JEE Advanced 2017 Paper 2 $Offline$

$54Questions$

Question 001 MCQ

QUESTION

Pure water freezes at

$$273$$

$$K$$

and

$$1$$

bar. The addition of

$$34.5$$

$$g$$

of ethanol to

$$500$$

$$g$$

of water changes the freezing point of the solution. Use the freezing point depression constant of water as

$$2$$

kg

$$mol^{-1}.$$

The figures shown below represent plots of vapor pressure

$$(V.P.)$$

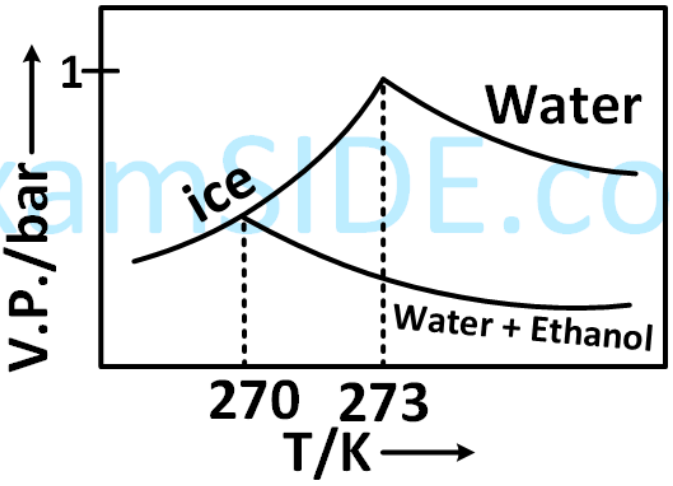
versus temperature

$$(T).$$

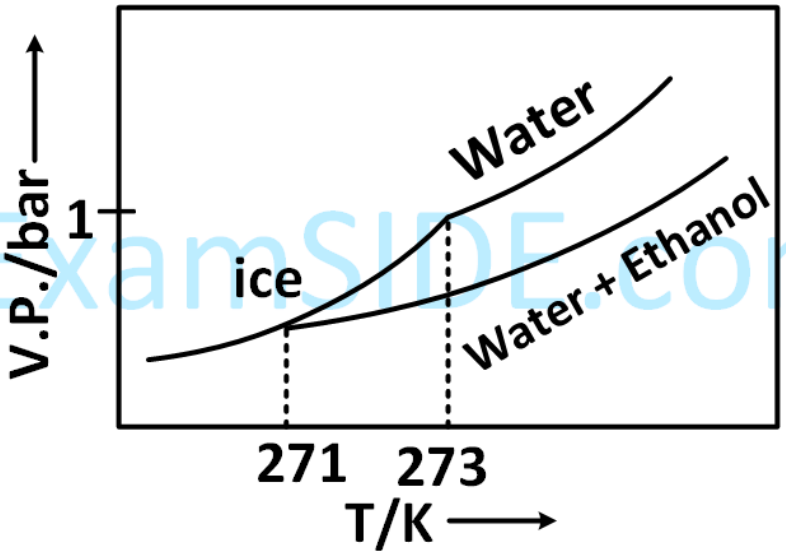
$$molecularweightofethanolis\$\$46\$\$\$\$g\$\$\$\$mol^{-1}.\$\$$$

Among the following, the option representing change in the freezing point is

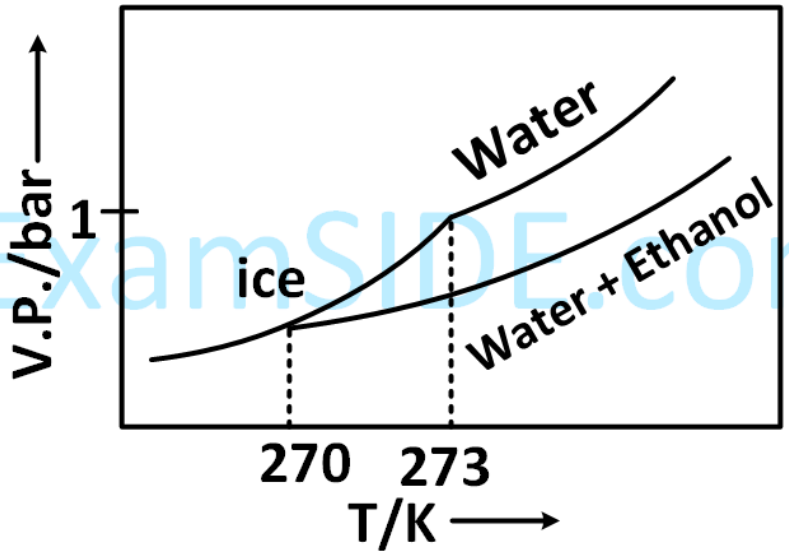
A



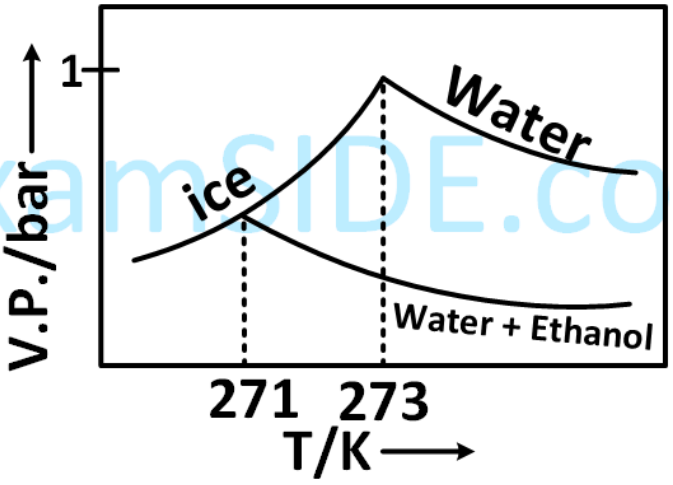
B



C

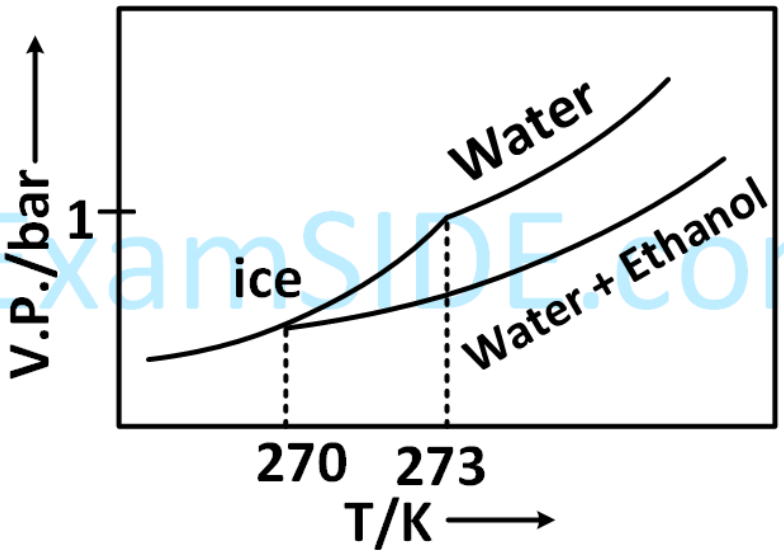


D



CORRECT OPTION

c



SOURCE

Chemistry • solutions

EXPLANATION

Depression in freezing point is given by

$$\Delta$$

$T_f = K_f m$

where

$$m = \frac{1000 \times w_{solute}}{M_{solute} \times w_{solvent}}$$

Substituting the values, we get

$$\Delta T_f = 2 \times \frac{1000 \times 34.5}{46 \times 500} = 3K$$

Therefore, T_f = 270 K and

$$T_f^o$$

= 273 K. Hence, option c indicates correct representation of the graph.

Question 002 MCQ

QUESTION

Y

and

Z

are, respectively

A and

N_2O_3

H_3PO_4

B and

N_2O_5

HPO_3

C and

N_2O_4

HPO_3

D and

N_2O_4

H_3PO_3

CORRECT OPTION

B and

$$N_2O_5$$

$$HPO_3$$

SOURCE

Chemistry • p-block-elements

EXPLANATION

Reaction of $\mathrm{P_4O_{10}}$ with pure nitric acid $(\mathrm{HNO_3})$ gives dinitrogen pentaoxide $(\mathrm{N_2O_5})$ and metaphosphoric acid $(\mathrm{HPO_3})$

$$\mathrm{P_4O_{10} + 4HNO_3 \rightarrow \underset{Z}{4HPO_3} + 2\,\underset{Y}{N_2O_5}}$$

The compound Y is dinitrogen pentaoxide $(\mathrm{N_2O_5})$ and compound Z is metaphosphoric $\mathrm{acid}\,(\mathrm{HPO_3})$.

Question 003 MCQ

QUESTION

The product

$$S$$

is

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

i Compound P reacts with $\mathrm{CH_3MgBr}$ in excess in $(\mathrm{C_2H_5})_2\mathrm{O}$ followed by addition of water to form alcohol derivative of the compound.

ii Compound Q undergoes dehydration in presence of $\mathrm{H_2SO_4}$ at $0^\circ\mathrm{C}$.

iii Compound R undergoes acylation of aromatic ring in presence of and anhydrous $\mathrm{AlCl_3}$.

The substitution should take place at ortho or para position but due to steric crowding at ortho and occupancy of para position. Meta position is substituted with acetyl group.

Question 004 MCQ

QUESTION

W

and

X

are, respectively

A O_3 and P_4O_6

B O_2 and P_4O_6

C O_2 and P_4O_{10}

D O_3 and P_4O_{10}

CORRECT OPTION

$$O_2$$

C and

$$P_4O_{10}$$

SOURCE

Chemistry • p-block-elements

EXPLANATION

i Potassium chlorate $(\mathrm{KClO_3})$ is decomposed in presence of $\mathrm{MnO_2}$ as catalyst to form potassium chloride and oxygen gas.

$$2\mathrm{KClO_3}(s) \xrightarrow[\mathrm{MnO_2}]{\Delta} 2\mathrm{KCl} + \underset{(w)}{3\mathrm{O_2}(g)}$$

The gas (w) is oxygen.

ii Reaction of white phosphorous with excess of gas w $i.e.,\$\mathrm{O_2}\$$ gives $\mathrm{P_4O_{10}}$ $adimerofphosphorouspentaoxide$.

$$\underset{\text{White}}{\mathrm{P_4}(s)} + \underset{(\text{excess})}{5\mathrm{O_2}(g)} \rightarrow \underset{(X)}{\mathrm{P_4O_{10}}(s)}$$

phosphorous oxygen

The compound (X) is $\mathrm{P_4O_{10}}$.

Question 005 MCQ

QUESTION

Compounds

$$P$$

and

$$R$$

upon ozonolysis produce

$$Q$$

and

$$S,$$

respectively. The molecular formula of

$$Q$$

and

$$S$$

is

$$C_8H_8O. Q$$

undergoes Canninzzaro reaction but not haloform reaction, whereas

$$S$$

undergoes haloform reaction but not Cannizzaro reaction

The option s with suitable combination of

$$P$$

and

$$R,$$

respectively, is are

A

B

C

D

CORRECT OPTION

A

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

Option A: Correct

Option B: Correct

Question 006 MCQ

QUESTION

The order of basicity among the following compounds is

A $\mathrm{II} > \mathrm{I} > \mathrm{IV} > \mathrm{III}$

B $\mathrm{IV} > \mathrm{II} > \mathrm{III} > \mathrm{I}$

C $\mathrm{IV} > \mathrm{I} > \mathrm{II} > \mathrm{III}$

D $\mathrm{I} > \mathrm{IV} > \mathrm{III} > \mathrm{II}$

CORRECT OPTION

C $\mathrm{IV} > \mathrm{I} > \mathrm{II} > \mathrm{III}$

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

d : Greater the electron density on nitrogen, more basic is the compound. Thus, order of basicity is :

IV > I > II > III

The conjugate acid of IV is stabilized by resonance with lone pairs on both

$$\ddot{N}H_2$$

groups.

The conjugate acid of I is stabilized by resonance with lone pair on

$$-$$

$\ddot{N}H_2$

group and by hyperconjugation of

$-$

CH_3 group.

The conjugate acid of II is stabilized by resonance with lone pair on

group.

In compound III, the lone pair of nitrogen is involved in aromaticity. So, it is least basic.

Question 007 MCQ

QUESTION

The major product of the following reaction is

A

B

C

D

CORRECT OPTION

C

SOURCE

Chemistry • compounds-containing-nitrogen

EXPLANATION

Question 008 MCQ

QUESTION

For the following compounds, the correct statement s with respect to nucleophilic substitution reaction is are

A

I

and

I

I

I

follow

S_N1

mechanism

I

and

B

$$\mathrm{II}$$

follow

$$S_N2$$

mechanism

Compound

C

$$\mathrm{IV}$$

undergoes inversion of configuration

The order of reactivity for

$$\mathrm{I},$$

$$\mathrm{III}$$

D

and

$$\mathrm{IV}$$

is :

$$\mathrm{I}V > \mathrm{I} > \mathrm{III}$$

CORRECT OPTION

$$\mathrm{I}$$

and

$$\mathrm{I}$$

A

$$\mathrm{I}$$

$$\mathrm{I}$$

follow

$$S_N1$$

mechanism

SOURCE

Chemistry • basics-of-organic-chemistry

EXPLANATION

a I is a benzylic halide, thus, it undergoes S_N1 reaction easily as benzylic carbocation is resonance stabilized. III also follows S_N1 mechanism as it is 3

$\circ$

alkyl halide.

b Compounds I and II are 1

$\circ$

alkyl halides, then they undergo S_N2 mechanism.

c Correct. In S_N1 reaction, the nucleophile approaches the substrate carbon from the back side with respect to the leaving group. Nucleophilic displacement of the leaving group in an S_N2 reaction causes inversion of configuration at the substrate carbon.

d Stability of carbocations follows the order:

$$\underset{(IV)}{2^\circ\, benzylic} > \underset{(III)}{3^\circ\, alkyl} > \underset{(I)}{1^\circ\, benzylic}$$

Question 009 MCQ

QUESTION

The options s with only amphoteric oxides is are

A Cr_2O_3, BeO, SnO, SnO_2

B Cr_2O_3, CrO, SnO, PbO

C NO, B_2O_3, PbO, SnO_2

D ZnO, Al_2O_3, PbO, PbO_2

CORRECT OPTION

A Cr_2O_3, BeO, SnO, SnO_2

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

Compounds which dissolve in both acids and bases are known as amphoteric oxides.

Amphoteric oxides are :

Cr_2O_3, BeO, SnO, SnO_2, ZnO, Al_2O_3 , PbO and PbO_2 Whereas, NO is a neutral oxide, B_2O_3 is an acidic oxide and CrO is a basic oxide.

Question 010 MCQ

QUESTION

Among the following, the correct statement s is are

A

$$Al(CH_3)_3$$

has the three-centre two-electron bonds in its dimeric structure

B

$$BH_3$$

has the three-center two-electron bonds in its dimeric structure

C

$$AlCl_3$$

has the three-center two-electron bonds in its dimeric structure

D

The Lewis acidity of

$$BCl_3$$

is greater than that of

$$AlCl_3$$

CORRECT OPTION

A

$$Al(CH_3)_3$$

has the three-centre two-electron bonds in its dimeric structure

SOURCE

Chemistry • p-block-elements

EXPLANATION

Option A: Correct.

The aluminium compounds are unusual because they have dimeric structures, and appear to have three-centre bonds involving sp^3 hybrid orbitals on Al and C in $\mathrm{Al-C-Al}$ bridges.

Option B: Correct.

In diborane $(\mathrm{BH_3})$ there are 12 valency electrons, three from each B atom and six from the H atoms. An sp^3 hybrid orbital from each boron atom overlaps with the $1s$ orbital of the hydrogen. This gives a delocalised molecular orbital covering all three nuclei, containing one pair of electrons and making up one of the bridges. This is a three-centre two-electron bond $(3c-2e)$.

Option D: Group 13 elements have only three valency electrons. When these are used to form three covalent bonds, the atom has a share in only six electrons. The compounds are therefore electron deficient. In $\mathrm{AlCl_3}$, effective π overlap takes place between p orbitals of Al and Cl due to their comparable size while in $\mathrm{BCl_3}$, the π overlap is not effective as p orbital of boron is smaller than that of p orbital of chlorine, hence, the acidity of $\mathrm{BCl_3}$ is greater than that of $\mathrm{AlCl_3}$.

Question 011 MCQ

QUESTION

In a bimolecular reaction, the steric factor

$$P$$

was experimentally determined to be

$$4.5.$$

The correct option s among the following is are

A The activation energy of the reaction is unaffected by the value of the steric factor

B Experimentally determined value of frequency factor is higher than that predicted by arrhenius equation

C Since

$$P = 4.5,$$

the reaction will not proceed unless an effective catalyst is used

D The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

CORRECT OPTION

B Experimentally determined value of frequency factor is higher than that predicted by arrhenius equation

SOURCE

Chemistry • chemical-kinetics-and-nuclear-chemistry

EXPLANATION

Arrhenius equation is

$$k = Ae^{-E_a/RT}$$

where, A = Frequency factor

Taking into account orientation factor,

$$k = PZ_{AB}e^{-E_a/RT}$$

where, P = steric factor, Z_{AB} = collision frequency

The value of steric factor lies between 0 and 1 predicted by Arrhenius equation. Thus, the experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation.

The activation energy of the reaction does not depend upon the value of the steric factor.

If P is very small, then catalyst is required to carry out the reaction at measurable rate.

Question 012 MCQ

QUESTION

Which of the following combination will produce

$$H_2$$

gas ?

A Fe metal and conc.

$$HNO_3$$

B

$$Cu$$

metal and conc.

$$HNO_3$$

C metal and

$$Zn$$

$$NaOH(aq)$$

D Au metal and

$$NaCN\,(aq)$$

in the presence of air

CORRECT OPTION

C metal and

$$Zn$$

$$NaOH(aq)$$

SOURCE

Chemistry • d-and-f-block-elements

EXPLANATION

a Conc. HNO_3 makes iron passive. Cold relatively concentrated HNO_3 will react with Fe.

$$\underset{Iron}{Fe} + 6\underset{Nitric\ acid}{HNO_3} \rightarrow \underset{Iron\ nitrate}{Fe(NO_3)_3} + \underset{Nitrogen\ dioxide}{3NO_2} + \underset{Water}{3H_2O}$$

b

$$\underset{Copper}{Cu} + 4\underset{Nitric\ acid}{HNO_3(conc.)} \rightarrow \underset{Copper\ nitrate}{Cu(NO_3)_2} + \underset{Nitrogen\ dioxide}{2NO_2} + \underset{Water}{2H_2O}$$

c

$$\underset{Zinc}{Zn} + \underset{Sodium\ hydroxide}{2NaOH_{(aq)}} \rightarrow \underset{Sodium\ zincate}{Na_2ZnO_2} + \underset{Hydrogen}{H_2}$$

d

$$\underset{Gold}{4Au} + \underset{Sodium\ cyanide}{8NaCN} + \underset{Oxygen}{O_2} + \underset{Water}{2H_2O} \rightarrow \underset{Sodium\ dicyanoaurate}{4Na[Au(CN)_2]} + \underset{Sodium\ hydroxide}{4NaOH}$$

Question 013 MCQ

QUESTION

The correct statement s about surface properties is are

A Adsorption is accompanied by decrease in ethalpy and decrease in entropy of the system

B The critical temperatures of ethane and nitrogen are

$$563$$
$$K$$

and

$$126$$
$$K,$$

respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature

C Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium

D Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution

CORRECT OPTION

A Adsorption is accompanied by decrease in ethalpy and decrease in entropy of the system

SOURCE

Chemistry • surface-chemistry

EXPLANATION

Option A : Correct. Adsorption is a surface phenomenon in which the concentration of the adsorbate increases only on the surface. It is always accompanied by a decrease in the entropy $$\Delta$$S. Since adsorption is an exothermic process, the enthalpy change $$\Delta$$H of the system is also negative.

Option B : Correct. Easily liquefiable gases, that is, gases with high critical temperatures, are more readily adsorbed because van der Waals forces are stronger near the critical temperature. Thus, the adsorption of ethane will be more than that of nitrogen.

Option C : Cloud is an aerosol in which liquid is dispersed phase and gas is dispersion medium. Whereas, emulsion is liquid in liquid colloidal system.

Option D : Incorrect. Brownian movement varies with the size of the particles and viscosity of the solution. Smaller particles with lower viscosity show faster movement.

Question 014 MCQ

QUESTION

For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant

$$K$$

in terms of change in entropy is described by

A

With increase in temperature, the value of

$$K$$

for exothermic reaction decreases because the entropy change of the system is positive

B

With increase in temperature, the value of

$$K.$$

for endothermic reaction increases because unfavorable change in entropy of the surroundings decreases

C

With increase in temperature, the value of

$$K$$

for endothermic reaction increases because the entropy change of the system is negative

D

with increase in temperature, the value of

$$K$$

for exothermic reaction decreases because favorable change in entropy of the surroundings decreases

CORRECT OPTION

B

With increase in temperature, the value of

$$K.$$

for endothermic reaction increases because unfavorable change in entropy of the surroundings decreases

SOURCE

Chemistry • thermodynamics

EXPLANATION

$$\Delta S_{surr} = -\frac{\Delta H}{T_{surr}}$$

For endothermic, if T_{surr} increases, unfavourable change in entropy of the surroundings decreases.

For exothermic, if T_{surr} increases, favourable change in entropy of the surroundings decreases.

Question 015 MCQ

QUESTION

The order of the oxidation state of the phosphorous atom in

$$H_3PO_2, H_3PO_4, H_3PO_3$$

and

$$H_4P_2O_6$$

is

A $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$

B $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$

C $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

D $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$

CORRECT OPTION

C $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

SOURCE

Chemistry • redox-reactions

EXPLANATION

Let oxidation states of phosphorus in H_3PO_2, H_3PO_4, H_3PO_3 and $H_4P_2O_6$ be p, q, r and s respectively.

Oxidation state of hydrogen = + 1

Oxidation state of oxygen =

$-$

2

Thus, in H_3PO_2 :

3

$\times$

$+1$ + p + 2

$\times$

$$ $$2 = 0

$\therefore$

p = +1

In H_3PO_4 :

3

$\times$

$+1$ + q + 4

$\times$

$\$\$ - \$\$2$ = 0

$\therefore$

q = +5

In H_3PO_3 :

3

$\times$

$+1$ + r + 3

$\times$

$\$\$ - \$\$2$ = 0

$\therefore$

r = + 3

In $H_4P_2O_6$:

4

$\times$

$+1$ + 2s + 6

$\times$

$\$\$ - \$\$2$ = 0

$\therefore$

s = +4

Thus, the order of oxidation state is :

$H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

Question 016 MCQ

QUESTION

The standard state Gibbs free energies of formation of

$$C$$

$graphite$ and

$$C$$

$diamond$ at

$$T = 298$$

$$K$$

are

$$\Delta_f G^0$$

$$\$\$C\$\$(graphite)$$

$$= 0kJmol^{-1}$$

$$\Delta_f G^0$$

$$\$\$C\$\$(diamond)$$

$$= 2.9kJmol^{-1}$$

The standard state means that the pressure should be

$$1$$

bar, and substance should be pure at a given temperature. The conversion of graphite

$$\$\$C\$\$(graphite)$$

to diamond

$$C$$(diamond)

reduces its volume by

$$2 \times 10^{-6} m^3 mol^{-1}$$

If

$$C$$

graphite is converted to

$$C$$

diamond isothermally at

$$T = 298$$

$$K,$$

the pressure at which

$$C$$

graphite is in equilibrium with

$$C$$

diamond, is

Usefulinformation : $$1$$$$J = 1$$$$kg\, m^2 s^{-2}; 1\, Pa = 1\, kg\, m^{-1} s^{-2}; $$$

A

$$14501$$

bar

B

$$58001$$

$$bar$$

C

1450

bar

D

29001

bar

CORRECT OPTION

A

14501

bar

SOURCE

Chemistry • thermodynamics

EXPLANATION

We have

C $graphite$

$\rightarrow$

C $diamond$

For solid at constant temperature, Gibbs energy can be calculated as

$$dG = \Delta V dp$$

$$\int_{\Delta G_1}^{\Delta G_2} d(\Delta G_T) = \int_{p_1}^{p_2} \Delta V dp$$

$$\Delta G_2 - \Delta G_1 = \Delta V(p_2 - p_1)$$

$$(2.9 \times 10^3 - 0) = 2 \times 10^{-6}(p_2 - 1)$$

$$(p_2 - 1) = \frac{2.9 \times 10^3}{2 \times 10^{-6}}$$

Pa

$$(p_2 - 1) = \frac{2.9 \times 10^3}{2 \times 10^{-6} \times 10^5}$$

bar

p_2 = 14501 bar

Question 017 MCQ

QUESTION

For the following cell,

$$Zn\,(s)\,|ZnSO_4\,(aq)|\,|CuSO_4\,(aq)|Cu\,(s)$$

when the concentration of

$$Zn^{2+}$$

is

$$10$$

times the concentration of

$$Cu^{2+},$$

the expression for

$$\Delta G$$

$in\$\$J\,mol^{-1}\$\$$ is

$$F$$ is Faraday constant; $$R$$ is gas constant; $$T$$ is temperature; $$E

A $1.1F$

B $2.303RT - 2.2F$

C $2.303RT + 1.1F$

D $-2.2F$

CORRECT OPTION

B $2.303RT - 2.2F$

SOURCE

Chemistry • electrochemistry

EXPLANATION

The reaction involved is

$$Zn(s) + Cu^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Cu(s)$$

$$\Delta G = \Delta G^{\circ} + 2.303RT \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

$$\Delta G = -nFE^{\circ} + 2.303RT \log \frac{[Zn^{2+}]}{[Cu^{2+}]}$$

..... 1

Substituting n = 2, E

$$\circ$$

= 1.1 and

$$[Zn^{2+}] = 10$$

and

$$[Cu^{2+}] = 1$$

in Eq. 1, we get

$$\Delta G = (-2 \times F \times 1.1) + 2.303RT \log \frac{10}{1}$$

= 2.303 RT

$$-$$

2.2 F

Question 018 MCQ

QUESTION

The reactions,

$$Q$$

to

$$R$$

and

$$R$$

to

$$S,$$

are

A
Dehydration and Friedel
—
Crafts acylation

B
Aromatic sulfonation and Friedel
—
Crafts acylation

C
Friedel
—
Crafts alkylation, dehydration and Friedel
—
Crafts acylation

D
Friedel
—
Crafts alkylation and Friedel
—
Crafts acylation

CORRECT OPTION

D
Friedel
—
Crafts alkylation and Friedel
—

Crafts acylation

SOURCE

Chemistry • aldehydes-ketones-and-carboxylic-acids

EXPLANATION

The reaction $\mathrm{Q} \rightarrow \mathrm{R}$ and $\mathrm{R} \rightarrow \mathrm{S}$ Friedel-Crafts alkylation and Friedel-Crafts alkylation is the introduction of alkyl group into benzene ring in presence of lewis acid catalyst such as anhydrous $\mathrm{AlCl_3}$. Friedel Crafts acylation is the introduction of acyl group into benzene ring in presence of lewis acid catalyst such as anhydrous $\mathrm{AlCl_3}$.

Question 019 MCQ

QUESTION

If f : R

$$\rightarrow$$

R is a twice differentiable function such that f"x > 0 for all x

$$\in$$

R, and

$$f\left(\frac{1}{2}\right) = \frac{1}{2}$$

, f1 = 1, then

A f'1 $\leq$ 0

B f'1 > 1

C 0 < f'1 $\leq \frac{1}{2}$

D $\frac{1}{2}$ < f'1 $\leq$ 1

CORRECT OPTION

B f'1 > 1

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

f'x is increasing

For some x in

$$\left(\frac{1}{2}, 1\right)$$

f'x = 1

$$LMVT$$

$$\therefore$$

f' 1 > 1

Question 020 MCQ

QUESTION

If y = y x satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1}$$

dx, x > 0 and y 0 =

$$\sqrt{7}$$

, then y 256 =

A 16

B 3

C 9

D 80

CORRECT OPTION

B 3

SOURCE

Mathematics • differential-equations

EXPLANATION

$$\frac{dy}{dx} = \frac{1}{8\sqrt{x} + \sqrt{9+\sqrt{x}}\sqrt{4+\sqrt{9+\sqrt{x}}}}$$

$$\Rightarrow y = \sqrt{4+\sqrt{+\sqrt{x}}} + c$$

Now,

$$y(0) = \sqrt{7} + c$$

$$\Rightarrow c = 0$$

$$y(256) = \sqrt{4+\sqrt{9+\sqrt{16}}} = \sqrt{4+5} = 3$$

Question 021 MCQ

QUESTION

How many 3

$$\times$$

3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of M^TM is 5?

A 198

B 162

C 126

D 135

CORRECT OPTION

A 198

SOURCE

Mathematics • matrices-and-determinants

EXPLANATION

Sum of diagonal entries of M^TM is

$$\sum a_i^2$$

$$\sum_{i=1}^{9} a_i^2 = 5$$

Possibilities

I. 2, 1, 0, 0, 0, 0, 0, 0, 0, which gives

$$\frac{9!}{7!}$$

matrices

II. 1, 1, 1, 1, 1, 0, 0, 0, 0, which gives

$$\frac{9!}{4! \times 5!}$$

matrices

Total matrices = 9

$\times$

8 + 9

$\times$

7

$\times$

2 = 198

Question 022 MCQ

QUESTION

Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is

A $\frac{1}{2}$

B $\frac{36}{55}$

C $\frac{6}{11}$

D $\frac{5}{11}$

CORRECT OPTION

C $\frac{6}{11}$

SOURCE

Mathematics • probability

EXPLANATION

To solve this problem, we need to find the total number of possible solutions to the equation $x + y + z = 10$ where $x, y,$ and z are nonnegative integers.

First, we employ the "stars and bars" method to determine the number of nonnegative integer solutions to this equation. This method states that the

number of ways to partition n identical items $oursum$ into k distinct groups $ourvariables$ is given by:

$$\binom{n+k-1}{k-1}$$

For $x+y+z=10$, we have $n=10$ and $k=3$. Plugging into the formula, we get:

$$\binom{10+3-1}{3-1} = \binom{12}{2}$$

We calculate the binomial coefficient:

$$\binom{12}{2} = \frac{12 \times 11}{2 \times 1} = 66$$

So, there are 66 possible solutions to the equation $x+y+z=10$.

Next, we need to find the number of solutions for which z is even. Let $z=2k$ where k is a nonnegative integer. Then the equation becomes:

$$x+y+2k=10$$

Rearranging it, we get:

$$x+y=10-2k$$

Here, $10-2k$ must be nonnegative, so k can take values $0,1,2,3,4,5$ making total 6 possible values of k.

For each value of k, $x+y$ must equal the corresponding $10-2k$. The number of nonnegative integer solutions to $x+y=m$ for any nonnegative integer m is given by:

$$\binom{m+1}{1} = m+1$$

We will sum the solutions for each valid k:

When $k=0$: $x+y=10$, the number of solutions is 11.

When $k=1$: $x+y=8$, the number of solutions is 9.

When $k = 2$: $x + y = 6$, the number of solutions is 7.

When $k = 3$: $x + y = 4$, the number of solutions is 5.

When $k = 4$: $x + y = 2$, the number of solutions is 3.

When $k = 5$: $x + y = 0$, the number of solutions is 1.

Summing these, we get:

$$11 + 9 + 7 + 5 + 3 + 1 = 36$$

Thus, there are 36 solutions where z is even out of a total of 66 solutions. Hence, the probability that z is even is given by:

$$\frac{36}{66} = \frac{6}{11}$$

Thus, the correct answer is:

Option C:

$$\frac{6}{11}$$

Question 023 MCQ

QUESTION

Let S = {1, 2, 3, ..., 9}. For k = 1, 2, ..., 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ =

A 210

B 252

C 126

D 125

CORRECT OPTION

C 126

SOURCE

Mathematics • functions

EXPLANATION

$$N_i = {}^5C_k \times {}^4C_{5-k}$$

$$N_1 = 5 \times 1$$
$$N_2 = 10 \times 4$$
$$N_3 = 10 \times 6$$
$$N_4 = 5 \times 4$$
$$N_5 = 1$$
$$N_1 + N_2 + N_3 + N_4 + N_5 = 126$$

Question 024 MCQ

QUESTION

Let O be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}$$

.

$$\overrightarrow{OQ}$$

+

$$\overrightarrow{OR}$$

.

$$\overrightarrow{OS}$$

=

$$\overrightarrow{OR}$$

.

$$\overrightarrow{OP}$$

+

$$\overrightarrow{OQ}$$

.

$$\overrightarrow{OS}$$

=

$$\overrightarrow{OQ}$$

.

$$\overrightarrow{OR}$$

+

$$\overrightarrow{OP}$$

.

$\overrightarrow{OS}$

Then the triangle PQR has S as its

A centroid

B orthocentre

C incentre

D circumcentre

CORRECT OPTION

B orthocentre

SOURCE

Mathematics • vector-algebra

EXPLANATION

$\overrightarrow{OP}$

.

$\overrightarrow{OQ}$

+

$\overrightarrow{OR}$

.

$$\overrightarrow{OS}$$

=

$$\overrightarrow{OR}$$

.

$$\overrightarrow{OP}$$

+

$$\overrightarrow{OQ}$$

.

$$\overrightarrow{OS}$$

$$\Rightarrow$$

$$\overrightarrow{OP}$$

$$\overrightarrow{OQ} - \overrightarrow{OR} +$$

$$\overrightarrow{OS}$$

$$\overrightarrow{OR} - \overrightarrow{OQ} = 0$$

$$\Rightarrow$$

$$\overrightarrow{OP} - \overrightarrow{OS} \overrightarrow{OQ} - \overrightarrow{OR} = 0$$

$$\Rightarrow$$

$$\overrightarrow{SP}$$

.

$$\overrightarrow{RQ}$$

= 0

Similarly

$$\overrightarrow{SR}$$

.

$$\overrightarrow{PQ}$$

= 0 and

$$\overrightarrow{SQ}$$

.

$$\overrightarrow{PR}$$

= 0

$$\therefore \overrightarrow{S}$$

is orthocentre.

Question 025 MCQ

QUESTION

The equation of the plane passing through the point $1, 1, 1$ and perpendicular to the planes 2x + y

2z = 5 and 3x — 6y — 2z = 7 is

A 14x + 2y — 15z = 1

B — 14x + 2y + 15z = 3

C 14x — 2y + 15z = 27

D 14x + 2y + 15z = 31

CORRECT OPTION

D 14x + 2y + 15z = 31

SOURCE

Mathematics • 3d-geometry

EXPLANATION

Let the equation of plane be ax + by + cz = 1. Then

a + b + c = 1

2a + b

$$-$$

2c = 0

3a

$$-$$

6b

$$-$$

2c = 0

$$\Rightarrow$$

a = 7b

c =

$$\frac{15b}{2}$$

b =

$$\frac{2}{31}$$

, a =

$$\frac{14}{31}$$

, c =

$$\frac{15}{31}$$

$\therefore$

14x + 2y + 15z = 31

Question 026 MCQ

QUESTION

f : R

$\rightarrow$

R is a differentiable function such that f'x > 2fx for all x

$\in$

R, and f0 = 1 then

A fx > e^{2x} in $0, \$\$\infty\$\$$

B f'x < e^{2x} in $0, \$\$\infty\$\$$

C fx is increasing in $0, \$\$\infty\$\$$

D fx is decreasing in $0, \$\$\infty\$\$$

CORRECT OPTION

A fx > e^{2x} in $0, \$\$\infty\$\$$

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

$$f'(x) > 2f(x)$$

$$\Rightarrow \frac{dy}{y} > 2dx$$

$$\Rightarrow \int_1^{f(x)} \frac{dy}{y} > 2\int_0^x dx$$

$$\ln(f(x)) > 2x$$

$$\therefore$$

$$f(x) > e^{2x}$$

Also, as

$$f'(x) > 2f(x)$$

$$\therefore$$

$$f'(x) > 2c^{2x} > 0$$

Question 027 MCQ

QUESTION

If

$$I=\sum_{k=1}^{98}\int_{k}^{k+1}\frac{k+1}{x(x+1)}dx$$

, then

A $I>\log_e 99$

B $I<\log_e 99$

C $I<\frac{49}{50}$

D $I>\frac{49}{50}$

CORRECT OPTION

B $I<\log_e 99$

SOURCE

Mathematics • definite-integration

EXPLANATION

$$I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{(k+1)}{x(x+1)} dx$$

Clearly,

$$I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{(k+1)}{x(x+1)^2} dx$$

$$\Rightarrow I > \sum_{k=1}^{98} (k+1) \int_{k}^{k+1} \frac{1}{(x+1)^2} dx$$

$$\Rightarrow I > \sum_{k=1}^{98} (-(k+1)) \left[\frac{1}{k+2} - \frac{1}{k+1} \right]$$

$$\Rightarrow I > \sum_{k=1}^{98} \frac{1}{k+2}$$

$$\Rightarrow I > \frac{1}{3} + \ldots + \frac{1}{100} > \frac{98}{100}$$

$$\Rightarrow I > \frac{49}{50}$$

Also,

$$I = \sum_{k=1}^{98} \int_{k}^{k+1} \frac{(k+1)}{x(x+1)} dx$$

$$= \sum_{k=1}^{98} [\log_e(k+1) - \log_e k]$$

$$I < \log_e 99$$

Question 028 MCQ

QUESTION

If the line x =

$$\alpha$$

divides the area of region R = {x, y

$$\in$$

R^2 : x^3

$$\leq$$

y

$$\leq$$

x, 0

$$\leq$$

x

$\leq$

1} into two equal parts, then

A

2

α

4

$-$

4

α

^{2} + 1 =0

B

α

^{4} + 4

α

2

$-$

1 =0

C

$\frac{1}{2} < \alpha < 1$

D

0 <

α

$\leq$

$\frac{1}{2}$

CORRECT OPTION

A $2\alpha^4 - 4\alpha^2 + 1 = 0$

SOURCE

Mathematics • application-of-integration

EXPLANATION

$$\int_0^1 (x - x^3)dx = 2\int_0^\alpha x - x^3)dx$$

$$\frac{1}{4} = 2\left(\frac{\alpha^2}{2} - \frac{\alpha^2}{4}\right)$$

$$2\alpha^2 - 4\alpha^2 + 1 = 0$$

$$\alpha^2 = \frac{4 - \sqrt{16 - 8}}{4}$$

$$ ∵ $$$$α$$$$ ∈ $$(0, 1)

$$∵$$

$$\alpha^2 = 1 - \frac{1}{\sqrt{2}}$$

Question 029 MCQ

QUESTION

Let

$$\alpha$$

and

$$\beta$$

be non zero real numbers such that

$$2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$$

. Then which of the following is/are true?

A $$\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 2$$

B $$\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$$

C $$\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$$

D $$\sqrt{3}\tan\left(\frac{\alpha}{2}\right)+\tan\left(\frac{\beta}{2}\right)=2$$

CORRECT OPTION

B $$\tan\left(\frac{\alpha}{2}\right)-\sqrt{3}\tan\left(\frac{\beta}{2}\right)=0$$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

$$2(\cos\beta-\cos\alpha)+\cos\alpha\cos\beta=1$$

or

$$4(\cos\beta-\cos\alpha)+2\cos\alpha\cos\beta=2$$

$$\Rightarrow 1-\cos\alpha+\cos\beta-\cos\alpha\cos\beta$$

$$=3+3\cos\alpha-3\cos\beta-3\cos\alpha\cos\beta$$

$$\Rightarrow(1-\cos\alpha)(1+\cos\beta)$$

$$= 3(1+\cos\alpha)(1-\cos\beta)$$

$$\Rightarrow \frac{(1-\cos\alpha)}{(1+\cos\alpha)} = \frac{3(1-\cos\beta)}{1+\cos\beta}$$

$$\Rightarrow \tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$$

$$\therefore$$

$$\tan\frac{\alpha}{2} \pm \sqrt{3}\tan\frac{\beta}{2} = 0$$

Question 030 MCQ

QUESTION

Let

$$f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$$

for x

$$\neq$$

1. Then

A

$\lim_{x \to 1^+} f(x)$

= 0

B

$\lim_{x \to 1^-} f(x)$

does not exist

C

$\lim_{x \to 1^-} f(x)$

= 0

D

$\lim_{x \to 1^+} f(x)$

does not exist

CORRECT OPTION

C

$\lim_{x \to 1^-} f(x)$

= 0

SOURCE

Mathematics • limits-continuity-and-differentiability

EXPLANATION

$$f(x) = \frac{1 - x(1 + |1 - x|)}{|1 - x|} \cos\left(\frac{1}{1 - x}\right)$$

Now,

$$\lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} \frac{1 - x(1 + 1 - x)}{1 - x} \cos\left(\frac{1}{1 - x}\right)$$

$$= \lim_{x \to 1^-} (1 - x) \cos\left(\frac{1}{1 - x}\right) = 0$$

and

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+}$$

$$\frac{1 - x(1 + 1 - x)}{x - 1} \cos\left(\frac{1}{1 - x}\right)$$

$$= \lim_{x \to 1^+} -(x + 1).\, cos\left(\frac{1}{x + 1}\right)$$

, which does not exist.

Question 031 MCQ

QUESTION

If

$$g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\, dt$$

, then

A $g'\left(-\frac{\pi}{2}\right) = 0$

B $g'\left(-\frac{\pi}{2}\right) = -2\pi$

C $g'\left(\frac{\pi}{2}\right) = 2\pi$

D $g'\left(\frac{\pi}{2}\right) = 0$

CORRECT OPTION

A $g'\left(-\frac{\pi}{2}\right) = 0$

SOURCE

Mathematics • differential-equations

EXPLANATION

$$g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\, dt$$

g'x = 2cos 2x sin

$-$

1 $sin2x$

$-$

cos x sin

$-$

1 $sinx$

$$g'\left(\frac{\pi}{2}\right) = -2\sin^{-1}(0) = 0$$

$$g'\left(-\frac{\pi}{2}\right) = -2\sin^{-1}(0) = 0$$

No option is matching.

Question 032 MCQ

QUESTION

If

$$f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$$

,

then

A fx attains its minimum at x = 0

B fx attains its maximum at x = 0

C f'x = 0 at more than three points in $$ − $$$$π$$, $$π$$

D f'x = 0 at exactly three points in $$ − $$$$π$$, $$π$$

CORRECT OPTION

B fx attains its maximum at x = 0

SOURCE

Mathematics • application-of-derivatives

EXPLANATION

$$f(x) = \begin{vmatrix} \cos 2x & \cos 2x & \sin 2x \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$$

$$\cos 2x(\cos^2 x + \sin^2 x) - \cos 2x(-\cos^2 x + \sin^2 x) + \sin 2x(-\sin 2x)$$

$$= \cos 2x + \cos 4x$$

$$f'(x) = -2\sin 2x - 4\sin 4x$$

$$= -2\sin 2x(1 + 4\cos 2x)$$

At x = 0

f' x = 0 and f x = 2

Also, f' x = 0 sin2x = 0 or

$$\cos 2x = \frac{-1}{4}$$

$$\Rightarrow x = \frac{n\pi}{2}$$

or

$$\cos 2x = -\frac{1}{4}$$

Question 033 MCQ

QUESTION

If the triangle PQR varies, then the minimum value of cos $P + Q$ + cos $Q + R$ + cos $R + P$ is

A $-\frac{3}{2}$

B $\frac{3}{2}$

C $\frac{5}{3}$

D $-\frac{5}{3}$

CORRECT OPTION

A $-\frac{3}{2}$

SOURCE

Mathematics • trigonometric-functions-and-equations

EXPLANATION

$\cos P + Q$ + $\cos Q + R$ + $\cos R + P$

=

$-$

$cosR + cosP + cosQ$

Max. of cosP + cosQ + cosR =

$$\frac{3}{2}$$

Min. of $\cos P + Q$ + $\cos Q + R$ + $\cos R + P$ is =

$$-\frac{3}{2}$$

Question 034 MCQ

QUESTION

|

$$\overrightarrow{OX} \times \overrightarrow{OY}$$

| = ?

A $\sin P + Q$

B $\sin P + R$

C $\sin Q + R$

D sin2R

CORRECT OPTION

A $\sin P + Q$

SOURCE

Mathematics • vector-algebra

EXPLANATION

Now,

$$\overrightarrow{OX} = \frac{\overrightarrow{QR}}{QR}$$

and

$$\overrightarrow{OY} = \frac{\overrightarrow{RP}}{RP}$$

Therefore,

$$(\overrightarrow{OX} \times \overrightarrow{OY}) = \frac{\overrightarrow{QR}}{QR} \times \frac{\overrightarrow{RP}}{RP} = \frac{\overrightarrow{QR} \times \overrightarrow{RP}}{PQ}$$

$$= \frac{PQ \sin R}{PQ} = \sin R = \sin(\pi - (P + Q) = \sin(P + Q))$$

Question 035 MCQ

QUESTION

a_{12} = ?

A $a_{11} + 2a_{10}$

B $2a_{11} + a_{10}$

C $\frac{a_{11}}{a_{10}}$

D $a_{11} + a_{10}$

CORRECT OPTION

D $a_{11} + a_{10}$

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

$\alpha^2 = \alpha + 1$

$\beta^2 = \beta + 1$

$a_n = p\alpha$

$n + q$

β

n

$= p($

α

n

$-$

$1 +$

α

n

$-$

$2) + q($

β

n

$-$

$1 +$

β

n

$-$

$2)$

$= a_n$

$-$

$1 + a_n$

$-$

2

$\therefore$

$a_{12} = a_{11} + a_{10}$

Question 036 MCQ

QUESTION

If $a_4 = 28$, then $p + 2q =$

A 14

B 7

C 21

D 12

CORRECT OPTION

D 12

SOURCE

Mathematics • quadratic-equation-and-inequalities

EXPLANATION

$$\alpha = \frac{1+\sqrt{5}}{2}$$

,

$$\beta = \frac{1-\sqrt{5}}{2}$$

$$a_4 = a_3 + a_2$$

$$= 2a_2 + a_1$$

$$= 3a_1 + 2a_0$$

$$28 = p(3\alpha + 2) + q(3\beta + 2)$$

$$28 = (p+q)\left(\frac{3}{2} + 2\right) + (p-q)\left(\frac{3\sqrt{5}}{2}\right)$$

$$\therefore$$

p

$$-$$

q = 0

and

$$(p+q)\times\frac{7}{2}=28$$

$$\Rightarrow$$

p + q = 8

$$\Rightarrow$$

p = q = 4

$$\therefore$$

p + 2q = 12

Question 037 MCQ

QUESTION

A point charge

$$+Q$$

is placed just outside an imaginary hemispherical surface of radius

$$R$$

as shown in the figure. Which of the following statements is/are correct?

A. The electric flux passing through the curved surface of the hemisphere is

$$-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

B. Total flux through the curved and the flat surfaces is

$$\frac{Q}{\varepsilon_0}$$

C. The component of the electric field normal to the flat surface is constant over the surface

D. The circumference of the flat surface is an equipotential

CORRECT OPTION

A. The electric flux passing through the curved surface of the hemisphere is

$$-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

SOURCE

Physics • electrostatics

EXPLANATION

Since charge Q is outside the hemispherical surface, the net flux passing through the curved surface of hemispherical surface and flat surface is zero.

Therefore,

$$\phi$$

curved +

$$\phi$$

flat = 0 1

Hence, option B is incorrect.

Now,

$$\phi_{flat} = \int \vec{E}.\, d\vec{A} = \int EdA\cos\theta$$

and

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\left(\sqrt{R^2+r^2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(R^2+r^2)}$$

Also,

$$\cos\theta = \frac{R}{\sqrt{R^2+r^2}}$$

and

$$A = 2\pi r \Rightarrow dA = 2\pi r dr$$

Therefore,

$$\phi_{flat} = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2+r^2}2\pi r dr\frac{R}{\sqrt{R^2+r^2}}$$

$$\phi_{flat} = \frac{QR}{2\varepsilon_0}\int_0^R \frac{rdr}{\left(R^2+r^2\right)^{3/2}}$$

Substituting $R^2 + r^2 = t$, we get

$$2rdr = dt$$

$$\Rightarrow \phi_{flat} = \frac{QR}{2\varepsilon_0}\int_0^R \frac{dt}{2}\frac{1}{t^{3/2}} = \frac{QR}{2\varepsilon_0}\left[\frac{1}{2}\frac{t^{-1/2}}{-1/2}\right]_0^R$$

Substituting

$$t = R^2 + r^2$$

, we get

$$\phi_{flat} = \frac{QR}{2\varepsilon_0}\left[\frac{1}{2}\frac{(R^2+r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$= \frac{QR}{2\varepsilon_0}\left[\frac{-1}{\sqrt{R^2+r^2}}\right]_0^R = \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{R^2+R^2}} + \frac{1}{\sqrt{R^2}}\right)$$

$$= \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2R^2}} + \frac{1}{\sqrt{R^2}}\right) = \frac{QR}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}R} + \frac{1}{R}\right)$$

$$= \frac{QR}{2\varepsilon_0}\frac{1}{R}\left(\frac{-1}{\sqrt{2}} + 1\right) = \frac{Q}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}} + 1\right)$$

Using Eq. 1, we get

$$\phi_{curved} = -\phi_{flat} = -\frac{Q}{2\varepsilon_0}\left(\frac{-1}{\sqrt{2}} + 1\right) = -\frac{Q}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Hence, option A is correct.

The potential at any point on the circumference of the flat surface is

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{2}R}$$

.

Thus, the circumference of flat surface is equipotential.

Question 038 MCQ

QUESTION

Two coherent monochromatic point sources

$$S_1$$

and

$$S_2$$

of wavelength

$$\lambda = 600\,nm$$

are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance

$$d = 1.8$$
$$mm.$$

This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is

$$\Delta\theta.$$

Which of the following options is/are correct?

A The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant.

B At P_2 the order of the fringe will be maximum.

C A dark spot will be formed at the point P_2.

D The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000.

CORRECT OPTION

B At P_2 the order of the fringe will be maximum.

SOURCE

Physics • wave-optics

EXPLANATION

At any point P on circumference, the path difference,

$$\Delta$$

x = d sin

$$\theta$$

At P_1,

$$\theta$$

= 0

$$^{\circ}$$

$$\Rightarrow$$

$$\Delta$$

x = 0

At P_2,

$$\theta$$

= 90

$$^{\circ}$$

$$\Rightarrow$$

Δ

x = d

For constructive interference,

Δ

x = n

λ

$\therefore$

$$n = \frac{d}{\lambda} = \frac{1.8\,mm}{600\,nm} = \frac{1.8 \times 10^{-3} m}{600 \times 10^{-9} m} = 3000$$

Since, n is integer, hence P_2 corresponds to bright spot and also it corresponds to maximum order of fringe.

Now,

Δ

x = d sin

θ

d

Δ

x = d cos

θ

d

θ

or R

λ

= d cos

θ

d

θ

$\therefore$

$$d\theta \propto \frac{1}{\cos\theta}$$

Hence, as

θ

increases, cos

θ

decreases and consequently d

θ

increases.

Question 039 MCQ

QUESTION

The instantaneous voltages at three terminals marked

X, Y

and

Z

are given by

$$V_x = V_0 \sin \omega t,$$

$$V_Y = V_0 \sin$$

$$\left(\omega t + \frac{2\pi}{3}\right)$$

and

$$Vz = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read

$$rms$$

value of the potential difference between its terminals. It is connected between points

$$X$$

and

$$Y$$

and then between

$$Y$$

and

$$Z.$$

The reading s of the voltmeter will be

A $$V_{xy}^{rms} = V_0\sqrt{\frac{3}{2}}$$

B $$V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$$

C $$V_{XY}^{rms} = V_0$$

D Independent of the choice of the two terminals

CORRECT OPTION

A $$V_{xy}^{rms} = V_0\sqrt{\frac{3}{2}}$$

SOURCE

Physics • alternating-current

EXPLANATION

The potential difference between point X and Y is given by

$V_{XY} = V_X$

$-$

V_Y

$= V_0 \sin \$\$\omega\$\$t$

$-$

$V_0 \sin \$\$\omega\$\$t + \$\$\frac{2\pi}{3}\$\$$

$= 2V_0 \cos \$\$\omega\$\$t + \$\$\frac{\pi}{3}\$\$ \sin \$\$ - \$\$\$\$\frac{\pi}{3}\$\$$

=

$-$

$\sqrt{3}$

$V_0 \cos \$\$\omega\$\$t + \$\$\frac{\pi}{3}\$\$ =$

$\sqrt{3}$

$V_0 \sin \$\$\omega\$\$t\$\$ - \$\$\$\$\frac{\pi}{6}\$\$.$

Similarly, the potential difference between point Y and Z is

$V_{YZ} = V_Y$

$-$

V_Z

$= V_0 \sin \$\$\omega\$\$t + \$\$\frac{2\pi}{3}\$\$$

$-$

$V_0 \sin \$\$\omega\$\$t + \$\$\frac{4\pi}{3}\$\$$

$= 2V_0 \cos \$\$\omega\$\$t + \$\$\pi\$\$ \sin \$\$ - \$\$\$\$\frac{\pi}{3}\$\$$

=

$\sqrt{3}$

$V_0 \cos \$\$\omega\$\$t$ =

$\sqrt{3}$

$V_0 \sin \$\$\omega\$\$t + \$\$\frac{\pi}{2}\$\$,$

and the potential difference between point Z and X is

$V_{ZX} = V_Z$

$-$

V_X

$= V_0 \sin \$\$\omega\$\$t + \$\$\frac{4\pi}{3}\$\$$

$-$

$V_0 \sin \$\$\omega\$\$t$

=

$\sqrt{3}$

$V_0 \cos \$\$\omega\$\$ + \$\$\frac{2\pi}{3}\$\$ =$

$$\sqrt{3}$$

$V_0 \sin \$\$\omega\$\$t + \$\$\frac{7\pi}{6}\$\$.$

The rms value of the potential $V = V_0 \sin \$\$\omega\$\$t + \$\$\phi\$\$$ is given by $V^{rms} = V_0$ /

$$\sqrt{2}$$

. Thus, rms values of the potentials are

$$V_{XY}^{rms} = V_{YZ}^{rms} = V_{ZX}^{rms} = V_0\sqrt{3/2}$$

. Hence, the reading of the voltmeter is independent of the two terminal i.e., reading is same whether it is connected across X-Y or Y-Z or Z-X.

Question 040 MCQ

QUESTION

A uniform magnetic field

$$B$$

exist in the region between

$$x = 0$$

and

$$x = \frac{3R}{2}$$

$region\$\$2\$\$in the figure$ pointing normally into the plane of the paper. A particle with charge

$$+Q$$

and momentum

p

directed along

x

-axis enters region

2

from region

1

$atpoint\ P_1\ (y) = -R).\$$ Which of the following option s is/are correct?

A

For

$$B > \frac{2}{3}\frac{p}{QR}$$

the particle well re-enter region

1

B

For

$$B = \frac{8}{13}\frac{p}{QR},$$

the particle will enter region

3

through the point

P_2

on

x

-axis

When the particle re-enters region

$$1$$

through the longest possible path in region

$$2,$$

the magnitude of the change in its linear momentum between point

C

$$P_1$$

and the farthest point from

$$y$$

-axis is

$$p/\sqrt{2}$$

For a fixed

$$B,$$

particles of same charge

$$Q$$

and same velocity

D

$$v,$$

the distance between the point

$$P_1$$

and the point of re-entry into region

$$1$$

is inversely proportional to the mass of the particle

CORRECT OPTION

For

$$B > \frac{2}{3}\frac{p}{QR}$$

A the particle well re-enter region

$$1$$

SOURCE

Physics • magnetism

EXPLANATION

As the magnetic field is perpendicular to motion of particle.

$$\therefore$$

Particle will move in curved path of radius,

$$r = \frac{mv}{QB} = \frac{p}{QB}$$

$$\$\$ \because \$\$p = mv$$

Particle will cross the region 2 if

$$r > \frac{3R}{2} \Rightarrow B < \frac{2p}{3QR}$$

and particle will re-enter region 1 if

$$r < \frac{3R}{2} \Rightarrow B > \frac{2p}{3QR}$$

Here a = 0, b = $$r\$\$ - \$\$R$$

Equation of trajectory,

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\therefore$$

To pass through region 2, at

$$P_2\left(\frac{3R}{2},0\right)$$

,

$$\left(\frac{3R}{2}\right)^2+(r-R)^2=r^2$$

or

$$\frac{9}{4}R^2+r^2+R^2-2rR=r^2$$

$$\frac{13}{4}R^2=2rR$$

or

$$r=\frac{13}{8}R\Rightarrow\frac{p}{QB}=\frac{13}{8}R$$

or

$$B=\frac{8}{13}\frac{p}{QR}$$

Hence, for

$$B=\frac{8}{13}\frac{p}{QR}$$

particle will enter region 3 through P_2

For

$$r<\frac{3R}{2}$$

, particle will re-enter region 1 through point $P_3\,x,y$.

$$y=(2r-R)=\left(\frac{2mv}{QB}-R\right)$$

, x = 0

$$\Rightarrow$$

y

$$\propto$$

m

At farthest point from y-axis, the momentum is perpendicular to initial momentum.

$$\therefore$$

$$\Delta p = \sqrt{2}mv = \sqrt{2}p$$

Question 041 MCQ

QUESTION

A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is

$$\delta T = 0.01$$

seconds and he measures the depth of the well to be

$$L = 20$$

meters. Take the acceleration due to gravity

$$g = 10ms^{-2}$$

and the velocity of sound is

$$300$$

$$ms^{-1}$$

. Then the fractional error in the measurement,

$$\delta L/L,$$

is closest to

A 0.2%

B 1%

C 3%

D 5%

CORRECT OPTION

B 1%

SOURCE

Physics • units-and-measurements

EXPLANATION

Let the time taken by stone to reach bottom of will be t_1 and time taken by sound to reach the observer be t_2 then

$$t_1 = \sqrt{\frac{2L}{g}}$$

.... Missing superscript or subscript argument

and

$$t_2 = \frac{L}{v_s}$$

.... 2

Where L is depth of well

g is acceleration due to gravity

v_s is velocity of sound

Total time taken,

$$T = t_1 + t_2 = \frac{\sqrt{2L}}{g} + \frac{L}{v_s}$$

..... 3

If

$$\delta$$

L is error in depth of well measured and

$$\delta$$

T is error is time measured then

$$T + \delta T = \sqrt{\frac{2(L + \delta L)}{g}} + \frac{(L + \delta L)}{v_s}$$

$$T + \delta T = \sqrt{\frac{2L}{g}\left(1 + \frac{\delta L}{L}\right)} + \frac{L}{v_s}\left(1 + \frac{\delta L}{L}\right)$$

expanding

$$\left(1 + \frac{\delta L}{L}\right)^{1/2}$$

using binomial approximation

$$T+\delta T=\sqrt{\frac{2L}{g}}\left(1+\frac{1}{2}\frac{\delta L}{L}\right)+\frac{L}{V_s}\left(1+\frac{\delta L}{L}\right)$$

$$=\sqrt{\frac{2L}{g}}+\sqrt{\frac{2L}{g}}\left(\frac{1}{2}\frac{\delta L}{L}\right)+\frac{L}{v_s}+\frac{L}{v_s}\frac{\delta L}{L}$$

$$=\sqrt{\frac{2L}{g}}+\frac{L}{v_s}+\left(\frac{1}{2}\sqrt{\frac{2L}{g}}+\frac{L}{v_s}\right)\frac{\delta L}{L}$$

given, L = 20 m, g = 10 m/s, v_s = 300 m/s and using equation 3 :

$$\sqrt{\frac{2L}{g}}+\frac{L}{v_s}=T$$

, above equation becomes

$$T+\delta T=T+\left(\frac{1}{2}\sqrt{\frac{2\times 20}{10}}+\frac{20}{300}\right)\frac{\delta L}{L}$$

$$\Rightarrow \delta T=\left(\frac{1}{2}\sqrt{4}+\frac{1}{15}\right)\frac{\delta L}{L}=\left(1+\frac{1}{15}\right)\frac{\delta L}{L}=\left(\frac{16}{15}\right)\frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta L}{L} = \delta T \left(\frac{15}{16}\right)$$

It is given that

$$\delta$$

T = 0.01 sec, therefore,

$$\left(\frac{\delta L}{L}\right) \times 100\% = \frac{15}{16} \times \frac{1}{100} \times 100\% = \frac{15}{16}\%$$

error

$$\simeq 1\%$$

Answer B

Question 042 MCQ

QUESTION

A rocket is launched normal to the surface of the Earth, away from the sun, along the line joining the Sun and the Earth. The Sun is

$$3 \times 10^5$$

times heavier than the earth and is at a distance

$$2.5 \times 10^4$$

times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is

$$V_c = 11.2km\,s^{-1}.$$

. The minimum initial velocity

$$(v_s)$$

required for the rocket to be able to leave the sun-earth system is closest to
$Ignorethetherotationandrevoluationoftheearthandthepresenceofanyothe$

A $v_s = 22\,km\,s^{-1}$

B $v_s = 42\,km\,s^{-1}$

C $v_s = 62km\,s^{-1}$

D $v_s = 72kms^{-1}$

CORRECT OPTION

B $v_s = 42\,km\,s^{-1}$

SOURCE

Physics • gravitation

EXPLANATION

Given : Mass of the sun, M_s = 3

$\times$

10^5

$\times$

mass of the earth = 3

$\times$

10^5 M_e

Distance between the sun and the earth,

d = 2.5

$\times$

10^4

$\times$

radius of the earth = 2.5

$\times$

10^4 R_e,

Escape speed, v_e = 11.2 km s

$-$

1

$\therefore$

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = 11.2$$

km s

$-$

1

Minimum velocity required for the rocket to escape the given earth

$-$

sun system = v_s

Using energy conservation for the given situation,

$$\frac{1}{2}mv_s^2 = \frac{GM_em}{R_e} + \frac{GM_sm}{d+R_e} = \frac{GM_em}{R_e} + \frac{G\times 3\times 10^5 M_em}{(2.5\times 10^4 R_e + R_e)}$$

$$\therefore$$

$$v_s^2 = \frac{2GM_e}{R_e} + \frac{24GM_e}{R_e}$$

(

$$\because$$

2.5

$$\times$$

10^4 >> 1)

$$\Rightarrow v_s^2 = v_e^2 + 12v_e^2 = 13v_e^2$$

or

$$v_s = \sqrt{13}$$

v_e = 40.38 km s

$$-$$

1

$$\approx$$

42 km s

$$-$$

1

Question 043 MCQ

QUESTION

Three vectors

$$\overrightarrow{P}, \overrightarrow{Q}$$

and

$$\overrightarrow{R}$$

are shown in the figure. Let

$$S$$

be any point on the vector

$$\overrightarrow{R}.$$

The distance between the points

$$P$$

and

$$S$$

is

$$b\left|\overrightarrow{R}\right|.$$

The general relation among vectors

$$\overrightarrow{P}, \overrightarrow{Q}$$

and

$$\overrightarrow{S}$$

is :

A $\overrightarrow{S} = (1-b)\overrightarrow{P} + b\overrightarrow{Q}$

B $\overrightarrow{S} = (b-1)\overrightarrow{P} + b\overrightarrow{Q}$

C $\overrightarrow{S} = (1-b^2)\overrightarrow{P} + b\overrightarrow{Q}$

D $\overrightarrow{S} = (1-b)\overrightarrow{P} + b^2\overrightarrow{Q}$

CORRECT OPTION

A $\overrightarrow{S} = (1-b)\overrightarrow{P} + b\overrightarrow{Q}$

SOURCE

Physics • motion

EXPLANATION

From figure we can write

$$\overrightarrow{P} = b\overrightarrow{R} = \overrightarrow{S}$$

It is given that

$$\overrightarrow{R} = \overrightarrow{Q} - \overrightarrow{P}$$

. Put in above equation we get

$$\vec{P} + b(\vec{Q} - \vec{P}) = \vec{S} \Rightarrow \vec{P} + b\vec{Q} - b\vec{P} = \vec{S}$$

$$\Rightarrow \vec{S} = \vec{P}(1 - \vec{b}) + b\vec{Q}$$

Question 044 MCQ

QUESTION

Consider regular polygons with number of sides

$$n = 3, 4, 5 \ldots$$

as shown in the figure. The center of mass of all the polygons is at height

$$h$$

from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is

$$\Delta$$

. Then

$$\Delta$$

depends on

$$n$$

and

$$h$$

as

A $$\Delta = h\sin^2\left(\frac{\pi}{n}\right)$$

B $$\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$$

C $$\Delta = h\sin\left(\frac{2\pi}{n}\right)$$

D $$\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$$

CORRECT OPTION

B $$\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$$

SOURCE

Physics • rotational-motion

EXPLANATION

Let n be the number of sides of a regular polygon. By symmetry, its centre of mass O will be equidistant from each vertex i.e., it lies at the centre of the circumscribed circle. Let r be the radius of circumscribed circle and h be the perpendicular distance of O from any side $see figure$. The angle subtended by any side on the centre O is 2

$$\pi$$

/n and

$$\angle$$

PON =

$$\pi$$

/n.

When polygon rolls about the vertex P $without slipping or sliding$, the point O moves in a circle of radius r centred at P. The point O reaches the maximum height $point O' in the figure$ when PO' is perpendicular to PQ. Thus, the maximum increase in height of the locus of the centre of mass O is given by

$$\Delta = r - h = \frac{h}{\cos(\pi/n)} - h = h\left(\frac{1}{\cos(\pi/n)} - 1\right)$$

.

Question 045 MCQ

QUESTION

A symmetric star shaped conducting wire loop is carrying a steady state current

$$\mathrm{I}$$

as shown in the figure. The distance between the diametrically opposite vertices of the star is

$$4a.$$

The magnitude of the magnetic field at the center of the loop is

A $\frac{\mu_0 1}{4\pi a} 6\left[\sqrt{3} - 1\right]$

B $\frac{\mu_0 1}{4\pi a} 6\left[\sqrt{3}+1\right]$

C $\frac{\mu_0 1}{4\pi a} 3\left[\sqrt{3}-1\right]$

D $\frac{\mu_0 1}{4\pi a} 3\left[2-\sqrt{3}\right]$

CORRECT OPTION

A $\frac{\mu_0 1}{4\pi a} 6\left[\sqrt{3}-1\right]$

SOURCE

Physics • magnetism

EXPLANATION

The star shape is composed of 12 wires. Thus, the total magnetic field at centre is 12 times of magnetic field due to one wire.

Let us first calculate the magnetic field due to element AB. Perpendicular distance of centre O from element AB, OP = a

Angle subtended by centre at element are

$$\theta$$

$_1$ = 30

$$\circ$$

,

$$\theta$$

$_2$ = 60

$$\circ$$

as shown in figure.

$$\therefore$$

$$\overrightarrow{B} = \frac{\mu_0 I}{4\pi a}(\cos 30^\circ - \cos 60^\circ)\odot$$

$$= \frac{\mu_0 I}{4\pi a}\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)\odot = \frac{\mu_0 I}{8\pi a}(\sqrt{3}-1)\odot$$

Since the direction of magnetic field due to each element side is out of the paper

$$\therefore$$

Net magnetic field at O

$$= 12 \times \frac{\mu_0 I}{8\pi a}(\sqrt{3}-1)$$

$$= \frac{\mu_0 I}{4\pi a}6(\sqrt{3}-1)$$

Question 046 MCQ

QUESTION

A photoelectric material having work-function

$$\phi_0$$

is illuminated with light of wavelength

$$\lambda\left(\lambda < \frac{he}{\phi_0}\right).$$

The fastest photoelectron has a de-Broglic wavelength

$$\lambda_d.$$

A change in wavelength of the incident light by

$$\Delta\lambda$$

result in a change

$$\Delta\lambda_d$$

in

$$\lambda_d.$$

Then the ratio

$$\Delta\lambda_d/\Delta\lambda$$

is proportional to

A λ_d/λ

B λ_d^2/λ^2

C λ_d^3/λ

D λ_d^3/λ^2

CORRECT OPTION

D λ_d^3/λ^2

SOURCE

Physics • dual-nature-of-radiation

EXPLANATION

Here,

$$\lambda$$

is the wavelength of incident light and

$$\lambda$$

d is the de Broglie wavelength of the fastest photoelectron. The fastest ejected photoelectron has the maximum kinetic energy which is given by

$$K_{\max} = \frac{hc}{\lambda} - \phi_0$$

..... 1

The de Broglie wavelength of the photoelectron having kinetic energy

$$K_{\max} = \frac{p^2}{2m}$$

is given by

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{2mK_{\max}}}$$

..... 2

where m is the mass of the electron and p is its linear momentum. Eliminate K_{max} from equations 1 and 2 to get

$$\frac{h^2}{2m\lambda_d^2} = \frac{hc}{\lambda} - \phi_0$$

..... 3

Differentiate equation 3 to get

$$(-2)\frac{h^2}{2m\lambda_d^3}\Delta\lambda_d = (-1)\frac{hc}{\lambda^2}\Delta\lambda$$

,

which gives

$$\frac{\Delta\lambda_d}{\Delta\lambda} = \frac{mc}{h}\frac{\lambda_d^3}{\lambda^2}$$

.

Question 047 MCQ

QUESTION

Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density

$$\rho$$

remains uniform throughout the volume. The rate of fractional change in density

$$\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$$

is constant. The velocity

$$v$$

of any point on the surface of the expanding sphere is proportional to

A R

B R^3

C $\frac{1}{R}$

D $R^{2/3}$

CORRECT OPTION

A R

SOURCE

Physics • motion

EXPLANATION

Density

$$\rho = \frac{mass\,(m)}{volume\,(V)}$$

Volume of sphere

$$= \frac{4}{3}\pi R^3 \Rightarrow \rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{3m}{4\pi R^3}$$

Rearranging it we get

$$\rho R^2 = \frac{3}{4\pi}m$$

It is given that mas m remains constant. Let the constant be k. Therefore,

$$\rho R^3 = k$$

Differentiating it w.r.t. time, we get

$$\frac{d}{dt}(\rho R^3) = 0$$

$Since, differentiation of constant is zero$

$$R^3 \frac{d\rho}{dt} + 3R^2 \rho \frac{dR}{dt} = 0$$

Now

$$\frac{dR}{dt}$$

, that is, rate of change of radius is equal to velocity. Therefore,

$$R^3 \frac{d\rho}{dt} + 3R^2 \rho v = 0 \Rightarrow 3R^2 \rho v = -R^3 \frac{d\rho}{dt}$$

$$\Rightarrow v = \frac{-1}{3} \frac{R^3}{R^2} \frac{1}{\rho} \frac{d\rho}{dt} = \frac{-1R}{3} \frac{1}{\rho} \frac{d\rho}{dt}$$

It is given that rate of fractional change in density

$$\left(\frac{1}{\rho} \frac{d\rho}{dt} \right)$$

is constant. Thus,

v

$$\propto$$

R

Thus velocity of any point on surface of expanding sphere is proportional to R.

Question 048 MCQ

QUESTION

A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque

$$\tau$$

about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?

A If the force is applied normal to the circumference at point P, then τ is zero

B If the force is applied tangentially at point S, then $\tau \neq$ 0 but the wheel never climbs the step

C If the force is applied at point P tangentially, then τ decreases continuously as the wheel climbs

D If the force is applied normal to the circumference at point X, then τ is constant

CORRECT OPTION

A If the force is applied normal to the circumference at point P, then τ is zero

SOURCE

Physics • rotational-motion

EXPLANATION

a If force is applied normal to surface at P, then line of action of force will pass from Q and thus

τ

= 0.

b Wheel can climb.

c

τ

= F $2Rcos\$\$\theta\$\$$

$-$

mgRcos

θ

$\Rightarrow$

τ

$\propto$

cos

θ

Hence, as

θ

increases,

τ

decreases. So its correct.

d

τ

= Fr

$\perp$

$-$

mgcos

θ

;

τ

increases with

θ

.

Question 049 MCQ

QUESTION

A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor $asshowninthefigure$. At some instant of time, the angle made by the bar with the vertical is

θ

. Which of the following statements about its motion is/are correct?

Instantaneous torque about the point in contact with the floor is proportional to sin

θ

B The trajectory of the point A is parabola

C The mid-point of the bar will fall vertically downward

D When the bar makes an angle

θ

with the vertical, the displacement of its mid-point from the initial position is proportional to $1$$ − $$cos$$θ$$

CORRECT OPTION

A Instantaneous torque about the point in contact with the floor is proportional to sin

θ

SOURCE

Physics • rotational-motion

EXPLANATION

We discuss the options as follows:

•

For option A : When the bar makes an angle

θ

with the floor, the height of the midpoint is

$$\frac{L}{2}\cos\theta$$

. this is height of centre of mass.

A force mg acts vertically downward. Thus, the midpoint of bar falls vertically downward. Hence, option A is correct.

•

For option B : Here, we have

$$x = \frac{L}{2}\sin\theta$$

and

$$y = L\cos\theta$$

; therefore,

$$\sin\theta = \frac{x}{L/2}$$

and

$$\cos\theta = \frac{y}{L}$$

Using the condition

$$\sin^2\theta + \cos^2\theta = 1$$

, we get

$$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$$

$equation of ellipse$

Thus, trajectory of point A is not parabola. Hence, option B is incorrect.

•

For option C : The torque acting about the point of contact with floor, that is, at point B is

$$\tau = \overrightarrow{F} \times \overrightarrow{r} = mg\frac{L}{2}\sin\theta$$

Thus, the torque is proportional to sin

θ

. Hence, option C is correct.

•

For option D : The displacement of midpoint is

$$\frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1 - \cos\theta)$$

That is,

Displacement

$\propto$

$1\$\$ - \$\$cos\$\$\theta\$\$$

Hence, option D is correct.

Question 050 MCQ

QUESTION

A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct?

A. After a long time, the current through L_1 will be

$$\frac{V}{R}\frac{L_2}{L_1+L_2}$$

B. After a long time, the current through L_2 will be

$$\frac{V}{R}\frac{L_1}{L_1+L_2}$$

C. The ratio of the currents through L_1 and L_2 is fixed at all times $t > 0$

D. At t = 0, the current through the resistance R is

$$\frac{V}{R}$$

CORRECT OPTION

A. After a long time, the current through L_1 will be

$$\frac{V}{R}\frac{L_2}{L_1+L_2}$$

SOURCE

Physics • electromagnetic-induction

EXPLANATION

Let i_1 be the current through L_1, i_2 be the current through L_2 and i be the current through R.

The inductors L_1 and L_2 are connected in parallel. Therefore, the self-induced emf in L_1 and L_2 are equal, which can be expressed as:

$$\varepsilon_1 = -L_1 \frac{di_1}{dt} = \varepsilon_2 = -L_2 \frac{di_2}{dt}$$

Upon integrating this equation, we get :

$$L_1 i_1 = L_2 i_2 \text{ .....} 1$$

This indicates that the ratio of the currents through L_1 and L_2 is constant.

Using Kirchhoff's current law at junction A, we have :

$$i = i_1 + i_2 \text{ .....} 2$$

At the moment the switch is closed $t = 0$, the inductors act as open circuits $since the inductive reactance \$X_L = \omega L \$ approaches infinity as \$\omega \to \infty \$$. Therefore, initially :

$$i = i_1 + i_2 = 0$$

and the current through the resistor is :

$$i = i_1 + i_2 = 0$$

After a long time, the inductive reactance of the inductors becomes zero. Applying Kirchhoff's loop law, the current through the resistor R is :

$$i = \frac{V}{R}$$

Substituting $i = \frac{V}{R}$ into equation 2 and solving equations 1 and 2, we find :

$$i_1 = \frac{V}{R} \frac{L_2}{L_1 + L_2}$$

$i_2 = \frac{V}{R}\frac{L_1}{L_1+L_2}$

Question 051 MCQ

QUESTION

In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by

A $E_C = E_D \ln 2$

B $E_C = E_D$

C $E_C = 2E_D$

D $E_C = \frac{1}{2} E_D$

CORRECT OPTION

B $E_C = E_D$

SOURCE

Physics • capacitor

EXPLANATION

Energy supplied to the circuit is CV

$$^{2}_{0}$$

Energy stored in capacitor, E_C =

$$\frac{1}{2}$$

CV

$$^{2}_{0}$$

Therefore,

Energy dissipated E_0 = Energy supplied

$$—$$

Energy stored

= CV

$$^{2}_{0}$$

$$—$$

$$\frac{1}{2}$$

CV

$$^{2}_{0}$$

=

$$\frac{1}{2}$$

CV

$$\frac{2}{0}$$

$$\Rightarrow$$

$E_C = E_D$

Question 052 MCQ

QUESTION

In Process 2, total energy dissipated across the resistance E_D is

A $$E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

B $$E_D = 3\left(\frac{1}{2}CV_0^2\right)$$

C $$E_D = 3CV_0^2$$

D $$E_D = \frac{1}{2}CV_0^2$$

CORRECT OPTION

A

$$E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

SOURCE

Physics • capacitor

EXPLANATION

Process 1 :

Voltage is set to $V_0/3$

Charge supplied =

$$\frac{CV_0}{3}$$

Energy supplied =

$$\frac{V_0}{3} \times \frac{CV_0}{3} = \frac{CV_0^2}{9}$$

Process 2 :

Voltage is raised to $2V_0/3$

additional charge supplied =

$$\frac{2V_0C}{3} - \frac{V_0C}{3} = \frac{CV_0}{3}$$

Energy supplied =

$$\frac{2V_0}{3} \times \frac{CV_0}{3} = \frac{2CV_0^2}{9}$$

Process 3 :

Voltage is raised to V_0

additional charge supplied =

$$V_0C - \frac{2V_0C}{3} = \frac{CV_0}{3}$$

Energy supplied =

$$V_0 \times \frac{CV_0}{3} = \frac{CV_0^2}{3}$$

Total energy supplied to circuit =

$$\frac{CV_0^2}{9} + \frac{2CV_0^2}{9} + \frac{CV_0^2}{3}$$

$$= \frac{6}{9}CV_0^2 = \frac{2}{3}CV_0^2$$

Final energy stored in capacitor =

$$\frac{1}{2}CV_0^2$$

Therefore,

Energy dissipated E_D = Energy supplied

$$-$$

Energy stored

$$= \frac{2}{3}CV_0^2 - \frac{1}{2}CV_0^2$$

$$= \frac{4CV_0^2 - 3CV_0^2}{6} = \frac{1}{6}CV_0^2$$

$$= \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$$

Question 053 MCQ

QUESTION

The total kinetic energy of the ring is

A $M\omega_0^2(R-r)^2$

B $\frac{1}{2}M\omega_0^2(R-r)^2$

C $M\omega_0^2 R^2$

D $\frac{1}{2}M\omega_0^2[(R-r)^2+R^2]$

CORRECT OPTION

D $\frac{1}{2}M\omega_0^2[(R-r)^2+R^2]$

SOURCE

Physics • rotational-motion

EXPLANATION

Let P be the contact point of the finger and the ring. The point P revolves with an angular velocity

ω

$_0$ in a circle of radius r centred at the point O $see figure$. The contact point P, the centre O and the centre of the ring C are in a straight line because P is common to both the circles $i.e., circular trajectory and circular ring$. Thus, the line CP will have same angular velocity as the line OP i.e., angular velocity of the line CP $or thering$ is

ω

0·

The velocity of the finger at the point P is

ω

$_0$r tangential to the circle in which it revolves. The ring rolls without slipping. Thus, velocity of the ring at the point P is also

$$\omega$$

$_0$r tangential to the circle in which the finger revolves. Let velocity of C be v_{cm}. Since P and C lie on the ring $arigidbody$, the velocities of C and P are related by

$$\overrightarrow{v_C} = \overrightarrow{v_P} + \overrightarrow{\omega} \times \overrightarrow{PC}$$

which gives

v_{cm} =

$$\omega$$

$_0 R\$\$ - \$\$r$.

The ring rotates with an angular velocity

$$\omega$$

=

$$\omega$$

$_0$ and its centre of mass translates with a velocity v_{cm} =

$$\omega$$

$_0 R\$\$ - \$\$r$. Thus, kinetic energy of the ring is given by

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

$$= \frac{1}{2} M \omega_0^2 (R - r)^2 + \frac{1}{2} M R^2 \omega_0^2$$

$$= \frac{1}{2} M\omega_0^2[(R-r)^2 + R^2]$$

.

Question 054 MCQ

QUESTION

The minimum value of

$$\omega$$

$_0$ below which the ring will drop down is

A $$\sqrt{\frac{g}{2\mu(R-r)}}$$

B $$\sqrt{\frac{3g}{2\mu(R-r)}}$$

C $$\sqrt{\frac{g}{\mu(R-r)}}$$

D $$\sqrt{\frac{2g}{\mu(R-r)}}$$

CORRECT OPTION

C $$\sqrt{\frac{g}{\mu(R-r)}}$$

SOURCE

Physics • rotational-motion

EXPLANATION

Let the ring makes an angle

$$\alpha$$

with the horizontal and the finger makes an angle

$$\beta$$

with the vertical $see figure$. The forces acting on the ring are its weight mg at the centre C, normal reaction N and the frictional force f at the contact point P. Resolve the forces in the horizontal and the vertical directions. The centre of mass C rotates with an angular velocity

$$\omega$$

$_0$ in a horizontal circle of radius $R\$\$ - \$\$r$ cos

$$\alpha$$

centred at O. Apply Newton's second law in the horizontal and the vertical directions to get

$$f\cos\beta - N\sin\beta - Mg = 0$$

..... 1

$$N\cos\beta + f\sin\beta = M\omega_0^2(R-r)\cos\alpha$$

.... 2

The minimum value of

ω

$_0$ occurs when the frictional force attains its limiting value i.e.,

$$f = \mu N$$

..... 3

Eliminate f and N from equations 1 - 3 to get

$$\omega_{0,\,\min} = \left[\frac{(\cos\beta + \mu\sin\beta)g}{(\mu\cos\beta - \sin\beta)(R-r)\cos\alpha}\right]^{1/2}$$

$$\approx \left[\frac{g}{\mu(R-r)}\right]^{1/2}$$

$\$\$ \because \$\$\$\$\alpha\$\$ = 0 and \$\$\beta\$\$ = 0.$