iit Jee 2009 Paper 2 Offline $57 Questions$

Question 001 MCQ

QUESTION

For a first-order reaction A

$$\rightarrow$$

P, the temperature T dependent rate constant k was found to follow the equation

$$\log k = -(2000)\frac{1}{T} + 6.0$$

. The pre-exponential factor A and activation energy

$$E_a$$

, respectively, are

$$1.0 \times 10^6 \text{ s}^{-1}$$

A and 9.2 kJ mol

$$-1$$

$$6.0 \text{ s}^{-1}$$

B and 16.6 kJ mol

$$-1$$

$$1.0 \times 10^6 \text{ s}^{-1}$$

C and 16.6 kJ mol

$$-1$$

$$1.0 \times 10^6 \text{ s}^{-1}$$

D and 38.3 kJ mol

$$-1$$

CORRECT OPTION

$$1.0 \times 10^6 \text{ s}^{-1}$$

D and 38.3 kJ mol

$$-1$$

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

Given that

$$\log k = 6 - \frac{2000}{T}$$

According to Arrhenius equation

$$k = Ae^{-E_a/RT}$$

$$\log K = \log A - \frac{E}{2.303RT}$$

Comparing with the given equation and solving, we get

$$A = 1.0 \times 10^{6}\, s^{-1}$$

$$E_a = 38.3$$

kJ/mol

Question 002 MCQ

QUESTION

The spin only magnetic moment value $in Bohr magneton units$ of $Cr(CO)_6$ is

A 0

B 2.84

C 4.90

D 5.92

CORRECT OPTION

A 0

SOURCE

Chemistry • coordination-compounds

EXPLANATION

In

$$Cr(CO)\$\$_6\$\$$$

: Cr24 =

$$Ar$$

3

$$d^5$$

4

$$s^1$$

Since CO is a strong field ligand, so pairing of electrons will take place and the configuration will become

$$3d^6$$

. There will be no unpaired electrons, so, the spin only magnetic moment is zero.

Question 003 MCQ

QUESTION

In the following carbocation, H/CH

$$3$$

that is most likely to migrate to the positively charged carbon is :

A CH

$$3$$

at C-4

B H at C-4

C CH$_3$ at C-2

D H at C-2

CORRECT OPTION

D H at C-2

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The hydride shift from C-2 will lead to a resonance stabilized secondary carbocation $conjugatedwithoxygen$.

Question 004 MCQ

QUESTION

The correct stability order of the following resonance structures is

A $I > II > IV > III$

B $I > III > II > IV$

C $II > I > III > IV$

D $III > I > IV > II$

CORRECT OPTION

B $I > III > II > IV$

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Among the resonance structures, the more stable ones are those with:

i more number of covalent bonds

ii all the atoms having octet of electrons complete,

iii less separation of opposite charges and more dispersal of charge.

Based on these, the order of stability of the resonance structures is $I > III > II > IV$.

Question 005 MCQ

QUESTION

For the reduction of NO

$$_{3}^{-}$$

ion in an aqueous solution, E

^{0}

is + 0.96 V. Values of E

^{0}

for some metal ions are given below:

$$V^{2+}(aq.) + 2e^- \rightarrow V \quad E^0 = -1.19\,V$$
$$Fe^{3+}(aq.) + 3e^- \rightarrow Fe \quad E^0 = -0.04\,V$$
$$Au^{3+}(aq) + 3e^- \rightarrow Au \quad E^0 = +1.40\,V$$
$$Hg^{2+}(aq) + 2e^- \rightarrow Hg \quad E^0 = +0.86\,V$$

The pair s of metals that is are oxidized by NO

$$_{3}^{-}$$

in aqueous solution is are

A V and Hg

B Hg and Fe

C Fe and Au

D Fe and V

CORRECT OPTION

A V and Hg

SOURCE

Chemistry • electrochemistry

EXPLANATION

The general criterion for a spontaneous redox reaction is that the reduction potential of the reducing agent $which gets oxidized$ must be lower than the reduction potential of the oxidizing agent $which gets reduced$. Here, the reduction of NO_3^- has an E^0 of +0.96 V. For a metal ion to be oxidized by NO_3^-, the E^0 of the metal ion's reduction must be less than +0.96 V.

Let's check each given ion to see if it can be oxidized:

- **V^{2+} (E^0 = -1.19 V)**: This potential is significantly lower than +0.96 V, thus vanadium can be oxidized by nitrate, as it is much easier to reduce NO_3^- than to reduce V^{2+} to V.
- **Fe^{3+} (E^0 = -0.04 V)**: This potential is also lower than +0.96 V, so iron can be oxidized by nitrate.
- **Au^{3+} (E^0 = +1.40 V)**: Since the potential for Au^{3+} is higher than the potential for nitrate reduction, gold cannot be oxidized by nitrate. It means nitrate cannot provide sufficient potential to reduce Au^{3+}.
- **Hg^{2+} (E^0 = +0.86 V)**: This potential is close but still lower than +0.96 V, therefore mercury can theoretically be oxidized by nitrate, though it is only slightly easier to reduce NO_3^- than to reduce Hg^{2+}.

Given this analysis:

- **Option A** $VandHg$: Correct, as both V and Hg have lower reduction potentials than the nitrate ion.
- **Option B** $HgandFe$: Correct, as analyzed above, both can indeed be oxidized by nitrate.
- **Option C** $FeandAu$: Incorrect because Au^{3+} has a higher potential and thus cannot be oxidized by nitrate.
- **Option D** $FeandV$: Correct, given that both V and Fe have lower potentials than nitrate reduction.

The correct answer includes **Option A**, **Option B**, and **Option D**.

Question 006 MCQ

QUESTION

Among the following, the state function s is are

A Internal energy.

B Irreversible expansion work.

C Reversible expansion work.

D Molar enthalpy.

CORRECT OPTION

A Internal energy.

SOURCE

Chemistry • thermodynamics

EXPLANATION

State functions are properties of a system that depend only on the current state of the system, not on the path used to get to that state. They are intrinsic properties of the system and include properties like pressure, temperature, volume, internal energy, enthalpy, entropy, and Gibbs free energy.

Option A: Internal Energy

Internal energy, denoted as

$$U$$

, is a state function. It is the total energy contained within the system, including kinetic and potential energy at the molecular level. Internal energy changes in a system only depend on the initial and final states of the system, regardless of the process or path taken to achieve these states. Therefore, it fits the criteria of a state function.

Option B: Irreversible Expansion Work

Irreversible expansion work is not a state function. Work, in general, is a path function because it depends on the path taken during a process. Whether the path involves irreversible or reversible processes, work includes energy transfer that depends significantly on how that transfer is carried out. Therefore, irreversible expansion work depends on the specific details of the process and is not solely determined by the initial and final states of the system.

Option C: Reversible Expansion Work

Similarly to irreversible expansion work, reversible expansion work is also a path function and not a state function. Even though reversible processes are ideal and involve quasi-static changes that maintain the system in near equilibrium throughout, the work done $orreversibleexpansionwork$ during such processes still depends on the specific path taken. This includes how slowly the process is carried out and the intermediate steps, distinguishing it from a state function.

Option D: Molar Enthalpy

Molar enthalpy, denoted as

$$H$$

, is a state function. It is defined as the sum of the internal energy

$$U$$

of a system plus the product of the pressure

$$P$$

and volume

$$V$$

of the system, multiplied by the number of moles

$$n$$

, expressed as:

$$H = U + PV$$

Since

$$U$$

,

$$P$$

, and

$$V$$

are all state functions, their combination into enthalpy continues to be dependent solely on the state of the system, not on the path taken to reach that state.

Conclusion:

Among the given options, **Internal energy** $OptionA$ and **Molar enthalpy** $OptionD$ are state functions, while both **Irreversible expansion work** $OptionB$ and **Reversible expansion work** $OptionC$ are not state functions. They are path functions.

Question 007 MCQ

QUESTION

In the reaction

$$2\mathrm{X} + \mathrm{B_2H_6} \rightarrow [\mathrm{BH_2(X)_2}]^+[\mathrm{BH_4}]^-$$

the amine s X is are :

A NH

3

B CH

3

NH

2

C $CH$$$_3$$$

2

NH

D $CH$$$_3$$$

3

N

CORRECT OPTION

A NH

3

SOURCE

Chemistry • p-block-elements

EXPLANATION

Tertiary amine will not react due to the bulkiness of trimethyl groups. Tertiary amines instead react with diborane to give simple addition compounds through

symmetrical cleavage of diborane.

Question 008 MCQ

QUESTION

The nitrogen oxide s that contain s N-N bond s is are

A N_2O

B N_2O_3

C N_2O_4

D N_2O_5

CORRECT OPTION

A N_2O

SOURCE

Chemistry • p-block-elements

EXPLANATION

The bond formation in the given oxides of nitrogen is as follows.

Thus N-N bond exists in N_2O, N_2O_3 and N_2O_4.

Question 009 MCQ

QUESTION

The correct statement s about the following sugar X and Y is are

A X is a reducing sugar and Y is a non-reducing sugar.

B X is a non-reducing sugar and Y is a reducing sugar.

C The glucosidic linkages in X and Y are α and β, respectively.

D The glucosidic linkages in X and Y are β and α, respectively.

CORRECT OPTION

B X is a non-reducing sugar and Y is a reducing sugar.

SOURCE

Chemistry • biomolecules

EXPLANATION

Reducing sugars contain cyclic hemiacetal or hemiketal groups in equilibrium with the open chain from having a free

—

CHO or

—

C = O group. Non-reducing sugars contain stable acetal or ketal structures their cyclic structures cannot be opened into an open chain form having a free carbonyl group. In X the glyco-sidic linkage is in between two anomeric carbon atoms while in Y it is only with one anomeric carbon, the other one is free. So, X is non-reducing while Y is reducing.

Question 010 MCQ

QUESTION

Match each of the reactions given in Column I with the corresponding product s given in Column II:

	Column I		Column II
A	$\mathrm{Cu} + \text{dil. } \mathrm{HNO_3}$	P	NO
B	$\mathrm{Cu} + \text{conc. } \mathrm{HNO_3}$	Q	$\mathrm{NO_2}$
C	$\mathrm{Zn} + \text{dil. } \mathrm{HNO_3}$	R	$\mathrm{N_2O}$
D	$\mathrm{Zn} + \text{conc. } \mathrm{HNO_3}$	S	$\mathrm{Cu(NO_3)_2}$
		T	$\mathrm{Zn(NO_3)_2}$

A

$\rightarrow$

$P, S; B$

$\rightarrow$

A $R, S; C$

$\rightarrow$

$R, T; D$

$\rightarrow$

R, T

A

$\rightarrow$

$T, S; B$

$\rightarrow$

B $P, S; C$

$\rightarrow$

$R, T; D$

$\rightarrow$

S, T

A

$\rightarrow$

$P, S; B$

$\rightarrow$

C $Q, S; C$

$\rightarrow$

$R, T; D$

$\rightarrow$

Q, T

A

$\rightarrow$

$R, S; B$

$\rightarrow$

D $Q, S; C$

$\rightarrow$

$R, T; D$

$\rightarrow$

S, T

CORRECT OPTION

A

$\rightarrow$

$P, S; B$

$\rightarrow$

C $Q, S; C$

$\rightarrow$

$R, T; D$

$\rightarrow$

Q, T

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reactions are as follows:

A 3Cu + 8HNO

3

$dil.$

$\rightarrow$

3Cu $NO\$\$_3\$\$$

2

+ 4H

2

O + 2NO

B Cu + 4HNO

3

$conc.$

$\rightarrow$

Cu $NO\$\$_3\$\$$

2

+ 4H

2

O + 2NO

2

C 4Zn + 10HNO

3

$dil.$

$\rightarrow$

4Zn $NO\$\$_3\$\$$

2

+ 5H

2

O + N

2

O

D Zn + HNO

3

$conc.$

$\rightarrow$

Zn $NO$$$_3$$$

2

+ 2H

2

O + 2NO

2

Question 011 MCQ

QUESTION

Match each of the compounds given in Column I with the reaction s, that they can undergo, given in Column II.

	Column I		Column II
A		P	Nucleophilic substitution
B		Q	Elimination

	Column I		Column II
C		R	Nucleophilic addition
D		S	Esterification with acetic anhydri
		T	Dehydrogenation

A

$\rightarrow$

$Q, T; B$

$\rightarrow$

A $P, Q, T; C$

$\rightarrow$

$P, S; D$

$\rightarrow$

P

A

$\rightarrow$

$P, T; B$

$\rightarrow$

B $S, T; C$

$\rightarrow$

$R, S; D$

$\rightarrow$

S

A

$\rightarrow$

$S, Q, T; B$

$\rightarrow$

C $P, Q, T; C$

$\rightarrow$

$R, S; D$

$\rightarrow$

T

A

$\rightarrow$

$P, Q, T; B$

$\rightarrow$

D $P, S, T; C$

$\rightarrow$

$R, S; D$

$\rightarrow$

P

CORRECT OPTION

A

$\rightarrow$

$P, Q, T; B$

$\rightarrow$

D $P, S, T; C$

$\rightarrow$

$R, S; D$

$\rightarrow$

P

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

A The compound undergoes nucleophilic substitution due to formation of stable carbocation and elimination reaction in which the alkene formed is stabilized due to conjugation with aromatic nucleus.

D The compound undergoes nucleophilic substitution reaction

Question 012 Numerical

QUESTION

In a constant volume calorimeter, 3.5 g of a gas with molecular weight 28 was burnt in excess oxygen at 298.0 K. The temperature of the calorimeter was found to increase from 298.0 K to 298.45 K due to the combustion process. Given that the heat capacity of the calorimeter is 2.5 kJ K

-1

, the numerical value for the enthalpy of combustion of the gas in kJ mol

-1

is _____.

SOURCE

Chemistry • thermodynamics

EXPLANATION

To find the numerical value for the enthalpy of combustion of the gas in kJ mol

$$-1$$

, we first need to determine the total heat released by the combustion of the gas within the calorimeter. We then convert this amount of heat into per mole of the gas. **Step 1: Calculate the total heat released,**

$$q$$

.

The heat released,

$$q$$

, due to combustion in the calorimeter can be calculated using the formula:

$$q = C \cdot \Delta T$$

where:

- C is the heat capacity of the calorimeter, and
- ΔT is the change in temperature.

In this problem:

- $C = 2.5 \text{ kJ K}^{-1}$
- $\Delta T = 298.45 \text{ K} - 298.0 \text{ K} = 0.45 \text{ K}$

Substituting these values into the equation gives:

$$q = 2.5 \text{ kJ K}^{-1} \times 0.45 \text{ K} = 1.125 \text{ kJ}$$

The total heat released by the process is therefore 1.125 kJ, where this amount of heat is a measure of energy released and absorbed by the calorimeter, therefore it is positive.

Step 2: Convert the heat released to a molar basis.

To convert the heat released into per mole of the gas, we first need to calculate the number of moles of the gas that was burnt. The number of moles,

$$n$$

, can be calculated from the mass of the gas and its molecular weight:

$$n = \frac{\text{mass}}{\text{molecular weight}}$$

In this problem:

- The mass of the gas = 3.5 g
- Molecular weight of the gas = 28 g mol

$$-1$$

Substituting these values gives:

$$n = \frac{3.5\ \text{g}}{28\ \text{g mol}^{-1}} = 0.125\ \text{mol}$$

Step 3: Calculate the enthalpy of combustion per mole.

The enthalpy of combustion per mole,

$$\Delta H$$

, is given by:

$$\Delta H = \frac{q}{n}$$

Substituting the values we obtained:

$$\Delta H = \frac{1.125\ \text{kJ}}{0.125\ \text{mol}} = 9\ \text{kJ mol}^{-1}$$

Therefore, the enthalpy of combustion of the gas is

$$-9\ \text{kJ mol}^{-1}$$

.

Note: The negative sign indicates that the process is exothermic $releasesheat$.

Question 013 Numerical

QUESTION

At 400 K, the root mean square rms speed of a gas X $molecularweight = 40$ is equal to the most probable speed of gas Y at 60 K. The molecular weight of the gas Y is _____.

SOURCE

Chemistry • gaseous-state

EXPLANATION

Let's start by writing down the equations for the root mean square rms speed and the most probable speed. The root mean square speed

$$v_{rms}$$

of a gas with molecular weight

$$M$$

at a temperature

$$T$$

in Kelvin is given by the formula:

$$v_{rms} = \sqrt{\frac{3kT}{M}}$$

where

$$k$$

is the Boltzmann constant.

The most probable speed

$$v_{mp}$$

of a gas is given by:

$$v_{mp} = \sqrt{\frac{2kT}{M}}$$

According to the problem statement, at 400 K, the

$$v_{rms}$$

speed of gas X $with molecular weight$$M_X = 40$$g/mol$ is equal to the

$$v_{mp}$$

of gas Y at 60 K. We set the equations equal to each other:

$$\sqrt{\frac{3k \times 400}{40}} = \sqrt{\frac{2k \times 60}{M_Y}}$$

We simplify this equation. First, we can cancel

$$k$$

from both sides:

$$\sqrt{\frac{3 \times 400}{40}} = \sqrt{\frac{2 \times 60}{M_Y}}$$

Simplify further:

$$\sqrt{\frac{1200}{40}} = \sqrt{\frac{120}{M_Y}}$$

$$\sqrt{30} = \sqrt{\frac{120}{M_Y}}$$

Squaring both sides gives:

$$30 = \frac{120}{M_Y}$$

Rearrange to solve for

$$M_Y$$

:

$$M_Y = \frac{120}{30} = 4$$

So, the molecular weight of gas Y is 4 g/mol.

Question 014 Numerical

QUESTION

The dissociation constant of a substituted benzoic acid at 25

$$\circ$$

C is 1.0

$$\times$$

10

$$-4$$

. The pH of a 0.01 M solution of its sodium salt is _____.

SOURCE

Chemistry • ionic-equilibrium

EXPLANATION

Given that

$$K_a(C_6H_5COOH) = 1 \times 10^{-4}$$

.

pH of 0.01 M

$$C_6H_5COONa$$

.

$$K_h = \frac{K_w}{K_a} = \frac{0.01\,h^2}{1-h}$$

$$\Rightarrow \frac{10^{-14}}{10^{-4}} = \frac{10^{-2}h^2}{1-h}$$

$$1-h$$

is approximately equal to 1.

$$OH\$\$^{-}\$\$$$

=

$$0.01h$$

= 0.01

$$\times$$

10

$$-4$$

= 10

$$-6$$

$$H\$\$^{+}\$\$$$

= 10

$$-8$$

pH = 8

Question 015 Numerical

QUESTION

The total number of

$$\alpha$$

and

$$\beta$$

particles emitted in the nuclear reaction

$$^{238}_{92}U \rightarrow ^{214}_{82}Pb$$

is _____.

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

The equation for the nuclear fission reaction is:

$$^{238}_{92}\mathrm{U} \longrightarrow ^{214}_{82}\mathrm{Pb}$$

It is also written as

So, to find the alpha particle you can solve it by atomic mass of Pb

$$= \text{Atomic mass of U} - 4a$$

Atomic mass of $U = 238$

Atomic mass of $\mathrm{Pb} = 214$

So, by putting the values in equation i, you will get

$$206 = 238 - 4a$$

$$\therefore \alpha = \frac{238 - 214}{4} = 6$$

Now, find the beta particle by using formula

Atomic number of U = Atomic number of $\mathrm{Pb} + 2\alpha + \beta$.

Where, atomic no. of U is 92 and atomic no. of Pb is 82 and the value of α that we have find is 6. By putting the values you will get.

$$92 = 82 + 2 \times 6 = \beta$$

Therefore, $\beta = 92 - 94 = 2$

Thus, the number of alpha and beta particles is 6 and 2.

Question 016 Numerical

QUESTION

The oxidation number of Mn in the product of alkaline oxidative fusion of MnO_2 is _____.

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

The reaction for alkaline oxidative fusion is

$2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$

In potassium manganite formed as product :

K_2MnO_4

2 $+1$ +

$$x$$

+ 4 $$ $ $$2$ = 0

$$2 + x - 8 = 0$$

$$x - 6 = 0 \Rightarrow x = +6$$

Question 017 Numerical

QUESTION

The number of water molecule s directly bonded to the centre in CuSO

$$4$$

. 5H

$$2$$

O is _____.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The structure is CuSO

$$4$$

. 5H

$$2$$

O

$$\rightarrow$$

$$Cu(H$$_2$$O)$$_4$$$$

SO

4

. H

2

O

so, H

2

O molecules directly attached to Cu are 4.

Question 018 Numerical

QUESTION

The coordination number of Al in the crystalline state of AlCl

3

is _____.

SOURCE

Chemistry • solid-state

EXPLANATION

$AlCl_3$ exists as a close packed lattice of chloride ions Cl^- with Al^{3+} occupying octahedral holes. Hence, coordination number of Al^{3+} is = 6.

Question 019 Numerical

QUESTION

The total number of cyclic structural as well as stereoisomers possible for a compound with the molecular formula C

$$5$$

H

$$10$$

is _____.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Cyclic C

$$5$$

H

$$10$$

For third structure, 2

$$cis - trans$$

and 1 optical isomer are possible. So, a total of 7 structures are.

Question 020 Numerical

QUESTION

Let

$$f : R \to R$$

be a continuous function which satisfies

$$f(x) = \int_0^x f(t)dt$$

. Then, the value of

$$f(\ln 5)$$

is _____.

SOURCE

Mathematics • definite-integration

EXPLANATION

We have

$$f(x) = \int_0^x f(t)dt \Rightarrow f(0) = 0$$

Also,

$$f'(x) = f(x), x > 0$$

. Therefore,

$$f(x) = k, x > 0$$

Hence,

$$f(0) = 0$$

and

$$f(x)$$

is continuous,

$$f(x) = 0 \forall x > 0$$

Since

$$f(\ln 5) = 0$$

.

Question 021 MCQ

QUESTION

For

$$0 < \theta < \frac{\pi}{2},$$

the solution s of

$$\$ \sum_{m=1}^{6} \operatorname{cosec}\left(\theta + \frac{(m-1)\pi}{4}\right) \operatorname{cosec}\left(\theta + \frac{m\pi}{4}\right) = 4\sqrt{2}$$

\$ is are

A $\frac{\pi}{4}$

B $\frac{\pi}{6}$

C $\frac{\pi}{12}$

D $\frac{5\pi}{12}$

CORRECT OPTION

C $\frac{\pi}{12}$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

Given solutions

$$\frac{1}{\sin(\pi/4)}\left[\frac{\sin(\theta+\pi/4-\theta)}{\sin\theta\,.\,\sin(\theta+\pi/4)}+\frac{\sin(\theta+\pi/2-(\theta+\pi/4))}{\sin(\theta+\pi/4)\,.\,(\theta+\pi/2)}+\ldots+\frac{\sin((\theta+\right.$$

$$\Rightarrow \sqrt{2}[\cos\theta-\cot(\theta+\pi/4)+\cot(\theta+\pi/4)-\cot(\theta+\pi/2)+\ldots+\cot(\theta+5$$

$$\Rightarrow \tan\theta+\cot\theta=4\Rightarrow\tan\theta=2\pm\sqrt{3}$$

$$\Rightarrow\theta=\frac{\pi}{12}$$

or

$$\frac{5\pi}{12}$$

Question 022 MCQ

QUESTION

Match the statements/expressions in Column I with the values given in Column II:

	Column I		Column II
A	The number of solutions of the equation $$xe^{\sin x} - \cos x = 0$$ in the interval $$\left(0, \frac{\pi}{2}\right)$$	P	1
B	Value s of $$k$$ for which the planes $$kx + 4y + z = 0, 4x + ky + 2z = 0$$ and $$2x + 2y + z = 0$$ intersect in a straight line	Q	2
C	Value s of $$k$$ for which $$\lvert x-1\rvert + \lvert x-2\rvert + \lvert x+1\rvert + \lvert x+2\rvert = 4k$$ has integer solution s	R	3
D	If $$y' = y + 1$$ and $$y(0) = 1$$ then value s of $$y(\ln 2)$$	S	4

	Column I		Column II
		T	5

A

$\rightarrow$

P; B

$\rightarrow$

A Q, S; C

$\rightarrow$

Q, R, S, T; D

$\rightarrow$

R

A

$\rightarrow$

T; B

$\rightarrow$

B Q, S; C

$\rightarrow$

Q, S, T; D

$\rightarrow$

Q

A

$\rightarrow$

S; B

$\rightarrow$

C

$Q, S; C$

$\rightarrow$

$P, R, S, T; D$

$\rightarrow$

R

A

$\rightarrow$

$P; B$

$\rightarrow$

D

$Q, S; C$

$\rightarrow$

$Q, R, T; D$

$\rightarrow$

S

CORRECT OPTION

A

$\rightarrow$

$P; B$

$\rightarrow$

A

$Q, S; C$

$\rightarrow$

$Q, R, S, T; D$

$\rightarrow$

R

SOURCE

Mathematics • differential-equations

EXPLANATION

A We have

$$f'(x) > 0, \forall x \in (0, \pi/2)$$

. Therefore,

$$f(0) < 0$$

and

$$f(\pi/2) > 0$$

Hence, there is no one solution.

B Let us consider that

$$(a, b, c)$$

is direction ratio of the intersected line. Therefore,

$$ak + 4b + c = 0$$

$$4a + kb + 2c = 0$$

$$\frac{a}{8-k} = \frac{b}{4-2k} = \frac{c}{k^2-16}$$

We need to have

$$2(8-k) + 2(4-2k) + (k^2-16) = 0$$

$$\Rightarrow k = 2, 4$$

.

C Let us consider

$$f(x) = |x+2| + |x+1| + |x-1| + |x-2|$$

Therefore,

$$k$$

can take values: 2, 3, 4, 5.

D

$$\int \frac{dy}{y+1} = \int dx$$

$$\Rightarrow f(x) = 2e^x - 1$$

$$\Rightarrow f(\ln 2) = 3$$

Question 023 MCQ

QUESTION

Match the statements/expressions in Column I with the values given in Column II:

	Column I		Column II
A	Root s of the expression $2\sin^2\theta + \sin^2 2\theta = 2$	P	$\frac{\pi}{6}$
B	Points of discontinuity of the function $f(x) = \left[\frac{6x}{\pi}\right] \cos\left[\frac{3x}{\pi}\right]$, where $[y]$ denotes the largest integer less than or equal to y	Q	$\frac{\pi}{4}$
C	Volume of the parallelopiped with its edges represented by the vectors $\hat{i} + \hat{j} + \hat{i} + 2\hat{j}$	R	$\frac{\pi}{3}$

	Column I		Column II
	and $\hat{i}+\hat{j}+\pi\hat{k}$		
D	Angle between vectors $\vec{a}$ and $\vec{b}$ where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $\vec{a}+\vec{b}+\sqrt{3}\vec{c}=\vec{0}$	S	$\frac{\pi}{2}$
		T	π

$A \rightarrow Q, S; B \rightarrow$

A $P, R, S, T; C \rightarrow$

$Q; D$

$\rightarrow$

T

A

$\rightarrow$

$R, S; B$

$\rightarrow$

B $P, R, S, T; C$

$\rightarrow$

$T; D$

$\rightarrow$

P

A

$\rightarrow$

$Q, S; B$

$\rightarrow$

C $P, R, S, T; C$

$\rightarrow$

$T; D$

$\rightarrow$

R

A

$\rightarrow$

$P, S; B$

$\rightarrow$

D $Q, R, S, T; C$

$\rightarrow$

$T; D$

$\rightarrow$

R

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

a We have

$$2\sin^2\theta + 4\sin^2\theta\cos^2\theta = 2$$

$$\sin^2\theta + 2\sin^2\theta(1 - \sin^2\theta) = 1$$

$$3\sin^2\theta - 2\sin^4\theta - 1 = 0$$

$$\Rightarrow \sin\theta = \pm\frac{1}{\sqrt{2}}, \pm 1$$

$$\Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{2}$$

B Let

$$y = \frac{3x}{\pi} \Rightarrow \frac{1}{2} \leq y \leq 3 \forall x \in \left[\frac{\pi}{4}, \pi\right]$$

Now,

$$f(y) = [2y]\cos[y]$$

.

The critical points are

$$y = \frac{1}{2}, y = 1, y = \frac{3}{2}$$

and

$$y = 3$$

$$\Rightarrow$$

points of discontinuity

$$\left\{\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \pi\right\}$$

.

C

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi \Rightarrow$$

volume of parallelepiped =

$$\pi$$

.

D We have

$$\left|\vec{a} + \vec{b}\right| = \sqrt{3}$$

$$\Rightarrow \sqrt{2 + 2\cos\alpha} = \sqrt{3}$$

$$\Rightarrow 2 + 2\cos\alpha = 3$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Question 024 MCQ

QUESTION

A line with positive direction cosines passes through the point P $2, \$\$ - \$\$1, 2$ and makes equal angles with the coordinate axes. The line meets the plane

$$2x + y + z = 9$$

at point Q. The length of the line segment PQ equals

A 1

B $\sqrt{2}$

C $\sqrt{3}$

D 2

CORRECT OPTION

C $\sqrt{3}$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

The D.C. of the line are

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

.

We find that any point on the line at a distance

$$t$$

from

$$P(2,-1,2)$$

is

$$\left(2+\frac{t}{\sqrt{3}},-1+\frac{t}{\sqrt{3}},2+\frac{t}{\sqrt{3}}\right)$$

which lies on

$$2x+y+z=9$$

$$\Rightarrow t=\sqrt{3}$$

.

Question 025 MCQ

QUESTION

If

$$I_n=\int_{-\pi}^{\pi}\frac{\sin nx}{(1+\pi^x)\sin x}dx, n=0,1,2,$$

.... then

A $I_n=I_{n+2}$

B $$\sum_{m=1}^{10} I_{2m+1} = 10\pi$$

C $$\sum_{m=1}^{10} I_{2m} = 0$$

D $$I_n = I_{n+1}$$

CORRECT OPTION

A $$I_n = I_{n+2}$$

SOURCE

Mathematics • definite-integration

EXPLANATION

A We have the integral

$$I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}\,dx$$

$$= \int_0^{\pi} \left(\frac{\sin nx}{(1+\pi^x)\sin x} + \frac{\pi^x \sin nx}{(1+\pi^x)\sin x} \right) dx = \int_0^{\pi} \frac{\sin nx}{\sin x}$$

Now,

$$I_{n+2} - I_n = \int_0^{\pi} \frac{\sin(n+2)x - \sin nx}{\sin x}\,dx$$

B Since

$$I_3 = I_5 = \ldots = I_{21}$$

, we have

$$\sum_{m=1}^{10} I_{2m+1} = 10I_3 = 10\int_0^{\pi} \frac{\sin 3x}{\sin x}dx = 10\int_0^{\pi} (3 - 4\sin^2 x)dx$$

$$= 10[3x - 2x + \sin 2x]_0^{\pi} = 2\pi$$

C Since

$$I_2 = I_4 = \ldots = I_{20}$$

, we have

$$\sum_{m=1}^{10} I_{2m} = 10\int_0^{\pi} \frac{\sin 2x}{\sin x}dx = 20[\sin x]_0^{\pi} = 0$$

Question 026 Numerical

QUESTION

The maximum value of the function

$$f(x) = 2x^3 - 15x^2 + 36x - 48$$

on the set

$$A = \{x | x^2 + 20 \leq 9x|\}$$

is _____.

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

We have,

$$f'(x) = 6(x-2)(x-3)$$

Hence,

$$f(x)$$

is increasing in

$$(3, \infty)$$

.

Also,

$$A = \{4 \leq x \leq 5\}$$

Therefore,

$$f_{\max} = f(5) = 7$$

Question 027 Numerical

QUESTION

Let

$$p(x)$$

be a polynomial of degree

$$4$$

having extremum at

$$x = 1, 2$$

and

$$\lim_{x\to 0}\left(1+\frac{p\left(x\right)}{x^2}\right)=2$$

.

Then the value of

$$p(2)$$

is

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Let us consider

$$P(x)=ax^4+bx^3+cx^2+dx+e$$

$$P'(1)=P'(2)=0$$

$$\lim_{x\to 0}\left(\frac{x^2+P(x)}{x^2}\right)=2$$

$$\Rightarrow P(0)=0\Rightarrow e=0$$

$$\lim_{x\to 0}\left(\frac{2x+P'(x)}{2x}\right)=2$$

$$\Rightarrow P'(0)=0\Rightarrow d=0$$

$$\lim_{x\to 0}\left(\frac{2+P''(x)}{2}\right)=2$$

$$\Rightarrow c=1$$

On solving, we get

$$a=1/4, b=-1$$

. Thus,

$$P(x) = \frac{x^4}{4} - x^3 + x^2 \Rightarrow P(2) = 0$$

Question 028 MCQ

QUESTION

For the function

$$\$f\left(x\right) = x\cos\frac{1}{x}, x \geq 1,$$

$

for at least one

$$x$$

in the interval

A

$$[1, \infty)$$

,

$$f\left(x+2\right) - f\left(x\right) < 2$$

B

$$\lim_{x \to \infty} f'\left(x\right) = 1$$

for all

$$x$$

C

in the interval

$$[1, \infty) f\left(x+2\right) - f\left(x\right) > 2$$

$$f'(x)$$

D is strictly decreasing in the interval

$$[1, \infty)$$

CORRECT OPTION

B $$\lim_{x \to \infty} f'(x) = 1$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

We have for

$$f(x) = x \cos\left(\frac{1}{x}\right), x \geq 1$$

$$f'(x) = \cos\left(\frac{1}{x}\right) + \frac{1}{x} \sin\left(\frac{1}{x}\right) \to 1$$

for

$$x \to \infty$$

Also,

$$f'(x) = \left(\frac{1}{x}\right) + \frac{1}{x} \sin\left(\frac{1}{x}\right) - \frac{1}{x^2} \sin\left(\frac{1}{x}\right) - \frac{1}{x^3} \cos\left(\frac{1}{x}\right)$$

$$= -\frac{1}{x^3} \cos\left(\frac{1}{x}\right) < 0$$

for

$$x \geq 1$$

$$\Rightarrow f'(x)$$

is decreasing for

$$[1, \infty)$$

$$\Rightarrow f'(x+2) < f'(x)$$

. Also,

$$\lim_{x\to\infty} f(x+2) - f(x) = \lim_{x\to\infty} \left[(x+2)\cos\frac{1}{x+2} - x\cos\frac{1}{x}\right] = 2$$

Hence,

$$f(x+2) - f(x) > 2 \forall x \geq 1$$

.

Question 029 Numerical

QUESTION

If the function

$$f(x) = x^3 + e^{x/2}$$

and

$$g(x) = f^{-1}(x)$$

, then the value of

$$g'(1)$$

is _____.

SOURCE

Mathematics • functions

EXPLANATION

We have

$$f(0)=1, f'(x)=3x^2+\frac{1}{2}e^{x/2}$$

$$\Rightarrow f'(g(x))g'(x)=1$$

Substituting

$$x=0 \Rightarrow g'(1)=\frac{1}{f'(0)}=2$$

.

Question 030 Numerical

QUESTION

The smallest value of

$$k$$

, for which both the roots of the equation

$$\$x^2-8kx+16\left(k^2-k+1\right)=0$$

$ are real, distinct and have values at least 4, is

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

We have

$$x^2-8kx+16(k^2-k+1)=0$$

$$D > 0 \Rightarrow k > 1$$

..... 1

$$\frac{-b}{2a} > 4 \Rightarrow \frac{8k}{2} > 4$$

$$\Rightarrow k > 1$$

..... 2

Now,

$$f(4) \geq 0 \Rightarrow 16 - 32k + 16(k^2 - k + 1) \geq 0$$

$$k^2 - 3k + 2 \geq 0$$

$$k \leq 1 \cup k \geq 2$$

..... 3

Using Eqs. 1, 2 and 3, we get

$$k_{\min} = 2$$

.

Question 031 Numerical

QUESTION

Let

$$(x, y, z)$$

be points with integer coordinates satisfying the system of homogeneous equation:

$$\begin{aligned} 3x - y - z &= 0 \\ -3x + z &= 0 \\ -3x + 2y + z &= 0 \end{aligned}$$

$

Then the number of such points for which

$$x^2 + y^2 + z^2 \leq 100$$

is

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

To solve this problem, we need to find the integer points

$$(x, y, z)$$

that satisfy the given system of homogeneous equations:

$$\begin{aligned} 3x - y - z &= 0 \\ -3x + z &= 0 \\ -3x + 2y + z &= 0 \end{aligned}$$

Firstly, let's solve for

$$z$$

in terms of

$$x$$

from the second equation:

$$-3x + z = 0 \Rightarrow z = 3x$$

Next, substitute

$$z = 3x$$

into the first equation:

$$3x - y - 3x = 0 \Rightarrow -y = 0 \Rightarrow y = 0$$

With

$$y = 0$$

and

$$z = 3x$$

, the third equation also should be satisfied. Let's substitute

$$y$$

and

$$z$$

back into the third equation to verify:

$$-3x + 2y + z = 0 \Rightarrow -3x + 2(0) + 3x = 0$$

This equation holds true, confirming that the solutions for

$$y$$

and

$$z$$

remain consistent. Therefore, the points that satisfy the given system are of the form:

$$(x,\, 0,\, 3x)$$

Additionally, we need

$$x^2 + y^2 + z^2 \leq 100$$

. Substituting

$$y = 0$$

and

$$z = 3x$$

, we get:

$$x^2 + 0^2 + (3x)^2 \leq 100$$

This further simplifies to:

$$x^2 + 9x^2 \leq 100$$

$$10x^2 \leq 100$$

$$x^2 \leq 10$$

Hence,

$$-\sqrt{10} \leq x \leq \sqrt{10}$$

Since

$$x$$

must be an integer, we evaluate acceptable values for

$$x$$

:

$$x \in \{-3,\, -2,\, -1,\, 0,\, 1,\, 2,\, 3\}$$

For each of these values, let's determine the corresponding points

$$(x,\, 0,\, 3x)$$

:

- $(-3,\, 0,\, -9)$
- $(-2,\, 0,\, -6)$
- $(-1,\, 0,\, -3)$
- $(0,\, 0,\, 0)$
- $(1,\, 0,\, 3)$
- $(2,\, 0,\, 6)$
- $(3,\, 0,\, 9)$

Thus, there are a total of 7 such points.

Therefore, the number of integer-coordinate points

$$(x,\, y,\, z)$$

satisfying the given system of equations and the condition

$$x^2 + y^2 + z^2 \leq 100$$

is **7**.

Question 032 MCQ

QUESTION

If the sum of first

$$n$$

terms of an A.P. is

$$cn^2$$

, then the sum of squares of these

$$n$$

terms is

A $$\frac{n\left(4n^2-1\right)c^2}{6}$$

B $$\frac{n\left(4n^2+1\right)c^2}{3}$$

C $$\frac{n\left(4n^2-1\right)c^2}{3}$$

D $$\frac{n\left(4n^2+1\right)c^2}{6}$$

CORRECT OPTION

C $$\frac{n\left(4n^2-1\right)c^2}{3}$$

SOURCE

Mathematics • sequences-and-series

EXPLANATION

We have

$$t_n = c\{n^2-(n-1)^2\}$$

$$= c(2n-1)$$

$$\Rightarrow t_n^2 = c^2(4n^2-4n+1)$$

$$\Rightarrow \sum_{n=1}^{n} t_n^2 = c^2\left\{\frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n\right\}$$

$$= \frac{c^2 n}{6}\{4(n+1)(2n+1) - 12(n+1) + 6\}$$

$$= \frac{c^2 n}{3}\{4n^2+6n+2-6n-6+3\} = \frac{c^2}{3}n(4n^2-1)$$

which is the sum of the square of

$$n$$

terms.

Question 033 Numerical

QUESTION

The centres of two circles

$$C_1$$

and

$$C_2$$

each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segement joining the centres of

$$C_1$$

and

$$C_2$$

and C a circle touching circles

$$C_1$$

and

$$C_2$$

externally. If a common tangent to

$$C_1$$

and passing through P is also a common tangent to

$$C_2$$

and C, then the radius of the circle C is

SOURCE

Mathematics • circle

EXPLANATION

We have

$$\cos\alpha = \frac{2\sqrt{2}}{3}$$

$$\sin\alpha = \frac{1}{3}$$

$$\tan\alpha = \frac{2\sqrt{2}}{R}$$

$$\Rightarrow R = \frac{2\sqrt{2}}{\tan\alpha} = 8$$

units.

Question 034 MCQ

QUESTION

The normal at a point

$$P$$

on the ellipse

$$x^2 + 4y^2 = 16$$

meets the

$$x$$

- axis

$$Q$$

. If

$$M$$

is the mid point of the line segment

PQ

, then the locus of

M

intersects the latus rectums of the given ellipse at the points

A $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$

B $\left(\pm\frac{3\sqrt{5}}{2}, \pm\sqrt{\frac{19}{4}}\right)$

C $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$

D $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

CORRECT OPTION

C $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$

SOURCE

Mathematics • ellipse

EXPLANATION

The normal is

$$4x\sec\phi - 2y\cos ec\phi = 12$$

Now, the points Q and M are given by

$$Q \equiv (3\cos\phi, 0)$$

$$M \equiv (\alpha, \beta)$$

Therefore,

$$\alpha = \frac{3\cos\phi + 4\cos\phi}{2} = \frac{7}{2}\cos\phi \Rightarrow \cos\phi = \frac{2}{7}\alpha$$

and

$$\beta = \sin\phi; \cos^2\phi + \sin^2\phi = 1$$

.

Therefore,

$$\frac{4}{49}\alpha^2 + \beta^2 = 1 \Rightarrow \frac{4}{49}x^2 + y^2 = 1$$

Hence, the rectum is

$$x = \pm 2\sqrt{3}$$

.

Hence,

$$\frac{48}{49} + y^2 = 1 \Rightarrow y = \pm\frac{1}{7}$$

$$\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$$

Hence, the locus of M intersects the latus rectum of the given ellipse at the points

Question 035 MCQ

QUESTION

The tangent

$$PT$$

and the normal

$$PN$$

to the parabola

$$y^2 = 4ax$$

at a point

$$P$$

on it meet its axis at points

$$T$$

and

$$N$$

, respectively. The locus of the centroid of the triangle

$$PTN$$

is a parabola whose

A vertex is

$$\left(\frac{2a}{3}, 0\right)$$

directrix is

B $x = 0$

latus rectum is

C $\frac{2a}{3}$

focus is

D $(a, 0)$

CORRECT OPTION

vertex is

A $\left(\frac{2a}{3}, 0\right)$

SOURCE

Mathematics • parabola

EXPLANATION

We have

$$G \equiv (h, k)$$

.

$$\Rightarrow h = \frac{2a + at^2}{3}, k = \frac{2at}{3}$$

$$\Rightarrow \left(\frac{3h - 2a}{a}\right) = \frac{9k^2}{4a^2}$$

Therefore, the required parabola is

$$\frac{9y^2}{4a^2} = \frac{(3x-2a)}{a} = \frac{3}{a}\left(x - \frac{2a}{3}\right)$$

$$\Rightarrow y^2 = \frac{4a}{3}\left(x - \frac{2a}{3}\right)$$

Hence, the vertex

$$\equiv \left(\frac{2a}{3}, 0\right)$$

; focus

$$\equiv (a, 0)$$

.

Question 036 MCQ

QUESTION

An ellipse intersects the hyperbola

$$2x^2 - 2y^2 = 1$$

orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes then

A equation of ellipse is

$$x^2 + 2y^2 = 2$$

B the foci of ellipse are

$$(\pm 1, 0)$$

C

equation of ellipse is

$$x^2 + 2y^2 = 4$$

D

the foci of ellipse are

$$\left(\pm\sqrt{2}, 0\right)$$

CORRECT OPTION

A

equation of ellipse is

$$x^2 + 2y^2 = 2$$

SOURCE

Mathematics • ellipse

EXPLANATION

The ellipse and hyperbola will be confocal. Therefore,

$$(\pm ae, 0) \equiv (\pm 1, 0)$$

$$\Rightarrow \left(\pm a \times \frac{1}{\sqrt{2}}, 0\right) \equiv (\pm 1, 0)$$

$$\Rightarrow a = \sqrt{2}$$

and

$$e = \frac{1}{\sqrt{2}}$$

$$\Rightarrow b^2 = a^2(1 - e^2) \Rightarrow b^2 = 1$$

Hence, the equation of ellipse

$$\frac{x^2}{2}+\frac{y^2}{1}=1$$

.

Question 037 Numerical

QUESTION

Let ABC and ABC' be two non-congruent triangles with sides AB = 4, AC = AC' = 2

$$\sqrt{2}$$

and angle B = 30

$$\circ$$

. The absolute value of the difference between the areas of these triangles is _____.

SOURCE

Mathematics • properties-of-triangle

EXPLANATION

We have,

$$\cos\beta=\frac{a^2+16-8}{2\times a\times 4}$$

$$\Rightarrow \frac{\sqrt{3}}{2}=\frac{a^2+8}{8a}$$

$$\Rightarrow a^2-4\sqrt{3}a+8=0$$

$$\Rightarrow a_1+a_2=4\sqrt{3}, a_1a_2=8$$

$$\Rightarrow |a_1 - a_2| = 4$$

$$\Rightarrow |\Delta_1 - \Delta_2| = \frac{1}{2} \times 4 \sin 30^\circ \times 4 = 4$$

Question 038 MCQ

QUESTION

The locus of the orthocentre of the triangle formed by the lines

$$(1+p)x - py + p(1+p) = 0,$$

$$(1+q)x - qy + q(1+q) = 0$$

and

$$y = 0$$

, where

$$p \neq q$$

, is :

A a hyperbola.

B a parabola.

an ellipse.

D a straight line.

CORRECT OPTION

D a straight line.

SOURCE

Mathematics • parabola

EXPLANATION

The intersection point of

$$y = 0$$

with first line is

$$B(-p, 0)$$

.

The intersection point of

$$y = 0$$

with second line is

$$A(-q, 0)$$

.

The intersection point of the two lines is

$$C(pq, (p+1)(q+1))$$

.

The altitude from C to AB is

$$x = pq$$

.

The altitude from B to AC is

$$y = -\frac{q}{1+q}(x+p)$$

Solving these two equations, we get

$$x = pq$$

and

$$y = -pq$$

.

Hence, the locus of orthocentre is

$$x + y = 0$$

.

Question 039 MCQ

QUESTION

A piece of wire is bent in the shape of a parabola y = kx^2 $y - axisvertical$ with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration

$$a$$

. The distance of the new equilibrium position of the bead, where the bead can stays at rest with respect to the wire, from the y-axis is

A $\frac{a}{gk}$

B $\frac{a}{2gk}$

C $\frac{2a}{gk}$

D $\frac{a}{4gk}$

CORRECT OPTION

B $\frac{a}{2gk}$

SOURCE

Physics • laws-of-motion

EXPLANATION

While the parabola is moving right side, the bead experiences a force with a magnitude

$$ma$$

to the left. That is, at equilibrium,

$$N\cos\theta = mg$$

$$N\sin\theta = ma$$

Now,

$$\tan\theta = \frac{a}{g}$$

Also,

$$\tan\theta = \frac{dy}{dx} = 2kx \Rightarrow x = \frac{a}{2gk}$$

Question 040 MCQ

QUESTION

Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions

$$\phi_p = 2.0 \text{ eV}$$

,

$$\phi_q = 2.5 \text{ eV}$$

and

$$\phi_r = 3.0 \text{ eV}$$

, respecticely. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is $Takehc = 1240eVnm$

A

B

C

D

CORRECT OPTION

A

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

We have

$$E = \left(\frac{hc}{\lambda}\right)$$

J

$$\Rightarrow E = \left(\frac{1240}{\lambda\, nm}\right)$$

eV

Therefore,

λ_1

= 550 nm,

E_1

= 2.25 eV

λ_2

= 450 nm,

E_2

= 2.75 eV

λ_3

= 350 nm,

$$E_3$$

= 3.5 eV

Also,

$$\phi_p = 2$$

eV, all

$$\lambda$$

's cause emissions.

$$\phi_q = 2.5$$

eV, last two

$$\lambda$$

's cause emissions.

$$\phi_r = 3$$

eV, only the last

$$\lambda$$

causes emissions.

That is,

$$I_p > I_q > I_r$$

.

Question 041 MCQ

QUESTION

The mass M shown in the figure below oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is

A $\frac{k_1 A}{k_2}$

B $\frac{k_2 A}{k_1}$

C $\frac{k_1 A}{k_1 + k_2}$

D $\frac{k_2 A}{k_1 + k_2}$

CORRECT OPTION

D $\frac{k_2 A}{k_1 + k_2}$

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Since the restoring force is same in both springs $which are being in series$, we have

$$k_1 x_1 = k_2 x_2$$

It is given that

$$x_1 + x_2 = A$$

$$\Rightarrow x_1 = \frac{Ak_2}{k_1 + k_2}$$

Question 042 MCQ

QUESTION

A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants

$$k$$

. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle

$$\theta$$

in one direction and released. The frequency of oscillation is

A $$\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$$

B $$\frac{1}{2\pi}\sqrt{\frac{k}{M}}$$

C $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

D $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

CORRECT OPTION

C $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

The restoring torque is

$$J = -2 \times kx\left(\frac{L}{2}\right)\cos\theta = I\left(\frac{d^2\theta}{dt^2}\right)$$

Now,

$$x = \frac{L}{2}\sin\theta$$

Therefore,

$$J = -k\left(\frac{L^2}{2}\right)\sin\theta\cos\theta = I\left(\frac{d^2\theta}{dt^2}\right)$$

$$\Rightarrow \left(\frac{-kL^2}{4}\right)\sin 2\theta = I\left(\frac{d^2\theta}{dt^2}\right)$$

For small

$$\theta$$

,

$$\sin 2\theta = 2\theta$$

. Therefore,

$$\frac{-kL^2\theta}{2} = I\left(\frac{d^2\theta}{dt^2}\right)$$

where

$$I = \frac{ML^2}{12}$$

. Therefore,

$$\frac{d^2\theta}{dt^2} = \left(\frac{-6k}{M}\right)\theta = -\omega^2\theta$$

SHM

$$\Rightarrow \omega = \sqrt{\frac{6k}{M}}$$

Hence, the frequency of oscillation is

$$\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$$

Question 043 MCQ

QUESTION

The figure shows the PV plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semicircle and CDA is half of an ellipse. Then,

A the process during the path A → B is isothermal.

B heat flows out of the gas during the path B → C → D.

C work done during the path A → B → C is zero.

D positive work is done by the gas in the cycle ABCDA.

CORRECT OPTION

B heat flows out of the gas during the path B → C → D.

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

We observe the following:

•

For the path B

→

C

→

D, the volume V decreases;

Δ

W < 0.

$$\Delta U < 0 \Rightarrow \Delta Q < 0$$

•

For the process A

→

B

→

C,

Δ

W is the area of the semicircle and

$$\Delta W \neq 0$$

.

•

For the process A

→

B $semicircle$, it cannot be an isothermal process $whosePV - diagramisarectangularhyperbola.$

For $clockwise$ process A

$\rightarrow$

B

$\rightarrow$

C

$\rightarrow$

D

$\rightarrow$

A,

Δ

W is the area enclosed, which is positive.

Question 044 MCQ

QUESTION

Under the influence of the Coulomb field of charge +Q, a charge

—

q is moving around it in an elliptical orbit. Find out the correct statement s:

A

The angular momentum of the charge

—

q is constant.

The linear momentum of the charge

—

q is constant.

C The angular velocity of the charge

—

q is constant.

D The linear speed of the charge

—

q is constant.

CORRECT OPTION

A The angular momentum of the charge

—

q is constant.

SOURCE

Physics • waves

EXPLANATION

We have,

$$\vec{J} = \vec{r} \times \vec{F}$$

$$|\vec{J}| = rF \sin 180^\circ = 0$$

Therefore, the angular momentum is conserved, that is,

$$mvr \sin\theta =$$

Constant

where

θ

is the angle between the velocity

$\overrightarrow{v}$

and position vector

$\overrightarrow{r}$

of

$-$

q with respect to Q.

Question 045 MCQ

QUESTION

Two metallic rings A and B, identical in shape and size but having different resistivities

ρ_A

and

ρ_B

, are kept on top of two identical solenoids as shown in the figure below. When current I is switched on in both the solenoids in identical manner, the rings A and B jump to heights

h_A

and

h_B

, respectively, with

$h_A > h_B$

. The possible relation s between their resistivities and their masses

$$m_A$$

and

$$m_B$$

is are

$$\rho_A$$

>

$$\rho_B$$

A and

$$m_A$$

=

$$m_B$$

$$\rho_A$$

<

$$\rho_B$$

B and

$$m_A$$

=

$$m_B$$

$$\rho_A$$

>

$$\rho_B$$

C and m_A > m_B

ρ_A < ρ_B

D and m_A < m_B

CORRECT OPTION

ρ_A < ρ_B

B and m_A = m_B

SOURCE

Physics • electromagnetic-induction

EXPLANATION

Induced emf is same in both the rings:

$$I = \frac{e}{R} = \frac{\rho A}{\rho l}$$

$$I \propto \frac{1}{\rho} \Rightarrow q \propto \frac{1}{\rho}$$

.... 1

Impulse is

$$J = \int Bil\, dt = mv = Bl \int I\, dl = mv$$

That is,

$$J = Blq = mv \Rightarrow v \left(\frac{q}{m}\right)$$

..... 2

and

$$v^2 \propto h$$

..... 3

From Eqs. 1, 2 and 3, we get

$$mv \propto \frac{1}{\rho}$$

$$m\sqrt{h} \propto \frac{1}{\rho}$$

$$m\rho \propto \frac{1}{\sqrt{h}}$$

Since

$$h_A > h_B$$

and for

$$m_A = m_B, \rho\sqrt{h} =$$

Constant. Therefore,

$$\rho_A < \rho_B$$

. Also if

$$m_A < m_B$$

and

$$\rho_A < \rho_B$$

,

$$m_A\rho_A < m_B\rho_B$$

$$\Rightarrow h_A > h_B$$

$already given$

Question 046 MCQ

QUESTION

A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure below, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then,

A $\overrightarrow{V_C} - \overrightarrow{V_A} = 2(\overrightarrow{V_B} - \overrightarrow{V_C})$

B $\overrightarrow{V_C} - \overrightarrow{V_B} = \overrightarrow{V_B} - \overrightarrow{V_A}$

C $$|\vec{V_C} - \vec{V_A}| = 2|\vec{V_B} - \vec{V_C}|$$

D $$|\vec{V_C} - \vec{V_A}| = 4|\vec{V_B}|$$

CORRECT OPTION

B $$\vec{V_C} - \vec{V_B} = \vec{V_B} - \vec{V_A}$$

SOURCE

Physics • rotational-motion

EXPLANATION

We have

$$\vec{V_C} = \vec{V_{CM}} + \vec{r\omega} = 2\vec{V_{CM}}$$

$$\vec{V_B} = \vec{V_{CM}}$$

$$\vec{V_A} = 0$$

Therefore,

$$\vec{V_C} - \vec{V_B} = \vec{r\omega} = \vec{V_{CM}}$$

$$\vec{V_B} - \vec{V_A} = \vec{V_{CM}}$$

$$\vec{V_C} - \vec{V_A} = \vec{V_{CM}} + \vec{r\omega} = 2\vec{V_{CM}}$$

$$\vec{V_B} - \vec{V_C} = -\vec{r\omega}$$

For pure rolling, we have

$$|\overrightarrow{V_{CM}}| = |\overrightarrow{r\omega}|$$

Therefore,

$$|\overrightarrow{V_C} - \overrightarrow{V_A}| = 2V_{CM}$$

$$\Rightarrow 2|\overrightarrow{V_B} - \overrightarrow{V_C}| = 2r\omega = 2V_{CM}$$

Question 047 MCQ

QUESTION

A student performed the experiment to measure the speed of sound in air using resonance air-column method. Two resonances in the air-column were obtained by lowering the water level. The resonance with the shorter air-column is the first resonance and that with the longer air-column is the second resonance. Then,

A the intensity of the sound heard at the first resonance was more than that at the second resonance.

B the prongs of the tuning fork were kept in a horizontal plane above the resonance tube.

C the amplitude of vibration of the ends of the prongs is typically around 1 cm.

D the length of the air-column at the first resonance was somewhat shorter than 1/4th of the wavelength of the sound in air.

CORRECT OPTION

A the intensity of the sound heard at the first resonance was more than that at the second resonance.

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

The sharpness of resonance decreases with increasing column length.

The first resonance happens when length of air column

$$\approx$$

$$\lambda/2$$

. The prongs of the tuning fork are always kept in a vertical plane and amplitude of vibration is typically in the mm range.

From the figure, we have

$$\frac{\lambda}{4} = l_1 + e$$

where e is the end-correction. Therefore,

$$l_1 + \left(\frac{\lambda}{4} - e\right) < \frac{\lambda}{4}$$

Question 048 MCQ

QUESTION

Column II gives certain systems undergoing a process. Column I suggests changes in some of the parameters related to the system. Match the statements in Column I to the appropriate process es from Column II:

	Column I		Column II
A	The energy of the system is increased.	P	System : A capacitor, initially uncharged. Process : It is connected to a ba
B	Mechanical energy is provided to the system, which is converted into energy of random motion of its parts.	Q	System : A gas in an adiabatic container filled with an adiabatic piston. Process : The gas is compresse pushing the piston.
C	Internal energy of the system is converted into its mechanical energy.	R	System : A gas in a rigid contain Process : The gas gets cooled d colder atmosphere surrounding i
D	Mass of the system is decreased.	S	System : A heavy nucleus, initial rest. Process : The nucleus fissions ir two fragments of nearly equal masses and some neutrons are emitted.
		T	System : A resistive wire loop. Process : The loop is placed in a time varying magnetic field perpendicular to its plane.

$A \rightarrow P, S, T; B \rightarrow$

A $Q; C \rightarrow S; D \rightarrow$

S, R

A

$\rightarrow$

P, Q, T; B

$\rightarrow$

B Q; C

$\rightarrow$

S; D

$\rightarrow$

S

A

$\rightarrow$

P, Q, T; B

$\rightarrow$

C T; C

$\rightarrow$

S; D

$\rightarrow$

S

A

$\rightarrow$

S, Q, T; B

$\rightarrow$

D Q; C

$\rightarrow$

S; D

$\rightarrow$

S

CORRECT OPTION

A

$\rightarrow$

P, Q, T; B

$\rightarrow$

B Q; C

$\rightarrow$

S; D

$\rightarrow$

S

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

A

$\rightarrow$

P, Q, T

•

Case P :

$$U = \frac{1}{2}CV^2$$

•

Case Q : Since the work is done on the system, the internal energy increases.

•

Case T : time varying

$$\overrightarrow{B}$$

creates an induced

$$\overrightarrow{E}$$

causing current to flow. This dissipates heat in the loop:

$$H = L^2(Rt)$$

.

B

$\rightarrow$

Q : As explained above.

C

$\rightarrow$

S

•

Case S : Mass defect is converted into energy which is released.

D

$\rightarrow$

S : As explained above.

Question 049 MCQ

QUESTION

Column I shows four situations of standard Young's double slit arrangement with the screen placed far away from the slits S

1

and S

2

. In each of these cases, S

1

P

0

= S

2

P

0

, S

1

P

1

—

S

2

P

1

=

$\lambda/4$

and S

1

P

2

—

S

2

P

2

=

$\lambda/3$

, where

λ

is the wavelength of the light used. In the cases B, C and D, a transparent sheet of refractive index

μ

and thickness t is pasted on slit S

2

. The thickness of the sheets are different in different cases. The phase difference between the light waves reaching a point P on the screen from the two slits is denoted by

δ

P and the intensity by I P. Match each situation given in Column I with the statement s in Column II valid for that situation:

	Column I		Column II
A		P	$\delta(P_0) = 0$
B	$(\mu - 1)t = \lambda/4$	Q	$\delta(P_1) = 0$

	Column I		Column II
C	$(\mu-1)t=\lambda/2$	R	$I(P_1)=0$
D	$(\mu-1)t=3\lambda/4$	S	$I(P_0)>I(P_1)$
		T	$I(P_2)>I(P_1)$

$A \rightarrow P, S; B \rightarrow Q; C \rightarrow T; D \rightarrow R, S, T$

A $A \rightarrow P, S; B \rightarrow Q; C \rightarrow T; D \rightarrow R, S, T$

$A \rightarrow P, S; B \rightarrow R; C \rightarrow T; D$

B $A \rightarrow P, S; B \rightarrow R; C \rightarrow T; D$

$\rightarrow$

R, S, T

A

$\rightarrow$

$P, S; B$

$\rightarrow$

C $Q; C$

$\rightarrow$

$S; D$

$\rightarrow$

R, S, T

A

$\rightarrow$

$P, R; B$

$\rightarrow$

D $Q; C$

$\rightarrow$

$T; D$

$\rightarrow$

R, S, T

CORRECT OPTION

A

$\rightarrow$

$P, S; B$

$\rightarrow$

A $Q;\ C$

$\rightarrow$

$T;\ D$

$\rightarrow$

$R,\ S,\ T$

SOURCE

Physics • wave-optics

EXPLANATION

A

$\rightarrow$

$P,\ S$

Since the path difference is

$$S_1P_0 = S_2P_0 = 0$$

the phase difference becomes

$$\delta(P_0) = 0$$

The intensity at any point is

$$I = I_{\max}\cos^2\left(\frac{8}{2}\right)$$

Now,

$$I(P_0) = I_{\max}$$

$$\$\$ \because \$\$\$\$\delta(P_0) = 0\$\$$$

$$I(P_1) = I_{\max}\cos^2\left(\frac{\pi}{4}\right) = \frac{I_{\max}}{2}$$

Therefore,

$$I(P_0) > I(P_1)$$

B

$\rightarrow$

Q

In this case, the path difference at P

1

is

$$P_1 = \frac{\lambda}{4} - (\mu - 1)t = \frac{\lambda}{4} - \frac{\lambda}{4} = 0$$

Therefore,

$$\delta(P_1) = 0$$

.

C

$\rightarrow$

T

In this case, the path difference at P

1

is

$$P_1 = \frac{\lambda}{4} - (\mu - 1)t = \frac{\lambda}{4} - \frac{\lambda}{2} = \frac{-\lambda}{4}$$

Therefore, the phase difference at P

1

is

$$P_1 = \frac{2\pi}{\lambda} \times \left(\frac{-\lambda}{4}\right) = -\frac{\pi}{2}$$

Therefore,

$$I(P_1) = I_{\max}\cos^2\left(\frac{\pi}{4}\right) = \frac{I_{\max}}{2}$$

The path difference at P

$$2$$

is

$$P_2 = \frac{\lambda}{3} - (\mu - 1)t = \frac{\lambda}{3} - \frac{\lambda}{2} = \frac{-\lambda}{6}$$

Therefore, the phase difference at P

$$1$$

is

$$P_2 = \frac{2\pi}{\lambda} \times \left(\frac{-\lambda}{6}\right) = -\frac{\pi}{3}$$

Therefore,

$$I(P_2) = I_{\max}\cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}I_{\max}$$

Therefore,

$$I(P_2) > I(P_1)$$

.

$$D$$

$$\rightarrow$$

$$R, S, T$$

In this case, the path difference at P

$$1$$

is

$$P_1 = \frac{\lambda}{4} - (\mu - 1)t = \frac{\lambda}{4} - \frac{3\lambda}{4} = \frac{-\lambda}{2}$$

Therefore, the phase difference at P

1

is

$$P_1 = \frac{2\pi}{\lambda} \times \left(-\frac{\lambda}{2}\right) = -\pi$$

Therefore,

$$I(P_1) = I_{\max}\cos^2\left(\frac{\pi}{2}\right) = 0$$

The path difference at P

0

is

$$P_0 = 0 - (\mu - 1)t = \frac{-3\lambda}{4}$$

Therefore, the phase difference at P

0

is

$$P_0 = \frac{2\pi}{\lambda} \times \left(\frac{-3\lambda}{4}\right) = \frac{-3\lambda}{2}$$

Therefore,

$$I(P_0) = I_{\max}\cos^2\frac{3\pi}{4} = \frac{I_{\max}}{2}$$

Now, the path difference at P

1

is

$$P_1 = \frac{\lambda}{4} - (\mu - 1)t = \frac{\lambda}{4} - \frac{3\lambda}{4} = -\frac{\lambda}{2}$$

The phase difference at P

1

is

$$P_1 = \frac{2\pi}{\lambda} \times \left(\frac{-\lambda}{2}\right) = -\pi$$

Therefore,

$$I(P_1) = I_{\max} \cos\left(\frac{\pi}{2}\right) = 0$$

Question 050 Numerical

QUESTION

A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking g = 10 m/s^2, find the work done $injoules$ by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

SOURCE

Physics • work-power-and-energy

EXPLANATION

We have,

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)g = \left(\frac{0.72 - 0.36}{0.72 - 0.36}\right) \times 10 = \frac{g}{3} = \frac{10}{3}$$

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 0.72 \times 0.36 \times 10}{0.72 + 0.36} = 4.8$$

N

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times \frac{10}{3} \times 1^2 = \frac{5}{3}$$

m

The work done by the rope on 0.36 kg is

$$W = Ts\cos 0^\circ = 4.8 \times \frac{5}{3} = +8$$

J

Question 051 Numerical

QUESTION

Two soap bubbles A and B are kept in a closed chamber where the air is maintained at pressure 8 N/m

$$2$$

. The radii of bubbles A and B are 2 cm and 4 cm, respectively. Surface tension of the soap-water used to make bubbles is 0.04 N/m. Find the ratio

$$n_B/n_A$$

, where

$$n_A$$

and

$$n_B$$

are the number of moles of air in bubbles A and B, respectively.
$Neglectthee ffectofgravity.$

SOURCE

Physics • properties-of-matter

EXPLANATION

We have,

$$P_A = P_0 + \frac{4S}{r_A} = 8 + \left[\frac{4(0.04)}{2 \times 10^{-2}}\right] = 16$$

N/m

$$2$$

$$P_B = P_0 + \frac{4S}{r_B} = 9 + \left[\frac{4(0.04)}{4 \times 10^{-2}}\right] = 12$$

N/m

$$2$$

Now,

$$PV = nRT$$

Therefore,

$$\frac{n_B}{n_A} = \frac{P_B V_B}{P_A V_A} - \frac{P_B r_B^3}{P_A r_A^3} = \frac{12}{16} \times \frac{64}{8} = 6$$

Question 052 Numerical

QUESTION

A steady current I goes through a wire loop PQR having shape of a right angle triangle wit6h PQ = 3, PR = 4x and QR = 5x. If the magnitude of the magnetic field at P due to this loop is

$$k\left(\frac{\mu_0 I}{48\pi x}\right)$$

, find the value of

$$k$$

.

SOURCE

Physics • magnetism

EXPLANATION

The segments PQ and PR, in the triangle shown here, cannot produce

$$\overrightarrow{B}$$

at point P since point P lies on them. Here, only QR creates

$$\overrightarrow{B}$$

at point P.

Applying

$$B_P = \frac{\mu_0 i}{4\pi R}(\cos\phi_1 + \cos\phi_2)$$

, where

$$\phi_1 = 53^\circ$$

and

$$\phi_2 = 37^\circ$$

, we get the magnitude of the magnetic field as follows:

$$B_P = \frac{\mu_0 I}{4\pi(4\pi\sin 37^\circ)}(\cos 53^\circ + \cos 37^\circ)$$

$$= \frac{\mu_0 I}{16\pi x(3/5)}\left(\frac{3x}{5x} + \frac{4x}{5x}\right)$$

$$= \frac{5\mu_0 I}{48\pi x}\left(\frac{7}{5}\right) = \frac{7\mu_0 I}{48\pi x} = k\left(\frac{\mu_0 I}{48\pi x}\right)$$

where

$$k = 7$$

.

Question 053 Numerical

QUESTION

A cylindrical vessel of height 500 mm has an orifice $smallhole$ at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in height $inmm$ of water level due to opening of the orifice.
$Takeatmosphericpressure = 1.0\$\$ \times \$\$10\$\$^5\$\$N/m\$\$^2\$\$, densityofwate$

SOURCE

Physics • properties-of-matter

EXPLANATION

We have

$$P_0V_0 = P_1V_1$$

..... 1

and the pressure at equilibrium is

$$P = P_0 - \rho gh$$

$$= (1.0 \times 10^5) - [(10^3)(10)(200 \times 10^{-3})]$$

$$= (98 \times 10^3)$$

N/m

2

Substituting in Eq. 1, we get

$$10^5[A(500-H)] = 98 \times 10^3[A(500-200)]$$

where A is the cross-section of water column.

Now, if H = 206 mm, we conclude that the level of water falls down by 6 mm. Hence, the correct answer is 6.

Question 054 Numerical

QUESTION

A 20 cm long string, having a mass of 1.0 g, is fixed at both the ends. The tension in the string is 0.5 N. The string is set into vibrations using an external vibrator of frequency 100 Hz. find the separation $incm$ between the successive nodes on the string.

SOURCE

Physics • waves

EXPLANATION

The distance between the successive nodes is

$$\lambda/2$$

. Therefore,

$$\frac{\lambda}{2} = \frac{V}{2v} = \frac{1}{2 \times 100}\sqrt{\frac{Tl}{m}} = \frac{1}{200}\sqrt{\frac{0.5 \times 0.2}{10^{-3}}} = \frac{1}{20}$$

m = 5 cm

Question 055 Numerical

QUESTION

A solid sphere of radius R has a charge Q distributed in its volume with a charge density

$$\rho = Kr^a$$

, where K and a are constants and r is the distance from its centre. If the electric field at

$$r = R/2$$

is 1/8 times than at

$$r = R$$

, find the value of

$$a$$

.

SOURCE

Physics • electrostatics

EXPLANATION

Applying Gauss's theorem, we get

$$E(4\pi r^2) = \frac{q_{encl}}{t_0} = \frac{1}{t}\int_0^r kx^2(4\pi x^2)dx$$

Now,

$$Er^2 = \frac{k}{t_0}\int_0^r x^{2+a}dx = \frac{k}{t_0}\left(\frac{r^{3+a}}{3+a}\right)$$

Therefore,

$$E = \frac{k}{t_0}\frac{r^{1+a}}{3+a}$$

That is,

$$E \propto r^{1+a}$$

Now,

$$E\left(\frac{R}{2}\right) = \frac{1}{8}E(R)$$

Therefore,

$$\left(\frac{R}{2}\right)^{1+a} = \frac{1}{8}(R)^{1+a} \Rightarrow 8 = 2^{1+a}$$

where

$$1 + a = 3$$

and hence

$$a = 2$$

.

Question 056 Numerical

QUESTION

A metal rod AB of length 10x has its one end A in ice at 0

$$^\circ$$

C and the other end B in water at 100

°

C. If a point P on the rod is maintained at 400

°

C, then it is found that equal amounts of water and ice evaporate and melt per unit time. The latent heat of evaporation of water is 540 cal/g and latent heat of melting of ice is 80 cal/g. If the point P is at a distance of

$$\lambda x$$

from the ice end A, find the value of

$$\lambda$$

. $Neglectanyheatlosstothesurrounding.$

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

The steady rate of flow of heat is

$$\frac{\Delta Q}{\Delta t} = kA\left(\frac{\Delta T}{\Delta x}\right)$$

Therefore,

$$\frac{kA(400-0^\circ)}{\lambda x} = m_{ice}\, l_{ice}$$

$$\frac{kA(400-100)}{(10-\lambda)x} = m_{water}\, l_{water}$$

where

$$m_{ice}$$

is the mass of ice melted per unit times and

$$m_{water}$$

is the mass of water evaporated per unit times. It is given that

$$m_{ice} = m_{water}$$

Therefore,

$$\frac{kA \times 400}{\lambda x(l_{ice})} = \frac{kA \times 300}{(10-\lambda)x\, l_{water}}$$

$$\Rightarrow 4(10-\lambda) \times 540 = 3\lambda \times 80$$

$$\Rightarrow 5400 - 540\lambda = 60\lambda$$

$$\Rightarrow \lambda = 9$$

Question 057 Numerical

QUESTION

Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed inm/s of the object C.

SOURCE

Physics • work-power-and-energy

EXPLANATION

Let

$$V_1$$

and

$$V_2$$

be the velocities of blocks A and B immediately after the elastic collision:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)m = \left(\frac{m - 2m}{m + 2m}\right) \times 9 = -3$$

m/s

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)m = \left(\frac{2m}{m + 2m}\right) \times 9 = 6$$

m/s

After the perfectly inelastic collision between blocks B and C, let

$$v$$

be the common velocity. Applying centre of mass concept, we get

$$2mv_2 = (2m + m)v$$

$$\Rightarrow v = \frac{2}{3} \times 6 = 4$$

m/s