# JEE Advanced 2017 Paper 1 $Offline$

$54Questions$

## Question 001 MCQ

**QUESTION**

The color of the

$$X_2$$

molecules of group

$$17$$

elements changes gradually from yellow to violet down the group. This is due to

**A** The physical state of $X_2$ at room temperature changes from gas to solid down the group

**B** Decrease in ionization energy down the group

**C** Decrease in $\pi^* - \sigma^*$ gap down the group

**D** Decrease in HOMO-LUMO gap down the group

**CORRECT OPTION**

C

Decrease in

$$\pi^* - \sigma^*$$

gap down the group

**SOURCE**

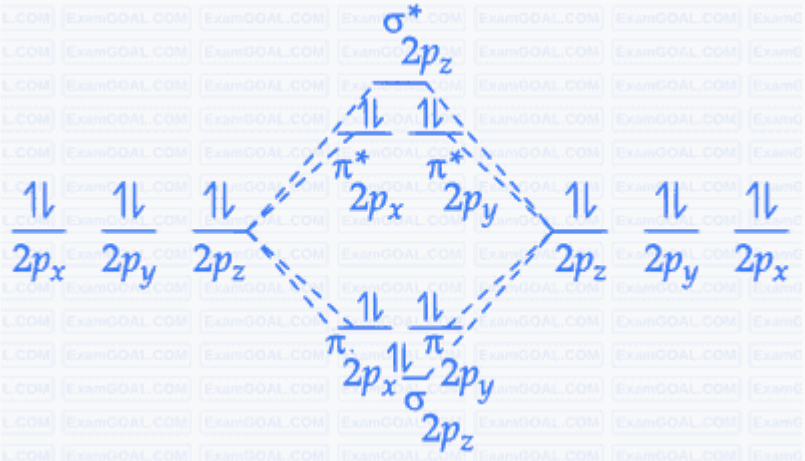
Chemistry • p-block-elements

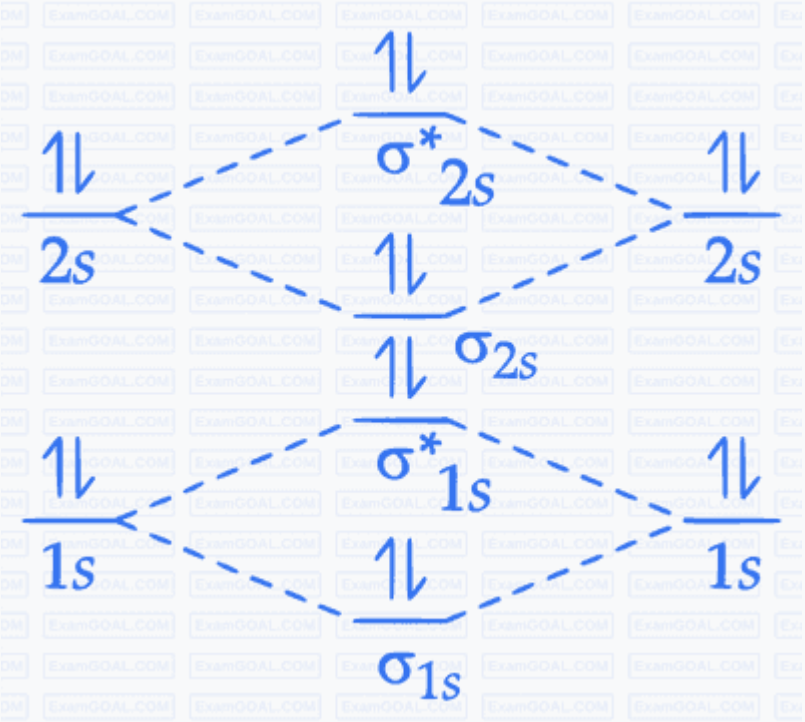
**EXPLANATION**

Halogens exist as diatomic molecule of different colours:

| S.No. | Halogen | Colour |
|---|---|---|
| 1. | Fluorine | Pale yellow |
| 2. | Chlorine | Yellow green |
| 3. | Bromine | Red brown |
| 4. | Iodine | Purple |

$i$ The molecular orbital energy level diagram explains the appearance of colour by halogens. The MOT for halogens is represented.






$ii$ It represents molecular orbital energy level diagram for fluorine. Similar molecular energy level diagram exist. Other halogens.

$iii$ Antibonding $\pi$ orbitals, i.e., $\pi^*_{2px}$ and $\pi^*_{2py}$ forms the highest occupied molecular orbital $HOMO$ and antibonding sigma* orbitals forms the lowest unoccupied molecular orbital $LUMO$ for halogens.

$iv$ Absorption of energy of suitable wavelength $orcolour$ results in transition of electron from HOMO to LUMO. As electron returns back to ground state, i.e, HOMO, it releases energy corresponding to a different wavelength $colourcomplementarytothecolourabsorbed$. This gives halogens their characteristic colour.

$v$ As we move down the group 17, size of halogen atom increases and nuclear force of attraction for the outermost shell electrons decrease. This affects the energy gap between $\mathrm{HOMO}$ and LUMO.

$$\mathrm{F_2} > \mathrm{Cl_2} > \mathrm{Br_2} > \mathrm{I_2}$$

HOMO-LUMO energy gap decreases $\rightarrow$

Wavelength of light emitted decreases $\rightarrow$

$vi$ The energy gap between HOMO and LUMO keeps on decreasing as we move down the group. As a result, the energy required for transition of electron from HOMO to LUMO decreases and less energy $orlightofloverwavelengthisabsorbed.$

$viii$ When electron moves back to HOMO, energy is emitted. It corresponds to the light of complementary colour to the colour of the light absorbed.

## Question 002 MCQ

**QUESTION**

For the synthesis of benzoic acid, the only CORRECT combination is

A $II\ i\ S$

B $IV\ ii\ P$

C $I\ iv\ Q$

D $III\ iv\ R$

**CORRECT OPTION**

A $II\ i\ S$

**SOURCE**

Chemistry • aldehydes-ketones-and-carboxylic-acids

**EXPLANATION**

The synthesis involving benzoic acid is

## Question 003 MCQ

**QUESTION**

The correct statement $s$ about the oxoacids,

$$HClO_4$$

and

$$HClO$$

is $are$

A The central atom in both $HClO_4$ and $HClO$ is $sp^3$ hybridized

B $HClO_4$ is more acidic than

$HClO$

because of the resonance stabilization of its anion

$HClO_4$

is formed in the reaction between

C $Cl_2$

and

$H_2O$

The conjugate base of

$HClO_4$

D is weaker base than

$H_2O$

**CORRECT OPTION**

The central atom in both

$HClO_4$

and

A $HClO$

is

$sp^3$

hybridized

**SOURCE**

Chemistry • p-block-elements

**EXPLANATION**

**Option** $A$: Correct. The structures of the ions formed are shown in below figure. All these structures are based on a tetrahedron. The $sp^3$ hybrid orbitals used for bonding form only weak $\sigma$ bonds, because the $s$ and $p$ levels differ appreciably in energy. The ions are stabilised by strong $p\pi - d\pi$ bonding between full $2p$ orbitals on oxygen with empty $d$ orbitals on the halogen atoms.

**Option** $B$: $\mathrm{HClO_4}$ is an extremely strong acid, while $\mathrm{HOCl}$ is a very weak acid. Oxygen is more electronegative than chlorine. The more oxygen atoms that are bonded, the more the electrons will be pulled away from the $\mathrm{O-H}$ bond, and the more this bond will be weakened. Thus $\mathrm{HClO_4}$ requires the least energy to break the $\mathrm{O-H}$ bond and form $\mathrm{H^+}$.

**Option** $C$: The reaction is $\mathrm{Cl_2 + H_2O \rightleftharpoons HOCl + HCl}$

**Option** $D$: As $\mathrm{HClO_4}$ is a stronger acid than $\mathrm{H_2O}$, therefore, its conjugate base $\left(\mathrm{ClO_4^-}\right)$ will be weaker than $\left(\mathrm{OH^-}\right)$ that of $\mathrm{H_2O}$.

$$\mathrm{HClO_4 + H_2O \rightarrow H_3O^+ + ClO_4^-}$$

## Question 004 MCQ

**QUESTION**

An ideal gas is expanded from

$$(p_1, V_1, T_1)$$

to

$$(p_2, V_2, T_2)$$

under different conditions. The correct statement $s$ among the following is $are$

A

The work done on the gas is maximum when it is compressed irreversibly from

$$(p_2, V_2)$$

to

$$(p_1, V_1)$$

against constant pressure

$$p_1$$

B

If the expansion is carried out freely, it is simultaneously both isothermal as well as adiabatic

C

The work done by the gas is less when it is expanded reversibly from

$$V_1$$

to

$$V_2$$

under adiabatic conditions as compared to that when expanded reverseibly from

$$V_1$$

to

$$V_2$$

under isothermal conditions

D

The change in internal energy of the gas is $i$ zero, if it is expanded reversibly with

$$T_1 = T_2$$

, and $ii$ positive, if it is expanded reversibly under adiabatic conditions with

$$T_1 \neq T_2$$

**CORRECT OPTION**

A

The work done on the gas is maximum when it is compressed irreversibly from

$$(p_2, V_2)$$

to

$$(p_1, V_1)$$

against constant pressure

$$p_1$$

**SOURCE**

Chemistry • thermodynamics

**EXPLANATION**

**Option** $A$: Correct. For compression of gas, $V_2 < V_1$, thus, from $w = -p\Delta V$, work done on the gas will be positive. Below graph shows the pressure-volume curve for work done during reversible and irreversible compression process. Area under curve during irreversible process is greater than the area under curve during reversible process. Therefore, work done on the gas in irreversible process is greater than in reversible process, that is, $w_{\text{irr}} > w_{\text{rev}}$.

**Option** $B$: Correct. For work done in free expansion, $p_{\text{ext}} = 0$, therefore, $w = 0$. For an adiabatic process, $q = 0$ and for isothermal process, $\Delta T = 0$. Therefore, from first law of thermodynamics, $\Delta U = q + w = 0$.

**Option** $C$: Correct. Below graph represents the expansion under adiabatic $AC$ and isothermal condition $AB$. Therefore, $|w_{\text{AB}}| > |w_{\text{AC}}|$.

## Question 005 MCQ

**QUESTION**

Addition of excess aqueous ammonia to a pink colored aqueous solution of

$$MCl_2, 6H_2O\,(X)$$

and

$$NH_4Cl$$

gives an octahedral complex

$$Y$$

in the presence of air. In aqueous solution, complex

$$Y$$

behaves as

$$1:3$$

electrolyte. The reaction of

$$X$$

with excess

$$HCl$$

at room temperature results in the formation of a blue colored complex

$$Z.$$

The calculated spin only magnetic moment of

$$X$$

and

$$Z$$

is

$$3.87$$

$$B.M.,$$

whereas it is zero for complex

$$Y.$$

Among the following options, which statement $s$ is $are$ correct?

A

Addition of silver nitrate to

$$Y$$

gives only two equivalents of silver chloride

B

The hybridization of the central metal ion in

$$Y$$

is

$$d^2sp^3$$

C

$$Z$$

is a tetrahedral complex

D

When

$$X$$

and

$$Z$$

are in equilibrium at

$$0^\circ C,$$

the color of the solution is pink

**CORRECT OPTION**

The hybridization of the central metal ion in

$Y$

**B** is

$$d^2sp^3$$

**SOURCE**

Chemistry • coordination-compounds

**EXPLANATION**

The pink coloured aqueous solution of $\mathrm{MCl_2 \cdot 6H_2O}$ has central metal atom bonded six water molecule by co-ordinate bonds and there are two primary valences of chloride ions. The formula and structure of the complex $X$ can be represented as :

Formula : $[\mathrm{M(H_2O)_6}]\mathrm{Cl_2}$

$$[\mathrm{M(H_2O)_6}]\mathrm{Cl_2} \rightarrow [\mathrm{M\,(H_2O)}]^{2+} + 2\mathrm{Cl}^-$$

Here, the six non ionisable water molecules are the secondary valence and two ionisable chloride ions are primary valence. Since, the hexa coordinated aqua complex of cobalt is pink in colour; hence, central metal is cobalt.

$i$ Reaction of the complex X with ammonium chloride in presence of air gives hexamminecobalt $III$ chloride which is a 1: 3 electrolyte.

$$\text{Option (A) :}$$

$$[\mathrm{Co(NH_3)_6}]\mathrm{Cl_3} + 3\mathrm{AgNO_3} \rightarrow [\mathrm{Co(NH_3)_6}](\mathrm{NO_3})_3 + 3\mathrm{AgCl}$$

Option $B$ : $\mathrm{NH_3}$ is a strong field ligand, therefore, pairing of electrons will take place.

Option $C$ :

Option $D$ : When X and Z are in equilibrium at 0°C

$$\underset{\text{Pink}}{[\mathrm{Co(H_2O)_6}]} \rightarrow \underset{\text{Blue}}{[\mathrm{CoCl_4}]^{2-}}$$

Since, formation of chlorine complex takes place in presence of excess chlorine at room temperature $(25^{\circ}\mathrm{C})$. At lower temperature $(0^{\circ}\mathrm{C})$, the equilibrium is more shifted towards left favouring the formation of pink coloured hexaaquacobalt $II$ chloride complex.

Option $D$ is correct.

## Question 006 Numerical

**QUESTION**

The conductance of a

$$0.0015$$

$$M$$

aqueous solution of a weak monobasic acid was determined by using a conductivity cell consisting of platinized

$$Pt$$

electrodes. The distance between the electrodes is

$$120$$
$$cm$$

with an area of cross section of

$$1$$
$$cm^2.$$

The conductance of this solution was found to be

$$5 \times 10^{-7} S.$$

The

$$pH$$

of the solution is

$$4.$$

The value of limiting molar conductivity

$$(\Lambda_m^o)$$

of this weak monobasic acid in aqueous solution is

$$Z \times 10^2 S$$
$$cm^2$$
$$mol^{-1}.$$

The value of

$$Z$$

is

**SOURCE**

Chemistry • electrochemistry

**EXPLANATION**

**Given**:

$i$ Concentration of weak monobasic $\mathrm{acid(C)} = 0.0015\mathrm{M}$

$ii$ Distance between the electrodes $d = 120\ \mathrm{cm}$

$iv$ Conductance of solution of monobasic acid $(G) = 5 \times 10^{-7}\ \mathrm{S}$

$v$ $\mathrm{pH}$ of the solution $= 4$

**To Find**: The value of $\mathrm{Z}$ in the limiting molar conductivity $(\Lambda.\ ^{\circ}\mathrm{m})\mathrm{Z} \times 10^{2}\ \mathrm{S\ cm^{-1}\ mol^{-1}}$ Formula used:

$i$

$$\mathrm{K} = \frac{\mathrm{G} \times l}{\mathrm{A}}$$

$ii$

$$\left[\mathrm{H_3O^+}\right]^{\mathrm{A}} = 10^{-\mathrm{pH}}$$

$iii$

$$\alpha = \frac{\Lambda_m^{\mathrm{C}}}{\Lambda_m^0}$$

Calculations: The conductivity of the aqueous solution of weak monobasic acid is represented as:

$$\mathrm{K} = \frac{\mathrm{G} \times l}{\mathrm{A}} = \frac{5 \times 10^{-7}\ \mathrm{S} \times 120\ \mathrm{cm}}{1\ \mathrm{cm}^2}$$

$$\mathrm{K} = 6 \times 10^{-5}\ \mathrm{S\ cm^{-1}}$$

The molar conductivity at concentration $(\mathrm{C} = 0.0015\mathrm{M})$ of weak monobasic acid is:

$$\Lambda_m^c = \frac{\mathrm{K} \times 1000}{\mathrm{C}} = \frac{6 \times 10^{-7}\ \mathrm{S\ cm^{-1}} \times 1000}{0.0015\mathrm{M}}$$

$$\Lambda_m^c = 40\ \mathrm{S\ cm^{-1}\ mol^{-1}} \quad \ldots . (i)$$

Weak monobasic acid dissociated to give monobasic anion and hydronium ion as follows:

$$\mathrm{HA}(aq) \rightarrow \mathrm{H_3^+O}(aq) + \mathrm{A^-}(aq)$$

$\alpha =$ degree of hydrolysis of weak monobasic acid

$$\mathrm{C}\alpha = \left[\mathrm{H_3O^+}\right] = 10^{-\mathrm{pH}}$$

$$= 10^{-\mathrm{H}}\mathrm{M}$$

$$\alpha = \frac{10^{-4}}{0.0015\mathrm{M}} \quad \ldots . (ii)$$

We know, degree of hydrolysis is the ratio of molar conductance at concentration $\mathrm{C}$ to molar conductance at infinite dilution.

$$\alpha = \frac{\Lambda_m^C}{\Lambda_m^0}$$

Substituting the value of and from equation $i$ and $ii$ respectively.

$$\frac{10^{-4}}{0.0015} = \frac{40}{\Lambda_m^0} \text{ S cm}^{-1} \text{ mol}^{-1}$$

$$\Lambda_m^0 = \frac{40 \times 0.0015}{10^{-4}} \text{ S cm}^{-1} \text{ mol}^{-1}$$

$$= 6 \times 10^2 \text{ S cm}^{-1} \text{ mol}^{-1}$$

The value of limiting molar conductivity is $6 \times 10^2 \text{ S cm}^{-1} \text{ mol}^{-1}$, where the value of $Z = 6$

## Question 007 Numerical

**QUESTION**

A crystalline solid of a pure substance has a face-centered cubic structure with a cell edge of

$$400$$
$$pm.$$

If the density of the substance in the crystal is

$$8$$
$$g\,cm^{-3},$$

then the number of atoms present in

$$256$$

$g$

of the crystal is

$$N \times 10^{24}.$$

The value of

$$N$$

is

**SOURCE**

Chemistry • solid-state

**EXPLANATION**

**Given**:

$i$ Edge length of face centred cubic $FCC$ unit cell $a = 400\text{pm}$

$ii$ Density of the substance $(\rho) = 8\ \text{g cm}^{-3}$

$iii$ Mass of the crystal $(m) = 256\ \text{g}$

**To Find**: The value of $\text{N}$ in $\text{N} \times 10^{24}$

**Formula**: Density of the cell $(\rho)$

$$= \frac{\text{Mass of crystal (M)} \times \text{ No. of atoms in unit cell}}{\text{Volume of unit cell } \times \text{ No. of atoms of pure substance in mass m}}$$

**Calculations**: A face-centred cubic $F.C.C.$ lattice has atoms of pure substance at the corner of the unit cell and/or atom each at the centre of the face of the unit cell.

Since there are eight atoms at the corners and one atom at each of the six faces

$i.e., six atoms in total$:

Total number of atoms present per unit cell $(z)$

$$= \frac{1}{8} \times 8 + \frac{1}{2} \times 6 = 4$$

Density of the cell $(\rho)$

$$= \frac{256 \text{ g} \times 4}{\left(400 \times 10^{-10}\right)^3 \times \text{N}^1}$$

$$8 \text{ g cm}^{-3} = \frac{256 \text{ g} \times 4}{\left(400 \times 10^{-10}\right) \times \text{N}^1}$$

$$\text{N}' = \frac{256 \text{ g} \times 4}{8 \text{ g cm}^{-3} \times (400)^3 \times 10^{-30}}$$

$$\mathbf{N}' = 2 \times 10^{24} \text{ atoms}$$

Hence, the crystal of pure substance of mass $256$ g contains $2 \times 10^{24}$ atoms.

Now, $\text{N}' = \text{N} \times 10^{24}$

Therefore, value of $\text{N} = 2$.

## Question 008 Numerical

**QUESTION**

The sum of the number of lone pairs of electrons on each central atom in the following species is

$$[TeBr_6]^{2-}, [BrF_2]^+, SNF_3,$$

and

$$[XeF_3]^-$$

$Atomic numbers : \$\$N = 7, F = 9, \$\$\$\$S = 16, Br = 35, \$\$\$\$Te = 52, Xe$

**SOURCE**

Chemistry • coordination-compounds

**EXPLANATION**

Sum of number of lone pairs = 1 + 2 + 0 + 3 = 6

## Question 009 Numerical

**QUESTION**

Among

$$H_2, He_2^+, Li_2,$$

$$Be_2, B_2, C_2, N_2, O_2^-$$

and

$$F_2,$$

the number of diamagnetic species is

$Atomic numbers: \$\$H = 1, He = 2, \$\$\$\$Li = 3, Be = 4, \$\$\$\$B = 5, C = 6$

**SOURCE**

Chemistry • chemical-bonding-and-molecular-structure

**EXPLANATION**

1 Electronic configuration of diatomic $\mathrm{H_2}$ on the the basis of molecular orbital theory:

$$(\sigma_{1\,s})^2(\sigma^*_{1s})^0$$

There are no unpaired electrons in the bonding and anti-bonding molecular orbitals; hence, it's a diamagnetic molecule.

2 Electronic configuration of diatomic $\mathrm{He^{2+}}$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma^*_{1s})^1$$

There is one unpaired electron in the sigma bonding molecular orbital; hence, it's a paramagnetic molecule.

3 Electronic configuration of diatomic $\mathrm{Li_2}$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2$$

There are no unpaired electrons in the bonding and anti-bonding molecular orbitals; hence, it's a diamagnetic molecule.

4 Electronic configuration of diatomic $\mathrm{Be_2}$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{1s})^2$$

There are no unpaired electrons in the bonding and anti-bonding molecular

orbitals; hence, it's a diamagnetic molecule.

$5$ Electronic configuration of diatomic $B_2$ on the basis of molecular orbitals theory:

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2\left(\pi_{2px}^1 \equiv \pi_{2py}^1\right)$$

There are two unpaired electrons in pi bonding molecular orbital; hence, it's a paramagnetic molecule.

$6$ Electronic configuration of diatomic $\mathrm{C}_2$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2\left(\pi_{2px}^2 \equiv \pi_{2py}^2\right)$$

There are no unpaired electrons in the bonding and anti-bonding molecular orbitals; hence, it's a diamagnetic molecule.

$7$ Electronic configuration of diatomic $\mathrm{N}_2$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2\left(\pi_{2px}^1 \equiv \pi_{2py}^1\right)\sigma_{2pz}^2$$

There are no unpaired electron in the bonding and anti-bonding molecular orbitals; hence, it's a diamagnetic molecule.

$8$ Electronic configuration of diatomic $\mathrm{O}_2$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2\left(\sigma^2_{p3}\right)^2\left(\pi^1_{2px}\equiv\pi^1_{2py}\right)$$

$$\left(\pi^*_{2px}\equiv\pi\right)$$

There is one unpaired electron in the $\mathrm{pi}^*$ anti-bonding molecular orbital; hence, it's a paramagnetic molecule.

$9$ Electronic configuration of diatomic $\mathrm{F}_2$ on the basis of molecular orbital theory:

$$(\sigma_{1s})^2(\sigma^*_{1s})^2(\sigma_{2s})^2(\sigma^*_{2s})^2(\sigma_{2pz})^2\left(\pi^2_{2px}\equiv\pi^2_{2py}\right)$$

$$\left(\pi^{2^*}_{2px}\equiv\pi^{2^*}_{2py}\right)$$

All the electrons are paired in the bonding and anti-bonding molecular orbitals; hence, $\mathrm{F}_2$ is a diamagnetic molecule

**Answer**. The are 6 diamagnetic species among the given diatomic molecules.

## Question 010 Numerical

**QUESTION**

Among the following, the number of aromatic compound $s$ is

**SOURCE**

Chemistry • basics-of-organic-chemistry

**EXPLANATION**

Huckel's rule states that a conjugated, planar, cyclic system will be aromatic if it contains $(4n+2)$ delocalised $\pi$ electrons, where $n$ is zero or integers like $1, 2, 3 \ldots$. Aromatic compounds are as follows:

Also

$i$ Both the rings in compound is cyclic.

$ii$ Due to the presence of alternate carboncarbon double bond in benzene ring all carbon are $sp^2$ hybridised and planner. But all carbons in second ring are not conjugated as carbons nine and ten are $sp^3$ hybridised. This makes second ring non-aromatic.

$iii$ There are $(4n+2)\pi$ electrons in aromatic ring $benzene$; hence, it's aromatic. Even if one ring in compound is aromatic, the entire molecule becomes aromatic.

Hence, there are total of 5 aromatic compounds in the given list of compounds.

## Question 011 MCQ

**QUESTION**

The correct statements $s$ for the following addition reactions is $are$

$O$

A and

$P$

are identical molecules

B $$M$$and$$O$$ and $$N$$and$$P$$ are two pairs of diastereomers

C $$M$$and$$O$$ and $$N$$and$$P$$ are two pairs of enantiomers

D Bromination proceeds through *trans*-addition in both the reactions

**CORRECT OPTION**

B $$M$$and$$O$$ and $$N$$and$$P$$ are two pairs of diastereomers

**SOURCE**

Chemistry • basics-of-organic-chemistry

**EXPLANATION**

Bromination of alkenes always proceeds via trans addition.

' $O$ ' and ' $P$ ' are enantiomers.

$(M$ and $O)$ and $N$and$P$ are two pairs of diastereomers.

## Question 012 MCQ

**QUESTION**

For a solution formed by mixing liquids

$$L$$

and

$$M,$$

the vapor pressure of

$$L$$

plotted against the mole fraction of

$$M$$

in solution is shown in the following figure. Here

$$X_L$$

and

$$X_M$$

represent mole fractions of

$$L$$

and

$$M,$$

respectively, in the solution. The correct statement $s$ applicable to this system is $are$

The point

$$Z$$

represents vapor pressure of pure liquid

$$M$$

A and Raoult's law is obeyed from

$$X_L = 0$$

to

$$X_L = 1$$

B

The point

$$Z$$

represents vapor pressure of pure liquid

$$L$$

and Raoult's law is obeyed when

$$X_L \to 1$$

C

The point

$$Z$$

represents vapor pressure of pure liquid

$$M$$

and Raoult's law is obeyed when

$$X_L \to 0$$

Attractive intermolecular interactions between

$$L$$

-

$$L$$

in pure liquid

$$L$$

and

$$M$$

-

$$M$$

in pure liquid

$$M$$

are stronger than those between

$$L - M$$

when mixed in solution.

**CORRECT OPTION**

B

The point

$$Z$$

represents vapor pressure of pure liquid

$$L$$

and Raoult's law is obeyed when

$$X_L \to 1$$

**SOURCE**

Chemistry • solutions

**EXPLANATION**

The vapour pressure plot for liquid $\mathrm{L}\left(\mathrm{P_L}\right)$ versus mole fraction of $\mathrm{M}\left(x_m\right)$ is represented as :

$i$ Point A on plot represents vapour pressure due to pure L $itsmolefractionbeing1$ and point $B$ represents vapour pressure due to pure $M$. Point $Z$ represents vapour pressure due to pure liquid $\mathrm{M}$ as mole fraction

of liquid $\mathrm{M}$ becomes maximum.

$ii$ The dotted line represents increase in the vapour of the pure liquid $M$ with increase in its concentration. Here, pure liquid $M$ follows Raoult's law.

$iii$ The red coloured curve on above the dotted line shows positive deviation shown by liquid M from the Raoult's law.

$a$ Since mole fraction of pure liquid $M$ is max $(x_m = 1)$ at point $Z$; hence, this point represents vapour pressure due to pure liquid $M$. There is no curve corresponding to pure liquid component L; hence, whether liquid L obey' Raoult's law or not cannot be predicted.

Option $A$ is incorrect.

$b$ Since mole fraction of liquid L is maximum at point B, which also corresponds to the maximum vapour pressure at point $Z$; hence, point $Z$ represents the vapour pressure of pure liquid $L$.

Option $B$ is correct.

$c$ At point Z, the molefraction of liquid component $\mathrm{L}$ and $\mathrm{M}$ lies above the vapour pressure curve for pure L $Thedottedline$. Therefore, the mixture obeys positive deviation from the Raoult's law. This means intermolecular force of attraction are lower between $M$ and L; then that between M-M and L-L.

Option $D$ is correct.

## Question 013 MCQ

**QUESTION**

The IUPAC name $s$ of the following compound is $are$

A -chloro- 1 4 -methylbenzene

B -chlorotoluene 4

C -methylchlorobenzene 4

D -methyl- 1 4 -chlorobenzene

**CORRECT OPTION**

A -chloro- 1 4 -methylbenzene

**SOURCE**

Chemistry • basics-of-organic-chemistry

**EXPLANATION**

The IUPAC name of the compound:

1. When two or more substituents such as alkane, halogen, nitro groups, etc., are attached to the aromatic ring $e.g.,benzene$, these substituents are named in the alphabetical order with substituents of lowest alphabet named first followed by the other.

2. These substituents are numbered in a way with lowest number given to substituent whose name starts with the alphabet that appear first in the series. The direction of numbering is chosen such that next substituent gets the lowest number.

3. After the numbering is complete, the substituents are numbered in alphabetical order.

Hence, the IUPAC name of the compound is 1-Chloro-4-methylbenzene.

4. If a substituent attached to benzene ring together has a common name that has been accepted as an IUPAC name, the compound is called by that common name with other group attached to it as its substituent.

Hence, methyl attached to benzene is commonly called toluene and chlorine is substituent at the fourth position.

The compound is named as 4-chlorotoluene.

## Question 014 MCQ

**QUESTION**

The only CORRECT combination in which the reaction proceeds through radical mechanism is :

A $III\ ii\ P$

B $IV\ i\ Q$

C $II\ iii\ R$

D $I\ ii\ R$

**CORRECT OPTION**

D $I\ ii\ R$

**SOURCE**

Chemistry • aldehydes-ketones-and-carboxylic-acids

**EXPLANATION**

The following reaction takes place by free radical mechanism :

The mechanism is as follows :

## Question 015 MCQ

**QUESTION**

The only CORRECT combination that gives two different carboxylic acids is :

A $II\ iv\ R$

B $IV\ iii\ Q$

C $III\ iii\ P$

D $I\ i\ S$

**CORRECT OPTION**

C $III\ iii\ P$

**SOURCE**

Chemistry • aldehydes-ketones-and-carboxylic-acids

**EXPLANATION**

Reaction of benzaldehyde with acetic anhydride in presence of potassium acetate results in formation of cinnamic acid. Cinnamic acid contains cis and trans forms; hence, two types of acids are generated. This is called Perkin reaction.

Mechanism of Perkin reaction is as follows :

$i$ The potassium acetate is a strong base and abstracts acidic $\alpha$-proton from acetic anhydride.

$ii$ The negatively charged carbanion attacks the carbon of carbonyl group of benzaldehyde.

$iii$ The intermediate formed undergoes rearrangement

$iv$ Loss of proton from $\alpha$-carbon gives cinnamic acid

$v$ Depending upon the orientation of carboxylic functional group cis and trans cinnamic acid are formed.

## Question 016 MCQ

**QUESTION**

For

$$He^{+}$$

ion, the only INCORRECT combination is

A $I\ i\ R$

B $II\ ii\ Q$

C $I\ iii\ R$

D $I\ i\ S$

**CORRECT OPTION**

C $I\ iii\ R$

**SOURCE**

Chemistry • structure-of-atom

**EXPLANATION**

The ion $\mathrm{He}^{+}$ having 1 electron $in\$1s\$orbital$ is hydrogen like species; hence, its $1s$ orbital will have:

$i$ Wave function same as that of hydrogen, i.e.,

$$\psi_{1,0,0} = \left(\frac{Z}{a_0}\right)^{3/2} e^{-Zr/a_0}$$

$ii$ The probability density for the electron in the $1s$ orbital is represented as:

For $1s$ orbital probability density is maximum at the nucleus.

$iii$ Probability density for $2s$ orbitals is given by

$$= \frac{1}{2a_0^{3/2}}\left(1 - \frac{r}{2a_0}\right)e^{-r/2a_0}$$

Probability density $\propto \frac{1}{a_0^3}$

$iv$ For electron in the $1s$ orbital, the wave function of electron is not the function of angular wave function i.e., $\theta$ and $\phi$. Hence, 1 s orbital cannot have the wave function

$$\psi_{n,l,m_l} \propto \left(\frac{Z}{a_0}\right)^{5/2} re^{(-Zr/2a_0)\cos\theta}$$

## Question 017 MCQ

**QUESTION**

For hydrogen atom, the only CORRECT combination is :

A $I\ i\ S$

B $II\ i\ Q$

C $I\ i\ P$

D $I\ iv\ R$

**CORRECT OPTION**

A $I\ i\ S$

**SOURCE**

Chemistry • structure-of-atom

**EXPLANATION**

$i$ Wave function for 1s orbital of hydrogen atom with $n, l$ and $m_l$ as quantum numbers is given

$$\psi_{n,l,m_l} = \frac{1}{\sqrt{\pi}} \left( \frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$

Where, $\mathrm{Z}$ is atomic number of the element, $r$ is the variable distance of electron from the nucleus, $a_0$ is the Bohr's radius

For $1\,\mathrm{s}$ orbital, $n = 1, l = 0, m_l = 0$ and wave function is represented as :

$$\psi_{1,0,0} = \frac{1}{\sqrt{\pi}} \left( \frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$

$$\Rightarrow \psi_{1,0,0} \propto \left( \frac{Z}{a_0} \right)^{3/2} e^{-Zr/a_0}$$

$ii$ Energy of excitation of electron from $n_1 = 2$ to $n_2 = 4$ is given as :

$$\mathrm{E}_{2\to4} \propto -\mathrm{R} \left( \frac{1}{n_2^2} - \frac{1}{n_1^2} \right)$$

$$\mathrm{E}_{2\to4} \propto -\mathrm{R} \left( \frac{1}{4^2} - \frac{1}{2^2} \right)$$

$$\mathrm{E}_{2\to4} \propto -\mathrm{R} \left( \frac{-3}{16} \right) = \frac{3R}{16}$$

$iii$ Energy of excitation of electron from $n_1 = 2$ to $n_2 = 4$ is given as:

$$\mathrm{E}_{2\to6} \propto -\mathrm{R}\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$$

$$\mathrm{E}_{2\to6} \propto -R\left(\frac{1}{6^2}-\frac{1}{2^2}\right)$$

$$= -\left(\frac{1}{36}-\frac{1}{4}\right)R$$

$$E_{2\to6} \propto \frac{+8R}{36}$$

The ratio of the energy of transition in the two cases is given by :

$$\frac{E_{2\to4}}{E_{2\to6}} = \frac{3R}{16}\times\frac{6}{8R} = \frac{27}{32}$$

$$E_{2\to4} = \frac{27}{32}\times E_{2\to6}$$

the energy of transition from 2 to 4 excited state is $\frac{27}{32}$ times the energy of transition from 2 to 6 excited state.

## Question 018 MCQ

**QUESTION**

For the given orbital in Column 1, the only **CORRECT** combination for any hydrogen-like species is :

A $I\ ii\ S$

B $IV\ iv\ R$

C $II\ ii\ P$

D $III\ iii\ P$

**CORRECT OPTION**

C $II\ ii\ P$

**SOURCE**

Chemistry • structure-of-atom

**EXPLANATION**

- Option $A$ : Incorrect. The total number of radial nodes is given by $n-l-1$. For $1s$ orbital, the radial node is zero $i.e.\$1-0-1=0\$$.

- Option $B$ : Incorrect. $d_{z^2}$ orbital has two lobes that point in opposite directions along $z$-axis plus a doughnut-shaped ring of electron density around the centre that lies in the $xy$-plane. Therefore, it does not have any nodal plane.

- Option $C$ : Correct. The total number of radial nodes is given by $n-l-1$.

For $2s$ : Radial nodes $=2-0-1=1$

The orbital 2s of hydrogen like species can be diagrammatically represented as :

There is a spherical region of zero electron density between $1s$ and the $2s$ orbital of hydrogen like species. This spherical region is called the node. Only

one radial node exist between $1s$ and $2s$.

The plot of wave function for $2s$ verses radial distance from the nucleus is represented as

Point $P$ represents the region of zero electron density at a distance $r$ from the nucleus. This represents spherical node.

- Option $D$ : Incorrect. The probability density at nucleus is for $2p$ orbital is zero.

## Question 019 MCQ

**QUESTION**

Let X and Y be two events such that

$$P(X) = \frac{1}{3}$$

,

$$P(X|Y) = \frac{1}{2}$$

and

$$P(Y|X) = \frac{2}{5}$$

. Then

A $$P(Y) = \frac{4}{15}$$

B $$P(X'|Y) = \frac{1}{2}$$

C $$P(X \cup Y) = \frac{2}{5}$$

D $$P(X \cap Y) = \frac{1}{5}$$

**CORRECT OPTION**

A $$P(Y) = \frac{4}{15}$$

**SOURCE**

Mathematics • probability

**EXPLANATION**

$$P(X) = \frac{1}{3}$$

$$P\left(\frac{X}{Y}\right) = \frac{P(X \cap Y)}{P(Y)} = \frac{1}{2}$$

$$P\left(\frac{Y}{X}\right) = \frac{P(X \cap Y)}{P(X)} = \frac{2}{5}$$

$$P(X \cap Y) = \frac{2}{15}$$

$$P(Y) = \frac{4}{15}$$

$$P\left(\frac{X'}{Y}\right) = \frac{P(Y) - P(X \cap Y)}{P(Y)}$$

$$= \frac{\frac{4}{15} - \frac{2}{15}}{\frac{4}{15}} = \frac{1}{2}$$

$$P(X \cup Y) =$$

$$\frac{1}{3} + \frac{4}{15} - \frac{2}{15}$$

$$= \frac{7}{15} = \frac{7}{15}$$

## Question 020 MCQ

**QUESTION**

Let f : R

$$\rightarrow$$

$0, 1$ be a continuous function. Then, which of the following function $s$ has $have$ the value zero at some point in the interval $0, 1$ ?

**A** $$e^x - \int_0^x f(t) \sin t \, dt$$

**B** $$f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$$

**C** $$f(x) - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$$

**D** x9

$$-$$

f $x$

**CORRECT OPTION**

**C** $$f(x) - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$$

**SOURCE**

Mathematics • limits-continuity-and-differentiability

**EXPLANATION**

$a$

$$\because$$

$$e^x \in (1, e)$$

in $0, 1$ and

$$\int_0^x f(t) \sin t \, dt \in$$

$0, 1$ in $0, 1$

$$\therefore$$

$$e^x - \int_0^x f(t) \sin t \, dt$$

cannot be zero.

So, option $a$ is incorrect.

$b$

$$f(x) + \int_0^{\frac{\pi}{2}} f(t) \sin t \, dt$$

always positive

$$\therefore$$

Option $b$ is incorrect.

$c$ Let

$$h(x) = x - \int_0^{\frac{\pi}{2}-x} f(t) \cos t \, dt$$

,

$$h(0) = -\int_0^{\frac{\pi}{2}} f(t)\cos t\,dt < 0$$

$$h(1) = 1 - \int_0^{\frac{\pi}{2}-1} f(t)\cos t\,dt > 0$$

$$\therefore$$

Option $c$ is correct.

$d$ Let

$$g(x) = x^9 - f(x)$$

$$g(0) = -f(0) < 0$$

,

$$g(1) = 1 - f(1) > 0$$

$$\therefore$$

Option $d$ is correct.

## Question 021 MCQ

**QUESTION**

Let a, b, x and y be real numbers such that a

$-$

b = 1 and y

$\neq$

0. If the complex number z = x + iy satisfies

$$\mathrm{Im}\left(\frac{az+b}{z+1}\right)=y$$

, then which of the following is $are$ possible value $s$ of x?

A $1-\sqrt{1+y^2}$

B $-1-\sqrt{1-y^2}$

C $1+\sqrt{1+y^2}$

D $-1+\sqrt{1-y^2}$

**CORRECT OPTION**

B $-1-\sqrt{1-y^2}$

**SOURCE**

Mathematics • complex-numbers

**EXPLANATION**

It is given that

$$z = x + iy$$

satisfies

$$\text{Im}\left(\frac{az+b}{z+1}\right) = y$$

.

Therefore,

$$\text{Im}\left(\frac{a(x+iy)+b}{(x+iy)+1}\right) = y$$

$$\Rightarrow Im\left(\frac{ax+iay+b}{x+iy+1}\right) = y$$

Rationalizing the above equation and multiplying and dividing LHS by $x + 1\$\$ - \$\$iy$, we get

$$\text{Im}\left(\frac{(ax+iay+b)}{(x+iy+1)} \times \frac{(x+1-iy)}{(x+1-iy)}\right) = y$$

Using

$$a^2 - b^2 = (a+b)(a-b)$$

, we get

$$\text{Im}\left(\frac{(ax+iay+b)}{(x+1)^2-(iy)^2}\right) = y$$

$$\text{Im}\left(\frac{ax^2+ax-iayx+iaxy-i^2ay^2+bx+b-iby}{(x+1)^2+y^2}\right) = y$$

$as\$\$i^2 = -1\$\$$

$$\text{Im}\left(\frac{(ax^2+ax+ay^2+bx+b)+i(axy-ayx+ay-by)}{(x+1)^2+y^2}\right) = y$$

Rearranging LHS, we get

$$\text{Im}\left(\frac{[(ax^2+bx)+(ax+b)+ay^2]+i(ay-by)}{(x+1)^2+y^2}\right)=y$$

$$\Rightarrow \frac{ay-by}{(x+1)^2+y^2}=y$$

$asimaginaryvalueinbracketiscoefficientofi$

$$\Rightarrow y(a-b)=y((x+1)^2+y^2)\Rightarrow (a-b)=(x+1)^2+y^2$$

It is given that a

$$-$$

b = 1 and y

$$\neq$$

0

$$\Rightarrow 1=(x+1)^2+y^2$$

$$\Rightarrow 1-y^2=(x+1)^2\Rightarrow (x+1)=\pm\sqrt{1-y^2}$$

$as\$\$x^2=b\Rightarrow x=\pm\sqrt{6}\$\$$

$$\Rightarrow 1=-1\pm\sqrt{1-y^2}$$

or,

$$x=-1+\sqrt{1-y^2}$$

and

$$x=-1-\sqrt{1-y^2}$$

## Question 022 MCQ

**QUESTION**

If

$$2x - y + 1 = 0$$

is a tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{16} = 1$$

then which of the following CANNOT be sides of a right angled triangle?

A a, 4, 1

B 2a, 4, 1

C a, 4, 2

D 2a, 8, 1

**CORRECT OPTION**

A a, 4, 1

**SOURCE**

Mathematics • hyperbola

**EXPLANATION**

Line y = mx + C is tangent to hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

if following condition is satisfied:

$$C^2 = a^2m^2 - b^2$$

..... 1

Given : The equation of line =

$$2x - y + 1 = 0$$

Hyperbola =

$$\frac{x^2}{a^2} - \frac{y^2}{16^2} = 1$$

Consider equation

$$2x - y + 1 = 0$$

$$\Rightarrow y = 2x + 1 \Rightarrow m = 2$$

and C = 1

From equation of hyperbola, we get $b^2 = 16$ and $a^2$

Substituting all values in Eq. $1$, we get

$$1^2 = a^2(2)^2 - 16$$

$$1 = 4a^2 - 16 \Rightarrow 4a^2 = 16 + 1 \Rightarrow 4a^2 = 17$$

$$\Rightarrow a^2 = \frac{17}{4} \Rightarrow a = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$$

Therefore,

$$a = \frac{\sqrt{17}}{2}$$

Option $A$ : a, 4, 1

$$\Rightarrow \frac{\sqrt{17}}{2}$$

, 4, 1 is not a right triangle $Since\$\$\sqrt{4^2 + 1^1} \neq \frac{\sqrt{17}}{2}\$\$$

Option $B$ : 2a, 4, 1

$$\sqrt{17}$$

, 4, 1 is a right angled triangle $since\$\$\sqrt{4^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}\$\$$

Option $C$ : a, 4, 2

$$\Rightarrow \frac{\sqrt{17}}{2}$$

, 4, 2 is not a right angled triangle $Since\$\$\sqrt{4^2+2^2} \neq \frac{\sqrt{17}}{2}\$\$$

Option $D$ : 2a, 8, 1

$$\sqrt{17}$$

, 8, 1 is not a right angled triangle $Since\$\$\sqrt{8^2+1^2} \neq \sqrt{17}\$\$$

## Question 023 MCQ

**QUESTION**

Let

$$x$$

be the greatest integer less than or equals to x. Then, at which of the following point $s$ the function

$$f(x) = x\cos(\pi(x+[x]))$$

is discontinuous?

A x = $-$ 1

B x = 1

C x = 0

D x = 2

**CORRECT OPTION**

A x = $-\frac{1}{2}$

**SOURCE**

Mathematics • limits-continuity-and-differentiability

**EXPLANATION**

$$f(x) = x\cos(\pi(x + [x]))$$

At x = 0

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x\cos(\pi(x + [x]) = 0$$

and f$x$ = 0

$$\therefore$$

It is continuous at x = 0 and clearly discontinuous at other integer points.

## Question 024 MCQ

**QUESTION**

Which of the following is $are$ NOT the square of a 3

$\times$

3 matrix with real entries?

A

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

B

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

C

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

D

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**CORRECT OPTION**

A

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

**SOURCE**

Mathematics • matrices-and-determinants

**EXPLANATION**

For a matrix to be square of matrix with real entries, its determinant should be positive.

Option $A$ :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

, determinant is NOT possible:

1$$ − $$1

−

00 + 00 =

−

1

Option $B$ :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

, determinant is possible:

11

−

00 + 00 = +1

Option $C$ :

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

, determinant is NOT possible :

$$-$$

$11$

$$-$$

$00 + 00 =$

$$-$$

1

Option $D$ :

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

, determinant is possible:

$1$

$$-$$

$00 + 00 = +1$

Thus, options $A$ and $C$ are NOT the square of a 3

$$\times$$

3 matrix with real entries.

## Question 025 MCQ

**QUESTION**

If a chord, which is not a tangent, of the parabola $y^2 = 16x$ has the equation $2x + y = p$, and mid-point $h, k$, then which of the following is $are$ possible value $s$ of p, h and k?

A

p = 1, h = 1, k = $\frac{-}{3}$

B

p = 2, h = 3, k = $\frac{-}{4}$

C

p = 2, h = 2, k = $\frac{-}{4}$

D

p = 5, h = 4, k = $\frac{-}{3}$

**CORRECT OPTION**

B

p = 2, h = 3, k = $\frac{-}{4}$

**SOURCE**

Mathematics • parabola

**EXPLANATION**

Given : The equation of parabola is $y^2 = 16x$

Equation chord is

$2x + y = p$ ..... $1$

Equation of chord with middle point $h, k$ is given as

$$ky - 16\left(\frac{x+h}{2}\right) = k^2 - 16h$$

$$\Rightarrow ky - 8(x+h) = k^2 - 16h$$

$$\Rightarrow ky - 8x - 8h = k^2 - 16h$$

$$\Rightarrow ky - 8x - 8h - k^2 + 16h = 0$$

$$\Rightarrow -8x + ky + 8h - k^2 = 0$$

$$\Rightarrow 8x - ky = 8h - k^2$$

..... $2$

Comparing above equation with equation of chord, we get

$$2x + y = p$$

Dividing Eq. $2$ by 4, we get

$$\frac{8x}{4} - \frac{ky}{4} = \frac{8h}{4} - \frac{k^2}{4}$$

$$2x - \frac{ky}{4} = 2h - \frac{k^2}{4}$$

On comparing, we get

$$\Rightarrow \frac{-k}{4} = 1$$

and

$$p = 2h - \frac{k^2}{4}$$

$$\Rightarrow k = -4$$

and

$$p = 2h - \frac{(-4)^2}{4} = 2h - 4$$

$$\Rightarrow p = 2h - 4 \Rightarrow 2h - p = 4$$

Only p = 2 and h = 3 satisfies this equation. Therefore, p = 2, h = 3 and k =

$$-$$

4.

## Question 026 Numerical

**QUESTION**

For a real number

$$\alpha$$

, if the system

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

of linear equations, has infinitely many solutions, then 1 +

$$\alpha$$

+

$$\alpha$$

$^2$ =

**SOURCE**

Mathematics • matrices-and-determinants

**EXPLANATION**

It is given that

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix} = 0$$

$$\Rightarrow 1(1-\alpha^2) - \alpha(\alpha - \alpha^3) + \alpha^2(\alpha^2 - \alpha^2) = 0$$

$$\Rightarrow \alpha(1-\alpha^2) - \alpha^2(1-\alpha^2) = 0$$

$$\Rightarrow (1-\alpha^2)(1-\alpha^2) = 0$$

$$\Rightarrow (1-\alpha^2)^2 = 0$$

$$\Rightarrow \alpha^2 = 1 \Rightarrow \alpha = \pm 1$$

For

$$\alpha$$

= 1, the given system of linear equations has no solution.

$$\begin{bmatrix} +1 & +1 & +1 \\ +1 & +1 & +1 \\ +1 & +1 & +1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$x + y + z = 1$$

$$x + y + z = -1$$

$$x + y + z = 1$$

Since two planes are parallel. So,

$$\alpha$$

= 1 is rejected for

$$\alpha$$

=

$$-$$

1 the given system of linear equations has coincident planes

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$x - y + z = 1$$

$$\Rightarrow -x + y - z = -1 \Rightarrow x - y + z = 1$$

$$x - y + 1 = 1$$

Therefore,

$$\alpha$$

=

$$-$$

1 is accepted. That is,

$$1 + \alpha + \alpha^2 = 1 + (-1) + (-1)^2 = 1 - 1 + 1 = 1$$

$$\Rightarrow 1 + \alpha + \alpha^2 = 1$$

## Question 027 Numerical

**QUESTION**

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

**SOURCE**

Mathematics • sequences-and-series

**EXPLANATION**

Let the sides be given by a

$$-$$

d, a, a + d, where $a, d > 0$. Also, d < a.

By the condition,

a$^2$ + $a\$\$ - \$\$d^2$ = $a + d^2$

$$\Rightarrow$$

a$^2$ = $a + d^2$

$$-$$

$a\$\$ - \$\$d^2$

$$\Rightarrow$$

a$^2$ = 4ad

$$\therefore$$

a = 4d

Thus the sides are 3d, 4d, 5d.

As area = 24, we have

$$\frac{1}{2}$$

. 3d . 4d = 24

$$\therefore$$

d = 2

The sides are 6, 8, 10./p>

## Question 028 Numerical

**QUESTION**

Let f : R

$$\rightarrow$$

R be a differentiable function such that f$0$ = 0,

$$f\left(\frac{\pi}{2}\right) = 3$$

and f'$0$ = 1.

If

$$g(x) = \int_{x}^{\pi/2} [f'(t)\operatorname{cosec} t - \cot t \operatorname{cosec} t\, f(t)]dt$$

for

$$x \in \left(0, \frac{\pi}{2}\right]$$

, then

$$\lim_{x \to 0} g(x)$$

=

**SOURCE**

Mathematics • limits-continuity-and-differentiability

**EXPLANATION**

Let

$$g(x) = \int_{x}^{\pi/2} [f'(t)\text{cosec}\, t - \cot t\, \text{cosec}\, t\, f(t)]dt$$

$$= \int_{x}^{\pi/2} \frac{d}{dt}(f(t)\cos ect))$$

So,

$$g(x) = f(\pi/2)\cos ec\frac{\pi}{2} - f(x)\cos ecx$$

$$= 3 - f(x)\cos ecx$$

$$\therefore$$

$$g(x) = 3 - \frac{f(x)}{\sin x}$$

$$\lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f(x)}{\sin x}$$

As the above is a 0/0 form, use L'Hospital's rule to get

$$\lim_{x\to 0} g(x) = 3 - \lim_{x\to 0}\frac{f'(x)}{\cos x} = 3 - f'(0) = 3 - 1 = 2$$

## Question 029 Numerical

**QUESTION**

For how many values of p, the circle $x^2 + y^2 + 2x + 4y$

—

p = 0 and the coordinate axes have exactly three common points?

**SOURCE**

Mathematics • ellipse

**EXPLANATION**

We will consider three cases

Case 1 : Circle passes through origin, that is, p = 0 the equation of circle becomes

$x^2 + y^2 + 2x + 4y = 0$

$x = 0$

$\Rightarrow$

$y^2 + 4y = 0$

$\Rightarrow$

$y\,y + 4 = 0$

$y = 0,$

$-$

$4$

$y = 0$

$\Rightarrow$

$x^2 + 2x = 0$

$\Rightarrow$

$x\,x + 2 = 0$

$\Rightarrow$

$x = 0,$

$-$

$2$

Case 2 : Circle touches y axis then circle will intersect x axis at two distinct points

Put y = 0 in equation of circle, we get

$x^2 + 2x$

—

p = 0

Now from $g^2$

—

c > 0 and $f^2$

—

c = 0

⇒

$1^2$

—

$$ − $$p > 0 and $2^2$

—

$$ − $$p = 0

1 + p > 0 and 4 + p = 0

p >

—

1 and p =

—

4

which is a contradiction. Also, for p =

—

4 we get

$x^2$

—

2x + 4 = 0

$$x = \frac{2 \pm \sqrt{4 - 4 \times 4}}{2} = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$$

They are imaginary roots.

Therefore, Case 2 is not possible.

Case 3 : Circle touches x axis. Then the circle will intersect y axis at two distinct points.

Substituting x = 0, we get

$y^2 + 4y$

—

p = 0

Now, from $g^2$

—

C = 0 and $f^2$

—

C > 0, we have

$1^2$

—

$\$\$ - \$\$p$ = 0 and $2^2$ = $\$\$ - \$\$p$ > 0

p =

—

1 and 4 + p > 0

⇒

p >

—

4

Therefore, the equation of circle becomes

$y^2 + 4y + 1 = 0$

$$\Rightarrow y = \frac{-4 \pm \sqrt{16-4}}{2} = \frac{-4 \pm \sqrt{12}}{2}$$

$$y = \frac{-4 \pm \sqrt{4 \times 3}}{2}$$

$$= \frac{-4 \pm 2\sqrt{3}}{2} = -2 \pm \sqrt{3}$$

real values

Thus, Case 3 is possible.

Thus, for p = 0 and p =

$$-$$

1 the circle and coordinates have exactly three points in common. Hence, the correct answer is 2.

## Question 030 Numerical

**QUESTION**

Words of length 10 are formed using the letters A, B, C, D, E, F, G, H, I, J. Let x be the number of such words where no letter is repeated; and let y be the number of such words where exactly one letter is repeated twice and no other letter is repeated. Then,

$$\frac{y}{9x}$$

= ?

**SOURCE**

Mathematics • permutations-and-combinations

**EXPLANATION**

The given, formed word is of length 10.

It is given that x is the number of words where no letter is repeated.

Also, it is given that y is the number of words where exactly one letter is repeated twice and no other letter is repeated. Therefore,

x = 10!

and y = $^{10}C_1$

$\times$

$^{10}C_2$

$\times$

$^{9}C_8$

$\times$

8!

Thus,

$$\frac{y}{9x} = \frac{^{10}C_1 \times {}^{10}C_2 \times {}^{9}C_8 \times 8!}{9 \times 10!}$$

Using

$$^{n}C_r = \frac{n!}{r!(n-r)!}$$

, we get

$$\frac{y}{9x} = \frac{\left[\frac{10!}{1!(10-1)!} \times \frac{10!}{2!(10-2)!} \times \frac{9!}{8!(9-8)!} \times 8!\right]}{9 \times 10!}$$

$$= \frac{10!}{9 \times 2! \times 8!} = \frac{10 \times 9 \times 8!}{9 \times 2 \times 8!} = \frac{10}{2} = 5$$

$$Using\$\$n! = n(n-1)(n-2).....1!\$\$$$

## Question 031 MCQ

**QUESTION**

For

$$a = \sqrt{2}$$

, if a tangent is drawn to a suitable conic $Column1$ at the point of contact $\$\$ - \$\$1, 1$, then which of the following options is the only CORRECT combination for obtaining its equation?

A $I\ ii$ Q)

B $I\ ii\ P$

C $III\ i\ P$

D $II\ ii\ Q$

**CORRECT OPTION**

A $I\ ii$ Q)

**SOURCE**

Mathematics • hyperbola

**EXPLANATION**

•

For the given equation

$$a = \sqrt{2}$$

and point $\$\$ - \$\$1, 1$, $x^2 + y^2 = a^2$ satisfies these conditions $i.e. I of Column 1$

•

Thus, equation of tangent is

$$-x + y = 2 \Rightarrow y = x + 2$$

; thus $ii$ of Column 2 satisfies this condition with m = 1. That is,

$$y = mx + a\sqrt{m^2 + 1}$$

•

It is given that point of constant $\$\$ - \$\$1, 1$ given m = 1 and

$$a = \sqrt{2}$$

Q; of Columns 3 :

$$\left(\frac{-ma}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$$

gives point of contact $\$\$ - \$\$1, 1$.

Thus, correct combination is $I\ II\ Q$; thus, option $A$ is correct.

## Question 032 MCQ

**QUESTION**

The tangent to a suitable conic $Column 1$ at

$$\left(\sqrt{3}, \frac{1}{2}\right)$$

is found to be

$$\sqrt{3}x + 2y = 4$$

, then which of the following options is the only CORRECT combination?

A $IV \ \ iv \ \ S$

B $II \ \ iv \ \ R$

C $IV \ \ iii \ \ S$

D $II \ \ ii \ \ R$

**CORRECT OPTION**

B $II \ \ iv \ \ R$

**SOURCE**

Mathematics • hyperbola

**EXPLANATION**

Given, tangent to conic is

$$\sqrt{3}x + 2y = 4$$

and point of contact is

$$\left(\sqrt{3}, \frac{1}{2}\right)$$

..... 1

$$\sqrt{3}x + 2y = 4 \Rightarrow 2y = 4 - \sqrt{3}x$$

$$\Rightarrow y = \frac{-\sqrt{3}}{2}x + 2$$

..... 2

On comparing this equation with options in Column 2, we get

$$m = \frac{-\sqrt{3}}{2}$$

Equation $1$ can also be written as

$$\Rightarrow \sqrt{3}x + 4\left(\frac{1}{2}\right)y = 4$$

On comparing with the option $II$, we get

$$a^2 = 4 \Rightarrow a = 2$$

Now, for a = 2,

$$m = \frac{-\sqrt{3}}{2}\left(\frac{-a^2 m}{\sqrt{a^2m^2+1}}, \frac{1}{\sqrt{a^2m^2+1}}\right)$$

gives

$$\left(\sqrt{3}, \frac{1}{2}\right)$$

as the point of contact.

Thus, the correct combination is $II\ iv\ R$; hence, option $B$ is correct.

## Question 033 MCQ

**QUESTION**

If a tangent to a suitable conic $Column1$ is found to be y = x + 8 and its point of contact is $8, 16$, then which of the following options is the only CORRECT combination?

A $III\ i\ P$

B $I\ ii\ Q$

C $II\ iv\ R$

D $III\ ii\ Q$

**CORRECT OPTION**

A $III\ i\ P$

**SOURCE**

Mathematics • parabola

**EXPLANATION**

The tangent is y = x + 8.

Satisfying $8, 16$ in $y^2 = 4ax$, we have

$16^2 = 4a \cdot 8.$

$\therefore$

a = 8

On comparing y = mx +

$$\frac{a}{m}$$

, we have m = 1

The point of contact is

$$\left(\frac{a}{m^2}, \frac{2a}{m}\right) = (8, 16)$$

So its verified.

## Question 034 MCQ

**QUESTION**

Which of the following options is the only INCORRECT combination?

A $I\ iii\ P$

B $II\ iv\ Q$

C $II\ ii\ P$

D $III\ i\ R$

**CORRECT OPTION**

D $III\ i\ R$

**SOURCE**

Mathematics • application-of-derivatives

**EXPLANATION**

$$f(x) = x + \ln x - x \ln x$$

$$f(1) = 1 > 0$$

$$f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$$

$$\Rightarrow f(x) = 0$$

for some

$$x \in (1,\, e^2)$$

$$\therefore$$

I is correct.

$$f'(x) = 1 + \frac{1}{x} - \ln x - 1$$

$$= \frac{1}{x} - \ln x$$

$$f'(x) > 0$$

for $0, 1$

$$f'(x) < 0$$

for

$$(e, \infty)$$

$$\therefore$$

P and Q are correct, II is correct, III is incorrect.

$$f''(x) = \frac{-1}{x^2} - \frac{1}{x}$$

$$f''(x) < 0$$

for

$$(0, \infty)$$

$$\therefore$$

S, is correct, R is incorrect.

IV is incorrect.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to \infty} f'(x) = -\infty$$

$$\lim_{x \to \infty} f''(x) = 0$$

$$\therefore$$

ii, iii, iv are correct.

## Question 035 MCQ

**QUESTION**

Which of the following options is the only CORRECT combination?

A $I \ ii \ R$

B $III \ iv \ P$

C $II \ iii \ S$

D $IV \ i \ S$

**CORRECT OPTION**

C $II \ iii \ S$

**SOURCE**

Mathematics • application-of-derivatives

**EXPLANATION**

$$f(x) = x + \ln x - x \ln x$$

$$f(1) = 1 > 0$$

$$f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$$

$$\Rightarrow f(x) = 0$$

for some

$$x \in (1,\, e^2)$$

$$\therefore$$

I is correct.

$$f'(x) = 1 + \frac{1}{x} - \ln x - 1$$

$$= \frac{1}{x} - \ln x$$

$$f'(x) > 0$$

for $0, 1$

$$f'(x) < 0$$

for

$$(e, \infty)$$

$$\therefore$$

P and Q are correct, II is correct, III is incorrect.

$$f''(x) = \frac{-1}{x^2} - \frac{1}{x}$$

$$f''(x) < 0$$

for

$$(0, \infty)$$

$$\therefore$$

S, is correct, R is incorrect.

IV is incorrect.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to \infty} f'(x) = -\infty$$

$$\lim_{x \to \infty} f''(x) = 0$$

$$\therefore$$

ii, iii, iv are correct.

## Question 036 MCQ

**QUESTION**

Which of the following options is the only CORRECT combination?

A $III \ iii \ R$

B $IV \ iv \ S$

C $II \ ii \ Q$

D $I0(i \ P$

**CORRECT OPTION**

C $II \ ii \ Q$

**SOURCE**

Mathematics • application-of-derivatives

**EXPLANATION**

$$f(x) = x + \ln x - x \ln x$$

$$f(1) = 1 > 0$$

$$f(e^2) = e^2 + 2 - 2e^2 = 2 - e^2 < 0$$

$$\Rightarrow f(x) = 0$$

for some

$$x \in (1,\, e^2)$$

$$\therefore$$

I is correct.

$$f'(x) = 1 + \frac{1}{x} - \ln x - 1$$

$$= \frac{1}{x} - \ln x$$

$$f'(x) > 0$$

for $0, 1$

$$f'(x) < 0$$

for

$$(e, \infty)$$

$$\therefore$$

P and Q are correct, II is correct, III is incorrect.

$$f''(x) = \frac{-1}{x^2} - \frac{1}{x}$$

$$f''(x) < 0$$

for

$$(0, \infty)$$

$$\therefore$$

S, is correct, R is incorrect.

IV is incorrect.

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to \infty} f'(x) = -\infty$$

$$\lim_{x \to \infty} f''(x) = 0$$

$$\therefore$$

ii, iii, iv are correct.

## Question 037 MCQ

**QUESTION**

In which case will the particle move in a straight line with constant velocity?

**A** $(\mathrm{III})\ (ii)\ (R)$

**B** $(\mathrm{IV})\ (i)\ (S)$

**C** $(\mathrm{III})\ (iii)\ (P)$

**D** $(\mathrm{II})\ (iii)\ (S)$

**CORRECT OPTION**

D $(\mathrm{II})\ (iii)\ (S)$

**SOURCE**

Physics • magnetism

**EXPLANATION**

$$\overrightarrow{F}$$

= q

$$\overrightarrow{E}$$

+ q $$\overrightarrow{v}$$$$ \times $$$$\overrightarrow{B}$$ ..... $1$

For a particle to move in a straight line with constant velocity,

$$\overrightarrow{F}$$

= 0. Therefore, from Eq. $1$, we get

$$q\overrightarrow{E} + q(\overrightarrow{V} \times \overrightarrow{B}) = 0$$

$$\Rightarrow q\overrightarrow{E} - q(\overrightarrow{v} \times \overrightarrow{B})$$

$$\Rightarrow \overrightarrow{E} = -(\overrightarrow{v} \times \overrightarrow{B})$$

For

$$\overrightarrow{v} = \frac{E_0}{B_0}\hat{y}$$

and

$$\overrightarrow{B} = B_0\hat{z}$$

, we get

$$\overrightarrow{v} \times \overrightarrow{B} = \left(\frac{E_0}{B}\hat{y}\right) \times (B_0\hat{z}) = \frac{E_0}{B_0} \times B_0(\hat{y} \times z)$$

Therefore,

$$\overrightarrow{v} \times \overrightarrow{B} = E_0\widehat{x}$$

$asy\$\$ \times \$\$z = x$

and

$$\overrightarrow{E} = -E_0\widehat{x}$$

Therefore,

$$v = \frac{E_0}{B_0}\hat{y}$$

;

$$\overrightarrow{E} = -E_0\widehat{x}$$

;

$$\overrightarrow{B} = B_0\hat{z}$$

Hence, option $D$ is correct.

## Question 038 MCQ

**QUESTION**

A block

$$M$$

hangs vertically at the bottom end of a uniform rope of constant mass per unit length. The top end of the rope is attached to fixed rigid support at

$$O.$$

A transverse wave pulse $Pulse1$ of wavelength

$$\lambda_0$$

is produced at point

$$O$$

on the rope. The pulse takes time

$$T_{OA}$$

to reach point

$$A.$$

If the wave pulse of wavelength

$$\lambda_0$$

is produced at point

$$A$$

$Pulse2$ without disturbing the position of

$$M$$

it takes time

$$T_{AO}$$

to reach point

$$O.$$

which of the following options is/are correct?

A The time

$$T_{AO} = T_{OA}$$

B The velocities of the two pulses $Pulse1 and Pulse2$ are the same at the midpoint of rope

C The wavelength of Pulse 1 becomes longer when it reaches point

$$A$$

D The velocity of any pulse along the rope is independent of its frequency and wavelength

**CORRECT OPTION**

A The time

$$T_{AO} = T_{OA}$$

**SOURCE**

Physics • waves

**EXPLANATION**

The velocity of the transverse wave is given by

$$v = \sqrt{\frac{T}{\mu}}$$

..... $i$

Where T is the tension in the rope and is given by

T = Mg +

$$\mu$$

yg

Where y = distance of point P on the rope from its bottom

L = total length of the rope

Put the value of T in eqn. $i$, we get

$$v = \sqrt{\frac{Mg + \mu yg}{\mu}}$$

..... $ii$

Also, speed of pulse 1 and pulse 2 are same at each point because speed is a characteristic of the medium. Hence, speed of the two pulses are same at the mid point $x = l/2$. However, since pulse 1 is moving in +x direction and pulse 2 is moving in −x direction, their velocities $whichisavectorquantity$ at the mid point are not same.

So, option $b$ is incorrect.

Since both the pulse travel in the same medium and velocity is medium dependent. The velocity of any pulse along the rope depends on tension in the rope and linear mass density so it is independent of its frequency $characteristicofthesourceofpulsegeneration$ and wavelength.

So, option $d$ is correct.

Also, v =

$$\nu$$

$$\lambda$$

or

$$\lambda$$

$$\propto$$

v

$\$\$ \because \$\$ frequencyofawavedependsonthesource$

For pulse 1, the velocity decreases at point A as tension decreases. So the wavelength of pulse 1 becomes shorter when it reaches point A.

So, option $c$ is incorrect.

From eqn. $ii$, the velocity is given as

$$v = \sqrt{\frac{Mg + \mu yg}{\mu}}$$

or

$$\frac{dy}{dt} = \sqrt{\frac{Mg + \mu yg}{\mu}} \Rightarrow dt = \frac{dy}{\sqrt{\frac{Mg + \mu yg}{\mu}}}$$

Integrating both sides,

$$\int_0^T dt = \int_0^L \frac{dy}{\sqrt{\frac{Mg}{\mu} + yg}}$$

;

$$\therefore$$

$$T = \frac{2}{g}\left(\sqrt{\frac{Mg}{\mu} + Lg} - \sqrt{\frac{Mg}{\mu}}\right)$$

Where T is time taken by pulse to travel distance L.

Since T depends upon g, M, L and

$$\mu$$

which are constant.

$$\therefore$$

Time taken by pulse 1 to reach point A and time taken by pulse 2 to reach point O is same.

$$\therefore$$

$T_{AO}$ = $T_{OA}$; So option $a$ is correct.

## Question 039 MCQ

**QUESTION**

A flat plate is moving normal to its plane through a gas under the reaction of a constant force

$$F.$$

The gas is kept at a very low pressure. The speed of the plate

$$v$$

is much less than the average speed u of the gas molecules.

Which of the following options is /are true?

**A** The pressure difference between the leading and trailing faces of the plate is proportional to uv

**B** The resistive force experimenced by the plate is proportional to

$$v$$

**C** The plate will continue to move with constant non-zero acceleration, at all times

**D** At a later time the external force

$$F$$

balances the resistive force

**CORRECT OPTION**

**A** The pressure difference between the leading and trailing faces of the plate is proportional to uv

**SOURCE**

Physics • work-power-and-energy

**EXPLANATION**

It is given that the average speed of gas molecules is u; the speed of the plate is v; the force on the plate if F.

•

Just before the collision : The gas molecules are approaching the leading and trailing faces of the plate with speed u.

•

Just after the collision : The gas molecules bounce back with speed $u_1$ and $u_2$.

•

At trailing face

The speed before collision is u

—

v.

The speed after collision is $u_2$ + v.

From law of conservation of linear momentum, we have

$u_2$ + v = u

—

v

That is,

$u_2$ = u

—

2v ..... 1

and

Δ

$u_2 = 2u$

$$-$$

2v ..... $2$

$$\bullet$$

At leading face

The speed before collision is u + v.

The speed after collision is $u_1$

$$-$$

v.

From law of conservation of linear momentum, we have

$u_1$

$$-$$

v = u + v

That is,

$u_1 = u + 2v$ ..... $3$

and

$$\Delta$$

$u_1 = 2u + 2v$ ..... $4$

Now, from Newton's second law of motion, we have

$$F = \frac{dp}{dt}$$

$$\bullet$$

For leading face :

$$F_1 = \frac{dp_1}{dt}$$

.

The volume swept by the plate is A$u+v$. Therefore,

$$F_1 = \frac{dp_1}{dt} = \rho A(u+v)\Delta u_1$$

$$F_1 = \rho A(u+v)(2u+2v)$$

..... 5

•

For trailing face :

$$F_2 = \frac{dp_2}{dt}$$

The volume swept by the plate is A$u\$\$ - \$\$v$. Therefore,

$$F_2 = \frac{dp_2}{dt} = \rho A(u-v)\Delta u_2$$

$$F_2 = \rho A(u-v)(2u-2v)$$

..... 6

The net force on the plate is

$$F_{net} = F_1 - F_2$$

$$= \rho A(u+v)(2u+2v) - \rho A(u-v)(2u-2v)$$

$$= 2\rho A(u+v)(u+v) - 2\rho A(u-v)(u-v)$$

$$= 2\rho A(u+v)^2 - 2\rho A(u-v)^2$$

$$= 2\rho A[(u+v)^2 - (u-v)^2]$$

$$= 2\rho A[u^2+v^2+2uv - (u^2+v^2-2uv)]$$

$$= 2\rho A(u^2+v^2+2uv - u^2 - v^2 + 2uv)$$

$$= 2\rho A(4uv)$$

$$= 8\rho Auv$$

Therefore, the pressure difference is

$$\frac{F_{net}}{A} = \frac{8\rho Auv}{A} = 8\rho uv$$

Thus, the pressure difference between the leading and trailing faces of the plate is proportional to uv. Hence, option $A$ is correct.

Now,

$$F_{net} = 8\rho Auv$$

The net resistive force is

$$F - (8\rho Au)v = \frac{mdv}{dt}$$

where F is the constant force applied by the gas molecules on the plate.

The resistive force experienced by the plate is proportional to v.

Hence, option $B$ is also correct.

At a later time, the velocity v becomes sufficient and the external force

$$F(= 8\rho Auv)$$

balances the resistive force.

Hence, option $D$ is also correct.

## Question 040 MCQ

**QUESTION**

A human body has a surface area of approximately

$$1\,m^2.$$

The normal body temperature is

$$10$$

$$K$$

above the surrounding room temperature

$$T_0.$$

Take the room temperature to be

$$T_0 = 300K.$$

For

$$T_0 = 300\,K,$$

the value of

$$\sigma T_0^4 = 460\,Wm^{-2}$$

$where\$\$\sigma\$\$istheStefan - Boltzmannconstant.$

Which of the following options is / are correct?

A The amount of energy radiated by the body in

$$1$$

second is close to

$$60$$

joules

B If the surrounding temperature reduces by a small amount

$$\Delta T_0 << T_0,$$

then to maintain the same body temperature the same $living$ human being needs to radiate

$$\Delta W = 4\sigma T_0^3 \Delta T_0$$

more energy per unit time

C Reducing the exposed surface area of the body $e.g.bycurlingup$ allows human to maintain the same body temperature while reducing the energy

lost by radiation

**D** If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

**CORRECT OPTION**

**C** Reducing the exposed surface area of the body $e.g. bycurlingup$ allows human to maintain the same body temperature while reducing the energy lost by radiation

**SOURCE**

Physics • properties-of-matter

**EXPLANATION**

The rate of radiation energy emitted by a body of surface area A, emissivity e, and kept at an absolute temperature T is given by Stefan's law

$dQ_e/dt =$

$\sigma$

$eAT^4,$

where

$\sigma$

is Stefan's constant. If this body is kept at a surrounding temperature $T_0$ then rate of radiation energy absorbed by the body is

$dQ_a/dt =$

$\sigma$

$eAT$

$4$
$0$

.

Thus, the net rate of energy loss by the body is

$$dQ_i/dt = dQ_e/dt - dQ_a/dt = \sigma eA(T^4 - T_0^4)$$

.

If surrounding temperature is reduced by

$$\Delta$$

$T_0$ (<< $T_0$) then net rate of energy lost by the body becomes

$$\frac{dQ_f}{dt} = \sigma eA\left[T^4 - (T_0 - \Delta T_0)^4\right]$$

$$= \sigma eA\left[T^4 - T_0^4\left(1 - \frac{\Delta T_0}{T_0}\right)^4\right]$$

$$\approx \sigma eA\left[T^4 - T_0^4\left(1 - 4\frac{\Delta T_0}{T_0}\right)\right]$$

$$= \frac{dQ_i}{dt} + 4\sigma eAT_0^3\Delta T_0$$

.

Thus, decrease in the surrounding temperature increases the rate of energy loss by

$$4\sigma eAT_0^3\Delta T_0$$

. To maintain the body temperature, the human being needs to increase the rate of energy generation $through chemical reactions in the body$ by

$$4\sigma eAT_0^3\Delta T_0$$

to balance the increase in rate of energy loss.

The human beings can also maintain the body temperature $without increasing the rate of energy generation$ by reducing the surface area $e.g., by curling up$ of their body. If surrounding temperature is reduced by

$$\Delta$$

$T_0$ and surface area of the body is reduced by

$$\Delta$$

A $<< A$ then net rate of energy loss by the body becomes

$$\frac{dQ_f}{dt} = 4\sigma e(A - \Delta A)\left[T^4 - (T_0 - \Delta T_0)^4\right]$$

$$\approx \frac{dQ_i}{dt} + 4\sigma e A T_0^3 \Delta T_0 - \sigma e \Delta A (T^4 - \Delta T_0^4)$$

.

Thus, the rate of energy loss by radiation remains same

$$(dQ_f/dt = dQ_i/dt)$$

if reduction in surface area is

$$\Delta A = \frac{4AT_0^3\Delta T_0}{T^4 - T_0^4} \approx A\frac{\Delta T_0}{\Delta T}$$

,

where, the body temperature is

$$T = T_0 + \Delta T$$

.

We assume body to be black i.e., e = 1. The amount of energy emitted by the body in one second is

$$dQ_e/dt = \sigma e A T^4 = \sigma e A (T_0 + \Delta T)^4$$

$$\approx \sigma e A T_0^4 (1 + 4\Delta T/T_0)$$

$$= (1)(1)(460)(1 + 4(10)/300) = 521$$

J.

The amount of energy lost by the body in one second is

$$dQ_i/dt = \sigma e A (T^4 - T_0^4) = \sigma e A (T_0 + \Delta T)^4 - T_0^4)$$

$$\approx eA\sigma T_0^4(4\Delta T/T_0) = 61$$

J.

## Question 041 MCQ

**QUESTION**

A circular insulated copper wire loop is twisted to form two loops of area

$$A$$

and

$$2A$$

as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane $ofthepaper$. A uniform magnetic field

$$\overrightarrow{B}$$

points into the plane of the paper. At

$$t = 0,$$

the loop starts rotating about the common diameter as axis with a constant angular velocity

$$\omega$$

in the magnetic field.

Which of the following options is/are correct?

**A** The emf induced in the loop is proportional to the sum of the areas of the two loops

B The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude if maximum emf induced in the smaller loop alone

C The net emf induced due to both the loops is proportional to

$$\cos \omega t$$

D The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

**CORRECT OPTION**

B The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude if maximum emf induced in the smaller loop alone

**SOURCE**

Physics • electromagnetic-induction

**EXPLANATION**

Consider the coordinate axes shown in the figure. The magnetic field is into the paper, i.e.,

$$\overrightarrow{B} = B\widehat{k}$$

. Let loop 1 be the loop of area A and loop 2 be the loop of area 2A. At t = 0, both loops are in the plane of the paper. They start rotating about the x-axis with a constant angular velocity

$$\overrightarrow{\omega} = \omega\hat{i}$$

. Let the area vectors of loop 1 and 2 be

$$\overrightarrow{A_1}$$

and

$$\overrightarrow{A_2}$$

, respectively. At t = 0,

$$\overrightarrow{A_1}$$

=

$$A\widehat{k}$$

and

$$\overrightarrow{A_2} = 2A\widehat{k}$$

$we chose area into the plane of the paper as positive$. In time t, the loops rotate by an angle

$$\theta$$

=

$$\omega$$

t. Their area vectors make an angle

$$\theta$$

=

$$\omega$$

t with z-axis $direction of the magnetic field$. Thus, the magnetic fluxes passing through the loop 1 and the loop 2 at time t are given by

$$\phi$$

$_1$ =

$$\overrightarrow{B}.\overrightarrow{A_1}$$

= B $A_1$ cos

$$\theta$$

= B A cos

$$\omega$$

t,

$$\phi$$

2 =

$$\vec{B}.\vec{A_2}$$

= B $A_2$ cos

$$\theta$$

= 2B A cos

$$\omega$$

t.

By Faraday's law of electromagnetic induction, induced emfs in the loop 1 and in the loop 2 are given by

$$\varepsilon_1 = -\frac{d\phi_1}{dt} = B\,A\omega \sin \omega t$$

,

$$\varepsilon_2 = -\frac{d\phi_2}{dt} = 2B\,A\omega \sin \omega t$$

.

The rate of change of flux is maximum when

$$\omega$$

t =

$$\pi$$

/2 i.e., when the plane of the loops is perpendicular to the plane to the paper.

Let T = 2

$$\pi$$

/

$$\omega$$

be the time period to complete one revolution. The fluxes

$$\phi$$

$_1$ and

$$\phi$$

$_2$ decrease with time if t

$$\in$$

$0, T/4$. By Lenz's law, the induced current should oppose reductions in

$$\phi$$

$_1$ and

$$\phi$$

$_2$. Thus, induced current in both the loops should be clockwise. Hence, the polarity of 'equivalent batteries' are as shown in the figure. These batteries are connected with reverse polarity. Thus, net emf induced due to both the loop is

$$\varepsilon = \varepsilon_2 - \varepsilon_1 = B\,A\omega \sin \omega t$$

.

Note that the net emf

$$\varepsilon$$

is proportional to the difference in areas of two loops and

$$|\varepsilon| = |\varepsilon_1|$$

.

## Question 042 MCQ

**QUESTION**

In the circuit shown,

$$L = 1\,\mu H, C = 1\,\mu F$$

and

$$R = 1\,k\Omega.$$

They are connected in series with an a.c. source

$$V = V_0 \sin \omega t$$

as shown.

Which of the following options is/are correct?

**A** The current will be in phase with the voltage if

$$\omega = 10^4$$

$$rad.\, s^{-1}$$

**B** The frequency at which the current will be in phase with the voltage is independent of

$$R$$

**C** At

$$\omega \sim 0$$

the current flowing through the circuit becomes nearly zero

**D** At

$$\omega >> 10^6 rad.\, s^{-1},$$

the circuit behaves like a capacitor

**CORRECT OPTION**

B The frequency at which the current will be in phase with the voltage is independent of

$$R$$

**SOURCE**

Physics • alternating-current

**EXPLANATION**

Given V = $V_0$ sin

$$\omega$$

t

At resonant frequency, current and voltage will be in same phase i.e.,

$$\omega L = \frac{1}{\omega C}$$

$$\therefore$$

Resonant frequency

$$\omega = \frac{1}{\sqrt{LC}}$$

..... $i$

Here, L = 1

$$\mu$$

H = 10

$$-$$

$^{6}$ H, C = 10

$$-$$

$^{6}$ F

$\therefore$

$$\omega = \frac{1}{\sqrt{10^{-12}}} = 10^6$$

rad s

$-$

1

So, option $a$ is incorrect.

Since,

$$X_C = \frac{1}{\omega C},\ X_L = \omega L$$

..... $ii$

For large

$\omega$

, $X_C$

$\sim$

0, so circuit will behave like a inductive circuit.

So, option $d$ is incorrect.

From Eqn. $i$

$$\omega = \frac{1}{\sqrt{LC}}$$

At resonant frequency or the frequency at which the current will be in phase with the voltage is independent of R.

$\therefore$

option $b$ is correct.

From Eqn. $ii$, at

$\omega$

$\sim$

0, $X_C$

$$\sim$$

$$\infty$$

and $X_L = 0$

$$\therefore$$

The current flowing through the circuit becomes nearly zero.

So, option $c$ is correct.

## Question 043 MCQ

**QUESTION**

For an isosceles prism of angle

$$A$$

and refractive index

$$\mu$$

, it is found that the angle of minimum deviation

$$\delta_m = A.$$

Which of the following options is/are correct?

A

For the angle of incidence

$$i_1 = A,$$

the ray inside the prism is parallel to the base of the prism

For this prism, the refractive index

$\mu$

and the angle of prism

B

$A$

are related as

$$A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$$

At minimum deviation, the incident angle

$i_1$

and the refracting angle

C

$r_1$

at the first refracting surface are related by

$$r_1 = (i_1/2)$$

For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

D

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

**CORRECT OPTION**

At minimum deviation, the incident angle

$i_1$

and the refracting angle

C

$r_1$

at the first refracting surface are related by

$$r_1 = (i_1/2)$$

**SOURCE**

Physics • geometrical-optics

**EXPLANATION**

We discuss the options as follows:

•

For option $A$ : For the angle of incidence $i_1$ = A, we have minimum deviation and the ray inside the prism is parallel to the base of the prism.

Hence, option $A$ is correct.

•

For option $C$ : The angle of deviation is given as

$\delta$

$= i_1 + i_2$

—

A

At minimum deviation, $i_1 = i_2$ and $r_1 = r_2$. Therefore,

$i_1$ = A ..... 1

Also, we know that

$r_1 + r_2$ = A

As $r_1 = r_2$, we get

$2r_1$ = A

$\Rightarrow$

$r_1$ =

$$\frac{A}{2}$$

From Eq. $1$, we get

$$r_1 = \frac{i_1}{2}$$

Hence, option $C$ is correct.

According to Snell's law, we have

$$\frac{\sin i_2}{\sin r_2} = \mu$$

For emergent ray to be tangential to the surface, we have

$i_2$ = 90

$$\circ$$

$$\Rightarrow$$

sin90

$$\circ$$

=

$$\mu$$

sin$r_2$

$$\Rightarrow$$

$$\mu$$

sin$r_2$ = 1

sin$r_2$ =

$$\frac{1}{\mu}$$

$$\Rightarrow$$

$r_2$ = sin

$$-$$

1

$$\left(\frac{1}{\mu}\right)$$

..... $2$

From the relation $r_1 + r_2 = A$, we get

$r_1 = A$

$-$

$r_2$

$\Rightarrow$

$\sin r_1 = \sin(A$

$-$

$r_2)$

Using $\sin a\$\$ - \$\$b$ = sinacosb

$-$

sinbcosa, we have

$\Rightarrow$

$\sin r_1 = \sin A \cos r_2$

$-$

$\sin r_2 \cos A$ ..... $3$

Using Eq. $2$, we have

$\sin r_2 =$

$$\frac{1}{\mu}$$

Squaring it, we get

$\sin^2 r_2 =$

$$\frac{1}{\mu^2}$$

Using the relation $\sin^2 x + \cos^2 x = 1$, we get

$\sin^2 x = 1$

$-$

$\cos^2 x$

$\Rightarrow$

1

$-$

$\cos^2 r_2 =$

$$\frac{1}{\mu^2}$$

$\Rightarrow$

$\cos^2 r_2 = 1$

$-$

$$\frac{1}{\mu^2}$$

That is,

$\cos r_2 =$

$$\sqrt{1 - \frac{1}{\mu^2}}$$

Substituting the values of $\sin r_2$ and $\cos r_2$ in Eq. $3$, we get

$\sin r_1 = \sin A$

$$\sqrt{1 - \frac{1}{\mu^2}}$$

$-$

$$\frac{1}{\mu}$$

cos A

According to Snell's law, we have

$$\frac{\sin i_1}{\sin r_1} = \mu$$

That is,

$$\sin i_1 = \mu \sin r_1 = \mu \left( \sin A \sqrt{1 - \frac{1}{\mu^2}} - \frac{1}{\mu} \cos A \right)$$

$$\sin i_1 = \mu \left( \sin A \frac{\sqrt{\mu^2 - 1}}{\mu} - \frac{\cos A}{\mu} \right)$$

$$\sin i_1 = \sin A \sqrt{\mu^2 - 1} - \cos A$$

..... 4

Now, at minimum deviation, we have A = $i_1$ and $r_1$ =

$$\frac{A}{2}$$

. Thus, from Snell's law, we get

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin(A/2)}$$

Using

$$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$$

, we get

$$\mu = \frac{2 \sin A/2 \cos A/2}{\sin A/2} \Rightarrow \mu = 2 \cos \frac{A}{2}$$

..... 5

Now, Eq. $4$ becomes

$$\sin i_1 = \sin A \sqrt{\left(2\cos\frac{A}{2}\right)^2 - 1} - \cos A$$

$$= \sin A \sqrt{4\cos^2\frac{A}{2} - 1} - \cos A$$

$$i_1 = \sin^{-1}\left[\sin A \sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

Hence, option $D$ is correct.

•

For option $B$ : From Eq. $5$, we have

$$\mu = 2\cos\frac{A}{2}$$

$$\cos\frac{A}{2} = \frac{\mu}{2}$$

$$\Rightarrow \frac{A}{2} = \cos^{-1}\left(\frac{\mu}{2}\right)$$

$$\Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$$

Hence, option $B$ is incorrect.

## Question 044 Numerical

**QUESTION**

An electron in a hydrogen atom undergoes a transition from an orbit with quantum number

$$n_i$$

to another with quantum number

$$n_f$$

.

$$V_i$$

and

$$V_f$$

are respectively the initial and final potential energies of the electron. If

$$\frac{V_i}{V_f} = 6.25$$

, then the smallest possible

$$n_f$$

is

**SOURCE**

Physics • atoms-and-nuclei

**EXPLANATION**

The energy of a hydrogen atom with an electron in $n^{th}$ orbit is given by

$$E_n = -\frac{13.6}{n^2} eV$$

.

The potential energy in $n^{th}$ orbit is related to $E_n$ by $V_n = E_n/2$. Thus, potential energies in orbits with quantum numbers $n_i$ and $n_f$ are given by

$$V_i = -\frac{13.6}{2n_i^2} eV$$

, and

$$V_f = -\frac{13.6}{2n_f^2} eV$$

, which gives

$$\frac{V_i}{V_f} = \frac{n_f^2}{n_i^2} = 6.25$$

$given.$

Take square root to get $n_f = 2.5n_i$. Since $n_i$ and $n_f$ are positive integers, smallest possible integral value of $n_f$ is 5 for $n_i = 2$.

## Question 045 Numerical

**QUESTION**

A drop of liquid of radius

$$R = 10^{-2}\, m$$

having surface tension

$$S = \frac{0.1}{4\pi} Nm^{-1}$$

divides itself into

$$K$$

identical drops. In this process the total change in the surface energy

$$\Delta U = 10^{-3}\, J.$$

If

$$K = 10^{\alpha}$$

then the value of

$$\alpha$$

is

**SOURCE**

Physics • properties-of-matter

**EXPLANATION**

Let a spherical drop of liquid of radius R divides itself into K spherical drops, each of radius r. The density of liquid does not change in this process. Hence, conservation of mass gives

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 K$$

, i.e.,

$$r = RK^{-1/3}$$

. ..... $1$

The surface energy of a drop of radius R is

$$U_i = 4\pi R^2 S$$

and total surface energy of K drops of radius r is

$$U_f = 4\pi r^2 K S$$

. Note that the surface energy increases when a bigger drop is divided into multiple smaller drops. Total change in surface energy is

$$\Delta U = U_f - U_i = 4\pi S(Kr^2 - R^2)$$

..... $2$

Eliminate r from equations $1$ and $2$ to get

$$K = \left(\frac{\Delta U}{4\pi S R^2} + 1\right)^3$$

.

Substitute

$$\Delta$$

U = 10

$$-$$

$^{3}$ J, S =

$$\frac{0.1}{4\pi}$$

N/m and R = 10

$$-$$

$^{2}$ m to get

$$K = (100 + 1)^3 \approx 10^6$$

.

## Question 046 Numerical

**QUESTION**

A monochromatic light is travelling in a medium of refractive index

$$n = 1.6.$$

It enters a stack of glass layers from the bottom side at an angle

$$\theta = 30^\circ.$$

The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as

$$n_m = n - m\Delta n,$$

where

$$n_m$$

is the refractive index of the

$$m^{th}$$

slab and

$$\Delta n = 0.1$$

$see the figure$. The ray is refracted out parallel to the interface between the

$$(m-1)^{th}$$

and

$$m^{th}$$

slabs from the right side of the stack. What is the value of

$$m$$

?

**SOURCE**

Physics • geometrical-optics

**EXPLANATION**

Applying Snell's law, we have nsin

$$\theta$$

= $n$$$ − $$$m$$$\Delta$$$n$sin90

$$\circ$$

it is given that n = 1.6,

$$\theta$$

= 30

$$\circ$$

,

$$\Delta$$

n = 0.1. The above equation becomes

1.6sin30 =

$$1.6\$\$ - \$\$m(0.1)$$

sin90

1.6

$$\times$$

$$\frac{1}{2}$$

= $$1.6\$\$ - \$\$0.1m$$

$$\times$$

1

0.8 = $$1.6\$\$ - \$\$0.1m$$

0.1m = 1.6

$$-$$

0.8

0.1m = 0.8

m = 8

## Question 047 Numerical

**QUESTION**

A stationary source emits sound of frequency

$$f_0 = 492\,Hz.$$

The sound is reflected by a large car approaching the source with a speed of

$$2\,ms^{-1}.$$

The reflected signal is received by the source and superposed with the original.

What will be the beat frequency of the resulting signal in

$$Hz$$

?
$Given that the speed of sound in air is $$330\,ms^{-1}$$ and the car reflects the sou$
.

**SOURCE**

Physics • waves

**EXPLANATION**

It is given that the source emits sound of frequency $f_0$ = 492 Hz. The car is approaching the source and the speed of car is v = 2 m/s.

Also, the speed of sound in air, $v_s$ = 330 m/s.

The frequency of sound received by car is given as

$$f_1 = \left(\frac{V_s + V}{V_s}\right) f_0 = \left(\frac{330 + 2}{330}\right) 492$$

Here, $f_1$ = 494.98 Hz, which is the frequency reflected by the car towards the source.

Therefore, now, the car acts as the source. The frequency of sound received by the source is

$$f_2 = \left(\frac{V_s}{V_s - v}\right) f_1 = \left(\frac{330}{330 - 2}\right) 494.98$$

Here, $f_2$ = 498 Hz. Therefore, the beat frequency of the resulting signal is

$$|f_0 - f_2| = |492 - 498| = 6$$

Hz

## Question 048 Numerical

**QUESTION**

$$^{131}\mathrm{I}$$

is an isotope of Iodine that

$$B$$

decays to an isotope of Xenon with a half-life of

$$8$$

days. A small amount of a serum labelled with

$$^{131}\mathrm{I}$$

is injected into the blood of a person. The activity of the amount of

$$^{131}\mathrm{I}$$

injected was

$$2.4 \times 10^5$$

Becquerel

$$(Bq).$$

It is known that the injected serum will get distributed uniformly in the blood stream in less than half an hour. After

$$11.5$$

hours,

$$2.5$$

ml of blood is drawn from person's body, and gives an activity of

$$115$$

$$Bq$$

. The total volume of blood in the person's body, in liters is approximately $$you may use $$e^x \approx 1 + x$$ $$for $$ |x| << 1 $$ and $$ \ln 2 \approx 0.7 .$$

**SOURCE**

Physics • atoms-and-nuclei

**EXPLANATION**

The decay rate is related to half life by

$$\lambda = \frac{\ln 2}{t_{1/2}}$$

. A sample having $N_0$ radioactive nuclei at time t = 0 will have

$$N = N_0 e^{-\lambda t}$$

radioactive nuclei at time t. Thus, activity $A = $$\lambda$$ $$N$ of a sample reduces from its initial value $A_0$ to a value

$$A = A_0 e^{-\lambda t}$$

at time t.

Let V be the total volume of blood and N be the total number of radioactive nuclei at time t. The nuclei are distributed uniformly in blood. A sample of volume v will have n = $v/V$ N radioactive nuclei in it. Thus, activity of this sample is

$$a = \lambda n = \lambda \left( \frac{v}{V} N \right) = \frac{v}{V} (\lambda N) = \frac{v}{V} A = \frac{v}{V} A_0 e^{-\lambda t}$$

.

Substitute a = 115 Bq, v = 2.5 ml, $A_0$ = 2.4

$$\times$$

$10^5$ Bq,

$$\lambda$$

= ln 2/$t_{1/2}$

$$\approx$$

0.7/8$$ \times $$24 hr

$$-$$

$^{1}$ and t = 11.5 hr to get

$$V = \frac{(2.5)(2.4 \times 10^5)}{115} e^{-\frac{0.7 \times 11.5}{8 \times 24}}$$

$$\approx \frac{(2.5)(2.4 \times 10^5)}{115} \left(1 - \frac{0.7 \times 11.5}{8 \times 24}\right)$$

= 4998 ml

$$\approx$$

5 litre.

## Question 049 MCQ

**QUESTION**

In which case will the particle describe a helical path with axis along the positive

$$z$$

direction?

A $(\mathrm{IV})\ (i)\ (S)$

B $(\mathrm{II})\ (ii)\ (R)$

C $(\mathrm{III})\,(iii)\,(P)$

D $(\mathrm{IV})\,(ii)\,(R)$

**CORRECT OPTION**

A $(\mathrm{IV})\,(i)\,(S)$

**SOURCE**

Physics • magnetism

**EXPLANATION**

The particle will describe a helical path with axis along +z direction if $i$ component of net force in x-y plane provides the centripetal acceleration and $ii$ the particle has a non-zero velocity and/or non-zero force along +z direction. A proton with initial velocity

$$\overrightarrow{v} = 2\frac{E_0}{B_0}\widehat{x}$$

will move in a helical path with axis along +z direction when placed in an electric field

$$\overrightarrow{E} = E_0\hat{z}$$

and magnetic field

$$\overrightarrow{B} = B_0\hat{z}$$

.

## Question 050 MCQ

**QUESTION**

In which case would the particle move in a straight line along the negative direction of

$$y$$

-axis $i.e., movealong\$\$ - \hat{y}\$\$$?

A $(\mathrm{II})\ (iii)\ (Q)$

B $(\mathrm{III})\ (ii)\ (R)$

C $(IV)\ (ii)\ (S)$

D $(\mathrm{III})\ (ii)\ (P)$

**CORRECT OPTION**

B $(\mathrm{III})\ (ii)\ (R)$

**SOURCE**

Physics • magnetism

**EXPLANATION**

Lorentz force on a charged particle having a charge q, moving with a velocity

$$\overrightarrow{v}$$

in a region of uniform electric field

$$\overrightarrow{E}$$

and uniform magnetic field

$$\overrightarrow{B}$$

, is given by

$$\overrightarrow{F_{net}} = \overrightarrow{F_E} + \overrightarrow{F_B} = q\overrightarrow{E} + q\overrightarrow{v} \times \overrightarrow{B}$$

. ..... 1

The particle will move along

$$-\hat{y}$$

axis if $i$ initial velocity

$$\overrightarrow{v}$$

is along

$$-\hat{y}$$

and net force is either zero or directed along

$$-\hat{y}$$

$ii$ initial velocity

$$\overrightarrow{v}$$

is zero and net force is directed along

$$-\hat{y}$$

. A proton with initial velocity

$$\overrightarrow{v} = \overrightarrow{0}$$

placed in an electric field

$$\vec{E} = -E_0\hat{y}$$

and magnetic field

$$\vec{B} = -B_0\hat{y}$$

will experience a net force

$$\vec{F}_{net} = -qE_0\hat{y}$$

. It moves in a straight line along

$$-\hat{y}$$

.

## Question 051 MCQ

**QUESTION**

Which one of the following options is the correct combination?

**A** $(\mathrm{IV})\ (ii)\ (S)$

**B** $(\mathrm{III})\ (ii)\ (S)$

**C** $(\mathrm{II})\ (iv)\ (P)$

**D** $(\mathrm{II})\ (iv)\ (R)$

**CORRECT OPTION**

B $(\mathrm{III})\ (ii)\ (S)$

**SOURCE**

Physics • heat-and-thermodynamics

**EXPLANATION**

$\because$

dW = PdV

and there is no change in volume in isochoric process.

$\therefore$

dW = 0

## Question 052 MCQ

**QUESTION**

Which of the following options is the only correct representation of a process in which

$$\Delta U = \Delta Q - P\Delta V$$

?

A $(\mathrm{II})\ (iv)\ (R)$

B $(\mathrm{III})\ (iii)\ (P)$

C $(\mathrm{II})\ (iii)\ (S)$

D $(\mathrm{II})\ (iii)\ (P)$

**CORRECT OPTION**

D $(\mathrm{II})\ (iii)\ (P)$

**SOURCE**

Physics • heat-and-thermodynamics

**EXPLANATION**

Compare the given relation,

$$\Delta$$

U =

$$\Delta$$

Q

$$-$$

p

$$\Delta$$

V, with the first law of thermodynamics,

$$\Delta$$

Q =

Δ

U +

Δ

W, to get work done by the gas as

Δ

W = p

Δ

V. This is the work done in an isobaric process. The p-V diagram of this process is shown in $P$. The work done in this process is

$$W_{1\to 2} = \int pdV = pV_1 - pV_2$$

.

## Question 053 MCQ

**QUESTION**

Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

**A** $(\mathrm{I})\ (ii)\ (Q)$

**B** $(\mathrm{IV})\ (ii)\ (R)$

C $(\mathrm{III})\ (iv)\ (R)$

D $(\mathrm{I})\ (iv)\ (Q)$

**CORRECT OPTION**

D $(\mathrm{I})\ (iv)\ (Q)$

**SOURCE**

Physics • heat-and-thermodynamics

**EXPLANATION**

Adiabatic process is used as a correction in the determination of the speed of sound in an ideal gas. The p-V diagram of this process is shown in $Q$. The work done on the gas in an adiabatic process is given by

$$W_{1\to 2} = \frac{p_2V_2 - p_1V_1}{\gamma - 1}$$

.

## Question 054 MCQ

**QUESTION**

A block of mass

$$M$$

has a circular cut with a frictionless surface as shown. The block resets on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at

$$x = 0,$$

in a co-ordinate system fixed to the table. A point mass

$$m$$

is released from rest at the topmost point of the path as shown and it slides down.

When the mass loses contact with the block, its position is

$$x$$

and the velocity is

$$v.$$

At that instant, which of the following options is/are correct?

**A**

The position of the point mass

$$m$$

is :

$$x = -\sqrt{2}\frac{mR}{M+m}$$

**B**

The velocity of the point mass

$$m$$

is :

$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

C

The $x$ component of displacement of the center of mass of the block $M$ is:

$$-\frac{mR}{M+m}$$

D

The velocity of the block $M$ is:

$$V = -\frac{m}{M}\sqrt{2gR}$$

**CORRECT OPTION**

B

The velocity of the point mass $m$ is :

$$v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

**SOURCE**

Physics • rotational-motion

**EXPLANATION**

We discuss the options as follows:

•

For option $B$ : From law of conservation of momentum, we have

mv = MV ..... 1

where v is the velocity of point mass m and V is velocity of block of mass M.

Now, by using law of conservation of energy, after the point mass is released, we have the following:

Loss in P.E. of mass m = Gain in K.E. of both mass M and the mass m.

$$\Rightarrow mgh = \frac{1}{2}mv^2 + \frac{1}{2}MV^2$$

Substituting

$$V = \frac{mu}{M}$$

$$from Eq.\,(1)$$

, we get

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{mv}{M}\right)^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}M\frac{m^2v^2}{M^2}$$

$$mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{m^2v^2}{M}$$

$$mgR = \frac{1}{2}mv^2\left(1 + \frac{m}{M}\right)$$

$$gR = \frac{1}{2}v^2\left(1+\frac{m}{M}\right)$$

Rearranging this equation, we get

$$2gR = v^2\left(1+\frac{m}{M}\right)$$

$$v^2 = \frac{2gR}{\left(1+\frac{m}{M}\right)}$$

$$\Rightarrow v = \sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

Hence, option $B$ is correct.

•

For option $C$ : Now, from Eq. $1$, we have

$$V = \frac{m}{M}v = \frac{m}{M}\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}}$$

$$V = \sqrt{\frac{m^2 2gR}{M^2\left(1+\frac{m}{M}\right)}} = \sqrt{\frac{m^2 2gR}{M^2\left(\frac{M+m}{M}\right)}}$$

$$V = \sqrt{\frac{m^2 2gR}{M(m+M)}} \Rightarrow V = m\sqrt{\frac{2gR}{M(m+M)}}$$

Since, there is no external force acting on the system, the centre of mass does not change.

Now, if the change in position of mass m is

$$\Delta$$

x and the change in position of mass M is

$$\Delta$$

X, then

m

$$\Delta$$

x + M

$\Delta$

x = 0 $since centre of mass will not change$

Now, we know that the change in position of point mass m is

$\Delta$

x = R

$-$

x

and the change in position of block of mass M is

$\Delta$

X =

$-$

x

Therefore,

m $R\$\$ - \$\$x$

$-$

Mx = 0

m $R\$\$ - \$\$x$ = Mx

mR

$-$

mx = Mx

mR = Mx + mx

mR = $M + m$x

$\Rightarrow$

x =

$$\frac{mR}{M+m}$$

The change in position of block of mass M is

$$\Delta$$

x =

$$-$$

x =

$$\frac{-mR}{M+m}$$

Thus, x-component of the displacement of the centre of mass of the block is

$$M = \frac{-mR}{M+m}$$

Hence, option $C$ is also correct.