

Some helping things

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(dbhoriya.github.io/teaching_F101/help.pdf) or SCAN:



Arc Length in Polar Coordinates

Derivation:

$$ds = \sqrt{dx^2 + dy^2}, \quad x = r \cos \theta, \quad y = r \sin \theta$$

$$dx = dr \cos \theta - r \sin \theta d\theta,$$

$$dy = dr \sin \theta + r \cos \theta d\theta,$$

$$dx^2 + dy^2 = (dr)^2 + r^2(d\theta)^2$$

$$\Rightarrow \boxed{ds = \sqrt{(dr)^2 + r^2(d\theta)^2}}$$

If $r = r(\theta)$:

$$ds = \sqrt{(r'(\theta))^2 + r(\theta)^2} d\theta, \quad L = \int_a^b \sqrt{(r'(\theta))^2 + r(\theta)^2} d\theta$$