QM - Chapter 2 - The Time-Independent Schrödinger Equation

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V(x,t)\Psi$$ in general. But consider the case where V is a function of x *only*.

V(x) constant in time... Separation of variables!

Posit: $\Psi(x,t) = \psi(x)\varphi(t)$ ..... return to shröd...

$$i\hbar\frac{d\varphi}{dt} = \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi\varphi$$

$$i\hbar\frac{1}{\varphi}\frac{d\varphi}{dt} = \frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V$$

Blah Blah ... x and t uncorrolated, both sides = constant...

$$i\hbar\frac{1}{\varphi}\frac{d\varphi}{dt} = E$$

$$\frac{d\varphi}{dt} = \frac{-i}{\hbar}\varphi$$

so...

$$\varphi = e^{-iEt/\hbar}$$

[use $\psi(x)$ for integration constant]

also note: Units of $\frac{\hbar}{E}$ = time

$$\frac{-\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V = E$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi}$$

which is known as the time independent schrödinger equation.

Now, $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ Essentially we find $\psi(x)$ which gives us $\Psi(x,0)$, and then we can just add on the extra time evolution.

Note: $\Psi(x,t)$ evolves in time but $|\Psi(x,t)|^2 = \Psi^*\Psi$ does NOT.

So $|\Psi(x,t)|^2 = \psi^*\psi$. ← probability density. Ψ is refered to as stationary state

Also... the sep. constant is the total Energy (classical energy).

let $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$ be the Hamiltonian Operator

$\Rightarrow$ $\hat{H}\psi = E\psi$ then $\langle\psi|\hat{H}\psi\rangle = \langle\psi|E\psi\rangle = E\langle\psi|\psi\rangle = E$

↑ time indep. shröd.