iit Jee 2008 Paper 2 Offline $66 Questions$

Question 001 MCQ

QUESTION

Match the entries in Column I with the correctly related quantum number s in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A	Orbital angular momentum of the electron in a hydrogen-like atomic orbital.	P	Principal quantum number
B	A hydrogen-like one-electron wave function obeying Pauli's principle.	Q	Azimuthal quantum number
C	Shape, size and orientation of hydrogen like atomic orbitals.	R	Magnetic quantum number
D	Probability density of electron at the nucleus in hydrogen-like atom.	S	Electron spin quantum number

A

$\rightarrow$

q,r; B

$\rightarrow$

A p,q,r; C

$\rightarrow$

p,q,r; D

$\rightarrow$

q,r;

A

$\rightarrow$

q,r; B

$\rightarrow$

B p,q,r,s; C

$\rightarrow$

p,q; D

$\rightarrow$

s,r;

A

$\rightarrow$

r,s; B

$\rightarrow$

C p,q,r; C

$\rightarrow$

p,q,r; D

$\rightarrow$

p,q;

A

$\rightarrow$

q,r; B

$\rightarrow$

D p,q,r,s; C

$\rightarrow$

p,q,r; D

$\rightarrow$

p,q;

CORRECT OPTION

A

$\rightarrow$

q,r; B

$\rightarrow$

D p,q,r,s; C

$\rightarrow$

p,q,r; D

$\rightarrow$

p,q;

SOURCE

Chemistry • structure-of-atom

EXPLANATION

A Orbital angular momentum of the electron in a hydrogen-like atomic orbital is represented by azimuthal quantum number.

B A hydrogen-like one-electron wave function obeying Pauli principle correspond to Electron spin quantum number.

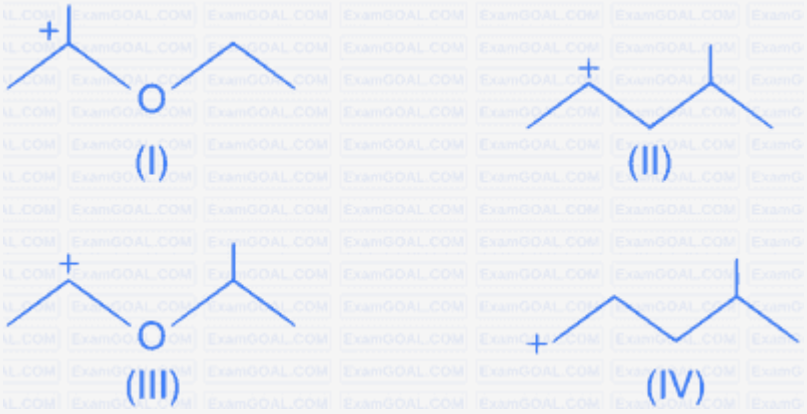
C Shape, size and orientation of hydrogen-like atomic orbitals are determined by Azimuthal, principal and Magnetic quantum number.

D Probability density of electron at the nucleus in hydrogen-like atom is determined by Principal quantum number, Azimuthal quantum number and Magnetic quantum number.

Question 002 MCQ

QUESTION

The correct stability order for the following species is :



A $II > IV > I > III$

B $I > II > III > IV$

C $II > I > IV > III$

D $I > III > II > IV$

CORRECT OPTION

D $I > III > II > IV$

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Stability of the following species depends upon the no. of

α

hydrogen which can undergoes hyperconjugation as well as resonance. Higher the no. of

α

hydrogen, higher will be the stability of the carbocation.

Stabilizes by resonance and have six

α

-hydrogen atoms $hyperconjugation$.

Stabilizes by resonance and have three

α

-hydrogen atoms.

It have five

α

-hydrogen atoms.

It have only two

α

-hydrogen atoms.

$\therefore$

$I > III > II > IV$

Question 003 MCQ

QUESTION

Cellulose upon acetylation with excess acetic anhydride/H

2

SO

4

$catalytic$ gives cellulose triacetate whose structure is :

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • biomolecules

EXPLANATION

Cellulose is linear chains of

β

-D-glucose molecules linked together by

β

-1, 4 glycosidic bonds. Cellulose has

β

-1, 4 linkage between

β

-D-glucose units. The cellulose is linear molecule.

A general reaction of alcohol with acetic anhydride is :

Acetic anhydride reacts with the alcoholic function group and converts the alcoholic function group into ester functional group. The

—

OH is known as an alcoholic functional group. The

—

OCO

—

is known as an ester functional group.

During the conversion hydrogen of alcoholic group is replaced by CH

3

CO

—

group of acetic anhydride. The above reaction is known as acetylation because acetyl group CH

3

CO

—

is attaching with the reactant alcohol.

So, upon acetylation of cellulose with excess acetic anhydride/H

2

SO

4

$catalytic$ all the

—

OH group will convert into acetyl group.

As the cellulose contains a large number of glucose units but our product is triacetate. It means the alcohol group of three glucose units of cellulose is converted into acetyl group. As the acetic anhydride is in excess, all alcoholic groups of three glucose units of cellulose will convert into acetyl groups.

So, the structure of the product is

Question 004 MCQ

QUESTION

In the following reaction sequence, the correct structure of E, F and G are :

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Question 005 MCQ

QUESTION

Among the following, the coloured compound is :

A CuCl

K

B

3

$Cu(CN)\$\$_4\$\$$

C

CuF

2

D

$Cu(CH\$\$_3\$\$CH)\$\$_4\$\$$

BF

4

CORRECT OPTION

C

CuF

2

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The electronic configuration of Cu

$2+$

is

Ar

3d

9

. It contains one unpaired electron. Hence, CuF

2

is coloured compound.

In all other compounds, Cu

$+$

ion is present. It has electronic configuration of Cu

$2+$

is

Ar

3d

10

. All electrons are paired.

Hence, they are colourless.

Question 006 MCQ

QUESTION

Both

$Ni(CO)$$_4$$

and

$Ni(CN)$$_4$$

$2-$

are diamagnetic. They hybridisations of nickel in these complexes, respectively, are :

sp

A

3

, sp

3

B

sp

3

, dsp

2

C

dsp

2

, sp

3

D

dsp

2

, dsp

2

CORRECT OPTION

B

sp

3

, dsp

2

SOURCE

Chemistry • coordination-compounds

EXPLANATION

In

$$Ni(CO)\$\$_4\$\$$$

, Ni is in 0 oxidation state, so there are 8 electrons in 3d subshell and 2 electrons in 4s. CO is the strong ligand, so it cause pairing of electrons in 3d subshell leaving one d subshell vacant. Now, the electron from s shell shifts to d creating vacant space in s subshell and thus the hybridisation is sp

$$3$$

.

Ni 28 =

$$Ar$$

4s

$$2$$

3d

$$8$$

CO is a strong ligand, causes coupling.

In

$$Ni(CN)\$\$_4\$\$$$

$$2-$$

, Ni is in +2 oxidation state so it has 8 electrons in d subshell and CN is also a strong ligand which pairs with d subshell atom and leaves one d orbital empty and thus its hybridisation is dsp

$$2$$

.

In

$$Ni(CN)\$\$_4\$\$$$

$$2-$$

, the oxidation state of Ni is +2

Ni

$$2+$$

=

$$Ar$$

3d

$$8$$

4s

$$0$$

CN

$$-$$

is strong ligand, causes coupling.

Question 007 MCQ

QUESTION

The IUPAC name of

$$Ni(NH\$\$_3\$\$)\$\$_4\$\$$$

$$NiCl\$\$_4\$\$$$

is :

A Tetrachloronickel II - tetraamminenickel II

B Tetraamminenickel II - tetrachloronickel II

C Tetraammnenickel II - tetrachloronickelate II

D Tetrachloronickel II - tetraamminenickelate 0

CORRECT OPTION

C Tetraammnenickel II - tetrachloronickelate II

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The IUPAC name for

$$Ni(NH\$\$_3\$\$)\$\$_4\$\$$$

$$NiCl\$\$_4\$\$$$

is : Tetraamminenickel II - tetrachloronickelate II

The name of the complex cation is written first followed by the name of the complex anion. Complex cation contains four ammine ligands and complex anion contains four chloride ligands.

Question 008 MCQ

QUESTION

Electrolysis of dilute aqueous NaCl solution was carried out by passing 10 milli ampere current. The time required to liberate 0.01 mol of H

$$2$$

gas at the cathode is (1 Faraday = 96500 C mol

$$-1$$

].

A 9.65×10^{4} sec

B 19.3×10^{4} sec

C 28.95×10^{4}

sec

38.6 × 10^4 sec

D

CORRECT OPTION

19.3 × 10^4 sec

B

SOURCE

Chemistry • electrochemistry

EXPLANATION

Given,

Current passed a 10 mA.

Moles of Hydrogen released are 0.01 moles.

In the electrolysis of aqueous solution sodium chloride, as the reduction potential of sodium ion is less than the hydrogen ion, the reaction that takes place at the cathode could be given as,

2H +

+ 2e

$$^{-}$$

$$\rightarrow$$

H

$$_{2}$$

0.01 moles of hydrogen would require 2

$$\times$$

0.01 = 0.02 moles of electrons or 0.02 Faradays.

We know the formula to calculate the amount of Faradays passed.

Amount of Faraday's passed =

$$\frac{I(A) \times t(s)}{96500 \text{ C mol}^{-1}}$$

On rearranging the equation to calculate the time taken we get,

t = Amount of Faraday's passed

$$\times$$

96500

$$\times$$

I

$$0.02 = \frac{\frac{10\,mA}{1000\,mA/A} \times t(s)}{96500\,C\,mol^{-1}}$$

$$\Rightarrow\ t = 0.02 \times 96500 \times 0.01$$

On multiplying we get,

$$\Rightarrow\ t = 19.3 \times 10^{4}$$

s

The time required to liberate 0.01 moles of H

$$_{2}$$

gas at the cathode is 19.3

$\times$

10

4

s.

Question 009 MCQ

QUESTION

Among the following, the surfactant that will form micelles in aqueous solution at the lowest molar concentration at ambient condition is:

A

CH

3

$CH\$\$_2\$\$$

15

N

$+$

$CH\$\$_3\$\$$

3

Br

$-$

CH

3

$CH\$\$_2\$\$$

11

B

OSO

$-$

3

Na

$+$

CH

3

$CH\$\$_2\$\$$

6

C

COO

$-$

Na

$+$

CH

3

$CH\$\$_2\$\$$

11

N

D

$+$

$CH\$\$_3\$\$$

3

Br

$-$

CORRECT OPTION

A

CH $_3$ CH_2 $_{15}$ N$^+$ CH_3 $_3$ Br$^-$

SOURCE

Chemistry • surface-chemistry

EXPLANATION

The surfactant CH $_3$ CH_2 $_{15}$ N$^+$ CH_3 $_3$ Br

$-$

forms micelles at lowest concentration.

Critical concentration for micelle formation decreases as the molecular weight of hydrocarbon chain of surfactant grows because its true solubility diminishes and the tendency of surfactant molecule to associate increases. The molecular weight is maximum in case of CH

3

$CH$$_2$$

15

N

$+$

$CH$$_3$$

3

Br

$-$

.

Question 010 MCQ

QUESTION

Solubility product constants $K$$_{sp}$$ of salts of types MX, MX

2

and M

3

X at temperature T are 4.0

$\times$

10 $^{-8}$, 3.2 $\times$ 10 $^{-14}$ and 2.7 $\times$ 10 $^{-15}$, respectively. Solubilities $moldm\$\$^{-3}\$\$$ of the salts at temperature 'T' are in the order:

A MX > MX $_2$ > M $_3$ X

B M $_3$ X > MX $_2$ > MX

MX $_2$

C > M

3

X > MX

MX > M

3

D X > MX

2

CORRECT OPTION

MX > M

3

D X > MX

2

SOURCE

Chemistry • ionic-equilibrium

EXPLANATION

MX > M

3

X > MX

2

$$\Rightarrow \quad \mathrm{MX} \rightarrow \mathrm{M}_{\mathrm{S}}^{+} + \mathrm{M}_{\mathrm{S}}^{-}$$

Solubiliity, $S\$\$_1\$\$$ of

$$MX = \sqrt{K_{sp}} = \sqrt{4 \times 10^{-8}}$$

$$= 2 \times 10^{-4}\, M$$

$$\Rightarrow MX_2 \rightarrow \underset{S}{M^+} + 2\underset{S}{X^-}$$

$$K_{sp} = (s)^1(2s)^2 = 4s^3$$

$$S = \left(\frac{3.2 \times 10^{-14}}{4}\right)^{\frac{1}{3}}$$

$$K_{sp} = 2 \times 10^{-5}\, M$$

$$\Rightarrow M_3X \rightarrow \underset{3S}{3M^+} + \underset{S}{X^{3-}}$$

$$K_{sp} = (3s)^3 s^1 = 27s^4$$

$$S = \left(\frac{2.7 \times 10^{-15}}{27}\right)^{\frac{1}{9}}$$

$$= (10^{-16})^{\frac{1}{2}}$$

$$S = 10^{-4}\, M = 1 \times 10^{-14}\, M$$

Hence, option D is correct.

MX > M

3

X > MX

2

Question 011 MCQ

QUESTION

Statement 1 : Aniline on reaction with NaNO

2

/HCl at 0

$\circ$

C followed by coupling with

β

-naphthol gives a dark blue coloured precipitate.

Statement 2 : The colour of the compound formed in the reaction of aniline with NaNO

2

/HCl at 0

$\circ$

C followed by coupling with

β

-naphthol is due to the extended conjugation.

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1.

C Statement 1 is True, Statement 2 is False.

D Statement 1 is False, Statement 2 is True.

CORRECT OPTION

D Statement 1 is False, Statement 2 is True.

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Let us discuss the given statements one by one.

Statement 1 : Aniline reacts with NaNO

$$2$$

/HCl at 273 K to form colourless diazonium salt.

Benzene diazonium chloride is then coupled with

$$\beta$$

-naphthol to yield a dark red or orange coloured dye as shown in the reaction below:

Thus, the statement 1 is false as the colour of the precipitate formed is orange red not blue. As we have already seen in the above mentioned reactions that the colour compound dye formed in the reaction of aniline with NaNO

$$2$$

/HCl at 0

$$\circ$$

C was due to the successive coupling of diazonium salt with

$$\beta$$

-naphthol. Hence, statement 2 is true.

As a result, Statement 1 is False and Statement 2 is true.

Question 012 MCQ

QUESTION

Statement 1 :

$$Fe(H\$\$_2\$\$O)\$\$_5\$\$NO$$

SO

$$4$$

is paramagnetic.

Statement 2 : The Fe in

$$Fe(H\$\$_2\$\$O)\$\$_5\$\$NO$$

SO

$$4$$

has three unpaired electrons.

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1.

C Statement 1 is True, Statement 2 is False.

D Statement 1 is False, Statement 2 is True.

CORRECT OPTION

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The oxidation state of Fe in the complex,

$$Fe(H\$\$_2\$\$O)\$\$_5\$\$NO$$

SO

$$4$$

is +1

$$NOhas + 1charge$$

Fe

$$+$$

:

$$Ar$$

3d

$$+$$

4s

$$1$$

When the weak ligand H

$$2$$

O and strong ligand NO attacks, the configuration changes.

NO

$$+$$

causes pairing of 4s electrons inside. Thus the configuration is 3d

$$7$$

and number of unpaired electrons = 3.

Fe

$$+$$

:

$$Ar$$

3d

$$7$$

4s

$$0$$

$$\therefore$$

Fe

$$+$$

has 3 unpaired electrons.

Question 013 MCQ

QUESTION

Statement 1 : The geometrical isomers of the complex

$$M(NH\$\$_3\$\$)\$\$_4\$\$Cl\$\$_2\$\$$$

are optically inactive.

Statement 2 : Both geometrical isomers of the complex

$$M(NH\$\$_3\$\$)\$\$_4\$\$Cl\$\$_2\$\$$$

possess axis of symmetry.

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1.

C Statement 1 is True, Statement 2 is False.

D Statement 1 is False, Statement 2 is True.

CORRECT OPTION

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

For a complex to be optically active, it should not possess either centre of symmetry or axis of symmetry or plane of symmetry. Here, the given complex is an octahedral complex that possesses plane of symmetry as well as axis of symmetry and thus is optically inactive.

The

$$cis$$

and

$$trans$$

both form of complex

$$M(NH\$\$_3\$\$)\$\$_4\$\$Cl\$\$_2\$\$$$

are optically inactive.

Question 014 MCQ

QUESTION

Statement 1 : There is a natural asymmetry between converting work to heat and converting heat to work.

Statement 2 : No process is possible in which the sole result is the absorption of heat from a reservoir and its complete conversion into work.

A. Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

B. Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1.

C. Statement 1 is True, Statement 2 is False.

D. Statement 1 is False, Statement 2 is True.

CORRECT OPTION

A. Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1.

SOURCE

Chemistry • thermodynamics

EXPLANATION

In reality, work can be converted to heat entirely but not vice versa. This is in accordance with second law of thermodynamics. So, no process is possible in practice where heat is entirely converted to work.

Take for example the heat engine. It has to reject some heat to the sink and convert the remainder into work. Thus, entire heat supplied is not converted into work. On the other hand, in a refrigerator, work has to be supplied to transfer heat from a colder body to hotter surroundings-in this case, the entire work input is converted to heat.

In practice, reversible isothermal processes are impossible because heat transfer always takes place across a finite temperature difference. This is an irreversibility. Thus, the entropy of the system will always increase if there is a heat transfer to the system. In that case, the amount of heat supplied to the system cannot entirely be converted to work $unlikereversibleisothermalprocesseswhereheatisentirelyconvertedtowork$.

So, both the statements are clearly true but the second one does not explain the first one.

Question 015 MCQ

QUESTION

Compound H is formed by the reaction of

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

Compound H $atertiaryalcohol$ is formed by the reaction of acetophenone PhCOCH

$$3$$

with benzyl magnesium bromide PhCH

$$2$$

MgBr.

Question 016 MCQ

QUESTION

The structure of compound I is :

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

The structure of compound I is -

Question 017 MCQ

QUESTION

The structures of compound J, K and L, respectively, are:

PhCOCH$_3$, PhCH$_2$

A

COCH$_3$ and PHCH$_2$COO$^-$K$^+$.

B

PhCHO, PhCH$_2$CHO and PhCOO$^-$K$^+$

C

PhCOCH$_3$, PhCH$_2$CHO and CH$_3$COO$^-$K$^+$

+

D

PhCHO, PhCOCH

3

and PhCOO

−

K

+

CORRECT OPTION

D

PhCHO, PhCOCH

3

and PhCOO

−

K

+

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

Question 018 MCQ

QUESTION

The number of atoms in this HCP unit cell is :

A 4

B 6

C 12

D 17

CORRECT OPTION

B 6

SOURCE

Chemistry • solid-state

EXPLANATION

The number of atoms in a HCP unit cells is 6.

3 atoms are at the middle of the unit cell and each atom contributes 1 atom to the unit cell.

2 atoms are at the two face centres of the unit cell and each atom contributes 1/2 atom to the unit cell.

12 atoms are at the 12 corners of the unit cell and each atom contributes 1/6 atom to the unit cell.

The number of atoms in one unit cell

$3$$ \times $$1 + 2$$ \times $$1/2 + 12$$ \times $$1/6 = 3 + 1 + 2 = 6

Question 019 MCQ

QUESTION

The volume of this HCP unit cell is :

A $24\sqrt{2}\,r^3$

B $16\sqrt{2}\,r^3$

C $12\sqrt{2}\,r^3$

D $\frac{64}{3\sqrt{3}}\,r^3$

CORRECT OPTION

A $24\sqrt{2}\,r^3$

SOURCE

Chemistry • solid-state

EXPLANATION

Volume of unit cell = Area of base

$$\times$$

Height of the unit cell

The HCP is a hexagonal prism structure. The base of HCP is a hexagon. We know that the hexagon is made of the 6 equilateral triangles.

Area of equilateral triangle

$$= \frac{\sqrt{3}}{4} a^2$$

Therefore, the area of the base is given as,

Area of base

$$= 6 \times \frac{\sqrt{3}}{4} a^2$$

Let's the 'h' be the height of the hexagonal unit

Therefore, we have

$$\frac{h}{a} = \sqrt{\frac{8}{3}}$$

Now, substitute these values in the volumes of HCP we have,

Volume of HCP = Area of base

$$\times$$

height of the hexagon

$$\therefore$$

Volume of HCP

$$= 6 \times \frac{\sqrt{3}}{4} a^2 \times \sqrt{\frac{8}{3}} a$$

Since, spheres touch each other, the edge length 'a' is equal to twice the radius of the atom. That is a = 2r

Substitute these values in the volume we get,

Volume of HCP

$$= 6 \times \frac{\sqrt{3}}{4} a^2 \times \sqrt{\frac{8}{3}} a$$

$$= 6 \times \frac{\sqrt{3}}{4} (2r)^2 \times \frac{\sqrt{8}}{\sqrt{2}} 2r = 24\sqrt{2} r^3$$

Question 020 MCQ

QUESTION

The empty space in this HCP unit cell is :

A 74%

B 47.6%

C 32%

D 26%

CORRECT OPTION

D 26%

SOURCE

Chemistry • solid-state

EXPLANATION

Packing fraction =

$$\frac{\text{Total volume of unit cell}}{\text{Volume occupied}}$$

Substituting the values of occupied sphere and volume of the unit cell we have:

$$\Rightarrow \frac{6 \times \frac{4}{3} \times \pi r^3}{24\sqrt{2}r^3} = 0.74$$

Rearranging and solving % occupied space = 74%

The space in the HCP unit cell = 100

$-$

74 = 26%

Question 021 MCQ

QUESTION

Match the compounds in Column I with their characteristic test s/reaction s given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A		P	sodium fusion extract of the compound gives Prussian blue colour with FeSO $_4$

	Column I		Column II
			.
B		Q	gives positive $FeCl_3$ test.
C		R	gives white precipitate with $AgNO_3$.
D		S	reacts with aldehydes to form the corresponding hydrazone derivative.

A $\rightarrow$ r, s; B $\rightarrow$

A p; C $\rightarrow$ p, q; D $\rightarrow$ s

A $\rightarrow$ r, s; B $\rightarrow$

p, q; C

$\rightarrow$

p, q, r; D

$\rightarrow$

p, s

A

$\rightarrow$

r; B

$\rightarrow$

C p, q; C

$\rightarrow$

p, r; D

$\rightarrow$

p, s

A

$\rightarrow$

r; B

$\rightarrow$

D p; C

$\rightarrow$

p, q, r; D

$\rightarrow$

s

CORRECT OPTION

A

$\rightarrow$

r, s; B

$\rightarrow$

B p, q; C

$\rightarrow$

p, q, r; D

$\rightarrow$

p, s

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

A, B, C, D will give NaCN in their sodium fusion extract. Adding $FeSO_4$ will give $Fe_4[Fe(CN)_6]_3$ which is Prussian blue in colour.

B and C have Phenol group which gave positive result to $FeCl_3$ test.

A, B, C, D both give precipitate with $AgNO_3$ but only A and C will give pure white precipitate.

A and D can only give hydrazone derivative in reaction with aldehyde.

Question 022 MCQ

QUESTION

Match the conversions in Column I with the type s of reaction s given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A	PbS $\rightarrow$ PbO	P	roasting
B	CaCO $\xrightarrow{3}$ CaO	Q	Calcination
C	ZnS $\rightarrow$ Zn	R	carbon reduction
D	Cu $\xrightarrow{2}$ S $\rightarrow$ Cu	S	self reduction

A

$\rightarrow$

p; B

$\rightarrow$

A q; C

$\rightarrow$

p; D

$\rightarrow$

s

A

$\rightarrow$

q; B

$\rightarrow$

B p; C

$\rightarrow$

r; D

$\rightarrow$

p, s

A

$\rightarrow$

p; B

$\rightarrow$

C q; C

$\rightarrow$

p, r; D

$\rightarrow$

p, s

A

$\rightarrow$

p; B

$\rightarrow$

D q, s; C

$\rightarrow$

p, r; D

$\rightarrow$

p

CORRECT OPTION

A

$\rightarrow$

p; B

$\rightarrow$

C q; C

$\rightarrow$

p, r; D

$\rightarrow$

p, s

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

A. For PbS

$$\rightarrow$$

O

$$2$$

PbO, roasting is done.

B. For CaCO

$$3$$

$$\rightarrow$$

CaO, calcinations is done in the absence of air.

C. For ZnS

$$\rightarrow$$

Zn, there is roasting is followed by reduction

2ZnS + 3O

$$2$$

$$\Delta$$

$$\longrightarrow$$

2ZnO + 2SO

$$2$$

$$\uparrow$$

2ZnSO

$$4$$

$$\Delta$$

$$\longrightarrow$$

ZnO + SO

$$2$$

+

$$\frac{1}{2}$$

O

$$2$$

$$\uparrow$$

ZnO

$$\xrightarrow{\Delta}$$

Zn + CO

$$\uparrow$$

D. For Cu

$$2$$

S

$$\rightarrow$$

Cu, roasting is done followed by self-reduction.

2Cu

$$2$$

S + 3O

$$2$$

$$\rightarrow$$

2Cu

$$2$$

O + 2SO

$$2$$

Cu

$$2$$

S + 2Cu

$$2$$

O

$$\rightarrow$$

6Cu + SO

$$2$$

Question 023 MCQ

QUESTION

Which of the following is true?

A $f(x)$ is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$

B $f(x)$ is increasing on $(-1, 1)$ and has a local minimum at $x = 1$

C $f(x)$ is increasing on $(-1, 1)$

but has neither a local maximum nor a local minimum at

$$x = 1$$

$$f(x)$$

is decreasing on

D $$(-1, 1)$$

but has neither a local maximum nor a local minimum at

$$x = 1$$

CORRECT OPTION

$$f(x)$$

is decreasing on

A $$(-1, 1)$$

and has a local minimum at

$$x = 1$$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

As when

$$x \in (-1, 1), f'(x) < 0$$

So, f x is decreasing on $$ $$ $-1, 1$$ $$ at x = 1

$$f''(1) = \frac{4a}{(a+2)^2} > 0$$

So, local minima at x = 1

Question 024 MCQ

QUESTION

A particle P stats from the point

$$z_0$$

= 1 +2i, where

$$i = \sqrt{-1}$$

. It moves horizontally away from origin by 5 unit and then vertically away from origin by 3 units to reach a point

$$z_1$$

. From

$$z_1$$

the particle moves

$$\sqrt{2}$$

units in the direction of the vector

$$\hat{i} + \hat{j}$$

and then it moves through an angle

$$\frac{\pi}{2}$$

in anticlockwise direction on a circle with centre at origin, to reach a point

$$z_2$$

. The point

$$z_2$$

is given by

A 6 + 7i

B -7 + 6i

C 7 + 6i

D - 6 + 7i

CORRECT OPTION

D - 6 + 7i

SOURCE

Mathematics • complex-numbers

EXPLANATION

given

$$\alpha$$

particle

In the direction of, or, Now rotation about origin through angle of means multiply by

$$z_0 = 1 + 2i = (1, 2) = (x_0, y_0)$$

$$z_1 = (x_0 + 5, y_0 + 3)$$

$$= (6, 5) = 6 + 5i$$

$$\Rightarrow 2$$

in the direction of

$$i = j$$

$$x_1 = 2\cos 45^\circ$$

$$y_1 = 2\sin 45^\circ$$

$$z_2(7 + 6i)$$

Now, rotation about origin through an angle of 2

$$\pi$$

means multiply z

$$2$$

by

$$\rightarrow e^{i\pi/2}$$

$$\cos 90^\circ + i\sin 90^\circ = i$$

$$z_3 = i(7 + 6i)$$

$$= -6 + 7i$$

Question 025 MCQ

QUESTION

The shortest distance between

$$L_1$$

and

$$L_2$$

is :

A 0

B $\frac{17}{\sqrt{3}}$

C $\frac{41}{5\sqrt{3}}$

D $\frac{17}{5\sqrt{3}}$

CORRECT OPTION

D $\frac{17}{5\sqrt{3}}$

SOURCE

Mathematics • vector-algebra

EXPLANATION

The shortest distance between L

$_1$

and L

$_2$

is

$$\frac{(\overrightarrow{a_2}-\overrightarrow{a_1})(\overrightarrow{b_1}\times\overrightarrow{b_2})}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}=(\overrightarrow{a_2}-\overrightarrow{a_1}).\,\widehat{n}$$

Where,

$$a_1 = -\hat{i} - 2\hat{j} - \hat{k}$$

$$a_2 = 2\hat{i} - 2\hat{j} + 3\hat{k}$$

$$\therefore$$

$$\overrightarrow{a_2} - \overrightarrow{a_1} = 3\hat{i} + 4\hat{k}$$

$$\therefore$$

$$(\hat{a}_2 - \hat{a}_1) \,.\, \hat{n}$$

$$= (3\hat{i} + 4\hat{k}) \,.\, \left(\frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}} \right)$$

$$= \frac{-3 + 20}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$$

Question 026 MCQ

QUESTION

The distance of the point

$$(1, 1, 1)$$

from the plane passing through the point

$$(-1, -2, -1)$$

and whose normal is perpendicular to both the lines

$$L_1$$

and

$$L_2$$

is :

A $\frac{2}{\sqrt{75}}$

B $\frac{7}{\sqrt{75}}$

C $\frac{13}{\sqrt{75}}$

D $\frac{23}{\sqrt{75}}$

CORRECT OPTION

C $\frac{13}{\sqrt{75}}$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

The equation of the plane passing through the point $\$\$ -1, -2, -1\$\$$ and whose normal is perpendicular to both the given lines L

1

and L

2

written as

$$(x+1)+7(y+2)-5(z+1)=0$$

i.e.,

$$x+7y-5z+10=0$$

The distance of the point $1,1,1$ from the plane

$$=\left|\frac{1+7-5+10}{\sqrt{1+49+25}}\right|$$

$$=\frac{13}{\sqrt{75}}$$

units

Question 027 MCQ

QUESTION

The unit vector perpendicular to both

$$L_1$$

and

$$L_2$$

is :

A $\dfrac{-\hat{i}+7\hat{j}+7\hat{k}}{\sqrt{99}}$

B $$\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$$

C $$\frac{-\hat{i}+7\hat{j}+5\hat{k}}{5\sqrt{3}}$$

D $$\frac{7\hat{i}-7\hat{j}-\hat{k}}{\sqrt{99}}$$

CORRECT OPTION

B $$\frac{-\hat{i}-7\hat{j}+5\hat{k}}{5\sqrt{3}}$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

Vector in the direction of

$$L_1 = \overrightarrow{n_1} = 3\hat{i}+\hat{j}+2\hat{k}$$

Vector in the direction of

$$L_2 = \overrightarrow{n_2} = \hat{i}+2\hat{j}+3\hat{k}$$

$$\therefore$$

Vector perpendicular to both L

and L

^{2}

$$\overrightarrow{n_1} \times \overrightarrow{n_2}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= -\hat{i} - 7\hat{j} + 5\hat{k}$$

$$\therefore$$

Required unit vector

$$= \hat{n} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{\sqrt{1 + 49 + 25}} = \frac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$$

Question 028 MCQ

QUESTION

Let two non-collinear unit vectors

$$\hat{a}$$

and

$$\hat{b}$$

form an acute angle. A point

$$P$$

moves so that at any time

$$t$$

the position vector

$$\overrightarrow{OP}$$

$where\$\$O\$\$istheorigin$ is given by

$$\widehat{a}\cos t + \hat{b}\sin t.$$

When

$$P$$

is farthest from origin

$$O,$$

let

$$M$$

be the length of

$$\overrightarrow{OP}$$

and

$$\widehat{u}$$

be the unit vector along

$$\overrightarrow{OP}$$

. Then :

A $$\widehat{u} = \frac{\widehat{a} + \hat{b}}{\left|\widehat{a} + \hat{b}\right|} \; and \; M = \left(1 + \widehat{a}.\,\hat{b}\right)^{1/2}$$

B $$\widehat{u} = \frac{\widehat{a} - \hat{b}}{\left|\widehat{a} - \hat{b}\right|} \; and \; M = \left(1 + \widehat{a}.\,\hat{b}\right)^{1/2}$$

C $$\hat{u}=\frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|} \text{ and } M=\left(1+2\hat{a}.\ \hat{b}\right)^{1/2}$$

D $$\hat{u}=\frac{\hat{a}-\hat{b}}{\left|\hat{a}-\hat{b}\right|} \text{ and } M=\left(1+2\hat{a}.\ \hat{b}\right)^{1/2}$$

CORRECT OPTION

A $$\hat{u}=\frac{\hat{a}+\hat{b}}{\left|\hat{a}+\hat{b}\right|} \text{ and } M=\left(1+\hat{a}.\ \hat{b}\right)^{1/2}$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

$$|\overrightarrow{OP}|=|\hat{a}\cos t+\hat{b}\sin t|$$

$$|\overrightarrow{OP}|^2=(\hat{a}\cos t+\hat{b}\sin t)\,.\,(\hat{a}\cos t+\hat{b}\sin t)$$

$$=|\hat{a}|^2\cos^2 t+2\hat{a}\,.\,\hat{b}\cos t\sin t+|\hat{b}|^2\sin^2 t$$

$$=\cos^2 t+\hat{a}\,.\,\hat{b}\sin 2t+\sin^2 t$$

$$=1+\hat{a}\,.\,\hat{b}\sin 2t$$

The greatest value of

$$|\overrightarrow{OP}|$$

is reached when

$$\sin 2t = 1$$

i.e.,

$$t = \frac{\pi}{4}$$

$$M = |\overrightarrow{OP}|$$

greatest

$$= \sqrt{1 + \hat{a} \,.\, \hat{b}}$$

The direction of

$$\overrightarrow{OP}$$

is given by

$$\hat{u} = \frac{\hat{a} + \hat{b}}{\sqrt{2} \,.\, \frac{|\hat{a}+\hat{b}|}{\sqrt{2}}} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

Question 029 MCQ

QUESTION

Let a solution

$$y = y(x)$$

of the differential equation,

$$x\sqrt{x^2 - 1}\, dy - y\sqrt{y^2 - 1}\, dx = 0$$

satify

$$y\,(2) = \frac{2}{\sqrt{3}}.$$

STATEMENT-1 :

$$y(x) = \sec\left(\sec^{-1}x - \frac{\pi}{6}\right)$$

and

STATEMENT-2 :

$$y(x)$$

given by

$$\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$$

A STATEMENT-1 is True, STATEMENT-2 is True;STATEMENT-2 is a correct explanation for STATEMENT-1

B STATEMENT-1 is True, STATEMENT-2 is True;STATEMENT-2 is **NOT** a correct explanation for STATEMENT-1

C STATEMENT-1 is True, STATEMENT-2 is False

D STATEMENT-1 is False , STATEMENT-2 is True

CORRECT OPTION

C STATEMENT-1 is True, STATEMENT-2 is False

SOURCE

Mathematics • differential-equations

EXPLANATION

The given differential equation is

$$x\sqrt{x^2-1}dy - y\sqrt{y^2-1}dx = 0$$

$$= \int \frac{dy}{y\sqrt{y^2-1}} = \int \frac{dx}{x\sqrt{x^2-1}}$$

$$\Rightarrow \sec^{-1}y = \sec^{-1}x + c$$

$$y = \sec[\sec^{-1}x + C]$$

$$\$\$ \because \$\$\$\$y(2) = \frac{2}{\sqrt{3}}\$\$$$

$$\frac{2}{\sqrt{3}} = \sec(\sec^{-1}2 + C)$$

$$= \sec^{-1}\frac{2}{\sqrt{3}} - \sec^{-1}2 = C$$

$$C = \frac{\pi}{6} - \frac{\pi}{3} = \frac{-\pi}{6}$$

Missing or unrecognized delimiter for \right

$$]

Statement 1 is true

Also,

$$\frac{1}{y} = \cos\left[\cos -\frac{\pi}{x} - \frac{\pi}{6}\right]$$

$$\cos\left(\cos^{-1}\frac{1}{x}\right)\cos\frac{\pi}{6} + \sin(\cos^{-1}\frac{1}{x})\sin\frac{\pi}{6}$$

$$= \frac{1}{y} = \frac{\sqrt{3}}{2x} + \frac{1}{2}\sqrt{1-\frac{1}{x^2}}$$

$$\therefore$$

Statement 2 is false.

Question 030 MCQ

QUESTION

Let

$$g(x) = \int_0^{e^x} \frac{f'(t)}{1+t^2}\, dt.$$

Which of the following is true?

$g'(x)$ is positive on $(-\infty, 0)$ and negative on $(0, \infty)$

A

$g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

B

$g'(x)$ changes sign on both

C

$(-\infty, 0)$

and

$(0, \infty)$

D

$g'(x)$

does not change sign on

$(-\infty, 0)$

CORRECT OPTION

$g'(x)$

is negative on

B

$(-\infty, 0)$

and positive on

$(0, \infty)$

SOURCE

Mathematics • definite-integration

EXPLANATION

Given

$$g'(x) = \int_0^{e^x} \frac{f'(t)dt}{1+t^2}$$

$$g''(x) = \frac{dg(x)}{d(e^x)} \cdot \frac{d(e^x)}{dx}$$

$$= \frac{f'(e^x)}{1+e^{2x}} \cdot e^x$$

$$= \frac{e^x}{1+e^{2x}} \cdot \frac{2a(e^{2x}-1)}{(e^{2x}+ae^x+1)^2} > 0$$

when

$$x \in (0, \infty)$$

$$g'(x)$$

< 0 when

$$x \in (-\infty, 0)$$

Question 031 MCQ

QUESTION

Which of the following is true?

A $(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$

B $(2-a)^2 f''(1) - (2+a)^2 f''(-1) = 0$

C $f'(1) f'(-1) = (2-a)^2$

D $f'(1) f'(-1) = -(2+a)^2$

CORRECT OPTION

A $$(2+a)^2 f''(1) + (2-a)^2 f''(-1) = 0$$

SOURCE

Mathematics • differentiation

EXPLANATION

$$f(x) = \frac{x^2 - ax + 1}{x^2 + ax + 1}$$

for differentiation, we can write fx as

$$f(x) = 1 - \frac{2ax}{x^2 + ax + 1}$$

Now, on differentiation

$$f'(x) = \frac{2a(x^2 - 1)}{(x^2 + ax + 1)^2}$$

..... i

$$f'(1) = 0 = f'(-1)$$

Then again differentiation 1

$$(x^2 + ax + 1)^2 \,.\, 2x - (x^2 - 1)$$

$$f''(x) = 2a.\ \frac{2(x^2 + ax + 1)(2x + a)}{(x^2 + ax + 1)^4}$$

$$f''(-1) = \frac{-4a}{(2-a)^2}$$

$$f''(1) = \frac{-4a}{(2-a)^2}$$

Combining both

$$f''(1)(2+a)^2 + f''(-1)(2-a)^2 = 0$$

Question 032 MCQ

QUESTION

The area of the region between the curves

$$y = \sqrt{\frac{1+\sin x}{\cos x}}$$

and

$$y = \sqrt{\frac{1-\sin x}{\cos x}}$$

bounded by the lines

$$x = 0$$

and

$$x = \frac{\pi}{4}$$

is

A $\displaystyle\int_0^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

B $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

C

$$\int_0^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$$

D

$$\int_0^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}dt$$

CORRECT OPTION

B

$$\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}dt$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

Let us take the required equation

$$y = \sqrt{\frac{1+\sin x}{\cos x}}$$

and

$$y = \sqrt{\frac{1-\sin x}{\cos x}}$$

$$\int_0^{\frac{\pi}{4}} \left(\sqrt{\frac{1+\sin x}{\cos x}} - \sqrt{\frac{1-\sin x}{\cos x}}\right)dx$$

$$\because$$

$$\left(\frac{1+\sin x}{\cos x} > \frac{1-\sin x}{\cos x} > 0\right)$$

$$= \int_0^{\frac{\pi}{4}} \left(\sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}} - \sqrt{\frac{1+\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}}{\frac{\left(1-\tan\frac{x}{2}\right)^2}{1-\tan\frac{x}{2}}}} \right) dx$$

$$= \int_0^{\frac{\pi}{4}} \left(\sqrt{\frac{\left(1+\tan^2\frac{x}{2}\right)^2}{1-\tan^2\frac{x}{2}}} - \sqrt{\frac{\left(1-\tan\frac{x}{2}\right)^2}{1-\tan^2\frac{x}{2}}} \right) dx$$

$$\int_0^{\frac{\pi}{4}} \frac{1+\tan\frac{x}{2}-1+\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}} dx$$

$$= \int_0^{\frac{\pi}{4}} \frac{2\tan\frac{x}{2}}{\sqrt{1-\tan^2\frac{x}{2}}} dx$$

Put

$$\tan\frac{x}{2} = t$$

$$dx = \frac{2\,dt}{1+t^2}$$

$$A = \int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}} dt$$

Question 033 MCQ

QUESTION

Let

$$I = \int \frac{e^x}{e^{4x}+e^{2x}+1} dx,\ J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1} dx.$$

Then

for an arbitrary constant

$$C$$

, the value of

$$J - I$$

equals :

A $$\frac{1}{2}\log\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+C$$

B $$\frac{1}{2}\log\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+C$$

C $$\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$$

D $$\frac{1}{2}\log\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+C$$

CORRECT OPTION

C $$\frac{1}{2}\log\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+C$$

SOURCE

Mathematics • indefinite-integrals

EXPLANATION

Given,

$$I = \int \frac{e^x}{e^{4x} + e^{2x} + 1} dx$$

$$J = \int \frac{e^{-x}}{e^{-4x} + e^{-2x} + 1} dx = \int \frac{e^{3x}}{e^{4x} + e^{2x} + 1} dx$$

$$\therefore$$

$$J - I = \int \frac{e^x(e^{2x} - 1)}{e^{4x} + e^{2x} + 1} dx$$

Let

$$e^x = t \Rightarrow e^x dx = dt$$

$$\therefore$$

$$J - I = \int \frac{t^2 - 1}{t^4 + t^2 + 1} dt$$

$$= \int \frac{1 - \frac{1}{t^2}}{t^2 + 1 + \frac{1}{t^2}} dt$$

Let,

$$t + \frac{1}{t} = u \Rightarrow \left(1 - \frac{1}{t^2}\right) dt = du$$

$$J - I = \int \frac{du}{u^2 - 1} = \frac{1}{2} \log \left| \frac{u - 1}{u + 1} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{\frac{t^2+1}{t} - 1}{\frac{t^2+1}{t} + 1} \right| + C$$

$$= \frac{1}{2} \log \left| \frac{e^{2x} - e^x + 1}{e^{2x} + e^x + 1} \right| + C$$

Question 034 MCQ

QUESTION

Let the function

$$g:(-\infty,\infty)\to\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

be given by

$$g(u)=2\tan^{-1}(e^{u})-\frac{\pi}{2}.$$

Then,

$$g$$

is

A even and is strictly increasing in

$$(0,\infty)$$

B odd and is strictly decreasing in

$$(-\infty,\infty)$$

C odd and is strictly increasing in

$$(-\infty,\infty)$$

D neither even nor odd, but is strictly increasing in

$$(-\infty,\infty)$$

CORRECT OPTION

C

odd and is strictly increasing in

$$(-\infty, \infty)$$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Given that,

$$g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$$

for

$$u \in (-\infty, \infty)$$

$$g(-u) = 2\tan^{-1}e^{-u} - \frac{\pi}{2} = 2\cot^{-1}(e^u) - \frac{\pi}{2}$$

$$= 2\left(\frac{\pi}{2} - \tan^{-1}(e^u) - \frac{\pi}{2}\right)$$

$$= -2\tan^{-1}(e^u) + \frac{\pi}{2}$$

$$= -g(u)$$

$$\therefore$$

$$g(u) = -g(u)$$

$$\Rightarrow g(u)$$

is an odd function

We have,

$$g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$$

$$g'(u) = \frac{2e^u}{1 + 2e^{2u}}$$

$$g'(u) > 0, \forall u \in R$$

$$\$\$ \because \$\$\$\$e^u > 0\$\$$$

So,

$$g'(u)$$

is increasing function.

Question 035 MCQ

QUESTION

Let

$$g(x) = \log f(x)$$

, where

$$f(x)$$

is a twice differentiable positive function on $0, \$\$\infty\$\$$ such that

$$f(x+1) = xf(x)$$

. Then for N = 1, 2, 3, ...,

$$g''\left(N + \frac{1}{2}\right) - g''\left(\frac{1}{2}\right) =$$

A $$-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \ldots\ldots + \frac{1}{(2N-1)^2}\right\}$$

B $$4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots\ldots+\frac{1}{(2N-1)^2}\right\}$$

C $$-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots\ldots+\frac{1}{(2N+1)^2}\right\}$$

D $$4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots\ldots+\frac{1}{(2N+1)^2}\right\}$$

CORRECT OPTION

A $$-4\left\{1+\frac{1}{9}+\frac{1}{25}+\ldots\ldots+\frac{1}{(2N-1)^2}\right\}$$

SOURCE

Mathematics • differentiation

EXPLANATION

Given that,

$$g(x)=\log f(x)$$

$$\Rightarrow g(x+1)=\log f(x+1)$$

$$\Rightarrow g(x+1)=\log xf(x)$$

$$\$\$\because \$\$\$\$f(x+1)=xf(x)\$\$$$

$$g(x+1)=\log x+\log f(n)$$

$$\Rightarrow g(x+1)-g(x)=\log(x)$$

$$g'(x+1)-g'(x)=\frac{1}{x}$$

$$\Rightarrow g''(x+1)-g''(x)=\frac{-1}{x^2}$$

Putting,

$$x=x-\frac{1}{2}$$

, we get

$$g''\left(x+\frac{1}{2}\right)-g''\left(x-\frac{1}{2}\right)=-\frac{1}{\left(x-\frac{1}{2}\right)^2}=\frac{-(2)^2}{(2x-1)^2}$$

Putting

$$x=1,2,3$$

....., N we get

$$g''\left(x+\frac{1}{2}\right)-g''\left(x-\frac{1}{2}\right)-=\frac{1}{\left(x-\frac{1}{2}\right)^2}=\frac{-(2)^2}{(2x-1)^2}$$

Putting

$$x=1,2,3$$

, ..., N we get

$$g''\left(\frac{3}{2}\right)-g''\left(\frac{1}{2}\right)=\frac{-2^2}{1^2}$$

.... i

$$g''\left(\frac{5}{2}\right)-g''\left(\frac{3}{2}\right)=\frac{-2^2}{1^2}$$

..... ii

$$g''\left(\frac{7}{2}\right)-g''\left(\frac{5}{2}\right)=\frac{-2^2}{5^2}$$

..... iii

$$g''\left(N+\frac{1}{2}\right)-g''\left(N-\frac{1}{2}\right)=\frac{-2^2}{(2N-1)^2}$$

..... iv

Adding all the above equations, we get

$$g''\left(N+\frac{1}{2}\right)-g''\left(\frac{1}{2}\right)=4$$

$$\left[1+\frac{1}{3^2}+\frac{1}{5^2}+\ldots+\frac{1}{(2N-1)^2}\right]$$

Question 036 MCQ

QUESTION

Consider a branch of the hyperbola

$$\$x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$$

$

with vertex at the point

$$A$$

. Let

$$B$$

be one of the end points of its latus rectum. If

$$C$$

is the focus of the hyperbola nearest to the point

$$A$$

, then the area of the triangle

$$ABC$$

is

A $1-\sqrt{\frac{2}{3}}$

B $\sqrt{\frac{3}{2}}-1$

C $1+\sqrt{\frac{2}{3}}$

D $\sqrt{\frac{3}{2}}+1$

CORRECT OPTION

B $\sqrt{\frac{3}{2}}-1$

SOURCE

Mathematics • hyperbola

EXPLANATION

We have,

$$x^2-2y^2-2\sqrt{2}x-4\sqrt{2}y-6=0$$

$$\frac{(x-\sqrt{2})^2}{4}-\frac{(y+\sqrt{2})^2}{2}=1$$

$$a=2, b=\sqrt{2}$$

$$\Rightarrow e=\sqrt{\frac{3}{2}}$$

Now,

$$e=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{3}{2}}$$

Area,

$$=\frac{1}{2}$$

$$\times$$

Base

$$\times$$

Height

$$=\frac{1}{2}a(e-1)\frac{b^2}{a}$$

$$=\frac{1}{2}\frac{(\sqrt{3}-\sqrt{2})\times 2}{\sqrt{2}}$$

$$=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}$$

$$=\left(\sqrt{\frac{3}{2}}-1\right)$$

Question 037 MCQ

QUESTION

Consider

$$L_1 : \ 2x \ + \ 3y + p \ - \ 3 = 0$$

$$L_2 : \ 2x \ + \ 3y + p \ + \ 3 = 0$$

where p is a real number, and

$$C : \ x^2 \ + \ y^2 \ + \ 6x \ - 10y + \ 30 = 0$$

STATEMENT-1 : If line

$$L_1$$

is a chord of circle C, then line

$$L_2$$

is not always a diameter of circle C
and

STATEMENT-2 : If line

$$L_1$$

is a diameter of circle C, then line

$$L_2$$

is not a chord of circle C.

A Statement-1 is True, Statement-2 is True; Statement-2 is a correct rexplanation for Statement-1

B Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct rexplanation for Statement-1

C Statement-1 is True, Statement-2 is False

D Statement-1 is False, Statement-2 is True

CORRECT OPTION

C Statement-1 is True, Statement-2 is False

SOURCE

Mathematics • circle

EXPLANATION

Equation of circle C is

$$(x+3)^2+(y-5)^2=9+25-30=4$$

$$\Rightarrow (x+3)^2+(y-5)^2=2^2$$

Centre = $3, \$\$ - \$\$5$

If L_1 is diameter, then it passes through the center $3, \$\$ - \$\$5,$

$$\therefore$$

$$2(3)+3(-5)+p-3=0 \Rightarrow p=12$$

$$\therefore$$

L_1 is

$$2x+3y+9=0$$

and L_2 is

$$2x + 3y + 15 = 0$$

Distance of centre of circle from

$$L_2(d) = \left| \frac{2(3) + 3(-5) + 15}{\sqrt{2^2 + 3^2}} \right| = \frac{6}{\sqrt{12}} < 2$$

$$radius of circle$$

$$\therefore$$

L_2 is a chord of circle C.

Statement 2 is false.

Statement - 1 is true. If L_1 is a chord, L_2 may lie outside the circle.

Question 038 MCQ

QUESTION

Suppose four distinct positive numbers

$$a_1, a_2, a_3, a_4$$

are in G.P. Let

$$b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3 \quad and \quad b_4 = b_3 + a_4$$

.

STATEMENT-1: The numbers

$$b_1, b_2, b_3, b_4$$

are neither in A.P. nor in G.P. and

STATEMENT-2 The numbers

$$b_1, b_2, b_3, b_4$$

are in H.P.

A STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

B STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

C STATEMENT-1 is True, STATEMENT-2 is False

D STATEMENT-1 is False, STATEMENT-2 is True

CORRECT OPTION

C STATEMENT-1 is True, STATEMENT-2 is False

SOURCE

Mathematics • sequences-and-series

EXPLANATION

Given,

$$a_1, a_2, a_3, a_4$$

are in G.P.

Then,

$$b_1, b_2, b_3, b_4$$

are the numbers.

$$a_1, a_1 + a_2, a_1 + a_2 + a_3, a_1 + a_2 + a_3 + a_4$$

or

$$a, a + ar, a + ar + ar^2, a + ar + ar^2 + ar^3$$

Clearly above numbers are neither in A.P. nor in G.P. and hence statement 1 is true.

Also,

$$\frac{1}{a}, \frac{1}{a + ar}, \frac{1}{a + ar + ar^2}, \frac{1}{a + ar + ar^2 + ar^3}$$

are not in H.P.

$$\therefore$$

$$b_1, b_2, b_3, b_4$$

are not in H.P.

$$\therefore$$

Statement 2 is false.

Question 039 MCQ

QUESTION

Consider all possible permutations of the letters of the word ENDEANOEL. Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

	Column I		Column II
A	The number of permutations containing the word ENDEA is	P	5!
B	The number of permutations in which the letter E occurs in	Q	2 $\times$

	Column I		Column II
	the first and the last position is		5!
C	The number of permutations in which none of the letters D, L, N occurs in the last five positions is	R	7 $\times$ 5!
D	The number of permutations in which the letters A, E, O occur only in odd positions is	S	21 $\times$ 5!

A A - p ; B - s; C - q ; D - q

B A - q ; B - q ; C - s ; D - p

C A - p ; B - s; C - p ; D - r

D A - p ; B - r ; C - q ; D - p

CORRECT OPTION

A A - p ; B - s; C - q ; D - q

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

A Considering ENDEA as one group, remaining letters are N, O, E, L.

So, no. of permutations = 5!

A - i

B E occurs in 1st and last positions. The remaining letters are N, N, D, A, O, E, L.

No. of permutation

$$= \frac{7!}{2!} = \frac{7 \times 6}{2} \times 5! = 21! \times 5!$$

B - iv

C D, L, N should not occur in last five positions

$$\Rightarrow$$

D, L, N should occur in 1st four positions, but we have D, L, N, N.

So, ways of arranging D, L, N, N in 1st four positions

$$= \frac{4!}{2!} = 12$$

Ways of arranging remaining E, E, A, O, E in last five positions

$$= \frac{5!}{3!} = 20$$

,

Total

$$= 12 \times 20 = 240 = 2 \times 5!$$

C - iv

D A, E, O occur in odd positions.

No. of odd positions = 5 and letters are E, E, E, A, O. i.e. 5

Ways of arranging those 5 letters in 5 odd positions

$$= \frac{5!}{3!} = 20$$

Remaining 4 letters D, L, N, N can be arranged in remaining 4 positions in

$$= \frac{4!}{2!} = 12$$

ways

Total no. of permutations

$$= 20 \times 12 = 240 = 2 \times 5!$$

D - iii

Question 040 MCQ

QUESTION

Let

$$a,\ b, c$$

,

$$p, q$$

be real numbers. Suppose

$$\alpha,\ \beta$$

are the roots of the equation

$$x^2 + 2px + q = 0$$

and

$$\alpha, \frac{1}{\beta}$$

are the roots of the equation

$$ax^2 + 2bx + c = 0,$$

where

$$\beta^2 \in \{-1, 0, 1\}$$

STATEMENT - 1 :

$$\left(p^2 - q\right)\left(b^2 - ac\right) \geq 0$$

and **STATEMENT - 2 :**

$$b \neq pa$$

or

$$c \neq qa$$

A. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1

B. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

C. STATEMENT - 1 is True, STATEMENT - 2 is False

D. STATEMENT - 1 is False, STATEMENT - 2 is True

CORRECT OPTION

B. STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

As a, b, c, p, q

$$\in$$

R and the two given equations have exactly one common root.

$$\Rightarrow$$

Either both equations have real roots.

Or both equations have imaginary roots.

$$\Rightarrow$$

Either

$$\Delta_1 \geq 0$$

and

$$\Delta_2 > 0$$

or

$$\Delta_1 < 0$$

and

$$\Delta_2 < 0$$

$$\Rightarrow\ p^2 - q \geq 0$$

and

$$b^2 - ac \geq 0$$

or

$$p^2 - q \leq 0$$

and

$$b^2 - ac \leq 0$$

$$\Rightarrow\ (p^2 - q)(b^2 - ac) \geq 0$$

$$\therefore$$

Statement 1 is true.

Also, we have

$$\alpha\beta = q$$

and

$$\frac{\alpha}{\beta} = \frac{c}{a}$$

$$\therefore$$

$$\frac{\alpha\beta}{\frac{\alpha}{\beta}} = \frac{q}{c} \times a \Rightarrow \beta^2 = \frac{qa}{c}$$

As

$$\beta \neq 1$$

or

$$-1$$

$$\Rightarrow \beta^2 \neq 1$$

$$\Rightarrow \frac{qa}{c} \neq 1$$

or

$$c \neq qa$$

Again as exactly one root

$$\alpha$$

is common and

$$\beta \neq 1$$

$$\therefore$$

$$\alpha + \beta \neq \alpha + \frac{1}{\beta} \Rightarrow \frac{-2b}{a} \neq -2P$$

$$\Rightarrow b \neq ap$$

$$\therefore$$

Statement 2 is correct.

But Statement 2 is not correct explanation of Statement 1.

Question 041 MCQ

QUESTION

Consider three points

$$P = (-\sin(\beta - \alpha), -cos\beta), Q = (cos(\beta - \alpha), \sin\beta)$$

and

$$R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$$

where

$$0 < \alpha, \beta, \theta < \frac{\pi}{4}$$

. Then :

A P lies on the line segment RQ

B Q lies on the line segment PR

C R lies on the line segment QP

D P, Q, R are non-collinear

CORRECT OPTION

D P, Q, R are non-collinear

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

$$\Delta = \begin{vmatrix} \cos(\beta-\alpha) & \sin\beta & 1 \\ -\sin(\beta-\alpha) & -\cos\beta & 1 \\ \cos(\beta-\alpha-\theta) & \sin(\beta-\theta) & 1 \end{vmatrix}$$

$$R_3 \to R(\cos\theta R_1 + \sin\theta R_2)$$

$$= \begin{vmatrix} \cos(\beta-\alpha) & \sin\beta & 1 \\ -\sin(\beta-\alpha) & -\cos\beta & 1 \\ 0 & 0 & 1-(\cos\theta+\sin\theta) \end{vmatrix}$$

$$= -[1-(\cos\theta+\sin\theta)]\cos(2\beta-\alpha) \neq 0$$

$$= 2\beta < \frac{\pi}{2}$$

and

$$\alpha > 0$$

$$\Rightarrow (2\beta-\alpha) < \frac{\pi}{2} \Rightarrow \cos(2\beta-\alpha) \neq 0$$

Also, since

$$0 < \theta < \frac{\pi}{2} \Rightarrow \cos\theta + \sin\theta > 1$$

$$\Delta \neq 0$$

$$\Rightarrow$$

P, Q and R are not collinear.

Question 042 MCQ

QUESTION

An experiment has 10 equally likely outcomes. Let A and B be two non-empty events of the experiment. If A consists of 4 outcomes, the number of outcomes that B must have so that A and B are independent is :

A 2, 4 or 8

B 3, 6 or 9

C 4 or 8

D 5 or 10

CORRECT OPTION

D 5 or 10

SOURCE

Mathematics • probability

EXPLANATION

Given,

An experiment has 10 equally likely outcomes,

Let B have n no of outcomes,

So,

$$P(B) = \frac{n}{10}$$

$$P(A) = \frac{4}{10}$$

$$P(A \cap B) = \frac{4}{10} \times \frac{n}{10} = \frac{\frac{2n}{5}}{10}$$

Since, A and B are independent

n is an integer

n = 5 or 10

Question 043 MCQ

QUESTION

Consider the lines given by:

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

Match the Statement/Expressions in Column I with the Statements/Expressions in Column II.

	Column I		Column II
A	L $_1$, L $_2$, L $_3$ are concurrent, if	P	$K = -9$

	Column I		Column II
B	One of L $_1$, L $_2$, L $_3$ is parallel to atleast one of the other two, if	Q	$K = -\frac{6}{5}$
C	L $_1$, L $_2$, L $_3$ form a triangle, if	R	$K = \frac{5}{6}$
D	L $_1$, L $_2$, L $_3$ do not form a triangle, if	S	$K = 5$

A A - iv; B - ii; C - iii; D - i, ii

B A - iv; B - i, ii; C - iii; D - i, ii, iv

C A - iv; B - i; C - iii; D - i, ii

D A - ii; B - i, iii; C - iii; D - i, ii, iv

CORRECT OPTION

B A - iv; B - i, ii; C - iii; D - i, ii, iv

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

We have,

A

$$L_1 : x + 3y - 5 = 0$$

$$L_2 : 3x - ky - 1 = 0$$

$$L_3 : 5x + 2y - 12 = 0$$

Point of intersection of L

$$1$$

and L

$$2$$

is $2, 1$

If lines are concurrent, then $2, 1$ will satisfy L

$$2$$

$$\Rightarrow 3(2) - k(1) - 1 = 0$$

$$k = 5$$

A

$$\rightarrow$$

iv

B If L

$$1$$

and L

$$2$$

are parallel

$$\frac{3}{1} = \frac{-k}{3} \neq \frac{-1}{-5} \Rightarrow k - 9$$

If L

$$2$$

and L

$$3$$

are parallel

$$\frac{3}{5} = \frac{-k}{2}$$

$$k = \frac{-6}{5}$$

B

$$\rightarrow$$

i, ii

C As lines form a triangle, they cannot be concurrent and no two of them are parallel.

$$\Rightarrow k \neq 5, -9, \frac{-6}{5}$$

$$\Rightarrow k = \frac{5}{6}$$

C

$$\rightarrow$$

iii

D If the lines do not form a triangle then

$$k = 5, -9, \frac{-6}{5}$$

D

$\rightarrow$

i, ii, iv

Question 044 MCQ

QUESTION

Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

	Column I		Column II
A	The minimum value of $\frac{x^2 + 2x + 4}{x + 2}$ is	P	0
B	Let A and B be 3 $\times$ 3 matrices of real numbers, where A is symmetric, B is skew-symmetric and $A + B$ $A\$\$ - \$\$B$ = $A\$\$ - \$\$B$ $A + B$. If AB^t = $\$\$ - 1\$\$^k$	Q	1

	Column I		Column II
	AB, where AB t is the transpose of the matrix AB, then the possible values of k are		
C	Let $a = \log_3 \log_3 2$. An integer k satisfying $1 < 2^{(-k+3-a)} < 2$, must be less than	R	2
D	If $\sin\theta = \cos\varphi$, then the possible values of $\frac{1}{\pi}\left(\theta + \varphi - \frac{\pi}{2}\right)$ are	S	3

A. A - iii; B - ii, iv; C - iii, iv; D - i, iii

B. A - iii; B - ii; C - iii, iv; D - i, iii

C. A - ii; B - ii, iv; C - iii, iv; D - i

D. A - ii; B - ii, iv; C - iii, iv; D - i, iii

CORRECT OPTION

A A - iii; B - ii, iv; C - iii, iv; D - i, iii

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

A Let

$$y = \frac{x^2 + 2x + 4}{x + 2}$$

$$\frac{dy}{dx} = \frac{x^2 + 4x}{(x+2)^2} = 0$$

$$x = 0, -4$$

$$\frac{d^2y}{dx^2} = \frac{8}{(x+2)^3}$$

At

$$x = 0, \frac{d^2y}{dx^2}$$

is true

$$\therefore$$

y is min. when

$$x = 0$$

$$\therefore$$

$$y_{\min} = 2$$

A - iii

B As A is symmetric and B is skew symmetric matrix

We should have

A

+

= A and B

+

=

—

B i

Also given that

$A + B\ \ A\$\$ - \$\$B$ = $A\$\$ - \$\$B\ \ A + B$

A

2

—

AB + BA

—

B

2

= A

2

+ AB

—

AB

—

B

2

2BA = 2AB or AB = BA ii

Now, given that

AB

t

= $$ − $$1

k

AB

BA

t

= $$ − $$1

k

AB $Usingequation(i)$

$\Rightarrow$

K should be an odd no.

$\therefore$

B - ii, iv

C Given that,

$$a = \log_3 \log_3 2$$

$$\Rightarrow \log_3 2 = 3^a \Rightarrow \frac{1_x}{\log_2{}^3} = 3^a$$

Or

$$\log_2{}^3 = 3^{-a}$$

$$3 = 2^{(3-a)}$$

Now,

$$< 2^{(-k+3-a)} < 2 \Rightarrow 1 < 2^{-2}.\ 2^{3-a} < 2$$

$$\Rightarrow 1 < 2^{-k}\ .\ 3 < 2$$

$usingeq.(i)$

$$= \frac{1}{3} . < 2^{-k} < \frac{2}{3} \Rightarrow \frac{3}{2} < 2^{k} < 3$$

$$\Rightarrow k = 1$$

$$\therefore$$

k is less than 2 and 3.

$$\therefore$$

C - iii, iv

D Given that,

$$\sin\theta = \cos\phi$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\phi$$

$$= \frac{\pi}{2} - \theta = 2n\pi \pm \phi, n \in Z$$

$$\Rightarrow \theta \pm \phi - \frac{\pi}{2} = -2n\pi$$

$$= \frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right) = -2n$$

$$\therefore$$

Here, possible value of

$$\frac{1}{\pi}\left(\theta \pm \phi - \frac{\pi}{2}\right)$$

are 0 and 2 for

$$n = 0, -1$$

$$\therefore$$

D - p, r

Question 045 MCQ

QUESTION

STATEMENT-1: For an observer looking out through the window of a fast moving train, the nearby objects appear to move in the opposite direction to the train, while the distant objects appear to be stationary.

STATEMENT-2: If the observer and the object are moving at velocities

$$\overrightarrow{v_1}$$

and

$$\overrightarrow{v_2}$$

respectively with reference to a laboratory frame, the velocity of the object with respect to the observer is

$$\overrightarrow{v_2}$$

-

$$\overrightarrow{v_1}$$

.

A STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT ,- 1

B STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

C STATEMENT - 1 is True, STATEMENT - 2 is False

D STATEMENT - 1 is False, STATEMENT - 2 is True

CORRECT OPTION

B STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is NOT a correct explanation for STATEMENT - 1

SOURCE

Physics • motion

EXPLANATION

Statement 1 is True.

For a moving observer, the nearly objects appear to move in the opposite direction at a large speed. This is because the angular speed of the nearby object w.r.t to observer is large. As the object moves away, the angular velocity decreases and therefore its speed seems to be less. The distant object almost remains stationary.

Statement 2 is True but related to relative velocity

$$\overline{V}_{21} = \overline{V}_{2G} - \overline{V}_{1G}$$

Where, G = Laboratory frame

Thus, both the statement are true but statement 2 is not the correct explanation of statement 1.

Question 046 MCQ

QUESTION

Consider a system of three charges

$$\frac{q}{3}, \frac{q}{3}$$

and

$$-\frac{2q}{3}$$

placed at points A, B and C, respectively, as shown in the figure. Take O to be the centre of the circle of radius R and angle CAB = 60

A The electric field at point O is

$$\frac{q}{8\pi\varepsilon_0 R^2}$$

directed along the negative x-axis

B The potential energy of the system is zero

C The magnitude of the force between the charges at C and B is

$$\frac{q^2}{54\pi\varepsilon_0 R^2}$$

D The potential at point O is

$$\frac{q}{12\pi\varepsilon_0 R}$$

CORRECT OPTION

C The magnitude of the force between the charges at C and B is

$$\frac{q^2}{54\pi\varepsilon_0 R^2}$$

SOURCE

Physics • electrostatics

EXPLANATION

Electric field at point O is vector sum of electric field due to all three point charges individually.

$$\overrightarrow{E} = \overrightarrow{E_A} + \overrightarrow{E_B} + \overrightarrow{E_C}$$

and

$$\overrightarrow{E_A} = -\overrightarrow{E_B}$$

So,

$$\overrightarrow{E} = \overrightarrow{E_C} = \frac{2q/3}{4\pi\varepsilon_0 . R^2}$$

along

$$\overrightarrow{OC}$$

$$= \frac{q}{6\pi\varepsilon_0^2 . R^2}$$

along

$$\overrightarrow{OC}$$

$$= \frac{+q'}{6\pi\varepsilon_0^2 . R^2}\hat{i}$$

$$= \frac{-q'}{6\pi\varepsilon_0 . R^2}\hat{j}$$

From geometry, we can find that

$$\angle$$

ABC = 30

$$\circ$$

and

$$\angle$$

ACB = 90

So, AB = 2R, AC = R, BC =

$$\sqrt{3}$$

R

Total potential energy of the system is,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q/3 \times q/3}{2R} - \frac{q/3 \times 2q/3}{R} - \frac{q/3 \times 2q/3}{\sqrt{3}R}\right] \neq 0$$

$$F = \frac{q/3 \times q/3}{4\pi\varepsilon_0 \times \left(\sqrt{3}R\right)^2} = \frac{q^2}{54\pi\varepsilon_0 R^2}$$

Potential at point O is,

$$V = \frac{q/3 \times q/3 - 2q/3}{4\pi\varepsilon_0 R} = 0$$

Question 047 MCQ

QUESTION

A radioactive sample S_1 having activity of 5

$$\mu$$

Ci has twice the number of nuclei as another sample S_2 which has an activity of 10

$$\mu$$

Ci. The half lives of S_1 and S_2 can be :

A 20 years and 5 years, respectively

B 20 years and 10 years, respectively

C 10 years each

D 5 years each

CORRECT OPTION

A 20 years and 5 years, respectively

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To solve this problem, let's begin with some fundamental concepts of radioactive decay.

The activity of a radioactive sample, which is its decay rate, is given by the equation:

$$A = \lambda N$$

where:

- A is the activity.
- λ is the decay constant.
- N is the number of radioactive nuclei present.

Given that sample S_1 has an activity of 5 μ

Ci and sample S_2 has an activity of 10

$$\mu$$

Ci, and that sample S_1 has twice the number of nuclei as sample S_2, we have:

$$A_1 = \lambda_1 N_1$$

$$A_2 = \lambda_2 N_2$$

We are also given that:

$$N_1 = 2N_2$$

Substitute

$$N_1 = 2N_2$$

into the first equation:

$$5\,\mu\text{Ci} = \lambda_1 (2N_2)$$

And for the second equation:

$$10\,\mu\text{Ci} = \lambda_2 N_2$$

Dividing the first equation by the second equation to eliminate

$$N_2$$

, we get:

$$\frac{5\,\mu\text{Ci}}{10\,\mu\text{Ci}} = \frac{\lambda_1 (2N_2)}{\lambda_2 N_2}$$

Simplifying this, we obtain:

$$\frac{1}{2} = \frac{2\lambda_1}{\lambda_2}$$

Therefore:

$$\lambda_2 = 4\lambda_1$$

Next, we can relate the decay constant to the half-life using the equation:

$$\lambda = \frac{\ln(2)}{T_{1/2}}$$

Using the relationship between the decay constants, we have:

$$\frac{\lambda_2}{\lambda_1} = \frac{T_{1/2,1}}{T_{1/2,2}}$$

Substituting

$$\lambda_2 = 4\lambda_1$$

, we get:

$$4 = \frac{T_{1/2,1}}{T_{1/2,2}}$$

Therefore:

$$T_{1/2,1} = 4T_{1/2,2}$$

Given the options, the half-lives that satisfy this relationship are:

Option A: 20 years and 5 years, respectively.

In this case:

$$\frac{20 \text{ years}}{5 \text{ years}} = 4$$

Thus, option A is correct. Therefore, the half-lives of S_1 and S_2 are 20 years and 5 years, respectively.

Question 048 MCQ

QUESTION

A transverse sinusoidal wave moves along a string in the positive x-direction at a speed of 10 cm/s. The wavelength of the waves is 0.5 m and its amplitude is 10

cm. At a particular time t, the snap-shot of the wave is shown in figure. The velocity of point P when its displacement is 5 cm is :

A $\frac{\sqrt{3\pi}}{50}\hat{j}$ m/s

B $-\frac{\sqrt{3\pi}}{50}\hat{j}$ m/s

C $\frac{\sqrt{3\pi}}{50}\hat{i}$ m/s

D $-\frac{\sqrt{3\pi}}{50}\hat{i}$ m/s

CORRECT OPTION

A $\frac{\sqrt{3\pi}}{50}\hat{j}$ m/s

SOURCE

Physics • waves

EXPLANATION

Velocity of point

$$p = dy/dt$$

=

$$-$$

$velocityofwave\$\$ \times \$\$\$\$dy/dx\$\$$

Here,

$$dy/dx =$$

negative $\$\$ - \$\$ve$. Therefore, velocity at point P is positive and is along X-axis only.

Equation of a wave moving in positive X-axis is given as,

$$y = A\sin(wt - \phi)$$

.... i

or,

$$V_P = Aw\cos(wt - \phi)$$

.... ii

Here,

$$y' = 5$$

cm,

$$A = 10$$

cm

$$\therefore$$

$$5 = 10\sin(wt - \phi) \Rightarrow wt - \phi = 30^\circ$$

.... iii

Put equation iii in equation ii,

$$V_P = 0.10 \times w \cos 30^\circ$$

Now,

$$V = V_\lambda$$

,

$$\therefore$$

$$v = V/\lambda = \frac{0.10}{0.5} = 0.2$$

$$\therefore$$

$$w = 2\pi v = 2\pi \times 0.2 = 0.4\pi$$

$$\Rightarrow V_P = 0.1 \times 0.4\pi \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{50}\pi\hat{j}$$

m/s

It is in positive Y-direction.

Question 049 MCQ

QUESTION

A block B is attached to two unstretched springs S1 and S2 with spring constants k and 4k respectively $see figure I$. The other ends are attached to identical supports M1 and M2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block displaced towards wall 1 by a small distance x $figure II$ and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio

$$\frac{y}{x}$$

is :

A 4

B 2

C $\frac{1}{2}$

D $\frac{1}{4}$

CORRECT OPTION

C $\frac{1}{2}$

SOURCE

Physics • work-power-and-energy

EXPLANATION

When the block B is displaced towards wall-1 only spring S

$_1$

is compressed and S

$_2$

is in natural state.

If spring S

$_2$

is not attached to wall but it is free but it gain momentum $$ \uparrow $$ and moves towards S

1

.

The spring S

1

comes to natural length and spring S

2

gets compressed.

So if there is no frictional force i.e.,

$$fr = 0$$

So potential energy of spring S

1

as the potential energy of spring.

Hence,

$$(PE)_{S_1} = (PE)_{S_2}$$

$$\Rightarrow \frac{1}{2}k_1x^2 = \frac{1}{2}k_2y$$

$$\Rightarrow 1kx^2 = (4k)y^2 \Rightarrow x^2 = 4y^2$$

Potential energy of spring.

Hence,

$$(PE)_{S_1} = (PE)_{S_2}$$

$$\Rightarrow \frac{1}{2}k_1x^2 = \frac{1}{2}k_2y^2$$

$$\Rightarrow \frac{1}{2}kx^2 = \frac{1}{2}(4k)y^2$$

$$\Rightarrow x^2 = 4y^2$$

$$\Rightarrow \frac{y}{x} = \frac{1}{2}$$

Question 050 MCQ

QUESTION

A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle

$$\theta$$

at which the speed of the bob is half of that at A, satisfies,

A $\theta = \frac{\pi}{4}$

B $\frac{\pi}{4} < \theta < \frac{\pi}{2}$

C $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$

D $\frac{3\pi}{4} < \theta < \pi$

CORRECT OPTION

D $$\frac{3\pi}{4} < \theta < \pi$$

SOURCE

Physics • work-power-and-energy

EXPLANATION

The bob of mass m requires velocity to reach at a point with vertical circular motion.

$$V = \sqrt{5gl}$$

Then,

$$h = L - L\cos\theta = L(1 - \cos\theta)$$

Applying law of conservation of energy at point A & C

Kinetic energy $K.E_A$ + Potential energy (U_A) = Kinetic energy ($K.E_C$) + Potential energy (U_C)

$$\Rightarrow \frac{1}{2}mV^2 + 0 = \frac{1}{2}m\left(\frac{\sqrt{5gl}}{2}\right)^2 + mgL(1-\theta)$$

$$\Rightarrow \frac{5L}{2} = \frac{5L}{8} + L(1 - \cos\theta)$$

$$\because V = \sqrt{5gl}$$

$$\Rightarrow \frac{5}{2} - \frac{5}{8} = 1 - \cos\theta$$

$$\Rightarrow \frac{20-5}{8} = 1 - \cos\theta$$

$$\Rightarrow \cos\theta = 1 - \frac{15}{8}$$

$$\Rightarrow \cos\theta = \frac{-7}{8}$$

$$\Rightarrow \theta = 152^\circ$$

$$\therefore$$

$$\frac{3\pi}{4} < \theta < \pi$$

Question 051 MCQ

QUESTION

A glass tube of uniform internal radius r has a valve separating the two identical ends. Initially, the valve is in a tightly closed position. End 1 has a hemispherical soap bubble of radius r. End 2 has sub-hemispherical soap bubble as shown in figure. Just after opening the valve,

A air from end 1 flows towards end 2. No change in the volume of the soap bubbles

B air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

C no change occurs

D air from end 2 flows towards end 1. Volume of the soap bubble at end 1 increases

CORRECT OPTION

B air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases

SOURCE

Physics • properties-of-matter

EXPLANATION

Given that,

$$r_2 > r_1$$

Just inside of soap bubble, the pressure

$$P_1 = P_0 + \frac{4T}{r_1}$$

..... i

$$P_2 = P_0 + \frac{4T}{r_2}$$

.... ii

Hence,

$$r_2 > r_1$$

then

$$P_1 > P_2$$

Therefore, air from end 1 flows towards end 2. Volume of the soap bubble at end 1 decreases.

Question 052 MCQ

QUESTION

A vibrating string of certain length 1 under a tension T resonates with a mode corresponding to the first overtone $thirdharmonic$ of an air column of length 75 cm inside a tube closed at one end. The string also generates 4 beats per second when excited along with a tuning fork of frequency n. Now when the tension of the string is slightly increased the number of beats reduces to 2 per

second. Assuming the velocity of sound in air to be 340 m/s, the frequency n of the tuning fork in Hz is:

A 344

B 336

C 117.3

D 109.3

CORRECT OPTION

A 344

SOURCE

Physics • waves

EXPLANATION

Let f be frequency of string

According to close open pipe, we have

$$\Rightarrow f = \frac{3V}{4l} = \frac{3 \times 340}{4 \times 75 \times 10^{-2}}$$

$$= 340$$

Hz

Where,

$$V = 340$$

m/s

$$l = 75$$

cm

$$\times 10^{-2}$$

m

$$= 75 \times 10^{-2}$$

m

And, n = frequency of tuning fork

$$= (340 + 4)$$

or

$$(340 - 4)$$

$$\Rightarrow n = 344$$

or 336

As tension T increase, beat frequency decrease

But frequency of tuning fork increases

$$\therefore$$

$$n = 344$$

Hz

Question 053 MCQ

QUESTION

A parallel plate capacitor C with plates of unit area and separation d is filled with a liquid of dielectric constant K = 2. The level of liquid is

$$\frac{d}{3}$$

initially. Suppose the liquid level decreases at a constant speed V, the time constant as a function of time t is:

A $$\frac{6\varepsilon_0 R}{5d+3Vt}$$

B $$\frac{(15d+9Vt)\varepsilon_0 R}{2d^2-3dVt-9V^2t^2}$$

C $$\frac{6\varepsilon_0 R}{5d-3Vt}$$

D $$\frac{(15d-9Vt)\varepsilon_0 R}{2d^2+3dVt-9V^2t^2}$$

CORRECT OPTION

A $$\frac{6\varepsilon_0 R}{5d+3Vt}$$

SOURCE

Physics • electrostatics

EXPLANATION

Time constant $$\tau$$ = RC

$$eq$$

Where R = Resistance

C

$$eq$$

= equivalent capacitance

And,

$$v = \frac{-dx}{dt} \Rightarrow dx = -vdt$$

$$\Rightarrow \int_{d/3}^{x} dx = -v \int_{0}^{t} dt$$

$$\Rightarrow [x]_{d/3}^{x} = -v[t]_{0}^{t}$$

$$x - \frac{d}{3} = -vt$$

$$x = \frac{d}{3} - vt$$

..... i

And also,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\varepsilon_0 A}{(d-x)}} + \frac{1}{\frac{\varepsilon_0 Ak}{x}}$$

$$\frac{1}{C_{eq}} = \frac{d-x}{\varepsilon_0 A} + \frac{x}{\varepsilon_0 Ak}$$

$$= \frac{1}{\varepsilon_0 A}\left[d - x + \frac{x}{k}\right]$$

$$= \frac{1}{\varepsilon_0 A}\left[\frac{(d-x)k + x}{k}\right]$$

$$\frac{1}{C_{eq}} = \frac{1}{\varepsilon_0 Ak}[dk - xk + x]$$

$$\therefore$$

$$C_{eq} = \frac{\varepsilon_0 A}{kd + x(1-k)}$$

.... ii

Given,

$$A = 1, k = 2, x = \frac{d}{3} - vt$$

Hence,

$$C_{eq} = \frac{\varepsilon_0 \times 1}{2d + \left[\frac{d}{3} - vt\right](1-2)}$$

$$= \frac{6\varepsilon_0}{5d + 3vt}$$

Therefore,

$$\tau = RC_{eq} = \frac{6R\varepsilon_0}{5d + 3vt}$$

Question 054 MCQ

QUESTION

A light beam is travelling from Region I to Region IV $Refer figure$. The refractive index in Regions I, II, III and IV are

$$n_0, \frac{n_0}{2}, \frac{n_0}{6}$$

and

$$\frac{n_0}{8}$$

, respectively. The angle of incidence

$$\theta$$

for which the beam just misses entering Region IV is

A $\sin^{-1}\left(\frac{3}{4}\right)$

B $\sin^{-1}\left(\frac{1}{8}\right)$

C $\sin^{-1}\left(\frac{1}{4}\right)$

D $\sin^{-1}\left(\frac{1}{3}\right)$

CORRECT OPTION

B $\sin^{-1}\left(\frac{1}{8}\right)$

SOURCE

Physics • geometrical-optics

EXPLANATION

From Snell's laws

$$\mu_1 \sin\theta = \mu_2 \sin\theta_2$$

Case I : Region I

$$n_0 \sin\theta = \frac{n_0}{2} \sin\alpha$$

..... i

Case II : Region II

$$\frac{n_0}{2}\sin\alpha = \frac{n_0}{6}\sin\theta_c$$

..... ii $$\therefore \beta = \theta_c$$

Case II : Region III

$$\frac{n_0}{6}\sin\theta_c = \frac{n_0}{8}\sin 90^\circ$$

$$\Rightarrow \frac{n_0}{6}\sin\theta_c = \frac{n_0}{8}$$

..... iii

From equation i, ii and iii, we have,

$$n_0 \sin\theta = \frac{n_0}{8}$$

$$\Rightarrow \sin\theta = \frac{1}{8}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{8}\right)$$

Question 055 MCQ

QUESTION

STATEMENT 1 : It is easier to pull a heavy object than to push it on a level ground.

and

STATEMENT 2 : The magnitude of frictional force depends on the nature of the two surfaces in contact.

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1

SOURCE

Physics • laws-of-motion

EXPLANATION

Case I : Free body diagram.

Frictional force $$fr$$ =

$$\mu$$

N

Normal force N = N

$$1$$

+

$$mg$$

Frictional force $$fr$$

$$I$$

=

$$\mu$$

$$N$$$$_1$$$$ + $$mg$$$$ ..... i$$

Where N = Normal force

$$\mu$$

= Coefficient of friction

$$m$$

= mass of object

$$g$$

= acc. due to gravity

N

$$1$$

= Normal force i.e., vertical component of force due to push.

N

$$2$$

= Parallel component of force due to push.

Case II : Free body diagram $$Pull$$

For balancing the force,

N = N

$$1$$

$$-$$

$$mg$$

Frictional force

$$(fr)$$

$$II$$

=

$$\mu$$

N ii

From eq. i and ii

$$(fr)$$

I

>

$$(fr)$$

II

In case of pull, it is easy to move object.

Statement 1 is true. Statement 2 is true.

Force of friction depends on roughness of two surfaces in contact. If roughness will increase, the force of friction will also increase.

Question 056 MCQ

QUESTION

STATEMENT 1 : For practical purposes, the earth is used as a reference at zero potential in electrical circuits.

and

STATEMENT 2 : The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by

$$\frac{Q}{4\pi\varepsilon_0 R}$$

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1

SOURCE

Physics • electrostatics

EXPLANATION

Statement 1 is True.

The earth is used as a reference with zero potential. The reasons are :

i Due to large size of the earth, its potential does not change even if a small amount of charge is given to the earth or taken away from it. Also, all conductors on the earth which are not given any external charge, are very nearly at the potential V_e. The choice of reference is quite arbitrary and is largely governed by convenience.

ii The earth is a good conductor. It is easily accessible for electrical circuits geographically distributed over it and hence it is used as a common reference with zero potentil.

Statement 2 is true.

V =

$$\frac{Q}{4\pi\varepsilon_0 R}$$

Therefore, statement 2 is correct explanation for statement 1.

Question 057 MCQ

QUESTION

STATEMENT 1 : The sensitivity of a moving coil galvanometer is increased by placing a suitable magnetic material as a core inside the coil.

and

STATEMENT 2 : Soft iron has a high magnetic permeability and cannot be easily magnetized or demagnetized.

A Statement 1 is True, Statement 2 is True; Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True; Statement 2 is a NOT CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

C Statement 1 is True, Statement 2 is False

SOURCE

Physics • magnetism

EXPLANATION

Statement 1 is true.

We know that,

Sensitivity

$$= \frac{\theta}{I} = \frac{NBA}{C}$$

If we place soft iron in magnetic field it get magnetise so magnetic field increases. Hence, sensitivity of a moving coil will also increase.

Therefore,

$$\overrightarrow{B}$$

the

$$\frac{\theta}{I}$$

also increases.

When we place any suitable magnetic material inside the core, the magnetic field will increase.

Statement 2 is wrong because soft iron can be easily magnetized and de magnetized.

Question 058 MCQ

QUESTION

The electric field at r = R is :

A independent of a

B directly proportional to a

C directly proportional to a

$$2$$

D inversely proportional to a

CORRECT OPTION

A independent of a

SOURCE

Physics • electrostatics

EXPLANATION

At

$$r = R$$

,

By applying Gauss law,

$$\phi = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow \oint E.ds = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E \oint ds = \frac{q_{enclosed}}{\varepsilon_0}$$

$$\Rightarrow E.4\pi R^2 = \frac{Ze}{\varepsilon_0}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Ze}{R^2}$$

i.e.,

$$E\,\mu\frac{1}{R^2}$$

Here, electric field is independent of 'a'.

Question 059 MCQ

QUESTION

For a = 0, the value of d $maximumvalueof\$\$\rho\$\$asshowninthefigure$ is

A $\frac{3Ze}{4\pi R^3}$

B $\frac{3Ze}{\pi R^3}$

C $\frac{4Ze}{3\pi R^3}$

D $$\frac{Ze}{3\pi R^3}$$

CORRECT OPTION

B $$\frac{3Ze}{\pi R^3}$$

SOURCE

Physics • electrostatics

EXPLANATION

For a = 0, the graph is as shown, the equation for the graph line is

The charge in the dotted element shown in figure is

$$dq = \sigma \times 4\pi r^2 dr$$

$$\therefore$$

$$dq = \left(d - \frac{d}{R}r\right)4\pi r^2 dr$$

$$\Rightarrow Ze = \int_0^R 4\pi d r^2 dr - \int_0^R \frac{4\pi d}{R} r^3 dr$$

$$Ze = 4\pi d\frac{R^3}{3} - \frac{4\pi d}{R}\frac{R^4}{4}$$

$$\therefore$$

$$\frac{Ze}{4\pi d R^3} = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\therefore$$

$$d = \frac{3Ze}{\pi R^3}$$

Question 060 MCQ

QUESTION

The electric field within the nucleus is generally observed to be linearly dependent on r. This implies

A $a = 0$

B $a = \frac{R}{2}$

C $a = R$

D $a = \frac{2R}{3}$

CORRECT OPTION

C $a = R$

SOURCE

Physics • electrostatics

EXPLANATION

The electric field inside a uniformly charged sphere is proportional to r. The volume charge density inside a uniformly charged sphere is constant and hence a = R.

Question 061 MCQ

QUESTION

The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is :

A $-kx$

B $-2kx$

C $-\frac{2kx}{3}$

D $-\frac{4kx}{3}$

CORRECT OPTION

D

$$-\frac{4kx}{3}$$

SOURCE

Physics • rotational-motion

EXPLANATION

When the disc is at a distance x from the mean position $equilibriumposition$, the force acting on the disc are give below.

$$\therefore -2kx + f = -M_{ac}$$

..... i

Where

$$a_c$$

= acceleration of centre of mass.

The torque acting on the disc about its centre of mass C is

$$\tau = f \times R = I \times a_c$$

$$\therefore$$

$$f = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2}{R} \times \frac{a_c}{R}$$

$$\$\$ \therefore \$\$\$\$I = \frac{1}{2}MR^2, a_c = Ra_c\$\$forrollingwithoutslipping$$

$$\therefore$$

$$f = \frac{1}{2}Ma_c$$

..... ii

From eq. i and ii,

$$-2kx + \frac{1}{2}Ma_c = -Ma_c$$

$$\Rightarrow \frac{3}{2}Ma_c = 2kx$$

$$\Rightarrow Ma_c = \frac{4kx}{3}$$

$$\Rightarrow$$

Net external force acting on the disc when its centre of mass is at displacement x with respect to the equilibrium position

$$= \frac{-4kx}{3}$$

directed towards the equilibrium.

Question 062 MCQ

QUESTION

The centre of mass of the disk undergoes simple harmonic motion with angular frequency

$$\omega$$

equal to:

A $\sqrt{\frac{k}{M}}$

B $\sqrt{\frac{2k}{M}}$

C $\sqrt{\frac{2k}{3M}}$

D $\sqrt{\frac{4k}{3M}}$

CORRECT OPTION

D $\sqrt{\frac{4k}{3M}}$

SOURCE

Physics • rotational-motion

EXPLANATION

When the disc is at a distance x from the mean position $equilibriumposition$, the force acting on the disc are give below.

$$\therefore -2kx + f = -M_{ac}$$

..... i

Where

$$a_c$$

= acceleration of centre of mass.

The torque acting on the disc about its centre of mass C is

$$\tau = f \times R = I \times a_c$$

$$\therefore$$

$$f = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2}{R} \times \frac{a_c}{R}$$

$$\$\$ \therefore \$\$\$\$I = \frac{1}{2}MR^2, a_c = Ra_c \$\$for rolling without slipping$$

$$\therefore$$

$$f = \frac{1}{2}Ma_c$$

..... ii

From eq. i and ii,

$$-2kx + \frac{1}{2}Ma_c = -Ma_c$$

$$\Rightarrow \frac{3}{2}Ma_c = 2kx$$

$$\Rightarrow Ma_c = \frac{4kx}{3}$$

$$\Rightarrow$$

Net external force acting on the disc when its centre of mass is at displacement x with respect to the equilibrium position

$$= \frac{-4kx}{3}$$

directed towards the equilibrium.

$$\therefore$$

$$|F_{net}| = \frac{4k}{3}x$$

For S.H.M

$$|F_{net}| = M\omega^2 x$$

$$\therefore$$

$$M\omega^2 = \frac{4k}{3}$$

$$\Rightarrow \omega = \sqrt{\frac{4k}{3M}}$$

.... iii

Question 063 MCQ

QUESTION

The maximum value of V

$$0$$

for which the disk will roll without slipping is:

A $\mu g\sqrt{\frac{M}{k}}$

B $\mu g\sqrt{\frac{M}{2k}}$

C $\mu g\sqrt{\frac{3M}{k}}$

D $\mu g\sqrt{\frac{5M}{2k}}$

CORRECT OPTION

C

$$\mu g\sqrt{\frac{3M}{k}}$$

SOURCE

Physics • rotational-motion

EXPLANATION

When the disc is at a distance x from the mean position $equilibriumposition$, the force acting on the disc are give below.

$$\therefore -2kx + f = -M_{ac}$$

..... i

Where

$$a_c$$

= acceleration of centre of mass.

The torque acting on the disc about its centre of mass C is

$$\tau = f \times R = I \times a_c$$

$$\therefore$$

$$f = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2}{R} \times \frac{a_c}{R}$$

$$\$\$ \therefore \$\$\$\$I = \frac{1}{2}MR^2, a_c = Ra_c\$\$forrollingwithoutslipping$$

$$\therefore$$

$$f = \frac{1}{2}Ma_c$$

..... ii

From eq. i and ii,

$$\Rightarrow -2kx + f = -2f$$

$$\Rightarrow f = \frac{2k}{3} \times x$$

Frictional force depends on x. As x increase friction increase. Frictional force f is maximum at

$$x = A$$

.

Where A = amplitude of S.H.M

Maximum frictional force,

$$f_{\max} = \frac{2k}{3} \times A$$

The force should be almost equal to the limiting friction $$$\mu mg$$$ for rolling without slipping.

$$\therefore$$

$$\mu mg = \frac{2k}{3} \times A$$

..... iv

For S.H.M

Velocity amplitude

$$= A\omega$$

$$\therefore$$

$$V_0 = A\omega$$

$$\therefore$$

$$V_0 = \frac{3m\,mg}{2k} w$$

$from (iv)$

$$\therefore$$

$$V_0 = \frac{3\mu mg}{2k} \times \sqrt{\frac{4k}{3M}}$$

$As\$\$\omega = \sqrt{\frac{4k}{3M}}\$\$$

$$\therefore$$

$$V_0 = mg\sqrt{\frac{3M}{k}}$$

Question 064 MCQ

QUESTION

Column I gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in Column II. Match the set of parameters given in Column I with the graphs given in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$$\times$$

4 matrix given in the ORS.

	Column I		Column II
A	Potential energy of a simple pendulum $y-axis$ as a function of displacement x axis	P	
B	Displacement $y-axis$ as a function of time $x-axis$ for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction	Q	
C	Range of a projectile $y-axis$ as a function of its velocity $x-axis$ when projected at a fixed angle	R	

	Column I		Column II
D	The square of the time period $y-axis$ of a simple pendulum as a function of its length $x-axis$	S	

A

$\rightarrow$

P, S; B

$\rightarrow$

A Q, S; C

$\rightarrow$

S; D

$\rightarrow$

Q

A

$\rightarrow$

S; B

$\rightarrow$

B Q, S; C

$\rightarrow$

S; D

$\rightarrow$

Q, S

A

$\rightarrow$

P, S; B

$\rightarrow$

C Q; C

$\rightarrow$

S; D

$\rightarrow$

Q, S

A

$\rightarrow$

S; B

$\rightarrow$

D Q, S; C

$\rightarrow$

S, P; D

$\rightarrow$

Q

CORRECT OPTION

A

$\rightarrow$

P, S; B

$\rightarrow$

A Q, S; C

$\rightarrow$

S; D

$\rightarrow$

Q

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

A

Potential energy at point A. U

A

= 0,

Kinetic Energy K

A

= maximum

Potential energy at point B, U_B = maximum,

$K_B = 0$

The graph of potential energy as function of displacement of a simple pendulum will be parabolic graph as given in option p. Hence A

$\rightarrow$

P. The choice S is also correct if the mean position of the pendulum is at the origin. Hence A

$\rightarrow$

$P, S.$

P

S

B

Slope of displacement time graph gives velocity.

$\therefore$

$$v = \frac{ds}{dt}$$

If

$$\vec{a} = 0$$

or constant

$$v = u + at$$

$$v = 0 + at$$

$$\therefore$$

$$a = v/t$$

$$a =$$

constant

S

$$(\mathrm{C}) \quad R = \frac{v^2 \sin 2\theta}{g}$$

$$\therefore \quad R \propto v^2 \rightarrow \text{option}(\mathrm{S})$$

D

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore$$

$$T \propto \sqrt{l}$$

$$T^2 \propto l$$

is a straight line

Question 065 MCQ

QUESTION

An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A		P	Real Image
B		Q	Virtual Image
C		R	Magnified Image
D		S	Image at infinity

A

$\rightarrow$

P, Q, R; B

$\rightarrow$

A Q; C

$\rightarrow$

P, R, S; D

$\rightarrow$

P, Q, R, S

A

$\rightarrow$

P, Q, R, S; B

$\rightarrow$

B Q; C

$\rightarrow$

P, Q, R, S; D

$\rightarrow$

P, Q, R, S

A

$\rightarrow$

P, Q, R; B

$\rightarrow$

C Q; C

$\rightarrow$

P, R, S; D

$\rightarrow$

P, Q, S

A

$\rightarrow$

P, Q; B

$\rightarrow$

D Q; C

$\rightarrow$

R, S; D

$\rightarrow$

P, Q, R, S

CORRECT OPTION

A

$\rightarrow$

P, Q, R, S; B

$\rightarrow$

B Q; C

$\rightarrow$

P, Q, R, S; D

$\rightarrow$

P, Q, R, S

SOURCE

Physics • geometrical-optics

EXPLANATION

A

$\rightarrow$

P, Q, R, S

$$v = \frac{f}{1 - \frac{f}{u}}, m = \frac{-v}{u}$$

For different values of u, we get different values of

v

$+ve, \$\$ - \$\$veor\$\$\infty\$\$$. Like wise

m

comes out to be positive, negative, greater than 1 etc.

B

$\rightarrow$

Q

In case of a convex mirror we always get virtual, diminished image, between P and F.

C

$\rightarrow$

P, Q, R, S

When object is at infinity, a real, inverted and diminished image is formed at F. When the object is brought closer to the lens. The image moves away from the lens and increase in size. $Imageisbetween Fand 2F$. when the object is at 2F', the image is formed at 2F. The image is real, inverted and is of the same size. When the object is moved closer between 2F' and F', the image is real, inverted and magnified. When the object is at F' the image is real, inverted, highly magnified and is formed at infinity.

D

$\rightarrow$

P, Q, R, S

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (\mu - 1)\left[\frac{R_2 - R_1}{R_1 R_2}\right]$$

Here

$$R_1 < R_2$$
$$\therefore$$
$$f$$

is positive.

Therefore, it behaves like a convex lens.

Question 066 MCQ

QUESTION

Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A	An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.	P	The temperature of the gas decreases
B	An ideal monatomic gas expands to twice its original volume such that its pressure P $\propto \frac{1}{V^2}$, where V is the volume of the gas	Q	The temperature of the gas increase or remains constant.
C	An ideal monoatomic gas expands to twice its original volume such that its pressure P $\propto \frac{1}{V^{4/3}}$, where V is its volume	R	The gas loses heat
D	An ideal monoatomic gas expands such that its	S	The gas gains heat

	Column I		Column II
	pressure P and volume V follows the behaviour shown in the graph		

A

$\rightarrow$

P; B

$\rightarrow$

A P, Q; C

$\rightarrow$

R, S; D

$\rightarrow$

P, R

A

$\rightarrow$

P; B

$\rightarrow$

B R, S; C

$\rightarrow$

P, Q; D

$\rightarrow$

Q, S

A

$\rightarrow$

Q; B

$\rightarrow$

C

R, S; C

$\rightarrow$

P, S; D

$\rightarrow$

P, R

A

$\rightarrow$

Q; B

$\rightarrow$

D P, Q; C

$\rightarrow$

P, S; D

$\rightarrow$

Q, S

CORRECT OPTION

A

$\rightarrow$

Q; B

$\rightarrow$

D P, Q; C

$\rightarrow$

P, S; D

$\rightarrow$

Q, S

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

As the ideal gas expands in vacuum, no work is done i.e., w = 0

Container is insulated, so no heat lost or gained i.e., Q = 0.

According to the first law of thermodynamics

$$\Delta$$

U = Q + w

$$\therefore$$

$$\Delta$$

U = 0

Therefore, no change in the temperature of the gas.

b

$$\rightarrow$$

$\$\$p, r\$\$$:

Given, PV

$$2$$

= constant ... i

For ideal gas,

$$\frac{PV}{T}$$

= constant ii

From i and ii V

μ

T = constant

As gas expands, its volume increase and temperature decreases.

$\therefore$

p is the correct.

Also, Q = nC

Δ

T i

Where C = molar specific heat

For a polytropic process,

$$C = C_V + \frac{R}{1-n}$$

and

PV

n

= constant

Here, PV

2

= constant, where n = 2

$\therefore$

$$C = C_V + \frac{R}{1.2} = C_V - R$$

For monoatomic gas,

$$C_V = \frac{3}{2}R$$

$\therefore$

$$C = \frac{3}{2}R - R = \frac{R}{2}$$

Substituting this value in 1, we get

$$Q = n \times \frac{R}{2} \times \Delta T$$

Temperature decreases i.e.,

$$\Delta$$

T is negative. Therefore, Q is negative. This is turn mean that heat is lost by the gas during the process.

So r is the correct option.

C

$$\rightarrow$$

P, S

V

$$1/3$$

$$\times$$

T = constant

$$\Rightarrow$$

As the gas expands and volume increases, the temperature decreases. Therefore P is the correct option.

In the process,

$$x = \frac{4}{3}$$

$$\therefore$$

$$C = C_V + \frac{R}{1 - \frac{4}{3}} = \frac{3}{2}R + \frac{3R}{-1}$$

$$= \frac{3}{2}R - 3R = -\frac{3R}{2}$$

$$\therefore$$

$$Q = n\left(-\frac{3}{2}R\right)\Delta t$$

As

$$\Delta t$$

=

$$-$$

negative $$ − $$ ve, Q = positive $+ve$. That means heat is gained by the gas during the process.

s is correct.

d

$$\rightarrow$$

q, s

$$\Delta T = \frac{\Delta(PV)}{nR}$$

$$\Delta(PV)$$

= Positive

$$\therefore$$

$$\Delta$$

T = positive

$$\therefore$$

Temperature increase Q is the correct option.

From graph it is clear that, during the process the pressure of the gas increases which shows that the internal energy of the gas has increased. Also, the volume increases which means work is done by the system which needs energy. We conclude from the above fact that gas gains heat during the process.

S is correct.