iit Jee 2012 Paper 1 Offline $60 Questions$

Question 001 Numerical

QUESTION

An organic compound undergoes first-order decomposition. The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of

$$\left[\frac{t_{1/8}}{t_{1/10}}\right] \times 10$$

? $$\log_{10} 2 = 0.3$$

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

The expression for first-order reaction is

$$t = \frac{2.303}{k} \log \frac{[\mathrm{A}_0]}{[\mathrm{A}]}$$

When the compound is decomposed to $1/8$ th of its initial value then the time taken is

$$t_{1/8} = \left(\frac{2.303}{k}\right) \log \frac{1}{(1/8)} = \left(\frac{2.303}{k}\right) \log 8$$

..... 1

When the compound is decomposed to $1/10$ th of its initial value then the time taken is

$$t_{1/10} = \left(\frac{2.303}{k}\right) \log \frac{1}{(1/10)} = \left(\frac{2.303}{k}\right) \log 10$$

..... 2

Dividing Eq. 1 by Eq. 2, we get

$$\frac{t_{1/8}}{t_{1/10}} = \frac{\log 8}{\log 10} = \log\left(2^3\right) = 3 \times 0.3 = 0.9$$

So, the value of

$$\frac{\left[t_{1/8}\right]}{\left[t_{1/10}\right]} \times 10 = 9$$

Question 002 Numerical

QUESTION

The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons undergoes a nuclear reaction yielding element X as shown below. To which group, element X belongs in the periodic table?

$$^{63}_{29}Cu$$

+

$$_{1}^{1}H$$

$$\rightarrow$$

$$6_{0}^{1}n$$

+

$$_{2}^{4}\alpha$$

+ 2

$$_{1}^{1}H$$

+ X

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

Considering the reaction $replace\$\alpha\$ - particleby\$_{2}^{4}\mathrm{He}\$$

$$_{29}^{63}\mathrm{Cu} + _{1}^{1}\mathrm{H} \rightarrow 6_{0}^{1}n + _{2}^{4}\mathrm{He} + 2_{1}^{1}\mathrm{H} + \mathrm{X}$$

Equating mass numbers on both the sides, we get

$$63 + 1 = 1 \times 6 + 4 \times 1 + 1 \times 2 + X$$

$$\Rightarrow$$

$$X = 64 - 12 = 52$$

Equating atomic numbers on both the sides, we get

$$29 + 1 = 6 \times 0 + 2 + 2 \times 1 + \mathrm{Y}$$

$$\Rightarrow \mathrm{Y} = 30 - 4 = 26$$

So, the element is $^{52}_{26}\mathrm{Fe}$ and iron is a d-block element, which belongs to Group 8 of the periodic table.

Question 003 Numerical

QUESTION

29.2 % w/w HCl stock solution has density of 1.25 g mL^{-1} . The molecular weight of HCl is 36.5 g mol^{-1} . The volume mL of stock solution required to prepare a 200 mL solution of 0.4 M HCl is

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

To calculate the volume of the 29.2% w/w HCl stock solution needed to prepare a 200 mL solution of 0.4 M HCl, we need to use several steps involving concentration and density conversions.

First, we calculate the mass of HCl that is contained in the 200 mL of a 0.4 M solution:

$$Mass = Molarity \times Volume \times Molecular\ Weight$$

$$Mass = 0.4\ mol/L \times 0.200\ L \times 36.5\ g/mol$$

Note that we convert the volume from mL to L to match the units of molarity mol/L.

Now, we calculate it:

$$Mass = (0.4 \times 0.200 \times 36.5)\ g$$

$$Mass = 0.08 \times 36.5\ g$$

$$Mass = 2.92\ g$$

The next step is to determine how much of the stock solution is needed to get 2.92 g of HCl. Since the stock solution is 29.2% w/w HCl, this means that in every 100 g of stock solution, there is 29.2 g of HCl. We can set up a proportion to find the mass of the stock solution needed:

$$\frac{29.2\ g\ HCl}{100\ g\ stock\ solution} = \frac{2.92\ g\ HCl}{x\ g\ stock\ solution}$$

Now we solve for

$$x$$

:

$$x = \frac{2.92\ g \times 100\ g\ stock\ solution}{29.2\ g\ HCl}$$

$$x = \frac{292}{29.2}\ g$$

$$x = 10\ g$$

So, we need 10 g of the stock solution to get 2.92 g of HCl.

The final step is to calculate the volume of the stock solution that has a mass of 10 g. We use the density to convert mass to volume:

$$Volume = \frac{Mass}{Density}$$

The density of the stock solution is given as 1.25 g/mL, so:

$$Volume = \frac{10\ g}{1.25\ g/mL}$$

$$Volume = 8\ mL$$

Therefore, to prepare a 200 mL solution of 0.4 M HCl, you would need to measure out 8 mL of the 29.2% HCl stock solution.

Question 004 MCQ

QUESTION

Choose the correct reason for the stability of the **lyophobic** colloidal particles

A Preferential adsorption of ions on their surface from the solution

B Preferential adsorption of solvent on their surface from the solution

C Attraction between different particles having opposite charges on their surface

D Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles.

CORRECT OPTION

A Preferential adsorption of ions on their surface from the solution

SOURCE

Chemistry • surface-chemistry

EXPLANATION

The stability of lyophobic colloidal particles is generally due to two main factors:

Preferential Adsorption: The same charge $either positive or negative$ is preferentially adsorbed onto the surface of lyophobic particles, preventing them from coming close to each other and precipitating. For instance, when a dilute solution of silver nitrate is mixed with potassium iodide, it forms a white precipitate of silver iodide. The iodide ions are preferentially adsorbed onto the silver iodide particles, making the sol negatively charged. These negatively charged particles repel each other, thus preventing precipitation.

Electrostatic Layers: Negatively charged sol particles attract positive charges from the surrounding medium, forming a second layer of positive charges around them. This results in two layers of opposite charges, known as the fixed layer and the diffused layer. The potential difference between these oppositely charged layers is referred to as electrokinetic potential or zeta potential.

Here is a visual illustration of the concept $known as Helmholtz double layer$:

This double layer of opposite charges around the colloidal particles is crucial in maintaining their stability.

Question 005 MCQ

QUESTION

In allene (C_3H_4), the type s of hybridisation of the carbon atoms is are:

A sp and sp^3

B sp and sp^2

C only sp^3

D sp^2 and sp^3

CORRECT OPTION

B sp and sp^2

SOURCE

Chemistry • hydrocarbons

EXPLANATION

The structure of allene is $CH_2 = C = CH_2$. Here both the terminal carbons are sp^2 hybridised while the central carbon is sp hybridised.

Question 006 MCQ

QUESTION

The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is

$$\$\$\alpha_0\$\$is Bohr radius$$

A $\frac{h^2}{4\pi^2 m\alpha_0^2}$

B $\frac{h^2}{16\pi^2 m\alpha_0^2}$

C $$\frac{h^2}{32\pi^2 m\alpha_0^2}$$

D $$\frac{h^2}{64\pi^2 m\alpha_0^2}$$

CORRECT OPTION

C $$\frac{h^2}{32\pi^2 m\alpha_0^2}$$

SOURCE

Chemistry • structure-of-atom

EXPLANATION

For Bohr orbit, angular momentum is

$$mvr_n = \frac{nh}{2\pi}$$

Velocity,

$$v = \frac{nh}{2\pi m r_n} \quad \ldots (\mathrm{i})$$

Kinetic energy,

$$K.E. = \frac{1}{2}mv^2 \quad \ldots (\mathrm{ii})$$

By putting the value of v from i into ii,

$$K.E. = \frac{1}{2}m \times \frac{n^2h^2}{4\pi^2 m^2 r_n^2} = \frac{n^2h^2}{8\pi^2 m r_n^2}$$

For second Bohr orbit, $n = 2$

$$r_n = a_0 \times n^2 \quad (a_0 = \text{Bohr radius})$$

$$r_n = 4a_0$$

$$K.E. = \frac{(2)^2 h^2}{8\pi^2 m(4a_0)^2}$$

Thus,

$$K.E. = \frac{h^2}{32\pi^2 m a_0^2}$$

Question 007 MCQ

QUESTION

A compound M_pX_q has cubic close packing ccp arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is

A MX

B MX_2

C M_2X

D M_5X_{14}

CORRECT OPTION

B MX_2

SOURCE

Chemistry • solid-state

EXPLANATION

8 X atoms present at the corners.

Atoms contribute to 1 unit cell

$$= \frac{1}{8} \times 8 = 1$$

6 X atoms present at the face centres.

Atoms contribute to 1 unit cell

$$= 6 \times \frac{1}{2} = 3$$

Total X atoms

$$= 3 + 1 = 4$$

4 M atoms present at edge centres.

Atoms present in 1 unit cell

$$= 4 \times \frac{1}{4} = 1$$

1 M atom present at body centre and it contribute completely to 1 unit cell.

Thus, total M atoms in one unit cell

$$= 1 + 1 = 2$$

Ratio is

$$M : X :: 2 : 4 :: 1 : 2$$

Thus, empirical formula is

$$MX_2$$

.

Question 008 MCQ

QUESTION

The carboxyl functional group $$ – $$COOH is present in

A picric acid

B barbituric acid

C ascorbic acid

D aspirin

CORRECT OPTION

D aspirin

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The structures of the given compounds are as follows. Only aspirin has

—

COOH group.

Question 009 MCQ

QUESTION

As per IUPAC nomenclature, the name of the complex

$$[Co(H_2O)_4(NH_3)_2]Cl_3$$

is

A Tetraaquadiaminecobalt III chloride.

B Tetraaquadiamminecobalt III chloride.

C Diaminetetraaquacobalt III chloride.

D Diamminetetraaquacobalt III chloride.

CORRECT OPTION

D Diamminetetraaquacobalt III chloride.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

There are two ligands in the coordination entity, that is, ammine and aqua, but according to the IUPAC rule, alphabetical order is followed, that is, ammine is named first. Now calculating the oxidation state of Co, the net charge on the coordination entity is +3 as Cl has

—

1 and there are three Cl

—

outside the square brackets. So the oxidation state of Co will be III as both ammine and aqua are neutral ligands. So, its correct name will be diamminetetraaquacobalt III chloride.

Question 010 MCQ

QUESTION

Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen?

A HNO_3, NO, NH_4Cl, N_2

B HNO_3, NO, N_2, NH_4Cl

C HNO_3, NH_4Cl, NO, N_2

D NO, HNO_3, NH_4Cl, N_2

CORRECT OPTION

B HNO_3, NO, N_2, NH_4Cl

SOURCE

Chemistry • p-block-elements

EXPLANATION

In HNO_3, the oxidation state of N is +5 while in NO, it is +2. In N_2, it is zero and in NH_4Cl, the oxidation state of nitrogen is

—

3. So, the decreasing order of oxidation state of nitrogen is

$HNO_3 > NO > N_2 > NH_4Cl$

Question 011 MCQ

QUESTION

For one mole of a van der Waals gas when b = 0 and T = 300 K, the PV vs. 1/V plot is shown below. The value of the van der Waals constant a (atm L^2 mol

—

^{2}) is

A 1.0

B 4.5

C 1.5

D 3.0

CORRECT OPTION

C 1.5

SOURCE

Chemistry • gaseous-state

EXPLANATION

The van der Waals equation is

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT$$

but it is given that b = 0. So, the equation reduces to

$$\left(p+\frac{a}{V^2}\right)V=RT \Rightarrow pV=-\frac{a}{V}+RT$$

Comparing it with a straight line equation we get slope as

$$-$$

a. Calculating the slope, we get

$$\frac{24.6-20.1}{3.0-0}=1.5 \Rightarrow a=1.5$$

Question 012 MCQ

QUESTION

The number of aldol reactions that occur in the given transformation are

A 1

B 2

C 3

D 4

CORRECT OPTION

C 3

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reactions involved in the transformation are:

Question 013 MCQ

QUESTION

The colour of light absorbed by an aqueous solution of $CuSO_4$ is

A orange-red

B blue-green

C yellow

D violet

CORRECT OPTION

A orange-red

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

When we consider the color of light absorbed by a substance, we are referring to the complementary color of the light that is transmitted or reflected by the substance. An aqueous solution of copper II sulfate ($CuSO_4$) appears blue to our eyes. This color results from the absorption of light in the complementary color range.

The visible spectrum of light can be divided into several regions, each corresponding to a different color. The complementary color of blue is orange-red. This means that the aqueous solution of $CuSO_4$ absorbs light in the orange-red region of the spectrum.

Therefore, the correct answer is:

Option A: orange-red

Question 014 MCQ

QUESTION

The number of optically active products obtained from the complete ozonolysis of the given compound is _____.

A 0

B 1

C 2

D 4

CORRECT OPTION

A 0

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction is

Question 015 MCQ

QUESTION

Which of the following hydrogen halides reacts with $AgNO_3\,aq.$ to give a precipitate that dissolves in $Na_2S_2O_3\ aq.$?

A HCl

B HF

C HBr

D HI

CORRECT OPTION

A HCl

SOURCE

Chemistry • p-block-elements

EXPLANATION

Only HF does not react with $AgNO_3$. All others react with $AgNO_3$ to give precipitate of AgX which is soluble in $Na_2S_2O_3$

$$HX + AgNO_3 \rightarrow AgX \downarrow + HNO_3 (X = Cl, Br, I)$$

$$AgX + Na_2S_2O_3 \rightarrow \underset{Soluble}{Na_3[Ag(S_2O_3)_2]} + NaX$$

Question 016 MCQ

QUESTION

Identify the binary mixtures that can be separated into individual compounds, by differential extraction, as shown in the given scheme.

A C_6H_5OH and C_6H_5COOH

B C_6H_5COOH and $C_6H_5CH_2OH$

C $C_6H_5CH_2OH$ and C_6H_5OH

D $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

CORRECT OPTION

B C_6H_5COOH and $C_6H_5CH_2OH$

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Both phenol and benzoic acid are soluble in NaOH, so they cannot be separated.

$C_6H_5OH + NaOH$

$\rightarrow$

$C_6H_5ONa + H_2O$

$C_6H_5COOH + NaOH$

$\rightarrow$

$C_6H_5COONa + H_2O$

Benzel alcohol $C_6H_5CH_2OH$ is not soluble in NaOH and $NaHCO_3$, while benzoic acid is soluble in both, so these can be separated. Both C_6H_5OH and $C_6H_5CH_2OH$ are insoluble in $NaHCO_3$, so they cannot be separated. $C_6H_5CH_2COOH$ is soluble in NaOH and $NaHCO_3$, whereas benzyl alcohol is not soluble in NaOH and $NaHCO_3$. So, they can be separated.

Question 017 MCQ

QUESTION

For an ideal gas, consider only P-V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice s is are correct?
$Take\$\$\Delta\$\$Saschangeinentropyand W asworkdone$

A $\Delta S_{X\rightarrow Z} = \Delta S_{X\rightarrow Y} + \Delta S_{Y\rightarrow Z}$

B $\Delta W_{X\rightarrow Z} = \Delta W_{X\rightarrow Y} + \Delta W_{Y\rightarrow Z}$

C $W_{X\rightarrow Y\rightarrow Z} = W_{X\rightarrow Y}$

D $$\Delta S_{X\to Y\to Z} = \Delta S_{X\to Y}$$

CORRECT OPTION

A $$\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$$

SOURCE

Chemistry • thermodynamics

EXPLANATION

As entropy is a state function and is additive

$$\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$$

On moving from Y to Z, the work done is zero as the volume is kept constant $isochoricprocess$, so

$$W_{X\to Y\to Z} = W_{X\to Y}$$

Question 018 MCQ

QUESTION

Which of the following molecules, in pure form, is are unstable at room temperature?

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

a The compound is monocyclic but non-planar due to presence of two sp^3 hybridised carbon atoms. As a result, delocalisation of pi (π) electrons $oraconjugatesystem$ is not possible. The compound is non-aromatic.

b The compound is monocyclic and planar. Carbon atoms are sp^2 hybridised and the pi (π) electrons are conjugated, i.e., there is delocalisation of pi (π) electrons. It follows $4n\,\pi$ electron system with 4π electrons. This makes the compound anti-aromatic and least stable.

c The compound is monocyclic and planar. The carbons including carbonyl carbon are sp^2 hybridised. There is no extended delocalisation of pi $\$\pi\$$ electrons $duetoexocycliccarbonyldoublebond$. It also follows $4n\pi$ electron system with 4π electrons inside the ring. This makes compound anti-aromatic and least stable.

d The compound is monocyclic and planar. The all carbons including carbonyl carbons are sp^2 hybridised. Though, there is no extended delocalisation of

electron inside the ring, but ring has $(4n+2)\pi$ electrons, i.e., 6π electrons. This makes compound aromatic and most stable.

Question 019 Numerical

QUESTION

The substituents R_1 and R_2 for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0 ?

Peptide	R_1	
I	H	
II	H	
III	CH_2COOH	
IV	CH_2CONH_2	
V	CH_2CONH_2	
VI	$(CH_2)_4NH_2$	
VII	CH_2COOH	
VIII	CH_2OH	

Peptide	R_1	
IX	$(CH_2)_4NH_2$	

SOURCE

Chemistry • biomolecules

EXPLANATION

For basic amino acids with pH > 7, peptides will exist as cations. For example, when the substituents are basic, that is $R_1 = CH_2CONH_2$ and $R_2 = (CH_2)_4NH_2$ or when $R_1 = (CH_2)_4NH_2$ and $R_2 = (CH_2)_4NH_2$ or when $R_1 = CH_2OH$ and $R_2 = (CH_2)_4NH_2$ or when $R_1 = (CH_2)_4NH_2$ and $R_2 = CH_3$.

Question 020 Numerical

QUESTION

When the following aldohexose exists in its D-configuration, the total number of stereoisomers in its pyranose form is _____.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The structure of aldohexose in D-configuration can be represented as

Question 021 MCQ

QUESTION

Let

$$\theta, \varphi \in [0, 2\pi]$$

be such that

$$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\varphi - 1, \tan(2\pi-\theta) > 0$$

and

$$-1 < \sin\theta < -\frac{\sqrt{3}}{2},$$

then

$$\varphi$$

cannot satisfy

A $$0 < \varphi < \frac{\pi}{2}$$

B $$\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$$

C $$\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$$

D $$\frac{3\pi}{2} < \varphi < 2\pi$$

CORRECT OPTION

C $$\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

Given,

$$\tan(2\pi - \theta) > 0$$

$$\Rightarrow 0 < (2\pi - \theta) < \frac{\pi}{2} \text{ or } \pi < (2\pi - \theta) < \frac{3\pi}{2}$$

$$\Rightarrow \frac{3\pi}{2} < \theta < 2\pi \text{ or } \frac{\pi}{2} < \theta < \pi \quad \ldots (\text{i})$$

Also,

$$-1 < \sin\theta < \frac{-\sqrt{3}}{2}$$

$$\Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3} \quad \ldots (\text{ii})$$

From i and ii,

$$\Rightarrow \theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \quad \ldots (\text{iii})$$

Now,

$$2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\phi-1$$

$$\Rightarrow 2\cos\theta(1-\sin\phi)$$

$$=\sin^2\theta\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}+\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}\right)\cos\phi-1$$

$$\Rightarrow 2\cos\theta(1-\sin\phi)$$

$$=\sin^2\theta\left(\frac{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\cos\phi-1$$

$$\Rightarrow \quad 2\cos\theta(1-\sin\phi)$$

$$=\sin\theta\cdot\sin\theta\left(\frac{1}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)\cos\phi-1$$

$$\Rightarrow \quad 2\cos(1-\sin\phi)=\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cdot\cos\phi\cdot\sin\theta}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}}-1$$

$$\Rightarrow \quad 2\cos\theta(1-\sin\phi)=2\sin\theta\cos\phi-1$$

$$\Rightarrow \quad 2\cos\theta+1=2\sin(\theta+\phi)$$

$$\text{As } \theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right) \quad [\text{from (iii)}]$$

$$\Rightarrow 2\cos\theta+1\in(1,2)$$

$$\Rightarrow 1<2\sin(\theta+\phi)<2$$

$$\Rightarrow \frac{1}{2}<\sin(\theta+\phi)<1$$

$$\text{Now, } \theta,\phi\in[0,2\pi]$$

$$\theta+\phi\in[0,4\pi]$$

$$\Rightarrow\theta+\phi\in\left(\frac{\pi}{6},\frac{5\pi}{6}\right)\text{ or }\theta+\phi\in\left(\frac{13\pi}{6},\frac{17\pi}{6}\right)$$

$$\Rightarrow\frac{\pi}{6}-\theta<\phi<\frac{5\pi}{6}-\theta\text{ or }\frac{13\pi}{6}-\theta<\phi<\frac{17\pi}{6}-\theta$$

$$\Rightarrow\phi\in\left(-\frac{3\pi}{2},\frac{-2\pi}{3}\right)\cup\left(\frac{2\pi}{3},\frac{7\pi}{6}\right)$$

$$\left[\because\theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)\right]$$

Question 022 MCQ

QUESTION

Let z be a complex number such that the imaginary part of z is non-zero and

$$a = z^2 + z + 1$$

is real. Then a cannot take the value

A - 1

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{3}{4}$

CORRECT OPTION

D $\frac{3}{4}$

SOURCE

Mathematics • complex-numbers

EXPLANATION

Given,

$$a = z^2 + z + 1$$

$$\Rightarrow z^2 + z + 1 - a = 0$$

Using Sridharacharya Rule

$$z = \frac{-1 \pm \sqrt{1^2 - 4(1-a)}}{2}$$

$$\Rightarrow \quad z = \frac{-1 \pm \sqrt{1 - 4 + 4a}}{2}$$

$$\Rightarrow \quad z = \frac{-1 \pm \sqrt{4a - 3}}{2}$$

Now, we can observe that if

$$\mathrm{a} = \frac{3}{4}$$

, then

$$z = \frac{-1}{2}$$

which will become purely real.

Hence, if

$$a \neq \frac{3}{4}$$

, otherwise

$$z$$

will be purely real.

i Make quadratic equation in

$$z$$

and apply Sridharacharya rule.

ii Analyse the value of '

$$a$$

' for which '

z

' becomes purely real.

Question 023 Numerical

QUESTION

If

$$\vec{a}, \vec{b}$$

and

$$\vec{c}$$

are unit vectors satisfying

$$\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9,$$

then

$$\left|2\vec{a}+5\vec{b}+5\vec{c}\right|$$

is

SOURCE

Mathematics • vector-algebra

EXPLANATION

$$\text{Given, } |\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$$
$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}+|\vec{b}|^2$$
$$|\vec{c}|^2-2\vec{b}\cdot\vec{c}+|\vec{c}|^2+|\vec{a}|^2-2\vec{c}\cdot\vec{a}=9$$
$$\Rightarrow 2\left(|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2\right)-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=9$$
$$\Rightarrow 2(1+1+1)-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=9$$
$$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=\frac{-3}{2} \quad \text{... (i)}$$
$$\text{Now, } |\vec{a}+\vec{b}+\vec{c}|^2\geq 0$$
$$\Rightarrow |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$$
$$\Rightarrow 1+1+1+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq 0$$
$$\Rightarrow (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})\geq\frac{-3}{2} \quad \text{... (ii)}$$

From i and ii, we get

$$\Rightarrow \quad (\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})=\tfrac{-3}{2}$$
$$\Rightarrow \quad |\vec{a}+\vec{b}+\vec{c}|=0$$
$$\Rightarrow \quad \vec{a}+\vec{b}+\vec{c}=0$$
$$\Rightarrow \quad \vec{b}+\vec{c}=-\vec{a} \quad \text{.... (iii)}$$

$$\therefore \quad |2\vec{a}+5\vec{b}+5\vec{c}|=|2\vec{a}+5(\vec{b}+\vec{c})|$$
$$=|2\vec{a}+5(-\vec{a})| \quad [\,[\text{from (iii)}]$$
$$=|-3\vec{a}|$$
$$=|3\vec{a}|$$
$$=3 \quad [\because \vec{a} \text{ is unit vector }]$$

Question 024 MCQ

QUESTION

A ship is fitted with three engines

$$E_1, E_2$$

and

$$E_3$$

. The engines function independently of each other with respective probabilities

$$\frac{1}{2}, \frac{1}{4}$$

and

$$\frac{1}{4}$$

. For the ship to be operational at least two of its engines must function. Let

$$X$$

denote the event that the ship is operational and Let

$$X_1, X_2$$

and

$$X_3$$

denote respectively the events that the engines

$$E_1, E_2$$

and

$$E_3$$

are functioning. Which of the following is are true?

A $$P\left[X_1^c | X\right] = \frac{3}{16}$$

B $$P$$

Missing or unrecognized delimiter for \right

= {7 \over 8}$$

C $$P\left[X|X_2\right]=\frac{5}{16}$$

D $$P\left[X|X_1\right]=\frac{7}{16}$$

CORRECT OPTION

D $$P\left[X|X_1\right]=\frac{7}{16}$$

SOURCE

Mathematics • probability

EXPLANATION

$$\text{Given, P}\left(\text{X}_1\right)=\frac{1}{2}\Rightarrow \text{P}\left(\overline{\text{X}}_1\right)=1-\frac{1}{2}=\frac{1}{2}$$

$$\text{P}\left(\text{X}_2\right)=\frac{1}{4}\Rightarrow \text{P}\left(\bar{X}_2\right)=1-\frac{1}{4}=\frac{3}{4}$$

$$\text{P}\left(\text{X}_3\right)=\frac{1}{4}\Rightarrow \text{P}\left(\bar{X}_3\right)=1-\frac{1}{4}=\frac{3}{4}$$

$$\text{Now, } P(X)=P\left(E_1E_2E_3\right)+P\left(\bar{E}_1E_2E_3\right)+P\left(E_1\overline{\text{E}}_2\text{E}_3\right)+\text{P}\left(\text{E}_1\text{E}_2\overline{\text{E}}_3\right)$$

$$=\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}$$

$$P(X)=\frac{1}{32}+\frac{1}{32}+\frac{3}{32}+\frac{3}{32}$$

$$P(X)=\frac{1}{4}$$

i

$$\mathrm{P}\left(\frac{\mathrm{X}_1^c}{\mathrm{X}}\right) = \frac{\mathrm{P}\left(\mathrm{X}_1^c \cap \mathrm{X}\right)}{\mathrm{P}(\mathrm{X})}$$

Now,

$$X_1^c \cap X$$

indicates the event that the ship is operational but

$$\mathrm{E}_1$$

is not functioning.

$$\therefore \quad \mathrm{P}\left(\mathrm{X}_1^c \cap \mathrm{X}\right) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$$

$$\therefore \mathrm{P}\left(\frac{\mathrm{X}_1^c}{\mathrm{X}}\right) = \frac{\frac{1}{32}}{\frac{1}{4}} = \frac{1}{8}$$

ii P $Exactlytwoenginesoftheshipare functioning\$\$ \mid X\$\$$

$$= \frac{\mathrm{P}\left(\overline{\mathrm{E}}_1\mathrm{E}_2\mathrm{E}_3\right) + \mathrm{P}\left(\mathrm{E}_1\overline{\mathrm{E}}_2\mathrm{E}_3\right) + 1\left(\mathrm{E}_1\mathrm{E}_2\overline{\mathrm{E}}_3\right)}{\mathrm{P}(\mathrm{X})}$$

$$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{4}}$$

$$= \frac{7}{8}$$

$$\text{(iii) } P\left(X \mid X_2\right) = \frac{P\left(X \cap X_2\right)}{P\left(X_2\right)}$$

$$= \frac{P\left(E_1E_2E_3\right) + P\left(\bar{E}_1E_2E_3\right) + P\left(E_1E_2\bar{E}_3\right)}{P\left(X_2\right)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{4}}$$

$$= \frac{5}{8}$$

$$
\begin{aligned}
\text{(iv) } P(X \mid X_1) &= \frac{P(X \cap X_1)}{P(X_1)} \\
&= \frac{P(E_1E_2E_3) + P\left(E_1\bar{E}_2E_3\right) + P\left(E_1E_2\bar{E}_3\right)}{P(X_1)} \\
&= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}}{\frac{1}{2}} \\
&= \frac{7}{16}
\end{aligned}
$$

Question 025 MCQ

QUESTION

The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

A 75

B 150

C 210

D 243

CORRECT OPTION

B 150

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

Here, 5 distinct balls are to be distributed amongst 3 persons so that each gets at least one ball. So, two possible cases arises

Case I : Two of the persons get one-one ball each and the third person gets three balls.

i.e.

A	B	C
1	1	3

Now, A can get the ball in

$$^5\mathrm{C}_1$$

ways. After that,

$$B$$

can get one ball in

$$^4C_1$$

ways and then after

$$C$$

can get three balls in

$$^3\mathrm{C}_3$$

ways.

Hence, Total number of ways

$$\begin{aligned}&= {}^5\mathrm{C}_1 \cdot {}^4\mathrm{C}_1 \cdot {}^3\mathrm{C}_3 \times \frac{3!}{2!} \\ &= 5 \times 4 \times 1 \times \frac{3 \times 2 \times 1}{2 \times 1} \\ &= 60\end{aligned}$$

Case II : Two of the persons get two-two balls each and the third person gets one ball.

i.e.

A	B	C
2	2	1

Now, A can get two balls in

$$^5\mathrm{C}_2$$

ways. After that,

$$B$$

can get 2 ball in

$$^3\mathrm{C}_2$$

ways and then after

$$C$$

can get 1 ball in

$$^1C_1$$

way. Hence, total number of ways

$$\begin{aligned}&= {}^5C_2 \cdot {}^3C_2 {}^1C_1 \cdot \frac{3!}{2!} \\ &= \frac{5 \times 4}{2 \times 1} \times \frac{3 \times 2}{2 \times 1} \times 1 \times \frac{3 \times 2 \times 1}{2 \times 1} \\ &= 90\end{aligned}$$

Hence, total number of ways to distribute 5 balls

$$= 60 + 90 = 150$$

Combination with Repetition

i Only two possible cases arises:

Case I :

A	B	C
1	1	3

Case II:

A	B	C
2	2	1

iii Use the concept of combination to find individual ways and then add up the total ways in each case.

Question 026 MCQ

QUESTION

The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line 4x - 5y = 20 to the circle

$$x^2 + y^2 = 9$$

is

A $20\left(x^2 + y^2\right) - 36x + 45y = 0$

B $20\left(x^2 + y^2\right) + 36x - 45y = 0$

C $36\left(x^2 + y^2\right) - 20x + 45y = 0$

D $36\left(x^2+y^2\right)+20x-45y=0$

CORRECT OPTION

A $20\left(x^2+y^2\right)-36x+45y=0$

SOURCE

Mathematics • circle

EXPLANATION

Equation of the chord AB is

$$\mathrm{T}=0$$

$$\Rightarrow \alpha x+\left(\frac{4\alpha-20}{5}\right)y=9 \quad \ldots(\mathrm{i})$$

Also, equation of the chord AB whose mid-point is

$$(h,k)$$

is

$$\mathrm{T}=\mathrm{S}_1$$

$$\begin{aligned}\Rightarrow \quad & hx+ky-9=h^2+k^2-9\\ \Rightarrow \quad & hx+ky=h^2+k^2 \quad \ldots(\mathrm{ii})\end{aligned}$$

$$\because$$

Equations i and ii both represent the same line

$$\therefore \frac{\alpha}{h}=\frac{\frac{4\alpha-20}{5}}{k}=\frac{9}{h^2+k^2}$$

$$\Rightarrow \alpha=\frac{9h}{h^2+k^2} \quad \text{and} \quad \frac{4\alpha-20}{5}=\frac{9k}{h^2+k^2}$$

$$\Rightarrow \alpha = \frac{9h}{h^2+k^2} \quad \text{and} \quad 4\alpha = \frac{45k}{h^2+k^2} + 20$$

$$\Rightarrow \alpha = \frac{9h}{h^2+k^2} \quad \text{and} \quad \alpha = \frac{45k + 20\left(h^2+k^2\right)}{4\left(h^2+k^2\right)}$$

$$\Rightarrow \quad \frac{9h}{h^2+k^2} = \frac{45k+20\left(h^2+k^2\right)}{4\left(h^2+k^2\right)}$$

$$\Rightarrow \quad 36h = 45k + 20\left(h^2+k^2\right)$$

$$\Rightarrow 20\left(x^2+y^2\right) - 36x + 45y = 0$$

, which is the required locus.

Question 027 MCQ

QUESTION

The ellipse

$$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$$

is inscribed in a rectangle

$$R$$

whose sides are parallel to the coordinate axes. Another ellipse

$$E_2$$

passing through the point

$$(0, 4)$$

circumscribes the rectangle

$$R$$

. The eccentricity of the ellipse

$$E_2$$

is

A $\frac{\sqrt{2}}{2}$

B $\frac{\sqrt{3}}{2}$

C $\frac{1}{2}$

D $\frac{3}{4}$

CORRECT OPTION

C $\frac{1}{2}$

SOURCE

Mathematics • ellipse

EXPLANATION

Let the equation of ellipse

$$E_2$$

is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

.

Now, it is given that

$$\mathrm{E}_2$$

passes through

$$(0,4)$$

$$\begin{aligned}\Rightarrow \quad \frac{0^2}{a^2}+\frac{4^2}{b^2}&=1\\ b^2&=16\end{aligned}$$

Also,

$$\mathrm{E}_2$$

passes through

$$(\pm 3,\pm 2)$$

$$\begin{aligned}\Rightarrow \quad \frac{3^2}{a^2}+\frac{2^2}{b^2}&=1\\ \Rightarrow \quad \frac{9}{a^2}+\frac{4}{16}&=1\\ \Rightarrow \quad \frac{9}{a^2}&=\frac{3}{4}\\ \Rightarrow \quad a^2&=12\\ \text{Now,} \quad b^2&>a^2\\ \Rightarrow \quad b&>a\\ \therefore \quad a^2&=b^2\left(1-e^2\right)\\ \Rightarrow \quad 12&=16\left(1-e^2\right)\\ \Rightarrow \quad \frac{12}{16}&=1-e^2\\ \Rightarrow \quad e^2&=1-\frac{3}{4}\\ \Rightarrow \quad e^2&=\frac{1}{4}\\ \Rightarrow \quad e&=\frac{1}{2}\end{aligned}$$

Question 028 MCQ

QUESTION

Tangents are drawn to the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{4} = 1,$$

parallel to the straight line

$$2x - y = 1,$$

The points of contact of the tangents on the hyperbola are

A $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

B $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

C $\left(3\sqrt{3}, -2\sqrt{2}\right)$

D $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

CORRECT OPTION

B

$$\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

SOURCE

Mathematics • hyperbola

EXPLANATION

Since, the tangents are parallel to the straight line

$$2x - y = 1$$

.

$$\therefore$$

Equation of the tangents is

$$\Rightarrow \quad \begin{aligned} 2x - y + \mathrm{C} &= 0 \\ y &= 2x + \mathrm{C} \end{aligned}$$

$$\Rightarrow$$

Slope of the tangents is

$$m = 2$$

.

Now, from the standard equation of given hyperbola.

$$\begin{aligned} a^2 = 9, b^2 = 4 \\ \therefore \quad C^2 &= a^2 m^2 - b^2 \\ \Rightarrow \quad C^2 &= 9 \times 2^2 - 4 \\ \Rightarrow \quad C^2 &= 32 \\ \Rightarrow \quad C &= \pm 4\sqrt{2} \end{aligned}$$

$$\therefore$$

Point of contact is

$$\left(\pm\frac{a^2m}{\mathrm{C}},\pm\frac{b^2}{\mathrm{C}}\right)$$

i.e.

$$\left(\pm\frac{9\times 2}{4\sqrt{2}},\pm\frac{4}{4\sqrt{2}}\right)$$

i.e.

$$\left(\pm\frac{9}{2\sqrt{2}},\pm\frac{1}{\sqrt{2}}\right)$$

Question 029 Numerical

QUESTION

Let

$$S$$

be the focus of the parabola

$$y^2=8x$$

and let

$$PQ$$

be the common chord of the circle

$$x^2+y^2-2x-4y=0$$

and the given parabola. The area of the triangle

$$PQS$$

is

SOURCE

Mathematics • parabola

EXPLANATION

Given, circle is

$$x^2 + y^2 - 2x - 4y = 0$$

and parabola

$$y^2 = 8x$$

.

$$\because$$

Both the curves intersect each other at P.

$$\begin{aligned}
\because \quad & x^2 + 8x - 2x - 4 \cdot 2\sqrt{2x} = 0 \\
\Rightarrow \quad & x^2 + 6x - 8\sqrt{2x} = 0 \\
\Rightarrow \quad & \sqrt{x}\left[x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - 8\sqrt{2}\right] = 0 \\
\Rightarrow \quad & \text{Let } \sqrt{x} = t \\
\therefore \quad & t\left[t^3 + 6t - 8\sqrt{2}\right] = 0 \\
\Rightarrow \quad & t(t - \sqrt{2})\left(t^2 - \sqrt{2}t + 4\right) = 0 \\
\Rightarrow \quad & t = 0 \text{ or } t = \sqrt{2} \text{ or } t = \frac{\sqrt{2} \pm \sqrt{2 - 4(4)}}{2}
\end{aligned}$$

$rejectedbecauseitisimaginary$

$$\begin{aligned}
&\Rightarrow t = 0 \quad \text{or } t = \sqrt{2} \\
&\Rightarrow x = 0 \quad \text{or } x = 2 \\
&\Rightarrow y = 0 \quad \text{or } y = 4
\end{aligned}$$

Hence, the required coordinates are

$$\mathrm{P}(2, 4), Q(0, 0)$$

and

$$S(2, 0)$$

.

$$\therefore$$

Area of

$$\triangle \mathrm{PQS} = \frac{1}{2} \times 2 \times 4 = 4$$

Question 030 Numerical

QUESTION

Let

$$f : IR \to IR$$

be defined as

$$f(x) = |x| + \left|x^2 - 1\right|.$$

The total number of points at which

$$f$$

attains either a local maximum or a local minimum is

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

$$\text{Given, } f(x) = |x| + \left|x^2 - 1\right|$$
$$\Rightarrow f(x) = |x| + |x-1||x+1|$$

$$
\begin{aligned}
f(x) &= x^2 + x - 1, x \geq 1 \\
&= 1 - x^2 + x, 0 \leq x < 1 \\
&= 1 - x^2 - x, -1 < x < 0 \\
&= x^2 - x - 1, x \leq -1 \\
f'(x) &= 2x + 1 \quad (+ve) \\
&= 1 - 2x \quad x > \frac{1}{2}(+ve) \\
&= -2x - 1 \quad x > -\frac{1}{2}(-ve); x < \frac{-1}{2}(+ve) \\
&= 2x - 1 \quad (-ve)
\end{aligned}
$$

From the figure, total number of points at which

$$f$$

attains either a local maximum or a local minimum is 5.

Question 031 Numerical

QUESTION

Let

$$p(x)$$

be a real polynomial of least degree which has a local maximum at

$$x = 1$$

and a local minimum at

$$x = 3$$

. If

$$p(1) = 6$$

and

$$p(3) = 2$$

, then

$$p'(0)$$

is

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Since,

$$p(x)$$

has a local maximum at

$$x = 1$$

and

$$a$$

local minimum at

$$x = 3$$

.

$$\begin{aligned}
&\text{Let } p'(x) = k(x-1)(x-3) \\
&\Rightarrow p'(x) = k\left(x^2 - 4x + 3\right) \\
&\Rightarrow p(x) = k\left(\frac{x^3}{3} - \frac{4x^2}{2} + 3x\right) + c \\
&\Rightarrow p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c
\end{aligned}$$

Now,

$$p(1) = 6$$

$$\Rightarrow k\left(\frac{1^3}{3} - 2(1)^2 + 3(1)\right) + c = 6$$

$$\begin{aligned}
\Rightarrow \quad k\left(\tfrac{1}{3} - 2 + 3\right) + c &= 6 \\
\Rightarrow \qquad \tfrac{4}{3}k + c &= 6
\end{aligned}$$

Also,

$$p(3) = 2$$

$$\Rightarrow \quad k\left(\frac{3^3}{3} - 2(3)^2 + 3(3)\right) + c = 2$$
$$\Rightarrow \quad k(0) + c = 2$$
$$\Rightarrow \quad c = 2$$

$$\text{Now,} \quad \frac{4}{3}k + 2 = 6$$
$$\Rightarrow \quad \frac{4}{3}k = 4$$

$$\Rightarrow \quad k = 3$$
$$\therefore \quad p'(0) = 3\left(0^2 - 4(0) + 3\right)$$
$$= 9$$

Question 032 MCQ

QUESTION

The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals $for some arbitrary constant \$\$K\$\$$

A $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

B $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

C $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

D $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

CORRECT OPTION

C $-\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

SOURCE

Mathematics • indefinite-integrals

EXPLANATION

We have

$$I=\int \frac{\sec^2 x}{(\sec x+\tan x)^{9/2}}dx$$

Let $\sec x+\tan x=t$

$\Rightarrow \sec x-\tan x=1/t.$

$\Rightarrow$

$\frac{1}{2}\left(t+\frac{1}{t}\right)=\sec x$

Now, $\left(\sec x\tan x+\sec^2 x\right)dx=dt$

$\Rightarrow$

$\sec x(\sec x+\tan x)dx=dt$

$\Rightarrow$

$\sec x dx=\frac{dt}{t}$

$I=\frac{1}{2}\int\frac{\left(t+\frac{1}{t}\right)}{t^{9/2}}\frac{dt}{t}$

$=\frac{1}{2}\int\left(t^{-9/2}+t^{-13/2}\right)dt$

$$= \frac{1}{2}\left[\frac{t^{-\frac{9}{2}+1}}{-\frac{9}{2}+1} + \frac{t^{-\frac{13}{2}+1}}{-\frac{13}{2}+1}\right]$$

$$= \frac{1}{2}\left[\frac{t^{-7/2}}{-\frac{7}{2}} + \frac{t^{-11/2}}{-\frac{11}{2}}\right]$$

$$= -\frac{1}{7}t^{-7/2} - \frac{1}{11}t^{-11/2}$$

$$= -\frac{1}{7}\frac{1}{t^{7/2}} - \frac{1}{11}\frac{1}{t^{11/2}}$$

$$= -\frac{1}{t^{11/2}}\left(\frac{1}{11} + \frac{t^2}{7}\right)$$

$$= -\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$$

Question 033 MCQ

QUESTION

Let

$$S$$

be the area of the region enclosed by

$$y = e^{-x^2}$$

,

$$y = 0$$

,

$$x = 0$$

, and

$$x = 1$$

; then

A $$S \geq \frac{1}{e}$$

B $$S \geq 1 - \frac{1}{e}$$

C $$S \leq \frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right)$$

D $$S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$$

CORRECT OPTION

A $$S \geq \frac{1}{e}$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

$$\begin{aligned}
&& S &= \int_0^1 e^{-x^2} dx \\
\text{Now,} && -x^2 &\leq 0 \\
\Rightarrow && e^{-x^2} &\leq 1 \\
\Rightarrow && \int_0^1 e^{-x^2} dx &\leq 1 \\
\text{Now,} && x &< 1 \\
\Rightarrow && x^2 &\leq x \\
\Rightarrow && -x^2 &\geq -x \\
\Rightarrow && e^{-x^2} &\geq e^{-x} \\
\Rightarrow && S &\geq \int_0^1 e^{-x} dx
\end{aligned}$$

$$\begin{aligned}
\Rightarrow && S &\geq -(e^{-x})_0^1 \\
\Rightarrow && S &\geq -\left(\tfrac{1}{e} - 1\right) \\
\Rightarrow && S &\geq 1 - \tfrac{1}{e} \Rightarrow (B) \text{ is correct.} \\
\text{Since,} && S &\geq 1 - \tfrac{1}{e} \\
\Rightarrow && S &> \tfrac{1}{e} \Rightarrow (\mathrm{A}) \text{ is correct.}
\end{aligned}$$

Now, Area of rectangle OAPQ + Area of rectangle QBRS > S

$$\Rightarrow \mathrm{S} < \frac{1}{\sqrt{2}}(1) + \left(1 - \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{e}}\right) \Rightarrow (\mathrm{D})$$

is correct.

Since,

$$\frac{1}{4}\left(1 + \frac{1}{\sqrt{e}}\right) < 1 - \frac{1}{e}$$

Hence,

$$(\mathrm{C})$$

is incorrect.

Question 034 MCQ

QUESTION

If

$$y(x)$$

satisfies the differential equation

$$y' - y\,tan\,x = 2x\,secx$$

and

$$y(0) = 0,$$

then

A

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$$

B

$$y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$$

C

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$$

D

$$y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

CORRECT OPTION

D

$$y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$$

SOURCE

Mathematics • differential-equations

EXPLANATION

Given,

$$\frac{dy}{dx} - y\tan x = 2x\sec x$$

Comparing with the liner differential form

$$\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q}$$
$$\Rightarrow \mathrm{P} = -\tan x \text{ and } \mathrm{Q} = 2x\sec x$$

Now, Integrating factor $I.F$

$$= e^{\int P dx}$$
$$= e^{\int -\tan x dx}$$
$$= e^{-\log\sec x}$$
$$= \cos x$$

$$\text{The solution will be } y\ (\text{I.F})\ = \int Q\ (\text{I.F})\ dx$$
$$\Rightarrow \quad y\cos x = \int 2x\sec x \cdot \cos x dx$$
$$\Rightarrow \quad y\cos x = 2\frac{x^2}{2} + c$$
$$\Rightarrow \quad y = x^2\sec x + c\sec x$$

Now,

$$y(0) = 0$$

$$\Rightarrow \quad 0 = 0^2 \sec 0 + \text{csec}0$$
$$\Rightarrow \quad c = 0$$
$$\therefore \quad y = x^2 \sec x$$
$$y' = x^2 \sec x \cdot \tan x + 2x \sec x \quad [\text{using Product Rule in Differentiation}]$$

Now,

$$y\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)^2 \sec\frac{\pi}{4} = \frac{\pi^2}{16}\sqrt{2} = \frac{\pi^2}{8\sqrt{2}}$$
$$y'\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4}\right)^2 \sec\frac{\pi}{4}\tan\frac{\pi}{4} + 2\cdot\frac{\pi}{4}\sec\frac{\pi}{4}$$
$$= \frac{\pi^2}{16}\cdot\sqrt{2}\cdot 1 + \frac{\pi}{2}\cdot\sqrt{2}$$
$$= \frac{\pi^2}{8\sqrt{2}} + \frac{\pi}{\sqrt{2}}$$
$$y\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)^2 \sec\frac{\pi}{3}$$
$$= \frac{\pi^2}{9}\cdot 2 = \frac{2\pi^2}{9}$$
$$y'\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)^2 \sec\frac{\pi}{3}\cdot\tan\frac{\pi}{3} + 2\cdot\frac{\pi}{3}\cdot\sec\frac{\pi}{3}$$
$$= \frac{\pi^2}{9}\cdot 2\cdot\sqrt{3} + \frac{2\pi}{3}\cdot 2$$
$$= \frac{2\pi^2}{3\sqrt{3}} + \frac{4\pi}{3}$$

Question 035 MCQ

QUESTION

The point

$$P$$

is the intersection of the straight line joining the points

$$Q(2,3,5)$$

and

$$R(1,-1,4)$$

with the plane

$$5x-4y-z=1.$$

If

$$S$$

is the foot of the perpendicular drawn from the point

$$T(2,1,4)$$

to

$$QR,$$

then the length of the line segment

$$PS$$

is

A $\frac{1}{\sqrt{2}}$

B $\sqrt{2}$

C 2

D $2\sqrt{2}$

CORRECT OPTION

A $$\frac{1}{\sqrt{2}}$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Equation of line QR is

$$\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-5}{1}=\lambda$$

General point on QR will be

$$(\lambda+2, 4\lambda+3, \lambda+5)$$

Let

$$P \equiv (\lambda+2, 4\lambda+3, \lambda+5)$$

Since, point P also lies on the plane.

$$\begin{aligned} \therefore \quad 5(\lambda+2)-4(4\lambda+3)-(\lambda+5) &= 1 \\ \Rightarrow \quad 5\lambda+10-16\lambda-12-\lambda-5 &= 1 \\ \Rightarrow \quad -12\lambda-7 &= 1 \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad \lambda &= \tfrac{-8}{12} \\ \Rightarrow \quad \lambda &= \tfrac{-2}{3} \end{aligned}$$

Hence,

$$\begin{aligned} P &\equiv \left(\frac{-2}{3}+2, 4\left(\frac{-2}{3}\right)+3, \frac{-2}{3}+5\right) \\ &\equiv \left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right) \end{aligned}$$

Now, for TS to be perpendicular to QR,

$$\begin{aligned} & \lambda + 4(4\lambda + 2) + (\lambda + 1) = 0 \\ \Rightarrow \quad & \lambda\lambda + 16\lambda + 8 + \lambda + 1 = 0 \\ \Rightarrow \quad & 18\lambda = -9 \\ \Rightarrow \quad & \lambda = \tfrac{-1}{2} \end{aligned}$$

$$\therefore \quad \text{Point } S \equiv \left(\frac{-1}{2} + 2, 4\left(\frac{-1}{2}\right) + 3, \frac{-1}{2} + 5\right)$$

$$\equiv \left(\frac{3}{2}, 1, \frac{9}{2}\right)$$

$$\therefore$$

Length of line segment PS

$$= \sqrt{\left(\frac{4}{3} - \frac{3}{2}\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{13}{3} - \frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{1}{36} + \frac{4}{9} + \frac{1}{36}}$$

$$= \sqrt{\frac{1 + 16 + 1}{36}}$$

$$= \sqrt{\frac{18}{36}}$$

$$= \frac{1}{\sqrt{2}}$$

Question 036 MCQ

QUESTION

If

$$\lim_{x \to \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b\right) = 4$$

, then

A a = 1, b = 4

B a = 1, b = $-\frac{1}{4}$

C a = 2, b = $-\frac{1}{3}$

D a = 2, b = 3

CORRECT OPTION

B a = 1, b = $-\frac{1}{4}$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2+x+1-ax^2-ax-bx-b}{x+1}=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2(1-a)+x(1-a-b)+(1-b)}{x+1}=4$$

Here, we make degree of N^r = degree of D^r

$$\therefore$$

$$1 - a = 0$$

and

$$\lim_{x \to \infty} \frac{x(1 - a - b) + (1 - b)}{x + 1} = 4$$

$$\Rightarrow 1 - a - b = 4$$

$$\Rightarrow b = -4$$

Question 037 MCQ

QUESTION

Let

$$P = [a_{ij}]$$

be a 3

$$\times$$

3 matrix and let

$$Q = [b_{ij}]$$

, where

$$b_{ij} = 2^{i+j} a_{ij}$$

for

$$1 \leq i, j \leq 3$$

. If the determinant of P is 2, then the determinant of the matrix Q is

A 2^{10}

B 2^{11}

C 2^{12}

D 2^{13}

CORRECT OPTION

D 2^{13}

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Here,

$$P = [a_{ij}]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$Q = [b_{ij}]_{3\times 3} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$$

where,

$$b_{ij} = 2^{i+j} a_{ij}$$

$$\therefore$$

$$|Q| = \begin{vmatrix} 4a_{11} & 8a_{12} & 16a_{13} \\ 8a_{21} & 16a_{22} & 32a_{23} \\ 16a_{31} & 32a_{32} & 64a_{33} \end{vmatrix}$$

$$= 4 \times 8 \times 16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2a_{22} & 2a_{23} \\ 4a_{31} & 4a_{32} & 4a_{33} \end{vmatrix}$$

$$= 2^9 \times 2 \times 4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= 2^{12}.|P| = 2^{12}.2 = 2^{13}$$

Question 038 MCQ

QUESTION

Let

$$f(x) = \begin{cases} x^2 \left|\cos \frac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

x

$$\in$$

R, then f is

A differentiable both at x = 0 and at x = 2.

B differentiable at x = 0 but not differentiable at x = 2.

C not differentiable at x = 0 but differentiable at x = 2.

D differentiable neither at x = 0 nor at x = 2.

CORRECT OPTION

B differentiable at x = 0 but not differentiable at x = 2.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

To check differentiable at x = 0

$$R\{f'(0)\} = \lim_{h\to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h\to 0} \frac{h^2 \left|\cos \frac{\pi}{h}\right| - 0}{h}$$

$$= \lim_{h\to 0} h. \left|\cos \frac{\pi}{h}\right| = 0$$

$$L\{f'(0)\} = \lim_{h\to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h\to 0} \frac{h^2 \left|\cos\left(-\frac{\pi}{h}\right)\right| - 0}{-h}$$

$$= 0$$

So, fx is differentiable at x = 0

To check differentiability at x = 2

$$R\{f'(2)\} = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 \left|\cos\left(\frac{\pi}{2+h}\right)\right| - 0}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 . \cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 . \sin\left(\frac{\pi}{2} - \frac{\pi}{2+h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{(2+h)^2 . \sin\left(\frac{\pi h}{2(2+h)}\right)}{h. \frac{\pi}{2(2+h)} . \frac{\pi}{2(2+h)}}$$

$$\Rightarrow R\left(f'(2)\right) = \pi$$

$$L\left(f'(2)\right) = \lim_{h\to 0} \frac{f(2-h) - f(2)}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2.\left|\cos\frac{\pi}{2-h}\right| - 2^2.\left|\cos\frac{\pi}{2}\right|}{-h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2 - \left(-\cos\frac{\pi}{2-h}\right) - 0}{-h}$$

$$= \lim_{h\to 0} \frac{-(2-h)^2.\sin\left(\frac{\pi}{2} - \frac{\pi}{2-h}\right)}{h}$$

$$= \lim_{h\to 0} \frac{(2-h)^2.\sin\left(-\frac{\pi h}{2(2-h)}\right)}{h \times \frac{-\pi}{2(2-h)} \times \frac{-\pi}{2(2-h)}}$$

$$\Rightarrow L(f'(2)) = -\pi$$

Thus, fx is differentiable at x = 0 but not at x = 2.

Question 039 MCQ

QUESTION

The function

$$f : [0, 3] \to [1, 29]$$

, defined by

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

, is

A one-one and onto.

B onto but not one-one.

C one-one but not onto.

D neither one-one nor onto.

CORRECT OPTION

B onto but not one-one.

SOURCE

Mathematics • functions

EXPLANATION

The function

$$f : [0, 3] \to [1, 29]$$

, defined by

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

, is :

First, calculate the derivative of f :

$$f'(x) = 6x^2 - 30x + 36$$

Which simplifies to :

$$f'(x) = 6(x^2 - 5x + 6) = 6(x - 2)(x - 3)$$

For the given domain $[0, 3]$, the function $f(x)$ is both increasing and decreasing. Therefore, $f(x)$ is many-to-one.

Setting $f'(x) = 0$ gives the critical points :

$$x = 2, 3$$

We now evaluate the function at these points within the domain to determine the range :

$$f(0) = 1$$

$$f(2) = 29$$

$$f(3) = 28$$

Thus, the range of f is $[1, 29]$, indicating that the function is onto.

Question 040 Numerical

QUESTION

The value of

$$6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$$

is _____.

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

$$6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}\cdots}}}\right)$$

Let

$$\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{\ldots}}}=y$$

$$\therefore$$

$$y=\sqrt{4-\frac{1}{3\sqrt{2}}y}$$

$$\Rightarrow y^2+\frac{1}{3\sqrt{2}}y-4=0$$

$$\Rightarrow 3\sqrt{2}y^2+y-12\sqrt{2}=0$$

$$\therefore$$

$$y=\frac{-1\pm 17}{6\sqrt{2}}$$

or

$$y=\frac{8}{3\sqrt{2}}$$

Now,

$$6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}.y\right)=6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}.\frac{8}{3\sqrt{2}}\right)$$

$$=6+\log_{\frac{3}{2}}\left(\frac{4}{9}\right)=6+\log_{\frac{3}{2}}\left(\frac{3}{2}\right)^{-2}$$

$$=6-2.\log_{\frac{3}{2}}\left(\frac{3}{2}\right)=4$$

Question 041 MCQ

QUESTION

A cubical region of side

$$a$$

has its center at the origin. It encloses three fixed point charges,

$$-q$$

at

$$(0, -a/4, 0), +3q$$

at

$$(0, 0, 0)$$

and

$$-q$$

at

$$(0, +a/4, 0).$$

Choose the correct option s

A

The net electric flux crossing the plane

$$x = +a/2$$

is equal to the net electric flux crossing the plane

$$x = -a/2$$

B

The net electric flux crossing the plane

$$y = +a/2$$

is more than the net electric flux crossing the plane

$$y = -a/2.$$

C

The net electric flux crossing the entire region is

$$\frac{q}{\varepsilon_0}$$

D

The net electric flux crossing the plane

$$z = +a/2$$

is equal to the net electric flux crossing the plane

$$x = +a/2.$$

CORRECT OPTION

A

The net electric flux crossing the plane

$$x = +a/2$$

is equal to the net electric flux crossing the plane

$$x = -a/2$$

SOURCE

Physics • electrostatics

EXPLANATION

The positions of all charges are symmetric about the planes

$$x = +\frac{a}{2}$$

and

$$x = -\frac{a}{2}$$

. Therefore, the net electric flux crossing the plane

$$x = +\frac{a}{2}$$

is equal to the net electric flux crossing the plane

$$x = -\frac{a}{2}$$

.

Similarly, the net electric flux crossing the plane

$$y = +\frac{a}{2}$$

is equal to the net electric flux crossing the plane

$$y = -\frac{a}{2}$$

.

According to Gauss's law, the net electric flux crossing the entire region is given by:

$$\phi = \frac{q_{\text{inside}}}{\varepsilon_0} = \frac{3q - q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

The charges are symmetrically placed about the planes

$$z = +\frac{a}{2}$$

and

$$x = +\frac{a}{2}$$

. Thus, the net electric flux crossing the plane

$$z = +\frac{a}{2}$$

is equal to the net electric flux crossing the plane

$$x = +\frac{a}{2}$$

.

Question 042 MCQ

QUESTION

Consider a thin spherical shell of radius

$$R$$

with center at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field

$$\left|\vec{E}(r)\right|$$

and the electric potential

$$V(r)$$

with the distance

$$r$$

from the center, best represented by which graph?

A

B

C

D

CORRECT OPTION

D

SOURCE

Physics • electrostatics

EXPLANATION

Consider a thin spherical shell of radius R with its center at the origin, carrying a uniform positive surface charge density. To understand how the electric field $\left|\overrightarrow{E}(r)\right|$ and the electric potential $V(r)$ vary with the distance r from the center, refer to the following details:

Electric Field due to a Uniformly Charged Thin Spherical Shell

Inside the Shell

For $r < R$, the electric field is given by:

$$E_{\text{inside}} = 0 \quad [r < R]$$

On the Surface of the Shell

For $r = R$, the electric field is:

$$E_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

Outside the Shell

For $r > R$, the electric field is:

$$E_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \quad [r > R]$$

Below is a graphical representation of the electric field E variation with the distance r from the center:

Electric Potential due to a Uniformly Charged Thin Spherical Shell

Inside the Shell

For $r < R$, the electric potential is:

$$V_{\text{inside}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

On the Surface of the Shell

For $r = R$, the electric potential is:

$$V_{\text{surface}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Outside the Shell

For $r > R$, the electric potential is:

$$V_{\text{outside}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Below is a graphical representation of the electric potential V variation with the distance r from the center:

Question 043 MCQ

QUESTION

Two large vertical and parallel metal plates having a separation of

$$1$$
$$cm$$

are connected to a

$$DC$$

voltage source of potential difference

$$X$$

. A proton is released at rest midway between the two plates. It is found to move at

$$45^\circ$$

to the vertical JUST after release. Then

$$X$$

is nearly

A $1 \times 10^{-5}\ V$

B $1 \times 10^{-7}\ V$

C $1 \times 10^{-9}\ V$

D $1 \times 10^{-10}\ V$

CORRECT OPTION

C $1 \times 10^{-9}\ V$

SOURCE

Physics • electrostatics

EXPLANATION

Given that the proton moves at a 45° angle to the vertical, we can derive the electric field E as follows :

$qE = mg$ or $E = \frac{mg}{q}$

Since $E = \frac{X}{d}$, we can substitute to find X :

$\frac{X}{d} = \frac{mg}{q}$ or $X = \frac{mgd}{q}$

Where :

- $m = 1.67 \times 10^{-27}\,\text{kg}$
- $g = 10\,\text{m/s}^2$
- $d = 1\,\text{cm} = 1 \times 10^{-2}\,\text{m}$
- $q = 1.6 \times 10^{-19}\,\text{C}$

Substituting these values, we get :

$$X = \frac{1.67 \times 10^{-27} \times 10 \times 1 \times 10^{-2}}{1.6 \times 10^{-19}}\,\text{V}$$

$$= \frac{1.67}{1.6} \times 10^{-9}\,\text{V}$$

$$= 1 \times 10^{-9}\,\text{V}$$

Question 044 MCQ

QUESTION

A mixture of 2 moles of helium gas $atomicmass = 4amu$ and 1 mole of argon gas $atomicmass = 40amu$ is kept at 300 K in a container. The ratio of the rms speeds

$$\left(\frac{v_{rms}\,(helium)}{v_{rms}\,(\text{arg}\,on)}\right)$$

is

A 0.32

B 0.45

C 2.24

D 3.16

CORRECT OPTION

D 3.16

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

The root mean square rms speed of a gas is given by the formula :

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

where :

- k is the Boltzmann constant
- T is the temperature in Kelvin
- m is the mass of one molecule of the gas

Given that the atomic mass of helium is 4 amu and that of argon is 40 amu, we can find the rms speed ratio by comparing the rms speeds of helium and argon, as temperature

$$T$$

and Boltzmann constant

$$k$$

are the same for both gases :

$$\frac{v_{rms}\,(\text{helium})}{v_{rms}\,(\text{argon})} = \sqrt{\frac{m_{\text{argon}}}{m_{\text{helium}}}}$$

Substituting the masses :

$$m_{\text{helium}} = 4 \text{ amu}$$

$$m_{\text{argon}} = 40 \text{ amu}$$

we get :

$$\frac{v_{rms}\,(\text{helium})}{v_{rms}\,(\text{argon})} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

Thus, the ratio of the rms speeds of helium to argon is approximately 3.16.

The correct answer is :

Option D : 3.16

Question 045 MCQ

QUESTION

In the determination of Young's modulus

$$\left(Y = \frac{4MLg}{\pi l d^2}\right)$$

by using Searle's method, a wire of length L = 2 m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of

divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

A due to the errors in the measurements of d and l are the same.

B due to the error in the measurement of d is twice that due to the error in the measurement of l.

C due to the error in the measurement of l is twice that due to the error in the measurement of d.

D due to the error in the measurement of d is four times that due to the error in the measurement of l.

CORRECT OPTION

A due to the errors in the measurements of d and l are the same.

SOURCE

Physics • units-and-measurements

EXPLANATION

To determine Young's modulus using Searle's method, we use the formula:

$$Y = \frac{4MLg}{\pi l d^2}$$

Given:
Length of wire, $L = 2\ \text{m}$
Diameter of wire, $d = 0.5\ \text{mm}$
Load, $M = 2.5\ \text{kg}$
Extension in length, $l = 0.25\ \text{mm}$

The quantities d and l are measured using a screw gauge and a micrometer, respectively, both having a pitch of 0.5 mm and 100 divisions on their circular scale. The least count, which is the smallest measurement that can be accurately read, is:

$$\text{Least count} = \frac{\text{Pitch}}{\text{Total number of divisions on the circular scale}} = \frac{0.5 \text{ mm}}{100} =$$

Hence, the least count, Δd and Δl, are both 0.005 mm.

Error in measurement of l $extensionlength$ is:

$$\frac{\Delta l}{l} = \frac{0.005}{0.25} = \frac{1}{50}$$

Error in measurement of d $diameter$ is:

$$\frac{\Delta d}{d} = \frac{0.005}{0.5} = \frac{1}{100}$$

Using the formula for Young's modulus, the combined error is:

$$Y = \frac{4MLg}{\pi l d^2}$$

The relative error in Y is given by:

$$\frac{\Delta Y}{Y} = \frac{\Delta l}{l} + 2\frac{\Delta d}{d}$$

As can be seen:

$$\frac{\Delta l}{l} = 2\frac{\Delta d}{d} = \frac{1}{50}$$

Therefore, the contributions to the error in the measurement of Y from errors in measuring l and d are the same.

Question 046 MCQ

QUESTION

A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the xy-plane with centre at O and constant angular speed

$$\omega$$

. If the angular momentum of the system, calculated about O and P are denoted by

$$\vec{L}_O$$

and

$$\vec{L}_P$$

, respectively, then

A $\vec{L}_O$ and $\vec{L}_P$ do not vary with time.

B $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

$\vec{L}_O$

C remains constant while

$$\overrightarrow{L_P}$$

varies with time.

$$\overrightarrow{L_O}$$

D and

$$\overrightarrow{L_P}$$

both vary with time.

CORRECT OPTION

$$\overrightarrow{L_O}$$

C remains constant while

$$\overrightarrow{L_P}$$

varies with time.

SOURCE

Physics • rotational-motion

EXPLANATION

Angular momentum of a particle about a point is given by:

L = r

$$\times$$

p = m $r$$$ \times $$$v$

For L_O

$$|L| = (mvr\sin\theta) = m(R\omega)(R)\sin 90^\circ$$

$$= mR^2\omega$$

= constant

Direction of L_O is always upwards. Therefore, complete L_O is constant, both in magnitude as well as direction.

For L_P

$$|L_P| = (mvr\sin\theta) = m(R\omega)(l)\sin 90^\circ$$

$$= (mRl\omega)$$

Magnitude of L_P will remain constant but direction of L_P keeps on changing.

Question 047 MCQ

QUESTION

A biconvex lens is formed with two planoconvex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both curved surface are of the same radius of curvature R = 14 cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be

A 280.0 cm

B 40.0 cm

C 21.5 cm

D 13.3 cm

CORRECT OPTION

B 40.0 cm

SOURCE

Physics • geometrical-optics

EXPLANATION

According to lens maker's formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the first lens

n = 1.5, R_1 = + 14 cm, R_2 =

$$\infty$$

$$\therefore$$

$$\frac{1}{f_1} = (1.5-1)\left(\frac{1}{14} - \frac{1}{\infty}\right) = \frac{0.5}{14}$$

For the second lens,

n = 1.2, R_1 =

$$\infty$$

, R_2 =

$$-$$

14 cm

$$\therefore$$

$$\frac{1}{f_2} = (1.2-1)\left(\frac{1}{\infty} - \frac{1}{-14}\right) = \frac{0.2}{14}$$

The focal length of the bi-convex lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{0.5}{14} + \frac{0.2}{14} = \frac{0.7}{14} = \frac{1}{20}$$

According to thin lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Here, u =

$$-$$

40 cm

$$\therefore$$

$$\frac{1}{v} - \frac{1}{-40} = \frac{1}{20}$$

or

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

or v = 40 cm

Question 048 MCQ

QUESTION

A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed

$$\omega$$

, as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v, with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains

$$\omega$$

throughout. The magnitude of the torque $|\$\$\tau\$\$|$ about O, as a function of time is best represented by which plot ?

A

B

C

D

CORRECT OPTION

B

SOURCE

Physics • rotational-motion

EXPLANATION

Let M and l be the mass and length of the rod respectively and m be the mass of the insect. Let the insect be at a distance x from O at any instant of time t.

$$\therefore$$

x = vt i

Angular momentum of the system about O,

$$L = \left[\frac{Ml^2}{12} + mx^2\right]\omega$$

$$= \left[\frac{Ml^2}{12} + m(vt)^2\right]\omega$$

$Using (i)$

$$= \left[\frac{Ml^2}{12} + mv^2t^2\right]\omega$$

As,

$$\left|\vec{\tau}\right| = \frac{dL}{dt} = \frac{d}{dt}\left[\frac{Ml^2}{12} + mv^2t^2\right]\omega$$

As

$$\omega$$

and v remain constant

$$\therefore$$

$$\left|\vec{\tau}\right| = 2m\omega v^2 t$$

$$\left|\vec{\tau}\right| \propto t$$

Hence, the graph and t is a straight line passing through the $0, 0$. Option b represents correct plot.

Question 049 MCQ

QUESTION

Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductively. The first and third plates are maintained at temperatures 2T and 3T, respectively. The temperature of the middle $i.e. second$ plate under steady state condition is

A $\left(\frac{65}{2}\right)^{1/4} T$

B $\left(\frac{97}{4}\right)^{1/4} T$

C $\left(\frac{97}{2}\right)^{1/4} T$

D $(97)^{1/4} T$

CORRECT OPTION

C $\left(\frac{97}{2}\right)^{1/4} T$

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Let the steady state temperature of the middle plate to T_0. In the steady state, the heat radiated per unit time by the middle plate is equal to the heat received per unit time by it. The middle plate radiates heat from both the surfaces and receives the heat radiated by the first and the third plates.

Stefan-Boltzmann law gives the rate of heat loss and heat gain by the middle plate as

$$dQ_{out}/dt = \sigma A T_0^4 + \sigma A T_0^4$$

,

$$dQ_{in}/dt = \sigma A(2T)^4 + \sigma A(3T)^4$$

.

The steady state condition,

$$dQ_{out}/dt = dQ_{in}/dt$$

$$\sigma A(2T)^4 + \sigma A(3T)^4 = \sigma 2A(T_0)^4$$

$$16T^4 + 81T^4 = 2(T_0)^4$$

$$97T^4 = 2(T_0)^4$$

$$(T_0)^4 = \frac{97}{2}T^4 = \left(\frac{97}{2}\right)^{1/4}T$$

Question 050 MCQ

QUESTION

A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring $see the figure$ is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency

$$\omega$$

= $$$$\pi$$$$/3 rad/s. Simultaneously, at t = 0, a small pebble is projected with speed v from point P at an angle of 45

$$\circ$$

as shown in the figure. Point O is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2)

A $\sqrt{50}$ m/s

B $\sqrt{51}$ m/s

C $\sqrt{52}$ m/s

D $\sqrt{53}$ m/s

CORRECT OPTION

A $\sqrt{50}$ m/s

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Since, the block starts executing simple harmonic motion from extreme position, we have

$$x = a\cos\omega t$$

At 1 s, we have

$$x = 0.2\cos\left(\frac{\pi}{3}\right) = 0.1\,m$$

That is, the block is at a distance 5.1

$$-$$

0.10 = 5 m from O, which is the range of the bebble as well. Now,

$$R = \frac{v^2 \sin 2\alpha}{g}$$

$$S = \frac{v^2 \sin 2(\pi/4)}{10}$$

Therefore,

$$v = \sqrt{50}$$

m/s

Alternate Method :

Since, pebble strikes the oscillating mass after 1 sec., its time of flight is $1\ \mathrm{s}$

$$1 = \frac{2v \sin 45^\circ}{\mathrm{g}} = \frac{\sqrt{2}v}{10}$$

$$v = \frac{10}{\sqrt{2}} = 5\sqrt{2} = \sqrt{50}\ \mathrm{m/s}$$

Question 051 MCQ

QUESTION

Young's double slit experiment is carried out by using green, red and blue light, one colour at a time. The fringe widths recorded are

$$\beta$$

G,

$$\beta$$

R and

β

B, respectively. Then,

A

β

G >

β

B >

β

R

B

β

B >

β

G >

β

R

C

β

R >

β

B >

β

G

D

β

R >

β

G >

β

B

CORRECT OPTION

D

β

R >

β

G >

β

B

SOURCE

Physics • wave-optics

EXPLANATION

Fringe width,

$$\beta = \frac{\lambda D}{d}$$

$$\therefore$$

$$\beta \propto \lambda$$

As,

$$\lambda_R > \lambda_G > \lambda_B$$

$$\therefore$$

$$\beta_R > \beta_G > \beta_B$$

Question 052 MCQ

QUESTION

Consider the motion of a positive point charge in a region, there are simultaneous uniform electric and magnetic fields

$$\overrightarrow{E} = E_0\hat{j}$$

and

$$\overrightarrow{B} = B_0\hat{j}$$

. At time t = 0, this charge has velocity

$$\overrightarrow{v}$$

in the xy-plane, making an angle

$$\theta$$

with the x-axis. Which of the following option s is are correct for time t > 0 ?

A If $\theta = 0^\circ$, the charge moves in a circular path in the xy-plane.

If $\theta = 0^\circ$, the charge undergoes helical motion with constant pitch along the y-axis.

If $\theta = 10^\circ$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

If $\theta = 90^\circ$, the charge undergoes linear but accelerated motion along the y-axis.

CORRECT OPTION

If $\theta = 10^\circ$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis.

SOURCE

Physics • magnetism

EXPLANATION

For θ = 90 $^\circ$:

$$\vec{v} || \vec{E}$$

and

$$\vec{v} || \vec{B}$$

. Therefore, magnetic field $\vec{B}$ exerts no force on the point charge :

$$\vec{F_B} = q(\vec{v} \times \vec{B}) = 0$$

$$\vec{F_E} = q\vec{E}$$

Therefore, the motion is along y-axis and the particle accelerates.

For θ = 0 $^\circ$ and

θ

= 10

$^{\circ}$

: The motion is helical with increasing pitch, along with increasing time $along y-axis$, as

$$p = (v\cos\theta)t$$

, where v increases due to

$$\overrightarrow{F_E}$$

. $For\$\$\theta\$\$ = 0\$\$^{\circ}\$\$, the change cannot move in a circular path.$

Question 053 MCQ

QUESTION

A person blows into the open end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe,

A. a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open.

B. a low-pressure pulse starts travelling up the pipe, it the other end of the pipe is open.

C. a low-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

D a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed.

CORRECT OPTION

B a low-pressure pulse starts travelling up the pipe, it the other end of the pipe is open.

SOURCE

Physics • properties-of-matter

EXPLANATION

The high pressure pulse is a compression pulse. When this gets reflected from a rigid boundary $closedpipe$, the reflected pressure wave has the same phase as the incident wave i.e., a compression pulse is reflected as a compression pulse. In case of open pipe, there is a phase change of 180

$$\circ$$

when it is reflected by an open end i.e., a compression pulse is reflected as a rarefaction pulse.

Question 054 MCQ

QUESTION

A small block of mass 0.1 kg lies on a fixed inclined plane PQ which makes an angle

$$\theta$$

with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2)

A = 45 θ $^\circ$

B > 45 θ $^\circ$

and a frictional force ats on the block towards P.

C > 45 θ $^\circ$

and a frictional force ats on the block towards Q.

D < 45 θ $^\circ$

and a frictional force ats on the block towards Q.

CORRECT OPTION

A = 45 θ $^\circ$

SOURCE

Physics • laws-of-motion

EXPLANATION

The forces acting on the block are F = 1 N towards the left, weight mg = 0.1

$$\times$$

10 = 1 N downwards, normal force N, and the frictional force f. Resolve F and mg along and perpendicular to the inclined plane.

When

$$\theta$$

= 45

$$\circ$$

, the net force that brings the block down is

$$F_d = mg\sin\theta - F\cos\theta = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$$

.

Thus, the block is stationary if

$$\theta = 45^\circ$$

. When

$$\theta > 45^\circ$$

, the force

$$F_d > 0$$

, and hence the block has a tendency to move down. Thus, the frictional force f acts on the block upwards i.e., towards Q.

Question 055 MCQ

QUESTION

For the resistance network shown in the figure, choose the correct option s.

A The current through PQ is zero.

B I_1 = 3A.

C The potential at S is less than that at Q.

D I_2 = 2A.

CORRECT OPTION

A The current through PQ is zero.

SOURCE

Physics • current-electricity

EXPLANATION

Due to symmetry on upper side and lower side, points P and Q are at same potentials. Similarly, points S and T are at same potentials. Therefore, the simple circuit can be drawn as shown below :

$$I_2 = \frac{12}{2+2+2} = 2A$$

$$I_3 = \frac{12}{4+4+4} = 1A$$

$$\therefore$$

$$I_1 = I_2 + I_3 = 3A$$

I_{PQ} = 0 because $V_P = V_Q$

Potential drop $fromleftto right$ across each resistance is

$$\frac{12}{3} = 4V$$

$$\therefore$$

$$V_{MS} = 2 \times 4 = 8V$$

$$V_{NQ} = 1 \times 4 = 4V$$

or,

$$V_S < V_Q$$

Question 056 Numerical

QUESTION

A circular wire loop of radius R is placed in the xy plane centred at the origin O. A square loop of side a $a << R$ having two turns is placed with its centre at z =

$$\sqrt{3}$$

R along the axis of the circular wire loop, as shown in the figure. The plane of the square loop makes an angle of 45

$$\circ$$

with respect to z-axis. If the mutual inductance between the loops is given by

$$\frac{\mu_0 a^2}{2^{p/2} R}$$

, then the value of p is _____.

SOURCE

Physics • electromagnetic-induction

EXPLANATION

The magnetic flux through a wire loop with n turns, an area vector $\vec{S}$, placed in a uniform magnetic field $\vec{B}$, is given by the equation $\phi = n\vec{B} \cdot \vec{S}$. For a circular loop of radius r that carries a current i, the magnetic field at an axial point located a distance z from the center of the loop can be expressed as :

$$|\vec{B}| = \frac{\mu_0 i r^2}{2(r^2+z^2)^{3/2}}$$

By substituting $r = R$ and $z = \sqrt{3}R$ into the expression for the magnetic field at an axial point of a circular loop, we get the magnetic field $|\vec{B}|$ as follows:

The formula for the magnetic field at an axial point z from a circular loop of radius r carrying a current i is given by:

$$B = \frac{\mu_0 i r^2}{2(r^2+z^2)^{3/2}}$$

Substituting $r = R$ and $z = \sqrt{3}R$ into this formula, we get:

$$B = \frac{\mu_0 i R^2}{2(R^2+(\sqrt{3}R)^2)^{3/2}}$$

Simplify the expression inside the parentheses:

$$B = \frac{\mu_0 i R^2}{2(R^2+3R^2)^{3/2}}$$

$$B = \frac{\mu_0 i R^2}{2(4R^2)^{3/2}}$$

$$B = \frac{\mu_0 i R^2}{2(4^{3/2}R^3)}$$

$$4^{3/2} = 8$$

$$B = \frac{\mu_0 i R^2}{2(8R^3)}$$

$$B = \frac{\mu_0 i R^2}{16R^3}$$

$B = \frac{\mu_0 i}{16R}$

Thus, the magnetic field at the axial point $z = \sqrt{3}R$ is:

$|\vec{B}| = \frac{\mu_0 i}{16R}$

This magnetic field is directed along the z-axis and can be considered uniform at the location of the square loop $since\$a \ll R\$$. Consequently, $\vec{B}$ forms an angle of 45° with the area vector $\vec{S}$ $where\$|\vec{S}| = a^2\$$ of the square loop. The magnetic flux through the square loop and the mutual inductance of the loops are given by:

$$\phi = n|\vec{B}||\vec{S}|\cos 45^\circ = 2 \cdot \frac{\mu_0 i}{16R} \cdot a^2 \cdot \frac{1}{\sqrt{2}} = \frac{\mu_0 i a^2}{2^{7/2} R},$$

$$M = \frac{\phi}{i} = \frac{\mu_0 a^2}{2^{7/2} R}.$$

Question 057 Numerical

QUESTION

An infinitely long solid cylinder of radius R has a uniform volume charge density

$$\rho$$

. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression

$$\frac{23\rho R}{16k\varepsilon_0}$$

. The value of k is _____.

SOURCE

Physics • electrostatics

EXPLANATION

Electric field at point P due to long uniformly charged solid cylinder is

$$E_1 = \frac{\rho R^2}{2\varepsilon_0(2R)} = \frac{\rho R}{4\varepsilon_0}$$

Electric field at point P due to spherical cavity is

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{(2R)^2} = \frac{\rho R}{96\varepsilon_0}$$

The electric field at the point P is

$$= E_1 - E_2$$

$$= \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} = \frac{\rho R}{4\varepsilon_0}\left[1 - \frac{1}{24}\right] = \frac{23\rho R}{96\varepsilon_0} = \frac{23\rho R}{(16)6\varepsilon_0} = \frac{23\rho R}{16k\varepsilon_0}$$

$$\therefore$$

$$k = 6$$

Question 058 Numerical

QUESTION

A proton is fired from very far away towards a nucleus with charge Q = 120e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength $inunitsoffm$ of the proton at its start is _____. $Taketheprotonmass, \$\$m_p = (5 \times 3$ \times {10^{ - 27}}

$$kg;$$

h/e = 4.2 \times {10^{ - 15}}

$$J.s/C;$$

{1 \over {4\pi {\varepsilon _0}}} = 9 \times {10^9}$$ m/F; 1 fm = 10^{15} m.)

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Let initial and final kinetic energies of the proton be K_i and K_f and corresponding potential energies be U_i and U_f. When proton is far away from the nucleus $r$$ \to $$$$\infty$$, its potential energy is

$$U_i = \lim_{r\to\infty} \frac{1}{4\pi\varepsilon_0}\frac{120e^2}{r} = 0$$

.

At closest distance, the proton comes to rest momentarily, giving $K_f = 0$. The potential energy at closest distance is

$$U_f = \frac{1}{4\pi\varepsilon_0}\frac{120e^2}{a}$$

,

where a is the distance of closest approach. Since electrostatic force is conservative, total energy is conserved i.e.,

$$K_i + U_i = K_f + U_f$$

. Substitute the values to get

$$K_i = U_f = \frac{1}{4\pi\varepsilon_0}\frac{120e^2}{a}$$

The de-Broglie wavelength of the proton is given by

$$\lambda_i = \frac{h}{p_i} = \frac{h}{\sqrt{2m_pK_i}} = \frac{h}{e}\sqrt{\frac{4\pi\varepsilon_0 a}{240m_p}}$$

$$= 4.2 \times 10^{-15} \left(\frac{10 \times 10^{-15}}{9 \times 10^{9} \times 240 \times (5/3) \times 10^{-27}} \right)^{1/2}$$

= 7 fm.

Question 059 Numerical

QUESTION

A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio I_O/I_P to the nearest integer is _____.

SOURCE

Physics • impulse-and-momentum

EXPLANATION

T = Total portion

R = Remaining portion and

C = Cavity and

Let

$$\sigma$$

= mass per unit area.

Then,

$$m_T = \pi(2R)^2\sigma = 4\pi R^2\sigma$$

$$m_C = \pi(R)^2\sigma = \pi R^2\sigma$$

For I_P

$$I_R = I_T - I_C$$

$$= \frac{3}{2}m_T(2R)^2 - \left[\frac{1}{2}m_C R^2 + m_C r^2\right]$$

$$= \frac{3}{2}(4\pi R^2\sigma)(4R^2) - \left[\frac{1}{2}(\pi R^2\sigma) + (\pi R^2\sigma)(5R^2)\right]$$

$$= (18.5\pi R^4\sigma)$$

For I_O

$$I_R = I_T - I_C$$

$$= \frac{1}{2}m_T(2R)^2 - \frac{3}{2}m_C R^2$$

$$= \frac{1}{2}(4\pi R^2\sigma)(4R^2) - \frac{3}{2}(\pi R^2\sigma)(R^2)$$

$$= 6.5\pi R^4\sigma$$

$$\therefore$$

$$\frac{I_P}{I_O} = \frac{18.5\pi R^4\sigma}{6.5\pi R^4\sigma} = 2.846$$

Therefore, the nearest integer is 3.

Question 060 Numerical

QUESTION

A cylinder cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by

$$\frac{N}{12}\mu_0 aJ$$

, then the value of N is _____.

SOURCE

Physics • magnetism

EXPLANATION

$B_R = B_T$

$-$

B_C

R = Remaining portion

T = Total portion and

C = cavity

$$B_R = \frac{\mu_0 I_T}{2a\pi} - \frac{\mu_0 I_C}{2(3a/2)\pi}$$

..... i

$$I_T = J(\pi a^2)$$

$$I_C = J\left(\frac{\pi a^2}{4}\right)$$

Substituting the values in Eq. i, we have

$$B_R = \frac{\mu_0}{a\pi}\left[\frac{I_T}{2} - \frac{I_C}{3}\right]$$

$$= \frac{\mu_0}{a\pi}\left[\frac{\pi a^2 J}{2} - \frac{\pi a^2 J}{12}\right]$$

$$= \frac{5\mu_0 aJ}{12}$$

$$\therefore$$

$$N = 5$$