Orthogonality & Fourier Coefficients on $[-\pi, \pi]$

Orthogonality of $1,\ \sin(nx),\ \cos(nx)$

$$\int_{-\pi}^{\pi} \cos(nx)\, dx = 0\ (n \neq 0), \qquad \int_{-\pi}^{\pi} \sin(nx)\, dx = 0$$

$$\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases} \qquad \int_{-\pi}^{\pi} \sin(mx)\sin(nx)\, dx = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\, dx = 0 \quad \text{(all } m, n\text{)}$$

Fourier Series: $\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx\right)}$

Euler Formulas (coefficients)

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx, \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx$$