1. $y'' - x^2 y' - xy = 0$
2. $y''' = y'' - x^2 (y')^2$
3. Aim to get this form: $\begin{cases} \dfrac{dx}{dt} = F(t, x, y), \\ \dfrac{dy}{dt} = G(t, x, y). \end{cases}$
4. The system $\begin{cases} \dfrac{dx}{dt} = 4x - y, \\ \dfrac{dy}{dt} = 2x + y \end{cases}$ has the independent solutions

$$\{x = e^{3t},\ y = e^{3t}\} \quad \text{and} \quad \{x = e^{2t},\ y = 2e^{2t}\}.$$

Will focus on the simple cases (Linear Systems):

5. Non-Homogeneous: $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t) \end{cases}$
6. Homogeneous: $\begin{cases} \dfrac{dx}{dt} = a_1(t)x + b_1(t)y + f_1(t), \\ \dfrac{dy}{dt} = a_2(t)x + b_2(t)y + f_2(t) \end{cases}$

Real-World Example 1: Biology (Predator–Prey Model)

Variables:

$$t = \text{time}, \quad x(t) = \text{rabbits}, \quad y(t) = \text{foxes}.$$

System:

$$\begin{cases} \dfrac{dx}{dt} = ax - bxy, \\ \dfrac{dy}{dt} = cxy - dy. \end{cases}$$

Meaning:

- Rabbits grow on their own.
- Rabbits decrease when foxes hunt them.
- Foxes grow when rabbits are available.
- Foxes decrease when food is scarce.

Real-World Example 2: Economics (Supply–Demand Model)

Variables:

$$t = \text{time}, \quad x = \text{price}, \quad y = \text{quantity supplied.}$$

System:

$$\begin{cases} \dfrac{dx}{dt} = A - By, \\ \dfrac{dy}{dt} = Cx - Dy. \end{cases}$$

Meaning:

- Price decreases when supply is high.
- Price increases due to cost/demand.
- Producers supply more when prices are high.
- Supply decreases as goods are used or sold.