iit Jee 2012 Paper 2 Offline $60 Questions$

Question 001 MCQ

QUESTION

In the cyanide extraction process of silver from argentite ore, the oxidising and reducing agents used are

A. O_2 and CO respectively

B. O_2 and Zn dust respectively

C. HNO_3 and Zn dust respectively

D. HNO_3 and CO respectively

CORRECT OPTION

B. O_2 and Zn dust respectively

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Silver ore is oxidized using oxygen from the air, as shown in the reaction below :

$$4\mathrm{Ag} + 8\mathrm{NaCN} + 2\mathrm{H_2O} + \mathrm{O_2(air)} \longrightarrow 4\mathrm{NaAg(CN)_2} + 4\mathrm{NaOH}$$

Here, silver $Ag(0)isundergoingoxidationtoform \text{Ag}+1 $:

$$\mathrm{Ag}(0) \xrightarrow{\text{oxidation}} \mathrm{Ag}(+1)$$

Silver is then precipitated from the solution by adding zinc dust in a finely divided form :

$$2\mathrm{NaAg(CN)_2} + \mathrm{Zn} \longrightarrow \mathrm{Na_2Zn(CN)_4} + 2\mathrm{Ag}$$

In this reaction, $\mathrm{Ag}(+1)$ is reduced back to $\mathrm{Ag}(0)$:

$$\mathrm{Ag}(+1) \xrightarrow{\text{reduction}} \mathrm{Ag}(0)$$

In summary, oxygen from the air acts as the oxidizing agent, and zinc dust serves as the reducing agent.

Question 002 MCQ

QUESTION

The electrochemical cell shown below is a concentration cell. M | M^{2+} (saturated solution of a sparingly soluble salt, MX_2) || M^{2+} (0.001 mol dm^{-3}) | M The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

The value of ΔG (kJ mol^{-1}) for the given cell is (take 1F = 96500 C mol^{-1})

A –5.7

B 5.7

C 11.4

D -11.4

CORRECT OPTION

D -11.4

SOURCE

Chemistry • electrochemistry

EXPLANATION

The given electrochemical cell is a concentration cell where both the electrodes are of the same metal but immersed in solutions of different concentrations of the same metal ion. The emf E generated by this cell can be calculated using the Nernst equation, which for this cell at 298 K $25^{\circ}C$ is given by:

$$E = E^{\circ} - \frac{0.059}{n} \log \frac{[C_1]}{[C_2]}$$

Where:

- E° is the standard electrode potential which is zero in a concentration cell because both electrodes are same.
- n is the number of moles of electrons transferred in the redox reaction $2 in this case, as it involves M^{2+} ions$.
- $[C_1]$ and $[C_2]$ are the concentrations of M^{2+} at the two electrodes.
- E is given to be 0.059 V.

Assuming the more concentrated solution is at the left hand electrode and the less concentrated solution $0.001M$ is at the right hand electrode, the Nernst equation simplifies to:

$$E = -\frac{0.059}{2} \log \frac{0.001}{[C_1]}$$

To find $[C_1]$, we solve for the argument of log such that the calculated emf matches the given emf $0.059V$:

$$0.059 = -\frac{0.059}{2}\log\frac{0.001}{[C_1]}$$

$$-2 = \log\frac{0.001}{[C_1]}$$

$$10^{-2} = \frac{0.001}{[C_1]}$$

$$[C_1] = 0.1\ \mathrm{M}$$

This $[C_1]$ value supports the direction of the redox reactions assumed. Now, to find the change in Gibbs free energy ΔG for this cell, we use the relationship:

$$\Delta G = -nFE$$

But E should be positive for the spontaneous reaction, hence:

$$\Delta G = -2 \times 96500 \times 0.059\,\mathrm{V} = -11381\,\mathrm{J/mol} = -11.381\,\mathrm{kJ/mol}$$

Therefore, ΔG for the cell is approximately -11.4 kJ/mol. The correct answer is Option D: -11.4.

Question 003 MCQ

QUESTION

The electrochemical cell shown below is a concentration cell. M | M^{2+} (saturated solution of a sparingly soluble salt, MX_2) || M^{2+} (0.001 mol dm^{-3}) | M The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059 V.

The solubility product (K_{sp}; $mol^3\ dm^{-9}$) of MX_2 at 298 K based on the information available for the given concentration cell is

$take 2.303\$\$ \times \$\$R\$\$ \times \$\$298/F = 0.059V$

A

1

×

10^{-15}

B

4

×

10^{-15}

C

1

×

10^{-12}

D

4

×

10^{-12}

CORRECT OPTION

B

4

×

10^{-15}

SOURCE

Chemistry • electrochemistry

EXPLANATION

To find the solubility product (K_{sp}) for the sparingly soluble salt, MX_2, in a concentration cell setup, we first analyze how the emf is related to the concentration differences across the cell. The reaction at each electrode involves the metal ion M^{2+} and the metal M, whereas the net cell reaction has no change in number of moles on both sides of the equation due to the symmetry of the cell. Thus, the notation for the cell reaction is:

$$M(s) \longleftrightarrow M^{2+}(aq) + 2e^-$$

Given that it's a concentration cell, the emf generated is due to the concentration difference of the M^{2+} ions at the two electrodes. The emf of the cell can be calculated by the Nernst equation:

$$E = E^\circ - \frac{RT}{nF}\ln\left(\frac{[M^{2+}]_{\text{cathode}}}{[M^{2+}]_{\text{anode}}}\right)$$

Since it's a concentration cell, $E^\circ = 0$. At 298 K, substituting from the provided conversion $2.303\$ \times \$R\$ \times \$298/F = 0.059V$ and given n = 2 $because two electrons are transferred per metal ion$:

$$E = -\frac{(0.059)}{2}\log\left(\frac{[M^{2+}]_{saturated}}{[M^{2+}]_{0.001\text{ M}}}\right)$$

Given $E = 0.059$ V, we can solve for the concentration of M^{2+} in the saturated solution:

$$0.059 = -0.0295\log\left(\frac{[M^{2+}]_{saturated}}{0.001}\right)$$

$$\log\left(\frac{[M^{2+}]_{saturated}}{0.001}\right) = -2$$

$$[M^{2+}]_{saturated} = 0.001 \times 10^{-2}$$

$$[M^{2+}]_{saturated} = 0.00001\text{ M}$$

The solubility product, K_{sp}, of MX_2 can now be computed. Let s be the solubility of MX_2 in mol/L. The dissolution of MX_2 is given by:

$$MX_2 \rightleftharpoons M^{2+} + 2X^-$$

$$s \qquad s \qquad 2s$$

The solubility product expression is:

$$K_{sp} = [M^{2+}][X^-]^2 = (s)(2s)^2$$

$$K_{sp} = 4s^3$$

Since $s = [M^{2+}]_{saturated} = 0.00001$ M,

$$K_{sp} = 4(0.00001)^3$$

$$K_{sp} = 4 \times 10^{-15}$$

Thus, the solubility product of MX_2 at 298 K is 4×10^{-15} mol^3 dm^{-9}, which corresponds to Option B.

Question 004 MCQ

QUESTION

For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water. the elevation in boiling point at 1 atm pressure is 2^oC. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure $mmofHg$ of the solution is (take K_b = 0.76 K Kg mol^{-1})

A 724

B 780

C 736

D 718

CORRECT OPTION

A 724

SOURCE

Chemistry • solutions

EXPLANATION

Using the boiling point elevation formula:

$$\Delta T_b = K_b \cdot m$$

,

where:

ΔT_b is the elevation in boiling point,

K_b is the ebullioscopic constant,

m is the molality.

Given:

$K_b = 0.76\,\mathrm{K\,kg\,mol^{-1}}$,

$\Delta T_b = 2^\circ\mathrm{C}$,

mass of solute $= 2.5\,\mathrm{g}$,

mass of solvent $water$ $= 100\,\mathrm{g} = 0.1\,\mathrm{kg}$.

First, find the molality m:

$$m = \frac{\Delta T_b}{K_b} = \frac{2}{0.76} = \frac{200}{76} \approx 2.63\,\mathrm{mol\,kg^{-1}}$$

.

Now calculate the moles of solute:

Given the molality m,

$$m = \frac{\text{moles of solute}}{\text{mass of solvent in kg}}$$

.

Thus,

$$\text{moles of solute} = m \times \text{mass of solvent in kg} = 2.63 \times 0.1 = 0.263\,\text{mol}$$

.

Next, determine the molecular weight M of the solute:

$$M = \frac{\text{mass of solute}}{\text{moles of solute}} = \frac{2.5\,\text{g}}{0.263\,\text{mol}} \approx 9.51\,\text{g/mol}$$

.

Using Raoult's Law for the vapour pressure of the solution:

$$P_{\text{solution}} = P_0 \cdot (1 - \chi_{\text{solute}})$$

,

where:

P_0 is the vapour pressure of pure solvent,

χ_{solute} is the mole fraction of the solute.

Approximate the mole fraction of the solute:

$$\chi_{\text{solute}} = \frac{\text{moles of solute}}{\nu_{\text{solvent}} + \nu_{\text{solute}}} \approx \frac{0.263}{5.55 + 0.263} \approx \frac{0.263}{5.813} \approx 0.0452$$

,

where ν_{solvent} are the moles of water $approximately 5.55 mol for 100 g of water$.

Therefore, the vapour pressure of the solution:

$$P_{\text{solution}} = 760 \times (1 - 0.0452) \approx 760 \times 0.9548 \approx 724\,\text{mm Hg}$$

.

Thus, the vapour pressure of the solution is 724 mm of Hg, corresponding to Option A.

Question 005 MCQ

QUESTION

The major product H of the given reaction sequence is

$$CH_3 - CH_2 - CO - CH_3 \xrightarrow{\overline{C}N} G \xrightarrow[Heat]{95\%\ H_2SO_4} H$$

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction is

Question 006 MCQ

QUESTION

$$NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$$

exhibits temperature-dependent magnetic behaviour $paramagnetic/diamagnetic$. The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are, respectively,

A tetrahedral and tetrahedral.

B square planar and square planar.

C tetrahedral and square planar.

D square planar and tetrahedral.

CORRECT OPTION

C tetrahedral and square planar.

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The configuration of Ni = $3d^8\ 4s^2$ and that of Ni^{2+} = $3d^8$.

In paramagnetic state, the hybridisation is sp^3 and geometry is tetrahedral.

In diamagnetic state, the hybridisation is dsp^2 and geometry is square planar.

Question 007 MCQ

QUESTION

The reaction of white phosphorous with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type; the oxidation states of phosphorus in phosphine and the other product are, respectively,

A. redox reaction;

—

3 and

—

5

B. redox reaction; +3 and +5

C. disproportionation reaction;

—

3 and +1

D. disproportionation reaction;

—

3 and +3

CORRECT OPTION

disproportionation reaction;

C

$-$

3 and +1

SOURCE

Chemistry • p-block-elements

EXPLANATION

According to the reaction

P_4 + NaOH + 3H_2O

$$\rightarrow$$

PH_3 + 3NaH_2PO_2

oxidation state of phosphorus in P_4 is zero while in PH_3 it is

$$-$$

3 and in NaH_2PO_2, it is +1. This shows that this is a type of disproportionation reaction because there is an increase as well as decrease in the oxidation state of phosphorus.

Question 008 MCQ

QUESTION

The shape of XeO_2F_2 molecule is

A trigonal bipyramidal.

B square planar.

C tetrahedral.

D see-saw.

CORRECT OPTION

D see-saw.

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

The electronic configuration of Xe is $5s^2\ 5p^6$; and that of Xe in excited state is $5s^2\ 5p^5\ 5d^1$. The hybridisation is sp^3d and geometry is see-saw. The actual shape is trigonal bipyramidal but due to the preaence of lone pair it gets distorted to see-saw structure.

Question 009 MCQ

QUESTION

The compound that undergoes decarboxylation most readily under mild condition is

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The

$$\beta$$

-keto acid given in option B undergoes decarboxylation most easily because it results in formation of stable product on rearrangement. The reaction taking place is

Question 010 MCQ

QUESTION

Using the data provided, calculate the multiple bond energy (kJ mol

$$-$$

^{1}) of a C=C bond in C_2H_2. That energy is (take the bond energy of C-H bond as

350 kJ mol

–

1).

$$\begin{aligned} 2C(s) + H_2(g) \rightarrow C_2H_2 \quad & \Delta H = 225\, kJ\, mol^{-1} \\ 2C(s) \rightarrow 2C(g) \quad & \Delta H = 1410\, kJ\, mol^{-1} \\ H_2(g) \rightarrow 2H(g) \quad & \Delta H = 330\, kJ\, mol^{-1} \end{aligned}$$

A
1165 kJ mol

–

1

B
837 kJ mol

–

1

C
865 kJ mol

–

1

D
815 kJ mol

–

1

CORRECT OPTION

D
815 kJ mol

–

1

SOURCE

Chemistry • thermodynamics

EXPLANATION

We know that

Binding energy = Total energy of reactants

$-$

Total energy of products

So, we have

225 = 2 $C$$ - $$C$ + 1 $H$$ - $$H$

$-$

2 $C$$ - $$H$ + 1 $C$$ \equiv $$C$

225 = 1410 + 330

$-$

2

$\times$

350 + $C$$ \equiv $$C$

Solving, we get the value of C

$\equiv$

C as 815 kJ mol

$-$

1

.

Question 011 MCQ

QUESTION

The compound I is

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

In the given reaction, an aromatic aldehyde is condensed with an acid anhydride $Perkincondensation$ in the presence of a base to form

$$\alpha$$

,

$$\beta$$

-unsaturated acid $cinnamicacid$. This gives effervescence on reaction with $NaHCO_3$ and positive test with Bayer's reagent (1% alkaline $KMnO_4$).

Question 012 MCQ

QUESTION

The compound K is

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction sequence is

Question 013 MCQ

QUESTION

Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

A Cl_2O

B Cl_2O_7

C ClO_2

D Cl_2O_6

CORRECT OPTION

A Cl_2O

SOURCE

Chemistry • p-block-elements

EXPLANATION

Bleaching powder is $CaOCl_2$ which contains HOCl as an oxoacid and the anhydride of which is Cl_2O.

Question 014 MCQ

QUESTION

25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is

A 0.48 M

B 0.96 M

C 0.24 M

D 0.024 M

CORRECT OPTION

C 0.24 M

SOURCE

Chemistry • p-block-elements

EXPLANATION

Consider the titration reaction

$CaOCl_2$ + 2KI

$\rightarrow$

I_2 + CaOH_2 + KCl

According to this reaction, 25 mL of $CaOCl_2$ reacts with 30 mL of 0.50 M KI

$I_2 + 2Na_2S_2O_3$

$$\rightarrow$$

$Na_2S_4O_6 + 2NaI$

Given that 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. So, the number of moles of I_2 produced = 48

$$\times$$

0.25/2 = 6.

According to the reaction,

Number of millimoles of bleaching powder = Number of moles of I_2 = $1/2$

$$\times$$

Number of moles of $Na_2S_2O_3$ = 6

So, the molarity of $CaOCl_2$ is

$$\frac{Number\, of\, moles\, of\, bleaching\, powder}{Volume\, of\, solution} = \frac{6\, mmol}{25\, mL} = 0.24M$$

Question 015 MCQ

QUESTION

The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement s is are correct?

A. $T_1 = T_2$

B $T_3 > T_1$

C $w_{isothermal} > w_{adiabatic}$

D $U_{isothermal} >$

Δ

Δ

$U_{adiabatic}$

CORRECT OPTION

A $T_1 = T_2$

SOURCE

Chemistry • thermodynamics

EXPLANATION

For isothermal process $T_1 = T_2$. Work done in isothermal process is less than adiabatic process. In case of isothermal process, the temperature remains constant so there is no change in the internal energy, whereas in case of adiabatic process expansion occurs through internal energy.

Question 016 MCQ

QUESTION

For the given aqueous reactions, which of the statement s is are true?

A The first reaction is a redox reaction.

B White precipitate is $Zn_3[Fe\,CN_6]_2$.

C Addition of filtrate to starch solution gives blue colour.

D White precipitate is soluble in NaOH solution.

CORRECT OPTION

A The first reaction is a redox reaction.

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

The first reaction is

$$K_3[Fe(CN)_6] + \underset{excess}{KI} \rightarrow K_4[Fe(CN)_6] + \underset{brownish\ yellow\ solution}{KI_3}$$

This is a redox reaction as both oxidation and reduction are taking place.

$$\underset{soluble}{K_4[Fe(CN)_6]} + ZnSO_4 \rightarrow \underset{white\ ppt}{K_2Zn_3[Fe(CN)_6]_3} + NaOH \rightarrow \underset{soluble}{Na_2[Zn(OH)_4}$$

$$2I_3^- + 2Na_2S_2O_3 \rightarrow \underset{clear\ solution}{Na_2S_4O_6} + 2NaI + \underset{(blue\ colour)}{2I_2}$$

Question 017 MCQ

QUESTION

With reference to the scheme given, which of the given statement s about T, U, V and W is are correct?

A T is soluble in hot aqueous NaOH.

B U is optically active.

C Molecular formula of W is $C_{10}H_{18}O_4$.

D V gives effervescence on treatment with aqueous $NaHCO_3$.

CORRECT OPTION

A T is soluble in hot aqueous NaOH.

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reactions involved are as follows:

Question 018 MCQ

QUESTION

Which of the given statement s about N, O, P and Q with respect to M is are correct?

A M and N are non-minor image stereoisomers.

B M and O are identical.

C M and P are enantiomers.

D None of these.

CORRECT OPTION

A M and N are non-minor image stereoisomers.

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The relation between the given compounds can be determined assigning them R and S configuration. The given structures can be represented as

M and N are diastereomers.

M and O are identical.

M and P are enantiomers $non-superimposablemirrorimages$.

M and Q are diastereomers.

Question 019 MCQ

QUESTION

With respect to graphite and diamond, which of the statement s given below is are correct?

- **A** Graphite is harder than diamond.
- **B** Graphite has higher electrical conductivity than diamond.
- **C** Graphite has higher thermal conductivity than diamond.
- **D** Graphite has higher C-C bond order than diamond.

CORRECT OPTION

B Graphite has higher electrical conductivity than diamond.

SOURCE

Chemistry • p-block-elements

EXPLANATION

Diamond is hard and graphite is soft. Graphite is a good conductor of electricity as it has one free electron which is responsible for the conduction.

Diamond has higher thermal conductivity than graphite because the structure of diamond is precise and thus the transfer of heat is faster in it.

In case of graphite, the C-C bond has a double bond character so its bond order becomes higher than that of diamond which has only single C-C bonds.

Question 020 MCQ

QUESTION

The given graphs/data I, II, III and IV represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice s about I, II, III and IV is are correct?

A. I is physisorption and II is chemisorption.

B. I is physisorption and III is chemisorption.

C. IV is chemisorption and II is chemisorption.

D. IV is chemisorption and III is chemisorption.

CORRECT OPTION

A. I is physisorption and II is chemisorption.

SOURCE

Chemistry • surface-chemistry

EXPLANATION

In the case of physiorption, with the increase of temperature and pressure the rate of adsorption decreases because according to Le Chatelier's principle, increase of temperature and pressure will shift the equilibrium to the left

Adsorbate + Adsorbent

$$\rightleftharpoons$$

Adsorption + Heat

This is shown in Graphs I and III; whereas in the case of chemisorption, there is a formation of strong bond between the adsorbate and the adsorbent and so the rate of adsorption increases with increase in temperature $Graphs II and IV$.

Question 021 MCQ

QUESTION

Let

$$a_n$$

denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0.Let

$$b_n$$

= the number of such n-digit integers ending with digit 1 and

$$c_n$$

=the number of such n-digit integers ending with digit 0.

The value of

$$b_6$$

is

A 7

B 8

C 9

D 11

CORRECT OPTION

B 8

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

Given,

$$b_n$$

denotes the number of

$$n$$

-digit integer formed by the digits 0, 1 or both such that

$$n$$

-digit integer ending with 1 and no consecutive digits are '0'.

$$\therefore \quad b_6 =$$

six digit number ending with 1.

Like 1 .... 1, and rest four places are filled as Case No. I : Use four ' 1 '

$$\text{Case No. (I) : Use four ' 1 '}$$

$$\underline{1}\underline{1}\underbrace{\underline{1}\,\underline{1}\,\underline{1}}\underline{1}$$

Number of ways = 1

Case No. II : Use three '1' and one '0'

$$\underline{1}\underline{1}\underbrace{\underline{1}\,\underline{1}\,\underline{0}}\underline{1}$$

Number of ways

$$=\frac{4!}{3!}=4$$

Case No. III : Use two '1' and two '0'

$$\underline{1}\underline{1}\underbrace{\underline{0}\,\underline{1}\,\underline{0}}\underline{1}$$

or

$$\underline{1}\underline{1}\underbrace{\underline{1}\,\underline{0}\,\underline{1}}\underline{1}$$

or

$$\underline{1}\underline{0}\underbrace{\underline{1}\,\underline{1}\,\underline{0}}\underline{1}$$

No. of ways = 3

Hence,

$$b_6=1+4+3=8$$

Question 022 MCQ

QUESTION

If the straight lines

$$\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$$

and

$$\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$$

are coplanar, then the plane s containing these two lines is are

A $y + 2z = -1$

B $y + z = -1$

C $y - z = -1$

D $y - 2z = -1$

CORRECT OPTION

B $y + z = -1$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Given, the lines

$$\frac{x-1}{2}=\frac{y+1}{k}=\frac{z}{2}$$

and

$$\frac{x+1}{5}=\frac{y+1}{2}=\frac{z}{k}$$

are coplanar.

Apply the scalar triple product of

$$(2\hat{i}+k\hat{j}+2\hat{k}),(5\hat{i}+2\hat{j}+k\hat{k})$$

and

$$(-2\hat{i}+0\hat{j}+0\hat{k})$$

is zero.

$$\begin{aligned}&\Rightarrow\left|\begin{array}{ccc}-2 & 0 & 0\\ 2 & k & 2\\ 5 & 2 & k\end{array}\right|=0\\ &\Rightarrow -2\left(k^2-4\right)=0\\ &\Rightarrow \quad k=\pm 2\end{aligned}$$

For the equation of plane containing given lines, apply scalar triple product of

$$(x-1)\hat{i}+(y+1)\hat{j}+z\hat{k},2\hat{i}$$

and

$$2\hat{i}+k\hat{j}+2\hat{k}$$

is zero.

$$\Rightarrow \quad \begin{vmatrix} x-1 & y+1 & z \\ -2 & 0 & 0 \\ 2 & \pm 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \quad 4(y+1) - 2kz = 0$$

$$\Rightarrow \quad 2(y+1) - (\pm 2)z = 0$$

$$\Rightarrow \quad y + 1 \mp z = 0$$

$$\Rightarrow \quad y - z + 1 = 0, y + z + 1 = 0$$

Question 023 MCQ

QUESTION

The equation of a plane passing through the line of intersection of the planes

$$x + 2y + 3z = 2$$

and

$$x - y + z = 3$$

and at a distance

$$\frac{2}{\sqrt{3}}$$

from the point

$$(3, 1, -1)$$

is

A
$$5x - 11y + z = 17$$

B
$$\sqrt{2}x + y = 3\sqrt{2} - 1$$

C

$$x + y + z = \sqrt{3}$$

D

$$x - \sqrt{2}y = 1 - \sqrt{2}$$

CORRECT OPTION

A

$$5x - 11y + z = 17$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

The equation of plane passing through the intersection of planes

$$x + 2y + 3z - 2 = 0$$

and

$$x - y + z - 3 = 0$$

is

$$\begin{aligned} &\Rightarrow (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0 \\ &\Rightarrow (\lambda + 1)x + (-\lambda + 2)y + (\lambda + 3)z - (3\lambda + 2) = 0 \quad \ldots \text{(i)} \end{aligned}$$

$$\text{Given, the distance from } (3, 1, -1) \text{ is } \frac{2}{\sqrt{3}}$$

$$\begin{aligned} &\Rightarrow \frac{2}{\sqrt{3}} = \frac{|3(\lambda + 1) + 1 \cdot (-\lambda + 2) - 1 \cdot (\lambda + 3) - (3\lambda + 2)|}{\sqrt{(\lambda + 1)^2 + (-\lambda + 2)^2 + (\lambda + 3)^2}} \\ &\Rightarrow \frac{2}{\sqrt{3}} = \frac{|-2\lambda|}{\sqrt{3\lambda^2 + 4\lambda + 14}} \\ &\Rightarrow 2\sqrt{3\lambda^2 + 4\lambda + 14} = 2|\lambda|\sqrt{3} \\ &\Rightarrow \sqrt{3x^2 + 4\lambda + 14} = \sqrt{3}|\lambda| \end{aligned}$$

On squaring both side

$$\Rightarrow 3\lambda^2 + 4\lambda + 14 = 3\lambda^2$$
$$\Rightarrow \lambda = -\frac{7}{2}$$

Put

$$\lambda = -\frac{7}{2}$$

in the equation i

$$\Rightarrow \left(-\frac{7}{2} + 1\right)x + \left(\frac{7}{2} + 2\right)y + \left(-\frac{7}{2} + 3\right)z + \frac{21}{2} - 2 = 0$$
$$\Rightarrow \frac{-5}{2}x + \frac{11}{2}y - \frac{1}{2}z + \frac{17}{2} = 0$$
$$\Rightarrow \quad 5x - 11y + z = 17$$

Question 024 MCQ

QUESTION

If

$$\vec{a}$$

and

$$\vec{b}$$

are vectors such that

$$\left|\vec{a} + \vec{b}\right| = \sqrt{29}$$

and

$$\vec{a} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \hat{b},$$

then a possible value of

$$\left(\overrightarrow{a}+\overrightarrow{b}\right).\left(-7\hat{i}+2\hat{j}+3\widehat{k}\right)$$

is

A 0

B 3

C 4

D 8

CORRECT OPTION

C 4

SOURCE

Mathematics • vector-algebra

EXPLANATION

$$\text{Given, } \vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$$
$$\Rightarrow \vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})-(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}=0$$
$$\Rightarrow \vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})+\vec{b}\times(2\hat{i}+3\hat{j}+4\hat{k})=0$$
$$\therefore (\overrightarrow{\mathrm{A}}\times\overrightarrow{\mathrm{B}}=-\overrightarrow{\mathrm{B}}\times\overrightarrow{\mathrm{A}})$$
$$\Rightarrow \quad (\vec{a}+\vec{b})\times(2\hat{i}+3\hat{j}+4\hat{k})=0$$

Hence,

$$\vec{a}+\vec{b}$$

is collinear to

$$2\hat{i}+3\hat{j}+4\hat{k}$$

Let

$$\vec{a}+\vec{b}=\lambda(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{... (i)}$$

$$\begin{aligned}
\Rightarrow \quad & |\vec{a}+\vec{b}| = |\lambda|\sqrt{2^2+3^2+4^2} \\
\Rightarrow \quad & \sqrt{29} = |\lambda|\sqrt{29} \quad (\text{Given, } |\vec{a}+\vec{b}| = \sqrt{29}) \\
\Rightarrow \quad & |\lambda| = 1 \\
\Rightarrow \quad & |\lambda| = \pm 1 \\
\therefore \quad & \vec{a}+\vec{b} = \pm(2\hat{i}+3\hat{j}+4\hat{k})
\end{aligned}$$

$$\begin{aligned}
\text{Now, } & (\vec{a}+\vec{b})\cdot(-7\hat{i}+2\hat{j}+3\hat{k}) \\
& = \pm(2\hat{i}+3\hat{j}+4\hat{k})\cdot(-7\hat{i}+2\hat{j}+3\hat{k}) \\
& = \pm(-14+6+12) \\
& = \pm 4
\end{aligned}$$

Question 025 MCQ

QUESTION

Let

$$X$$

and

$$Y$$

be two events such that

$$P\left(X|Y\right)=\frac{1}{2},$$

$$P(Y|X) = \frac{1}{3}$$

and

$$P(X \cap Y) = \frac{1}{6}.$$

Which of the following is are correct ?

A

$$P(X \cup Y) = \frac{2}{3}$$

B

and

$$X$$

$$Y$$

are independent

C

and

$$X$$

$$Y$$

are not independent

D

$$P(X^c \cap Y) = \frac{1}{3}$$

CORRECT OPTION

A

$$P(X \cup Y) = \frac{2}{3}$$

SOURCE

Mathematics • probability

EXPLANATION

Let's analyze the given conditions and evaluate which options are correct.

Given:

1. $$P(X|Y) = \frac{1}{2}$$

2. $$P(Y|X) = \frac{1}{3}$$

3. $$P(X \cap Y) = \frac{1}{6}$$

Now, let's proceed step by step through each option.

Option A:

$$P(X \cup Y) = \frac{2}{3}$$

We can use the formula for the union of two events:

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

To find

$$P(X)$$

and

$$P(Y)$$

, we use the definitions of conditional probability:

$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$

$$\frac{1}{2} = \frac{\frac{1}{6}}{P(Y)}$$

$$P(Y) = \frac{1}{6} \times 2 = \frac{1}{3}$$

Similarly,

$$P(Y|X) = \frac{P(X \cap Y)}{P(X)}$$

$$\frac{1}{3} = \frac{\frac{1}{6}}{P(X)}$$

$$P(X) = \frac{1}{6} \times 3 = \frac{1}{2}$$

Now, substituting these values into the union formula:

$$P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6}$$

To add these fractions, we need a common denominator. The common denominator is 6:

$$P(X \cup Y) = \frac{3}{6} + \frac{2}{6} - \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Hence, **Option A is correct**.

Option B:

$$X$$

and

$$Y$$

are independent

For two events

$$X$$

and

$$Y$$

to be independent, the following condition should hold:

$$P(X \cap Y) = P(X) \times P(Y)$$

We already found that:

$$P(X) = \frac{1}{2}$$

$$P(Y) = \frac{1}{3}$$

$$P(X \cap Y) = \frac{1}{6}$$

Checking the independence condition:

$$P(X) \times P(Y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Since this is equal to

$$P(X \cap Y)$$

,

$$X$$

and

$$Y$$

are indeed independent.

Hence, **Option B is correct**.

Option C:

$$X$$

and

$$Y$$

are not independent

From the previous analysis, we have shown that

$$X$$

and

$$Y$$

are independent.

Hence, **Option C is not correct**.

Option D:

$$P((X^c) \cap Y) = \frac{1}{3}$$

To find

$$P((X^c) \cap Y)$$

, we use the fact that:

$$P(Y) = P((X \cap Y) \cup (X^c \cap Y))$$

Since

$$(X \cap Y)$$

and

$$(X^c \cap Y)$$

are disjoint events, we can write:

$$P(Y) = P(X \cap Y) + P(X^c \cap Y)$$

Given:

$$P(Y) = \frac{1}{3}$$

$$P(X \cap Y) = \frac{1}{6}$$

Substituting these,

$$\frac{1}{3} = \frac{1}{6} + P(X^c \cap Y)$$

Solving for

$$P(X^c \cap Y)$$

:

$$P(X^c \cap Y) = \frac{1}{3} - \frac{1}{6} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$$

This contradicts Option D, as it says

$$\frac{1}{3}$$

.

Hence, **Option D is not correct**.

In conclusion:

Option A and Option B are correct.

Option C and Option D are not correct.

Question 026 MCQ

QUESTION

Four fair dice

$$D_1,$$

$$D_2,$$

$$D_3$$

and

$$D_4$$

; each having six faces numbered

$$1, 2, 3, 4, 5$$

and

$$6$$

are rolled simultaneously. The probability that

$$D_4$$

shows a number appearing on one of

$$D_1,$$

$$D_2$$

and

$$D_3$$

is

A $\frac{91}{216}$

B $\frac{108}{216}$

C $\frac{125}{216}$

D $\frac{127}{216}$

CORRECT OPTION

A $\frac{91}{216}$

SOURCE

Mathematics • probability

EXPLANATION

For the given condition, the sample space

$$= 6^4$$

For favourable condition

Case I : For

$$D_4$$

there are

$$^6C_1$$

way. Now, it appears on any one of

$$D_1, D_2, D_3$$

i.e.

$$^3C_1 \cdot 1$$

, for other two there are

$$5 \times 5$$

ways.

$$\Rightarrow {}^6\mathrm{C}_1 \cdot {}^3\mathrm{C}_1 \cdot 1 \cdot 5^2 = 450 \text{ Ways}$$

Case II : For

$$\mathrm{D}_4$$

there are

$$^6\mathrm{C}_1$$

ways. Now, it appears on any two of

$$D_1, D_2, D_3$$

i.e.

$$^3\mathrm{C}_2.1.1$$

, for other one there are 5 ways.

$$\Rightarrow {}^6\mathrm{C}_1 \cdot {}^3\mathrm{C}_2 \cdot 1^2 \cdot 5 = 90 \text{ Ways}$$

Case III : For

$$\mathrm{D}_4$$

there are

$$^6\mathrm{C}_1$$

ways. Now, it appears on all i.e.

$$1^3$$

.

$$\Rightarrow {}^6\mathrm{C}_1 \cdot 1^3 = 6$$

Total number of favourable cases

$$\begin{aligned} &= 450 + 90 + 6 \\ &= 546 \text{ ways} \end{aligned}$$

Thus, Probability

$$= \frac{546}{6^4} = \frac{91}{216}$$

Question 027 MCQ

QUESTION

The value of the integral

$$\int_{-\pi/2}^{\pi/2} \left(x^2 + 1n \frac{\pi + x}{\pi - x} \right) \cos x dx$$

is

A

$$0$$

B

$$\frac{\pi^2}{2} - 4$$

C

$$\frac{\pi^2}{2} + 4$$

D

$$\frac{\pi^2}{2}$$

CORRECT OPTION

B

$$\frac{\pi^2}{2} - 4$$

SOURCE

Mathematics • definite-integration

EXPLANATION

We know the property

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{when } f(x) \text{ is an even function} \\ 0, & \text{when } f(x) \text{ is an odd function} \end{cases}$$

Let,

$$\mathrm{I} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^2 + \ln\left(\frac{\pi + x}{\pi - x}\right)\right) \cos x \cdot dx$$

$$\Rightarrow \mathrm{I} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^2 \cdot \cos x dx + \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos x \cdot \ln\left(\frac{\pi + x}{\pi - x}\right) dx$$

Here,

$$x^2 \cos x$$

is an even function and

$$\cos x \cdot \ln\left(\frac{\pi + x}{\pi - x}\right)$$

is an odd function.

$$\therefore \mathrm{I} = 2\int_0^{\frac{\pi}{2}} x^2 \cos x dx + 0$$

$$\Rightarrow \mathrm{I} = 2\left[\left(\sin x \cdot x^2\right)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2x \sin x \cdot dx\right]$$

$$\Rightarrow \mathrm{I} = \frac{\pi^2}{2} - 4\int_0^{\frac{\pi}{2}} x \sin x \cdot dx$$

$$\Rightarrow \mathrm{I} = \frac{\pi^2}{2} - 4\left[\left(x(-\cos x)\right)_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 1 \cdot (-\cos x) dx\right]$$

$$\Rightarrow \mathrm{I} = \frac{\pi^2}{2} - 4\int_0^{\frac{\pi}{2}} \cos x dx$$

$$\Rightarrow \mathrm{I} = \frac{\pi^2}{2} - 4[\sin x]_0^{\frac{\pi}{2}}$$

$$\Rightarrow \mathrm{I} = \frac{\pi^2}{2} - 4$$

Question 028 MCQ

QUESTION

Let

$$f\left(x\right)=\left(1-x\right)^{2}\sin^{2}x+x^{2}$$

for all

$$x\in IR$$

and let

$$g\left(x\right)=\int_{1}^{x}\left(\frac{2\left(t-1\right)}{t+1}-In\,t\right)f\left(t\right)dt$$

for all

$$x\in\left(1,\,\infty\right)$$

.

Consider the statements:

$$P:$$

There exists some

$$x\in R$$

such that

$$f\left(x\right)+2x=2\left(1+x^{2}\right)$$

$$Q:$$

There exists some

$$x\in R$$

such that

$$2\,f\,(x) + 1 = 2x\,(1 + x)$$

Then

A both

$$P$$

and

$$Q$$

are true

B

$$P$$

is true and

$$Q$$

is false

C

$$P$$

is false and

$$Q$$

is true

D both

$$P$$

and

$$Q$$

are false

CORRECT OPTION

C

$$P$$

is false and

$$Q$$

is true

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

For statement

$$P$$

$$\begin{aligned}
& f(x)+2x=2\left(1+x^2\right) \\
\Rightarrow & (1-x)^2 \sin ^2 x+x^2+2x=2+2x^2 \\
\Rightarrow & (1-x)^2 \sin ^2 x=x^2-2x+1+1 \\
\Rightarrow & (1-x)^2 \sin ^2 x=(1-x)^2+1 \\
\Rightarrow & -(1-x)^2\left(1-\sin ^2 x\right)=1 \\
\Rightarrow & -\cos ^2 x=\frac{1}{(1-x)^2}>0 \\
\Rightarrow & \cos ^2 x<0
\end{aligned}$$

Here, no value of

$$x$$

that satisfy the above equation.

$$\therefore$$

Statement P is false

For Statement Q

$$
\begin{aligned}
& 2f(x)+1=2x(1+x) \\
\Rightarrow \quad & 2(1-x)^2\sin^2 x+2x^2+1=2x+2x^2 \\
\Rightarrow \quad & 2(x-1)^2\sin^2 x=2x-1 \\
\Rightarrow \quad & \sin^2 x=\frac{2x-1}{2(x-1)^2} \\
\Rightarrow \quad & \sin^2 x=\frac{1}{x-1}+\frac{1}{2(x-1)^2} \quad \ldots \text{(i)}
\end{aligned}
$$

$$
\begin{aligned}
& \text{Let } B(x)=\frac{1}{x-1}+\frac{1}{2(x-1)^2} \\
\Rightarrow \quad & B'(x)=\frac{-1}{(x-1)^2}-\frac{1}{(x-1)^3}=\frac{-x}{(x-1)^3} \text{ and} \\
& B''(x)=\frac{2x+1}{(x-1)^4}
\end{aligned}
$$

Hence,

$$\mathrm{B}(x)$$

is increasing function on

$$x\in(0,1)$$

and decreasing on

$$x\in(-\infty,0)\cup(1,\infty),\mathrm{B}''(x)=0$$

at

$$x=-\frac{1}{2}$$

and

$$\mathrm{B}''(x)\geq 0$$

for

$$x\in\mathrm{R}-\left\{\frac{-1}{2},1\right\}.$$

Draw the graph of the

$$y=\sin^2 x$$

and

$$y = \frac{1}{x+1} + \frac{1}{2(x-1)^2}$$

The graph of

$$y = \sin^2 x$$

and

$$y = \frac{1}{x+1} + \frac{1}{2(x-1)^2}$$

intersect at some

$$x$$

.

Hence, statement

$$Q$$

is true.

Question 029 MCQ

QUESTION

Let

$$f(x) = (1-x)^2 \sin^2 x + x^2$$

for all

$$x \in IR$$

and let

$$g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - In\, t\right) f(t)dt$$

for all

$$x \in (1,\ \infty)$$

.

Which of the following is true?

A

$$g$$

is increasing on

$$(1, \infty)$$

B

$$g$$

is decreasing on

$$(1, \infty)$$

C

$$g$$

is increasing on

$$(1, 2)$$

and decreasing on

$$(2, \infty)$$

$$g$$

is decreasing on

D

$$(1, 2)$$

and increasing on

$$(2, \infty)$$

CORRECT OPTION

$$g$$

B is decreasing on

$$(1, \infty)$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given,

$$\begin{aligned} g(x) &= \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) f(t) dt \\ \text{and } f(x) &= (1-x)^2 \sin^2 x + x^2 \\ \Rightarrow g(x) &= \int_1^x \left(\frac{2(t-1)}{t+1} - \ln t\right) \left((1-t)^2 \sin^2 t + t^2\right) dt \end{aligned}$$

On differentiating the above equation w.r.t.

$$x$$

$$\begin{aligned} \Rightarrow g'(x) &= \left(\frac{2(x-1)}{x+1} - \ln x\right) \left((1-x)^2 \sin^2 x + x^2\right) \\ \Rightarrow g'(x) &= \left(2 - \frac{4}{x+1} - \ln x\right) \left((1-x)^2 \sin^2 x + x^2\right) \end{aligned}$$

Here

$$(1-x)^2 \sin^2 x + x^2 > 0$$

for all

$$x \in \mathrm{R}$$

Now, draw the graph of

$$y = 2 - \frac{4}{x + 1}$$

and

$$y = -\ln x$$

For

$$x > 0$$

$$\text{Add the graph of } y = -\ln x \text{ and } y = 2 - \frac{4}{x + 1}$$

$$\Rightarrow g'(x) < 0 \text{ for } x \in (1, \infty)$$

Hence,

$$g(x)$$

is decreasing function for

$$x \in (1, \infty)$$

Question 030 MCQ

QUESTION

If

$$f(x) = \int_0^x e^{t^2} (t - 2)(t - 3) dt$$

for all

$$x \in (0, \infty),$$

then

A

$$f$$

has a local maximum at

$$x = 2$$

B

$$f$$

is decreasing on

$$(2, 3)$$

C

there exists some

$$c \in (0, \infty),$$

such that

$$f'(c) = 0$$

D

$$f$$

has a local minimum at

$$x = 3$$

CORRECT OPTION

$$f$$

B is decreasing on

$$(2,3)$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

$$\text{Given, } f(x)=\int_0^x e^{t^2}(t-2)(t-3)dt, x\in(0,\infty)$$
$$\Rightarrow \quad f'(x)=e^{x^2}(x-2)(x-3)$$

Here,

$$f'(x)$$

changes its sign

$$+ve$$

to

$$-ve$$

about

$$x=2$$

and

$$f'(x)$$

changes its sign

$$-ve$$

to

$$+ve$$

about

$$x = 3$$

Hence,

$$x = 2$$

is the point of local maxima and

$$x = 3$$

is the point of local minima

$$\because f'(x) < 0$$

for

$$x \in (2, 3)$$

$$\therefore f(x)$$

is decreasing on

$$x \in (2, 3)$$

$$\because \quad f'(x)$$

is continuous and differentiable for all

$$x(0, \infty)$$

and

$$f'(2) = f'(3) = 0$$

$$\therefore$$

According to Rolle's theorem,

$$f''(c) = 0$$

must have at least one root

$$\in (2, 3)$$

Question 031 MCQ

QUESTION

Let

$$PQR$$

be a triangle of area

$$\Delta$$

with

$$a = 2$$

,

$$b = \frac{7}{2}$$

and

$$c = \frac{5}{2}$$

; where

$$a, b,$$

and

$$c$$

are the lengths of the sides of the triangle opposite to the angles at

$$P. Q$$

and

$$R$$

respectively. Then

$$\frac{2 \sin P - \sin 2P}{2 \sin P + \sin 2P}$$

equals.

A $\dfrac{3}{4\Delta}$

B $\dfrac{45}{4\Delta}$

C $\left(\dfrac{3}{4\Delta}\right)^2$

D $\left(\dfrac{45}{4\Delta}\right)^2$

CORRECT OPTION

C $\left(\dfrac{3}{4\Delta}\right)^2$

SOURCE

Mathematics • properties-of-triangle

EXPLANATION

Given,

$$\Delta$$

be the area of

$$\triangle \mathrm{PQR}$$

of side length

$$a=2, b=\frac{7}{2} \text{ and } c=\frac{5}{2}$$

$$\Rightarrow s=\frac{a+b+c}{2}=\frac{2+\frac{7}{2}+\frac{5}{2}}{2}=4$$

$$\text{Now, } \frac{2 \sin \mathrm{P}-\sin 2 \mathrm{P}}{2 \sin \mathrm{P}+\sin 2 \mathrm{P}}$$

$$=\frac{2 \sin \mathrm{P}-2 \sin \mathrm{P} \cdot \cos \mathrm{P}}{2 \sin \mathrm{P}+2 \sin \mathrm{P} \cdot \cos \mathrm{P}}$$

$$(\sin 2 \theta=2 \sin \theta \cdot \cos \theta)$$

$$=\frac{1-\cos \mathrm{P}}{1+\cos \mathrm{P}}$$

$$=\frac{1-\left(1-2 \sin ^2 \frac{\mathrm{P}}{2}\right)}{1+\left(2 \cos ^2 \frac{\mathrm{P}}{2}-1\right)}$$

$$\left(\cos 2 \theta=2 \cos ^2 \theta-1=1-2 \sin ^2 \theta\right)$$

$$=\tan ^2 \frac{\mathrm{P}}{2}$$

$$=\left(\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}\right)^2$$

$$=\left(\frac{(s-b)(s-c)}{\sqrt{s(s-a)(s-b)(s-c)}}\right)^2$$

$$=\left(\frac{(s-b)(s-c)}{\Delta}\right)^2$$

$$\therefore \quad (\Delta=\sqrt{s(s-a)(s-b)(s-c)})$$

$$=\frac{\left(4-\frac{7}{2}\right)^2\left(4-\frac{5}{2}\right)^2}{\Delta^2}$$

$$=\frac{9}{16 \Delta^2}$$

$$=\left(\frac{3}{4 \Delta}\right)^2$$

Question 032 MCQ

QUESTION

A tangent PT is drawn to the circle

$$x^2 + y^2 = 4$$

at the point P

$$\left(\sqrt{3}, 1\right)$$

. A straight line L, perpendicular to PT is a tangent to the circle

$$(x-3)^2$$

+

$$y^2$$

= 1

A common tangent of the two circles is

A x = 4

B y = 2

C $$x + \sqrt{3}\,y = 4$$

D $$x + 2\sqrt{2}\,y = 6$$

CORRECT OPTION

D

$$x + 2\sqrt{2}\, y = 6$$

SOURCE

Mathematics • circle

EXPLANATION

The equation of tangent of the circle

$$x^2 + y^2 = 4$$

is

$$y = mx \pm 2\sqrt{1+m^2} \quad .... \text{(i)}$$

Let

$$y = mx \pm 2\sqrt{1+m^2}$$

also touches

$$(x-3)^2 + y^2 = 1$$

$$\Rightarrow (x-3)^2 + \left(mx \pm 2\sqrt{1+m^2}\right)^2 = 1$$

$$\Rightarrow x^2 - 6x + 9 + m^2x^2 + 4\left(1+m^2\right) \pm 4m\sqrt{1+m^2}x = 1$$

$$\Rightarrow (1+m^2)x^2 + \left(-6 \pm 4m\sqrt{1+m^2}\right)x + 4\left(m^2+3\right) = 0$$

Apply

$$\left(-6 \pm 4m\sqrt{1+m^2}\right)^2 - 4\left(1+m^2\right) \cdot 4\left(m^2+3\right) = 0$$

$$\begin{aligned} \Rightarrow \quad & 36 + 16m^2\left(1+m^2\right) \pm 48m\sqrt{1+m^2} \\ & -16\left(m^4 + 4m^2 + 3\right) = 0 \\ \Rightarrow \quad & 4m^2 + 1 = \pm 4m\sqrt{1+m^2} \end{aligned}$$

On squaring both side

$$\Rightarrow \quad 16m^4 + 1 + 8m^2 = 16m^2 + 16m^4$$
$$\Rightarrow \quad m^2 = \frac{1}{8}$$
$$\Rightarrow \quad m = \pm\frac{1}{2\sqrt{2}}$$

$$\text{Put} \quad m = \pm\frac{1}{2\sqrt{2}} \text{ in the equation (i)}$$
$$\Rightarrow \quad y = \pm\frac{x}{2\sqrt{2}} \pm \frac{6}{2\sqrt{2}}$$
$$\Rightarrow \quad 2\sqrt{2}y = \pm x \pm 6$$

Hence, the equation of common tangent of given circles are

$$2\sqrt{2}y = -x + 6, 2\sqrt{2}y = x + 6, 2\sqrt{2}y = -x - 6$$

and

$$2\sqrt{2}y = x - 6$$

.

Question 033 MCQ

QUESTION

A tangent PT is drawn to the circle

$$x^2 + y^2 = 4$$

at the point P

$$\left(\sqrt{3}, 1\right)$$

. A straight line L, perpendicular to PT is a tangent to the circle

$$(x - 3)^2$$

+

$$y^2$$

= 1.

A possible equation of L is

A

$$x-\sqrt{3}\,y=1$$

B

$$x+\sqrt{3}\,y=1$$

C

$$x-\sqrt{3}\,y=-1$$

D

$$x+\sqrt{3}\,y=5$$

CORRECT OPTION

A

$$x-\sqrt{3}\,y=1$$

SOURCE

Mathematics • circle

EXPLANATION

Equation of tangent PT of the circle

$$x^2+y^2=4$$

at

$$\mathrm{P}(\sqrt{3},1)$$

is

$$\Rightarrow \quad \sqrt{3}x + y = 4$$

Given, L is a line perpendicular to PT

$$\therefore \quad \mathrm{L} \equiv x - \sqrt{3}y = \lambda$$

Also given L is the tangent of circle

$$(x-3)^2 + y^2 = 1$$

$$\begin{aligned} &\Rightarrow \quad 1 = \frac{|3-\sqrt{3}\cdot 0-\lambda|}{\sqrt{1^2+(-\sqrt{3})^2}} \\ &\Rightarrow \quad |3-\lambda| = 2 \\ &\Rightarrow \quad 3-\lambda = \pm 2 \\ &\Rightarrow \quad \lambda = 1, 5 \end{aligned}$$

Hence, the possible equation of line L are

$$x - \sqrt{3}y = 1$$

and

$$x - \sqrt{3}y = 5$$

.

Question 034 MCQ

QUESTION

Let

$$a_1, a_2, a_3, \ldots..$$

be in harmonic progression with

$$a_1 = 5$$

and

$$a_{20} = 25.$$

The least positive integer

$$n$$

for which

$$a_n < 0$$

is

A 22

B 23

C 24

D 25

CORRECT OPTION

D 25

SOURCE

Mathematics • sequences-and-series

EXPLANATION

Given:

$$a_1 = 5$$

and

$$a_{20} = 25$$

Also given,

$$a_1, a_2, a_3, \dots\dots\dots$$

are in H.P.

$$\Rightarrow \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, \dots\dots\dots$$

are in A.P.

Let D be the common difference of above A. P.

$$\begin{aligned} \therefore \quad & \tfrac{1}{a_{20}} = \tfrac{1}{a_1} + (20-1)d \\ \Rightarrow \quad & \tfrac{1}{25} = \tfrac{1}{5} + 19d \\ \Rightarrow \quad & d = \tfrac{-4}{475} \end{aligned}$$

$$\begin{aligned} \text{Now,} \quad & \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \\ \Rightarrow \quad & \frac{1}{a_n} = \frac{1}{5} + (n-1) \cdot \left(\frac{-4}{475}\right) \\ \Rightarrow \quad & \frac{1}{a_n} = \frac{95 - 4n + 4}{475} \\ \Rightarrow \quad & a_n = \frac{475}{99 - 4n} \end{aligned}$$

Apply

$$a_n < 0$$

$$\Rightarrow \quad \frac{475}{99 - 4n} < 0$$

The least positive integral value of

$$n$$

is 25 which satisfy the above condition.

Question 035 MCQ

QUESTION

Let

$$a_n$$

denote the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0.Let

$$b_n$$

= the number of such n-digit integers ending with digit 1 and

$$c_n$$

=the number of such n-digit integers ending with digit 0.

Which of the following is correct?

A $$a_{17} = a_{16} + a_{15}$$

B $$c_{17} \neq c_{16} + c_{15}$$

C $$b_{17} \neq b_{16} + c_{16}$$

D $$a_{17} = c_{17} + b_{16}$$

CORRECT OPTION

A $$a_{17} = a_{16} + a_{15}$$

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

For

$$a_n$$

Case I : If the unit digit is 1, and rest

$$(n-1)$$

places are filled as

$$a_{n-1}$$

$$\underbrace{1\_\_\_\ldots\ldots\_1}_{\text{(n−1) places}}$$

Case II : If the unit digit is 0, then tenth place must be 1 and rest

$$(n-2)$$

places are filled as

$$a_{n-2}$$

$$\underbrace{1\_\_\_\_\ldots\ldots\_10}_{\text{(n−2) place}}$$

$$\begin{aligned} &\text{Hence, } a_n = a_{n-1} + a_{n-2} \\ \Rightarrow \quad & a_{17} = a_{16} + a_{15} \end{aligned}$$

Question 036 MCQ

QUESTION

If P is a 3

$$\times$$

3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3

$$\times$$

3 identity matrix, then there exists a column matrix

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

such that

A
$$PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

B PX = X

C PX = 2X

D PX =

—

X

CORRECT OPTION

D PX =

—

X

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

We have

$$P^T = 2P + I$$

We get

$$P^T - 2P = I$$

..... i

Taking transpose, we have

$$(P^T - 2P)^T = I^T$$

$$\Rightarrow P - 2P^T = I$$

..... ii

From i and ii on eliminating P^T we have

$$-4P + P = 3I \Rightarrow P = -I$$

$$\therefore$$

$$P + I = 0$$

Thus,

$$(P + 1)X = 0 \Rightarrow PX = -X$$

Question 037 MCQ

QUESTION

Let

$$\alpha$$

a and

$$\beta$$

a be the roots of the equation

$$(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$$

where

$$a > -1$$

. Then

$$\lim_{a \to 0^+} \alpha(a)$$

and

$$\lim_{a \to 0^+} \beta(a)$$

are

A

$$-\frac{5}{2}$$

B

$$-\frac{1}{2}$$

C

$$-\frac{7}{2}$$

D

$$-\frac{9}{2}$$

CORRECT OPTION

B

$$-\frac{1}{2}$$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

Let a + 1 = t^6. Thus, when a

$$\rightarrow$$

0, t

$$\rightarrow$$

1.

$$\therefore$$

$$(t^2-1)x^2+(t^3-1)x+(t-1)=0$$

$$\Rightarrow (t-1)\{(t+1)x^2+(t^2+t+1)x+1\}=0$$

,

as t

$$\rightarrow$$

1

$$2x^2+3x+1=0$$

$$\Rightarrow 2x^2+2x+x+1=0$$

$$\Rightarrow (2x+1)(x+1)=0$$

Thus, x =

$$-$$

1,

$$-$$

1/2

or,

$$\lim_{a\to 0^+}\alpha(a)=-\frac{1}{2}$$

and

$$\lim_{a \to 0^+} \beta(a) = -1$$

Question 038 MCQ

QUESTION

For every integer n, let a_n and b_n be real numbers. Let function f : R

$\rightarrow$

R be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & for\, x \in [2n, 2n+1] \\ b_n + \cos \pi x, & for\, x \in (2n-1, 2n) \end{cases}$$

, for all integers n. If f is continuous, then which of the following hold s for all n ?

A

a_n

$-$

1

$-$

b_n

$-$

$_1 = 0$

B

a_n

$-$

$b_n = 1$

a_n

—

C b_n

+

$_1 = 1$

a_n

—

1

D

—

$b_n =$

—

1

CORRECT OPTION

a_n

B

—

$b_n = 1$

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

We have at the points x = 2n

$f 2n = a_n + \sin 2n$

π

$= a_n$

Also for the L.H.L., we have

L.H.L. =

$$\lim_{h \to 0}(b_n + \cos \pi(2n - h)) = b_{n+1}$$

R.H.L. =

$$\lim_{h \to 0}(a_n + \sin \pi(2n + h)) = a_n$$

For continuity

$$b_{n+1} = a_n$$

Again at

$$x = 2n + 1$$

L.H.L. =

$$\lim_{h \to 0}(a_n + \sin(\pi(2n + 1 - h))) = a_n$$

R.H.L. =

$$\lim_{h \to 0}(b_{n+1} + \cos(\pi(2n + 1) - h)) = b_{n+1} - 1$$

Also,

$$f(2n + 1) = a_n$$

For continuity we require

$$b_n + 1 = a_n$$

$$\therefore$$

$$a_n - b_n = 1$$

Also,

$$a_n = b_{n+1} - 1$$

$$\therefore$$

$$a_{n-1} - b_n = -1$$

Question 039 MCQ

QUESTION

If the ad joint of a 3

$\times$

3 matrix P is

$$\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$$

, then the possible value s of the determinant of P is are

A —

2

B —

1

C 1

D 2

CORRECT OPTION

A —

2

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Concept Involved If

$$|A_{n\times n}| = \Delta$$

, then

$$|adj\, A| = \Delta^{n-1}$$

Here,

$$P_{3\times 3} = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow |adj\, P| = |P|^2$$

$$\therefore$$

$$|adj\, P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 1(3-7) - 4(6-7) + 4(2-1)$$

$$= -4 + 4 + 4 = 4$$

$$\Rightarrow |P| = \pm 2$$

Question 040 MCQ

QUESTION

Let

$$f : (-1, 1) \rightarrow R$$

be such that

$$f(\cos 4\theta) = \frac{2}{2 - \sec^2\theta}$$

for

$$\theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

. Then the value s of

$$f\left(\frac{1}{3}\right)$$

is are

A

$$1 - \sqrt{\frac{3}{2}}$$

B

$$1 + \sqrt{\frac{3}{2}}$$

C

$$1 - \sqrt{\frac{2}{3}}$$

D

$$1 + \sqrt{\frac{2}{3}}$$

CORRECT OPTION

A

$$1-\sqrt{\frac{3}{2}}$$

SOURCE

Mathematics • functions

EXPLANATION

$$f(\cos 4\theta)=\frac{2}{2-\sec^2\theta}$$

Let

$$\cos 4\theta=t$$

$$\Rightarrow 2\cos^2 2\theta-1=t\Rightarrow \cos^2 2\theta=\frac{2}{3}$$

For

$$t=\frac{1}{3}$$

we have

$$\cos^2 2\theta=\frac{2}{3}$$

$$\cos 2\theta=\sqrt{\frac{2}{3}}$$

or

$$\cos 2\theta=-\sqrt{\frac{2}{3}}$$

$$f(\cos 4\theta)=\frac{2}{2-\frac{1}{\cos^2\theta}}=\frac{2\cos^2\theta}{2\cos^2\theta-1}=\frac{1+\cos 2\theta}{\cos 2\theta}=1+\frac{1}{\cos 2\theta}$$

Hence,

$$f\left(\frac{1}{3}\right) = 1 + \sqrt{\frac{3}{2}}$$

or

$$1 - \sqrt{\frac{3}{2}}$$

Question 041 MCQ

QUESTION

Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed

$$\omega$$

. The discs are in the same horizontal plane. At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period T of rotation of the discs, v_r as a function of time is best represented by

A

B

C

D

CORRECT OPTION

A

SOURCE

Physics • rotational-motion

EXPLANATION

The relative velocity of the point P w.r.t. the point Q is given by

$$\vec{v}_r = \vec{v}_P - \vec{v}_Q$$

.........1

It is easy to see that $\left|\vec{v}_P\right| = \left|\vec{v}_Q\right| = \omega R$ and angle traversed in time t is ωt. Thus, velocities of P and Q are

$$\begin{aligned} \vec{v}_P &= \omega R(-\sin\omega t\hat{\imath} - \cos\omega t\hat{\jmath}) \\ \vec{v}_Q &= \omega R(\sin\omega t\hat{\imath} - \cos\omega t\hat{\jmath}) \end{aligned}$$

Substitute $\vec{v}_P$ and $\vec{v}_Q$ in equation 1 to get $\vec{v}_r = -2\omega R\sin\omega t\hat{\imath}$ and thus $v_r = 2\omega R|\sin\omega t|$.

Question 042 MCQ

QUESTION

Six point charges are kept at the vertices of a regular hexagon of side

$$L$$

and center

$$O,$$

as shown in the figure. Given that

$$K = \frac{1}{4\pi\varepsilon_0} \frac{q}{L^2},$$

which of the following statement s is are correct ?

A

The electric field at

$$O$$

is

$$6K$$

along

$$OD$$

B

The potential at

$$O$$

is zero

C

The potential at all points on the line

$$PR$$

is same

D

The potential at all points on the line

$$ST$$

is same

CORRECT OPTION

The electric field at

$$O$$

is

A

$$6K$$

along

$$OD$$

SOURCE

Physics • electrostatics

EXPLANATION

The electric field at point O due to the charges at vertices A and D is $4K$ along the direction OD. Similarly, due to the charges at vertices B and E, the electric field is $2K$ along the direction OE, and due to the charges at vertices C and F, it is $2K$ along the direction OC. Given the uniform geometry of this setup, the resulting electric field is $6K$ along OD.

The potential at point O is calculated as :

$$V_{\mathrm{O}} = \sum \frac{1}{4\pi\epsilon_0}\frac{q_i}{L} = \frac{1}{4\pi\epsilon_0 L}\sum q_i = 0$$

For any point on the line PR, we observe that there are pairs of equal and opposite charges equidistant from these points, making the potential at any point on PR zero. If we consider points on OS, the potential is positive, while for points on OT, the potential is negative. The potential at points on the line ST, at a distance x from O $with\$x\$consideredpositivetowardstheright$, can be shown to be :

$$V(x) = \frac{q}{4\pi\epsilon_0}\left[\frac{2}{\sqrt{L^2+x^2+xL}} - \frac{2}{\sqrt{L^2+x^2-xL}} - \frac{4x}{L^2-x^2}\right].$$

Question 043 MCQ

QUESTION

In the given circuit, a charge of

$$+80$$

$$\mu C$$

is given to the upper plate of the

$$4$$

$$\mu F$$

capacitor. Then in the steady state, the charge on the upper plate of the

$$3$$

$$\mu F$$

capacitor is

A $+32\,\mu C$

B $+40\,\mu C$

C $+48\,\mu C$

D $+80\,\mu C$

CORRECT OPTION

C $+48\,\mu C$

SOURCE

Physics • capacitor

EXPLANATION

Let the charges on the $3\mu\mathrm{F}$ and $2\mu\mathrm{F}$ capacitors be q and q' respectively. The charge on the lower plate of the $4\mu\mathrm{F}$ capacitor is $-80\mu\mathrm{C}$. The lower plate of the $4\mu\mathrm{F}$ capacitor and the upper plates of the $2\mu\mathrm{F}$ and $3\mu\mathrm{F}$ capacitors form an isolated system. Therefore, the net charge on this system must be zero, which can be represented as :

$q + q' - 80\mu\mathrm{C} = 0 \quad ....(\mathrm{i})$

The potentials $\$V = q/C\$$ across these two capacitors are equal, which gives :

$\frac{q}{3} = \frac{q'}{2} \quad ....(\mathrm{ii})$

By substituting q' from equation ii into equation i, we can solve for q and find :

$q = 48\mu\mathrm{C}$

Question 044 MCQ

QUESTION

A student is performing the experiment of resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

A 14.0 cm

15.2 cm

C 16.4 cm

D 17.6 cm

CORRECT OPTION

B 15.2 cm

SOURCE

Physics • waves

EXPLANATION

A student is conducting a resonance column experiment. The tube has a diameter of 4 cm, and the tuning fork vibrates at 512 Hz. The air temperature is 38^{o}C, where the speed of sound is 336 m/s. The zero mark on the meter stick aligns with the top of the tube. For the first resonance, the water level reads :

During the first resonance :

$$\frac{\lambda}{4} = l_1 + e$$

where l_1 is the length of the air column at resonance

Given that :

$$\text{end correction} = e = 0.6r = 0.6 \times 2 = 1.2\,\text{cm}$$

where r is the radius of the tube

The wavelength λ can be expressed as :

$$\lambda = 4(l_1 + e)$$

The frequency f is related to the wavelength λ by :

$$f = \frac{v}{\lambda} = \frac{v}{4(l_1 + e)}$$

Rearranging for l_1 :

$$4(l_1 + e) = \frac{v}{f}$$

Solving for l_1 :

$$l_1 = \frac{v}{4f} - e$$

Substitute the given values :

$$l_1 = \frac{336\,\mathrm{m/s}}{4 \times 512\,\mathrm{Hz}} - 0.012\,\mathrm{m}$$

$$l_1 = \frac{336}{2048} - 0.012$$

$$l_1 = 0.164\,\mathrm{m} - 0.012\,\mathrm{m}$$

$$l_1 = 0.152\,\mathrm{m}$$

Converting to cm :

$$l_1 = 15.2\,\mathrm{cm}$$

Question 045 MCQ

QUESTION

Two moles of ideal helium gas are in a rubber balloon at 30^{o}C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35^{o}C. The amount of heat required in raising the temperature is nearly $takeR = 8.31J/mol.K$

A 62 J

B 104 J

C 124 J

D 208 J

CORRECT OPTION

D 208 J

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Here's a breakdown of how to solve this problem:

Understanding the Concepts

- **Ideal Gas:** An ideal gas is a theoretical gas that follows the ideal gas law perfectly. The ideal gas law is a relationship between pressure $$P$$, volume $$V$$, temperature $$T$$, and the number of moles $$n$$ of a gas:

$$PV = nRT$$

, where

$$R$$

is the ideal gas constant.

- **Heat Capacity:** Heat capacity is the amount of heat energy required to raise the temperature of a substance by 1 degree Celsius $or 1 Kelvin$. For an ideal gas, we can use the specific heat capacity at constant volume $$C_v$$ or at constant pressure $$C_p$$.

- **Constant Volume Process:** In a constant volume process, the volume of the gas remains constant. The heat required to raise the temperature is given by:

$$Q = nC_v\Delta T$$

- **Constant Pressure Process:** In a constant pressure process, the pressure of the gas remains constant. The heat required to raise the temperature is given by:

$$Q = nC_p\Delta T$$

- **Molar Heat Capacities of Helium:** Helium is a monatomic gas. For monatomic gases, we have:

- $$C_v = \frac{3}{2}R$$

- $$C_p = \frac{5}{2}R$$

Solving the Problem

1. **Identify the Process:** Since the balloon is fully expandable, the pressure remains constant. This is a constant pressure process.

2. **Calculate the Heat:** Use the formula for heat at constant pressure:

$$Q = nC_p\Delta T$$

1. **Substitute the Values:**

- $$n = 2 \text{ moles}$$

- $$C_p = \frac{5}{2}R = \frac{5}{2}(8.31 \text{ J/mol.K})$$

- $$\Delta T = 35^\circ C - 30^\circ C = 5K$$

1. **Solve for Q:**

$$Q = (2 \text{ moles}) \times \left(\frac{5}{2} \times 8.31 \text{ J/mol.K}\right) \times (5K)$$

$$Q = 207.75 \text{ J}$$

Answer:

The amount of heat required to raise the temperature of the helium gas is approximately **208 J**. Therefore, the correct answer is **Option D**.

Question 046 MCQ

QUESTION

A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If

$$\rho_c$$

is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

A. more than half-filled if

$$\rho_c$$

is less than 0.5.

B. more than half-filled if

$$\rho_c$$

is more than 1.0.

C. half-filled if

$$\rho_c$$

is more than 0.5.

D

less than half-filled if

$$\rho_c$$

is less than 0.5.

CORRECT OPTION

A

more than half-filled if

$$\rho_c$$

is less than 0.5.

SOURCE

Physics • properties-of-matter

EXPLANATION

Consider a thin, uniform cylindrical shell that is closed at both ends and partially filled with water. This cylindrical shell is floating vertically in water, with exactly half of its height submerged. The relative density of the shell's material compared to water is denoted by

$$\rho_c$$

.

The inner radius of the cylindrical shell is r_1, the outer radius is r_2, the height is h, and the material density is ρ. The shell is filled with water, which has a density of ρ_w, up to a height of x. Given

$$\rho_c = \frac{\rho}{\rho_w}$$

, we know that the cylinder is in a half-submerged state, meaning $h/2$ of the cylinder's height is submerged in water.

In equilibrium, the weight of the cylindrical shell, including the water inside it, must equal the buoyant force $upthrust$ acting on it. This relationship can be expressed as follows:

$$\left(\pi r_2^2-\pi r_1^2\right) h \rho g+\pi r_1^2 x \rho_w g=\pi r_2^2\left(\frac{h}{2}\right) \rho_w g$$

By simplifying this equation, we get:

$$x=h\left[\frac{r_2^2}{r_1^2}\left(0.5-\rho_c\right)+\rho_c\right]=h\left[0.5+\frac{r_2^2-r_1^2}{r_1^2}\left(0.5-\rho_c\right)\right] \ldots \ldots \ldots \ldots(i)$$

From equation i, we can determine the condition under which x is greater than $0.5h$. This occurs if $\rho_c<0.5\left(\because r_2>r_1\right)$.

Question 047 MCQ

QUESTION

Two spherical planets P and Q have the same uniform density r, masses M_P and M_Q and surface areas A and 4A respectively. A spherical planet R also has uniform density r and its mass is (M_P + M_Q). The escape velocities from the planets P, Q and R are V_P, V_Q and V_R, respectively. Then

A. $V_Q > V_R > V_P$

B. $V_R > V_Q > V_P$

C. $$\frac{V_R}{V_P}=3$$

D $$\frac{V_P}{V_Q} = \frac{1}{2}$$

CORRECT OPTION

B $V_R > V_Q > V_P$

SOURCE

Physics • gravitation

EXPLANATION

Escape velocity, expressed as

$$v_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G \cdot \frac{4}{3}\pi R^3 \rho}{R}}$$

can be simplified to

$$\sqrt{\frac{8\pi G\rho}{3}}R.$$

The surface area, A_s, is given by

$$A_s = 4\pi R^2.$$

Considering planet Q's surface area is 4 times that of planet P:

$$A_Q = 4A = 4\pi R_Q^2$$

$$A_P = A = 4\pi R_P^2$$

therefore,

$$4R_P^2 = R_Q^2.$$

Solving for R_Q :

$$\Rightarrow \quad R_Q = 2R_P \quad ....(i).$$

For planet R, with mass $\mathrm{M_R = M_P + M_Q}$:

$$\rho \cdot \frac{4}{3}\pi \mathrm{R_R^3} = \rho \cdot \frac{4}{3}\pi(\mathrm{R_P^3 + R_Q^3})$$

Since

$$\mathrm{R_Q = 2R_P},$$

we have :

$$\mathrm{R_R^3 = R_P^3 + 8R_P^3 = 9R_P^3}.$$

Thus,

$$\mathrm{R_R = (9)^{1/3}R_P} \quad \text{........(ii)}.$$

From i and ii, it follows that :

$$R_R > R_Q > R_P.$$

Since

$$V_e \propto \mathrm{R},$$

it implies that :

$$\mathrm{V_R > V_Q > V_P}.$$

Also,

$$\frac{\mathrm{V_R}}{\mathrm{V_P}} = \frac{\mathrm{R_R}}{\mathrm{R_P}} = 9^{1/3},$$

and

$$\frac{\mathrm{V_P}}{\mathrm{V_Q}} = \frac{\mathrm{R_P}}{\mathrm{R_Q}} = \frac{1}{2}.$$

Question 048 MCQ

QUESTION

The figure shows a system consisting of i a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed

$$\omega$$

and ii an inner disc of radius 2R rotating anti-clockwise with angular speed

$$\frac{\omega}{2}$$

. The ring and disc are separated by frictionless ball bearings. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of

$$30^{\circ}$$

with the horizontal. Then with respect to the horizontal surface,

A. the point O has linear velocity

$$3R\omega\hat{i}$$

B. the point P has linear velocity

$$\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\widehat{k}$$

C. the point P has linear velocity

$$\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\widehat{k}$$

D. the point P has linear velocity

$$\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\widehat{k}$$

CORRECT OPTION

A

the point O has linear velocity

$$3R\omega\hat{i}$$

SOURCE

Physics • rotational-motion

EXPLANATION

The outer ring rolls without slipping, meaning the point on the ring in contact with the ground $denotedaspointC$ is stationary, i.e., $\vec{v}_{\mathrm{C}} = \overrightarrow{0}$. Therefore, the velocity of point O is :

$$\vec{v}_{\mathrm{O}} = \vec{v}_{\mathrm{C}} + \vec{\omega}_{\mathrm{o}} \times \vec{r}_{\mathrm{CO}} = \overrightarrow{0} + \omega\hat{\jmath} \times 3R\hat{k} = 3R\omega\hat{\imath}$$

Point P is located on the inner disc, which has an angular velocity $\vec{\omega}_{\mathrm{i}} = -\omega/2\hat{\jmath}$. The position vector from O to P is given by :

$$\vec{r}_{\mathrm{OP}} = R\cos 30^{\circ}\hat{\imath} + R\sin 30^{\circ}\hat{k} = \frac{\sqrt{3}R}{2}\hat{\imath} + \frac{R}{2}\hat{k}$$

Thus, the velocity of point P is calculated as follows :

$$\begin{aligned}\vec{v}_{\mathrm{P}} &= \vec{v}_{\mathrm{O}} + \vec{\omega}_{\mathrm{i}} \times \vec{r}_{\mathrm{OP}} \\ &= 3R\omega\hat{\imath} + \left(-\frac{\omega}{2}\hat{\jmath}\right) \times \left(\frac{\sqrt{3}R}{2}\hat{\imath} + \frac{R}{2}\hat{k}\right) \\ &= \frac{11\omega R}{4}\hat{\imath} + \frac{\sqrt{3}\omega R}{4}\hat{k}.\end{aligned}$$

Question 049 MCQ

QUESTION

Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement s is are correct?

A Both cylinders P and Q reach the ground at the same time.

B Cylinders P has larger linear acceleration than cylinder Q.

C Both cylinders reach the ground with same translational kinetic energy.

D Cylinder Q reaches the ground with larger angular speed.

CORRECT OPTION

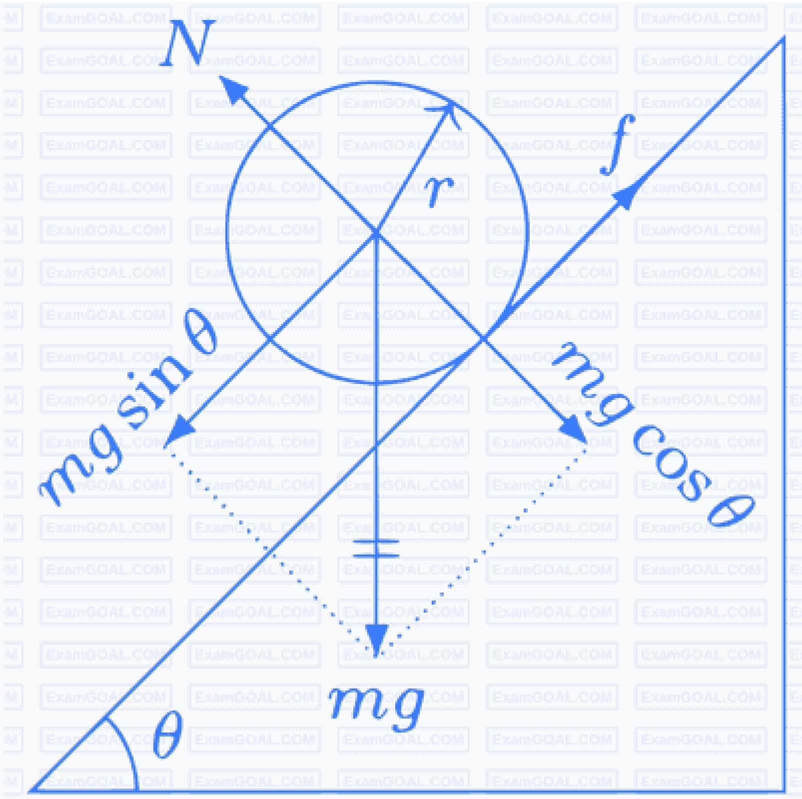
D Cylinder Q reaches the ground with larger angular speed.

SOURCE

Physics • rotational-motion

EXPLANATION

Since Cylinder P has most of its mass concentrated near its surface, its moment of inertia $aboutthecylinderaxis$ is greater than that of Cylinder Q, i.e., $I_P > I_Q$. The forces acting on the cylinder include its weight mg, the normal reaction N, and the frictional force f.



In the case of rolling without slipping, we have :

$$v = \omega r$$

$$a = \alpha r$$

The torque about the center of mass is related to α by :

$$\tau = rf = I\alpha \quad (1)$$

The force along the plane is related to a by :

$$mg\sin\theta - f = ma \quad (2)$$

By solving equations 1 and 2, we get :

$$a = \frac{g\sin\theta}{1+\frac{I}{mr^2}} \quad (3)$$

From equation 3, it follows that $a_P < a_Q \ since\$I_P > I_Q\$$. Therefore, Cylinder P reaches the ground later, has a lower velocity, lower angular velocity $\$\omega = \frac{v}{r}\$$, and lower translational kinetic energy.

For verification, the conservation of energy can be used :

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

This yields :

$\frac{1}{2}mv_P^2 < \frac{1}{2}mv_Q^2$

Thus :

$v_P < v_Q,\ \omega_P < \omega_Q,\ a_P < a_Q$

And the time relationships :

$t_P > t_Q$

Question 050 MCQ

QUESTION

Consider a disc rotating in the horizontal plane with a constant angular speed

$$\omega$$

about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y - z plane and is same for both pebbles with respect to the disc. Assume that i they land back on the disc before the disc has completed

$$\frac{1}{8}$$

rotation, ii their range is less than half the disc radius, and iii

$$\omega$$

remains constant throughout. Then

A P lands in the shaded region and Q in the unshaded region.

B P lands in the unshaded region and Q in the shaded region.

C Both P and Q land in the unshaded region.

D Both P and Q land in the shaded region.

CORRECT OPTION

C Both P and Q land in the unshaded region.

SOURCE

Physics • rotational-motion

EXPLANATION

$\left(\frac{1}{8}\right)^{\text{th}}$ rotation of the disc with a constant Ω implies that the disc has turned through

$$\frac{1}{8} \times 360^\circ = 45^\circ$$

The orientation of the disc now is

For the particle thrown from Q towards R, will have only the velocity given to it with respect to disc i.e., it possess a velocity only in the Y-Z plane and covers a range along OR. Hence, it can be seen to land on the disc in the unshaded region. Actually point O of the disc has zero velocity and Q being very close to O, will have only the velocity given to it w.r.t disc.

For the particle thrown from P , will have an additional velocity $= R\Omega$ along +X direction i.e., it will cover a range $= \frac{R}{2}$ along PO as well as a distance $= (R\omega)\frac{T}{8} = \frac{R}{8} \times 2\pi = \frac{\pi R}{4}$ along $+X$ direction

The resultant displacement of particle P is thus obtained as

$$\Rightarrow \quad d=\frac{R}{2} \sqrt{1+\frac{\pi^2}{4}}$$

$$\Rightarrow \quad \pi^2=10$$

$$d=\frac{\mathrm{R}}{4} \sqrt{14}$$

$$\tan \theta=\frac{\pi \mathrm{R} / 4}{\mathrm{R} / 2}=\frac{\pi}{2}$$

In the diagram shown,

$$\begin{aligned} PS &= R \cos \theta \\ &= R \times \frac{2}{\sqrt{14}} \\ &= \frac{2R}{\sqrt{14}} \end{aligned}$$

Since, $d>$ PS PS $=\frac{R}{7} \sqrt{14}$

Hence, the particle from P also lands in the unshaded region.

Question 051 MCQ

QUESTION

A loop carrying current

$$l$$

lies in the xy-plane as shown in the figure. The unit vector

$$\widehat{k}$$

is coming out of the plane of the paper. The magnetic moment of the current loop is

A $a^2 I\widehat{k}$

B $\left(\frac{\pi}{2}+1\right)a^2 I\widehat{k}$

C $-\left(\frac{\pi}{2}+1\right)a^2 I\widehat{k}$

D $(2\pi+1)a^2 I\widehat{k}$

CORRECT OPTION

B $\left(\frac{\pi}{2}+1\right)a^2 I\widehat{k}$

SOURCE

Physics • magnetism

EXPLANATION

Area of the loop

$$A=\left[a^2+4\times\frac{\pi\left(\frac{a}{2}\right)^2}{2}\right]\widehat{k}=\left[a^2+\frac{\pi a^2}{2}\right]\widehat{k}$$

Therefore, the magnetic moment of the current loop is

$$M = I \times A = I\left[a^2 + \frac{\pi a^2}{2}\right]\widehat{k} = \left[1 + \frac{\pi}{2}\right] I a^2 \widehat{k}$$

Question 052 MCQ

QUESTION

An infinite long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field,

$$\left|\overrightarrow{B}\right|$$

as a function of the radial distance r from the axis is best represented by

A

B

C

D

CORRECT OPTION

D

SOURCE

Physics • magnetism

EXPLANATION

r = distance of a point from centre.

For r

$$\leq$$

R/2 Using Ampere's circuital law,

$$\oint B.\ dl$$

or,

$$Bl = \mu_0(I_{in})$$

$$B(2\pi r) = \mu_0(I_{in})$$

$$B = \frac{\mu_0}{2\pi}\frac{I_{in}}{r}$$

...... i

Since,

$$I_{in} = 0$$

$$\therefore$$

$$B = 0$$

For

$$\frac{R}{2} \leq r \leq R$$

$$I_{in} = \left[\pi r^2 - \pi\left(\frac{R}{2}\right)^2\right]\sigma$$

Here

$$\sigma$$

= current per unit area.

Substituting in Eq. i, we have

$$B = \frac{\mu_0}{2\pi}\frac{\left[\pi r^2 - \pi\frac{R^2}{2}\right]\sigma}{r}$$

$$= \frac{\mu_0\sigma}{2r}\left(r^2 - \frac{R^2}{4}\right)$$

At

$$r = \frac{R}{2}, B = 0$$

At

$$r = R, B = \frac{3\mu_0\sigma R}{8}$$

For r

$$\geq$$

R

$$I_{in} = I_{Total} = I$$

say

Therefore, substituting in Eq. i, we have

$$B = \frac{\mu_0}{2\pi}.\frac{I}{r}$$

or,

$$B \propto \frac{1}{r}$$

Question 053 MCQ

QUESTION

Which of the following statements about the instantaneous axis $passingthroughthecentreofmass$ is correct?

A It is vertical for both Cases a and b.

B It is vertical for Case a; and is at 45

∘

to the xz-plane and lies in the plane of the disc for Case b.

C It is horizontal for Case a; and is 45

∘

to the xz-plane and is normal to the plane of the disc for Case b.

D It is vertical for Case a; and is 45

∘

to the xz-plane and is normal to the plane of the disc for Case b.

CORRECT OPTION

A It is vertical for both Cases a and b.

SOURCE

Physics • rotational-motion

EXPLANATION

As depicted in the figure shown here, when the system, as a whole, turns by 180

∘

, the disc also turns by 180

about its vertical axis. Hence, the instantaneous axis that passes through the centre of mass is vertical in both cases a and b.

Question 054 MCQ

QUESTION

Which of the following statements regarding the angular speed about the instantaneous axis $passingthroughthecentreofmass$ is correct?

A

It is

$$\sqrt{2}$$

$$\omega$$

for both cases.

B

It is

$$\omega$$

for case a; and

$$\omega$$

/

$$\sqrt{2}$$

for case b.

C

It is

$$\omega$$

for case a; and

$$\sqrt{2}$$

$$\omega$$

for case b.

D

It is

$$\omega$$

for both cases.

CORRECT OPTION

D

It is

$$\omega$$

for both cases.

SOURCE

Physics • rotational-motion

EXPLANATION

As depicted in the figure shown here, when the system, as a whole, turns by 180

$$\circ$$

, the disc also turns by 180

$$\circ$$

about vertical axis. Hence, for both cases a and b, the angular speed about the instantaneous axis that passes through the centre of mass is

$$\omega$$

.

Question 055 MCQ

QUESTION

What is the maximum energy of the anti-neutrino?

A. Zero.

B. Much less than 0.8 $\times$ 10^6 eV.

C. Nearly 0.8 $\times$ 10^6 eV.

D. much larger than 0.8 $\times$ 10^6 eV.

CORRECT OPTION

C. Nearly 0.8 $\times$ 10^6 eV.

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

$$K_p + K_{\overline{e}} + K_{\overline{v}}$$

= 0.8

$$\times$$

10^6 eV

When electron has zero kinetic energy is shared by antineutrino and proton.

Then,

$$K_p + K_{\overline{v}}$$

= 0.8

$$\times$$

10^6 eV

As antineutrino is very light mass in comparison to proton so it will have almost contribution in total energy.

$$\therefore$$

Its energy is almost 0.8

$$\times$$

10^6 eV

Question 056 MCQ

QUESTION

If the anti-neutrino had a mass of 3 eV/c^2 $wherecisthespeedoflight$ instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

A. $0 \leq K \leq 0.8 \times 10^6$ eV

B. 3.0 eV $\leq K \leq 0.8 \times 10^6$ eV

C. 3.0 eV $\leq K < 0.8 \times 10^6$ eV

D. $0 \leq K < 0.8 \times$

10^6 eV

CORRECT OPTION

D $0 \leq K < 0.8 \times 10^6$ eV

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

Total energy remains conserved. Energy is shared by antineutrino, proton and electron. Kinetic energy of electron has continuous spectrum and it is maximum when antineutrino does not share any kinetic energy.

So total energy is shared with proton and electron only.

$$\therefore K \leq 0.8 \times 10^6 \text{ eV}$$

and kinetic energy of electron will be minimum or zero when total energy is shared by proton and antineutrino.

$$\therefore 0$$

$\leq$

K

$\leq$

0.8

$\times$

10^6 eV

Question 057 MCQ

QUESTION

For light incident from air on a meta-material, the appropriate ray diagram is

A

B

C

D

CORRECT OPTION

C

SOURCE

Physics • geometrical-optics

EXPLANATION

For meta-material, the refractive index is negative. Let n_1 is refractive index of air and n_2 is refractive index of meta-material.

$$\therefore$$

From Snell's law,

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$$

Since, n_2 is negative, therefore

$$\theta$$

$_2$ is also negative. Hence, appropriate diagram c is correct.

Question 058 MCQ

QUESTION

Choose the correct statement.

A. The speed of light in the meta-material is v = c|n|.

B. The speed of light in the meta-material is

$$v = \frac{c}{|n|}$$

.

C. The speed of light in the meta-material is v = c.

D

The wavelength of the light in the meta-material (

$$\lambda$$

m) is given by

$$\lambda_m = \lambda_{air}|n|$$

, where

$$\lambda_{air}$$

is wavelength of the light in air.

CORRECT OPTION

B

The speed of light in the meta-material is

$$v = \frac{c}{|n|}$$

.

SOURCE

Physics • geometrical-optics

EXPLANATION

Refractive index for a medium

$$n = \left(\frac{c}{v}\right)$$

For meta-material,

$$n = |n|$$

$$\therefore$$

$$v = \frac{c}{|n|}$$

Question 059 MCQ

QUESTION

In the given circuit, the AC source has

$$\omega$$

= 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice s is are

A. The current through the circuit, I is 0.3 A.

B. The current through the circuit, I is 0.3

$$\sqrt{2}$$

A.

C. The voltage across 100

$$\Omega$$

resistor = 10

$$\sqrt{2}$$

V.

D. The voltage across 50

$$\Omega$$

resistor = 10 V.

CORRECT OPTION

A. The current through the circuit, I is 0.3 A.

SOURCE

Physics • alternating-current

EXPLANATION

Here,

$$\Omega$$

= 100 rad/s, L = 0.5 H, C = 100

$$\mu$$

F, V = 20 V

$$\therefore$$

$$X_L = \omega L = 100 \times 0.5 = 50\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\,\Omega$$

Impedance across capacitor,

$$Z_1 = \sqrt{R^2 + X_C^2}$$

$$= \sqrt{(100)^2 + (100)^2}$$

$$Z_1 = 100\sqrt{2}\,\Omega$$

$$\therefore$$

$$I_1 = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}\,A$$

Voltage across 100

$$\Omega$$

$$V = I_1 \times 100 = \frac{1}{5\sqrt{2}} \times 100 = 10\sqrt{2}\,V$$

Impedance across inductance,

$$Z_2 = \sqrt{R^2 + (X_L)^2} = \sqrt{(50)^2 + (50)^2}$$

$$Z_2 = 50\sqrt{2}\,\Omega$$

$$\therefore$$

$$I_2 = \frac{20}{50\sqrt{2}} = \frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5}$$

Now, voltage across

$$50\,\Omega = \frac{\sqrt{2}}{5} \times 50 = 10\sqrt{2}$$

$$I_1 = \frac{1}{5\sqrt{2}} A$$

at 45

$$\circ$$

leading

$$I_2 = \frac{\sqrt{2}}{5} A$$

at 45

$$\circ$$

lagging

$$\therefore$$

Current through circuit

$$I_{net} = \sqrt{I_1^2 + I_2^2} = \sqrt{\left(\frac{1}{5\sqrt{2}}\right)^2 + \left(\frac{\sqrt{2}}{5}\right)} = 0.3\,A$$

Question 060 MCQ

QUESTION

A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it, the correct statement s is are

A. the emf induced in the loop is zero if the current is constant.

B. the emf induced in the loop is finite if the current is constant.

C. the emf induced in the loop is zero if the current decreases at a steady state

D. the emf induced in the loop is finite if the current decreases at a steady state.

CORRECT OPTION

A. the emf induced in the loop is zero if the current is constant.

SOURCE

Physics • electromagnetic-induction

EXPLANATION

The magnetic field due to an infinitely long conductor is circumferential.

Since this conductor is placed along the diameter of circular loop $see figure$, the total magnetic flux through the circular wire loop is always zero.