Stream Cipher

CUI TINGTING

School of Cypberspace, Hangzhou Dianzi University

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Contents

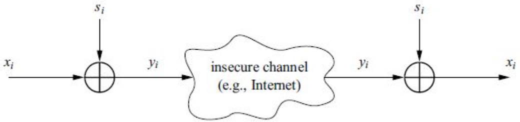
what is OTP

One-Time Pad (OTP)

A cipher for which

- Encryption where a keystream is bitwise added to plaintext
- Keystream is generated perfect randomly
- Keystream is only known to the legitimate communicating parties
- Every keystream bit k_i is only used once

is called a **one-time pad**.



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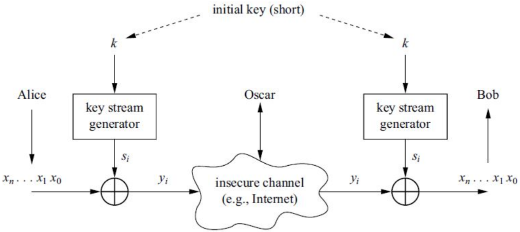
Practical Stream Cipher



- Keystream is generated by PRNG
- Hope Stream Cipher is computational security

Computational Security

A cryptosystem is computationally secure if the best known algorithm for breaking it requires at least t operations.

True Random Number Generators (TRNG)

- Output CANNOT be predicted or reproduced. e.g. flip a coin 100 times.
- TRNGs are based on physical processes. e.g. coin flipping, rolling of dice, semiconductor noise, clock jitter in digital circuits and radioactive decay.
- TRNGs are often needed for generating session keys.

Pseudorandom Number Generators (PRNG)

Pseudorandom number generators (PRNGs)

- Generate sequences from an initial seed value.
- Often they are computed recursively:

$$
\begin{aligned}
s_0 =& seed, \\
s_{i+1} =& f(s_i, s_{i-1}, \ldots, s_{i-t}), i = 0, 1, \ldots.
\end{aligned}
$$

where t is a fixed integer.

- Typically, output stream has good statistical properties.
- Output can be reproduced and can be predicted.
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Pseudorandom Number Generators (PRNG)

Example 1 (Linear congruential generator)

$$s_0 = seed,$$

$$s_{i+1} = as_i + b \mod m, i = 0, 1, \ldots$$

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Example 2 (rand() function used in ANSI C)

$$s_0 = 12345,$$

$$s_{i+1} = 1103515245 s_i + 12345 \mod 2^{31}, i = 0, 1, \ldots$$

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- The need for unpredictability of CSPRNGs is unique to cryptography.

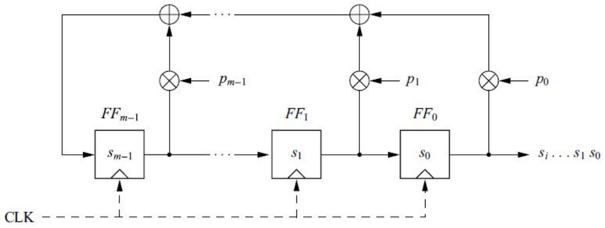
1 One-Time Pad (OTP)

2 Random number generators (RNGs)

3 Linear feedback shift registers (LFSRs)

4 Trivium: a modern stream cipher

Linear feedback shift registers (LFSRs)



- An LFSR: storage elements (flip-flops) and a feedback path.
- Degree of the LFSR: #storage elements.
- The feedback computes fresh FF as XOR-sum of certain FFs.
- If $p_i = 1$ (closed switch), the feedback is active. Otherwise, there is not feedback from this flip-flop (open switch).

$$s_{m+i} = s_{m+i-1}p_{m-1} + \ldots + s_{i+1}p_1 + s_i p_0 \mod 2,$$

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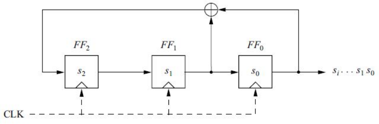
Example 3 (LFSR)

Initial state is $s_2 = 1, s_1 = 0, s_0 = 0$, what is the complete output sequence?



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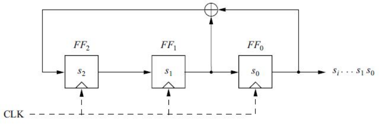

clk	FF_2	FF_1	$FF_0 = s_i$
0	1	0	0
1	0	1	0
2	1	0	1
3	1	1	0
4	1	1	1
5	0	1	1
6	0	0	1
7	1	0	0

$$s_{i+3} = s_{i+1} + s_i \mod 2$$

0010111 0010111 0010111...

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Example 4

Given an LFSR of degree $m = 4$ and the feedback path $(p_3 = 0, p_2 = 0, p_1 = 1, p_0 = 1)$, the output sequence of the LFSR has a period of $2^m - 1 = 15$, i.e., it is a maximum-length LFSR.

Attack on LFSR: Exhaustive Key Search

A stream cipher using LFSR with degree n as the keystream generator. Assume initial key K is n bits.

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  - Compute $P' = C + S'$ and check if P' is meaningful.
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Upper bound to the security strength s of a cipher

Security strength s of a cipher with a k-bit key is at most k.

Attack on LFSR: state reconstruction using linear algebra

Linearity

A function f is linear (over $\mathbb{Z} = 2\mathbb{Z}$) if $f(x+y) = f(x) + f(y)$ If f_1 and f_2 are linear, $f_2 \circ f_1$ is linear.

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  - Assume we know the state S^t of clock t
  - $S^t \leftarrow M \cdot S^{t-1}$, $S^{t-1} \leftarrow M \cdot S^{t-2}$, ...
  - Hence, $S^t = M^t S^0$, while $S^0 = K$
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Need for non-linearity

Purely linear ciphers offer no security.

Specifications of Trivium

Trivium is one finial cipher of eSTREAM Stream Cipher Project.

Parameters	
Key size	80 bits
IV size	80 bits
Internal state size	288 bits
Keystream size	2^{64}

Desgin Document:
https://link.springer.com/chapter/10.1007/11836810_13

Trivium: Key and IV Setup

$(s_1, s_2, \ldots, s_{93}) \leftarrow (K_{80}, \ldots, K_1, 0, \ldots, 0)$
$(s_{94}, s_{95}, \ldots, s_{177}) \leftarrow (IV_{80}, \ldots, IV_1, 0, \ldots, 0)$
$(s_{178}, s_{179}, \ldots, s_{288}) \leftarrow (0, \ldots, 0, 1, 1, 1)$
for $i = 1$ to $4 \cdot 288$ **do**
  $t_1 \leftarrow s_{66} + s_{91} \cdot s_{92} + s_{93} + s_{171}$
  $t_2 \leftarrow s_{162} + s_{175} \cdot s_{176} + s_{177} + s_{264}$
  $t_3 \leftarrow s_{243} + s_{286} \cdot s_{287} + s_{288} + s_{69}$
  $(s_1, s_2, \ldots, s_{93}) \leftarrow (t_3, s_1, \ldots, s_{92})$
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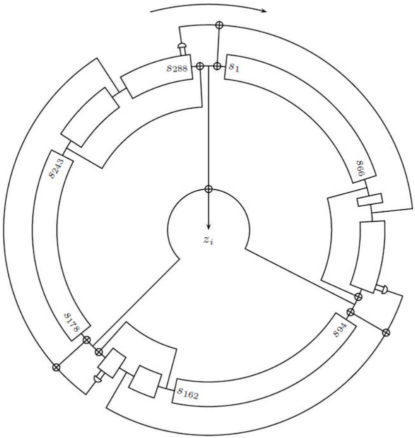
end for



Trivium: Key Stream Generation

for $i = 1$ **to** N **do**

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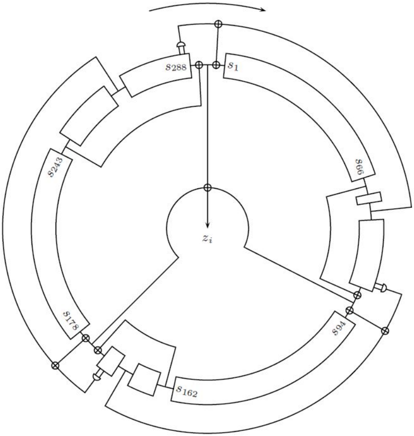

Thanks & Questions