iit Jee 2008 Paper 1 Offline $69 Questions$

Question 001 MCQ

QUESTION

STATEMENT - 1 : The plot of atomic number $y-axis$ versus number of neutrons $x-axis$ for stable nuclei shows a curvature towards x-axis from the line of 45^{o} slope as the atomic number is increased.

STATEMENT - 2 : Proton-proton electrostatic repulsions begin to overcome attractive forces involving protons and neutrons in heavier nuclides

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B. Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C. Statement - 1 is true, Statement - 2 is False

D. Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

SOURCE

Chemistry • structure-of-atom

EXPLANATION

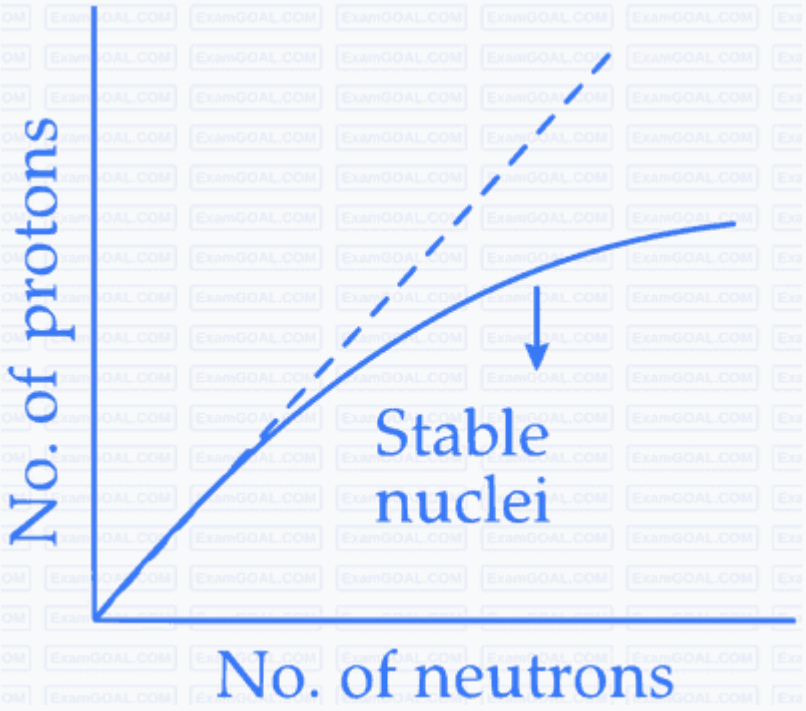
With increase in the atomic number, the proton-proton electrostatic repulsion begins to overcome attractive forces involving proton and neutrons in heavier nuclides. The stability relationship can be represented by a line with a slope of 45

$^{\circ}$

, i.e., the maximum stability is attained when N = Z. Right of the curve a radioactive nuclide would be neutron rich and would decay by

β

-emission to produce a daughter nucleus with a lower n/p ratio. For heavier nuclides, p-p repulsions start to offset the attractive forces and an excess of neutrons over protons, is required for stability.



Question 002 MCQ

QUESTION

Hyperconjugation involves overlapping of the following orbitals:

A $\sigma - \sigma$

B $\sigma - p$

C $p - p$

D $\pi - \pi$

CORRECT OPTION

B $\sigma - p$

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Hyperconjugation effect is one of the type of permanent effect in which delocalisation of

$$\sigma$$

electrons of C-H bond of an alkyl group directly attached to an atom of the unsaturated system or to an atom with an unshared p orbital. The main role of hyperconjugation is to stabilise the carbocation because it helps in the dispersal

of positive charge. Higher the number of alkyl groups attached to a positively charged carbon atom, greater is the hyperconjugative effect and stability of carbonation. It generally involves the interaction of electrons in a C-H sigma orbital with nearby non-bonding p or anti-bonding

$$\sigma^*$$

or

$$\pi^*$$

orbitals to provide extended conjugation which further increases the stability of the carbocation. Sometimes, low lying anti-bonding

$$\sigma^*$$

also interact with filled orbitals of lone pair character. This type of conjugation is termed as negative hyperconjugation.

Question 003 MCQ

QUESTION

The major product of the following reaction is:

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • haloalkanes-and-haloarenes

EXPLANATION

Alkyl halides undergo nucleophilic substitution more readily than aryl halides due to partial double bond character between aryl carbon and halogen. Hence, substitution occurs more readily in the side chain rather than in the ring. PhS

$$-$$

is a strong nucleophile and dimethylformamide DMF is a highly polar aprotic solvent. Hence, nucleophilic substitution $S\$\$_N2\$\$$ takes place at 2

$$\circ$$

benzylic carbon. Stereochemically, it involves inversion of configuration at benzylic carbon atom.

Alkyl group exhibit neighbouring group participation or anchimeric assistance from aryl group but deactivation by

$$-$$

NO

$$2$$

and

$$-$$

F makes S

$$_N1$$

difficult and retention of configuration at chiral carbon does not takes place.

Question 004 MCQ

QUESTION

Aqueous solutions of $Na_2S_2O_3$ on reaction with Cl_2 gives:

A $Na_2S_4O_6$

B $NaHSO_4$

C NaCl

D NaOH

CORRECT OPTION

B NaHSO$_4$

SOURCE

Chemistry • s-block-elements

EXPLANATION

The product formed the reaction is used for chlorination of water bodies. The compound Na$_2$S$_2$O$_3$ gets oxidised by chlorine to form NaHSO$_3$. The balanced chemical reaction is,

Na$_2$S$_2$O$_3$ + 4Cl

$$2$$

+ 5H

$$2$$

O

$$\rightarrow$$

2NaHSO

$$4$$

+ 8HCl

Question 005 MCQ

QUESTION

Native silver metal forms a water soluble complex with a dilute aqueous solution of NaCN in the presence of:

A nitrogen

B oxygen

C carbon dioxide

D argon

CORRECT OPTION

B oxygen

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Reaction involving leaching of silver ores is reversible add, so to get the maximum yield of the product the reaction is carried out in presence of oxygen. Also, oxygen oxidises silver which then produces soluble complex with CN

$$-$$

ions.

4Ag + 8NaCN + 2H

$$2$$

O + O

$$2$$

$$\rightarrow$$

4Na

$$Ag(CN)\$\$_2\$\$$$

+ 4NaOH

Question 006 MCQ

QUESTION

Under the same reaction conditions, initial concentration of 1.386 mol dm

$$-3$$

of a substance becomes half in 40 seconds and 20 seconds through first order and zero order kinetics, respectively. Ratio

$$\left(\frac{k_1}{k_0}\right)$$

of the rate constants for first order $$k_1$$ and zero order $$k_0$$ of the reactions is:

A

0.5 mol $$dm^{-1}$$ ^{3}

B

1.0 mol $$dm^{-3}$$

C

1.5 mol $$dm^{-3}$$

D

2.0 mol $$dm^{-1}$$ ^{3}

CORRECT OPTION

0.5 mol

$^{-1}$

A dm

^{3}

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

Rate constant for first order kinetics:

$$k_1 = \frac{0.693}{t_{1/2}} = \frac{0.693}{40\ s^{-1}}$$

..... i

Rate constant for zero order kinetics:

$$k_0 = \frac{A_0}{2t_{1/2}} = \frac{1.386}{2 \times 20}$$

mol dm

$^{-3}$

s

$^{-1}$

..... ii

Divide i by ii

$$\frac{k_1}{k_0} = \frac{0.693}{40} \times \frac{40}{1.386} = \frac{0.693}{1.386}$$

mol

$^{-1}$

dm

-3

= 0.5 mol

-1

dm

3

Question 007 MCQ

QUESTION

2.5 mL of

$$\frac{2}{5}$$

M weak monoacidic base $K\$\$_b\$\$ = 1\$\$ \times \$\$10\$\$^{-12}\$\$at25\$\$^\circ\$\$C$ is titrated with

$$\frac{2}{15}$$

M HCl in water at 25

$\circ$

C. The concentration of H

$+$

at equivalence point is $K\$\$_w\$\$ = 1\$\$ \times \$\$10\$\$^{-14}\$\$at25\$\$^\circ\$\$C$.

3.7

$\times$

A 10 -13 M

3.2 $\times$

B 10 -7 M

3.2 $\times$

C 10 -2 M

2.7 $\times$

D 10 -2 M

CORRECT OPTION

2.7 $\times$

D 10 -2

M

SOURCE

Chemistry • ionic-equilibrium

EXPLANATION

Weak monoacidic base, e.g., BOH is neutralised as follows:

BOH + HCl

$$\rightarrow$$

BCl + H

$$2$$

O

At the equivalence point, all BOH get converted into the salt. The concentration of H

$$+$$

$orpHofsolution$ is due to hydrolysis of the resultant salt $BCl, cationichydrolysishere.$

Volume of HCl used up,

$$V_a = \frac{N_b V_b}{N_a}$$

$$V_a = \frac{2.5 \times 2 \times 15}{2 \times 5}$$

$$V_a = 7.5$$

Concentration of salt,

$$[BCl] = \frac{conc.\ of\ base}{total\ volume}$$

$$= \frac{2 \times 2.5}{5(7.5 + 2.5)} = \frac{1}{10} = 0.1$$

$$K_h = \frac{Ch^2}{1-h} = \frac{K_w}{K_b} = \frac{10^{-14}}{10^{-12}}$$

$$= 10^{-2} = \frac{0.1 \times h^2}{1-h}$$

..... i

$hshouldbeestimatedwhetherthatcanbeneglectedornot.$

On calculating,

$$h = 0.27$$

$significant, notnegligible$

$$[H^+] = Ch = 0.1 \times 0.27 = 2.7 \times 10^{-2}$$

M

Question 008 MCQ

QUESTION

The correct statement s about the compound given below is are:

A The compound is optically active.

B The compound possesses centre of symmetry.

C The compound possesses plane of symmetry.

D The compound possesses axis of symmetry.

CORRECT OPTION

A The compound is optically active.

SOURCE

Chemistry • haloalkanes-and-haloarenes

EXPLANATION

We know that following are the cases when a compound is optically inactive:

1. When it does not have a chiral carbon atom e.g., Methyl bromide

2. When it has a plane or axis of symmetry e.g., Tartaric acid
$thesecompoundsarecalledasMesocompounds$

3. When it forms a racemic mixture e.g.: Equimolar mixture of d-Lactic acid and l-Lactic acid.

The given compound possesses an axis of symmetry

Question 009 MCQ

QUESTION

The correct statement s concerning the structures E, F and G is are :

A E, F and G are resonance structures.

B E, F and E, G are tautomers.

C F and G are geometrical isomers.

D F and G are diasteromers.

CORRECT OPTION

B E, F and E, G are tautomers.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The correct statements concerning the structures E, F and G are:

B E, F and G are tautomers.

E is in keto form $C = O$ and F and G are in enol form $C = C\$\$ - \$\$OH$

C F and G are geometrical isomers. F is Z isomer and E is E isomer.

D F and G are diastereomers. They are not mirror images of each other and non-superimposable.

Question 010 MCQ

QUESTION

A solution of colourless salt H on boiling with excess NaOH produces a non-flammable gas. The gas evolution ceases after sometime. Upon addition of Zn dust to the same solution, the gas evolution restarts. The colourless salt s H is are:

A

NH

4

NO

3

B

NH

4

NO

2

C

NH

4

Cl

D

$NH\$\$_4\$\$$

2

SO

4

CORRECT OPTION

A

NH

4

NO

3

SOURCE

Chemistry • p-block-elements

EXPLANATION

The colourless salt H is either NH_4NO_3 or NH_4NO_2. The colourless salt produces NH_3 gas $non-inflammable$ on boiling with excess of NaOH. On addition of zinc dust $areducingagent$ to this solution, sodium nitrite or sodium nitrate will liberate NH_3 gas again.

$NH_4NO_3 + NaOH \rightarrow NH_3 + NaNO_3$

+ H

2

O

7NaOH + NaNO

3

+ 4Zn

$\rightarrow$

4Na

2

ZnO

2

+ NH

3

+ 2H

2

O

NH

4

NO

2

+ NaOH

$\rightarrow$

NaNO

2

+ NH

3

+ H

$$2$$

O

3Zn + 5NaOH + NaNO

$$2$$

$$\rightarrow$$

3Na

$$2$$

ZnO

$$2$$

+ NH

$$3$$

Question 011 MCQ

QUESTION

A gas described by van Der Waals equation

A behaves similar to an ideal gas in the limit of large molar volumes.

B behaves similar to an ideal gas in the limit of large pressures.

C is characterized by van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature.

D has the pressure that is lower than the pressure exerted by the same gas behaving ideally.

CORRECT OPTION

A behaves similar to an ideal gas in the limit of large molar volumes.

SOURCE

Chemistry • gaseous-state

EXPLANATION

Van der Waals equation is

$$\left(P+\frac{an^2}{V^2}\right)(V-nb)=nRT$$

, where a is a constant whose value depends on the attraction between the gas molecules and b is the volume that is occupied by one mole of gas.

In option A, it behaves similar to an ideal gas in the limit of large molar volumes. When the volume is very large then the attraction between the gas molecules can be neglected, so a

$\approx$

0 and the volume occupied by one mole of gas compared to large volume of container will also be neglected. So, b

$\approx$

0. Now, we can neglect a and b and the gas behaves ideally. Thus, option A is correct.

In option B, it behaves similar to an ideal gas in limit of low pressure and high temperatures. Now, the pressure is very low. So, the attraction between the particles is negligible.

Van der Waals coefficients that are dependent on the identity of the gas but are independent of the temperature. Van der Waals coefficient depends on the

attraction between the gas molecules $eachofwhichischaracteristicofagas$ and b is the volume that is occupied by one mole of the molecules. So, this option is correct. Since, molecules of Van der Waals gas exert force of attraction on each other; hence, the force which they exert on the walls of the container is less than the force exerted by it if the gas would have been ideal.

Thus, option D is correct.

Question 012 MCQ

QUESTION

Statement 1 : Bromobenzene upon reaction with Br

$$2$$

/Fe gives 1, 4-dibromobenzene as the major product.

and

Statement 2: In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile.

A. Statement 1 is True, Statement 2 is True; Statement 2 is a correct explanation for Statement 1.

B. Statement 1 is True, Statement 2 is True; Statement 2 is a NOT correct explanation for Statement 1.

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

CORRECT OPTION

C Statement 1 is True, Statement 2 is False.

SOURCE

Chemistry • haloalkanes-and-haloarenes

EXPLANATION

Bromine orientation is controlled by its mesomeric effect. Its stabilisation of arenium ion by +M activity.

In bromobenzene, the inductive effect of the bromo group is more dominant than the mesomeric effect in directing the incoming electrophile at para position. As a result, only mono substitution of bromobenzene takes place.

Question 013 MCQ

QUESTION

Statement 1 : Pb

$$4+$$

compounds are stronger oxidising agents than Sn

$$4+$$

compounds.

and

Statement 2 : The higher oxidation states for the group 14 elements are more stable for the heavier members of the group due to 'inert pair effect'.

A Statement 1 is True, Statement 2 is True; Statement 2 is correct explanation for Statement 1.

B Statement 1 is True, Statement 2 is True; Statement 2 is NOT correct explanation for Statement 1.

C Statement 1 is True, Statement 2 is False.

D Statement 1 is False, Statement 2 is True.

CORRECT OPTION

C Statement 1 is True, Statement 2 is False.

SOURCE

Chemistry • p-block-elements

EXPLANATION

In p-block elements, inert pair effect is observed, due to which lower oxidation state becomes more stable on going down the group. In 14^{th} group C, Si, Ge, Sn, Pb Pb is placed at lower position than Sn, that's why lower oxidation state is more stable for lead $Pb$$$^{2+}$$ and Sn

$+4$

is more stable than Pb

$+4$

. So, Pb

$+4$

compounds are stronger oxidising agents than Sn

$+4$

compounds. Lower oxidation for the group 14^{th} elements are more stable for the heavier members.

Question 014 MCQ

QUESTION

Statement 1 : For every chemical reaction at equilibrium, standard Gibbs energy of reaction is zero.

and

Statement 2 : At constant temperature and pressure, chemical reactions are spontaneous in the direction of decreasing Gibbs energy.

A Statement 1 is True, Statement 2 is True; Statement 2 is correct explanation for Statement 1.

B Statement 1 is True, Statement 2 is True; Statement 2 is NOT correct explanation for Statement 1.

C Statement 1 is True, Statement 2 is False.

D Statement 1 is False, Statement 2 is True.

CORRECT OPTION

D Statement 1 is False, Statement 2 is True.

SOURCE

Chemistry • chemical-equilibrium

EXPLANATION

The Gibbs free energy varies from standard value as a function of temperature and equilibrium constant.

$$\Delta$$

G =

$$\Delta$$

G

$$\circ$$

+ RT ln K

$$eq$$

The standard Gibbs energy for a reaction is given by

$$\Delta$$

G

$$\circ$$

.

At equilibrium,

$$\Delta$$

G = 0whereas

$$\Delta$$

G

$$\circ$$

for a reaction may or may not be zero.

$$\Delta$$

G

$$\circ$$

=

$$-$$

RT ln K

$$eq$$

For a spontaneous process, Gibbs energy for a reaction is always negative,

$$\Delta$$

G < 0.

Question 015 MCQ

QUESTION

The structure of the product I is :

A

B

C

D

CORRECT OPTION

D

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Hex-3-ynal on reduction with sodium borohydride produces an alcohol.

The

—

OH group of alcohol reacts with phosphorus tribromide to form alkyl bromide $compound I$.

Question 016 MCQ

QUESTION

The structure of compounds J and K, respectively, are:

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The alkyl bromide reacts with Mg to form alkyl magnesium bromide $Grignardreagent$ which attacks carbon dioxide followed by hydrolysis to form a carboxylic acid $compoundJ$.

It reacts with thionyl chloride $compoundK$ to form acid chloride.

Question 017 MCQ

QUESTION

The structure of product L is :

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Acid chloride which on hydrogenation with Pd supported on barium sulphate partially poisoned with quinoline to form compound L. Here, a triple bond is selectively reduced to a double bond which has

$$cis$$

configuration.

Question 018 MCQ

QUESTION

Among the following, the correct statement is:

A Phosphates have no biological significance in humans.

B Between nitrates and phosphates, phosphates are less abundant in earth's crust.

C Between nitrates and phosphates, nitrates are less abundant in earth's crust.

D Oxidation of nitrates is possible in soil.

CORRECT OPTION

C Between nitrates and phosphates, nitrates are less abundant in earth's crust.

SOURCE

Chemistry • p-block-elements

EXPLANATION

Due to greater solubility and nature to be prone to microbial action, nitrates are less abundant in earth's crust. Nitrates are soluble. In nitrates, nitrogen has maximum oxidation state and cannot be further oxidised.

Question 019 MCQ

QUESTION

Among the following, the correct statement is :

A Between NH_3 and PH_3, NH_3 is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional.

Between NH_3

and PH

3

B , PH

3

is a better electron donor because the lone pair of electrons occupies sp

3

orbital and is more directional.

Between NH

3

and PH

3

C , NH

3

is a better electron donor because the lone pair of electrons occupies sp

3

orbital and is more directional.

Between NH

3

and PH

3

D , PH

3

is better electron donor because the lone pair of electrons occupies spherical 's' orbital and is less directional.

CORRECT OPTION

C

Between NH_3 and PH_3, NH_3 is a better electron donor because the lone pair of electrons occupies sp^3 orbital and is more directional.

SOURCE

Chemistry • p-block-elements

EXPLANATION

Both NH_3 and PH_3 have one lone pair of electron on it. The lone pair in case of ammonia occupy sp^3 orbital is directional.

Hence, the lone pair in case of ammonia is easily available and can be easily donated. But for PH_3 the lone pair of electrons occupy spherical s-orbital which is less directional and thus its lone pair is not easily available neither it can be donated easily.

Also the lone pair of electrons in sp

$$3$$

orbital has lesser s-character and less attracted by nucleus and available for easy donation than in PH

$$3$$

with pure s orbital, the lone pair in which lone pair are more strongly attracted by nucleus. As we go down a group, the p-character in bond pair increases and the s-character in lone pair increases.

Thus ammonia is a better base than PH

$$3$$

.

Question 020 MCQ

QUESTION

White phosphorus on reaction with NaOH gives PH

$$3$$

as one of the products. This is a:

A dimerisation reaction

B disproportionation reaction

C condensation reaction

D precipitation reaction

CORRECT OPTION

B disproportionation reaction

SOURCE

Chemistry • p-block-elements

EXPLANATION

White phosphorus on reaction with NaOH gives PH_3 as one of the products.

$P_4 + 3NaOH + 3H_2O \rightarrow 3NaH_2PO_2 + PH_3$

The oxidation state of P in P_4, NaH_2PO_2

PO

$_2$

and PH

$_3$

is 0, +1 and

—

3 respectively. It is an example of disproportionation reaction.

Question 021 MCQ

QUESTION

The freezing point of the solution M is :

A 268.7 K

B 268.5 K

C 234.2 K

D 150.9 K

CORRECT OPTION

D 150.9 K

SOURCE

Chemistry • solutions

EXPLANATION

For solution M, water is solute and ethanol is solvent.

$$\because$$

$$\Delta T_f = (K_f)_{ethanol} \times m$$

$$2 \times \frac{0.1}{0.9 \times \frac{46}{1000}} = \frac{2000}{9 \times 46} = 4.8$$

$$\therefore$$

$$T_f = T_f^0 - \Delta T_f = 155.7 - 4.8 = 150.9\,K$$

Question 022 MCQ

QUESTION

The vapour pressure of the solution M is :

A 39.3 mm Hg

B 36.0 mm Hg

C 29.5 mm Hg

D 28.8 mm Hg

CORRECT OPTION

B 36.0 mm Hg

SOURCE

Chemistry • solutions

EXPLANATION

Total vapour pressure

$$P = P_A^0 x_A$$

$$\$\$ \because \$\$ Solute is to be taken as non - volatile$$

$$= 40 \times 0.9 = 36$$

mm Hg

Question 023 MCQ

QUESTION

Water is added to the solution M such that the fraction of water in the solution becomes 0.9 mole. The boiling point of this solution is:

A 380.4 K

B 376.2 K

C 375.5 K

D 354.7 K

CORRECT OPTION

B 376.2 K

SOURCE

Chemistry • solutions

EXPLANATION

$$\underset{(\text{Solute}\equiv\text{B})}{x_{ethanol}} = 0.1 \quad : \quad \underset{(\text{Solvent}\equiv\text{A})}{x_{water}} = 0.9$$

$$\Delta T_b = K_b \,.\, m = K_b \,.\, \frac{x_B}{1 - x_B} \times \frac{1000}{M_{w_A}}$$

$$= 0.52 \times \frac{0.1}{0.9} \times \frac{1000}{18} = 3.2\,K$$

$$T_b = T_b^0 + \Delta T_b = 373 + 3.2 = 376.2\,K$$

Question 024 MCQ

QUESTION

Let z be any point in

$$A \cap B \cap C$$

Then,

$$|z + 1 - i|^2 + |z - 5 - i|^2$$

lies between :

A 25 and 29

B 30 and 34

C 35 and 39

D 40 and 44

CORRECT OPTION

C 35 and 39

SOURCE

Mathematics • complex-numbers

EXPLANATION

$$|z+1-i|^2+|z-5-i|^2$$

The points $\$\$ -1,1\$\$$ and $5,1$ are the extremities of the diameter of the given circle of radius 3

Hence, PA

$$2$$

+ PB

$$2$$

= AB

$$2$$

= 36

Question 025 MCQ

QUESTION

The number of elements in the set

$$A \cap B \cap C$$

is

A 0

B 1

C 2

D ∞

CORRECT OPTION

B 1

SOURCE

Mathematics • complex-numbers

EXPLANATION

In the Cartesian coordinates sets A, B and C defined the regions given by

$$A : y \geq 1, B : (x-2) + (y-1)^2 = 9$$

B and C being a circle and a straight line intersect in two points, out of which only one satisfies

$$y > -1$$

.

Thus, the no. of elements in the set A

$$\cap$$

B

$$\cap$$

C is 1.

Question 026 MCQ

QUESTION

Consider three planes

$$\$P_1 : x - y + z = 1$$

$$P_2 : x + y - z = 1$$

$$P_3 : x - 3y + 3z = 2$$

$

Let

$$L_1,$$

$$L_2,$$

$$L_3$$

be the lines of intersection of the planes

$$P_2$$

and

$$P_3,$$

$$P_3$$

and

$$P_1,$$

$$P_1$$

and

$$P_2,$$

respectively.

STATEMENT - 1Z: At least two of the lines

$$L_1,$$

$$L_2$$

and

$$L_3$$

are non-parallel and

STATEMENT - 2: The three planes doe not have a common point.

A STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

B STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

C STATEMENT - 1 is True, STATEMENT - 2 is False

D STATEMENT - 1 is False, STATEMENT - 2 is True

CORRECT OPTION

D STATEMENT - 1 is False, STATEMENT - 2 is True

SOURCE

Mathematics • 3d-geometry

EXPLANATION

We have,

$$P_1 : x - y + z = -1$$

$$P_2 : x + y - z = -1$$

$$P_3 : x - 3y + 3z = 2$$

Let dr's of the lines of L

$$1$$

, L

$$2$$

and L

$$3$$

are

$$a_1, b_1, c_1 : a_2, b_2, c_2$$

and

$$a_3, b_3, c_3$$

respectively.

Therefore,

$$a_1 + b_1 - c_1 = 0$$

$$a_1 - 3b_1 + 3c_1 = 0$$

$$\Rightarrow \frac{a_1}{0} = \frac{b_1}{-4} = \frac{c_1}{-4}$$

$$a_1, b_1, c_1 = 0, 1, 1$$

again

$$a_2 - b_2 + c_2 = 0$$

$$a_2 - 3b_2 + 3c_2 = 0$$

$$\frac{a_2}{0} = \frac{b_2}{-2} = \frac{c_2}{-2}$$

$$a_2, b_2, c_2 = 0, 1, 1$$

Again

$$a_3 - b_3 + c_3 = 0$$

$$a_3 + b_3 - c_3 = 0$$

$$\Rightarrow \frac{a_3}{0} = \frac{b_3}{2} = \frac{c_3}{2} \Rightarrow a_3, b_3, c_3 = 0, 1, 1$$

L

$$1$$

, L

$$2$$

and L

$$3$$

are parallel.

Question 027 MCQ

QUESTION

The edges of a parallelopiped are of unit length and are parallel to non-coplanar unit vectors

$$\vec{a}, \vec{b}, \vec{c}$$

such that

$$\hat{a}\,.\,\hat{b} = \hat{b}\,.\,\hat{c} = \hat{c}\,.\,\hat{a} = \frac{1}{2}.$$

Then, the volume of the parallelopiped is :

A $\frac{1}{\sqrt{2}}$

B $\frac{1}{2\sqrt{2}}$

C $\frac{\sqrt{3}}{2}$

D $\frac{1}{\sqrt{3}}$

CORRECT OPTION

A $\frac{1}{\sqrt{2}}$

SOURCE

Mathematics • vector-algebra

EXPLANATION

The important thing to remember in this is the formula

$$\left[\vec{x}.\vec{y}.\vec{z}\right]^2 = \begin{vmatrix} \vec{x}.\vec{x} & \vec{x}.\vec{y} & \vec{x}.\vec{z} \\ \vec{y}.\vec{x} & \vec{y}.\vec{y} & \vec{y}.\vec{z} \\ \vec{z}.\vec{x} & \vec{z}.\vec{y} & \vec{z}.\vec{z} \end{vmatrix}$$

Volume of the parallelopiped

$$v = \left[\hat{a} \quad \hat{b} \quad \hat{c}\right]$$

$$\therefore$$

$$v^2 = \left[\hat{a} \quad \hat{b} \quad \hat{c}\right]^2 = \begin{vmatrix} \hat{a}.\hat{a} & \hat{a}.\hat{b} & \hat{a}.\hat{c} \\ \hat{b}.\hat{a} & \hat{b}.\hat{b} & \hat{b}.\hat{c} \\ \hat{c}.\hat{a} & \hat{c}.\hat{b} & \hat{c}.\hat{c} \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{vmatrix} = \frac{1}{2}$$

$onevaluation$

$$\Rightarrow v = \left[\hat{a} \quad \hat{b} \quad \hat{c}\right] = \frac{1}{\sqrt{2}}$$

Question 028 MCQ

QUESTION

Consider the system of equations

$$ax + by = 0; cx + dy = 0,$$

where

$$a, b, c, d$$

$$\in \{0, 1\}$$

STATEMENT - 1 : The probability that the system of equations has a unique solution is

$$\frac{3}{8}.$$

and

STATEMENT - 2 : The probability that the system of equations has a solution is

$$1.$$

A. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

B. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

C. STATEMENT - 1 is True, STATEMENT - 2 is False.

D. STATEMENT - 1 is False, STATEMENT - 2 is True.

CORRECT OPTION

B. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

SOURCE

Mathematics • probability

EXPLANATION

We have,

$$ax + by = 0$$

$$cx + dy = 0$$

since, the system of homogeneous equation is always consistent and has a solution.

Therefore, statement 2 is true.

Now,

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

and

$$a, b, c, d \in \{0, 1\}$$

$$= ad - bc$$

No. of ways of selecting

$$a, b, c, d$$

from the set {0, 1} is 2

$$\times$$

2

$$\times$$

2

$$\times$$

2 = 16

If the system has unique solution,

Then

$$\Delta \neq 0$$

$$\Rightarrow$$

either

$$ad = 1, bc = 0$$

or

$$ad = 0, bc = 1$$

favourable case = 6

Therefore, probability that system of equation has unique solution is

$$\frac{6}{16} = \frac{3}{8}$$

. Statement 1 is True.

Hence, the Statement 2 is True but is not a correct explanation of statement 1.

Question 029 MCQ

QUESTION

$$\int_{-1}^{1} g'(x)dx =$$

A $2g(-1)$

B 0

C $-2g(1)$

D $2g(1)$

CORRECT OPTION

D $2g(1)$

SOURCE

Mathematics • definite-integration

EXPLANATION

$$y' = \frac{1}{3[1 - f(x)^2]}$$

Clearly

$$f(x)$$

is an odd function then

$$g'(x)$$

is an even function

So,

$$\int_{-1}^{1} g'(x) = 2\int_{0}^{1} g'(x)dx$$

$$= 2[g(x)]_0^1 = 2[g(1) - g(0)]$$

$$= 2g(1)$$

Question 030 MCQ

QUESTION

The area of the region bounded by the curve

$$y = f(x),$$

the

$$x$$

-axis, and the lines

$$x = a$$

and

$$x = b$$

, where

$$-\infty < a < b < -2,$$

is :

A $$\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)} dx + bf\left(b\right) - af\left(a\right)$$

B $$-\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)} dx + bf\left(b\right) - af\left(a\right)$$

C $$\int_a^b \frac{x}{3\left(\left(f(x)\right)^2 - 1\right)} dx - bf\left(b\right) + af\left(a\right)$$

D $$-\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx - bf(b) + af(a)$$

CORRECT OPTION

A $$\int_a^b \frac{x}{3\left((f(x))^2-1\right)}dx + bf(b) - af(a)$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

Required area

$$\int_a^b ydx = \int_a^b f(x)dx$$

$$= [f(x).x]_a^b - \int_a^b f'(x)xdx$$

$$= bf(b) - af(a) - \int_a^b f'(x)xdx$$

$$= bf(b) - af(a) + \int_a^b \frac{xdx}{3[\{f(x)\}^2-1]}$$

$$\because$$

$$f'(x) = \frac{dy}{dx} = \frac{-1}{3(y^2-1)} = \frac{-1}{3[\{f(x)\}^2-1]}$$

Question 031 MCQ

QUESTION

If

$$f\left(-10\sqrt{2}\right) = 2\sqrt{2},$$

then

$$f''\left(-10\sqrt{2}\right) =$$

A $\frac{4\sqrt{2}}{7^3 3^2}$

B $-\frac{4\sqrt{2}}{7^3 3^2}$

C $\frac{4\sqrt{2}}{7^3 3}$

D $-\frac{4\sqrt{2}}{7^3 3}$

CORRECT OPTION

B

$$-\frac{4\sqrt{2}}{7^3 3^2}$$

SOURCE

Mathematics • differentiation

EXPLANATION

We have

$$y^3 - 3y + x = 0$$

Differentiate both sides we get

$$3y^2 . y' - 3y' + 1 = 0$$

..... i

Put

$$y = 2\sqrt{2}, x = -10\sqrt{2}$$

then

$$y'(-10\sqrt{2}) = \frac{-1}{21}$$

Differentiate eq. i, we get

$$3y^2 y'' + 6y(y')^2 - 3y'' = 0$$

Put

$$y = 2\sqrt{2}, x = -10\sqrt{2}, y' = \frac{-1}{21}$$

then

$$y''(-10\sqrt{2}) = \frac{-4\sqrt{2}}{7^3 \cdot 3^2}$$

Question 032 MCQ

QUESTION

Let

$$f(x)$$

be a non-constant twice differentiable function defined on

$$(-\infty, \infty)$$

such that

$$f(x) = f(1-x)$$

and

$$f'\left(\frac{1}{4}\right) = 0.$$

Then,

$$f''(x)$$

A vanishes at least twice on

$$[0, 1]$$

B $$f'\left(\frac{1}{2}\right)=0$$

C $$\int_{-1/2}^{1/2} f\left(x+\frac{1}{2}\right)\sin x\,dx=0$$

D $$\int_{0}^{1/2} f(t)e^{\sin\,\pi t}dt=\int_{1/2}^{1} f(1-t)e^{\sin\,\pi t}dt$$

CORRECT OPTION

$$f''(x)$$

A vanishes at least twice on

$$[0,1]$$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$f(x)$$

is a non constant twice differential function such that

$$f(x)=f(1-x)\Rightarrow f'(x)=-f'(1-x)$$

.... i

For

$$x = \frac{1}{2}$$

We get

$$f'\left(\frac{1}{2}\right) = -f'\left(1 - \frac{1}{2}\right)$$

$$= f'\left(\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) = 0 \Rightarrow f'\left(\frac{1}{2}\right) = 0$$

For

$$x = \frac{1}{4}$$

, we get

$$f'\left(\frac{1}{4}\right) = -f'\left(\frac{3}{4}\right)$$

But given that

$$f'\left(\frac{1}{4}\right) = 0 : f'\left(\frac{1}{4}\right) = f'\left(\frac{3}{4}\right) = 0$$

Hence

$$f'(x)$$

satisfies all conditions of roller's theorem for

$$x \in \left[\frac{1}{4}, \frac{1}{2}\right]$$

and

$$\left[\frac{1}{2}, \frac{3}{4}\right]$$

, so there exist at least one point

$$C_1 \in \left(\frac{1}{4}, \frac{1}{2}\right)$$

and at least one point

$$C_2 \in \left(\frac{1}{2}, \frac{3}{4}\right)$$

such that

$$f''(G) = 0$$

and

$$f''(C)_2 = 0$$

$$\therefore$$

$$f''(x)$$

vanishes at least twice on

$$0, 1$$

Also using

$$f(x) = f(1-x)$$

$$f\left(x + \frac{1}{2}\right) = f\left(1 - x - \frac{1}{2}\right) = f\left(-x + \frac{1}{2}\right)$$

$$f'\left(x + \frac{1}{2}\right)$$

is an odd function

$$\Rightarrow \sin x \, f\left(x + \frac{1}{x}\right)$$

is an even function

$$\int_{\frac{-1}{2}}^{\frac{1}{2}} f\left(x + \frac{1}{x}\right) \sin x \, dx = 0$$

Question 033 MCQ

QUESTION

If

$$0 < x < 1$$

, then

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right)+\sin\left(\cot^{-1}x\right)\right\}^2-1\right]^{1/2}=$$

A $\dfrac{x}{\sqrt{1+x^2}}$

B x

C $x\sqrt{1+x^2}$

D $\sqrt{1+x^2}$

CORRECT OPTION

C $x\sqrt{1+x^2}$

SOURCE

Mathematics • inverse-trigonometric-functions

EXPLANATION

Given that,

$$0 < x < 1$$

,

$$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{\frac{1}{2}}$$

Find : value of this expression

Here,

$$0 < x < 1$$

,

$$\cot^{-1}x = \sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$$

Now,

$$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x)+\sin(\cot^{-1}x)\}^2-1]^{\frac{1}{2}}$$

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cos^{-1}\left(\frac{x}{1+x^2}\right)\right)+\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}\left[\left\{x\,.\,\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}\left[\left\{\frac{x^2+1}{\sqrt{1+x^2}}\right\}^2-1\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}[x^2+1-1]^{\frac{1}{2}}$$

$$=x\sqrt{1+x^2}$$

which is the value of the expression.

Question 034 MCQ

QUESTION

Consider the two curves

$$C_1 : y^2 = 4x,\ C_2 : x^2 + y^2 - 6x + 1 = 0$$

. Then,

A C_1 and C_2 touch each other only at one point.

B C_1 and C_2 touch each other exactly at two points

C C_1 and C_2 intersect $butdonottouch$ at exactly two points

D C_1 and C_2

neither intersect nor touch each other

CORRECT OPTION

B

$$C_1$$

and

$$C_2$$

touch each other exactly at two points

SOURCE

Mathematics • ellipse

EXPLANATION

Given that,

$$C_1 : y^2 = 4x$$

$$C^2 : x^2 + y^2 - 6x + 1 = 0$$

Putting

$$y^2 = 4x$$

in

$$x^2 + y^2 - 6x + 1 = 0$$

, we get

$$x^2 + 4x - 6x + 1 = 0 \Rightarrow (x-1)^2 = 0$$

$$x = 1$$

putting

$$x = 1$$

in

$$y^2 = 4x$$

we get

$$y = \pm 2$$

So, the curves touches each other at two points $1, 2$ and $1, \$\$ - \$\$2,$

Question 035 MCQ

QUESTION

Let z be any point

$$A \cap B \cap C$$

and let w be any point satisfying

$$|w - 2 - i| < 3$$

. Then,

$$|z| - |w| + 3$$

lies between :

A - 6 and 3

B - 3 and 6

C - 6 and 6

D - 3 and 9

CORRECT OPTION

D - 3 and 9

SOURCE

Mathematics • complex-numbers

EXPLANATION

Since,

$$|w-(2+i)|<3$$

$$|w|-|2+i|<3$$

$$-3+\sqrt{5}<|w|<3+\sqrt{5}$$

$$-3-\sqrt{5}<|w|<3-\sqrt{5}$$

Also,

$$|z-(2+i)|=3$$

$$-3+\sqrt{5}\leq|z|\leq 3+\sqrt{5}$$

$$\therefore$$

$$-3<|z|-|w|+3<9$$

Question 036 MCQ

QUESTION

Let

$$S_n=\sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$$

and

$$T_n = \sum_{k=0}^{n-1} \frac{n}{n^2 + kn + k^2}$$

for

$$n = 1, 2, 3, \ldots\ldots\ldots\ldots$$

Then,

A $S_n < \frac{\pi}{3\sqrt{3}}$

B $S_n > \frac{\pi}{3\sqrt{3}}$

C $T_n < \frac{\pi}{3\sqrt{3}}$

D $T_n > \frac{\pi}{3\sqrt{3}}$

CORRECT OPTION

A $S_n < \frac{\pi}{3\sqrt{3}}$

SOURCE

Mathematics • sequences-and-series

EXPLANATION

$$S_n < \lim_{x\to\infty} S_n = \lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n} \frac{1}{1+\frac{k}{n}+\left(\frac{k}{n^2}\right)}$$

$$= \int_0^1 \frac{dx}{1+x+x^2} = \frac{\pi}{3\sqrt{3}}$$

As

$$h\sum_{k=0}^{n} f(kh) > \int_0^1 f(x)dx > h$$

$$\sum_{k=1}^{n} f(kh)$$

So,

$$T_n > \frac{\pi}{3\sqrt{3}}$$

Question 037 MCQ

QUESTION

A straight line through the vertex p of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then :

A $\frac{1}{PS}+\frac{1}{ST}<\frac{2}{\sqrt{QS\times SR}}$

B $$\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$$

C $$\frac{1}{PS}+\frac{1}{ST}<\frac{4}{QR}$$

D $$\frac{1}{PS}+\frac{1}{ST}>\frac{4}{QR}$$

CORRECT OPTION

B $$\frac{1}{PS}+\frac{1}{ST}>\frac{2}{\sqrt{QS\times SR}}$$

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

As S is not the centre of the circumcircle, there for PS

$$\neq$$

ST and QS

$$\neq$$

SR

Also, $PS\ ST$ = $QS\ SR$

$$\$\$\ \therefore \$\$ from the properties of two intersecting chords in a circle$$

$\Rightarrow$

A.M

$\geq$

G.M

We get

$$\frac{\frac{1}{PS}+\frac{1}{ST}}{2} \geq \sqrt{\frac{1}{PS} \times \frac{1}{ST}}$$

or

$$\frac{1}{PS}+\frac{1}{ST} \geq \frac{2}{\sqrt{QS \times SR}}$$

$$\frac{QS+SR}{2} \geq \sqrt{QS \times SR}$$

..... i

$$\frac{1}{\sqrt{QS \times SR}} \geq \frac{2}{QR}$$

.... ii

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST} \geq \frac{4}{QR}$$

From i and ii

Question 038 MCQ

QUESTION

The equation of circle C is

A $\left(x-2\sqrt{3}\right)^2+(y-1)^2=1$

B $\left(x-2\sqrt{3}\right)^2+(y+\frac{1}{2})^2=1$

C $\left(x-\sqrt{3}\right)^2+(y+1)^2=1$

D $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$

CORRECT OPTION

D $\left(x-\sqrt{3}\right)^2+(y-1)^2=1$

SOURCE

Mathematics • circle

EXPLANATION

Let centre of circle C be h, k then,

$$\left|\frac{\sqrt{3}h+k-6}{\sqrt{3+1}}\right|=1$$

$$\sqrt{3}h+k-6=2,-2$$

$$\sqrt{3}h+k=4$$

Missing or unrecognized delimiter for \right $$, satisfies eq.)

eq. of circle C is

$$\left(x-\sqrt{3}\right)^2+(y-1)^2=1$$

Clearly point E and F satisfy the equation in given option D.

As

$$\left(x-2\sqrt{3}\right)^2+(y-1)^2=1$$

not possible.

Question 039 MCQ

QUESTION

Points E and F are given by

A $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\sqrt{3},0\right)$

B $\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right),\left(\sqrt{3},0\right)$

C $\left(\frac{\sqrt{3}}{2},\frac{3}{2}\right),\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

D $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

CORRECT OPTION

A $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\sqrt{3}, 0\right)$

SOURCE

Mathematics • circle

EXPLANATION

Slope of line

$$PQ = -\sqrt{3}$$

Therefore, PQ make 120

$^{\circ}$

angle with x-axis. So, side PR lies along X-axis.

Therefore,

$$F = \left(\sqrt{3}, 0\right)$$

Now, equation CE is

$$\frac{x - \sqrt{3}}{\frac{-\sqrt{3}}{2}} = \frac{y - 1}{\frac{1}{2}} = 1$$

$$E = \left(\frac{\sqrt{3}}{2}, \frac{3}{2} \right)$$

Question 040 MCQ

QUESTION

Equations of the sides QR, RP are

A $y = \frac{2}{\sqrt{3}} x + 1,\ y = -\frac{2}{\sqrt{3}} x - 1$

B $y = \frac{1}{\sqrt{3}} x,\ y = 0$

C $y = \frac{\sqrt{3}}{2} x + 1,\ y = -\frac{\sqrt{3}}{2} x - 1$

D $y = \sqrt{3}\, x,\ y = 0$

CORRECT OPTION

D $y = \sqrt{3}\, x,\ y = 0$

SOURCE

Mathematics • circle

EXPLANATION

Equation of PR is X-axis, that is

$$y = 0$$

and the equation of side QR is

$$\left(y - \frac{3}{2}\right) = \sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$$

$$y = \sqrt{3}x$$

Question 041 MCQ

QUESTION

Let

$$P\left(x_1, y_1\right)$$

and

$$Q\left(x_2, y_2\right), y_1 < 0, y_2 < 0,$$

be the end points of the latus rectum of the ellipse

$$x^2 + 4y^2 = 4.$$

The equations of parabolas with latus rectum

$$PQ$$

are :

A $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$

B $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

C $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$

D $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

CORRECT OPTION

B $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

SOURCE

Mathematics • ellipse

EXPLANATION

The ellipse is

$$x^2 + 4y^2 = 4$$

,

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

We have

$$b^2 - a^2(1 - e^2)$$

$$e = \frac{\sqrt{3}}{2}$$

P and Q are obtained as

$$P\left(\sqrt{3}, \frac{-1}{2}\right), Q = \left(-\sqrt{3}, \frac{-1}{2}\right)$$

We have

$$PQ = 2\sqrt{3}$$

With PQ as latus rectum two parabolas are possible their vertices being

$$\left(0, \frac{-\sqrt{3}-1}{2}\right)$$

and

$$\left(0, \frac{\sqrt{3}-1}{2}\right)$$

The equation of the parabola s can be obtained as

$$x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$$

and

$$x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$$

Question 042 MCQ

QUESTION

Let

$$f$$

and

$$g$$

be real valued functions defined on interval

$$(-1, 1)$$

such that

$$g''(x)$$

is continuous,

$$g(0) \neq 0.$$

$$g'(0) = 0$$

,

$$g''(0) \neq 0$$

, and

$$f(x) = g(x) \sin x$$

STATEMENT - 1:

$$\lim_{x \to 0} [g(x) \cot x - g(0) \cos ec\, x] = f''(0)$$

and

STATEMENT - 2:

$$f'(0) = g(0)$$

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B Statement - 1 is True, Statement - 2 is True; Statement - 2 is **NOT** a correct explanation for Statement - 1

C Statement - 1 is True, Statement -2 is False

D Statement - 1 is False, Statement -2 is True

CORRECT OPTION

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

SOURCE

Mathematics • differentiation

EXPLANATION

$$\lim_{x\to 0}\frac{g(x)\cos x - g(0)}{\sin x}$$

$$= \lim_{x\to 0}\frac{g'(x)\cos x - g(x)\sin x}{\cos x}$$

$Applying L - Hospital rule$

$$g'(0) - 0 = 0 = f''(0)$$

Statement - 1 is True.

$$f'(x) = g(x)\cos x + g'(x)\sin x$$

$$f'(0) = g(0)$$

Statement - 2 is True.

Question 043 MCQ

QUESTION

Let a and b be non-zero real numbers. Then, the equation

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$$

represents :

A four straight lines, when c = 0 and a, b are of the same sign

B two straight lines and a circle, when a = b, and c is of sign opposite to that of a

C two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

D a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

CORRECT OPTION

B two straight lines and a circle, when a = b, and c is of sign opposite to that of a

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

Let a and b be non-zero real numbers.

Therefore, the given equation

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$$

implies either

$$x^2 - 5xy + 6y^2 = 0$$

$$(x - 2y)(x - 3y) = 0$$

$$x = 2y$$

and

$$x = 3y$$

represent two straight line passing through origin or

$$ax^2 + by^2 + c = 0$$

when c = 0 and a and b are of same signs then

$$ax^2 + by^2 + c = 0$$

$$y = 0$$

Which is a point specified as the origin. When a = b and c is of sign opposite to that of a

$$ax^2 + by^2 + c = 0$$

represent a circle.

Hence, the given equation,

$$(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$$

May represent two straight lines and a circle.

Question 044 MCQ

QUESTION

Let

$$g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}; 0 < x < 2, m$$

and

$$n$$

are integers,

$$m \neq 0, n > 0$$

, and let

$$p$$

be the left hand derivative of

$$|x - 1|$$

at

$$x = 1$$

. If

$$\lim_{x \to 1^+} g(x) = p$$

, then

A $n = 1, m = 1$

B $n = 1, m = -1$

C $n = 2, m = 2$

D $n > 2, m = n$

CORRECT OPTION

C $n = 2, m = 2$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Given,

$$g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}, 0 < x < 2, m \neq 0, n > 0, m, n$$

are integer.

Left hand derivative $L.H.D.$ of

$$|x-1|$$

at

$$x = 1$$

is P.

As

$$|x-1| = \begin{cases} x-1, & x \geq 1 \\ -(1-x), & x < 1 \end{cases}$$

$$\therefore$$

L.H.D. at

$$x = 1$$

will be

$$-1$$

.

$$\$\$ \therefore \$\$L.H.D. at \$\$x = 1\$\$is\$\$ \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h} \$\$$$

$$p = -1$$

Also,

$$\lim_{x \to 1^+} g(x) = p = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{(1+h-1)^n}{\log \cos^m (1+h-1)} = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{h^n}{\log \cos^m h} = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{h^n}{m \log \cos\ h} = -1$$

$$\Rightarrow \lim_{h\to 0} \frac{n\,.\,h^{n-1}}{m \log \cos\ h} = -1$$

$$using L.Hospital rule$$

$$\Rightarrow \lim_{h\to 0} \left(\frac{-n}{m}\right) . \frac{h^{n-2}}{\left(\frac{\tan\ h}{h}\right)} = -1$$

$$\Rightarrow \left(\frac{n}{m}\right) \lim_{h\to 0} \frac{h^{n-2}}{\left(\frac{\tan\ h}{h}\right)} = 1$$

$$n-2=0$$

and

$$\frac{n}{m} = 1 \Rightarrow m = n = 2$$

or

$$[f(x) = |x-1| = -x+1, x < 1 f'(x) = -1$$

So, L.H.D. at

$$x = -1$$

is

$$-1$$

$$\therefore$$

$$P = -1$$

Now,

$$\lim_{x\to 1^+} \frac{(x-1)^n}{\log \cos^m (x-1)} = p$$

.... i

Let

$$x - 1 = t$$

in L.H.S. of eq. i

$$\lim_{t\to} \frac{t^n}{m \log \cos\, t}, \left(\frac{0}{0}\,\text{form}\right)$$

$$usingL'Hospitalrule$$

We get,

$$\lim_{t\to 0} \frac{nt^{n-1}}{m \tan\, t} = \frac{-n}{m} \lim_{t\to 0} \frac{t^{n-1}}{\tan\, t} \left(\frac{0}{0}\,\text{form}\right)$$

Again using L' Hospital rule,

$$\frac{-n}{m} \lim_{t\to 0} \frac{(n-1)t^{n-2}}{\sec^2 t} = P$$

$$\Rightarrow \frac{+n}{m} \lim_{t\to 0} \frac{(n-1)t^{n-2}}{\sec^2 t} = -1$$

For non-zero answer

$$n - 2 = 0 \Rightarrow n = 2$$

Also

$$m = n = 2$$

.

Question 045 MCQ

QUESTION

The total number of local maxima and local minima of the function

$$f(x)=\begin{cases}(2+x)^3, & -3<x\leq -1\\ x^{2/3}, & -1<x<2\end{cases}$$

is

A 0

B 1

C 2

D 3

CORRECT OPTION

C 2

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given that,

$$f(x)=\begin{cases}(2+x)^3, & -3<x\leq -1\\ x^{2/3}, & -1<x<2\end{cases}$$

$$f'(x)=\begin{cases}3(2+x)^2, & -3<x<-1\\ \frac{2}{3}x^{\frac{-1}{3}}, & -1<x<2\end{cases}$$

Clearly,

$$f'(x)$$

changes signs from positive to negative as x passes through x

$$= -1$$

$$f'(x) < 0$$

for all

$$x \in (-3, -1)$$

Also,

$$f'(x) > 0$$

for all

$$x \in$$

$$f'(x) < 0$$

for all

$$x \in (-1, 0)$$

But

$$f'(0)$$

does not exist

So,

$$f(x)$$

attains a local minimum at x = 0

Hence, the total number of local maxima and local minima is 2.

Question 046 MCQ

QUESTION

Consider the system of equations:

$$x - 2y + 3z = -1$$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1$$

Statement - 1 : The system of equations has no solution for

$$k \neq 3$$

.

and

Statement - 2 : The determinant

$$\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix} \neq 0$$

, for

$$k \neq 3$$

.

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C Statement - 1 is True, Statement - 2 is False

D Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

The given equations are

$$x - 2y + 3z = -1$$

$$-x + y - 2z = k$$

$$x - 3y + 4z = 1$$

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$$

$$D = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = k - 3 \neq 0$$

If

$$k \neq 3$$

, the system has no solutions.

Hence, Statement - 1 is True Statement - 2 is correct explanation for Statement - 1.

Question 047 MCQ

QUESTION

Student I, II and III perform an experiment for measuring the acceleration due to gravity g using a simple pendulum. They use different length of the pendulum and/or record time for different number of oscillations. The observations area shown in the table.

Least count for length = 0.1 cm

Least count for time = 0.1 s

Student	Length of the pendulum cm	No. of oscillations n	Total time for n oscillations s	Time periods s
I	64.0	8	128.0	16.0
II	64.0	4	64.0	16.0
III	20.0	4	36.0	9.0

If E_I, E_{II} and E_{III} are the percentage errors in g, i.e.,

$$\left(\frac{\triangle g}{g} \times 100\right)$$

for students I, II and III, respectively,then

A $E_I = 0$

B E_I is minimum

C $E_I = E_{II}$

D E_{II} is maximum

CORRECT OPTION

B E_I is minimum

SOURCE

Physics • units-and-measurements

EXPLANATION

Time period T

$$= 2\pi\sqrt{\frac{l}{g}}$$

or

$$\frac{t}{n} = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore$$

$$g = \frac{(4\pi^2)(n^2)l}{t^2}$$

% error in

$$g = \frac{\Delta g}{g} \times 100$$

$$= \left(\frac{\Delta l}{l} \times \frac{2\Delta l}{l}\right) \times 100$$

$$E_I = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{128}\right) \times 100 = 0.3125\%$$

$$E_{II} = \left(\frac{0.1}{64} + \frac{2 \times 0.1}{64}\right) \times 100 = 0.46875\%$$

$$E_{III} = \left(\frac{0.1}{20} + \frac{2 \times 0.1}{36}\right) \times 100 = 1.055\%$$

Hence,

E_I

is minimum.

Question 048 MCQ

QUESTION

Figure shows three resistor configurations R1, R2 and R3 connected to 3 V battery. If the power dissipated by the configuration R1, R2 and R3 is P1, P2 and P3, respectively, then

A P1 > P2 > P3

B P1 > P3 > P2

C P2 > P1 > P3

D P3 > P2 > P1

CORRECT OPTION

C P2 > P1 > P3

SOURCE

Physics • current-electricity

EXPLANATION

We know that

$$P = \frac{V^2}{R}$$

If potential is constant we have

$$P \propto \frac{1}{R}$$

Case I

Hence, this is a clear case of wheat stone bridge R

1

= 1

Ω

Case II

Equivalent resistance $R\$\$_2\$\$$

$\therefore$

$$\frac{1}{R_2} = \frac{1}{2} + \frac{1}{1} + \frac{1}{2} = \frac{1+2+1}{2} = \frac{4}{2} = 2\,\Omega$$

Hence,

$$R_2 = \frac{1}{2} = 0.5\,\Omega$$

It is clear that the equivalent resistance $R\$\$_2\$\$$ will be less than 1

Ω

.

Case III

Hence, R

3

= 2

Ω

Since, R

2

< R

1

< R

2

$\therefore$

P

2

> P

1

> P

3

$$\because P \propto \frac{1}{R}$$

Question 049 MCQ

QUESTION

Which one of the following statements is WRONG in the context of X-rays generated from a X-ray tube?

A Wavelength of characteristic X-rays decreases when the atomic number of the target increases.

B Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

C Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.

D Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

CORRECT OPTION

B Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

The cut-off wavelength is given by

$$\lambda_{\min} = \frac{hc}{eV}$$

The cut-off wavelength depends on the energy eV of the accelerated electrons and is independent of the atomic number of target. Thus, greater the accelerating voltage for electrons, higher will be kinetic energy it attains before striking the target, higher will be the frequency of X-rays and smaller will be wavelength.

Question 050 MCQ

QUESTION

Two beams of red and violet colours are made to pass separately through a prism $angleofthepriismis60$$^\circ$$$. In the position of minimum deviation, the angle of refraction will be :

A 30 ° for both the colours

B greater for the violet colour

C greater for the red colour

D equal but not 30 ° for both the colours

CORRECT OPTION

A 30 ° for both the colours

SOURCE

Physics • geometrical-optics

EXPLANATION

For minimum deviation, the ray in the prism is parallel to base of the prism. This condition does not depend on the colour (i.e., wavelength of incident radiation.

So, both the cases, by geometry,

$$r = 30^{\circ}$$

.

Question 051 MCQ

QUESTION

An ideal gas is expanding such that PT

$$2$$

= constant. The coefficient of volume expansion of the gas is

A $\frac{1}{T}$

B $\frac{2}{T}$

C $\frac{3}{T}$

D $\frac{4}{T}$

CORRECT OPTION

C $\frac{3}{T}$

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Given, PT

2

= constant i

For an ideal gas,

$$\frac{PV}{T}$$

= constant ii

From above two equation, after eliminating P.

$$\frac{V}{T^3}$$

= Constant

$\Rightarrow$

V = KT

3

, where

k

= constant.

$$\frac{dV}{V} = 3\frac{dT}{T}$$

$$\Rightarrow dV = \left(\frac{3}{T}\right)VdT$$

..... iii

Change in volume due to thermal expansion is given by

$$dV = Vydt$$

..... iv

Where,

$$y$$

= coefficient of volume expansion

From equation iii and iv, we have,

$$VydT = \left(\frac{3}{T}\right)VdT$$

$$\Rightarrow y = \frac{3}{T}$$

Question 052 MCQ

QUESTION

A spherically symmetric gravitational system of particles has a mass density

$$\rho = \begin{cases} \rho_0 & for \quad r \leq R \\ 0 & for \quad r > R \end{cases}$$

Where

$$\rho_0$$

is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance

$$r(0 < r < \infty)$$

from the centre of the system is represented by

A

B

C

D

CORRECT OPTION

C

SOURCE

Physics • gravitation

EXPLANATION

For

$$r \geq R$$

,

Force on test mass m is

$$F = m \times |E_g|$$

where,

$$E_g$$

= Gravitational field intensity at the point of observation

$$\therefore$$

$$\frac{mv^2}{r} = m \times \left[\frac{GM}{r^2}\right]$$

,

Where, M = Total mass of spherical system

$$\therefore$$

$$v \propto \frac{1}{\sqrt{r}}$$

For

$$r < R$$

,

$$F' = m|E'_g|$$

$$\therefore$$

$$\frac{mV^2}{r} = m\left[\frac{GM}{R^3} \times r\right]$$

Hence,

$$V \propto r$$

Question 053 MCQ

QUESTION

Two balls, having linear momenta

$$\overrightarrow{p_1} = p\hat{i}$$

and

$$\overrightarrow{p_2} = -p\hat{i}$$

, undergo a collision in free space. There is no external force acting on the balls. Let

$$\overrightarrow{p'}_1$$

and

$$\overrightarrow{p'}_2$$

be their final momenta. The following option s is are **NOT ALLOWED** for any non-zero value of

$$p, a_1, a_2, b_1, b_2, c_1$$

and

$$c_2$$

:

A $\overrightarrow{p'}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}; \overrightarrow{p'}_2 = a_2\hat{i} + b_2\hat{j}$

B $\overrightarrow{p'}_1 = c_1\hat{k}; \overrightarrow{p'}_2 = c_2\hat{k}$

C $\overrightarrow{p'}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}; \overrightarrow{p'}_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

D $\overrightarrow{p'}_1 = a_1\hat{i} + b_1\hat{j}; \overrightarrow{p'}_2 = a_2\hat{i} + b_1\hat{j}$

CORRECT OPTION

A

$$\overrightarrow{p'}_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}; \overrightarrow{p'}_2 = a_2\hat{i} + b_2\hat{j}$$

SOURCE

Physics • impulse-and-momentum

EXPLANATION

In free space, there is no external force. Hence linear momentum of the system is conserved. Initial and final linear momentum of the system are

$$\vec{p}_i = \vec{p}_1 + \vec{p}_2 = \hat{p} - \hat{i}\hat{i} = \overrightarrow{0},$$

$$\vec{p}_f = \vec{p}'_1 + \vec{p}'_2.$$

The conservation of linear momentum, $\vec{p}_i = \vec{p}_f$ gives

$$\vec{p}'_1 + \vec{p}'_2 = \overrightarrow{0}.$$

In case (A),

$$\vec{p}'_1 + \vec{p}'_2 = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$$

$$\neq \overrightarrow{0}, \quad (\because c_1 \neq 0).$$

In case (B),

$$\vec{p}_1' + \vec{p}_2' = (c_1 + c_2)\hat{k}$$

$$= \vec{0}, \quad (\text{ if } c_2 = -c_1).$$

In case (C),

$$\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{\imath} + (b_1 + b_2)\hat{\jmath}$$

$$= \vec{0}, \quad (\text{ if } a_2 = -a_1 \text{ and } b_2 = -b_1).$$

In case D,

$$\vec{p}_1' + \vec{p}_2' = (a_1 + a_2)\hat{\imath} + 2b_1\hat{\jmath}$$

$$\neq \vec{0}, \quad (\because b_1 \neq 0).$$

Question 054 MCQ

QUESTION

Assume that the nuclear binding energy per nucleon B/A versus mass number A is as shown in the figure. Use this plot to choose the correct choice s given below.

A Fusion of two nuclei with mass number lying in the range of 1 < A < 50 will release energy

B Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy

C Fission of a nucleus lying in the mass range of 100 < A < 200 will release energy when broken into two equal fragments

D Fission of a nucleus lying in the mass range of 200 < A < 260 will release energy when broken into two equal fragments

CORRECT OPTION

B Fusion of two nuclei with mass numbers lying in the range of 51 < A < 100 will release energy

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

When binding energy per nucleon increases for a nuclear process, energy is released.

a For 1 < A < 50, on fusion, mass number of the resulting nucleus will be less than 100. No energy will be released.

b For 51 < A < 100, on fusion, mass number of the resulting nucleus will be between 100 and 200. As BE increases, energy will be released.

c On fission for 100 < A < 200, the mass number for fission nuclei will be between 50 and 100. As BE decreases, no energy will be released.

d On fission for 200 < A < 260, the mass number for the fission nuclei will be between 100 and 130. Since BE will increase, energy will be released.

Question 055 MCQ

QUESTION

A particle of mass m and charge q, moving with velocity v enters Region II normal to the boundary as shown in the figure. Region II has a uniform magnetic field B perpendicular to the plane of the paper. The length of the Region II is

$$l$$

. Choose the correct choice s.

A The particle enters Region III only if its velocity

$$v > \frac{qlB}{m}$$

B The particle enters Region III only if its velocity

$$v < \frac{qlB}{m}$$

C Path length of the particle in Region II is maximum when velocity

$$v = \frac{qlB}{m}$$

D Time spent in Region II is same for any velocity v as long as the particle returns to Region I

CORRECT OPTION

A The particle enters Region III only if its velocity

$$v > \frac{qlB}{m}$$

SOURCE

Physics • magnetism

EXPLANATION

As the particle enters the magnetic field, a force acts on it due to the magnetic field which moves the particle in a circular path of radius.

$$r = \frac{mv}{qB}$$

For the particle entering in region III, r >

$$l$$

$$\Rightarrow \frac{mv}{qB} > l$$

$$\Rightarrow v > qlB/m$$

For maximum path length in region II, r =

$$l$$

$$\therefore$$

$$l = \frac{mv}{qB}$$

$$\Rightarrow v = \frac{qlB}{m}$$

$$\$\$ \because \$\$\$\$v_{\max} = \frac{qBl}{m}, T = \frac{2\pi r}{v} = \frac{2\pi m}{q.B}\$\$$$

The time taken by the particle to move in region II before coming back in region I is given by

$$t = \pi m$$

, which is independent of v.

Question 056 MCQ

QUESTION

In a Young's double slit experiment, the separation between the two slits is d and the wavelength of the light is

$$\lambda$$

. The intensity of light falling on slit 1 is four times the intensity of light falling on slit 2. Choose the correct choice s.

A If

$$d = \lambda$$

, the screen will contain only one maximum

B If

$$\lambda < d < 2\lambda$$

, at least one more maximum $besidesthecentralmaximum$ will be observed on the screen

C If the intensity of light falling on slit 1 is reduced so that it becomes equal to that of slit 2, the intensities of the observed dark and bright fringes will increase

D If the intensity of light falling on slit 2 is increased so that it becomes equal to that of slit 1, the intensities of the observed dark and bright fringes will increase

CORRECT OPTION

If

A $$d = \lambda$$

, the screen will contain only one maximum

SOURCE

Physics • wave-optics

EXPLANATION

If

$$d = \lambda$$

, the maximum path difference $path difference is given by$$d \sin \theta$$ will be less than

$$\lambda$$

. So there will be only central maximum on the screen, because in the equation

$$d \sin \theta = n\lambda, n$$

can take only one value.

If

$$\lambda < d < 2\lambda$$

, then the maximum path difference will be less than 2

$$\lambda$$

. So there will be two more maximum on the screen in addition to the central maximum.

Intensity of dark fringes becomes zero when intensities at the two slits are equal. Initial intensity at both the slits is unequal so there will some brightness at dark

fringes. Hence, when intensity of both slits is made equal, the intensity at dark fringes on screen will reduces to zero.

Question 057 MCQ

QUESTION

STATEMENT - 1

In a Meter Bridge experiment, null point for an unknown resistance is measured. Now, the unknown resistance is put inside an enclosure maintained at a higher temperature. The null point can be obtained at the same point as before by decreasing the value of the standard resistance.

and

STATEMENT - 2

Resistance of a metal increases with increase in temperature.

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B. Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C. Statement - 1 is True, Statement - 2 is False

D. Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

D Statement - 1 is False, Statement - 2 is True

SOURCE

Physics • current-electricity

EXPLANATION

When the temperature of a metal increases, its resistance will increase.

Therefore statement-2 is correct.

For meter bridge, when null point N is obtained, we get

$$\frac{X}{l} = \frac{R}{100 - l}$$

When the unknown resistance is put inside an enclosure, maintained at a high temperature, then X increases. To maintain the ratio of null point,

$$l$$

should also increase. But if we want to keep the null point at the initial position $i.e., if we want to change the value of $$l$$ to maintain the ratio, R should be increased.

Therefore, statement - 1 is false.

Question 058 MCQ

QUESTION

STATEMENT - 1

An astronaut in an orbiting space station above the Earth experiences weightlessness.

and

STATEMENT - 2

An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C Statement - 1 is True, Statement - 2 is False

D Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

A Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

SOURCE

Physics • gravitation

EXPLANATION

1st Method : The normal force exerted by the astronaut on the orbiting space station is zero. Therefore, the apparent weight of astronaut in the orbiting space station is zero. Astronaut is called in a state of weightlessness. Why because astronaut as well as space ship are free falling bodies. Statement - I is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - 1.

2nd method : For the body to follow circular path, there must be a centripetal force. Here the astronaut is inside the satellites which is revolving around the

earth under the influence of the earth's gravitation. Thus, the earth's gravitation acts as a centripetal force and the net force on the astronaut is zero.

Statement - 2 is right explanation of 1.

Question 059 MCQ

QUESTION

STATEMENT - 1 :

Two cylinders, one hollow $metal$ and the other solid $wood$ with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

STATEMENT - 2 :

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B. Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C. Statement - 1 is True, Statement - 2 is False

D. Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

D Statement - 1 is False, Statement - 2 is True

SOURCE

Physics • rotational-motion

EXPLANATION

The acceleration of a body rolling down an inclined plane in given by

$$a = \frac{g \sin \theta}{1 + \frac{I}{MR^2}}$$

For hollow cylinder,

$$\frac{I}{MR^2} = \frac{MR^2}{MR^2} = 1$$

For solid cylinder,

$$\frac{1}{MR^2} = \frac{\frac{1}{2}MR^2}{MR^2} = \frac{1}{2}$$

$$\therefore$$

Acceleration of the solid cylinder is more than that of hollow cylinder and therefore solid cylinder will reach the bottom of the inclined plane first.

$$\therefore$$

Statement - 1 is False.

Statement - 2 : In case of rolling, there will be no heat losses. Therefore, total mechanical energy remains conserved. The potential energy, therefore gets converted into kinetic energy. In both the cases, since the initial potential energy is same, the final kinetic energy will also be same. Therefore, statement - 2 is correct.

Question 060 MCQ

QUESTION

STATEMENT - 1 :

The stream of water flowing at high speed from a garden hose pipe tends to spread line a fountain when held vertically up, but tends to narrow down when held vertically down.

and

STATEMENT - 2 :

In any steady flow of an incompressible fluid, the volume flow rate of the fluid remains constant.

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

B. Statement - 1 is True, Statement - 2 is True; Statement - 2 is NOT a correct explanation for Statement - 1

C. Statement - 1 is True, Statement - 2 is False

D. Statement - 1 is False, Statement - 2 is True

CORRECT OPTION

A. Statement - 1 is True, Statement - 2 is True; Statement - 2 is a correct explanation for Statement - 1

SOURCE

Physics • properties-of-matter

EXPLANATION

Volume flow rate V of an incompressible fluid in the steady flow remains constant.

$$V = a \times v$$

Where

$$a$$

= area of cross-section and

$$v$$

= velocity

If '

$$v$$

' decreases, '

$$a$$

' increases and vice-versa.

When steam of water moves up, its speed v decreases and therefore 'a' increases, i.e., the water spreads out like a fountain. When steam of water from hose pipe moves down, its speed increases and as a result, area of cross-section decreases.

Therefore, statement - 1 is true.

Statement - 2 is the correct explanation of statement - 1.

Question 061 MCQ

QUESTION

As the bubble moves upwards, besides the buoyancy force the following forces are acting on it

A Only the force of gravity

B The force due to gravity and the force due to the pressure of the liquid

C The force due to gravity, the force due to the pressure of the liquid and the force due to viscosity of the liquid

D The force due to gravity and the force due to viscosity of the liquid

CORRECT OPTION

D The force due to gravity and the force due to viscosity of the liquid

SOURCE

Physics • properties-of-matter

EXPLANATION

As the bubble moves upwards, besides the buoyancy force $thecauseofwhichispressuredifference$, only the force of gravity and the force of viscosity will act.

Question 062 MCQ

QUESTION

When the gas bubble is at a height y from the bottom, its temperature is :

A $$T_0\left(\frac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{\frac{2}{5}}$$

B $$T_0\left(\frac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{\frac{2}{5}}$$

C $$T_0\left(\frac{P_0+\rho_l gH}{P_0+\rho_l gy}\right)^{\frac{3}{5}}$$

D $$T_0\left(\frac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{\frac{3}{5}}$$

CORRECT OPTION

B $$T_0\left(\frac{P_0+\rho_l g(H-y)}{P_0+\rho_l gH}\right)^{\frac{2}{5}}$$

SOURCE

Physics • properties-of-matter

EXPLANATION

Since the process is adiabatic,

$$PV^{\gamma}$$

= constant for gas inside bubble.

Thus,

$$PV^{(1-\gamma)}.\ T^{\gamma} =$$

constant

$$\Rightarrow P^{1-\gamma}{}_{bottom} T^{\gamma}{}_{bottom} = P_y^{1-\gamma} T_y^{\gamma} =$$

constant

$$\Rightarrow (P_0 + \rho_1 g H)^{-2/3} (T_0^{5/3}) = (P_0 + \rho_1 g (H-y)^{-2/3}) T^{5/3}$$

$$\Rightarrow T_0 = \left[\frac{P_0 + \rho_1 g (H-y)}{P_0 + \rho_1 g H}\right]^{2/5}$$

Question 063 MCQ

QUESTION

The buoyancy force acting on the gas bubble is
$Assume R is the universal gas constant$

A

$$\rho_l n R g T_0 \frac{(P_0 + \rho_l g H)^{\frac{2}{5}}}{(P_0 + \rho_l g y)^{\frac{7}{5}}}$$

B

$$\frac{\rho_l n R g T_0}{(P_0 + \rho_l g H)^{\frac{2}{5}} [P_0 + \rho_l g (H-y)]^{\frac{3}{5}}}$$

C

$$\rho_l nRgT_0 \frac{(P_0+\rho_l gH)^{\frac{3}{5}}}{(P_0+\rho_l gy)^{\frac{8}{5}}}$$

D

$$\frac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{\frac{3}{5}}[P_0+\rho_l g(H-y)^{\frac{2}{5}}}$$

CORRECT OPTION

B

$$\frac{\rho_l nRgT_0}{(P_0+\rho_l gH)^{\frac{2}{5}}[P_0+\rho_l g(H-y)]^{\frac{3}{5}}}$$

SOURCE

Physics • properties-of-matter

EXPLANATION

Buoyancy force = Weight of fluid displaced

= $mass of fluid displaced$ g

$$= V\rho_l g$$

.... i

Where V = Volume of fluid displaced

= Volume of the bubble

$$PV = nRT$$

$$\Rightarrow V = \frac{nRT}{P} = \frac{nRT}{P_0(H-y)\rho_l g}$$

.... ii

Where P is the pressure of the bubble at an arbitrary location at a distance 'y' from the bottom.

Put the value of temperature from eq. i

$$V = \frac{nR}{[P_0 + (H-y)\rho_l g]} \times \frac{T_0[P_0 + (H-y)\rho_l g]^{2/5}}{[P_0 + H\rho_l g]^{2/5}}$$

$$= \frac{nRT_0}{[P_0 + (H-y)\rho_l g]^{3/5}[P_0 + H\rho_l g]^{2/5}}$$

.... iii

From eq. i and iii Buoyance force

$$= \frac{nRT_0\rho_1 g}{[P_0 + (H-y)\rho_l g]^{3/5}[P_0 + H\rho_l g]^{2/5}}$$

Question 064 MCQ

QUESTION

The quantum number n of the state finally populated in He

+

ions is :

A 2

B 3

C 4

D 5

CORRECT OPTION

C 4

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

For H atom,

$$E_1 = -\frac{13.6}{1^2} = -13.6\,eV$$

$$E_2 = -\frac{13.6}{2^2} = -3.4\,eV$$

Energy released by H atom = E

$$2$$

$$-$$

E

$$1$$

$$= -3.4 - (-13.6) = 10.2\,eV$$

This energy will be absorbed by He atom. Thus, for He atom

$$10.2 = -13.6 \times 2^2 \left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$0.1875 = \frac{1}{4} - \frac{1}{n^2}$$

$$n^2 = 16$$

$$n = 4$$

Question 065 MCQ

QUESTION

The wavelength of light emitted in the visible region by He

+

ions after collisions with H atoms is

A 6.5×10^{-7} m

B 5.6×10^{-7} m

C 4.8×10^{-7} m

D 4.0×10^{-7} m

CORRECT OPTION

C 4.8×10^{-7} m

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

$$\frac{hc}{\lambda} = E_4 - E_3$$

$$\frac{hc}{\lambda} = -13.68 \times 2^2 \left(\frac{1}{4^2} - \frac{1}{3^2}\right)$$

$$\frac{hc}{\lambda} = 13.6 \times 2^2 \left(\frac{1}{9} - \frac{1}{16}\right)$$

$$\frac{4.1356 \times 15^{-15}\, eV.s \times 3 \times 10^8\, ms^{-1}}{1} = 2.644$$

$$\Rightarrow \lambda \cong 4.8 * 10^{-7}\, m$$

Question 066 MCQ

QUESTION

The ratio of the kinetic energy of the

$$n = 2$$

electron for the H atom to that of He

$$+$$

ion is

A $\frac{1}{4}$

B $\frac{1}{2}$

C 1

D 2

CORRECT OPTION

A $\frac{1}{4}$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To determine the ratio of the kinetic energy of the

$$n = 2$$

electron for the H atom to that of the He

$$+$$

ion, we need to consider the Bohr model of the atom. According to the Bohr model, the kinetic energy KE of an electron in a hydrogen-like ion in the nth orbit can be given by:

$$KE_n = \frac{1}{2}mv_n^2 = \frac{Z^2e^4m}{8\epsilon_0^2h^2n^2}$$

where:

- m is the mass of the electron
- Z is the atomic number of the nucleus

- e is the charge of the electron
- ϵ_0 is the permittivity of free space
- h is Planck's constant
- n is the principle quantum number, which is 2 in this case

For a hydrogen atom H $\$Z = 1\$$ in the $n = 2$ state:

$$KE_{H(n=2)} = \frac{1^2 \cdot e^4 m}{8\epsilon_0^2 h^2 \cdot 2^2} = \frac{e^4 m}{32\epsilon_0^2 h^2}$$

For a singly ionized helium ion $He\$\$^+\$\$$ $\$Z = 2\$$ in the $n = 2$ state:

$$KE_{He^+(n=2)} = \frac{2^2 \cdot e^4 m}{8\epsilon_0^2 h^2 \cdot 2^2} = \frac{4e^4 m}{32\epsilon_0^2 h^2} = \frac{e^4 m}{8\epsilon_0^2 h^2}$$

To find the ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of the He

+

ion, we divide the kinetic energies:

$$\text{Ratio} = \frac{KE_{H(n=2)}}{KE_{He^+(n=2)}} = \frac{\frac{e^4 m}{32\epsilon_0^2 h^2}}{\frac{e^4 m}{8\epsilon_0^2 h^2}} = \frac{1}{4}$$

Therefore, the ratio of the kinetic energy of the $n = 2$ electron for the H atom to that of the He

+

ion is:

Option A:

$$\frac{1}{4}$$

Question 067 MCQ

QUESTION

The speed of the block at point B immediately after it strikes the second incline is

A $\sqrt{60}$ m/s

B $\sqrt{45}$ m/s

C $\sqrt{30}$ m/s

D $\sqrt{75}$ m/s

CORRECT OPTION

B $\sqrt{45}$ m/s

SOURCE

Physics • laws-of-motion

EXPLANATION

Velocity along the plane just before collision,

$$v^2 - u^2 = 2gs \Rightarrow v = \sqrt{2g \times 3} = \sqrt{60}\, m/s$$

Velocity along the plane just after collision

$$v_B - v\cos 30 = \sqrt{45}\, ms^{-1}$$

Question 068 MCQ

QUESTION

The speed of the block at point C, immediately before it leaves the second incline is

A $\sqrt{120}$ m/s

B $\sqrt{105}$ m/s

C $\sqrt{90}$ m/s

D $\sqrt{75}$ m/s

CORRECT OPTION

B $\sqrt{105}$ m/s

SOURCE

Physics • laws-of-motion

EXPLANATION

$$v_C^2 - v_B^2 = 2as$$

$$v_C^2 - 45 = 2 \times 10 \times 3$$

$$v_c = \sqrt{60 + 45} = \sqrt{105}\, ms^{-1}$$

is the velocity of block just before leaving incline.

Question 069 MCQ

QUESTION

If collision between the block and the incline is completely elastic, then the vertical $upward$ component of the velocity of the block at point B, immediately after it strikes the second incline is

A $\sqrt{30}$ m/s

B $\sqrt{15}$ m/s

C 0

D $-\sqrt{15}$ m/s

CORRECT OPTION

C 0

SOURCE

Physics • impulse-and-momentum

EXPLANATION

$$v_V = v \sin 30^\circ \cos 30^\circ - v \cos 30^\circ \cos 60^\circ = 0$$