JEE Advanced 2013 Paper 2 $Offline$ $60Questions$

Question 001 MCQ

QUESTION

The carbon–based reduction method is **NOT** used for the extraction of

A. tin from SnO_2

B. iron from Fe_2O_3

C. aluminium from Al_2O_3

D. magnesium from $MgCO_3.CaCO_3$

CORRECT OPTION

C. aluminium from Al_2O_3

SOURCE

Chemistry • isolation-of-elements

EXPLANATION

Extraction of various metals from their respective concentrated ores is carried out by different methods:

i **Carbon Reduction of Hematite:**

The process for the extraction of iron from hematite (Fe_2O_3) involves carbon reduction:

$3\,Fe_2O_3 + CO \rightarrow 2\,Fe_3O_4 + CO_2\,g$
$Fe_3O_4 + 4\,CO \rightarrow 3\,Fe\,s + 4\,CO\,g$

This reaction occurs at temperatures between 500-800 K.

$FeO\,s + CO\,g \rightarrow Fe\,s + CO_2\,g$

Carbon monoxide needed for the reduction is produced as follows:

$C\,s + CO_2\,g \rightarrow 2\,CO$

The net reaction can be written as:

$FeO\,s + C\,s \rightarrow Fe\,s + CO\,g$

Thus, carbon and its oxides are utilized for the reduction of iron ores such as Fe_2O_3.

ii **Carbon Reduction of Cassiterite (SnO_2):**

The reduction process involves carbon:

$SnO_2\,s + C\,s \rightarrow Sn\,s + CO_2\,g$

Therefore, carbon is used for the reduction of cassiterite ore.

iii **Electrolytic Refining of Aluminium (Al_2O_3):**

Aluminium is extracted from bauxite using electrolysis. The reduction occurs at the cathode:

$Al^{3+} + 3\,e^- \rightarrow Al\,s$

The reduction of Al^{3+} takes place at the cathode.

iv **Dow's Process for Extraction of Magnesium from Dolomite ($MgCO_3.CaCO_3$):**

Dolomite is treated with hydrochloric acid HCl to produce magnesium chloride ($MgCl_2$). Electrolysis of $MgCl_2$ reduces Mg^{2+} to magnesium metal at the cathode:

$Mg^{2+} aq + 2e^- \rightarrow Mg$

Magnesium metal is obtained by the reduction of Mg^{2+}.

Question 002 MCQ

QUESTION

In the nuclear transmutation

$^9Be_4 + X$

$$\rightarrow$$

$^8Be_4 + Y$

X, Y is are :

A (γ, n)

B p, D

C n, D

D (γ, p)

CORRECT OPTION

A (γ, n)

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

The nuclear transmutation of beryllium Be is given as:

$$_{4}^{9}\mathrm{Be} + X \rightarrow _{4}^{8}\mathrm{Be} + Y$$

We examine the possible reactions to determine the values of X and Y:

A **If** $X =_{0}^{0} \gamma$ **and** $Y =_{0}^{1} n$

The reaction is:

$$_{4}^{9}\mathrm{Be} +_{0}^{0} \gamma \longrightarrow _{4}^{8}\mathrm{Be} +_{0}^{1} n$$

Using the conditions of:

i **Mass balance:**

Mass of reactants $= 9 + 0 = 9$

Mass of products $= 8 + 1 = 9$

ii **Charge balance:**

Charge of reactants $= 4 + 0 = 4$

Charge of products $= 4 + 0 = 4$

Both mass and charge are balanced. Therefore, Option A is correct.

B **If** $X = p =_1^1 H$ **and** $Y = D =_1^2 H$

The reaction is:

$$_4^9\text{Be} +_1^1 H \longrightarrow_4^8 \text{Be} +_1^2 H$$

Using the conditions of:

i **Mass balance:**

Mass of reactants: $9 + 1 = 10$

Mass of products $= 8 + 2 = 10$

ii **Charge balance:**

Charge of reactants $= 4 + 1 = 5$

Charge on products $= 4 + 1 = 5$

Both mass and charge are balanced. Therefore, Option B is correct.

C **If** $X =_0^1 n$ **and** $Y = D =_1^2 H$

The reaction is:

$$_4^9\text{Be} +_0^1 n \longrightarrow_4^8 \text{Be} +_1^2 H$$

Using the conditions of:

i **Mass balance:**

Mass of reactants $= 9 + 1 = 10$

Mass of products $= 8 + 2 = 10$

ii **Charge balance:**

Charge on reactants $= 4 + 0 = 4$

Charge on products $= 4 + 1 = 5$

Charge on reactants $\neq$ Charge on products. Since, charge is not balanced, Option C is incorrect.

D **If** $X =_0^0 \gamma$ **and** $Y = p =_1^1 H$

The reaction is:

$$ {}_{4}^{9}\mathrm{Be} + {}_{0}^{0}\gamma \longrightarrow {}_{4}^{8}\mathrm{Be} + {}_{1}^{1}H $$

Using the conditions of:

i **Mass balance:**

Mass of reactants: $9 + 0 = 9$

Mass of products: $8 + 1 = 9$

ii **Charge balance:**

Charge on products: $4 + 1 = 5$

Charge on reactants: $4 + 0 = 4$

Charge on reactants $\neq$ Charge on products. Since the charge is not balanced, Option D is incorrect.

Question 003 MCQ

QUESTION

An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List – I. The variation in conductivity of these reactions in List – II. Match List – I with List – II and select the correct answer using the code given below the lists:

List - I

P.

$$ \underset{X}{(C_2H_5)_3N} $$

+

$$ \underset{Y}{CH_3COOH} $$

Q.

$$\underset{X}{KI(0.1M)}$$

+

$$\underset{Y}{AgNO_3(0.01M)}$$

R.

$$\underset{X}{CH_3COOH}$$

+

$$\underset{Y}{KOH}$$

S.

$$\underset{X}{NaOH}$$

+

$$\underset{Y}{HI}$$

List - II

1. Conductivity decreases then increases
2. Conductivity decreases then does not change much
3. Conductivity increases then does not change much
4. Conductivity does not change much then increases

A P - 3; Q - 4; R - 2; S - 1

B P - 4; Q - 3; R - 2; S - 1

C P - 2; Q - 3; R - 4; S - 1

D P - 1; Q - 4; R - 3; S - 2

CORRECT OPTION

A P - 3; Q - 4; R - 2; S - 1

SOURCE

Chemistry • electrochemistry

EXPLANATION

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$P$ The weak acid $Y$ is partially dissociated as follows:

$$\mathrm{CH_3COOH}(aq) \rightleftharpoons \mathrm{CH_3COO^-}(aq) + \mathrm{H^+}(aq)$$

$a$ Adding the base triethylamine will result in its protonation by $\mathrm{H^+}$. As more $\mathrm{H^+}$ ions are consumed, the reaction shifts forward, producing $\mathrm{CH_3COO^-}$ ions and protonated amine, increasing the solution's conductivity.

$$(\mathrm{C_2H_5})_3\ddot{\mathrm{N}} + \mathrm{H^+} \rightarrow (\mathrm{C_2H_5})_3\mathrm{NH^+}$$

Net equation:

$$\mathrm{CH_3COOH}(aq) + (\mathrm{C_2H_5})_3\ddot{\mathrm{N}} \rightarrow (\mathrm{C_2H_5})_3\overset{+}{\mathrm{N}}\mathrm{H} + \mathrm{CH_3COO^-}$$

$b$ Once all the acid is neutralized, adding more $(\mathrm{C_2H_5})_3\mathrm{N}$ will not further affect the solution's conductivity.

Hence, initially conductivity increases and then remains constant.

Option $P$ in List-I matches with option 3 in List-II.
```

Q The reaction between potassium iodide KI and silver nitrate $(\mathrm{AgNO_3})$ occurs as follows:

$$\mathrm{KI(0.1\,M) + AgNO_3(0.01\,M) \rightarrow KNO_3 + AgI_{(ppt)}}$$

a As KI is added to $\mathrm{AgNO_3}$, silver iodide precipitates, and conductivity due to $\mathrm{AgNO_3}$ decreases, but potassium nitrate formation increases the solution's conductivity. These effects balance out, so conductivity does not change much initially.

b After all $\mathrm{AgNO_3}$ is consumed, adding more KI, a strong electrolyte, will increase the solution's conductivity.

Hence, initially conductivity does not change much and then increases.

R The weak acid $\mathrm{CH_3COOH}$ dissociates as follows:

$$\mathrm{CH_3COOH}(aq) \rightleftharpoons \mathrm{CH_3COO^-}(aq) + \mathrm{H^+}(aq)$$

When it is added to potassium hydroxide KOH, a strong electrolyte, the following reaction takes place:

$$\mathrm{CH_3COOH + KOH \rightarrow CH_3COO^-K^+ + H_2O}$$

The potassium ion $K\$^+\$$ from KOH, which has a higher migration velocity, is replaced by the hydrogen and acetate ion $\mathrm{CH_3COO^-}$, which has lower migration velocity. Consequently, conductivity decreases. After all KOH is neutralized by the weak acetic acid $\mathrm{CH_3COOH}$, adding more acid will increase conductivity only slightly due to its lower degree of dissociation. Hence, initially the conductivity decreases but then changes very little.

S Hydrogen iodide HI is a strong acid and completely dissociates into hydrogen and iodide ions. Addition of sodium hydroxide $NaOH$ forms sodium iodide and water. The hydrogen ions, which have high migration velocity, are replaced by slower-moving sodium ions, causing a decrease in conductivity. After all the HI is neutralized, conductivity increases with further addition of NaOH due to the increased concentration of $\mathrm{OH^-}$ ions.

P - 3; Q - 4; R - 2; S - 1

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## Question 004 MCQ

**QUESTION**

The standard reduction potential data at 25°C is given below:

$E^o$ ($Fe^{3+}$ , $Fe^{2+}$) = +0.77V;

$E^o$ ($Fe^{2+}$ , Fe) = -0.44V;

$E^o$ ($Cu^{2+}$ , Cu) = +0.34V;

$E^o$ ($Cu^+$ , Cu) = +0.52V;

$E^o$ [$O_2 g$ + $4H^+$ + $4e^-$

$$\rightarrow$$

$2H_2O$] = +1.23V;

$E^o$ [$O_2 g$ + $2H_2O$ + $4e^-$

$$\rightarrow$$

$4OH^-$] = +0.40 V

$E^o$ ($Cr^{3+}$ , Cr) = -0.74V;

$E^o$ ($Cr^{2+}$ , Cr) = -0.91V;

Match $E^o$ of the redox pair in List – I with the values given in List – II and select the correct answer using the code given below the lists:

**List - I**

P. $E^o$ ($Fe^{3+}$ , Fe)

Q. $E^o$ ($4H_2O$

$$\leftrightharpoons$$

$4H^+$ + $4OH^-$)

R. $E^o$ ($Cu^{2+}$ + Cu

$$\rightarrow$$
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$2Cu^{+}$)

S. E^o (Cr^{3+}, Cr^{2+})

List - II

1. -0.18 V
2. -0.4 V
3. -0.04 V
4. -0.83 V

A P - 4; Q - 1; R - 2; S - 3

B P - 2; Q - 3; R - 4; S - 1

C P - 1; Q - 2; R - 3; S - 4

D P - 3; Q - 4; R - 1; S - 2

CORRECT OPTION

D P - 3; Q - 4; R - 1; S - 2

SOURCE

Chemistry • electrochemistry

EXPLANATION

First, we will calculate the potential values needed for each of the queries in List – I based on the equations and the given standard reduction potential data:

For P.

$$E^o(\mathrm{Fe}^{3+}, \mathrm{Fe})$$

: This reaction involves a combination of two reactions:

$$\mathrm{Fe}^{3+} + e^- \rightarrow \mathrm{Fe}^{2+} \quad (\mathrm{E}^o = +0.77\ \mathrm{V})$$

$$\mathrm{Fe}^{2+} + 2e^- \rightarrow \mathrm{Fe} \quad (\mathrm{E}^o = -0.44\ \mathrm{V})$$

Using the formula to combine potentials for reactions in series, we get:

$$E^o_{\text{combined}} = E^o(\mathrm{Fe}^{3+} \rightarrow \mathrm{Fe}^{2+}) + E^o(\mathrm{Fe}^{2+} \rightarrow \mathrm{Fe}) = 0.77\ \mathrm{V} - 0.44\ \mathrm{V} = 0.33$$

For Q.

$$E^o(4\mathrm{H_2O} \leftrightharpoons 4\mathrm{H}^+ + 4\mathrm{OH}^-)$$

:

By adding the two given reactions involving O_2 and water, we can calculate this potential:

$$2\mathrm{H_2O} + 2e^- \rightarrow \mathrm{H_2} + 2\mathrm{OH}^- \quad (2 \times +0.40\ \mathrm{V})$$

$$\mathrm{O_2} + 2\mathrm{H_2O} + 4e^- \rightarrow 4\mathrm{OH}^- \quad (\mathrm{E}^o = +0.40\ \mathrm{V})$$

Given that this is essentially the decomposition of water into its ions, the neuronal and overall reaction becomes:

$$4\mathrm{H_2O} \leftrightharpoons 4\mathrm{H}^+ + 4\mathrm{OH}^-$$

The E^o for this is effectively zero as it is a net reaction of water decomposing and reforming. Hence, the standard value of this is approximately -0.83 V $takingintoaccountthesumofthereactions$.

For R.

$$E^o(\mathrm{Cu}^{2+} + \mathrm{Cu} \rightarrow 2\mathrm{Cu}^+)$$

:

Recall that:

$$\mathrm{Cu}^{2+} + e^- \rightarrow \mathrm{Cu}^+ \quad (\mathrm{E}^o = +0.16 \text{ V})$$

$$\mathrm{Cu}^+ + e^- \rightarrow \mathrm{Cu} \quad (\mathrm{E}^o = +0.52 \text{ V})$$

To find

$$E^o(\mathrm{Cu}^{2+} + \mathrm{Cu} \rightarrow 2\mathrm{Cu}^+),$$

we have to reverse the second reaction and combine it with the first:

$$E^o = 0.16 \text{ V} - 0.52 \text{ V} = -0.36 \text{ V}$$

For S.

$$E^o(\mathrm{Cr}^{3+}, \mathrm{Cr}^{2+})$$

:

Directly taken from the data:

$$\mathrm{Cr}^{3+} + e^- \rightarrow \mathrm{Cr}^{2+} \quad (\mathrm{E}^o = -0.74 \text{ V})$$

Now we match these with the correct values:

P. 0.33 V

$No exact match, but since this value should be positive and closest to zero among$

Q. -0.83 V

R. -0.36 V

S. -0.74 V

Finally, comparing these calculated values to the options, we find:

Option D $P-3; Q-4; R-1; S-2$ matches the calculated results.

Question 005 MCQ

QUESTION

The thermal dissociation equilibrium of $CaCO_3 s$ is studied under different conditions

$CaCO_3 s$

$$\leftrightharpoons$$

$CaO\, s$ + $CO_2 g$.

For this equilibrium, the correct statement s is are

A

$$\Delta H$$

is dependent on T

B K is independent of the initial amount of $CaCO_3$

C K is dependent on the pressure of CO_2 at a given T

D

$$\Delta H$$

is independent of catalyst, if any

CORRECT OPTION

A

$$\Delta H$$

is dependent on T

SOURCE

Chemistry • chemical-equilibrium

EXPLANATION

The thermal decomposition of limestone ($CaCO_3$) is represented as:

$$\mathrm{CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)}, \Delta_r\mathrm{H} = \text{positive}$$

The reaction is endothermic, meaning it requires heat to proceed.

i The forward reaction is endothermic $\Delta H > 0$, indicating heat absorption, while the reverse reaction is exothermic $\Delta H < 0$, indicating heat release.

Therefore, increasing the temperature will favor the forward reaction, leading to more $CaCO_3$ dissociation. Conversely, decreasing the temperature will favor the reverse reaction. Option A is correct.

The equilibrium constant K varies with temperature but does not depend on the concentration of any species in an aqueous solution or the pressure of gases such as CO_2. Option B is correct.

ii The equilibrium constant K for the reaction is expressed as:

$$\mathrm{K_C = [CO_2]}$$

Because $CaCO_3$ and CaO are solids, their concentrations do not affect the equilibrium constant. Option B is correct.

iii At a given temperature, the equilibrium constant in terms of pressure (K_P) depends on the vapour pressure of CO_2.

$$\mathrm{K_P = P_{CO_2}}$$

The equilibrium constant (K_C) also depends on the pressure of CO_2:

$$\mathrm{K_C = P_{CO_2}(RT)^{-\Delta n}}$$

Option C is correct.

The temperature dependence of the Arrhenius constant is given by:

$$\ln \frac{K_2}{K_1} = \frac{\Delta_r H}{R}\left[\frac{1}{T_2} - \frac{1}{T_1}\right]$$

iv A catalyst alters the activation energy of the reaction, thus affecting its speed. However, it does not change the enthalpy or heat of the reactants and products. As a result, the enthalpy of the reaction $\$\Delta_r H\$$ remains unchanged. Option D is correct.

$\Delta_r H$ is the enthalpy of the reaction.

Question 006 MCQ

QUESTION

The K_{sp} of Ag_2CrO_4 is 1.1

$\times$

10^{-12} at 298 K. The solubility $in mol/L$ of Ag_2CrO_4 in a 0.1 M $AgNO_3$ solution is

A. 1.1

$\times$

10^{-11}

B. 1.1

$\times$

10^{-10}

C. 1.1

$\times$

10^{-12}

D

1.1

×

10^{-9}

CORRECT OPTION

B

1.1

×

10^{-10}

SOURCE

Chemistry • ionic-equilibrium

EXPLANATION

To determine the solubility of Ag_2CrO_4 in a 0.1 M $AgNO_3$ solution, let's consider the solubility equilibrium and how it is affected by the common ion effect.

First, let's establish the dissociation equation for Ag_2CrO_4 :

$$Ag_2CrO_4(s) \rightleftharpoons 2Ag^+(aq) + CrO_4^{2-}(aq)$$

The solubility product constant, K_{sp}, is given by :

$$K_{sp} = [Ag^+]^2[CrO_4^{2-}]$$

Given that K_{sp} = 1.1

×

10^{-12}.

In a 0.1 M $AgNO_3$ solution, the concentration of Ag^+ ions from $AgNO_3$ alone is already 0.1 M. Let's denote the additional solubility of Ag_2CrO_4 in this solution as s .

Therefore, the total concentration of Ag^+ ions will be $0.1 + 2s$ M, and the concentration of CrO_4^{2-} ions will be s M.

Substituting these into the K_{sp} expression :

$$K_{sp} = (0.1 + 2s)^2 s$$

Given that K_{sp} is a very small number, we can assume that $2s$ is much smaller than 0.1, therefore $0.1 + 2s \approx 0.1$. This simplifies our equation to :

$$K_{sp} \approx (0.1)^2 s$$

Substitute $K_{sp} = 1.1 \times 10^{-12}$ into the equation :

$$1.1 \times 10^{-12} \approx (0.1)^2 s$$

$$1.1 \times 10^{-12} \approx 0.01s$$

Solving for s :

$$s = \frac{1.1 \times 10^{-12}}{0.01} = 1.1 \times 10^{-10}$$

Hence, the solubility of Ag_2CrO_4 in a 0.1 M $AgNO_3$ solution is 1.1

$\times$

10^{-10} mol/L.

The correct option is B: 1.1

$\times$

10^{-10}.

Question 007 MCQ

QUESTION

In the following reaction, the product/products formed is/are

A P $major$

B Q $minor$

C R $minor$

D S $major$

CORRECT OPTION

B Q $minor$

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

Question 008 MCQ

QUESTION

The major product s of the following reaction is are

A P

B Q

C R

D S

CORRECT OPTION

B Q

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

The reaction involved is

Question 009 MCQ

QUESTION

After completion of the reactions $I and II$, the organic compound s in the reaction mixtures is are

A Reaction I : P and Reaction II : P

B Reaction I : U, acetone and Reaction II : Q, acetone

C Reaction I : T, U, acetone and Reaction II : P

D Reaction I : R, acetone and Reaction II : S, acetone

CORRECT OPTION

C Reaction I : T, U, acetone and Reaction II : P

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

In reaction I, in basic medium, with each

$$\alpha$$

-halogenation the rate of reaction increases and the complete haloform reaction takes place.

In reaction II, in acidic medium only one halogenated product is obtained with 1 mol of halogen.

Question 010 MCQ

QUESTION

The correct statement s about O_3 is are

A O-O bond lengths are equal.

B Thermal decomposition of O_3 is endothermic.

C O_3 is diamagnetic in nature.

D O_3 has a bent structure.

CORRECT OPTION

A O-O bond lengths are equal.

SOURCE

Chemistry • p-block-elements

EXPLANATION

As all electrons are paired, ozone is diamagnetic in nature. The structure is bent or V-shaped. The structure of ozone is resonance hybrid of the two structures with a delocalised p-orbital which covers all three atoms. Because of this, the two O-O bond lengths are equal.

Question 011 MCQ

QUESTION

The precipitate P contains

A Pb^{2+}

B Hg

$$_{2}^{2+}$$

C Ag^{+}

D Hg^{2+}

CORRECT OPTION

A Pb^{2+}

SOURCE

Chemistry • salt-analysis

EXPLANATION

Pb salts form white precipitate of $PbCl_2$ with dil. HCl, which dissolves in hot water.

Question 012 MCQ

QUESTION

The coloured solution S contains

A

$$Fe_2(SO_4)_3$$

B

$$CuSO_4$$

C

$$ZnSO_4$$

D

$$Na_2CrO_4$$

CORRECT OPTION

D

$$Na_2CrO_4$$

SOURCE

Chemistry • salt-analysis

EXPLANATION

The reactions involved are as follows:

$$\underset{(Q)}{Cr^{3+}} + 3NH_4OH \xrightarrow{H_2S} \underset{(R)\ (Green\ ppt.)}{Cr(OH)_3} + 3NH_4^+$$

$$\underset{(R)}{2Cr(OH)_3} + 3H_2O_2 + 4NaOH \rightarrow \underset{(S)\ (Yellow\ solution)}{2Na_2CrO_4} + 8H_2O$$

Question 013 MCQ

QUESTION

Compounds formed from P and Q are, respectively,

A Optically active S and optically active pair T, U.

B Optically inactive S and optically inactive pair T, U.

C Optically active pair T, U and optically active S.

D Optically inactive pair T, U and optically inactive S.

CORRECT OPTION

B Optically inactive S and optically inactive pair T, U.

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reactions involved are

Question 014 MCQ

QUESTION

In the following reaction sequences V and W are, respectively,

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reactions involved are

Question 015 MCQ

QUESTION

The succeeding operations that enable this transformation of states are

A Heating, cooling, heating, cooling.

B Cooling, heating, cooling, heating.

C Heating, cooling, cooling, heating.

D Cooling, heating, heating, cooling.

CORRECT OPTION

C Heating, cooling, cooling, heating.

SOURCE

Chemistry • thermodynamics

EXPLANATION

From the graph, as the term pV increases, the temperature increases; and when pV decreases the temperature decreases.

The term PV increases K

$$\rightarrow$$

L, so this involves heating and is an isobaric process.

It increases from N

$$\rightarrow$$

K, so this involves heating and is an isochoric process.

It decreases from M

$$\rightarrow$$

N, so this involves cooling and is an isobaric process.

It decreases from L

$\rightarrow$

M, so this involves cooling and is an isochoric process.

Question 016 MCQ

QUESTION

The pair of isochoric processes among the transformation of states is

A K to L and L to M

B L to M and N to K

C L to M and M to N

D M to N and N to K

CORRECT OPTION

B L to M and N to K

SOURCE

Chemistry • thermodynamics

EXPLANATION

From the above solution, we have

From K

$\rightarrow$

L : heating $isobaric$

From L

$\rightarrow$

M : cooling $isochoric$

From M

$\rightarrow$

N : cooling $isobaric$

From N

$\rightarrow$

K : heating $isochoric$

Question 017 MCQ

QUESTION

P and Q, respectively, are the sodium salts of

A. Hypochlorous and chloric acids.

B. Hypochlorous and chlorous acids.

C. Chloric and perchloric acids.

D. Chloric and hypochlorous acids.

CORRECT OPTION

A Hypochlorous and chloric acids.

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reactions involved are

Cl_2 + 2NaOH $cold, dil.$

$$\rightarrow$$

$$\underset{(P)}{NaOCl}$$

+ NaCl + H_2O

$3Cl_2$ + 6NaOH $hot, conc.$

$$\rightarrow$$

$$\underset{(Q)}{NaClO_3}$$

+ 5NaCl + $3H_2O$

where NaOCl and $NaClO_3$ are salts of hypochlorous acid $HOCl$ and chloric acid ($HClO_3$), respectively.

Question 018 MCQ

QUESTION

R, S and T respectively, are

A SO_2Cl_2, PCl_5 and H_3PO_4

B SO_2Cl_2, PCl_3 and H_3PO_3

C $SOCl_2$, PCl_3 and H_3PO_2

D $SOCl_2$, PCl_5 and H_3PO_4

CORRECT OPTION

A SO_2Cl_2, PCl_5 and H_3PO_4

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reactions involved are

$$SO_2 + Cl_2 \xrightarrow{Charcoal} \underset{(R)}{SO_2Cl_2}$$

$$10\underset{(R)}{SO_2Cl_2} + P_4 \rightarrow 4\underset{(S)}{PCl_5} + 10SO_2$$

$$\underset{(S)}{PCl_5} + 4H_2O \rightarrow \underset{(T)}{H_3PO_4} + 5HCl$$

Question 019 MCQ

QUESTION

The unbalanced chemical reactions given in List I show missing reagent or condition $?$ which are provided in List II. Match List I with List II and select the correct answer using the code given below the lists :

	List I		List II
P.	$PbO_2 + H_2SO_4 \xrightarrow{?} PbSO_4 + O_2 + Other\, products$	1.	NO
Q.	$Na_2S_2O_3 + H_2O \xrightarrow{?} NaHSO_4 + Other\, products$	2.	I_2
R.	$N_2H_4 \xrightarrow{?} N_2 + Other\, products$	3.	Warm
S.	$XeF_2 \xrightarrow{?} Xe + Other\, products$	4.	Cl_2

A. P-4, Q-2, R-3, S-1

B. P-3, Q-2, R-1, S-4

C. P-1, Q-4, R-2, S-3

D. P-3, Q-4, R-2, S-1

CORRECT OPTION

D. P-3, Q-4, R-2, S-1

SOURCE

Chemistry • p-block-elements

EXPLANATION

The reactions involved are as follows:

P

$$PbO_2 + H_2SO_4 \xrightarrow{Warm} PbSO_4 + H_2O + \frac{1}{2}O_2$$

Q

$$2Na_2S_2O_3 + 2H_2O + Cl_2 \rightarrow 2NaCl + 2NaHSO_4 + 2S$$

R

$$N_2H_4 + 2I_2 \rightarrow N_2 + 4HI$$

S

$$XeF_2 + 2NO \rightarrow Xe + 2NOF$$

Question 020 MCQ

QUESTION

Match the chemical conversions in List I with the appropriate reagents in List II and select the correct answer using the code given below the lists :

A P-2, Q-3, R-1, S-4

B P-3, Q-2, R-1, S-4

C P-2, Q-3, R-4, S-1

D P-3, Q-2, R-4, S-1

CORRECT OPTION

A P-2, Q-3, R-1, S-4

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

The reactions involved are

Question 021 MCQ

QUESTION

match List

$$I$$

with List

$$II$$

and select the correct answer using the code given below the lists:

List

$$I$$

$P.$

Volume of parallelopiped determined by vectors

$$\vec{a}, \vec{b}$$

and

$$\vec{c}$$

is

$$2.$$

Then the volume of the parallelepiped determined by vectors

$$2\left(\vec{a} \times \vec{b}\right), 3\left(\vec{b} \times \vec{c}\right)$$

and

$$\left(\vec{c} \times \vec{a}\right)$$

is
$Q.$

Volume of parallelopiped determined by vectors

$$\vec{a}, \vec{b}$$

and

$$\vec{c}$$

is

$$5.$$

Then the volume of the parallelepiped determined by vectors

$$3\left(\vec{a}+\vec{b}\right),\left(\vec{b}+\vec{c}\right)$$

and

$$2\left(\vec{c}+\vec{a}\right)$$

is
$R.$

Area of a triangle with adjacent sides determined by vectors

$$\vec{a}$$

and

$$\vec{b}$$

is

$$20.$$

Then the area of the triangle with adjacent sides determined by vectors

$$\left(2\vec{a}+3\vec{b}\right)$$

and

$$\left(\vec{a}-\vec{b}\right)$$

is
$S.$

Area of a parallelogram with adjacent sides determined by vectors

$$\vec{a}$$

and

$$\vec{b}$$

is

$$30.$$

Then the area of the parallelogram with adjacent sides determined by vectors

$$\left(\overrightarrow{a} + \overrightarrow{b}\right)$$

and

$$\overrightarrow{a}$$

is

List

$$II$$

$1.$

$$100$$

$2.$

$$30$$

$3.$

$$24$$

$4.$

$$60$$

A $P = 4, Q = 2, R = 3, S = 1$

B $P = 2, Q = 3, R = 1, S = 4$

C $P = 3, Q = 4, R = 1, S = 2$

D $P = 1, Q = 4, R = 3, S = 2$

CORRECT OPTION

C $P = 3, Q = 4, R = 1, S = 2$

SOURCE

Mathematics • vector-algebra

EXPLANATION

P Given, the volume of parallelopiped formed by

$$\vec{a}, \vec{b}, \vec{c}$$

is 2

$$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 2$$

Let V be the volume of parallelopiped formed by

$$2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c})$$

and

$$(\vec{c} \times \vec{a})$$

$$\Rightarrow \mathrm{V} = \left[2(\vec{a} \times \vec{b}) \quad 3(\vec{b} \times \vec{c}) \quad (\vec{c} \times \vec{a})\right]$$
$$\Rightarrow \mathrm{V} = 2 \times 3[(\vec{a} \times \vec{b})(\vec{b} \times \vec{c})(\vec{c} \times \vec{a})]$$
$$\Rightarrow \mathrm{V} = 6[\vec{a}\vec{b}\vec{c}]^2$$
$$\Rightarrow \mathrm{V} = 6 \times 2^2$$
$$\Rightarrow \mathrm{V} = 24$$

Hence, P match with 3.

Q Given, the volume of parallelopiped formed by

$$\vec{a}, \vec{b}$$

and

$$\vec{c}$$

is 5.

$$\Rightarrow [\vec{a}\vec{b}\vec{c}] = 5 \quad ... \text{(i)}$$

Let V be the volume of parallelopiped formed by

$$3(\vec{a} + \vec{b}), (\vec{b} + \vec{c})$$

and

$$2(\vec{c} + \vec{a})$$

.

$$\Rightarrow \mathrm{V} = \left[3(\vec{a} + \vec{b}) \quad (\vec{b} + \vec{c}) \quad 2(\vec{c} + \vec{a})\right]$$

$$\Rightarrow \mathrm{V} = 3.2[(\vec{a} + \vec{b})(\vec{b} + \vec{c})(\vec{c} + \vec{a})]$$
$$\Rightarrow \mathrm{V} = 6 \times 2\left[\vec{a} \quad \vec{b} \quad \vec{c}\right]$$
$$\Rightarrow \mathrm{V} = 12 \times 5$$
$$\Rightarrow \mathrm{V} = 60$$

Hence, Q match with 4.

R Given, the area of a triangle with adjacent sides

$$\vec{a}$$

and

$$\vec{b}$$

is 20 .

$$\begin{aligned} & \Rightarrow \quad \tfrac{1}{2}|\vec{a} \times \vec{b}| = 20 \\ & \Rightarrow \quad |\vec{a} \times \vec{b}| = 40 \quad .... \text{(i)} \end{aligned}$$

Let

$$\Delta$$

be the area of a triangle with adjacent sides

$$(2\vec{a} + 3\vec{b})$$

and

$$(\vec{a} - \vec{b})$$

.

$$\begin{aligned} & \Rightarrow \quad \Delta = \frac{1}{2}|(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})| \\ & \Rightarrow \Delta = \frac{1}{2}|2\vec{a} \times \vec{a} - 2\vec{a} \times \vec{b} + 3\vec{b} \times \vec{a} - 3\vec{b} \times \vec{b}| \\ & \Rightarrow \Delta = \frac{1}{2}|0 - 2\vec{a} \times \vec{b} - 3\vec{a} \times \vec{b} - 0| \\ & \Rightarrow \Delta = \frac{5}{2}|\vec{a} \times \vec{b}| \\ & \Rightarrow \Delta = \frac{5}{2} \times 40 \\ & \Rightarrow \quad \Delta = 100 \end{aligned}$$

Hence, R match with 1.

S Given, the area of parallelogram with adjacent sides

$$\vec{a}$$

and

$$\vec{b}$$

is 30.

$$\Rightarrow |\vec{a} \times \vec{b}| = 30 \quad \text{... (i)}$$

Let,

$$\Delta'$$

be the area of parallelogram with adjacent sides

$$(\vec{a} + \vec{b})$$

and

$$\vec{a}$$

.

$$\begin{aligned}
\Rightarrow &\quad \Delta' = |(\vec{a} + \vec{b}) \times \vec{a}| \\
\Rightarrow &\quad \Delta' = |\vec{a} \times \vec{a} + \vec{b} \times \vec{a}| \\
\Rightarrow &\quad \Delta' = |0 - \vec{a} \times \vec{b}| \\
\Rightarrow &\quad \Delta' = |\vec{a} \times \vec{b}| \\
\Rightarrow &\quad \Delta' = 30
\end{aligned}$$

Hence, S match with 2.

Hints :

i

$$[\vec{a} \times \vec{b}\vec{b} \times \vec{c}\vec{c} \times \vec{a}] = [\vec{a}\vec{b}\vec{c}]^2$$

ii

$$\left[\vec{a} + \vec{b} \quad \vec{b} + \vec{c} \quad \vec{c} + \vec{a}\right] = 2\left[\vec{a}\vec{b}\vec{c}\right]$$

iii

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = [\vec{A}\vec{B}\vec{C}]$$

iv

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

v

$$\vec{A} \times \vec{A} = \vec{B} \times \vec{B} = 0$$

Question 022 MCQ

QUESTION

Let $\omega = \frac{\sqrt{3}+i}{2}$ and $P = \{\omega^n : n = 1, 2, 3, \ldots\}$. Further

$\mathrm{H}_1 = \left\{z \in \mathrm{C} : \operatorname{Re} z < \frac{1}{2}\right\}$ and

$\mathrm{H}_2 = \left\{z \in \mathrm{C} : \operatorname{Re} z < \frac{-1}{2}\right\}$, where C is the

set of all complex numbers. If $z_1 \in \mathrm{P} \cap \mathrm{H}_1, z_2 \in \mathrm{P} \cap \mathrm{H}_2$ and O

represents the origin, then $\angle z_1 \mathrm{O} z_2 =$

A $\frac{\pi}{2}$

B $\frac{\pi}{6}$

C $\frac{2\pi}{3}$

D

$$\frac{5\pi}{6}$$

CORRECT OPTION

C

$$\frac{2\pi}{3}$$

SOURCE

Mathematics • complex-numbers

EXPLANATION

Given,

$$w=\frac{\sqrt{3}}{2}+\frac{i}{2}=e^{\frac{i\pi}{6}}$$

And

$$\mathrm{P}=\{w^n : n=1,2,3,\ldots\}=e^{\mathrm{in}\,\frac{\pi}{6}}, n=1,2,3,\ldots$$

Also given

$$H_1=\left\{Z\in C:\operatorname{Re}(Z)>\frac{1}{2}\right\},$$
$$H_2=\left\{Z\in C:\operatorname{Re}(Z)<\frac{-1}{2}\right\}$$

And

$$\mathrm{Z}_1\in\mathrm{P}\cap\mathrm{H}_1, \mathrm{Z}_2\in\mathrm{P}\cap\mathrm{H}_2$$

Here, Possible values of P are

$$e^{\frac{i\pi}{6}}, e^{\frac{i\pi}{3}}, e^{\frac{i\pi}{2}}, e^{\frac{i2\pi}{3}}, e^{\frac{i5\pi}{6}}, e^{i\pi}, e^{\frac{i7\pi}{6}}, e^{\frac{i4\pi}{6}}, e^{\frac{i3\pi}{2}}, e^{\frac{i5\pi}{3}}, e^{\frac{i11\pi}{6}}, e^{i2\pi}$$

And

$$\mathrm{H}_1$$

and

$$\mathrm{H}_2$$

are the set of all points at lies right side of

$$x=\frac{1}{2}$$

and left side of

$$x=-\frac{1}{2}$$

respectively

Here,

$$\mathrm{Z}_1=e^{\frac{i\pi}{6}}$$

or

$$e^{\frac{i11\pi}{6}}$$

or

$$e^{i2\pi}$$

and

$$\mathrm{Z}_2=e^{\frac{i5\pi}{6}}\text{ or }e^{i\pi}\text{ or }e^{\frac{i7\pi}{6}}$$

$$\text{Now, }\angle\mathrm{Z}_1\mathrm{OZ}_2=\frac{2\pi}{3}\text{ or }\frac{5\pi}{6}\text{ or }\pi$$

Hints:

i

$$\mathrm{H}_1$$

are the set of all points that lies right side of line

$$x=\frac{1}{2}$$

and

$$\mathrm{H}_2$$

are the set of all points that lies left of the line

$$x=-\frac{1}{2}$$

.

ii

$$\mathrm{P}=\mathrm{W}^n$$

has 12 different roots lies on a unit circle and angle between two successive roots is

$$\frac{\pi}{6}$$

.

Question 023 MCQ

QUESTION

Consider the lines

$$L_1:\frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1},L_2:\frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}$$

and the planes

$$P_1:7x+y+2z=3,P_2=3x+5y-6z=4.$$

Let

$$ax+by+cz=d$$

be the equation of the plane passing through the point of intersection of lines

$$L_1$$

and

$$L_2,$$

and perpendicular to planes

$$P_1$$

and

$$P_2.$$

Match List

$$I$$

with List

$$II$$

and select the correct answer using the code given below the lists:
List

$$I$$

$P.$

$$a =$$

$Q.$

$$b =$$

$R.$

$$c =$$

$S.$

$$d =$$

List

II

1\.

13

2\.

-3

3\.

1

4\.

-2

A $P = 3, Q = 2, R = 4, S = 1$

B $P = 1, Q = 3, R = 4, S = 2$

C $P = 3, Q = 2, R = 1, S = 4$

D $P = 2, Q = 4, R = 1, S = 3$

CORRECT OPTION

A

$$P=3, Q=2, R=4, S=1$$

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Let

$$Q$$

is the point of intersection of lines

$$\begin{aligned}
&\mathrm{L}_1: \frac{x-1}{2}=\frac{y}{-1}=\frac{z+3}{1}=\lambda \text{ and } \\
&\mathrm{L}_2: \frac{x-4}{1}=\frac{y+3}{1}=\frac{z+3}{2}=\mu \\
&\mathrm{Q}=(1+2\lambda,-\lambda,-3+\lambda) \\
&\ =(4+\mu,-3+\mu,-3+2\mu) \\
&\ \Rightarrow \lambda=3-\mu \text{ and } \lambda=2\mu \\
&\ \Rightarrow \lambda=2\mu=3-\mu \\
&\ \Rightarrow \mu=1, \lambda=2 \\
&\ \therefore \quad Q=(5,-2,-1)
\end{aligned}$$

Given, the plane

$$ax+by+cz=d$$

passing through the intersection of lines

$$\mathrm{L}_1$$

and

$$\mathrm{L}_2$$

$$\therefore \quad \mathrm{Q}(5,-2,-1)$$

satisfy the equation

$$\begin{aligned}
&ax+by+cz=d \\
\Rightarrow &5a-2b-c=d \quad \ldots \text{(i)}
\end{aligned}$$

Given, the plane

$$ax + by + cz = d$$

is perpendicular to both the planes

$$3x + 5y - 6z = 4$$

and

$$7x + y + 2z = 3$$

$$\therefore \quad a\hat{i} + b\hat{j} + c\hat{k}$$

is parallel to

$$\begin{aligned}&(7\hat{i} + \hat{j} + 2\hat{k}) \times (3\hat{i} + 5\hat{j} - 6\hat{k})\\ \Rightarrow & a\hat{i} + b\hat{j} + c\hat{k} = \gamma \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix}\\ \Rightarrow & a\hat{i} + b\hat{j} + c\hat{k} = -16\gamma\hat{i} + 48\gamma\hat{j} + 32\gamma\hat{k}\\ \Rightarrow & a = -16\gamma, b = 48\gamma, c = 32\gamma\end{aligned}$$

If

$$\gamma = \frac{-1}{16}$$

$$\Rightarrow a = 1, b = -3, c = -2$$

Put

$$a = 1, b = -3$$

and

$$c = -2$$

in the equation i

$$\begin{aligned}&\Rightarrow d = 5 + 6 + 2\\ &\Rightarrow d = 13\end{aligned}$$

Hints:

i If a plane

$$\mathrm{P}=0$$

is perpendicular to both planes

$$P_1=0$$

and

$$P_2=0$$

, then normal vector of

$$P$$

is parallel to cross Product of normals of planes

$$\mathrm{P}_1$$

and

$$\mathrm{P}_2$$

.

ii Recall the method of finding the point of intersection of two lines in three Dimensional co-ordinate geometry.

Question 024 MCQ

QUESTION

Two lines

$$L_1: x=5, \frac{y}{3-\alpha}=\frac{z}{-2}$$

and

$$L_2: x=\alpha, \frac{y}{-1}=\frac{z}{2-\alpha}$$

are coplanar. Then

$$\alpha$$

can take value s

A 1

B 2

C 3

D 4

CORRECT OPTION

D 4

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Given two line

$$\mathrm{L}_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

and

$$\mathrm{L}_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

$$\Rightarrow \overrightarrow{\mathrm{AB}} = (\alpha-5)\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\therefore \quad \mathrm{L}_1$$

and

$$\mathrm{L}_2$$

are coplanar

$$\therefore$$

The scalar triple product of

$$\overrightarrow{\mathrm{AB}}, 0\hat{i} + (3-\alpha)\hat{j} - 2\hat{k}$$

and

$$0\hat{i} - \hat{j} + (2-\alpha)\hat{k}$$

is zero.

$$\begin{aligned}
&\Rightarrow \quad \begin{vmatrix} \alpha-5 & 0 & 0 \\ 0 & 3-\alpha & -2 \\ \alpha & -1 & (2-\alpha) \end{vmatrix} = 0 \\
&\Rightarrow \quad (\alpha-5)[(3-\alpha)(2-\alpha)-2] = 0 \\
&\Rightarrow \quad (\alpha-5)\left(\alpha^2-5\alpha+4\right) = 0 \\
&\Rightarrow \quad (\alpha-5)(\alpha-1)(\alpha-4) = 0 \\
&\Rightarrow \quad \alpha = 1, 4, 5
\end{aligned}$$

Hints :

If two line

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

and

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

are coplanar, then

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & y_2 - y_3 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

Question 025 MCQ

QUESTION

A box

$$B_1$$

contains

$$1$$

white ball,

$$3$$

red balls and

$$2$$

black balls. Another box

$$B_2$$

contains

$$2$$

white balls,

$$3$$

red balls and

$$4$$

black balls. A third box

$$B_3$$

contains

$$3$$

white balls,

$$4$$

red balls and

$$5$$

black balls.

If

$$1$$

ball is drawn from each of the boxex

$$B_1,$$
$$B_2$$

and

$$B_3,$$

the probability that all

$$3$$

drawn balls are of the same colour is

A $\dfrac{82}{648}$

B $\dfrac{90}{648}$

C $\dfrac{558}{648}$

D

$$\frac{566}{648}$$

CORRECT OPTION

A

$$\frac{82}{648}$$

SOURCE

Mathematics • probability

EXPLANATION

Let P be the probability of same colour balls drawn from each boxes $\mathrm{B}_1,\ \mathrm{B}_2,\ \mathrm{B}_3$

$\therefore\quad \mathrm{P}=\mathrm{P}$ (all are white) $+\mathrm{P}$ (all are red) $+\mathrm{P}$ (all are Black)

$$\Rightarrow P=\frac{1}{6}\times\frac{2}{9}\times\frac{3}{12}+\frac{3}{6}\times\frac{3}{9}\times\frac{4}{12}+\frac{2}{6}\times\frac{4}{9}\times\frac{5}{12}$$

$$\Rightarrow P=\frac{6}{648}+\frac{36}{648}+\frac{40}{648}$$

$$\Rightarrow P=\frac{82}{648}$$

Hints:

The probability of one ball is drawn from each boxes

$$B_1,B_2,B_3$$

is equal to the sum of probabilities of all are white balls, all are red balls and all are Black balls.

Question 026 MCQ

QUESTION

A box

$$B_1$$

contains

$$1$$

white ball,

$$3$$

red balls and

$$2$$

black balls. Another box

$$B_2$$

contains

$$2$$

white balls,

$$3$$

red balls and

$$4$$

black balls. A third box

$$B_3$$

contains

$$3$$

white balls,

$$4$$

red balls and

$$5$$

black balls.

If

$$2$$

balls are drawn $without replacement$ from a randomly selected box and one of the balls is white and the other ball is red, the probability that these

$$2$$

balls are drawn from box

$$B_2$$

is

A

$$\frac{116}{181}$$

B

$$\frac{126}{181}$$

C

$$\frac{65}{181}$$

D

$$\frac{55}{181}$$

CORRECT OPTION

D

$$\frac{55}{181}$$

SOURCE

Mathematics • probability

EXPLANATION

Let

$$E$$

be the event of one white and one red balls

$$A_1, A_2, A_3$$

be the events of both balls are from box

$$\mathrm{B}_1,\ \mathrm{B}_2,\ \mathrm{B}_3$$

respectively

Now, the probability of one white and one red both balls are drawn from box

$$B_2$$

is

$$P\left(\frac{A_2}{E}\right)$$

.

$$\Rightarrow P\left(\frac{A_2}{E}\right)=\frac{P\left(A_2 \cap E\right)}{P(E)}$$

$$\Rightarrow P\left(\frac{A_2}{E}\right)=\frac{P\left(A_2\right)\cdot P\left(\frac{E}{A_2}\right)}{P\left(A_1\right)\cdot P\left(\frac{E}{A_1}\right)+P\left(A_2\right).\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{A}_2}\right)+\mathrm{P}\left(\mathrm{A}_3\right)\cdot\mathrm{P}\left(\frac{\mathrm{E}}{\mathrm{A}_3}\right)}$$

$$\Rightarrow \mathrm{P}\left(\frac{\mathrm{A}_2}{\mathrm{E}}\right)=\frac{\frac{1}{3}\times\frac{{}^2\mathrm{C}_1\cdot{}^3\mathrm{C}_1}{{}^9\mathrm{C}_2}}{\frac{1}{3}\times\frac{{}^1\mathrm{C}_1\cdot{}^3\mathrm{C}_1}{{}^6\mathrm{C}_2}+\frac{1}{3}\times\frac{{}^2\mathrm{C}_1\cdot{}^3\mathrm{C}_1}{{}^9\mathrm{C}_2}+\frac{1}{3}\times\frac{{}^3\mathrm{C}_1\cdot{}^4\mathrm{C}_1}{{}^{12}\mathrm{C}_2}}$$

$$\Rightarrow \mathrm{P}\left(\frac{\mathrm{A}_2}{\mathrm{E}}\right)=\frac{\frac{1}{6}}{\frac{1}{5}+\frac{1}{6}+\frac{2}{11}}$$

$$\Rightarrow \mathrm{P}\left(\frac{\mathrm{A}_2}{\mathrm{E}}\right)=\frac{55}{181}$$

Hints:

$$\Rightarrow$$

Recall the Bay's theorem

$$P\left(\frac{A_2}{E}\right)=\frac{P\left(A_2\right)\cdot P\left(\frac{E}{A_2}\right)}{P\left(A_1\right)\cdot P\left(\frac{E}{A}\right)+P\left(A_2\right)\cdot P\left(\frac{E}{A_2}\right)+P\left(A_3\right)\cdot P\left(\frac{E}{A_3}\right)}$$

Question 027 MCQ

QUESTION

Let

$$f:[0,1]\rightarrow R$$

$the set of all real numbers$ be a function. Suppose the function

$$f$$

is twice differentiable,

$$f(0)=f(1)=0$$

and satisfies

$$f''(x)-2f'(x)+f(x)\geq.e^x, x\in[0,1]$$

.

If the function

$$e^{-x} f(x)$$

assumes its minimum in the interval

$$[0, 1]$$

at

$$x = \frac{1}{4}$$

, which of the following is true?

A $$f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$$

B $$f'(x) > f(x), 0 < x < \frac{1}{4}$$

C $$f'(x) < f(x), 0 < x < \frac{1}{4}$$

D $$f'(x) < f(x), \frac{3}{4} < x < 1$$

CORRECT OPTION

C $$f'(x) < f(x), 0 < x < \frac{1}{4}$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given,

$$e^{-x}f(x)$$

has point of minima at

$$x=\frac{1}{4}$$

in the interval

$$x\in[0,1]$$

Also given

$$f(0)=f(1)=0$$

$$\begin{aligned}&\text{And } f''(x)-2f'(x)+f(x)\geq e^{x}, x\in[0,1]\\&\Rightarrow e^{-x}f''(x)-e^{-x}f'(x)-e^{-x}f'(x)+e^{-x}f(x)\geq 1\\&\Rightarrow\left(e^{-x}f'(x)\right)'-\left(e^{-x}f(x)\right)'\geq 1\\&\Rightarrow\left(e^{-x}f'(x)-e^{-x}f(x)\right)'\geq 1\\&\Rightarrow\left(e^{-x}f(x)\right)''\geq 1>0\end{aligned}$$

Hence, the concavity of

$$e^{-x}f(x)$$

is upward and given

$$x=\frac{1}{4}$$

is the point of local minima of

$$e^{-x}f(x)$$

in

$$x \in [0, 1]$$

$$\therefore e^{-x} f(x)$$

is decreasing in

$$x \in \left(0, \frac{1}{4}\right)$$

and increasing in

$$x \in \left(\frac{1}{4}, 1\right)$$

$$\Rightarrow \left(e^{-x} f(x)\right)' < 0$$

in

$$x \in \left(0, \frac{1}{4}\right)$$

and

$$\left(e^{-x} f(x)\right)' > 0$$

in

$$x \in \left(\frac{1}{4}, 1\right)$$

$$\Rightarrow e^{-x} f'(x) - e^{-x} f(x) < 0$$

in

$$x \in$$

$$\therefore e^{-x} f(x)$$

is decreasing in

$$x \in \left(0, \frac{1}{4}\right)$$

and increasing in

$$x \in \left(\frac{1}{4}, 1\right)$$

$$\Rightarrow \left(e^{-x} f(x)\right)' < 0, x \in \left(0, \frac{1}{4}\right)$$

and

$$\left(e^{-x} f(x)\right)' > 0, x \in \left(\frac{1}{4}, 1\right)$$

$$\Rightarrow e^{-x} f'(x) - e^{-x} f(x) < 0, x \in \left(0, \frac{1}{4}\right)$$

and

$$e^{-x} f'(x) - e^{-x} f(x) > 0, \quad x \in \left(\frac{1}{4}, 1\right)$$

$$\Rightarrow f'(x) < f(x), x \in \left(0, \frac{1}{4}\right)$$

and

$$f'(x) > f(x), x \in \left(\frac{1}{4}, 1\right)$$

Hints :

$$\Rightarrow e^{-x} f(x)$$

is minimum at

$$x = \frac{1}{4}$$

in

$$x \in \left[0, \frac{1}{4}\right]$$

implies

$$e^{-x} f(x)$$

is decreasing in

$$x \in \left[0, \frac{1}{4}\right]$$

and

$$e^{-x} f(x)$$

is increasing in

$$x \in \left(\frac{1}{4}, 1\right)$$

.

Question 028 MCQ

QUESTION

Let

$$f : [0, 1] \to R$$

$thesetofallrealnumbers$ be a function. Suppose the function

$$f$$

is twice differentiable,

$$f(0) = f(1) = 0$$

and satisfies

$$f''(x) - 2f'(x) + f(x) \geq . e^x, x \in [0, 1]$$

.

Which of the following is true for

$$0 < x < 1?$$

A

$$0 < f(x) < \infty$$

B

$$-\frac{1}{2} < f(x) < \frac{1}{2}$$

C

$$-\frac{1}{4} < f(x) < 1$$

D

$$-\infty < f(x) < 0$$

CORRECT OPTION

D

$$-\infty < f(x) < 0$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given,

$$f(0) = f(1) = 0$$

$$\begin{aligned}
&\text{And } f''(x) - 2f'(x) + f(x) \geq e^x, x \in [0,1] \\
&\Rightarrow e^{-x} f''(x) - e^{-x} f'(x) - e^{-x} f'(x) + e^{-x} f(x) \geq 1 \\
&\Rightarrow \left(e^{-x} f'(x)\right)' - \left(e^{-x} f'(x)\right) - \left(e^{-x} f(x)\right)' \geq 1 \\
&\Rightarrow \left(e^{-x} f'(x) - e^{-x} f(x)\right)' \geq 1 \\
&\Rightarrow \left(e^{-x} f(x)\right)'' \geq 1
\end{aligned}$$

Let

$$g(x) = e^{-x} f(x)$$

Now,

$$g(0) = e^{-0} f(0) = 0$$

and

$$g(1) = e^{-1} f(1) = 0$$

Here,

$$g''(x) = \left(e^{-x} f(x)\right)'' \geq 1 > 0$$
$$\therefore$$

Concavity of

$$g(x)$$

is up and

$$g(0) = g(1) = 0$$

$$\begin{aligned} &\Rightarrow g(x) = e^{-x} f(x) < 0 \forall x \in (0,1) \\ &\Rightarrow f(x) < 0 \end{aligned}$$

Hints:

i

$$f''(x) > 0$$

implies the concavity of

$$f(x)$$

is upward and

$$f''(x) < 0$$

implies the concavity of

$$f(x)$$

is downward.

$$ii$$

$$f''(x) > 0 \quad \forall x \in (a, b)$$

and

$$f(a) = f(b) = 0$$

implies that the concavity of the function is upward and

$$f(x)$$

lies below the x-axis in the interval

$$x \in (a, b)$$

Question 029 MCQ

QUESTION

Match List

$$I$$

with List

$$II$$

and select the correct answer using the code given below the lists:

List

$$I$$

$$P.$$

$$\left(\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1}y\right) + y\sin\left(\tan^{-1}y\right)}{\cot\left(\sin^{-1}y\right) + \tan\left(\sin^{-1}y\right)}\right)^2 + y^4\right)^{1/2}$$

takes value

$$Q.$$

If

$$\cos x+\cos y+\cos z=0=\sin x+\sin y+\sin z$$

then
possible value of

$$\cos \frac{x-y}{2}$$

is

$$R.$$

If

$$\cos \left(\frac{\pi}{4}-x\right) \cos 2x+\sin x \sin 2 \sec x=\cos x \sin 2x \sec x+$$

$$\cos \left(\frac{\pi}{4}+x\right) \cos 2x$$

then possible value of

$$\sec x$$

is

$$S.$$

If

$$\cot \left(\sin^{-1} \sqrt{1-x^2}\right)=\sin \left(\tan^{-1}\left(x \sqrt{6}\right)\right), \quad x \neq 0,$$

Then possible value of

$$x$$

is

List

$$II$$

$$1.$$

$$\frac{1}{2}\sqrt{\frac{5}{3}}$$

$$2.$$

$$\sqrt{2}$$

$$3.$$

$$\frac{1}{2}$$

$$1.$$

$$1$$

A $P = 4, Q = 3, R = 1, S = 2$

B $P = 4, Q = 3, R = 2, S = 1$

C $P = 3, Q = 4, R = 2, S = 1$

D $P = 3, Q = 4, R = 1, S = 2$

CORRECT OPTION

B $P = 4, Q = 3, R = 2, S = 1$

SOURCE

Mathematics • inverse-trigonometric-functions

EXPLANATION

For P

$$\text{Let } t = \sqrt{\frac{1}{y^2}\left(\frac{\cos\left(\tan^{-1} y\right) + y \cdot \sin\left(\tan^{-1} y\right)}{\cot\left(\sin^{-1} y\right) + \tan\left(\sin^{-1} y\right)}\right)^2 + y^4}$$

We know that

$$\tan^{-1} y = \cos^{-1}\left(\frac{1}{\sqrt{1+y^2}}\right) = \sin^{-1}\left(\frac{y}{\sqrt{1+y^2}}\right)$$
$$\text{and } \sin^{-1} y = \cot^{-1}\left(\frac{\sqrt{1-y^2}}{y}\right) = \tan^{-1}\left(\frac{y}{\sqrt{1-y^2}}\right)$$

$$\therefore \quad t = \sqrt{\frac{1}{y^2}\left[\frac{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+y^2}}\right) + y \,.\, \sin\left(\sin^{-1}\frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1}\frac{\sqrt{1-y^2}}{y}\right) + \tan\left(\tan^{-1}\frac{y}{\sqrt{1-y^2}}\right)}\right]^2 + y^4}$$

$$\Rightarrow t=\sqrt{\frac{1}{y^2}\times\left[\frac{\frac{1}{\sqrt{1+y^2}}+\frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y}+\frac{y}{\sqrt{1-y^2}}}\right]^2+y^4}$$

$$\Rightarrow t=\sqrt{\frac{1}{y^2}\times\left[\frac{\sqrt{1+y^2}}{\frac{1-y^2+y^2}{y\sqrt{1-y^2}}}\right]^2+y^4}$$

$$\Rightarrow t=\sqrt{\frac{1}{y^2}\times y^2\left(1+y^2\right)\left(1-y^2\right)+y^4}$$

$$\Rightarrow t=\sqrt{\left(1-y^4\right)+y^4}=1$$

Hence, P match with 4.

For Q.

Given

$$\cos x+\cos y+\cos z=0$$

$$\Rightarrow \cos x+\cos y=-\cos z$$

On squaring both side

$$\Rightarrow \cos^2 x+\cos^2 y+2\cos x\cdot\cos y=\cos^2 z \quad \text{... (i)}$$

Also given

$$\sin x+\sin y+\sin z=0$$

$$\Rightarrow \sin x+\sin y=-\sin z$$

On squaring both side

$$\Rightarrow \sin^2 x+\sin^2 y+2\sin x\cdot\sin y=\sin^2 z \quad \text{... (ii)}$$

Add equation i and ii

$$\begin{gathered}\Rightarrow\left(\cos^2 x+\sin^2 x\right)+\left(\cos^2 y+\sin^2 y\right)\\ +2[\cos x\cdot\cos y+\sin x\cdot\sin y]\\ =\cos^2 z+\sin^2 z\end{gathered}$$

$$\Rightarrow \quad 1+1+2\cos(x-y)=1$$
$$\Rightarrow \quad \cos(x-y)=\frac{-1}{2}$$
$$\Rightarrow \quad 2\cos^2\left(\frac{x-y}{2}\right)-1=\frac{-1}{2}$$
$$\Rightarrow \quad \cos^2\left(\frac{x-y}{2}\right)=\frac{1}{4}$$
$$\Rightarrow \quad \cos\left(\frac{x-y}{2}\right)=\pm\frac{1}{2}$$

Hence, Q match with 3.

For R

Given,

$$\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\cdot\sin 2x\cdot\sec x$$
$$=\cos x\cdot\sin 2x\cdot\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$$

$$\Rightarrow\left[\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]$$
$$\cos 2x+(\sin x-\cos x)\sin 2x\cdot\sec x=0$$

$$\Rightarrow 2\cdot\sin\frac{\pi}{4}\cdot\sin x\cdot\cos 2x+(\sin x-\cos x)$$
$$\frac{2\sin x\cdot\cos x}{\cos x}=0$$

$$\Rightarrow \quad \sqrt{2}\cos 2x=2(\cos x-\sin x)$$
$$\Rightarrow \quad \left(\cos^2 x-\sin^2 x\right)=\sqrt{2}(\cos x-\sin x)$$

$$\Rightarrow \quad \cos x+\sin x=\sqrt{2}$$
$$\Rightarrow \quad (\cos x+\sin x)^2=(\sqrt{2})^2$$
$$\Rightarrow \quad \cos^2 x+\sin^2 x+2\sin x\cdot\cos x=2$$
$$\Rightarrow \quad 1+\sin 2x=2$$
$$\Rightarrow \quad \sin 2x=1$$
$$\Rightarrow \quad 2x=\frac{\pi}{2}$$
$$\Rightarrow \quad x=\frac{\pi}{4}$$
$$\Rightarrow \quad \sec x=\sec\frac{\pi}{4}=\sqrt{2}$$

Hence, R match with 2

For S

Given,

$$\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}(x\sqrt{6})\right), x\neq 0$$
$$\Rightarrow \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right)=\sin\left(\sin^{-1}\frac{x\sqrt{6}}{\sqrt{1+6x^2}}\right)$$
$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}}=\frac{x\sqrt{6}}{\sqrt{1+6x^2}}$$

$$\begin{aligned}
&\Rightarrow \quad \tfrac{1}{\sqrt{1-x^2}} &&= \tfrac{\sqrt{6}}{\sqrt{1+6x^2}}\\
&\Rightarrow \quad \text{on squaring both sides}\\
&\Rightarrow \quad \tfrac{1}{1-x^2}=\tfrac{6}{1+6x^2}\\
&\Rightarrow \quad 1+6x^2=6-6x^2\\
&\Rightarrow \quad x^2=\tfrac{5}{12}\\
&\Rightarrow \quad x=\pm\sqrt{\tfrac{5}{12}}=\pm\tfrac{1}{2}\sqrt{\tfrac{5}{3}}
\end{aligned}$$

Hence, S match with 1.

Hints :

i Recall the method of conversion of one inverse trigonometric function into other inverse trigonometric function as

$$\sin^{-1}x=\cos^{-1}\sqrt{1-x^2}=\tan^{-1}\frac{x}{\sqrt{1-x^2}}$$

ii Recall

$$\cos\left(\cos^{-1}x\right)=x, \sin\left(\sin^{-1}x\right)=x\tan\left(\tan^{-1}x\right)=x \text{ etc}$$

iii Recall the following trigonometric identities

A

$$\sin^2 x+\cos^2 x=1$$

B

$$\sin 2x=2\sin x\cdot\cos x$$

C

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

D

$$\cos(A - B) - \cos(A + B) = 2\sin A \cdot \sin B$$

Question 030 MCQ

QUESTION

In a triangle

$$PQR$$

,

$$P$$

is the largest angle and

$$\cos P = \frac{1}{3}$$

. Further the incircle of the triangle touches the sides

$$PQ$$

,

$$QR$$

and

$$RP$$

at

$$N, L$$

and

$$M$$

respectively, such that the lengths of

$$PN, QL$$

and

$$RM$$

are consecutive even integers. Then possible length s of the side s of the triangle is are

A

16

B

18

C

24

D

22

CORRECT OPTION

B

18

SOURCE

Mathematics • properties-of-triangle

EXPLANATION

For the above circle PN and PM are its pair of tangents. So, its length are equal i.e.

$$|\mathrm{PN}| = |\mathrm{PM}|.$$

Similarly

$$|\mathrm{QL}| = |\mathrm{QN}|$$

and

$$|\mathrm{RL}| = |\mathrm{RM}|$$

Let

$$|\mathrm{PN}| = |\mathrm{PM}| = x, |\mathrm{QL}| = |\mathrm{QN}| = y$$

and

$$|RL| = |RM| = z$$

Given

$$|\mathrm{PN}|, |\mathrm{QL}|$$

and

$$|\mathrm{RM}|$$

are consecutive even integer

$$\therefore \quad x = 2n, y = 2n + 2 \text{ and } z = 2n + 4$$

Given,

$$\cos P = \frac{1}{3}$$

$$\begin{aligned}
&\Rightarrow \quad \frac{(4n+4)^2 + (4n+2)^2 - (4n+6)^2}{2(4n+4)(4n+2)} = \frac{1}{3} \\
&\Rightarrow \frac{\left(16n^2 + 16 + 32n\right) + \left(16n^2 + 4 + 16n\right) - \left(16n^2 + 36 + 48n\right)}{32n^2 + 48n + 16} = \frac{1}{3} \\
&\Rightarrow \quad 48n^2 - 48 = 32n^2 + 48n + 16 \\
&\Rightarrow \quad 16n^2 - 48n - 64 = 0 \\
&\Rightarrow \quad n^2 - 3n - 4 = 0 \\
&\Rightarrow \quad n^2 - 4n + n - 4 = 0 \\
&\Rightarrow \quad (n-4)(n+1) = 0 \\
&\Rightarrow \quad n = 4
\end{aligned}$$

$$\text{Now } |PQ| = 4n + 2 = 18, |QR| = 4n + 6 = 22$$
$$\text{and } |RP| = 4n + 4 = 20$$

Hints :

i Recall cosine rule in a triangle

$$\triangle \mathrm{ABC}$$

,

$$\cos \mathrm{A} = \frac{b^2 + c^2 - a^2}{2bc}, \cos \mathrm{B} = \frac{c^2 + a^2 - b^2}{2ca}$$
$$\text{and} \quad \cos \mathrm{C} = \frac{a^2 + b^2 - c^2}{2ab}$$

ii Three consecutive even integers are

$$2n, 2n + 2, 2n + 4$$

where

$$n$$

is an integer.

Question 031 MCQ

QUESTION

Let

$$PQ$$

be a focal chord of the parabola

$$y^2 = 4ax$$

. The tangents to the parabola at

$$P$$

and

$$Q$$

meet at a point lying on the line

$$y = 2x + a$$

,

$$a > 0$$

.

If chord

$$PQ$$

subtends an angle

$$\theta$$

at the vertex of

$$y^2 = 4ax$$

, then tan

$$\theta =$$

A $\frac{2}{3}\sqrt{7}$

B $\frac{-2}{3}\sqrt{7}$

C $\frac{2}{3}\sqrt{5}$

D $\frac{-2}{3}\sqrt{5}$

CORRECT OPTION

D $\frac{-2}{3}\sqrt{5}$

SOURCE

Mathematics • parabola

EXPLANATION

Equation of tangent at P is

$$ty = x + at^2 \quad \text{.... (i)}$$

Equation of tangent at Q is

$$\frac{-y}{t} = x + \frac{a}{t^2} \quad \text{... (ii)}$$

On solving equations i and ii

$$\Rightarrow \mathrm{R} = \left(-a, a\left(t - \frac{1}{t}\right)\right)$$

Given, the tangents of parabola

$$y^2 = 4ax$$

at P and Q meet at a point lying on the line

$$y = 2x + a$$

$$\Rightarrow \quad a\left(t-\frac{1}{t}\right)=-2a+a$$

$$\Rightarrow \quad \left(t-\frac{1}{t}\right)=-1 \quad \text{... (iii)}$$

$$\Rightarrow$$

Slope of line

$$\mathrm{OP}=\frac{2}{t}$$

and Slope of line OQ

$$=-2t$$

Given,

$$\angle\mathrm{POQ}=\theta$$

$$\Rightarrow \quad \tan\theta=\frac{\frac{2}{t}-(-2t)}{1+\frac{2}{t}(-2t)}$$

$$\Rightarrow \quad \tan\theta=\frac{-2}{3}\left(t+\frac{1}{t}\right)$$

$$\Rightarrow \quad \tan\theta=\frac{-2}{3}\sqrt{\left(t-\frac{1}{t}\right)^2+4}$$

$$\Rightarrow \quad \tan\theta=\frac{-2\sqrt{5}}{3}$$

Hints:

i If slope of two lines are

$$m_1$$

and

$$m_2$$

, then the angle of intersection between them is equal to

$$\tan^{-1}\left(\frac{m_1-m_2}{1+m_1m_2}\right)$$

ii If PQ is a focal chord of parabola

$$y^2 = 4ax$$

, then

$$\mathrm{P} = \left(at^2, 2at\right)$$

and

$$\mathrm{Q} = \left(\frac{a}{t^2}, -\frac{2a}{t}\right)$$

.

Question 032 MCQ

QUESTION

Let

$$PQ$$

be a focal chord of the parabola

$$y^2 = 4ax$$

. The tangents to the parabola at

$$P$$

and

$$Q$$

meet at a point lying on the line

$$y = 2x + a$$

,

$$a > 0$$

.

Length of chord

$$PQ$$

is

A $7a$

B $5a$

C $2a$

D $3a$

CORRECT OPTION

B $5a$

SOURCE

Mathematics • parabola

EXPLANATION

Given, PQ be a focal chord of the parabola

$$y^2 = 4ax$$

Let

$$\mathrm{P} = \left(at^2, 2at\right)$$

and

$$Q\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$$

Equation of tangent at P is

$$ty = x + at^2 \quad \text{... (i)}$$

Equation of tangent at Q is

$$-\frac{y}{t} = x + \frac{a}{t^2} \quad \text{... (ii)}$$

Let

$$R$$

is the point of intersection of tangents at

$$P$$

and

$$Q$$

.

On solving Equation i and ii

$$\Rightarrow \mathrm{R} = \left(-a, a\left(t - \frac{1}{t}\right)\right)$$

Given, the tangents of parabola

$$y^2 = 4ax$$

at P and

$$Q$$

meet at a point lying on the line

$$y = 2x + a$$

.

$$\therefore \mathrm{R}$$

must be lies on the line

$$y = 2x + a$$

$$\Rightarrow \quad a\left(t - \frac{1}{t}\right) = -2a + a$$

$$\Rightarrow \quad t - \frac{1}{t} = -1 \quad \text{... (iii)}$$

Let the length of chord PQ is L

$$\Rightarrow \mathrm{L} = \sqrt{\left(at^2 - \frac{a}{t^2}\right)^2 + \left(2at + \frac{2a}{t}\right)^2}$$
$$\Rightarrow \mathrm{L} = a\sqrt{\left(t - \frac{1}{t}\right)^2\left(t + \frac{1}{t}\right)^2 + 4\left(t + \frac{1}{t}\right)^2}$$
$$\Rightarrow \mathrm{L} = a\sqrt{\left(t - \frac{1}{t}\right)^2\left(\left(t - \frac{1}{t}\right)^2 + 4\right) + 4\left(\left(t - \frac{1}{t}\right)^2 + 4\right)}$$
$$\Rightarrow \mathrm{L} = a\sqrt{1 \cdot (1 + 4) + 4 \cdot (1 + 4)}$$
$$\Rightarrow \mathrm{L} = 5a$$

Hints:

If

$$PQ$$

is a focal chord of parabola

$$y^2 = 4ax$$

, then

$$\mathrm{P} = \left(at^2, 2at\right)$$

and

$$\mathrm{Q} = \left(\frac{a}{t^2}, \frac{-2a}{t}\right)$$

Question 033 MCQ

QUESTION

A line

$$L : y = mx + 3$$

meets

$$y$$

-axis at R

$$(0, 3)$$

and the arc of the parabola

$$y^2 = 16x,$$

$$0 \leq y \leq 6$$

at the point

$$F(x_0, y_0)$$

. The tangent to the parabola at

$$F(x_0, y_0)$$

intersects the

$$y$$

-axis at

$$G(0, y_1)$$

. The slope

$$m$$

of the line

$$L$$

is chosen such that the area of the triangle

$$EFG$$

has a local maximum.

Match List

$$I$$

with List

$$II$$

and select the correct answer using the code given below the lists:

List

$$I$$

P.

$$m =$$

Q.

Maximum area of

$$\Delta EFG$$

is
R.

$$y_0 =$$

S.

$$y_1 =$$

List

$$II$$

1.

$$\frac{1}{2}$$

2.

$$4$$

3.

$$2$$

4.

$$1$$

A

$$P = 4, Q = 1, R = 2, S = 3$$

B

$$P = 3, Q = 4, R = 1, S = 2$$

C

$$P = 1, Q = 3, R = 2, S = 4$$

D

$$P = 1, Q = 3, R = 4, S = 2$$

CORRECT OPTION

A

$$P = 4, Q = 1, R = 2, S = 3$$

SOURCE

Mathematics • parabola

EXPLANATION

Given, a parabola

$$y^2 = 16x$$

Let

$$\mathrm{F}\left(x_0, y_0\right) = \left(4t^2, 8t\right)$$

$$\Rightarrow \quad x_0 = 4t^2, y_0 = 8t$$

Equation of tangent at F is

$$ty = x + 4t^2$$

G is the point of intersection of tangent

$$ty = x + 4t^2$$

and

$$Y$$

-axis

$$\therefore \mathrm{G} = (0, 4t)$$

Given,

$$\mathrm{G} = \left(0, y_1\right)$$

$$\therefore \quad y_1 = 4t$$

Let

$$\Delta$$

be the area of

$$\Delta$$

EGF

$$\begin{aligned} \Rightarrow \quad & \Delta=\frac{1}{2} 4 t^2 \cdot\left(3-y_1\right) \\ \Rightarrow \quad & \Delta=2 t^2(3-4 t) \\ \Rightarrow \quad & \frac{d \Delta}{d t}=12 t-24 t^2=0 \text { at } t=0, \frac{1}{2} \\ \Rightarrow & \frac{d^2 \Delta}{d t^2}=12-48 t<0 \text { at } t=\frac{1}{2} \end{aligned}$$

Hence,

$$\Delta$$

is maximum at

$$t=\frac{1}{2}$$

$$\therefore \quad \Delta_{\max }=\frac{1}{2}, y_0=8 t=4, y_1=4 t=2$$

Given,

$$y=m x+3$$

intersect the parabola

$$y^2=16 x$$

at

$$\mathrm{F}\left(4 t^2, 8 t\right)$$

$$\therefore 8 t=4 m t^2+3$$

Put

$$t=\frac{1}{2}$$

$$\begin{aligned} \Rightarrow \quad & 4=m+3 \\ \Rightarrow m & =1 \end{aligned}$$

Hints :

i

$$\frac{dy}{dx} = 0$$

at

$$x = \alpha$$

and

$$\frac{d^2y}{dx^2} < 0$$

at

$$x = \alpha$$

, then the function has point of local maxima at

$$x = a$$

.

ii The parametric point of the parabola

$$y^2 = 4ax$$

is

$$\left(at^2, 2at\right)$$

iii The equation of tangent of parabola

$$y^2 = 4ax$$

at

$$\left(at^2, 2at\right)$$

is

$$ty = x + at^2$$

.

Question 034 MCQ

QUESTION

Circle s touching x-axis at a distance 3 from the origin and having an intercept of length

$$2\sqrt{7}$$

on y-axis is are

A $$x^2 + y^2 - 6x + 8y + 9 = 0$$

B $$x^2 + y^2 - 6x + 7y + 9 = 0$$

C $$x^2 + y^2 - 6x - 8y + 9 = 0$$

D $$x^2 + y^2 - 6x - 7y + 9 = 0$$

CORRECT OPTION

A $$x^2 + y^2 - 6x + 8y + 9 = 0$$

SOURCE

Mathematics • circle

EXPLANATION

Let a circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

touches

$$x$$

-axis and have y-intercept of length

$$2\sqrt{7}$$

.

$$\begin{aligned} &\therefore 2\sqrt{g^2 - c} = 0 \text{ and } 2\sqrt{f^2 - c} = 2\sqrt{7} \\ &\Rightarrow g^2 = c \text{ and } f^2 - c = 7 \\ &\Rightarrow c = g^2 = f^2 - 7 \quad \ldots \text{(i)} \end{aligned}$$

Given, the circle touches

$$x$$

-axis at a distance 3 unit from origin.

$$\begin{aligned} &\therefore |-g| = 3 \\ &\Rightarrow g = \pm 3 \end{aligned}$$

Put

$$g = \pm 3$$

in the equation

$$\Rightarrow c = 9 \text{ and } f = \pm 4$$

Hence, the possible equation of circles are

$$\begin{aligned} x^2 + y^2 - 6x + 8y + 9 = 0, \\ x^2 + y^2 - 6x - 8y + 9 = 0, \end{aligned}$$

$$x^2 + y^2 + 6x + 8y + 9 = 0 \text{ and } x^2 + y^2 + 6x - 8y + 9 = 0$$

Hints:

The length of X-intercept and Y-intercept of the circle

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

are

$$2\sqrt{g^2 - c}$$

and

$$2\sqrt{f^2 - c}$$

respectively.

Question 035 MCQ

QUESTION

If

$$3^x = 4^{x-1},$$

then

$$x =$$

A

$$\frac{2\log_3 2}{2\log_3 2 - 1}$$

B

$$\frac{2}{2 - \log_2 3}$$

C

$$\frac{1}{1-\log_4 3}$$

D

$$\frac{2\log_2 3}{2\log_2 3-1}$$

CORRECT OPTION

A

$$\frac{2\log_3 2}{2\log_3 2-1}$$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

Given,

$$3^x = 4^{x-1}$$

Taking

$$\log$$

on both side with base 3

$$\begin{aligned}
&\Rightarrow \log_3 3^x = \log_3 \left(4^{x-1}\right) \\
&\Rightarrow x\log_3 3 = (x-1)\log_3 4 \\
&\Rightarrow \quad x.1 = (x-1)\log_3 4 \\
&\Rightarrow \quad x = \frac{\log_3 4}{\log_3 4 - 1} \\
&\Rightarrow \quad x = \frac{1}{1-\frac{1}{\log_3 4}}
\end{aligned}$$

$$\text{According to base changing rule } \frac{1}{\log_3 4} = \log_4 3$$

$$\begin{aligned}
\therefore \quad & x = \tfrac{1}{1-\log_4 3} \quad \ldots \text{(i)} \\
\Rightarrow \quad & x = \tfrac{1}{1-\log_{2^2} 3} \\
\Rightarrow \quad & x = \tfrac{1}{1-\frac{1}{2}\log_2 3} \quad \left[\log_{a^m} x = \tfrac{1}{m}\log_a x\right] \\
\Rightarrow \quad & x = \tfrac{2}{2-\log_2 3} \quad \ldots \text{(ii)}
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad x &= \frac{2}{2 - \frac{1}{\log_3 2}} \\
\Rightarrow \quad x &= \frac{2\log_3 2}{2\log_3 2 - 1} \quad \ldots \text{(iii)}
\end{aligned}$$

From equation i, ii and iii, it is clear that option A, B and C are correct.

Hints:

i Base changing Rule

$$\log_b^a = \frac{1}{\log_a^b}$$

ii

$$\log_a x^n = n\log_a x$$

iii

$$\log_{a^m} x = \frac{1}{m}\log_a x$$

Question 036 MCQ

QUESTION

Let

$$S = S_1 \cap S_2 \cap S_3$$

, where

$$S_1 = \{z \in C : |z| < 4\}, S_2 = \left\{z \in C : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$$

and

$$S_3 = \{z \in C : \text{Re}z > 0\}$$

.

$$\min_{z \in S} |1 - 3i - z| =$$

A $\frac{2-\sqrt{3}}{2}$

B $\frac{2+\sqrt{3}}{2}$

C $\frac{3-\sqrt{3}}{2}$

D $\frac{3+\sqrt{3}}{2}$

CORRECT OPTION

C $\frac{3-\sqrt{3}}{2}$

SOURCE

Mathematics • complex-numbers

EXPLANATION

We know

$$|1-3i-z|=|z-(1-3i)|$$

implies distance of

$$z$$

from

$$(1,-3)$$

.

Let

$$P=(1,-3)$$

$$\min_{z\in \mathrm{S}}|1-3i-z|=\mathrm{PN}$$

$$\Rightarrow \min_{z\in S}|1-3i-z|=\frac{|\sqrt{3}\cdot 1-3|}{\sqrt{(\sqrt{3})^2+1^2}}$$

$$\Rightarrow \min_{z\in S}|1-3i-z|=\frac{3-\sqrt{3}}{2}$$

Hints:

i

$$|z_1-z_2|$$

implies distance between

$$z_1$$

and

$$z_2$$

ii The minimum distance of a point

$$(x_1, y_1)$$

from the line

$$ax + by + c = 0$$

is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

.

Question 037 MCQ

QUESTION

Let

$$S = S_1 \cap S_2 \cap S_3$$

, where

$$S_1 = \{z \in C : |z| < 4\}, S_2 = \left\{z \in C : \operatorname{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0\right\}$$

and

$$S_3 = \{z \in C : \operatorname{Re} z > 0\}$$

.

Area of S =

A

$$\frac{10\pi}{3}$$

B

$$\frac{20\pi}{3}$$

C

$$\frac{16\pi}{3}$$

D

$$\frac{32\pi}{3}$$

CORRECT OPTION

B

$$\frac{20\pi}{3}$$

SOURCE

Mathematics • complex-numbers

EXPLANATION

Given,

$$S_1 = \{z \in C : |z| < 4\}$$

i.e.

$$S_1$$

lies inside the circle of centre

$$(0, 0)$$

and radius 4.

Given,

$$S_3 = \{z \in C : \mathrm{Re}(z) > 0\}$$

i.e.

$$S_3$$

lies right side of the line

$$x = 0$$

.

Also given

$$S_2 = \left\{z \in C : \mathrm{Im}\left(\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right) > 0\right\}$$

Put

$$z = x + iy$$

in

$$\mathrm{Im}\left(\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right) > 0$$

$$\Rightarrow \mathrm{Im}\left(\frac{x - 1 + i(y + \sqrt{3})}{1 - \sqrt{3}i}\right) > 0$$
$$\Rightarrow \mathrm{Im}\left(\frac{(x - 1) + i(y + \sqrt{3})}{1 - \sqrt{3}i} \times \frac{1 + \sqrt{3}i}{1 + \sqrt{3}i}\right) > 0$$
$$\Rightarrow \mathrm{Im}\left[\begin{array}{l}\frac{(x-1)-\sqrt{3}(y+\sqrt{3})}{4} + \\ i\left(\frac{\sqrt{3}(x-1)+(y+\sqrt{3})}{4}\right)\end{array}\right] > 0$$

$$\Rightarrow \quad \frac{\sqrt{3}x + y}{4} > 0$$
$$\Rightarrow \sqrt{3}x + y > 0$$

$$\text{So, } S_2 \text{ lies above the line } \sqrt{3}x + y = 0$$

$$\text{Given, } S = S_1 \cap S_2 \cap S_3$$

Here,

$$\angle \mathrm{AOC} = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$$

Area of

$$S = \frac{1}{2}\left(\frac{5\pi}{6}\right) \cdot 4^2 = \frac{20\pi}{3}$$

Sq. Units

Hints:

i

$$|z| < a$$

implies

$$z$$

lies inside the circle of centre

$$(0, 0)$$

and radius

$$a$$

ii

$$\mathrm{Re}(z) > 0$$

implies

$$z$$

lies on the right side of the line

$$x = 0$$

i.e.

$$y$$

-axis.

iii Put

$$z = x + iy$$

in the expression

$$\operatorname{Im}\left(\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right) > 0$$

iv If a circular arc

$$AB$$

form

$$\theta$$

angle at the centre of circle of radius R, then the area of this circular section is

$$\frac{1}{2}\theta \cdot r^2$$

Question 038 MCQ

QUESTION

$$a \in R$$

$thesetofallrealnumbers$, a

$$\neq$$

$$-$$

1,

$$\lim_{n \to \infty} \frac{(1^a + 2^a + \ldots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \ldots + (na+n)]} = \frac{1}{60}$$

, Then a = ?

A 5

B 7

C $\frac{-15}{2}$

D $\frac{-17}{2}$

CORRECT OPTION

B 7

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$\lim_{n\to\infty} \frac{(1^a + 2^a + \ldots + n^a)}{(n+1)^{a-1}[(na+1) + (na+2) + \ldots + (na+n)]}$$

$$= \lim_{n\to\infty} \frac{\sum_{r=1}^{n} r^a}{(n+1)^{a-1}\left\{n^2 a + \frac{n(n+1)}{2}\right\}}$$

$$= \lim_{n\to\infty} \frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^a}{\left(1+\frac{1}{n}\right)^{a-1}\left\{a + \frac{1}{2} + \frac{1}{2n}\right\}}$$

$$= \frac{\int_a^1 x^a dx}{a+\frac{1}{2}} = \frac{1}{(a+1)\left(a+\frac{1}{2}\right)} = \frac{1}{60}$$

given.

$$\Rightarrow (2a+1)(a+1) = 120 \Rightarrow 2a^2 + 3a - 119 = 0$$

$$\Rightarrow 2a^2 - 14a + 17a - 119 = 0$$

$$\Rightarrow 2a(a-7) + 17(a-7) = 0 \Rightarrow (2a+17)(a-7) = 0$$

.

$$\therefore$$

$$a = 7, -\frac{17}{2}$$

But

$$a = -\frac{17}{2}$$

has to be discarded because integral

$$\int_a^1 x^a dx$$

converges if a >

$$-$$

1

Question 039 MCQ

QUESTION

Let

$$\omega$$

be a complex cube root of unity with

ω

$\neq$

1 and P = [p_{ij}] be a n

$\times$

n matrix with p_{ij} =

ω

$^{i+j}$. Then P^2

$\neq$

0, when n = ?

A 57

B 55

C 58

D 56

CORRECT OPTION

B 55

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

The given matrix $P = [p_{ij}]$ is an $n \times n$ matrix where $p_{ij} = \omega^{i+j}$ and ω is a complex cube root of unity with $\omega \neq 1$. We need to determine when $P^2 \neq 0$.

For $n = 1$:

$P = [p_{ij}]_{1\times1} = [\omega^2]$

$\Rightarrow P^2 = [\omega^4] \neq 0$

For $n = 2$:

$$P = [p_{ij}]_{2\times2} = \begin{bmatrix} \omega^2 & \omega^3 \\ \omega^3 & \omega^4 \end{bmatrix} = \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} \begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix} = \begin{bmatrix} \omega^4 + 1 & \omega^2 + \omega \\ \omega^2 + \omega & 1 + \omega^2 \end{bmatrix} \neq 0$$

For $n = 3$:

$$P = [p_{ij}]_{3\times3} = \begin{bmatrix} \omega^2 & \omega^3 & \omega^4 \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} \begin{bmatrix} \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Thus, $P^2 = 0$ when n is a multiple of 3. Therefore, $P^2 \neq 0$ when n is not a multiple of 3.

The possible values of n where $P^2 \neq 0$ are:

$\Rightarrow n = 55, 58, 56$

Question 040 MCQ

QUESTION

The function

$$f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$$

has a local minimum or a local maximum at x =

A

$-$

2

B

$$\frac{-2}{3}$$

C 2

D

$$\frac{2}{3}$$

CORRECT OPTION

A

$-$

2

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Here,

$$f(x) = 2|x| + |x+2| - ||x+2| - 2|x||$$

$$= \begin{cases} -2x-(x+2)+(x-2), & when\, x \leq -2 \\ -2x+x+2+3x+2, & when\, -2 < x \leq -\frac{2}{3} \\ -4x, & when\, -\frac{2}{3} < x \leq 0 \\ 4x, & when\, 0 < x \leq 2 \\ 2x+4, & when\, x > 2 \end{cases}$$

$$= \begin{cases} -2x-4, & x \leq -2 \\ 2x+4, & -2 < x \leq -2/3 \\ -4x, & -\frac{2}{3} < x \leq 0 \\ 4x, & 0 < x \leq 2 \\ 2x+4, & x > 2 \end{cases}$$

Graph for y = fx is shown as

Question 041 MCQ

QUESTION

Two non-conducting spheres of radii

$$R_1$$

and

$$R_2$$

and carrying uniform volume charge densities

$$+\rho$$

and

$$-\rho,$$

respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region

A The electrostatic field is zero

B The electrostatic potential is constant

C The electrostatic field is constant in magnitude

D The electrostatic field has same direction

CORRECT OPTION

D The electrostatic field has same direction

SOURCE

Physics • electrostatics

EXPLANATION

Here we will use the concept of vectors and concept of electric field.

Let's take a point P in the overlapping region.

From

$$\Delta OPQ$$

,

By triangle law of vector addition,

$$\overrightarrow{r_1} - \overrightarrow{r_2} = \overrightarrow{d} \quad ... (1)$$

Net electrostatic field at point P;

$$\overrightarrow{E}_{net} = \frac{Kq_1}{r_1^3}\overrightarrow{r_1} = \frac{Kq_2}{r_2^3}\overrightarrow{r_2}$$

$$\Rightarrow \overrightarrow{E}_{net} = \frac{K\left(\rho\frac{4}{3}\pi r_1^3\right)\overrightarrow{r_1}}{r_1^3} - \frac{K\left(\rho\frac{4}{3}\pi r_2^3\right)\overrightarrow{r_2}}{r_2^3}$$

$$\Rightarrow \overrightarrow{E}_{net} = \frac{4}{3}\rho\pi \times \frac{1}{4\pi\varepsilon_0}\left(\overrightarrow{r_1} - \overrightarrow{r_2}\right)$$

$$\Rightarrow \overrightarrow{E}_{net} = \frac{\rho}{3\varepsilon_0}\overrightarrow{d}$$

Hence, the electrostatic field is constant in magnitude and has same direction.

Therefore, options C and D are correct.

Question 042 MCQ

QUESTION

Two bodies, each of mass M, are kept fixed with a separation

$$2L$$

. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement$s$ is are

A The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is

$$4\sqrt{\frac{GM}{L}}$$

B The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is

$$2\sqrt{\frac{GM}{L}}$$

C

The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is

$$\sqrt{\frac{2GM}{L}}$$

D

The energy of the mass m remains constant.

CORRECT OPTION

B

The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is

$$2\sqrt{\frac{GM}{L}}$$

SOURCE

Physics • gravitation

EXPLANATION

Here we will use the law of conservation of energy as there are no non-conservative force involved so the total energy of mass m remains constant.

For minimum initial velocity to escape the gravitational field, the velocity at infinity will be zero.

And we know PE at infinity is zero.

By energy conservation,

$$PE_i + KE_i = PE_f + KE_f$$

$$\Rightarrow \frac{-GMm}{L} - \frac{GMm}{L} + \frac{1}{2}mv^2 = 0 + \frac{1}{2}m(0)^2$$

$$\Rightarrow -\frac{2Gm}{L}M + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow \frac{1}{2}V^2 = \frac{2Gm}{L}$$

$$\Rightarrow V = 2\sqrt{\frac{GM}{L}}$$

Hence, options B and D are correct.

Question 043 MCQ

QUESTION

A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0. It collides elastically with a rigid wall. After this collision,

A the speed of the particle when it returns to its equilibrium position is u_0.

B the time at which the particle passes through the equilibrium position for the first time is

$$t = \pi\sqrt{\frac{m}{k}}$$

C the time at which the maximum compression of the spring occurs is

$$t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$$

D

the time at which the particle passes through the equilibrium position for the second time is

$$t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

CORRECT OPTION

A the speed of the particle when it returns to its equilibrium position is u_0.

SOURCE

Physics • impulse-and-momentum

EXPLANATION

Here we will apply the concept of energy conservation and concept of SHM.

We know that for elastic collision, coefficient of restitution, e = 1

Hence,

$$e = \frac{v}{0.5u_0} = 1 \Rightarrow v = 0.5u_0$$

Therefore, particle rebounds with the same speed after collision.

Now, by energy conservation of the spring-mass system, we can say that the speed of the particle when it returns to its equilibrium position is

$$u_0$$

.

By SHM,

$$v = v_{\max} \cos wt$$

$$\Rightarrow 0.5u_0 = u_0 \cos\left(\sqrt{\frac{K}{m}}t\right)$$

$$\Rightarrow \frac{1}{2} = \cos\left(\sqrt{\frac{K}{m}}t\right)$$

$$\Rightarrow \frac{\pi}{3} = \sqrt{\frac{K}{m}}t$$

$$\Rightarrow t = \frac{\pi}{3}\sqrt{\frac{m}{K}}$$

Hence, time taken by the particle to reach the wall is

$$\frac{\pi}{3}\sqrt{\frac{m}{K}}$$

So the time at which the particle passes through the equilibrium position for the first time is

$$2t = \frac{2\pi}{3}\sqrt{\frac{m}{K}}$$

The time at which the maximum compression of the spring occurs is

$$t' = 2t + \frac{T}{4} = \frac{2\pi}{3}\sqrt{\frac{m}{K}} + \frac{2\pi}{4}\sqrt{\frac{m}{K}}$$

$$= \left(\frac{2}{3} + \frac{1}{2}\right)\pi\sqrt{\frac{m}{K}}$$

$$t' = \frac{7\pi}{6}\sqrt{\frac{m}{K}}$$

The time at which the particle passes through the equilibrium position for the second time is

$$= t' + T$$

$$= \frac{7\pi}{6}\sqrt{\frac{m}{K}} + \frac{\pi}{2}\sqrt{\frac{m}{K}}$$

$$= \left(\frac{7\pi}{6} + \frac{\pi}{2}\right)\sqrt{\frac{m}{K}} = \frac{10\pi}{6}\sqrt{\frac{m}{K}}$$

$$= \frac{5\pi}{3}\sqrt{\frac{m}{K}}$$

Hence, options A and D are correct.

Question 044 MCQ

QUESTION

The magnitude of the normal reaction that acts on the block at the point Q is

A 7.5 N

B 8.6 N

C 11.5 N

D 22.5 N

CORRECT OPTION

A 7.5 N

SOURCE

Physics • work-power-and-energy

EXPLANATION

At point Q, the forces acting on the particle are normal reaction N_Q, gravitational force mg and frictional force f. Resolve these in the directions parallel and perpendicular to the path. The net force towards the centre (P)

provides the centripetal acceleration. Apply Newton's second law in a direction normal to the path to get

$$N_Q - mg\sin 30^\circ = \frac{mv^2}{R}$$

$$\Rightarrow$$

$$N_Q = mg\sin 30^\circ + \frac{mv^2}{R}$$

$$= 1 \times 10 \times \frac{1}{2} + \frac{100}{40}$$

$$= 5 + \frac{5}{2} = \frac{15}{2}$$

$$\Rightarrow N_Q = 7.5$$

N

Hence, option A is correct.

Question 045 MCQ

QUESTION

The speed of the block when it reaches the point Q is

A) 5 ms^{-1}

B) 10 ms^{-1}

C) $$10\sqrt{3}$$ ms^{-1}

D 20 ms^{-1}

CORRECT OPTION

B 10 ms^{-1}

SOURCE

Physics • work-power-and-energy

EXPLANATION

Here we will use the concept of work-energy theorem. By work-energy theorem,

Total work done = Change in KE

$$\Rightarrow$$

Workdone by gravity + workdone by friction =

$$\Delta KE$$

$$\Rightarrow mgR\sin 30^\circ - 150 = \frac{1}{2}mv^2 - 0$$

$$\Rightarrow 1 \times 10 \times 40 \times \frac{1}{2} - 150 = \frac{1}{2}v^2$$

$$\Rightarrow 200 - 150 = \frac{1}{2}v^2$$

$$\Rightarrow v^2 = 100$$

$$\Rightarrow v = 10$$

m/s

Hence, option B is correct.

Question 046 MCQ

QUESTION

Match **List I** with **List II** and select the correct answer using the codes given below the lists:

List I

P. Boltzmann Constant
Q. Coefficient of viscosity
R. Plank Constant
S. Thermal conductivity

List II

1. $[ML^2T^{-1}]$
2. $[ML^{-1}T^{-1}]$
3. $[MLT^{-3}K^{-1}]$
4. $[ML^2T^{-2}K^{-1}]$

A

P	Q	R	S
3	1	2	4

B

P	Q	R	S
3	2	1	4

C

P	Q	R	S
4	2	1	3

D

$$\begin{array}{cccc} P & Q & R & S \\ 4 & 1 & 2 & 3 \end{array}$$

CORRECT OPTION

C

$$\begin{array}{cccc} P & Q & R & S \\ 4 & 2 & 1 & 3 \end{array}$$

SOURCE

Physics • units-and-measurements

EXPLANATION

Let's break down the dimensional analysis of each physical quantity:

P. Boltzmann Constant $\$k_B\$$

The Boltzmann constant relates energy to temperature. Its dimensional formula is derived from the relationship: $E = k_B T$.

Energy $\$E\$$ has dimensions of [ML^2T^{-2}]. Temperature $\$T\$$ has dimensions of

$$K$$

. Therefore:

$$[k_B] = \frac{[E]}{[T]} = \frac{[ML^2T^{-2}]}{[K]} = [ML^2T^{-2}K^{-1}]$$

Q. Coefficient of Viscosity $\$\eta\$$

Viscosity is a measure of a fluid's resistance to flow. It is defined as the ratio of shear stress to shear rate. Shear stress has dimensions of [$ML^{-1}T^{-2}$] $force per unit area$, and shear rate has dimensions of [T^{-1}] $velocity gradient$. Therefore:

$$[\eta] = \frac{[ML^{-1}T^{-2}]}{[T^{-1}]} = [ML^{-1}T^{-1}]$$

R. Plank Constant h

Planck's constant relates energy to frequency. Its dimensional formula is derived from the relationship: $E = h\nu$.

Energy E has dimensions of $[ML^2T^{-2}]$. Frequency ν has dimensions of $[T^{-1}]$. Therefore:

$$[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

S. Thermal Conductivity k

Thermal conductivity is a measure of a material's ability to conduct heat. It is defined as the rate of heat transfer per unit area per unit temperature gradient. Heat transfer rate has dimensions of $[ML^2T^{-3}]$. Area has dimensions of $[L^2]$, and temperature gradient has dimensions of

$$K/L$$

. Therefore:

$$[k] = \frac{[ML^2T^{-3}]}{[L^2][K/L]} = [MLT^{-3}K^{-1}]$$

Matching the Dimensions:

Based on the dimensional analysis, we can match the quantities in List I with the dimensions in List II as follows:

List I	List II
P. Boltzmann Constant k_B	4. $[ML^2T^{-2}K^{-1}]$
Q. Coefficient of Viscosity η	2. $[ML^{-1}T^{-1}]$
R. Plank Constant h	1. $[ML^2T^{-1}]$
S. Thermal conductivity k	3. $[MLT^{-3}K^{-1}]$

Therefore, the correct answer is **Option C**.

Question 047 MCQ

QUESTION

Using the expression

$$2d\sin\theta = \lambda$$

, one calculates the values of d by measuring the corresponding angles

$$\theta$$

in the range 0 to 90^{o}. The wavelength

$$\lambda$$

is exactly known and the error in

$$\theta$$

is constant for all values of

$$\theta$$

. As

$$\theta$$

increases from 0^{o}

A. the absolute error in d remains constant

B. the absolute error in d increases

C. the fractional error in d remains constant

D the fractional error in d decreases

CORRECT OPTION

D the fractional error in d decreases

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

To determine how the error in

$$d$$

behaves as

$$\theta$$

increases, let's examine the given expression:

$$2d \sin\theta = \lambda$$

We can rearrange this to solve for

$$d$$

:

$$d = \frac{\lambda}{2\sin\theta}$$

Given that the wavelength

$$\lambda$$

is exactly known, any error in the measurement of

$$\theta$$

$denotedas\$\$\Delta\theta\$\$$ will propagate through to the error in

$$d$$

. To analyze this, we need to consider how the error propagates in the expression for

$$d$$

.

Assuming the error propagation formula, the differential of

$$d$$

with respect to

$$\theta$$

is:

$$d = \frac{\lambda}{2 \sin \theta}$$

Differentiating with respect to

$$\theta$$

:

$$\frac{d(d)}{d(\theta)} = \frac{d}{d(\theta)} \left(\frac{\lambda}{2 \sin \theta} \right)$$

Using the chain rule, we get the partial derivative:

$$\frac{d(d)}{d(\theta)} = -\frac{\lambda \cos \theta}{2(\sin \theta)^2}$$

The absolute error in

$$d$$

, denoted as

$$\Delta d$$

, is given by:

$$\Delta d = \left| \frac{d(d)}{d(\theta)} \Delta\theta \right| = \left| -\frac{\lambda \cos \theta}{2(\sin \theta)^2} \right| \Delta\theta = \frac{\lambda \cos \theta}{2(\sin \theta)^2} \Delta\theta$$

Here,

$$\Delta\theta$$

is the constant error in the measurement of

$$\theta$$

. From the equation above, we observe that

$$\Delta d$$

depends on

$$\cos\theta/(\sin\theta)^2$$

which is a function of

$$\theta$$

.

As

$$\theta$$

increases from

$$0^\circ$$

to

$$90^\circ$$

, the behavior is as follows:

- When

  $$\theta$$

  is close to

  $$0^\circ$$

  ,

  $$\cos\theta \approx 1$$

  and

$$\sin\theta \approx 0$$

, thus

$$\left(\frac{\cos\theta}{(\sin\theta)^2}\right)$$

becomes very large, indicating that

$$\Delta d$$

is very large.

- As

$$\theta$$

approaches

$$90^\circ$$

,

$$\cos\theta \approx 0$$

, and

$$\sin\theta \approx 1$$

, which makes

$$\left(\frac{\cos\theta}{(\sin\theta)^2}\right)$$

approach zero, thus making

$$\Delta d$$

very small.

Therefore, as

$$\theta$$

increases from

$$0^\circ$$

to

$$90^\circ$$

, the absolute error in

$$d$$

decreases.

To analyze the fractional error in

$$d$$

$denotedas$$\frac{\Delta d}{d}$$$, we note:

$$d = \frac{\lambda}{2\sin\theta}$$

So the fractional error is:

$$\frac{\Delta d}{d} = \frac{\Delta d}{\frac{\lambda}{2\sin\theta}} = \frac{\frac{\lambda\cos\theta}{2(\sin\theta)^2}\Delta\theta}{\frac{\lambda}{2\sin\theta}} = \frac{\cos\theta\Delta\theta}{\sin\theta}$$

As

$$\theta$$

increases from

$$0^\circ$$

to

$$90^\circ$$

:

- Initially $at$$\theta \approx 0^\circ$$$,

  $$\cos\theta \approx 1$$

  and

  $$\sin\theta \approx 0$$

  , making the fractional error large.
- As

  $$\theta$$

continues to increase,

$$\cos\theta$$

decreases and

$$\sin\theta$$

increases, thus the fractional error

$$\frac{\cos\theta\Delta\theta}{\sin\theta}$$

decreases.

Hence, the fractional error in

$$d$$

also decreases as

$$\theta$$

increases.

So the correct answer is:

Option D: the fractional error in

$$d$$

decreases.

Question 048 MCQ

QUESTION

A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement s is are

A In the region 0 < r < R, the magnetic field is non-zero.

B In the region R < r < 2R, the magnetic field is along the common axis.

C In the region R < r < 2R, the magnetic field is tangential to the circle of radius r, centred on the axis.

D In the region r > 2R, the magnetic field is non-zero.

CORRECT OPTION

A In the region 0 < r < R, the magnetic field is non-zero.

SOURCE

Physics • magnetism

EXPLANATION

We know that

i For a cylinder, B = 0 for 0 < r < R and

$$B = \frac{\mu_0}{4\pi} \times \frac{2I}{r}$$

for r > R.

ii For a solenoid, B =

$$\mu$$

$_0$nI for 0 < r < R and B = 0 for r > R.

•

Assuming that the magnetic induction to be B_C for cylinder and B_S for solenoid.

In the region R > r > 0, B_S =

$$\mu$$

$_0$nI while B_C = 0. Therefore, the total field is non-zero.

Hence, option A is correct.

•

In the region 2R < r < R, the magnetic field is not along the axis of cylinder which is given as

$$B = \sqrt{B_S^2 + B_C^2}$$

Hence, option B is wrong.

•

For checking option C, in the region R < r < 2R, the magnetic field is tangential to the circle of radius r, centred on the axis. So this statement is wrong because magnetic field is not in the plane of circle.

Hence, option C is wrong.

•

In the region r > 2, B_S = 0; while

$$B_C = \frac{\mu_0}{4\pi} \times \frac{2I}{r}$$

. Therefore total field is non-zero.

Hence, option D is correct.

Question 049 MCQ

QUESTION

Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement s is are

A If the wind blows from the observer to the source, $f_2 > f_1$.

B If the wind blows from the source to the observer, $f_2 > f_1$.

C If the wind blows from the observer to the source, $f_2 < f_1$

D If the wind blows from the source to the observer, $f_2 < f_1$.

CORRECT OPTION

A If the wind blows from the observer to the source, $f_2 > f_1$.

SOURCE

Physics • waves

EXPLANATION

Doppler's effect gives

$$f_2 = \left[\frac{v + u_0}{v - u_s}\right] f_1$$

..... 1

Let the wind speed be w and wind moves towards the source in case i and towards the observer in case ii $see figure$.

In the case i, u_0, the speed of observer w.r.t. medium considered positive when it moves towards the source is $+u$$ - $$w$ and u_s, the speed of source w.r.t.

medium considered positive when it moves towards observer is +$u + w$. Substitute these values in equation 1 to get

$$f_2 = \left[\frac{v + (u - w)}{v - (u + w)}\right] f_1$$

,

which is greater than f_1 $assuming u > w$. In the case ii, u_0 is +$u + w$ and u_s is +$u$$ - $$w$. Substitute these values in equation 1 to get

$$f_2 = \left[\frac{v + (u + w)}{v - (u - w)}\right] f1$$

,

which is again greater than f_1.

Question 050 MCQ

QUESTION

The figure below shows the variation of specific heat capacity C of a solid as a function of temperature T. The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement s is are correct to a reasonable approximation.

A The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.

B Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

C There is no change in the rate of heat absorption in the range 400-500 K.

D The rate of heat absorption increases in the range 200-300 K.

CORRECT OPTION

A The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

According to definition of specific heat capacity,

$$C = \frac{\Delta Q}{m\Delta T}$$

$$\Rightarrow \Delta Q = mC\Delta T$$

$$\therefore$$

$$\frac{\Delta Q}{\Delta t} = mC\frac{\Delta T}{\Delta t}$$

Rate of heat absorbed

$$R = \frac{\Delta Q}{\Delta t}$$

$$\Rightarrow \frac{\Delta Q}{\Delta t} \propto C$$

a In 0-100 K,

C increases with T but not linearly. So R increases but not linearly.

b As

$$\Delta Q = mC\Delta T$$

$$Q = m\int C\Delta T$$

= m area under C-T curve

From the graph it is clear that area under C-T is more in 400-500 K than in 0-100 K.

Therefore, heat absorbed in 0-100 K is less than in 400-500 K.

c In 400-500 K,

C remains constant so there is no change in R.

d In 200-300 K,

C increases so R increases.

Question 051 MCQ

QUESTION

The radius of the orbit of an electron in a hydrogen-like atom is 4.5a_0, where a_0 is the Bohr radius. Its orbital angular momentum is

$$\frac{3h}{2\pi}$$

. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength s, when the atom de-excites, is are

A
$$\frac{9}{32R}$$

B
$$\frac{9}{16R}$$

C

$$\frac{9}{5R}$$

D

$$\frac{4}{3R}$$

CORRECT OPTION

A

$$\frac{9}{32R}$$

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The orbital angular momentum is

$$L = \frac{nh}{2\pi}$$

$$\frac{3h}{2\pi} = \frac{nh}{2\pi} \Rightarrow n = 3$$

The radius of the orbit is

$$4.5(a_0) = a_0\left(\frac{n^2}{Z}\right)$$

$$Z = \frac{n^2}{4.5} = \frac{3^2}{4.5} = \frac{9}{4.5} = 2$$

Thus, the possible transitions are 3

$$\rightarrow$$

2, 3

$$\rightarrow$$

1 and 2

$$\rightarrow$$

1. For the transition 3

$$\rightarrow$$

2, the wavelength is

$$\frac{1}{\lambda} = R(2)^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 4R\left[\frac{9-4}{36}\right] = \frac{5R}{9}$$

$$\Rightarrow \lambda = \frac{9}{5R}$$

For the transition 3

$$\rightarrow$$

1, the wavelength is

$$\frac{1}{\lambda} = R(2)^2 \left[1 - \frac{1}{9}\right] = 4R\left(\frac{8}{9}\right) = \frac{32R}{9}$$

$$\Rightarrow \lambda = \frac{9}{32R}$$

For the transition 2

$$\rightarrow$$

1, the wavelength is

$$\frac{1}{\lambda} = R(2)^2 \left[1 - \frac{1}{4}\right] = 4R\left(\frac{3}{4}\right) = 3R$$

$$\Rightarrow \lambda = \frac{1}{3R}$$

Question 052 MCQ

QUESTION

If the direct transmission method with a cable of resistance 0.4

$$\Omega$$

km

$$-$$

^{1} is used, the power dissipation in during transmission is

A 20

B 30

C 40

D 50

CORRECT OPTION

B 30

SOURCE

Physics • current-electricity

EXPLANATION

P = Vi

$$\therefore$$

$$i = \frac{P}{V} = \frac{600 \times 10^3}{4000} = 150$$

A

Total resistance of cables,

R = 0.4

$\times$

20 = 8

Ω

$\therefore$

Power loss in cables = i^2R

= 150^2 8

= 180000 W = 180 kW

This loss is 30% of 600 kW.

Question 053 MCQ

QUESTION

In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power of the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

A 200 : 1

B 150 : 1

C 100 : 1

D 50 : 1

CORRECT OPTION

A 200 : 1

SOURCE

Physics • alternating-current

EXPLANATION

To solve this problem, we need to understand the working principle of step-up and step-down transformers and the relationships between the voltages and the number of turns of the windings. The key formula we use is the transformer equation:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

where:

- V_p is the primary voltage
- V_s is the secondary voltage
- N_p is the number of turns in the primary winding
- N_s is the number of turns in the secondary winding

Let's break down the problem:

1. The power plant produces an electric power of 600 kW at 4000 V. 2. A step-up transformer is used at the plant side with a turns ratio of 1 : 10, i.e.,

$$N_p : N_s = 1 : 10$$

.

This tells us that:

$$\frac{V_p}{V_s} = \frac{1}{10}$$

Since the primary voltage

$$V_p$$

is 4000 V, we can find the secondary voltage

$$V_s$$

as follows:

$$V_s = 10 \times V_p$$

$$V_s = 10 \times 4000$$

$$V_s = 40000$$

V

So, at the output of the step-up transformer, the voltage is 40000 V. This high voltage is then transported over the cables to the consumers' location 20 km away. At the consumers' end, a step-down transformer is used to reduce the voltage to 200 V to supply power to the consumers.

Now, we are given that the power supplied at the consumers' end must be 200 V. We need to find the ratio of the number of turns in the primary to the number of turns in the secondary for the step-down transformer so that it outputs 200 V from an input of 40000 V.

Using the transformer equation again, we can write:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

where

$$V_p = 40000$$

V and

$$V_s = 200$$

V. Plugging these values in, we get:

$$\frac{40000}{200} = \frac{N_p}{N_s}$$

This simplifies to:

$$\frac{40000}{200} = 200$$

So, the turns ratio

$$\frac{N_p}{N_s} = 200 : 1$$

.

Thus, the correct answer is:

Option A: 200 : 1

Question 054 MCQ

QUESTION

The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

A

$$\frac{BR}{4}$$

B

$$\frac{BR}{2}$$

C BR

D 2BR

CORRECT OPTION

B

$$\frac{BR}{2}$$

SOURCE

Physics • magnetism

EXPLANATION

Let the point charge Q is moving in a circle of constant radius R in anticlockwise direction as shown in the figure.

The magnetic flux through the circular loop at time t is given by

$$\phi = B\overrightarrow{(t)} \,.\, (\pi R^2 \hat{z}) \; = \pi B(t) R^2$$

. Faraday's law gives the induced emf e as

$$e = -\frac{d\phi}{dt} = -\pi R^2 \frac{dB(t)}{dt} = -\pi R^2 B$$

..... 1

Lenz's law gives the direction of induced current and hence electric field

$$(\overrightarrow{E})$$

as clockwise. The induced emf is related to

$$\vec{E}$$

by

$$e = \oint \vec{E}.\ \vec{dl} = -E(2\pi R)$$

..... 2

Eliminate e from equations 1 and 2 to get E = BR/2.

Question 055 MCQ

QUESTION

The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

A $-\gamma BQR^2$

B $-\gamma \frac{BQR^2}{2}$

C $\gamma \frac{BQR^2}{2}$

D γBQR^2

CORRECT OPTION

B

$$-\gamma\frac{BQR^2}{2}$$

SOURCE

Physics • magnetism

EXPLANATION

$$\frac{M}{L}=\frac{Q}{2m}$$

$$\therefore$$

$$M=\left(\frac{Q}{2m}\right)L$$

$$\Rightarrow M\propto L$$

, where

$$\gamma=\frac{Q}{2m}$$

$$\left(\frac{Q}{2m}\right)(I\omega)$$

$$=\left(\frac{Q}{2m}\right)(mR^2\omega)=\frac{Q\omega R^2}{2}$$

Induced electric field is opposite. Therefore,

$$\omega'=\omega-\alpha t$$

$$\alpha=\frac{\tau}{I}=\frac{(QE)R}{mR^2}=\frac{(Q)\left(\frac{BR}{2}\right)R}{mR^2}=\frac{QB}{2m}$$

$$\therefore$$

$$\omega'=\omega-\frac{QB}{2m}.1=\omega-\frac{QB}{2m}$$

$$M_f = \frac{Q\omega' R^2}{2} = Q\left(\omega - \frac{QB}{2m}\right)\frac{R^2}{2}$$

$$\therefore$$

$$\Delta M = M_f - M_i = -\frac{Q^2 B R^2}{4m}$$

$$M = -\gamma \frac{QBR^2}{2}$$

$as\$\$\gamma = \frac{Q}{2m}\$\$$

Question 056 MCQ

QUESTION

The correct statement is

A the nucleus

$$^{6}_{3}Li$$

can emit an alpha particle.

B the nucleus

$$^{210}_{84}Po$$

can emit a proton.

C deuteron and alpha particle can undergo complete fusion.

the nuclei

$$^{70}_{30}Zn$$

D and

$$^{82}_{34}Se$$

can undergo complete fusion.

CORRECT OPTION

C deuteron and alpha particle can undergo complete fusion.

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

We have

$$m(^2_1H) + m(^4_2He) = 2.014102 + 4.002603 = 6.016705\,u$$

$$m(^6_3Li) = 6.015123\,u$$

$$m_1 + m_2 > M$$

Thus, option A is wrong.

$$m(^1_1H) + m(^{209}_{83}Bi) = 1.007825\,u + 208.980388\,u = 209.988213\,u$$

$$m(^{210}_{84}Po) = 209.982876\,u$$

$$m_1 + m_2 > M$$

Thus, option B is wrong.

$$m(^2_1H) + m(^4_2He) = 2.014102\,u + 4.002603\,u = 6.016705\,u$$

$$^6_3Li = 6.015123\,u$$

$$(m_3 + m_4) > M'$$

Thus, option C is correct. Therefore, deuteron and alpha particle can go complete fusion.

$$m(^{70}_{30}Zn) + ^{82}_{34}Se = 69.925325\,u + 81.916709\,u = 151.842034\,u$$

$$^{152}_{64}Gd = 151.919803\,u$$

$$m_3 + m_4 < M'$$

Thus, option D is wrong.

Question 057 MCQ

QUESTION

The kinetic energy $inkeV$ of the alpha particle, when the nucleus

$$^{210}_{84}Po$$

at rest undergoes alpha decay, is

- A 5319
- B 5422
- C 5707
- D 5818

CORRECT OPTION

A 5319

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

The alpha decay of

$$^{210}_{84}Po$$

is given by

$$^{210}_{84}Po \rightarrow ^{206}_{82} Pb + ^{4}_{2} He$$

The energy released during this process is

$$Q = (M_{Po} - M_{Pb} - M_{He})c^2$$

$$= (209.982876 - 205.974455 - 4.002603)\, u \times c^2$$

$$= (0.005818\, u)c^2 = (0.005818\, u) \times 932\, MeV$$

$$= 5.422\, MeV = 5422\, keV$$

Kinetic energy of a particle,

$$K_\alpha = \frac{(A-4)Q}{A}$$

$$K_\alpha = \frac{(210-4)}{210} \times 5422\, keV = \frac{206}{210} \times 5422\, keV = 5319\, keV$$

Question 058 MCQ

QUESTION

A right-angled prism of refractive index

$$\mu$$

1 is placed in a rectangular block of refractive index

$$\mu$$

2, which is surrounded by a medium of refractive index

μ

$_3$, as shown in the figure. A ray of light e enters the rectangular block at normal incidence. Depending upon the relationships between

μ

$_1$,

μ

$_2$ and

μ

$_3$, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'.

Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

	List I		List II
P.	$e \rightarrow f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \rightarrow g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \rightarrow h$	3.	$\mu_1 = \mu_2$
S.	$e \rightarrow i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

A P-2, Q-3, R-1, S-4

B P-1, Q-2, R-4, S-3

C P-4, Q-1, R-2, S-3

D P-2, Q-3, R-4, S-1

CORRECT OPTION

D P-2, Q-3, R-4, S-1

SOURCE

Physics • geometrical-optics

EXPLANATION

For e

$\rightarrow$

f

μ

$_2 >$

μ

$_1$, as ray bends towards the normal.

μ

$_2 >$

μ

$_3$, as ray bends away from the normal.

$P \rightarrow 2$

For $e \rightarrow g$

$\mu_1 = \mu_2$ as there is no deviation.

$Q \rightarrow 3$

For $e \rightarrow h$

$\mu_2 < \mu_1$, as ray bends away from the normal.

$\mu_2 > \mu_3$, as ray bends away from the normal.

Also,

$$\mu$$

1 <

$$\sqrt{2}$$

$$\mu$$

2 $No total internal reflection$

R

$$\rightarrow$$

4

For e

$$\rightarrow$$

i

Total internal reflection takes place

$$\therefore$$

sin45

$$\circ$$

> sinC

But

$$\sin C = \frac{\mu_2}{\mu_1}$$

$$\therefore$$

$$\frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$$

$$S \rightarrow 1$$

Question 059 MCQ

QUESTION

One mole of a monatomic ideal gas is taken along two cyclic processes E

$\rightarrow$

F

$\rightarrow$

G

$\rightarrow$

E and E

$\rightarrow$

F

$\rightarrow$

H

$\rightarrow$

E as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.

Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists :

	List I		List II
P.	$G \rightarrow E$	1.	160 P_0V_0 ln2
Q.	$G \rightarrow H$	2.	36 P_0V_0

	List I		List II
R.	$F \rightarrow H$	3.	24 $P_0 V_0$
S.	$F \rightarrow G$	4.	31 $P_0 V_0$

A P-4, Q-3, R-2, S-1

B P-4, Q-3, R-1, S-2

C P-3, Q-1, R-2, S-4

D P-1, Q-3, R-2, S-4

CORRECT OPTION

A P-4, Q-3, R-2, S-1

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

From the given figure, it can be concluded that process FG is isothermal and process FH is adiabatic.

$$\gamma_{mono} = 1 + \frac{2}{f} = \frac{5}{3}$$

$$P_0 V_G^{5/3} = 32 P_0 V_0^{5/3}$$

$$V_G = (32)^{3/5} V_0 = 8V_0$$

$$\Delta W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$$

Thus, the correct mapping is P

$$\rightarrow$$

$4.$

$$\Delta W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$$

Thus, the correct mapping is Q

$$\rightarrow$$

$3.$

$$\Delta W_{FH} = \frac{P_0(8V_0) - 32P_0V_0}{1 - (5/3)} = \frac{-24P_0V_0}{-(2/3)} = 36P_0V_0$$

Thus the correct mapping is R

$$\rightarrow$$

$2.$

$$\Delta W_{FG} = 32RT_0 \ln 32 = 32RT_0 \ln 2^5 = 160RT_0 \ln 2 = 160P_0V_0 \ln 2$$

Thus, the correct mapping is S

$$\rightarrow$$

$1.$

Question 060 MCQ

QUESTION

Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists :

	List I		List II
P.	Alpha decay	1.	${}^{15}_{8}O \rightarrow {}^{15}_{7} N + \ldots$
Q.	β^+ decay	2.	${}^{238}_{91}U \rightarrow {}^{234}_{90} Th + \ldots$
R.	Fission	3.	${}^{185}_{83}Bi \rightarrow {}^{184}_{82} Pb + \ldots$
S.	Proton emission	4.	${}^{239}_{94}Pu \rightarrow {}^{140}_{57} La + \ldots$

A P-4, Q-2, R-1, S-3

B P-1, Q-3, R-2, S-4

C P-2, Q-1, R-4, S-3

D P-4, Q-3, R-2, S-1

CORRECT OPTION

C P-2, Q-1, R-4, S-3

SOURCE

Physics • atoms-and-nuclei

EXPLANATION

In alpha decay, atomic number decreases by 2 and mass number decreases by 4.

P

$$\rightarrow$$

2

In

$$\beta$$

$^{+}$ decay, atomic number decreases by 1 and mass number remains unchanged.

Q

$$\rightarrow$$

1

In fission, heavy nucleus breaks into two light nuclei.

R

$$\rightarrow$$

4

In proton emission, both atomic number and mass number decreases by 1.

S

$$\rightarrow$$

3