iit Jee 2009 Paper 1 Offline $60 Questions$

Question 001 MCQ

QUESTION

The compound s formed upon combustion of sodium metal in excess air is are

A Na_2O_2

B Na_2O

C NaO_2

D NaOH

CORRECT OPTION

A Na_2O_2

SOURCE

Chemistry • s-block-elements

EXPLANATION

The compound s formed upon the combustion of sodium metal in the excess air is sodium peroxide (Na_2O_2) and sodium oxide (Na_2O).

$$4Na + O_2 \rightarrow 2Na_2O$$

$$2Na + O_2 \rightarrow Na_2O_2$$

Question 002 MCQ

QUESTION

Given that the abundances of isotopes ^{54}Fe, ^{56}Fe and ^{57}Fe are 5%, 90% and 5%, respectively, the atomic mass of Fe is :

A 55.85

B 55.95

C 55.75

D 56.05

CORRECT OPTION

B 55.95

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

To calculate the atomic mass of an element based on its isotopic abundances, we use a weighted average. The atomic mass reported on the periodic table is a reflection of the weighted averages of all the naturally occuring isotopes of that

element. The formula to calculate the average atomic mass M of an element when given the abundance and atomic mass of each isotope is:

$$M = \sum(f_i \times m_i)$$

where:

- f_i is the fractional percent abundance $asadecimal$ of the i-th isotope.
- m_i is the mass number of the i-th isotope.

For iron Fe, we have three isotopes $^{54}Fe, ^{56}Fe, ^{57}Fe$ with abundances 5%, 90%, and 5%, respectively. To convert these percentages into decimals, we divide each by 100.

Let's calculate the weighted average:

$$M = (0.05 \times 54) + (0.90 \times 56) + (0.05 \times 57)$$

Multiplying each fractional abundance by its respective mass we get:

$$M = (0.05 \times 54) + (0.90 \times 56) + (0.05 \times 57)$$

$$M = (2.7) + (50.4) + (2.85)$$

$$M = 55.95$$

Therefore, the calculated atomic mass of Fe is 55.95, which corresponds to Option B.

Question 003 MCQ

QUESTION

The term that corrects for the attractive forces present in a real gas in the van der Waals equation is

nb

B $\frac{an^2}{V^2}$

C $-\frac{an^2}{V^2}$

D $-nb$

CORRECT OPTION

B $\frac{an^2}{V^2}$

SOURCE

Chemistry • gaseous-state

EXPLANATION

The van der Waals equation is a modified version of the ideal gas law which accounts for the non-ideal behavior of real gases. This adjustment is done through two correction terms, each addressing a specific factor where real gases deviate from ideal behavior:

- Intermolecular attraction
- Volume occupied by the gas particles themselves

The van der Waals equation is given by:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

Let's break down the components:

1. $$\left(P + \frac{an^2}{V^2}\right)$$

   : This term adds a correction to the pressure P of the gas. The parameter

   $$a$$

   provides a correction for the intermolecular forces. As these forces are attractive, they effectively reduce the pressure exerted by the gas. Thus,

   $$\frac{an^2}{V^2}$$

   is added to the actual pressure to account for this decrease due to attraction.

2. $$(V - nb)$$

   : The corrected volume term where

   $$b$$

   accounts for the volume occupied by the gas particles themselves, which is otherwise not considered in the ideal gas law. The subtraction by

   $$nb$$

   $where\$\$n\$\$isthenumberofmolesofthegasand\$\$b\$\$isaconstantrelate$
   corrects for the volume unavailable to the gas particles for movement.

From the options provided:

- Option A $\$\$nb\$\$$ and Option D $\$\$ - nb\$\$$ relate to the volume correction for the space that the gas molecules occupy.
- Option B $\$\$\frac{an^2}{V^2}\$\$$ and Option C $\$\$ - \frac{an^2}{V^2}\$\$$ relate to the intermolecular forces.

The enhancement factor

$$\frac{an^2}{V^2}$$

is added to the pressure to counteract the lowering effect of the attractions on the measured pressure, thus the term correcting for the attractive forces and increasing the effective pressure would be positively stated, not negatively. Therefore:

Option B $$\frac{an^2}{V^2}$$ is the correct answer, as it accounts for the attractive forces in the modification of the ideal gas law in the van der Waals equation.

Question 004 MCQ

QUESTION

Among the electrolytes Na

$$2$$

SO

$$4$$

, CaCl

$$2$$

, Al

$$2$$

$$SO$$$$_4$$$$

$$3$$

and NH

$$4$$

Cl, the most effective coagulating agent for Sb

$$2$$

S

$$3$$

sol is

A Na $_2$ SO $_4$

B CaCl $_2$

C Al $_2$ $SO$$$_4$$$ $_3$

D NH $_4$ Cl

CORRECT OPTION

C Al $_2$ $SO$$$_4$$$ $_3$

SOURCE

Chemistry • surface-chemistry

EXPLANATION

As Sb

$$2$$

S

$$3$$

is a negative sol, so, Al

$$2$$

$$SO$$$$_4$$$$

$$3$$

will be the most effective coagulant due to higher charge density on Al

$$3+$$

in accordance with Hardy-Schulze rule.

Order of effectiveness of cations: Al

$$3+$$

> Ca

$$2+$$

> Na

$$+$$

> NH

$$\begin{matrix}+\\4\end{matrix}$$

Question 005 MCQ

QUESTION

The Henry's law constant for the solubility of N_2 gas in water at 298 K is 1.0×10^5 atm. The mole fraction of N_2 in air is 0.8. The number of moles of N_2 from air dissolved in 10 moles of water at 298 K and 5 atm pressure is

A 4.0×10^{-4}

B 4.0×10^{-5}

C 5.0×10^{-4}

D

4.0×10^{-6}

CORRECT OPTION

A

4.0×10^{-4}

SOURCE

Chemistry • solutions

EXPLANATION

According to Henry's law, we have

$$p_{N_2} = K_H x_{N_2}$$

$$0.8 \times 5 = 1 \times 10^5 \times x_{N_2}$$

$$x_{N_2} = 4 \times 10^{-5}$$

Thus one mole of solution contains

$$4 \times 10^{-5}$$

moles of N

$_2$

and

$$1 - 4 \times 10^{-5} \approx 1$$

mole of water. Therefore, N

$$2$$

from air dissolved in 10 moles of water is

$$10 \times 4 \times 10^{-5}$$

moles

$$= 4 \times 10^{-4}$$

.

Question 006 MCQ

QUESTION

The reaction of P

$$4$$

with X leads selectively to P

$$4$$

O

$$6$$

. The X is

A Dry O

$$2$$

.

A mixture of O

B and N $_2$.

C Moist O_2 .

D O_2 in the presence of aqueous NaOH.

CORRECT OPTION

B A mixture of O_2 and N_2 .

SOURCE

Chemistry • p-block-elements

EXPLANATION

Phosphorus reacts with limited supply of oxygen to form the given product.

$P_4 + 3O_2$

$$\xrightarrow{2}$$

P

$$\xrightarrow{4}$$

O

$$\xrightarrow{6}$$

N

$$\xrightarrow{2}$$

is used to retard the further oxidation.

Question 007 MCQ

QUESTION

The correct acidity order of the following is

A $III > IV > II > I$

B $IV > III > I > II$

C $III > II > I > IV$

D $II > III > IV > I$

CORRECT OPTION

A $III > IV > II > I$

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

We know that, carboxylic acids are more acidic than phenols. Further, presence of electron withdrawing groups on the ring increases the acidic nature and electron releasing group decreases the acidic strength. So, structure III, due to the presence of carboxylic acid is the strongest acid

$$(pK_a = 4.17)$$

.

In structure IV, the +I effect of alkyl group reduces the acidic strength

$$(pK_a = 4.37)$$

of the carboxylic acid. Structure III, due to the presence of electron withdrawing substituent Cl, is more acidic than phenol

$$(pK_a = 9.38)$$

. Phenol $structure I$ is the weakest acid amongst all

$$(pK = 9.98)$$

.

Question 008 MCQ

QUESTION

Among cellulose, poly $vinylchloride$, nylon and natural rubber, the polymer in which the intermolecular force of attraction is weakest is

A Nylon

B Poly $vinylchloride$

C Cellulose

D Natural Rubber

CORRECT OPTION

D Natural Rubber

SOURCE

Chemistry • polymers

EXPLANATION

Natural rubber is an elastomer in which the chains are held together by weak van der Waals forces of attraction. Cellulose and nylon are fibres and PVC is a thermoplastic. Both these classes of polymers have stronger forces of attraction in the order

Fibre > Thermoplastic > Elastomer.

Question 009 MCQ

QUESTION

The IUPAC name of the following compound is

A 4-Bromo-3-cyanophenol

B 2-Bromo-5-hydroxybenzonitrile

C 2-Cyano-4-hydroxybromobenzene

D 6-Bromo-3-hydroxybenzonitrile

CORRECT OPTION

B 2-Bromo-5-hydroxybenzonitrile

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The priority order of the substituents on the ring is

—

CN >

—

Br >

—

OH

Cyanide group is given the highest priority.

Hence, it is written as suffix benzonitrile and other functional groups behave as substituents. The numbering of substituents is done in such a way that sum of

the locants is least.

The names of substituents are written in alphabetical order.

Question 010 MCQ

QUESTION

The correct statement s regarding defects in solids is are

A. Frenkel defect is usually favoured by a very small difference in the sizes of cation and anion.

B. Frenkel defect is a dislocation defect.

C. Trapping of an electron in the lattice leads to the formation of F-centre.

D. Schottky defects have no effect on the physical properties of solids.

CORRECT OPTION

B. Frenkel defect is a dislocation defect.

SOURCE

Chemistry • solid-state

EXPLANATION

Frenkel defect occurs in compounds in which the anions are much larger in size than cations. It is a dislocation effect. The presence of Schottky defects lowers the density of the crystal, it hence affects the physical properties of the crystal. In the metal excess defects, the electrons trapped in anion vacancies are known as F-centres.

Question 011 MCQ

QUESTION

The compound s that exhibit s geometrical isomerism is are

A $[Pt(en)Cl_2]$

B $[Pt(en)_2]Cl_2$

C $[Pt(en)_2Cl_2]Cl_2$

D $[Pt(NH_3)_2Cl_2]$

CORRECT OPTION

C $[Pt(en)_2Cl_2]Cl_2$

SOURCE

Chemistry • coordination-compounds

EXPLANATION

Among the given compounds, the compounds of the type

$$MA\$\$_2\$\$X\$\$_2\$\$$$

and

$$M(AA)\$\$_2\$\$X\$\$_2\$\$$$

will exhibit geometrical isomerism.

Question 012 MCQ

QUESTION

The correct statement s about the compound

$$\mathrm{H_3C(HO)HC-CH=CH-CH(OH)CH_3}\ \ \mathrm{(X)}$$

is are

A The total number of stereoisomers possible for X is 6.

B The total number of diastereomers possible for X is 3.

C If the stereochemistry about the double bond in X in $trans$, the number of enantiomers possible for X is 4.

D If the stereochemistry about the double bond in X is cis, the number of enantiomers possible for X is 2.

CORRECT OPTION

A The total number of stereoisomers possible for X is 6.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The possible stereoisomers of the given compounds are as follows:

Question 013 MCQ

QUESTION

The compound X is

A NaNO $_3$

B NaCl

Na $_2$

C SO

4

D Na

2

S

CORRECT OPTION

D Na

2

S

SOURCE

Chemistry • salt-analysis

EXPLANATION

The compound **X** is H

2

S:

$$\underset{(X)}{Na_2S} + 2H^+ \rightarrow H_2S + 2Na^+$$

Thus, the compound **Y** is FeCl

3

.

Compound **Y** on reaction with potassium hyxacyanoferrate II forms intense blue precipitate which dissolves on addition of reagent

$$4FeCl_3 + 3K_4\,[Fe(II)(CN)_6] \rightarrow \underset{\text{Intense blue ppt.}}{Fe_4[Fe(CN)_6]_3} + 12KCl$$

Compound **Y** on reaction with potassium hexacyanoferrate III forms brown colouration due to

$$Fe[Fe(CN)_6]$$

Question 014 MCQ

QUESTION

The compound Y is

A MgCl

$$2$$

B FeCl

$$2$$

C FeCl

$$3$$

D ZnCl

$$2$$

CORRECT OPTION

C FeCl

3

SOURCE

Chemistry • salt-analysis

EXPLANATION

The compound **X** is H

2

S:

$$\underset{(X)}{Na_2S} + 2H^+ \rightarrow H_2S + 2Na^+$$

Thus, the compound **Y** is FeCl

3

.

Compound **Y** on reaction with potassium hyxacyanoferrate II forms intense blue precipitate which dissolves on addition of reagent

$$4FeCl_3 + 3K_4\left[Fe(II)(CN)_6\right] \rightarrow \underset{\text{Intense blue ppt.}}{Fe_4[Fe(CN)_6]_3} + 12KCl$$

Compound **Y** on reaction with potassium hexacyanoferrate III forms brown colouration due to

$$Fe[Fe(CN)_6]$$

Question 015 MCQ

QUESTION

The compound Z is

A $Mg_2[Fe(CN)_6]$

B $Fe[Fe(CN)_6]$

C $Fe_4[Fe(CN)_6]_3$

D $K_2Zn_3[Fe(CN)_6]_2$

CORRECT OPTION

B $Fe[Fe(CN)_6]$

SOURCE

Chemistry • salt-analysis

EXPLANATION

The compound **X** is H

$$_{2}$$

S:

$$\underset{(X)}{Na_2S} + 2H^+ \rightarrow H_2S + 2Na^+$$

Thus, the compound **Y** is FeCl

3

.

Compound **Y** on reaction with potassium hyxacyanoferrate II forms intense blue precipitate which dissolves on addition of reagent

$$4FeCl_3 + 3K_4\left[Fe(II)(CN)_6\right] \rightarrow \underset{\text{Intense blue ppt.}}{Fe_4[Fe(CN)_6]_3} + 12KCl$$

Compound **Y** on reaction with potassium hexacyanoferrate III forms brown colouration due to

$$Fe[Fe(CN)_6]$$

Question 016 MCQ

QUESTION

The structure of the carbonyl compound P is

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The structure of carbonyl compound P is

Question 017 MCQ

QUESTION

The structures of the products Q and R, respectively, are

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The structures for compounds Q and R are

Question 018 MCQ

QUESTION

The structure of the product S is

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The structure of compound S is

Question 019 MCQ

QUESTION

Match each of the diatomic molecules in Column I with its property/properties in Column II:

	Column I		Column II
A	B_2	P	Paramagnetic
B	N_2	Q	Undergoes oxidation
C	O_2^-	R	Undergoes reduction
D	O_2	S	Bond order $\geq$ 2
		T	Mixing of s and p orbitals

A $(\mathrm{A}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{R}), (\mathrm{T}); (\mathrm{B}) \rightarrow (\mathrm{S}), (\mathrm{T}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}); (\mathrm{D}) \rightarrow (\mathrm{P}), ($

B $(A) \rightarrow (P), (S), (R), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P), (Q); (D) \rightarrow (P), ($

C $(A) \rightarrow (Q), (R), (T); (B) \rightarrow (P), (T); (C) \rightarrow (P), (Q); (D) \rightarrow (T), (Q)$

D $(A) \rightarrow (P), (R), (T); (B) \rightarrow (Q), (T); (C) \rightarrow (S), (Q); (D) \rightarrow (P), (Q)$

CORRECT OPTION

A $(A) \rightarrow (P), (Q), (R), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P), (Q); (D) \rightarrow (P), ($

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

A

$$B_2 \rightarrow \sigma 1s^2\, \sigma * 1s^2\, \sigma 2s^2\, \sigma * 2s^2 \pi 2p_x^1 = \pi 2p_y^1$$

It is paramagnetic due to two unpaired electrons. The bond order is

$$\frac{N_b - N_a}{2} = \frac{6-4}{2} = 1$$

The gain of electron increases bond order, so reduction is possible.

B

$$N_2 \rightarrow \sigma 1s^2\, \sigma * 1s^2\, \sigma 2s^2\, \sigma * 2s^2 \pi 2p_x^2 = \pi 2p_y^2 \sigma 2p_z^2$$

There are no unpaired electrons. Bond order is

$$\frac{N_b - N_a}{2} = \frac{10-4}{2} = \frac{6}{2} = 3$$

Mixing of 2s and 2p orbitals is possible because of similar energies.

C

$$O_2^- \rightarrow \sigma 1s^2\, \sigma * 1s^2\, \sigma 2s^2\, \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2\, \pi * 2p_x^2\, \pi * 2p_y^1$$

The molecule is paramagnetic due to presence of unpaired electron. Bond order is less than 2.

$$\frac{N_b - N_a}{2} = \frac{10 - 7}{2} = \frac{3}{2}$$

Loss of electron increases the bond order so oxidation is possible.

D

$$O_2 \rightarrow \sigma 1s^2\, \sigma * 1s^2\, \sigma 2s^2\, \sigma * 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2\, \pi * 2p_x^1\, \pi * 2p_y^1$$

The molecule is paramagnetic due to presence of unpaired electrons. Bond order is

$$\frac{N_b - N_a}{2} = \frac{10 - 6}{2} = \frac{4}{2} = 2$$

Loss of electron causes increase in bond order, so it undergoes oxidation.

Question 020 MCQ

QUESTION

Match each of the compounds in Column I with its characteristic reaction s in Column II.

	Column I		Column II
A	$CH_3CH_2CH_2CN$	P	Reduction with $Pd - C/H_2$

	Column I		Column II
B	$CH_3CH_2OCOCH_3$	Q	Reduction with $SnCl_2/HCl$
C	$CH_3-CH=CH-CH_2OH$	R	Development of foul smell on treatment w chloroform and alcoholic KOH
D	$CH_3CH_2CH_2CH_2NH_2$	S	Reduction with diisobutylaluminium hydrid $DIBAL-H$
		T	Alkaline hydrolysis

A $(A) \rightarrow (P), (Q), (S), (T); (B) \rightarrow (Q), (T); (C) \rightarrow (P); (D) \rightarrow (S)$

B $(A) \rightarrow (Q), (R), (S), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P); (D) \rightarrow (R)$

C $(A) \rightarrow (P), (R), (S), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P); (D) \rightarrow (R), (T)$

D $(A) \rightarrow (P), (Q), (S), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P); (D) \rightarrow (R)$

CORRECT OPTION

D $(A) \rightarrow (P), (Q), (S), (T); (B) \rightarrow (S), (T); (C) \rightarrow (P); (D) \rightarrow (R)$

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

A

B Reaction with DIBAL-H and alkaline hydrolysis:

C Reduction with Pd-C/H

$$2$$

:

$$CH_3 - CH = CH - CH_2OH \overset{Pd-C/CH_2}{\longrightarrow} CH_3 - CH_2 - CH_2 - CH_2OH$$

D Foul smell on treatment with

$$CHCl_3 + KOH$$

:

$$RNH_2 \overset{CHCl_3+KOH}{\longrightarrow} \underset{Foul\ smell}{RNC} + KCl + H_2O$$

Question 021 MCQ

QUESTION

Let

$$z = x + iy$$

be a complex number where x and y are integers. Then the area of the rectangle whose vertices are the roots of the equation

$$\bar{z}z^3 + z\bar{z}^3 = 350$$

is

A 48

B 32

C 40

D 80

CORRECT OPTION

A 48

SOURCE

Mathematics • complex-numbers

EXPLANATION

We have

$$\bar{z}z^3 + z\bar{z}^3 = 350$$

Substituting

$$z = x + iy$$

, we get

$$(x^2 + y^2)(x^2 - y^2) = 175$$

$$(x^2 + y^2)(x^2 - y^2) = 5 \times 5 \times 7$$

$$x^2 + y^2 = 25$$

$$x^2 - y^2 = 7$$

whose solutions are

$$x = \pm 4$$

and

$$y = \pm 3; x, y \in I$$

.

Therefore, that is, area is found as

$$8 \times 6 = 48$$

sq. unit.

Question 022 MCQ

QUESTION

Let

$$z = \cos\,\theta + i\,\sin\,\theta$$

. Then the value of

$$\sum_{m=1}^{15} \mathrm{Im}(z^{2m-1})\,at\,\theta\, = 2^{\circ}$$

is

A $\dfrac{1}{\sin\,2^{\circ}}$

B $\dfrac{1}{3\sin\,2^{\circ}}$

C $\dfrac{1}{2\sin\,2^{\circ}}$

D $\dfrac{1}{4\sin 2^\circ}$

CORRECT OPTION

D $\dfrac{1}{4\sin 2^\circ}$

SOURCE

Mathematics • complex-numbers

EXPLANATION

We have

$$X = \sin\theta + \sin 3\theta + \ldots + \sin 29\theta$$

$$2(\sin\theta)X = 1 - \cos 2\theta + \cos 2\theta - \cos 4\theta + \ldots + \cos 28\theta - \cos 30\theta$$

$$X = \frac{1-\cos 30\theta}{2\sin\theta} = \frac{1}{4\sin 2^\circ}$$

Question 023 MCQ

QUESTION

Let

$$P(3,2,6)$$

be a point in space and

$$Q$$

be a point on the line

$$\$\hat{r} = \left(\hat{i} - \hat{j} + 2\widehat{k}\right) + \mu\left(-3\hat{i} + \hat{j} + 5\widehat{k}\right)$$

$

Then the value of

$$\mu$$

for which the vector

$$\overrightarrow{PQ}$$

is parallel to the plane

$$x - 4y + 3z = 1$$

is :

A $\frac{1}{4}$

B $-\frac{1}{4}$

C $\frac{1}{8}$

D $-\frac{1}{8}$

CORRECT OPTION

A

$$\frac{1}{4}$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

We see that any point on the line is given as

$$Q \equiv \{(1-3\mu), (\mu-1), (5\mu+2)\}$$

$$\overrightarrow{PQ} \equiv \{-3\mu-2, \mu-3, 5\mu-4\}$$

Now, we have

$$1(-3\mu-2) - 4(\mu-3) + 3(5\mu-4) = 0$$

$$\Rightarrow -3\mu - 2 - 4\mu + 12 + 15\mu - 12 = 0$$

That is,

$$8\mu = 2 \Rightarrow \mu = \frac{1}{4}$$

Question 024 MCQ

QUESTION

If

$$\vec{a}, \vec{b}, \vec{c}$$

and

$$\vec{d}$$

are unit vectors such that

$$(\vec{a} \times \vec{b}) \,.\, (\vec{c} \times \vec{d}) = 1$$

and

$$\vec{a} \,.\, \vec{c} = \frac{1}{2}$$

, then

A $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar

B $\vec{b}, \vec{c}, \vec{d}$ are non-coplanar

C $\vec{b}, \vec{d}$ are non-parallel

D $\vec{a}, \vec{d}$ parallel and $\vec{b}, \vec{c}$ are parallel

CORRECT OPTION

C

$\overrightarrow{b}, \overrightarrow{d}$

are non-parallel

SOURCE

Mathematics • vector-algebra

EXPLANATION

The given equation,

$$(\overrightarrow{a} \times \overrightarrow{b}) \,.\, (\overrightarrow{c} \times \overrightarrow{d}) = 1$$

, is possible only when

$$|\overrightarrow{a} \times \overrightarrow{b}| = |\overrightarrow{c} \times \overrightarrow{d}| = 1$$

and

$$(\overrightarrow{a} \times \overrightarrow{b}) || (\overrightarrow{c} \times \overrightarrow{d})$$

.

Since

$$\overrightarrow{a} \,.\, \overrightarrow{c} = 1/2$$

and

$$\overrightarrow{b} || \overrightarrow{d}$$

, we get

$$|\overrightarrow{c} \times \overrightarrow{d}| \neq 1$$

; hence, we conclude that the vectors

$$\overrightarrow{b}$$

and

$$\overrightarrow{d}$$

are non-parallel.

Question 025 MCQ

QUESTION

The conditional probability that

$$X \geq 6$$

given

$$X > 3$$

equals :

A $\frac{125}{216}$

B $\frac{25}{216}$

C $\frac{5}{36}$

D $\frac{25}{36}$

CORRECT OPTION

D $\frac{25}{36}$

SOURCE

Mathematics • probability

EXPLANATION

$$X \geq 6$$

: The probability for

$$\frac{5^5}{6^6} + \frac{5^6}{6^7} + \ldots + \infty = \frac{5^5}{6^6}\left(\frac{1}{1-5/6}\right) = \left(\frac{5}{6}\right)^5$$

For

$$X > 3$$

, we have

$$\frac{5^3}{6^4} + \frac{5^4}{6^5} + \frac{5^5}{6^6} + \ldots + \infty = \left(\frac{5}{6}\right)^3$$

Hence, the conditional probability is

$$\frac{(5/6)^6}{(5/6)^3} = \frac{25}{36}$$

Question 026 MCQ

QUESTION

The probability that

$$X \geq 3$$

equals :

A $\frac{125}{216}$

B $\frac{25}{36}$

C $\frac{5}{36}$

D $\frac{25}{216}$

CORRECT OPTION

B $\frac{25}{36}$

SOURCE

Mathematics • probability

EXPLANATION

The probability

$$P(X \geq 3)$$

is nothing but the probability of

$$P(X \leq 2)$$

:

$$\frac{1}{6} + \frac{5}{6} \times \frac{1}{6} = \frac{11}{36}$$

The required probability is

$$1 - \frac{11}{36} = \frac{25}{36}$$

Question 027 MCQ

QUESTION

The probability that X = 3 equals

A $\frac{25}{216}$

B $\frac{25}{36}$

C $\frac{5}{36}$

D $\frac{125}{216}$

CORRECT OPTION

A $\frac{25}{216}$

SOURCE

Mathematics • probability

EXPLANATION

The required probability is

$$P(X = 3)$$

$$\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\frac{1}{6} = \frac{25}{216}$$

Question 028 MCQ

QUESTION

Area of the region bounded by the curve

$$y = e^x$$

and lines

$$x = 0$$

and

$y = e$

is

A $e - 1$

B $\int_1^e \ln(e + 1 - y)dy$

C $e - \int_0^1 e^x dx$

D $\int_1^e \ln y \, dy$

CORRECT OPTION

B $\int_1^e \ln(e + 1 - y)dy$

SOURCE

Mathematics • application-of-integration

EXPLANATION

The required area is obtained as follows:

$$\int_1^e \ln y\, dy = (y \ln y - y)_1^e = (e - e) - \{-1\} = 1$$

.

Also,

$$\int_1^e \ln y\, dy = \int_1^e \ln(e + 1 - y) dy$$

Further, the area bounded by the region is

$$= e \times 1 - \int_0^e e^x dx$$

Question 029 MCQ

QUESTION

Let

$$f$$

be a non-negative function defined on the interval

$$[0, 1]$$

.

If

$$\int_0^x \sqrt{1 - (f'(t))^2 dt} = \int_0^x f(t) dt, 0 \leq x \leq 1$$

, and

$$f(0) = 0$$

, then

A $$f\left(\frac{1}{2}\right) < \frac{1}{2}$$ and $$f\left(\frac{1}{3}\right) > \frac{1}{3}$$

B $$f\left(\frac{1}{2}\right) > \frac{1}{2}$$ and $$f\left(\frac{1}{3}\right) > \frac{1}{3}$$

C $$f\left(\frac{1}{2}\right) < \frac{1}{2}$$ and $$f\left(\frac{1}{3}\right) < \frac{1}{3}$$

D $$f\left(\frac{1}{2}\right) > \frac{1}{2}$$ and $$f\left(\frac{1}{3}\right) < \frac{1}{3}$$

CORRECT OPTION

$$f\left(\frac{1}{2}\right) < \frac{1}{2}$$

C and

$$f\left(\frac{1}{3}\right) < \frac{1}{3}$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

We have,

$$f' = \pm\sqrt{1 - f^2}$$

$$\Rightarrow f(x) = \sin x$$

or

$$f(x) = -\sin x$$

$notpossible$

$$\Rightarrow f(x) = \sin x$$

Also,

$$x > \sin x \forall x > 0$$

.

Question 030 MCQ

QUESTION

Match the conics in Column I with the statements/expressions in Column II :

	Column I		Column II
A	Circle	P	The locus of the point $\$\$h, k\$\$$ for which the line $$hx + ky = 1$$ touches the circle $$x^2 + y^2 = 4$$.
B	Parabola	Q	Points z in the complex plane satisfying $$\|z+2\| - \|z-2\| = \pm 3$$.
C	Ellipse	R	Points of the conic have parametric representation $$x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$$
D	Hyperbola	S	The eccentricity of the conic lies in the interval $$1 \leq x \leq \infty$$.
		T	Points z in the complex plane satisfying $$\mathrm{Re}(z+1)^2 = \|z\|^2 + 1$$.

A

$\rightarrow$

$P;\ B$

$\rightarrow$

A $S,\ T;\ C$

$\rightarrow$

$R;\ D$

$\rightarrow$

$R,\ S$

A

$\rightarrow$

$P;\ B$

$\rightarrow$

B $S,\ T;\ C$

$\rightarrow$

$R;\ D$

$\rightarrow$

$Q,\ S$

A

$\rightarrow$

$P;\ B$

$\rightarrow$

C $S,\ T;\ C$

$\rightarrow$

$S;\ D$

$\rightarrow$

R, S

A

$\rightarrow$

$P; B$

$\rightarrow$

D $P, T; C$

$\rightarrow$

$R; D$

$\rightarrow$

Q, S

CORRECT OPTION

A

$\rightarrow$

$P; B$

$\rightarrow$

B $S, T; C$

$\rightarrow$

$R; D$

$\rightarrow$

Q, S

SOURCE

Mathematics • ellipse

EXPLANATION

P We have

$$\frac{1}{k^2} = 4\left(1 + \frac{h^2}{k^2}\right) \Rightarrow 1 = 4(k^2 + h^2)$$

Hence,

$$h^2 + k^2 = \left(\frac{1}{2}\right)^2$$

, which is a circle.

Q If

$$|z - z_1| - |z - z_2| = k$$

, where

$$k < |z_1 - z_2|$$

, the locus is a hyperbola.

R Let

$$t = \tan\alpha$$

. Hence,

$$x = \sqrt{3}\cos 2\alpha$$

and

$$y = \sin 2\alpha$$

or

$$\cos 2\alpha = \frac{x}{\sqrt{3}}$$

and

$$\sin 2\alpha = y$$

$$\frac{x^2}{3} + y^2 = \sin^2 2\alpha + \cos^2 2\alpha = 1$$

, which is an ellipse.

S If eccentricity is

$$[1, \infty]$$

, then the conic can be a parabola $if\$\$e = 1\$\$$ and a hyperbola if

$$e \in (1, \infty)$$

.

T Let

$$z = x + iy; x, y \in R$$

. Hence,

$$(x+1)^2 - y^2 = x^2 + y^2 + 1$$

$$\Rightarrow y^2 = x;$$

which is a parabola

Question 031 MCQ

QUESTION

In a triangle

$$ABC$$

with fixed base

$$BC$$

, the vertex

$$A$$

moves such that

$$\$ \cos\ B + \cos\ C = 4\sin^2 \frac{A}{2}.$$

$

If

$$a, b$$

and

$$c$$

denote the lengths of the sides of the triangle opposite to the angles

$$A, B$$

and

$$C$$

, respectively, then

A $$b + c = 4a$$

B $$b + c = 2a$$

C locus of point $$A$$ is an ellipse

D locus of point $$A$$ is a pair of straight lines

CORRECT OPTION

B $$b + c = 2a$$

SOURCE

Mathematics • ellipse

EXPLANATION

From this given data, we can write as

$$2\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right) = 4\sin^2\frac{A}{2}$$

$$\cos\left(\frac{B-C}{2}\right) = 2\sin(A/2)$$

$$\Rightarrow \frac{\cos\left(\frac{B-C}{2}\right)}{\sin A/2} = 2$$

$$\Rightarrow \frac{\sin B + \sin C}{\sin A} = 2$$

$$\Rightarrow b + c = 2a$$

where a is a constant. Therefore, the locus of point A is an ellipse.

Question 032 MCQ

QUESTION

The line passing through the extremity

$$A$$

of the major axis and extremity

$$B$$

of the minor axis of the ellipse

$$x^2 + 9y^2 = 9$$

meets its auxiliary circle at the point

$$M$$

. Then the area of the triangle with vertices at

$$A$$

,

$$M$$

and the origin

$$O$$

is

A $\frac{31}{10}$

B $\frac{29}{10}$

C $\frac{21}{10}$

D $\frac{27}{10}$

CORRECT OPTION

D $\frac{27}{10}$

SOURCE

Mathematics • ellipse

EXPLANATION

Equation of line AM is

$$x + 3y - 3 = 0$$

.

Perpendicular distance of line from origin is

$$3/\sqrt{10}$$

.

Length of AM is

$$2\sqrt{9 - \frac{9}{10}} = 2 \times \frac{9}{\sqrt{10}}$$

The required area of the rectangle is

$$\frac{1}{2} \times 2 \times \frac{9}{\sqrt{10}} \times \frac{3}{\sqrt{10}} = \frac{27}{10}$$

sq. unit

Question 033 MCQ

QUESTION

Tangents drawn from the point P $1, 8$ to the circle

$$x^2 + y^2 - 6x - 4y - 11 = 0$$

touch the circle at the points A and B. The equation of the cirumcircle of the triangle PAB is

A $x^2 + y^2 + 4x - 6y + 19 = 0$

B $x^2 + y^2 - 4x - 10y + 19 = 0$

C $x^2 + y^2 - 2x + 6y - 29 = 0$

D $x^2 + y^2 - 6x - 4y + 19 = 0$

CORRECT OPTION

B $x^2 + y^2 - 4x - 10y + 19 = 0$

SOURCE

Mathematics • circle

EXPLANATION

From the given data, the centre of the circle is C $3, 2$.

Since, CA and CB are perpendicular to PA and PB, CP is the diameter of the circumcircle of triangle PAB. Its equation is

$$(x - 3)(x - 1) + (y - 2)(y - 8) = 0$$

or

$$x^2 + y^2 - 4x - 10y + 19 = 0$$

.

Question 034 MCQ

QUESTION

The number of seven digit integers, with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only, is

A 55

B 66

C 77

D 88

CORRECT OPTION

C 77

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

The two possible cases are as follows:

Case 1 : There are five 1's; one 2; one 3. Therefore, the number of numbers is 7!/5! = 42.

Case 2 : There are four 1's; three 2's. Therefore, the number of numbers is 7!/4! 3! = 35. Hence, the total number of numbers is 42 + 35 = 77

Question 035 MCQ

QUESTION

If

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5},$$

then

A $\tan^2 x = \frac{2}{3}$

B $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$

C $\tan^2 x = \frac{1}{3}$

D $\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{2}{125}$

CORRECT OPTION

A $$\tan^2 x = \frac{2}{3}$$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

It is given that

$$\frac{\sin^4 x}{2} + \frac{\cos^4 x}{3} = \frac{1}{5}$$

$$3\sin^4 x + 2(1 - \sin^2 x)^2 = \frac{6}{5}$$

$$\Rightarrow 25\sin^4 x - 20\sin^2 x + 4 = 0$$

$$\Rightarrow \sin^2 x = \frac{2}{5}$$

and

$$\cos^2 x = \frac{3}{5}$$

Hence,

$$\tan^2 x = \frac{2}{3}$$

Therefore,

$$\frac{\sin^8 x}{8} + \frac{\cos^8 x}{27} = \frac{1}{125}$$

Question 036 MCQ

QUESTION

Match the statements/expressions in Column I with the open intervals in Column II :

	Column I		Column II
A	Interval contained in the domain of definition of non-zero solutions of the differential equation $$(x-3)^2y' + y = 0$$	P	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
B	Interval containing the value of the integral $$\int_1^5 (x-1)(x-2)(x-3)(x-4)(x-5)dx$$	Q	$\left(0, \frac{\pi}{2}\right)$
C	Interval in which at least one of the points of local maximum of $$\cos^2 x + \sin x$$ lies	R	$\left(\frac{\pi}{8}, \frac{5\pi}{4}\right)$
D	Interval in which $$\tan^{-1}(\sin x + \cos x)$$ is increasing	S	$\left(0, \frac{\pi}{8}\right)$
		T	$(-\pi, \pi)$

A $\rightarrow$

$P, Q, S; B$

$\rightarrow$

A $P, T, S; C$

$\rightarrow$

$P, Q, R, T; D$

$\rightarrow$

S

A

$\rightarrow$

$P, Q, S; B$

$\rightarrow$

B $P, T, R; C$

$\rightarrow$

$P, Q, R, T; D$

$\rightarrow$

R

A

$\rightarrow$

$P, Q, S; B$

$\rightarrow$

C $P, T, S; C$

$\rightarrow$

$S, Q, R, T; D$

$\rightarrow$

S

A

$\rightarrow$

$P, T, S; B$

$\rightarrow$

D $P, T, S; C$

$\rightarrow$

$P, Q, R, T; D$

$\rightarrow$

S

CORRECT OPTION

A

$\rightarrow$

$P, Q, S; B$

$\rightarrow$

A $P, T, S; C$

$\rightarrow$

$P, Q, R, T; D$

$\rightarrow$

S

SOURCE

Mathematics • differential-equations

EXPLANATION

A We have

$$(x-3)^2\frac{dy}{dx}+y=0$$

$$\int\frac{dx}{(x-3)^2}=-\int\frac{dy}{y}$$

$$\Rightarrow\frac{1}{x-3}=\ln|y|+c$$

So the domain is

$$R\to\{3\}$$

.

B On substituting

$$x=t+3$$

, we get

$$\int_{-2}^{2}(t+2)(t+1)t(t-1)(t-2)dt=\int_{-2}^{2}t(t^2-1)(t^2-4)dt=0$$

$beingoddfunction$

C

$$f(x)=\frac{5}{4}-\left(\sin x-\frac{1}{2}\right)^2$$

The maximum value occurs when

$$\sin x=1/2$$

.

D

$$f'(x)>0$$

if

$$\cos x>\sin x$$

.

Question 037 MCQ

QUESTION

Let

$$L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}, a > 0$$

. If L is finite, then

A $a = 2$

B $a = 1$

C $L = \frac{1}{64}$

D $L = \frac{1}{32}$

CORRECT OPTION

A $a = 2$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

The given limit is

$$L = \lim_{x \to 0} \frac{a - \sqrt{a^2 - x^2} - \frac{x^2}{4}}{x^4}$$

$$= \lim_{x \to 0} \frac{1}{x^2(a + \sqrt{a^2 - x^2})} - \frac{1}{4x^2}$$

$$= \lim_{x \to 0} \frac{(4 - a) - \sqrt{a^2 - x^2}}{4x^2(a + \sqrt{a^2 - x^2})}$$

The numerator

$$\rightarrow$$

0 if

$$a = 2$$

; therefore,

$$L = \frac{1}{64}$$

.

Question 038 MCQ

QUESTION

The number of matrices in A is

A 12

B 6

C 9

D 3

CORRECT OPTION

A 12

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Here,

Matrix

$$A$$

3×3 is symmetric

Here, 5 be the number of 1's and 4 be the 0's entry in matrix A

Let x be the number of 1's on main diagonal.

Since A is symmetric so, y be the number of 1's below the main diagonal

So , x + 2y = 5

x = 1 and y = 2

or x = 3 and y = 1

Case I : x = 1 and y = 2

Here, Main diagonal entries in 0, 0 and 1. Hence, we can choose main diagonal in 3 ways, and element above the diagonal is 1, 1, 0 . Hence, we can choose it element above the main diagonal in 3 ways , element of below diagonal depends on above the main diagonal.

Hence, total way = 3 × 3 = 9

Case II : x = 3 and y = 1

Here, Main diagonal entries in 1, 1 and 1.
Hence, we can choose main diagonal 1 ways and element above the diagonal is 1, 0, 0. Hence, we can choose it element above the main diagonal in 3 ways, element of below diagonal depends on above the main diagonal.

Hence, total ways = 3 × 4 = 12

So, Total matrices = 9 + 3 = 12

Question 039 MCQ

QUESTION

The number of matrices A in A for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

has a unique solution, is

A less than 4

B at least 4 but less than 7

C at least 7 but less than 10

D at least 10

CORRECT OPTION

B at least 4 but less than 7

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

We have

$$\begin{bmatrix} 0 & a & b \\ a & 0 & c \\ b & c & 1 \end{bmatrix}$$

We can see that either

$$b = 0$$

or

$$c = 0 \Rightarrow |A| \neq 0$$

; therefore two matrices.

$$\begin{bmatrix} 0 & a & b \\ a & 1 & c \\ b & c & 0 \end{bmatrix}$$

Now, either

$$a = 0$$

or

$$c = 0 \Rightarrow |A| \neq 0$$

; therefore, two matrices

$$\begin{bmatrix} 1 & a & b \\ a & 0 & c \\ b & c & 0 \end{bmatrix}$$

Now, either

$$a = 0$$

or

$$b = 0 \Rightarrow |A| \neq 0$$

; therefore two matrices.

$$\begin{bmatrix} 1 & a & b \\ a & 1 & c \\ b & c & 1 \end{bmatrix}$$

When

$$a = b = 0 \Rightarrow |A| = 0$$

When

$$a = c = 0 \Rightarrow |A| = 0$$

When

$$b = c = 0 \Rightarrow |A| = 0$$

Therefore, there are only six matrices.

Question 040 MCQ

QUESTION

The number of matrices A in A for which the system of linear equations

$$A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

is inconsistent, is

A 0

B more than 2

C 2

D 1

CORRECT OPTION

B more than 2

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

The six matrices A for which

$$|A| = 0$$

are as follows:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow$$

inconsistent.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow$$

inconsistent.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow$$

infinite solutions.

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow$$

inconsistent.

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \Rightarrow$$

inconsistent.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \Rightarrow$$

infinite solutions.

Question 041 MCQ

QUESTION

A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is

$$\sqrt{3}$$

. The inclination θ of this inclined plane from the horizontal plane is gradually increased from

$$0^\circ$$

. Then

A at θ =

$$60^\circ$$

, the block will start sliding down the plane

B the block will remain at rest on the plane up to certain θ and then it will topple

C at θ =

$$60^\circ$$

, the block will start sliding down the plane and continue to do so at higher angles

D at θ =

$$60^\circ$$

, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

CORRECT OPTION

B the block will remain at rest on the plane up to certain θ and then it will topple

SOURCE

Physics • laws-of-motion

EXPLANATION

The block slides on the incline when

$$mg\sin\theta \geq \mu mg\cos\theta$$

$$\Rightarrow \tan\theta \geq \mu$$

$$\Rightarrow \tan\theta \geq \sqrt{3} = 1.732$$

For the block to topple, we have

$$f\left(\frac{15}{2}\right) \geq N(5)$$

$about centre of mass$

$$\Rightarrow \mu N\left(\frac{15}{2}\right) \geq N(5)$$

$$\Rightarrow \mu \geq \frac{2}{3} = 0.67$$

or

$$\tan\theta \geq 0.67$$

That is, the block neither slides nor topples till

$$\theta \leq \tan^{-1}(0.67)$$

$i.e. it remains at rest$ and on exceeding this, it topples first before sliding.

Question 042 MCQ

QUESTION

Look at the drawing given in the figure below which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is

$$m$$

. the mass of the ink used to draw the outer circle is

$$6m$$

. The coordinates of the centres of the different parts are: outer circle $0, 0$, left inner circle $\$\$ - a, a\$\$$, right inner circle $\$\$a, a\$\$$, vertical line $0, 0$ and horizontal line $\$\$0, -a\$\$$. The y-coordinate of the centre of mass of the ink in this drawing is

A $\frac{a}{10}$

B $\frac{a}{8}$

C $\frac{a}{12}$

D $\frac{a}{3}$

CORRECT OPTION

A

$$\frac{a}{10}$$

SOURCE

Physics • impulse-and-momentum

EXPLANATION

The coordinates of centre of mass is defined as

$$R_{CM} = \frac{\sum_i m_i r_i}{\sum_i m_i}$$

Thus,

$$Y_{CM} = \frac{(6m \times 0) + (m \times a) + (m \times a) + (m \times 0) + (m \times -a)}{6m + m + m + m + m} = \frac{a}{10}$$

Question 043 MCQ

QUESTION

The figure shows certain wire segments joined together to form a coplanar loop. The loop is placed in a perpendicular magnetic field in the direction going into the plane of the figure. The magnitude of the field increases with time.

$$I_1$$

and

$$I_2$$

are the currents in the segments ab and cd. Then,

A $I_1 > I_2$

B $I_1 < I_2$

C I_1 is in the direction ba and I_2 is in the direction cd.

D I_1 is in the direction ab and I_2 is in the direction dc.

CORRECT OPTION

D I_1 is in the direction ab and I_2 is in the direction dc.

SOURCE

Physics • electromagnetic-induction

EXPLANATION

Since

$$\phi_B$$

increases in the downward direction, according to Lenz's law, the induced current flows in such a manner to create an upward going flux, that is, a flux in anticlockwise direction. Since the outer loop has a bigger enclosed area than inner loop, the induced current flow is as follows:

- Induced current,

$$I_2$$

: Flows from d to c

- Induced current,

$$I_1$$

: Flows from a to b

Question 044 MCQ

QUESTION

Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are

$$v$$

and 2

$$v$$

, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A?

A 4

B 3

C 2

D 1

CORRECT OPTION

C 2

SOURCE

Physics • impulse-and-momentum

EXPLANATION

The faster particle covers twice the distance covered by the slower one and they meet each other whenever the slower one covers an angular displacement of

$$2\pi/3$$

. since the masses are same at each collision, they interchange the velocity. The number of meeting points is

$$\frac{2\pi}{2\pi/3} = 3$$

points; hence, they meet at two points other than A.

Question 045 MCQ

QUESTION

A disk of radius

$$\frac{a}{4}$$

having a uniformly distributed charge 6C is placed in the xy-plane with its centre at $\$\$ - \$\$a/2, 0, 0$. A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from x = a/4 to x = 5a/4. Two points charges

$$-$$

7C and 3C are placed at $a/4, \$\$ - \$\$a/4, 0$ and $\$\$ - \$\$3a/4, 3a/4, 0$, respectively. Consider a cubical surface formed by six surfaces

$$x = \pm a/2, y = \pm a/2, z = \pm a/2$$

. The electric flux through this cubical surface is

A $\frac{-2c}{\varepsilon_0}$

B $\frac{2c}{\varepsilon_0}$

C $\frac{10c}{\varepsilon_0}$

D $\frac{12c}{\varepsilon_0}$

CORRECT OPTION

A $\dfrac{-2c}{\varepsilon_0}$

SOURCE

Physics • electrostatics

EXPLANATION

According to Gauss's law, we have

$$\text{Flux} = \frac{\text{Charge enclosed}}{\varepsilon_0}$$

The enclosed charges are half of the charges in the disc, point charge at

$$(a/4, -a/4, 0)$$

, and charge in the rod from point

$$a/4$$

to

$$a/2$$

, that is, 8 C/4. Therefore,

Flux

$$= \frac{3C - 7C + (8C/4)}{\varepsilon_0} = -\frac{2C}{\varepsilon_0}$$

Question 046 MCQ

QUESTION

Three concentric metallic spherical shells of radii

$$R, 2R, 3R$$

are given charges

$$Q_1, Q_2, Q_3$$

, respectively. It is found that the surface charge densities on the outer surfaces of the shells are equal. Then, the ratio of the charges given to the shells,

$$Q_1 : Q_2 : Q_3$$

, is

A 1 : 2 : 3

B 1 : 3 : 5

C 1 : 4 : 9

D 1 : 8 : 18

CORRECT OPTION

B 1 : 3 : 5

SOURCE

Physics • electrostatics

EXPLANATION

Surface charge density is

$$\rho = \frac{Q}{4\pi R^2}$$

where R is the radius of the sphere. For cancelling the field inside the cavity due to the shell of radius R, the shell with radius 2R induces a charge

$$-$$

Q in its inner surface and hence the total charge on the outer surface is

$$Q_1 + Q_2$$

. Similarly, the charge on the outermost shell is

$$Q_1 + Q_2 + Q_3$$

It is given that the surface charge densities

$$\rho_1, \rho_2$$

and

$$\rho_3$$

are equal, that is,

$$\frac{Q_1}{4\pi R^2} = \frac{Q_1 + Q_2}{4\pi (2R)^2} = \frac{Q_1 + Q_2 + Q_3}{4\pi (3R)^2}$$

$$\Rightarrow Q_1 = \frac{Q_1 + Q_2}{4} = \frac{Q_1 + Q_2 + Q_3}{9}$$

That is,

$$Q_1 = \frac{Q_1}{4} + \frac{Q_2}{4}$$

$$\Rightarrow Q_2 = 3Q_1$$

Therefore,

$$Q_3 = 5Q_1$$

Hence, the ratio of the charges given to the shells

$$Q_1 : Q_2 : Q_3$$

is 1 : 3 : 5.

Question 047 MCQ

QUESTION

The

$$x$$

-

$$t$$

graph of a particle undergoing simple harmonic motion is shown in the figure. The acceleration of the particle at

$$t = 4/3$$

s is

A $$\frac{\sqrt{3}}{32}\pi^2$$ cm/s

$$2$$

B $\frac{-\pi^2}{32}$ cm/s^2

C $\frac{\pi^2}{32}$ cm/s^2

D $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

CORRECT OPTION

D $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

The equation of single harmonic motion is

$$x = A\sin\left(\frac{2\pi}{T}t\right)$$

, where A is amplitude and T is the time period of the motion.

From the given figure,

$$A = 1$$

and

$$T = 8$$

. Thus,

$$x = \sin\left(\frac{\pi}{4}t\right)$$

The acceleration is

$$a = \frac{d^2x}{dt^2} = \frac{-\pi^2}{16}\sin\left(\frac{\pi}{4}t\right)$$

At time

$$t = \frac{4}{3}s$$

, the acceleration is

$$a = \frac{-\pi^2}{16}\sin\left(\frac{\pi}{4}\times\frac{4}{3}\right) = -\frac{\sqrt{3}}{32}\pi^2$$

cm/s

^{2}

Question 048 MCQ

QUESTION

A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is 4/3. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as $Takeg = 10m/s^{2}

A 9 m/s

B 12 m/s

C 16 m/s

D 21.33 m/s

CORRECT OPTION

C 16 m/s

SOURCE

Physics • geometrical-optics

EXPLANATION

The velocity of the ball when it has fallen through 7.2 m is

$$V = \sqrt{2g(7.2)} = 12$$

m/s

For a general height

h

above the water surface, we have

$$\frac{h'}{h} = \frac{4}{3} \Rightarrow h' = \frac{4}{3}h$$

Therefore,

$$V' = \frac{4}{3}V = \frac{4}{3} \times 12 = 16$$

m/s

where

V'

is the velocity of ball with respect to the fish.

Question 049 MCQ

QUESTION

For the circuit shown in the figure

A The current I through the battery is 7.5 mA.

B The potential difference across R_L, is 18 V.

Ratio of powers dissipated in R_1

C and

$$R_2$$

is 3.

If

$$R_1$$

and

D

$$R_2$$

are interchanged, magnitude of the power dissipated in

$$R_L$$

will decrease by a factor of 9.

CORRECT OPTION

A The current I through the battery is 7.5 mA.

SOURCE

Physics • current-electricity

EXPLANATION

The current in the net circuit is

$$I = \frac{V}{R_{effective}}$$

$$= \frac{V}{R_1 + [(R_2 R_L)/(R_1 + R_L)]}$$

$$= \frac{24}{2 \times 10^{-3} + [(6 \times 1.5 \times 10^{-6})/(7.5 \times 10^{-3})]} = 7.5$$

mA

The potential across

$$R_L$$

is

$$V_L = I \times \frac{R_2 R_L}{R_2 + R_L}$$

$$= 7.5 \times 10^{-3} \times 1.2 \times 10^3 = 9$$

V

Therefore,

$$I = \frac{9}{R_L} = 6$$

mA

The ratio of power dissipated in

$$R_1$$

and

$$R_2$$

is

$$\frac{IR_1}{(I-i)R_2} = \frac{(7.5 \times 10^{-3})^2 \times 2 \times 10^3}{(1.5 \times 10^{-3})^2 6 \times 10^3} = 0.75$$

J

The power dissipated in

$$R_L$$

is

$$\frac{V_L^2}{R_L} = \frac{9^2}{1.5 \times 10^3} = 54$$

mJ

If

$$R_1$$

and

$$R_2$$

are interchanged, then

$$I' = \frac{V}{R_{effective}}$$

$$= \frac{V}{R_2 + [(R_1 R_L)/(R_L + R_1)]}$$

$$= \frac{24}{6 \times 10^{-3} + [(2 \times 1.5 \times 10^{-6})/3.5 \times 10^{-3}]} = 3.5$$

mA

Hence, the voltage drop across the load is

$$V_L' = I \times \frac{R_1 R_L}{R_1 + R_L} = 1$$

V

Therefore,

$$i' = 2$$

mA. The magnitude of the power dissipated is the ratio between

$$i^2$$

and

$$(i')^2$$

:

$$\frac{P_1}{P_2} = \frac{i^2}{(i')^2} = \frac{6^2}{2^2} = 9$$

Question 050 MCQ

QUESTION

$$C_V$$

and

$$C_P$$

denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

A

$$C_P - C_V$$

is larger for a diatomic ideal gas than for a monoatomic ideal gas.

B

$$C_P + C_V$$

is larger for a diatomic ideal gas than for a monoatomic ideal gas.

C

$$C_P/C_V$$

is larger for a diatomic ideal gas than for a monoatomic ideal gas.

D

$$C_P \, . C_V$$

is larger for a diatomic ideal gas than for a monoatomic ideal gas.

CORRECT OPTION

B

$$C_P + C_V$$

is larger for a diatomic ideal gas than for a monoatomic ideal gas.

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

We know that

$$C_P - C_V = R$$

is same for all gases. For a diatomic gas, we have

$$C_V = \frac{5R}{2}$$

Therefore,

$$C_P = C_V + R = \frac{7R}{2}$$

For a monoatomic gas, we have

$$C_V = \frac{3R}{2}$$

$$C_P = \frac{5R}{2}$$

Therefore,

$$C_P + C_V = 6R$$

$diatomic$

$$C_P + C_V = 4R$$

$monoatomic$

$$\frac{C_P}{C_V} = \frac{7}{5} = 1.4$$

$diatomic$

$$\frac{C_P}{C_V} = \frac{5}{3} = 1.67$$

$monoatomic$

$$C_P \, . \, C_V = \frac{35R^2}{4}$$

$diatomic$

$$C_P \, . \, C_V = \frac{15R^2}{4}$$

$monoatomic$

Question 051 MCQ

QUESTION

A student performed the experiment of determination of focal length of a concave mirror by

$$u$$

-

$$v$$

method using an optical bench of length 1.5 m. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of $$u, v$$ values recorded by the student $incm$ are : $42, 56$, $48, 48$, $60, 40$, $66, 33$, $78, 39$. The data set s that cannot come from experiment and is are incorrectly recorded, is are

A $42, 56$

B $48, 48$

C $66, 33$

D $78, 39$

CORRECT OPTION

C $66, 33$

SOURCE

Physics • geometrical-optics

EXPLANATION

Using mirror formula for the concave mirror, we have

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Option A : When the value is $42, 56$, we have

$$\frac{1}{-56}+\frac{1}{-42}=\frac{-98}{56\times 42}\Rightarrow f=\frac{-56\times 42}{98}=-24$$

cm

Option B : When the value is $48, 48$, we have

$$\frac{1}{-48}+\frac{1}{-48}=\frac{-1}{24}\Rightarrow f=-24$$

cm

Option C : When the value is $66, 33$, we have

$$\frac{1}{-33}+\frac{1}{-66}=\frac{-1}{22}\Rightarrow f=-22$$

cm

Option D : When the value is $78, 39$, we have

$$\frac{1}{-39}+\frac{1}{-78}=\frac{-1}{26}\Rightarrow f=-26$$

cm

Since options A and B give

$$f=-24$$

cm, which is the same as that given in the problem, we conclude that the options C and D as the correct options.

Question 052 MCQ

QUESTION

If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that

A Linear momentum of the system does not change in time.

B Kinetic energy of the system does not change in time.

C Angular momentum of the system does not change in time.

D Potential energy of the system does not change in time.

CORRECT OPTION

A Linear momentum of the system does not change in time.

SOURCE

Physics • rotational-motion

EXPLANATION

According to Newton's second law, we have

$$\sum \overrightarrow{f_{ext}} = \frac{d\overrightarrow{p}}{dt}$$

If

$$\sum \overrightarrow{f_{ext}} = 0$$

, we find that the linear momentum

$$\overrightarrow{p}$$

as constant.

Question 053 MCQ

QUESTION

The allowed energy for the particle for a particular value of

$$n$$

is proportional to

A a^{-2}

B $a^{-3/2}$

C a^{-1}

D a^{2}

CORRECT OPTION

A a^{-2}

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

We have,

$$a = n\left(\frac{\lambda}{2}\right) \Rightarrow \lambda = \frac{2a}{n}$$

From de Broglie relation, we have

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Therefore,

$$\frac{2a}{n} = \frac{h}{p} \Rightarrow p = \frac{nh}{2a}$$

Now,

$$E = \frac{p^2}{2m} = \frac{n^2h^2}{8ma^2} \Rightarrow E \propto a^{-2}$$

Question 054 MCQ

QUESTION

If the mass of the particle is

$$m = 1.0 \times 10^{-30}$$

kg and

$$a = 6.6$$

nm, the energy of the particle in its ground state is closest to

A 0.8 meV

B 8 meV

C 80 meV

D 800 meV

CORRECT OPTION

B 8 meV

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

For ground state,

$$n = 1$$

. Therefore,

$$E_1 = \frac{h^2}{8ma^2} = \left[\frac{(6.6 \times 10^{-34})^2}{8 \times 10^{-30} \times (6.6 \times 10^{-9})^2}\right]$$

J

Therefore,

$$\frac{E_1}{e} = \left(\frac{E_1}{1.6 \times 10^{-19}}\right)$$

meV = 8 meV

Question 055 MCQ

QUESTION

The speed of the particle, that can take discrete values, is proportional to

A $n^{-3/2}$

B n^{-1}

C $n^{1/2}$

D n

CORRECT OPTION

D n

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To find the relationship between the speed of the particle and the quantum number

$$n$$

, we need to follow a few steps using the given information and fundamental quantum mechanical principles.

Firstly, for a particle moving in a one-dimensional box of length

$$a$$

, the allowed wavelengths

$$\lambda$$

of standing waves can be given by:

$$\lambda = \frac{2a}{n}$$

for $n = 1, 2, 3, \ldots$.

The de Broglie relation links the wavelength

$$\lambda$$

of a particle to its momentum

$$p$$

:

$$\lambda = \frac{h}{p}$$

From this, we can express the momentum

p

in terms of the quantum number

n

:

$$\frac{h}{p} = \frac{2a}{n}$$

Solving for

p

, we get:

$$p = \frac{nh}{2a}$$

The kinetic energy

E

of the particle is related to its momentum

p

by the equation:

$$E = \frac{p^2}{2m}$$

Substituting

$$p = \frac{nh}{2a}$$

into the energy expression, we get:

$$E = \frac{(nh/2a)^2}{2m} = \frac{n^2h^2}{8ma^2}$$

The kinetic energy can also be written in terms of the speed

v

of the particle:

$$E = \frac{1}{2}mv^2$$

Equating the two expressions for energy, we have:

$$\frac{1}{2}mv^2 = \frac{n^2h^2}{8ma^2}$$

Solving for the speed

$$v$$

, we get:

$$v^2 = \frac{n^2h^2}{4m^2a^2}$$

Therefore, the speed

$$v$$

of the particle is:

$$v = \frac{nh}{2ma}$$

From the above expression, we see that the speed

$$v$$

of the particle is directly proportional to

$$n$$

:

$$v \propto n$$

Therefore, the speed of the particle is proportional to:

Option D:

$$n$$

Question 056 MCQ

QUESTION

In the core of nuclear fusion reactor, the gas becomes plasma because of

A strong nuclear force acting between the deuterons.

B Coulomb force acting between the deuterons.

C Coulomb force acting between deuteron-electrons pairs.

D the high temperature maintained inside the reactor core.

CORRECT OPTION

D the high temperature maintained inside the reactor core.

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

In a nuclear fusion reactor, the gas becomes plasma primarily because of the high temperature maintained inside the reactor core. At such high temperatures, typically in the range of millions of degrees Kelvin, the thermal energy is sufficient to ionize the atoms, meaning electrons are stripped from the nuclei, resulting in a collection of positively charged deuteron nuclei and negatively charged electrons. This ionized state of matter is known as plasma.

The strong nuclear force and Coulomb force $betweendeuteronsandbetweendeuteron-electronpairs$ play roles in the

fusion process and the behavior of particles. However, the ionization into plasma is primarily due to the extremely high temperatures.

Therefore, the correct option is:

Option D: the high temperature maintained inside the reactor core.

Question 057 MCQ

QUESTION

Assume that two deuteron nuclei in the core of fusion reactor at temperature T are moving towards each other, each with kinetic energy 1.5 kT, when the separation between them is large enough to neglect Coulomb potential energy. Also neglect any interaction from other particles in the core. The minimum temperature T required for them to reach a separation of 4

$$\times$$

10

$$-15$$

m is in the range

A $1.0 \times 10^9 K < T < 2.0 < 10^9 K$

B $2.0 \times 10^9 K < T < 3.0 < 10^9 K$

C $3.0 \times 10^9 K < T < 4.0 < 10^9 K$

D $4.0 \times 10^9 K < T < 5.0 < 10^9 K$

CORRECT OPTION

A $1.0 \times 10^9 K < T < 2.0 < 10^9 K$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To determine the minimum temperature required for two deuteron nuclei to reach a separation of

$$4 \times 10^{-15}$$

m, we need to equate the initial kinetic energy of the deuterons with the potential energy at the specified separation distance.

The initial kinetic energy of each deuteron is given as :

$$KE = 1.5\,kT$$

Since there are two deuterons, the total kinetic energy is :

$$KE_{\text{total}} = 2 \times 1.5\,kT = 3\,kT$$

The Coulomb potential energy PE between two deuterons at a separation distance

$$r = 4 \times 10^{-15}$$

m is given by :

$$PE = \frac{e^2}{4\pi\epsilon_0 r}$$

Using the given value:

$$\frac{e^2}{4\pi\epsilon_0} = 1.44 \times 10^9 \,\text{eV} \cdot \text{m}$$

So, the potential energy becomes :

$$PE = \frac{1.44 \times 10^9 \,\text{eV} \cdot \text{m}}{4 \times 10^{-15} \,\text{m}}$$

Simplifying :

$$PE = \frac{1.44 \times 10^9}{4 \times 10^{-15}} \,\text{eV}$$

$$PE = 0.36 \times 10^{24} \,\text{eV}$$

Now, we equate the total kinetic energy to the potential energy to find the temperature T :

$$3\,k\,T = 0.36 \times 10^{24} \,\text{eV}$$

Solving for T :

$$T = \frac{0.36 \times 10^{24}}{3 \times 8.6 \times 10^{-5}} \,\text{K}$$

$$T = \frac{0.36 \times 10^{24}}{2.58 \times 10^{-4}} \,\text{K}$$

$$T = 1.395 \times 10^9 \,\text{K}$$

This temperature is in the range :

Option A:

$$1.0 \times 10^9 K < T < 2.0 < 10^9 K$$

So, the correct answer is :

Option A :

$$1.0 \times 10^9 K < T < 2.0 < 10^9 K$$

Question 058 MCQ

QUESTION

Results of calculations for four different designs of a fusion reactor using D-D reaction are given below. Which of these is most promising based on Lawson criterion?

A Deuteron density = $2.0 \times 10^{12}\ \mathrm{cm}^{-3}$; Confinement time = $5.0 \times 10^{-3}\ \mathrm{s}$.

B Deuteron density = $8.0 \times 10^{14}\ \mathrm{cm}^{-3}$; Confinement time = $9.0 \times 10^{-1}\ \mathrm{s}$.

C Deuteron density = $4.0 \times 10^{23}\ \mathrm{cm}^{-3}$; Confinement time = $1.0 \times 10^{-11}\ \mathrm{s}$.

Deuteron density =

$$1.0 \times 10^{24}\ \mathrm{cm}^{-3}$$

D ; Confinement time =

$$4.0 \times 10^{-12}\ \mathrm{s}$$

.

CORRECT OPTION

Deuteron density =

$$8.0 \times 10^{14}\ \mathrm{cm}^{-3}$$

B ; Confinement time =

$$9.0 \times 10^{-1}\ \mathrm{s}$$

.

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To determine the most promising design based on the Lawson criterion, we need to calculate the Lawson number $$$nt_0$$$ for each design and check if it exceeds

$$5 \times 10^{14}$$

s/cm

$$3$$

. We will do this for each of the given options:

Option A:

Deuteron density,

$$n = 2.0 \times 10^{12}\ \mathrm{cm}^{-3}$$

Confinement time,

$$t_0 = 5.0 \times 10^{-3}\ \mathrm{s}$$

Lawson number,

$$nt_0 = n \cdot t_0 = (2.0 \times 10^{12}) \cdot (5.0 \times 10^{-3}) = 1.0 \times 10^{10}\ \mathrm{s/cm^3}$$

Option B:

Deuteron density,

$$n = 8.0 \times 10^{14}\ \mathrm{cm^{-3}}$$

Confinement time,

$$t_0 = 9.0 \times 10^{-1}\ \mathrm{s}$$

Lawson number,

$$nt_0 = n \cdot t_0 = (8.0 \times 10^{14}) \cdot (9.0 \times 10^{-1}) = 7.2 \times 10^{14}\ \mathrm{s/cm^3}$$

Option C:

Deuteron density,

$$n = 4.0 \times 10^{23}\ \mathrm{cm^{-3}}$$

Confinement time,

$$t_0 = 1.0 \times 10^{-11}\ \mathrm{s}$$

Lawson number,

$$nt_0 = n \cdot t_0 = (4.0 \times 10^{23}) \cdot (1.0 \times 10^{-11}) = 4.0 \times 10^{12}\ \mathrm{s/cm^3}$$

Option D:

Deuteron density,

$$n = 1.0 \times 10^{24}\ \mathrm{cm^{-3}}$$

Confinement time,

$$t_0 = 4.0 \times 10^{-12}\ \mathrm{s}$$

Lawson number,

$$nt_0 = n \cdot t_0 = (1.0 \times 10^{24}) \cdot (4.0 \times 10^{-12}) = 4.0 \times 10^{12}\ \mathrm{s/cm^3}$$

Based on the calculations, the Lawson numbers for each option are:

- Option A:

$$1.0 \times 10^{10}\ \mathrm{s/cm^3}$$

- Option B:

$$7.2 \times 10^{14}\ \mathrm{s/cm^3}$$

- Option C:

$$4.0 \times 10^{12}\ \mathrm{s/cm^3}$$

- Option D:

$$4.0 \times 10^{12}\ \mathrm{s/cm^3}$$

The most promising design based on the Lawson criterion, which requires the Lawson number to be greater than

$$5 \times 10^{14}\ \mathrm{s/cm^3}$$

, is **Option B** because its Lawson number exceeds the threshold.

Question 059 MCQ

QUESTION

Column II shows five systems in which two objects are labelled as X and Y. Also in each case a point P is shown. Column I gives some statements about X and/or Y. Match these statements to the appropriate system s from Column II:

	Column I		Column II
A	The force exerted by X on Y has a magnitude Mg .	P	Block Y of mass M left on a fixed inclined plane X, slides on it with a constant veloci
B	The gravitational potential energy of X is continuously increasing.	Q	Two rings magnets Y and Z, each of mass are kept in frictionless vertical plastic stan so that they repel each other. Y rests on th base X and Z hangs in air in equilibrium. F the topmost point of the stand on the common axis of the two rings. The whole system is in a lift that is going up with a constant velocity.
C	Mechanical energy of the system X + Y is continuously decreasing.	R	A pulley Y of mass m_0 is fixed to a table through a clamp X. A blo of mass M hangs from a string that goes o the pulley and is fixed at point P of the tab The whole system is kept in a lift that is go down with a constant velocity.
D	The torque of the weight of Y about point is zero.	S	A sphere Y of mass M is put in a non-visc liquid X kept in a container at rest. The sphere is released and it moves down in t liquid.
		T	A sphere Y of mass M is falling with its terminal velocity in a viscous liquid X kept a container.

A $(A) \rightarrow (T), (S); (B) \rightarrow (Q), (T); (C) \rightarrow (P), (R), (T); (D) \rightarrow (Q)$

B $(A) \rightarrow (T), (P); (B) \rightarrow (Q), (S), (T); (C) \rightarrow (P), (R), (T); (D) \rightarrow (Q)$

C $(A) \rightarrow (T), (Q); (B) \rightarrow (Q), (S), (T); (C) \rightarrow (P), (R), (T); (D) \rightarrow (S)$

D $(A) \rightarrow (P); (B) \rightarrow (S), (T); (C) \rightarrow (P), (R), (T); (D) \rightarrow (T)$

CORRECT OPTION

B $(A) \rightarrow (T), (P); (B) \rightarrow (Q), (S), (T); (C) \rightarrow (P), (R), (T); (D) \rightarrow (Q)$

SOURCE

Physics • gravitation

EXPLANATION

Case P : Since

$$v$$

= Constant, we have

$$Mg\sin\theta = \mu Mg\cos\theta = f \Rightarrow \mu = \tan\theta$$

Now,

$$N = Mg\cos\theta$$

. Therefore,

$$R = \sqrt{N^2 + f^2} = Mg\sqrt{\cos^2\theta + \sin^2\theta}$$

Case T : We have

$$Mg = F_v + F_b$$

, where

$$F_v$$

is the viscous force and

$$F_b$$

is the buoyant force.

Case S : We have

$$Mg - F_b = Ma$$

.

Case R : We have

$$Mg = T$$

. Therefore,

$$T + M_0 g = F_y$$

$by the clamp$

$$T = F_X$$

$by the clamp$

Therefore, the force exerted by the clamp X on pulley Y is

$$F = \sqrt{F_X^2 + F_Y^2} = g\sqrt{M^2 + (M + m_0)^2}$$

Case Q : We have

$$Mg = F_m$$

$for Z$

where

$$F_m$$

is the magnetic repulsion. Also,

$$Mg + F_m = N$$

$forY$

Therefore,

$$N = 2Mg$$

.

Note :

Option A : For option T, if

$$F_b$$

is ignored, then

$$Mg = F_v$$

; otherwise, no case matches for option A.

Hence, A

$$\rightarrow$$

T, P.

Option B : In option Q, it is mentioned that the lift is moving up continuously; therefore, the gravitational potential energy of X goes on increasing. In option B , as Y comes down, X goes up $displaced$. The same is applicable for option T.

Hence, B

$$\rightarrow$$

Q, S, T.

Option C : For option P, since Y moves down with a constant

$$v$$

, the gravitational potential energy of the system X + Y goes on decreasing, similar is the case in Options R and T.

Hence, C

$\rightarrow$

$P, R, T.$

Option D : For option S, the mass moves down with acceleration. Therefore, the kinetic energy goes on increasing. Since the line of action of Mg of Y phases through point P, as mentioned in option Q, its torque about P is zero.

Hence, D

$\rightarrow$

$Q.$

Question 060 MCQ

QUESTION

Six point charges, each of the same magnitude q, are arranged in different manners as shown in Column II. In each case, a point M and a line PQ passing through M are shown. Let E be the electric field and V be the electric potential at M $potentialatinfinityiszero$ due to the given charge distribution when it is at rest. Now, the whole system is set into rotation with a constant angular velocity about the line PQ. Let B be the magnetic field at M and

μ

be the magnetic moment of the system in this condition. Assume each rotating charge to be equivalent to a steady current.

	Column I		Column II
A	$E = 0$	P	Charge are at the corners of a regular hexagon. M is at the centre of the hexago PQ is perpendicular to the plane of the hexagon.

	Column I		Column II
B	$V \neq 0$	Q	Charges are on a line perpendicular to PQ equal intervals. M is the midpoint betweer the two innermost charges.
C	$B = 0$	R	Charges are placed on two coplanar insulating rings at equal intervals. M is the common centre of the rings. PQ is perpendicular to the plane of the rings.
D	$\mu \neq 0$	S	Charges are placed at the corners of a rectangle of sides a and 2a and at the mic points of the longer sides. M is at the cent of the rectangle. PQ is parallel to the long sides.
		T	Charges are placed on two coplanar, identical insulating rings are equal interval M is the midpoint between the centres of t rings. PQ is perpendicular to the line joinir the centres and coplanar to the rings.

A $(\mathrm{A}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{B}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{T}); (\mathrm{D}) \rightarrow (\mathrm{T}), (\mathrm{S})$

B $(\mathrm{A}) \rightarrow (\mathrm{P}), (\mathrm{R}), (\mathrm{S}); (\mathrm{B}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{S}); (\mathrm{D}) \rightarrow (\mathrm{R}), (\mathrm{S}$

C $(\mathrm{A}) \rightarrow (\mathrm{P}), (\mathrm{R}), (\mathrm{S}); (\mathrm{B}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{T}); (\mathrm{D}) \rightarrow (\mathrm{R}), (\mathrm{S}$

D $(\mathrm{A}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{S}); (\mathrm{B}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{T}); (\mathrm{D}) \rightarrow (\mathrm{R}), (\mathrm{S}$

CORRECT OPTION

C $(\mathrm{A}) \rightarrow (\mathrm{P}), (\mathrm{R}), (\mathrm{S}); (\mathrm{B}) \rightarrow (\mathrm{R}), (\mathrm{S}); (\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q}), (\mathrm{T}); (\mathrm{D}) \rightarrow (\mathrm{R}), ($

SOURCE

Physics • electrostatics

EXPLANATION

Case P :

$$E = 0, V = 0, B = 0, \mu = 0$$

.

Case Q :

$$E \neq 0, V = 0, B = 0, \mu = 0$$

.

Case R :

$$E = 0, V \neq 0, B \neq 0, \mu \neq 0$$

.

Case S :

$$E = 0, V \neq 0, B \neq 0, \mu \neq 0$$

.

Case T :

$$E \neq 0, V = 0, B = 0, \mu = 0$$

.

Hence,

$(A) \rightarrow (P), (R), (S); (B) \rightarrow (R), (S); (C) \rightarrow (P), (Q), (T); (D) \rightarrow (R), (S)$