

# Survey of Scientific Computing (SciComp 301)

Dave Biersach
Brookhaven National
Laboratory
dbiersach@bnl.gov

Exam 1
Total of 100 points

## 1. Implement Heron's Method

In the **q01** folder, edit the C++ console application to calculate and display the square root of a random integer greater than one million, using **Heron's Method**, to 8 digits of precision to the right of the decimal point.

Specifically you must implement the missing code in the function **heron**() to return the estimate of the square root of the parameter **s**.

You may stop iterating when your current  $x^2$  estimate is within  $1 \times 10^{-06}$  of s

$$x = initial \ guess \ of \ \sqrt{S}$$

$$S = (x + \Delta_x)^2 = x^2 + (2x)\Delta_x + \Delta_x^2$$

$$\Delta_x = \frac{S - x^2}{2x + \Delta_x}$$

$$Assume \ \Delta_x \ll x \therefore \Delta_x \approx \frac{S - x^2}{2x}$$

$$x_{revised} = x + \Delta_x$$
 $x_{revised} \approx x + \frac{S - x^2}{2x}$ 
 $\approx \left[\frac{2x^2}{(2x)} + \frac{S - x^2}{2x}\right] \approx \frac{1}{2} \left(\frac{S + x^2}{x}\right)$ 
 $x_{revised} \approx Mean\left(\frac{S}{x}, x\right)$ 

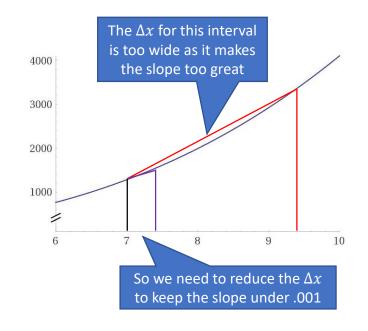
### 2. Adaptive Quadrature

In the **q02** folder, edit the C++ console application to calculate the following integral using the **midpoint rule** 

$$F(x) = \int_0^{10} 5x^3 - 9x^2 - 11$$

Specifically you must complete the function **midPointAdaptive**() that while using the midpoint rule will ensure that the maximum slope of any given interval on the curve will be less than **0.001** 

The function **midPointFixed**() is set to use 1 million fixed width intervals. How does the adaptive quadrature compare to this in terms of relative % error?



#### Adaptive quadrature

Adaptive quadrature is a numerical integration method in which the integral of a function f(x) is approximated using static quadrature rules on adaptively refined subintervals of the integration domain. Generally, adaptive algorithms are just as efficient and effective as traditional algorithms for "well behaved" integrands, but are also effective for "badly behaved" integrands for which traditional algorithms fail.

## 3. Sum of Multiples

In the **q03** folder, edit the C++ console application to calculate and display the sum of all natural numbers less than 1,900 that are fully (cleanly, evenly) divisible by both 7 and 11

### 4. Temperature Converter

In the **q04** folder, edit the C++ console application copied from Session 02 Lab 03 to display the same range in temperatures as before, but this time converting from Celsius degrees to Fahrenheit degrees

Be sure to make the human readable output make sense because you are flipping which scale is on which side of the equality symbol

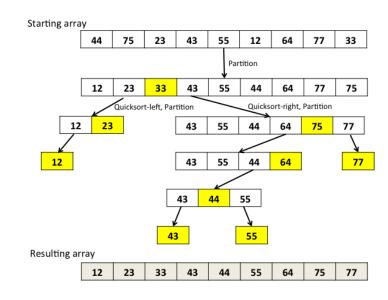
Your code changes must successfully compile and run as expected

### 5. Quicksort Optimization

Research Hoare's Quicksort algorithm and understand the benefit of partitioning using the *median of three* for the pivot element

In the **q05** folder, edit the C++ console application to implement the **MedianOfThree**() function: a, b, c are the values, ai, bi, ci are the indexes

Your function should return the *index* of the median of the three passed in values



### **6. Lowest Common Multiple**

In the **q06** folder, edit the C++ console application to calculate the lowest common multiple (LCM) of two integers a & b, using only **basic** arithmetic (*no looping*) and the greatest common divisor (**GCD**) function which is provided

```
// lcm-gcd.cpp
#include "stdafx.h"
using namespace std;
int GCD(uint64_t a, uint64_t b)
    return b == 0 ? a : GCD(b, a % b);
int LCM(uint64_t a, uint64_t b)
  // Implement your code here
int main()
    uint64 t a = 447618;
    uint64 t b = 2011835;
    cout.imbue(locale(""));
    cout << "LCM of " << a
         << " and " << b << " is "
         << LCM(a, b) << endl;
    return 0;
```

#### 7. Vector Addition

In the q07 folder, edit the C++
console application to
calculate the addition of two
vectors. In particular,
complete the function
SumVectors() by providing
values for each element of
vec3 using some form of
looping construct

```
vector-addition.cpp
#include "stdafx.h"
using namespace std;
template <typename T>
void DisplayVector(string name, vector<T> vec)
    cout << "Vector " << name << " = {";</pre>
    for (size t i{}; i < vec.size() - 1; i++)</pre>
        cout << vec.at(i) << ", ";</pre>
    cout << vec.back() << "}" << endl << endl;</pre>
template <typename T>
vector<T> AddVectors(vector<T> vec1, vector<T> vec2)
    vector<T> vec3(vec1.size());
    // Implement your code here
    return vec3;
int main()
   vector<int> a{ -2, 3, 6, 113, 49, 0, 123 };
   vector<int> b{ 18, 13, 990, 2, -55, -9, 14 };
    DisplayVector("a", a);
    DisplayVector("b", b);
   DisplayVector("c", AddVectors(a, b));
    return 0;
```

### 8. Hamming Weight

In the **q08** folder, edit the C++ console application to calculate the Base 2 *Hamming Weight* of a given natural number.

You must write the function **PopCount()** 

```
/ hamming-weight.cpp
#include "stdafx.h"
#include <bitset>
using namespace std;
int PopCount(int n)
    int onebits = 0;
    // Insert your code here
    return onebits:
int main()
    int n = 95601;
    cout << "The Hamming weight of " << n</pre>
         << " in base 2 is " << PopCount(n)</pre>
         << endl << endl;
    bitset<17> b(n);
    cout << n << " in base 10 is "
         << b.to string() << " in base 2" << endl;</pre>
    cout << "Therefore the popcount is " << b.count()</pre>
         << endl << endl;
    return 0;
```

#### 9. Multimodal Sets

Some lists can be multimodal.

If two or more elements appear an equal number of times within a list, the list is said to be multimodal.

In the **q09** folder, edit the C++ console application to return all of the modes of a list.

Specifically, you must implement the **FindModes**() function by populating the **modes** vector from the passed in **data** vector

```
multi-modal.cpp
#include "stdafx.h"
using namespace std;
vector<int> FindModes(vector<int> data)
   // Prepare return vector
   vector<int> modes:
   // Implement your code here
   return modes;
int main()
   // Prepare data vector
   vector<int> data {7,1,2,3,7,4,5,6,7,1,2,1,1,9,7,10,2,2};
   cout << "The modes of the data are: " << endl;</pre>
    for (auto& n : FindModes(data))
        cout << n << endl;
    return 0:
```

#### **10. Circle Lattice Points**

In the **q10** folder, edit the C++ console application to calculate the integer lattice points in a circle.

Specifically you must write the LatticePoints() function, which receives an integer radius, and must count all the lattice points within that radius.

This is known as the Gauss Circle Problem, and Gauss was the first one to prove the number of lattice points is bounded by this expression:

$$N(r) = \pi r^2 + E(r)$$

where

$$|E(r)| \le 2\sqrt{2\pi r}$$

