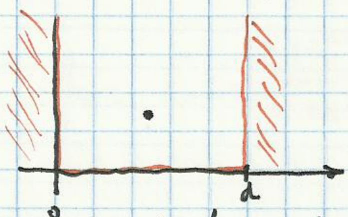
QM ~~The~~ Problems of a specific $V(x)$

Infinite Square Well

$$V(x) = \begin{cases} 0 \text{ between } x=0 \text{ and } a & 0 \le x \le a \\ \infty \text{ elsewhere} \end{cases}$$



"Particles don't like to be confined". This results in QM things, quantized energies etc.

since $x>a$ and $x<0 \Rightarrow \infty$, $\psi = 0$ outside of the well. So, we must solve TISE inside.

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi \Rightarrow \frac{\partial^2\psi}{\partial x^2} = -k^2\psi \quad \text{where } k = \frac{\sqrt{2mE}}{\hbar}$$

This has the general solution $\psi(x) = A\sin(kx) + B\cos(kx)$. Must fix A, B.

in order for this to make sense, $\frac{\partial^2\psi}{\partial x^2}$ must be "nice" (a smoothness condition):

ψ is continuous, and $\frac{\partial\psi}{\partial x}$ is continuous. [for ISW, we only need the first one]

$\psi(0) = \psi(a) = 0$, due to the boundary conditions! Thus, $B=0$. ~~Then~~ Also,

$\psi(a) = A\sin(ka) = 0$. The non-trivial solution is to set $\sin(ka)=0$ w/ $k \neq 0$: $k = \frac{n\pi}{a}$ $n \in \mathbb{Z}^+$

so, $\psi(x) = A\sin(k_n x)$. $\Rightarrow \underline{k_n = \frac{n\pi}{a}}$

finally, $\int_0^a A^2 \sin^2(k_n x)\,dx = 1$ (normalize) $\Rightarrow$ ~~A~~ $A^2\frac{a}{2} = 1$ $A^2 = \frac{2}{a}$; $\underline{A = \sqrt{\frac{2}{a}}}$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)} \quad \text{and} \quad \boxed{E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{n^2\pi^2\hbar^2}{2ma^2}}$$

Notes: There is an infinite set of solutions (one for each n), and the energies are quantized! The spacings $\propto n^2$.

The infinite square well is perfectly fine approximation if E doesn't get higher than the "true height" of a real well.

Also note, the energy is bounded from below by a non-zero ground state!

With respect to the well's center, ψ_n are alternatingly even and odd. Each ψ_n has $n-1$ nodes where $\psi_n(x)$ is zero.

$\langle \psi_m | \psi_n \rangle = \delta_{mn}$ (the wavefunctions are orthonormal) :)

$f(x) = \sum_{n=1}^{\infty} c_n \psi_n(x) = \sqrt{\frac{2}{a}} \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{a}\right)$ ...looks like fourier series... ψ_n's are complete.

how do you get c_n? $\Rightarrow$ $c_n = \langle \psi_n | f \rangle = \int \psi_n^*(x) f(x)\,dx$