CSCI 5352 Network Analysis and Modeling Fall 2020
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Problem Set 3

1. (20 pts) Consider the following simple and rather unrealistic model of a network: each of n vertices belongs to one of g groups. The mth group has n_m vertices and each vertex in that group is connected to others in the group with independent probability $p_m = A(n_m - 1)^{-\beta}$, where A and β are constants, but not to any vertices in other groups. Thus, this network takes the form of a set of disjoint groups of communities.
   (a) Calculate the expected degree $\langle k \rangle$ of a vertex in group m.
   (b) Calculate the expected value $\langle C_m \rangle$ of the local clustering coefficient for vertices in group m.
   (c) Hence show that $\langle C_m \rangle \propto \langle k \rangle^{-\beta/(1-\beta)}$. What value would β have to assume for the expected value of the local clustering coefficient to fall off as $\langle k \rangle^{-0.75}$, as has been conjectured by some researchers?

2. (20 pts) Consider the random graph $G(n, p)$ with average degree c.
   (a) Show that in the limit of large n the expected number of triangles in the network is $\frac{1}{6}c^3$. In other words, show that the number of triangles is constant, neither growing nor vanishing in the limit of large n.
   (b) Show that the expected number of connected triples in the network, as in Eq. (7.28) in *Networks* [v1: 7.41], is $\frac{1}{2}nc^2$.
   (c) Hence, calculate the clustering coefficient C, as defined in Eq. (7.28) in *Networks* [v1: 7.41], and confirm that it agrees for large n with the value given in Eq. (11.11) in *Networks* [v1: 12.11].