JEE Advanced 2016 Paper 2 $Offline$ $54Questions$

Question 001 MCQ

QUESTION

Extraction of copper from copper pyrite ($CuFeS_2$) involves

A crushing followed by concentration of the ore by froth flotation

B removal of iron as slag

C self-reduction step to produce ‘blister copper’ following evolution of SO_2

D refining of ‘blister copper’ by carbon reduction

CORRECT OPTION

A crushing followed by concentration of the ore by froth flotation

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

a In the extraction of copper from copper pyrite ($CuFeS_2$), after crushing, concentration of ore is done by froth floatation process.

b Iron is removed as slag.

$$\underset{}{FeO} + SiO_2 \longrightarrow \underset{\text{(Slag)}}{FeSiO_3}$$

c Auto-reduction :

$$2Cu_2O + Cu_2S \longrightarrow \underset{\text{(Blister copper)}}{6Cu} + SO_2$$

d Blister copper is finally purified by electrolytic refining.

Question 002 MCQ

QUESTION

For the following electrochemical cell at 298 K

Pts | H_2 $g, 1bar$ | H^+ $aq, 1M$ || M^{4+} aq, M^{2+} aq | Pt s

E_{cell} = 0.092 V when

$$\frac{[M^{2+}(aq)]}{[M^{4+}(aq)]}$$

= 10^x

Give,

$$E^o_{M^{4+}/M^{2+}}$$

= 0.151 V; 2.303 RT/F = 0.059 V

The value of x is

A -2

B -1

C 1

D 2

CORRECT OPTION

D 2

SOURCE

Chemistry • electrochemistry

EXPLANATION

For the given electrochemical cell, the half-cell reactions are

- At anode : $H_2 g$

$\rightleftharpoons$

$2H^+ aq$ + 2e

–

- At cathode : $M^{4+} aq$ + 2e

–

$\rightleftharpoons$

$M^{2+}aq$.

The overall cell reaction is

$M^{4+}aq$ + H_2g

$$\rightarrow$$

$M^{2+}aq$ + $2H^+aq$

From Nernst equation, we have

$$E_{cell} = E^o_{cell} - \frac{2.303RT}{nF}\log\frac{[M^{2+}][H^+]^2}{[M^{4+}]p_{H_2}}$$

Substituting the given values, we get

$$E_{cell} = (E^o_{M^{4+}/M^{2+}} - E^o_{H^+/H_2})\frac{0.059}{2}\log\frac{[M^{2+}][H^+]^2}{[M^{4+}]p_{H_2}}$$

$$0.092 = (0.151 - 0) - \frac{0.059}{2}\log 10^x$$

$$0.092 = 0.151 - 0.0245\log 10^x$$

$$-0.059 = -0.0245x \Rightarrow x = 2$$

Question 003 MCQ

QUESTION

Mixture s showing positive deviation from Raoult's law at 35°C is are

A carbon tetrachloride + methanol

B carbon disulphide + acetone

C benzene + toluene

D phenol + aniline

CORRECT OPTION

A carbon tetrachloride + methanol

SOURCE

Chemistry • solutions

EXPLANATION

When intermolecular attraction between two components A and B in the mixture is same as between A and A or B and B, hence it is a case of ideal solution. When intermolecular attraction between A and B in a mixture is smaller than that between A and A or B and B, then mixture is more vaporised, bp is lowered. It is a case of positive deviation from Raoult's law. When intermolecular attraction between A and B is higher than that between A and A or B and B, then mixture is less vaporised, bp is increased. It is a case of negative deviation.

a Methanol molecules CH_3OH are hydrogen bonded. In a mixture of CCl_4 and CH_3OH, extent of H-bonding is decreased. Mixture is more vaporised thus, positive deviation from Raoult's law.

b Acetone molecules have higher intermolecular attraction due to dipole-dipole interaction. With CS_2 , this interaction is decreased thus, positive deviation.

c Mixture of benzene and toluene forms ideal solution.

d Phenol and aniline have higher interaction due to intermolecular H-bonding. Hence, negative deviation.

Question 004 MCQ

QUESTION

The CORRECT statement$s$ for cubic close packed ccp three dimensional structure is are :

A The number of the nearest neighbours of an atom present in the topmost layer is 12

B The efficiency of an atom packing is 74%

C The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively

D The unit cell edge length is

$$2\sqrt{2}$$

times the radius of the atom

CORRECT OPTION

C The number of octahedral and tetrahedral voids per atom are 1 and 2, respectively

SOURCE

Chemistry • solid-state

EXPLANATION

Coordination number cannot be 12, for any atom in the topmost layer, as there is no layer above it. Thus, each atom is in contact with six atoms in the same layer and three atoms from the layer below it.

For cubic close packing, we have

Packing fraction =

$$\frac{Volume\, of\, four\, spheres\, in\, the\, unit\, cell}{Total\, volume\, of\, the\, unit\, cell}$$

$$= 4 \times \frac{(4/3)\pi r^3}{16\sqrt{2}r^3} = \frac{\pi}{3\sqrt{2}} = 0.74 = 74\%$$

In fcc unit cell, the effective number of atoms is

One atom at each corner = 8 corner atoms

$$\times \frac{1}{8} = 1$$

Atoms at each of the six face centres :

6 face centered atoms

$$\times \frac{1}{2} = 3$$

.

Number of octahedral voids = 4.

Number of tetrahedral voids = 8.

Therefore, per atom, there is one octahedral void and two tetrahedral voids.

In fcc $orccp$, the unit edge length is given by

$$\sqrt{2}a = 4r \Rightarrow a = \frac{4r}{\sqrt{2}} = 2\sqrt{2}r$$

Question 005 MCQ

QUESTION

Paragraph

Thermal decomposition of gaseous X_2 to gaseous X at 298 K takes place according to the following equations:

X_2 g

$$\leftrightharpoons$$

2X g

The standard reaction Gibbs energy,

$$\Delta_r G^o$$

, of this reaction is positive. At the start of the reaction, there is one mole of X_2 and no X. As the reaction proceeds, the number of moles of X formed is given by

$$\beta$$

. Thus,

$$\beta_{equilibrium}$$

is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally. (Given R = 0.083 L bar K^{-1} mol^{-1})

Question

The INCORRECT statement among the following for this reaction, is

A Decrease in the total pressure will result in formation of more moles of gaseous X

B At the start of the reaction, dissociation of gaseous X_2 takes place spontaneously

C

$$\beta_{equilibrium}$$

= 0.7

D $K_c < 1$

CORRECT OPTION

C

$$\beta_{equilibrium}$$

= 0.7

SOURCE

Chemistry • chemical-equilibrium

EXPLANATION

On decreasing total pressure, the reaction will move in the forward direction where number of gaseous molecules is less.

Hence, option A is correct.

At the start of reaction Q = 0. From

$$\Delta$$

G =

$$\Delta$$

G^o + RT lnQ, we have that for Q = 0 at the start of reaction

$$\Delta$$

$_rG$ is negative, this causes dissociation of X_2 to take place spontaneously.

Hence, option B is correct.

If $b_{equilibrium}$ = 0.7, then value of equilibrium constant is

$$K_p = \frac{8\beta_{eq}^2}{4 - \beta_{eq}^2} = \frac{8(0.7)^2}{7 - (0.7)^2}$$

The value of K_p is greater than 1, which is not possible as given that

$$\Delta$$

$G^o > 0$ for the reaction.

Hence, option C is incorrect.

As

$$\Delta$$

$G^o > 0$ and

$$\Delta$$

$G^o =$

$$-$$

RT $\ln(K_p)$

$$\Delta$$

$G^o > 1$, so K_p should be less than 1.

We know that

$$K_p = K_c(RT)^{\Delta n_g}$$

$$K_c = \frac{K_p}{(RT)^{\Delta n_g}}$$

Therefore, $K_c < K_p$. As K_p is less than 1, so K_c is also less than 1.

Hence, option D is correct.

Question 006 MCQ

QUESTION

Paragraph

Thermal decomposition of gaseous X_2 to gaseous X at 298 K takes place according to the following equations:

X_2 g

$$\leftrightharpoons$$

2X g

The standard reaction Gibbs energy,

$$\Delta_r G^o$$

, of this reaction is positive. At the start of the reaction, there is one mole of X_2 and no X. As the reaction proceeds, the number of moles of X formed is given by

$$\beta$$

. Thus,

$$\beta_{equilibrium}$$

is the number of moles of X formed at equilibrium. The reaction is carried out at a constant total pressure of 2 bar. Consider the gases to behave ideally. (Given R = 0.083 L bar K^{-1} mol^{-1})

Question

The equilibrium constant K_p for this reaction at 298 K, in terms of

$$\beta_{equilibrium}$$

, is

A

$$\frac{8\beta^2_{equilibrium}}{2 - \beta_{equilibrium}}$$

B

$$\frac{8\beta^2_{equilibrium}}{4 - \beta^2_{equilibrium}}$$

C

$$\frac{4\beta^2_{equilibrium}}{2 - \beta_{equilibrium}}$$

D

$$\frac{4\beta^2_{equilibrium}}{4 - \beta^2_{equilibrium}}$$

CORRECT OPTION

B

$$\frac{8\beta^2_{equilibrium}}{4 - \beta^2_{equilibrium}}$$

SOURCE

Chemistry • chemical-equilibrium

EXPLANATION

Total number of moles at equilibrium = $1 +$ $$\alpha$$ = 1 + (b_{eq} / 2)

Therefore,

$$p_x = \left(\frac{\beta_{eq}}{1 + (\beta_{eq}/2)} p_T\right)$$

$$p_{x_2} = \left(\frac{1 - (\beta_{eq}/2)}{1 + (\beta_{eq}/2)} p_T\right)$$

Therefore, the equilibrium constant K_p is given by

$$K_p = \frac{p_x^2}{p_{x_2}} = \frac{\left(\frac{\beta_{eq}}{1+(\beta_{eq}/2)} p_T\right)}{\left(\frac{1-(\beta_{eq}/2)}{1+(\beta_{eq}/2)} p_T\right)}$$

$$= \frac{\beta_{eq}^2}{1 - (\beta_{eq}^2/4)} p_T = \frac{\beta_{eq}^2}{1 - (\beta_{eq}^2/4)} \times 2 = \frac{8\beta_{eq}^2}{4 - \beta_{eq}^2}$$

Question 007 MCQ

QUESTION

According to Molecular Orbital Theory, which of the following statements is are correct?

A

$$C_2^{2-}$$

is expected to be diamagnetic

B

$$O_2^{2+}$$

expected to have a longer bond length than O_2

C

$$N_2^+$$

and

$$N_2^-$$

have the same bond order

D

$$He_2^+$$

has the same energy as two isolated He atoms

CORRECT OPTION

A

$$C_2^{2-}$$

is expected to be diamagnetic

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

Option A : According to molecular orbital theory, the arrangement of the electrons in the molecular orbitals is as follows :

For

$$C_2^{2-}$$

, the total number of electrons is 14. The molecular orbital configuration is

$$\sigma_{1s^2}\,\sigma^*_{1s^2}\,\sigma_{2s^2}\,\sigma^*_{2s^2}\,\pi_{2p_x^2} = \pi_{2p_y^2}\,\sigma_{2p_z^2}$$

There is no unpaired electron and thus it is diamagnetic.

Option B : For

$$O_2^{2+}$$

, the total number of electrons is 14. The molecular orbital configuration is

$$\sigma_{1s^2}\ \sigma^*_{1s^2}$$
$$\sigma_{2s^2}\ \sigma^*_{2s^2}$$
$$\sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2p_y^2}\ \pi^*_{2p_x^0}\ = \pi^*_{2p_y^0}$$

Thus the bond order = $10\$\$ - \$\$4$/2 = 3;

Molecular orbital configuration of O_2 $16 electrons$ is

$$\sigma_{1s^2}\ \sigma^*_{1s^2}$$
$$\sigma_{2s^2}\ \sigma^*_{2s^2}$$
$$\sigma_{2p_z^2}\ \pi_{2p_x^2} = \pi_{2p_y^2}\ \pi^*_{2p_x^1}\ = \pi^*_{2p_y^1}$$

So O

$$2$$

has 2 unpaired electrons. for O_2, bond order = 2.

As bond order is inversely proportional to bond length, thus the bond length of

$$O_2^{2+}$$

is less than the bond length of O_2.

Option C : For

$$N_2^+$$

, the total number of electrons is 13. The molecular orbital configuration is

$$\sigma_{1s^2}\,\sigma^*_{1s^2}\,\sigma_{2s^2}\,\sigma^*_{2s^2}\,\pi_{2p_x^2} = \pi_{2p_y^2}\,\sigma_{2p_z^1}$$

Bond order of

$$N_2^+$$

= $9$$ - $$4$/2 = 2.5

$$N_2^-$$

has 15 electrons.

Moleculer orbital configuration of

$$N_2^-$$

is

$$\sigma_{1s^2}\,\sigma^*_{1s^2}\,\sigma_{2s^2}\,\sigma^*_{2s^2}\,\pi_{2p_x^2} = \pi_{2p_y^2}\,\sigma_{2p_z^2}\,\pi^*_{2p_x^1} = \pi^*_{2p_y^0}$$

$$\therefore$$

$N_a = 5$

$N_b = 10$

$$\therefore$$

BO =

$$\frac{1}{2}[10-5]=2.5$$

Thus, the bond orders are the same.

Option D : As some energy is released during the formation of

$$He_2^+$$

from two isolated He atoms, thus it has lesser energy as compared to two isolated He atoms.

Question 008 MCQ

QUESTION

The geometries of the ammonia complexes of Ni^{2+}, Pt^{2+} and Zn^{2+}, respectively, are

A octahedral, square planar and tetrahedral

B square planar, octahedral and tetrahedral

C tetrahedral, square planar and octahedral

D octahedral, tetrahedral and square planar

CORRECT OPTION

A octahedral, square planar and tetrahedral

SOURCE

Chemistry • coordination-compounds

EXPLANATION

The hybridisation of central atoms and the geometries of ammonia complex are tabulated as follows:

Metal Ion	Hybridisation	Coordination Number	Compl
$Ni^{2+} : [Ar]3d^8 4s^2$	d^2sp^3	6	[Ni(
$Pt^{2+} : [Xe]4f^4 5d^9 6s^1$	dsp^2	4	[Pt(
$Zn^{2+} : [Ar]3d^{10} 4s^2$	sp^3	4	[Zn(

Question 009 MCQ

QUESTION

The correct order of acidity for the following compounds is

A I > II > III > IV

B III > I > II > IV

C III > IV > II > I

D I > III > IV > II

CORRECT OPTION

A I > II > III > IV

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Due to ortho-effect, I and II are stronger acid than III and IV. Due to two ortho hydroxyl groups in I, it is stronger acid than II. III is a stronger acid than IV because at m-position,

—

OH group cannot exert its +R effect but can only exert its

—

I effect while at p-position,

—

OH group exerts its strong +R effect. Thus, the correct order of acidity is :

$I > II > III > IV$

Question 010 MCQ

QUESTION

The major product of the following reaction sequence is:

A

B

C

D

CORRECT OPTION

D

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The major product of the given reaction sequence is :

Question 011 MCQ

QUESTION

In the following reaction, sequence in aqueous solution, the species X, Y and Z, respectively, are

$$S_2O_3^{2-} \xrightarrow{Ag^+} \underset{Clear\ solution}{X} \xrightarrow{Ag^+} \underset{White\ precipitate}{Y} \xrightarrow{With\ time} \underset{Black\ precipitate}{Z}$$

A

$[Ag(S_2O_3)_2]^3$

−

, $Ag_2S_2O_3$, Ag_2S

B

$[Ag(S_2O_3)_2]^5$

−

, Ag_2SO_3, Ag_2S

C

$[Ag(SO_3)_2]^3$

−

, $Ag_2S_2O_3$, Ag

D

$[Ag(SO_3)_3]^3$

−

, Ag_2SO_4, Ag

CORRECT OPTION

$[Ag(S_2O_3)_2]^3$

A

$-$

, $Ag_2S_2O_3$, Ag_2S

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

$$\underset{(Excess)}{Ag^{+}2S_2O_3^{2-}} \rightarrow \underset{Argentothiosulphate\ (X)\ (Soluble)}{[Ag(S_2O_3)_2]^{3-}} \xrightarrow{Ag^{+}} \underset{(Y)\ (White\ ppt.)}{Ag_2S_2O_3} \underset{(due\ to\ hydrolysis)}{\xrightarrow{With\ time}} (Z)$$

Question 012 MCQ

QUESTION

The qualitative sketches I, II and III given below show the variation of surface tension with molar concentration of three different aqueous solutions of KCl, CH_3OH and $CH_3(CH_2)_{11}$ OSO

$\overline{3}$

Na^+ at room temperature. The correct assignment of the sketches is

A

I : KCl

II : CH_3OH

III : $CH_3(CH_2)_{11}$ OSO

$\overline{3}$

Na^+

B

I. $CH_3(CH_2)_{11}$ OSO

$\bar{3}$

Na^+

II. CH_3OH

III. KCl

C

I. KCl

II. $CH_3(CH_2)_{11}$ OSO

$\bar{3}$

Na^+

III. CH_3OH

D

I. CH_3OH

II. KCl

III. $CH_3(CH_2)_{11}$ OSO

$\bar{3}$

Na^+

CORRECT OPTION

I. CH_3OH

II. KCl

D III. $CH_3(CH_2)_{11}$ OSO

$\bar{3}$

Na^+

SOURCE

Chemistry • solutions

EXPLANATION

I. (CH_3OH) : Surface tension decreases as concentration increases.

II. KCl : Surface tension increases with concentration for ionic salt.

III. [$CH_3(CH_2)_{11}$ OSO

$\bar{3}$

Na^+] : It is an anionic detergent.

There is decrease in surface tension before micelle formation, and after CMC $Critical Micelle Concentration$ is attained, no change in surface tension.

Question 013 MCQ

QUESTION

For "invert sugar", the correct statement s is are

$Given: specificrotationsof(+$-sucrose, $+$-maltose, L-$\$\$ - \$\$$-glucose and L-$+$-fructose in aqueous solution are +66

$\circ$

, +140

$\circ$

,

$-$

52

$\circ$

and +92

$\circ$

, respectively.)

A "invert sugar" is prepared by acid catalyzed hydrolysis of maltose.

B "invert sugar" is an equimolar mixture of D-$+$-glucose and D-$\$\$ - \$\$$-fructose.

C specific rotation of "invert sugar" is

$-$

20

$\circ$

.

D on reaction with Br_2 water, "invert sugar" forms saccharic acid as one of the products.

CORRECT OPTION

B "invert sugar" is an equimolar mixture of D-$+$-glucose and D-$$-$$-fructose.

SOURCE

Chemistry • biomolecules

EXPLANATION

Invert sugar is prepared by acid catalyzed hydrolysis of sucrose.

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{HCl} \underbrace{\underset{D-(+)-Glu\cos e}{C_6H_{12}O_6} + \underset{D-(-)-Fructose}{C_6H_{12}O_6}}_{Invert\ sugar}$$

Specific rotation of invert sugar is

$$\alpha$$

mix = 0.5

$\times$

$+52$ + 0.5

$\times$

$$-$$92 = + 26

$-$

46 =

$-$

20

$\circ$

On reaction with Br_2 water, invert sugar forms gluconic acid as one of the products. Br_2 water oxidises glucose into gluconic acid and fructose is not oxidised by it.

Question 014 MCQ

QUESTION

Among the following reaction s, which gives $give$ tert-butyl benzene as the major product is are

A

B

C

D

CORRECT OPTION

B

SOURCE

Chemistry • hydrocarbons

EXPLANATION

Question 015 MCQ

QUESTION

Reagent s which can be used to bring about the following transformation is are

A $LiAlH_4$ in $(C_2H_5)_2O$

B BH_3 in THF

C $NaBH_4$ in C_2H_5OH

D Raney Ni/H_2 in THF

CORRECT OPTION

C $NaBH_4$ in C_2H_5OH

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Option A : $LiAlH_4$ in $(C_2H_5)_2$ reduces ester $$ - $$COOR$, carboxylic acid $$ - $$COOH$ epoxide, aldehydes and ketones.

Option B : BH_3 in THF reduces

—

COOH and aldehydes into alcohols. However, it does not reduces esters and epoxides.

Option C : NABH$_4$ in C_2H_5OH reduces only aldehydes and ketones into alcohols, but it does not reduce to others.

Option D : Raney Ni/H_2 in THF does not reduce to

—

COOH,

—

COOR and epoxide but it can reduce aldehyde into alcohols.

Hence, options C and D are correct.

Question 016 MCQ

QUESTION

The nitrogen containing compound produced in the reaction of HNO_3 with P_4O_{10}

A can also be prepared by reaction of P_4 and HNO_3.

B is diamagnetic.

C contains one N

—

N bond.

D reacts with Na metal producing a brown gas.

CORRECT OPTION

B is diamagnetic.

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reaction of HNO_3 and P_4O_{10} produces N_2O_5.

$4HNO_3$ + P_4O_{10}

$$\rightarrow$$

$2N_2O_5$ + $4HPO_3$

The reaction of HNO_3 with P_4 does not yield N_2O_5.

P_4 + 20 HNO_3

$$\rightarrow$$

$4H_3PO_4$ + 20 NO_2 + 4 H_2O

The structure of N_2O_5 has one N

$$-$$

O

$$-$$

N bond, but no N

$$-$$

N bond. It is diamagnetic in nature.

N_2O_5 reacts with sodium metal to produce NO_2 $brown gas$

N_2O_5 + Na

$$\rightarrow$$

$NaNO_3$ +

$$\underset{Brown\ gas}{NO_2 \uparrow}$$

Question 017 MCQ

QUESTION

The compound R is

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Question 018 MCQ

QUESTION

The compound T is

A Glycine

B Alanine

C Valine

D Serine

CORRECT OPTION

B Alanine

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Question 019 MCQ

QUESTION

Let

$$a,\, b \in R\, and\, a^2\, + b^2 \neq 0$$

. Suppose

$$S = \left\{ Z \in C : Z = \frac{1}{a + ibt}, + \in R, t \neq 0 \right\}$$

, where

$$i = \sqrt{-1}$$

. Ifz = x + iy and z

$$\in$$

S, then x, y lies on

A

the circle with radius

$$\frac{1}{2a}$$

and centre

$$\left\{ \frac{1}{2a}, 0 \right\} for\, a > 0\, , b \neq\, 0$$

B

the circle with radius

$$-\frac{1}{2a}$$

and centre

$$\left\{ -\frac{1}{2a}, 0 \right\} for\, a < 0\, , b \neq\, 0$$

C

the x-axis for

$$a \neq \ 0, b \neq \ 0$$

D

the y-axis for

$$a = \ 0, b \neq \ 0$$

CORRECT OPTION

D

the y-axis for

$$a = \ 0, b \neq \ 0$$

SOURCE

Mathematics • complex-numbers

EXPLANATION

Here,

$$x + iy = \frac{1}{a + ibt} \times \frac{a - ibt}{a - ibt}$$

$$\therefore$$

$$x + iy = \frac{a - ibt}{a^2 + b^2t^2}$$

Let a

$$\neq$$

0, b

$$\neq$$

0

$$\therefore$$

$$x = \frac{a}{a^2 + b^2t^2}$$

and

$$y = \frac{-bt}{a^2 + b^2t^2}$$

$$\Rightarrow \frac{y}{x} = \frac{-bt}{a} \Rightarrow t = \frac{ay}{bx}$$

On putting

$$x = \frac{a}{a^2 + b^2t^2}$$

, we get

$$x\left(a^2 + b^2 . \frac{a^2y^2}{b^2x^2}\right) = a$$

$$\Rightarrow a^2(x^2 + y^2) = ax$$

or,

$$x^2 + y^2 - \frac{x}{a} = 0$$

...... i

or,

$$\left(x - \frac{1}{2a}\right)^2 + y^2 = \frac{1}{4a^2}$$

$$\therefore$$

Option a is correct.

For a

$$\neq$$

0 and b = 0,

$$x + iy = \frac{1}{a} \Rightarrow x = \frac{1}{a}, \; y = 0$$

$$\Rightarrow$$

z lies on X-axis.

$$\therefore$$

Option c is correct.

For a = 0 and b

$$\neq$$

0,

$$x + iy = \frac{1}{ibt}$$

$$\Rightarrow x = 0,\ y = -\frac{1}{bt}$$

$$\Rightarrow$$

z lies on Y-axis.

$$\therefore$$

Option d is correct.

Question 020 MCQ

QUESTION

The value of

$$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4} + \frac{(k-1)\pi}{6}\right) \sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)}$$

is equal to

A

$$3 - \sqrt{3}$$

B

$$2\left(3-\sqrt{3}\right)$$

C

$$2\left(\sqrt{3}-1\right)$$

D

$$2\left(2-\sqrt{3}\right)$$

CORRECT OPTION

C

$$2\left(\sqrt{3}-1\right)$$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

It is given that,

$$\sum_{k=1}^{13} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{(k-1)\pi}{6}\right)\sin\left(\frac{\pi}{4}+\frac{k\pi}{6}\right)}$$

Let

$$\alpha=\frac{\pi}{4}$$

and

$$\beta=\frac{\pi}{6}$$

. Therefore,

$$\sum_{k=1}^{13} \frac{1}{\sin(\alpha + k\beta) sin(\alpha + (k-1)\beta)}$$

$$= \frac{1}{\sin\beta} \sum_{k=1}^{13} \frac{\sin((\alpha + k\beta) - (\alpha + (k-1)\beta))}{\sin(\alpha + k\beta)\sin(\alpha + (k-1)\beta)}$$

$$= \frac{1}{\sin\beta} \sum_{k=1}^{13} (\cot(\alpha + (k-1)\beta) - \cot(\alpha + k\beta))$$

$$= \frac{1}{\sin\beta}\{[\cot(\alpha) - \cot(\alpha+\beta)] + [\cot(\alpha+\beta) - \cot(\alpha+2\beta)] + \ldots\ldots + [\cot(\alpha$$

$$= \frac{1}{\sin\beta}(\cot\alpha - \cot(\alpha + 13\beta))$$

$$= \frac{1}{\sin(\pi/6)}\left(\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right)$$

$$= 2(1 - 2 + \sqrt{3}) = 2(\sqrt{3} - 1)$$

Question 021 MCQ

QUESTION

Let

$$\widehat{u} = u_1\hat{i} + u_2\hat{j} + u_3\widehat{k}$$

be a unit vector in

$$R^3$$

and

$$\widehat{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\widehat{k}\right).$$

Given that there exists a vector

$$\overrightarrow{v}$$

in

$$R^3$$

such that

$$\left|\widehat{u} \times \overrightarrow{v}\right| = 1$$

and

$$\widehat{w}.\left(\widehat{u} \times \overrightarrow{v}\right) = 1.$$

Which of the following statement s is are correct?

A

There is exactly one choice for such

$$\overrightarrow{v}$$

B

There are infinitely many choices for such

$$\overrightarrow{v}$$

C

If

$$\widehat{u}$$

lies in the

$$xy$$

-plane then

$$|u_1| = |u_2|$$

If

$$\widehat{u}$$

D lies in the

$$xz$$

-plane then

$$2|u_1| = |u_3|$$

CORRECT OPTION

C If

$$\widehat{u}$$

lies in the

$$xy$$

-plane then

$$|u_1| = |u_2|$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

We have

$$\widehat{u} = u_1\hat{i} + u_2\hat{j} + u_3\widehat{k}$$

That is,

$$|\widehat{u}| = 1 = \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$\Rightarrow u_1^2 + u_2^2 + u_3^2 = 1$$

Also, it is given that

$$\widehat{\omega} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\widehat{k})$$

That is,

$$\left|\widehat{\omega}\right| = 1$$

Now,

$$\left|\widehat{u} \times \overrightarrow{v}\right| = 1$$

That is,

$$\left|\widehat{u}\right| \left|\overrightarrow{v}\right| \sin\theta = 1 \Rightarrow \left|\overrightarrow{v}\right| = \frac{1}{\sin\theta}$$

which shows that there are infinitely many possible values exist for

$$\overrightarrow{v}$$

$here\$\$\theta\$\$isanglebetweenthevectors\$\$\overrightarrow{v}\$\$and\$\$\widehat{u}\$\$.$

Hence, option B is correct.

Now,

$$\widehat{\omega}\,.\,(\widehat{u} \times \overrightarrow{v}) = 1$$

$$\left|\widehat{\omega}\,.\,(\widehat{u} \times \overrightarrow{v})\right| = 1$$

That is,

$$\left|\widehat{\omega}\right| \left|\widehat{u} \times \overrightarrow{v}\right| \cos\alpha = 1$$

where

$$\alpha$$

is the angle between

$$\widehat{\omega}$$

and

$$\widehat{u} \times \overrightarrow{v}$$

.

Therefore, 1 1 cos

$$\alpha$$

= 1

$$\Rightarrow$$

a = 0

which means that

$$\widehat{\omega}$$

and

$$\widehat{u} \times \overrightarrow{v}$$

are parallel vector or

$$\widehat{\omega}$$

is perpendicular vector to

$$\widehat{u}$$

and

$$\overrightarrow{v}$$

.

$$\widehat{u} \,.\, \widehat{\omega} = 0$$

$$(u_1\hat{i} + u_2\hat{j} + u_3\widehat{k}) \,.\, \left(\frac{(\hat{i} + \hat{j} + 2\widehat{k})}{\sqrt{6}}\right) = 0$$

$$u_1 + u_2 + 2u_3 = 0$$

If

$$\widehat{u}$$

lies in xy-plane then u_3 = 0. Therefore,

$$u_1 + u_2 = 0 \Rightarrow u_1 = -u_2 \Rightarrow |u_1| = |u_2|$$

Hence, option C is correct.

Question 022 MCQ

QUESTION

Let

$$P$$

be the image of the point

$$(3, 1, 7)$$

with respect to the plane

$$x - y + z = 3.$$

Then the equation of the plane passing through

$$P$$

and containing the straight line

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$

is

A $x + y - 3z = 0$

B $3x + z = 0$

C $x - 4y + 7z = 0$

D

$$2x - y = 0$$

CORRECT OPTION

C

$$x - 4y + 7z = 0$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Let P(x_1, y_1, z_1) image of Q$3, 1, 7$ w.r.t. the plane x

$$-$$

y + z = 3.

Let R be the point on plane which is midpoint of the line joining P and Q.

The equation of the line PQ is

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-7}{1} = \lambda$$

Therefore, x, y, z = $3 + \$\$\lambda\$\$, 1\$\$ - \$\$\$\$\lambda\$\$, 7 + \$\$\lambda\$\$$ lies on plane.

$$3 + \lambda - 1 + \lambda + 7 + \lambda = 3$$
$$3\lambda + 6 = 0 \Rightarrow \lambda = -2$$

The point R is $3\$\$ - \$\$2, 3, 7\$\$ - \$\$2$ = $1, 3, 5$.

Now,

$$\frac{x_1 + 3}{2} = 1 \Rightarrow x_1 = -1$$

$$\frac{y_1 + 1}{2} = 3 \Rightarrow y_1 = 5$$

$$\frac{z_1 + 7}{2} = 5 \Rightarrow z_1 = +3$$

That is, the point P is P$\$\$ - \$\$1, 5, +3$.

Now, the equation of the plane passing through P is

$$a(x+1) + b(y-5) + c(z-3) = 0$$

This plane contains the line:

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$

This is,

$$a(0+1) + b(0-5) + c(0-3) = 0 \Rightarrow a = 5b + 3c$$

$$a + 2b + c = 0 \Rightarrow 7b + 4c = 0 \Rightarrow b = \frac{-4c}{7}$$

$$a = -\frac{20c}{7} + 3c = \frac{c}{7}$$

Now, the equation of the plane is obtained as follows:

$$\frac{c}{7}(x+1) - \frac{4c}{7}(y-5) + c(z-3) = 0$$

$$(x+1) - 4(y-5) + 7(z-3) = 0$$

$$x + 1 - 4y + 20 + 7z - 21 = 0$$

$$x - 4y + 7z = 0$$

Question 023 MCQ

QUESTION

Football teams

$$T_1$$

and

$$T_2$$

have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of

$$T_1$$

winning, drawing and losing a game against

$$T_2$$

are

$$\frac{1}{2}, \frac{1}{6}$$

and

$$\frac{1}{3}$$

respectively. Each team gets

$$3$$

points for a win,

$$1$$

point for a draw and

$$0$$

point for a loss in a game. Let

$$X$$

and

$$Y$$

denote the total points scored by teams

$$T_1$$

and

$$T_2$$

respectively after two games.

$$P\,(X = Y)$$

is

A $\frac{11}{36}$

B $\frac{1}{3}$

C $\frac{13}{36}$

D $\frac{1}{2}$

CORRECT OPTION

C $\frac{13}{36}$

SOURCE

Mathematics • probability

EXPLANATION

•

Probability of wining of T_1 against T_2 = 1/2.

- Probability of drawing of T_1 against T_2 is = 1/6.
- Probability of losing of T_1 against T_2 is = 1/3.
- 3 points for win.
- 1 point for draw.
- 0 point for loss.

P

$$X = Y$$

= P$draw$. P$draw$ + P(T_1 win) P(T_2 win) + P(T_2 win) . P(T_1 win)

= $1/6$$ \times $$1/6$ + $1/2$$ \times $$1/3$ + $1/3$$ \times $$1/2$ = 13/36

Question 024 MCQ

QUESTION

Football teams

$$T_1$$

and

$$T_2$$

have to play two games against each other. It is assumed that the outcomes of the two games are independent. The probabilities of

$$T_1$$

winning, drawing and losing a game against

$$T_2$$

are

$$\frac{1}{2}, \frac{1}{6}$$

and

$$\frac{1}{3}$$

respectively. Each team gets

$$3$$

points for a win,

$$1$$

point for a draw and

$$0$$

point for a loss in a game. Let

$$X$$

and

$$Y$$

denote the total points scored by teams

$$T_1$$

and

$$T_2$$

respectively after two games.

$$P\left(X > Y\right)$$

is

A

$$\frac{1}{4}$$

B

$$\frac{5}{12}$$

C

$$\frac{1}{2}$$

D

$$\frac{7}{12}$$

CORRECT OPTION

B

$$\frac{5}{12}$$

SOURCE

Mathematics • probability

EXPLANATION

- Probability of wining of T_1 against T_2 is = 1/2.
- Probability of drawing of T_1 against T_2 is = 1/6.
- Probability of losing of T_1 against T_2 is = 1/3.

•

3 points for win.

•

1 point for draw.

•

0 point for loss.

Here, P $X > Y$ = P(T_1 win) P(T_1 win) + P(T_1 win) P $draw$ + P $draw$ P(T_1 win)

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) + \left(\frac{1}{6} \times \frac{1}{2}\right) = \frac{5}{12}$$

Question 025 MCQ

QUESTION

Let

$$f(x) = \lim_{n \to \infty} \left(\frac{n^n (x+n)\left(x + \frac{n}{2}\right) \ldots \left(x + \frac{n}{n}\right)}{n!\left(x^2 + n^2\right)\left(x^2 + \frac{n^2}{4}\right) \ldots \left(x^2 + \frac{n^2}{n^2}\right)} \right)^{\frac{x}{n}},$$

for

all

$$x > 0.$$

Then

A $f\left(\frac{1}{2}\right) \geq f(1)$

B

$$f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$$

C

$$f'(2) \leq 0$$

D

$$\frac{f'(3)}{f(3)} \geq \frac{f'(2)}{f(2)}$$

CORRECT OPTION

B

$$f\left(\frac{1}{3}\right) \leq f\left(\frac{2}{3}\right)$$

SOURCE

Mathematics • definite-integration

EXPLANATION

$$f(x) = \lim_{n\to\infty}\left\{\frac{n^{2n}\left(\frac{x}{n}+1\right)\left(\frac{x}{n}+\frac{1}{2}\right)\ldots\left(\frac{x}{n}+\frac{1}{n}\right)}{n!n^{2n}\left(\frac{x^2}{n^2}+1\right)\left(\frac{x^2}{n^2}+\frac{1}{2^2}\right)\ldots.\left(\frac{x^2}{n^2}+\frac{1}{n^2}\right)}\right\}^{\frac{x}{n}}$$

By taking logarithm we have

$$\ln f(x) = \lim_{n\to\infty}\frac{x}{n}\left\{\sum_{r=1}^{n}\ln\left(1+\frac{rx}{n}\right)-\sum_{r=1}^{n}\ln\left(1+\frac{r^2x^2}{n^2}\right)\right\}$$

By definite integration, we can write it as

$$\ln f(x) = x\int_0^1 \ln(1+xy)dy - x\int_0^1 \ln(1+x^2y^2)dy$$

Let xy = u

Then,

$$\ln f(x) = \int_0^x \ln(1+u)du - \int_0^x \ln(1+u^2)du$$

Using Newton-Leibnitz formula, we get

$$\frac{1}{f(x)} \cdot f'(x) = log\left(\frac{1+x}{1+x^2}\right)$$

...... i

Here, at x = 1,

$$\frac{f'(1)}{f(1)} = \log(1) = 0$$

$$\therefore$$

$$f'(1) = 0$$

Now, sign scheme of f' x is shown below

$$\therefore$$

At x = 1, function attains maximum. Since, f x increases on $0, 1$.

$$\therefore$$

f 1 > f $1/2$

$$\therefore$$

Option a is incorrect.

f $1/3$ < f $2/3$

$$\therefore$$

Option b is correct.

Also, f' x < 0, when x > 1

$$\Rightarrow$$

f' 2 < 0

$$\therefore$$

Option c is correct.

Also,

$$\frac{f'(x)}{f(x)} = \log\left(\frac{1+x}{1+x^2}\right)$$

$$\therefore$$

$$\frac{f'(3)}{f(3)} - \frac{f'(2)}{f(2)} = \log\left(\frac{4}{10}\right) - \log\left(\frac{3}{5}\right)$$

$$= \log(2/3) < 0$$

$$\Rightarrow \frac{f'(3)}{f(3)} < \frac{f'(2)}{f(2)}$$

$$\therefore$$

Option d is incorrect.

Question 026 MCQ

QUESTION

Area of the region

$$\left\{(x, y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq x+9 \leq 15\right\}$$

is equal to

A

$$\frac{1}{6}$$

B

$$\frac{4}{3}$$

C

$$\frac{3}{2}$$

D

$$\frac{5}{3}$$

CORRECT OPTION

C

$$\frac{3}{2}$$

SOURCE

Mathematics • application-of-integration

EXPLANATION

Here,

$$\{(x,y) \in R^2 : y \geq \sqrt{|x+3|}, 5y \leq (x+9) \leq 15\}$$

$$\therefore$$

$$y \geq \sqrt{x+3}$$

$$\Rightarrow y \geq \begin{cases} \sqrt{x+3},\ when\, x \geq -3 \\ \sqrt{-x-3},\ when\, x \leq -3 \end{cases}$$

or,

$$y^2 \geq \begin{cases} x+3,\ when\, x \geq -3 \\ -3-x,\ when\, x \leq -3 \end{cases}$$

Shown as

Also,

$$5y \leq (x+9) \leq 15$$

$$\Rightarrow (x+9) \geq 5y$$

and

$$x \leq 6$$

Shown as

$$\therefore$$

$$\{(x,y) \in R^2 : y \geq \sqrt{|x+3|},\, 5y \leq (x+9) \leq 15\}$$

$$\therefore$$

Required area = Area of trapezium ABCD

$$-$$

Area of ABE under parabola

$$-$$

Area of CDE under parabola

$$= \frac{1}{2}(1+2)(5) - \int_{-4}^{-3} \sqrt{-(x+3)}dx - \int_{-3}^{1} \sqrt{(x+3)}dx$$

$$= \frac{15}{2} - \left[\frac{(-3-x)^{3/2}}{-\frac{3}{2}}\right]_{-4}^{-3} - \left[\frac{(x+3)^{3/2}}{\frac{3}{2}}\right]_{-3}^{1}$$

$$= \frac{15}{2} + \frac{2}{3}[0-1] - \frac{2}{3}[8-0]$$

$$= \frac{15}{2} - \frac{2}{3} - \frac{16}{3} = \frac{15}{2} - \frac{18}{3} = \frac{3}{2}$$

Question 027 MCQ

QUESTION

The value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$$

is equal to

A $\frac{\pi^2}{4} - 2$

B $\frac{\pi^2}{4} + 2$

C $\pi^2 - e^{\frac{\pi}{2}}$

D $\pi^2 + e^{\frac{\pi}{2}}$

CORRECT OPTION

A

$$\frac{\pi^2}{4} - 2$$

SOURCE

Mathematics • definite-integration

EXPLANATION

Let

$$I = \int_{-\pi/2}^{\pi/2} \frac{x^2 \cos x}{1+e^x} dx$$

...... i

$$\$\$ \because \$\$\$\$ \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\$\$$$

$$\Rightarrow I = \int_{-\pi/2}^{\pi/2} \frac{x^2 \cos(-x)}{1+e^{-x}} dx$$

...... ii

On adding Eqs. i and ii, we get

$$2I = \int_{-\pi/2}^{\pi/2} x^2 \cos x \left[\frac{1}{1+e^x} + \frac{1}{1+e^{-x}}\right] dx$$

$$= \int_{-\pi/2}^{\pi/2} x^2 \cos x \,.\, (1) dx$$

$$\$\$ \because \$\$\$\$ \int_{-a}^{a} f(x)dx\$\$\$\$ = 2\int_0^a f(x)dx\$\$, when \$\$ f(-x) = f(x)\$\$$$

$$\Rightarrow 2I = 2\int_0^{\pi/2} x^2 \cos x \, dx$$

Using integration by parts, we get

$$2I = 2[x^2(\sin x) - (2x)(-\cos x) + (2)(-\sin x)]_0^{\pi/2}$$

$$\Rightarrow 2I = 2\left[\frac{\pi^2}{4} - 2\right]$$

$$\therefore$$

$$I = \frac{\pi^2}{4} - 2$$

Question 028 MCQ

QUESTION

Let f: R

$$\rightarrow (0, \infty)$$

and g : R

$$\rightarrow$$

R be twice differentiable functions such that f'' and g'' are continuous functions on R. Suppose f'

$$(2)$$

$$=$$

g

$$(2) = 0$$

, f''

$$(2)$$

$$\neq 0$$

and g'

$$(2)$$

$$\neq 0$$

. If

$$\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1,$$

then

A

f

has a local minimum at

$x = 2$

B

f

has a local maximum at

$x = 2$

C

$f''(2) > f(2)$

D

$f(x) - f''(x) = 0$

for at least one

$x \in R$

CORRECT OPTION

A

f

has a local minimum at

$x = 2$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Let f : R

$\rightarrow$

$0,$ $$\infty$$ and g : R

$\rightarrow$

R.

fx > 0

$\forall$

x

$\in$

R

It is given that f'2 = 0, g2 = 0, f''2

$\neq$

0 and g'2

$\neq$

0.

It is also given that

$$\lim_{x \to 2} \frac{f(x)g(x)}{f'(x)g'(x)} = 1$$

$$\left(\frac{0}{0}\right)$$

Applying L' Hospital rule, we get

$$\lim_{x \to 2} \frac{f'(x)g(x) + g'(x)f(x)}{f''(x)g'(x) + f'(x)g''(x)} = 1$$

For finite limit, we get

$$\frac{f'(2)g(2) + g'(2)f(2)}{f''(2)g'(2) + f'(2)g''(2)} = 1$$

$$\frac{g'(2)f(2)}{f''(2)g'(2)} = 1$$

$$\frac{f(2)}{f''(2)} = 1 \Rightarrow f''(2) = f(2) > 0$$

and f' 2 = 0 which means that f x has local minima at x = 2.

Hence, option A is correct.

$$f(2) - f''(2) = 0$$

Therefore, we can say that

$$f(x) - f''(x) = 0$$

has at least one solution in x

$$\in$$

R.

Hence, option D is correct.

Question 029 MCQ

QUESTION

Let

$$F_1(x_1, 0)$$

and

$$F_2\left(x_2, 0\right)$$

for

$$x_1 < 0$$

and

$$x_2 > 0$$

, be the foci of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

. Suppose a parabola having vertex at the origin and focus at

$$F_2$$

intersects the ellipse at point

$$M$$

in the first quadrant and at point

$$N$$

in the fourth quadrant.

The orthocentre of the triangle

$$F_1 M N$$

is

A $$\left(-\frac{9}{10}, 0\right)$$

B $$\left(\frac{2}{3}, 0\right)$$

C

$$\left(\frac{9}{10}, 0\right)$$

D

$$\left(\frac{2}{3}, \sqrt{6}\right)$$

CORRECT OPTION

A

$$\left(-\frac{9}{10}, 0\right)$$

SOURCE

Mathematics • ellipse

EXPLANATION

$F_1 x, 0$ and $F_2(x_2, 0)$ are the foci of the ellipse:

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

Therefore, $a^2 = 9$ and $b^2 = 8$.

$$b^2 = a^2(1 - e^2)$$

$$1 - e^2 = \frac{8}{9} \Rightarrow e^2 = 1 - \frac{8}{9} = \frac{1}{9} \Rightarrow e = \frac{1}{3}$$

The focus is

$$F_1\left(-3 \times \frac{1}{3}, 0\right)$$

and

$$F_2\left(3 \times \frac{1}{3}, 0\right)$$

That is, F_1 $\$\$ - \$\$1, 0$ and $F_2 1, 0$.

The equation of parabola is

$$y^2 = 4(OF_2)x$$

$$y^2 = 4x(OF_2 = 1)$$

The point of intersection of ellipse and parabola is

$$\frac{x^2}{9} + \frac{4x}{8} = 1 \Rightarrow \frac{x^2}{9} + \frac{x}{2} = 1$$

$$\Rightarrow 2x^2 + 9x - 18 = 0$$

$$\Rightarrow 2x^2 + 12x - 3x - 18 = 0$$

$$\Rightarrow 2x(x+6) - 3(x+6) = 0$$

$$\Rightarrow x = \frac{3}{2}$$

$x\$\$ - \$\$6 is rejected$

Now,

$$y^2(4)\frac{3}{2} = 6$$

$$y = \pm\sqrt{6}$$

That is, the points M and N are, respectively,

$$M\left(\frac{3}{2}, \sqrt{6}\right)$$

and

$$N\left(\frac{3}{2}, -\sqrt{6}\right)$$

.

Let the orthocenter be h, k.

The slope of

$$OM = \frac{k - \sqrt{6}}{h - (3/2)}$$

The slope of

$$ON = \frac{\sqrt{6}}{-1 - (3/2)} = \frac{-2\sqrt{6}}{5}$$

Now,

$$\left(\frac{k - \sqrt{6}}{h - (3/2)}\right)\left(\frac{-2\sqrt{6}}{5}\right) = -1$$

$$2\sqrt{6}k - 12 = 5h - \frac{15}{2}$$

$$5h - 2\sqrt{6}k = \frac{15}{2} - 12 = \frac{-9}{2}$$

The slope of

$$ON = \frac{k + \sqrt{6}}{h - (3/2)}$$

The slope of

$$F_1M = \frac{\sqrt{6}}{1 + (3/2)} = \frac{2\sqrt{6}}{5}$$

$$\frac{k + \sqrt{6}}{h - (3/2)} \times \frac{2\sqrt{6}}{5} = -1$$

$$2\sqrt{6}k + 12 = -5h + \frac{15}{2}$$

$$5h + 2\sqrt{6}k = \frac{15}{2} - 12 = \frac{-9}{2}$$

$$5h + 2\sqrt{6}k = \frac{-9}{2}$$

.... 1

$$5h - 2\sqrt{6}k = \frac{-9}{2}$$

.... 2

Solving Eqs. 1 and 2, we get

$$10h = -9 \Rightarrow h = \frac{-9}{10}$$

and k = 0

Hence, the orthocentre of the triangle F_1MN is

$$\left(\frac{-9}{10}, 0\right)$$

.

Question 030 MCQ

QUESTION

Let

$$F_1\left(x_1, 0\right)$$

and

$$F_2\left(x_2, 0\right)$$

for

$$x_1 < 0$$

and

$$x_2 > 0$$

, be the foci of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

. Suppose a parabola having vertex at the origin and focus at

$$F_2$$

intersects the ellipse at point

$$M$$

in the first quadrant and at point

$$N$$

in the fourth quadrant.

If the tangents to the ellipse at

$$M$$

and

$$N$$

meet at

$$R$$

and the normal to the parabola at

$$M$$

meets the

$$x$$

-axis at

$$Q$$

, then the ratio of area of the triangle

$$MQR$$

to area of the quadrilateral

$$MF_1NF_2$$

is

A $3:4$

B $4:5$

C $5:8$

D $2:3$

CORRECT OPTION

C $5:8$

SOURCE

Mathematics • ellipse

EXPLANATION

Equation of tangent at M$3/2, \$\$\sqrt{6}\$\$$ to

$$\frac{x^2}{9} + \frac{y^2}{8} = 1$$

is

$$\frac{3}{2} \cdot \frac{x}{9} + \sqrt{6} . \frac{y}{8} = 1$$

.... i

which intersect X-axis at $6, 0$.

Also, equation of tangent at N $3/2, \$\$ - \$\$\$\$\sqrt{6}\$\$$ is

$$\frac{3}{2} \cdot \frac{x}{9} - \sqrt{6} \cdot \frac{y}{8} = 1$$

.... ii

Eqs. i and ii intersect on X-axis at R $6, 0$. iii

Also, normal at

$$M(3/2, \sqrt{6})$$

is

$$y - \sqrt{6} = \frac{-\sqrt{6}}{2}\left(x - \frac{3}{2}\right)$$

On solving with y = 0, we get Q $7/2, 0$ iv

The area of MQR is

$$\left|\frac{1}{2}\right| \begin{vmatrix} 3/2 & \sqrt{6} & 1 \\ 6 & 0 & 1 \\ 7/2 & 0 & 1 \end{vmatrix} = \left|\frac{\sqrt{6}}{2}\left(6 - \frac{7}{2}\right)\right| = \frac{5\sqrt{6}}{4}$$

The area of the quadrilateral MF_1NF_2 is

$$2(\Delta m_1 F_1 F_2) = 2\sqrt{6}$$

and the required ratio is

$$\frac{5\sqrt{6}}{4 \, . \, 2\sqrt{6}} = \frac{5}{8}$$

Question 031 MCQ

QUESTION

Let

$$P$$

be the point on the parabola

$$y^2 = 4x$$

which is at the shortest distance from the center

$$S$$

of the circle

$$x^2 + y^2 - 4x - 16y + 64 = 0$$

. Let

$$Q$$

be the point on the circle dividing the line segment

$$SP$$

internally. Then

A $$SP = 2\sqrt{5}$$

B $$SQ : QP = \left(\sqrt{5} + 1\right) : 2$$

C the

$$x$$

-intercept of the normal to the parabola at

$$P$$

is

$$6$$

the slope of the tangent to the circle at

$$Q$$

D is

$$\frac{1}{2}$$

CORRECT OPTION

A

$$SP = 2\sqrt{5}$$

SOURCE

Mathematics • parabola

EXPLANATION

Tangent to $y^2 = 4x$ at $(t^2, 2t)$ is

$y2t = 2(x + t^2)$

$$\Rightarrow$$

$yt = x + t^2$ i

Equation of normal at $P(t^2, 2t)$ is

$y + tx = 2t + t^3$

Since, normal at P passes through centre of circle $S2, 8$.

$$\therefore$$

$8 + 2t = 2t + t^3$

$$\Rightarrow$$

$t = 2$, i.e., $P4, 4$

$$\therefore$$

$$SP=\sqrt{(4-2)^2+(4-8)^2}=2\sqrt{5}$$

$$\therefore$$

Option a is correct.

Also, SQ = 2

$$\therefore$$

PQ = SP

$$-$$

SQ = 2

$$\sqrt{5}$$

$$-$$

2

Thus,

$$\frac{SQ}{QP}=\frac{1}{\sqrt{5}-1}=\frac{\sqrt{5}+1}{4}$$

$$\therefore$$

Option b is incorrect.

Now, x-intercept of normal is

x = 2 + 2^2 = 6

$$\therefore$$

Option c is correct.

Slope of tangent

$$=\frac{1}{t}=\frac{1}{2}$$

$$\therefore$$

Option d is correct.

Question 032 MCQ

QUESTION

Let

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$$

and I be the identity matrix of order 3. If

$$Q = [q_{ij}]$$

is a matrix such that

$$P^{50} - Q = I$$

and

$$\frac{q_{31} + q_{32}}{q_{21}}$$

equals

A. 52

B. 103

C. 201

D. 205

CORRECT OPTION

B 103

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Here,

$$P=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$$

$$\therefore$$

$$P^2=\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 4+4 & 1 & 0\\ 16+32 & 4+4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 4\times 2 & 1 & 0\\ 16(1+2) & 4\times 2 & 1\end{bmatrix}$$

...... i

and

$$P^3=\begin{bmatrix}1 & 0 & 0\\ 4\times 2 & 1 & 0\\ 16(1+2) & 4\times 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 4 & 1 & 0\\ 16 & 4 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0 & 0\\ 4\times 3 & 1 & 0\\ 16(1+2+3) & 4\times 3 & 1\end{bmatrix}$$

..... ii

From symmetry,

$$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 4 \times 50 & 1 & 0 \\ 16(1+2+3+....+50) & 4 \times 50 & 1 \end{bmatrix}$$

$$\because$$

$$P^{50} - Q = I$$

$$given$$

$$\therefore$$

$$\begin{bmatrix} 1 - q_{11} & -q_{12} & -q_{13} \\ 200 - q_{21} & 1 - q_{22} & -q_{23} \\ 16 \times \frac{50}{2}(51) - q_{31} & 200 - q_{32} & 1 - q_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 200 - q_{21} = 0$$

,

$$\frac{16 \times 50 \times 51}{2} - q_{31} = 0$$

,

$$200 - q_{32} = 0$$

$$\therefore$$

$$q_{21} = 200$$

,

$$q_{32} = 200$$

,

$$q_{31} = 20400$$

Thus,

$$\frac{q_{31}+q_{32}}{q_{21}}=\frac{20400+200}{200}$$

$$=\frac{20600}{200}=103$$

Question 033 MCQ

QUESTION

Let $b_i > 1$ for I = 1, 2, ..., 101. Suppose $\log_e b_1$, $\log_e b_2$, ..., $\log_e b_{101}$ are in Arithmetic Progression $A.P.$ with the common difference $\log_e 2$. Suppose a_1, a_2, ..., a_{101} are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a_{51}$, then

A. $s > t$ and $a_{101} > b_{101}$

B. $s > t$ and $a_{101} < b_{101}$

C. $s < t$ and $a_{101} > b_{101}$

D. $s < t$ and $a_{101} < b_{101}$

CORRECT OPTION

B. $s > t$ and $a_{101} < b_{101}$

SOURCE

Mathematics • sequences-and-series

EXPLANATION

If $\log b_1$, $\log b_2$, ..., $\log b_{101}$ are in A.P. with common difference $\log_e 2$, then b_1, b_2, ..., b_{101} are in G.P., with common ratio 2.

$$\therefore$$

$b_1 = 2^0 b_1$

$b_2 = 2^1 b_1$

$b_3 = 2^2 b_1$

$$\vdots \quad \vdots \quad \vdots$$

$b_{101} = 2^{100} b_1$ i

Also, a_1, a_2, ..., a_{101} are in A.P.

Given, $a_1 = b_1$ and $a_{51} = b_{51}$

$$\Rightarrow$$

$a_1 = b_1$ and $a_{51} = b_{51}$

$$\Rightarrow$$

$a_1 + 50D = 2^{50}\, b_1$

$$\Rightarrow$$

$a_1 + 50D = 2^{50}\, a_1$ [

$$\because$$

$a_1 = b_1$] ii

Now, $t = b_1 + b_2 + ... + b_{51}$

$$\Rightarrow t = b_1 \frac{(2^{51} - 1)}{2 - 1}$$

..... iii

and s = $a_1 + a_2 + ... + a_{51}$

$$= \frac{51}{2}(2a_1 + 50D)$$

...... iv

$$\therefore$$

t = $a_1(2^{51}$

$$-$$

1) [

$$\because$$

$a_1 = b_1$]

or t = 2^{51} a_1

$$-$$

$a_1 < 2^{51}$ a_1 v

and

$$s = \frac{51}{2}[a_1 + (a_1 + 50D)]$$

$$fromEq.\,(ii)$$

$$= \frac{51}{2}[a_1 + 2^{50}a_1] = \frac{51}{2}a_1 + \frac{51}{2}2^{50}a_1$$

$$\therefore$$

s > 2^{51} a_1 vi

From Eqs. v and vi,

we get s > t

Also, $a_{101} = a_1 + 100$ D

and $b_{101} = 2^{100}$ b_1

$$\therefore$$

$$a_{101} = a_1 + 100\left(\frac{2^{50}a_1 - a_1}{50}\right)$$

and $b_{101} = 2^{100} a_1$

$$\Rightarrow$$

$a_{101} = a_1 + 2^{51} a_1$

$$-$$

$2a_1 = 2^{51} a_1$

$$-$$

a_1

$$\Rightarrow$$

$a_{101} < 2^{51} a_1$

and $b_{101} > 2^{51} a_1$

$$\Rightarrow$$

$b_{101} > a_{101}$

Question 034 MCQ

QUESTION

Let a, b

$$\in$$

R and f : R

$$\rightarrow$$

R be defined by

$$f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$$

. Then f is

A differentiable at x = 0 if a = 0 and b = 1.

B differentiable at x = 1 if a = 1 and b = 0.

C NOT differentiable at x = 0 if a = 1 and b = 0.

D NOT differentiable at x = 1 if a = 1 and b = 1.

CORRECT OPTION

A differentiable at x = 0 if a = 0 and b = 1.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$f(x) = a\cos(|x^3 - x|) + b|x|\sin(|x^3 + x|)$$

If

$$x^3 - x \geq 0$$

$$\Rightarrow \cos\left|x^3 - x\right| = \cos(x^3 - x)$$

and if

$$x^3 - x \leq 0$$

$$\Rightarrow \cos|x^3 - x| = \cos(x^3 - x)$$

$$\therefore$$

$$\cos(|x^3 - x|) = \cos(x^3 - x), \forall x \in R$$

..... i

Again, if

$$x^3 + x \geq 0$$
$$\Rightarrow |x| \sin(|x^3 + x|) = x \sin(x^3 + x)$$

and if

$$x^3 + x \leq 0$$
$$\Rightarrow |x| \sin(|x^3 + x|) = -x \sin\{-(x^3 + x)\}$$
$$\therefore$$
$$|x| \sin(|x^3 + x|) = x \sin(x^3 + x), \forall x \in R$$

...... ii

$$\Rightarrow f(x) = a \cos(|x^3 - x|) + b|x| \sin(|x^3 + x|)$$
$$\therefore$$
$$f(x) = a \cos(x^3 - x) + bx \sin(x^3 + x)$$

..... iii

For a function to be differentiable at x = 0, the function must be continuous.

$$f(0) = a \cos(0 - 0) + b(0) \sin(0) = a$$

Therefore,

$$f(0^+) = \lim_{h \to \infty} [a \cos(h^3 - h) + bh \sin(h^3 + h)] = 0$$
$$f(0^-) = \lim_{h \to \infty} [a(\cos(-h^3 + h) + b(-h) \sin(-h^3 - h)]$$
$$= \lim_{h \to \infty} [a \cos(h^3 - h) + bh \sin(h^3 + h)] = 0$$

which is continuous at x = 0; hence, fx is differentiable for all values of a and b.

Therefore,

$$f(1) = a \cos(1 - 1) + 1 \sin(1 + 1) = a + b \sin 2$$

$$f(1^{+}) = \lim_{h \to 0} a\cos{(1+h)^3} - (1+h) + b(1+h)\sin{(1+h)^3} + (1+h)$$

$$= f(1)$$

$$f(1^{-}) = \lim_{h \to 0} a\cos{(1-h)^3} - (1-h) + b(1-h)\sin{(1-h)^3} + (1-h)$$

$$= f(1)$$

Thus, fx is continuous and we can also see that f is differentiable at x = 0 and x = 1.

Question 035 MCQ

QUESTION

Let a,

$$\lambda$$

, m

$$\in$$

R. Consider the system of linear equations

ax + 2y =

$$\lambda$$

3x

$$-$$

2y =

$$\mu$$

Which of the following statements is are correct?

If a =

$-$

3, then the system has infinitely many solutions for all values of

A

λ

and

μ

.

If a

$\neq$

$-$

3, then the system has a unique solution for all values of

B

λ

and

μ

.

If

λ

+

C

μ

= 0, then the system has infinitely many solutions for a =

$-$

3.

If

λ

+

D

μ

$\neq$

0, then the system has no solution for a = -3.

CORRECT OPTION

B

If a

$\neq$

$-$

3, then the system has a unique solution for all values of

λ

and

μ

.

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

The given system of linear equation is

ax + 2y =

λ

3x

$-$

2y =

μ

By Cramer's rule, we have

$$\Delta = \begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6 = -2(a+3)$$

$$\Delta_1 = \begin{vmatrix} \lambda & 2 \\ \mu & -2 \end{vmatrix} = -2\lambda - 2\mu = -2(\lambda + \mu)$$

$$\Delta_2 = \begin{vmatrix} a & \lambda \\ 3 & \mu \end{vmatrix} = (a\mu - 3\lambda)$$

For unique solution:

$$\Delta$$

$$\neq$$

0

$$\Rightarrow$$

a + 3

$$\neq$$

0

$$\Rightarrow$$

a

$$\neq$$

$$-$$

3, where

$$\lambda$$

and

$$\mu$$

can take any values.

Hence, option B is correct.

For infinite solution:

$$\Delta$$

$= 0$, Δ_1 and Δ_2 are zero.

That is, a $= -3$ and $\lambda + \mu = 0$ or a$\mu - 3\lambda = 0$.

Hence, options C are correct.

For no solution:

$\Delta = 0$ and either Δ_1 or Δ_2 is non-zero.

That is, a =

$$-$$

3 and

$$\lambda$$

+

$$\mu$$

$$\neq$$

0.

Hence, option D is correct.

Question 036 MCQ

QUESTION

Let

$$f:\left[-\frac{1}{2},2\right]\rightarrow R$$

and

$$g:\left[-\frac{1}{2},2\right]\rightarrow R$$

be function defined by

$$f(x)=[x^2-3]$$

and

$$g(x)=|x|f(x)+|4x-7|f(x)$$

, where

$$y$$

denotes the greatest integer less than or equal to y for

$$y \in R$$

. Then

A

f is discontinuous exactly at three points in

$$\left[-\frac{1}{2}, 2\right]$$

.

B

f is discontinuous exactly at four points in

$$\left[-\frac{1}{2}, 2\right]$$

.

C

g is NOT differentiable exactly at four points in

$$\left(-\frac{1}{2}, 2\right)$$

.

D

g is NOT differentiable exactly at five points in

$$\left(-\frac{1}{2}, 2\right)$$

.

CORRECT OPTION

B

f is discontinuous exactly at four points in

$$\left[-\frac{1}{2}, 2\right]$$

.

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$f(x) = [x^2 - 3] = [x^2] - 3$$

$$= \begin{cases} -3, & -1/2 \leq x < 1 \\ -2, & 1 \leq x < \sqrt{2} \\ -1, & \sqrt{2} \leq x < \sqrt{3} \\ 0, & \sqrt{3} \leq x < 2 \\ 1, & x = 2 \end{cases}$$

and

$$g(x) = |x| f(x) + |4x - 7| f(x)$$

$$= (|x| + |4x - 7|) f(x)$$

$$= (|x| + |4x - 7|)[x^2 - 3]$$

$$= \begin{cases} (-x - 4x - 7)(-3), & -1/2 \leq x < 0 \\ (x - 4x + 7)(-3), & 0 \leq x < 1 \\ (x - 4x + 7)(-2), & 1 \leq x < \sqrt{2} \\ (x - 4x + 7)(-1), & \sqrt{2} \leq x < \sqrt{3} \\ (x - 4x + 7)(0), & \sqrt{3} \leq x < 7/4 \\ (x + 4x - 7)(0), & 7/4 \leq x < 2 \\ (x + 4x - 7)(1), & x = 2 \end{cases}$$

$$\therefore$$

$$g(x) = \begin{cases} 15x + 21, & -1/2 \leq x < 0 \\ 9x - 21, & 0 \leq x < 1 \\ 6x - 14, & 1 \leq x < \sqrt{2} \\ 3x - 7, & \sqrt{2} \leq x\sqrt{3} \\ 0, & \sqrt{3} \leq x < 2 \\ 5x - 7, & x = 2 \end{cases}$$

Now, the graphs of fx and gx are shown below.

Graph for fx

Clearly, fx is discontinuous at 4 points.

$$\therefore$$

Option b is correct.

Graph for gx

Clearly, gx is not differentiable at 4 points, when

x

$$\in$$

$$\$\$ - \$\$1/2, 2.$$

$$\therefore$$

Option c is correct.

Question 037 MCQ

QUESTION

A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases:
i when the block is at x_0; and
ii when the block is at $x = x_0 + A$.
In both cases, a particle with mass m$< M$ is softly placed on the block after which they stick on each other. Which of the following statement$s$ isare true about the motion after the mass m is placed on the mass M?

A

The amplitude of oscillation in the first case changes by a factor of

$$\sqrt{\frac{M}{m+M}}$$

, whereas in the second case it remains unchanged.

B

The final time period of oscillation in both the cases is same

C

The total energy decreases in both the cases

D

The instantaneous speed at x_0 of the combined masses decreases in both the cases

CORRECT OPTION

A

The amplitude of oscillation in the first case changes by a factor of

$$\sqrt{\frac{M}{m+M}}$$

, whereas in the second case it remains unchanged.

SOURCE

Physics • simple-harmonic-motion

EXPLANATION

Case - I

Case - II

In case 1,

$$Mv_1 = (M+m)v_2 \Rightarrow v_2 = \left(\frac{M}{M+m}\right)v_1$$

$$\sqrt{\frac{k}{M+m}}A_2 = \left(\frac{M}{M+m}\right)\sqrt{\frac{k}{M}}A_1$$

$$A_2 = \sqrt{\frac{k}{M+m}}A_1$$

In case - 2,

$$A_2 = A_1$$

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

in both cases.

Total energy decreases in first case whereas remain same in 2nd case. Instantaneous speed at x_0 decreases in both cases.

Question 038 MCQ

QUESTION

A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at P_f =

$$\left(\frac{1}{32}\right) \times 10^5\ Pa$$

and V_f = 8

$$\times$$

10^{-3} m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric $isovolumetric$ process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately

A 112 J

B 294 J

C 588 J

D 813 J

CORRECT OPTION

C 588 J

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

In the first process :

$$p_i p_i^\gamma = p_f V_f^\gamma$$

$$\Rightarrow \frac{p_i}{p_f} = \left(\frac{V_f}{V_i}\right)^\gamma \Rightarrow 32 = 8^\gamma$$

$$\gamma = \frac{5}{3}$$

... i

For the two step process,

$$W = p_i(V_f - V_i) = 10^5(7 \times 10^{-3})$$

$$W = 7 \times 10^2 J$$

$$\Delta U = \frac{f}{2}(p_f V_f - p_i V_i)$$

$$= \frac{1}{\gamma - 1}\left(\frac{1}{4} \times 10^2 - 10^2\right)$$

$$\Delta U = -\frac{3}{2} \cdot \frac{3}{4} \times 10^2 = -\frac{9}{8} \times 10^2 J$$

$$Q - W = \Delta U$$

$$\Rightarrow Q = 7 \times 10^2 - \frac{9}{8} \times 10^2$$

$$= \frac{47}{8} \times 10^2 J = 588J$$

Question 039 MCQ

QUESTION

The ends Q and R of two thin wires, PQ and RS, are soldered $joined$ together. Initially each of the wires has a length of 1 m at 10^{o}C. Now the end P is maintained at 10^{o}C, while the end S is heated and maintained at 400^{o}C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is 1.2

$$\times$$

10^{-5} K^{-1} , the change in length of the wire PQ is

A 0.78 mm

B 0.90 mm

C 1.56 mm

D 2.34 mm

CORRECT OPTION

A 0.78 mm

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

Rate of heat flow from P to Q,

$$\frac{dQ}{dt} = \frac{2KA(T - 10)}{1}$$

Rate of heat flow from Q to S,

$$\frac{dQ}{dt} = \frac{KA(4000 - T)}{1}$$

At steady state, state rate of heat flow is same

$$\therefore$$

$$\frac{2KA(T-10)}{1} = KA(400-T)$$

or

$$2T - 20 = 400 - T$$

or

$$3T = 420$$

$$\therefore$$

$$T = 140^{\circ}$$

Temperature of junction is

$$140^{\circ}$$

C.

Temperature at a distance x from end P is

$$T_x = (130x + 10^{\circ})$$

Change in length dx is suppose dy.

Then,

$$dy = \alpha dx(T_x - 10)$$

$$\int_0^{\Delta y} dy = \int_0^1 \alpha dx(130x + 10 - 10)$$

$$\Delta y = \left[\frac{\alpha x^2}{2} \times 130\right]_0^1$$

$$\Delta y = 1.2 \times 10^{-5} \times 65$$

$$\Delta y = 78.0 \times 10^{-5}$$

m = 0.78 mm

Question 040 MCQ

QUESTION

Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length

$$l = \sqrt{24}a$$

through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is

$$\omega$$

. The angular momentum of the entire assembly about the point ‘O’ is

$$\vec{L}$$

$see the figure$. Which of the following statement s is are true?

A

The center of mass of the assembly rotates about the z-axis with an angular speed of

$$\frac{\omega}{5}$$

B

The magnitude of angular momentum of center of mass of the assembly about the point O is

$$81\,ma^2\omega$$

C

The magnitude of angular momentum of the assembly about its center of mass is

$$\frac{17ma^2\omega}{2}$$

D

The magnitude of the z-component of

$$\vec{L}$$

is

$$55ma^2\omega$$

CORRECT OPTION

A

The center of mass of the assembly rotates about the z-axis with an angular speed of

$$\frac{\omega}{5}$$

SOURCE

Physics • rotational-motion

EXPLANATION

The following figure depicts the rolling condition of a circular disc:

For the smaller disc, we have the velocity expressed as

v = a

$$\omega$$

For the bigger disc, we have the velocity expressed as

v = $2a$

$$\omega$$

= 2a

$$\omega$$

Now, let us consider that both discs roll and also rotate about point O.

•

The angular momentum of a rotating body is given by

$$\overrightarrow{L} = \overrightarrow{r} \times \overrightarrow{p}$$

About point O, the combined angular momentum of the discs is given by

L = *l*ma

$$\omega$$

+ 2*l*$4m$ $2a$$$\omega$$

where lma

$$\omega$$

is due to the smaller disc and 2*l*$4m$ $2a$$$\omega$$ is due to the bigger disc.

Now, substituting the values in Eq. 1, we get

$$L = ma^2\omega[\sqrt{24} + 16\sqrt{24}]$$

$$= ma^2\omega(34\sqrt{6}) \approx 83ma^2\omega$$

Hence, option B is incorrect.

-

Now, let us consider that z-component of

$$\overrightarrow{L}$$

:

$$\overrightarrow{L} = L\cos\theta$$

$$= 34\sqrt{6}ma^2\omega\left[\frac{1}{\sqrt{l^2 + a^2}}\right] = 34\sqrt{6}ma^2\omega\left(\frac{\sqrt{24}}{6}\right)$$

= 81 ma^2

$$\omega$$

Hence, option D is incorrect.

•

The centre of mass of the assembly about its centre of mass

$$i.e. theaxislavelledas(I)inthefigureshownhere$$

is

$$L_{(I)} = \frac{ma^2}{2}\omega + \frac{4m(2a)^2}{2}\omega = \frac{17ma^2}{2}\omega$$

Hence, option C is correct.

•

About z-axis, the centre of mass of the assembly rotates with an angular speed

$$\omega_z = \omega\sin\theta = \omega\left[\frac{a}{\sqrt{l^2 + a^2}}\right] = \frac{\omega}{5}$$

Hence, option A is correct.

Question 041 MCQ

QUESTION

In an experiment to determine the acceleration due to gravity g, the formula used for the time period of a periodic motion is

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

. The values of R and r are measured to be $60\$\$ \pm \$\$1$ mm and $10\$\$ \pm \$\$1$ mm, respectively. In five successive measurements, the time period is found to be 0.52 s, 0.56 s, 0.57 s, 0.54 s and 0.59 s. The least count of the watch used for the measurement of time period is 0.01 s. Which of the following statement$s$ isare true?

A. The error in the measurement of r is 10 %

B. The error in the measurement of T is 3.57 %

C. The error in the measurement of T is 2 %

D. The error in the determined value of g is 11 %

CORRECT OPTION

B. The error in the measurement of T is 3.57 %

SOURCE

Physics • units-and-measurements

EXPLANATION

The time period is measured for five successive measurements as follows :

T s	0.52	0.56	0.57	0.54	0.59
Δ T	0.04	0	0.01	0.02	0.03

Therefore,

$$T_{mean} = \frac{0.52 + 0.56 + 0.57 + 0.54 + 0.59}{5}$$

$$= \frac{2.78}{5} = 0.556 \approx 0.56$$

and we know

$$\Delta$$

T $absoluteerror$ =

$$|T_{mean} - T|$$

Error in reading =

$$|T_{mean} - T_1| = 0.04$$

$$|T_{mean} - T_2| = 0.00$$

$$|T_{mean} - T_3| = 0.01$$

$$|T_{mean} - T_4| = 0.02$$

$$|T_{mean} - T_5| = 0.03$$

$$\Rightarrow \Delta T_{mean} = \frac{0.04 + 0 + 0.01 + 0.02 + 0.03}{5} = 0.02$$

Thus, the error in the measurement of T is

$$\frac{\Delta T}{T} \times 100 = \frac{0.02}{0.56} \times 100 = 3.57\%$$

.... 1

Hence, option B is correct.

The error in the measurement of r is

$$\frac{\Delta r}{r} \times 100 = \frac{1}{10} \times 100 = 10\%$$

..... 2

Hence, option A is correct.

Now, it is given that

$$T = 2\pi\sqrt{\frac{7(R-r)}{5g}}$$

$$\Rightarrow T^2 = 4\pi^2\left(\frac{7(R-r)}{5g}\right)$$

$$\Rightarrow g = \frac{28\pi^2}{5}\left(\frac{R-r}{T^2}\right)$$

Taking log and differentiating on both the sides of this equation, we get

$$\ln g = \ln\left(\frac{28\pi^2}{5}\right) + \ln(R-r) - 2\ln T$$

$$\Rightarrow \frac{\Delta g}{g} = \left(\frac{\Delta R + \Delta r}{R-r}\right) + \frac{2\Delta T}{T}$$

$$= \left(\frac{1+1}{50}\right) + 2 \times 0.0357$$

$$\Rightarrow \frac{\Delta g}{g} \times 100 = \left(\frac{1}{25} + 2 \times 0.0357\right) \times 100$$

$$= (4 + 7.14)\% = 11.14\% = 11\%$$

Therefore, the error in the determined value g is 11%.

Answer A, B, D

Question 042 MCQ

QUESTION

There are two Vernier calipers both of which have 1 cm divided into 10 equal divisions on the main scale. The Vernier scale of one of the calipers (C_1) has 10 equal divisions that correspond to 9 main scale divisions. The Vernier scale of the other caliper (C_2) has 10 equal divisions that correspond to 11 main scale divisions. The readings of the two calipers are shown in the figure. The measured values $incm$ by calipers C_1 and C_2 respectively, are

A. 2.85 and 2.82

B. 2.87 and 2.83

C. 2.87 and 2.86

D. 2.87 and 2.87

CORRECT OPTION

B 2.87 and 2.83

SOURCE

Physics • units-and-measurements

EXPLANATION

In both calipers C_1 and C_2, 1 cm is divided into 10 equal divisions on the main scale. Thus, 1 division on the main scale is equal to $x_{m1} = x_{m2} = 1$ cm/10 = 0.1 cm.

In calipers C_1, 10 equal divisions on the Vernier scale are equal to 9 main scale divisions.
Thus, 1 division on the Vernier scale of C_1 is equal to $x_{v1} = 9x_{m1}/10 = 0.09$ cm.

In calipers C_2, 10 equal divisions on the Vernier scale are equal to 11 main scale divisions.
Thus, 1 division on the Vernier scale of C_2 is equal to $x_{v2} = 11x_{m2}/10 = 0.11$ cm.

Let main scale reading be MSR and v^{th} division of the Vernier scale coincides with m^{th} division of the main scale $miscountedbeyondMSR$. The value measured by this calipers is

X = MSR + x = MSR + mx_m

$-$

vx_v 1

In calipers C_1, MSR_1 = 2.8 cm, m_1 = 7 and v_1 = 7 and in calipers C_2, MSR_2 = 2.8 cm, m_2 = 8 and v_2 = 7. Substitute these values in equation 1 to get

$X_1 = MSR_1 + m_1 x_{m1}$

—

$v_1 x_{v1}$

$= 2.8 + 7 0.1$

—

$7 0.09$ = 2.87 cm.

$X_2 = MSR_2 + m_2 x_{m2}$

—

$v_2 x_{v2}$

$= 2.8 + 8 0.1$

—

$7 0.11$ = 2.83 cm.

Question 043 MCQ

QUESTION

An accident in a nuclear laboratory resulted in deposition of a certain amount of radioactive material of half-life 18 days inside the laboratory. Tests revealed that the radiation was 64 times more than the permissible level required for safe operation of the laboratory. What is the minimum number of days after which the laboratory can be considered safe for use?

A 64

B 90

C 108

D 120

CORRECT OPTION

C 108

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

To determine the minimum number of days after which the laboratory can be considered safe for use, we need to understand how the radioactive material decays over time.

The half-life of a radioactive material is the time it takes for half of the material to decay. For our given material, the half-life is 18 days. This means every 18 days, the amount of radioactive material is reduced to half of its previous amount.

Given that the initial radiation level is 64 times more than the permissible level, we can represent this situation mathematically. Let R be the initial radiation level and P be the permissible level of radiation. We are given:

$$R = 64P$$

After each half-life period of 18 days, the amount of radioactive material will be halved. If n is the number of half-life periods required for the radiation to reduce to the permissible level, we can write:

$$\frac{R}{2^n} = P$$

Using the given information $\$R = 64P\$$, we get:

$$\frac{64P}{2^n} = P$$

Dividing both sides by P, we get:

$$\frac{64}{2^n} = 1$$

This implies:

$$2^n = 64$$

Knowing that 64 is a power of 2:

$$64 = 2^6$$

Therefore, $n = 6$. This means it will take six half-life periods for the radiation to decay to a safe level.

Since each half-life period is 18 days, the total number of days required is:

$$6 \times 18 = 108$$

Hence, the minimum number of days after which the laboratory can be considered safe for use is 108 days.

Answer: Option C 108

Question 044 MCQ

QUESTION

The electrostatic energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E = \frac{3}{5} \frac{Z(Z-1)e^2}{4\pi\varepsilon_0 R}$$

The measured masses of the neutron,

$$^{1}_{1}H$$

,

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

are 1.008665u, 1.007825u, 15.000109u and 15.003065u, respectively. Given that the radii of both the

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

nuclei are same, 1 u = 931.5 MeV/c^2 $cisthespeedoflight$ and e^2/ $4\$\$\pi\$\$\$\$\varepsilon_0\$\$$ = 1.44 MeV fm. Assuming that the difference between the binding energies of

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

is purely due to the electrostatic energy, the radius of either of the nuclei is (1 fm = 10

$$-$$

^{15} m)

A 2.85 fm

B 3.03 fm

C 3.42 fm

D 3.80 fm

CORRECT OPTION

C 3.42 fm

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The binding energies of

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

are given by

$$BE_N = \Delta m_N c^2 = (8m_n + 7m_p - M_N)c^2$$

$$= (8 \times 1.008665 + 7 \times 1.007825 - 15.000109 \times 931.5$$

= 115.49 MeV,

$$BE_O = \Delta m_O c^2 = (7m_n + 8m_p - M_O)c^2$$

$$= (7 \times 1.008665 + 8 \times 1.007825 - 15.003065 \times 931.5$$

= 111.95 MeV.

The difference in binding energies of

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

is

$$\Delta BE = BE_N - BE_O = 115.49 - 111.95$$

= 3.54 MeV. 1

The electrostatic energies of

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

are given by

$$E_N = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\in_0 R} = \frac{3}{5}\frac{7(7-1)1.44}{R}$$

$$= \frac{36.288}{R}$$

MeV-fm

$$E_O = \frac{3}{5}\frac{8(8-1)1.44}{R} = \frac{48.384}{R}$$

MeV-fm.

The difference in electrostatic energies of

$$^{15}_{7}N$$

and

$$^{15}_{8}O$$

is

$$\Delta E = E_O - E_N = (12.096/R)$$

...... 2

Since,

$$\Delta E = \Delta BE$$

, equations 1 and 2 give

$$R = 12.096/3.54 = 3.42$$

fm.

Question 045 MCQ

QUESTION

A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle

$$\theta$$

= 30

$$\circ$$

to the axis of the lens, as shown in the figure.

If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates $incm$ of the point x, y at which the image is formed are

A $125/3, 25/\$\$\sqrt{3}\$\$$

B $50\$\$-\$\$25\$\$\sqrt{3}\$\$, 25$

C $0, 0$

D 25, 25$$\sqrt{3}$$

CORRECT OPTION

D 25, 25$$\sqrt{3}$$

SOURCE

Physics • geometrical-optics

EXPLANATION

For convex lens, u =

$$-$$

50 cm, f = 30 cm

$$\therefore$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{30} - \frac{1}{50} = \frac{5-3}{150} = \frac{2}{150}$$

or v = 75 cm

Image formed by convex lens acts as a virtual object for mirror.

If we consider that the mirror is not tilted, then for mirror,

$$u_1 = 75 - 50 = 25$$

cm,

$$f_1 = \frac{R}{2} = \frac{100\,cm}{2} = 50$$

cm

$$\therefore$$

$$\frac{1}{v_1} = \frac{1}{f_1} - \frac{1}{u_1} = \frac{1}{50} - \frac{1}{25} = \frac{-1}{50}$$

or

$$v_1 = 50$$

cm

So, the co-ordinates of final image formed (I_2) if the mirror is not tilted are $0cm, 0cm$

If mirror is not tilted then this ray starts from the object, refracts through the lens without deviation, incident normally on the mirror and finally comes to the image I_2 after reflection from the mirror. If the mirror is tilted by an angle

$$\theta$$

then this ray is incident on the mirror at an angle of reflection

$$\theta$$

and finally forms a new image at I'_2 $see figure$. Thus, the axis on which image lies makes an angle 2

$$\theta$$

= 60

$$\circ$$

with the principal axis of the lens. Also, the image distance will remain same upto the first order of approximation $image distance will remain exactly same in case of plane mirror$. Thus, x, y coordinates of the image are

$$x = 50 - 50 \cos 60^\circ = 25$$

cm,

$$y = 50 \sin 60^\circ = 25\sqrt{3}$$

cm.

Question 046 MCQ

QUESTION

Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement$s$ about anyone of the galvanometers isare true?

A. The maximum voltage range is obtained when all the components are connected in series

B. The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

C. The maximum current range is obtained when all the components are connected in parallel

D. The maximum current range is obtained when the two galvanometers are connected in series, and the combination is connected in parallel with both the resistors

CORRECT OPTION

A. The maximum voltage range is obtained when all the components are connected in series

SOURCE

Physics • current-electricity

EXPLANATION

Let i_g be the current that gives full deflection of the galvanometer. When all components are connected in series $see figure$, effective resistance of the circuit is

$$R_e = 2R + 2R_c$$

and maximum current allowed in the circuit is i_g.

Thus, the voltage between A and B is

$$V_{AB} = i_g R_e = 2i_g(R + R_c)$$

.

Consider the case when two resistors and one galvanometer are connected in series and the second galvanometer is connected in parallel.

The maximum current through each galvanometer is i_g and the maximum current through the resistors is $2i_g$. Apply Kirchhoff's law to get the voltage between A and B as

$$\begin{aligned} V'_{AB} &= 2i_g R + 2i_g R + i_g R_c \\ &= 2i_g(R + R_c) + 2i_g(R - R_c/2) \\ &= V_{AB} + 2i_g(R - R_c/2) \\ &> V_{AB} \end{aligned}$$

$\$\$ \because \$\$\$\$R_c < R/2\$\$$.

Consider the case when all four components are connected in parallel.

Let i and i_g be the currents through the resistors and the galvanometers. By Kirchhoff's law,

$$iR = i_g R_c$$

, which gives

$$i = i_g R_c / R$$

. The current between A and B is

$$I_{AB} = 2i + 2i_g = 2i_g(1 + R_c/R)$$

.

Consider the case when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

$$I'_{AB} = i_g + 2i = i_g + 2i_g(2R_c/R)$$
$$= 2i_g(1 + R_c/R) - (1 - 2R_c/R)i_g$$
$$= I_{AB} - (1 - 2R_c/R)i_g$$
$$< I_{AB}$$

$\because$ $R_c < R/2$.

Question 047 MCQ

QUESTION

A rigid wire loop of square shape having side of length L and resistance R is moving along the X-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let vx, Ix and Fx represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive.

Which of the following schematic plot$s$ is are correct? $Ignoregravity$

A

B

C

D

CORRECT OPTION

C

SOURCE

Physics • electromagnetic-induction

EXPLANATION

We discuss the following cases :

•

For x = 0

$$\rightarrow$$

L : The induced emf in the loop is

$$\varepsilon = vB_0L$$

Therefore, the induced current is

$$I = \frac{vB_0L}{R}$$

.... 1

As the flux is increasing, the current passes in counterclockwise direction and oppose the existing magnetic field $i.e., Lenz's law$.

Let us consider the loop which is labelled as shown in the following figure:

Force on the side CD of the loop is

$$\overrightarrow{F} = i(\overrightarrow{L} \times \overrightarrow{B}) = \frac{vB_0L}{R}[L\hat{j} \times B_0(-\widehat{k})]$$

$$\overrightarrow{F} = -\frac{vB_0^2L^2}{R}\hat{i}$$

$towardsleft$ 2

$$\Rightarrow \frac{mdv}{dt} = -\frac{B_0^2L^2}{R}v$$

$$\Rightarrow \frac{mdv}{dx}.\frac{dx}{dt} = -\frac{-B_0^2L^2}{R}v$$

$$\Rightarrow \int_{v_0}^{v} dv = -\frac{B_0^2L^2}{mR}\int_{0}^{x} dx$$

$Since$$\frac{dx}{dt} = v$$

$$\Rightarrow v = v_0 - \frac{B_0^2L^2}{mR}x$$

.... 3

From Eqs. 1 and 3, we get

$$I = \frac{v_0B_0L}{R} - \frac{B_0^3L^3}{mR^2}x$$

.... 4

From Eqs. 2 and 3, we get

$$\overrightarrow{F} = -\frac{v_0B_0^2L^2}{R} + \frac{B_0^4L^4}{mR^2}x$$

-

For x = L

$$\rightarrow$$

3L: Here, there is no change in flux and hence there is no induced current and thereby there is no force and no change in velocity.

-

For x = 3L

$\rightarrow$

4L :

As the flux is decreasing, the current passes in clockwise direction and supports the existing magnetic field $i.e., Lenz's law$.

Hence, option A is incorrect.

Consequently, the magnetic force experienced by the arm AB of the loop is

$$\overrightarrow{F} = i(\overrightarrow{L} \times \overrightarrow{B}) = \frac{vB_0L}{R}[L\hat{j} \times B_0(-\widehat{k})]$$

$$= -\frac{vB_0^2L^2}{R}\hat{i}$$

$towardsleft$

Hence, option B is incorrect.

Similarly, now we have

$$\int_{v_{3L}}^{v} dv = -\frac{B_0^2L^2}{mR}\int_{3L}^{x} dx \Rightarrow v = v_{3L} - \frac{B_0^2L^2}{mR}(x - 3L)$$

When the loop is completely within the magnetic field, the force and current is zero and velocity is constant. However, while coming out, the current passes in clockwise direction, the force exists in backwards and the velocity decreases linearly.

Question 048 MCQ

QUESTION

While conducting the Young's double slit experiment, a student replaced the two slits with a large opaque plate in the XY-plane containing two small holes that act as two coherent point sources (S_1, S_2) emitting light of wavelength 600 mm. The student mistakenly placed the screen parallel to the XZ-plane $for z > 0$ at a distance D = 3 m from the mid-point of S_1S_2, as shown schematically in the figure. The distance between the source d = 0.6003 mm. The origin O is at the intersection of the screen and the line joining S_1S_2.

Which of the following is are true of the intensity pattern on the screen?

A Semi circular bright and dark bands centered at point O

B The region very close to the point O will be dark

C Straight bright and dark bands parallel to the X-axis.

D Hyperbolic bright and dark bands with foci symmetrically placed about O in the x-direction

CORRECT OPTION

A Semi circular bright and dark bands centered at point O

SOURCE

Physics • wave-optics

EXPLANATION

At point O, the path difference is

$$\frac{2\pi}{\lambda}(S_1O - S_2O) = \frac{2\pi}{\lambda}\left[\left(D + \frac{d}{2}\right) - \left(D - \frac{d}{2}\right)\right]$$

$$= \frac{2\pi d}{\lambda} = \frac{2\pi(0.6003 \times 10^{-3})}{600 \times 10^{-9}} = 2001\pi = (2n + 1)\pi$$

Therefore, the dark fringe forms at O.

Hence, option A is correct.

The shape of wavefronts from both sources for D >> d is semi-circular, which are concentric with each other with the difference just in their phase resulting in concentric semicircular interference fringes. That is, the region nearby point O is dark.

Hence, option B is correct.

Question 049 MCQ

QUESTION

In the circuit shown below, the key is pressed at time t = 0. Which of the following statement$s$ is are true?

A The voltmeter display

−

5V as soon as the key is pressed and displays +5 V after a long time

B The voltmeter will display 0 V at time t = ln 2 seconds

C The current in the ammeter becomes 1/e of the initial value after 1 second

D The current in the ammeter becomes zero after a long time

CORRECT OPTION

The voltmeter display

A

$-$

5V as soon as the key is pressed and displays +5 V after a long time

SOURCE

Physics • current-electricity

EXPLANATION

When the key is just pressed, there is no charge on capacitors

$$\therefore$$

$$5 = 25000 I_1 \Rightarrow I_1 = \frac{5}{25000} = 0.2\,mA$$

and

$$5 = 50000 I_2 \Rightarrow I_2 = \frac{5}{50000} = 0.1\,mA$$

Reading of voltmeter

$$= V_Q - V_P = -25000 I_1$$

$$= -25000 \times 0.2 \times 10^{-3} V = -5V$$

After a very long time, steady state is reached, i.e.

$$I_1 - I_2 = 0$$

$$\therefore$$

$$5 = \frac{q_1}{40 \times 10^{-6}}$$

or

$$q_1 = 200 \mu C$$

and

$$\therefore$$

$$5 = \frac{q_2}{20 \times 10^{-6}}$$

or

$$q_2 = 100\mu C$$

Now, reading of voltmeter

$$= V_Q - V_P$$

$$= \frac{q_2}{20\mu F} = \frac{100\mu C}{20\mu F} = 5V$$

For capacitor of 40

$$\mu$$

F,

$$\tau_1 = RC = 25 \times 10^3 \times 40 \times 10^{-6} = 1\,s$$

and for capacitor of 20

$$\mu$$

F,

$$\tau_2 = RC = 50 \times 10^3 \times 20 \times 10^{-6} = 1\,s$$

$$\therefore$$

At any instant t,

$$q_1 = 200[1 - e^{-t}]\mu C$$

,

$$q_2 = 100[1 - e^{-t}]\mu C$$

$$I_1 = 0.2\,e^{-t}$$

mA,

$$I_2 = 0.1\,e^{-t}$$

mA

$$V_Q - V_P = \frac{100(1-e^{-t})\mu C}{20\mu F} - 25k\Omega \times 0.2\, e^{-t}$$

mA

$$= 5(1-e^{-t}) - 5e^{-t} = 5 - 10e^{-t}$$

At t = ln 2 s;

$$V_Q - V_P = 5 - 10e^{-\ln 2} = 5 - \frac{10}{2} = 0$$

V

Initially, I_0 = 0.1 mA + 0.2 mA = 0.3 mA

At t = 1 s, I = I_1 + I_2

$$= (0.2e^{-1} + 0.1e^{-1}) = \frac{0.3}{e}\, mA = \frac{I_0}{e}$$

After a long time, i.e. t

$$\rightarrow$$

$$\infty$$

$$I = I_1 + I_2 = 0.2e^{-\infty} + 0.1e^{-\infty} = 0$$

Question 050 MCQ

QUESTION

Light of wavelength

$$\lambda$$

ph falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is

$$\phi$$

and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de-Broglie wavelength of the electrons passing through the anode is

$$\lambda$$

e, which of the following statement s is are true?

A

$$\lambda$$

e increases at the same rate as

$$\lambda$$

ph for

$$\lambda$$

ph < hc/

$$\phi$$

B

$$\lambda$$

e is approximately halved, if d is doubled

C

$$\lambda$$

e decreases with increase in

$$\phi$$

and

$$\lambda$$

ph

D

For large potential difference $V >> $$\phi$$/e$,

$$\lambda$$

e is approximately halved if V is made four times

CORRECT OPTION

D

For large potential difference $V >> $$\phi$$/e$,

$$\lambda$$

e is approximately halved if V is made four times

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

In photo-electric effect, the maximum kinetic energy of the photo-electron ejected at the cathode, is given by

$$K_{\text{max, }c} = \frac{hc}{\lambda_{ph}} - \phi$$

,

where

$$\lambda$$

ph is the wavelength of the incident light and

$$\phi$$

is the work function of the material. The ejected photoelectrons are accelerated from the cathode to the anode by a potential V. Thus, the maximum kinetic energy of the photo-electron at the anode is

$$K_{\text{max, }a} = K_{\text{max, }c} + eV = \frac{hc}{\lambda_{ph}} - \phi + eV$$

.

Now, minimum de-Broglie wavelength of electrons passing through anode,

$$\lambda_e = \frac{h}{\sqrt{2m\,K_{\text{max, }a}}} = \frac{h}{\sqrt{2m\left(\frac{hc}{\lambda_{ph}} - \phi + eV\right)}}$$

...... i

D : For

$$V >> \phi e$$

$$\lambda_e \approx \frac{h}{\sqrt{2meV}} \propto \frac{1}{\sqrt{V}}$$

$$\therefore$$

If V is made four times,

$$\lambda$$

e becomes

$$\frac{1}{\sqrt{4}} = \frac{1}{2}$$

, i.e. halved.

C : When

$$\phi$$

and

$$\lambda$$

ph increases,

$$\lambda$$

e also increases.

A : From i

$$\frac{d\lambda_e}{dt} = \frac{-h}{2}\left[2m\left(\frac{hc}{\lambda_{ph}} - \phi + eV\right)\right]^{-3/2} \times 2m\left(\frac{-hc}{\lambda_{ph}^2}\right)\frac{d\lambda_{ph}}{dt}$$

$$\Rightarrow \frac{d\lambda_e}{dt} \neq \frac{d\lambda_{ph}}{dt}$$

I :

$$\lambda$$

e does not depend on d.

Question 051 MCQ

QUESTION

A frame of the reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity

$$\omega$$

is an example of a non-inertial frame of reference. The relationship between the force

$$\vec{F}$$

rot experienced by a particle of mass m moving on the rotating disc and the force

$$\vec{F}$$

in experienced by the particle in an inertial frame of reference is,

$$\vec{F}$$

rot =

$$\vec{F}$$

in + 2m (

$$\vec{v}$$

rot

$$\times$$

$$\vec{\omega}$$

) + m $\vec{\omega} \times \vec{r}$

$$\times$$

$$\vec{\omega}$$

,

where, v_{rot} is the velocity of the particle in the rotating frame of reference and r is the position vector of the particle with respect to the centre of the disc.

Now, consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed

$$\omega$$

about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the X-axis along the slot, the Y-axis perpendicular to the slot and the Z-axis along the rotation axis $\omega = \omega\widehat{k}$. A small block of mass m is gently placed in the slot at r = $R/2$

$$\hat{i}$$

at t = 0 and is constrained to move only along the slot.

The distance r of the block at time t is

A

$$\frac{R}{2}\cos 2\omega t$$

B $$\frac{R}{2}\cos\omega t$$

C $$\frac{R}{2}(e^{\omega t}+e^{-\omega t})$$

D $$\frac{R}{2}(e^{2\omega t}+e^{-2\omega t})$$

CORRECT OPTION

C $$\frac{R}{2}(e^{\omega t}+e^{-\omega t})$$

SOURCE

Physics • rotational-motion

EXPLANATION

The given particle of mass m experiences a centrifugal force:

F = ma = mr

$$\omega$$

2

$$\Rightarrow$$

a = r

$$\omega$$

2

$$\Rightarrow$$

$$\frac{dv}{dt} = r\omega^2$$

or,

$$\left(\frac{dv}{dr}\right)\left(\frac{dr}{dt}\right) = r\omega^2$$

$\$\$ \because \$\$\$\$ \frac{dr}{dt} = v\$\$$

$$\Rightarrow vdv = r\omega^2 dr$$

$$\int_0^v vdv = \omega^2 \int_{R/2}^r rdr$$

$\$\$ \because \$\$att = 0, \$\$r = \frac{R}{2}\$\$$

$$\Rightarrow \frac{v^2}{2} = \omega^2 \left(\frac{r^2}{2}\right)_{R/2}^{r}$$

$$\Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

Now,

$$v = \frac{dr}{dt}$$

Therefore,

$$\frac{dr}{dt} = \omega\sqrt{r^2 - \frac{R^2}{4}}$$

or,

$$\int_{R/2}^{r} \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \omega \int_0^t dt$$

...... 1

To solve this integral, we substitute

$$r = \frac{R}{2}\sec\theta$$

; therefore,

$$dr = \frac{R}{2}\sec\theta\tan\theta d\theta$$

$$\Rightarrow \int\limits_{\frac{\pi}{2}}^{\sec^{-1}(2r/R)} \frac{(r/2)\sec\theta\tan\theta d\theta}{\sqrt{\frac{R^2}{4}\tan^2\theta}} = \omega t$$

$$\Rightarrow \omega t = \int\limits_{\pi/2}^{\sec^{-1}(2r/R)} \sec\theta d\theta$$

$$= [\ln|\sec\theta + \tan\theta|]^{\sec^{-1}(2r/R)}$$

$$= \left[\ln\left|\sec\theta + \sqrt{\sec^2\theta - 1}\right|\right]_{\pi/2}^{\sec^{-1}(2r/R)}$$

$$\Rightarrow \omega t = \ln\left[\frac{2r}{R} + \frac{\sqrt{4r^2 - R^2}}{R}\right]$$

Let

$$\frac{2r}{R} = x$$

. Therefore,

$$e^{\omega} = x + \sqrt{x^2 - 1}$$

$$\Rightarrow (e^{\omega t} - x)^2 = x^2 - 1$$

$$\Rightarrow e^{2\omega t} - 2xe^{\omega t} + x^2 = x^2 - 1$$

$$\Rightarrow e^{2\omega t} - 2xe^{\omega t} + 1 = 0$$

$$\Rightarrow x = \frac{e^{2\omega t} + 1}{2e^{\omega t}} = \frac{e^{\omega t} + e^{-\omega t}}{2}$$

On replacing x by

$$\frac{2r}{R}$$

, we get

$$\frac{2r}{R} = \frac{e^{\omega t} + e^{-\omega t}}{2}$$

or,

$$r = \frac{R}{4}[e^{\omega t} + e^{-\omega t}]$$

Hence, option c is correct.

Question 052 MCQ

QUESTION

A frame of the reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity

$$\omega$$

is an example of a non-inertial frame of reference. The relationship between the force

$$\vec{F}$$

rot experienced by a particle of mass m moving on the rotating disc and the force

$$\vec{F}$$

in experienced by the particle in an inertial frame of reference is,

$$\vec{F}$$

rot =

$$\vec{F}$$

in + 2m (

$$\vec{v}$$

rot

$$\times$$

$$\vec{\omega}$$

) + m $$\vec{\omega}$$$$ \times $$$$\vec{r}$$

$$\times$$

$$\vec{\omega}$$

,

where, v_{rot} is the velocity of the particle in the rotating frame of reference and r is the position vector of the particle with respect to the centre of the disc.

Now, consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed

$$\omega$$

about its vertical axis through its centre. We assign a coordinate system with the origin at the centre of the disc, the X-axis along the slot, the Y-axis perpendicular to the slot and the Z-axis along the rotation axis $$\omega$$ $=$ $$\omega$$$$\hat{k}$$. A small block of mass m is gently placed in the slot at r = $R/2$

$$\hat{i}$$

at t = 0 and is constrained to move only along the slot.

The net reaction of the disc on the block is

A

$$m\omega^2 R\sin\omega t\hat{j} - mg\widehat{k}$$

B

$$\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\widehat{k}$$

C

$$\frac{1}{2}m\omega^2 R(e^{2\omega t} - e^{-2\omega t})\hat{j} + mg\widehat{k}$$

D

$$-m\omega^2 R\cos\omega r\hat{j} - mg\widehat{k}$$

CORRECT OPTION

B

$$\frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\widehat{k}$$

SOURCE

Physics • rotational-motion

EXPLANATION

The rotational quantity of velocity given by

$$v_{rot} = \frac{dr}{dt} = \frac{R}{4}[\omega e^{\omega t} - \omega e^{-\omega t}]$$

$$v_{rot} = \frac{\omega R}{4}[e^{\omega t} - e^{-\omega t}]$$

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The force experienced by the block in the inertial frame is

$$\overrightarrow{F}_{in} = -m\omega^2 r\hat{i} - mg\widehat{k}$$

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Therefore, the force experienced by the block in the rotating frame is

$$\overrightarrow{F}_{rot} = \overrightarrow{F}_{in} + 2m(\overrightarrow{v}_{rot} \times \omega\widehat{k}) + m(\omega\widehat{k} \times r\hat{i}) \times \omega\widehat{k}$$

$$\overrightarrow{F}_{rot} = (-m\omega^2 r\hat{i} - mg\widehat{k}) + 2m\frac{\omega R}{r}[e^{\omega t} - e^{-\omega t}]\hat{i} \times \omega\widehat{k} + m\omega^2 r\hat{i}$$

$$= -\frac{m\omega^2 R}{2}[e^{\omega t} - e^{-\omega t}]\hat{j} - mg\widehat{k}$$

Therefore, the net reaction of the disc on the block is

$$\overrightarrow{F}_{reaction} - \overrightarrow{F}_{rot} = \frac{m\omega^2 R}{2}[e^{\omega t} - e^{\omega t}]\hat{j} + mg\widehat{k}$$

Hence, option B is correct.

Question 053 MCQ

QUESTION

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r << h. Now, a high voltage source HV connected across the conducting plates such that the bottom plate is at +V_0 and the top plate at

$$-$$

V_0. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. $Ignoregravity$

Which one of the following statement is correct?

A The balls will execute simple harmonic motion between the two plates

B The balls will bounce back to the bottom plate carrying the same charge they went up with

C The balls will stick to the top plate and remain there

D The balls will bounce back to the bottom plate carrying the opposite charge they went up with

CORRECT OPTION

D The balls will bounce back to the bottom plate carrying the opposite charge they went up with

SOURCE

Physics • electrostatics

EXPLANATION

The distance between the two plates is h. The potential of the bottom plate is V_0 and that of the top plates is

$$-$$

V_0. The electric field between the plates is E = $2V_0/h$ $directedupwards$. The radius of each ball is r $<< h$. Let m be the mass and C be the capacitance of each ball.

When the ball touches the bottom plate, it gets a positive charge q = CV_0 $weassumethatthechargetransferisinstantaneous$. This positively charged ball experiences an upward force, F = qE = 2CV

$$\begin{matrix} 2 \\ 0 \end{matrix}$$

/h, which accelerates the ball upwards. Since the force is constant, the ball cannot do SHM $forSHM, theforceshouldbeproportionaltothedisplacementanddirectedto$
.

When the ball hits the top plate, it transfers the positive charge to the plate and gets negative charge q =

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CV_0. This negatively charged ball again experience a force F = qE $downward$ and starts accelerating downwards. Thus, the ball keeps moving between the bottom and the top plates carrying a charge +q upwards and

—

q downwards.

Question 054 MCQ

QUESTION

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r << h. Now, a high voltage source HV connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at

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V_0. Due to their conducting surface, the balls will get charge, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collisions between the balls and the interaction between them is negligible. $Ignore gravity$

The average current in the steady state registered by the ammeter in the circuit will be

A proportional to

$$V_0^2$$

B proportional to the potential

$$V_0$$

C zero

D proportions to

$$V_0^{1/2}$$

CORRECT OPTION

A proportional to

$$V_0^2$$

SOURCE

Physics • electrostatics

EXPLANATION

As the balls keep on oscillating between plates, current will flow even in steady state.

Let a ball attains charge q at bottom plate. Then

$$\frac{Kq}{r} = V_0 \Rightarrow q = \frac{V_0 r}{K}$$

Electric field inside the chamber,

$$E = [V_0 - (-V_0)]/h$$
$$= 2V_0/h$$
$$\therefore$$

Acceleration of each ball,

$$a = \frac{qE}{m} = \frac{2V_0^2 r}{mhK}$$
$$\therefore$$

Time taken by a ball to reach the other plate

$$t = \sqrt{\frac{2h}{a}} = \frac{h}{V_0}\sqrt{\frac{mK}{r}}$$
$$\therefore$$

Average current due to n such balls,

$$I_{av} = \frac{nq}{t} = \frac{nV_0 r}{K} \times \frac{V_0}{h}\sqrt{\frac{r}{mK}} \Rightarrow I_{av} \propto V_0^2$$