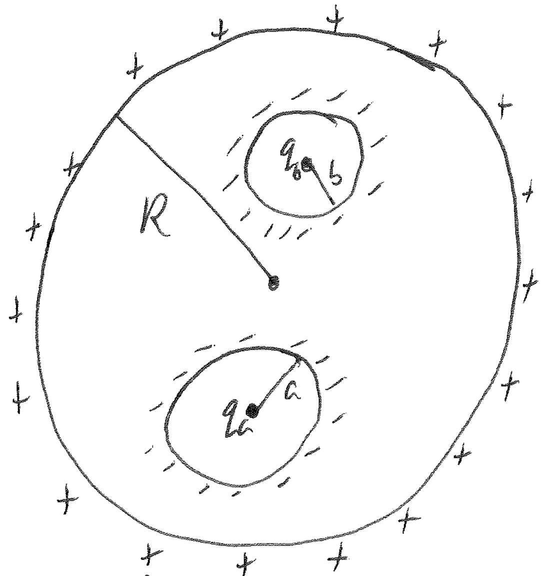
2.36) Theres a lot of physics ~~happe~~ happening in this problem.

Two spherical cavities are carved out from a neutral conducting sphere of radius R
At the center of each cavity a point charge is placed q_a and q_b.



suppose q_a and q_b are positive (this is for convenience only)

a) find σ_a σ_b and σ_R.

Remember, conductor was neutral, and its a conductor. this means all the charges that are induced will be on the surfaces at a, b, and R

we also know that the field inside needs to be cancelled exactly. this is crucial to the problem!

Since E just outside each sperical hole is zero $\sigma_{a,b}$ must cancel $q_{a,b}$

$\therefore$ it follows imediately $\boxed{\sigma_a = \frac{-q_a}{4\pi a^2} \quad \Big| \quad \sigma_b = \frac{-q_b}{4\pi b^2}}$

2.36 a) now that σ_a and σ_b are cancelling the field for us, there's a net positive charge left over. It has to be on the outer surface and it has to balance the induced charges σ_a, σ_b $\therefore$ 10/20

$$\sigma_R = \frac{q_a + q_b}{4\pi R^2}$$

b) notice σ_R is uniform and has an effective $Q = q_a + q_b$ it follows imediately (from Gauss's Law or Coulomb's Law) that

$$E = \frac{(q_a + q_b)}{4\pi \epsilon_0 r^2} \hat{r}$$

(the origin is at the center of the sphere)

c) ~~there's no electric field between the hollow spheres! this means the force between them.~~

for a) put origin at q_a:

$$E_a = \frac{q_a \hat{r}_a}{4\pi\epsilon_0 r_a^2}$$

for b put origin at q_b:

$$E_b = \frac{q_b \hat{r}_b}{4\pi\epsilon_0 r_b^2}$$

specifying the origin is essential! its not a complete answer without it.

d) remember the field between the spheres is zero! $\therefore$ the force is zero.

if the force was non-zero, the charges inside will move and σ_a, σ_b will no longer be uniform, the field in the interior will be non-zero but its a conductor so this can't happen!

e) ~~which charge is~~ what would change if q_c is put near the conductor?

Remember $\vec{E} \equiv \vec{0}$ inside the conductor!!

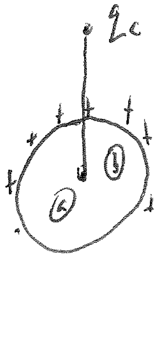
this means no matter what σ_a, σ_b E_a E_b all remain the same!

<u>but</u> $E \equiv 0$ inside the conductor!

therefore the surface charge at $r = R$ must cancel the field from q_c.

What is this charge distribution?

before we effectively had $q_a + q_b$ at the origin exposed to the world. now we have q_c near our sphere.



suppose q_c is at $\vec{p} = (0, 0, d)$ (origin at center of the sphere)

then $$\boxed{\vec{E}_{all} = \frac{1}{4\pi\epsilon_0}\left(\frac{q_c}{\mathscr{r}_c^2}\hat{\mathscr{r}}_c + \frac{(q_a+q_b)}{r^2}\hat{r}\right)} \qquad \vec{\mathscr{r}}_c = \vec{r} - \vec{p}$$

but we know that for $|\vec{r}| < R$ $\quad \vec{E} = 0$

2.36 e) So using the boundary condition

$$E^{\perp}_{above} - E^{\perp}_{below} = \frac{\sigma}{\varepsilon_0}$$

$$E_{\perp below} = 0$$

$$\therefore \quad \sigma = E^{\perp}_{above}\,\varepsilon_0$$

$$\sigma\Big|_{r=R} = \frac{1}{4\pi}\left(\frac{q_c}{\text{ᴙ}_c^2}\hat{\text{ᴙ}}_c + \frac{(q_a+q_b)}{\text{ᴙ}^2}\hat{\text{ᴙ}}\right)\cdot\hat{r}$$

$$\text{ᴙ}^2 = r^2 \quad \hat{\text{ᴙ}} = \hat{r}$$

$$\boldsymbol{\text{ᴙ}}_c = \mathbf{r} - \mathbf{p}$$

$$\text{ᴙ}_c^2 = x^2 + y^2 + (z-d)^2$$

$$\boldsymbol{\text{ᴙ}}_c = (x, y, z-d)$$

$$\sigma = \underbrace{\frac{1}{4\pi}\frac{q_c}{\text{ᴙ}_c^2}\hat{\text{ᴙ}}_c\cdot\hat{r}}_{\text{newly } \sigma \text{ induced by } q_c} + \underbrace{\frac{q_a+q_b}{4\pi R^2}}_{\text{old } \sigma_R}$$

2.48) In a vacuum diode electrons are boiled off from a hot cathode across a gap to an anode held at positive potential V_0. The cloud of moving electrons within the gap (called the space charge) quickly builds to the point where it reduces the field at the cathode to zero. From then on a steady current I flows between the plates. Suppose the plates are large enough ($A >> d$) so that edge effects can be ignored