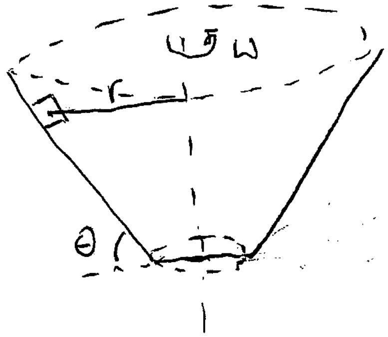
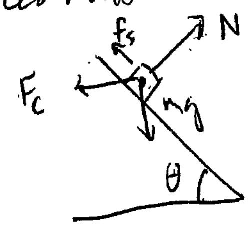


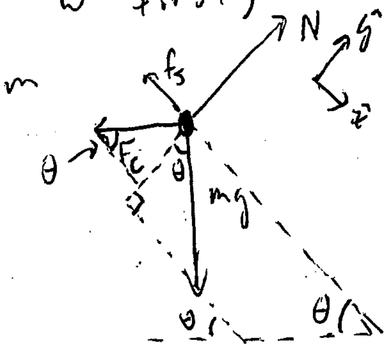
A plug of mass m sits in a funnel. the coefficient of static friction is μ_s. What are the min and max values of ω that will keep the plug fixed?

$(F_c \cos\theta < mg\sin\theta$

redraw with forces; (min ω first)



free body diagram $\Rightarrow$



in $\hat{y}$ direction:

$f_s = \mu_s N$

① $\quad N - mg\cos\theta - F_c \sin\theta = 0$

$F_c = m\omega^2 r$

in $\hat{x}$ direction:

② $\quad -f_s - F_c\cos\theta + mg\sin\theta = 0$

① gives: $N = mg\cos\theta + F_c\sin\theta$

plug ① into ② using $f_s = \mu_s N$

$$-\mu_s(mg\cos\theta + F_c\sin\theta) - F_c\cos\theta + mg\sin\theta = 0$$

solve for F_c

$$-\mu_s mg\cos\theta + mg\sin\theta = \mu_s F_c\sin\theta + F_c\cos\theta$$

$$F_c = mg\frac{(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}$$

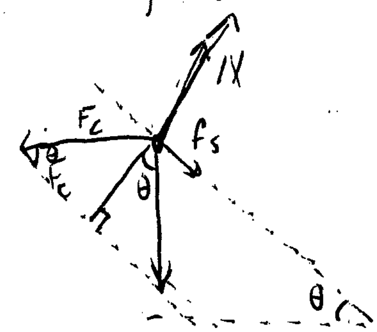
now $F_c = m\omega^2 r$

so our min ω is

$$\omega_{min} = \pm\sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}}$$

for max ω switch sign of f_s (think about why f_s was that way before)

force equations are:



$$y: \quad N - mg\cos\theta - F_c\sin\theta = 0$$

$$x: \quad +f_s - F_c\cos\theta + mg\sin\theta = 0$$

solve for F_c again:

$$F_c = mg\,\frac{(\sin\theta + \mu_s\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$

and our max ω is:

$$\omega_{max} = \pm\sqrt{\frac{g}{r}}\sqrt{\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$$

does this work? $\omega_{max} > \omega_{min}$ ✓

what about $\theta = 0$ $\quad$ ω_{min} isn't defined ✓

$$\omega_{max} = \pm\sqrt{\frac{g\mu_s}{r}}$$

what about $\theta = \pi/2$

$$\omega_{min} = \pm\sqrt{\frac{g}{r\mu_s}}$$

ω_{max} isn't defined ✓