Birla Institute of Technology & Science, Pilani (Raj.)

First Semester 2025-2026, MATH F101 (Multivariable Calculus)

Tutorial Sheet 6

Q.1 Compute the double integral

$$\iint_R \frac{y}{x^2y^2+1}\, dA,$$

where $R = \{(x, y) : 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$.

Q.2 Sketch the region of integration and evaluate the integral

$$\int_0^{\frac{3}{2}} \int_1^{4-2u} \frac{4-2u}{v^2}\, dvdu.$$

Q.3 Sketch the region of integration, reverse the order, and evaluate the resulting integral

$$\int_0^4 \int_y^4 e^{-x^2}\, dxdy.$$

Q.4 The following sum of integrals give the area of region in the xy-plane. Sketch the region and then find the area of the region

$$\int_{-1}^0 \int_{-2x}^{1-x} dydx + \int_0^2 \int_{-\frac{x}{2}}^{1-x} dydx$$

Q.5 Evaluate the integral by changing the order of integration in an appropriate way

$$\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{ln\ 3} \frac{\pi e^{2x} sin \pi y^2}{y^2} dxdydz.$$

Q.6 Find the average value of $F(x, y, z) = x + y - z$ over the rectangular solid in the first octant bounded by the coordinate planes and the planes $x = 1, y = 1$ and $z = 2$.

Q.7 Sketch the region of integration and convert this polar integral to a Cartesian integral or sum of integrals

$$\int_0^{\frac{\pi}{2}} \int_0^1 r^3 \sin\theta \cos\theta\ drd\theta.$$

Q.8 Convert the following iterated triple integral I to an iterated triple integral in

(a) cylindrical co-ordinates in the order $dzdrd\theta$,

(b) spherical co-ordinates in the order $d\rho d\phi d\theta$, where

$$I = \int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^1 dzdydx.$$

Please turn over...

Q.9 Let D be the region in $xyz-$space defined by the inequalities

$$1 \leq x \leq 2, \quad 0 \leq xy \leq 2, \quad 0 \leq z \leq 1.$$

Evaluate

$$\iiint_D (x^2y + 3xyz)\, dxdydz$$

applying by transformation $u = x, \quad v = xy, \quad w = 3z$, and integrating over an appropriate region in uvw-plane.

Q.10 Evaluate the volume of the solid which is

(i) bounded below by the $xy-$plane, on the sides by the sphere $\rho = 2$, and above by the plane $z = 1$;

(ii) bounded below by the $xy-$plane, on the sides by the sphere $\rho = 2$, and above by the cone $\phi = \frac{\pi}{3}$.