

# Sine & Cosine at Key Angles

with Cofunction Identities

([dbhoriya.github.io/teaching\\_F101/tutorial\\_2.pdf](https://dbhoriya.github.io/teaching_F101/tutorial_2.pdf)) or SCAN:

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## Some Trigonometry identities:

### Exact Values at Common Angles

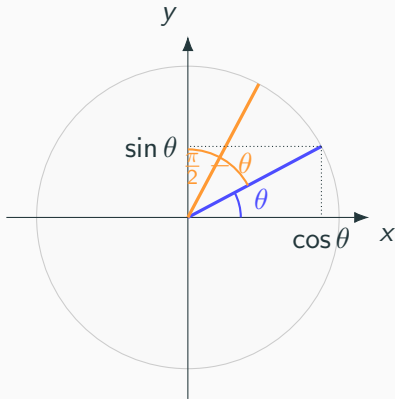
$\theta$	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

### Some identities:

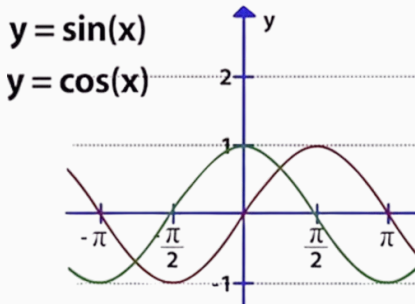
$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta,$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta.$$

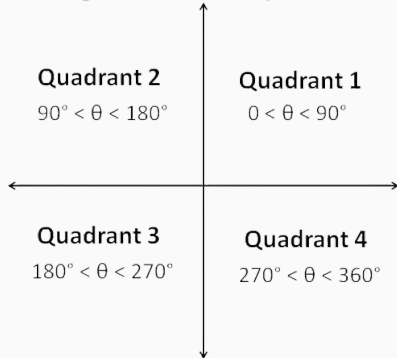
**Example:**  $\sin 30^\circ = \cos 60^\circ = \frac{1}{2}, \quad \cos 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}.$



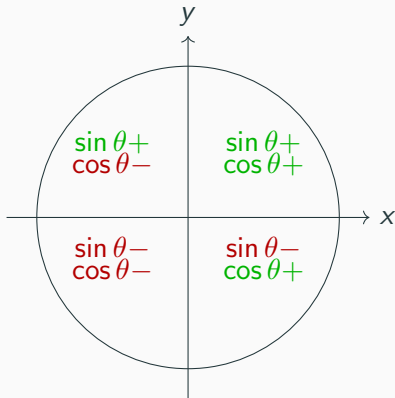
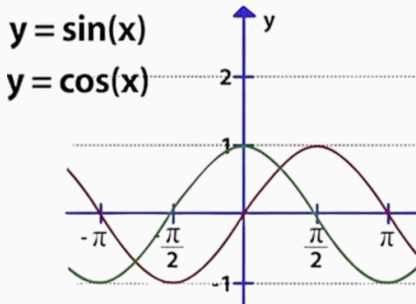
# Sin-Cos in various quadrants



## Angles in different quadrants



# Sin-Cos in various quadrants



# **Polar co-ordinates**

Definition and motivation

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# Polar Coordinates: Definition & Formulas

- A point  $(x, y)$  is represented as  $(r, \theta)$ :

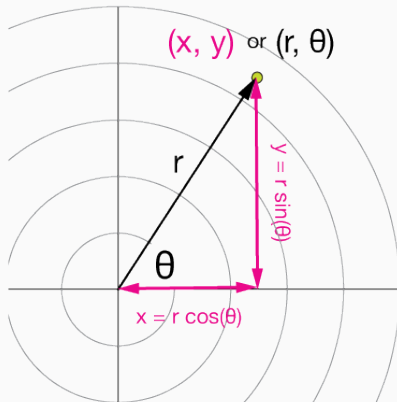
- $r$  = distance from origin
- $\theta$  = angle from positive x-axis

- Conversion to Cartesian coordinates:

$$x = r \cos \theta, \quad y = r \sin \theta$$

- Conversion from Cartesian coordinates:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \arctan \frac{y}{x} = \tan^{-1} \left( \frac{y}{x} \right)$$



# Why Do We Need Polar Coordinates? Real-Life Motivation

- Some situations are easier to describe with **distance + angle** than  $(x, y)$ .
- **Examples:**
  - GPS: “5 km at 30° NE”
  - Radar/Sonar: Track objects by angle & range
  - Lighthouses: Light beam at a specific distance & direction
  - CCTV: Camera coverage via rotation & reach



# Engineering & Math Usefulness

- **Engineering:**

- AC circuits: voltages/currents as polar vectors
- Mechanical systems: gears, turbines, rotating parts
- Robotics: arm positions via joint angles
- Civil structures: domes, arches, tunnels

- **Mathematics:**

- Circle equation:  $x^2 + y^2 = a^2 \Rightarrow r = a$  (simpler in polar)
- Ideal for problems with circular or rotational symmetry



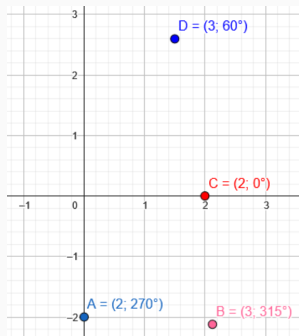
## Polar Coordinates: Tutorial Sheet-2

Plot the following points, given in polar coordinates, and find all polar coordinates for each point.

$$\mathbf{A:} (-2, \pi/2), \quad \mathbf{C:} (-2, -5\pi)$$

$$\mathbf{B:} (3, -\pi/4), \quad \mathbf{D:} (3, \pi/3)$$

*Hint: Remember that a point  $(r, \theta)$  is equivalent to  $(-r, \theta + \pi)$  and that angles can be expressed modulo  $2\pi$ .*



**Answer:**

$$\mathbf{A:} (-2, \pi/2 + 2n\pi), (2, -\pi/2 + 2n\pi) \text{ where } n \in \mathbb{Z}$$

$$\mathbf{B:} (3, -\pi/4 + 2n\pi), (-3, 3\pi/4 + 2n\pi) \text{ where } n \in \mathbb{Z}.$$

$$\mathbf{C:} (-2, -\pi + 2n\pi), (2, 2n\pi) \text{ where } n \in \mathbb{Z}.$$

$$\mathbf{D:} (3, \pi/3 + 2n\pi), (-3, -2\pi/3 + 2n\pi) \text{ where } n \in \mathbb{Z}.$$

## Polar Coordinates: Tutorial Sheet-2

**Question 2:** Find the polar coordinates of the following points.

(a)  $(\sqrt{3}, -1)$ , with  $0 \leq \theta < 2\pi$  and  $r \leq 0$ .

(b)  $(\sqrt{3}, -1)$ , with  $-\pi \leq \theta < \pi$  and  $r \geq 0$ .

**Answer:** (a)  $(-2, \pi - \pi/6)$  (b)  $(2, -\pi/6)$

**Question 3:** Convert the following into Cartesian equations:

(a)  $r^2 + 2r^2 \cos \theta \sin \theta = 1$  (b)  $r \sin \theta = \ln r + \ln \cos \theta$ .

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**Answer:**

(a)  $x^2 + y^2 + 2r \cos \theta r \sin \theta = 1 \iff x^2 + y^2 + 2xy = 1$

(b)  $r \sin \theta = \ln r + \ln \cos \theta \implies r \sin \theta = \ln r \cos \theta$ .

This gives  $y = \ln x$

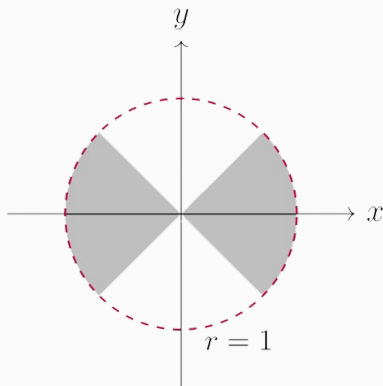
## Polar Coordinates: Tutorial Sheet-2

**Question 5:** Sketch the region defined by the inequalities  
 $-1 \leq r \leq 1$  and  $-\pi/4 \leq \theta \leq \pi/4$

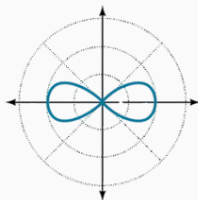
## Polar Coordinates: Tutorial Sheet-2

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**Answer:**

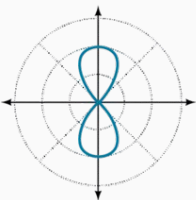


## Polar Coordinates: lemniscate (a polar curve resembling the infinity symbol $\infty$ )



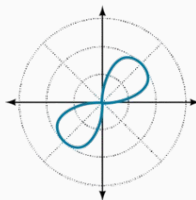
$$r^2 = a^2 \cos(2\theta)$$

(a)



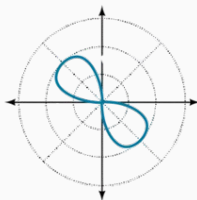
$$r^2 = -a^2 \cos(2\theta)$$

(b)



$$r^2 = a^2 \sin(2\theta)$$

(c)



$$r^2 = -a^2 \sin(2\theta)$$

(d)

**Question 4:** Draw the graph of the curve  $r^2 = -\sin 2\theta$ .