**Optimization Project**

All work is to be done in project teams. Each team will turn in one final report. The project is due by December 10 at 11:59PM.

Your report should be presented in a professional manner and will be graded on correctness, completeness, clarity, presentation, creativeness, etc.

Your task is to analyze the capacity expansion, production, and distribution for a new product at XYZ Manufacturing, and write a report on your conclusions and recommendations. When deciding what information to include in your report, keep in mind that the report is intended for management personnel. That is, management would tend to be much more interested in the system requirements and analysis than in the details of the mathematical model.

A basic report will include (i) a brief overview of the problem to establish a baseline for management, (ii) a summary of results and recommendations, (iii) any additional analyses that you deem appropriate, and (iv) appendices that include your Python program(s), data, and model statistics. Contents should be put into a single folder, with a “Read Me” file to describe the contents, and then ZIPPED and uploaded to Blackboard.

Try to be clear and concise rather than long and wordy. That is, given a choice, present results in clearly laid-out tables, charts, and graphics rather than in prose form. Your report should provide the manager with a valuable decision making tool.

A complete description of the problem follows.

**Problem Description**

XYZ Manufacturing plans to distribute a new product, the Flugel and would like to do some long-range planning regarding its production and distribution. There are 8 Retail Centers where the product will be sold and the Forecasting Department has estimated that the year-one demand for Flugels will be as follows:

|  |  |
| --- | --- |
| Retail Center | Year-1 Demand |
| 1 | 1000 |
| 2 | 1200 |
| 3 | 1800 |
| 4 | 1200 |
| 5 | 1000 |
| 6 | 1400 |
| 7 | 1600 |
| 8 | 1000 |

Forecasting has also estimated that demand at Retail Centers 1, 2, 4, 5, and 8 will grow yearly at a rate of 20% of the year-1 demand. For example, the demand at Retail Center 1 in year *t* is estimated to be *1000 + 0.20(1000)(t – 1)*. The corresponding percentage for Retail Centers 3, 6, and 7 is 25%.

The company has several existing plants and has a choice of building (and then operating) a new production line at the plants with the specifications and estimated year-1 costs shown in the following table. In addition to construction costs and yearly operating costs, XYZ may shut down a line that is operating or reopen a line that has been previously shut down. You can envision a line that is shut down as being converted for other plant purposes. Similarly, a line that is reopened is being converted back to Flugel production. For simplicity, assume that a line may be reopened at the beginning of each year or shut down at the end of each year. Also assume that in the initial construction year, a line incurs construction costs as well as start-up (i.e., reopening) costs. However, if the line has already been constructed and is then shut down at the end of some year, the plant only incurs the reopening cost to begin production again. Of course, operating costs are only realized if production actually occurs in a given year. And, clearly, a production line cannot be allowed to remain idle in a given year.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Plant | Capacity (units) | Construction Cost (1000's of $) | Annual Operating Cost (1000's of $) | Reopening Cost (1000's of $) | Shutdown Cost (1000s of $) |
| 1 | 16000 | 2000 | 420 | 190 | 170 |
| 2 | 12000 | 1600 | 380 | 150 | 120 |
| 3 | 14000 | 1800 | 460 | 160 | 130 |
| 4 | 10000 | 900 | 280 | 100 | 80 |
| 5 | 13000 | 1500 | 340 | 130 | 110 |

Flugels will be stored and distributed using XYZ’s existing Warehouses. There is no specific per unit charge for storage at a Warehouse. However, since stored items require considerable capital investment XYZ would like to limit the number of units stored. Over the course of a year, obviously the inventory level fluctuates as items flow into and out of the warehouses. To simplify modeling, XYZ has decided to model an average inventory level based on the inventory level at the beginning of the year and at the end of the year. The average inventory level for the year will be the average of these two inventory levels. XYZ has determined, based on a simple average cost model, that they can keep the capital investment within reasonable bounds if they maintain the average inventory in any year to be no more than 4000 items (among all Warehouses).

The existing warehouse system handles many other products and services, and XYZ estimates that the maximum that any warehouse can realistically handle is an average of 1000 Flugels per month (i.e., 12000 per year). Since some units may be stored in a warehouse, both the flow into a warehouse and the flow out of a warehouse should not exceed an average of 1000 units per month.

In addition to the operating expenses (which include overhead items such as electrical power, salaries, etc.), the production of Flugels requires two key resources. Each Flugel requires 4.7 pounds of alloy, and a given plant can acquire a maximum of 60000 pounds of alloy per year. The year-1 cost for one pound of alloy is estimated to be 0.02 (in 1000’s of $). Also, each Flugel requires 3 Widget subassemblies. The year-1 cost of each Widget subassembly is 0.15 (in 1000’s of $), however, at a given plant, it is estimated that the cost will drop to 0.12 (in 1000’s of $) after the first 9000 Widget subassemblies have been purchased in a particular year. Each plant can purchase as many Widget subassemblies as needed, and it is assumed that both resources will be supplied on a just-in-time basis.

The estimated year-1 cost of shipping a Flugel from Plant *i* to Warehouse *j* is given in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Plant | Warehouse 1 (1000's of $) | Warehouse 2 (1000's of $) | Warehouse 3 (1000's of $) | Warehouse 4 (1000's of $) |
| 1 | 0.12 | 0.13 | 0.08 | 0.05 |
| 2 | 0.1 | 0.03 | 0.1 | 0.09 |
| 3 | 0.05 | 0.07 | 0.06 | 0.03 |
| 4 | 0.06 | 0.03 | 0.07 | 0.07 |
| 5 | 0.06 | 0.02 | 0.04 | 0.08 |

Finally, the estimated year-1 unit shipping cost from each Warehouse to each Retail Center is given in the following table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Warehouse | Retail 1 (1000's of $) | Retail 2 (1000's of $) | Retail 3 (1000's of $) | Retail 4 (1000's of $) |
| 1 | 0.09 | 0.1 | 0.06 | 0.05 |
| 2 | 0.05 | 0.07 | 0.12 | 0.04 |
| 3 | 0.06 | 0.09 | 0.07 | 0.09 |
| 4 | 0.07 | 0.08 | 0.09 | 0.06 |
|  |  |  |  |  |
|  |  |  |  |  |
| Warehouse | Retail 5 (1000's of $) | Retail 6 (1000's of $) | Retail 7 (1000's of $) | Retail 8 (1000's of $) |
| 1 | 0.08 | 0.09 | 0.02 | 0.12 |
| 2 | 0.03 | 0.09 | 0.03 | 0.08 |
| 3 | 0.09 | 0.04 | 0.11 | 0.07 |
| 4 | 0.1 | 0.07 | 0.06 | 0.09 |

Costs are expected to rise at the rate of 3% per year. That is, if *c* is a particular cost in year 1, then the cost in year *t* is estimated to be *c(1 + 0.03)t-1*.

Determine the strategy that minimizes the total cost of meeting the expected demand over the next 10 years.

**Indexed Sets:**

R = (1,2,3,4,5,6,7,8) number of retail centers

W = (1,2,3,4) number of warehouses

P = (1,2,3,4,5) number of plants

Y = (1,2,3,4,5,6,7,8,9,10) number of years

**Data:**

**Demand:**

DRY = Demand for retailer R in year Y

GR = growth rate for retailer R

**Plants:**

PCP = production capacity of plant P

CCPY = construction cost of plant P in year Y

OCPY = operation cost of plant P in year Y

RCPY = reopening cost of plant P in year Y

SCPY = shutdown cost of plant P in year Y

**Warehouses:**

IMax = maximum average total inventory (and also a type of movie theater)

FMax = maximum average monthly flugels

**Flugel Production**

APF = lbs of alloy per flugel

APPMax = maximum number of alloys any plant can order

ACY = alloy cost in year Y

WPF = number of widgets per flugel

OWCY = original cost of widgets in year Y

DWCY = discounted cost of widgets in year Y

WTD = number of widgets needed to purchase before discount applied (segment break)

**Shipping:**

PWSCPWY = shipping cost from plant P to warehouse W in year Y

WRSCWRY = shipping cost from warehouse W to retailer R in year Y

C = annual cost growth rate for all costs

**Decision Variables:**

OPY = 0 or 1 – binary variable indicating whether plant P was used in year Y

PWFPWY = number of flugels sent from plant P to warehouse W in year Y

A1PY = 0 or 1 – binary indicating whether to use non discount function for widgets for plant P in year Y

A2PY = 0 or 1 – binary indicating whether to use discount function for widgets for plant P in year Y

λ1PY, λ2PY, λ3PY = breakpoint variables (0,WTD/WPF,PCPY)

WRFWRY = number of flugels sent from warehouse W to retailer R in year Y

CDPY = 0 or 1 – binary variable indicating whether or not construction cost (forced by constraints)

RDPY = 1 or 0 – binary variable indicating whether or not reopening cost (forced by constraints)

SDPY = 0 or 1 – binary variable indicating whether or not shutdown cost (forced by constraints)

EIWY = remaining inventory in warehouse W in year Y

**Objective: Minimize Costs of meeting expected demand over 10 years**

Cost functions for yearly growth:

Alloy Costs:

{Total alloy cost of all flugels across all years}

Shipping Costs:

{Ttl ship cost from plant to warehouse all years}

{Ttl ship cost from warehouse to retail all years}

Plant Costs:

{Operating costs for plants all years}

{Construction costs for plants at all years}

{Reopening costs for plants at all years}

{Shutdown costs for plants at all years}

Widget costs:

f(PWFPWY) = PWFPWY × WPF × OWCY × (1 – C)Y-1 0 ≤ PWFPWY ≤

WTD× OWCY × (1 – C)Y-1 + (PWFPWY × WPF – ) × DWCY × (1 – C)Y-1  ≤ PWFPWY ≤

=

Plug in PWF into f(PWF):

Cost =

Minimize: sum of all the cost structures above

**Constraints:**

Widget Costs:

λ1 + λ2 + λ3 = 1

λ1 <= A1

λ2 <= Y1 + Y2

λ3 <= A2

A1 + A2 = 1

A’s binary

λi >= 0 for all I

Production and Shipping:

{Flugels in plant P must not exceed capacity each year}

{Flugels entering warehouse must not exceed capacity}

{Flugels exiting warehouse must not exceed capacity}

{Flow into warehouse must meet flow out of warehouse each year}

{Average inventory cannot exceed limits}

{Flugels sent to retailers must meet demand}

{Each plant cannot exceed alloy limit}

for all PWY pairs; for all WRY pairs; EIWY for all WY pairs {Non negative}

Plant Operation:

OPY × PCP ≥ PWFPWY {Operation Costs on when units produced}

RDPY ≥OPY  - OP(Y-1) {Reopening costs}

SDPY ≥OP(Y-1)  - OPY {Shutdown costs}

CDPY ≥ {Construction Costs on when units produced}

OPY – binary variable

RDPY – binary variable

SDPY – binary variable

CDPY – binary variable