Further notes on Ψ

- Normalized $\int_{-\infty}^{\infty} |\Psi|^2 = 1$
- Normalized applies for all time if normalized at a specific time

Notes on Probability Distributions

- $\langle x - \langle x \rangle \rangle = 0$
- RMS: $\left(\langle (x - \langle x \rangle)^2 \rangle\right)^{1/2} = \left[\langle x^2 - 2x\langle x \rangle + \langle x \rangle^2 \rangle\right]^{1/2}$

$$= \left[\langle x^2 \rangle - 2\langle x \rangle\langle x \rangle + \langle x \rangle^2\right]^{1/2}$$

$$= \left[\langle x^2 \rangle - \langle x \rangle^2\right]^{1/2}$$

- $\langle x \rangle$: Expectation Value
- In general $\langle f(x) \rangle = \int_{\text{all space}} f(x)\rho(x)\,dx$

$\int_{-\infty}^{\infty} |\Psi|^2 dx = 1$ That is $\int_{-\infty}^{\infty} \Psi^* \Psi \, dx = 1$

Ψ lives in a vector space. The vector space of square-integrable functions

$\Rightarrow f(x) = \int_a^b |f(x)|^2 dx < \infty$ Hilbert space

Notation: inner product of 2 functions is $\langle f | g \rangle = \int_a^b f^*(x) g(x) dx$

So $\langle g | f \rangle = \{op\} \langle f | g \rangle$

$\Rightarrow \langle g | f \rangle = \langle f | g \rangle^*$

Then $\langle \Psi | \Psi \rangle = 1$

a set of functions is orthonormal if $\langle f_m | f_n \rangle = \delta_{mn}$. The set is complete if you can write any other function as $f(x) = \sum_m c_m f_m(x)$

$|f\rangle = \sum_m c_m |f_m\rangle$

if want a c_m, then $\langle f_n | f \rangle = \sum_m \langle f_n | c_m | f_m \rangle = c_n$

$c_n = \langle f_n | f \rangle$ fourier's trick