JEE Advanced 2014 Paper 1 $Offline$

$60Questions$

Question 001 MCQ

QUESTION

Upon heating with Cu_2S, the reagent s that give copper metal is/are

A $CuFeS_2$

B CuO

C Cu_2O

D $CuSO_4$

CORRECT OPTION

B CuO

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

To understand which of the given reagents will give copper metal upon heating with Cu_2S, let's discuss each one based on their chemical reactions during the process:

Option A: $CuFeS_2$

$CuFeS_2$ $Chalcopyrite$ when heated with Cu_2S, it can react to form copper metal as part of the copper extraction process. However, the direct reaction between $CuFeS_2$ and Cu_2S is more involved and typically requires a series of steps, including smelting and conversion to get to the copper metal. Therefore, while $CuFeS_2$ is a source of copper, it's not directly because of its reaction with Cu_2S, but through a complex metallurgical process.

Option B: CuO

$$4\mathrm{CuO} \xrightarrow{\Delta} 2\mathrm{Cu_2O} + \mathrm{O_2}$$

$$\mathrm{Cu_2\ S} + 2\mathrm{Cu_2O} \xrightarrow{\Delta} 6\mathrm{Cu} + \mathrm{SO_2}$$

Option C: Cu_2O

Copper I oxide $\$Cu_2O\$$ can also react with Cu_2S to produce copper metal directly. The equation for this reaction is:

$$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$$

Here, Cu_2O serves as the oxidizing agent, reducing Cu_2S to copper metal, while itself being reduced to copper, and sulfur is oxidized to sulfur dioxide. This reaction is similar in principle to the reaction with CuO, but involves copper I oxide instead.

Option D: $CuSO_4$

$$\mathrm{CuSO_4} \xrightarrow{720^\circ\mathrm{C}} \mathrm{CuO} + \mathrm{SO_2} + \tfrac{1}{2}\mathrm{O_2}$$

$$4\mathrm{CuO} \xrightarrow{1100^\circ\mathrm{C}} 2\mathrm{Cu_2O} + \mathrm{O_2}$$

$$2\mathrm{Cu_2O} + \mathrm{Cu_2\ S} \xrightarrow{\Delta} 6\mathrm{Cu} + \mathrm{SO_2}$$

Question 002 MCQ

QUESTION

In a galvanic cell, the salt bridge

A does not participate chemically in the cell reaction.

B stops the diffusion of ions from one electrode to another.

C is necessary for the occurrence of the cell reaction.

D ensures mixing of the two electrolytic solutions.

CORRECT OPTION

A does not participate chemically in the cell reaction.

SOURCE

Chemistry • electrochemistry

EXPLANATION

In a galvanic cell, also known as a voltaic cell, a salt bridge plays a crucial role in maintaining electrical neutrality within the internal circuit, which is critical for the ongoing electrochemical reaction and thus the production of electrical energy. Here, we will analyze each of the given options in the context of the function of a salt bridge in a galvanic cell:

Option A: does not participate chemically in the cell reaction.

This statement is correct. The salt bridge in a galvanic cell does not directly participate in the chemical reactions occurring at the electrodes. Its primary function is to complete the electrical circuit between the cathode and anode by allowing the transfer of ions. The salt bridge contains a salt solution (usually KNO_3, KCl, or NH_4NO_3), where the ions migrate to oppose and balance the charge buildup due to the migration of electrons through the external circuit.

Option B: stops the diffusion of ions from one electrode to another.

This option can be misleading. The salt bridge does not stop the diffusion of ions from diffusing from one electrode to another but rather provides a pathway for ions to flow back and forth, which is essential to maintain the charge balance. Hence, this statement is not the best descriptor of the salt bridge's function.

Option C: is necessary for the occurrence of the cell reaction.

This option is correct. A salt bridge is necessary for the occurrence of the cell reaction because it maintains electrical neutrality in the electrochemical cell. Without the salt bridge, the flow of electrons through the external circuit would soon cease as the solutions in the anode and cathode compartments become respectively positively and negatively charged, which would stop the electrochemical reaction.

Option D: ensures mixing of the two electrolytic solutions.

This statement is incorrect. The purpose of the salt bridge is not to mix the two electrolytic solutions; in fact, it prevents them from mixing, which could otherwise result in a direct neutralization reaction that could interfere with the proper functioning of the cell. The ions in the salt bridge only migrate enough to balance the charges in the separate solutions.

In summary, the correct answers are Option A $''does not participate chemically in the cell reaction''$ and Option C $''is necessary for the occurrence of the cell reaction''$.

Question 003 Numerical

QUESTION

A compound H_2X with molar weight of 80g is dissolved in a solvent having density of 0.4 gml^{-1} . Assuming no change in volume upon dissolution, the molality of a 3.2 molar solution is

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

Understanding the Concepts

- **Molarity** M**:** Moles of solute per liter of solution.
- **Molality** m**:** Moles of solute per kilogram of solvent.
- **Density:** Mass per unit volume.

Steps to Calculate Molality

1. **Find the Mass of Solute** H_2X**:**

- Molarity = moles of solute / volume of solution $inliters$
- 3.2 M solution means 3.2 moles of H_2X are present in 1 liter of solution.
- Mass of H_2X = moles

$\times$

molar mass = 3.2 moles

$\times$

80 g/mole = 256 g

1. **Find the Mass of Solvent:**

- Assuming no change in volume, the volume of the solution remains 1 liter.
- Density = mass / volume
- Mass of solvent = density

  $\times$

  volume = 0.4 g/mL

  $\times$

  1000 mL = 400 g

1. **Convert Mass of Solvent to Kilograms:**

- 1 kg = 1000 g
- Mass of solvent = 400 g

  $\times$

  $1kg/1000g$ = 0.4 kg

1. **Calculate Molality:**

- Molality = moles of solute / mass of solvent $inkg$
- Molality = 3.2 moles / 0.4 kg = 8 mol/kg

Answer:

The molality of the 3.2 molar solution is 8 mol/kg.

Question 004 Numerical

QUESTION

MX_2 dissociates in M^{2+} and X^- ions in an aqueous solution, with a degree of dissociation $$\alpha$$ of 0.5. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation is

SOURCE

Chemistry • solutions

EXPLANATION

The depression of freezing point is an important colligative property, which is influenced by the number of solute particles in a solution. When a solute dissociates or ionizes in a solution, it increases the number of particles in the solution, thereby affecting colligative properties such as the depression of freezing point. The degree of dissociation $$\alpha$$ gives us a measure of the extent to which a compound dissociates into its ions. For our case, MX_2 dissociates into one M^{2+} ion and two X^- ions.

Let's initially consider 1 mole of MX_2 is present. Since the degree of dissociation $$\alpha$$ is 0.5, this means half of the MX_2 dissociates into its ions, and half remains undissociated.

For the dissociation reaction:

$$\mathrm{MX_2} \rightarrow \mathrm{M^{2+}} + 2\mathrm{X^-}$$

If the initial amount of MX_2 is 1 mole, then:

- The amount of MX_2 that dissociates =

$$\alpha = 0.5$$

  moles
- The amount of M^{2+} formed =

$$\alpha = 0.5$$

moles (since for every mole of MX_2 that dissociates, 1 mole of M^{2+} is formed)

- The amount of X^- formed =

$$2\alpha = 2 \times 0.5 = 1$$

mole (since for every mole of MX_2 that dissociates, 2 moles of X^- are formed)

The Van't Hoff factor i quantifies the effect of solute particles on the colligative properties of a solution. It is defined as the ratio of the actual number of particles in solution after dissociation to the number of formula units initially dissolved in the solvent.

$$i = \frac{\text{Total number of particles after dissociation}}{\text{Number of moles of solute originally dissolved}}$$

Considering the dissociation and the amounts calculated:

- Total number of particles after dissociation = (undissociated MX_2) + (M^{2+}) + (X^-) =

$$(1 - \alpha) + \alpha + 2\alpha$$

- Putting the value of

$$\alpha = 0.5$$

, we get:

$$i = \frac{(1 - 0.5) + 0.5 + 2(0.5)}{1} = \frac{1 + 1}{1} = 2$$

Now, the depression in freezing point $$\Delta T_f$$ is directly proportional to the molal concentration of the solute particles and Van't Hoff factor i:

$$\Delta T_f = i \cdot K_f \cdot m$$

Where

$$K_f$$

is the cryoscopic constant and

$$m$$

is the molality of the solution.

Without ionic dissociation, the value of

$$i$$

would have been 1
$since the solute would not have dissociated into multiple particles.$
However, due to the dissociation, the value of

$$i$$

has increased to 2. The ratio of the observed depression of freezing point of the aqueous solution to the value of the depression of freezing point in the absence of ionic dissociation, therefore, would be directly equal to the ratio of the Van't Hoff factors:

$$\frac{\text{Observed } \Delta T_f}{\text{In the absence of ionic dissociation}} = \frac{2 \cdot K_f \cdot m}{1 \cdot K_f \cdot m} = \frac{2}{1} = 2$$

This ratio shows that due to the ionic dissociation of MX_2 into M^{2+} and X^- ions with a degree of dissociation $$\alpha$$ of 0.5, the observed depression in freezing point is twice the value it would have been in the absence of ionic dissociation.

Question 005 Numerical

QUESTION

A list of species having the formula XZ_4 is given below.

XeF_4, SF_4 ,SiF_4,

$$BF_4^-$$

,

BrF_4^-

, $[Cu(NH_3)_4]^{2+}$, $[FeCl_4]^{2-}$, $[CoCl_4]^{2-}$ and $[PtCl_4]^{2-}$

Defining shape on the basis of the location of X and Z atoms, the total number of species having a square planar shape is

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

$XeF_4, BrF_4^-, [Cu(NH_3)_4]^{2+}, [PtCl_4]^{2-}$ are square planar as shown below :

$SF_4(See-saw)$ as shown below:

$SiF_4, BF_4^-, [FeCl_4]^{2-}, [CoCl_4]^{2-}$ are tetrahedral as shown below :

Question 006 Numerical

QUESTION

In an atom, the total number of electrons having quantum numbers n = 4, $|m_l|$ = 1 and m_s = –1/2 is

SOURCE

Chemistry • structure-of-atom

EXPLANATION

As n = 4, so l = 0, 1, 2, 3 which implies the orbitals are 4s, 4p, 4d and 4f.

Now, $|m_l| = 1$ implies $m_l = +1$ and -1. Therefore, l can be 3, 2, 1, as for l = 0, $m_l = 0$.

For l = 1, $m_l = -1, 0, +1$

For l = 2, $m_l = -2, -1, 0, +1, +2$

For l = 3, $m_l = -3, -2, -1, 0, +1, +2, +3$

Therefore, there are six possible orbitals with $|m_l| = 1$. Also, given that $m_s = -1/2$ so six electrons are possible with $m_s = -1/2$.

Question 007 Numerical

QUESTION

If the value of Avogadro number is 6.023

$\times$

10^{23} mol^{-1} and the value of Boltzmann constant is 1.380

$\times$

10^{-23} J K^{-1}, then the number of significant digits in the calculated value of the universal gas constant is

SOURCE

Chemistry • some-basic-concepts-of-chemistry

EXPLANATION

The universal gas constant, denoted by R, can be calculated using the Avogadro number (N_A) and the Boltzmann constant (k_B) by the following relationship:

$$R = N_A \times k_B$$

Given that:

$$N_A = 6.023 \times 10^{23}\ \text{mol}^{-1}$$

$$k_B = 1.380 \times 10^{-23}\ \text{J K}^{-1}$$

Let's multiply these values to find R:

$$R = (6.023 \times 10^{23}\ \text{mol}^{-1}) \times (1.380 \times 10^{-23}\ \text{J K}^{-1})$$

$$R = (6.023 \times 1.380) \times (10^{23} \times 10^{-23})\ \text{J mol}^{-1}\text{K}^{-1}$$

$$R = 8.31174 \times 10^{0}\ \text{J mol}^{-1}\text{K}^{-1}$$

To determine the number of significant digits in the calculated value of R, we must consider the number of significant digits in the given values of N_A and k_B.

The value for N_A has four significant digits 6.023, and the value for k_B also has four significant digits 1.380. When multiplying or dividing numbers, the number of significant digits in the result is determined by the number with the smallest amount of significant digits used in the calculation.

In this case, since both constants have four significant digits, the value of R calculated from their multiplication will also contain four significant digits:

$$R \approx 8.314\ \text{J mol}^{-1}\text{K}^{-1}$$

Therefore, the calculated value of the universal gas constant R has four significant digits.

Question 008 MCQ

QUESTION

For the reaction,

$$I^- + ClO_3^- + H_2SO_4 \rightarrow Cl^- + HSO_4^- + I_2$$

the correct statement s in the balanced equation is/are

A stoichiometric coefficient of HSO $\frac{}{4}$ is 6

B iodide is oxidised

C sulphur is reduced

D H_2O is one of the products

CORRECT OPTION

A stoichiometric coefficient of HSO $\frac{}{4}$ is 6

SOURCE

Chemistry • redox-reactions

EXPLANATION

Oxidation half-reaction,

$$2I^- \rightarrow I_2 + 2e^-$$

.... i

Here, I

$$-$$

is converted into I_2. Oxidation number of I is increasing from

$$-$$

1 to 0 hence, this is a type of oxidation reaction.

Reduction half-reaction

$$6H^+ + ClO_3^- + 6e^- \rightarrow Cl^- + 3H_2O$$

..... ii

Here, H_2O releases as a product. Hence, option d is correct.

Multiplying equation i by 3 and adding in equation ii

Stoichiometric coefficient of

$$HSO_4^-$$

is 6.

Hence, option a, b and d are correct.

Question 009 MCQ

QUESTION

The pair s of reagents that yield paramagnetic species is/are

A Na and excess of NH_3

B K and excess of O_2

C Cu and dilute HNO_3

D O_2 and 2-ethylanthraquinol

CORRECT OPTION

A Na and excess of NH_3

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

Reaction of alkali metals with ammonia depends upon the physical state of ammonia whether it is in gaseous state or liquid state. If ammonia is considered as a gas then reaction will be

a

$$Na + \underset{(Excess)}{NH_3} \rightarrow NaNH_2 + \frac{1}{2}H_2$$

($NaNH_2$ + 1/2H_2 are diamagnetic) If ammonia is considered as a liquid then reaction will be

$$M + (x+y)NH_3 \rightarrow [M(NH_3)_x]^+ + [e(NH_3)_y]^-$$

ammoniated electron

- blue colour
- paramagnetic
- very strong reducing agent

b

$$K + \underset{(Excess)}{O_2} \rightarrow \underset{Potassium\ \text{sup}eroxide\ Paramagnetic}{KO_2(K^+, O_2^-)}$$

c

$$3Cu + 8HNO_3 \rightarrow \underset{Paramagnetic}{3Cu(NO_3)_2} + \underset{Paramagnetic}{2NO} + 4H_2O$$

d

Hence, option a, b and c are correct choices.

Question 010 MCQ

QUESTION

In the reaction shown below, the major product s formed is/are

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Regioselectivity means which group will react selectively in the presence of two or more than two functional groups. Here, among two functional group

NH_2 and

$CONH_2$, NH_2 is more nucleophilic hence, NH_2 group will undergo reaction faster than $CONH_2$.

Hence, correct choice is a.

Question 011 MCQ

QUESTION

Hydrogen bonding plays a central role in the following phenomena

A ice floats in water

B higher Lewis basicity of primary amines than tertiary amines in aqueous solutions

C formic acid is more acidic than acetic acid

D dimerisation of acetic acid in benzene

CORRECT OPTION

A ice floats in water

SOURCE

Chemistry • chemical-bonding-and-molecular-structure

EXPLANATION

a Ice floats in water due to the low density of ice as compare to water which is due to open cage like structure $formedbyintermolecularH-bonding.$

b Basic strength of RNH_2 > R_3N. It is also explained by hydrogen bonding.

c

d Dimerisation of acetic acid in benzene is due to intermolecular hydrogen

bonding.

Question 012 MCQ

QUESTION

The reactivity of compound Z with different halogens under appropriate conditions is given below

The observed pattern of electrophilic substitution can be explained by

A the steric effect on the halogen

B the steric effect of the tert-butyl group

C the electronic effect of the phenolic group

D the electronic effect of the tert-butyl group

CORRECT OPTION

A the steric effect on the halogen

SOURCE

Chemistry • alcohols-phenols-and-ethers

EXPLANATION

Steric effect of halogens are as follows $Cl_2 < Br_2 < I_2$

Electronic effect of phenolic group directs the approaching electrophile towards ortho and para positions. Tertiary butyl group has large size so it causes steric effect around aromatic nucleus. On the basis of above factors the products of the given reactions are as follows

Hence, orientation in electrophilic substitution reaction is decided by

a The steric effect of the halogen

b The steric effect of the tert-butyl group

c The electronic effect of the phenolic group

So, a, b and c are correct choices.

Question 013 MCQ

QUESTION

The correct combination of names for isomeric alcohols with molecular formula $C_4H_{10}O$ is/are

A tert-butanol and 2-methylpropan-2-ol

B tert-butanol and 1, 1-dimethylethan-1-ol

C n-butanol and butan-1-ol

iso-butyl alcohol and 2-methylpropan-1-ol

CORRECT OPTION

A tert-butanol and 2-methylpropan-2-ol

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

Question 014 MCQ

QUESTION

An ideal gas in thermally insulated vessel at internal pressure = p_1, volume = V_1 and absolute temperature = T_1 expands irreversibly against zero external pressure, as shown in the diagram.

The final internal pressure, volume and absolute temperature of the gas are p_2, V_2 and T_2, respectively. For this expansion

A $q = 0$

B $T_2 = T_1$

C $p_2V_2 = p_1V_1$

D $p_2V^{\frac{\gamma}{2}} = p_1V^{\frac{\gamma}{1}}$

CORRECT OPTION

A q = 0

SOURCE

Chemistry • gaseous-state

EXPLANATION

As the vessel is thermally insulated $so, q = 0$. Therefore,

ΔU = 0,

ΔT = 0

$\Rightarrow$

$T_1 = T_2$

According to combined gas law equation,

$$\frac{P_2V_2}{T_2} = \frac{P_1V_1}{T_1} \Rightarrow P_2V_2 = P_1V_1$$

Question 015 MCQ

QUESTION

The correct statement s for orthoboric acid is/are

A it behaves as a weak acid in water due to self ionisation

B acidity of its aqueous solution increases upon addition of ethylene glycol

C it has a three dimensional structure due to hydrogen bonding

D it is a weak electrolyte in water

CORRECT OPTION

B acidity of its aqueous solution increases upon addition of ethylene glycol

SOURCE

Chemistry • p-block-elements

EXPLANATION

a It does not undergo self ionisation in water but accepts an electron pair from water, so it behaves as weak monobasic acid

$H_3BO_3 + H_2O$

$\rightleftharpoons$

BOH

$\overline{4}$

+ H^+

Hence, a is incorrect.

b When treated with 1, 2-dihydroxy or polyhydroxy compounds, they form chelate $ringcomplex$ which effectively remove [BOH_4]$^-$ species from solution and thereby produce maximum number of H_3O^+ or H^+ ions, i.e., results in increased acidity.

c Baric acid crystallises in a layer structure in which planar triangular BO_3^{3-} ions are bonded together through hydrogen bonds.

d In water the pK_a value of H_3BO_3 is 9.25

$H_3BO_3 + H_2O \rightleftharpoons BOH_4^- + H^+$; $pK_a = 9.25$

So, it is weak electrolyte in water.

Question 016 Numerical

QUESTION

The total number of distinct naturally occurring amino acids obtained by complete acidic hydrolysis of the peptide shown below is

SOURCE

Chemistry • biomolecules

EXPLANATION

Chemical reaction and product formed after hydrolysis of given peptide can be represented as

A is glycine which is only naturally occurring amino acid. While B, C and D are not the naturally occurring amino acids. Hence, correct integer is 1.

Question 017 Numerical

QUESTION

Consider the following list of reagents, acidified $K_2Cr_2O_7$, alkaline $KMnO_4$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$, HNO_3 and $Na_2S_2O_3$. The total number of reagents that can oxidise aqueous iodide to iodine is

SOURCE

Chemistry • redox-reactions

EXPLANATION

Acidified $K_2Cr_2O_7$, $CuSO_4$, H_2O_2, Cl_2, O_3, $FeCl_3$ and HNO_3 oxidise aq. iodide to iodine. Alkaline $KMnO_4$ oxidise aq. iodide to IO

$$\overline{3}$$

.

$Na_2S_2O_3$ is a strong reducing agent which on reaction with I_2 produces I

$$-$$

.

$Na_2S_2O_3 + I_2$

$\rightarrow$

2I

–

$+ Na_2S_4O_6$

Therefore, no reaction takes place between $Na_2S_2O_3$ and iodide ion. Hence, correct integer is 7.

Question 018 Numerical

QUESTION

The total number s of stable conformers with non-zero dipole moment for the following compound is are

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

The conformations of the given compound are as follows

These three have non-zero dipole moment due to non-cancellation of all dipole moment created by C

–

Cl and C

–

Br bond.

Question 019 Numerical

QUESTION

Among PbS, CuS, HgS, MnS, Ag_2S, NiS, CoS, Bi_2S_3 and SnS_2 the total number of black coloured sulphides is

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

i Lead in +2 oxidation state as Pb^{2+} reacts with hydrogen sulphide gas $(\mathrm{H_2\ S})$ to form black coloured precipitate of lead sulphide (PbS).

$$\mathrm{Pb}^{2+}(aq) + \mathrm{H_2\ S}(g) \rightarrow \mathrm{PbS}(s) \downarrow\ (\mathrm{Black\ \ ppt})\ + 2\mathrm{H}^{+}(aq)$$

PbS is black coloured precipitate.

ii Silver in +1 oxidation state as Ag^{+} reacts with hydrogen sulphide in neutral or acidic medium to form black coloured precipitate of silver sulphide $(\mathrm{Ag_2\ S})$.

$$2\mathrm{Ag}^{+}(aq) + \mathrm{H_2\ S}(g) \xrightarrow[\text{or acidic medium}]{\text{Neutral}} \mathrm{Ag_2\ S}(s)$$

$Blackppt$

$\mathrm{Ag_2\ S}$ **is black coloured precipitate.**

iii Mercury in +2 oxidation state as Hg^{2+} reacts with hydrogen sulphide in dilute hydrochloric acid to form black coloured precipitate of mercury sulphide HgS.

$$\mathrm{Hg^{2+}}(aq) + \underset{\text{(saturated aq. solution or gas)}}{\mathrm{H_2\,S}} \xrightarrow{\text{dil. HCl}} \underset{\text{Black ppt}}{\mathrm{HgS}(s)}$$

HgS is black coloured precipitate.

iv Copper in +2 oxidation state as $\mathrm{Cu^{2+}}$ reacts with dilute hydrochloric acid to form black coloured precipitate of copper sulphide CuS.

$$\mathrm{Cu^{2+}}(aq) + \mathrm{H_2\,S(\,g)\ (saturated\ aq.\ solution)} \xrightarrow[\mathrm{HCl}]{\text{dil}} \mathrm{CuS(s)\ (black\ ppt)}$$

CuS is black coloured precipitate.

v Cobalt in +2 oxidation state as $\mathrm{Co^{2+}}$ reacts with hydrogen sulphide in neutral or alkaline solution to form black coloured precipitate of cobalt sulphide.

$$\mathrm{Co^{2+}}(aq)\ \mathrm{(ammonical\ hydrogen\ sulphide)} + \mathrm{H_2\,S} \longrightarrow \mathrm{CoS(s)\ (Black\ ppt.)}$$

$\mathrm{(CoS)}$ **is black coloured precipitate.**

vi Nickel in +2 oxidation state as $\mathrm{Ni^{2+}}$ reacts with hydrogen sulphide in neutral or slightly alkaline solution to form, black coloured precipitate of nickel sulphide.

$$\mathrm{Ni^{2+}}(aq) + \mathrm{H_2\,S} \longrightarrow \underset{\text{Black ppt.}}{\mathrm{NiS(s)}}$$

NiS is black coloured precipitate.

vii Bismuth in +3 oxidation state as $\mathrm{Bi^{3+}}$ reacts with hydrogen sulphide in cold dilute hydrochloric acid to form crystalline dark brown coloured precipitate, but appears black coloured solid of $\mathrm{Bi_2\,S_3}$.

$$\mathrm{Bi^{3+}}(aq)\ \mathrm{(Saturated\ solution)} + \mathrm{H_2\,S} \longrightarrow \mathrm{Bi_2\,S_3(\,s)\ (Dark\ brown\ or\ Black}$$

$\mathrm{Bi_2\,S_3}$ **may appear black in colours.**

Either compounds of sulphur are black in colour.

$viii$ In mildly acidic, medium tin in +2 state, i.e., $\mathrm{Sn^{2+}}$ reacts with Hydrogen sulphide $\mathrm{(H_2\,S)}$ to form brown coloured tin II sulphide which further reacts with excess of hydrogen sulphide to form light yellow coloured tin IV sulphide $\mathrm{(SnS_2)}$.

$$\mathrm{Sn}^{2+}(aq) + \mathrm{H}_2\,\mathrm{S} \rightleftharpoons \underset{\text{Brown}}{\mathrm{SnS}(s)}$$

$$\mathrm{SnS(s)} + \mathrm{H}_2\,\mathrm{S} \rightleftharpoons \underset{\text{Yellow}}{\mathrm{SnS}_2(\,\mathrm{s})} + 2\mathrm{H}^+$$

Hence, tin IV sulphide or SnS_2 is yellow in colour.

ix MnS is known to be dirty pink coloured.

Question 020 Numerical

QUESTION

Consider all possible isomeric ketones including stereoisomers of MW = 100. All these isomers are independently reacted with $NaBH_4$
$Note: stereoisomers are also reacted separately$. The total number of ketones that give a racemic product s is/are

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Molecular weight of the ketone is 100. So, molecular formula = $C_6H_{12}O$. The possible number of ketones with molecular formula $C_6H_{12}O$ is six. Out of these, the total number of ketones that give a racemic product s on reaction with $NaBH_4$ is 5. The structure with chiral carbon will not give racemic product on reduction.

Question 021 Numerical

QUESTION

Let

$$n_1 < n_2 < n_3 < n_4 < n_5$$

be positive integers such that

$$n_1 + n_2 + n_3 + n_4 + n_5$$

= 20. Then the number of such destinct arrangements

$$(n_1, n_2, n_3, n_4, n_5)$$

is

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

As, $n_1 \geq 1, n_2 \geq 2, n_3 \geq 3, n_4 \geq 4, n_5 \geq 5$

Let $n_1 - 1 = x_1 \geq 0, n_2 - 2 = x_2 \geq 0, \ldots, n_5 - 5 = x_5 \geq 0$

$\Rightarrow$ New equation will be

$$x_1 + 1 + x_2 + 2 + \ldots + x_5 + 5 = 20$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 20 - 15 = 5$$

Now, $x_1 \leq x_2 \leq x_3 \leq x_4 \leq x_5$

So the cases can be listed giving -

x_1	x_2	x_3	x_4	x_5
0	0	0	0	5
0	0	0	1	4
0	0	0	2	3
0	0	1	1	3
0	0	1	2	2
0	1	1	1	2
1	1	1	1	1

So there are 7 distinct arrangements.

Question 022 MCQ

QUESTION

A circle S passes through the point $0, 1$ and is orthogonal to the circles

$$(x-1)^2 + y^2 = 16 \; and \; x^2 + y^2 = 1$$

. Then

A radius of S is 8

B radius of S is 7

C centre of S is $-7, 1$

D centre of S is $-8, 1$

CORRECT OPTION

C centre of S is $-7, 1$

SOURCE

Mathematics • circle

EXPLANATION

Let, the equation of the required circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

..... 1

Circle I cuts the circle

$$(x-1)^2 + y^2 = 16$$

i.e.,

$$x^2 + y^2 - 2x = 15$$

orthogonally

$$\therefore$$

$$2(-g + 0) = -15 + c$$

or,

$$-2g = -15 + c$$

The circle 1 also cuts the circle

$$x^2 + y^2 = 1$$

orthogonally.

$$\therefore$$

0 =

$$-$$

1 + c or, c = 1

$$\therefore$$

g = 7

Now, the circle 1 passes through the point $0, 1$.

$$\therefore$$

$$2f + 1 + c = 0$$

or,

$$2f + 1 + 1 = 0$$

or, f =

$$-$$

1

$$\therefore$$

the equation of the required circle is

$$x^2 + y^2 + 14x - 2y + 1 = 0$$

whose centre is $\$\$ - \$\$7, 1$ and radius

$$= \sqrt{49 + 1 - 1} = 7$$

units

Therefore, B and C are the correct option.

Note :

The condition of the circle

$$x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$$

cuts orthogonally to the circle

$$x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$$

is

$$2g_1g_2 + 2f_1f_2 = c_1 + c_2$$

Question 023 Numerical

QUESTION

Let

$$n \geq 2$$

be an integer. Take n distinct points on a circle and join each pair of points by a line segment. Colour the line segment joining every pair of adjacent points by blue and the rest by red. If the number of red and blue line segments are equal, then the value of n is

SOURCE

Mathematics • permutations-and-combinations

EXPLANATION

Number of blue lines $= n =$ number of sides of polygon so formed.

Number of red lines $= {}^{n}C_2 - n$.

Thus, by joining n points $notmorethan2onaline$ there are ${}^{n}C_2$ lines formed because for each line two points are required.

Also, red lines come after excluding sides of polygon.

Therefore, $n = {}^{n}C_2 - n$

or ${}^{n}C_2 = 2n$

$\frac{n(n-1)}{2} = 2n$

or $n - 1 = 4 (\because n \neq 0)$

$\therefore n = 5$

Question 024 Numerical

QUESTION

Let a, b, c be positive integers such that

$$\frac{b}{a}$$

is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of

$$\frac{a^2 + a - 14}{a + 1}$$

is

SOURCE

Mathematics • sequences-and-series

EXPLANATION

Let $a = a, b = ar$ and $c = ar^2$, where r is integer since

$$\frac{b}{a}$$

is an integer.

According to the question, we have

$\frac{a+b+c}{3} = b + 2$

$$\$\$ \because \$\$\$(\$A.M.\$) = (b + 2)\$$$

$$\Rightarrow$$

$$\frac{a+ar+ar^2}{3}=ar+2$$

$a+ar+ar^2=3ar+6$

$\Rightarrow ar^2-2r+a=6$

$\Rightarrow \underbrace{r^2-2r+1}_{\text{integer}}=\underbrace{\frac{6}{a}}_{\text{integer}}$

$\Rightarrow (r-1)^2=\frac{6}{a}$

If $a=1,2,3,4,5,6$, then it is not a perfect square and integer.

Therefore, the only possibility is that $a=6$. Thus,

$\frac{a^2+a-14}{a+1}=\frac{36+6-14}{6+1}=\frac{284}{7}=4$

Question 025 Numerical

QUESTION

For a point

$$P$$

in the plane, Let

$$d_1\left(P\right)$$

and

$$d_2\left(P\right)$$

be the distance of the point

$$P$$

from the lines

$$x-y=0$$

and

$$x+y=0$$

respectively. The area of the region

$$R$$

consisting of all points

$$P$$

lying in the first quadrant of the plane and satisfying

$$2 \leq d_1\left(P\right)+d_2\left(P\right) \leq 4$$

, is

SOURCE

Mathematics • straight-lines-and-pair-of-straight-lines

EXPLANATION

Let $P(X, Y)$ is the point in first quadrant.

Now, $2 \leq \left|\frac{x-y}{\sqrt{2}}\right|+\left|\frac{x+y}{\sqrt{2}}\right| \leq 4$

$$2\sqrt{2} \leq |x-y|+|x+y| \leq 4\sqrt{2}$$

Case I : $x \geq y$

$$2\sqrt{2} \leq (x-y)+(x+y) \leq 4\sqrt{2}$$

$$\Rightarrow x \in [\sqrt{2}, 2\sqrt{2}]$$

Case II : $x < y$

$$2\sqrt{2} \leq y-x+(x+y) \leq 4\sqrt{2}$$

$$y \in [\sqrt{2}, 2\sqrt{2}]$$

$\Rightarrow A = (2\sqrt{2})^2 - (\sqrt{2})^2 = 6$ sq units

Question 026 MCQ

QUESTION

Let

$$f : (0, \infty) \to R$$

be given by

$$f(x)$$

=

$$\int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

. Then

$$f(x)$$

A is monotonically increasing on

$$[1, \infty)$$

$f(x)$

B is monotonically decreasing on

$(0, 1)$

C $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$

D $f\left(2^x\right)$ is an odd function of x on R

CORRECT OPTION

$f(x)$

A is monotonically increasing on

$[1, \infty)$

SOURCE

Mathematics • definite-integration

EXPLANATION

Given,

$$f(x)$$

=

$$\int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

Therefore, $\frac{d}{dx} f(x) = \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} \frac{d}{dx}(x) - \frac{e^{-\left(\frac{1}{x}+x\right)}}{1/x} \times \frac{d}{dx}\left(\frac{1}{x}\right)$

$$= \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} + xe^{-\left(x+\frac{1}{x}\right)} \times \left(-\frac{1}{x^2}\right)$$

$$= \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} + \frac{1}{x} e^{-\left(x+\frac{1}{x}\right)}$$

$$= \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x} > 0$$

As, $f'(x) > 0, \forall x \in (0, \infty)$

$\therefore f(x)$ is monotonically increasing on $(0, \infty)$.

$\Rightarrow$ options a is correct and b is wrong.

Now,

$$f(x)+f\left(\frac{1}{x}\right)=\int_{1/x}^{x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt+\int_{x}^{\frac{1}{x}}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$$

$$=\int_{1/x}^{1/x}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt=0$$

Now, let

$$g(x)=f\left(2^{x}\right)=\int_{2^{-x}}^{2^{x}}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$$

$$g(-x)=f\left(2^{-x}\right)$$

$$=\int_{2^{x}}^{2^{-x}}\frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$$

$$=-g(x)$$

$\therefore f\left(2^{x}\right)$ is an odd function.

Question 027 Numerical

QUESTION

The slope of the tangent to the curve

$$\left(y-x^{5}\right)^{2}=x\left(1+x^{2}\right)^{2}$$

at the point

$$(1,3)$$

is

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

Given curve

$$\left(y-x^5\right)^2=x\left(1+x^2\right)^2$$

$$\Rightarrow 2\left(y-x^5\right)\left(\frac{dy}{dx}-5x^4\right)=\left(1+x^2\right)^2+2x\left(1+x^2\right)\cdot 2x$$

Now putting $(1,3)$ in it, we get

$$2(3-1)\left(\frac{dy}{dx}-5\right)=1\{2(2)2\}+(1+1)^2$$

$$\Rightarrow 4\left(\frac{dy}{dx}-5\right)=8+4\Rightarrow \frac{dy}{dx}=8$$

Thus, the slope at $(1,3)$ is 8 .

Question 028 Numerical

QUESTION

The value of

$$\int_0^1 4x^3 \left\{ \frac{d^2}{dx^2}\left(1-x^2\right)^5 \right\} dx$$

is

SOURCE

Mathematics • definite-integration

EXPLANATION

$$\int_0^1 \underbrace{4x^3}_{\text{I}} \underbrace{\frac{d^2}{dx^2}\left(1-x^2\right)^5}_{\text{II}} dx$$

Integrating by parts:

$$I = 4x^3\left[\frac{d}{dx}\left(1-x^2\right)^5\right]_0^1 - \int_0^1 12x^2 \frac{d}{dx}\left(1-x^2\right)^5 dx$$

$$= 4x^3\left[5\left(1-x^2\right)^4(-2x)\right]_0^1 - 12\left[\left[x^2\left(1-x^2\right)^5\right]_0^1 - \int_0^1 2x\left(1-x^2\right)^5 dx\right.$$

$$= 0 - 0 + 12\int_0^1 2x\left(1-x^2\right)^5 dx$$

Now, putting $1-x^2 = t$, we get $-2xdr = dt$. Therefore,

$$I = -12\int_1^0 t^5 dt$$

When $x = 0, t = 1.$

When $x = 1, t = 0.$

$$I = 12 \times \int_0^1 t^5 dt = 12 \times \left[\frac{t^6}{6}\right]_0^1 = 12 \times \frac{1}{6} = 2$$

Question 029 MCQ

QUESTION

From a point

$$P\,(\lambda, \lambda, \lambda),$$

perpendicular

$$PQ$$

and

$$PR$$

are drawn respectively on the lines

$$y = x, z = 1$$

and

$$y = -x, z = -1.$$

If

$$P$$

is such that

$$\angle QPR$$

is a right angle, then the possible value s of

λ

is/are

A $\sqrt{2}$

B 1

C -1

D $-\sqrt{2}$

CORRECT OPTION

C -1

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Line L_1 given by $y = x; z = 1$ can be expressed

$$L_1 : \frac{X}{1} = \frac{Y}{1} = \frac{z-1}{0}$$

$$\frac{x}{1} = \frac{y}{1} = \frac{z-1}{0} = \alpha(say)$$

$$\Rightarrow x = \alpha, y = \alpha, z = 1$$

Let the coordinates of Q on L_1 be $(\alpha, \alpha, 1)$

Line L_2 given by $y = -x, z = -1$ can be expressed as

$L_2 : \frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0}$

$\frac{x}{1} = \frac{y}{-1} = \frac{z+1}{0} = \beta \ say$

$$\Rightarrow \quad x = \beta, y = -\beta, z = -1$$

Let the coordinates of R on L_2 be $(\beta, -\beta, -1)$.

Direction ratios of PQ are $\lambda - \alpha, \lambda - \alpha, \lambda - 1$.

Now, $PQ \perp L_1$

$\therefore 1(\lambda - \alpha) + 1 \cdot (\lambda - \alpha) + 0 \cdot (\lambda - 1) = 0$

$\Rightarrow \lambda = \alpha$

$\therefore Q(\lambda, \lambda, 1)$

Direction ratio of PR are

$$\lambda - \beta, \lambda + \beta, \lambda + 1.$$

Now, $PR \perp L_2$

$\therefore 1(\lambda-\beta)+(-1)(\lambda+\beta)+0(\lambda+1)=0$

$\lambda-\beta-\lambda-\beta=0$

$\Rightarrow \beta=0$

$\therefore R(0,0,-1)$

Now, as $\angle QPR=90^\circ$

$$(\text{as } a_1a_2+b_1b_2+c_1c_2=0,$$

if two lines with DR's $a_1,b_1,c_1;a_2,b_2,c_2$ are perpendicular)

$$\therefore \quad (\lambda-\lambda)(\lambda-0)+(\lambda-\lambda)(\lambda-0)+(\lambda-1)(\lambda+1)=0$$

$$\Rightarrow (\lambda-1)(\lambda+1)=0$$

$$\Rightarrow \lambda=1 \text{ or } \lambda=-1$$

$\lambda=1$, rejected as P and Q are different points.

$$\Rightarrow \lambda=-1$$

Question 030 MCQ

QUESTION

Let

$$\vec{x},\vec{y}$$

and

$$\vec{z}$$

be three vectors each of magnitude

$$\sqrt{2}$$

and the angle between each pair of them is

$$\frac{\pi}{3}$$

. If

$$\vec{a}$$

is a non-zero vector perpendicular to

$$\vec{x}$$

and

$$\vec{y} \times \vec{z}$$

and

$$\vec{b}$$

is a non-zero vector perpendicular to

$$\vec{y}$$

and

$$\vec{z} \times \vec{x},$$

then

A $$\vec{b} = \left(\vec{b} . \vec{z}\right)\left(\vec{z} - \vec{x}\right)$$

B $$\vec{a} = \left(\vec{a} . \vec{y}\right)\left(\vec{y} - \vec{z}\right)$$

C $$\vec{a}.\vec{b} = -\left(\vec{a}.\vec{y}\right)\left(\vec{b}.\vec{z}\right)$$

D $$\vec{a} = \left(\vec{a}.\vec{y}\right)\left(\vec{z}-\vec{y}\right)$$

CORRECT OPTION

A $$\vec{b} = \left(\vec{b}.\vec{z}\right)\left(\vec{z}-\vec{x}\right)$$

SOURCE

Mathematics • vector-algebra

EXPLANATION

$$\angle \mathrm{AOB} = \angle \mathrm{BOC} = \angle \mathrm{CCOA} = \frac{\pi}{3}$$

According to question, we have

$$\vec{a} = \lambda\{(\vec{x}\cdot\vec{z})\vec{y} - (\vec{x}\cdot\vec{y})\vec{z}\}$$

$$= \lambda\left\{\left(2\cos\frac{\pi}{3}\right)\vec{y} - \left(2\cos\frac{\pi}{3}\right)\vec{z}\right\} = \lambda(\vec{y}-\vec{z})$$

Thus,

$$\vec{\alpha} \times (\vec{\beta} \times \vec{\gamma}) = (\vec{\alpha} \cdot \vec{\gamma})\vec{\beta} - (\vec{\alpha} \cdot \vec{\beta})\vec{\gamma}$$

$$\vec{b} = \mu\{(\vec{y} \cdot \vec{x})\vec{z} - (\vec{y} \cdot \vec{z})\vec{x}\}$$

$$= \mu\{\vec{z} - \vec{x}\}$$

Now, $\vec{a} \cdot \vec{y} = \lambda\{2 - 1\}$; therefore, $\lambda = \vec{a} \cdot \vec{y}$.

Hence, $\vec{a} = \vec{a} \cdot \vec{y}(\vec{y} - \vec{z})$ 1

Similarly,

$$\vec{b} = \vec{b} \cdot \vec{z}(\vec{z} - \vec{x})$$

.....2

Now,

$$\vec{a} \cdot \vec{b} = (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})\{\vec{y} \cdot \vec{z} - \vec{y} \cdot \vec{x} - \vec{z} \cdot \vec{z} + \vec{z} \cdot \vec{x}\}$$

$$= (\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z})\{1 - 1 - 2 + 1\}$$

$$= -(\vec{a} \cdot \vec{y})(\vec{b} \cdot \vec{z}).....(3)$$

Hence, from Eqs. 1, 2 and 3, we can conclude that the correct options are $(\mathrm{A}), (\mathrm{B})$ and (C).

Question 031 Numerical

QUESTION

Let

$$\overrightarrow{a}, \overrightarrow{b}$$

and

$$\overrightarrow{c}$$

be three non-coplanar unit vectors such that the angle between every pair of them is

$$\frac{\pi}{3}.$$

If

$$\overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{b} \times \overrightarrow{c} = p\overrightarrow{a} + q\overrightarrow{b} + r\overrightarrow{c},$$

where

$$p, q$$

and

$$r$$

are scalars, then the value of

$$\frac{p^2 + 2q^2 + r^2}{q^2}$$

is

SOURCE

Mathematics • vector-algebra

EXPLANATION

Given $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ 1

Taking dot product with $\vec{a}$:

Hence,

$$0 + \vec{a} \cdot \vec{b} \times \vec{c} = p(1 \cdot 1 \cdot \cos 0) + q\left(1 \cdot 1 \cdot \cos \frac{\pi}{3}\right) + r\left(1 \cdot 1 \cdot \cos \frac{\pi}{3}\right)$$

$$\Rightarrow \vec{a} \cdot \vec{b} \times \vec{c} = p + \tfrac{q}{2} + \tfrac{r}{2} \quad \ldots\ldots\ldots 2$$

Taking the dot product of 1 with $\vec{b}$:

$$0 + 0 = \frac{p}{2} + q + \frac{r}{2}$$

$\ldots\ldots\ldots\ldots 3$

Taking the dot product of 1 with $\vec{c}$:

$$\vec{c} \cdot \vec{a} \times \vec{b} + 0 = \frac{p}{2} + \frac{q}{2} + r$$

$\ldots\ldots\ldots\ldots 4$

From 2 and 4, we get

$$p + \frac{q}{2} + \frac{r}{2} = \frac{p}{2} + \frac{q}{2} + r$$

$$\frac{p}{2} = \frac{r}{2} \Rightarrow p = r$$

Now, from Eq. 3, we get $0 = \tfrac{r}{2} + q + \tfrac{r}{2} \Rightarrow q = -r$.

Now, $\frac{p^2+2q^2+r^2}{q^2} = \frac{r^2+2(-r)^2+r^2}{(-r)^2} = \frac{4r^2}{r^2} = 4$.

Question 032 MCQ

QUESTION

Let

$$f : (a, b) \to [1, \infty)$$

be a continuous function and g : R

$$\to$$

R be defined as

$$g(x) = \begin{cases} 0 & , \quad x < a \\ \int_a^x f(t)dt & , \quad a \leq x \leq b \\ \int_a^b f(t)dt & , \quad x > b \end{cases}$$

Then,

A g x is continuous but not differentiable at a

B g x is differentiable on R

C g x is continuous but not differentiable at b

D g x is continuous and differentiable at either a or b but not both

CORRECT OPTION

A g x is continuous but not differentiable at a

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Given that

$$f:(a,b)\to[1,\infty)$$

and

$$g(x)=\begin{cases}0 & , & x<a\\ \int_a^x f(t)dt & , & a\le x\le b\\ \int_a^b f(t)dt & , & x>b\end{cases}$$

Now,

$$g(a^-)=0=g(a^+)=g(a)$$

$$as\$\$g(a^+)=\lim_{x\to a^+}\int_a^x f(t)dt=0\$\$and\$\$g(a)=\int_a^a f(t)dt=0\$\$$$

$$g(b^-)=g(b^+)=g(b)=\int_a^b f(t)dt$$

$$\Rightarrow$$

g is continuous,

$$\forall$$

x

$$\in$$

R.

Now,

$$g'(x) = \begin{cases} 0 & , \quad x < a \\ f(x) & , \quad a < x < b \\ 0 & , \quad x > b \end{cases}$$

g'(a

$$-$$

) = 0 but g'(a$^+$) = fa

$$\geq$$

1

$$\$\$ \because \$\$Rangeoff(x)is[1, \$\$\infty\$\$), \$\$\forall\$\$x\$\$ \in \$\$[a, b$$

]

$$\Rightarrow$$

g is non-differentiable at x = a

and g'(b$^+$) = 0

but g'(b

$$-$$

) = fb

$$\geq$$

1

$$\Rightarrow$$

g is not differentiable at x = b.

Question 033 MCQ

QUESTION

For every pair of continuous function f, g :

$$0, 1$$

$$\rightarrow$$

R such that max {f x : x

$$\in$$

$$0, 1$$

} = max {g x : x

$$\in$$

$$0, 1$$

}. The correct statement s is are

A

$$f(c)$$

^{2} + 3f c =

$$g(c)$$

^{2} + 3g c for some c

$$\in$$

$$0, 1$$

$$f(c)$$

2 + fc =

B

$g(c)$

2 + 3gc for some c

$\in$

$0, 1$

$f(c)$

2 + 3fc =

C

$g(c)$

2 + gc for some c

$\in$

$0, 1$

$f(c)$

2 =

D

$g(c)$

2 for some c

$\in$

$0, 1$

CORRECT OPTION

$f(c)$

2 + 3fc =

A

$g(c)$

^{2} + 3g c for some c

$\in$

$0, 1$

SOURCE

Mathematics • functions

EXPLANATION

Suppose f x is maximum at c_1 and g x is maximum at c_2. When f x is maximum g x may or may not be maximum.

Therefore, in the function h x = f x

$-$

g x, we get

$$h(c_1) = f(c_1) - g(c_1) \geq 0$$

and

$$h(c_2) = f(c_2) - g(c_2) \geq 0$$

.

Therefore, h x = 0 for some c

$\in$

$0, 1$

.

Therefore,

$$h(c) = 0 \Rightarrow f(c) - g(c) = 0$$

.

Therefore,

$$f(c) = g(c)$$

.

Option a

$$\Rightarrow f^2(c) - g^2(c) + 3[f(c) - g(c)] = 0$$

which is true from Eq. i.

Option d

$$\Rightarrow f^2(c) - g^2(c) = 0$$

which is true from Eq. i

Now, if we take

fx = 1 and gx = 1,

$$\forall$$

x

$$\in$$

$$0, 1$$

Option b and c does not hold. Hence, option a and d are correct.

Question 034 MCQ

QUESTION

Let M be a 2

$\times$

2 symmetric matrix with integer entries. Then, M is invertible, if

A the first column of M is the transpose of the second row of M

B the second row of M is the transpose of the first column of M

C M is a diagonal matrix with non-zero entries in the main diagonal

D the product of entries in the main diagonal of M is not the square of an integer

CORRECT OPTION

C M is a diagonal matrix with non-zero entries in the main diagonal

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Note : A square matrix M is invertible if det M or | M |

$\neq$

0.

Let

$$M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

a Given that

$$\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$$

$$\Rightarrow a = b = c = \alpha$$

let

$$\Rightarrow M = \begin{bmatrix} \alpha & \alpha \\ \alpha & \alpha \end{bmatrix}$$

$$\Rightarrow$$

| M | = 0

$$\Rightarrow$$

M is non-invertible.

b Given that

$$bc$$

=

$$ab$$

$$\Rightarrow a = b = c = \alpha$$

let

Again | M | = 0

$$\Rightarrow$$

M is non-invertible.

c As given

$$M = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$$

$$\Rightarrow |M| = ac \neq 0$$

$\$\$ \because \$\$ a and c are non - zero$

$$\Rightarrow$$

M is invertible.

d

$$M = \begin{bmatrix} a & b \\ b & c \end{bmatrix} \Rightarrow |M| = ac - b^2 \neq 0$$

$$\because$$

ac is not equal to square of an integer.

$$\therefore$$

M is invertible.

Question 035 MCQ

QUESTION

Let M and N be two 3

$\times$

3 matrices such that MN = NM. Further, if M

$\neq$

N^2 and $M^2 = N^4$, then

A determinant of $(M^2 + MN^2)$ is 0

B there is a 3

$\times$

3 non-zero matrix U such that $(M^2 + MN^2)$ U is zero matrix

C determinant of $(M^2 + MN^2)$

$\geq$

1

D for a 3

$\times$

3 matrix U, if $(M^2 + MN^2)$ U equals the zero matrix, then U is the zero matrix

CORRECT OPTION

A determinant of $(M^2 + MN^2)$ is 0

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Given MN = NM. Therefore, a^2

$-$

b^2 = $a + b\ \ a\$\$ - \$\$b$ of algebra of numbers is applicable.

Now, $M^2 = N^4$

$\Rightarrow$

M^2

$-$

$N^4 = 0\ Nullmatrix$

$\Rightarrow$

$(M + N^2)\ (M$

$-$

$N^2) = 0$

Since M

$\neq$

$N^2\ given$, the possibilities are

$(M + N^2) = 0$ and M

$-$

N^2

$\neq$

0 1

or (M + N^2)

$\neq$

0 and M + N^2

$\neq$

0 2

Now, we know if A and B are non-null square matrix nd AB = 0, then A and B both are singular, that is, | A | = 0 and | B | = 0 and AB = 0.

Note : For example, let A be non-singular. Therefore, B = In 2 = A

$-$

^{1} AB = 0 $since AB = 0 is assumed.$

Hence, B is singular, which is a contradiction and A has to be singular. Similarly, B also has to be singular.

Therefore, from Eqs. 1 and 2, we conclude the only possibility is | M + N^2 | = 0.

Now checking options :

A | M^2 + MN^2 | = | M || M + N^2 | = 0

Hence, option A is correct.

B (M^2 + MN^2) U = 0

Since, M^2 + MN^2 is singular. Therefore, U has infinitely many possible values $non-trivial solutions$. Hence, option B is true.

C False since | M^2 + MN^2 | = 0.

D Since | $M^2 + MN^2$ | = 0, U is not a necessarily a zero matrix.

Question 036 MCQ

QUESTION

Let

$$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$$

be given by

$$f(x) = [\log(\sec x + \tan x)]^3$$

. Then,

A fx is an odd function

B fx is a one-one function

C fx is an onto function

D fx is an even function

CORRECT OPTION

A fx is an odd function

SOURCE

Mathematics • functions

EXPLANATION

$$f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$$

$$f(x)=[\log(\sec x+\tan x)]^3$$

$$f(-x)=[\log(\sec x-\tan x)]^3$$

$$=\left[\log\left(\frac{(\sec x-\tan x)(\sec x+\tan x)}{\sec x+\tan x}\right)\right]^3$$

$$=\left[\log\left(\frac{1}{\sec x+\tan x}\right)\right]^3=[-\log(\sec x+\tan x)]^3$$

$$=-[\log(\sec x+\tan x)]^3=-f(x)$$

$$\therefore$$

f is an odd function. a is correct and d is not correct.

Also,

$$f'(x) = 3[\log(\sec x + \tan x)]^2 \,.\, \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

$$= 3 \sec x [\log(\sec x + \tan x)]^2 > 0 \, \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\therefore$$

f is increasing on

$$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

We know that strictly increasing function is one one.

$$\therefore$$

f is one one

$$\therefore$$

b is correct.

$$(\sec x + \tan x) = \tan\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$$

as

$$x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

, then

$$0 < \tan\left(\frac{\pi}{4} + \frac{\pi}{2}\right) < \infty$$

$$0 < \sec x + \tan x < \infty$$

$$\Rightarrow -\infty < \ln(\sec x + \tan x) < \infty$$

$$-\infty < [\ln(\sec x + \tan x)]^3 < \infty$$

$$\Rightarrow -\infty < f(x) < \infty$$

Range of fx is R and thus fx is an onto function.

$$\therefore$$

c is correct.

Question 037 MCQ

QUESTION

Let a

$$\in$$

R and f : R

$$\rightarrow$$

R be given by fx = x^5

—

5x + a. Then,

A fx has three real roots , if a > 4

B fx has only one real root, if a > 4

C fx has three real roots, if a <

—

4

D fx has three real roots, if

—

4 < a < 4

CORRECT OPTION

B fx has only one real root, if a > 4

SOURCE

Mathematics • definite-integration

EXPLANATION

In the equation

$$f(x) = x^5 - 5x + a$$

, there are different polynomials depending on the parameter a. Now, for the roots of each of these, in general, fx = 0.

That is,

$$a = 5x - x^5 = x(5 - x^5)$$

Hence, the parameter a is a function of x. That is,

$$a(x) = x(5 - x^5)$$

Now,

$$a'(x) = 5 - 5x^4$$

Therefore, extrema occurs at a'x = 0

That is, when x^4 = 1 or x = 1 and x =

$$-$$

1 $only real roots considered$

Here,

$$a''(x) = -20x^3$$

$$a''(1) < 0$$

$maximum$

$$a''(-1) > 0$$

$minimum$

Hence, the maximum value is

a 1 = 4

The minimum value is

a $$ − $$1 =

−

4

Hence, when

−

4 < a < 4, there are three points that is, x values where f x = 0, that is, three roots of f x for any value of a lying in $$ − $$4, 4}. 1

When | a | > 4, there is only one x for which f x = 0 2

Hence, from statements 1 and 2, we can conclude that options B and D are correct.

Question 038 Numerical

QUESTION

Let f :

$$0, 4\$\$\pi\$\$$$

$$\rightarrow$$

$$0, \$\$\pi\$\$$$

be defined by f x = cos

−

[1] $cosx$. The number of points x

$$\in$$

$$0, 4\$\$\pi\$\$$$

satisfying the equation

$$f(x) = \frac{10 - x}{10}$$

is

SOURCE

Mathematics • inverse-trigonometric-functions

EXPLANATION

Concept :

The number of solutions of equations involving trigonometric functions and algebraic functions are found using graphs of the curves.

We know,

$$\cos^{-1}(\cos x) = \begin{cases} x, \, if \, x \in [0, \pi] \\ 2\pi - x, \, if \, x \in [\pi, 2\pi] \\ -2\pi + x, \, if \, x \in [2\pi, 3\pi] \\ 4\pi - x, \, if \, x \in [3\pi, 4\pi] \end{cases}$$

$$y = \frac{10 - x}{10} = 1 - \frac{x}{10}$$

From above figure, it is clear that

$$y = \frac{10 - x}{10}$$

and

$$y = \cos^{-1}(\cos x)$$

intersect at three distinct points, so number of solutions is 3.

Question 039 Numerical

QUESTION

The largest value of the non-negative integer a for which

$$\lim_{x \to 1} \left\{ \frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1} \right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$$

is

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Given,

$$\lim_{x \to 1} \left\{ \frac{\sin(x-1) + a(1-x)}{(x-1) + \sin(x-1)} \right\}^{\frac{(1-\sqrt{x})(1-\sqrt{x})}{1-\sqrt{x}}} = \frac{1}{4}$$

$$\Rightarrow \lim_{x \to 1} \left\{ \frac{\frac{\sin(x-1)}{(x-1)} - a}{1 + \frac{\sin(x-1)}{(x-1)}} \right\}^{1+\sqrt{x}} = \frac{1}{4}$$

$$\Rightarrow \left(\frac{1-a}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow (a-1)^2 = 1$$

$\Rightarrow$

a = 2 or 0

But for a = 2, base of above limit approaches

$-$

1/2 and exponent approaches to 2 and since base cannot be negative, hence limit does not exist.

Question 040 Numerical

QUESTION

Let f : R

$\rightarrow$

R and g : R

$\rightarrow$

R be respectively given by f x = | x | + 1 and g x = x^2 + 1. Define h : R

$\rightarrow$

R by

$$h(x) = \begin{cases} \max\{f(x), g(x)\}, & if\, x \leq 0. \\ \min\{f(x), g(x)\}, & if\, x > 0. \end{cases}$$

The number of points at which h x is not differentiable is

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

Concept :

The points at which the curve taken a sharp turn, are the points of non-differentiability.

Curve of f x and g x are

h x is not differentiable at x =

$$\pm$$

1 and 0.

As h x take sharp turns at x =

$$\pm$$

1 and 0.

Hence, number of points of non-differentiability of h x is 3.

Question 041 Numerical

QUESTION

Airplanes A and B are flying with constant velocity in the same vertical plane at angles

$$30^\circ$$

and

$$60^\circ$$

with respect to the horizontal respectively as shown in the figure. The speed of A is

$$100\sqrt{3}$$

m/s. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at t = t_0, A just escapes being hit by B, t_0 in seconds is

SOURCE

Physics • motion

EXPLANATION

Since A observes B as moving normal to it

$$v_B \cos 30^\circ = v_A$$

$$v_B \frac{\sqrt{3}}{2} = 100\sqrt{3}$$

$$v_B = 200$$

m/s

Therefore,

$$t_0 = \frac{500}{200 \sin 30^\circ} = 5$$

s

Question 042 MCQ

QUESTION

A parallel plate capacitor has a dielectric slab of dielectric constant

$$K$$

between its plates that covers

$$1/3$$

of the area of its plates, as shown in the figure. The total capacitance of the capacitor is

$$C$$

while that of the portion with dielectric in between is

$$C_1.$$

When the capacitor is charged, the plate area covered by the dielectric gets charge

$$Q_1$$

and the rest of the area gets charge

$$Q_2.$$

The electric field in the dielectric is

$$E_1$$

and that in the other portion is

$$E_2.$$

Choose the correct option/ options, ignoring edge effects.

A $\frac{E_1}{E_2} = 1$

B $\frac{E_1}{E_2} = \frac{1}{K}$

C $\frac{Q_1}{Q_2} = \frac{3}{K}$

D $\frac{C}{C_1} = \frac{2+K}{K}$

CORRECT OPTION

A $\frac{E_1}{E_2} = 1$

SOURCE

Physics • capacitor

EXPLANATION

Let A be area of each plate and d is the distance between the plates.

The given capacitor is equivalent to two capacitors in parallel with capacitances

$$C_1 = \frac{K\varepsilon_0(A/3)}{d} = \frac{K\varepsilon_0 A}{3d}$$

$$C_2 = \frac{\varepsilon_0(2A/3)}{d} = \frac{2\varepsilon_0 A}{3d}$$

$$\therefore$$

$$C = C_1 + C_2$$

$$= \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d} = \frac{\varepsilon_0 A}{3d}(K+2)$$

$$\therefore$$

$$\frac{C}{C_1} = \frac{K+2}{K}$$

Hence, option d is correct.

Let V be potential difference between the plates. Then

$$E_1 = \frac{V}{d}$$

and

$$E_2 = \frac{V}{d}$$

$$\therefore$$

$$\frac{E_1}{E_2} = 1$$

Hence, option a is correct and option b is incorrect.

$$Q_1 = C_1 V = \frac{K\varepsilon_0 A}{3d} V$$

and

$$Q_2 = C_2 V = \frac{2\varepsilon_0 A}{3d} V$$

$$\therefore$$

$$\frac{Q_1}{Q_2} = \frac{K}{2}$$

Hence, option c is incorrect.

Question 043 MCQ

QUESTION

Let

$$E_1(r), E_2(r)$$

and

$$E_3(r)$$

be the respective electric field at a distance

$$r$$

from a point charge

$$Q,$$

an infinitely long wire with constant linear charge density

$$\lambda,$$

and an infinite plane with uniform surface charge density

$$\sigma.$$

If

$$E_1(r_0) = E_2(r_0) = E_3(r_0)$$

at a given distance

$$r_0.$$

then

A $$Q = 4\sigma\pi r_0^2$$

B $$r_0 = \frac{\lambda}{2\pi\sigma}$$

C $E_1\left(r_0/2\right) = 2E_2\left(r_0/2\right)$

D $E_2\left(r_0/2\right) = 4E_3\left(r_0/2\right)$

CORRECT OPTION

C $E_1\left(r_0/2\right) = 2E_2\left(r_0/2\right)$

SOURCE

Physics • electrostatics

EXPLANATION

$$E_1(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$E_2(r) = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E_3(r) = \frac{\sigma}{2\varepsilon_0}$$

At

$$r = r_0$$

,

$$E_1(r_0) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2}$$

$$E_2(r_0) = \frac{\lambda}{2\pi\varepsilon_0 r_0}$$

$$E_3(r_0) = \frac{\sigma}{2\varepsilon_0}$$

As

$$E_1(r_0) = E_2(r_0) = E_3(r_0)$$

$Given$

$$\therefore$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = \frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

Then

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = \frac{\sigma}{2\varepsilon_0}$$

or

$$Q = \sigma\pi r_0^2$$

Hence, option a is incorrect.

Now,

$$\frac{\lambda}{2\pi\varepsilon_0 r_0} = \frac{\sigma}{2\varepsilon_0}$$

or

$$r_0 = \frac{\lambda}{\pi\sigma}$$

Hence, option b is incorrect.

At

$$r = r_0/2$$

,

$$E_1(r_0/2) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(r_0/2)^2} = \frac{4}{4\pi\varepsilon_0}\frac{Q}{r_0^2} = 4E_1(r_0)$$

or,

$$E_1(r_0) = \frac{1}{4}E_1(r_0/2)$$

$$E_2(r_0/2) = \frac{\lambda}{2\pi\varepsilon_0(r_0/2)} = \frac{2\lambda}{2\pi\varepsilon_0 r_0} = 2E_2(r_0)$$

or,

$$E_2(r_0) = \frac{1}{2}E_2(r_0/2)$$

$$\because$$

$$E_1(r_0) = E_2(r_0)$$

$$\therefore$$

$$\frac{1}{4}E_1(r_0/2) = \frac{1}{2}E_2(r_0/2)$$

or,

$$E_1(r_0/2) = 2E_2(r_0/2)$$

Hence, option c is correct.

$$E_3(r_0/2) = \frac{\sigma}{2\varepsilon_0} = E_3(r_0)$$

$$\because$$

$$E_2(r_0) = E_3(r_0)$$

$$\therefore$$

$$\frac{1}{2}E_2(\frac{r_0}{2}) = E_3(\frac{r_0}{2})$$

or,

$$E_2(r_0/2) = 2E_3(r_0/2)$$

Hence, option d is incorrect.

Question 044 Numerical

QUESTION

Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4 m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ $see the figure given$. Assuming no frictional losses, the kinetic energy of the block when it reaches Q is $n \times 10$ Joules. The value of n is (take acceleration due to gravity = 10 ms^{-2})

SOURCE

Physics • work-power-and-energy

EXPLANATION

Work done by the gravitational force is

$$W_g = mgh \cos 180^\circ$$

=

−

mgh =

−

1

×

10

×

4 =

−

40 J

Work done by the applied force F

$$W_F = Fd \cos 0^\circ = Fd = 18 \times 5 = 90$$

J

According to work-energy theorem

$$\Delta$$

$K = W_g + W_F$

$$\Delta$$

$K =$

$$-$$

40 J + 90 J = 50 J = $5\$\$ \times \$\$10$ J

$$\therefore$$

n = 5

Question 045 Numerical

QUESTION

A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction $see figure$. The length of a chamber inside the rocket is 4 m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in - x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is

SOURCE

Physics • motion

EXPLANATION

Consider the motion of two balls with respect to rocket.

Distance travelled by ball A from left end of the chamber is

$$= \frac{u^2}{2a} = \frac{(0.3)^2}{2 \times 2} = \frac{0.09}{4} \approx 0.02$$

m

So, collision of two balls will take place very near to left end of the chamber.

For ball B

$$S = ut\frac{1}{2}at^2$$

$$-4 = -0.2 \times t - \frac{1}{2} \times 2 \times t^2$$

$$t^2 + 0.2t - 4 = 0$$

$$t = \frac{-0.2 \pm \sqrt{(0.2)^2 - 4(1)(-4)}}{2} = \frac{-0.2 \pm \sqrt{0.04 + 16}}{2}$$

t = 1.9 s,

$$-$$

2.1 s

Since t can't be negative

$$\therefore$$

t = 1.9 s

Nearest integer is 2 s.

Also, from

$$S = ut + \frac{1}{2}at^2$$

$$S_A = 0.3t + \frac{1}{2}(-2)t^2 = 0.3t - t^2$$

$$S_B = 0.2t + \frac{1}{2}(2)t^2 = 0.2t + t^2$$

$$\because$$

$$S_A + S_B = 4$$

$$\Rightarrow$$

0.5 t = 4 or t = 8 s

Question 046 Numerical

QUESTION

During Searle's experiment, zero of the Vernier scale lies between 3.20

$\times$

10^{-2} m and 3.25

$\times$

10^{-2} m of the main scale. The 20^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20

$\times$

10^{-2} m and 3.25

$\times$

10^{-2} m of the main scale but now the 45^{th} division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is 8

$\times$

10^{-7} m^2. The least count of the Vernier scale is 1.0

$\times$

10^{-5} m. The maximum percentage error in the Young's modulus of the wire is

SOURCE

Physics • units-and-measurements

EXPLANATION

The difference between the two measurements by Vernier scale gives elongation of the wire caused by the additional load of 2 kg. In the first measurement, main scale reading is MSR = 3.20

$\times$

10

$-$

^{2} m and Vernier scale reading is VSR = 20. The least count of Vernier scale is LC = 1

$\times$

10

$-$

^{5} m. Thus, the first measurement by Vernier scale is

L_1 = MSR + VSR

$\times$

LC

= 3.20

$\times$

10

$-$

^{2} + 20(1

$\times$

10

$-$

^{5})

= 3.220

$\times$

10

−

2 m.

In the second measurement, MSR = 3.20

×

10

−

2 m and VSR = 45. Thus, the second measurement by Vernier scale is

L_2 = 3.20

×

10

−

2 + 45(1

×

10

−

5)

= 3.245

×

10

−

2 m.

The elongation of the wire due to force F = 2g is

l = L_2

−

L_1 = 0.025

$\times$

10

$-$

^{2} m.

The maximum error in the measurement of l is

Δ

l = LC = 1

$\times$

10

$-$

^{5} m. Young's modulus is given by Y =

$$\frac{FL}{lA}$$

. The maximum percentage error in the measurement of Y is

$$\frac{\Delta Y}{Y} \times 100 = \frac{\Delta l}{l} \times 100 = \frac{1 \times 10^{-5}}{0.025 \times 10^{-2}} \times 100 = 4\%$$

.

Question 047 Numerical

QUESTION

To find the distance d over which a signal can be seen clearly in foggy conditions, a railways engineer uses dimensional analysis and assumes that the distance depends on the mass density

ρ

of the fog, intensity $power/area$ S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is

SOURCE

Physics • units-and-measurements

EXPLANATION

Given

$$d \propto \rho^a S^b b^c$$

$$M^0LT^0 \propto (ML^{-3})^a (MT^{-3})^b (T^{-1})^c$$

$$M^0LT^0 \propto M^{(a+b)} L^{-3a} T^{-3b-c}$$

Equating the coefficients, we get

$$a + b = 0 - 3a = 1 - 3b - c = 0$$

$$b = -a$$

$$a = -\frac{1}{3} - c = 3b$$

$$b = \frac{1}{3} c = -3b \Rightarrow c = 1$$

Therefore,

$$b = \frac{1}{n} = \frac{1}{3} \Rightarrow n = 3$$

.

Question 048 MCQ

QUESTION

At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current I t = I_0 cos $$ω$$$t$, with I_0 = 1 A and

$$\omega$$

= 500 rad s^{-1} starts flowing in it with the initial direction shown in the figure. At

$$t = \frac{7\pi}{6\omega}$$

, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20

$$\mu$$

F, R = 10

$$\Omega$$

and the battery is ideal with emf of 50 V, identify the correct statement s.

A

Magnitude of the maximum charge on the capacitor before

$$t = \frac{7\pi}{6\omega}$$

is 1

$$\times$$

10

$$-$$

^{3} C

B

The current in the left part of the circuit just before

$$t = \frac{7\pi}{6\omega}$$

is clockwise

C Immediately after A is connected to D, the current in R is 10 A

D Q = 2 × 10 − 3 C

CORRECT OPTION

C Immediately after A is connected to D, the current in R is 10 A

SOURCE

Physics • alternating-current

EXPLANATION

The current flowing through the capacitor and its charge are given by

$$i = i_0 \cos \omega t$$

, ... 1

$$Q = \int i dt = \int i_0 \cos \omega t \, dt = \frac{i_0}{\omega} \sin \omega t + k$$

$$= \frac{i_0}{\omega} \sin \omega t$$

, ... 2

where we have used the initial condition, Q = 0 at t = 0, to get the integration constant k = 0. The charge attains the maximum value Q_{max} at t =

$$\pi$$

/ 2$$ω$$,

$Q_{max} = i_0/\omega = 1/500 = 2 \times 10^{-3}$ C.

The key is switched from B to D at the time t = 7$\pi/6\omega$. Substitute t = 7$\pi/6\omega$ in equations 1 and 2 to get current and charge

$i = i_0 \cos 7\$\$\pi\$\$/6 = \cos \$\$\pi\$\$ + \$\$\pi\$\$/6 = -\cos \$\$\pi\$\$/6 = -\sqrt{3}/2$

A,

$Q_i = (i_0/\omega$

) sin 7$$π$$/6 = 2

×

10

−

3 sin $$π$$ + $$π$$/6

=

−

1

×

10

−

3 C.

Note that negative sign in i indicates the counterclockwise direction of current and negative sign in Q_i indicates the negative charge on the upper plate.

The potential across the capacitor is

$$V_C = \frac{Q_i}{C} = -\frac{1 \times 10^{-3}}{20 \times 10^{-6}}$$

=

−

50 V $upperplateatlowerpotential$. Immediately after A is connected to D, the potential across the resistor is V_R = 100 V and hence the current through it is i = V_R/R = 100/10 = 10 A $counterclockwise$. The potential across the capacitor when it is fully charged is equal to the battery emf. Thus, the charge on the capacitor in fully charged condition is

Q_f = V/C = 50/(20

×

10

−

6) = 1

×

10

−

3 C.

The sign of Q_f indicates the positive charge on the upper plate. Hence, charge that flows through the battery is

$Q = Q_f$

−

$Q_i = 1$

×

10

−

3

−

(

−

1

×

10

−

3) = 2

×

10

–

3 C.

Question 049 MCQ

QUESTION

A light source, width emits two wavelengths

λ

1 = 400 nm and

λ

2 = 600 nm, is used in a Young's double-slit experiment. If recorded fringe widths for

λ

1 and

λ

2 are

β

1 and

β

2 and the number of fringes for them within a distance y on one side of the central maximum are m_1 and m_2, respectively, then

β

A $_2 > \beta_1$

B $m_1 > m_2$

C from the central maximum, 3rd maximum of λ_2 overlaps with 5th minimum of λ_1

D the angular separation of fringes of λ_1 is greater than λ_2

CORRECT OPTION

A $\beta_2 > \beta_1$

SOURCE

Physics • wave-optics

EXPLANATION

Fringe width

$$\beta = \frac{\lambda D}{d}$$

;

$$\lambda$$

1 = 400 nm;

$$\lambda$$

2 = 400 nm

Since

$$\lambda$$

2 >

$$\lambda$$

1,

$$\beta$$

2 >

$$\beta$$

1

Number of fringes within a distance y is given by

$$m_1 = \frac{y}{\beta_1}$$

$$m_2 = \frac{y}{\beta_2}$$

Since,

$$\beta$$

$_2$ >

$$\beta$$

$_1$, we get $m_2 < m_1$.

Position of 3rd maxima of

$$\lambda$$

$_2$:

$$y' = \frac{D}{d}(3\lambda_2) = 1800\frac{D}{d}$$

Position of 5th minima of

$$\lambda$$

$_1$:

$$y'' = \frac{D}{d}\left(\frac{9\lambda_1}{2}\right) = 1800\frac{D}{d}$$

Hence, y' = y''.

Angular fringe width is

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

Since,

$$\beta_2 > \beta_1$$

$$\Rightarrow \theta_2 > \theta_1$$

Question 050 MCQ

QUESTION

One end of a taut string of length 3 m along the x-axis is fixed at x = 0. The speed of the waves in the string is 100 ms^{-1}. The other end of the string is vibrating in the y-direction so that stationary waves are set up in the string. The possible waveform s of these stationary wave is are

A $y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$

B $$y(t) = A \sin \frac{\pi x}{3} \cos \frac{100\pi t}{3}$$

C $$y(t) = A \sin \frac{5\pi x}{6} \cos \frac{250\pi t}{3}$$

D $$y(t) = A \sin \frac{5\pi x}{2} \cos 250\pi t$$

CORRECT OPTION

A $$y(t) = A \sin \frac{\pi x}{6} \cos \frac{50\pi t}{3}$$

SOURCE

Physics • waves

EXPLANATION

The displacement of a stationary wave is given by

$$y(x,t) = A \sin\left(\frac{2\pi x}{\lambda}\right) \cos(2\pi v t)$$

.

The boundary conditions give node at x = 0 and anti-node at x = 3 m i.e.,

y $0, t$ = 0, ... 1

y $3, t$ =

$$\pm$$

A. ..... $$2$$

The fundamental frequency is given by

$$v_0 = \frac{v}{\lambda} = \frac{v}{4l} = \frac{100}{4(3)} = \frac{25}{3}$$

Hz.

Thus, the waveform will satisfy equations $$1$$ and $$2$$, and the permissible frequencies will be odd multiples of v_0.

Question 051 MCQ

QUESTION

A student is performing an experiment using a resonance column and a tuning fork of frequency 244 s

$$-$$

[1]. He is told that the air in the tube has been replaced by another gas $$assumethatthecolumnremainsfilledwiththegas$$. If the minimum height at which resonance occurs is $$0.350\$\$ \pm \$\$0.005$$ m, the gas in the tube is

(Useful information :

$$\sqrt{167RT}$$

= 640 $J^{1/2}$ mol

$$-$$

1/2;

$$\sqrt{140RT}$$

= 590 $J^{1/2}$ mol

–

$^{1/2}$. The molar mass M in grams is given in the options. Take the values of

$$\sqrt{10/M}$$

for each gas as given there.)

Neon

A $\left(M = 20, \sqrt{\frac{10}{20}} = \frac{7}{10}\right)$

Nitrogen

B $\left(M = 28, \sqrt{\frac{10}{28}} = \frac{3}{5}\right)$

Oxygen

C $\left(M = 32, \sqrt{\frac{10}{32}} = \frac{9}{16}\right)$

Argon

D $\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$

CORRECT OPTION

Argon

D

$$\left(M = 36, \sqrt{\frac{10}{36}} = \frac{17}{32}\right)$$

SOURCE

Physics • waves

EXPLANATION

Minimum length

$$= \frac{\lambda}{4} \Rightarrow \lambda = 4l$$

Now, v = f

$$\lambda$$

= 244

$$\times$$

4

$$\times$$

l

as l = 0.350

$$\pm$$

0.005

$$\Rightarrow$$

v lies between 336.7 m/s to 346.5 m/s

Now,

$$v = \sqrt{\frac{\gamma RT}{M \times 10^{-3}}}$$

, here M is molecular mass in gram

$$= \sqrt{100\gamma RT} \times \sqrt{\frac{10}{m}}$$

.

For monoatomic gas,

$$\gamma$$

= 1.67

$$\Rightarrow$$

$$v = 640 \times \sqrt{\frac{10}{m}}$$

For diatomic gas,

$$\gamma = 1.4 \Rightarrow v = 590 \times \sqrt{\frac{10}{m}}$$

$$\therefore$$

$$v_{Ne} = 640 \times \frac{7}{10} = 448$$

m/s

$$v_{Ar} = 640 \times \frac{17}{32} = 340$$

m/s

$$v_{O_2} = 590 \times \frac{9}{16} = 331.8$$

m/s

$$v_{N_2} = 590 \times \frac{3}{5} = 354$$

m/s

$$\therefore$$

Only possible answer is Argon.

Question 052 MCQ

QUESTION

Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K?

A 4, if wires are in parallel

B 2, if wires are in series

C 1, if wires are in series

D 0.5, if wires are in parallel

CORRECT OPTION

B 2, if wires are in series

SOURCE

Physics • properties-of-matter

EXPLANATION

Resistance of initially given kettle

$$R = \rho\frac{l}{A}$$

$$= \rho\frac{L}{\pi(d/2)^2} = \frac{4\rho L}{\pi d^2}$$

Resistance of two replaced kettles

$$R_1 = \frac{\rho L}{\pi d^2}$$

and

$$R_2 = \frac{\rho L}{\pi d^2}$$

So,

$$R_1 = R_2 = \frac{R}{4}$$

If wires are in parallel then equivalent resistance

$$R_P = \frac{R_1 R_2}{R_1 + R_2} = \frac{R}{8}$$

..... 1

If wires are in series then equivalent resistance

$$R_S = R_1 + R_2 = \frac{R}{2}$$

..... 2

Let V be the applied voltage, m = 0.5 kg be the mass of the water, and S be the specific heat of the water. Initially, the heat produced by a resistance R_1 in time t_1 = 4 min is V^2t_1/R_1. This heat is used to raise the temperature of the water by

$$\Delta$$

T = 40 K. Thus,

$$V^2 t_1 / R_1 = mS\Delta T$$

. 3

Let t_s and t_p be the time taken to raise the temperature of same amount of water by

$$\Delta$$

T when the resistances are connected in series and parallel. Thus,

$$V^2 t_s / R_s = mS\Delta T$$

..... 4

$$V^2 t_p / R_p = mS\Delta T$$

..... 5

Divide equation 4 by 3 and use the equation 1 to get

$$t_s = \frac{R_s}{R_1} t_1 = \frac{1}{2} \times 4 = 2$$

min. Similarly, divide equation 5 by 3 and use the equation 2 to get

$$t_p = \frac{R_p}{R_1} t_1 = \frac{1}{8} \times 4 = 0.5$$

min.

Question 053 MCQ

QUESTION

In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle

$$\theta$$

with the horizontal floor. The coefficient of friction between the wall and the ladder is

$$\mu$$

$_1$ and that between the floor and the ladder is

$$\mu$$

$_2$. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then

μ

$_1$ = 0,

μ

A $_2$

$\neq$

0 and

$$N_2 \tan\theta = \frac{mg}{2}$$

μ

$_1$

$\neq$

B 0,

μ

$_2$ = 0 and

$$N_1 \tan\theta = \frac{mg}{2}$$

μ

$_1$

$\neq$

0,

C μ

$_2$

$\neq$

0 and

$$N_2 = \frac{mg}{1 + \mu_1\mu_2}$$

μ

1 = 0,

μ

D 2

$\neq$

0 and

$$N_1 \tan\theta = \frac{mg}{2}$$

CORRECT OPTION

μ

1

$\neq$

0,

μ

C 2

$\neq$

0 and

$$N_2 = \frac{mg}{1 + \mu_1\mu_2}$$

SOURCE

Physics • laws-of-motion

EXPLANATION

As the rod is about to slip, wall and floor exert limiting friction on the ladder.

Case 1 : If

$$\mu$$

$_1$ = 0,

$$\sum \overrightarrow{\tau_A} = \overrightarrow{0}$$

$$mg\left(\frac{l}{2}\cos\theta\right) = N_1(l\sin\theta)$$

$$N_1 = \frac{mg\cot\theta}{2}$$

or

$$N_1\tan\theta = \frac{mg}{2}$$

and

$$N_2 = mg$$

Case 2 : If

$$\mu$$

$_2$ = 0, N_1 remain unbalanced and rod can never be in equilibrium.

Case 3 : If

$$\mu$$

1

$$\neq$$

0,

$$\mu$$

2

$$\neq$$

0

$$N_1 = f_2 = \mu_2 N_2$$

$$N_2 + f_1 = mg$$

or

$$N_2 + \mu_1 N_1 = mg$$

or

$$N_2 + \mu_1(\mu_2 N_2) = mg$$

or

$$N_2 = \frac{mg}{1 + \mu_1\mu_2}$$

Question 054 MCQ

QUESTION

A transparent thin film of uniform thickness and refractive index n_1 = 1.4 is coated on the convex spherical surface of radius R at one end of a long solid glass cylinder of refractive index n_2 = 1.5, as shown in the figure. Rays of light parallel to the axis of the cylinder traversing through the film from air to glass get focused at distance f_1 from the film, while rays of light traversing from glass to air get focused at distance f_2 from the film. Then

A $|f_1| = 3R$

B $|f_1| = 2.8R$

C $|f_2| = 2R$

D $|f_2| = 1.4R$

CORRECT OPTION

A $$|f_1| = 3R$$

SOURCE

Physics • geometrical-optics

EXPLANATION

As the film has $R_1 = R_2 = R$ say, its focal length

$$\frac{1}{f} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right)$$

$$\frac{1}{f} = 0$$

or

$$f = \infty$$

. It will not cause refraction.

Refraction will take place at air and glass cylinder. Given u =

$$\infty$$

$$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$$

$$-\frac{n_1}{\infty} + \frac{1.5}{f_1} = \frac{1.5 - 1}{R}$$

or

$$f_1 = 3R$$

For glass-air refraction, we have

$$-\frac{n_2}{u} + \frac{n_1}{v} = \frac{n_1 - n_2}{R}$$

$$-\frac{n_2}{\infty} + \frac{1}{b_2} = \frac{1 - 1.5}{R}$$

or

$$f_2 = -2R$$

or

$$|f_2| = 2R$$

.

Question 055 MCQ

QUESTION

Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if

A $V_1 = V_2$ and $R_1 = R_2 = R_3$

B $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

C $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$

D $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

CORRECT OPTION

A $V_1 = V_2$ and $R_1 = R_2 = R_3$

SOURCE

Physics • current-electricity

EXPLANATION

Let i_1 and i_2 be the currents as shown in the figure.

Apply Kirchhoff's law in the loop ABCDA and CEFDC to get

$$i_1R_1 + (i_1 - i_2)R_2 = V_1$$

..... 1

$$i_2R_3 - (i_1 - i_2)R_2 = V_2$$

..... 2

Multiply equation 1 by R_3 and 2 by R_1 and then subtract to get the current through R_2 as

$$(i_1 - i_2) = \frac{V_1 R_3 - V_2 R_1}{R_1 R_3 + R_2 R_3 - R_1 R_2}$$

.

The current through R_2 becomes zero when $V_1R_3 = V_2R_1$.

Question 056 Numerical

QUESTION

A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc $see figure$. One second after applying the forces, the angular speed of the disc in rad s^{-1} is

SOURCE

Physics • rotational-motion

EXPLANATION

In triangle OZP, the distances OZ = R and OP = R sin30

$\circ$

= R/2.

The total torque by the three forces about the centre of mass O is

τ

= 3F| OP | = 3F R/2.

The monent of inertia of the disc about the axis of rotation is

$$I = \frac{1}{2}MR^2$$

. Using

$$\tau$$

= I

$$\alpha$$

, we get

$$\alpha = \frac{\tau}{I} = \frac{3F}{MR} = \frac{3 \times 0.5}{1.5 \times 0.5} = 2$$

rad/s^2.

The angular velocity after t = 1 is given by

$$\omega = \omega_0 + \alpha t = 0 + 2 \times 1 = 2$$

rad/s.

Question 057 Numerical

QUESTION

Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If

$$\frac{X_0}{X_1} = 3$$

, and value of R_1/R_2 is

SOURCE

Physics • magnetism

EXPLANATION

When current in wires is in same direction, the magnetic fields due to two wires all in opposite direction.

From

$$B = \frac{\mu_0}{4\pi}.\frac{2I}{r}$$

, we get

$$B_1 = \frac{\mu_0}{4\pi}.2I\left[\frac{1}{x_1} - \frac{1}{(x_0 - x_1)}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - x_1 - x_1}{x_1(x_0 - x_1)}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - 2x_1}{x_1(x_0 - x_1)}\right]$$

..... 1

When the direction of current in two wires is opposite, field will be in the same direction.

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x_1} + \frac{1}{(x_0 - x_1)}\right]$$

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{x_0 - x_1 + x_1}{x_1(x_0 - x_1)}\right]$$

$$B_2 = \frac{\mu_0 I}{2\pi}\left[\frac{x_0}{x_1(x_0 - x_1)}\right]$$

From

$$\frac{mv^2}{r} = qvB$$

or

$$v = \frac{qBr}{m}$$

or

$$r = \frac{mv}{qB}$$

$$B \propto \frac{1}{r}$$

Therefore,

$$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{x_0}{(x_0 - 2x_1)}$$

$$\frac{R_1}{R_2} = \frac{x_0/x_1}{(x_0/x_1) - (2x_1/x_1)} = \frac{3}{3-2} = 3$$

Question 058 Numerical

QUESTION

A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990

$$\Omega$$

resistance, it can be converted into a voltmeter of range 0-30V. If connected to a

$$\frac{2n}{249}\Omega$$

resistance, it becomes an ammeter of range 0-1.5 A. The value of n is

SOURCE

Physics • current-electricity

EXPLANATION

$$i_g(G + 4990) = V$$

$$\Rightarrow \frac{6}{1000}(G + 4990) = 30$$

$$\Rightarrow G + 4990 = \frac{30,000}{6} = 5000$$

$$\Rightarrow G = 10\,\Omega$$

$$V_{ab} = V_{cd}$$

$$\Rightarrow i_g G = (1.5 - i_g) S$$

$$\Rightarrow \frac{6}{1000} \times 10 = \left(1.5 - \frac{6}{1000}\right) S$$

$$\Rightarrow S = \frac{60}{1494} = \frac{2n}{249}$$

$$\Rightarrow n = \frac{249 \times 30}{1494} = \frac{2490}{498} = 5$$

Question 059 Numerical

QUESTION

A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter $see figure$. Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

SOURCE

Physics • rotational-motion

EXPLANATION

Consider the balls and the platform together as a system. There is no external torque on the system about its centre. Hence, angular momentum of the system about its centre is conserved. Initial and final angular momentum of the system are

$$L_i = 0$$

,

$$L_f = mvr + mvr + I\omega = 2mvr + \frac{1}{2}MR^2\omega$$

.

The conservation of angular momentum, $L_i = L_f$, gives

$$\omega = -\frac{4mvr}{MR^2} = -\frac{4(0.05)(9)(0.25)}{0.45(0.5)^2} = -4$$

rad/s.

Question 060 Numerical

QUESTION

A thermodynamic system is taken from an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf} / Q_{ib} is

SOURCE

Physics • heat-and-thermodynamics

EXPLANATION

In a thermodynamics process, the heat supplied to the system, the increase in internal energy of the system, and the work done by the system are related by the first law of thermodynamics,

$$\Delta Q = \Delta U + \Delta W$$

.

The first law of thermodynamics for the process iaf gives

$$Q_{iaf} = U_{iaf} + W_{iaf} = (U_f - U_i) + (W_{ia} + W_{af})$$

..... 1

Substitute Q_{iaf} = 500 J, U_i = 100 J, W_{ia} = 0 $constantvolume$, and W_{af} = 200 J in equation 1 to get U_f = 400 J.

In the process ib,

$$Q_{ib} = U_{ib} + W_{ib} = (U_b - U_i) + W_{ib}$$

..... 2

Substitute U_b = 200 J, U_i = 100 J, and W_{ib} = 50 J in equation 2 to get Q_{ib} = 150 J.

In the process bf,

$$Q_{bf} = U_{bf} + W_{bf} = (U_f - U_b) + W_{bf}$$

..... 3

Substitute U_f = 400 J, U_b = 200 J and W_{bf} = 100 J in equation 3 to get Q_{bf} = 300 J. Thus, Q_{bf}/Q_{ib} = 300/150 = 2.