QM

(Those vertical bars are optional)

The order of operations matters! $\langle\psi|\hat{x}\hat{p}|\psi\rangle \neq \langle\psi|\hat{p}\hat{x}|\psi\rangle$

Beause $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$, so $\hat{p}\hat{x}\psi = \frac{i}{\hbar}\frac{\partial}{\partial x}(x\psi) \neq \hat{x}\hat{p}\psi = x\frac{i}{\hbar}\left(\frac{\partial}{\partial x}\psi\right)$

This hints at the uncertainty principle...

$\hat{x}\hat{p}-\hat{p}\hat{x}=$

It's also important to note the connection between p and x! "Canonically conjugate"

~~Commutator~~ $\boxed{\text{Commutator } [\hat{q},\hat{p}] = \hat{q}\hat{p}-\hat{p}\hat{q}\text{; so } [\hat{x},\hat{p}] = i\hbar}$

~~Commutator~~ operator algebra

- Non commuting algebra
- Order of Measurements matters, in general (though sometimes it doesn't).
- $[\hat{A}\hat{B},\hat{C}] = \hat{A}[\hat{B},\hat{C}] + [\hat{A},\hat{C}]\hat{B}$

Schwartz Inequality

$$\boxed{\langle f|f\rangle\langle g|g\rangle \geq \langle f|g\rangle^2}$$

two operations $\hat{A}$ and $\hat{B}$, gives $\left(\sigma_x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}\right.$

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\langle[\hat{A},\hat{B}]\rangle\right)^2$$

[Pg 110 Griffiths]

This is the general uncertainty relation.

So, let $\hat{A}=\hat{x}$, $\hat{B}=\hat{p}$ ... Then $[\hat{A},\hat{B}] = [\hat{x},\hat{p}] = i\hbar$

so $\sigma_x^2\sigma_p^2 \geq \left(\frac{1}{2i}i\hbar\right)^2$, or

$$\boxed{\sigma_x\sigma_p \geq \frac{\hbar}{2}}$$

first heisenberg uncertainty relation, of position and momentum.

Note that if $\hat{x}$ and $\hat{p}$ commuted, there would be no uncertainty relation between them!