iit Jee 2010 Paper 2 Offline $57 Questions$

Question 001 MCQ

QUESTION

The species having pyramidal shape is

A SO_3

B BrF_3

C SiO_3^{2-}

D OSF_2

CORRECT OPTION

D OSF_2

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

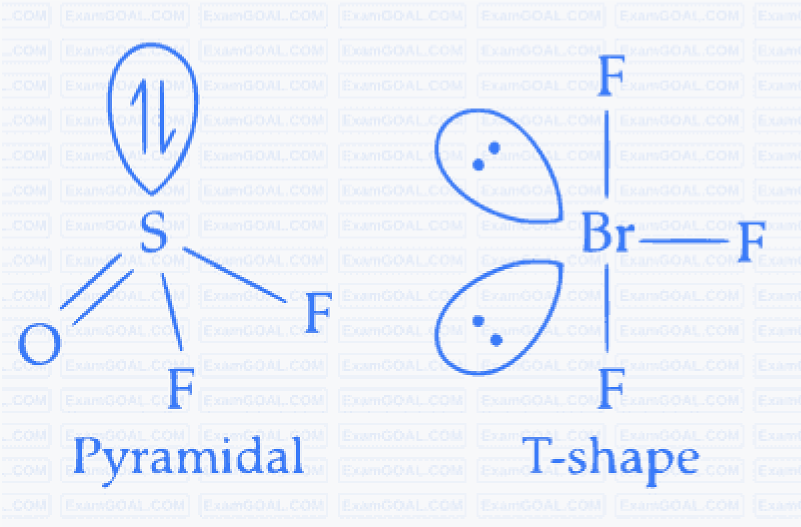
EXPLANATION

$\mathrm{OSF_2}$ has sp^3 hybridisation. According to VSEPR theory, we can conclude that the shape of $\mathrm{OSF_2}$ is a trigonal pyramid.

Sulphur has a non-bonding pair of electrons and three sulphur-oxygen $S - O$ *bondpairs*.\mathrm{SO}_2 *is* s p^2$ hybridised with three bond pairs *twosigmaandonepi* and one lone pair. It is replace trigonal by bent.

BrF_3 is sp^3d hybridized with three bond pair and two lone pairs. It is T shaped.

SiO_3^{2-} is sp^2 hybridised with four bond pairs *threesigmaandonepi* and no lone pair. It has trigonal planar shape.



Question 002 Numerical

QUESTION

Silver (atomic weight = 108 g mol^{-1}) has a density of 10.5 $g.cm^{-3}$. The number of silver atoms on a surface of area 10^{-12} m^2 can be expressed in scientific notation as y

$$\times$$

10^x. The value of x is?

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

Given, atomic weight $= 108 \text{ g mol}^{-1}$

Density $= 10.5 \text{ g cm}^{-3}$

Surface area $= 10^{-12} \text{ m}^2$

Volume of one silver atom $= 4/3\pi r^3$

$$\because \text{ Density } = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \text{ Volume } = \frac{\text{Mass}}{\text{Density}}$$

$$\text{or } \frac{4}{3}\pi r^3 = \frac{108}{6.023 \times 10^{23} \times 10.5}$$

$$r^3 = \frac{108 \times 3}{6.023 \times 10^{23} \times 10.5 \times 4 \times 3.14}$$

$$r^3 = 0.40 \times 10^{-23} = 4 \times 10^{-24}$$

$$\text{or } r = 1.58 \times 10^{-8} \text{ cm}$$

No. of silver atoms on a surface area of 10^{-12} m^2 can be given by $10^{-12} = \text{p}r^2 \times n$

$$n = \frac{10^{-12}}{3.14 \times \left(1.58 \times 10^{-10}\right)^2} = 0.127 \times 10^8$$

$$\Rightarrow n = 1.27 \times 10^7 \text{ or } x = 7$$

Question 003 MCQ

QUESTION

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

The state S_1 is :

A 1s

B 2s

C 2p

D 3s

CORRECT OPTION

B 2s

SOURCE

Chemistry • structure-of-atom

EXPLANATION

For hydrogen-like species, the energy levels depend on the principal quantum number n and are inversely proportional to the square of n, and they can be characterized by their nuclear charge Z. The energy for a hydrogen-like ion can be expressed as:

$$E_n = -\frac{Z^2 R_H}{n^2}$$

where R_H is the Rydberg constant for hydrogen, Z is the atomic number (for Li^{2+}, $Z = 3$), and n is the principal quantum number associated with the energy level.

Given that S_2 has energy equal to the ground state energy of the hydrogen atom $\$E_n\$forhydrogenwhen\$n = 1\$and\$Z = 1\$is\$E_1 = -R_H\$$, and given that S_2 has one radial node. The radial node information tells us about the principal quantum number n, as the number of radial nodes is given by $n - l - 1$, where l is the azimuthal quantum number.

The ground state of hydrogen corresponds to $n = 1$. For the given species Li^{2+} to have the same energy as the ground state of the hydrogen atom but for state S_2, we can use the energy relation. Since the energy is specified to be the same as hydrogen's ground state, let's set up the equality according to the equation given and solve for n specific to the condition (Li^{2+} is considered here, but the condition is about energy equivalence).

Given that S_2 has one radial node, it cannot be the ground state energy level for Li^{2+} $whichwoulddirectlycorrespondto\$n = 1\$$, it implies a different n. For one radial node, the condition $n - l - 1 = 1$ must be met.

The options presented are:

- 1s $\$n = 1\$, \$l = 0\$$ - No radial nodes
- 2s $\$n = 2\$, \$l = 0\$$ - One radial node
- 2p $\$n = 2\$, \$l = 1\$$ - No radial nodes, due to the different l
- 3s $\$n = 3\$, \$l = 0\$$ - Two radial nodes

S_1 is described as having one radial node. Based on the rule for the number of radial nodes $\$n - l - 1\$$, the only states that fits this condition directly from the options provided are $2s$ $sincefora\$2s\$orbital, \$n = 2\$, \$l = 0\$, yielding\$2 - 0 - 1 = 1\$radialnode$.

Hence, the correct option for S_1 is:

Option B: 2s

Question 004 MCQ

QUESTION

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

Energy of the state S_1 in units of the hydrogen atom ground state energy is:

- A 0.75
- B 1.50
- C 2.25
- D 4.50

CORRECT OPTION

C 2.25

SOURCE

Chemistry • structure-of-atom

EXPLANATION

For a hydrogen-like ion, the energy levels can be given by the formula:

$$E_n = -\frac{Z^2}{n^2} E_0$$

where E_n is the energy of the nth level, Z is the atomic number $for Li^{2+}, $Z = 3$$, n is the principal quantum number, and E_0 is the ground state energy of the hydrogen atom $\$ - 13.6\, eV\$$.

Given that state S_2 has energy equal to the ground state energy of the hydrogen atom and one radial node, we identify that S_2 corresponds to $n = 2$ for a hydrogen atom. This is because for hydrogen-like species, the number of radial nodes is given by $n - 1$, where n is the principal quantum number. The ground state $\$n = 1\$$ has 0 nodes, the first excited state $\$n = 2\$$ has 1 radial node, etc.

Since the energy of S_2 is equal to the ground state energy of the hydrogen atom, we can directly compare the energies. For hydrogen $\$Z = 1\$$, the ground state energy $\$n = 1\$$ is:

$$E = -E_0$$

For the Li $^{2+}$ ion in state S_2 $whichweestablishedisequivalentto\$n = 2\$intermsofenergyforhydrogen$, we use the formula $E_n = -\frac{Z^2}{n^2} E_0$. Since $Z = 3$ for Li $^{2+}$, and given that S_2 has the energy equivalent to the ground state of hydrogen $\$E_0\$$, we solve for the energy ratio rather than the specific energy of S_2.

We then look at S_1, which we know must be the ground state for Li $^{2+}$ since it is the state before S_2 and has one radial node $indicating\$n = 2\$for\$S_1\$$.

Thus, for S_1, which actually corresponds to $n = 2$ for Li $^{2+}$, the energy in units of the hydrogen atom ground state energy is:

$$E_{S_1} = -\frac{Z^2}{n^2} E_0 = -\frac{3^2}{2^2} E_0 = -\frac{9}{4} E_0$$

Now, to express this in units of the hydrogen atom ground state energy $\$ - E_0\$$:

$$\frac{E_{S_1}}{E_0} = -\frac{9}{4} = -2.25$$

So, considering the provided options and the fact that energy levels are usually considered in positive values when comparing magnitudes, the correct answer is:

Option C: 2.25.

Question 005 MCQ

QUESTION

The hydrogen like species Li^{2+} is in a spherically symmetric state S_1 with one radial node. Upon absorbing light the ion undergoes transition to a state S_2. The state S_2 has one radial node and its energy is equal to the ground state energy of the hydrogen atom.

The orbital angular momentum quantum number of the state S_2 is

A 0

B 1

C 2

D 3

CORRECT OPTION

B 1

SOURCE

Chemistry • structure-of-atom

EXPLANATION

To identify the orbital angular momentum quantum number, l, for the state S_2 of a hydrogen-like species such as Li^{2+}, we can refer to the given information and the known equations for the energy levels of hydrogen-like atoms. Hydrogen-like atoms or ions have only one electron and their energy in a particular state is given by the equation:

$$E_n = -\frac{Z^2}{n^2}E_0$$

where:

- E_n is the energy of the electron in the nth energy level,
- Z is the atomic number of the species $for\$Li\$, \$Z = 3\$$,
- n is the principal quantum number,
- E_0 is the energy of the ground state of hydrogen $\$-13.6\$eV$.

The problem states that S_2 has energy equal to the ground state energy of the hydrogen atom. The ground state of hydrogen corresponds to $n = 1$ and $E_0 = -13.6$ eV. However, for Li^{2+}, with $Z = 3$, the same energy level could be attained at a different value of n since the Z^2 factor magnifies the energy levels with increasing atomic number. Let's calculate the principal quantum number, n, for the Li^{2+} ion that would give it an energy equal to E_0:

For Li^{2+} ion to have the ground state energy of a hydrogen atom, we can set the energies equal and solve for n:

$$-\frac{Z^2}{n^2}E_0 = E_0$$

Substituting $Z = 3$ and simplifying:

$$-\frac{9}{n^2} = 1$$

From which we find, $n^2 = 9$ and thus $n = 3$.

Furthermore, the problem states that S_1 has one radial node and upon absorbing light, it transitions to S_2 which also has one radial node. Radial nodes are related to the principal quantum number n and the angular quantum number l by the formula:

Number of radial nodes = $n - l - 1$

Given that S_2 has one radial node, we can plug $n = 3$ into the radial node formula to solve for l:

$$1 = 3 - l - 1$$

This simplifies to:

$$l = 3 - 2$$
$$l = 1$$

Therefore, the orbital angular momentum quantum number of the state S_2 is 1, which corresponds to option B.

Question 006 Numerical

QUESTION

Among the following, the number of elements showing only one non-zero oxidation state is :
O, Cl, F, N, P, Sn, Tl, Na, Ti

SOURCE

Chemistry • periodic-table-and-periodicity

EXPLANATION

Answer: 2

Detailed Reasoning:

We are looking for elements from the given list that exhibit only one non-zero oxidation state. The elements given are:

O $Oxygen$

Cl $Chlorine$

F $Fluorine$

N $Nitrogen$

P $Phosphorus$

Sn Tin

Tl $Thallium$

Na $Sodium$

Ti $Titanium$

Let's analyze each:

Oxygen O:

Oxygen typically shows multiple non-zero oxidation states, such as -2 $mostcompounds$, -1 $peroxides$, and even positive oxidation states in compounds like OF_2 $+2$. Hence, it has more than one non-zero oxidation state.

Chlorine Cl:

Chlorine exhibits a wide range of oxidation states: -1 HCl, +1 $HOCl$, +3 $HClO_2$, +5 $HClO_3$, and +7 $HClO_4$. Clearly multiple non-zero states.

Fluorine F:

Fluorine is the most electronegative element and almost always exhibits an oxidation state of -1 in its compounds. It does not show other stable non-zero oxidation states. Thus, fluorine has only one non-zero oxidation state: -1.

Nitrogen N:

Nitrogen shows many oxidation states: from -3 $ammonia, NH_3$ to +5 $innitrates, NO_3^-$, passing through several intermediate states $+1, +2, +3, +4$. Many non-zero oxidation states.

Phosphorus P:

Phosphorus can exist in -3 PH_3, +3 PCl_3, and +5 PCl_5 states. Multiple non-zero oxidation states.

Tin Sn:

Tin commonly shows +2 and +4 oxidation states. Hence more than one non-zero oxidation state.

Thallium Tl:

Thallium typically shows +1 and +3 oxidation states. Again, multiple non-zero oxidation states.

Sodium Na:

Sodium, in stable compounds, is almost always present as Na^+ $oxidation state + 1$. It does not commonly display any other non-zero oxidation state. So sodium has only one non-zero oxidation state.

Titanium Ti:

Titanium can exhibit +2, +3, and +4 oxidation states, among others. Multiple non-zero oxidation states.

Conclusion:

Only fluorine F and sodium Na have exactly one non-zero oxidation state.

Number of such elements = 2

Question 007 MCQ

QUESTION

Assuming that Hund's rule is violated, the bond order and magnetic nature of the diatomic molecule B_2 is

A 1 and diamagnetic

B 0 and diamagnetic

C 1 and paramagnetic

D 0 and paramagnetic

CORRECT OPTION

A 1 and diamagnetic

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

Molecular orbital configuration of B_2 $10 electrons$ is

$$\sigma_{1s^2}\, \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \pi_{2p_x^1} = \pi_{2p_y^1}$$

Here in B_2, 2 unpaired electrons present.

Since the Hund's rule is violated, two electrons are placed in $\pi 2p_x$ molecular orbital, so

$$\mathrm{B}_2(10) : \sigma_{1s^2}\, \sigma^*_{1s^2}\ \sigma_{2s^2}\ \sigma^*_{2s^2}\ \pi_{2p_x^2}$$

Thus, bond order $= \frac{6-4}{2} = 1$.

As there are no unpaired electrons, the nature is diamagnetic.

Question 008 Numerical

QUESTION

The total number of diprotic acids among the following is:

H_3PO_4, H_2SO_4, H_3PO_3, H_2CO_3, $H_2S_2O_7$, H_3BO_3, H_3PO_2, H_2CrO_4 and H_2SO_3

SOURCE

Chemistry • p-block-elements

EXPLANATION

A diprotic acid is an acid that contains within its molecular structure two hydrogen atoms per molecule capable of dissociating $i.e., ionisableprotons$ in water.

$$\mathrm{H_2SO_4, H_2CO_3, H_2\ S_2O_7, H_2CrO_4, H_3PO_3, H_2SO_3}$$

Structure of each of the compounds is given below :

Question 009 MCQ

QUESTION

The compounds $\mathbf{P}, \mathbf{Q}$ and $\mathbf{S}$ were separately subjected to nitration using $\mathrm{HNO_3/H_2SO_4}$ mixture. The major product formed in each case respectively, is :

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

$$\text{The products obtained on nitration of } \mathbf{P}, \mathbf{Q} \text{ and } \mathbf{S} \text{ are}$$

Here strongly activating and ortho-, para directing -OH group determines the site for electrophilic substitution.

Here $-\mathrm{OCH_3}$ is a stronger activator than $-\mathrm{CH_3}$ and both are ortho-, para directing.

Here, the substitution takes place on the activated ring $withsubstituent$PhCOO^-$ at the sterically unhindered para-position.

Question 010 MCQ

QUESTION

The packing efficiency of the twodimensional square unit cell shown below is

A 39.27%

B 68.02%

C 74.05%

D 78.54%

CORRECT OPTION

D 78.54%

SOURCE

Chemistry • solid-state

EXPLANATION

The diagonal is given as 4r, where r is the radius of atom or sphere forming close packed structure.

$$\text{Diagonal } = \sqrt{\mathrm{L}^2+\mathrm{L}^2}=\sqrt{2\,\mathrm{L}^2}$$

$$4r=\sqrt{2\,\mathrm{L}^2} \text{ or } \mathrm{L}=\frac{4r}{\sqrt{2}}=2\sqrt{2}r$$

$$\text{or } \text{ Total area } =\mathrm{L}^2$$

$$(2\sqrt{2}r)^2=8r^2$$

Number of spheres inside the square is

$$1+4\left(\frac{1}{4}\right)=2$$

Area of each sphere $=\pi r^2$

Total area of spheres $= 2 \times \pi r^2$

$$\text{Packing fraction } = \frac{\text{Total area of spheres}}{\text{Total area}}$$

$$= \frac{2 \times \pi r^2}{8r^2} = \frac{\pi}{4} = 0.785$$

So, the percentage fraction is 78.5%.

Question 011 MCQ

QUESTION

The complex showing a spin-only magnetic moment of 2.82 B.M. is :

A $\mathrm{Ni(CO)_4}$

B $[\mathrm{NiCl_4}]^{2-}$

C $\mathrm{Ni(PPh_3)_4}$

D $[\mathrm{Ni(CN)_4}]^{2-}$

CORRECT OPTION

B $[\mathrm{NiCl_4}]^{2-}$

SOURCE

Chemistry • coordination-compounds

EXPLANATION

$[\mathrm{NiCl_4}]^{2-}$

Oxidation state of $[\mathrm{NiCl_4}]^{2-} = +2$

$\mathrm{Ni}(28) = [\mathrm{Ar}]3d^8 4s^2$

$\mathrm{Ni}^{2+} = [\mathrm{Ar}]3d^8$

Among the given complexes of Ni , only Cl^- is a weak ligand and does not cause pairing of electrons to take place.

Thus, the spin only magnetic moment, with two unpaired electrons $(n = 2)$ configuration is

$$\sqrt{n(n+2)} = \sqrt{2(2+2)} = 2.82\mathrm{BM} = 2.82\mathrm{BM}$$

Question 012 MCQ

QUESTION

In the reaction,

The structure of the product **T** is :

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

The reaction taking place is Hoffmann bromamide degradation followed by benzoylation.

Question 013 Numerical

QUESTION

One mole of an ideal gas is taken from $\mathbf{a}$ to $\mathbf{b}$ along two paths denoted by the solid and the dashed lines as shown in the graph below. If the work done along the solid line path is W_s and that dotted line path is W_d, then the integer closest to the ratio $W_\mathrm{d}/W_\mathrm{s}$ is

SOURCE

Chemistry • thermodynamics

EXPLANATION

For calculating work done, we need to calculate the area under curve for solid and dotted lines.

Let ' w_d and ' w ' be work done along the dotted and solid path respectively.

$$\mathrm{W}_d = \text{ Area ABCD } + \text{ Area EFGC} + \text{Area FGIH}$$

$$w_d = 4 \times 1.5 + 1 \times 1 + 2.5 \times 2/3$$

$$= 8.65$$

$$\text{Process of work done } (w_s) \text{ is isothermal}$$

$$w_s = 2 \times 2.303 \log \frac{5.5}{0.5}$$

$$= 2 \times 2.303 \times \log 11$$

$$= 2 \times 2.303 \times 1.0414 = 4.79$$

$$\frac{w_d}{w_s} = \frac{8.65}{4.79} = 1.80 \simeq 2$$

Question 014 Numerical

QUESTION

Total number of geometrical isomers for the complex $[\mathrm{RhCl(CO)}\,(\mathrm{PPh_3})\,(\mathrm{NH_3})]$ is ____________.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The number of geometrical isomers possible for the given complex is three.

Question 015 MCQ

QUESTION

The compounds **P** and **Q** respectively are :

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The given product is an ester, obtained by condensation of a hydroxy acid obtained through hydrolysis of a cyanohydrin :

Acid above is obtained by acid hydrolysis of cyanohydrin S as

S is obtained by nucleophile addition of HCN on R, hence R is

R is obtained by treatment of P and Q with aqueous K_2CO_3 through aldol condensation reaction as

Question 016 MCQ

QUESTION

The compound **R** is :

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The given product is an ester, obtained by condensation of a hydroxy acid obtained through hydrolysis of a cyanohydrin :

Acid above is obtained by acid hydrolysis of cyanohydrin S as

S is obtained by nucleophile addition of HCN on R, hence R is

R is obtained by treatment of P and Q with aqueous $\mathrm{K_2CO_3}$ through aldol condensation reaction as

Question 017 MCQ

QUESTION

The compound **S** is :

A

B

C

D

CORRECT OPTION

D

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The given product is an ester, obtained by condensation of a hydroxy acid obtained through hydrolysis of a cyanohydrin :

Acid above is obtained by acid hydrolysis of cyanohydrin S as

S is obtained by nucleophile addition of HCN on R, hence R is

R is obtained by treatment of P and Q with aqueous $\mathrm{K_2CO_3}$ through aldol condensation reaction as

Question 018 MCQ

QUESTION

Match the reactions in **Column I** with appropriate options in **Column II**.

Column I	**Column II**
A	P Racemic mixture
B	Q Addition reaction
C	R Substitution reaction
D	S Coupling reaction

A $A \rightarrow R$; $B \rightarrow T$; $C \rightarrow P, Q$; $D \rightarrow R$

B $A \rightarrow S$; $B \rightarrow T$; $C \rightarrow P$; $D \rightarrow R$

C $A \rightarrow R,\ S;\ B \rightarrow T;\ C \rightarrow\ Q;\ D \rightarrow R$

D $A \rightarrow R,\ S;\ B \rightarrow T;\ C \rightarrow\ P,\ Q;\ D \rightarrow R$

CORRECT OPTION

D $A \rightarrow R,\ S;\ B \rightarrow T;\ C \rightarrow\ P,\ Q;\ D \rightarrow R$

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

i It is an example of electrophilic substitution reaction which results in the formation of a coupled product. The nucleophilic nitrogen attacks at electron rich carbon of phenol at para position as follows:

$$A \rightarrow R \text{ and } S$$

ii The reaction represents Pinacole-pinacolone rearrangement. In this reaction, the intermediate is carbocation.

The reaction is represented as follows:

$$B \rightarrow T$$

iii It is an example of addition reaction to carbonyl group, where lithium aluminium hydride $\left(\mathrm{LiAlH_4}\right)$ adds hydrogen across carbon and oxygen of carbonyl group thus reducing it to an alcohol. Both R and S enantiomers will be formed. Hence, racemic mixture will be obtained as carbonyl carbon becomes chiral.

C

$$\rightarrow$$

P, Q

iv It is an example of nucleophilic substitution, where electron rich sulphur $duetolonepairs$ displaces chlorine on the ring forming a bicyclic compound.

$$D \rightarrow R$$

Question 019 MCQ

QUESTION

All the compounds listed in **Column I** react with water. Match the result of the respective reactions with the appropriate options listed in **Column II**.

Column I	Column II
A $(CH_3)_2SiCl_2$	P Hydrogen halide formation
B XeF_4	Q Redox reaction
C Cl_2	R Reacts with glass
D VCl_5	S Polymerisation
	T O_2 formation

A $A \rightarrow P$; $B \rightarrow P, Q, R, T$; $C \rightarrow P$; $D \rightarrow P$

B $A \rightarrow P, S$; $B \rightarrow P, Q, R, T$; $C \rightarrow P, Q$; $D \rightarrow P$

C $A \rightarrow P, S$; $B \rightarrow P, R, T$; $C \rightarrow P, Q$; $D \rightarrow P$

D $A \rightarrow P, S$; $B \rightarrow P, Q, R$; $C \rightarrow P, Q$; $D \rightarrow P$

CORRECT OPTION

B $A \rightarrow P, S; B \rightarrow P, Q, R, T; C \rightarrow P, Q; D \rightarrow P$

SOURCE

Chemistry • p-block-elements

EXPLANATION

A Dimethyl silyl chloride undergoes hydration followed by polymerisation owing to the loss of water molecule.

A → P, S

$$(\mathrm{B})\ 6\mathrm{XeF}_4 + 12\mathrm{H}_2\mathrm{O} \rightarrow 4\mathrm{Xe} + 2\mathrm{XeO}_3 + 24\mathrm{HF}\ (\text{Hydrogen fluoride}) + 3\mathrm{O}_2$$

Here, xenon undergoes disproportionation reaction $whichisakindofredoxreaction$ with xenon $in + 4oxidationstate$ gets oxidized to xenon trioxide (XeO_3) and reduced to xenon Xe at the same time.

HF reacts with glass as shown :

$$\mathrm{Na}_2\mathrm{SiO}_3 + 6\mathrm{HF} \rightarrow \mathrm{Na}_2\mathrm{SiF}_6 + 3\mathrm{H}_2\mathrm{O}$$

B → P, Q, R, T

C

$$2\mathrm{Cl}_2 + 2\mathrm{H}_2\mathrm{O} \longrightarrow 4\mathrm{HCl}\ (\text{Hydrogen halide}) + \mathrm{O}_2$$

C → P, Q

$$D\ \mathrm{VCl}_5 + \mathrm{H}_2\mathrm{O} \longrightarrow \mathrm{VOCl}_3 + 2\mathrm{HCl}\ Hydrogenchloride$$

$D \rightarrow P$

Question 020 MCQ

QUESTION

Match the statement in **Column-**

$$I$$

with the values in **Column-**

$$II$$

Column-

$$I$$

A

A line from the origin meets the lines

$$\frac{x-2}{1}=\frac{y-1}{-2}=\frac{z+1}{1}$$

and

$$\frac{x-\frac{8}{3}}{2}=\frac{y+3}{-1}=\frac{z-1}{1}$$

at

$$P$$

and

$$Q$$

respectively. If length

$$PQ=d,$$

then

$$d^2$$

is

B

The values of

$$x$$

satisfying

$$\tan^{-1}(x+3)-\tan^{-1}(x-3)=\sin^{-1}\left(\frac{3}{5}\right)$$

are

C

Non-zero vectors

$$\overrightarrow{a},\overrightarrow{b}$$

and

$$\overrightarrow{c}$$

satisfy

$$\overrightarrow{a}\,.\,\overrightarrow{b}=0.$$

$$\left(\overrightarrow{b}-\overrightarrow{a}\right).\left(\overrightarrow{b}+\overrightarrow{c}\right)=0$$

and

$$2\left|\overrightarrow{b}+\overrightarrow{c}\right|=\left|\overrightarrow{b}-\overrightarrow{a}\right|.$$

If

$$\overrightarrow{a} = \mu\overrightarrow{b} + 4\overrightarrow{c},$$

then the possible values of

$$\mu$$

are

D

Let

$$f$$

be the function on

$$[-\pi, \pi]$$

given by

$$f(0) = 9$$

and

$$f(x) = \sin\left(\frac{9x}{2}\right) / \sin\left(\frac{x}{2}\right)$$

for

$$x \neq 0$$

The value of

$$\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

is

Column-

II

p

-4

q

0

r

4

s

5

t

6

A $(A) \to t;\ (B) \to p, r;\ (C) \to q, s;\ (D) \to r$

B $(A) \to r;\ (B) \to p;\ (C) \to q, s;\ (D) \to r$

C $(A) \to t;\ (B) \to p, r;\ (C) \to q;\ (D) \to r$

D $(A) \rightarrow t;\ (B) \rightarrow r;\ (C) \rightarrow q, s;\ (D) \rightarrow r$

CORRECT OPTION

A $(A) \rightarrow t;\ (B) \rightarrow p, r;\ (C) \rightarrow q, s;\ (D) \rightarrow r$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

A Equation of the line passing through origin is

$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$$

$$\therefore$$

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -2 & 1 \\ a & b & c \end{vmatrix} = 0$$

$$\Rightarrow a(-1) - b(3) + c(-5) = 0$$

$$\Rightarrow -a - 3b - 5c = 0$$

$$\Rightarrow a + 3b + 5c = 0$$

.... i

Also,

$$\begin{vmatrix} \frac{8}{3} & -3 & 1 \\ 2 & -1 & 1 \\ a & b & c \end{vmatrix} = 0$$

$$\therefore$$

$$a(-2) - b\left(\frac{2}{3}\right) + c\left(\frac{10}{3}\right) = 0$$

$$\Rightarrow 2a + \frac{2b}{3} - \frac{10c}{3} = 0$$

$$3a + b - 5c = 0$$

...... ii

From Eqs. i and ii,

$$\frac{a}{-20} = \frac{b}{20} = \frac{c}{-8}$$

$$\frac{a}{5} = \frac{b}{-5} = \frac{c}{4}$$

Equation of line is

$$\frac{x}{5} = \frac{y}{-5} = \frac{z}{4} = \lambda$$

say iii

Also,

$$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z+1}{1} = k_1$$

say iv

Now,

$$\frac{x-\frac{8}{3}}{2} = \frac{y+3}{-1} = \frac{z-1}{1} = k_2$$

say v

Point on iii is

$$(5\lambda,\ -5\lambda,\ +4\lambda)$$

Point on iv is

$$(2+k_1,\ 1-2k_1,\ -1+k_1)$$

Point on v is

$$\left(\frac{8}{3}+2k_2,\ -3-k_2,\ 1+k_2\right)$$

On solving,

$$2+k_1+1-2k_1=0$$

$$-k_1+3=0$$

$$k_1=3$$

$$P\equiv(5,\ -5,\ 2)$$

Again, for Q

$$\frac{8}{3}+2k_2-3-k_2=0$$

$$k_2-\frac{1}{3}=0$$

$$k_2=\frac{1}{3}$$

$$Q\equiv\left(\frac{10}{3},\frac{-10}{3},\frac{4}{3}\right)$$

Now,

$$PQ=\sqrt{\left(\frac{5}{3}\right)^2+\left(\frac{5}{3}\right)^2+\left(\frac{2}{3}\right)^2}$$

$$=\frac{\sqrt{54}}{3}$$

$$PQ^2=d^2=\frac{54}{9}=6$$

B

$$\tan^{-1}\left(\frac{x+3-x+3}{1+(x^2-9)}\right)=\tan^{-1}\left(\frac{3}{4}\right)$$

$$\Rightarrow\frac{6}{x^2-8}=\frac{3}{4}$$

$$\Rightarrow 3x^2 = 48$$

$$\Rightarrow x = \pm 4$$

C

$$\left(\vec{b} - \vec{a}\right) . \left(\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right) = 0$$

$$\Rightarrow \left(\vec{b} - \vec{a}\right) . \left(4\vec{b} + \vec{a} - \mu\vec{b}\right) = 0$$

$$(4 - \mu)\vec{b}^2 - \vec{a}^2 = 0$$

...... i

Also,

$$2\left|\vec{b} + \frac{\vec{a} - \mu\vec{b}}{4}\right| = \left|\vec{b} - \vec{a}\right|$$

$$\Rightarrow 2\left|\frac{(4 - \mu)\vec{b} + \vec{a}}{4}\right| = \left|\vec{b} - \vec{a}\right|$$

Question 021 MCQ

QUESTION

Match the statements in **Column I** with those in **Column II.**

$Note : Hereztakesvalueinthecomplexplane and Imz and Rez denotes, respec$

Column I

A The set of points z satisfying

$$|z - i| z || = |z + i| z ||$$

is contained in or equal to

B The set of points z satisfying

$$|z+4|+\ |z-4|=10$$

is contained in or equal to

C If

$$|w|$$

= 2, then the set of points

$$z=w-\frac{1}{w}$$

is contained in or equal to

D If

$$|w|$$

= 1, then the set of points

$$z=w+\frac{1}{w}$$

is contained in or equal to.

Column II

p an ellipse with eccentricity

$$\frac{4}{5}$$

q the set of points z satisfying Im z = 0

r the set of points z satisfying

$$|\mathrm{Im\,z}|\leq 1$$

s the set of points z satisfying

$$|\mathrm{Re}\ z|<2$$

t the set of points z satisfying

$$|z| \leq 3$$

A) A - q, s ; B - p ; C - p, t ; D - q, r, s, t

B) A - q, r ; B - p ; C - p, s, t ; D - q, r, s, t

C) A - p, r ; B - p ; C - p, t ; D -q, r, s, t

D) A - p ; B - q ; C - r, s ; D -q, r, s, t

CORRECT OPTION

B) A - q, r ; B - p ; C - p, s, t ; D - q, r, s, t

SOURCE

Mathematics • complex-numbers

EXPLANATION

A z is equidistant from the points

$$i|z|$$

and

$$-i|z|$$

, whose perpendicular bisector is

$$\text{Im}(z) = 0$$

.

B Sum of distance of z from $4, 0$ and $\$\$-\$\$4, 0$ is a constant 10, hence locus of z is ellipse with semi-major axis 5 and focus at $\$\$\pm\$\$4, 0$, ae = 4.

$$\therefore$$

$$e = \frac{4}{5}$$

C

$$|z| \le |w| + \left|\frac{1}{w}\right| = \frac{5}{2} < 3$$

D

$$|z| \le |w| + \left|\frac{1}{w}\right| = 2$$

$$\therefore$$

$$\mathrm{Re}(z) \le |z| \le 2$$

Question 022 Numerical

QUESTION

Let

$$f$$

be a function defined on

$$R$$

$theset of all real numbers$

such that

$$f'(x) = 2010\,(x-2009)(x-2010)^2(x-2011)^3(x-2012)^4$$

for all

$$x \in$$

$$R$$

If

$$g$$

is a function defined on

$$R$$

with values in the interval

$$(0, \infty)$$

such that

$$\$ f\left(x\right) = ln\ \left(g\left(x\right)\right),\ \ for\ all\ x \in R$$

$thenthenumberofpointsin\ R\ atwhich\ g$$ has a local maximum is

____________.

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Let

$$g(x) = e^{f(x)}, \forall x \in R$$

$$\Rightarrow g'\left(x\right) = e^{f\left(x\right)}\ .\ f'\left(x\right)$$

$$\Rightarrow$$

f' x changes its sign from positive to negative in the neighbourhood of x = 2009

$$\Rightarrow$$

f x has local maxima at x = 2009

So, the number of local maximum is one.

Question 023 MCQ

QUESTION

If the distance of the point

$$P(1, -2, 1)$$

from the plane

$$x + 2y - 2z$$
$$= \alpha,$$

where

$$\alpha > 0,$$

is

$$5,$$

then the foot of the perpendicular from

$$P$$

to the planes is

A $$\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$$

B $$\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$$

C $$\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$$

D

$$\left(\frac{2}{3},-\frac{1}{3},\frac{5}{3}\right)$$

CORRECT OPTION

A

$$\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Distance of point P from plane = 5

$$\therefore$$

$$5=\left|\frac{1-4-2-\alpha}{3}\right|$$

$$\alpha=10$$

Foot of perpendicular

$$\frac{x-1}{1}=\frac{y+2}{2}=\frac{z-1}{-2}=\frac{5}{3}$$

$$\Rightarrow x=\frac{8}{3},y=\frac{4}{3},z=-\frac{7}{3}$$

Thus, the foot of the perpendicular is

$$A\left(\frac{8}{3},\frac{4}{3},-\frac{7}{3}\right)$$

Question 024 Numerical

QUESTION

Two parallel chords of a circle of radius 2 are at a distance

$$\sqrt{3}+1$$

apart. If the chords subtend at the center , angles of

$$\frac{\pi}{k}$$

and

$$\frac{2\pi}{k},$$

where

$$k>0,$$

then the value of

$$[k]$$

is

[Note :

$$k$$

denotes the largest integer less than or equal to k]

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

Let

$$\theta=\frac{\pi}{2k}$$

$$\cos\theta = \frac{x}{2}$$

$$\Rightarrow \cos 2\theta = \frac{\sqrt{3}+1-x}{2}$$

$$\Rightarrow 2\cos^2\theta - 1 = \frac{\sqrt{3}+1-x}{2}$$

$$\Rightarrow 2\left(\frac{x^2}{4}\right) - 1 = \frac{\sqrt{3}+1-x}{2}$$

$$\Rightarrow x^2 + x - 3 - \sqrt{3} = 0$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+12+4\sqrt{3}}}{2}$$

$$= \frac{-1 \pm \sqrt{13+4\sqrt{3}}}{2}$$

$$= \frac{-1+2\sqrt{3}+1}{2} = \sqrt{3}$$

$$\therefore$$

$$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$$

$$\therefore$$

Required angle

$$= \frac{\pi}{k} = 2\theta = \frac{\pi}{3}$$

$$\Rightarrow k = 3$$

Question 025 MCQ

QUESTION

For

$$r = 0,\, 1, \ldots,$$

let

$$A_r,\ B_r$$

and

$$C_r$$

denote, respectively, the coefficient of

$$X^r$$

in the expansions of

$$(1+x)^{10},$$

$$(1+x)^{20}$$

and

$$(1+x)^{30}.$$

Then

$$\sum_{r=1}^{10} A_r\left(B_{10}B_r - C_{10}A_r\right)$$

is equal to

A $(B_{10} - C_{10})$

B $A_{10}\left({B^2}_{10}C_{10}A_{10}\right)$

C 0

D

$$C_{10} - B_{10}$$

CORRECT OPTION

D

$$C_{10} - B_{10}$$

SOURCE

Mathematics • mathematical-induction-and-binomial-theorem

EXPLANATION

A_r = Coefficient of x^r in

$$(1+x)^{10} = {}^{10}C_r$$

B_r = Coefficient of x^r in

$$(1+x)^{20} = {}^{20}C_r$$

C_r = Coefficient of x^r in

$$(1+x)^{30} = {}^{30}C_r$$

$$\therefore$$

$$\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$$

$$= \sum_{r=1}^{10} A_r B_{10} B_r - \sum_{r=1}^{10} A_r C_{10} A_r$$

$$= \sum_{r=1}^{10} {}^{10}C_r\, {}^{20}C_{10}\, {}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_r\, {}^{30}C_{10}\, {}^{10}C_r$$

$$= \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{20}C_{10}\, {}^{20}C_r - \sum_{r=1}^{10} {}^{10}C_{10-r}\, {}^{30}C_{10}\, {}^{10}C_r$$

$$= {}^{20}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r} \cdot \ {}^{20}C_r - {}^{30}C_{10} \sum_{r=1}^{10} {}^{10}C_{10-r} \cdot \ {}^{10}C_r$$

$$= {}^{20}C_{10} \left({}^{30}C_{10} - 1\right) - {}^{30}C_{10} \left({}^{20}C_{10} - 1\right)$$

$$= {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$$

Question 026 Numerical

QUESTION

Let

$$a_1, \ a_2 \ , \ a_3$$

......,

$$a_{11}$$

be real numbers satisfying

$$a_1 = 15, 27 - 2a_2 > 0 \ and \ a_k = 2a_{k-1} - a_{k-2} \ for \, k = 3, 4, \ldots\ldots\ldots 11$$

. if

$$\frac{a_1^2 + a_2^2 + \ldots . + a_{11}^2}{11} = 90$$

, then the value of

$$\frac{a_1 + a_2 + \ldots . + a_{11}}{11}$$

is equal to :

SOURCE

Mathematics • sequences-and-series

EXPLANATION

$$a_k = 2a_{k-1} - a_{k-2}$$

$$\Rightarrow a_1, a_2, \ \ldots\ldots, \ a_{11}$$

are in AP

$$\therefore$$

$$\frac{a_1^2 + a_2^2 + \ \ldots. \ + a_{11}^2}{11}$$

$$= \frac{11a^2 + 35 \times 11d^2 + 10ad}{11} = 90$$

$$\Rightarrow 225 + 35d^2 + 150d = 90$$

$$35d^2 + 150d + 135 = 0$$

$$\Rightarrow d = -3, -\frac{9}{7}$$

Given,

$$a_2 < \frac{22}{7}$$

$$\therefore$$

$$d = -3$$

and

$$d \neq -\frac{9}{7}$$

$$\Rightarrow \frac{a_1 + a_2 + \ \ldots \ + a_{11}}{11}$$

$$= \frac{11}{2}[30 - 10 \times 3] = 0$$

Question 027 MCQ

QUESTION

Tangents are drawn from the point

$$P(3,4)$$

to the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

touching the ellipse at points

$$A$$

and

$$B$$

.

The coordinates of

$$A$$

and

$$B$$

are

$$(3,0)$$

A and

$$(0,2)$$

$$\left(-\frac{8}{5},\frac{2\sqrt{161}}{15}\right)$$

B and

$$\left(-\frac{9}{5}, \frac{8}{5}\right)$$

C

$$\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$$

and

$$(0, 2)$$

D

$$(3, 0)$$

and

$$\left(-\frac{9}{5}, \frac{8}{5}\right)$$

CORRECT OPTION

D

$$(3, 0)$$

and

$$\left(-\frac{9}{5}, \frac{8}{5}\right)$$

SOURCE

Mathematics • ellipse

EXPLANATION

Equation of chord of contact is

$$\frac{x}{3} + y = 1$$

, i.e.

$$x = 3(1 - y)$$

Solving it with the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

$$(1 - y)^2 + \frac{y^2}{4} = 1 \Rightarrow 4(y^2 - 2y + 1) + y^2 = 4$$

$$\therefore$$

$$y = 0 \Rightarrow 5y^2 - 8y = 0$$

$$\therefore$$

$$y = 0,\ 8/5$$

Correspondingly

$$x = 3,\ -9/5$$

Points are

$$(3, 0)$$

and

$$\left(-\frac{9}{5}, \frac{8}{5}\right)$$

Question 028 MCQ

QUESTION

Tangents are drawn from the point

$$P(3, 4)$$

to the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

touching the ellipse at points

$$A$$

and

$$B$$

.

The orthocentre of the triangle

$$PAB$$

is

A $\left(5, \frac{8}{7}\right)$

B $\left(\frac{7}{5}, \frac{25}{8}\right)$

C $\left(\frac{11}{5}, \frac{8}{5}\right)$

D $\left(\frac{8}{25}, \frac{7}{5}\right)$

CORRECT OPTION

C

$$\left(\frac{11}{5}, \frac{8}{5}\right)$$

SOURCE

Mathematics • ellipse

EXPLANATION

Equation of AB is

$$y - 0 = \frac{\frac{8}{5}}{-\frac{9}{5} - 3}(x - 3) = \frac{8}{-24}(x - 3)$$

$$\Rightarrow y = -\frac{1}{3}(x - 3)$$

$$\Rightarrow x + 3y = 3$$

...... i

Equation of the straight line perpendicular to AB through P is

$$3x - y = 5$$

.

Equation of PA is

$$x - 3 = 0$$

.

The equation of straight line perpendicular to PA through

$$B\left(\frac{-9}{5}, \frac{8}{5}\right)$$

is

$$y = \frac{8}{5}$$

.

Hence, the orthocentre is

$$\left(\frac{11}{5}, \frac{8}{5}\right)$$

.

Question 029 MCQ

QUESTION

Tangents are drawn from the point

$$P(3,4)$$

to the ellipse

$$\frac{x^2}{9}+\frac{y^2}{4}=1$$

touching the ellipse at points

$$A$$

and

$$B$$

.

The equation of the locus of the point whose distances from the point

$$P$$

and the line

$$AB$$

are equal, is

A $9x^2+y^2-6xy-54x-62y+241=0$

B $x^2+9y^2+6xy-54x+62y-241=0$

C $9x^2+9y^2-6xy-54x-62y-241=0$

D $x^2+y^2-2xy+27x+31y-120=0$

CORRECT OPTION

A $9x^2+y^2-6xy-54x-62y+241=0$

SOURCE

Mathematics • ellipse

EXPLANATION

From the given condition,

$$\sqrt{(x-3)^2-(y-4)^2}=\frac{|x+3y-3|}{\sqrt{1+9}}$$

$$\Rightarrow 10\{(x^2-6x+9)+(y^2-8y+16)\}=(x+3y-3)^2$$

$$\Rightarrow 10x^2+10y^2-60x-80y+250=x^2+9y^2+9+6xy-6x-18y$$

$$\Rightarrow 9x^2+y^2-6xy-54x-62y+241=0$$

Question 030 Numerical

QUESTION

Consider a triangle

$$ABC$$

and let

$$a, b$$

and

$$c$$

denote the lengths of the sides opposit to vertices

$$A, B$$

and

$$C$$

respectively. Suppose

$$a = 6, b = 10$$

and the area of the triangle is

$$15\sqrt{3}$$

, if

$$\angle ACB$$

is obtuse and if

$$r$$

denotes the radius of the incircle of the triangle, then r^2 is equal to :

SOURCE

Mathematics • properties-of-triangle

EXPLANATION

$$\sin C = \frac{\sqrt{3}}{2}$$

and C is given to be obtuse

$$\Rightarrow C = \frac{2\pi}{3} = \sqrt{a^2 + b^2 - 2ab\cos C}$$

$$= \sqrt{6^2 + 10^2 - 2 \times 6 \times 10 \times \cos\frac{2\pi}{3}} = 14$$

$$\therefore$$

$$r = \frac{\Delta}{s} \Rightarrow r^2 = \frac{225 \times 3}{\left(\frac{6+10+14}{2}\right)^2} = 3$$

Question 031 MCQ

QUESTION

Let

$$f$$

be a real-valued function defined on the interval

$$(-1, 1)$$

such that

$$e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\, dt,$$

for all

$$x \in (-1, 1)$$

,

and let

$$f^{-1}$$

be the inverse function of

$$f$$

. Then

$$\left(f^{-1}\right)'(2)$$

is equal to

A

$$1$$

B

$$\frac{1}{3}$$

C

$$\frac{1}{2}$$

D

$$\frac{1}{e}$$

CORRECT OPTION

B

$$\frac{1}{3}$$

SOURCE

Mathematics • definite-integration

EXPLANATION

We have,

$$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\,dt\, x \in (-1, 1)$$

On differentiating w.r.t. x, we get

$$e^{-x}(f'(x) - f(x)) = \sqrt{x^4+1}$$

$$\Rightarrow f'(x) = f(x) + \sqrt{x^4+1}\,e^x$$

$$\because$$

$$f^{-1}$$

is the inverse of f

$$\therefore$$

$$f^{-1}(f(x)) = x$$

$$\Rightarrow f^{-1'}(f(x))f'(x) = 1$$

$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f(x) + \sqrt{x^4+1}\,e^x}$$

At

$$x = 0$$

,

$$f(x) = 2$$

$$f^{-1'}(2) = \frac{1}{2+1} = \frac{1}{3}$$

Question 032 Numerical

QUESTION

Let k be a positive real number and let

$$A = \begin{bmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{bmatrix} \text{ and}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 2k-1 & \sqrt{k} \\ 1-2k & 0 & 2\sqrt{k} \\ -\sqrt{k} & -2\sqrt{k} & 0 \end{bmatrix}.$$

If $\det(\text{adj } A) + \det(\text{adj } B) = 10^6$, then $[k]$

is equal to __________.

[**Note :** adj M denotes the adjoint of a square matrix M and $[k]$ denotes the largest integer less than or equal to k].

CORRECT OPTION

C C

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

$$|A| = (2k+1)^3, |B| = 0$$

$ \therefore $Bisaskew - symmetricmatrixoforder3$

$$\text{Let } (\text{adj A}) = |\text{A}|^{n-1}$$

$$\left((2k+1)^3\right)^2 = 10^6$$

$$(2k+1)^6 = 10^6 \Rightarrow 2k+1 = 10$$

$$2k = 9 \Rightarrow [k] = 4$$

Question 033 MCQ

QUESTION

Let $S = \{1, 2, 3, 4\}$. The total number of unordered pairs of disjoint subsets of S is equal to :

A 25

B 34

C 42

D 41

CORRECT OPTION

D 41

SOURCE

Mathematics • functions

EXPLANATION

To solve this problem, we need to determine the total number of **unordered pairs** of **disjoint subsets** of the set $S = \{1, 2, 3, 4\}$.

Understanding the Problem:

Disjoint Subsets: Two subsets A and B are disjoint if they have no elements in common, i.e., $A \cap B = \emptyset$.

Unordered Pairs: Pairs where $\{A, B\}$ is considered the same as $\{B, A\}$.

Approach:

Counting Ordered Pairs of Disjoint Subsets:

For each element in S, it can be in:

Subset A only,

Subset B only,

Neither A nor B.

Note: An element cannot be in both A and B because A and B are disjoint.

Therefore, each element has **3 choices**.

Total number of **ordered pairs** (A, B) is $3^4 = 81$.

Identifying Ordered Pairs where $A = B$:

Since A and B are disjoint and $A = B$, the only possibility is when both are the **empty set**.

So, there's **1 ordered pair** where $A = B = \emptyset$.

Calculating Unordered Pairs:

Ordered Pairs with $A \neq B$: $81 - 1 = 80$.

Unordered Pairs from Ordered Pairs with $A \neq B$: Each unordered pair corresponds to **2 ordered pairs** $since\$(A, B \ \ and \ \ B, A$ $ are different but represent the same unordered pair).

Number of unordered pairs from $A \neq B$ is $\frac{80}{2} = 40$.

Include the pair where $A = B = \emptyset$: Add 1.

Total Unordered Pairs: $40 + 1 = 41$.

Conclusion:

The total number of unordered pairs of disjoint subsets of S is **41**.

Answer: Option D

Question 034 MCQ

QUESTION

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3.$$

Let

$$s$$

be the sum of all distinct real roots of

$$f(x)$$

and let

$$t = |s|.$$

The area bounded by the curve

$$y = f(x)$$

and the lines

$$x = 0,$$

$$y = 0$$

and

$$x = t,$$

lies in the interval

A

$$\left(\frac{3}{4}, 3\right)$$

B

$$\left(\frac{21}{64}, \frac{11}{16}\right)$$

C

$$(9, 10)$$

D

$$\left(0, \frac{21}{64}\right)$$

CORRECT OPTION

A

$$\left(\frac{3}{4}, 3\right)$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

$$\int_0^{1/2} f(x)dx < \int_0^t f(x)dx < \int_0^{3/4} f(x)dx$$

Now,

$$\int f(x)dx$$

$$= \int (1 + 2x + 3x^2 + 4x^3)dx$$

$$= x + x^2 + x^3 + x^4$$

$$\Rightarrow \int_0^{1/2} f(x)dx = \frac{15}{16} > \frac{3}{4}$$

$$\int_0^{3/4} f(x)dx = \frac{530}{256} < 3$$

Question 035 MCQ

QUESTION

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3.$$

Let

$$s$$

be the sum of all distinct real roots of

$$f(x)$$

and let

$$t = |s|.$$

The real numbers lies in the interval

A $\left(-\frac{1}{4}, 0\right)$

B $\left(-11, -\frac{3}{4}\right)$

C $\left(-\frac{3}{4}, -\frac{1}{2}\right)$

D $\left(0, \frac{1}{4}\right)$

CORRECT OPTION

C $\left(-\frac{3}{4}, -\frac{1}{2}\right)$

SOURCE

Mathematics • functions

EXPLANATION

Given,

$$f(x) = 4x^3 + 3x^2 + 2x + 1$$

$$f'(x) = 2(6x^2 + 3x + 1)$$

$$D = 9 - 24 < 0$$

Hence, fx = 0 has only one real root.

$$f\left(-\frac{1}{2}\right) = 1 - 1 + \frac{3}{4} - \frac{4}{8} > 0$$

$$f\left(-\frac{3}{4}\right) = 1 - \frac{6}{4} + \frac{27}{16} - \frac{108}{64}$$

$$= \frac{64 - 96 + 108 - 108}{64} < 0$$

fx changes its sign in

$$\left(-\frac{3}{4}, \frac{-1}{2}\right)$$

,

hence fx = 0 has a root in

$$\left(-\frac{3}{4}, \frac{-1}{2}\right)$$

.

Question 036 MCQ

QUESTION

Consider the polynomial

$$f(x) = 1 + 2x + 3x^2 + 4x^3.$$

Let

$$s$$

be the sum of all distinct real roots of

$$f(x)$$

and let

$$t = |s|.$$

The function

$$f'(x)$$

is

A

increasing in

$$\left(-t, -\frac{1}{4}\right)$$

and decreasing in

$$\left(-\frac{1}{4}, t\right)$$

B

decreasing in

$$\left(-t, -\frac{1}{4}\right)$$

and increasing in

$$\left(-\frac{1}{4}, t\right)$$

C

increasing in

$$(-t, t)$$

D

decreasing in

$$(-t, t)$$

CORRECT OPTION

B

decreasing in

$$\left(-t, -\frac{1}{4}\right)$$

and increasing in

$$\left(-\frac{1}{4}, t\right)$$

SOURCE

Mathematics • functions

EXPLANATION

Question 037 MCQ

QUESTION

A signal which can be green or red with probability

$$\frac{4}{5}$$

and

$$\frac{1}{5}$$

respectively, is received by station A and then transmitted to station

$$B$$

. The probability of each station receving the signal correctly is

$$\frac{3}{4}$$

. If the signal received at atation

$$B$$

is green, then the probability that the original signal was green is

A $\frac{3}{5}$

B $\frac{6}{7}$

C $\frac{20}{23}$

D $\frac{9}{20}$

CORRECT OPTION

C $\frac{20}{23}$

SOURCE

Mathematics • probability

EXPLANATION

From the tree-diagram it follows that

$$P(B_G) = \frac{46}{80}$$

$$P(B_G|G) = \frac{10}{16} = \frac{5}{8}$$

$$\therefore$$

$$P(B_G \cap G) = \frac{5}{8} \times \frac{4}{5} = \frac{1}{2}$$

$$P(G|B_G) = \frac{\frac{1}{2}}{P(B_G)} = \frac{1}{2} \times \frac{80}{46} = \frac{20}{23}$$

Question 038 MCQ

QUESTION

Two adjacent sides of a parallelogram

$$ABCD$$

are given by

$$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\widehat{k}$$

and

$$\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\widehat{k}$$

The side

$$AD$$

is rotated by an acute angle

$$\alpha$$

in the plane of the parallelogram so that

$$AD$$

becomes

$$AD'.$$

If

$$AD'$$

makes a right angle with the side

$$AB,$$

then the cosine of the angle

$$\alpha$$

is given by

A $\frac{8}{9}$

B $\frac{\sqrt{17}}{9}$

C $\frac{1}{9}$

D $\frac{4\sqrt{5}}{9}$

CORRECT OPTION

B

$$\frac{\sqrt{17}}{9}$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

$$\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\widehat{k}$$

$$\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\widehat{k}$$

Angle '

$$\theta$$

' between

$$\overrightarrow{AB}$$

and

$$\overrightarrow{AD}$$

is

$$\cos(\theta) = \left|\frac{\overrightarrow{AB} \cdot \overrightarrow{AD}}{\left|\overrightarrow{AB}\right| \left|\overrightarrow{AD}\right|}\right|$$

$$= \left|\frac{-2 + 20 + 22}{(15)(3)}\right| = \frac{8}{9}$$

$$\Rightarrow \sin(\theta) = \frac{\sqrt{17}}{9}$$

Since,

$$\alpha + \theta = 90^\circ$$

$$\therefore$$

$$\cos(\alpha) = \cos(90^\circ - \theta)$$

$$= \sin(\theta) = \frac{\sqrt{17}}{9}$$

Question 039 MCQ

QUESTION

A vernier calipers has 1 mm marks on the main scale. It has 20 equal divisions on the Vernier scale which match with 16 main scale divisions. For this Vernier calipers, the least count is

- **A** 0.02 mm
- **B** 0.05 mm
- **C** 0.1 mm
- **D** 0.2 mm

CORRECT OPTION

D 0.2 mm

SOURCE

Physics • units-and-measurements

EXPLANATION

In Vernier scale, 16 mm is divided into 20 parts, that is,

1 VSD = 16/20 mm.

Least count = 1 MSD

$$-$$

1 VSD

$$LC = \left(1 - \frac{16}{20}\right)$$

mm = 0.22 mm

Question 040 MCQ

QUESTION

When liquid medicine of density

$$\rho$$

is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

If the radius of the opening of the dropper is

$$r$$

, the vertical force due to the surface tension on the drop of radius R $assuming$$r$$ << R$ is

A

$$2\pi rT$$

B

$$2\pi RT$$

C

$$\frac{2\pi r^2 T}{R}$$

D

$$\frac{2\pi R^2 T}{r}$$

CORRECT OPTION

C

$$\frac{2\pi r^2 T}{R}$$

SOURCE

Physics • properties-of-matter

EXPLANATION

Consider a small element of length dl on the drop-dropper interface. The force on this element is

$$dF = T\,dl$$

, and it makes an angle

$$\theta$$

from the horizontal. Resolve

$$dF$$

in the horizontal and the vertical directions to get,

$$dF_h = T\,dl\cos\theta$$

and

$$dF_v = T\,dl\sin\theta$$

. By symmetry, the force

$$dF_h$$

on two diametrically opposite elements on circular interface is equal and opposite. Thus, total force in the horizontal direction,

$$F_h = \int dF_h = 0$$

. Integrate the vertical component over the circular interface to get

$$F_v = T(2\pi r)\sin\theta = 2\pi r T(r/R) = 2\pi r^2 T/R$$

.

Question 041 MCQ

QUESTION

When liquid medicine of density

$$\rho$$

is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

If r = 5

$\times$

10^{-4} m,

ρ

$= 10^3$ kg m^{-3} , g = 10 m/s^2 , T = 0.11 Nm^{-1} , the radius of the drop when it detaches from the dropper is approximately

A

1.4 $\times$ 10^{-3} m

B

3.3 $\times$ 10^{-3} m

C

2.0 $\times$ 10^{-3} m

D

4.1 $\times$ 10^{-3} m

CORRECT OPTION

A

1.4 $\times$ 10^{-3} m

SOURCE

EXPLANATION

The drop falls down when the weight of the drop becomes larger than the surface tension. Hence,

$$mg = \frac{2\pi r^2 T}{R}$$

That is,

$$\frac{4}{3}\pi R^3 \rho g = \frac{2\pi r^2 T}{R}$$

where

r = 5

$\times$

10

$-$

4 m ;

ρ

= 10^3 kg m

$-$

3 ; g = 10 m/s^2 ; T = 0.11 Nm

$-$

1

Therefore,

$$R^4 = \frac{3r^2 t}{2\rho g} = \frac{3 \times (5 \times 10^{-4})^2 \times 0.11}{2 \times 10^3 \times 10} = 4.125 \times 10^{-12}$$

m^4

$$\Rightarrow R = 1.425 \times 10^{-3}$$

m

Question 042 MCQ

QUESTION

When liquid medicine of density

$$\rho$$

is to be put in the eye, it is done with the help of a dropper. As the bulb on the top of the dropper is pressed, a drop forms at the opening of the dropper. We wish to estimate the size of the drop. We first assume that the drop formed at the opening is spherical because that requires a minimum increase in its surface energy. To determine the size, we calculate the net vertical force due to the surface tension T when the radius of the drop is R. When the force becomes smaller than the weight of the drop, the drop gets detached from the dropper.

After the drop detaches, its surface energy is

A.
1.4

$$\times$$

10^{-6} J

B.
2.7

$$\times$$

10^{-6} J

C.
5.4

$$\times$$

10^{-6} J

8.1

D

$\times$

10^{-6} J

CORRECT OPTION

2.7

B

$\times$

10^{-6} J

SOURCE

Physics • properties-of-matter

EXPLANATION

We have

Surface energy = Tension

$\times$

Area

That is, $E_{surface}$ = T

$\times$

4

π

R^2

where T = 0.11 N/m and R = 1.4

$\times$

10

$-$

4 m. Therefore, the surface energy is

$$E_{surface} = T \times 4\pi R^2 = 0.11 \times 4 \times 3.14 \times (1.4 \times 10^{-3})^2 = 2.7 \times 10^{-6} J$$

Question 043 Numerical

QUESTION

A diatomic ideal gas is compressed adiabatically

$$\frac{1}{32}$$

of its initial volume. If the initial temperature of the gas is T_i $in Kelvin$ and the final temperature is a T_i, the value of

$$a$$

is

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

The diatomic gas has five degrees of freedom i.e., f = 5. Thus, the internal energy per mole, specific heat at constant volume, specific heat at constant pressure, and the ratio of specific heats for a diatomic gas, are given by

$$U = (f/2)RT = (5/2)RT$$

,

$$C_v = dU/dT = 5R/2$$

,

$$C_p = C_v + R = 7R/2$$

,

$$\gamma = C_p/C_v = 7/5$$

.

For an adiabatic process,

$$TV^{\gamma-1}$$

= constant

$$T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$$

Substituting the given values, we get

$$T_iV_i^{\gamma-1} = aT_i\left(\frac{V_i}{32}\right)^{\gamma-1} \Rightarrow a = 32^{\gamma-1}$$

For diatomic gas,

$$\gamma = \frac{7}{5}$$

$$a = 32^{\frac{7}{5}-1} = 32^{2/5} = 2^2 = 4$$

Question 044 MCQ

QUESTION

A hollow pipe of length 0.8 m is closed at one end. At its open end a 0.5 m long uniform string is vibrating in its second harmonic and it resonates with the fundamental frequency of the pipe. If the tension in the wire is 50 N and the speed of sound is 320 ms^{-1}, the mass of the string is

A 5 grams

B 10 grams

C 20 grams

D 40 grams

CORRECT OPTION

B 10 grams

SOURCE

Physics • waves

EXPLANATION

The fundamental mode in a pipe closed at one end and the second harmonic in a string are shown in the figure.

Fundamental frequency of a hollow pipe closed at one end is

$$\upsilon_p = \frac{v}{4L}$$

where, v = speed of sound, L = length of a pipe

Frequency of second harmonic of a string is

$$\upsilon_s = \frac{2}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{l}\sqrt{\frac{T}{\mu}}$$

where, T = tension of the string, m = mass per unit length of the string, l = length of the string

$$\upsilon_s = \frac{1}{l}\sqrt{\frac{T}{\frac{m}{l}}} = \frac{1}{l}\sqrt{\frac{Tl}{m}} = \sqrt{\frac{T}{ml}}$$

...... i

where m is the mass of the string

According to given problem,

$$v_p = v_s$$

$$\therefore$$

$$\frac{v}{4L} = \sqrt{\frac{T}{ml}}$$

or

$$m = \frac{16L^2T}{v^2l}$$

Substituting the given values, we get

$$m = \frac{16 \times (0.8)^2 \times (50)}{(320)^2 \times 0.5} = 0.01$$

kg = 10 gram

Question 045 MCQ

QUESTION

A tiny spherical oil drop carrying a net charge

$$q$$

is balanced in still air with a vertical uniform electric field of strength

$$\frac{81\pi}{7} \times 10^5 \ Vm^{-1}.$$

When the field is switched off, the drop is observed to fall with terminal velocity

$$2 \times 10^{-3} \ ms^{-1}.$$

Given

$$g = 9.8 \, m \, s^{-2},$$

viscosity of the air

$$= 1.8 \times 10^{-5}\ Ns\,m^{-2}$$

and the density of coil

$$= 900$$

$$kg$$

$$m^{-3},$$

the magnitude of

$$q$$

is

A $1.6 \times 10^{-19} C$

B $3.2 \times 10^{-19} C$

C $4.8 \times 10^{-19} C$

D $8.0 \times 10^{-19} C$

CORRECT OPTION

D $8.0 \times 10^{-19} C$

SOURCE

Physics • electrostatics

EXPLANATION

The forces acting on the oil drop are its weight, buoyant force, and electrostatic force. The buoyant force on the oil drop is very small as compared to other two forces. Thus, the weight of the spherical oil drop is balanced by the electrostatic force

$$qE = mg$$

$$qE = \frac{4}{3}\pi r^3 \rho g$$

..... i

where, r = radius of oil drop,

$$\rho$$

= density of oil, q = charge on the oil drop

IN the absence of electric field,

IN equilibrium,

Viscous force on the drop = Weight of the drop

$$6\pi\eta r v_T = mg$$

$$6\pi\eta r v_T = \frac{4}{3}\pi r^3 \rho g$$

...... ii

where, v_T = terminal velocity,

$$\eta$$

= coefficient of viscosity of air

or

$$r^2 = \frac{18 \times \eta \times v_T}{4 \times \rho \times g}$$

Substituting the given values, we get

$$r^2 = \frac{18 \times 1.8 \times 10^{-5} \times 2 \times 10^{-3}}{4 \times 900 \times 9.8}$$

or

$$r = \frac{3}{7} \times 10^{-5}$$

m

From equation i, we get

$$q = \frac{4\pi r^3 \rho g}{3E}$$

Substituting the given values, we get

$$q = \frac{4 \times \pi \times \left(\frac{3}{7} \times 10^{-5}\right)^3 \times 900 \times 9.8}{3 \times \left(\frac{81\pi}{7} \times 10^5\right)}$$

$$= \frac{4 \times \pi \times 7 \times 3 \times 3 \times 3 \times 10^{-15} \times 900 \times 9.8}{3 \times 81\pi \times 10^5 \times 7 \times 7 \times 7} = 8 \times 10^{-19} C$$

Question 046 MCQ

QUESTION

A uniformly charged thin spherical shell of radius

$$R$$

carries uniform surface charge density of

$$\sigma$$

per unit area. It is made of two hemispherical shells, held together by pressing them with force

$$F$$

$see figure.$

$$F$$

is proportional to

A $\frac{1}{\varepsilon_0}\sigma^2 R^2$

B $\frac{1}{\varepsilon_0}\sigma^2 R$

C $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$

D $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

CORRECT OPTION

A $\frac{1}{\varepsilon_0}\sigma^2 R^2$

SOURCE

Physics • electrostatics

EXPLANATION

Electrostatic pressure is

$$\left(\frac{\sigma^2}{2\varepsilon_0}\right)$$

N/m^2

The force is

$$F = \frac{\sigma^2}{2\varepsilon_0} \times \pi R^2$$

Therefore,

$$F \propto \frac{\sigma^2 R^2}{\varepsilon_0}$$

Question 047 MCQ

QUESTION

A block of mass 2 kg is free to move along the x-axis. It is at rest and from t = 0 onwards, it is subjected to a time-dependent force F t in the x-direction. The force F t varies with t as shown in the figure. The kinetic energy of the block after 4.5 s is

A 4.50 J

B 7.50 J

C 5.06 J

D 14.06 J

CORRECT OPTION

C 5.06 J

SOURCE

Physics • work-power-and-energy

EXPLANATION

The t - F diagram is a straight line passing through $0, 4$ and $3, 0$. The equation of this straight line is

$$F = -\frac{4}{3}t + 4$$

.

Newton's second law gives acceleration of the block as

$$a = F/m = -\frac{2}{3}t + 2$$

.

Integrate to get the velocity v at 4.5 s

$$v = \int_0^{4.5} a\,dt = \int_0^{4.5} \left(-\frac{2}{3}t + 2\right) dt = 2.25$$

m/s.

Thus, kinetic energy of the block at 4.5 s is

$$K = \frac{1}{2}mv^2 = 5.06$$

J.

Question 048 MCQ

QUESTION

A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is

- **A** virtual and at a distance of 16 cm from the mirror.
- **B** real and at a distance of 16 cm from the mirror.
- **C** virtual and at a distance of 20 cm from the mirror.
- **D** real and at a distance of 20 cm from the mirror.

CORRECT OPTION

B real and at a distance of 16 cm from the mirror.

SOURCE

Physics • geometrical-optics

EXPLANATION

First refraction through lens

Here, u =

—

30 cm, f = +15 cm

Using

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{+15}$$

or

$$\frac{1}{v} = \frac{1}{15} - \frac{1}{30}$$

v = +30 cm

$$\therefore$$

Image I_1 is real and formed at a distance of 30 cm on the right side of the lens.

Distance of image I_1 from the mirror is

30 cm

$$-$$

10 cm = 20 cm

The image I_1 acts as an virtual object for the mirror. The mirror forms an image I_2 at a distance of 20 cm in front of it.

The image I_2 acts as an object for the lens.

Second refraction through lens

Here, u = +10 cm, f = +15 cm

$$\frac{1}{v} - \frac{1}{+10} = \frac{1}{+15}$$

or

$$\frac{1}{v} = \frac{1}{15} + \frac{1}{10}$$

$$\Rightarrow$$

v = +6 cm

The final image I_3 is real and at a distance of 16 cm from the mirror.

Question 049 Numerical

QUESTION

A large glass slab $\$\$\mu\$\$ = 5/3$ of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R?

SOURCE

Physics • geometrical-optics

EXPLANATION

From the figure shown here, we have

$$\tan i_c = \frac{R}{t}$$

$$\sin i_c = \frac{R}{\sqrt{R^2 + t^2}} = \frac{1}{\mu} = \frac{3}{5}$$

$$25R^2 = 9R^2 + 9t^2$$

$$16R^2 = 9t^2 \Rightarrow R = \frac{3t}{4} = \frac{3 \times 8}{4} = 6$$

cm

Question 050 Numerical

QUESTION

Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from

$$\frac{25}{3}$$

m to

$$\frac{50}{7}$$

m in 30 s. What is the speed of the object in km per hour?

SOURCE

Physics • geometrical-optics

EXPLANATION

Focal length of a convex mirror,

$$f = \frac{R}{2} = \frac{20}{2}$$

m = 10 m

For first object,

$$v_1 = +\frac{25}{3}$$

m,

$$f = +10$$

m

Using mirror formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\therefore$$

$$\frac{1}{(25/3)}+\frac{1}{u_1}=\frac{1}{10}$$

or

$$\frac{1}{u_1}=\frac{1}{10}-\frac{3}{25}$$

or

$$u_1=-50$$

m

For second object,

$$v_2=+\frac{50}{7}$$

m,

$$f=+10$$

m

$$\therefore$$

$$\frac{1}{v_2}+\frac{1}{u_2}=\frac{1}{f}$$

$$\frac{1}{(50/7)}+\frac{1}{u_2}=\frac{1}{10}$$

or

$$\frac{1}{u_2}=\frac{1}{10}-\frac{7}{50}$$

or

$$u_2=-25$$

m

Speed of the object

$$= \frac{25}{30}$$

m s

–

1

$$= \frac{25}{30} \times \frac{18}{5}$$

km h

–

1 = 3 km h

–

1

Question 051 Numerical

QUESTION

To determine the half-life of a radioactive element, a student plots a graph of

$$\ln \left| \frac{dN(t)}{dt} \right|$$

versus t. Here,

$$\frac{dN(t)}{dt}$$

is the rate of radioactive decay at time t. If the number of radioactive nuclei of this element decreases by a factor of p after 4.16 years, the value of p is

____________.

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The activity of a radioactive substance, having a decay constant

$$\lambda$$

and number of nuclei N at time t, is given by

$$A = |dN/dt| = \lambda N = \lambda N_0 e^{-\lambda t}$$

..... 1

Take logarithm on both sides of equation 1 to get

$$\ln|dN/dt| = \ln(\lambda N_0) - \lambda t$$

...... 2

Thus, the graph between t and

$$|dN/dt|$$

is a straight line with slope

$$-\lambda$$

.

Slope

$$= -\lambda = \frac{3-4}{6-4}$$

$From graph$ or

$$\lambda = \frac{1}{2}$$

year

$$-$$

1

Half life

$$T_{1/2} = \frac{0.693}{\lambda} = 2 \times 0.693$$

years = 1.386 years

4.16 years is approximately 3 half-lives

Nuclei will decay by a factor of 2^3 = 8

$$\therefore$$

p = 8

Question 052 Numerical

QUESTION

At time t = 0, a battery of 10 V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time $inseconds$ does the voltage across them becomes 4 V? $Takeln5 = 1.6, ln3 = 1.1$

SOURCE

Physics • capacitor

EXPLANATION

The equivalent resistance of the two parallel resistors is

$$R = \frac{(2\,M\Omega)(2\,M\Omega)}{(2\,M\Omega) + (2\,M\Omega)} = 1M\Omega$$

The equivalent capacitance of the two parallel capacitors is

$$C = 2\mu F + 2\mu F = 4\mu F$$

This corresponding equivalent diagram is as shown in the figure.

The voltage across the equivalent capacitor is same as the voltage across the individual capacitors $parallel combination$. Thus, we need to find time t at which the voltage across C become 4 V in the equivalent circuit $charging of a capacitor$. The voltage across C at time t is

$$V = V_0 \left[1 - e^{-t/(RC)}\right]$$

,

which simplifies to

$$t = RC \ln\left(\frac{V_0}{V_0 - V}\right)$$

.

Substitute

$$V_0 = 10$$

V,

$$V = 4$$

V,

$$R = 1 \times 10^6 \Omega$$

and

$$C = 4 \times 10^{-6} F$$

to get

$$t = 4 \ln(5/3) = 4(\ln 5 - \ln 3) = 2\,s$$

.

Question 053 MCQ

QUESTION

A diatomic molecule has moment of inertia I. By Bohr's quantization condition, its rotational energy in the nth level $n = 0 isnotallowed$ is

A $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$

B $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

C $n\left(\frac{h^2}{8\pi^2 I}\right)$

D $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

CORRECT OPTION

D $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To find the quantized rotational energy of a diatomic molecule, we can use Bohr's quantization condition for angular momentum. Bohr's quantization condition states that the angular momentum L of an electron in a hydrogen atom is quantized and given by:

$$L = n\hbar$$

where n is a positive integer $knownasthequantumnumber$ and $\hbar$ is the reduced Planck's constant, given by $\hbar = \frac{\hbar}{2\pi}$.

For a diatomic molecule, we extend this concept to its rotational motion. A rigid diatomic molecule can be approximated as a rigid rotor with a moment of inertia I. The angular momentum L for the molecule in the nth level is given by:

$$L = n\hbar$$

where n is the quantum number $notethat\$n = 0\$isnotallowed$.

The rotational kinetic energy for a rotating body with moment of inertia I is given by:

$$E = \frac{1}{2} I\omega^2$$

where ω is the angular velocity. The angular momentum L is related to the angular velocity ω by:

$$L = I\omega$$

Solving for ω, we get:

$$\omega = \frac{L}{I}$$

Substituting this back into the expression for energy, we get:

$$E = \frac{1}{2} I \left(\frac{L}{I}\right)^2 = \frac{L^2}{2I}$$

Using Bohr's quantization condition $L = n\hbar$, the energy expression becomes:

$$E = \frac{(n\hbar)^2}{2I}$$

Substituting $\hbar = \frac{\hbar}{2\pi}$, we obtain:

$$E = \frac{n^2}{2I}\left(\frac{\hbar^2}{4\pi^2}\right) = n^2\left(\frac{\hbar^2}{8\pi^2 I}\right)$$

Therefore, the quantized rotational energy of the diatomic molecule in the nth level is:

$$E_n = n^2\left(\frac{\hbar^2}{8\pi^2 I}\right)$$

So, the correct option is:

Option D

$$n^2\left(\frac{h^2}{8\pi^2 I}\right)$$

Question 054 MCQ

QUESTION

It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to

$$\frac{4}{\pi} \times 10^{11}$$

Hz. Then, the moment of inertia of CO molecule about its centre of mass is close to (Take h = 2

$$\pi$$

$$\times$$

10

$$-$$

^{34} J-s)

A. 2.76×10^{-46} kg m^2

B. 1.87×10^{-46} kg m^2

C. 4.67×10^{-47} kg m^2

D. 1.17×10^{-47} kg m^2

CORRECT OPTION

1.87

×

B 10

−

46 kg m^2

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The energy of photon is equal to the energy difference between the ground level and first excited level.

$$hv = E_2 - E_1$$

$$hv = \frac{(4-1)h^2}{8\pi^2 I} \Rightarrow I = \frac{3h}{8\pi^2 v}$$

$$I = \frac{3 \times 2\pi \times 10^{-34}}{[(8\pi^2 4)/\pi] \times 10^{11}} = \frac{3}{16} \times 10^{-45}$$

kg-m^2 = 1.87

×

10

−

46 kg-m^2

Question 055 MCQ

QUESTION

In a CO molecule, the distance between C $mass = 12amu$ and O $mass = 16amu$, where 1 amu

$$= \frac{5}{3} \times 10^{-27}$$

kg, is close to :

A

2.4

×

10

−

10 m

B

1.9

×

10

−

10 m

C

1.3

×

10

−

10 m

D

4.4

×

10

−

11 m

CORRECT OPTION

1.3

×

C 10

−

10 m

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The moment of inertia of CO molecule is

I =

μ

r^2 i

where,

μ

= reduced mass of the CO molecule, r = distance between C and O or bond length

The reduced mass

μ

of the CO molecule is

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \left[\frac{(12)(16)}{12+16}\right] \times \frac{5}{3} \times 10^{-27}$$

kg

But

$$I = 1.87 \times 10^{-46}$$

kg m^2 $From the above question$

From equation i, we get

$$r^2 = \frac{I}{\mu}$$

Substituting the values of I and

$$\mu$$

in above equation, we get

$$r^2 = \frac{1.87 \times 10^{-46}}{\left[\frac{12 \times 16}{28} \times \frac{5}{3} \times 10^{-27}\right]}$$

or

$$r^2 = \frac{1.87 \times 10^{-46} \times 28 \times 3}{12 \times 16 \times 5 \times 10^{-27}}$$

or

$$r = 1.3 \times 10^{-10}$$

m

Question 056 MCQ

QUESTION

Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2 and μ_3 are given. Match them to the ray diagram shown in Column II :

A $A \rightarrow P,\ R;\ B \rightarrow Q,\ S,\ T;\ C \rightarrow P,\ R,\ T;\ D \rightarrow Q,\ S$

B $A \rightarrow R;\ B \rightarrow Q,\ S;\ C \rightarrow P,\ R,\ T;\ D \rightarrow Q,\ S$

C $A \rightarrow P,\ R;\ B \rightarrow S,\ T;\ C \rightarrow P,\ R;\ D \rightarrow Q,\ S$

D $A \rightarrow P;\ B \rightarrow Q,\ S,\ T;\ C \rightarrow P,\ T;\ D \rightarrow Q$

CORRECT OPTION

A $A \rightarrow P,\ R;\ B \rightarrow Q,\ S,\ T;\ C \rightarrow P,\ R,\ T;\ D \rightarrow Q,\ S$

SOURCE

Physics • geometrical-optics

EXPLANATION

If $\mu_1 < \mu_2$ then the ray bends towards the normal after refraction at $\mu_1 - \mu_2$ interface. Incident rays parallel to the optic axis bend towards the optic axis by the convex lens and away from the optic axis by the concave lens. If $\mu_1 > \mu_2$ then the ray bends away from the normal. If $\mu_2 = \mu_3$ then the ray goes straight without bending at the $\mu_2 - \mu_3$ interface. If $\mu_2 > \mu_3$ then the ray bends away from the normal.

Question 057 MCQ

QUESTION

You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source $thefirsttwocircuits$ or an AC voltage source of 50 Hz frequency $thenextthreecircuits$ in different ways as shown in Column II. When a current I $steadystateforDCorrmsforAC$ flows through the circuit, the corresponding voltage V_1 and V_2 $indicatedincircuits$ are related as shown in Column I. Match the two :

A $A \rightarrow R,\ S,\ T;\ B \rightarrow Q,\ R,\ S,\ T;\ C \rightarrow P,\ Q;\ D \rightarrow Q,\ R,\ S,\ T$

B $A \rightarrow R,\ S;\ B \rightarrow Q,\ R,\ S,\ T;\ C \rightarrow P,\ Q;\ D \rightarrow Q,\ R,\ T$

C $A \rightarrow R,\ S,\ T;\ B \rightarrow Q,\ R,\ S;\ C \rightarrow P,\ Q;\ D \rightarrow Q,\ R,\ S$

D $A \rightarrow S,\ T;\ B \rightarrow Q,\ R,\ S,\ T;\ C \rightarrow P;\ D \rightarrow Q,\ R,\ S,\ T$

CORRECT OPTION

A $A \rightarrow R,\ S,\ T;\ B \rightarrow Q,\ R,\ S,\ T;\ C \rightarrow P,\ Q;\ D \rightarrow Q,\ R,\ S,\ T$

SOURCE

Physics • alternating-current

EXPLANATION

In circuit (P), under steady state, capacitor will act like an infinite impedance and inductor will act like a zero impedance. Thus, $I = 0, V_1 = 0$, and $V_2 = V$.

In circuit (Q), inductor will act as zero resistance in steady state giving us $I = V/R = V/2, V_1 = 0$, and $V_2 = V$.

In circuit (R), the inductive reactance X_L and impedance Z are

$$X_L = \omega L = 2\pi\nu L = 1.88\Omega, \quad \text{and}$$

$$Z = \sqrt{X_L^2 + R^2} = 2.75\Omega$$

Thus, $I = V/Z \neq 0, V_1 = X_L I = 1.88I, V_2 = RI = 2I$, and $V_2 > V_1$.

In circuit S, inductive reactance X_L, capacitive reactance X_C, and impedance Z are

$$X_L = 1.88\Omega$$

$$X_C = 1/(\omega C) = 1061\Omega, \quad \text{and}$$

$$Z = X_C - X_L = 1059\Omega$$

Thus the current in the circuit $I = V/Z \neq 0, V_1 = X_L I = 1.88I, V_2 = X_C I = 1061I$, and $V_2 > V_1$.

In circuit $(T), X_C, R$, and Z are

$$X_C = 1/(\omega C) = 1061\Omega$$

$$R = 1000\Omega, \quad \text{and}$$

$$Z = \sqrt{R^2 + X_C^2} = 1458\Omega$$

Thus, $I = V/Z \neq 0, V_1 = RI = 1000I, V_2 = X_C I = 1061I$, and $V_2 > V_1$.