JEE Advanced 2015 Paper 1 $Offline$ $60Questions$

Question 001 MCQ

QUESTION

Match the anionic species given in **Column-I** that are present in the ore s given in **Column-II**

Column - I	Column - II
A Carbonate	p Siderite
B Sulphide	q Malachite
C Hydroxide	r Bauxite
D Oxide	s Calamine
	t Argentite

A

A

$\rightarrow$

p,q,s; B

$\rightarrow$

t; C

$\rightarrow$

q,r; D

$\rightarrow$

r

B

$A \rightarrow$ s; $B \rightarrow$ s,t; $C \rightarrow$ r; $D \rightarrow$ r,s

C

$A \rightarrow$ p,s; $B \rightarrow$ s,t; $C \rightarrow$ r; $D \rightarrow$ s

D

$A \rightarrow$ p,q,s; $B \rightarrow$ s,t; $C \rightarrow$

p,r; D

$\rightarrow$

t

CORRECT OPTION

A

$\rightarrow$

p,q,s; B

$\rightarrow$

A t; C

$\rightarrow$

q,r; D

$\rightarrow$

r

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Carbonate ores are
P Siderite : $FeCO_3$
Q Malachite : $CuCO_3.Cu(OH)_2$
S Calamine : $ZnCO_3$

Sulphide ore is T Argentite : Ag_2S.

Hydroxide ion is present in
Q Malachite : $CuCO_3.Cu(OH)_2$
R Bauxite : $Al_2O_3.2H_2O$ or $AlO_x(OH)_{3-2x}$ where $0 < x < 1$

Oxide ore is bauxite R only

Note :
$Rememberallthoseoresnames. Anyoneofthosecanbeaskedintheexam.$

Oxides Ores :

1 ZnO - Zincite

2 Fe_2O_3 - Haematite

3 Fe_3O_4 - Magnetite (FeO + Fe_2O_3 mixture)

4 $Fe_2O_3 . 3H_2O$ - Limonite

5 MnO_2 - Pyrolusite

6 Cu_2O - Cuprite or Ruby Copper

7 TiO_2 - Rutile

8 $FeCr_2O_4$ - Chromite (FeO + Cr_2O_3)

9 $FeTiO_3$ - Illmenite (FeO + TiO_2)

10 $Na_2B_4O_7 . 10H_2O$ - Borax or Tincal

11 U_3O_8 - Pitch Blende

12 SnO_2 - Tin Stone or Cassiterite

13 $Ca_2B_6O_{11} . 5H_2O$ - Colemanite (2 Cao + 3 B_2O_3)

14 $Al_2O_3 . 2H_2O$ - Bauxite

15 $Al_2O_3 . H_2O$ - Diaspore

16 Al_2O_3 - Corundum

Sulphides Ores :

1 ZnS - Zinc Blende or Sphalerite

2 PbS - Galena

3 Ag_2S - Argentite or Silver Glance

4 HgS - Cinnabar

5 Cu_2S - Chalcocite or Copper glance

6 $CuFeS_2$ - Copper pyrites or Chalco pyrites ($Cu_2S + Fe_2S_3$ mixture)

7 FeS_2 - Iron pyrites or Fool's Gold

8 $3Ag_2S . Sb_2S_2$ - Pyrargyrite or ruby silver

Halides Ores :

1 NaCl - Rock Salt

2 KCl - Sylvine

3 Na_3AlF_6 - Cryolite [3NaF + AlF_6]

4 CaF_2 - Fluorspar

5 KCl . $MgCl_2 . 6H_2O$ - Carnalite

6 AgCl - Horn Silver

Carbonates Ores :

1 $CaCO_3$ - Limestone

2 $MgCO_3$ - Magnesite

3 $CaCO_3 . MgCO_3$ - Dolomite

4 $ZnCO_3$ - Calamine

5 $PbCO_3$ - Cerrusite

6 $FeCO_3$ - Siderite

7 $CuCO_3 . Cu OH_2$ or $Cu_2CO_3 OH_2$ - Malachite or Basic Copper Carbonates

8 2 $CuCO_3 . Cu OH_2$ - Azurite

Sulphates Ores :

1 $CuSO_4 . 2H_2O$ - Gypsum

2 $MgSO_4 . 7H_2O$ - Epson Salt

3 $Na_2SO_4 . 10 H_2O$ - Glauber's Salt

4 $PbSO_4$ - Anglesite

5 $ZnSO_4 . 7H_2O$ - White Vitriol

6 $FeSO_4 . 7H_2O$ - Green Vitriol

7 $CuSO_4 . 5H_2O$ - Blue Vitriol or Chalcanthite

Nitrate Ores :

1 KNO_3 - Indian Saltpetre

2 $NaNO_3$ - Chile Saltpetre

Arsenides Ores :

1 NiAs - Kupfernickel

2 NiAsS - Nickel glance

Question 002 MCQ

QUESTION

Copper is purified by electrolytic refining of blister copper. The correct statement s about this process is are

A Impure Cu strip is used as cathode

B Acidified aqueous $CuSO_4$ is used as electrolyte

C Pure Cu deposits at cathode

D Impurities settle as anode – mud

CORRECT OPTION

B Acidified aqueous $CuSO_4$ is used as electrolyte

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

a is wrong statement. Impure copper is set as anode where copper is oxidized to Cu^{2+} and goes into electrolytic solutions.

b $CuSO_4$ is used as an electrolyte in purification process.

c Pure copper is deposited at cathode as : Cu^{2+} + 2e

$$-$$

$$\rightarrow$$

Cu : $Atcathode$

d Less active metals like Ag, Au etc. settle down as anode mud.

Question 003 Numerical

QUESTION

All the energy released from the reaction

$$X \rightarrow Y, \Delta_t G^o$$

= -193 kJ mol^{-1} is used for oxidizing M^+ as M^+

$$\rightarrow$$

M^{3+} + $2e^-$, E^o = -0.25 V

Under standard conditions, the number of moles of M^+ oxidized when one mole of X is converted to Y is [F = 96500 C mol^{-1}]

SOURCE

Chemistry • electrochemistry

EXPLANATION

Given :

$X \rightarrow Y; \Delta_r G^{\circ} = -193 \text{ kJ mol}^{-1}$

$M^{+} \rightarrow M^{3+} + 2e^{-}; E^{\circ} = -0.25 \text{ V}$

$F = 96500 \text{ C mol}^{-1}$

Let 193 kJ is used for oxidising x moles of M^+.

For 1 mole of M^+,

$\Delta G^\circ = -nFE^\circ = -2 \times 96500 \times -0.25$

= 48250 J mol^{-1} = 48.25 kJ mol^{-1}

Thus, no. of moles of M^+ oxidized when one mole of X is converted to Y =

$$\frac{193}{48.25} = 4$$

.

Question 004 Numerical

QUESTION

If the freezing point of a 0.01 molal aqueous solution of a cobalt III chloride-ammonia complex $which behaves as a strong electrolyte$ is – 0.0558°C, the number of chloride s in the coordination sphere of the complex is [K_f of water = 1.86 K kg mol^{-1}]

SOURCE

Chemistry • solutions

EXPLANATION

The depression in freezing point is given by

Δ

$T_f = K_f$

$\times$

m

$\times$

i

0.0558 = 1.86

$\times$

0.01

$\times$

i

i = 3

Therefore, one mole of complex gives three moles of ions in solution.

Hence, the complex is $[Co(NH_3)_5Cl]Cl_2$ and the number of Cl

$-$

ions inside the coordination sphere is 1.

Question 005 MCQ

QUESTION

If the unit cell of a mineral has cubic close packed ccp array of oxygen atoms with m fraction of octahedral holes occupied by aluminium ions and n fraction of tetrahedral holes occupied by magnesium ions, m and n, respectively, are

A 1/2, 1/8

B 1, 1/4

C 1/2, 1/2

D 1/4, 1/8

CORRECT OPTION

A 1/2, 1/8

SOURCE

Chemistry • solid-state

EXPLANATION

For ccp, Z = 4 = no. of O-atoms

No. of octahedral voids = 4

No. of tetrahedral voids = 2

$$\times$$

4 = 8

No. of Al^{3+} ions = m

$$\times$$

4

No. of Mg^{2+} ions = n

$$\times$$

8

Thus, the formula of the mineral is Al_{4m} $Mg_{8n}O_4$

4m$+3$ + 8n$+2$ + 4$\$\$ - \$\$2$ = 0

12m + 16n

$$-$$

8 = 0

4$3m + 4n\$\$ - \$\$2$ = 0

3m + 4n = 2

Possible values of m and n are

$$\frac{1}{2}$$

and

$$\frac{1}{8}$$

respectively.

Question 006 MCQ

QUESTION

Match the thermodynamics processes given under **column I** with expression given under **column II**

Column I
A Freezing water at 273 K and 1 atm
B Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions.
C Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container.
D Reversible heating of $H_2 g$ at 1 atm from 300K to 600K, followed by reversible cooling to 300K at 1 atm

Column II
p q = 0
q w = 0
r

$$\Delta S_{sys}$$

< 0
s

$$\Delta U$$

= 0
t

$$\Delta G$$

= 0

A

→

r,t; B

→

A q,s; C

→

p,s; D

→

p,q,s;

A

→

r,t; B

→

B p,s; C

→

s; D

→

q,s,t;

A

→

r,t; B

→

C p,q,s; C

→

p,q,s; D

→

p,q,s,t;

A

$\rightarrow$

r,t; B

$\rightarrow$

D p,s; C

$\rightarrow$

p,q,s; D

$\rightarrow$

p,s,t;

CORRECT OPTION

A

$\rightarrow$

r,t; B

$\rightarrow$

C p,q,s; C

$\rightarrow$

p,q,s; D

$\rightarrow$

p,q,s,t;

SOURCE

Chemistry • thermodynamics

EXPLANATION

A

$$\rightarrow$$

$RandT$

Freezing of water,

H_2O_l

$$\underset{273\ K}{\overset{1\ atm}{\rightleftharpoons}}$$

H_2O_s

The system is cooled i.e.; heat is released during the process

so, q < 0

$$\underset{(Less\ volume)}{Water}$$

$$\rightleftharpoons$$

$$\underset{(More\ volume)}{Ice}$$

+ heat

Volume is increased i.e.;

$$\Delta$$

V = +ve.

w =

$$-$$

P

$$\Delta$$

V =

$$-$$

ve

i.e.; w < 0 $expansion$

Entropy of system is decreased,

$$\Delta$$

$S_{sys} < 0$.

$$\Delta$$

U = q + w

As q < 0, w < 0 so,

$$\Delta$$

U < 0.

At equilibrium,

$$\Delta$$

G = 0.

B

$$\rightarrow$$

$P, QandS$

Expansion of 1 mol of an ideal gas into a vacuum under isolated conditions,

w = 0, q = 0 so,

$$\Delta$$

U = 0

For expansion,

$$\Delta$$

$S_{sys} > 0$ as entropy increases.

$$\Delta G = -nRT \ln \frac{V_2}{V_1}$$

For expansion, $V_2 > V_1$

$$\Delta$$

G =

$-$

ve i.e.;

Δ

G < 0.

C

$\rightarrow$

$P, QandS$

Mixing of equal volumes of two ideal gases at constant temperature and pressure in an isolated container.

q = 0 $isolated$

w =

$-$

P

Δ

V

w = 0 $$ \because $$$$\Delta$$V = 0$

Δ

S_{sys} > 0 $mixingofgases$

Δ

U = q + w = 0

Δ

G =

Δ

H

$-$

T

Δ

S

Δ

G = q_p

$-$

T

Δ

S $at constant P, T$

Δ

G = 0

$-$

T

Δ

S =

$-$

T

Δ

S

Δ

G < 0 (

$\because$

Δ

S_{sys} > 0)

D

$\rightarrow$

$P, Q, SandT$

$$\underset{(300\ K)}{H_{2(g)}} \underset{Cool}{\overset{Heat,\ 1\ atm}{\rightleftharpoons}} \underset{(600\ K)}{H_{2(g)}}$$

Internal energy U, entropy S and free energy G are state functions which depend only upon the state of the system and do not depend upon the path by which the state is attained.

Thus,

$$\Delta$$

U = 0,

$$\Delta$$

S = 0 and

$$\Delta$$

G = 0

Work and heat are path functions but the same path is retraced so, q = 0 and w = 0.

Question 007 Numerical

QUESTION

Among the triatomic molecules/ions, $BeCl_2$,

$$N_3^-$$

, N_2O,

$$NO_2^+$$

, O_3, SCl_2,

$$ICl_2^-$$

,

$$I_3^-$$

and XeF_2, the total number of linear molecule s /ion s where the hybridization of the central atom does not have contribution from the d-orbital s is

$$Atomicnumber : S = 16, Cl = 17, I = 53 and Xe = 54$$

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

Question 008 Numerical

QUESTION

Not considering the electronic spin, the degeneracy of the second excited state $n = 3$ of H atom is 9, while the degeneracy of the second excited state of H^- is ____________.

SOURCE

Chemistry • structure-of-atom

EXPLANATION

i Number of electrons in hydride ion (H^-) is $= 2$

ii Electronic configuration of ground state in H^- ion $G.S$ = $1s^2$

iii Electronic configuration of first excited state of H^- ion $(\mathrm{E}S_1)$

iv Electronic configuration of second excited state of H^- (E.S $_2$)

v The electron in $2p$ orbital can occupy any three $2p$ orbitals $2p_{x'}2p_y$ and $2p_z$ as follows :

Hence, three degenerate orbitals represents second excited state of H^-.

Question 009 Numerical

QUESTION

The total number of stereoisomers that can exist for M is ___________.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The total number of stereo isomers = $2^n = 2^2 = 4$, where n is the number of chiral centres.

However, in bridge/bicycle compounds, the number of stereo-isomers is equal to the number of chiral centres because no carbon centres rotation is possible. Therefore, two stereoisomers would be possible for the given compound.

Question 010 Numerical

QUESTION

The number of resonance structure for N is __________.

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

The possible resonance structures for the given compound on loss of proton are as follows:

Hence, the number of possible resonance structures is nine.

Question 011 Numerical

QUESTION

The number of lone pairs of electrons in N_2O_3 is ____________.

SOURCE

Chemistry • p-block-elements

EXPLANATION

The structure of N_2O_3 is

Therefore, the total number of lone pairs is 8.

Question 012 Numerical

QUESTION

For the octahedral complexes of Fe^{3+} in SCN

$$-$$

$thiocyana - to - S$ and in CN

$$-$$

ligand environments, the difference between the spin-only magnetic moments in Bohr magnetons $whenapproximatedtothenearestinteger$ is __________.

$$AtomicnumberFe = 26$$

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The spin only magnetic moment is given by

$$\mu = \sqrt{n(n+1)}$$

, where n is the number of unpaired electrons.

Fe^{3+} complex with weak field ligand SCN

$$-$$

contains five unpaired electrons.

Therefore,

$$\mu$$

= 5.9 BM.

Fe^{3+} complex with strong field ligand CN

$$-$$

contains one unpaired electron.

Therefore,

$$\mu$$

= 1.73 BM.

Thus, the difference in spin only magnetic moment is

$$\approx$$

4.

Question 013 MCQ

QUESTION

Compound$s$ that on hydrogenation produce$s$ optically inactive compound$s$ is are

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • haloalkanes-and-haloarenes

EXPLANATION

Question 014 MCQ

QUESTION

The major product of the following reaction is

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction proceeds via intramolecular aldol condensation of the given diketone. The enolate formed on abstraction of proton adds to the second ketone group followed by cyclization to a six-membered ring.

Question 015 MCQ

QUESTION

In the following reaction, the major product is

A

B

C

D

CORRECT OPTION

D

SOURCE

Chemistry • hydrocarbons

EXPLANATION

1, 4-addition is the major product as it is the result of the formation of a stable allylic carbocation.

Question 016 MCQ

QUESTION

The structure of D-$+$-glucose is

The structure of L-$$ — $$-glucose is

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • biomolecules

EXPLANATION

In D-$+$-glucose the OH group attached to the last stereocentre is on the right hand side while in L-$$ - $$-glucose it is on left hand side. The structure of L-$$ - $$-glucose is

Question 017 MCQ

QUESTION

The major product of the reaction is

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Question 018 MCQ

QUESTION

The correct statements about Cr^{2+} and Mn^{3+} is are
$Atomic numbers of Cr = 24 and Mn = 25$

A. Cr^{2+} is a reducing agent.

B. Mn^{3+} is an oxidizing agent.

C. both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration.

D. when Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration.

CORRECT OPTION

A. Cr^{2+} is a reducing agent.

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

The correct statements are as follows:

A Cr^{3+} is more stable than Cr^{2+}, thus, Cr^{2+} act as reducing agent.

B Mn^{2+} is more stable than Mn^{3+}, thus, Mn^{3+} act as an oxidizing agent.

C Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration.

Question 019 MCQ

QUESTION

Fe^{3+} is reduced to Fe^{2+} by using

- **A** H_2O_2 in presence of NaOH.
- **B** Na_2O_2 in water.
- **C** H_2O_2 in presence of H_2SO_4.
- **D** Na_2O_2 in presence of H_2SO_4.

CORRECT OPTION

A H_2O_2 in presence of NaOH.

SOURCE

Chemistry • redox-reactions

EXPLANATION

Fe^{3+} is reduced to Fe^{2+} by H_2O_2/NaOH and Na_2O_2/H_2O.

$$2Fe^{3+} + H_2O_2 + OH^- \rightarrow 2Fe^{2+} + 2H_2O + O_2$$

$$Na_2O_2 + H_2O \rightarrow H_2O_2 + NaOH$$

Question 020 MCQ

QUESTION

The % yield of ammonia as a function of time in the reaction

$N_2 g$ + $3H_2 g$

$$\rightleftharpoons$$

$2NH_3 g$,

$$\Delta$$

H < 0 at (P, T_1) is given below:

If this reactions is conducted at (P, T_2), with $T_2 > T_1$, the % yield of ammonia as a function of time is represented by

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

The equilibrium yield of ammonia gets lowered with the increase of temperature. However, at higher temperature the initial rate of forward reaction would be greater than at lower temperature that is why the percentage yield of NH_3 would be more initially.

Question 021 Numerical

QUESTION

Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of

$$\frac{m}{n}$$

is

SOURCE

EXPLANATION

Given: 5 boys and 5 girls

$n =$ number of ways of arranging them in a queue such that all the girls stand consecutively.

Let us consider 5 girls as one set

So, we have to arrange 5 boys and one set of girls. They can be arranged in 6 ! ways.

Also, the girls in the set can be arranged in 5 ! ways

So, total number of ways $= 6! \times 5!$

$$\Rightarrow n = 6! \times 5!$$

Now, $m =$ number of ways of arranging them in a queue, such that exactly four girls stand consecutively.

$\because$ Exactly four girls can stand together so the remaining one girl must not stand consecutively with four girls.

Let us consider 2 cases:

Case I : The set of four girls is at the corner. Firstly, four girls are selected out of five girls in ${}^5\mathrm{C}_4$ ways. These girls are arranged in 4 ! ways.

Also, these girls can be placed in any of the two corners and the remaining one girl cannot stand next to the set of girls placed at the corner. So, the 5^{th} girl can stand at $7 - 1 - 1 = 5 ways$ And the boys can be arranged in 5 ! ways.

$$\text{So, number of ways } = 4! \times 2 \times {}^5\mathrm{C}_4 \times 5! \times 5$$
$$= 2 \times 5 \times 5! \times 5!$$

Case II: The set of four girls are not placed at the corner.

So, four girls can be selected and arranged among themselves in ${}^5C_4 \times 4! = 5$! ways These girls are not at the corner so they can be arranged at 5 places.

The 5^{th} girl can stand at $7 - 2 - 1 = 4$ ways. $\{$ As she cannot stand at places near the set of four girls $\}$ and the boys can be arranged in 5 ! ways.

So, number of ways $= 5! \times 5 \times 5 \times 4 \times 5!$

$$\Rightarrow m = (2 \times 5 \times 5! \times 5!) + (5 \times 4 \times 5! \times 5!)$$
$$= 5! \times 5!(10 + 20)$$
$$= 30 \times 5! \times 5!$$
$$\therefore \frac{m}{n} = \frac{30 \times 5! \times 5!}{6! \times 5!}$$
$$\Rightarrow \frac{m}{n} = \frac{30 \times 5!}{6 \times 5!} \qquad \because n! = n(n-1)!$$
$$\Rightarrow \frac{m}{n} = 5$$

Question 022 Numerical

QUESTION

The number of distinct solutions of the equation

$$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$$

in the interval

$$[0,\ 2\pi]$$

is

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

Given: $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \ \sin^6 x = 2$

$$\Rightarrow \frac{5}{4}\cos^2 2x + \left(\cos^2 x\right)^2 + \left(\sin^2 x\right)^2 + \left(\cos^2 x\right)^3$$

$$+ \left(\sin^2 x\right)^3 = 2 \qquad \ldots(i)$$

As we know, $a^2 + b^2 + 2ab = (a+b)^2$

$$\Rightarrow a^2 + b^2 = (a+b)^2 - 2ab$$

And $a^3 + b^3 + 3ab(a+b) = (a+b)^3$

$$\Rightarrow a^3 + b^3 = (a+b)^3 - 3ab(a+b)$$

So, equation i can be written as

$$\frac{5}{4}\cos^2 2x+\left(\cos^2 x+\sin^2 x\right)^2-2\left(\cos^2 x\right)$$

$$\left(\sin^2 x\right)+\left(\cos^2 x+\sin^2 x\right)^3-3\cos^2 x\sin^2 x$$

$$\left(\cos^2 x+\sin^2 x\right)=2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x+(1)^2-2\cos^2 x\sin^2 x+(1)^3$$

$$-3\cos^2 x\sin^2 x(1)=2 \quad \left\{\because \cos^2 x+\sin^2 x=1\right\}$$

$$\Rightarrow \frac{5}{4}\cos^2 2x+2-5\cos^2 x\sin^2 x=2$$

$$\Rightarrow \frac{5}{4}\cos^2 2x-5\cos^2 x\sin^2 x=0$$

As we know, $\sin 2\theta=2\sin\theta\cos\theta$

$$\Rightarrow \frac{5}{4}\cos^2 2x - \frac{5}{4}\sin^2 2x = 0$$

$$\because \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\Rightarrow \frac{5}{4}\cos 4x = 0$$

$$\Rightarrow \cos 4x = 0$$

$$\Rightarrow 4x = 2(n+1)\frac{\pi}{2}, n \in \mathrm{I}$$

$$\because x \in [0, 2\pi]$$

So, possible distinct values of x are $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}$ and $\frac{15\pi}{8}$.

So, the number of distinct solutions of the given equation are 8 .

Question 023 MCQ

QUESTION

Match the following :

Column

I

A

In

$$R^2,$$

If the magnitude of the projection vector of the vector

$$\alpha\hat{i}+\beta\hat{j}$$

on

$$\sqrt{3}\hat{i}+\hat{j}$$

and If

$$\alpha=2+\sqrt{3}\beta,$$

then possible value of

$$|\alpha|$$

is/are
B

Let

$$a$$

and

$$b$$

be real numbers such that the function

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ bx+a^2, & x\geq 1\end{cases}$$

if differentiable for all

$$x\in R$$

. Then possible value of

$$a$$

is are
C

Let

$$\omega \neq 1$$

be a complex cube root of unity. If

$$\left(3-3\omega+2\omega^2\right)^{4n+3}+\left(2+3\omega-3\omega^2\right)^{4n+3}+\left(-3+2\omega+3\omega^2\right)^{4n+3}=0,$$

then possible value s of

$$n$$

is are
D

Let the harmonic mean of two positive real numbers

$$a$$

and

$$b$$

be

$$4.$$

If

$$q$$

is a positive real nimber such that

$$a, 5, q, b$$

is an arithmetic progression, then the value s of

$$|q-a|$$

is are

Column

$$II$$

p

1

q

2

r

3

s

4

t

5

A $(A) \rightarrow p, q;\ (B) \rightarrow p, q;\ (C) \rightarrow p, q, s, t;\ (D) \rightarrow q, t$

B $(A) \rightarrow q;\ (B) \rightarrow q;\ (C) \rightarrow p, q, s, t;\ (D) \rightarrow q, t$

C $(A) \rightarrow q;\ (B) \rightarrow p, q;\ (C) \rightarrow p, t;\ (D) \rightarrow q, t$

D $(A) \to q;\ (B) \to p, q;\ (C) \to p, q, s, t;\ (D) \to q$

CORRECT OPTION

A $(A) \to p, q;\ (B) \to p, q;\ (C) \to p, q, s, t;\ (D) \to q, t$

SOURCE

Mathematics • vector-algebra

EXPLANATION

Option A: Let $\vec{a} = \alpha\hat{i} + \beta\hat{j}$ and $\vec{b} = \sqrt{3}\hat{i} + \hat{j}$.

Therefore, the magnitude of projection of $\vec{a}$ on $\vec{b}$ is

$$\vec{b} = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|}$$

$$= \frac{|\sqrt{3}\alpha + \beta|}{\sqrt{3+1}} = \sqrt{3}$$

$$\Rightarrow \sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$$

$$\Rightarrow \sqrt{3}(2 + \sqrt{3}\beta) + \beta = \pm 2\sqrt{2}$$

$$\Rightarrow \beta = 0 \text{ or } \beta = -\sqrt{3} \Rightarrow \alpha = 2 \text{ or } \alpha = -1$$

$$\Rightarrow |\alpha| = 2 \text{ or } 1$$

Hence, $(\mathrm{A}) \to (\mathrm{P}), (\mathrm{Q})$.

Option B: $f(x)=\begin{cases}-3ax^2-2, x<1\\ bx+a^2, x>1\end{cases}$

Since $f(x)$ is differentiable $\forall x\in\mathbb{R}$, we have $f\left(1^{-}\right)=f\left(1^{+}\right)$

$$\Rightarrow -3a-2=b+a^2$$

$$\Rightarrow a^2+3a+2=-b$$

$$\Rightarrow (a+2)(a+1)=-b \qquad \ldots(1)$$

Also, $f'(x)=\begin{cases}-6ax; & x<1\\ b; & x>1\end{cases}$

$$=f'\left(1^{-}\right)=f'\left(1^{+}\right)$$

$$\Rightarrow \quad -6a=b \qquad \ldots(2)$$

Therefore, from Eqs. 1 and 2, we get $2\ \ a^2+3a+2\ =6a$

$$\Rightarrow a=1 \text{ or } a=2$$

Hence, $B\ \rightarrow(\mathrm{P})$, Q.

Also,

$$f'(x)=\begin{cases}-6ax, & x<1 \\ b, & x\geq 1\end{cases}$$

$$\Rightarrow f'\left(1^{-}\right)=f'\left(1^{+}\right)$$

$$\Rightarrow -6a=b$$

$$f'\left(1^{-}\right)=f'\left(1^{+}\right), a^2+3a+2=6a$$

$$\Rightarrow a=1 \text{ or } a=2$$

Hence, $(\mathrm{B}) \rightarrow (\mathrm{P}),(\mathrm{Q})$

$\mathbb{Option}$ **($\mathbb{C}$):** $\left(3-3\omega+2\omega^2\right)^{4x+3}+\left(2+3\omega-3\omega^2\right)^{4x+3}+$

$\left(-3+2\omega+3\omega^2\right)^{4x+3}=0$

$\Rightarrow \left[1-3\omega+2\left(1+\omega^2\right)\right]^{4x+3}+\left[2(1+\omega)+\omega-3\omega^2\right]+$

$\left[-3+\omega^2+2\left(\omega+\omega^2\right)\right]^{4x+3}=0$

$$\Rightarrow [1-3\omega-2\omega]^{4x+3}+\left[-5\omega^2+\omega\right]^{4x+3}$$

$$\left(\omega^2\right)^{4x+3}(1-5\omega)^{4x+3}=0$$

$$\Rightarrow (1-5\omega)^{4x+3}\left(1+\omega^n+\omega^{2x}\right)=0$$

$$\Rightarrow \omega(1-5\omega)^{4x+3}\neq 0$$

$$\Rightarrow 1+\omega^x+\omega^{2x}=0$$

$$\Rightarrow x=3k+1 \text{ or } x=3k+2; k\in\mathbb{Z}$$

$$\Rightarrow x\in\{1,2,4,5\}$$

Hence, $(\mathrm{C})\rightarrow(\mathrm{P}),(\mathrm{Q}),(\mathrm{S}),(\mathrm{T})$.

Option D: HM of a and $b=\frac{2ab}{a+b}=4$, where $a,b>0$. Now, $a,5,q,b$ are in AP, where $q>0$.

$$\Rightarrow a+b=5+q$$

$$\Rightarrow \frac{ab}{2}=5+q \qquad (1)$$

Also

$$a+q=10 \text{ and } q=\frac{5+b}{2} \qquad (2)$$

$$\Rightarrow b=2q-5(3)$$

Therefore, from Eqs. 1, 2 and 3, $a = \frac{5}{2}$ or $a = 6$.

$$\Rightarrow q = \frac{15}{2} \text{ or } 4 \Rightarrow |q - a| = 5 \text{ or } 2$$

Hence, $D \rightarrow Q, T$.

Question 024 MCQ

QUESTION

Let

$$P$$

and

$$Q$$

be distinct points on the parabola

$$y^2 = 2x$$

such that a circle with

$$PQ$$

as diameter passes through the vertex

$$O$$

of the parabola. If

$$P$$

lies in the first quadrant and the area of the triangle

$$\Delta OPQ$$

is

$$3\sqrt{2},$$

then which of the following is are the coordinates of

$$P$$

?

A $\left(4, 2\sqrt{2}\right)$

B $\left(9, 3\sqrt{2}\right)$

C $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$

D $\left(1, \sqrt{2}\right)$

CORRECT OPTION

A $\left(4, 2\sqrt{2}\right)$

SOURCE

Mathematics • parabola

EXPLANATION

Let

$$P\left(\frac{t_1^2}{2}, t_1\right)$$

and

$$Q\left(\frac{t_2^2}{2}, t_2\right)$$

be two distinct points on the parabola

$$y^2 = 2x$$

.

The circle with PQ as diameter passes through the vertex O $0, 0$ of the parabola.

Clearly, PO

$$\perp$$

OQ

So, slope of PO

$$\times$$

slope of OQ =

$$-$$

1

or,

$$\frac{t_1 - 0}{\frac{t_1^2}{2} - 0} \times \frac{t_2 - 0}{\frac{t_2^2}{2} - 0} = -1$$

or,

$$\frac{2}{t_1} \times \frac{2}{t_2} = -1$$

or,

$$t_1 t_2 = -4$$

By question, area of

$$\Delta OPQ = 3\sqrt{2}$$

or,

$$\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{t_1^2}{2} & t_1 & 1 \\ \frac{t_2^2}{2} & t_2 & 1 \end{vmatrix} = 3\sqrt{2}$$

or,

$$\frac{1}{2}\left|\frac{t_1^2 t_2}{2} - \frac{t_1 t_2^2}{2}\right| = 3\sqrt{2}$$

or,

$$|t_1 t_2|\,|t_1 - t_2| = 12\sqrt{2}$$

or,

$$\left|t_1 + \frac{4}{t_1}\right| = 3\sqrt{2}$$

$$\$\$ \because \$\$\$\$ t_1 t_2 = -4 \$\$$$

or,

$$t_1 + \frac{4}{t_1} = 3\sqrt{2}$$

$$\$\$ \because \$\$ Pliesinfirstquadrant$$

or,

$$t_1^2 - 3\sqrt{2}t_1 + 4 = 0$$

or,

$$t_1 = \frac{3\sqrt{2} \pm \sqrt{18 - 4 \times 1 \times 4}}{2}$$

$$= \frac{3\sqrt{2} \pm \sqrt{2}}{2}$$

$$= 2\sqrt{2}, \sqrt{2}$$

$$\therefore$$

coordinates of

$$P = (4, 2\sqrt{2})$$

or

$$(1, \sqrt{2})$$

.

Therefore, A and D are correct options.

Question 025 Numerical

QUESTION

If the normals of the parabola

$$y^2 = 4x$$

drawn at the end points of its latus rectum are tangents to the circle

$$(x - 3)^2 + (y + 2)^2 = r^2$$

, then the value of

$$r^2$$

is

SOURCE

EXPLANATION

Given: A parabola

$$y^2 = 4x$$

Comparing the given equation of parabola with the standard equation of parabola

$$y^2 = 4ax$$

, we get

$$a = 1$$

Also, the end points of latus Rectum are

$$(a, \pm 2a)$$
$$\Rightarrow$$

The end points of latus rectum are

$$(1, 2)$$

and

$$(1, -2)$$

Also we know that the equation of normal to the parabola at point

$$\begin{aligned} & \left(am^2, -2am\right) \text{ is } y = mx - 2am - am^3 \\ \Rightarrow \quad & \left(am^2, -2am\right) = (1, 2) \\ \Rightarrow \quad & \left(m^2, -2m\right) = (1, 2) \\ \Rightarrow \quad & m^2 = 1 \text{ and } m = -1 \\ \Rightarrow \quad & m = -1 \end{aligned}$$

So, the equation of the normal at

$$(1, 2)$$

is,

$$
\begin{aligned}
& y = (-1)x - 2(1)(-1) - (1)(-1)^3 \\
\Rightarrow \quad & y = -x + 3 \\
\Rightarrow \quad & x + y - 3 = 0
\end{aligned}
$$

As the normal is tangent to the circle

$$(x-3)^2 + (y+2)^2 = r^2$$

$$\Rightarrow$$

The perpendicular distance of the tangent from the centre of the circle is equal to the radius of the circle.

Now, comparing the equation of the circle with the general form of the circle we get Coordinates of centre

$$\equiv (3, -2)$$

$$\Rightarrow$$

Perpendicular distance from

$$(3, -2)$$

to

$$x + y - 3 = r$$

$$
\begin{aligned}
\Rightarrow \quad & \left| \frac{3+(-2)-3}{\sqrt{1^2+1^2}} \right| = r \\
\Rightarrow \quad & \frac{2}{\sqrt{2}} = r \\
\Rightarrow \quad & r^2 = 2
\end{aligned}
$$

Hint :

i The equation of the normal to the parabola at point

$$\left(am^2, -2am\right)$$

is

$$y = mx - 2am - am^3$$

.

ii The perpendicular distance of a point

$$(h, k)$$

from the line

$$ax + by + c = o$$

is

$$\left| \frac{ah + bk + c}{\sqrt{a^2 + b^2}} \right|$$

units.

Question 026 Numerical

QUESTION

Let the curve

$$C$$

be the mirror image of the parabola

$$y^2 = 4x$$

with respect to the line

$$x + y + 4 = 0$$

. If

$$A$$

and

$$B$$

are the points of intersection of

$$C$$

with the line

$$y = -5$$

, then the distance between

$$A$$

and

$$B$$

is

SOURCE

Mathematics • parabola

EXPLANATION

Let, P(t^2, 2t) be any point on the parabola y^2 = 4x. C be the mirror image of the parabola y^2 = 4x with respect to the line UV : x + y + 4 = 0.

The curve C cuts the line KL : y =

$$-$$

5 at A and B.

Let, B $$\alpha$$, $$\beta$$ be the image of the point P(t^2, 2t).

Clearly, PB

$$\perp$$

UV and PQ = QB.

$$\therefore$$

$$\frac{\alpha - t^2}{\beta - 2t} \times (-1) = -1$$

or,

$$\alpha - t^2 = \beta - 2t$$

...... 1

The point of intersection of the lines UV and KL is R.

Let us join P and R.

From

$$\Delta$$

PQR and

$$\Delta$$

BQR,

i BQ = PQ

$$\$\$ \because \$\$ B is the image of P$$

ii

$$\angle$$

PQR =

$$\angle$$

RQB = 90

$$\circ$$

$$\$\$ \because \$\$ PB \$\$ \perp \$\$ UV$$

iii QR common

$$\therefore$$

$$\Delta$$

PQR

$$\cong$$

$$\Delta$$

BQR

$$by SAS congruence criterion$$

$$\therefore$$

$$\angle$$

QRP =

$$\angle$$

BRQ

$$CPCT$$

$$\because$$

slope of x + y + 4 = 0 is

$$-$$

1,

$$\therefore$$

$$\angle$$

UTO = 135

$$\circ$$

$$\therefore$$

$$\angle$$

OTR = 45

$$\circ$$

Again, X'X || KL and UV transversal.

$$\therefore$$

$$\angle$$

OTR =

$$\angle$$

TRB = 45

$$\circ$$

$$\therefore$$

$$\angle$$

BRQ =

$$\angle$$

QRP = 45

$\circ$

$\therefore$

$\angle$

PRB = 90

$\circ$

$\therefore$

PR

$\perp$

KL

$\therefore$

coordinates of R are (t^2,

β

).

$\because$

the point R lies on KL,

$\therefore$

β

=

$-$

5

Again, the point R lies on the straight line x + y + 4 = 0.

$\therefore$

t^2 +

β

+ 4 = 0

or, t^2

$$-$$

5 + 4 = 0

$$ \$\$ \because \$\$\$\$\beta\$\$ = \$\$ - \$\$5 $$

or, t^2 = 1 or, t =

$$\pm$$

1

when t = 1,

$$\beta$$

=

$$-$$

5, then 1

$$\Rightarrow$$

$$\alpha$$

$$-$$

1 =

$$-$$

5

$$-$$

2 or

$$\alpha$$

=

$$-$$

6

when, t =

$$-$$

1,

$$\beta$$

=

$$-$$

5, then 1

$$\Rightarrow$$

$$\alpha$$

$$-$$

1 =

$$-$$

5 + 2 or,

$$\alpha$$

=

$$-$$

2

So, the coordinates of A and B are $\$\$ - \$\$6, \$\$ - \$\$5$ and $\$\$ - \$\$2, \$\$ - \$\$5$ respectively.

$$\therefore$$

AB = 4 units

So, the distance between A and B is 4 units.

Question 027 Numerical

QUESTION

A cylindrical container is to be made from certain solid material with the following constraints: It has a fixed inner volume of

$$V$$

$$mm^3$$

, has a

$$2$$

mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness

$$2$$

mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is

$$10$$

mm,
then the value of

$$\frac{V}{250\pi}$$

is

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given: The inner volume of cylinder $= \mathrm{V mm}^3$

Thickness of wall $= 2 \mathrm{~mm}$

Thickness of bottom circular disc $= 2 \mathrm{~mm}$

Let the inner radius of cylinder $= r \mathrm{~mm}$ and height of the inner cylinder $= h \mathrm{~mm}$.

$$\Rightarrow \mathrm{V} = \pi r^2 h$$

Now, volume of the material used $=$ volume of outer cylinder - volume of the inner cylinder + Volume of the circular disc

$$\Rightarrow \mathrm{V_m} = \pi(r+2)^2 h - \pi r^2 h + \pi(r+2)^2 2$$

$$\Rightarrow \mathrm{V_m} = \pi h\left\{(r+2)^2 - r^2\right\} + 2\pi(r+2)^2$$

$$\Rightarrow \mathrm{V_m} = \pi h(4r+4) + 2\pi(r+2)^2$$

$$\left\{\because (a+b)^2 = a^2 + b^2 + 2ab\right\}$$

$$\Rightarrow \mathrm{V_m} = 2\pi\left\{2h(r+1) + (r+2)^2\right\}$$

$$\Rightarrow \mathrm{V_m} = 2\pi\left\{\frac{2\,\mathrm{V}}{\pi r^2}(r+1) + (r+2)^2\right\}$$

$$\left\{\because \mathrm{V} = \pi r^2 h\right\}$$

For $\mathrm{V_m}$ to be minimum, $\frac{dv_m}{dr} = 0$

Differentiating the above equation w.r.t.r,

$$\frac{d\,\mathrm{V}_m}{dr} = 2\pi\left\{\frac{2\,\mathrm{V}}{\pi}\frac{d}{dr}\left\{\frac{r+1}{r^2}\right\} + \frac{d}{dr}(r+2)^2\right\}$$

$$\Rightarrow \frac{d\,\mathrm{V}_m}{dr} = 2\pi\left\{\frac{2\,\mathrm{V}}{\pi}\cdot\frac{r^2(1)-(r+1)(2r)}{r^4} + 2(r+2)\right\}$$

$$\Rightarrow \frac{d\,\mathrm{V}_m}{dr} = 2\pi\left\{\frac{2\,\mathrm{V}}{\pi}\cdot\frac{r^2-2r^2-2r}{r^4} + 2(r+2)\right\}$$

$$\Rightarrow \frac{d\,\mathrm{V}_m}{dr} = 4\pi\left\{\frac{\mathrm{V}}{\pi r^3}(-r-2) + (r+2)\right\}$$

$$\Rightarrow \frac{d\,\mathrm{V}_m}{dr} = 4\pi(r+2)\left(-\frac{\mathrm{V}}{\pi r^3} + 1\right)$$

$$\because \frac{d\,\mathrm{V}_m}{dr} = 0$$

$$\Rightarrow 4\pi(r+2)\left(\frac{-\mathrm{V}}{\pi r^3} + 1\right) = 0$$

Also, given that $\mathrm{V_m}$ is minimum at $r = 10\ \mathrm{mm}$

$$\Rightarrow \quad 4\pi(10+2)\left(\frac{-\mathrm{V}}{10^3\pi} + 1\right) = 0$$

$$\Rightarrow \quad 48\pi\left(\frac{-\mathrm{V}}{10^3\pi} + 1\right) = 0$$

$$\Rightarrow \quad \frac{\mathrm{V}}{10^3\pi} = 1$$

$$\Rightarrow \quad \frac{\mathrm{V}}{250\pi} = 4$$

Question 028 MCQ

QUESTION

Let

$$y(x)$$

be a solution of the differential equation

$$(1+e^x)y' + ye^x = 1.$$

If

$$y(0) = 2$$

, then which of the following statement is are true?

A

$$y(-4) = 0$$

B

$$y(-2) = 0$$

C

$$y(x)$$

has a critical point in the interval

$$(-1, 0)$$

D

$$y(x)$$

has no critical point in the interval

$$(-1, 0)$$

CORRECT OPTION

A $$y(-4)=0$$

SOURCE

Mathematics • differential-equations

EXPLANATION

Given:

$$(1+e^x)y'+ye^x=1$$

$$(1+e^x)\frac{dy}{dx}+ye^x=1$$

$$\frac{dy}{dx}+\frac{e^x}{1+e^x}y=\frac{1}{1+e^x}$$

, which is liner differential equation in

$$y$$

.

Comparing above equation with

$$\frac{dy}{dx}+\mathrm{P}y=Q$$

, we get

$$Q$$

, we get

$$\begin{aligned} \mathrm{P}=\frac{e^x}{1+e^x}&\text{ and }\mathrm{Q}=\frac{1}{1+e^x}\\ \text{So, I.F. }&=e^{\int \mathrm{P}\cdot dx}\\ \text{I.F. }&=e^{\int \frac{e^x}{1+e^x}dx}\\ \text{I.F. }&=e^{\ln(1+e^x)} \end{aligned}$$

$$\left\{\because \int \frac{f'(x)}{f(x)} dx = \ln f(x)\right\}$$

$$\text{I.F } = 1 + e^x$$

So, solution of given differential equation is given by

$$y\,(\text{I.F.}) \; = \int \text{Q.(I.F.)}\, dx$$

$$\begin{aligned} y \cdot (1+e^x) &= \int \frac{1}{1+e^x} \cdot (1+e^x) dx \\ y\,(1+e^x) &= \int 1 dx \\ y\,(1+e^x) &= x + \text{C} \\ \because \quad y(0) &= 2 \\ \Rightarrow \quad 2\left(1+e^0\right) &= 0 + \text{C} \\ c &= 4 \end{aligned}$$

So,

$$y(x) = \frac{x+4}{1+e^x} \quad \text{... (i)}$$

Put

$$x = -4$$

in the equation, we get

$$y(-4) = 0$$

Put

$$x = -2$$

in the above equation i, we get

$$y(-2) = \frac{2}{1+e^{-2}} \neq 0$$

For critical points,

$$y' = 0$$

From e.q, i,

$$y\left(1+e^{x}\right)=x+4$$

Differentiating the above equation w.r.t.

$$x$$

, we get

$$\begin{aligned} y'\left(1+e^{x}\right)+y\left(e^{x}\right)&=1 \\ 0\left(1+e^{x}\right)+ye^{x}&=1 \quad \{\because y'=0\} \\ ye^{x}-1&=0 \end{aligned}$$

$$\begin{aligned} \text{Now, let } g(x)&=ye^{x}-1 \\ g(x)&=\frac{(x+4)e^{x}}{1+e^{x}}-1 \\ g(x)&=\frac{(x+3)e^{x}-1}{1+e^{x}} \end{aligned}$$

$$\text{Now,} \quad g(-1)=\frac{2e^{-1}-1}{1+e^{-1}}=\frac{2-e}{1+e}<0 \quad \{\because e=2\cdot 7\cdot 8\}$$

$$\text{And,} \quad g(0)=\frac{3e^{0}-1}{1+e^{0}}=1>0$$

So, there exists one value of

$$x$$

in

$$(-1,0)$$

for which

$$\begin{aligned} g(x)&=0 \\ \Rightarrow \quad y'&=0 \end{aligned}$$

There exist a critical point of

$$y(x)$$

in

$$(-1,0)$$

Hint :

i Use solution of liner differential equation

$$\frac{dy}{dx} + \mathrm{P}y = \mathrm{Q} \text{ is given by}$$

$$y\,(\text{I.F.}) = \int \mathrm{Q} \cdot \ (\text{I.F})\ dx,\ \text{where I.F} = e^{\int p \cdot dx}$$

ii For critical points,

$$y' = 0$$

Question 029 MCQ

QUESTION

Consider the family of all circles whose centres lie on the straight line

$$y = x,$$

If this family of circle is represented by the differential equation

$$Py'' + Qy' + 1 = 0,$$

where

$$P, Q$$

are functions of

$$x, y$$

and

$$y'$$

$$\left(here\ \ y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}\right)$$

then which of the following statements is are true?

A $$P = y + x$$

B

$$P = y - x$$

C

$$P + Q = 1 - x + y + y' + (y')^2$$

D

$$P - Q = 1 - x + y - y' - (y')^2$$

CORRECT OPTION

B

$$P = y - x$$

SOURCE

Mathematics • differential-equations

EXPLANATION

Let equation of circle whose centre lie on straight line $y = x$ be

$(x-k)^2 + (y-k)^2 = r^2 \quad \ldots(i)$

Differentiating the above equation w.r.t. x, we get

$$2(x-k) + 2(y-k)y' = 0$$

$$\Rightarrow x + yy' = k\left(1 + y'\right) \quad \ldots(ii)$$

$$\Rightarrow k = \frac{x + yy'}{1 + y'}$$

Differentiating the eq. ii w.r.t. x, we get

$$\Rightarrow 1 + yy'' + \left(y'\right)^2 = ky'' \quad \{\because (uv)' = uv' + vu'\}$$

$$\Rightarrow 1 + yy'' + \left(y'\right)^2 = \left(\frac{x + yy'}{1 + y'}\right)y''$$

$$\Rightarrow 1 + yy'' + \left(y'\right)^2 + y' + yy'y'' + \left(y'\right)^3$$

$$= xy'' + yy'y''$$

$$\Rightarrow y''(y - x) + \left(y'\right)^2\left(1 + y'\right) + 1 + y' = 0$$

$$\Rightarrow y''(y - x) + y'\left(y' + \left(y'\right)^2 + 1\right) + 1 = 0$$

Comparing the above equation with $py'' + \mathrm{Q}y' \ +1 = 0$, we get

$$P = y - x \text{ and } \mathrm{Q} = y' + \left(y'\right)^2 + 1$$

$$\therefore P + Q = 1 - x + y + y' + \left(y'\right)^2$$

Question 030 Numerical

QUESTION

Let

$$f : R \to R$$

be a function defined by

$$f(x) = \begin{cases} [x], & x \leq 2 \\ 0, & x > 2 \end{cases}$$

where

$$[x]$$

is the greatest integer less than or equal to

$$x$$

, if

$$I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx,$$

then the value of

$$(4I - 1)$$

is

SOURCE

Mathematics • definite-integration

EXPLANATION

Given:

$$f : \mathrm{R} \to \mathrm{R} f(x) = \begin{cases} [x], x \leq 2 \\ o, x > 2 \end{cases}$$

$$\text{And} \quad I = \int_{-1}^{2} \frac{x f(x^2)}{2 + f(x+1)} dx$$

So,

$$f(x^2) = \begin{cases} [x^2], & x^2 \leq 2, & x \in [-\sqrt{2}, \sqrt{2} \\ 0, & x^2 > 2, & x \in (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty) \end{cases}$$

$$\text{And } f(x+1) = \begin{cases} [x+1], & x+1 \le 2, \quad x \le 1 \\ 0, & x+1 > 2, \quad x > 1 \end{cases}$$

$$\text{So, I} = \int_{-1}^{0} \frac{xf\left[x^2\right]}{2+f(x+1)}dx + \int_{0}^{1} \frac{xf\left(x^2\right)}{2+f(x+1)}dx + \int_{1}^{\sqrt{2}} \frac{xf\left(x^2\right)}{2+f(x+1)}dx + \int_{2}$$

$$\Rightarrow \text{I} = \int_{-1}^{0} \frac{x\left[x^2\right]}{2+[x+1]}dx + \int_{0}^{1} \frac{x\left[x^2\right]}{2+[x+1]}dx + \int_{1}^{\sqrt{2}} \frac{x\left[x^2\right]}{2+0}dx + \int_{\sqrt{2}}^{2} \frac{x \cdot 0}{2+0}dx$$

$$\Rightarrow \text{I} = \int_{-1}^{0} \frac{x\left[x^2\right]}{2+[x+1]}dx + \int_{0}^{1} \frac{x\left[x^2\right]}{2+[x+1]}dx + \int_{1}^{\sqrt{2}} \frac{x\left[x^2\right]}{2}dx$$

Using the property of greatest integer function, For

$$x \in (-1,0), [x+1] = 0$$

and

$$\left[x^2\right] = 0$$

and, For

$$x \in (0,1), [x+1] = 1$$

and

$$\left[x^2\right] = 0$$

and, For

$$x \in (1,\sqrt{2}), \left[x^2\right] = 1$$

$$\Rightarrow \quad \mathrm{I}=\int_{-1}^{0} \frac{x.0}{2+0} dx+\int_{0}^{1} \frac{x.0}{2+1} dx+\int_{1}^{\sqrt{2}} \frac{x.1}{2} dx$$

$$\Rightarrow \quad \mathrm{I}=\int_{1}^{\sqrt{2}} \frac{x}{2} dx$$

$$\Rightarrow \quad \mathrm{I}=\frac{1}{2}\left[\frac{x^2}{2}\right]_1^{\sqrt{2}}$$

$$\Rightarrow \quad \mathrm{I}=\frac{1}{2}\left[\frac{2}{2}-\frac{1}{2}\right]$$

$$\Rightarrow \quad \mathrm{I}=\frac{1}{4}$$

$$\Rightarrow \quad 4\mathrm{I}=1$$

$$\Rightarrow 4\mathrm{I}-1=0$$

Hint:

i Find

$$f\left(x^2\right)$$

and

$$f(x+1)$$

using composite function.

ii Split the given integral using the property of the greatest integer function.

iii Find the value of the definite integral using, if

$$\int g(x) dx=\mathrm{G}(x) \Rightarrow \int_a^b g(x) dx=[\mathrm{G}(b)-\mathrm{G}(a)]$$

Question 031 Numerical

QUESTION

Let

$$F(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,(dt)$$

for all

$$x \in R$$

and

$$f : \left[0, \frac{1}{2}\right] \to [0, \infty]$$

be a continuous function. For

$$a \in \left[0, \frac{1}{2}\right],$$

$$F'(a) + 2$$

is the area of the region bounded by

$$x = 0, y = 0, y = f(x)$$

and

$$x = a,$$

then

$$f(0)$$

is

SOURCE

Mathematics • application-of-integration

EXPLANATION

$$\text{Given, } f(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt \forall x \in \mathrm{R}$$

$$\text{As we know, if } \mathrm{I}(x) = \int\limits_{g(x)}^{h(x)} \phi(t)dt, \text{ then}$$

$$\mathrm{I}'(x) = \phi\{h(x)\}h'(x) - \phi\{g(x)\}g'(x)$$

$$\Rightarrow f(x) = 2\left\{\cos\left(x^2 + \frac{\pi}{6}\right)\right\}^2 \cdot \frac{d}{dx}\left(x^2 + \frac{\pi}{6}\right) - 2\cos^2 x \cdot \frac{dx}{dx}$$

$$\Rightarrow f(x) = 4x\left\{\cos\left(x^2 + \frac{\pi}{6}\right)\right\}^2 - 2\cos^2 x$$

Putting

$$x = a$$

in the above equation, we get

$$f(a) = 4a\left\{\cos\left(a^2 + \frac{\pi}{6}\right)\right\}^2 - 2\cos^2 a$$

Also, the area of the region bounded by

$$x = 0$$

,

$$y = 0, y = f(x) \text{ and } x = a \text{ is } \int_0^a f(x)dx$$

$$\Rightarrow f(a) + 2 = \int\limits_0^a f(x)dx$$

$$\Rightarrow 4a\left\{\cos\left(a^2 + \frac{\pi}{6}\right)\right\}^2 - 2\cos^2 a + 2 = \int\limits_0^a f(x)dx$$

Differentiating above equation w.r.t. a, we get

$$\Rightarrow -4a \cdot 2\cos\left(a^2 + \frac{\pi}{6}\right) \cdot \sin\left(a^2 + \frac{\pi}{6}\right)$$
$$2a + 4\left\{\cos\left(a^2 + \frac{\pi}{6}\right)\right\}^2$$
$$-4\cos a(-\sin a) = f(a)$$
$$\Rightarrow -8a^2\sin\left(2a^2 + \frac{\pi}{3}\right) + 4\left\{\cos\left(a^2 + \frac{\pi}{6}\right)\right\}^2$$
$$+2\sin 2a = f(a)\{\because 2\sin x\cos x = \sin 2x\}$$

Putting

$$a = 0$$

in the above equation, we get.

$$\begin{aligned}&0 + 4\cos^2\left(\frac{\pi}{6}\right) + 2\sin(0) = f(0)\\ &\Rightarrow f(0) = 4\left(\frac{\sqrt{3}}{2}\right)^2 \quad \left\{\because \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}\right\}.\\ &\Rightarrow f(0) = 3\end{aligned}$$

i Use if

$$I(x) = \int_{g(x)}^{h(x)} \phi(t)dt$$

, then

$$\mathrm{I}'(x) = \phi\{h(x)\}h'(x) - \phi\left\{g(x)g'(x)\right.$$

ii Use the area of the region bounded by

$$x = 0, y = 0, y = g(x)$$

and

$$x = k$$

is

$$\int_0^k g(x)dx$$

iii Use the product rule of differentiation for further simplification.

Question 032 Numerical

QUESTION

The minimum number of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least

$$0.96,$$

is

SOURCE

Mathematics • probability

EXPLANATION

Let the coin is tossed

$$n$$

times.

$$\because p$$

$$atleast2heads$$

$$= 1 - [p$$

$$oneheads$$

$$+p$$

$$Noheads$$

$$\}$$

As we know, by binominal probability theorem the probability of getting

$$r$$

success in

$$n$$

trials with

$$p$$

being the probability of success and

$$q$$

be the probability of failure, is given by

$${}^{n}\mathrm{C}_{r}(p)^{r}(q)^{n-r}$$

.

Let head be considered as the success and tail be the failure probability of getting head in a toss

$$=p=\frac{1}{2}$$

and probability of getting tail in a toss

$$=q=\frac{1}{2}$$

$$\begin{aligned}
&\therefore \mathrm{P}(\text{ one head })={}^{n}\mathrm{C}_{1}\left(\frac{1}{2}\right)^{1}\left(\frac{1}{2}\right)^{n-1}\\
&={}^{n}\mathrm{C}_{1}\left(\frac{1}{2}\right)^{n}\\
&\Rightarrow \mathrm{P}\text{ (one head) }=n\left(\frac{1}{2}\right)^{n}\\
&\text{Similarly, P(No heads)}={}^{n}\mathrm{C}_{0}\left(\frac{1}{2}\right)^{0}\left(\frac{1}{2}\right)^{n}\\
&\Rightarrow \mathrm{P}(\text{ No heads })=\left(\frac{1}{2}\right)^{n} \quad \{\because n_{c_{o}}=1\}\\
&\therefore \mathrm{P}\text{ (at least 2 heads) }=1-\left\{n\left(\frac{1}{2}\right)^{n}+\left(\frac{1}{2}\right)^{n}\right\}
\end{aligned}$$

$$= 1 - \frac{(n+1)}{2^n}$$

$$\because P(\text{ at least 2 heads }) \geq 0.96$$

$$\Rightarrow 1 - \frac{(n+1)}{2^n} \geq 0.96$$

$$\Rightarrow 0.04 \geq \frac{n+1}{2^n}$$

$$\Rightarrow \frac{1}{25} \geq \frac{n+1}{2^n}$$

$$\Rightarrow \frac{2^n}{n+1} \geq 25$$

$$\Rightarrow n \geq 8$$

So, the minimum number of times a fair coin to be tossed is 8.

Hint:

i P $atleast2heads$

$$= 1 - \{\mathrm{P}$$

Extra close brace or missing open brace}

ii Use binomial probability theorem and simplify it.

Question 033 MCQ

QUESTION

In

$$R^3,$$

consider the planes

$$P_1 : y = 0$$

and

$$P_2 : x + z = 1.$$

Let

$$P_3$$

be the plane, different from

$$P_1$$

and

$$P_2$$

, which passes through the intersection of

$$P_1$$

and

$$P_2.$$

If the distance of the point

$$(0, 1, 0)$$

from

$$P_3$$

is

$$1$$

and the distance of a point

$$(\alpha, \beta, \gamma)$$

from

$$P_3$$

is

$$2,$$

then which of the following relations is are true?

A

$$2\alpha + \beta + 2\gamma + 2 = 0$$

B

$$2\alpha - \beta + 2\gamma + 4 = 0$$

C

$$2\alpha + \beta - 2\gamma - 10 = 0$$

D

$$2\alpha - \beta + 2\gamma - 8 = 0$$

CORRECT OPTION

B

$$2\alpha - \beta + 2\gamma + 4 = 0$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

$$\text{Given, } \mathrm{P}_1 : y = 0 \quad \text{... (i)}$$
$$\mathrm{P}_2 : x + z - 1 = 0 \quad \text{... (ii)}$$

Equation of plane

$$P_3$$

passing through the intersection of plane

$$P_1$$

and

$$P_2$$

is given by

$$\mathrm{P}_3 : x + z - 1 + \lambda y = 0 \quad \text{... (iii)}$$
$$\because$$

Distance of the point

$$(0,1,0)$$

from

$$P_3$$

is 1.

$$\begin{aligned} \therefore \quad & \left|\frac{0+\lambda+0-1}{\sqrt{1^2+\lambda^2+1^2}}\right|=1 \\ \Rightarrow \quad & \left|\frac{\lambda-1}{\sqrt{2+\lambda^2}}\right|=1 \\ \Rightarrow \quad & (\lambda-1)^2=1\left(\sqrt{2+\lambda^2}\right)^2 \\ \Rightarrow \quad & \lambda^2+1-2\lambda=\lambda^2+2 \\ \Rightarrow \quad & \lambda=-\frac{1}{2} \end{aligned}$$

Put the value of

$$\lambda$$

in equation iii, we get

$$\begin{aligned} \mathrm{P}_3: \quad & x+z-1-\frac{1}{2}y=0 \\ \Rightarrow \quad & 2x-y+2z-2=0 \end{aligned}$$

$$\because$$

Distance of a point

$$(\alpha,\beta,\gamma)$$

from

$$\mathrm{P}_3$$

is 2

$$\begin{aligned}
\therefore \quad & \left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{2^2+1^2+2^2}}\right| = 2 \\
\Rightarrow \quad & \left|\frac{2\alpha-\beta+2\gamma-2}{3}\right| = 2 \\
\Rightarrow \quad & 2\alpha-\beta+2\gamma-2 = \pm 6 \\
\Rightarrow \quad & 2\alpha-\beta+2\gamma = 8 \text{ or } -4 \\
\Rightarrow \quad & 2\alpha-\beta+2\gamma-8 = 0 \text{ or } 2\alpha-\beta+2\gamma+4 = 0
\end{aligned}$$

Hint :

i Equation of plane passing through the intersection of two plane

$$P_1$$

and

$$P_2$$

is given by

$$\mathrm{P}_1 + \lambda \mathrm{P}_2 = 0$$

; where

$$\lambda$$

is a constant.

ii Distance of a point

$$(x_1 y_1 z_1)$$

from plane

$$ax + by + cz + d = 0$$

is

$$\left|\frac{ax_1 + by_1 + cz_1 + d}{\sqrt{a^2 + b^2 + c^2}}\right|$$

Question 034 MCQ

QUESTION

In

$$R^3,$$

let

$$L$$

be a straight lines passing through the origin. Suppose that all the points on

$$L$$

are at a constant distance from the two planes

$$P_1 : x + 2y - z + 1 = 0$$

and

$$P_2 : 2x - y + z - 1 = 0.$$

Let

$$M$$

be the locus of the feet of the perpendiculars drawn from the points on

$$L$$

to the plane

$$P_1.$$

Which of the following points lie s on

$$M$$

?

A

$$\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$$

B $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

C $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

D $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

CORRECT OPTION

B $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Let the equation of the line passing through the origin be

$$\mathrm{L} : \frac{x}{l} = \frac{y}{m} = \frac{z}{n} \quad \ldots \text{(i)}$$

Given, planes

$$\mathrm{P}_1 : x + 2y - z + 1 = 0 \quad \ldots \text{(ii)}$$
$$\mathrm{P}_2 : 2x - y + z - 1 = 0 \quad \ldots \text{(iii)}$$

since, all the points on

$$\mathrm{L}$$

are at a constant distance from the planes

$$\mathrm{P}_1$$

and

$$\mathrm{P}_2$$

$$\therefore$$

Line

$$\mathrm{L}$$

is perpendicular to normal of plane

$$\mathrm{P}_1$$

and

$$\mathrm{P}_2$$

As we know if two lines are perpendicular then the sum of products of their respective direction ratios is zero.

$$\begin{aligned}&\Rightarrow l+2m-n=0\ldots\ldots\text{ (iv)} \quad \{\because L\perp P_1\}\\&\Rightarrow 2l-m+n=0\ldots\ldots\text{ (v)} \quad \{\because L\perp P_2\}\end{aligned}$$

On solving equation iv and equation v, we get

$$\frac{l}{1}=\frac{m}{-3}=\frac{n}{-5}$$

$$\therefore$$

Equation of line L is

$$\frac{x}{1}=\frac{y}{-3}=\frac{z}{-5}$$

Let any point on L is

$$\mathrm{A}(k,-3k,-5k)$$

Now foot of perpendicular from

$$A$$

to plane

$$P_1$$

is given by

$$\begin{aligned} & \frac{x-k}{1}=\frac{y+3k}{2}=\frac{z+5k}{-1}=-\frac{(k-6k+5k+1)}{1^2+2^2+1^2} \\ \Rightarrow \quad & \frac{x-k}{1}=\frac{y+3k}{2}=\frac{z+5k}{-1}=-\frac{1}{6} \\ \Rightarrow \quad & x=k-\frac{1}{6}, y=-3k-\frac{1}{3}, z=-5k+\frac{1}{6} \end{aligned}$$

So, coordinates of foot of perpendicular is

$$\left(k-\frac{1}{6},-3k-\frac{1}{3},-5k+\frac{1}{6}\right)$$

For

$$k=0$$

, foot of perpendicular

$$\left(-\frac{1}{6},-\frac{1}{3},\frac{1}{6}\right)$$

For

$$k=\frac{1}{6}$$

, foot of perpendicular

$$\left(0,-\frac{5}{6},-\frac{2}{3}\right)$$

Hint :

i since all points on L are at constant distance from the planes

$$\mathrm{P}_1$$

and

$$\mathrm{P}_2$$

. So line

$$L$$

is perpendicular to normal of

$$P_1$$

and

$$P_2$$

.

ii Find equation of line L using above condition and assume any point on L .

iii Use coordinates of foot of perpendicular drawn from a point

$$(x_1 y_1 z_1)$$

to the plane

$$ax + by + cz + d = 0$$

is given by

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = -\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}$$

Question 035 MCQ

QUESTION

Let

$$\Delta PQR$$

be a triangle. Let

$$\vec{a} = \overrightarrow{QR}, \vec{b} = \overrightarrow{RP}$$

and

$$\overrightarrow{c} = \overrightarrow{PQ}.$$

If

$$\left|\vec{a}\right| = 12, \ \left|\vec{b}\right| = 4\sqrt{3}, \ \vec{b}.\vec{c} = 24,$$

then which of the following is are true?

A

$$\frac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 12$$

B

$$\frac{\left|\vec{c}\right|^2}{2} + \left|\vec{a}\right| = 30$$

C

$$\left|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}\right| = 48\sqrt{3}$$

D

$$\vec{a}.\vec{b} = -72$$

CORRECT OPTION

A

$$\frac{\left|\vec{c}\right|^2}{2} - \left|\vec{a}\right| = 12$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

For a triangle, we have $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$

$$\Rightarrow \vec{a}=-(\vec{b}+\vec{c})$$

$$\Rightarrow |\vec{a}|^2=|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}$$

$$\Rightarrow |\vec{a}|^2=48+|\vec{c}|^2+48$$

$$\Rightarrow |\vec{c}|^2=|\vec{a}|^2-96=144-96$$

$$\Rightarrow |\vec{c}|^2=48$$

$$\Rightarrow |\vec{c}|=\sqrt{48}=4\sqrt{3}$$

Therefore, $\frac{|\vec{c}|^2}{2}-|\vec{a}|=24-12=12$

Option A is correct.

$$\frac{|\vec{c}|^2}{2}+|\vec{a}|=24+12=36$$

Also, $\vec{a}+\vec{b}=-\vec{c}\Rightarrow\vec{a}\cdot\vec{b}=-72$

From Eq. 1, we get

$$\vec{a} \times \vec{a} = -(\vec{a} \times \vec{b} + \vec{a} \times \vec{c}) \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

$$|\vec{a} \times \vec{b} + \vec{a} \times \vec{c}| = |2(\vec{a} \times \vec{b})| = 2|\vec{a}||\vec{b}| \sin\theta$$

$$= 96\sqrt{3}\sqrt{1 - \left(\frac{-72}{48\sqrt{3}}\right)^2}$$

$$= 96\sqrt{3}\sqrt{1 - \left(\frac{\sqrt{3}}{12}\right)^2} = 48\sqrt{3}$$

Question 036 MCQ

QUESTION

Match the following :

	Column I		Column I
A	In a triangle ΔXYZ, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $2\left(a^2 - b^2\right) = c^2$ and $\lambda = \frac{\sin(X-Y)}{\sin Z}$, then possible values of n for which $\cos(n\lambda) = 0$ is (are)	P	1
B	In a triangle $\triangle XYZ$, let a, b and c be the lengths of the sides opposite to the angles X, Y and Z, respectively. If $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$, then possible value(s) of $\frac{a}{b}$ is (are)	Q	2
C	In $\mathbb{R}^2$, let $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$ and $\beta\hat{i} + (1-\beta)\hat{j}$ be the position vectors of X, Y and Z with respect of the origin O, respectively. If the distance of Z from the bisector of the acute angle of $\overrightarrow{\mathrm{OX}}$ with $\overrightarrow{\mathrm{OY}}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $\lvert\beta\rvert$ is (are)	R	3

	Column I		Column I
D	Suppose that $F(\alpha)$ denotes the area of the region bounded by $x=0, x=2, y^2=4x$ and $y=\|\alpha x-1\|+\|\alpha x-2\|+\alpha x$, where, $\alpha \in \{0,1\}$. Then the value(s) of $F(\alpha)+\frac{8}{2}\sqrt{2}$, when $\alpha=0$ and $\alpha=1$, is (are)	S	5
		T	6

A $(A) \rightarrow P, R;\ (B) \rightarrow P;\ (C) \rightarrow P, Q;\ (D) \rightarrow S, T$

B $(A) \rightarrow P, R, S;\ (B) \rightarrow P;\ (C) \rightarrow P, Q;\ (D) \rightarrow S, T$

C $(A) \rightarrow P, R, S;\ (B) \rightarrow P;\ (C) \rightarrow P;\ (D) \rightarrow S, T$

D $(A) \rightarrow S;\ (B) \rightarrow P;\ (C) \rightarrow P;\ (D) \rightarrow S, T$

CORRECT OPTION

B $(A) \rightarrow P, R, S;\ (B) \rightarrow P;\ (C) \rightarrow P, Q;\ (D) \rightarrow S, T$

SOURCE

Mathematics • properties-of-triangle

EXPLANATION

Option $A:$

$$2\left(a^{2}-b^{2}\right)=c^{2} \qquad (1)$$

$$\lambda=\frac{\sin(x-y)}{\sin z} \qquad (2)$$

$$\cos(n\pi\lambda)=0 \qquad (3)$$

$$\Rightarrow n\lambda=\frac{(2m+1)}{2} \qquad (4)$$

From Eq. 2, we have

$$\lambda=\frac{\sin x(0)y-\cos x\sin y}{\sin z}$$

$$\Rightarrow \lambda=\frac{a\cos y-b\cos x}{c}$$

$$\Rightarrow \lambda=\frac{a\left(\frac{a^{2}+c^{2}-b^{2}}{2ac}\right)-b\left(\frac{b^{2}+c^{2}-a^{2}}{2bc}\right)}{2c} \qquad \text{(by sine formula)}$$

$$\Rightarrow \lambda=\frac{2\left(a^{2}-b^{2}\right)}{2c^{2}}=\frac{1}{2} \qquad (5)$$

Therefore, from Eqs. 4 and 5, we get

$$\frac{x}{2}=\frac{2m+1}{2}\Rightarrow x=(2m+1)$$

Hence, $A\rightarrow P$, R, S.

Option B:

$$
\begin{aligned}
& 1+\cos 2x-2\cos 2y=2\sin x\sin y \\
\Rightarrow & 2\cos^2 x-2\left(2\cos^2 y-1\right)=2\sin x\sin y \\
\Rightarrow & 2\cos^2 x-4\cos^2 y+2=2\sin x\sin y \\
\Rightarrow & 2\sin^2 y-2\sin x\sin y+\sin x\sin y-\sin^2 x=0 \\
\Rightarrow & 2\sin y(\sin y-\sin x)+\sin x(\sin y-\sin x)=0 \\
\Rightarrow & (\sin y-\sin x)(2\sin y+\sin x)=0 \\
\Rightarrow & b=a \text{ or } 2b=-a(\text{ which is impossible }) \\
\Rightarrow & \frac{a}{b}=1
\end{aligned}
$$

Hence, $(\mathrm{B}) \rightarrow (\mathrm{P})$

Option C:

Vector along the bisector of acute angle between $\overrightarrow{OX}$ and $\overrightarrow{OY}$ is

$$
\frac{\sqrt{3}\hat{i}+\hat{j}}{2}+\frac{\hat{i}+\sqrt{3}\hat{j}}{2}=\frac{(\sqrt{3}+1)}{2}(\hat{i}+\hat{j})
$$

Slope of $\overrightarrow{OB}=\tan\left(\frac{\pi}{4}\right)=1$

Equation of OB is $y = x$.

Since

$$ZL = \frac{3}{\sqrt{3}} \Rightarrow \frac{|\beta - (1-\beta)|}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$\Rightarrow |2\beta - 1| = 3$$

$$\Rightarrow (2\beta - 1) = \pm 3$$

$$\Rightarrow \beta = 2 \text{ or } \beta = -1$$

$$\Rightarrow |\beta| = 1 \text{ or } 2$$

$(\mathrm{C}) \rightarrow (\mathrm{P}), (\mathrm{Q})$

Option D:

$$\Rightarrow y = |\alpha x - 1| + |\alpha x - 2| + \alpha x; \alpha \in \{0, 1\}$$

Case I: For $\alpha = 0, y = 3$

Case II: For $\alpha = 1, y = |x-1| + |x-2| + x$

$$\Rightarrow y = \begin{cases} 3 - x; & x \leq 1 \\ x + 1; & 1 < x < 2 \\ 3x - 3; & x \geq 2 \end{cases}$$

Hence,

$$F(0) = \int_0^2 (3 - 2\sqrt{x})dx = \left[3x - \frac{4}{3}x^{3/2}\right]_0^2$$

$$= \left[6 - \frac{4}{3}(2\sqrt{2})\right] = 6 - \frac{8}{3}\sqrt{2}$$

$$\Rightarrow F(0) + \frac{8}{3}\sqrt{2} = 6 \Rightarrow (T)$$

and

$$F(1) = F(0) - \text{ area of } \triangle ACD$$

$$= \left(6 - \frac{8}{3}\sqrt{2}\right) - \frac{1}{2}(2)(1) = 5 - \frac{8}{3}\sqrt{2}$$

$$\Rightarrow F(1) + \frac{8}{3}\sqrt{2} = 5 \Rightarrow (S)$$

Hence, $(\mathrm{D}) \rightarrow (\mathrm{T}), (\mathrm{S})$

Question 037 MCQ

QUESTION

Let X and Y be two arbitrary, 3

$$\times$$

3, non-zero, skew-symmetric matrices and Z be an arbitrary 3

$$\times$$

3, non-zero, symmetric matrix. Then which of the following matrices is are skew symmetric?

A

Y^3Z^4

—

Z^4Y^3

B $X^{44} + Y^{44}$

C

X^4Z^3

—

Z^3X^4

D $X^{23} + Y^{23}$

CORRECT OPTION

C

X^4Z^3

—

Z^3X^4

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given,

$$X^T = -X, Y^T = -Y, Z^T = Z$$

a Let

$$P = Y^3 Z^4 - Z^4 Y^3$$

Then,

$$P^T = (Y^3 Z^4)^T - (Z^4 Y^3)^T$$

$$= (Z^T)^4 (Y^T)^3 - (Y^T)^3 (Z^T)^4$$

$$= -Z^4 Y^3 + Y^3 Z^4 = P$$

$$\therefore$$

P is symmetric matrix.

b Let

$$P = X^{44} + Y^{44}$$

Then,

$$P^T = (X^T)^{44} + (Y^T)^{44}$$

$$= X^{44} + Y^{44} = P$$

$$\therefore$$

P is symmetric matrix.

c Let

$$P = X^4 Z^3 - Z^3 X^4$$

Then,

$$P^T = (X^4 Z^3)^T - (Z^3 X^4)^T$$

$$= (Z^T)^3 (X^T)^4 - (X^T)^4 (Z^T)^3$$

$$= Z^3 X^4 - X^4 Z^3$$

$$= -P$$

$$\therefore$$

P is skew-symmetric matrix.

d Let

$$P = X^{23} + Y^{23}$$

Then,

$$P^T = (X^T)^{23} + (Y^T)^{23}$$
$$= -X^{23} - Y^{23}$$
$$= -P$$
$$\therefore$$

P is skew-symmetric matrix.

Question 038 MCQ

QUESTION

Which of the following values of

$$\alpha$$

satisfy the equation

$$\begin{vmatrix} (1-\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$$

?

A

$-$

4

B 9

C

$-$

9

D 4

CORRECT OPTION

B 9

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

$$\begin{vmatrix} (1-\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$$

Applying

$$R_3 \to R_3 - R_2, R_2 \to R_2, R_1$$

, we get

$$\begin{vmatrix} (1-\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

Applying

$$R_3 \to R_3 - R_2$$

, we get

$$\begin{vmatrix} (1-\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha$$

Applying

$$C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$$

, we get

$$\begin{vmatrix} (1-\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 2\alpha & 2\alpha \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

Expanding,

$$2\alpha^2(3\alpha+2) - 2\alpha^2(5\alpha+2) = -324\alpha$$

$$\Rightarrow -4\alpha^3 = -324\alpha \Rightarrow \alpha(\alpha^2-81) = 0$$

$$\therefore$$

$$\alpha = 0, -9, 9$$

Question 039 MCQ

QUESTION

Let

$$g : R \to R$$

be a differentiable function with

$$g(0) = 0$$

,

$$g'(0) = 0$$

and

$$g'(1) \neq 0$$

. Let

$$f(x) = \begin{cases} \frac{x}{|x|} g(x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

and

$$h(x) = e^{|x|}$$

for all

$$x \in R$$

. Let

$$(f \circ h)(x)$$

denote

$$f(h(x))$$

and

$$(h \circ f)(x)$$

denote

$$f(f(x))$$

. Then which of the following is are true?

A f is differentiable at x = 0.

B h is differentiable at x = 0.

C

$$f \circ h$$

is differentiable at x = 0.

D

$$h \circ f$$

is differentiable at x = 0.

CORRECT OPTION

A f is differentiable at x = 0.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Rewrite f as

$$f(x)=\begin{cases} g(x), & x>0 \\ 0 & x=0 \\ -g(x), & x<0 \end{cases}$$

We have,

$$f'(x)=\begin{cases} g'(x), & x>0 \\ -g'(x), & x<0 \end{cases}$$

At x = 0

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{g(h)-0}{h}$$

$$=\lim_{h\to 0}\frac{g(h)-g(0)}{h}=g'(0)$$

$$f'(x)=\begin{cases} g'(x), & x\geq 0 \\ -g'(x), & x<0 \end{cases}$$

f is differentiable at x = 0.

$$\therefore$$

Option a is correct.

b

$$h(x) = e^{|x|} = \begin{cases} e^x, & x \geq 0 \\ e^{-x}, & x < 0 \end{cases}$$

$$\Rightarrow h'(x) = \begin{cases} e^x, & x \geq 0 \\ -e^{-x}, & x < 0 \end{cases}$$

$$\Rightarrow h'(0^+) = 1$$

and

$$h'(0^-) = -1$$

So, hx is not differentiable at x = 0.

$$\therefore$$

Option b is not correct.

c

$$(foh)(x) = f\{h(x)\}$$

, as

$$h(x) > 0$$

$$= \begin{cases} g(e^x), & x \geq 0 \\ g(e^{-x}), & x < 0 \end{cases}$$

$$\Rightarrow (foh)'(x) = \begin{cases} e^x g'(e^x), & x \geq 0 \\ -e^x g'(e^{-x}), & x < 0 \end{cases}$$

$$\Rightarrow (foh)'(0^+) = g'(1), (foh)'(0^-)$$

$$= -g'(1)$$

So,

$$(hof)(x)$$

is not differentiable at

$$x = 0$$

.

$$\therefore$$

Option c is not correct.

d

$$(hof)(x) = e^{|f(x)|} = \begin{cases} e^{|g(x)|}, & x \neq 0 \\ e^0 = 1, & x = 0 \end{cases}$$

Now,

$$(hof)'(0) = \lim_{h \to 0} \frac{e^{|g(x)|} - 1}{x}$$

$$= \lim_{h \to 0} \frac{e^{|g(x)|} - 1}{|g(x)|} \cdot \frac{|g(x)|}{x}$$

$$= \lim_{h \to 0} \frac{e^{|g(x)|} - 1}{|g(x)|} \cdot \lim_{h \to 0} \frac{|g(x)| - 0|}{|x|} \cdot \lim_{h \to 0} \frac{|x|}{x}$$

$$= 1 \,.\, g'(0) \,.\, \lim_{h \to 0} \frac{|x|}{x}$$

= 0, as g'0 = 0

$$\therefore$$

Option d is correct.

Question 040 MCQ

QUESTION

Let

$$f(x) = \sin\left(\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right)$$

for all

$$x \in R$$

and gx =

$$\frac{\pi}{2} \sin x$$

for all x

$$\in$$

R. Let

$$(f \circ g)(x)$$

denote f$g(x)$ and

$$(g \circ f)(x)$$

denote g $f(x)$. Then which of the following is/are true?

A

Range of f is

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

.

B

Range of f

$$\circ$$

g is

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

.

C

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \frac{\pi}{6}$$

.

There is an x

D

$\in$

R such that $g$$ \circ $$f$ x = 1.

CORRECT OPTION

Range of f is

A

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

.

SOURCE

Mathematics • functions

EXPLANATION

a

$$f(x) = \sin\left[\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right], x \in R$$
$$= \sin\left(\frac{\pi}{6}\sin\theta\right), \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

,

where,

$$\theta = \frac{\pi}{2}\sin x$$
$$= \sin\alpha, \alpha \in \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$$

,

where,

$$\alpha = \frac{\pi}{6}\sin\theta$$
$$\therefore$$

$$f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

Hence, range of

$$f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

So, option a is correct.

b

$$f\{g(x)\} = f(t), t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$
$$\Rightarrow f(t) \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$
$$\therefore$$

Option b is correct.

c

$$\lim_{x \to 0} \frac{f(x)}{g(x)}$$

$$= \lim_{x \to 0} \frac{\sin\left[\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right]}{\frac{\pi}{2}(\sin x)}$$

$$= \lim_{x \to 0} \frac{\sin\left[\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)\right]}{\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)} \cdot \frac{\frac{\pi}{6} \sin\left(\frac{\pi}{2} \sin x\right)}{\left(\frac{\pi}{2} \sin x\right)}$$
$$= 1 \times \frac{\pi}{6} \times 1 = \frac{\pi}{6}$$
$$\therefore$$

Option c is correct.

d

$$g\{f(x)\} = 1$$

$$\Rightarrow \frac{\pi}{2} \sin\{f(x)\} = 1$$

$$\Rightarrow \sin\{f(x)\} = \frac{2}{\pi}$$

..... i

But,

$$f(x) \in \left[-\frac{1}{2}, \frac{1}{2}\right] \subset \left[-\frac{\pi}{6}, \frac{\pi}{6}\right]$$

$$\therefore$$

$$\sin\{f(x)\} \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

..... ii

$$\Rightarrow \sin\{f(x)\} \neq \frac{2}{\pi}$$

,

$$fromEqs.\,(i)and(ii)$$

i.e. No solution.

$$\therefore$$

Option d is not correct.

Question 041 MCQ

QUESTION

The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density

$$\lambda$$

are kept parallel to each other. In their resulting electric field, point charges

$$q$$

and

$$-q$$

are kept in equilibrium between them. The point charges are confined to move in the

$$x$$

direction only. If they are given a small displacement about their equilibrium positions, then the correct statement s is are

A Both charges execute simple harmonic motion

B Both charges will continue moving in the direction of their displacement

C Charge

$$+q$$

executes simple harmonic motion while charge

$$-q$$

continues moving in the direction of its displacement

D Charge

$$-q$$

executes simple harmonic motion while charge

$$+q$$

continues moving in the direction of its displacement

CORRECT OPTION

Charge

$$+q$$

C executes simple harmonic motion while charge

$$-q$$

continues moving in the direction of its displacement

SOURCE

Physics • electrostatics

EXPLANATION

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Case 1 : If q is shifted towards right by x, we get

$$F = F_2 - F_1 = \frac{\lambda q}{2\pi\varepsilon_0}\left(\frac{1}{\frac{d}{2} - x} - \frac{1}{\frac{d}{2} + x}\right)$$

$$towards left$$

Case 2 : If

$$-$$

q is shifted towards right by x

$$F = F_2 - F_1 = \frac{\lambda q}{2\pi\varepsilon_0}\left(\frac{1}{\frac{d}{2} - x} - \frac{1}{\frac{d}{2} + x}\right)$$

$$towards right$$

Thus, +q exhibits SHM while

$$-$$

q continues to move towards rightwards.

Question 042 Numerical

QUESTION

Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio

$$\left(\frac{\lambda_A}{\lambda_B}\right)$$

of their wavelengths

$$\lambda_A$$

and

$$\lambda_B$$

at which the peaks occur in their respective radiation curves is

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Power,

$$P = (\sigma T^4 A) = \sigma T^4 (4\pi R^2)$$

or,

$$P \propto T^4 R^2$$

..... i

According to Wien's law,

$$\lambda \propto \frac{1}{T}$$

$$\$\$\lambda\$\$ is the wavelength at which peak occurs$$

$$\therefore$$

Eq. i will become,

$$P \propto \frac{R^2}{\lambda^4}$$

or,

$$\lambda \propto \left[\frac{R^2}{P}\right]^{1/4}$$

$$\Rightarrow \frac{\lambda_A}{\lambda_B} = \left[\frac{R_A}{R_B}\right]^{1/2}\left[\frac{P_B}{P_A}\right]^{1/4}$$

$$= [400]^{1/2}\left[\frac{1}{10^4}\right]^{1/4} = 2$$

Question 043 MCQ

QUESTION

A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement s is are

A The average energy per mole of the gas mixture is 2RT

B The ratio of speed of sound in the gas mixture to that in helium gas is

$$\sqrt{\frac{6}{5}}$$

C The ratio of the rms speed of helium atoms to that of hydrogen molecules is

$$\frac{1}{2}$$

D The ratio of the rms speed of helium atoms to that of hydrogen molecules is

$$\frac{1}{\sqrt{2}}$$

CORRECT OPTION

A The average energy per mole of the gas mixture is 2RT

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

The internal energy of one mole of an ideal gas at temperature T is given by

$$U = \frac{f}{2}RT$$

, where f is the degrees of freedom of the gas molecule. The degress of freedom of the gas molecule. The degrees of freedom for hydrogen $diatomic$ and helium $monatomic$ gases are f_{H_2} = 5 and f_{He} = 3, respectively. Thus,

$$U_{H_2} = \frac{5}{2}RT$$

and

$$U_{He} = \frac{3}{2}RT$$

. The total internal energy of the gas mixture is

$$U_{total} = U_{H_2} + U_{He} = \frac{5}{2}RT + \frac{3}{2}RT = 4RT$$

.

The mixture contains two moles of the gases. The internal energy per mole of the mixture is

$$U_{mix} = U_{total}/2 = 2RT$$

.

The specific heat at constant volume is given by

$$C_v = dU/dT$$

. Thus, the specific heats at constant volume for helium and the mixture are

$$C_{v,He} = dU_{He}/dT = \frac{3}{2}R$$

, and

$$C_{v,mix} = dU_{mix}/dT = 2R$$

The specific heats at constant pressure,

$$C_p = C_v + R$$

, for these gases are

$$C_{p,He} = C_{v,He} + R = \frac{5}{2}R$$

, and

$$C_{p,mix} = C_{v,mix} + R = 3R$$

.

The ratio of specific heats,

$$\gamma = C_p/C_v$$

, are

$$\gamma_{He} = 5/3$$

and

$$\gamma_{mix} = 3/2$$

. The speed of sound, in a gas of molecular mass M, is given by

$$v_s = \sqrt{\gamma RT/M}$$

. The molecular mass of the gas mixture is

$$M_{mix} = \frac{n_{H_2}M_{H_2} + n_{He}M_{He}}{n_{H_2} + n_{He}}$$

$$= \frac{(1)(2) + (1)(4)}{1+1} = 3$$

g/mol,

where n_{H_2} = 1 and n_{He} = 1 are the number of moles of hydrogen and helium in the gas mixture. The ratio of the speeds of sound in the gas mixture and helium is

$$\frac{v_{s,mix}}{v_{s,He}} = \frac{\sqrt{\gamma_{mix}RT/M_{mix}}}{\sqrt{\gamma_{He}RT/M_{He}}} = \sqrt{\frac{\gamma_{mix}}{\gamma_{He}}\frac{M_{He}}{M_{mix}}}$$

$$= \sqrt{\frac{(3/2)(4)}{(5/3)(3)}} = \sqrt{\frac{6}{5}}$$

The rms speed of the atoms/molecules is given by

$$v_{rms} = \sqrt{3RT/M}$$

. The ratio of the rms speed of helium atoms to that of hydrogen molecules is

$$\frac{v_{rms,He}}{v_{rms,H_2}} = \frac{\sqrt{3RT/M_{He}}}{\sqrt{3RT/M_{H_2}}} = \sqrt{\frac{M_{H_2}}{M_{He}}} = \sqrt{\frac{2}{4}} = \sqrt{\frac{1}{2}}$$

.

Question 044 Numerical

QUESTION

A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is

$$\left(\frac{1}{4}\right)^{th}$$

of its value at the surface of the planet. If the escape velocity from the planet is

$$v_{esc} = v\sqrt{N}$$

, then the value of N is $ignore energy loss due to atmosphere$

SOURCE

Physics • gravitation

EXPLANATION

Given situation is shown in the figure. Let acceleration due to gravity at the surface of the planet be g. At height h above planet's surface v = 0.

According to question, acceleration due to gravity of the planet at height h above its surface becomes g/4.

$$g_h = \frac{g}{4} = \frac{g}{\left(1 + \frac{h}{R}\right)^2}$$

$$4 = \left(1 + \frac{h}{R}\right)^2 \Rightarrow 1 + \frac{h}{R} = 2$$

$$\frac{h}{R} = 1 \Rightarrow h = R$$

.

So, velocity of the bullet becomes zero at h = R.

Also,

$$v_{esc} = v\sqrt{N} \Rightarrow \sqrt{\frac{2GM}{R}} = v\sqrt{N}$$

...... i

Applying energy conservation principle,

Energy of bullet at surface of earth = Energy of bullet at highest point

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = \frac{-GMm}{2R}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2R}$$

$$\therefore$$

$$v = \sqrt{\frac{GM}{R}}$$

Putting this value in eqn. i, we get

$$\sqrt{\frac{2GM}{R}} = \sqrt{\frac{NGM}{R}}$$

$$\therefore$$

N = 2

Question 045 MCQ

QUESTION

Consider a Vernier callipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Vernier callipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

A

If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

B

If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

C

If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.01 mm.

D

If the least count of the linear scale of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

CORRECT OPTION

B

If the pitch of the screw gauge is twice the least count of the Vernier callipers, the least count of the screw gauge is 0.005 mm.

SOURCE

Physics • units-and-measurements

EXPLANATION

In given Vernier callipers, each 1 cm is equally divided into 8 main scale divisions MSD. Thus, 1 MSD =

$$\frac{1}{8}$$

= 0.125 cm. Further, 4 main scale divisions coincide with 5 Vernier scale divisions VSD i.e., 4 MSD = 5 VSD. Thus, 1 VSD = 4/5 MSD = 0.8

$$\times$$

0.125 = 0.1 cm. The least count of the Vernier callipers is given by

LC = 1 MSD

$-$

1 VSD = 0.125

$-$

0.1 = 0.025 cm.

In screw gauge, let l be the distance between two adjacent divisions on the linear scale. The pitch p of the screw gauge is the distance travelled on the linear scale when it makes one complete rotation. Since circular scale moves by two divisions on the linear scale when it makes one complete rotation, we get p = 2l. The least count of the screw gauge is defined as ratio of the pitch to the number of divisions on the circular scale n i.e.,

$$LC' = \frac{p}{n} = \frac{2l}{100} = \frac{l}{50}$$

.... 1

If p = 2 LC = 2$0.025$ = 0.05 cm, then

$$l = \frac{p}{2} = 0.025$$

cm. Substitute l in equation 1 to get the least count of the screw gauge

$$LC' = \frac{0.025}{50} = 5 \times 10^{-4}$$

cm = 0.005 mm.

If l = 2 LC = 2$0.025$ = 0.05 cm then equation 1 gives

$$LC' = \frac{0.05}{50} = 1 \times 10^{-3}$$

cm = 0.01 mm.

Question 046 MCQ

QUESTION

Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option s is are

A

$$M \propto \sqrt{c}$$

B

$$M \propto \sqrt{G}$$

C

$$L \propto \sqrt{h}$$

D

$$L \propto \sqrt{G}$$

CORRECT OPTION

D

$$L \propto \sqrt{G}$$

SOURCE

Physics • units-and-measurements

EXPLANATION

The dimensions of Planck's constant is

$$h = [M^1 L^3 T^{-2}]$$

Speed of light is

$$c = [L^1 T^{-1}]$$

Gravitational constant is

$$G = [M^{-1}L^{3}T^{-2}]$$

Let

$$L \propto h^{x}c^{y}G^{z}$$

.

$$[L] = [M^{1}L^{2}T^{-1}]^{x}[L^{1}T^{-1}]^{y}[M^{-1}L^{3}T^{-2}]^{z}$$

Comparing, we get

$$\left.\begin{aligned} x - z = 0 \\ 2x + y + 3z = 1 \\ -x - y - 2z = 0 \end{aligned}\right\}$$

Solving, we get

$$x = z$$

$$y + 5x = 1$$

$$-y - 3x = 0$$

or,

$$2x = 1$$

$$x = \frac{1}{2} = z$$

$$y = -\frac{3}{2}$$

Therefore,

$$L \propto h^{1/2}C^{-3/2}G^{1/2}$$

$$L \propto \sqrt{h}$$

$$L \propto \sqrt{G}$$

Let

$$M \propto h^{a}C^{b}G^{c}$$

$$[M] = [M^1L^2T^{-1}]^a[L^1T^{-1}]^b[M^{-1}L^3T^{-2}]^c$$

Comparing, we get

$$a - c = 1$$

.

$$\left.\begin{array}{r} 2a + b + 3c = 0 \\ -a - b - 2c = 0 \end{array}\right\} a + c = 0$$

Therefore,

$$2a = 1$$

$$a = \frac{1}{2}$$

$$c = -\frac{1}{2}$$

$$b = \frac{1}{2}$$

Therefore,

$$M \propto h^{1/2}c^{1/2}G^{-1/2}$$

$$M \propto \sqrt{c}$$

Question 047 Numerical

QUESTION

A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water $refractiveindex = 4/3$ as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2$

$$\lambda$$

2

$$-$$

d^2, where

$$\lambda$$

is the wavelength of light in air $refractiveindex = 1$. 2d is the separation between the slits and m is an integer. The value of p is

SOURCE

Physics • wave-optics

EXPLANATION

$$\mu(S_2P) - S_1P = m\lambda$$

$$\Rightarrow \mu\sqrt{d^2 + x^2} - \sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow (\mu - 1)\sqrt{d^2 + x^2} = m\lambda$$

$$\Rightarrow \left(\frac{4}{3} - 1\right)\sqrt{d^2 + x^2} = m\lambda$$

or,

$$\sqrt{d^2 + x^2} = 3m\lambda$$

Squaring this equation we get,

$$x^2 = 9m^2\lambda^2 - d^2$$

$$\Rightarrow p^2 = 9$$

or

$$p = 3$$

Question 048 Numerical

QUESTION

Consider a concave mirror and a convex lens $refractiveindex = 1.5$ of focal length 10 cm each, separated by a distance of 50 cm in air $refractiveindex = 1$ as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index

$$\frac{7}{6}$$

, the magnification becomes M_2. The magnitude

$$\left|\frac{M_2}{M_1}\right|$$

is

SOURCE

Physics • geometrical-optics

EXPLANATION

Case I

Reflection from mirror

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15}$$

$$\Rightarrow v = -30$$

For lens

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} - \frac{1}{-20}$$

$$v = 20$$

$$|M_1| = \left|\frac{v_1}{u_1}\right|\left|\frac{v_2}{u_2}\right|$$

$$= \left(\frac{30}{15}\right)\left(\frac{20}{20}\right)$$

$$= 2 \times 1 = 2$$

$inair$

Case II :

Now, consider the setup placed in a medium of refractive index

$$\mu_1'$$

= 7/6. The focal length of the mirror does not change. Thus, the distance of the image formed by the mirror and its magnification does not change. The focal length of the lens changes. The refractive index of the lens material is

$$\mu$$

2 = 1.5. Apply lens maker's formula to get the new focal length of the lens

$$\frac{1}{f'} = \left(\frac{\mu_2 - \mu_1'}{\mu_1'}\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \frac{\mu_2 - \mu_1'}{\mu_1'} \frac{\mu_1}{\mu_2 - \mu_1} \frac{\mu_2 - \mu_1}{\mu_1}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$= \left(\frac{\mu_2 - \mu_1'}{\mu_2 - \mu_1}\right)\left(\frac{\mu_1}{\mu_1'}\right)\frac{1}{f}$$

$$= \left(\frac{1.5 - 7/6}{1.5 - 1}\right)\left(\frac{1}{7/6}\right)\frac{1}{10} = \frac{2}{35}$$

. 1

Again using lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_l{}'}$$

$$\frac{1}{v} - \frac{1}{-20} = \frac{2}{35} \Rightarrow \frac{1}{v} = \frac{2}{35} - \frac{1}{20} = \frac{1}{140}$$

$$\therefore$$

v = 140 cm

Magnification,

$$m_2' = \frac{v}{u} = \frac{140}{-20} = -7$$

Magnification produced by the combination,

$$M_2 = m_1 \times m_2' = (-2) \times (-7) = 14$$

$$\therefore$$

$$\left|\frac{M_2}{M_1}\right| = \frac{14}{2} = 7$$

Question 049 Numerical

QUESTION

An infinitely long uniform line charge distribution of charge per unit length

$$\lambda$$

lies parallel to the y-axis in the y-z plane at

$$z = \frac{\sqrt{3}}{2}$$

a $see figure$. If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is

$$\frac{\lambda L}{n\varepsilon_0}$$

$$\varepsilon_0$$ $= permittivity of free space$, then the value of n is

SOURCE

Physics • electrostatics

EXPLANATION

ANBP is cross-section of a cylinder of length L. The line charge passes through the centre O and perpendicular to paper.

$$AM = \frac{a}{2}$$

,

$$MO = \frac{\sqrt{3}a}{2}$$

$$\therefore$$

$$\angle AOM = \tan^{-1}\left(\frac{AM}{OM}\right)$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

Electric flux passing from the whole cylinder

$$\phi_1 = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$\therefore$$

Electric flux passing through ABCD plane surface $shownonlyAB$ = Electric flux passing through cylindrical surface ANB

$$= \left(\frac{60^\circ}{360^\circ}\right)(\phi_1) = \frac{\lambda L}{6\varepsilon_0}$$

$$\therefore$$

n = 6

Question 050 Numerical

QUESTION

Consider a hydrogen atom with its electron in the nth orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is $hc = 1242 eV nm$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

Energy of the incident photon

$$= hf = \frac{hc}{\lambda} = \frac{1242}{90} = 13.8$$

eV. Since after ionisation, electron is ejected with some kinetic energy. By energy conservation, we get

Energy $photon$ = Kinetic energy $electron$ +

$$\Delta$$

E

Transition energy from nth orbit to n

$$\rightarrow$$

$$\infty$$

. Therefore,

13.8 = 10.4 +

$$\Delta$$

E

$$\Rightarrow$$

$$\Delta$$

E = 3.4 eV

From Bohr's theory,

$$E_n = \frac{-13.6}{n^2} = -3.4 \Rightarrow n = 2$$

Question 051 Numerical

QUESTION

Two identical uniform discs roll without slipping on two different surfaces AB and CD $see figure$ starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s^2)

SOURCE

Physics • rotational-motion

EXPLANATION

Suppose mass and radius of each disc are m and R respectively. Also potential energy at points B and D is zero i.e., they are on reference line.

Given final kinetic energy for each disc is same, say it is K.

Applying energy conservation principle,

For surface AB,

$$\frac{1}{2} I_2 \omega_1^2 + mg \times 30 = K$$

..... i

For surface CD,

$$\frac{1}{2} I_2 \omega_2^2 + mg \times 27 = K$$

..... ii

From eqns. i and ii, we get

$$\frac{1}{2} I_2 \omega_1^2 + mg \times 30 = \frac{1}{2} I_2 \omega_2^2 + mg \times 27$$

...... iii

Here,

$$\omega_1 = \frac{v_1}{R}$$

,

$$\omega_2 = \frac{v_2}{R}$$

, v_1 = 3 m s

$$-$$

[1], v_2 = ?

$I_1 = I_2$ = Moment of inertia of disc about the point of contact

$$= \frac{1}{2} mR^2 + mR^2 = \frac{3}{2} mR^2$$

From eqn. iii,

$$\frac{1}{2}\left(\frac{3}{2} mR^2\right) \times \left(\frac{3}{R}\right)^2 + m \times 10 \times 30$$

$$= \frac{1}{2}\left(\frac{3}{2} mR^2\right) \times \left(\frac{v_2}{R}\right)^2 + m \times 10 \times 27$$

$$\frac{27}{4} + 300 = \frac{3}{4} v_2^2 + 270$$

$$\frac{3}{4} v_2^2 = \frac{27}{4} + 30 \Rightarrow 3v_2^2 = 147$$

$$v_2^2 = 49$$

$$\therefore$$

$v_2 = 7$ m s

—

1

Question 052 Numerical

QUESTION

A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel.

The amount of fuel at the beginning is such that the total power requirement of the village is 12.5 % of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, then the value of n is

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

Half life of radioactive material = T years

Let amount of radioactive material as fuel at the beginning be N_0 and corresponding power produced by it be P_0.

According to question,

Power requirement of the village

= 12.5% of

$$P_0 = \frac{P_0}{8}$$

Since, after each T year, power will be half, i.e.,

$$P_0 \xrightarrow{T} \frac{P_0}{2} \xrightarrow{T} \frac{P_0}{4} \xrightarrow{T} \frac{P_0}{8}$$

Total time upto which the plant can meet the village's need = 3T years = nT years

$$\therefore$$

n = 3

Question 053 MCQ

QUESTION

A ring of mass M and radius R is rotating with angular speed

$$\omega$$

about a fixed vertical axis passing through its centre O with two point masses each of mass

$$\frac{M}{8}$$

at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant, the angular speed of the system is

$$\frac{8}{9}$$

$$\omega$$

and one of the masses is at a distance of

$$\frac{3}{5}$$

R from O. At this instant, the distance of the other mass from O is

A

$$\frac{2}{3}$$

R

B

$$\frac{1}{3}$$

R

C

$$\frac{3}{5}$$

R

D

$$\frac{4}{5}$$

R

CORRECT OPTION

D

$$\frac{4}{5}$$

R

SOURCE

Physics • rotational-motion

EXPLANATION

Ring has mass M and radius R. Initial angular speed of ring =

$$\omega$$

. Two point masses, each of mass are at rest at O. Initial angular momentum of ring and point masses system,

$$L_i = I_R\omega + I_m\omega + I_m\omega$$

$$= MR^2\omega + 0 + 0 = MR^2\omega$$

After some time, situation is changed as shown in the figure.

Angular speed of the system,

$$\omega' = \frac{8}{9}\omega$$

;

$$OA = \frac{3R}{5}$$

; OB = r = ?

Moment of inertia about O of point mass at A,

$$I_A = \frac{M}{8} \times \frac{9R^2}{25}$$

Moment of inertia about O of point mass at B,

$$I_B = \frac{M}{8}r^2$$

Fina angular momentum of the system

$$L_f = MR^2\omega' + I_A\omega' + I_B\omega'$$

$$= MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8}r^2 \times \frac{8\omega}{9}$$

As there is no external torque acting on the system so its angular momentum will be conserved, $L_i = L_f$

$$MR^2\omega = MR^2 \times \frac{8\omega}{9} + \frac{M}{8} \times \frac{9R^2}{25} \times \frac{8\omega}{9} + \frac{M}{8}r^2 \times \frac{8\omega}{9}$$

$$R^2 = \frac{8R^2}{9} + \frac{R^2}{25} + \frac{r^2}{9} \Rightarrow \frac{r^2}{9} = \frac{16}{225}R^2$$

$$\therefore$$

$$r = \frac{4}{5}R$$

Question 054 MCQ

QUESTION

Two identical glass rods S_1 and S_2 $refractiveindex = 1.5$ have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes $shownbythedashedline$ aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

- **A** 60 cm
- **B** 70 cm
- **C** 80 cm
- **D** 90 cm

CORRECT OPTION

B 70 cm

SOURCE

Physics • geometrical-optics

EXPLANATION

Apply the formula for the refraction at the spherical surface of S_1,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

, to get

$$\frac{1.0}{v} - \frac{1.5}{(-50)} = \frac{1 - 1.5}{(-10)}$$

.

Solve to get the image distance v = 50 cm $point Q in the figure$.

The image Q acts as an object for refraction at the spherical surface of S_2. The object distance is

$$u = -(d - 50)$$

. The image distance is v =

$$\infty$$

(because rays are parallel in S_2). Apply the formula for refraction at the spherical surface of S_2, to get

$$\frac{1.5}{\infty} - \frac{1}{-(d - 50)} = \frac{1.5 - 1}{(10)}$$

.

Solve to get d = 70 cm.

Question 055 MCQ

QUESTION

A conductor $shown in the figure$ carrying constant current I is kept in the x-y plane in a uniform magnetic field B. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statements is/are

A

if B is along

$\hat{z}$

, F

$\propto$

$L + R$

B

if B is along

$\hat{x}$

, F = 0

C

if B is along

$\hat{y}$

, F

$\propto$

$L + R$

D

if B is along

$\hat{z}$

, F = 0

CORRECT OPTION

if B is along

$\hat{z}$

A , F

$$\propto$$

$L + R$

SOURCE

Physics • magnetism

EXPLANATION

The force on a conducting element of length

$$\overrightarrow{dl}$$

, carrying a current I in a magnetic field

$$\overrightarrow{B}$$

, is given by d

$$\overrightarrow{F}$$

= I d

$$\overrightarrow{l}$$

$$\times$$

$$\overrightarrow{B}$$

. If the field is uniform, the total force on the conductor is given by

$$\overrightarrow{F} = \int_a^g I\overrightarrow{dl} \times \overrightarrow{B} = I\left(\int_a^g \overrightarrow{dl}\right) \times \overrightarrow{B} = I\,\overrightarrow{ag} \times \overrightarrow{B}$$

Note that

$$\overrightarrow{B}$$

is taken out of the integral sign because it is constant. The vector

$$\overrightarrow{ag} = 2(L+R)\widehat{x}$$

. The magnetic forces on the conductor in the given cases are

Case A :

$$\begin{aligned}\overrightarrow{F} &= I(2(L+R)\widehat{x}) \times (B\hat{z}) \\ &= -2IB(L+R)\hat{y}\end{aligned}$$

Case B :

$$\overrightarrow{F} = I(2(L+R)\widehat{x}) \times (B\widehat{x}) = \overrightarrow{0}$$

Case C :

$$\begin{aligned}\overrightarrow{F} &= I(2(L+R)\widehat{x}) \times (B\hat{y}) \\ &= 2IB(L+R)\hat{z}\end{aligned}$$

Question 056 MCQ

QUESTION

In an aluminium Al bar of square cross section, a square hole is drilled and is filled with iron Fe as shown in the figure. The electrical resistivities of Al and Fe are 2.7

$\times$

10

$-$

8

Ω

m and 1.0

$\times$

10

—

7

Ω

m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

A $\frac{2475}{64}\mu\Omega$

B $\frac{1875}{64}\mu\Omega$

C $\frac{1875}{49}\mu\Omega$

D $\frac{2475}{132}\mu\Omega$

CORRECT OPTION

B $\frac{1875}{64}\mu\Omega$

SOURCE

Physics • current-electricity

EXPLANATION

Resistance of a wire,

$$R = \frac{\rho l}{A}$$

For iron Fe bar,

$$\rho$$

= 10

$$-$$

7

$$\Omega$$

m, l = 50 mm = 50

$$\times$$

10

$$-$$

3 m

A = $2mm$

$$\times$$

$2mm$ = 4 mm^2 = 4

$$\times$$

10

$$-$$

6 m^2

$$R_1 = \frac{10^{-7} \times 50 \times 10^{-3}}{4 \times 10^{-6}} = 1250 \times 10^{-6}$$

$$\Omega$$

= 1250

μ

W

For aluminium Al bar,

ρ

= 2.7

$\times$

10

$-$

8

Ω

m, l = 50 mm = 50

$\times$

10

$-$

3 m

A = (7^2

$-$

2^2) mm^2 = 45 mm^2 = 45

$\times$

10

$-$

6 m^2

$\therefore$

$$R_2 = \frac{2.7 \times 10^{-8} \times 50 \times 10^{-3}}{45 \times 10^{-6}}$$

$$= \frac{27 \times 50}{45} \times 10^{-6} = 30 \times 10^{-6}$$

$$\Omega$$

= 30

$$\mu$$

$$\Omega$$

Potential difference across both bars $resistors$ is same so they are in parallel combination.

Equivalent resistance between P and Q is given by

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{1250 \times 30}{1250 + 30} = \frac{125 \times 30}{128} = \frac{1875}{64} \mu\Omega$$

.

Question 057 MCQ

QUESTION

For photo-electric effect with incident photon wavelength

$$\lambda$$

, the stopping potential is V_0. Identify the correct variation s of V_0 with

$$\lambda$$

and

$$\frac{1}{\lambda}$$

.

A

B

C

D

CORRECT OPTION

A

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Stopping potential (V_0) is given by

$$eV_0 = \frac{hc}{\lambda} - \phi$$

Graph between V_0 and

$$\lambda$$

:

$$eV_0 + \phi = \frac{hc}{\lambda}$$

$$(eV_0 + \phi)\lambda = hc$$

$$(eV_0 + \phi)\lambda$$

= constant

Here, both e and

$$\phi$$

are also constant. It represents a hyperbola.

For, $V_0 = 0$,

$$\lambda = \frac{constant}{\phi} =$$

constant

So correct option is a.

Graph between V_0 and

$$\frac{1}{\lambda}$$

:

$$V_0 = \left(\frac{hc}{e}\right)\left(\frac{1}{\lambda}\right) - \left(\frac{\phi}{e}\right)$$

It represents a straight line with slope

$$\left(\frac{hc}{e}\right)$$

and intercept

$$\left(-\frac{\phi}{e}\right)$$

on V_0 axis.

Question 058 MCQ

QUESTION

Two independent harmonic oscillators of equal masses are oscillating about the origin with angular frequencies

$$\omega$$

$_1$ and

ω

2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If

$$\frac{a}{b} = n^2$$

and

$$\frac{a}{R} = n$$

, then the correct equations is/are

A
E_1

ω

$_1 = E_2$

ω

2

B
$$\frac{\omega_2}{\omega_1} = n^2$$

C
$$\omega_1 \omega_2 = n^2$$

D
$$\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

CORRECT OPTION

D

$$\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

1st Particle

P = 0 at x = a

$$\Rightarrow$$

'a' is the amplitude of oscillation 'A_1'.

At x = 0, P = b $atmeanposition$

$$\Rightarrow m v_{\max} = b \Rightarrow v_{\max} = \frac{b}{m}$$

$$E_1 = \frac{1}{2} m v_{\max}^2 = \frac{m}{2}\left[\frac{b}{m}\right]^2 = \frac{b^2}{2m}$$

$$A_1 \omega_1 = v_{\max} = \frac{b}{m}$$

$$\Rightarrow \omega_1 = \frac{b}{ma} = \frac{1}{mn^2}\left(A = a, \frac{a}{b} = n^2\right)$$

2nd Particle

P = 0 at x = R

$$\Rightarrow$$

A_2 = R

At x = 0, P = R

$$\Rightarrow$$

$$v_{\max} = \frac{R}{m}$$

$$E_2 = \frac{1}{2}mv_{\max}^2 = \frac{m}{2}\left[\frac{R}{m}\right]^2 = \frac{R^2}{2m}$$

$$A_2\omega_2 = \frac{R}{m} \Rightarrow \omega_2 = \frac{R}{mR} = \frac{1}{m}$$

b

$$\frac{\omega_2}{\omega_1} = \frac{1/m}{1/mn^2} = n^2$$

c

$$\omega_1\omega_2 = \frac{1}{mn^2} \times \frac{1}{m} = \frac{1}{m^2n^2}$$

d

$$\frac{E_1}{\omega_1} = \frac{b^2/2m}{1/mn^2} = \frac{b^2n^2}{2} = \frac{a^2}{2n^2} = \frac{R^2}{2}$$

$$\frac{E_2}{\omega_2} = \frac{R^2/2m}{1/m} = \frac{R^2}{2}$$

$$\Rightarrow \frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$$

Question 059 MCQ

QUESTION

Match the nuclear processes given in Column I with the appropriate option s in Column II:

A $A \rightarrow R$ or RT, T; $B \rightarrow P$, S; $C \rightarrow Q$, T; $D \rightarrow R$

B $A \rightarrow R, T; B \rightarrow Q, S; C \rightarrow Q, T; D \rightarrow R$

C $A \rightarrow R$ or $RT, T; B \rightarrow P, S; C \rightarrow S, T; D \rightarrow R$

D $A \rightarrow P, T; B \rightarrow P, S; C \rightarrow Q, T; D \rightarrow R$

CORRECT OPTION

A $A \rightarrow R$ or $RT, T; B \rightarrow P, S; C \rightarrow Q, T; D \rightarrow R$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The nuclear fusion is responsible for energy production in stars via fusion of hydrogen nuclei into helium nuclei. In sun, the fusion takes place dominantly by proton-proton cycle,

$$4{}_1^1H \rightarrow {}_2^4He + 2e^{+} + 2\nu + 2\gamma$$

. The neutrino $$\nu$$ and

$$\gamma$$

-rays emissions are parts of this fusion reaction.

The uranium based fission reactions involve absorption of thermal neutrons by

$$^{235}_{92}$$

U nuclei to produce the highly fissionable

$$^{236}_{92}$$

U nuclei. This nuclei then fissions into two parts e.g.,

$$^{236}_{92}$$

U

$$\rightarrow$$

$$\begin{matrix}137\\63\end{matrix}$$

I +

$$\begin{matrix}97\\39\end{matrix}$$

Y + 2n. The fission fragments are unstable and undergo

$$\beta$$

-decay to reduce their neutron to proton ratio. The fragments are generally formed in excited states and consequently emit

$$\gamma$$

-rays. The heavy water (D_2O) is used as a moderator to slow down the fast moving neutrons.

In

$$\beta$$

-decay, a neutron is converted into a proton. In this process, an electron and an antineutrino are created and emitted from the nucleus, n

$$\rightarrow$$

p + e

$$-$$

+

$$\overline{v}$$

. The

$$\beta$$

-decay in

$$\begin{matrix}60\\27\end{matrix}$$

Co

$$\rightarrow$$

$$^{60}_{28}$$

Ni + e

$$-$$

+

$$\overline{v}$$

. The daughter nuclei

$$^{60}_{28}$$

Ni is formed in excited state and comes to ground state by

$$\gamma$$

-ray emission.

The

$$\gamma$$

-rays are high energy electromagnetic rays. These rays are generally emitted when a nuclei in excited state $highenergy$ makes a transition to a lower state $lowenergy$.

Question 060 MCQ

QUESTION

A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in Column I (a and U_0 are constants). Match the potential energies in Column I to the corresponding statement$s$ in Column II:

A $A \rightarrow P, Q, R; B \rightarrow Q, S; C \rightarrow P, Q, R, S; D \rightarrow P, R, T$

B $A \rightarrow P, Q, R, T; B \rightarrow Q, S; C \rightarrow P, Q, R, S; D \rightarrow P, R, T$

C $A \rightarrow P, Q, R, T; B \rightarrow Q; C \rightarrow P, Q, R, S; D \rightarrow P, R, T$

D $A \rightarrow P, Q, R, T; B \rightarrow Q, S; C \rightarrow P, Q, R, S; D \rightarrow P, R$

CORRECT OPTION

B $A \rightarrow P, Q, R, T; B \rightarrow Q, S; C \rightarrow P, Q, R, S; D \rightarrow P, R, T$

SOURCE

Physics • work-power-and-energy

EXPLANATION

$A \rightarrow P, Q, R, T; B \rightarrow Q, S; C \rightarrow P, Q; D \rightarrow P, R, T$