Capability Analysis for Data from a Low Discrimination Gauge

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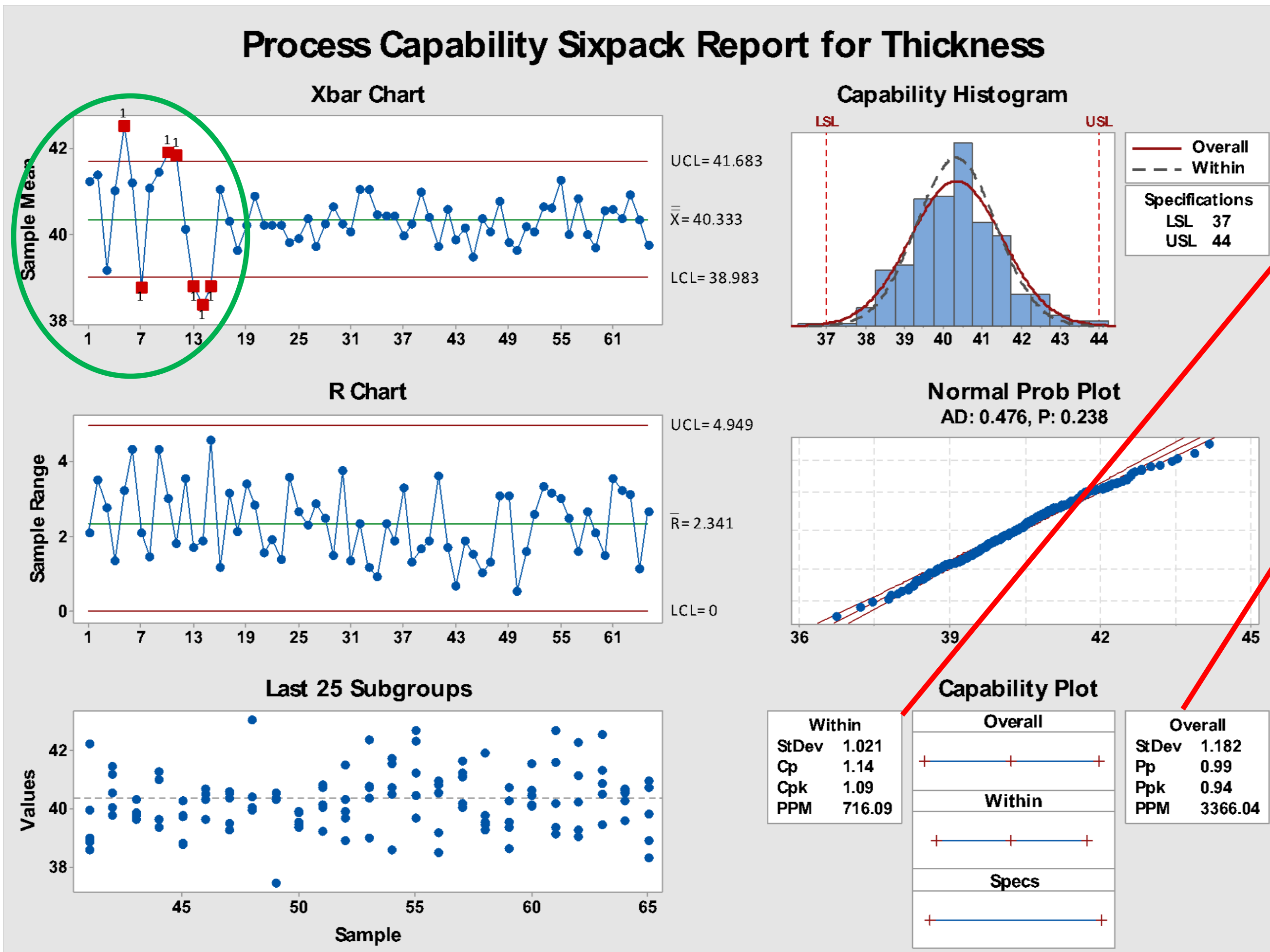
Learning Objectives

1. Quick overview of Normal Capability Analysis
2. Non-Normal (NN) Capability Analysis
3. Determine what makes data Non-Normal
4. Assess the weaknesses of Normality tests
5. How to deal with data coming from a measurement system with low discrimination

Is the Data Stable Over Time?



Within	
StDev	1.021
Cp	1.14
Cpk	1.09
PPM	716.09

Overall	
StDev	1.182
Pp	0.99
Ppk	0.94
PPM	3366.04

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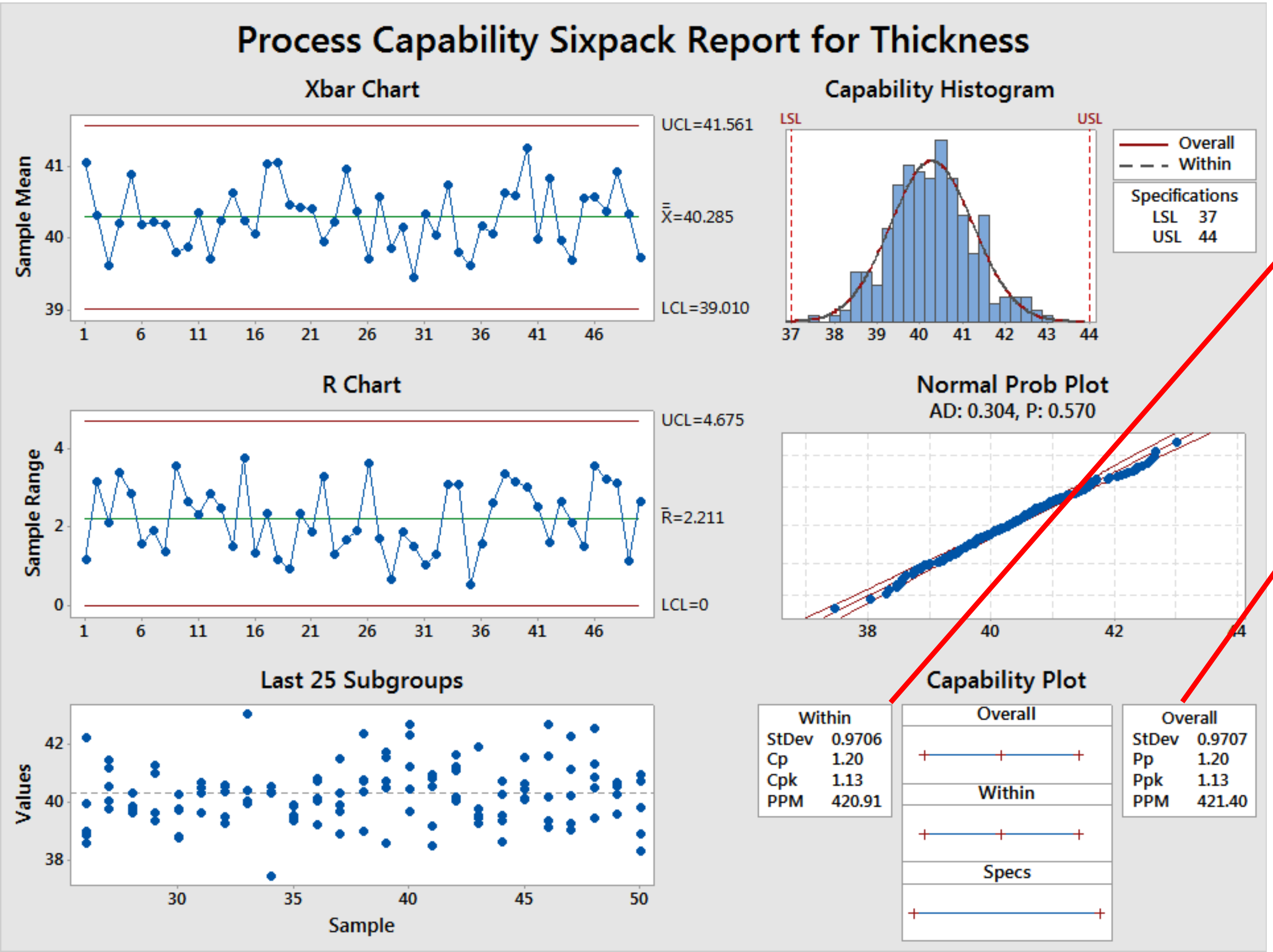
Is the Data Stable Over Time?



Within	
StDev	0.9706
Cp	1.20
Cpk	1.13
PPM	420.91

Overall	
StDev	0.9707
Pp	1.20
Ppk	1.13
PPM	421.40

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What to do when Data are not Normal?

1. Use a transformation (Johnson, Box-Cox)
2. Find a Non-Normal (NN) distribution to model the data
3. Use a nonparametric method
4. Options above do not apply or are not feasible, ask yourself why is the data not normal?

Why are the Data not Normal?

Case I. Nature of the beast – process near a boundary, naturally produces data that are skewed.

Case II. Mixture of distributions or a few outliers – process may not be in statistical control.

Why are the Data not Normal?

Case III. Large sample sizes – power of normality tests detects small departures from “perfect” normality.

Case IV. The gauge has low discrimination.

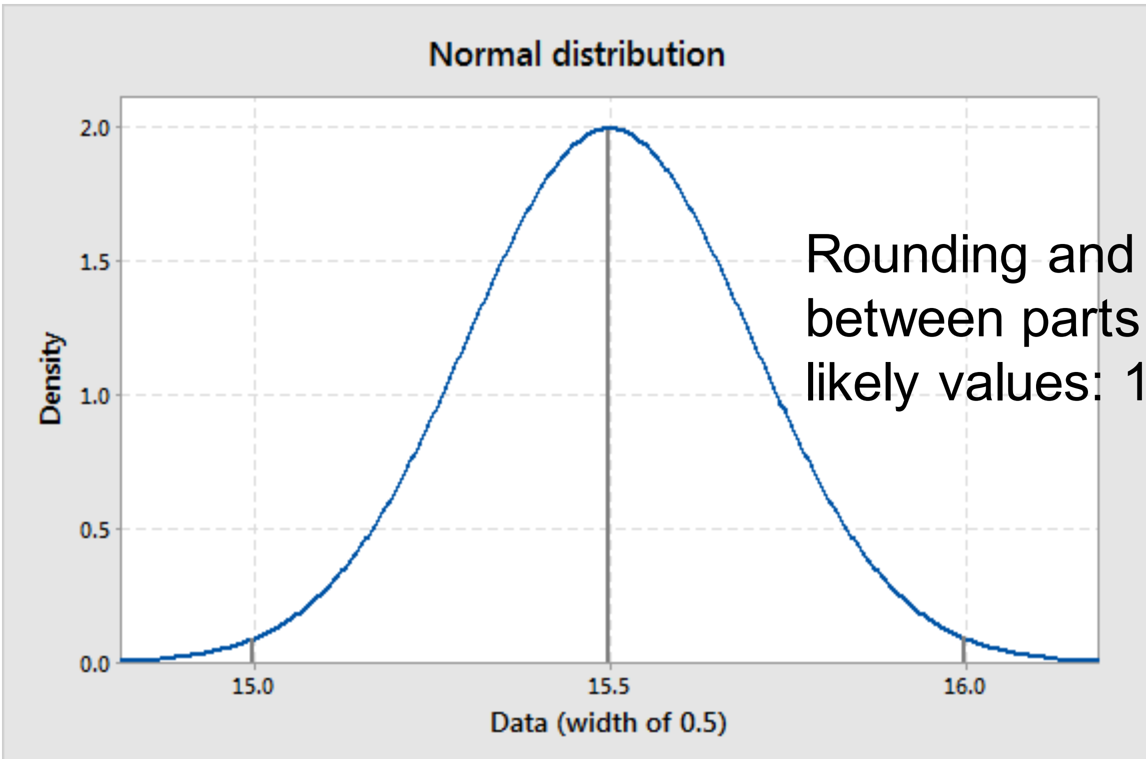
What is Low Discrimination?

A gauge with low discrimination is one that can not differentiate between parts that are different due to:

1. The gauge not having enough decimal places to distinguish parts from each other. It could be the gauge, or the variation of the process is so small no gauge will be good enough.
2. Digital gauge rounds to the nearest integer, or for simplicity the operator collects the data by dropping several decimal places.

Normality Test Failed (discrimination)

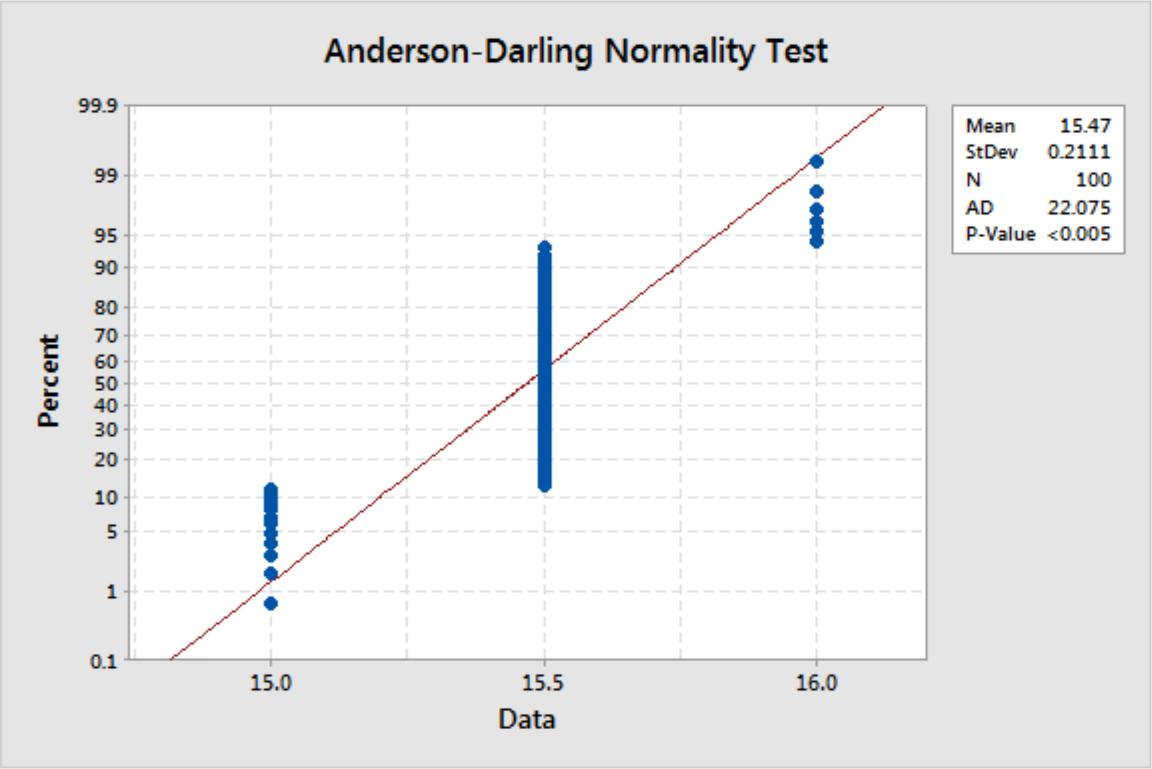
► Low discrimination of the measurement system.



Rounding and low discrimination between parts generates only three likely values: 15, 15.5, 16.

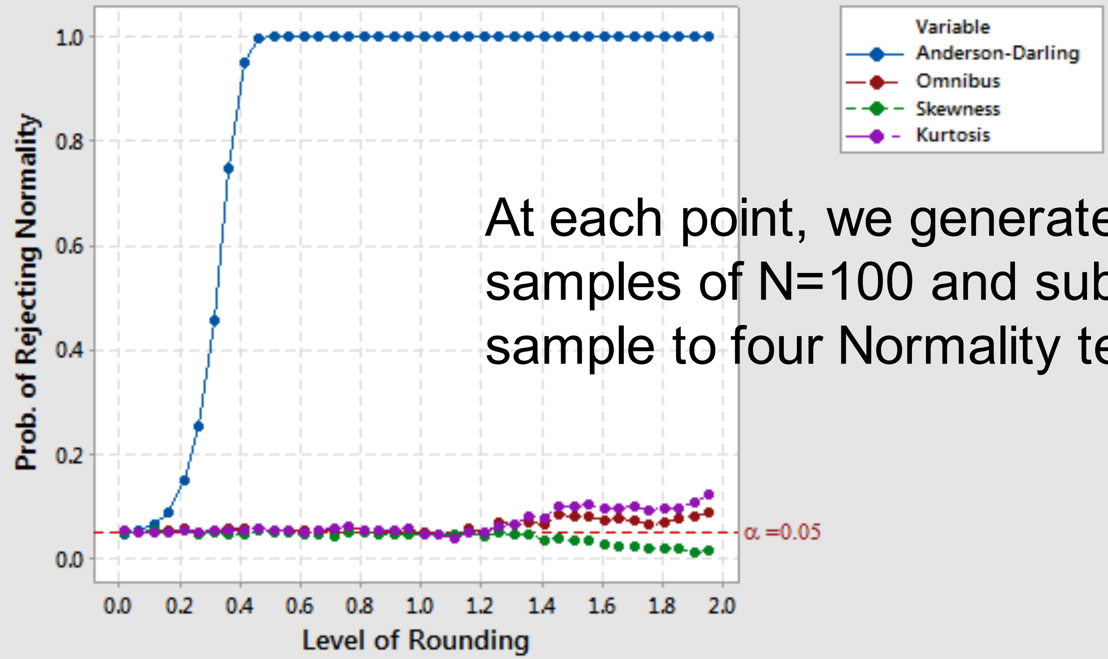
Normality Test Failed (discrimination)

- Ties lead to the Anderson-Darling (AD) test failing normality.



Normality Test Failed (discrimination)

► For heavily rounded datasets the Skewness (SK), Kurtosis (KT), Omnibus (OB) tests may be preferred.

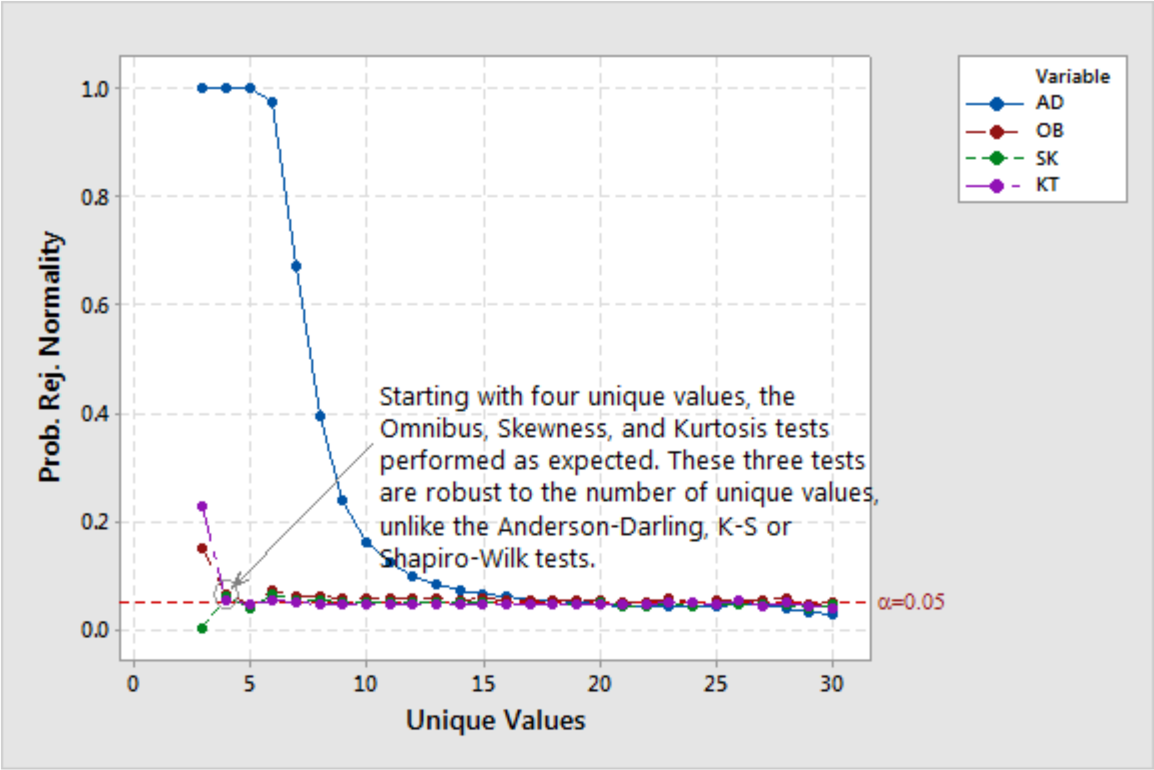


Note: Each simulation was made with a sample size of 100 normally distributed observations.

At each point, we generated 5000 normal samples of N=100 and subject each sample to four Normality tests.

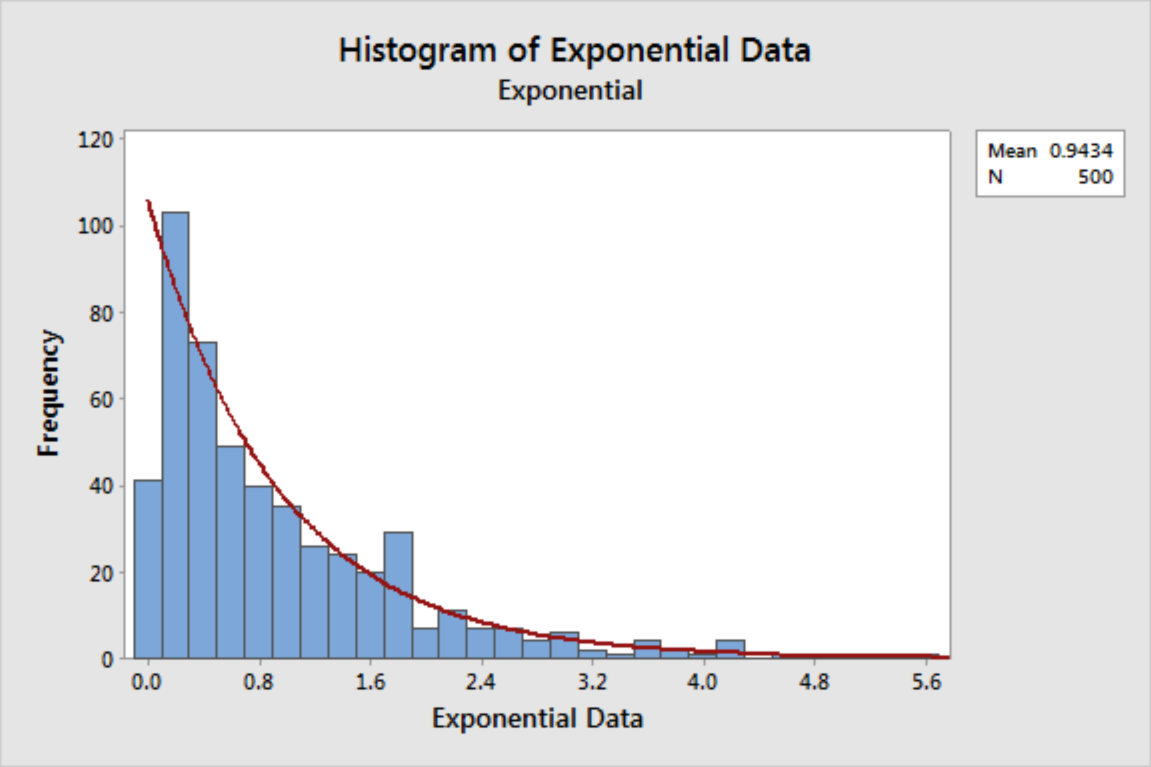
Normality Test Failed (discrimination)

- ► For practitioners it is easier to visualize and think about the x-axis as the number of distinct categories or values.



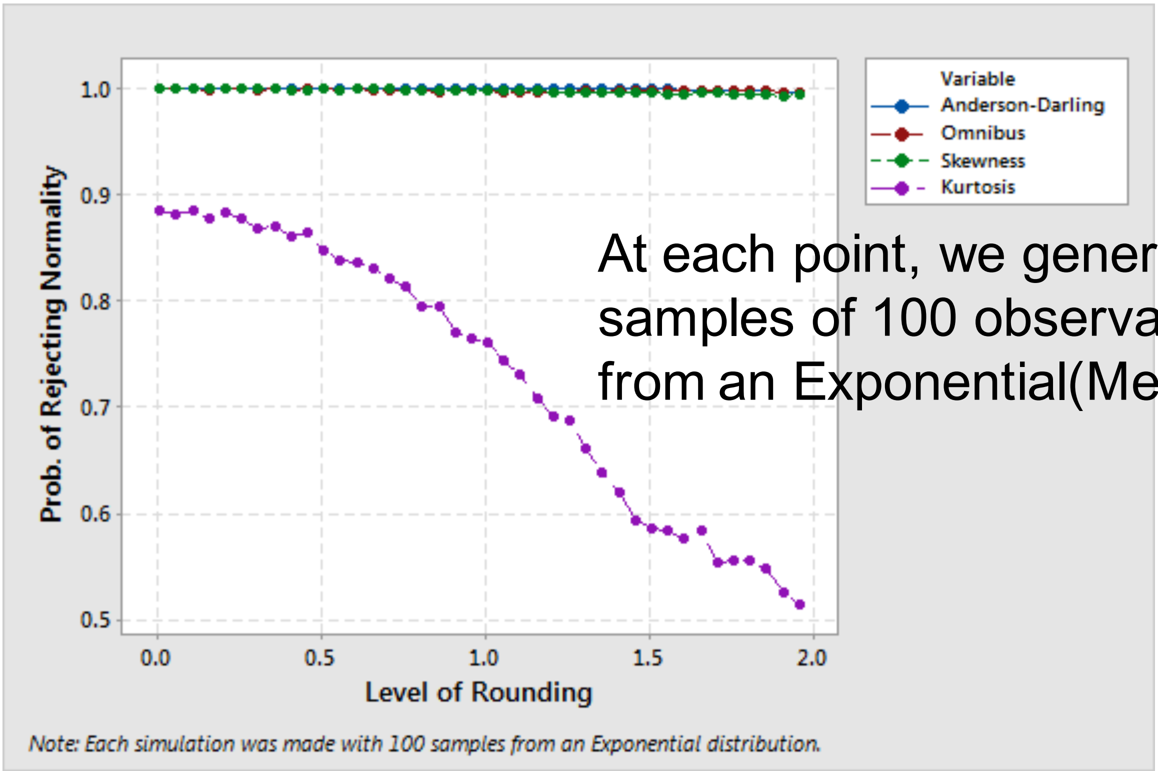
Normality Test Failed (discrimination)

► Let's consider now a highly skewed distribution, namely the exponential distribution.



Normality Test Failed (discrimination)

► The AD seems to perform as well as the OB and SK tests, but its detection of non-normal data comes as a consequence of rounding.



At each point, we generated 5000 samples of 100 observations each from an Exponential(Mean=1).

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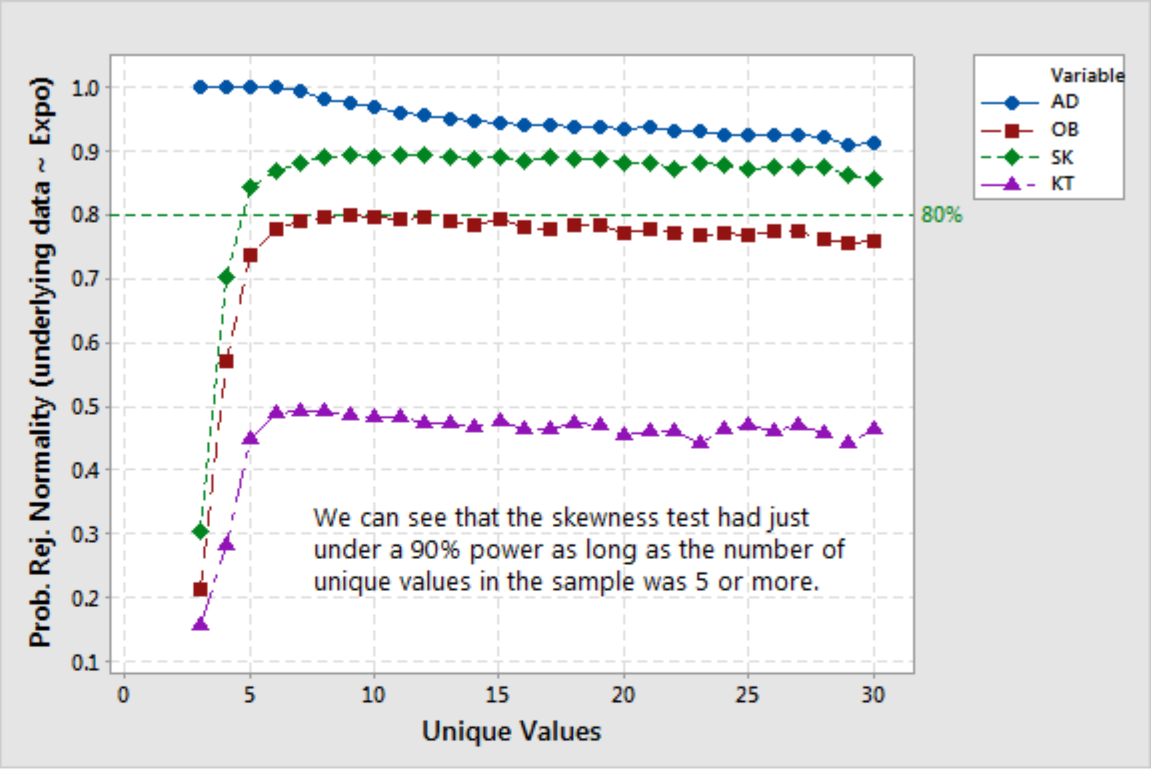
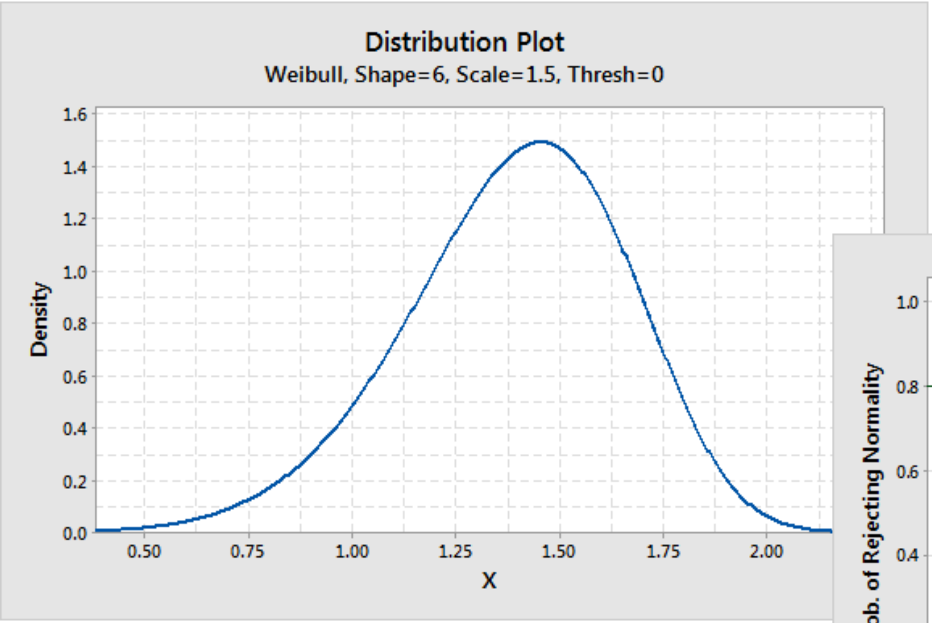
Normality Test Failed (discrimination)

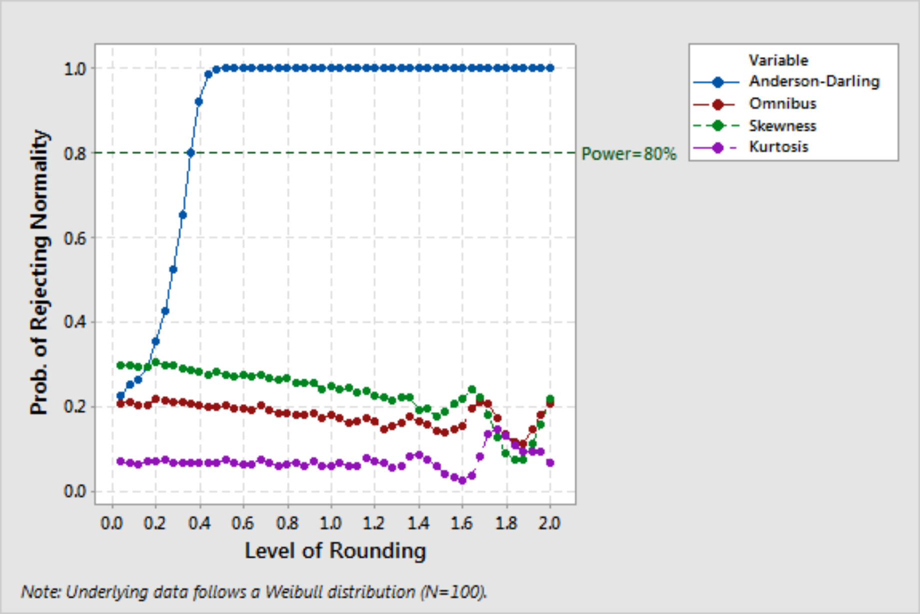
- Once again, as a simple rule of thumb we can confidently identify the data is not normal if the number of distinct values is four or higher.



Normality Test Failed (discrimination)

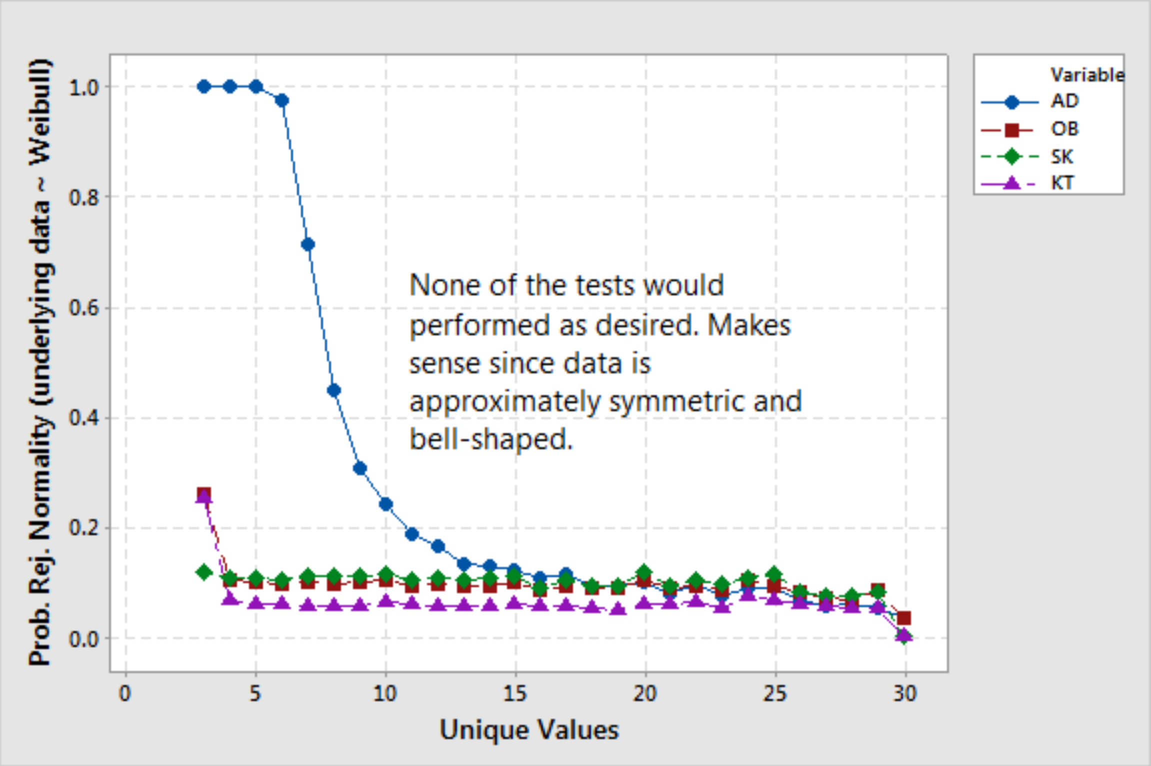
► What if the data is not severely skewed but still not normally distributed?





Normality Test Failed (discrimination)

► Finally, any kind of rounding would render the tests ineffective. The false detection from the AD test is due to rounding.



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What Normality Test to Use?

- Skewness fails to reject normality at the expected α level when the data are normal.
- Skewness has good power and is less sensitive to the degree of rounding.
- The Omnibus and Skewness tests have similar behavior.

Capability with Rounded Data

- There a few approaches to estimate the capability of a process:

1. Classic approach
2. Adjust the estimates considering the bias induced by the measurement system
3. Handle the data as being interval-censored

Classic Capability Estimates

- ► The rounded data, denoted *Y**, is assumed to be normally distributed:

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} Y_i^{\,*} \qquad \hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i^{\,*} - \bar{Y})^2}{n-1}}$$

- ► Proceed to estimate Cpk as usual.

Adjust the Standard Deviation

- Sheppard [5] describes the bias in the estimation of the standard deviation when the data is rounded.

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i{}^{*} - \bar{Y})^2}{n-1} - \frac{w^2}{12}}$$

where *w* is the width (incremental unit) of the measurement system.

Estimates from Interval-Censored Data

► Suppose we don't know the exact value that corresponds to the true measurement but we know the interval in which it must be.

Observed Values
5.1
5.1
5.0
5.2
5.1
5.0

Start	End	Frequency
4.95	5.05	2
5.05	5.15	3
5.15	5.25	1

Estimates from Interval-Censored Data

- We get the following estimates:

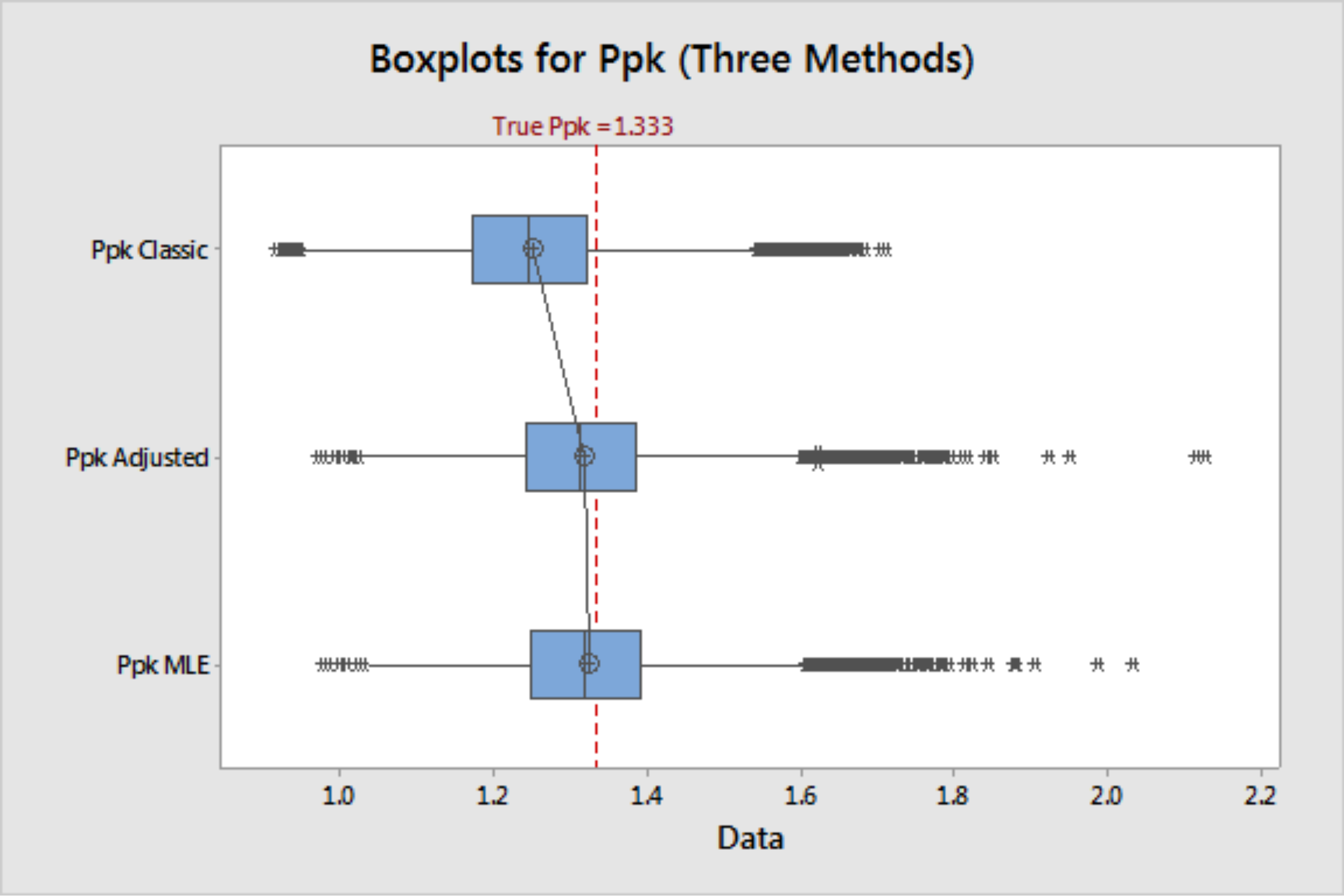
```
Variable Start: Start  End: End
Frequency: Frequency

Censoring Information    Count
Interval censored value      6
```

Estimation Method: Maximum Likelihood (MLE)

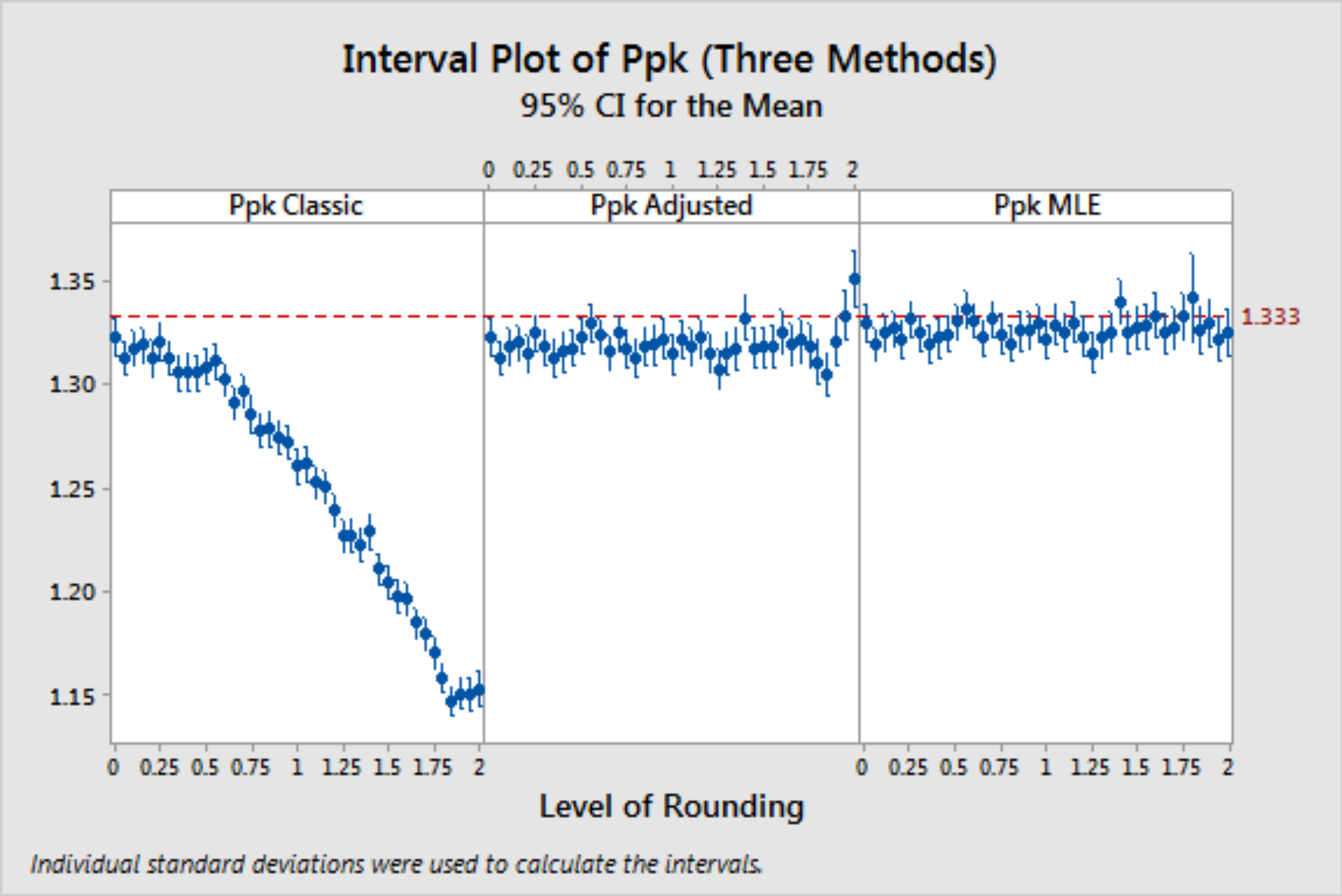
```
Distribution:   Normal

Parameter Estimates
```

Parameter	Estimate	Standard Error	95.0% Normal CI Lower	95.0% Normal CI Upper
Mean	**5.08345**	0.0278668	5.02883	5.13807
StDev	**0.0619204**	0.0218490	0.0310086	0.123648

Capability with Rounded Data



Capability with Rounded Data



Conclusions

- The ultimate goal of a capability analysis is to estimate the defective level of a process.
- When the interest of an analysis is on the estimation of defects, the distribution assumption will be an important one.
- Another important assumption is ensuring the process is stable and in control.

Conclusions

- Of the two assumptions, normality is the one typically violated in practice.
- Non-normal (NN) capability analysis requires:
  - Using a transformation
  - Finding an alternative distribution that fits the data
  - Using a nonparametric approach which requires a large sample size

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Conclusions

- Classic normality tests (AD, KS, SW) typically reject normality when the data is heavily-rounded regardless of the underlying distribution.
- When using a gauge with low discrimination, use different tests to check for normality, e.g. Skewness or the Omnibus test.

Conclusions

- If no evidence exists of the rounded data not being normal, assume normality.
- Utilize interval-censoring (MLE) to estimate the mean and standard deviation.

References

1. Juran, J.M., Godfrey, A.M. “Juran’s Quality Handbook”. 5th edition, McGraw-Hill. New York, 1999.
2. Kane, V.E. (1986) “Process Capability Indices”. *Journal of Quality Technology*, 18, 41-52.
3. McComack, D.W., Harris, I.R., Hurwitz, A.M., and Spagon, P.D. (2000) “Capability Indices for Non-normal data”, *Quality Engineering*. 12(4), 489-495.
4. Schneeweiss, H., Komlos, J., and Ahmad, A.S. (2006) “Symmetric and Assymetric Rounding.” Working paper.
5. Sheppard, W.F. (1898). “On the calculation of the most probable values of frequency constants for data arranged according to equidistant division of a scale.” *Proceedings of the London Mathematical Society.* 29, 231-258.
6. Tricker, A.R. (1984) “Effects of Rounding on the Moments of a Probability Distribution.” *Journal of the Royal Statistical Society. Series D (The Statistician).* 33(4), 381-390.

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ADDITIONAL SLIDES

Case Study

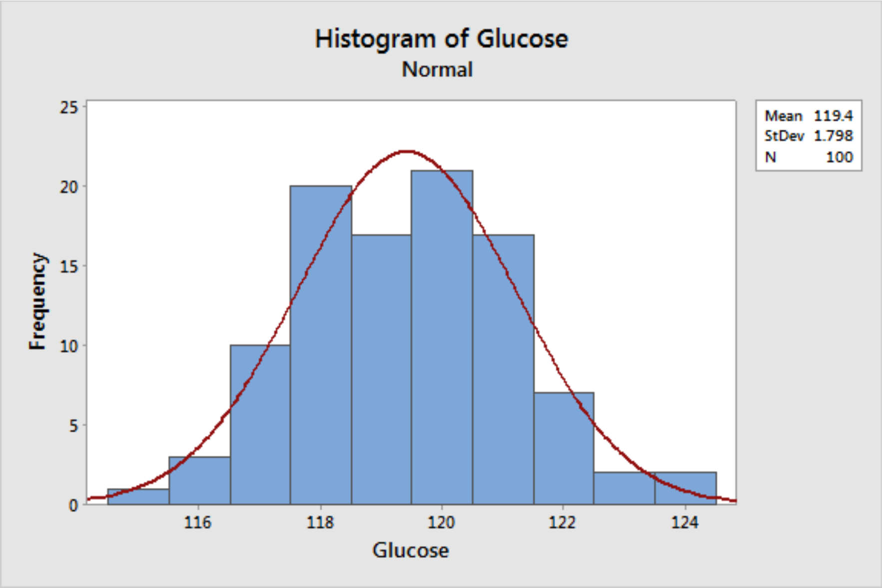
A medical device manufacturer builds a blood glucose measurement apparatus for diabetics to use at home. The reading has to be truncated so that it is easy for the customer to read and understand. They measure a standard solution on 100 devices to set a baseline. The specs are [99, 136].

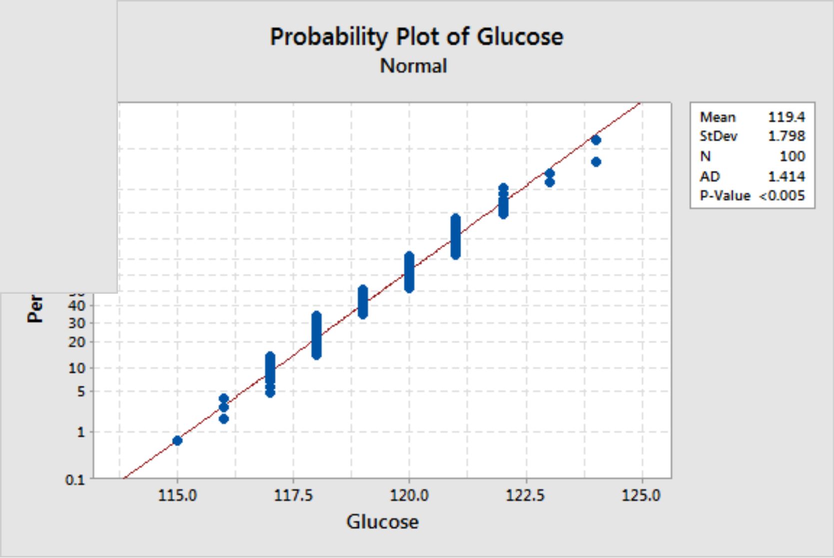
Data

122	121	119	119	123	116	119	120	119	121
118	120	118	120	117	116	120	118	121	120
117	118	119	120	118	118	120	119	120	123
120	117	119	121	120	121	118	117	119	118
120	120	120	122	118	120	117	119	121	117
121	118	117	118	122	119	120	120	120	118
122	119	121	118	118	119	118	121	119	120
116	122	120	117	124	117	120	121	120	115
124	121	118	119	118	121	119	118	122	121
117	118	122	121	121	121	121	119	118	119

Case Study

- Classic Normality tests fail.





Case Study

- Try an alternative normality test instead, such as the Skewness test.

```
Total number of observations in Glucose = 100
```

Data Display

```
Z        0.600407
P-value  0.548235
```

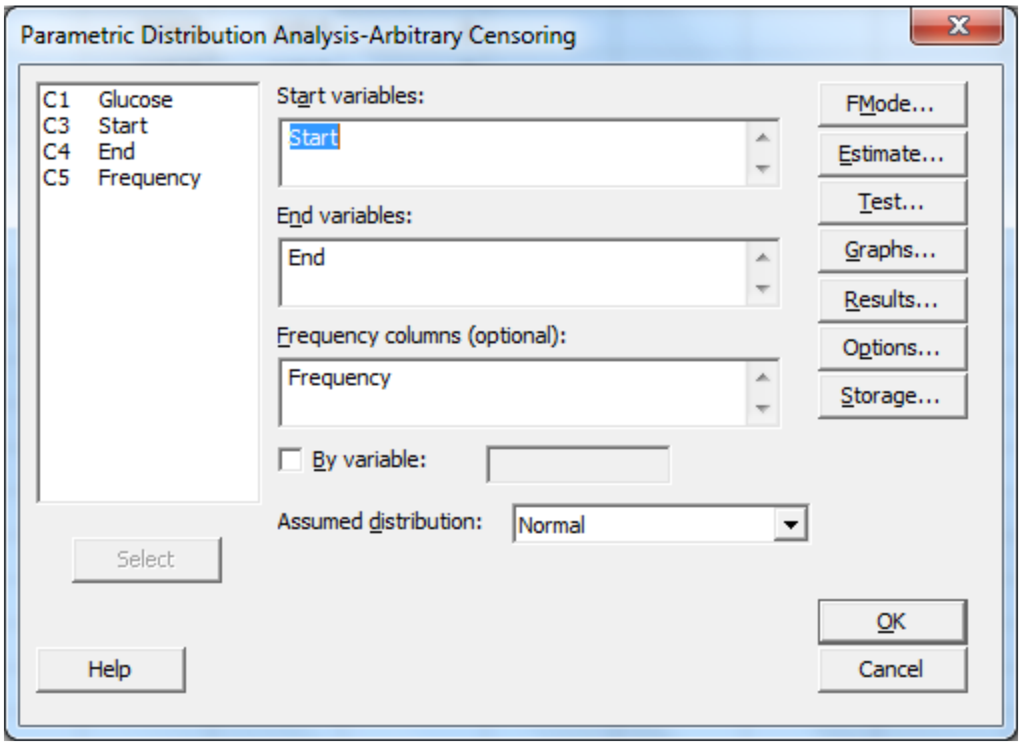
Case Study

► Convert the data to the following format.

Start	End	Frequency
114.5	115.5	1
115.5	116.5	3
116.5	117.5	10
117.5	118.5	20
118.5	119.5	17
119.5	120.5	21
120.5	121.5	17
121.5	122.5	7
122.5	123.5	2
123.5	124.5	2

Case Study

- Treat the data as interval-censored and analyze it with Parametric Distribution Analysis to get the estimates of ! and ?



Case Study

- Finally, with the estimates of 119.41 for the mean and 1.766 for the standard deviation proceed to estimate Ppk as usual.

$$Ppk = min\left[\frac{USL - \hat{\mu}}{3\hat{\sigma}}, \frac{\hat{\mu} - LSL}{3\hat{\sigma}}\right] = 3.13$$