Birla Institute of Technology and Science Pilani (Raj.)

First Semester 2025 – 2026

Multivariable Calculus (MATH F101)

Tutorial Sheet - 5

Q1. Consider the function

$$f(x,y) = \begin{cases} x^2 \tan^{-1}\left(\dfrac{y}{x}\right) + y^2 \tan^{-1}\left(\dfrac{x}{y}\right), & \text{for } xy \neq 0, \\ 0, & \text{for } xy = 0. \end{cases}$$

Find $f_{xy}(0,0)$. Examine the continuity of f at $(0,0)$. Without using polar form, examine the continuity of f_y at $(0,0)$.

Q2. Let $f(x,y) = \dfrac{1}{2}\left(\Big||x| - |y|\Big| - |x| - |y|\right)$. Find all the directions in which the directional derivative of f exists at the origin. Is f differentiable at the origin? Justify.

Q3. Is there a direction u in which the rate of change of $f(x,y,z) = xy^2 - yz^2 + zx^2$ at $P(1,-1,-1)$ is -4? Justify.

Q4. Let $f(x,y) = x^2 + y^2$. Find all unit vectors $\mathbf{u}$ so that the directional derivative $D_{\mathbf{u}}f(1,-1) = 0$.

Q5. Find the parametric equation of the tangent line to the curve of intersection of the surfaces $z = x^2 + y^2$ and $z = 4 - y$ at the point $P_0(2,-1,5)$.

Q6. Let $f(x,y,z) = \ln(x^2 + y^2 + z^2)$. Find

(a) the equation of the level surface that passes through the point $(1,0,2)$.

(b) the equation of the tangent plane to the level surface at the point $(1,0,2)$.

(c) the parametric equations of the normal line at the point $(1,0,2)$.

Q7. Find the linear approximation of $f(x,y) = \ln(x^2 + y^2)$ at the point $(1,1)$.

Q8. Examine the following functions for local maxima, local minima and saddle points:

(a) $f(x,y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x$

(b) $f(x,y) = x^3 + 3x^2 + y^2 + 4xy$

Q9. Find the absolute maxima of $f(x,y) = xy$ on the unit disc $\{(x,y) : x^2 + y^2 \leq 1\}$.

Q10. A ring in the form of a circle $x^2 + y^2 = 1$ is heated in such a way that its temperature at (x,y) is $T = x^2 + 2y^2 - x$. Find the hottest and coldest points of the ring.