# JEE Advanced 2013 Paper 1 $Offline 60Questions$

## Question 001 Numerical

### QUESTION

The atomic masses of He and Ne are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of He gas at –73$^{o}$C is “M” times that of the de Broglie wavelength of Ne at 727$^{o}$C. M is

### SOURCE

Chemistry • structure-of-atom

### EXPLANATION

To solve this problem, we will use the de Broglie wavelength formula, which is given by

$$\lambda = \frac{h}{mv}$$

where $\lambda$ is the de Broglie wavelength, $h$ is Planck's constant, $m$ is the mass of the particle, and $v$ is the velocity of the particle.

However, it's more relevant to use the form of the de Broglie equation that involves temperature, given that kinetic energy $\$KE\$$ at a particular temperature is linked to the velocity of the gas particles. The kinetic energy for a gas particle can be calculated using the equation:

$$KE = \frac{3}{2}k_B T$$

where $k_B$ is the Boltzmann constant and $T$ is the temperature in Kelvin. The velocity $v$ of a gas particle can be related to its kinetic energy by the equation:

$$KE = \frac{1}{2}mv^2$$

Solving for $v$ gives:

$$v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{3k_BT}{m}}$$

Substituting $v$ in the de Broglie wavelength formula gives:

$$\lambda = \frac{h}{m\sqrt{\frac{3k_BT}{m}}} = \frac{h}{\sqrt{3mk_BT}}$$

Given that we are comparing helium $He$ and neon $Ne$ gases, the relative de Broglie wavelengths can be related by the mass of the particles and their temperatures. For helium and neon, respectively, this becomes:

$$\lambda_{He} = \frac{h}{\sqrt{3m_{He}k_BT_{He}}}$$

$$\lambda_{Ne} = \frac{h}{\sqrt{3m_{Ne}k_BT_{Ne}}}$$

Given the temperatures are –73°C for He and 727°C for Ne, we first convert these temperatures to Kelvin:

$$T_{He} = 200K \quad (\text{-}73^\circ\mathrm{C} + 273)$$

$$T_{Ne} = 1000K \quad (727^\circ\mathrm{C} + 273)$$

The ratio of the de Broglie wavelength of He to Ne, $\frac{\lambda_{He}}{\lambda_{Ne}}$, would thus be:

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \frac{\frac{h}{\sqrt{3m_{He}k_BT_{He}}}}{\frac{h}{\sqrt{3m_{Ne}k_BT_{Ne}}}} = \sqrt{\frac{m_{Ne}T_{Ne}}{m_{He}T_{He}}}$$

Substituting the masses
$\$m_{He} = 4\$a.m.u.and\$m_{Ne} = 20\$a.m.u.,with\$1\$a.m.u.approximatelye$
and the temperatures:

$$\frac{\lambda_{He}}{\lambda_{Ne}} = \sqrt{\frac{20 \times 1000}{4 \times 200}} = \sqrt{\frac{20000}{800}} = \sqrt{25} = 5$$

Therefore, the value of $M$, which is the multiplier for the de Broglie wavelength of He gas at –73°C compared to that of Ne at 727°C, is 5.

## Question 002 Numerical

**QUESTION**

The total number of lone-pairs of electrons in melamine is

**SOURCE**

Chemistry • biomolecules

**EXPLANATION**

The structure of melamine is :

Each nitrogen has one lone pair of electrons. Number $no.$ of nitrogen in a molecule $= 6$

Total no. of lone pairs in melamine

$$= \text{ No. of nitrogen } \times \text{ lone pair}$$

$$= 6 \times 1 = 6$$

Hence, total number of lone pair on nitrogen is 6.

## Question 003 MCQ

QUESTION

The standard enthalpies of formation of $CO_2\,g$, $H_2O\,l$ and glucose $s$ at 25°C are –400 kJ/mol, –300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25°C is

A +2900 kJ

B – 2900 kJ

C –16.11 kJ

D +16.11 kJ

CORRECT OPTION

C –16.11 kJ

SOURCE

Chemistry • thermodynamics

EXPLANATION

To find the standard enthalpy of combustion of glucose, we will first write the balanced chemical equation for the combustion of glucose. The chemical formula for glucose is $C_6H_{12}O_6$. Combustion involves the reaction of a substance with oxygen to produce carbon dioxide ($CO_2$) and water ($H_2O$) typically. The reaction for the combustion of glucose can be represented as:

$$C_6H_{12}O_6(s) + 6O_2(g) \rightarrow 6CO_2(g) + 6H_2O(l)$$

Each molecule of glucose $C6H12O6$ produces six molecules of $CO_2$ and six molecules of $H_2O$. Now, we use the given standard enthalpies of formation ($\Delta H_f°$) for these substances to find the enthalpy of the reaction. The enthalpy of a reaction can be calculated using the formula:

$$\Delta H_{\text{reaction}} = \sum \Delta H_{f,\text{products}} - \sum \Delta H_{f,\text{reactants}}$$

For the given reaction:

- $\Delta H_f°$ of $CO_2 g$ = –400 kJ/mol
- $\Delta H_f°$ of $H_2O l$ = –300 kJ/mol
- $\Delta H_f°$ of glucose $s$ = –1300 kJ/mol

The sum of the standard enthalpies of formation of the products:

$$[6 \times (-400) \text{ kJ/mol}] + [6 \times (-300) \text{ kJ/mol}] = (-2400 + -1800) \text{ kJ} = -42($$

The sum of the standard enthalpies of formation of the reactants $glucoseasonlypartcontributingwithenthalpyvalueotherthanzero$:

$$[-1300 \text{ kJ/mol}] + [6 \times 0 \text{ kJ/mol for } O_2]$$

Therefore, the standard enthalpy of combustion of glucose is:

$$\Delta H_{\text{reaction}} = (-4200 \text{ kJ}) - (-1300 \text{ kJ}) = -2900 \text{ kJ}$$

So, -2900 kJ of energy is released per mole of glucose combusted at standard conditions. The standard enthalpy of combustion per gram of glucose can be calculated using the molar mass of glucose. Glucose has a molar mass of approximately 180 g/mol.

$$\Delta H_{\text{combustion per gram}} = \frac{-2900 \text{ kJ/mol}}{180 \text{ g/mol}} = -16.11 \text{ kJ/g}$$

Thus, the correct answer is -16.11 kJ per gram of glucose, which corresponds to option C –16.11 kJ.

**Question 004** MCQ

**QUESTION**

The initial rate of hydrolysis of methyl acetate $1M$ by a weak acid $HA, 1M$ is 1/100$^{th}$ of that of a strong acid $HX, 1M$, at 25$^{o}$C. The $K_a$ of HA is

A

1

$10^{-4}$

×

B

1

$10^{-5}$

×

C

1

$10^{-6}$

×

D

1

$10^{-3}$

×

**CORRECT OPTION**

A

1

$10^{-4}$

×

**SOURCE**

Chemistry • ionic-equilibrium

**EXPLANATION**

Let's break down this problem step-by-step:

**Understanding the Reaction:**

The hydrolysis of methyl acetate ($MeCOOCH_3$) is catalyzed by both strong acids $like HX$ and weak acids $like HA$. The reaction proceeds as follows:

$$MeCOOCH_3 + H_2O \xrightarrow[or\ HA]{H^+\ or\ HX} MeCOOH + CH_3OH$$

**Rate of Reaction and Acid Strength:**

The rate of acid-catalyzed hydrolysis depends on the concentration of $H^+$ ions.

- **Strong acids** ionize completely. So, a 1M solution of HX provides 1M $H^+$ ions.

- **Weak acids** ionize partially. So, a 1M solution of HA will provide significantly less than 1M $H^+$ ions.

**Information from the Problem:**

We are told that the initial rate of hydrolysis with HA is $1/100^{th}$ of that with HX. Since the rate is directly proportional to $[H^+]$, this means the $[H^+]$ from HA is $1/100^{th}$ of that from HX.

**Calculating $K_a$:**

1. **$[H^+]$ from HX:** Since HX is a strong acid and fully ionizes, $[H^+]$ = 1M.

2. **$[H^+]$ from HA:** This is $1/100^{th}$ the $[H^+]$ from HX, so $[H^+]$ = $1/100$ * 1M = $10^{-2}$ M.

3. **Setting up the $K_a$ expression:**

$$K_a = \frac{[H^+][A^-]}{[HA]}$$

Since the initial concentration of HA is 1M and it ionizes to a small extent, we can approximate:

- $$HA$$
  ≈ 1 M
- $[H^+] = [A^-] = 10^{-2}$ M

1. **Calculating $K_a$:**

$$K_a = \frac{(10^{-2})(10^{-2})}{1} = 1 \times 10^{-4}$$

**Answer:**

Therefore, the $K_a$ of the weak acid HA is **$1 \times 10^{-4}$** $OptionA$.

## Question 005 MCQ

**QUESTION**

The hyperconjugative stabilities of tert-butyl cation and 2-butene, respectively, are due to

A
$$\sigma \to p$$
$empty$ and
$$\sigma \to \pi^*$$
electron delocalisations

B
$$\sigma \to \sigma^*$$
and
$$\sigma \to \pi$$

electron delocalisations

C

$$\sigma \rightarrow p$$

$filled$ and

$$\sigma \rightarrow \pi$$

electron delocalisations

D

p $filled$

$$\rightarrow \sigma^*$$

and

$$\sigma \rightarrow \pi^*$$

electron delocalisations

**CORRECT OPTION**

A

$$\sigma \rightarrow p$$

$empty$ and

$$\sigma \rightarrow \pi^*$$

electron delocalisations

**SOURCE**

Chemistry • basics-of-organic-chemistry

**EXPLANATION**

$i$ The structure of tert-butyl cation $\left[\left(\mathrm{CH}_3\right)_3 \mathrm{C}^{\oplus}\right]$ is as follows :

The carbocation is highly stable due to positive hyperconjugative $(+\mathrm{H})$ and inductive $(+\mathrm{I})$ effect.

$ii$ The hyperconjugative effect is due to delocalication of $\sigma$ $sigma$ electrons of $\mathrm{C-H}$ sigma $\$\sigma\$$ bond on carbon adjacent to carbocation with an empty P orbital $oncarbcation$.

Hence, carbocation is stabilised due to formation of pi bond with adjacent carbon $duetodelocalised$ electrons of $\mathrm{C-H}$ sigma bond). Its a s to p $empty$ delocalisation.

$iii$ There are eight more protons on carbons adjacent to carbocation. This gives eight more hyperconjugative structures. This impart stability to carbocation.

$iv$ The structure of 2-butene is as follows :

$$\mathrm{CH_3-C=C-CH_3}$$

The hyperconjugative effect is due to the delocalisation of sigma $\$\sigma\$$ electrons of $\mathrm{C-H}$ bond with adjacent carbon containing pi bond.

The electrons of $\mathrm{C-H}$ $\sigma$ bond are donated to anti-bonding $(\pi^*)$ orbitals of carbon-carbon double bond. Its $\sigma$ $\mathrm{s}$ to $\pi^*$ delocalisation.

There are five more hydrogens from methyl groups attached to carbon-carbon double bond. Hence, five more hyperconjugative structures.

## Question 006 MCQ

**QUESTION**

Methylene blue, from its aqueous solution, is adsorbed on activated charcoal at $25^{o}$C. For this process, the correct statement is

**A** The adsorption requires activation at $25^{o}$C.

B The adsorption is accompanied by a decrease in enthalpy

C The adsorption increases with increase of temperature

D The adsorption is irreversible

**CORRECT OPTION**

B The adsorption is accompanied by a decrease in enthalpy

**SOURCE**

Chemistry • surface-chemistry

**EXPLANATION**

Methylene blue adsorbs onto activated charcoal from its aqueous solution at 25°C in a process known as physical adsorption $physisorption$.

$i$ Physical adsorption of methylene blue on activated charcoal can occur at all temperatures, but it is more pronounced at lower temperatures. This means Option $A$ is incorrect.

$ii$ In the bulk, forces between molecules are balanced. However, molecules on the surface experience unbalanced or residual attractive forces, leading to higher surface energy. These unbalanced forces are responsible for van der Waals attractions between the adsorbent $activatedcharcoal$ and the adsorbate $methyleneblue$. The formation of these weak attractions reduces the energy of surface molecules, resulting in a decrease in the system's enthalpy. Therefore, Option $B$ is correct.

$iii$ The extent of physisorption decreases as temperature increases because the weak van der Waals forces weaken at higher temperatures, reducing the adsorbate's ability to adhere to the adsorbent surface. Consequently, Option $C$ is incorrect.

$iv$ Physical adsorption is a reversible process. As the temperature decreases, the attractive forces between the adsorbent and adsorbate strengthen, increasing the extent of physisorption. Thus, Option $D$ is incorrect.

## Question 007 MCQ

**QUESTION**

Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement $s$ is $are$

A
$$\Delta G$$
is positive

B
$$\Delta S_{system}$$
is positive

C
$$\Delta S_{surroundings}$$
= 0

D
$$\Delta H$$
= 0

**CORRECT OPTION**

B
$$\Delta S_{system}$$
is positive

**SOURCE**

Chemistry • thermodynamics

**EXPLANATION**

When dealing with solutions and thermodynamics, particularly with ideal solutions, several key aspects must be considered to understand the behavior of the system and its interactions with the surroundings. Let's analyze each option in the context of forming an ideal solution from benzene and naphthalene at room temperature:

**Option A:**

$$\Delta G$$

**is positive**

For an ideal solution, the Gibbs free energy change $\$\$\Delta G\$\$$ upon mixing can be described as follows:

$$\Delta G = \Delta H - T\Delta S$$

An ideal solution has the characteristic that there is no enthalpy change upon mixing $\$\$\Delta H = 0\$\$$ because the intermolecular forces between unlike molecules are assumed to be equal to those between like molecules. Additionally, when two or more components form an ideal solution, the entropy of the system $\$\$\Delta S\$\$$ increases due to the mixing of different molecules, which is a manifestation of increased randomness or disorder.

Since

$$\Delta H = 0$$

and

$$\Delta S > 0$$

for an ideal solution, applying these to the Gibbs free energy equation yields:

$$\Delta G = 0 - T\Delta S = -T\Delta S$$

Given that

$$T > 0$$

and

$$\Delta S > 0$$

,

$$\Delta G$$

will ultimately be negative, indicating a spontaneous process. Therefore, **Option A is false.**

**Option B:**

$$\Delta S_{system}$$

**is positive**

As mentioned earlier, the entropy of the system increases in the formation of an ideal solution. This is because the different molecules of benzene and naphthalene mix, increasing the randomness or disorder of the system. Therefore, **Option B is true.**

**Option C:**

$$\Delta S_{surroundings}$$

**= 0**

In the formation of an ideal solution, there is no heat exchange with the surroundings since

$$\Delta H = 0$$

. Without a heat exchange, there is no change in the entropy of the surroundings, as entropy change in the surroundings is generally associated with heat exchange according to the relation

$$\Delta S = \frac{q}{T}$$

for reversible processes. Therefore, **Option C is true.**

**Option D:**

$$\Delta H$$

**= 0**

In the context of an ideal solution, it is assumed that the enthalpy change $$\Delta H$$ is zero because the energy required to break interactions among similar molecules is exactly balanced by the energy released from the formation of new interactions between different types of molecules. This implies no net heat effect. Therefore, **Option D is true.**

In summary, Options B, C, and D are true, while Option A is false for the ideal solution of benzene and naphthalene at room temperature.

## Question 008 MCQ

**QUESTION**

Sulphide ores are common for the metals

A. Ag, Cu and Pb

B. Ag, Mg and Pb

C. Ag, Cu and Sn

D. Al, Cu and Pb

**CORRECT OPTION**

A. Ag, Cu and Pb

**SOURCE**

Chemistry • isolation-of-elements

**EXPLANATION**

The correct answer is **Option A: Ag, Cu and Pb**. Here's why :

Sulphide ores are minerals where the metal is combined with sulfur. Many metals are found in this form due to the chemical reactivity of sulfur.

- **Ag** $Silver$**:** Silver is often found as the sulfide mineral **argentite ($Ag_2S$)**.
- **Cu** $Copper$**:** Copper is commonly found as **chalcopyrite ($CuFeS_2$)** and **chalcocite ($Cu_2S$)**.
- **Pb** $Lead$**:** Lead is extracted from the sulfide mineral **galena** $PbS$.

Let's look at why the other options are incorrect :

- **Option B: Ag, Mg and Pb** - Magnesium $Mg$ is typically found in oxide ores, not sulfide ores.
- **Option C: Ag, Cu and Sn** - Tin $Sn$ is primarily found in oxide ores, not sulfide ores.
- **Option D: Al, Cu and Pb** - Aluminum $Al$ is mainly extracted from bauxite ore, an oxide mineral.

## Question 009 MCQ

**QUESTION**

KI in acetone, undergoes $S_N2$ reaction with each of P, Q, R and S. The rates of the reaction vary as

A. P > Q > R > S

B. S > P > R > Q

C P > R > Q > S

D R > P > S > Q

**CORRECT OPTION**

B S > P > R > Q

**SOURCE**

Chemistry • haloalkanes-and-haloarenes

**EXPLANATION**

The bulky groups cause steric hindrance in the formation of transition state. Therefore, higher homologues $Q$ are less reactive than lower homologues $P$. In compound $S$, the transition state is highly stabilized by Ph-$C=O$- group, so it has highest rate of reaction towards $S_N2$.

The relative reactivities towards $S_N2$ reaction S > P > R > Q.

## Question 010 MCQ

**QUESTION**

The compound that does NOT liberate $CO_2$, on treatment with aqueous sodium bicarbonate solution, is

A Benzoic acid

B Benzenesulphonic acid

C Salicylic acid

D Carbolic acid $Phenol$

**CORRECT OPTION**

D Carbolic acid $Phenol$

**SOURCE**

Chemistry • alcohols-phenols-and-ethers

**EXPLANATION**

Comparatively phenol is a weaker acidic than carbonic acid ($H_2CO_3$), and hence does not liberate $CO_2$ on treatment with aqueous $NaHCO_3$ solution. The rest are more acidic than $H_2CO_3$, and so liberate $CO_2$.

## Question 011 MCQ

**QUESTION**

Consider the following complex ions : P, Q and R.

$$P = [FeF_6]^{3-}$$

,

$$Q = [V(H_2O)_6]^{2+}$$

and

$$R = [Fe(H_2O)_6]^{2+}$$

The correct order of the complex ions, according to their spin-only magnetic moment values $inB.M.$ is

**A** R < Q < P

**B** Q < R < P

**C** R < P < Q

**D** Q < P < R

**CORRECT OPTION**

**B** Q < R < P

**SOURCE**

Chemistry • coordination-compounds

**EXPLANATION**

The electronic configurations of the central metal ions are as follows:

The number of unpaired electrons is P = 5, Q = 3 and R = 4. From the relation

$$\mu = \sqrt{n(n+2)}$$

, where n is the number of unpaired electrons, we have the order of spin only magnetic moment as Q < R < P.

## Question 012 MCQ

### QUESTION

In the reaction, P + Q

$\rightarrow$

R + S, the time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is

A 2

B 3

C 0

D 1

### CORRECT OPTION

D 1

### SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

### EXPLANATION

This is a first-order reaction because $t_{75\%}$ = 2

$\times$

$t_{50\%}$. The graph shows that the order with respect to Q is 0, so we can write the rate expression as

Rate = k

$$P$$

1

$$Q$$

0

## Question 013 MCQ

**QUESTION**

Concentrated nitric acid, upon long standing, turns yellow-brown due to the formation of

A. NO

B. $NO_2$

C. $N_2O$

D. $N_2O_4$

**CORRECT OPTION**

B. $NO_2$

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reaction involved is $4HNO_3$

$$\rightarrow$$

$4NO_2 + 2H_2O + O_2$. The yellow-brown color appears due to the formation of $NO_2$.

## Question 014 MCQ

QUESTION

The arrangement of X

$$-$$

ions around $A^+$ ion in solid AX is given in the figure $not drawn to scale$. If the radius of X

$$-$$

is 250 pm, the radius of $A^+$ is

A 104 pm

B 125 pm

C 183 pm

D 57 pm

**CORRECT OPTION**

A 104 pm

**SOURCE**

Chemistry • solid-state

**EXPLANATION**

From the figure, it can be seen that the cation $A^+$ occupies octahedral void formed by the anion X

$-$

. The radius ratio for an octahedral void is $r_{A^+} / r_X$

$-$

= 0.414. Now, given that the radius of anion X

$-$

is 250 pm, so the radius of $A^+$ is

$r_{A^+}$ = 0.414

$\times$

250 = 103.50

$\simeq$

104 pm

## Question 015 MCQ

**QUESTION**

Upon treatment with ammoniacal $H_2S$, the metal ion that precipitates as a sulphide is

A. Fe$III$

B. Al$III$

C. Mg$II$

D. Zn$II$

**CORRECT OPTION**

D. Zn$II$

**SOURCE**

Chemistry • d-and-f-block-elements

**EXPLANATION**

On treatment with $H_2S$, Zn$II$ gets precipitated as ZnS. Mg$II$ does not get precipitated, while $Fe^{3+}$ and $Al^{3+}$ get precipitated as hydroxides.

## Question 016 MCQ

**QUESTION**

The pair $s$ of coordination complexes/ions exhibiting the same kind of isomerism is $are$

**A** and

$$[Cr(NH_3)_5Cl]Cl_2$$

$$[Cr(NH_3)_4Cl_2]Cl$$

**B** and

$$[Co(NH_3)_4Cl_2]^+$$

$$[Pt(NH_3)_2(H_2O)Cl]^+$$

**C** and

$$[CoBr_2Cl_2]^{2-}$$

$$[PtBr_2Cl_{12}]^{2-}$$

**D** and

$$[Pt(NH_3)_3](NO_3)Cl$$

$$[Pt(NH_3)_3Cl]Br$$

**CORRECT OPTION**

$$[Co(NH_3)_4Cl_2]^+$$

B and

$$[Pt(NH_3)_2(H_2O)Cl]^+$$

**SOURCE**

Chemistry • coordination-compounds

**EXPLANATION**

$$[Co(NH_3)_4Cl_2]^+$$

and

$$[Pt(NH_3)_2(H_2O)Cl]^+$$

are octahedral and square planar complexes, which show geometrical isomerism.

$$[Pt(NH_3)_3(NO_3)]Cl$$

and

$$[Pt(NH_3)_3Cl]Br$$

show ionisation isomerism.

## Question 017 MCQ

**QUESTION**

Among P, Q, R and S, the aromatic compound $s$ is $are$

A P

B Q

C R

D S

**CORRECT OPTION**

A P

**SOURCE**

Chemistry • hydrocarbons

**EXPLANATION**

The reactions involved are

$A$

$B$

$C$

$$(NH_4)_2CO_3 \xrightarrow{\Delta} 2NH_3 + CO_2 + H_2O$$

$D$

## Question 018 Numerical

QUESTION

The total number of carboxylic acid groups is the product P is _________.

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction involved is

The number of carboxylic acid groups in the product is 2.

## Question 019 Numerical

QUESTION

$EDTA^4$

$$-$$

is ethylenediaminetetraacetate ion. The total number of N

$$-$$

Co

$$-$$

O bond angles in

$$Co(EDTA)$$

1

$$-$$

complex ion is ________.

**SOURCE**

Chemistry • coordination-compounds

**EXPLANATION**

EDTA is a multidentate ligand as it can donate six pairs of electrons – two pair from the two nitrogen atoms and four pair from the four terminal oxygens of the

—

$COO^-$ groups.

The structure of the complex is

Therefore, the number of N

—

Co

—

O bonds are 8.

## Question 020 Numerical

**QUESTION**

A tetrapeptide has

—

COOH group on alanine. This produces glycine $Gly$, valine $Val$, phenyl alanine $Phe$ and alanine $Ala$, on complete hydrolysis. For this tetrapeptide, the number of possible sequences $primarystructures$ with

—

$NH_2$ group attached to a chiral center is _______.

**SOURCE**

Chemistry • biomolecules

**EXPLANATION**

The possible combinations with C-terminal as alanine and N-terminal with chiral carbon $i.e. excludingglycine$ are four.

Val

—

Phe

—

Gly

—

Ala

Val

—

Gly

—

Phe

—

Ala

Phe

—

Val

—

Gly

—

Ala

Phe

—

Gly

—

Val

—

Ala

## Question 021 MCQ

**QUESTION**

Let

$$f\left(x\right)=x\sin\,\pi x,\;x>0.$$

Then for all natural numbers

$$n,\;f'\left(x\right)$$

vanishes at

A

A unique point in the interval

$$\left(n,\;n+\frac{1}{2}\right)$$

B

A unique point in the interval

$$\left(n+\frac{1}{2}, n+1\right)$$

C

A unique point in the interval

$$(n,\, n+1)$$

D

Two points in the interval

$$(n,\, n+1)$$

**CORRECT OPTION**

B

A unique point in the interval

$$\left(n+\frac{1}{2}, n+1\right)$$

**SOURCE**

Mathematics • trigonometric-functions-and-equations

**EXPLANATION**

Given, ,

$$f(x) = x \cdot \sin \pi x, x > 0$$

$$\Rightarrow \quad f'(x) = 1 \cdot \sin \pi x + x \cdot \pi \cos \pi x$$

Apply

$$f'(x) = 0$$

$$\Rightarrow \sin \pi x + \pi x \cos \pi x = 0$$
$$\Rightarrow \sin \pi x = -\pi x \cos \pi x$$
$$\Rightarrow \frac{\sin \pi x}{\cos \pi x} = -\pi x$$
$$\Rightarrow \tan \pi x = -\pi x$$

Draw the graph of

$$y = \tan \pi x$$

and

$$y = -\pi x$$

for

$$x > 0$$

It is clear that

$$y = \tan \pi x$$

and

$$y = -\pi x$$

intersect at a unique point if

$$\frac{1}{2} < x < 1$$

as

$$\frac{3}{2} < x < 2$$

as

$$\frac{5}{2} < x < 3$$

or.....

So,

$$y = \tan \pi x$$

and

$$y = -\pi x$$

intersect at a unique point is

$$x \in \left(n + \frac{1}{2}, n + 1\right)$$

as

$$(n, n+1)$$

.

Hints:

The number of point of intersection of graphs of

$$y = f(x)$$

and

$$y = g(x)$$

is equal to the number of solution of

$$f(x) = g(x)$$

## Question 022 Numerical

**QUESTION**

A pack contains

$$n$$

cards numbered from

$$1$$

to

$$n.$$

Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is

$$1224.$$

If the smaller of the numbers on the removed cards is

$$k,$$

then

$$k - 20 =$$

**SOURCE**

Mathematics • sequences-and-series

**EXPLANATION**

Let number of removed cards are

$$k$$

and

$$k + 1$$

.

Given, the sum of numbers on the cards after removing

$$k$$

and

$$k + 1$$

is 1224.

$$
\begin{aligned}
&\therefore (1+2+3+\ldots.+n)-(k+(k+1))=1224\\
&\Rightarrow \quad \frac{n(n+1)}{2}-2k-1=1224\\
&\Rightarrow \quad \frac{n(n+1)}{2}-2k=1225\\
&\Rightarrow \quad n^2+n-4k=2450\\
&\Rightarrow \quad n^2+n-2450=4k\\
&\Rightarrow (n+50)(n-49)=4k\\
&\therefore \quad n>49\\
&\text{Let } n=50\\
&\Rightarrow \quad 100\times 1=4k
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad k=25\\
\Rightarrow \quad k-20=5
\end{aligned}
$$

Hints :

$i$ Recall

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$

$ii$ If

$$k$$

is the smallest number on two consecutive numbers, then the second number is

$$k+1$$

.

## Question 023 MCQ

**QUESTION**

Let complex numbers

$$\alpha\, and\, \frac{1}{\overline{\alpha}}$$

lie on circles

$$(x - x_0)^2 + \ (y - y_0)^2 = r^2$$

and

$$(x - x_0)^2 + \ (y - y_0)^2 = 4r^2$$

respextively. If

$$z_0 = x_0 + iy_0$$

satisfies the equation

$$2|z_0|^2 \ = r^2 + 2, \, then \ |a| =$$

A $\frac{1}{\sqrt{2}}$

B $\frac{1}{2}$

C $\frac{1}{\sqrt{7}}$

D $\frac{1}{3}$

**CORRECT OPTION**

C $\frac{1}{\sqrt{7}}$

**SOURCE**

Mathematics • complex-numbers

**EXPLANATION**

If

$$z = x + iy$$

, then

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

and

$$(x - x_0)^2 + (y - y_0)^2 = 4r^2$$

can be written as

$$|Z - Z_0|^2 = r^2$$

and

$$|Z - Z_0|^2 = 4r^2$$

respectively where,

$$\mathrm{Z}_0 = x_0 + iy_0$$

$given$

$$\Rightarrow |Z - Z_0|^2 = r^2$$

and

$$|Z - Z_0|^2 = 4r^2$$

$$\Rightarrow (Z - Z_0)\left(\bar{Z} - \bar{Z}_0\right) = r^2$$

and

$$(Z - Z_0)\left(\bar{Z} - \bar{Z}_0\right) = 4r^2\left(|Z|^2 = Z\bar{Z}\right)$$

$$\text{Given, } \alpha \text{ and } \frac{1}{\bar{\alpha}} \text{ line on circles } (x-x_0)^2+(y-y_0)^2$$
$$= r^2 \text{ and } (x-x_0)^2+(y-y_0)^2 = 4r^2 \text{ respectively}$$
$$\therefore \quad (\alpha - Z_0)\left(\bar{\alpha}-\bar{Z}_0\right) = r^2 \text{ and}$$
$$\left(\tfrac{1}{\bar{\alpha}} - Z_0\right)\left(\tfrac{1}{\alpha}-\bar{Z}_0\right) = 4r^2$$

$$\Rightarrow |\alpha|^2 - \alpha\bar{Z}_0 - \bar{\alpha}Z_0 + |Z_0|^2 = r^2 \text{ and}$$
$$\frac{1}{|\alpha|^2} - \frac{1}{\bar{\alpha}}\bar{Z}_0 - \frac{1}{\alpha}Z_0 + \left|\bar{Z}_0\right|^2 = 4r^2$$
$$\Rightarrow \alpha Z_0 + \bar{\alpha}Z_0 = |\alpha|^2 + |Z_0|^2 - r^2 \text{ and}$$
$$\frac{\alpha\bar{Z}_0}{|\alpha|^2} + \frac{\bar{\alpha}Z_0}{|\alpha|^2} = \frac{1}{|\alpha|^2} + |Z_0|^2 - 4r^2$$
$$\Rightarrow \alpha\bar{Z}_0 + \bar{\alpha}Z_0 = |\alpha|^2 + |Z_0|^2 - r^2 = |\alpha|^2\left(\frac{1}{|\alpha^2|} + \left|Z_0^2\right| - 4r^2\right)$$

$$\Rightarrow |\alpha|^2 + |Z_0|^2 - r^2 = 1 + |\alpha|^2|Z_0|^2 - 4|\alpha|^2r^2 \quad \text{... (i)}$$

Given,

$$2|Z_0|^2 = r^2 + 2$$

Put

$$|Z_0|^2 = \frac{r^2}{2} + 1$$

in the equation $i$

$$\Rightarrow |\alpha|^2 + \frac{r^2}{2} + 1 - r^2 = 1 + |\alpha|^2\left(\frac{r^2}{2}+1\right) - 4|\alpha|^2r^2$$
$$\Rightarrow |\alpha|^2 - \frac{r^2}{2} = \frac{1}{2}|\alpha|^2r^2 + |\alpha|^2 - 4|\alpha|^2r^2$$

$$\Rightarrow \quad -\tfrac{r^2}{2} = \tfrac{1}{2}|\alpha|^2r^2 - 4|\alpha|^2r^2$$
$$\Rightarrow \quad -\tfrac{1}{2} = \tfrac{1}{2}(\alpha)^2 - 4|\alpha|^2$$
$$\Rightarrow \quad \tfrac{-1}{2} = -\tfrac{7}{2}|\alpha|^2$$
$$\Rightarrow \quad |\alpha|^2 = \tfrac{1}{7}$$
$$\Rightarrow \quad |\alpha| = \tfrac{1}{\sqrt{7}}$$

Hints :

$i$ Apply the property

$$Z \cdot \bar{Z} = |\bar{Z}|^2$$

$ii$ If

$$Z = (x + iy)$$

and

$$Z_0 = (x_0 + iy_0)$$

then

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

and

$$(x - x_0)^2 + (y - y_0)^2 = 4r^2$$

can be written as

$$|Z - Z_0|^2 = r^2$$

and

$$|Z - Z_0|^2 = 4r^2$$

respectively.

## Question 024 MCQ

**QUESTION**

The number of points in

$$(-\infty \, \infty),$$

for which

$$x^2 - x \sin x - \cos x = 0,$$

is

A 6

B 4

C 2

D 0

**CORRECT OPTION**

C 2

**SOURCE**

Mathematics • trigonometric-functions-and-equations

**EXPLANATION**

$$\text{We know the graph of } y = x \sin x \text{ is}$$

And the graph of

$$y = \cos x$$

is

Now add the graph of

$$y = x \sin x$$

and

$$y = \cos x$$

and also draw the graph of

$$y = x^2$$

It is clear that the graph of

$$y = x^2$$

and

$$y = x \sin x + \cos x$$

intersect at two points

$$\therefore$$

The equation

$$x^2 = x \sin x + x$$

satisfy for two values of

$$x$$

.

Hints:

$i$ Recall the graph of

$$y = x \sin x, y = \cos x$$

and

$$y = x^2$$

.

$ii$ Recall the method of addition of two graphs.

$iii$ The number of point of intersection of graphs of

$$y = f(x)$$

and

$$y = g(x)$$

is equal to the number of solution of

$$f(x) = g(x)$$

## Question 025 Numerical

**QUESTION**

The coefficient of three consecutive terms of

$$(1 + x)^{n+5}$$

are in the ratio

$$5 : 10 : 14.$$

Then

$$n$$

=

**SOURCE**

Mathematics • mathematical-induction-and-binomial-theorem

**EXPLANATION**

We know

$$(1 + x)^{n+5} = \sum_{r=0}^{n+5} {}^{n+5}\mathrm{C}_r \cdot x^r$$

Given, the coefficients of three consecutive terms of

$$(1 + x)^{n+5}$$

are in the ratio

$$5 : 10 : 14$$

.

$$\begin{aligned}
&\Rightarrow {}^{n+5}\mathrm{C}_r : {}^{n+5}\mathrm{C}_{r+1} : {}^{n+5}\mathrm{C}_{r+2} = 5 : 10 : 14 \\
&\Rightarrow \frac{{}^{n+5}\mathrm{C}_{r+1}}{{}^{n+5}\mathrm{C}_r} = \frac{10}{5} \text{ and } \frac{{}^{n+5}\mathrm{C}_{r+2}}{{}^{n+5}\mathrm{C}_{r+1}} = \frac{14}{10} \\
&\Rightarrow {}^{n+5}\mathrm{C}_{r+1} = 2.\,{}^{n+5}\mathrm{C}_r \text{ and } 5.\,{}^{n+5}\mathrm{C}_{r+2} = 7.\,{}^{n+5}\mathrm{C}_{r+1}
\end{aligned}$$

$$\Rightarrow \frac{\underline{|n+5}}{\underline{|r+1}\ \underline{|n-r+4}} = 2\frac{\underline{|n+5}}{\underline{|r}\ \underline{|n-r+5}}$$

and

$$\frac{5\,\underline{|n+5}}{\underline{|r+2}\ \underline{|n-r+3}} = \frac{7\,\underline{|n+5}}{\underline{|r+1}\ \underline{|n-r+4}}$$

$$\begin{aligned}
&\Rightarrow \quad \frac{1}{r+1} = \frac{2}{n-r+5} \text{ and } \frac{5}{r+2} = \frac{7}{n-r+4} \\
&\Rightarrow \quad n-r+5 = 2r+2 \text{ and } \\
&\qquad 5n-5r+20 = 7r+14 \\
&\Rightarrow \quad n = 3r-3 = \frac{12r-6}{5} \\
&\Rightarrow \quad n = 3(r-1) \text{ and } 15r-15 = 12r-6 \\
&\Rightarrow \quad r = 3 \text{ and } n = 6 \\
&\Rightarrow \quad n = 6
\end{aligned}$$

Hints :

$i$ Recall

$$(1+x)^m = \sum_{r=1}^{m} {}^mC_r \cdot x^r$$

$ii$ The coefficients of three consecutive terms

$$(1+x)^m$$

are

$${}^m\mathrm{C}_r, {}^m\mathrm{C}_{r+1}, {}^m\mathrm{C}_{r+2}$$

.

## Question 026 Numerical

### QUESTION

Consider the set of eight vectors

$$V = \left\{a\,\hat{i} + b\,\hat{j} + c\hat{k} : a,\ b,\ c\ \in \{-1,\ 1\}\right\}$$

. Three non-coplanar vectors can be chosen from v in

$$2^p$$

ways. Then p is

### SOURCE

Mathematics • permutations-and-combinations

### EXPLANATION

Given, the set of eight vectors

$$\mathrm{V} = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}.$$

Now, the eight vectors are

$$\hat{i} + \hat{j} + \hat{k}, \hat{i} + \hat{j} - \hat{k}$$

,

$$\begin{gathered}\hat{i} - \hat{j} + \hat{k}, -\hat{i} + \hat{j} + \hat{k}, \hat{i} - \hat{j} - \hat{k}, \\ -\hat{i} + \hat{j} - \hat{k}, -\hat{i} - \hat{j} + \hat{k} \text{ and } -\hat{i} - \hat{j} - \hat{k}.\end{gathered}$$

Here,

$$\hat{i} + \hat{j} + \hat{k}$$

and

$$-\hat{i} - \hat{j} - \hat{k}, \hat{i} + \hat{j} - \hat{k}$$

and

$$-\hat{i}-\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}$$

and

$$-\hat{i}+\hat{j}-\hat{k}, -\hat{i}+\hat{j}+\hat{k}$$

and

$$\hat{i}-\hat{j}-\hat{k}$$

are collinear vectors.

$$\begin{aligned} \text{Let} \quad & \mathrm{S}_1=\{\hat{i}+\hat{j}+\hat{k}, -\hat{i}-\hat{j}-\hat{k}\}, \\ & \mathrm{S}_2=\{\hat{i}+\hat{j}-\hat{k}, -\hat{i}-\hat{j}+\hat{k}\}, \\ & \mathrm{S}_3=\{\hat{i}-\hat{j}+\hat{k}, -\hat{i}+\hat{j}-\hat{k}\} \text{ and} \\ & \mathrm{S}_4=\{-\hat{i}+\hat{j}+\hat{k}, \hat{i}-\hat{j}-\hat{k}\} \end{aligned}$$

For the set of three non - coplanar vector, we have to select three set out of

$$\mathrm{S}_1,\ \mathrm{S}_2,\ \mathrm{S}_3,\ \mathrm{S}_4$$

and select one vector in every selected set of

$$\mathrm{S}_1, \mathrm{S}_2, \mathrm{S}_3,\ \mathrm{S}_4$$

$$\begin{aligned} & \Rightarrow {}^4C_3 \cdot {}^2C_1 \cdot {}^2C_1 {}^2C_1 = 2^p \\ & \Rightarrow \quad 4.2.2.2 = 2^p \\ & \Rightarrow \quad 2^5 = 2^p \\ & \Rightarrow \quad p = 5 \end{aligned}$$

Hints :

Recall that

$$\hat{i}+\hat{j}+\hat{k}$$

and

$$-\hat{i}-\hat{j}-\hat{k}, \hat{i}+\hat{j}-\hat{k}$$

and

$$-\hat{i}-\hat{j}+\hat{k}, \hat{i}-\hat{j}+\hat{k}$$

and

$$-\hat{i}+\hat{j}-\hat{k}, -\hat{i}+\hat{j}+\hat{k}$$

and

$$\hat{i}-\hat{j}-\hat{k}$$

are collinear vectors.

## Question 027 MCQ

**QUESTION**

Let

$$S_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}k^2.$$

Then

$$S_n$$

can take value $s$

A 1056

B 1088

C 1120

D 1332

CORRECT OPTION

A 1056

SOURCE

Mathematics • sequences-and-series

EXPLANATION

$$\text{Given, } \mathrm{S}_n=\sum_{k=1}^{4n}(-1)^{\frac{k(k+1)}{2}}\cdot \mathrm{K}^2$$

$$\Rightarrow \mathrm{S}_n=-1^2-2^2+3^2+4^2-5^2-6^2+7^2+8^2-9^2-10^2+11^2+12^2\ldots$$

$$\begin{aligned}\Rightarrow \mathrm{S}_n=&-\left[1^2+5^2+9^2+\ldots .n \text{ terms }\right]\\&-\left[2^2+6^2+10^2+\ldots n \text{ terms }\right]\\&+\left[3^2+7^2+11^2+\ldots n \text{ terms }\right]\\&+\left[4^2+8^2+12^2+\ldots n \text{ terms }\right]\end{aligned}$$

$$\Rightarrow \mathrm{S}_n=-\sum_{r=1}^n(4r-3)^2-\sum_{r=1}^n(4r-2)^2+\sum_{r=1}^n(4r-1)^2+\sum_{r=1}^n(4r)^2$$

$$\begin{aligned}&\Rightarrow \mathrm{S}_n=\sum_{r=1}^n\left((4r)^2+(4r-1)^2-(4r-2)^2-(4r-3)^2\right)\\&\Rightarrow \mathrm{S}_n=\sum_{r=1}^n(32r-12)\\&\Rightarrow \mathrm{S}_n=32\sum_{r=1}^n r-\sum_{r=1}^n 12\\&\Rightarrow \mathrm{S}_n=32\cdot\frac{n(n+1)}{2}-12n\\&\Rightarrow \mathrm{S}_n=16n^2+16n-12n\\&\Rightarrow \mathrm{S}_n=4n(4n+1)\end{aligned}$$

If

$$n=9$$

, then

$$S_9 = 1332$$

If

$$n = 8$$

, then

$$S_8 = 1056$$

Hints :

$i$ Recall

$$\sum_{K=1}^{4n}(-1)^{\frac{k(k+1)}{2}} \cdot K^2 = -\sum_{r=1}^{n}(4r-3)^2 - \sum_{r=1}^{n}(4r-2)^2 + \sum_{r=1}^{n}(4r-1)^2 + \sum_{r=1}^{n}(4r$$

$ii$

$$\sum_{r=1}^{n} a \cdot f(r) + b \cdot g(r) - c \cdot h(r) = \sum_{r=1}^{n} a \cdot f(r) + \sum_{r=1}^{n} b \cdot g(r) - \sum_{r=1}^{n} c \cdot h(r)$$

$$= a\sum_{r=1}^{n} f(r) + b\sum_{r=1}^{n} g(r) - c\sum_{r=1}^{n} h(r)$$

$iii$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}, \sum_{r=1}^{n} a = ar$$

where is a constant.

## Question 028 MCQ

**QUESTION**

Perpendiculars are drawn from points on the line $\frac{x+2}{2} = \frac{y+1}{-1} = \frac{z}{3}$ to the plane $x + y + z = 3$. The foot of perpendiculars lie on the line

A $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$

B $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$

C $\frac{x}{4}=\frac{y-1}{3}=\frac{z-2}{-7}$

D $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

**CORRECT OPTION**

D $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

**SOURCE**

Mathematics • 3d-geometry

**EXPLANATION**

Let $P$ be a general point on the line :

$$\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}=\lambda$$

Therefore, the coordinates of $P$ can be expressed as :

$$P=(-2+2\lambda,-1-\lambda,3\lambda)$$

Now, let $(h,k,w)$ represent the foot of the perpendicular from $P$ to the plane $x+y+z=3$.

We use the formula for the foot of the perpendicular from a point to a plane :

$$h=\frac{-1\cdot(-2+2\lambda-1-\lambda+3\lambda-3)}{1^2+1^2+1^2}+(-2+2\lambda), k=\frac{-1\cdot(-2+2\lambda-1}{1^2+1^2}$$

Simplifying these, we get :

$$h=\frac{+2\lambda}{3}, k=\frac{-7\lambda}{3}+1, w=\frac{5\lambda}{3}+2$$

So, we can express $\lambda$ and the coordinates $(h, k, w)$ as :

$$\frac{h}{2} = \frac{k-1}{-7} = \frac{w-2}{5} = \frac{\lambda}{3}$$

Therefore, the locus of the foot of the perpendiculars is :

$$\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5} = \frac{\lambda}{3}$$

Thus, the line on which the feet of the perpendiculars lie is represented as :

$$\frac{x}{2} = \frac{y-1}{-7} = \frac{z-2}{5}$$

## Question 029 MCQ

**QUESTION**

For

$$a > b > c > 0,$$

the distance between

$$(1, 1)$$

and the point of intersection of the lines

$$ax + by + c = 0$$

and

$$bx + ay + c = 0$$

is less than

$$\left(2\sqrt{2}\right)$$

. Then

A

$$a+b-c>0$$

B

$$a-b+c<0$$

C

$$a-b+c=>0$$

D

$$a+b-c<0$$

**CORRECT OPTION**

A

$$a+b-c>0$$

**SOURCE**

Mathematics • straight-lines-and-pair-of-straight-lines

**EXPLANATION**

Let P is the point of intersection of line

$$ax+by+c=0$$

and

$$bx+ay-c=0$$

$$\mathrm{P}=\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$$

Given, the distance between

$$(1,1)$$

and

$$P$$

is less than

$$2\sqrt{2}$$

$$\therefore \sqrt{\left(1+\frac{c}{a+b}\right)^2+\left(1+\frac{c}{a+b}\right)^2}<2\sqrt{2}$$
$$\Rightarrow \sqrt{2}\left|\frac{a+b+c}{a+b}\right|<2\sqrt{2}$$
$$\Rightarrow \left|\frac{a+b+c}{a+b}\right|<2$$
$$\therefore a>b>c>0$$
$$\therefore \frac{a+b+c}{a+b}<2$$
$$\Rightarrow a+b+c<2a+2b$$
$$\Rightarrow a+b-c>0$$

Hence, option $A$ correct

$$\therefore$$

Given

$$a>b>c>0$$
$$\therefore a-b$$

is positive and c is also positive

$$\Rightarrow a-b+c>0$$

Hence, option $C$ is also true.

Hints :

Given,

$$a>b>c>0$$

So,

$$a-b, a-c, b$$

and

$$c$$

all are positive

$$\therefore \quad a-b+c>0, a-c+b>0$$

## Question 030 Numerical

**QUESTION**

A vertical line passing through the point

$$(h, 0)$$

intersects the ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

at the points

$$P$$

and

$$Q$$

. Let the tangents to the ellipse at

$$P$$

and

$$Q$$

meet at the point

$$R$$

. If

$$\Delta(h)$$

$$=$$

area of the triangle

$$PQR$$

,

$$\Delta_1$$

$$= \max_{1/2 \le h \le 1} \Delta(h)$$

and

$$\Delta_2$$

$$= \min_{1/2 \le h \le 1} \Delta(h)$$

, then

$$\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$$

**SOURCE**

Mathematics • ellipse

**EXPLANATION**

A vertical line passing through

$$(h, 0)$$

is

$$x = h$$

and this vertical line intersect the ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

at P and Q.

Put

$$x = h$$

in the ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$\Rightarrow \frac{h^2}{4}+\frac{y^2}{3}=1$$

$$\Rightarrow y=\pm\sqrt{3-\frac{3h^2}{4}}$$

So

$$\mathrm{P}=\left(h,\sqrt{3-\frac{3h^2}{4}}\right)$$

and

$$\mathrm{Q}=\left(h,-\sqrt{3-\frac{3h^2}{4}}\right)$$

Equation of tangent of ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

at P is

$$\frac{hx}{4}+\frac{y}{3}\sqrt{3-\frac{3h^2}{4}}=1 \quad \text{... (i)}$$

Equation of tangent of ellipse

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

at Q is

$$\frac{hx}{4}-\frac{y}{3}\sqrt{3-\frac{3h^2}{4}}=1 \quad \text{... (ii)}$$

Given,

$$R$$

is the point of intersection of tangents at

$$P$$

and at

$$Q$$

.

On solving the equations $i$ and $ii$

$$\Rightarrow \quad \mathrm{R}=\left(\frac{8}{h}, 0\right)$$

Given,

$$\Delta(h)=$$

Area of

$$\Delta \mathrm{PQR}$$

$$\Rightarrow \quad \Delta(h)=\frac{1}{2}\left(\frac{8}{h}-h\right) \cdot 2 \sqrt{3-\frac{3 h^{2}}{4}}$$

$$\Rightarrow \quad \Delta(h)=\frac{\sqrt{3}}{2}\left(\frac{8}{h}-h\right) \sqrt{4-h^{2}}$$

$$\Rightarrow \quad \frac{d \Delta(h)}{d h}=\frac{\sqrt{3}}{2}\left(\frac{-8}{h^{2}}-1\right) \sqrt{4-h^{2}}+\frac{\sqrt{3}}{2}\left(\frac{8}{h}-h\right) \cdot \frac{(-2 h)}{2 \sqrt{4-h^{2}}}$$

$$\Rightarrow \quad \frac{d \Delta(h)}{d h}=\frac{-\sqrt{3}}{2 \sqrt{4-h^{2}}}\left[\left(\frac{8}{h^{2}}+1\right)\left(4-h^{2}\right)+\left(\frac{8}{h}-h\right) h\right]$$

$$\Rightarrow \quad \frac{d\Delta(h)}{dh} = \frac{-\sqrt{3}}{2\sqrt{4-h^2}}\left[\frac{32}{h^2} - 8 + 4 - \hbar^2 + 8 - h^2\right]$$

$$\Rightarrow \quad \frac{d\Delta(h)}{dh} = \frac{-\sqrt{3}}{2\sqrt{4-h^2}}\left[-2h^2 + \frac{32}{h^2} + 4\right]$$

$$\Rightarrow \quad \frac{d\Delta(h)}{dh} = \frac{\sqrt{3}}{h^2\sqrt{4-h^2}}\left[h^4 - 2h^2 - 16\right]$$

$$\Rightarrow \quad \frac{d\Delta(h)}{dh} = \frac{\sqrt{3}}{h^2\sqrt{4-h^2}}\left[\left(h^2-1\right)^2 - 17\right]$$

For,

$$\frac{1}{2} \leq h \leq 1, \quad \frac{d\Delta(h)}{dh} < 0$$

$$\therefore \quad \Delta(h)$$

is a decreasing function for

$$x \in \left[\frac{1}{2}, 1\right]$$

$$\Rightarrow \Delta(1) \leq \Delta(h) \leq \Delta\left(\frac{1}{2}\right)$$

Given

$$\Delta_1 = \max_{\frac{1}{2} \leq h \leq 1} \Delta(h)$$

and

$$\Delta_2 = \min_{\frac{1}{2} \leq h \leq 1} \Delta(h)$$

$$\Rightarrow \quad \Delta_1 = \Delta\left(\frac{1}{2}\right) \text{ and } \Delta_2 = \Delta(1)$$

$$\Rightarrow \quad \Delta_1 = \frac{\sqrt{3}}{2}\left(16 - \frac{1}{2}\right)\sqrt{4 - \frac{1}{4}} \text{ and}$$

$$\Delta_2 = \frac{\sqrt{3}}{2}(8-1)\sqrt{4-1}$$

$$\Rightarrow \quad \Delta_1 = \frac{\sqrt{3}}{2} \times \frac{31}{2} \times \frac{\sqrt{5} \cdot \sqrt{3}}{2} \text{ and } \Delta_2 = \frac{21}{2}$$

$$\Rightarrow \quad \frac{8\Delta_1}{\sqrt{5}} = 93 \text{ and } 8\Delta_2 = 84$$

$$\Rightarrow \frac{8\Delta_1}{\sqrt{5}} - 8\Delta_2 = 9$$

Hints:

$i$ If

$$f(x)$$

is a decreasing function in

$$x \in [a, b]$$

, then

$$f(b) \leq f(x) \leq f(a)$$

$ii$ If

$$(x_1, y_1)$$

is a point on the curve

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

, then equation of tangent at

$$(x_1, y_1)$$

is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

.

## Question 031 MCQ

**QUESTION**

The value of

$$\cot\left(\sum_{n=1}^{23} \cot^{-1}\left(1 + \sum_{k=1}^{n} 2k\right)\right)$$

is

A $\frac{23}{25}$

B $\frac{25}{23}$

C $\frac{23}{24}$

D $\frac{24}{23}$

**CORRECT OPTION**

**B**

$$\frac{25}{23}$$

**SOURCE**

Mathematics • inverse-trigonometric-functions

**EXPLANATION**

$$\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right)$$

$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(1+2\times\frac{n(n+1)}{2}\right)\right)$$

$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}(1+n(n+1))\right)$$

$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}\left(\frac{n(n+1)+1}{(n+1)-n}\right)\right)$$

$$=\cot\left(\sum_{n=1}^{23}\cot^{-1}n-\cot^{-1}(n+1)\right)$$

$$=\cot((\cot^{-1}1+\cot^{-1}2+\cot^{-1}3+....+\cot^{-1}23)-(\cot^{-1}2+\cot^{-1}3+....$$

$$=\cot(\cot^{-1}1-\cot^{-1}24)$$

$$=\cot\left(\cot^{-1}\frac{24\times1+1}{24-1}\right)=\frac{25}{23}$$

## Question 032 MCQ

**QUESTION**

A rectangular sheet of fixed perimeter with sides having their lengths in the ratio

$$8 : 15$$

is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is

$$100$$

, the resulting box has maximum volume. Then the lengths of the vsides of the rectangular sheet are

A

$$24$$

B

$$32$$

C

$$45$$

D

$$60$$

**CORRECT OPTION**

A

$$24$$

**SOURCE**

Mathematics • application-of-derivatives

**EXPLANATION**

Let

$$a$$

rectangular sheet with sides

$$15a$$

and

$$8a$$

and four squares of side length

$$b$$

removing from each corners of the rectangular sheet

Given, the perimeter of rectangular sheet is constant.

$$\therefore 15a + 8a =$$

Constant

$$\Rightarrow a$$

is constant

Let

$$V$$

be the volume of open rectangular box

$$\begin{aligned}
&\Rightarrow \quad \mathrm{V} = (15a - 2b)b(8a - 2b) \\
&\Rightarrow \quad \mathrm{V} = 4b^3 - 46ab^2 + 120a^2b \\
&\Rightarrow \quad \frac{d\,\mathrm{V}}{db} = 12b^2 - 92ab + 120a^2 = 0 \\
&\Rightarrow \quad \frac{d\,\mathrm{V}}{db} = 4(3b - 5a)(b - 6a) = 0 \\
&\Rightarrow \quad b = \frac{5a}{3}, 6a
\end{aligned}$$

Now

$$\frac{d^2v}{db^2} = 24b - 92a < 0$$

at

$$b = \frac{5a}{3}$$

So,

$$b = \frac{5a}{3}$$

is the point of local maxima

Given, the area of removed squares is 100

$$\begin{aligned} \therefore \qquad 4b^2 &= 100 \\ \Rightarrow \qquad b^2 &= 25 \\ \Rightarrow \qquad \left(\frac{5a}{3}\right)^2 &= 25 \\ \Rightarrow \qquad a &= 3 \end{aligned}$$

Hence, the side length of the given rectangular sheet are

$$15a = 45$$

and

$$8a = 24$$

.

Hints:

$$\Rightarrow$$

If

$$x = x_0$$

is the point of local maxima of a function

$$y = f(x)$$

, then

$$f'(x_0) = 0$$

and

$$f''(x_0) < 0$$

## Question 033 MCQ

**QUESTION**

For 3 × 3 matrices M and N, which of the following statement $s$ is $are$ NOT correct?

A. $N^TMN$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric.

B. MN – NM is skew symmetric for all symmetric matrices M and N.

C. MN is symmetric for all symmetric matrices M and N.

D. $adjM \cdot adjN$ = adj $MN$ for all invertible matrices M and N.

**CORRECT OPTION**

C. MN is symmetric for all symmetric matrices M and N.

**SOURCE**

Mathematics • matrices-and-determinants

**EXPLANATION**

In case of option $A$,

$$(N^TMN)^T = N^TM^T(N^T)^T = N^TM^TN$$

Now,

$$N^T M^T N = \begin{cases} N^T MN, \; when \; M^T = M \\ -N^T MN, \; when \; M^T = -M \end{cases}$$

$$\therefore$$

$N^T MN$ is symmetric or skew symmetric according as M is symmetric or skew symmetric.

In case of option $B$,

$$(MN - NM)^T = (MN)^T - (NM)^T$$

$$= N^T M^T - M^T N^T$$

$$= NM - MN$$

$$\$\$ \because \$\$\$\$ M^T = M, \; N^T = N\$\$$$

$$= -(MN - NM)$$

$$\therefore$$

$$MN - NM$$

is skew symmetric matrix.

In case of option $C$,

$$(MN)^T = N^T M^T = NM$$

$$\$\$ \because \$\$\$\$ M^T = M, \; N^T = N\$\$$$

Now, MN cannot always be equal to NM

$$\therefore$$

MN is not symmetric matrix.

In case of option $D$,

$$(Adj\, M)(Adj\, N) = Adj(NM) \neq Adj(MN)$$

Therefore, $C$ and $D$ are the correct options.

**Question 034** MCQ

**QUESTION**

The area enclosed by the curves

$$y = \sin x + \text{cosx}$$

and

$$y = |\cos x - \sin x|$$

over the interval

$$\left[0, \frac{\pi}{2}\right]$$

is

A $4\left(\sqrt{2} - 1\right)$

B $2\sqrt{2}\left(\sqrt{2} - 1\right)$

C $2\left(\sqrt{2} + 1\right)$

D $2\sqrt{2}\left(\sqrt{2} + 1\right)$

**CORRECT OPTION**

B $2\sqrt{2}\left(\sqrt{2} - 1\right)$

**SOURCE**

Mathematics • application-of-integration

**EXPLANATION**

Draw the graph of

$$y = \sin x + \cos x$$

and

$$y = |\cos x - \sin x|$$

$$\text{For } x \in \left[0, \frac{\pi}{2}\right]$$

Let A be the area bounded by curves

$$y = \sin x + \cos x$$

and

$$y = |\cos x + \sin x|$$

for

$$x \in \left[0, \frac{\pi}{2}\right]$$

$$\mathrm{A} = 2\int_0^{\frac{\pi}{4}} ((\sin x + \cos x) - (\cos x - \sin x))dx$$

$$\mathrm{A} = 4\int_0^{\frac{\pi}{4}} \sin x dx$$

$$\Rightarrow \mathrm{A} = 4[-\cos x]_0^{\frac{\pi}{4}}$$

$$\Rightarrow \mathrm{A} = 4\left[\frac{-1}{\sqrt{2}} + 1\right]$$

$$\Rightarrow \mathrm{A} = 2\sqrt{2}(\sqrt{2} - 1) \text{ sq. units}$$

Hints:

$i$ the area bounded by curves

$$y = f(x)$$

and

$$y = g(x)$$

and the lines

$$x = a$$

and

$$x = b(b > a)$$

is

$$\int_a^b |f(x) - g(x)| dx$$

$ii$ Recall the graph of

$$y = \sin x + \cos x$$

and

$$y = \cos x - \sin x$$

$iii$ Recall the graphical transformation

$$y = f(x)$$

in to

$$y = |f(x)|$$

.

## Question 035 MCQ

QUESTION

Let

$$f$$

$$:\left[\frac{1}{2},1\right]\to R$$

$the set of all real number$ be a positive, non-constant and differentiable function such that

$$f'\left(x\right)<2f\left(x\right)$$

and

$$f\left(\frac{1}{2}\right)=1.$$

Then the value of

$$\int_{1/2}^{1} f\left(x\right)dx$$

lies in the interval

A $(2e-1, 2e)$

B $(e-1,\ 2e-1)$

C $\left(\frac{e-1}{2}, e-1\right)$

D $\left(0, \frac{e-1}{2}\right)$

**CORRECT OPTION**

D $\left(0, \frac{e-1}{2}\right)$

**SOURCE**

Mathematics • definite-integration

**EXPLANATION**

Given,

$$f(x)$$

be a positive, non-constant and differentiable function and

$$f\left(\frac{1}{2}\right) = 1$$

.

Also given

$$f'(x) < 2f(x)$$

$$\Rightarrow f'(x) - 2f(x) < 0$$

Multiply both side by

$$e^{-2x}$$

$$\Rightarrow e^{-2x} f'(x) - 2e^{-2x} f(x) < 0$$
$$\Rightarrow d\left(e^{-2x} f(x)\right) < 0$$

Let

$$g(x) = e^{-2x} f(x) \forall x \in \left[\frac{1}{2}, 1\right]$$

$$\Rightarrow d(g(x)) < 0$$

Hence,

$$g(x)$$

is a decreasing function in the Given, domain

$$x \in \left[\frac{1}{2}, 1\right]$$

$$\begin{aligned}
&\because \quad g(x) < g\left(\frac{1}{2}\right) \\
&\Rightarrow e^{-2x} f(x) < e^{-1} \cdot f\left(\frac{1}{2}\right) \\
&\Rightarrow f(x) < e^{2x-1} \cdot 1 \\
&\therefore \quad f(x) \text{ is always positive} \\
&\therefore \quad 0 < f(x) < e^{2x-1} \\
&\Rightarrow 0 < \int_{\frac{1}{2}}^{1} f(x) dx < \int_{\frac{1}{2}}^{1} e^{2x-1} dx \\
&\Rightarrow 0 < \int_{\frac{1}{2}}^{1} f(x) dx < \left[\frac{e^{2x-1}}{2}\right]_{\frac{1}{2}}^{1} \\
&\Rightarrow 0 < \int_{\frac{1}{2}}^{1} f(x) dx < \frac{e-1}{2} \\
&\Rightarrow \int_{\frac{1}{2}}^{1} f(x) dx \in \left(0, \frac{e-1}{2}\right)
\end{aligned}$$

Hints:

$$\Rightarrow$$

$i$ if

$$f(x)$$

is a decreasing function for

$$x \in [a, b]$$

, then

$$f(b) < f(x) < f(a)$$

$ii$ Recall that

$$e^{-2x}f'(x) - 2e^{-2x}f(x) = d\left(e^{-2x}f(x)\right)$$

## Question 036 MCQ

**QUESTION**

A curve passes through the point

$$\left(1, \frac{\pi}{6}\right)$$

. Let the slope of
the curve at each point

$$(x, y)$$

be

$$\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0.$$

Then the equation of the curve is

A

$$sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$$

B

$$cos\,ec\left(\frac{y}{x}\right) = \log x + 2$$

C

$$s\,ec\left(\frac{2y}{x}\right)=\log x+2$$

D

$$cos\left(\frac{2y}{x}\right)=\log x+\frac{1}{2}$$

**CORRECT OPTION**

A

$$sin\left(\frac{y}{x}\right)=\log x+\frac{1}{2}$$

**SOURCE**

Mathematics • differential-equations

**EXPLANATION**

Given, the slope of curve at

$$(x, y)$$

is

$$\begin{aligned}&\frac{y}{x}+\sec\left(\frac{y}{x}\right), x>0.\\ \therefore\ &\frac{dy}{dx}=\frac{y}{x}+\sec\left(\frac{y}{x}\right)\quad \ldots\text{(i)}\end{aligned}$$

Put

$$y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$$

in the equation $i$

$$
\begin{aligned}
\Rightarrow \quad & v+x\frac{dv}{dx}=\frac{vx}{x}+\sec\left(\frac{vx}{x}\right) \\
\Rightarrow \quad & v+x\frac{dv}{dx}=v+\sec(v) \\
\Rightarrow \quad & x\frac{dv}{dx}=\sec v \\
\Rightarrow \quad & \cos v dv=\frac{dx}{x} \\
\Rightarrow \quad & \int \cos v dv=\int \frac{dx}{x} \\
\Rightarrow \quad & \sin v=\ln x+c \\
\Rightarrow \quad & \sin\left(\frac{y}{x}\right)=\ln x+c \quad \text{... (i)}
\end{aligned}
$$

Put

$$x=1$$

and

$$y=\frac{\pi}{6}$$

in the equation $ii$

$$
\begin{aligned}
\Rightarrow \qquad \sin\frac{\pi}{6} &= \log 1+c \\
\Rightarrow \qquad \frac{1}{2} &= 0+c \\
\Rightarrow \qquad c &= \frac{1}{2}
\end{aligned}
$$

Put

$$c=\frac{1}{2}$$

in the equation $ii$

$$\Rightarrow \quad \sin\frac{y}{x}=\log x+\frac{1}{2}$$

Hints:

$i$ the slope of a curve at

$$(x,y)$$

is equal to

$$\frac{dy}{dx}$$

$ii$ For the solution of homogeneous differential equation

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

put

$$y = vx$$

i.e.

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

.

## Question 037 MCQ

**QUESTION**

Four persons independently solve a certain problem correctly with probabilities

$$\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}.$$

Then the probability that the problem is solved correctly by at least one of them is

A $$\frac{235}{256}$$

B $$\frac{21}{256}$$

C

$$\frac{3}{256}$$

D

$$\frac{253}{256}$$

**CORRECT OPTION**

A

$$\frac{235}{256}$$

**SOURCE**

Mathematics • probability

**EXPLANATION**

Let A, B, C, D are the events of solving a problem.

Given:

$$\mathrm{P(A)}=\frac{1}{2}, \mathrm{P(B)}=\frac{3}{4}, \mathrm{P(C)}=\frac{1}{4}, \mathrm{P(D)}=\frac{1}{8}$$

Also given A, B, C, D are independent events.

Let P is the probability of solved by at least one person.

$$\Rightarrow \mathrm{P}=1-$$

$$Probability that none of the person can solve the problem$$

$$\Rightarrow \mathrm{P} = 1 - \mathrm{P}(\overline{\mathrm{A}} \cap \overline{\mathrm{B}} \cap \overline{\mathrm{C}} \cap \overline{\mathrm{D}})$$
$$\Rightarrow \mathrm{P} = 1 - \mathrm{P}(\overline{\mathrm{A}}) \cdot \mathrm{P}(\overline{\mathrm{B}}) \cdot \mathrm{P}(\overline{\mathrm{C}}) \cdot \mathrm{P}(\overline{\mathrm{D}})$$
$$\Rightarrow \mathrm{P} = 1 - (1 - \mathrm{P}(\mathrm{A}))(1 - \mathrm{P}(\mathrm{B}))(1 - \mathrm{P}(\mathrm{C}))(1 - \mathrm{P}(\mathrm{D}))$$
$$\Rightarrow \mathrm{P} = 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{3}{4}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{8}\right)$$
$$\Rightarrow \mathrm{P} = 1 - \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}$$
$$\Rightarrow \mathrm{P} = 1 - \frac{21}{256}$$
$$\Rightarrow \mathrm{P} = \frac{235}{256}$$

Hints:

$$\Rightarrow$$

$i$ If

$$A$$

and

$$B$$

are independent events, then

$$\rightarrow \mathrm{P}(\mathrm{A} \cap \mathrm{B}) = \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\mathrm{B})$$
$$\rightarrow \mathrm{P}(\overline{\mathrm{A}} \cap \overline{\mathrm{B}}) = \mathrm{P}(\overline{\mathrm{A}}) \cdot \mathrm{P}(\overline{\mathrm{B}}) = (1 - \mathrm{P}(\mathrm{A}))(1 - \mathrm{P}(\mathrm{B}))$$
$$\rightarrow \mathrm{P}(\overline{\mathrm{A}} \cap \mathrm{B}) = \mathrm{P}(\overline{\mathrm{A}}) \cdot \mathrm{P}(\mathrm{B}) = (1 - \mathrm{P}(\mathrm{A}))\mathrm{P}(\mathrm{B})$$
$$\rightarrow \mathrm{P}(\mathrm{A} \cap \overline{\mathrm{B}}) = \mathrm{P}(\mathrm{A}) \cdot \mathrm{P}(\overline{\mathrm{B}}) = \mathrm{P}(\mathrm{A}) \cdot (1 - \mathrm{P}(\mathrm{B}))$$

$ii$ Probability of at least one person can solve the problem

$$= 1 -$$

Probability that none of the persons can solve that problem.

## Question 038 Numerical

**QUESTION**

Of the three independent events

$$E_1, E_2$$

and

$$E_3,$$

the probability that only

$$E_1$$

occurs is

$$\alpha,$$

only

$$E_2$$

occurs is

$$\beta$$

and only

$$E_3$$

occurs is

$$\gamma.$$

Let the probability

$$p$$

that none of events

$$E_1, E_2$$

or

$$E_3$$

occurs satisfy the equations

$$(\alpha - 2\beta)p = \alpha\beta$$

and

$$(\beta - 3\gamma)p = 2\beta\gamma.$$

All the given probabilities are assumed to lie in the interval

$$(0, 1)$$

.

Then

$$\frac{\Pr obability\ of\ occurrence\ of\ E_1}{\Pr obability\ of\ occurrence\ of\ E_3}$$

**SOURCE**

Mathematics • probability

**EXPLANATION**

Given, three independent events

$$\mathrm{E}_1, \mathrm{E}_2$$

and

$$\mathrm{E}_3$$

.

Probability that only

$$\mathrm{E}_1 \text{ occurs } = \mathrm{P}\left(\mathrm{E}_1 \cap \overline{\mathrm{E}}_2 \cap \overline{\mathrm{E}}_3\right) = \alpha$$

Probability that only

$$\mathrm{E}_2 \text{ occurs } = \mathrm{P}\left(\mathrm{E}_1 \cap \overline{\mathrm{E}}_2 \cap \overline{\mathrm{E}}_3\right) = \beta$$

Probability that only

$$\mathrm{E}_3 \text{ occurs } = P\left(\bar{E}_1 \cap \bar{E}_2 \cap E_3\right) = \gamma$$

Probability that none of

$$E_1, E_2$$

or

$$E_3$$

occurs

$$= P\left(\overline{\mathrm{E}}_1 \cap \overline{\mathrm{E}}_2 \cap \overline{\mathrm{E}}_3\right) = \rho$$

$$\begin{aligned}
&\text{Let } \mathrm{P}\left(\mathrm{E}_1\right) = x, \mathrm{P}\left(\mathrm{E}_2\right) = y \text{ and } \mathrm{P}\left(\mathrm{E}_3\right) = z \\
&\alpha = \mathrm{P}\left(\mathrm{E}_1\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_2\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_3\right) \\
&\Rightarrow \quad \alpha = x(1-y)(1-z) \quad \ldots \text{(i)} \\
&\beta = \mathrm{P}\left(\overline{\mathrm{E}}_1\right) \cdot \mathrm{P}\left(\mathrm{E}_2\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_3\right) \\
&\Rightarrow \quad \beta = (1-x) \cdot (y)(1-z) \quad \ldots \text{(ii)} \\
&\gamma = \mathrm{P}\left(\overline{\mathrm{E}}_1\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_2\right) \cdot \mathrm{P}\left(\mathrm{E}_3\right) \\
&\Rightarrow \quad \gamma = (1-x)(1-y)z \quad \ldots \text{(iii)} \\
&\rho = \mathrm{P}\left(\overline{\mathrm{E}}_1\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_2\right) \cdot \mathrm{P}\left(\overline{\mathrm{E}}_3\right) \\
&\Rightarrow \quad \rho = (1-x)(1-y)(1-z) \quad \ldots \text{(iv)}
\end{aligned}$$

Given

$$(\alpha - 2\beta)\rho = a\beta$$

and

$$(\beta - 3\gamma)\rho = 2b\gamma$$

$$
\begin{aligned}
\Rightarrow\quad & [x(1-y)(1-z)-2(1-x)y(1-z)](1-x)\\
& (1-y)(1-z)\\
& =x(1-y)(1-z)\cdot(1-x)y(1-z) \text{ and}\\
& [(1-x)y(1-z)-3(1-x)(1-y)z](1-x)\\
& (1-y)(1-z)\\
& =2(1-x)y(1-z)\cdot(1-x)(1-y)z\\
\Rightarrow\quad & [x-xy-2y+2xy](1-x)(1-y)(1-z)^2\\
& =xy(1-x)(1-y)(1-z)^2 \text{ and}\\
& [y-yz-3z+3yz](1-x)^2(1-y)(1-z)\\
& =2yz(1-x)^2(1-y)(1-z)\\
\Rightarrow\quad & (x-2y+xy)=xy \text{ and } (y-3z+2yz)=2yz\\
\Rightarrow\quad & x=2y \text{ and } y=3z\\
\Rightarrow\quad & \frac{x}{2}=y=3z\\
\Rightarrow\quad & \frac{x}{Z}=6
\end{aligned}
$$

$$
\Rightarrow \frac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}=\frac{x}{Z}=6
$$

Hints :

If

$$
\mathrm{A, B, C}
$$

are independent events, then probability of occurrence of only event A

$$
=\mathrm{P}(\mathrm{A}\cap\overline{\mathrm{B}}\cap\overline{\mathrm{C}})=\mathrm{P}(\mathrm{A})\cdot\mathrm{P}(\overline{\mathrm{B}})\cdot\mathrm{P}(\overline{\mathrm{C}})
$$

.

## Question 039 MCQ

**QUESTION**

A line

$$
l
$$

passing through the origin is perpendicular to the lines

$$\$ l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\widehat{k}, \quad -\infty < t < \infty$$

$$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\widehat{k}, \quad -\infty < s < \infty$$

$Then, the coordinate(s) of the points(s) on\ l_2\ at a distance of\ \sqrt{17}$ $from the point of intersection of\ l\ and\ l_1$ \$ is $are$

A

$$\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$$

B

$$(-1, -1, 0)$$

C

$$(1, 1, 1)$$

D

$$\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$$

**CORRECT OPTION**

B

$$(-1, -1, 0)$$

**SOURCE**

Mathematics • 3d-geometry

**EXPLANATION**

Given two lines.

$$l_1 : (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k} \text{ and}$$
$$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$
$$\Rightarrow l_1 : \frac{x-3}{1} = \frac{y+1}{2} = \frac{z-4}{2} = t \text{ and}$$
$$l_2 : \frac{x-3}{2} = \frac{y-3}{2} = \frac{z-2}{1} = s$$

Given, a line

$$l$$

is perpendicular to

$$l_1$$

and

$$l_2$$

$$\therefore \quad l$$

is parallel to

$$l_1 \times l_2$$

$$\Rightarrow \vec{l}_1 \times \vec{l}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$$
$$\Rightarrow \vec{l}_1 \times \vec{l}_2 = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

The equation of line

$$l$$

passing through origin and parallel to

$$-2\hat{i} + 3\hat{j} - 2\hat{k}$$

is

$$l : \frac{x}{-2} = \frac{y}{3} = \frac{z}{-2} = \lambda$$

Let P is the point of intersection of

$$l$$

and

$$l_1$$

$$\Rightarrow \mathrm{P}=(-2\lambda, 3\lambda, -2\lambda)=(3+t, -1+2t, 4+2t)$$
$$\Rightarrow \lambda=\frac{3+t}{-2}=\frac{-1+2t}{3}=\frac{4+2t}{-2}$$
$$\Rightarrow t=-1, \lambda=-1$$

$$\text{So, } \mathrm{P}=(2,-3,2)$$

Let Q is a point on the line

$$l_2$$

$$Q=(3+2s, 3+2s, 2+s)$$

Apply

$$|\mathrm{PQ}|=\sqrt{17}$$

$$\Rightarrow \sqrt{(3+2s-2)^2+(3+2s+3)^2+(2+s-2)^2}$$
$$=\sqrt{17}$$
$$\Rightarrow \sqrt{9s^2+28s+37}=\sqrt{17}$$
$$\Rightarrow \quad 9s^2+28s+37=17$$
$$\Rightarrow \quad 9s^2+28s+20=0$$
$$\Rightarrow \mathrm{s}=\frac{-28 \pm \sqrt{28^2-4 \times 9 \times 20}}{2.9}$$
$$\Rightarrow \mathrm{s}=\frac{-28 \pm 8}{18}$$
$$\Rightarrow \mathrm{s}=\frac{-10}{9},-2$$

If

$$s=-2$$

, then

$$Q=(-1,-1,0)$$

$$\text{If } s=-\frac{10}{9}, \text{ then } Q=\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$$

Hints :

$i$ If

$$l$$

is perpendicular to

$$\vec{l}_1$$

and

$$\vec{l}_2$$

, then

$$l$$

is parallel to

$$\vec{l}_1 \times \vec{l}_2$$

.

$ii$ A general point on the line

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = \lambda \text{ is } (x_1 + \lambda a, y_1 + \lambda b, z_1 + \lambda c).$$

## Question 040 MCQ

**QUESTION**

Let $\overrightarrow{\mathrm{PR}} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{\mathrm{SQ}} = \hat{i} - 3\hat{j} - 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $\overrightarrow{\mathrm{PT}} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of the parallelopiped determined by the vectors $\overrightarrow{\mathrm{PT}}, \overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PS}}$ is :

A 5 units

B 20 units

C 10 units

D 30 units

**CORRECT OPTION**

C 10 units

**SOURCE**

Mathematics • vector-algebra

**EXPLANATION**

Given that $\overrightarrow{\mathrm{PR}} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{\mathrm{SQ}} = \hat{i} - 3\hat{j} - 4\hat{k}$ are the diagonals of the parallelogram $PQRS$,

we have the following relationships:

$$\overrightarrow{\mathrm{PR}} = \overrightarrow{\mathrm{PQ}} + \overrightarrow{\mathrm{QR}}$$
$$\overrightarrow{\mathrm{SQ}} = \overrightarrow{\mathrm{PQ}} - \overrightarrow{\mathrm{PS}}$$

Since $\overrightarrow{\mathrm{QR}} = \overrightarrow{\mathrm{PS}}$,

$$\overrightarrow{\mathrm{PQ}} = \frac{\overrightarrow{\mathrm{PR}} + \overrightarrow{\mathrm{SQ}}}{2}$$

and

$$\overrightarrow{\mathrm{PS}} = \frac{\overrightarrow{\mathrm{PR}} - \overrightarrow{\mathrm{SQ}}}{2}.$$

Substituting the values of $\overrightarrow{\mathrm{PR}}$ and $\overrightarrow{\mathrm{SQ}}$,

$$\overrightarrow{\mathrm{PQ}} = \frac{(3\hat{i}+\hat{j}-2\hat{k})+(\hat{i}-3\hat{j}-4\hat{k})}{2}$$
$$= \frac{4\hat{i}-2\hat{j}-6\hat{k}}{2}$$
$$= 2\hat{i}-\hat{j}-3\hat{k},$$

$$\overrightarrow{\mathrm{PS}} = \frac{(3\hat{i}+\hat{j}-2\hat{k})-(\hat{i}-3\hat{j}-4\hat{k})}{2}$$
$$= \frac{2\hat{i}+4\hat{j}+2\hat{k}}{2}$$
$$= \hat{i}+2\hat{j}+\hat{k}.$$

Given $\overrightarrow{\mathrm{PT}} = \hat{i}+2\hat{j}+3\hat{k}$,

To find the volume $V$ of the parallelepiped formed by $\overrightarrow{\mathrm{PT}}, \overrightarrow{\mathrm{PQ}},$ and $\overrightarrow{\mathrm{PS}}$, we calculate the determinant of the following matrix:

$$V = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -1 & -3 \\ 1 & 2 & 1 \end{vmatrix}.$$

Calculating this determinant:

$$V = 1(-1\times 1+6)-2(2+3)+3(4+1)$$

$$= 1\times 5-2\times 5+3\times 5$$

$$= 5-10+15$$

$$= 10.$$

Thus, the volume of the parallelepiped is 10 units.

## Question 041 Numerical

**QUESTION**

A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad $s^{-1}$ about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad $s^{-1}$ ) of the system is

**SOURCE**

Physics • rotational-motion

**EXPLANATION**

Consider the disc and the rings together as a system. Since the external torque $\vec{\Gamma}_{\text{ext}}$ is zero, the angular momentum of the system about the vertical axis through $O$ is conserved, which can be expressed as:

$$I_i\omega_i = I_f\omega_f$$

Initially, the moment of inertia is due to the disc alone, and its value is:

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2} \times 50 \times 0.4^2 = 4\ \mathrm{kg} \cdot \mathrm{m}^2$$

The moment of inertia of a ring with mass $m$ and radius $r$ about an axis perpendicular to its center is $mr^2$. Using the theorem of parallel axes, the ring's moment of inertia about the axis of rotation is given by:

$$I_{\text{ring}} = mr^2 + md^2 = 2mr^2$$

where $d = r$ is the distance between the parallel axes. Thus, the moment of inertia of the complete system in the final configuration is:

$$\begin{aligned} I_f &= \frac{1}{2}MR^2 + 2mr^2 + 2mr^2 \\ &= \frac{1}{2}(50)(0.4)^2 + 2\left(2(6.25)(0.2)^2\right) = 5\ \mathrm{kg} \cdot \mathrm{m}^2 \end{aligned}$$

With no external torque acting on the system, the angular momentum is conserved:

$$I_i\omega_i = I_f\omega_f$$
$$4 \times 10 = 5\omega_f$$
$$\omega_f = 8\ \mathrm{rad/s}$$

## Question 042 MCQ

**QUESTION**

The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is

$$\frac{2}{3}$$

times the wavelength in free space. The radius of the curved surface of the lens is

A. 1 m

B. 2 m

C. 3 m

D. 6 m

**CORRECT OPTION**

C. 3 m

**SOURCE**

Physics • geometrical-optics

**EXPLANATION**

The speed of light in a medium is given by the equation :

$$\frac{C}{V} = \frac{\lambda_0}{\lambda} = \mu$$

Where :

- $V$ is the velocity of light in the medium
- $\lambda$ is the wavelength of light in the medium
- $\lambda_0$ is the wavelength in free space
- $\mu$ is the refractive index of the medium

Given :

$$\lambda = \frac{2}{3}\lambda_0$$

Using the given data :

$$u = -24 \text{ m},$$

$$h_i = \frac{1}{3}h_0$$

To find the magnification ($m$) :

$$m = \frac{h_i}{h_0} = \frac{1}{3} = \frac{v}{u}$$

Hence,

$$v = \frac{u}{3}$$

Given :

$$v = 8 \text{ m}, \ u = -24 \text{ m}$$

Applying the lens formula :

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Since the plane surface has an infinite radius :

$$\frac{1}{\infty} = 0$$

Therefore :

$$\frac{1}{8} - \frac{1}{-24} = (1.5 - 1)\left(\frac{1}{R} - 0\right)$$

$$\Rightarrow \frac{1}{6} = \frac{0.5}{R}$$

Solving for $R$ :

$$\Rightarrow R = 3 \text{ m}$$

## Question 043 MCQ

**QUESTION**

The work done on a particle of mass m by a force

$$\$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

$(Kbeingaconstantofappropriatedimensions), whentheparticleistakenfro$
$(a, 0)$ $tothepoint$ $(0, a)$ \$ along a circular path of radius a about the origin in the x-y plane is

**A** $\frac{2K\pi}{a}$

B $\frac{K\pi}{a}$

C $\frac{K\pi}{2a}$

D 0

**CORRECT OPTION**

D 0

**SOURCE**

Physics • work-power-and-energy

**EXPLANATION**

The work done on a particle of mass *m* by a force

$$\vec{F} = K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i} + \frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(with *K* being a constant with appropriate dimensions), when the particle is taken from the point $a, 0$ to the point $0, a$ along a circular path of radius *a* about the origin in the x-y plane is:

Let $\vec{r} = x\hat{i} + y\hat{j}$

Therefore, $d\vec{r} = dx\hat{i} + dy\hat{j}$ and $r = \sqrt{x^2 + y^2}$

Work done by the force in moving a particle from A to B is

$$W = \int_{r_A}^{r_B} \vec{F} \cdot d\vec{r}$$

$$= \int_{r_A}^{r_B} K \left[ \frac{x}{(x^2+y^2)^{3/2}} \hat{i} + \frac{y}{(x^2+y^2)^{3/2}} \hat{j} \right] \cdot (dx\hat{i} + dy\hat{j})$$

$$= K \int_{r_A}^{r_B} \frac{xdx}{(x^2+y^2)^{3/2}} + \frac{ydy}{(x^2+y^2)^{3/2}}$$

$$= K \int_{r_A}^{r_B} \frac{1}{(x^2+y^2)^{3/2}} \left[ d\left(\frac{x^2}{2}\right) + d\left(\frac{y^2}{2}\right) \right]$$

$$= K \int_{r_A}^{r_B} \frac{1}{2(x^2+y^2)^{3/2}} d(x^2+y^2)$$

$$= K \int_{r_A}^{r_B} \frac{1}{2r^3} d(r^2) = K \int_{r_A}^{r_B} \frac{2rdr}{2r^3} = K \int_{r_A}^{r_B} \frac{dr}{r^2} \quad (\because r = \sqrt{x^2+y^2})$$

$$= K \left[ -\frac{1}{r} \right]_{r_A}^{r_B} = K \left[ \frac{1}{r_A} - \frac{1}{r_B} \right]$$

Here, $r_A = a$ and $r_B = a$

Therefore, $W = 0$

## Question 044 Numerical

**QUESTION**

A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed $inm/s$ of the particle is zero, the speed $inm/s$ after 5 s is

**SOURCE**

Physics • work-power-and-energy

**EXPLANATION**

To determine the speed of a particle after $t = 5$ seconds, given a constant power, the following steps should be taken :

**Power Equation:** Power is the rate at which work is done, and work is the change in kinetic energy :

$$P = \frac{d(W)}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2\right)$$

**Given Data :**

Mass of the particle $\$m\$$ = 0.2 kg

Constant power $\$P\$$ = 0.5 W

Initial speed $\$v_0\$$ = 0 m/s

**Differentiating Kinetic Energy with Respect to Time :**

$$P = \frac{d}{dt}\left(\frac{1}{2}mv^2\right) = mv\frac{dv}{dt}$$

**Rearranging to Solve for $v\frac{dv}{dt}$ :**

$$0.5 = 0.2 \cdot v \cdot \frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{5}{2v}$$

**Integrating Both Sides :**

$$v\,dv = \frac{5}{2}\,dt$$

$$\int v\,dv = \int \frac{5}{2}\,dt$$

$$\frac{v^2}{2} = \frac{5t}{2} + C$$

**Applying Initial Condition $when\$t = 0\$, \$v = 0\$$ :**

$$0 = \frac{5 \cdot 0}{2} + C$$

$$C = 0$$

**Solving for $v$ :**

$$\frac{v^2}{2} = \frac{5t}{2}$$

$$v^2 = 5t$$

$$v = \sqrt{5t}$$

**Speed After 5 Seconds :**

$$v = \sqrt{5 \cdot 5} = \sqrt{25} = 5\,\mathrm{m/s}$$

Thus, the speed of the particle after 5 seconds is **5 m/s**.

## Question 045 MCQ

**QUESTION**

A particle of mass m is projected from the ground with an initial speed $u_0$ at an angle

$$\alpha$$

with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed $u_0$. The angle that the composite system makes with the horizontal immediately after the collision is

A $\frac{\pi}{4}$

B

$$\frac{\pi}{4} + \alpha$$

C

$$\frac{\pi}{2} - \alpha$$

D

$$\frac{\pi}{2}$$

**CORRECT OPTION**

A

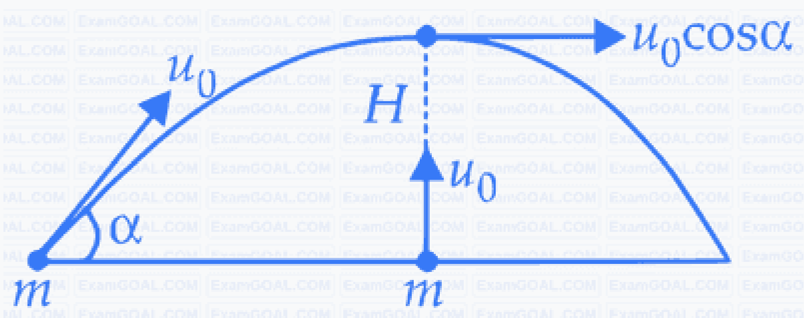
$$\frac{\pi}{4}$$

**SOURCE**

Physics • impulse-and-momentum

**EXPLANATION**

The situation is depicted in the figure below.




At the highest point of the first particle's trajectory, its speed is given by :

$$\text{Speed of the first particle} = u_0 \cos \alpha$$

For the second particle, which was thrown vertically upward, the speed at its highest point is calculated as :

$$\text{Speed of the second particle} = \sqrt{u_0^2 - 2gH}$$

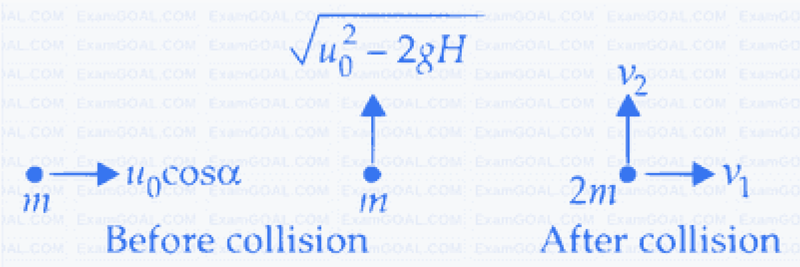
Where $H$ is the maximum height reached by the second particle :

$$H = \frac{u_0^2 \sin^2 \alpha}{2g}$$

Therefore, the speed of the second particle at the highest point becomes :

$$= \sqrt{u_0^2 - \frac{2g(u_0^2 \sin^2 \alpha)}{2g}} = \sqrt{u_0^2 - u_0^2 \sin^2 \alpha} = u_0 \cos \alpha$$

Thus, at the highest point :




According to the law of conservation of linear momentum in the horizontal direction :

$$mu_0 \cos \alpha = 2mv_1$$

$$\implies v_1 = \frac{u_0 \cos \alpha}{2} \quad \ldots \text{(i)}$$

In the vertical direction :

$$mu_0 \cos \alpha = 2mv_2$$

$$\implies v_2 = \frac{u_0 \cos \alpha}{2} \quad \ldots \text{(ii)}$$

Let $\theta$ be the angle that the composite system makes with the horizontal immediately after the collision. Then :

$$\tan \theta = \frac{v_2}{v_1} = 1 \quad \text{(Using (i) and (ii))}$$

$$\therefore \theta = \tan^{-1} 1 = 45^\circ = \frac{\pi}{4}$$

## Question 046 Numerical

**QUESTION**

A bob of mass m, suspended by a string of length $l_1$ is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length $l_2$, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio

$$\frac{l_1}{l_2}$$

is

**SOURCE**

Physics • work-power-and-energy

**EXPLANATION**

To determine the minimum velocity required for the bob of mass $m$ to complete a full circle in the vertical plane, we start with the condition at the bottommost point:

$$u = \sqrt{5gl_1} \text{ (at point A)}$$

At the highest point B, using the conservation of energy, we get:

$$\frac{1}{2}mu_1^2 = mg(2l_1) + \frac{1}{2}mv_1^2$$

Simplifying, we have:

$$v_1^2 = u_1^2 - 4gl_1$$

Substituting $u_1 = \sqrt{5gl_1}$:

$$v_1^2 = 5gl_1 - 4gl_1$$

$v_1 = \sqrt{gl_1}$

When the bob at point B elastically collides with another identical mass that is initially at rest, the velocities swap due to the nature of elastic collisions. Thus, the second bob attains the velocity:

$u_2 = v_1 = \sqrt{gl_1}$

For the second bob to complete its circular path, the minimum velocity requirement is:

$u_2 = \sqrt{5gl_2} = \sqrt{gl_1}$

$\therefore \frac{l_1}{l_2} = 5$

## Question 047 MCQ

**QUESTION**

One end of a horizontal thick copper wire of length 2L and radius 2R is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

A 0.25

B 0.50

C 2.00

D 4.00

CORRECT OPTION

C 2.00

SOURCE

Physics • properties-of-matter

EXPLANATION

To solve this problem, we need to use Hooke's Law, which states that the elongation $or stretch$ of a material is directly proportional to the force applied and the length of the material, and inversely proportional to the cross-sectional area and the Young's modulus of the material. Mathematically, this relation can be expressed as:

$$\Delta L = \frac{F \cdot L}{A \cdot Y}$$

where:

- $$\Delta L$$ is the elongation
- $$F$$ is the force applied
- $$L$$ is the original length of the material
- $$A$$ is the cross-sectional area of the material
- $$Y$$ is the Young's modulus of the material

In this problem, the Young's modulus $\$\$Y\$\$$ and the force $\$\$F\$\$$ are the same for both wires because they are made of the same material $copper$ and the forces applied are equal. Therefore, the elongation is dependent on the length and cross-sectional area of each wire.

For the thick wire with length

$$2L$$

and radius

$$2R$$

, the cross-sectional area

$$A_1$$

is given by:

$$A_1 = \pi(2R)^2 = 4\pi R^2$$

For the thin wire with length

$$L$$

and radius

$$R$$

, the cross-sectional area

$$A_2$$

is given by:

$$A_2 = \pi R^2$$

Using the formula for elongation for both wires, we get:

Elongation of the thick wire $\$\$\Delta L_{\text{thick}}\$\$$:

$$\Delta L_{\text{thick}} = \frac{F \cdot 2L}{4\pi R^2 \cdot Y} = \frac{F \cdot 2L}{4\pi R^2 Y} = \frac{FL}{2\pi R^2 Y}$$

Elongation of the thin wire $\$\$\Delta L_{\text{thin}}\$\$$:

$$\Delta L_{\text{thin}} = \frac{F \cdot L}{\pi R^2 \cdot Y} = \frac{FL}{\pi R^2 Y}$$

To find the ratio of the elongation in the thin wire to that in the thick wire, we divide

$$\Delta L_{\text{thin}}$$

by

$$\Delta L_{\text{thick}}$$

:

$$\text{Ratio} = \frac{\Delta L_{\text{thin}}}{\Delta L_{\text{thick}}} = \frac{\frac{FL}{\pi R^2 Y}}{\frac{FL}{2\pi R^2 Y}} = \frac{FL}{\pi R^2 Y} \cdot \frac{2\pi R^2 Y}{FL} = 2$$

Therefore, the ratio of the elongation in the thin wire to that in the thick wire is 2. The correct answer is:

Option C: 2.00

**Question 048** MCQ

**QUESTION**

A solid sphere of radius R and density

$$\rho$$

is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3

$$\rho$$

. The complete arrangement is placed in a liquid of density 2

$$\rho$$

and is allowed to reach equilibrium. The correct statement$s$ is $are$

The net elongation of the spring is

A

$$\frac{4\pi R^3 \rho g}{3k}$$

B The net elongation of the spring is

$$\frac{8\pi R^3 \rho g}{3k}$$

C The light sphere is partially submerged

D The light sphere is completely submerged

**CORRECT OPTION**

A The net elongation of the spring is

$$\frac{4\pi R^3 \rho g}{3k}$$

**SOURCE**

Physics • properties-of-matter

**EXPLANATION**

Considering FBDs of spheres A and B, we get

Here, $x$ is the extension produced in the spring

At equilibrium $\Sigma \mathrm{F_A} = 0$ and $\Sigma \mathrm{F_B} = 0$

$$\frac{4}{3}\pi R^3(2\rho)g = kx + \frac{4}{3}\pi R^3\rho g \ldots\ldots\ldots\ldots (i)$$

$$kx + \frac{4}{3}\pi R^3(2\rho)g = \frac{4}{3}\pi R^3(3\rho)g \ldots\ldots\ldots (ii)$$

Subtract $ii$ from $i$, we get

$$-kx = kx - \frac{4}{3}\pi \mathrm{R}^3(2\rho)g$$

$$x = \frac{4\pi \mathrm{R}^3\rho g}{3k}$$

$Option A is correct$

If we consider both the spheres as a system, then at equilibrium,

$$\begin{aligned} \text{total weight } &= \text{buoyant force} \\ \text{Total weight (A + B)} &= \frac{4}{3}\pi \mathrm{R}^3(4\rho)g \\ \text{Total buoyant force } &= \frac{4}{3}\pi \mathrm{R}^3(4\rho)g \end{aligned}$$

Hence, sphere A $light sphere$ is completely submerged $option D is correct$

## Question 049 MCQ

**QUESTION**

Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

A. 1 : 4

B. 1 : 2

C 6 : 9

D 8 : 9

**CORRECT OPTION**

D 8 : 9

**SOURCE**

Physics • heat-and-thermodynamics

**EXPLANATION**

To determine the ratio of the densities of the two non-reactive monoatomic ideal gases, we can use the ideal gas law and the given information. The ideal gas law is given by:

$$PV = nRT$$

Where:

- **P** is the pressure
- **V** is the volume
- **n** is the number of moles
- **R** is the gas constant
- **T** is the temperature

Given that the gases are monoatomic and non-reactive, their molecular masses are directly proportional to their atomic masses. Let the atomic masses of the two gases be $M_1$ and $M_2$, with $M_1 : M_2 = 2 : 3$.

The density $\$\rho\$$ of a gas is given by the formula:

$$\rho = \frac{m}{V}$$

Where $m$ is the mass and $V$ is the volume. Using the ideal gas law, we can relate density to pressure and temperature.

From the ideal gas law:

$$PV = nRT$$

Since $n = \frac{m}{M}$, where $M$ is the molar mass, we have:

$$P = \frac{mRT}{MV}$$

Rearranging to solve for density $\$\rho = \frac{m}{V}\$$:

$$\rho = \frac{PM}{RT}$$

Thus, the density of a gas is directly proportional to its pressure and molar mass, and inversely proportional to the temperature. Given that the ratio of their partial pressures $P_1 : P_2 = 4 : 3$, we can write:

$$\rho_1 = \frac{P_1 M_1}{RT}$$

$$\rho_2 = \frac{P_2 M_2}{RT}$$

Taking the ratio of the densities:

$$\frac{\rho_1}{\rho_2} = \frac{\left(\frac{P_1 M_1}{RT}\right)}{\left(\frac{P_2 M_2}{RT}\right)} = \frac{P_1 M_1}{P_2 M_2}$$

Substituting the given ratios, $P_1 : P_2 = 4 : 3$ and $M_1 : M_2 = 2 : 3$, into the equation, we get:

$$\frac{\rho_1}{\rho_2} = \frac{4 \times 2}{3 \times 3} = \frac{8}{9}$$

Therefore, the ratio of their densities is $\frac{8}{9}$, which corresponds to option D. Consequently, the answer is:

**Option D: 8 : 9**

## Question 050 MCQ

**QUESTION**

Two non-conducting solid spheres of radii

$$R$$

and

$$2R,$$

having uniform volume charge densities

$$\rho_1$$

and

$$\rho_2$$

respectively, touch each other. The net electric field at a distance

$$2$$
$$R$$

from the center of the smaller sphere, along the line joining the centers of the spheres, is zero. The ratio

$$\frac{\rho_1}{\rho_2}$$

can be

A $$-4$$

B $$-\frac{32}{25}$$

C $$\frac{32}{25}$$

D

$$4$$

CORRECT OPTION

B

$$-\frac{32}{25}$$

SOURCE

Physics • electrostatics

EXPLANATION

The electric field at $Q$

$$\vec{E}_Q = \frac{1}{4\pi\epsilon_0}\left[\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}\hat{\imath} - \frac{\frac{4}{3}\pi R^3\rho_2}{R^2}\hat{\imath}\right] = \overrightarrow{0}$$

gives $\rho_1/\rho_2 = 4$

The electric field at $P$,

$$\vec{E}_P = -\frac{1}{4\pi\epsilon_0}\left[\frac{\frac{4}{3}\pi R^3\rho_1}{(2R)^2}\hat{\imath} + \frac{\frac{4}{3}\pi(2R)^3\rho_2}{(5R)^2}\hat{\imath}\right] = \overrightarrow{0},$$

gives $\rho_1/\rho_2 = -32/25$.

## Question 051 MCQ

QUESTION

In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance

$$C.$$

The switch

$$S_1$$

is pressed first to fully charge the capacitor

$$C_1$$

and then released. The switch

$$S_2$$

is then pressed to charge the capacitor

$$C_2.$$

After some time,

$$S_2$$

is released and then

$$S_3$$

is pressed. After some time

A

The charge on the upper plate of

$$C_1$$

is

$$2CV_0$$

B

The charge on the upper plate of

$$C_1$$

is

$$CV_0$$

C

The charge on the upper plate of

$$C_2$$

is

$$0$$

D

The charge on the upper plate of

$$C_2$$

is

$$-CV_0$$

**CORRECT OPTION**

B

The charge on the upper plate of

$$C_1$$

is

$$CV_0$$

**SOURCE**

Physics • capacitor

**EXPLANATION**

$$\mathrm{C}_1=\mathrm{C}_2=\mathrm{C}$$

* When $\mathrm{S}_1$ is closed. charge on $\mathrm{C}_1=+2\mathrm{VC}$ on upper plate and $-2\mathrm{Vc}$ on lower plate.

* When $S_1$ is released and $S_2$ is closed. charge on $C_1$ and $C_2=\mathrm{VC}$

* When $\mathrm{S}_2$ is released and $\mathrm{C}_3$ is closed. charge on $\mathrm{C}_2 = \mathrm{CV}$

* When $\mathrm{S}_2$ is closed $\$\mathrm{S}_1\$open$

As $\mathrm{C}_1 = \mathrm{C}_2$ the charge gets distributed equal. The upper plates of $C_1$ and $C_2$ now take charge to $\mathrm{CV}_0$ each and lower plate $-\mathrm{CV}_0$ each.

## Question 052 MCQ

**QUESTION**

The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The $24^{\text{th}}$ division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

A. 5.112 cm

B. 5.124 cm

C. 5.136 cm

D. 5.148 cm

**CORRECT OPTION**

B. 5.124 cm

**SOURCE**

Physics • units-and-measurements

**EXPLANATION**

From the given data :

- One main scale division $MSD$ is $5.15 \text{ cm} - 5.10 \text{ cm} = 0.05 \text{ cm}$.
- One Vernier scale division $VSD$ is $\frac{2.45 \text{ cm}}{50} = 0.049 \text{ cm}$.

The least count $LC$ of the Vernier calipers is:

$$\text{LC} = 1 \text{ MSD} - 1 \text{ VSD} = 0.05 \text{ cm} - 0.049 \text{ cm} = 0.001 \text{ cm}$$

Given the main scale reading $MSR$ is 5.10 cm and the Vernier scale reading $VSR$ is 24. The diameter $D$ of the cylinder is calculated as:

$$D = \text{MSR} + \text{VSR} \times \text{LC}$$

$$= 5.10 \text{ cm} + 24 \times 0.001 \text{ cm}$$

$$= 5.124 \text{ cm}$$

## Question 053 MCQ

**QUESTION**

A ray of light travelling in the direction

$$\frac{1}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

is incident on a plane mirror. After reflection, it travels along the direction

$$\frac{1}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$$

. The angle of incidence is

A

30

$^\circ$

B

45

$^\circ$

C

60

$^\circ$

D

75

$^\circ$

**CORRECT OPTION**

A

30

$^\circ$

**SOURCE**

Physics • geometrical-optics

**EXPLANATION**

Let

$$\hat{v}_i = \frac{1}{2}\left(\hat{i} + \sqrt{3}\hat{j}\right)$$

and

$$\hat{v}_r = \frac{1}{2}\left(\hat{i} - \sqrt{3}\hat{j}\right)$$

be the unit vectors along the incident and the reflected rays. From Snell's law, angle of incidence $say$$\theta$$$ is equal to the angle of reflection. Thus, angle between

$$-\hat{v}_i$$

and

$$\hat{v}_r$$

is 2

$$\theta$$

.

Using, dot products of vectors, we get

$$\cos(2\theta) = -\hat{v}_i\,.\,\hat{v}_r = 1/2$$

.

Hence,

$$\theta = 30^\circ$$

.

## Question 054 MCQ

**QUESTION**

Two rectangular blocks, having identical dimensions, can be arranged in either configuration-I or configuration-II as shown in the figure. One of the blocks has thermal conductivity

$$\kappa$$

and the other 2

$$\kappa$$

. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in configuration-I. The time to transport the same amount of heat in configuration-II is

A 2.0 s

B 3.0 s

C 4.5 s

D 6.0 s

**CORRECT OPTION**

A 2.0 s

**SOURCE**

Physics • heat-and-thermodynamics

**EXPLANATION**

Let L and A be length and area of cross-section of each block.

$$\therefore$$

Thermal resistance of block 1 is

$$\therefore$$

$$R_1 = \frac{L}{\kappa A}$$

Thermal resistance of block 2 is

$$R_2 = \frac{L}{2\kappa A}$$

In configuration I, two blocks are connected in series. So, their equivalent thermal resistance is

$$R_S = R_1 + R_2 = \frac{L}{\kappa A} + \frac{L}{2\kappa A} = \frac{3}{2}\frac{L}{\kappa A}$$

...... $i$

$$\therefore$$

Rate of heat flow in configuration I is

$$\therefore$$

$$\frac{Q}{t} = \frac{T_1 - T_2}{R_s}$$

..... $ii$

In configuration II, two blocks are connected in parallel. So, their equivalent thermal resistance is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\frac{L}{\kappa A}} + \frac{1}{\frac{L}{2\kappa A}} = \frac{3\kappa A}{L}$$

...... $iii$

$$R_P = \frac{1}{3}\frac{L}{\kappa A}$$

$$\therefore$$

Rate of same amount of heat flow in configuration II is

$$\therefore$$

$$\frac{Q}{t'} = \frac{T_1 - T_2}{R_P}$$

...... $iv$

Divide $ii$ by $iv$, we get

$$\frac{t'}{t} = \frac{R_P}{R_S} = \frac{1}{3}\frac{L}{\kappa A} \times \frac{2\kappa A}{3L} = \frac{2}{9}$$

$Using(i$ and $iii)$

$$t' = \frac{2}{9}t = \frac{2}{9} \times 9s = 2s$$

## Question 055 MCQ

**QUESTION**

A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3

$\times$

$10^8$ ms

$-$

1. The final momentum of the object is

0.3

$\times$

10

A

$-$

17 kg-ms

$-$

1

1.0

$\times$

B

10

–

17 kg-ms

–

1

3.0

×

10

C

–

17 kg-ms

–

1

9.0

×

10

D

–

17 kg-ms

–

1

**CORRECT OPTION**

1.0

×

10

B

–

17 kg-ms

−

1

**SOURCE**

Physics • electromagnetic-waves

**EXPLANATION**

Duration of pulse, t = 100 ns = 100

×

10

−

9 s.

Power of the pulse, P = 30 mW = 30

×

10

−

3 W.

Speed of light, c = 3

×

$10^8$ ms

−

1

The final momentum of the object is

$$p = \frac{E}{c} = \frac{P \times t}{c} = \frac{30 \times 10^{-3} \times 100 \times 10^{-9}}{3 \times 10^{8}} = 10^{-17}$$

kg-ms

$$-$$

1

## Question 056 MCQ

**QUESTION**

In the Young's double-slit experiment using a monochromatic light of wavelength

$$\lambda$$

, the path difference $intermsofanintegern$ corresponding to any point having half the peak intensity is

A $$(2n+1)\frac{\lambda}{2}$$

B $$(2n+1)\frac{\lambda}{4}$$

C $$(2n+1)\frac{\lambda}{8}$$

D $$(2n+1)\frac{\lambda}{16}$$

**CORRECT OPTION**

**B**

$$(2n+1)\frac{\lambda}{4}$$

**SOURCE**

Physics • wave-optics

**EXPLANATION**

Wavelength of light is

$$\lambda$$

.

Now, using the relation for intensity, we have

$$I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)$$

$$\frac{1}{2} = \cos^2\left(\frac{\phi}{2}\right) \Rightarrow \cos\frac{\phi}{2} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \frac{\phi}{2} = \frac{\pi}{4} \Rightarrow \phi = \frac{\pi}{2}$$

$$\Rightarrow \phi = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$\phi = \frac{2\pi}{\lambda}\Delta x \Rightarrow \frac{\pi}{2} = \frac{2\pi}{2}\Delta x \Rightarrow \Delta x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$$

$$\Rightarrow \Delta x = (2n+1)\frac{\lambda}{4}$$

## Question 057 MCQ

**QUESTION**

A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation

y $x, t$ = $0.01m$ sin[(62.8 m

—

$^{1}$)x] cos[(628 s

—

$^{1}$)t]

Assuming

$\pi$

= 3.14, the correct statement $s$ is $are$

A The number of nodes is 5.

B The length of the string is 0.25 m.

C The maximum displacement of the mid-point of the string, from its equilibrium position is 0.01 m.

D The fundamental frequency is 100 Hz.

**CORRECT OPTION**

B The length of the string is 0.25 m.

**SOURCE**

Physics • waves

**EXPLANATION**

Number of nodes = 6

From the given equation, we can see that

$$k = \frac{2\pi}{\lambda} = 62.8$$

m

$$-$$

1

$$\therefore$$

$$\lambda = \frac{2\pi}{62.8}$$

m = 0.1 m

$$1 = \frac{5\lambda}{2}$$

= 0.25 m

The mid-point of the string is P, an antinode

$$\therefore$$

maximum displacement = 0.01 m

$$\omega$$

= 2

$$\pi$$

f = 628 s

$$-$$

1

$$\therefore$$

$$f = \frac{628}{2\pi} = 100$$

Hz

But this is fifth harmonic frequency.

$$\therefore$$

Fundamental frequency $f_0$

$$= \frac{f}{5} = 20$$

Hz.

## Question 058 MCQ

**QUESTION**

A particle of mass M and positive charge Q, moving with a constant velocity

$$\overrightarrow{u_1} = 4\hat{i}$$

ms

$$-$$

[1] enters a region of uniform static magnetic field, normal to the xy plane. The region of the magnetic field extends from x = 0 to x = L for all values of y. After passing through this region, the particle emerges on the other side after 10 ms with a velocity

$$\overrightarrow{u_2} = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

ms

$$-$$

[1]. The correct statement $s$ is $are$

A. The direction of the magnetic field is

$$-$$

z direction.

B The direction of the magnetic field is +z direction.

C The magnitude of the magnetic field is

$$\frac{50\pi M}{3Q}$$

units.

D The magnitude of the magnetic field is

$$\frac{100\pi M}{3Q}$$

units.

**CORRECT OPTION**

A The direction of the magnetic field is

$$-$$

z direction.

**SOURCE**

Physics • magnetism

**EXPLANATION**

The situation is as shown in the figure.

Here,

$$\overrightarrow{u_1} = 4\hat{i}$$

ms

$$-$$

1

$$\overrightarrow{u}_2 = 2\left(\sqrt{3}\hat{i} + \hat{j}\right)$$

ms

$$-$$

1

When the charged particle enters in an uniform magnetic field, it travels a circular path.

Component of final velocity of particle is in positive y direction.

Centre of circle is present on positive y-axis.

So, magnetic field is present in negative z-direction.

Angle of deviation

$$\tan\theta = \frac{u_{2y}}{u_{2x}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ = \frac{\pi}{6}$$

$$\omega = \frac{\theta}{t}$$

$$\therefore$$

$$t = \frac{\theta}{\omega} = \frac{\theta}{(QB/M)} = \frac{M\theta}{QB}$$

(

$$\because$$

$$\omega = \frac{QB}{M}$$

$$\therefore$$

$$B = \frac{M\theta}{Qt} = \frac{M\pi}{6Q \times 10 \times 10^{-3}}$$

(.... t = 10 ms = 10

–

3 s)

$$= \frac{100\pi M}{6Q} = \frac{50\pi M}{3Q}$$

## Question 059 Numerical

### QUESTION

The work functions of silver and sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is ___________.

### SOURCE

Physics • dual-nature-of-radiation

### EXPLANATION

We have,

$$V = \frac{hf}{e} - \frac{\phi}{e}$$

Slope is h/e.

Slope is the same for both silver and sodium.

Therefore, the ratio of the slope of the stopping potential versus frequency plot for silver to that of sodium is 1 : 1.

## Question 060 Numerical

### QUESTION

A freshly prepared sample of a radioisotope of half-life 1386 s has activity $10^3$ disintegrations per second. Given that ln2 = 0.693, the fraction of the initial number of nuclei $expressedinnearestintegerpercentage$ that will decay in the first 80 s after preparation of the sample is ___________.

### SOURCE

Physics • atoms-and-nuclei

### EXPLANATION

Decay constant,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1386\,s} = 5 \times 10^{-4}\,s$$

According to radioactive decay,

$$N = N_0 e^{-\lambda t}$$

$$\frac{N}{N_0} = e^{-5\times10^{-4}\times80}$$

or

$$\frac{N}{N_0} = e^{-0.04}$$

Fraction of nuclei decayed

$$= \frac{N_0 - N}{N_0} = 1 - \frac{N}{N_0}$$

$$= 1 - e^{-0.04} = 1 - 0.96 = 0.04 = 4\%$$