QM

Full Schröd can be written this way too, $\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t}$. Well at least in the T-I case, $\hat{H}$ is the total energy operator. But you notice that it's related to time here... Another canonically conjugate.

$\langle H\rangle = \langle\psi|\hat{H}\psi\rangle = E\langle\psi|\psi\rangle = E$. These separated states of definite total energy. $\Rightarrow$ $\underline{\sigma_H = 0}$.

$$\sigma_H^2 = \langle H^2\rangle - \langle H\rangle^2 = E^2 - E^2 = 0. \quad [\text{next time}]$$

Note that this only works for the time-indep. version! for the full Schröd, energy isn't sharply defined.

Superpose

$$\Psi(x,t) = \sum_n c_n\psi_n(x)e^{-iE_nt/\hbar} \leftarrow \text{standard finding PDE solutions method.}$$

Ψ you get by solving TISE for ψ, and then find c_n's based on initial conditions, at $t=0$:

$$\Psi(x,0) = \sum c_n\psi_n(x) \quad \Big(\text{Each } \underset{\text{eigen fn}}{\psi_n} \text{ has its own } \underset{\text{eigen val}}{E_n}\Big) \Rightarrow \hat{H}|\psi_n\rangle = E_n|\psi_n\rangle$$

The c_n's indicate "mixture". It's normal for solving PDE's, but what do they mean if a particle has a specific allowed energy level, in which case only one $c_n \neq 0$. But if a particle has a superposition of wave functions... c_n indicates the probability of being in one state or another.

c_n's are calculated w/ fourier trick: (ψ_n orthogonal)

$$\text{if } f(x) = \sum c_n\psi_n(x)\text{, then } c_n = \int\psi_n^*(x)f(x)\,dx = \langle\psi_n|f\rangle$$

the $|c_n|^2$ tells a probability for measuring particular eigenvalues.

[There emerges an abstract concept of a state of a particle or system. the wavefunction is a representation of that state, but there are alternative representations, for example, if you take $\Psi(x)$ and do a fourier transform, you get some function as a function of $k = \frac{2\pi}{\lambda}$. But the particle remains unchanged.]

Time evolution $\frac{\partial}{\partial t}\langle Q\rangle$

$$\frac{\partial}{\partial t}\langle Q\rangle = \frac{\partial}{\partial t}\langle\Psi|\hat{Q}\Psi\rangle = \left\langle\left(\frac{\partial\Psi}{\partial t}\right)\middle|\hat{Q}\Psi\right\rangle + \underline{\left\langle\Psi\middle|\left(\frac{\partial\hat{Q}}{\partial t}\right)\Psi\right\rangle} + \left\langle\Psi\middle|\hat{Q}\frac{\partial\Psi}{\partial t}\right\rangle$$

The middle term we originally threw out ($\hat{x}$ doesn't change w/ time!). but now...

use Schröd: $\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial t} \Rightarrow \frac{\partial}{\partial t}\langle Q\rangle = -\frac{1}{i\hbar}\langle\hat{H}\Psi|\hat{Q}\Psi\rangle + \frac{1}{i\hbar}\langle\Psi|\hat{Q}\hat{H}\Psi\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle$

But $\hat{H}$ is hermitian! $\langle f|\hat{Q}g\rangle = \langle\hat{Q}f|g\rangle$, let $g = \hat{Q}\Psi$

$$\boxed{\frac{\partial}{\partial t}\langle Q\rangle = \frac{i}{\hbar}\langle\Psi|\hat{H}\hat{Q} - \hat{Q}\hat{H}|\Psi\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle = \frac{i}{\hbar}\langle[\hat{H},\hat{Q}]\rangle + \left\langle\frac{\partial\hat{Q}}{\partial t}\right\rangle}$$