Monte Carlo normalization

If we generate $\mathrm{N_{tot}}$ Monte Carlo events for a given physics process with cross-section σ and integrated luminosity $\mathcal{L}$, the total expected number of events $\mathrm{n_{sel}}$ after all selections is given by:

$$\mathrm{n_{sel}} = \sigma \mathcal{L} \frac{\mathrm{N_{sel}}}{\mathrm{N_{tot}}}, \tag{1}$$

with $\mathrm{N_{sel}}$ the number of Monte Carlo events after all the selections. We use the notation of Monte Carlo events (raw events) as N and the number of expected events as n (normalized events).

Cross-section calculation

The cross-section is defined as:

$$\sigma = \frac{\mathrm{n_{obs}} - \mathrm{n_{bkg}}}{\mathrm{A}\epsilon\mathcal{L}} \tag{2}$$

The five parameters are independently obtained:

1. $\mathrm{n_{obs}}$ is the total number of observed events in the data (i.e. measured in our detector). We don't have the data yet, so we take the observed number of events as the sum of signal and background events estimated from the Monte Carlo normalized events: $\mathrm{n_{obs}} = \mathrm{n_{sig}} + \mathrm{n_{bkg}}$.

2. $\mathrm{n_{bkg}}$ is the total number of background events. This is estimated from the Monte Carlo simulation.

3. The acceptance $\mathrm{A} = \frac{\mathrm{N_{sel}}}{\mathrm{N_{tot}}}$ is the ratio of selected over total signal Monte Carlo events. It is computed using signal events only.

4. The efficiency ϵ represent small detector inefficiencies. For now, we take ϵ equal to 1 (perfect detector).

5. The integrated luminosity $\mathcal{L}$ is a measure of the total amount of collisions delivered by the accelerator.

Cross-section statistical uncertainty calculation

All the parameters in the cross-section equation are uncorrelated, therefore the total statistical uncertainty on the cross-section $\delta\sigma$ (units in pb) can be calculated using error propagation (i.e. 5 terms):

$$\begin{aligned}\delta\sigma^2 = {} & \left(\frac{1}{\mathrm{A}\epsilon\mathcal{L}}\right)^2 \delta\mathrm{n}_{\mathrm{obs}}^2 + \left(\frac{1}{\mathrm{A}\epsilon\mathcal{L}}\right)^2 \delta\mathrm{n}_{\mathrm{bkg}}^2 + \left(\frac{\mathrm{n_{obs}} - \mathrm{n_{bkg}}}{\epsilon\mathcal{L}\mathrm{A}^2}\right)^2 \delta\mathrm{A}^2 \\ & + \left(\frac{\mathrm{n_{obs}} - \mathrm{n_{bkg}}}{\mathrm{A}\mathcal{L}\epsilon^2}\right)^2 \delta\epsilon^2 + \left(\frac{\mathrm{n_{obs}} - \mathrm{n_{bkg}}}{\mathrm{A}\epsilon\mathcal{L}^2}\right)^2 \delta\mathcal{L}^2.\end{aligned} \tag{3}$$

We need to evaluate the different uncertainties of each term (i.e. the δ's). Currently, we only take into account the contributions from $\mathrm{n_{obs}}$, $\mathrm{n_{bkg}}$, A, and $\mathcal{L}$ (the efficiency ϵ we take as 1 and zero uncertainty).

1. $\delta \mathrm{n_{obs}}$ is given by the Poissionian uncertainty of the total number of expected events:

$$\delta \mathrm{n_{obs}} = \delta(\mathrm{n_{sig}} + \mathrm{n_{bkg}}) = \sqrt{\mathrm{n_{sig}} + \mathrm{n_{bkg}}}. \tag{4}$$

2. The statistical uncertainty on the acceptance is computed from Binomial statistics as the numerator ($\mathrm{N_{sel}}$) and denominator ($\mathrm{N_{tot}}$) are correlated. The statistical error is given by:

$$\delta \mathrm{A} = \sqrt{\frac{\mathrm{A}(1-\mathrm{A})}{\mathrm{N_{tot}}}}. \tag{5}$$

3. The statistical uncertainty of the background $\Delta \mathrm{n_{bkg}}$ is purely related to the amount of generated Monte Carlo events:

$$\delta \mathrm{n_{bkg}} \equiv \delta(\sigma \mathcal{L} \frac{\mathrm{N_{sel}}}{\mathrm{N_{tot}}}) \approx \sigma \mathcal{L} \Delta(\frac{\mathrm{N_{sel}}}{\mathrm{N_{tot}}}), \tag{6}$$

where we assume the error on the cross-section luminosity for the background calculation to be negligible (moreover they are not from a statistical nature). The latter term is similar to the acceptance and is therefore described by a Binomial uncertainty:

$$\delta \mathrm{n_{bkg}} = \sigma \mathcal{L} \sqrt{\frac{\mathrm{N_{sel}}}{\mathrm{N_{tot}^3}}(\mathrm{N_{tot}} - \mathrm{N_{sel}})}. \tag{7}$$

For several background processes considered, the second term in Equation 3 changes to treat each background as statistically independent:

$$\left(\frac{1}{\mathrm{A}\epsilon\mathcal{L}}\right)^2 \delta \mathrm{n_{bkg}^2} \to \Sigma_{i \in \mathrm{backgrounds}} \left(\frac{1}{\mathrm{A}\epsilon\mathcal{L}}\right)^2 \delta \mathrm{n_{bkg,\ i}^2} \tag{8}$$

4. The uncertainty on the luminosity is not statistical, but relies on a separate measurement, and the uncertainty depends on the method used. The target at the Z-pole run is a relative uncertainty of 10^{-4} (corresponding to $\delta\mathcal{L} = 0.015$ $\mathrm{ab^{-1}}$). At the Higgs pole, we can estimate to first order an uncertainty of $\delta\mathcal{L} = 0.3\ \mathrm{ab^{-1}}$.