Uncertainty Quantification in a Model Electric Energy Bidding Problem

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ABSTRACT

Deciding what to bid in an electricity auction typifies the large class of problems requiring decisions despite the possibility of severe uncertainty about important problem parameters. An important form of such uncertainty occurs when details about a probability distribution are unknown, so that information about a problem input is sufficient only to specify a family of distributions. This input uncertainty must be propagated through the solution process to obtain an answer. If the desired answer is a decision about what to bid in an auction, various decision criteria may be employed to determine or constrain this bid in a rational manner. The choice of which among the decision criteria to use should be made based on the match between the varied premises that underlie them and the bidder's objectives.

This paper explores the bidding problem when severe uncertainty about inputs requires expressing them as bounded families of distributions. The set of different decision criteria examined illustrates the variety of different approaches possible, the different conclusions these approaches produce, and the bidder's need to understand goals and information availability in order to rationally decide on a bid to best meet the goals of the organization.

1. INTRODUCTION

Motivation. New ideas for innovative solutions to problems in finance, electric power engineering, and other fields are arising from recent activity in uncertainty quantification (also called 2^{nd}-order uncertainty, epistemic uncertainty, and other terms). Traditionally, models in many fields, including electric power engineering, have represented uncertainty about continuous variables using probability distributions. This enables well-known techniques like Monte Carlo simulation. Other approaches to uncertainty have also been applied to electric energy industry and other problems, most notably fuzzy methods and intervals. Methods consistent with the classical, mathematically well-founded framework of probability, however, have stood the test of time and are undoubtedly here to stay. Yet traditional probabilistic methods cannot flexibly account for uncertainty about uncertainty such as, for example, uncertainty about the specific forms of the probability distributions describing variables in electric energy auction problems. This is a serious shortcoming because faithful representation of the limited knowledge that may be available about problem parameters can require expressing such uncertainties.

Because knowledge that supports bidding decisions is often limited by what information is available, inference under conditions of epistemic uncertainty is important to do in order to support optimized bidding. In this paper, such uncertainty is quantified as bounded families of probability distributions. These are applied to the important problem of bidding, in this case to sell blocks of electric power. Thus GenCos (electric generation companies) and others may eventually apply results such as those presented here to obtain guidance in making bids, as the results generalize to other similar bidding problems.

The interplay among problem inputs, simplifying assumptions, and inferential method defines an important yet incompletely charted landscape. One rationale for its exploration is the need for better bidding under severe uncertainty. Because the value of electric power traded in the US is in the hundreds of billions of dollars per year (EIA 2002), and because of the trend to deregulate the electricity marketplace, research addressing financial problems in the field has the potential for significant impact. Deregulation is making the buying and selling of power increasingly auction-centered, so improved decision-making related to bidding in these auctions has become a key class of financial problems in the electric industry.

State of the art. Applying bounded families of distributions to electric power bidding problems exemplifies what may be expected in engineering from applying uncertainty quantification. The growth in interest in uncertainty quantification engendered by such expectations is illustrated, for example, by four recent journal special issues (Berleant 2003; Helton and Oberkampf 2004; Nesterov and Berleant 2004; Ghanem and Wojtkiewicz 2004). Applications to problems in electric power (Dancre and Monclar 2004, Sheblé and Berleant 2002, Berleant et al. 2002, Cheong et al. 2003) follow naturally from the insights and investigations of other researchers, who have found that uncertainty quantification has applicability to power problems characterized by severe uncertainty. Prominent techniques include intervals (e.g. Wang and Alvarado 1992; Shaalan and Broadwater 1993; Shaalan 2000), and fuzzy methods (Ng 2000; Ng and Sheblé 2000a,b,c; Richter et al. 1998a,b, 1999; Bhattacharyya and Crow 1996; Guan et al. 1996; Yan and Luh 1997; see Hiyama and Tomsovic 1999 for a review). The well-recognized need for decisions in the presence of severe uncertainty, coupled with the grounding of our approach in the mathematically well-founded theory of probability, support its use in addressing important problems in electric power. Guidance can then be obtained regarding bidding decisions under conditions of uncertainty in which standard methods would require extra, unjustified assumptions. The ability to avoid such assumptions enables GenCos and others to obtain guidance in bidding of a type that they could not otherwise obtain.

2. MODELING THE PROBLEM

We wish to optimize bidding in the presence of uncertainty about the applicable distribution. Figure 1 shows an example. In a figure of this type, the left- and rightmost cumulative distribution functions (CDFs) could arise for at least three reasons: (1) as confidence limits (Kolmogoroff 1941); (2) as bounds on the distribution of a function of two (or more) random variables whose dependency relationship(s) are unspecified, e.g. using the DEnv algorithm (Berleant and Zhang 2004b); or (3) as bounds on the set of different distributions estimated by different experts. The stepped appearance in the figure could be due to the graininess of data in the Kolmogoroff case, or to discretization in the computations in the case of the DEnv algorithm. In the case of composition of expert estimates the curves would probably be smooth instead of stepped. However whether it is stepped or smooth does not impact the subsequent discussion.

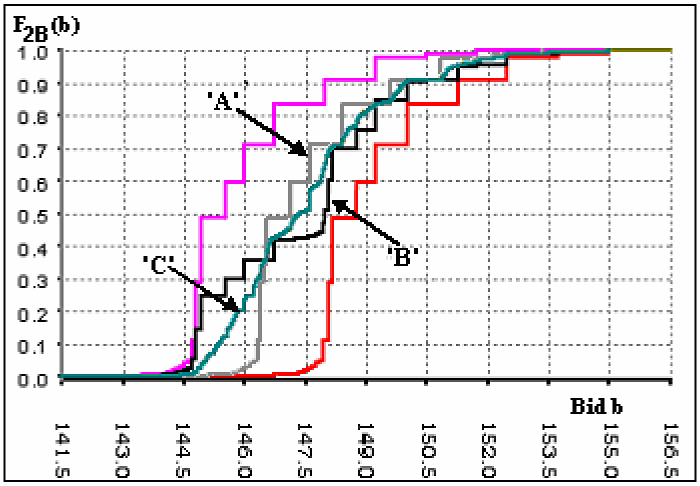

Figure 1. Bounds on the cumulative distribution for the bid that will be made by GenCo 2, for power from its generator G_{2B}. Curve A is the horizontal average of the left- and rightmost curves, curve B is the vertical average, and curve C is the fixed-point average.

Consider an example incorporating Figure 1. We take the perspective of GenCo 1, which is bidding against GenCo 2 to sell electricity with the following problem parameters.

Demand X_D=1000MWh

Cost to GenCo 1=$40/MWh

GenCo 2 has two generators, G_{2A} and G_{2B}

Capacity of G_{2A} is X_{2A}=300MWh

Capacity of G_{2B} is X_{2B}>700MWh

The distribution describing what GenCo 2 will bid to sell power from G_{2A} is within the left- and rightmost curves shown in Figure 2, while the bid for power from the more expensive generator G_{2B} is within the left- and rightmost curves of Figure 1.

Careful inspection shows that underbidding G_{2A} with an offer of 1000MWh will result in selling the entire 1000MWh, with GenCo 2 unable to sell from either G_{2A} or G_{2B}. However, this will have a lower expected monetary value (EMV) to GenCo 1 than attempting to underbid G_{2A} with a block of 300MWh and G_{2B} with a block of 700MWh. A still better strategy is to underbid G_{2B} and sell 700MWh at a relatively high price while permitting GenCo 2 to sell 300MWh at the relatively low price characterized by Figure 2 (Cheong et al. 2003). However, this strategy is consistent with a range of different bid prices. Given this strategy, we wish to determine what price GenCo 1 should bid to its best advantage. This is addressed next.

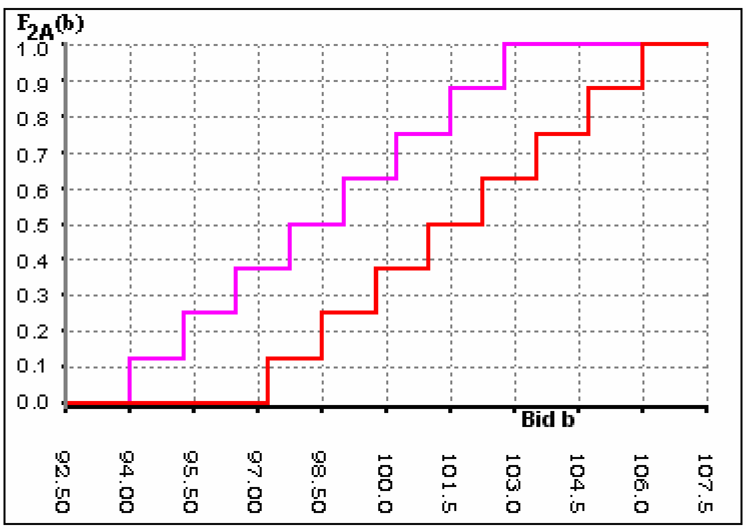

Figure 2. Bounds on the cumulative distribution for the bid predicted to be made by competitor GenCo 2 for power from its generator G_{2A}.

2.1 Implications for Bidding

If GenCo 1 bids below the left tail of the left bounding curve in Figure 1, it underbids GenCo 2's G_{2B}, therefore selling 700 MWh with probability 1. For bids between the left tail of the leftmost curve and the right (upper) tail of the rightmost curve, GenCo 1's EMV for a bid is

$$EMV=p_{win}(b)*700*(b-40) \quad (1)$$

where b is the bid amount and 40 is the per-MWh production cost given above. The probability of winning the auction is $p_{win}(b)=1-F_{2B}(b)$, where $F_{2B}(.)$ is the cumulative distribution for the bid predicted from competitor GenCo 2 for power from G_{2B}. The form of this distribution is uncertain, however it must fall between the left and right curves of Figure 1. Each possible curve for $F_{2B}(b)$ implies a different curve for EMV(b). Some of these are shown in Figure 3.

If the correct EMV curve is known, then the horizontal axis coordinate of its highest point defines the optimum bid. Figure 3 shows that the optimum bid differs for different EMV curves. Example optimum bids shown in Figure 3 are the horizontal axis coordinates of triangular point A, x-shaped point B, and point C. Thus, although the best bid may be easily found for a particular EMV curve, the situation is less clear when the proper EMV curve is unknown. Some other points in Figure 3 also deserve mention. Point E (below the visible portion of the graph) is the point on the low curve with the same horizontal axis value as point C on the high curve. Point F is where the curves start to diverge. And finally, if the top and bottom EMV curves are vertically averaged, curve M is obtained.

Uncertainty about which EMV curve applies. In order to address bidding when the right EMV curve to use is unknown, we start by observing that the family of possible

EMV curves (i.e. those that are implied by cumulative distributions within the bounds shown in Figure 2) is bounded from below by one such curve. We term this the pessimistic curve. Similarly the family is bounded from above by another EMV curve termed the optimistic curve.

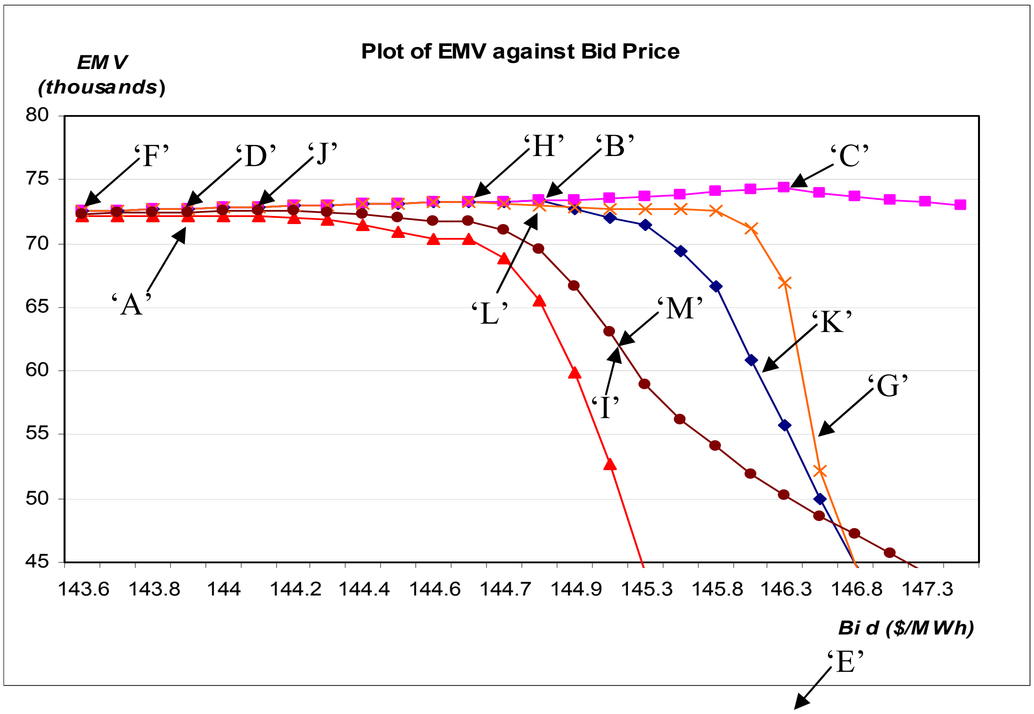

Figure 3. Some possible curves for the EMV as a function of bid value. Each EMV curve corresponds to some distribution that predicts the competing bid of GenCo 2 for its generator G_{2B} (see Fig. 1).

Assuming the pessimistic EMV curve. Suppose the best bid is chosen under the assumption that the pessimistic curve applies. The maximum point of that curve, whose coordinates state the best bid and its EMV, is triangular point A in Figure 3. If instead the optimistic curve applies, then the EMV of the same bid is higher (square point D). While this higher EMV is welcome, it is not as high as it would have been had we only known the optimistic curve applied. Then we could have chosen a different bid with an even higher EMV (point C). Hence the suboptimality of the bid is potentially as great as the difference in the heights of points D and C.

Assuming the optimistic EMV curve. One alternative to basing bid choice on the pessimistic curve is to base it on the optimistic curve. The best bid in that case is the horizontal axis coordinate of point C. However this bid is problematic because of the severe consequences if the pessimistic curve turns out to apply. Then, the EMV of the bid will be the height of the pessimistic curve at point E, which is so low it is not even visible in the figure. Had the pessimistic curve been known to apply then the bid would have been determined by point A. Therefore the suboptimality of a bid based on the optimistic curve is potentially as much as the relatively large difference in height between points A and E.

Assuming an intermediate EMV curve. Another alternative is to base the bid on some EMV curve between the pessimistic and optimistic curves. Then, the suboptimality of the bid in the case where the pessimistic (optimistic) curve actually turns out to apply is less severe than before, e.g. for curve K, the difference in height between diamond point B and triangular point A (B and C).

2.2 Deciding on a Bid Under Severe Uncertainty

The following twelve decision criteria are presented in related groups. Each criterion has its own conditions of applicability – and its own answer (see Discussion section).

A. Decision Criteria Based on Analysis of Extreme Scenarios

The three decision criteria that follow determine bids based on extreme scenarios.

Decision Criterion #1: Minimize Potential Suboptimality

The principle here is to minimize the maximum possible suboptimality. By suboptimality is meant the worst case amount by which the EMV of a bid might fall short of what it would be if the actual EMV curve were known. Figure 4 illustrates the bid resulting from such an analysis. In the figure, the length of a heavy vertical line segment indicates the worst case suboptimality of the corresponding bid, which is indicated by the placement of a vertical dotted line. Optimizing the EMV of the low EMV curve results in a bid whose worst case suboptimality is the length of line segment B. Bidding to optimize the EMV of the high EMV curve results in line segment C. Both B and C are longer than either of the two heavy line segments associated with bid A. For an EMV curve family like this, a graphical way to apply this decision criterion is to find the bid for which the suboptimality, if the low EMV curve applies, equals the suboptimality given that the high EMV curve applies. This decision criterion may be particularly appropriate in high-stakes scenarios in which it is desired to minimize risk. An example related to the present problem would be determining a policy for generating many bids governed by the same, unknown, EMV curve.

Decision Criterion #2: Maximize the Minimum Possible EMV

If the EMV of a bid is estimated from the lowest (most pessimistic) curve in the family of plausible EMV curves, then the estimated best bid and its EMV is the maximum of that curve. In Figure 3 that is the bid of 143.92 (point A). The actual EMV of that bid, which may be governed by a higher EMV curve, will then be at least as high. This is conservative in the sense that the actual EMV of the bid is at least as high. This strategy however is problematic for two reasons. First, the actual EMV curve is probably one of the infinitely numerous ones that are higher than the lowest one, and they have maxima at bids above 143.92. Therefore, a bid of 143.92+ε probably will have a higher EMV than 143.92. Second, even a very risk averse bidder for whom winning has overriding importance would not bid 143.92. Such a player would instead place a bid just below the bottom tail of the leftmost curve in Figure 1. This is approximately 143.2.

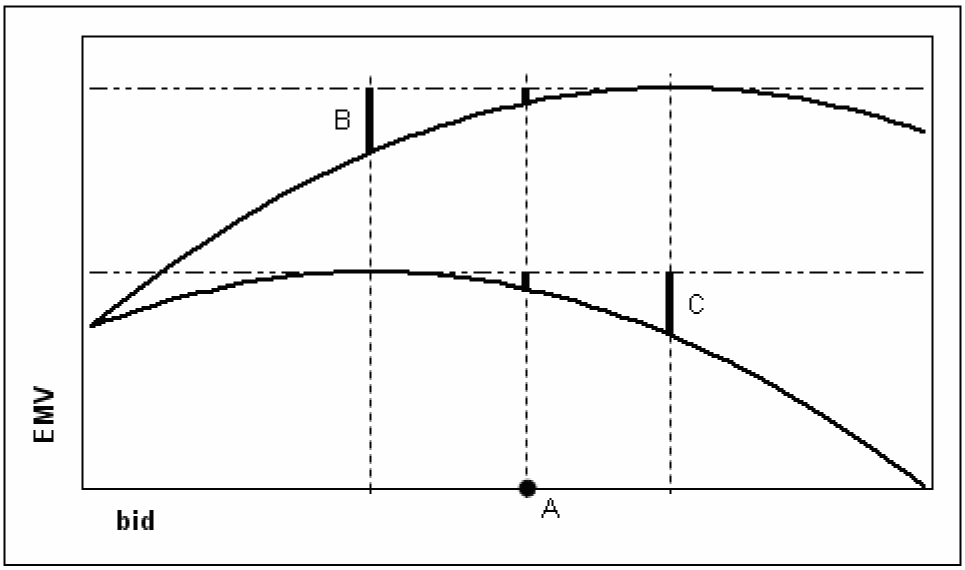

Figure 4. A family of EMV curves (lowest and highest shown), and three bids represented by vertical dotted lines. The length of the longest solid line segment associated with a bid is that bid's worst case suboptimality.

Decision Criterion #3: Maximize the Maximum Possible EMV

This approach is the conceptual opposite of the one just described. Here the bidder is guided by the optimistic EMV curve instead of the pessimistic curve. In this strategy, the optimal bid is 146.15, the horizontal axis coordinate of point C in Figure 3. However, the consequences if another EMV curve much below the top one applies could be serious. Many of the EMV curves in the family are dropping off precipitously at that bid value, leading to a serious risk of a very low EMV if one of the lower EMV curves is the correct one, such as the EMV of point E. Even a very risk-seeking bidder might not use this strategy because yet higher bids could still be successful (as long as they are below \$155/MWh, the end of the upper tail of the rightmost curve in Figure 1).

B. Decision Criteria Based on Averaging Scenarios

This category of decision criteria identifies a single curve within the bounding left- and rightmost CDF curves (e.g. Figures 1 & 2). This curve's corresponding EMV curve then has a maximum point whose horizontal axis value specifies a value to bid. Thus, we wish to determine bids from a curve that is somehow representative of the entire family (rather than using the most extreme curves to determine bids, as done earlier). This suggests applying an averaging technique to the family of plausible cumulative distributions predicting the competitor's bid (Figure 1). Decision criteria #4-6 are of this type. Averaging can also be applied directly to the family of EMV curves (Figure 3). Decision criterion #7 is of this type. Criterion #8 applies to families of functions more generally.

Decision Criterion #4: Use Horizontal Averaging

This method computes a curve estimating the CDF of the competitor's bid by connecting points that are calculated as follows. Any given value of F_{2B} in Figure 1 intersects the left- and rightmost curves at corresponding bid values, call them b_l and b_r. Specify a new value $b=(b_l+b_r)/2$ as a coordinate of a new point (b, F_{2B}). Done enough times, the result is curve A in Figure 1. This curve corresponds to EMV curve G in Figure 3, for which the best bid and its EMV are shown as point H.

Decision Criterion #5: Use Vertical Averaging

The horizontal averaging strategy just described has a vertical dual: for any given value of *b* on the horizontal axis, the vertical axis values of the left- and rightmost curves are both identified. Their mean, and *b*, form the coordinates of a point on the vertically averaged curve. The result is curve B in Figure 1. This curve implies curve I in Figure 3, with a maximum at point J that defines its best bid and corresponding EMV.

Decision Criterion #6: Use Fixed-Point Averaging

The CDF curves predicting the competitor's bid implied by horizontal vs. vertical averaging of the left- and rightmost curves are different (Figure 1, curves A and B). These average curves have complementary desirable and undesirable features. The horizontally averaged curve has an S-shape but the vertically averaged curve has a wavy form that seems counter-intuitive (though, in principle, possible). On the other hand, the vertically averaged curve has tails that reflect the plausibility of competitor bids that are extreme but within the bounding curves, while the horizontally averaged curve does not. Thus a third form of averaging was designed that has the advantages of both.

This averaging method takes the horizontal and vertical average curves as inputs, and averages those curves. However, if this new averaging step was done by vertical averaging, the result would be different than if it was done by horizontal averaging. Therefore, the average curves (A and B in Figure 1) are averaged both ways to obtain two new curves, these new curves are then again averaged both ways, and the process iterated until the two curves resulting from each iteration converge. The result (curve C in Figure 1) is both S-shaped and has the desired tail property. This is termed the fixed-point average curve, and the procedure, fixed-point averaging.

The fixed-point average curve implies a corresponding EMV curve (labeled K in Figure 3). This curve has a maximum point (labeled L in Figure 3) which gives the maximum EMV and its corresponding bid amount, as estimated through fixed-point averaging.

Decision Criterion #7: Use Vertical Averaging of the EMV Curves

Because CDFs are monotonic, both vertical and horizontal averaging are possible. However EMV and other curves (e.g Figure 3) in general are not. In such cases, horizontal averaging as described above may not be possible. This is because horizontal averaging requires taking the mean of the horizontal axis values of two curves for various heights on the vertical axis. If either curve is (continuous and) non-monotonic it will have vertical axis value(s) with two or more horizontal axis values. Furthermore if one curve stays below the maximum height reached by the other, there will be vertical axis values for which no horizontal average exists. Such properties prevent application of horizontal averaging, at least as it was defined above.

The EMV curves (Figure 3) are examples for which horizontal averaging is inapplicable, so horizontal averaging was not considered. However vertical averaging may be readily applied. Vertical averaging of the top and bottom EMV curves in Figure 3 results in curve M. Curve M is identical to curve I, due to properties of Equation (1). For other equations, the two curves might not be identical. Since the CDF bounding curves are more directly connected to the problem inputs than the EMV curves, averaging based on CDF bounding curves is generally preferable.

Decision Criterion #8: Use More Information for Better Averaging

Until now, we have not accounted for the possibility that some members of a bounded family of curves might be more likely to hold than others. Information about these relative likelihoods, if available, enables more informed computation of an average curve. Given a specification of the relative likelihoods of curves in a family, averaging can be done by weighting each member of the family by its probability, in the discrete case, or probability density in the continuous case.

The relatively extreme knowledge states of (1) knowing only the left- and rightmost bounding members of a curve family, and (2) knowing relative likelihoods of all family members, are not the only possibilities. Intermediate degrees of knowledge are also possible. The forms such information can take and how to use it continues to be an important problem.

C. Decision Criteria Based on Risk

The avoidance of calamity is often a powerful factor in decisions. The remaining decision criteria directly address risk.

Decision Criterion #9: Use Bid Utility Instead of Bid EMV

The goal in choosing a bid price is ultimately to optimize its utility to the business. For bids to sell blocks of power this might mean maximizing the expected monetary value (EMV) of a bid. However, a full analysis would need to recognize that the goal of maximizing EMV may be tempered by risk position. For example, individuals who make bids may wish to avoid jeopardizing their bidding records with risky bids. As another example, risk aversion may affect higher level decisions of sufficiently high value, such as those relevant to value at risk (VaR), profit at risk (PaR), capital budgeting, etc. To account for risk, the utility of bids rather than their EMV should be maximized. This may require transforming EMV curves (Figure 3) into utility curves based on risk profile. Then the solution approaches described earlier, which refer to EMV curves, can often instead be applied to the corresponding utility curves.

EMV curves may be transformed into corresponding utility curves by transforming the points on those curves (the EMVs of bids) into utilities of bids. The first step in converting an EMV to a utility is specifying the risk profile of the player, GenCo 1 in this case, as a utility function. Figure 5 shows representative examples of utility functions.

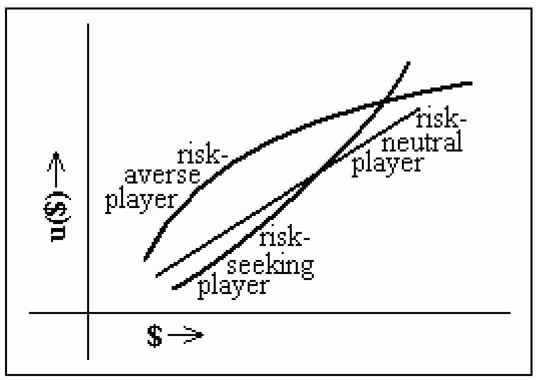

Figure 5. Three risk profiles. Risk neutrality gives a constant slope utility function, risk aversion a concave-down one, and risk attraction a concave-up one.

A utility function, $u(x)$, describes utility as a function of monetary quantity. It expresses the player's subjective perception of the value of objective monetary amounts. A utility function can model the fact that, for example, a risk averse player prefers getting $10,000 to a 50% chance of getting $20,000, even though both options have the same expected monetary value (EMV). Thus the utility of $10,000 to this player exceeds 50% of the utility of $20,000 + 50% of the utility of $0. In general, the utility of a gamble is the average of the utilities of the possible outcomes weighted by their probabilities. Thus for this player, $u(\$10{,}000) > (½)u(\$20{,}000)$. This implies a utility function of decreasing slope and a risk averse player (Figure 5). Risk aversion is more common than risk attraction. A risk averse salesman, for example, might prefer working at a steady salary to working for uncertain commissions, even if the EMV of working for commissions is higher.

For the example bidding problem, a given CDF in the family of CDFs (Figure 1) predicting competitor GenCo 2's bid associates each bid amount b with a cumulative probability $p=F_{2B}(b)$. Thus if GenCo 1 bids at b the probability is p that GenCo 2's bid is lower. Therefore GenCo 1 will win the auction with probability $1-p$. If GenCo 1 wins, the resulting sale will have a certain monetary value v_b. If GenCo 1 loses, the model assigns that result a value of 0. The utility of the bid, u_b, is thus $u_b=(1-p)*u(v_b)+p*u(0)$. Its monetary equivalent is then $u^{-1}(u_b)$. If risk is an insignificant consideration then risk-neutrality applies, so $u(v)=v$. Then the utility of a bid equals its EMV, that is, $u_b=(1-p)*u(v_b)+p*u(0)=(1-p)*v_b+p*0=\mathrm{EMV}_b$. Taking risk into consideration then has no effect on the decision of what to bid. Risk neutrality might apply if the amount of money involved is small from the bidder's standpoint, such as one auction out of many occurring over time. On the other hand, risk would be a consideration for a large enough bid, for example, if it was for a significant capital investment like building a windmill farm.

Decision Criterion #10: Use EMV Utility Instead of Bid Utility.

Given a bid amount b and the associated $EMV(b)$, the monetary value of n such bids will be close enough to $n*EMV(b)$ to within an acceptable probability for sufficient n. Thus $n*EMV(b)$ approaches not an EMV but an actual monetary value, as a statistical property of a large enough number of bids. The risk in this scenario is not losing a given auction. Rather, the risk is of choosing bids with relatively low EMVs due to basing them on the wrong CDF in the CDF family.

Given plans for n auctions before a re-examination of bidding policy, an single-bid EMV of m has a utility of $u(n*m)/n$ irrespective of the probability of winning. Since any EMV can thus be converted into a utility value, any EMV curve can be transformed into a utility curve. This enables transforming a family of EMV curves such as those in Figure 3 into utility curves. These utility curves might normally be expected to have some resemblance to their original EMV curves. For example, the maximum of each utility curve will be at the same bid value as the maximum of the EMV curve of which it is a transformation. Yet the utility curves will not be identical to the EMV curves (unless risk neutrality applies). Therefore various decision criteria described earlier, if given utility curves as inputs, will likely produce recommendations for bids that differ to some degree from the recommendations they make when applied to the corresponding EMV curves.

Decision Criterion #11: Use Information Gap Theory

Information Gap Theory (Ben-Haim 2001) is useful for making decisions in cases where uncertainty is described with bounds, but the probabilistic structure within those bounds is not specified. Figures 1 & 2 showed examples. Rather than attempting a form of optimization as was done in earlier decision criteria, the appropriate goal in using information gap analysis here is to ensure that the EMV of a bid meets or exceeds a given minimum value. An information gap model for this results in a *robustness function* that helps identify bids meeting that requirement. An information gap model can also identify the additional information that would be needed to reduce the uncertainty enough to ensure that other, more desirable bids, meet that requirement.

For example, various bid amounts in Figure 3 have high EMVs for EMV curves near the highest EMV curve, making them potentially desirable bids. However some of these bids also lead to unacceptably low EMVs for EMV curves near the lowest EMV curve, making them too risky. If additional information was obtained that ruled out such low EMV curves, some overly risky bids would then become feasible. The cost of obtaining such information could be weighed against its potential benefit and a decision then made about whether to obtain it. However such information, once obtained, might rule out high EMV curves instead, thereby also ruling out bids that it was hoped would become feasible. Results of an information gap analysis for the bidding problem are shown in Figure 6.

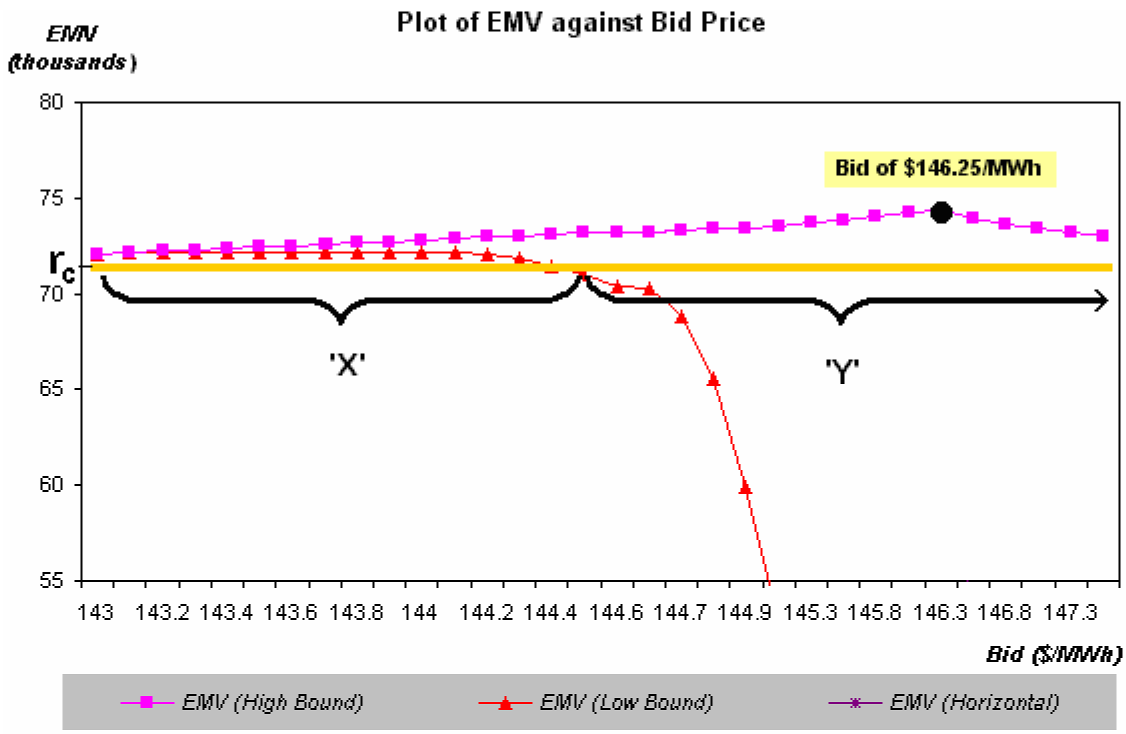

Figure 6. Bounding EMV curves (from Fig. 3), a given minimum acceptable reward level r_c, and two ranges of bids, X and Y. Bids in range X guarantee EMVs of at least r_c, while bids in range Y might have EMVs of at least r_c, but also might not.

More formally, an information gap model requires defining a number of items. For this example these are as follows.

1. *Decision variable*. This is the amount of the bid, b.
2. *Uncertain variable*. Use some averaging method to define a CDF predicting the competitor's bid (e.g. Figure 1) that will fill the role of nominal best guess. While the averaging computations described earlier weight the left- and rightmost curves equally, any pair of positive weights summing to 1 are possible. Define the uncertain variable as the weight w of the leftmost curve (the weight of the rightmost curve is then $1-w$).

3. *Nominal value of uncertain variable*. If there is no reason to prefer weighting one bounding CDF curve more than the other *a priori*, the default nominal value of weight w would be $\tilde{w}$=0.5. Otherwise an analyst could choose a different weight for $\tilde{w}$.

4. *Uncertainty parameter*. The amount of uncertainty in the model, α, is the amount of deviation from the nominal value of the uncertain variable that is to be considered plausible. If $\tilde{w}$=0.5, this might be as much as α=0.5, which would give a range of plausible weights for w of $0=\tilde{w}-\alpha\leq w\leq\tilde{w}+\alpha=1$. Further information might shrink uncertainty parameter α, and it might be necessary to obtain such information to ensure the organization's goals are met.

5. *Uncertainty model*. This is a function $\mathbf{u}(\alpha,\tilde{w})$ that describes the amount of uncertainty in uncertain variable w in terms of its nominal value $\tilde{w}$ and the uncertainty parameter α.

6. *Reward function*. The reward is the EMV of some bid. Therefore the reward function, r, may be defined by $r(b, w)=EMV(b, wL(b)+(1-w)R(b))$, where L and R are the left and right bounding curves respectively, in Figure 1, and $wL(b)+(1-w)R(b)$, a weighted average of L and R is the CDF predicting the competitor's bid.

7. *Critical reward*. This is the minimum acceptable value of the reward function, call it r_c. The results of an information gap analysis differ depending on the value assigned to r_c.

8. *Robustness function*. This function, $\hat{\alpha}(b,r_c)$, returns the greatest value of uncertainty parameter α for which, given bid b, falling below the critical reward r_c is not possible in the model. It therefore measures the ability of the model to

deliver a reward that will be at least minimally acceptable despite the presence of uncertainty, hence the term *robustness*.

From Figure 6 some facts may be deduced about $\hat{\alpha}(b, r_c)$ for a range of bid amounts, given a minimally acceptable reward r_c. For bid amounts in region '**X**' the EMV is above r_c for all EMV curves corresponding to CDF *L*, *R*, or any average of *L* & *R*, weighted or not. Therefore weight *w* could be from 0 to 1, meaning that $\hat{\alpha}(b, r_c)$, the maximum allowable deviation *α* from nominal weight $\tilde{w}$, is $\tilde{w}$ in the downward direction (in which case $0 \leq w \leq \tilde{w}$) and $1-\tilde{w}$ in the upward direction (in which case $\tilde{w} \leq w \leq 1$). Thus, r_c will be safely met for any bid in region '**X**.' However it may be desired to consider bidding higher (in range '**Y**') in order to attempt to reap the benefits of possibly greater EMVs. Bids in range '**Y**' in Figure 6 would be guaranteed to have an EMV of at least r_c if new information is obtained which rules out values of *w* that are too close to 1 (thereby moving the worst-case EMV curve upward). Thus, new information about *w* may be sought that would permit bids above region '**X**.' This information, once obtained, might or might not do this, depending on what values of *w* the new information rules out.

To summarize, the result of an information gap analysis is a description of what new information (if any) would be required, for any specified bid amount, to guarantee at least a given EMV. Thus guidance is provided about both acquiring new information and making bids.

Decision criterion #12: Apply VaR and PaR With Bernoulli Processes

Bernoulli processes may be used constrain acceptable bid amounts based on Value at Risk (VaR) or Profit at Risk (PaR). Given a maximum loss (VaR) or minimum profit (PaR) *v*, a corresponding certainty factor *p* representing the minimum tolerable probability of meeting requirement v by a time t, and a function F relating a bid amount to the probability of winning the auction, an acceptable range for bids may be deduced. The outcome of a particular auction will change the parameters of the problem because there are fewer bids remaining until time t; also, winning an auction changes the amount of money that may be risked over the course of the remaining bids. Consequently, the range of acceptable bid amounts will tend to vary from one auction to the next.

Under severe uncertainty, exact probabilities of exceeding specific values of loss (Value at Risk, or VaR), or failing to meet specific values of profit (Profit at Risk, or PaR) can be impossible to compute. Applying the Bernoulli process approach in the presence of 2^{nd}-order uncertainty about CDFs requires accounting for that uncertainty in deciding on bid values (Figures 1 & 3). While uncertainty in the EMV of a given bid has been addressed using various decision criteria described earlier, the 2^{nd}-order uncertainty in the problem description can alternatively play out as uncertainty in the probability of winning an auction for a given bid. Viewing each auction as a Bernoulli trial, this uncertainty about the probability of winning it leads to a range of possible probabilities of meeting VaR or PaR requirement v in the remaining Bernoulli trials.

If every value in the range of probabilities is below the desired certainty factor p, then the bid is allowable. If no bid is allowable, acquiring more information might reveal a range of acceptable bids. If not, still more information might be sought. If it is possible to show that additional information could not lead to any acceptable bids, then the conclusion is that the VaR or PAR objective cannot be achieved with the desired certainty factor p, necessitating some form of damage control. If analysis does provide a range of acceptable bids for the next auction, acquiring more information might expand the range. The details of what information is required may be determined by an information gap analysis (one example of which was given earlier). Ultimately, a bid within the acceptable range must be chosen. This may be done using one of the other applicable optimization methods described earlier.

3. DISCUSSION AND CONCLUSION

When different decision criteria give different answers, a decision may still be necessary. One way to identify one is to examine the premises of the criteria and rule out those whose premises are not consistent with the goals and perspectives of the decision-maker. Utility-based criteria might be eliminated from consideration in favor of EMV-based criteria, or vice versa. Worst-case scenarios may be of paramount concern or average-case scenarios may be. Information-seeking activities may be on the table or decisions may need to be made strictly on the basis of existing knowledge. It may be possible to identify a criterion to use based on such considerations. In general however, more than one criteria recommending different decisions may stubbornly remain plausible. This is in the nature of severe uncertainty. Yet a decision may nevertheless be required. Ultimately, decisions may have to depend in part on considerations outside of the problem definition.

We have illustrated bounded families of CDFs, exemplified by Figures 1 & 2, as a representation for limited information about an important type of problem in electric power. Manipulating such CDF families has become increasingly visible in the field of uncertainty quantification both because it shows promise for solving problems characterized by severe uncertainty and because understanding of this information structure is improving. The work presented here forms a natural point of departure for work on related but more realistically complex problems. These include the following.

- *Multiple Bidders*. The example we have discussed had two bidders, but often there are more. To address this, one major new issue that will need to be handled is how to deal with competitors' bids whose probability boxes (the region between the left- and rightmost CDF curves) overlap.
- *Bidding Histories*. Real electric power bidding problems typically occur in the context of a history of bids by the same players. Information about those bids should be used to improve future binds. One approach to this is accounting for information about dependencies among random variables in the computations. Correlation is one form of such information. We have recently shown how to use information about correlation to compute better bounds on the distribution of a function of random variables (Berleant and Zhang 2004a,c).
- *Complex bidding and other financial decisions*. While a single bid for a block of electric power may not have major consequences for a player, other financial decisions can. Examples include decisions about large capital purchases, about strategic responses to perennially controversial projections of fossil fuel production and reserves (Deffeyes 2001), and about bids for building transmission lines and other costly capital items. Risk position is an important factor in quantitative analysis of such problems.
- The bounded families of distributions typified by Figures 1 & 2 illustrate one way to incompletely specify probability distributions. However there are other ways as well. Flexibility in how to provide partial characterizations of distributions will

allow flexibility in using available partial information about a problem to its maximum potential. Varieties of partial specification include (1) intervals bounding the probabilities of given regions of a probability density function (pdf), (2) numbers stating the probabilities of sets of regions of a pdf, and (3) unimodality constraints that rule out, for example, cumulative distributions whose corresponding density functions are bimodal.

Applying uncertainty quantification to electric power and related financial problems is an essential step in developing an apparent potential for synergy between the fields. Resulting efforts to assist pragmatic decision-making under severe uncertainty illustrate the applicability of uncertainty quantification while motivating continued theoretical progress. Advances in uncertainty quantification and its application to electric power problems are likely to lead to solutions in other areas of engineering, as well as in science, finance, business, and economics. Effective inference and decision-making in problems characterized by considerations of risk and reliability under severe uncertainty are likely to impact a wide range of both scholarly fields and practical endeavors.

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