

Survey of Scientific Computing (SciComp 301)

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Exam 1
Total of 100 points

1. Implement Heron's Method

In the **q01** folder, edit the C++ console application to calculate and display the square root of a random integer greater than one million, using **Heron's Method**, to 8 digits of precision to the right of the decimal point

Specifically you must implement the missing code in the function **heron**() to return the estimate of the square root of the parameter **S**

You may stop iterating when your current x^2 estimate is within 1×10^{-06} of **S**

$$x = initial \ guess \ of \ \sqrt{S} = s/2$$

$$S = (x + \Delta_x)^2 = x^2 + (2x)\Delta_x + \Delta_x^2$$

$$\Delta_x = \frac{S - x^2}{2x + \Delta_x}$$

$$Assume \ \Delta_x \ll x : \Delta_x \approx \frac{S - x^2}{2x}$$

$$x_{revised} = x + \Delta_x$$

$$x_{revised} \approx x + \frac{S - x^2}{2x}$$

$$\approx \left[\frac{2x^2}{(2x)} + \frac{S - x^2}{2x}\right] \approx \frac{1}{2} \left(\frac{S + x^2}{x}\right)$$

$$x_{revised} \approx Mean \left(\frac{S}{x}, x\right)$$

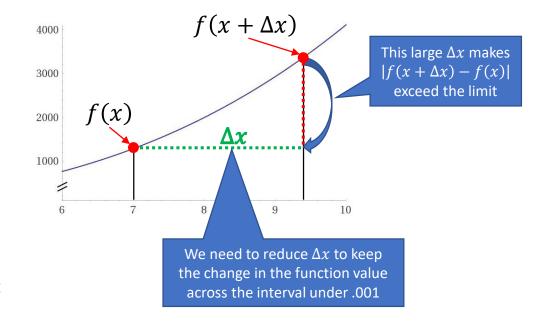
2. Adaptive Quadrature

In the **q02** folder, edit the C++ console application to calculate the following integral using the **midpoint rule**

$$f(x) = \int_0^{10} 5x^3 - 9x^2 + 11$$

Specifically you must complete the function **midPointAdaptive**() that, while using the midpoint area, will ensure that the maximum *change* in the function value between the left and right side of any interval is under the limit of **0.001**

The function **midPointFixed**() is set to use 1 million fixed width intervals. How does the adaptive quadrature compare in terms of relative % error and overall execution time?



Adaptive quadrature

Adaptive quadrature is a numerical integration method in which the integral of a function f(x) is approximated using static quadrature rules on adaptively refined subintervals of the integration domain. Generally, adaptive algorithms are just as efficient and effective as traditional algorithms for "well behaved" integrands, but are also effective for "badly behaved" integrands for which traditional algorithms fail.

3. Sum of Multiples

In the **q03** folder, edit the C++ console application to calculate and display the sum of all natural numbers less than 1,900 that are fully (cleanly, evenly) divisible by <u>both</u> 7 and 11

```
main.cpp 🗷
          #include "stdafx.h"
          using namespace std;
          int main()
              int termsMax = 1900;
              int sum = 0;
   10
              // Implement your code here
  11
              cout.imbue(locale(""));
  12
  13
              cout << "The sum of all natural numbers less than " << termsMax << endl</pre>
  14
  15
                   << "that are fully divisible by both 7 and 11 is " << sum << endl << endl;</pre>
  16
  17
              return 0;
  18
  19
   20
```

4. Temperature Converter

In the **q04** folder, edit the C++ console application copied from Session 02 Lab 03 to display the same range in temperatures as before, but this time converting from Celsius degrees to Fahrenheit degrees

Be sure to make the human readable output make sense because you are flipping which scale is on which side of the equality symbol

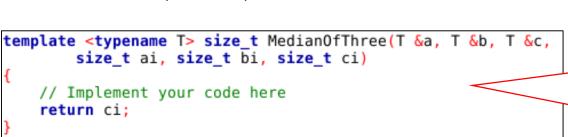
Your code changes must successfully compile and run as expected

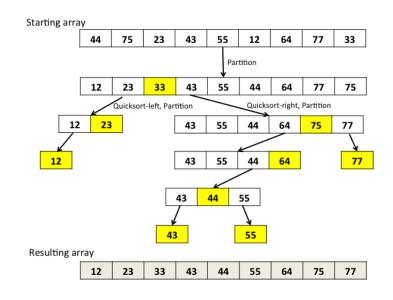
5. Quicksort Optimization

Research Hoare's Quicksort algorithm and understand the benefit of partitioning using the *median of three* for the pivot element - how does this effect the **total run time**?

In the **q05** folder, edit the C++ console application to implement the **MedianOfThree**() function: a, b, c are the values, ai, bi, ci are the indexes

Your function should return the *index* (ai, bi, or ci) corresponding to the **median** of the three values (a, b, or c)





The code initially is not using median of three.
Run it as-is, then see how the total time changes once you implement the correct function.

6. Lowest Common Multiple

In the **q06** folder, edit the C++ console application to calculate the lowest common multiple (LCM) of two integers a & b, using only basic arithmetic operators (*no looping*) and the greatest common divisor (**GCD**) function which is already provided

```
main.cpp 🗷
          #include "stdafx.h"
          using namespace std;
          int GCD(uint64 t a, uint64 t b)
   6
              return b == 0 ? a : GCD(b, a % b);
   8
          int LCM(uint64_t a, uint64_t b)
   10
   11
             // Implement your code here
  13
  14
  15
          int main()
  16
              uint64 t a = 447618;
   17
              uint64 t b = 2011835;
   18
   19
   20
              cout.imbue(locale(""));
   21
              cout << "LCM of " << a
                   << " and " << b << " is "
   22
   23
                   << LCM(a, b) << endl;
   24
   25
              return 0;
  26
   27
```

7. Vector Addition

In the **q07** folder, edit the C++ console application to calculate the addition of two vectors

In particular, complete the function **SumVectors**() by providing values for each element of **vec3** using some form of looping construct

```
main.cpp 🗷
   1
          #include "stdafx.h"
          using namespace std;
          template <typename T>
          void DisplayVector(const string& name, const vector<T>& vec)
   8
              cout << "Vector " << name << " = {";</pre>
   9
              for (size_t i{}; i < vec.size() - 1; i++)</pre>
  10
                  cout << vec.at(i) << ", ";
  11
              cout << vec.back() << "}" << endl << endl;</pre>
  12
  13
  14
          template <typename T>
  15
          vector<T> AddVectors(const vector<T>& vec1. const vector<T>& vec2)
  16
  17
              vector<T> vec3(vec1.size());
              // Implement your code here
  19
              return vec3;
  20
  21
  22
          int main()
  23
  24
              vector<int> a{ -2, 3, 6, 113, 49, 0, 123 };
  25
              vector<int> b{ 18, 13, 990, 2, -55, -9, 14 };
  26
  27
              DisplayVector("a", a);
  28
              DisplayVector("b", b);
  29
              DisplayVector("c", AddVectors(a, b));
  30
  31
              return 0;
  32
  33
```

8. Hamming Weight

In the **q08** folder, edit the C++ console application to calculate the Base 2 *Hamming Weight* of a given natural number

You must write the correct code to find the **PopCount()** for the passed in variable **n**

```
main.cpp 🗷
          #include "stdafx.h"
          #include <bitset>
          using namespace std;
          int PopCount(int n)
              int onebits = 0;
               // Insert your code here
  11
              return onebits;
   12
   13
  14
          int main()
   15
   16
              int n = 95601;
   17
   18
              cout << "The Hamming weight of " << n</pre>
   19
                    << " in base 2 is " << PopCount(n)</pre>
   20
                    << endl << endl:
   21
   22
              bitset<17> b(n):
   23
              cout << n << " in base 10 is "
   24
   25
                    << b.to string() << " in base 2" << endl;
   26
              cout << "Therefore the popcount is " << b.count()</pre>
   27
                    << endl << endl:
   28
   29
              return 0:
   30
   31
```

9. Multimodal Sets

If two or more elements appear an equal number of times within a list, the list is said to be multimodal

In the **q09** folder, edit the C++ console application to return all of the modes of a list

Specifically, you must implement the **FindModes**() function by populating the **modes** vector from the passed in **data** vector

```
main.cpp 🗷
          #include "stdafx.h"
          using namespace std;
          vector<int> FindModes(vector<int> data)
        \square{
              // Prepare return vector
    8
              vector<int> modes:
   9
              // Implement your code here
  10
  11
  12
              return modes;
  13
  14
  15
          int main()
  16
  17
              // Prepare data vector
  18
              vector<int> data {7,1,2,3,7,4,5,6,7,1,2,1,1,9,7,10,2,2};
  19
  20
              cout << "The modes of the data are: " << endl;</pre>
  21
              for (auto& n : FindModes(data))
  22
                   cout << n << endl;
  23
  24
              return 0:
  25
   26
```

10. Circle Lattice Points

In the **q10** folder, edit the C++ console application to calculate the integer lattice points in a circle

Specifically you must write the LatticePoints() function, which receives an integer radius, and must count all the lattice points within that radius

This is known as the Gauss Circle Problem, and Gauss was the first one to prove the number of lattice points is bounded by this expression:

$$N(r) = \pi r^2 + E(r)$$

where

$$|E(r)| \le 2\sqrt{2\pi r}$$

