iit Jee 2011 Paper 2 Offline $60 Questions$

Question 001 MCQ

QUESTION

Oxidation states of the metal in the minerals haematite and magnetite, respectively are

A II, III in haematite and III in magnetite

B II, III in haematite and II in magnetite

C II in haematite and II, III in magnetite

D III in haematite and II, III in magnetite

CORRECT OPTION

D III in haematite and II, III in magnetite

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

The molecular formula of the mineral haematite is $\mathrm{Fe_2O_3}$ and that of magnetite is $\mathrm{Fe_3O_4}$ respective. Both contain iron Fe but in different oxidation states.

Oxidation state of iron Fe in :

a Haematite $(\mathrm{Fe_2O_3})$

Let the oxidation state of iron Fe be x

$$2x + 3 \times (-2) = 0$$

$$2x = 6$$

$$x = +3$$

Iron exist as Fe III.

b Magnetite $(\mathrm{Fe_3O_4})$

The mineral magnetite is made by two iron oxides $\mathrm{FeO}.\mathrm{Fe_2O_3}$.

The oxidation state of iron in $\mathrm{Fe_2O_3}$ is +3 and oxidation state of iron in FeO can be calculated as :

$$x + 2 \times (-2) = 0$$

$$x = +2$$

Iron exist as Fe II.

Hence, the oxidation state of iron in haematite is III and that in magnetite is II, III respectively.

Question 002 MCQ

QUESTION

For the first order reaction

$2N_2O_5\ g$

$$\rightarrow$$

$4NO_2\ g + O_2\ g$

A the concentration of the reactant decreases exponentially with time

B the half-life of the reaction decreases with increasing temperature

C the half-life of the reaction depends on the initial concentration of the reactant

D the reaction proceeds to 99.6% completion in eight half-life duration

CORRECT OPTION

A the concentration of the reactant decreases exponentially with time

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

The given chemical equation represents a first-order reaction:

$$2N_2O_5(g) \rightarrow 4NO_2(g) + O_2(g)$$

In first-order reactions, the rate of reaction is directly proportional to the concentration of the reactant. The rate law can be expressed as:

$$Rate = k[N_2O_5]$$

where *k* is the rate constant.

The integrated rate equation for a first-order reaction is:

$$\ln[N_2O_5] = -kt + \ln[N_2O_5]_0$$

where

$$N_2O_5$$

_*0* is the initial concentration and

$$N_2O_5$$

is the concentration at time *t*.

Rearranging gives:

$$[N_2O_5] = [N_2O_5]_0 e^{-kt}$$

Now, let's analyze each option:

Option A: "the concentration of the reactant decreases exponentially with time".

This matches the expression

$$[N_2O_5] = [N_2O_5]_0 e^{-kt}$$

which shows an exponential decline in the concentration of the reactant. Therefore, option A is **correct**.

Option B: "the half-life of the reaction decreases with increasing temperature".

In first-order reactions, the half-life is determined by the equation

$$t_{1/2} = \frac{0.693}{k}.$$

Increases in temperature typically increase the rate constant *k*, thus decreasing the half-life. However, it is not directly related to the half-life itself but rather to the rate constant via the Arrhenius equation. Nonetheless, the essence of the statement is **correct** as increasing temperature generally decreases the half-life.

Option C: "the half-life of the reaction depends on the initial concentration of the reactant".

The half-life for a first-order reaction given by

$$t_{1/2} = \frac{0.693}{k}$$

is independent of the initial concentration

$$N_2O_5$$

_0. Therefore, option C is **incorrect**.

Option D: "the reaction proceeds to 99.6% completion in eight half-life durations".

Completion to a certain level can be found using the formula for decay over multiple half-lives, given as

$$[N_2O_5] = [N_2O_5]_0 \times \left(\frac{1}{2}\right)^n$$

where *n* is the number of half-lives. For eight half-lives, we have:

$$Final\,Concentration = [N_2O_5]_0 \times \left(\frac{1}{2}\right)^8 = [N_2O_5]_0 \times \frac{1}{256}$$

The reaction proceeds to

$$\frac{255}{256} \approx 99.6\%$$

completion. Thus, option D is **correct**.

In summary, options A, B, and D are correct, while option C is incorrect.

Question 003 MCQ

QUESTION

Consider the following cell reaction:

$2Fe\,s + O_2\,g + 4H^+\,aq \rightarrow 2Fe^{2+}\,aq + 2H_2O\,l$; $E^o = 1.67$ V

At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is

A 1.47 V

B 1.77 V

C 1.87 V

D 1.57 V

CORRECT OPTION

D 1.57 V

SOURCE

Chemistry • electrochemistry

EXPLANATION

To find the cell potential under non-standard conditions, we can use the Nernst equation, which is given as:

$$E = E^o - \frac{RT}{nF}\ln(Q)$$

Where:

- E^o is the standard cell potential $1.67V$.
- R is the gas constant $8.314J/mol \cdot K$.
- T is the temperature in Kelvin $298Kfor25^\circ C$.
- n

is the number of moles of electrons transferred per mole of reaction $4 in this case$.

- F

  is the Faraday constant $96485 C/mol$.

- Q

  is the reaction quotient.

The reaction quotient,

$$Q$$

, can be calculated based on the given conditions and the reaction:

$$Q = \frac{[\mathrm{Fe}^{2+}]^2}{[\mathrm{H}^+]^4 \cdot P(O_2)}$$

Substituting the given values:

- $$[\mathrm{Fe}^{2+}] = 10^{-3}\ \mathrm{M}$$
- $$[\mathrm{H}^+] = 10^{-\mathrm{pH}} = 10^{-3}\ \mathrm{M}$$
- $$P(O_2) = 0.1\ \mathrm{atm}$$

$$Q = \frac{(10^{-3})^2}{(10^{-3})^4 \cdot 0.1}$$

$$Q = \frac{10^{-6}}{10^{-12} \cdot 0.1}$$

$$Q = \frac{10^{-6}}{10^{-13}}$$

$$Q = 10^7$$

Now, substituting the values into the Nernst equation:

$$E = 1.67\ \mathrm{V} - \frac{8.314 \times 298}{4 \times 96485} \ln(10^7)$$

Calculating the term

$$\frac{RT}{nF}$$

;

$$\frac{RT}{nF} = \frac{8.314 \times 298}{4 \times 96485}$$

$$\approx \frac{2476}{385940}$$

$$\approx 0.0064 \text{ V}$$

The natural logarithm of

$$10^7$$

is about 16.1 $since$$$ \ln(10 \approx 2.303

$$, so$$

\ln 10^7 = 7 \times \ln 10 \approx 16.1$$).

Thus,

$$E = 1.67 \text{ V} - 0.0064 \times 16.1$$

$$E = 1.67 \text{ V} - 0.103 \text{ V}$$

$$E = 1.567 \text{ V}$$

Therefore, the cell potential under the given conditions is approximately 1.57 V, so option D $1.57V$ is the correct answer.

Question 004 MCQ

QUESTION

The freezing point (in ^{o}C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ $Mol.wt.329$ in 100 g of water (K_f = 1.86 K kg mol^{-1}) is

-2.3

A

-5.7×10^{-2}

B

-5.7×10^{-2}

C

-5.7×10^{-3}

D

-1.2×10^{-2}

CORRECT OPTION

A

-2.3×10^{-2}

SOURCE

Chemistry • solutions

EXPLANATION

To find the freezing point of the solution, we will use the formula for freezing point depression, which is given by:

$$\Delta T_f = i \cdot K_f \cdot m$$

where:

- ΔT_f

  is the freezing point depression.

- i

  is the van't Hoff factor $number of particles the solute splits into or forms in solution$.

- K_f

  is the cryoscopic constant $freezing point depression constant$, for water it's 1.86 K kg mol^{-1}.

- m

  is the molality of the solution.

Given:

- Solute is

  $$K_3[Fe(CN)_6]$$

  , and molecular weight M = 329 g/mol.

- Mass of solute = 0.1 g.

- Mass of solvent $water$ = 100 g.

First, we calculate the molality

$$m$$

of the solution:

$$m = \frac{\text{Moles of solute}}{\text{Kilograms of solvent}}$$

Moles of solute:

$$\text{Moles of solute} = \frac{\text{Mass of solute}}{\text{Molecular weight}} = \frac{0.1}{329}\text{mol}$$

$$\text{Moles of solute} = 3.04 \times 10^{-4}\ \text{mol}$$

Kilograms of solvent:

$$\text{Kilograms of solvent} = 100\ \text{g} = 0.1\ \text{kg}$$

Now molality:

$$m = \frac{3.04 \times 10^{-4}\ \text{mol}}{0.1\ \text{kg}} = 0.00304\ \text{mol/kg}$$

Next, calculate the van't Hoff factor

$$i$$

:

- The solute

  $$K_3[Fe(CN)_6]$$

  disassociates into 4 ions: 3

  $$K^+$$

  and 1

  $$[Fe(CN)_6]^{3-}$$

  . Therefore,

  $$i = 4$$

  .

Substituting these into the freezing point depression formula:

$$\Delta T_f = i \cdot K_f \cdot m = 4 \cdot 1.86 \cdot 0.00304$$

$$\Delta T_f = 0.02259\ \text{K}$$

Convert to degrees Celsius and remember that this will lower the freezing point, so it should be negative

$since the freezing point depression means the solution freezes at a lower temp$
:

$$\Delta T_f = -0.02259\ ^\circ\mathrm{C}$$

So,

$$\Delta T_f = -2.259 \times 10^{-2}\ ^\circ\mathrm{C}$$

From the given options, the closest to this calculated value of

$$\Delta T_f$$

is:

Option A: -2.3

$$\times$$

10^{-2}°C. Hence, Option A is the correct answer.

Question 005 Numerical

QUESTION

The number of hexagonal faces that are present in a truncated octahedron is _____.

SOURCE

Chemistry • solid-state

EXPLANATION

i The shape of a regular octahedron :

When the octahedron is truncated from all edges.

ii Each face of truncated octahedron is hexagonal.

Since, there are 8 faces, there are eight hexagons.

Question 006 Numerical

QUESTION

In 1 L saturated solution of AgCl [$K_{sp}\ AgCl$ = 1.6

$\times$

10^{-10}], 0.1 mol of CuCl [$K_{sp}\ CuCl$ = 1.0

$\times$

10^{-6}] is added. The resultant concentration of Ag^+ in the solution is 1.6

$\times$

10^{-x}. The value of "x" is

SOURCE

Chemistry • ionic-equilibrium

EXPLANATION

i The solubility of $\mathrm{AgCl(s)}$ in saturated solution is expressed as :

$$\mathrm{H_2O + AgCl_{(s)} \rightarrow \underset{x}{Ag^+}(aq) + \underset{x}{Cl^-}(aq)}$$

Though $\mathrm{AgCl(s)}$ has low solubility with $\mathrm{K}_{sp} = 1.6\ \times 10^{-10}$ still some silver $\left(\mathrm{Ag}^+\right)$ and chloride $\left(\mathrm{Cl}^-\right)$ are dissolved in solution. Let the concentration of these ions be $x\ \mathrm{mol}^{-1}\ insolution$.

$$\begin{aligned}[\mathrm{Ag}]^+ &= \left[\mathrm{Cl}^-\right] = x \\ &= 1.6 \times 10^{-10} \\ \mathrm{CuCl}(s) + \mathrm{H_2O}(l) &\rightleftharpoons \mathrm{Cu}^+(aq) + \mathrm{Cl}^-(aq)\end{aligned}$$

ii Similarly though CuCl has low solubility in aqueous solution $\left[K_{sp} = 1.0 \times 10^{-6}\right]$, still some copper $\left(\mathrm{Cu}^{+}\right)$ and chloride $\left(\mathrm{Cl}^{-}\right)$ are dissolved in the solution. Let the concentration of these ions be $y \,\mathrm{mol\ L}^{-1}$ $insolution$.

$$\left[\mathrm{Cu}^{+}\right] = \left[\mathrm{Cl}^{-}\right] = y$$

iii The salts $\mathrm{AgCl(s)}$ and $\mathrm{CuCl(s)}$ are present in equilibrium with their ions as follows :

$$\mathrm{AgCl}(s) + \mathrm{H_2O} \rightleftharpoons \underset{x}{\mathrm{Ag}^{+}}(aq) + \underset{x+y}{\mathrm{Cl}^{-}}(aq)$$

$$K_{sp}(\mathrm{AgCl}) = 1.6 \times 10^{-10} = x(x+y) \quad \ldots(\mathrm{i})$$

$$\mathrm{CuCl}_{(s)} + \mathrm{H_2O} \rightleftharpoons \underset{x}{\mathrm{Cu}^{+}}(aq) + \underset{y+x}{\mathrm{Cl}^{-}}(aq)$$

$$K_{sp}(\mathrm{CuCl}) = 1.0 \times 10^{-6} = y \times (x+y) \quad \ldots(\mathrm{ii})$$

Dividing equation ii by i :

$\frac{1.0\times10^{-6}}{1.6\times10^{-10}} = \frac{y}{x}$

$\frac{y}{x} = \frac{1.0}{1.6} \times 10^{4} \ \ldots iii$

No. of moles of CuCl = $0.1 \,\mathrm{mol}$

Volume of solution = $1 \,\mathrm{L}$

$$\text{Concentration of CuCl} = \frac{\text{No. of moles of CuCl}}{\text{Volume of solution}}$$

$$\text{Concentration of CuCl} = \frac{0.1 \,\mathrm{mol}}{1 \,\mathrm{L}} = 0.1$$

$$\mathrm{K}_{sp} = 1.0 \times 10^{-6} = y \times y$$

$$y = \sqrt{10^{-6}} = 10^{-3} \,\mathrm{mol\ L}^{-1}$$

Substituting value of y in equation iii,

$$\frac{\left[\mathrm{Cu}^{+}\right]}{\left[\mathrm{Ag}^{+}\right]}=\frac{y}{x}=\frac{10^{-3}\ \mathrm{mol\ L}^{-1}}{x}=10^{4}\frac{1.0}{1.6}$$

$$x=1.6\times10^{-7}$$

The concentration of silver ion, $\left[\mathrm{Ag}^{+}\right]=x=1.6\times10^{-7}\ \mathrm{mol\ L}$.

Hence, the value of x in 1.6 $\times10^{-x}$ is 7 .

Question 007 MCQ

QUESTION

Match the transformations in **column I** with appropriate options in **column II**

Column I

A $\mathrm{CO}_2\,s$

$\rightarrow$

$\mathrm{CO}_2\,g$

B $\mathrm{CaCO}_3\,s$

$\rightarrow$

$\mathrm{CaO}\,s$ + $\mathrm{CO}_2\,g$

C 2H

$\rightarrow$

$\mathrm{H}_2\,g$

D $\mathrm{P}_{white,\,solid}$

$\rightarrow$

$\mathrm{P}_{red,\,solid}$

Column II

p phase transition

q allotropic change

r

$$\Delta H$$

is positive

s

$$\Delta S$$

is positive

t

$$\Delta S$$

is negative

A

$\rightarrow$

p,r,s; B

$\rightarrow$

A r,s; C

$\rightarrow$

t; D

$\rightarrow$

p,q,t

A

$\rightarrow$

r,s; B

$\rightarrow$

B p,s; C

$\rightarrow$

t; D

→

p,q,t

A

→

p,r,s; B

→

C r,s; C

→

r; D

→

p,t

A

→

p,r,s; B

→

D r,s; C

→

t; D

→

q,t

CORRECT OPTION

A

→

p,r,s; B

$\rightarrow$

A

r,s; C

$\rightarrow$

t; D

$\rightarrow$

p,q,t

SOURCE

Chemistry • thermodynamics

EXPLANATION

Analysis of each transformation:

A $\mathbf{CO_2}\,s \rightarrow \mathbf{CO_2}\,g$

This transformation represents the sublimation of solid carbon dioxide $dryice$ into gaseous carbon dioxide. This process is:

Phase transition p: The substance changes from solid to gas phase.

$$\Delta H$$

is positive r: Sublimation is an endothermic process, meaning it absorbs heat from the surroundings.

$$\Delta S$$

is positive s: The entropy increases as the substance moves from a more ordered solid state to a less ordered gaseous state.

B $\mathbf{CaCO_3}\,s \rightarrow \mathbf{CaO}\,s + \mathbf{CO_2}\,g$

This transformation is the thermal decomposition of calcium carbonate to form calcium oxide and carbon dioxide. This process involves:

$$\Delta H$$

is positive r: Decomposition reactions are typically endothermic, requiring energy to break chemical bonds.

$$\Delta S$$

is positive s: The process results in an increase in entropy, primarily because a gas (CO_2) is produced from a solid compound.

C **2H** $\rightarrow$ **H**$_2 g$

This is the formation of hydrogen gas (H_2) from atomic hydrogen H. This transformation:

$$\Delta S$$

is negative t: Two individual hydrogen atoms combine into a diatomic molecule, decreasing the system's entropy since fewer particles are free to move independently.

D **P**$_{white,\ solid}$ $\rightarrow$ **P**$_{red,\ solid}$

Crystalline solid white phosphorous is converted to amorphous red phosphorous. This process involves a phase change. Since, white and red phosphorous are allotropic form of phosphorous, i.e., both are composed of phosphorous atoms but the connectivity between atom differ, it is therefore regarded as allotropic change.

The white form of phosphorous has lot of angular strain as compared to stable red phosphorous. Hence, randomness of system due to angular strain decreases as red phosphorus is formed. The entropy change is negative $(\Delta S < 0)$

Option Evaluation:

Upon reviewing each transformation and comparing the mappings:

Option A agrees completely with the above analysis with proper mapping for each statement.

Thus, **Option A** is correct:

A $\rightarrow$ p,r,s; B $\rightarrow$ r,s; C $\rightarrow$ t; D $\rightarrow$ p,q,t

Question 008 MCQ

QUESTION

Among the following complexes $K - P$,

$K_3[Fe(CN)_6]$ K, $[Co(NH_3)_6]Cl_3$ L, $Na_3[Co(oxalate)_3]$ M, $[Ni(H_2O)_3]Cl_2$ N, $K_2[Pt(CN)_4]$ O and $[Zn(H_2O)_6(NO_3)_2]$ P

The diamagnetic complexes are

A K, L, M, N

B K, M, O, P

C L, M, O, P

D L, M, N, O

CORRECT OPTION

C L, M, O, P

SOURCE

Chemistry • coordination-compounds

EXPLANATION

In $K_3Fe(CN)_6$:

CN

–

being a strong field ligand, it causes pairing of electrons, so only one unpaired electron. The complex is paramagnetic.

In $Co[(NH_3)_6]Cl_3$:

NH_3 being a strong field ligand it causes pairing of spins. There is no unpaired electron, hence the complex is diamagnetic.

In $Na_3[Co\,oxalate_3]$:

Oxalate being a strong field ligand it causes pairing of electron spins. There is no unpaired electron, hence the complex is diamagnetic.

In $[Ni(H_2O)_6]Cl_2$:

Since H_2O is a weak field ligand, no pairing takes place. There are two unpaired electrons, so the complex is paramagnetic.

In $K_2[Pt\,CN_4]$:

CN

–

being a strong field ligand, it causes pairing of electrons spins, so no unpaired electron. Hence, the complex is diamagnetic.

In $[Zn(H_2O)_6(NO_3)_2]$: Zn^{2+} has $3d^{10}$ configuration. Therefore, it is a diamagnetic complex.

Question 009 MCQ

QUESTION

The major product of the following reaction is

A a hemiacetal

B an acetal

C an ether

D an ester

CORRECT OPTION

B an acetal

SOURCE

Chemistry • hydrocarbons

EXPLANATION

In presence of mineral acid $\mathrm{H}^{+}\ with no water$ positively charged centre is generated at second position on the ring.

Since, oxygen is electronegative, it shifts pi electron towards itself generating a carbocation which attack one lone pair of electron on oxygen of alcohol.

Loss of proton $\left(\mathrm{H}^{+}\right)$ from oxygen generates acetal.

Question 010 MCQ

QUESTION

Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

A CuS and HgS

B MnS and CuS

C MnS and NiS

D NiS and HgS

CORRECT OPTION

A CuS and HgS

SOURCE

Chemistry • salt-analysis

EXPLANATION

Cu^{2+} and Hg^{2+} ions belong to Group II of inorganic salt analysis and their sulphides are very less soluble in aqueous medium.

Hence, CuS and HgS will be precipitated out even in acidic medium by passing H_2S gas.

Question 011 MCQ

QUESTION

Amongst the compounds given, the one that would from a brilliant coloured dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of

β

-naphthlol is

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Azo dye test is given by aromatic primary amines.

Question 012 MCQ

QUESTION

The following carbohydrate is

A a ketohexose

B an aldohexose

C an α-furanose

D an α-pyransose

CORRECT OPTION

B an aldohexose

SOURCE

Chemistry • biomolecules

EXPLANATION

i The cyclic structure of pyranose ring

This is 3D structure of glucose formed as a result of cyclisation of an aldohexose.

ii There are 2 types of groups in cyclic structure; groups that lie perpendicular to the plane of pyranose ring are axial and other are axial groups.

The fischer form of pyranose ring has groups above the ring on left and groups below the ring on right.

The cyclic form of pyranose ring is obtained as a result of reaction of hydroxyl group $-OH$ at fifth carbon to the electropositive carbon of aldehydic group. The reaction happens as follows :

Question 013 MCQ

QUESTION

The equilibrium

$$2Cu^{+} \rightarrow Cu^{\circ} + Cu^{2+}$$

In aqueous medium at 25

$$^{\circ}$$

C shifts towards the left in the presence of

A NO

$$_{3}^{-}$$

B Cl

$$^{-}$$

C SCN$^-$

D CN$^-$

CORRECT OPTION

B Cl$^-$

SOURCE

Chemistry • chemical-equilibrium

EXPLANATION

For the equilibrium reaction in aqueous medium :

$$2\mathrm{Cu}^{+} \rightleftharpoons \mathrm{Cu}^{0} + \mathrm{Cu}^{2+}$$

i If concentration of copper I is reduced, then according to Le-chatelier's principle the equilibrium reaction will shift backward.

ii The copper II ion reacts with chloride $\left(\mathrm{Cl}^{-}\right)$ ion forming copper III chloride, which reacts with copper to produce the precipitate of CuCl .

$$\mathrm{Cu}^{2+} + 2\mathrm{Cl}^{-} \rightarrow \mathrm{CuCl}_2$$

$$\mathrm{CuCl}_2 + \mathrm{Cu} \rightarrow 2\mathrm{CuCl}$$

Since, the end product involves consumption of copper I ion to form CuCl , hence, reactive shifts backward.

iii The copper II ion also reacts with cyanide $\left(\mathrm{CN}^{-}\right)$ ion forming copper II cyanide which on further reactions form $\left[\mathrm{Cu}(\mathrm{CN})_4\right]^{3-}$ with copper in I oxidation state shifting reaction backward.

$$Cu^{2+} + 2CN^- \rightarrow Cu(CO)_2$$

$$2Cu(CN)_2 \rightarrow 2CuCN + (CN)_2$$

$$CuCN + 3CN^- \rightarrow [Cu(CN)_4]^{3-}$$

iv Reaction of copper II ion with thiocyanide (SCN^-) gives a complex $(Cu(SCN)_4]^{3-}$ with copper in +1 oxidation state. This shifts reaction backward.

$$Cu^{2+} + 4SCN^- \rightarrow [Cu(SCN)_4]^{3-}$$

Hence, cyanide, thiocyanide and chloride forms complexes with copper in I oxidation state. Hence, reaction shifts backward to generate more copper I ions.

Question 014 MCQ

QUESTION

The correct functional group X and the reagent/reaction condition Y in the following scheme are

A X = $COOCH_3$, Y = H_2/Ni/heat

B X = $CONH_2$, Y = H_2/Ni/heat

C X = $CONH_2$, Y = Br_2/NaOH

D X = CN, Y = H_2/Ni/heat

CORRECT OPTION

B X = $CONH_2$, Y = H_2/Ni/heat

SOURCE

Chemistry • polymers

EXPLANATION

$i\ \ a$ Hexa-1, 6-diamide undergoes Hoffmann bromamide degradation to form tetra-1, 4-diammine.

If

$$\mathrm{X} = -\mathrm{CONH}_2$$

$$\mathrm{H_2NOC} - (\mathrm{CH_2})_4 - \mathrm{CONH_2} + 2\mathrm{Br_2} + 4\mathrm{NaOH}(aq) \rightarrow$$

$$\mathrm{H_2\ N} - (\mathrm{CH_2})_4 - \mathrm{NH_2} + 2\mathrm{Na_2CO_3} + 4\mathrm{NaBr}(aq) + 4\mathrm{H_2O}$$

b Hexa-1, 6-diammine reacts with hexa-1, 6-dioic acid to form an amide with release water.

Since, water is lost during the reaction between amine and carboxylic acid, the reaction is condensation reaction.

Option C is correct.

$ii\ \ a$ Hexa-1, 6-dinitrile undergoes reduction to amine in pressure of same hydrogen and nickel as catalyst to form hexane-1, 6-diammine. If $X\ = \mathrm{CN}$

$$\mathrm{NC} - (\mathrm{CH_2})_4 - \mathrm{CN} \xrightarrow[\mathrm{Ni(\ s)}]{\mathrm{H_2(\ g)}} \quad \mathrm{H_2\ N} - (\mathrm{CH_2})_6 - \mathrm{NH_2}\ (\text{hexane-1, 6-diammine}$$

b Hexane-1, 6-diammine reacts with hexane-1, 4-dioic acid to form an amide with the loss of water molecule. This reaction is condensation reaction.

Since, water molecule is lost during the reaction between amine and carboxylic acid the reaction is condensation reaction.

Option D is correct.

iii The compound contains carbonyl functional group of ester and cannot be reduced by hydrogen in presence of nickel. Hence, no condensation reaction is possible between ester and an acid.

If $\mathrm{X} = \mathrm{COOCH_3}$

$$\mathrm{H_3COOC-(CH_2)_4-COOCH_3 + H_{2(g)} \xrightarrow{Ni(s)}}$$

No reaction

$$\mathrm{H_3COOC-(CH_2)_4-COOCH_3\ (hexan\text{-}1,4\text{-}dioic\ acid)\ + HOOC-(CH_2)}$$

Option A is not correct.

iv Hexan-1, 4-diamide is not affected by the reducing agent hydrogen in presence of catalyst nickel. Reaction between the hexan-1, 4-diamine with basic functional group $(-\mathrm{NH_2})$ and acidic proton of hexane-1, 4-dioic acid gives salt.

Option B is correct.

Question 015 MCQ

QUESTION

Reduction of the metal centre in aqueous permanganate ion involves

A 3 electrons in neutral medium.

B 5 electrons in neutral medium.

C 3 electrons in alkaline medium.

D 5 electrons in acidic medium.

CORRECT OPTION

A 3 electrons in neutral medium.

SOURCE

Chemistry • redox-reactions

EXPLANATION

In an aqueous solution, the reduction of the metal center in permanganate ions $\$MnO_4^-\$$ varies depending on the medium :

Acidic Medium : Permanganate acts as a strong oxidizing agent. The reaction can be represented as :

$$8H^+ + 5e^- + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$$

This indicates that 5 electrons are involved in the reduction process in an acidic medium.

Neutral Medium : Permanganate acts as a moderate oxidizing agent. The reaction in a neutral medium is :

$$2H_2O + 3e^- + MnO_4^- \rightarrow MnO_2 + 4OH^-$$

Here, 3 electrons are involved in the reduction process in a neutral medium.

Alkaline Medium : In alkaline conditions, the reaction is similar to that in a neutral medium :

$$2H_2O + 3e^- + MnO_4^- \rightarrow MnO_2 + 4OH^-$$

Again, 3 electrons are involved in the reduction process in an alkaline medium.

Question 016 Numerical

QUESTION

The total number of contributing structure showing hyper-conjugation $involving C-H bonds$ for the following carbocation is _____.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

i Hyper conjugation involves the electrons of $\mathrm{C}-\mathrm{H}$ sigma $\$\sigma\$$ bond, of alkyl group is delocalised with an atom containing empty p-orbital $i.e., a carbocation$ or unsaturated system.

ii For the given carbocation, hypercojugation involves the delocalisation of $\mathrm{C}-\mathrm{H}$ sigma $\$\sigma\$$ bond, of alkyl group with the adjacent atom containing unshared p-orbital.

iii The different kinds of hydrogen that will be involved in hyperconjugation are as follows :

There are three different kinds of hydrogen; $\mathrm{H}_{a'} \mathrm{H}_b$ and H_c.

iv Hyperconjugative structure due to different kind of hydrogens are as follows :

a Hyperconjugation due to $\mathrm{C}-\mathrm{H}_a$ sigma $or \$\sigma\$$ bond.

Similarly, two more hyperconjugative structures are possible due to two other hydrogen atoms.

A total of three hyperconjugative structures of carbocation are possible due to delocalisation of $\mathrm{C}-\mathrm{H}_a$ $\$\sigma\$ bond$ with empty p-orbital $on carbocation$.

b Hyperconjugation due to $\mathrm{C}-\mathrm{H}_b$ sigma $or \$\sigma\$$ bond.

Similarly, one more hyperconjugative structure is possible due to other hydrogen (H_b). A total of two hyperconjugative structures of carbocations are possible due to delocalisation of $\mathrm{C}-\mathrm{H}_b$ $\$\sigma\$bond$ with empty p-orbital $oncarbocation$.

c Hyperconjugation due to $\mathrm{C}-\mathrm{H}_c$ sigma $or\$\sigma\$$ bond.

There is only one H^c hydrogen; hence, one hyperconjugative structure is possible due to delocalisation of $\mathrm{C}-\mathrm{H}_c$ $\$\sigma\$bond$ with empty p-orbital $oncarbocation$.

Question 017 Numerical

QUESTION

Among the following, the number of compounds that can react with PCl_5 to given $POCl_3$ is _____.

O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}

SOURCE

Chemistry • p-block-elements

EXPLANATION

The following reactions show how PCl_5 reacts with different compounds to form $POCl_3$:

$$CO_2 + PCl_5 \rightarrow POCl_3 + COCl_2$$

$$H_2O + PCl_5 \rightarrow POCl_3 + 2HCl$$

$$SO_2 + PCl_5 \rightarrow POCl_3 + SOCl_2$$

$$P_4O_{10} + 6PCl_5 \rightarrow 10POCl_3$$

$$H_2SO_4 + PCl_5 \rightarrow POCl_3 + HSO_3Cl + HCl$$

From these reactions, it is clear that CO_2, H_2O, SO_2, P_4O_{10}, and H_2SO_4 can react with PCl_5 to give $POCl_3$.

There are 5 compounds that meet this criterion.

Question 018 Numerical

QUESTION

The volume $inmL$ of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of

$$[Cr(H_2O)_5Cl]Cl_2$$

, as silver chloride is close to _____.

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

The reaction taking place is

$$2AgNO_3 + [Cr(H_2O)_5Cl]Cl_2 \rightarrow 2AgCl + [Cr(H_2O)_5Cl](NO_3)_2$$

Using molarity equation

$$(M \times n \times V)_{AgNO_3} = (M \times n \times V)_{[Cr(H_2O)_5Cl]Cl_2}$$

$$0.1 \times 1 \times V = 0.01 \times 2 \times 30 \Rightarrow V = 6$$

Question 019 Numerical

QUESTION

The maximum number of isomers $including stereoisomers$ that are possible on mono-chlorination of the following compound, is _____.

SOURCE

Chemistry • hydrocarbons

EXPLANATION

i The monochlorination products obtained depends upon the kinds of hydrogen present in the molecule.

ii There are four different types of hydrogen in the molecule; $\mathrm{H}_a, \mathrm{H}_b, \mathrm{H}_c$ and H_d.

iii The monochlorination products obtained on replacement of these hydrogen are as follows :

a Replacing H_a with chlorine :

Since, the carbon is a chiral; only one monochlorination isomer is possible when H_a is replaced by chlorine.

b Replacing H_b with chlorine :

Since all the carbons are a chiral; only one monochlorination product is possible by when H_b replaced by Cl .

c Replacing H_c with chlorine

The carbon on which hydrogen is replaced by chlorine becomes chiral; hence R/S isomers exist.

Also, carbon adjacent to the carbon on which hydrogen (H_c) is replaced, is also chiral; hence, it will also exist as a pair of enantiomer R/S four monochlorination products $which are stereoisomers or optical isomers$ are possible.

d Replacing H^d by chlorine :

The central carbon $(^*)$ becomes chiral when H^d is replaced by chlorine; hence R/S isomers exist. Two monochlorination products are possible $stereoisomers or optical isomers$ are possible.

Total number of isomers possible are 8.

Question 020 MCQ

QUESTION

Match the reactions in Column I with appropriate types of steps/reactive intermediate involved in these reactions as given in Column II :

Column I		Column II	
A		P	Nucleophilic substitution
B		Q	Electrophilic substitution
C		R	Dehydration
D		S	Nucleophilic
		T	Carbanion

$A \rightarrow R, S, T; B$

$\rightarrow$

A $P;\ C$

$\rightarrow$

$R,\ S;\ D$

$\rightarrow$

Q

A

$\rightarrow$

$R,\ S,\ T;\ B$

$\rightarrow$

B $P,\ S;\ C$

$\rightarrow$

$R,\ S;\ D$

$\rightarrow$

$Q,\ R$

A

$\rightarrow$

$R,\ S,\ T;\ B$

$\rightarrow$

C $P,\ S;\ C$

$\rightarrow$

$R;\ D$

$\rightarrow$

$Q,\ R$

D $A \rightarrow R, S, T; B \rightarrow P, S; C \rightarrow R, S; D \rightarrow Q$

CORRECT OPTION

B $A \rightarrow R, S, T; B \rightarrow P, S; C \rightarrow R, S; D \rightarrow Q, R$

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

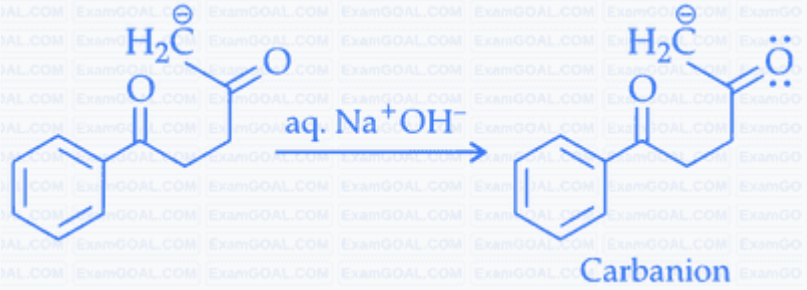
A

The compound undergoes an intramolecular aldol condensation reaction in the presence of aqueous NaOH.

Steps Involved :

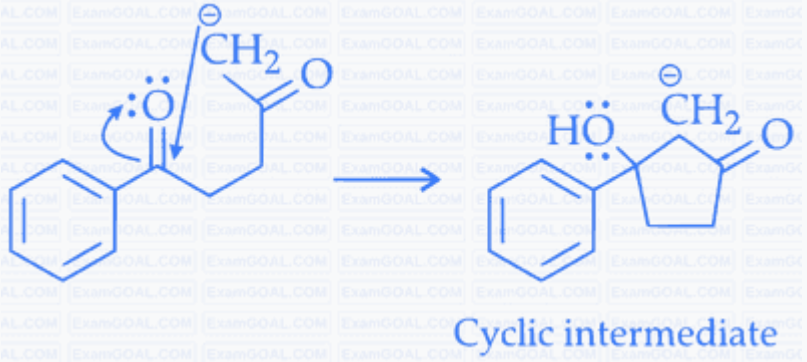
Abstraction of Proton :

The proton attached to the alpha carbon is abstracted, generating a carbanion.



Nucleophilic Addition :

The carbanion attacks the electropositive carbonyl carbon, forming a cyclic intermediate.



Dehydration :

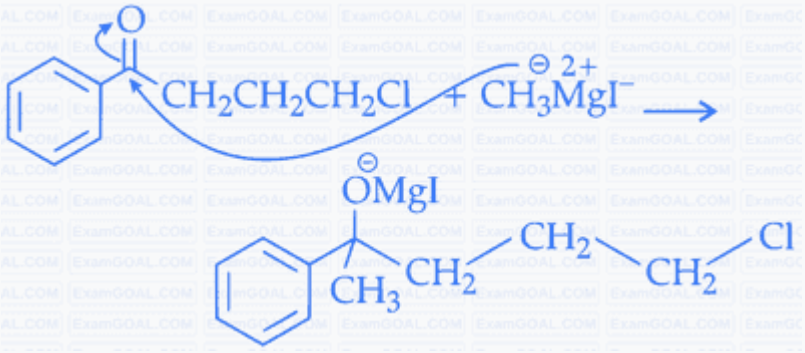
The cyclic intermediate undergoes dehydration to yield the final product.

Option A in Column I matches with R, S, and T in Column II.

B

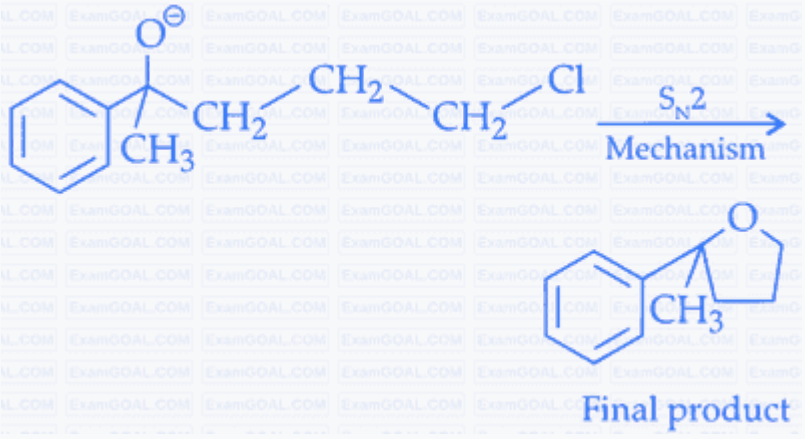
Nucleophilic Addition :

The ketone reacts with methyl magnesium bromide, forming a secondary alcohol. This is a nucleophilic addition reaction.



Nucleophilic Substitution :

The highly nucleophilic oxygen attacks the C_1 carbon of the alkyl chain from the back, displacing chlorine to form the final product.

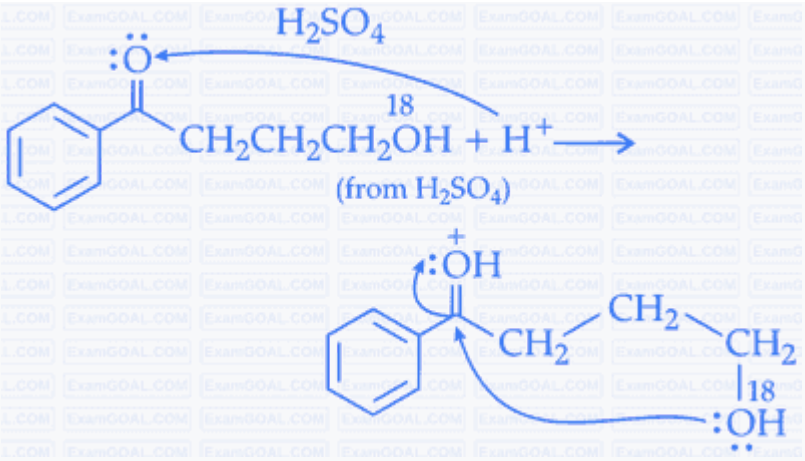


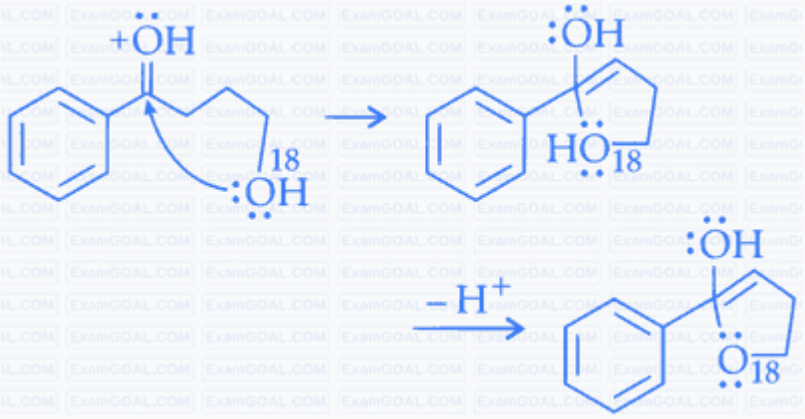
Option B in Column I matches with P and S in Column II.

C

Nucleophilic Addition :

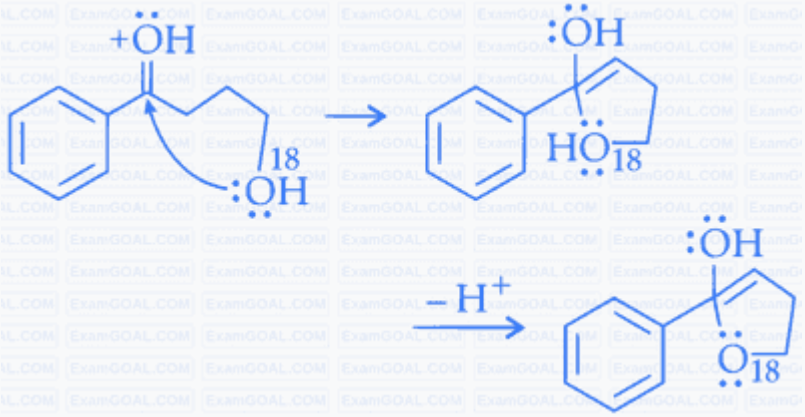
The compound undergoes a nucleophilic addition reaction in the presence of H_2SO_4.





Dehydration :

The molecule loses water, forming an alkene.

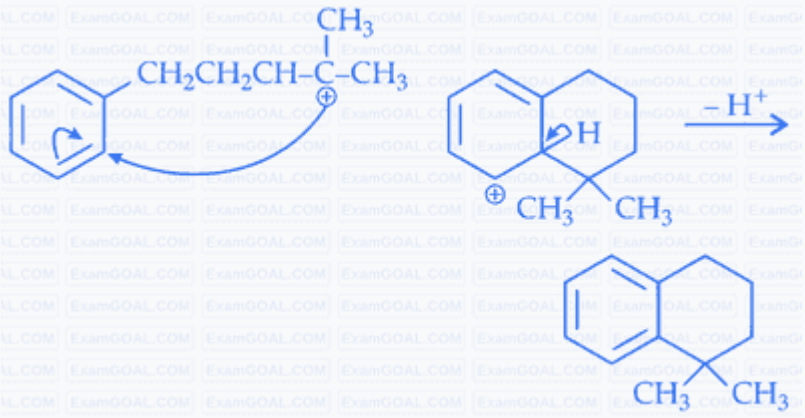


Option C in Column I matches with R and S in Column II.

D

The compound reacts with sulfuric acid to form a carbocation, which undergoes electrophilic substitution.

Reaction :



Option D in Column I matches with Q and R in Column II.

Question 021 Numerical

QUESTION

The number of distinct real roots of

$$x^4 - 4x^3 + 12x^2 + x - 1 = 0$$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

Let

$$f(x) = x^4 - 4x^3 + 12x^2 + x - 1 = 0$$

$$f'(x) = 4x^3 - 12x^2 + 24x + 1 = 4(x^3 - 3x^2 + 6x) + 1$$

$$f''(x) = 12x^2 - 24x + 24 = 12(x^2 - 2x + 2)$$

f'x has 0 real roots.

fx has maximum two distinct real roots as f0 =

$$-$$

1.

Question 022 Numerical

QUESTION

Let

$$\omega = e^{\frac{i\pi}{3}}$$

, and a, b, c, x, y, z be non-zero complex numbers such that

$$a + b + c = x$$

$$a + b\omega + c\omega^2 = y$$

$$a + b\omega^2 + c\omega = z$$

Then the value of

$$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$$

is

SOURCE

Mathematics • complex-numbers

EXPLANATION

The expression may not attain integral value for all a, b, c.

If we consider a = b = c, then

x = 3a

y = a(1 +

$$\omega$$

+

$$\omega$$

^{2}) = a$1 + i\$\$\sqrt{3}\$\$$

z = a(1 +

$$\omega$$

^{2} +

$$\omega$$

) = a$1 + i\$\$\sqrt{3}\$\$$

Therefore,

$$|x|^2 + |y|^2 + |z|^2 = 9|a|^2 + 4|a|^2 + 4|a|^2 = 17|a|^2$$

Hence,

$$\frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = \frac{17}{13}$$

Note : However, if

$$\omega = e^{i(2\pi/3)}$$

, then the value of the expression is 3.

Question 023 MCQ

QUESTION

A value of

$$b$$

for which the equations

$$\$ x^2 + bx - 1 = 0$$
$$x^2 + x + b = 0$$

$

have one root in common is

A $-\sqrt{2}$

B $-i\sqrt{3}$

C $i\sqrt{5}$

D $\sqrt{2}$

CORRECT OPTION

B $-i\sqrt{3}$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

The given equations are

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0$$

..... 1

Common root is

$$(b-1)x - 1 - b = 0$$

.

$$\Rightarrow x = \frac{b+1}{b-1}$$

This value of x satisfies Eq. 1, we get

$$\frac{(b+1)^2}{(b-1)^2} + \frac{b+1}{b-1} + b = 0$$

$$\Rightarrow b = i\sqrt{3},\ -i\sqrt{3},\ 0$$

Question 024 MCQ

QUESTION

The circle passing through the point $-1, 0$ and touching the y-axis at $0, 2$ also passes through the point.

A $\left(-\frac{3}{0}, 0\right)$

B $\left(-\frac{5}{2}, 2\right)$

C $\left(-\frac{3}{0}, \frac{5}{2}\right)$

D $-4, 0$

CORRECT OPTION

D $-4, 0$

SOURCE

Mathematics • circle

EXPLANATION

Circle touching y-axis at $0, 2$ is

$$(x-0)^2 + (y-2)^2 + \lambda x = 0$$

passes through $\$\$ - \$\$1, 0.$

Therefore,

$$1 + 4 - \lambda = 0 \Rightarrow \lambda = 5$$

Hence,

$$x^2 + y^2 + 5x - 4y + 4 = 0$$

Substituting

$$y = 0 \Rightarrow x = -1, -4$$

.

Hence, the Circle passes through $\$\$ - \$\$4, 0$.

Question 025 Numerical

QUESTION

The straight line 2x - 3y = 1 divides the circular region

$$x^2 + y^2 \leq 6$$

into two parts.
If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4} - \frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\}$$

then the number of points s in S lying inside the smaller part is

SOURCE

Mathematics • circle

EXPLANATION

$$L : 2x - 3y - 1$$

$$S : x^2 + y^2 - 6$$

If

$$L_1 > 0$$

and

$$S_1 < 0$$

The point lies in the smaller part. Therefore,

$$\left(2, \frac{3}{4}\right)$$

and

$$\left(\frac{1}{4}, -\frac{1}{4}\right)$$

lie inside.

Question 026 MCQ

QUESTION

Let

$$(x, y)$$

be any point on the parabola

$$y^2 = 4x$$

. Let

$$P$$

be the point that divides the line segment from

$$(0, 0)$$

to

$$(x, y)$$

in the ratio

$$1 : 3$$

. Then the locus of

$$P$$

is

A $x^2 = y$

B $y^2 = 2x$

C $y^2 = x$

D $x^2 = 2y$

CORRECT OPTION

C $y^2 = x$

SOURCE

Mathematics • parabola

EXPLANATION

Now,

$$y^2 = 4x$$

and Q will lie on it.

$$\Rightarrow (4k)^2 = 4 \times 4h$$

$$\Rightarrow k^2 = h$$

$$\Rightarrow y^2 = x$$

$replacinghbyxandkbyy$

Question 027 MCQ

QUESTION

Let

$$P(6,3)$$

be a point on the hyperbola

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

. If the normal at the point

$$P$$

intersects the

$$x$$

-axis at

$$(9,0)$$

, then the eccentricity of the hyperbola is

A $\sqrt{\frac{5}{2}}$

B $\sqrt{\frac{3}{2}}$

C $\sqrt{2}$

D $\sqrt{3}$

CORRECT OPTION

B $\sqrt{\frac{3}{2}}$

SOURCE

Mathematics • hyperbola

EXPLANATION

Equation of normal is

$$(y-3)=\frac{-a^2}{2b^2}(x-6)\Rightarrow\frac{a^2}{2b^2}=1\Rightarrow e=\sqrt{\frac{3}{2}}$$

Question 028 MCQ

QUESTION

Let f

$$:$$

$$[-1,2]\to[0,\infty]$$

be a continuous function such that

$$f(x)=f(1-x)$$

for all

$$x \in [-1, 2]$$

Let

$$R_1 = \int_{-1}^{2} xf(x)dx,$$

and

$$R_2$$

be the area of the region bounded by

$$y = f(x),$$

$$x = -1,$$

$$x = 2,$$

and the

$$x$$

-axis. Then

A $R_1 = 2R_2$

B $R_1 = 3R_2$

C $2R_1 = R_2$

D $3R_1 = R_2$

CORRECT OPTION

C

$$2R_1 = R_2$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

$$R_1 = \int_{-1}^{2} xf(x)dx = \int_{-1}^{2} (2-1-x)f(2-1-x)dx$$

$$= \int_{-1}^{2} (1-x)f(1-x)dx = \int_{-1}^{2} (1-x)f(x)dx$$

Hence,

$$2R_1 = \int_{-1}^{2} f(x)dx = R_2$$

.

Question 029 MCQ

QUESTION

Let

$$E$$

and

$$F$$

be two independent events. The probability that exactly one of them occurs is

$$\frac{11}{25}$$

and the probability of none of them occurring is

$$\frac{2}{25}$$

. If

$$P(T)$$

denotes the probability of occurrence of the event

$$T,$$

then

A $$P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$$

B $$P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$$

C $$P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$$

D $$P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$$

CORRECT OPTION

A

$$P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$$

SOURCE

Mathematics • probability

EXPLANATION

Let

$$P(E)=e$$

and

$$P(F)=f$$

.

$$P(E \cup F)-P(E \cap F)=\frac{11}{25}$$

$$\Rightarrow e+f-2ef=\frac{11}{25}$$

..... 1

$$P(\overline{E} \cap \overline{F})=\frac{2}{25}$$

$$\Rightarrow (1-e)(1-f)=\frac{2}{25}$$

$$\Rightarrow 1-e-f+ef=\frac{2}{25}$$

..... 2

From Eqs. 1 and 2, we get

$$ef=\frac{12}{25}$$

and

$$e+f=\frac{7}{5}$$

Solving, we get

$$e=\frac{4}{5}, f=\frac{3}{5}$$

or

$$e=\frac{3}{5}, f=\frac{4}{5}$$

Question 030 MCQ

QUESTION

Match the statements given in Column -

$$I$$

with the values given in Column-

$$II.$$

Column-

$$I$$

$$A$$

If

$$\overrightarrow{a}=\hat{j}+\sqrt{3}\hat{k}, \overrightarrow{b}=-\hat{j}+\sqrt{3}\hat{k}$$

and

$$\overrightarrow{c} = 2\sqrt{3}\widehat{k}$$

form a triangle, then the internal angle of the triangle between

$$\overrightarrow{a}$$

and

$$\overrightarrow{b}$$

is
B

If

$$\int_{a}^{b} (f(x) - 3x)dx = a^2 - b^2,$$

then the value of

$$f$$

$$\left(\frac{\pi}{6}\right)$$

is
C

The value of

$$\frac{\pi^2}{\ell n 3} \int_{7/6}^{5/6} \sec(\pi x)dx$$

is
D

The maximum value of

$$\left|Arg\left(\frac{1}{1-z}\right)\right|$$

for

$$|z|=1,\, z\neq 1$$

is given by

Column-

$$II$$

p

$$\frac{\pi}{6}$$

q

$$\frac{2\pi}{3}$$

r

$$\frac{\pi}{3}$$

s

$$\pi$$

t

$$\frac{\pi}{2}$$

A $(A) \to q;\ (B) \to p;\ (C) \to s;\ (D) \to t$

B $(A) \to q;\ (B) \to p;\ (C) \to t;\ (D) \to s$

C $(A) \to p;\ (B) \to q;\ (C) \to s;\ (D) \to t$

D $(A) \to q;\ (B) \to s;\ (C) \to p;\ (D) \to t$

CORRECT OPTION

A $(A) \to q;\ (B) \to p;\ (C) \to s;\ (D) \to t$

SOURCE

Mathematics • vector-algebra

EXPLANATION

A We have,

$$\vec{a} - \vec{b} = -1 + 3 = 2$$

, where

$$|\vec{a}| = 2$$

and

$$\overrightarrow{|b|} = 2$$

.

$$\cos\theta = \frac{2}{2\times 2} = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$$

; however, it is

$$\frac{2\pi}{3}$$

as its opposite to side of maximum length.

B

$$\int_a^b (f(x) - 3x)dx = a^2 - b^2$$

$$\int_a^b f(x)dx = \frac{3}{2}(b^2 - a^2) + a^2 - b^2 = \frac{-a^2 + b^2}{2} \Rightarrow f(x) = x$$

Therefore,

$$f(\pi/6) = (\pi^2/6)$$

C

$$\frac{\pi^2}{\ln 3}\left(\frac{\ln|(\sec\pi x + \tan\pi x)|_{7/6}^{5/6}}{\pi}\right)$$

$$= \frac{\pi}{\ln 3}\left(\ln\left|\sec\frac{5\pi}{6} + \tan\frac{5\pi}{6}\right| - \ln\left|\sec\frac{7\pi}{6} + \tan\frac{7\pi}{6}\right|\right) = \pi$$

D Let us consider,

$$u = \frac{1}{1-z} \Rightarrow z = 1 - \frac{1}{u}$$

$$|z| = 1 \Rightarrow \left|1 - \frac{1}{u}\right| = 1 \Rightarrow |u - 1| = |u|$$

Hence, the locus of u is perpendicular bisector of line segment joining 0 and 1.

Therefore, the maximum arg u approaches

$$\pi$$

/2, but it will not attain.

Question 031 Numerical

QUESTION

Let

$$\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$$

and

$$\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$$

be three given vectors. If

$$\vec{r}$$

is a vector such that

$$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$$

and

$$\vec{r}.\vec{a} = 0,$$

then the value of

$$\vec{r}.\vec{b}$$

is

SOURCE

Mathematics • vector-algebra

EXPLANATION

Since it is given that

$$\overrightarrow{r} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{b}$$

, taking cross product with

$$\overrightarrow{a} \times (\overrightarrow{r} \times \overrightarrow{b}) = \overrightarrow{a} \times (\overrightarrow{c} \times \overrightarrow{b})$$

$$(\overrightarrow{a} . \overrightarrow{b})\overrightarrow{r} - (\overrightarrow{a} . \overrightarrow{r})\overrightarrow{b} = \overrightarrow{a} \times (\overrightarrow{c} \times \overrightarrow{b})$$

$$\Rightarrow \overrightarrow{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\overrightarrow{r} . \overrightarrow{b} = 3 + 6 = 9$$

Question 032 Numerical

QUESTION

Let

$$y'(x) + y(x)g'(x) = g(x), g'(x), y(0) = 0, x \in R,$$

where

$$f'(x)$$

denotes

$$\frac{df(x)}{dx}$$

and

$$g(x)$$

is a given non-constant differentiable function on

$$R$$

with

$$g(0) = g(2) = 0.$$

Then the value of

$$y(2)$$

is

SOURCE

Mathematics • differential-equations

EXPLANATION

It is given that

$$y'(x) + y(x)g'(x) = g(x)g'(x)$$

$$\Rightarrow e^{g(x)}y'(x) + e^{g(x)}g'(x)y(x) = e^{g(x)}g(x)g'(x)$$

$$\Rightarrow \frac{d}{dx}(y(x)e^{g(x)}) = e^{g(x)}g(x)g'(x)$$

Therefore,

$$y(x) = e^{g(x)} = \int e^{g(x)}g(x)g'(x)dx$$

$$= \int e^t t\, dt$$

$$where g(x) = t$$

$$= (t-1)e^t + c$$

Therefore,

$$y(x)e^{g(x)} = (g(x) - 1)e^{g(x)} + c$$

Substituting

$$x = 0 \Rightarrow 0 = (0 - 1) \times 1 + c \Rightarrow c = 1$$

Substituting

$$x = 2 \Rightarrow y(2) \times 1 = (0 - 1) \times (1) + 1$$

Hence,

$$y(2) = 0$$

.

Question 033 MCQ

QUESTION

If

$$\lim_{x \to 0} [1 + x \ln(1 + b^2)]^{1/x} = 2b\sin^2\theta$$

,

$$b > 0$$

and

$$\theta \in (-\pi, \pi]$$

, then the value of

$$\theta$$

is

A $\pm\frac{\pi}{4}$

B $\pm\frac{\pi}{3}$

C $\pm\frac{\pi}{6}$

D $\pm\frac{\pi}{2}$

CORRECT OPTION

D $\pm\frac{\pi}{2}$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Here,

$$\lim_{x\to 0}\{1+x\log(1+b^2)\}^{1/x}$$

[1

$$\infty$$

from]

$$\Rightarrow e^{\lim_{x\to 0}\{x\log(1+b^2)\}\cdot\frac{1}{x}}$$

$$\Rightarrow e^{\log(1+b^2)}=(1+b)^2$$

..... i

Given,

$$\lim_{x \to 0} \{1 + x \log(1 + b^2)\}^{1/x} = 2b\sin^2\theta$$

$$\Rightarrow (1 + b^2) = 2b\sin^2\theta$$

$$\therefore$$

$$\sin^2\theta = \frac{1 + b^2}{2b^2}$$

..... ii

By

$$AM \geq GM$$

$$\frac{b + \frac{1}{b}}{2} \geq \left(b \,.\, \frac{1}{b}\right)^{1/2} \Rightarrow \frac{b^2 + 1}{2b} \geq 1$$

..... iii

From Eqs. ii and iii, we get

$$\sin^2\theta = 1$$

$$\Rightarrow \theta = \pm\frac{\pi}{2}$$

as

$$\theta \in (-\pi, \pi]$$

Question 034 MCQ

QUESTION

Let fx = x^2 and gx = sin x for all x

$$\in$$

R. Then the set of all x satisfying

$$(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$$

, where

$$(f \circ g)(x) = f(g(x))$$

, is

A $\pm\sqrt{n\pi},\ n \in \{0, 1, 2, \dots\}$

B $\pm\sqrt{n\pi},\ n \in \{1, 2, \dots\}$

C $\frac{\pi}{2} + 2n\pi,\ n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

D $2n\pi, n \in \{\dots, -2, -1, 0, 1, 2, \dots\}$

CORRECT OPTION

A $\pm\sqrt{n\pi},\ n \in \{0, 1, 2, \dots\}$

SOURCE

Mathematics • functions

EXPLANATION

$$f(x) = x^2$$

,

$$g(x) = \sin x$$

$$(g \circ f)(x) = \sin x^2$$

$$g \circ (g \circ f)(x) = \sin(\sin x^2)$$

$$(f \circ g \circ g \circ f)(x) = (\sin(\sin x^2))^2$$

..... i

Again,

$$(g \circ f)(x) = \sin x^2$$

$$(g \circ g \circ f)(x) = \sin(\sin x^2)$$

..... ii

Given,

$$(f \circ g \circ g \circ f)(x) = (g \circ g \circ f)(x)$$

$$\Rightarrow (\sin(\sin x^2))^2 = \sin(\sin x^2)$$

$$\Rightarrow \sin(\sin x^2)\{\sin(\sin x^2) - 1\} = 0$$

$$\Rightarrow \sin(\sin x^2) = 0$$

or

$$\sin(\sin x^2) = 1$$

$$\Rightarrow \sin x^2 = 0$$

or

$$\sin x^2 = \frac{\pi}{2}$$

$$\therefore$$

$$x^2 = n\pi$$

$i.e. not possible as \$\$ -1 \leq \sin\theta \leq 1 \$\$$

$$x = \pm\sqrt{n\pi}$$

Question 035 MCQ

QUESTION

Let

$$\omega$$

$$\neq$$

1 be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$$

, where each of a, b, and c is either

$$\omega$$

or

$$\omega$$

^{2}. Then the number of distinct matrices in the set S is

A 2

B 6

C 4

D 8

CORRECT OPTION

A 2

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

$$|A| \neq 0$$

, as non-singular.

$$\therefore$$

$$\begin{vmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix} \neq 0$$

$$\Rightarrow 1(1 - c\omega) - a(\omega - c\omega^2) + b(\omega^2 - \omega^2) \neq 0$$

$$\Rightarrow 1 - c\omega - a\omega + ac\omega^2 \neq 0$$

$$\Rightarrow (1 - c\omega)(1 - a\omega) \neq 0$$

$$\Rightarrow a \neq \frac{1}{\omega}, c \neq \frac{1}{\omega} \Rightarrow a = \omega, c = \omega$$

and

$$b \in \{\omega, \omega^2\} \Rightarrow 2$$

solutions

Question 036 MCQ

QUESTION

If

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \leq -\frac{\pi}{2} \\ -\cos x & -\frac{\pi}{2} < x \leq 0 \\ x - 1 & 0 < x \leq 1 \\ \ln x & x > 1 \end{cases}$$

, then

fx is continuous at x =

A

$$-$$

$$\pi$$

/2.

B fx is not differentiable at x = 0.

C fx is differentiable at x = 1.

fx is differentiable at x =

D

$$-$$

3/2.

CORRECT OPTION

fx is continuous at x =

A

$$-$$

$$\pi$$

/2.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$\lim_{x \to \frac{\pi^-}{2}} f(x) = 0 = f(-\pi/2)$$

$$\lim_{x \to \frac{\pi^+}{2}} f(x) = \cos\left(-\frac{\pi}{2}\right) = 0$$

$$f'(x) = \begin{cases} -1, & x \leq -\pi/2 \\ \sin x, & -\pi/2 < x \leq 0 \\ 1, & 0 < x \leq 1 \\ 1/x, & x > 1 \end{cases}$$

Clearly, fx is not differentiable at x = 0 as f'(0

$$-$$

) = 0 and f'(0^+) = 1.

fx is differentiable at x = 1 as

$$f'(1^{-1}) = f'(1^+) = 1$$

.

Question 037 MCQ

QUESTION

Let

$$f : (0, 1) \to R$$

be defined by

$$f(x) = \frac{b - x}{1 - bx}$$

, where b is a constant such that

$$0 < b < 1$$

. Then

A f is not invertible on $0, 1$.

B f $\neq$ f $-$ ^{1} on $0, 1$ and

$$f'(b) = \frac{1}{f'(0)}$$

.

C f = f $-$ ^{1} on $0, 1$ and

$$f'(b) = \frac{1}{f'(0)}$$

.

D f $-$ ^{1} is differentiable on $0, 1$.

CORRECT OPTION

A f is not invertible on $0, 1$.

SOURCE

Mathematics • functions

EXPLANATION

Here,

$$f(x) = \frac{b - x}{1 - bx}$$

where, 0 < b < 1, 0 < x < 1

For function to be invertible it should be one-one onto.

$$\therefore$$

Check range :

Let

$$f(x) = y \Rightarrow y = \frac{b - x}{1 - bx}$$

$$\Rightarrow y - bxy = b - x \Rightarrow x(1 - by) = b - y$$

$$\Rightarrow x = \frac{b - y}{1 - by}$$

where, 0 < x < 1

$$\therefore$$

$$0 < \frac{b - y}{1 - by} < 1$$

$$\frac{b - y}{1 - by} > 0$$

and

$$\frac{b - y}{1 - by} < 1$$

$$\Rightarrow y < b$$

or

$$y > \frac{1}{b}$$

.... i

$$\frac{(b-1)(y+1)}{1-by} < -1 < y < \frac{1}{b}$$

..... ii

From Eqs. i and ii, we get

$$y \in \left(-1, \frac{1}{b}\right) \subset$$

Codomain

Thus, fx is not invertible.

Question 038 MCQ

QUESTION

Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $9, 6$, then L is given by

A

y

—

x + 3 = 0

B

y + 3x

—

33 = 0

C

$y + x$

$$\frac{}{15 = 0}$$

D

7

$$\frac{}{2x + 12 = 0}$$

CORRECT OPTION

A

y

$$\frac{}{x + 3 = 0}$$

SOURCE

Mathematics • parabola

EXPLANATION

The equation of normal is

$y = mx$

$$\frac{}{2m}$$

$$\frac{}{m^3}$$

As $9, 6$ lies on it, $6 = 9m$

$$\frac{}{2m}$$

$m^3 3$

—

7m + 6 = 0

The roots are m = 1, 2,

—

3. So the normal are

y = x

—

3, y = 2x

—

12, y =

—

3x + 33.

Question 039 Numerical

QUESTION

Let M be a 3

×

3 matrix satisfying

$$M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

,

$$M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

and

$$M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

. Then the sum of the diagonal entries of M is _____.

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Let

$$M=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$$

$$M=\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}\Rightarrow b=-1,\ e=2,\ h=3$$

$$M=\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}\Rightarrow a=0,\ d=3,\ g=2$$

$$M=\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}\Rightarrow g+h+i=12\Rightarrow i=7$$

Hence, the sum of diagonal elements is 9.

Question 040 MCQ

QUESTION

Match the statements given in Column I with the intervals/union of intervals given in Column II :

A

$A \rightarrow S, B \rightarrow T, C \rightarrow P, D \rightarrow Q$

B

$A \rightarrow S, B \rightarrow T, C \rightarrow R, D \rightarrow P$

A

$\rightarrow$

S, B

$\rightarrow$

C T, C

$\rightarrow$

R, D

$\rightarrow$

R

A

$\rightarrow$

P, B

$\rightarrow$

D Q, C

$\rightarrow$

R, D

$\rightarrow$

R

CORRECT OPTION

A

$\rightarrow$

S, B

$\rightarrow$

C T, C

$\rightarrow$

R, D

$\rightarrow$

R

SOURCE

Mathematics • functions

EXPLANATION

A

$$z = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{2i(x+iy)}{1-(x^2-y^2+2ixy)}$$

Using

$$1-x^2 = y^2$$

, we get

$$Z = \frac{2ix-2y}{2y^2-2ixy} = -\frac{1}{y}$$

Since

$$-1 \le y \le 1 \Rightarrow -\frac{1}{y} \le -1$$

or

$$-\frac{1}{y} \ge 1$$

.

B For domain :

$$-1 \le \frac{8(3^{x-2})}{1-3^{2(x-1)}} \le 1 \Rightarrow -1 \le \frac{3^x-3^{x-2}}{1-3^{2x-2}} \le 1$$

Case 1 :

$$\frac{3^x - 3^{x-2}}{1 - 3^{2x-2}} - 1 \leq 0$$

.

$$\Rightarrow \frac{(3^x - 1)(3^{x-2} - 1)}{(3^{2x-2} - 1)} \geq 0$$

$$\Rightarrow x \in (-\infty, 0] \cup (1, \infty)$$

Case 2 :

$$\frac{3^x - 3^{x-2}}{1 - 3^{2x} - 2} + 1 \geq 0$$

$$\Rightarrow \frac{(x^{x-2} - 1)(3^x + 1)}{(3^x \,.\, 3^{x-2} - 1)} \geq 0$$

$$\Rightarrow x \in (-\infty, 1) \cup [2, \infty)$$

So,

$$x \in (-\infty, 0] \cup [2, \infty)$$

.

C R_1

$$\rightarrow$$

$R_1 + R_3$:

$$f(\theta) = \begin{vmatrix} 0 & 0 & 2 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix} = 2(\tan^2\theta + 1) = 2\sec^2\theta$$

D

$$f'(x) = \frac{3}{2}(x)^{1/2}(3x - 10) + (x)^{3/2} \times 3 = \frac{15}{2}(x)^{1/2}(x - 2)$$

Increasing, when

$$x \geq 2$$

.

Question 041 MCQ

QUESTION

Which of the following statement s is/are correct?

A If the electric field due to a point charge varies as

$$r^{-2.5}$$

instead of

$$r^{-2},$$

then the Gauss law will still be valid.

B The Gauss law can be used to calculate the field distribution around an electric dipole.

C If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

D The work done by the external force in moving a unit positive charge from point

$$A$$

at potential

$$V_A$$

to point

$$B$$

at potential

$$V_B$$

is

$$(V_B - V_A).$$

CORRECT OPTION

C If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

SOURCE

Physics • electrostatics

EXPLANATION

Gauss's law is valid only if Coulomb's law holds, i.e. if

$$E \propto r^{-2}$$

. Hence, choice a is wrong. Gauss's law cannot be used to calculate a non-uniform field distribution around an electric dipole. So choice b is also wrong.

Choice c is correct because the directions of electric fields are opposite at a point between two similar charges.

Work done

$$W_{A \to B} = q(V_B - V_A) = (V_B - V_A)$$

$$\because q = +1\, C$$

Hence, choice d is correct.

So the correct choices are c and d.

Question 042 MCQ

QUESTION

A wooden block performs

$$SHM$$

on a frictionless surface with frequency,

$$v_0.$$

The block carries a charge

$$+Q$$

on its surface . If now a uniform electric field

$$\vec{E}$$

is switched- on as shown, then the

$$SHM$$

of the block will be

A of the same frequency and with shifted mean position.

B of the same frequency and with the same mean position

C of changed frequency and with shifted mean position.

D of changed frequency and with the same mean position.

CORRECT OPTION

A of the same frequency and with shifted mean position.

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

The force exerted on charge +Q by the electric field

$$\vec{E}$$

is

$$\vec{F} = Q\vec{E}$$

in the direction of

$$\vec{E}$$

. Since

$$\vec{F}$$

is constant, a constant force is added to the applied force. Hence only the mean position will change.

The frequency will be same.

As

$$v_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

does not depend on the constant external force.

Question 043 MCQ

QUESTION

Which of the field patterns given below is valid for electric field as well as for magnetic field ?

A

B

C

D

CORRECT OPTION

C

SOURCE

Physics • electrostatics

EXPLANATION

The magnetic fields and the induced electric fields form the closed loops.

Question 044 MCQ

QUESTION

A point mass is subjected to two simultaneous sinusoidal displacements in x-direction,

$$x_1(t) = A \sin \omega t$$

and

$$x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$$

. Adding a third sinusoidal displacement

$$x_3(t) = B\sin(\omega t + \phi)$$

brings the mass to a complete rest. The values of B and

$$\phi$$

are

A $\sqrt{2}A, \frac{3\pi}{4}$

B $A, \frac{4\pi}{3}$

C $\sqrt{3}A, \frac{5\pi}{6}$

D $A, \frac{\pi}{3}$

CORRECT OPTION

B $A, \frac{4\pi}{3}$

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Here,

$$x_1 = A\sin\omega t$$

$$x_2 = A\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$\therefore$$

$$x_1 + x_2 = A\sin\omega t + A\sin\left(\omega t + \frac{2\pi}{3}\right)$$

$$= A\sin\omega t + A\left[\sin\omega t\cos\frac{2\pi}{3} + \cos\omega t\sin\frac{2\pi}{3}\right]$$

$$= A\sin\omega t + A\left[\sin\omega t\left(-\frac{1}{2}\right) + \cos\omega t\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{A}{2}\sin\omega t + \frac{\sqrt{3}}{2}A\cos\omega t = A\left[\sin\omega t\cos\frac{\pi}{3} + \cos\omega t\sin\frac{\pi}{3}\right]$$

$$= A\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$\because$$

$$x_1 + x_2 + x_3 = 0$$

$$\Rightarrow x_3 = -(x_1 + x_2) = -A\sin\left(\omega t + \frac{\pi}{3}\right)$$

$$= A\sin\left(\omega t + \pi + \frac{\pi}{3}\right)$$

$$x_3 = A\sin\left(\omega t + \frac{4\pi}{3}\right)$$

$$\because$$

$$x_3 = B\sin(\omega t + \phi)$$

Hence,

$$B = A,\ \phi = \frac{4\pi}{3}$$

Question 045 MCQ

QUESTION

A satellite is moving with a constant speed ‘V’ in a circular orbit about the earth. An object of mass ‘m’ is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

A $\frac{1}{2}mV^2$

B mV^2

C $\frac{3}{2}mV^2$

D $2mV^2$

CORRECT OPTION

B mV^2

SOURCE

Physics • gravitation

EXPLANATION

A particle escapes from the gravitational pull if its total energy T i.e., sum of kinetic energy K and potential energy U, is greater than or equal to zero. The condition for just escape T = K + U = 0 i.e.,

K =

$$-$$

U. 1

In a circular orbit of radius r, gravitational attraction provides the centripetal acceleration, $mV^2/r = GMm/r^2$, which gives

$$r = \frac{GM}{V^2}$$

..... 2

From equations 1 and 2, the kinetic energy of the particle at the time of injection is given by

$$K = -U = -\left(-\frac{GMm}{r}\right) = \frac{GMm}{GM/V^2} = mV^2$$

.

Question 046 MCQ

QUESTION

A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the center of the ball. After the collision, the ball and bullet travel independently. The ball hits the

ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is

A 250 m/s

B $250\sqrt{2}$ m/s

C 400 m/s

D 500 m/s

CORRECT OPTION

D 500 m/s

SOURCE

Physics • impulse-and-momentum

EXPLANATION

The time of flight for the bullet is same as that of the ball and is given by

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} = 1\,s$$

Just after the collision, the velocity of bullet (v_1) is related to its range (R_1) by $v_1 t = R_1$ which gives v_1 = 100 m/s. Similarly, the velocity of the ball is v_2 = 20 m/s. Consider the bullet and the ball together as a system. Along the direction of collision, there is no external force on the system. Hence, linear momentum of

the system in the direction of collision is conserved. The linear momentum of the system before and after the collision are

$$p_i = m_1 V$$

,

$$p_f = m_1 v_1 + m_2 v_2$$

.

The conservation of linear momentum,

$$p_i = p_f$$

, gives

$$V = \frac{m_1 v_1 + m_2 v_2}{m_1}$$

$$= \frac{(0.01)(100) + (0.2)(20)}{0.01} = 500$$

m/s.

Question 047 Numerical

QUESTION

A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of

$$60^\circ$$

to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

SOURCE

Physics • motion

EXPLANATION

u = 10 ms

$$-$$

1,

$$\theta$$

= 60

$$\circ$$

Time of flight is

$$t = \frac{2u\sin\theta}{g} = \frac{2 \times 10 \times \sin 60^\circ}{10} = \sqrt{3}\, s$$

Let v be the velocity of the train. The horizontal velocity of ball at the instant it is thrown

$$= (v + u_x) = (v + u\cos\theta)$$

. Therefore, the horizontal range of the ball with respect to the ground is

$$R = (v + u\cos\theta)t$$

, where

$$t = \sqrt{3}\, s$$

It is clear that

Distance travelled by ball in time

$$t + 1.15 = R$$

i.e.

$$vt + \frac{1}{2}at^2 + 1.15 = (v + u\cos\theta)t$$

$$\Rightarrow \frac{1}{2}at^2 + 1.15 = (u\cos\theta)t$$

$$\Rightarrow \frac{1}{2}a \times (\sqrt{3})^2 + 1.15 = (10\cos 60^\circ) \times \sqrt{3}$$

$$\Rightarrow a = 5$$

ms

$$-$$

2

Question 048 MCQ

QUESTION

The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2 %, the relative percentage error in the density is

A 0.9 %

B 2.4 %

C 3.1 %

D 4.2 %

CORRECT OPTION

C 3.1 %

SOURCE

Physics • units-and-measurements

EXPLANATION

Least count of screw gauge

$$= \frac{Pitch}{No.\ of\ divisions\ on\ the\ circular\ scale}$$

$$= \frac{0.5\,mm}{50}$$

= 0.01 mm

Diameter of ball,

$$D = MSR + CSR \times LC$$

= 2.5 mm + $20\ 0.01mm$ = 2.7 mm

Density,

$$\rho = \frac{Mass}{Volume} = \frac{M}{\frac{4\pi}{3}\left(\frac{D}{2}\right)^3}$$

The relative error in the density is

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta D}{D}$$

The relative percentage in the density is

$$\frac{\Delta\rho}{\rho} \times 100 = \left[\frac{\Delta M}{M} + \frac{3\Delta D}{D}\right] \times 100$$

$$= 2\% + \frac{3 \times 0.01}{2.7} \times 100 = 2\% + 1.11\% = 3.11\%$$

Question 049 Numerical

QUESTION

A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is

SOURCE

Physics • work-power-and-energy

EXPLANATION

Loss of kinetic energy

$$= \frac{1}{2}mV^2$$

Work done against friction

$$= \mu mgx$$

Gain in potential energy

$$= \frac{1}{2}kx^2$$

From work-energy principle,

$$\frac{1}{2}mV^2 = \mu mgx + \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2} \times 0.18 \times V^2 = 0.1 \times 0.18 \times 10 \times 0.06 + \frac{1}{2} \times 2 \times (0.06)^2$$

$$\Rightarrow V = 0.4 = \frac{4}{10}$$

ms

$$-$$

^{1}. Hence, N = 4.

Question 050 MCQ

QUESTION

A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence

$$\theta$$

. The reflected R and transmitted T intensities, both as function of

$$\theta$$

, are plotted. The correct sketch is

A

B

C

D

CORRECT OPTION

C

SOURCE

Physics • wave-optics

EXPLANATION

If I is the intensity of the incident light then R + T = I. Since the incident ray is travelling in a denser medium $glass$, it will be totally reflected at a certain critical angle

$$\theta$$

$_c$. For

$$\theta$$

<

$$\theta$$

$_c$ a part of the incident intensity is reflected and the remaining part is transmitted. But for

$$\theta$$

>

$$\theta$$

$_c$, there is no refracted ray. Hence for

$$\theta$$

>

$$\theta$$

$_c$, the value of R is 100%. Hence the only correct option is c.

Question 051 MCQ

QUESTION

A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the xy-plane and a steady current I flows through the wire. The z-component of the magnetic field at the centre of the spiral is

A $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$

B $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b+a}{b-a}\right)$

C $\frac{\mu_0 NI}{2b} \ln\left(\frac{b}{a}\right)$

D $\frac{\mu_0 NI}{2b} \ln\left(\frac{b+a}{b-a}\right)$

CORRECT OPTION

A $\frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$

SOURCE

Physics • magnetism

EXPLANATION

Magnetic field at the centre of a circular loop of radius r and carrying a current

$$I = \frac{\mu_0 I}{2r}$$

. The direction of the field is along z-direction if the current is anticlockwise.

Consider a small element of width dr. The current through the element is

$$dI = \frac{total\,current\,in\,spiral}{total\,width\,of\,spiral} \times width\,of\,element$$

$$= \frac{Idr}{(b-a)}$$

$$\therefore$$

$$B = \int_a^b \frac{\mu_0 N dI}{2r} = \int_a^b \frac{\mu_0 NI}{2(b-a)} \frac{dr}{r}$$

$$= \frac{\mu_0 NI}{2(b-a)} \int_a^b \frac{dr}{r} = \frac{\mu_0 NI}{2(b-a)} \ln\left(\frac{b}{a}\right)$$

Question 052 MCQ

QUESTION

Two solid spheres A and B of equal volumes but of different densities d_A and d_B are connected by a string. They are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if

A $d_A < d_F$

B $d_B > d_F$

C $d_A > d_F$

D $d_A + d_B = 2d_F$

CORRECT OPTION

A $d_A < d_F$

SOURCE

Physics • properties-of-matter

EXPLANATION

Let V be the volume of each sphere and let T be the tension in the string.

Buoyant force on sphere A is $U_A = d_F Vg$

Buoyant force on sphere B is $U_B = d_F Vg$

Weight of A is $W_A = d_A Vg$

Weight of B is $W_B = d_B Vg$

The free body diagrams of A and B are as follows. $see Fig.\ 11.34$

For equilibrium,

$U_A = T + W_A$ and $U_B + T = W_B$

i.e. $d_F Vg = T + d_A Vg$ i

and $d_F Vg + T = d_B Vg$ ii

From Eq. i

$$d_F = \frac{T}{Vg} + d_A$$

. Hence, $d_F > d_A$. So choice a is correct.

From Eq. ii

$$d_B = \frac{T}{Vg} + d_F$$

. Hence $d_B > d_F$. So choice b is also correct.

Eliminating T from Eqs. i and ii we get

$$2d_F = d_A + d_B$$

, which is choice d.

So, the correct choices are a, b and d.

Question 053 MCQ

QUESTION

A thin ring of mass 2 kg and radius 0.5 m is rolling without on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision,

A the ring has pure rotation about its stationary CM.

B the ring comes to a complete stop.

C friction between the ring and the ground is to the left.

D there is no friction between the ring and the ground.

CORRECT OPTION

A the ring has pure rotation about its stationary CM.

SOURCE

Physics • work-power-and-energy

EXPLANATION

Let M be the mass of the ring and m that of the ball and let V and

$$v$$

be their velocity before collision. The initial momentum of the system $ringandball$ in the horizontal direction is

$$\overrightarrow{p_i} = M\overrightarrow{V} + m\overrightarrow{v}$$

$$= 2 \times 1 + 0.1 \times (-20)$$

$$= 2 - 2 = 0$$

From conservation of momentum, the final momentum of the system

$$\overrightarrow{p_f} = 0$$

in the horizontal direction. Hence

$$V_{cm} = 0$$

for the ring, i.e. the ring has pure rotation about its centre of mass. So choice a is correct.

The total initial angular momentum of the system about the point of collision is

$$L_i = mvr - I\omega$$

$$= mvr - MR^2\frac{V}{R}$$

$$= mvr - MRV$$

$$= 0.1 \times 20 \times 0.75 - 2 \times 0.5 \times 1$$

$$= 1.5 - 1 = 0.5$$

kg m^2 s

$$-$$

1

From the conservation of angular momentum, the final angular velocity must be anticlockwise. Hence the friction between the ring and the ground is to the left. So the correct choices are a and c.

Question 054 MCQ

QUESTION

A series RC-current is connected to AC voltage source. Consider two cases : A When C is without a dielectric medium and B when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

A $I_R^A > I_R^B$

B $I_R^A < I_R^B$

C $V_C^A > V_C^B$

D $V_C^A < V_C^B$

CORRECT OPTION

B $I_R^A < I_R^B$

SOURCE

Physics • alternating-current

EXPLANATION

In case A,

The capacitive reactance is

$$X_C^A = \frac{1}{\omega C}$$

Impedance of the circuit is

$$Z_A = \sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$I_R^A = \frac{V}{\sqrt{(R)^2 + \left(\frac{1}{\omega C}\right)^2}}$$

..... i

$$V_C^A = \frac{I_R^A}{\omega C} = \frac{V}{\sqrt{(R\omega C)^2 + 1}}$$

..... ii

In case B,

The capacitive reactance is

$$X_C^B = \frac{1}{\omega(4C)} = \frac{1}{4\omega C}$$

Impedance of the circuit is

$$Z_B = \sqrt{R^2 + \left(\frac{1}{4\omega C}\right)^2}$$

$$I_R^B = \frac{V}{\sqrt{R^2 + \left(\frac{1}{4\omega C}\right)^2}}$$

..... iii

$$V_C^B = \frac{V}{\sqrt{(4R\omega C)^2 + 1}}$$

..... iv

From i and iii, we conclude that

$$I_R^A < I_R^B$$

From ii and iv, we conclude that

$$V_C^A > V_C^B$$

Question 055 Numerical

QUESTION

A series RC combination is connected to an AC voltage of angular frequency

$$\omega$$

= 500 rad/s. If the impedance of the RC circuit is R

$$\sqrt{1.25}$$

, the time constant $inmillisecond$ of the circuit is _____.

SOURCE

Physics • alternating-current

EXPLANATION

We have impedance in the circuit

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

However,

$$Z = R\sqrt{1.25}$$

and

$$R\sqrt{1.25} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\Rightarrow 0.25R^2 = \frac{1}{(\omega C)^2}$$

The time constant is

$$RC = \sqrt{\frac{1}{0.25 \times 500^2}} = 4$$

ms

Question 056 Numerical

QUESTION

A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the spheres is A

$$\times$$

$10^{Z}\ where 1 < A < 10$. The value of Z is _____.

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

The silver sphere gets positively charged due to emission of photoelectrons. This positively charged sphere attracts $binds$ the emitted photoelectrons. The emitted photoelectrons cannot escape if their kinetic energies $hc/\$\$\lambda\$\$\$\$ - \$\$\$\$\phi\$\$$ are less than or equal to their potential energies

$$\left(\frac{1}{4\pi\varepsilon_0}\frac{ne^2}{r}\right)$$

. Thus, in limiting case,

$$\frac{hc}{\lambda} - \phi = \frac{1}{4\pi\varepsilon_0}\frac{ne^2}{r}$$

..... 1

Substitute the values of various parameters in equation 1,

$$\frac{1242}{200} - 4.7 = \frac{n(9\times10^{9})(1.6\times10^{-19})}{10^{-2}}$$

,

to get n = 1.04

$\times$

10^7.

$$We have used hc = 1242 eV - nm.$$

Question 057 Numerical

QUESTION

Two batteries of different emfs and different internal resistance are connected as shown. The voltage across AB in volts is _____.

SOURCE

Physics • current-electricity

EXPLANATION

Applying Kirchhoff's second law for closed loop CDEFC we get

$$-3 - 2I - I + 6 = 0$$

$$I = \frac{6-3}{3} = 1A$$

For the lower path

$$V_A - 3 - 2 \times 1 = V_B$$

$$\therefore$$

$$V_A - V_B = 5V$$

We can also find the V_{AB} by considering the upper path

For the upper path,

$$V_A - 6 + 1 \times 1 = V_B$$

$$V_A - V_B = 5V$$

Question 058 Numerical

QUESTION

Water $withrefractiveindex = 4/3$ in a tank is 18 cm deep. Oil of refractive index 7/4 lies on water making a convex surface of radius of curvature R = 6 cm as shown. Consider oil to act a thin lens. An object S is placed 24 cm above water surface. The location of its image is at x cm above the bottom of the tank. Then x is _____.

SOURCE

Physics • geometrical-optics

EXPLANATION

We have

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_1 - n_2}{R}$$

For the first refracting surface $air - oil$, we have n_2 = 7/4; n_1 = 1; R = 6 cm. Therefore,

$$\frac{7}{4v_1} - \frac{1}{24} = \frac{-(7/4)}{6}$$

or v_1 = 21 cm and for the second interface $water - oil$, we have

$$n_1 = \frac{7}{4}; n_2 = \frac{4}{3} u = v_1; R = \infty$$

Therefore,

$$\frac{4}{3v_2} - \frac{7}{4 \times 21} = 0$$

v_2 = 16 cm and v_2 + x = height of water.

Therefore,

$$x = 18 - 16 = 2$$

Question 059 MCQ

QUESTION

One mole of a monatomic gas is taken through a cycle ABCDA as shown in the PV diagram. Column II give the characteristics involved in the cycle. Match them with each of the processes given in Column I.

Column I		Column II	
Process A A B	$\rightarrow$	P	Internal energy decreases.
Process B B C	$\rightarrow$	Q	Internal energy increase.
Process C C D	$\rightarrow$	R	Heat is lost.
Process D D A	$\rightarrow$	S	Heat is gained.

Column I	Column II
T	Work is done on the gas.

A

$\rightarrow$

$P, R, T; B$

$\rightarrow$

A P, R; C

$\rightarrow$

$Q, S; D$

$\rightarrow$

R, T

A

$\rightarrow$

$P, T; B$

$\rightarrow$

B $P, R; C$

$\rightarrow$

$Q, S; D$

$\rightarrow$

R

A

$\rightarrow$

$R, T; B$

$\rightarrow$

C

$P, R; C$

$\rightarrow$

$S; D$

$\rightarrow$

R, T

A

$\rightarrow$

$P, R, T; B$

$\rightarrow$

D

$P, R; C$

$\rightarrow$

$Q; D$

$\rightarrow$

R, T

CORRECT OPTION

A

$\rightarrow$

$P, R, T; B$

$\rightarrow$

A

P, R; C

$\rightarrow$

$Q, S; D$

$\rightarrow$

R, T

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Process A

$\rightarrow$

B,

It is a isobaric process

P = constant, V

$\propto$

T

$\because$

$V_B < V_A$

$\Rightarrow$

$T_B < T_A$

Δ

$U = nC_V$

Δ

T =

$-$

ve

Hence internal energy decreases.

Δ

$Q = nC_P$

Δ

T =

$-$

ve

Hence heat is lost.

Δ

W = nR

Δ

T =

$-$

ve

Hence, work is done on the gas.

Process, B

$\rightarrow$

C,

It is a isochoric process.

V = constant, P

$\propto$

T

$\because$

$P_C < P_B$

$\Rightarrow$

$T_C < T_B$

Δ

$U = nC_V$

Δ

T =

$-$

ve

Hence, internal energy decreases.

Δ

W = 0,

Δ

Q =

Δ

U =

$-$

ve

Hence, heat is lost.

Process C

$\rightarrow$

D,

It is a isobaric process.

P = constant, V

$\propto$

T

$\because$

$V_D > V_C$

$\Rightarrow$

$T_D > T_C$

Δ

$U = nC_V$

Δ

T = +ve

Hence internal energy increases.

Δ

$Q = nC_P$

Δ

T = +ve

Hence, heat is gained.

Δ

W = nR

Δ

T = +ve

Hence, work is done by the gas.

Process, D

$\rightarrow$

A

According to ideal gas equation

$$\frac{P_A V_A}{T_A} = \frac{P_D V_D}{T_D} \Rightarrow \frac{(3P)(3V)}{T_A} = \frac{(P)(9V)}{T_D} \Rightarrow T_D = T_A$$

Hence, it is a isothermal process.

$\therefore$

Δ

U = 0

$\because$

$V_A < V_D$

Δ

W =

$-$

ve, hence work done is done on the gas.

Δ

Q =

Δ

W =

$-$

ve, hence heat is lost.

Question 060 MCQ

QUESTION

Column I shows four systems, each of the same length L, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as

λ

$_f$. Match each system with statements given in Column II describing the nature and wavelength of the standing waves :

A

$\rightarrow$

$T; B$

$\rightarrow$

A $P, S; C$

$\rightarrow$

$Q, S; D$

$\rightarrow$

Q

A

$\rightarrow$

$P, T; B$

$\rightarrow$

B $P; C$

$\rightarrow$

$Q, S; D$

$\rightarrow$

Q

A

$\rightarrow$

$P; B$

$\rightarrow$

C $P, S; C$

$\rightarrow$

$Q; D$

$\rightarrow$

Q, R

A

$\rightarrow$

$P, T; B$

$\rightarrow$

D $P, S; C$

$\rightarrow$

$Q, S; D$

$\rightarrow$

Q, R

CORRECT OPTION

A

$\rightarrow$

$P, T; B$

$\rightarrow$

D $P, S; C$

$\rightarrow$

$Q, S; D$

$\rightarrow$

Q, R

SOURCE

Physics • waves

EXPLANATION

1 For a pipe closed at one end, we have

$$\frac{\lambda_f}{4} = L$$

or

$$\lambda_f = 4L$$

Therefore, the sound waves are longitudinal.

2 For a pipe open at both ends, we have

$$\frac{\lambda_f}{2} = L$$

or

$$\lambda_f = 2L$$

Therefore, the sound waves are longitudinal.

3 For a stretched wire clamped at both ends, we have

$$\frac{\lambda_f}{2} = L$$

Vibration on the string is transverse.

4 For a stretched wire clamped at both ends and at mid-point, we have

$$\frac{\lambda_f}{2} = \frac{L}{2} \Rightarrow \lambda_f = L$$

Therefore, the vibration on the string is transverse.