Introduction to Astronomy
Group Project References
Group Members: Ge Gao, Robert Goodfellow, Derek Benson
Date: Dec. 10th

Project Overview

This orbital simulation game is an simulation of a problem in classical physics called the "Three-Body Problem". The three-body problem can be phrased in the form of a mathematical challenge: given three bodies with some masses and initial velocities, is it possible to determine the exact location of those bodies in space at some later time?

Unfortunately for those who prefer to attempt such problems with pen and paper, there is no known general-form solution for the three body problem, and a solution for the problem can only be approximated with numerical methods. Fortunately, computers are fantastic at performing numerical computations quickly enough to get meaningful approximations. Those perpetual approximations are what this game demonstrates as its system evolves.

At the outset of this project, it was desired to show to the user the changes in the orbit that their ship had around the planet, and demonstrate visually the elliptical nature of orbits. However, computational challenges associated with the three-body problem were discovered that would cause the instability of orbital trajectories over time, it was determined that rendering such an trajectory in a stable manner would be impossible.

When modeling a three-body problem, there are generally two different avenues for implementation. The first is to reduce the scope of the problem by modeling the whole three-body problem as two different instances of the two-body problem. As an example, instead of modeling how the Sun exerts gravity on the Earth and Moon as individual bodies, the implementation would treat the Earth and the Moon as part of the same system. Thus, there are two distinct two-body problems - one for the Earth and Moon's influence on each other, and one for the Sun's influence on the Earth-Moon system as a whole. This is generally the simpler method to implement and conceptualize.

The second approach is to treat all three bodies distinctly, and use numerical methods of integration to determine the approximate location of each body as the system evolves. This implementation - though more accurate - involves a larger amount of computational resources, which in the context of a video game means the introduction of latency between screen updates (commonly referred to as "lag"). This is not acceptable in a video game that is designed to be interactive, especially when the lag-inducing operation needs to be performed frequently.

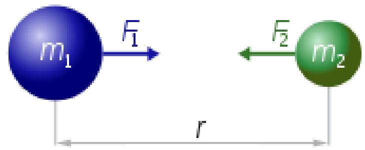
Regardless of methodology, however, any approximation of the three-body problem is at least NP-Hard [3]. This means a solution to the problem cannot be found efficiently, even by approximation. For the sake of reducing time complexity and reducing the overall challenge of implementing the physics calculations correctly, neither method was deemed usable, and the trajectory-display feature was abandoned.

Interestingly, we found it possible to model three-body interactions *without* having to resort to complex numerical methods, purely because the time steps we were concerned with were so small. The discrete nature of our modeling - perpetually aggregating changes in velocity at every given small timestep - is inherently an approximation, but it approximates so frequently on such a small scale that the roundoff errors incurred are quite small. Our model even necessarily accounts for the complicating factor that one of the bodies - the player's ship - can change its momentum under its own power.

Though the aesthetics of the game may not be the most pleasant, the aesthetics were not at all the primary focus of the game's construction. The primary focus was to demonstrate an understanding of the underlying physical interactions between orbiting bodies, and to create a rudimentary visualization of those interactions. In this respect, we believe our game was a success.

Facts List:

1. Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them:



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Where:

- F is the force between the masses;
- G is the gravitational constant (6.674×10^{-11} N · $(\mathrm{m/kg})^2$);
- m_1 is the first mass;
- m_2 is the second mass;
- r is the distance between the centers of the masses.

<u>In Project Code Snippet</u>:

```
let rawForce = (curr.mass * target.mass) / Math.pow(this.distanceBetween(curr.loc, target.loc), 2);
let angle = this.getAngleTo(curr.loc, target.loc);
let dforceX = rawForce * Math.cos(angle);
let dforceY = rawForce * Math.sin(angle);
forceX += dforceX;
forceY += dforceY;
```

2. In Cartesian coordinates, if p = (p1, …, pn) and q = (q1, ..., qn), then the distance between p, q is defined as:

$$\begin{aligned}\mathrm{d}(\mathbf{q},\mathbf{p}) &= \sqrt{(q_1-p_1)^2+(q_2-p_2)^2+\cdots+(q_n-p_n)^2}\\ &= \sqrt{\sum_{i=1}^{n}(q_i-p_i)^2}.\end{aligned}$$

In Project Code Snippet:

```
public collisionSurvivable(otherBody: GravityBody): boolean {
    let dvx = (this.vx - otherBody.vx);
    let dvy = (this.vy - otherBody.vy);
    let relv = Math.sqrt((dvx * dvx) + (dvy * dvy));
    //5.0 Meters per second is the impact survivability limit.
    return relv < 5.0;
}
```

3. A general-form solution to the three body problem does not exist. Further, an efficient approximation of the time-evolved three body problem does not exist, which implies that accurate orbital trajectories would be impossible to display without inducing intolerable lag.

Sources / References:

1. Brackenridge, J. Bruce. "ISAAC NEWTON. The Principia: Mathematical Principles of Natural Philosophy, 3rd Edition (1726). Newly Translated by I. Bernard Cohen and Anne Whitman. With a Supplement by I. Bernard Cohen. Berkeley: University of California Press, 1999." *The British Journal for the History of Science*, vol. 33, no. 2, 2000, pp. 231–254.

2. Fitzpatrick, Patrick. *Advanced Calculus*. American Mathematical Society, 2009.
3. Vasiliev, N. N. and Pavlov, D. A. "The Computational Complexity of the Initial Value Problem for the Three Body Problem" *Journal of Mathematical Sciences*, vol. 224, no. 2, July, 2017