Notes on Polchinski's String Theory

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1 A first look at strings

We start by reviewing the action of a point-like particle. If we let $X^\mu(\tau)$ be a parametrization of the particle's worldline, the simplest Poincare-invariant action is

$$S_{\text{pp}} = -m\int \mathrm{d}\tau(-\dot{X}^\mu\dot{X}_\mu)^{1/2} \implies \delta S_{\text{pp}} = -m\int \mathrm{d}\tau \dot{u}_\mu \delta X^\mu, \quad u^\mu = \dot{X}^\mu(-\dot{X}^\nu\dot{X}_\nu)^{-1/2}. \tag{1.1}$$

Evidently, the equation of motion is $\dot{u}^\mu = 0$. If we let the world-line metric be $\gamma_{\tau\tau}(\tau)$ and define $\eta(\tau) = (-\gamma_{\tau\tau})^{1/2}$, then

$$S'_{\text{pp}} = \frac{1}{2}\int \mathrm{d}\tau \left(\eta^{-1}\dot{X}^\mu\dot{X}_\mu - \eta m^2\right) \implies \eta^2 = -\dot{X}^\mu\dot{X}_\mu/m^2. \tag{1.2}$$

From the equation of motion, we find $S' = S$.