qquad \text{for } z<0$$

$$\bar{E}^s = \hat{x}(Ae^{-jkz} + Be^{jkz}) \qquad \bar{H}^s = \frac{\hat{y}}{\eta}(Ae^{-jkz} - Be^{jkz}) \qquad \text{for } 0<z<d$$

$$\bar{E}^t = \hat{x}\,T e^{-jk_0(z-d)} \qquad \bar{H}^t = \frac{\hat{y}}{\eta_0} T e^{-jk_0(z-d)} \qquad \text{for } z>d$$

Now match E_x and H_y at $z=0$ and $z=d$ to obtain four equations for Γ, T, A, B:

$$z=0: \qquad 1+\Gamma = A+B \qquad \frac{1}{\eta_0}(1-\Gamma) = \frac{1}{\eta}(A-B)$$

$$z=d: \qquad j(-A+B) = T \qquad \frac{j}{\eta}(-A-B) = \frac{T}{\eta_0} \qquad (\text{since } d = \lambda_0/4\sqrt{\epsilon_r})$$

Solving for Γ gives

$$\Gamma = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2} \quad \checkmark$$

CHECK:

$$\lambda/4 \text{ TRANSFORMER} \Rightarrow Z_{in} = \eta^2/\eta_0 \;,\; \Gamma = \frac{\eta^2/\eta_0 - \eta_0}{\eta^2/\eta_0 + \eta_0} = \frac{\eta^2 - \eta_0^2}{\eta^2 + \eta_0^2}.$$

1.6 The incident, reflected, and transmitted fields can be written as,

$$\bar{E}^i = E_0(\hat{x} - j\hat{y})e^{-jk_0 z} \qquad \bar{H}^i = j\frac{E_0}{\eta_0}(\hat{x} - j\hat{y})e^{-jk_0 z} \qquad \text{(RHCP)}$$

$$\bar{E}^r = E_0\Gamma(\hat{x} - j\hat{y})e^{jk_0 z} \qquad \bar{H}^r = -j\frac{E_0}{\eta_0}\Gamma(\hat{x} - j\hat{y})e^{jk_0 z} \qquad \text{(LHCP)}$$

$$\bar{E}^t = E_0 T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \bar{H}^t = j\frac{E_0}{\eta}T(\hat{x} - j\hat{y})e^{-\gamma z} \qquad \text{(RHCP)}$$

Matching fields at $z=0$ gives

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0}, \qquad T = \frac{2\eta}{\eta + \eta_0}$$

The Poynting vectors are: $(\hat{x} - j\hat{y}) \times (\hat{x} - j\hat{y})^* = 2j\hat{z}$

For $z<0$: $\bar{S}^- = (\bar{E}^i + \bar{E}^r) \times (\bar{H}^i + \bar{H}^r)^* = \dfrac{2\hat{z}|E_0|^2}{\eta_0}\left(1 - |\Gamma|^2 + \Gamma e^{2jk_0 z} + \Gamma^* e^{-2jk_0 z}\right)$ ✓

For $z>0$: $\bar{S}^+ = \bar{E}^t \times \bar{H}^{t*} = \dfrac{2\hat{z}|E_0|^2|T|^2}{\eta^*}e^{-2\alpha z}$ ✓

at $z=0$,

$$\bar{S}^- = \frac{2\hat{z}|E_0|^2}{\eta_0}\left(1 - |\Gamma|^2 + \Gamma - \Gamma^*\right) = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

$$\bar{S}^+ = 2\hat{z}|E_0|^2 \frac{4\eta}{|\eta + \eta_0|^2} \qquad \left(\text{using } T = \frac{2\eta}{\eta + \eta_0}\right)$$

$$= \frac{2\hat{z}|E_0|^2}{\eta_0}\left(\frac{2\eta}{\eta + \eta_0}\right)\left(\frac{2\eta_0}{\eta + \eta_0}\right)^* = \frac{2\hat{z}|E_0|^2}{\eta_0}(1+\Gamma)(1-\Gamma^*) \checkmark$$

Thus $\bar{S}^- = \bar{S}^+$ at $z=0$, and power is conserved.

1.7 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = 2\pi j f\sqrt{\mu_0\epsilon_0}\sqrt{5-j2} = j\frac{2\pi(1000)}{300}\sqrt{5.385\angle{-22^\circ}}$$

$$= 48.5\angle{79^\circ} = 9.25 + j47.6 = \alpha + j\beta \quad (\text{neper/m, rad/m})$$

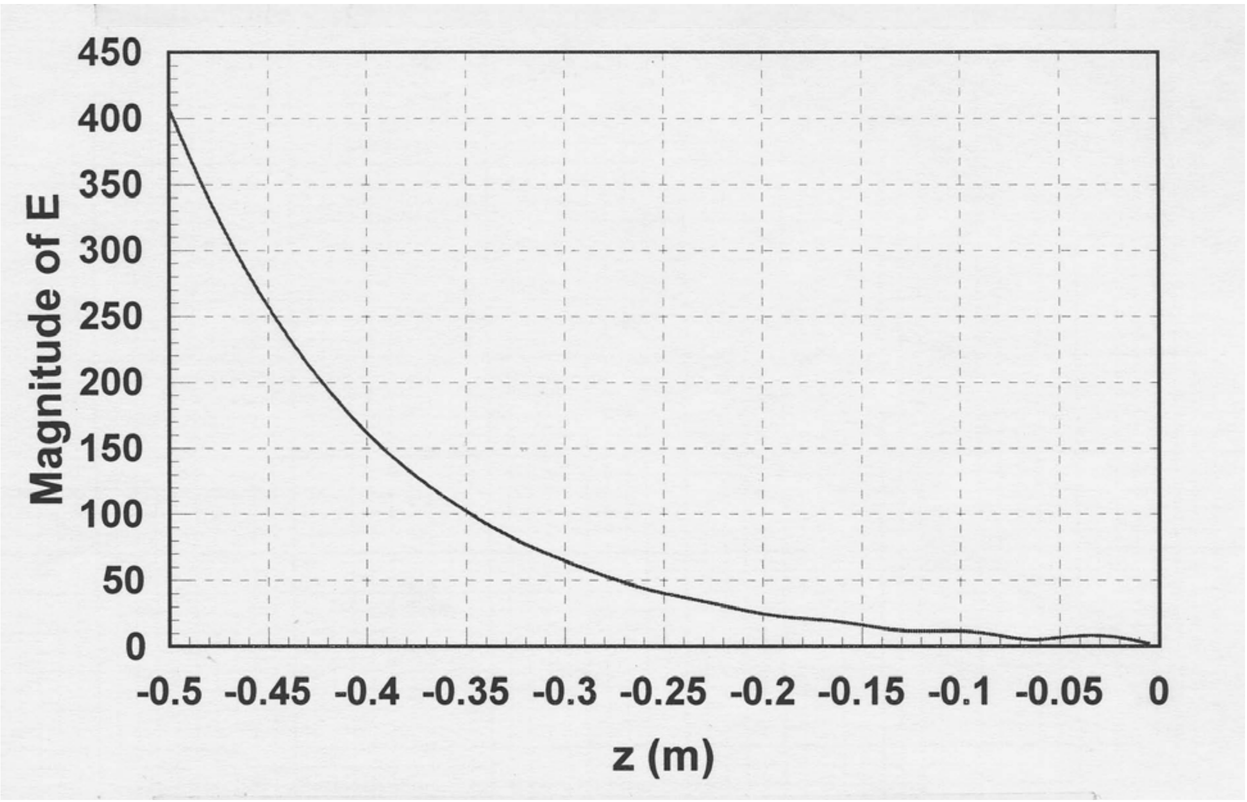
$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\sqrt{\mu_0\epsilon_0}}{j\omega\sqrt{\mu_0\epsilon}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{\eta_0}{\sqrt{5-j2}} = \frac{377}{2.32\angle{-11^\circ}} = 163\angle{11^\circ}\ \Omega$$

$$\Gamma = -1$$

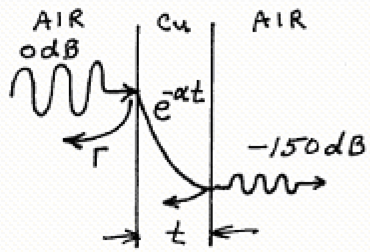
For $z<0$, $\quad \bar{E} = \bar{E}^i + \bar{E}^r = 4\hat{x}\left(e^{-\gamma z} - e^{\gamma z}\right)$

$$|\bar{E}| = 4\left|e^{-\alpha z}e^{-j\beta z} - e^{\alpha z}e^{j\beta z}\right|$$

$|\bar{E}|$ vs z is plotted below.



1.8 The total loss through the sheet is the product of the transmission losses at the air-copper and copper-air interfaces, and the exponential loss through the sheet.



$$\delta_s = \sqrt{\frac{2}{\omega\mu\sigma}} = 2.09\times10^{-6}\,m = \frac{1}{\alpha}$$

$$\eta_c = \frac{(1+j)}{\sigma\delta_s} = 8.2\times10^{-3}(1+j)\,\Omega$$

a) Power transfer from air into copper is given by,

$$1-|\Gamma|^2 \quad , \quad \Gamma = \frac{\eta_c-\eta_o}{\eta_c+\eta_o} \simeq \frac{8.2\times10^{-3}(1+j)-377}{377} = -0.999956+j\,4.35E\text{-}5$$

This yields a power transfer of -40.6 dB into the copper. By symmetry, the same transfer occurs for the copper-air interface.

b) the attenuation within the copper sheet is,

$$\text{copper att.} = 150\,dB - 40.6\,dB - 40.6\,dB = 68.8\,dB$$
$$= -20\log e^{-t/\delta_s} \Rightarrow t = \underline{0.017\ mm}\ ✓$$

(J. Mead provided this correction on 9/04)

1.9 From Table 1.1,

$$\gamma = j\omega\sqrt{\mu_0\epsilon} = j\frac{2\pi(3000)}{300}\sqrt{3(1-j.1)} = 5.435 + j108.964 = \alpha + j\beta \ \mathrm{m}^{-1}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r(1-j.1)}} = 217.121 \angle 2.855^\circ$$

a) $$S_i = \mathrm{Re}\left\{\frac{|\bar{E}_i(z=0)|^2}{\eta^*}\right\} = 46.000 \ \mathrm{W/m^2} \ \checkmark$$

$\Gamma = -1$ at $z = \ell = 20$ cm

$$\bar{E}_r = \Gamma \bar{E}_i(z=\ell)e^{\gamma(z-\ell)} = -100\hat{x}\,e^{-2\gamma\ell}e^{\gamma z}$$

$$S_r = \mathrm{Re}\left\{\frac{|\bar{E}_r(z=0)|^2}{\eta^*}\right\} = 0.595 \ \mathrm{W/m^2} \ \checkmark$$

b) $\bar{E}_t = \bar{E}_i + \bar{E}_r$

$$\bar{E}_t(z=0) = 100\hat{x}(1-e^{-2\gamma\ell}), \quad \bar{H}_t(z=0) = \frac{100\hat{y}}{\eta}(1+e^{-2\gamma\ell})$$

$$S_{in} = \mathrm{Re}\left\{\bar{E}_t \times \bar{H}_t^* \cdot \hat{z}\right\} = 45.584 \ \mathrm{W/m^2}$$

But $S_i - S_r = 45.405 \ \mathrm{W/m^2} \neq S_{in}$. This is because S_i and S_r individually are not physically meaningful in a lossy medium.

(The above were computed using a FORTRAN program, with 6 digit precision. The error between $S_i - S_r$ and S_{in} is only about 0.4% - this would be larger if the loss were greater.)

1.10 As in Example 1.3, assume outgoing plane wave fields in each region. To get J_{sx}, we need H_y, since $\hat{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s$ $(\hat{n} = \hat{z})$. Then we must have E_x to get $\bar{S} = \bar{E} \times \bar{H}^* = \pm S \hat{z}$. So the form of the fields must be,

for $z<0$, $\bar{E}_1 = \hat{x} A e^{jk_0 z}$ $\quad$ for $z>0$, $\bar{E}_2 = \hat{x} B e^{-jkz}$

$$\bar{H}_1 = \frac{-\hat{y}}{\eta_0} A e^{jk_0 z} \qquad \bar{H}_2 = \hat{y} \frac{B}{\eta} e^{-jkz}$$

with $k_0 = \omega\sqrt{\mu_0 \epsilon_0}$, $k = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_r}$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\eta = \sqrt{\mu_0/\epsilon_0 \epsilon_r}$, and

A and B are unknown amplitudes to be determined.

The boundary conditions at $z=0$ are, from (1.36) and (1.37),

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = 0 \quad \Rightarrow \quad A = B$$

$$\hat{z} \times (\bar{H}_2 - \bar{H}_1) = \bar{J}_s \quad \Rightarrow \quad -\left(\frac{B}{\eta} + \frac{A}{\eta_0}\right) = J_0$$

$$\therefore \quad A = B = \frac{-J_0 \eta \eta_0}{\eta + \eta_0}$$

1.11 This current sheet will generate obliquely propagating plane waves. From (1.132)-(1.133), assume

$$\left.\begin{aligned} \bar{E}_1 &= A(\hat{x}\cos\theta_1 + \hat{z}\sin\theta_1)\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)} \\ \bar{H}_1 &= \frac{-A}{\eta_0}\hat{y}\, e^{-jk_0(x\sin\theta_1 - z\cos\theta_1)} \end{aligned}\right\} \text{for } z<0$$

$$\left.\begin{aligned} \bar{E}_2 &= B(\hat{x}\cos\theta_2 - \hat{z}\sin\theta_2)\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \\ \bar{H}_2 &= \frac{B}{\eta}\hat{y}\, e^{-jk(x\sin\theta_2 + z\cos\theta_2)} \end{aligned}\right\} \text{for } z>0$$

with $k_0 = \omega\sqrt{\mu_0\epsilon_0}$, $k = \sqrt{\epsilon_r}\,k_0$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$, $\eta = \eta_0/\sqrt{\epsilon_r}$.

Apply boundary conditions at $z=0$:

$$\hat{z}\times(\bar{E}_2 - \bar{E}_1) = 0 \Rightarrow A\cos\theta_1 e^{-jk_0 x\sin\theta_1} - B\cos\theta_2 e^{-jkx\sin\theta_2} = 0$$

$$\hat{z}\times(\bar{H}_2 - \bar{H}_1) = J_s \Rightarrow \frac{A}{\eta_0}e^{-jk_0 x\sin\theta_1} + \frac{B}{\eta}e^{-jkx\sin\theta_2} = -J_0 e^{-j\beta x}$$

For phase matching we must have $k_0\sin\theta_1 = k\sin\theta_2 = \beta$

$\therefore\ \theta_1 = \sin^{-1}\beta/k_0$ ✓ $\quad \theta_2 = \sin^{-1}\beta/k$ (must have $\beta < k_0$)

Then, $\quad A\cos\theta_1 = B\cos\theta_2 \quad , \quad \dfrac{A}{\eta_0} + \dfrac{B}{\eta} = -J_0$

$$A = \frac{-J_0\eta\eta_0\cos\theta_2}{\eta\cos\theta_2 + \eta_0\cos\theta_1}, \quad B = \frac{-J_0\eta\eta_0\cos\theta_1}{\eta\cos\theta_2 + \eta_0\cos\theta_1}$$

Check: If $\beta = 0$, then $\theta_1 = \theta_2 = 0$, and $A = B = \dfrac{-J_0\eta\eta_0}{\eta + \eta_0}$,

which agrees with Problem 1.10 ✓

1.12 This solution is identical to the parallel polarized dielectric case of Section 1.8, except for the definitions of k_1, k_2, η_1, and η_2. Thus,

$$k_0 \sin\theta_i = k_0 \sin\theta_r = k \sin\theta_t \quad ; \; k = k_0\sqrt{\mu_r}$$

$$\Gamma = \frac{\eta\cos\theta_t - \eta_0\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i} \qquad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_t + \eta_0\cos\theta_i}$$

$$\eta = \eta_0\sqrt{\mu_r}$$

There will be a Brewster angle if $\Gamma = 0$. This requires that,

$$\eta\cos\theta_t = \eta_0\cos\theta_i$$

$$\sqrt{\mu_r}\sqrt{1-\left(\frac{k_0}{k}\right)^2\sin^2\theta_i} = \cos\theta_i = \sqrt{1-\sin^2\theta_i}$$

$$\mu_r\left(1-\frac{1}{\mu_r}\cancel{\sin^2\theta_i}\right) = 1-\cancel{\sin^2\theta_i}$$

or, $\mu_r = 1$. This implies a uniform region, so there is no Brewster angle for $\mu_r \neq 1$. ✓

1.13 Again, this solution is similar to the perpendicular polarized case of Section 1.8, except for the definition of k_1, k_2, η_1, η_2. Thus,

$$\Gamma = \frac{\eta\cos\theta_i - \eta_0\cos\theta_t}{\eta\cos\theta_i + \eta_0\cos\theta_t} \quad , \quad T = \frac{2\eta\cos\theta_i}{\eta\cos\theta_i + \eta_0\cos\theta_t}$$

A Brewster angle exists if

$$\eta\cos\theta_i = \eta_0\cos\theta_t$$

$$\sqrt{\mu_r}\sqrt{1-\sin^2\theta_i} = \sqrt{1-\frac{1}{\mu_r}\sin^2\theta_i}$$

$$\mu_r^2 - \mu_r^2\sin^2\theta_i = \mu_r - \sin^2\theta_i$$

$$\mu_r = (\mu_r+1)\sin^2\theta_i$$

$$\sin\theta_i = \sin\theta_b = \sqrt{\frac{\mu_r}{1+\mu_r}} < 1 \quad \checkmark$$

Thus, a Brewster angle does exist for this case.

1.14 $\bar{E} = 3\hat{x} - 2\hat{y} + 5\hat{z}$

$$\bar{D} = [\epsilon]\bar{E} = \begin{bmatrix} 1 & 3j & 0 \\ -3j & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3-6j \\ -4-9j \\ 20 \end{bmatrix} = (3-6j)\hat{x} + (-4-9j)\hat{y} + 20\hat{z}$$

1.15

$$D_x = \epsilon_0(\epsilon_r E_x + j\kappa E_y)$$
$$D_y = \epsilon_0(-j\kappa E_x + \epsilon_r E_y)$$
$$D_z = \epsilon_0 E_z$$

Then,

$$D_+ = D_x - jD_y = \epsilon_0(\epsilon_r - \kappa)E_x - j\epsilon_0(\epsilon_r - \kappa)E_y = \epsilon_0(\epsilon_r - \kappa)E_+$$
$$D_- = D_x + jD_y = \epsilon_0(\epsilon_r + \kappa)E_x + j\epsilon_0(\epsilon_r + \kappa)E_y = \epsilon_0(\epsilon_r + \kappa)E_-$$

OR,

$$\begin{bmatrix} D_+ \\ D_- \\ D_z \end{bmatrix} = \begin{bmatrix} (\epsilon_r - \kappa) & 0 & 0 \\ 0 & (\epsilon_r + \kappa) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_+ \\ E_- \\ E_z \end{bmatrix}$$

From Maxwell's equations,

$$\left.\begin{aligned} \nabla \times \bar{E} &= -j\omega\mu \bar{H} \\ \nabla \times \bar{H} &= j\omega[\epsilon]\bar{E} \end{aligned}\right\} \quad \begin{aligned} \nabla \times \nabla \times \bar{E} &= -j\omega\mu \nabla \times \bar{H} = \omega^2\mu[\epsilon]\bar{E} \\ \nabla^2 \bar{E} + \omega^2\mu[\epsilon]\bar{E} &= 0 \quad \text{(CARTESIAN)} \end{aligned}$$

Expanding this wave equation gives,

$$\nabla^2 E_x + \omega^2\mu\epsilon_0(\epsilon_r E_x + j\kappa E_y) = 0 \qquad (1)$$
$$\nabla^2 E_y + \omega^2\mu\epsilon_0(-j\kappa E_x + \epsilon_r E_y) = 0 \qquad (2)$$
$$\nabla^2 E_z + k_0^2 E_z = 0 \qquad (3)$$

adding (1) +j(2) gives $\nabla^2(E_x + jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r + \kappa)E_x + j(\epsilon_r + \kappa)E_y] = 0$

$$\nabla^2 E^+ + \omega^2\mu\epsilon_0(\epsilon_r + \kappa)E^+ = 0$$
$$\therefore \beta_+ = k_0\sqrt{\epsilon_r + \kappa} \checkmark$$

adding (1) −j(2) gives $\nabla^2(E_x - jE_y) + \omega^2\mu\epsilon_0[(\epsilon_r - \kappa)E_x - j(\epsilon_r - \kappa)E_y] = 0$

$$\nabla^2 E^- + \omega^2\mu\epsilon_0(\epsilon_r - \kappa)E^- = 0$$
$$\therefore \beta_- = k_0\sqrt{\epsilon_r - \kappa} \checkmark$$

Note that the wave equations for E^+, E^- must be satisfied simultaneously. Thus, for E^+ we must have $E^- = 0$. This implies that $E_y = jE_x = jE_0$. The actual electric field is then, $\bar{E}^+ = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} + j\hat{y})e^{-j\beta_+ z}$ (LHCP)

This is a LHCP wave. Similarly for $\bar{E}^-$ we must have $E^+ = 0$: $\bar{E}^- = \hat{x}E_x + \hat{y}E_y = E_0(\hat{x} - j\hat{y})e^{-j\beta_- z}$ (RHCP)

1.16 Comparing (1.118), (1.125), and (1.129) shows that

$$E_t = \frac{J_t}{\sigma} = \frac{J_s}{\sigma \delta_s} = R_s J_s.$$

Thus $\bar{E}_t = R_s \bar{J}_s = R_s \hat{n} \times \bar{H}$ is the desired surface impedance relation. Applying this to the surface integral of (1.155) gives, on S,

$$[(\bar{E}_1 \times \bar{H}_2) - (\bar{E}_2 \times \bar{H}_1)] \cdot \hat{n} = R_s [(\hat{n} \times \bar{H}_{1t}) \times \bar{H}_{2t} - (\hat{n} \times \bar{H}_{2t}) \times \bar{H}_{1t}]$$

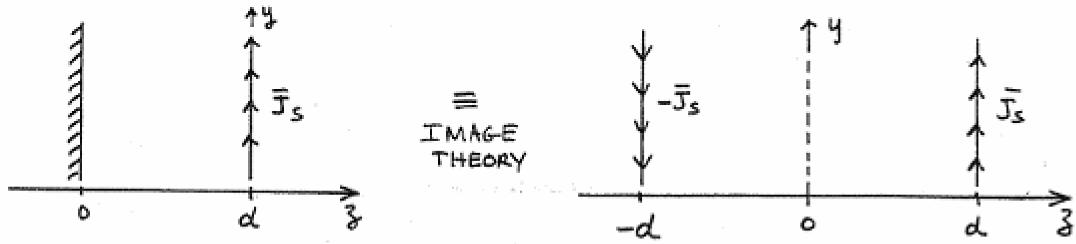
(USING B.5)

$$= R_s [(\cancel{\bar{H}_{2t} \cdot \hat{n}}) \bar{H}_{1t} - (\cancel{\bar{H}_{2t} \cdot \bar{H}_{1t}}) \hat{n} - (\cancel{\bar{H}_{1t} \cdot \hat{n}}) H_{2t} + (\cancel{\bar{H}_{1t} \cdot \bar{H}_{2t}}) \hat{n}]$$

$$= 0$$

So (1.157) is obtained.

1.17



First find the fields due to the source at $z=d$. From (1.139)–(1.140),

For $z<d$, $\bar{E}_1 = A\hat{y}\, e^{-jk_o(x\sin\theta - z\cos\theta)}$

$\bar{H}_1 = \frac{A}{\eta_o}(\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta - z\cos\theta)}$

For $z>d$, $\bar{E}_2 = B\hat{y}\, e^{-jk_o(x\sin\theta + z\cos\theta)}$

$\bar{H}_2 = \frac{B}{\eta_o}(-\hat{x}\cos\theta + \hat{z}\sin\theta)\, e^{-jk_o(x\sin\theta + z\cos\theta)}$

Apply boundary conditions at $z=d$:

$$\hat{z}\times[\bar{E}(d^+) - \bar{E}(d^-)] = 0 \Rightarrow A e^{jk_o d\cos\theta} = B e^{-jk_o d\cos\theta}$$

$$\hat{z}\times[\bar{H}(d^+) - \bar{H}(d^-)] = \bar{J}_s \Rightarrow [-B\cos\theta\, e^{-jk_o d\cos\theta} - A\cos\theta\, e^{jk_o d\cos\theta}]\cdot e^{-jk_o x\sin\theta} = \eta_o J_o e^{-j\beta x}$$

For phase matching, $k_o \sin\theta = \beta$

Then, $A = \frac{-\eta_o J_o}{2\cos\theta} e^{-jk_o d\cos\theta}$ $\quad B = \frac{-\eta_o J_o}{2\cos\theta} e^{jk_o d\cos\theta}$

$$\bar{E} = \frac{-\eta_o J_o \hat{y}}{2\cos\theta}\begin{cases} e^{-jk_o[x\sin\theta - (z-d)\cos\theta]} & z<d \\ e^{-jk_o[x\sin\theta + (z-d)\cos\theta]} & z>d \end{cases}$$

The fields due to the source at $z=-d$ can then be found by replacing d with $-d$, and J_o with $-J_o$:

$$\bar{E} = \frac{\eta_0 J_0 \hat{y}}{2\cos\theta}\begin{cases} e^{-jk_0[x\sin\theta - (z+d)\cos\theta]} & z < -d \\ e^{-jk_0[x\sin\theta + (z+d)\cos\theta]} & z > -d \end{cases}$$

Combining these results gives the total fields:

$$\bar{E} = \frac{-j\eta_0 J_0 \hat{y}}{\cos\theta}\begin{cases} e^{-jk_0 x\sin\theta}\, e^{-jk_0 d}\, \sin(k_0 z\cos\theta) & 0 < z < d \\ e^{-jk_0 x\sin\theta}\, e^{-jk_0 z}\, \sin(k_0 d\cos\theta) & z > d \end{cases}$$

CHECK: If $\beta = 0$, then $\theta = 0$ and we have,

$$\bar{E} = -j\eta_0 J_0 \hat{y}\begin{cases} e^{-jk_0 d}\sin k_0 z & \text{for } 0 < z < d \\ e^{-jk_0 z}\sin k_0 d & \text{for } z > d \end{cases}$$

This agrees with the results in (1.161) – (1.162).

1.18

$$\nabla\times\bar{E} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial E_z}{\partial\phi} - \frac{\partial E_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho}\right) + \hat{z}\,\frac{1}{\rho}\left(\frac{\partial(\rho E_\phi)}{\partial\rho} - \frac{\partial E_\rho}{\partial\phi}\right)$$

$$\nabla\times\nabla\times\bar{E} = \hat{\rho}\left[\frac{-1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} - \frac{\partial^2 E_\rho}{\partial z^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi}\right]$$

$$+\hat{\phi}\left[-\frac{\partial^2 E_\phi}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} - \frac{\partial^2 E_\phi}{\partial\rho^2} - \frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{E_\phi}{\rho^2} - \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\phi\partial\rho}\right]$$

$$+\hat{z}\left[\frac{-\partial^2 E_z}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z} - \frac{1}{\rho}\frac{\partial E_z}{\partial\rho}\right]$$

$$\nabla(\nabla\cdot\bar{E}) = \hat{\rho}\left[\frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{\partial^2 E_z}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\rho\partial\phi} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} - \frac{1}{\rho^2}\frac{\partial E_\phi}{\partial\phi} - \frac{E_\rho}{\rho^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial^2 E_z}{\partial\phi\partial z} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{1}{\rho}\frac{\partial^2 E_\rho}{\partial\rho\partial\phi} + \frac{1}{\rho^2}\frac{\partial E_\rho}{\partial\phi}\right]$$

$$+\hat{z}\left[\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{\rho}\frac{\partial^2 E_\phi}{\partial\phi\partial z} + \frac{\partial^2 E_\rho}{\partial\rho\partial z} + \frac{1}{\rho}\frac{\partial E_\rho}{\partial z}\right]$$

If we apply ∇^2 to the cylindrical components of $\bar{E}$ we get:

$$\nabla^2\bar{E} \stackrel{?}{=} \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z \qquad \text{(THIS IS NOT A VALID STEP!)}$$

$$= \hat{\rho}\left[\frac{1}{\rho}\frac{\partial E_\rho}{\partial\rho} + \frac{\partial^2 E_\rho}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\rho}{\partial\phi^2} + \frac{\partial^2 E_\rho}{\partial z^2}\right]$$

$$+\hat{\phi}\left[\frac{1}{\rho}\frac{\partial E_\phi}{\partial\rho} + \frac{\partial^2 E_\phi}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_\phi}{\partial\phi^2} + \frac{\partial^2 E_\phi}{\partial z^2}\right]$$

$$+\hat{z}\left[\frac{1}{\rho}\frac{\partial E_z}{\partial\rho} + \frac{\partial^2 E_z}{\partial\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial\phi^2} + \frac{\partial^2 E_z}{\partial z^2}\right]$$

Note that the $\hat{\rho}$ and $\hat{\phi}$ components of $\nabla\times\nabla\times\bar{E}$ and $\nabla(\nabla\cdot\bar{E}) - \nabla^2\bar{E}$ do not agree. This is because $\hat{\rho}$ and $\hat{\phi}$ are not constant vectors, so $\nabla^2\bar{E} \neq \hat{\rho}\,\nabla^2 E_\rho + \hat{\phi}\,\nabla^2 E_\phi + \hat{z}\,\nabla^2 E_z$. The $\hat{z}$ components are equal.

Chapter 2

2.1

$$i(t,z) = 1.8 \cos(3.77\times10^9 t - 18.13z) \text{ mA}$$

$$\omega = 3.77\times10^9 \text{ rad/sec}, \quad \beta = 18.13 \text{ m}^{-1}, \quad Z_0 = 75\,\Omega$$

a) $f = \omega/2\pi = 3.77\times10^9/2\pi = 600 \text{ MHz}$

b) $v_p = \omega/\beta = 2.08\times10^8 \text{ m/sec}$

c) $\lambda = 2\pi/\beta = 0.346 \text{ m}$

d) $\epsilon_r = (c/v_p)^2 = 2.08$ (Teflon)

e) $I(z) = 1.8\, e^{-j\beta z}$ (mA)

f) $v(t,z) = 0.135 \cos(\omega t - \beta z)$ V.

2.2

$R = 4.0\,\Omega/\text{m}$, $G = 0.02$ S/m, $L = 0.5\,\mu$H/m, $C = 200$ pF/m

$f = 800$ MHz, $\ell = 30$ cm

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = 0.540 + j\,50.268 \quad /\text{m}$$

$$Z_0 = \sqrt{(R+j\omega L)/(G+j\omega C)} = 49.99 + j\,0.46\,\Omega$$

with $R = G = 0$, $\beta = \omega\sqrt{LC} = 50.265$ rad/m

$$Z_0 = \sqrt{L/C} = 50.0\,\Omega$$

Note that β, Z_0 w/o loss are very close to values with loss.

For $\ell = 30$ cm, att $= \alpha\ell = 0.162$ np $\left(\frac{8.686\text{dB}}{\text{np}}\right) = 1.4$dB

2.3 From Table 2.1:

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.00825\,\Omega$$

$$L = \frac{\mu_0}{2\pi}\ln\frac{b}{a} = 2.40\times10^{-7}\ \text{H/m}$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln b/a} = 9.64\times10^{-11}\ \text{Fd/m}$$

$$R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right) = 3.76\ \Omega/\text{m}$$

$$G = \frac{2\pi\omega\epsilon_0\epsilon_r\tan\delta}{\ln b/a} = 2.42\times10^{-4}\ \text{S/m}$$

For small loss, $Z_0 = \sqrt{L/C} = 49.9\,\Omega$ ✓

From (2.85a), $\alpha \simeq \frac{1}{2}\left(\frac{R}{Z_0}+GZ_0\right) = 0.044\ \text{np/m} = 0.38\ \text{dB/m}$ ✓

From RG-402U cable data: $Z_0 = 50\,\Omega$, $\alpha = 13\ \text{dB}/100\ \text{ft}$
$= 0.426\ \text{dB/m}$

Check using formulas from Example 2.7: (difference due to braided outer conductor, not solid)

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\,\frac{\ln b/a}{2\pi} = 49.9\,\Omega \ ✓$$

$$\alpha_c = \frac{R_s}{2\eta\ln b/a}\left(\frac{1}{a}+\frac{1}{b}\right) = 0.0376\ \text{np/m} = 0.326\ \text{dB/m} \ ✓$$

$$\alpha_d = \frac{\omega\epsilon_0\epsilon_r}{2}\eta\tan\delta = 0.00605\ \text{np/m} = 0.052\ \text{dB/m}$$

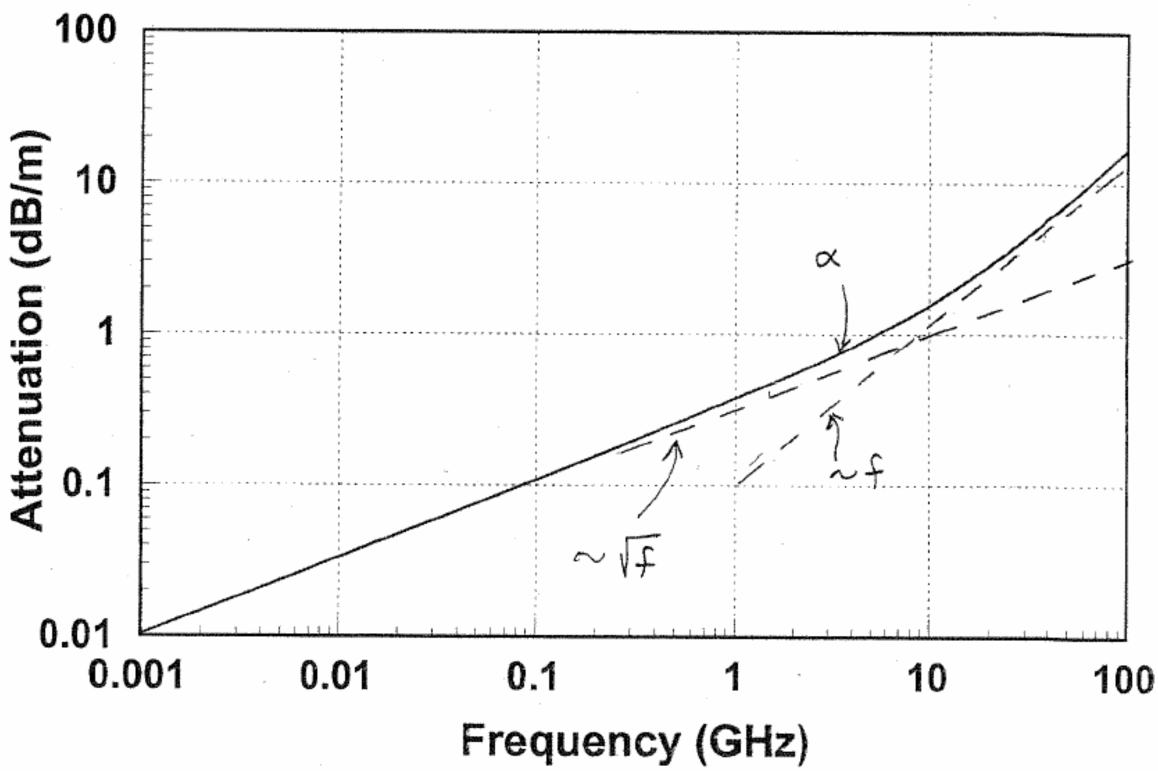
$$\alpha_T = 0.378\ \text{dB/m} \ ✓$$

also verified with Serenade.

2.4 Using the formulas of Problem 2.3, with $\alpha \simeq \frac{1}{2}(R/Z_0 + G Z_0)$:

f	R_s (Ω)	R (Ω)	G (S)	α (Np/m)	α (dB/m)
1 MHz	2.6×10^{-4}	0.118	2.42×10^{-7}	1.19×10^{-3}	0.0103
10 MHz	8.25×10^{-4}	0.376	2.42×10^{-6}	3.82×10^{-3}	0.0332
100 MHz	2.6×10^{-3}	1.18	2.42×10^{-5}	1.24×10^{-3}	0.1078
1 GHz	8.25×10^{-3}	3.76	2.42×10^{-4}	4.365×10^{-2}	0.379
10 GHz	2.6×10^{-2}	11.8	2.42×10^{-3}	1.785×10^{-1}	1.55
100 GHz	8.25×10^{-3}	37.6	2.42×10^{-2}	1.96	17.0

Results are plotted below (with additional data points). Note that the frequency dependence is between $\sqrt{f}$ ($R \sim \sqrt{f}$), and f ($G \sim f$), at low and high frequencies.



2.5 Ignoring fringing fields, $\bar{E}$ and $\bar{H}$ can be assumed as,

$$E_y = \frac{-V_o}{d} \;\; \text{V/m}, \quad H_x = \frac{V_o}{d\eta} = \frac{I_o}{W} \;\; \text{A/m}, \quad \eta = \sqrt{\mu/\epsilon}.$$

Then $\bar{E} \times \bar{H}^* = \hat{z}\,|s|$ ✓ and $I_o = V_o\left(\frac{W}{\eta d}\right)$.

From (2.17) - (2.20),

$$L = \frac{\mu_o}{I_o^2}\int_s |\bar{H}|^2 ds = \frac{\mu_o}{I_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{I_o}{W}\right)^2 dx\,dy = \frac{\mu_o d}{W} \quad \text{H/m} \quad ✓$$

$$C = \frac{\epsilon}{V_o^2}\int_s |\bar{E}|^2 ds = \frac{\epsilon}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\epsilon W}{d} \quad \text{Fd/m} \quad ✓$$

$$R = \frac{R_s}{I_o^2}\int_{C_1+C_2} |\bar{H}|^2 dl = \frac{2R_s}{I_o^2}\int_{x=0}^{W}\left(\frac{I_o}{W}\right)^2 dx = \frac{2R_s}{W} \quad \Omega/\text{m} \quad ✓$$

$$G = \frac{\omega\epsilon''}{V_o^2}\int_s |\bar{E}|^2 ds = \frac{\omega\epsilon''}{V_o^2}\int_{x=0}^{W}\int_{y=0}^{d}\left(\frac{-V_o}{d}\right)^2 dx\,dy = \frac{\omega\epsilon'' W}{d} \quad \text{S/m} \quad ✓$$

These results agree with those in Table 2.1

2.6 Assume $E_z = H_z = 0$, $\partial/\partial x = \partial/\partial y = 0$.

Then Maxwell's curl equations reduce to,

$$-\frac{\partial E_y}{\partial z} = -j\omega\mu H_x \quad (1) \qquad -\frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \quad (3)$$

$$\frac{\partial E_x}{\partial z} = -j\omega\mu H_y \quad (2) \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y \quad (4)$$

Since $E_x = 0$ at $y=0$ and $y=d$, and $\partial/\partial y = 0$, we must have $E_x \equiv 0$. Then (3) implies $H_y \equiv 0$. So we have,

$$\frac{\partial E_y}{\partial z} = j\omega\mu H_x \qquad \frac{\partial H_x}{\partial z} = j\omega\epsilon E_y$$

Now let $E_y = \frac{1}{d} V(z)$ and $H_x = \frac{-1}{W} I(z)$.

Then the voltage and current are,

$$V(z) = \int_{y=0}^{d} E_y \, dy \qquad I(z) = \int_{x=0}^{W} (\hat{y} \times \bar{H}) \cdot \hat{z} \, dx = -\int_{x=0}^{W} H_x \, dx$$

Then,

$$\left.\begin{aligned} \frac{\partial V}{\partial z} &= -j \frac{\omega\mu d}{W} I(z) \implies L = \frac{\mu d}{W} \\ \frac{\partial I(z)}{\partial z} &= -j \frac{\omega\epsilon W}{d} V(z) \implies C = \frac{\epsilon W}{d} \end{aligned}\right\} \text{agree with Table 2.1}$$

2.7 Using KVL:

$$-v(z) + R\frac{\Delta z}{2} i(z) + L\frac{\Delta z}{2}\frac{\partial i(z)}{\partial t} + R\frac{\Delta z}{2} i(z+\Delta z) + L\frac{\Delta z}{2}\frac{\partial i(z+\Delta z)}{\partial t} + v(z+\Delta z) = 0$$

divide by Δz and let $\Delta z \to 0$:

$$\frac{\partial v(z)}{\partial z} = -R\,i(z) - L\frac{\partial i(z)}{\partial t} \quad ✓$$

Using KCL:

$$i(z) - \Delta z\left[G + C\frac{\partial}{\partial t}\right]\left[v(z) - \frac{\Delta z}{2}\left(R + L\frac{\partial}{\partial t}\right)i(z)\right] - i(z+\Delta z) = 0$$

divide by Δz and let $\Delta z \to 0$:

$$\frac{\partial i(z)}{\partial z} = -G\,v(z) - C\frac{\partial v(z)}{\partial t} \quad ✓$$

These results agree with (2.2a,b).

2.8 $z_L = Z_L/Z_0 = 0.400 - j0.267$

From Smith chart,

$\Gamma_L = 0.461\angle 215°$

$SWR = 2.71$

$\Gamma_{in} = 0.461\angle 359°$ ✓

$Z_{in} = 203 - j5.2\ \Omega$

2.9 $\lambda_g = \lambda_0/\sqrt{\epsilon_r} = \dfrac{300}{3000\sqrt{2.56}} = 6.25\text{ cm}$

$\ell = \dfrac{2.0\text{ cm}}{6.25\text{ cm}/\lambda_g} = 0.320\lambda_g \qquad \beta\ell = \dfrac{2\pi}{\lambda_g}(.32\lambda_g) = 115.2°$

Smith chart solution:

$Z_{in} = 18.98 - j20.55\ \Omega$ ✓

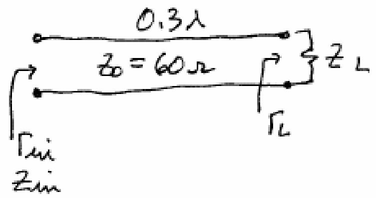
$\Gamma_{in} = 0.62\angle 212°$ ✓

$\Gamma_L = 0.62\angle 83°$ ✓

$SWR = 4.27$ ✓

These results were verified with the analytical formulas.

2.10



$\Gamma_L = 0.4\angle 60^\circ = 0.2 + j\,0.3464$

$Z_L = Z_0 \dfrac{1+\Gamma_L}{1-\Gamma_L} = 60 \dfrac{1.2 + j.3464}{.8 - j.3464} = \dfrac{74.94\angle 16.1^\circ}{.8718\angle -23.4^\circ} = 66.3 + j\,54.7\,\Omega$ ✓

$\Gamma_{in} = \Gamma_L e^{-2j\beta\ell} = .4\angle 60-216 = .4\angle -156^\circ = .4\angle 204^\circ$ ✓

$Z_{in} = 26.6 - j\,10.3\,\Omega$ ✓ (Smith Chart)

2.11

C: $Z_{oc} = -j/\omega C = -j\,12.73\,\Omega = -jZ_0 \cot\beta\ell$ $C = 5\,pF$

$\tan\beta\ell = 100/12.73 \Rightarrow \beta\ell = 82.74^\circ$ ✓

$\lambda_0 = 0.12\,m,\ \beta = 2\pi\sqrt{\epsilon_e}/\lambda_0 = 3854^\circ/m \Rightarrow \ell = \underline{\underline{2.147\,cm}}$ ✓

L: $Z_{oc} = j\omega L = +j\,78.5\,\Omega = -jZ_0 \cot\beta\ell$ $L = 5\,nH$

$\tan\beta\ell = -100/78.5 \Rightarrow \beta\ell = 128.1^\circ$ ✓ $\Rightarrow \ell = \underline{\underline{3.324\,cm}}$ ✓

These results were verified with Serenade.

2.12 $|\Gamma| = \dfrac{S-1}{S+1} = \dfrac{0.5}{2.5} = 0.2$

$|\Gamma| = \left|\dfrac{Z_L - Z_0}{Z_L + Z_0}\right| = \left|\dfrac{100 - Z_0}{100 + Z_0}\right|$ (Z_0 real)

So either, $\dfrac{100 - Z_0}{100 + Z_0} = 0.2 \Rightarrow Z_0 = Z_L \dfrac{1-\Gamma}{1+\Gamma} = 100\left(\dfrac{.8}{1.2}\right) = 66.7\,\Omega$ ✓

or

$\dfrac{100 - Z_0}{100 + Z_0} = -0.2 \Rightarrow Z_0 = Z_L \dfrac{1-\Gamma}{1+\Gamma} = 100\left(\dfrac{1.2}{.8}\right) = 150\,\Omega$ ✓

2.13

$$Z_{sc} = jZ_0 \tan\beta l \quad , \quad Z_{oc} = -jZ_0 \cot\beta l$$

$$Z_{sc} \cdot Z_{oc} = Z_0^2 \Rightarrow Z_0 = \sqrt{Z_{sc} Z_{oc}}$$

2.14

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 + j40}{130 + j40} = \frac{50\angle 53^\circ}{136\angle 17^\circ} = 0.367\angle 36^\circ \checkmark$$

$$P_{LOAD} = P_{INC} - P_{REF} = P_{INC}(1 - |\Gamma|^2) = 30[1 - (.367)^2] = 25.9 \text{ W} \checkmark$$

2.15

$$RL = -20 \log |\Gamma|$$

$$SWR = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

$$|\Gamma| = 10^{-RL/20}$$

$$|\Gamma| = \frac{SWR - 1}{SWR + 1}$$

SWR	$\lvert\Gamma\rvert$	RL(dB)
1.00	0.0	∞
1.01	.005	46.0
1.02	.01	40.0
1.05	.024	32.3
1.07	.0316	30.0
1.10	.0476	26.4
1.20	.091	20.8
1.22	.100	20.0
1.50	.200	14.0
1.92	.316	10.0
2.00	.333	9.5
2.50	.429	7.4

2.16 $V_g = 15\text{v RMS}, \; Z_g = 75\Omega, \; Z_0 = 75\Omega, \; Z_L = 60 - j40\Omega, \; \ell = 0.7\lambda.$

a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-15 - j40}{135 - j40} = \frac{42.7\angle -110.6^\circ}{140.8\angle -16.5^\circ} = 0.303\angle -94^\circ = -0.021 - j0.302$$

$$P_L = \left(\frac{V_g}{2}\right)^2 \frac{1}{Z_0}\left(1 - |\Gamma|^2\right) = 0.681 \text{ W} \checkmark$$

This method is actually based on $P_L = P_{inc}(1 - |\Gamma|^2)$. It is the simplest method, but only applies to lossless lines.

b) $$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell} = 75\,\frac{60 + j190.8}{198.1 + j184.7} = 75\,\frac{200\angle 72.5^\circ}{270.8\angle 43^\circ}$$

$$= 55.4\angle 29.5^\circ = 48.2 + j27.3\ \Omega$$

$$P_L = \left|\frac{V_g}{Z_g + Z_{in}}\right|^2 Re(Z_{in}) = \left|\frac{15}{123.2 + j27.3}\right|^2 (48.2) = 0.681 \text{ W} \checkmark$$

This method computes $P_L = P_{in} = |I_{in}|^2 R_{in}$, and also applies only to lossless lines.

c) $$V(z) = V^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$V_L = V(0) = V^+(1 + \Gamma) \qquad V^+ = \frac{V_g}{2} = 7.5\text{v}$$

$$= 7.5(1 - .021 - j.302)$$
$$= 7.68\angle -17^\circ$$

$$P_L = \left|\frac{V_L}{Z_L}\right|^2 Re(Z_L) = \left(\frac{7.68}{72.1}\right)^2 (60) = 0.681 \text{ W} \checkmark$$

This method computes $P_L = |I_L|^2 R_L$, and applies to lossy as well as lossless lines. Note the concept that $V^+ = V_g/2$ requires a good understanding of the transmission line equations, and only applies here because $Z_g = Z_0$.

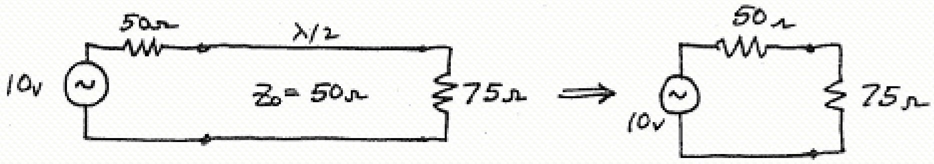
2.17

$$Z_L = jX$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{jX - Z_0}{jX + Z_0}$$

$$|\Gamma|^2 = \Gamma\Gamma^* = \frac{(jX - Z_0)}{(jX + Z_0)}\frac{(-jX - Z_0)}{(-jX + Z_0)} = \frac{X^2 - j\cancel{Z_0 X} + j\cancel{Z_0 X} + Z_0^2}{X^2 + Z_0^2} = 1 \checkmark$$

2.18



POWER DELIVERED BY SOURCE $= \frac{1}{2}\frac{(10)^2}{50+75} = 0.400\ W$ ✓

POWER DISSIPATED IN 50Ω LOAD $= \frac{1}{2}(50)\left(\frac{10}{50+75}\right)^2 = 0.160\ W$ ✓

POWER TRANSMITTED DOWN LINE $= \frac{1}{2}(75)\left(\frac{10}{50+75}\right)^2 = 0.240\ W$ ✓

INCIDENT POWER $= \frac{1}{2}(50)\left(\frac{10}{50+50}\right)^2 = 0.250\ W$ ✓

REFLECTED POWER $= P_{INC}|\Gamma|^2 = .250\left|\frac{75-50}{75+50}\right|^2 = 0.010\ W$ ✓

$P_{INC} - P_{REF} = .250 - .010 = 0.240 = P_{TRANS}$ ✓

$P_{DISS} + P_{TRANS} = .160 + .240 = 0.400 = P_{SOURCE}$ ✓

2.19

$$\Gamma = \frac{-20-j40}{180-j40} = \frac{44.7\angle -116.6^\circ}{184.4\angle -12.5^\circ} = 0.24\angle -104^\circ = -0.058 - j0.233 \checkmark$$

$$V_L = 10\,\frac{80-j40}{180-j40} = 10\,\frac{89.4\angle -26^\circ}{184\angle -12.5^\circ} = 4.86\angle -13.5^\circ$$

$$V(z) = V^+\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right] \qquad V^+ = 10\,\frac{100}{100+100} = 5\,\text{V} \checkmark$$

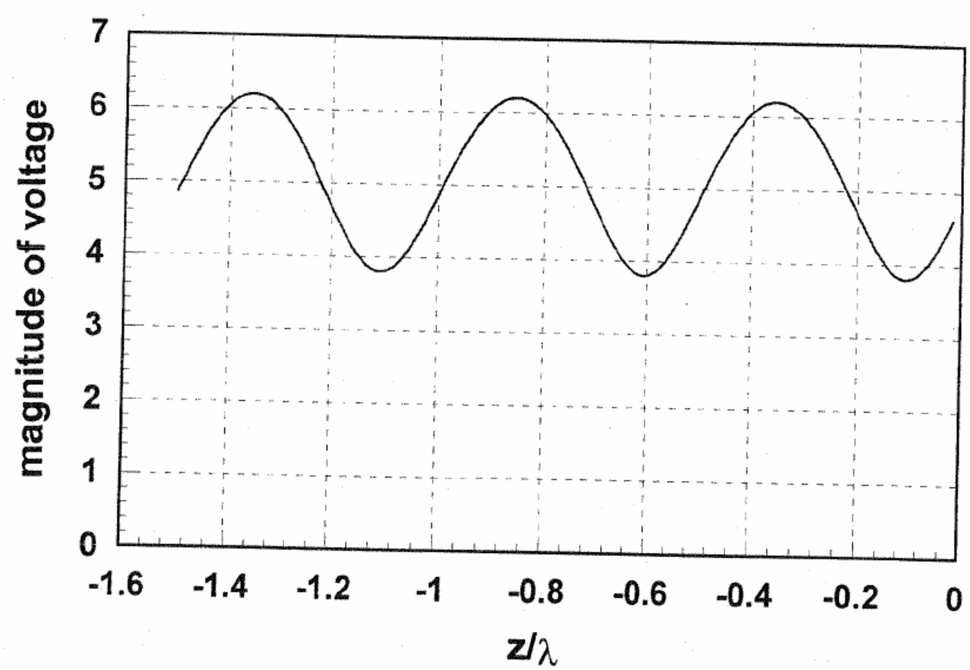
So $\quad V(z) = 5\left[e^{-j\beta z} + \Gamma e^{j\beta z}\right]$

$V_{MAX} = 5(1+|\Gamma|) = 5(1.24) = 6.2$ at $z = -0.355\lambda$

$V_{MIN} = 5(1-|\Gamma|) = 5(.76) = 3.8$ at $z = -0.105\lambda$

These results repeat every $\lambda/2$.

$|V(z)|$ is plotted below:



2.20 $Z_0 = 50\Omega$, $Z_L = 60 + j50\Omega$, $l = 0.4\lambda$

From Smith chart, ($z_L = 1.2 + j1.0$)

a) $SWR = 2.46$ ✓

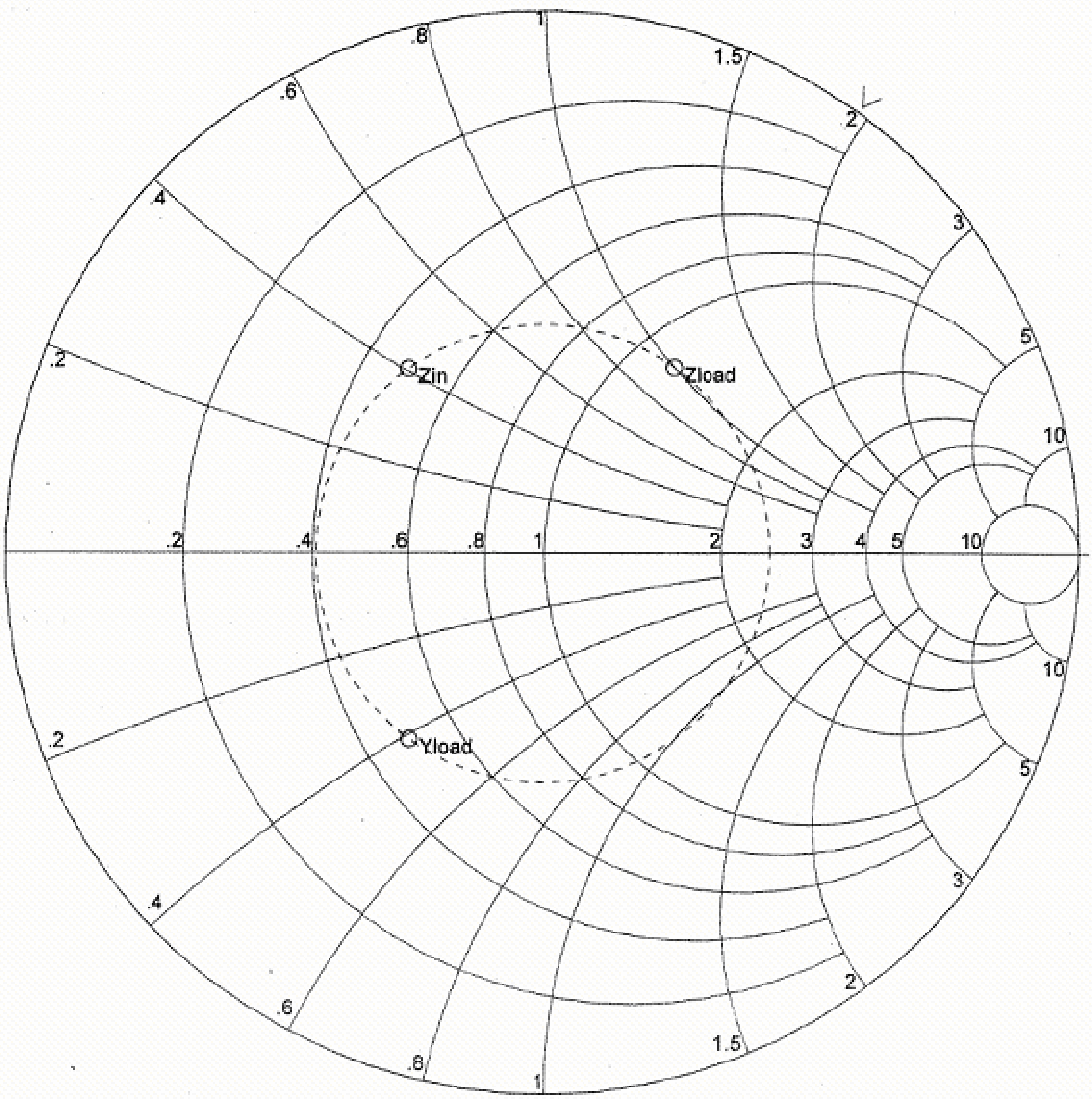
b) $\Gamma = 0.422 \angle 54^\circ$ ✓

c) $Y_L = (.492 - j.410)/50 = 9.84 - j\,8.2$ mS ✓

d) $Z_{in} = 24.5 + j\,20.3\ \Omega$

e) $l_{MIN} = 0.325\lambda$

f) $l_{max} = 0.075\lambda$



2.21

a) $\ell = 0$ or $\ell = 0.5\lambda$ ✓

b) $\ell = 0.25\lambda$ ✓

c) $\ell = 0.125\lambda$ ✓

d) $\ell = 0.406\lambda$ ✓

e) $\ell = 0.021\lambda$ ✓

These results check with $Z_{in} = jZ_0 \tan\beta\ell$.

2.22

a) $\ell = 0.25\lambda$ ✓

b) $\ell = 0\lambda$ or 0.5λ ✓

c) $\ell = 0.375\lambda$ ✓

d) $\ell = 0.656\lambda - 0.5\lambda = 0.156\lambda$ ✓

e) $\ell = 0.271\lambda$ ✓

(add $\lambda/4$ to results of P.2.22)

(also check with $Z_{in} = -jZ_0 \cot\beta\ell$)

2.23

$\lambda = 4.2$ cm. From the Smith chart, $\ell_{MIN} = .9/4.2 = 0.214\lambda$ from the load, So $z_L = 2 - j.9 \Rightarrow \underline{\underline{Z_L = 100 - j45\ \Omega}}$ ✓

Analytically, using (2.58)-(2.60),

$$\Gamma = |\Gamma| e^{j\theta}, \quad |\Gamma| = \frac{2.5-1}{2.5+1} = 0.428$$

$$\theta = \pi + 2\beta\ell_{MIN} = 180 + 2(360)(.214) = -26° \checkmark$$

Then,

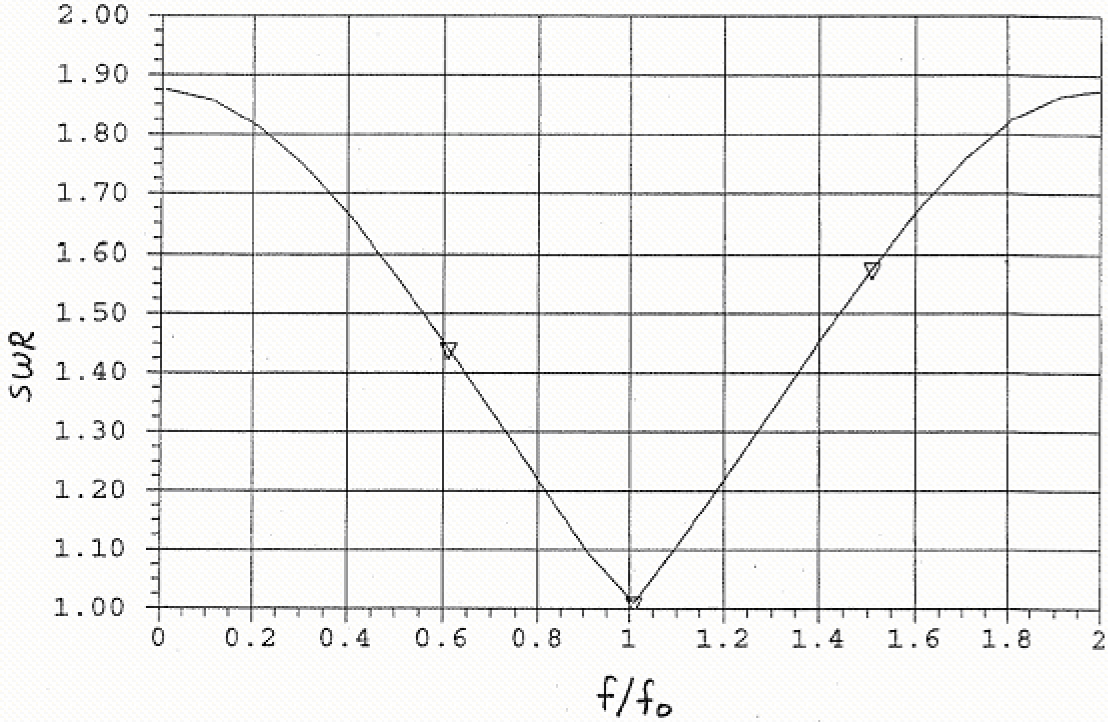
$$Z_L = \frac{1 + .428\angle{-26°}}{1 - .428\angle{-26°}}(50) = 50\frac{1.4\angle{-7.7°}}{.643\angle{17°}} = 109\angle{-25°}$$

$$= \underline{\underline{99 - j46\ \Omega}}$$

2.24

$$Z_t = \sqrt{40(75)} = 54.77\ \Omega$$

The VSWR is plotted vs f/f_0 below:



2.25

On the $\lambda/4$ transformer, the voltage can be expressed as,

$$V(z) = V^+ e^{-j\beta z} + \Gamma V^+ e^{j\beta z} \quad , \quad \Gamma = \frac{R_L - \sqrt{Z_0 R_L}}{R_L + \sqrt{Z_0 R_L}}$$

at $z = -\ell$, $\quad V(-\ell) = V^i = V^+\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]$

$$V^+ = \frac{V^i}{\left[e^{j\beta\ell} + \Gamma e^{-j\beta\ell}\right]} \quad , \quad V^- = \Gamma V^+$$

(assuming V^i with a phase reference at $z = -\ell$.)

2.26 From (2.70),

$$V_0^+ = V_g \frac{Z_{in}}{Z_{in}+Z_g} \frac{1}{(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})}$$

From (2.67),

$$Z_{in} = Z_0 \frac{1+\Gamma_\ell e^{-2j\beta\ell}}{1-\Gamma_\ell e^{-2j\beta\ell}}$$

Then,

$$\frac{Z_{in}}{Z_{in}+Z_g} = \frac{Z_0(1+\Gamma_\ell e^{-2j\beta\ell})}{Z_0(1+\Gamma_\ell e^{-2j\beta\ell})+Z_g(1-\Gamma_\ell e^{-2j\beta\ell})}$$

$$= \frac{Z_0(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})\, e^{-j\beta\ell}}{(Z_0+Z_g)+\Gamma_\ell(Z_0-Z_g)e^{-2j\beta\ell}}$$

$$= \frac{Z_0(e^{j\beta\ell}+\Gamma_\ell e^{-j\beta\ell})\, e^{-j\beta\ell}}{(Z_0+Z_g)\left[1+\Gamma_\ell \frac{Z_0-Z_g}{Z_0+Z_g} e^{-2j\beta\ell}\right]}$$

Thus,

$$V_0^+ = V_g \frac{Z_0 e^{-j\beta\ell}}{(Z_0+Z_g)(1-\Gamma_\ell\Gamma_g e^{-2j\beta\ell})}, \text{ since } \Gamma_g = \frac{Z_g-Z_0}{Z_g+Z_0}.$$

2.27

$$\frac{\partial \alpha_c}{\partial a} = \frac{R_s}{2\eta}\left[\frac{1}{a}\left(\frac{1}{\ln b/a}\right)^2\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{1}{\ln b/a}\left(\frac{-1}{a^2}\right)\right] = 0$$

$$a\left(\frac{1}{a}+\frac{1}{b}\right) = \ln b/a$$

$$(1+b/a) = b/a \ \ln b/a$$

If $x = b/a$, then $1 + x = x \ln x$.

(If $\frac{\partial \alpha_c}{\partial b}$ is taken, the same result is obtained if $x = a/b$)

Now solve this equation for x:

Using interval-halving method:

x	$x \ln x - x - 1$
1	-2.0
2	-1.6
3	-.704
4	.545
3.5	-.115
3.6	.011
3.55	-.052
→ 3.59	-.01

For $x = \frac{b}{a} = 3.59$,

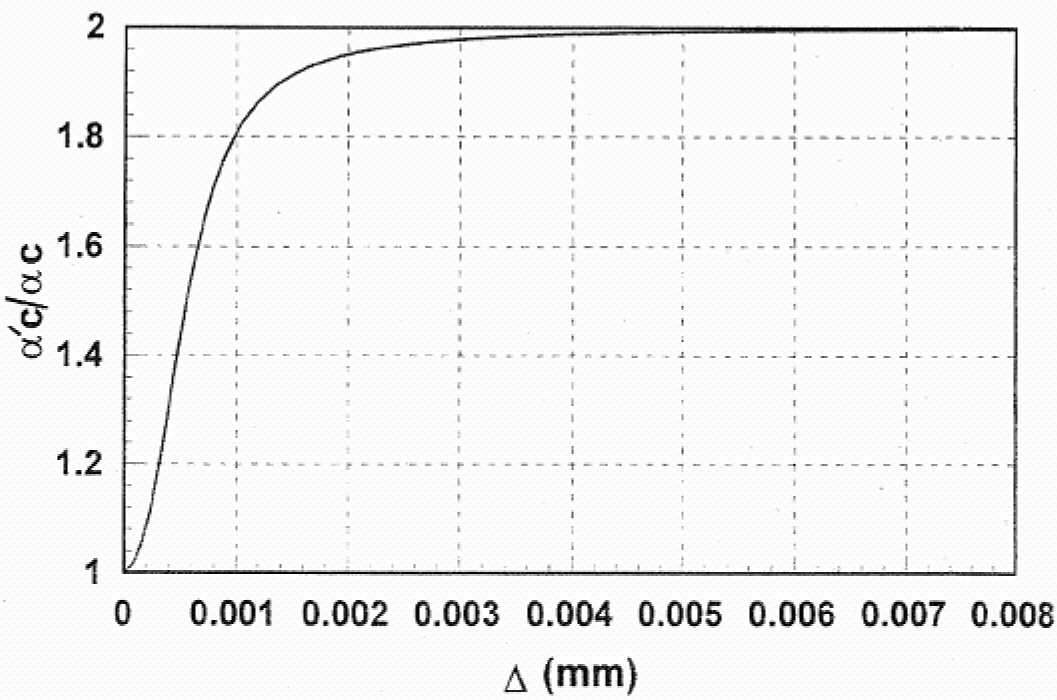
$$Z_o = \frac{\eta}{2\pi}\ln\frac{b}{a} = \frac{377}{\sqrt{\epsilon_r}}\ln(3.59) = \frac{76.7}{\sqrt{\epsilon_r}} \simeq 77\,\Omega \text{ for } \epsilon_r = 1.$$

Thus, for an air dielectric, minimum attenuation occurs for a characteristic impedance near $77\,\Omega$.

2.28 The skin depth of copper at 10GHz is $\delta_s = 6.60 \times 10^{-7}$ m.

Then, compute $\frac{\alpha_c'}{\alpha_c} = 1 + \frac{2}{\pi} \tan^{-1} 1.4 \left(\frac{\Delta}{\delta_s}\right)^2$ (2.107)

The results are plotted below.



2.29 Since the generator is matched to the line,

$V_o^+ = \frac{V_g}{2} e^{-\gamma \ell}$ (phase reference at $z=0$)

$\alpha = 0.5\, dB/\lambda = 0.0576$ neper$/\lambda$

$\gamma\ell = (\alpha + j\beta)\ell = 0.1325 + j\,108°$ ✓

Thus $|V_o^+| = \frac{10}{2} e^{-\alpha\ell} = 4.38\,v.$

$\Gamma = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100-50}{100+50} = 0.333$, $\Gamma(\ell) = \Gamma e^{-2\gamma\ell}$

From (2.92)-(2.94) we then have,

$P_{in} = \frac{|V_o^+|^2}{2Z_o}\left[1 - |\Gamma(\ell)|^2\right] e^{2\alpha\ell} = \frac{(4.38)^2}{100}\left[e^{2(.1325)} - (.333)^2 e^{-2(.1325)}\right]$

$= \underline{0.2337\ w}$ (power delivered to line)

$P_L = \frac{|V_o^+|^2}{2Z_o}\left(1 - |\Gamma|^2\right) = \frac{(4.38)^2}{100}\left[1 - (.333)^2\right] = \underline{0.1706\,w}$ (power to load

$P_{Loss} = P_{in} - P_L = 0.2337 - 0.1706 = \underline{0.0631\,w}$

The input impedance is,

$Z_{in} = Z_o \frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell} = 50\,\frac{100 + 50(.845 + j2.19)}{50 + 100(.845 + j2.19)} = 32.5 - j\,12.4\ \Omega$

The input current is,

$I_{in} = \frac{V_g}{R_g + Z_{in}} = \frac{10}{82.5 - j12.4} = 0.1199\angle 8.5°$ A

The generator power is,

$P_s = \frac{1}{2} V_g |I_{in}| = 5(.1199) = \underline{0.600}\ w$

Power lost in R_g is,

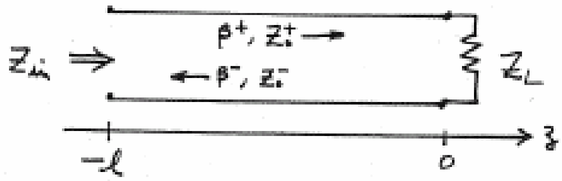
$P_{R_g} = \frac{1}{2}|I_{in}|^2 R_g = \frac{1}{2}(.1199)^2(50) = \underline{0.3594\ w}$

CHECK:

$P_L + P_{Loss} + P_{R_g} = .1706 + .0631 + .3594 = 0.5931\ w \simeq P_s$ ✓

$P_{in} + P_{R_g} = .2337 + .3594 = 0.5931\ w \simeq P_s$ ✓

2.30



$$V(z) = V_0^+ e^{-j\beta^+ z} + V_0^- e^{j\beta^- z}$$

$$I(z) = \frac{V_0^+}{Z_0^+} e^{-j\beta^+ z} - \frac{V_0^-}{Z_0^-} e^{j\beta^- z}$$

at $z=0$ (load), $\quad V(0) = V_0^+ + V_0^-$

$$I(0) = \frac{V_0^+}{Z_0^+} - \frac{V_0^-}{Z_0^-}$$

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_0^+ + V_0^-}{V_0^+/Z_0^+ - V_0^-/Z_0^-} = \frac{1 + V_0^-/V_0^+}{\frac{1}{Z_0^+} - \frac{V_0^-}{V_0^+}\frac{1}{Z_0^-}}$$

as usual, let $\Gamma(0) = V_0^-/V_0^+$. Then,

$$Z_L\left(\frac{1}{Z_0^+} - \Gamma\frac{1}{Z_0^-}\right) = 1 + \Gamma$$

$$\frac{Z_L}{Z_0^+} - 1 = \Gamma\left(1 + \frac{Z_L}{Z_0^-}\right)$$

$$\Gamma = \Gamma(0) = \frac{Z_L - Z_0^-}{Z_L + Z_0^+} \qquad \text{(at load)}$$

The input impedance is,

$$Z_{in} = \frac{V(-\ell)}{I(-\ell)} = \frac{\cancel{V_0^+}\left[e^{j\beta^+\ell} + \Gamma e^{-j\beta^-\ell}\right]}{\cancel{V_0^+}\left[\frac{1}{Z_0^+} e^{j\beta^+\ell} - \Gamma\frac{1}{Z_0^-} e^{-j\beta^-\ell}\right]}$$

$$= \frac{(Z_L + Z_0^+) e^{j\beta^+\ell} + (Z_L - Z_0^-) e^{-j\beta^-\ell}}{\frac{1}{Z_0^+}(Z_L + Z_0^+) e^{j\beta^+\ell} - \frac{1}{Z_0^-}(Z_L - Z_0^-) e^{-j\beta^-\ell}}$$

This result does not simplify much further. From (2.42),

$\Gamma(-\ell) = \Gamma(0)\, e^{-j(\beta^- + \beta^+)\ell} \qquad$ (reflection coefficient at the input)

2.31 The incident wave amplitude is $v_1^+ = 10 \frac{50}{25+50} = 6.67$ v

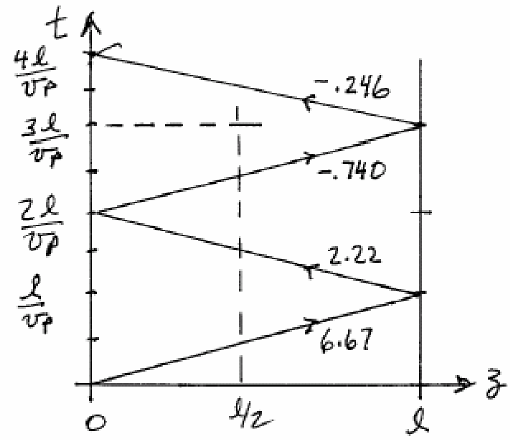
The reflection coefficients are,

$$\Gamma_L = \frac{100-50}{100+50} = 0.333 \quad , \quad \Gamma_g = \frac{25-50}{25+50} = -0.333$$

$v_1^- = \Gamma_L v_1^+ = 2.22$ v

$v_2^+ = v_1^- \Gamma_g = -.740$ v

$v_2^- = v_2^+ \Gamma_L = -.246$ v



at $z = \ell/2$ and $t = 3\ell/v_p$,

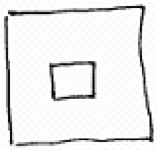
$v_T = 6.67 + 2.22 - 0.740$

$= \underline{8.15 \text{ v}}$

Chapter 3

3.1

<u>Variation on coax:</u>

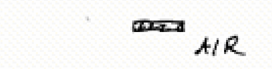
#1. 

Square Coax (actually used in micromachined circuits). Easier to fabricate in micromachined form, using deposition and vias. TEM mode, so dispersion is low. Losses can be low if dielectric loss is small and metalization is good. Higher order modes will exist above cutoff frequency.

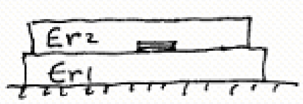
#2. 

coax with two dielectric cores. Will not be strictly TEM, thus slightly dispersive. May also allow control of Z_0 without changing a, b, possibly for use as $\lambda/4$ matching section without discontinuity in diameters. More expensive than standard coax.

<u>Variation on microstrip:</u>

#1 

air-filled microstrip may be supported with foam spacers. No dielectric loss, light weight, pure TEM mode, low dispersion, fewer higher order modes. Lower Cost.

#2 

covered microstrip - provides physical protection of metalization. More expensive, higher dielectric loss, heavier. These drawbacks would be minimized if the top layer was very thin

3.2 Let $k_c^2 = k^2 - \beta^2$

H_x: multiply (3.3a) by $\omega\epsilon$, multiply (3.4b) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial y} - j\beta^2 H_x - \beta \frac{\partial H_z}{\partial x} = -j\omega^2\mu\epsilon H_x$$

$$H_x = \frac{j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial y} - \beta \frac{\partial H_z}{\partial x}\right] \checkmark$$

H_y: multiply (3.3b) by $-\omega\epsilon$, multiply (3.4a) by β, and add:

$$\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y} + j\beta^2 H_y = j\omega^2\mu\epsilon H_y$$

$$H_y = \frac{-j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial x} + \beta \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_x: multiply (3.3b) by $-\beta$, multiply (3.4a) by $\omega\mu$, and add:

$$j\beta^2 E_x + \beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y} = j\omega^2\mu\epsilon E_x$$

$$E_x = \frac{-j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial x} + \omega\mu \frac{\partial H_z}{\partial y}\right] \checkmark$$

E_y: multiply (3.3a) by β, multiply (3.4b) by $\omega\mu$, and add:

$$\beta \frac{\partial E_z}{\partial y} + j\beta^2 E_y - \omega\mu \frac{\partial H_z}{\partial x} = j\omega^2\mu\epsilon E_y$$

$$E_y = \frac{j}{k_c^2}\left[\beta \frac{\partial E_z}{\partial y} - \omega\mu \frac{\partial H_z}{\partial x}\right] \checkmark$$

3.3 From (3.66)–(3.67),

$$H_z = B_n \cos \frac{n\pi y}{d} e^{-j\beta z}$$

$$H_y = \frac{j\beta}{k_c} B_n \sin \frac{n\pi y}{d} e^{-j\beta z}$$

From (3.71),

$$P_o = \frac{\omega\mu d W \beta}{4 k_c^2} |B_n|^2 \quad \text{for } n>0,\ \beta \text{ real}$$

From (2.97), the power lost in both plates is,

$$P_\ell = 2\left(\frac{R_s}{2}\right)\int_s |\bar{H}_t|^2 ds = R_s \int_{z=0}^{1}\int_{x=0}^{w}\left[|H_y(y=0)|^2 + |H_z(y=0)|^2\right] dx\,dz$$

$$= R_s W |B_n|^2$$

Then, $\alpha_c = \dfrac{P_\ell}{2P_o} = \dfrac{2R_s k_c^2}{kd\eta\beta}$. (agrees with (3.72)) ✓

3.4 From Appendix I, $a = 1.07$ cm, $b = 0.43$ cm.

$f_{c_{10}} = \frac{c}{2a} = 14.02$ GHz ✓
$f_{c_{20}} = \frac{c}{a} = 28.04$ GHz ✓ } LOWEST ORDER MODES
$f_{c_{01}} = \frac{c}{2b} = 34.88$ GHz ✓

The fractional BW from $f_{c_{10}}$ to $f_{c_{20}}$ is $\dfrac{(28-14)}{(28+14)/2} = 67\%$

The fractional BW of the recommended operating range of 18.0–26.5 GHz is $\dfrac{(26.5-18.0)}{(26.5+18.0)/2} = 38\%$ (reduction of 29%)

3.5 K-band guide, $\ell = 10$ cm, $\epsilon_r = 2.55$, $\tan\delta = 0.0015$
copper, $f = 15$ GHz. $a = 1.07$ cm, $b = 0.43$ cm, $\sigma = 5.8\times10^7$

$$f_{c_{10}} = \frac{c}{2a\sqrt{\epsilon_r}} = 8.78 \text{ GHz} \checkmark \quad , \quad f_{c_{20}} = 17.6 \text{ GHz} \text{ (one prop. mode)}$$

$$k = \sqrt{\epsilon_r}\,k_0 = 501.67 \text{ m}^{-1} \qquad R_S = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.03195\,\Omega$$

$$\beta_{10} = \sqrt{k^2 - (\pi/a)^2} = 406.78 \text{ m}^{-1} \checkmark \qquad \eta = 377/\sqrt{\epsilon_r} = 236.1\,\Omega$$

From (3.29) $$\alpha_d = \frac{k^2 \tan\delta}{2\beta} = 0.464 \text{ np/m} = 4.03 \text{ dB/m} \checkmark$$

From (3.96) $$\alpha_c = \frac{R_s}{a^3 b \beta k \eta}\left(2b\pi^2 + a^3 k^2\right) = 0.0495 \text{ np/m} = 0.430 \text{ dB/m} \checkmark$$

$$\text{Loss} = (\alpha_c + \alpha_d)\ell = 0.446 \text{ dB}$$
$$\Delta\phi = \beta\ell = 2330.7^\circ$$

3.6 In the section of guide of width $a/2$, the TE_{10} mode is below cutoff (evanescent), with an attenuation constant α:

$$k = \frac{2\pi\,(12{,}000)}{300} = 251.3 \text{ m}^{-1} \checkmark$$

$$\alpha = \sqrt{\left(\frac{\pi}{a/2}\right)^2 - k^2} = \sqrt{\left(\frac{2\pi}{.02286}\right)^2 - (251.3)^2} = 111.3 \text{ nepers/m} \checkmark$$

To obtain 100 dB attenuation (ignoring reflections),

$$-100 \text{ dB} = 20 \log e^{-\alpha\ell}$$
$$10^{-5} = e^{-\alpha\ell}$$
$$\ell = \frac{11.5}{111.3} = 0.103 \text{ m} = \underline{10.3 \text{ cm}} \checkmark$$

3.7 The TE_{10} H-fields from (3.89) are:

$$H_x = \frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta x}$$

$$H_y = 0$$

$$H_z = A \cos\frac{\pi x}{a} e^{-j\beta x}$$

$\bar{J}_s = \hat{n} \times \bar{H}$, so the surface currents are,

ON BOTTOM WALL: $\hat{n} = \hat{y}$; $\bar{J}_s = -\hat{z}\frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} + \hat{x} A \cos\frac{\pi x}{a} e^{-j\beta z}$

ON TOP WALL: $\hat{n} = -\hat{y}$; $\bar{J}_s = \hat{z}\frac{j\beta a A}{\pi} \sin\frac{\pi x}{a} e^{-j\beta z} - \hat{x} A \cos\frac{\pi x}{a} e^{-j\beta z}$

ON LEFT SIDE WALL: $\hat{n} = \hat{x}$, $x = 0$; $\bar{J}_s = -\hat{y} A e^{-j\beta z}$

ON RIGHT SIDE WALL: $\hat{n} = -\hat{x}$, $x = a$; $\bar{J}_s = -\hat{y} A e^{-j\beta z}$

Note that the top and bottom currents are the negative of each other.

Along the centerline of the top or bottom (broad) walls, $x = a/2$, so the surface currents can be reduced to,

$$\bar{J}_s = \pm\hat{z}\frac{j\beta a A}{\pi} e^{-j\beta z},$$

which shows that current flow is only in the longitudinal direction. Thus a narrow longitudinal slot will not break any current lines, and will have a negligible effect on the operation of the waveguide.

3.8 From (3.101),

$$\bar{E} \times \bar{H}^* \cdot \hat{z} = E_x H_y^* - E_y H_x^*$$

$$= \frac{\omega \epsilon \beta m^2 \pi^2}{a^2 k_c^4} |B|^2 \cos^2 \frac{m\pi x}{a} \sin^2 \frac{n\pi y}{b}$$

$$+ \frac{\omega \epsilon \beta n^2 \pi^2}{b^2 k_c^4} |B|^2 \sin^2 \frac{m\pi x}{a} \cos^2 \frac{n\pi y}{b}$$

So the power flow down the guide is,

$$P_o = \frac{1}{2} \int_{x=0}^{a} \int_{y=0}^{b} \bar{E} \times \bar{H}^* \cdot \hat{z} \, dx\,dy = \frac{\omega \epsilon \beta \pi^2 |B|^2}{2 k_c^4} \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \frac{ab}{4} = \frac{\omega \epsilon \beta a b}{8 k_c^2} |B|^2$$

The power loss in the walls is,

$$P_\ell = \frac{R_s}{2} \int_c |\bar{H}_t|^2 ds = R_s \left\{ \int_{x=0}^{a} |H_x(y=0)|^2 dx + \int_{y=0}^{b} |H_y(x=0)|^2 dy \right\}$$

$$= R_s \left\{ \frac{\omega^2 \epsilon^2 n^2 \pi^2}{b^2 k_c^4} |B|^2 \frac{a}{2} + \frac{\omega^2 \epsilon^2 m^2 \pi^2}{a^2 k_c^4} |B|^2 \frac{b}{2} \right\}$$

$$= R_s \frac{\omega^2 \epsilon^2 \pi^2}{2 k_c^4} |B|^2 \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right)$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{R_s \omega^2 \epsilon^2 \pi^2 4 k_c^2}{2 k_c^4 \omega \epsilon \beta a b} \left(\frac{n^2 a}{b^2} + \frac{m^2 b}{a^2} \right)$$

$$= \frac{2 R_s k \pi^2}{k_c^2 \beta \eta} \left(\frac{n^2}{b^3} + \frac{m^2}{a^3} \right) \quad \text{neper/m} \checkmark$$

3.9 From (3.109), the propagation constant is a solution of,

$$k_a \tan k_d t + k_d \tan k_a (a-t) = 0,$$

where

$$\beta = \sqrt{k_0^2 - k_a^2} = \sqrt{\epsilon_r k_0^2 - k_d^2} \qquad (3.106)$$

Since $\beta = 0$ at cutoff, we have that $k_a = k_0$, and $k_d = \sqrt{\epsilon_r}\, k_0$. Thus we must find the root of the following equation:

$$f(k_0) = k_0 \tan \sqrt{\epsilon_r}\, k_0 t + \sqrt{\epsilon_r}\, k_0 \tan k_0 t = 0 \quad (\text{since } t = a/2)$$

We know that $k_c = k_0$ must be between k_c of the empty guide, and k_c for the completely filled guide:

$k_c(\text{EMPTY}) = \frac{\pi}{a} = 137.\ m^{-1}$

$k_c(\text{FILLED}) = \frac{\pi}{\sqrt{\epsilon_r}\, a} = 92.\ m^{-1}$

k_0	$f(k_0)$
95	−1366
100	−362
105	−44
110	171
106	2.3
105.9	−2.25
✓ → 105.95	.017

This result is accurate to at least four figures, and agrees with a result given in the Waveguide Handbook.

The cutoff frequency is,

$$f_c = \frac{k_c c}{2\pi} = 5.06 \text{ GHz}$$

3.10 The lowest order mode will have an H_z component which is even in x, and no variation in y. Thus, h_z can be written as,

$$h_z(x,y) = \begin{cases} A \cos k_d x & \text{for } |x| < W/2 \quad (k_c = k_d) \\ B e^{-k_a |x|} & \text{for } |x| > W/2 \quad (k_c = j k_a) \end{cases}$$

where k_d and k_a are the cutoff wavenumbers in the dielectric and air regions, respectively, satisfying

$$\beta = \sqrt{\epsilon_r k_o^2 - k_d^2} = \sqrt{k_o^2 - k_a^2} \qquad \text{(phase matching)}$$

Next, we need e_y, from (3.19d):

$$e_y(x,y) = \frac{j\omega\mu}{k_c^2} \frac{\partial h_z}{\partial x} = \begin{cases} -j\dfrac{\omega\mu A}{k_d} \sin k_d x & \text{for } |x| < W/2 \\ \dfrac{j\omega\mu B}{k_a} e^{-k_a x} & \text{for } x > W/2 \end{cases}$$

Matching h_z and e_y at $x = W/2$ gives,

$$A \cos k_d W/2 = B e^{-k_a W/2}$$

$$-\frac{A}{k_d} \sin k_d W/2 = \frac{B}{k_a} e^{-k_a W/2}$$

Setting the determinant of these equations to zero gives,

$$k_a \tan k_d W/2 + k_d = 0.$$

A TEM mode cannot exist by itself because of the impossibility of phase matching at $x = W/2$. (For a TEM mode, $\beta = k$ in both regions, which is not possible.)

3.11 Maxwell's curl equations are,

$$\nabla \times \bar{E} = -j\omega\mu\bar{H} \quad , \quad \nabla \times \bar{H} = j\omega\epsilon\bar{E}$$

The ρ and ϕ components in cylindrical form are,

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z} = -j\omega\mu H_\rho \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} = j\omega\epsilon E_\rho$$

$$\frac{\partial E_\rho}{\partial z} - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \qquad \frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi$$

Now assume $\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi)e^{-j\beta z}$

$\bar{H}(\rho,\phi,z) = \bar{h}(\rho,\phi)e^{-j\beta z}$

Then $\partial/\partial z \to -j\beta$, and the above equations reduce to:

$$\frac{1}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta E_\phi = -j\omega\mu H_\rho \quad (1) \qquad \frac{1}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta H_\phi = j\omega\epsilon E_\rho \quad (3)$$

$$-j\beta E_\rho - \frac{\partial E_z}{\partial \rho} = -j\omega\mu H_\phi \quad (2) \qquad -j\beta H_\rho - \frac{\partial H_z}{\partial \rho} = j\omega\epsilon E_\phi \quad (4)$$

Multiply (2) by $-\beta$, multiply (3) by $\omega\mu$, and add:

$$j\beta^2 E_\rho + \beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi} = j\omega^2\mu\epsilon E_\rho$$

$$E_\rho = \frac{-j}{k_c^2}\left[\beta\frac{\partial E_z}{\partial \rho} + \frac{\omega\mu}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

multiply (1) by β, multiply (4) by $\omega\mu$, and add:

$$\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} + j\beta^2 E_\phi - \omega\mu\frac{\partial H_z}{\partial \rho} = j\omega^2\mu\epsilon E_\phi$$

$$E_\phi = \frac{-j}{k_c^2}\left[\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} - \omega\mu\frac{\partial H_z}{\partial \rho}\right]$$

multiply (1) by $\omega\epsilon$, multiply (4) by β, and add:

$$\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - j\beta^2 H_\rho - \beta\frac{\partial H_z}{\partial \rho} = -j\omega^2\mu\epsilon H_\rho$$

$$H_\rho = \frac{j}{k_c^2}\left[\frac{\omega\epsilon}{\rho}\frac{\partial E_z}{\partial \phi} - \beta\frac{\partial H_z}{\partial \rho}\right]$$

$$\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi} + j\beta^2 H_\phi = j\omega^2\mu\epsilon H_\phi$$

$$H_\phi = \frac{-j}{k_c^2}\left[\omega\epsilon \frac{\partial E_z}{\partial \rho} + \frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi}\right]$$

with $k_c^2 = k^2 - \beta^2$.

These results agree with those of (3.110). ✓

3.12 Let $A=1$, $B=0$ in (3.141). Then the transverse fields are,

$$E_\rho = \frac{-j\beta}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$$E_\phi = \frac{-j\beta n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2 \rho} \cos n\phi \, J_n(k_c\rho)\, e^{-j\beta z}$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} \sin n\phi \, J_n'(k_c\rho)\, e^{-j\beta z}$$

$$\bar{E}\times\bar{H}^*\cdot\hat{z} = E_\rho H_\phi^* - E_\phi H_\rho^*.$$

The power flow down the guide is, for $n>0$,

$$P_o = \frac{1}{2}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[\frac{\beta\omega\epsilon}{k_c^2}\sin^2 n\phi\, J_n'^2(k_c\rho) + \frac{\beta\omega\epsilon n^2}{k_c^4\rho^2}\cos^2 n\phi\, J_n^2(k_c\rho)\right]\rho\, d\phi\, d\rho$$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^2}\int_{\rho=0}^{a}\left[J_n'^2(k_c\rho) + \frac{n^2}{k_c^2\rho^2}J_n^2(k_c\rho)\right]\rho\, d\rho \qquad \text{Let } x = k_c\rho,\ dx = k_c d\rho,\ k_c a = p_{nm}$$

$$= \frac{\beta\omega\epsilon\pi}{2k_c^4}\int_{x=0}^{p_{nm}}\left[J_n'^2(x) + \frac{n^2}{x^2}J_n^2(x)\right]x\, dx = \frac{\beta\omega\epsilon\pi}{4k_c^4}p_{nm}^2 J_n'^2(p_{nm}) \quad \text{(SEE C.16)}$$

The power lost in the conducting wall is,

$$P_\ell = \frac{R_s}{2}\int_{z=0}^{1}\int_{\phi=0}^{2\pi}|H_\phi(\rho=a)|^2 a\, d\phi\, dz = \frac{aR_s}{2}\frac{\omega^2\epsilon^2}{k_c^2}J_n'^2(p_{nm})\int_{\phi=0}^{2\pi}\sin^2 n\phi\, d\phi$$

$$= \frac{aR_s\omega^2\epsilon^2\pi}{2k_c^2}J_n'^2(p_{nm})$$

The attenuation is then,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{aR_s\omega^2\epsilon^2\pi\, 4k_c^4}{4k_c^2\beta\omega\epsilon\pi p_{nm}^2} = \frac{aR_s\omega\epsilon k_c^2}{\beta p_{nm}^2} = \frac{kR_s}{\beta\eta a} \text{ neper/m} ✓$$

3.13 $a = 0.4$ cm, $\epsilon_r = 1.50$, copper, $\tan\delta = 0.0002$

TE_{11}: $f_c = \dfrac{p'_{11} c}{2\pi a\sqrt{\epsilon_r}} = 17.94$ GHz ✓

TM_{01}: $f_c = \dfrac{p_{01} c}{2\pi a\sqrt{\epsilon_r}} = 23.44$ GHz ✓

TE_{21}: $f_c = \dfrac{p'_{21} c}{2\pi a\sqrt{\epsilon_r}} = 29.76$ GHz ✓

TE_{01}: $f_c = \dfrac{p'_{01} c}{2\pi a\sqrt{\epsilon_r}} = 37.35$ GHz. ✓

at 20 GHz, $k_0 = 418.88\ m^{-1}$, $\beta = \sqrt{\epsilon_r k_0^2 - (p'_{11}/a)^2} = 226.63\ m^{-1}$ ✓

$$\alpha_d = \frac{\epsilon_r k_0^2 \tan\delta}{2\beta} = 0.116 \text{ np/m} = 1.01 \text{ dB/m} \quad ✓$$

$R_s = 0.0369\,\Omega$

$$\alpha_c = \frac{R_s}{a k_0 \eta_0 \beta}\left(k_c^2 + \frac{k^2}{p'^2_{11} - 1}\right) = 0.083 \text{ np/m} = 0.721 \text{ dB/m} ✓$$

3.14 From (3.153), $\Phi(\rho,\phi) = \dfrac{V_0 \ln b/\rho}{\ln b/a}$

From (3.13) and Appendix,

$$\bar{e}(\rho,\phi) = -\nabla_t \Phi(\rho,\phi) = -\left(+\hat{\rho}\frac{\partial \Phi}{\partial \rho} + \hat{\phi}\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\right) = \frac{V_0 \hat{\rho}}{\rho \ln b/a}$$

Then,

$$\bar{E}(\rho,\phi,z) = \bar{e}(\rho,\phi)e^{-j\beta z} = \frac{V_0 \hat{\rho} e^{-j\beta z}}{\rho \ln b/a} \checkmark \qquad (4.155) \text{ of 1st Ed.}$$

From (3.18),

$$\bar{h}(\rho,\phi) = \frac{1}{\eta}\hat{z}\times\bar{e}(\rho,\phi) = \frac{V_0 \hat{\phi}}{\eta \rho \ln b/a}$$

Then,

$$\bar{H}(\rho,\phi,z) = \frac{V_0 \hat{\phi} e^{-j\beta z}}{\eta \rho \ln b/a} \checkmark \qquad (4.157) \text{ of 1st Ed.}$$

The potential between the two conductors is,

$$V_{ab} = \int_{\rho=a}^{b} E_\rho(\rho,\phi,z)\,d\rho = V_0 e^{-j\beta z} \checkmark \qquad (4.158) \text{ of 1st Ed.}$$

The current on the inner conductor is,

$$I_a = \int_{\phi=0}^{2\pi} H_\phi(a,\phi,z)\,a\,d\phi = \frac{2\pi V_0 e^{-j\beta z}}{\eta \ln b/a} \checkmark \qquad (4.159) \text{ of 1st Ed.}$$

The characteristic impedance is,

$$Z_0 = \frac{V_{ab}}{I_0} = \frac{\eta \ln b/a}{2\pi} \checkmark \qquad (4.162) \text{ of 1st Ed.}$$

3.15 The solution is similar to the TE mode case for the coax, but with e_z in place of h_z:

$$e_z(\rho,\phi) = (A \sin n\phi + B \cos n\phi)\left[C J_n(k_c\rho) + D Y_n(k_c\rho)\right]$$

Then the boundary condition that $e_z = 0$ at $\rho = a$ and at $\rho = b$ yields two equations:

$$C J_n(k_c a) + D Y_n(k_c a) = 0$$
$$C J_n(k_c b) + D Y_n(k_c b) = 0$$

or,

$$J_n(k_c a) Y_n(k_c b) = J_n(k_c b) Y_n(k_c a)$$

For the TM_{01} mode, $n=0$. Let $x = k_c a$. Then for $b = 2a$, we have that $k_c b = 2x$, and so the above equation can be written as,

$$f(x) = J_0(x) Y_0(2x) - J_0(2x) Y_0(x) = 0$$

We know that k_c should be greater than k_c for a circular waveguide of radius b, for which $k_{c_{01}} = p_{01}/b = 2.405/2a$, which implies that $x = 1.2$. So we can begin the root search at $x = 1.2$. Using a table of Bessel functions gives the following results in only a few minutes:

x	$J_0(x)$	$Y_0(x)$	$J_0(2x)$	$Y_0(2x)$	$f(x)$
1.2	.671	.228	.003	.510	.342
1.5	.512	.382	−.260	.377	.292
2.0	.224	.510	−.397	−.017	.198
3.1	−.292	.343	.202	−.248	.003
3.2	−.320	.307	.243	−.200	−.011

Linear interpolation between $x = 3.1$ and 3.2 gives a more accurate value for the root:

$$f(x) \simeq .003 + \frac{.003 - (-.011)}{3.1 - 3.2}(x - 3.1)$$

$$\simeq .437 - .14x = 0$$

$$x = \frac{.437}{.14} = \underline{\underline{3.12 = k_c a}}$$

3.16 From (3.175),

$$\bar{E}\times\bar{H}^*\cdot\hat{z} = -E_y H_x^* = \begin{cases} \dfrac{\omega\mu_0\beta|B|^2}{k_c^2}\sin^2 k_c x & \text{for } 0\le x\le d \\[2ex] \dfrac{\omega\mu_0\beta|B|^2}{h^2}\cos^2 k_c d\, e^{-2h(x-d)} & \text{for } d\le x<\infty \end{cases}$$

The power flow is,

$$P_o = \frac{1}{2}\int_{x=0}^{\infty}\int_{y=0}^{1}\bar{E}\times\bar{H}^*\cdot\hat{z}\,dy\,dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2k_c^2}\int_{x=0}^{d}\sin^2 k_c x\,dx + \frac{\omega\mu_0\beta|B|^2}{2h^2}\cos^2 k_c d\int_{x=d}^{\infty}e^{-2h(x-d)}dx$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{x}{2}-\frac{\sin 2k_c x}{4k_c}\right)\Big|_0^d + \frac{\cos^2 k_c d}{h^2}\left(\frac{e^{-2h(x-d)}}{-2h}\right)\Big|_d^{\infty}\right]$$

$$= \frac{\omega\mu_0\beta|B|^2}{2}\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right) + \frac{\cos^2 k_c d}{2h^3}\right]$$

The power loss is,

$$P_\ell = \frac{R_s}{2}\int_S|\bar{H}_t|^2 ds = \frac{R_s}{2}\int_{y=0}^{1}\int_{z=0}^{1}\left[\overbrace{|H_x(x=0)|^2}^{0} + |H_z(x=0)|^2\right]dz\,dy$$

$$= \frac{R_s}{2}|B|^2$$

So the attenuation is,

$$\alpha_c = \frac{P_\ell}{2P_o} = \frac{2R_s}{4\omega\mu_0\beta\left[\frac{1}{k_c^2}\left(\frac{d}{2}-\frac{\sin 2k_c d}{4k_c}\right)+\frac{\cos^2 k_c d}{2h^3}\right]}$$

$$= \frac{R_s}{k_0\eta_0\beta\left[\frac{d}{k_c^2}-\frac{\sin 2k_c d}{2k_c^3}+\frac{\cos^2 k_c d}{h^3}\right]}$$

3.17 Following the derivation in Section 3.6 for the TM surface waves of a dielectric slab:

$$k_c^2 = \mu_r k_0^2 - \beta^2 \qquad \text{for } 0 \le y \le d$$
$$h^2 = \beta^2 - k_0^2 \qquad \text{for } y \ge d$$

Then,

$$e_z(x,y) = \begin{cases} A \sin k_c y & \text{for } 0 \le y \le d \\ B e^{-hy} & \text{for } y > d \end{cases}$$

This form of e_z is selected to satisfy $e_z = 0$ at $y = 0$, and to have exponential decay for $y \to \infty$ (radiation condition). next, we need H_x ($H_y = E_x = H_z = 0$): From (3.23a),

$$H_x = \frac{j\omega\epsilon_0}{k_c^2}\frac{\partial E_z}{\partial y} = \begin{cases} \frac{j\omega\epsilon_0}{k_c} A \cos k_c y & \text{for } 0 \le y \le d \\ \frac{j\omega\epsilon_0}{h} B e^{-hy} & \text{for } y > d \end{cases}$$

at $y = d$:

E_z continuous $\Rightarrow$ $A \sin k_c d = B e^{-hd}$

H_x continuous $\Rightarrow$ $\frac{A}{k_c} \cos k_c d = \frac{B}{h} e^{-hd}$

or,

$$h \cos k_c d = k_c \sin k_c d$$

$$\underline{h = k_c \tan k_c d} \quad ✓$$

and,

$$\underline{h^2 + k_c^2 = (\mu_r - 1) k_0^2} \quad ✓$$

These two equations must be solved simultaneously to find h and k_c.

3.18 TM_{om} mode. $H_z = 0$ $\quad E_z(\rho,\phi,z) = e_z(\rho,\phi)\, e^{-j\beta z}$

(No TEM mode can be supported by this line because of the impossibility of phase matching at $\rho = b$)

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k_c^2\right) e_z(\rho,\phi) = 0$$

$\partial/\partial\phi = 0$ for $n=0$ modes $\Rightarrow E_\phi = H_\rho = 0$.

Thus,

$$e_z(\rho,\phi) = \begin{cases} A\, J_0(k_d\rho) + B Y_0(k_d\rho) & \text{for } a \le \rho \le b \\ C\, J_0(k_a\rho) + D Y_0(k_a\rho) & \text{for } b \le \rho \le c \end{cases}$$

where $\quad \beta^2 = \epsilon_r k_o^2 - k_d^2 = k_o^2 - k_a^2$.

The boundary conditions are:

$e_z = 0$ at $\rho = a$ and $\rho = c$,

e_z and H_ϕ are continuous at $\rho = b$.

From (3.110d),

$$H_\phi = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial\rho},$$

So we get the following four equations:

$$A\, J_0(k_d a) + B\, Y_0(k_d a) = 0$$
$$C\, J_0(k_a c) + D\, Y_0(k_a c) = 0$$
$$A\, J_0(k_d b) + B Y_0(k_d b) = C J_0(k_a b) + D\, Y_0(k_a b)$$
$$\epsilon_r k_d\left[A\, J_0'(k_d b) + B\, Y_0'(k_d b)\right] = k_a\left[C J_0'(k_a b) + D Y_0'(k_a b)\right]$$

k_a and k_d can be expressed in terms of β, and β can be found so that the determinant of the above system of equations vanishes. This is as far as we can go without actual values for $a, b, c,$ and ϵ_r.

3.19 STRIPLINE: 100Ω, $b = 1.02$ mm, $\epsilon_r = 2.2$, copper, $\tan\delta = 0.001$
$f = 5$ GHz.

$$\lambda_g = \lambda_o/\sqrt{\epsilon_r} = c/\sqrt{\epsilon_r}f = 4.045 \text{ cm}$$

$$\sqrt{\epsilon_r}\, Z_o = 148.3 > 120\,\Omega$$

$$x = \frac{30\pi}{\sqrt{\epsilon_r}\, Z_o} - 0.441 = 0.194$$

$$W/b = .85 - \sqrt{.6 - x} = 0.213 \Rightarrow W = 0.2174 \text{ mm}$$

(PCAAD: $W = 0.218$ mm)

$$\left.\begin{array}{l}\alpha_d = 0.0067 \text{ dB/cm} \\ \alpha_c = 0.044 \text{ dB/cm}\end{array}\right\} \text{PCAAD}$$

3.20 MICROSTRIP: 100Ω, $d = 0.51$ mm, $\epsilon_r = 2.2$, copper
$\tan\delta = 0.001$, $f = 5$ GHz.

First try $W/d < 2$: $A = \frac{Z_o}{60}\sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1}(.23 + .11/\epsilon_r) = 2.213$

$$W/d = \frac{8e^A}{e^{2A} - 2} = .896 \Rightarrow W = 0.457 \text{ mm } \checkmark$$

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2}\frac{1}{\sqrt{1 + 12d/W}} = 1.758 \quad \checkmark$$

$$\lambda_g = c/\sqrt{\epsilon_r}f = 4.525 \text{ cm}$$

$$\left.\begin{array}{l}\alpha_d = 0.0048 \text{ dB/cm} \\ \alpha_c = 0.017 \text{ dB/cm}\end{array}\right\} \text{PCAAD}$$

3.21 $\lambda_0 = 0.12\,m$, $\beta = 2\pi\sqrt{\epsilon_e}/\lambda_0 = 3986.7°/m$ $(f = 2.5\,GHz)$

$C = 5\,pF$: From P2.11, $\beta\ell = 82.74° \Rightarrow \ell = 2.0754\,cm$ $(Z_{in} = -j12.73\,\Omega)$

$L = 5\,nH$: From P2.11, $\beta\ell = 128.1° \Rightarrow \ell = 3.2132\,cm$ $(Z_{in} = +j78.5\,\Omega)$

From SERENADE:

LOSSLESS: C: $Z_{in} = -j12.63\,\Omega$ ✓
L: $Z_{in} = +j78.7\,\Omega$ ✓

LOSSY: C: $Z_{in} = 0.27 - j12.82\,\Omega$
$(t = 0.5\,mil)$ L: $Z_{in} = 0.66 + j76.7\,\Omega$

3.22 $k_o = \frac{2\pi f}{c} = 104.7\ m^{-1}$ ✓ ; $R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{2\pi(5\times10^9)(4\pi\times10^{-7})}{2(5.813\times10^7)}} = 0.018\Omega$ ✓

MICROSTRIP CASE:

First try $W/d > 2$: From (3.197), $B = \frac{377\pi}{2Z_o\sqrt{\epsilon_r}} = 8.0$

$$W/d = \frac{2}{\pi}\left[B-1-\ln(2B-1)+\frac{\epsilon_r-1}{2\epsilon_r}\left\{\ln(B-1)+.39-\frac{.61}{\epsilon_r}\right\}\right] = 3.09 > 2 \checkmark$$

$W = 3.09(.16\ cm) = \underline{0.494\ cm}$

From (3.195), $\epsilon_e = \frac{\epsilon_r+1}{2}+\frac{\epsilon_r-1}{2}\frac{1}{\sqrt{1+12d/W}} = 1.87 \checkmark \Rightarrow \lambda_g = \frac{c}{\sqrt{\epsilon_e}f} = \underline{4.38\ cm}$ ✓

From (3.198),

$$\alpha_d = \frac{k_o\epsilon_r(\epsilon_e-1)}{2\sqrt{\epsilon_e}(\epsilon_r-1)}\tan\delta = \underline{0.061}\ \text{neper/m} \checkmark$$

From (3.199),

$$\alpha_c = \frac{R_s}{Z_oW} = \underline{0.073}\ \text{neper/m} \checkmark$$

Total MS Loss:

$$\text{LOSS} = (.061+.073)\left(\frac{np}{m}\right)(16\lambda_g)\left(.0438\frac{m}{\lambda_g}\right)\left(\frac{8.686\ dB}{neper}\right)$$
$$= 0.82\ dB \checkmark$$

STRIPLINE CASE:

From (3.180), $\sqrt{\epsilon_r}Z_o = \sqrt{2.2}(50) = 74 < 120$. $x = \frac{30\pi}{\sqrt{\epsilon_r}Z_o} - .441 = 0.833$

$W/b = x = 0.833 \Rightarrow W = .833(.32\ cm) = \underline{0.267\ cm}$ ✓

$\lambda_g = \frac{c}{\sqrt{\epsilon_r}f} = \underline{4.045\ cm}$ ✓ $\quad A = 1+\frac{2W}{b-t}+\frac{1}{\pi}\frac{b+t}{b-t}\ln\left(\frac{2b-t}{t}\right) = 4.73$

From (3.181),

$$\alpha_c = \frac{2.7\times10^{-3}R_s\epsilon_rZ_o}{30\pi b}A = \underline{0.084}\ \text{neper/m} \checkmark$$

From (3.30),

$$\alpha_d = \frac{k\tan\delta}{2} = \frac{\sqrt{2.2}(104.7)(.001)}{2} = \underline{0.078}\ \text{neper/m} \checkmark$$

Total S.L. Loss:

$$\text{LOSS} = (.084+.078)\left(\frac{np}{m}\right)(16\lambda_g)\left(.04045\frac{m}{\lambda_g}\right)\left(\frac{8.686\ dB}{neper}\right)$$
$$= \underline{0.91\ dB} \checkmark$$

Thus the microstrip line should be used.

3.23 $H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$; h_z real, β real

From (3.19),

$$H_x = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial x}e^{-j\beta z} \qquad H_y = \frac{-j\beta}{k_c^2}\frac{\partial h_z}{\partial y}e^{-j\beta z}$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial y}e^{-j\beta z} \qquad E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial h_z}{\partial x}e^{-j\beta z}$$

$$\bar{E}\times\bar{H}^* = (E_x H_y^* - E_y H_x^*)\hat{z} - E_x H_z^*\hat{y} + E_y H_z^*\hat{x}$$

$$= \frac{\omega\mu\beta}{k_c^4}\left[\left(\frac{\partial h_z}{\partial y}\right)^2 + \left(\frac{\partial h_z}{\partial x}\right)^2\right]\hat{z} + \frac{j\omega\mu}{k_c^2}\left(\frac{\partial h_z}{\partial y}\hat{y} + \frac{\partial h_z}{\partial x}\hat{x}\right)h_z$$

So if h_z is real (or a real function times a complex constant), there is real power flow only in the z-direction.

3.24 Write the incident, reflected, and transmitted TE_{10} fields as follows:

$$E_y^i = E_o \sin\frac{\pi x}{a}e^{-j\beta_a z} \qquad E_y^r = E_o\Gamma \sin\frac{\pi x}{a}e^{j\beta_a z}$$

$$H_x^i = \frac{-E_o}{Z_a}\sin\frac{\pi x}{a}e^{-j\beta_a z} \qquad H_x^r = \frac{E_o\Gamma}{Z_a}\sin\frac{\pi x}{a}e^{j\beta_a z}$$

$$E_y^t = E_o T \sin\frac{\pi x}{a}e^{-j\beta_d z}$$

$$H_x^t = \frac{-E_o T}{Z_d}\sin\frac{\pi x}{a}e^{-j\beta_d z}$$

where $\beta_a = \sqrt{k_o^2 - (\pi/a)^2}$ $\qquad Z_a = k_o\eta_o/\beta_a$

$\beta_d = \sqrt{\epsilon_r k_o^2 - (\pi/a)^2}$ $\qquad Z_d = k_o\eta_o/\beta_d$

Match fields at $z=0$ to obtain:

$$1 + \Gamma = T \qquad (E_y \text{ continuous})$$

$$\frac{1}{Z_a}(-1+\Gamma) = \frac{-T}{Z_d} \qquad (H_x \text{ continuous})$$

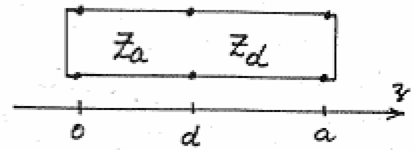
Solving for Γ gives,

$$\Gamma = \frac{Z_d - Z_a}{Z_d + Z_a} \checkmark$$

which agrees with the transmission line theory result if Z_{TE} is used as Z_o in each region. ✓ 50

3.25

$$Z_{TM} = \eta\beta/k = \eta_0\beta/k_0\sqrt{\epsilon_r}$$



for $0 < x < d$, $Z_a = \eta_0 k_{xa}/k_0$

for $d < x < a$, $Z_d = \eta_0 k_{xd}/\sqrt{\epsilon_r}k_0$

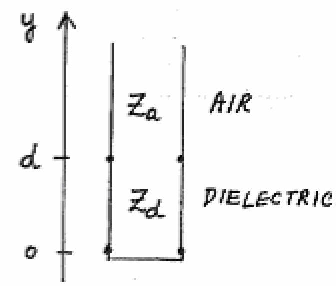
$$\beta = \sqrt{\epsilon_r k_0^2 - k_{xd}^2 - (n\pi/b)^2} = \sqrt{k_0^2 - k_{xa}^2 - (n\pi/b)^2}$$

Applying (3.215):

$$Z_a \tan\beta_a d + Z_d \tan\beta_d (a-b) = 0$$

The m-th root of this equation applies to the TM_{mn} mode.

3.26



$$Z_a = k_{ya}\eta_0/k_0 = -jh\eta_0/k_0$$

$$Z_d = k_{yd}\eta/k = k_{yd}\eta_0/k_0$$

$$\beta = \sqrt{\mu_r k_0^2 - k_{yd}^2} = \sqrt{k_0^2 - k_{ya}^2} = \sqrt{k_0^2 + h^2}$$

Applying (3.215):

$$Z_a + jZ_d \tan k_{yd} d = 0$$

$$h = k_{yd} \tan k_{yd} = 0$$

This agrees with the solution to Problem 3.17, with $k_c = k_{yd}$. ✓

3.27 For X-band guide, $a = 2.286$ cm.

$$k = \frac{2\pi f\sqrt{\epsilon_r}}{c} = \frac{2\pi(9500)\sqrt{2.08}}{300} = 287.\ \text{m}^{-1} \checkmark$$

$$\beta = \sqrt{k^2 - (\pi/a)^2} = 252.\ \text{m}^{-1} \checkmark$$

$$\text{speed of light in Teflon} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{2.08}} = \underline{2.08\times10^8}\ \text{m/sec} \checkmark$$

$$\text{phase velocity} = v_p = \frac{\omega}{\beta} = \frac{2\pi(9.5\times10^9)}{252.} = \underline{2.37\times10^8}\ \text{m/sec} \checkmark$$

From (3.231),

$$\text{group velocity} = v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} = \left(\frac{d\beta}{dk}\frac{dk}{d\omega}\right)^{-1} = \left(\frac{k}{\beta}\sqrt{\mu\epsilon}\right)^{-1}$$

$$= \frac{\beta}{k\sqrt{\mu\epsilon}} = \frac{252(2.08\times10^8)}{287.} = 1.83\times10^8\ \text{m/sec}$$

Note that $v_g < \frac{c}{\sqrt{\epsilon_r}} < v_p$.

3.28

$$P_{MAX} = c\,a^2 \ln\frac{b}{a}$$

$$\frac{dP_{MAX}}{da} = 2a\ln\frac{b}{a} - \frac{a^2}{a} = 0$$

$$2\ln\frac{b}{a} - 1 = 0$$

$$2\ln x = 1$$

$$\ln x = 0.5$$

$$x = 1.65$$

$$Z_0 = \frac{377}{2\pi}\ln\frac{b}{a} = \frac{120\pi}{2\pi}\left(\frac{1}{2}\right) = \underline{30\,\Omega}$$

3.29 alumina, $\epsilon_r = 9.9$, $d = 2.0$ mm, $W = 1.93$ mm

$Z_0 = 50\Omega$. $\epsilon_e = 6.771$

$$f_{T1} = \frac{c}{2\pi d}\sqrt{\frac{2}{\epsilon_r - 1}}\tan^{-1}\epsilon_r = 11.3 \text{ GHz}$$

$$f_{T2} = \frac{c}{4d\sqrt{\epsilon_r - 1}} = 12.5 \text{ GHz}.$$

$$f_{T3} = \frac{c}{\sqrt{\epsilon_r}(2W + d)} = 16.3 \text{ GHz}$$

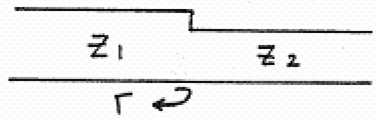
$$f_{T4} = \frac{c}{2d\sqrt{\epsilon_r}} = 23.8 \text{ GHz}.$$

It would be advisable to keep the operating frequency below 10 GHz for this line.

Chapter 4

4.1 Using a transmission line analogy gives,

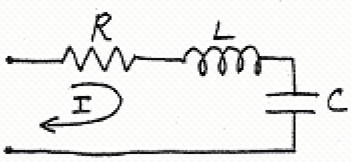
$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$



where $Z_1 = k_0\eta_0/\beta_1$, $Z_2 = k_0\eta_0/\beta_2$.

But $\beta_1 = \beta_2 = \sqrt{k_0^2 - (\pi/a)^2}$ in both regions, since only the height (b) of the guide changes. Thus, $\Gamma = 0$ from above. This is obviously not correct, as E_y should be zero for $b/2 < y < b$. Higher order TE_{1n} modes must be considered, in a mode matching procedure. This will result in a solution where $\Gamma \neq 0$. Consideration of only the dominant mode is not adequate.

4.2



$$P_\ell = \tfrac{1}{2}|I|^2 R \implies R = \frac{P_\ell}{\frac{1}{2}|I|^2}$$

$$W_m = \tfrac{1}{4}L|I|^2 \implies L = \frac{2W_m}{\frac{1}{2}|I|^2}$$

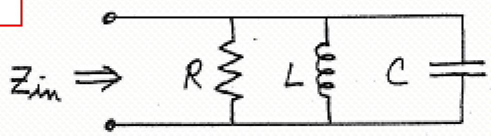
$$W_e = \tfrac{1}{4}C|V_c|^2 = \frac{1}{4\omega^2 C}|I|^2 \implies \frac{1}{\omega^2 C} = \frac{2W_e}{\frac{1}{2}|I|^2}$$

The input impedance is,

$$Z_{in} = R + j\left(\omega L + \frac{1}{\omega C}\right) = \frac{P_\ell + 2j\omega(W_m - W_e)}{\frac{1}{2}|I|^2} \quad \checkmark$$

In agreement with (4.17)

4.3



$$Z = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C} = \frac{1}{\frac{1}{R} + j\omega\left(C - \frac{1}{\omega^2 L}\right)}$$

$$Z(-\omega) = \frac{1}{\frac{1}{R} - j\omega\left(C - \frac{1}{\omega^2 L}\right)} = Z^*(\omega) \quad \checkmark$$

4.4

$$V_1 = 10\angle 90^\circ \qquad I_1 = 0.2\angle 90^\circ \qquad Z_0 = 50\,\Omega$$

$$V_2 = 8\angle 0 \qquad I_2 = 0.16\angle -90^\circ$$

$$V_n^+ = (V_n + Z_0 I_n)/2$$

$$V_n^- = (V_n - Z_0 I_n)/2$$

$$V_1^+ = \tfrac{1}{2}\left[10j + 50(.2j)\right] = 10\angle 90^\circ$$

$$V_1^- = \tfrac{1}{2}\left[10j - 50(.2j)\right] = 0$$

$$V_2^+ = \tfrac{1}{2}\left[8 + 50(-.16j)\right] = 4 - 4j = 5.66\angle -45^\circ$$

$$V_2^- = \tfrac{1}{2}\left[8 - 50(-.16j)\right] = 4 + 4j = 5.66\angle +45^\circ$$

$$Z_{in}^{(1)} = \frac{V_1}{I_1} = \frac{10j}{.2j} = 50\,\Omega$$

$$Z_{in}^{(2)} = \frac{V_2}{I_2} = \frac{8}{-.16j} = 50j = 50\angle 90^\circ\,\Omega$$

4.5

$$P_{in} = \frac{1}{2}[V]^t[I]^* = \frac{1}{2}[V]^t[Y]^*[V]^*$$

$$= \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N} V_m Y_{mn}^* V_n^*$$

If lossless, $Re\{P_{in}\}=0$. Since the V_n's are independent, we first let all $V_m=0$, except for V_n. Then,

$$P_{in} = \frac{1}{2} V_n Y_{nn}^* V_n^* = \frac{1}{2}|V_n|^2 Y_{nn}^*$$

$$\therefore \quad Re\{Y_{nn}^*\} = Re\{Y_{nn}\} = 0 \quad \checkmark$$

Now let all port voltages be zero except for V_m and V_n. Then,

$$P_{in} = \frac{1}{2} V_m Y_{mn}^* V_n^* + \frac{1}{2} V_n Y_{nm}^* V_m^*$$

So,

$$Re\{V_m Y_{mn}^* V_n^* + V_n Y_{nm}^* V_m^*\} = 0$$

If $Y_{mn}=Y_{nm}$ (reciprocal), then

$$Re\{Y_{mn}^*(V_m V_n^* + V_n V_m^*)\} = Re\{Y_{mn}^*[(V_m V_n^* + (V_m V_n^*)^*]\} = 0$$

Since $A+A^*$ is real, we must have $Re\{Y_{mn}\}=0$ ✓

4.6 Let $[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$, and show that Z_{ij}'s can be found such that $P_{in}=0$, but not all Z_{ij}'s are pure imaginary.

$$P_{in} = \frac{1}{2}[I]^t[Z]^t[I]^* = \frac{1}{2}(I_1 Z_{11} I_1^* + I_1 Z_{21} I_2^* + I_2 Z_{12} I_1^* + I_2 Z_{22} I_2^*)$$

$$= \frac{1}{2}(Z_{11}|I_1|^2 + Z_{22}|I_2|^2 + Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*)$$

To be lossless, we must have $Re\{Z_{11}\} = Re\{Z_{21}\} = 0$.

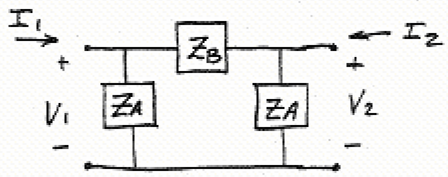
also, $Re\{Z_{21} I_1 I_2^* + Z_{12} I_2 I_1^*\} = 0$.

This will occur if $Z_{12} = -Z_{21}^*$ (since $Re\{A-A^*\}=0$).

For example, if $Z_{12} = a+jb$, then $Z_{21} = -a+jb$.

Thus, $[Z]$ is not symmetric, and the answer is NO.

4.7



From (4.28),

$$Z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0} = \frac{V_1}{V_1\left(\frac{2Z_A+Z_B}{Z_A(Z_A+Z_B)}\right)} = \frac{Z_A(Z_A+Z_B)}{2Z_A+Z_B} \checkmark = Z_{22} \quad \text{(BY SYMMETRY)}$$

$$Z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0} = \frac{I_1 Z_{11}\left(\frac{Z_A}{Z_A+Z_B}\right)}{I_1} = \frac{Z_A^2}{2Z_A+Z_B} \checkmark = Z_{12} \quad \text{(BY RECIPROCITY)}$$

From (4.29),

$$Y_{11} = \frac{I_1}{V_1}\bigg|_{V_2=0} = \frac{I_1}{I_1\left(\frac{Z_A Z_B}{Z_A+Z_B}\right)} = \frac{Z_A+Z_B}{Z_A Z_B} \checkmark = Y_{22} \quad \text{(BY SYMMETRY)}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0} = \frac{-V_1/Z_B}{V_1} = \frac{-1}{Z_B} \checkmark = Y_{12} \quad \text{(BY RECIPROCITY)}$$

CHECK: $[Z][Y]=[U]$?

$$Z_{11}Y_{11} + Z_{12}Y_{21} = \frac{(Z_A+Z_B)^2}{Z_B(2Z_A+Z_B)} - \frac{Z_A^2}{Z_B(2Z_A+Z_B)} = \frac{2Z_A Z_B + Z_B^2}{Z_B(2Z_A+Z_B)} = 1 \checkmark$$

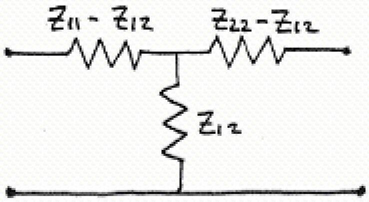
$$Z_{11}Y_{12} + Z_{12}Y_{22} = \frac{-Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} + \frac{Z_A(Z_A+Z_B)}{Z_B(2Z_A+Z_B)} = 0 \checkmark$$

Similarly for the T-network. The results are,

$$Z_{11} = Z_{22} = \frac{Y_A+Y_B}{Y_A Y_B} \checkmark \qquad Z_{12} = Z_{21} = \frac{-1}{Y_B} \checkmark$$

$$Y_{11} = Y_{22} = \frac{Y_A(Y_A+Y_B)}{2Y_A+Y_B} \checkmark \qquad Y_{12} = Y_{21} = \frac{Y_A^2}{2Y_A+Y_B} \checkmark$$

4.8 Model the two-port as below:



Then,
$$Z_{sc}^{(1)} = Z_{11} - Z_{12} + \frac{Z_{12}(Z_{22} - Z_{12})}{Z_{22}} = Z_{11} - Z_{12}^2/Z_{22}$$

$$Z_{sc}^{(2)} = Z_{22} - Z_{12}^2/Z_{11}$$

$$Z_{oc}^{(1)} = Z_{11} \quad \checkmark$$

$$Z_{oc}^{(2)} = Z_{22} \quad \checkmark$$

From the first equation,
$$Z_{12}^2 = -(Z_{sc}^{(1)} - Z_{11}) Z_{22} = (Z_{oc}^{(1)} - Z_{sc}^{(1)}) Z_{oc}^{(2)}.$$

4.9 From Table 4.1 the ABCD parameters for a transmission line section are,

$A = D = \cos\beta\ell$, $B = jZ_0 \sin\beta\ell$, $C = jY_0 \sin\beta\ell$.

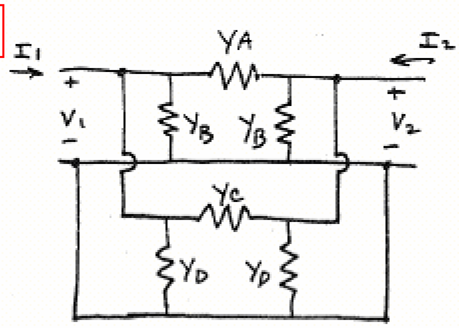
Now use Table 4.2 to convert to Z-parameters:

$$Z_{11} = \frac{A}{C} = \frac{\cos\beta\ell}{jY_0 \sin\beta\ell} = -jZ_0 \cot\beta\ell \quad \checkmark$$

$$Z_{12} = Z_{21} = \frac{1}{C} = -jZ_0 \csc\beta\ell \quad \checkmark$$

$$Z_{22} = \frac{D}{C} = \frac{\cos\beta\ell}{jY_0 \sin\beta\ell} = -jZ_0 \cot\beta\ell \quad \checkmark$$

4.10



$$[Y_1] = \begin{bmatrix} Y_A + Y_B & -Y_A \\ -Y_A & Y_A + Y_B \end{bmatrix}$$

$$[Y_2] = \begin{bmatrix} Y_C + Y_D & -Y_C \\ -Y_C & Y_C + Y_D \end{bmatrix}$$

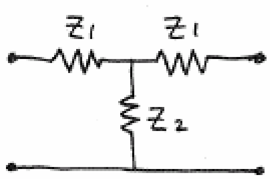
adding [Y] matrices gives:

$$[Y] = [Y_1] + [Y_2] = \begin{bmatrix} Y_A + Y_B + Y_C + Y_D & -Y_A - Y_C \\ -Y_A - Y_C & Y_A + Y_B + Y_C + Y_D \end{bmatrix}$$

By direct calculation, we obtain similar results:

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = Y_A + Y_B + Y_C + Y_D \checkmark \qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -(Y_A + Y_C) \checkmark$$

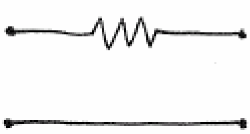
Now apply to bridged-T network (Example 5.7 of 1st edition)



$$[Z_A] = \begin{bmatrix} Z_1 + Z_2 & Z_2 \\ Z_2 & Z_1 + Z_2 \end{bmatrix}$$

$$[Y_A] = \frac{1}{D} \begin{bmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_1 + Z_2 \end{bmatrix} \checkmark$$

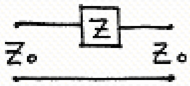
$$D = (Z_1 + Z_2)^2 - Z_2^2 = Z_1^2 + 2 Z_1 Z_2 \checkmark$$



$$[Y_B] = \begin{bmatrix} 1/Z_3 & -1/Z_3 \\ -1/Z_3 & 1/Z_3 \end{bmatrix} \checkmark$$

$$[Y_{TOT}] = [Y_A] + [Y_B] = \begin{bmatrix} \frac{1}{Z_3} + \frac{Z_1 + Z_2}{D} & -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) \\ -\left(\frac{1}{Z_3} + \frac{Z_2}{D}\right) & \frac{1}{Z_3} + \frac{Z_1 + Z_2}{D} \end{bmatrix} \checkmark$$

4.11

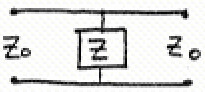


From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

convert to [S] using Table 4.2:

$$S_{11} = \frac{1 + Z/Z_0 - 1}{1 + Z/Z_0 + 1} = \frac{Z}{2Z_0 + Z} \quad ; \quad S_{12} = \frac{2}{1 + Z/Z_0 + 1} = \frac{2Z_0}{2Z_0 + Z}$$

$$1 - S_{11} = \frac{2Z_0}{2Z_0 + Z} = S_{12} \checkmark$$



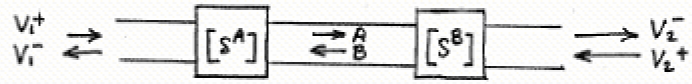
From Table 4.1, $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$

convert to [S]:

$$S_{11} = \frac{1 - Z_0/Z - 1}{1 + Z_0/Z + 1} = \frac{-Z_0}{2Z + Z_0} \quad ; \quad S_{12} = \frac{2}{1 + Z_0/Z + 1} = \frac{2Z}{2Z + Z_0}$$

$$1 + S_{11} = \frac{2Z}{2Z + Z_0} = S_{12} \checkmark$$

4.12 Define wave amplitudes as shown:



Then,

$$\begin{bmatrix} V_1^- \\ A \end{bmatrix} = [S^A] \begin{bmatrix} V_1^+ \\ B \end{bmatrix} \qquad \begin{bmatrix} B \\ V_2^- \end{bmatrix} = [S^B] \begin{bmatrix} A \\ V_2^+ \end{bmatrix} \qquad \begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = [S] \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$S_{21} = \left.\dfrac{V_2^-}{V_1^+}\right|_{V_2^+=0}$. When $V_2^+=0$, we have $B = S_{11}^B A$, $V_2^- = S_{21}^B A$.

Then,

$$A = S_{21}^A V_1^+ + S_{22}^A B = S_{21}^A V_1^+ + S_{22}^A S_{11}^B A$$

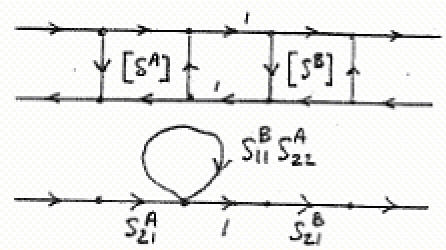
$$\frac{V_2^-}{S_{21}^B} = S_{21}^A V_1^+ + S_{22}^A S_{11}^B \frac{V_2^-}{S_{21}^B}$$

$$V_2^- \left(\frac{1 - S_{22}^A S_{11}^B}{S_{21}^B} \right) = S_{21}^A V_1^+$$

So,

$$S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{22}^A S_{11}^B} \checkmark$$

SIGNAL FLOWGRAPH SOLUTION:



$$\therefore \; S_{21} = \frac{S_{21}^A S_{21}^B}{1 - S_{11}^B S_{22}^A} \checkmark$$

4.13

a) $[S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{21} & S_{22} \end{bmatrix}$ $S_{12} = S_{21}$ since reciprocal

[S] is unitary if lossless, so

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$ (or 1st col.)

$|S_{21}|^2 = 1 - |S_{11}|^2$ ✓

b) $[S] = \begin{bmatrix} S_{11} & S_{21} \\ 0 & S_{22} \end{bmatrix}$ $S_{12} \neq S_{21}$ since nonreciprocal

1st Row: $|S_{11}|^2 + |S_{21}|^2 = 1$

1st Col.: $|S_{11}|^2 = 1$

$\therefore |S_{21}| = 0$

4.14

a) to be lossless, [S] must be unitary. From 1ST row:

$$|S_{11}|^2+|S_{12}|^2+|S_{13}|^2+|S_{14}|^2=(.178)^2+(.6)^2+(.4)^2=0.552\neq 1$$

so the network is not lossless.

b) The [S] matrix is symmetric, so it is reciprocal.

c) When ports 2, 3, 4 are matched, $\Gamma = S_{11}$.

so $RL = -20\log|\Gamma| = -20\log(.178) = 15.0\text{ dB}$

d) For ports 1 and 3 terminated with Z_0, we have

$V_1^+=0,\ V_3^+=0$, so $V_4^- = S_{42}V_2^+$

$$IL = -20\log|S_{42}| = -20\log(.3) = 10.5\text{ dB}$$

$$\text{phase delay} = +45^\circ$$

e) For a short at port 3, Z_0 on other ports, we have

$$V_2^+ = V_4^+ = 0$$

$$V_3^+ = -V_3^-$$

$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+ = S_{11}V_1^+ - S_{13}V_3^-$$

$$V_3^- = S_{31}V_1^+$$

Then,

$$\Gamma^{(1)} = \frac{V_1^-}{V_1^+} = S_{11} - S_{13}S_{31} = 0.178j - (.4\angle 45)(.4\angle 45)$$

$$= 0.178j - .16j = 0.018j = 0.018\angle 90^\circ$$

4.15 A matched, reciprocal, 3-port network has an [S] matrix of the following form:

$$[S] = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

If the network is lossless, then [S] must be unitary:

$$|S_{12}|^2 + |S_{13}|^2 = 1 \quad (1) \qquad S_{13}S_{23}^* = 0 \quad (4)$$

$$|S_{12}|^2 + |S_{23}|^2 = 1 \quad (2) \qquad S_{12}S_{13}^* = 0 \quad (5)$$

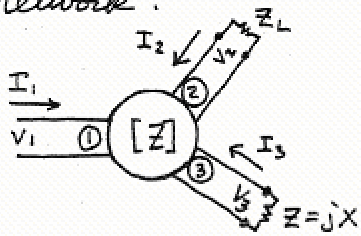
$$|S_{13}|^2 + |S_{23}|^2 = 1 \quad (3) \qquad S_{12}S_{23}^* = 0 \quad (6)$$

To show that a contradiction exists, assume that $S_{12}=0$, in order to satisfy (5) and (6). Then from (1), $|S_{13}|^2=1$, and from (3), $|S_{23}|=0$. But then (2) will be contradicted. Similarly, a contradiction will follow if we let $S_{13}=0$, or $S_{23}=0$.

A circulator is an example of a nonreciprocal, lossless, matched 3-port network.

4.16 For this problem it is easiest to use the z-matrix for a lossless reciprocal 3-port network:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} jX_{11} & jX_{12} & jX_{13} \\ jX_{12} & jX_{22} & jX_{23} \\ jX_{13} & jX_{23} & jX_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



If we terminate port 3 in a reactance jX,

then $V_3 = -jX I_3$. Then we must find jX so that $V_2 = 0$ for $V_1 \neq 0$. If $V_2 = 0$, then $I_2 = 0$:

$$V_3 = jX_{13} I_1 + jX_{33} I_3 = -jX I_3$$

$$I_3 = \frac{-X_{13} I_1}{X_{33} + X}$$

$$V_2 = jX_{12} I_1 + jX_{23} I_3 = \left(jX_{12} - \frac{jX_{23}X_{13}}{X_{33}+X}\right) I_1 = 0$$

So,
$$X_{12}X_{33} + XX_{12} - X_{13}X_{23} = 0$$

$$X = \frac{X_{13}X_{23} - X_{12}X_{33}}{X_{12}} \checkmark$$

CHECK: The input impedance at Port 1 is,

$$Z_{in}^{(1)} = \frac{V_1}{I_1} = \frac{jX_{11}I_1 + jX_{13}I_3}{I_1} = jX_{11} + jX_{13}\left(\frac{-X_{13}}{X_{33}+X}\right)$$

$$= j\left(X_{11} - \frac{X_{13}^2}{X_{33}+X}\right)$$ which is pure imaginary ✓

4.17

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{11} & S_{13} \\ S_{13} & S_{13} & S_{33} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ V_3^+ \end{bmatrix}$$

Assume the network is fed at port 1, so $V_1^+ \neq 0$. Port 2 is terminated in a matched load, so $V_2^+ = 0$. Port 3 is terminated in a reactive load, so $V_3^+ = e^{j\phi} V_3^-$. We must find $e^{j\phi}$ so that $V_1^-/V_1^+ = 0$.

$$V_3^- = S_{13} V_1^+ + S_{33} V_3^+ = e^{-j\phi} V_3^+$$

$$V_3^+ = \frac{S_{13} V_1^+}{e^{-j\phi} - S_{33}}$$

$$V_1^- = S_{11} V_1^+ + S_{13} V_3^+ = S_{11} V_1^+ + \frac{S_{13}^2 V_1^+}{e^{-j\phi} - S_{33}}$$

$$\frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{13}^2}{e^{-j\phi} - S_{33}} = 0 \Rightarrow e^{-j\phi} = S_{33} - \frac{S_{13}^2}{S_{11}}. \checkmark$$

We should also verify that this quantity has unit magnitude:

$$|S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 - |S_{13}|^4 - S_{11}^* S_{33}^* S_{13}^2 - S_{11} S_{33} S_{13}^{*2}}{|S_{11}|^2}$$

The unitary properties of [S] lead to four equations:

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1 \qquad S_{11} S_{12}^* + S_{12} S_{11}^* + |S_{13}|^2 = 0$$

$$2|S_{13}|^2 + |S_{33}|^2 = 1 \qquad S_{12} S_{13}^* + S_{11} S_{13}^* + S_{13} S_{33}^* = 0$$

Eliminating S_{12} from the two equations on the right yields,

$$-2|S_{11}|^2 - \frac{S_{11} S_{13}^* S_{33}}{S_{13}} - \frac{S_{11}^* S_{13} S_{33}^*}{S_{13}^*} + |S_{13}|^2 = 0$$

or,

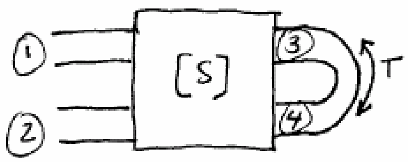
$$-S_{11} S_{13}^{*2} S_{33} - S_{11}^* S_{13}^2 S_{33}^* = 2|S_{11}|^2 |S_{13}|^2 - |S_{13}|^4$$

Then,

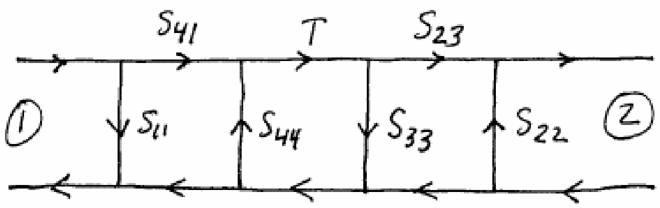
$$|S_{33} - S_{13}^2/S_{11}|^2 = \frac{|S_{11}|^2 |S_{33}|^2 + \cancel{|S_{13}|^4} + 2|S_{11}|^2 |S_{13}|^2 - \cancel{|S_{13}|^4}}{|S_{11}|^2}$$

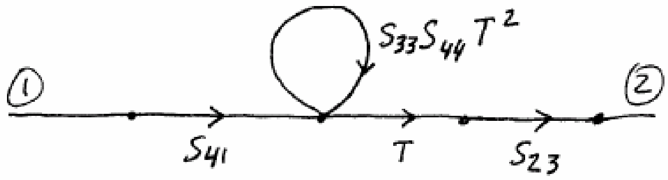
$$= |S_{33}|^2 + 2|S_{13}|^2 = 1 \checkmark$$

4.18 signal flow graph solution:



$T = 1\angle{-45^\circ}$





$$T_{21} = \frac{S_{41}S_{23}}{1 - S_{33}S_{44}T^2} = \frac{(.4\angle{45})(.7\angle{-45})}{1 - (.6\angle{45})(.5\angle{45})(1\angle{-90})} = 0.4\angle{-90^\circ}$$

$IL = -20\log(.4) = 7.96\ dB$

delay $= +90^\circ$.

4.19 From (4.62),

$$S'_{ij} = \frac{\sqrt{Z_{0j}}}{\sqrt{Z_{0i}}} S_{ij}$$

So,

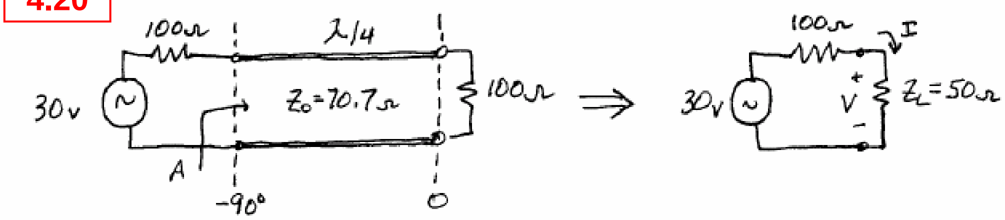
$S'_{11} = S_{11}$ ✓

$S'_{12} = \sqrt{\frac{Z_{02}}{Z_{01}}}\, S_{12}$ ✓

$S'_{21} = \sqrt{\frac{Z_{01}}{Z_{02}}}\, S_{21}$ ✓

$S'_{22} = S_{22}$ ✓

4.20



Z_{in} (at A) $= (70.7)^2/100 = 50\Omega$

with reference at A : Let $Z_R = Z_L^* = 50\Omega$. Then $\Gamma_p = 0$ (not conj. mat.)

$$V = 30\,\frac{50}{150} = 10\text{ v}$$

$$I = \frac{30}{150} = 0.2\text{ A}$$

$$a = \frac{1}{2\sqrt{R_R}}\left(V + Z_R I\right) = \frac{1}{2\sqrt{50}}(10+10) = 1.414$$

$$P_L = \tfrac{1}{2}|a|^2 = 1\text{ W} \quad \checkmark$$

with reference at B : Let $Z_R = Z_L^* = 100\Omega$. Then $\Gamma_p = 0$

$$\Gamma = \frac{100 - Z_0}{100 + Z_0} = 0.1716$$

$$V(z) = V_0^+\left(e^{-j\beta z} + \Gamma e^{+j\beta z}\right)$$

$$V(-90°) = 10 = V_0^+(j - .1716j)$$

$$V_0^+ = -j\,12.07$$

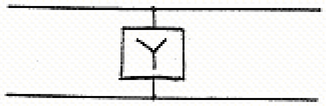
$$V(0) = V_0^+(1+\Gamma) = -j\,14.14$$

$$I(0) = -j\,.1414$$

$$a = \frac{1}{2\sqrt{100}}\left(-j14.14 - j14.14\right) = -j\,1.414$$

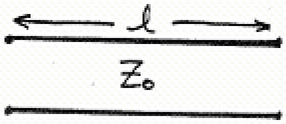
$$P_L = \tfrac{1}{2}|a|^2 = 1\text{ W} \quad \checkmark$$

4.21



$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1$ ✓ $\quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = Y$ ✓

$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0$ ✓ $\quad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$ ✓



for $I_2=0$, $\quad V_1 = V^+(e^{j\beta\ell}+e^{-j\beta\ell}) = V^+ 2\cos\beta\ell$

$V_2 = 2V^+ = V_1/\cos\beta\ell$

So,

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \cos\beta\ell \ ✓$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{V_1}{Z_{in}V_2} = \frac{\cos\beta\ell}{-jZ_0\cot\beta\ell} = jY_0\sin\beta\ell \ ✓$$

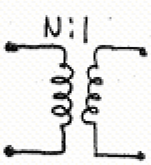
for $V_2=0$, $\quad V_1 = V^+(e^{j\beta\ell}-e^{-j\beta\ell}) = V^+ 2j\sin\beta\ell$

$I_2 = \frac{2V^+}{Z_0}$

So,

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = jZ_0\sin\beta\ell \ ✓$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{V_1}{Z_{in}I_2} = \frac{B}{Z_{in}} = \frac{jZ_0\sin\beta\ell}{jZ_0\tan\beta\ell} = \cos\beta\ell \ ✓$$



$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{NV_2}{V_2} = N \ ✓$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = 0 \ ✓$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \ ✓$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{1}{N} \ ✓$$

4.22

NOTE: Difference in signs for Z and ABCD.

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{V_2}\frac{V_2}{I_1}\right|_{I_2=0} = A/C \checkmark$$

for $I_1 = 0$,

$$V_1 = AV_2 - BI_2$$

$$0 = CV_2 - DI_2 \Rightarrow V_2 = DI_2/C$$

$$V_1 = \left(\frac{AD}{C} - B\right) I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = \frac{AD-BC}{C} \checkmark$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 1/C \checkmark$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = D/C \checkmark$$

4.23

DIRECT CALCULATION:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{V_1}{V_1 \frac{1/Y}{Z+1/Y}} = 1+YZ$$

$$B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = \frac{I_1}{I_1/Y} = Y$$

$$D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = 1$$

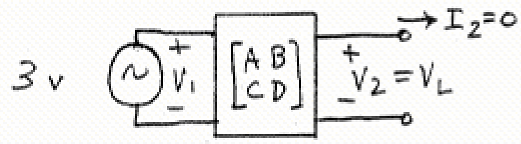
CHECK: $AD - BC = 1 + YZ - ZY = 1 \checkmark$

CALCULATION USING CASCADE: (From Table 4.1)

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix} \checkmark$$

4.24 Using Table 4.1, the ABCD matrix of the cascade of four components (including load) is,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & j50 \\ j/50 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/25 & 1 \end{bmatrix} = \begin{bmatrix} 3j & 25j \\ j/25 & 0 \end{bmatrix}$$

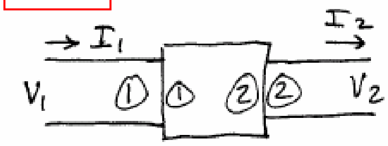


$$V_1 = AV_2 + BI_2 = AV_2 = AV_L$$

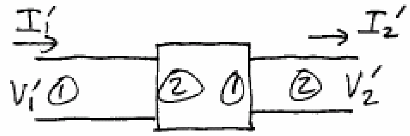
$$V_L = \frac{V_1}{A} = \frac{3}{3j} = 1\angle -90^\circ \checkmark$$

(verified with Serenade)

4.25



$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$



$\begin{bmatrix} D & B \\ C & A \end{bmatrix}$

$$I_1' = -I_2 \,, \quad I_2' = -I_1$$
$$V_1' = V_2 \,, \quad V_2' = V_1$$

$$\begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1' \\ -I_1' \end{bmatrix}$$

inverting: $$\begin{bmatrix} V_1' \\ -I_1' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

since $AD - BC = 1$ if reciprocal.

Rewriting: $$\begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} D & B \\ C & A \end{bmatrix} \begin{bmatrix} V_2' \\ I_2' \end{bmatrix}$$

Example:

<u>original</u> 

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1+ZY & Z \\ Y & 1 \end{bmatrix}$$

<u>reversed</u> 

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & Z \\ Y & 1+ZY \end{bmatrix}$$

so $A' = D, \; B' = B, \; C' = C, \; D' = A$ ✓

4.26

$$V_1 = AV_2 - BI_2 \qquad V_n = V_n^+ + V_n^-$$
$$I_1 = CV_2 - DI_2 \qquad I_n = (V_n^+ - V_n^-)/Z_0$$

So,

$$V_1^+ + V_1^- = A(V_2^+ + V_2^-) - B(V_2^+ - V_2^-)/Z_0$$
$$V_1^+ - V_1^- = C(V_2^+ + V_2^-)Z_0 - D(V_2^+ - V_2^-)$$

For $V_2^+ = 0$,

$$V_1^+ + V_1^- = (A + B/Z_0)V_2^-$$
$$V_1^+ - V_1^- = (CZ_0 + D)V_2^-$$

eliminate V_2^+:

$$V_1^+ + V_1^- = \frac{A + B/Z_0}{CZ_0 + D}(V_1^+ - V_1^-)$$

$$V_1^-(CZ_0 + D + A + B/Z_0) = V_1^+(A + B/Z_0 - CZ_0 - D)$$

$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} \checkmark$$

eliminate V_1^-:

$$2V_1^+ = (A + B/Z_0 + CZ_0 + D)V_2^-$$

$$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0} = \frac{2}{A + B/Z_0 + CZ_0 + D} \checkmark$$

for $V_1^+ = 0$ the above set reduces to,

$$V_1^- = (A - B/Z_0)V_2^+ + (A + B/Z_0)V_2^-$$

$$-V_1^- = (CZ_0 - D)V_2^+ + (CZ_0 + D)V_2^-$$

eliminate V_1^-:

$$(A - B/Z_0 + CZ_0 - D)V_2^+ + (A + B/Z_0 + CZ_0 + D)V_2^- = 0$$

$$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{-A + B/Z_0 - CZ_0 + D}{A + B/Z_0 + CZ_0 + D} \checkmark$$

eliminate V_2^-:

$$\frac{V_1^-}{A + B/Z_0} - \frac{A - B/Z_0}{A + B/Z_0}V_2^+ = \frac{-V_1^-}{CZ_0 + D} - \frac{CZ_0 - D}{CZ_0 + D}V_2^+$$

$$V_1^-\left(\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}\right) = V_2^+\left(\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}\right)$$

$$S_{12} = \left.\frac{V_1^-}{V_2^+}\right|_{V_1^+=0} = \frac{\frac{A - B/Z_0}{A + B/Z_0} - \frac{CZ_0 - D}{CZ_0 + D}}{\frac{1}{A + B/Z_0} + \frac{1}{CZ_0 + D}} = \frac{2(AD - BC)}{A + B/Z_0 + CZ_0 + D} \checkmark$$

These results agree with Table 4.2.

4.27

a) Using the S-parameters, the transmission coefficient from Port 1 to Port 4 is,

$$T = \frac{V_4^-}{V_1^+} = \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(V_2^+ + jV_3^+\right) = \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(\Gamma V_2^- + j\Gamma V_3^-\right)$$

$$= \frac{1}{V_1^+}\left(\frac{-1}{\sqrt{2}}\right)\left(\frac{-1}{\sqrt{2}}\right)(\Gamma)(j+j)V_1^+ = j\Gamma \checkmark$$

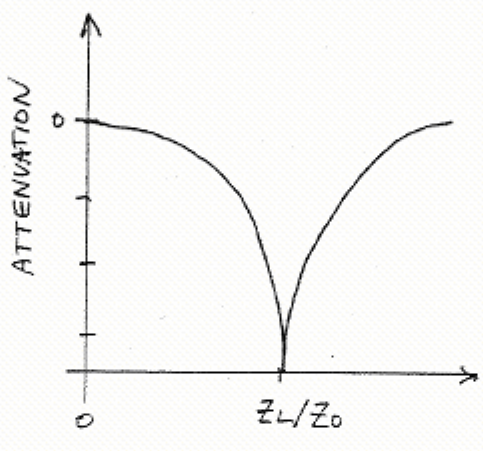
$$\text{attenuation} = |T| = |\Gamma| = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

at port 1 the reflected wave is,

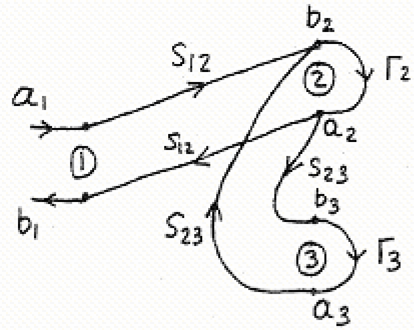
$$V_1^- = \frac{-1}{\sqrt{2}}\left(jV_2^+ + V_3^+\right) = \frac{-1}{\sqrt{2}}\Gamma\left(jV_2^- + V_3^-\right) = \frac{1}{2}\Gamma(-1+1)V_1^+ = 0 \checkmark$$

b)

Z_L/Z_0	atten. (dB)
0	0
0.172	3
1	∞
5.83	3
∞	0

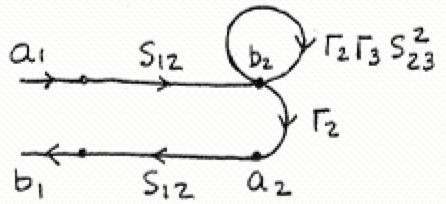


4.28 The signal flowgraph is as follows:



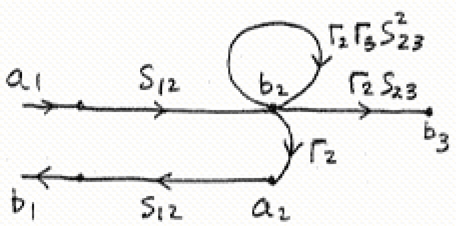
Let $\Gamma_{in} = \frac{b_1}{a_1}$

Using the reduction rules:



$$b_2 = a_1 \frac{S_{12}}{1-\Gamma_2\Gamma_3 S_{23}^2} \quad \checkmark$$

$$b_1 = a_1 \frac{S_{12}^2\Gamma_2}{1-\Gamma_2\Gamma_3 S_{23}^2} \quad \checkmark$$



$$b_3 = b_2\,\Gamma_2 S_{23}$$

Then,
$$\frac{P_2}{P_1} = \frac{b_2^2 - a_2^2}{a_1^2 - b_1^2} = \frac{b_2^2(1-|\Gamma_2|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{\left|1-\Gamma_2\Gamma_3 S_{23}^2\right|^2\left(1-\frac{|S_{12}^2|^2|\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2(1-|\Gamma_2|^2)}{\left|1-\Gamma_2\Gamma_3 S_{23}^2\right|^2 - \left|S_{12}^2\Gamma_2\right|^2} \quad \checkmark$$

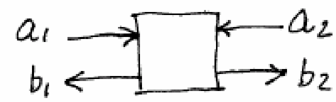
$$\frac{P_3}{P_1} = \frac{b_3^2 - a_3^2}{a_1^2 - b_1^2} = \frac{b_3^2(1-|\Gamma_3|^2)}{a_1^2(1-|\Gamma_{in}|^2)} = \frac{|S_{12}|^2|\Gamma_2 S_{23}|^2(1-|\Gamma_3|^2)}{\left|1-\Gamma_2\Gamma_3 S_{23}^2\right|^2\left(1-\frac{|S_{12}^2\Gamma_2|^2}{|1-\Gamma_2\Gamma_3 S_{23}^2|^2}\right)}$$

$$= \frac{|S_{12}|^2|S_{23}|^2|\Gamma_2|^2(1-|\Gamma_3|^2)}{\left|1-\Gamma_2\Gamma_3 S_{23}^2\right|^2 - \left|S_{12}^2\Gamma_2\right|^2} \quad \checkmark$$

(verified by direct calculation using S-parameters)

4.29

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$



$$\left.\begin{aligned} a_1 &= T_{11} b_2 + T_{12} a_2 \\ b_1 &= T_{21} b_2 + T_{22} a_2 \end{aligned}\right\} \text{T-parameters}$$

$$\left.\begin{aligned} b_1 &= S_{11} a_1 + S_{12} a_2 \\ b_2 &= S_{21} a_1 + S_{22} a_2 \end{aligned}\right\} \text{S-parameters}$$

$$T_{11} = \left.\frac{a_1}{b_2}\right|_{a_2=0} = 1/S_{21} \quad \checkmark$$

$$T_{12} = \left.\frac{a_1}{a_2}\right|_{b_2=0} = -S_{22}/S_{21} \quad \checkmark$$

$$T_{21} = \left.\frac{b_1}{b_2}\right|_{a_2=0} = S_{11}/S_{21} \quad \checkmark$$

$$T_{22} = \left.\frac{b_1}{a_2}\right|_{b_2=0} = S_{11}\frac{a_1}{a_2} + S_{12} = S_{11}\left(\frac{-S_{22}}{S_{21}}\right) + S_{12} = S_{12} - S_{11}S_{22}/S_{21}$$

$$= \frac{S_{12}S_{21} - S_{11}S_{22}}{S_{21}} \quad \checkmark$$

4.30

$$Z_{oc} = -jZ_o \cot \beta\Delta \simeq \frac{-jZ_o}{\beta\Delta} = \frac{-jZ_o c}{\omega\sqrt{\epsilon_e}\,\Delta} = \frac{-j}{\omega C_f}$$

$$\therefore\ \Delta \simeq \frac{Z_o c C_f}{\sqrt{\epsilon_e}} \quad \text{(agrees with T. Edwards, p. 123)}$$

For $C_f = 0.075$ pF, $\epsilon_e = 1.894$, $Z_o = 50\ \Omega$,

this gives $\underline{\Delta = 0.082 \text{ cm}}$

(Using $\epsilon_r = 2.2$, $d = 0.158$ cm, $W = 0.487$ cm)

The Hammerstad & Bekkadal approximation gives

$$\Delta = 0.412d\left(\frac{\epsilon_e + .3}{\epsilon_e - .258}\right)\left(\frac{W + .262d}{W + .813d}\right) = \underline{0.075 \text{ cm}}$$

4.31 The complex reflected power can be computed using (4.88):

$$P_r = \int_S \bar{E}^r \times \bar{H}^{r*} \cdot \hat{z}\, ds = -\int_{x=0}^{a}\int_{y=0}^{b} E_y^r H_x^{r*}\, dx\, dy$$

$$= -b\int_{x=0}^{a}\left[\sum_n A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z}\right]\left[\sum_m \frac{A_m^*}{Z_m^{a*}} \sin\frac{m\pi x}{a} e^{-j\beta_m^{a*} z}\right]dx$$

$$= \frac{-ab}{2}\sum_{n=1}^{\infty}\frac{|A_n|^2}{Z_n^{a*}} e^{j(\beta_n^a - \beta_n^{a*})z}$$

The only propagating mode is the $n=1$ (TE_{10}) mode, so β_1^a is real, and β_n^a is imaginary for $n>1$. Let $\alpha_n = j\beta_n = \sqrt{(n\pi/a)^2 - k_o^2}$ for $n>1$. Then $Z_1^a = k_o\eta_o/\beta_1^a$, and $Z_n^a = k_o\eta_o/\beta_n^a = jk_o\eta_o/\alpha_n$ for $n>1$.

$$\text{Then}\quad P_r = \frac{-ab}{2}\left[\frac{|A_1|^2\beta_1^a}{k_o\eta_o} - j\sum_{n=2}^{\infty}\frac{|A_n|^2\alpha_n}{k_o\eta_o}e^{2\alpha_n z}\right] \quad \text{for } z<0.$$

So we see that $\text{Im}\{P_r\} > 0$, indicating an inductive load.

4.32 This solution is essentially the same as the analysis in Section 4.6. Let $d = (a-c)/2$

$$E_y^i = \sin\frac{\pi x}{a} e^{-j\beta_1^a z} \qquad H_x^i = \frac{-1}{Z_1^a}\sin\frac{\pi x}{a} e^{-j\beta_1^a z}$$

$$E_y^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} A_n \sin\frac{n\pi x}{a} e^{j\beta_n^a z} \qquad H_x^r = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{A_n}{Z_n^a}\sin\frac{n\pi x}{a} e^{j\beta_n^a z}$$

$$E_y^t = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} B_n \sin\frac{n\pi}{c}(x-d) e^{-j\beta_n^c z} \qquad H_x^t = -\sum_{\substack{n=1\\ \text{odd}}}^{\infty} \frac{B_n}{Z_n^c}\sin\frac{n\pi}{c}(x-d) e^{-j\beta_n^c z}$$

where $\beta_n^a = \sqrt{k_0^2 - (n\pi/a)^2}$, $\beta_n^c = \sqrt{k_0^2 - (n\pi/c)^2}$

The solution has the same form as (4.97):

$$\frac{a}{2}A_m + \sum_{\substack{n=1\\ \text{odd}}}^{\infty}\sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2 Z_k^c I_{km} I_{kn}}{c Z_n^a} A_n = \sum_{\substack{k=1\\ \text{odd}}}^{\infty} \frac{2 Z_k^c I_{km} I_{k1}}{c Z_1^a} - \frac{a}{2}\delta_{m1}$$

for $m = 1, 3, 5 \ldots$,

and,

$$I_{mn} = \int_{x=d}^{d+c} \sin\frac{m\pi}{c}(x-d) \sin\frac{n\pi x}{a}\, dx$$

$$\delta_{mn} = \begin{cases} 1 & \text{for } m = n \\ 0 & \text{for } m \neq n \end{cases}$$

4.33 From (4.110) the source current is,

$$\bar{J}_s = \hat{x}\,\frac{2B^+ m\pi}{a}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + \hat{y}\,\frac{2B^+ n\pi}{b}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$

From Table 3.2, the transverse fields for $\pm$ traveling TM_{mn} modes are,

$$E_x = \frac{\mp j\beta m\pi}{k_c^2 a} C^\pm \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$E_y = \frac{\mp j\beta n\pi}{k_c^2 b} C^\pm \sin\frac{m\pi x}{a}\cos\frac{m\pi y}{b} e^{\mp j\beta z}$$

$$H_x = \frac{j\omega\epsilon n\pi}{k_c^2 b} C^\pm \sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} e^{\mp j\beta z}$$

$$H_y = \frac{-j\omega\epsilon m\pi}{k_c^2 a} C^\pm \cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} e^{\mp j\beta z}$$

where $C^\pm$ are the unknown amplitudes. At $z=0$, E_t is continuous, so $C^+ = -C^-$. Also, $\hat{z}\times(\bar{H}^+ - \bar{H}^-) = \bar{J}_s$, or $H_y^+ - H_y^- = J_{sx}$ and $-H_x^+ + H_x^- = J_{sy}$. So,

$$J_{sx}: \quad \frac{-j\omega\epsilon m\pi}{k_c^2 a}(C^+ - C^-) = 2B^+\frac{m\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon}$$

$$J_{sy}: \quad \frac{j\omega\epsilon n\pi}{k_c^2 b}(-C^+ + C^-) = 2B^+\frac{n\pi}{a} \Rightarrow C^+ = -C^- = \frac{k_c^2 B^+}{-j\omega\epsilon} \checkmark$$

Since these fields satisfy Maxwell's equations and the boundary conditions, they must form the unique solution.

4.34 Following Example 4.8:

$$\bar{J}(x,y,z) = I(y)\,\delta(x-a/2)\,\delta(z) \quad \text{for } 0<y<d.$$

$$\bar{e}_1 = \hat{y}\sin\frac{\pi x}{a}, \quad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a}, \quad Z_1 = k_0\eta_0/\beta,$$

From (4.119),

$$P_1 = \frac{ab}{Z_1}$$

From (4.118),

$$A_1^+ = \frac{-1}{P_1}\int_v \sin\frac{\pi x}{a}\, e^{j\beta_1 z}\, I(y)\,\delta(x-a/2)\,\delta(z)\,dx\,dy\,dz$$

$$= \frac{-I_0}{P_1}\int_{y=0}^{d}\frac{\sin k(d-y)}{\sin kd}\,dy = \frac{-I_0}{P_1\sin kd}\int_0^d \sin kw\,dw$$

$$(\text{let } w = d-y)$$

$$= \frac{I_0 Z_1(\cos kd - 1)}{kab\sin kd}$$

The total power flow in the TE_{10} mode is,

$$P = \frac{ab\,|A_1^+|^2}{2Z_1},$$

for both + and − traveling waves, since $|A_1^+| = |A_1^-|$. Then the radiation resistance is,

$$R_{in} = \frac{2P}{I_0^2} = \frac{ab|A_1^+|^2}{I_0^2 Z_1} = \frac{Z_1}{ab}\,\frac{(1-\cos kd)^2}{k^2\sin^2 kd}.$$

$$= \frac{Z_1}{k^2ab}\,\frac{(2\sin^2\frac{kd}{2})^2}{4\sin^2\frac{kd}{2}\cos^2\frac{kd}{2}} = \frac{Z_1}{k^2ab}\tan^2\frac{kd}{2} \quad \checkmark$$

4.35 Following Example 4.8:

$$\bar{J}(x,y,z) = I\,\delta(z)\left[\delta(x-a/4) - \delta(x-3a/4)\right]\hat{y} \quad \text{for } 0<y<b$$

From Table 3.2,

$$TE_{10}: \quad \bar{e}_1 = \hat{y}\sin\frac{\pi x}{a} \qquad \bar{h}_1 = \frac{-\hat{x}}{Z_1}\sin\frac{\pi x}{a} \qquad Z_1 = k_o\eta_o/\beta_1$$

$$TE_{20}: \quad \bar{e}_2 = \hat{y}\sin\frac{2\pi x}{a} \qquad \bar{h}_2 = \frac{-\hat{x}}{Z_2}\sin\frac{2\pi x}{a} \qquad Z_2 = k_o\eta_o/\beta_2$$

$$P_1 = ab/Z_1 \qquad \beta_1 = \sqrt{k_o^2-(\pi/a)^2}$$

$$P_2 = ab/Z_2 \qquad \beta_2 = \sqrt{k_o^2-(2\pi/a)^2}$$

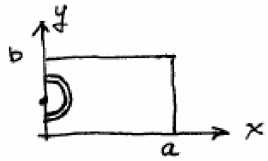
From (4.118):

$$A_1^+ = \frac{-1}{P_1}\int_v \bar{E}_1^- \cdot \bar{J}\,dv = -\frac{Ib}{P_1}\left(\sin\frac{\pi}{4} - \sin\frac{3\pi}{4}\right) = 0 \quad \checkmark$$

$$A_2^+ = \frac{-1}{P_2}\int_v \bar{E}_2^- \cdot \bar{J}\,dv = \frac{-Ib}{P_2}\left(\sin\frac{\pi}{2} - \sin\frac{3\pi}{2}\right) = \frac{-2Ib}{a}$$

Since the excitation has an odd symmetry about the center of the guide, it will only excite modes that have an electric field with an odd symmetry about $x=a/2$. This implies the TE_{m0} modes, for m even, will be excited. The TE_{10} mode is not excited.

4.36 By image theory, the half-loop on the side wall can be replaced with a full loop without the wall. For a small loop, the equivalent magnetic dipole moment is,



$$\bar{P}_m = \hat{z}\, I_o \pi r_o^2\, \delta(x)\,\delta(y-b/2)\,\delta(z)$$

$$\bar{M} = j\omega\mu_o \bar{P}_m$$

$$= \hat{z}\, j\omega\mu_o I_o \pi r_o^2\, \delta(x)\delta(y-b/2)\,\delta(z) \quad V/m^2$$

For the TE_{10} mode,

$$\bar{e}_1 = \hat{y} \sin \pi x/a$$

$$\bar{h}_1 = \frac{-\hat{x}}{Z_1} \sin \pi x/a$$

$$h_{z1} = \frac{j\pi}{k_o \eta_o a} \cos \pi x/a$$

where $Z_1 = k_o\eta_o/\beta_1$, $P_1 = ab/Z_1$

From (4.128),

$$A_1^+ = \frac{1}{P_1} \int_v (-\bar{h}_1 + \hat{z}\, h_{z1}) \cdot \bar{M}\, e^{j\beta_1 z}\, dv$$

$$= \frac{Z_1}{ab} \int_v h_{z1} M \, dv = \frac{-\pi^2 Z_1 I_o r_o^2}{a^2 b} = A_1^-$$

These results are for a <u>full</u> loop - reduce by $\frac{1}{2}$ for half-loop.

4.37

FIRST SOLUTION: (all fields and currents are TE_{10})

$$E_y = B \sin\frac{\pi x}{a}\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jB \sin\frac{\pi x}{a} \sin\beta z \qquad 0<z<d$$

$$H_x = \frac{B}{Z_1} \sin\frac{\pi x}{a}\left[-e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2B}{Z_1} \sin\frac{\pi x}{a} \cos\beta z \qquad 0<z<d$$

This satisfies $E_y = 0$ at $z=0$.

$$E_y = C \sin\frac{\pi x}{a} e^{-j\beta(z-d)} \qquad z>d$$

$$H_x = \frac{C}{Z_1} \sin\frac{\pi x}{a} e^{-j\beta(z-d)} \qquad z>d$$

at $z=d$, E_y is continuous, so

$$-2jB \sin\beta d = C$$

at $z=d$, $\hat{z}\times(\bar{H}^+ - \bar{H}^-) = \bar{J}_s$, or

$$\frac{C}{Z_1} + \frac{2B}{Z_1}\cos\beta d = \frac{2\pi A}{a}$$

Solving for B, C:

$$B = \frac{\pi Z_1 A}{a} e^{-j\beta d}, \qquad C = \frac{\pi Z_1 A}{a}\left(e^{-2j\beta d} - 1\right)$$

SECOND SOLUTION: (Using (4.105) and (4.106b)):

E_y due to J_{sy} at $z=d$:

$$E_y^{\pm} = \frac{-\pi Z_1 A}{a} \sin\frac{\pi x}{a} e^{\mp j\beta(z-d)}$$

E_y due to $-J_{sy}$ at $z=-d$:

$$E_y^{\pm} = \frac{\pi Z_1 A}{a} \sin\frac{\pi x}{a} e^{\mp j\beta(z+d)}$$

For $0<z<d$,

$$E_y = \frac{\pi Z_1 A}{a} \sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a} e^{-j\beta d} \sin\frac{\pi x}{a} \sin\beta z \checkmark$$

For $z>d$,

$$E_y = \frac{\pi Z_1 A}{a} \sin\frac{\pi x}{a}\left[e^{-j\beta(z+d)} - e^{-j\beta(z-d)}\right] = \frac{-2j\pi Z_1 A}{a} \sin\beta d \sin\frac{\pi x}{a} e^{-j\beta z} \checkmark$$

These results agree with those from the first solution.

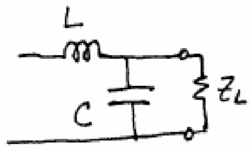
Chapter 5

5.1

a) $Z_L = 150 - j200\,\Omega$

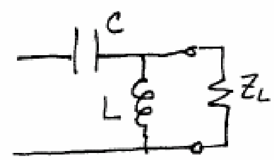
$z_L = 1.5 - j2$ inside $1+jx$ circle

#1 $\quad b_1 = 0.107 \Rightarrow C = \frac{b}{2\pi f Z_0} = 0.0568\,pF$ ✓



$x_1 = 1.78 \Rightarrow L = \frac{x Z_0}{2\pi f} = 9.44\,nH$ ✓

#2 $\quad b_2 = -.747 \Rightarrow L = \frac{-Z_0}{2\pi f b} = 7.10\,nH$ ✓

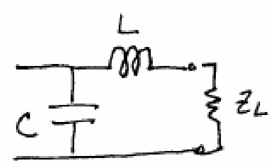


$x_2 = -1.78 \Rightarrow C = \frac{-1}{2\pi f x Z_0} = 0.298\,pF$ ✓

b) $Z_L = 20 - j90\,\Omega$

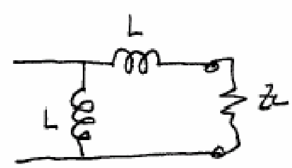
$z_L = 0.2 - j.9$ outside $1+jx$ circle

#1 $\quad x_1 = 1.30 \Rightarrow L = \frac{x Z_0}{2\pi f} = 6.90\,nH$ ✓



$b_1 = 2.00 \Rightarrow C = \frac{b}{2\pi f Z_0} = 1.06\,pF$ ✓

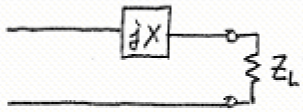
#2 $\quad x_2 = 0.5 \Rightarrow L = \frac{-Z_0}{2\pi f} = 2.65\,nH$ ✓



$b_2 = -2.00 \Rightarrow L = \frac{-Z_0}{2\pi f b} = 2.65\,nH$ ✓

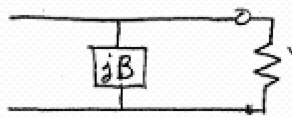
verified w/ PCAAD 7.0

5.2 a)



matching is possible if $Z_L = Z_0 - jX$ ✓

b)



matching is possible if $Y_L = \frac{1}{Z_0} - jB$ ✓

5.3 Analytical Solutions:

From (5.9), $t = \dfrac{80 \pm \sqrt{100[(75-100)^2+(80)^2]/75}}{100-75} = 3.2 \pm 3.871$

$t_1 = 7.071 \quad , \quad t_2 = -0.671$

From (5.10) the possible stub positions are,

$d_1 = \frac{\lambda}{2\pi}\tan^{-1} t_1 = 0.2276\,\lambda$ ✓

$d_2 = \frac{\lambda}{2\pi}(\pi + \tan^{-1} t_2) = 0.4059\,\lambda$ ✓

From (5.8b) the required stub susceptances are,

$$B = \frac{R_L^2 t - (Z_0 - X_L t)(X_L + Z_0 t)}{Z_0[R_L^2 + (X_L + Z_0 t)^2]}$$

$B_1 = 0.0129 \quad , \quad B_2 = -0.0129$

From (5.11a) the o.c. stub lengths are,

$\ell_1 = \frac{-\lambda}{2\pi}\tan^{-1}(B_1 Z_0) = 0.3776\,\lambda$ ✓ ($\lambda/2$ added to get $\ell_1 > 0$)

$\ell_2 = \frac{-\lambda}{2\pi}\tan^{-1}(B_2 Z_0) = 0.1224\,\lambda$ ✓

5.4

Use B_1, B_2 from Problem 5.3 with (5.11b):

$$\ell_1 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_1} = 0.1276\lambda \checkmark$$

$$\ell_2 = \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{Z_0 B_2} = 0.3724\lambda \checkmark \quad (\lambda/2 \text{ added to get } \ell_2 > 0)$$

5.5

Smith chart solutions:

The normalized load impedance is $z_L = 1.2 + j0.8$

The stub positions and required reactances are,

$$d_1 = 0.346 - 0.172 = 0.174\lambda \checkmark \quad , \quad x_1 = +j0.753$$

$$d_2 = (.5 - .172) + 0.153 = 0.481\lambda \checkmark, \quad x_2 = -j0.753$$

open ckt stub lengths are,

$$\ell_1 = .25 + .103 = 0.353\lambda \quad \checkmark$$

$$\ell_2 = .397 - .25 = 0.147\lambda \quad \checkmark$$

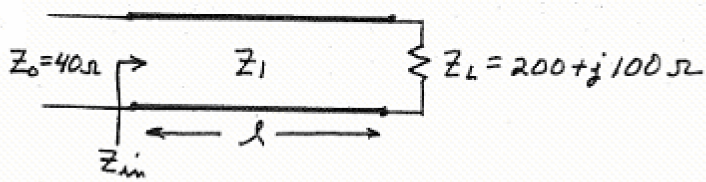
verified w/ PCAAD 7.0

5.6 The required stub lengths for s.c. stubs are $\lambda/4$ longer (or shorter) than the o.c. stub lengths:

$$\ell_1 = .353 - .25 = 0.103\lambda \checkmark$$

$$\ell_2 = .147 + .25 = 0.397\lambda \checkmark$$

5.7



To match this load, we must find Z_1 and ℓ so that $Z_{in} = Z_0 = 40\,\Omega$:

$$Z_{in} = 40 = Z_1 \frac{(200+j100)+jZ_1 t}{Z_1 + j(200+j100)t}, \quad \text{with } t = \tan\beta\ell.$$

$$(40Z_1 - 4000t) + j\,8000t = 200Z_1 + j(100 + Z_1 t)Z_1$$

Equating real and imaginary parts gives two equations for the two unknowns, Z_1 and t: (if they exist!)

$$\text{Re:} \quad 40Z_1 - 4000t = 200Z_1 \Rightarrow Z_1 = -25t$$

$$\text{Im:} \quad 8000t = Z_1(100 + Z_1 t)$$

$$8000t = -25t(100 - 25t^2)$$

$$t = \pm\sqrt{16.8} = \pm 4.10 \quad (\text{use } -4.10 \text{ so that } Z_1 > 0)$$

Then, $\beta\ell = \tan^{-1}(-4.10) = -76.3^\circ \equiv 104^\circ \Rightarrow \underline{\ell = 0.288\lambda}$

The characteristic impedance is then,

$$Z_1 = -25(-4.10) = 102.5\,\Omega$$

(Note: Not all load impedances can be matched in this way – a good exam problem to determine which impedances can be matched using this technique!)

5.8 From (2.91) the impedance of a terminated lossy line is,

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l}, \quad \gamma l = \alpha l + j\beta l$$

For $Z_L = \infty$ (o.c.), the normalized input admittance is,

$$y_{in} = \tanh \gamma l = \frac{\tanh \alpha l + j \tan \beta l}{1 + j \tanh \alpha l \tan \beta l}$$

The normalized input susceptance is,

$$b_{in} = \frac{\tan \beta l (1 - \tanh^2 \alpha l)}{1 + \tanh^2 \alpha l \tan^2 \beta l}$$ (at this point, we could find max. b_{in} by calculating b_{in} vs. l)

Since maximum susceptance for a lossless line is obtained for $\beta l = \pi/2$, we expect βl to be close to $\pi/2$ for the lossy case. So let $\beta l = \pi/2 + \Delta$, where Δ is small. Also, αl is small, so we have $\tanh \alpha l \simeq \alpha l$, and $\tan \beta l = -\cot \Delta \simeq -1/\Delta$. Then,

$$b_{in} \simeq \frac{\frac{-1}{\Delta}(1 - \alpha^2 l^2)}{1 + \alpha^2 l^2 / \Delta^2} \simeq \frac{-1}{\Delta + \alpha^2 l^2 / \Delta}$$

To maximize b_{in}, we can minimize $\Delta + \alpha^2 l^2 / \Delta$ with respect to l:

$$\frac{d}{dl}\left(\Delta + \alpha^2 l^2 / \Delta\right) = \frac{d\Delta}{dl} + \frac{2\alpha^2 l}{\Delta} + \alpha^2 l^2 \left(\frac{-1}{\Delta^2}\right) \frac{d\Delta}{dl} = 0$$

or, since $\frac{d\Delta}{dl} = \beta$,

$$\beta + \frac{2\alpha^2 l}{\Delta} - \frac{\alpha^2 l^2}{\Delta^2} \beta = 0$$

since $\Delta = \beta l - \pi/2$, we have,

$$l^2 \beta (\alpha^2 + \beta^2) - \pi l (\alpha^2 + \beta^2) + \beta \frac{\pi^2}{4} = 0$$

Solve for l:

$$\ell = \frac{\pi(\alpha^2+\beta^2) \pm \sqrt{\pi^2(\alpha^2+\beta^2)^2 - \beta^2\pi^2(\alpha^2+\beta^2)}}{2\beta(\alpha^2+\beta^2)}$$

$$= \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta\sqrt{\alpha^2+\beta^2}} \simeq \frac{\pi}{2\beta} \pm \frac{\pi\alpha}{2\beta^2} \qquad (\text{since } \alpha^2 << \beta^2)$$

Then,

$$\Delta = \beta\ell - \pi/2 \simeq \frac{\pi\alpha}{2\beta} \simeq \alpha\ell \qquad (\text{since } \beta \simeq \pi/2\ell)$$

The corresponding value of b_{in} is,

$$b_{in}^{MAX} = \frac{\pm 1}{\alpha\ell + \alpha\ell} = \frac{\pm 1}{2\alpha\ell} = \frac{\pm 2}{\alpha\lambda} \qquad (\text{since } \ell \simeq \lambda/4)$$

For $\alpha = 0.01$ neper/λ, $b_{in}^{MAX} = \frac{\pm 2}{.01} = \underline{\pm 200}$

(This checks with direct calculation of y_{in} vs. ℓ.)

The reactance of a short-circuited line is the dual case of the above problem, so $x_{in}^{MAX} = \pm 200$.

5.9 Smith chart solution:

1. plot $y_L = 0.4 + j1.2$ on admittance chart
2. plot rotated $1+jb$ circle
3. add a stub susceptance of $j0.6$ or $-j1.0$ to move to rotated $1+jb$ circle
4. move $\lambda/8$ toward generator, to $1+jb$ circle
5. add a stub susceptance of $+j3.0$ or $-j1.0$ to move to center of chart.
6. the o.c. stub lengths are,

$$\ell_1 = 0.086\lambda \checkmark \quad \text{or} \quad \ell_1 = 0.375\lambda \checkmark$$
$$\ell_2 = 0.199\lambda \checkmark \quad \text{or} \quad \ell_2 = 0.375\lambda \checkmark$$

(see attached Smith chart for first solution)

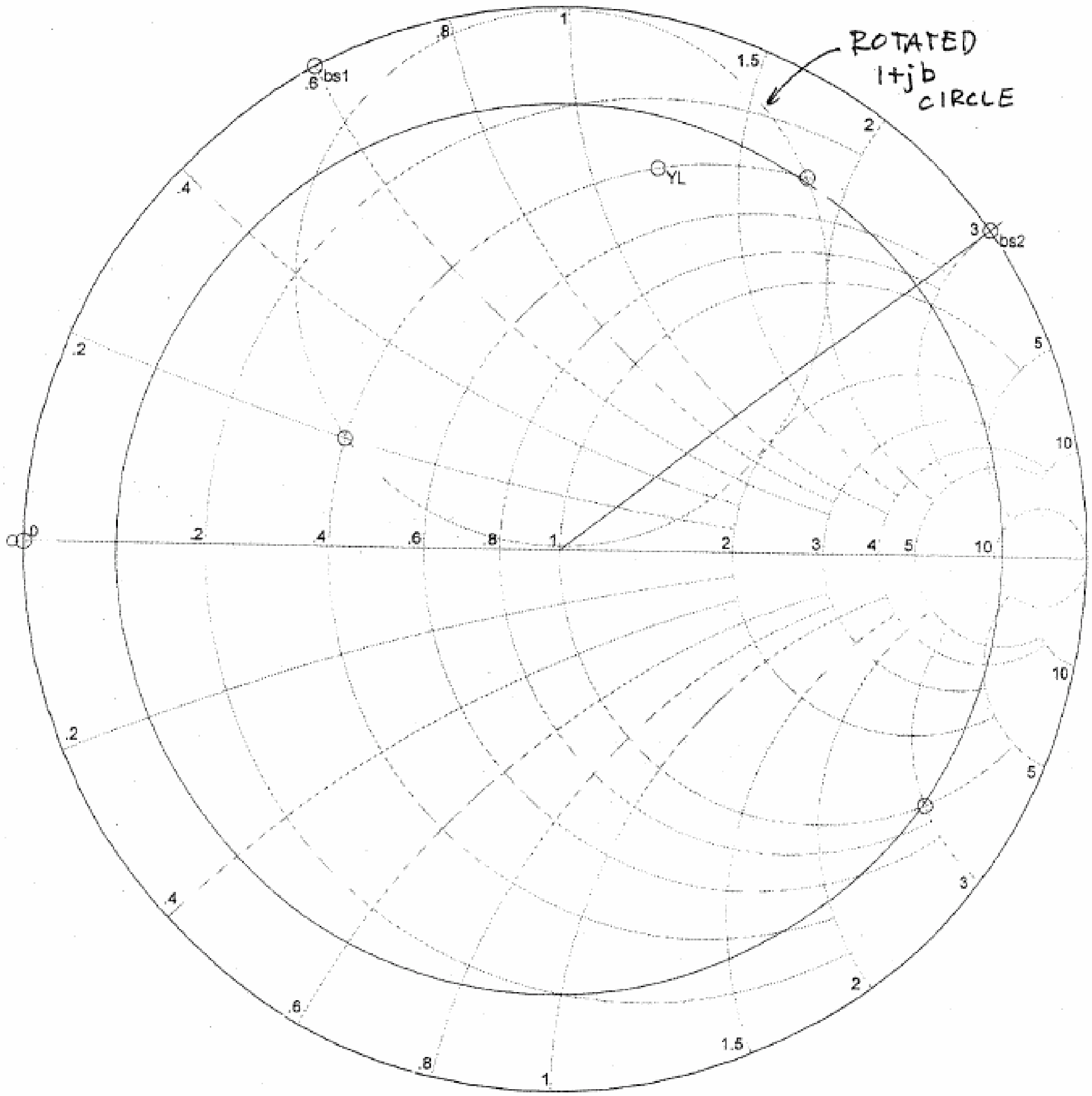
Analytic solution: $t = \tan \beta d = 1$

$$b_1 = -b_L + 1 \pm \sqrt{2g_L - g_L^2} = 0.6,\ -1.0$$

$$b_2 = \frac{1}{g_L}\left[\pm\sqrt{2g_L - g_L^2} + g_L\right] = 3.0,\ -1.0$$

$$\ell_1 = \frac{\lambda}{2\pi}\tan^{-1} b_1 = 0.086\lambda \quad \text{or} \quad \ell_1 = 0.375\lambda$$

$$\ell_2 = \frac{\lambda}{2\pi}\tan^{-1} b_2 = 0.198\lambda \quad \text{or} \quad \ell_2 = 0.375\lambda$$



Smith chart for P. 5.9 (#1)

5.10 Analytic Solution: let $t = \tan\beta d = \tan 135° = -1.0$

From (5.22) the first stub susceptance is

$$b_1 = -b_L + \frac{1 \pm \sqrt{(1+t^2)g_L - g_L^2 t^2}}{t} = -3 \text{ or } -1.4 \quad \checkmark$$

From (5.23) the second stub susceptance is

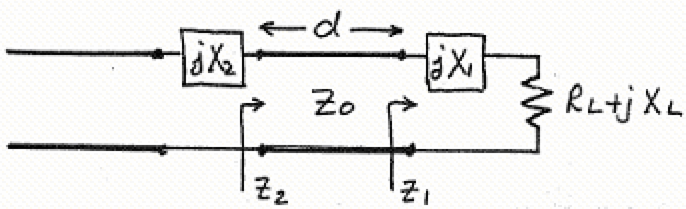
$$b_2 = \frac{\pm\sqrt{(1+t^2)g_L - g_L^2 t^2} + g_L}{g_L t} = -3 \text{ or } 1.0 \quad \checkmark$$

The S.C. stub lengths are, from (5.24b),

$$\ell_1 = 0.051\lambda \quad \text{or } 0.0987\lambda$$

$$\ell_2 = 0.051\lambda \quad \text{or } 0.375\lambda$$

5.11



$$Z_1 = R_L + j(X_L + X_1)$$

$$Z_2 = Z_0 \frac{R_L + j(X_L + X_1 + Z_0 t)}{Z_0 + jt(R_L + jX_L + jX_1)} = Z_0 \qquad t = \tan\beta d$$

Solving for R_L:

$$R_L = Z_0 \frac{1+t^2}{2t^2}\left[1 \pm \sqrt{\frac{1-4t^2(Z_0 - X_L t - X_1 t)^2}{Z_0(1+t^2)^2}}\right]$$

So we must have,

$$0 \le R_L \le Z_0 \frac{1+t^2}{2t^2} = \frac{Z_0}{\sin^2\beta d}$$

The first stub reactance is,

$$X_1 = -X_L + \frac{Z_0 \pm \sqrt{(1+t^2)R_L Z_0 - R_L^2 t^2}}{t}$$

The second stub reactance is,

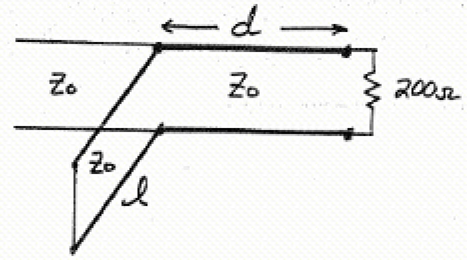
$$X_2 = \frac{\pm Z_0\sqrt{Z_0 R_L(1+t^2) - R_L^2 t^2} + R_L Z_0}{R_L t}$$

The stub lengths are given by,

$$l_{oc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{Z_0}{X}\right), \qquad l_{sc} = \frac{1}{2\pi}\tan^{-1}\left(\frac{X}{Z_0}\right)$$

5.12 Using the Smith chart $(Z_0 = 100\,\Omega)$

a) a single short-circuited shunt stub:



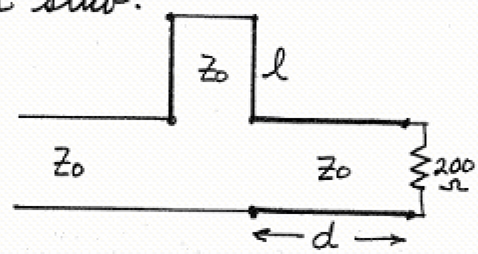
at f_0, $d_1 = 0.152\lambda$ ✓ $\quad d_2 = 0.348\lambda$ ✓

$b_1 = -0.7 \quad b_2 = +0.7$

$\ell_1 = 0.153\lambda$ ✓ $\quad \ell_2 = 0.347$ ✓

$|\Gamma_1| = 0 \quad |\Gamma_2| = 0$

b) a single short-circuited series stub:



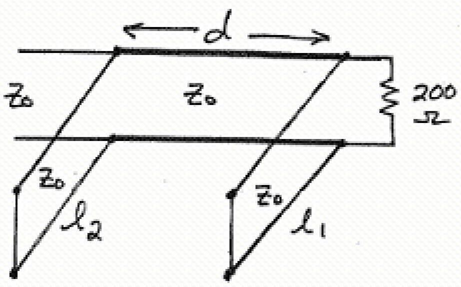
at f_0, $d_1 = 0.098\lambda$ ✓ $\quad d_2 = 0.402\lambda$ ✓

$X_1 = 0.7 \quad X_2 = -0.7$

$\ell_1 = 0.097\lambda$ ✓ $\quad \ell_2 = 0.403\lambda$ ✓

$|\Gamma_1| = 0 \quad |\Gamma_2| = 0$

c) a double short-circuited shunt stub: (let $d = \lambda/8$)



at f_0,

$b_1 = 0.14 \quad b_1' = 1.85$

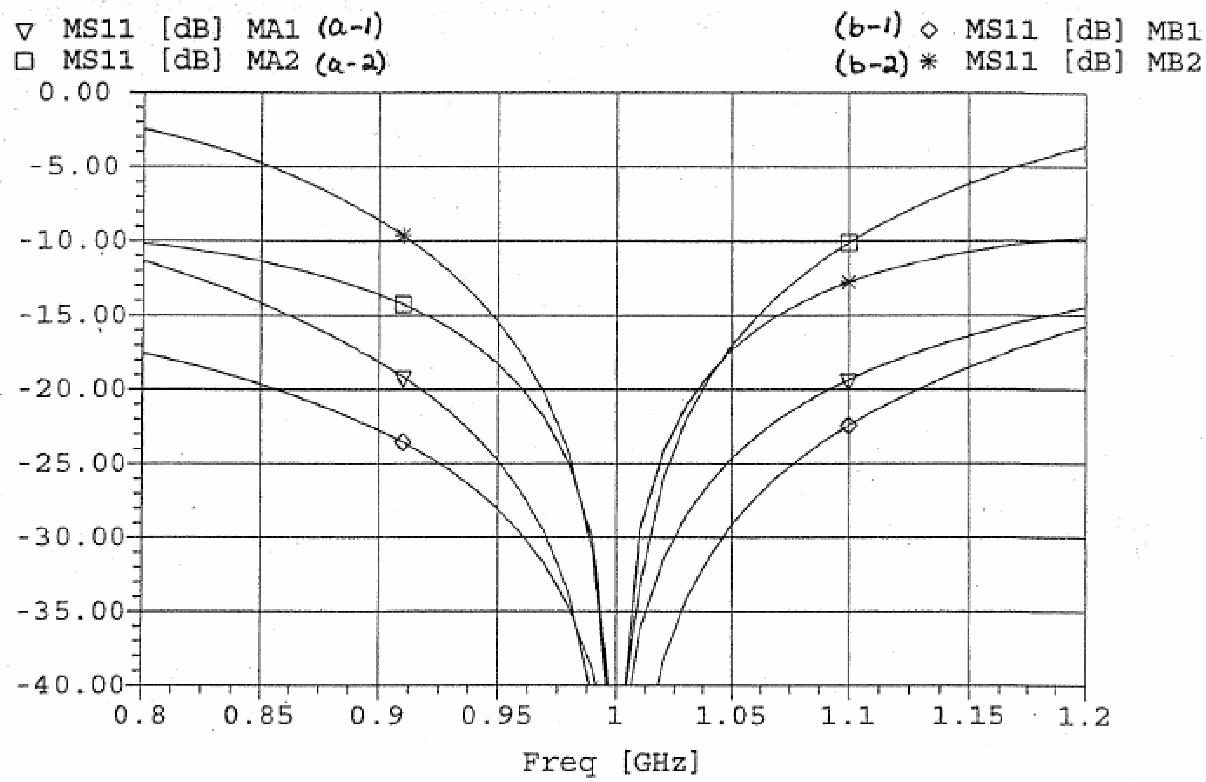
$\ell_1 = 0.272\lambda$ ✓ $\quad \ell_1' = 0.421\lambda$ ✓

$b_2 = -0.73 \quad b_2' = 2.75$

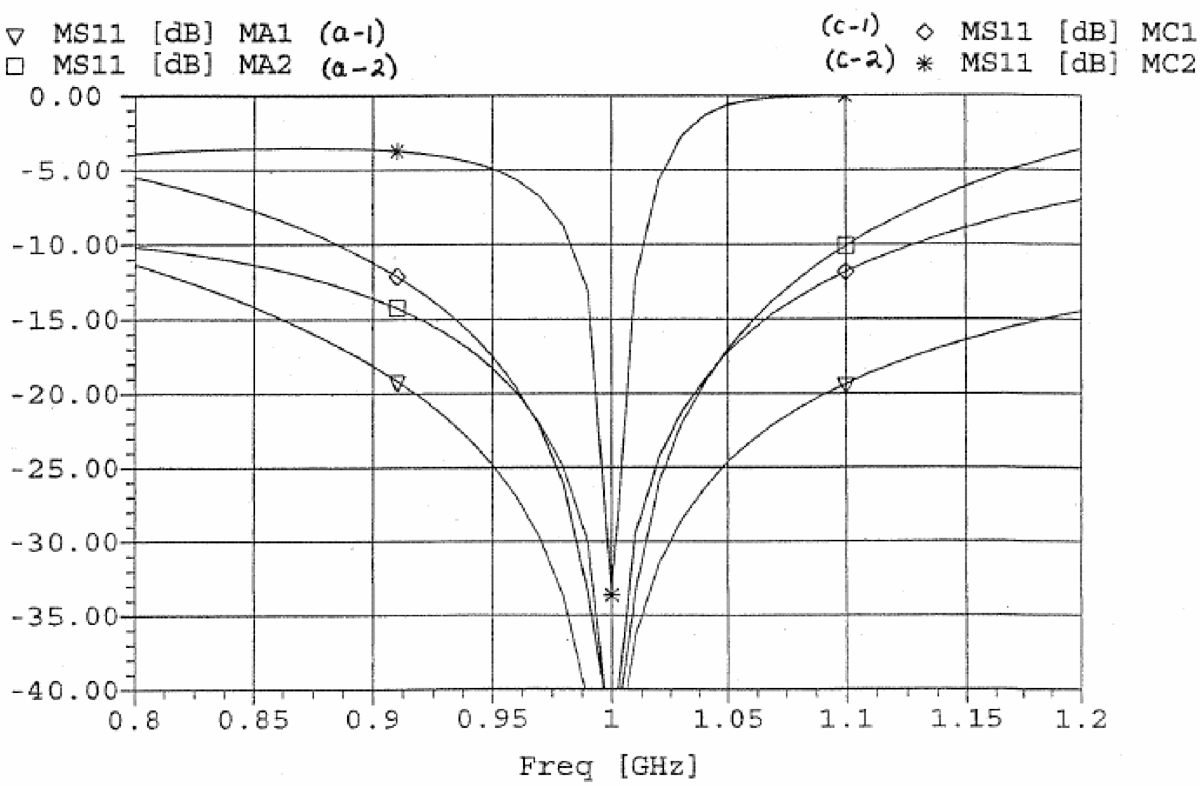
$\ell_2 = 0.15\lambda$ ✓ $\quad \ell_2' = 0.444\lambda$ ✓

$|\Gamma| = 0 \quad |\Gamma'| = 0$

Plots of return loss vs. f/f_0 for these six solutions are shown on the following page. (only 4 curves could be plotted per graph). These results show that the tuner of solution (b-1), the series stub tuner, gives the best bandwidth. This is probably because the stub length and line length are shortest for this case, giving the smallest frequency variation.

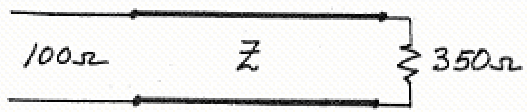


▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(b-1) ◇ MS11 [dB] MB1
(b-2) * MS11 [dB] MB2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Freq [GHz]



▽ MS11 [dB] MA1 (a-1)
□ MS11 [dB] MA2 (a-2)
(c-1) ◇ MS11 [dB] MC1
(c-2) * MS11 [dB] MC2
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
-40.00
0.8
0.85
0.9
0.95
1
1.05
1.1
1.15
1.2
Freq [GHz]

5.13



$Z = \sqrt{100(350)} = 187.1\,\Omega$ ✓

An SWR of 2 corresponds to a reflection coefficient magnitude of,

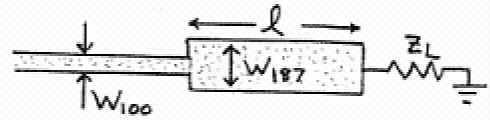
$$\Gamma_m = \frac{S-1}{S+1} = 1/3$$

Then from (5.33) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{\Gamma_m}{\sqrt{1-\Gamma_m^2}}\,\frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}\right] = 71\%$$

MICROSTRIP LAYOUT:

$\epsilon_r = 2.2$, $d = 0.159$ cm, $f = 4$ GHz



First try $w/d < 2$:

for W_{100}, $A_{100} = 2.213$, $W_{100}/d = 0.896 < 2$ (OK), $W_{100} = 0.142$ cm

for W_{187}, $A_{187} = 4.047$, $W_{187}/d = 0.140 < 2$ (OK), $W_{187} = 0.022$ cm

From (3.195), ϵ_e for W_{187} is $\epsilon_e = 1.66$.

Then the physical length of the $\lambda/4$ transformer is,

$$\ell = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_e}\,f} = 1.455 \text{ cm} ✓$$

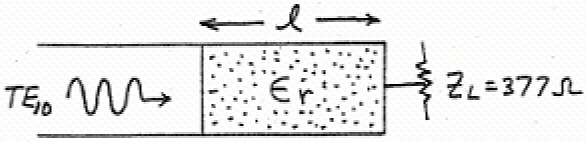
5.14

From (5.34) and (5.36), the partial reflection coefficients are,

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{150-100}{150+100} = 0.2 \quad ; \quad \Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2} = \frac{225-150}{225+150} = 0.2$$

Since the approximate expression for Γ in (5.42) is identical to the numerator for the exact expression in (5.41), the greatest error will occur when the denominator of (5.41) departs from unity to the greatest extent. This occurs for $\theta = 0$ or $180°$. Then (5.41) gives the exact Γ as 0.384, while (5.42) gives the approximate $\Gamma = 0.4$. Thus the error is about 4%.

5.15



$a = 2.286$ cm

$k_o = \frac{2\pi f}{c} = 209.4 \ m^{-1}$

In the air-filled guide,

$$\beta_a = \sqrt{k_o^2 - (\pi/a)^2} = 158.0 \ m^{-1} \ ✓$$

$$Z_a = \frac{k_o \eta_o}{\beta_a} = \frac{(209.4)(377)}{158} = 499.6 \ \Omega \ ✓$$

So the matching section impedance must be,

$$Z_m = \sqrt{Z_a Z_L} = \sqrt{(499.6)(377)} = 434.0 \ \Omega$$

$$= \frac{k_m \eta_m}{\beta_m} = \frac{k_o \eta_o}{\beta_m}$$

So the propagation constant of the matching section must be,

$$\beta_m = \frac{k_o \eta_o}{Z_m} = \frac{(209.4)(377)}{434} = 181.9 \ m^{-1}$$

$$= \sqrt{\epsilon_r k_o^2 - (\pi/a)^2}$$

Solving for ϵ_r:

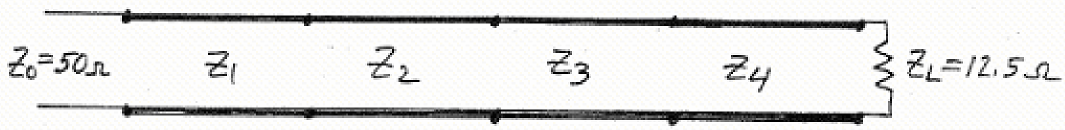
$$\epsilon_r = \frac{\beta_m^2 + (\pi/.02286)^2}{(209.4)^2} = \underline{1.185} \ ✓$$

The physical length of the matching section is,

$$\ell = \frac{\lambda_g}{4} = \frac{2\pi}{4\beta_m} = \frac{\pi}{2\beta_m} = 0.86 \text{ cm} \ ✓$$

(Note that this type of matching is not possible if $Z_L > Z_a$.)

5.16



a) Using (5.53):

$n=0$: $\ln Z_1/Z_0 = 2^{-4} C_0^4 \ln 12.5/50 \Rightarrow Z_1 = 45.85\Omega$

$n=1$: $\ln Z_2/Z_1 = 2^{-4} C_1^4 \ln 12.5/50 \Rightarrow Z_2 = 32.42\Omega$

$n=2$: $\ln Z_3/Z_2 = 2^{-4} C_2^4 \ln 12.5/50 \Rightarrow Z_3 = 19.28\Omega$

$n=3$: $\ln Z_4/Z_3 = 2^{-4} C_3^4 \ln 12.5/50 \Rightarrow Z_4 = 13.63\Omega$

Check: $n=4$: $\ln Z_5/Z_4 = 2^{-4} C_4^4 \ln 12.5/50 \Rightarrow Z_5 = 12.50\Omega = Z_L$ ✓

Can also check with data in Table 5.1, using $Z_L/Z_0 = 4$, which gives $Z_1 = 13.65\Omega$, $Z_2 = 19.30\Omega$, $Z_3 = 32.38\Omega$, $Z_4 = 45.79\Omega$
(source and load are reversed in this case)

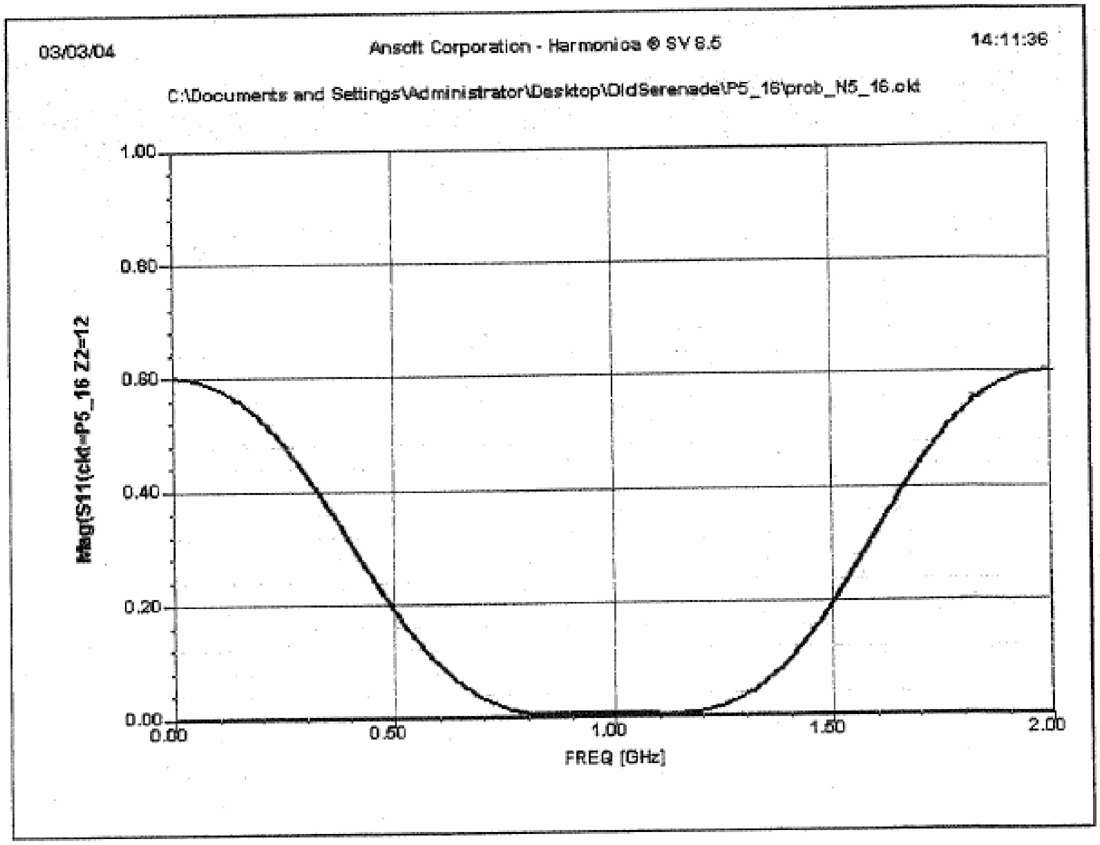
From (5.55), $A \simeq \frac{1}{2^{N+1}} \ln Z_L/Z_0 = -0.0433$

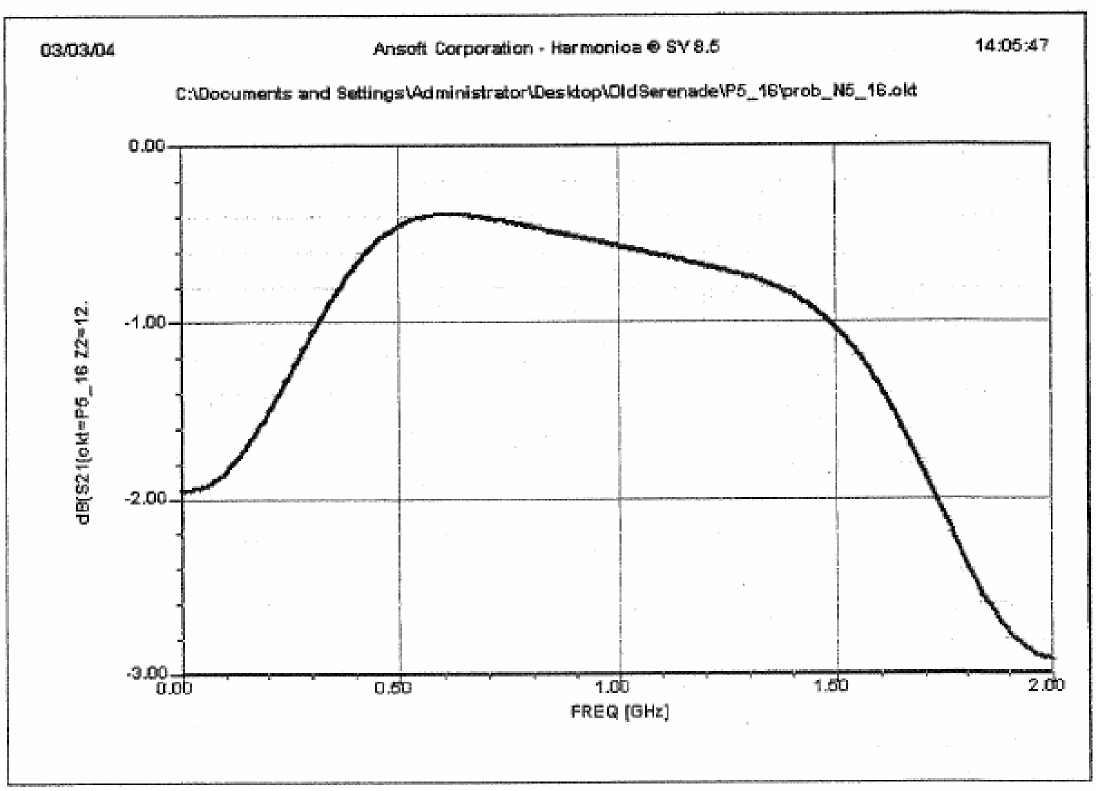
$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \cos^{-1}\left[\frac{1}{2}\left|\frac{\Gamma_m}{A}\right|^{1/N}\right] = 69\%$$ (agrees with plot)

b) Microstrip line widths and lengths:

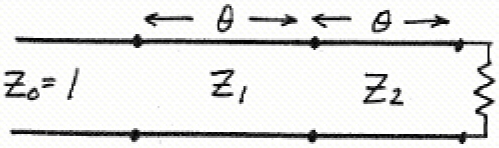
Z_c	W (cm)	ϵ_e	$\lambda_g/4$ (cm)
45.85	0.356	3.239	4.16
32.42	0.597	3.402	4.06
19.28	1.175	3.627	3.94
13.63	1.781	3.756	3.87

Results from Serenade modeling of parts a) and b) are shown on the following page. Note the good match, and the insertion loss of about 0.5dB.


03/03/04
Ansoft Corporation - Harmonica ® SV 8.5
14:11:36
C:\Documents and Settings\Administrator\Desktop\OldSerenade\P5_16\prob_N5_16.ckt
Mag(S11(ckt=P5_16 Z2=12
1.00
0.80
0.60
0.40
0.20
0.00
0.00
0.50
1.00
1.50
2.00
FREQ [GHz]


03/03/04
Ansoft Corporation - Harmonica ® SV 8.5
14:05:47
C:\Documents and Settings\Administrator\Desktop\OldSerenade\P5_16\prob_N5_16.ckt
dB(S21(ckt=P5_16 Z2=12.
0.00
-1.00
-2.00
-3.00
0.00
0.50
1.00
1.50
2.00
FREQ [GHz]

5.17



From (5.50) the desired input reflection coefficient response is ($N=2$):

$$\Gamma(\theta) = 2A(1+\cos 2\theta)$$

From the above circuit, we have that $\Gamma(0) = \frac{R-1}{R+1} = 0.2$, so $A = 0.2/4 = 0.05$.

Now we calculate the input reflection coefficient of the above circuit using ABCD matrices and conversion to S-parameters:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & jZ_1\sin\theta \\ jY_1\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & jZ_2\sin\theta \\ jY_2\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta - Z_1Y_2\sin^2\theta & j(Z_1+Z_2)\cos\theta\sin\theta \\ j(Y_1+Y_2)\sin\theta\cos\theta & \cos^2\theta - Y_1Z_2\sin^2\theta \end{bmatrix}$$

Using Table 4.2 to convert to S-parameters gives the input reflection coefficient as,

$$\Gamma(\theta) = S_{11} + \frac{S_{12}S_{21}\Gamma_\ell}{1-S_{22}\Gamma_\ell} = \frac{A+B-C-D}{S} + \frac{4\Gamma_\ell/S^2}{1-\frac{-A+B-C+D}{S}\Gamma_\ell}$$

$$= \frac{(A+B-C-D)\left[S-\Gamma_\ell(-A+B-C+D)\right] + 4\Gamma_\ell}{S\left[S-\Gamma_\ell(-A+B-C+D)\right]}$$

where $S = A+B+C+D$, $\Gamma_\ell = \frac{R-1}{R+1}$.

This result can be equated to $2A(1+\cos 2\theta)$, and solved for Z_1 and Z_2, but this is a <u>very</u> lengthy procedure. Instead, we will first evaluate both expressions at $\theta = 90°$:

$\Gamma(90°) = 0$, and $\left.\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right|_{\theta=90°} = \begin{bmatrix} -Z_1Y_2 & 0 \\ 0 & -Y_1Z_2 \end{bmatrix}$

So $\Gamma(\theta)$ reduces to the following equation:

$$(-Z_1Y_2 + Y_1Z_2)\left[-(Y_1Z_2 + Z_1Y_2) - \Gamma_\ell\,(Z_1Y_2 - Y_1Z_2)\right] + 4\Gamma_\ell = 0$$

$$(Z_1^2Y_2^2 - Y_1^2Z_2^2) + \Gamma_\ell\,(Z_1^2Y_2^2 + Y_1^2Z_2^2 + 2) = 0$$

$$(Z_1^4 - Z_2^4) + \Gamma_\ell\,(Z_1^4 + Z_2^4 + 2Z_1^2Z_2^2) = 0$$

$$(Z_1^2 - Z_2^2) + \Gamma_\ell\,(Z_1^2 + Z_2^2) = 0$$

$$Z_2^2 = Z_1^2\,\frac{1+\Gamma_\ell}{1-\Gamma_\ell} = Z_1^2 R \;\Rightarrow\; \underline{\underline{Z_2 = Z_1\sqrt{R}}}\ \checkmark \quad (\text{for } Z_0 = 1)$$

another equation is harder to find, so we will make use of the fact that the transformer will be symmetric:

$$\frac{Z_1 - 1}{Z_1 + 1} = \frac{R - Z_2}{R + Z_2} = \frac{R/Z_2 - 1}{R/Z_2 + 1}$$

Thus, $\frac{R}{Z_2} = Z_1$ or $\underline{\underline{Z_1 = R^{1/4}}}$ (for $Z_0 = 1$)

If $R = 1.5$, these results reduce to,

$$Z_1 = (1.5)^{1/4} = 1.1067 \ \checkmark$$

$$Z_2 = 1.1067\sqrt{1.5} = 1.3554 \ \checkmark$$

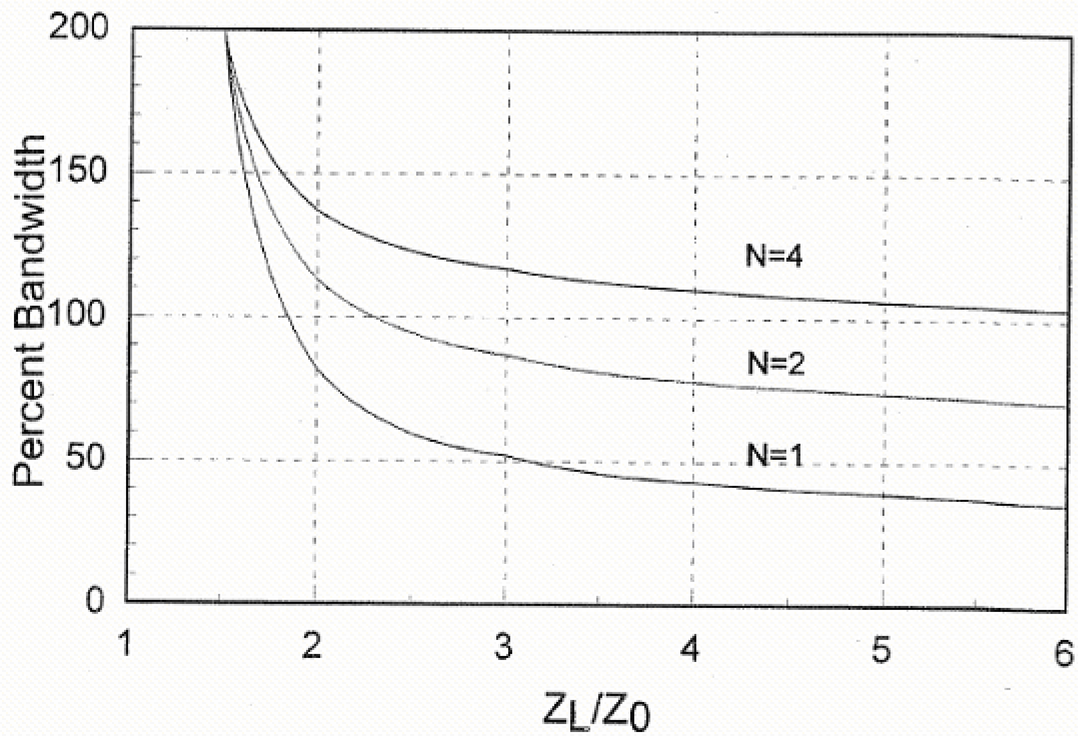
which agree with Table 5.1

5.18 From (5.55), the fractional bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi}\cos^{-1}\left[\frac{1}{2}\left(\frac{\Gamma_m}{A}\right)^{1/N}\right], \text{ with } A = 2^{-N}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|$$

Z_L/Z_0	N=1		N=2		N=4	
	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%	A	$\Delta f/f_0$-%
1.5	0.1000	200	0.0500	200	0.0125	200
2.0	0.1667	82	0.0833	113	0.0208	137
3.0	0.2500	52	0.1250	87	0.0313	117
4.0	0.3000	43	0.1500	78	0.0375	110
5.0	0.3333	39	0.1667	74	0.0417	106
6.0	0.3571	36	0.1786	71	0.0446	104

This data is plotted in the graph below.



5.19 $Z_0 = 50\,\Omega, \quad Z_L = 30\,\Omega, \quad SWR_{MAX} = 1.25$

$$|\Gamma_m| = \frac{SWR_m - 1}{SWR_m + 1} = 0.111 = A$$

From (5.61), $\Gamma(\theta) = 2e^{-j4\theta}\left[\Gamma_0 \cos 4\theta + \Gamma_1 \cos 2\theta + \tfrac{1}{2}\Gamma_2\right]$

$$= A e^{-j4\theta}\, T_4(\sec\theta_m \cos\theta)$$

$$= A e^{-j4\theta}\left[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m \cdot (\cos 2\theta + 1) + 1\right]$$

From (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\left|\frac{\ln Z_L/Z_0}{2\Gamma_m}\right|\right)\right]$$

$$= \cosh\left[\frac{1}{4}\cosh^{-1}\left(\frac{1}{2(.111)}\ln 30/50\right)\right] = 1.0687$$

so,

$$\theta_m = \cos^{-1}(1/\sec\theta_m) = 20.66^\circ$$

Equate $\cos 4\theta$ terms:

$$2\Gamma_0 = A\sec^4\theta_m \Rightarrow \Gamma_0 = 0.07246 = \Gamma_4$$

Equate $\cos 2\theta$ terms:

$$2\Gamma_1 = A(4\sec^4\theta_m - 4\sec^2\theta_m) \Rightarrow \Gamma_1 = 0.03607 = \Gamma_3$$

Equate constant terms:

$$\Gamma_2 = A(3\sec^4\theta_m - 4\sec^2\theta_m + 1) = 0.03831$$

Compute Z_n's: (reverse Z_L and Z_0)

$$Z_1 = Z_L\frac{1+\Gamma_0}{1-\Gamma_0} = 34.69\,\Omega$$

$$Z_2 = Z_1\frac{1+\Gamma_1}{1-\Gamma_1} = 37.29\,\Omega$$

$$Z_3 = Z_2\frac{1+\Gamma_2}{1-\Gamma_2} = 40.26\,\Omega$$

$$Z_4 = Z_3 \frac{1+\Gamma_3}{1-\Gamma_3} = 43.27\,\Omega$$

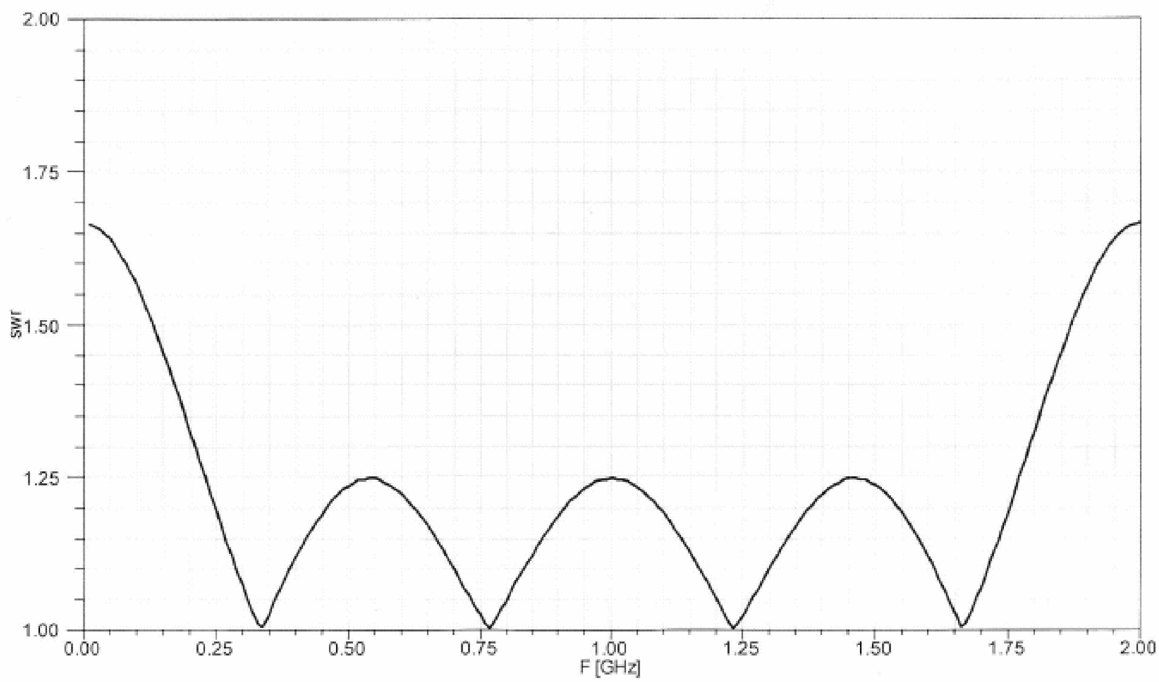
check: $Z_5 = Z_4 \frac{1+\Gamma_4}{1-\Gamma_4} = 50.03\,\Omega \simeq Z_0$ ✓

From (5.64) the bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 154\%$$

From the graph,

$$\frac{\Delta f}{f_0} \simeq \frac{1.77 - .225}{1} = 154.5\%$$ ✓



5.20 From (5.61) and (5.60b),

$$\Gamma(\theta) = A e^{-2j\theta} T_2(\sec\theta_m \cos\theta) = A e^{-2j\theta}\left[\sec^2\theta_m(1+\cos 2\theta) - 1\right]$$

$$\Gamma(0) = A\, T_2(\sec\theta_m) = \left|\frac{Z_L - Z_0}{Z_L + Z_0}\right| = \frac{R-1}{R+1} = 0.2 \quad ; \quad A = \Gamma_m = 0.05$$

As in Problem 5.17, we will evaluate $\Gamma(\theta)$ for $\theta = 90°$. Then $\Gamma(90°) = \Gamma_m$. Also, as in Problem 5.17, from symmetry we have that $Z_1 Z_2 = R$. Then,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -Z_1^2/R & 0 \\ 0 & -R/Z_1^2 \end{bmatrix} \qquad (Z_0 = 1)$$

$$\Gamma(90°) = \Gamma_m = \frac{\left(-Z_1^2/R + R/Z_1^2\right)\left[-\left(Z_1^2/R + R/Z_1^2\right) - \Gamma_\ell\left(Z_1^2/R - R/Z_1^2\right)\right] + 4\Gamma_\ell}{-\left(Z_1^2/R + R/Z_1^2\right)\left[-\left(Z_1^2/R + R/Z_1^2\right) - \Gamma_\ell\left(Z_1^2/R - R/Z_1^2\right)\right]}$$

$$= \frac{(R^2 - Z_1^4)\left[-(Z_1^4 + R^2) - \Gamma_\ell(Z_1^4 - R^2)\right] + 4\Gamma_\ell R^2 Z_1^4}{(Z_1^4 + R^2)\left[(Z_1^4 + R^2) + \Gamma_\ell(Z_1^4 - R^2)\right]}$$

$$\Gamma_m(Z_1^4 + R^2)^2 + \Gamma_m\Gamma_\ell(Z_1^4 + R^2)(Z_1^4 - R^2) = -(R^2 - Z_1^4)(Z_1^4 + R^2) + \Gamma_\ell(Z_1^4 - R^2)^2 + 4\Gamma_\ell R^2 Z_1^4$$

$$Z_1^8(\Gamma_m - 1)(\Gamma_\ell + 1) + 2Z_1^4R^2(\Gamma_m - \Gamma_\ell) - R^4(\Gamma_m + 1)(\Gamma_\ell - 1) = 0$$

For $\Gamma_m = 0.05$, $\Gamma_\ell = 0.2$, $R = 1.5$:

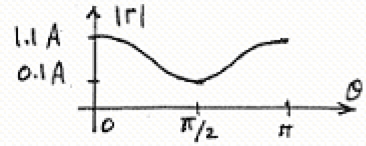
$$-1.140\, Z_1^8 - 0.6750\, Z_1^4 + 4.2525 = 0$$

$$Z_1^4 = \frac{0.675 \pm 4.455}{-2.280} = 1.65789 \Rightarrow Z_1 = 1.1347\, Z_0 \checkmark$$

$$Z_2 = R/Z_1 = 1.3219\, Z_0 \checkmark$$

These results agree with Table 5.2.

5.21 $|\Gamma(\theta)| = A(0.1 + \cos^2\theta)\ , \quad 0 < \theta < \pi$



From (5.46a), for $N=2$,

$$|\Gamma(\theta)| = 2\left(\Gamma_0 \cos 2\theta + \tfrac{1}{2}\Gamma_1\right) = A(0.1 + \cos^2\theta)$$

$$= A(0.6 + 0.5\cos 2\theta)$$

When $\theta = 0$, $|\Gamma(0)| = 1.1A = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{1.5-1}{1.5+1} = 0.2 \Rightarrow A = 0.182$

Equating coefficients of $\cos 2\theta$:

$$2\Gamma_0 = 0.5A \quad \Rightarrow \quad \Gamma_0 = 0.0455$$

Equating constant terms:

$$\Gamma_1 = 0.6A = 0.109$$

so the characteristic impedances are,

$$Z_1 = Z_0 \frac{1+\Gamma_0}{1-\Gamma_0} = 1.095\, Z_0$$

$$Z_2 = Z_1 \frac{1+\Gamma_1}{1-\Gamma_1} = 1.245\, Z_1 = 1.363\, Z_0$$

CHECK: at $\theta = \pi/2$, the input impedance to the transformer will be,

$$Z_{in} = \frac{Z_1^2}{(Z_2^2/Z_L)} = \frac{Z_L Z_0 Z_1^2}{Z_2^2} = 0.968\, Z_0$$

So the input reflection coefficient is,

$$\Gamma_{in} = \left|\frac{Z_{in} - Z_0}{Z_{in} + Z_0}\right| = 0.016$$

which is reasonably close to $|\Gamma(\pi/2)| = 0.1A = 0.018$

5.22

$$\frac{d(\ln Z/Z_0)}{dz} = A \sin\frac{\pi z}{L}$$

$$\ln(Z/Z_0) = B - \frac{LA}{\pi}\cos\frac{\pi z}{L}$$

$$Z(z) = C e^{-\frac{LA}{\pi}\cos\frac{\pi z}{L}}$$

$$Z(0) = Z_0 = C e^{-LA/\pi} \quad , \quad Z(L) = Z_L = C e^{+LA/\pi}$$

Solve for C, A to get,

$$C = \sqrt{Z_0 Z_L}$$

$$A = \frac{-\pi}{2L}\ln(Z_0/Z_L) \checkmark$$

From (5.67),

$$\Gamma(\theta) = \frac{1}{2}\int_{z=0}^{L} e^{-2j\beta z}\frac{d}{dz}(\ln Z/Z_0)\,dz$$

$$= \frac{1}{2}\int_{z=0}^{L} A \sin\frac{\pi z}{L} e^{-2j\beta z}\,dz$$

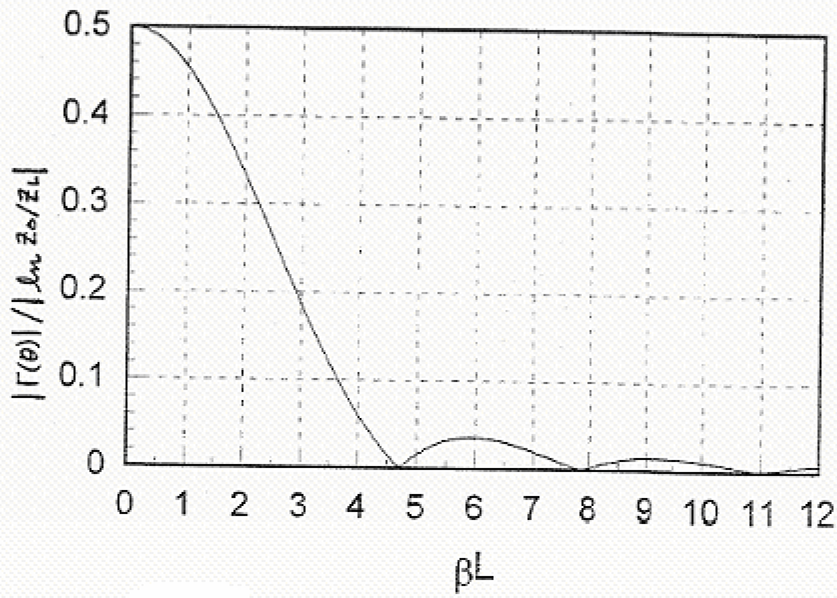
$$= \frac{A}{2}\left.\frac{e^{-2j\beta z}\left[-2j\beta\sin\frac{\pi z}{L} - \frac{\pi}{L}\cos\frac{\pi z}{L}\right]}{(\pi/L)^2 - 4\beta^2}\right|_0^L$$

$$= \frac{\pi A}{2L} e^{-j\beta L}\frac{(e^{-j\beta L} + e^{j\beta L})}{(\pi/L)^2 - 4\beta^2}$$

So,

$$|\Gamma(\theta)| = \frac{\pi^2}{2}\left|\ln\frac{Z_0}{Z_L}\right|\left|\frac{\cos\beta L}{\pi^2 - (2\beta L)^2}\right| \checkmark$$

This result is plotted as shown:



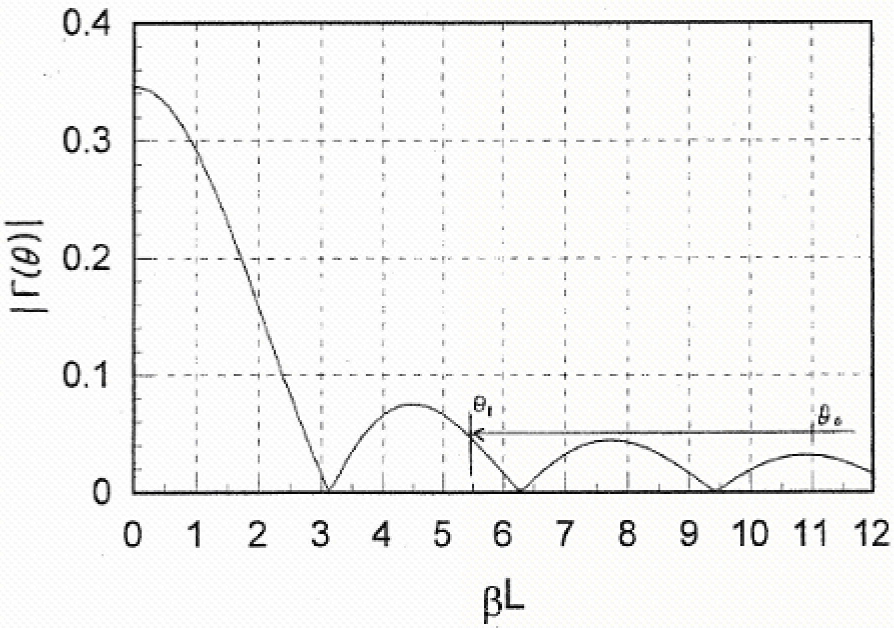
5.23 From (5.68), $Z(z) = Z_0 e^{az}$ for $0 < z < L$.

$$a = \frac{1}{L} \ln \frac{Z_L}{Z_0} = \frac{0.693}{L}$$

From (5.70),

$$|\Gamma(\theta)| = \frac{1}{2} \left| \ln \frac{Z_L}{Z_0} \right| \left| \frac{\sin \beta L}{\beta L} \right| = 0.346 \left| \frac{\sin \beta L}{\beta L} \right| \checkmark$$

This result is plotted in the graph shown below:



We see that the lower frequency limit for $|\Gamma| \leq 0.05$ is $\theta_1 = 5.5$. To obtain 100% bandwidth, we must have,

$$\frac{\theta_2 - \theta_1}{(\theta_1 + \theta_2)/2} = 1 \quad , \text{or} \quad \theta_2 = 3\theta_1 = 16.5$$

Then at the center frequency,

$$\theta_0 = \frac{\theta_1 + \theta_2}{2} = 11.0 = \beta L$$

So,

$$L = \frac{11 \lambda_0}{2\pi} = 1.75 \lambda_0 \checkmark$$

From (5.64), θ_m for a Chebyshev transformer with 100% bandwidth is,

$$\frac{\Delta f}{f_0} = 2 - \frac{4\theta_m}{\pi} = 1 \implies \theta_m = \pi/4 .$$

Then from (5.63),

$$\sec\theta_m = \cosh\left[\frac{1}{N}\cosh^{-1}\left(\frac{1}{\Gamma_m}\left|\frac{Z_L - Z_0}{Z_L + Z_0}\right|\right)\right]$$

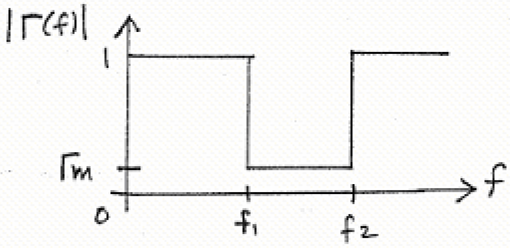
$$1.414 = \cosh\left[\frac{1}{N}(2.5846)\right] \implies N = 2.93 \Rightarrow \underline{N=3}$$

So $N=3$ sections would be required, for a length of $3\lambda_0/4$ at the center frequency. ✓
This is much shorter than the exponential taper matching section.

5.24 From Figure 5.22 the Bode-Fano limit for a parallel RC load is,

$$\int_0^\infty \ln\frac{1}{|\Gamma(\omega)|}\,d\omega \le \frac{\pi}{RC}$$

The optimum reflection coefficient magnitude response will be as shown:



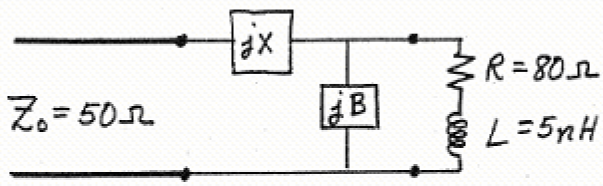
$$\text{Thus, } \ln\frac{1}{\Gamma_m} \le \frac{\pi}{\Delta\omega RC} = \frac{\pi}{2\pi(10.6-3.1)\times10^9(75)(0.6\times10^{-12})}$$

$$\le 1.48$$

$$\Gamma_m > 0.228 \implies RL < \underline{6.4\,dB}$$

5.25

L-section matching solution:



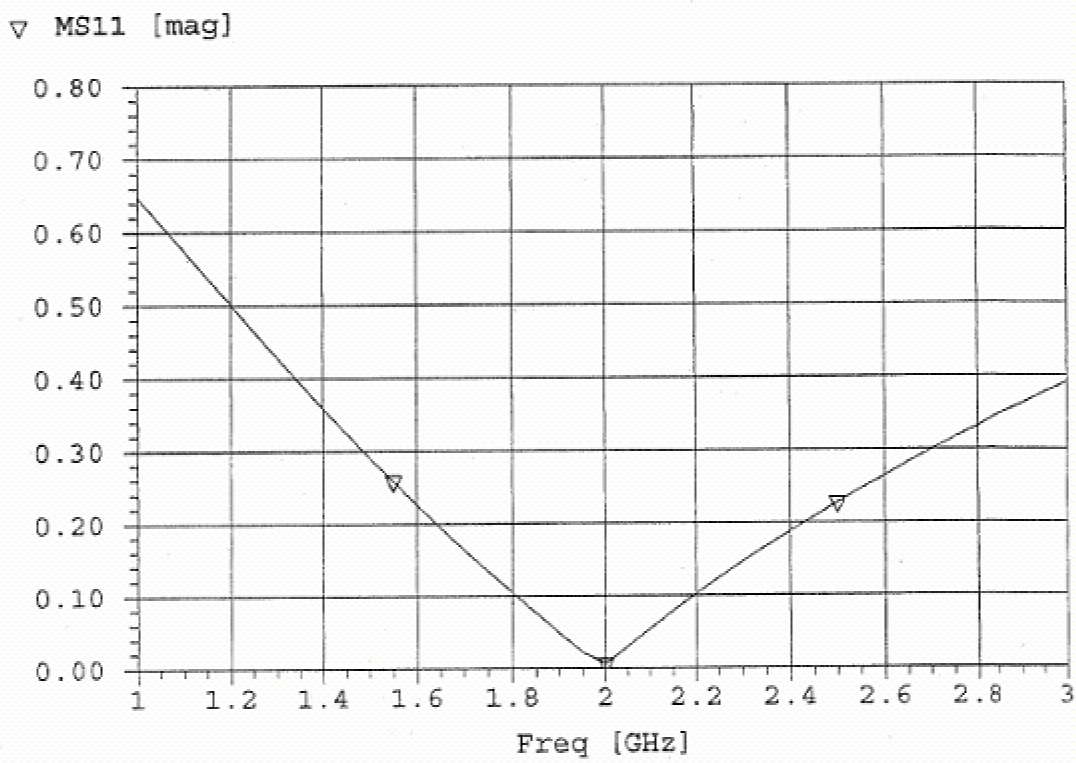
at $f = 2\,GHz$, $Z_L = 80 + j63\,\Omega$, $z_L = 1.6 + j1.26$ (INSIDE $1 + jx$)

a Smith chart solution gives,

$jb = -j1.8 \Rightarrow$ INDUCTOR with $L = 22.1\,nH$. ✓

$jx = -j1.25 \Rightarrow$ CAPACITOR with $C = 1.27\,pF$. ✓

The input reflection coefficient magnitude is plotted below, where it is seen that the bandwidth for $|\Gamma| < 0.1$ is 20%.



Bode-Fano limit:

From Figure 5.22d, the Bode-Fano criteria gives a bandwidth limit of

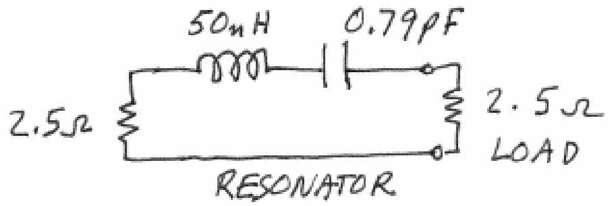
$$\Delta\omega = \frac{\pi R}{L} \frac{1}{\ln 1/\Gamma_m} = 2.18 \times 10^{10} = \omega_2 - \omega_1$$

$$\frac{\Delta f}{f_0} = \frac{f_2 - f_1}{f_0} = \frac{2.18 \times 10^{10}}{2\pi\,(2 \times 10^9)} = 174\%$$

This is considerably more than the bandwidth of the L-section match.

Chapter 6

6.1



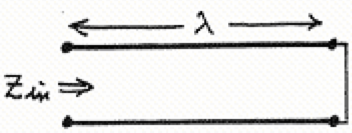
$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 800\,MHz$$

$$Q_0 = \frac{\omega_0 L}{R} = 100$$

$$Q_e = \frac{\omega_0 L}{R_L} = 100$$

$$Q_L = 50$$

6.2



$\ell = \lambda = \frac{2\pi v_p}{\omega_0}$ for $\omega = \omega_0$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ resonator. Thus, let

$$\beta\ell = \frac{\omega_0 \ell}{v_p} + \frac{\Delta\omega\,\ell}{v_p} = 2\pi\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then from (6.24) the input impedance is,

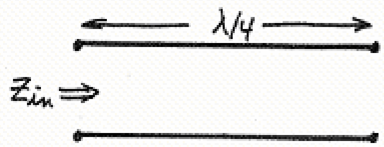
$$Z_{in} \simeq Z_0 \frac{\alpha\ell + j2\pi\frac{\Delta\omega}{\omega_0}}{1 + j2\pi\frac{\Delta\omega}{\omega_0}} \simeq Z_0\left(\alpha\ell + j2\pi\frac{\Delta\omega}{\omega_0}\right) = R + 2jL\Delta\omega$$

Thus, $\quad R = Z_0\alpha\ell \quad , \quad L = \frac{\pi Z_0}{\omega_0}$.

and,

$$Q = \frac{\omega_0 L}{R} = \frac{\pi Z_0}{Z_0 \alpha\ell} = \frac{\pi}{\alpha\ell} = \frac{\beta}{2\alpha} \quad \left(\text{since } \ell = \lambda = \frac{2\pi}{\beta} \text{ at res.}\right)$$

6.3



$$\ell = \frac{\lambda}{4} = \frac{\pi v_p}{2\omega_0} \quad \text{for } \omega = \omega_0$$

This circuit has a series-type resonance, like the short-circuited $\lambda/2$ line. So let,

$$\beta\ell = \frac{\omega_0 \ell}{v_p} + \frac{\Delta\omega \ell}{v_p} = \frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right)$$

Then,

$$\tan\beta\ell = \tan\frac{\pi}{2}\left(1 + \frac{\Delta\omega}{\omega_0}\right) = -\cot\frac{\Delta\omega\pi}{2\omega_0} \simeq \frac{-2\omega_0}{\pi\Delta\omega}$$

The input impedance is,

$$Z_{in} = Z_0 \frac{1 + j\tan\beta\ell \tanh\alpha\ell}{\tanh\alpha\ell + j\tan\beta\ell} \simeq \frac{1 - j\frac{2\omega_0}{\pi\Delta\omega}\alpha\ell}{\alpha\ell - j\frac{2\omega_0}{\pi\Delta\omega}}$$

$$\simeq Z_0 \frac{\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}}{1 + j\frac{\pi\Delta\omega}{2\omega_0}\alpha\ell} \simeq Z_0\left(\alpha\ell + j\frac{\pi\Delta\omega}{2\omega_0}\right) = R + 2jL\Delta\omega$$

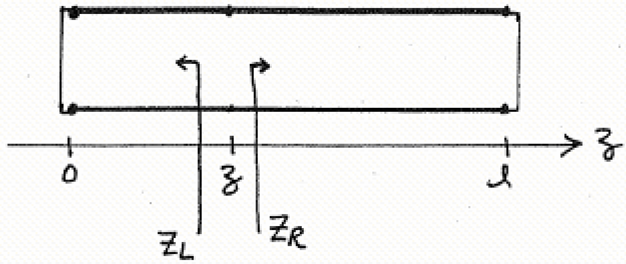
$$\therefore\ R = Z_0\alpha\ell \ \checkmark \quad , \quad L = \frac{\pi Z_0}{4\omega_0} \ \checkmark$$

Then,

$$Q = \frac{\omega_0 L}{R} = \frac{\pi}{4\alpha\ell} = \frac{\beta}{2\alpha} \ \checkmark$$

(since $\ell = \frac{\lambda}{4} = \frac{\pi}{2\beta}$ at resonance)

6.4

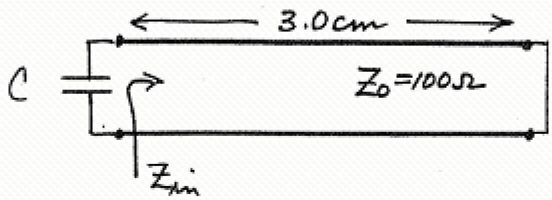


$\beta l = \pi$

$Z_L = j Z_0 \tan \beta z$

$Z_R = j Z_0 \tan \beta(l-z) = j Z_0 \tan(\pi - \beta z) = -j Z_0 \tan \beta z = Z_L^*$ ✓

6.5



$f_0 = 6$ GHz

$\beta = \frac{2\pi f}{c} = 125.7\ m^{-1}$ for an air-filled line

$\beta l = (125.7)(0.03) = 216°$ ✓

$Z_{in} = j Z_0 \tan \beta l = j(100) \tan 216° = j\,72.6\Omega = j\omega L$ ✓

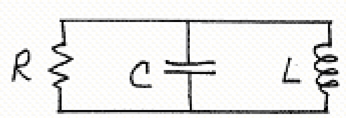
To achieve resonance we must have,

$$Z_{in} = (jX_c)^* = \frac{j}{\omega C}$$

So,

$$C = \frac{1}{\omega X_{in}} = 0.365\,pF \checkmark$$

The equivalent circuit at 6 GHz, with the shunt resistor, is as follows:



$R = 10{,}000\ \Omega$

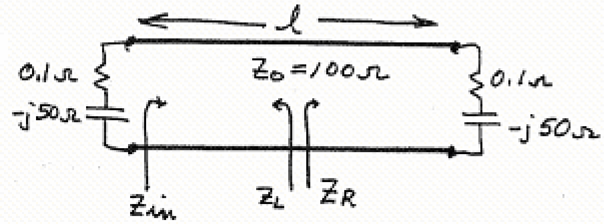
$C = 0.365\ pF$

$L = \frac{X_{in}}{\omega} = \frac{72.6}{2\pi(6 \times 10^9)} = 1.93\ nH$ ✓

So the Q is,

$$Q = \omega RC = 2\pi(6\times10^9)(10{,}000)(0.365\times10^{-12}) = 138. \checkmark$$

6.6



Since the resonator is symmetrical, at the midpoint of the line we must have, $Z_L = Z_R^* = Z_R$, or $\text{Im}\{Z_R\} = 0$:

Let $t = \tan \beta l/2$ and $Z_L = R_L + jX_L$. $(R_L = 0.1,\ X_L = -50.)$

$$Z_R = Z_0 \frac{Z_L + jZ_0 t}{Z_0 + jZ_L t} = Z_0 \frac{R_L + j(X_L + Z_0 t)}{(Z_0 - X_L t) + jR_L t}$$

$$= Z_0 \frac{R_L(Z_0 - X_L t) + R_L t(X_L + Z_0 t) + j(X_L + Z_0 t)(Z_0 - X_L t) - jR_L^2 t}{(Z_0 - X_L t)^2 + (R_L t)^2}$$

$$\text{Im}\{Z_R\} = 0 \Rightarrow \quad (X_L + Z_0 t)(Z_0 - X_L t) - R_L^2 t = 0$$

$$-X_L Z_0 t^2 + (Z_0^2 - X_L^2 - R_L^2)t + Z_0 X_L = 0$$

$$5000t^2 + 7500t - 5000 = 0$$

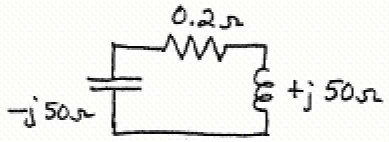
$$t^2 + 1.5t - 1 = 0$$

$$t = \frac{-1.5 \pm \sqrt{(1.5)^2 + 4}}{2} = -0.75 \pm 1.25 = \begin{cases} 0.50 \Rightarrow \beta l = 53.1^\circ \\ -2.00 \Rightarrow \beta l = -126.9^\circ = 53.1^\circ \end{cases}$$

So,

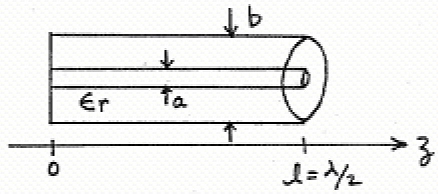
$$l = \frac{53.1^\circ}{360^\circ}\lambda = 0.148\lambda \qquad \tan \beta l = 1.332$$

CHECK:
$$Z_{in} = 100 \frac{(.1 - j50) + j133.2}{100 + j(.1 - j50)(1.332)} = 0.1 + j50\,\Omega \ \checkmark$$



$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{50}{0.2} = 250.$$

6.7



$\beta \ell = \pi$

$\beta = \pi/\ell$

$\eta = \sqrt{\mu_0/\epsilon}$

From Section 2.2 the TEM fields of a coaxial line are,

$$\bar{E}^{\pm} = \hat{\rho}\frac{V_0}{\rho \ln b/a} e^{\mp j\beta z} \quad , \quad \bar{H}^{\pm} = \pm\hat{\phi}\frac{V_0}{\eta \ln b/a} e^{\mp j\beta z}$$

$E_\rho = 0$ at $z = 0$ in the resonator, so the standing wave fields can be written as,

$$E_\rho = \frac{V_0}{\rho \ln b/a}\left[e^{-j\beta z} - e^{j\beta z}\right] = \frac{-2jV_0}{\rho \ln b/a} \sin \beta z$$

$$H_\phi = \frac{V_0}{\eta \ln b/a}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0}{\eta \ln b/a} \cos \beta z$$

From (1.84) and (1.86) the time-average stored electric and magnetic energies are,

$$W_e = \frac{\epsilon}{4}\int_v |\bar{E}|^2 dv = \frac{\epsilon}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\rho \ln b/a}\right)^2 \sin^2\frac{\pi z}{\ell}\, \rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi \epsilon V_0^2}{\ln b/a}$$

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell}\left(\frac{2V_0}{\eta \ln b/a}\right)^2 \cos^2\frac{\pi z}{\ell}\, \rho\, dz\, d\phi\, d\rho$$

$$= \frac{\pi \mu_0 V_0^2}{\eta^2 \ln b/a} = \frac{\pi \epsilon V_0^2}{\ln b/a} = W_e \ \checkmark$$

6.8

$$Z_{in} = \frac{Z_0^2}{Z_L} = \frac{Z_0^2}{R + j(\omega L - \frac{1}{\omega C})} = \frac{1}{\frac{R}{Z_0^2} + j\omega\left(\frac{L}{Z_0^2} - \frac{1}{\omega^2 C Z_0^2}\right)}$$

The input impedance of a parallel RLC circuit is,

$$Z_{in} = \frac{1}{\frac{1}{R'} + \frac{1}{j\omega L'} + j\omega C'} = \frac{1}{\frac{1}{R'} + j\omega(C' - \frac{1}{\omega^2 L'})}$$

Thus the original circuit acts as a parallel $R'L'C'$ resonator with $R' = Z_0^2/R$, $C' = L/Z_0^2$, $L' = C Z_0^2$.
(This is the basis for using $\lambda/4$ lines as impedance and admittance inverters.)

6.9

air-filled, aluminium, X-band,
$d = 2.0$ cm, $a = 2.286$ cm, $b = 1.016$ cm
$\sigma_{al} = 3.816 \times 10^7$ S/m

$$f_{101} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = 9.965 \text{ GHz} \checkmark \quad R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0321\,\Omega$$
$$k = 208.7 \text{ m}^{-1}$$

$$f_{202} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2 + \left(\frac{2\pi}{d}\right)^2} = 16.372 \text{ GHz} \checkmark \quad R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.0412\,\Omega$$
$$k = 342.9 \text{ m}^{-1}$$

From (6.46), $(2\ell^2 a^3 b + 2bd^3 + \ell^2 a^3 d + ad^3) =$
$= 34.54 + 48.17\,\ell^2$ cm^4

$$Q_{101} = \frac{k^3 a^3 d^3 b \eta_0}{2\pi^2 R_s}\,\frac{1}{(34.54 + 48.17)10^{-8}} = 6349. \checkmark$$

$$Q_{102} = \frac{k^3 a^3 d^3 b \eta_0}{2\pi^2 R_s}\,\frac{1}{(34.54 + 4 \times 48.17)10^{-8}} = 7987. \checkmark$$

verified w/ RECCAVITY.FOR

6.10 From Table 3.2, the magnetic fields of the TM_{11} waveguide mode are,

$$H_x^{\pm} = \frac{B^{\pm}}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} e^{\mp j\beta z}$$

$$H_y^{\pm} = \frac{B^{\pm}}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} e^{\mp j\beta z}$$

To have current maxima at $z=0,d$ the cavity fields must be,

$$H_x = \frac{A}{b} \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

$$H_y = \frac{A}{a} \cos\frac{\pi x}{a} \sin\frac{\pi y}{b} \cos\frac{\pi z}{d}$$

The stored magnetic energy is,

$$W_m = \frac{\mu_0}{4}\int_v |\bar{H}|^2 dv = \frac{\mu_0}{4} A^2 \frac{a}{2}\frac{b}{2}\frac{d}{2}\left(\frac{1}{b^2}+\frac{1}{a^2}\right) = \frac{abd\mu_0 A^2}{32}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$$

The power lost in the walls is,

$$P_\ell = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds = R_s\left\{\int_{x=0}^{a}\int_{z=0}^{d} |H_x(y=0)|^2 dz\,dx + \int_{y=0}^{b}\int_{z=0}^{d} |H_y(x=0)|^2 dy\,dz + \right.$$

$$\left. + \int_{x=0}^{a}\int_{y=0}^{b}\left[|H_x(z=0)|^2 + |H_y(z=0)|^2\right] dx\,dy\right\}$$

$$= \frac{A^2 R_s}{4}\,\frac{a^3 d + b^3 d + a^3 b + a b^3}{a^2 b^2}$$

Then,

$$Q = \frac{\omega_0 (W_e + W_m)}{P_\ell} = \frac{2\omega_0 W_m}{P_\ell} = \frac{k_0 \eta_0}{4 R_s}\,\frac{abd\,(a^2+b^2)}{(a^3 d + b^3 d + a^3 b + a b^3)} \checkmark$$

6.11 From Section 3.3 the transverse fields of the TE_{10} mode in the two regions can be written as,

$$E_y = \begin{cases} A \sin \frac{\pi x}{a} \sin \beta_a z & \text{for } 0<z<d-t \\ B \sin \frac{\pi x}{a} \sin \beta_d (d-z) & \text{for } d-t<z<d \end{cases}$$

$$H_x = \begin{cases} -j \frac{A}{Z_a} \sin \frac{\pi x}{a} \cos \beta_a z & \text{for } 0<z<d-t \\ -j \frac{B}{Z_d} \sin \frac{\pi x}{a} \cos \beta_d (d-z) & \text{for } d-t<z<d \end{cases}$$

where $\beta_a = \sqrt{k_o^2 - (\pi/a)^2}$, $\beta_d = \sqrt{\epsilon_r k_o^2 - (\pi/a)^2}$

$Z_a = k_o \eta_o / \beta_a$, $Z_d = k\eta/\beta_d = k_o \eta_o / \beta_d$

Continuity of E_y, H_x at $z=d-t$:

E_y: $\quad A \sin \beta_a (d-t) = B \sin \beta_d t$

H_x: $\quad \frac{A}{Z_a} \cos \beta_a (d-t) = \frac{B}{Z_d} \cos \beta_d t$

Divide to obtain:

$$Z_a \tan \beta_a (d-t) = Z_d \tan \beta_d t$$

$$\beta_d \tan \beta_a (d-t) = \beta_a \tan \beta_d t$$

This equation can be solved for k_o. β_a and β_d are functions of k_o as given above.

6.12 TM modes: $(\nabla^2 + k^2)E_z = 0$

Let $E_z(x,y,z) = X(x)Y(y)Z(z)$.
Substitute into wave equation and divide by XYZ:

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + \frac{1}{Z}\frac{d^2Z}{dz^2} + k^2 = 0$$

By the separation of variables argument,

$$\frac{1}{X}\frac{d^2X}{dx^2} = -k_x^2 \quad \Rightarrow X(x) = A\cos k_x x + B\sin k_x x$$

$$\frac{1}{Y}\frac{d^2Y}{dy^2} = -k_y^2 \quad \Rightarrow Y(y) = C\cos k_y y + D\sin k_y y$$

$$\frac{1}{Z}\frac{d^2Z}{dz^2} = -k_z^2 \quad \Rightarrow Z(z) = E\cos k_z z + F\sin k_z z$$

with $k^2 = k_x^2 + k_y^2 + k_z^2$.

Now, $E_z = 0$ for $x = 0, a$ and $y = 0, b$. Therefore, $A = C = 0$ and $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$. To enforce the remaining boundary conditions, we need E_x or E_y: From Maxwell's equations,

$$E_x = \frac{1}{k^2 - k_z^2}\frac{\partial^2 E_z}{\partial x \partial z} = \frac{1}{k^2 - k_z^2}(Bk_x\cos k_x x)(D\sin k_y y)\cdot$$

$$\cdot(-k_z E\sin k_z z + k_z F\cos k_z z)$$

For $E_x = 0$ at $z = 0, d$ we must have $F = 0$, and $k_z = \frac{\ell\pi}{d}$.

Thus, $$k^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{\ell\pi}{d}\right)^2,$$

which determines the resonant frequencies. The solution for TE modes is similar.

6.13 From Table 3.5 the fields of the TM_{nm0} mode are ($\beta=0$):

$$E_z = A \sin n\phi \, J_n(k_c\rho)$$

$$H_\rho = \frac{j\omega\epsilon n}{k_c^2 \rho} A \cos n\phi \, J_n(k_c\rho)$$

$$H_\phi = \frac{-j\omega\epsilon}{k_c} A \sin n\phi \, J_n'(k_c\rho) \,, \qquad k_c = p_{nm}/a = k$$

The stored electric energy is,

$$W_e = \frac{\epsilon}{4}\int_v |\bar{E}|^2 dv = \frac{A^2\epsilon}{4}\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \sin^2 n\phi \, J_n^2(k_c\rho)\,\rho \, d\rho \, d\phi \, dz$$

$$= \frac{A^2\epsilon}{4}\pi d \frac{a^2}{2} J_n'^2(p_{nm}) = \frac{A^2 a^2 \pi d \epsilon}{8} J_n'^2(p_{nm}) \quad \text{(using C.14}$$

The power loss due to finite conductivity is,

$$P_\ell = \frac{R_s}{2}\int_s |\bar{H}_t|^2 ds$$

$$= \frac{R_s}{2}\left\{\int_{\phi=0}^{2\pi}\int_{z=0}^{d} |H_\phi(\rho=a)|^2 a \, d\phi \, dz + 2\int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\left[|H_\rho|^2 + |H_\phi|^2\right]\rho \, d\rho \, d\phi\right\}$$

$$= \frac{A^2 R_s}{2}\left\{\frac{\pi a d}{\eta^2} J_n'^2(p_{nm}) + \frac{2\pi}{\eta^2}\frac{p_{nm}^2}{2k_c^2} J_n'^2(p_{nm})\right\}$$

$$= \frac{A^2 R_s \pi}{2\eta^2}(ad + a^2) J_n'^2(p_{nm})$$

Then, $$Q_c = \frac{2\omega W_e}{P_\ell} = \frac{\omega a^{\cancel{2}} \pi d \epsilon (2\eta^2)}{4 R_s \pi \cancel{a}(d+a)} = \frac{a d k \eta}{2 R_s (d+a)} \checkmark$$

The power lost in the dielectric is,

$$P_d = \frac{\omega\epsilon''}{2}\int_v |\bar{E}|^2 dv = \frac{\omega\epsilon}{2}\tan\delta \int_v |\bar{E}|^2 dv = \frac{2kW_e}{\eta\epsilon}\tan\delta$$

So, $$Q_d = \frac{2\omega W_e}{P_\ell} = \frac{1}{\tan\delta} \checkmark \qquad \text{(as in (6.48))}$$

6.14 From Figure 6.10, maximum Q for the TE_{111} mode occurs for $2a/d \simeq 1.7$. From (6.53a) the resonant frequency is,

$$f_{111} = \frac{c}{2\pi\sqrt{\epsilon_r}}\sqrt{\left(\frac{p'_{11}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2} = \frac{3\times10^8}{2\pi\sqrt{1.5}}\sqrt{\left(\frac{1.841}{a}\right)^2 + \left(\frac{1.7\pi}{2a}\right)^2}$$

$$= \frac{1.264\times10^8}{a} = 6\times10^9\,Hz \Rightarrow \underline{a = 2.107\ cm}$$

$$d = \frac{2a}{1.7} = \underline{2.479\ cm}$$

$\sigma_{AU} = 4.1\times10^7$ S/m , $R_S = \sqrt{\frac{\omega\mu_0}{2\sigma}} = 0.024\ \Omega$

$k = 2\pi f\sqrt{\epsilon_r}/c = 153.9\ m^{-1}$

$\beta = \frac{\pi}{d} = 126.7\ m^{-1}$

From (6.57) the unloaded Q is, (due to conductor losses)

$$Q_c = \frac{(ka)^3\eta ad\left[1-\left(\frac{1}{p'_{11}}\right)^2\right]}{4(p'_{11})^2 R_s\left\{\frac{ad}{2}\left[1+\left(\frac{\beta a}{p'^2_{11}}\right)^2\right]+\left(\frac{\beta a^2}{p'_{11}}\right)^2\left(1-\frac{1}{p'^2_{11}}\right)\right\}}$$

$= 10{,}985$ ✓

The unloaded Q due to dielectric loss is

$$Q_d = \frac{1}{\tan\delta} = 2{,}000 \ ✓$$

Then the total Q is,

$$Q = \frac{1}{\frac{1}{Q_d}+\frac{1}{Q_c}} = 1{,}692 \ ✓$$

(results checked with FORTRAN program CIRCAVITY.FOR)

6.15 Choose coordinate system so that $b<a<d$. Then the dominant resonant mode is the TE_{101} mode:

$$f_{101} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{d}\right)^2} = 5.2\ GHz$$

or, $$\frac{1}{a^2}+\frac{1}{d^2} = \left(\frac{2f_{101}}{c}\right)^2 = (34.7)^2$$

The next two higher modes must be either the TM_{110}, TE_{102}, or TE_{011} modes:

$$\left(\frac{2f_{110}}{c}\right)^2 = \frac{1}{a^2}+\frac{1}{b^2} = (34.7)^2 + \frac{1}{b^2}-\frac{1}{d^2}$$

$$\left(\frac{2f_{102}}{c}\right)^2 = \frac{1}{a^2}+\frac{4}{d^2} = (34.7)^2+\frac{3}{d^2}$$

$$\left(\frac{2f_{011}}{c}\right)^2 = \frac{1}{b^2}+\frac{1}{d^2}$$

Since $d>a$, $f_{011} < f_{110}$

Try $f_{011} = 6.5\ GHz$; $f_{110} = 7.2\ GHz$

Then we have, $$\frac{1}{b^2}-\frac{1}{d^2} = 1100.$$

$$\frac{1}{b^2}+\frac{1}{d^2} = 1878.$$

Solving gives,

$b = 2.60$ cm ✓

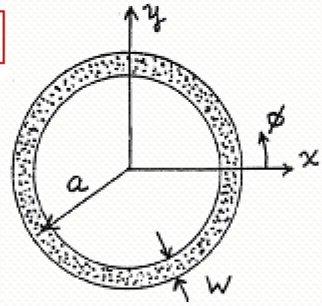
$d = 5.00$ cm ✓

$a = 3.53$ cm ✓

CHECK: $b<a<d$ OK ✓

$f_{102} = 7.35\ GHz > f_{110} = 7.2\ GHz$ ✓

6.16



$$e^{\pm j\beta a\phi} = e^{\pm jn\phi} \quad , n=1,2,3\ldots$$

FOR PERIODICITY

So, $$\beta a = \frac{2\pi a}{\lambda_g} = \frac{2\pi a\sqrt{\epsilon_e}\, f}{c} = n$$

$$f = \frac{nc}{2\pi a\sqrt{\epsilon_e}} \; ; \quad n=1,2,3\ldots$$

(The ring circumference is $2\pi a = n\lambda_g$)

The above result assumes $a \gg W$, so that curvature effects can be neglected. This type of resonator is most often coupled using a gap feed to a microstripline.

6.17

For TM_{nm0} modes we have $H_z=0$ and $\frac{\partial}{\partial z}=0$. The wave equation for E_z is,

$$\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + k^2\right)E_z = 0 \qquad \text{(from 3.134)} \; (\beta = k)$$

The general solution is,

$$E_z = (A_n \cos n\phi + B_n \sin n\phi)\, J_n(k\rho) \qquad \text{(finite at } \rho=0\text{)}$$

Since the choice of $\sin n\phi$ or $\cos n\phi$ (or any combination) depends only on the choice of the $\phi=0$ reference, we can let $B_n=0$.

Then,

$$E_z = A_n \cos n\phi \, J_n(k\rho)$$

We can find H_ϕ from (3.110d):

$$H_\phi = \frac{-j\omega\epsilon}{k^2}\frac{\partial E_z}{\partial\rho} = \frac{-j\omega\epsilon}{k} A_n \cos n\phi\, J_n'(k\rho)$$

For $H_\phi=0$ at $\rho=a$ we require $J_n'(ka)=0$, or $ka = p'_{nm}$.

So the resonant frequency is,

$$f_{nm0} = \frac{ck}{2\pi\sqrt{\epsilon_r}} = \frac{c\,p'_{nm}}{2\pi a\sqrt{\epsilon_r}}$$

and,

$$f_{110} = \frac{c\,p'_{11}}{2\pi a\sqrt{\epsilon_r}} = \frac{1.841c}{2\pi a\sqrt{\epsilon_r}} \; ✓$$

This solution neglects the effect of fringing fields.

6.18 From (6.70), $\tan \beta L/2 = \alpha/\beta$,

with
$$\alpha = \sqrt{\left(\frac{2.405}{a}\right)^2 - k_o^2}$$
$$\beta = \sqrt{\epsilon_r k_o^2 - (2.405/a)^2}$$

The value of k_o at resonance must lie between $k_o = \frac{2.405}{a} = 602$, and $k_o = \frac{2.405}{a\sqrt{\epsilon_r}} = 100$.

We carry out a trial-and-error numerical search as follows:

k_o	α	β	$\tan \beta L/2 - \alpha/\beta$
110	592	275	−1.8
120	590	399	−1.02
150	583	672	.008
145	584	631	−.12
→ 149	583	664	−.0018

Thus, the resonant frequency is,

$$f_o = \frac{c k_o}{2\pi} = 7.11 \text{ GHz} \checkmark$$

(measured value is 7.8 GHz)

6.19 Following the analysis of Section 6.5, for $TE_{01\delta}$ mode:

$$H_z = H_0 J_0(k_c\rho)e^{\pm j\beta z}$$

$$E_\phi = \frac{j\omega\mu_0 H_0}{k_c} J_0'(k_c\rho)e^{\pm j\beta z} = A J_0'(k_c\rho)e^{\pm j\beta z}$$

$$H_\rho = \frac{\mp j\beta H_0}{k_c} J_0'(k_c\rho)e^{\pm j\beta z} = \frac{\mp A}{Z_{TE}} J_0'(k_c\rho)e^{\pm j\beta z}$$

for $|z| < L/2$, $\beta = \sqrt{\epsilon_r k_0^2 - k_c^2} = \sqrt{\epsilon_r k_0^2 - (p_{01}/a)^2}$; $Z_{TE} = \frac{\omega\mu_0}{\beta} = Z_d$

for $|z| > L/2$, $j\beta = \alpha = \sqrt{k_c^2 - k_0^2} = \sqrt{(p_{01}/a)^2 - k_0^2}$; $Z_{TE} = \frac{j\omega\mu_0}{\alpha} = Z_a$

So the standing wave fields can be written as,

$$E_\phi = \begin{cases} A J_0'(k_c\rho)\left[e^{-j\beta z} - e^{j\beta z}\right] = -2jA J_0'(k_c\rho)\sin\beta z & \text{for } |z| < L/2 \\ B J_0'(k_c\rho)e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

$$H_\rho = \begin{cases} \frac{A}{Z_d} J_0'(k_c\rho)\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2A}{Z_d} J_0'(k_c\rho)\cos\beta z & \text{for } |z| < L/2 \\ \frac{B}{Z_a} J_0'(k_c\rho)e^{-\alpha z} & \text{for } z > L/2 \end{cases}$$

Continuity of E_ϕ and H_ρ at $z = L/2$ gives:

$$E_\phi: \quad -2jA\sin\beta L/2 = Be^{-\alpha L/2}$$

$$H_\rho: \quad \frac{2A}{Z_d}\cos\beta L/2 = \frac{B}{Z_a}e^{-\alpha L/2}$$

dividing gives:

$$-jZ_d\tan\beta L/2 = Z_a$$

$$\frac{-j}{\beta}\tan\beta L/2 = j/\alpha$$

$$\tan\beta L/2 + \beta/\alpha = 0 \quad ✓$$

6.20 assume $a>b$

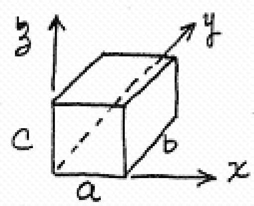
Because of the magnetic wall boundary conditions on the sidewalls, a rectangular dielectric waveguide along the z-axis would support TE modes with an H_z field of the form,

$$H_z = H_0 \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b},$$

so the lowest order TE mode would be the TE_{11} mode. But $H_z \equiv 0$ for TM modes, so the lowest order TM mode would have,

$$H_x = H_0 \sin\frac{\pi x}{a}, \quad H_y = 0 \quad (\text{if } a>b)$$

So the dominant mode of this resonator must be the $TM_{10\delta}$ mode. Thus we can write,



$$E_y = E_0 \sin\frac{\pi x}{a} e^{\pm j\beta z}$$

$$H_x = \frac{\pm E_0}{Z_{TM}} \sin\frac{\pi x}{a} e^{\pm j\beta z},$$

where,

$$\beta = \sqrt{\epsilon_r k_0^2 - (\pi/a)^2} \quad \text{for } |z| < c/2$$

$$j\beta = \alpha = \sqrt{(\pi/a)^2 - k_0^2} \quad \text{for } z > c/2,$$

and $Z_{TM} = Z_d = \beta\eta/k = \beta\eta_0/\epsilon_r k_0$ for $|z|<c/2$,

$Z_{TM} = Z_a = -j\alpha\eta_0/k_0$ for $z > c/2$

Then the standing wave fields can be written as,

$$E_y = \begin{cases} A \sin\frac{\pi x}{a}\left[e^{-j\beta z} + e^{j\beta z}\right] = 2A \sin\frac{\pi x}{a}\cos\beta z & \text{for } |z|<c/2 \\ B \sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z > c/2 \end{cases}$$

$$H_x = \begin{cases} \frac{A}{Z_d} \sin\frac{\pi x}{a}\left[-e^{-j\beta z} + e^{j\beta z}\right] = \frac{2jA}{Z_d}\sin\frac{\pi x}{a}\sin\beta z & \text{for } |z|<c/2 \\ \frac{-B}{Z_a}\sin\frac{\pi x}{a} e^{-\alpha z} & \text{for } z>c/2 \end{cases}$$

Continuity of E_y, H_x at $z = c/2$:

$$2A\cos\beta c/2 = B e^{-\alpha c/2}$$

$$\frac{2jA}{Z_d}\sin\beta c/2 = \frac{-B}{Z_a} e^{-\alpha c/2}$$

divide to get:

$$\alpha\epsilon_r \tan\beta c/2 + \beta = 0$$

6.21

a) $d = \ell\lambda_o/2 = \frac{\ell}{2}\frac{c}{f_o} \Rightarrow f_o = \frac{\ell c}{2d}$ ✓

b) $E_x = E_o \sin k_o z$

$H_y = \frac{jE_o}{\eta_o} \cos k_o z$

$$W_e = \frac{\epsilon_o}{4}\int_{z=0}^{d} |E_x|^2 dz = \frac{\epsilon_o |E_o|^2}{4}\int_{z=0}^{d} \sin^2 \frac{\ell\pi z}{d} dz = \frac{\epsilon_o |E_o|^2 d}{8}$$

$$W_m = \frac{\mu_o}{4}\int_{z=0}^{d} |H_y|^2 dz = \frac{\mu_o |E_o|^2}{4\eta_o^2}\int_{z=0}^{d} \cos^2 \frac{\ell\pi z}{d} dz = \frac{\mu_o |E_o|^2 d}{8\eta_o^2} = \frac{\epsilon_o |E_o|^2 d}{8}$$

Thus $W_e = W_m$ at resonance ✓

$$P_c = 2\left(\frac{R_s}{2}\right)|H_y(z=0)|^2 = \frac{R_s |E_o|^2}{\eta_o^2}, \quad R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}}$$

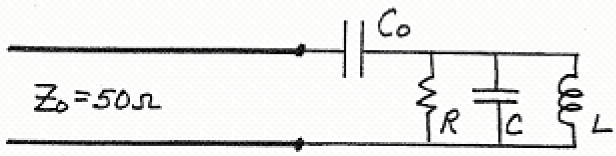
$$Q_c = \omega(W_e + W_m)/P_c = \frac{\omega\epsilon_o d \eta_o^2}{4R_s} = \frac{c\pi\ell\epsilon_o\eta_o^2}{4R_s} = \frac{\pi\ell\eta_o}{4R_s}$$

c) $f_o = \frac{(25)(3\times10^8)}{2(.04)} = \underline{93.8}$ GHz

$R_s = 0.08\ \Omega$

$Q_c = \frac{\pi(25)(377)}{4(.08)} = 9\underline{2,500}$

6.22



$R = 1000\,\Omega$
$L = 1.26\text{ nH}$
$C = 0.804\text{ pF}$

The simplest way to solve this problem is graphically, with a Smith chart. The admittance of the resonator at frequencies near resonance is,

$$Y_R = \frac{1}{R} + j\frac{2Q\Delta\omega}{R\omega_o},$$

where $\omega_o = \frac{1}{\sqrt{LC}} = 3.142 \times 10^{10}$ RPS ; $f_o = \frac{\omega_o}{2\pi} = 5.00$ GHz

$$Q = \frac{R}{\omega_o L} = 25.3$$

Normalized to Z_o, we have $y_R = Z_o Y_R = 0.05 + j2.53\frac{\Delta\omega}{\omega_o}$.
We can plot y_R on a Smith chart, versus $\Delta\omega/\omega_o$. For $\Delta\omega = 0$, $y_R = 0.05$. For $\Delta\omega = \pm 0.1\omega_o$, $y_R = 0.05 \pm j0.253$.
Next, convert this locus to z_R, an impedance locus.
Then we see that a series capacitive reactance of $-jX_{C_o} = -j4.2$ will yield an input impedance of $z_{in} = 1$. This corresponds to a resonator admittance $y_R = 0.05 - j0.22$. So the resonant frequency will be,

$$\Delta\omega = \frac{-0.22\,\omega_o}{2.53} = -0.0869\,\omega_o$$

$$\omega_r = \omega_o + \Delta\omega = (1 - .0869)\omega_o = 0.913\,\omega_o$$

so, $f_r = \frac{\omega_r}{2\pi} = 4.566$ GHz (note lowering from f_o)

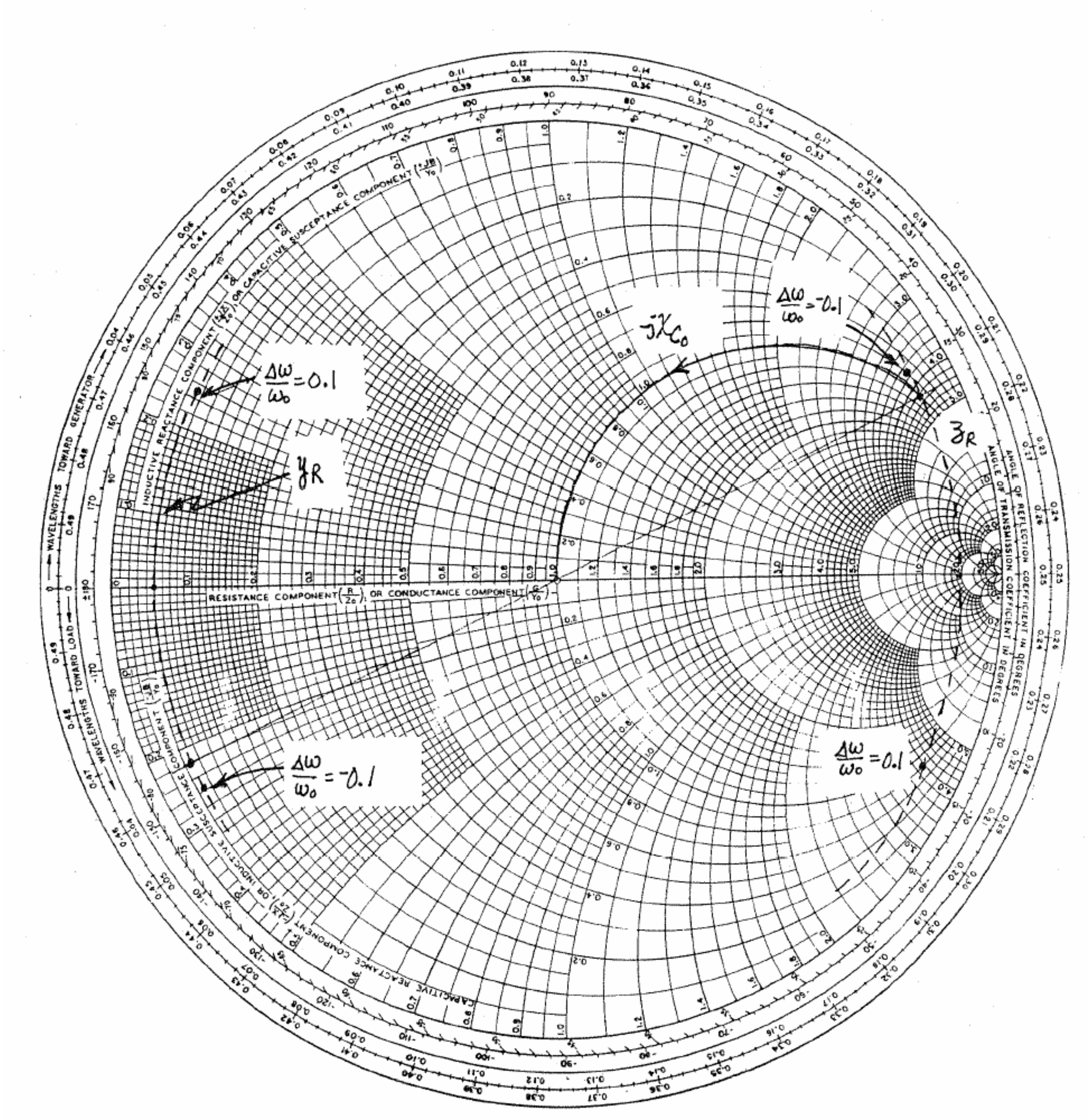
The coupling capacitor value is,

$$C_o = \frac{1}{4.2 Z_o \omega_r} = 0.166\text{ pF}.$$

CHECK: at 4.566 GHz, $Y_R = (1 - j4.39) \times 10^{-3}$ S

$$Z_R = 49.2 + j216.5\,\Omega \simeq 50 + jX_{C_o}$$

$$\frac{1}{j\omega C_o} = -j210.$$


Δω/ω₀ = -0.1
-jX_{C_0}
Δω/ω₀ = 0.1
y_R
z_R
Δω/ω₀ = -0.1
Δω/ω₀ = 0.1
RESISTANCE COMPONENT (R/Z₀), OR CONDUCTANCE COMPONENT (G/Y₀)
INDUCTIVE REACTANCE COMPONENT (+jX/Z₀), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Y₀)
CAPACITIVE REACTANCE COMPONENT (-jX/Z₀), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Y₀)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

6.23 Assume TE_{101} mode, as in Section 6.6.

At 9 GHz, $k_0 = 188.\ m^{-1}$; $\beta_0 = 140.5\ m^{-1}$; $\ell = \frac{\lambda_g}{2} = \frac{\pi}{\beta_0} = 2.24$ cm.

$\frac{\omega_0}{2\pi} = f_0 = 9$ GHz is the resonant frequency of the closed cavity, and does not include the effect of the coupling aperture. For a high-Q cavity, the actual resonant frequency, ω_1, will be close to ω_0. So we can approximately compute x_L using ω_0. From (6.89),

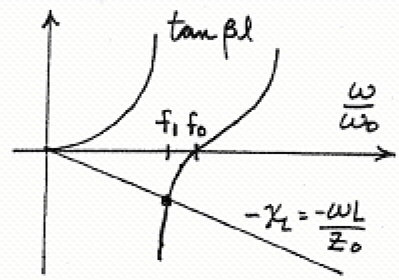
$$x_L = \sqrt{\frac{\pi k_0 \omega_1}{2Q\beta^2 C}} = 0.016 = \frac{\omega L}{Z_0} \implies \frac{L}{Z_0} = 2.83 \times 10^{-13}$$

Then solve (6.85) for ω:

$$\tan \beta\ell + x_L = 0$$

Numerical trial-and-error:

f	β	x_L	$\tan\beta\ell + x_L$
9	140.	.0160	.01
8.9	137.7	.0158	−.04
8.97	139.65	.0159	.0025



Thus, $f_1 = 8.97$ GHz

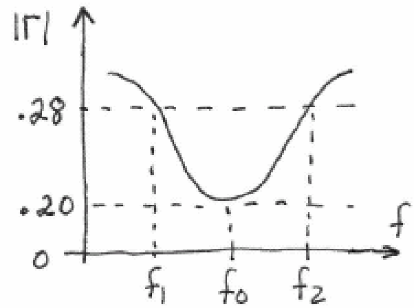
6.24

$f_1 = 2.9985$ GHz
$f_2 = 3.0015$ GHz

so $f_0 = 3.0000$ GHz, $BW = 0.1\%$, $Q_L = \frac{1}{BW} = 1000.$

at resonance, $RL = 14\,dB \Rightarrow \Gamma = 0.200 \Rightarrow r = \frac{1+\Gamma}{1-\Gamma} = 1.5$

at f_1 or f_2, $RL = 11\,dB \Rightarrow \Gamma = 0.282$



assuming a series resonance,
from (6.91), $g = \frac{Z_0}{R} = \frac{1}{r} = 0.667$

$Q_0 = (1+g)Q_L = \underline{\underline{1667}}$

assuming parallel resonator : $g = \frac{R}{Z_0} = r = 1.5$

$Q_0 = (1+g)\,Q_L = \underline{\underline{2500}}$

6.25

f (GHz)	IL (dB)	$\lvert S_{21}\rvert$ (dB)	$\lvert S_{21}\rvert$
3.0000	1.94	−1.94	0.800
2.9925	4.95	−4.95	
3.0075	4.95	−4.95	

$Q_L = \frac{3}{.015} = 200$, $g = \frac{S}{1-S} = \frac{.8}{1-.8} = 4.0$

From (6.91) $Q_0 = Q_L(1+g) = 1000$ ✓

Chapter 7

7.1

From (7.17) an <u>ideal</u> symmetric coupler has

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

a non-ideal matched symmetric coupler has

$$[S] = \begin{bmatrix} 0 & 1/L & j/C & 1/I \\ 1/L & 0 & 1/I & j/C \\ j/C & 1/I & 0 & 1/L \\ 1/I & j/C & 1/L & 0 \end{bmatrix}$$

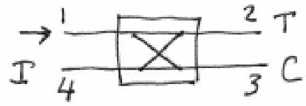
From (7.18) an <u>ideal</u> anti-symmetric coupler has

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

a non-ideal matched anti-symmetric coupler has

$$[S] = \begin{bmatrix} 0 & 1/L & 1/C & 1/I \\ 1/L & 0 & 1/I & -1/C \\ 1/C & 1/I & 0 & 1/L \\ 1/I & -1/C & 1/L & 0 \end{bmatrix}$$

7.2



$P_{in} = 20\,dBm,\; C = 20\,dB,\; D = 35\,dB,\; L = 0.5\,dB,\; RL = \infty\,dB.$

$C = 10\log P_1/P_3,\; D = 10\log P_3/P_4,\; I = 10\log P_1/P_4,\; L = 10\log P_1/P_2$

in dB,

Through : $P_2 = P_1 - L = 20 - .5 = 19.5\,dBm$

Coupled : $P_3 = P_1 - C = 20 - 20 = 0\,dBm$

Isolated : $P_4 = P_3 - D = 0 - 35 = -35\,dBm$

7.3

$$[S] = \begin{bmatrix} 0.1\angle 40^\circ & 0.944\angle 90^\circ & 0.178\angle 180^\circ & 0.0056\angle 90^\circ \\ 0.944\angle 90^\circ & 0.1\angle 40^\circ & 0.0056\angle 90^\circ & 0.178\angle 180^\circ \\ 0.178\angle 180^\circ & 0.0056\angle 90^\circ & 0.1\angle 40^\circ & 0.944\angle 90^\circ \\ 0.0056\angle 90^\circ & 0.178\angle 180^\circ & 0.944\angle 90^\circ & 0.1\angle 40^\circ \end{bmatrix}$$

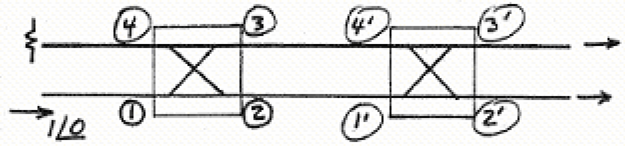
$RL = -20\log|S_{11}| = -20\log 0.1 = 20\,dB$

$C = -20\log|S_{13}| = -20\log 0.178 = 15\,dB$

$D = 20\log\left|\frac{S_{13}}{S_{14}}\right| = 20\log\left|\frac{.178}{.0056}\right| = 30\,dB$

$L = -20\log|S_{12}| = -20\log .944 = +0.5\,dB$

7.4



$$C = 8.34\,dB \Rightarrow \beta = |S_{13}| = 0.383$$

$$\alpha = \sqrt{1-\beta^2} = 0.924$$

If $V_1^+ = 1\angle 0°$, then from (7.17),

$$V_3^- = j\beta V_1^+ = 0.383\angle 90°$$

$$V_2^- = \alpha V_1^+ = 0.924\angle 0°$$

Then the outputs of the second coupler are,

$$V_3'^- = j\beta V_1'^+ + \alpha V_4'^+ = j\beta V_2^- + \alpha V_3^-$$

$$= (0.383)(0.924)\angle 90° + (0.924)(0.383)\angle 90° = 0.707\angle 90° \checkmark$$

$$V_2'^- = \alpha V_1'^+ + j\beta V_4'^+ = \alpha V_2^- + j\beta V_3^-$$

$$= (0.924)(0.924)\angle 0 - (0.383)(0.383)\angle 0 = 0.707\angle 0° \checkmark$$

Thus the outputs are identical to those for a single 3dB hybrid.

7.5 This is a special case of a lossless reciprocal 3-port network; it was shown in general that such a network could not be matched at all ports (using the [S] matrix). Alternatively, we can argue as follows: If the input to each port is to be matched to its respective characteristic impedance, we must have,

$$\frac{1}{Z_1} = \frac{1}{Z_2} + \frac{1}{Z_3} \qquad \text{(port 1)}$$

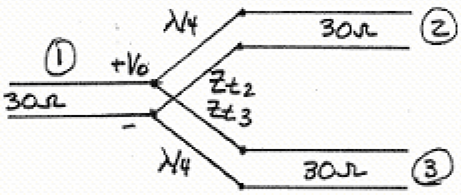
$$\frac{1}{Z_2} = \frac{1}{Z_1} + \frac{1}{Z_3} \qquad \text{(port 2)}$$

$$\frac{1}{Z_3} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad \text{(port 3)}$$

It is not possible to satisfy these three equations simultaneously:

$$\det\begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = -4 \neq 0$$

7.6



$$\frac{P_2}{P_3} = 3$$

$$P_1 = \frac{1}{2}\frac{V_0^2}{Z_0}$$

$$P_2 = \frac{1}{2}\frac{V_0^2}{Z_2} = \frac{3}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{3}{4Z_0}\right)$$

$$P_3 = \frac{1}{2}\frac{V_0^2}{Z_3} = \frac{1}{4}P_1 = \frac{1}{2}V_0^2\left(\frac{1}{4Z_0}\right)$$

So, $Z_2 = 4Z_0/3 = 40\Omega$ ✓

$Z_3 = 4Z_0 = 120\Omega$ ✓

The $\lambda/4$ matching transformers have impedances,

$$Z_{t2} = \sqrt{30(40)} = 34.6\,\Omega \quad ✓$$

$$Z_{t3} = \sqrt{30(120)} = 60.0\,\Omega \quad ✓$$

Then the S-parameters are, (phase ref. at 30Ω ports)

$$S_{11} = \frac{30-30}{30+30} = 0$$

$$S_{22} = \frac{30\|120 - 40}{30\|120 + 40} = \frac{24-40}{24+40} = -0.25 \quad ✓$$

$$S_{33} = \frac{30\|40 - 120}{30\|40 + 120} = \frac{17.1-120.}{17.1+120.} = -0.75 \quad ✓$$

$$S_{21} = S_{12} = \sqrt{P_2/P_1}\,e^{j\theta} = \sqrt{3/4}\,\underline{/-90^\circ} = 0.866\,\underline{/-90^\circ} \quad ✓$$

$$S_{31} = S_{13} = \sqrt{P_3/P_1}\,e^{j\theta} = \sqrt{1/4}\,\underline{/-90^\circ} = 0.50\,\underline{/-90^\circ} \quad ✓$$

Since the network is lossless, we should have,

$$|S_{21}|^2 + |S_{22}|^2 + |S_{23}|^2 = 1$$

so, $S_{23} = S_{32} = \sqrt{1-(.25)^2-(.866)^2}\,e^{-2j\theta} = 0.433\,\underline{/-180^\circ}$ ✓

7.7

T- NETWORK: From Table 4.1 the ABCD parameters are

$$A = 1 + R_1/R_2 \qquad B = 2R_1 + R_1^2/R_2$$
$$C = 1/R_2 \qquad D = 1 + R_1/R_2$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \Rightarrow \cancel{1} + \cancel{\frac{R_1}{R_2}} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0R_2} - \frac{Z_0}{R_2} - \cancel{1} - \cancel{\frac{R_1}{R_2}} = 0$$

$$R_1^2 + 2R_1R_2 - Z_0^2 = 0$$

$$R_2 = \frac{Z_0^2 - R_1^2}{2R_1}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \Rightarrow 2 + \frac{2R_1}{R_2} + \frac{2R_1}{Z_0} + \frac{R_1^2}{Z_0R_2} + \frac{Z_0}{R_2} = \frac{2}{\alpha}$$

$$2Z_0R_2 + 2Z_0R_1 + \underbrace{2R_1R_2 + R_1^2}_{=Z_0^2} + Z_0^2 = \frac{2}{\alpha}Z_0R_2$$

$$R_2 + R_1 + Z_0 = R_2/\alpha$$

$$\left(\frac{1}{\alpha} - 1\right)(Z_0 - R_1) = 2R_1$$

$$Z_0\left(\frac{1}{\alpha} - 1\right) = R_1\left(1 + \frac{1}{\alpha}\right)$$

$$R_1 = Z_0\frac{1-\alpha}{1+\alpha} \checkmark$$

$$R_2 = \frac{2\alpha}{1-\alpha^2}Z_0 \checkmark$$

For $Z_0 = 50\Omega$:

α (dB)	α	$R_1 (\Omega)$	$R_2 (\Omega)$
3	.708	8.6	141.9
10	.316	26.0	35.1
20	.100	40.9	10.1

π – NETWORK: From Table 4.1 the ABCD parameters are,

$$A = 1 + R_2/R_1 \qquad\qquad B = R_2$$

$$C = \frac{2}{R_1} + \frac{R_2}{R_1^2} \qquad\qquad D = 1 + R_2/R_1$$

Convert to S-parameters using Table 4.2:

$$S_{11} = \frac{A + B/Z_0 - CZ_0 - D}{A + B/Z_0 + CZ_0 + D} = 0 \implies \frac{R_2}{Z_0} - \frac{2Z_0}{R_1} - \frac{R_2 Z_0}{R_1^2} = 0$$

$$R_2 R_1^2 - 2Z_0^2 R_1 - R_2 Z_0^2 = 0$$

$$R_2 = \frac{2Z_0^2 R_1}{R_1^2 - Z_0^2}$$

$$S_{12} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \alpha \implies 2 + \frac{2R_2}{R_1} + \frac{R_2}{Z_0} + \underbrace{\frac{2Z_0}{R_1} + \frac{Z_0 R_2}{R_1^2}}_{= R_2/Z_0} = \frac{2}{\alpha}$$

$$1 + \frac{R_2}{R_1} + \frac{R_2}{Z_0} = \frac{1}{\alpha}$$

$$R_1 Z_0 + R_2(Z_0 + R_1) = \frac{1}{\alpha} Z_0 R_1$$

$$2 Z_0^{\cancel{2}} \cancel{R_1} = \cancel{R_1} \cancel{Z_0} \left(\frac{1}{\alpha} - 1\right)(R_1 - Z_0)$$

$$\frac{2Z_0 \alpha}{1-\alpha} = R_1 - Z_0$$

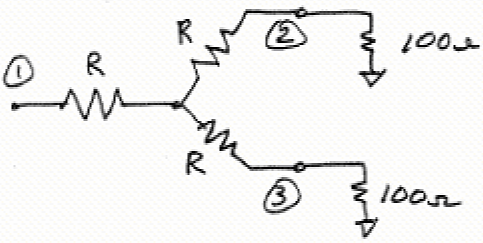
$$R_1 = Z_0\left(1 + \frac{2\alpha}{1-\alpha}\right) = \frac{1+\alpha}{1-\alpha} Z_0 \quad \checkmark$$

$$R_2 = \frac{1-\alpha^2}{2\alpha} Z_0 \quad \checkmark$$

For $Z_0 = 50\,\Omega$:

α (dB)	α	R_1 (Ω)	R_2 (Ω)
3	.708	292.5	17.6
10	.316	96.2	71.2
20	.100	61.1	247.5

7.8



DESIGN:

$Z_0 = 100\Omega$

$R = 33.3\,\Omega$ ✓

$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

CASE a) ports 2 & 3 matched to 100Ω ($V_2^+ = V_3^+ = 0$);

If $V_1^+ = 1$,

$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}$$

$$V_3 = V_3^+ + V_3^- = \tfrac{1}{2}$$

$$P_3 = V_3^2/Z_0 = 0.25/Z_0$$

CASE b) port 3 matched, $\Gamma = 0.3$ at port 2 ($V_3^+ = 0$):

If $V_1^+ = 1$,

$$V_2^- = \tfrac{1}{2}[V_1^+ + V_3^+] = \tfrac{1}{2}$$

$$V_2^+ = \Gamma V_2^- = (0.3)(\tfrac{1}{2}) = 0.15$$

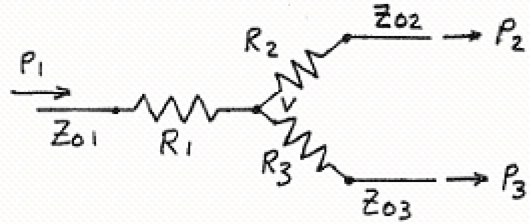
$$V_3^- = \tfrac{1}{2}[V_1^+ + V_2^+] = \tfrac{1}{2}(1.15) = 0.575\text{ v}$$

$$V_3 = V_3^+ + V_3^- = 0.575\text{ v}$$

$$P_3 = V_3^2/Z_0 = 0.331/Z_0$$

$$\frac{P_3\text{ (PORT 2 MISMATCHED)}}{P_3\text{ (PORT 2 MATCHED)}}(dB) = 10\log\left(\frac{0.331}{0.25}\right) = \underline{1.2\,dB}$$

7.9



$\alpha = \dfrac{P_2}{P_3}$

let $Z_1 = R_1 + Z_{01}$
$Z_2 = R_2 + Z_{02}$
$Z_3 = R_3 + Z_{03}$

For $\alpha = \dfrac{P_2}{P_3}$ we must have,

$$\frac{Z_{02}\, Z_3^2}{Z_2^2\, Z_{03}} = \alpha \qquad \ldots (1)$$

For all three ports to be matched we must have,

$$Z_{01} = R_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \qquad \ldots (2)$$

$$Z_{02} = R_2 + \frac{Z_1 Z_3}{Z_1 + Z_3} \qquad \ldots (3)$$

$$Z_{03} = R_3 + \frac{Z_1 Z_2}{Z_1 + Z_2} \qquad \ldots (4)$$

We assume that Z_{01} and α are given; then we have 5 unknowns ($R_1, R_2, R_3, Z_{02}, Z_{03}$), and 4 equations. So we need one more equation: we will choose the condition that,

$$Z_{02} Z_{03} = Z_{01}^2 \qquad \ldots (5)$$

which will ensure that when $\lambda/4$ transformers are used to match Z_{02} and Z_{03} to a final output impedance of Z_{01}, phase tracking will be maintained for ports 2 and 3. (See Problem 7.10). Now use (1) and (5) to eliminate Z_3 and Z_{03} from (2), (3), (4). Then (2) reduces to,

$$R_1 = \frac{Z_{01}\left[Z_{02} + \sqrt{\alpha}\,(Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

$$Z_1 = \frac{Z_{01}\left[2 Z_{02} + \sqrt{\alpha}\,(2 Z_{01} - Z_2)\right]}{Z_{02} + \sqrt{\alpha}\, Z_{01}}$$

and (3) and (4) then reduce to,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-\alpha Z_{01}Z_{02}\right] = 2Z_{02}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

and,

$$Z_2\left[(Z_{02}+\sqrt{\alpha}\,Z_{01})^2-Z_{01}Z_{02}\right] = 2Z_{01}^2\,(Z_{02}+\sqrt{\alpha}\,Z_{01})$$

Then we obtain a quartic equation for Z_{02}:

$$Z_{02}^4+Z_{01}(2\sqrt{\alpha}-1)Z_{02}^3+Z_{01}^2(\alpha-1)Z_{02}^2+Z_{01}^3(\alpha-2\sqrt{\alpha})Z_{02}-\alpha Z_{01}^4=0$$

after finding Z_{02} (either numerically or using the formula for a quartic equation), the above equations can be used to find Z_{03}, R_2, R_3, and R_1.

EXAMPLE: let $Z_{01}=1$, $\alpha=2$:

Then, $\quad Z_{02}^4+1.828\,Z_{02}^3+Z_{02}^2-0.828\,Z_{02}-2=0$

The solution for Z_{02} was computed numerically using an HP-15C calculator to be,

$$Z_{02}=0.8935$$

Then,

$Z_2=1.041$; $R_2=0.1478$

$Z_3=1.6477$; $R_3=0.5285$; $Z_{03}=1.1192$

$R_1=0.3621$

CHECK: $\quad Z_{in}=R_1+Z_2\|Z_3=1.000$ ✓

NOTE: There are other choices for the relation between Z_{02} and Z_{03}, or other pairs of variables. One possibility that may give a simpler solution would be to let $Z_1=\alpha^{1/3}Z_2$.

7.10 From (7.37), $K^2 = P_3/P_2 = 1/3 \Rightarrow K = 0.577$

$$Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}} = 131.7\ \Omega \checkmark$$

$$Z_{02} = K^2 Z_{03} = 43.9\ \Omega \checkmark$$

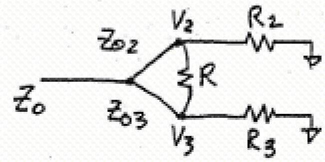
$$R = Z_0(K + 1/K) = 115.5\ \Omega \checkmark$$

The output impedances are,

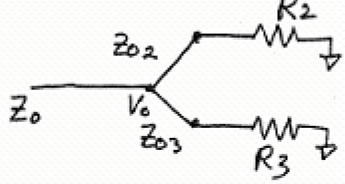
$$R_2 = Z_0 K = 28.9\ \Omega \checkmark$$

$$R_3 = Z_0/K = 86.7\ \Omega \checkmark$$

7.11



Assuming the output ports are matched, no power should be dissipated in resistor R (for lossless power division). Therefore $V_2 = V_3$, and the resistor R can be removed:



Input matching requires that,

$$\frac{1}{Z_0} = \frac{R_2}{Z_{02}^2} + \frac{R_3}{Z_{03}^2}$$

Power division requires that $P_3 = K^2 P_2$:

$$P_2 = \tfrac{1}{2}|V_0|^2 R_2 / Z_{02}^2$$

$$P_3 = \tfrac{1}{2}|V_0|^2 R_3 / Z_{03}^2 = K^2 P_2$$

Thus,

$$K^2 \frac{R_2}{Z_{02}^2} = \frac{R_3}{Z_{03}^2}$$

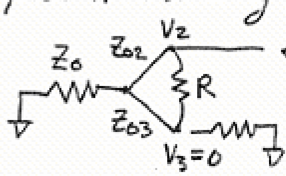
and,

$$Z_{02}^2 = Z_0 R_2 (1+K^2) \qquad \ldots (1)$$

$$Z_{03}^2 = Z_0 R_3 (1+1/K^2) \qquad \ldots (2)$$

For output matching,

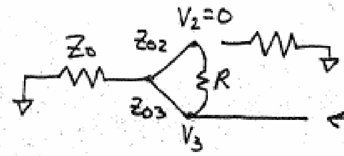
PORT 2:



$$\leftarrow R_2 = R \,\|\, \frac{Z_{02}^2}{Z_0} = \frac{R R_2(1+K^2)}{R + R_2(1+K^2)}$$

$$R_2(1+K^2) = R K^2 \qquad \ldots (3)$$

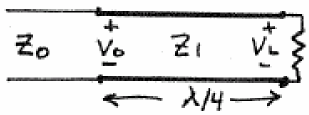
PORT 3:



$$\leftarrow R = R \,\|\, \frac{Z_{03}^2}{Z_0} = \frac{R R_3(1+1/K^2)}{R + R_3(1+1/K^2)}$$

$$R_3(1+K^2) = R \qquad \ldots (4)$$

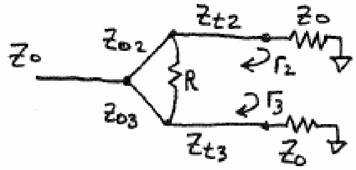
From (3) and (4) we have that $R_2 = K^2 R_3$. We need one more condition to obtain the design equations. This condition is that the output impedances R_2, R_3 be chosen so that output matching transformers have the same transfer phase at both ports:



It is easy to show that,

$$\frac{V_L}{V_0} = -j\,\frac{1+\Gamma}{1-\Gamma} \;;\quad \Gamma = \frac{Z_L - Z_1}{Z_L + Z_1}$$

So the phase of V_L/V_0 does not change when Γ is replaced by $-\Gamma$, or when Z_L and Z_1 are interchanged (Z_L, Z_1 real).



$$\Gamma_2 = \frac{Z_0/Z_{t2} - 1}{Z_0/Z_{t2} + 1}$$

$$\Gamma_3 = \frac{Z_0/Z_{t3} - 1}{Z_0/Z_{t3} + 1}$$

So we choose $Z_{t2} Z_{t3} = Z_0^2 = \sqrt{Z_0 R_2}\,\sqrt{Z_0 R_3} = K Z_0 R_3$, which leads to the design equations,

$$R_2 = K Z_0 \;\checkmark \qquad Z_{02} = Z_0\sqrt{K(1+K^2)} \;\checkmark$$

$$R_3 = Z_0/K \;\checkmark$$

$$R = Z_0(K + 1/K) \;\checkmark \qquad Z_{03} = Z_0\sqrt{\frac{1+K^2}{K^3}} \;\checkmark$$

7.12 Setting $A_{10}^- = 0$ from (7.40b):

$$\left(\epsilon_0 \alpha_e + \frac{\mu_0 \alpha_m}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{\mu_0 \pi^2 \alpha_m}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

For a round aperture, $\alpha_e = 2r_0^3/3$; $\alpha_m = 4r_0^3/3$:

$$\left(\epsilon_0 + \frac{2\mu_0}{Z_{10}^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\mu_0 \pi^2}{\beta^2 a^2 Z_{10}^2}\cos^2\frac{\pi s}{a} = 0$$

Since $Z_{10} = k_0\eta_0/\beta$, this simplifies as follows:

$$(k_0^2 + 2\beta^2)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\cos^2\frac{\pi s}{a} = 0$$

$$\left(3k_0^2 - \frac{2\pi^2}{a^2}\right)\sin^2\frac{\pi s}{a} - \frac{2\pi^2}{a^2}\left(1 - \sin^2\frac{\pi s}{a}\right) = 0$$

$$3k_0^2 \sin^2\frac{\pi s}{a} = \frac{2\pi^2}{a^2}$$

$$\frac{6a^2}{\lambda_0^2}\sin^2\frac{\pi s}{a} = 1, \text{ or } \sin\frac{\pi s}{a} = \frac{\lambda_0}{\sqrt{6}\,a} < 1 \text{ for } a > \lambda_0/2 \checkmark$$

7.13 at $f = 11$ GHz, $k_0 = 230.4\ \text{m}^{-1}$; $\beta = 116.4\ \text{m}^{-1}$; $Z_{10} = k_0\eta_0/\beta = 746.2\ \Omega$

$P_{10} = ab/Z_{10} = 1.67 \times 10^{-7}$ W

From (7.41) the position of the coupling aperture is,

$$\sin\frac{\pi s}{a} = \pi\sqrt{\frac{2}{4\pi^2 - k_0^2 a^2}} = 0.867 \Rightarrow S = 0.334a = \underline{0.528\text{ cm}}$$

Then, $\sin^2\frac{\pi s}{a} = 0.752$, $\cos^2\frac{\pi s}{a} = 0.248$

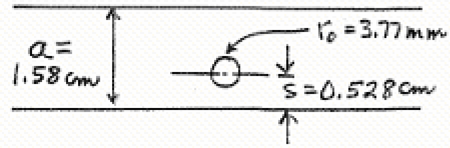
For $C = 20$dB, $\left|\frac{A_{10}^-}{A}\right| = 0.1$. From (7.40b) we have,

$$0.1 = \frac{\omega}{P_{10}}\left[\frac{2}{3}\epsilon_0 \sin^2\frac{\pi s}{a} + \frac{4\mu_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] r_0^3$$

$$0.1 P_{10} = \left[\frac{2}{3}\frac{k_0}{\eta_0}\sin^2\frac{\pi s}{a} + \frac{4k_0\eta_0}{3Z_{10}^2}\left(\sin^2\frac{\pi s}{a} - \frac{\pi^2}{\beta^2 a^2}\cos^2\frac{\pi s}{a}\right)\right] r_0^3$$

$$1.67 \times 10^{-8} = 3.12 \times 10^{-1} r_0^3$$

$$r_0 = \underline{3.77\text{ mm}}$$



7.14 at $f = 17$ GHz, $k_0 = 356.\ m^{-1}$, $\beta = 295.3\ m^{-1}$,
$Z_{10} = k_0\eta_0/\beta = 454\ \Omega$, $P_{10} = ab/Z_{10} = 2.75\times10^{-7}$ W

From (7.44) the necessary skew angle is, for $s = a/2$,
$$\cos\theta = k_0^2/2\beta^2 = 0.728 \implies \underline{\theta = 43^\circ} \checkmark$$

From (7.45) the aperture radius is,
$$\left|\frac{A_{10}^-}{A}\right| = 0.0316 = \frac{4k_0^2 r_0^3}{3ab\beta} = 4.58\times10^6 r_0^3 \implies \underline{r_0 = 1.90\text{ mm}} \checkmark$$

7.15 $N=4$ (5-holes)
$k_0 = 366.5\ m^{-1}$, $\beta = 307.9\ m^{-1}$, $Z_{10} = k_0\eta_0/\beta = 448.8\ \Omega$
$P_{10} = ab/Z_{10} = 2.78\times10^{-7}\ \Omega$

From (7.40a), with $s = a/2$,
$$|K_f| = \frac{2k_0}{3\eta_0 P_{10}}\left|1 - 2\beta^2/k_0^2\right| = 9.59\times10^5$$

From (7.55),
$$C = 20dB = -20\log|K_f| - 20\log k - 20\log\sum_{n=0}^{N} C_n^N$$

For $N=4$, $\sum_{n=0}^{N} C_n^N = 1+4+6+4+1 = 16$, so
$$20 = -119.6 - 20\log k - 24.1$$
$$k = 6.53\times10^{-9}$$

From (7.54) the aperture radii are,
$$r_0 = k^{1/3} = 1.87\text{ mm} = r_4$$
$$r_1 = (4k)^{1/3} = 2.97\text{ mm} = r_3$$
$$r_2 = (6k)^{1/3} = 3.97\text{ mm}$$

The spacing between the apertures is $\lambda_g/4 = 5.1$ mm.
(Smaller apertures would result if $s = a/4$ were used.)

7.16

$N=4$ (5 hole)

$k_0 = 366.5\ m^{-1}$, $\beta = 307.9\ m^{-1}$, $Z_{10} = 448.8\ \Omega$

$P_{10} = 2.78 \times 10^{-7}\ W$

From (7.40a,b) with $s = a/2$,

$$|K_f| = \frac{2 k_0}{3 \eta_0 P_{10}} \left| 1 - \frac{2\beta^2}{k_0^2} \right| = 9.59 \times 10^5$$

From (7.59),

$$30dB = D_{min} = 20 \log T_N(\sec\theta_m)$$

$$T_4(\sec\theta_m) = \cosh[4 \cosh^{-1}(\sec\theta_m)] = 31.6$$

$$\sec\theta_m = 1.587$$

From (7.57),

$$C = 20dB = -20\log|K_f| - 20\log k - 30$$

$$k = 3.30 \times 10^{-9}$$

From (7.56) and (6.60d):

$$2[r_0^3 \cos 4\theta + r_1^3 \cos 2\theta + r_2^3] = k[\sec^4\theta_m(\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2\theta_m(\cos 2\theta + 1) + 1]$$

Then,

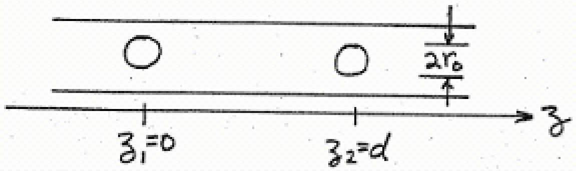
$$2r_0^3 = k \sec^4\theta_m \Rightarrow r_0 = r_4 = 2.19\ mm$$

$$2r_1^3 = k[4\sec^4\theta_m - 4\sec^2\theta_m] \Rightarrow r_1 = r_3 = 2.93\ mm$$

$$2r_2^3 = k[3\sec^4\theta_m - 4\sec^2\theta_m + 1] \Rightarrow r_2 = 2.54\ mm.$$

The spacing between the apertures is $\lambda_g/4 = 5.1\ mm$.

7.17



The incident TE_{10} fields are, for $x=0$, $y=b/2$, $z=z_n$:

$$E_y = A \sin\frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_x = \frac{-A}{Z_{10}} \sin\frac{\pi x}{a} e^{-j\beta z} = 0$$

$$H_z = \frac{j\pi A}{\beta a Z_{10}} \cos\frac{\pi x}{a} e^{-j\beta z} = \frac{j\pi A}{\beta a Z_{10}} e^{-j\beta z_n}$$

$P_{10} = ab/Z_{10}$; $Z_{10} = k_0\eta_0/\beta$.

From (4.124)–(4.125) the equivalent polarization currents are: ($\hat{n}=\hat{z}$)

$$\bar{P}_e = 0$$

$$\bar{P}_m = -\alpha_m H_z \delta(x)\,\delta(y-b/2)\,\delta(z-z_n)\,e^{-j\beta z_n}$$

Then the amplitudes of the forward and reverse coupled waves from a single aperture are,

$$A_{10}^{+} = \frac{1}{P_{10}}\int_v \bar{H}_{10}^{-}\cdot j\omega\mu_0 \bar{P}_m\, dv = \frac{-j\omega\mu_0\alpha_m}{P_{10}}\left(\frac{j\pi A}{\beta a Z_{10}}\right) e^{-j\beta z_n} H_z^{-} = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}$$

$$A_{10}^{-} = \frac{1}{P_{10}}\int_v \bar{H}_{10}^{+}\cdot j\omega\mu_0 \bar{P}_m\, dv = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0} e^{-2j\beta z_n}$$

Then the total forward and backward wave amplitudes from two apertures at $z_1=0$ and $z_2=d$ are,

$$A_{10}^{+} = \frac{j\pi^2\alpha_m A(2)}{P_{10}a^2 k_0\eta_0} \qquad A_{10}^{-} = \frac{j\pi^2\alpha_m A}{P_{10}a^2 k_0\eta_0}\left(1+e^{-2j\beta d}\right)$$

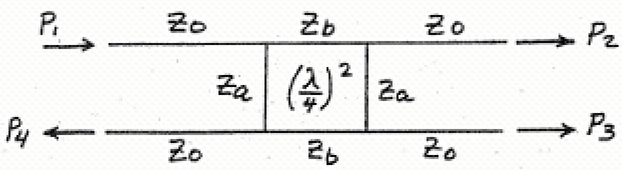
For $A_{10}^{-}=0$, we must have $(1+e^{-2j\beta d})=0$, or $\underline{d=\lambda_g/4.}$

Then the coupling factor is,

$$C = 20\log\left|\frac{A}{A_{10}^{+}}\right| = 20\log\left|\frac{P_{10}a^2k_0\eta_0}{2\pi^2\alpha_m}\right|$$

$$= 20\log\left|\frac{3a^3b\beta}{8\pi^2 r_0^3}\right| \text{ dB}$$

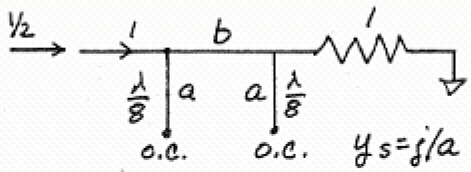
7.18



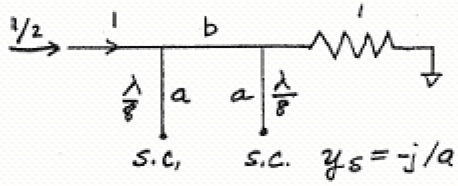
$\alpha = P_2/P_3$

let $a = \frac{Z_a}{Z_0}$, $b = \frac{Z_b}{Z_0}$

Following the analysis of Section 7.5, the even and odd circuits are: (in normalized form)



EVEN MODE



ODD MODE

The ABCD matrices are,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} \begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ j/a & 1 \end{bmatrix} = \begin{bmatrix} -b/a & jb \\ j/b - jb/a^2 & -b/a \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} \begin{bmatrix} 0 & jb \\ j/b & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -j/a & 1 \end{bmatrix} = \begin{bmatrix} b/a & jb \\ j/b - jb/a^2 & b/a \end{bmatrix}$$

$$\Gamma_e = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{-2b/a + j(b + 1/b - b/a^2)}$$

$$\Gamma_o = \frac{A+B-C-D}{A+B+C+D} = \frac{j(b - 1/b + b/a^2)}{2b/a + j(b + 1/b - b/a^2)}$$

The reflection at port 1 is then,

$$B_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right) \frac{-2j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)}{\left(\frac{2b}{a}\right)^2 + \left(b + \frac{1}{b} - \frac{b}{a^2}\right)^2} = 0$$

Thus, $b - \frac{1}{b} + \frac{b}{a^2} = 0$ (can't have $b + \frac{1}{b} - \frac{b}{a^2} = 0$, or else $B_2 = B_3 = 0$)

So, $a = \frac{b}{\sqrt{1-b^2}}$ Then $\frac{b}{a^2} = \frac{1}{b} - b$

Then,

$$T_e = \frac{2}{A+B+C+D} = \frac{2}{-2\frac{b}{a} + j\left(b + \frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{\frac{-b}{a} + jb}$$

$$T_o = \frac{2}{A+B+C+D} = \frac{2}{\frac{2b}{a} + j\left(b+\frac{1}{b} - \frac{b}{a^2}\right)} = \frac{1}{\frac{b}{a}+jb}$$

So the output wave amplitudes at ports 2 and 3 are,

$$B_2 = \tfrac{1}{2}(T_e + T_o) = \tfrac{1}{2}\left[\frac{1}{-b/a+jb} + \frac{1}{b/a+jb}\right] = \frac{-j}{b(1+1/a^2)}$$

$$B_3 = \tfrac{1}{2}(T_e - T_o) = \tfrac{1}{2}\left[\frac{1}{-b/a+jb} - \frac{1}{b/a+jb}\right] = \frac{-1/a}{b(1+1/a^2)}$$

This shows a 90° phase shift between ports 2 and 3.
For $P_2/P_3 = \alpha$,

$$P_2 = \alpha P_3$$

$$|B_2|^2 = \alpha |B_3|^2$$

$$1 = \frac{\alpha}{a^2} \implies a = \sqrt{\alpha} \quad \text{or} \quad Z_a = \sqrt{\alpha}\, Z_0 \ \checkmark$$

Then, $$b = \frac{a}{\sqrt{1+a^2}} = \frac{\sqrt{\alpha}}{\sqrt{1+\alpha}} = \sqrt{\frac{\alpha}{1+\alpha}}, \quad \text{or} \quad Z_b = \sqrt{\frac{\alpha}{1+\alpha}}\, Z_0 \ \checkmark$$

CHECK: When $\alpha = 1$, $Z_a = Z_0$ ✓, $Z_b = Z_0/\sqrt{2}$ ✓

at the isolated port,

$$B_4 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) = \frac{j}{2}\left(b - \frac{1}{b} + \frac{b}{a^2}\right)(\cdot) = 0$$

So there is isolation.

EXAMPLE: $\alpha = 3$ (6dB), $Z_0 = 50\Omega$

Then $Z_a = 87\Omega$ ✓

$Z_b = 43\Omega$ ✓

7.19 $b = 2.0\text{ mm}, \quad \epsilon_r = 4.2$

say $S/b = 0.1$, $W/b = 0.3 \Rightarrow S = 0.2\text{ mm}$, $W = 0.6\text{ mm}$

Ansoft: $Z_{oe} = 80.0\,\Omega$, $Z_{oo} = 39.0\,\Omega$

graph: $\sqrt{\epsilon_r}\, Z_{oe} = 168 \Rightarrow Z_{oe} = 82.0\,\Omega$

(Fig 7.29) $\sqrt{\epsilon_r}\, Z_{oo} = 83 \Rightarrow Z_{oo} = 40.5\,\Omega$

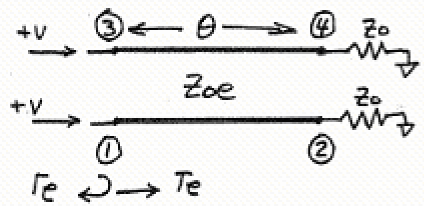
7.20 $d = 2.0\text{ mm}, \quad \epsilon_r = 10$ (to use graph of Fig 7.30)

say $S/d = 0.4$, $W/d = 0.1 \Rightarrow S = 0.8\text{ mm}$, $W = 0.2\text{ mm}$.

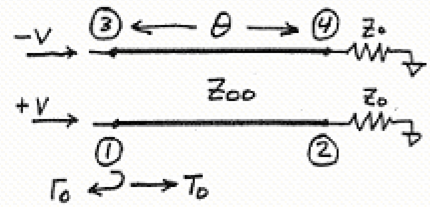
Ansoft: $Z_{oe} = 133\,\Omega$, $Z_{oo} = 71.5\,\Omega$

graph: $Z_{oe} = 138\,\Omega$, $Z_{oo} = 78\,\Omega$

7.21



EVEN MODE



ODD MODE

For $\theta = \pi/2$,

$$\Gamma_e = \frac{Z_{0e}^2/Z_0 - Z_0}{Z_{0e}^2/Z_0 + Z_0} = \frac{Z_{0e}^2 - Z_0^2}{Z_{0e}^2 + Z_0^2}$$

$$\Gamma_o = \frac{Z_{0o}^2/Z_0 - Z_0}{Z_{0o}^2/Z_0 + Z_0} = \frac{Z_{0o}^2 - Z_0^2}{Z_{0o}^2 + Z_0^2}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} \cos\theta & jZ_{0e}\sin\theta \\ \frac{j}{Z_{0e}}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} \cos\theta & jZ_{0o}\sin\theta \\ \frac{j}{Z_{0o}}\sin\theta & \cos\theta \end{bmatrix}$$

$$T_e = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{0e}}{Z_0} + \frac{Z_0}{Z_{0e}}\right)\sin\theta}$$

$$T_o = S_{21} = \frac{2}{2\cos\theta + j\left(\frac{Z_{0o}}{Z_0} + \frac{Z_0}{Z_{0o}}\right)\sin\theta}$$

For a unit amplitude wave incident at port 1, the output wave amplitudes are,

$$B_1 = \tfrac{1}{2}(\Gamma_e + \Gamma_o) \qquad B_2 = \tfrac{1}{2}(T_e + T_o)$$

$$B_3 = \tfrac{1}{2}(\Gamma_e - \Gamma_o) \qquad B_4 = \tfrac{1}{2}(T_e - T_o)$$

So the reflection at port 1 is,

$$B_1 = \frac{1}{2}\left[\frac{Z_{oe}^2 - Z_o^2}{Z_{oe}^2 + Z_o^2} + \frac{Z_{oo}^2 - Z_o^2}{Z_{oo}^2 + Z_o^2}\right] = \frac{Z_{oo}^2 Z_{oe}^2 - Z_o^4}{(Z_o^2 + Z_{oe}^2)(Z_o^2 + Z_{oo}^2)} = 0$$

Thus, $\underline{Z_o = \sqrt{Z_{oe} Z_{oo}}}$ ✓ (so all ports are matched ✓)

Then, $\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right) = \left(\frac{Z_o}{Z_{oo}} + \frac{Z_{oo}}{Z_o}\right)$

So, $T_e = T_o$, and $\underline{B_4 \equiv 0}$ ✓

The output waves at ports 2 and 3 are,

$$B_2 = \frac{2}{2\cos\theta + j\left(\frac{Z_{oe}}{Z_o} + \frac{Z_o}{Z_{oe}}\right)\sin\theta} = \frac{1}{\cos\theta + j\,\frac{Z_{oe}+Z_{oo}}{2Z_o}\sin\theta}$$

$$B_3 = \frac{1}{2}\left[\frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}} - \frac{Z_{oo} - Z_{oe}}{Z_{oo} + Z_{oe}}\right] = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$$

Let $C = B_3 = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$

Then $\sqrt{1-C^2} = \frac{2Z_o}{Z_{oe} + Z_{oo}}$, and so,

$$B_2 = \frac{\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta}$$

For $\theta = \pi/2$, the midband responses are,

$B_3 = C$ ✓

$B_2 = -j\sqrt{1-C^2}$ ✓

which agree with (7.85) – (7.86).

7.22 $C = 10^{-19.1/20} = 0.1109$; $f = 8\text{GHz}$; $Z_0 = 60\Omega$

From (7.87),

$$Z_{oe} = Z_0\sqrt{\frac{1+C}{1-C}} = 67.1\Omega \qquad Z_{oo} = Z_0\sqrt{\frac{1-C}{1+C}} = 53.7\Omega$$

For a stripline with $\epsilon_r = 2.2$, $b = 0.32$ cm,

$$\sqrt{\epsilon_r}\, Z_{oe} = 99.5\Omega\ , \quad \sqrt{\epsilon_r}\, Z_{oo} = 79.7\Omega$$

From Figure 7.29,

$$S/b = 0.36 \implies S = 1.15 \text{ mm}$$
$$W/b = 0.60 \implies W = 1.92 \text{ mm}$$

The line lengths are,

$$\ell = \frac{\lambda_g}{4} = \frac{c}{4\sqrt{\epsilon_r}\, f} = 6.32 \text{ mm}$$

7.23 $C = 5\text{dB} = 10^{-5/20} = 0.562$; $f_o = 8\text{GHz}$; $Z_0 = 60\Omega$

From (7.87),

$$Z_{oe} = Z_0\sqrt{\frac{1+C}{1-C}} = 113.3\Omega \qquad Z_{oo} = Z_0\sqrt{\frac{1-C}{1+C}} = 31.8\Omega$$

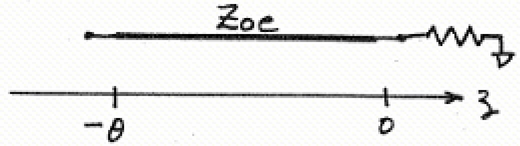
Then, $\sqrt{\epsilon_r}\, Z_{oe} = 168.1\Omega$ $\qquad \sqrt{\epsilon_r}\, Z_{oo} = 47.2\Omega$ $\qquad (\epsilon_r = 2.2)$

From Figure 7.29,

$$S/b \cong 0.009 \implies S = 0.029 \text{ mm } (!)$$
$$W/b \cong 0.34 \implies W = 1.09 \text{ mm}$$

This design is probably not practical due to the extremely close spacing of the lines.

7.24 For V_{1e} or V_{1o} at port 1, we first find V_{2e} or V_{2o}:



$$\Gamma_e = \frac{Z_o - Z_{oe}}{Z_o + Z_{oe}}$$

$$1+\Gamma_e = \frac{2Z_o}{Z_o + Z_{oe}}$$

$$V(z) = V^+\left(e^{-j\beta z} + \Gamma_e e^{j\beta z}\right)$$

$$V_{1e} = V(-\theta) = V^+\left(e^{j\theta} + \Gamma_e e^{-j\theta}\right)$$

$$V_{2e} = V(0) = V^+(1+\Gamma_e)$$

So,

$$V_{2e} = \frac{V_{1e}(1+\Gamma_e)}{e^{j\theta}+\Gamma_e e^{-j\theta}} = \frac{2Z_o V_{1e}}{(Z_o+Z_{oe})e^{j\theta} + (Z_o - Z_{oe})e^{-j\theta}}$$

$$= \frac{Z_o V_{1e}}{Z_o\cos\theta + jZ_{oe}\sin\theta} = \frac{Z_o V_{1e}\sec\theta}{Z_o + jZ_{oe}\tan\theta}$$

Similarly,

$$V_{2o} = \frac{Z_o V_{1o}\sec\theta}{Z_o + jZ_{oo}\tan\theta}$$

Then using (7.74) and the results following (7.79) gives,

$$V_4 = V_{2e} - V_{2o} = \frac{Z_o V\sec\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta}\left[\frac{Z_o + jZ_{oe}\tan\theta}{Z_o + jZ_{oe}\tan\theta} - \frac{Z_o + jZ_{oo}\tan\theta}{Z_o + jZ_{oo}\tan\theta}\right]$$

$$V_2 = V_{2e} + V_{2o} = \frac{2Z_o V\sec\theta}{2Z_o + j(Z_{oe}+Z_{oo})\tan\theta} = \frac{\frac{2Z_o}{Z_{oe}+Z_{oo}}V}{\frac{2Z_o}{Z_{oe}+Z_{oo}}\cos\theta + j\sin\theta}$$

$$= \frac{V\sqrt{1-C^2}}{\sqrt{1-C^2}\cos\theta + j\sin\theta} \checkmark \qquad \text{since } \sqrt{1-C^2} = \frac{2Z_o}{Z_{oe}+Z_{oo}}$$

7.25 $N = 3$, $C = 20$ dB, maximally flat, $Z_0 = 50\Omega$

(a) From (7.90) and Example

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \tfrac{1}{2}C_2\right] \qquad ;\ C_3 = C_1$$

$$= C_1\sin 3\theta + (C_2 - C_1)\sin\theta \qquad \text{for } \theta = \pi/2,\ C = C_0 = C_2 - 2C_1$$

$$\frac{dC}{d\theta} = \left[3C_1\cos 3\theta + (C_2 - C_1)\cos\theta\right]\Big|_{\pi/2} \equiv 0$$

$$\frac{d^2C}{d\theta^2} = \left[-9C_1\sin 3\theta - (C_2 - C_1)\sin\theta\right]\Big|_{\pi/2} = 10C_1 - C_2 = 0$$

at midband ($\theta = \pi/2$), $C_0 = 10^{-20/20} = 0.1 = C_2 - 2C_1$

$$\therefore\ C_1 = C_3 = 0.1/8 = 0.0125$$

$$C_2 = 10C_1 = 0.125$$

Using (7.87) gives Z_{0e}, Z_{0o}:

$$Z_{0e}^{(1)} = Z_{0e}^{(3)} = Z_0\sqrt{\frac{1+C_1}{1-C_1}} = 50.63\Omega$$

$$Z_{0o}^{(1)} = Z_{0o}^{(3)} = Z_0\sqrt{\frac{1-C_1}{1+C_1}} = 49.38\Omega$$

$$Z_{0e}^{(2)} = Z_0\sqrt{\frac{1+C_2}{1-C_2}} = 56.69\Omega$$

$$Z_{0o}^{(2)} = Z_0\sqrt{\frac{1-C_2}{1+C_2}} = 44.10\Omega$$

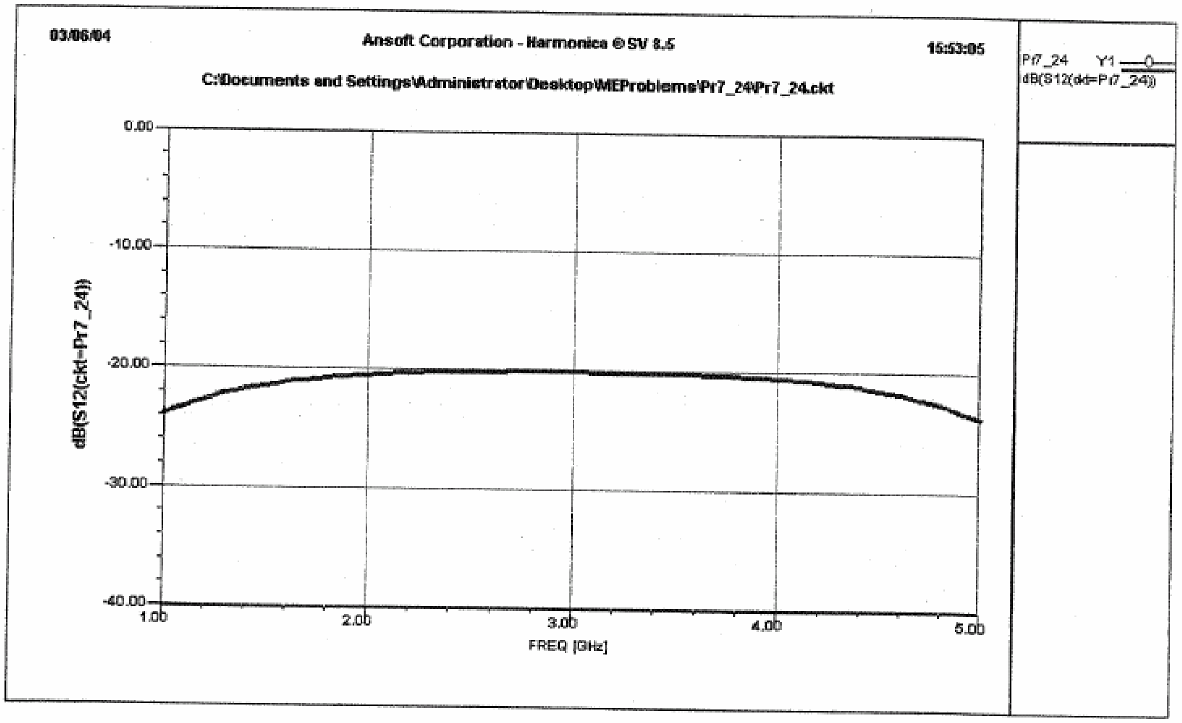
(b) $\epsilon_r = 4.2$, $d = 0.158$ cm, $\tan\delta = 0.02$, Cu, 0.5 mil $= t$.

From Serenade,

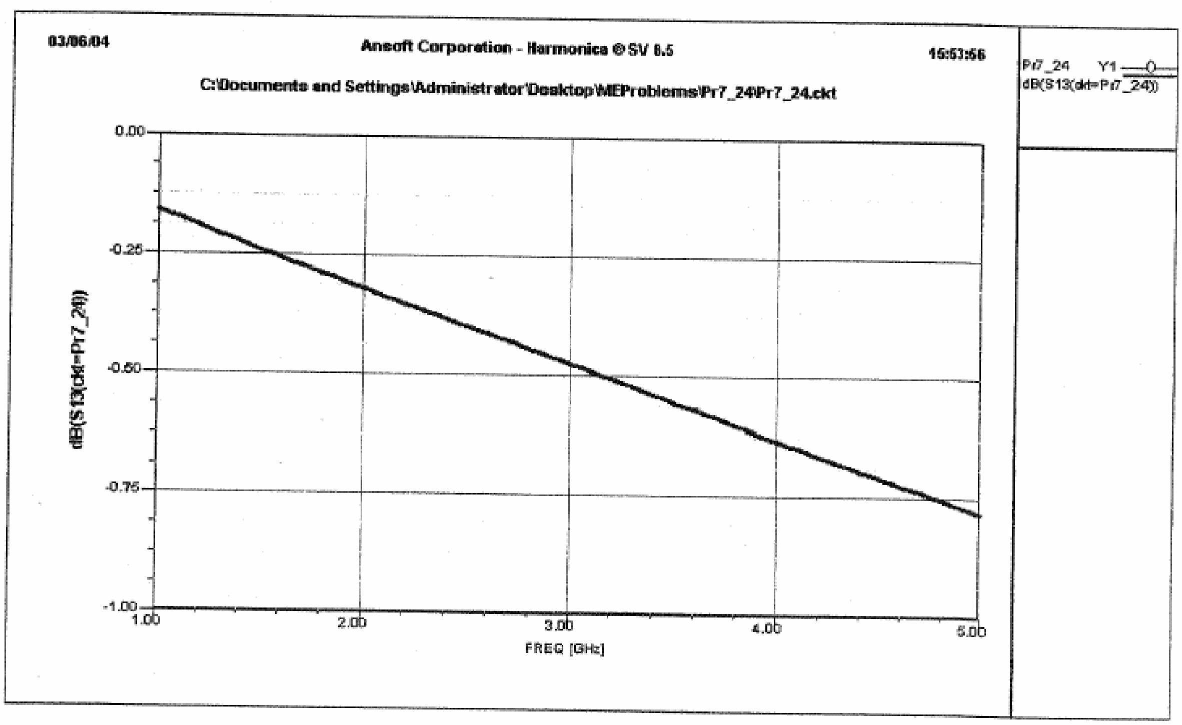
$Z_{0e} = 50.63$, $Z_{0o} = 49.38 \Rightarrow W = 3.125$ mm, $S = 9.941$ mm, $\ell = 1.38$ cm

$Z_{0e} = 56.69$, $Z_{0o} = 44.10 \Rightarrow W = 3.054$ mm, $S = 1.601$ mm, $\ell = 1.39$ cm

The resulting coupling and insertion loss are plotted on the following page.



03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
15:53:05
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_24\Pr7_24.ckt
Pr7_24 Y1
dB(S12(ckt=Pr7_24))
0.00
-10.00
-20.00
-30.00
-40.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]



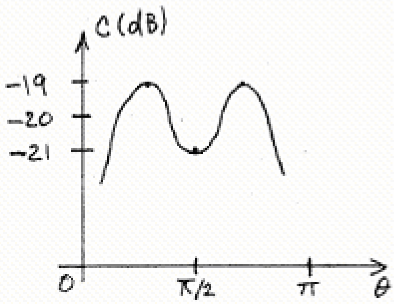
03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
15:53:56
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_24\Pr7_24.ckt
Pr7_24 Y1
dB(S13(ckt=Pr7_24))
0.00
-0.25
-0.50
-0.75
-1.00
1.00
2.00
3.00
4.00
5.00
FREQ [GHz]

7.26 $C = 20\,dB$, equal-ripple (1dB), $N=3$, $Z_0 = 50\,\Omega$

(a) From (7.90) and Example 7.8:

$$C = \left|\frac{V_3}{V_1}\right| = 2\sin\theta\left[C_1\cos 2\theta + \tfrac{1}{2}C_2\right]$$
$$= C_1 \sin 3\theta + (C_2 - C_1)\sin\theta$$

We cannot equate this to a Chebyshev polynomial, since C is a polynomial in $\sin\theta$. The desired response is as shown:



So at $\theta = \pi/2$, $C_0 = -21\,dB = 0.08913$
$= C_2 - 2C_1$

$C_{MAX} = -19\,dB = 0.1122$

We can use trial-and-error to find C_1:

C_1	C_{MAX}
.05	.134
.06	.147
.03	.106
$\boxed{.035}$	$\boxed{.1125}$

Thus $C_1 = 0.035$, $C_2 = 0.1591$

Then,

$Z_{0e}^{(1)} = Z_{0e}^{(3)} = 51.78\,\Omega$

$Z_{0o}^{(1)} = Z_{0o}^{(3)} = 48.28\,\Omega$

$Z_{0e}^{(2)} = 58.70\,\Omega$

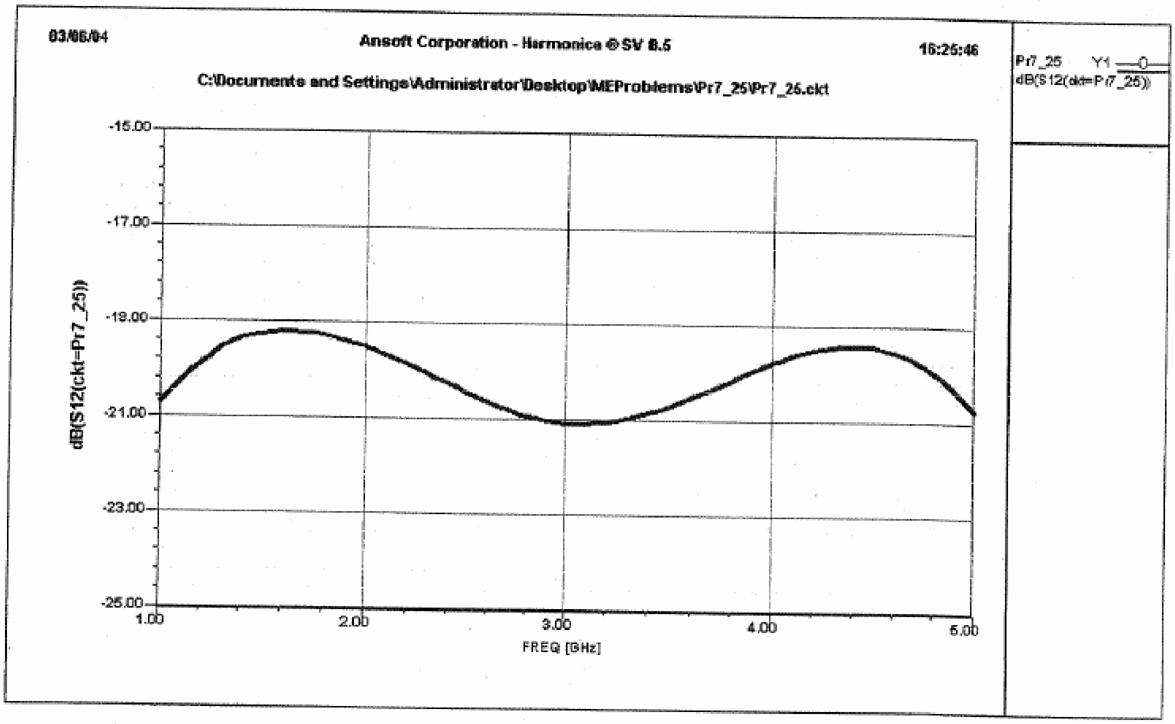
$Z_{0o}^{(2)} = 42.59\,\Omega$

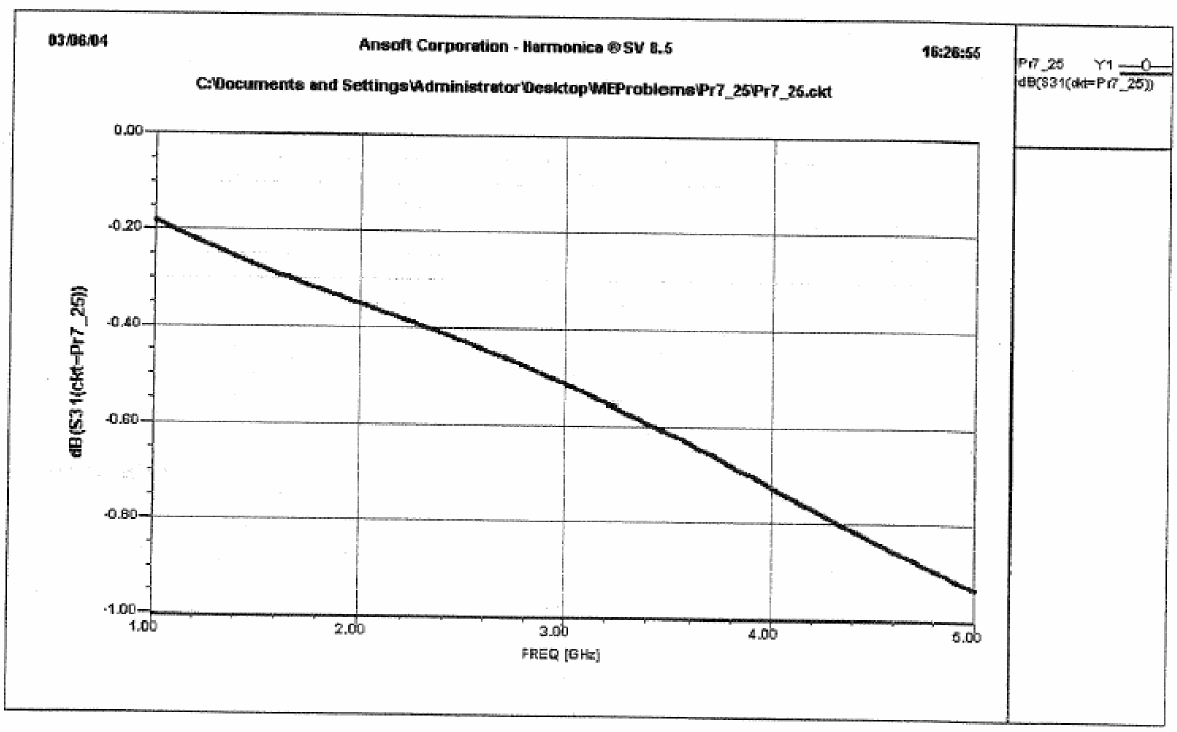
(b) From Serenade,

$Z_{0e} = 51.78$, $Z_{0o} = 48.28 \Rightarrow W = 3.121\,mm$, $S = 4.842\,mm$, $\ell = 1.38\,cm$

$Z_{0e} = 58.70$, $Z_{0o} = 42.59 \Rightarrow W = 3.002\,mm$, $S = 1.173\,mm$, $\ell = 1.39\,cm$

The coupling and insertion loss are plotted on the following page.


03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
16:25:46
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_25\Pr7_25.ckt
Pr7_25 Y1
dB(S12(ckt=Pr7_25))
dB(S12(ckt=Pr7_25))
FREQ [GHz]


03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
16:26:55
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_25\Pr7_25.ckt
Pr7_25 Y1
dB(S31(ckt=Pr7_25))
dB(S31(ckt=Pr7_25))
FREQ [GHz]

7.27 From (7.98) and (7.99) we can show that,

$$Z_{e4} = Z_0 \sqrt{\frac{1+C}{1-C}}$$

Equating this to (7.97a) gives:

$$Z_0 \sqrt{\frac{1+C}{1-C}} = \frac{Z_{oo} + Z_{oe}}{3Z_{oo} + Z_{oe}} = \frac{1 + \frac{Z_{oe}}{Z_{oo}}}{3 + \frac{Z_{oe}}{Z_{oo}}} Z_{oe}$$

Now solve (7.99) for Z_{oe} in terms of Z_{oo}:

$$3C\,(Z_{oe}^2 + Z_{oo}^2) + 2C\,Z_{oe}Z_{oo} = 3(Z_{oe}^2 - Z_{oo}^2)$$

$$3(C-1)\,Z_{oe}^2 + 2C\,Z_{oe}Z_{oo} + 3(C+1)\,Z_{oo}^2 = 0$$

$$Z_{oe} = \frac{-2C\,Z_{oo} \pm \sqrt{4C^2 Z_{oo}^2 - 36(C^2-1)Z_{oo}^2}}{6(C-1)} = \frac{-C - \sqrt{9-8C^2}}{3(C-1)} Z_{oo}$$

(choose negative root since Z_{oe} and Z_{oo} are positive, and $C<1$)
Substituting for Z_{oe}/Z_{oo} in the above expression gives,

$$Z_0 \sqrt{\frac{1+C}{1-C}} = \frac{2C - 3 - \sqrt{9-8C^2}}{8C - 9 - \sqrt{9-8C^2}} Z_{oe}$$

or

$$Z_{oe} = Z_0 \sqrt{\frac{1+C}{1-C}} \frac{[8C - 9 - \sqrt{9-8C^2}][2C - 3 + \sqrt{9-8C^2}]}{(2C-3)^2 - (9-8C^2)}$$

$$= Z_0 \sqrt{\frac{1+C}{1-C}} \frac{(24C^2 - 42C + 18) + 6(C-1)\sqrt{9-8C^2}}{12C(C-1)}$$

$$= Z_0 \sqrt{\frac{1+C}{1-C}} \frac{4C - 3 + \sqrt{9-8C^2}}{2C} \checkmark$$

To find Z_{oo}, simply replace C by $-C$ (because of symmetry of (7.98) and (7.99):

$$Z_{oo} = Z_0 \sqrt{\frac{1-C}{1+C}} \frac{4C + 3 - \sqrt{9-8C^2}}{2C} \checkmark$$

7.28

$f = 5\,GHz$, $\epsilon_r = 10$, $d = 1\,mm$

$C = 10^{-3/20} = 0.708$

assume $Z_0 = 50\,\Omega$ (not stated in problem!)

From (7.100) we have,

$$Z_{oe} = \frac{4C - 3 + \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1-C}{1+C}}} Z_o = 176.4\,\Omega$$

$$Z_{oo} = \frac{4C + 3 - \sqrt{9 - 8C^2}}{2C\sqrt{\frac{1+C}{1-C}}} Z_o = 52.5\,\Omega$$

(These results are very approximate; SuperCompact gives $Z_{oe} = 121\,\Omega$ and $Z_{oo} = 21\,\Omega$ for this design)

From Figure 7.30,

$$S/d \cong 0.075 \Rightarrow S = 0.075\,mm$$

$$W/d \cong 0.07 \Rightarrow W = 0.07\,mm$$

(SuperCompact gives 0.071 mm and 0.075 mm for S and W, respectively, starting with the values calculated above for Z_{oe} and Z_{oo}. These are reasonably close to our values for S and W, even though Z_{oe}, Z_{oo} are not close to the SuperCompact values.)

7.29 From (7.101) the [S] matrix of a 180° (3dB) hybrid is,

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

For V_1 at port 1 and V_4 at port 4, the output voltages are (note: hybrid is matched, so $V_1 = V_1^+$, $V_4 = V_4^+$)

$$\begin{bmatrix} V_1^- \\ V_2^- \\ V_3^- \\ V_4^- \end{bmatrix} = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ 0 \\ 0 \\ V_4 \end{bmatrix}$$

$$= \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 \\ V_1 - V_4 \\ V_1 + V_4 \\ 0 \end{bmatrix} \begin{matrix} \checkmark \\ \checkmark \quad \text{(difference)} \\ \checkmark \quad \text{(sum)} \\ \checkmark \end{matrix}$$

7.30

$\alpha = \beta = \sqrt{2}/2$ for $C = 3\,dB$

From (7.115a), $\beta = \dfrac{2\sqrt{k}}{k+1} \Longrightarrow k = 0.1716$ ✓

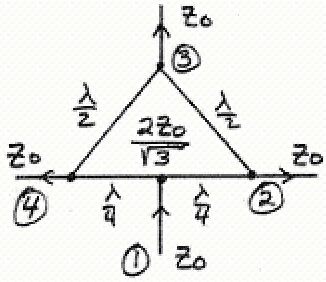
From (7.115b), $\alpha = \dfrac{1-k}{1+k} \Longrightarrow k = 0.1716$ ✓

Then,

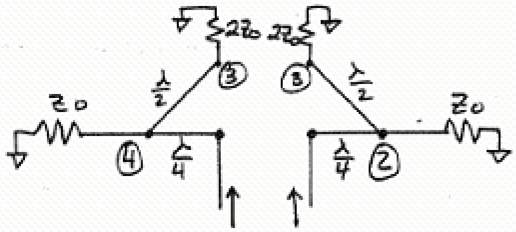
$$Z_{oe}(0) = Z_{oo}(0) = Z_0 = 50\,\Omega$$
$$Z_{oe}(L) = Z_0/k = 291\,\Omega$$
$$Z_{oo}(L) = k Z_0 = 8.6\,\Omega$$

A Klopfenstein taper can be used for these taper variations.

7.31



First, let $V_1^+ = 1v$ at port 1, with matched loads at other ports. Then we can bisect the network as follows:



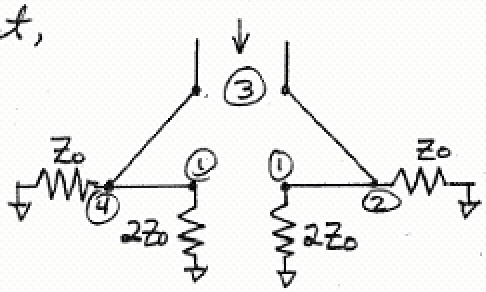
The input impedance of one of these halves is,

$$\left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{(Z_0+2Z_0)}{2Z_0^2} = 2Z_0 \text{, so } Z_{in}^{(1)} = Z_0 \text{, and } S_{11}=0. \checkmark$$

Because of the $\lambda/2$ line, the voltage magnitude at port 3 is equal to the voltage magnitude at port 4. By power conservation, $P_2 = P_4 = P_3 = P_{in}/3$. Thus,

$$S_{41} = S_{21} = 1/\sqrt{3}\,\underline{/-90^\circ}, \quad S_{31} = 1/\sqrt{3}\,\underline{/-270^\circ}. \quad (|S_{11}|^2+|S_{21}|^2+|S_{31}|^2+|S_{41}|^2=1)$$

Next, let $V_3^+ = 1v$ at port 3, with other ports matched, and bisect,

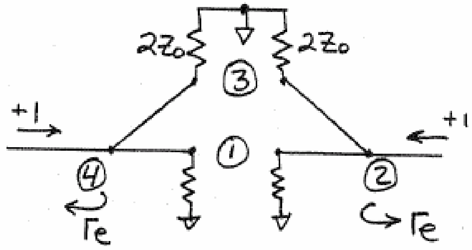


The input impedance of one of these halves is

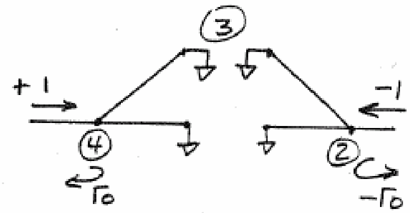
$$Z_0 \,\|\, \left(\frac{2Z_0}{\sqrt{3}}\right)^2 \frac{1}{2Z_0} = Z_0 \,\|\, \frac{2Z_0}{3} = \frac{2Z_0}{5} \text{, so } Z_{in}^{(3)} = Z_0/5 \text{, and } S_{33} = -2/3.$$

So the power delivered to each half is $\frac{1}{2}P_{inc}(1-|S_{33}|^2) = 5/18\, P_{inc}$. Of this, $2/5$ goes to port 4 and $3/5$ goes to port 1. So, $S_{43} = S_{23} = 1/3\,\underline{/-180^\circ}$. The total power to port 1 is then $1/3\, P_{inc}$, so $S_{13} = 1/\sqrt{3}\,\underline{/-270^\circ}$. (Then $|S_{13}|^2+|S_{23}|^2+|S_{33}|^2+|S_{43}|^2 = \frac{1}{3}+\frac{1}{9}+\frac{4}{9}+\frac{1}{9}=1$)

Now drive ports 2 and 4 with even and odd excitations:



EVEN MODE



ODD MODE

$$Z^e_{in} = 2Z_0 \| \frac{2Z_0}{3} = Z_0/2 \qquad\qquad Z^o_{in} = 0$$

$$\Gamma_e = \frac{1/2 - 1}{1/2 + 1} = -1/3 \qquad\qquad \Gamma_o = -1$$

Then, $S_{22} = S_{44} = \frac{1}{2}(\Gamma_e + \Gamma_o) = \frac{1}{2}(-1/3 - 1) = -2/3$

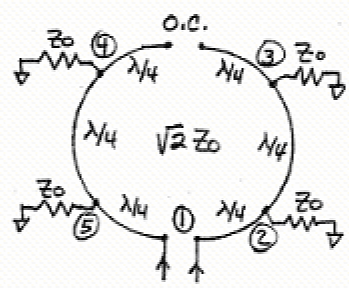
$S_{24} = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{1}{2}(-1/3 + 1) = 1/3$

So the complete S-matrix is,

$$[S] = \begin{bmatrix} 0 & 1/\sqrt{3}\angle{-90} & 1/\sqrt{3}\angle{-270} & 1/\sqrt{3}\angle{-90} \\ 1/\sqrt{3}\angle{-90} & -2/3 & -1/3 & 1/3 \\ 1/\sqrt{3}\angle{-270} & -1/3 & -2/3 & -1/3 \\ 1/\sqrt{3}\angle{-90} & 1/3 & -1/3 & -2/3 \end{bmatrix} \quad \checkmark$$

This checks with an analysis using SuperCompact.

7.32 Let $V_1^+ = 1\angle 0$



Bisecting the network places an effective short circuit at ports 3 and 4, due to the $\lambda/4$ O.C. stubs. Thus $S_{41} = S_{31} = 0$. Then there is an effective open circuit in parallel with the Z_0 loads at ports 2 and 5. So the input impedance of one of the halves is $(\sqrt{2}Z_0)^2/Z_0 = 2Z_0$. The total input impedance at port 1 is then Z_0, so $S_{11} = 0$, and the input power divides evenly to ports 2 and 5. Thus,

$V_1^- = 0; \quad V_2^- = V_5^- = 0.707\angle{-90°}; \quad V_3^- = V_4^- = 0.$

CHECK: $|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 0 + \frac{1}{2} + 0 + \frac{1}{2} = 1 \quad \checkmark$

7.33

$$[S] = \frac{-1}{\sqrt{2}}\begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix} \quad \text{for hybrid}$$

a) let $b = \lambda/4$ at $f_0 \Rightarrow \beta b = \pi/2$

Assume V_0^+ incident at T-junction. Then,

$$V_1^+ = V_0^+ e^{-j\beta(b-a)} = e^{-j\pi/2} e^{j\beta a}$$

$$V_4^+ = V_0^+ e^{-j\beta a}$$

$$V_2^- = \frac{-V_0^+}{\sqrt{2}}(jV_1^+ + V_4^+) = \frac{-V_0^+}{\sqrt{2}}(e^{j\beta a} + e^{-j\beta a}) = -\sqrt{2}\, V_0^+ \cos\beta a$$

$$P_2 = \tfrac{1}{2}|V_2^-|^2 = |V_0^+|^2 \cos^2\beta a$$

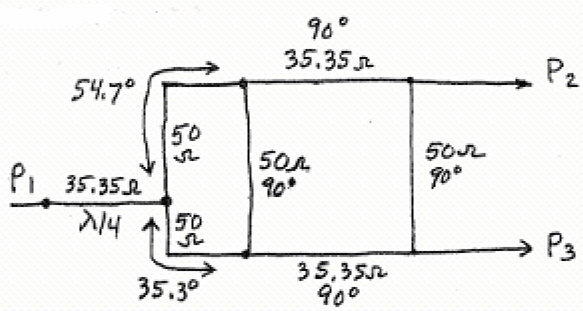
$$V_3^- = \frac{-V_0^+}{\sqrt{2}}(V_1^+ + jV_4^+) = \frac{-V_0^+}{\sqrt{2}}(-je^{j\beta a} + je^{-j\beta a}) = -\sqrt{2}\, V_0^+ \sin\beta a$$

$$P_3 = \tfrac{1}{2}|V_3^-|^2 = |V_0^+|^2 \sin^2\beta a$$

So $\dfrac{P_3}{P_2} = \tan^2\beta a = \tan^2\dfrac{\pi a}{2b}$ ✓ $\quad$ (since $\beta = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{4b} = \dfrac{\pi}{2b}$)

b) For $\dfrac{P_3}{P_2} = 0.5$, $\; a = 0.098\lambda = 35.3°$; $(b-a) = 0.152\lambda = 54.7°$

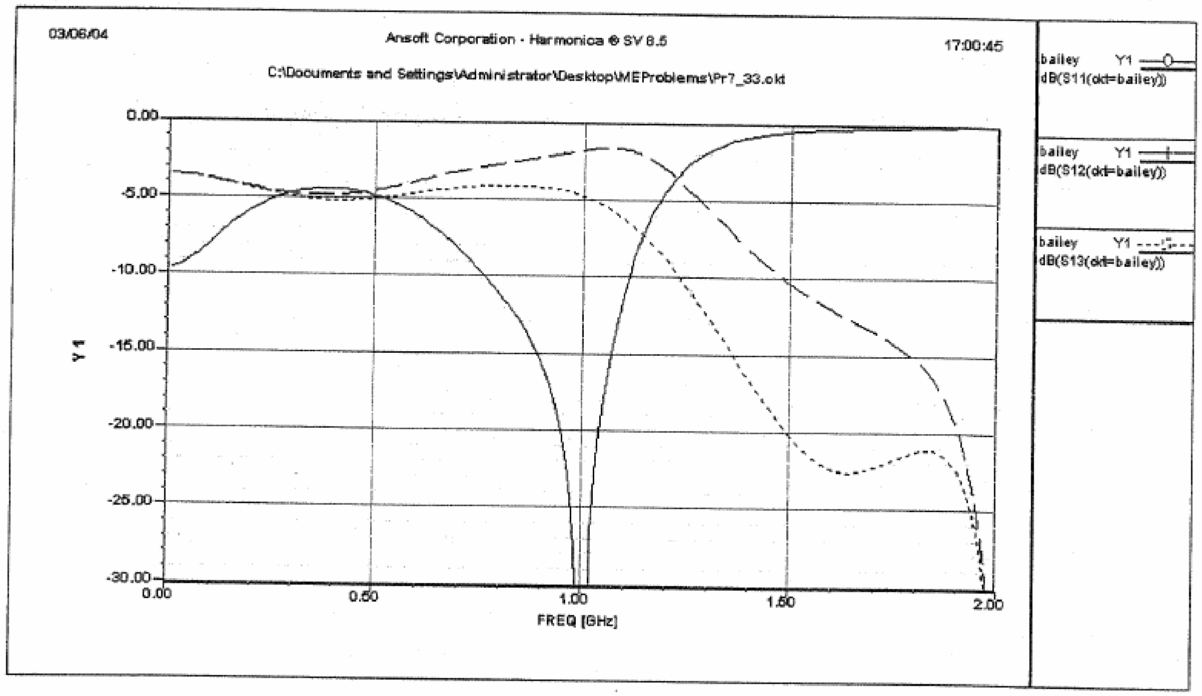
CIRCUIT:



The S-parameters are plotted on the following page, for a center frequency of 1 GHz. Note that the power output ratios,

$$\frac{P_2}{P_1} = \frac{2}{3} = -1.76\,dB \quad \text{and} \quad \frac{P_3}{P_1} = \frac{1}{3} = -4.77\,dB$$

are verified.


03/06/04
Ansoft Corporation - Harmonica ® SV 8.5
17:00:45
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr7_33.ckt
Y1
FREQ [GHz]
0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
0.50
1.00
1.50
2.00
bailey Y1
dB(S11(ckt=bailey))
bailey Y1
dB(S12(ckt=bailey))
bailey Y1
dB(S13(ckt=bailey))

Chapter 8

8.1

$Z_0 = 75\,\Omega$, $L_0 = 1.25\,\text{mH}$, $d = 1.0\,\text{cm}$

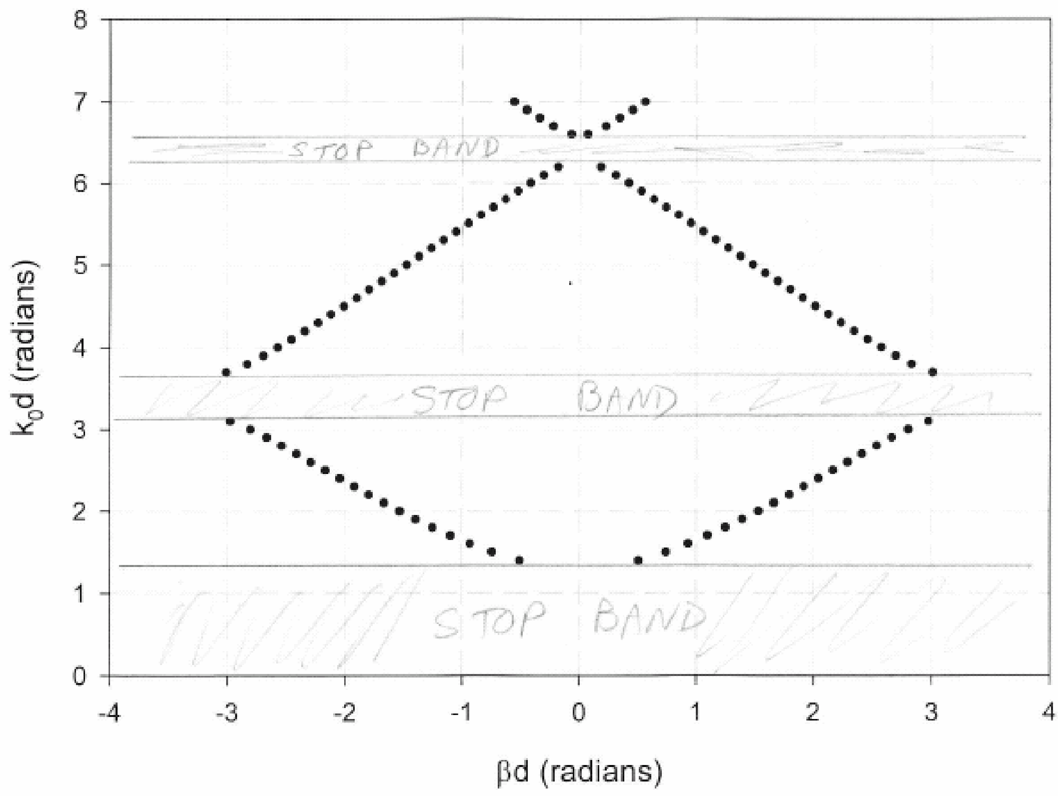
let $\theta = k_0 d$.

$$b = \frac{-Z_0}{\omega L_0} = \frac{-Z_0}{c\,k_0 L_0} = \frac{-200}{k_0}$$

Passband when $|\cos\beta d| = |\cos\theta - \frac{b}{2}\sin\theta| \leq 1$

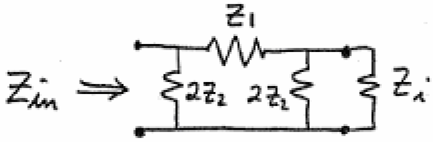
Stopband when $\cosh\alpha d = |\cos\theta - \frac{b}{2}\sin\theta| \geq 1$

a short program was used to compute βd, with results plotted below.



8.2 $Z_{i\pi}$ can easily be derived from $Z_i = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{B}{C}}$.

Verify as follows:



$$Z_i = \frac{\sqrt{Z_1 Z_2}}{\sqrt{1 + Z_1/4Z_2}} = \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2 + Z_1/4}}$$

let $Z = 2Z_2 \| Z_i = \dfrac{\dfrac{2Z_2^2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}}{2Z_2 + \dfrac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}}} = \dfrac{2Z_2\sqrt{Z_1}}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}$

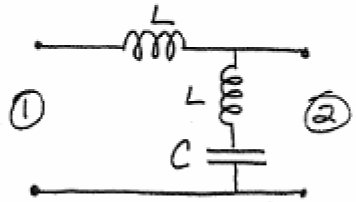
$$Z_{in} = 2Z_2 \| (Z_1 + Z) = \frac{\dfrac{4Z_1Z_2\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}}{2Z_2 + \dfrac{2Z_1\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}(2Z_2+Z_1)}{2\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}}}$$

$$= \frac{4Z_1Z_2\sqrt{Z_2+Z_1/4} + Z_2\sqrt{Z_1}(2Z_2+Z_1)(2)}{4Z_2\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_1} + 2Z_1\sqrt{Z_2+Z_1/4} + \sqrt{Z_1}(2Z_2+Z_1)}$$

$$= \frac{Z_2\left[\sqrt{Z_1}(2Z_2+Z_1) + 2Z_1\sqrt{Z_2+Z_1/4}\right]}{2\sqrt{Z_1}(Z_2+Z_1/4) + Z_1\sqrt{Z_2+Z_1/4} + 2Z_2\sqrt{Z_2+Z_1/4}}$$

$$= \frac{Z_2\sqrt{Z_1}}{\sqrt{Z_2+Z_1/4}} = Z_i \quad \checkmark$$

8.3



$$\frac{1}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega C}{1-\omega^2 LC}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j\omega L \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1-2\omega^2 LC}{1-\omega^2 LC} & j\omega L \\ \frac{j\omega C}{1-\omega^2 LC} & 1 \end{bmatrix} \checkmark$$

From (8.27),

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{\frac{j\omega L(1-2\omega^2 LC)}{j\omega C}} = \sqrt{\frac{L}{C}(1-2\omega^2 LC)} \checkmark$$

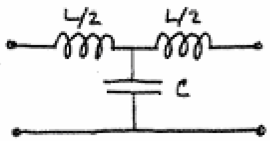
$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{j\omega L}{j\omega C(1-2\omega^2 LC)}} = \frac{1}{1-\omega^2 LC}\sqrt{\frac{L}{C(1-2\omega^2 LC)}}$$

From (8.31), $\cosh\gamma = \sqrt{AD} = \sqrt{\frac{1-2\omega^2 LC}{1-\omega^2 LC}}$.

8.4 $R_0 = 50\,\Omega$, $f_c = 50$ MHz, $f_\infty = 52$ MHz, LOW-PASS

Design equations from Table 8.2:

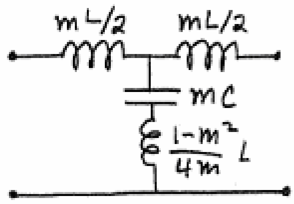
CONSTANT-k SECTION:



$L = 2R_0/\omega_c = 3.18\times10^{-7}$ H ; $L/2 = 159$ nH ✓

$C = 2/\omega_c R_0 = 127.$ pF ✓

m-DERIVED SECTION:



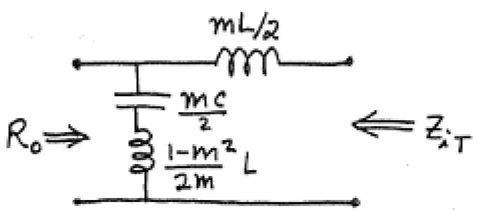
$m = \sqrt{1-(f_c/f_\infty)^2} = 0.275$ ✓

$\frac{mL}{2} = 43.7$ nH ✓

$mC = 34.9$ pF ✓

$\frac{1-m^2}{4m}L = 267.$ nH ✓

MATCHING SECTIONS: ($m = 0.6$)

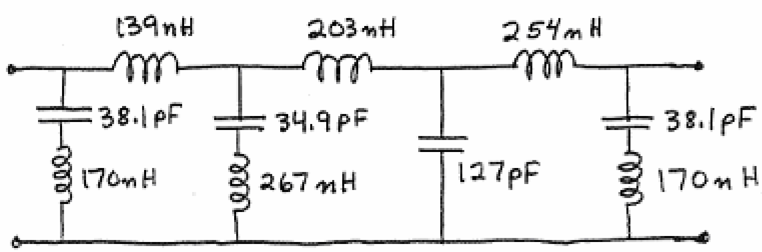


$\frac{mL}{2} = 95.4$ nH ✓

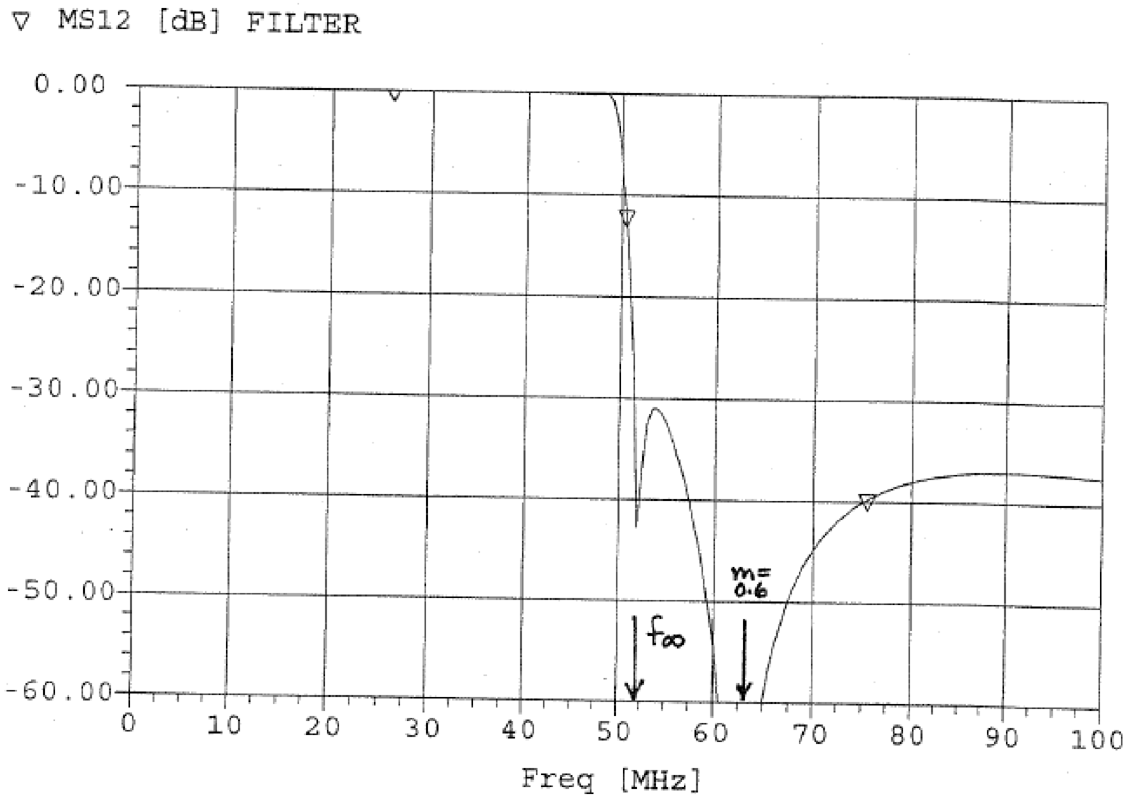
$\frac{mC}{2} = 38.1$ pF ✓

$\frac{1-m^2}{2m}L = 170.$ nH ✓

COMPLETE FILTER:



The calculated response is shown on the following page.



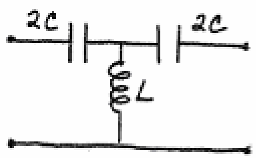
▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-60.00
0
10
20
30
40
50
60
70
80
90
100
Freq [MHz]
f_∞
m= 0.6

8.5

$R_0 = 75\,\Omega$, $f_c = 50\text{ MHz}$, $f_\infty = 48\text{ MHz}$, HIGH-PASS

Design equations are given in Table 8.2

CONSTANT-k SECTION:

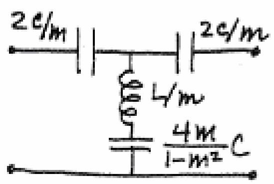


$L = R_0/2\omega_c = 119.\text{ nH}$ ✓

$C = 1/2R_0\omega_c = 21.2\text{ pF}$ ✓

$2C = 42.4\text{ pF}$ ✓

m-DERIVED SECTION:



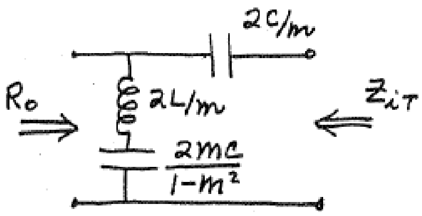
$m = \sqrt{1-(f_\infty/f_c)^2} = 0.280$ ✓

$\frac{2C}{m} = 151.\text{ pF}$ ✓

$L/m = 425.\text{ nH}$ ✓

$\frac{4mC}{1-m^2} = 25.8\text{ pF}$ ✓

MATCHING SECTION: $(m=0.6)$

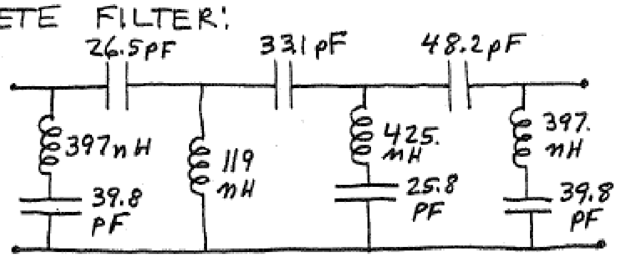


$\frac{2C}{m} = 70.7\text{ pF}$ ✓

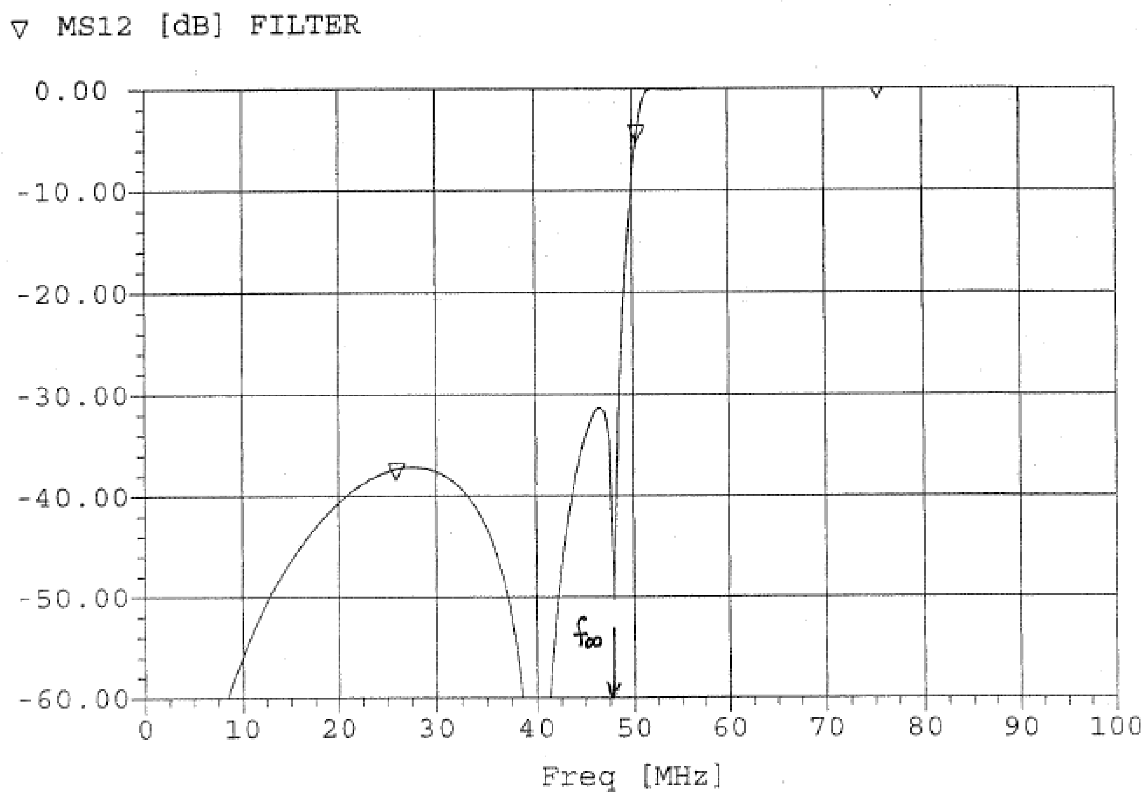
$\frac{2L}{m} = 397.\text{ nH}$ ✓

$\frac{2mC}{1-m^2} = 39.8\text{ pF}$ ✓

COMPLETE FILTER:



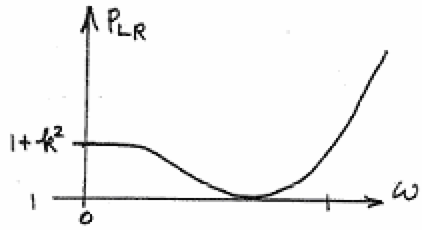
The calculated filter response is shown below.



▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
-50.00
-60.00
0
10
20
30
40
50
60
70
80
90
100
Freq [MHz]
f∞

8.6 From (8.61),

$$P_{LR} = 1 + k^2 T_N^2(\omega) = 1 + k^2(2\omega^2 - 1)^2 \quad \text{for } N = 2$$



$P_{LR} = 1 = 0\,dB$

$P_{LR} = 1 + k^2 = 1\,dB = 1.259$

so $k = \pm 0.509$

(We must choose $k = -0.509$, otherwise L, C are not real.)

Then from (8.63),

$$R = 1 + 2k^2 - 2k\sqrt{1+k^2} = 2.66 \checkmark$$

We also have that,

$$4k^2 = \frac{1}{4R} L^2 C^2 R^2 \implies L = \frac{-4k}{C\sqrt{R}}$$

$$-4k^2 = \frac{1}{4R}(R^2C^2 + L^2 - 2LCR^2)$$

$$-16k^2 R = R^2C^2 + \frac{16k^2}{C^2 R} + 8kR\sqrt{R}$$

$$R^2C^4 + (16k^2R + 8kR\sqrt{R})C^2 + \frac{16k^2}{R} = 0$$

$$7.08C^4 - 6.64C^2 + 1.56 = 0$$

Thus,

$$C = 0.685 \checkmark$$

$$L = 1.822 \checkmark$$

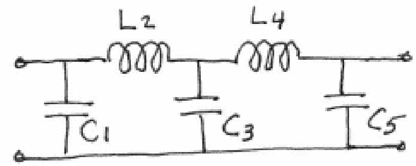
(R, L, C check with results given in Matthai, Young, and Jones.)

8.7 maximally flat, 0-2 GHz, $\alpha > 20$ dB at 3.4 GHz
$Z_0 = 50\,\Omega$

$$\left|\frac{\omega}{\omega_c}\right| - 1 = \frac{3.4}{2} - 1 = 0.7$$ From Fig 8.26, $N=5$

From Table 8.3

$g_1 = 0.6180$
$g_2 = 1.6180$
$g_3 = 2.000$
$g_4 = 1.6180$
$g_5 = 0.6180$



Using (8.67),

$$C_1 = \frac{g_1}{R_0 \omega_c} = 0.984\ \text{pF}$$

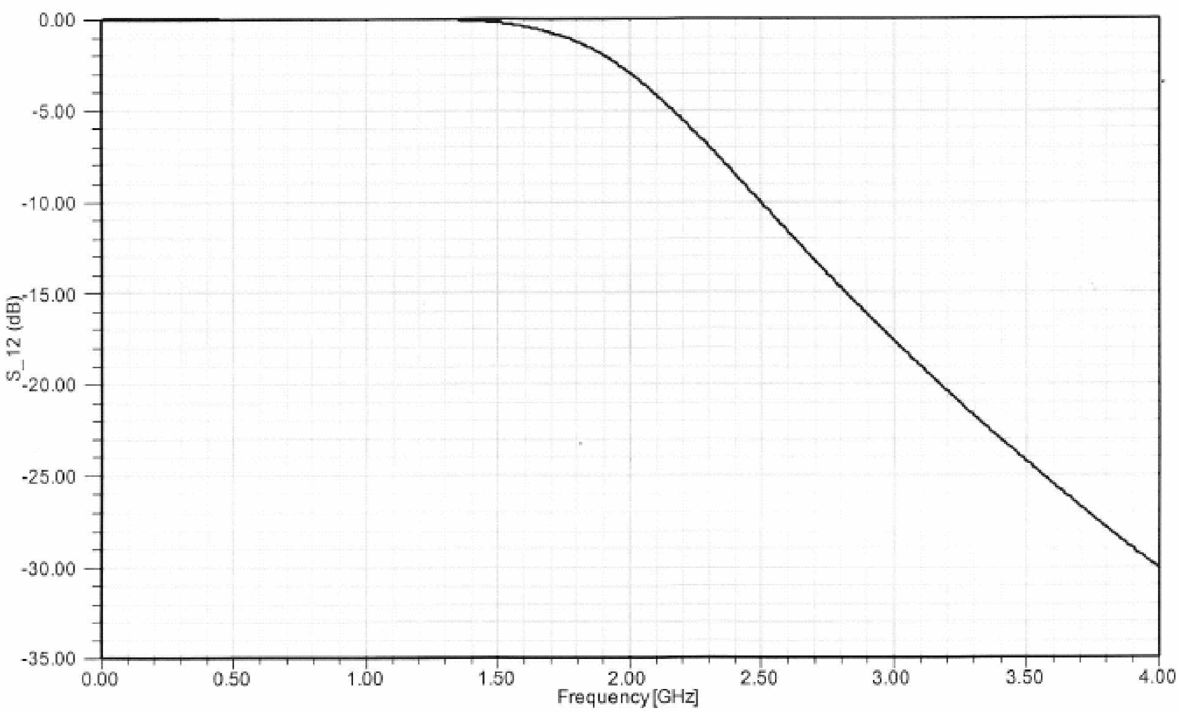
$$L_2 = \frac{R_0 g_2}{\omega_c} = 6.44\ \text{nH}$$

$$C_3 = \frac{g_3}{R_0 \omega_c} = 3.183\ \text{pF}$$

$$L_4 = \frac{R_0 g_4}{\omega_c} = 6.44\ \text{nH}$$

$$C_5 = \frac{g_5}{R_0 \omega_0} = 0.984\ \text{pF}$$

filter response is plotted below.

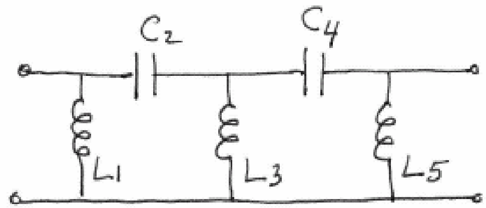

0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
-35.00
S_12 (dB)
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
4.00
Frequency [GHz]

8.8 high-pass, 3dB equal-ripple, $f_c = 3\,GHz$, $\alpha > 30\,dB$ at $2.0\,GHz$, $Z_0 = 75\,\Omega$

at 2.0 GHz, $\left|\frac{\omega_c}{\omega}\right| - 1 = \frac{3}{2} - 1 = 0.5 \Rightarrow N = 5$

From Table 8.4,

$g_1 = 3.4817$
$g_2 = 0.7618$
$g_3 = 4.4381$
$g_4 = 0.7618$
$g_5 = 3.4817$



Using (8.70),

$$L_1 = \frac{Z_0}{\omega_c g_1} = 1.143\,nH$$

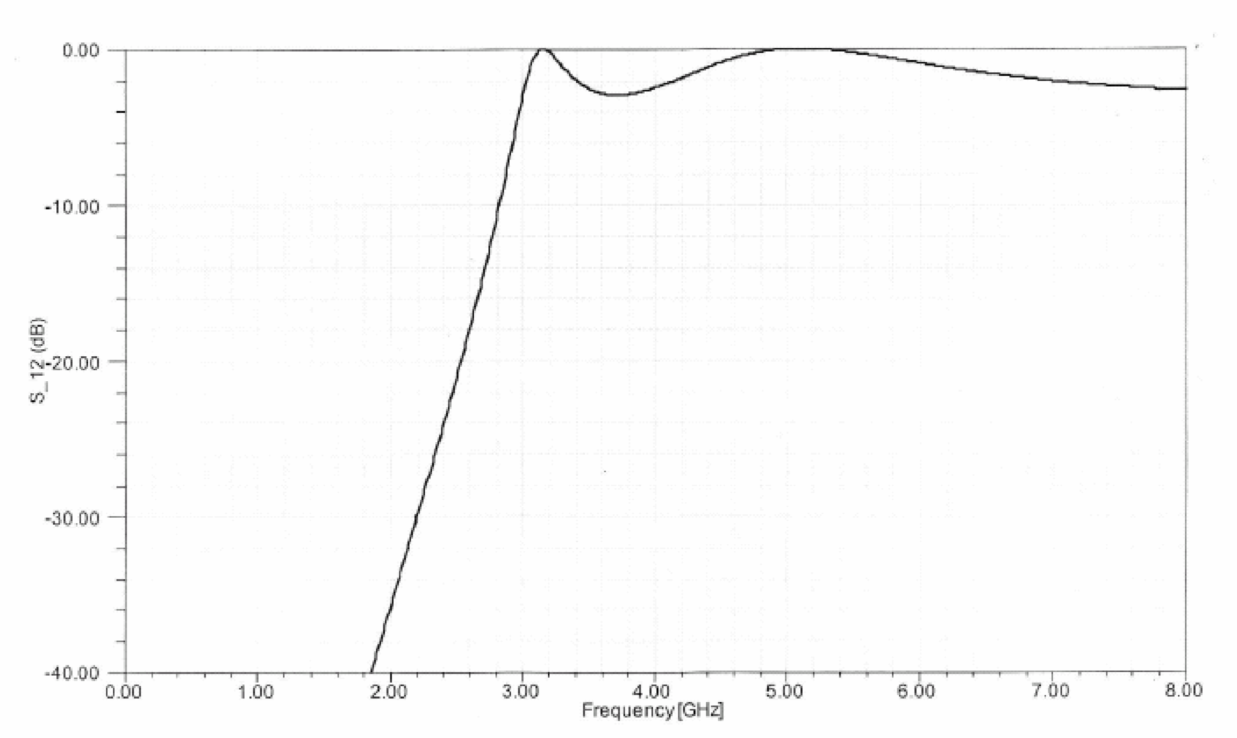
$$C_2 = \frac{1}{Z_0 \omega_c g_2} = 0.928\,pF$$

$$L_3 = \frac{Z_0}{\omega_c g_3} = 0.877\,nH$$

$$C_4 = \frac{1}{Z_0 \omega_c g_4} = 0.928\,pF$$

$$L_5 = \frac{Z_0}{\omega_c g_5} = 1.143\,nH$$

filter response is shown below.


0.00
-10.00
-20.00
-30.00
-40.00
S_12 (dB)
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
Frequency [GHz]

8.9

$f_0 = 2\,GHz$, B.P., M.F.G.D., $\Delta = 0.05$, $N = 4$, $Z_0 = 50\,\Omega$

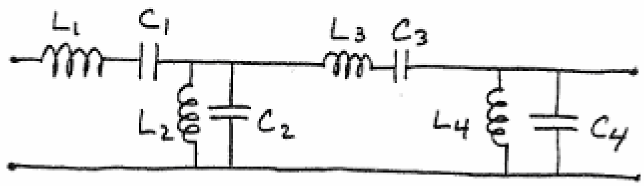
From Table 8.5 the prototype element values are,

$g_1 = 1.0598$

$g_2 = 0.5116$

$g_3 = 0.3181$

$g_4 = 0.1104$



From Table 8.6 and (8.64) the scaled element values are

$L_1 = \dfrac{g_1 Z_0}{\omega_0 \Delta} = 84.3\ nH$ ✓

$C_1 = \dfrac{\Delta}{\omega_0 g_1 Z_0} = 0.075\ pF$ ✓

$L_2 = \dfrac{\Delta Z_0}{\omega_0 g_2} = 0.388\ nH$ ✓

$C_2 = \dfrac{g_2}{\omega_0 \Delta Z_0} = 16.3\ pF$ ✓

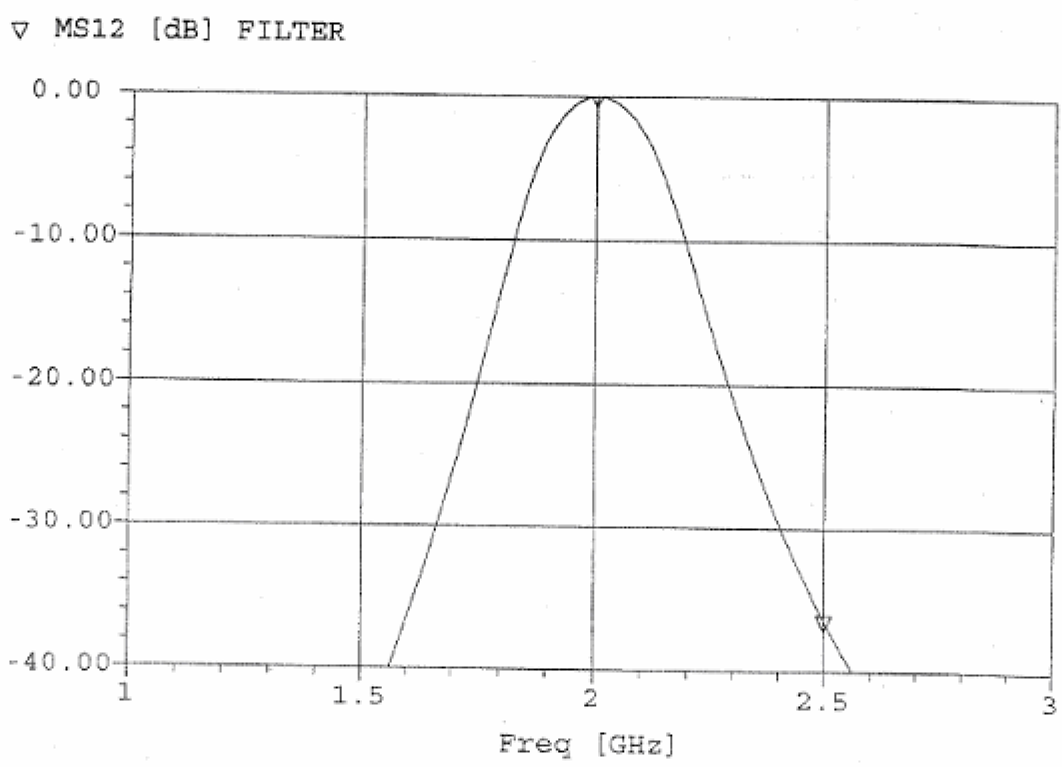
$L_3 = \dfrac{g_3 Z_0}{\omega_0 \Delta} = 25.3\ nH$ ✓

$C_3 = \dfrac{\Delta}{\omega_0 g_3 Z_0} = 0.25\ pF$ ✓

$L_4 = \dfrac{\Delta Z_0}{\omega_0 g_4} = 1.80\ nH$ ✓

$C_4 = \dfrac{g_4}{\omega_0 \Delta Z_0} = 3.51\ pF$ ✓

The calculated filter response is shown below.



8.10 $f_0 = 3\text{GHz}$, $Z_0 = 75\Omega$, $N=3$, B.S., 0.5dB E.R.

First use (8.75) to transform 3.1 GHz to a L.P. prototype response frequency:

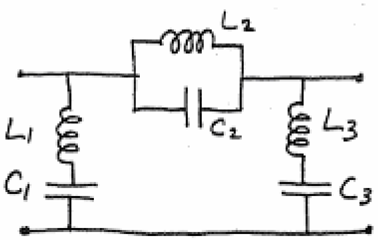
$$\omega \leftarrow \Delta\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)^{-1} = 0.1\left(\frac{3.1}{3} - \frac{3}{3.1}\right)^{-1} = 1.52$$

So, $\left|\frac{\omega}{\omega_c}\right| - 1 = 0.52$, and Figure 8.27a gives an attenuation of 11 dB for $N=3$. From Table 8.4, the prototype values are,

$g_1 = 1.5963$

$g_2 = 1.0967$

$g_3 = 1.5963$



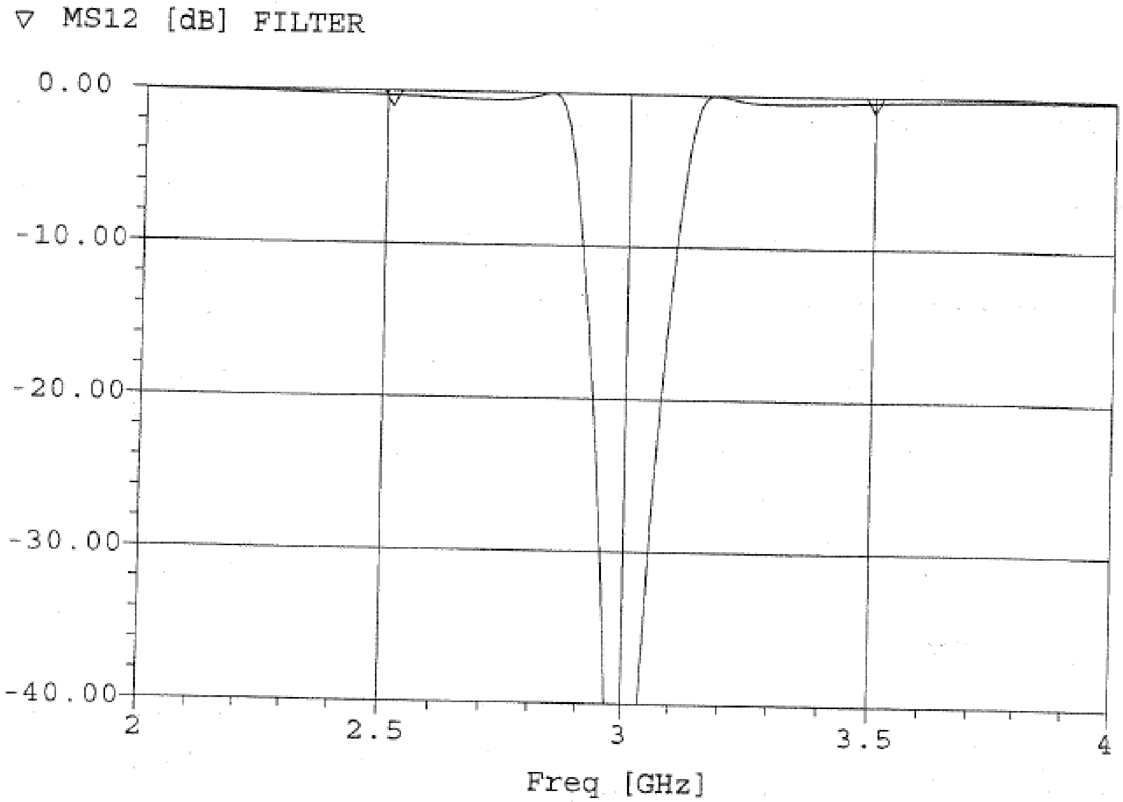
From Table 8.6 and (8.64) the scaled element values are,

$$L_1 = \frac{Z_0}{\omega_0 g_1 \Delta} = 24.9\ \text{nH} \quad \checkmark \qquad C_1 = \frac{g_1 \Delta}{\omega_0 Z_0} = 0.113\ \text{pF} \quad \checkmark$$

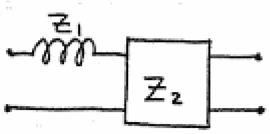
$$L_2 = \frac{g_2 \Delta Z_0}{\omega_0} = 0.436\ \text{nH} \quad \checkmark \qquad C_2 = \frac{1}{Z_0 \omega_0 g_2 \Delta} = 6.45\ \text{pF} \quad \checkmark$$

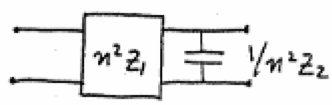
$$L_3 = \frac{Z_0}{\omega_0 g_3 \Delta} = 24.9\ \text{nH} \quad \checkmark \qquad C_3 = \frac{g_3 \Delta}{Z_0 \omega_0} = 0.113\ \text{pF} \quad \checkmark$$

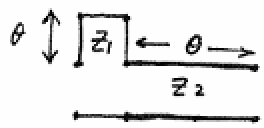
The calculated response for this filter is shown on the following page. Note that the insertion loss at 3.1 GHz is about 10 dB.



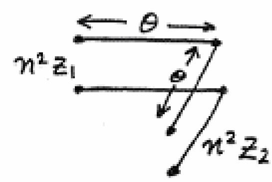
8.11







$Z_S = j Z_1 \tan\theta$



$Y_S = \frac{j}{n^2 Z_2} \tan\theta$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & j Z_1 \tan\theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & j Z_2 \sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j(Z_1+Z_2)\sin\theta \\ \frac{j}{Z_2}\sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta & j n^2 Z_2 \sin\theta \\ \frac{j\sin\theta}{n^2 Z_1} & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{j\tan\theta}{n^2 Z_2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta - \frac{Z_1}{Z_2}\frac{\sin^2\theta}{\cos\theta} & j n^2 Z_1 \sin\theta \\ \frac{j}{n^2}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)\sin\theta & \cos\theta \end{bmatrix}$$

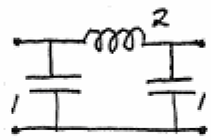
So these two matrices are equal if,

$$Z_1 + Z_2 = n^2 Z_1$$

or, $\quad n^2 = 1 + Z_2/Z_1$ ✓

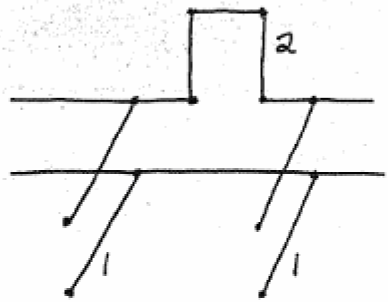
8.12 $f_o = 6$ GHz, $N = 3$, M.F., $Z_o = 50\Omega$

From Table 8.3 the L.P. prototype is,

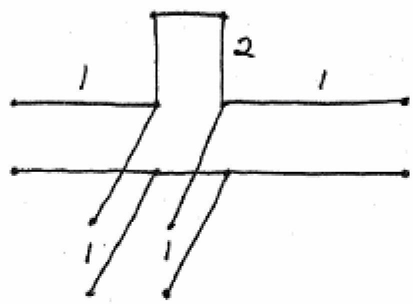


(choosing a π-circuit simplifies the problem)

Richards' transform:



add unit elements at ends:



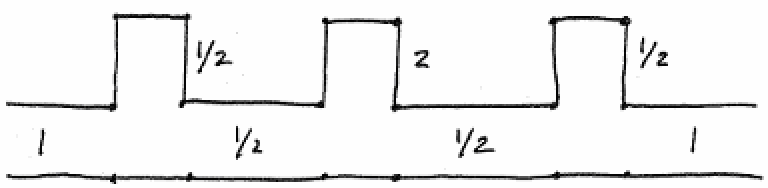
apply first Kuroda identity (twice):

$Z_1 = 1$

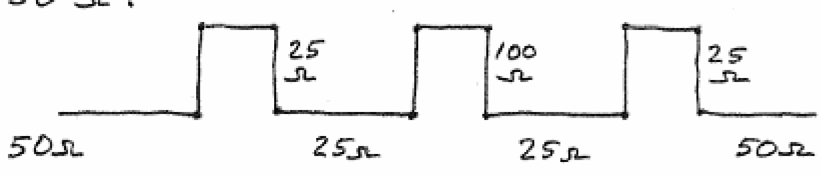
$Z_2 = 1$

$n^2 = 1 + \frac{Z_2}{Z_1}$

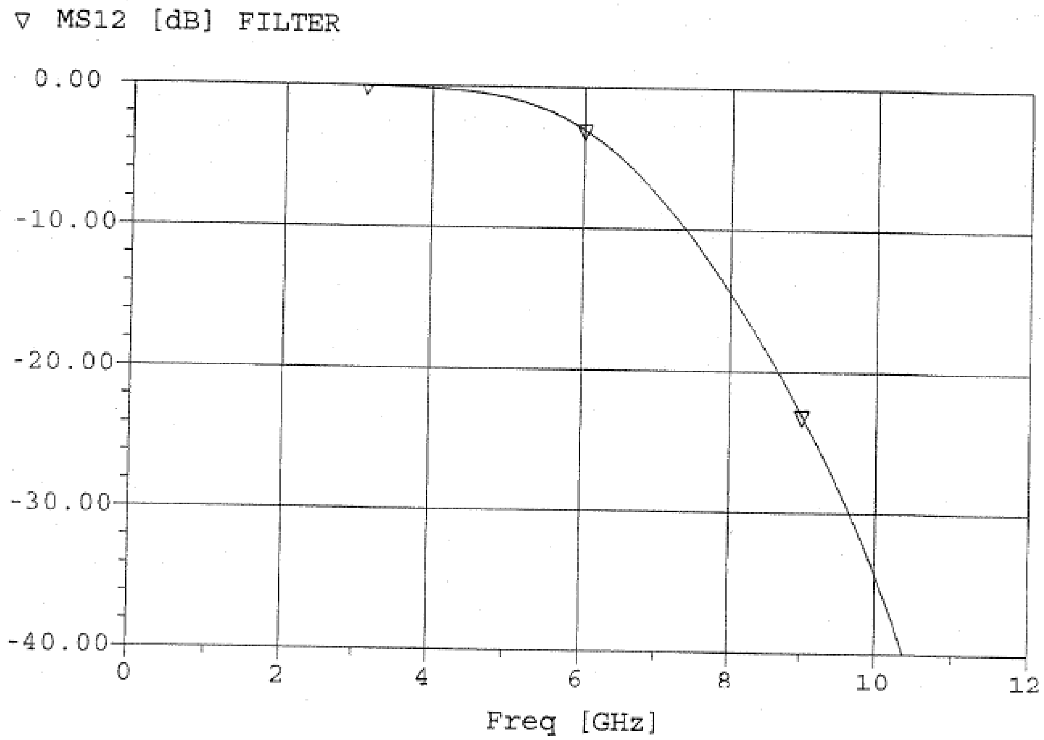
$= 2$



Scale to 50Ω:

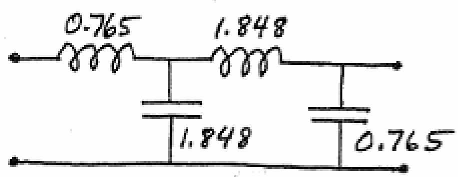


all line lengths and stub lengths are $\lambda/8$ long at 6 GHz. The calculated filter response is shown below.

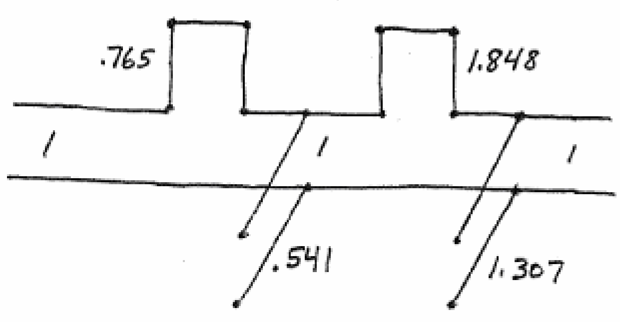


8.13 $f_o = 8\,GHz$, $N=4$, L.P., M.F., $Z_o = 50\,\Omega$

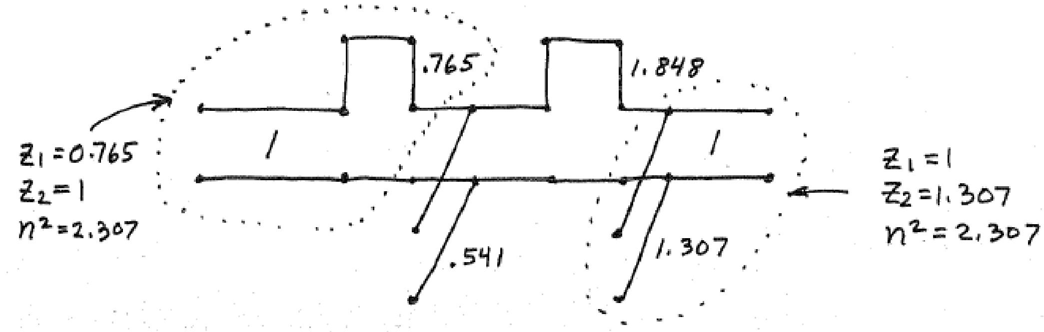
From Table 8.3 the L.P. prototype is,



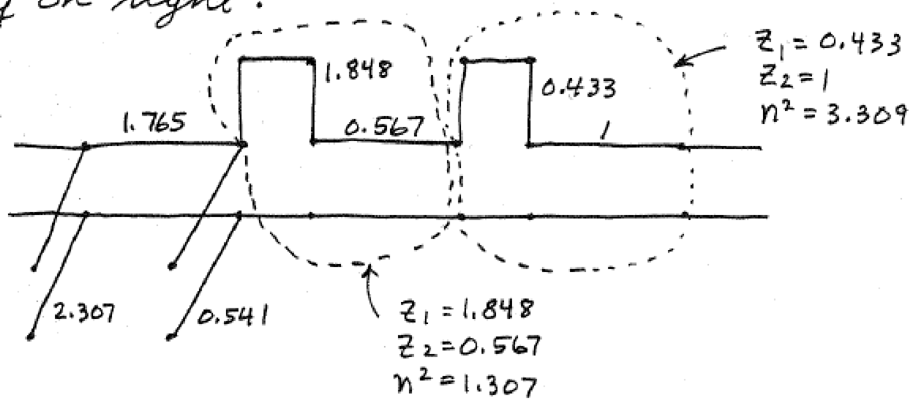
applying Richard's transform:



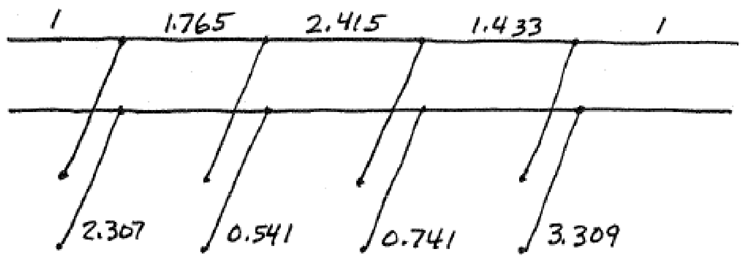
add unit elements:



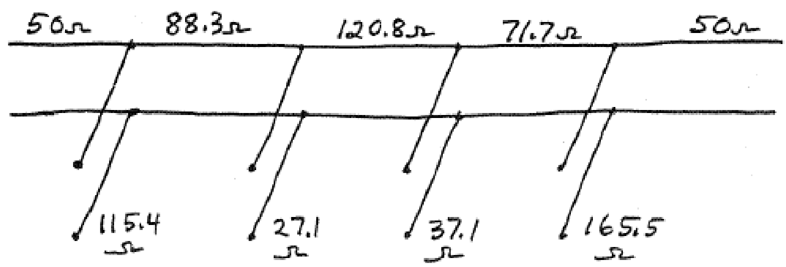
use the second Kuroda identity on left; first Kuroda identity on right:



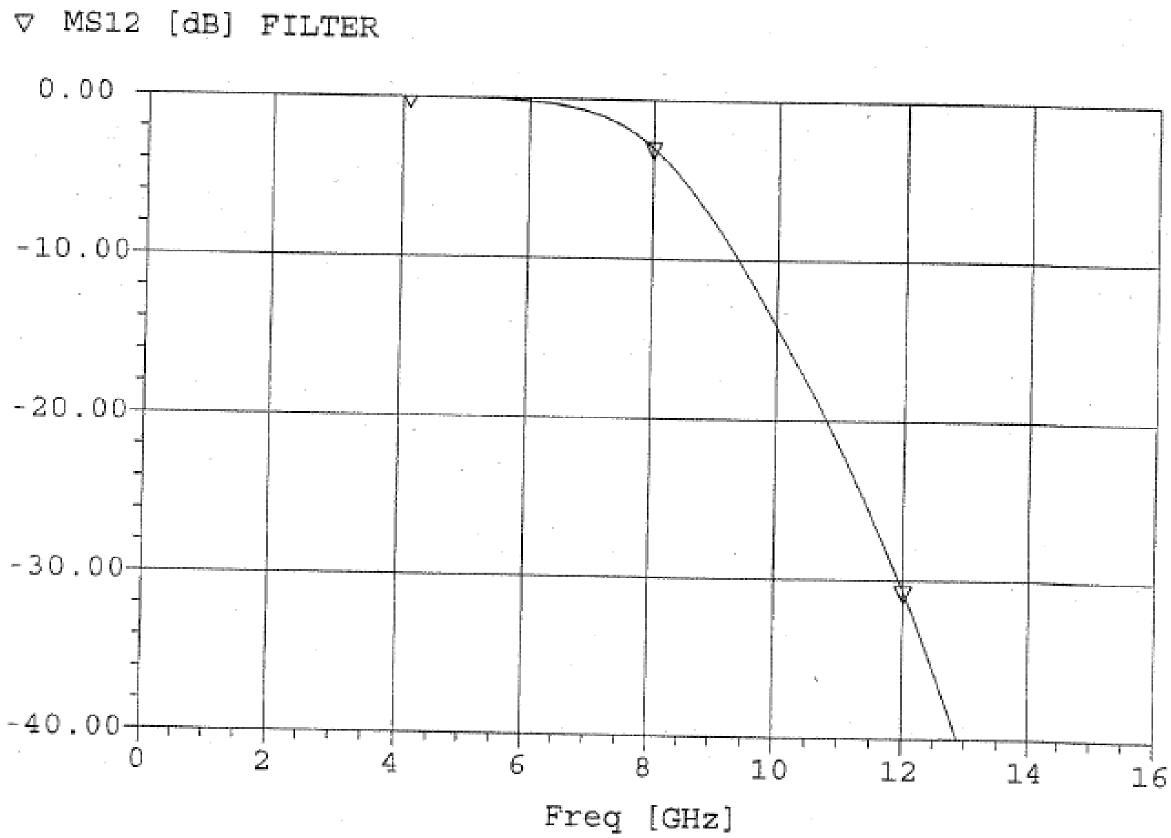
Now use the second Kuroda identity twice:



Scale to 50Ω:

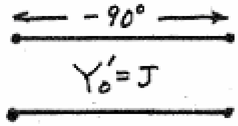


all lines are λ/8 long at 8GHz. The calculated filter response is shown on the following page.



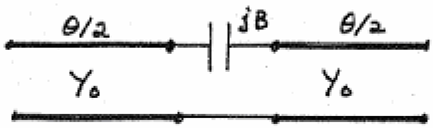
▽ MS12 [dB] FILTER
0.00
-10.00
-20.00
-30.00
-40.00
0
2
4
6
8
10
12
14
16
Freq [GHz]

8.14 Quarter-wave line ($-\lambda/4$ long, since $\theta < 0$)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & -j/J \\ -jJ & 0 \end{bmatrix}$$

ADMITTANCE INVERTER:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix} \begin{bmatrix} 1 & -j/B \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta/2 & j/Y_0 \sin\theta/2 \\ jY_0 \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) & j\left(\frac{1}{Y_0}\sin\theta - \frac{1}{B}\cos^2\frac{\theta}{2}\right) \\ jY_0\left(\sin\theta + \frac{Y_0}{B}\sin^2\frac{\theta}{2}\right) & \left(\cos\theta + \frac{Y_0}{2B}\sin\theta\right) \end{bmatrix} \checkmark$$

Equivalence requires the following conditions:

A, D: $\cos\theta + \frac{Y}{2B}\sin\theta = 0 \Rightarrow \theta = -\tan^{-1}\left(\frac{2B}{Y_0}\right) < 0$ ✓

B: $\frac{Y_0}{J} = -\sin\theta + \frac{Y_0}{B}\cos^2\theta/2$

C: $\frac{J}{Y_0} = -\sin\theta - \frac{Y_0}{B}\sin^2\theta/2$

$\ominus$ $\frac{Y_0}{J} - \frac{J}{Y_0} = \frac{Y_0}{B}$

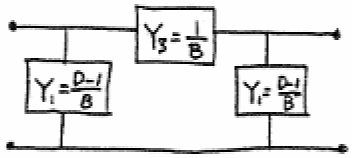
or, $B = \dfrac{Y_0}{\frac{Y_0}{J} - \frac{J}{Y_0}} = \dfrac{J}{1-(J/Y_0)^2}$ ✓

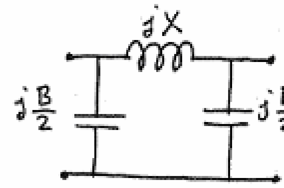
also, $\tan|\theta| = \dfrac{2B}{Y_0} = \dfrac{2(J/Y_0)}{1-(J/Y_0)^2} = \dfrac{2\tan\left|\frac{\theta}{2}\right|}{1-\tan^2\left|\frac{\theta}{2}\right|}$

so,

$\tan\left|\frac{\theta}{2}\right| = J/Y_0$ ✓

8.15 The easiest way to do this problem is to use the π-network of Table 4.1, with the shunt and series element values given in terms of the ABCD parameters:





Then using the ABCD parameters for a transmission line gives the equivalent circuit elements as,

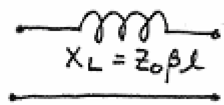
$$jX = B = jZ_0 \sin \beta \ell$$

$$j\frac{B}{2} = \frac{D-1}{B} = \frac{\cos\beta\ell - 1}{jZ_0 \sin\beta\ell} = \frac{j}{Z_0} \tan \beta\ell/2$$

For $\beta\ell < \pi/4$ and large Z_0, these results reduce to:

$$X \simeq Z_0 \beta \ell$$

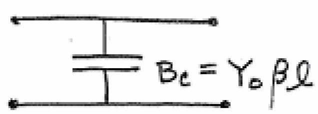
$$\frac{B}{2} \simeq 0$$



For $\beta\ell < \pi/4$ and small Z_0, these results reduce to:

$$X \simeq 0$$

$$\frac{B}{2} \simeq \frac{\beta\ell}{2Z_0}$$



(NOTE: This problem can also be done using y-parameters, but a sign change for y_{12} will be required because of the reference direction for I_2 for the ABCD parameters.)

8.16 $f_0 = 3\text{GHz}$, $N=5$, 0.5dB equal ripple, $Z_0 = 50\Omega$, $Z_\ell = 15\Omega$, $Z_h = 120\Omega$

a) From Table 8.4 and (8.86):

$g_1 = 1.7058 = C_1 \Rightarrow \beta\ell_1 = g_1 Z_\ell/Z_0 = 29.3°$

$g_2 = 1.2296 = L_2 \Rightarrow \beta\ell_2 = g_2 Z_0/Z_h = 29.4°$

$g_3 = 2.5408 = C_3 \Rightarrow \beta\ell_3 = g_3 Z_\ell/Z_0 = 43.7°$

$g_4 = 1.2296 = L_4 \rightarrow \beta\ell_4 = g_4 Z_0/Z_h = 29.4°$

$g_5 = 1.7058 = C_5 \Rightarrow \beta\ell_5 = g_5 Z_\ell/Z_0 = 29.3°$

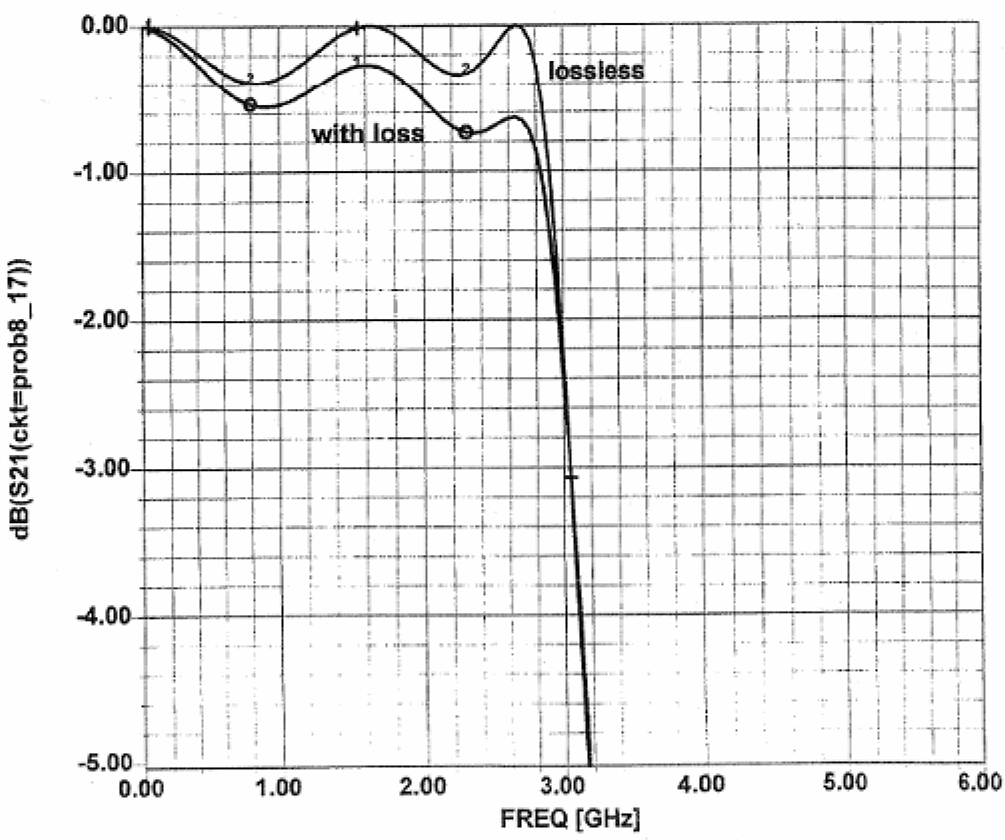
Observe that $\beta\ell_i < 45°$ for all lines

b) $\epsilon_r = 4.2$, $d = 0.079$ cm, $\tan\delta = 0.02$, Cu, $t = 0.5$ mil

$W(15\Omega) = 7.98$ mm ; $\ell_1 = 0.42$ cm; $\ell_3 = 0.63$ cm; $\ell_5 = 0.42$ cm

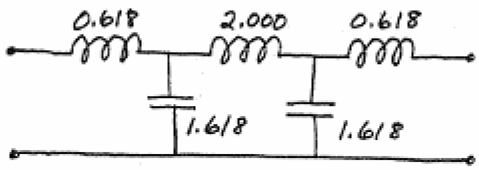
$W(120\Omega) = 0.213$ mm ; $\ell_2 = 0.48$ cm ; $\ell_4 = 0.48$ cm

The response, with and without loss, is shown below.



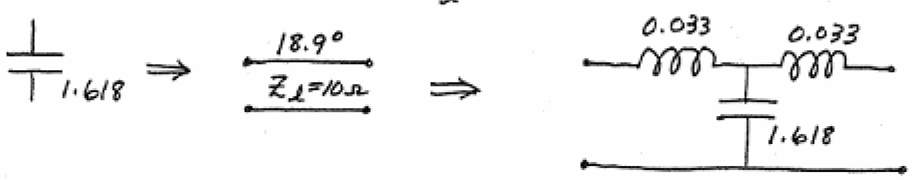
8.17 $f_o = 2\,GHz$, L.P., M.F., $Z_o = 50\,\Omega$

From Table 8.3 the LP prototype is,



The shunt capacitors can be implemented with a length of Z_ℓ line. Using (8.83b) gives,

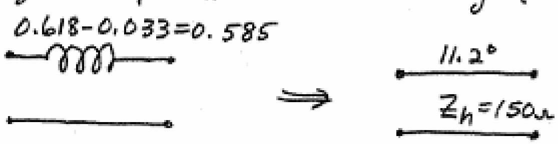
$$B = \frac{1.618}{Z_o} = \frac{\sin \beta \ell}{Z_\ell} \Rightarrow \beta\ell = 18.9°$$



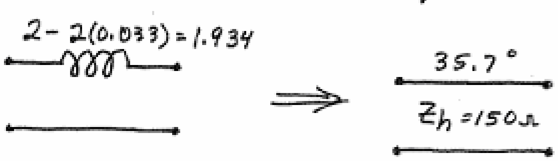
The model for this line (see Figure 8.39) shows inductors on either side, with values given by (8.83a):

$$\frac{X}{2Z_o} = \frac{Z_\ell}{Z_o} \tan \beta\ell/2 = 0.033$$

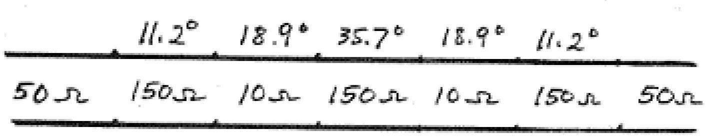
Then the end inductors of value 0.618 can be implemented as lengths of Z_h line. Using (8.83a) gives,



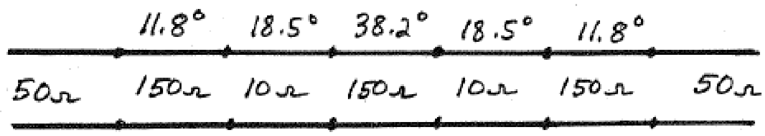
and the middle inductor of value 2.000 can be implemented as,



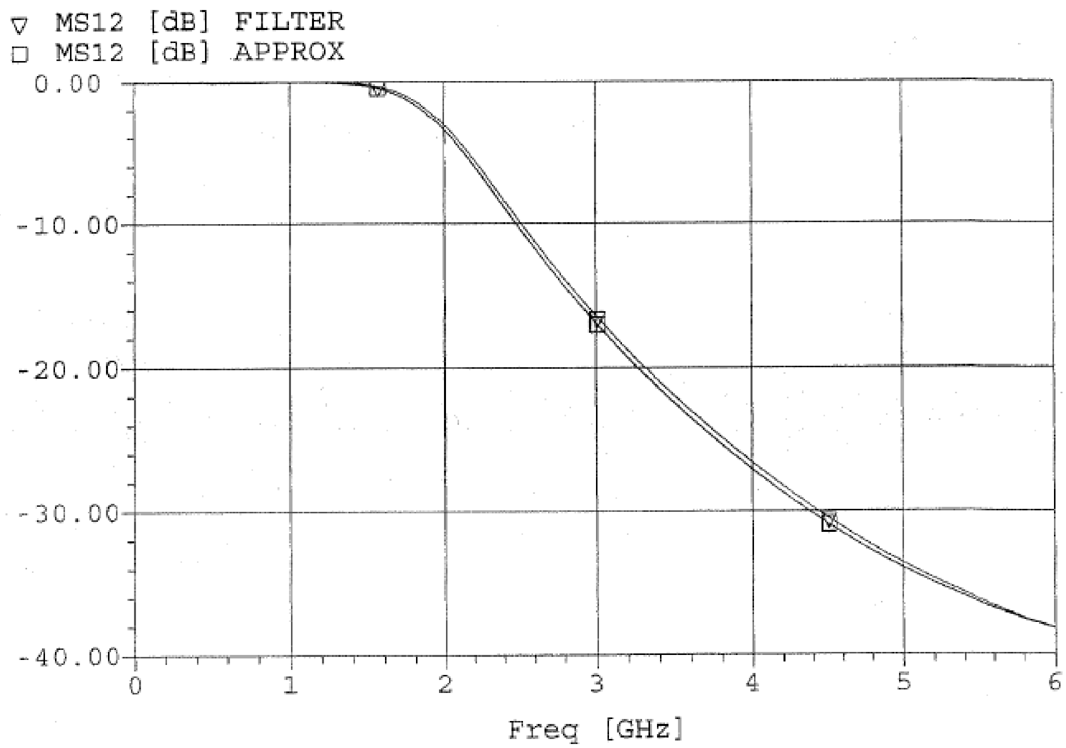
The final filter:



For comparison, the design using the approximations of (8.84) and (8.85) is,



Note that only the middle section differs very much in these two designs. The calculated filter response is shown below for both designs. Note that there is very little difference.



8.18 $f_0 = 2.45$ GHz, BW = 10%, EQUAL-RIPPLE (0.5 dB), $N=3$, $Z_0 = 50\,\Omega$

a) Use (8.71) to transform 2.1 GHz to normalized L.P. form:

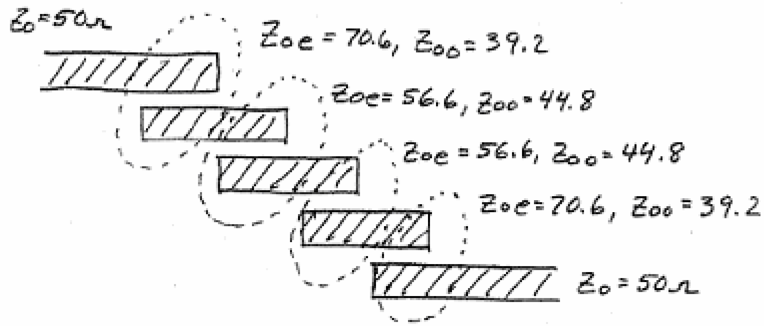
$$\omega \leftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{0.1}\left(\frac{2.1}{2.45} - \frac{2.45}{2.1}\right) = -3.09$$

Then, $\left|\frac{\omega}{\omega_c}\right| - 1 = 2.09 \Rightarrow$ Fig 8.27 gives $\alpha \simeq 30$ dB

The L.P. prototype values are given in Table 8.4

$Z_0 J_n$ is found from (8.121), and Z_{0e}, Z_{0o} from (8.108):

n	g_n	$Z_0 J_n$	$Z_{0e}\,(\Omega)$	$Z_{0o}\,(\Omega)$
1	1.5963	0.3137	70.6	39.2
2	1.0967	0.1187	56.6	44.8
3	1.5963	0.1187	56.6	44.8
4	1.0000	0.3137	70.6	39.2



all lines are $\lambda/4$ long at 2.45 GHz.

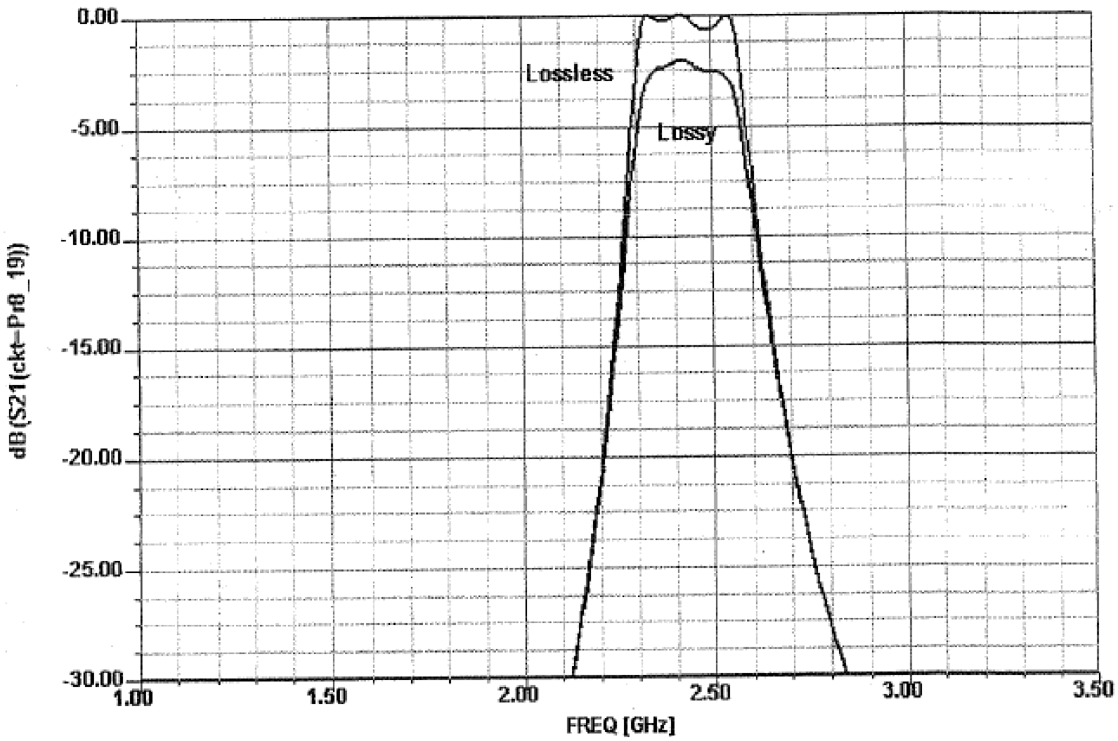
b) $d = 0.158$ cm, $\epsilon_r = 4.2$, $\tan\delta = 0.01$, Cu, $t = 0.5$ mil

From Serenade,

$Z_{0e} = 70.6$, $Z_{0o} = 39.2 \Longrightarrow W = 2.484$ mm, $S = 0.415$ mm, $\ell = 1.74$ cm

$Z_{0e} = 56.6$, $Z_{0o} = 44.8 \Longrightarrow W = 3.026$ mm, $S = 1.723$ mm, $\ell = 1.71$ cm

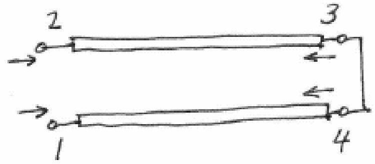
The calculated response, from Serenade, is shown on the following page.



0.00
-5.00
-10.00
-15.00
-20.00
-25.00
-30.00
dB(S21(ckt=Pr8_19))
Lossless
Lossy
1.00
1.50
2.00
2.50
3.00
3.50
FREQ [GHz]

8.19 Schiffman Phase Shifter (see MTT April 1958)

First consider the shorted coupled line section:



$I_3 = -I_4$
$V_3 = V_4$

$$V_1 = Z_{11} I_1 + Z_{12} I_2 + Z_{13} I_3 + Z_{14} I_4$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 + Z_{23} I_3 + Z_{24} I_4$$
$$V_3 = Z_{31} I_1 + Z_{32} I_2 + Z_{33} I_3 + Z_{34} I_4$$
$$V_4 = Z_{41} I_1 + Z_{42} I_2 + Z_{43} I_3 + Z_{44} I_4$$

Using the terminal conditions that $I_3 = -I_4$, $V_3 = V_4$, allow this to be reduced to

$$V_1 = \left[Z_{11} - \frac{(Z_{13} - Z_{14})^2}{2(Z_{11} - Z_{12})} \right] I_1 + \left[Z_{12} + \frac{(Z_{13} - Z_{14})^2}{2(Z_{11} - Z_{12})} \right] I_2$$

$$V_2 = \left[Z_{12} + \frac{(Z_{14} - Z_{13})^2}{2(Z_{11} - Z_{12})} \right] I_1 + \left[Z_{11} - \frac{(Z_{14} - Z_{13})^2}{2(Z_{11} - Z_{12})} \right] I_2$$

where we have used symmetry and relations such as $Z_{11} = Z_{22} = Z_{33} = Z_{44}$. From (8.99), the Z_{ij} are given for the coupled line. For a 2-port, the image impedance is,

$$Z_i = \sqrt{Z_1 Z_2} \sqrt{1 + \frac{Z_1}{4Z_2}} = \sqrt{2Z_{12}(Z_{11} - Z_{12})} \sqrt{1 + \frac{2(Z_{11} - Z_{12})}{4Z_{12}}}$$

$$= \sqrt{(Z_{11} - Z_{12})(Z_{11} + Z_{12})} \quad (Z_{11}, Z_{12} \text{ are the equiv. 2-port Z params})$$

we can show that

$$Z_{11}-Z_{12} = -j\, Z_{0o} \cot\theta + j\, Z_{0o} \sec\theta \csc\theta$$

$$Z_{11}+Z_{12} = -j\, Z_{0e} \cot\theta$$

Then $Z_i = \sqrt{Z_{0e} Z_{0o}}$ ✓ (agrees w/ result in Table 8.8)

The propagation constant is,

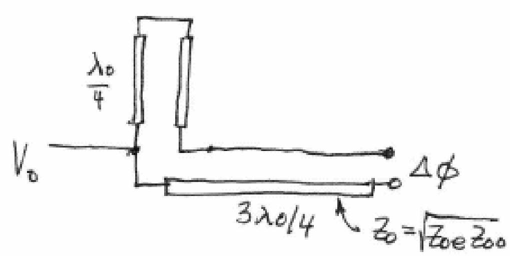
$$e^{\gamma} = 1 + Z_1/2Z_2 + \sqrt{Z_1/Z_2 + Z_1^2/4Z_2^2}$$

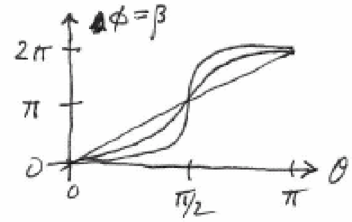
$$= \frac{Z_{11}}{Z_{12}} + \frac{1}{Z_{12}}\sqrt{(Z_{11}+Z_{12})(Z_{11}-Z_{12})} = \frac{Z_{11}}{Z_{12}} + \frac{1}{Z_{12}} Z_i$$

$$= \frac{Z_{0e}\cot\theta - Z_{0o}\tan\theta + 2j\sqrt{Z_{0e}Z_{0o}}}{Z_{0e}\cot\theta + Z_{0o}\tan\theta} = (\cos\beta + j\sin\beta)\, e^{\alpha}$$

so, $$\cos\beta = \frac{Z_{0e}\cot\theta - Z_{0o}\tan\theta}{Z_{0e}\cot\theta + Z_{0o}\tan\theta} = \frac{Z_{0e} - Z_{0o}\tan^2\theta}{Z_{0e} + Z_{0o}\tan^2\theta}$$ ✓

at midband, $\theta = \pi/2$, $\cos\beta = -1$, $\beta = 180^\circ$ ✓



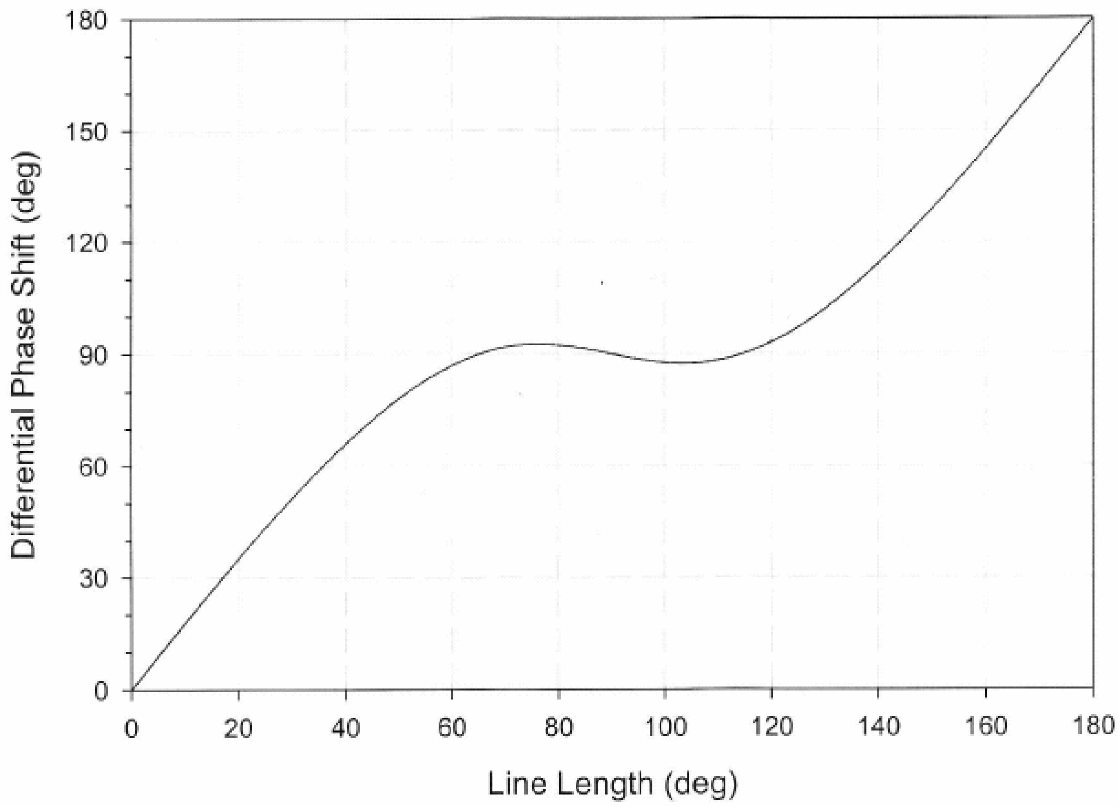


$$\text{let } \phi = \cos^{-1}\frac{Z_{0e} - Z_{0o}\tan^2\theta}{Z_{0e} + Z_{0o}\tan^2\theta}, \quad \text{let } \rho = \frac{Z_{0e}}{Z_{0o}}$$

Then $\Delta\phi = \phi - 3\theta$

Using $\rho = 2.7$ gives BW from 61.2° to 118.8° for $\Delta\phi = 90^\circ \pm 2.5^\circ$, or 1.93:1.

$\Delta\phi$ is plotted vs θ below.



Differential Phase Shift (deg)
Line Length (deg)
0
30
60
90
120
150
180
0
20
40
60
80
100
120
140
160
180

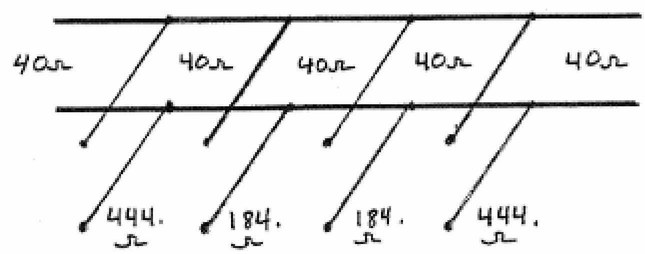
8.20 $f_0 = 3$ GHz, B.S., M.F., $N=4$, $\Delta = 0.15$, $Z_0 = 40\Omega$.

We find the g_n values from Table 8.3. Then the $\lambda/4$ o.c. stub characteristic impedances can be found from (8.130):

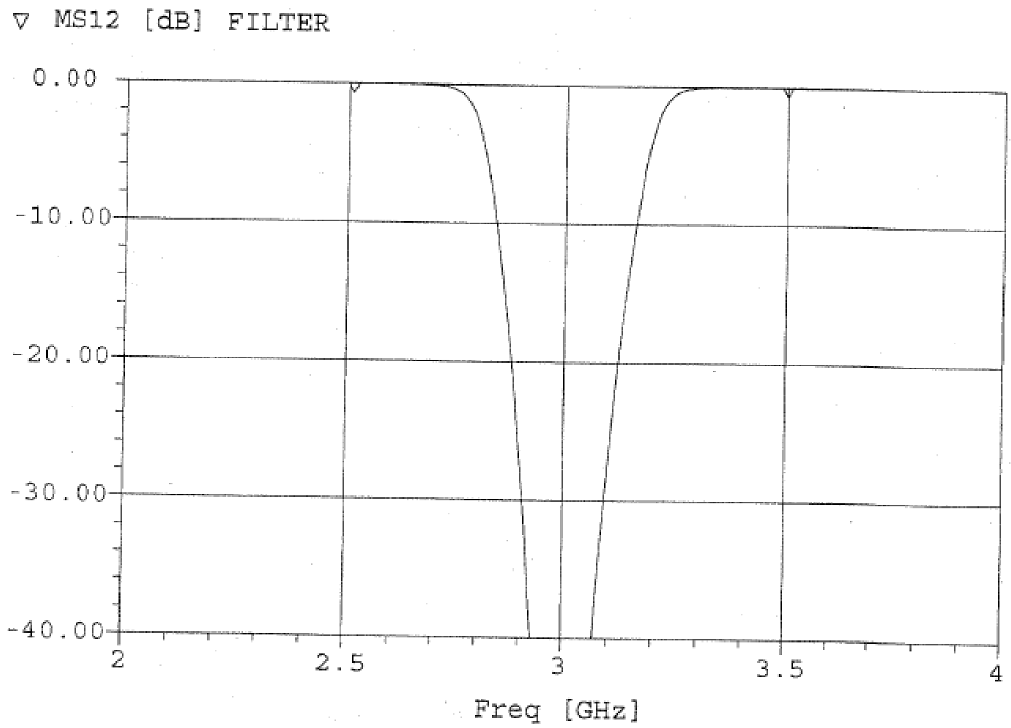
n	g_n	$Z_{0n}(\Omega)$
1	0.765	444.
2	1.848	184.
3	1.848	184.
4	0.765	444.

(Z_{01} and Z_{04} may be too high to be practical)

The final filter is shown below:



all lines and stubs are λ/4 long at 3 GHz. The calculated filter response is shown below:



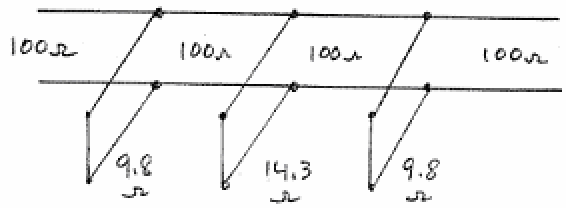
8.21 BANDPASS, $N=3$, EQUAL RIPPLE (0.5 dB), $\lambda/4$ S.C. stubs, $f_0 = 3$ GHz, 20% BW

a) Find g_n from Table 8.4. Then the $\lambda/4$ short-circuited stub characteristic impedances can be found as,

n	g_n	Z_{0n} (Ω)
1	1.5963	9.8
2	1.0967	14.3
3	1.5963	9.8

$\Delta = 0.20$

$$Z_{0n} = \frac{\pi Z_0 \Delta}{4 g_n}$$

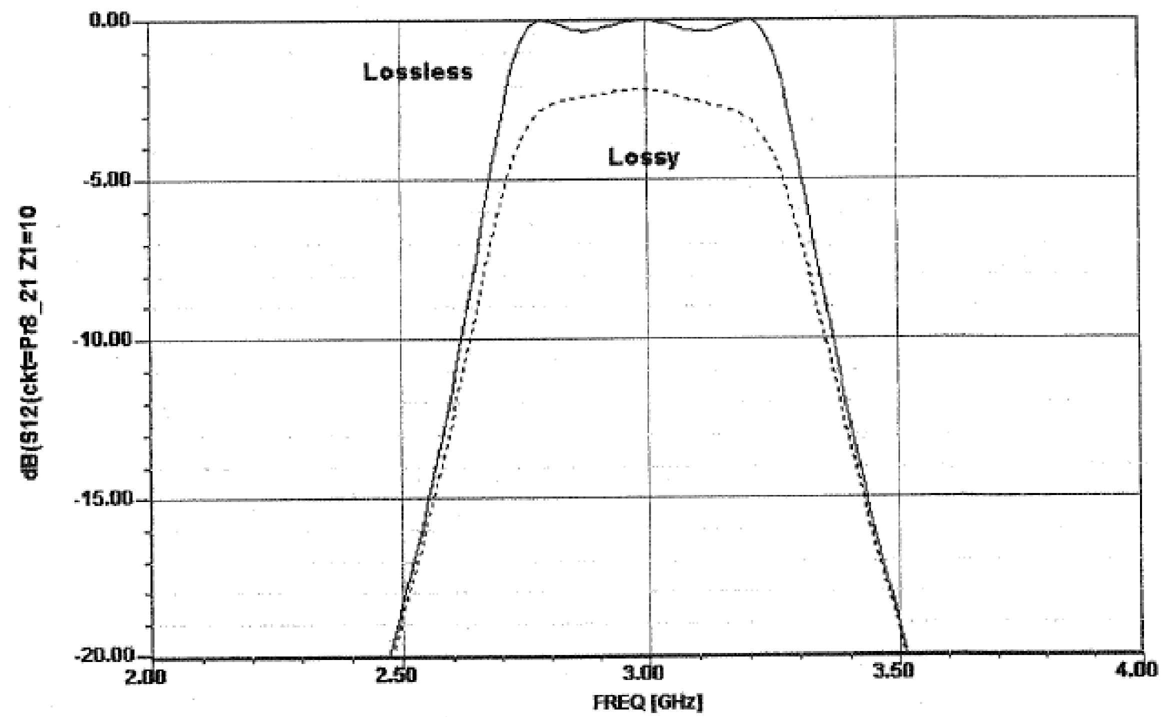


b) FR-4, $\epsilon_r = 4.2$, $d = 0.079$ cm, $\tan\delta = 0.02$, Cu, $t = 0.5$ mil

$W(9.8) = 1.307$ cm, $90° = 1.27$ cm

$W(14.3) = 0.845$ cm, $90° = 1.29$ cm

$W(100) = 0.367$ mm, $90° = 1.46$ cm



8.22 The bandpass $\lambda/4$ resonator filter uses short-circuited stubs, which have an input admittance of,

$$Y = -jY_{on} \cot\theta \quad , \text{ where } \theta = \pi/2 \text{ for } \omega = \omega_o .$$

Let $\omega = \omega_o + \Delta\omega$. Then $\theta = \pi/2 \, (1 + \Delta\omega/\omega_o)$, and

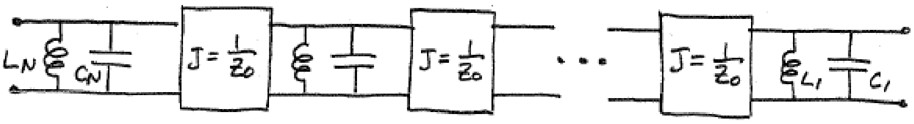
$Y = jY_{on} \tan \frac{\pi \Delta\omega}{2\omega_o} \simeq \frac{j\pi Y_{on}(\omega - \omega_o)}{2\omega_o}$. Now the admittance of a parallel LC resonator is, from Table 6.1,

$$Y \simeq 2jC_n(\omega - \omega_o).$$

So the characteristic impedance of the stub is,

$$Z_{on} = \frac{1}{Y_{on}} = \frac{\pi}{4\omega_0 C_n}.$$

The filter circuit can be redrawn as follows:



This is the same as the circuit in part (e) of Figure 8.45, for coupled line bandpass filters (with $Z_0 J_n = 1$). Correspondance with the lumped-element bandpass filter requires that,

$$\sqrt{\frac{C_1}{L_1}} = \sqrt{\frac{C_1'}{L_1'}}$$

$$Z_0^2\sqrt{\frac{C_2}{L_2}} = \sqrt{\frac{L_2'}{C_2'}}$$

where C_n' and L_n' are the lumped element filter values, and $L_n C_n = L_n' C_n' = 1/\omega_0^2$. Solving for C_n gives,

$$C_1 = C_1'$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2 C_2'}.$$

Using Table 8.6 to transform back to LP prototype values gives,

$$C_1 = C_1' = \frac{g_1}{\Delta\omega_0 Z_0}$$

$$C_2 = \frac{1}{\omega_0^2 Z_0^2}\left(\frac{\omega_0 g_2 Z_0}{\Delta}\right) = \frac{g_2}{\Delta\omega_0 Z_0}$$

So the characteristic impedances are,

$$Z_{on} = \frac{\pi}{4\omega_0}\left(\frac{\Delta\omega_0 Z_0}{g_1}\right) = \frac{\pi\Delta Z_0}{4 g_n} \checkmark$$

which agrees with (8.131).

8.23 $f_0 = 4$ GHz, $\Delta = 0.12$, B.P., M.F., $Z_0 = 50\Omega$

First transform 3.6 GHz to L.P. prototype form:

$$\omega \longleftarrow \frac{1}{\Delta}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \frac{1}{.12}\left(\frac{3.6}{4} - \frac{4}{3.6}\right) = -1.76$$

Then,

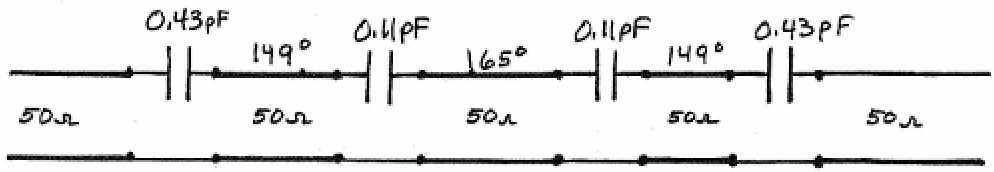
$$\left|\frac{\omega}{\omega_c}\right| - 1 = 0.76$$

So from Figure 8.26 we see that $N=3$ is required to achieve $\alpha > 12$ dB at 3.6 GHz. (or, analytically, $P_{LR} = 1 + (\omega/\omega_c)^{2N} = 14.9$ dB for $N=3$)

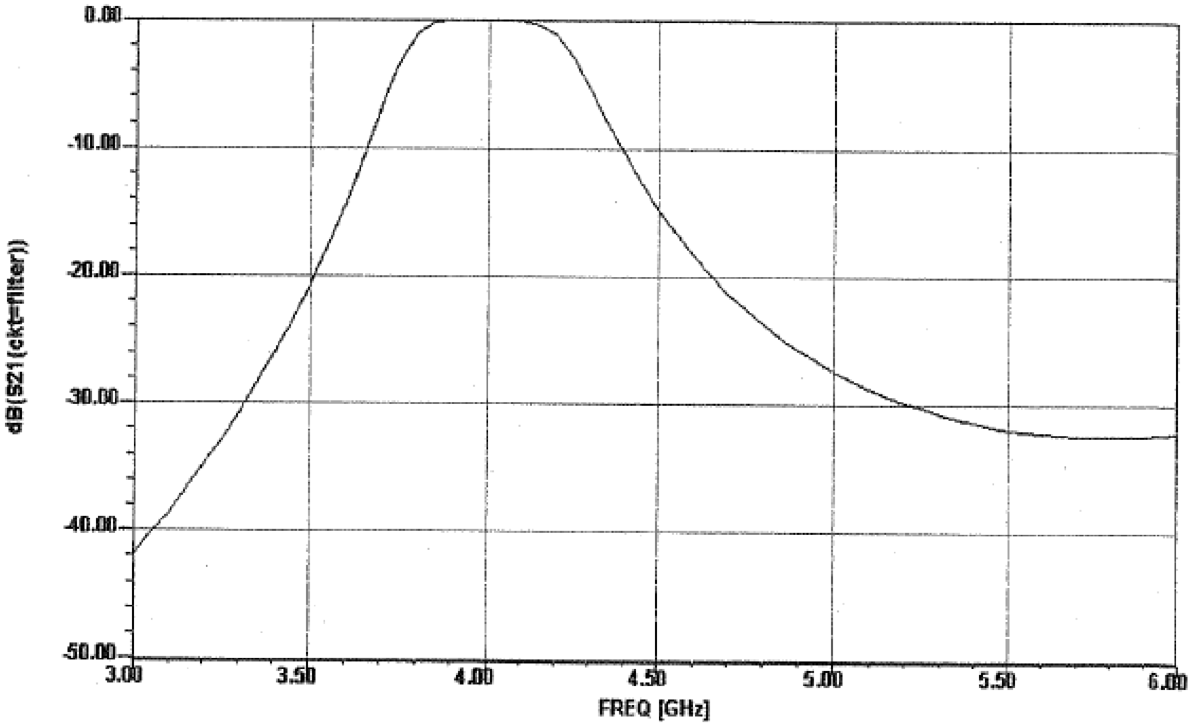
The prototype values are given in Table 8.3. Then $Z_0 J_n$ can be found from (8.121). Then, $Z_0 B_n = \dfrac{J_n Z_0}{1-(J_n Z_0)^2}$, and $\theta_n = \pi - \frac{1}{2}\left[\tan^{-1}(2Z_0 B_n) + \tan^{-1}(2Z_0 B_{n+1})\right]$. Also, $C_n = B_n/\omega_0$.

n	g_n	$Z_0 J_n$	$Z_0 B_n$	C_n (pF)	θ_n @ 4GHz
1	1.000	0.434	0.535	0.43	149°
2	2.000	0.133	0.135	0.11	165°
3	1.000	0.133	0.135	0.11	149°
4	1.000	0.434	0.535	0.43	—

The filter circuit is shown below:



The calculated filter response is shown on the following page.



8.24 $f_0 = 836.5$ MHz, $\Delta = 0.03$, BANDPASS, 1.0 dB EQUAL RIPPLE, $Z_0 = 50\Omega$

Use 0.5 dB ripple design to allow for approximation errors. Use (8.136)–(8.137) to find inverter constants and coupling capacitor values:

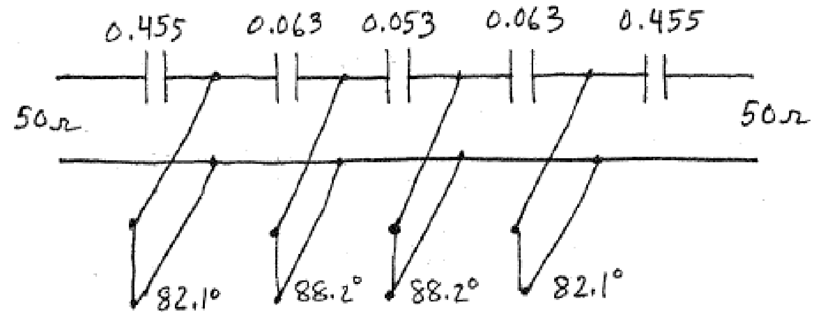
n	g_n	$Z_0 J_{n-1,n}$	$C_{n-1,n}$ (PF)
1	1.6703	0.1188	0.455
2	1.1926	0.0167	0.063
3	2.3661	0.0140	0.053
4	0.8419	0.0167	0.063
5	1.9841	0.1188	0.455

Then use (8.138) and (8.141) to find the resonator lengths:

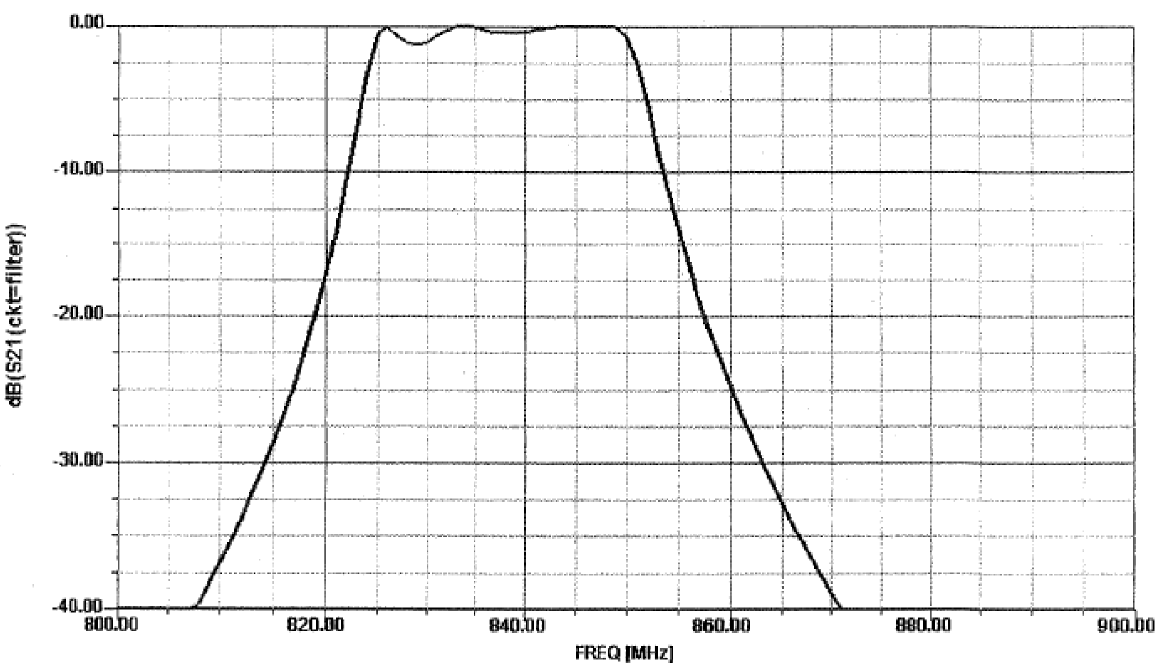
$C_n' = C_n + \Delta C_n$

n	ΔC_n (PF)	ℓ_n (λ)	$\ell°$
1	−0.518	0.228	82.1
2	−0.116	0.245	88.2
3	−0.116	0.245	88.2
4	−0.518	0.228	82.1

Final filter circuit:



The resulting response is shown below.

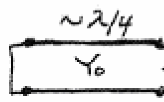


8.25 From Matthai, Young & Jones, P.482, we have the general results that

$$J_1 = \sqrt{\frac{G_A b_1 \Delta}{g_0 g_1 \omega_1'}} \;, \quad J_n = \frac{\Delta}{\omega_1'}\sqrt{\frac{b_n b_{n+1}}{g_n g_{n+1}}} \;, \quad J_N = \sqrt{\frac{G_B b_N \Delta}{\omega_1' g_N g_{N+1}}}$$

with $G_A = G_B = 1/Z_0$, $\omega_1' = 1$, $g_0 = 1$, and where

$$b_j = \frac{\omega_0}{2}\frac{dB_j}{d\omega}\bigg|_{\omega=\omega_0} = \frac{\pi}{4Z_0}$$ is the admittance slope parameter for the s.c. stub resonator:



$$\leftarrow Y = -jY_0 \cot\theta \approx \frac{j\pi(\omega-\omega_0)}{2Z_0\omega_0}$$

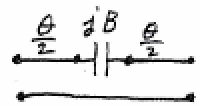
Then,

$$Z_0 J_1 = \sqrt{\frac{\pi\Delta}{4g_1}} \quad ✓$$

$$Z_0 J_n = \frac{\pi\Delta}{4\sqrt{g_n g_{n+1}}} \quad ✓$$

$$Z_0 J_N = \sqrt{\frac{\pi\Delta}{4 g_N g_{N+1}}} \quad ✓$$

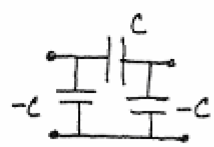
The end inverters are modeled as



(Fig. 8.38c)

for which $C_{i,i+1} = \frac{B}{\omega} = \frac{J_{i,i+1}}{\omega_0\sqrt{1-(Z_0 J_{i,i+1})^2}}$ ✓ for C_{01}, J_{01} and $C_{N,N+1}$, $J_{N,N+1}$.

The middle inverters are modeled as



(Fig. 8.38d)

for which $C_{n,n+1} = \frac{J_{n,n+1}}{\omega}$ ✓

Chapter 9

9.1 $\bar{H} = (0.5\hat{x} + j\,0.5\hat{y})$, $4\pi M_s = 900\,G$

a) $H_0 = M_s = 0$. So $\omega_0 = \omega_m = 0 \Rightarrow \mu = \mu_0$, $\kappa = 0$

$$\bar{B} = [\mu]\bar{H} = \begin{bmatrix} \mu_0 & 0 & 0 \\ 0 & \mu_0 & 0 \\ 0 & 0 & \mu_0 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5j \\ 0 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.5j \\ 0 \end{bmatrix} \mu_0$$

(isotropic, non-magnetic)

b) $H_0 = 800\,Oe$. $\quad f_0 = 2.8\,\frac{MHz}{Oe}(800\,Oe) = 2.24\,GHz$

$$f_m = 2.8\,\frac{MHz}{Oe}(900\,G) = 2.52\,GHz$$

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 6.55\,\mu_0$$

$$\kappa = \mu_0 \frac{f\,f_m}{f_0^2 - f^2} = 4.95\,\mu_0$$

$$\bar{B} = [\mu]\bar{H} = \mu_0 \begin{bmatrix} 6.55 & j4.95 & 0 \\ -j4.95 & 6.55 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5j \\ 0 \end{bmatrix}$$

$$= (3.275 - 2.475)\,\mu_0\hat{x} + (-j\,2.475 + j\,3.275)\,\mu_0\hat{y}$$

$$= 0.8\,\mu_0\hat{x} + j\,0.8\,\mu_0\hat{y}$$

check: since LHCP, $\mu_e = \mu - \kappa = 1.6$, $\quad 1.6 \times 0.5 = 0.8$ ✓

9.2

$$B_x = \mu H_x + j\kappa H_y$$
$$B_y = -j\kappa H_x + \mu H_y$$
$$B_z = \mu_o H_z$$

Then,

$$B^+ = \frac{1}{2}(B_x + jB_y) = \frac{1}{2}[\mu H_x + j\kappa H_y + \kappa H_x + j\mu H_y]$$
$$= \frac{1}{2}(\mu+\kappa)(H_x + jH_y) = (\mu+\kappa)H^+$$
$$B^- = \frac{1}{2}(B_x - jB_y) = \frac{1}{2}[\mu H_x + j\kappa H_y - \kappa H_x - j\mu H_y]$$
$$= \frac{1}{2}(\mu-\kappa)(H_x - jH_y) = (\mu-\kappa)H^-$$

Thus,

$$\begin{bmatrix} B^+ \\ B^- \\ B_z \end{bmatrix} = \begin{bmatrix} (\mu+\kappa) & 0 & 0 \\ 0 & (\mu-\kappa) & 0 \\ 0 & 0 & \mu_o \end{bmatrix} \begin{bmatrix} H^+ \\ H^- \\ H_z \end{bmatrix} \checkmark$$

9.3

$$4\pi M_s = 1780 \text{ G}$$

internal field $= \bar{H} = \bar{H}e - N\bar{M}$

From Table 9.1 the demagnetization factors for a sphere are $N_x = N_y = N_z = 1/3$, so

$$H_{e_z} = H_z = N_z M_z = 700 + \frac{1}{3}(1780) = \underline{1293 \text{ Oe}}$$

9.4 $4\pi M_s = 800 G$

From Table 9.1 the demagnetization factors for a thin rod are $N_x = N_y = 1/2$, $N_z = 0$. Eq. (9.46) gives the gyromagnetic resonance frequency as

$$\omega_r = \mu_0 \gamma \sqrt{(H_a + \tfrac{1}{2}M_s)(H_a + \tfrac{1}{2}M_s)}$$
$$= \mu_0 \gamma (H_a + \tfrac{1}{2}M_s)$$

$$H_a = \frac{\omega_r}{\mu_0 \gamma} - \frac{1}{2}M_s = f_r\left(\frac{1\ Oe}{2.8\ MHz}\right) - \frac{M_s}{2}$$

$$= 2520\left(\frac{1\ Oe}{2.8}\right) - \frac{800}{2} = \underline{500\ Oe}$$

9.5 $4\pi M_s = 1200\,G$, $\epsilon_r = 10$, $H_0 = 500\,Oe$, $f = 8\,GHz$

(FARADAY ROTATION)

$$f_0 = (2.8\ \frac{MHz}{Oe})(500\,Oe) = 1.4\,GHz$$

$$f_m = (2.8\ \frac{MHz}{Oe})(1200\,G) = 3.36\,GHz$$

$$k_0 = 167.6\ m^{-1}$$

Then from (9.25),

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.924\,\mu_0 \checkmark$$

$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = -0.433\,\mu_0 \checkmark$$

From (9.52) the propagation constants of the CP waves are,

RHCP: $\beta_+ = \omega\sqrt{\epsilon(\mu+\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924-0.433} = 371.4\ m^{-1}$ ✓

LHCP: $\beta_- = \omega\sqrt{\epsilon(\mu-\kappa)} = k_0\sqrt{\epsilon_r}\sqrt{0.924+0.433} = 617.4\ m^{-1}$ ✓

Then, $\Delta\beta = \beta_+ - \beta_- = -246.0\ m^{-1}$

From (9.57) the polarization rotation of an LP wave is,

$$\phi = -(\beta_+ - \beta_-)z/2$$

so,

$$z = \frac{2\phi}{\beta_- - \beta_+} = \frac{2(\pi/2)}{246\ rad/m} = 12.8\ mm \checkmark$$

9.6 $4\pi M_s = 1780$ G, $\epsilon_r = 13$, $H_0 = 2000$ Oe, $f = 5$ GHz

$$f_0 = (2.8\text{ MHz/Oe})(2000\text{ Oe}) = 5.60\text{ GHz}$$
$$f_m = (2.8\text{ MHz/Oe})(1780\text{ Oe}) = 4.98\text{ GHz}$$
$$k_0 = 2\pi f/c = 104.7\text{ m}^{-1}$$

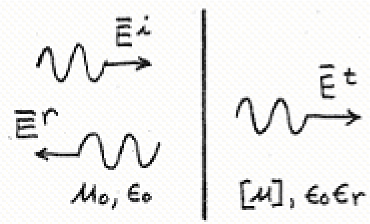
From (9.25),
$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 5.385\,\mu_0$$
$$\kappa = \mu_0 \frac{f f_m}{f_0^2 - f^2} = 3.915\,\mu_0$$

The $\hat{x}$-polarized wave has $\bar{H} = \hat{y} H_y$, and is the extraordinary wave. From (9.64)–(9.65),
$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 2.539\,\mu_0$$
$$\beta_e = \omega\sqrt{\epsilon\mu_e} = k_0\sqrt{\epsilon_r \mu_e/\mu_0} = 601.5\text{ m}^{-1}$$

The $\hat{y}$-polarized wave has $\bar{H} = \hat{x} H_x$, and is the ordinary wave. Thus,
$$\beta_o = \sqrt{\epsilon_r}\, k_0 = 377.5\text{ m}^{-1}$$

So the distance required for a differential phase shift of 180° is,
$$L = \frac{\pi}{\beta_e - \beta_o} = \underline{\underline{1.403\text{ cm}}}$$

9.7



The incident, reflected, and transmitted fields for a RHCP wave can be written as,

$$\bar{E}^i = E_0(\hat{x} - j\hat{y})e^{-j\beta_0 z} \qquad \bar{H}^i = \frac{E_0}{\eta_0}(\hat{y} + j\hat{x})e^{-j\beta_0 z}$$

$$\bar{E}^r = \Gamma^+ E_0(\hat{x} - j\hat{y})e^{j\beta_0 z} \qquad \bar{H}^r = \frac{-E_0}{\eta_0}\Gamma^+(\hat{y} + j\hat{x})e^{j\beta_0 z}$$

$$\bar{E}^t = T^+ E_0(\hat{x} - j\hat{y})e^{-j\beta_+ z} \qquad \bar{H}^t = \frac{E_0}{\eta_0}T^+(\hat{y} + j\hat{x})e^{-j\beta_+ z}$$

where, $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$, $\beta_+ = \omega\sqrt{\epsilon(\mu + \kappa)}$, $\eta_+ = \frac{1}{Y_+} = \sqrt{\frac{\mu+\kappa}{\epsilon}}$

matching fields at $z=0$ gives (for both $\hat{x}$ and $\hat{y}$ components)

$$1 + \Gamma^+ = T^+$$

$$\eta_+(1 - \Gamma^+) = \eta_0 T^+$$

Solving gives,

$$\Gamma^+ = \frac{\eta_+ - \eta_0}{\eta_+ + \eta_0} \quad , \quad T^+ = \frac{2\eta_+}{\eta_+ + \eta_0} \quad \checkmark$$

Similarly, for a LHCP wave we obtain,

$$\Gamma^- = \frac{\eta_- - \eta_0}{\eta_- + \eta_0} \quad , \quad T^- = \frac{2\eta_-}{\eta_- + \eta_0} \quad \checkmark$$

where, $\eta_- = \frac{1}{Y_-} = \sqrt{\frac{\mu - \kappa}{\epsilon}}$.

9.8 $4\pi M_s = 1200\,G$, $\bar{H}_0 = \hat{x} H_0$, $f = 4\ GHz$, $\bar{E} = \hat{x} E_0$

$f_m = (2.8\,MHz/Oe)(1200\,G) = 3.36\ GHz$

This is a case of birefringence. From (9.64)–(9.65),

$$\beta_e = \omega\sqrt{\mu_e \epsilon}$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}$$

The wave will be cutoff when $\mu_e \le 0$:

$$\frac{\mu^2 - \kappa^2}{\mu} < 0$$

$$\frac{\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right)^2 - \left(\frac{f f_m}{f_0^2 - f^2}\right)^2}{1 + \frac{f_0 f_m}{f_0^2 - f^2}} < 0$$

$$\frac{(f_0^2 - f^2)^2 + 2 f_0 f_m (f_0^2 - f^2) + f_m^2 (f_0^2 - f^2)}{(f_0^2 - f^2)\left[(f_0^2 - f^2) + f_0 f_m\right]} < 0$$

$$\frac{(f_0^2 - f^2) + 2 f_0 f_m + f_m^2}{f_0^2 - f^2 + f_0 f_m} < 0$$

If $f_0^2 - f^2 + f_0 f_m > 0$, then $(f_0 + f_m)^2 - f^2 < 0$

$$f_0 + f_m < f$$

$$f_0 < f - f_m = 4.0 - 3.36 = 0.64\ GHz \Rightarrow H_0 = 229\ Oe\ \checkmark$$

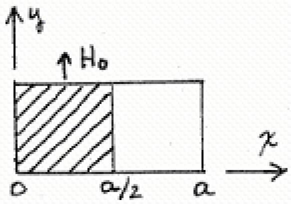
If $f_0^2 - f^2 + f_0 f_m < 0$, then $(f_0 + f_m)^2 - f^2 > 0$

$$f_0 = \frac{-f_m \pm \sqrt{f_m^2 + 4 f^2}}{2} = -1.68 \pm 4.34 = 2.66\ GHz$$

$$H_0 = 950\ Oe\ \checkmark$$

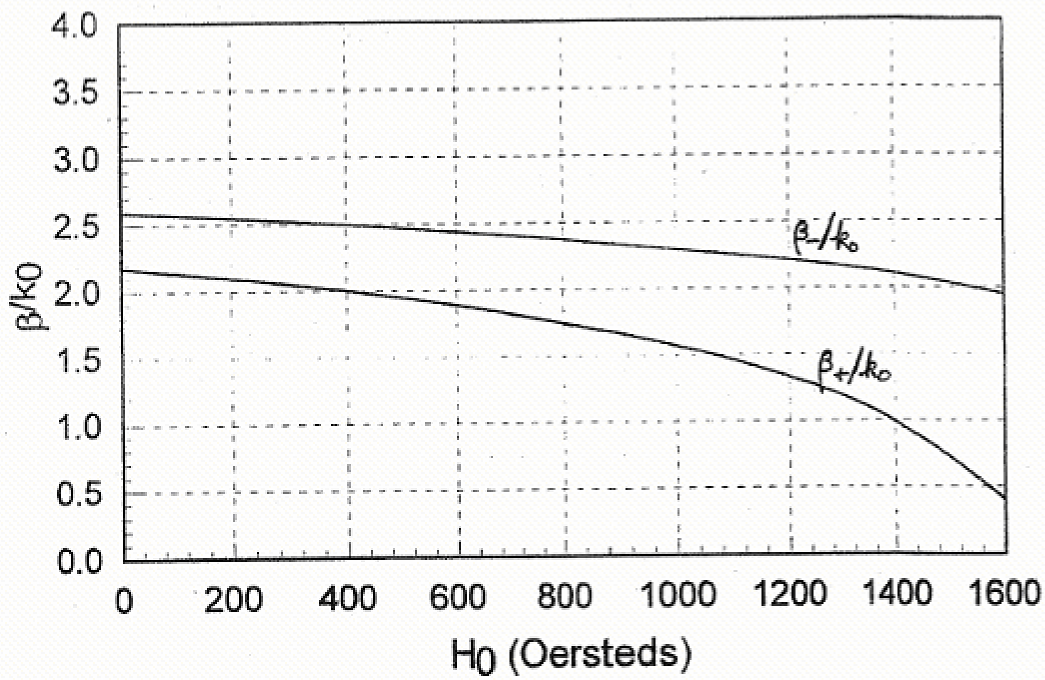
So the cutoff range is between 229 Oe and 950 Oe.

9.9

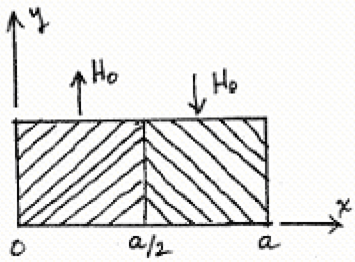


$a = 1.0$ cm $\quad (c=0,\ t=a/2)$
$f = 10$ GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants are found by solving (9.79) numerically. This was done using the FORTRAN program FLW1S.FOR, with the results shown below:

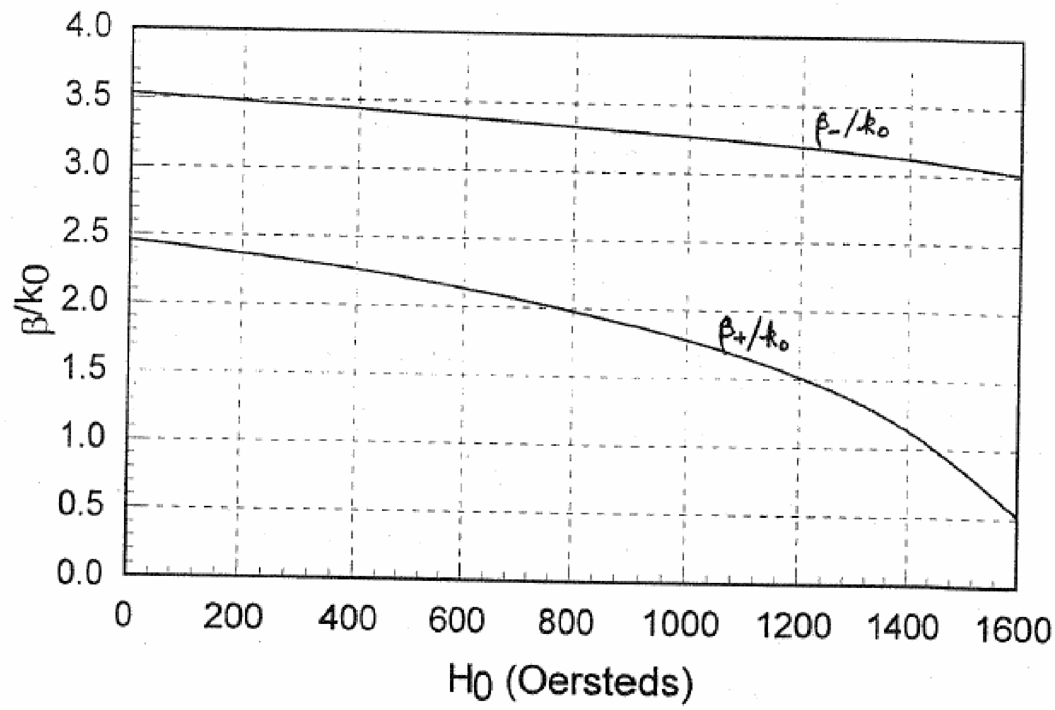


9.10

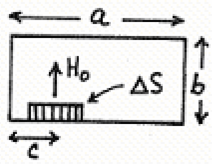


a = 1.0 cm (c = 0, t = a/2)
f = 10 GHz
$4\pi M_s = 1700$ G
$\epsilon_r = 13$

The propagation constants are found by solving (9.84) numerically. This was done using the FORTRAN program FLW2S.FOR, with the results shown below.



9.11



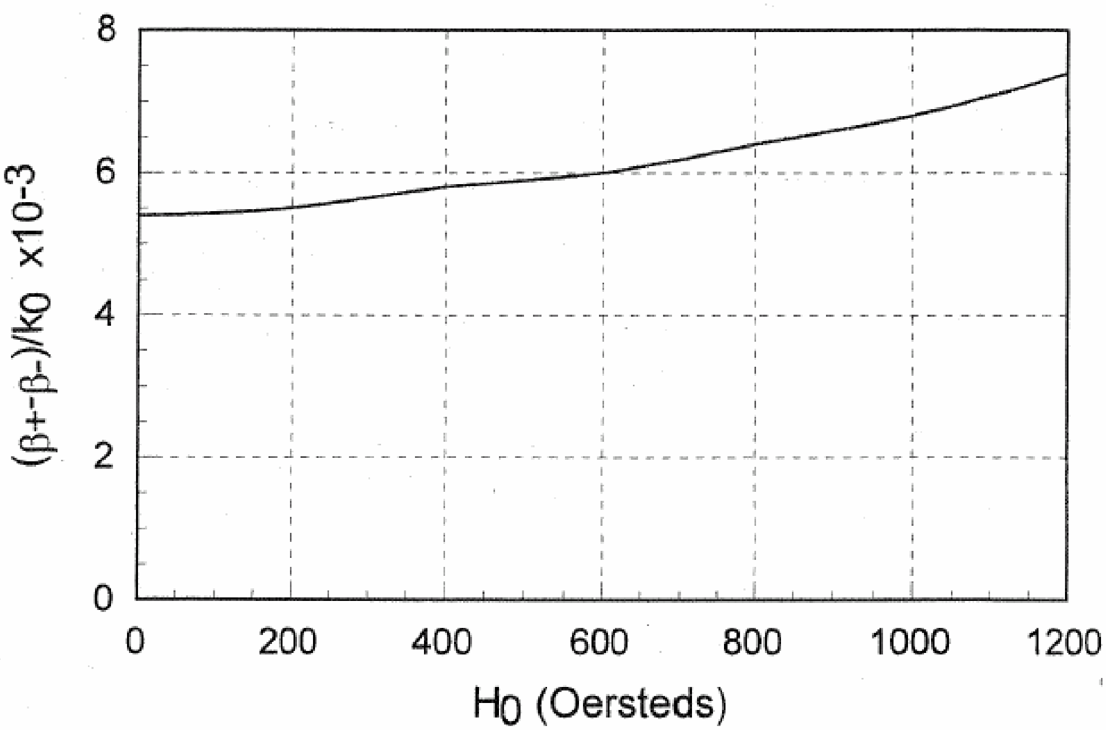
$f = 10$ GHz, $a = 2.286$ cm, $b = 1.016$ cm
$4\pi M_s = 1700$ G, $c = a/4$, $\Delta S = 2\ mm^2$
$k_0 = 209.4\ m^{-1}$, $S = 232.3\ mm^2$, $f_m = 4.76$ GHz ✓

From (9.80) the differential phase shift is,

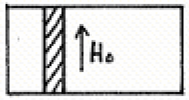
$$(\beta_+ - \beta_-)/k_0 = -2\,\frac{k_c}{k_0}\,\frac{\kappa}{\mu}\,\frac{\Delta S}{S}\,\sin 2k_c c = -0.0113\ \kappa/\mu$$

H_0 (Oe)	f_0 (GHz)	κ/μ_0	μ/μ_0	$(\beta_+ - \beta_-)/k_0$
0	0	−0.476	1.000	0.0054 ✓
200	0.56	−0.477	0.973	0.0055
400	1.12	−0.482	0.946	0.0058
600	1.68	−0.489	0.918	0.0060
800	2.24	−0.501	0.891	0.0064
1000	2.80	−0.516	0.855	0.0068
1200	3.36	−0.537	0.820	0.0074 ✓

This data is plotted on the following page



9.12



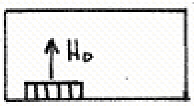
$f = 8$ GHz, $4\pi M_s = 1500$ G, $f_m = 4.2$ GHz

Gyromagnetic resonance for this geometry is given approximately by (9.87): $(N_x = 1,\ N_y = N_z = 0)$

$$f = \sqrt{f_o(f_o + f_m)}$$

Solve for f_o: $\quad f_o^2 + 4.2 f_o - 64 = 0 \Rightarrow f_o = -2.1 \pm 8.27 = 6.17$ GHz

Thus, $\quad H_o = \dfrac{6170 \text{ MHz}}{2.8 \text{ MHz/Oe}} = 2204$ Oe. ✓

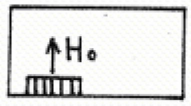


Gyromagnetic resonance for this geometry is given by the condition,

$f_o = f = 8$ GHz $\quad (N_x = N_z = 0,\ N_y = 1)$

Thus, $\quad H_o = \dfrac{8000 \text{ MHz}}{2.8 \text{ MHz/Oe}} = 2857.$ Oe ✓

9.13



$f = 10\ \mathrm{GHz}$, $4\pi M_s = 1700\ \mathrm{G}$, $\Delta H = 200\ \mathrm{Oe}$

$\frac{\Delta S}{S} = 0.01$

Gyromagnetic resonance is given by the condition,

$$f_0 = f = 10\ \mathrm{GHz} \quad (\text{since } N_x = N_z = 0,\ N_y = 1)$$

$$H_0 = \frac{10{,}000\ \mathrm{MHz}}{2.8\ \mathrm{MHz/Oe}} = 3571.\ \mathrm{Oe}$$

The position of the ferrite slab is given by (9.86), since the RF magnetic fields, h_x and h_z, have demagnetization factors of zero. Thus,

$$\tan k_c x = \pm \frac{k_c}{\beta_o}$$

$k_0 = 209.4\ \mathrm{m}^{-1}$ ✓

$k_c = \pi/a = 137.4\ \mathrm{m}^{-1}$.

$\beta_o = \sqrt{k_0^2 - k_c^2} = 158.\ \mathrm{m}^{-1}$

$$s = x = \frac{1}{k_c}\tan^{-1}\frac{k_c}{\beta_o} = \underline{0.521\ \mathrm{cm}}$$

The perturbation result of (9.81) must be used to find the attenuation constants, since this geometry cannot be analyzed exactly:

$$\alpha_\pm = \frac{\Delta S}{S\beta_o}\left(\beta_o^2 \chi''_{xx}\sin^2 k_c x + k_c^2 \chi''_{zz}\cos^2 k_c x \mp \chi''_{xy} k_c \beta_o \sin 2k_c x\right)$$

where the susceptibilities are given by (9.39):

$f_0 = f = 10\ \mathrm{GHz}$

$f_m = 1700(2.8) = 4.76\ \mathrm{GHz}$

$$\alpha = \frac{\Delta H}{2\omega\mu_0\gamma} = \frac{(200)(2.8\ \mathrm{MHz})}{2(10{,}000\ \mathrm{MHz})} = 0.028$$

$$\chi''_{xx} = \frac{\alpha f f_m\left[f_0^2 + f^2(1+\alpha^2)\right]}{\left[f_0^2 - f^2(1+\alpha^2)\right]^2 + 4 f_0^2 f^2 \alpha^2} = 8.50$$

$$\chi''_{zz} = \chi''_{xx} = 8.50$$

$$\chi''_{xy} = \frac{2 f_0 f_m f^2 \alpha}{\left[f_0^2 - f^2(1+\alpha^2)\right]^2 + 4 f_0^2 f^2 \alpha^2} = 8.49$$

Then, $\alpha_\pm = 6.3\times10^{-5}(9.139\times10^4 + 9.136\times10^4 \mp 1.825\times10^5)$

$$\alpha_+ = 0.0158 \text{ neper/m} = 0.137 \text{ dB/m}$$

$$\alpha_- = 23.0 \text{ neper/m} = 200 \text{ dB/m}$$

For 30 dB reverse attenuation, the required length is,

$$L = \frac{30\text{dB}}{200\text{dB/m}} = 0.15 \text{ m} = 15. \text{ cm}$$

Then the forward insertion loss is,

$$IL = (0.137)(0.15) = 0.02 \text{ dB}$$

Note: the calculation of $\alpha_\pm$ is numerically sensitive.

9.14 The magnetic fields for the TE_{10} waveguide mode can be written as,

$$H_x = \frac{j\beta A}{k_c} \sin k_c x \, e^{-j\beta z}$$

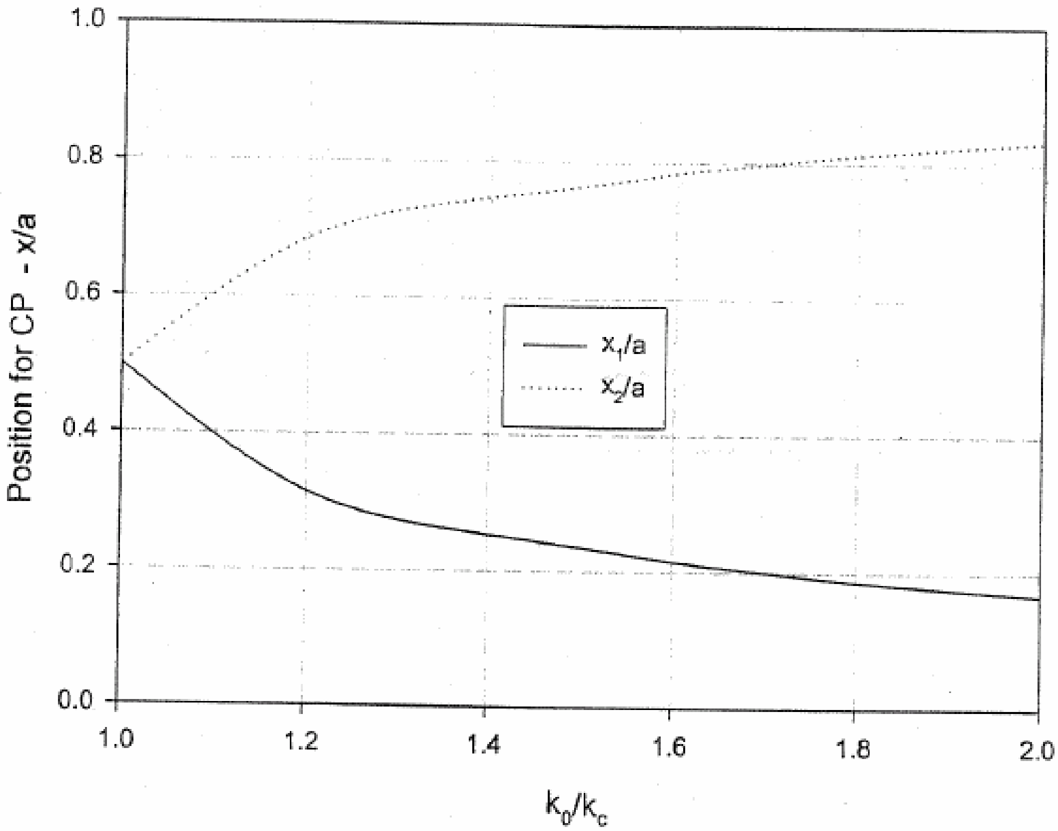
$$H_z = A \cos k_c x \, e^{-j\beta z}$$

with $k_c = \pi/a$, $\beta = \sqrt{k_0^2 - k_c^2}$. Circular polarization occurs when,

$$\frac{H_x}{H_z} = \pm j = \frac{j\beta}{k_c} \tan k_c x, \text{ or } \tan k_c x = \pm \frac{k_c}{\beta} \qquad (9.86)$$

k_0/k_c	β/k_c	x_1/a	x_2/a
1.0	0	0.500	0.500
1.2	0.663	0.314	0.686
1.4	0.980	0.253	0.747
1.6	1.249	0.215	0.785
1.8	1.497	0.187	0.813
2.0	1.732	0.167	0.833

These results are plotted on the following page.



1.0
0.8
0.6
0.4
0.2
0.0
1.0
1.2
1.4
1.6
1.8
2.0
Position for CP - x/a
k_0/k_c
x_1/a
x_2/a

9.15 $f = 10\text{ GHz},\ \epsilon_r = 12,\ 4\pi M_r = 1500.\ \Delta\phi = 90°$

$$f_m = (2.8\text{ MHz/Oe})(1500\text{ Oe}) = 4.2\text{ GHz}$$
$$k_0 = 2\pi f/c = 209.44\text{ m}^{-1}$$

In the remanent state, $H_0 = f_0 = 0$. Then,

$$\mu = \mu_0,\quad \kappa = \frac{-f_m}{f}\mu_0 = -0.420\mu_0,\quad \mu_e = \frac{\mu^2 - \kappa^2}{\mu} = 0.824\mu_0$$

State #1: $\bar{M} = M_r\hat{x}$ and $\bar{H} = \hat{y}H_y$, so this is an extraordinary wave: $\beta_e = k_0\sqrt{\epsilon_r\mu_e/\mu_0} = 658.6\text{ m}^{-1}$

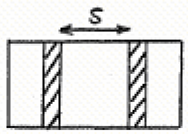
State #2: $\bar{M} = M_r\hat{y}$ and $\bar{H} = \hat{y}H_y$, so this is an ordinary wave: $\beta_o = \sqrt{\epsilon_r}\,k_0 = 725.5\text{ m}^{-1}$

Differential phase shift is

$$\Delta\phi = (\beta_o - \beta_e)L = 90°$$

$$L = \frac{90°(\pi/180°)}{\beta_o - \beta_e} = \underline{23.5\text{ mm}}$$

9.16



$s = 2\ mm$, $f = 10\ GHz$, $4\pi M_r = 1000\ G$

From Figure 9.17, maximum differential phase shift for $s = 2\ mm$ occurs for $t/a \cong 0.112$, so $t = 0.112a = 2.6\ mm$ ✓.
Then $(\beta_+ - \beta_-)/k_o = 0.24$, for $4\pi M_r = 1786\ G$. If we assume $(\beta_+ - \beta_-)$ is proportional to κ (and so M_r), then for $4\pi M_r = 1000\ G$ we have

$$\frac{(\beta_+ - \beta_-)}{k_o} = 0.24\left(\frac{1000}{1786}\right) = 0.134,$$

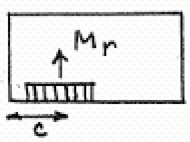
so, $(\beta_+ - \beta_-) = 0.134 k_o = 16.1°/cm$ ✓

Then the slab lengths for 180° and 90° sections are,

$$L = \frac{180°}{16.1°/cm} = 11.2\ cm \checkmark$$

$$L = \frac{90°}{16.1°/cm} = 5.6\ cm \checkmark$$

9.17



$a = 2.286\ cm$, $b = 1.016\ cm$, $f = 9\ GHz$
$4\pi M_r = 1200\ G$, $c = a/4$, $\Delta S = 2\ mm^2$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a}\,\frac{\kappa}{\mu}\,\frac{\Delta S}{S}\,\sin\frac{2\pi c}{a}$$

Now for $H_o = 0$,

$$\frac{\kappa}{\mu} = -\frac{f_m}{f} = \frac{-2.8(1200)}{9000} = -0.373$$

and,

$$S = ab = 232.3\ mm^2, \text{ so}$$

$$\beta_+ - \beta_- = 0.883\ rad/m = 0.506°/cm \checkmark$$

So the required length is,

$$L = \frac{22.5°}{0.506°/cm} = 44.5\ cm \checkmark$$

(a bit long!)

9.18

$f = 9$ GHz, $4\pi M_s = 1700$ G, $\Delta S = 6\ mm^2$
$a = 2.286$ cm, $b = 1.016$ cm, $H_a = 4000$ Oe

From (9.41) the internal bias field is ($N_z = 1$)

$$H_0 = H_a - NM_s = 4000 - 1700 = \underline{2300\ \text{Oe}}$$

From (9.80) the (approximate) differential phase shift is,

$$\beta_+ - \beta_- = \frac{-2\pi}{a}\frac{\kappa}{\mu}\frac{\Delta S}{S}\sin\frac{2\pi c}{a}$$

Maximum phase shift will occur for $c = a/4 = \underline{0.572\text{ cm}}$.
Then, $f_m = 2.8(1700) = 4.76$ GHz, $f_0 = 2.8(2300) = 6.44$ GHz.
So,

$$\mu = \mu_0\left(1 + \frac{f_0 f_m}{f_0^2 - f^2}\right) = 0.224\,\mu_0$$

$$\kappa = \mu_0\frac{f f_m}{f_0^2 - f^2} = -1.08\,\mu_0$$

Thus,

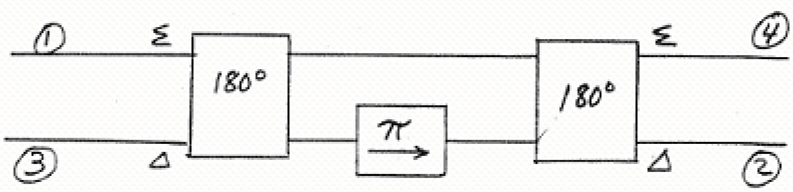
$$\beta_+ - \beta_- = 0.342\text{ rad/cm} = 19.6°/\text{cm}$$

So the required length for a 180° phase shift is,

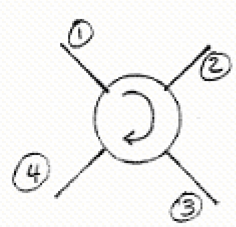
$$L = \frac{180°}{19.6°/\text{cm}} = \underline{9.2\text{ cm}}$$

9.19

A four-port circulator can be made using a gyrator and two 180° hybrid couplers:



A three port circulator can be obtained by shorting any one of the ports.



9.20 $RL = 10 dB, 20 dB.$

From (9.92) the scattering matrix of a mismatched circulator is,

$$[S] = \begin{bmatrix} \Gamma & \beta & \alpha \\ \alpha & \Gamma & \beta \\ \beta & \alpha & \Gamma \end{bmatrix}$$

Then $|\beta| \approx |\Gamma|$

$|\alpha| \simeq 1 - |\Gamma|^2$

For $RL = 10 dB$, $I = |\beta| = 10 dB$

For $RL = 20 dB$, $I = |\beta| = 20 dB$

Chapter 10

10.1 $ENR = 22\,dB \ (T_1)$, $T_2 = 77K$, $Y = 15.83 dB$

$$ENR = 10 \log \frac{T_1 - T_0}{T_0} \Rightarrow \frac{T_1 - T_0}{T_0} = 10^{22/10} = 158.5$$

$$T_1 = T_0(1 + 158.5) = 4.63 \times 10^4 K$$

$$Y = 10^{15.83/10} = 38.28$$

$$T_e = \frac{T_1 - Y T_2}{Y - 1} = 1.163 \times 10^3 K$$

$$F = 1 + \frac{T_e}{T_0} = 5.01 = \underline{7.0\,dB} \quad \checkmark$$

10.2

$$T_e = \frac{T_1 - \Upsilon T_2}{\Upsilon - 1}$$

$$T_e + \Delta T_e = \frac{T_1 - (\Upsilon + \Delta\Upsilon)T_2}{(\Upsilon + \Delta\Upsilon) - 1}$$

$$\Delta T_e = \frac{T_1 - (\Upsilon + \Delta\Upsilon)T_2}{(\Upsilon + \Delta\Upsilon) - 1} - \frac{T_1 - \Upsilon T_2}{\Upsilon - 1}$$

$$= \frac{T_1 - (\Upsilon + \Delta\Upsilon)T_2}{(\Upsilon - 1)\left(1 + \frac{\Delta\Upsilon}{\Upsilon - 1}\right)} - \frac{T_1 - \Upsilon T_2}{\Upsilon - 1}$$

$$\simeq \frac{[T_1 - (\Upsilon + \Delta\Upsilon)T_2]\left[1 - \frac{\Delta\Upsilon}{\Upsilon - 1}\right] - (T_1 - \Upsilon T_2)}{\Upsilon - 1}$$

$$\simeq \frac{-\frac{T_1}{\Upsilon - 1} + \frac{\Upsilon T_2}{\Upsilon - 1} - T_2}{\Upsilon - 1}\Delta\Upsilon = \frac{(T_2 - T_1)}{(\Upsilon - 1)^2}\Delta\Upsilon$$

$$\frac{\Delta T_e}{T_e} = \frac{(T_2 - T_1)\Upsilon}{(\Upsilon - 1)^2 T_e}\frac{\Delta\Upsilon}{\Upsilon} = \frac{(T_1 + T_e)(T_2 + T_e)}{T_e(T_2 - T_1)}\frac{\Delta\Upsilon}{\Upsilon}$$

minimize with respect to T_e:

$$\frac{d}{dT_e}\left[\frac{\Delta T_e}{T_e}\right] = \frac{\left(\frac{T_1}{T_e} + 1\right)\left(\frac{T_2}{T_e} + 1\right) + T_e\left(\frac{-T_1}{T_e^2}\right)\left(\frac{T_2}{T_e} + 1\right) + T_e\left(\frac{T_1}{T_e} + 1\right)\left(\frac{-T_2}{T_e^2}\right)}{T_2 - T_1} = 0$$

Thus,

$$T_e = \sqrt{T_1 T_2} \checkmark$$

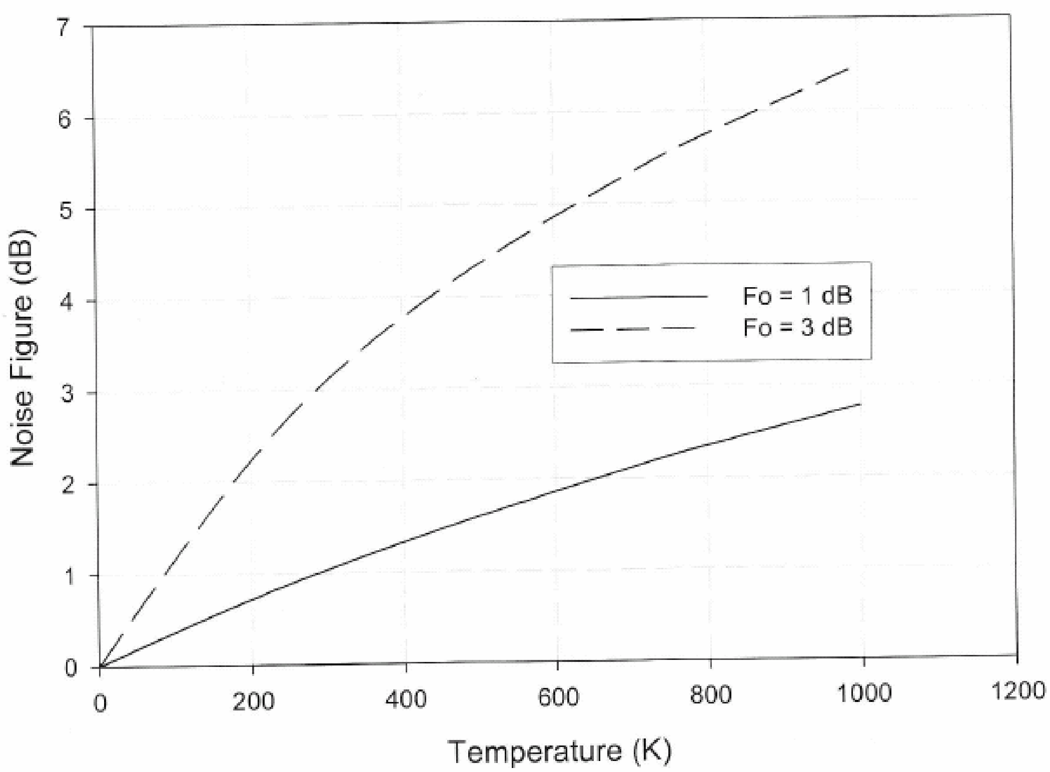
10.3 $T = 0$ to $1000\,K$, $F_0 = 1\,dB$, $3\,dB$.

From (10.16) $F = 1 + (L-1)\,T/T_0$

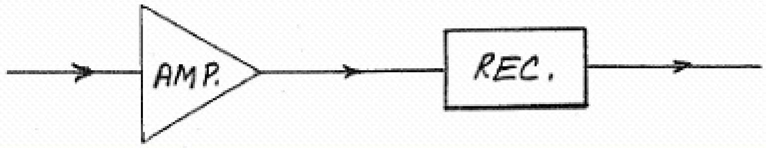
When $F_0 = 1\,dB$, $L = 1\,dB = 1.259$

$F_0 = 3\,dB$, $L = 3\,dB = 2.00$

T (K)	F ($F_0 = 1\,dB$)	F ($F_0 = 3\,dB$)
0	0	0
250	0.88	2.70
500	1.60	4.35
750	2.23	5.55
1000	2.77	6.48



10.4



$G = 12\,dB = 15.8$ $\qquad$ $T_e = 900\,K$

$BW = 150\,MHz$

$F = 4\,dB = 2.51$

The noise figure of the receiver is, from (10.11),

$$F_2 = 1 + \frac{T_e}{T_0} = 1 + \frac{900}{290} = 4.10$$

Then the noise figure of the cascade is, from (10.21),

$$F_{cas} = F_1 + \frac{1}{G_1}(F_2 - 1) = 2.51 + \frac{4.10 - 1}{15.8} = 2.71 = 4.3\,dB \checkmark$$

10.5

a) $$T_e = \frac{P}{kB} = \frac{(.001)\times 10^{-9.5}}{(1.38\times 10^{-23})(75\times 10^{6})} = 305.5\,K \checkmark$$

b) $$F_\ell = 1 + (L-1)\frac{T}{T_0} = 1 + (1.413 - 1)\frac{300}{290} = 1.43, \quad F_a = 1 + \frac{T_e}{T_0} = 1.62 = 2.1\,dB$$

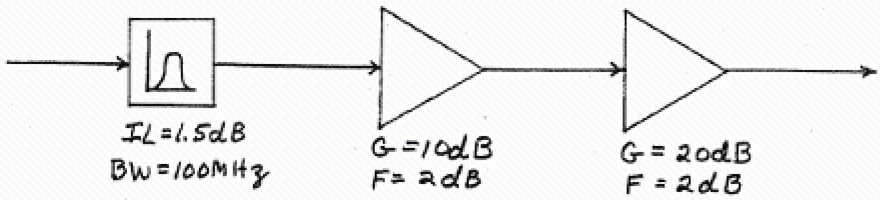
c) $$F_c = F_\ell + \frac{F_a - 1}{G_\ell} = 1.43 + \frac{1.62 - 1}{1/1.413} = 2.30 = \underline{3.6\,dB} \checkmark$$

$$T_c = (F_c - 1)T_0 = (2.30 - 1)(290) = \underline{378\,K} \checkmark$$

d) $$N_o = k(T_c + T_i)BG = (1.38\times 10^{-23})(378 + 305.5)(75\times 10^{6})\left(\frac{15.8}{1.413}\right)$$

$$= 7.9\times 10^{-12}\,W = 7.9\times 10^{-9}\,mW = \underline{-81.0\,dBm} \checkmark$$

10.6



From (10.23) the noise figure of the cascade is ($F_1 = IL = 1.5dB$)

$$F_{CAS} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} = 1.41 + (1.41)(1.58 - 1) + \frac{1.41}{10}(1.58 - 1)$$

$$= 2.31 = 3.64dB$$

If $P_{in} = -90$ dBm, then $P_{out} = -90dBm - 1.5dB + 10dB + 20dB = -61.5dBm$

The noise power output is then,

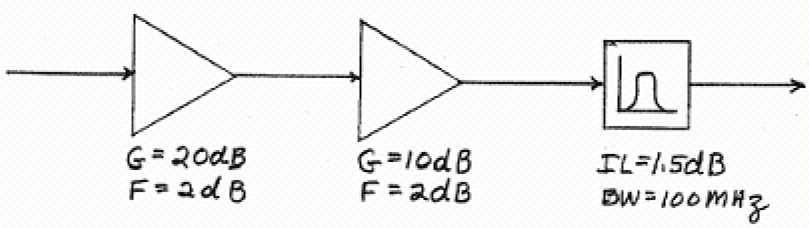
$$P_n = G_{cas} k T_{e_{cas}} B = k(F_{cas} - 1) T_o B G_{cas}$$

$$= (1.38 \times 10^{-23})(2.31 - 1)(290)(10^8)(10^{28.5/10}) = 3.71 \times 10^{-10} \text{ W}$$

$$= -64.3 \text{ dBm}$$

Thus, $\quad \frac{S_o}{N_o} = -61.5 + 64.3 = 2.8dB$

The best noise figure would be achieved with the arrangement shown below:



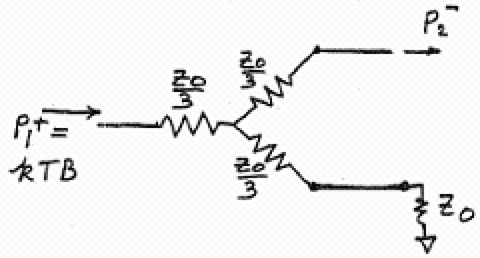
Then,

$$F_{CAS} = 1.58 + \frac{(1.58 - 1)}{100} + \frac{(1.41 - 1)}{1000} = 1.586 = 2.0dB$$

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)

10.7

a) RESISTIVE DIVIDER



$$[S] = \frac{1}{2}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

When the input noise power at port 1 is kTB, and the divider is at temperature T, the system is in thermodynamic equilibrium. Thus the output noise power at port 2 must be kTB. We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$P_2^- = kTB = \frac{kTB}{4} + \frac{N_{added}}{4}$$

$$\therefore N_{added} = 3kTB$$

The equivalent noise temperature is then,

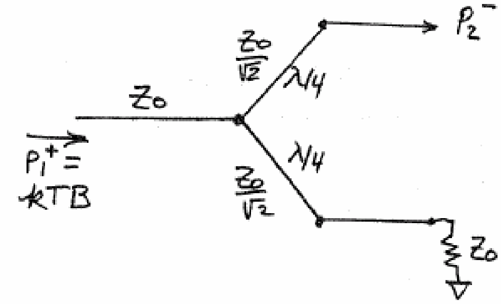
$$T_e = \frac{N_{added}}{kB} = 3T$$

and the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

at room temperature, $T = T_0$, so $F = 4 = 6\text{dB}$.
(this result checks with that obtained using the available gain method)

b) WILKINSON DIVIDER



$$[S] = \begin{bmatrix} 0 & -j/\sqrt{2} & -j/\sqrt{2} \\ -j/\sqrt{2} & 1/2 & -1/2 \\ -j/\sqrt{2} & -1/2 & 1/2 \end{bmatrix}$$

In this case, if the input noise power is kTB, and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{4}kTB$, because of the mismatch of the output ports (1/4 of output power is reflected). Then we have,

$$P_2^- = \frac{3}{4}kTB = \frac{kTB}{2} + \frac{N_{added}}{2} \qquad (N_{added} \text{ ref. at input})$$

$$\therefore N_{added} = \frac{1}{2}kTB$$

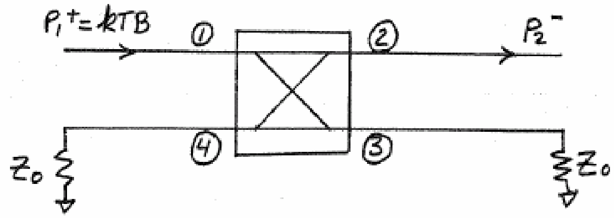
$$T_e = \frac{N_{added}}{kB} = \frac{T}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{2T_0}$$

If $T = T_0$, $F = \frac{3}{2} = 1.76$ dB.

(Result verified with HP-MDS, calculations using available gain, and direct measurement)

c) QUADRATURE HYBRID



$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Using the same thermodynamic arguments as above, the output noise power is kTB (outputs are matched). Thus,

$$P_2^- = kTB = \frac{kTB}{2} + \frac{N_{added}}{2}$$

$$\therefore N_{added} = kTB$$

$$T_e = \frac{N_{added}}{kB} = T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{T}{T_0}$$

If $T = T_0$, we have $F = 2 = 3dB$

10.8 From (10.33), $T_e = \dfrac{(L-1)(L+|\Gamma_s|^2)}{L(1-|\Gamma_s|^2)} T$

let $x^2 = |\Gamma_s|^2$; $C = (L-1)T/L$. Then $T_e = C\dfrac{L+x^2}{1-x^2}$

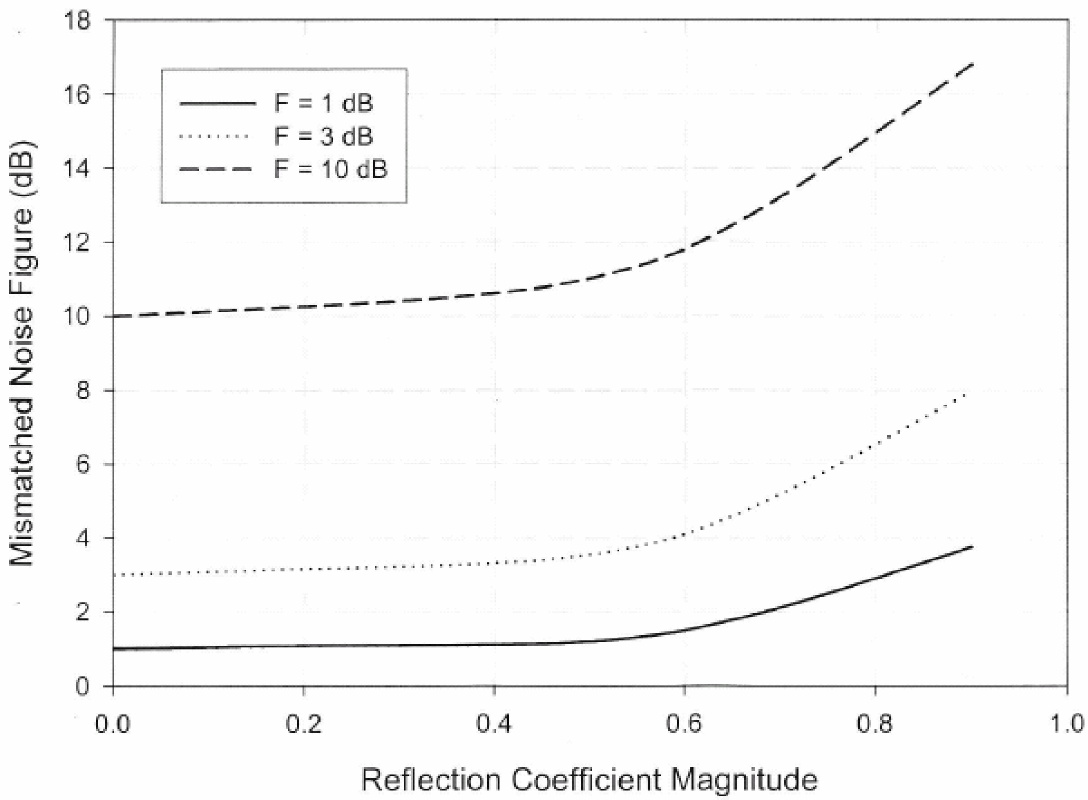
$$\frac{dT_e}{dx} = C\frac{(1-x^2)(2x)+(2x)(L+x^2)}{(1-x^2)^2} = \frac{2x(1+L)}{(1-x^2)^2} = 0$$

Thus $x = 0$, so $|\Gamma_s| = 0$ minimizes T_e ✓

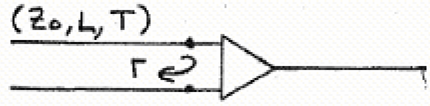
10.9 $|\Gamma| = 0$ to 0.9, $F = 1\,dB, 3\,dB, 10\,dB$.

From (10.36) $\quad F_m = 1 + \dfrac{F-1}{1-|\Gamma|^2}$

$\lvert\Gamma\rvert$	F=1dB	F=3dB	F=10dB
0	1.00	3.00	10.0
0.3	1.09	3.22	10.4
0.6	1.48	4.09	11.8
0.9	3.74	7.97	16.8



10.10



Solution using noise temperature:

Let $N_i = kT_oB$

Then $N_o = \underbrace{\frac{kT_oBG}{L}(1-|\Gamma|^2)}_{\text{INPUT NOISE}} + \underbrace{\frac{(L-1)}{L}kTB(1-|\Gamma|^2)G}_{\text{NOISE ADDED BY LINE}} + \underbrace{kT_o(F-1)GB}_{\text{NOISE ADDED BY AMP}}$

also,
$$S_o = \frac{G(1-|\Gamma|^2)}{L}S_i$$

So,
$$F_{CAS} = \frac{S_iN_o}{S_oN_i} = \frac{L}{\cancel{G}(1-|\Gamma|^2)} \cdot \frac{\frac{\cancel{kT_oBG}}{L}(1-|\Gamma|^2)+\left(\frac{L-1}{L}\right)\cancel{G}\cancel{k}T\cancel{B}(1-|\Gamma|^2)+\cancel{k}T_o(F-1)\cancel{G}\cancel{B}}{\cancel{kT_oB}}$$

$$= 1+(L-1)\frac{T}{T_o}+\frac{L(F-1)}{1-|\Gamma|^2} \checkmark$$

Solution using cascade formula:

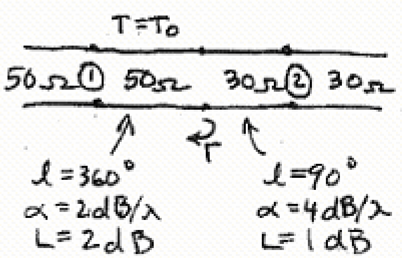
$$T_{e(LINE)} = (L-1)T$$
$$F_{(LINE)} = 1+(L-1)T/T_o$$
$$G_{(LINE)} = \frac{1}{L}(1-|\Gamma|^2)$$

$$F_{CAS} = F_{(LINE)} + \frac{F_{AMP}-1}{G_{(LINE)}} = 1+(L-1)\frac{T}{T_o}+\frac{L}{1-|\Gamma|^2}(F-1) \checkmark$$

CHECK: IF $\Gamma=0$, $F_{CAS} = 1+(L-1)\frac{T}{T_o}+L(F-1)$ ✓

IF $\Gamma=0$ AND $T=T_o$, $F_{CAS} = 1+(L-1)+L(F-1) = LF$ ✓

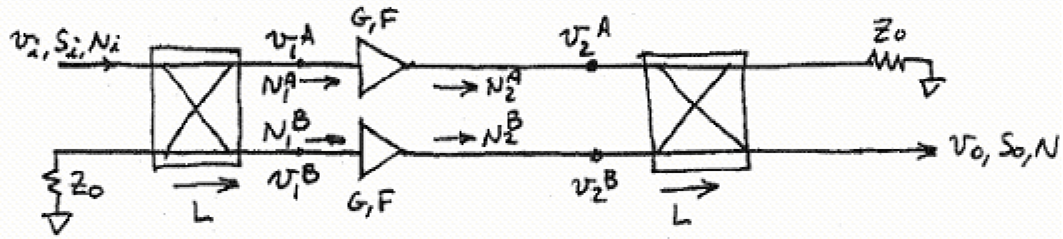
NUMERICAL CHECK:



$$\Gamma = \frac{30-50}{30+50} = \frac{-1}{4}$$

$F_{CAS} = 3.06\,dB$ – AGREES WITH SERENADE.

10.11



$$S_i = v_i^2/2$$

$$v_1^A = \frac{v_i}{\sqrt{2L}} \qquad v_1^B = -j\frac{v_i}{\sqrt{2L}}$$

$$v_2^A = \frac{v_i\sqrt{G}}{\sqrt{2L}} \qquad v_2^B = -j\frac{v_i\sqrt{G}}{\sqrt{2L}}$$

$$v_o = -j\frac{v_2^A}{\sqrt{2L}} + \frac{v_2^B}{\sqrt{2L}} = -j\frac{v_i\sqrt{G}}{2L} - j\frac{v_i\sqrt{G}}{2L} = -j\frac{v_i\sqrt{G}}{L}$$

$$S_o = \frac{v_o^2}{2} = \frac{v_i^2 G}{2L^2} = \frac{G S_i}{L^2} \quad ✓$$

$$N_1^A = N_1^B = kT_oB$$

$$N_2^A = N_2^B = kT_oBG + kT_eBG = kT_oBG(1+F-1) = kT_oBGF$$

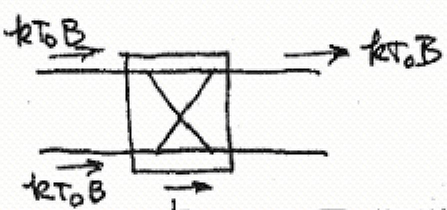
$$N_o = \frac{N_2^A}{2L} + \frac{N_2^B}{2L} + \frac{N_{ADDED}}{2L} = \frac{kT_oBG}{L}F + \underbrace{\frac{kT_oB}{2L}(2L-2)}_{\text{SEE BELOW}}$$

$$= \frac{kT_oBG}{L}F + kT_oB\left(1-\frac{1}{L}\right)$$

$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L^2}{G}\left[\frac{GF}{L} + \left(1-\frac{1}{L}\right)\right] = LF + \frac{L}{G}(L-1) \quad ✓$$

CHECK: IF $L=1$, $F_{TOT} = F$ ✓

N_{ADDED} FOR HYBRID:



$$N_o = \frac{kT_oB}{2L} + \frac{kT_oB}{2L} + \frac{N_{ADDED}}{2L} = kT_oB$$

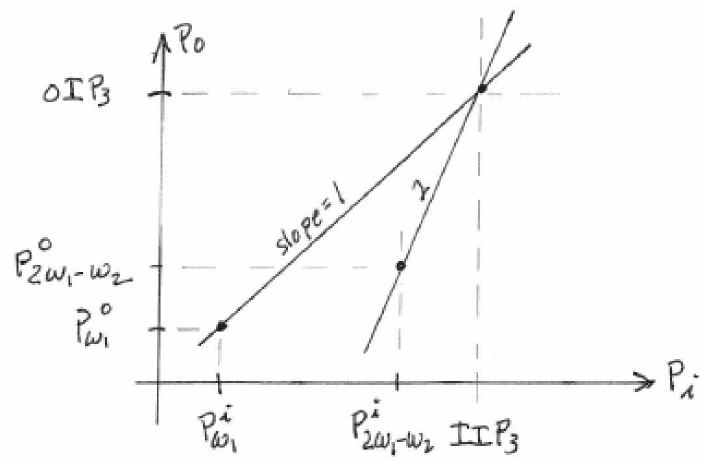
REF. AT INPUT

$\therefore$ $N_{ADDED} = 2kT_oB(L-1)$ (REF. AT INPUT)

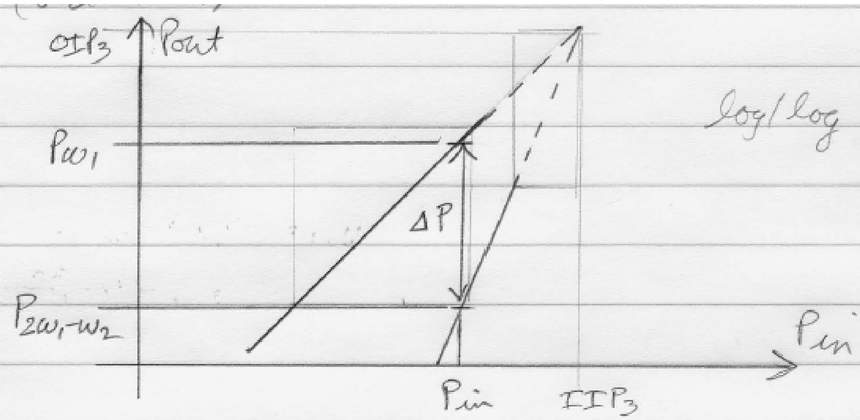
10.12

The ratio $\dfrac{OIP_3 - P^o_{\omega_1}}{IIP_3 - P^i_{\omega_1}} = 1$ defines the slope of the fundamental (1st order response)

The ratio $\dfrac{OIP_3 - P^o_{2\omega_1-\omega_2}}{IIP_3 - P^i_{2\omega_1-\omega_2}} = 3$ defines the slope of the third order response.



10.13



$P_{\omega_1} = P_{in} + b_1$ (eq. of line, slope = 1)

$P_{2\omega_1-\omega_2} = 3P_{in} + b_2$ (eq. of line, slope = 3)

subtract:

$$\Delta P = P_{\omega_1} - P_{2\omega_1-\omega_2} = -2P_{in} + b_1 - b_2$$

Now,

$IIP_3 = P_{in}$ when $\Delta P = 0$, so

$$\Delta P = -2P_{in} + b_1 - b_2$$

$$0 = -2(IIP_3) + b_1 - b_2$$

so $\Delta P = -2P_{in} + 2(IIP_3)$

or,

in dB $\begin{cases} IIP_3 = P_{in} + \Delta P/2 & \text{(ref. at input)} \\ OIP_3 = P_{\omega_1} + \Delta P/2 & \text{(ref at output)} \end{cases}$

Example: $P_{\omega_1} = 5\,dBm$, $P_{2\omega_1-\omega_2} = -27\,dBm$, $P_{in} = -4\,dBm$ (Egan, P. 100)

Then $\Delta P = 32\,dB$

$IIP_3 = -4 + 32/2 = 12\,dBm$

$OIP_3 = 5 + 32/2 = 21\,dBm$

check: $G = P_{\omega_1}/P_{in} = 5 + 4 = 9\,dB$

$OIP_3 = G(IIP_3) = 9 + 12 = 21\,dBm$ ✓

10.14

Retaining only the terms that give rise to the third order intermodulation products:

$$v_0 \sim k\,(v_1 \cos\omega_1 t + v_2 \cos\omega_2 t)^3$$

$$\sim k\,(v_1^2 v_2 \cos^2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos^2\omega_2 t)$$

$$\sim k\,(v_1^2 v_2 \cos 2\omega_1 t \cos\omega_2 t + v_1 v_2^2 \cos\omega_1 t \cos 2\omega_2 t)$$

$$\sim \frac{k}{2}\left[v_1^2 v_2 \cos(2\omega_1 - \omega_2)t + v_1 v_2^2 \cos(2\omega_2 - \omega_1)t\right]$$

So the ratio of the powers in the two outputs is,

$$\left(\frac{v_1^2 v_2}{v_1 v_2^2}\right)^2 = \left(\frac{v_1}{v_2}\right)^2 = 6\,dB$$

Note that the individual output powers vary as P_{in}^3.

10.15 Moving the reference for IP_3' to the output of the mixer gives

$$OIP_3' = 13\ dBm - 6\ dB = 7\ dBm$$

numerical values:

$$OIP_3'' = 22\ dBm = 158\ mW \qquad (AMP)$$

$$OIP_3' = 7\ dBm = 5\ mW \qquad (MIXER)$$

$$G_2 = 20\ dB = 100 \qquad (AMP)$$

Assuming coherent products, using (10.53),

$$OIP_3 = \left(\frac{1}{G_2(OIP_3')} + \frac{1}{OIP_3''}\right)^{-1} = \left(\frac{1}{(100)(5)} + \frac{1}{158}\right)^{-1} = 120\ mW$$

$$= \underline{20.8\ dBm}$$

Assuming non-coherent products, using (10.54),

$$OIP_3 = \left[\frac{1}{G_2^2\,(OIP_3')^2} + \frac{1}{(OIP_3'')^2}\right]^{-1/2} = \left[\frac{1}{(100)^2(5)^2} + \frac{1}{(158)^2}\right]^{-1/2}$$

$$= 150.7\ mW = \underline{21.8\ dBm}$$

10.16

For $v_i = V_o \cos \omega_o t$

$$V_{\omega_o} = a_1 V_o + \frac{3}{4} a_3 V_o^3 \qquad (10.40)$$

$$V_{3\omega_o} = \frac{1}{4} a_3 V_o^3$$

For $v_i = V_o(\cos \omega_1 t + \cos \omega_2 t)$

$$V_{2\omega_1 - \omega_2} = \frac{3}{4} a_3 V_o^3 \qquad (10.43)$$

assume a_3 is of opposite sign to a_1 (for compression).
Now let V_o be input voltage where 3rd order term reduces 1st order term by 1 dB:

$$\frac{|a_1|V_o - \frac{3}{4}|a_3|V_o^3}{|a_1|V_o} = 1 - \frac{3}{4}\left|\frac{a_3}{a_1}\right| V_o^2 = 10^{-1/20} = 0.8913$$

$$\frac{3}{4}\left|\frac{a_3}{a_1}\right| V_o^2 = 0.10875$$

From (10.44) the input voltage at IP_3 is

$$V_{IP}^2 = \frac{4a_1}{3a_3}, \text{ so } \frac{V_o^2}{V_{IP}^2} = 0.10875 = \frac{IP_{1dB}}{IIP_3} = -9.64\,dB$$

Then, $OP_{1dB} = G + IP_{1dB} - 1\,dB = G + IIP_3 - 1\,dB - 9.64\,dB$

$= OIP_3 - 10.64\,dB$ ✓ (see Egan, p.103)

10.17 $OP_{1dB} = 5\text{ dBm}, \quad G = 15\text{ dB}, \quad B = 1\text{ GHz}, \quad T_e = 250\text{K}$

$$N_i = kT_eB = (1.38\times10^{-23})(250)(10^9) = 3.5\times10^{-12}\text{ W} = -84.5\text{ dBm}$$
$$N_o = GN_i = 15\text{dB} + (-84.5\text{dBm}) = -69.5\text{ dBm}$$

From (10.55), $LDR = OP_{1dB} - N_o = 5-(-69.5) = \underline{74.5\text{ dB}}$

(Note: using input levels will result in a value 1dB higher, but LDR is usually defined as output levels)

10.18 $F = 6\text{dB} = 4, \quad G = 30\text{dB} = 10^3, \quad B = 20\text{MHz},$
$OIP_3 = 33\text{dBm}, \quad OP_{1dB} = 21\text{ dBm}, \quad SNR = 8\text{ dB}.$

$$N_i = -105\text{ dBm} = 3.16\times10^{-14}\text{ W}$$
$$T_e = (F-1)T_0 = 3(290) = 870\text{K} \quad ✓$$
$$N_o = G(N_i + kT_eB)$$
$$= 10^3[3.16\times10^{-14} + (1.38\times10^{-23})(870)(20\times10^6)]$$
$$= 2.72\times10^{-10}\text{ W} = -65.7\text{ dBm} \quad ✓$$

From (10.55) $LDR = OP_{1dB} - N_o = 21-(-65.7) = \underline{86.7\text{ dB}}$ ✓

From (10.60) $SFDR = \frac{2}{3}(OIP_3 - N_o) - SNR$
$$= \frac{2}{3}(33+65.7) - 8 = \underline{57.8\text{ dB}}$$

Chapter 11

11.1

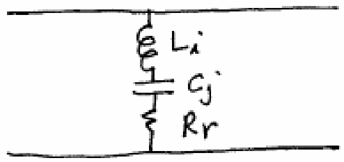
$C_j = 0.38\,pF$, $R_S = 4\,\Omega$, $I_S = 0.3\,\mu A$

$I_0\,(\mu A)$	$R_j\,(\Omega)$	$\beta_v\,(V/mW)$
0	8.3×10^4	8.7
20	1.23×10^3	6.4
50	4.97×10^2	4.6

11.2 INFINEON BA592 PIN: $R_f = 0.36\,\Omega$, $C_j = 1.4\,pF$

assume $R_r = 0.5\,\Omega$, $L_i = 0.5\,nH$

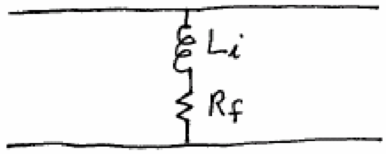
SWITCH ON (diode OFF)



$Z_d = 0.5 - j15.8\,\Omega$

$Y_d = 1.98 + j63.01\,mS$

SWITCH OFF (diode ON)



$Z_d = 0.36 + j12.6\,\Omega$

$Y_d = 2.28 - j79.5\,mS$

Use stub with $Y_s = -j0.06301 = -j3.15/Z_0 \Rightarrow \ell_s = 0.30\lambda$.

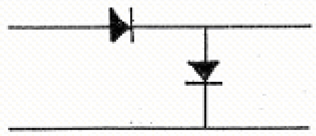
ON: $Z = 1/.00198 = 505.1\,\Omega$

$$IL(ON) = -20\log\left|\frac{2Z}{2Z+Z_0}\right| = 0.42\,dB \checkmark$$

OFF: $Z = 0.112 + j7.01$

$$IL(OFF) = -20\log\left|\frac{2Z}{2Z+Z_0}\right| = 11.4\,dB \checkmark$$

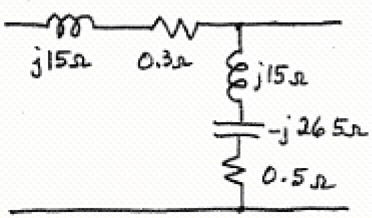
11.3



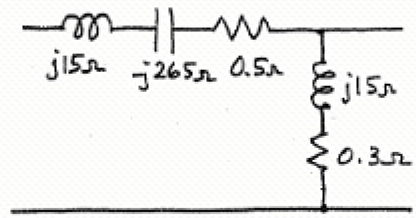
$\omega L_i = 15\,\Omega$

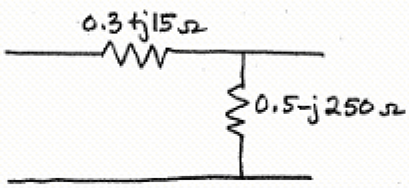
$1/\omega C_j = 265\,\Omega$

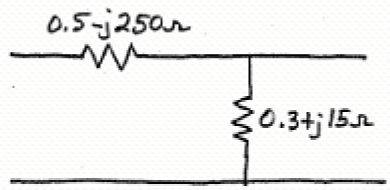
SWITCH ON:



SWITCH OFF:

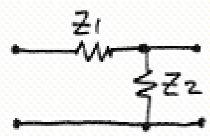






ABCD matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z_1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1/Z_2 & 1 \end{bmatrix} = \begin{bmatrix} 1+Z_1/Z_2 & Z_1 \\ 1/Z_2 & 1 \end{bmatrix}$$



Convert to S_{21}:

$$S_{21} = \frac{2}{A+B/Z_0+CZ_0+D} = \frac{2}{1+Z_1/Z_2+Z_1/Z_0+Z_0/Z_2+1} = \frac{2}{2+\frac{Z_1}{Z_2}+\frac{Z_1}{Z_0}+\frac{Z_0}{Z_2}}$$

ON STATE: $Z_1 = 0.3+j15\,\Omega$, $Z_2 = 0.5-j250\,\Omega$

$S_{21} = 0.995\angle{-14^\circ}$

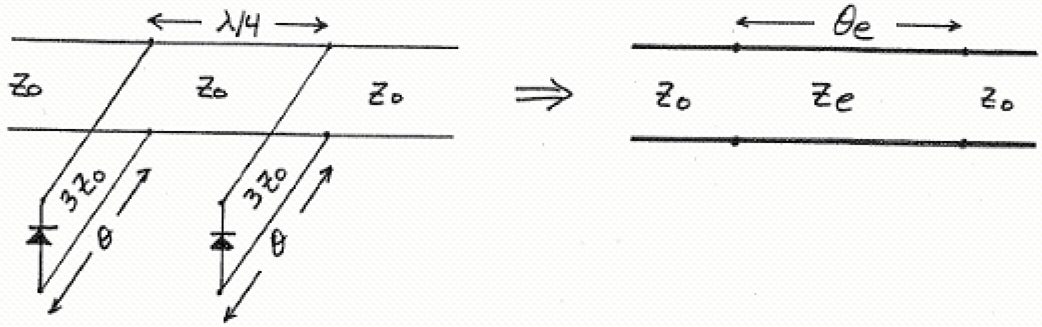
$IL = 0.044\text{ dB}$

OFF STATE: $Z_1 = 0.5-j250\,\Omega$, $Z_2 = 0.3+j15\,\Omega$

$S_{21} = 0.118\angle{149^\circ}$

$IL = 18.6\text{ dB}$

11.4



From (10.74), $\cos\theta_e = -b$

$$Z_e = Z_0/\sqrt{1-b^2}$$

where b is the normalized stub susceptance.

For diodes ON, $b = -\frac{1}{3}\cot\theta$

$$\cos\theta_e = \frac{1}{3}\cot\theta$$

For diodes OFF, $b = 1/3 \tan\theta$

$$\cos\theta_e = \frac{-1}{3}\tan\theta$$

So $\Delta\phi = 45° = \cos^{-1}\left(\frac{1}{3}\cot\theta\right) - \cos^{-1}\left(\frac{-1}{3}\tan\theta\right)$ (ON−OFF)

Solving this equation numerically:

θ	$\Delta\phi$
110°	73°
120°	46°
130°	40°
122°	44.3°
121°	45.2°

So we choose $\theta = 121°$. (Using $\theta = 31°$ gives $\Delta\phi = -45°$)

Insertion loss for $\theta = 121°$:

Using (10.73), $b = BZ_0$

$$S_{21} = \frac{2}{A + B/Z_0 + CZ_0 + D} = \frac{2}{-BZ_0 + j(1 - B^2Z_0^2) - BZ_0} = \frac{2}{-2b + j(2-b^2)}$$

$$|S_{21}|^2 = \frac{4}{4b^2 + (2-b^2)^2}$$

DIODES ON:

$$b = \frac{-1}{3}\cot\theta = 0.20$$

$$|S_{21}|^2 = 0.9996$$

$$IL = 0.0017\,dB \sim 0\,dB \checkmark$$

DIODES OFF:

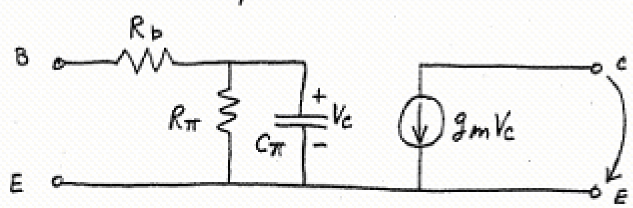
$$b = \frac{1}{3}\tan\theta = -0.555$$

$$|S_{21}|^2 = 0.977$$

$$IL = 0.102\,dB \checkmark$$

(SuperCompact analysis gives $IL_{ON} = 0\,dB$, $\phi_{ON} = -101.5°$, $IL_{OFF} = 0.10\,dB$, $\phi_{OFF} = -56.7°$, thus $\Delta\phi = 44.8°$)

11.5 Unilateral bipolar transistor model:



From (10.77) the short-circuit current gain is,

$$G_i^{sc} = \left|\frac{I_c}{I_b}\right|\Bigg|_{V_{ce}=0} = \frac{g_m V_c}{I_b} = \frac{g_m I_b \left|\frac{R_\pi / j\omega C_\pi}{R_\pi + 1/j\omega C_\pi}\right|}{I_b}$$

$$= g_m \frac{R_\pi}{|1 + j\omega R_\pi C_\pi|} = \frac{g_m}{\left|\frac{1}{R_\pi} + j\omega C_\pi\right|} \simeq \frac{g_m}{\omega C_\pi} \quad \text{since } R_\pi \gg 1/\omega C_\pi$$

(e.g., if $R_\pi = 110\Omega$, $C_\pi = 18pF$, $f = 1GHz$, then $1/\omega C_\pi = 9\Omega$)

11.6 First find Z_{ij}, then convert to S_{ij}:

$$Z_{11} = R_i - j/\omega C_{gs}$$

$$Z_{22} = \left(\frac{1}{R_{ds}} + j\omega C_{ds}\right)^{-1}$$

$$Z_{12} = 0$$

$$Z_{21} = -g_m Z_{22}/j\omega C_{gs} = j g_m Z_{22}/\omega C_{gs}$$

$$S_{11} = \frac{(Z_{11}-Z_0)(Z_{22}+Z_0)}{\Delta Z} \quad , \quad \Delta Z = (Z_{11}+Z_0)(Z_{22}+Z_0)$$

$$= \frac{Z_{11}-Z_0}{Z_{11}+Z_0} \checkmark$$

$$S_{12} = 0$$

$$S_{21} = \frac{2Z_{12}Z_0}{\Delta Z} = \frac{2jZ_0 g_m Z_{22}/\omega C_{gs}}{(Z_{11}+Z_0)(Z_{22}+Z_0)}$$

$$S_{22} = \frac{(Z_{11}+Z_0)(Z_{22}-Z_0)}{\Delta Z} = \frac{Z_{22}-Z_0}{Z_{22}+Z_0} \checkmark$$

11.7 From 11.6,

$$Z_{11} = Z_0 \frac{1+S_{11}}{1-S_{11}} = R_i - j/\omega C_{gs}$$

$$R_i = \mathrm{Re}\{Z_{11}\} \;,\quad C_{gs} = -1/\omega\, \mathrm{Im}\{Z_{11}\}$$

$$Z_{22} = Z_0 \frac{1+S_{22}}{1-S_{22}} = \left(\frac{1}{R_{ds}} + j\omega C_{ds}\right)^{-1}$$

$$R_{ds} = 1/\mathrm{Re}\{Z_{22}\} \;,\quad C_{ds} = \mathrm{Im}\{1/Z_{22}\}/\omega$$

$$g_m = -j\omega C_{gs} Z_{21}/Z_{22}, \quad Z_{21} = \frac{S_{21}\,\Delta Z}{2Z_0} = \frac{S_{21}(Z_{11}+Z_0)(Z_{22}+Z_0)}{2Z_0}$$

From Table 11.7, MESFET @ 2GHz:

$$S_{11} = 0.9 \angle -55^\circ$$
$$S_{21} = 3.56 \angle 129^\circ$$
$$S_{12} = 0.08 \angle 54^\circ \sim 0$$
$$S_{22} = 0.65 \angle -37^\circ$$

Then,

$$R_i = 12.2\ \Omega, \quad C_{gs} = 0.84\ \mathrm{pF}$$
$$R_{ds} = 213\ \Omega, \quad C_{ds} = 0.51\ \mathrm{pF}$$
$$g_m = 54\ \mathrm{mS}.$$

Chapter 12

12.1 The [S] matrix for a 3-dB matched attenuator is,

$$[S] = \begin{bmatrix} 0 & 0.707 \\ 0.707 & 0 \end{bmatrix}$$

For $Z_L = 50\Omega$: $\quad \Gamma_L = \Gamma_{in} = 0, \; \Gamma_S = 0, \; \Gamma_{out} = 0$

Then from (11.12), (11.13), and (11.8) we have

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2(1-|\Gamma_{out}|^2)} = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

$$G = \frac{|S_{21}|^2 \; (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2} = |S_{21}|^2 = 0.5 \checkmark$$

For $Z_L = 25\Omega$: $\quad \Gamma_L = -1/3, \; \Gamma_S = \Gamma_{out} = 0, \; \Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = -1/6$

$$G_A = |S_{21}|^2 = 0.5 \checkmark$$

$$G_T = |S_{21}|^2(1-|\Gamma_L|^2) = 0.444 \checkmark$$

$$G = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)} = 0.457 \checkmark$$

For $Z_S = 25\Omega$, $Z_L = 50\Omega$: $\quad \Gamma_L = \Gamma_{in} = 0, \; \Gamma_S = -1/3, \; \Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} = -1/6$

$$G_A = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)} = 0.457$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{1} = 0.444$$

$$G = |S_{21}|^2 = 0.5$$

12.2 Infineon BFP SiGe HBT at 1 GHz

(a) $\Gamma_S = \Gamma_L = 0$, so $\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$

$$(12.8) \quad G = \frac{|S_{21}|^2 (1 - |\Gamma_L|^2)}{(1 - |\Gamma_{in}|^2)|1 - S_{22}\Gamma_L|^2} = \frac{|S_{21}|^2}{(1 - |S_{11}|^2)} = \frac{(3.92)^2}{1 - (.91)^2} = 89.4 \checkmark$$

$$(12.12) \quad G_A = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)}{(1 - |\Gamma_{out}|^2)|1 - S_{11}\Gamma_S|^2} = \frac{|S_{21}|^2}{1 - |S_{22}|^2} = \frac{(3.92)^2}{1 - (.93)^2} = 113.7 \checkmark$$

$$(12.13) \quad G_T = \frac{|S_{21}|^2 (1 - |\Gamma_S|^2)(1 - |\Gamma_L|^2)}{|1 - \Gamma_S \Gamma_{in}|^2 |1 - S_{22}\Gamma_L|^2} = |S_{21}|^2 = (3.92)^2 = 15.4 \checkmark$$

(b) for G, let $\Gamma_L = S_{22}^*$, $\Gamma_S = 0$ (G does not depend on Γ_S)
Then, for $S_{12} = 0$, $\Gamma_{in} = S_{11}$

$$G = \frac{|S_{21}|^2}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = 661.7 \checkmark$$

for G_A, let $\Gamma_S = S_{11}^*$, $\Gamma_L = 0$ (G_A does not depend on Γ_L)
Then, with $S_{12} = 0$, $\Gamma_{out} = S_{22}$

$$G_A = \frac{|S_{21}|^2}{(1 - |S_{22}|^2)(1 - |S_{11}|^2)} = 661.7 \checkmark$$

For G_T, let $\Gamma_S = S_{11}^*$, $\Gamma_L = S_{22}^*$.
Then, with $S_{12} = 0$, $\Gamma_{in} = S_{11}$, $\Gamma_{out} = S_{22}$.

$$G_T = \frac{|S_{21}|^2}{(1 - |S_{11}|^2)(1 - |S_{22}|^2)} = 661.7 \checkmark$$

12.3

$S_{11} = 0.61\angle{-170^\circ}$, $S_{12} = 0.06\angle{70^\circ}$, $S_{21} = 2.3\angle{80^\circ}$, $S_{22} = 0.72\angle{-25^\circ}$

$Z_S = 25\,\Omega$, $Z_L = 100\,\Omega$, $V_S = 2V$.

a)

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = 0.641\angle{-174^\circ} \quad ✓$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} = 0.777\angle{-25^\circ} \quad ✓$$

$$G = 12.8 = \underline{11.1\,dB} \quad ✓$$

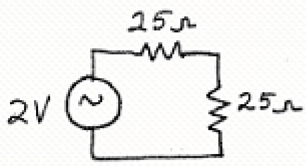
$$G_A = 18.4 = \underline{12.6\,dB} \quad ✓$$

$$G_T = 10.8 = \underline{10.3\,dB} \quad ✓$$

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2\,|1-S_{22}\Gamma_L|^2} = 10.4 = \underline{10.2\,dB} \quad ✓$$

(verified with Serenade)

b)



$$P_{AVS} = \frac{1}{2}\left(\frac{2}{2}\right)^2\frac{1}{25} = \underline{0.02\,W}$$

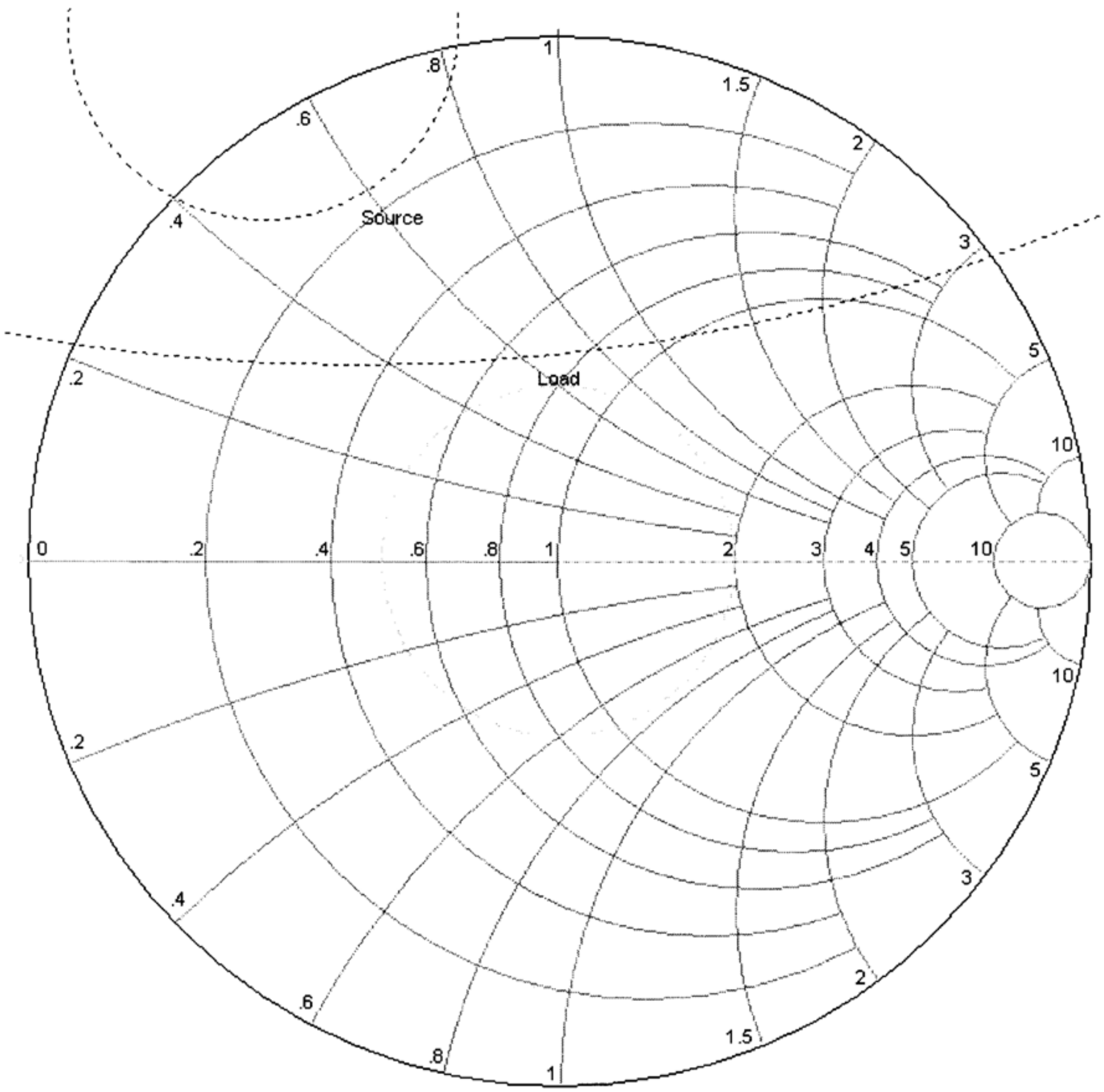
$$P_L = G_T P_{AVS} = (10.8)(.02) = \underline{0.216\,W}$$

12.4

$S_{11} = 0.880\angle{-115}$, $S_{12} = 0.029\angle{31}$, $S_{21} = 9.4\angle{110}$, $S_{22} = 0.328\angle{-67}$

$C_L = 4.0\angle{96^\circ}$, $R_L = 3.60$, $C_S = 1.16\angle{119^\circ}$, $R_S = 0.367$

$K = 0.275$



12.5

GaN HEMT

S-parameters from Table 11.8

f(GHz)	K	Δ	STABILITY
0.5	0.604	0.686	pot. unstable
1.0	0.452	0.725	pot. unstable
2.0	1.030	0.705	uncond. stable
4.0	3.262	0.770	uncond. stable

12.6

Using (12.30) to compute μ:

DEVICE	S_{11}	S_{12}	S_{21}	S_{22}	μ	
A	$0.34\angle{-170°}$	$0.06\angle{70°}$	$4.3\angle{80°}$	$0.45\angle{-25°}$	1.193	UNC. STABLE
B	$0.75\angle{-60°}$	$0.2\angle{70°}$	$5.0\angle{90°}$	$0.5\angle{60°}$	0.283	POT. UNSTABLE
C	$0.65\angle{-140°}$	$0.04\angle{60°}$	$2.4\angle{50°}$	$0.7\angle{-65°}$	1.057	UNC. STABLE

Device A has the best stability.

12.7 From (11.30) the μ-parameter test is,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-S_{11}^*\Delta| + |S_{21}S_{12}|} > 1$$

If $S_{12}=0$ (unilateral) then we have,

$$\Delta = S_{11}S_{22}$$

So,

$$\mu = \frac{1-|S_{11}|^2}{|S_{22}-|S_{11}|^2 S_{22}|} = \frac{1-|S_{11}|^2}{|S_{22}|\,|1-|S_{11}|^2|} > 1$$

Since the denominator is positive and μ is positive, the numerator must also be positive, thus $|S_{11}|<1$. Then the above reduces to,

$$\mu = \frac{1}{|S_{22}|} > 1 \;,$$

so,

$$|S_{22}| < 1 .$$

12.8 From the definitions of (11.41),

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad , \quad C_1 = S_{11} - \Delta S_{22}^* .$$

Similar to the expansion used after (11.35), it can be verified by direct expansion that,

$$|C_1|^2 = |S_{11}-\Delta S_{22}^*|^2 = |S_{12}S_{21}|^2 + (1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

So the condition that $B_1^2 - 4|C_1|^2 > 0$ implies that,

$$(1+|S_{11}|^2-|S_{22}|^2-|\Delta|^2)^2 > 4|S_{12}S_{21}|^2 + 4(1-|S_{22}|^2)(|S_{11}|^2-|\Delta|^2)$$

$$1+2|S_{11}|^2-2|S_{22}|^2-2|\Delta|^2+|S_{11}|^4-2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$+2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2+4(|S_{11}|^2-|\Delta|^2-|S_{11}|^2|S_{22}|^2+|\Delta|^2|S_{22}|^2)$$

$$1-2|S_{11}|^2-2|S_{22}|^2+2|\Delta|^2+|S_{11}|^4+2|S_{11}|^2|S_{22}|^2-2|S_{11}|^2|\Delta|^2+|S_{22}|^4$$
$$-2|\Delta|^2|S_{22}|^2+|\Delta|^4 > 4|S_{12}S_{21}|^2$$

$$(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2 > 4|S_{12}S_{21}|^2$$

or,

$$K^2 = \frac{(1-|S_{11}|^2-|S_{22}|^2+|\Delta|^2)^2}{4|S_{12}S_{21}|^2} > 1 \quad \checkmark$$

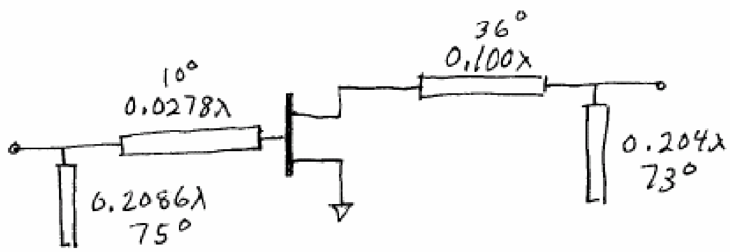
12.9 Data from Table 11.7 GaAs MESFET

Conjugate matching for maximum gain (at 8 GHz):

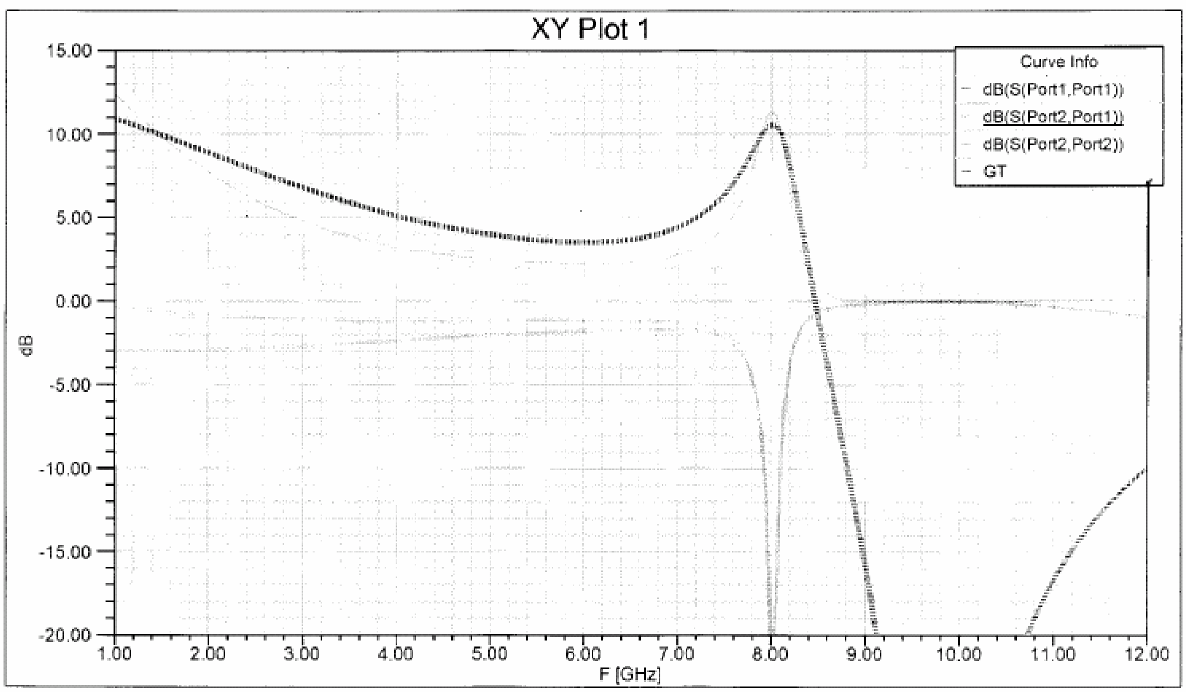
$$\Gamma_S = \Gamma_{in}^* = 0.883\angle{-172^\circ}$$

$$\Gamma_L = \Gamma_{out}^* = 0.859\angle{139^\circ}$$

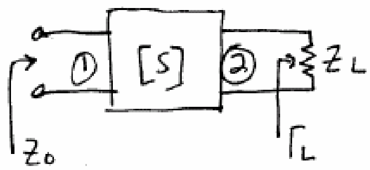
$G_S = 4.53$, $G_o = 4.00$, $G_L = 0.623 \Rightarrow G_T = 11.29 = 10.53\text{dB}$

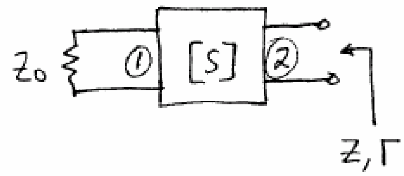


CAD modeling gives gain = 10.5 dB; no stability problems from 1–12 GHz. Gain is plotted below.



12.10





Assuming a lossless reciprocal network,

$$S_{12} = S_{21}, \quad |S_{11}|^2 + |S_{12}|^2 = |S_{22}|^2 + |S_{12}|^2 = 1$$

$$S_{11} S_{12}^* + S_{12} S_{22}^* = 0$$

For the second circuit, $\Gamma = S_{22} + \dfrac{S_{12} S_{21} \Gamma_S}{1 - S_{11}\Gamma_S} = S_{22}$ since $\Gamma_S = 0$.

For the first circuit, at port 1, $\Gamma_L = \Gamma^* = S_{22}^*$, so

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} + \frac{S_{12}^2 S_{22}^*}{1 - |S_{22}|^2}$$

$$= S_{11} + \frac{S_{12}^2 S_{22}^*}{|S_{12}|^2} = \frac{S_{12}(S_{11} S_{12}^* + S_{12} S_{22}^*)}{|S_{12}|^2} = 0 \quad ✓$$

12.11 $S_{11} = 0.61\angle -170°$, $S_{21} = 2.24\angle 32°$, $S_{12} = 0$, $S_{22} = 0.72\angle -83°$

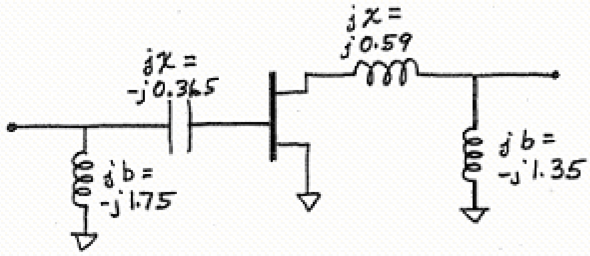
The transistor is unconditionally stable since $K = \infty$ and $|\Delta| < 1$. Since the transistor is unilateral,

$$\Gamma_S = S_{11}^* = 0.61\angle 170° \checkmark \quad , \quad \Gamma_L = S_{22}^* = 0.72\angle 83° \checkmark$$

and the maximum gain is, from (11.45),

$$G_{TU_{MAX}} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2} = 16.6 = 12.2\,dB$$

Matching was done with a Smith chart. The final circuit is:



The matching element values are, at 6 GHz,

$$C = \frac{-1}{\omega Z_0 x_c} = 1.45\,pF \checkmark \qquad L = \frac{Z_0 x_L}{\omega} = 0.78\,mH \checkmark$$

$$L = \frac{-Z_0}{\omega b_L} = 0.76\,mH \checkmark \qquad L = \frac{-Z_0}{\omega b_L} = 0.98\,mH \checkmark$$

SuperCompact analysis gives $|S_{11}| = 0.035$, $|S_{22}| = 0.008$, and $G = 12.2\,dB$ ✓

12.12 $S_{11} = 0.61\angle{-170°}$, $S_{21} = 2.24\angle{32°}$; $S_{12} = 0$, $S_{22} = 0.72\angle{-83°}$

$G = 10\,dB$, $G_S = 1\,dB$, $G_L = 2\,dB$

Since $K = \infty$ and $|\Delta| < 1$, the transistor is unconditionally stable. From (11.45), we have

$$G_{S_{MAX}} = \frac{1}{1-|S_{11}|^2} = 1.59 \checkmark, \qquad G_{L_{MAX}} = \frac{1}{1-|S_{22}|^2} = 2.08 \checkmark$$

So for $G_S = 1\,dB = 1.26$, and $G_L = 2\,dB = 1.58$, we have from (11.46),

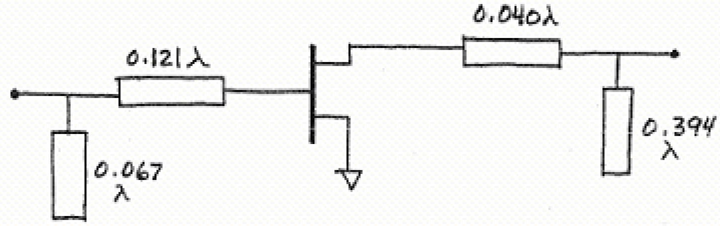
$$g_S = \frac{G_S}{G_{S_{MAX}}} = 0.792, \qquad g_L = \frac{G_L}{G_{L_{MAX}}} = 0.760$$

Then the centers and radii of the constant gain circles can be found from (11.49)-(11.50) :

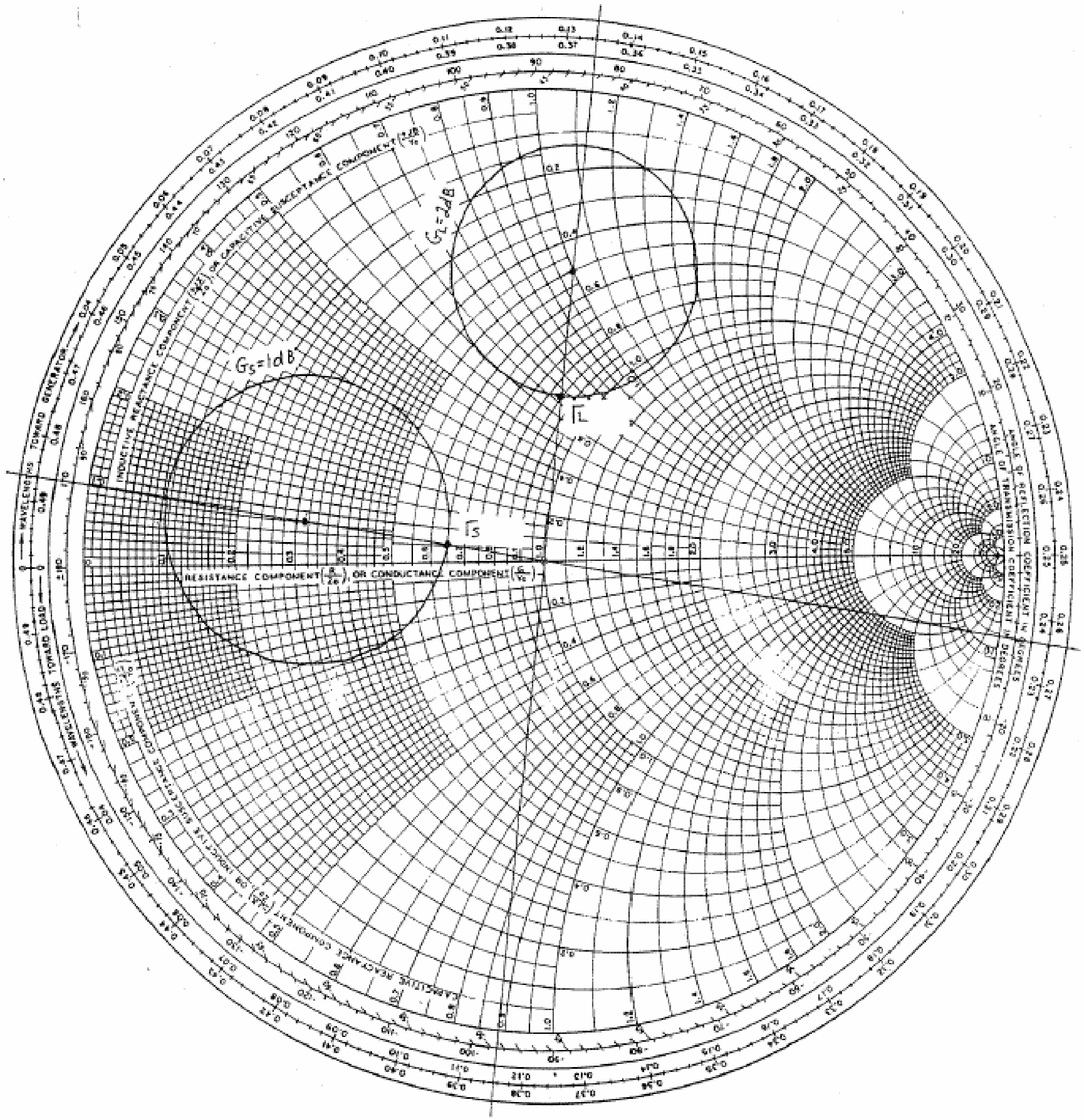
$$C_S = 0.524\angle{170°} \checkmark \qquad C_L = 0.625\angle{83°} \checkmark$$

$$R_S = 0.310 \checkmark \qquad R_L = 0.269 \checkmark$$

Since $G_o = 10 \log |S_{21}|^2 = 7.0\,dB$, using the $G_S = 1\,dB$ and the $G_L = 2\,dB$ gain circles will give an overall gain of 10dB. We plot these circles on the Smith chart, and choose $\Gamma_S = 0.215\angle{170°} \checkmark$ and $\Gamma_L = 0.361\angle{83°} \checkmark$ to minimize the magnitude of these values. after matching, we have the following amplifier circuit:



SuperCompact analysis gives $|S_{11}| = 0.45$, $|S_{22}| = 0.48$, $G = 10.05\,dB \checkmark$ (reflections at input and output serve to reduce the gain to 10dB). Smith chart shown on following page.



G_L=2dB
G_S=1dB
Γ_L
Γ_S
RESISTANCE COMPONENT (R/Z0), OR CONDUCTANCE COMPONENT (G/Y0)
INDUCTIVE REACTANCE COMPONENT (+jX/Z0), OR CAPACITIVE SUSCEPTANCE COMPONENT (+jB/Y0)
CAPACITIVE REACTANCE COMPONENT (-jX/Z0), OR INDUCTIVE SUSCEPTANCE COMPONENT (-jB/Y0)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

12.13

$S_{11} = .88\angle{-115^\circ}$, $S_{12} = .029\angle{31^\circ}$, $9.40\angle{110^\circ}$, $S_{22} = .328\angle{-67^\circ}$

From (12.46) the unilateral figure of merit is

$$U = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = \frac{(.029)(9.4)(.88)(.328)}{[1-(.88)^2][1-(.328)^2]} = 0.391$$

From (12.45) this bounds the error in G_T/G_{TU} by

$$.517 = \frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2} = 2.70$$

in dB, $\quad -2.9\,dB < G_T(dB) - G_{TU}(dB) < 4.3\,dB$

An assumption that the device is unilateral is not justified in this case.

12.14 From (11.48) and (11.47), when $G_S = 1$ we have,

$$g_s = \frac{1}{G_{S_{MAX}}} = 1 - |S_{11}|^2 \quad , \quad 1 - g_s = |S_{11}|^2$$

so (11.51) reduces to,

$$C_S = \frac{(1-|S_{11}|^2)\, S_{11}^*}{1-|S_{11}|^4} = \frac{S_{11}^*}{1+|S_{11}|^2}$$

$$R_S = \frac{|S_{11}|(1-|S_{11}|^2)}{1-|S_{11}|^4} = \frac{|S_{11}|}{1+|S_{11}|^2}$$

So the equation of the constant gain circle becomes,

$$\left| \Gamma_S - \frac{S_{11}^*}{1+|S_{11}|^2} \right| = \frac{|S_{11}|}{1+|S_{11}|^2}$$

one solution to this equation occurs for $\Gamma_S = 0$, so the circle must pass through the center of the Smith chart.

12.15 $S_{11} = 0.7\angle{-110°}$, $S_{12} = 0.02\angle{60°}$, $S_{21} = 3.5\angle{60°}$, $S_{22} = 0.8\angle{-70°}$

$F_{MIN} = 2.5\text{dB}$, $\Gamma_{OPT} = 0.7\angle{120°}$, $R_N = 15\Omega$

First check stability: $K = 1.07$, $|\Delta| = 0.53$

Since $K > 1$ and $|\Delta| < 1$ the device is unconditionally stable.

Minimum noise figure occurs for $\Gamma_S = \Gamma_{OPT} = 0.7\angle{120°}$. Then we maximize gain by conjugate matching the output. From (11.41b),

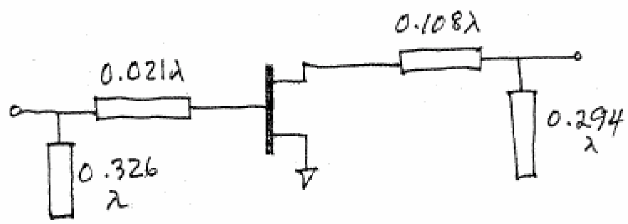
$$\Gamma_L = \left(S_{22} + \frac{S_{12}S_{21}\Gamma_S^*}{1 - S_{11}\Gamma_S}\right)^* = 0.873\angle{74°} \checkmark$$

So the noise figure will be $F = F_{min} = 2.5\text{dB}$, and the gain will be,

$$G_T = \frac{1 - |\Gamma_S|^2}{|1 - S_{11}\Gamma_S|^2} |S_{21}|^2 \frac{1 - |S_{22}|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$= (1.85)(12.25)(3.81) = 86.3 = 19.4\text{ dB}$$

Impedance matching is done with a Smith chart; the final amplifier circuit is shown below.



Serenade analysis of this amplifier gives

$|S_{11}| = 0.33$, $|S_{22}| = 0.13$,

$G = 19.7\text{dB}$, $F = 2.5\text{dB}$ ✓.

The solution is simpler if S_{12} is set to zero, resulting in $G = 18$ dB.

12.16 $S_{11} = 0.6\angle{-60°}$, $S_{21} = 2.\angle{81°}$, $S_{12} = 0$, $S_{22} = 0.7\angle{-60°}$

$F_{MIN} = 2.0 dB$, $\Gamma_{OPT} = 0.62\angle{100°}$, $R_N = 20\,\Omega$

Since $S_{12}=0$ and $|S_{11}||S_{22}| < 1$, the device is unconditionally stable. The overall gain is, $G_{TU} = G_S G_0 G_L$, where $G_0 = |S_{21}|^2 = 4 = 6 dB$. ✓ So $G_S + G_L = 0 dB$.

Plot noise figure circles for $F = 2.0, 2.05, 2.2$, and $3.0 dB$:

F(dB)	N	C_F	R_F
2.05	0.0134	$0.61\angle{100°}$	0.09
2.20	0.055	$0.59\angle{100°}$	0.18
3.00	0.30	$0.48\angle{100°}$	0.40
2.00	0.	$0.62\angle{100°}$	0

Now plot constant gain circles for $G_S = G_L = 0 dB$:

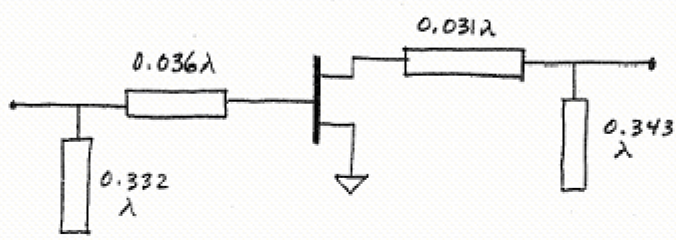
$G_{S_{MAX}} = 1.56$ ✓ $\qquad$ $G_{L_{MAX}} = 1.96$ ✓

$g_s = 0.641$ $\qquad$ $g_L = 0.510$

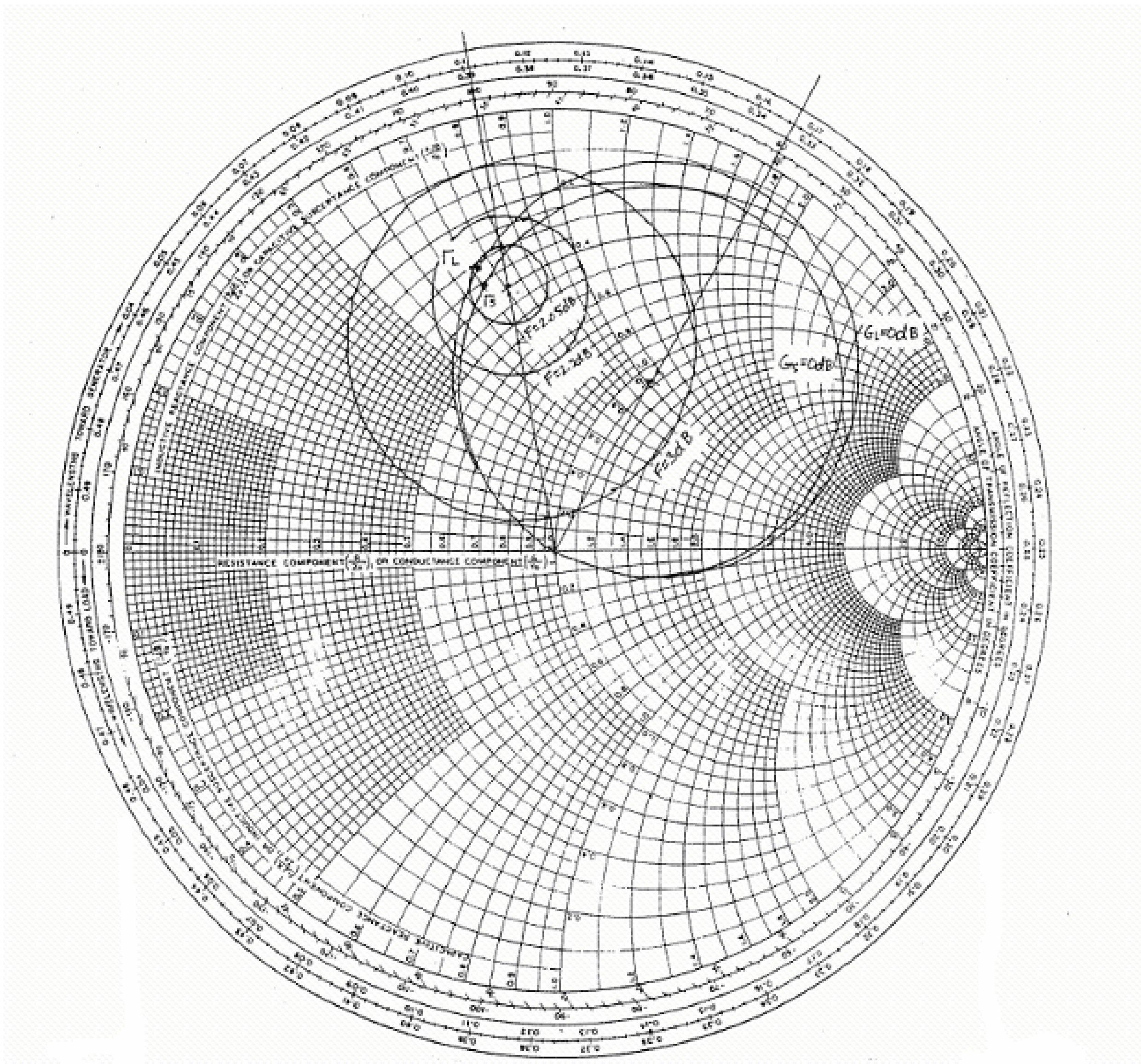
$C_S = 0.44\angle{60°}$ $\qquad$ $C_L = 0.47\angle{60°}$

$R_S = 0.44$ $\qquad$ $R_L = 0.47$

These two circles are close together near the $F = 2dB$ point. We choose $\Gamma_L = 0.66\angle{105°}$, $\Gamma_S = 0.62\angle{105°}$. Then we should obtain $F \simeq 2.04 dB$. The final AC amplifier circuit is:



Super Compact analysis gives $|S_{11}| = 0.62$, $|S_{22}| = 0.67$, $G = 6.1 dB$, and $F = 2.04 dB$ ✓ The gain and noise circles are shown below.



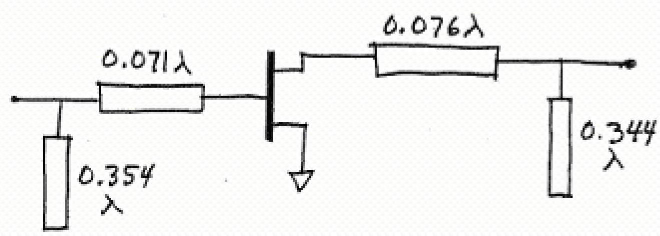
ΓL
ΓS
F=2.05dB
F=2.2dB
F=3dB
GL=0dB
GS=0dB

12.17 S-parameters and noise parameters of Problem 11.14
Plot the F = 2.5 dB constant noise figure circle:
$N = 0.141$, $C_F = 0.543\angle 100^\circ$, $R_F = 0.286$

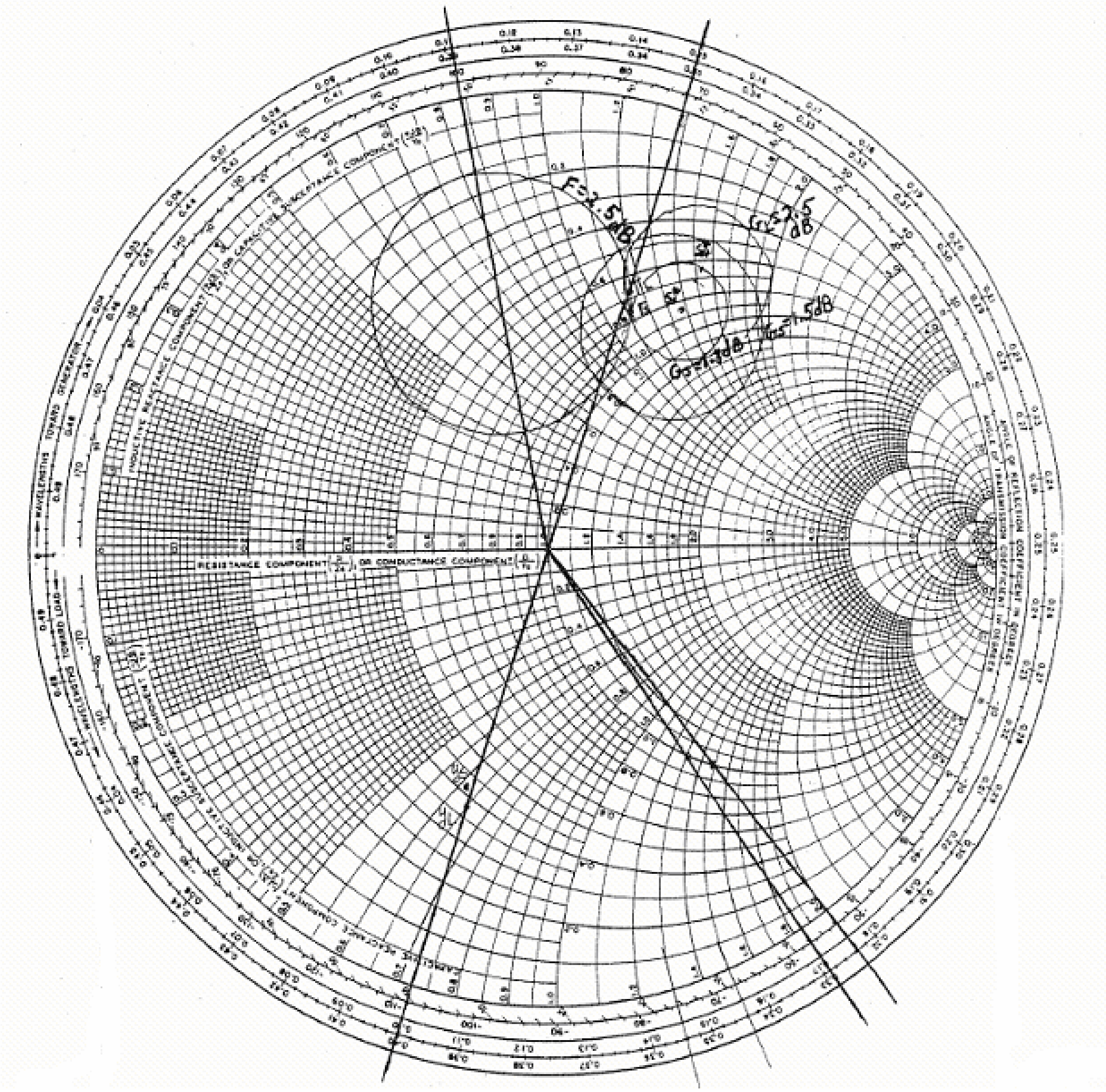
Now, $G_{S_{MAX}} = 1.56 = 1.93$ dB, $G_{L_{MAX}} = 1.96 = 2.92$ dB
But these points (S_{11}^*, S_{22}^*) do not lie on the F = 2.5 dB circle. We can plot some gain circles to just give intersections with the F = 2.5 dB noise circle:

$G_S = 1.5$ dB	$g_S = 0.905$	$C_S = 0.56\angle 60^\circ$	$R_S = 0.204$
$G_L = 2.5$ dB	$g_L = 0.907$	$C_L = 0.67\angle 60^\circ$	$R_L = 0.163$
$G_S = 1.7$ dB	$g_S = 0.948$	$C_S = 0.58\angle 60^\circ$	$R_S = 0.149$
$G_S = 1.8$ dB	$g_S = 0.970$	$C_S = 0.59\angle 60^\circ$	$R_S = 0.112$

The $G_S = 1.8$ dB and $G_L = 2.5$ dB circles are close to optimum (the F = 2.5 dB noise circle). Thus we have $\Gamma_S = 0.545\angle 70^\circ$, $\Gamma_L = 0.59\angle 72^\circ$, which will yield a gain of $G_T = 1.8 + 2.5 + 6 = 10.3$ dB.
The final AC amplifier circuit is shown below:



SuperCompact analysis of this circuit gives $|S_{11}| = 0.20$, $|S_{22}| = 0.28$, G = 10.3 dB ✓, and F = 2.4 dB ✓. The noise and gain circles are shown on the following page.



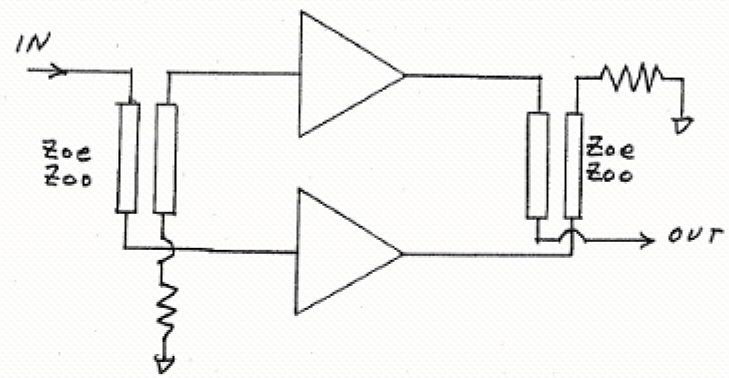
F=2.5dB

12.18 FOR THE COUPLED LINE COUPLERS:

$$C = 10^{-3/20} = 0.708$$

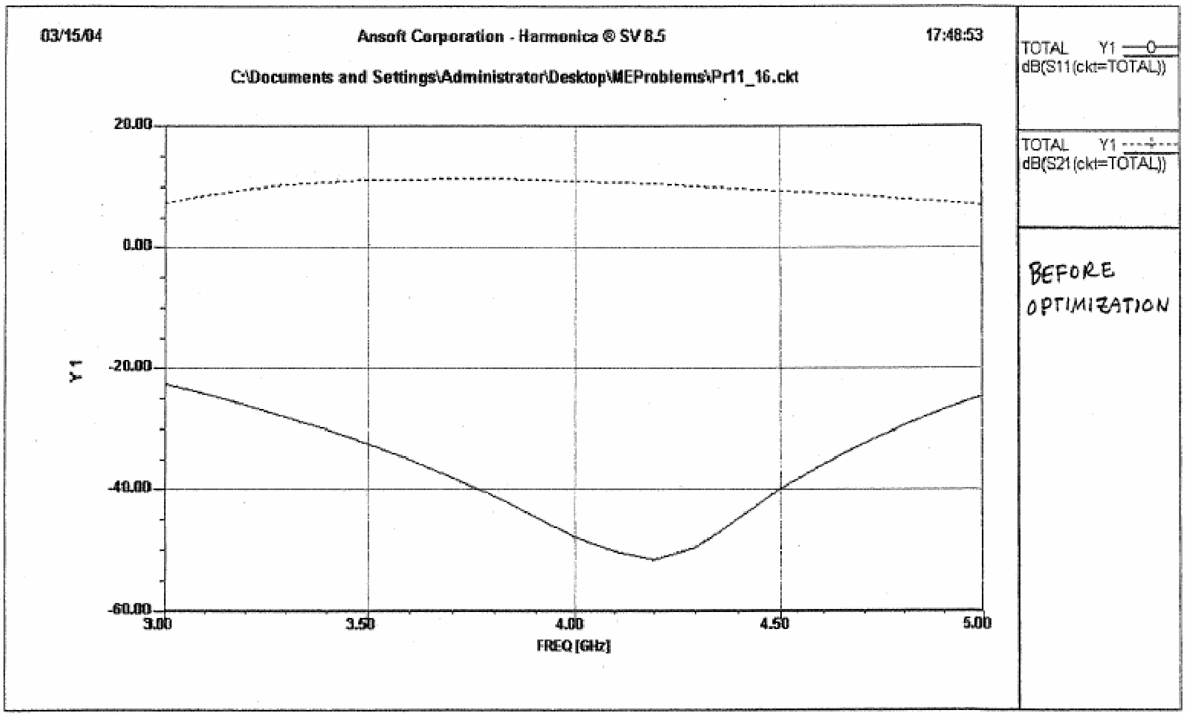
$$Z_{oe} = 50\sqrt{\frac{1+c}{1-c}} = 121\,\Omega$$

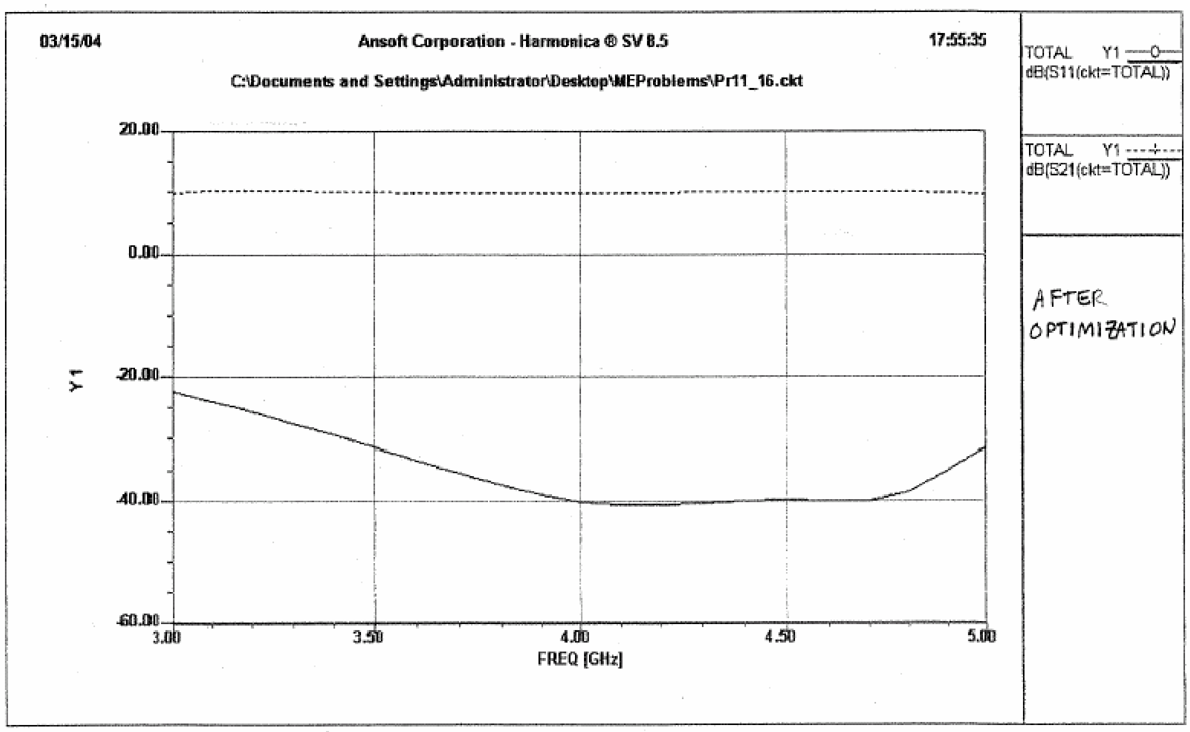
$$Z_{oo} = 50\sqrt{\frac{1-c}{1+c}} = 21\,\Omega$$



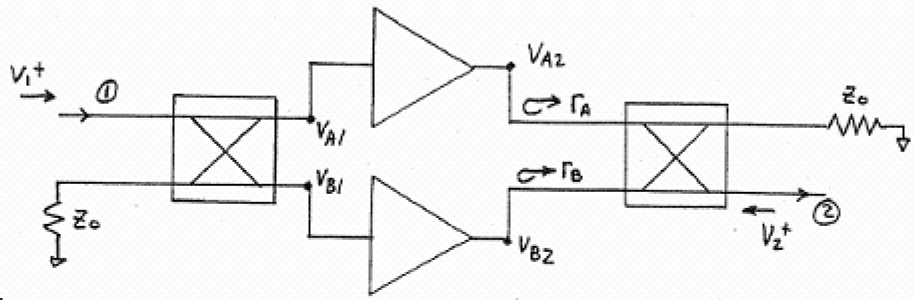
The amplifier circuit of Example 11.4 was used for both amplifiers here. As in Example 11.6, the amplifier matching networks were optimized using SuperCompact to give a flat 10dB gain response, with good input matching. Results of the optimization are given below, including the line and stub lengths before and after optimization, the SuperCompact data file, and the calculated gain and input return loss of the balanced amplifier before and after optimization. Results seem to be a bit better than those of Example 11.6.

PARAMETER	BEFORE OPT.	AFTER OPT.
INPUT SECTION STUB LENGTH	0.100λ	0.125λ
INPUT SECTION LINE LENGTH	0.179λ	0.119λ
OUTPUT SECTION LINE LENGTH	0.045λ	0.089λ
OUTPUT SECTION LINE LENGTH	0.432λ	0.458λ


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:48:53
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
BEFORE OPTIMIZATION
20.00
0.00
-20.00
-40.00
-60.00
Y1
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]


03/15/04
Ansoft Corporation - Harmonica ® SV 8.5
17:55:35
C:\Documents and Settings\Administrator\Desktop\MEProblems\Pr11_16.ckt
TOTAL Y1
dB(S11(ckt=TOTAL))
TOTAL Y1
dB(S21(ckt=TOTAL))
AFTER OPTIMIZATION
20.00
0.00
-20.00
-40.00
-60.00
Y1
3.00
3.50
4.00
4.50
5.00
FREQ [GHz]

12.19



The analysis for S_{22} is identical to that for S_{11} in eqs (11.61)–(11.65), but with input V_2^+ at port 2.

Thus, if the input at port 2 is V_2^+, then the voltages incident at the amplifiers are,

$$V_{A2}^- = \frac{1}{\sqrt{2}} V_2^+$$

$$V_{B2}^- = \frac{-j}{\sqrt{2}} V_2^+$$

Then the reflected output voltage at port 2 is,

$$V_2^- = \frac{1}{\sqrt{2}} V_{A2}^+ + \frac{-j}{\sqrt{2}} V_{B2}^+ = \frac{1}{\sqrt{2}} \Gamma_A V_{A2}^- + \frac{-j}{\sqrt{2}} \Gamma_B V_{B2}^-$$

$$= \frac{1}{2} V_2^+ (\Gamma_A - \Gamma_B)$$

Thus, $$S_{22} = \left.\frac{V_2^-}{V_2^+}\right|_{V_1^+=0} = \frac{1}{2}(\Gamma_A - \Gamma_B) \checkmark$$

12.20 From (11.77), $G = \dfrac{g_m^2 Z_d Z_g}{4} \dfrac{(e^{-N\alpha_g l_g} - e^{-N\alpha_d l_d})^2}{(e^{-\alpha_g l_g} - e^{-\alpha_d l_d})^2}$

differentiating with respect to N and setting to zero gives,

$$\alpha_g l_g e^{-N\alpha_g l_g} - \alpha_d l_d e^{-N\alpha_d l_d} = 0$$

$$\ln \alpha_g l_g - N\alpha_g l_g = \ln \alpha_d l_d - N\alpha_d l_d$$

$$\ln \frac{\alpha_g l_g}{\alpha_d l_d} = N(\alpha_g l_g - \alpha_d l_d)$$

$$N = \frac{\ln(\alpha_g l_g / \alpha_d l_d)}{\alpha_g l_g - \alpha_d l_d} \quad \checkmark$$

12.21 $R_i = 5\,\Omega$, $R_{ds} = 200\,\Omega$, $C_{gs} = 0.3\,pF$, $g_m = 40\,mS$

$f = 0-20\,GHz$, opt G at 16 GHz. $N = 4, 8, 16$.

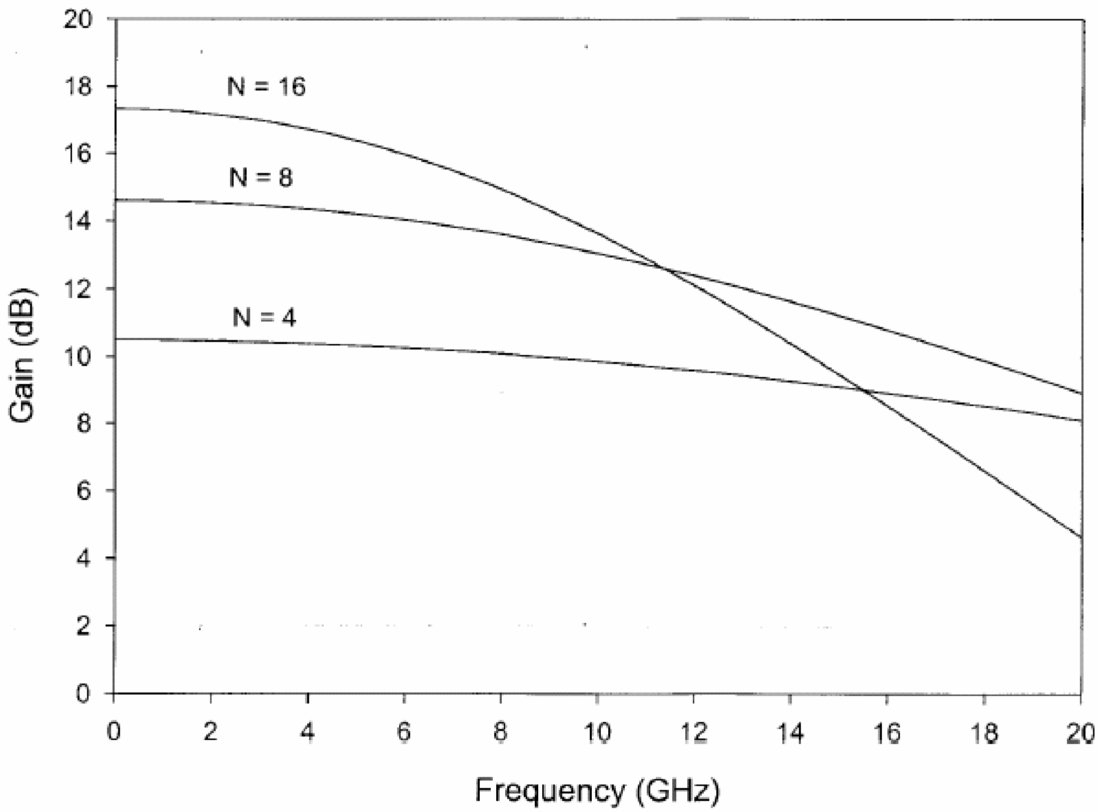
at $f = 16\,GHz$, $\alpha_g l_g = 0.1137$

$\alpha_d l_d = 0.125$

$$N_{opt} = \frac{\ln \alpha_g l_g / \alpha_d l_d}{(\alpha_g l_g - \alpha_d l_d)} = 8.4$$

gain vs. frequency is plotted below for $N = 4, 8, 16$.

graph shows $N_{opt} \sim 8$ at 16 GHz ✓



12.22 $S_{11} = 0.76\angle 169°$, $S_{12} = 3.08\angle 67°$, $S_{21} = 0.079\angle 53°$, $S_{22} = 0.36\angle -169°$

$\Gamma_{SP} = 0.797\angle -187°$, $\Gamma_{LP} = 0.491\angle 185°$, $G_P = 10\,dB$. $f = 1\,GHz$.

Check stability using small-signal S-parameters:

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.452\angle -27°$$

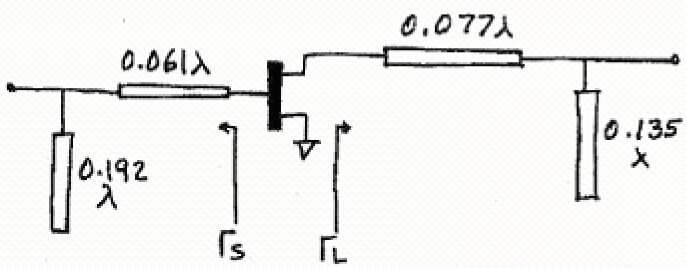
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} = 1.02$$

Since $|\Delta| < 1$ and $K > 1$, the device is unconditionally stable at this frequency.

Using the given large-signal source and load reflection coefficients gives,

$$\Gamma_S = 0.797\angle -187°, \quad \Gamma_{LP} = 0.491\angle 185°$$

Then the matching circuits can be designed, resulting in the following AC circuit:

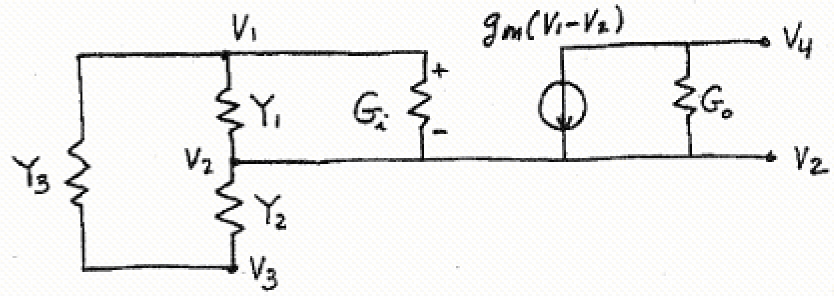


Since the gain with this Γ_S, Γ_P is 10 dB, the input power for a 1 W output is,

$$P_{in} = P_{out} - G_P = 30\,dBm - 10 = 20\,dBm = 100\,mW.$$

Chapter 13

13.1



Writing KCL for nodes V_1, V_2, V_3, V_4:

V_1: $(V_3-V_1)Y_3 + (V_2-V_1)Y_1 + (V_2-V_1)G_i = 0$

V_2: $(V_1-V_2)Y_1 + (V_3-V_2)Y_2 + (V_1-V_2)G_i + g_m(V_1-V_2) + (V_4-V_2)G_o = 0$

V_3: $(V_1-V_3)Y_3 + (V_2-V_3)Y_2 = 0$

V_4: $(V_2-V_4)G_o - g_m(V_1-V_2) = 0$

Rearranging:

$$V_1(Y_1+Y_3+G_i) + V_2(-Y_1-G_i) + V_3(-Y_3) + V_4(0) = 0$$
$$V_1(-Y_1-G_i-g_m) + V_2(Y_1+Y_2+G_i+G_o+g_m) + V_3(-Y_2) + V_4(-G_o) = 0$$
$$V_1(-Y_3) + V_2(-Y_2) + V_3(Y_2+Y_3) + V_4(0) = 0$$
$$V_1(g_m) + V_2(-G_o-g_m) + V_3(0) + V_4 G_o = 0$$

which agrees with the matrix of (12.3).

13.2 From (13.4),

$$\det\begin{bmatrix}(Y_1+Y_3+G_i) & -Y_3\\ (g_m-Y_3) & (Y_2+Y_3)\end{bmatrix}=0 \quad \text{for oscillation.}$$

For a Colpitts oscillator, let $Y_1=j\omega C_1$, $Y_2=j\omega C_2$, $Z_3=R+j\omega L_3$.

Then,

$$\det[\cdot]=\left(j\omega C_1+\frac{1}{R+j\omega L_3}+G_i\right)\left(j\omega C_2+\frac{1}{R+j\omega L_3}\right)+\left(\frac{1}{R+j\omega L_3}\right)\left(g_m-\frac{1}{R+j\omega L_3}\right)=0$$

$$\left[1+(G_i+j\omega C_1)(R+j\omega L_3)\right]\left[1+j\omega C_2(R+j\omega L_3)\right]+g_m(R+j\omega L_3)-1=0$$

$$\cancel{1}+j\omega C_2\cancel{(R+j\omega L_3)}+(G_i+j\omega C_1)\cancel{(R+j\omega L_3)}+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)^{\cancel{2}}+$$
$$+g_m\cancel{(R+j\omega L_3)}-\cancel{1}=0$$

$$j\omega C_2+G_i+j\omega C_1+j\omega C_2(G_i+j\omega C_1)(R+j\omega L_3)+g_m=0$$

Re: $\quad G_i+g_m-\omega^2 L_3 G_i C_2-\omega^2 C_1 C_2 R=0$

Im: $\quad \omega C_2+\omega C_1+\omega C_2 G_i R-\omega^3 C_1 C_2 L_3=0$

$$C_1+C_2+C_2 G_i R-\omega^2 C_1 C_2 L_3=0$$

$$\omega=\sqrt{\frac{C_1+C_2+C_2G_iR}{C_1C_2L_3}}=\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{G_iR}{C_1}\right)} \quad ✓$$

13.3

From (13.4):

$$\begin{vmatrix} (Y_1+Y_2+g_m+G_o) & -(Y_2+G_o) \\ -(G_o+g_m+Y_2) & (Y_2+Y_3+G_o) \end{vmatrix} = 0$$

$$(Y_1+Y_2+g_m+G_o)(Y_2+Y_3+G_o)-(Y_2+G_o)(G_o+g_m+Y_2) = 0$$

Simplifying gives

$$Y_1Y_2+Y_1Y_3+Y_1G_o+Y_2Y_3+g_mY_3+G_oY_3 = 0$$

$$\frac{1}{Y_3}+\frac{1}{Y_2}+\frac{G_o}{Y_2Y_3}+\frac{g_m}{Y_1Y_2}+\frac{G_o}{Y_1Y_2} = 0$$

For Colpitts, let $\frac{1}{Y_3} = Z_3 = R+j\omega L$; $\frac{1}{Y_1} = 1/j\omega C_1$; $\frac{1}{Y_2} = 1/j\omega C_2$.

$$R+j\omega L_3-\frac{j}{\omega C_2}+G_o(R+j\omega L_3)\left(\frac{1}{j\omega C_2}\right)-\frac{j}{\omega C_1}-\frac{(g_m+G_o)}{\omega^2C_1C_2} = 0$$

Re: $$R+G_oL_3/C_2-\frac{(g_m+G_o)}{\omega^2C_1C_2} = 0 \quad \checkmark$$

Im: $$\omega L_3-\frac{1}{\omega C_2}-G_oR/\omega C_2-1/\omega C_1 = 0$$

$$\omega L_3 = \frac{1}{\omega}\left(\frac{1}{C_2}+\frac{G_oR}{C_2}+\frac{1}{C_1}\right)$$

$$\omega = \sqrt{L_3\left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{G_oR}{C_2}\right)} \quad \checkmark$$

Design: $f = 200$ MHz common gate FET, $g_m = 20$ mS, $R_o = 1/G_o = 200\,\Omega$, $L_3 = 15$ nH, $Q = 50$.

Then, $$\frac{1}{\omega_o^2L_3} = \frac{C_1C_2'}{C_1+C_2'} = 42.2\text{ pF}, \quad C_2' = \frac{C_2}{1+G_oR}$$

$$R = \frac{\omega_0 L_3}{Q} = 0.38\ \Omega$$

Assume $C_1 = C_2'$, then $C_1 = 84.4\ pF = C_2'$

$$C_2 = C_2'(1 + G_0 R) = 84.4\ pF$$

$$R < \frac{g_m + G_0}{\omega^2 C_1 C_2} - \frac{G_0 L_3}{C_2} = 1.33\ \Omega$$

$$Q_{MIN} = \frac{\omega_0 L_3}{R_{MAX}} = \underline{14.}$$

13.4 Let $Z = R + jX$, $R > 0$ (POSITIVE RESISTANCE)

Then $z = Z/Z_0 = r + jx$, $r > 0$

$$\Gamma = \frac{z-1}{z+1} = \frac{(r-1) + jx}{(r+1) + jx}$$

Now let $Z = -R + jX$, $R > 0$ (NEGATIVE RESISTANCE)

$$z = -r + jx, \quad r > 0$$

Then,
$$\Gamma = \frac{z-1}{z+1} = \frac{-(r+1) + jx}{(-r+1) + jx} = \frac{(r+1) - jx}{(r-1) - jx}$$

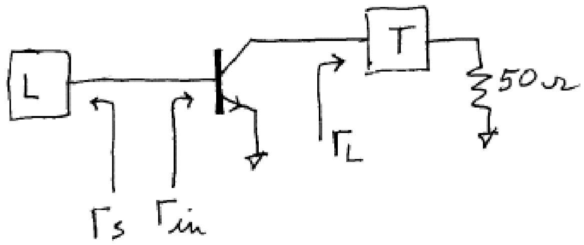
So,
$$\frac{1}{\Gamma^*} = \frac{(r-1) + jx}{(r+1) + jx}$$

which has the same form as Γ for positive resistance. So we can read the resistance circles as negative, and interpret the "reflection coefficient" read from the Smith chart as $1/\Gamma^*$.

13.5

$f = 1.9\ \text{GHz}$, Si BJT

$S_{11} = .72\angle 157°$, $S_{12} = .15\angle 56°$, $S_{21} = 1.9\angle 52°$, $S_{22} = .63\angle -63°$

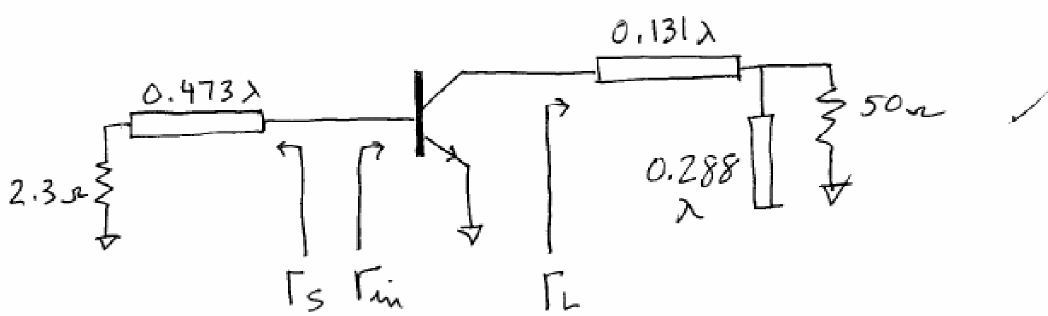


From 13.3a, $\Gamma_{in} = S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$

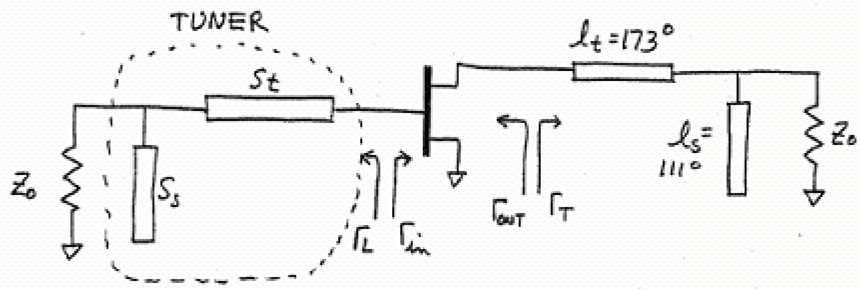
Choose Γ_L so that $|\Gamma_L| < 1$ and $|\Gamma_{in}|$ is large.
By trial and error,

Γ_L	Γ_{in}	Z_{in}
$.6\angle 60$	$.99\angle 160$	$.2 + j.9$
$.8\angle 60$	$1.17\angle 160$	$-4.2 + j8.7$
$.9\angle 50$	$1.26\angle 150$	$-6.2 + j12.9$
$.9\angle 60$	$1.31\angle 160$	$-6.8 + j8.6$

Select $\Gamma_L = 0.9\angle 60°$. Then $\Gamma_{in} = 1.31\angle 160°$, $Z_{in} = -6.8 + j8.6\,\Omega$
Let $Z_s = -R_{in}/3 - jX_{in} = 2.3 - j8.6\,\Omega$. Stub matching circuits were designed as shown below.



13.6

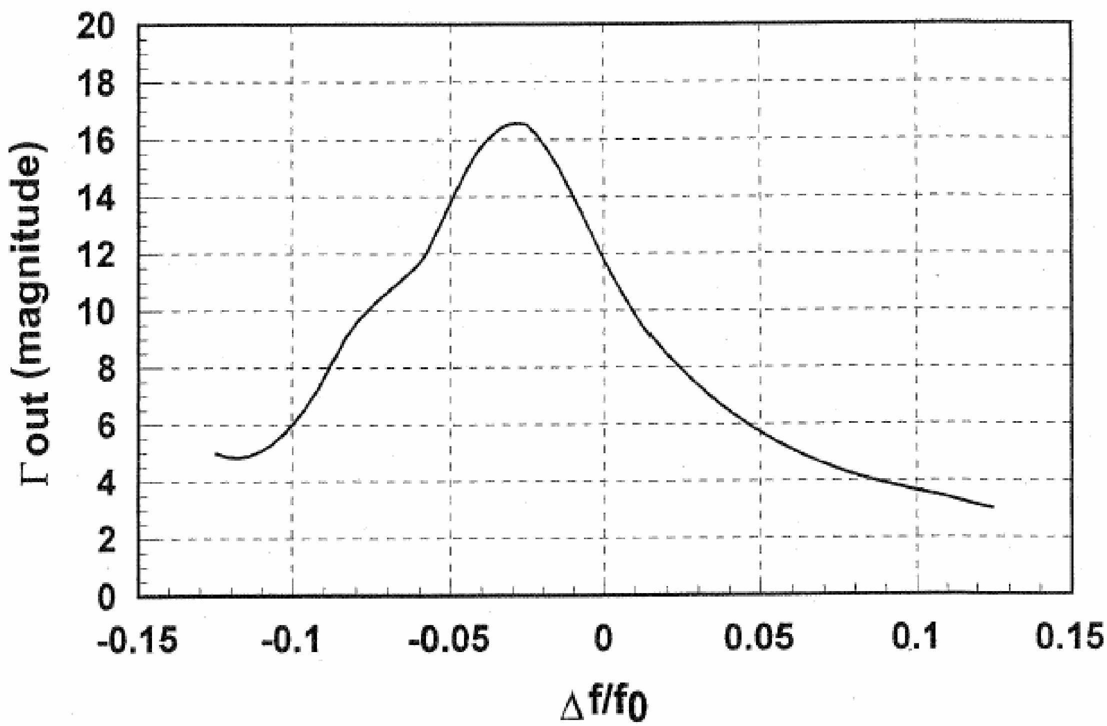


As in Example 12.4, choose $\Gamma_L = 0.6\angle -130°$. The Γ_{out}, Z_{out}, Z_T, ℓ_t, and ℓ_s are unchanged. Then we have the simple matching problem of using the stub tuner to match 50Ω to Γ_L. The stub susceptance is $jb_s = +j1.56$, or a stub length of $S_s = 0.158\lambda$. The line length is $S_t = 0.18 - 0.176 = 0.004\lambda$.

We then analyze the above circuit to compute Γ_{out} versus frequency:

f (GHz)	$\Delta f/f_0$	$\lvert\Gamma_{out}\rvert$
2.10	−0.125	5.0
2.18	−0.092	7.2
2.20	−0.083	9.1
2.26	−0.058	11.9
2.30	−0.042	15.4
2.34	−0.025	16.5
2.38	−0.008	13.6
2.40	0	11.8
2.42	0.008	10.2
2.46	0.025	7.9
2.50	0.042	6.3
2.60	0.083	4.1
2.66	0.110	3.4
2.70	0.125	3.0

The maximum of $\lvert\Gamma_{out}\rvert$ does not occur at $\Delta f = 0$ because the tuner is not resonant at f_0. The "Q" is much lower than in Example 12.4. This problem shows the advantage of using a high-Q resonator for the oscillator. $\lvert\Gamma_{out}\rvert$ vs f is plotted on the following page.



13.7

$S_{11} = 1.2\angle 150°$, $S_{12} = 0.2\angle 120°$, $S_{21} = 3.7\angle -72°$, $S_{22} = 1.3\angle -67°$

as in Example 12.4, maximize $|\Gamma_{out}|$ by choosing $S_{11}\Gamma_L \simeq 1$, since

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{11}\Gamma_L}$$

Thus let $\Gamma_L = 0.8\angle -150°$ ✓. Then $\Gamma_{out} = 15.88\angle -99.3°$, and

$$Z_{out} = Z_0 \frac{1+\Gamma_{out}}{1-\Gamma_{out}} = -7.6 + j1.9\,\Omega$$

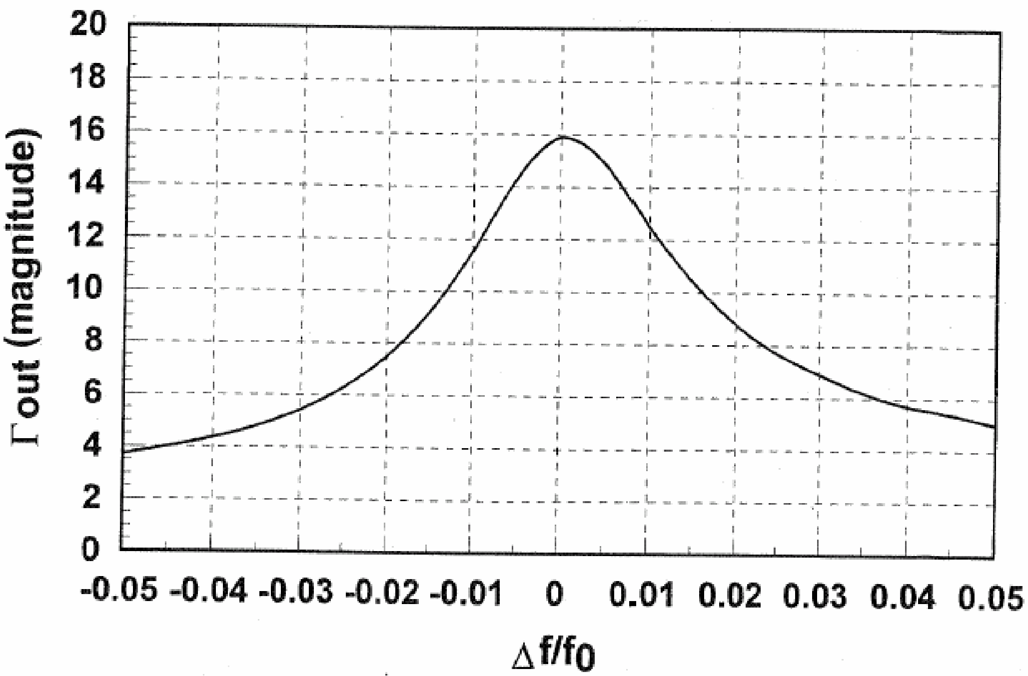
$$Z_T = \frac{-R_{out}}{3} - jX_{out} = 2.53 - j1.9\,\Omega \quad (z_T = 0.0506 - j0.038)$$

matching Z_T to the load impedance gives $l_t = 0.031\lambda$ with a required stub susceptance of $+j4$. Thus $l_s = 0.21\lambda$.

at the dielectric resonator, $\Gamma_L' = \Gamma_L e^{2j\beta l_r} = (0.8\angle -150°)e^{2j\beta l_r} = 0.8\angle 180°$. Thus $l_r = 0.4583\lambda$.

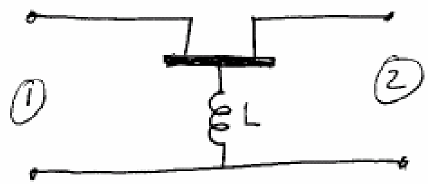
$$Z_L' = Z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'} = 5.55\,\Omega = N^2 R$$

$|\Gamma_{out}|$ vs f was calculated with SuperCompact, and is plotted below:



13.8

common gate: $S_{11} = .46\angle 178$, $S_{12} = .045\angle 73$,
$S_{21} = 1.41\angle -19$, $S_{22} = 1.02\angle -12$



Find L to minimize μ,
using CAD.

L (nH)	μ
0	.922
1	.088
2	−.928
3	−.927
5	−.89
1.5	−.88
2.5	−.931

13.9 From (12.49),

$$S_\phi = \frac{kT_0F}{P_0}\left(\frac{K\omega_\alpha \omega_h^2}{\Delta\omega^3} + \frac{\omega_h^2}{\Delta\omega^2} + \frac{K\omega_\alpha}{\Delta\omega} + 1\right)$$

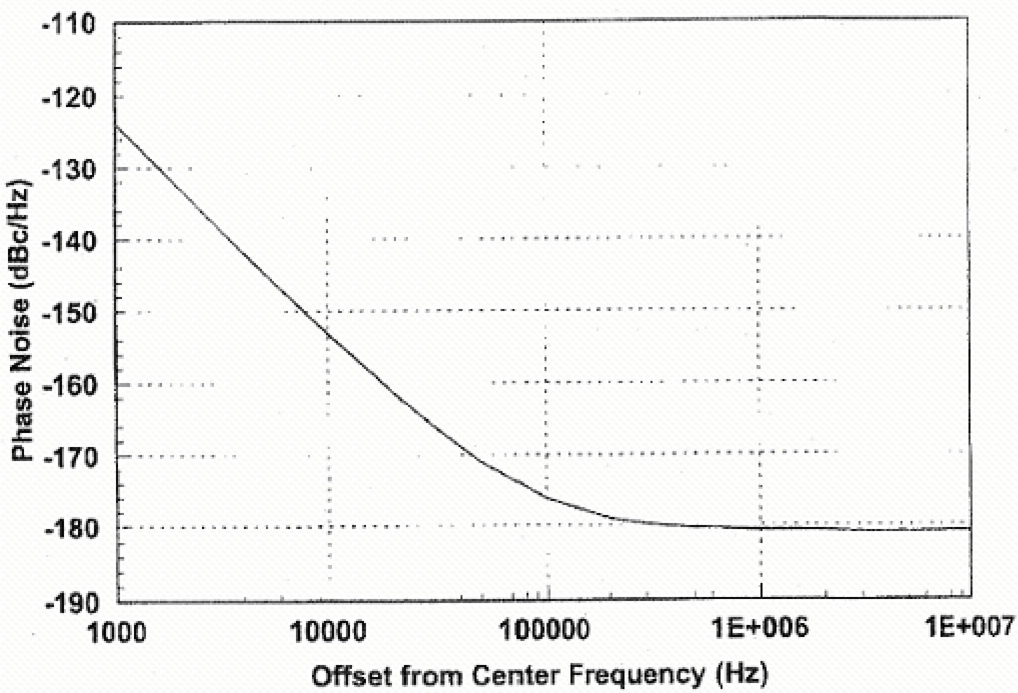
$$\mathcal{L}(f) = S_\phi/2.$$

For $F = 6\text{dB} = 4$, $f_0 = 100\text{MHz}$, $Q = 500$, $P_0 = 10\text{dBm} = 10\text{mW}$, $K=1$,
$f_\alpha = 50\text{kHz}$, $f_h = f_0/2Q = 100\text{kHz}$, $\Delta f = f - f_0$

a short computer program was written to compute data for the plot shown below.

(a) $\Delta f = 1\text{MHz}$, $S_\phi = -178\text{dBm}$, $\mathcal{L}(1\text{MHz}) = -181\text{dBc/Hz}$

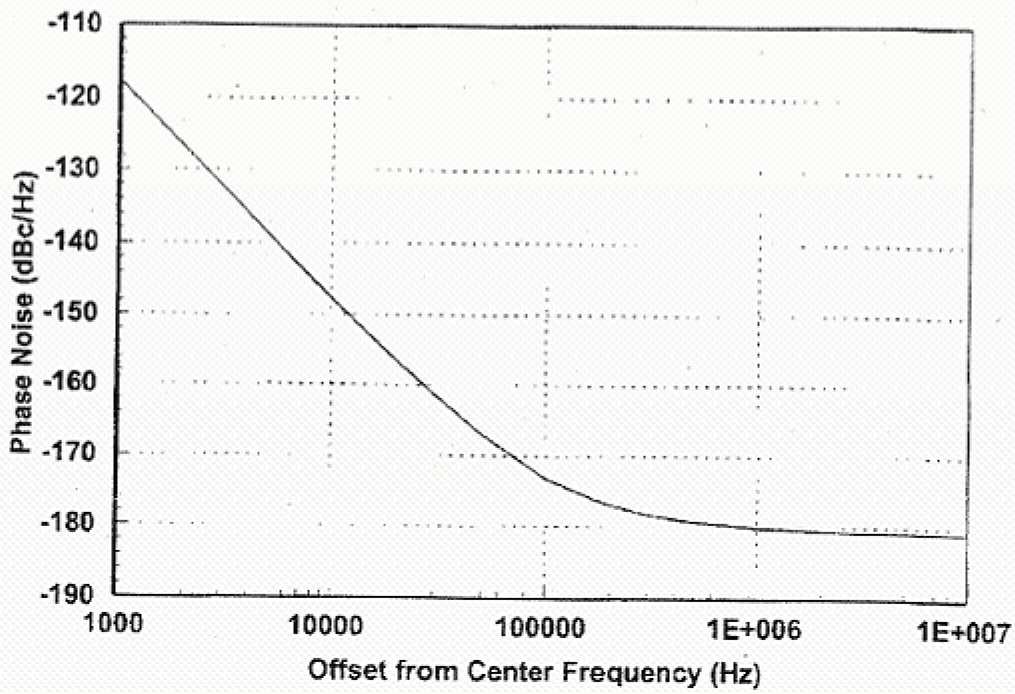
(b) $\Delta f = 10\text{kHz}$, $S_\phi = -150\text{dBm}$, $\mathcal{L}(10\text{kHz}) = -153\text{dBc/Hz}$



13.10 This calculation is similar to that of Problem 13.9, but with $f_\alpha = 200\,\text{kHz}$. Plot shown below.

(a) $\Delta f = 1\,\text{MHz}$, $S_\phi = -177\,\text{dBm}$, $\mathcal{L}(1\,\text{MHz}) = -180\,\text{dBc/Hz}$

(b) $\Delta f = 10\,\text{kHz}$, $S_\phi = -144\,\text{dBm}$, $\mathcal{L}(10\,\text{kHz}) = -147\,\text{dBc/Hz}$.



13.11

If C is the desired signal level, I is the undesired signal level, S is the desired rejection ratio, $\mathcal{L}(f)$ the phase noise, and B the filter bandwidth, then

$$S = \frac{C}{I B \mathcal{L}(f)}$$

In dB,

$$\mathcal{L}(f) = C(dBm) - I(dBm) - S(dB) - 10\log(B). \checkmark$$

13.12 $B = 12$ kHz, $S = 80$ dB, $C = I$

From (13.50),

$$\mathcal{L}(30\text{kHz}) = C(dBm) - I(dBm) - S(dB) - 10\log(B)$$
$$= -80dB - 10\log(12\times10^3)$$
$$= -121 \text{ dBc/Hz}.$$

13.13 Assume excitation at f_1, f_2; o.c. at all other frequencies except $f_3 = f_1 + f_2$. Then all power terms are zero except for $n = \pm1, m=0$; $n=0, m=\pm1$; and $n=m=\pm1$. So the Manley-Rowe relations give,

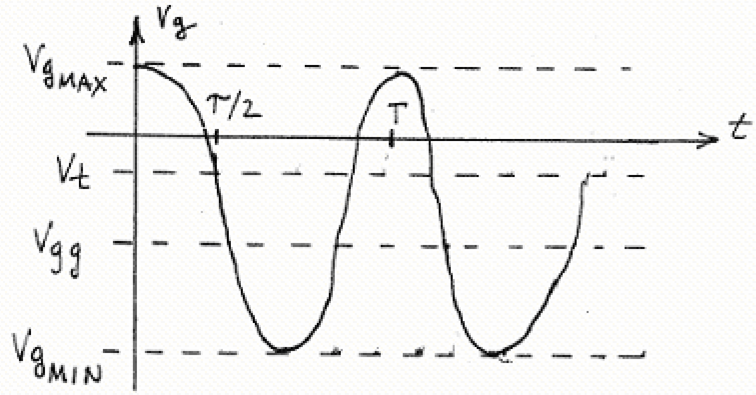
$$\frac{P_{10}}{\omega_1} + \frac{P_{11}}{\omega_1+\omega_2} = 0$$

$$\frac{P_{01}}{\omega_2} + \frac{P_{11}}{\omega_1+\omega_2} = 0$$

For sources at f_1, f_2, we have $P_{10} > 0$ and $P_{01} > 0$. Then $P_{11} < 0$, representing power at $f_3 = f_1 + f_2$ ($m=n=1$). Conversion gain is then,

$$G_C = \frac{-P_{11}}{P_{10}} = \frac{\omega_1+\omega_2}{\omega_1} = 1 + \omega_2/\omega_1. \checkmark$$

13.14



$$V_g = V_{gg} + V_g \cos\frac{2\pi t}{T}$$

$$V_g = (V_{g_{MAX}} - V_{g_{MIN}})/2 \qquad \text{(PEAK)}$$

$$V_{gg} = (V_{g_{MAX}} + V_{g_{MIN}})/2 \qquad \text{(AVG)}$$

$$V_t = V_{gg} + V_g \cos\frac{\pi\tau}{T} \qquad \text{solve for } \cos\frac{\pi\tau}{T}:$$

$$\cos\frac{\pi\tau}{T} = \frac{V_t - V_{gg}}{V_g} = \frac{2V_t - V_{g_{MAX}} - V_{g_{MIN}}}{V_{g_{MAX}} - V_{g_{MIN}}} \quad \checkmark$$

13.15

$$v_{RF}(t) = V_{RF}\left[\cos(\omega_{LO} - \omega_{IF})t + \cos(\omega_{LO} + \omega_{IF})t\right]$$

$$v_{LO}(t) = V_{LO}\cos\omega_{LO}t$$

after mixing and LPF:

$$v_{OUT}(t) = \frac{K V_{RF} V_{LO}}{2}\left[\cos\omega_{IF}t + \cos\omega_{IF}t\right] = K V_{RF} V_{LO}\cos\omega_{IF}t$$

(both sidebands convert to same IF)

13.16 $i(t) = e^{3v(t)} - 1$, $v(t) = 0.1\cos\omega_1 t + 0.1\cos\omega_2 t$

$$i(t) = i\Big|_{v=0} + \frac{di}{dv}\Big|_{v=0} v + \frac{d^2 i}{dv^2}\Big|_{v=0}\frac{v^2}{2} + \frac{d^3 i}{dv^3}\Big|_{v=0}\frac{v^3}{6} + \cdots$$

$$i\Big|_{v=0} = 0 \;;\; \frac{di}{dv}\Big|_{v=0} = 3 \;;\; \frac{d^2 i}{dv^2}\Big|_{v=0} = 9 \;;\; \frac{d^3 i}{dv^3}\Big|_{v=0} = 27.$$

So,

$$i(t) = 3v + 4.5v^2 + 4.5v^3 + \cdots$$

$$v^2 = .01\left[\cos^2\omega_1 t + 2\cos\omega_1 t\cos\omega_2 t + \cos^2\omega_2 t\right]$$

$$= .01\left[1 + \tfrac{1}{2}\cos^2\omega_1 t + \tfrac{1}{2}\cos 2\omega_2 t + \cos(\omega_1+\omega_2)t + \cos(\omega_1-\omega_2)t\right]$$

$$v^3 = .001\left[\cos^3\omega_1 t + 3\cos^2\omega_1 t\cos\omega_2 t + 3\cos\omega_1 t\cos^2\omega_2 t + \cos^3\omega_2 t\right]$$

$$= .001\Big[\tfrac{1}{4}\cos 3\omega_1 t + \tfrac{3}{4}\cos\omega_1 t + \tfrac{3}{2}\cos\omega_2 t + \tfrac{3}{4}\cos(2\omega_1-\omega_2)t$$
$$+ \tfrac{3}{4}\cos(2\omega_1+\omega_2)t + \tfrac{3}{2}\cos\omega_1 t + \tfrac{3}{4}\cos(\omega_1-2\omega_2)t$$
$$+ \tfrac{3}{4}\cos(\omega_1+2\omega_2)t + \tfrac{3}{4}\cos\omega_2 t + \tfrac{1}{4}\cos 3\omega_2 t\Big]$$

ω	I	
0	$(4.5)(.01)$	$=0.045$ ✓
ω_1, ω_2	$(3)(.1)+(4.5)(.001)\left(\frac{3}{2}+\frac{3}{4}\right)$	$=0.3101$ ✓
$2\omega_1, 2\omega_2$	$(4.5)(.01)\left(\frac{1}{2}\right)$	$=0.0225$ ✓
$3\omega_1, 3\omega_2$	$(4.5)(.001)(1/4)$	$=0.00113$ ✓
$\omega_1+\omega_2$	$(4.5)(.01)(1)$	$=0.045$ ✓
$\omega_1-\omega_2$	$(4.5)(.01)(1)$	$=0.045$ ✓
$2\omega_1-\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$2\omega_1+\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$\omega_1-2\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓
$\omega_1+2\omega_2$	$(4.5)(.001)(3/4)$	$=0.003375$ ✓

13.17 $f_{RF} = 1800$ MHz , $f_{IF} = 87$ MHz

possible LO frequencies are

$$f_{LO} = f_{RF} \pm f_{IF} = 1887 \text{ MHz}, \ 1713 \text{ MHz}.$$

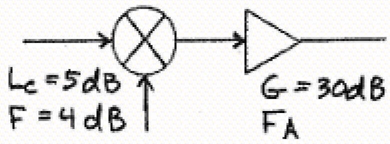
image frequency for $f_{LO} = 1887$ MHz is

$$f_{IM} = f_{LO} + f_{IF} = 1974 \text{ MHz}$$

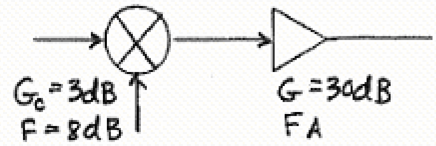
image frequency for $f_{LO} = 1713$ MHz is

$$f_{IM} = f_{LO} - f_{IF} = 1626 \text{ MHz}$$

13.18



CASE #1



CASE #2

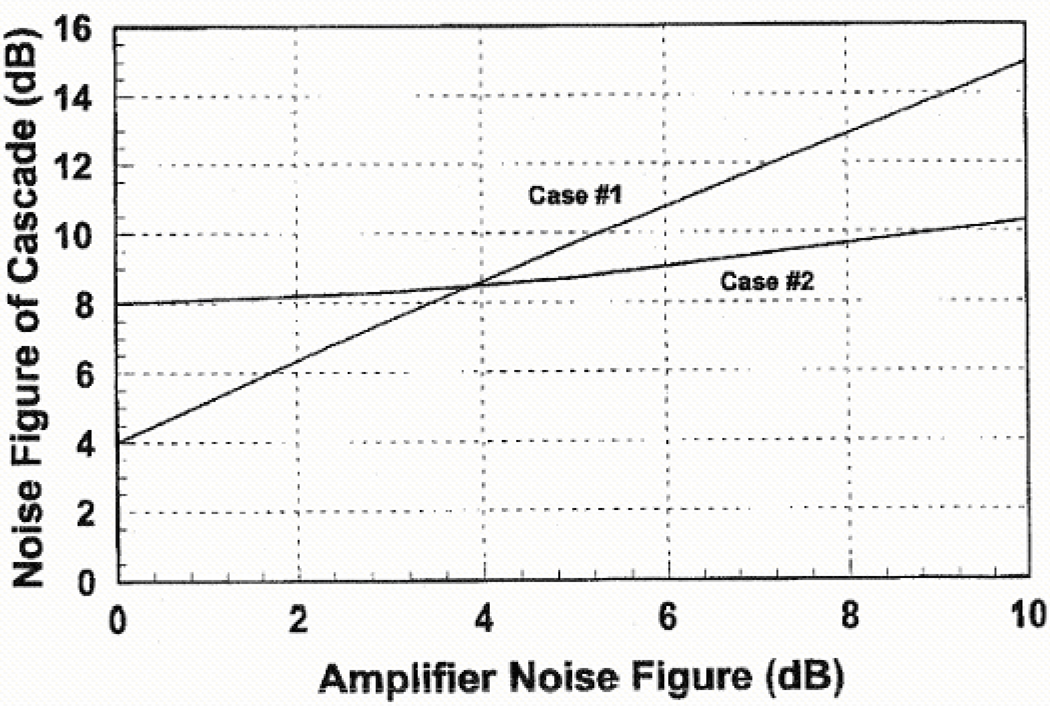
$$F_C = 2.51 + \frac{F_A - 1}{1/3.16}$$

$$F_C = 6.31 + \frac{F_A - 1}{2.0}$$

F_A (dB)	F_C (#1) dB	F_C (#2) dB
0	4.0	8.0
3	7.5	8.3
5	9.7	8.7
10	14.9	10.3

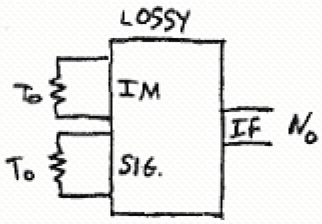
3dB = 2.0
4dB = 2.51
5dB = 3.16
8dB = 6.31

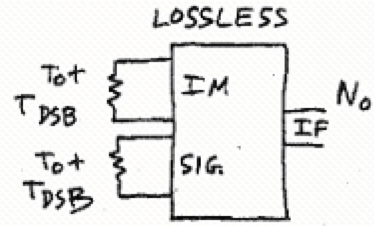
RESULTS ARE PLOTTED BELOW:



13.19

DSB





$N_0 = N_{ADDED} + \frac{kT_0B}{L} + \frac{kT_0B}{L}$

(B in SSB)

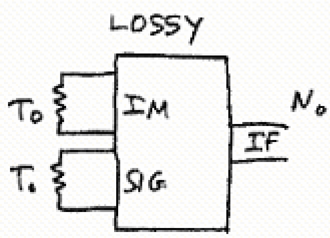
$N_0 = \frac{2kB}{L}(T_0 + T_{DSB})$

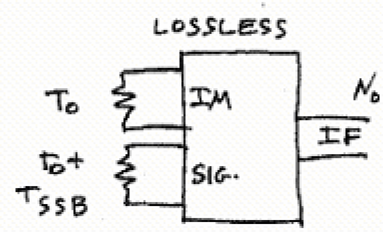
$$\therefore \quad N_{ADDED} = \frac{2kB}{L} T_{DSB}$$

$$F_{DSB} = \frac{S_i N_0}{S_0 N_i} = \frac{N_0 L}{2kT_0 B} = 1 + \frac{T_{DSB}}{T_0}$$

(INPUT NOISE – $N_i = 2kT_0B$)

SSB





$N_0 = N_{ADDED} + \frac{2kT_0B}{L}$

$N_0 = \frac{2kBT_0}{L} + \frac{kBT_{SSB}}{L}$

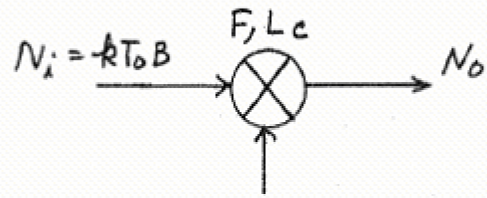
$$\therefore \quad T_{SSB} = \frac{L\, N_{ADDED}}{kB}$$

$$\underline{T_{SSB} = 2T_{DSB}} \quad \checkmark$$

$$F_{SSB} = \frac{S_i N_0}{S_0 N_i} = \frac{N_0 L}{kT_0 B} = 2 + \frac{T_{SSB}}{T_0} = 2\left(1 + \frac{T_{DSB}}{T_0}\right) = 2F_{DSB} \quad \checkmark$$

(INPUT NOISE – $N_i = kT_0B$)

13.20



$$N_o = \underbrace{\frac{kT_0B}{L_c}}_{\text{INPUT NOISE}} + \underbrace{\frac{kT_0B(F-1)}{L_c}}_{\text{MIXER NOISE}} = \frac{kT_0BF}{L_c} \checkmark$$

13.21 $v_1 = V_0 \cos\omega t \quad ; \quad v_2 = V_0 \cos(\omega t + \theta)$

as in (12.112)-(12.113), the diode currents in a mixer using a quadrature hybrid will be,

$$i_1 = kV_0^2[\cos(\omega t - \pi/2) + \cos(\omega t + \theta - \pi)]^2$$
$$= kV_0^2[\sin\omega t - \cos(\omega t + \theta)]^2$$
$$i_2 = -kV_0^2[\cos(\omega t - \pi) + \cos(\omega t + \theta - \pi/2)]^2$$
$$= -kV_0^2[-\cos\omega t + \sin(\omega t + \theta)]^2$$

Low-pass filtering leaves the following DC components:

$$i_1 = kV_0^2(1 + \tfrac{1}{2}\sin\theta)$$
$$i_2 = -kV_0^2(1 - \tfrac{1}{2}\sin\theta)$$

So the output is $i_1 + i_2 = kV_0^2 \sin\theta$ ✓

If a mixer with a 180° hybrid is used, the diode currents become,

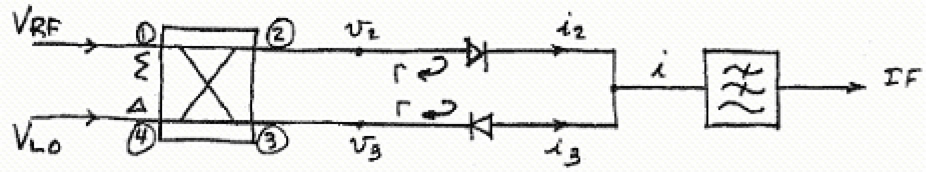
$$i_1 = kV_0^2[\cos\omega t + \cos(\omega t + \theta)]^2$$
$$i_2 = -kV_0^2[\cos\omega t - \cos(\omega t + \theta)]^2$$

Then low-pass filtering leaves the following DC components:

$$i_1 = kV_0^2(1 + \tfrac{1}{2}\cos\theta)$$
$$i_2 = -kV_0^2(1 - \tfrac{1}{2}\cos\theta)$$

So the output is $i_1 + i_2 = kV_0^2 \cos\theta$ ✓

13.22



$$[S] = \frac{-j}{\sqrt{2}}\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

let $v_{RF}(t) = V_{RF} \cos \omega_{RF} t = v_1(t)$

$v_{LO}(t) = V_{LO} \cos \omega_{LO} t = v_4(t)$

Then the diode voltages are,

$$v_2(t) = \frac{1}{\sqrt{2}} V_{RF} \cos(\omega_{RF} t - 90°) + \frac{1}{\sqrt{2}} V_{LO} \cos(\omega_{LO} t + 90°)$$

$$v_3(t) = \frac{1}{\sqrt{2}} V_{RF} \cos(\omega_{RF} t - 90°) + \frac{1}{\sqrt{2}} V_{LO} \cos(\omega_{LO} t - 90°)$$

Assume $i_2 = k v_2^2$, $i_3 = -k v_2^2$. $\omega_{IF} = \omega_{RF} - \omega_{LO}$.

Then, after LP filtering, the diode currents are,

$$i_2 = \frac{k}{4} V_{RF} V_{LO} \cos(\omega_{RF} t - 90° - \omega_{LO} t - 90°) = \frac{k}{4} V_{RF} V_{LO} \cos \omega_{IF} t.$$

$$i_3 = \frac{-k}{4} V_{RF} V_{LO} \cos(\omega_{RF} t - 90° - \omega_{LO} t + 90°) = \frac{-k}{4} V_{RF} V_{LO} \cos \omega_{IF} t.$$

So the IF output current is $i(t) = -\frac{k}{2} V_{RF} V_{LO} \cos \omega_{IF} t$ ✓

AT RF INPUT:

$V_2^+ = \Gamma V_2^- = \frac{-j}{\sqrt{2}} \Gamma V_{RF}^+$; $V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}} \Gamma V_{RF}^+$

$V_{RF}^{\Sigma} = V_1^- = V_2^+ (-j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{-\Gamma V_{RF}^+}$ ✓

$V_{RF}^{\Delta} = V_4^- = V_2^+ (j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{0}$ ✓

AT LO INPUT:

$V_2^+ = \Gamma V_2^- = \frac{j}{\sqrt{2}} \Gamma V_{LO}^+$; $V_3^+ = \Gamma V_3^- = \frac{-j}{\sqrt{2}} \Gamma V_{LO}^+$.

$V_{LO}^{\Sigma} = V_1^- = V_2^+ (-j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{0}$ ✓

$V_{LO}^{\Delta} = V_4^- = V_2^+ (j/\sqrt{2}) + V_3^+ (-j/\sqrt{2}) = \underline{-\Gamma V_{LO}^+}$ ✓

Assume now that

$$v_{LO}(t) = V_{LO}^{(2)} \cos 2\omega_{LO} t.$$

Then after LPF,

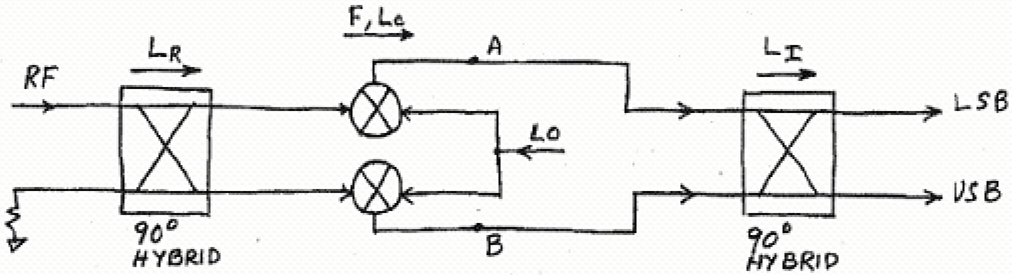
$$v_2^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t + 2\omega_{Lo} t + 90^\circ) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{Lo} t + 90^\circ)$$

$$v_3^2(t) = \frac{1}{\sqrt{2}} V_{RF} V_{Lo}^{(2)} \cos(\omega_{RF} t + 2\omega_{Lo} t + 90^\circ) + \frac{1}{\sqrt{2}} V_{RF} V_{LO}^{(2)} \cos(\omega_{RF} t - 2\omega_{Lo} t + 90^\circ)$$

Then forming

$$i(t) = k(v_2^2 - v_3^2)\Big|_{LPF} = 0 \text{ for } \omega_{RF} \pm 2\omega_{LO} \text{ frequencies.}$$

13.23



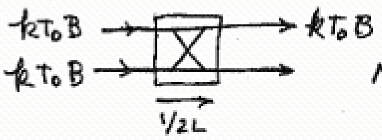
The noise power due to the RF hybrid and the mixer, ref. to IF output of mixer, is

$$N_A = N_B = \frac{kB}{L_c}\left[T_0 + (F-1)T_0\right] = \frac{kBT_0F}{L_c},$$

since the noise power output of the matched hybrid is kT_0B. The total noise power output is (at either LSB or USB),

$$N_o = \frac{N_A}{2L_I} + \frac{N_B}{2L_I} + \frac{N_{added}}{2L_I} = \frac{kBT_0F}{L_IL_c} + \frac{N_{added}}{2L_I}$$

Nadded is the output noise power of the IF hybrid when not terminated at second input port:



$$N_o = \frac{kT_0B}{2L} + \frac{kT_0B}{2L} + \frac{N_{added}}{2L} = kT_0B$$

Thus $N_{added} = 2kT_0B(L-1)$

So, $N_o = \dfrac{kBT_0F}{L_IL_c} + kT_0B\left(1 - \dfrac{1}{L_I}\right)$; $S_o = \dfrac{4S_i}{L_c}\dfrac{1}{4L_IL_R} = \dfrac{S_i}{L_cL_IL_R}$

$N_i = kT_0B$

and then,

$$F_{TOT} = \frac{S_iN_o}{S_oN_i} = \frac{L_cL_IL_R}{kT_0B}\left[\frac{kBFT_0}{L_IL_c} + kT_0B\left(1-\frac{1}{L_I}\right)\right] = \underline{FL_R + L_cL_RL_I - L_cL_R} \checkmark$$

CHECK: if $L_R = L_I = 1$, $F_{TOT} = F + 2L_c - 2L_c = F$ ✓ (mixer noise only)

CHECK: if $F = L_c$ (passive mixer loss only), $F_{TOT} = L_cL_IL_R$ ✓

(The cascade noise figure formula can be used to obtain the same result if we set $F_R = L_R$, $F_I = L_I$.)

Chapter 14

14.1

Data on satellite fading at L-band in various environments can be found in "Handbook of Propagation Effects for Vehicular and Personal Mobile Satellite Systems", by J. Goldhirsh and W. Vogel, and in "Satellite Systems for Personal and Broadband Communications", by E. Lutz, M. Werner, and A. Jahn, as well as from various other sources. Typically, one can expect fading levels of 15 to 20 dB for domestic and commercial buildings, for 95% link availability. For vehicles, the fading levels can be 20 dB or more. On the other hand, a line-of-sight system (as when the handset is used outdoors with little or no blockage to the satellite) would require a link margin of 0 dB in principal, although a few dB of margin would provide a more robust system. In view of this data, it is not clear why the Iridium system was designed with a link margin of 16 dB.

14.2

$$F_\theta(\theta,\phi) = A \sin^2\theta \cos\phi$$

Main beam at $\theta = 90°$, $\phi = 0°$ or $180°$

3 dB points at $\theta = 90°$ (az) plane:

$$\cos\phi = 0.707 \Rightarrow \phi = 45° \text{ or } 135°$$

$$HPBW_\phi = 135 - 45 = 90° \quad \checkmark$$

3 dB points in $\phi = 0°$ (el) plane:

$$\sin^2\theta = 0.707 \Rightarrow \theta = \sin^{-1}\sqrt{.707} = 57.2°, \ 122.8°$$

$$HPBW_\theta = 122.8 - 57.2 = 65.6°$$

$$\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F^2(\theta,\phi)\sin\theta\, d\theta\, d\phi = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sin^5\theta \cos^2\phi\, d\theta\, d\phi$$

$$= \left(\frac{16}{15}\right)\pi$$

$$D = \frac{4\pi F^2_{MAX}}{\iint} = \frac{15}{4} = 3.75 = \underline{5.74\ dB}$$

linearly polarized in vertical direction.

14.3 $F_\theta(\theta,\phi) = \begin{cases} A\sin\theta & \text{for } 0 \le \theta \le \pi/2 \\ 0 & \text{otherwise} \end{cases}$

$$D = \frac{4\pi F_{MAX}^2}{\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} F_\theta^2(\theta,\phi)\sin\theta\, d\theta\, d\phi} = \frac{4\pi}{2\pi\int_{\theta=0}^{\pi/2}\sin^3\theta\, d\theta} = \frac{4\pi}{2\pi\left(\frac{2}{3}\right)}$$

$$= 3 = \underline{4.8\,dB} \qquad \text{(verified with PCAAD)}$$

14.4 $f = 12.4$ GHz, Diam $= 18'' = 0.457$ m, $\eta_{ap} = 65\%$

$\lambda = \frac{c}{f} = 0.0242$ m ✓

$A = \pi R^2 = \pi\left(\frac{Diam}{2}\right)^2 = 0.164\, m^2$

From (14.13)

$$D = \eta_{ap}\frac{4\pi A}{\lambda^2} = (0.65)\frac{4\pi(0.164)}{(0.0242)^2} = 2287 = \underline{33.6\,dB} \checkmark$$

14.5 $f = 38\,GHz$, $G = 39.0\,dB$, $Diam = 12.0''$, $\eta_{rad} = 90\%$

a) From (14.11), $D = \dfrac{G}{\eta_{rad}} = \dfrac{10^{39/10}}{0.9} = 8{,}826.$

From (14.13), $D = \dfrac{4\pi A}{\lambda^2}\eta_{ap}$

$$\eta_{ap} = \frac{\lambda^2 D}{4\pi A} = \left(\frac{c}{\pi f\, Diam}\right)^2 D = \underline{\underline{60\%}}$$

b) From (14.9), $D = \dfrac{32{,}400}{\theta_3^2}$

$$\theta_3 = \sqrt{\frac{32{,}400}{8826}} = \underline{\underline{1.9^\circ}}$$

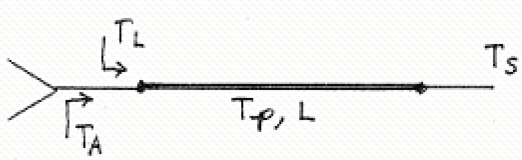
14.6 From (14.18), $T_A = \eta_{rad} T_b + (1-\eta_{rad}) T_p$

$$= (T_b - T_p)\eta_{rad} + T_p$$

Thus,

$$\eta_{rad} = \frac{T_A - T_p}{T_b - T_p} = \frac{105-290}{5-290} = \underline{65\%}$$

14.7



$T_A = \eta_{rad} T_b + (1-\eta_{rad}) T_p$ (at antenna) (14.18)

$T_L = (L-1) T_p$ (at antenna) (10.15)

$T_S = \frac{1}{L}(T_A + T_L)$ (at output)

$= \frac{1}{L}[\eta_{rad} T_b + (1-\eta_{rad}) T_p] + \frac{(L-1)}{L} T_p$ (14.20) ✓

14.8

$$T = \eta_{rad} T_b + (1-\eta_{rad}) T_p + T_R$$

$T_b = 50K$
$T_p = 290K$
$F = 1.1dB = 1.29$
$L = 2.5dB = 1.78$
$G = 33.5dB = 2240$

The noise temperature of the receiver is,

$$T_R = (F-1)T_o = (1.29-1)(290) = 84K$$

The efficiency of the array is,

$$\eta_{rad} = \frac{1}{L} = \frac{1}{1.78} = 0.56$$

Thus,

$$T = (0.56)(50) + (1-0.56)(290) + 84 = \underline{240K}$$

Then,

$$\frac{G}{T}(dB) = 10 \log \frac{2240}{240} = 9.7 \underline{dB/K}$$

This value is well below the desired minimum of 12 dB/K.

14.9 Solving (14.23) for G gives,

$$G = \frac{4\pi S R^2}{P_t} = \frac{4\pi(7.5\times10^{-3})(300)^2}{85} = 100 = \underline{20 dB}$$

14.10

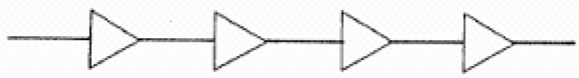
1) RADIO LINK: $f = 28\,GHz \Longrightarrow \lambda = 0.0107\,m$; $G_t = G_r = 25\,dB = 316$.

Let $P_r = 1\,W$. Then,

$$P_t = \frac{(4\pi R)^2}{P_r G_t G_r \lambda^2} = \frac{(4\pi)^2(5000)^2}{(1)(316)^2(0.0107)^2} = 3.45\times10^8\,W$$

$$\text{ATTENUATION} = 10\log\frac{P_r}{P_t} = 10\log\frac{1}{3.45\times10^8} = \underline{-85.4\,dB}\ ✓$$

2) WIRED LINK: $\alpha = 0.05\,dB/m = 0.0057$ neper/m; $4\times30\,dB$ REPEATERS.



$$\begin{aligned}\text{ATTENUATION OF LINE} &= 10\log e^{-2\alpha R}\\ &= 10\log e^{-2(0.0057)(5000)}\\ &= -250\,dB\end{aligned}$$

$$\text{TOTAL LOSS} = -250 + 4(30) = \underline{-130\,dB}\ ✓$$

The radio link has much less link loss than the wired link, and will thus require less transmit power.

14.11 GSM downlink, 935-960 MHz, EIRP = 20 W, $G_r = 0$ dBi, $T = 450$ K, SNR = 10 dB, LM = 30 dB, $B = 200$ kHz, $F_R = 8$ dB.

$f = 947.5$ MHz $\Rightarrow \lambda = 0.317$ m, $G_r = 0$ dBi $= 1$

$F_R = 8$ dB $= 6.31$, SNR = 10 dB = 10.

$$T_{sys} = T_A + T_R = T_A + (F_R - 1)T_0 = 450 + (6.31-1)(290)$$
$$= 1990\text{ K}$$
$$N_o = kT_{sys}B = (1.38\times10^{-23})(1990)(200\times10^3)$$
$$= 5.5\times10^{-15}\text{ W} = -112.6\text{ dBm (at rec. input)}$$

$$S_o(\text{dBm}) = \left(\frac{S_o}{N_o}\right) + N_o + LM = 10 - 112.6 + 30 = -72.6\text{ dBm}$$
$$= 5.5\times10^{-11}\text{ W}$$

$$R = \sqrt{\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 S_o}} = \sqrt{\frac{(20)(1)(.317)^2}{(4\pi)^2(5.5\times10^{-11})}} = \underline{15.2\text{ km}}$$

(stated base station range for GSM 900 is 2-20 km)

14.12 Carrier power at receiver:

$$C = S_i G_A G/L \qquad (S_i \text{ ref. to antenna w/ 0 dBi})$$

at input to amplifier:

$$T_e = T_A + (F-1)T_o + (L-1)T_o/G$$

The noise power at input to receiver:

$$N = kT_e G/L = \frac{C}{(C/N)}$$

$L = 25\,dB = 316.2$
$S_i = 1\times10^{-16}$
$G_A = 5\,dB = 3.16$
$\frac{C}{N} = 32\,dB = 1580.$
$G = 10\,dB = 10$

So,

$$T_e = \frac{CL}{k\left(\frac{C}{N}\right)G} = \frac{S_i G_A}{k\left(\frac{C}{N}\right)}$$

$$F = 1 + \frac{T_e}{T_o} - \frac{T_A}{T_o} - \frac{(L-1)}{G} = 1 + \frac{S_i G_A}{kT_o\left(\frac{C}{N}\right)} - \frac{T_A}{T_o} - \frac{(L-1)}{G}$$

$$= 1 + \frac{(1\times10^{-16})(3.16)}{(1.38\times10^{-23})(290)(1.58\times10^{3})} - \frac{300}{290} - \frac{(316.2-1)}{10}$$

$$= 18.4 = \underline{12.6\,dB}$$

14.13 $N_o = kT_bB = S_o$ for $S_o/N_o = 0\,dB$

$$kT_bB = P_r = \frac{P_t G^2 \lambda^2}{(4\pi R)^2}$$

$$R = \sqrt{\frac{P_t G^2 \lambda^2}{16\pi^2 k T_b B}} = \sqrt{\frac{(1000)(2.51)^2(4.48)^2}{16\pi^2(1.38\times10^{-23})(4)(4\times10^{6})}} = \underline{1.9\times10^{9}}\,m$$

If SNR = 30 dB = 1000, $R = 6.0\times10^{7}\,m$

$R_{VENUS} = 4.2\times10^{10}\,m$, so the signal will not even reach the nearest planet.

14.14 Mariner 10, PSK, $P_b = 0.05$, $(E_b/n_o) = 1.4dB$,
$R = 1.6\times10^8 km$, $f = 2.295 GHz$. $G_t = 27.6dB$
$P_t = 16.8W$, $G_r = 61.3dB$, $T_{sys} = 13.5K$

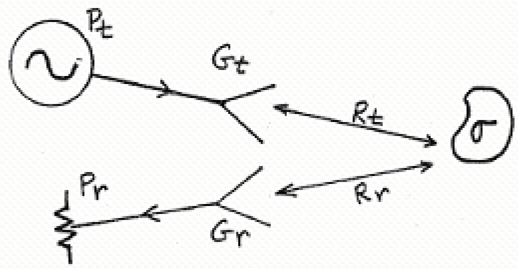
$\lambda = 0.1307 m$, $P_t = 12.25dBW$, $E_b/n_o = 1.38$

$L_o = 20\log(4\pi R/\lambda) = 263.7dB$.
$P_r = P_t + G_t - L_o + G_r = -162.6\,dBW = 5.56\times10^{-17}\,W$ ✓
$n_o = kT_{sys}$

From (14.36), $R_b = \left(\frac{P_r}{n_o}\right)\left(\frac{n_o}{E_b}\right) = \frac{5.56\times10^{-17}}{(1.38\times10^{-23})(13.5)}\left(\frac{1}{1.38}\right)$

$= \underline{216\ kbps}$ ✓

14.15



From (14.23) the power density incident on the target is,

$$S = \frac{P_t G_t}{4\pi R_t^2}$$

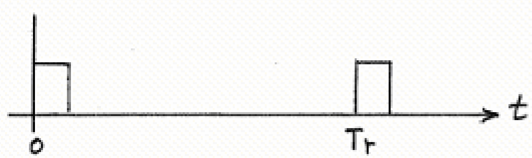
The scattered power density at the receiver is, from (14.40),

$$S_r = \frac{P_t G_t \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^2 R_t^2 R_r^2},$$

where $\sigma(\theta_t, \phi_t; \theta_r, \phi_r)$ is the radar cross-section of the target seen at θ_r, ϕ_r with an incident wave at θ_t, ϕ_t. Then the received power can be found using (14.14):

$$P_r = P_t \frac{G_t G_r \lambda^2 \sigma(\theta_t, \phi_t; \theta_r, \phi_r)}{(4\pi)^3 R_t^2 R_r^2} \quad ✓$$

14.16



When a pulse is transmitted at $t=0$, the return pulse must come back before the next pulse in transmitted at $t=T_r$, to avoid an ambiguity in range. The round-trip time for a pulse return is,

$$T = 2R/c,$$

so the maximum unambiguous range is,

$$R_{MAX} = \frac{cT_r}{2} = \frac{c}{2f_r} \quad ✓$$

14.17 From (14.43) the doppler frequencies are,

$$f_{d(MIN)} = \frac{2 v_{MIN} f_0}{c} = \frac{2(1\,\mathrm{m/sec})(12\,\mathrm{GHz})}{3\times 10^{8}\,\mathrm{m/sec}} = 80\ \mathrm{Hz}$$

$$f_{d(MAX)} = \frac{2 v_{MAX} f_0}{c} = \frac{2(20\,\mathrm{m/sec})(12\,\mathrm{GHz})}{3\times 10^{8}\,\mathrm{m/sec}} = 1.6\ \mathrm{kHz}$$

So the necessary passband is 80–1600 Hz.

14.18 From (14.41) the received power is,

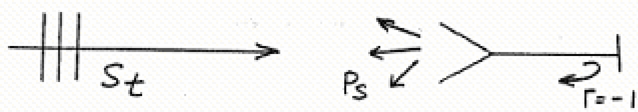
$$P_r = P_t \frac{G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} = 1000 \frac{(1000)^2 (0.15)^2 (20)}{(4\pi)^3 (10^4)^4} = 2.27\times 10^{-11}\ W$$

$$= -76\ \mathrm{dBm}$$

The transmitter power is $10 \log (10^4)(10^3) = 70$ dBm. So the isolation between receiver and transmitter must be,

$$I = 70\ \mathrm{dBm} - (-76\ \mathrm{dBm}) + 10\ \mathrm{dB} = 156\ \mathrm{dB}$$

14.19



assume an incident plane wave with power density S_t. Then the received power of the antenna is, from (13.14) and (14.15)

$$P_\ell = S_t A_e = S_t \frac{\lambda^2 G}{4\pi}$$

Because of the short-circuit termination, all of this power is re-transmitted (assuming a lossless antenna), giving a radiated power in the main beam direction of, $P_s = G P_\ell$. Then the RCS can be found from (14.39):

$$\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2}{4\pi} \checkmark$$

14.20

$$\Delta T_{TRUE} = T_p - T_2$$

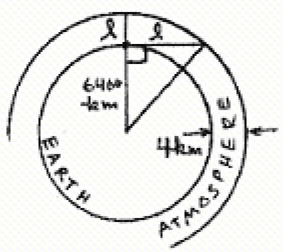
$$\Delta T_S = T_S\Big|_{T_B=T_1} - T_S\Big|_{T_B=T_2}$$

From (14.19), with $L=1$, $T_S = (1-\Gamma^2)\left[\eta_{rad} T_b + (1-\eta_{rad})T_p\right]$

$$\Delta T_S = (1-\Gamma^2)\left[\eta_{rad}\cancel{T_p} + \cancel{(1-\eta_{rad})T_p}\right] - (1-\Gamma^2)\left[\eta_{rad} T_2 + \cancel{(1-\eta_{rad})T_p}\right]$$

$$\frac{\Delta T_S}{\Delta T_{TRUE}} = \frac{(1-\Gamma^2)\eta_{rad}(T_p - T_2)}{T_p - T_2} = (1-\Gamma^2)\eta_{rad} \quad \checkmark$$

14.21

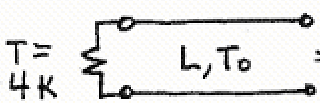


LOOKING TOWARD ZENITH, $\ell = 4000$ m. $= 4$ km.

LOOKING TOWARD HORIZON,

$$\ell = \sqrt{(6404)^2 - (6400)^2} = 226 \text{ km}.$$

$\alpha = 0.005$ dB/km



$$\Rightarrow T_e = \frac{4}{L} + (L-1)T_0$$

AT ZENITH: $L = (0.005 \text{ dB/km})(4 \text{ km}) = 0.02 \text{dB} = 1.0046$

$\underline{\underline{T_e = 5.3\text{K}}}$

AT HORIZON: $L = (0.005 \text{dB/km})(226 \text{ km}) = 1.13 \text{dB} = 1.297$

$\underline{\underline{T_e = 89\text{K}}}$

14.22 $f = 28\ GHz \Rightarrow \lambda_0 = 0.0107\ m,\ G = 32\ dB = 1585\ ,\ P_t = 5\ W$

a) $R = \sqrt{\dfrac{G P_t}{4\pi S}} = \sqrt{\dfrac{(1585)(5)}{4\pi(.01 W/cm^2)}} = 251\ cm = \underline{2.51\ m}$

b) $G = 32 - 10 = 22\ dB = 158.5$

$R = \sqrt{\dfrac{(158.5)(5)}{4\pi\ (.01 W/cm^2)}} = 79.5\ cm = \underline{0.8\ m}$

c) $d = \sqrt{\dfrac{\lambda^2 D}{\pi^2}} = 0.175\ m$ (use $D = \dfrac{G}{\eta_{ap}} = 2641.$)

$R_{ff} = \dfrac{2d^2}{\lambda} = \underline{5.7 m}$

(neither distance is in the far-field of the antenna)

14.23 $S = 1300\ W/m^2 = \frac{1}{2}|\bar{E}||\bar{H}| = \frac{1}{2\eta_0}|\bar{E}|^2 = \frac{\eta_0}{2}|\bar{H}|^2$

$|\bar{E}| = \sqrt{2\eta_0 S} = \sqrt{2(377)(1300)} = 990\ V/m$

$|\bar{H}| = \sqrt{\dfrac{2S}{\eta_0}} = \sqrt{\dfrac{2(1300)}{377}} = 2.6\ A/m$

check: $\frac{1}{2}|\bar{E}||\bar{H}| = \frac{1}{2}(990)(2.6) = 1300\ W/m^2$ ✓

6.26 The unperturbed TE_{101} cavity fields are,

$$E_y = A \sin\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

$$H_x = \frac{-jA}{Z} \sin\frac{\pi x}{a} \cos\frac{\pi z}{d} \qquad ; \; Z = k\eta/\beta$$

$$H_z = \frac{j\pi A}{k\eta a} \cos\frac{\pi x}{a} \sin\frac{\pi z}{d}$$

Then the numerator in (6.95) is,

$$\int_{v_0} \left(\Delta\epsilon |\bar{E}_0|^2 + \Delta\mu |\bar{H}_0|^2\right) dv = (\mu_r - 1)\mu_0 \int_{x=0}^{a} \int_{y=0}^{b} \int_{z=0}^{t} \left(|H_x|^2 + |H_z|^2\right) dz\,dy\,dx$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \int_{z=0}^{t} \left(\frac{1}{Z^2}\cos^2\frac{\pi z}{d} + \frac{\pi^2}{k^2\eta^2 a^2}\sin^2\frac{\pi z}{d}\right) dz$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{1}{Z^2}\left(\frac{z}{2} + \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Big|_0^t + \frac{\pi^2}{k^2\eta^2 a^2}\left(\frac{z}{2} - \frac{\sin\frac{2\pi z}{d}}{4\pi/d}\right)\Big|_0^t\right]$$

$$= \mu_0(\mu_r - 1)\frac{ab}{2} A^2 \left[\frac{t}{2\eta^2} + \frac{\beta^2 - \pi^2/a^2}{k^2\eta^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right]$$

The denominator in (6.95) is $\frac{abd\,\epsilon_0 A^2}{2}$, so

$$\frac{\omega - \omega_0}{\omega_0} = \frac{-(\mu_r - 1)ab\,\eta^2[\bullet]}{abd}$$

$$= \frac{-(\mu_r - 1)}{d}\left(\frac{t}{2} + \frac{\beta^2 - \pi^2/a^2}{k^2}\frac{d}{4\pi}\sin\frac{2\pi t}{d}\right)$$

For $t \ll d$ this simplifies to,

$$\frac{\omega - \omega_0}{\omega_0} \simeq -(\mu_r - 1)\left(\frac{t}{d}\right)\left(\frac{\beta^2}{k^2}\right)$$

6.27 Following Example 6.8:

at $x = a/2$, $z = 0$: $E_y = 0$

$$H_x = \frac{-jA}{Z} \quad , \; Z = k_0\eta_0/\beta$$

$$H_y = 0$$

Then,

$$\int_{\Delta v} (\mu|\bar{H}_0|^2 - \epsilon|\bar{E}_0|^2)\,dv = \mu_0 \frac{A^2}{Z^2}\Delta V \quad ; \; \Delta V = \pi l r_0^2$$

$$\int_{\Delta v} (\mu|\bar{H}_0|^2 + \epsilon|\bar{E}_0|^2)\,dv = \frac{V_0 \epsilon_0 A^2}{2}$$

So (6.102) reduces to,

$$\frac{\omega - \omega_0}{\omega_0} = \frac{2\mu_0 \Delta V}{Z^2 \epsilon_0 V_0} = \frac{2\eta_0^2 \Delta V \beta^2}{k_0^2 \eta_0^2 V_0} = \frac{2\beta^2}{k_0^2}\frac{\Delta V}{V_0}$$

(an increase in resonant frequency)