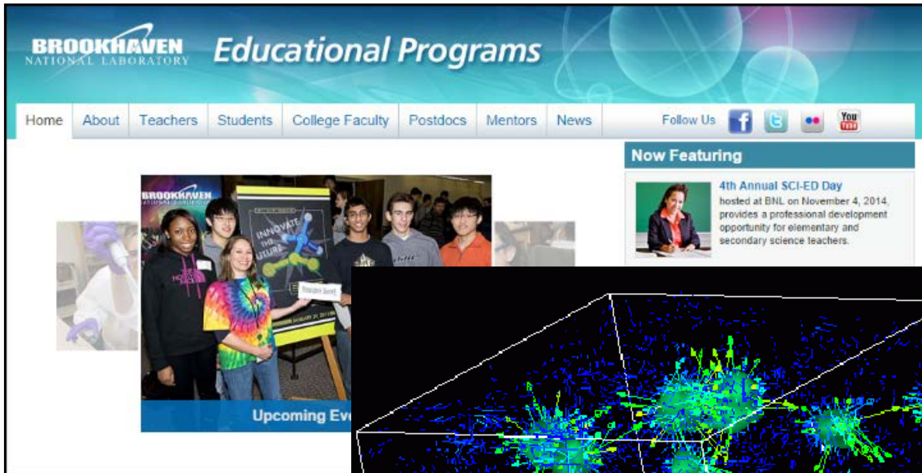
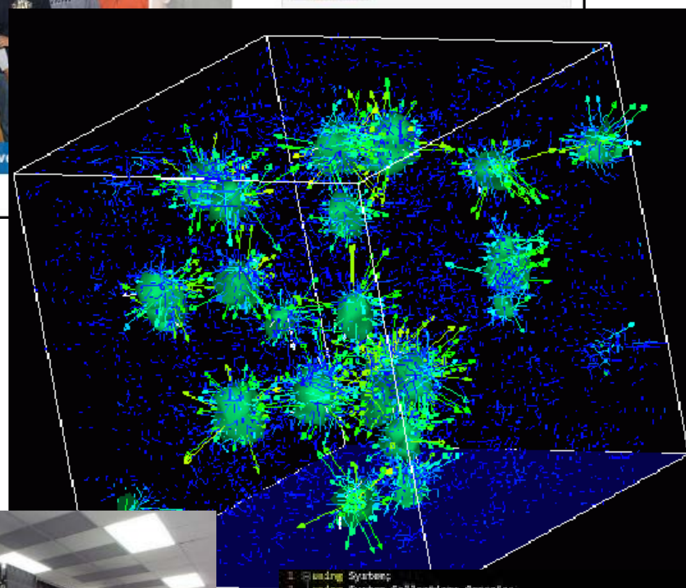
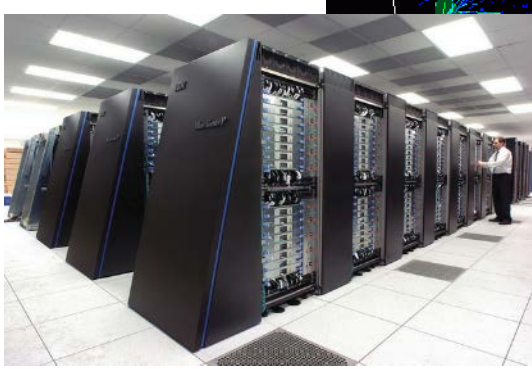






Survey of Scientific Computing

(SciComp 301)

Dave Biersach
Brookhaven National Laboratory
dbiersach@bnl.gov

Session 23

Difference Tables, Least Squares

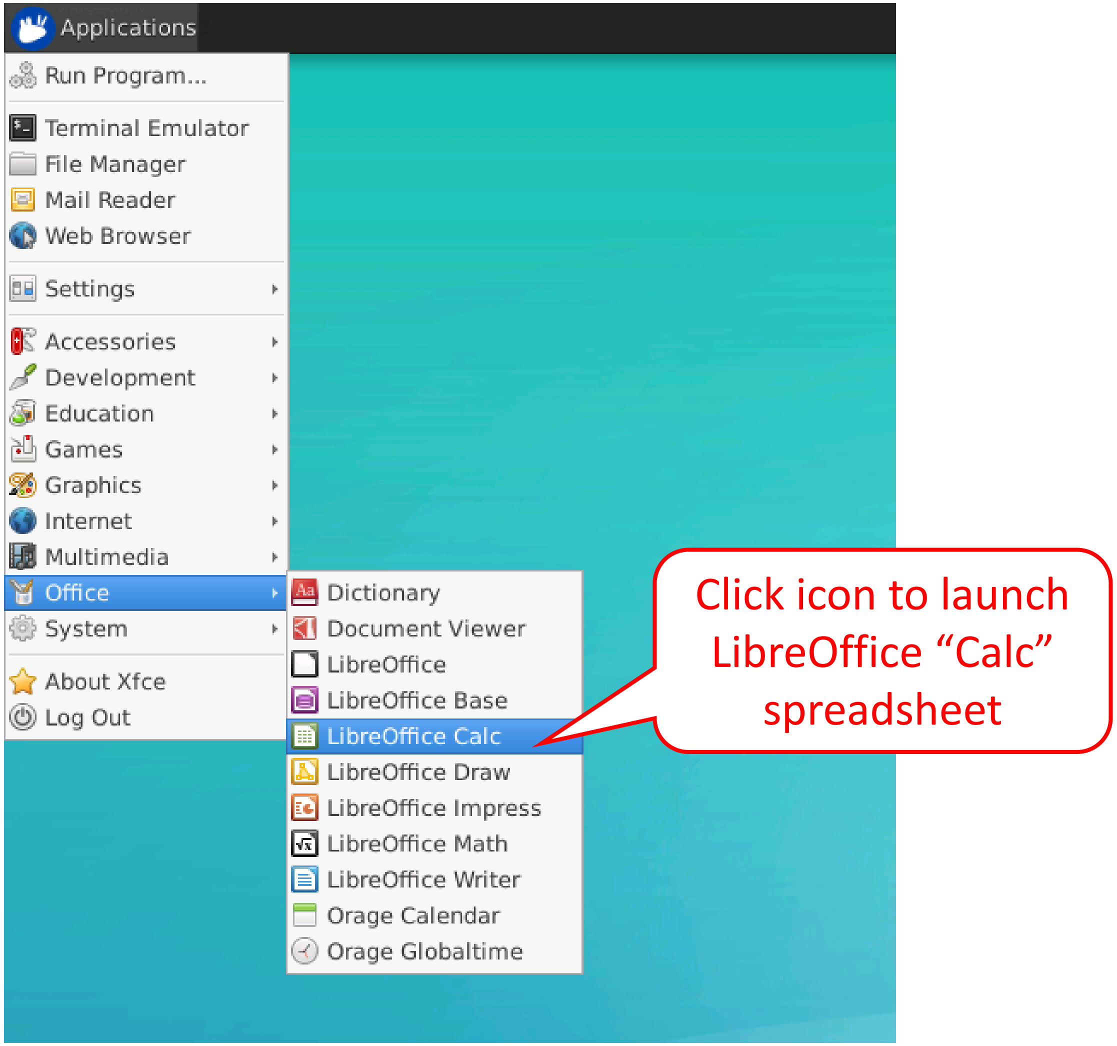
Session Goals

- Determine the exact formula of the underlying **generator** for a sequence of integers (a ***functional equation***)
  - Fit a **quadratic** curve to a set of observations to **interpolate** resulting values that lie between the observations
  - Understand how to create difference tables in the open source (and free) LibreOffice **Calc** spreadsheet program
  - Appreciate **% relative error** as a measure of **goodness of fit** of a model to experimental data
- Fit a curve using the **Method of Least Squares**
  - Derive the least squares equations using the **partial derivatives** for each coefficient of the unknown quadratic

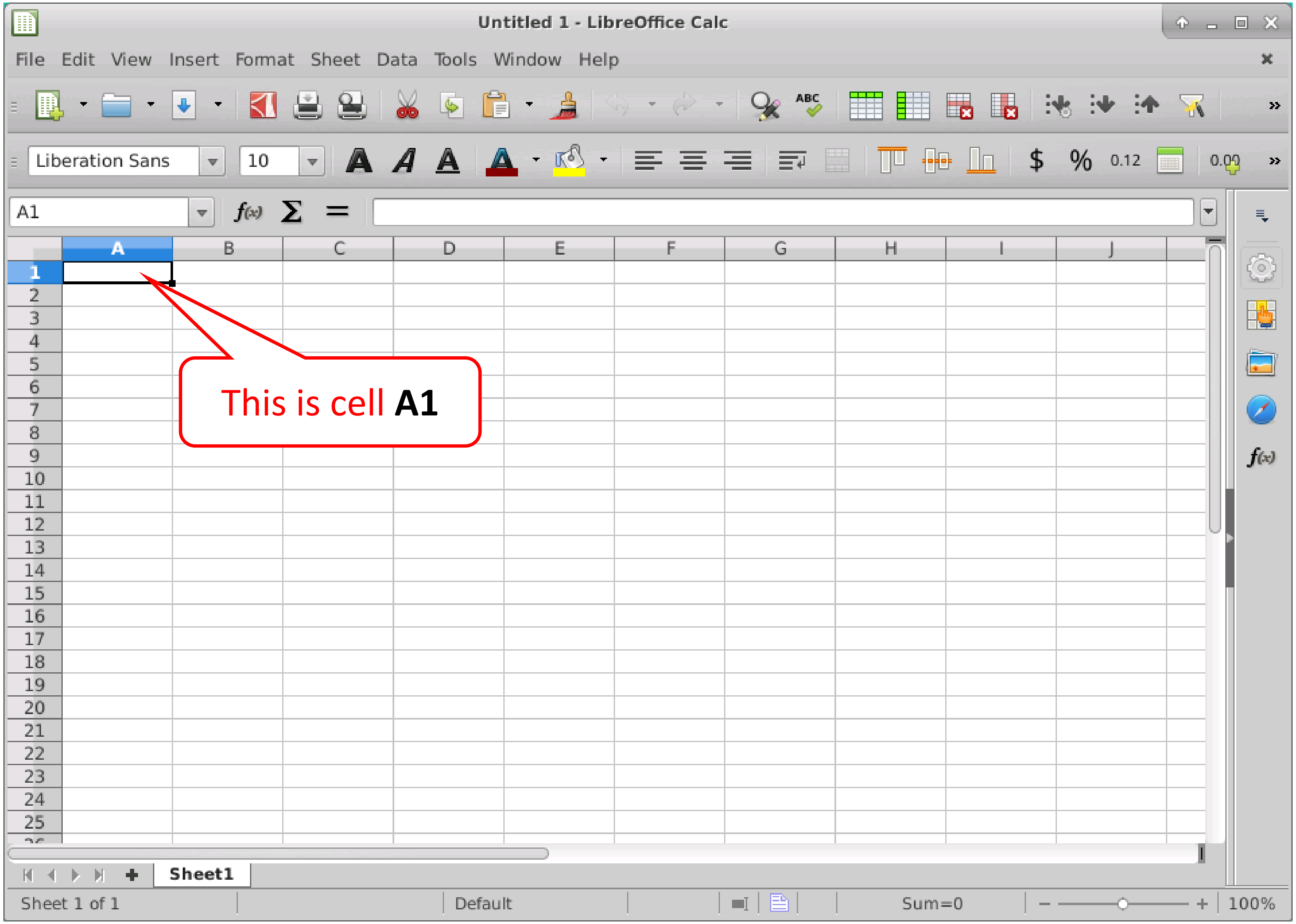
Creating a Spreadsheet

- A **spreadsheet** is a flexible computing tool that allows you to enter data and write formulas to operate on that data
- Everything is based on the concept of a "**cell**" that has a unique **column** (letter) and **row** (number) address
- A formula entered in one cell can reference data in one ore more other cells by using a cell addresses, or by using a **range of cell addresses**
- When source data cells are update, the spreadsheet *automatically recalculates all dependent formula cells*
- Graphs can be created to depict the values in cells

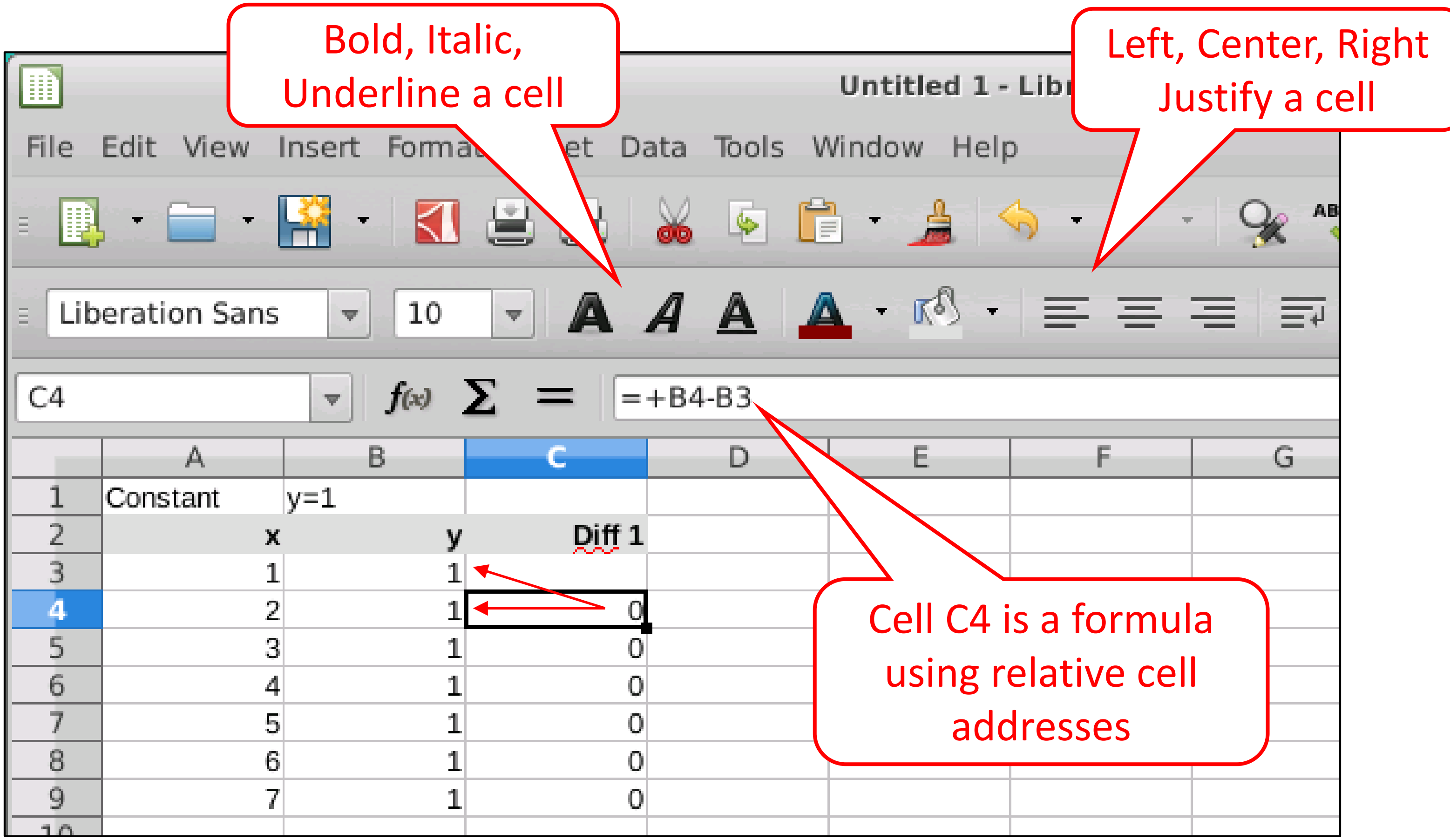
Creating a Spreadsheet



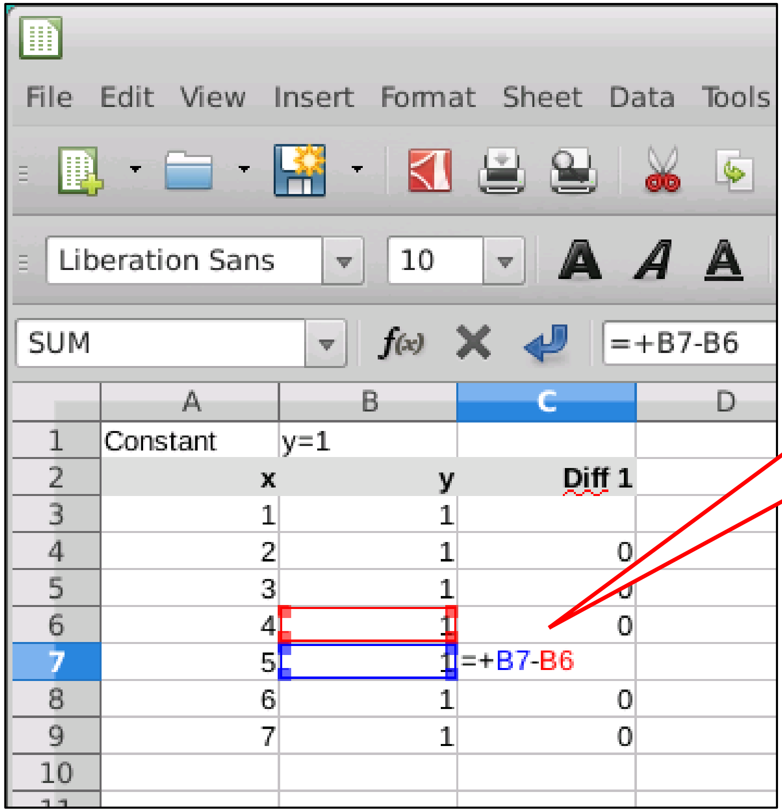
Lab 1 - Creating a Spreadsheet



Lab 1 - Creating a Spreadsheet

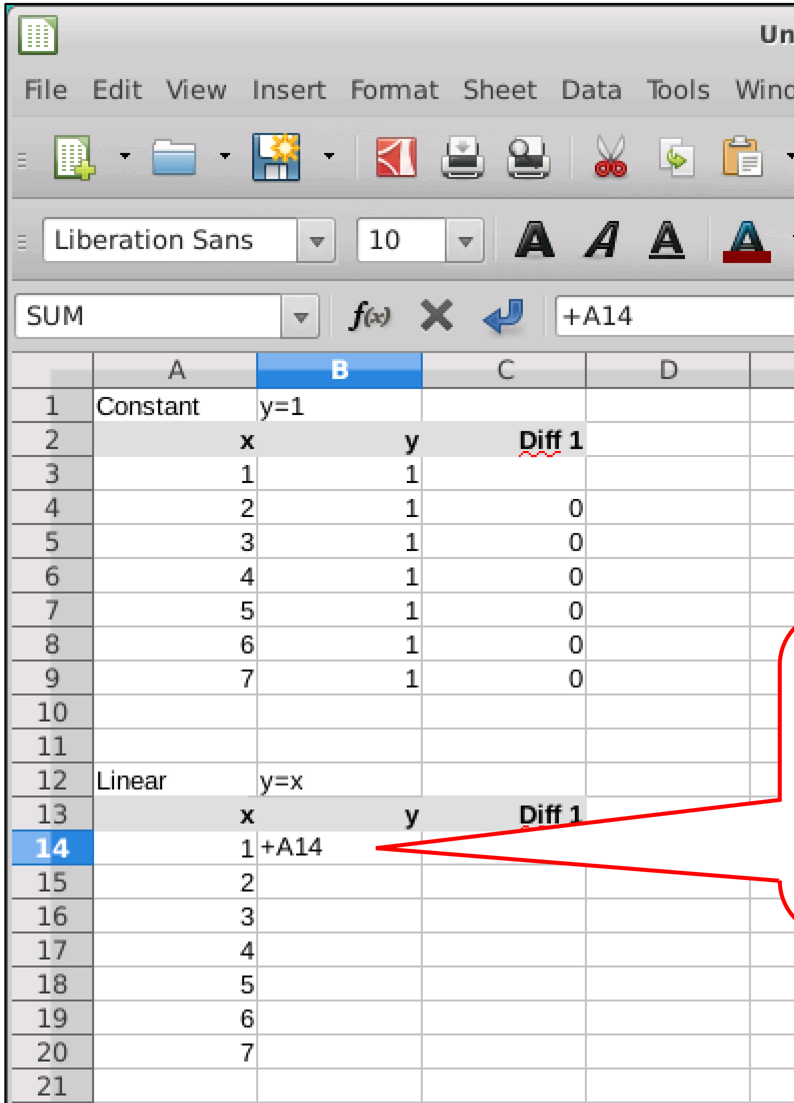


Lab 1 - Creating a Spreadsheet



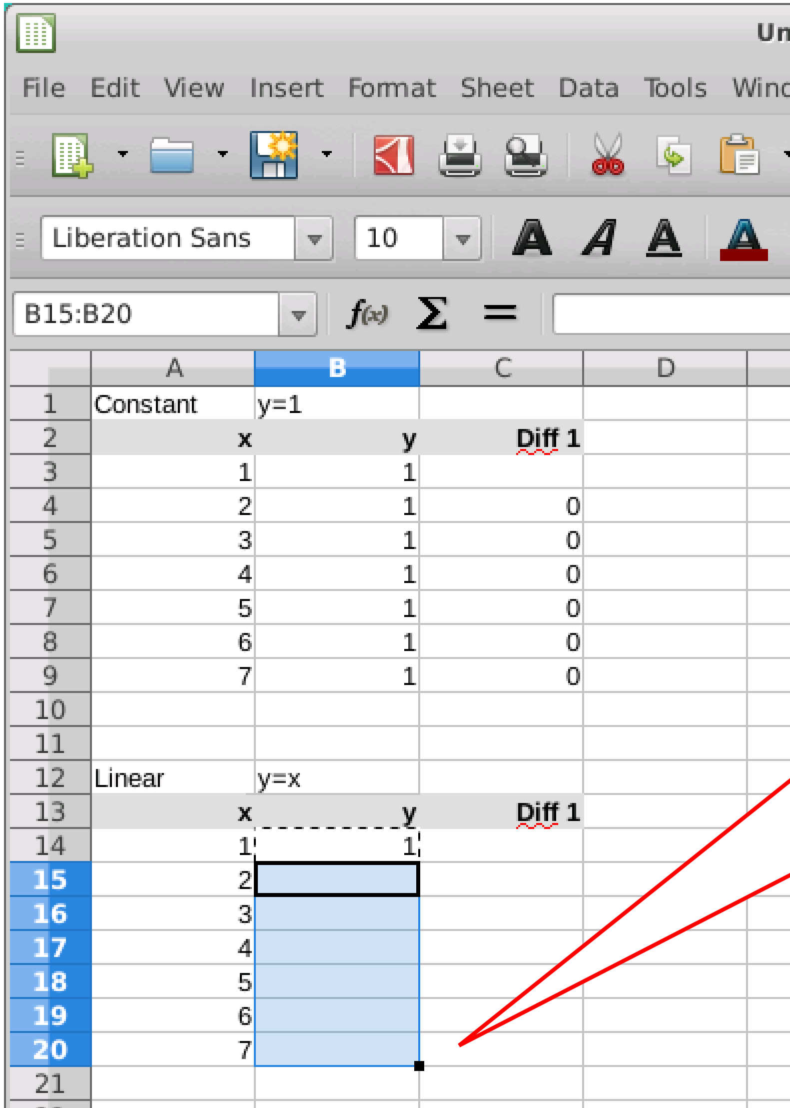
To check a formula cell, hit **F2** to edit the formula – color coding identifies the cells

Lab 1 – Creating a Spreadsheet



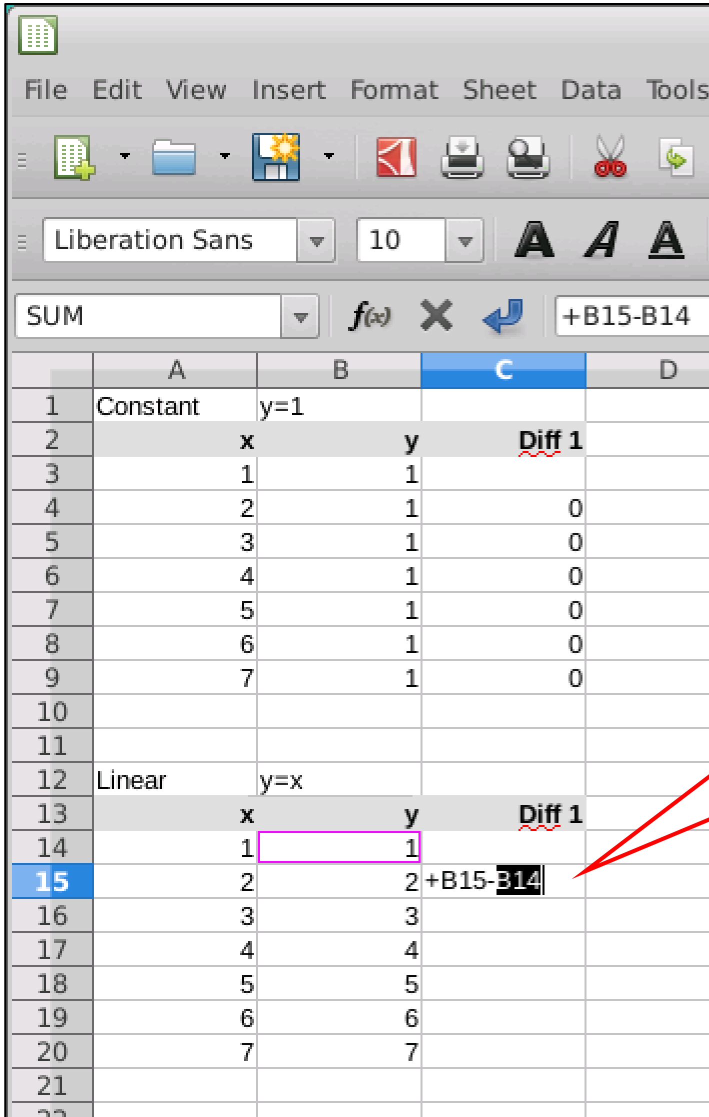
To enter a formula cell, press **+**, then use the arrow keys to select the source data cells. Press ENTER when done with the formula

Lab 1 - Creating a Spreadsheet



To copy a formula to other cells, highlight the source cell, press "Control + C" to copy, then use arrow keys + SHIFT to highlight a destination **range**, and press ENTER to paste

Lab 1 - Creating a Spreadsheet



Create the **Diff 1** column using the difference between the left-adjacent cell and the cell above that one. Then copy that formula down the remaining cells in the **Diff 1** column.

Lab 1 - Creating a Spreadsheet

	A	B	C	D
22				
23	Quadratic	y=x^2		
24	x	y	Diff 1	Diff 2
25	1	+A25^2		
26	2			
27	3			
28	4			
29	5			
30	6			
31	7			
32				

Create the y column with the **exponent 2** and then copy down

	A	B	C	D
23	Quadratic	y=x^2		
24	x	y	Diff 1	Diff 2
25	1	1		
26	2	4	3	
27	3	9	5	+C27-C26
28	4	16	7	
29	5	25	9	
30	6	36	11	
31	7	49	13	

Add the **Diff 1** & **Diff 2** columns and formulas and then copy down

	A	B	C	D
23	Quadratic	y=x^2		
24	x	y	Diff 1	Diff 2
25	1	1		
26	2	4	3	
27	3	9	5	2
28	4	16	7	2
29	5	25	9	2
30	6	36	11	2
31	7	49	13	2
32				

The completed **quadratic** difference table

Lab 1 - Creating a Spreadsheet

34	Cubic	y=x^3				
35	x	y	Diff 1	Diff 2	Diff 3	
36	1	1				
37	2	8	7			
38	3	27	19	12		
39	4	64	37	18	6	
40	5	125	61	24	6	
41	6	216	91	30	6	
42	7	343	127	36	6	
43						
44						
45	Quadratic	y=x^4				
46	x	y	Diff 1	Diff 2	Diff 3	Diff 4
47	1	1				
48	2	16	15			
49	3	81	65	50		
50	4	256	175	110	60	
51	5	625	369	194	84	24
52	6	1296	671	302	108	24
53	7	2401	1105	434	132	24
54						

Finish the **Cubic** and **Quartic** **Difference Tables**

Difference Tables

- A difference table calculates the gap between **successive** values of a given function
- For every higher power of the *independent* variable (the domain) we add **another** difference column
- Difference column #2 is the gap between successive values of difference column #1
- We keep adding difference columns until the row values achieve a **steady state**: they are the same for each row
- The **cubic** table needed **3** difference columns – the **quartic** table needed **4** difference columns to achieve **steady state**

Difference Tables

34	Cubic	y=x^3				
35	x	y	Diff 1	Diff 2	Diff 3	
36	1	1				
37	2	8	7			
38	3	27	19	12		
39	4	64	37	18	6	
40	5	125	61	24	6	
41	6	216	91	30	6	
42	7	343	127	36	6	
43						
44						
45	Quadratic	y=x^4				
46	x	y	Diff 1	Diff 2	Diff 3	Diff 4
47	1	1				
48	2	16	15			
49	3	81	65	50		
50	4	256	175	110	60	
51	5	625	369	194	84	24
52	6	1296	671	302	108	24
53	7	2401	1105	434	132	24
54						

Keep adding difference columns until the **rightmost** column reaches a **steady** value

Difference Tables – What They Reveal

		Steady Difference
Constant	$y = 1$	0
Linear	$y = x$	1
Quadratic	$y = x^2$	2
Cubic	$y = x^3$	6
Quartic	$y = x^4$	24

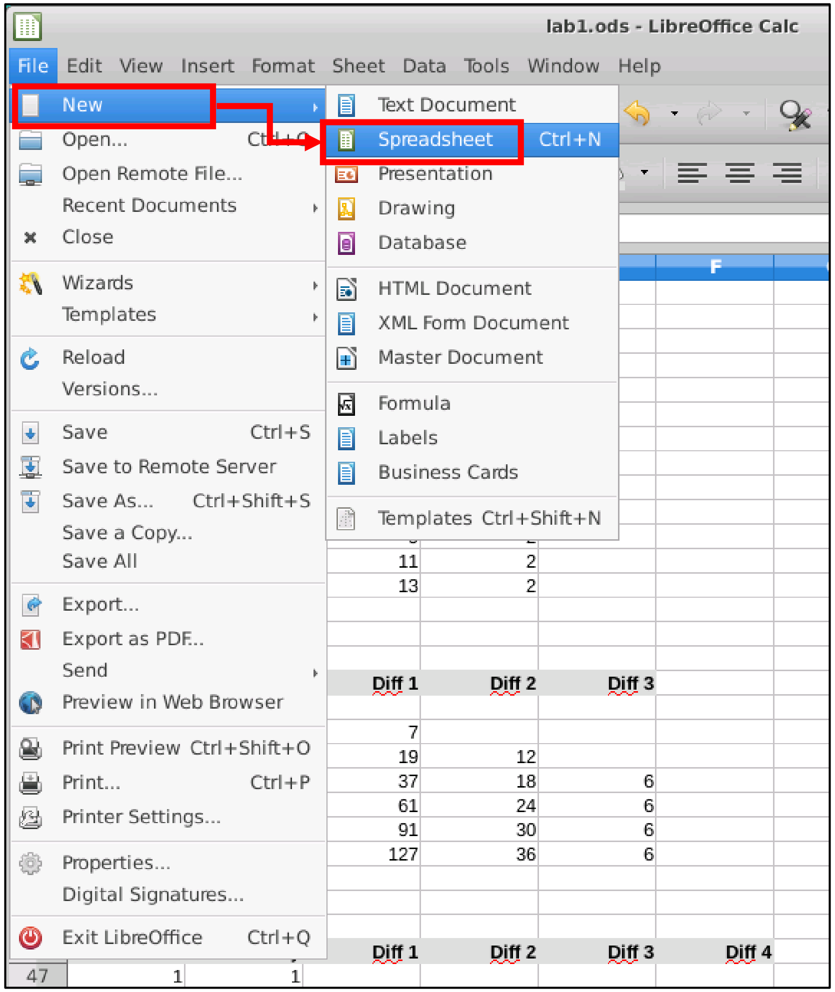
Difference Tables – What They Reveal

- If we believe a given a set of observations obeys a general underlying law but remains an unknown specific formula, we can use difference tables to systematically reveal the set's hidden functional equation (the generator)
- We start by "guessing" the highest power of x that would likely be in the underlying generator
- We compare the expected steady state values for each power of x with the our observed values
- As we determine the coefficient of each decreasing power of x, we begin to expose the underlying functional equation that originally generated the sequence

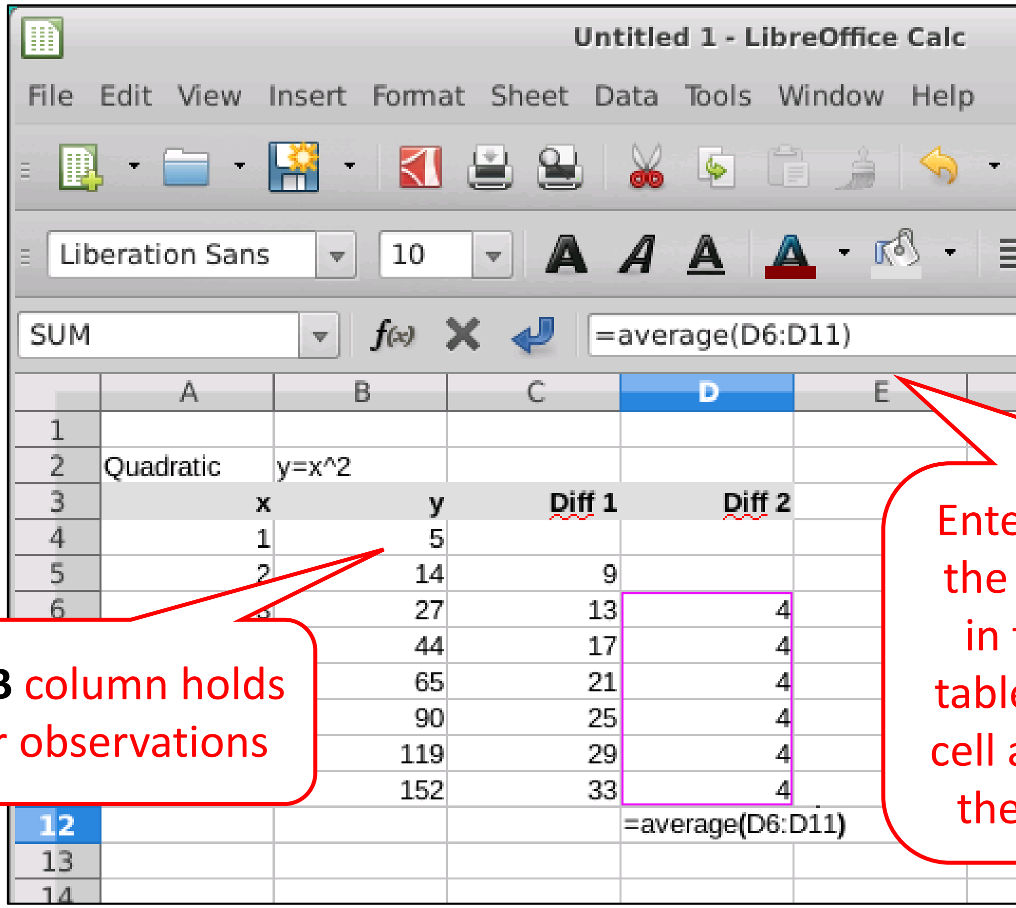
Lab 2

- Find an equation to generate this sequence: **5, 14, 27, 44, 65, 90, 119, 152 …**
- We will guess this data set is generated by a quadratic formula: $\boldsymbol{y = ax^2 + bx + c}$
- We have to figure out the values for $\boldsymbol{a, b, c}$
- We then create difference tables, working ***backwards***, starting with a **quadratic**, then a linear table, and then a constant table (if necessary)
- We stop when our model produces **values that match our observations** to a reasonable level of ***accuracy***

Lab 2 – Create a new Spreadsheet



Calculating the Mean Steady Difference



The **B** column holds your observations

Enter **=AVERAGE** then select the range of cells to include in the average – for every table always put the average cell at the bottom row under the **last** difference column

Mean Difference = Term's Coefficient

2	Quadratic	y=x^2			
3	x	y	Diff 1	Diff 2	
4	1	5			
5	2	14	9		
6	3	27	13	4	
7	4	44	17	4	
8	5	65	21	4	
9	6	90	25	4	
10	7	119	29	4	
11	8	152	33	4	
12			mean:	4	
13					

- The $y = x^2$ table had an expected steady difference of **2**
- In our sequence we have steady difference **mean** of **4**
- This means the coefficient for the x^2 term is $a = \frac{\mathbf{4}}{\mathbf{2}} = \mathbf{2}$
- So we know the est. generator so far is $y = \mathbf{2}x^2 + bx + c$

Delta = Observed - Expected

15	Linear	y=x				
16		**x**	**y Obs**	**y Est**	**delta**	**Diff 1**
17		1	5	=+2*A17^2		
18		2	14			
19		3	27			
20		4	44			
21		5	65			
22		6	90			
23		7	119			
24		8	152			

15	Linear	y=x				
16		**x**	**y Obs**	**y Est**	**delta**	**Diff 1**
17		1	5	2	=+B17-C17	
18		2	14			
19		3	27			
20		4	44			
21		5	65			
22		6	90			
23		7	119			
24		8	152			

- $y_{obs} = given\ values$
- $y_{est} = 2x^2$
- $\Delta = y_{obs} - y_{est}$

15	Linear	y=x				
16		**x**	**y Obs**	**y Est**	**delta**	**Diff 1**
17		1	5	2	3	
18		2	14	8	6	=+D18-D17
19		3	27	18	9	
20		4	44	32	12	
21		5	65	50	15	
22		6	90	72	18	
23		7	119	98	21	
24		8	152	128	24	

Add the **Diff 1** column and formula, then copy down all rows

Delta = Observed - Expected

15	Linear	y=x				
16	x	y Obs	y Est	delta	Diff 1	
17	1	5	2	3		
18	2	14	8	6	3	
19	3	27	18	9	3	
20	4	44	32	12	3	
21	5	65	50	15	3	
22	6	90	72	18	3	
23	7	119	98	21	3	
24	8	152	128	24	3	
25				mean:	=+AVERAGE(E18:E24)	
26						

- $y_{obs} = given\ values$
- $y_{est} = 2x^2$
- $\Delta = y_{obs} - y_{est}$
- Observed mean steady value = **3**
- Expected linear steady value = **1**
- $b = \frac{\mathbf{3}}{\mathbf{1}} = \mathbf{3}$
- Est. generator is now $y = 2x^2 + \mathbf{3}x$

% Relative Error

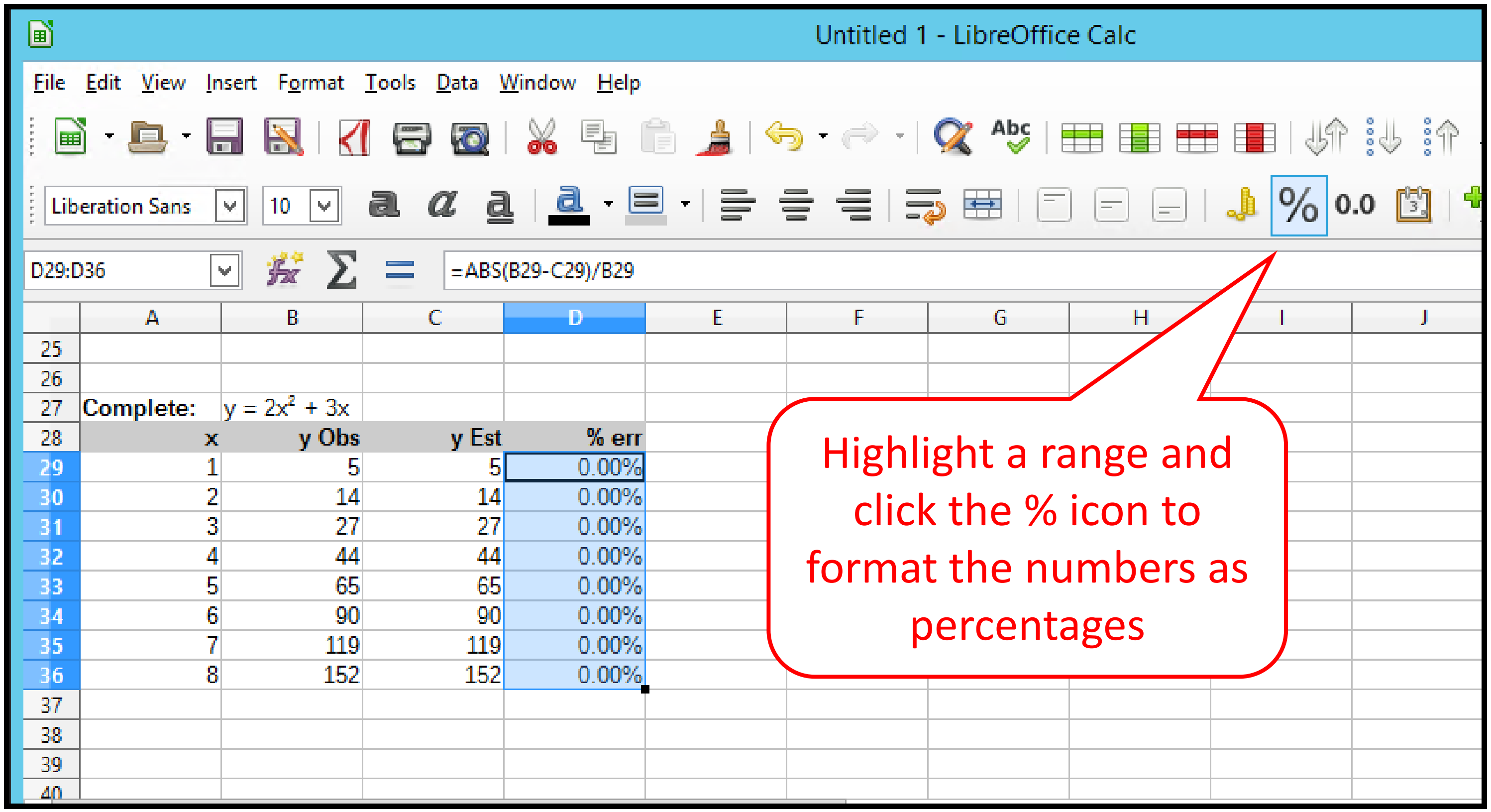
28	Complete	y=2*x^2+3*x			
29		x	y Obs	y Est	% Err
30		1	5	=+2*A30^2+3*A30	
31		2	14	14	0.00%
32		3	27	27	0.00%
33		4	44	44	0.00%
34		5	[illegible]	65	0.00%
35		6	[illegible]	90	0.00%
36		7	[illegible]	119	0.00%
37		8	[illegible]	152	0.00%
38					

Remember to add any final constant term to the **estimated** y function

- ABS = Absolute Value
- $\%_{err} = \left|\frac{(y_{obs}-y_{est})}{y_{obs}}\right|$
- 0% error = perfect fit
- $y = 2x^2 + 3x$

28	Complete	y=2*x^2+3*x			
29		x	y Obs	y Est	% Err
30		1	5	5	=+(B30-C30)/B30
31		2	14	14	0.00%
32		3	27	27	0.00%
33		4	44	44	0.00%
34		5	65	65	0.00%
35		6	90	90	0.00%
36		7	119	119	0.00%
37		8	152	152	0.00%
38					

Changing Cell Number Format



Lab 3

- Create a new Lab 3 spreadsheet and generate the difference tables to find the underlying equation that generate this sequence: **36, 103, 244, 489, 868, 1411, 2148, 3109, 4324…**
- Hint: This data set is generated by a cubic formula:

$$\boldsymbol{y = ax^3 + bx^2 + cx + d}$$

- You have to figure out the values for $\boldsymbol{a, b, c, d}$
- Create difference tables, starting with a **cubic**, then quadratic, then linear, then constant table (if necessary)
- Stop when your model produces **values that match our observations** with 0% relative error
- Create a new worksheet named **Lab 3.ods**

Lab 3

	A	B	C	D	E
1	Cubic	y=x^3			
2	**x**	**y Obs**	**Diff 1**	**Diff 2**	**Diff 3**
3	1	36			
4	2	103	67		
5	3	244	141	74	
6	4	489	245	104	30
7	5	868	379	134	30
8	6	1411	543	164	30
9	7	2148	737	194	30
10	8	3109	961	224	30
11	9	4324	1215	254	30
12				**mean:**	30

		Steady Difference
Constant	$y = 1$	0
Linear	$y = x$	1
Quadratic	$y = x^2$	2
Cubic	$y = x^3$	6
Quartic	$y = x^4$	[illegible]4

Expected difference for a cubic = 6, while observed steady diff <u>mean</u> = 30, so coefficient of x^3 must be

$$\frac{30}{6} = 5$$

So far … $y = 5x^3 + bx^2 + cx + d$

Lab 3

	A	B	C	D	E	F
1	Cubic	y=x^3				
2	x	y Obs	Diff 1	Diff 2	Diff 3	
3	1	36				
4	2	103	67			
5	3	244	141	74		
6	4	489	245	104	30	
7	5	868	379	134	30	
8	6	1411	543	164	30	
9	7	2148	737	194	30	
10	8	3109	961	224	30	
11	9	4324	1215	254	30	
12				mean:	30	
13						
14	Quadratic	y=x^2				
15	x	y Obs	y Est	delta	Diff 1	Diff 2
16	1	36	=+5*A16^3	31		
17	2	103	40	63	32	
18	3	244	135	109	46	14
19	4	489	320	169	60	14
20	5	868	625	243	74	14
21	6	1411	1080	331	88	14
22	7	2148	1715	433	102	14
23	8	3109	2560	549	116	14
24	9	4324	3645	679	130	14
25					mean:	14
26						

		Steady Difference
Constant	y = 1	0
Linear	y = x	1
Quadratic	y = x^2	2
Cubic	y = x^3	
Quartic	y = x^4	

Expected difference for a quadratic = 2, while observed steady diff <u>mean</u> = 14, so coefficient of x^2 must be

$$\frac{14}{2} = 7$$

Now so far … $y = \mathbf{5}x^3 + \mathbf{7}x^2 + cx + d$

Lab 3

Remember to add the new quadratic term to the estimated y function

27	Linear	y=x			
28	x	y Obs	y Est	delta	Diff 1
29	1	36	=+5*A29^3+7*A29^2		
30	2	103	68	35	11
31	3	244	198	46	11
32	4	489	432	57	11
33	5	868	800	68	11
34	6	1411	1332	79	11
35	7	2148	2058	90	11
36	8	3109	3008	101	11
37	9	4324	4212	112	11
38				mean:	11
39					

		Steady Difference
Constant	y = 1	0
Linear	y = x	1
Quadratic	y = x^2	
Cubic	y = x^3	
Quartic	y = x^4	

Steady difference for a linear = 1, while observed steady diff mean = 11, so coefficient of x^2 must be

$$\frac{11}{1} = 11$$

Now so far … $y = \mathbf{5}x^3 + \mathbf{7}x^2 + \mathbf{11}x + d$

Lab 3

40	Constant	y=c			
41		x	y Obs	y Est	delta
42		1	36	23	13
43		2	103	90	13
44		3	244	231	13
45		4	489	476	13
46		5	868	855	13
47		6	1411	1398	13
48		7	2148	2135	13
49		8	3109	3096	13
50		9	4324	4311	13
51				mean:	13
52					
53	Complete				
54		x	y Obs	y Est	% Error
55		1	36	=+5*A55^3+7*A55^2+11*A55+13	
56		2	103	103	0.00%
57		3	244	244	0.00%
58		4	489	489	0.00%
59		5	868	868	0.00%
60		6	1411	1411	0.00%
61		7	2148	2148	0.00%
62		8	3109	3109	0.00%
63		9	4324	4324	0.00%
64					

If delta values are all the same, then that value is the final constant term

Functional Equation

$$y = 5x^3 + 7x^2 + 11x + 13$$

$$with\ x \in \mathbb{Z}^+$$

Generates the sequence:
36, 103, 244, 489, 868, 1411, 2148, 3109, 4324…

Lab 4

Elapsed Time (10 mins each)	Observed Counter
1	205
2	430
3	677
4	945
5	1233
6	1542
7	1872
8	2224

- This case study will use data from an experiment that measured the **counter on a tape machine vs. the elapsed time** the tape had been played
- Ideally this would be a linear relationship – but due to the changing circumference of a tape reel as it is played, the drive motor does not maintain consistent timing, so it is **non-linear**

Lab 4

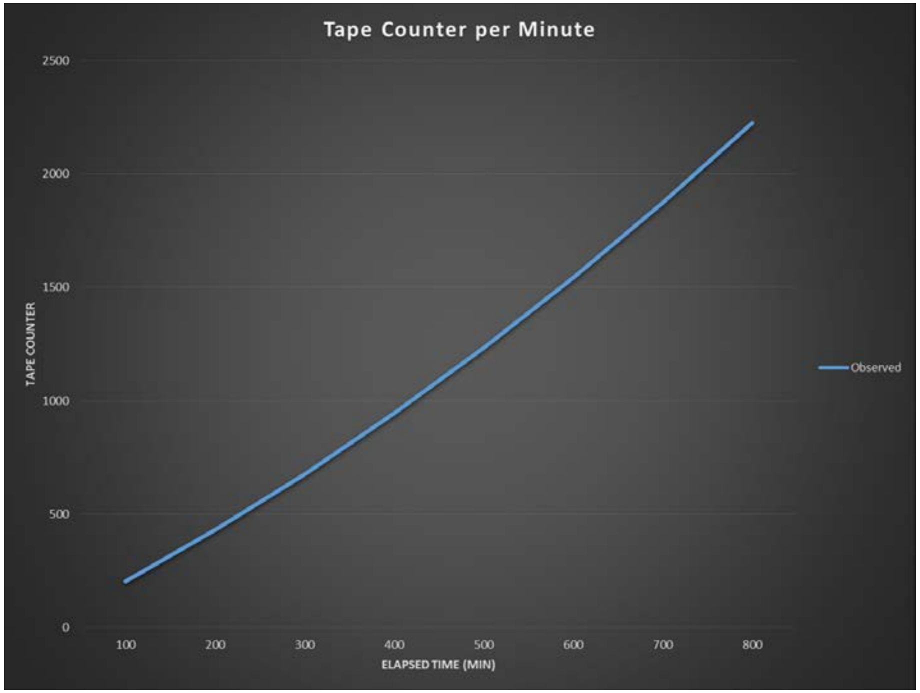
Elapsed Time (10 mins each)	Observed Counter
1	205
2	430
3	677
4	945
5	1233
6	1542
7	1872
8	2224

- Find an equation to model a tape counter as a function of playing time
  - X value is the number of 10 minute blocks the tape has been playing from the beginning
  - Y value is the counter on the tape player (linear feet)
- With a good fitting model we can answer questions like: Where should we stop the tape to be **65** minutes into the recording?

Lab 4

$$Assume$$
$$y = ax^2 + bx + c$$

Elapsed Time (10 mins each)	Observed Counter
1	205
2	430
3	677
4	945
5	1233
6	1542
7	1872
8	2224



Lab 4

1	Quadratic	y=x^2		
2	x	y Obs	Diff 1	Diff 2
3	1	205		
4	2	430	225	
5	3	677	247	22
6	4	945	268	21
7	5	1233	288	20
8	6	1542	309	21
9	7	1872	330	21
10	8	2224	352	22
11			mean:	21.1667

13	Linear	y=x			
14	x	y Obs	y Est	delta	Diff 1
15	1	205	10.5833	194.4167	
16	2	430	42.3333	387.6667	193.2500
17	3	677	95.2500	581.7500	194.0833
18	4	945	169.3333	775.6667	193.9167
19	5	1233	264.5833	968.4167	192.7500
20	6	1542	381.0000	1,161.0000	192.5833
21	7	1872	518.5833	1,353.4167	192.4167
22	8	2224	677.3333	1,546.6667	193.2500
23				mean:	193.1786

25	Constant	y=c		
26	x	y Obs	y Est	delta
27	1	205	203.7619	1.2381
28	2	430	428.6905	1.3095
29	3	677	674.7857	2.2143
30	4	945	942.0476	2.9524
31	5	1233	1,230.4762	2.5238
32	6	1542	1,540.0714	1.9286
33	7	1872	1,870.8333	1.1667
34	8	2224	2,222.7619	1.2381
35			mean:	1.8214

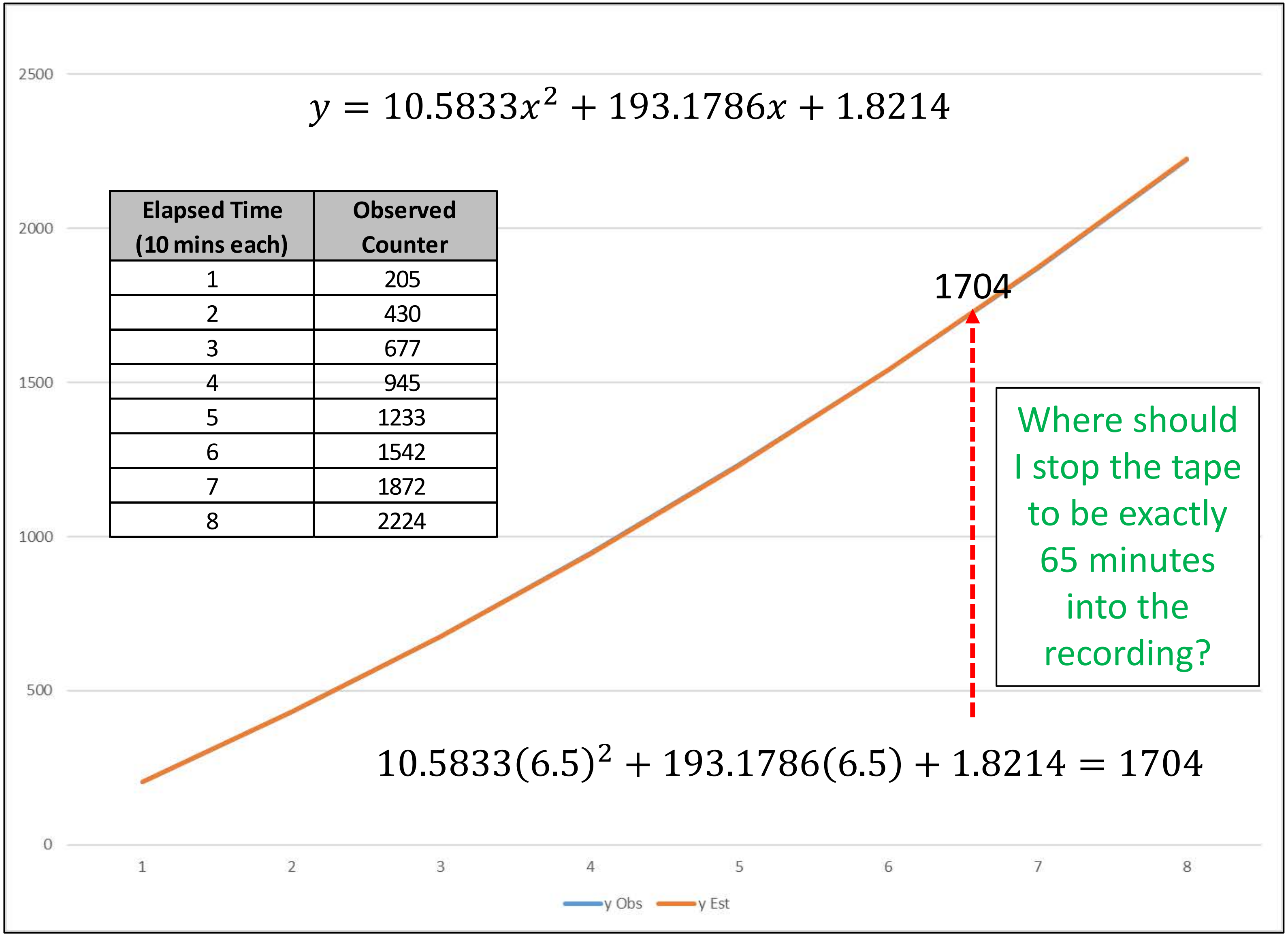
38	Complete			
39	x	y Obs	y Est	% Err
40	1	205	205.5833	-0.2846%
41	2	430	430.5119	-0.1190%
42	3	677	676.6071	0.0580%
43	4	945	943.8690	0.1197%
44	5	1233	1,232.2976	0.0570%
45	6	1542	1,541.8929	0.0069%
46	7	1872	1,872.6548	-0.0350%
47	8	2224	2,224.5833	-0.0262%

$$y_{est} = 10.5833x^2 + 193.1786x + 1.8214$$

Lab 4

$$y = 10.5833x^2 + 193.1786x + 1.8214$$

Elapsed Time (10 mins each)	Observed Counter
1	205
2	430
3	677
4	945
5	1233
6	1542
7	1872
8	2224



$$10.5833(6.5)^2 + 193.1786(6.5) + 1.8214 = 1704$$

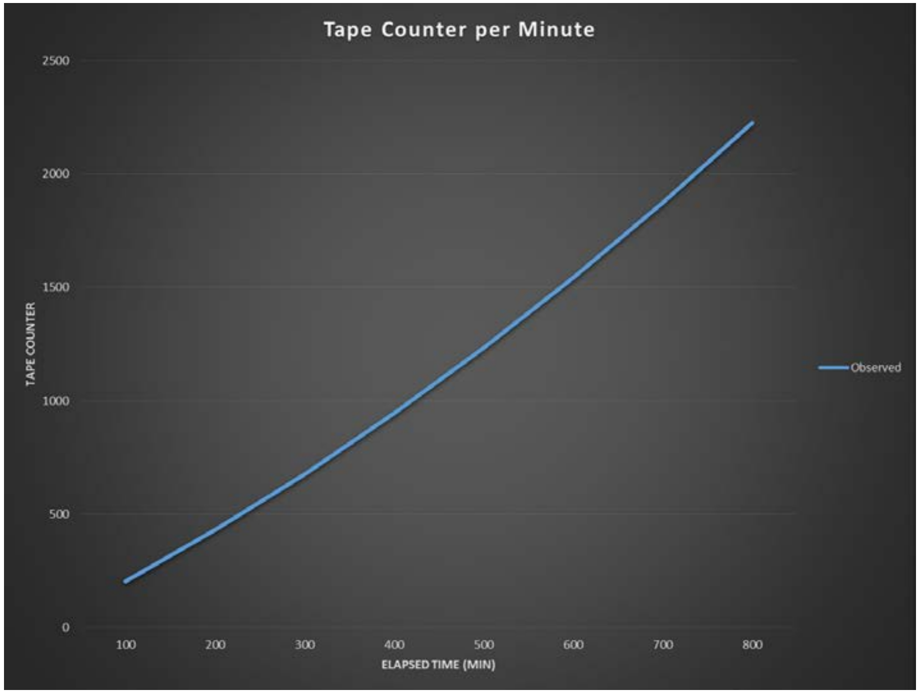
Lab 5 - Method of Least Squares

- Developed by Gauss, least squares finds the coefficients of a polynomial of a given degree that **approximates** a set of observations with **minimal variance** from the data
- The modeler selects a curve whose **general shape** matches the trend of the data
- The **partial derivatives** of the chosen polynomial are then determined which leads to a system of linear equations that can be solved using matrices and **Cramer's Rule**
- The coefficients of each term of the model polynomial can then be determined to allow us to estimate the unknown function values

Lab 5 - Method of Least Squares

$$Assume$$
$$y = ax^2 + bx + c$$

Elapsed Time (10 mins each)	Observed Counter
1	205
2	430
3	677
4	945
5	1233
6	1542
7	1872
8	2224



Lab 5 - Method of Least Squares

$$Minimize\ S = \sum_{i=1}^{n}\left[y_i - \left(ax_i^2 + bx_i + c\right)\right]^2$$

$$(y - ax^2 - bx - c)(y - ax^2 - bx - c)$$

$$\boxed{\begin{array}{l} y^2 - ax^2y - bxy - cy \\ \quad -ax^2y \quad + a^2x^4 + abx^3 + acx^2 \\ \quad\quad -bxy \quad + abx^3 \quad + b^2x^2 + bcx \\ \quad\quad\quad -cy \quad + acx^2 \quad + bcx + c^2 \end{array}}$$

$$\boxed{S = y^2 - 2ax^2y - 2bxy - 2cy + a^2x^4 + 2abx^3 + 2acx^2 + b^2x^2 + 2bcx + c^2}$$

Lab 5 - Method of Least Squares

$$Minimize\ S = \sum_{i=1}^{n} \left[y_i - \left(ax_i^2 + bx_i + c\right)\right]^2$$

$$S = y^2 - 2ax^2y - 2bxy - 2cy + a^2x^4 + 2abx^3 + 2acx^2 + b^2x^2 + 2bcx + c^2$$

For S to have a minimum, these partial derivatives must exist:

$$\frac{\partial S}{\partial a} = 0, \frac{\partial S}{\partial b} = 0, \frac{\partial S}{\partial c} = 0$$

$$\frac{\partial S}{\partial a} = -2x^2y + 2ax^4 + 2bx^3 + 2cx^2 = 0$$

$$\frac{\partial S}{\partial b} = -2xy + 2ax^3 + 2bx^2 + 2cx = 0$$

$$\frac{\partial S}{\partial c} = -2y + 2ax^2 + 2bx + 2c = 0$$

Lab 5 - Method of Least Squares

$$Minimize\ S = \sum_{i=1}^{n} \left[y_i - \left(ax_i^2 + bx_i + c\right)\right]^2$$

$$\frac{\partial S}{\partial a} = -2x^2y + 2ax^4 + 2bx^3 + 2cx^2 = 0$$

$$(x^4)a + (x^3)b + (x^2)c = x^2y$$

$$\frac{\partial S}{\partial b} = -2xy + 2ax^3 + 2bx^2 + 2cx = 0$$

$$(x^3)a + (x^2)b + (x)c = xy$$

$$\frac{\partial S}{\partial c} = -2y + 2ax^2 + 2bx + 2c = 0$$

$$(x^2)a + (x)b + c = y$$

Lab 5 - Method of Least Squares

$$Minimize\ S = \sum_{i=1}^{n}\left[y_i - \left(ax_i^2 + bx_i + c\right)\right]^2$$

This is a system of 3 linear equations and 3 unknowns!

$$\boxed{\begin{aligned}\left(\sum x_i^4\right)a + \left(\sum x_i^3\right)b + \left(\sum x_i^2\right)c \\ \left(\sum x_i^3\right)a + \left(\sum x_i^2\right)b + \left(\sum x_i\right)c \\ \left(\sum x_i^2\right)a + \left(\sum x_i\right)b + (n)c\end{aligned}} \begin{aligned} = \\ = \\ = \end{aligned} \boxed{\begin{aligned}\left(\sum x_i^2 y_i\right) \\ \left(\sum x_i y_i\right) \\ \left(\sum y_i\right)\end{aligned}}$$

Open Lab 5 - Method of Least Squares

$$Minimize\ S = \sum_{i=1}^{n} \left[y_i - \left(ax_i^2 + bx_i + c\right)\right]^2$$

$$\left(\sum x_i^4\right)a + \left(\sum x_i^3\right)b + \left(\sum x_i^2\right)c = \left(\sum x_i^2 y_i\right)$$

$$\left(\sum x_i^3\right)a + \left(\sum x_i^2\right)b + \left(\sum x_i\right)c = \left(\sum x_i y_i\right)$$

$$\left(\sum x_i^2\right)a + \left(\sum x_i\right)b + (n)c = \left(\sum y_i\right)$$

```
double sumX = PowerSum(vecX, 1);
double sumX2 = PowerSum(vecX, 2);
double sumX3 = PowerSum(vecX, 3);
double sumX4 = PowerSum(vecX, 4);

double sumY = PowerSum(vecY, 1);

double sumXY = PowerSum(vecX, 1, vecY);
double sumX2Y = PowerSum(vecX, 2, vecY);

double coeffMatrix[3][3]{
    { sumX4, sumX3, sumX2},
    { sumX3, sumX2, sumX },
    { sumX2, sumX, 8 } };

double valueVector[3]{ sumX2Y,sumXY,sumY };
```

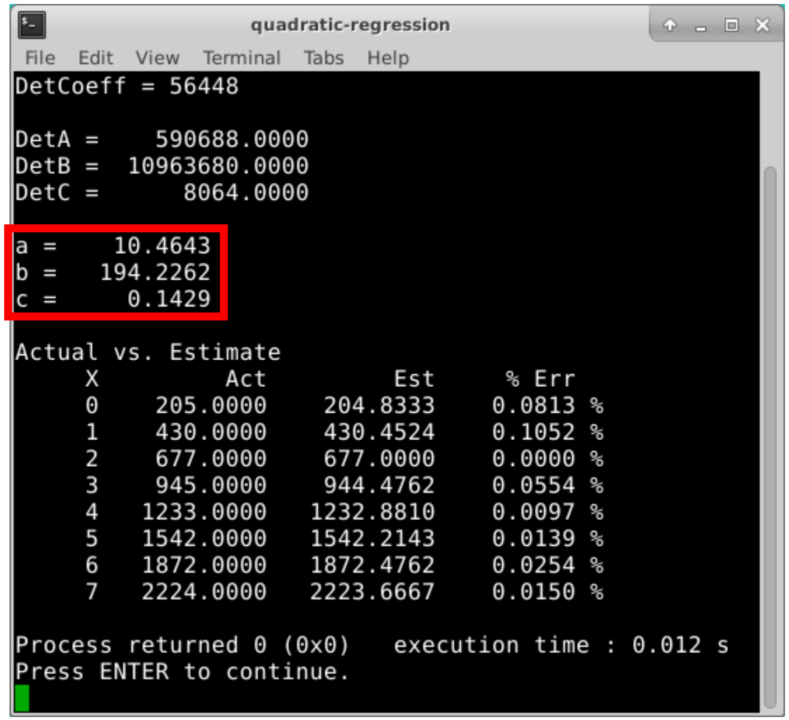
We have 8 sample points in the Lab 5 data set

View Lab 5 - Method of Least Squares

$$y = ax^2 + bx + c$$

```
double a = detA / detCoeff;
double b = detB / detCoeff;
double c = detC / detCoeff;

cout << "a = " << setw(10) << setprecision(4) << fixed << a << endl;
cout << "b = " << setw(10) << setprecision(4) << fixed << b << endl;
cout << "c = " << setw(10) << setprecision(4) << fixed << c << endl;

cout << endl << "Actual vs. Estimate" << endl;
cout << setw(6) << "X";
cout << setw(12) << "Act";
cout << setw(12) << "Est";
cout << setw(10) << "Err" << endl;

for (int i{}; i < 8; ++i) {
    double yp = a * pow(vecX[i], 2) + b * vecX[i] + c;
    double err = abs(vecY[i] - yp) / vecY[i];
    cout << setw(6) << i;
    cout << setw(12) << vecY[i];
    cout << setw(12) << yp;
    cout << setw(10) << setprecision(4) << err << endl;
```

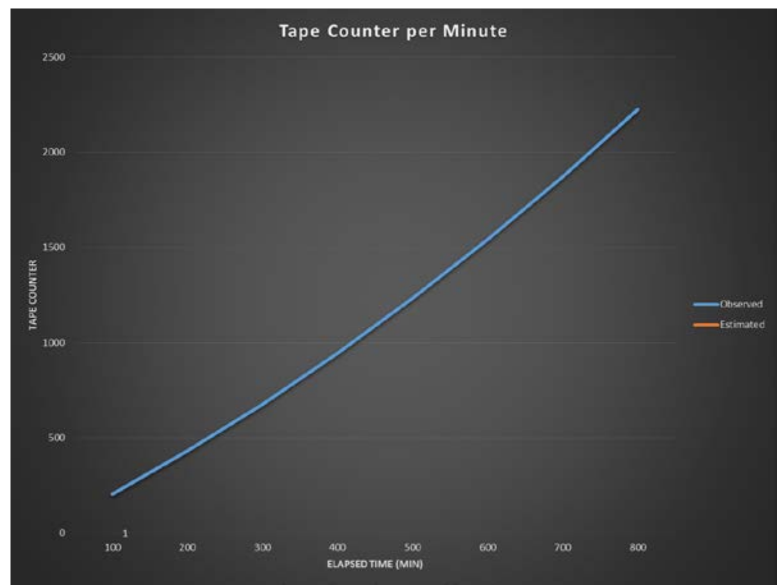
Run Lab 5 - Method of Least Squares



```
quadratic-regression
File  Edit  View  Terminal  Tabs  Help
DetCoeff = 56448

DetA =     590688.0000
DetB =   10963680.0000
DetC =       8064.0000

a =    10.4643
b =   194.2262
c =     0.1429

Actual vs. Estimate
     X         Act          Est      % Err
     0    205.0000     204.8333    0.0813 %
     1    430.0000     430.4524    0.1052 %
     2    677.0000     677.0000    0.0000 %
     3    945.0000     944.4762    0.0554 %
     4   1233.0000    1232.8810    0.0097 %
     5   1542.0000    1542.2143    0.0139 %
     6   1872.0000    1872.4762    0.0254 %
     7   2224.0000    2223.6667    0.0150 %

Process returned 0 (0x0)   execution time : 0.012 s
Press ENTER to continue.
```

$$y = 10.4643x^2 + 194.2262x + 0.1429$$

Elapsed Time (10 mins each)	Observed Counter	Estimated Counter
1	205	204.8333
2	430	430.4524
3	677	677.0000
4	945	944.4762
5	1233	1232.8810
6	1542	1542.2143
7	1872	1872.4762
8	2224	2223.6667



Verify Lab 5 - Method of Least Squares

via Difference Tables

$$y = 10.5833x^2 + 193.1786x + 1.8214$$

via Least Squares

$$y = 10.4643x^2 + 194.2262x + 0.1429$$

x	y Obs	y Est DT	% Err	y Est LS	% Err
1	205	205.5833	-0.2846%	204.8334	0.0813%
2	430	430.5119	-0.1190%	430.4525	-0.1052%
3	677	676.6071	0.0580%	677.0002	0.0000%
4	945	943.8690	0.1197%	944.4765	0.0554%
5	1233	1,232.2976	0.0570%	1232.8814	0.0096%
6	1542	1,541.8929	0.0069%	1542.2149	-0.0139%
7	1872	1,872.6548	-0.0350%	1872.477	-0.0255%
8	2224	2,224.5833	-0.0262%	2223.6677	0.0149%
		mean:	-0.0279%		0.0021%

The **method of least squares** guarantees the tightest possible fit between the observed data and the polynomial form chosen to model the data

Now you know…

- Model fitting starts by assuming the degree of an appropriate curve that reasonably matches the data
- How to create **difference tables** for constant, linear, quadratic, cubic, and quartic terms along with their expected average
- Gauss's "**Method of Least Squares**" refers to shaping the approximating curve to ***minimize the total deviations*** between the observed data and points on that curve
  - The function to minimize must be expanded, and then **partial derivatives** must be found for each coefficient of the curve
- The **% relative error** measures the "goodness of fit" of a model to experimental observations