iit Jee 2007 Paper 1 Offline $66 Questions$

Question 001 MCQ

QUESTION

The species having bond order different from that in CO is :

A NO $-$

B NO $+$

C CN $-$

D N 2

CORRECT OPTION

A NO $-$

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

Species having the same number of electrons will have the same bond order.

Number of electrons in CO are 14 $8 from oxygen and 6 from carbon$.

NO

$^{-}$

has 16 electrons $7 from nitrogen, 8 from oxygen, add 1 for negative charge$.

NO

$^{+}$

has 14 electrons $7 from nitrogen, 8 from oxygen, subtract 1 for positive charge$.

CN

$^{-}$

has 14 electrons $7 from nitrogen, 6 from carbon, add 1 for negative charge$.

N

$_{2}$

has 14 electrons $7 from each nitrogen$.

Question 002 MCQ

QUESTION

Among the following the paramagnetic compound is:

A. Na_2O_2

B. O_3

C. NO_2

D. KO_2

CORRECT OPTION

D. KO_2

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

(a) $Na_2O_2 \rightarrow Na^+$ $8e^-$ all shells are filled.

$O_2^{2-} \rightarrow \begin{matrix} 1s^2 & 2s^2 & 2p^5 \\ 1s^2 & 2s^2 & 2p^5 \end{matrix}\Big] 10e^-$

c N_2O

$$: N = N \rightarrow \ddot{O} :$$

all electrons are paired.

d $KO_2 \rightarrow K^+O_2^-$

$$O_2^- = \begin{matrix} 1s^2 & 2s^2 & 2p^4 \\ 1s^2 & 2s^2 & 2p^5 \end{matrix}\Big] 9e^-$$

Question 003 MCQ

QUESTION

Extraction of zinc from zinc blende is achieved by

A electrolytic reduction

B roasting followed by reduction with carbon

C roasting followed by reduction with another metal

D roasting followed by self-reduction

CORRECT OPTION

B roasting followed by reduction with carbon

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Zinc from zinc blende is obtained by first roasting of zinc. After roasting, a reduction process is carried out in the presence of carbon which leads to the formation of free metal that is zinc. The molecular formula of zinc blende is ZnS. Zinc blende is an ore. Any metal can be extracted by following two major steps:

1. Conversion of the ore into metallic oxide

2. Reduction of the metallic oxide into free metal

In order to get zinc from ZnS, first convert the zinc blende ore to zinc oxide $metallicoxide$. The conversion of ore into metallic oxide can be done by calcination and roasting.

Calcination is generally utilised to convert hydrated oxides or hydroxides and carbonates into respective oxides while roasting is utilised to convert sulphide ores into respective oxides. Zinc blende is sulphide ore. Thus, the roasting process will be carried out to first convert zinc blende to Zinc oxide ZnO.

2ZnS + 3O

$$2$$

$$\rightarrow$$

2ZnO + 2SO

$$2$$

Now, reduction of metallic oxide to free metal is carried out. The reduction of zinc oxide in presence of carbon leads to the formation of zinc metal.

ZnO + C

$$\rightarrow$$

Zn + CO

Question 004 MCQ

QUESTION

In the following reaction,

The structure of the major 'X' is

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • hydrocarbons

EXPLANATION

Concentrated HNO

3

and concentrated H

$$2$$

SO

$$4$$

will cause nitration of the ring to introduce

$$—$$

NO

$$2$$

in one of the rings.

The ring which is adjacent to N atom is more electron rich than the other. This is due to +R effect of NH group.

Further, it will attack on more activated ortho or para positions. Since, ortho attack is not possible due to bulky ring present adjacent to it, there will be an attack on para position.

Question 005 MCQ

QUESTION

The reagent s for the following conversion,

A alcoholic KOH

B alcoholic KOH followed by NaNH

$$2$$

C aqueous KOH followed by NaNH

2

D Zn/CH

3

OH

CORRECT OPTION

B alcoholic KOH followed by NaNH

2

SOURCE

Chemistry • hydrocarbons

EXPLANATION

Alcoholic KOH helps in beta elimination in 1, 2-dibromoethane to form ethene.

The proton of ethene cannot be abstracted by a second molecule of ethene; hence, a much stronger base such as sodium amide is used to carry out another elimination to form ethyne.

Question 006 MCQ

QUESTION

The number of structural isomers for C

6

H

14

is :

A 3

B 4

C 5

D 6

CORRECT OPTION

C 5

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Structural Isomers

Question 007 MCQ

QUESTION

The percentage of p-character in the orbitals forming P-P bonds in P

4

is

A 25

B 33

C 50

D 75

CORRECT OPTION

D 75

SOURCE

Chemistry • p-block-elements

EXPLANATION

P = 5 Value electrons

Steric number = 3 + 1 = 4

Geometry - Tetrahedral hybridisation

—

sp

3

P character

$$= \frac{3}{4} \times 100 = 75\%$$

Question 008 MCQ

QUESTION

When 20 g of naphthoic acid $C\$\$_{11}\$\$H\$\$_8\$\$O\$\$_2\$\$$ is dissolved in 50 g of benzene in 50 g of benzene $K\$\$_f\$\$ = 1.72 K kg mol\$\$^{-1}\$\$$, a freezing point depression of 2 K is observed. The van't Hoff factor i is :

A 0.5

B 1

C 2

D 3

CORRECT OPTION

A 0.5

SOURCE

Chemistry • solutions

EXPLANATION

Step 1: Formula for Van't Hoff Factor

The Van't Hoff factor $$i$$ tells us how particles behave in a solution. We can find it using this formula:

$$i = \frac{\text{Actual molecular weight}}{\text{Calculated molecular weight}}$$

.

Step 2: Find Actual Molecular Weight

The real molecular weight of naphthoic acid $C$$_{11}$$H$$_{8}$$O$$_{2}$$ is 172 g/mol.

Step 3: Find Calculated Molecular Weight

We use the formula:

$$\text{Calculated molecular weight} = \frac{1000 \times w \times K_f}{W \times \Delta T_f}$$

Here,

$$\Delta T_f$$

= Drop in freezing point = 2 K

$$K_f$$

= Freezing point constant = 1.72 K kg mol

$$-1$$

$$w$$

= Mass of naphthoic acid used = 20 g

$$W$$

= Mass of benzene $solvent$ = 50 g

Put the values in:

$$= \frac{1000 \times 1.72 \times 20}{50 \times 2} = 344$$

g/mol

The calculated molecular weight is 344 g/mol.

Step 4: Calculate Van't Hoff Factor

Now, use the formula from Step 1:

$$i = \frac{\text{Actual molecular weight}}{\text{Calculated molecular weight}}$$

$$i = \frac{172}{344} = 0.5$$

Question 009 MCQ

QUESTION

The value of log

10

K for a reaction $A \rightleftharpoons B$ is

$Given : \$\$\Delta_r H^\circ_{298\ K} = -54.07\$\$kJmol\$\$^{-1}\$\$, \$\$\Delta_r S^\circ_{298\ K} = 10\$\$JK\$\$^{-1}\$$

A 5

B 10

C 95

D 100

CORRECT OPTION

B 10

SOURCE

Chemistry • thermodynamics

EXPLANATION

$$(\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ)$$

$$= -54.07 \times 1000 - 298 \times 10$$

J/mol

$$= -54070 - 2980 = 57050$$

$$\Delta G^\circ = -2.303\, RT \log_{10} K$$

$$-57050 = -2.303 \times 298 \times 8.314 \log_{10} K$$

$$\log K = 10$$

Question 010 MCQ

QUESTION

Statement 1 : Boron always forms covalent bond.

Statement 2 : The small size of B

$$3+$$

favours formation of covalent bond.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

Statement 1: *Boron always forms covalent bonds.*

→ Boron has a small size and high ionization energy. It does not easily lose electrons to form B^{3+} ions, since removing three electrons requires a lot of energy.

→ Therefore, boron compounds $likeBF_3, BCl_3, BH_3, etc.$ are **covalent** in nature.

✅ **Statement 1 is True.**

Statement 2: *The small size of B^{3+} favors formation of covalent bond.*

→ According to Fajan's rules, small cations with high charge density have strong polarizing power, which distorts the electron cloud of the anion, leading to covalency.

→ B^{3+} is extremely small and highly charged, therefore its compounds are predominantly covalent.

✅ **Statement 2 is True.**

Furthermore, **Statement 2 gives the correct reason** why boron compounds are covalent — due to the small size and high charge of B^{3+}.

✅ **Correct Option: A**

Answer: Option A — Both statements are true, and Statement 2 is the correct explanation for Statement 1.

Question 011 MCQ

QUESTION

Statement 1 : In water, orthoboric acid behaves as a weak monobasic acid.

Statement 2 : In water, orthoboric acid acts as a proton donor.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

C Statement 1 is True, Statement 2 is False

SOURCE

Chemistry • p-block-elements

EXPLANATION

H

3

BO

3

$orthoboricacid$ is a weak Lewis acid.

H

3

BO

3

+ H

2

O

$\rightleftharpoons$

BOH

$-$
4

+ H

$\oplus$

It does not donate proton rather it accepts OH

$-$

form water.

So, Statement 1 is True, Statement 2 is False.

Question 012 MCQ

QUESTION

Statement 1 : p-Hydroxybenzoic acid has a lower boiling point than o-hydroxybenzoic acid.

Statement 2 : o-Hydroxybenzoic acid has intramolecular hydrogen bonding.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

D Statement 1 is False, Statement 2 is True

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

We know that hydrogen are of two types :

1. Intermolecular hydrogen bonding

2. Intramolecular hydrogen bonding

In any organic compound, if there is ortho substituent and hydrogen bonding is possible in that compound then always the bonding will be intramolecular hydrogen bonding. If the substituent is at para position, then intermolecular hydrogen bonding exists. Therefore, we can say that o-Hydroxybenzoic acid will show intramolecular benzoic acid and p-hydroxybenzoic acid will show intermolecular hydrogen bonding.

The strength of intermolecular hydrogen bonding is more than intramolecular hydrogen bonding. Therefore we can say that p-hydroxybenzoic acid because of intermolecular hydrogen bonding has a higher boiling point than o-hydroxybenzoic acid. Therefore, statement 1 is false but statement 2 is correct.

Now, if we look at the reason it states that o-hydroxybenzoic acid has intramolecular hydrogen bonding. From the above explanation we can say that this statement is true.

Question 013 MCQ

QUESTION

Statement 1 : Micelles are formed by surfactant molecules above the critical micellar concentration CMC.

Statement 2 : The conductivity of a solution having surfactant molecules decreases sharply at the CMC.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

SOURCE

Chemistry • surface-chemistry

EXPLANATION

In colloidal surfactants, critical micelle concentration CMC is the concentration of surfactants above which micelles. Once micelles are formed, the polar surfactants are not free to conduct electricity. Also, a micelle is much larger than a monomer it diffuses more slowly through solution and so is a less efficient charge carrier.

Question 014 MCQ

QUESTION

Argon is used in arc welding because of its

A low reactivity with metal

B ability to lower the melting point of metal

C flammability

D high calorific value

CORRECT OPTION

A low reactivity with metal

SOURCE

Chemistry • p-block-elements

EXPLANATION

Argon is an inert gas; hence, less reactive. It shield metal during welding. Argon is frequently blended with carbon dioxide $CO$$_2$$, hydrogen $H$$_2$$, helium He or oxygen $O$$_2$$ to enhance the arc characteristics or facilitate metal transfer in gas metal arc welding.

Question 015 MCQ

QUESTION

The structure of XeO

3

A linear

B planar

C pyramidal

D T-shaped

CORRECT OPTION

C pyramidal

SOURCE

Chemistry • p-block-elements

EXPLANATION

Hybridisation, H =

$$\frac{1}{2}$$

$V + X\$\$ - \$\$C + A$ =

$$\frac{1}{2}$$

$8 + 0\$\$ - \$\$0 + 0$ = 4. So, sp

$$3$$

hybridised and has tetrahedral structure.

No. of lone pair = H

$$-$$

X

—

D = 4

—

0

—

3 = 1. As lone pair is present the electron pair geometry will be distorted tetrahedral and structure is pyramidal.

H = hybridisation no. $if2thensp, if3thensp^{2}etc.$

V = valence electrons of central atom.

X = no. of monovalent atom.

C = cationic charge, A = anionic charge.

D = no. of divalent atom

Question 016 MCQ

QUESTION

XeF

4

and XeF

6

are expected to be

A oxidizing

B reducing

C unreactive

D strongly basic

CORRECT OPTION

A oxidizing

SOURCE

Chemistry • p-block-elements

EXPLANATION

$6XeF_4 + 12H_2O \rightarrow 4Xe + 2XeO_3 + 24HF + 3O_2$

XeF_6

+ 3H

$$2$$

O

$$\rightarrow$$

XeO

$$3$$

+ 6HF

XeF

$$4$$

and XeF

$$6$$

are expected to be oxidizing because in both the reactions XeF

$$4$$

and XeF

$$6$$

are themselves reduced to xenon Xe and oxidises water to oxygen.

Question 017 MCQ

QUESTION

The total number of moles of chlorine gas evolved is :

A 0.5

B 1.0

C 2.0

D 3.0

CORRECT OPTION

B 1.0

SOURCE

Chemistry • electrochemistry

EXPLANATION

Moles of NaCl = 4

$\times$

0.5 = 2 moles $Since 500ml = 0.5L$

2 moles of NaCl = 2 moles of Cl

$-$

2Cl

$-$

$\rightarrow$

Cl

2

+ 2e

$-$

This equation clears that 2 moles of Cl

yields 1 mole of Cl_2 gas

Therefore, 1 mole of Cl_2 gas is evolved.

Question 018 MCQ

QUESTION

If the cathode is a Hg electrode, the maximum weight g of amalgam formed from this solution is:

A 200

B 225

C 400

D 446

CORRECT OPTION

D 446

SOURCE

Chemistry • electrochemistry

EXPLANATION

Number of moles of Na

$+$

= 2

2NaCl = 2Na

2Na + 2Hg

$\rightarrow$

2Na Hg

By electrolysis we get 2 moles of sodium which can combine with exactly 2 moles of mercury to give 2 moles of amalgam.

$\therefore$

Maximum weight of Na amalgam $assumingequimolarNaandHg$ = 46 + 400 = 446 g

Question 019 MCQ

QUESTION

The total charge $coulombs$ required for complete electrolysis is:

A 24125

B 48250

C 96500

D 193000

CORRECT OPTION

D 193000

SOURCE

Chemistry • electrochemistry

EXPLANATION

Na

$+$

+ e

$-$

$\rightarrow$

Na

Moles of Na discharged at cathode = 2

$\therefore$

The number of electron required for this purpose = 2 moles

$\therefore$

Total charge required = 2 faraday = 2

$\times$

96500 = 193000 coulombs.

Question 020 MCQ

QUESTION

Match the complexes in Column I with their properties listed in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A	$[Co(NH_3)_4(H_2O)_2]Cl_2$	P	geometrical isomers
B	$[Pt(NH_3)_2Cl_2]$	Q	paramagnetic
C	$[Co(H_2O)_5Cl]Cl$	R	diamagnetic
D	$[Ni(H_2O)_6]Cl_2$	S	metal ion with +2 oxidation state

A. A - p, q, s; B - p, r, s; C - q, s; D - q, s

B. A - q, s; B - p, r, s; C - q; D - q, s

C. A - p, q, s; B - p, r, s; C - q; D - s

D. A - p, q, s; B - s; C - q, s; D - q

CORRECT OPTION

A A - p, q, s; B - p, r, s; C - q, s; D - q, s

SOURCE

Chemistry • coordination-compounds

EXPLANATION

A The complex

$$Co(NH\$\$_3\$\$)\$\$_4\$\$(H\$\$_2\$\$O)\$\$_2\$\$$$

Cl

$$2$$

shows geometrical isomers $cis - transisomers$. It is paramagnetic due to presence of unpaired electrons. The central metal Co ion has +2 oxidation state.

B The complex

$$Pt(NH\$\$_3\$\$)\$\$_2\$\$Cl\$\$_2\$\$$$

shows geometrical isomers $cis - transisomers$. The complex is diamagnetic as all the electrons are paired. The Pt metal ion has +2 oxidation state.

C The complex

$$Co(H\$\$_2\$\$O)\$\$_5\$\$Cl$$

Cl cannot show geometrical isomerism. It is paramagnetic due to presence of unpaired electrons. The cobalt ion has +2 oxidation state.

D The complex

$$Ni(H\$\$_2\$\$O)\$\$_6\$\$$$

Cl

$$2$$

is paramagnetic in nature due to presence of unpaired electrons. In

$$Ni(H\$\$_2\$\$O)\$\$_6\$\$$$

Cl

$$2$$

, oxidation state of nickel is +2

Ni

$$2+$$

:

$$Ar$$

3d

$$8$$

4s

$$2$$

;

2 unpaired electrons.

Question 021 MCQ

QUESTION

Match the chemical substances in Column I with type of polymers/type of bonds in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$$\times$$

4 matrix given in the ORS.

	Column I		Column II
A	cellulose	P	natural polymer

	Column I		Column II
B	nylon-6, 6	Q	synthetic polymer
C	protein	R	amide linkage
D	sucrose	S	glycoside linkage

A A - p, s; B - q; C - p, r; D - s

B A - p, s; B - q, r; C - r; D - s

C A - p, s; B - q, r; C - p, r; D - s

D A - s; B - q, r; C - p, r; D - s

CORRECT OPTION

C A - p, s; B - q, r; C - p, r; D - s

SOURCE

Chemistry • polymers

EXPLANATION

A Cellulose - a natural polymer of

$$\alpha$$

-D-glucose, linked by glycoside linkage.

B Nylon-6, 6 is a synthetic polymer of adipic acid and 1, 6-diaminohexane. The diacid is linked with diamine through amide linkage.

C Protein- a natural polymer of

$$\alpha$$

-amino acids where individual amino acid units are linked by amide linkage.

D Sucrose- has glycoside linkage, a disaccharide

Question 022 MCQ

QUESTION

Match gases under specified conditions listed in Column I with their properties/laws in Column II. Indicate your answer by darkening the appropriate bubbles of the 4

$$\times$$

4 matrix given in the ORS.

	Column I		Column II
A	hydrogen gas $P = 200atm, T = 273K$	P	Compressibility factor $\neq$ 1
B	hydrogen gas $P\$\$ \sim \$\$0, T = 273K$	Q	attractive forces are dominant
C	CO $_2$ $P = 1atm, T = 273K$	R	PV = nRT
D	real gas with very large molar volume	S	$P(V - nb) = nRT$

A - s; B - r; C - q; D - p, s

B - p, s; B - r, q; C - p, q; D - s

C - p; B - r; C - p, q; D - s

D - p, s; B - r; C - p, q; D - p, s

CORRECT OPTION

D - p, s; B - r; C - p, q; D - p, s

SOURCE

Chemistry • gaseous-state

EXPLANATION

A

$$Z = PV_m/RT$$

at high pressure and low temperature. The value of Z tends towards infinity at very high pressure such as 200 bar.

Equation

$$(P + an^2/V^2)(V - nb) = nRT$$

reduces to

$$P(V - nb) = nRT$$

.

B For hydrogen gas value of Z = 1 at P = 0 and it increase continuously on increasing pressure. At such low pressure the gas behaves ideally, the van der Waals equation is reduced to :

$$PV = nRT$$

C As can be seen from the plot of compressibility factor Z versus pressure $inbar$, the compressibility factor for carbon dioxide gas is less than 0 at 1 atm $or 1.013 bar$. This is because CO

$$2$$

molecules have larger attractive forces, under normal conditions.

D

$$Z = PV_m/RT$$

, at very large molar volume Z

$$\neq$$

1.

The van Der Waals equation for real gas:

$$\left(P + \frac{an^2}{V^2}\right)(v - nb) = nRT$$

All very large molar volume,

$$\frac{an^2}{V^2}$$

almost neglibible, hence, equation is reduced to

$$P(v - nb) = nRT$$

.

Question 023 MCQ

QUESTION

Let

$$\alpha, \beta$$

be the roots of the equation

$$x^2 - px + r = 0$$

and

$$\frac{\alpha}{2}, 2\beta$$

be the roots of the equation

$$x^2 - qx + r = 0$$

. Then the value of r is

A $\frac{2}{9}(p-q)(2q-p)$

B $\frac{2}{9}(q-p)(2p-q)$

C $\frac{2}{9}(q-2p)(2q-p)$

D $\frac{2}{9}(2p-q)(2q-p)$

CORRECT OPTION

D $$\frac{2}{9}(2p-q)(2q-p)$$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

Since

$$\alpha, \beta$$

are the roots of

$$x^2 - px + r = 0$$

$$\therefore$$

$$\alpha + \beta = p$$

and

$$\alpha\beta = r$$

We know that

$$\frac{\alpha}{2}$$

and 2

$$\beta$$

are the roots of the equation

$$x^2 - qx + r = 0$$

$$\therefore$$

$$\frac{\alpha}{2} + 2\beta = q$$

and

$$\frac{\alpha}{2} \times 2\beta = r$$

Solving

$$\alpha + \beta = p$$

and

$$\frac{\alpha}{2} + 2\beta = q$$

We get

$$\alpha = \frac{2}{3}(2p - q)$$

and

$$\beta = \frac{1}{3}(2q - p)$$

$$\alpha\beta = r$$

$$r = \frac{2}{9}(2p - q)(2q - p)$$

Question 024 MCQ

QUESTION

Let

$$f(x)$$

be differentiable on the interval $0, \$\$\infty\$\$$ such that

$$f(1) = 1$$

, and

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

for each

$$x > 0$$

. Then

$$f(x)$$

is

A $\frac{1}{3x} + \frac{2x^2}{3}$

B $-\frac{1}{3x} + \frac{4x^2}{3}$

C $-\frac{1}{x} + \frac{2}{x^2}$

D $\frac{1}{x}$

CORRECT OPTION

A $\frac{1}{3x} + \frac{2x^2}{3}$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

We have,

$$\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$$

$$\Rightarrow \lim_{t \to x} \frac{2tf(x) - x^2 f'(t)}{1 - 0} = 1$$

$$\Rightarrow 2xf(x) - x^2 f'(x) = 1$$

$$\Rightarrow \frac{x^2 f'(x) - 2xf(x)}{x^4} = \frac{-1}{x^4}$$

$$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right) = \frac{-1}{x^4}$$

..... i

On integrating both sides of this equation, we get

$$\frac{f(x)}{x^2} = \frac{1}{3x^3} + c$$

Now, since

$$f(1) = 1 \Rightarrow c = \frac{2}{3}$$

We get

$$f(x) = \frac{1}{3x} + \frac{2}{3}x^2$$

Question 025 MCQ

QUESTION

One Indian and four American men and their wives are to be seated randomly around a circular table. Then the conditional probability that the Indian man is

seated adjacent to his wife given that each American man is seated adjacent to his wife is

A $\frac{1}{2}$

B $\frac{1}{3}$

C $\frac{2}{5}$

D $\frac{1}{5}$

CORRECT OPTION

C $\frac{2}{5}$

SOURCE

Mathematics • probability

EXPLANATION

Let E = event when each American man is seated adjacent to his wife and

A = event when Indian man is seated adjacent to his wife.

Now,

$$n(A \cap E) = (4!) \times (2!)^5$$

Event when each American man is seated adjacent to his wife.

Again

$$n(E) = (5!) \times (2!)^4$$

$$p\left(\frac{A}{E}\right) = \frac{n(A \cap E)}{n(E)}$$

$$= \frac{(4!) \times (2!)^5}{(5!) \times (2!)^4} = \frac{2}{5}$$

Question 026 MCQ

QUESTION

The tangent to the curve

$$y = e^x$$

drawn at the point $$c, e^c$$ intersects the line joining the points $$c-1, e^{c-1}$$ and $$c+1, e^{c+1}$$

A on the left of

$$x = c$$

B on the right of

$$x = c$$

C at no point

D at all points

CORRECT OPTION

A on the left of

$$x = c$$

SOURCE

Mathematics • parabola

EXPLANATION

The tangent to the curve

$$y = e^x$$

drawn at the point $$c, e^c$$ is given as:

$$(y - e^c) = e^c(x - c)$$

..... i

The equation of the line joining the points

$$(c - 1), (e^{c-1})$$

and

$$(c + 1, e^{c+1})$$

$$y - e^{c-1} = \frac{e^c(e - e^{-1})}{2}(x - c + 1)$$

..... ii

On subtracting i from ii

$$e^c - e^{c-1} = \frac{e^c(e - e^{-1})}{2}(x - c + 1) - e^c(x - c)$$

$$e^c - e^{c-1} = (x-c)e^c\left(\frac{e-e^{-1}}{2} - 1\right) + e^c\left(\frac{e-e^{-1}}{2}\right)$$

$$x - c = \frac{(e-1)^2}{2-(e-1)^2} < 0$$

$$\Rightarrow x < c$$

Question 027 MCQ

QUESTION

$$\lim_{x \to \frac{\pi}{4}} \frac{\int_2^{\sec^2 x} f(t)\,dt}{x^2 - \frac{\pi^2}{16}}$$

equal

A $\frac{8}{\pi} f(2)$

B $\frac{2}{\pi} f(2)$

C $\frac{2}{\pi} f\left(\frac{1}{2}\right)$

D $4f(2)$

CORRECT OPTION

A $\frac{8}{\pi}f(2)$

SOURCE

Mathematics • definite-integration

EXPLANATION

$$\lim_{x\to\frac{\pi}{4}}\frac{\int_{2}^{\sec^2 x} f(t)\,dt}{x^2-\frac{\pi^2}{16}}$$

$\$\$\frac{0}{0}\$\$form$

$$=\lim_{x\to\frac{\pi}{4}}\frac{f(\sec^2 x)2\sec x\sec x\tan x}{2x}$$

$ApplyingL'HospitalRule$

$$=\frac{2f(2)}{\frac{\pi}{4}}=\frac{8f(2)}{\pi}$$

Question 028 MCQ

QUESTION

A hyperbola, having the transverse axis of the length

$$2\sin\theta$$

, is confocal with the ellipse

$$3x^2+4y^2=12$$

. Then its equation is

A $x^2 \cos ec^2\theta - y^2\sec^2\theta = 1$

B $x^2\sec^2\theta - y^2 \cos ec^2\theta = 1$

C $x^2\sin^2\theta - y^2\cos^2\theta = 1$

D $x^2\cos^2\theta - y^2\sin^2\theta = 1$

CORRECT OPTION

A $x^2 \cos ec^2\theta - y^2\sec^2\theta = 1$

SOURCE

Mathematics • hyperbola

EXPLANATION

The length of transverse axis

$$= 2\sin\theta = 2a$$

$$a = \sin\theta$$

also for ellipse

$$3x^2 + 4y^2 = 12$$

or

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

$$a^2=4, b^3=3$$

$$e=\sqrt{1-\frac{b^2}{a^2}}$$

$$\therefore$$

$$e=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$$

$$\therefore$$

Focus of ellipse

$$=\left(2\times\frac{1}{2},0\right)\Rightarrow(1,0)$$

As hyperbola is confocal with ellipse, focus of hyperbola is = $1,0$

$$ae=1\Rightarrow\sin\theta\times e=1$$

$$e=\cos ec\theta$$

$$\therefore$$

$$b^2=a^2(e^2-1)$$

$$\sin^2\theta(\cos ec^2\theta-1)=\cos^2\theta$$

$$\therefore$$

equation of hyperbola is

$$\frac{x^2}{\sin^2\theta}-\frac{y^2}{\cos^2\theta}=1$$

$$x^2\cos ec^2\theta-y^2\sec^2\theta=1$$

Question 029 MCQ

QUESTION

The number of distinct real values of

$$\lambda$$

, for which the vectors

$$-\lambda^2\hat{i}+\hat{j}+\hat{k}, \hat{i}-\lambda^2\hat{j}+\hat{k}$$

and

$$\hat{i}+\hat{j}-\lambda^2\hat{k}$$

are coplanar, is :

A zero

B one

C two

D three

CORRECT OPTION

C two

SOURCE

Mathematics • vector-algebra

EXPLANATION

We know that for coplanar of vector

$$\begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$R_1 \to R_1 + R_2 + R_3$$

$$\begin{vmatrix} 2-\lambda^2 & 2-\lambda^2 & 2-\lambda^2 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$(2-\lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$(2-\lambda^2)\begin{vmatrix} 1 & 1 & 1 \\ 0 & -(1+\lambda^2) & 0 \\ 0 & 0 & -(1+\lambda^2) \end{vmatrix} = 0$$

$$(R_1 - R_1, R_3 - R_1)$$

$$\Rightarrow (2-\lambda^2)(1+\lambda^2)^2 = 0$$

$$\lambda = \pm\sqrt{2}$$

$$\therefore$$

Two real solutions.

Question 030 MCQ

QUESTION

A man walks a distance of 3 units from the origin towards the north-east $N45\$\$^\circ\$\$E$ direction. From there, he walks a distance of 4 units towards the north-west $N45\$\$^\circ\$\$W$ direction to reach a point P. Then the position of P in the Argand plane is

A $3e^{i\pi/4} + 4i$

B $(3 - 4i)e^{i\pi/4}$

C $(4 + 3i)e^{i\pi/4}$

D $(3 + 4i)e^{i\pi/4}$

CORRECT OPTION

D $(3 + 4i)e^{i\pi/4}$

SOURCE

Mathematics • complex-numbers

EXPLANATION

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OP}$$

$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$$

$$\overrightarrow{OP} = 3e^{\frac{i\pi}{4}} + 4e^{i\left(\frac{\pi}{2}+\frac{\pi}{4}\right)}$$

$$= 3e^{\frac{i\pi}{4}} + 4e^{\frac{i\pi}{2}}.\ e^{\frac{i\pi}{4}}$$

$$= 3e^{\frac{i\pi}{4}} + 4ie^{\frac{i\pi}{4}}$$

$$= e^{\frac{i\pi}{4}}(3+4i)$$

Question 031 MCQ

QUESTION

The number of solutions of the pair of equations

$$2\sin^2\theta - \cos 2\theta = 0$$

$$2\cos^2\theta - 3\sin\theta = 0$$

in the interval

$$[0, 2\pi]$$

is

A zero

B one

C two

D four

CORRECT OPTION

C two

SOURCE

EXPLANATION

Given,

$$2\sin^2\theta - \cos 2\theta = 0$$

and

$$2\cos^2\theta - 3\sin\theta = 0$$

$$2\sin^2\theta - \cos 2\theta = 0$$

..... i

$$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$$

$$4\sin^2\theta - 1 = 0$$

$$4\sin^2\theta = 1$$

$$\sin^2\theta = \frac{1}{4}$$

$$\sin\theta = \sqrt{\frac{1}{4}} \Rightarrow \sin\theta = \pm\frac{1}{2}$$

And,

$$2\cos^2\theta - 3\sin\theta = 0$$

..... ii

$$2(1 - \sin^2\theta) - 3\sin\theta = 0$$

$$2 - 2\sin^2\theta - 3\sin\theta = 0$$

$$2\sin^2\theta + 3\sin\theta - 2 = 0$$

$$\sin\theta = \frac{-3 \pm \sqrt{3^2 - 4 \times 2 \times (-2)}}{2 \times 2}$$

$$\sin\theta = \frac{-3 \pm \sqrt{9 + 16}}{4}$$

$using quadratic formula$

$$\sin\theta = \frac{-3 \pm 5}{4}$$

or

$$\sin\theta = \frac{-3+5}{4}, \frac{-3-5}{4}$$

$$\sin\theta = \frac{1}{2}$$

or

$$\sin\theta = -2$$

$$-1 \leq \sin\theta \leq 1$$

$$\therefore$$

$$\sin\theta = \frac{1}{2}$$

So, the solution of the pair of equation is

$$\sin\theta = \frac{1}{2}$$

$$\sin\theta = \sin\frac{\pi}{6}$$

$$\sin\theta = \sin\left(\pi - \frac{\pi}{6}\right)$$

$$\because \sin(\pi - x) = \sin x$$

$$\theta = \frac{\pi}{6}$$

or

$$\frac{5\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \theta \in [0, 2\pi]$$

So, the number of solutions are 2.

Question 032 MCQ

QUESTION

Let H

$$1$$

, H

$$2$$

, ..., H

$$n$$

be mutually exclusive and exhaustive events with P $H\$\$_i\$\$$ > 0, i = 1, 2, ..., n. Let E be any other event with 0 < P E < 1.

Statement 1 : P $H\$\$_i\$\$|E$ > P $E|H\$\$_i\$\$$. P $H\$\$_i\$\$$ for

$$i = 1, 2, \ldots, n$$

.

Statement 2 :

$$\sum_{i=1}^{n} P(H_i) = 1$$

.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

D Statement 1 is False, Statement 2 is True

SOURCE

Mathematics • probability

EXPLANATION

Statement 1 : If P $H\$\$_i\$\$\$\$ \cap \$\$E$ = O for some, then

$$P\left(\frac{H_i}{E}\right) = P\left(\frac{E}{H_i}\right) = 0$$

If P $H\$\$_i\$\$\$\$ \cap \$\$E$

$$\neq o \; \forall i = 1, 2, 3..... , n$$

then

$$P\left(\frac{H_i}{E}\right) = \frac{P(H_i \cap E)}{P(H_i)} \times \frac{P(H_i)}{P(E)}$$

$$= \frac{P\left(\frac{E}{H_i}\right) \times P(H_i)}{P(E)}$$

$$> P\left(\frac{E}{H_i}\right) \times P(H_i)$$

Hence, statement 1 may not always be true.

Statement 2 : Clearly, we can write as

H

$$1$$

$$\cup$$

H

$$2$$

$$\cup$$

H

$$1$$

$$\cup$$

$$\cup$$

H

$$n$$

= S

$$\Rightarrow$$

P $H$$$_1$$ + P $H$$$_2$$ + ---------- + P $H$$$_n$$ = 1

Hence, statement 2 is true.

Question 033 MCQ

QUESTION

Tangents are drawn from the point $17, 7$ to the circle

$$x^2 + y^2 = 169$$

.

Statement 1 : The tangents are mutually perpendicular.

Statement 2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is

$$x^2 + y^2 = 338$$

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

SOURCE

Mathematics • circle

EXPLANATION

Locus of the points of intersections of perpendicular tangents to the circles

$$x^2 + y^2 = a^2$$

$$x^2 + y^2 = 2a^2$$

$$\therefore$$

director circle of

$$x^2 + y^2 = 169$$

is the circle of

$$x^2 + y^2 = (169)(2) = 338$$

The point $17, 7$ lies of on the circle

$$x^2 + y^2 = 338$$

. Thus, the tangent drawn from $17, 7$ to the circle

$$x^2 + y^2 = 169$$

are perpendicular.

Question 034 MCQ

QUESTION

Let the vector

$$\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TU}$$

and

$$\overrightarrow{UP}$$

, represent the sides of a regular hexagon.

Statement 1 :

$$\overrightarrow{PQ} \times \left(\overrightarrow{RS} + \overrightarrow{ST}\right) \neq \overrightarrow{0}$$

Statement 2 :

$$\overrightarrow{PQ} \times \overrightarrow{RS} = \overrightarrow{0}$$

and

$$\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$$

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

C Statement 1 is True, Statement 2 is False

SOURCE

Mathematics • vector-algebra

EXPLANATION

Statement 1 :

$$\overrightarrow{PQ} \times (\overrightarrow{RS} + \overrightarrow{ST}) \neq 0$$

since

$$\overrightarrow{PQ}$$

is not parallel to

$$\overrightarrow{RT}$$

,

We get,

$$\overrightarrow{PQ} \times \overrightarrow{RT}$$

That is statement 1 is true.

$$\overrightarrow{PQ} \times \overrightarrow{RT} \neq 0$$

Statement 2 :

$$\overrightarrow{PQ} \times \overrightarrow{RS} \neq \overrightarrow{0}$$

and

$$\overrightarrow{PQ} \times \overrightarrow{ST} \neq \overrightarrow{0}$$

since

$$\overrightarrow{PQ}$$

is not parallel to

$$\overrightarrow{RS}$$

.

Statement 2 is false.

Question 035 MCQ

QUESTION

Let F x be an indefinite integral of

$$\sin^2 x$$

.

Statement 1 : The function F x satisfies F $$$x + \pi$$$ = F $$$x$$$ for all real x.

Statement 2 :

$$\sin^2(x+\pi) = \sin^2 x$$

for all real x.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

D Statement 1 is False, Statement 2 is True

SOURCE

Mathematics • inverse-trigonometric-functions

EXPLANATION

$$f(x) = \int \sin^2 x dx = \frac{1}{2}\int (1-\cos 2x)dx$$

$$= \frac{1}{4}(2x - \sin 2x) + c$$

Now,

$$f(x+\pi) = \frac{1}{4}(2x + 2\pi - \sin(2x+2\pi)) + c$$

$$= \frac{1}{4}[2x + 2\pi - \sin 2x] + c \neq f(x)$$

$$\therefore$$

Statement 1 is false.

Also

$$\sin^2(x + \pi) = \sin^2 x, \forall x \in R$$

$$\therefore$$

Statement 2 is true.

Question 036 MCQ

QUESTION

The sum V

$$1$$

+ V

$$2$$

+ ... + V

$$n$$

is

A $$\frac{1}{12}n(n+1)(3n^2 - n + 1)$$

B $$\frac{1}{12}n(n+1)(3n^2 + n + 2)$$

C $\frac{1}{2}n(2n^2 - n + 1)$

D $\frac{1}{3}(2n^3 - 2n + 3)$

CORRECT OPTION

B $\frac{1}{12}n(n+1)(3n^2 + n + 2)$

SOURCE

Mathematics • sequences-and-series

EXPLANATION

$$\sum_{r=1}^{n} v_r = \sum_{r=1}^{n} \left(\frac{r}{2}(2r + (r-1)(2r-1)) \right)$$

$$= \sum_{r=1}^{n} \left(r^3 - \frac{r^2}{2} + \frac{r}{2} \right)$$

$$= \sum n^3 - \frac{1}{2}\sum n^2 + \frac{1}{2}\sum n$$

$$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{2 \times 2}$$

$$= \frac{n(n+1)}{4}\left[n(n+1) - \frac{(2n+1)}{3} + 1 \right]$$

$$= \frac{n(n+1)[3n^2 + n + 2]}{12}$$

$$= \frac{1}{12} n(n+1)[3n^2 + n + 2]$$

Question 037 MCQ

QUESTION

T

r

is always

A an odd number

B an even number

C a prime number

D a composite number

CORRECT OPTION

D a composite number

SOURCE

Mathematics • sequences-and-series

EXPLANATION

We have,

$$v_r = \frac{1}{2}(2r^3 - r^2 + r)$$

$$v_{r+1} = \frac{1}{2}[2(r+1)^3 - (r+1)^2 + (r+1)]$$

Now,

$$T_r = v_{r+1} - v_r - 2$$

$$= [(r+1)^3 - r^3] - \frac{1}{2}[(r+1)^2 - r^2] + \frac{1}{2}(1) - 2$$

$$= 3r^2 + 2r - 1$$

$$= (r+1)(3r-1)$$

Which is a composite number

Question 038 MCQ

QUESTION

Which one of the following is a correct statement?

A

Q$_1$, Q$_2$, Q$_3$, ... are in A.P. with common difference 5

Q$_1$

, Q

B

2

, Q

3

, ... are in A.P. with common difference 6

Q

1

, Q

C

2

, Q

3

, ... are in A.P. with common difference 11

Q

1

= Q

D

2

= Q

3

, ...

CORRECT OPTION

Q

1

, Q

B

2

, Q

$$3$$

, ... are in A.P. with common difference 6

SOURCE

Mathematics • sequences-and-series

EXPLANATION

We have

$$T_r = 3r^2 + 2r - 1$$

$$T_{r+1} = 3(r+1)^2 + 2(r+1) - 1$$

Now,

$$Q_r = T_{r+1} - T_r = 3[(r+1)^2 - r^2] + 2(1)$$

$$= 6r + 5$$

$$Q_{r+1} = 6(r+1) + 5$$

$$Q_{r+1} - Q_r = 6 =$$

constant

Therefore, Q

$$1$$

, Q

$$2$$

and Q

$$3$$

..... are in A.P. with common difference 6.

Question 039 MCQ

QUESTION

The ratio of the areas of the triangles PQS and PQR is

A 1 : $\sqrt{2}$

B 1 : 2

C 1 : 4

D 1 : 8

CORRECT OPTION

C 1 : 4

SOURCE

Mathematics • parabola

EXPLANATION

We first obtain the coordinate of P & Q.

Towards this end we solve

$$x^2 + y^2 = 9$$

..... i

$$y^2 = 8x$$

..... ii

from i and ii, we get

$$x^2 + 8x - 9 = 0$$

$$(x+9)(x-1) = 0 \Rightarrow x = 1$$

$$\$\$ \therefore \$\$\$\$x + 9 > 0\$\$$$

$$y^2 = 8 \Rightarrow y = \pm 2\sqrt{2}$$

Thus, coordinates of P are

$$(1, 2\sqrt{2})$$

and Q are

$$(1, -2\sqrt{2})$$

equation of tangents to the circle at P and Q respectively.

$$x + 2\sqrt{2}y = 9$$

$$x - 2\sqrt{2}y = 9$$

On solving these we get coordinates of R as $9, 0$

Equations of tangents to parabola at P and Q are

$$2\sqrt{2}y = 4(x+1)$$

$$-2\sqrt{2}y = 4(x+1)$$

These lines intersect each other at S $\$\$ - 1, 0\$\$$

Let

$$\Delta_1$$

= area of

$$\Delta$$

PQS =

$$\frac{1}{2}$$

$PQ\ ST$

$$= \frac{1}{2}(2)(4\sqrt{2})$$

$$= 4\sqrt{2}$$

And

$$\Delta_2$$

= area of

$$\Delta$$

PQR =

$$\frac{1}{2}$$

$PQ\ TR$

$$= \frac{1}{2}(4\sqrt{2})8$$

$$= 16\sqrt{2}$$

$$\Delta_1 : \Delta_2 = 4\sqrt{2} : 16\sqrt{2}$$

= 1 : 4

Question 040 MCQ

QUESTION

The radius of the circumcircle of the triangle PRS is

A 5 units

B 3 $\sqrt{3}$ units

C 3 $\sqrt{2}$ units

D 2 $\sqrt{3}$ units

CORRECT OPTION

B 3 $\sqrt{3}$ units

SOURCE

Mathematics • parabola

EXPLANATION

Equation of perpendicular bisector of SR is x = 4

Midpoint of PR is $5, \$\$\sqrt{2}\$\$$ i

Slope of PR is

$$\frac{0-2\sqrt{2}}{9-1} = \frac{-\sqrt{2}}{4}$$

$$\therefore$$

Equation of perpendicular bisector of PR is

$$y - \sqrt{2} = \frac{4}{\sqrt{2}}(x-5)$$

..... ii

Solving equation i and ii we get

$$x = 4, y = -\sqrt{2}$$

Circumcentre is $4, \$\$ - \sqrt{2}\$\$$

Radius of circumcircle is = 3

$$\sqrt{3}$$

units

Question 041 MCQ

QUESTION

The radius of the incircle of the triangle PQR is

A 4 units

B 3 units

C $\frac{8}{3}$

units

D 2 units

CORRECT OPTION

D 2 units

SOURCE

Mathematics • parabola

EXPLANATION

We have

PR = RQ = 6

$$\sqrt{2}$$

PQ = 4

$$\sqrt{2}$$

$$\Delta_2$$

= area of

$$\Delta$$

PQR

= 16

$$\sqrt{2}$$

$$r = \frac{\Delta_2}{s}$$

$$= \frac{16\sqrt{2}}{\frac{1}{2}(6\sqrt{2} + 6\sqrt{2} + 4\sqrt{2})} = 2$$

units

Radius of incircle of

$$\Delta$$

PQR = 2 units

Question 042 MCQ

QUESTION

Consider the following linear equations

$$ax + by + cz = 0$$

$$bx + cy + az = 0$$

$$cx + ay + bz = 0$$

Match the conditions/expressions in Column I with statements in Column II.

	Column I		Column II
A	$a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	P	the equations represent planes meeting only at a single point.
B	$a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	Q	the equations represent the line $x = y = z$.

	Column I		Column II
C	$a+b+c \neq 0$ and $a^2+b^2+c^2 \neq ab+bc+ca$	R	the equations represent identical planes.
D	$a+b+c=0$ and $a^2+b^2+c^2=ab+bc+ca$	S	the equations represent the whole of the three dimensional space.

A - q, B - r, C - p, D - s

B - r, B - q, C - s, D - p

C - r, B - p, C - q, D - s

D - r, B - q, C - p, D - s

CORRECT OPTION

D - r, B - q, C - p, D - s

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

The given system can be written as

A AX = 0

Where

$$A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}, X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

R

$$2$$

$$\rightarrow$$

R

$$2$$

$$-$$

R

$$1$$

, R

$$3$$

$$\rightarrow$$

R

$$3$$

$$-$$

R

$$1$$

$$= (a+b+c)(bc+ac+ab-a^2-b^2-c^2)$$

$$= \frac{1}{2}(a+b+c)[(b-c)^2+(c-a)^2+(a-b)^2]$$

If

$$a+b+c \neq 0$$

and

$$a^2+b^2+c^2 = bc+ca+ab$$

then

$$a=b=c \neq 0$$

Thus, three equation represent identical planes

$$x+y+z=0$$

B If

$$a+b+c=0$$

we get two equations as

$$ax+by=(a+b)z$$

$$bx+cy=(b+c)z$$

Eliminating y, we have

$$(ac-b^2)x=(ac-b^2)z$$

$$x=z$$

putting

$$x=z$$

in first equation, we get

$$ax+by+cx=0$$

$$by=-(a+c)x=bx$$

$$y=x$$

hence,

$$x = y = z$$

C If

$$a + b + c \neq 0$$

$$a^2 + b^2 + c^2 \neq ab + bc + ca$$

then

$$|A| \neq 0$$

So, only solution is

$$x = y = z = 0$$

D If

$$a + b + c = 0, a^2 + b^2 + c^2 = ab + bc + ca$$

$$(b - c)^2 + (c - a)^2 + (a - b)^2 = 0$$

$$a = b = c$$

$$a = b = c = 0$$

Thus, the system represents the whole three dimensional space.

Question 043 MCQ

QUESTION

In the following

$$x$$

denotes the greatest integer less than or equal to x.

Match the functions in Column I with the properties Column II.

	Column I		Column II
A	$x\|x\|$	P	continuous in $$-1,1$$.
B	$\sqrt{\|x\|}$	Q	differentiable in $$-1,1$$
C	$x+[x]$	R	strictly increasing in $$-1,1$$
D	$\|x-1\|+\|x+1\|$	S	not differentiable at least at one point in $$-1,1$$

A A - p, q, r, B - p, s, C - r, s, D - p, q

B A - p, q, B - p, s, C - r, s, D - p

C A - p, q, r, B - p, C - r, D - p, q

D A - p, r, B - p, s, C - r, D - p, q

CORRECT OPTION

A A - p, q, r, B - p, s, C - r, s, D - p, q

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

A

$$y = x|x| = \begin{cases} -x^2 & if\, x < 0 \\ x^2 & if\, x \geq 0 \end{cases}$$

From graph

$$y = x|x|$$

is $continuousin(\$\$ -1, 1\$\$\ p$ differentiable in $\$\$ -1, 1\$\$\ q$ Strictly increasing in $\$\$ -1, 1\$\$\ r$

B

$$y = \sqrt{|x|}$$

$$= \begin{cases} \sqrt{-x} & if\, x < 0 \\ \sqrt{x} & if\, x \geq 0 \end{cases}$$

$$y^2 = -x, x < 0$$

{where y can take only +ve values}

and

$$y^2 = x, x \geq 0$$

$$\therefore$$

Graph is as follows:

From graph

$$y = \sqrt{|x|}$$

is continuous in $\$\$ -1, 1\$\$\ P$ not differentiable at x = 0 s

C Note this step

$$y = x + [x] = \begin{cases} x - 1, & -1 \leq x < 0 \\ x, & 0 \leq x < 1 \\ x + 1, & x + 1 \end{cases}$$

$$\therefore$$

Graph of

$$y = x + [x]$$

is

From graph,

$$y = x + [x]$$

is neither continuous, nor differentiable at x = 0 and hence in $\$\$ -1, 1\$\$$ s.

D

$$= |x-1| + |x+1|$$

$$= \begin{cases} -2x & x < -1 \\ 2 & -1 \le x < 1 \\ 2x & x \ge 1 \end{cases}$$

Graph is as follows:

From graph,

$$y = f(x)$$

is continuous p differentiable q in $\$\$ -1, 1\$\$$ but not strictly increasing in $\$\$ -1, 1\$\$$.

Question 044 MCQ

QUESTION

Match the integrals in Column I with the values in Column II.

	Column I		Column II
A	$\int_{-1}^{1} \frac{dx}{1+x^2}$	P	$\frac{1}{2}\log\left(\frac{2}{3}\right)$
B	$\int_{0}^{1} \frac{dx}{\sqrt{1+x^2}}$	Q	$2\log\left(\frac{2}{3}\right)$
C	$\int_{2}^{3} \frac{dx}{1+x^2}$	R	$\frac{\pi}{3}$
D	$\int_{1}^{2} \frac{dx}{x\sqrt{x^2-1}}$	S	$\frac{\pi}{2}$

A. A - s, B - s, C - r, D - p

B. A - s, B - q, C - p, D - r

C. A - s, B - s, C - p, D - r

D. A - s, B - q, C - s, D - r

CORRECT OPTION

C. A - s, B - s, C - p, D - r

SOURCE

Mathematics • definite-integration

EXPLANATION

A

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \tan^{-1}x]_{-1}^{1}$$

$$= \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right)$$

$$= \frac{2\pi}{4} = \frac{\pi}{2}$$

B

$$\int_{0}^{1} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x]_{0}^{1}$$

$$= \sin^{-1}(1) - \sin^{-1}(0)$$

$$= \frac{\pi}{2} - 0$$

$$= \frac{\pi}{2}$$

C

$$\int_{2}^{3} \frac{dx}{1-x^2} = \frac{1}{2}\log\left|\frac{1+x}{1-x}\right|$$

$$= \left[\log\left|\frac{1+3}{1-3}\right| - \log\frac{1+2}{1-2}\right]$$

$$= \frac{1}{2}\log\left(\frac{2}{3}\right)$$

D

$$\int_1^2 \frac{dx}{x\sqrt{x^2-1}} [\sec^{-1}x]_1^2$$

$$= \sec^{-1}2 - \sec^{-1}1$$

$$= \frac{\pi}{3} - 0$$

$$= \frac{\pi}{3}$$

Question 045 MCQ

QUESTION

A resistance of 2

$$\Omega$$

is connected across one gap of a metre-bridge $the length of the wire is 100 cm$ and an unknown resistance, greater than 2

$$\Omega$$

, is connected across the other gap. When these resistance are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

A 3

$$\Omega$$

B 4

$$\Omega$$

C 5 Ω

D 6 Ω

CORRECT OPTION

A 3 Ω

SOURCE

Physics • current-electricity

EXPLANATION

Given, that x is greater than 2

$$\Omega$$

, when the bridge is balanced then,

$$\frac{R}{l} = \frac{x}{100 - l}$$

..... i

$frommetrebridgebalancedbridgeformula$

$$\Rightarrow 100R - Rl = lx$$

$$\Rightarrow 200 - 2l = lx \ \ (\because R = 2\Omega)$$

$$l = \frac{200}{x + 2}$$

..... ii

Where,

$$l$$

= length of segment of wire from one end where null point is obtained.

Now, these resistances are interchanged, the Jockey shifts 20 cm. So,

$$\frac{x}{l+20} = \frac{2}{80-l}$$

..... iii

From equation iii we have,

$$80x = l(x+2) + 40$$

$$\Rightarrow 80x = \left(\frac{200}{x+2}\right)(x+2) + 40$$

$$\Rightarrow x = \frac{240}{80} = 3\Omega$$

$$\therefore x = 3\Omega$$

Question 046 MCQ

QUESTION

In an experiment to determine the focal length f of a concave mirror by the u-v method, a student places the object pin A on the principal axis at a distance x form the pole P. The student looks at the pin and its inverted image form a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right, oh the object pin. Then,

A $x < f$

B $f < x < 2f$

C $x = 2f$

D $x > 2f$

CORRECT OPTION

B $f < x < 2f$

SOURCE

Physics • geometrical-optics

EXPLANATION

About forming an inverted image: A concave mirror will make an upside-down $inverted$ image only when the object is placed farther from the mirror than its focal point. This means the distance x from the object to the pole should be bigger than the focal length f $so, \$x > f\$$.

When the object and image overlap: If you place the object at $x = 2f$ $twicethefocallength$, the image will appear at the same spot as the object. At this exact spot, moving your eye side to side does not make the image and object swap positions. This means you cannot see “parallax.”

Understanding parallax with two points: Imagine you have two points in space, A and B. A is closer to your eye, B is farther away. If your eye is at point E on the line joining A and B, the following image shows this:

How parallax works: If you move your eye in one direction $say, tothelef t$, the point that is closer to you $\$A\$$ will appear to move in the opposite direction

$totheright$ compared to the background $\$B\$$. The point that is farther away $\$B\$$ will seem to move in the same direction as your eye $totheleft$.

For example, if you move your eye from E to E_1 $leftwards$, the line from your eye to the far point B changes from EB to E_1B. The near point A looks like it shifts to the right compared to this line. The line from your eye to the near point A changes from EA to E_1A, and the far point B now seems to move to the left relative to A.

You can try this! Put two fingers in front of your eyes $onecloser, onefarther$, and move your head. You'll see the closer finger appears to shift more, in the opposite direction you move your eye.

If the two points $\$A\$and\$B\$$ are at the same spot $they\backslash coincide''$, you won't see any parallax. There is no shifting effect between them.

What happens in the mirror experiment: In this experiment, when you move your eye to the left, the image of the pin seems to move to the right of the real object pin. This shows the image is between your eye and the object pin. That means the object is closer to the mirror's pole P than its image is. For this to happen, the object must be between the focal length f and twice the focal length $2f$. In other words, $f < x < 2f$.

Question 047 MCQ

QUESTION

Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' form the centre P $asshowninthefigure$. Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surface. The magnitude of acceleration, with the separation between them becomes 2x, is

A $\frac{F}{2m}\frac{a}{\sqrt{a^2-x^2}}$

B $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

C $\frac{F}{2m}\frac{x}{a}$

D $\frac{F}{2m}\frac{\sqrt{a^2-x^2}}{x}$

CORRECT OPTION

B $\frac{F}{2m}\frac{x}{\sqrt{a^2-x^2}}$

SOURCE

Physics • laws-of-motion

EXPLANATION

The acceleration of mass 'm' is due to the force

$$T\cos\theta$$

, from 2nd law of motion we have,

$$T\cos\theta = ma$$

$$\Rightarrow a = \frac{T\cos\theta}{m}$$

..... i

and also,

$$F = 2T\sin\theta$$

$$\Rightarrow T = \frac{F}{2\sin\theta}$$

..... ii

From i & ii, we have,

$$a = \left(\frac{F}{2\sin\theta}\right) \cdot \frac{\cos\theta}{m}$$

$$a = \frac{F}{2m\tan\theta}$$

$$\therefore$$

$$a = \frac{F}{2m\frac{\sqrt{a^2-x^2}}{x}}$$

$$\therefore$$

$$a = \frac{F}{2m} \cdot \frac{x}{\sqrt{a^2-x^2}}$$

Question 048 MCQ

QUESTION

A long, hollow conducting cylinder is kept coaxially inside another long, hollow conducting cylinder of larger radius. Both the cylinder are initially electrically neutral.

A. A potential difference appears between the two cylinders when a charge density is given to the inner cylinder

B. A potential difference appears between the two cylinders when a charge density is given to the outer cylinder

C. No potential difference appears between the two cylinders when a uniform line charge is kept along the axis of the cylinders

D. No potential difference appears between the two cylinders when same charge density is given to both the cylinders

CORRECT OPTION

A. A potential difference appears between the two cylinders when a charge density is given to the inner cylinder

SOURCE

Physics • electrostatics

EXPLANATION

Let the radii of the inner and outer cylinders be r_1 and r_2, respectively. The problem has cylindrical symmetry and Gauss's law can be applied to find the electric field E.

In the case A, surface charge density σ is given to the inner cylinder. Take a cylinder of radius $r\left(r_1<\right. r

$$E(2\pi rl) = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{\sigma\,(2\pi r_1 l)}{\epsilon_0}, \quad \text{i.e.,} \quad E = \frac{\sigma r_1}{\epsilon_0 r}.$$

Potential difference between the two cylinders is given by

$$\Delta V = -\int_{r_1}^{r_2} E\,\mathrm{d}r = -\frac{\sigma r_1}{\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

In the case B, charge density σ is given to the outer cylinder. Apply Gauss's law to get $E = 0$ $because \$q_{\text{enc}} = 0\$$ and hence $\Delta V = 0$.

In the case C, uniform linear charge density λ is kept along the common axis of cylinders. Apply Gauss's law as in case A to get the electric field

$$E(2\pi rl) = \lambda l/\epsilon_0, \quad \text{i.e.,} \quad E = \lambda/(2\pi\epsilon_0 r).$$

Potential difference between the two cylinders is

$$\Delta V = -\int_{r_1}^{r_2} E\,\mathrm{d}r = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$$

In the case D charge density σ is given to both the cylinders. Using the argument similar to the case A, we get, $\Delta V = -\frac{\sigma r_1}{\epsilon_0}\ln\left(\frac{r_2}{r_1}\right)$.

Question 049 MCQ

QUESTION

Consider a neutral conducting sphere. A positive point charge is placed outside the sphere. The net charge on the sphere is then,

A negative and distributed uniformly over the surface of the sphere

B negative and appears only at the point on the sphere closest to the point charge

C negative and distributed non-uniformly over the entire surface of the sphere

D zero

CORRECT OPTION

D zero

SOURCE

Physics • electrostatics

EXPLANATION

Net charge on sphere =

$$-20 + 20$$

= 0

When we brought +q charge near neutral conducting sphere induction no produced.

So, net charge on sphere should be zero.

When +q is placed option d.

Outside the conducting sphere, rearrange of charge taken place on sphere.

Question 050 MCQ

QUESTION

A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is

A 0

B 54 μ C

C 27 μ C

D 81 μ C

CORRECT OPTION

C 27 μ C

SOURCE

Physics • capacitor

EXPLANATION

Step 1: Initial State $SwitchOpen$

Let q_1 and V_1 be the charge and voltage on capacitor $C_1 = 3\,\mu\text{F}$. Let q_2 and V_2 be the charge and voltage on capacitor $C_2 = 6\,\mu\text{F}$.

When the switch S is open, the two capacitors are connected in series. This means that the charge stored on each capacitor is the same. So, $q_1 = q_2$.

The voltages on the capacitors add up to the battery voltage:

$$V_1 + V_2 = 9\,\mathrm{V}$$

The charges and voltages on capacitors in series also follow this equation:

$$C_1 V_1 = C_2 V_2$$

This means the charge on each capacitor is the same since $q_1 = C_1 V_1$ and $q_2 = C_2 V_2$.

Solving these two equations gives:

- $V_1 = 6\,\mathrm{V}$, $q_1 = 3\,\mu\mathrm{F} \times 6\,\mathrm{V} = 18\,\mu\mathrm{C}$
- $V_2 = 3\,\mathrm{V}$, $q_2 = 6\,\mu\mathrm{F} \times 3\,\mathrm{V} = 18\,\mu\mathrm{C}$

Total charge on the side connected to X is $-q_1 + q_2 = 0$. No net charge has moved yet.

Step 2: After Closing the Switch

When the switch is closed, the resistors $R_1 = 3\,\Omega$ and $R_2 = 6\,\Omega$ are connected in series with the capacitors and battery.

Total resistance is $R_1 + R_2 = 9\,\Omega$.

The current from the battery is:

$$I = frac9\,\mathrm{V}9\,\Omega = 1\,\mathrm{A}$$

The voltage drop across each resistor is:

- $V_1' = I \times R_1 = 1\,\mathrm{A} \times 3\,\Omega = 3\,\mathrm{V}$
- $V_2' = I \times R_2 = 1\,\mathrm{A} \times 6\,\Omega = 6\,\mathrm{V}$

The new charges on the capacitors are:

- $q_1' = C_1 \times V_1' = 3\,\mu\mathrm{F} \times 3\,\mathrm{V} = 9\,\mu\mathrm{C}$
- $q_2' = C_2 \times V_2' = 6\,\mu\mathrm{F} \times 6\,\mathrm{V} = 36\,\mu\mathrm{C}$

Final Calculation

The total charge on the plates connected to X is now $-q_1' + q_2' = -9 + 36 = 27\,\mu\mathrm{C}$.

This means that $27\,\mu\mathrm{C}$ of charge flows from Y to X when the switch is closed.

Question 051 MCQ

QUESTION

A ray of light travelling in water in incident on its surface open to air. The angle of incidence is

$$\theta$$

, which is less than the critical angle. Then there will be

A only a reflected ray and no refracted ray

B only a refracted ray and no reflected ray

C a reflected ray and a refracted ray and the angle between them would be less than 180

$$^{\circ}$$

- 2

$$\theta$$

D a reflected ray and the angle between them would be greater than 180

$$^{\circ}$$

- 2

$$\theta$$

CORRECT OPTION

a reflected ray and a refracted ray and the angle between them would be less than 180

C

- 2

θ

SOURCE

Physics • geometrical-optics

EXPLANATION

a reflected ray and refracted ray and the angle b/w them would be less than $180\$\$^{\circ}\$\$ - 2\$\$\theta\$\$$.

Question 052 MCQ

QUESTION

In the option given below, let E denote the rest mass energy of a nucleus and n a neutron. The correct option is

A $E\left({}_{92}^{236}U\right) > E\left({}_{53}^{137}I\right) + E\left({}_{39}^{97}Y\right) + 2E(n)$

B $E\left({}_{92}^{236}U\right) < E\left({}_{53}^{137}I\right) + E\left({}_{39}^{97}Y\right) + 2E(n)$

C $E\left({}_{92}^{236}U\right) < E\left({}_{56}^{140}Ba\right) + E\left({}_{36}^{94}Kr\right) + 2E(n)$

D $$E\left({}_{92}^{236}U\right) = E\left({}_{56}^{140}Ba\right) + E\left({}_{36}^{94}Kr\right) + 2E(n)$$

CORRECT OPTION

A $$E\left({}_{92}^{236}U\right) > E\left({}_{53}^{137}I\right) + E\left({}_{39}^{97}Y\right) + 2E(n)$$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

Iodine and Yttrium are medium sized nuclei and therefore, have more binding energy per nucleon as compared to uranium which has a big nuclei and less binding energy per nucleon. So that Iodine and Yttrium are more stable and therefore possess less energy and less restmass. When uranium nuclei explodes, it will convert into I and Y nuclei having kinetic energies.

Question 053 MCQ

QUESTION

The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum $to the nearest integer$ is

A 802 nm

B 823 nm

C 1882 nm

D 1648 nm

CORRECT OPTION

B 823 nm

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

For largest

$$\lambda$$

in UV

$$\rightarrow$$

122 nm

For

$$\Delta$$

E

$$\min$$

,

$$\frac{1}{\lambda_{lyman}} = R_c\left(1 - \frac{1}{4}\right)$$

$$\$$ \because \$\$\$\$ \frac{1}{\lambda} = R_c \left(\frac{1}{n_2^2} - \frac{1}{n_1^2} \right) \$\$$$

$$\frac{1}{\lambda_{lyman}} = \frac{3R_c}{4}$$

..... i

R

c

= Rydber constant = 1.097

$\times$

10

7

m

-1

For min

λ

in IR region, n = D

$\rightarrow$

n = 3

$$\frac{1}{\lambda_{IR}} = \frac{R_c}{9}$$

..... ii

On dividing eq. i by ii

$$\frac{1}{\lambda_{lyman}} \times \lambda_{IR} = \frac{1}{\frac{3R_c}{4}} \times \frac{9}{R_c} = \frac{3R_c}{4} \times \frac{9}{R_c} = \frac{27}{4}$$

$$\therefore$$

$$\lambda_{IR} = \frac{27}{4} \times 122$$

nm = 823.5 nm

Question 054 MCQ

QUESTION

Statement 1 :

A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30

$^{\circ}$

with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

Statement 2 :

The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C. Statement 1 is True, Statement 2 is False

D. Statement 1 is False, Statement 2 is True

CORRECT OPTION

C Statement 1 is True, Statement 2 is False

SOURCE

Physics • work-power-and-energy

EXPLANATION

Case I:

Case II : Tilted at 30

$$\circ$$

.

$$N = mg\cos 30^\circ$$

$$= mg\frac{\sqrt{3}}{2}$$

$$f_2 = \mu B = rmg\frac{\sqrt{3}}{2}$$

$$f_1 = \frac{l}{N} = \frac{l}{mg\frac{\sqrt{3}}{2}}$$

Case III :

$$\Delta$$

E

$$= \frac{1}{2}mv^2 - mgh$$

= Decreases

Option C is correct because friction is depend on surface.

Case I :

Statement 1 : In the 1st case the mechanical energy is completely converted into heat energy because of friction.

Case II :

M.E. is converted into heat due to friction but another part of M.E. is retained in the form of P.E. of block. Therefore statement I is correct.

Statement 2 : This is a wrong statement because the coefficient of friction between block and surface does not depend on angle of inclination.

Question 055 MCQ

QUESTION

Statement 1 :

In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision.

Statement 2 :

In an elastic collision, the linear momentum of the system is conserved.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

D Statement 1 is False, Statement 2 is True

SOURCE

Physics • impulse-and-momentum

EXPLANATION

Statement 1 : For an elastic collision, the coefficient of restitution = 1

$$e = \left| \frac{v_2 - v_1}{u_1 - u_2} \right|$$

$$\Rightarrow |v_2 - v_1| = |u_1 - u_2|$$

Relative velocity after collision is equal to relative velocity before collision. But in the statement relative speed is given.

Statement 2 : Linear momentum remains conserved in an elastic collision. This statement is true.

Hint :

Relative velocity before collision

$$= |u_1 - u_2|$$

Relative velocity after collision

$$= |v_1 - v_2|$$

Elastic collision

$$= e = 1$$

$$e = \left| \frac{v_1 - v_2}{u_1 - u_2} \right| = 1$$

$$\Rightarrow |v_1 - v_2| = |u_1 - u_2|$$

So statement 1 false

$$\sum F_{ext} = 0$$

, So momentum should be conserved.

Statement 2 is true.

Question 056 MCQ

QUESTION

The ratio

$$\frac{x_1}{x_2}$$

is

A 2

B $\frac{1}{2}$

C $\sqrt{2}$

D $\frac{1}{\sqrt{2}}$

CORRECT OPTION

C $\sqrt{2}$

SOURCE

Physics • rotational-motion

EXPLANATION

For disc A,

$$\frac{1}{2}kx_1^2 = \frac{1}{2}I(2\omega)^2$$

$$\Rightarrow kx_1^2 = 2I\omega^2$$

..... i

For disc B,

$$\frac{1}{2}kx_2^2 = \frac{1}{2}2I\omega^2$$

$$\Rightarrow kx_2^2 = I\omega^2$$

..... ii

On dividing eq. i by ii

$$\frac{kx_1^2}{kx_2^2} = \frac{2I\omega^2}{I\omega^2}$$

$$\frac{x_1}{x_2} = \sqrt{2}$$

Question 057 MCQ

QUESTION

When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during

this period is

A $\frac{2I\omega}{3t}$

B $\frac{9I\omega}{2t}$

C $\frac{9I\omega}{3t}$

D $\frac{3I\omega}{2t}$

CORRECT OPTION

A $\frac{2I\omega}{3t}$

SOURCE

Physics • rotational-motion

EXPLANATION

When disc B is brought in contact with disc A. Let w' be the final angular velocity of both the disc rotating together.

Applying conservation of angular momentum for the two disc system.

$$\Rightarrow I(2\omega) + 2I(\omega) = (I + 2I)\omega'$$

$$\Rightarrow \omega' = \frac{4}{3}\omega$$

Torque on disc A,

$$T_A = \frac{\Delta L_A}{t} = \frac{L_f - L_i}{t} = \frac{I \times \frac{4}{3}\omega - I \times 2\omega}{t}$$

$$= \frac{-2I\omega}{3t}$$

Note : The negative sign represents that the torque creates angular retardation.

Question 058 MCQ

QUESTION

The loss of kinetic energy during the above process is :

A $\frac{I\omega^2}{2}$

B $\frac{I\omega^2}{3}$

C $\frac{I\omega^2}{4}$

D $\frac{I\omega^2}{6}$

CORRECT OPTION

B

$$\frac{I\omega^2}{3}$$

SOURCE

Physics • rotational-motion

EXPLANATION

Loss in kinetic energy

= $K.E.$

$$initial$$

$$-$$

$K.E.$

$$final$$

$$= \left[\frac{1}{2}I(2\omega)^2 + \frac{1}{2}(2I)\omega^2\right] - \left[\frac{1}{2}(I+2I)\left(\frac{4}{3}\omega\right)^2\right]$$

$$= [2I\omega^2 + I\omega^2] - \left[\frac{3}{2}I \times \frac{4}{3} \times \frac{4}{3}\omega^2\right]$$

$$= 3I\omega^2 - \frac{8}{3}I\omega^2 = \frac{I\omega^2}{3}$$

Question 059 MCQ

QUESTION

The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

A P 0

B $\frac{P_0}{2}$

C $\frac{P_0}{2} + \frac{Mg}{\pi R^2}$

D $\frac{P_0}{2} - \frac{Mg}{\pi R^2}$

CORRECT OPTION

A P 0

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

When the piston is pulled out slowly, the pressure drop produced inside the cylinder is almost instantaneously neutralized by the air entering from outside into the cylinder. So, the pressure inside the cylinder is P 0

throughout the slow pulling process.

Question 060 MCQ

QUESTION

While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is :

A $$\left(\frac{2P_0\pi R^2}{\pi R^2 P_0 + Mg}\right)(2L)$$

B $$\left(\frac{P_0\pi R^2 - Mg}{\pi R^2 P_0}\right)(2L)$$

C $$\left(\frac{P_0\pi R^2 + Mg}{\pi R^2 P_0}\right)(2L)$$

D $$\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$$

CORRECT OPTION

D $$\left(\frac{P_0\pi R^2}{\pi R^2 P_0 - Mg}\right)(2L)$$

SOURCE

Physics • motion

EXPLANATION

Also condition for equilibrium of the piston is

Mg = $P\$\$_0\$\$\$\$ - \$\$P$

π

R

2

$$\Rightarrow P = \frac{-Mg}{\pi R^2} + P_0$$

Cylinder is thermally conducting, the temperature remains the same.

$$\Rightarrow P_0 \times (2L \times \pi R^2) = Py \times \pi R^2$$

$$\Rightarrow y = \frac{P_0}{P} \times 2L$$

$$= \frac{P_0}{\left[P_0 - \frac{Mg}{\pi R^2}\right]} \times 2L$$

$$= \left[\frac{P_0 \times \pi R^2}{P_0\pi R^2 - Mg}\right] \times 2L$$

Question 061 MCQ

QUESTION

The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is

$$\rho$$

. In equilibrium, the height H of the water column in the cylinder satisfies

A $$\rho g(L_0 - H)^2 + P_0(L_0 - H) + L_0P_0 = 0$$

B $$\rho g(L_0 - H)^2 - P_0(L_0 - H) - L_0P_0 = 0$$

C $$\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$$

D $$\rho g(L_0 - H)^2 - P_0(L_0 - H) + L_0P_0 = 0$$

CORRECT OPTION

C $$\rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$$

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

At equilibrium, P = P

$$\Rightarrow$$

P = P

$$0$$

+ $L\$\$_0\$\$\$\$ - \$\$H$

$$\rho$$

g i

$$\Rightarrow P_0\pi(\pi R^2 L_0) = P[\pi R^2(L_0 - H)]$$

$$\Rightarrow P = \frac{L_0 P_0}{L_0 - H}$$

..... ii

From i as ii

$$\Rightarrow \frac{L_0 P_0}{L_0 - H} = P_0 + (L_0 - H)\rho g$$

$$\Rightarrow L_0 P_0 = P_0(L_0 - H) + (L_0 - H)^2 \rho g$$

$$\Rightarrow \rho g(L_0 - H)^2 + P_0(L_0 - H) - L_0 P_0 = 0$$

Question 062 MCQ

QUESTION

Some physical quantities are given in Column I and some possible SI units in which these quantities may be expressed are given in Column II. Match the physical quantities in Column I with the units in Column II and indicate your answer by darkening appropriate bubbles in the 4

$$\times$$

4 matrix given in the ORS.

	Column I		Column II
A	GM $_e$ M $_s$ G - universal gravitational constant, M $_e$ - mass of the earth, M $_s$ - mass of the Sun	P	$volt$ $coulomb$ $metre$
B	$\frac{3RT}{M}$ R - universal gas constant, T - absolute temperature, M - molar mass	Q	$kilogram$ $metre^{3}$ $second^{-2}$
C	$\frac{F^2}{q^2B^2}$ F - force, q - charge, B - magnetic field	R	$metre^{2}$ $second^{-2}$
D	$\frac{GM_e}{R_e}$ G - universal gravitational	S	$farad$ $volt^{2}$ kg

	Column I		Column II
	constant, M e - mass of the earth R e - radius of the earth		-1

A $A \rightarrow P, Q; B \rightarrow R, S; C \rightarrow R, S; D \rightarrow R, S$

B $A \rightarrow P; B \rightarrow R, S; C \rightarrow R, S; D \rightarrow R$

C $A \rightarrow P, Q; B \rightarrow S; C \rightarrow R, S; D \rightarrow S$

D $A \rightarrow Q; B \rightarrow R, S; C \rightarrow S; D \rightarrow R, S$

CORRECT OPTION

A $A \rightarrow P, Q; B \rightarrow R, S; C \rightarrow R, S; D \rightarrow R, S$

SOURCE

Physics • gravitation

EXPLANATION

Reason : Unit of

$$GM_eM_s = Fr^2 = Nm^2$$

$$= kg\frac{m}{s^2} \times m^2$$

$$= Kgm^3s^{-2}$$

Also $volt\ coulomb\ metre$ = $Joule\ metre$

= $N\$\$ - \$\$m\ m$ = Nm

$$2$$

= Kgm

$$3$$

s

$$-2$$

A

$$\rightarrow$$

P, Q

Reason :

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\Rightarrow v_{rms}^2 = \frac{3RT}{M}$$

Unit of

$$\frac{3RT}{M}$$

is m

$$2$$

s

$$-2$$

Also $farad\ volt$

$$2$$

kg

-1

= $joule\ kg$$$^{-}1$$

= N-mkg

-1

= kgms

-2

mkg

-1

= m

2

s

-2

B

$\rightarrow$

R, S

Reason :

$$F = qvB \Rightarrow v^2 = \frac{F^2}{q^2B^2}$$

Unit of v

2

is m

2

s

-2

which to further equal to FV

2

kg

-1

C

$\rightarrow$

R, S

Reason : Escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$v_e^2 = \frac{2GM}{R}$$

The unit of

$$\frac{GM}{R}$$

is

$$m^2 s^{-2}$$

D

$\rightarrow$

R, S

Question 063 MCQ

QUESTION

Column I gives certain situations in which a straight metallic wire of resistance R is used and Column II gives some resulting effects. Match the statements in Column I with the statements in Column II and indicate your answer by darkening appropriate bubbles in the 4

4 matrix given in the ORS.

	Column I		Column II
A	A charged capacitor is connected to the ends of the wire	P	A constant current flows through the wire
B	The wire is moved perpendicular to its length with a constant velocity in a uniform magnetic field perpendicular to the plane of motion	Q	Thermal energy is generated in the wire
C	The wire is placed in a constant electric field that has a direction along the length of the wire.	R	A constant potential difference develops between the ends of the wire
D	A battery of constant emf is connected to the ends of the wire	S	Charges of constant magnitude appear at the ends of the wire

A

$\rightarrow$

Q; B

$\rightarrow$

A R; C

$\rightarrow$

R, S; D

$\rightarrow$

P, Q

A

$\rightarrow$

Q; B

$\rightarrow$

B R, S; C

$\rightarrow$

R, S; D

$\rightarrow$

P, Q, R

A

$\rightarrow$

Q; B

$\rightarrow$

C R, S; C

$\rightarrow$

R; D

$\rightarrow$

Q, R

A

$\rightarrow$

Q, R; B

$\rightarrow$

D R, S; C

$\rightarrow$

R; D

$\rightarrow$

P, Q, R

CORRECT OPTION

A

$\rightarrow$

Q; B

$\rightarrow$

B R, S; C

$\rightarrow$

R, S; D

$\rightarrow$

P, Q, R

SOURCE

Physics • capacitor

EXPLANATION

A When capacitor is shorted, current will begin to flow and capacitor will start discharging. Because of discharging, magnitude of current in circuit and potential across the two ends will reduce continually until becomes zero. Thermal energy will be generated due to flow of charges.

B When wire is moved perpendicularly in magnetic field, due to change in flux associated with wire, a potential difference will be developed across its ends, but not current will flow as circuit is not complete and no heat will be generated.

C Since wire is placed in electric field, electrons inside the wire will align in such a way that net electric field inside the metal is zero. Net charges will be developed across the two ends of wire, creating a potential difference. Here, circuit is incomplete, so no current will flow and no heat will be generated.

D When a battery is connected, across two ends of wire, there will be constant potential difference. In this case circuit is complete, so current will start flowing and heat will be generated.

Question 064 MCQ

QUESTION

Some laws/processes are given in Column I. Match these with the physical phenomena given in Column II and indicate your answer by darkening appropriate bubbles in the 4

$\times$

4 matrix given in the ORS.

	Column I		Column II
A	Transition between two atomic energy levels	P	Characteristic X-rays
B	Electron emission from a material	Q	Photoelectric effect
C	Mosley's law	R	Hydrogen spectrum
D	Change of photon energy into kinetic energy of electrons	S	β -decay

A

$\rightarrow$

P; B

$\rightarrow$

Q; C

$\rightarrow$

P; D

$\rightarrow$

Q

A

$\rightarrow$

R; B

$\rightarrow$

B Q, S; C

$\rightarrow$

P; D

$\rightarrow$

Q, P

A

$\rightarrow$

P, R; B

$\rightarrow$

C Q, S; C

$\rightarrow$

P; D

$\rightarrow$

Q

A

$\rightarrow$

P, R; B

$\rightarrow$

D Q, S; C

$\rightarrow$

Q; D

$\rightarrow$

P

CORRECT OPTION

A

$\rightarrow$

P, R; B

$\rightarrow$

C Q, S; C

$\rightarrow$

P; D

$\rightarrow$

Q

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Reason : Characteristics x-ray are produced due to transition of electron from one energy level to another. Similarly, the line in the hydrogen spectrum is obtained due to transition of electron from one energy level to another.

A

$\rightarrow$

P, R

Reason : In photoelectric effect electron from the metal surface are emitted upon the incidence of light of appropriate freaquency.

Note : In

β

-decay, electrons are emitted from the nucleus of atom.

B

$\rightarrow$

Q, S

Moseley gave a law which related frequency of emitted X-ray with the atomic number of the target material.

$$\sqrt{v} = a(z - b)$$

C

$\rightarrow$

P

In photo electric effect, energy of photons of incident ray gets converted into K.E of emitted electron.

D

$\rightarrow$

Q

Question 065 MCQ

QUESTION

Statement 1 :

The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

Statement 2 :

Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

A. Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B. Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C. Statement 1 is True, Statement 2 is False

D. Statement 1 is False, Statement 2 is True

CORRECT OPTION

C. Statement 1 is True, Statement 2 is False

SOURCE

Physics • geometrical-optics

EXPLANATION

Statement 1

The formula connecting u, v and f for a spherical mirror is valid only for mirrors whose sizes are very small compared to their radii of curvature.

✅ **True.**

This is due to the **paraxial approximation**—the mirror formula $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ holds only when the mirror aperture is small so that rays make small angles with the principal axis. This avoids spherical aberration.

Statement 2

Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces.

❌ **False.**

The **laws of reflection** $angleofincidence = angleofreflection$ are fundamental and apply to all reflecting surfaces—plane, spherical, or irregular. What changes in large spherical mirrors is that reflected rays do not all converge at a single focus due to geometry, but the law of reflection itself **still holds locally** at every point on the surface.

Conclusion

- Statement 1: **True**
- Statement 2: **False**

✅ **Correct Option: C**

Option C: Statement 1 is True, Statement 2 is False.

Question 066 MCQ

QUESTION

Statement 1 :

If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change.

Statement 2 :

When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.

A Statement 1 is True, Statement 2 is True, Statement 2 is a CORRECT explanation for Statement 1

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

C Statement 1 is True, Statement 2 is False

D Statement 1 is False, Statement 2 is True

CORRECT OPTION

B Statement 1 is True, Statement 2 is True, Statement 2 is NOT a CORRECT explanation for Statement 1

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Since, Potential increases

If we give the constant energy release we observe constant wavelength. So the statement 1 is true.

From Einstein equation,

$$\frac{hc}{\lambda} = W + K.E$$

W = work function 2.4 eV

If we given energy above 2.4 eV it will release photon.

Statement 2 is true.

Hint : $$\lambda_{ray}$$$$\mu$$$target material$ does not depend on V

$$0$$

$$accelerating potential$$

Statement 1 is True

Statement 2 is True, it follows the energy conservatioin.

But statement 2 is not correct explanation of statement 1.