$\boxed{QM}$

if $\langle \frac{\partial \hat{Q}}{\partial t} \rangle = 0$ and $[\hat{H}, \hat{Q}] = 0$, then $\langle Q \rangle$ is constant in time. Any such [illegible] is a constant, conserved quantity.

$$\boxed{\frac{\partial}{\partial t}\langle Q \rangle = \frac{i}{\hbar}\langle [\hat{H}, \hat{Q}] \rangle + \langle \frac{\partial \hat{Q}}{\partial t} \rangle}$$

(note that since $\hat{H}$ commutes w/ itself, Energy is conserved)

recall generalized uncertainty relation:

$$\sigma_H^2 \sigma_Q^2 \geq \underbrace{\left(\langle \frac{1}{2i}[\hat{H}, \hat{Q}] \rangle\right)^2}$$

. But note that $[\hat{H}, \hat{Q}]$ is in above equation!

$$= \left(\frac{1}{2i} \frac{\hbar}{i} \frac{\partial \langle Q \rangle}{\partial t}\right)^2 = \left(\frac{\hbar}{2}\right)^2 \left(\frac{\partial \langle Q \rangle}{\partial t}\right)^2$$

Then if $\frac{\partial}{\partial t}\langle Q \rangle$ is non-zero, there is an uncertainty associated w/ energy.

$$\sigma_H \sigma_Q \geq \frac{\hbar}{2} \left| \frac{\partial \langle Q \rangle}{\partial t} \right|$$

(get rid of the squares).

define $\boxed{\Delta t = \frac{\sigma_Q}{\left(\frac{d \langle Q \rangle}{dt}\right)}}$: given an uncertainty in $\langle Q \rangle$, Δt is how long it takes for that value to change by σ_Q.

define $\Delta E = \sigma_H$, then $\boxed{\Delta E \Delta t \geq \frac{\hbar}{2}}$ (second uncertainty relation of energy and time)

Okay, but what does that really mean? If something is around for a finite time, it's energy is uncertain... Additionally, particles may "borrow" energy for some small amount of time.

Note that if we don't care about time (time indep), $\Delta t \to \infty$, so ΔE can $\to 0$.

If you isolate a particle in time, it's precise energy becomes uncertain.