iit Jee 2011 Paper 1 Offline $69 Questions$

Question 001 MCQ

QUESTION

The correct statement s pertaining to the adsorption of a gas on a solid surface is are

A. Adsorption is always exothermic

B. Physisorption may transform into chemisorption at high temperature

C. Physisorption increases with increasing temperature but chemisorption decreases with increasing temperature

D. Chemisorption is more exothermic than physisorption, however it is very slow due to higher energy of activation

CORRECT OPTION

A. Adsorption is always exothermic

SOURCE

Chemistry • surface-chemistry

EXPLANATION

A

Adsorption, whether it is chemisorption or physisorption, is an exothermic process. In physisorption, gas molecules are adsorbed onto the surface of the adsorbent by van der Waals forces. When the attractive forces between the gas and the solid adsorbent are stronger than those between the gas molecules themselves, energy is released, making the process exothermic.

In chemisorption, a chemical bond is formed between the adsorbent and the adsorbate. This bond formation releases energy, making the process exothermic.

Thus, **Option A is correct**.

B

At low temperatures, gases such as hydrogen or oxygen can accumulate on the solid surface $adsorbent$ through a process called physisorption, which has a low enthalpy of adsorption $20 - 40kJ/mol$. With increasing temperature, oxygen or hydrogen can react with metal surfaces $adsorbent$ to form metal oxides or metal hydrides, a process known as chemisorption, with an enthalpy of chemisorption between 80-240 kJ/mol.

Thus, **Option B is correct**.

C

Physisorption is a low enthalpy process that is favorable at low temperatures, while chemisorption has a high enthalpy process and occurs more readily at higher temperatures. Therefore, the statement that physisorption increases with increasing temperature and chemisorption decreases with increasing temperature is incorrect.

Thus, **Option C is incorrect**.

D

Chemisorption occurs at higher temperatures because the energy of the reactant molecules $adsorbateandadsorbent$ must be sufficient to overcome the potential energy barrier to form the product. However, once chemical bonds are formed between the reactant molecules, a significant amount of energy is released. Therefore, chemisorption is more exothermic than physisorption.

Thus, **Option D is correct**.

Note : Physisorption arises due to weak forces of attraction between the adsorbent and the absorbent molecules.

Question 002 MCQ

QUESTION

Geometrical shapes of the complexes formed by the reaction of Ni^{2+} with Cl^-, CN^- and H_2O respectively are

A octahedral, tetrahedral and square planar

B tetrahedral, square planar and octahedral

C square planar, tetrahedral, and octahedral

D octahedral, square planar and octahedral

CORRECT OPTION

B tetrahedral, square planar and octahedral

SOURCE

Chemistry • coordination-compounds

EXPLANATION

i Nickel in +2 oxidation state $\left(Ni^{2+}\right)$ prefers to form tetrahedral complexes with chlorine, e.g., $\left[NiCl_4\right]^{2-}$.

The oxidation state of nickel in $[NiCl_4]^{2-}$

$$x + 4 \times (-1) = -2$$

$$x = -2 + 4 = +2$$

Electronic configuration of $Ni^{2+} = [Ar]3d^8 4s^0$

Since, chlorine is a weak ligand, electrons in Ni^{2+} do not pair up. One $4s$ and three $4p$ orbitals undergo hybridisation to form four sp^3 hybrid orbitals. These vacant orbitals accept a pair of electron from each of the chloride ion.

The geometry of sp^3 hybridised nickel $\left[Ni^{2+}\right]$ is tetrahedral

ii Nickel in +2 oxidation state $\left(\right.$ as $\left.Ni^{2+}\right)$ forms square planner complex with cyanide ligands, e.g., $[Ni(CN)_4]^{2-}$.

Oxidation state of Ni in $[Ni(CN)_4]^{2-}$

$$x + 4 \times (-1) = -2$$
$$x = -2 + 4 = +2$$

The oxidation state of nickel in $[Ni(CN)_4]^{2-}$ is $+2$

Electronic configuration of $Ni^{2+} = [Ar]3d^8 4s^0$

Since, cyanide is a strong ligand, electrons in $3d$-orbitals are paired up.

One $3d$, one $4s$ and two $4p$ orbitals undergoes hybridisation to form four dsp^2 hybrid orbitals. These vacant orbitals accept a pair of electrons from each of

cyanide ligand.

The geometry of dsp^2 hybridised nickel complex $\left[\mathrm{Ni(CN)_4}\right]^{2-}$ is square planner.

iii Nickel in +2 oxidation state as Ni^{2+} forms octahedral complex with water as ligands, e.g., $\left[\mathrm{Ni(H_2O)_6}\right]^{2+}$

Oxidation state of nickel in $\left[\mathrm{Ni(H_2O)_6}\right]^{2+}$ is:

$$x + 6 \times 0 = +2$$

$$x = +2$$

Electronic configuration of $\mathrm{Ni}^{2+} =$

Since, water is a weak ligand, electrons in $3d$ orbitals of Nickel remains unpaired. One $4s$, three $4p$ and two $4d$ orbitals undergo hybridisation to form sp^3d^2 hybrid orbitals. These vacant orbitals accept lone pair of electron from each of the water molecule.

The geometry of sp^3d^2 hybridised nickel complex $\left[\mathrm{Ni(H_2O)_6}\right]^{3+}$ is octahedral.

The geometries of different Ni^{2+} complexes are as follows:

$$[NiCl_4]^{2-} \rightarrow \text{tetrahedral}$$

$$[NiCN_4]^{2-} \rightarrow \text{square planner}$$

$$\left[Ni_{(}(H_2O)_6\right]^{2+} \rightarrow \text{octahedral}$$

Question 003 MCQ

QUESTION

According to kinetic theory of gases

A collisions are always elastic

B heavier molecules transfer more momentum to the wall of the container

C only a small number molecules have very high velocity

D between collisions, the molecules move in straight line with constant velocity

CORRECT OPTION

A collisions are always elastic

SOURCE

Chemistry • gaseous-state

EXPLANATION

A When gas molecules collide with each other, high energy molecule transfers some of its energy to less energetic molecule, but the total energy remains conserved. This type of collision between gas molecules is called elastic collision. According to kinetic theory of gas, no loss of energy is observed on collisions and kinetic energy is proportional to absolute temperature of gas.

Option A is correct.

B Momentum of any particle for example, gas molecules depends upon the mass as well as the velocity of the gas. But when there is huge increase in the mass of the gas, its velocity decreases considerably. Hence, heavier gas molecules collides the walls of container with less impact. The transfer of momentum to the walls is very less.

Option B is incorrect.

C Maximum number $no.$ of gas molecules have speed distributed around the most probable speed $\$v_{\mathrm{mps}}\$$. This speed corresponds to the maximum peak of Maxwell-Boltzmann distribution curve.

There are few molecules which have very high speed $the shaded area under the curve$.

Option (C) is correct.

D Molecules are assumed to be moving in straight line with constant velocities.

Option D is correct.

Question 004 Numerical

QUESTION

To an evacuated vessel with movable piston under external pressure of 1 atm, 0.1 mol of He and 1.0 mol of an unknown compound (vapour pressure 0.68 atm,

at 0°C) are introduced. Considering the ideal gas behaviour, the total volume $inlitre$ of the gases at 0°C is close to

SOURCE

Chemistry • gaseous-state

EXPLANATION

Given: External pressure $(\mathrm{P}_{\mathrm{ext}}) = 1\ \mathrm{atm}$

Number of mole of helium $(n_{\mathrm{He}}) = 0.1\ \mathrm{mol}$

No. of mole of unknown compound

$(n_{\text{unknown compound}}) = 1.0\ \mathrm{mol}$

Vapour pressure of unknown compound

$\left(p^0_{\text{unknown}}\right) = 0.68\ \mathrm{atm}$

Temperature of the mixture $0^\circ\mathrm{C} = 273\ \mathrm{K}$

To Find: The volume of gas $inlitre = v_{\mathrm{gas}}$

Formula: i Vapour pressure of helium $(\mathrm{P}_{\mathrm{He}}) =$

$P_{\mathrm{ext}} - P_{\text{unknown compound}}$

$ii\ \mathrm{V}_{\mathrm{He}} = \frac{n_{\mathrm{He}} \times \mathrm{R} \times \mathrm{T}}{\mathrm{P}_{\mathrm{He}}}$

Since, the evacuated vessel $withfitted$ piston in equilibrium with its surroundings. Hence, external pressure $orpressureoutsidethevessel$ is equal to pressure inside the vessel.

$$P_{ext} = P_{internal} = P_T$$

$$P_{ext} = P_{He} + P_{unknwon\ compound}$$

$$1\ atm = P_{He} + 0.68\ atm$$

$$P_{He} = 0.32\ atm$$

$[P_{He} P_{unknown\ compound}$ are partial pressures of helium and unknown gas respectively]

According to ideal gas equation:

$$P_{He} \times V_{He} = n_{He} \times R \times T$$

$$V_{He} = \frac{0.1 \times 0.0821 \times 273}{0.32}$$

$$V_{He} = 7.004\ L$$

Question 005 MCQ

QUESTION

Among the following compounds, the most acidic is

A p-nitrophenol

B p-hydroxybenzoic acid

o-hydroxybenzoic acid

D p-toluic acid

CORRECT OPTION

C o-hydroxybenzoic acid

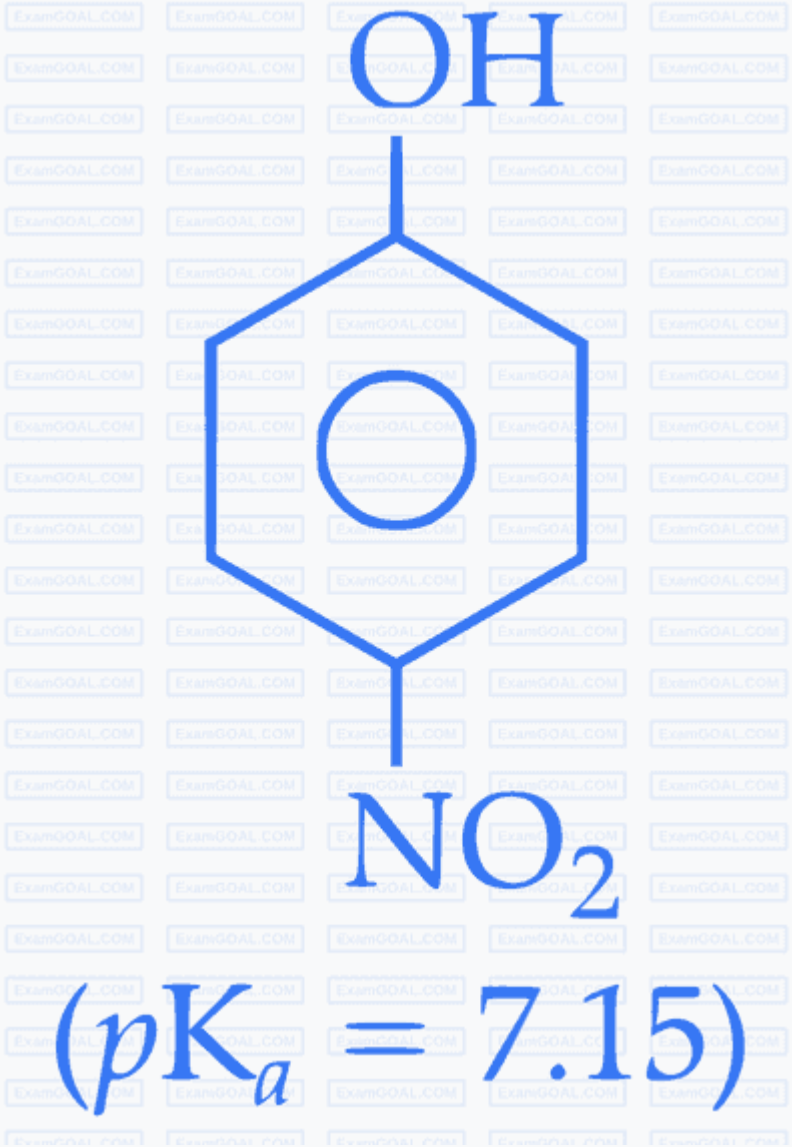
SOURCE

Chemistry • basics-of-organic-chemistry

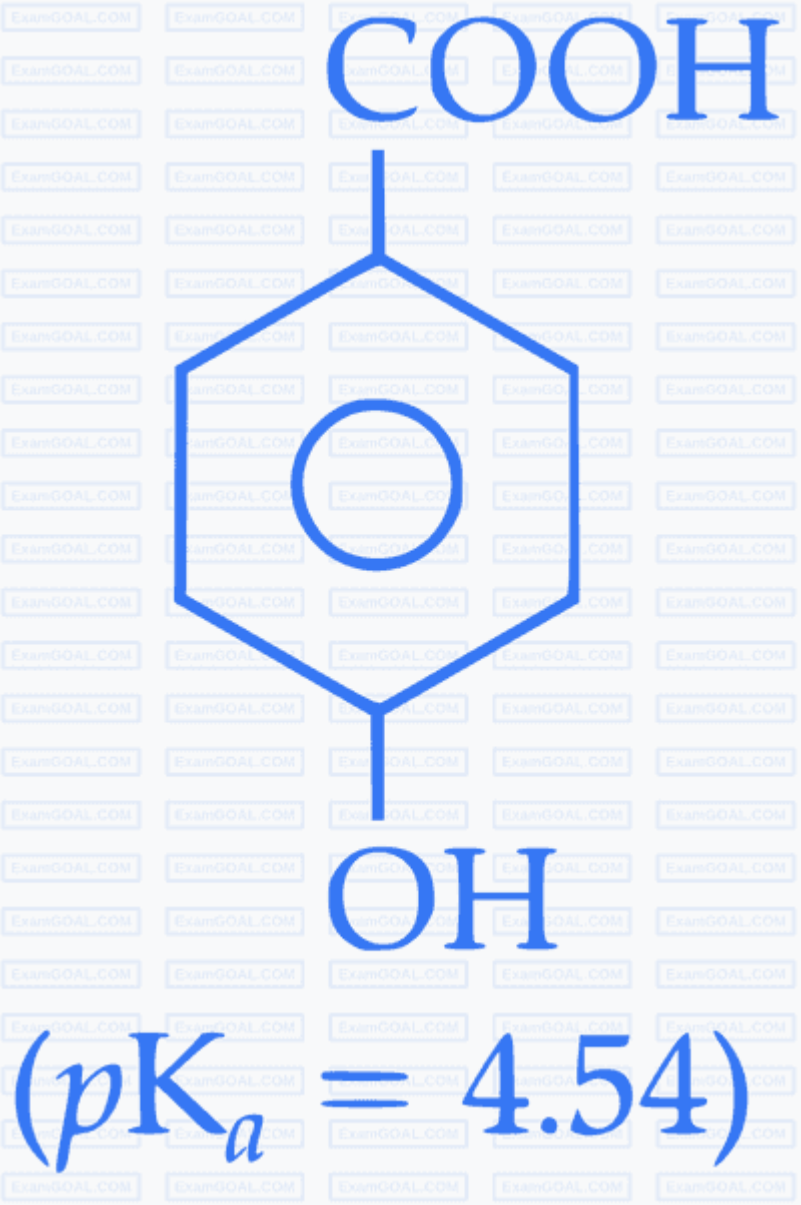
EXPLANATION

i The structures of the different organic compounds are as follows :

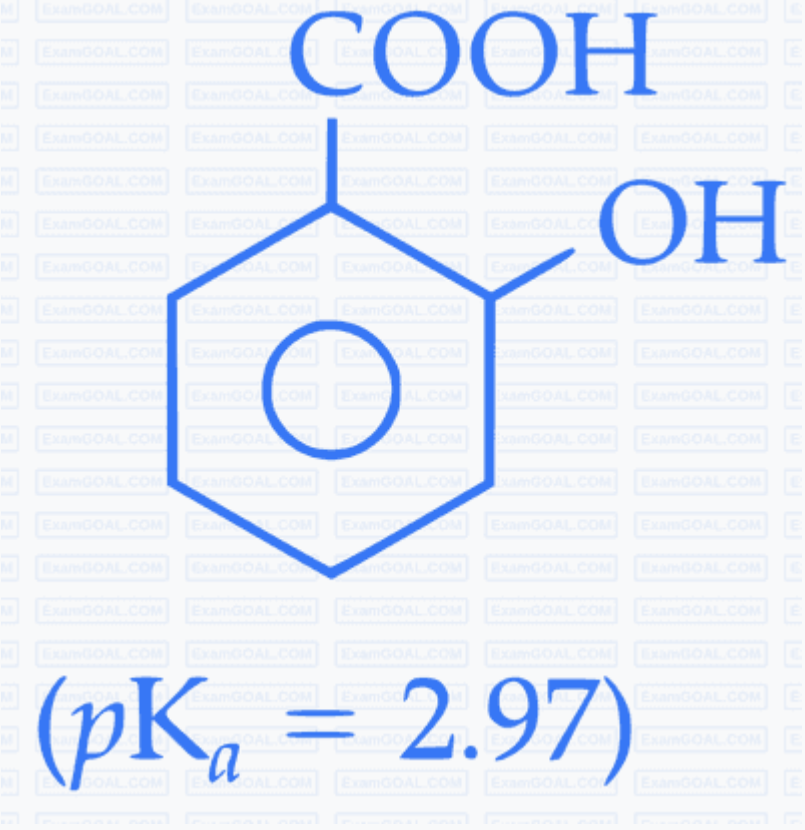
A **p-Nitrophenol**



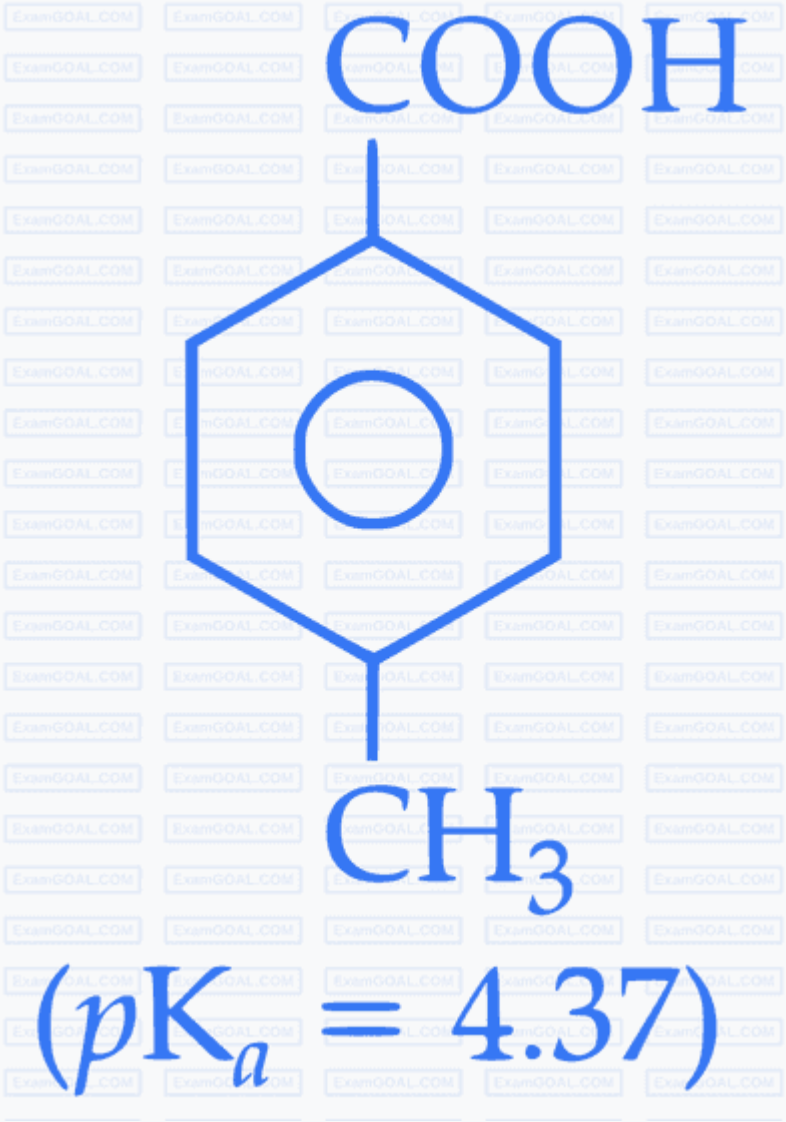
B **p-Hydroxybenzoic Acid**



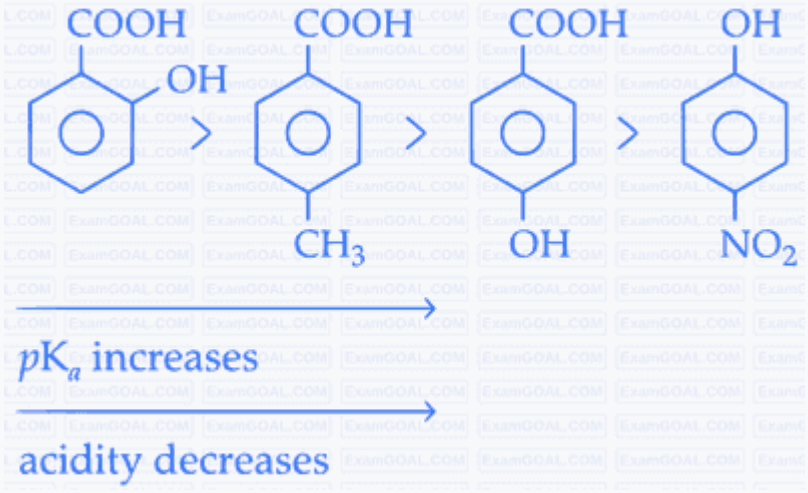
C **o-Hydroxybenzoic Acid**



D **p-Toluic Acid**



ii According to the pK_a values, the order of acidity of these compounds is as follows :

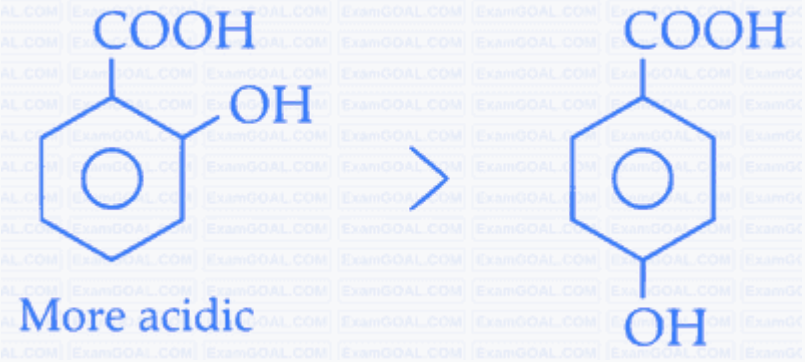


iii Regardless of the substitution, phenols are generally less acidic than carboxylic acids. Therefore, p-nitrophenol is the least acidic, despite the

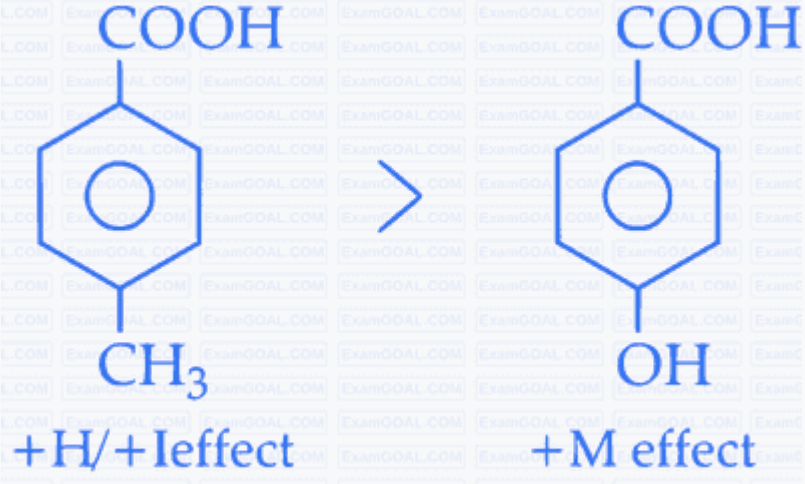
presence of an electron-withdrawing nitro group.

iv The "ortho effect," i.e., the presence of an electron-withdrawing or releasing substituent at the ortho position relative to the carboxylic acid group, increases the acidity of substituted benzoic acids.

Therefore, o-hydroxybenzoic acid is more acidic than p-hydroxybenzoic acid.



v The presence of an -OH group at the para position decreases the acidity of carboxylic acids due to its electron-donating nature $via the positive mesomeric effect, +M$. This effect is stronger compared to the electron-donating nature of a methyl group via the positive inductive effect $+I$ or hyperconjugative effect $+H$.



Hence, the order of acidity of the compounds is :

C o-Hydroxybenzoic Acid

D p-Toluic Acid

B p-Hydroxybenzoic Acid

A p-Nitrophenol

Option C has the lowest pK_a or the highest K_a.

Question 006 MCQ

QUESTION

Dissolving 120 g of urea $mol.wt.60$ in 1000 g of water gave a solution of density 1.15 g/mL. The molarity of the solution is

A 1.78 M

B 2.00 M

C 2.05 M

D 2.22 M

CORRECT OPTION

C 2.05 M

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

To find the molarity of the urea solution, we will follow these steps:

1. Calculate the number of moles of urea using its molecular weight.

2. Determine the volume of the solution using its mass and density.

3. Use the number of moles and volume of the solution to calculate the molarity.

Step 1: Calculate the number of moles of urea.

The number of moles of a substance n is determined by dividing the mass of the substance $\$m\$$ by its molecular weight $\$M\$$:

$$n = \frac{m}{M}$$

For urea, $m = 120\ \text{g}$ and $M = 60\ \text{g/mol}$, thus:

$$n_{urea} = \frac{120\ \text{g}}{60\ \text{g/mol}}$$

$$n_{urea} = 2\ \text{moles}$$

Step 2: Determine the volume of the solution.

The volume V of the solution can be found by dividing the mass of the solution by its density ρ:

$$V = \frac{mass_{solution}}{\rho}$$

The mass of the solution is the sum of the mass of urea and the mass of water. Thus:

$$mass_{solution} = mass_{urea} + mass_{water} = 120\ \text{g} + 1000\ \text{g}$$

$$mass_{solution} = 1120\ \text{g}$$

The density of the solution ρ is given as 1.15 g/mL. Therefore, the volume in milliliters $which is equivalent to cubic centimeters$ is:

$$V = \frac{1120\ \text{g}}{1.15\ \text{g/mL}}$$

$$V = 973.91\ \text{mL}$$

To convert milliliters to liters $since molarity is defined in terms of liters$, we divide by 1000:

$$V = 0.97391\ \text{L}$$

Step 3: Calculate the molarity.

Molarity $\$M\$$ is defined as the number of moles of solute divided by the volume of solution in liters:

$$M = \frac{n_{solute}}{V_{solution}}$$

$$M = \frac{2 \text{ moles}}{0.97391 \text{ L}}$$

$$M \approx 2.05 \text{ M}$$

Therefore, the molarity of the urea solution is approximately 2.05 M, which corresponds to Option C.

Question 007 MCQ

QUESTION

Extraction of metal from the ore cassiterite involves

A carbon reduction of an oxide ore

B self-reduction of a sulphide ore

C removal of copper impurity

D removal of iron impurity

CORRECT OPTION

A carbon reduction of an oxide ore

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Tin is obtained by reducing the ore cassiterite with coal in a reverberatory furnace. Limestone is added to produce a slag with the impurities, which can be separated.

$$\mathrm{SnO_2 + 2C \longrightarrow Sn + 2CO}$$

Crude tin obtained from this process is contaminated with iron, copper, lead, and other metals. To purify it, the crude tin is remelted in an inclined furnace—a process known as liquation. During liquation, the easily fusible tin melts away, leaving the less fusible impurities behind.

Finally, the molten tin is refined using green poles of wood in contact with air. This step, known as poling, helps to oxidize any remaining metal impurities. These impurities form a scum on the surface, which can be removed.

This step-by-step process ensures the extraction and purification of tin from cassiterite.

Question 008 Numerical

QUESTION

The maximum number of electrons that can have principal quantum number, n = 3, and spin quantum number, $m_s = -1/2$, is

SOURCE

Chemistry • structure-of-atom

EXPLANATION

To answer this question, we need to consider the quantum numbers that describe electrons in atoms. Each electron in an atom is described by four quantum numbers:

1. The principal quantum number n dictates the energy level and size of the electron orbital.
2. The azimuthal $angular momentum$ quantum number l defines the shape of the orbital. For a given n, l can take any integer value from 0 to $n-1$.
3. The magnetic quantum number m_l describes the orientation of the orbital in space. For a given l, m_l can take values from $-l$ to $+l$, including zero.
4. The spin quantum number m_s describes the direction of the electron's spin and can have a value of $+\frac{1}{2}$ or $-\frac{1}{2}$.

Given the principal quantum number, $n = 3$, and the spin quantum number, $m_s = -\frac{1}{2}$, we need to calculate the maximum number of electrons that can fit this criteria.

For $n = 3$, the possible values of l are 0, 1, and 2 $s, p, and d orbitals respectively$. Here's how the orbitals break down:

- When $l = 0$ $the 3s orbital$, $m_l = 0$, so there's 1 orbital.
- When $l = 1$ $the 3p orbitals$, m_l can be -1, 0, or +1, providing 3 orbitals.
- When $l = 2$ $the 3d orbitals$, m_l can be -2, -1, 0, +1, or +2, giving 5 orbitals.

Each orbital can hold 2 electrons with opposite spins. Since we are specifically looking for electrons with $m_s = -\frac{1}{2}$, we can only count one electron per orbital. Therefore, we sum the total number of orbitals across the s, p, and d sublevels for $n = 3$:

3s: 1 orbital $\times$ 1 electron $with m_s = -\frac{1}{2}$ = 1 electron

3p: 3 orbitals $\times$ 1 electron $with m_s = -\frac{1}{2}$ = 3 electrons

3d: 5 orbitals $\times$ 1 electron $with m_s = -\frac{1}{2}$ = 5 electrons

If we sum these, we get:

1 + 3 + 5 = 9 electrons

Thus, the maximum number of electrons that can have the principal quantum number $n = 3$ and the spin quantum number $m_s = -\frac{1}{2}$ is 9.

Question 009 Numerical

QUESTION

The work function φ of some metals is listed below. The number of metals which will show photoelectric effect when light of 300 nm wavelength falls on the metal is

Metal	Li	Na	K	Mg	Cu	Ag	Fe	Pt	W
$\Phi\ eV$	2.4	2.3	2.2	3.7	4.8	4.3	4.7	6.3	4.75

SOURCE

Chemistry • structure-of-atom

EXPLANATION

To determine the number of metals that will show the photoelectric effect when light of 300 nm wavelength falls on them, we first need to calculate the energy $ineV$ of the incident photons. The energy $\$E\$$ of a photon can be calculated using the formula:

$$E = \frac{hc}{\lambda}$$

where

- h is Planck's constant $\$6.626 \times 10^{-34}\$J \cdot s$,
- c is the speed of light $\$3.00 \times 10^{8}\$m/s$,
- λ is the wavelength of the light $inmeters$.

First, convert the wavelength from nm to meters:

$$300\,\text{nm} = 300 \times 10^{-9}\,\text{m}$$

Then, calculate the energy of the photons:

$$E = \frac{(6.626 \times 10^{-34}\,\text{J·s}) \times (3.00 \times 10^{8}\,\text{m/s})}{300 \times 10^{-9}\,\text{m}}$$

$$E = \frac{(6.626 \times 3.00) \times 10^{-19}}{300}\,\text{J}$$

$$E = \frac{19.878 \times 10^{-19}}{300}\,\text{J}$$

$$E \approx 6.626 \times 10^{-19}\,\text{J}$$

To convert the energy from joules to electron volts eV, we divide by the charge of an electron $\$1.602 \times 10^{-19}\$C$:

$$E \approx \frac{6.626 \times 10^{-19}\,\text{J}}{1.602 \times 10^{-19}\,\text{C/e\^-}} \approx 4.14\,\text{eV}$$

Now, to determine whether the photoelectric effect will occur, we compare the energy of the incident photons to the work function $\$\phi\$$ of each metal. The photoelectric effect occurs if the photon energy is greater than or equal to the work function of the metal:

- For Li $\$\phi = 2.4\,\text{eV}\$$: Yes, $4.14\,\text{eV} > 2.4\,\text{eV}$
- For Na $\$\phi = 2.3\,\text{eV}\$$: Yes, $4.14\,\text{eV} > 2.3\,\text{eV}$
- For K $\$\phi = 2.2\,\text{eV}\$$: Yes, $4.14\,\text{eV} > 2.2\,\text{eV}$
- For Mg $\$\phi = 3.7\,\text{eV}\$$: Yes, $4.14\,\text{eV} > 3.7\,\text{eV}$
- For Cu $\$\phi = 4.8\,\text{eV}\$$: No, $4.14\,\text{eV} < 4.8\,\text{eV}$
- For Ag $\$\phi = 4.3\,\text{eV}\$$: No, $4.14\,\text{eV} < 4.3\,\text{eV}$
- For Fe $\$\phi = 4.7\,\text{eV}\$$: No, $4.14\,\text{eV} < 4.7\,\text{eV}$
- For Pt $\$\phi = 6.3\,\text{eV}\$$: No, $4.14\,\text{eV} < 6.3\,\text{eV}$
- For W $\$\phi = 4.75\,\text{eV}\$$: No, $4.14\,\text{eV} < 4.75\,\text{eV}$

Thus, the number of metals which will show the photoelectric effect when light of 300 nm wavelength falls on the metal is 4 Li, Na, K, Mg.

Question 010 Numerical

QUESTION

The difference in the oxidation numbers of the two types of sulphur atoms in $Na_2S_4O_6$ is

SOURCE

Chemistry • redox-reactions

EXPLANATION

i The structure of compound containing sulphur in $\mathrm{Na_2\ S_4O_6}$ is :

Let the oxidation state of sulpher be x :

$$4 \times x + 2 \times (+1) + 6 \times (-2) = 0$$
$$4x = 12 - 2 = 0$$
$$4x = 10$$
$$x = \frac{5}{2} = 2.5$$

ii Each of the corner sulphurs utilises five valence electrons to form bond with oxygen atoms.

Their oxidation state is +5.

iii Oxidation state of central sulphur atom is zero 0.

Difference between two types of sulphur = 5 – 0 = 5.

Question 011 MCQ

QUESTION

Bombardment of aluminium by

$$\alpha$$

-particle leads to its artificial disintegration in two ways : i and ii as shown. Products X, Y and Z, respectively, are

A proton, neutron, positron.

B neutron, positron, proton.

C proton, positron, neutron.

D positron, proton, neutron, neutron.

CORRECT OPTION

A proton, neutron, positron.

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

Bombardment of aluminum by an

$$\alpha$$

-particle leads to its artificial disintegration in two ways, as shown in the reactions below. The resultant products X, Y, and Z are identified as follows :

Reaction i

Disintegration of aluminum into silicon :

$$ {}_{2}^{4}\mathrm{He} + {}_{13}^{27}\mathrm{Al} \rightarrow {}_{14}^{30}\mathrm{Si} + {}_{Z}^{A}X $$

Conservation conditions :

a **Charge balance :**

$$ 2 + 13 = 14 + Z \implies Z = 1 $$

b **Mass balance :**

$$ 4 + 27 = 30 + A \implies A = 1 $$

The particle ${}_{Z}^{A}X$ is ${}_{1}^{1}\mathrm{H}$, a proton $anisotopeofhydrogen$.

Reaction ii

Disintegration of aluminum into phosphorus :

$$ {}_{2}^{4}\mathrm{He} + {}_{13}^{27}\mathrm{Al} \rightarrow {}_{15}^{30}\mathrm{P} + {}_{Z}^{A}Y $$

Conservation conditions :

a **Charge balance :**

$$ 2 + 13 = 15 + Z \implies Z = 0 $$

b **Mass balance :**

$$ 4 + 27 = 30 + A \implies A = 1 $$

The particle Y has one unit mass and no charge, which is a neutron ${}_{0}^{1}n$.

Reaction iii

Disintegration of phosphorus into silicon :

$$ {}_{15}^{30}\mathrm{P} \longrightarrow {}_{14}^{30}\mathrm{Si} + {}_{Z}^{A}\mathrm{Z} $$

Conservation conditions :

a **Mass balance :**

$$30 = 30 + A \implies A = 0$$

b **Charge balance :**

$$15 = 14 + Z \implies Z = 1$$

The particle has zero mass but one unit of positive charge. Hence, the particle is a positron $\left({}^{0}_{+1}\beta\right)$.

Summary

The particles are :

X = ${}^{1}_{1}\mathrm{H}$ $proton$

Y = ${}^{1}_{0}n$ $neutron$

Z = ${}^{0}_{+1}\beta$ $positron$

Question 012 MCQ

QUESTION

$AgNO_3\,aq.$ was added to an aqueous KCl solution gradually and the conductivity of the solution was measured. The plot of conductance $\$\$\Lambda\$\$$ versus the volume of $AgNO_3$ is

A P

B Q

C R

D S

CORRECT OPTION

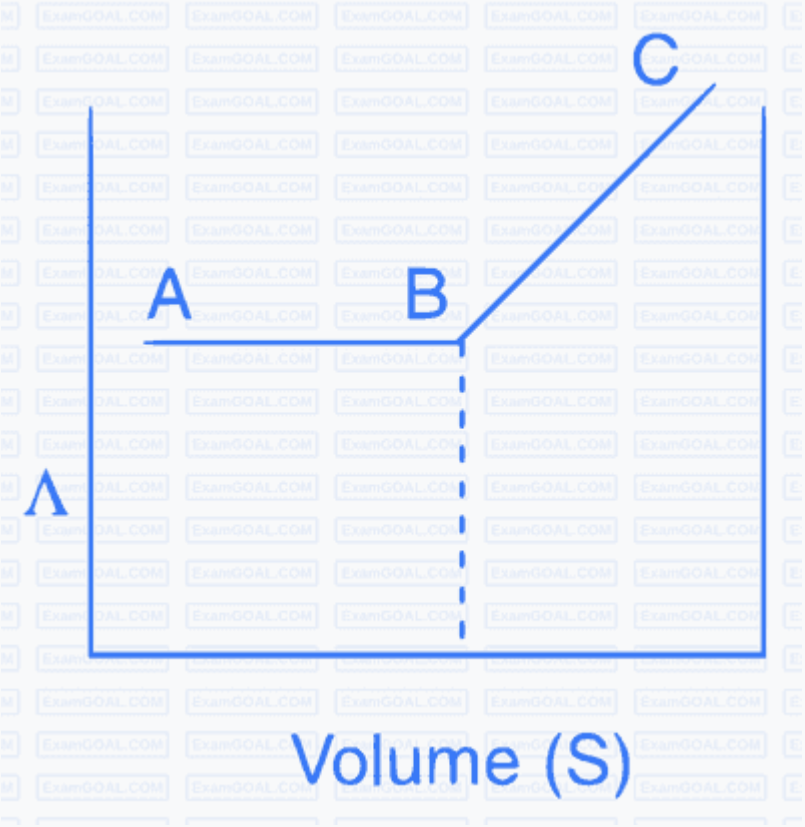
D S

SOURCE

Chemistry • electrochemistry

EXPLANATION

The plot obtained from adding $AgNO_3\,aq.$ to a solution of KCl is as follows :



The reaction occurring is :

$$Ag^+ + NO_3^- + K^+ + Cl^- \rightarrow AgCl + K^+ + NO_3^-$$

Or more succinctly :

$$AgNO_3(aq.) + K^+Cl^-(aq.) \rightarrow AgCl(s) + KNO_3$$

Upon the gradual addition of aqueous $AgNO_3$, precipitation does not begin immediately. The precipitation of AgCl starts only when the ionic product of AgCl exceeds its solubility product. During this initial phase, $AgNO_3$ precipitates as AgCl, simultaneously adding NO_3^- ions to the solution. Since the total number of ions remains constant, the conductance does not change, represented by the flat segment AB in the figure. Once precipitation is complete, any further addition of $AgNO_3$ increases the ion concentration in the solution, thus increasing the conductance, shown by the rising segment BC.

Question 013 MCQ

QUESTION

The major product of the following reaction is

A

B

C

D

CORRECT OPTION

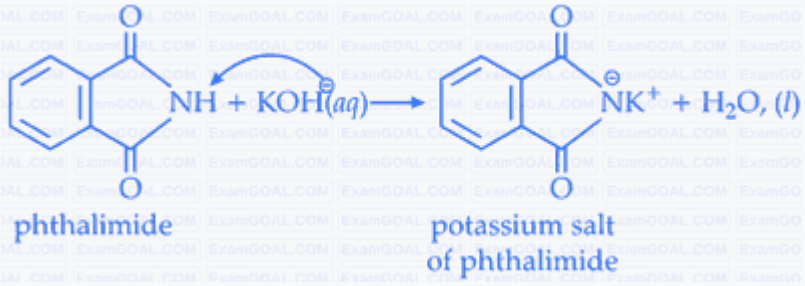
A

SOURCE

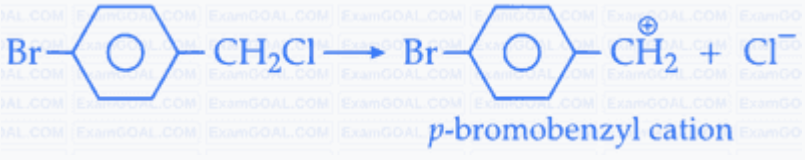
Chemistry • compounds-containing-nitrogen

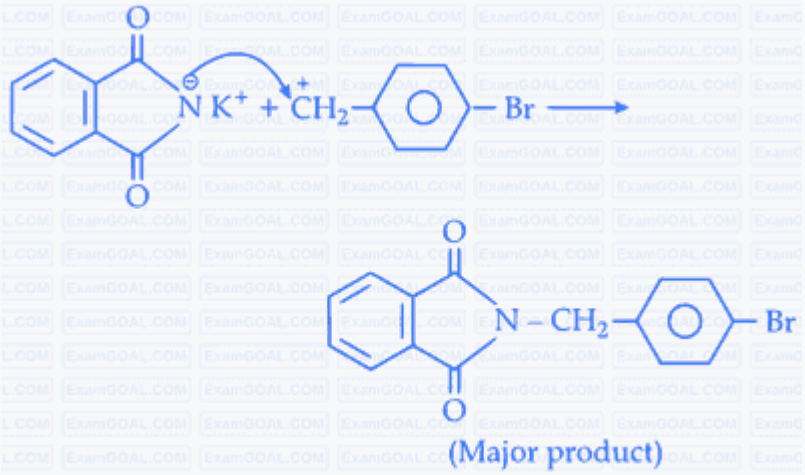
EXPLANATION

The reactant, phthalimide, undergoes an acid-base reaction with KOH. In this reaction, the proton from the nitrogen reacts with the OH^- from the base to form water.



The resulting salt then undergoes a substitution reaction with p-bromobenzyl chloride to form the major product. This reaction proceeds via an S_N1 mechanism due to the stability of the p-bromobenzyl cation.





Question 014 MCQ

QUESTION

Extra very pure N_2 can be obtained by heating

A NH_3 with CuO

B NH_4NO_3

C $(NH_4)_2CrO_7$

D $Ba(N_3)_2$

CORRECT OPTION

D $Ba(N_3)_2$

SOURCE

Chemistry • p-block-elements

EXPLANATION

Extra pure N_2 can be obtained by heating $Ba(N_3)_2$.

Explanation

Thermal Decomposition of Barium Azide

Thermal decomposition of barium azide produces very pure nitrogen gas (N_2).

$$\mathrm{Ba(N_3)_2} \xrightarrow{\Delta} \mathrm{Ba}(s) + 3\mathrm{N_2}(g)$$

This reaction yields pure nitrogen gas.

Decomposition of Ammonium Dichromate

Decomposition of ammonium dichromate ($(NH_4)_2Cr_2O_7$) produces chromium III oxide as impurities.

$$(\mathrm{NH_4})_2\mathrm{Cr_2O_7} \xrightarrow{\Delta} \mathrm{N_2}(g) + 4\mathrm{H_2O} + \mathrm{Cr_2O_3}(s)$$

While nitrogen gas is produced, chromium oxide impurities also form.

Heating Ammonium Nitrate

Heating ammonium nitrate (NH_4NO_3) results in the formation of nitrous oxide and water, not pure nitrogen gas.

$$\mathrm{NH_4NO_3} \xrightarrow{\Delta} \mathrm{N_2O}(g) + \mathrm{H_2O}$$

No nitrogen gas is produced in this reaction.

Reaction of Ammonia with Copper Oxide

The reaction between ammonia (NH_3) and copper oxide CuO gives nitrogen gas along with copper as an impurity.

$$2\mathrm{NH_3}(g) + 3\mathrm{CuO}(s) \rightarrow \mathrm{N_2}(g) + 3\mathrm{Cu}(s) + 3\mathrm{H_2O}(l)$$

While nitrogen gas is produced, copper impurities remain.

Question 015 MCQ

QUESTION

Among the given options, the compound s in which all the atoms are in one plane in all the possible conformations $if any$ is are

A

B

C $H_2C = C = 0$

D $H_2C = C = CH_2$

CORRECT OPTION

B

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

For compound in option A : Only two of the conformers $cis iod and trans oid$ have all the atom in the same plane.

For compound in option B : The terminal hydrogen of allene will be perpendicular to each other plane.

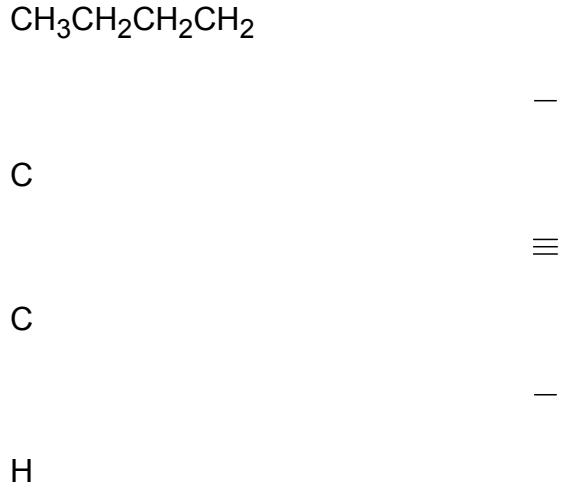
For compound in option C : All the atoms are in one plane in all the possible conformations. There is no atom on oxygen.

Question 016 MCQ

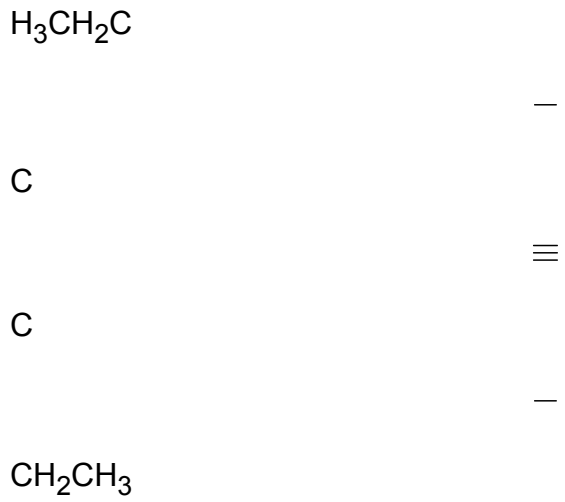
QUESTION

The structure of compound P is

A



B



C

D

CORRECT OPTION

D

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

i When 3,3-dimethyl but-1-yne reacts with dil. $\mathrm{H_2SO_4}$ in presence of $\mathrm{HgSO_4}$, ketone is formed.

ii The ketone is reduced to secondary alcohol in presence of sodium borohydride $(\mathrm{NaBH_4})$ ethanol).

iii Reaction of acid with alcohol results in dehydration forming an alkene. During the process, methyl shift results in rearrangement of carbocation which further leads to the formation of alkene.

The product 2,3-dimethyl but-2-ene undergoes ozonolysis to form two moles acetone.

Two moles of acetone is the final product of the reaction.

Compound P is 3,3-dimethyl butyne.

Question 017 MCQ

QUESTION

The structure of the compound Q is

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

i When 3,3-dimethyl but-1-yne reacts with dil. $\mathrm{H_2SO_4}$ in presence of $\mathrm{HgSO_4}$, ketone is formed.

ii The ketone is reduced to secondary alcohol in presence of sodium borohydride $(\mathrm{NaBH_4})$ ethanol).

Question 018 MCQ

QUESTION

The metal rod M is

A Fe

B Cu

C Ni

D Co

CORRECT OPTION

B Cu

SOURCE

Chemistry • salt-analysis

EXPLANATION

M is copper and N is silver nitrate according to the equation

$$\underset{(M)}{Cu} + 2\underset{(N)}{AgNO_3} \rightarrow \underset{Blue}{Cu(NO_3)_2} + 2Ag$$

The solution of copper nitrate is blue in colour which is formed by the reaction of copper with silver nitrate.

$$AgNO_3 + NaCl \rightarrow AgCl + NaNO_3$$

silver chloride precipitates out $white colour ppt.$ when silver nitrate reacts with sodium chloride $double displacement reaction.$

$$AgCl + 2NH_4OH \rightarrow [Ag(NH_3)_2]Cl + 2H_2O$$

AgCl is soluble in NH_4OH.

$$Cu(NO_3)_2 + 4NH_4OH \rightarrow [Cu(NH_3)_4](NO_3)_2 + 4H_2O$$

Copper nitrate on reaction with ammonia or ammonium hydroxide gives intense blue solution which is due to

$$[Cu(NH_3)_4]$$

.

Question 019 MCQ

QUESTION

The compound N is

A $AgNO_3$

B $Zn(NO_3)_2$

C $Al(NO_3)_3$

D $Pb(NO_3)_2$

CORRECT OPTION

A $AgNO_3$

SOURCE

Chemistry • salt-analysis

EXPLANATION

M is copper and N is silver nitrate according to the equation

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Copper nitrate on reaction with ammonia or ammonium hydroxide gives intense blue solution which is due to

$$[Cu(NH_3)_4]$$

.

Question 020 MCQ

QUESTION

The final solution contains :

A $[Pb(NH_3)_4]^{2+}$ and $[CoCl_2]^{2-}$

B and

$[Al(NH_3)_4]^{3+}$

$[Cu(NH_3)_4]^{2+}$

C and

$[Ag(NH_3)_2]^{+}$

$[Cu(NH_3)_4]^{2+}$

D and

$[Ag(NH_3)_2]^{+}$

$[Ni(NH_3)_6]^{2+}$

CORRECT OPTION

C and

$[Ag(NH_3)_2]^{+}$

$[Cu(NH_3)_4]^{2+}$

SOURCE

Chemistry • salt-analysis

EXPLANATION

M is copper and N is silver nitrate according to the equation

$$\underset{(M)}{Cu} + 2\underset{(N)}{AgNO_3} \rightarrow \underset{Blue}{Cu(NO_3)_2} + 2Ag$$

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$$Cu(NO_3)_2 + 4NH_4OH \rightarrow [Cu(NH_3)_4](NO_3)_2 + 4H_2O$$

Copper nitrate on reaction with ammonia or ammonium hydroxide gives intense blue solution which is due to

$$[Cu(NH_3)_4]$$

.

Question 021 Numerical

QUESTION

Reaction of Br_2 with Na_2CO_3 in aqueous solution gives sodium bromide and sodium bromate with evolution of CO_2 gas. The number of sodium bromide molecules involved in the balanced chemical equation is _____.

SOURCE

Chemistry • s-block-elements

EXPLANATION

The balanced chemical equation is

$3Na_2CO_3 + 3Br_2$

$\rightarrow$

$5NaBr + NaBrO_3 + 3CO_2$

Question 022 Numerical

QUESTION

The total number of alkenes possible by dehydrobromination of 3-bromo-3-cyclopentylhexane using alcoholic KOH is _____.

SOURCE

Chemistry • hydrocarbons

EXPLANATION

1 The structure of 3-bromo-3-cyclopentylhexane -

2 Since, bromide (Br^-) is a good leaving group, elimination using strong base takes place using strong base KOH take via E_2 mechanism. This is also called

β

elimination.

3 There are 3 different types of protons :

iv The strong base abstracts

β

hydrogen (H_1 or H_2 or H_3) with simultaneous loss of bromide ion forming an alkene. This alkene can exist in 2 conformations, E and Z.

a Elimination of H_1 proton :

This product can exist in 2 conformations, E and Z.

b Elimination of H_2 proton :

Since, the groups about sp^2 hybridised carbon of cyclopentane is fixed no E and Z forms are possible. This molecule has only one conformation.

c Elimination of H_3 proton :

This product can exist in 2 conformations, E and Z.

Hence, a total of 5 products are possible.

Question 023 Numerical

QUESTION

A decapeptide $mol.wt.796$ on complete hydrolysis gives glycine $mol.wt.75$, alanine and phenylalanine. Glycine contributes 47.0% to the total weight of the hydrolysed products. The number of glycine units present in the decapeptide is _____.

SOURCE

Chemistry • biomolecules

EXPLANATION

i A decapeptide has nine peptide bonds which hydrolyzes to give ten amino acids. Each peptide bond hydrolyses, to form one molecule of water. Hence, nine molecules of water are required to hydrolysis nine peptide bonds.

$$\text{Decapeptide} \xrightarrow{\text{hydrolyse}+9\ H_2O} \text{Amino acids}$$

ii On hydrolysis a molecule of water $equivalentto18g$ is added across each amino acid.

Mass of hydrolysed decapeptide = Mass of decapeptide + 9 \times \text{mass of each water molecule}

$= 796\,\text{g mol}^{-1} + 9 \times 18\,\text{g mol}^{-1}$

$= (796 + 162)\,\text{g mol}^{-1}$

$= 958\,\text{g mol}^{-1}$

Mass of glycine in hydrolysed decapeptide

$= \frac{47}{100} \times 958\,\text{g mol}^{-1}$

$= 450.26\,\text{g mol}^{-1}$

Mass of each glycine = 75 g mol^{-1}

Number of glycine units

$= \frac{\text{Mass of hydrolysed decapeptide}}{\text{Mass of each glycine}}$

$n = \frac{450.26\ \text{g mol}^{-1}}{75\ \text{g mol}^{-1}}$

$n = 6.00$

Hence, there are six molecules of water in decapeptide.

Question 024 Numerical

QUESTION

If z is any complex number satisfying

$$|z - 3 - 2i| \leq 2$$

, then the minimum value of

$$|2z - 6 + 5i|$$

is

SOURCE

Mathematics • complex-numbers

EXPLANATION

Length

$$AB = \frac{5}{2} \Rightarrow$$

Minimum value = 5.

Question 025 Numerical

QUESTION

The positive integer value of

$$n > 3$$

satisfying the equation

$$\frac{1}{\sin\left(\frac{\pi}{n}\right)} = \frac{1}{\sin\left(\frac{2\pi}{n}\right)} + \frac{1}{\sin\left(\frac{3\pi}{n}\right)}$$

is

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

We have,

$$\frac{1}{\sin(\pi/n)} - \frac{1}{\sin(3\pi/n)} = \frac{1}{\sin(2\pi/n)}$$

$$\Rightarrow \frac{\sin(3\pi/n) - \sin(\pi/n)}{\sin(\pi/n)\sin(3\pi/n)} = \frac{1}{\sin(2\pi/n)} \frac{(2\sin(\pi/n)\cos(2\pi/n))\sin(2\pi/n)}{\sin(\pi/n)\sin(3\pi/n)} =$$

$$\Rightarrow \sin\frac{4\pi}{n} = \sin\frac{3\pi}{n} \Rightarrow \frac{4\pi}{n} + \frac{3\pi}{n} = \pi \Rightarrow n = 7$$

Question 026 Numerical

QUESTION

The minimum value of the sum of real numbers

$$a^{-5},\ a^{-4},\ 3a^{-3},\ 1,\ a^{8}$$

and

$$a^{10}$$

where

$$a > 0$$

is

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

We have

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + a^{8} + a^{10} + 1}{8} \geq 1$$

Therefore, the minimum value is 8.

Question 027 MCQ

QUESTION

Let

$$\alpha$$

and

$$\beta$$

be the roots of

$$x^2 - 6x - 2 = 0,$$

with

$$\alpha > \beta.$$

If

$$a_n = \alpha^n - \beta^n$$

for

$$n \geq 1$$

then the value of

$$\frac{a_{10} - 2a_8}{2a_9}$$

is

A 1

B 2

C 3

D 4

CORRECT OPTION

C 3

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

We have,

$$a_n = \alpha^n - \beta^n$$

$$\alpha^2 - 6\alpha - 2 = 0$$

Multiplying with

$$\alpha$$

^{8} on both sides, we get

$$\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$$

..... 1

Similarly,

$$\beta^{10} - 6\beta^9 - 2\beta^8 = 0$$

..... 2

From Eqs. 1 and 2, we get

$$\alpha^{10} - \beta^{10} - 6(\alpha^9 - \beta^9) = 2(\alpha^8 - \beta^8)$$

$$\Rightarrow a_{10} - 6a_9 = 2a_8 \Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$$

.

Question 028 MCQ

QUESTION

Let

$$(x_0, y_0)$$

be the solution of the following equations

$$\begin{aligned} (2x)^{\ell n2} &= (3y)^{\ell n3} \\ 3^{\ell nx} &= 2^{\ell ny} \end{aligned}$$

Then

$$x_0$$

is

A $\frac{1}{6}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D 6

CORRECT OPTION

C $\frac{1}{2}$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

We have,

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

..... 1

$$3^{\ln x} = 2^{\ln y}$$

..... 2

$$\Rightarrow (\log x)(\log 3) = (\log y)\log 2$$

$$\Rightarrow \log y = \frac{(\log x)(\log 3)}{\log 2}$$

..... 3

Taking log both sides of Eq. 1 , we get

$$(\log 2)\{\log 2 + \log x\} = \log 3\{\log 3 + \log y\}$$

$$(\log 2)^2 + (\log 2)(\log x) = (\log 3)^2 + \frac{(\log 3)^2(\log x)}{\log 2}$$

from Eq. 3

$$\Rightarrow (\log 2)^2 - (\log 3)^2 = \frac{(\log 3)^2 - (\log 2)^2}{\log 2}(\log x)$$

$$\Rightarrow -\log 2 = \log x$$

$$\Rightarrow x = \frac{1}{2} \Rightarrow x_0 = \frac{1}{2}$$

.

Question 029 Numerical

QUESTION

Let

$$a_1$$

,

$$a_2$$

,

$$a_3$$

.....

$$a_{100}$$

be an arithmetic progression with

$$a_1$$

= 3 and

$$S_p = \sum_{i=1}^{p} a_i, 1 \leq p \leq 100$$

. For any integer n with

$$1 \leq n \leq 20$$

, let m = 5n. If

$$\frac{S_m}{S_n}$$

does not depend on n, then

$$a_2$$

is

SOURCE

Mathematics • sequences-and-series

EXPLANATION

It is given that a_1, a_2, a_3, ..., a_{100} is an A.P.

$$a_1 = 3,\ S_p = \sum_{i=1}^{p} a_1,\ 1 \leq p \leq 100$$

$$\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}(6 + (5n-1)d)}{\frac{n}{2}(6 - d + nd)}$$

$$\frac{S_m}{S_n}$$

is independent of n of

$$6 - d = 0 \Rightarrow d = 6$$

.

Therefore,

$$a_2 = a_1 + d = 3 + 6 = 9$$

.

Question 030 MCQ

QUESTION

A straight line

$$L$$

through the point

$$(3, -2)$$

is inclined at an angle

$$60^\circ$$

to the line

$$\sqrt{3x} + y = 1.$$

If

$$L$$

also intersects the x-axis, then the equation of

$$L$$

is

A $$y + \sqrt{3x} + 2 - 3\sqrt{3} = 0$$

B $y - \sqrt{3x} + 2 + 3\sqrt{3} = 0$

C $\sqrt{3y} - x + 3 + 2\sqrt{3} = 0$

D $\sqrt{3y} + x - 3 + 2\sqrt{3} = 0$

CORRECT OPTION

B $y - \sqrt{3x} + 2 + 3\sqrt{3} = 0$

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

We have

$$\left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right| = \sqrt{3}$$

.

$$\Rightarrow m + \sqrt{3} = \pm(\sqrt{3} - 3m)$$

$$\Rightarrow 4m = 0 \Rightarrow m = 0$$

or

$$2m = 2\sqrt{3} \Rightarrow m = \sqrt{3}$$

Therefore, the equation is

$$y + 2 = \sqrt{3}(x - 3)$$

$$\Rightarrow \sqrt{3}x - y - (2 + 3\sqrt{3}) = 0$$

Question 031 MCQ

QUESTION

Let the eccentricity of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

be reciprocal to that of the ellipse

$$x^2 + 4y^2 = 4$$

. If the hyperbola passes through a focus of the ellipse, then

A the equation of the hyperbola is

$$\frac{x^2}{3} - \frac{y^2}{2} = 1$$

B a focus of the hyperbola is

$$(2, 0)$$

C theeccentricity of the hyperbola is

$$\sqrt{\frac{5}{3}}$$

D The equation of the hyperbola is

$$x^2 - 3y^2 = 3$$

CORRECT OPTION

D

The equation of the hyperbola is

$$x^2 - 3y^2 = 3$$

SOURCE

Mathematics • hyperbola

EXPLANATION

Ellipse is

$$\frac{x^2}{2^2} + \frac{y^2}{1^2} = 1$$

.

$$1^2 = 2^2(1 - e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$$

Therefore, the eccentricity of the hyperbola is

$$\frac{2}{\sqrt{3}} \Rightarrow b^2 = a^2\left(\frac{4}{3} - 1\right) \Rightarrow 3b^2 = a^2$$

Foci of the ellipse are

$$(\sqrt{3},\ 0)$$

and

$$(-\sqrt{3},\ 0)$$

.

Hyperbola passes through

$$(\sqrt{3},\ 0)$$

.

$$\frac{3}{a^2} = 1 \Rightarrow a^2 = 3$$

and

$$b^2 = 1$$

Therefore, the equation of hyperbola is

$$x^2 - 3y^2 = 3$$

.

Focus of hyperbola is

$$(ae,\ 0) \equiv \left(\sqrt{3} \times \frac{2}{\sqrt{3}},\ 0\right) \equiv (2, 0)$$

.

Question 032 Numerical

QUESTION

Consider the parabola

$$y^2 = 8x$$

. Let

$$\Delta_1$$

be the area of the triangle formed by the end points of its latus rectum and the point

$$P\left(\frac{1}{2}, 2\right)$$

on the parabola and

$$\Delta_2$$

be the area of the triangle formed by drawing tangents at

$$P$$

and at the end points of the latus rectum. Then

$$\frac{\Delta_1}{\Delta_2}$$

is

SOURCE

Mathematics • parabola

EXPLANATION

The area of triangle formed by the three points on the parabola is twice the area of the triangle formed by the respective tangents. That is,

$$\Delta LPM = 2\times$$

$$Areaof\$\$\Delta ABC\$\$$$

$$y^2 = 8x = 4 \times 2 \times x$$

$$\frac{\Delta LPM}{\Delta ABC} = 2$$

$$\frac{\Delta_1}{\Delta_2} = 2$$

Question 033 Numerical

QUESTION

Let

$$f(\theta)=\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right),$$

where

$$-\frac{\pi}{4}<\theta<\frac{\pi}{4}.$$

Then the value of

$$\frac{d}{d(\tan\theta)}(f(\theta))$$

is

SOURCE

Mathematics • differentiation

EXPLANATION

$$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$$

, where

$$\theta\in\left(-\frac{\pi}{4},\frac{\pi}{4}\right)$$

$$\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{2\cos^2\theta-1}}\right)\right)=\sin(\sin^{-1}(\tan\theta))=\tan\theta$$

$$\frac{d(\tan\theta)}{d(\tan\theta)}=1$$

Question 034 MCQ

QUESTION

The value of

$$\int_{\sqrt{\ell n 2}}^{\sqrt{\ell n 3}} \frac{x \sin x^2}{\sin x^2 + \sin\left(\ell n 6 - x^2\right)} dx$$

is

A $\frac{1}{4} \ell n \frac{3}{2}$

B $\frac{1}{2} \ell n \frac{3}{2}$

C $\ell n \frac{3}{2}$

D $\frac{1}{6} \ell n \frac{3}{2}$

CORRECT OPTION

A $\frac{1}{4} \ell n \frac{3}{2}$

SOURCE

Mathematics • definite-integration

EXPLANATION

$$x^2 = t \Rightarrow 2x\,dx = dt$$

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin t}{\sin t + \sin(\ln 6 - t)}dt$$

and

$$I = \frac{1}{2}\int_{\ln 2}^{\ln 3} \frac{\sin(\ln 6 - t)}{\sin(\ln 6 - t) + \sin t}dt$$

$$2I = \frac{1}{2}\int_{\ln 2}^{\ln 3} 1dt \Rightarrow I = \frac{1}{4}\ln\frac{3}{2}$$

.

Question 035 MCQ

QUESTION

Let the straight line

$$x = b$$

divide the area enclosed by

$$y = (1-x)^2, y = 0,$$

and

$$x = 0$$

into two parts

$$R_1\,(0 \le x \le b)$$

and

$$R_2\,(b \le x \le 1)$$

such that

$$R_1 - R_2 = \frac{1}{4}.$$

Then

$$b$$

equals

A $\frac{3}{4}$

B $\frac{1}{2}$

C $\frac{1}{3}$

D $\frac{1}{4}$

CORRECT OPTION

B $\frac{1}{2}$

SOURCE

Mathematics • application-of-integration

EXPLANATION

We can write the integral

$$\int_0^b (1-x)^2 dx - \int_0^1 (1-x)^2 dx = \frac{1}{4}$$

$$\Rightarrow \frac{(x-1)^3}{3}\Big|_0^b - \frac{(x-1)^3}{3}\Big|_b^1 = \frac{1}{4}$$

$$\Rightarrow \frac{(b-1)^3}{3} + \frac{1}{3} - \left(0 - \frac{(b-1)^3}{3}\right) = \frac{1}{4}$$

$$\Rightarrow \frac{2(b-1)^3}{3} = -\frac{1}{12} \Rightarrow (b-1)^3 = -\frac{1}{8} \Rightarrow b = \frac{1}{2}$$

Question 036 MCQ

QUESTION

The probability of the drawn ball from

$$U_2$$

being white is

A $\frac{13}{30}$

B $\frac{23}{30}$

C $\frac{19}{30}$

D $\frac{11}{30}$

CORRECT OPTION

B $\frac{23}{30}$

SOURCE

Mathematics • probability

EXPLANATION

H

$\rightarrow$

One ball from U_1 to U_2.

T

$\rightarrow$

Two balls from U_1 to U_2.

E : One ball drawn from U_2.

P/W from

$$U_2 = \frac{1}{2} \times \left(\frac{3}{5} \times 1\right) + \frac{1}{2} \times \left(\frac{2}{5} \times \frac{1}{2}\right) + \frac{1}{2} \times \left(\frac{{}^3C_2}{{}^5C_2} \times 1\right) + \frac{1}{2} \times \left(\frac{{}^2C_2}{{}^5C_2} \times$$

Question 037 MCQ

QUESTION

Given that the drawn ball from

$$U_2$$

is white, the probability that head appeared on the coin is

A $\frac{17}{23}$

B $\frac{11}{23}$

C $\frac{15}{23}$

D $\frac{12}{23}$

CORRECT OPTION

D $$\frac{12}{23}$$

SOURCE

Mathematics • probability

EXPLANATION

$$P\left(\frac{H}{W}\right) = \frac{P(W/H) \times P(H)}{P(W/T)\,.\,P(T) + (W/H)\,.\,P(H)}$$

$$= \frac{\frac{1}{2}\left(\frac{3}{5} \times 1 + \frac{2}{5} \times \frac{1}{2}\right)}{23/30} = \frac{12}{23}$$

Question 038 MCQ

QUESTION

Let

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} - \hat{j} + \hat{k}$$

and

$$\overrightarrow{c} = \hat{i} - \hat{j} - \hat{k}$$

be three vectors. A vector

$$\overrightarrow{v}$$

in the plane of

$$\overrightarrow{a}$$

and

$\overrightarrow{b},$

whose projection on

$\overrightarrow{c}$

is

$\frac{1}{\sqrt{3}}$

, is given by

A $\hat{i} - 3\hat{j} + 3\hat{k}$

B $-3\hat{i} - 3\hat{j} - \hat{k}$

C $3\hat{i} - \hat{j} + 3\hat{k}$

D $\hat{i} + 3\hat{j} - 3\hat{k}$

CORRECT OPTION

C $3\hat{i} - \hat{j} + 3\hat{k}$

SOURCE

Mathematics • vector-algebra

EXPLANATION

We have,

$$\overrightarrow{v} = \lambda\overline{a} + \mu\overline{b}$$

$$= \lambda(\hat{i} + \hat{j} + \widehat{k}) + \mu(\hat{i} - \hat{j} + \widehat{k})$$

Projection of

$$\overrightarrow{v}$$

on

$$\overline{c}$$

$$\frac{\overline{v} \,.\, \overline{c}}{|\overline{c}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\left[(\lambda+\mu)\hat{i} + (\lambda-\mu)\hat{j} + (\lambda+\mu)\widehat{k}\right] \,.\, \left(\hat{i} - \hat{j} - \widehat{k}\right)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \lambda + \mu - \lambda + \mu - \lambda - \mu = 1 \Rightarrow \mu - \lambda = 1 \Rightarrow \lambda = \mu - 1$$

$$\overline{v} = (\mu-1)(\hat{i} + \hat{j} + \widehat{k}) + \mu(\hat{i} - \hat{j} + \widehat{k}) = \mu(2\hat{i} + 2\widehat{k}) - \hat{i} - \hat{j} - \widehat{k}$$

$$\overline{v} = (2\mu - 1)\hat{i} - \hat{j} + (2\mu - 1)\widehat{k}$$

At

$$\mu = 2$$

,

$$\overline{v} = 3\hat{i} - \hat{j} + 3\widehat{k}$$

.

Question 039 MCQ

QUESTION

The vector s which is/are coplanar with vectors

$$\hat{i}+\hat{j}+2\hat{k}$$

and

$$\hat{i}+2\hat{j}+\hat{k},$$

and perpendicular to the vector

$$\hat{i}+\hat{j}+\hat{k}$$

is/are

A $\hat{j}-\hat{k}$

B $-\hat{i}+\hat{j}$

C $\hat{i}-\hat{j}$

D $-\hat{j}+\hat{k}$

CORRECT OPTION

D $-\hat{j}+\hat{k}$

SOURCE

Mathematics • vector-algebra

EXPLANATION

Let

$$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$$

,

$$\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$$

and

$$\vec{c} = \hat{i} + \hat{j} + \hat{k}$$

.

Any vector in the plane of

$$\hat{i} + \hat{j} + 2\hat{k}$$

and

$$\hat{i} + 2\hat{j} + \hat{k}$$

is given by

$$\vec{r} = \lambda\vec{a} + \mu\vec{b}$$

$$= \lambda(\hat{i} + \hat{j} + 2\hat{k}) + \mu(\hat{i} + 2\hat{j} + \hat{k})$$

$$= (\lambda + \mu)\hat{i} + (\lambda + 2\mu)\hat{j} + (2\lambda + \mu)\hat{k}$$

Also,

$$\vec{r} \,.\, \vec{c} = 0$$

$$\Rightarrow (\lambda + \mu) \,.\, 1 + (\lambda + 2\mu) \,.\, 1 + (2\lambda + \mu) \,.\, 1 = 0$$

$$\Rightarrow 4\lambda + 4\mu = 0$$

$$\Rightarrow \lambda + \mu = 0$$

$$\Rightarrow \left[\vec{r} \;\; \vec{a} \;\; \vec{b}\right] = 0$$

So, vectors

$$\hat{j} - \hat{k}$$

and

$$-\hat{j} + \hat{k}$$

satisfy this.

Question 040 MCQ

QUESTION

Let

$$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$$

and

$$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$$

be two sets. Then

A $P \subset Q$ and $Q - P \neq \emptyset$

B $Q \not\subset P$

C $P \not\subset Q$

D $P = Q$

CORRECT OPTION

D $P = Q$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

$$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$$

$$\Rightarrow \cos\theta\left(\sqrt{2}+1\right) = \sin\theta$$

$$\Rightarrow \tan\theta = \sqrt{2}+1$$

..... i

$$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$$

$$\Rightarrow \sin\theta\left(\sqrt{2}-1\right) = \cos\theta$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \left(\sqrt{2}+1\right)$$

..... ii

$$\therefore$$

$$P = Q$$

Question 041 MCQ

QUESTION

Let f : R

$$\rightarrow$$

R be a function such that

$$f(x+y) = f(x) + f(y),\ \forall x, y \in R$$

. If f x is differentiable at x = 0, then

A f x is differentiable only in a finite interval containing zero.

B f x is continuous

$$\forall x \in R$$

.

C f' x is constant

$$\forall x \in R$$

.

D f x is differentiable except at finitely many points.

CORRECT OPTION

B f x is continuous

$$\forall x \in R$$

.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Set x = 0 in the functional equation to obtain

$$f(0) = f(0) + f(0)$$

$$\therefore$$

$$f(0) = 0$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x+0)}{h} = \lim_{h \to 0} \frac{f(x) + f(h) - f(x) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - f(0)}{h} = f'(0)$$

Thus,

$$f'(x) = \lambda$$

say. Also

$$f(x) = \lambda x + \mu$$

As

$$f(x) = 0$$

we have

$$\mu = 0$$

$$\therefore$$

$$f(x) = \lambda x$$

.

Question 042 MCQ

QUESTION

Let M and N be two 3

×

3 non-singular skew symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2N^2(M^TN)$

–

^{1}(MN

–

$^1)^T$ is equal to

A M^2

B –

N^2

C –

M^2

D MN

CORRECT OPTION

C –

M^2

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given,

$$M^T = -M$$

,

$$N^T = -N$$

and

$$MN = NM$$

..... i

$$\therefore$$

$$M^2N^2(M^TN)^{-1}(MN^{-1})^T$$

$$= M^2N^2N^{-1}(M^T)^{-1}(N^{-1})^T.M^T$$

$$= M^2N(NM^{-1})(-M)^{-1}(N^T)^{-1}(-M)$$

$$= M^2NI(-M^{-1})(-N)^{-1}(-M)$$

$$= -M^2NM^{-1}N^{-1}M$$

$$= -M.(MN)M^{-1}N^{-1}M$$

$$= -M(NM)M^{-1}N^{-1}M$$

$$= -MN(NM^{-1})N^{-1}M$$

$$= -M(NN^{-1})M = -M^2$$

Note : This question is wrong, as given. An odd order skew symmetric matrix can't be invertible. Had the matrix be of even order, it could have been correct.

Question 043 MCQ

QUESTION

If the point P a, b, c, with reference to E, lies on the plane 2x + y + z = 1, then the value of 7a + b + c is

A 0

B 12

C 7

D 6

CORRECT OPTION

D 6

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given,

$$[a \quad b \quad c]_{1\times 3}\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}_{3\times 3} = [0 \quad 0 \quad 0]$$

$$\Rightarrow \begin{bmatrix} a+8b+7c \\ 9a+2b+3c \\ 7a+7b+7c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow a+8b+7c=0$$

..... i

$$\Rightarrow 9a+2b+3c=0$$

.... ii

$$\Rightarrow a+b+c=0$$

..... iii

On multiplying Eq. iii by 2, then subtract from Eq. ii, we get

$$7a+c=0$$

..... iv

Again multiplying Eq. iii by 3, then subtract from Eq. ii, we get

$$6a-b=0$$

..... v

$$\therefore$$

$$b=6a$$

and

$$c=-7a$$

As a, b, c lies on

$$2x+y+z=1$$

$$\Rightarrow 2a+b+c=1$$

$$\Rightarrow 2a+6a-7a=1$$

$$\Rightarrow$$

a = 1, b = 6 and c =

$$-$$

7

$$\therefore$$

$$7a + b + c = 7 + 6 - 7 = 6$$

Question 044 MCQ

QUESTION

Let

$$\omega$$

be a solution of

$$x^3 - 1 = 0$$

with

$$\mathrm{Im}(\omega) > 0$$

. If a = 2 with b and c satisfying E, then the value of

$$\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$$

is equal to

A

$$-$$

2

B 2

C 3

D

3

CORRECT OPTION

A

2

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given,

$$[a \quad b \quad c]_{1\times 3}\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}_{3\times 3} = [0 \quad 0 \quad 0]$$

$$\Rightarrow \begin{bmatrix}a+8b+7c\\ 9a+2b+3c\\ 7a+7b+7c\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\Rightarrow a+8b+7c=0$$

..... i

$$\Rightarrow 9a+2b+3c=0$$

.... ii

$$\Rightarrow a+b+c=0$$

.... iii

On multiplying Eq. iii by 2, then subtract from Eq. ii, we get

$$7a+c=0$$

..... iv

Again multiplying Eq. iii by 3, then subtract from Eq. ii, we get

$$6a - b = 0$$

..... v

$$\therefore$$

b = 6a and c =

$$-$$

7a

If a = 2, b = 12 and c =

$$-$$

14

$$\therefore$$

$$\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$$

$$\Rightarrow \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + \frac{3}{\omega^{-14}} = \frac{3}{\omega^2} + 1 + 3\omega^2$$

$$= 3\omega + 1 + 3\omega^2$$

$$= 1 + 3(\omega + \omega^2)$$

$$= 1 - 3 = -2$$

Question 045 MCQ

QUESTION

Let b = 6, with a and c satisfying E. If

$$\alpha$$

and

$$\beta$$

are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$\sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n$$

is

A 6

B 7

C $\frac{6}{7}$

D ∞

CORRECT OPTION

B 7

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given,

$$[a \quad b \quad c]_{1\times 3}\begin{bmatrix}1 & 9 & 7\\ 8 & 2 & 7\\ 7 & 3 & 7\end{bmatrix}_{3\times 3} = [0 \quad 0 \quad 0]$$

$$\Rightarrow \begin{bmatrix} a+8b+7c \\ 9a+2b+3c \\ 7a+7b+7c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow a+8b+7c=0$$

..... i

$$\Rightarrow 9a+2b+3c=0$$

.... ii

$$\Rightarrow a+b+c=0$$

.... iii

On multiplying Eq. iii by 2, then subtract from Eq. ii, we get

$$7a+c=0$$

..... iv

Again multiplying Eq. iii by 3, then subtract from Eq. ii, we get

$$6a-b=0$$

..... v

$$\therefore$$

b = 6a and c =

$$-$$

7a

If b = 6, a = 1 and c =

$$-$$

7

$$\therefore$$

$$ax^2+bx+c=0$$

$$\Rightarrow x^2+6x-7=0$$

$$\Rightarrow (x+7)(x-1)=0$$

$$\therefore$$

$$x = 1, 7$$

$$\Rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{1} - \frac{1}{7}\right)^n \Rightarrow \sum_{n=0}^{\infty} \left(\frac{6}{7}\right)^n$$

$$\Rightarrow 1 + \frac{6}{7} + \left(\frac{6}{7}\right)^n + \ldots \infty$$

$$= \frac{1}{1 - \frac{6}{7}} = \frac{1}{1/7} = 7$$

Question 046 Numerical

QUESTION

Let

$$f : [1, \infty) \to [2, \infty)$$

be a differentiable function such that

$$f(1) = 2$$

. If

$$6 \int_1^x f(t)dt = 3xf(x) - x^3 - 5$$

for all

$$x \geq 1$$

, then the value of f2 is _____.

SOURCE

Mathematics • differential-equations

EXPLANATION

It is given that

$$6\int_1^x f(t)dt = 3xf(x) - x^3 - 5$$

$$\Rightarrow 6f(x) = 3f(x) + 3xf'(x) - 3x^2$$

$$\Rightarrow 3f(x) = 3xf'(x) - 3x^2 \Rightarrow xf'(x) - f(x) = x^2$$

$$\Rightarrow x\frac{dy}{dx} - y = x^2 \Rightarrow \frac{dy}{dx} - \frac{1}{x}y = x$$

.... 1

Now,

$$I.F. = e^{\int -\frac{1}{x}dx} = e^{-\log_e x}$$

Multiplying Eq. 1 both sides by

$$\frac{1}{x}$$

, we get

$$\frac{1}{x}\frac{dy}{dx} - \frac{1}{x^2}y = 1 \Rightarrow \frac{d}{dx}\left(y.\frac{1}{x}\right) = 1$$

Integrating, we get

$$\frac{y}{x} = x + c$$

Substituting x = 1 and y = 2, we get

$$\Rightarrow 2 = 1 + c \Rightarrow c = 1 \Rightarrow y = x^2 + x$$

$$\Rightarrow f(x) = x^2 + x \Rightarrow f(2) = 6$$

Question 047 MCQ

QUESTION

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency '

$$\omega_p$$

' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency

$$\omega$$

, where a part of the energy is absorbed and a part of it is reflected. As

$$\omega$$

approaches

$$\omega_p$$

all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

Taking the electronic charge as 'e' and the permittivity as

$$'\varepsilon_0'$$

. Use dimensional analysis to determine the correct expression for

$$\omega_p$$

.

A $\sqrt{\frac{Ne}{m\varepsilon_0}}$

B $\sqrt{\frac{m\varepsilon_0}{Ne}}$

C $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$

D $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

CORRECT OPTION

C $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$

SOURCE

Physics • units-and-measurements

EXPLANATION

We have

$$[\omega] = T^{-1}$$

and

$$[\varepsilon_0] = \frac{[QT]^2}{ML^3}$$

Therefore,

$$\left[\frac{1}{m\varepsilon_0}\right] = \frac{L^3}{[QT]^2}$$

Now,

$$\left[\frac{e^2}{m\varepsilon_0}\right] = \frac{L^3}{[T]^2}$$

Also, the number N is defined as number of particle in unit volume, that is, N = n/V.

$$[N] = \frac{1}{[V]} = \frac{1}{L^3}$$

Therefore,

$$\left[\frac{Ne^2}{m\varepsilon_0}\right] = \frac{1}{[T]^2}$$

Therefore, the quantity

$$\sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

has the dimension of

$$\omega$$

.

Question 048 MCQ

QUESTION

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency '

$$\omega_p$$

' which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency

$$\omega$$

, where a part of the energy is absorbed and a part of it is reflected. As

$$\omega$$

approaches

$$\omega_p$$

all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals.

Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons N

$$\approx$$

4

$$\times$$

10^{27} m^{-3}. Taking

$$\varepsilon_0$$

= 10^{-11} and m

$$\approx$$

10^{-30}, where these quantities are in proper SI units.

A 800 nm

B 600 nm

C 300 nm

D 200 nm

CORRECT OPTION

B 600 nm

SOURCE

Physics • magnetism

EXPLANATION

For resonance

$$\omega = \omega_p = \sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

$$Frompreviousquestion$$

$$\upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{Ne^2}{m\varepsilon_0}}$$

$$\lambda = \frac{c}{\upsilon} = 2\pi c\sqrt{\frac{m\varepsilon_0}{Ne^2}}$$

Substituting the given values, we get

$$\lambda = 2 \times 3.14 \times 3 \times 10^8 \times \sqrt{\frac{10^{-30} \times 10^{-11}}{4 \times 10^{27} \times (1.6 \times 10^{-19})^2}}$$

$$= \frac{2 \times 3.14 \times 3 \times 10^{8}}{1.6 \times 10^{-19}} \sqrt{\frac{10^{-41}}{4 \times 10^{27}}} = \frac{9.42}{1.6} \times 10^{27} \times 10^{-34}$$

m

$$\approx 6.00 \times 10^{-7}$$

m = 600 nm

Question 049 MCQ

QUESTION

A ball of mass m 0.5 kg is attached to the end of a string having length L 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball $in radian/s$ is

A 9

B 18

C 27

D 36

CORRECT OPTION

D 36

SOURCE

Physics • laws-of-motion

EXPLANATION

Resolve T along the horizontal and the vertical directions. As the particle moves in a horizontal plane, net vertical fore on it is zero. Net horizontal force on it provides the necessary centripetal acceleration for circular motion. Apply Netwon's second law in the horizontal direction to get

$$T\sin\theta = m\omega^2 r = m\omega^2(l\sin\theta)$$

$$T = m\omega^2 l$$

$$\omega = \sqrt{\frac{T}{ml}}$$

Substituting the given values, we get

$$\omega = \sqrt{\frac{324}{0.5\times 0.5}} = 36$$

rad/s

Question 050 Numerical

QUESTION

A block is moving on an inclined plane making an angle

$$45^\circ$$

with the horizontal and the coefficient of friction is

$$\mu$$

. The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define N = 10

μ

, then N is

SOURCE

Physics • laws-of-motion

EXPLANATION

The pushing force

$$F_1 = mg\sin\theta + f$$

$$\therefore$$

$$F_1 = mg\sin\theta + \mu mg\cos\theta = mg(\sin\theta + \mu\cos\theta)$$

The force required to just prevent it from sliding down

$$F_2 = mg\sin\theta - \mu N = mg(\sin\theta - \mu\cos\theta)$$

Given,

$$F_1 = 3F_2$$

$$\therefore$$

$$\sin\theta + \mu\cos\theta = 3(\sin\theta - \mu\cos\theta)$$

$$\therefore$$

$$1 + \mu = 3(1 - \mu)$$

$$\$\$ \because \$\$\$\$ \sin\theta = \mu\cos\theta \$\$$$

$$\therefore$$

$$4\mu = 2$$

$$\therefore$$

$$\mu = 0.5$$

$$\therefore$$

$$N = 10\mu = 5$$

Question 051 Numerical

QUESTION

Four solid spheres each of diameter

$$\sqrt{5}$$

cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is N

$$\times$$

10^{-4} kg-m^2, then N is

SOURCE

Physics • rotational-motion

EXPLANATION

The moment of inertia of each sphere about an axis passing through its centre is

$$\frac{2}{5}mr^2$$

.

The moment of inertia of sphere B and sphere D about X

$$-$$

X' is

$$I_B = I_D = \frac{2}{5}mr^2$$

.

Using parallel axes theorem, the moment of inertia of sphere A and sphere C about X

$$-$$

X' is

$$I_A = I_C = \frac{2}{5}mr^2 + md^2$$

The moment of inertia of the system about the diagonal is

$$I = I_A + I_B + I_C + I_D = \frac{8}{5}mr^2 + 2md^2$$

$$m = 0.5$$

kg

$$d = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

cm

$$r = \frac{\sqrt{5}}{2}$$

cm

$$\therefore$$

$$I = \frac{8}{5} \times 0.5 \times \left(\frac{\sqrt{5}}{2} \times 10^{-2}\right)^2 + 2 \times 0.5 \times \left(\frac{4}{\sqrt{2}} \times 10^{-2}\right)^2$$

$$= 9 \times 10^{-4}$$

kg m^2

Hence,

$$N = 9$$

Question 052 MCQ

QUESTION

5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

A $\frac{9}{8}RT_1$

B $\frac{3}{2}RT_1$

C $\frac{15}{8}RT_1$

D $\frac{9}{2}RT_1$

CORRECT OPTION

A $\frac{9}{8}RT_1$

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Initially

V_1 = 5.6 l, T_1 = 273 K, P_1 = 1 atm,

$$\gamma = \frac{5}{3}$$

$For monoatomic gas$

The number of moles of gas is

$$n = \frac{5.6l}{22.4l} = \frac{1}{4}$$

Finally $after adiabatic compression$

V_2 = 0.7 l

For adiabatic compression

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore$$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1} = T_1(8)^{2/3} = 4T_1$$

Work done during an adiabatic process is

$$W = \frac{nR[T_1 - T_2]}{(\gamma-1)} = \frac{\frac{1}{4}R[T_1 - 4T_1]}{\left[\frac{5}{3}-1\right]} = -\frac{9}{8}RT_1$$

Negative sign shows that work is done on the gas.

Question 053 Numerical

QUESTION

Steel wire of lenght ‘L’ at 40^{o}C is suspended from the ceiling and then a mass ‘m’ is hung from its free end. The wire is cooled down from 40^{o}C to 30^{o}C to regain its original length ‘L’. The coefficient of linear thermal expansion of the steel is

10^{-5} /°C, Young's modulus of steel is 10^{11} N/m^2 and radius of the wire is 1 mm. Assume that L >> diameter of the wire. Then the value of 'm' in kg is nearly

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Change in length

$$\Delta L = L\,\alpha\,\Delta T$$

..... i

Also

$$Y = \frac{mgL}{A\Delta L} \Rightarrow \Delta L = \frac{mgL}{YA}$$

..... ii

Equation i and ii we get $$ $\because$ $$$$A = \pi r^2$$

$$m = \frac{\alpha \Delta T Y \times \pi r^2}{g}$$

$$= \frac{(10^{-5}) \times (10) \times (10^{11}) \times 3.14 \times (1 \times 10^{-3})^2}{9.8}$$

= 3.2 kg

$$\simeq$$

3 kg

Question 054 MCQ

QUESTION

A police car with a siren of frequency 8 kHz is moving with uniform velocity 36 km/hr towards a tall building which reflects the sound waves. The speed of sound in air is 320 m/s. The frequency of the siren heard by the car driver is

A 8.50 kHz

B 8.25 kHz

C 7.75 kHz

D 7.50 kHz

CORRECT OPTION

A 8.50 kHz

SOURCE

Physics • waves

EXPLANATION

u = 36 km h

$^{-1}$ = 10 ms

$^{-1}$,

v

= 320 ms

1,

$$\nu$$

= 8 kHz

The sound reflected from the building may be imagined to be coming from the mirror image. The driver is approaching the image-source which is also approaching him with the same speed. Hence the frequency of sound heard by the driver is

$$\nu$$

' =

$$\nu$$

$$\left(\frac{v+u}{v-u}\right)$$

= 8 kHz

$$\times$$

$$\left(\frac{320+10}{320-10}\right)$$

= 8.5 kHz

Question 055 MCQ

QUESTION

Consider an electric field

$$\overrightarrow{E} = E_0\widehat{x}$$

where

$$E_0$$

is a constant. The flux through the shaded area $as shown in the figure$ due to this field is

A $2E_0a_2$

B $\sqrt{2}E_0a^2$

C $E_0\,a^2$

D $\frac{E_0a^2}{\sqrt{2}}$

CORRECT OPTION

C $E_0\,a^2$

SOURCE

Physics • electrostatics

EXPLANATION

The flux through an area

$\vec{S}$

due to an electric field

$\vec{E}$

is given by

$$\phi = \oint \vec{E} . d\vec{S}$$

$$= \vec{E} . \oint d\vec{S} = \vec{E} . \vec{S}$$

$$\because \$\$\$\$\vec{E}\$\$ is a constant. \ldots\ldots 1$$

The area of the shaded region is the cross product of vectors representing the two sides i.e.,

$$\vec{S} = (a\hat{j}) \times (a\hat{i} + a\hat{k}) = a^2(\hat{i} - \hat{k})$$

$\ldots\ldots 2$

Use equations 1 and 2 to get

$$\phi = (E_0\hat{i}) \, . \, a^2(\hat{i} - \hat{k}) = E_0 a^2$$

.

Question 056 MCQ

QUESTION

A

$$2$$

$$\mu F$$

capacitor is charged as shown in the figure. The percentage of its stored energy dissipated after the switch

$$S$$

is turned to position

$$2$$

is

A 0%

B 20%

C 75%

D 80%

CORRECT OPTION

D 80%

SOURCE

Physics • capacitor

EXPLANATION

When switch S is connected to terminal 1, the potential difference across the 2

$$\mu$$

F capacitor is V volt. Therefore, energy stored in the system is

$$U_1 = \frac{1}{2}C_1V^2 = \frac{1}{2} \times 2 \times V^2$$

$$= V^2\,\mu J$$

When switch S is turned to terminal 2, the charge will flow from 2

μ

F capacitor to 8

μ

F capacitor until their potentials are equalized. The common potential is

$$V^2 = \frac{q}{C_1 + C_2} = \frac{C_1 V}{C_1 + C_2}$$

$$= \frac{2V}{(2+8)} = \frac{V}{5}$$

volt

$\therefore$

Energy stored in the system now will be

$$U_2 = \frac{1}{2}(C_1 + C_2)V_2^2$$

$$= \frac{1}{2}(2+8) \times \left(\frac{V}{5}\right)^2 = \frac{V^2}{5}\mu J$$

$\therefore$

Percentage loss of energy is

$$\frac{U_1 - U_2}{U_1} \times 100 = \frac{\left(V^2 - \frac{V^2}{5}\right)}{V^2} \times 100 = 80\%$$

Question 057 MCQ

QUESTION

A spherical metal shell A of radius

R_A

and a solid metal sphere

B

of radius

$$R_B\,(<R_A)$$

are kept far apart and each is given charge

$$'+Q'.$$

Now they are connected by a thin metal wire. Then

A $$E_A^{inside}=0$$

B $$Q_A>Q_B$$

C $$\frac{\sigma_A}{\sigma_B}=\frac{R_B}{R_A}$$

D $$E_A^{on\ surface}<E_B^{on\ surface}$$

CORRECT OPTION

A $$E_A^{inside}=0$$

SOURCE

Physics • electrostatics

EXPLANATION

When connected by a wire, charges on A and B are redistributed until potential on both becomes equal. After the charge redistribution, A and B are not

influenced by each other because they are far apart. Thus,

$$E_A^{inside} = 0$$

as field inside a conducting shell is zero.

So choice a is correct.

Let Q_A and Q_B are the charges on metal shell A and metal sphere B after they are connected by a wire. Since their electric potentials will be equal,

$V_A = V_B$

$$\Rightarrow \frac{Q_A}{4\pi\varepsilon_0 R_A} = \frac{Q_B}{4\pi\varepsilon_0 R_B} \Rightarrow \frac{Q_A}{Q_B} = \frac{R_A}{R_B}$$

Since

$$R_B < R_A$$

,

$$Q_A > Q_B$$

. So choice b is correct.

Now,

$$\sigma_A = \frac{Q_A}{4\pi R_A^2}$$

and

$$\sigma_B = \frac{Q_B}{4\pi R_B^2}$$

$$\therefore$$

$$\frac{\sigma_A}{\sigma_B} = \frac{Q_A}{Q_B} \times \left(\frac{R_B}{R_A}\right)^2 = \frac{R_A}{R_B} \times \left(\frac{R_B}{R_A}\right)^2 = \frac{R_B}{R_A}$$

Hence, choice c is also correct.

Electric fields on the surface of shell and sphere are

$$E_A = \frac{\sigma_A}{\varepsilon_0}$$

and

$$E_B = \frac{\sigma_B}{\varepsilon_0}$$

$$\therefore$$

$$\frac{E_A}{E_B} = \frac{\sigma_A}{\sigma_B} < 1$$

, i.e.

$$E_A < E_B$$

So, choice d is also correct. All the four choices are correct.

Question 058 MCQ

QUESTION

The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561

$$\overset{o}{A}$$

. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is

A 1215 $\overset{o}{A}$

B 1640 $\overset{o}{A}$

C 2430 $\overset{o}{A}$

D 4687 $\overset{o}{A}$

CORRECT OPTION

A 1215 $\overset{o}{A}$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

$$\frac{1}{\lambda} = R_H Z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For singly-ionized helium atom, Z = 2. For hydrogen atom Z = 1.

For Balmer series $n_1 = 2$.

For first spectral line of hydrogen :

$$\frac{1}{6561} = R_H \times (1)^2 \left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36}$$

$$\Rightarrow R_H = \frac{36}{5 \times 6561}$$

For second spectral line of helium,

$$\frac{1}{\lambda} = R_H \times (2)^2 \left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3R_H}{4}$$

$$= \frac{3}{4} \times \frac{36}{5 \times 6561}$$

$$\Rightarrow \lambda = 1215 \overset{o}{A}$$

Question 059 MCQ

QUESTION

A meter bridge is set up as shown, to determine an unknown resistance X using a standard 10

$$\Omega$$

resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm, respectively, for the ends A and B. The determined value of X is

A 10.2 Ω

B 10.6 Ω

C 10.8 Ω

11.1

D

Ω

CORRECT OPTION

B

10.6

Ω

SOURCE

Physics • current-electricity

EXPLANATION

Corrected length L_1 AJ = 52 + 1 = 53 cm

Corrected length L_2 BJ = $100\$\$ - \$\$52$ + 2 = 50 cm

For a balanced Wheatstone bridge,

$$\frac{X}{10} = \frac{L_1}{L_2} = \frac{53}{50}$$

$$\Rightarrow$$

X = 10.6

Ω

Question 060 MCQ

QUESTION

A metal rod of length L and mass m is pivoted at one end. A thin disk of mass M and radius R $< L$ is attached at its centre to the free end of the rod. Consider two ways the disc is attached : $caseA$. The disc is not free to rotate about its centre and $caseB$ the disc is free to rotate about its centre. The rod-disc system

performs SHM in vertical plane after being released from the same displaced position. Which of the following statement s is are true?

A Restoring torque in case A = Restoring torque in case B.

B Restoring torque in case A < Restoring torque in case B.

C Angular frequency for case A > Angular frequency for case B.

D Angular frequency for case A < Angular frequency for case B.

CORRECT OPTION

A Restoring torque in case A = Restoring torque in case B.

SOURCE

Physics • rotational-motion

EXPLANATION

We have

Restoring torque = Force of gravity on disc and rod which is same in both cases.

•

Case A : The moment of inertia is

$$I_A = \frac{MR^2}{2} + ML^2 + \frac{ml^3}{3}$$

Therefore,

$$\tau_A = -I_A\omega_A^2\theta = -\left(\frac{MR^2}{2} + ML^2 + \frac{ml^3}{3}\right)\omega_A^2\theta$$

Case B : The moment of inertia is

$$I_B = \frac{ml^3}{3} + ML^2$$

Therefore,

$$\tau_B = -I_B\omega_B^2\theta = -\left(\frac{ml^3}{3} + ML^2\right)\omega_B^2\theta$$

since

$$\tau_A = \tau_B \Rightarrow \omega_A < \omega_B$$

.

Question 061 MCQ

QUESTION

A composite block is made of slabs A, B, C, D and E of different thermal conductivities $given in terms of a constant K$ and sizes $given in terms of length, L$ as shown in the figure. All slabs are of same width. Heat Q flows only from left to right through the blocks. Then, in steady-state

A heat flow through A and E slabs are same.

B heat flow through slab E is maximum.

C temperature difference across slab E is smallest.

D heat flow through C = heat flow through B + heat flow through D.

CORRECT OPTION

A heat flow through A and E slabs are same.

SOURCE

Physics • properties-of-matter

EXPLANATION

Let T_1, T_2, T_3 and T_4 be the temperatures at slab interfaces as shown in the figure.

The rate of heat flow across a slab, with thermal conductivity

$$\kappa$$

, width w, thickness l, length x, and temperature difference

$$\Delta$$

T, is given by

$$dQ/dt = \kappa lw(\Delta T/x)$$

.

Thus, the heat flow rates through the given slabs are

$$\frac{dQ_A}{dt} = \frac{(2K)(4Lw)(T_1 - T_2)}{L} = 8Kw(T_1 - T_2)$$

..... 1

$$\frac{dQ_B}{dt} = \frac{(3K)(Lw)(T_2 - T_3)}{4L} = \frac{3}{4}Kw(T_2 - T_3)$$

..... 2

$$\frac{dQ_C}{dt} = \frac{(4K)(2Lw)(T_2 - T_3)}{4L} = 2Kw(T_2 - T_3)$$

..... 3

$$\frac{dQ_D}{dt} = \frac{(5K)(Lw)(T_2 - T_3)}{4L} = \frac{5}{4}Kw(T_2 - T_3)$$

..... 4

$$\frac{dQ_E}{dt} = \frac{(6K)(4Lw)(T_3 - T_4)}{L} = 24Kw(T_3 - T_4)$$

..... 5

In the steady state, heat flow into the system is equal to the heat flow out of the system i.e.,

$$dQ_A/dt = dQ_E/dt$$

.

Use equations 1 and 5 to get

$$T_1 - T_2 = 3(T_3 - T_4)$$

..... 6

Similarly, the steady state condition for the interface at temperature T_2 is

$$dQ_A/dt = dQ_B/dt + dQ_C/dt + dQ_D/dt$$

,

which gives $by using equations (1-4)$

$$2(T_1 - T_2) = T_2 - T_3$$

..... 7

Use equations 6 and 7 to get

$$T_3 - T_4 = (1/3)(T_1 - T_2) = (1/6)(T_2/T_3)$$

.

Also, equations $2-4$ give

$$dQ_C/dt = dQ_B/dt + dQ_D/dt$$

.

Question 062 MCQ

QUESTION

An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement s is/are true?

A They will never come out of the magnetic field region.

B They will come out travelling along parallel paths.

C They will come out at the same time.

D They will come out at different times.

CORRECT OPTION

B They will come out travelling along parallel paths.

SOURCE

Physics • magnetism

EXPLANATION

$$r = \frac{mv}{Bq}$$

or,

$$r \propto m$$

$$\therefore$$

$$r_e < r_p$$

as

$$m_e < m_p$$

Further,

$$T = \frac{2\pi m}{Bq}$$

or

$$T \propto m$$

$$\therefore$$

$$T_e < T_p$$

$$t_e = \frac{T_e}{2}$$

and

$$t_p = \frac{T_p}{2}$$

or

$$t_e < t_p$$

$$\therefore$$

Correct options are b and d.

Question 063 MCQ

QUESTION

The phase space diagram for a ball thrown vertically up from ground is

A

B

C

D

CORRECT OPTION

D

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Let the ball of mass m be thrown up with an initial velocity u. Its velocity v and displacement x are related by v^2

—

u^2 =

—

2gx, where g is the acceleration due to gravity. The momentum $p = mv$ is given by

$p^2 = m^2u^2$

—

$2m^2gx$,

which gives

$$p = \pm\sqrt{m^2u^2 - 2m^2gx}$$

.

At x = 0, the momentum is mu when the ball starts going up and it becomes

$-$

mu when the ball comes back. At the maximum height, $x = u^2/2g$, the momentum becomes zero.

Question 064 MCQ

QUESTION

The phase space diagram for simple harmonic motion is a circle centred at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

A $\frac{E_1}{E_2} = \sqrt{2}$

B $E_1 = 2E_2$

C $E_1 = 4E_2$

D $E_1 = 16E_2$

CORRECT OPTION

C $E_1 = 4E_2$

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Energy of simple harmonic oscillator is

$$E = \frac{1}{2}kA^2$$

where k is the force constant and A the amplitude of the oscillator. Since the oscillator is the same, the value of k is the same. Hence

$$E_1 = \frac{1}{2}kA_1^2$$

and

$$E_2 = \frac{1}{2}kA_2^2$$

$$\therefore$$

$$\frac{E_1}{E_2} = \left(\frac{A_1}{A_2}\right)^2$$

Now, A_1 = maximum value of displacement of oscillator having energy E_1 = 2a and A_2 = a. Therefore

$$\frac{E_1}{E_2} = \left(\frac{2a}{a}\right)^2 = 4$$

. So,

$$E_1 = 4E_2$$

Question 065 MCQ

QUESTION

Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is

A

B

C

D

CORRECT OPTION

B

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Due to upthrust, the spring will be compressed. Due to damping by the liquid, the final position will be smaller than the initial position. Hence choices c and d are not possible. Due to buoyancy, the block will move upwards. Hence, according to

the given sign convention, position x is positive initially. When the system is released, x will decrease and momentum p will increase becoming maximum when the system reaches the mean position $x = 0$ after which the momentum will decrease to zero when the oscillator reaches the extreme position, after which the momentum becomes negative. Hence the correct graph is b.

Question 066 Numerical

QUESTION

A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2 N on the ring and rolls it without slipping with an acceleration of 0.2 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is $P/10$. The value of P is _____.

SOURCE

Physics • rotational-motion

EXPLANATION

f =

$$\mu$$

mg

The net torque about point P is

F

$$\times$$

R

$$-$$

fR = I_p

$$\alpha$$

Where, $I_p = mR^2 + mR^2 = 2mR^2$ $parallelaxestheorem$

and, a = R

$$\alpha$$

Also, f =

$$\mu$$

mg

$$\therefore$$

$$F \times R - \mu mgR = (2\,mR^2) \times \left(\frac{a}{R}\right) = 2maR$$

$$\Rightarrow F - \mu mg = 2\,ma$$

$$\Rightarrow 2 - \mu \times 2 \times 10 = 2 \times 2 \times 0.3$$

which gives

$$\mu = \frac{0.8}{2 \times 10} = \frac{0.4}{10}$$

. Hence, P = 4

Question 067 Numerical

QUESTION

Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side a. The surface tension of the soap film is

$$\gamma$$

. The system of charges and planar film are in equilibrium, and

$$a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$$

, where k is a constant. Then N is _____.

SOURCE

Physics • electrostatics

EXPLANATION

The net force on one of the charges due to other charges is

$$F = \frac{2kq^2}{a^2} + \frac{kq^2}{2a^2} = \frac{5}{2}\left(\frac{kq^2}{a^2}\right)$$

where

$$k = \frac{1}{4\pi\varepsilon}$$

. Here, as shown in the figure, line AB divided the soap film into two equal parts. The free-body diagram of half part is also depicted in the figure here.

At equilibrium, the surface tension balances the force.

Therefore,

$$F_{surface} = 2\sqrt{2}a\gamma$$

That is,

$$2\sqrt{2}a\gamma = \frac{5}{2}\left(\frac{kq^2}{a^2}\right)$$

$$\Rightarrow a^3 = \frac{5}{4\sqrt{2}}\left(\frac{q^2}{\gamma}\right)$$

Therefore,

a = Any constant

$$\times\left(\frac{q^2}{\gamma}\right)^{1/3}$$

Hence, N = 3.

Question 068 Numerical

QUESTION

The activity of a freshly prepared radioactive sample is 10^{10} disintegrations per second, whose mean life is 10^9 s. The mass of an atom of this radioisotope is 10^{-25} kg. The mass $inmg$ of the radioactive sample is _____.

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Activity

$$A = \lambda N$$

, where

$$\lambda$$

is decay constant and N is number of particles present. Therefore,

$$N = \frac{A}{\lambda} = A\tau$$

where

$$\tau$$

= 1 /

λ

is the mean life of the sample. The mass of the sample is

$$M = mN = mA\tau$$

where m is mass of an atom. Therefore, the mass of the radioactive sample is

M = 10

$-$

25

$\times$

10^{10}

$\times$

10^{9} = 10

$-$

^{6} kg = 1 mg

Question 069 Numerical

QUESTION

A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005

Ω

and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as

$$I = I_0 \cos(300t)$$

, where I_0 is constant. If the magnetic moment of the loop is

$$N\mu_0 I_0 \sin(300t)$$

, then N is _____.

SOURCE

Physics • magnetism

EXPLANATION

The flux through the ring is

$$\phi = B\pi r^2$$

Assuming the cylinder as a solenoid with close winding, we have

$$B = \frac{\mu_0 I}{L}$$

Therefore,

$$\phi = \left(\frac{\mu_0 I}{L}\right)\pi r^2 \cos 300t$$

The induced emf is

$$\varepsilon = \frac{-d\phi}{dt} = 300\left(\frac{\mu_0 I}{L}\right)\pi r^2 \sin 300t$$

Therefore, the current induced is

$$i = \frac{\varepsilon}{R} = \left(\frac{\pi r^2 300}{RL}\right)\mu_0 I_0 \sin 300t$$

The magnetic moment is

M = Current

$\times$

Area of loop

Therefore,

$$m = \left(\frac{(3.14)^2 \times (0.1)^4 \times 300}{0.005 \times 10} \right) \mu_0 I_0 \sin 300t$$

$$= 6\mu_0 I_0 \sin 300t$$

Hence, N = 6.