

# Survey of Scientific Computing (SciComp 301)

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Exam 2
Total of 100 points

## 1. Solve a given 4x4 system

In the **q01** folder, edit the C++ console application to calculate the four unknowns for this system of equations:

$$x_1 + 2x_2 + x_3 - x_4 = 5$$

$$3x_1 + 6x_2 + 4x_3 + 4x_4 = 16$$

$$4x_1 + 8x_2 + 3x_3 + 4x_4 = 22$$

$$2x_1 + 4x_2 + x_3 + 5x_4 = 15$$

#### 2. Create and solve a random 10x10 system

In the **q02** folder, edit the C++ console application to **generate** and solve a system of 10 random linear equations having 10 unknowns using Cramer's Rule

The coefficients and values should be taken from a uniform real distribution between [-10,10) with initial seed of 2016

Display all numbers with **four** digits of precision to the right of the decimal point

Copy any needed relevant code from the q01 solution files

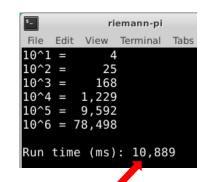
```
// solve10x10-random.cpp
        #include "stdafx.h"
        using namespace std;
        typedef vector<double> matrix1D;
 8
        typedef vector<matrix1D> matrix2D:
 9
10
        seed seq seed{ 2016 };
11
        default random engine generator{ seed };
12
        uniform real distribution  

→ distribution(-10, 10);
13
14
        matrix1D CreateRandomVector(size t rows)
15
16
17
            matrix1D A(rows);
            for (size t row{); row < rows; row++)</pre>
18
                A.at(row) = distribution(generator);
19
20
21
22
23
24
25
26
27
            return move(A);
        matrix2D CreateRandomMatrix(size t rows, size t cols)
            matrix2D A:
            A.resize(rows, matrix1D(cols));
            for (size t row{); row < rows; row++)</pre>
                for (size t col{); col < cols; col++)</pre>
28
                     A.at(row).at(col) = distribution(generator);
29
            return move(A);
30
```

# 3. Calculate Riemann's $\pi(x)$

In the **q03** folder, edit the C++ console application to calculate and display  $\pi(x)$  for each successive power of 10 from 1 to 6 inclusive

| X               | $\pi(x)$ |
|-----------------|----------|
| 10              | 4        |
| 10 <sup>2</sup> | 25       |
| 10 <sup>3</sup> | 168      |
| 10 <sup>4</sup> | 1,229    |
| 10 <sup>5</sup> | 9,592    |
| 10 <sup>6</sup> | 78,498   |



Special bonus points if your code runs faster than mine!

```
// riemann-pi.cpp
       #include "stdafx.h"
       using namespace std;
       using namespace chrono;
      vector<int> primes{
                                                 You must write
       void FindPrimesUnder(int limit)
11
12
           int n = primes.back() + 2;
                                                   this function
13
           while (n < limit)
14
15
               // Add your code here
16
17
18
19
       int main()
20
21
           cout.imbue(locale(""));
22
           cout << right;</pre>
23
24
           auto startTime = system clock::now();
25
26
           for (int i{ 1 }; i <= 6; i++)
27
28
               FindPrimesUnder((int)pow(10, i));
29
               cout << "10^" << to string(i) << " ="
30
                    << setw(7) << primes.size() << endl:
31
32
33
           auto stopTime = system clock::now();
34
35
           auto totalTime = duration cast<milliseconds>(stopTime - startTime);
36
           cout << endl << "Run time (ms): " << totalTime.count() << endl;</pre>
37
38
           return 0;
39
```

#### 4. Calculate Gamma from Eta

$$\eta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x + 1} dx$$

```
double Eta(double s)
{
    double eta = 1;
    for (int n{ 2 }; n <= 1e5; n++)
    {
        double term = 1. / pow(n, s);
        eta += (n % 2) == 0 ? -term : term;
    }
    return eta;
}</pre>
```

In the **q04** folder, edit the C++ console application to display 5! using only  $\eta(s)$  and the value of the integral

```
inline double f(double x, double s)
{

You must write
this function

double Simpsons(double s)
{

double a{ 0 };
double b{ 1e3 };
int intervals = 1e5;

double dx{ (b - a) / intervals };
double sum{ f(a,s) + f(b,s) };
a += dx;
for (int i{ 1 }; i < intervals; ++i, a += dx)
    sum += f(a, s)*(2 * (i % 2 + 1));
return (dx / 3)*sum;
}</pre>
```

## 5. Expand a Standard CF

In the **q05** folder, edit the C++ console application to display the first **seven** terms of the standard continued fraction encoding for each value of  $x_n$ :

$$x = \frac{1 + \sqrt{4n^2 - 4n + 5}}{2}$$

$$n \in \mathbb{Z}^+, n < 10$$

```
string EncodeCF(double x)
{
    const int maxTerms = 7;
    string cf{to_string(int(x)) + "," };
    x = x - int(x);
    for (int terms = 1; terms < maxTerms; terms++)
    {
        cf += to_string((int)(1 / x));
        if (terms < maxTerms - 1) cf += ",";
        x = 1 / x - (int)(1 / x);
    }
    return move(cf);
}</pre>
```

This is something I personally discovered. It reminds me of the Golden Ratio

#### 6. Integrate the Standard Normal

In the **q06** folder, edit the C++ console application to use Simpson's Rule to reasonably estimate the area under the standard normal curve from -1 to 1

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Is this area the same as the probability of a normally distributed random variable falling within the **first** standard deviation away from the mean?

# 7. Decrypt Ciphertext

In the **q07** folder, edit the C++ console application to decrypt the file "ciphertext.txt"

Use the tools & techniques from Session 14

## 8. Finding an Open Reading Frame (ORF)

https://en.wikipedia.org/wiki/Open reading frame

http://vlab.amrita.edu/?sub=3&brch=273&sim=1432&cnt=1

**DNA** (Deoxyribonucleic acid) is the genetic material that contains all the genetic information in a living organisms. The information is stored as genetic codes using adenine (A), guanine (G), cytosine(C) and thymine (T). During the transcription process, DNA is copied to mRNA. Each of these base pairs will bond with a sugar and phosphate molecule to form a nucleotide. **Three nucleotides** that code for a particular amino acid during translation is called as a **codon**. An Open Reading Frame (ORF) goes from the **start codon** to a **stop codon** and encodes a specific protein. By analyzing the ORF we can predict the possible amino acids that might be produced during translation.

Given a sequence, create 6 different reading frames labeled +1, +2, +3, -1, -2 and -3:

- The first reading frame is obtained by considering the sequence in words of 3
- The second reading frame is formed after skipping the first nucleotide and then grouping the remaining sequence into words of 3 nucleotides
- The third reading frame is formed after skipping the first 2 nucleotides and then grouping the remaining sequence into words of 3 nucleotides
- The other negative 3 reading frames can be found only after finding the reverse complement of the sequence
- The same process as for the +1, +2 and +3 strands is repeated for the -1, -2 and -3 strands, but now using the reverse complement sequence

To identify each open reading frame (ORF):

- Find the start codon and stop codons in each reading frame
- Display the sequence stretch beginning with a start codon and ending in a stop codon

# 8. Finding an Open Reading Frame (ORF)

http://vlab.amrita.edu/?sub=3&brch=273&sim=1432&cnt=1

```
int main()
   string seq{ "CGCTACGTCTTACGCTGGAGCTCTCATGGATCGGTTCGGTAGGGCTCGATCACATCGCTAGCCAT" };
   cout << seq << endl << endl;</pre>
   string seqRevComp = ReverseComplement(seq);
   for (auto offset : { 1,2,3,-1,-2,-3 })
       if (offset > 0)
           FindORF(seq, offset);
                                                             // find-orf.cpp
       else
           FindORF(seqRevComp, offset);
                                                             #include "stdafx.h"
   system("pause");
   return 0;
                                                             using namespace std;
                                                             string ReverseComplement(string seq)
                                                                                    You must write
                                                                                      this function
```

In the **q08** folder, edit the existing C++ console application to generate the **reverse complement** of a given DNA sequence

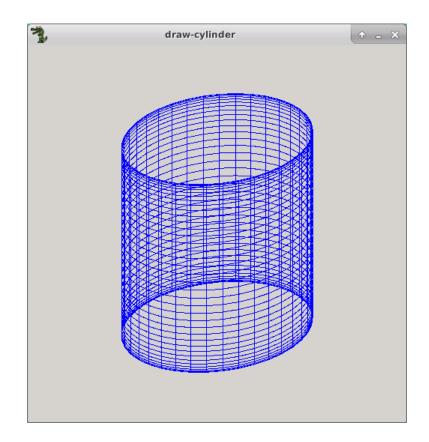
## 8. Finding an Open Reading Frame (ORF)

```
vector<string> CreateReadingFrame(const string& seq, int offset)
                                                                          size t i = abs(offset) - 1;
                                                                          vector<string> frame{};
                                                                          while (i < seq.length() - 2)</pre>
void FindORF(const string& seq, int offset)
                                                                              frame.push back(seq.substr(i, 3));
   vector<string> frame = CreateReadingFrame(seq, offset);
                                                                              i += 3;
   size t posStart{};
                                                                          return move(frame);
   if (!FindCodon(frame, { "ATG" }, 0, posStart))
       return;
   size t posStop{};
   if (!FindCodon(frame, { "TAA", "TAG", "TGA" }, posStart + 1, posStop)
       return;
   if (posStart + 1 == posStop)
                                                                      bool FindCodon(const vector<string>& frame,
       return;
                                                                          const vector<string>& codons, size t start, size t& pos)
   cout << "Frame ";</pre>
                                                                          pos = start;
   if (offset > 0) cout << "+";</pre>
                                                                          bool found = false;
   cout << offset << ": ";</pre>
                                                                          while (!found && pos < frame.size())</pre>
   for (size_t i{ posStart }; i <= posStop; i++)</pre>
       cout << frame.at(i) << " ";</pre>
                                                                               for (const string& codon : codons)
   cout << endl:</pre>
                                                                                   if (frame.at(pos) == codon)
                                                                                        found = true;
                                                                               if (!found)
                                                                                   pos++;
                                                                          return found;
```

# 9. Draw a 3D Cylinder

In the **q09** folder, edit the C++ Allegro application to draw an open-ended 3D cylinder with radius 100 and length 200

Make it a wireframe drawing. No back face culling or facet shading is required.



# 10. Sample a Sinusoid

In the **q10** folder, edit the C++ ROOT application to draw a sine wave with exactly **7 crests** per every **13 units**, over the domain  $0 \le x \le 26$ 

