Answer Key
Unit 2: Functions

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1. A function maps each element in the domain to exactly one element in the range. Example: $f(x) = x^3 + 1$ is neither even nor odd.
2. Even, because $f(-x) = |-x| = |x| = f(x)$.
3. $f(-2) = 3(-2) + 2 = -6 + 2 = -4$.
4. If $f(x_1) = f(x_2)$ implies $x_1 = x_2$ for all x_1, x_2 in the domain.
5. No, because $f(2) = f(-2) = 4$.
6. All real numbers except $x = 3$.
7. Range: $[0, \infty)$.
8. $x + 4 > 0 \rightarrow x > -4$.
9. $-3 \leq x \leq 3$.
10. Range: $[-1, 1)$.
11. $(f \circ g)(x) = f(g(x)) = 2x^2 + 3$.
12. $(g \circ f)(x) = g(\sqrt{x}) = \sqrt{x} - 2$.
13. $(f \circ g)(2) = 1/(2 + 5) = 1/7$.
14. $(g \circ f)(x) = (3 - x)^2$.
15. Not necessarily. In general, $f \circ g \neq g \circ f$.
16. $f(x) = 2x^3 - x^2 + 5x - 7$.
17. $f(x) = 5^x$.
18. $f(x) = (x^2 + 1)/(x - 4)$.
19. $f(x) = \{x^2, x < 0; 2x + 1, x \geq 0\}$.
20. $f(x) = \sin(x)$.
21. The inverse of f, denoted f^{-1}, is a function such that $f(f^{-1}(x)) = x$.
22. $y = 2x + 5 \rightarrow x = (y - 5)/2 \rightarrow f^{-1}(x) = (x - 5)/2$.
23. Yes, because it is strictly increasing on $\mathbb{R}$.
24. $y = (x - 3)/(x + 2) \rightarrow xy + 2y = x - 3 \rightarrow xy - x = -3 - 2y \rightarrow x(y - 1) = -(3 + 2y) \rightarrow x = -(3 + 2y)/(y - 1)$.
25. $f(f^{-1}(x)) = e^{(\ln(x))} = x$, and $f^{-1}(f(x)) = \ln(e^x) = x$.
26. $(x - 3)(x + 3)$.
27. $(x - 2)(x^2 + 2x + 4)$.
28. $(x + 2)(x + 3)$.

29. $(x + 3)(x^2 - 3x + 9)$.
30. $(x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$.
31. $x = 2$ or $x = 3$.
32. $x = (-3 \pm \sqrt{(9 + 16)})/4 = (-3 \pm 5)/4 \rightarrow x = 1/2$ or $x = -2$.
33. $D = (-4)^2 - 4(1)(4) = 0$, so one repeated real root.
34. $x = (-1 \pm i\sqrt{3})/2$.
35. $x = 1/2$ (repeated root).
36. $x > 3$.
37. $x \leq 2$.
38. $-5 < x - 2 < 5 \rightarrow -3 < x < 7$.
39. $x \geq 11$.
40. $-2x < 4 \rightarrow x > -2$.
41. Because by definition, a function assigns exactly one output to each input.
42. No, by the vertical line test.
43. $2 + 2 = 4$.
44. $x^3 - x = -x^3 + x \rightarrow 2x^3 - 2x = 0 \rightarrow 2x(x^2 - 1) = 0 \rightarrow x = 0, \pm 1$.
45. $(2 + 1/2) + (1/2 + 2) = 2.5 + 2.5 = 5$.
46. $\sqrt{(x + 1)} - \sqrt{x} = 1 \rightarrow$ Let $\sqrt{x} = t \rightarrow \sqrt{(t^2 + 1)} - t = 1 \rightarrow \sqrt{(t^2 + 1)} = t + 1 \rightarrow t^2 + 1 = t^2 + 2t + 1 \rightarrow 2t = 0 \rightarrow t = 0 \rightarrow x = 0$.
47. Inverse formula: $f^{-1}(x) = (x - b)/a = 2x - 3 \rightarrow 1/a = 2 \rightarrow a = 1/2$, and $-b/a = -3 \rightarrow b = 3/2$.
48. Not over $\mathbb{R}$, only over $[0, \infty)$ because f is not one-to-one over $\mathbb{R}$.
49. Equal roots when $k^2 - 16 = 0 \rightarrow k = \pm 4$.
50. $f(1) = 2 - 3 + 1 = 0$, $f(-1) = 2 + 3 + 1 = 6 \rightarrow$ sum $= 6$.