Networks & Hierarchies

Daniel Larremore
Assistant Professor
Dept. of Computer Science
& BioFrontiers Institute

CSCI 5352 - Fall 2019

University of Colorado **Boulder**

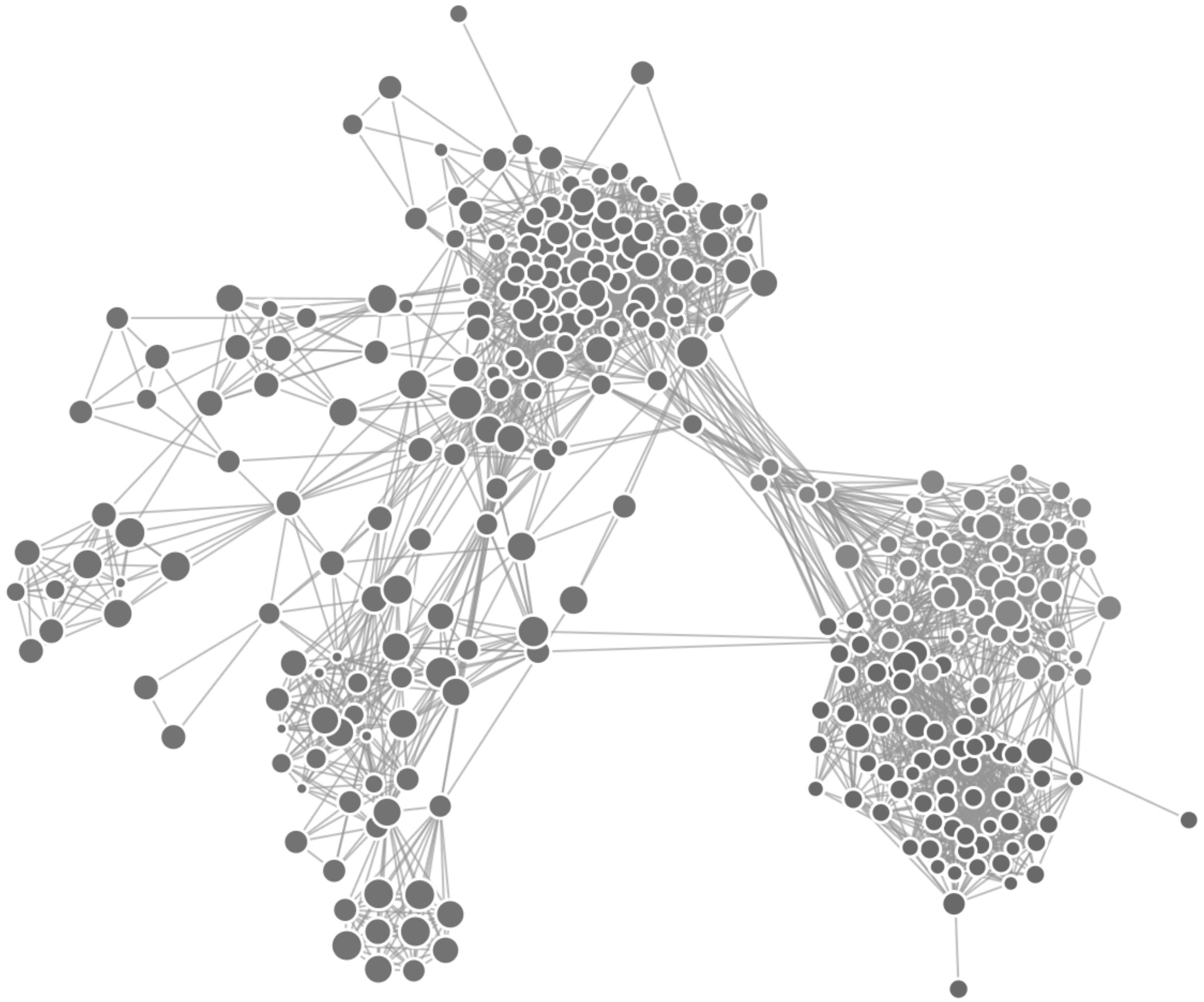
daniel.larremore@colorado.edu
@danlarremore

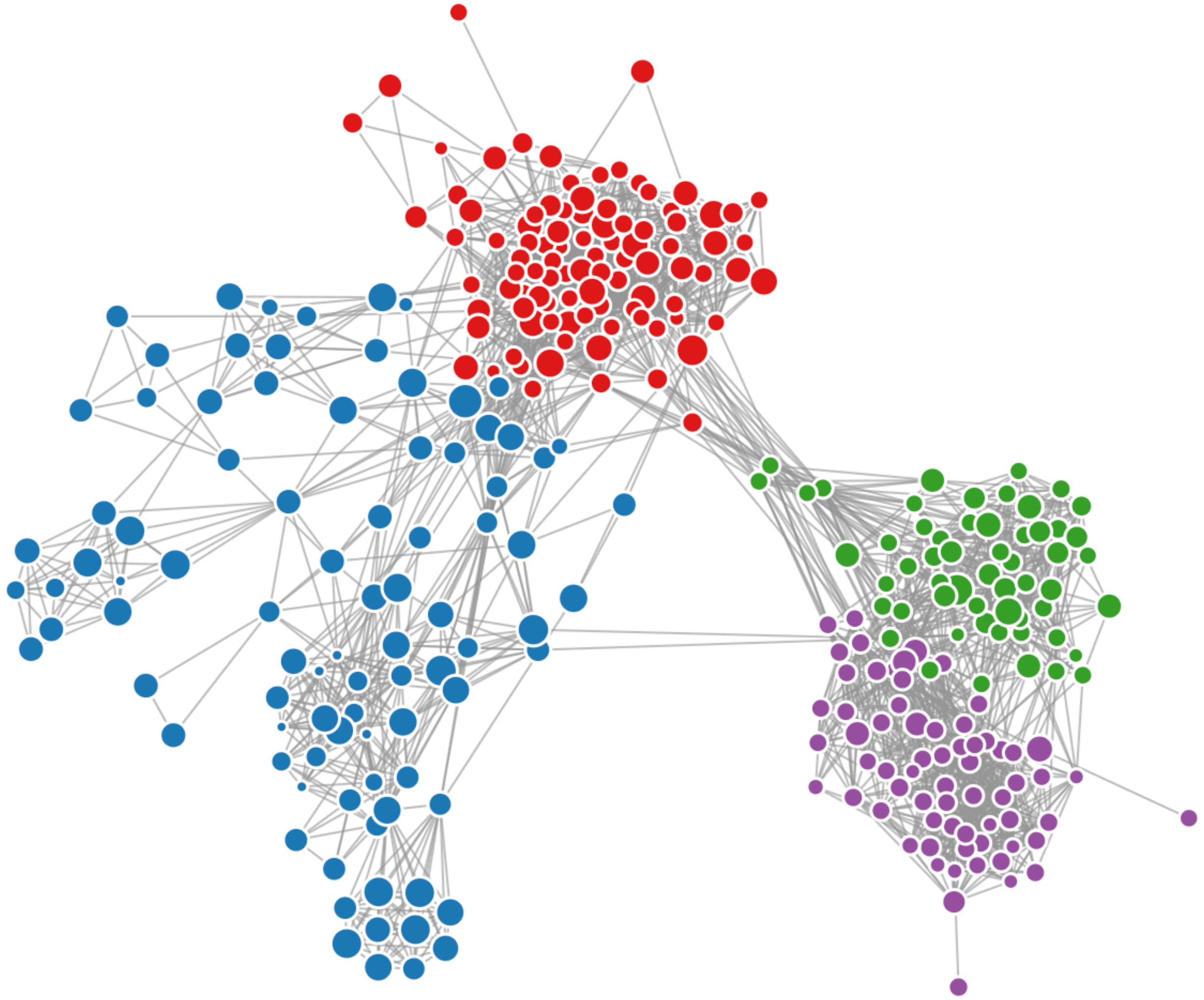
Goals for these two lectures:

1. **Why** do we look for large-scale structure? 🤔
2. **How** do we find linear hierarchies? 🧐
3. **Where** can we read more details? 📚

Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.

E. W. Dijkstra





X

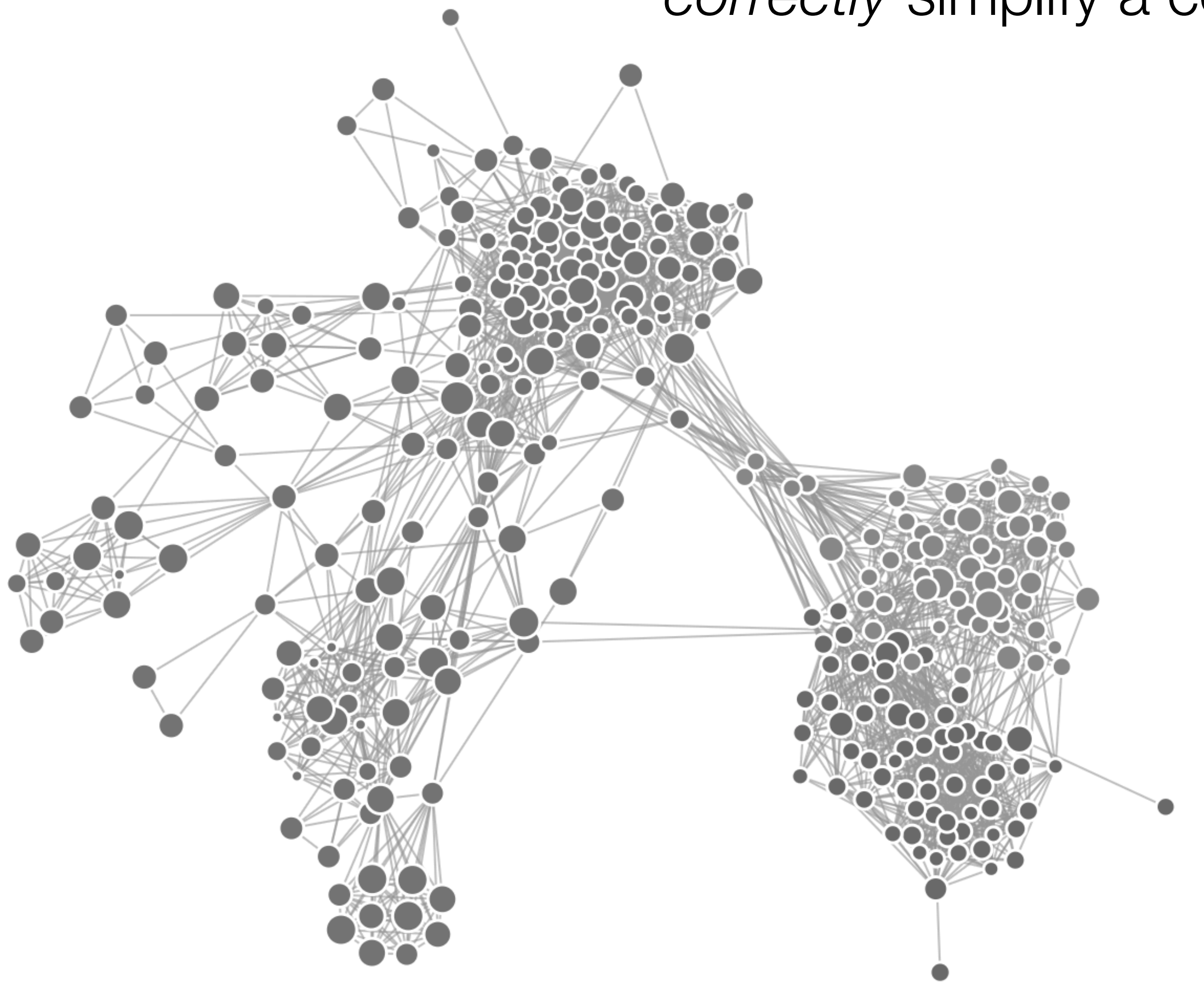
Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.

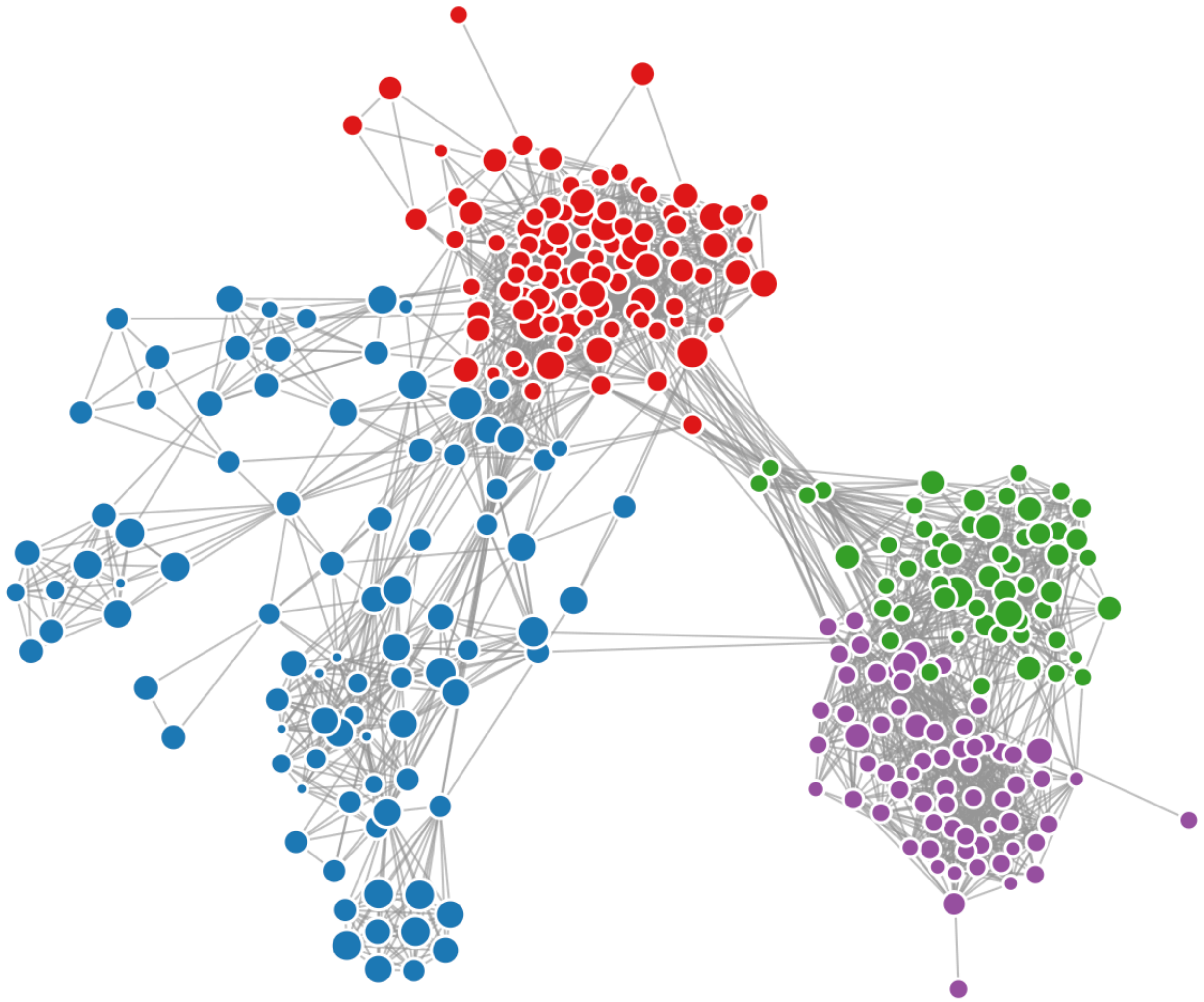
E. W. Dijkstra

We can interpret this in two ways:

The Cynic: Pictures of networks can be *really cool* but our goal is to do good science, not make pretty pictures.

The Scientist: The most beautiful science is when we *correctly* simplify a complex system.





X

What do we mean by “large-scale structure” ?

Structure is what makes data different from noise.

It’s what makes a network different from a random graph.

What do we mean by “large-scale structure” ?

Structure is what makes data different from noise.
It’s what makes a network different from a random graph.

Networks are often too large and complex to be adequately summarized by a few scalars, like the number of nodes, the number of edges, or the mean degree.

However, they are also often too large and complex to be analyzed *without* some kind of simplification!

What do we mean by “large-scale structure” ?

Structure is what makes data different from noise.
It’s what makes a network different from a random graph.

Networks are often too large and complex to be adequately summarized by a few scalars, like the number of nodes, the number of edges, or the mean degree.

However, they are also often too large and complex to be analyzed *without* some kind of simplification!

Therefore, understanding what the network *means* requires that we identify key structures.

Searching for large-scale structures in a network reflects a belief that in all the complexity there are patterns that make the network less complicated.

What do we mean by "large-scale structure" ?

Structure is what makes data different from noise.
It's what makes a network different from a random graph.

Networks are often too large and complex to be adequately summarized by a few scalars, like the number of nodes, the number of edges, or the mean degree.

However, they are also often too large and complex to be analyzed *without* some kind of simplification!

Therefore, understanding what the network *means* requires that we identify key structures.

Searching for large-scale structures in a network reflects a belief that in all the complexity there are patterns that make the network less complicated.

We define these large-scale structures—models, really—to compress complex networks.

Goal: understanding, not a list of parts and dimensions

Finding large-scale structures
is the same as anything else:

We want a **simplified model** of
something very complicated.

We want to know what the
important pieces are,
and **how they fit together**.

slide more or less stolen from Aaron Clauset

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).

Interpolation. Identify missing links.

Generalization. Nodes of this type are like others of the same type.

intuition: compare this list with the list you would write for regression

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).

Interpolation. Identify missing links.

Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?

Explanations. Coarse-graining or compression.

intuition: compare this list with the list you would write for regression

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in "space" or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

intuition: compare this list with the list you would write for regression

Rankings and linear hierarchies

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in "space" or time).

Interpolation. Identify missing links.

Generalization. Nodes of this type are like others of the same type.

intuition: compare this list with the list you would write for regression

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

intuition: compare this list with the list you would write for regression

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in “space” or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

intuition: compare this list with the list you would write for regression

The idea of rankings—pervasive!

Assumptions:

1. Competitors have some intrinsic quality (or vector of qualities).
2. Interactions can (stochastically) reveal differences in qualities.
3. Competitions are pair-wise. (Lee Sedol vs. AlphaGo; Astros vs. Dodgers)

The idea of rankings—pervasive!

Assumptions:

1. Competitors have some intrinsic quality (or vector of qualities).
2. Interactions can (stochastically) reveal differences in qualities.
3. Competitions are pair-wise. (Lee Sedol vs. AlphaGo; Astros vs. Dodgers)

In other words: outcomes are generated by a stochastic process, which is some function of the positions of the competitors.

The idea of rankings—pervasive!

Assumptions:

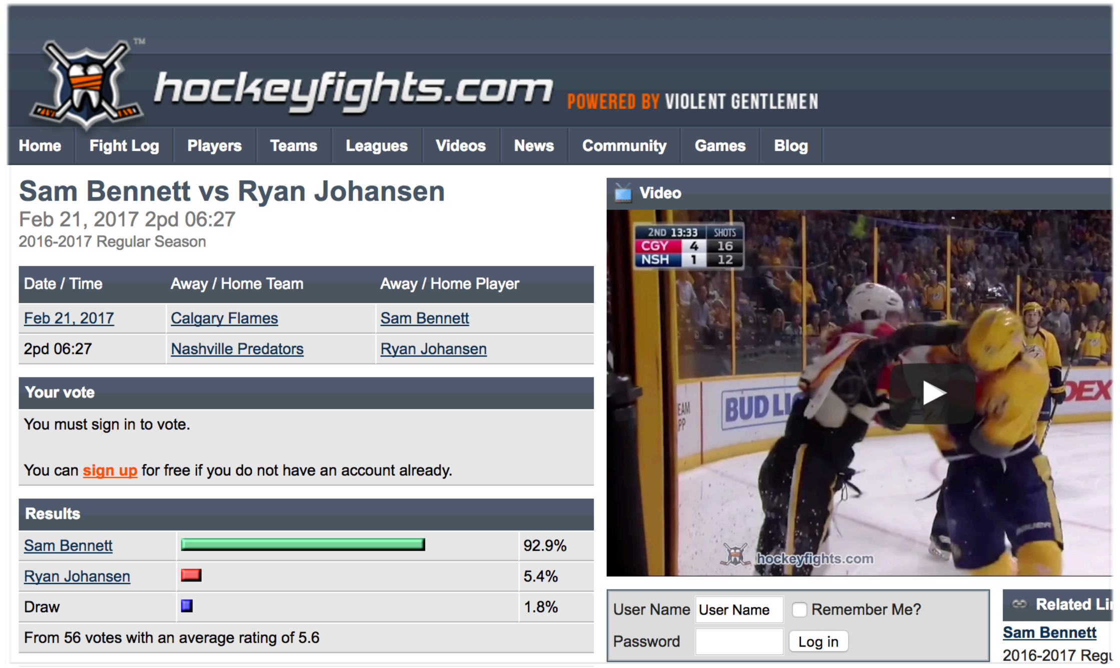
1. Competitors have some intrinsic quality (or vector of qualities).
2. Interactions can (stochastically) reveal differences in qualities.
3. Competitions are pair-wise. (Lee Sedol vs. AlphaGo; Astros vs. Dodgers)

In other words: outcomes are generated by a stochastic process, which is some function of the positions of the competitors.

Systems of dominance

social

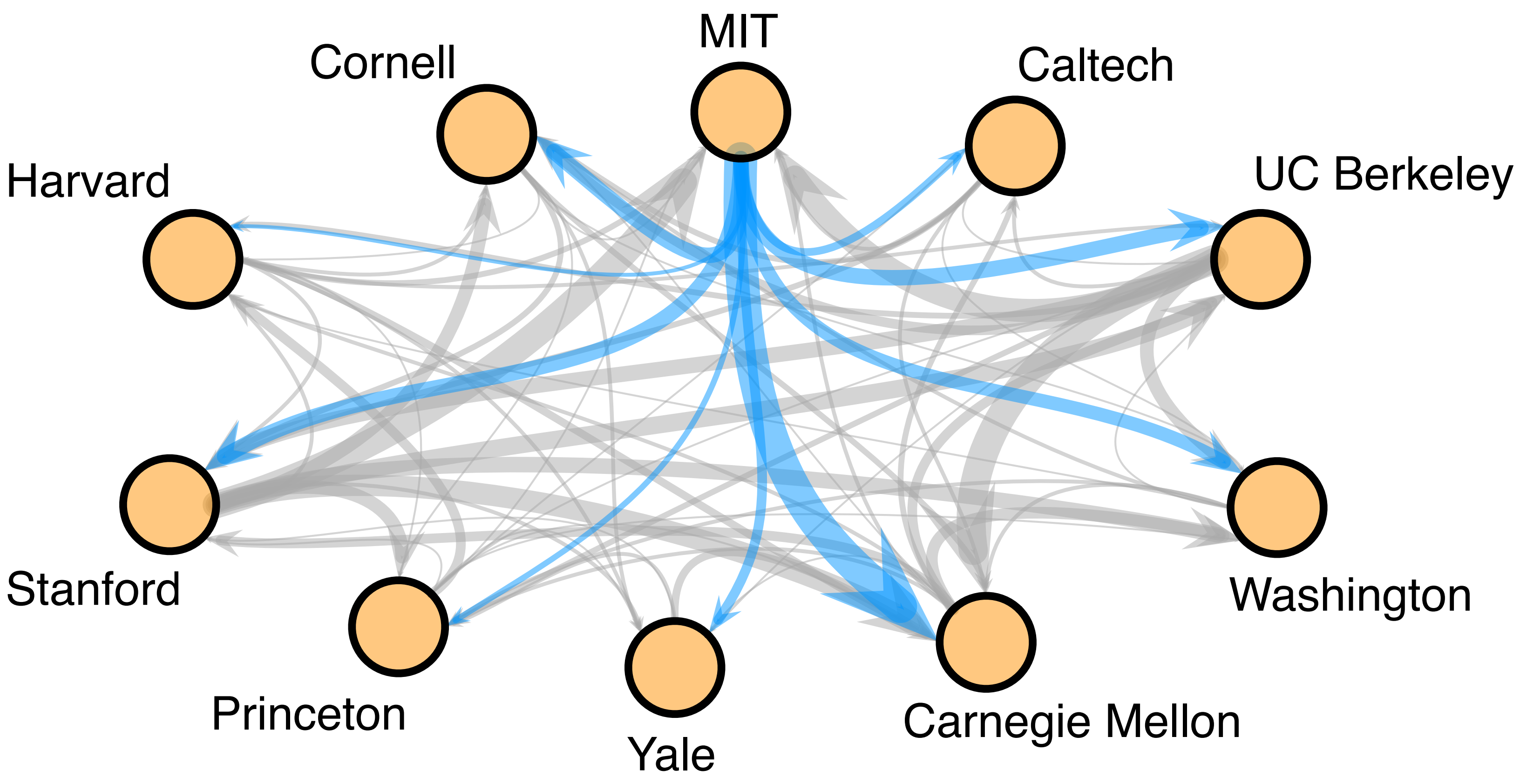
physical



mental

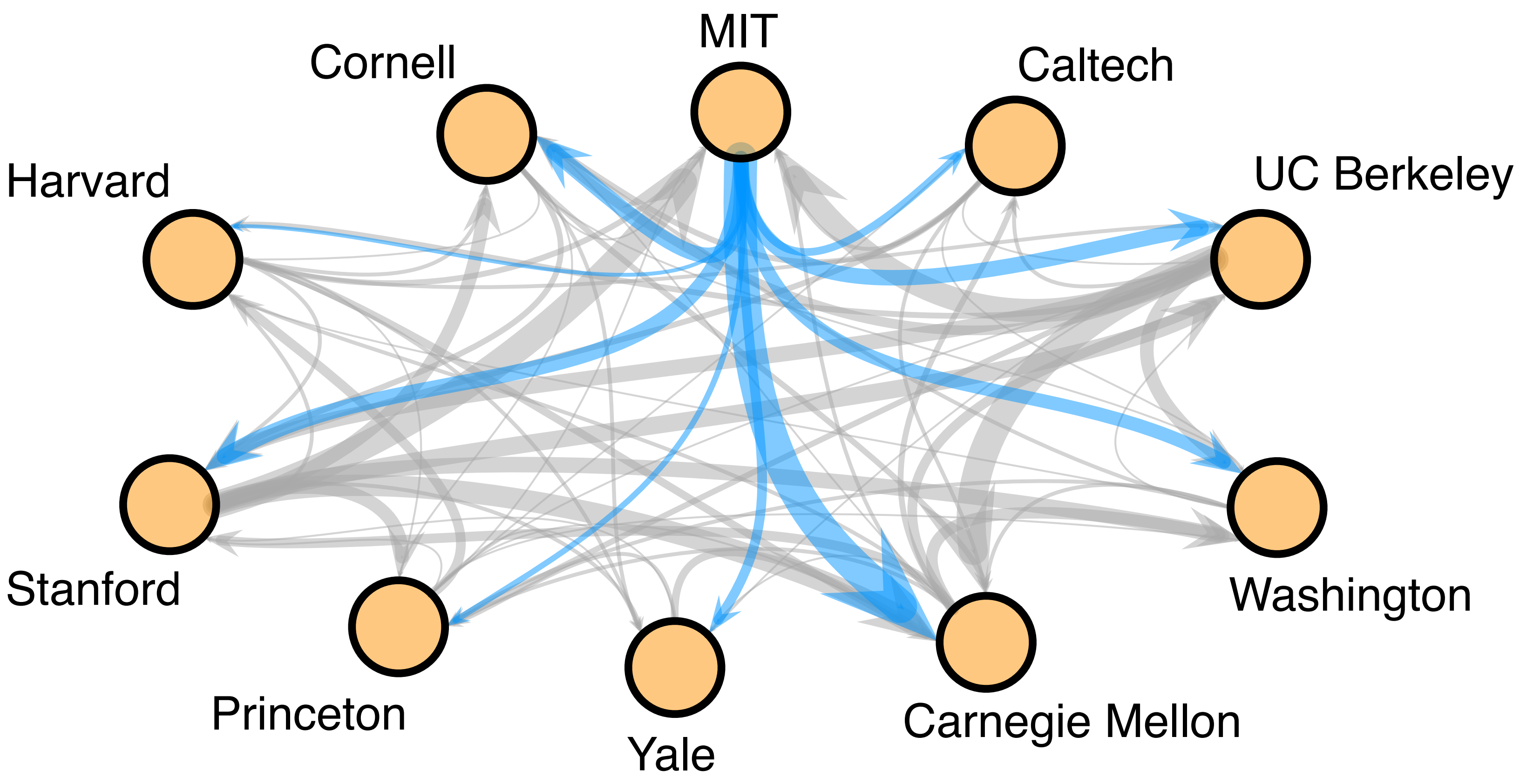
financial

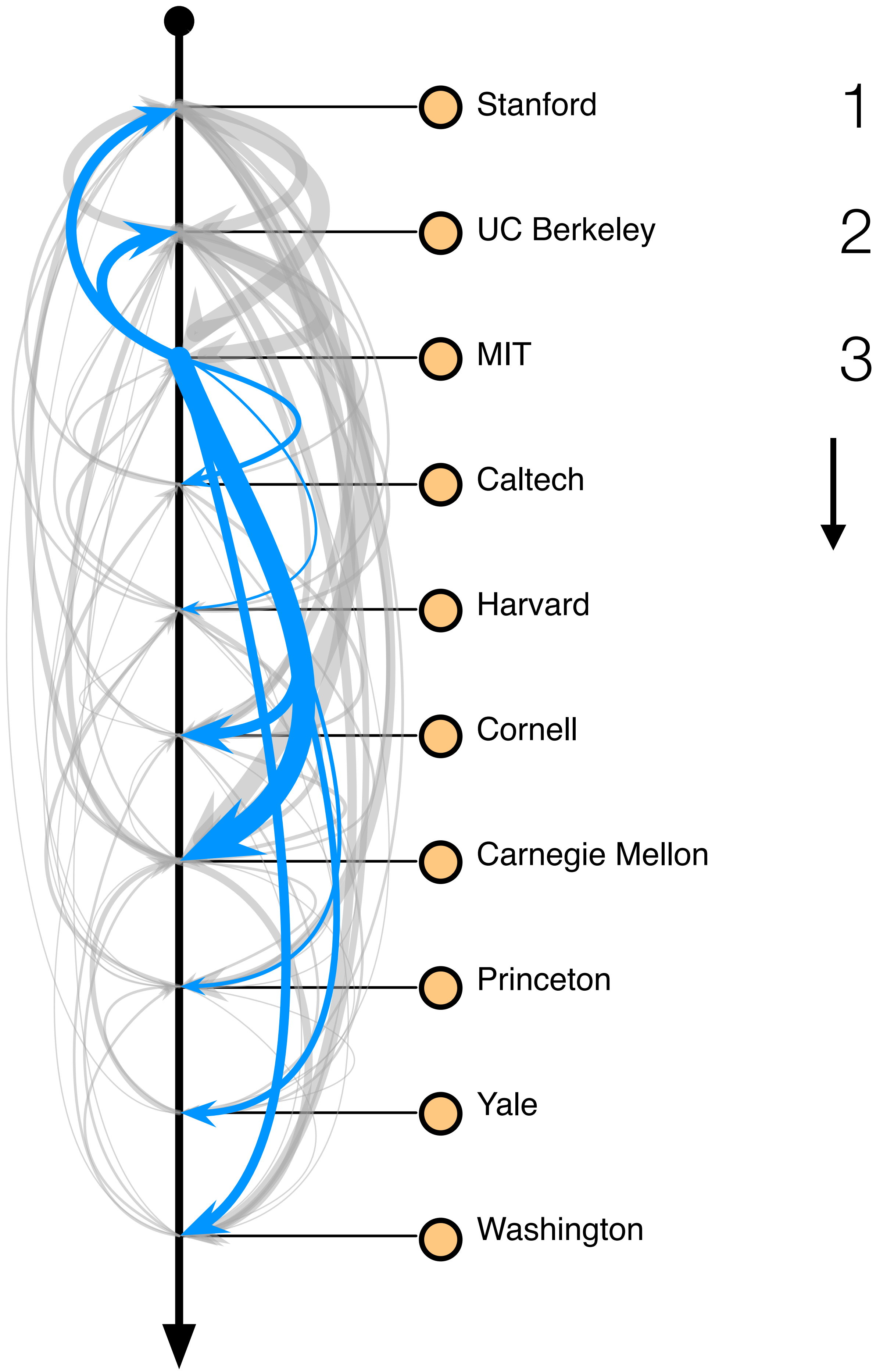
Systems of endorsement



Clauset, Arbesman, Larremore. *Science Advances* 1, e1400005 (2015).

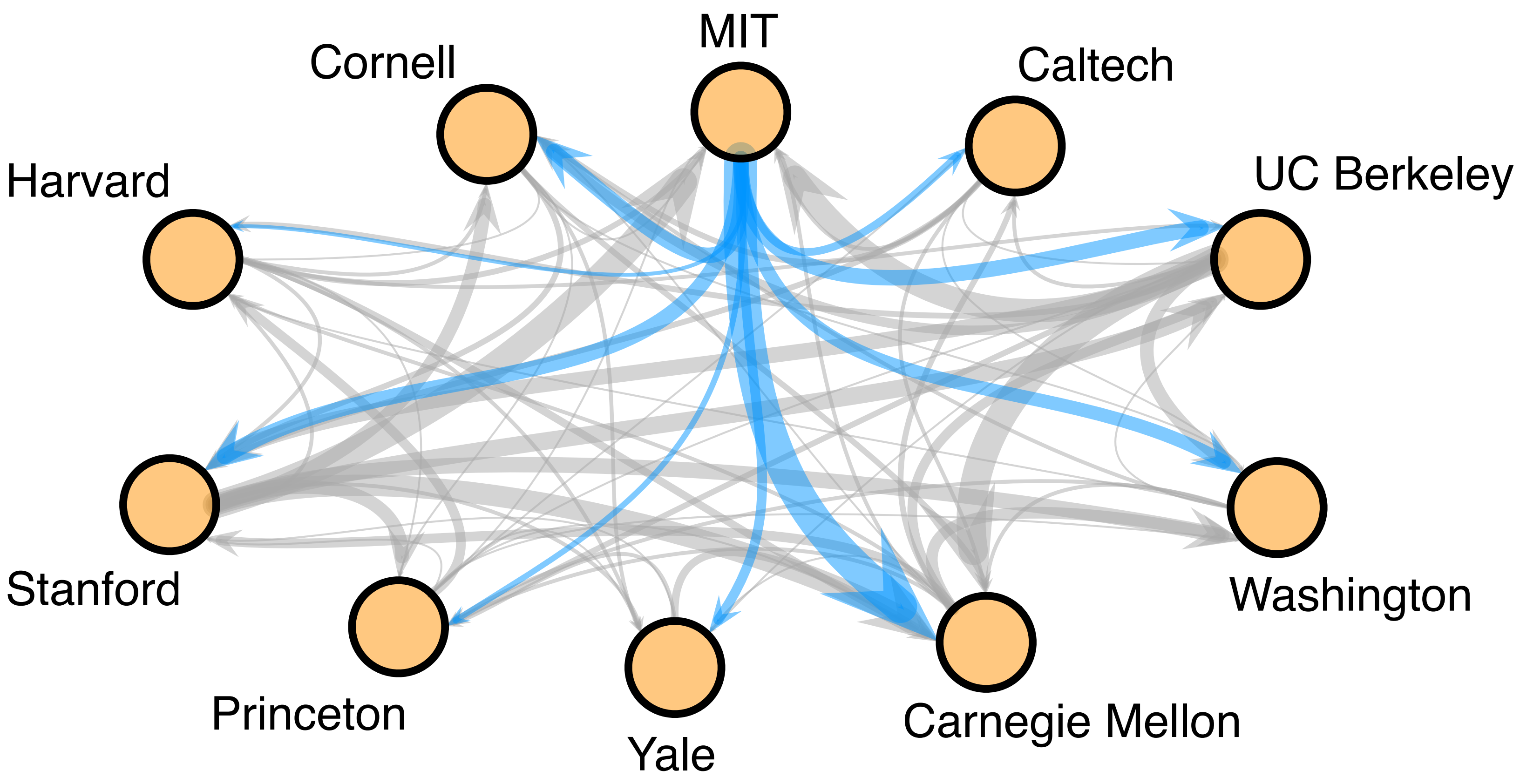
Systems of endorsement

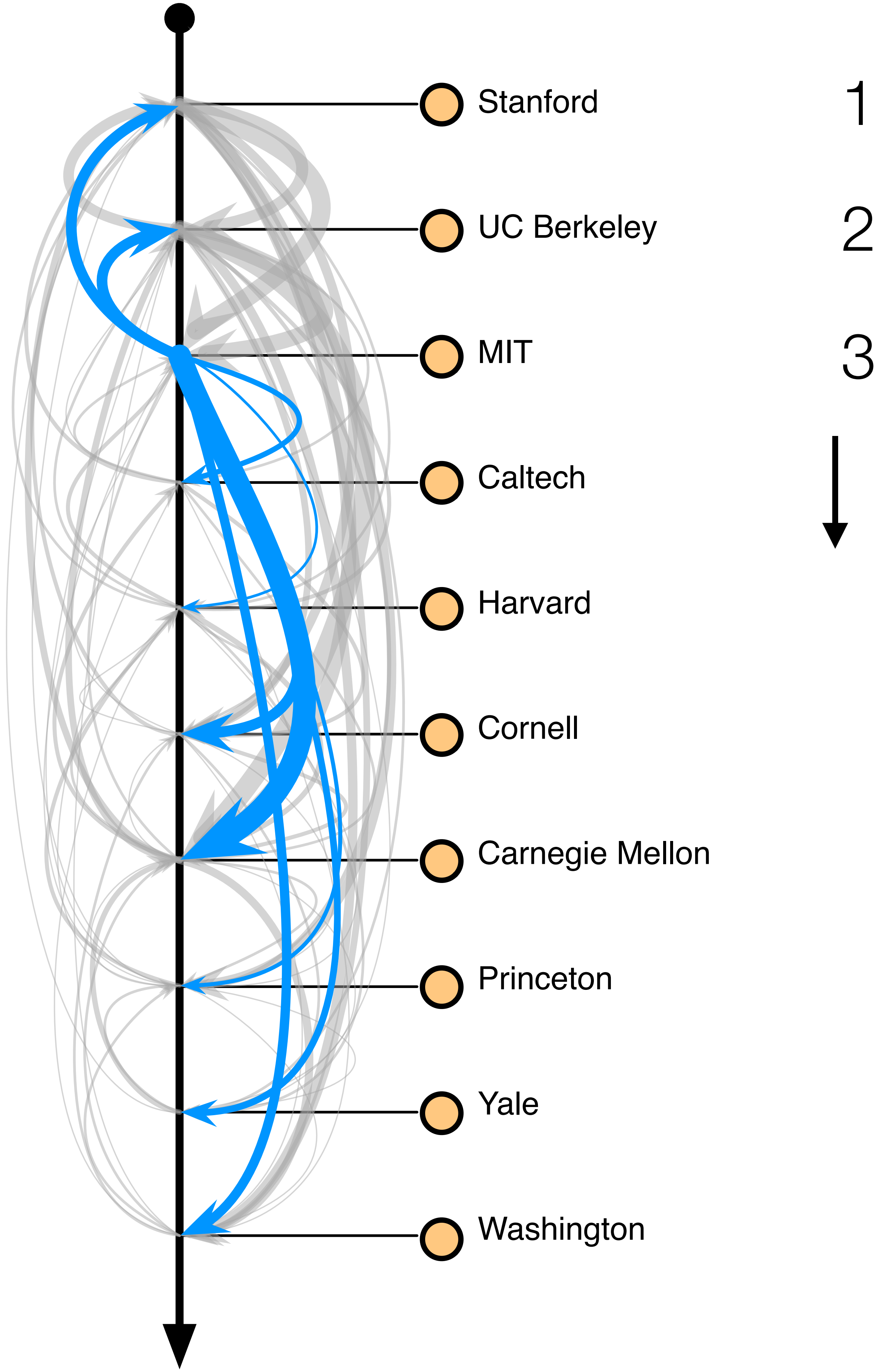




Clauset, Arbesman, Larremore. *Science Advances* 1, e1400005 (2015).

Systems of endorsement



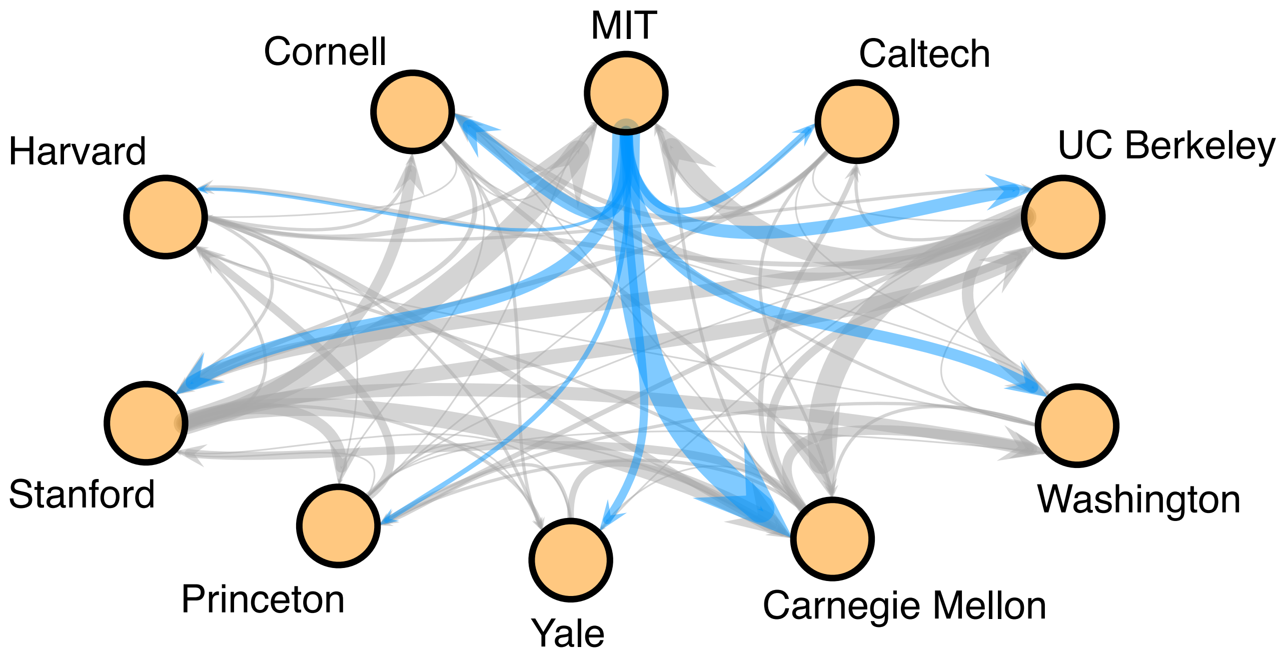


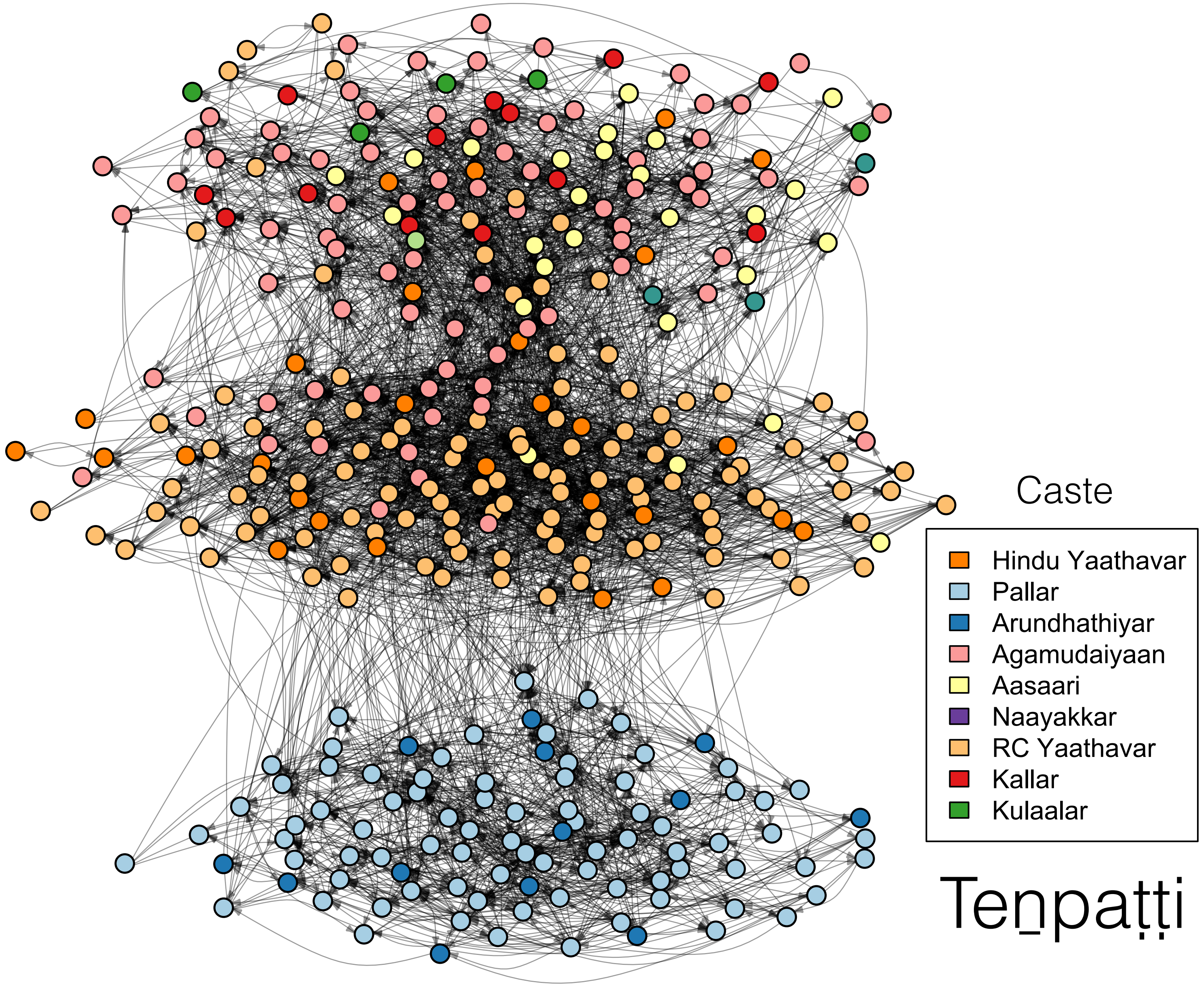
Assumptions:

1. Endorsers have some intrinsic quality.
2. Interactions can reveal differences in qualities.
3. Endorsements are pair-wise.

Clauset, Arbesman, Larremore. *Science Advances* 1, e1400005 (2015).

Systems of endorsement





Latent position can be revealed by **dominance** or **endorsement** interactions.

Figure: Larremore, Hébert-Dufresne, Power. *Draft.* Data: Power. *Nature Human Behaviour* 1, 0057 (2017)

The setup: suppose we have a *directed* network.

Its adjacency matrix is A .

$A_{ij} = A_{i \to j}$ means i beat j or i was endorsed by j

The problem: Rank the nodes.

The setup: suppose we have a *directed* network.

Its adjacency matrix is A .

$A_{ij} = A_{i\to j}$ means i beat j or i was endorsed by j

The problem: Rank the nodes.

Alternative view: there might be no network here. In some cases we're just seeing a network in pairwise comparison data because networks are a convenient data structure.

The setup: suppose we have a *directed* network.

Its adjacency matrix is A .

$A_{ij} = A_{i \to j}$ means i beat j or i was endorsed by j

The problem: Rank the nodes.

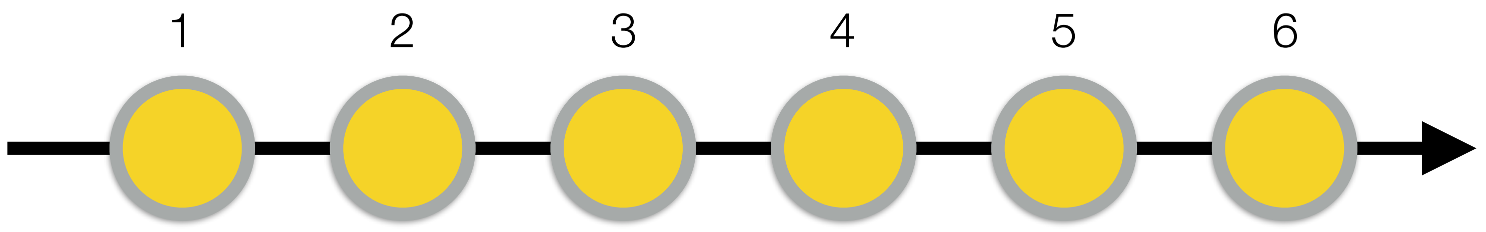
Alternative view: there might be **no network here.** In some cases we're just seeing a network in pairwise comparison data because **networks are a convenient data structure.**

Alternative problem: Which items should be compared next in order to most/best resolve our estimate of the ranks? (sequential tournament design)

Embeddings vs Orderings

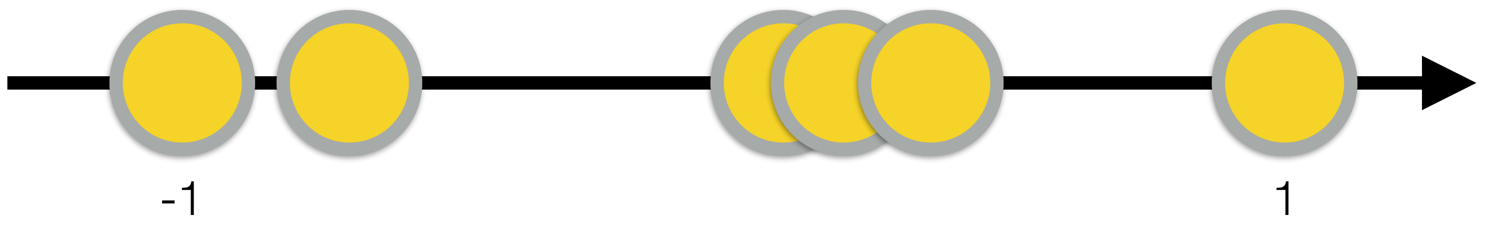
Ordering place the nodes in order:

1, 2, 3, …



Embedding assigns a position to each node:

1, 1.2, 7, 20, 21, 21.2, …



Which one should I use?

> Depends on the use case.

> Is it possible for two nodes to occupy the same rank or position? If so, an embedding is more appropriate. Also better when meaning of 1-rank Δ varies.

> Consider that you can always go from an embedding to an ordering, if you have a rule for breaking ties.

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

Win and loss tallies don't take this "schedule difficulty" into account. Put differently, win-loss records leave information on the table.

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

Win and loss tallies don't take this "schedule difficulty" into account. Put differently, win-loss records leave information on the table.

One way to make use of this information:

i beats j implies $s_i > s_j$

Therefore if we have a whole list of outcomes, we can try to find a total ordering that breaks as few of these implications as possible.

Win-Loss is not satisfactory: schedule matters

Beating the grandmaster counts for more than beating a novice.

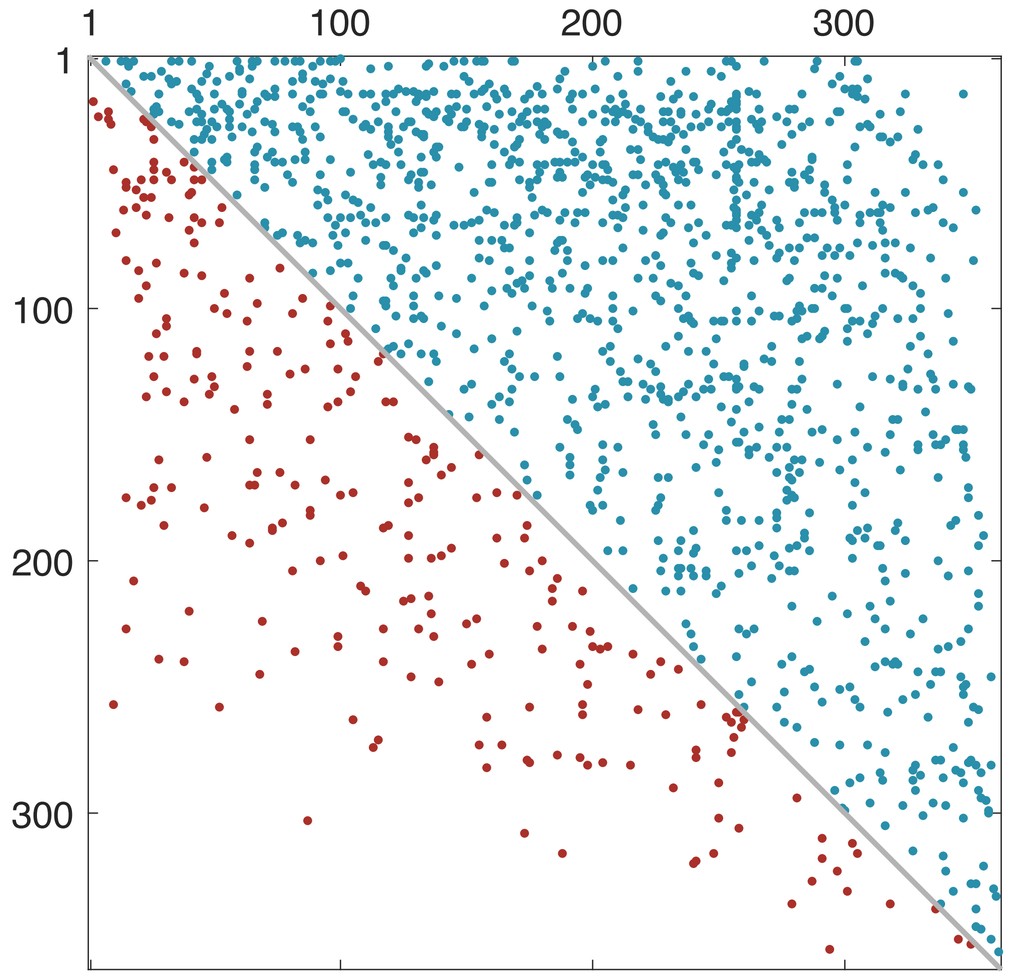
Win and loss tallies don't take this "schedule difficulty" into account. Put differently, win-loss records **leave information on the table.**

One way to make use of this information:

i **beats** *j* implies $s_i > s_j$

Therefore if we have a whole list of outcomes, we can try to find a total ordering that breaks as few of these implications as possible.

$A_{ij} =$ number of times that *i* beat *j*.



minimum violation ranking: sort *A*.

Win-Loss is not satisfactory: schedule matters

How do we find an ordering that minimizes the number of violations (or upsets) ?

Win-Loss is not satisfactory: schedule matters

How do we find an ordering that minimizes the number of violations (or upsets) ?

Recipe (MCMC):

1. Order the nodes randomly.
2. Compute the number of violations. In expectation, this should be 50% of edges.

Win-Loss is not satisfactory: schedule matters

How do we find an ordering that minimizes the number of violations (or upsets) ?

Recipe (MCMC):

1. Order the nodes randomly.
2. Compute the number of violations. In expectation, this should be 50% of edges.

3. Pick two nodes at random and propose to swap their positions.
4. Compute the number of violations in this scenario.

Win-Loss is not satisfactory: schedule matters

How do we find an ordering that minimizes the number of violations (or upsets) ?

Recipe (MCMC):

1. Order the nodes randomly.
2. Compute the number of violations. In expectation, this should be 50% of edges.
3. Pick two nodes at random and propose to swap their positions.
4. Compute the number of violations in this scenario.
5. If #violations decreases or stays the same, keep the swap. Otherwise, reject.
6. Repeat until….?

Win-Loss is not satisfactory: schedule matters

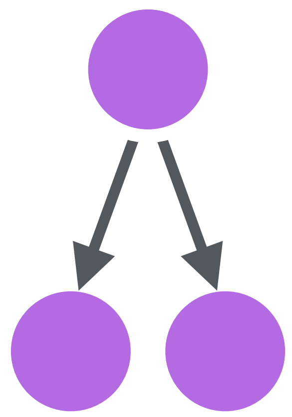
How do we find an ordering that minimizes the number of violations (or upsets) ?

Recipe (MCMC):

1. Order the nodes randomly.
2. Compute the number of violations. In expectation, this should be 50% of edges.
3. Pick two nodes at random and propose to swap their positions.
4. Compute the number of violations in this scenario.
5. If #violations decreases or stays the same, keep the swap. Otherwise, reject.
6. Repeat until….?

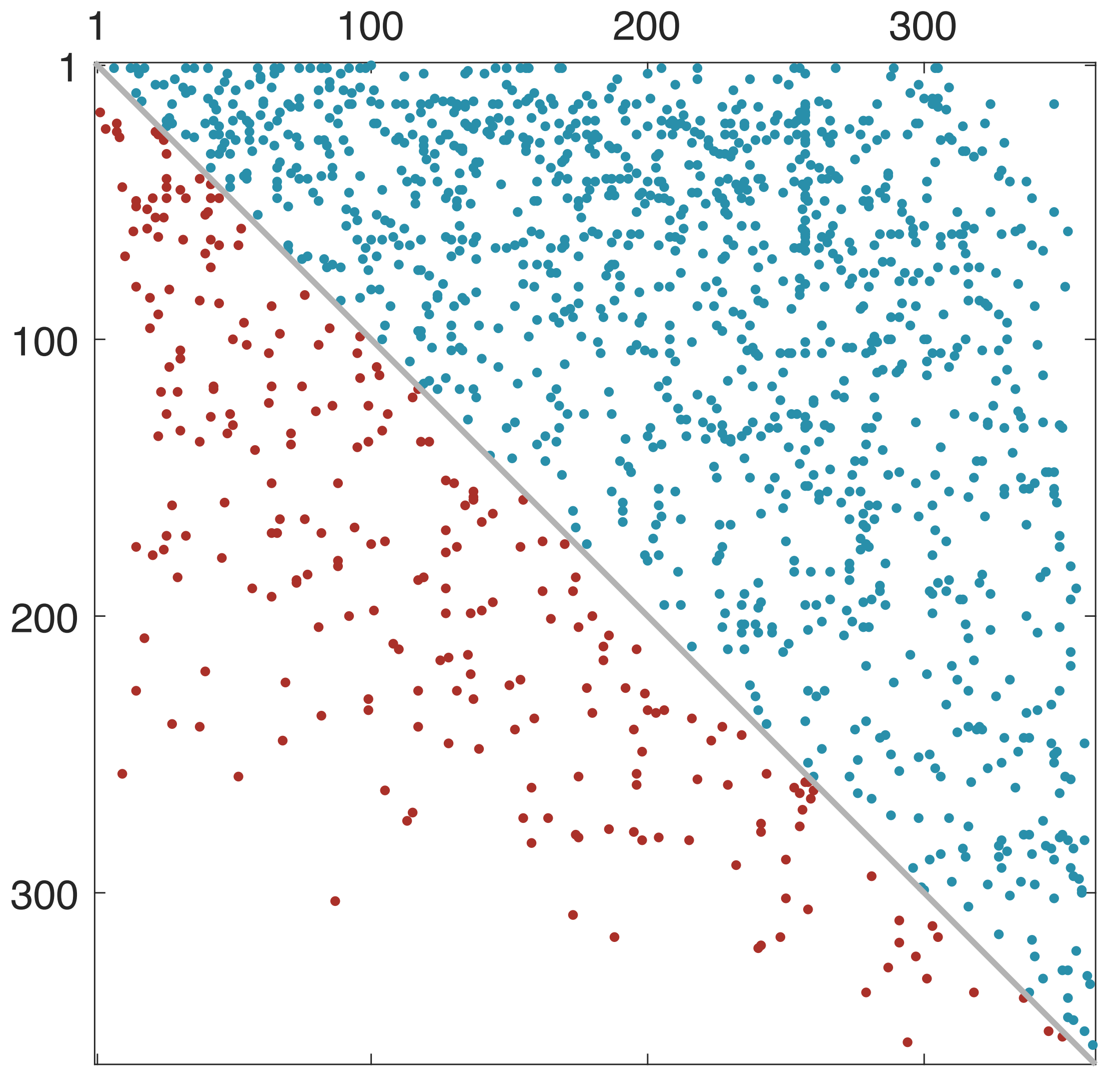
Notes:

* The number of violations is non-increasing over time.
* There may be no unique minimum. Consider this scenario:



Embeddings & Orderings 0: MVR & Agony

* There is no guarantee of a unique minimizing ranking s.
* Space of ordinal rankings has n! elements, requiring slow search algorithms (e.g. MCMC).
* Ordinal. No ties. No interpretability of rank differences.

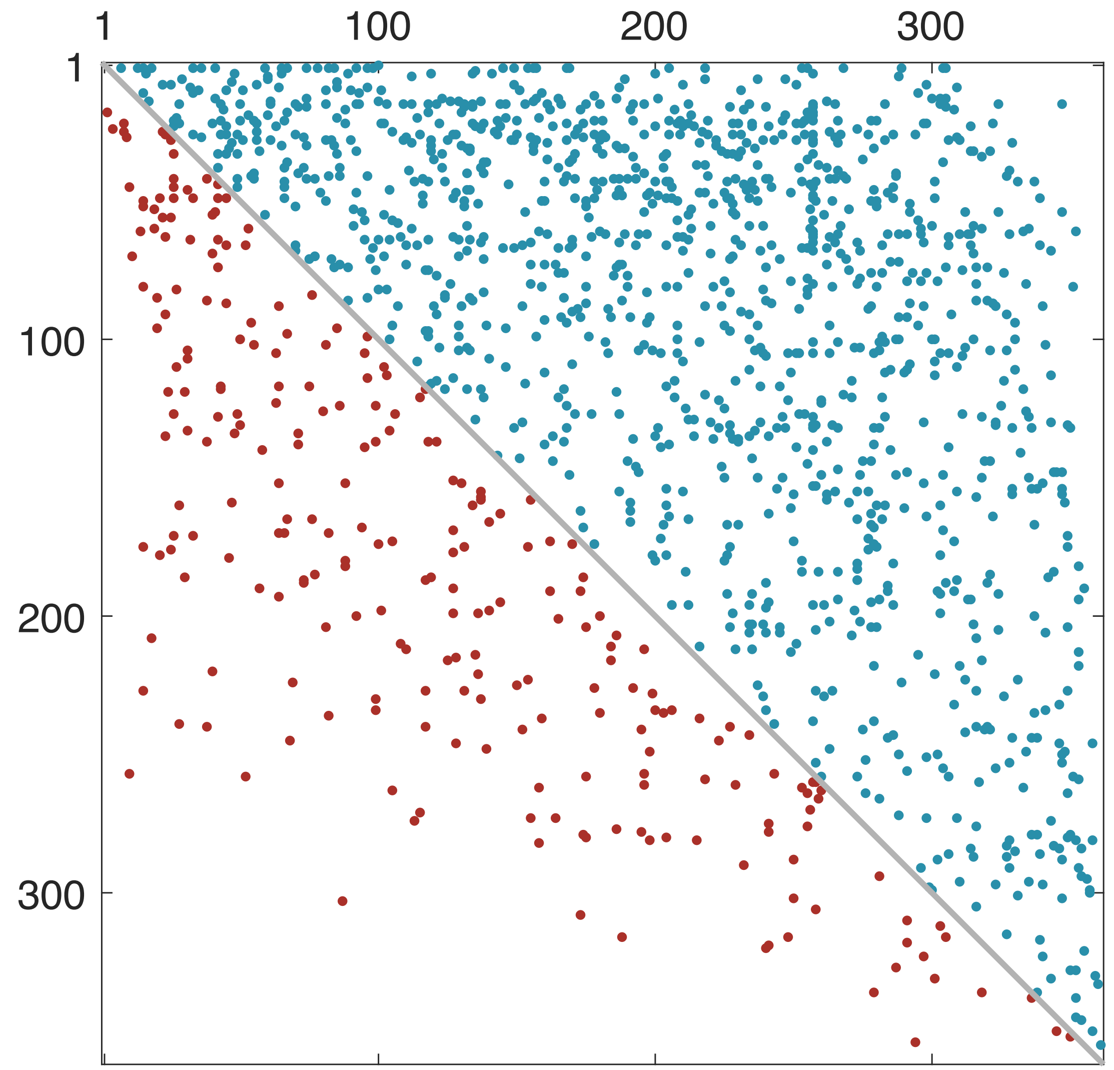


minimum violation ranking: sort *A*.

Embeddings & Orderings 0: MVR & Agony

* There is no guarantee of a unique minimizing ranking s.
* Space of ordinal rankings has n! elements, requiring slow search algorithms (e.g. MCMC).
* Ordinal. No ties. No interpretability of rank differences.

What if you allowed for **ties** and then ran Minimum Violation Ranking (MVR)? What would happen?



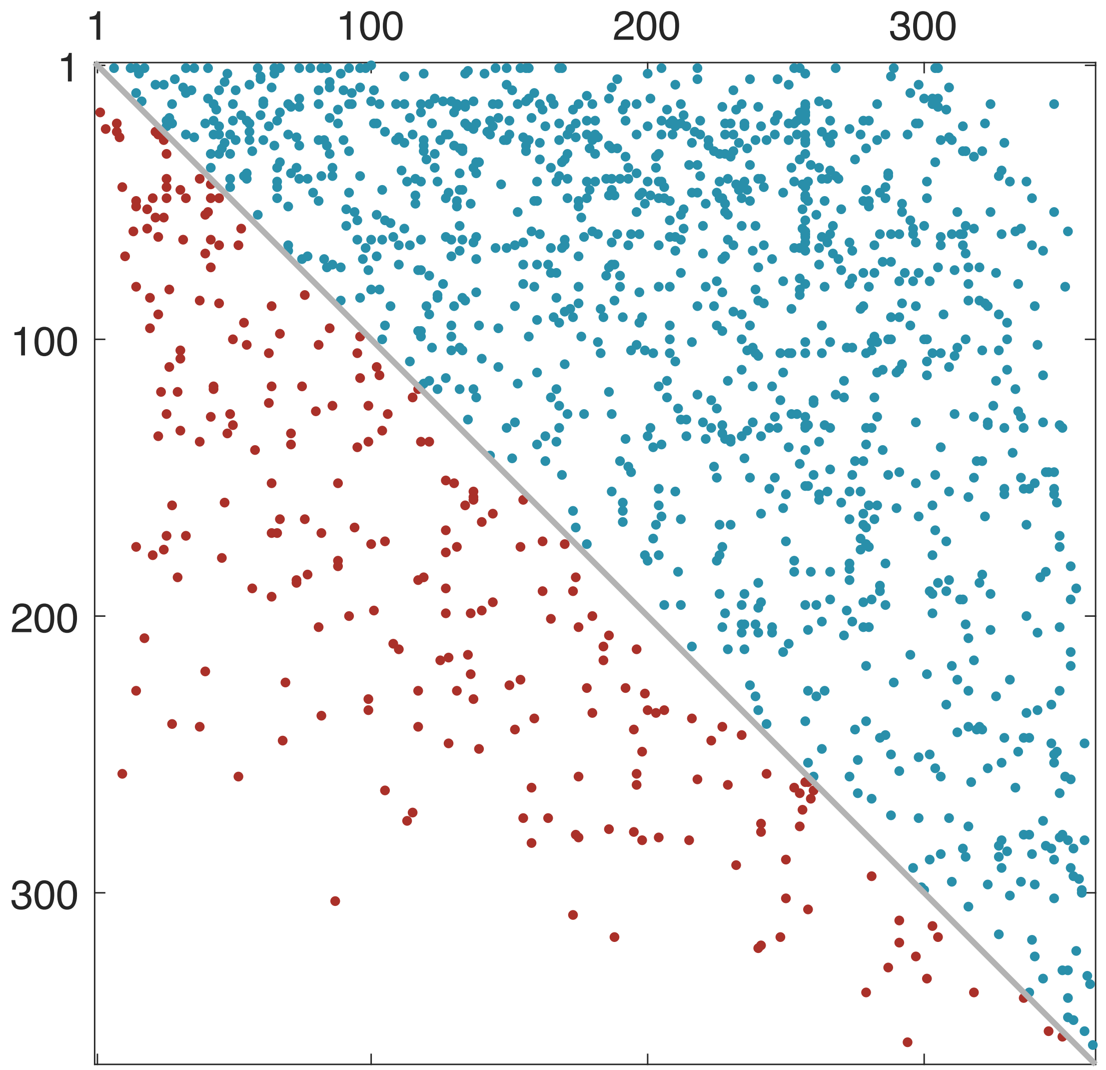
minimum violation ranking: sort *A*.

Embeddings & Orderings 0: MVR & Agony

* There is no guarantee of a unique minimizing ranking s.
* Space of ordinal rankings has n! elements, requiring slow search algorithms (e.g. MCMC).
* Ordinal. No ties. No interpretability of rank differences.

What if you allowed for **ties** and then ran Minimum Violation Ranking (MVR)? What would happen?

MVR: uniform cost (1 per edge).
Agony: generic cost function.
for example, difference in ranks.



minimum violation ranking: sort *A*.

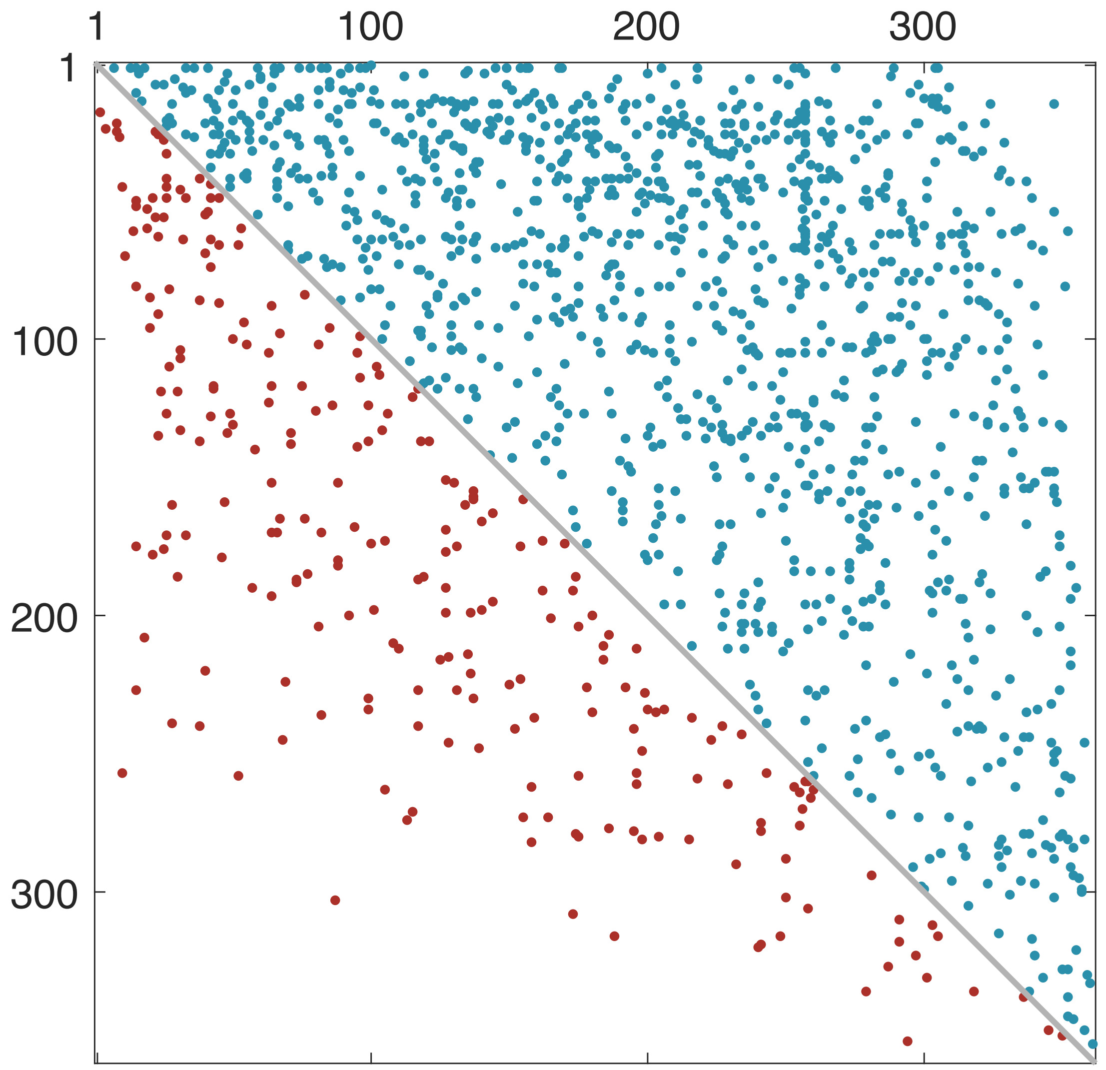
Gupte et al. Finding hierarchy in directed online social networks WWW (2011)

Embeddings & Orderings 0: MVR & Agony

* There is no guarantee of a unique minimizing ranking s.
* Space of ordinal rankings has n! elements, requiring slow search algorithms (e.g. MCMC).
* Ordinal. No ties. No interpretability of rank differences.

What if you allowed for **ties** and then ran Minimum Violation Ranking (MVR)? What would happen?

MVR: uniform cost (1 per edge).
Agony: generic cost function.
for example, difference in ranks.



minimum violation ranking: sort *A*.

What are other premises on which we can base a ranking model?

Gupte et al. Finding hierarchy in directed online social networks WWW (2011)

Embeddings and Orderings 1: Discrete choice models

Louis Leon Thurstone and Thelma Thurstone

Thelma: Prof. of Education & Psych UNC Chapel Hill. Louis: Worked with Edison.

Embeddings and Orderings 1: Discrete choice models

https://xkcd.com/883/

Embeddings and Orderings 1: Discrete choice models

Instead of rating everything from 1 to 10, try *paired comparisons*.

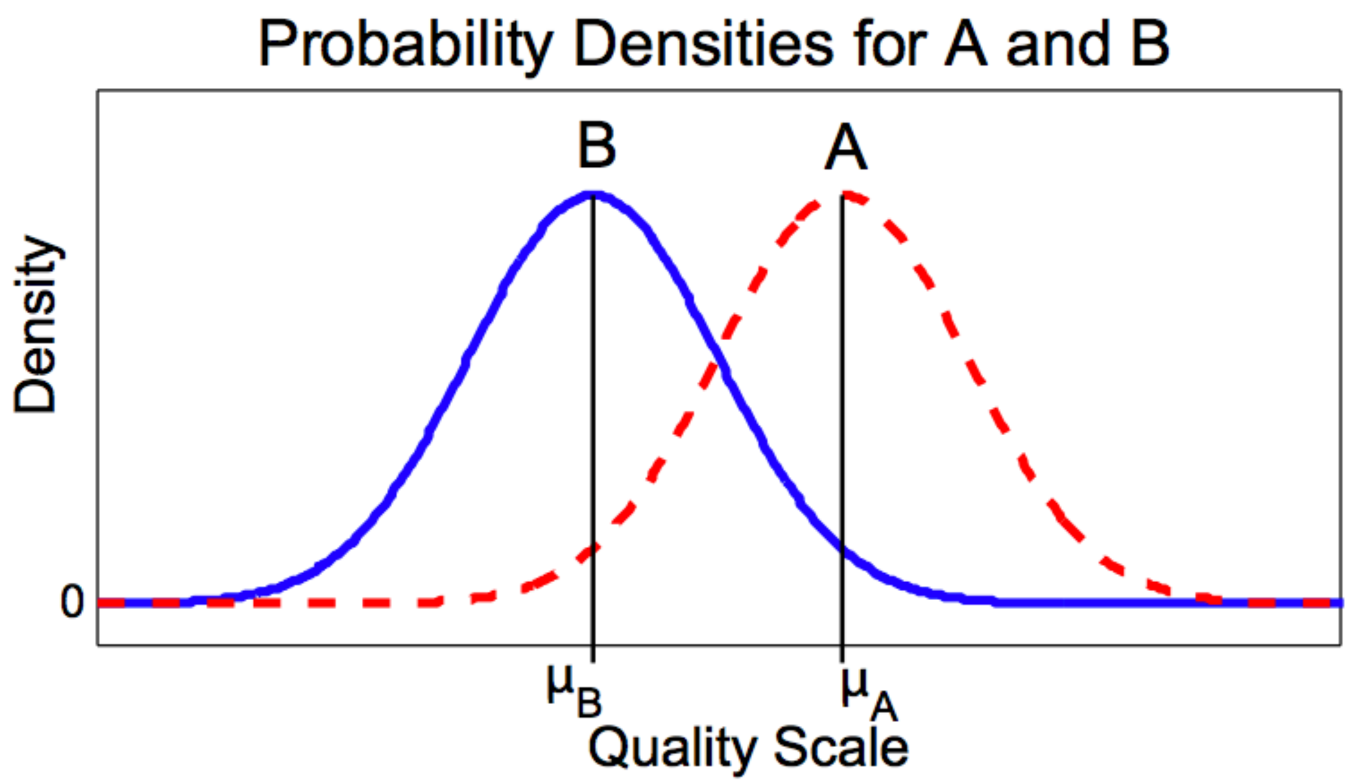
Do you prefer *i* or *j* ?

Why? Consider: My 3 is not your 3. What is 1 and what is 10?

https://xkcd.com/883/

Embeddings and Orderings 1: Discrete choice models

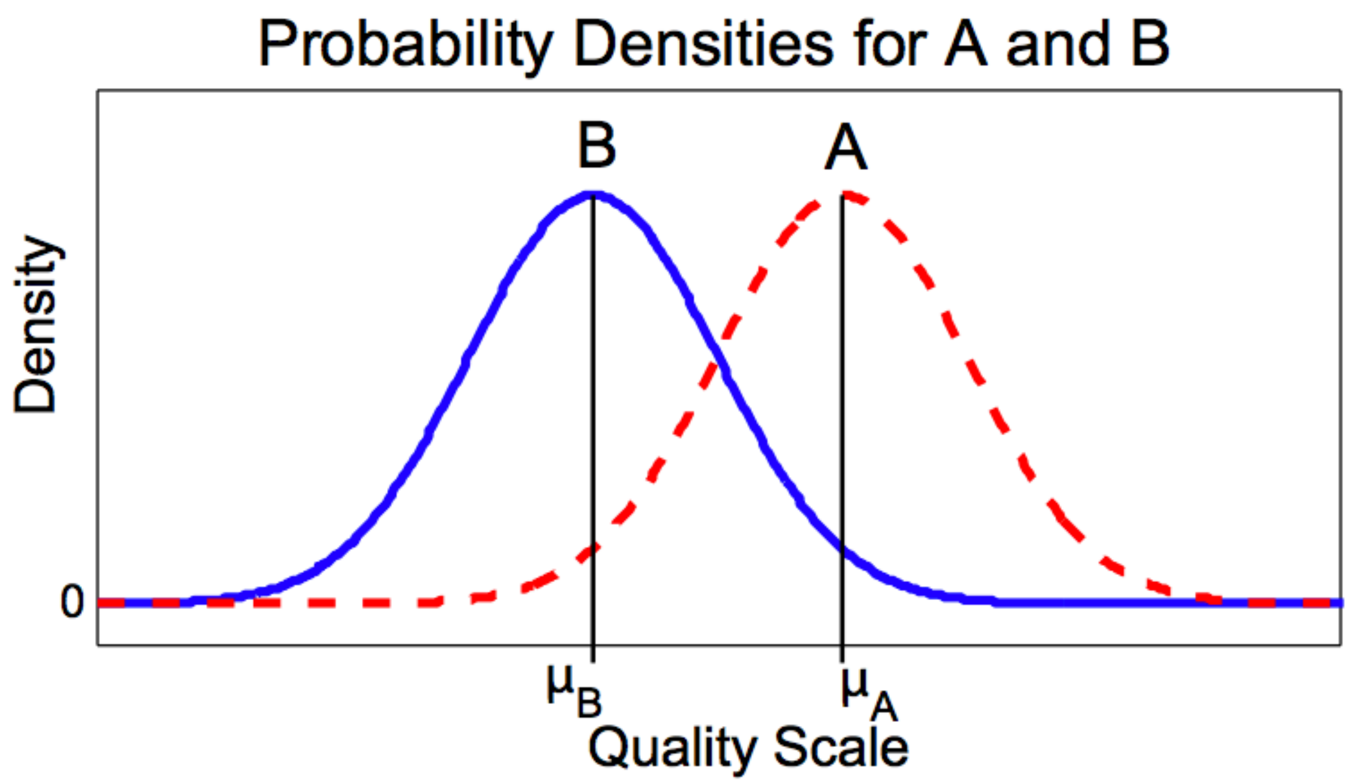
Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



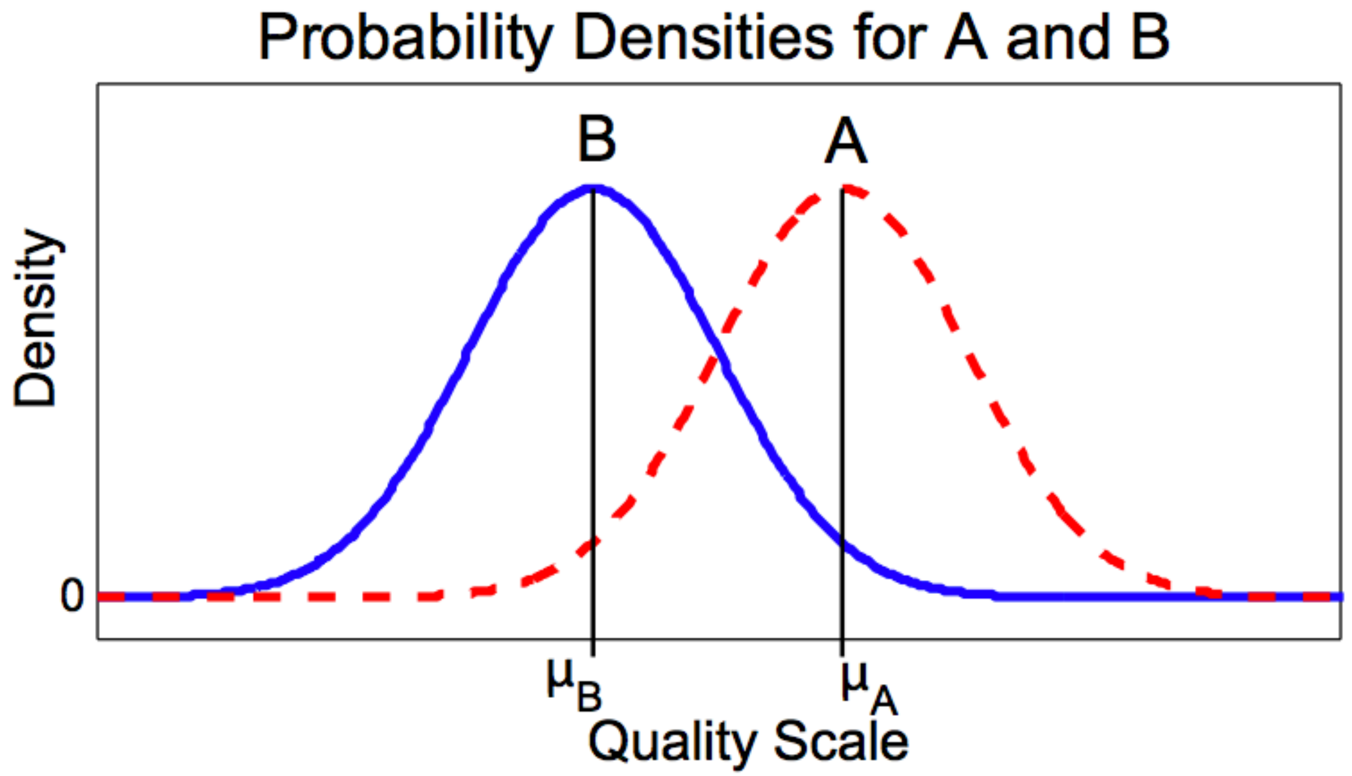
Thurstone modeled these as Gaussians.

$$P(A > B) = P(A - B > 0)$$

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



Thurstone modeled these as Gaussians.

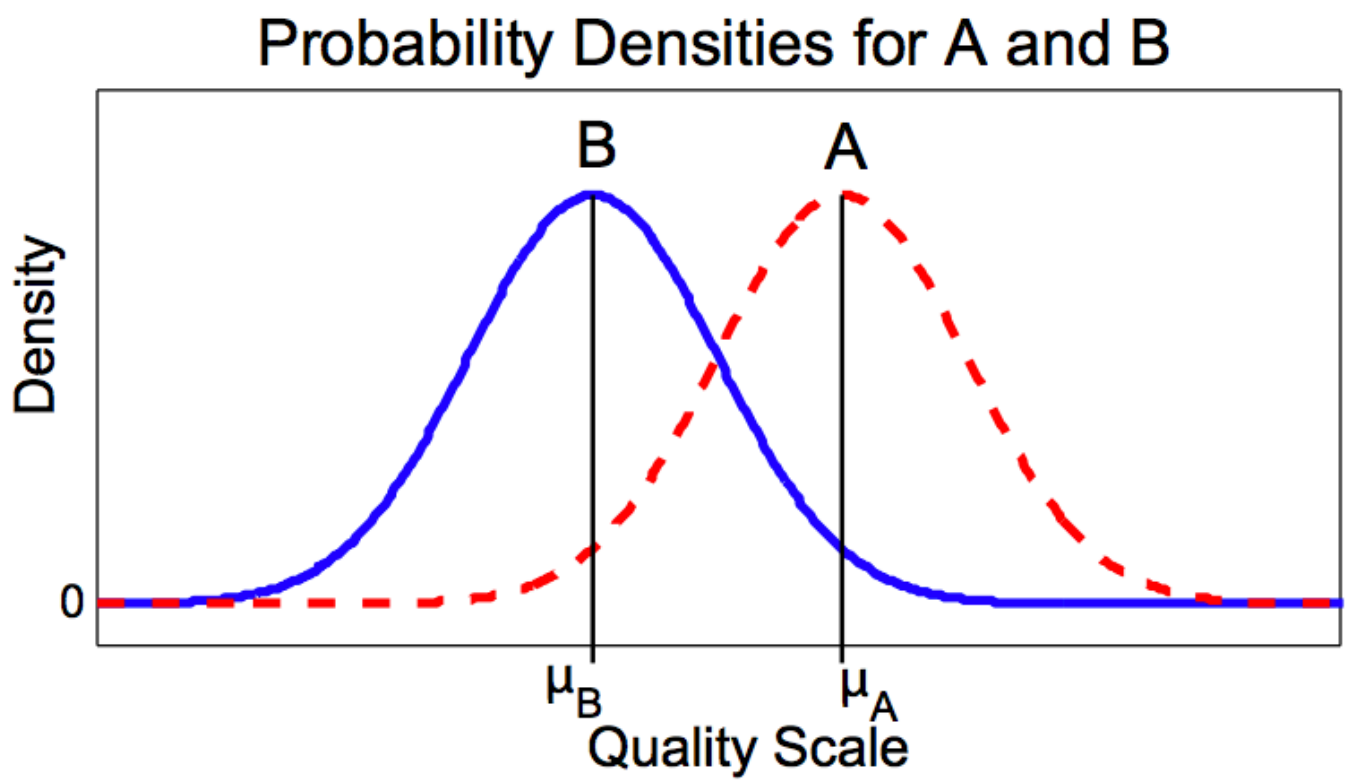
$$P(A > B) = P(A - B > 0)$$

Difference of Gaussians is Gaussian.

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



Thurstone modeled these as Gaussians.

$$P(A > B) = P(A - B > 0)$$

Difference of Gaussians is Gaussian.

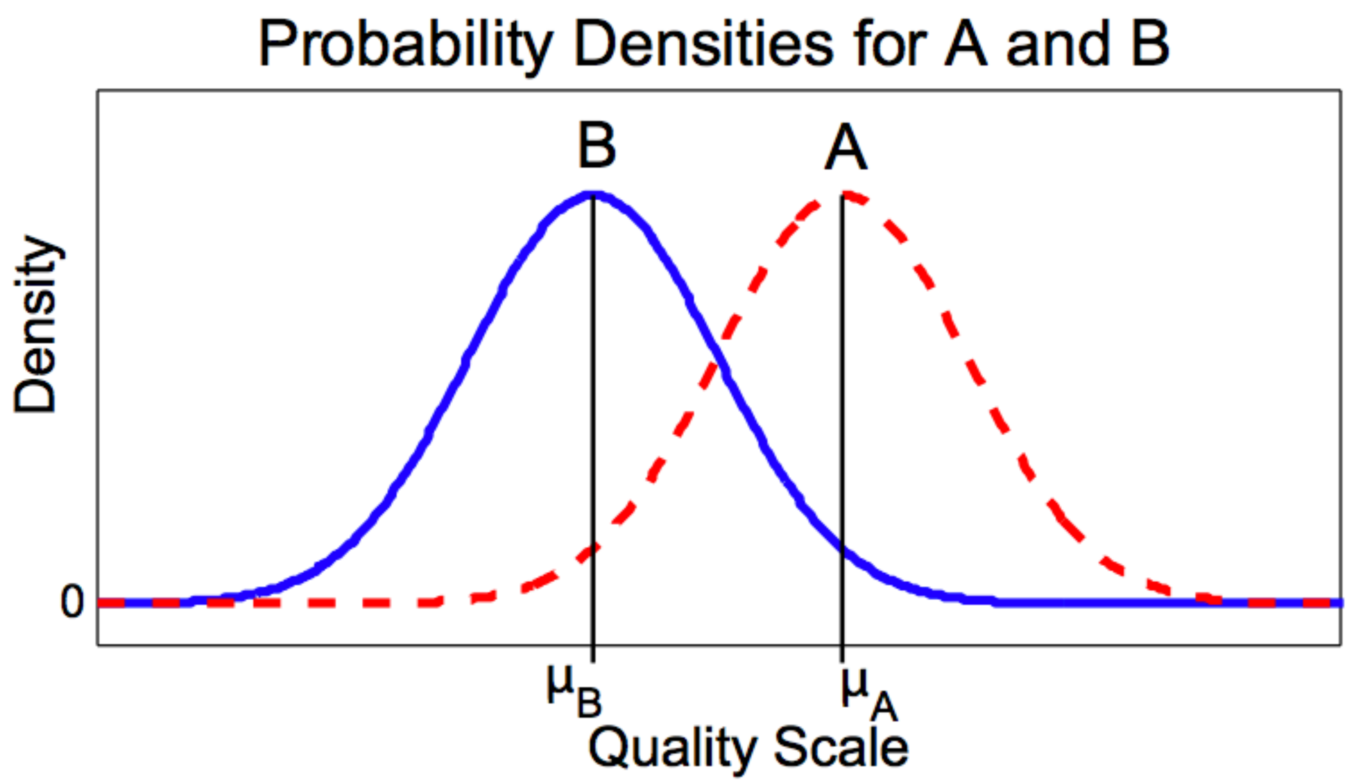
$$\hat{\mu}_{AB} = \Phi^{-1}\left(\frac{C_{A\to B}}{C_{A\to B} + C_{B\to A}}\right)$$

Where $\Phi^{-1}(x)$ is the inverse CDF of standard normal, a.k.a. the *probit*.

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Thurstone: items have quality distributions. When a person judges whether A is better than B they draw from A's distribution and from B's distribution and see which is higher.



Thurstone modeled these as Gaussians.

$$P(A > B) = P(A - B > 0)$$

Difference of Gaussians is Gaussian.

$$\hat{\mu}_{AB} = \Phi^{-1}\left(\frac{C_{A\to B}}{C_{A\to B} + C_{B\to A}}\right)$$

Where $\Phi^{-1}(x)$ is the inverse CDF of standard normal, a.k.a. the *probit*.

Powerful idea: lots of pairwise comparisons = estimates of all the qualities! An embedding!

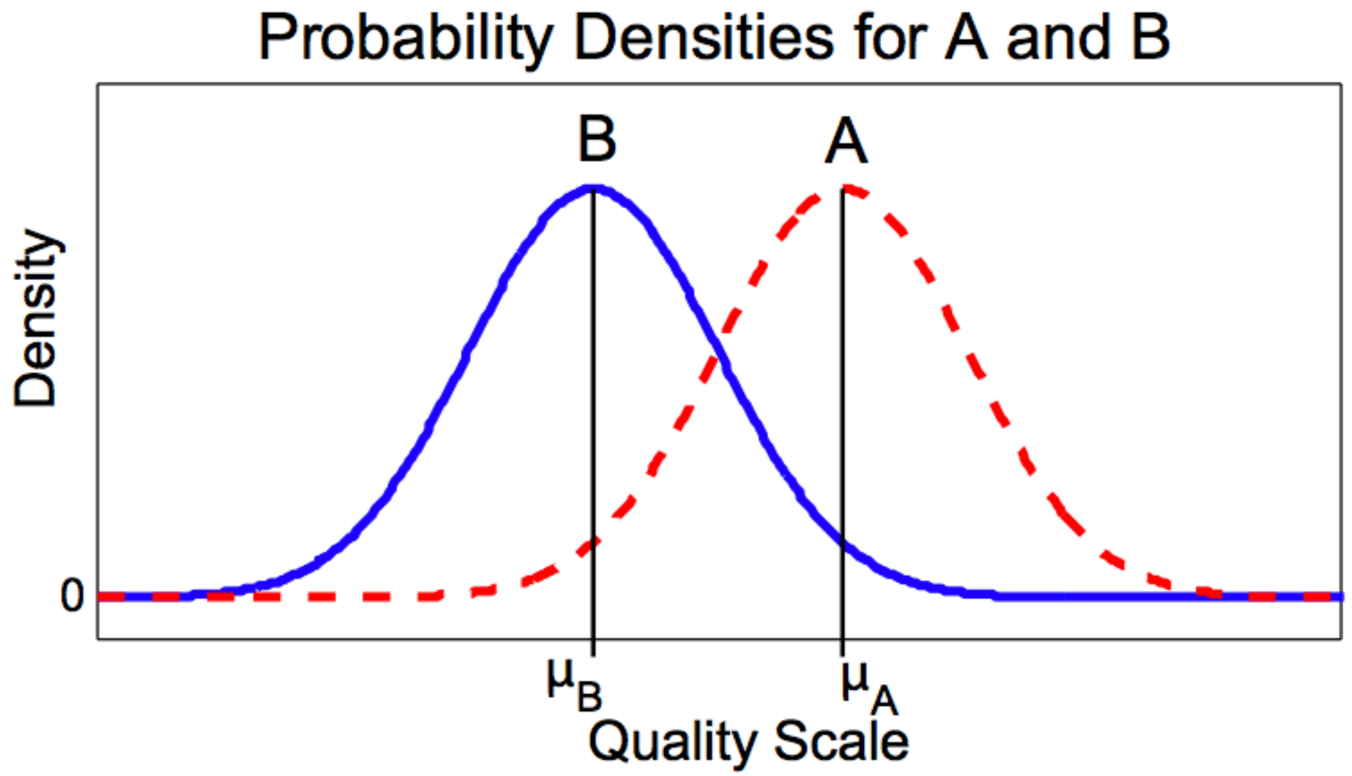
Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Key: pairwise comparisons = directed network. i preferred to j $=$ $i \to j$

Finding the qualities of items from pairwise comparisons = Finding embedding of nodes.

Embeddings and Orderings 1: Discrete choice models

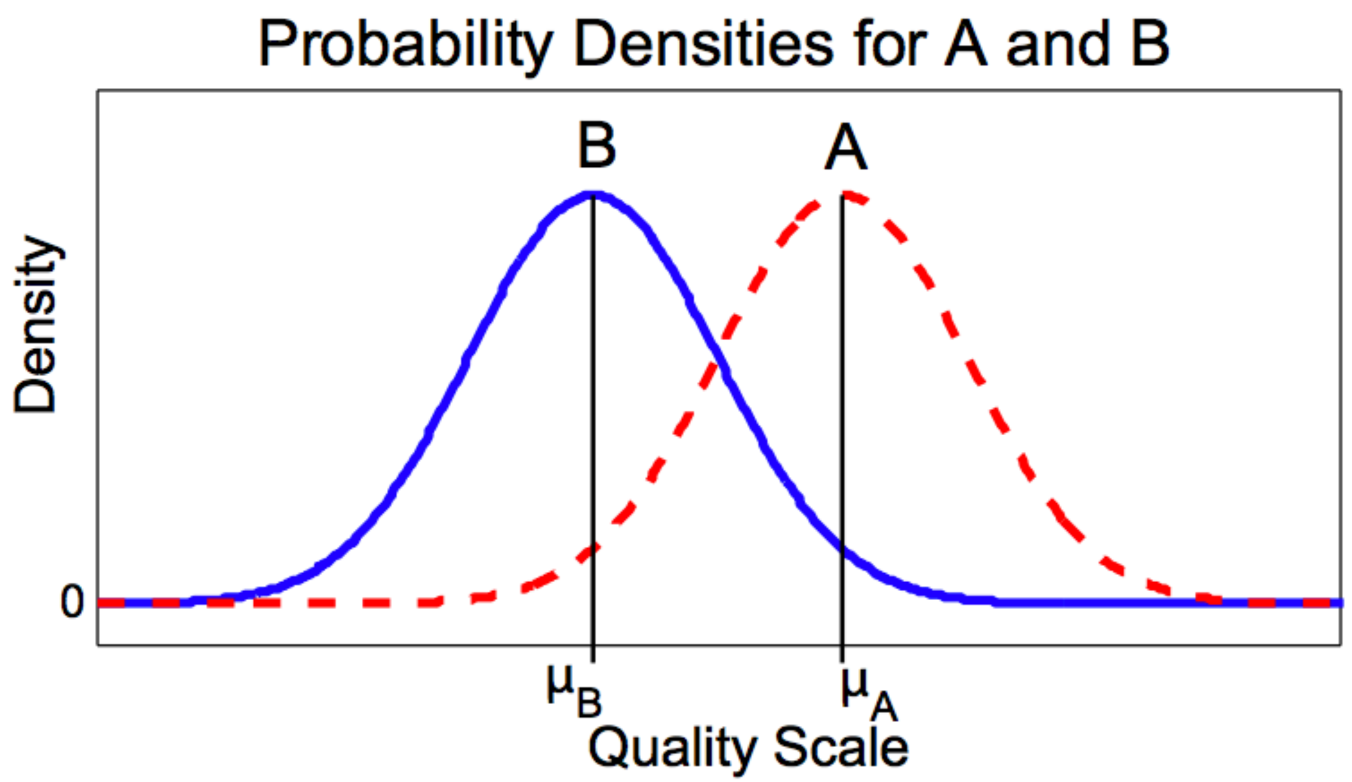
Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



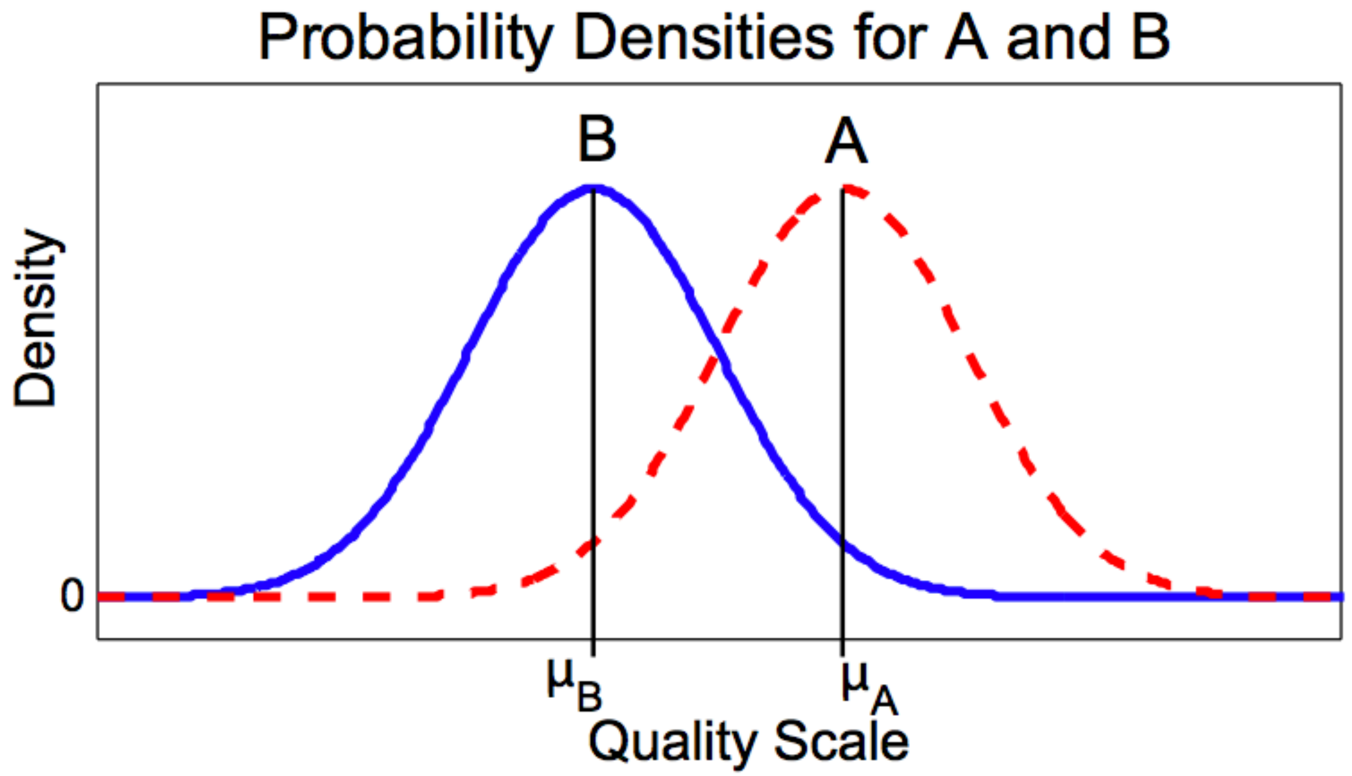
BTL

$$P(A > B) = \frac{\pi_A}{\pi_A + \pi_B}$$

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



BTL

$$P(A > B) = \frac{\pi_A}{\pi_A + \pi_B}$$

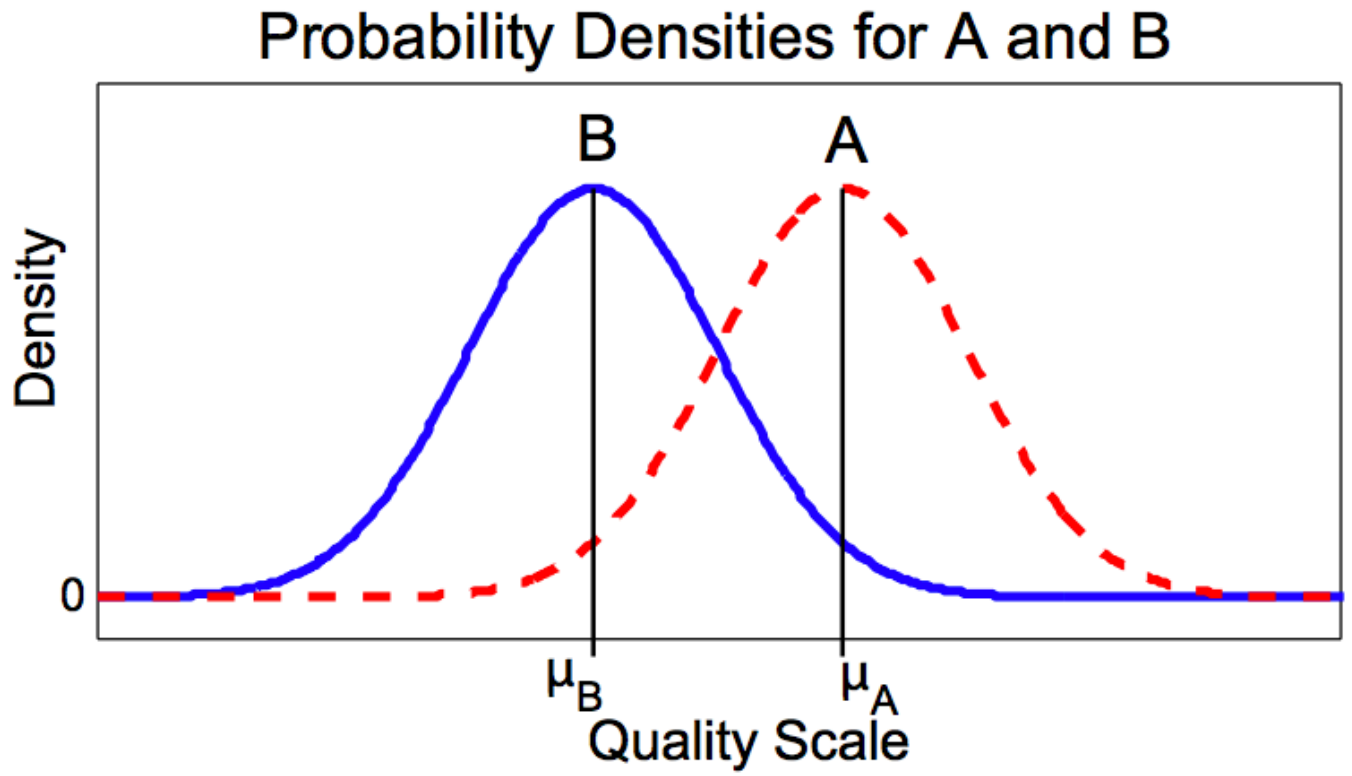
Or usually:

$$P(A > B) = \frac{e^{\mu_A/s}}{e^{\mu_A/s} + e^{\mu_B/s}} = \frac{1}{1 + e^{-(\mu_A - \mu_B)/s}}$$

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



BTL

$$P(A > B) = \frac{\pi_A}{\pi_A + \pi_B}$$

Or usually:

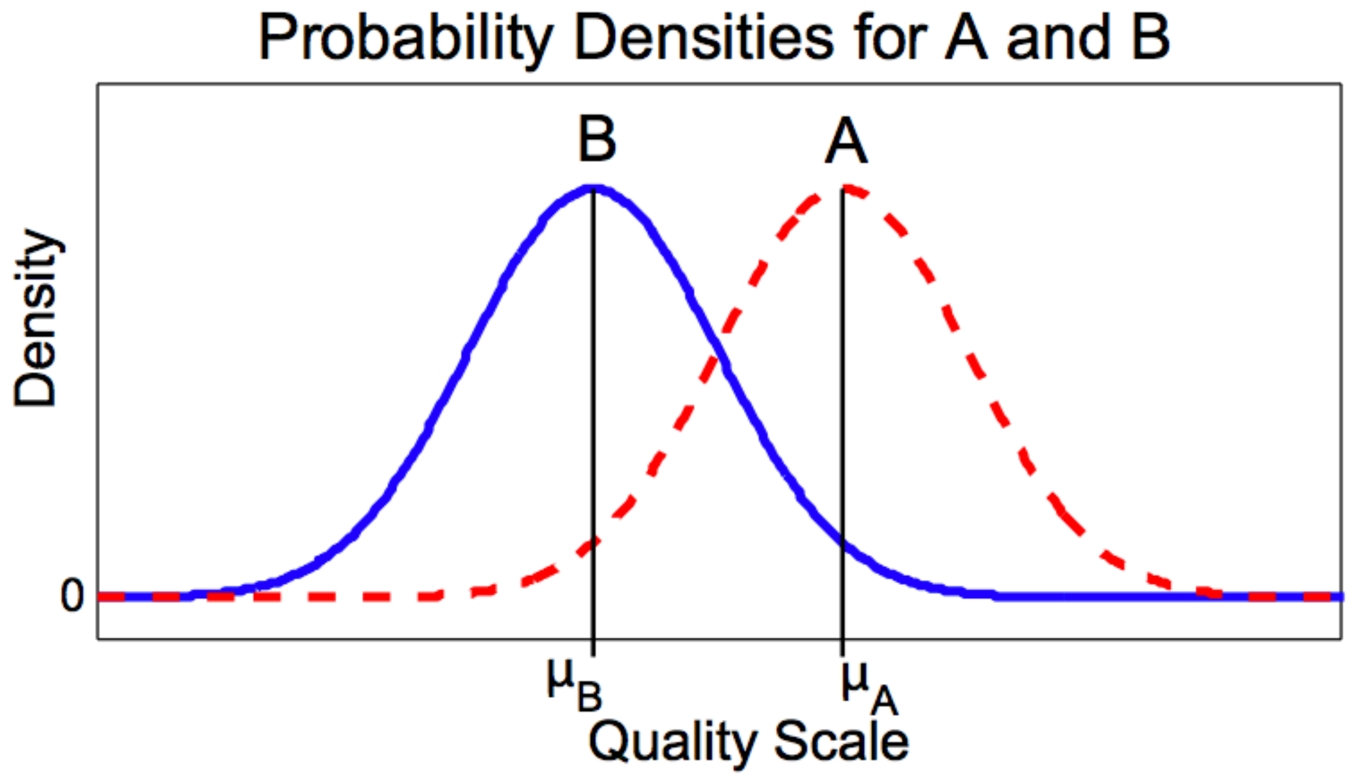
$$P(A > B) = \frac{e^{\mu_A/s}}{e^{\mu_A/s} + e^{\mu_B/s}} = \frac{1}{1 + e^{-(\mu_A - \mu_B)/s}}$$

Same idea; different distribution. (*logit* instead of *probit*; *Gumbel* instead of *Gaussian*)

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

Embeddings and Orderings 1: Discrete choice models

Bradley-Terry & Luce: items have quality distributions. When a person judges whether A is better than B they draw from A's and from B's distribution and see which is higher.



BTL

$$P(A > B) = \frac{\pi_A}{\pi_A + \pi_B}$$

Or usually:

$$P(A > B) = \frac{e^{\mu_A/s}}{e^{\mu_A/s} + e^{\mu_B/s}} = \frac{1}{1 + e^{-(\mu_A - \mu_B)/s}}$$

Same idea; different distribution. (*logit* instead of *probit*; *Gumbel* instead of *Gaussian*)

Powerful idea: lots of pairwise comparisons = estimates of all the qualities! An embedding!

Choice modelers would instead say that we model utility, which has an observable component and a random component. They call these Random Utility Models.

BTL avoids non-transitivities (aka rock-paper-scissors)

BTL avoids non-transitivities (aka rock-paper-scissors)

Introducing: **non-transitive dice**!

BTL avoids non-transitivities (aka rock-paper-scissors)

Introducing: **non-transitive dice**!

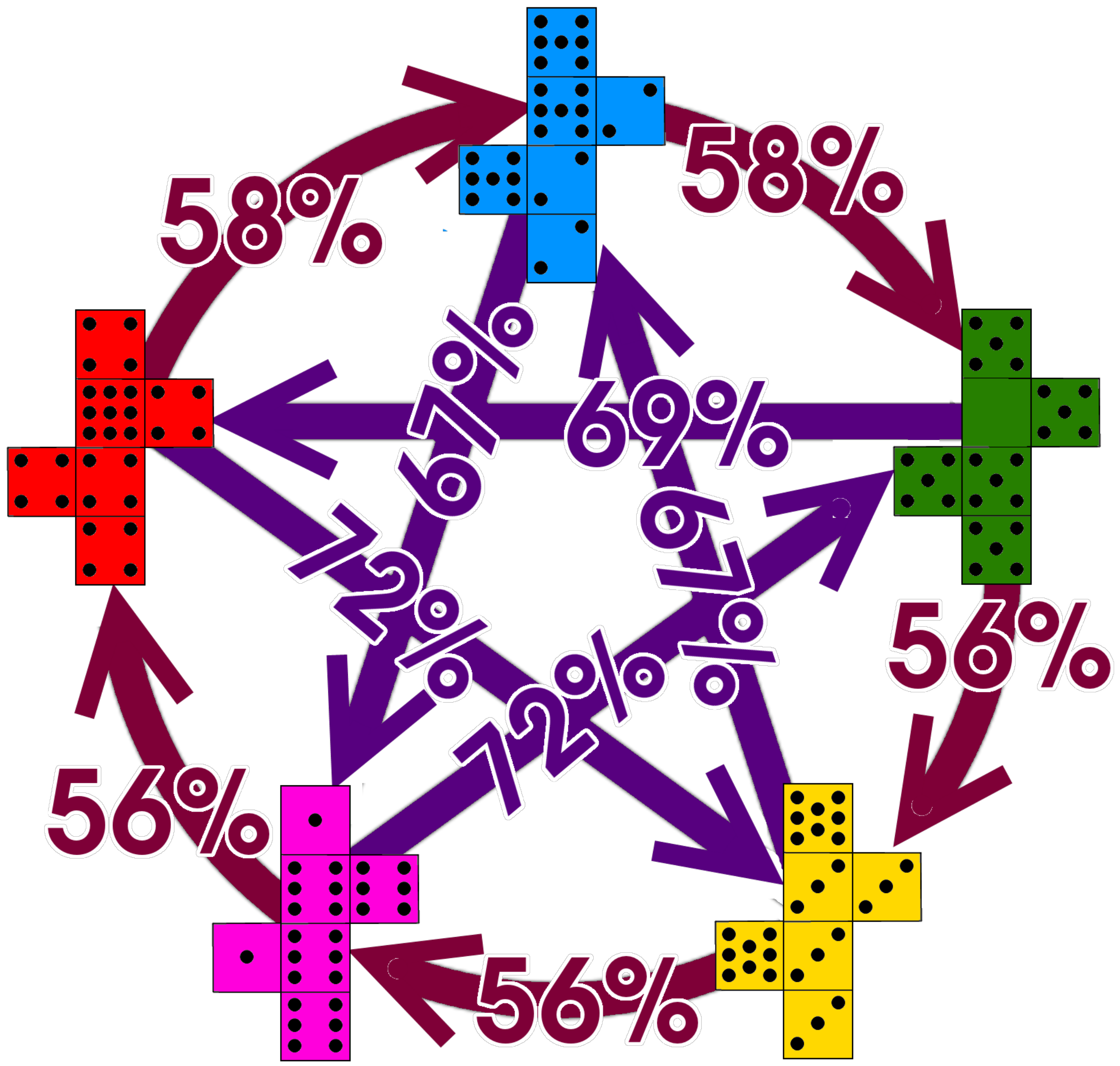
- 3 (or more) dice {A,B,C}
- faces chosen so that they have the property:
  - A>B more than half the time.
  - B>C more than half the time.
  - C>A more than half the time (?!)

BTL avoids non-transitivities (aka rock-paper-scissors)

Introducing: **non-transitive dice**!

- 3 (or more) dice {A,B,C}
- faces chosen so that they have the property:
  - A>B more than half the time.
  - B>C more than half the time.
  - C>A more than half the time (?!)

https://en.wikipedia.org/wiki/Nontransitive_dice



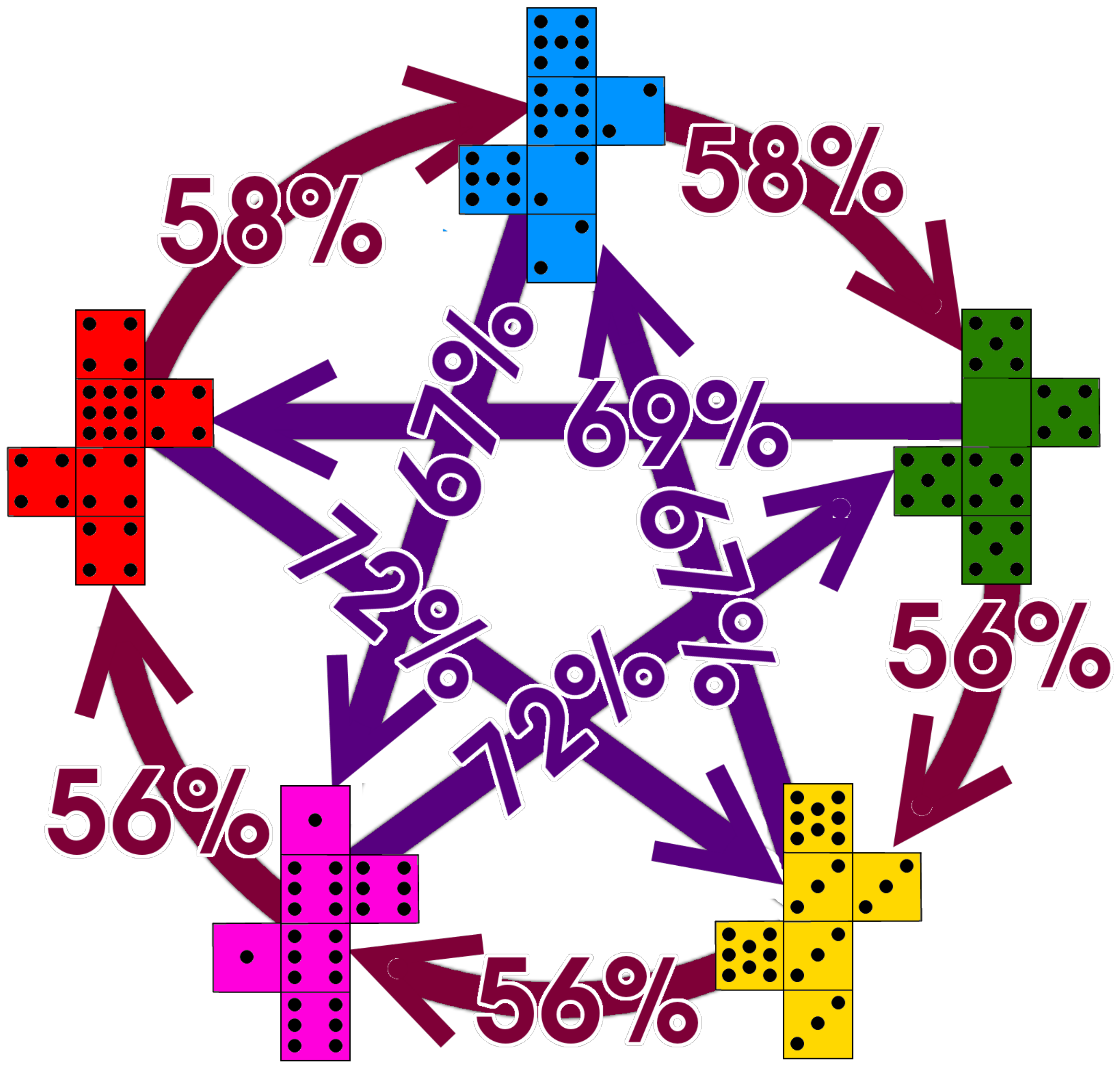
BTL avoids non-transitivities (aka rock-paper-scissors)

Introducing: **non-transitive dice**!

- 3 (or more) dice {A,B,C}
- faces chosen so that they have the property:
  - A>B more than half the time.
  - B>C more than half the time.
  - C>A more than half the time (?!)

https://en.wikipedia.org/wiki/Nontransitive_dice

A great gift for your favorite nerd's desk!
Go to the makerspace and laserbeam your own!



Bradley-Terry-Luce

These methods embed items or players in a 1D space.

- Provably avoids non-transitive properties
- Great when lots of data per interaction.

Pairwise ranking is really nice for ordering large sets of preferences too, and this model specifically models the probability that the preference will be for *i* over *j*.

Iterative algorithms exist. Needs a little regularization so the winningest winners don't fly off to infinity. [why?]

$$P(A \to B) = \frac{\pi_A}{\pi_A + \pi_B} = \frac{e^{\gamma_A}}{e^{\gamma_A} + e^{\gamma_B}}$$

Embeddings and Orderings 1: Discrete choice models

Introductory tutorial (Gupta):
http://mayagupta.org/publications/PairedComparisonTutorialTsukidaGupta.pdf

Discrete choice today (Ugander):
https://web.stanford.edu/~jugander/papers/nips16-pcmc-slides.pdf

The textbook (Train):
https://eml.berkeley.edu/books/train1201.pdf

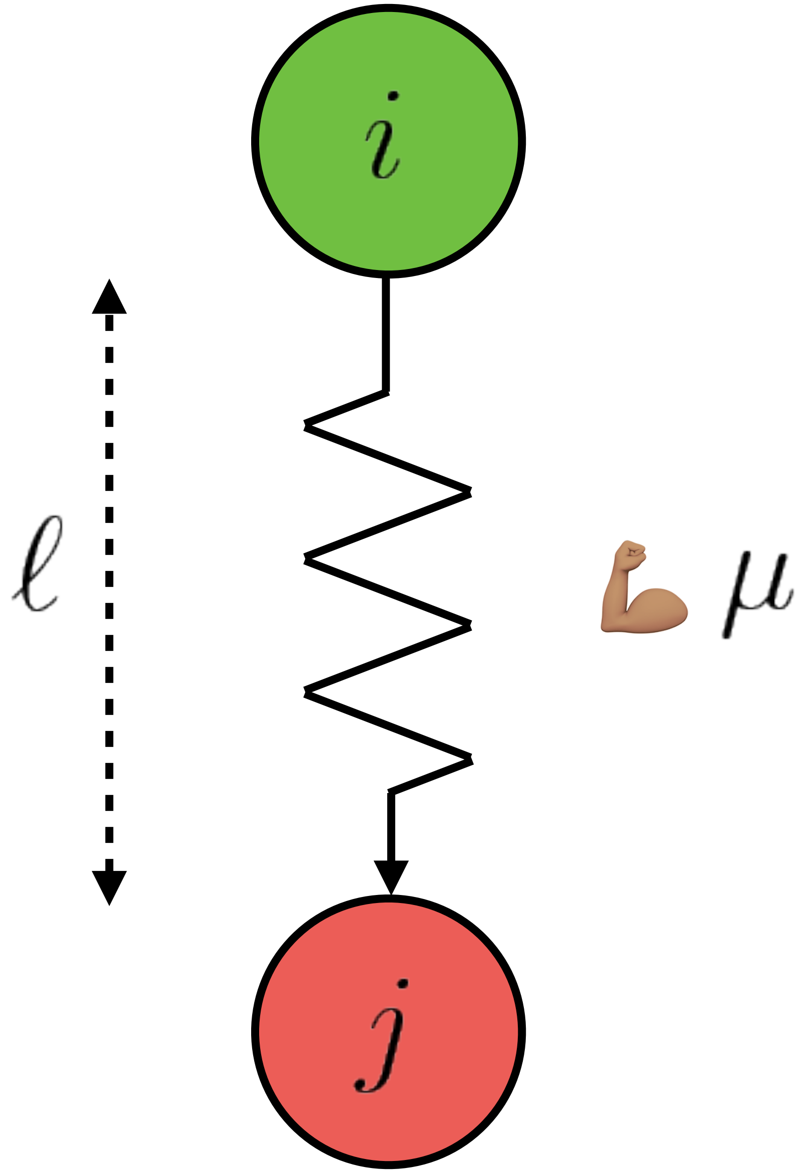
Nobel Lecture for roadtrip when you're out of podcasts (McFadden)
https://www.nobelprize.org/prizes/economic-sciences/2000/mcfadden/facts/

This is such good stuff, people! Lots of problems can be viewed through the lens of Choice Modeling, and powerful Random Utility Models. Dig in!

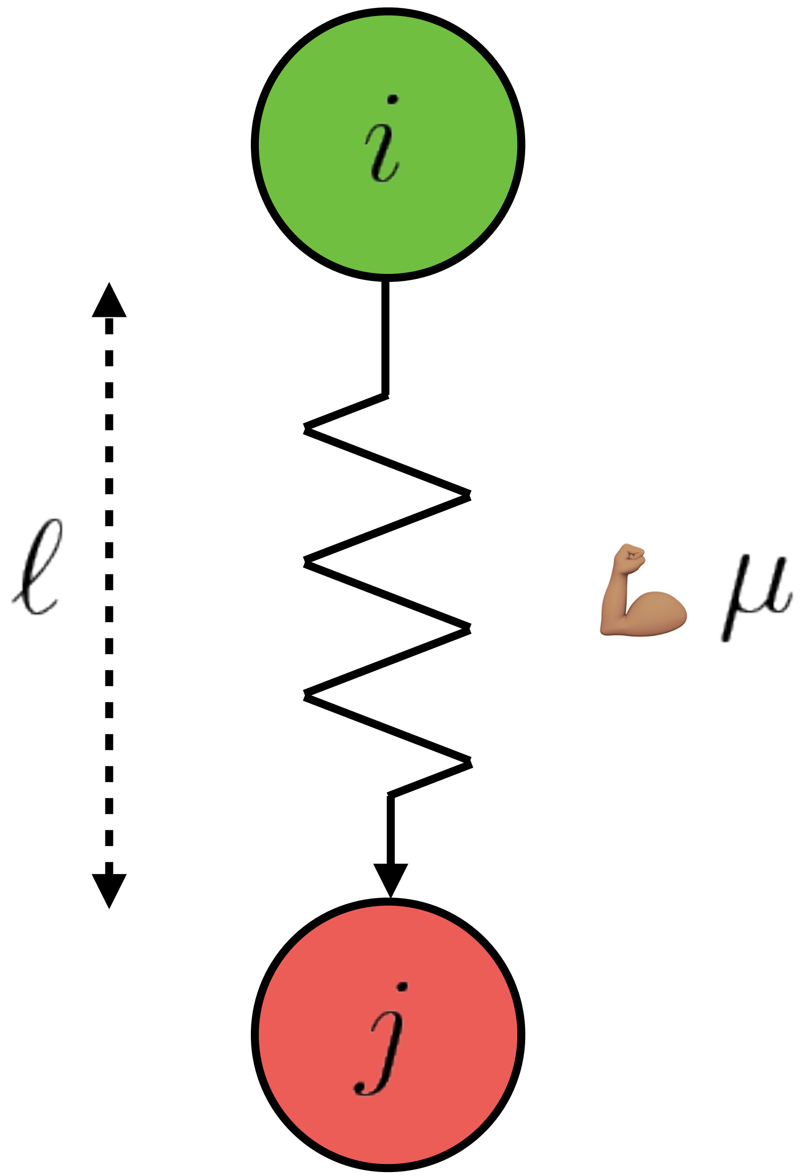
Embeddings & Orderings 2: SpringRank

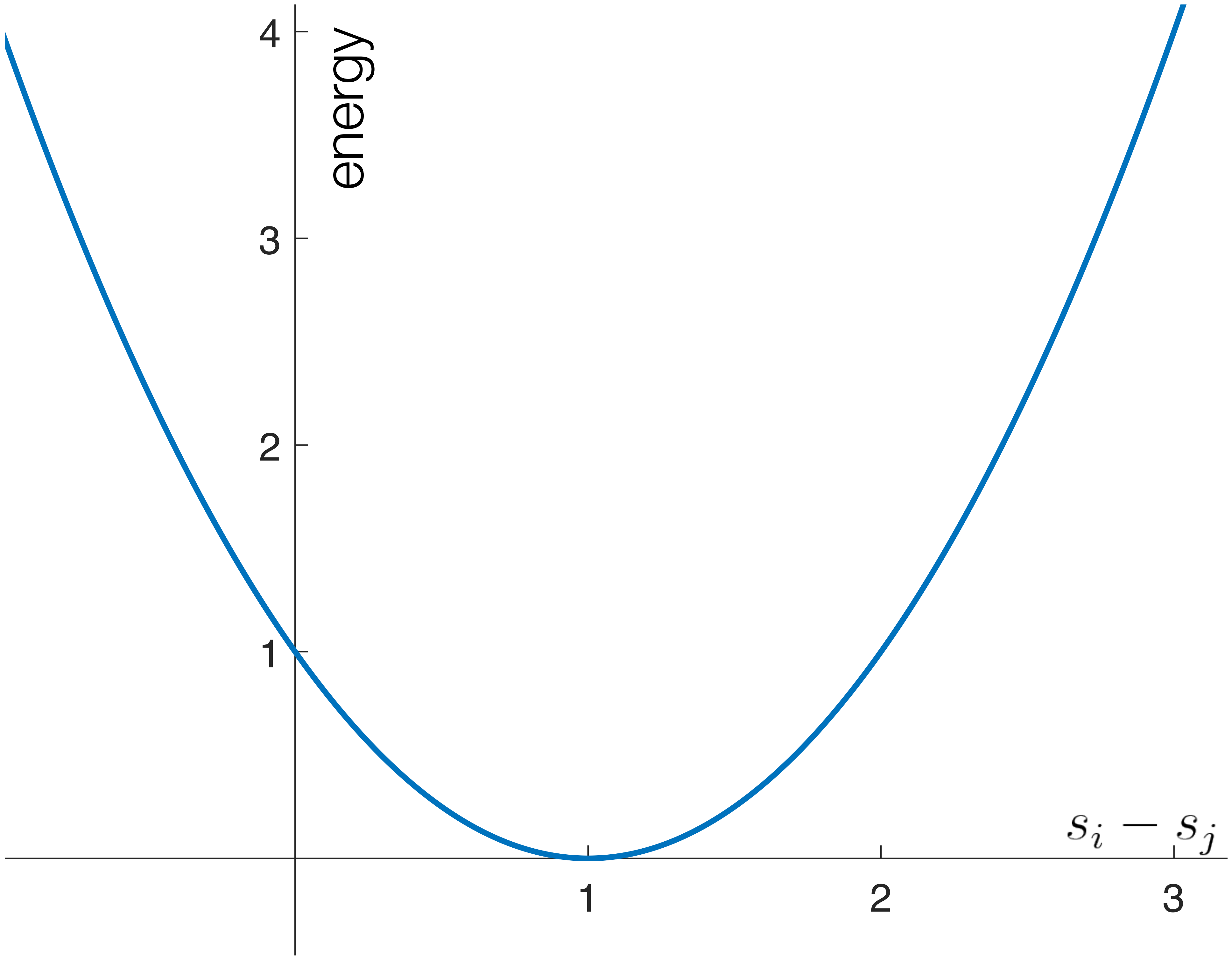
Each directed edge = directed spring

Each directed edge = directed spring

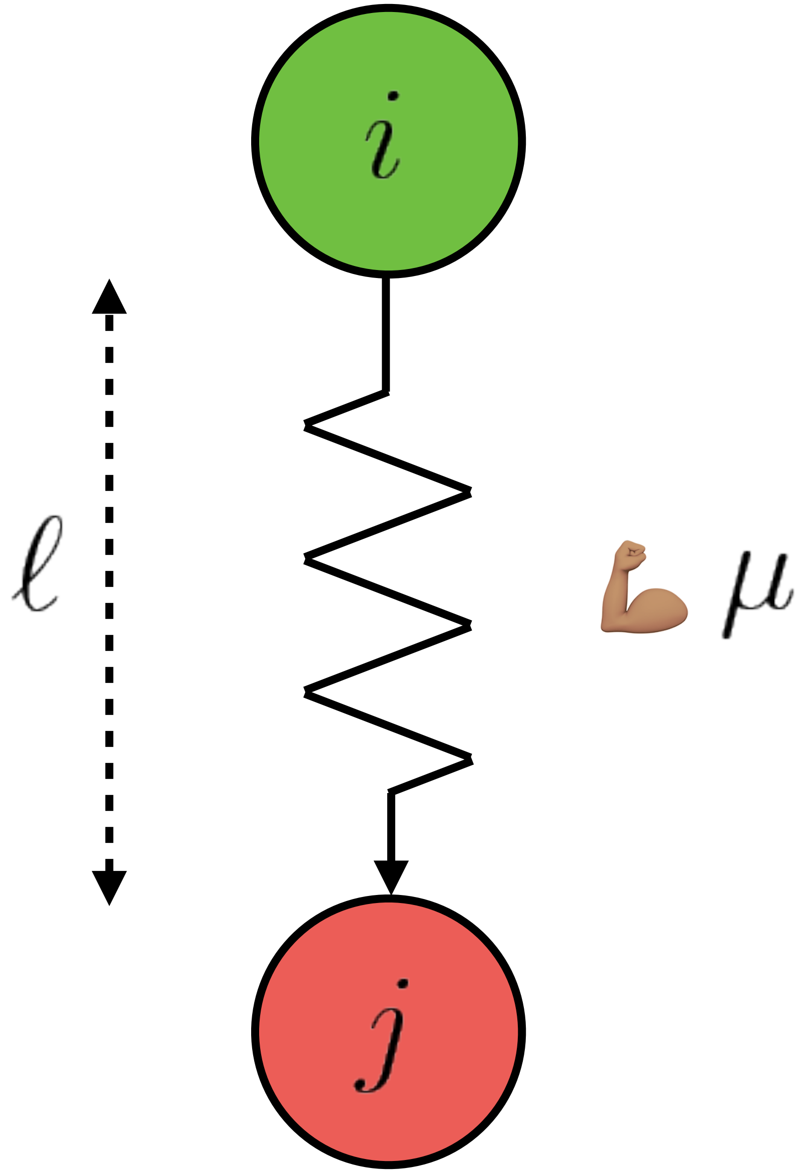


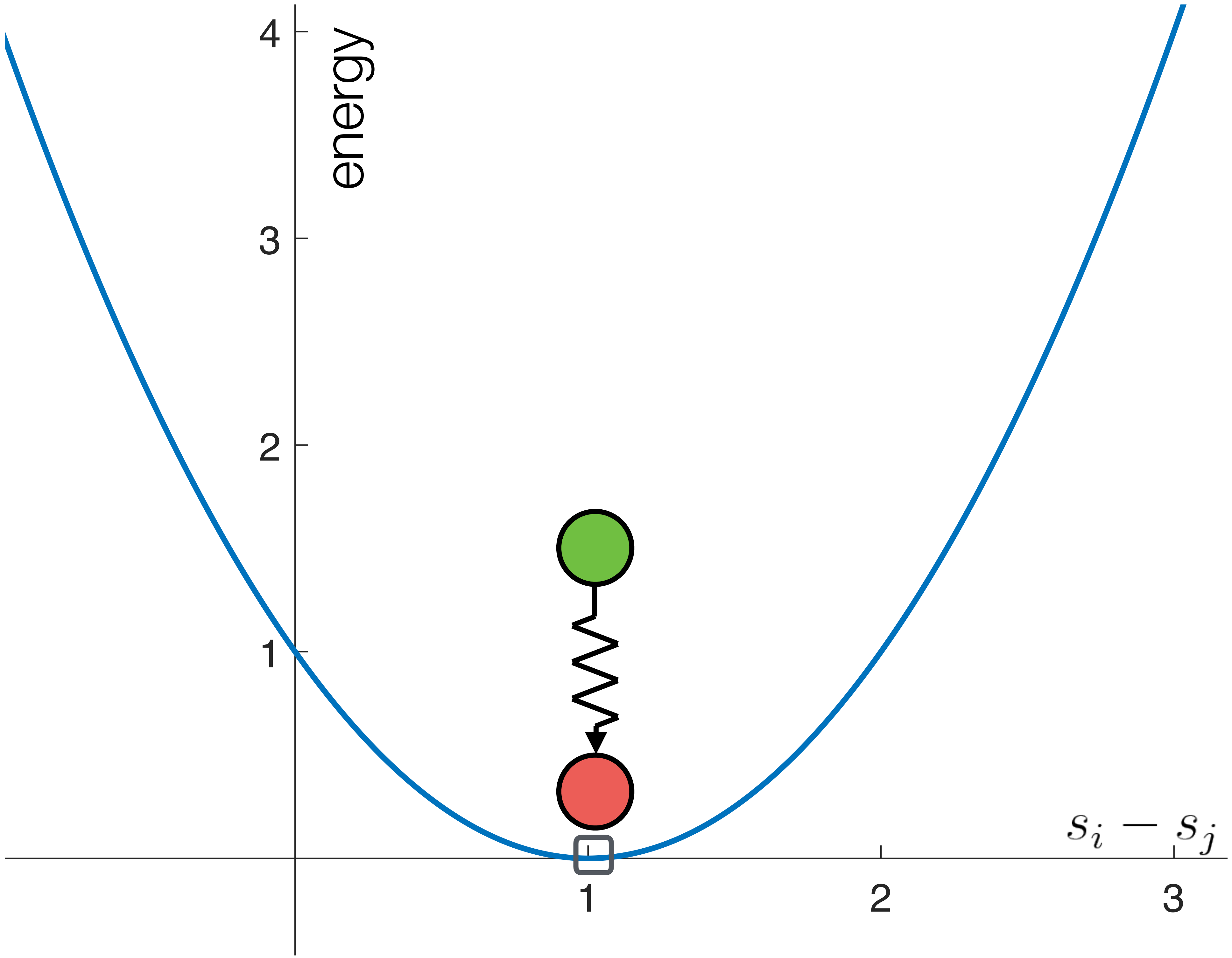
Each directed edge = directed spring



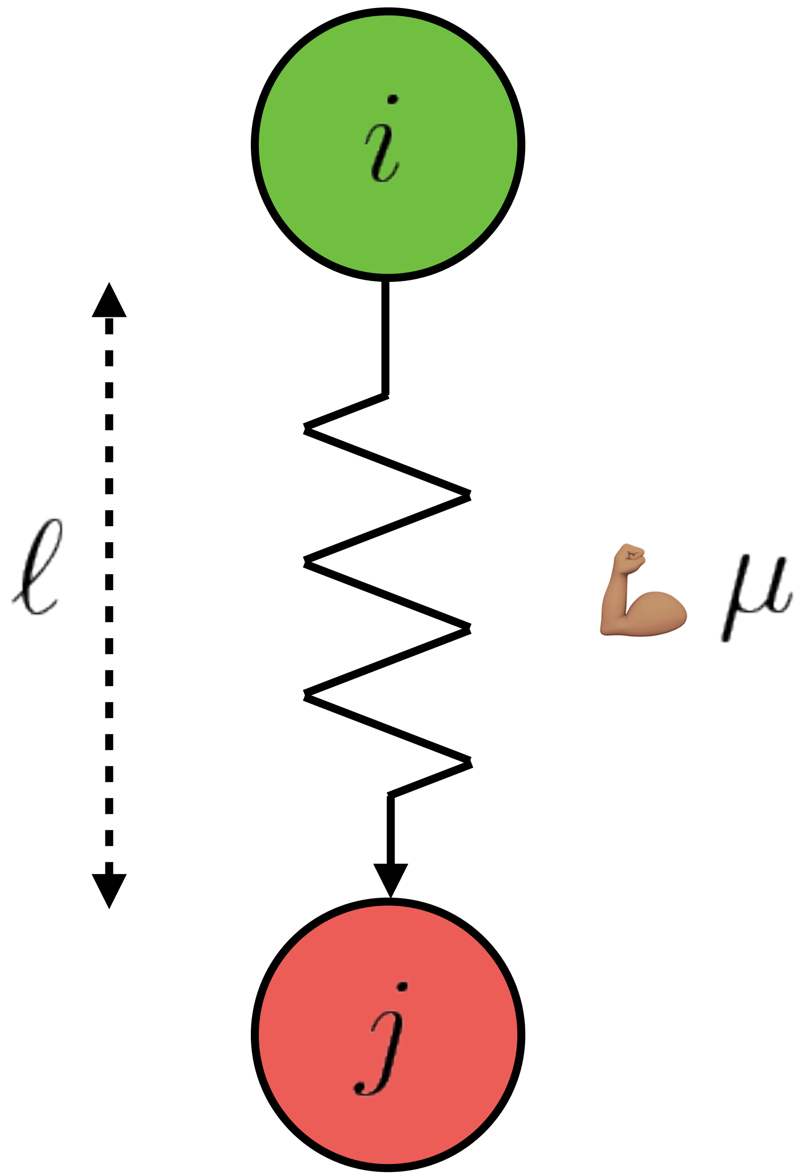


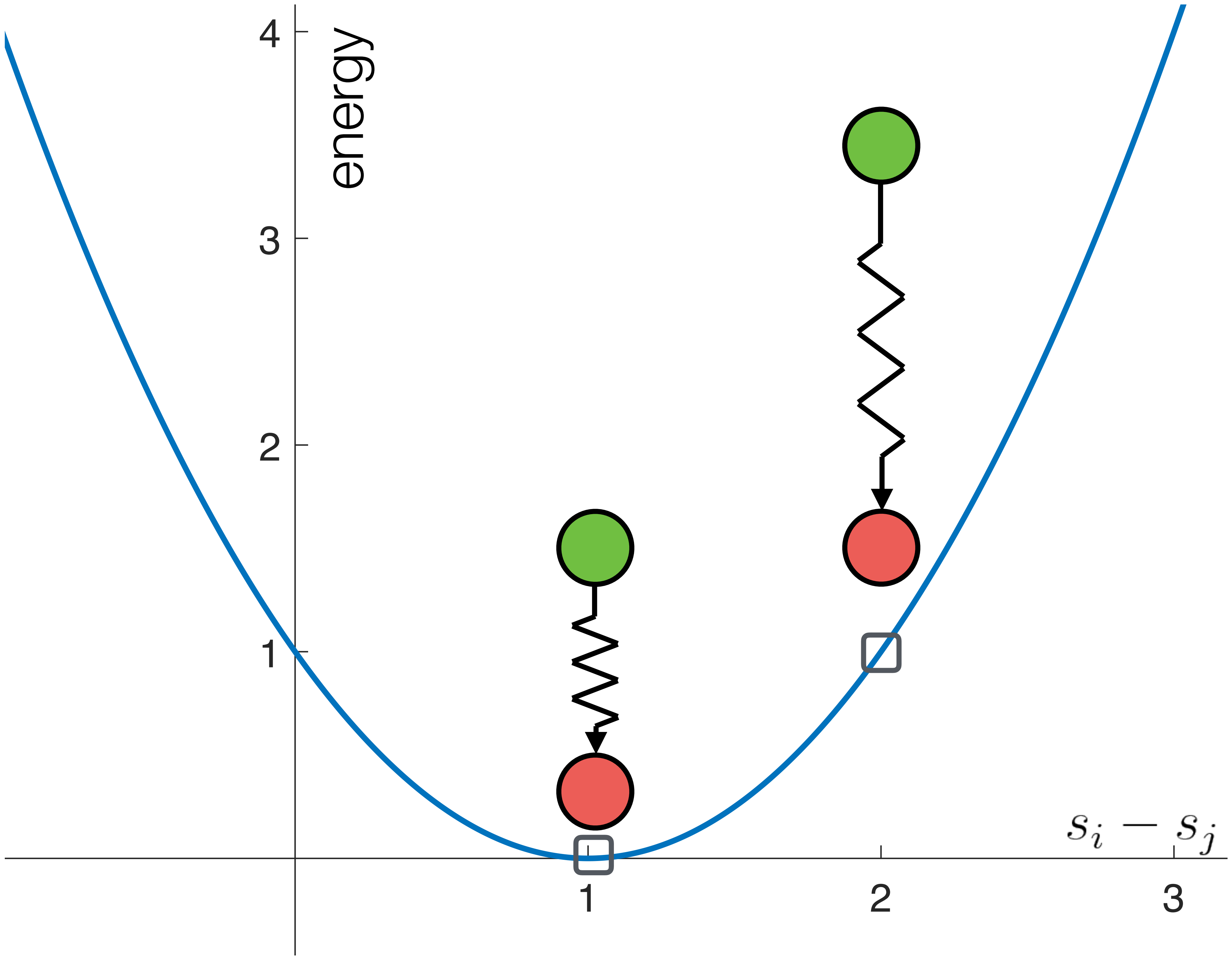
Each directed edge = directed spring



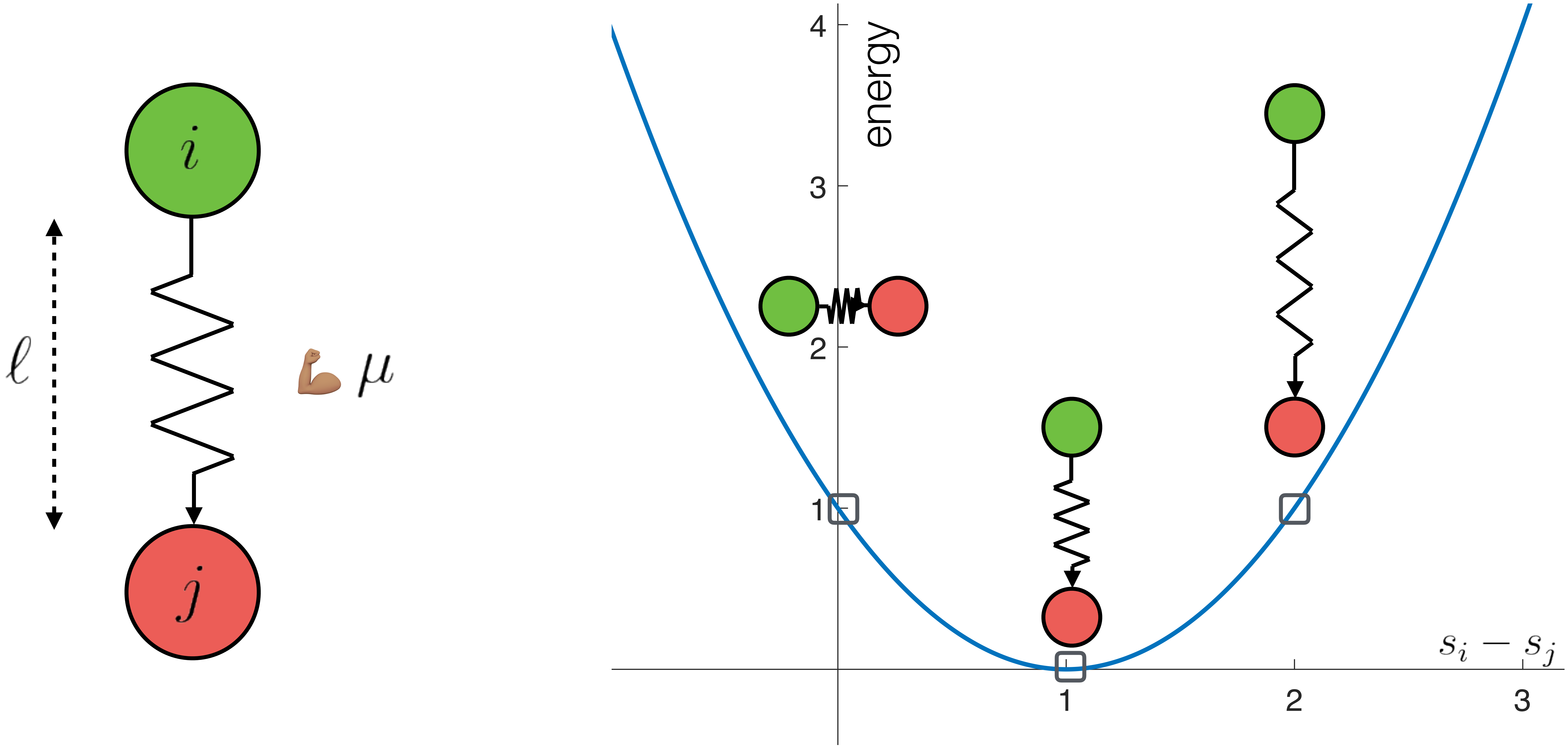


Each directed edge = directed spring

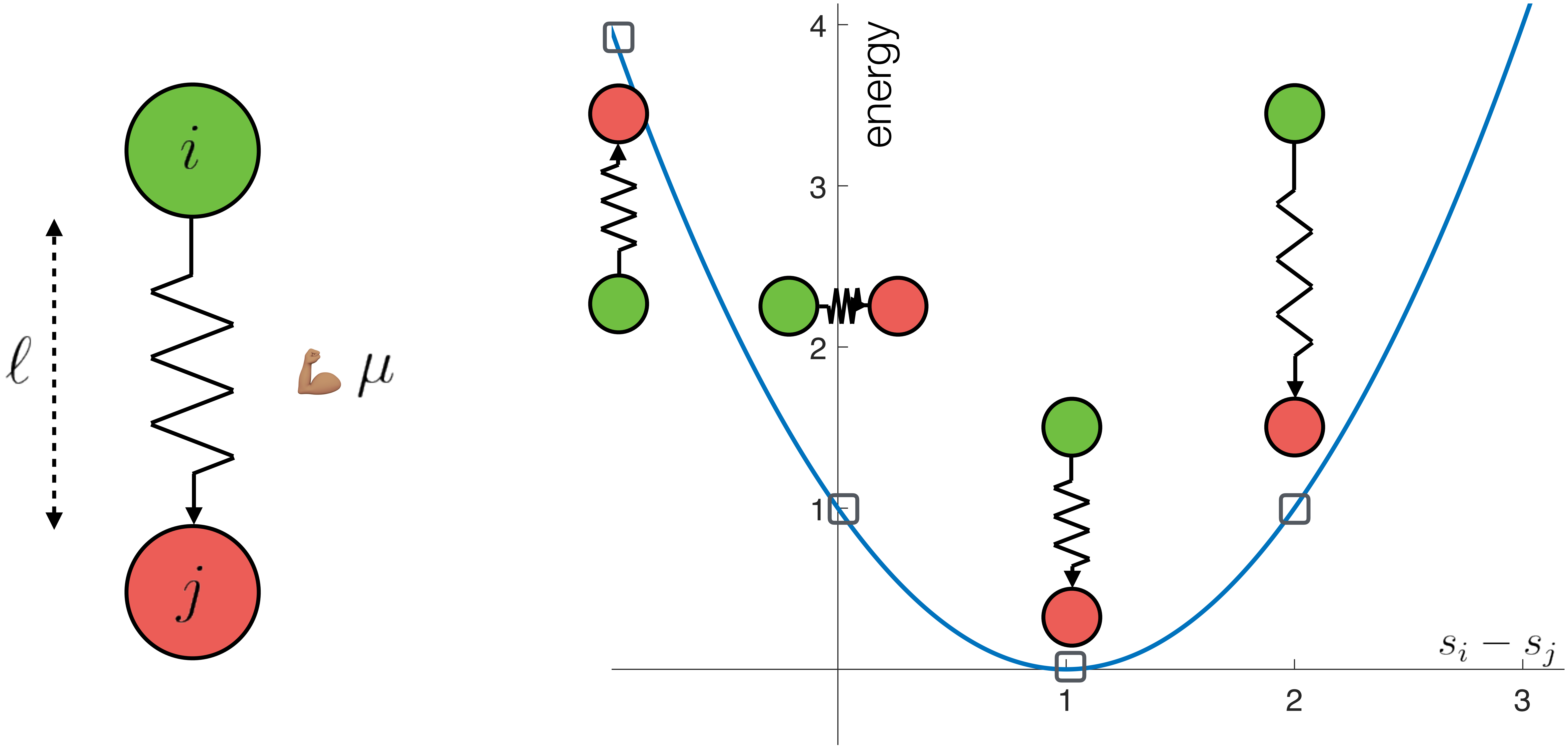




Each directed edge = directed spring



Each directed edge = directed spring



How much energy is this system of springs?

How much energy is this system of springs?

$$(s_i - s_j - 1)^2$$

How much energy is this system of springs?

$$A_{ij}(s_i - s_j - 1)^2$$

How much energy is this system of springs?

$$\sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

How much energy is this system of springs?

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

How much energy is this system of springs?

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

SpringRank Hamiltonian = energy of the system, given the node positions *s.*

Relax and let the springs decide the ranks

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

SpringRank Hamiltonian = energy of the system, given the node positions *s.*

Because the springs are linear, the potential is quadratic.

Relax and let the springs decide the ranks

$$H(s) = \frac{1}{2} \sum_{i,j=1}^{N} A_{ij}(s_i - s_j - 1)^2$$

SpringRank Hamiltonian = energy of the system, given the node positions *s.*

Because the springs are linear, the potential is quadratic.

The SR Hamiltonian is *convex* in s.

$$\nabla H(s) = 0$$

The solution is unique…up to an additive constant. (Why?)

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

We know *a priori* that the matrix on the left is singular: translational invariance of H(s).

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

We know *a priori* that the matrix on the left is singular: translational invariance of H(s).

[if *s* is a solution, then *s* + *k* is a solution for any constant *k;* eigenvalue 0, eigenvector **1**]

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

We know *a priori* that the matrix on the left is singular: translational invariance of H(s).

[if *s* is a solution, then *s* + *k* is a solution for any constant *k;* eigenvalue 0, eigenvector **1**]

Notice: the matrix on the left is the *graph Laplacian* of the *undirected* network.

Derivatives work out nicely

$$0 = \frac{\partial H}{\partial s_i} = \sum_j A_{ij}(s_i - s_j - 1) - A_{ji}(s_j - s_i - 1)$$

Rewrite as a linear algebra problem.

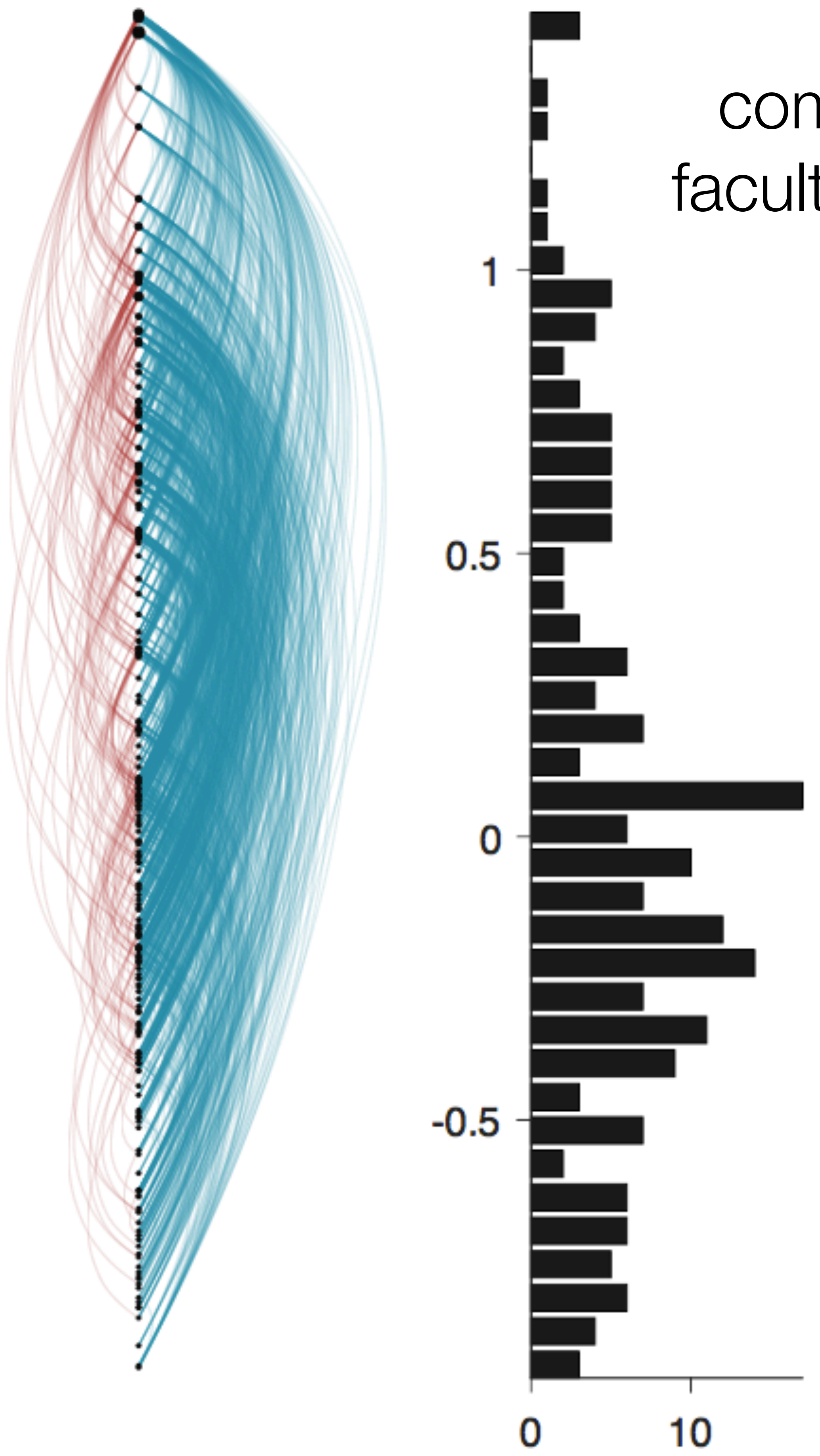
$$\left[D^{\text{out}} + D^{\text{in}} - \left(A + A^T\right)\right] s^* = \left[D^{\text{out}} - D^{\text{in}}\right] \mathbf{1}$$

We know *a priori* that the matrix on the left is singular: translational invariance of H(s).
[if *s* is a solution, then *s* + *k* is a solution for any constant *k;* eigenvalue 0, eigenvector **1**]

Notice: the matrix on the left is the *graph Laplacian* of the *undirected* network.

Uniqueness: Set s_1=0, min(*s*)=0, or mean(*s*)=0. Or use a pseudoinverse. Or regularize.

It works!

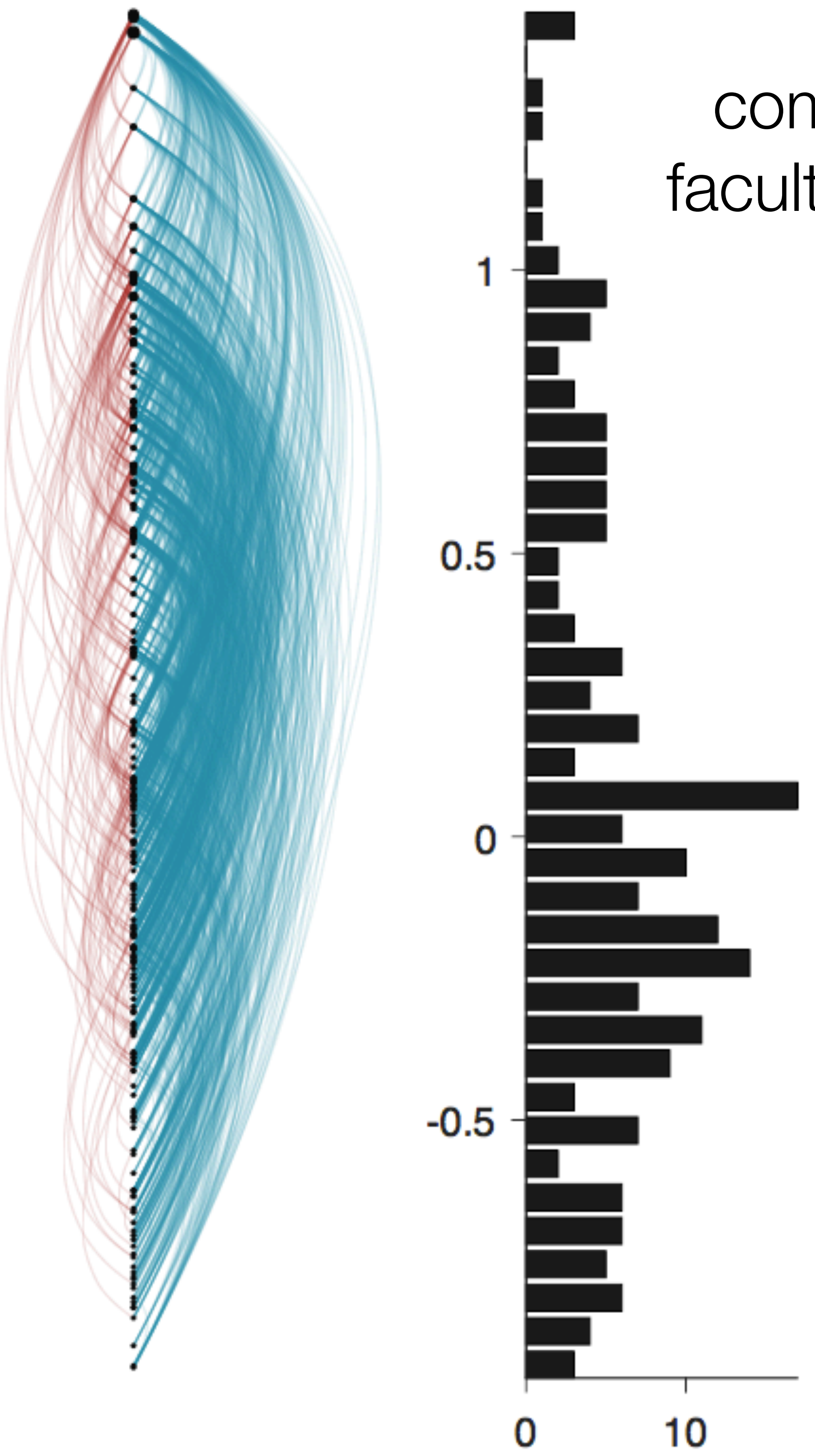


computer science
faculty hiring network

It works!

Real networks tend to be sparse…
  our linear algebra problem is sparse…
    we can use sparse iterative solvers…
      millions of edges in **seconds**.

Even better: it's a linear-Laplacian system.
🚀 Near-linear-time (in |edges|) solutions.



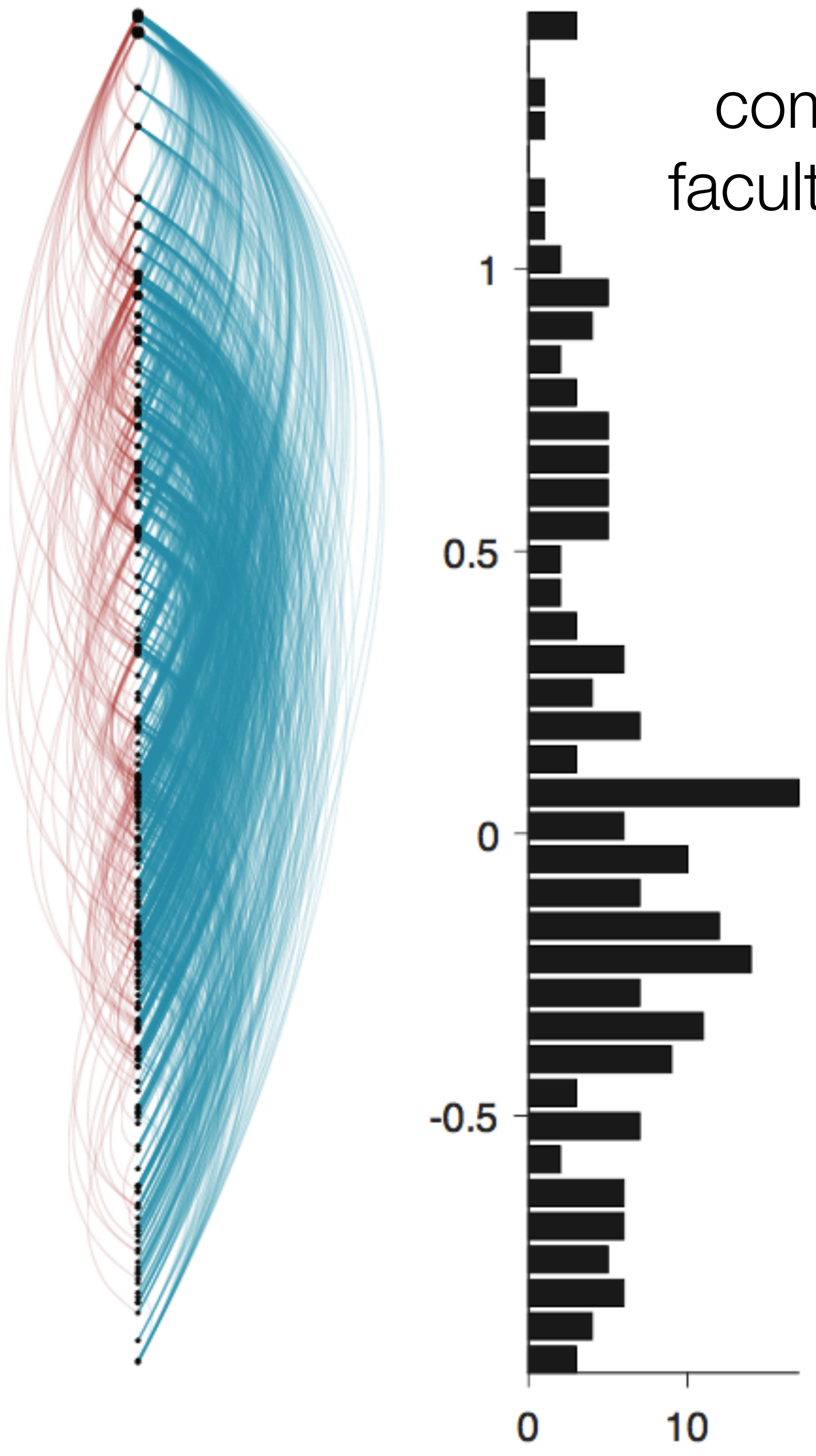
computer science
faculty hiring network

It works!

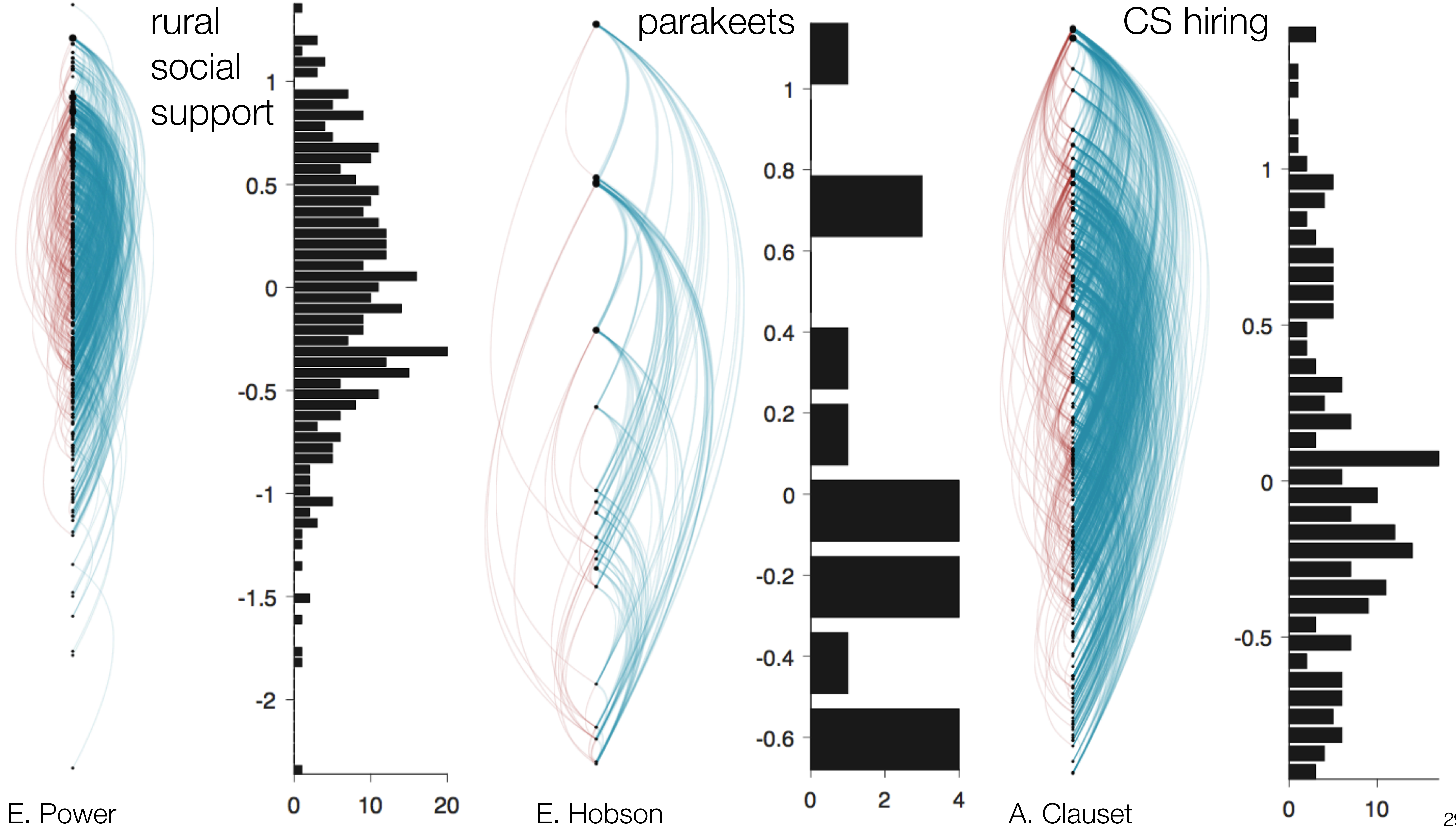
Real networks tend to be sparse…
our linear algebra problem is sparse…
we can use sparse iterative solvers…
millions of edges in **seconds**.

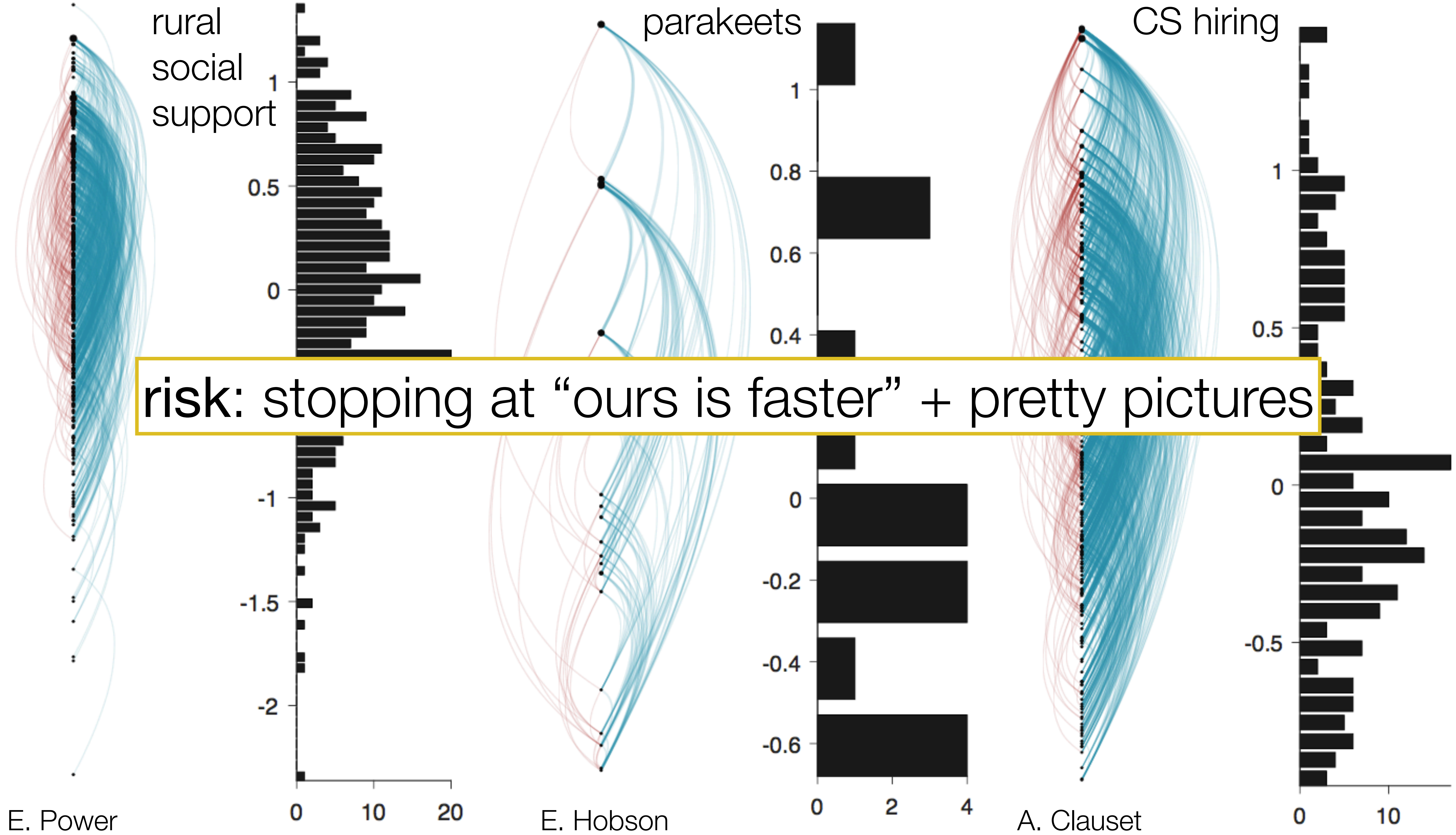
Even better: it's a linear-Laplacian system.
🚀 Near-linear-time (in |edges|) solutions.

Note that node positions can be clumpy, since this is an *embedding*.



computer science faculty hiring network


rural
social
support
1
0.5
0
-0.5
-1
-1.5
-2
0
10
20
E. Power
parakeets
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0
2
4
E. Hobson
CS hiring
1
0.5
0
-0.5
0
10
A. Clauset



risk: stopping at “ours is faster” + pretty pictures

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If i and j were to face off, who would win?

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If *i* and *j* were to face off, who would win?

I'll give you *undirected(A)*, and you predict *directed(A).*

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If i and j were to face off, who would win?

I'll give you *undirected(A)*, and you predict *directed(A).*

Setup: learn s from 80% of A. Then predict edge directions for remaining 20% of A.

Cross validation: train on 80%, predict 20%

In a linear hierarchy the key quantity to predict is *edge direction,* given *edge existence*.

If *i* and *j* were to face off, who would win?

I'll give you *undirected(A)*, and you predict *directed(A).*

Setup: learn *s* from 80% of *A*. Then predict edge directions for remaining 20% of *A.*

SpringRank predicts edge direction based on the relative direction probabilities:

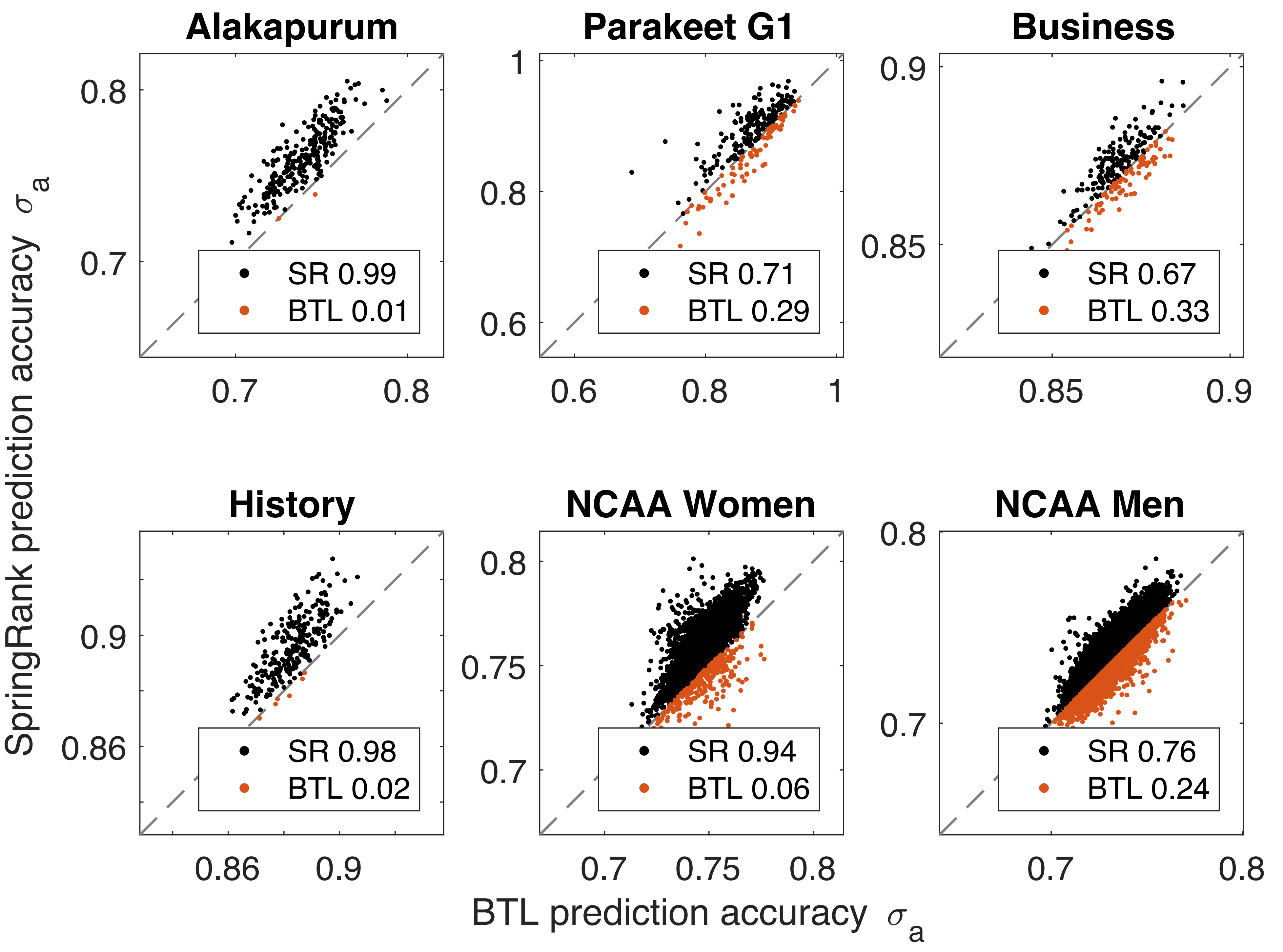
$$P_{ij}(\beta) = \frac{e^{-\beta H_{ij}}}{e^{-\beta H_{ij}} + e^{-\beta H_{ji}}} = \frac{1}{1 + e^{-2\beta(s_i - s_j)}}$$

Cross validation vs BTL: SR makes better predictions

Accuracy:

$$\sigma_a = 1 - \frac{1}{2M} \sum_{i,j} |A_{ij} - (A_{ij} + A_{ji})\, P_{ij}|$$

Goal: maximize the number of correctly predicted edge directions.



Sync Rank, Cucuringu et al. https://ieeexplore.ieee.org/document/7395367

50 independent trials of 5-fold cross validation (250 folds)

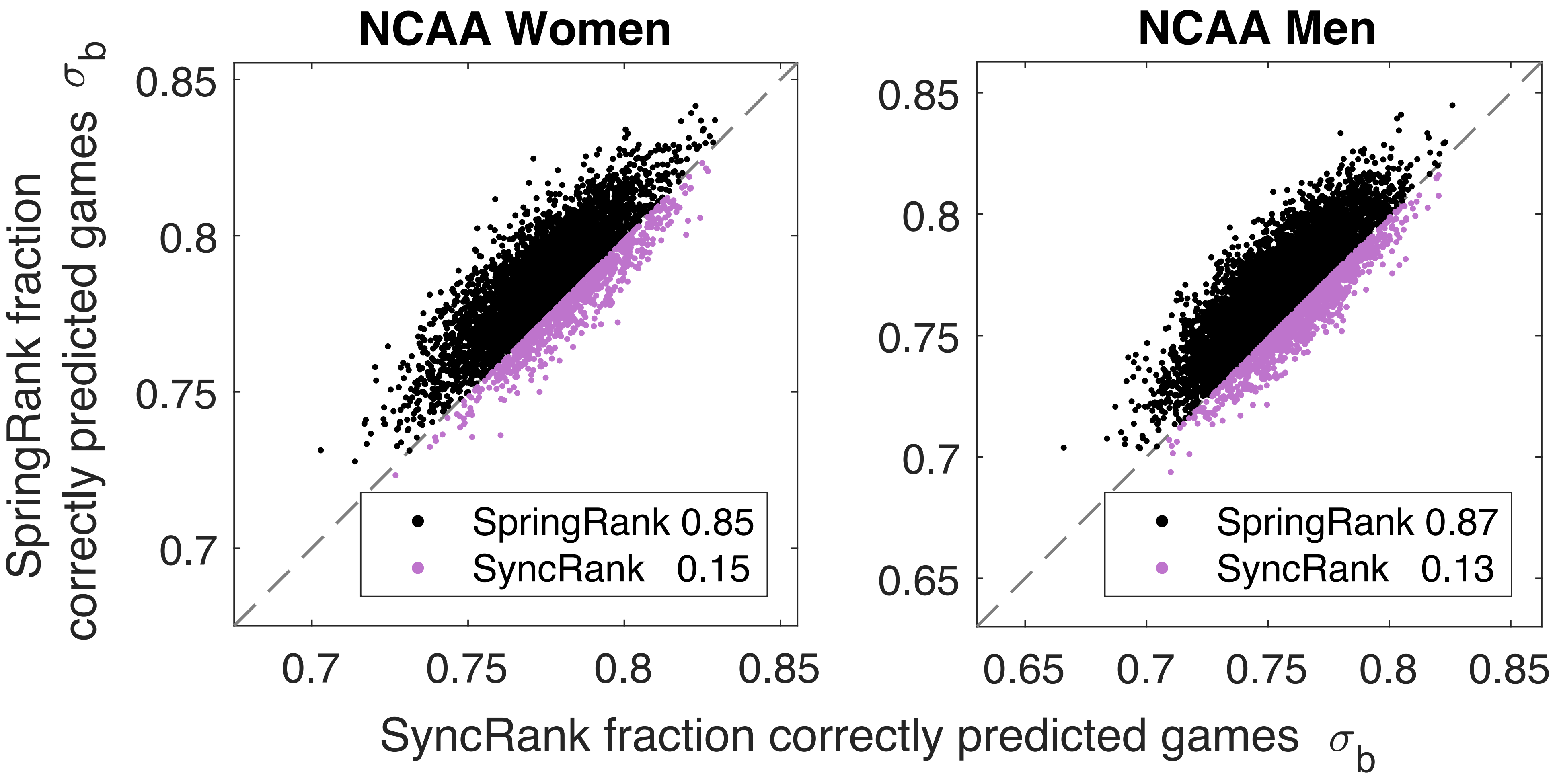
Cross validation vs SyncRank: SR makes better predictions

"One-bit" Accuracy:

Higher ranked player always wins.

- No probabilistic prediction.
- Bad for gambling.

Goal: maximize the number of correctly predicted edge directions.



50 independent trials of 5-fold cross validation (250 folds)

Why/when would a model of springs make better predictions than a model of the choices themselves?🧐

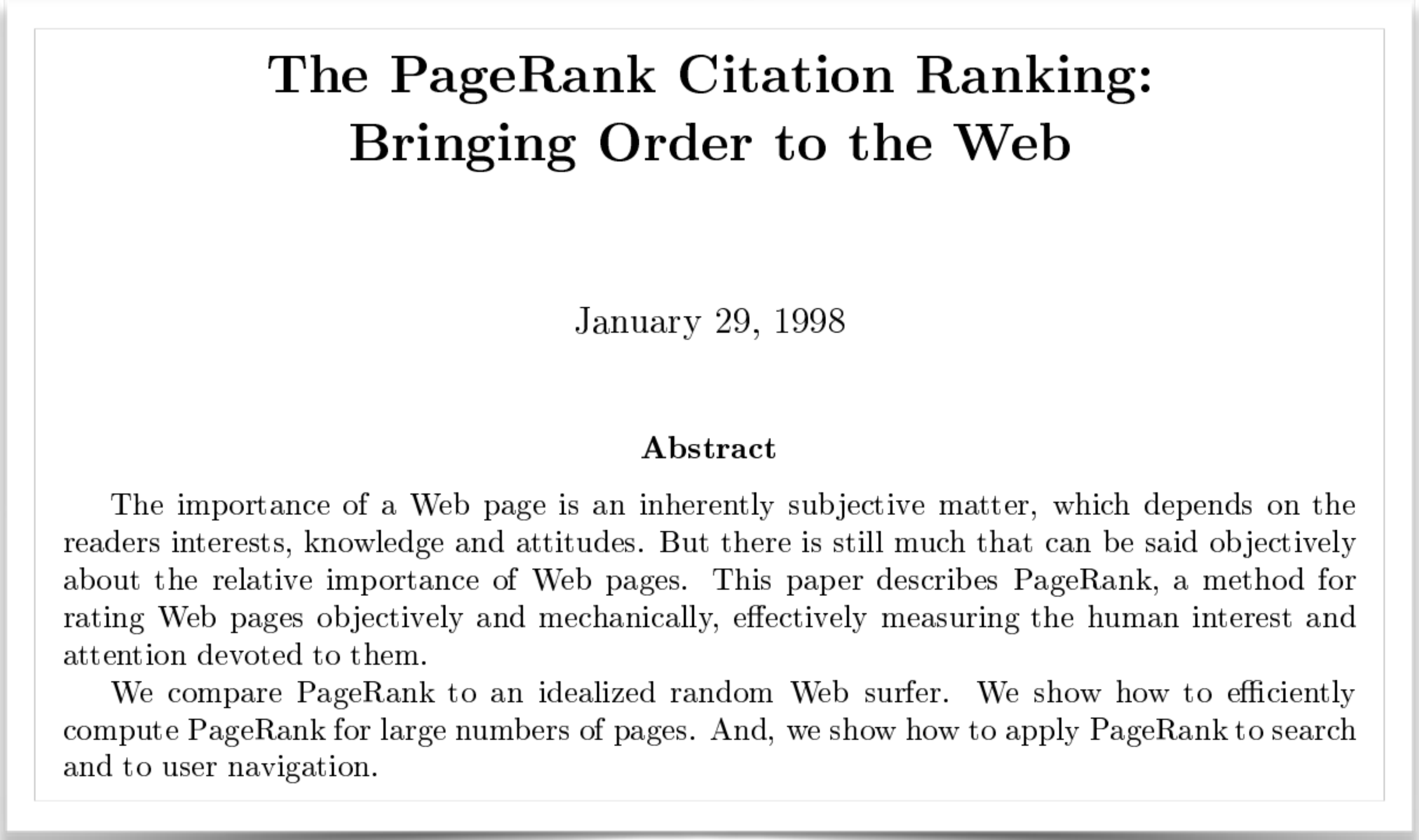
It’s unclear *why* we get this result!
Both BTL and SpringRank make logistic predictions about preference.
Key Idea: SpringRank makes different regularization assumptions.

Embeddings and Orderings 3: PageRank

PageRank defines scalar rank recursively:

important pages are those that are linked to by important pages.

- Great at finding the top 3 but limited predictions available using the PageRank scores.



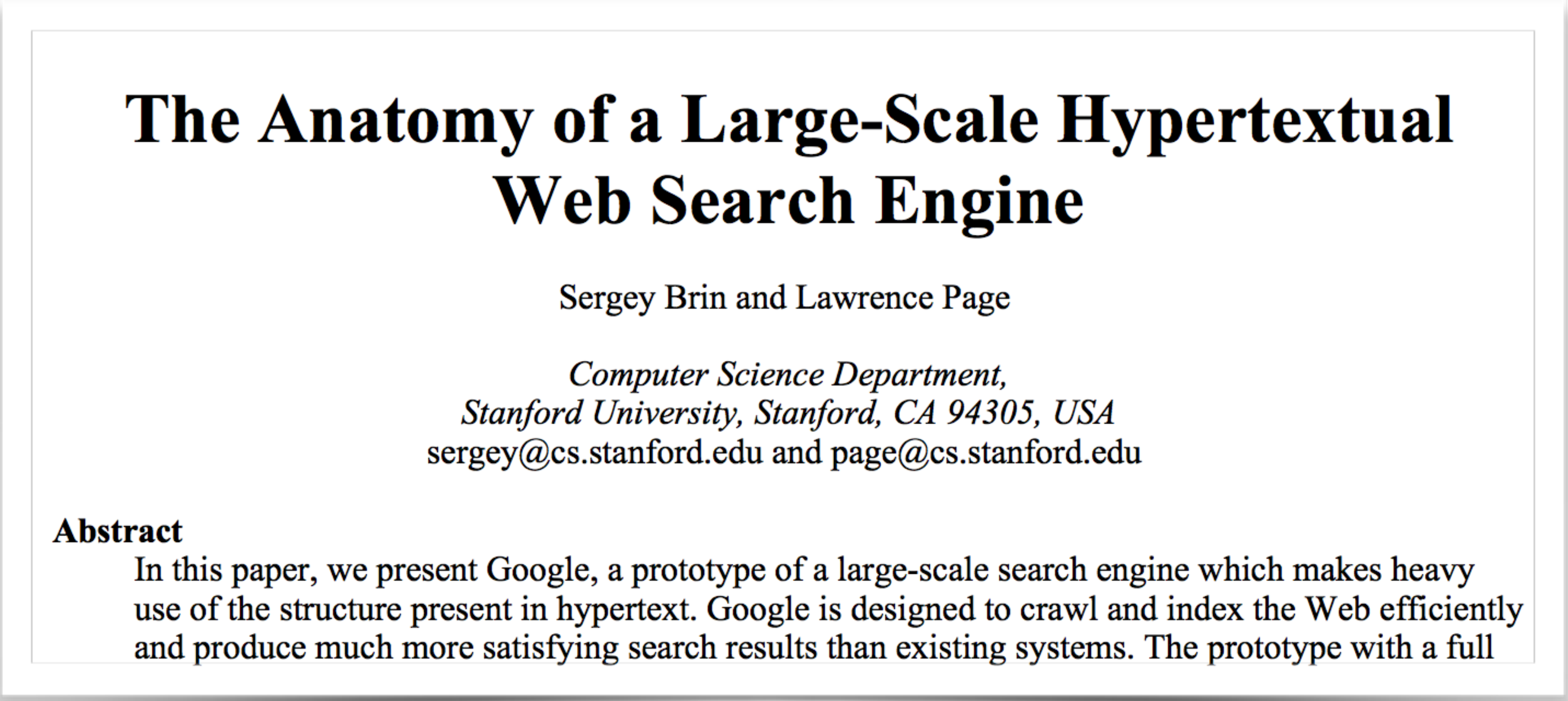
The PageRank Citation Ranking:
Bringing Order to the Web

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.



The Anatomy of a Large-Scale Hypertextual Web Search Engine

Sergey Brin and Lawrence Page

Computer Science Department,
Stanford University, Stanford, CA 94305, USA
sergey@cs.stanford.edu and page@cs.stanford.edu

Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d\sum_j p_j \pi_{ji}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d\sum_j p_j \pi_{ji}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d\sum_j p_j \pi_{ji}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d \sum_j p_j \pi_{ji}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N}\right) \mathbf{1} + d\pi^T \mathbf{p}$$

matrix-vector form

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \boxed{\frac{1-d}{N}} + \boxed{d \sum_j p_j \pi_{ji}}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N}\right)\mathbf{1} + d\pi^T\mathbf{p}$$

matrix-vector form

Alternative: stationary distribution of random walk on the network + weak all-to-all links

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d\sum_j p_j \pi_{ji}$$

write the equation

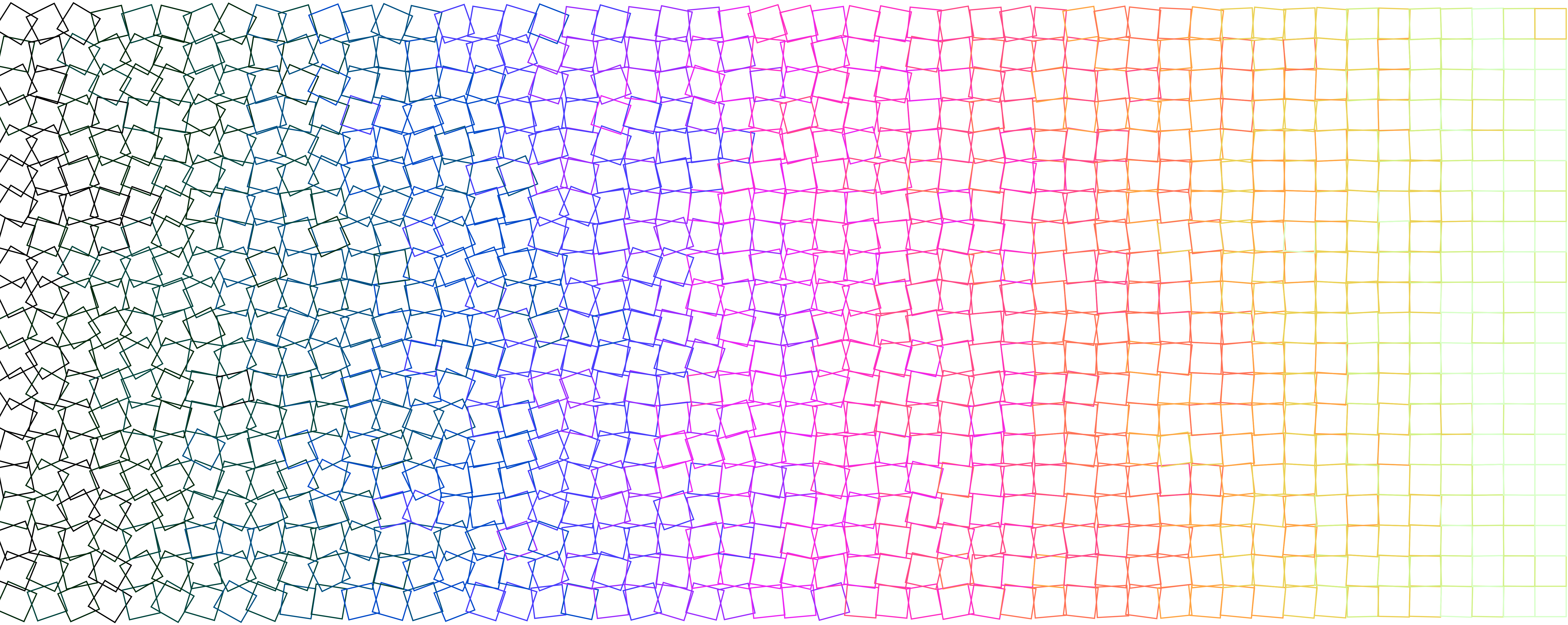
$$\mathbf{p} = \left(\frac{1-d}{N}\right)\mathbf{1} + d\pi^T \mathbf{p}$$

matrix-vector form

Alternative: stationary distribution of random walk on the network + weak all-to-all links

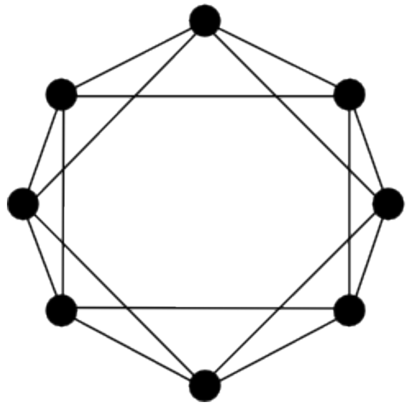
Jeremy Kun: http://www.infinitelooper.com/?v=K3pT0gTaDec&p=n

sin/cos: generative models for networks



Stochastic models, sets, and distributions

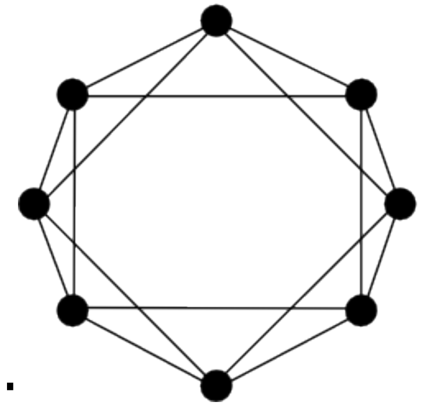
- a model is just a recipe:
  choose parameters→make the network

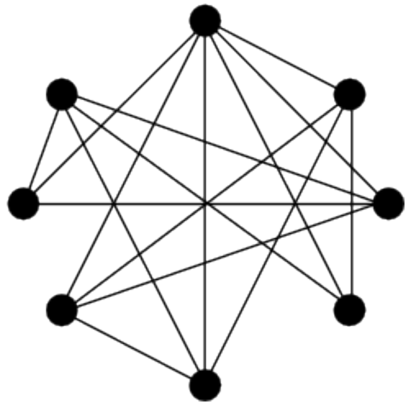


Consider the configuration model.

Stochastic models, sets, and distributions

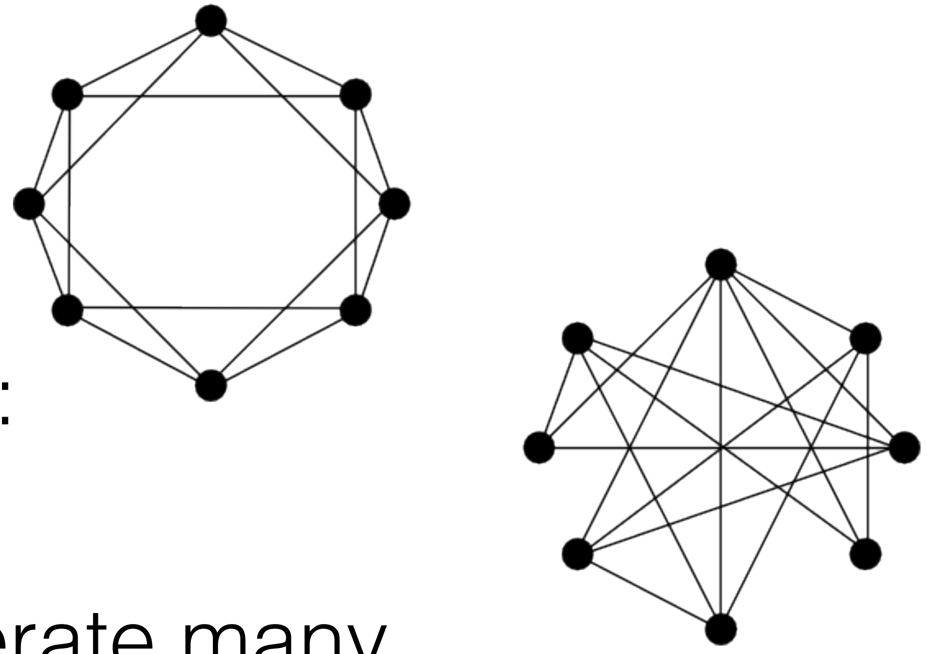
- a model is just a recipe:
  choose parameters→make the network
- a **stochastic** (generative) model is also just a recipe:
  choose parameters→draw **a** network





Consider the configuration model.

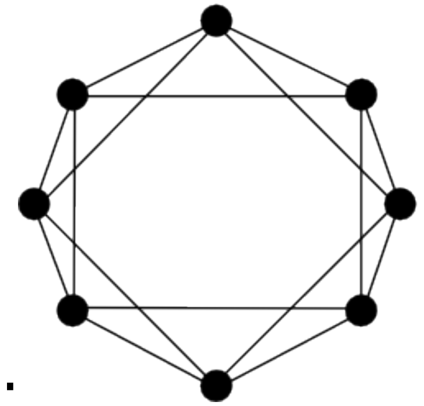
Stochastic models, sets, and distributions

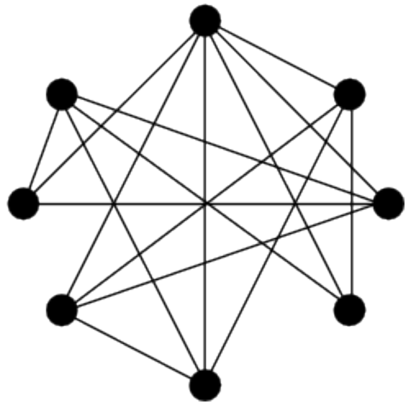


- a model is just a recipe:
  choose parameters→make the network
- a **stochastic** (generative) model is also just a recipe:
  choose parameters→draw **a** network
- since a single stochastic generative model can generate many networks, the model itself corresponds to a **set of networks**.

Consider the configuration model.

Stochastic models, sets, and distributions

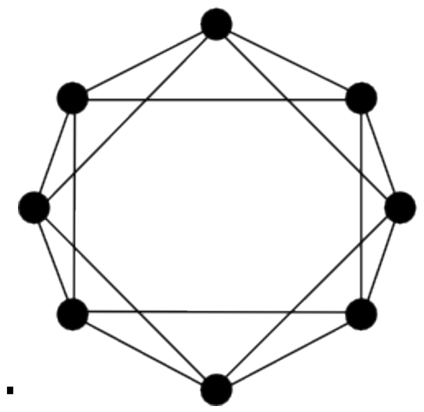


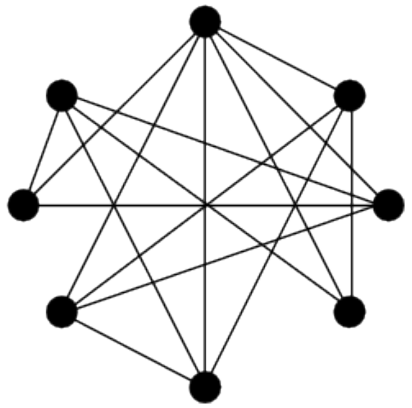


- a model is just a recipe:
  choose parameters→make the network
- a **stochastic** (generative) model is also just a recipe:
  choose parameters→draw **a** network
- since a single stochastic generative model can generate many networks, the model itself corresponds to a **set of networks**.
- and since the generative model itself is some combination or composition of random variables, a **random graph model** is a set of possible networks, each with an associated probability, i.e., a distribution.

Consider the configuration model.

Stochastic models, sets, and distributions





- a model is just a recipe:
  choose parameters→make the network
- a **stochastic** (generative) model is also just a recipe:
  choose parameters→draw **a** network
- since a single stochastic generative model can generate many networks, the model itself corresponds to a **set of networks**.
- and since the generative model itself is some combination or composition of random variables, a **random graph model** is a set of possible networks, each with an associated probability, i.e., a distribution.

By changing the recipe, we can change the *support* of the distribution and the *probability masses* themselves.

Consider the configuration model.

Generative models for network structure

Generate the structure you wish to infer.

We like generative models because they open the door to inference:

Generative models for network structure

Generate the structure you wish to infer.

We like generative models because they open the door to inference:

$$\text{GM + parameters} \xrightarrow{\text{stochastic draw}} \text{Data}$$

Generative models for network structure

Generate the structure you wish to infer.

We like generative models because they open the door to inference:

$$\text{GM + parameters} \xrightarrow{\text{stochastic draw}} \text{Data}$$

$$\text{GM + parameters} \xleftarrow{\text{inference}} \text{Data}$$

Generative models for network structure

Generate the structure you wish to infer.

We like generative models because they open the door to inference:

$$\text{GM + parameters} \xrightarrow{\text{stochastic draw}} \text{Data}$$

$$\text{GM + parameters} \xleftarrow{\text{inference}} \text{Data}$$

In other words: let's write down a model whose ensemble's distribution is not uniform but **highly peaked** around networks with structures that we want to see.

e.g. The stochastic block model

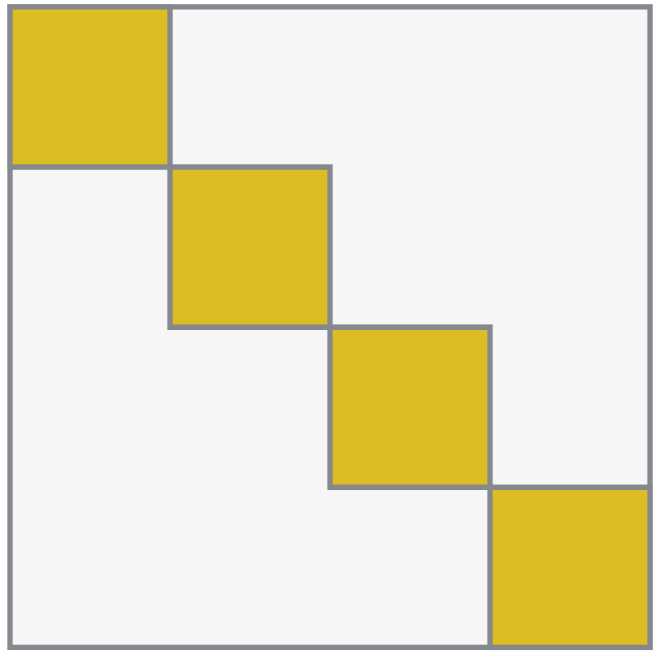
GM + parameters $\xrightarrow{\text{stochastic draw}}$ Data

Assign each node to one of *B* blocks. b_i

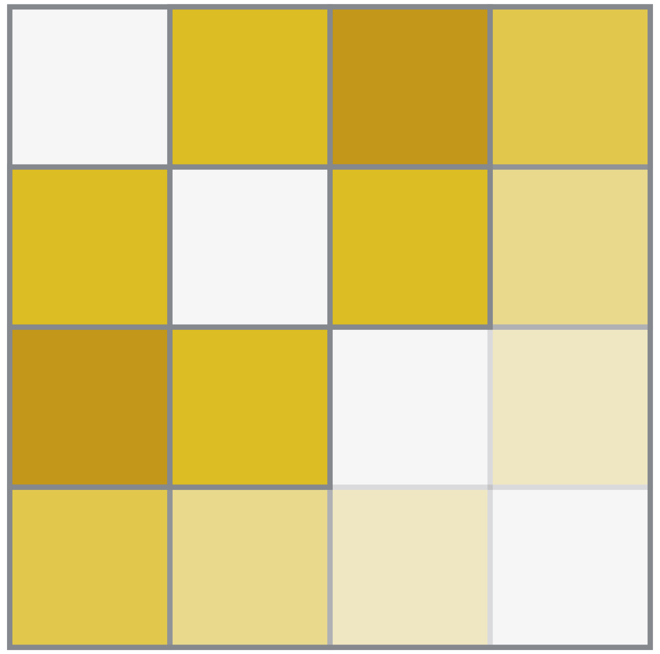
Let the probability that two nodes connect depend *only* on their blocks:

$$Pr(A_{ij}|b_i, b_j) = \omega_{b_i, b_j}$$

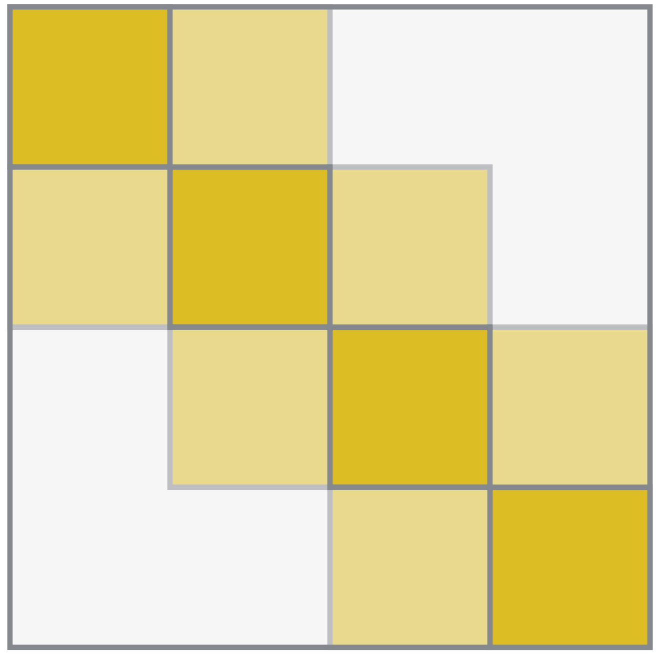
Then we can choose the matrix ω to have whatever structure we want!



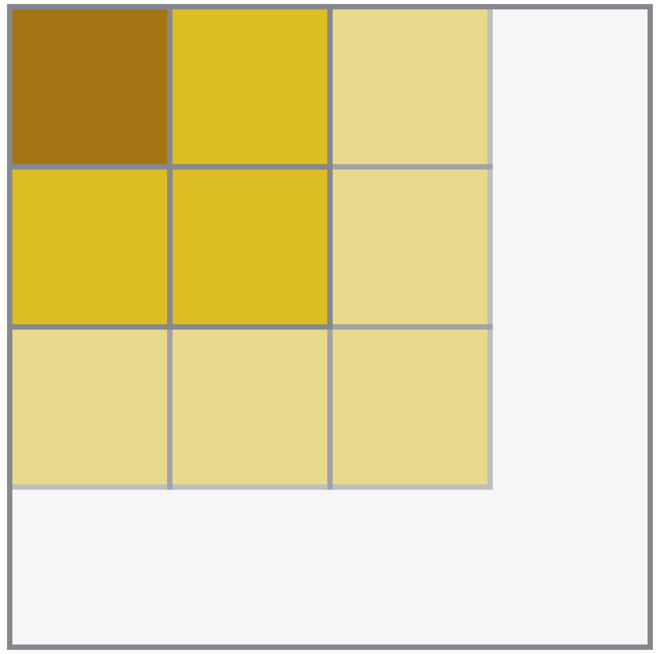
Assortative



Disassortative



Ordered



Core-periphery

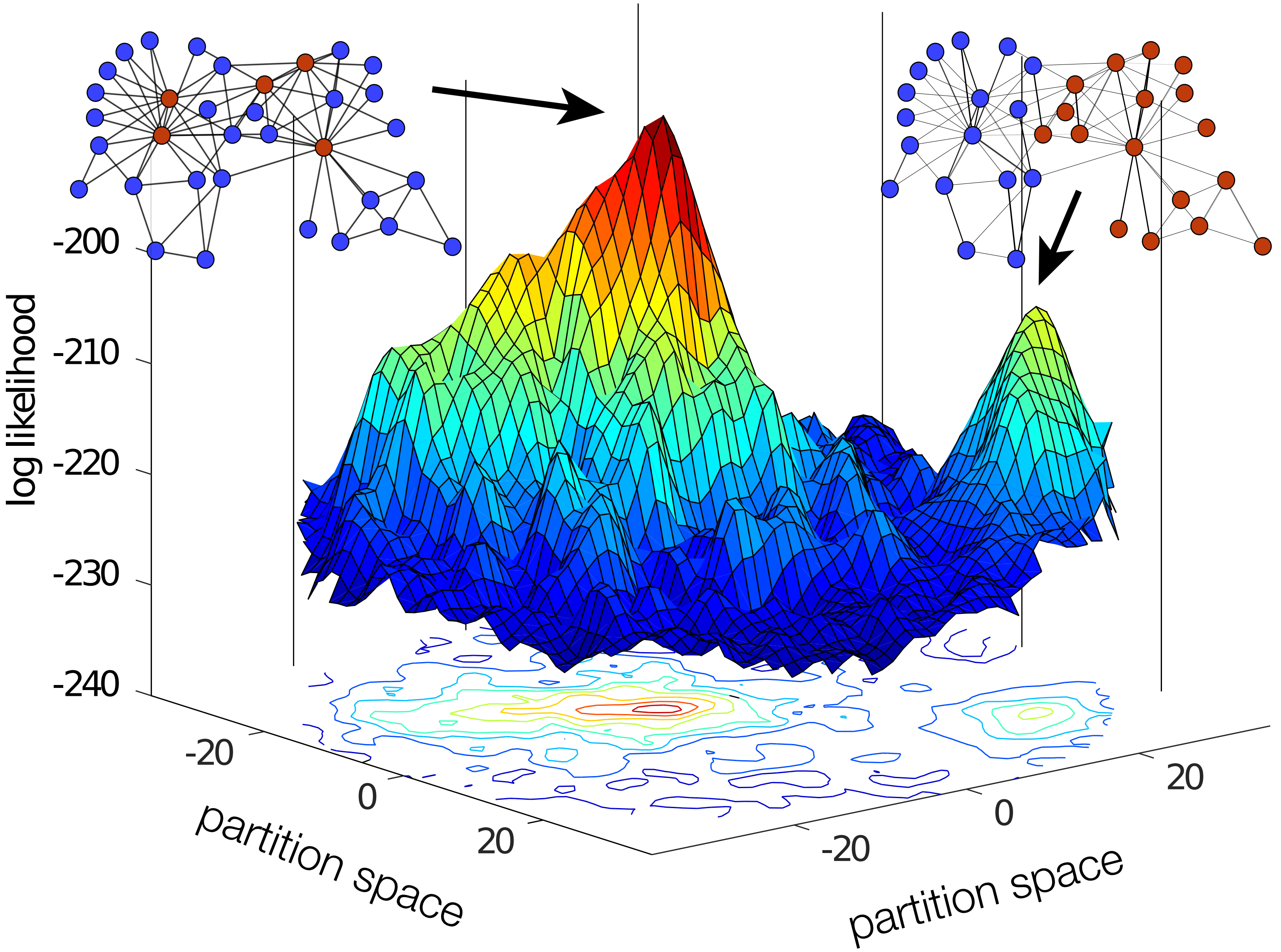
e.g. The stochastic block model

GM + parameters ← inference — Data

When we run the generative process in reverse (aka inference), we find community structure.

This is nothing more than a statistically principled approach to **fitting a model to data.**

But instead of fitting a line to a scatter of (x,y) data, we're fitting a model for networks with community structure to data.



Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:

$$P(i \leftrightarrow j) \;=\; \alpha(r_i - r_j)$$

$$P(i \rightarrow j) \;=\; \beta(r_i - r_j)$$

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:

$$P(i \leftrightarrow j) \;=\; \alpha(r_i - r_j)$$

A gaussian centered at 0

$$P(i \rightarrow j) \;=\; \beta(r_i - r_j)$$

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:

$$P(i \leftrightarrow j) \;=\; \alpha(r_i - r_j)$$

A gaussian centered at 0

$$P(i \rightarrow j) \;=\; \beta(r_i - r_j)$$

Fourier cosine series, keeping five terms & squaring to enforce nonnegativity, plus an additional Gaussian peak at the origin.

Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

Generative model:
Generate the patterns that you want to identify.

Create N nodes.
Assign each node an integer rank *r*, from 1 to N.

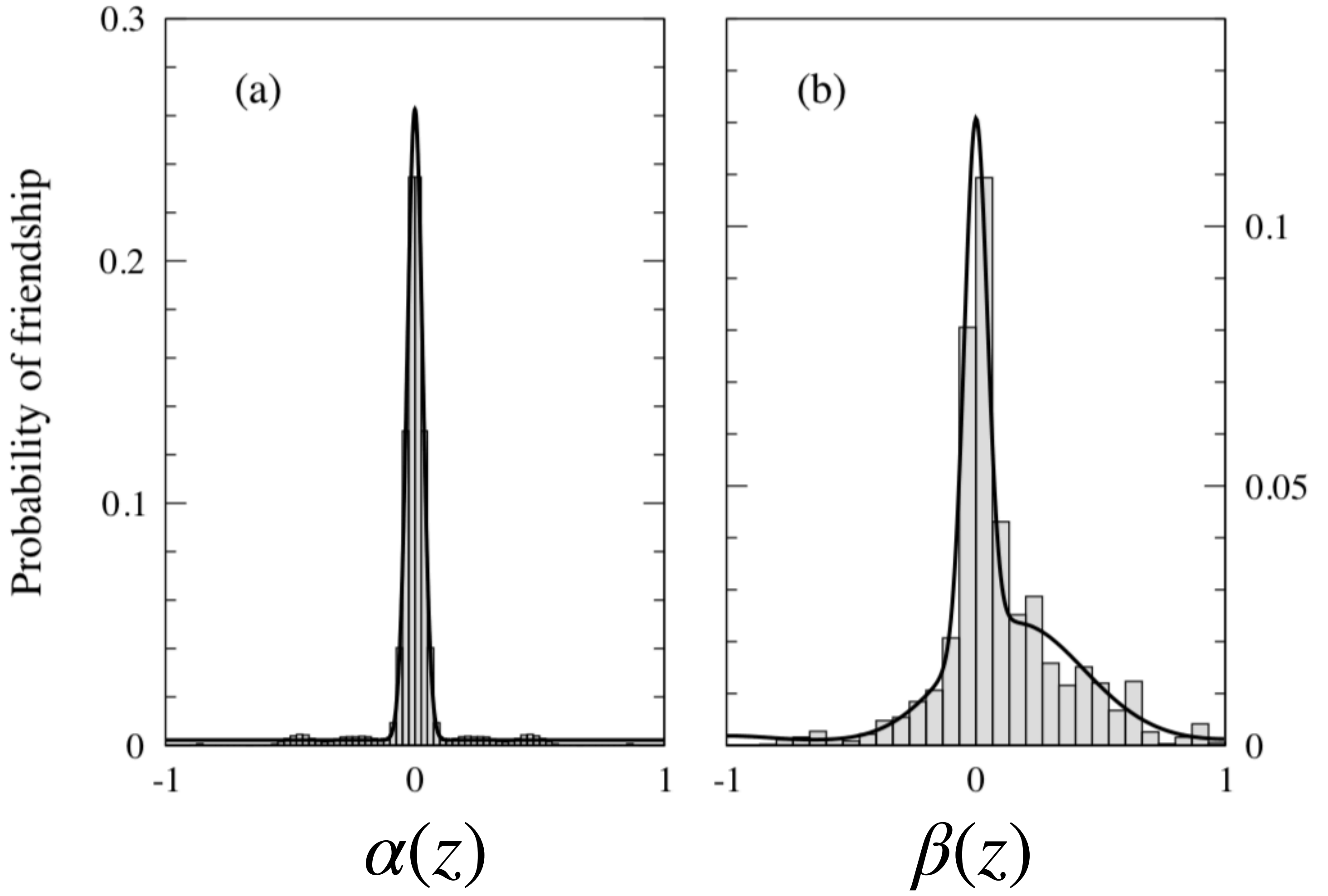
IRL, not all friendships are reciprocated 😭
So let's generate undirected AND directed edges:

$$P(i \leftrightarrow j) \;=\; \alpha(r_i - r_j)$$

A gaussian centered at 0

$$P(i \to j) \;=\; \beta(r_i - r_j)$$

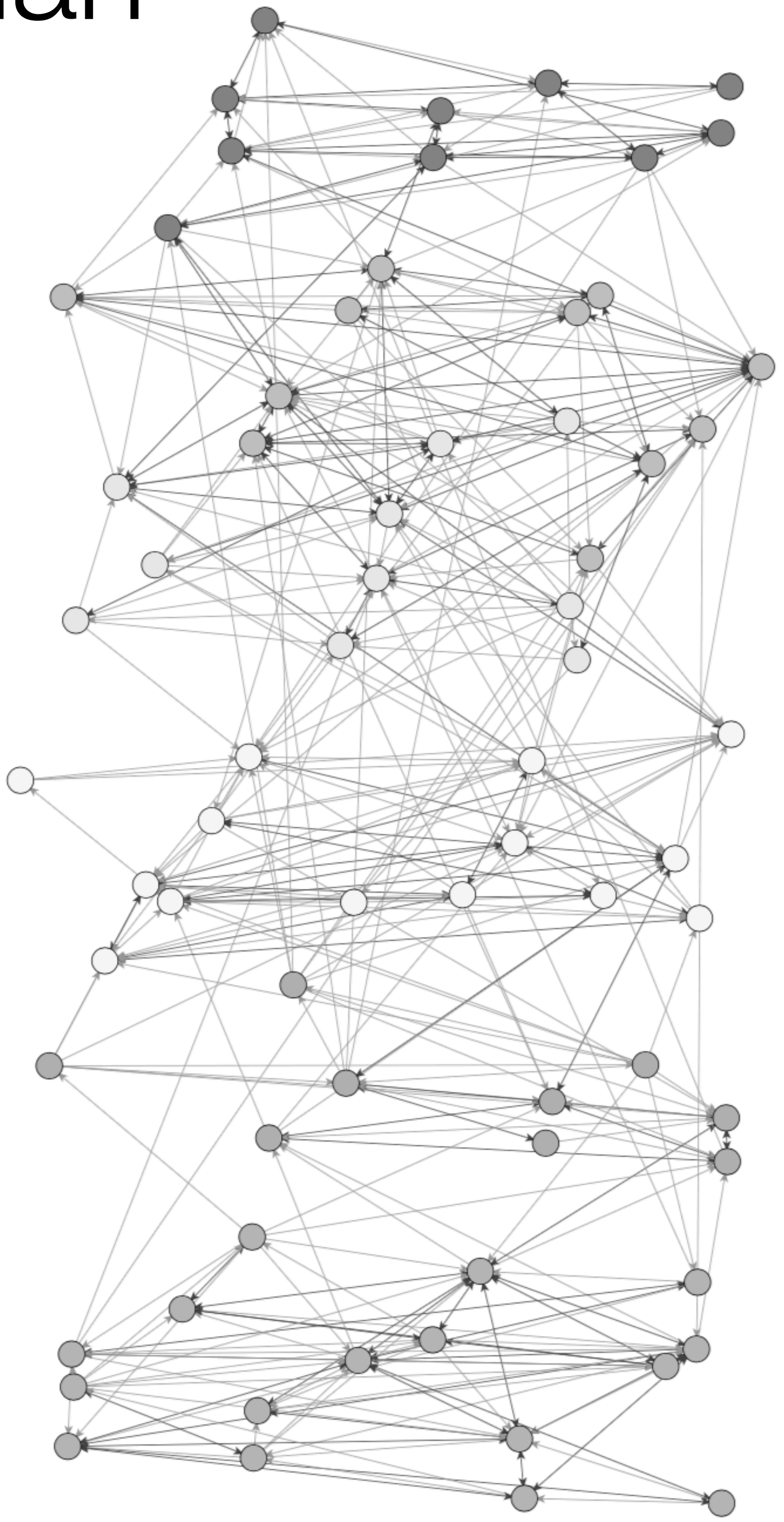
Fourier cosine series, keeping five terms & squaring to enforce nonnegativity, plus an additional Gaussian peak at the origin.



Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

GM + parameters ← inference — Data



Inferred parameters of people's attachment preferences & ranks.

- Identified the need to learn from reciprocated friendships.
- Found that in AddHealth data, teens link to others of *nearby* social status.

Ball, Newman. Network Science 1, 16-30 (2013)

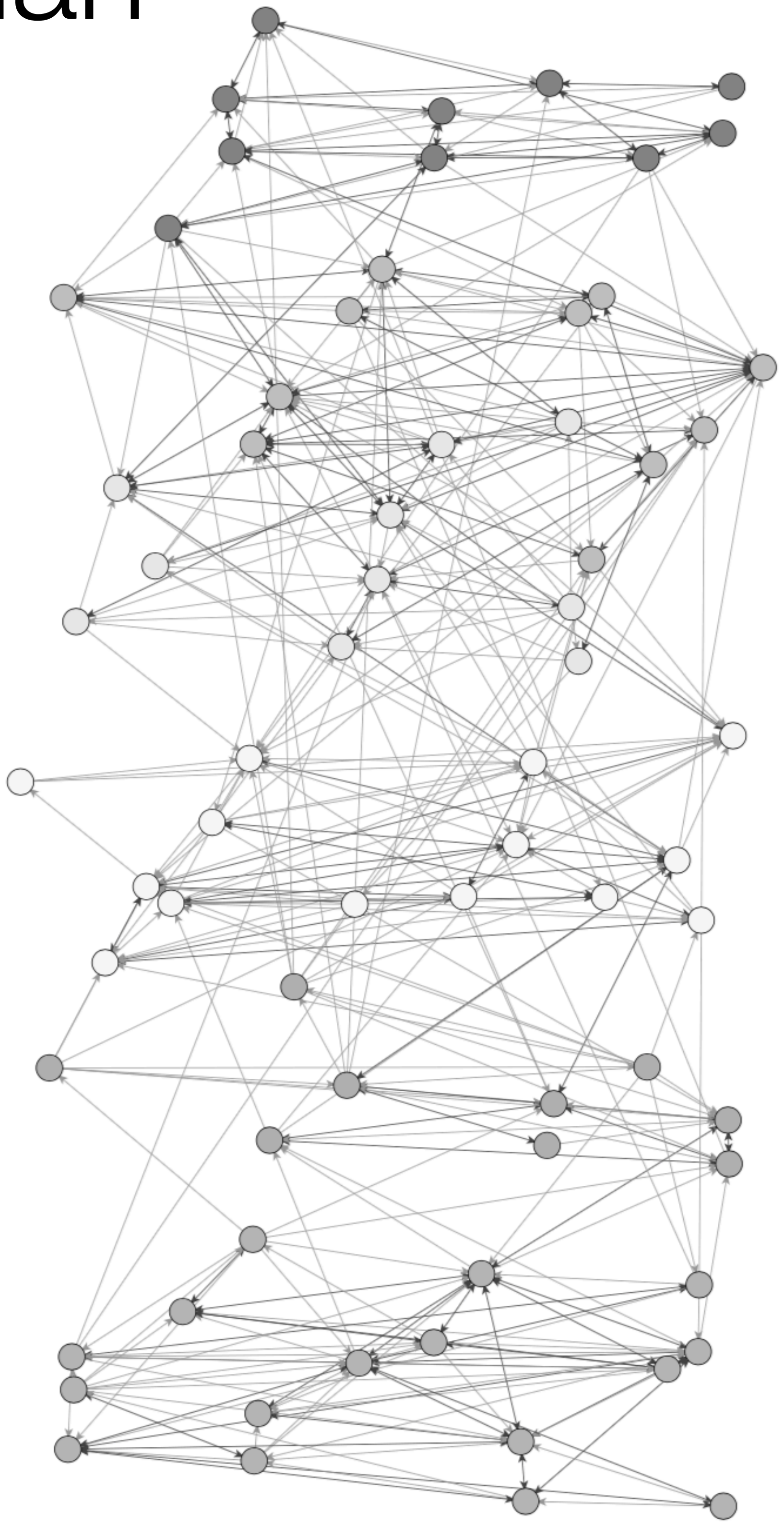
Embeddings and Orderings 4: Ball & Newman

GM + parameters $\xleftarrow{\text{inference}}$ Data

α

β

ranks



Inferred parameters of people's attachment preferences & ranks.

- Identified the need to learn from reciprocated friendships.
- Found that in AddHealth data, teens link to others of *nearby* social status.

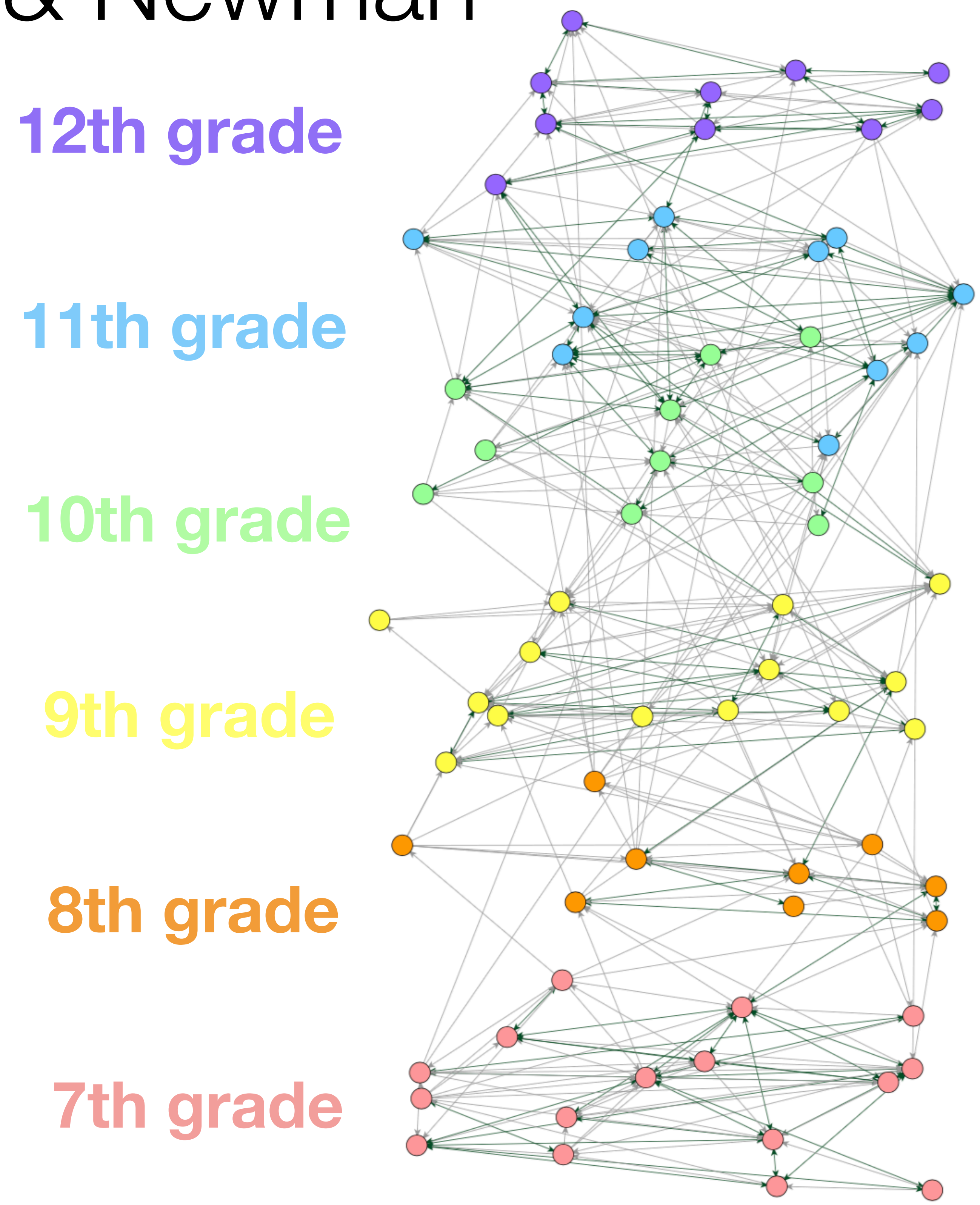
Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 4: Ball & Newman

GM + parameters α β ranks $\xleftarrow{\text{inference}}$ Data

Inferred parameters of people's attachment preferences & ranks.

- Identified the need to learn from reciprocated friendships.
- Found that in AddHealth data, teens link to others of *nearby* social status.



Ball, Newman. Network Science 1, 16-30 (2013)

Embeddings and Orderings 5: Niche Models

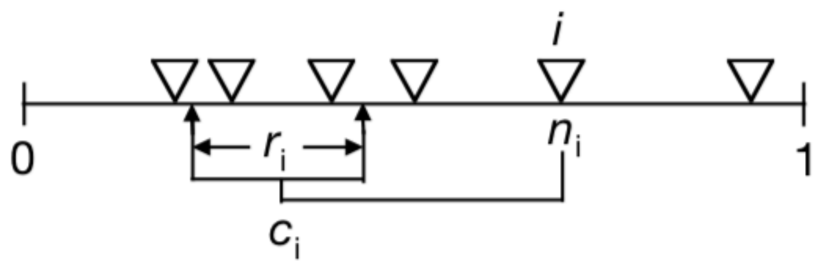


Figure 1 Diagram of the niche model. Each of ***S*** species (for example, ***S*** = 6, each shown as an inverted triangle) is assigned a 'niche value' parameter (n_i) drawn uniformly from the interval [0,1]. Species ***i*** consumes all species falling in a range (r_i) that is placed by uniformly drawing the centre of the range (c_i) from [$r_i/2$, n_i]. This permits looping and cannibalism by allowing up to half of r_i to include values $\geqslant n_i$. The size of r_i is assigned by using a beta function to randomly draw values from [0,1] whose expected value is 2***C*** and then multiplying that value by n_i [expected $E(n_i) = 0.5$] to obtain the desired ***C***. A beta distribution with $\alpha = 1$ has the form $f(x|1, \beta) = \beta(1-x)^{\beta-1}$, $0 < x < 1$, 0 otherwise, and $E(X) = 1/(1+\beta)$. In this case, $x = 1-(1-y)^{1/\beta}$ is a random variable from the beta distribution if y is a uniform random variable and β is chosen to obtain the desired expected value. We chose this form because of its simplicity and ease of calculation. The fundamental generality of species i is measured by r_i. The number of species falling within r_i measures realized generality. Occasionally, model-generated webs contain completely disconnected species or trophically identical species. Such species are eliminated and replaced until the web is free of such species. The species with the smallest n_i has $r_i = 0$ so that every web has at least one basal species.

Embeddings and Orderings 5: Niche Models

Niche Models embed species in a latent space based on feeding preferences:

most species feed from narrow range in a 1-dim. space (~body size).

- Great for food webs. Inference models v slow for all but small networks.

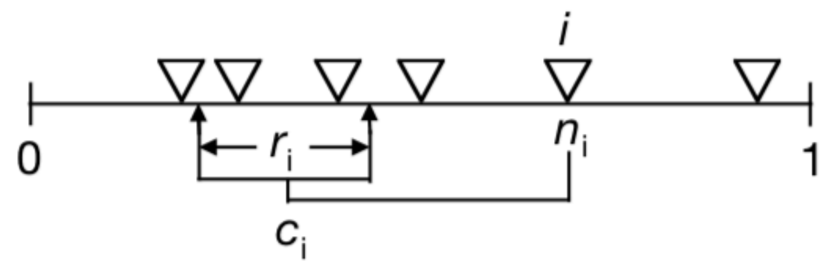


Figure 1 Diagram of the niche model. Each of ***S*** species (for example, ***S*** = 6, each shown as an inverted triangle) is assigned a 'niche value' parameter (n_i) drawn uniformly from the interval [0,1]. Species ***i*** consumes all species falling in a range (r_i) that is placed by uniformly drawing the centre of the range (c_i) from [r_i/2, n_i]. This permits looping and cannibalism by allowing up to half of r_i to include values $\geqslant n_i$. The size of r_i is assigned by using a beta function to randomly draw values from [0,1] whose expected value is 2***C*** and then multiplying that value by n_i [expected $E(n_i) = 0.5$] to obtain the desired ***C***. A beta distribution with $\alpha = 1$ has the form $f(x|1, \beta) = \beta(1-x)^{\beta-1}$, $0 < x < 1$, 0 otherwise, and $E(X) = 1/(1+\beta)$. In this case, $x = 1-(1-y)^{1/\beta}$ is a random variable from the beta distribution if y is a uniform random variable and β is chosen to obtain the desired expected value. We chose this form because of its simplicity and ease of calculation. The fundamental generality of species i is measured by r_i. The number of species falling within r_i measures realized generality. Occasionally, model-generated webs contain completely disconnected species or trophically identical species. Such species are eliminated and replaced until the web is free of such species. The species with the smallest n_i has $r_i = 0$ so that every web has at least one basal species.

Williams & Martinez. Nature 404.6774 (2000).

Embeddings and Orderings 5: Niche Models

Niche Models embed species in a latent space based on feeding preferences:

most species feed from narrow range in a 1-dim. space (~body size).

- Great for food webs. Inference models v slow for all but small networks.

Want more? Jen Dunne, Cris Moore

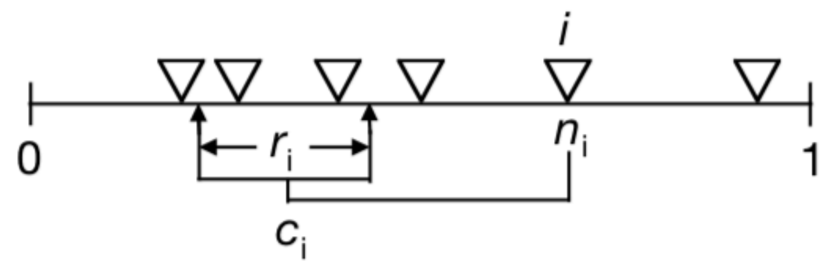


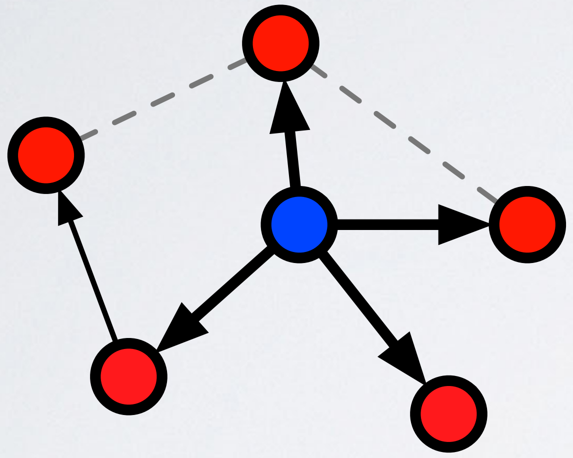
Figure 1 Diagram of the niche model. Each of ***S*** species (for example, ***S*** = 6, each shown as an inverted triangle) is assigned a 'niche value' parameter (n_i) drawn uniformly from the interval [0,1]. Species ***i*** consumes all species falling in a range (r_i) that is placed by uniformly drawing the centre of the range (c_i) from [r_i/2, n_i]. This permits looping and cannibalism by allowing up to half of r_i to include values $\geqslant n_i$. The size of r_i is assigned by using a beta function to randomly draw values from [0,1] whose expected value is 2***C*** and then multiplying that value by n_i [expected $E(n_i) = 0.5$] to obtain the desired ***C***. A beta distribution with $\alpha = 1$ has the form $f(x|1, \beta) = \beta(1-x)^{\beta-1}$, $0 < x < 1$, 0 otherwise, and $E(X) = 1/(1+\beta)$. In this case, $x = 1-(1-y)^{1/\beta}$ is a random variable from the beta distribution if y is a uniform random variable and β is chosen to obtain the desired expected value. We chose this form because of its simplicity and ease of calculation. The fundamental generality of species i is measured by r_i. The number of species falling within r_i measures realized generality. Occasionally, model-generated webs contain completely disconnected species or trophically identical species. Such species are eliminated and replaced until the web is free of such species. The species with the smallest n_i has $r_i = 0$ so that every web has at least one basal species.

Williams & Martinez. Nature 404.6774 (2000).

Embeddings and Orderings 6: Centrality Redux!

[Centrality Week]

describing networks



position = centrality:
structural vs. dynamical importance

geometric	harmonic centrality
	closeness centrality
	betweenness centrality
connectivity	degree centrality
	eigenvector centrality
	PageRank
	Katz centrality
	many many more…

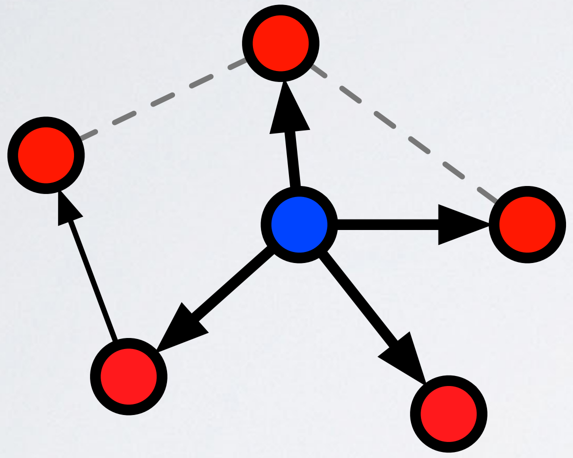
structural importance = cheap estimate of dynamical importance (aka "influence")

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

Which nodes are important… we've heard this before!

[Centrality Week]

describing networks



position = centrality:
harmonic, closeness centrality

importance = being in "center" of the network

harmonic $$c_i = \frac{1}{n-1} \sum_{j \neq i} \frac{1}{d_{ij}}$$

length of shortest path

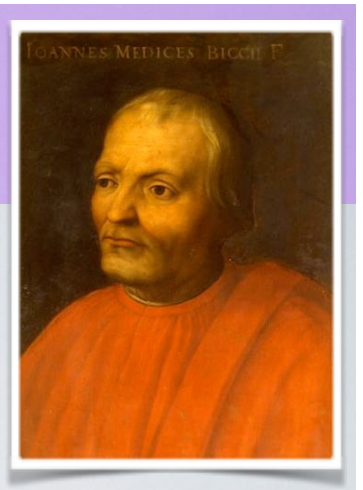
distance: $$d_{ij} = \begin{cases} \boxed{\ell_{ij}} & \text{if } j \text{ reachable from } i \\ \infty & \text{otherwise} \end{cases}$$

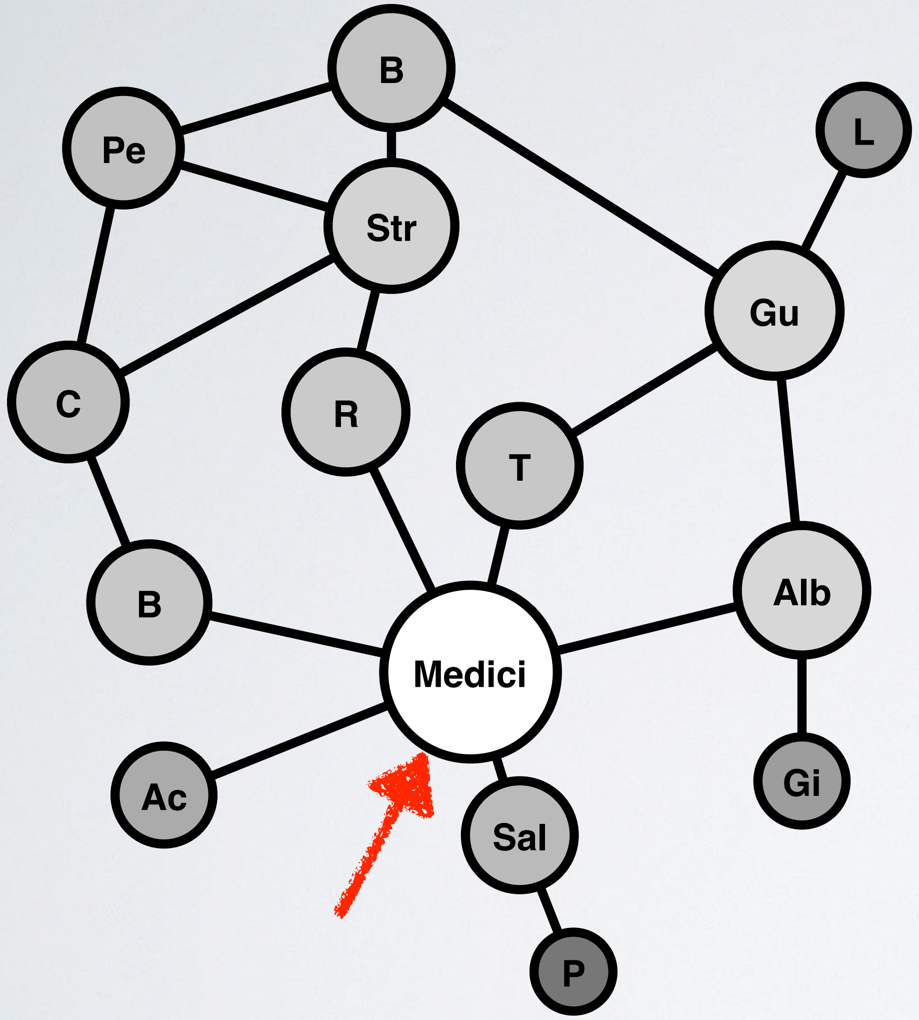
Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

Which nodes are important… we've heard this before!

[Centrality Week]

network position: harmonic



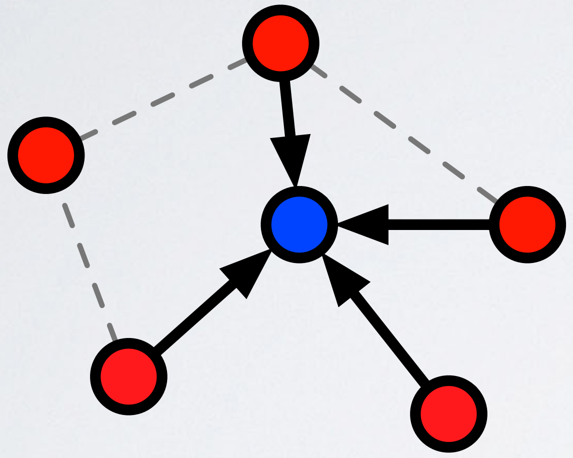


Medici	9.5
Guadagni	7.92
Albizzi	7.83
Strozzi	7.67
Ridolfi	7.25
Bischeri	7.2
Tornabuoni	7.17
Barbadori	7.08
Peruzzi	6.87
Castellani	6.87
Salviati	6.58
Acciaiuoli	5.92
Ginori	5.33
Lamberteschi	5.28
Pazzi	4.77

Which nodes are important… we've heard this before!

[Centrality Week]

describing networks



position = centrality:
PageRank, Katz, eigenvector centrality

importance = sum of importances* of nodes that point at you

$$I_i = \sum_{j \to i} \frac{I_j}{k_j}$$

or, the left eigenvector of

$$\mathbf{A}\mathbf{x} = \lambda\,\mathbf{x}$$

Boldi & Vigna, arxiv:1308.2140 (2013)
Borgatti, *Social Networks* **27**, 55–71 (2005)

*modulo several technical details

Embeddings and Orderings 3: PageRank

PageRank defines scalar rank recursively:

important pages are those that are linked to by important pages.

- Great at finding the top 3 but limited predictions available using the PageRank scores.

The PageRank Citation Ranking: Bringing Order to the Web

January 29, 1998

Abstract

The importance of a Web page is an inherently subjective matter, which depends on the readers interests, knowledge and attitudes. But there is still much that can be said objectively about the relative importance of Web pages. This paper describes PageRank, a method for rating Web pages objectively and mechanically, effectively measuring the human interest and attention devoted to them.

We compare PageRank to an idealized random Web surfer. We show how to efficiently compute PageRank for large numbers of pages. And, we show how to apply PageRank to search and to user navigation.

The Anatomy of a Large-Scale Hypertextual Web Search Engine

Sergey Brin and Lawrence Page

Computer Science Department,
Stanford University, Stanford, CA 94305, USA
sergey@cs.stanford.edu and page@cs.stanford.edu

Abstract

In this paper, we present Google, a prototype of a large-scale search engine which makes heavy use of the structure present in hypertext. Google is designed to crawl and index the Web efficiently and produce much more satisfying search results than existing systems. The prototype with a full

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d \sum_j p_j \pi_{ji}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + \boxed{d \sum_j p_j \pi_{ji}}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \boxed{\frac{1-d}{N}} + \boxed{d \sum_j p_j \pi_{ji}}$$

write the equation

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d \sum_j p_j \pi_{ji}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N}\right) \mathbf{1} + d\pi^T \mathbf{p}$$

matrix-vector form

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d\sum_j p_j \pi_{ji}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N}\right)\mathbf{1} + d\pi^T\mathbf{p}$$

matrix-vector form

Alternative: stationary distribution of random walk on the network + weak all-to-all links

Embeddings and Orderings 3: PageRank

We imagine a web surfer who choose a starting webpage at random.

From that webpage, she looks at the links on the page, and either
(a) clicks on a random link or
(b) stops surfing; when she returns, she starts at a new random page.

What's the probability that she's at a particular page? *That's PageRank*.

$$\pi_{ji} = \frac{A_{ji}}{k_j}$$

define a transition matrix

$$p_i = \frac{1-d}{N} + d \sum_j p_j \pi_{ji}$$

write the equation

$$\mathbf{p} = \left(\frac{1-d}{N} \right) \mathbf{1} + d\pi^T \mathbf{p}$$

matrix-vector form

Alternative: stationary distribution of random walk on the network + weak all-to-all links

Jeremy Kun: http://www.infinitelooper.com/?v=K3pT0gTaDec&p=n

</methods>

<applications>

Many uses for the same techniques. cf regression

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in "space" or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

Structure and inequality in academic hiring

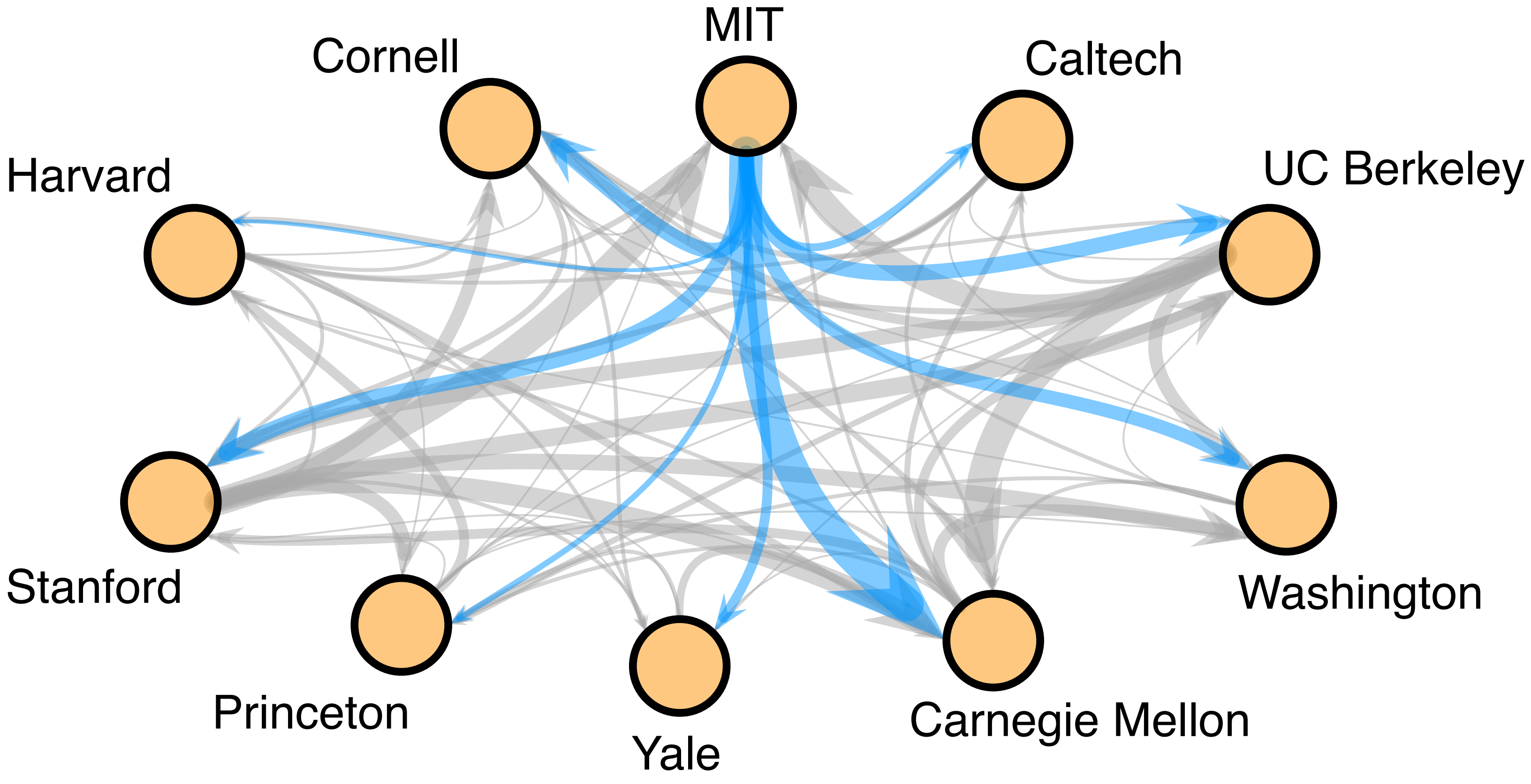
Collect the data (by hand 😭)

CVs of all US & Canadian tenure-track faculty in CS, Business, History: 2011-2013.

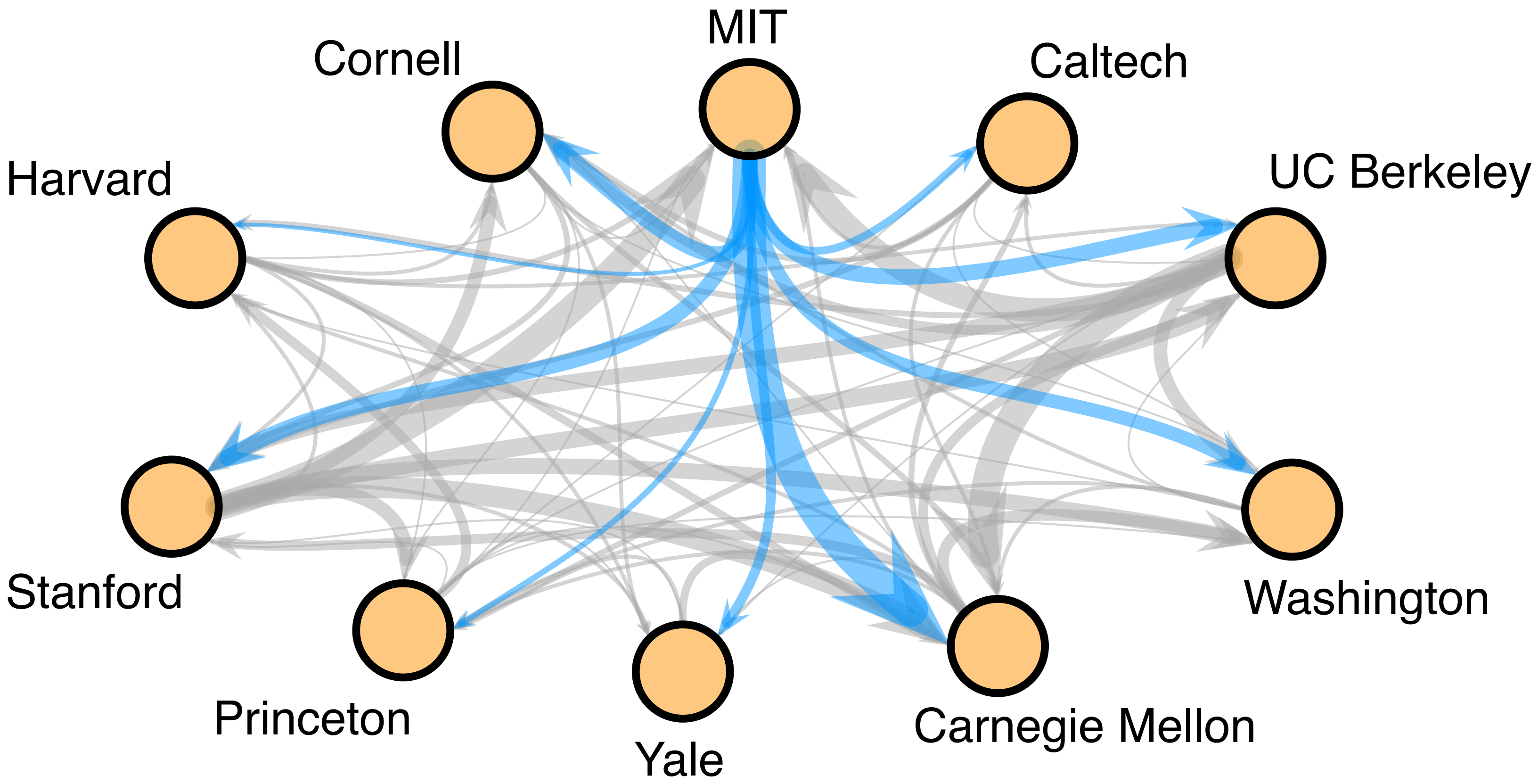
	Computer Science	Business	History
institutions	205	112	144
tenure-track faculty	5032	9336	4556
mean size	25	83	32
female	15%	22%	36%

total: **18,924** CVs

Faculty hiring networks



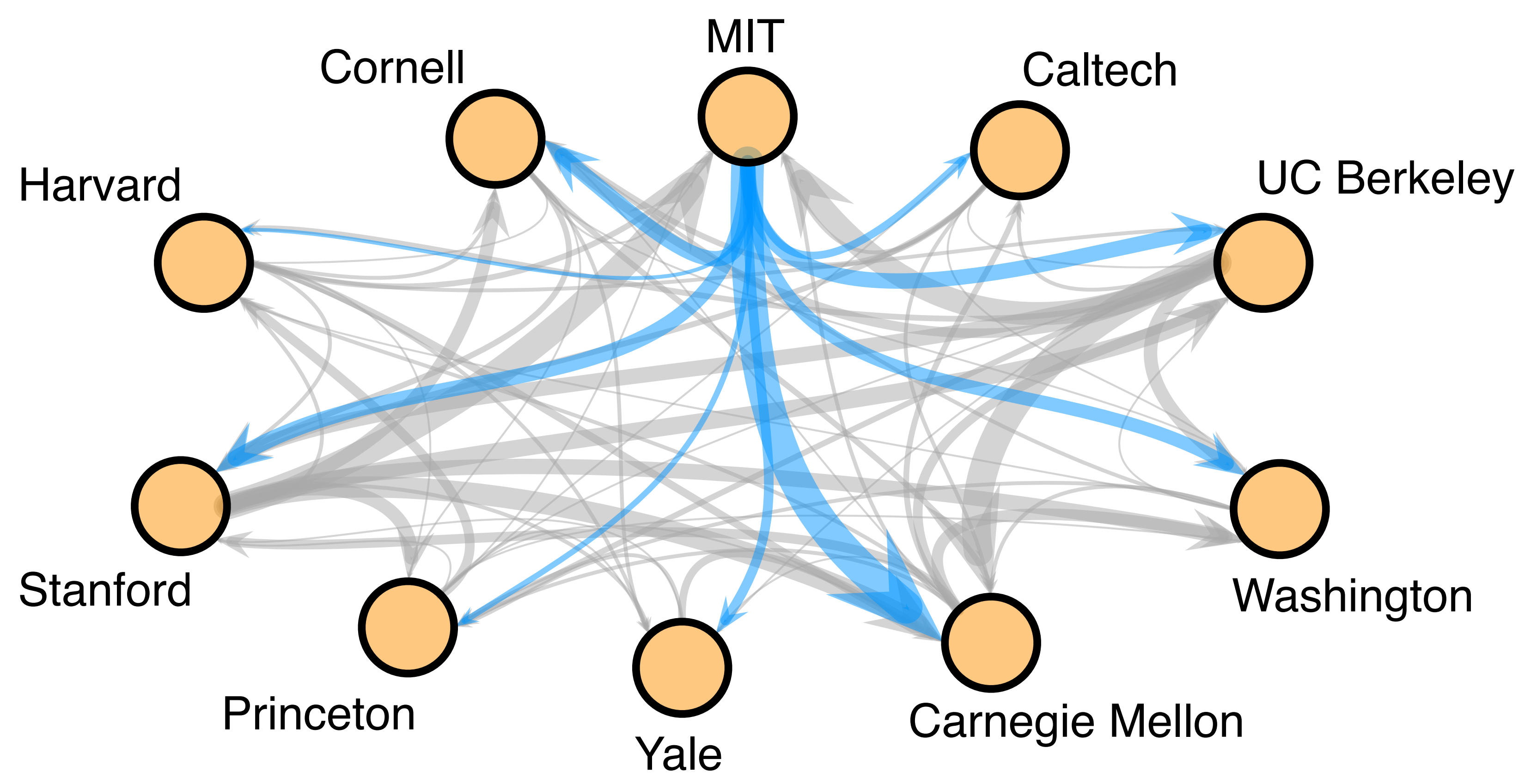
Faculty hiring networks

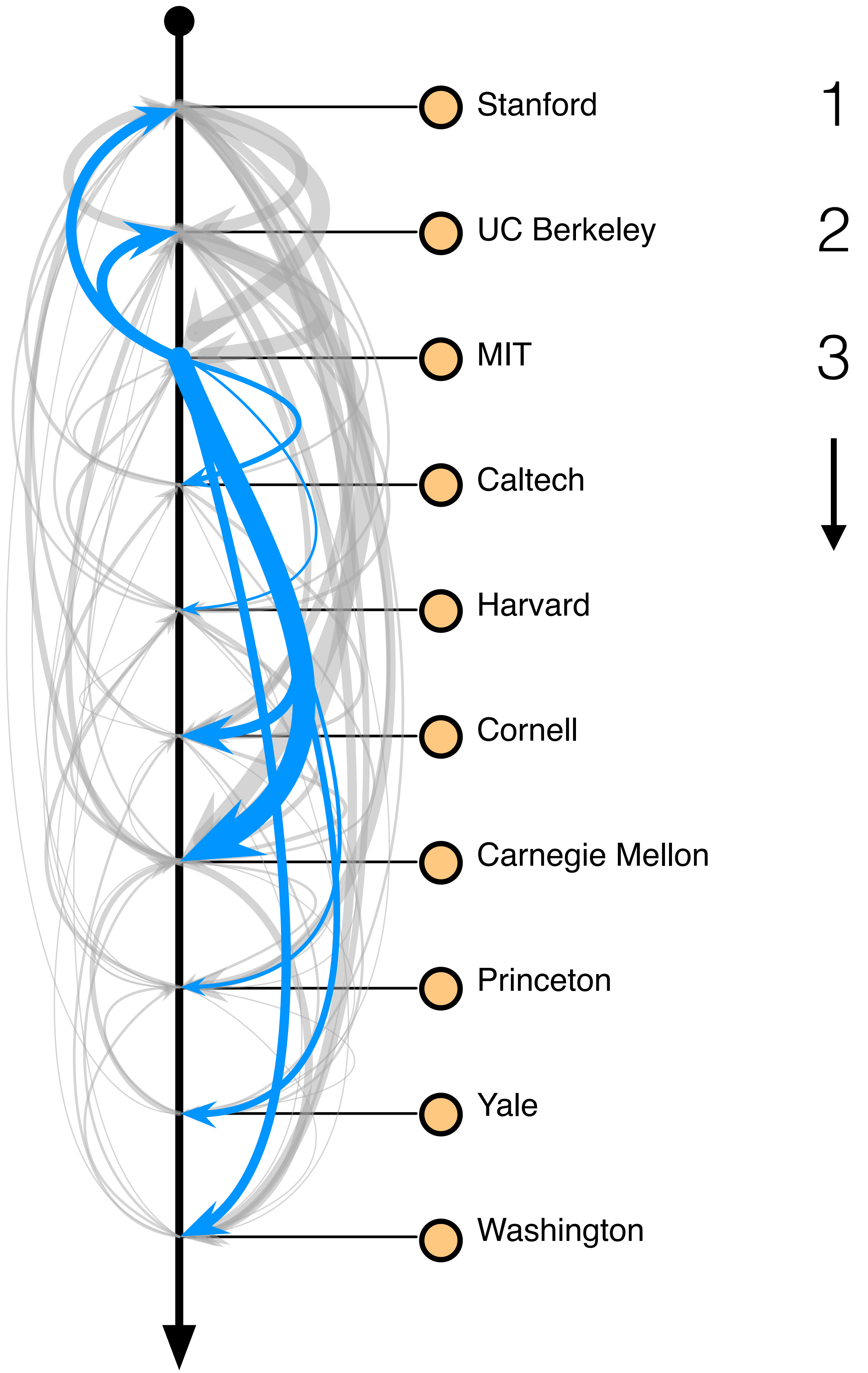


Premises:

1. Each hiring committee wants to hire **the best**.
2. Entire network reveals **collective preferences**.

Faculty hiring networks





Premises:

1. Each hiring committee wants to hire **the best**.
2. Entire network reveals **collective preferences**.

Faculty hiring networks

systematic

90% of hiring movement
is “down” the hierarchy

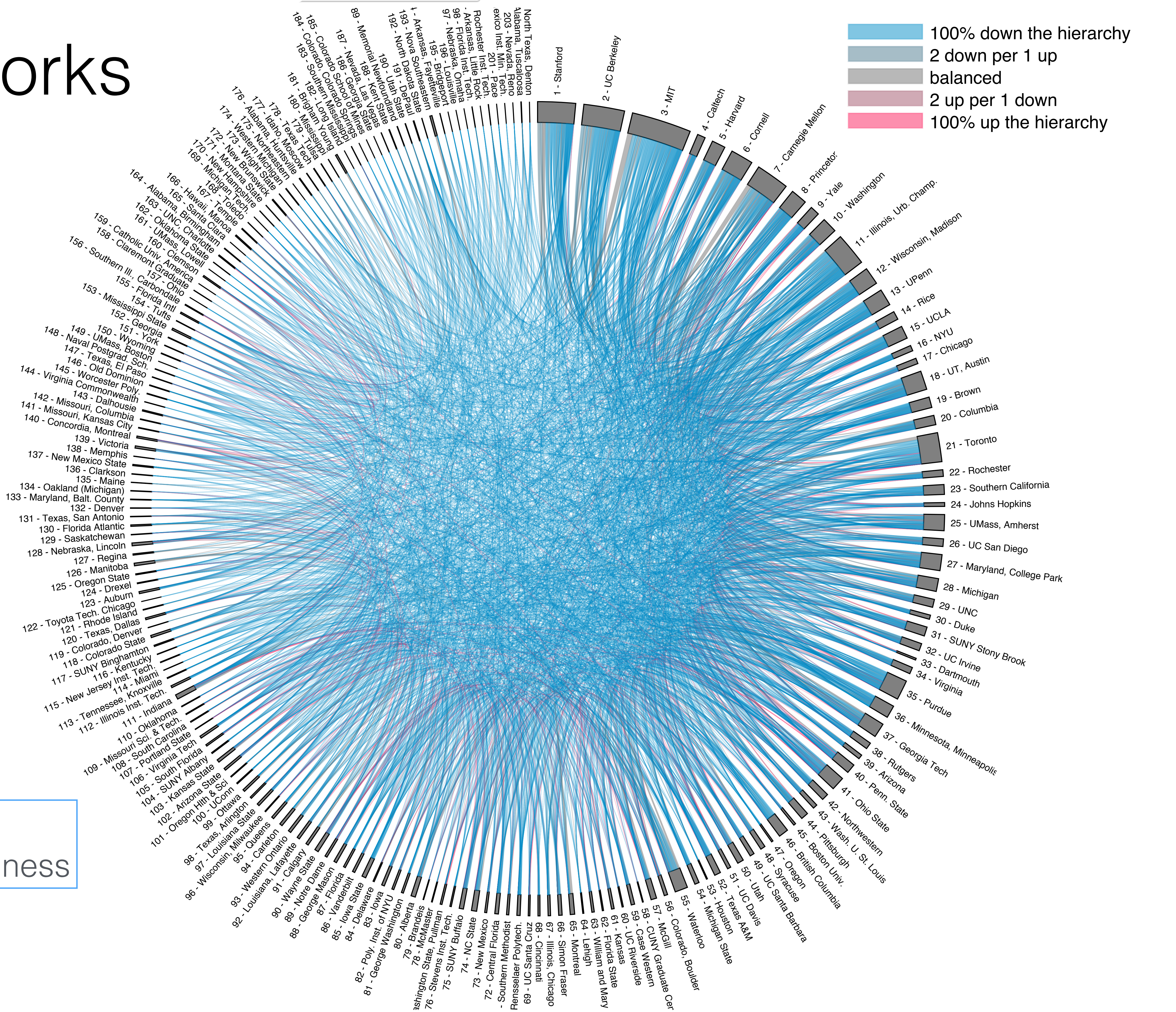
steep

< 7% of faculty have PhD
from lower 75% of universities

biased

median change for women
~3 ranks worse than men

danlarremore.com/**faculty/**
explore 19,000 hires for History, CS, Business

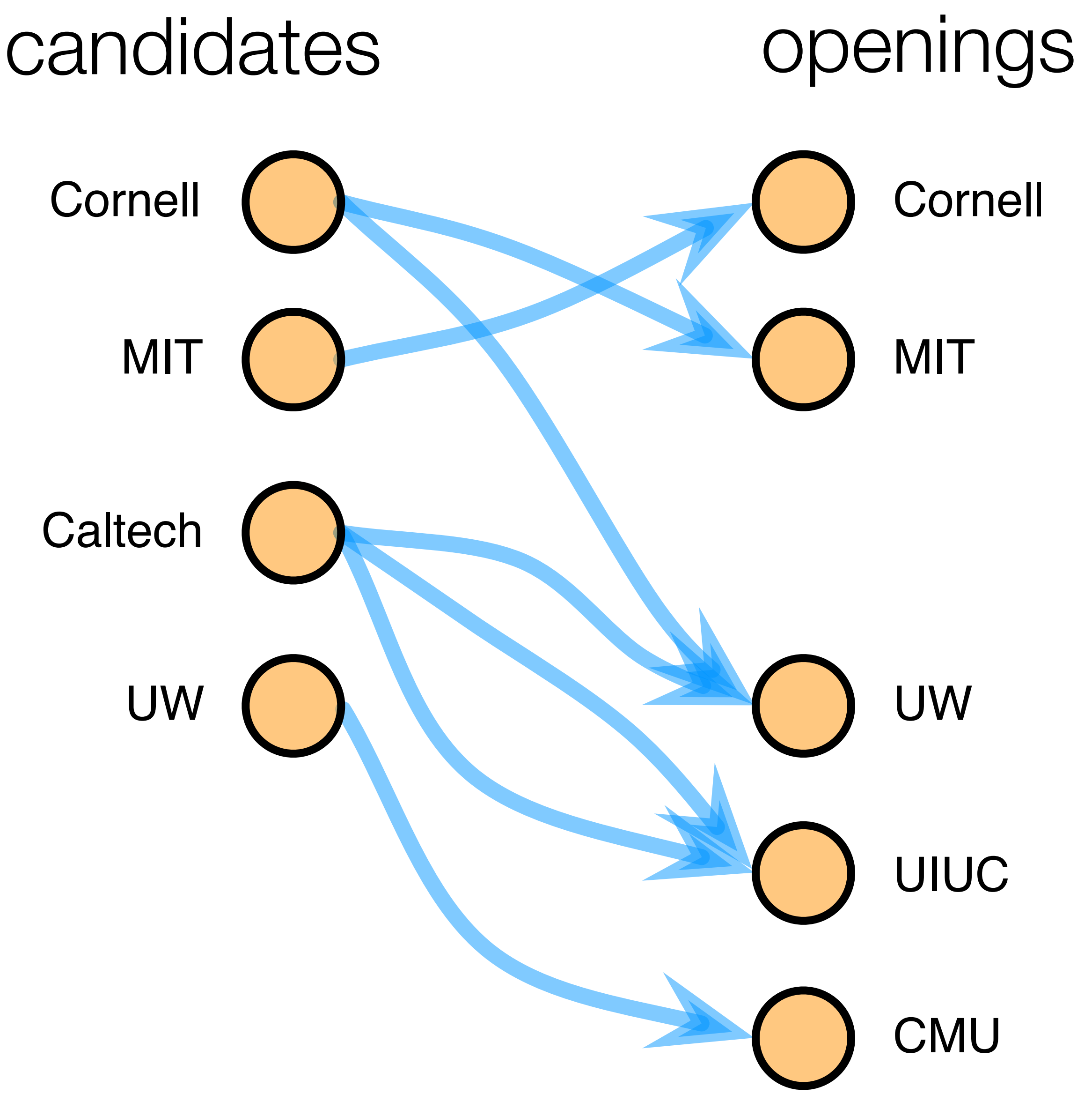


What else explains movement in this labor market?

Generative model:

prestige
productivity
postdoc experience

gender
geography

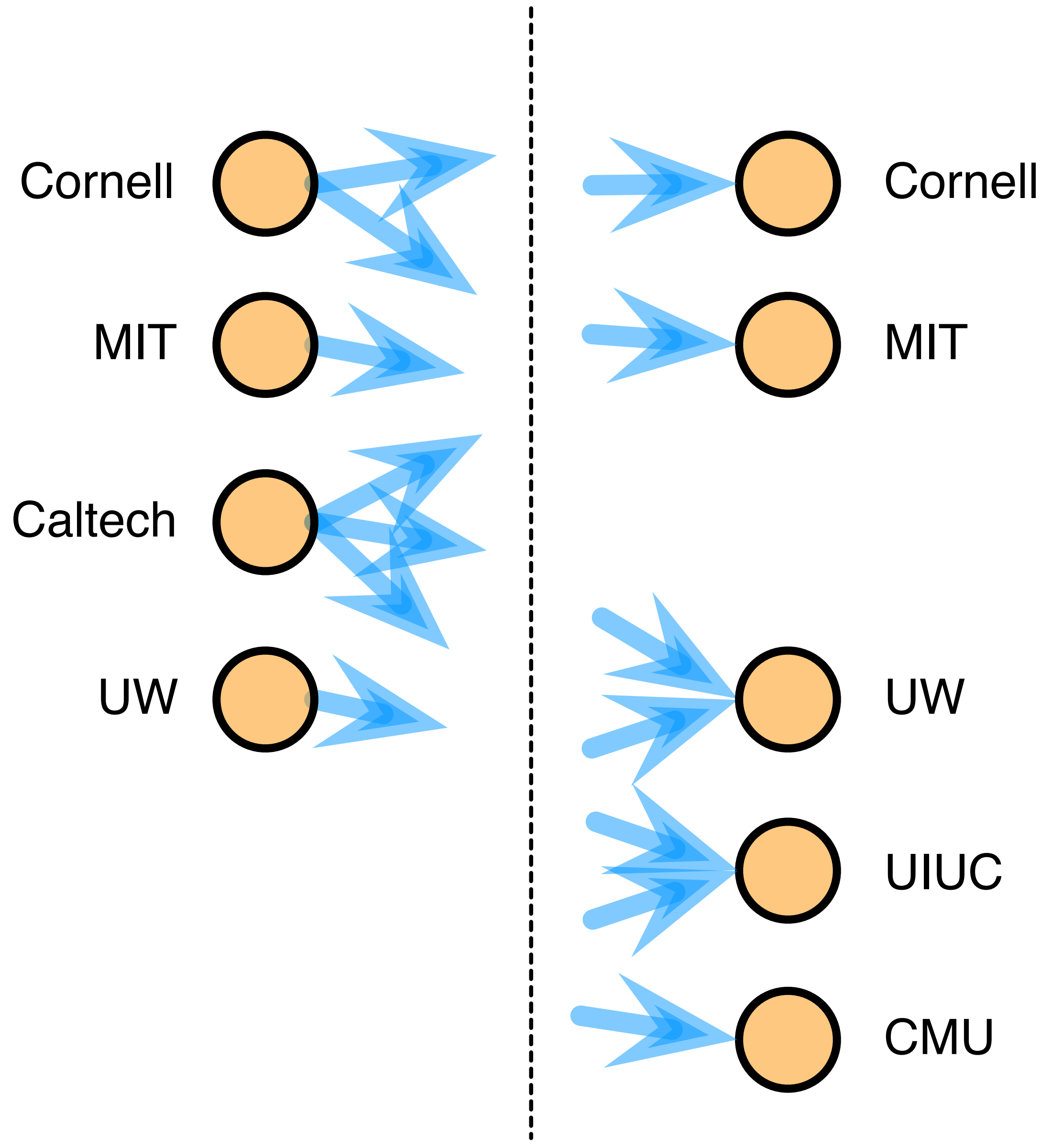


What else explains movement in this labor market?

Generative model:

prestige
productivity
postdoc experience

gender
geography



accurately generate the links!

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD

2. **Productivity**

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**
4. **Postdoc experience** + **geography** (together)

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**
4. **Postdoc experience** + **geography** (together)
5. **Gender**.

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**
4. **Postdoc experience** + **geography** (together)
5. **Gender**.

Gender bias is not **uniformly**, systematically affecting all hires. But…

What else explains movement in this labor market?

1. **Prestige difference**: Faculty Job vs PhD
2. **Productivity**
3. **Prestige of Faculty Job**
4. **Postdoc experience** + **geography** (together)
5. **Gender**.

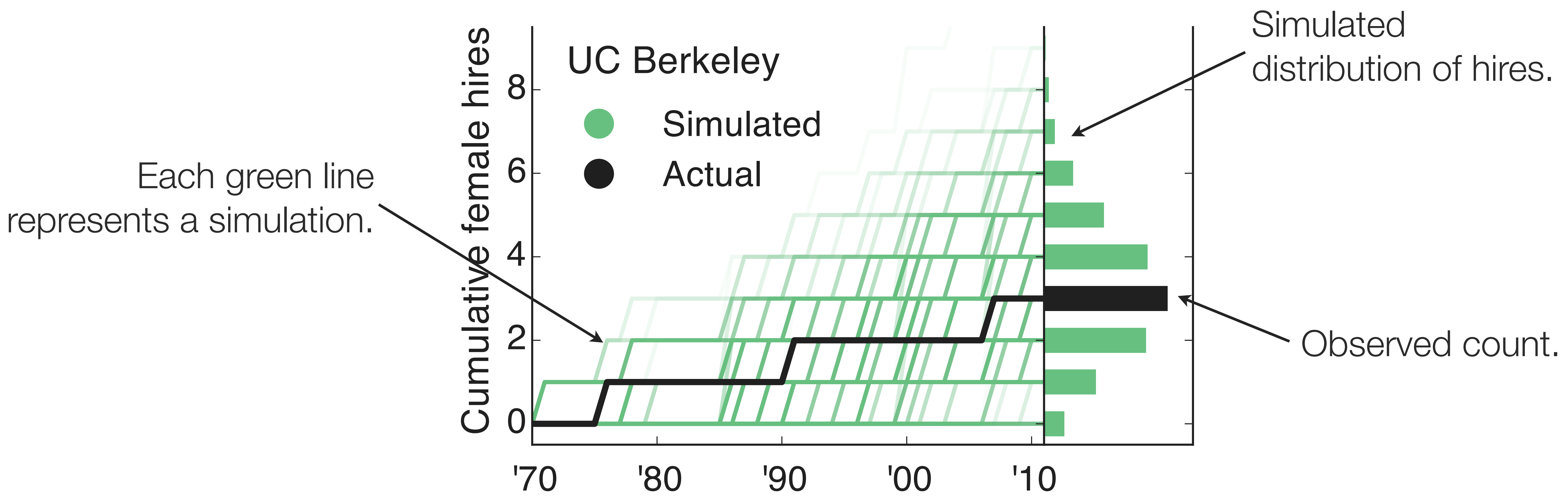
Gender bias is not **uniformly**, systematically affecting all hires. But…

a woman on the job market must have published ~1 additional paper to be placed the same as an equally qualified man.

Institution-level results

Using 40 years of actual hiring data, **simulate** hiring patterns for **each institution**.

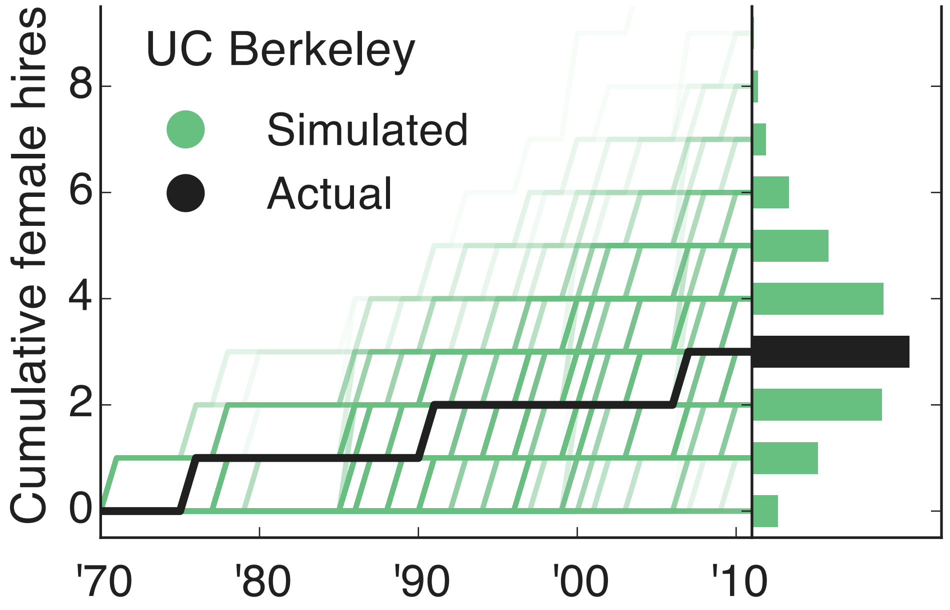
Compare actual vs. expected number of female hires.



Institution-level results

Using 40 years of actual hiring data, **simulate** hiring patterns for **each institution.**

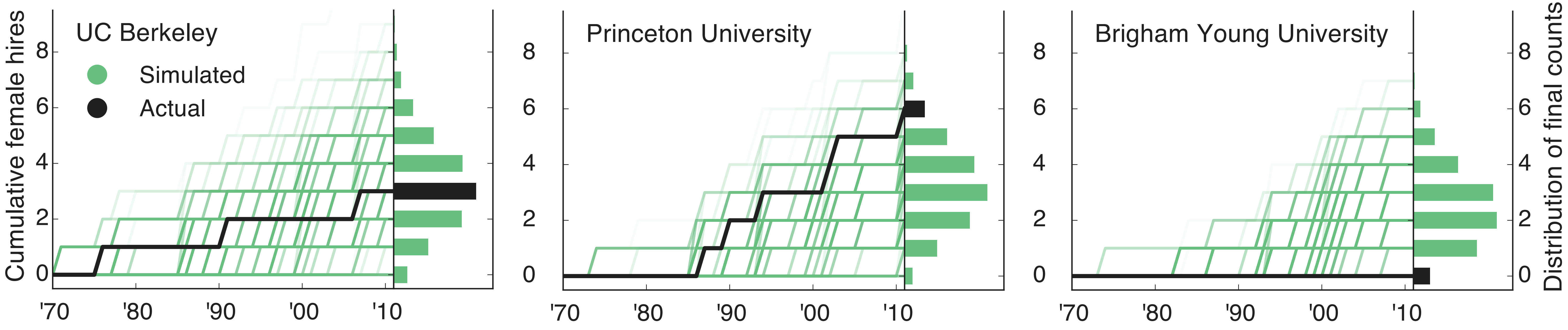
Compare actual vs. expected number of female hires.



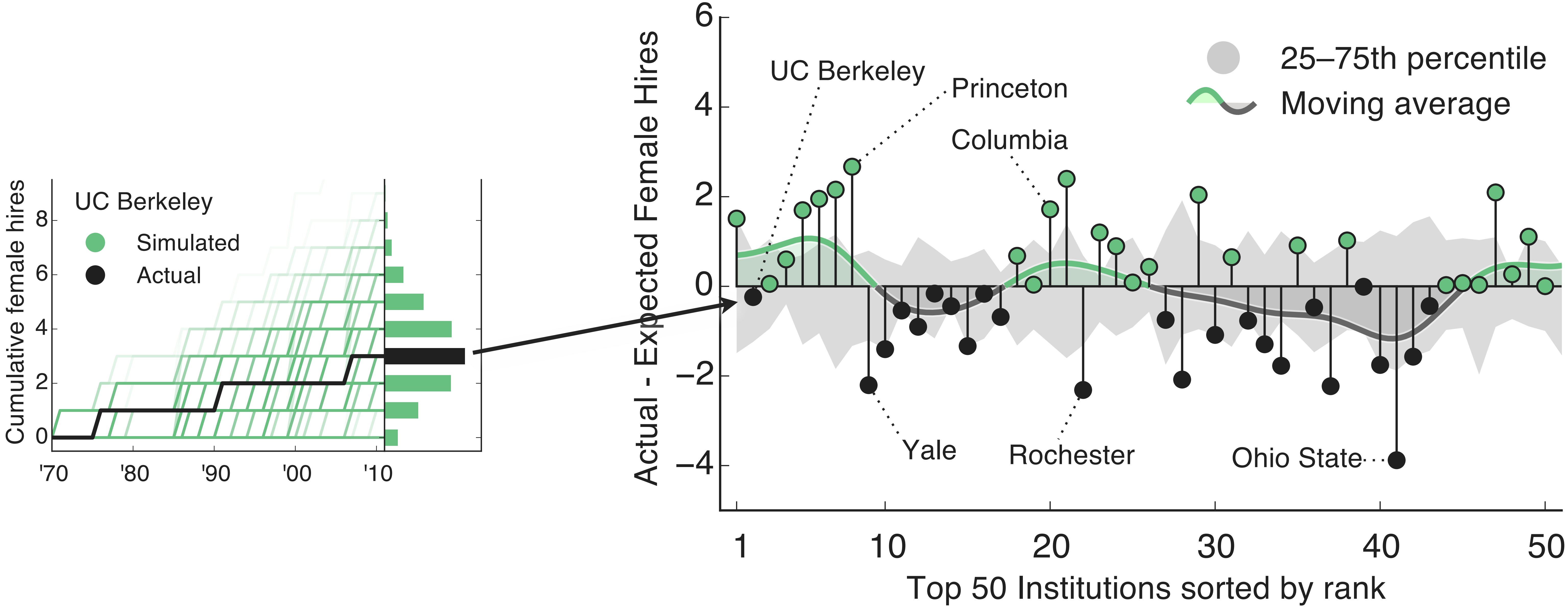
Institution-level results

Using 40 years of actual hiring data, **simulate** hiring patterns for **each institution**.

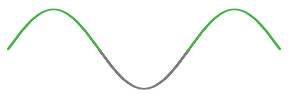
Compare actual vs. expected number of female hires.



Institution-level results



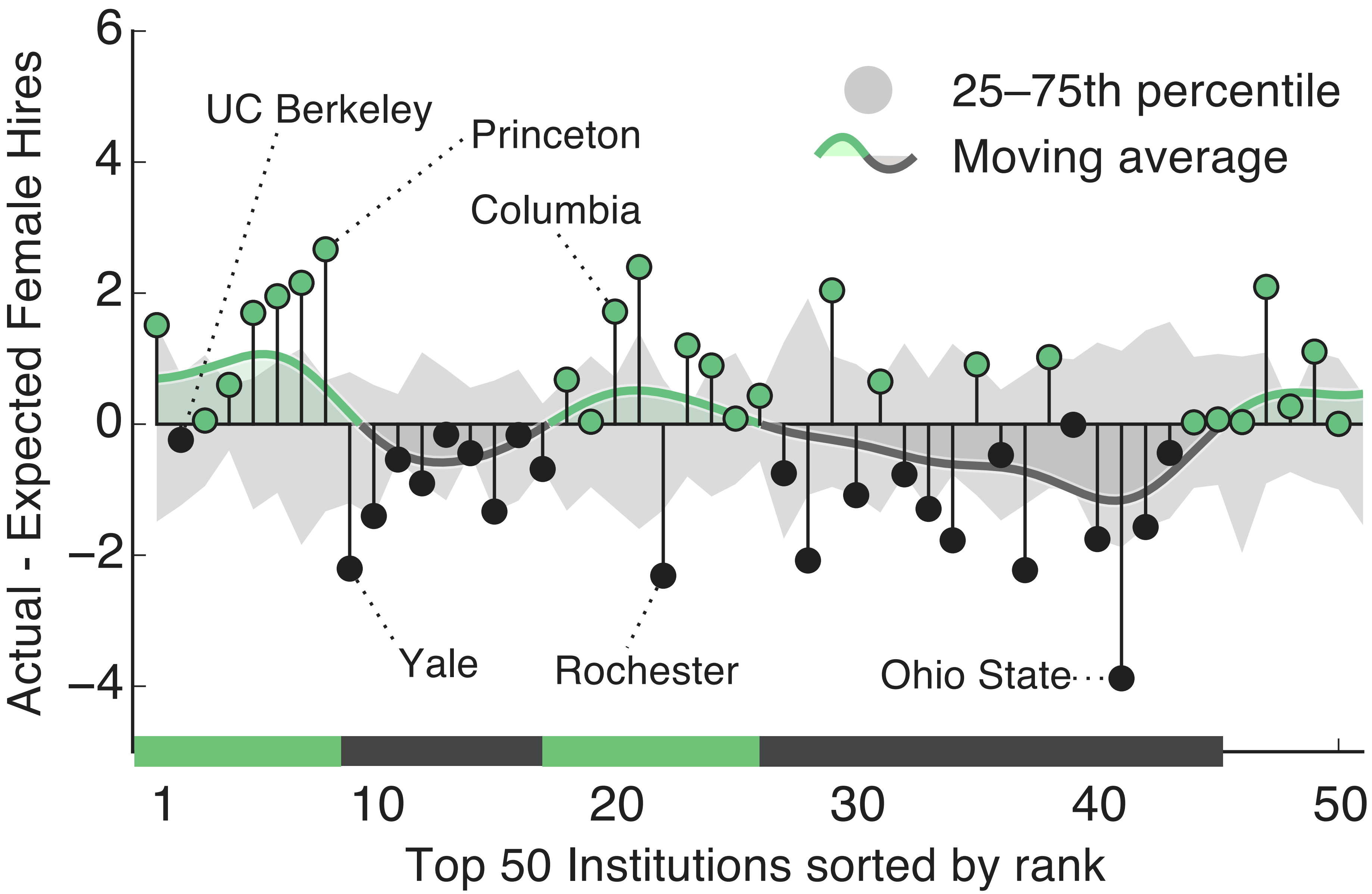
Institution-level results

For the top 50 institutions,
we see an **oscillation.** 

Why?

An interference effect?
Two distinct candidate pools?

Is it real?

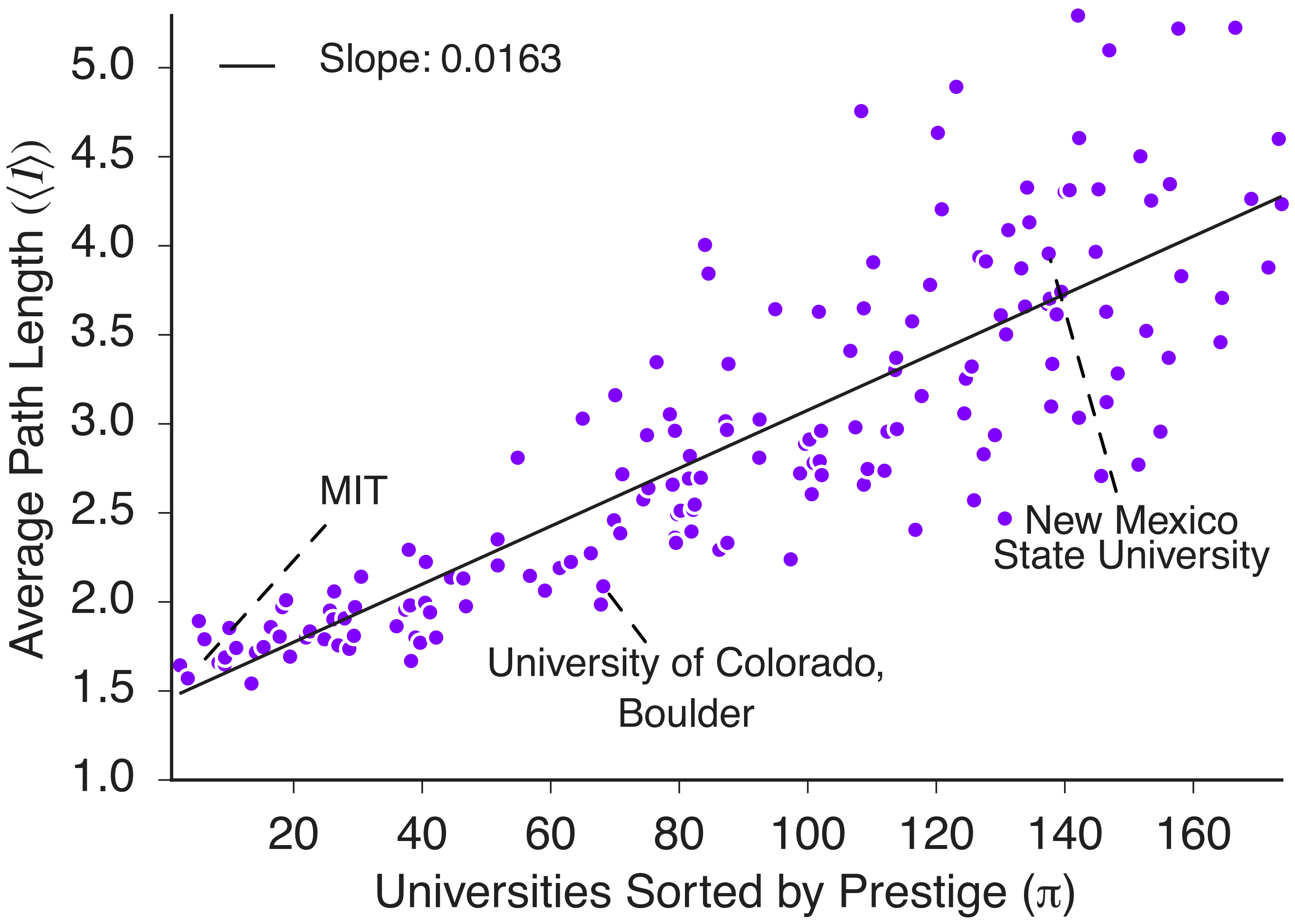


Does the structure of this network affect *ideas?*

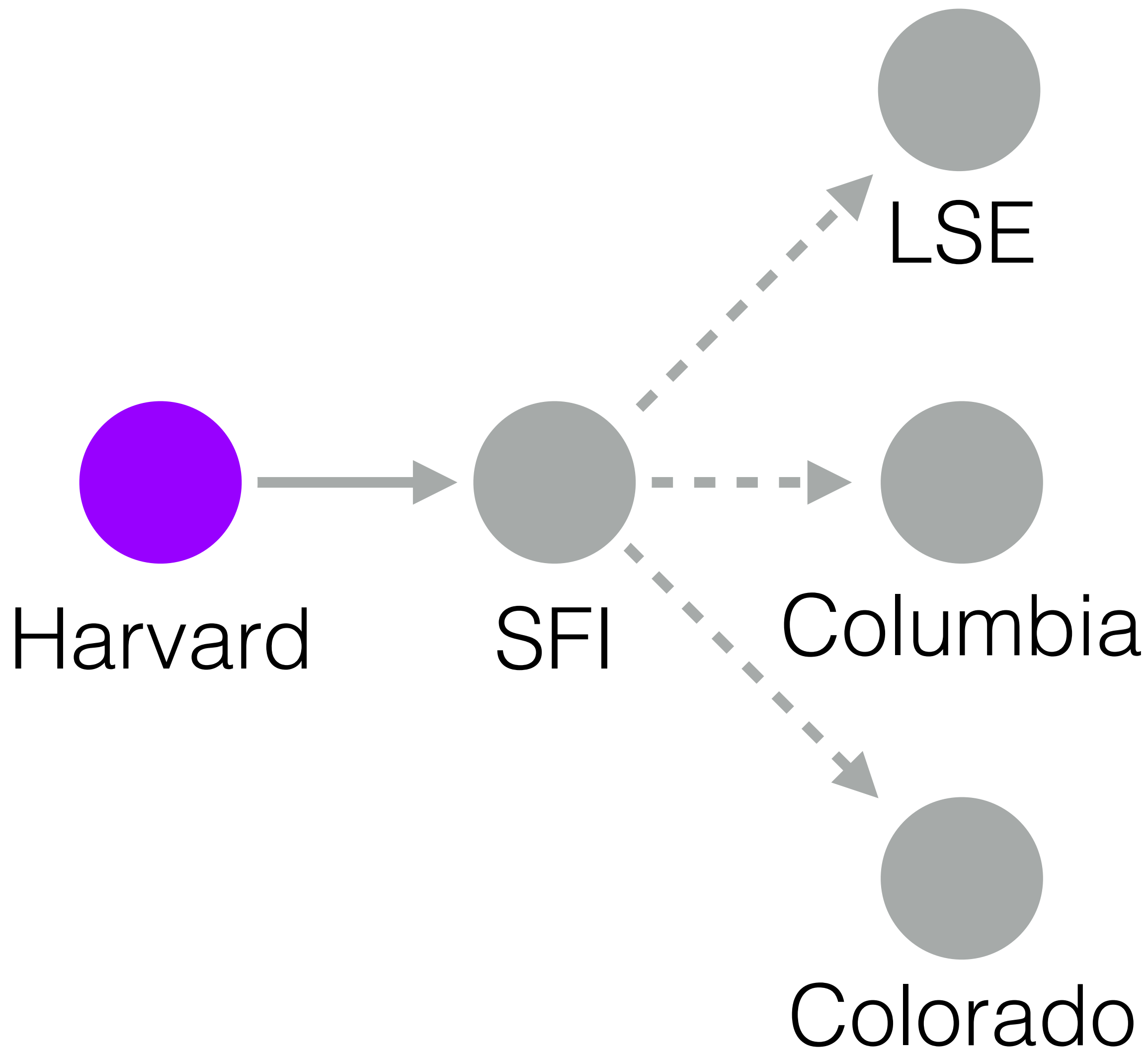
Prestigious institutions are **closer** to all other institutions.

What implications does this have for the **exchange** & **filtration** of ideas?

Does the prestige hierarchy lead to **epistemic inequality**?



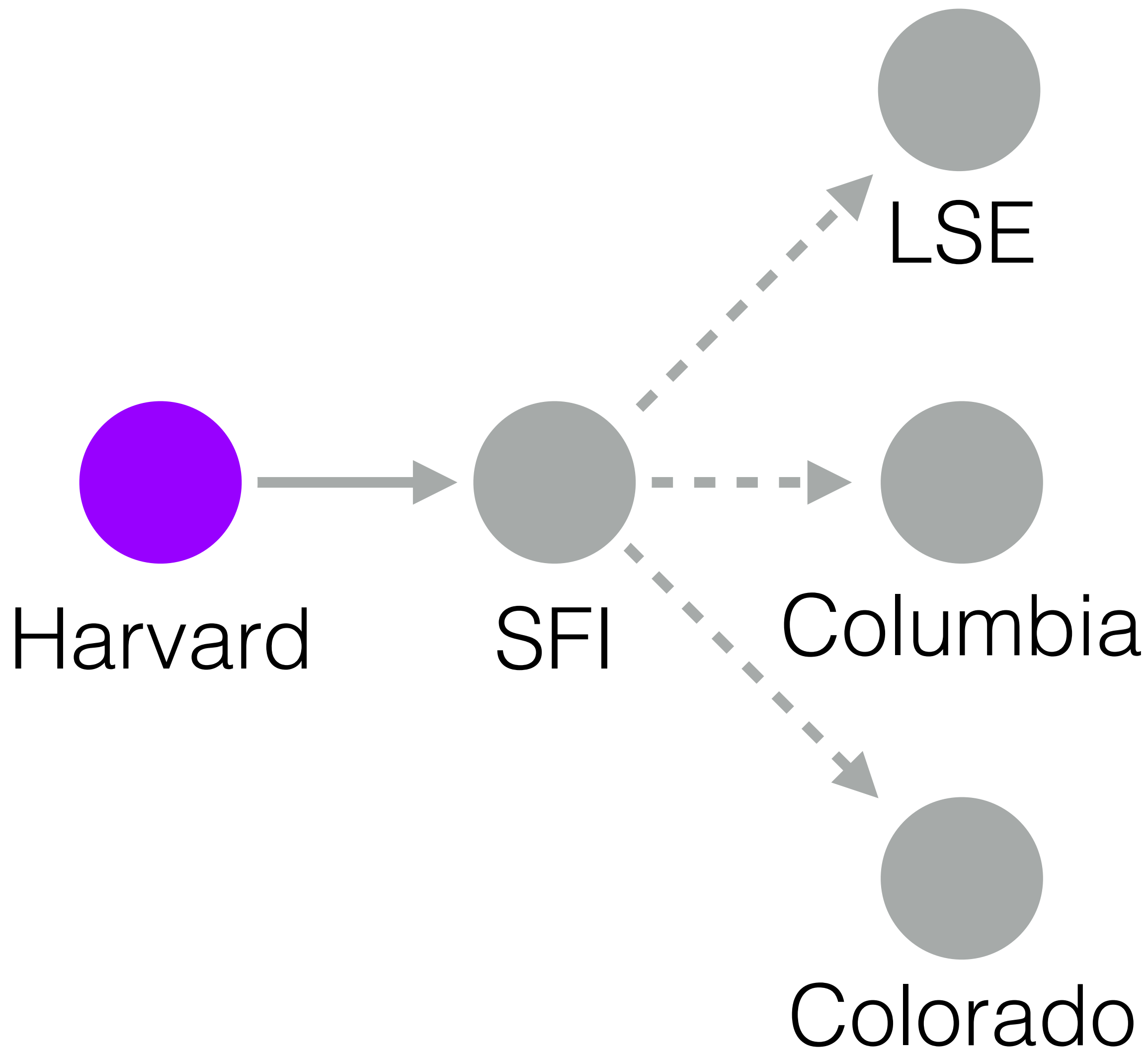
New hires as vectors for infectious ideas?



Do new hires *actually* bring ideas with them?
[or would popular topics get there anyway?]

Are some universities better idea exporters?

New hires as vectors for infectious ideas?



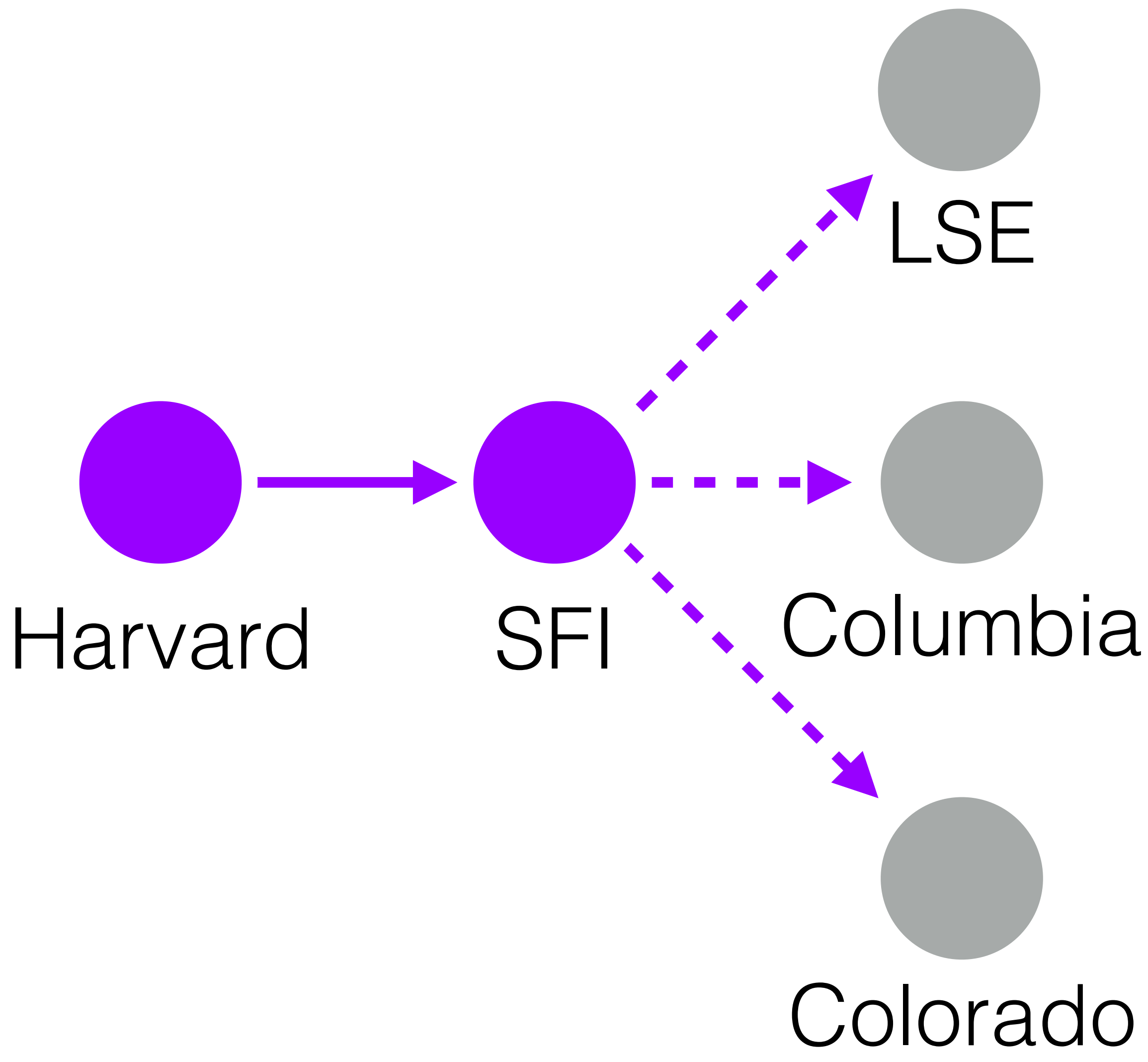
Do new hires *actually* bring ideas with them?
[or would popular topics get there anyway?]

Are some universities better idea exporters?

Epidemic model: treat the idea as an infection, and a new hire as "infectious."

The probability that a hire transmits the idea to an uninfected university: p [idea quality]

New hires as vectors for infectious ideas?



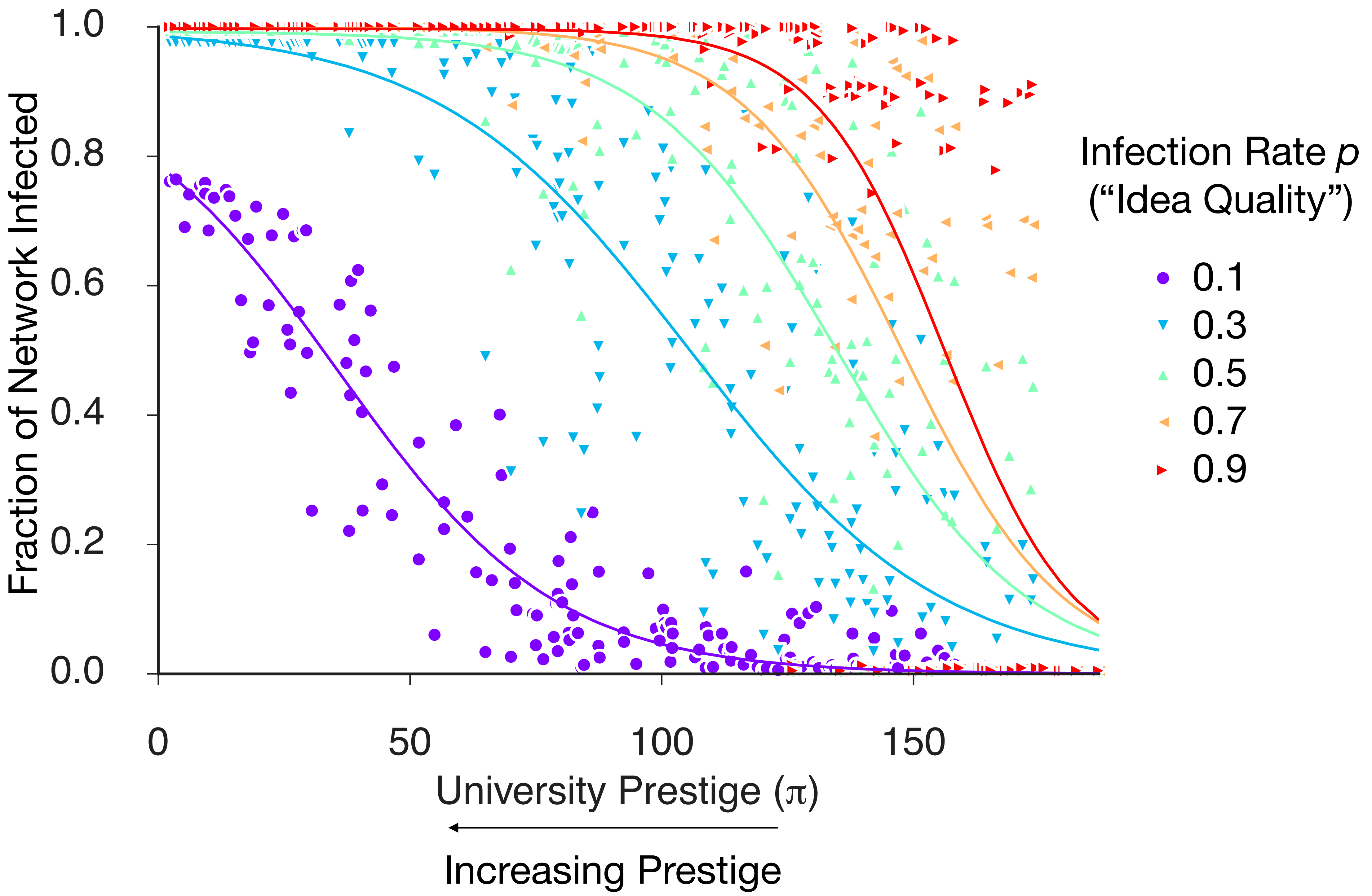
Do new hires *actually* bring ideas with them?
[or would popular topics get there anyway?]

Are some universities better idea exporters?

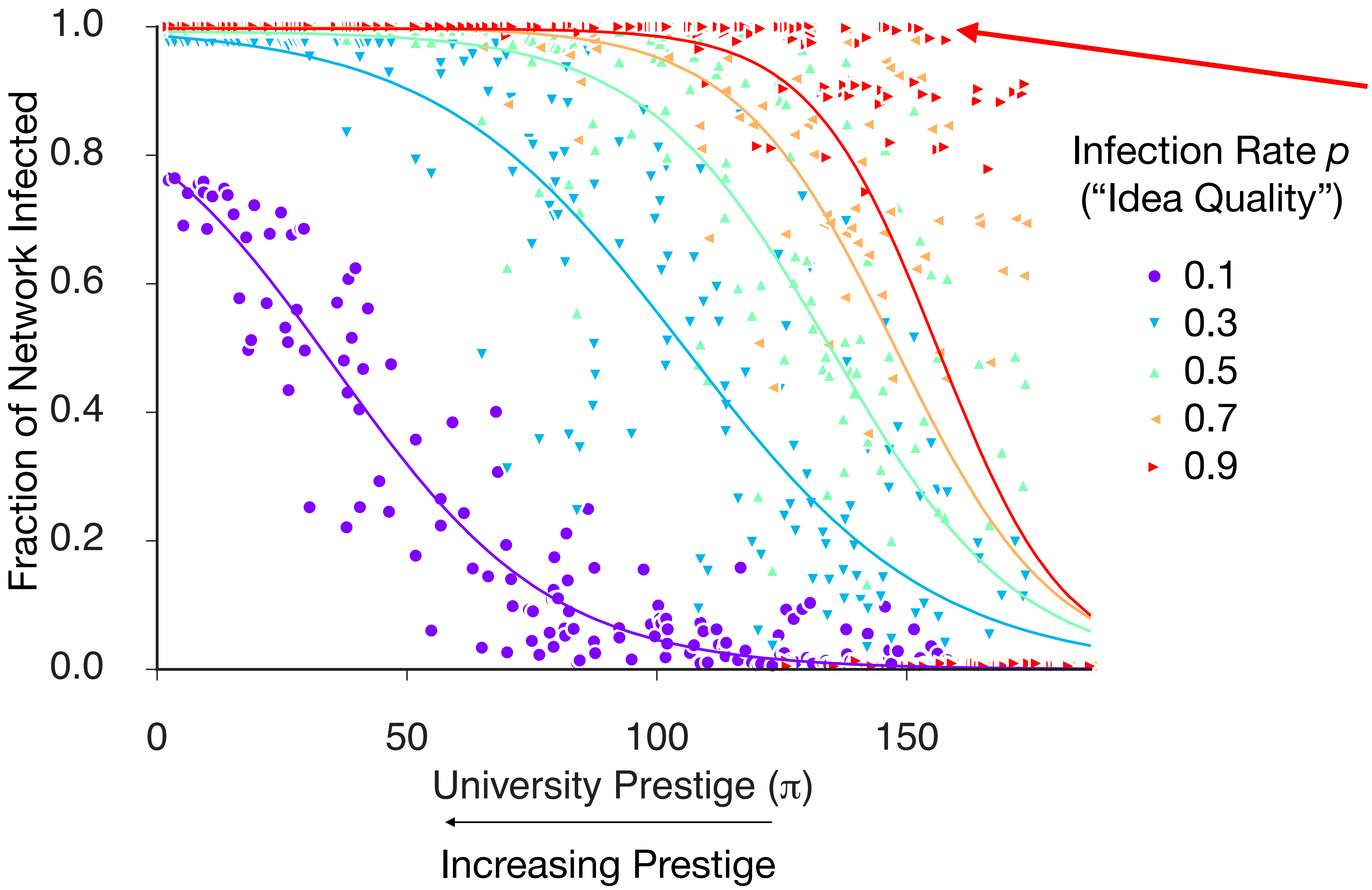
Epidemic model: treat the idea as an infection, and a new hire as "infectious."

The probability that a hire transmits the idea to an uninfected university: p [idea quality]

Network position & the spread of ideas

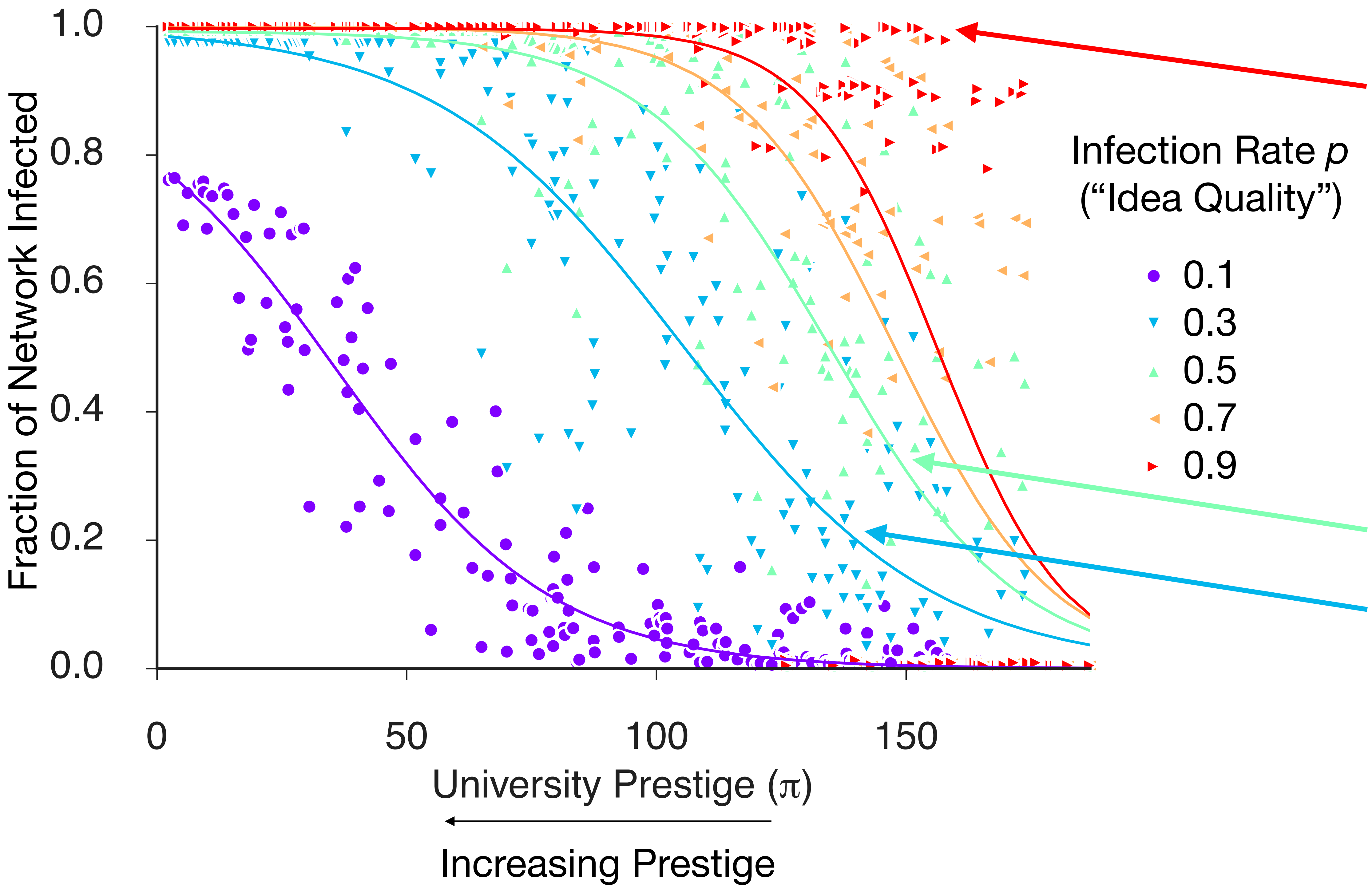


Network position & the spread of ideas



great ideas take root, regardless of starting place

Network position & the spread of ideas



great ideas take root, regardless of starting place

good ideas spread easily from high-prestige univs, but poorly otherwise.

Do *real* ideas spread along hiring links?

Analyzed over 200,000 computer science publications and over 2,500 hires.

Flagged publications on topic modeling, incremental computing, deep learning.

Do *real* ideas spread along hiring links?

Analyzed over 200,000 computer science publications and over 2,500 hires.
Flagged publications on topic modeling, incremental computing, deep learning.

Identified faculty who brought [topic] with them when they were hired.
Identified faculty who began working on [topic] only 2+ yrs after being hired.

Do *real* ideas spread along hiring links?

Analyzed over 200,000 computer science publications and over 2,500 hires.
Flagged publications on topic modeling, incremental computing, deep learning.

Identified faculty who brought [topic] with them when they were hired.
Identified faculty who began working on [topic] only 2+ yrs after being hired.

Compared relative rates of hiring-link spread vs spontaneous spread. (vs random)

Do *real* ideas spread along hiring links?

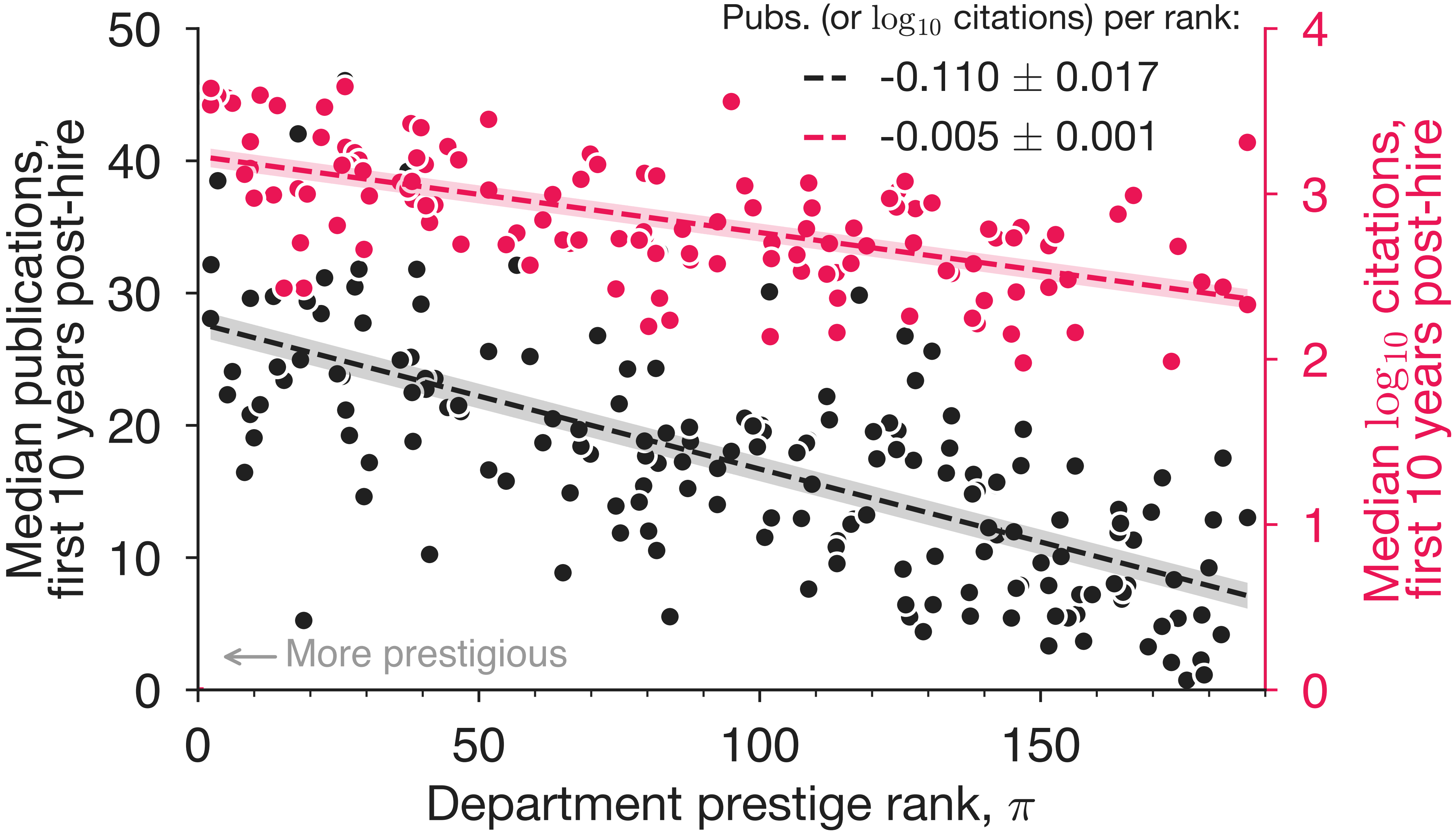
Analyzed over 200,000 computer science publications and over 2,500 hires.
Flagged publications on **topic modeling, incremental computing, deep learning**.

Identified faculty who brought [topic] with them when they were hired.
Identified faculty who began working on [topic] only 2+ yrs after being hired.

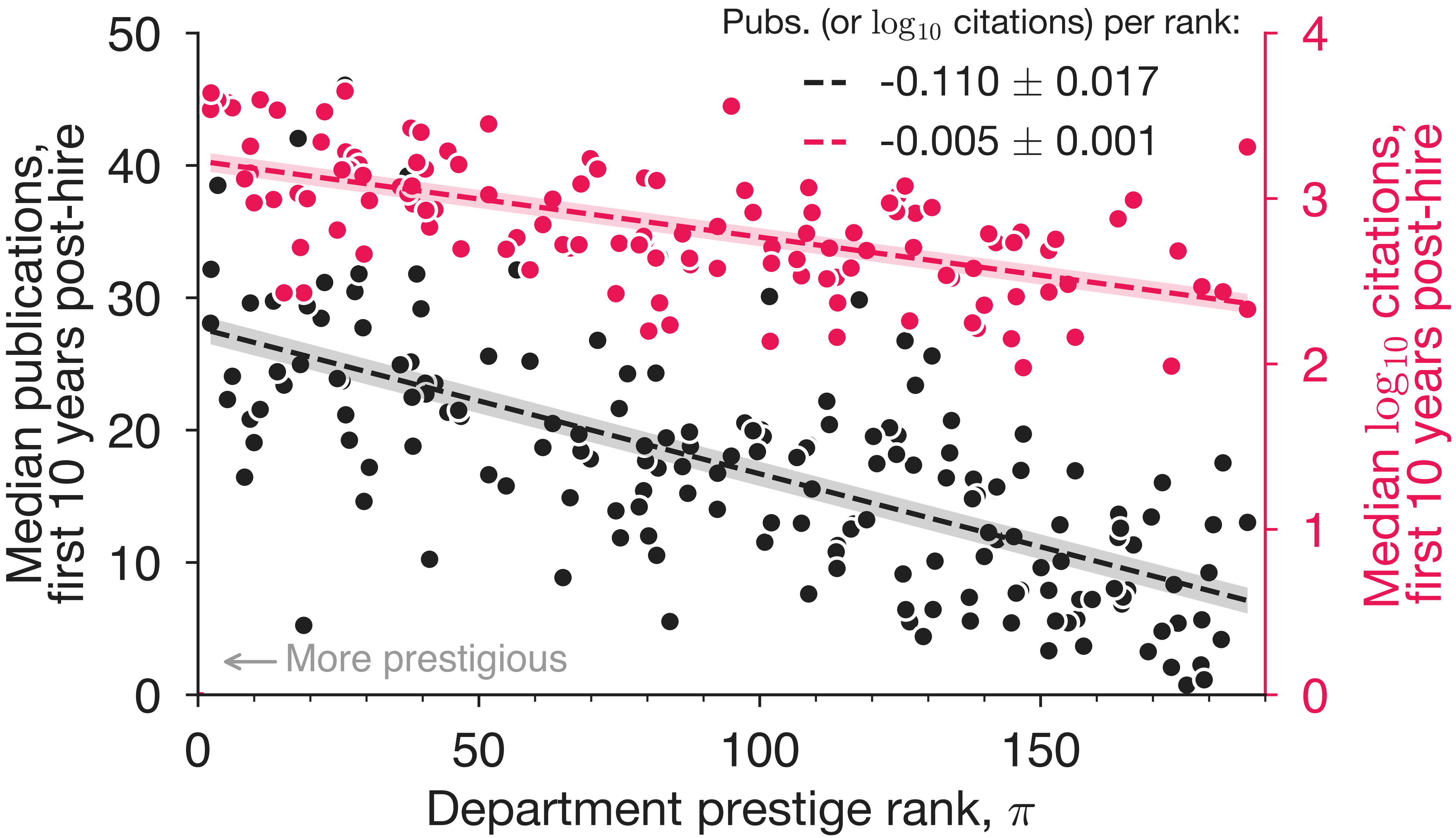
Compared relative rates of **hiring-link spread** vs **spontaneous spread**. (vs random)

Spread of topic modeling (p=0.01) & incremental computing (p=0.01) significantly tied to infection via hiring. Spread of deep learning (p=0.2) *not* significantly linked to hiring.

Prestige correlates with productivity. Why?



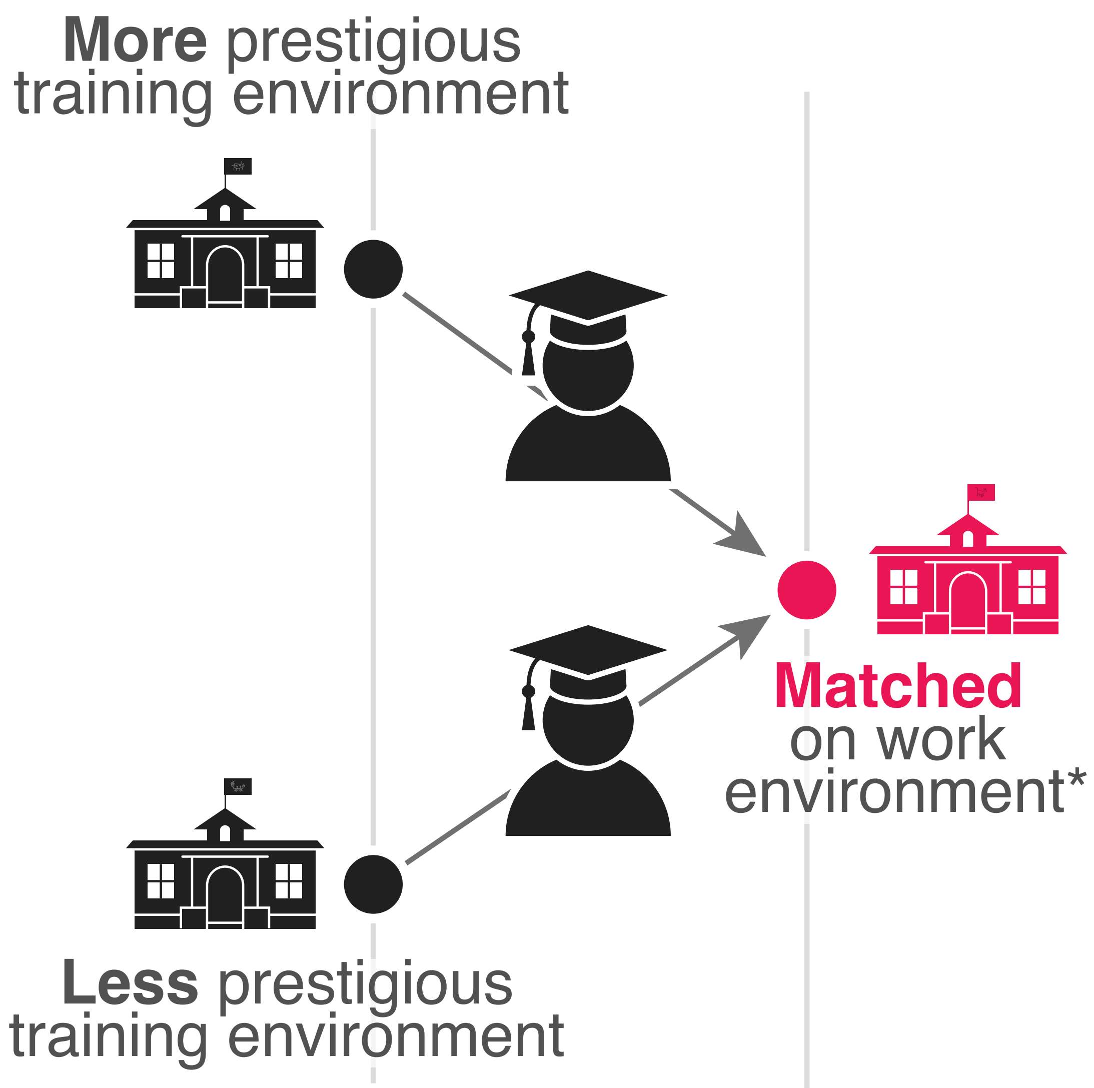
Prestige correlates with productivity. Why?

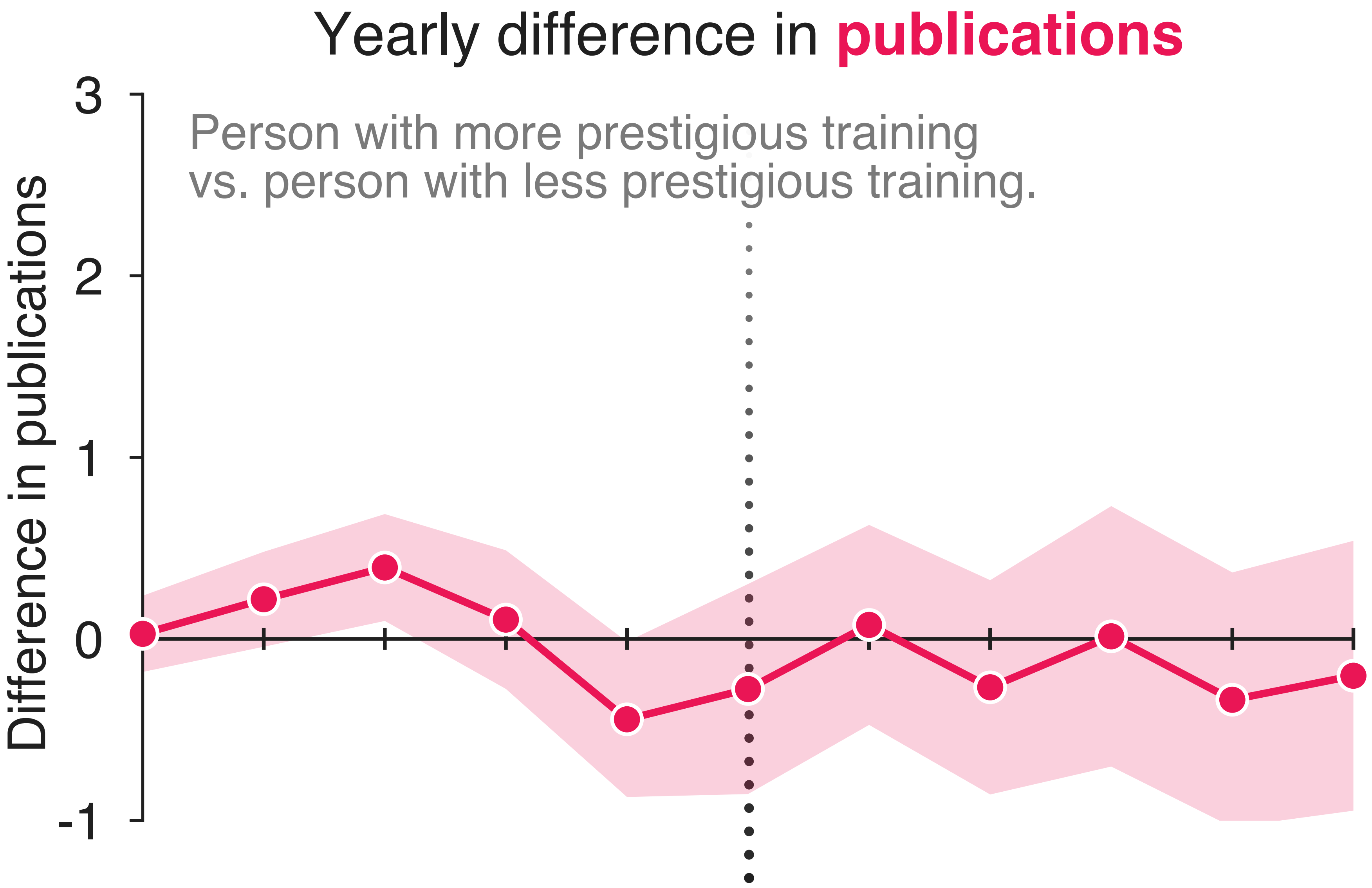


But faculty prestige is predicted by PhD prestige.

Is this effect due to training or Environment?

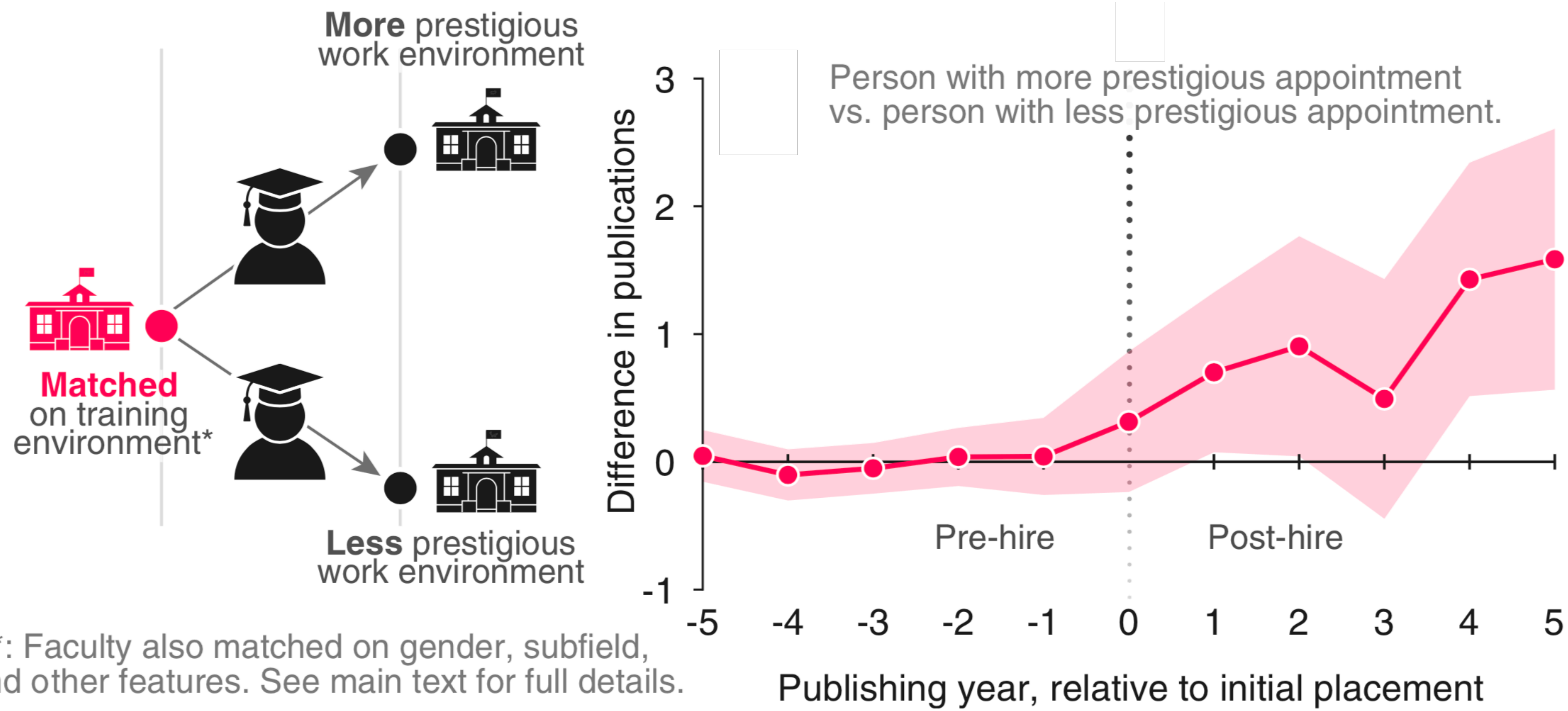
Matching pairs of "academic twins": faculty twins





→Pedigree is not destiny. Once you get a job, previous prestige has no clear impact.

Matching pairs of “academic twins”: grad school twins



*: Faculty also matched on gender, subfield, and other features. See main text for full details.

→Environments matter, and prestigious environments somehow lead to productivity.

Why does environmental prestige predict productivity?

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

H [Selection]: prestigious institutions hire only productive people.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

H [Selection]: prestigious institutions hire only productive people.

→Not supported by the [observable*] data.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

H [Selection]: prestigious institutions hire only productive people.

→Not supported by the [observable*] data.

H [Facilitation]: prestigious institutions provide resources that lead to productivity.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

H [Selection]: prestigious institutions hire only productive people.

→Not supported by the [observable*] data.

H [Facilitation]: prestigious institutions provide resources that lead to productivity.

Worked with the Computing Research Association to use their data on department covariates as predictors of publications and prominence.

Why does environmental prestige predict productivity?

H [Retention]: prestigious institutions grant tenure only to highly productive profs.

→Not supported by the data.

H [Norms]: prestigious institutions shape productivity via adaptation to norms.

→Not supported by the data.

H [Selection]: prestigious institutions hire only productive people.

→Not supported by the [observable*] data.

H [Facilitation]: prestigious institutions provide resources that lead to productivity.

Worked with the Computing Research Association to use their data on department covariates as predictors of publications and prominence.

* hiring committees may (and, if they're doing their jobs correctly, *do*) select on things that are not on CVs. This is an important caveat to keep in mind, and highlights the difficulty of causal inference!

University of Colorado **Boulder**

Colorado
Sam Way
Aaron Clauset
Allison Morgan
Dimitrios Economou

Kauffman / Lux
Sam Arbesman

luxcapital

Ewing Marion KAUFFMAN Foundation

SANTA FE INSTITUTE

NSF

2018

Allie Morgan

2015

Sam Way

2003

Aaron Clauset

Productivity, prominence, and the effects of academic environment
Way, Morgan, Larremore, Clauset. *PNAS* (2019).

Prestige drives epistemic inequality in the diffusion of scientific ideas
Morgan, Economou, Way, Clauset. *EPJ Data Science* (2018).

The misleading narrative of the canonical faculty productivity trajectory
Way, Morgan, Clauset, Larremore. *PNAS* (2017).

Data-driven predictions in the science of science
Clauset, Larremore, Sinatra. *Science* (2017).

Gender, productivity, and prestige in computer science faculty hiring networks
Way, Larremore, Clauset. *Proc. WWW* (2016).

Systematic inequality and hierarchy in faculty hiring networks
Clauset, Arbesman, Larremore. *Science Advances*. (2015).

Postdoc Opening for 2020 PhDs

Aaron Clauset and are looking for a postdoc to help us investigate the dynamics of the scientific ecosystem.

→ daniel.larremore@colorado.edu

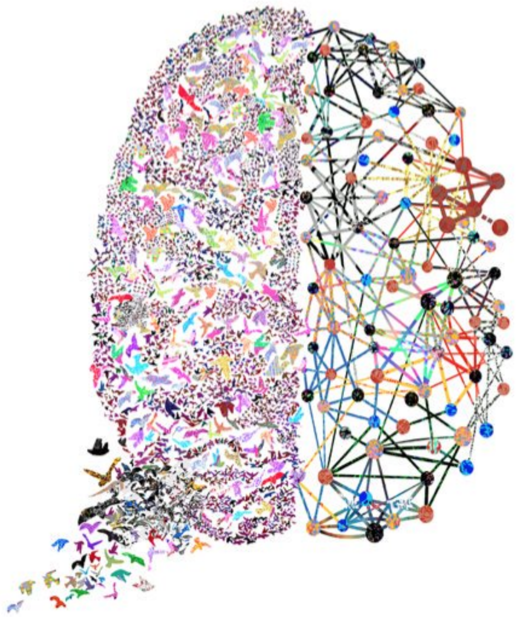
- faculty hiring networks, dynamics of prestige
- productivity and career trajectories
- impacts of inequality on the spread of ideas
- role identification in the scientific ecosystem

University of Colorado **Boulder**

Conference on Complex Networks

COMPLENET '18

Hosted by Northeastern University Network Science Institute



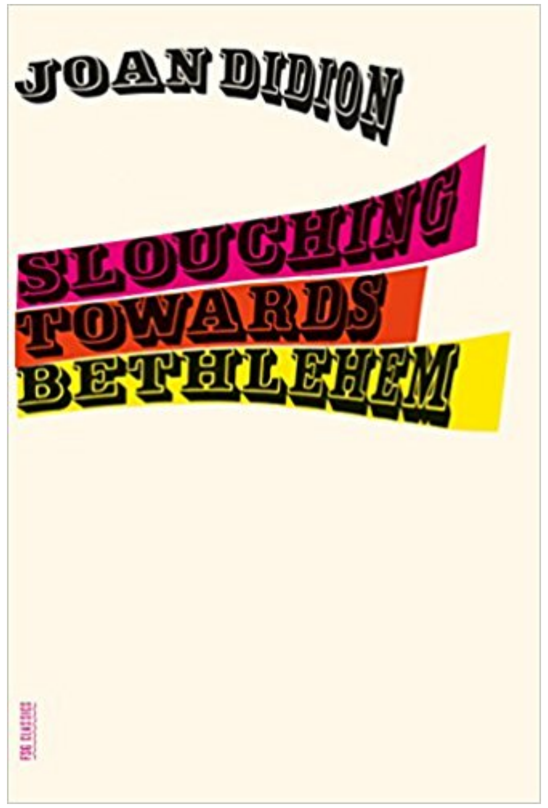
BOSTON, MA APRIL 2018

Xindi Wang

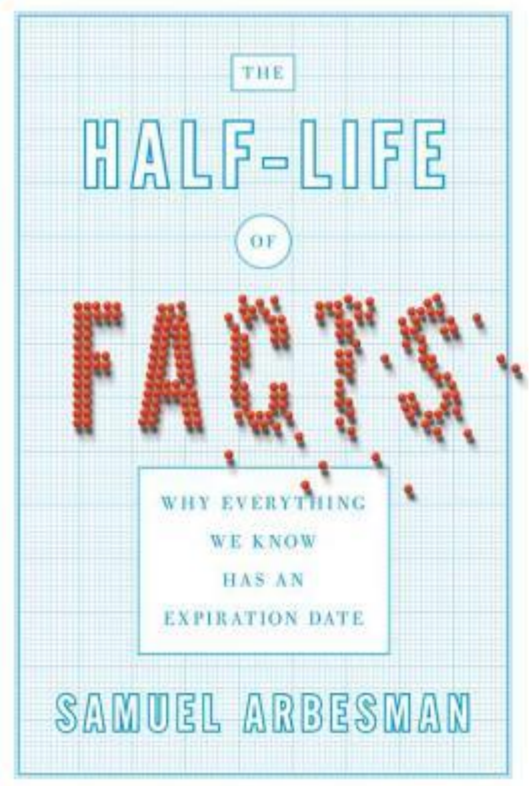
Suppose I give you a book. Predict its sales.

Existing data: books and their sales.

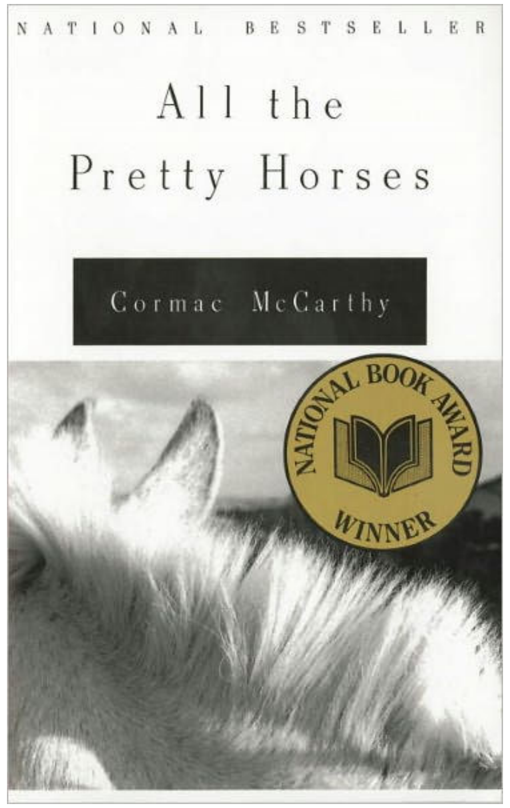
1. turn books into feature vectors.



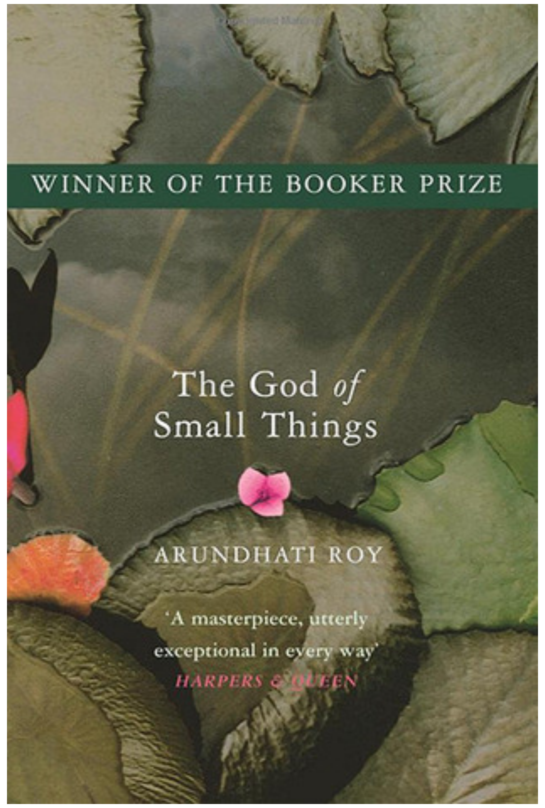
$\overrightarrow{x}_1$



$\overrightarrow{x}_2$



$\overrightarrow{x}_3$



$\overrightarrow{x}_4$

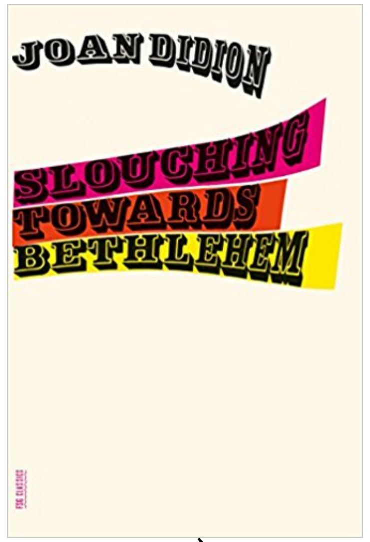
2. Train a model:

$P(\text{book } i > \text{book } j \mid \overrightarrow{x}_i, \overrightarrow{x}_j, \theta)$

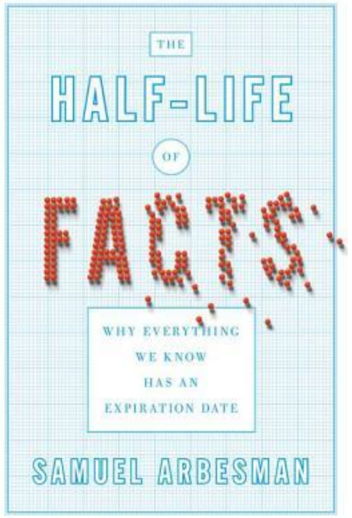
Suppose I give you a book. Predict its sales.

Existing data: books and their sales.

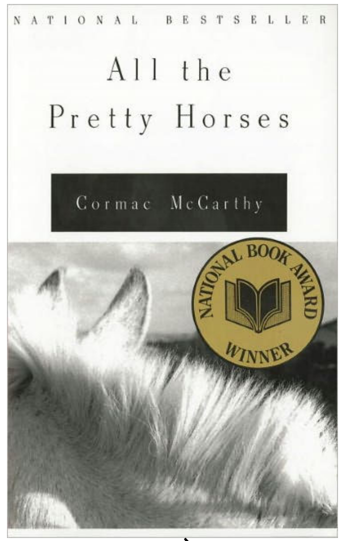
1. turn books into feature vectors.



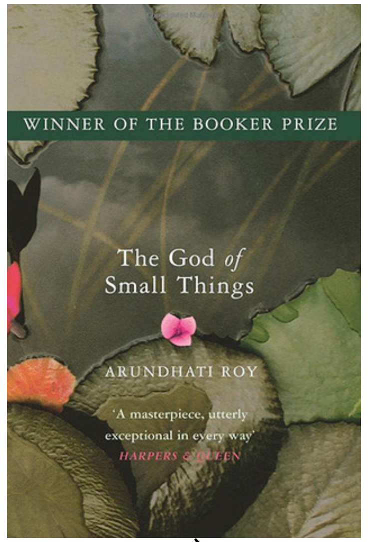
$\vec{x}_1$



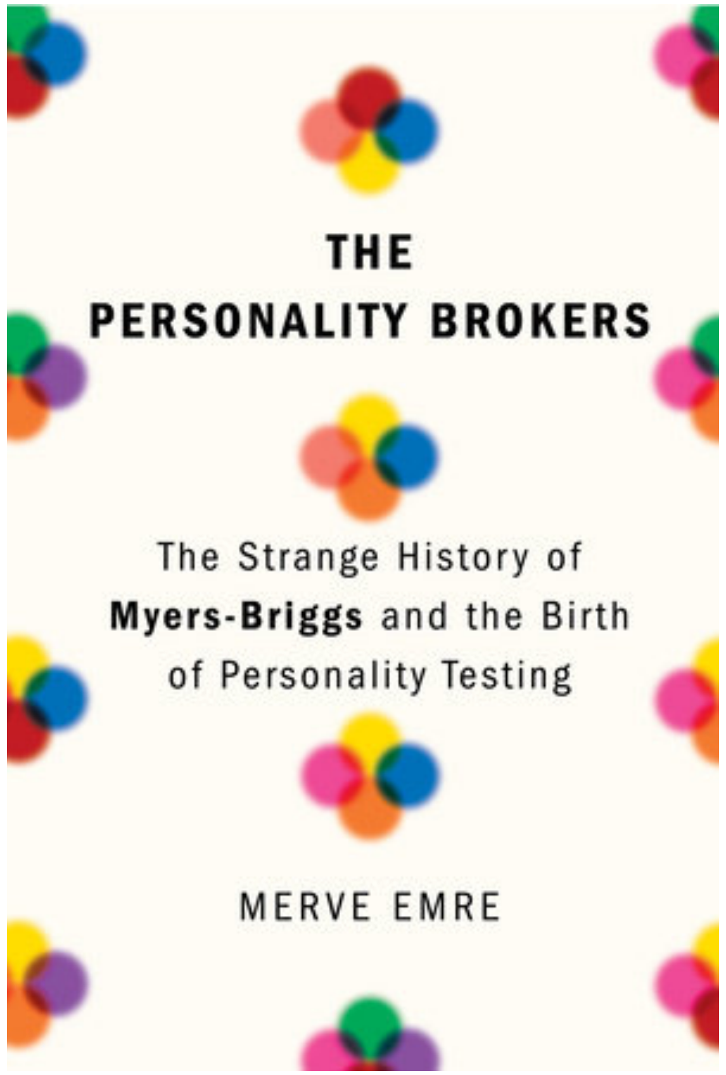
$\vec{x}_2$



$\vec{x}_3$



$\vec{x}_4$



$\vec{x}_5$

2. Train a model.

3. Use the model to simulate pairwise competitions.

   $P(\text{book } i > \text{book } 5 \mid \vec{x}_i, \vec{x}_5, \theta)$

4. Use [**your favo(u)rite algorithm**] to infer rank_5 from pairwise comparisons.

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	**Cohen et al.**	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	Cohen et al.	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**
	SpringRank	**0.908**	**0.893**

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	Cohen et al.	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**
	SpringRank	**0.908**	**0.893**

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	Cohen et al.	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**
	SpringRank	3 **0.908**	1 **0.893**

Rankings rankings

Area under the receiver-operator curve (AUC)

Method		AUC on Fiction	AUC on Biography
	KNN	0.759	0.815
Pairwise +	Cohen et al.	0.892	0.871
	WTG wave	**0.910**	**0.892**
	Voting	**0.915**	**0.891**
	FAS-PIVOT	**0.907**	**0.892**
	SpringRank	**0.908**	**0.893**

But actually… many methods performed well!

Now the question is: why do the top four algorithms perform similarly?

What does that tell us about the **structure of the problem** and the **structure of the space** over which we are ranking?

But actually… many methods performed well!

Now the question is: why do the top four algorithms perform similarly?

What does that tell us about the **structure of the problem** and the **structure of the space** over which we are ranking?

Use the consistency of the ranking results across algorithms to learn about the system itself.

What does it mean for a space or problem or set to be easily ordered or rankable?

Hierarchies of Desirability in Online Dating


Hierarchies of Desirability in Online Dating
me
outdated memes
CSSS
2019

Hierarchies are encoded in language about courtship

“She’s out of your league.”

[How can we find them in data?]

[What behavior? And how noisy is the prediction?]

Hierarchies are encoded in language about courtship

"She's out of your league."

asserts that:

1. Leagues or hierarchies of desirability exist.

    [How can we find them in data?]

[What behavior? And how noisy is the prediction?]

Hierarchies are encoded in language about courtship

"She's out of your league."

asserts that:

1. Leagues or hierarchies of desirability exist.

   [How can we find them in data?]

2. The relative positions of individuals can be estimated.

   [What behavior? And how noisy is the prediction?]

Hierarchies are encoded in language about courtship

“She’s out of your league.”

asserts that:

1. Leagues or hierarchies of desirability exist.
   [How can we find them in data?]
2. The relative positions of individuals can be estimated.
3. Positions are predictive of something.
   [What behavior? And how noisy is the prediction?]

Hierarchies are encoded in language about courtship

“She’s out of your league.”

asserts that:

1. Leagues or hierarchies of desirability exist.

   [How can we find them in data?]

2. The relative positions of individuals can be estimated.

3. Positions are predictive of something.

   [What behavior? And how noisy is the prediction?]

Data: a popular online dating service

Characteristics of the online dating service:

- Free.
- Around 4 million active self-identified heterosexual users.
- Ethnically diverse, urban, youngish.
- Approved and highly restricted data access through collaboration.

Data: a popular online dating service

Characteristics of the online dating service:
- Free.
- Around 4 million active self-identified heterosexual users.
- Ethnically diverse, urban, youngish.
- Approved and highly restricted data access through collaboration.

Characteristics of the data I work with:
- One month of anonymized and timestamped messaging.
- Self-identified heterosexual users with genders declared as M or F.

Data: a popular online dating service

Characteristics of the online dating service:

- Free.
- Around 4 million active self-identified heterosexual users.
- Ethnically diverse, urban, youngish.
- Approved and highly restricted data access through collaboration.

Characteristics of the data I work with:

- One month of anonymized and timestamped messaging.
- Self-identified heterosexual users with genders declared as M or F.

How can we use messaging data to answer these questions?

1. Do desirability hierarchies exist?
2. Are they predictive of behavior?

Intuition: Something like PageRank?

SCIENCE ADVANCES | RESEARCH ARTICLE

SOCIAL SCIENCES

Aspirational pursuit of mates in online dating markets

Elizabeth E. Bruch[1,2]* and M. E. J. Newman[2,3]

- Bruch & Newman 2018 used PageRank to analyze messaging network.
- Sorted individuals by **percentile PageRank scores**.
- **Aspirational pursuit** patterns: people message "up" the sorted network.

Recommended reading if you are interested in this subject!

Intuition: Something like PageRank?

SCIENCE ADVANCES | RESEARCH ARTICLE

SOCIAL SCIENCES

Aspirational pursuit of mates in online dating markets

Elizabeth E. Bruch[1,2]* and M. E. J. Newman[2,3]

- Bruch & Newman 2018 used PageRank to analyze messaging network.
- Sorted individuals by **percentile PageRank scores**.
- **Aspirational pursuit** patterns: people message "up" the sorted network.

Recommended reading if you are interested in this subject!

Downside: PageRank scores aren't *usefully* interpretable.
[Stationary distrib. of random walk + teleportation, on a network of message passing?]

Intuition: Something like PageRank?

SCIENCE ADVANCES | RESEARCH ARTICLE

SOCIAL SCIENCES

Aspirational pursuit of mates in online dating markets

Elizabeth E. Bruch[1,2]* and M. E. J. Newman[2,3]

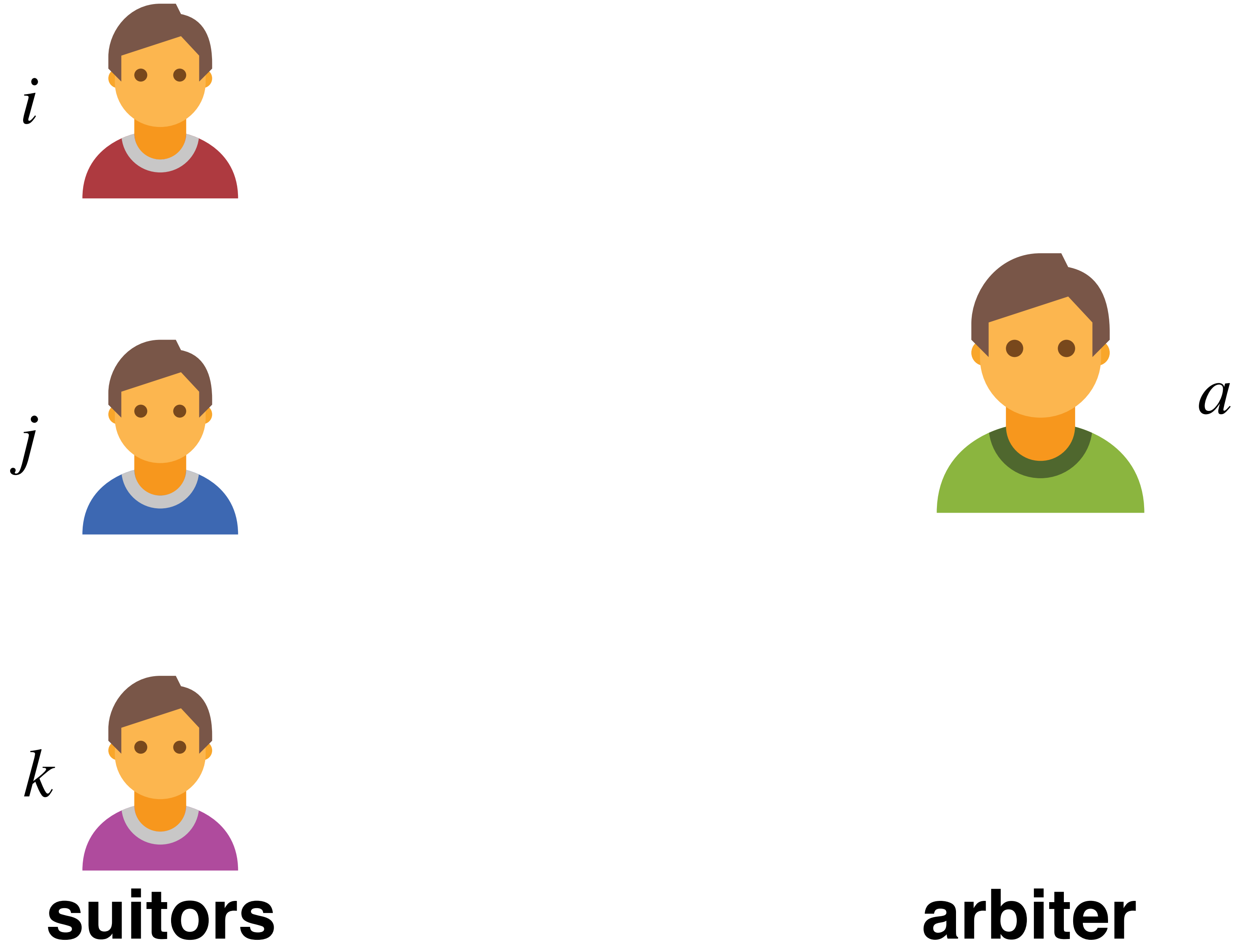
- Bruch & Newman 2018 used PageRank to analyze messaging network.
- Sorted individuals by **percentile PageRank scores**.
- **Aspirational pursuit** patterns: people message “up” the sorted network.

Recommended reading if you are interested in this subject!

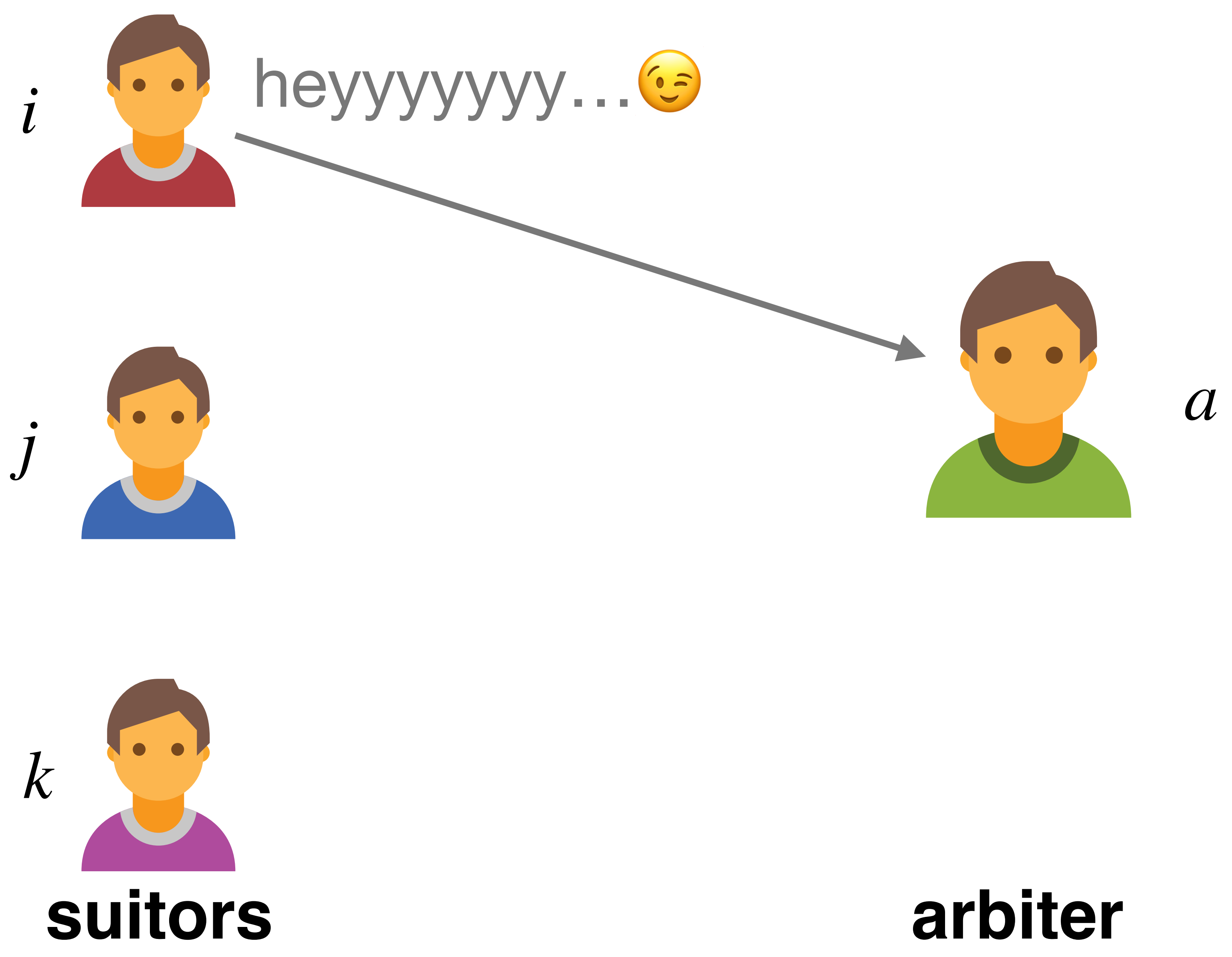
Downside: PageRank scores aren’t *usefully* interpretable.
[Stationary distrib. of random walk + teleportation, on a network of message passing?]

Can we use the messaging data to find more **meaningfully interpretable ranks**?

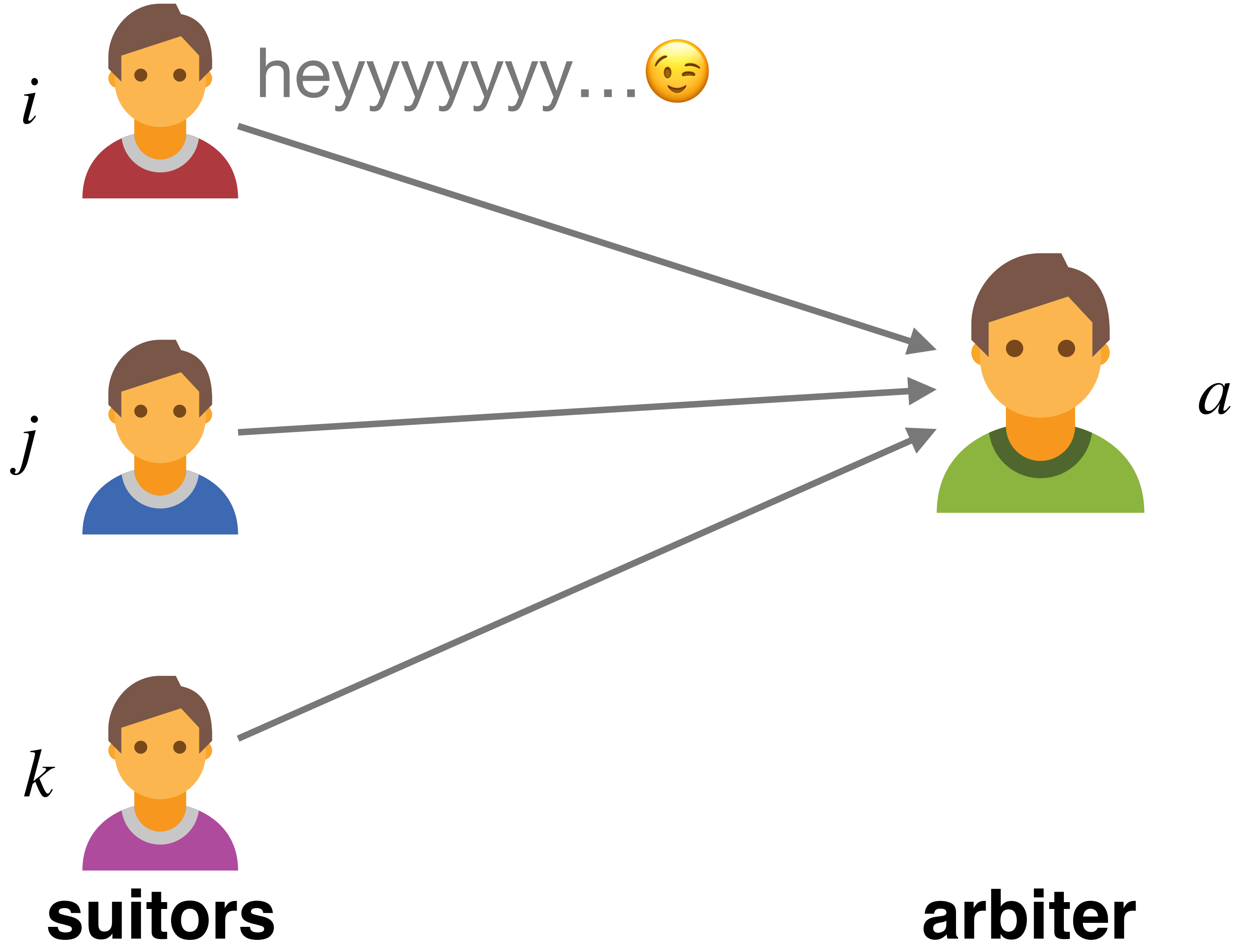
Insight: messaging is a competition for attention



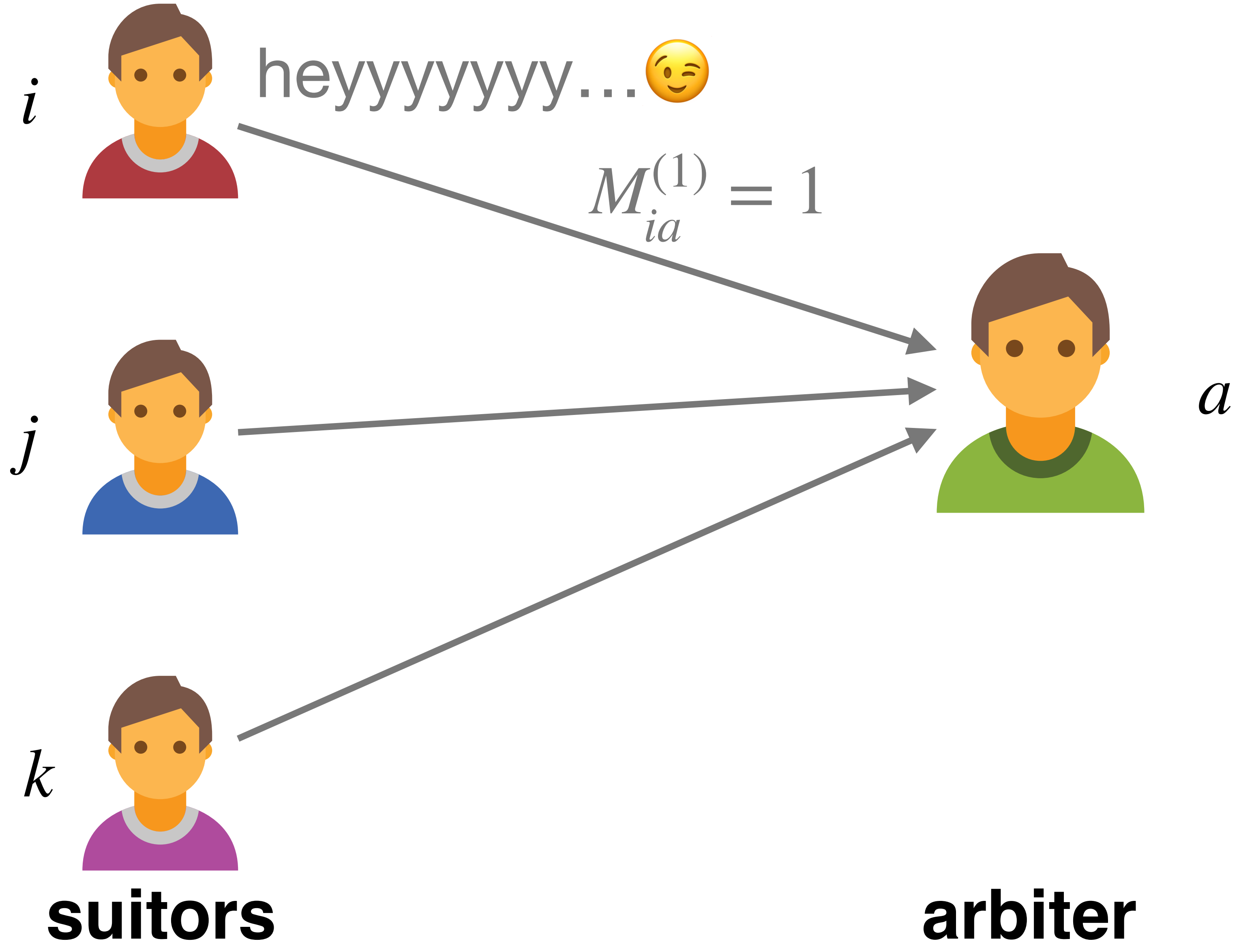
Insight: messaging is a competition for attention



Insight: messaging is a competition for attention

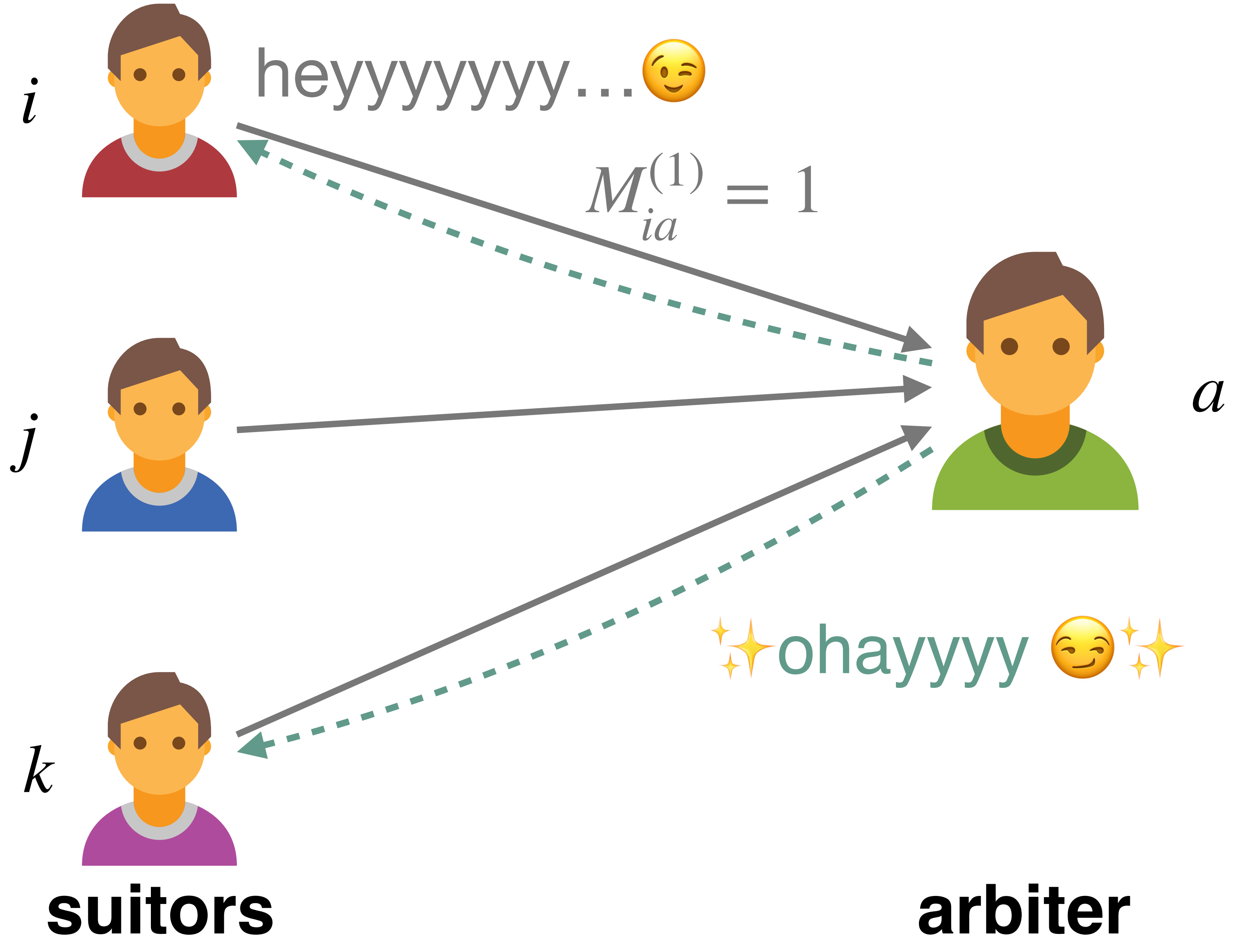


Insight: messaging is a competition for attention



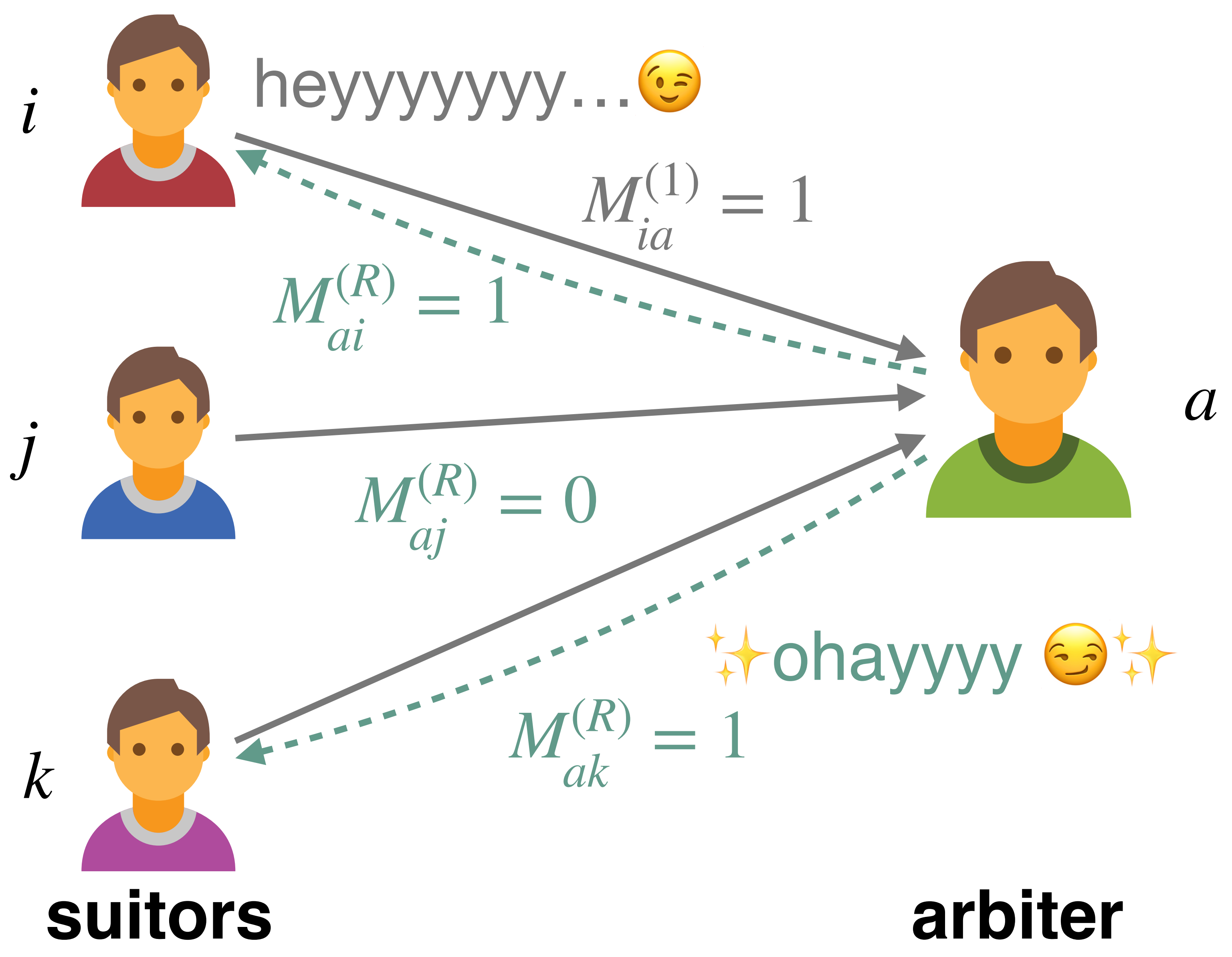
$M^{(1)}$: network layer of first messages

Insight: messaging is a competition for attention



$M^{(1)}$: network layer of first messages

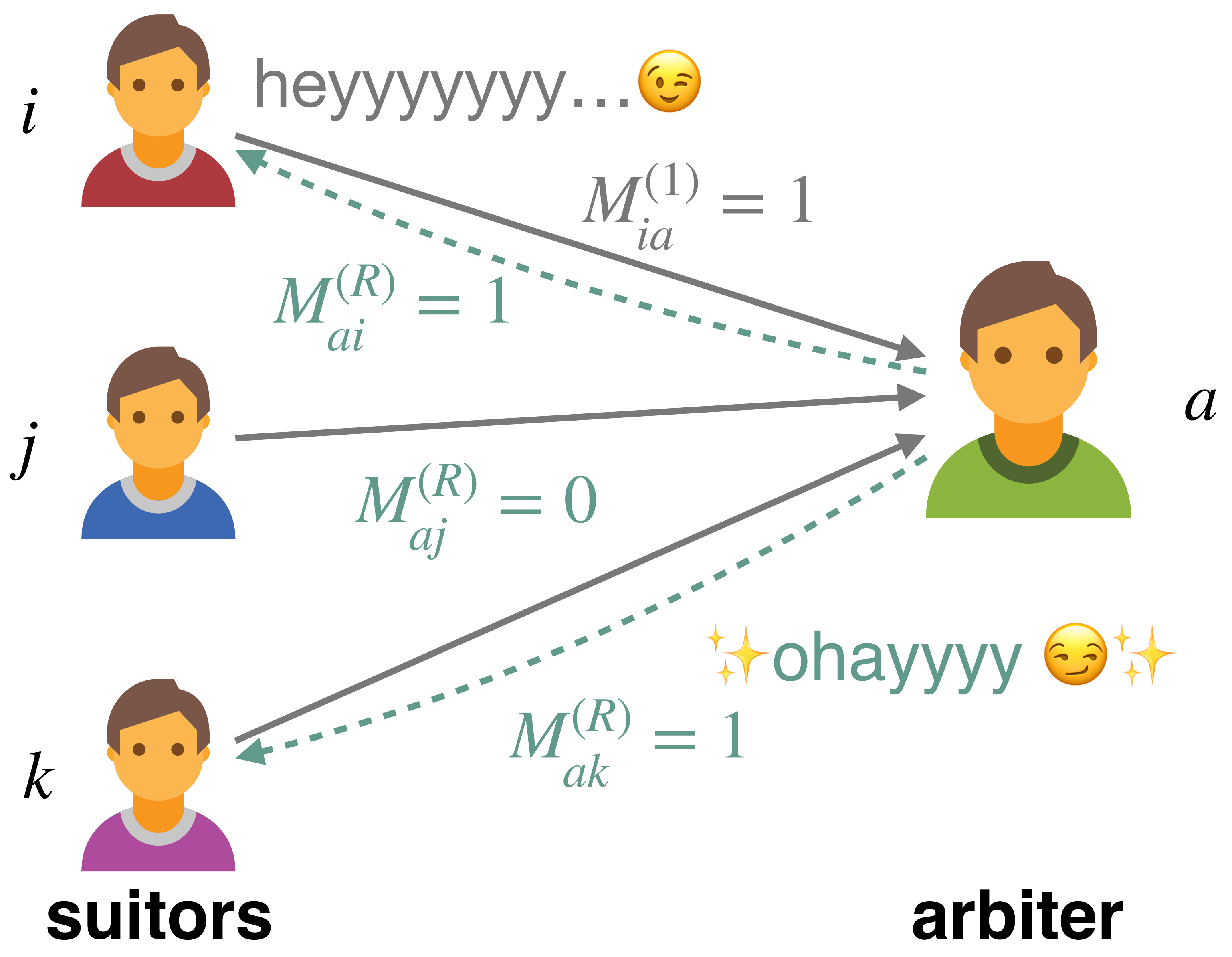
Insight: messaging is a competition for attention



$M^{(1)}$: network layer of first messages

$M^{(R)}$: network layer of replies

Insight: messaging is a competition for attention

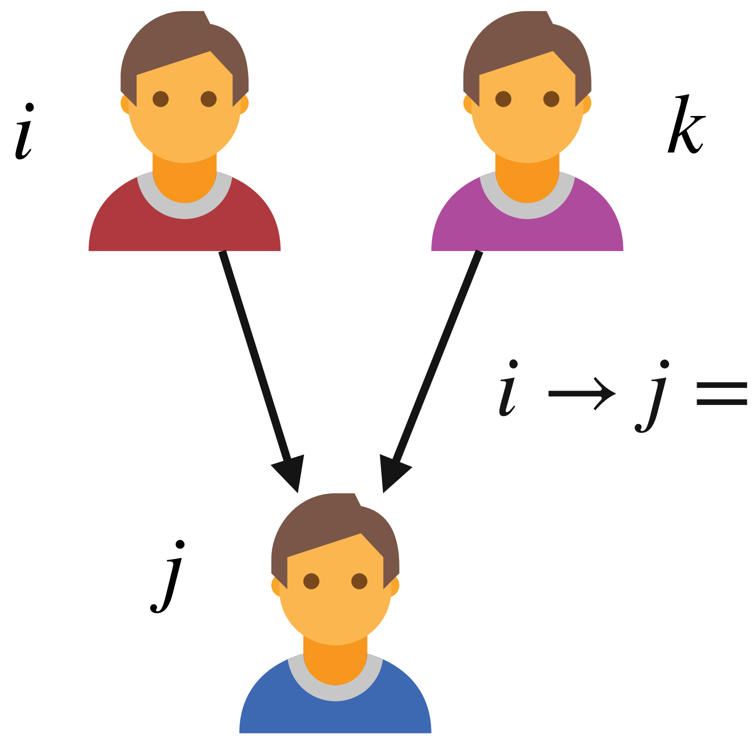


$M^{(1)}$: network layer of first messages

$M^{(R)}$: network layer of replies

Who won/lost this competition for attention?

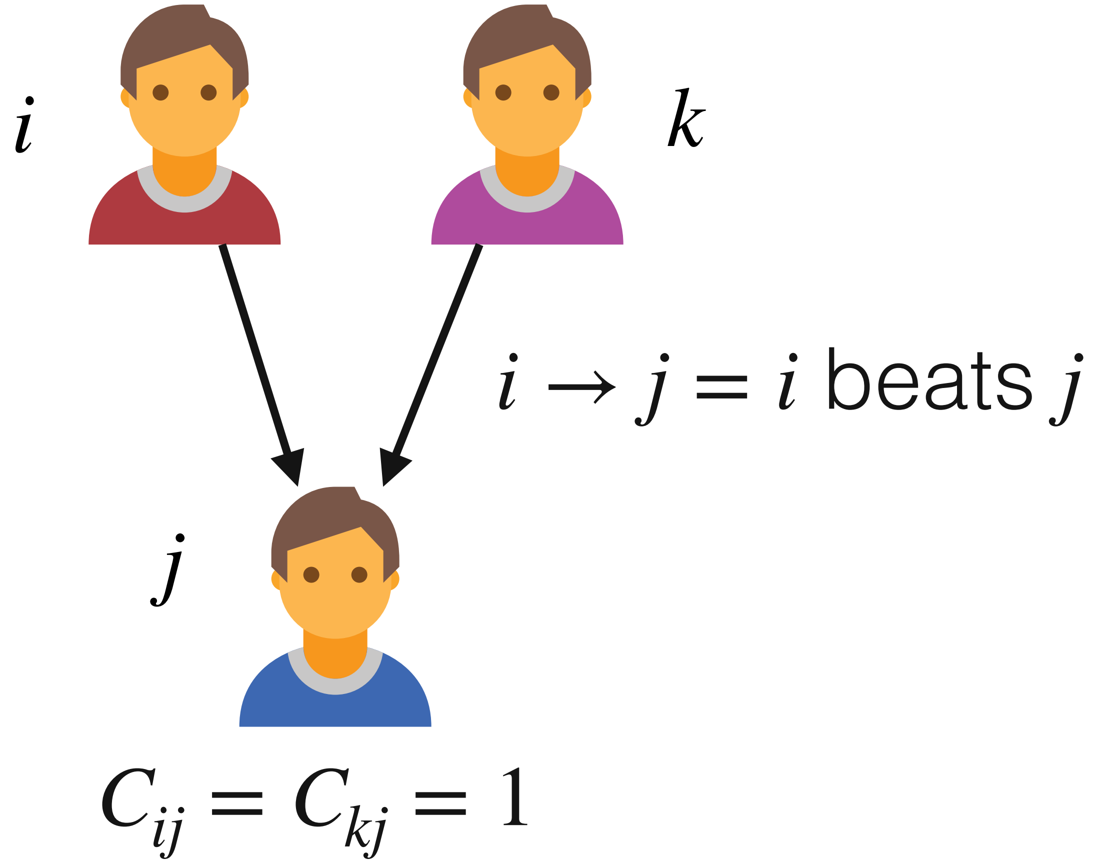
Hierarchies in the competition for attention



$i \rightarrow j = i$ beats j

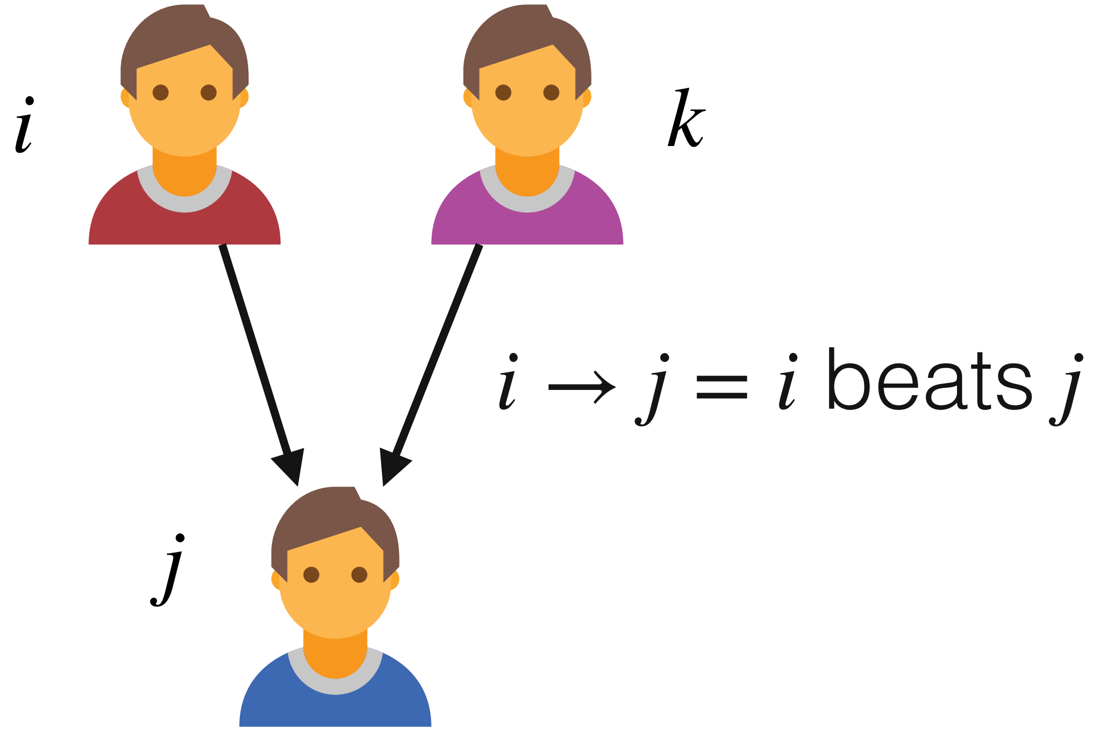
$C_{ij} = C_{kj} = 1$

Hierarchies in the competition for attention



Let C_{ij} be the number of times that i outcompetes j.

Hierarchies in the competition for attention

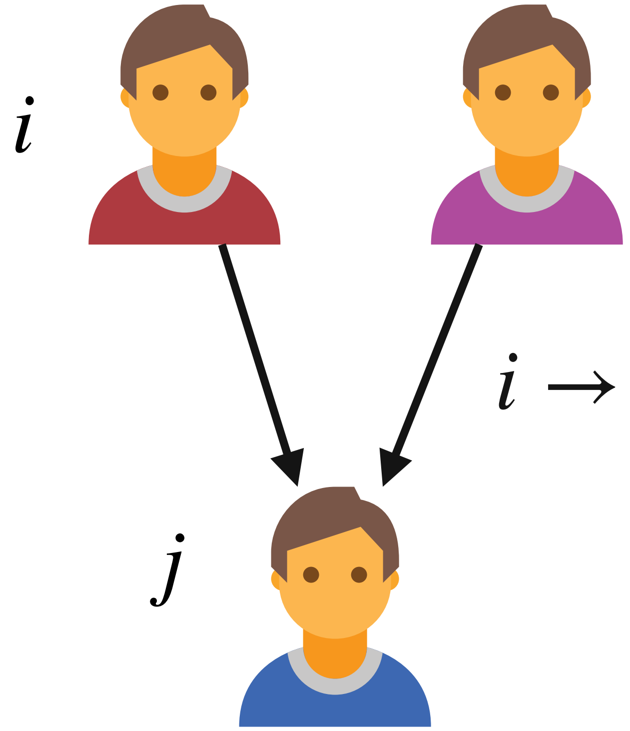


$$C_{ij} = C_{kj} = 1$$

Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

Hierarchies in the competition for attention



$i \to j = i$ beats j

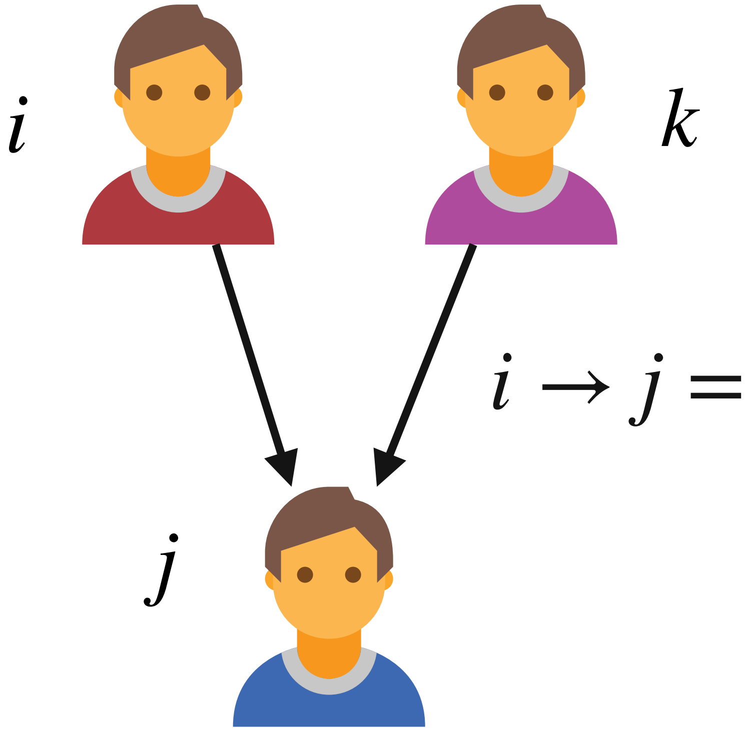
$C_{ij} = C_{kj} = 1$

Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

$i \mapsto a$
$j \mapsto a$

Hierarchies in the competition for attention



$i \rightarrow j = i$ beats j

$C_{ij} = C_{kj} = 1$

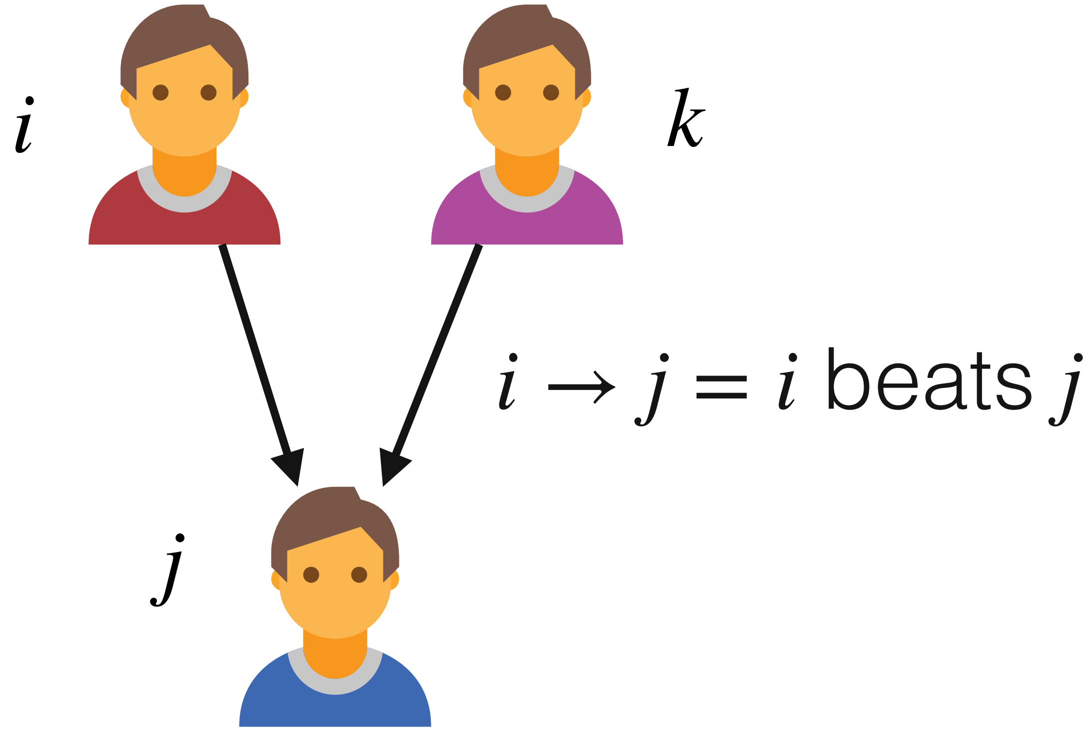
Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M_{ia}^{(1)} M_{ja}^{(1)} M_{ai}^{(R)} \left(1 - M_{aj}^{(R)}\right)$$

$i \mapsto a$
$j \mapsto a$

$i \hookleftarrow a$

Hierarchies in the competition for attention



$$C_{ij} = C_{kj} = 1$$

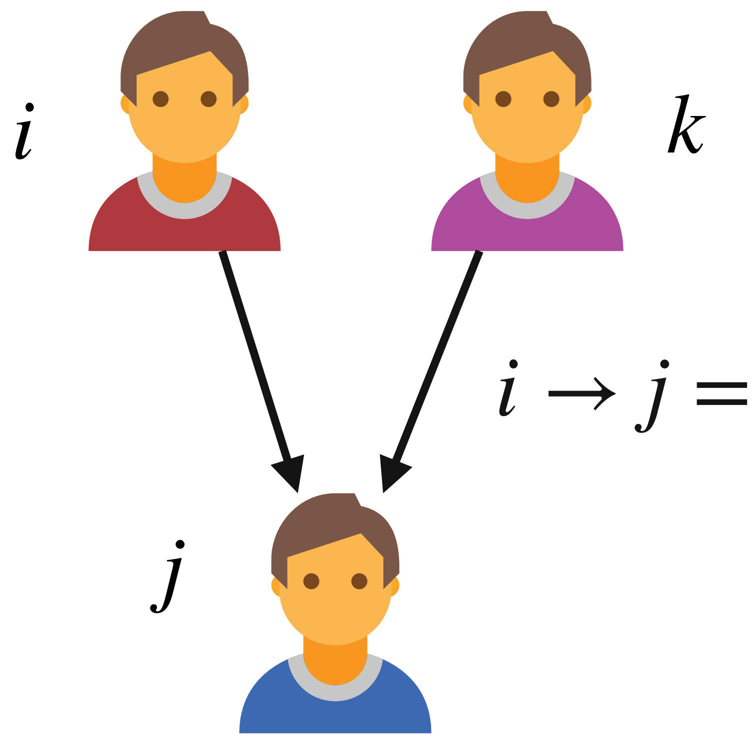
Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

$i \mapsto a$
$j \mapsto a$

$i \hookleftarrow a$ $\quad j \not\hookleftarrow a$

Hierarchies in the competition for attention



$i \rightarrow j = i$ beats j

$C_{ij} = C_{kj} = 1$

Let C_{ij} be the number of times that i outcompetes j.

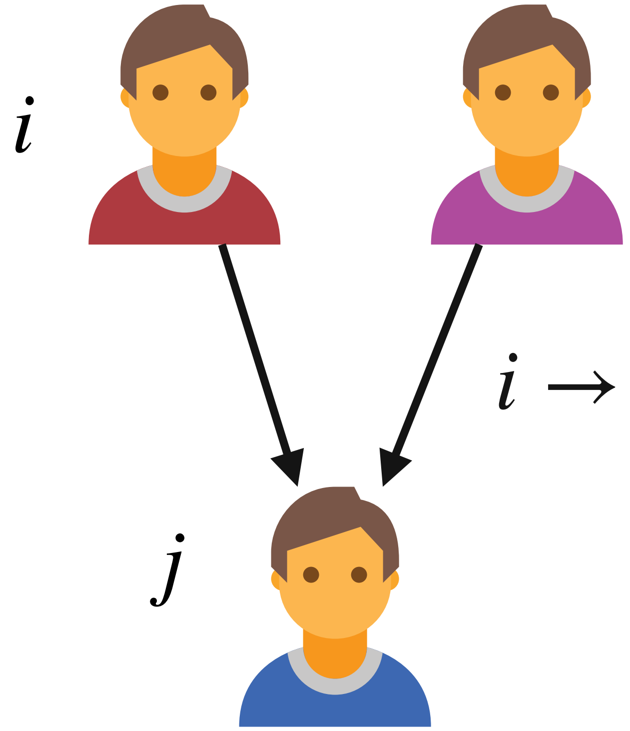
$$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

sum over all arbiters
(i.e. first-message receivers)

$i \mapsto a$
$j \mapsto a$

$i \hookleftarrow a$ $\quad j \not\hookleftarrow a$

Hierarchies in the competition for attention



$i \rightarrow j = i$ beats j

$C_{ij} = C_{kj} = 1$

Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M_{ia}^{(1)} M_{ja}^{(1)} M_{ai}^{(R)} \left(1 - M_{aj}^{(R)}\right)$$

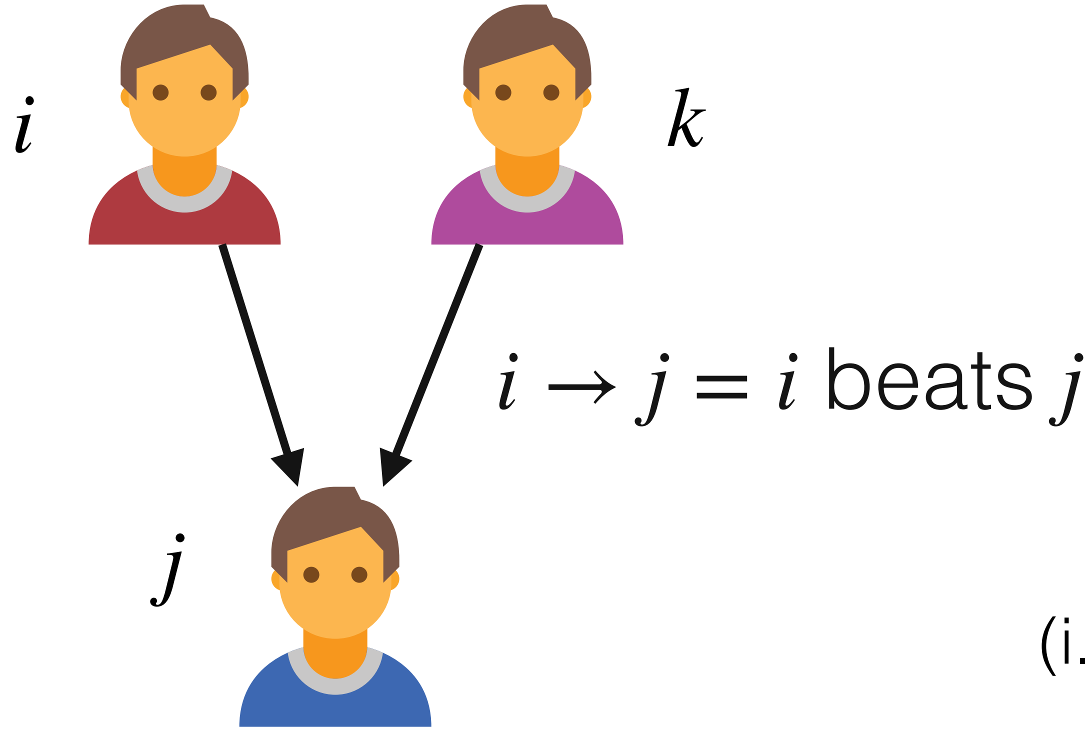
sum over all arbiters
(i.e. first-message receivers)

$i \mapsto a$
$j \mapsto a$

$i \hookleftarrow a$ $\quad j \not\hookleftarrow a$

Each arbiter is presented with a choice set of suitors.
Collectively, arbiters' choices provide many *partial orderings* of suitors.

Hierarchies in the competition for attention



$C_{ij} = C_{kj} = 1$

Let C_{ij} be the number of times that i outcompetes j.

$$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

sum over all arbiters
(i.e. first-message receivers)

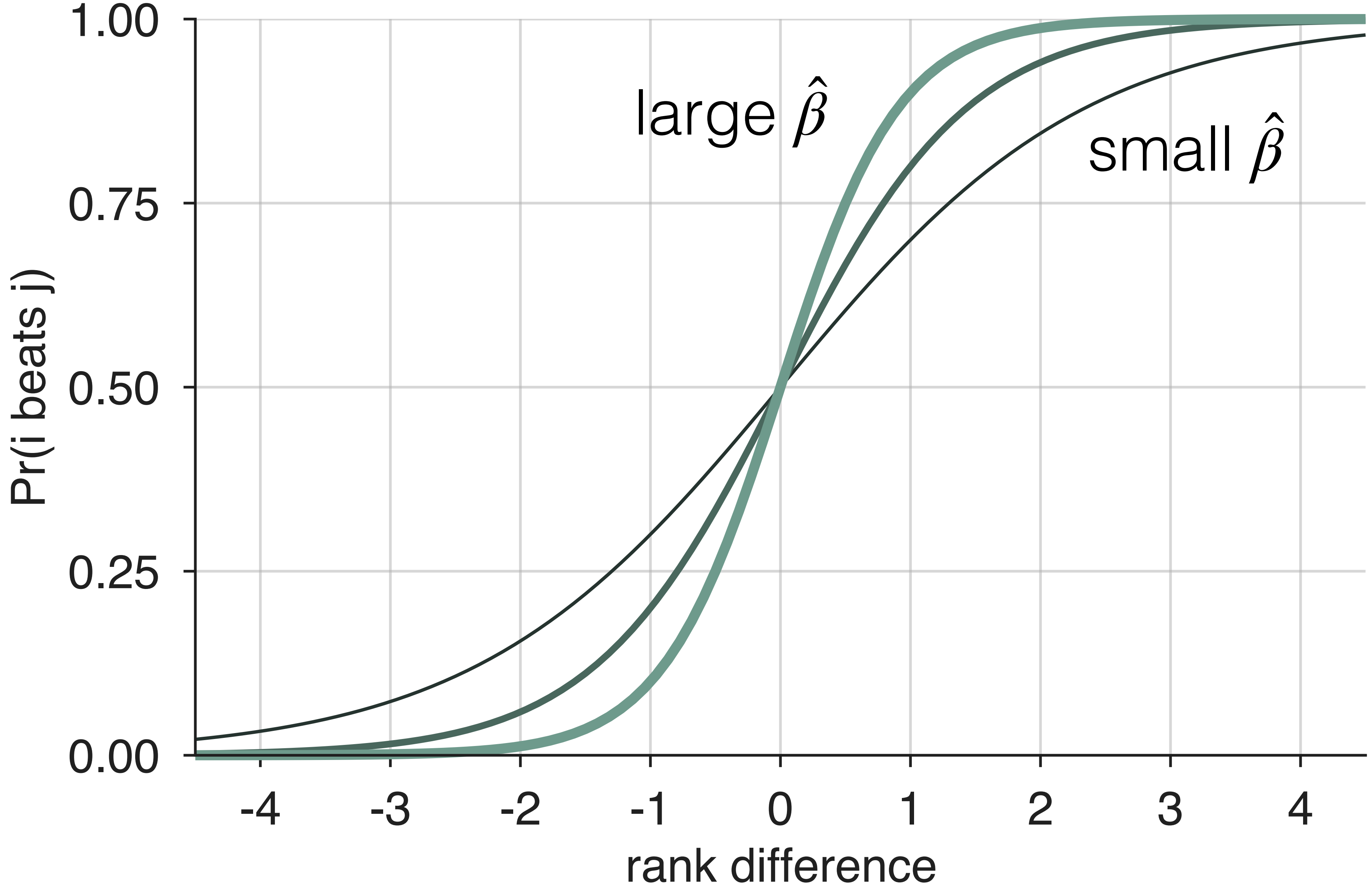
$i \mapsto a$
$j \mapsto a$

$i \hookleftarrow a$ $\quad j \not\hookleftarrow a$

Each arbiter is presented with a choice set of suitors.
Collectively, arbiters' choices provide many *partial orderings* of suitors.

C is a one-component, directed network representing pairwise comparisons.
We will use **SpringRank** to find people's latent positions to *predict future behavior*.

Beta tells us how to interpret rank differences



$$P(i \to j \mid i \leftrightarrow j) = \frac{1}{1 + e^{-2\hat{\beta}(c_i - c_j)}}$$

$\hat{\beta}$ is the MLE inverse temperature of the SpringRank Boltzmann

It tells us the sensitivity/scale for predictions in the ranking space.

The Depths of Leagues

Dear Editors,

There are some questions I would like to ask. Firstly, how complex is backgammon compared to other games of skill such as chess or bridge?

Robertie, 1980 *Inside Backgammon.* h/t Johan Ugander, Brendan Tracey

The Depths of Leagues

Dear Editors,

There are some questions I would like to ask. Firstly, how complex is backgammon compared to other games of skill such as chess or bridge?

Let's start with chess, which has evolved a well-developed rating system over the past 40 years. Chess ratings range from a high of about 2800 to theoretical lows of about 0 (a complete beginner who has just learned the moves). Chess ratings are also designed so that a 200-point rating difference between two players anywhere on the scale means that the higher-rated player has a 70-75% chance of defeating a lower-rated player (discounting draws, which are possible in chess but not in most of the other games we'll consider).

Robertie, 1980 *Inside Backgammon.* h/t Johan Ugander, Brendan Tracey

The Depths of Leagues

Dear Editors,

There are some questions I would like to ask. Firstly, how complex is backgammon compared to other games of skill such as chess or bridge?

Let's start with chess, which has evolved a well-developed rating system over the past 40 years. Chess ratings range from a high of about 2800 to theoretical lows of about 0 (a complete beginner who has just learned the moves). Chess ratings are also designed so that a 200-point rating difference between two players anywhere on the scale means that the higher-rated player has a 70-75% chance of defeating a lower-rated player (discounting draws, which are possible in chess but not in most of the other games we'll consider).

Now consider the following experiment:

(1) Take the best player in the world (in the case of chess, it's Gary Kasparov). Call him player 1.

(2) Find someone that the best player beats 70-75% of the time. Call him player 2.

(3) Call the difference between players 1 and 2 ***one skill differential.***

(4) Find someone that player 2 can beat 70-75% of the time. Call him player 3. The difference between players 2 and 3 is another skill differential.

(5) Continue this process until you have taken the chain down to an absolute beginner.

(6) Count the number of skill differentials involved. This is the ***complexity number*** *of the game.*

In the case of chess, this number is about 14.

Robertie, 1980 *Inside Backgammon.* h/t Johan Ugander, Brendan Tracey

The Depths of Leagues

COMPLEXITY NUMBERS

Go	*40*
Chess	*14*
Scrabble	*10*
Poker	*10*
Backgammon	*8*
Checkers	*8*
Hearts	*5*
Blackjack	*2*
Craps	*0.001*
Lotteries	*0.0000001*
Roulette	*0*

Now consider the following experiment:

(1) Take the best player in the world (in the case of chess, it's Gary Kasparov). Call him player 1.

(2) Find someone that the best player beats 70-75% of the time. Call him player 2.

(3) Call the difference between players 1 and 2 ***one skill differential.***

(4) Find someone that player 2 can beat 70-75% of the time. Call him player 3. The difference between players 2 and 3 is another skill differential.

(5) Continue this process until you have taken the chain down to an absolute beginner.

(6) Count the number of skill differentials involved. This is the ***complexity number*** *of the game.*

In the case of chess, this number is about 14.

Robertie, 1980 *Inside Backgammon.* h/t Johan Ugander, Brendan Tracey

The Depths of Leagues

Now consider the following experiment:

(1) Take the best player in the world (in the case of chess, it's Gary Kasparov). Call him player 1.

(2) Find someone that the best player beats 70-75% of the time. Call him player 2.

(3) Call the difference between players 1 and 2 ***one skill differential.***

(4) Find someone that player 2 can beat 70-75% of the time. Call him player 3. The difference between players 2 and 3 is another skill differential.

(5) Continue this process until you have taken the chain down to an absolute beginner.

(6) Count the number of skill differentials involved. This is the ***complexity number*** *of the game.*

In the case of chess, this number is about 14.

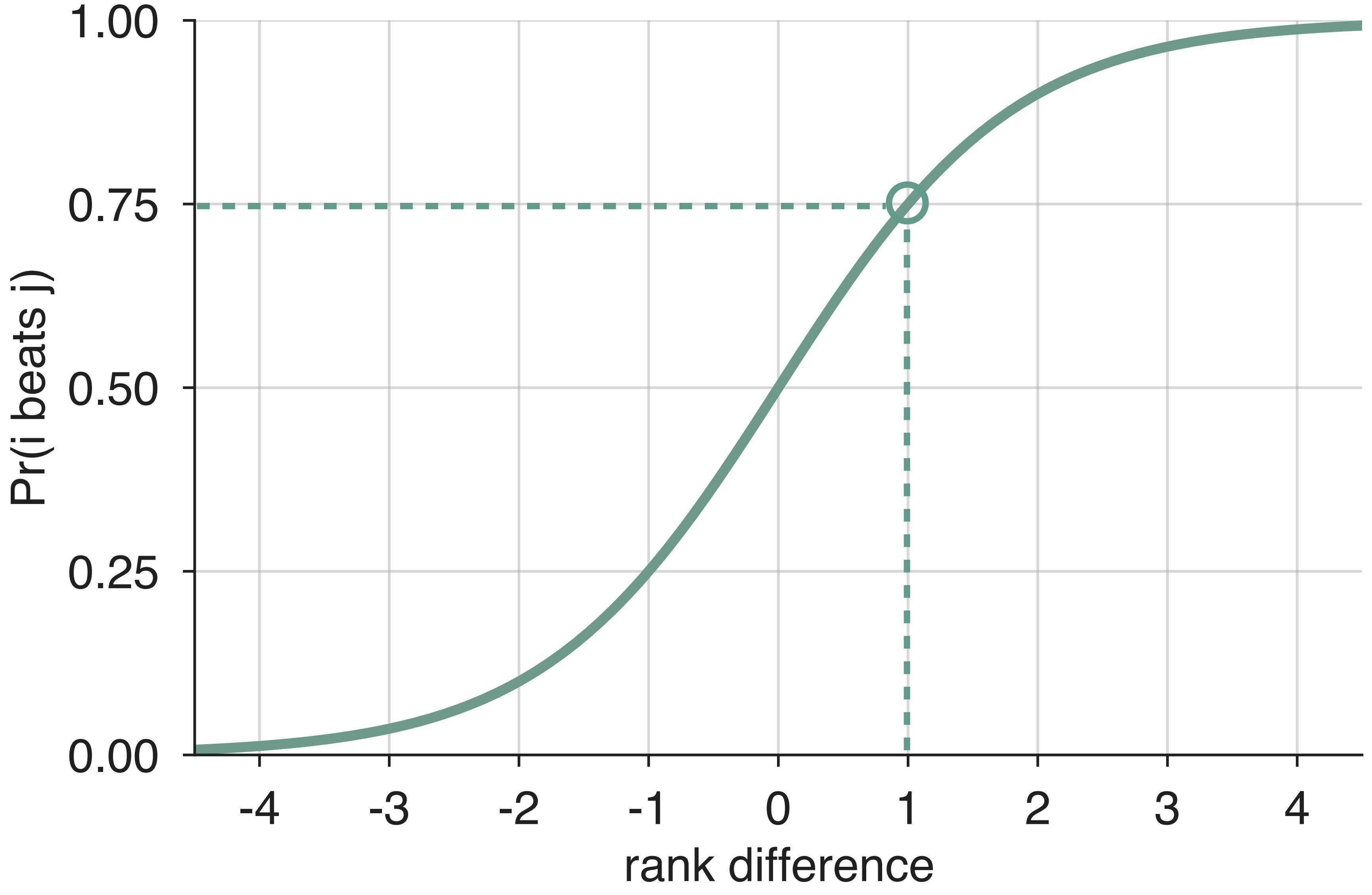
COMPLEXITY NUMBERS

Go	*40*
Chess	*14*
Scrabble	*10*
Poker	*10*
Backgammon	*8*
Checkers	*8*
Hearts	*5*
Blackjack	*2*
Craps	*0.001*
Lotteries	*0.0000001*
Roulette	*0*
Online Dating	**???**

Robertie, 1980 *Inside Backgammon.* h/t Johan Ugander, Brendan Tracey

Choosing a scale for interpretability

scaling that we want

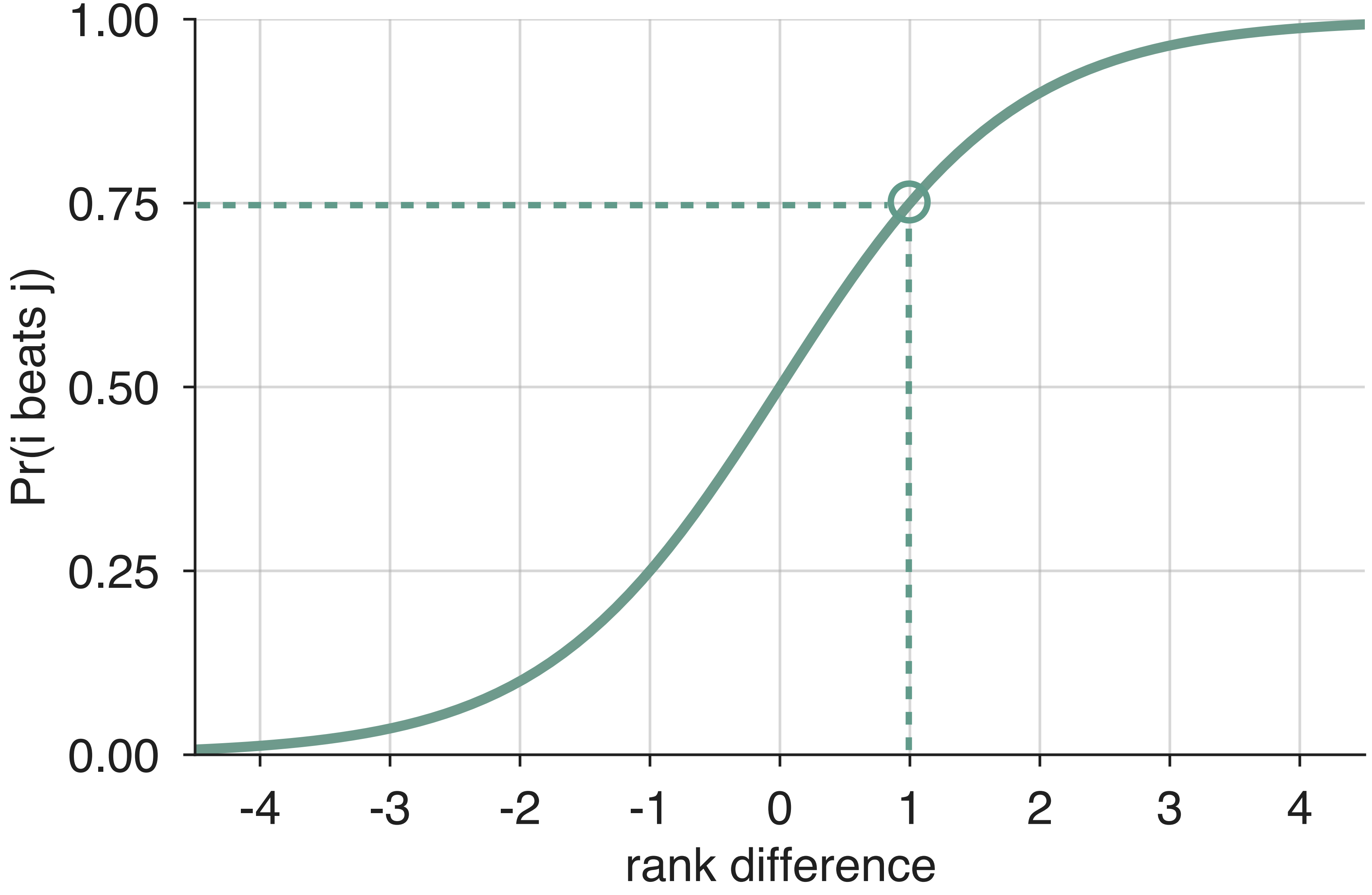


scaling that we have

$$P(i \to j \mid i \leftrightarrow j) = \frac{1}{1 + e^{-2\hat{\beta}(c_i - c_j)}}$$

Choosing a scale for interpretability

scaling that we want



scaling that we have

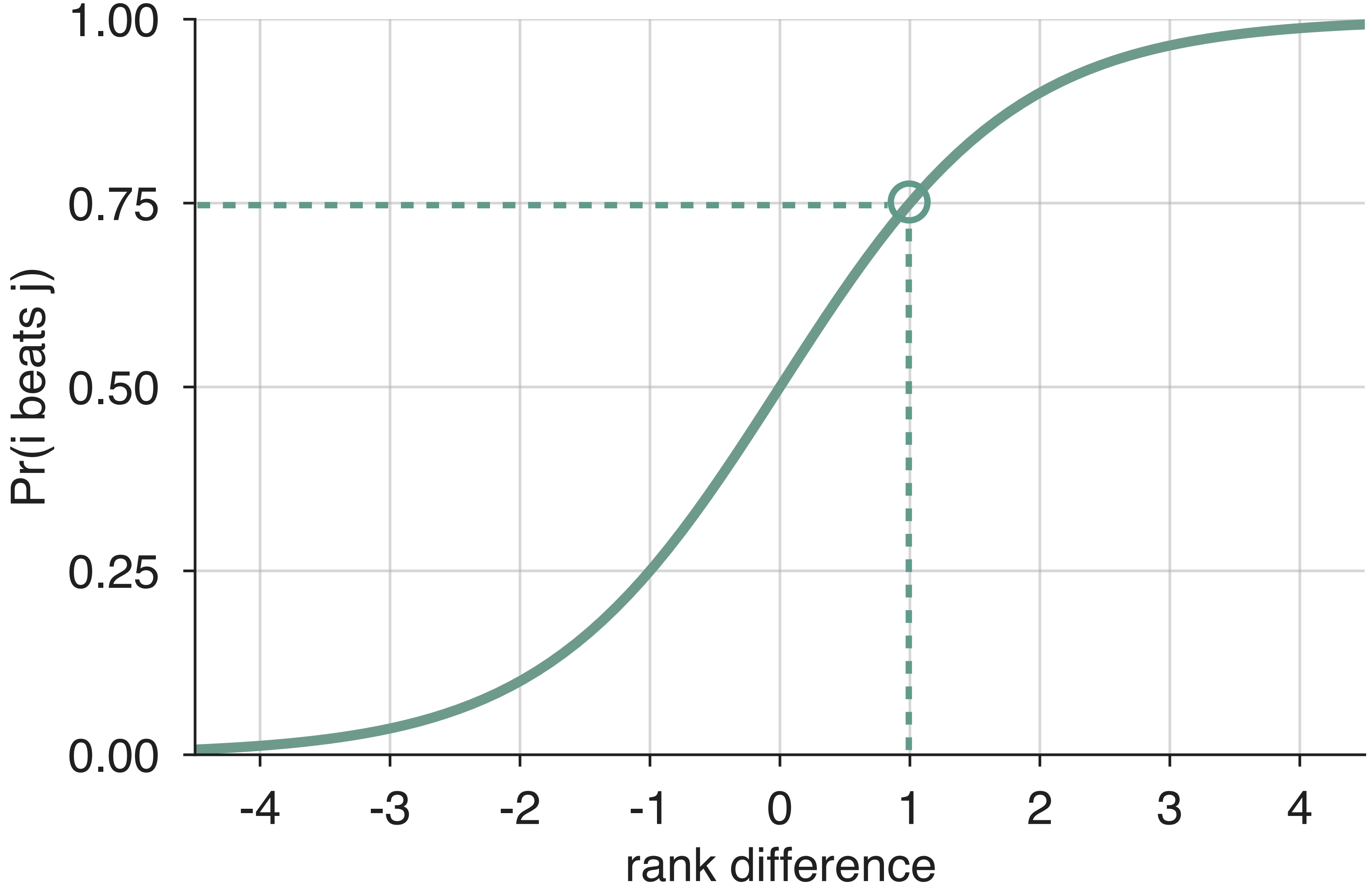
$$P(i \to j \mid i \leftrightarrow j) = \frac{1}{1 + e^{-2\hat{\beta}(c_i - c_j)}}$$

enforce the desired scale,
then solve for a rescaling constant *k*

let $c = k\bar{c}$

Choosing a scale for interpretability

scaling that we want



scaling that we have

$$P(i \rightarrow j \mid i \leftrightarrow j) = \frac{1}{1 + e^{-2\hat{\beta}(c_i - c_j)}}$$

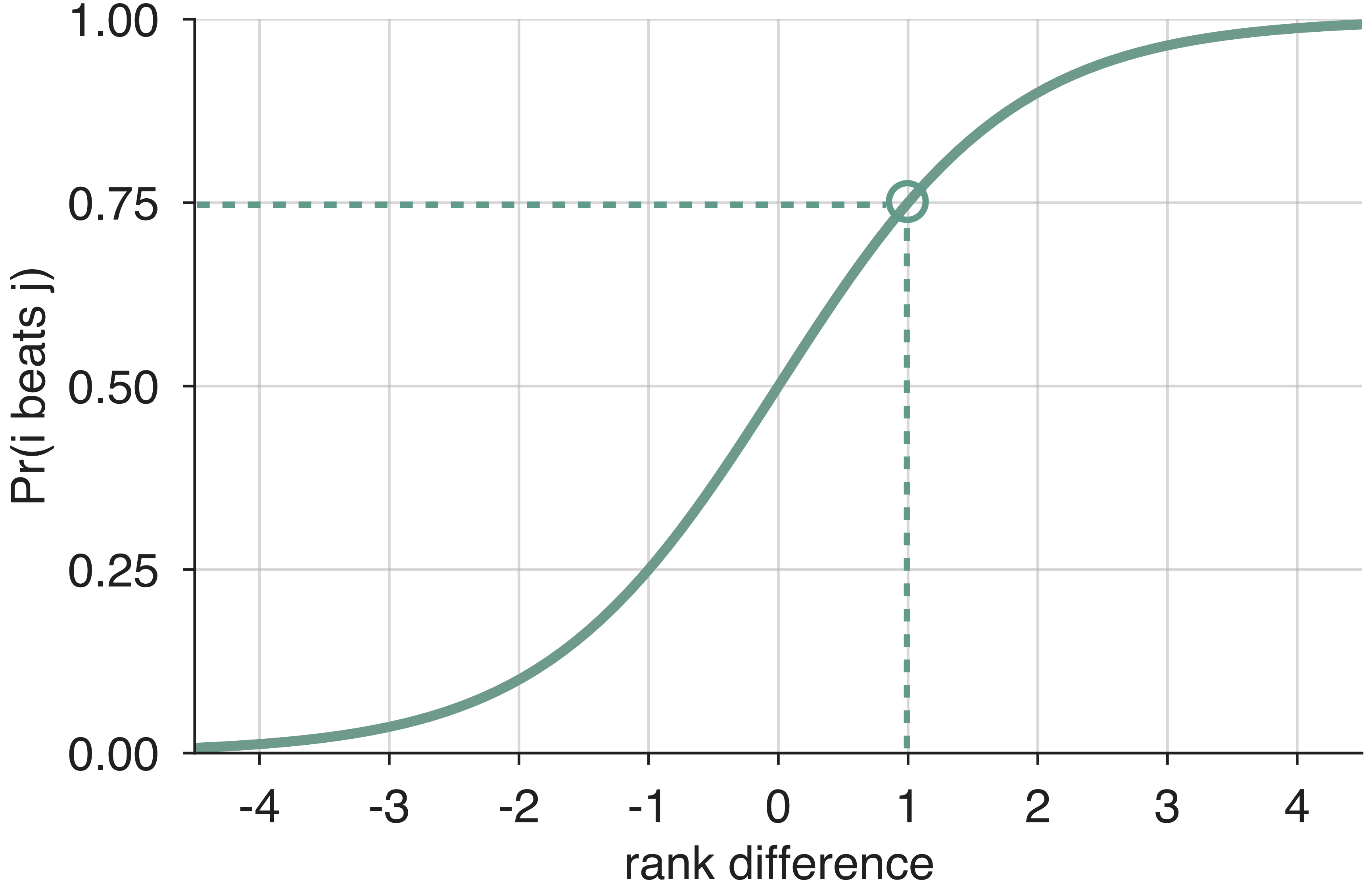
enforce the desired scale,
then solve for a rescaling constant *k*

let $c = k\bar{c}$

$$\bar{c}_i - \bar{c}_j = 1, \quad 0.75 = \frac{1}{1 + e^{-2\hat{\beta}(k\bar{c}_i - k\bar{c}_j)}}$$

Choosing a scale for interpretability

scaling that we want



scaling that we have

$$P(i \to j \mid i \leftrightarrow j) = \frac{1}{1 + e^{-2\hat{\beta}(c_i - c_j)}}$$

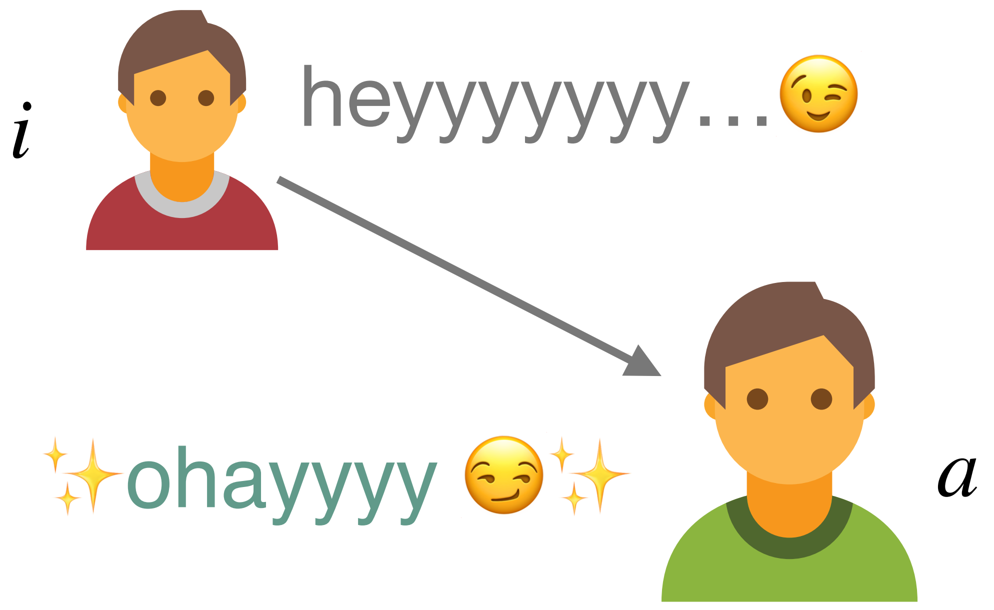
enforce the desired scale,
then solve for a rescaling constant *k*

let $c = k\bar{c}$

$$\underline{\bar{c}_i - \bar{c}_j = 1}, \quad \underline{0.75} = \frac{1}{1 + e^{-2\hat{\beta}(k\bar{c}_i - k\bar{c}_j)}}$$

$$k = \frac{\text{log odds } 0.75}{2\hat{\beta}}$$

Reviewing the approach.



1 $M^{(1)}$: network layer of first messages

$M^{(R)}$: network layer of replies

2 $$C_{ij} = \sum_a M^{(1)}_{ia} M^{(1)}_{ja} M^{(R)}_{ai} \left(1 - M^{(R)}_{aj}\right)$$

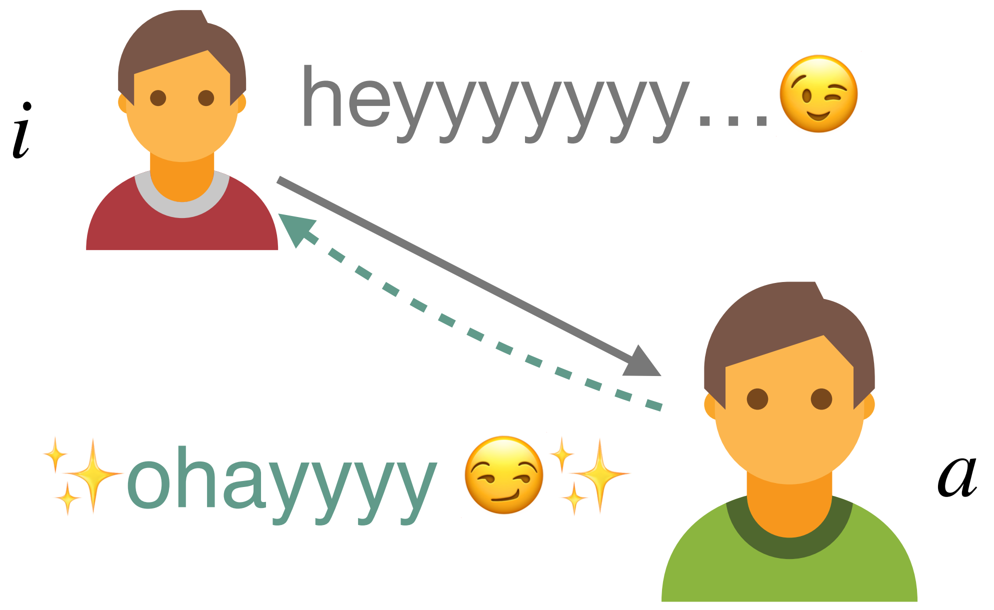
3 $$H(c) = \frac{1}{2} \sum_{ij} C_{ij} \left(c_i - c_j - 1\right)^2$$

4 $$\bar{c} = c \frac{\text{log odds } 0.75}{2\hat{\beta}}$$

Result:

An embedding of individuals in a linear hierarchy, such that a one-unit difference predicts a 75% “win rate” in the competition for attention.

Reviewing the approach.



1 $M^{(1)}$: network layer of first messages

$M^{(R)}$: network layer of replies

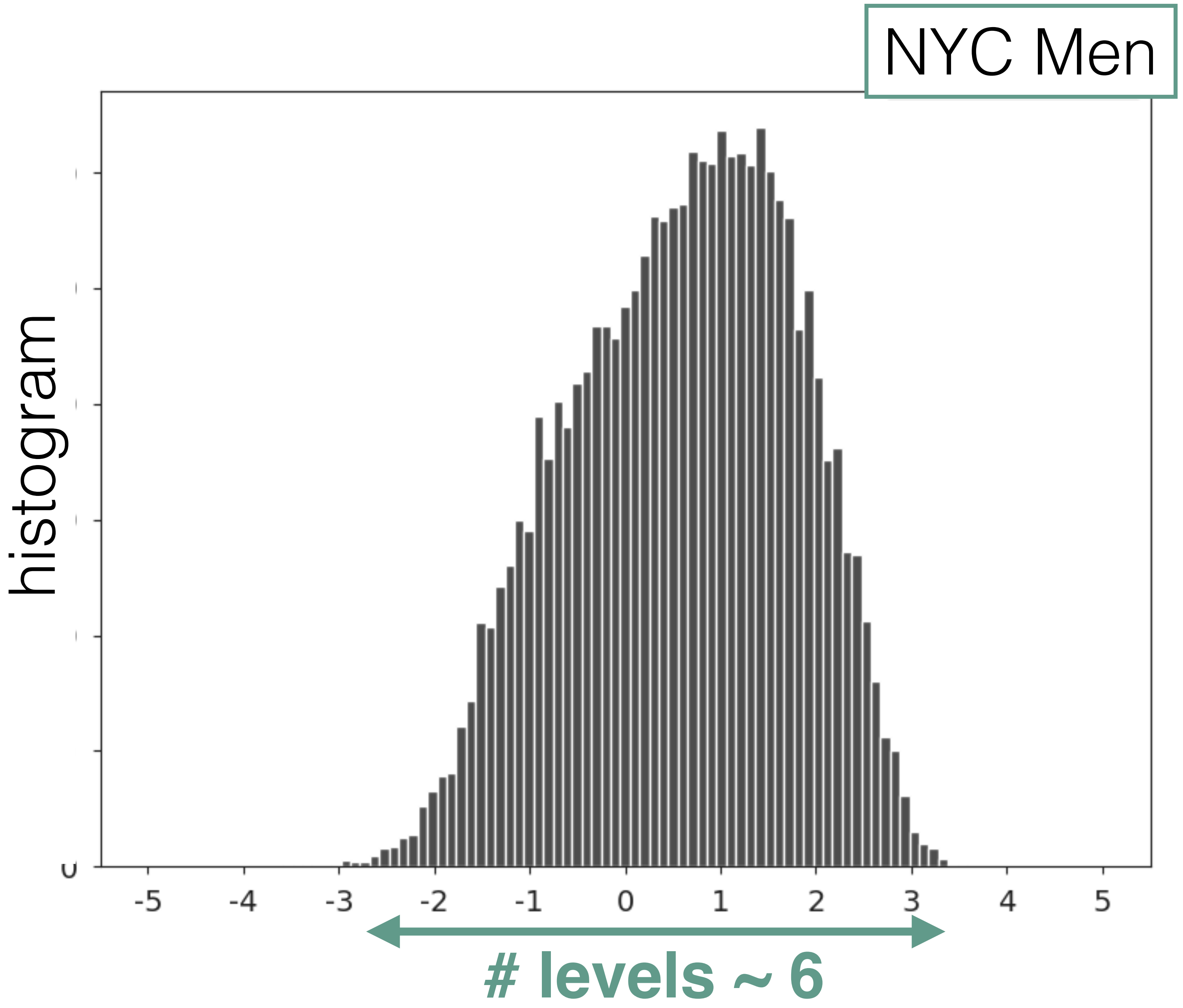
2 $$C_{ij} = \sum_a M_{ia}^{(1)} M_{ja}^{(1)} M_{ai}^{(R)} \left(1 - M_{aj}^{(R)}\right)$$

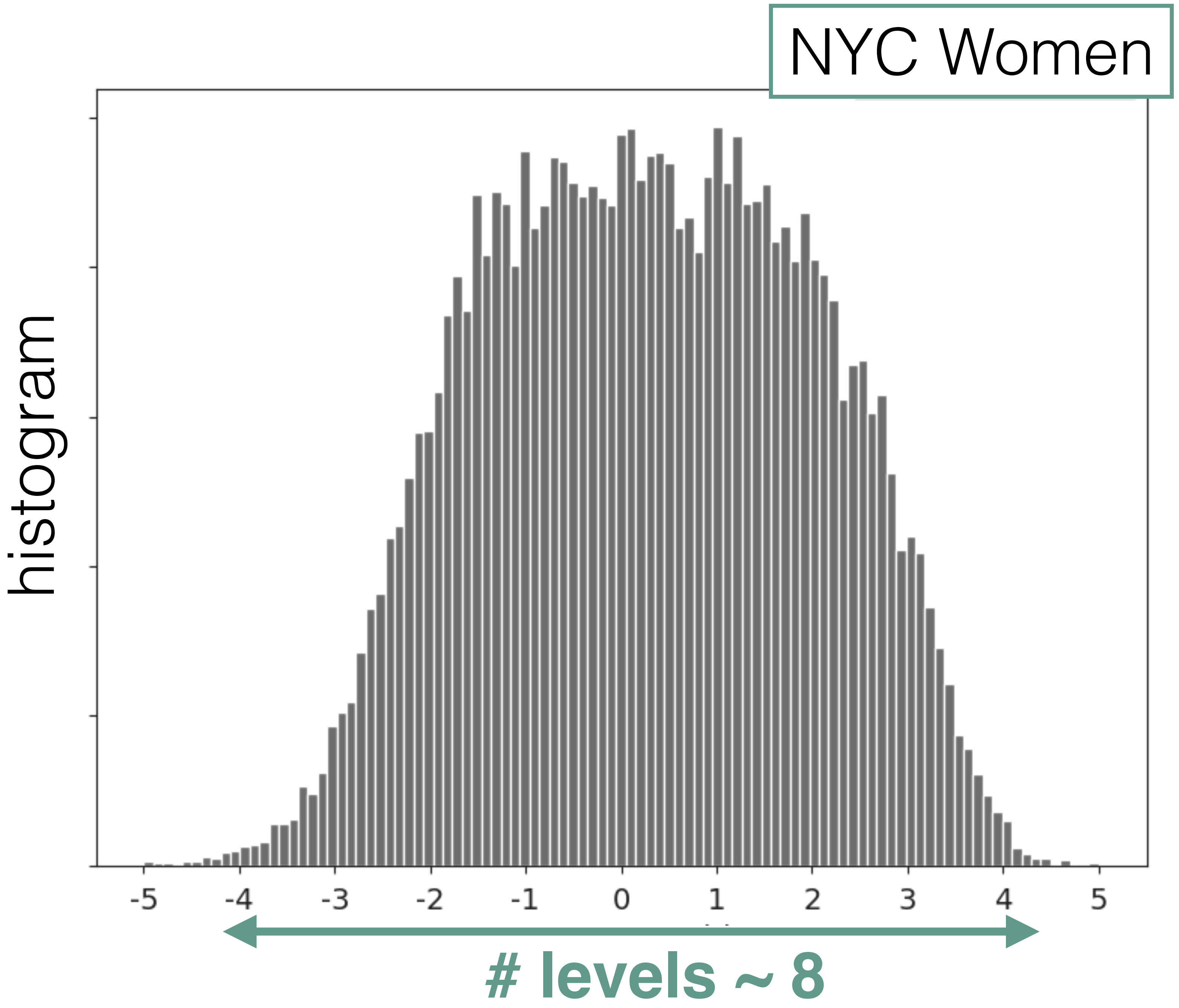
3 $$H(c) = \frac{1}{2} \sum_{ij} C_{ij} \left(c_i - c_j - 1\right)^2$$

4 $$\bar{c} = c \frac{\text{log odds } 0.75}{2\hat{\beta}}$$

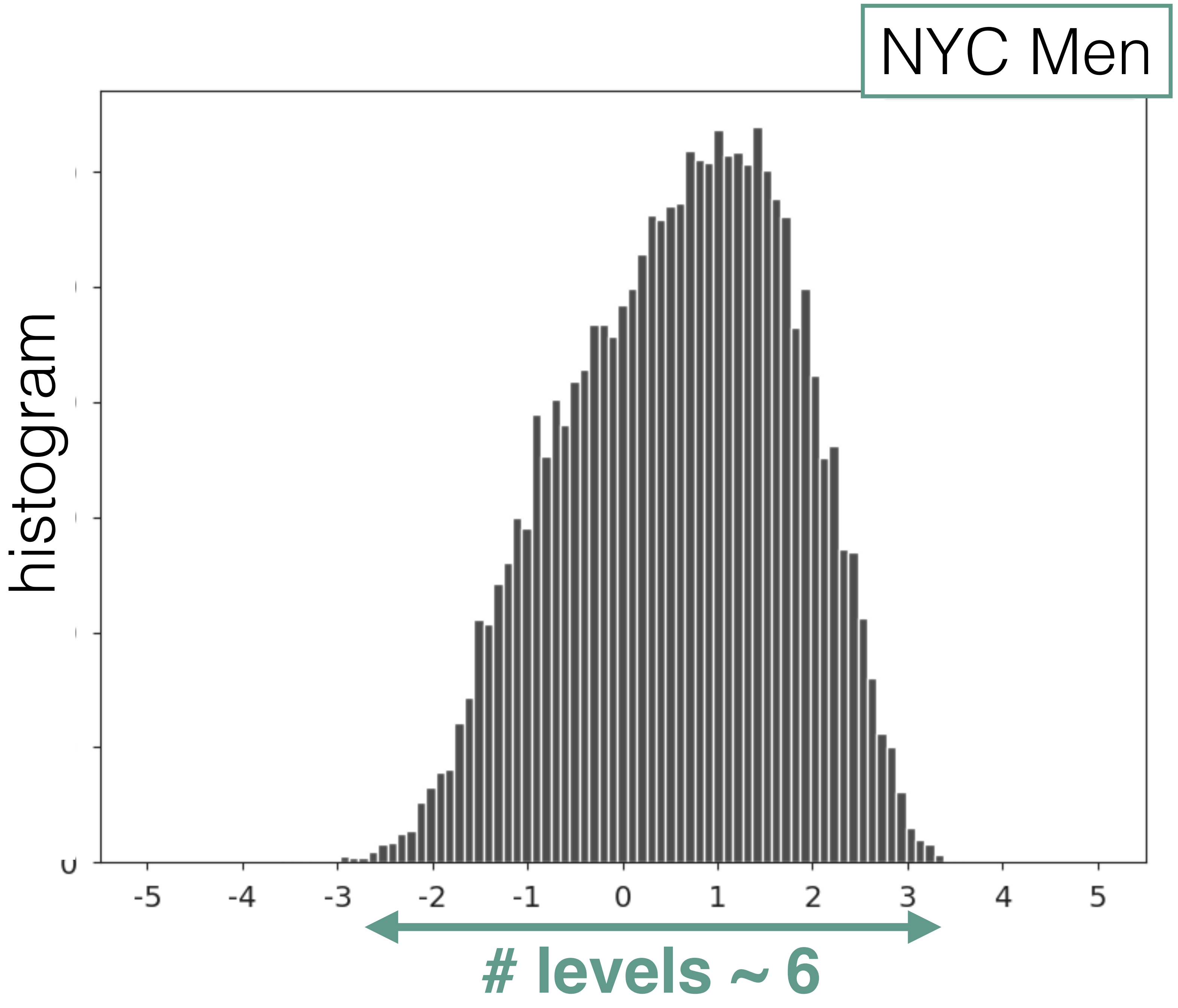
Result:
An embedding of individuals in a linear hierarchy, such that a one-unit difference predicts a 75% “win rate” in the competition for attention.

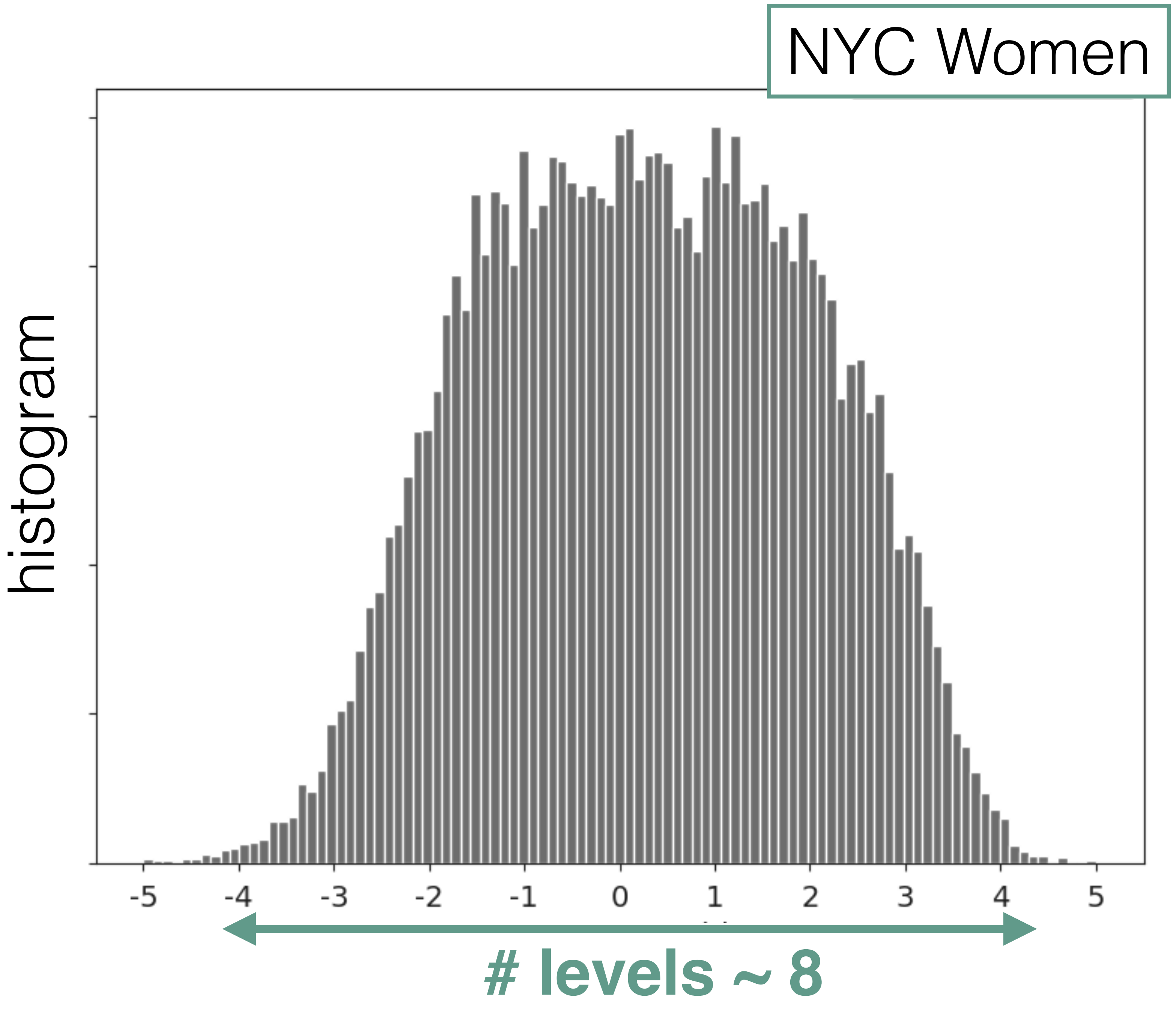
Real data: New York City rank distributions



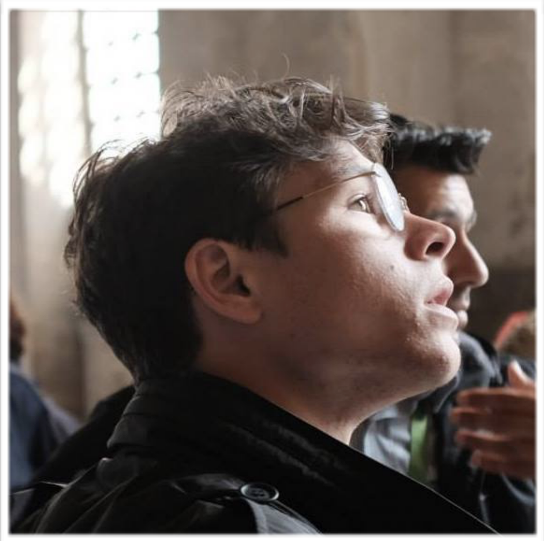


Real data: New York City rank distributions





- Those who had 100% or 0% reply rates not shown.
- NYC Women's competition space has a deeper "strategic complexity" than NYC Men's.



Elizabeth Bruch   Swapnil Gavade   K. Hunter Wapman

[empirical analyses]   [simulations]

UNIVERSITY OF MICHIGAN   University of Colorado **Boulder**

SCIENCE ADVANCES | RESEARCH ARTICLE

SOCIAL SCIENCES

Aspirational pursuit of mates in online dating markets

Elizabeth E. Bruch[1,2]* and M. E. J. Newman[2,3]

Recommended reading if you are interested in this subject!


Hierarchy and cognition

What are the mechanisms that create large-scale patterns from many small interactions?

Hobson and DeDeo. *PLoS Computational Biology* (2015)

What are the mechanisms that create large-scale patterns from many small interactions?

Confront models with data to reveal cognitively-accessible social mechanisms in parakeets.

Hobson and DeDeo. *PLoS Computational Biology* (2015)

1

2

3

elapsed time < 1 second

Hobson and DeDeo. *PLoS Computational Biology* (2015)

1

2

3

aggressor (winner)

target (loser)

elapsed time < 1 second

Hobson and DeDeo. *PLoS Computational Biology* (2015)

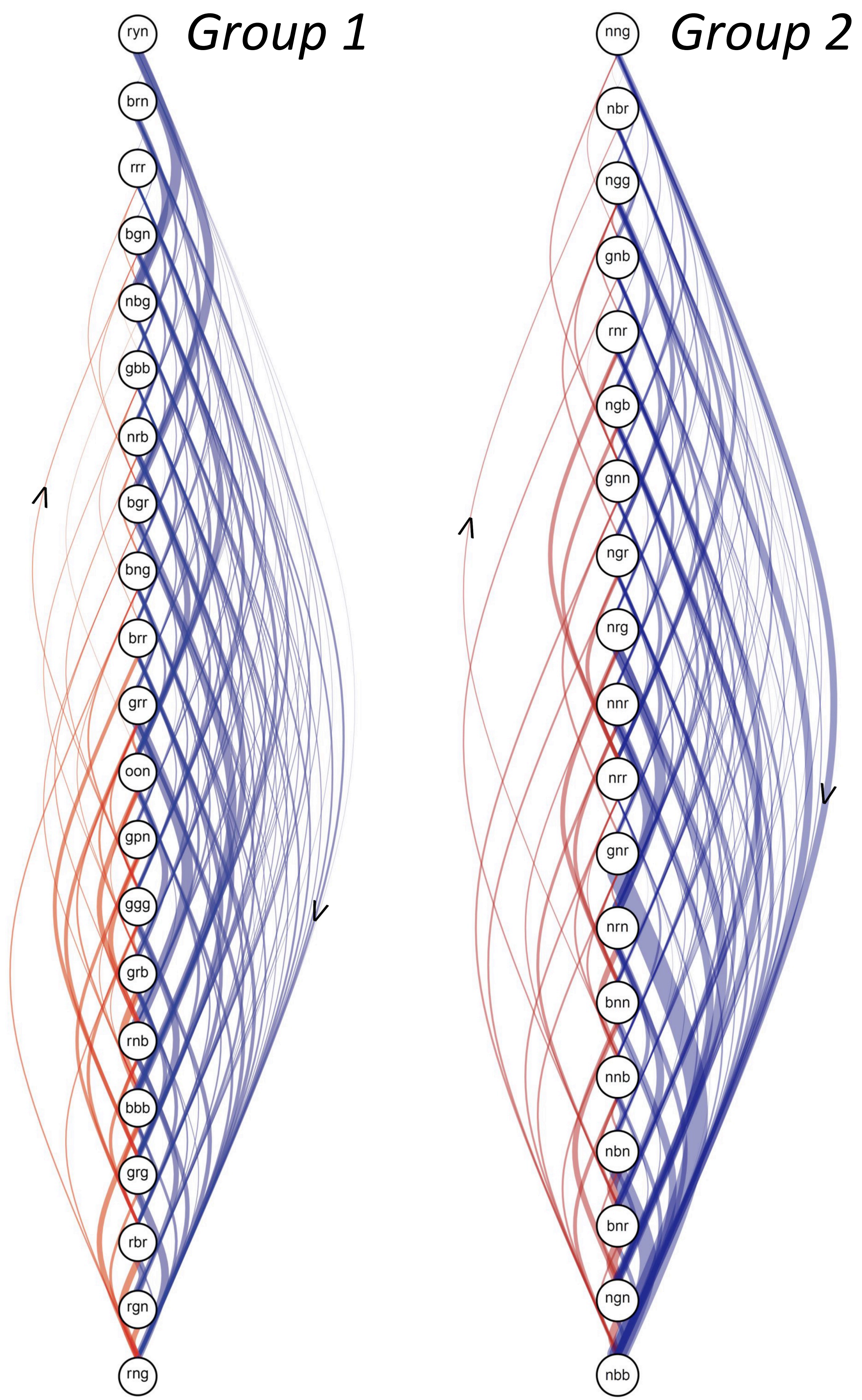
aggressor (winner)

target (loser)

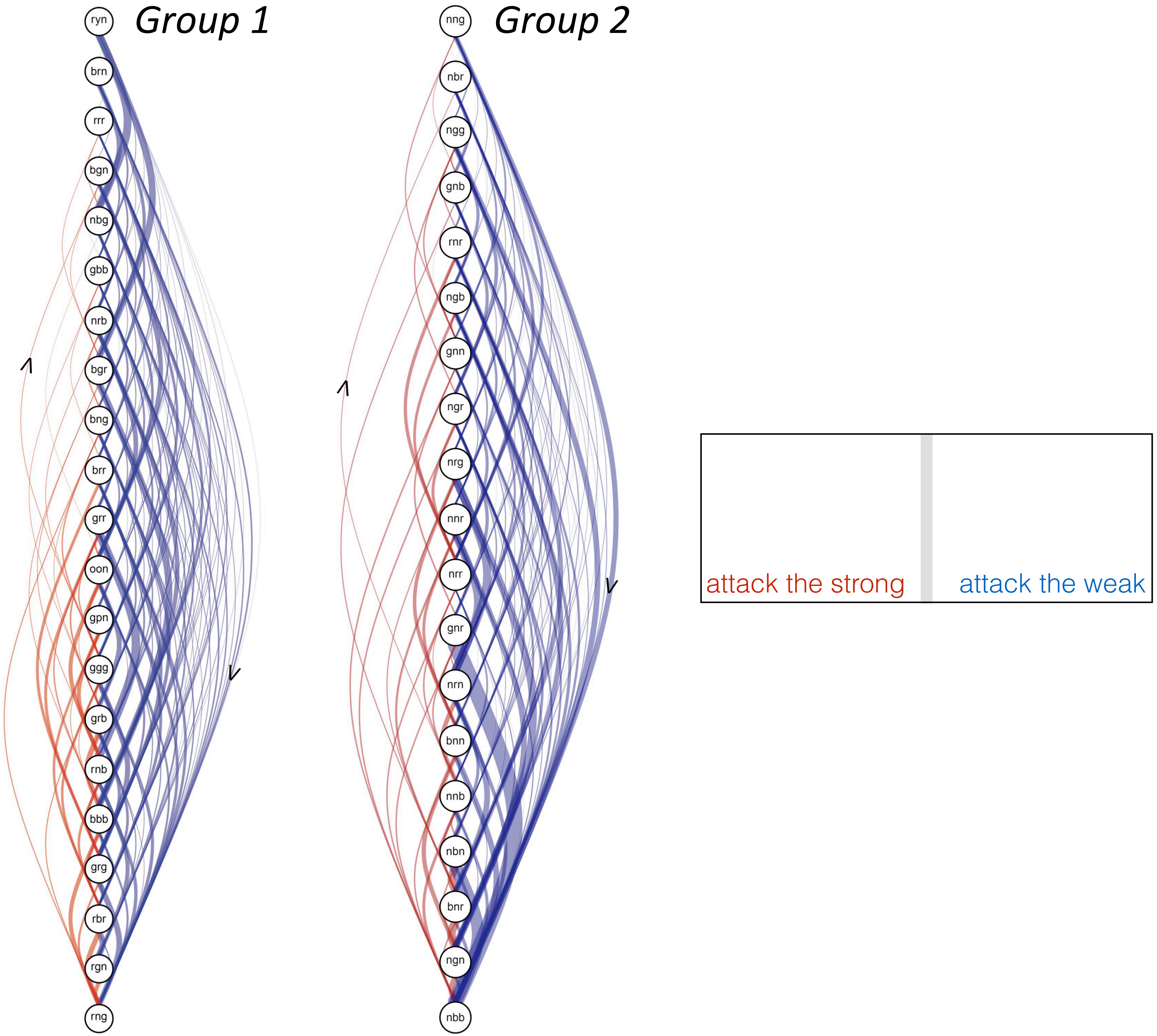
gawking, staring, shameless witnesses!

elapsed time < 1 second

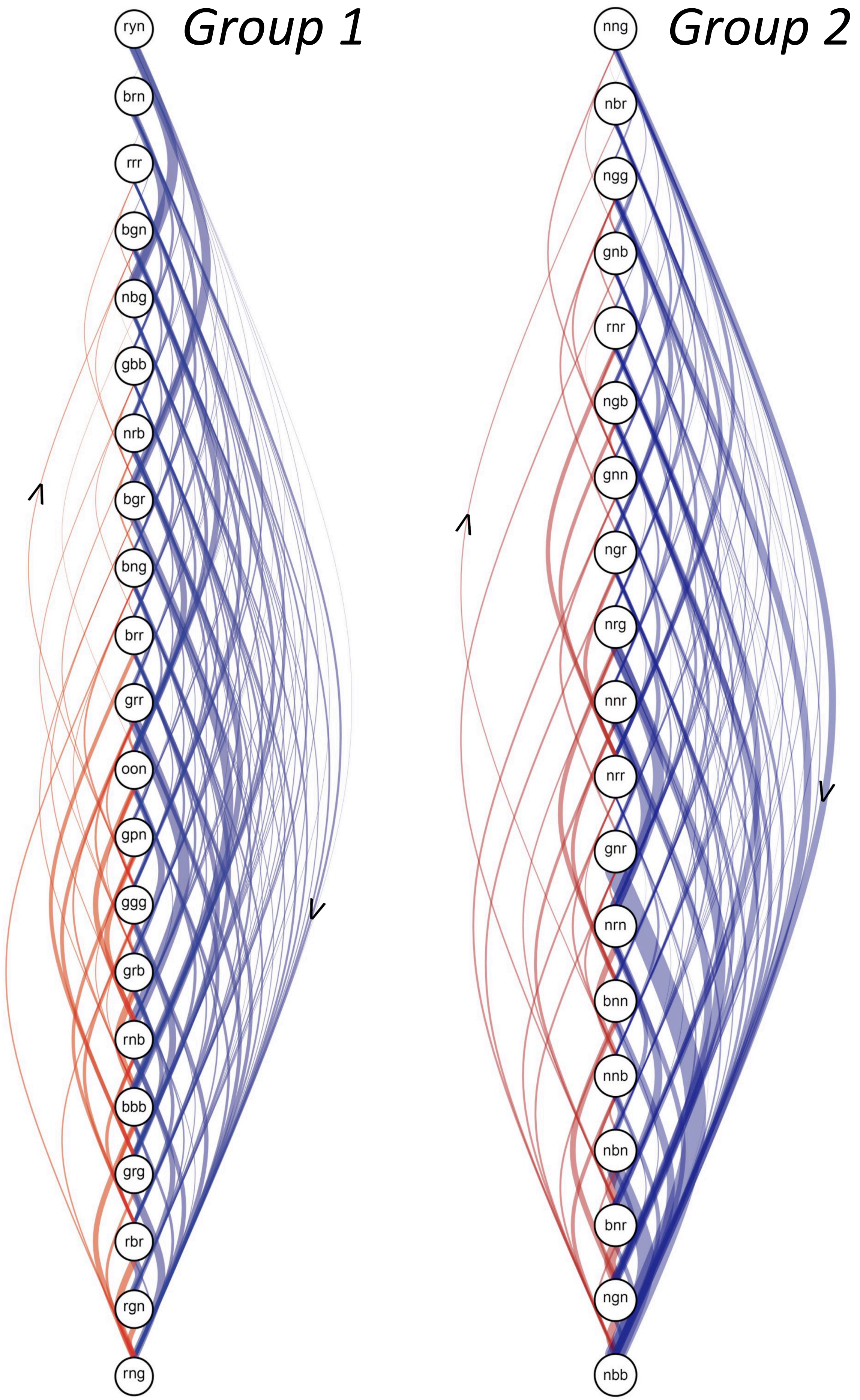
Hobson and DeDeo. *PLoS Computational Biology* (2015)

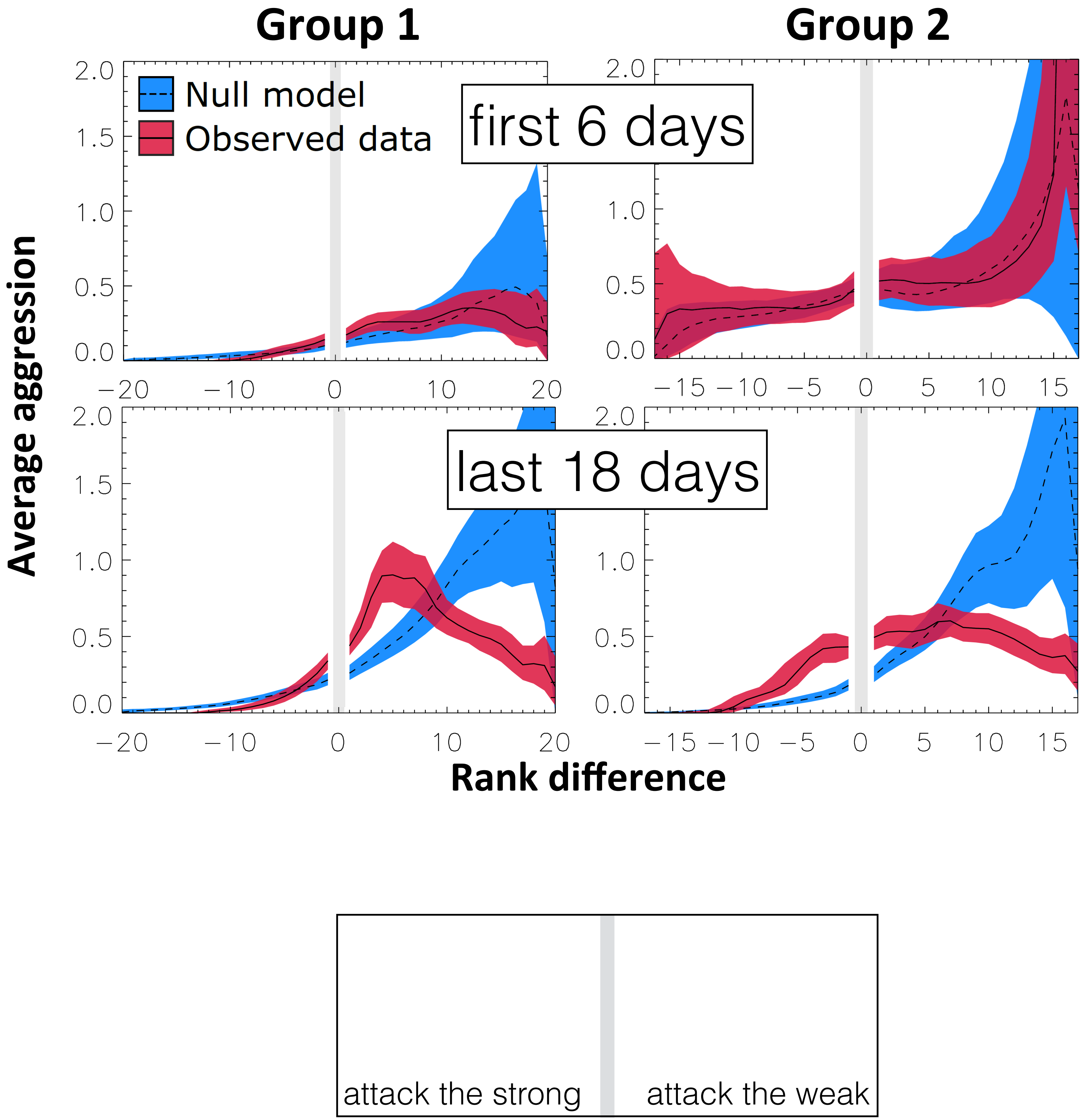


Hobson and DeDeo. *PLoS Computational Biology* (2015)



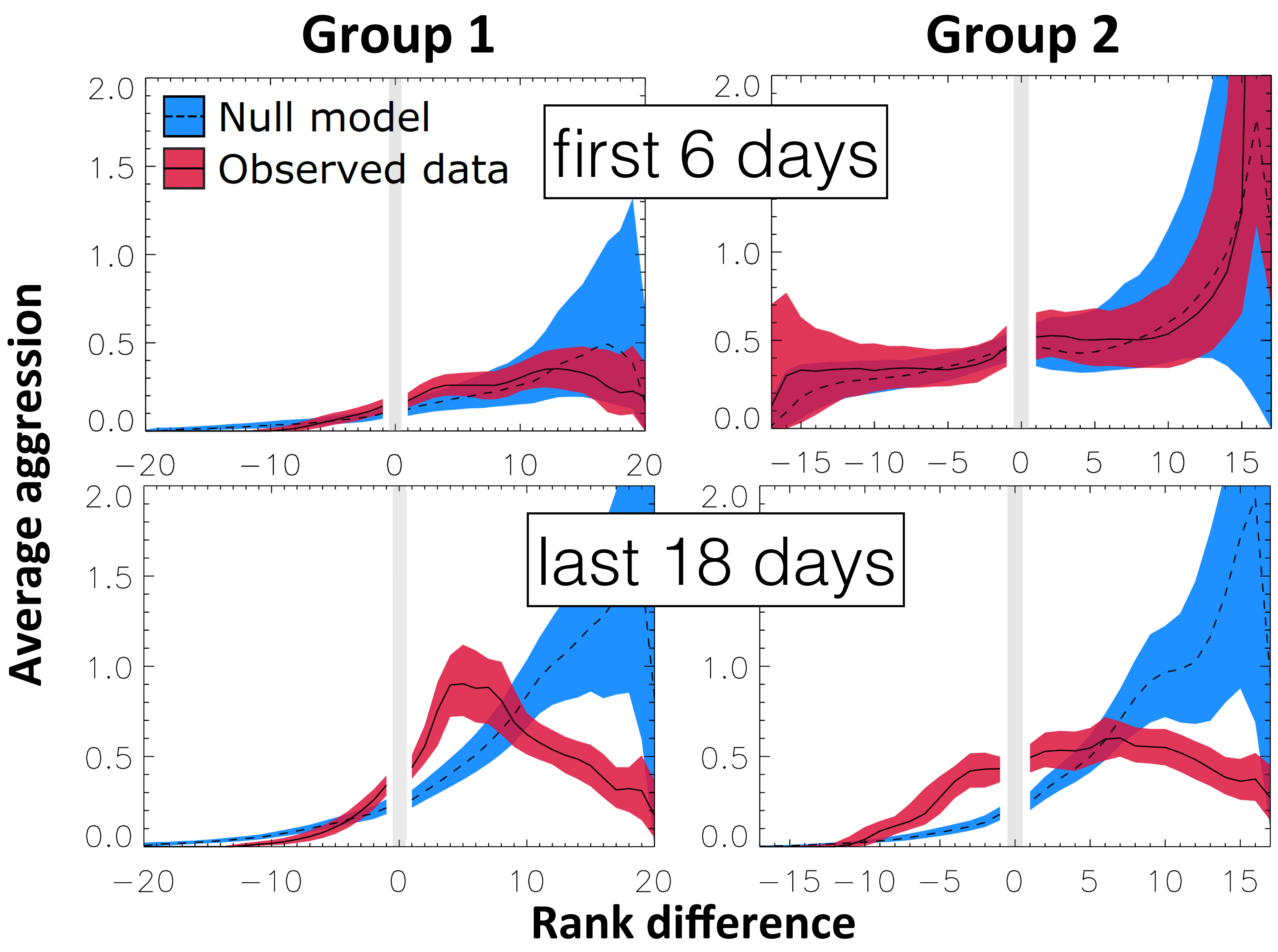
Hobson and DeDeo. *PLoS Computational Biology* (2015)


Group 1
Group 2
ryn brn rrr bgn nbg gbb nrb bgr bng brr grr oon gpn ggg grb rnb bbb grg rbr rgn rng
nng nbr ngg gnb rnr ngb gnn ngr nrg nnr nrr gnr nrn bnn nnb nbn bnr ngn nbb


Group 1
Group 2
Null model
Observed data
first 6 days
last 18 days
Average aggression
Rank difference
attack the strong
attack the weak

Parakeets know which individuals are ranked above and below themselves.

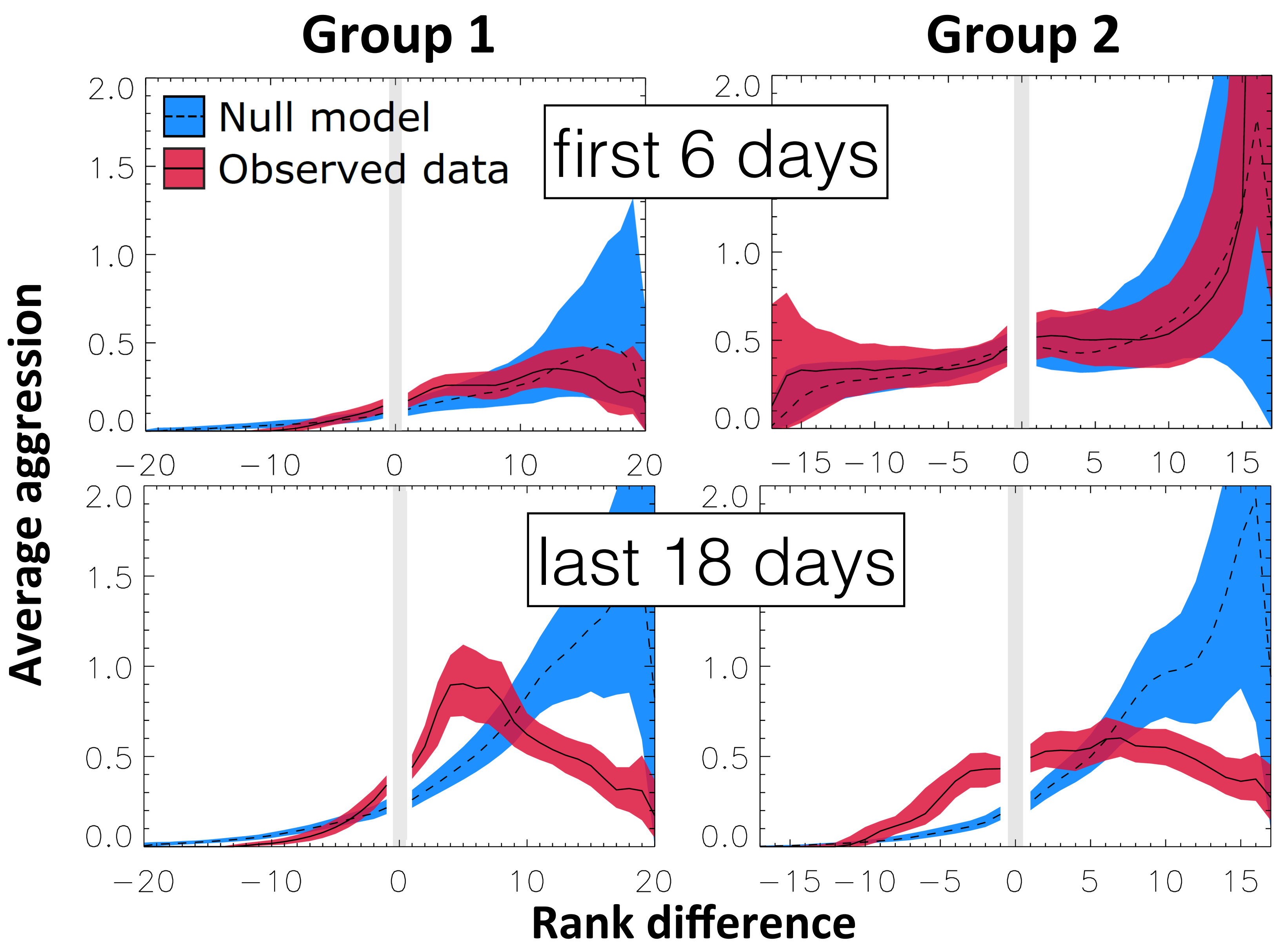
Parakeets know their own rank and the ranks of others.



Hobson and DeDeo. *PLoS Computational Biology* (2015)

Parakeets know which individuals are ranked above and below themselves.

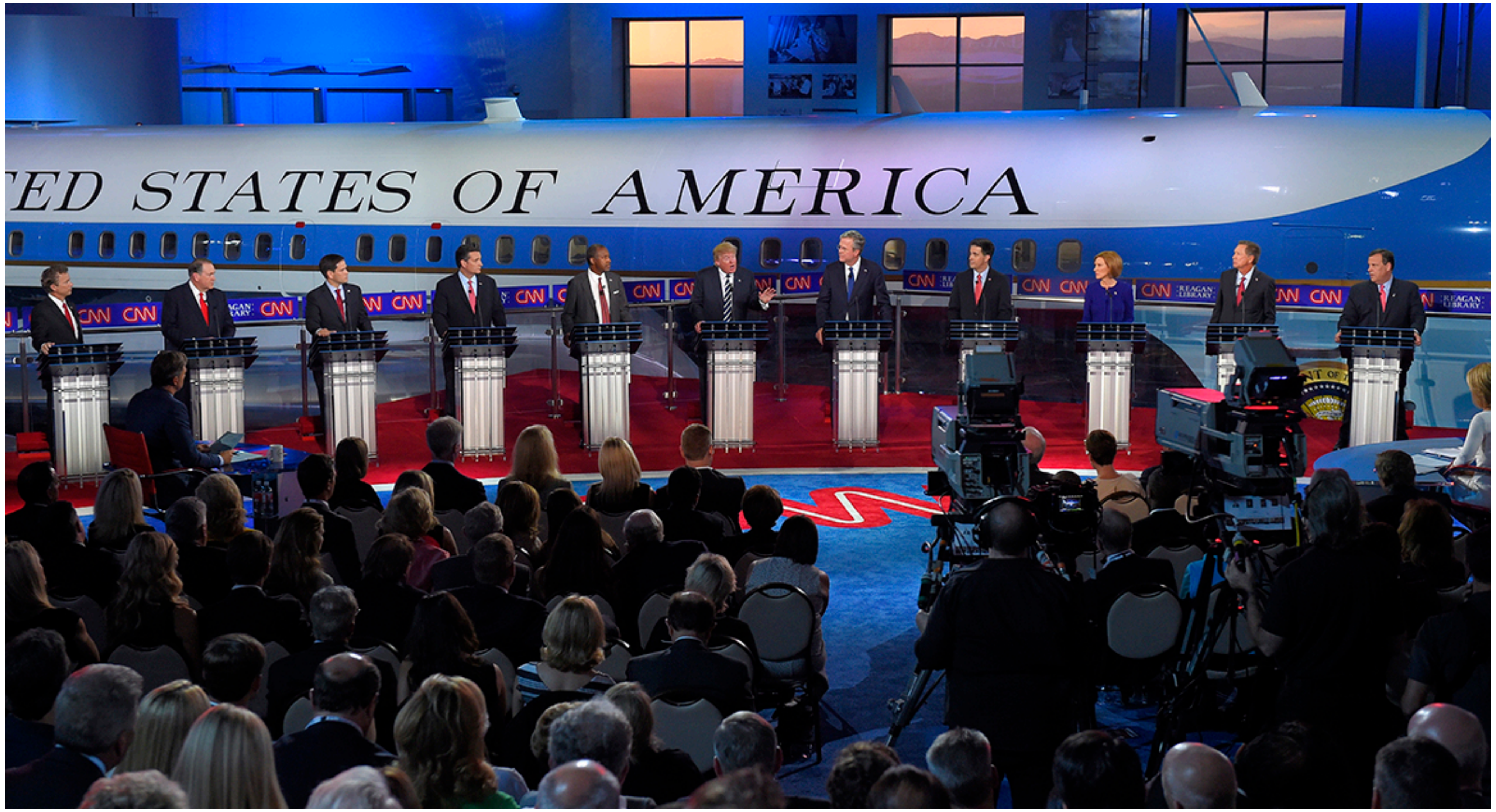
Parakeets know their own rank and the ranks of others.



Confronting models, which incorporate different complexities of bird-knowledge, with meticulous data, reveals clues about mechanisms of hierarchy formation.

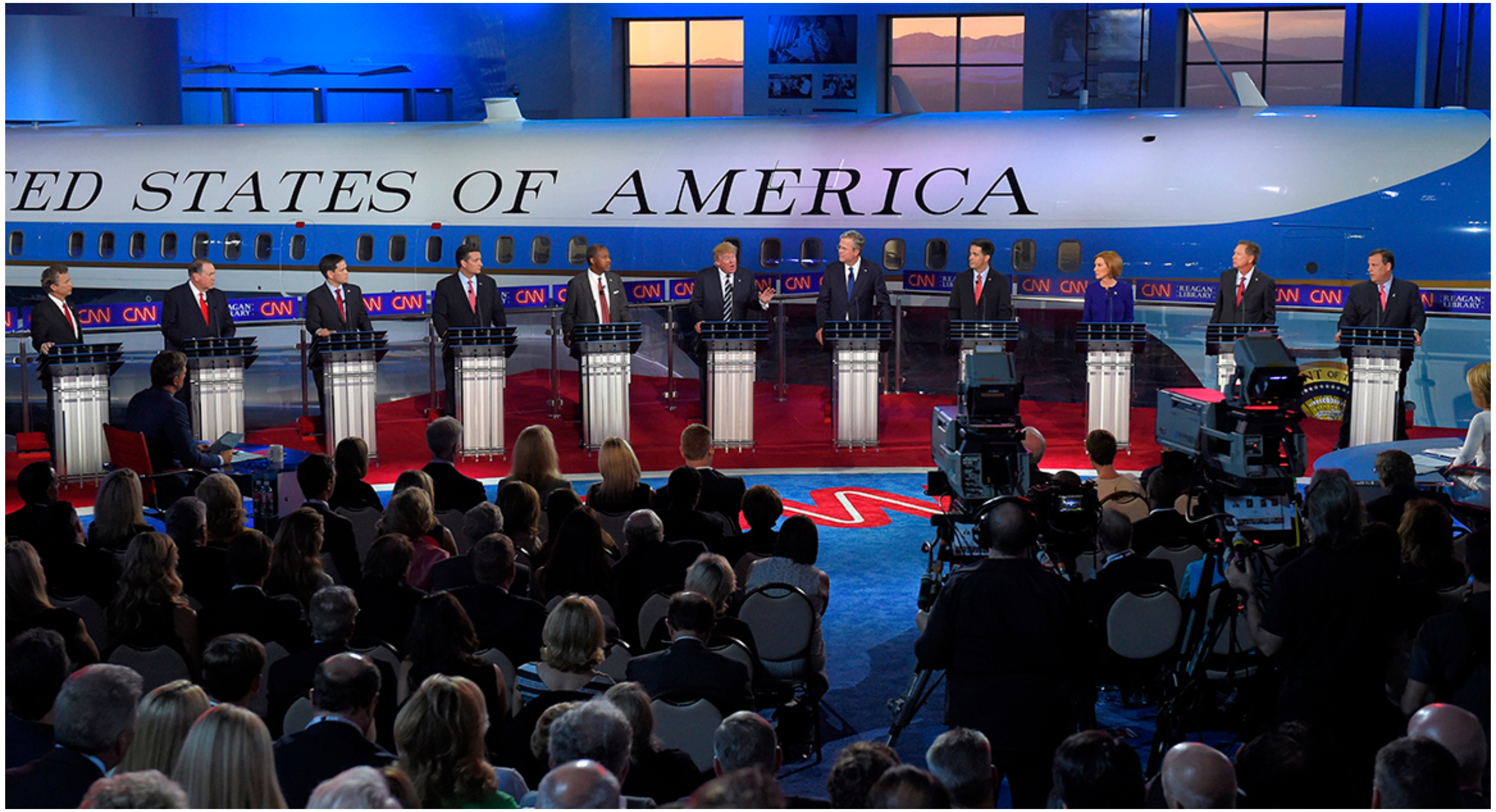
Hobson and DeDeo. *PLoS Computational Biology* (2015)

Complex models reveal complex behaviors



Hobson and DeDeo. *PLoS Computational Biology* (2015)

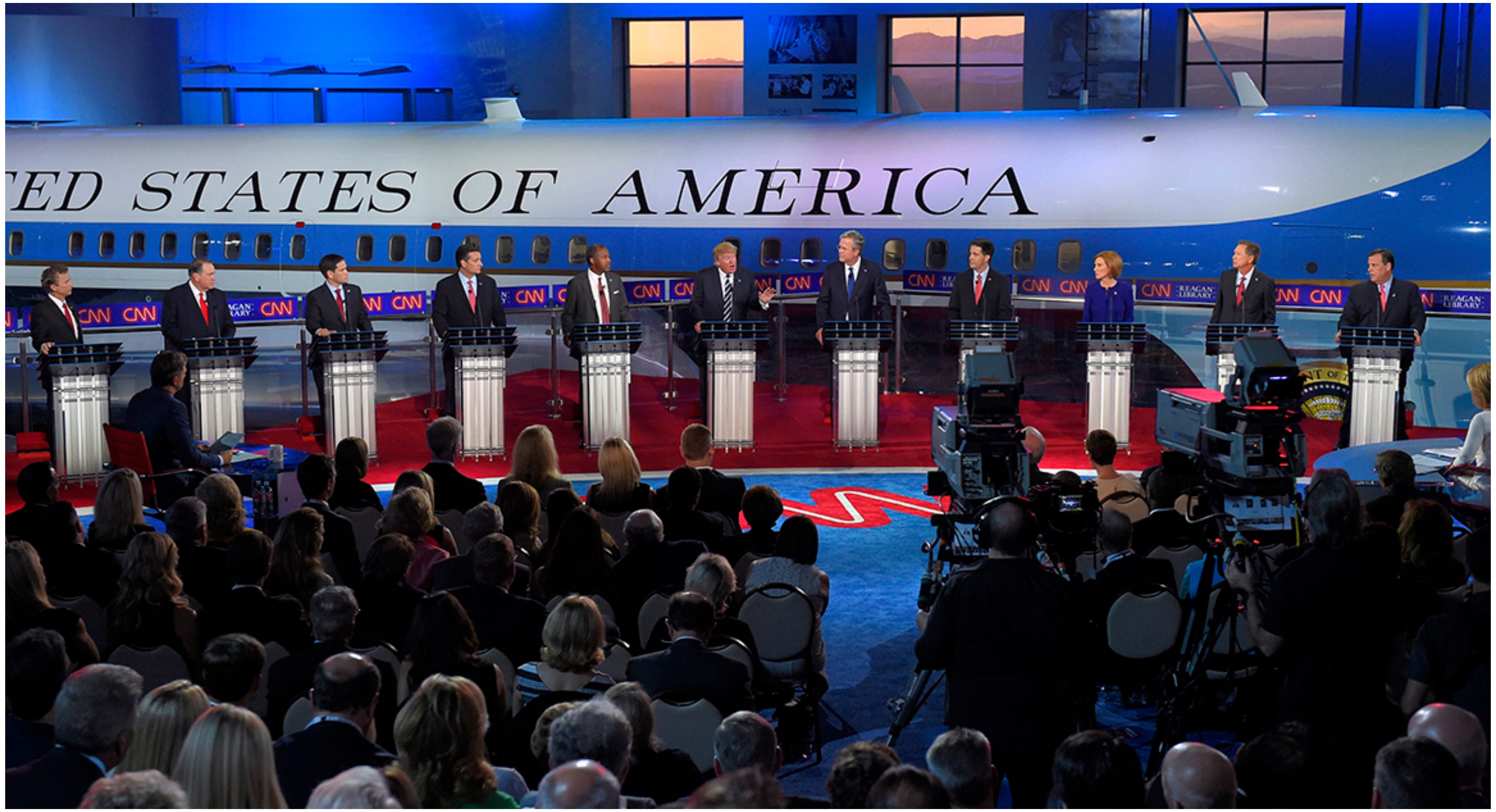
Complex models reveal complex behaviors



Pile-on Target the most recent loser. [kick 'em while they're down]

Hobson and DeDeo. *PLoS Computational Biology* (2015)

Complex models reveal complex behaviors

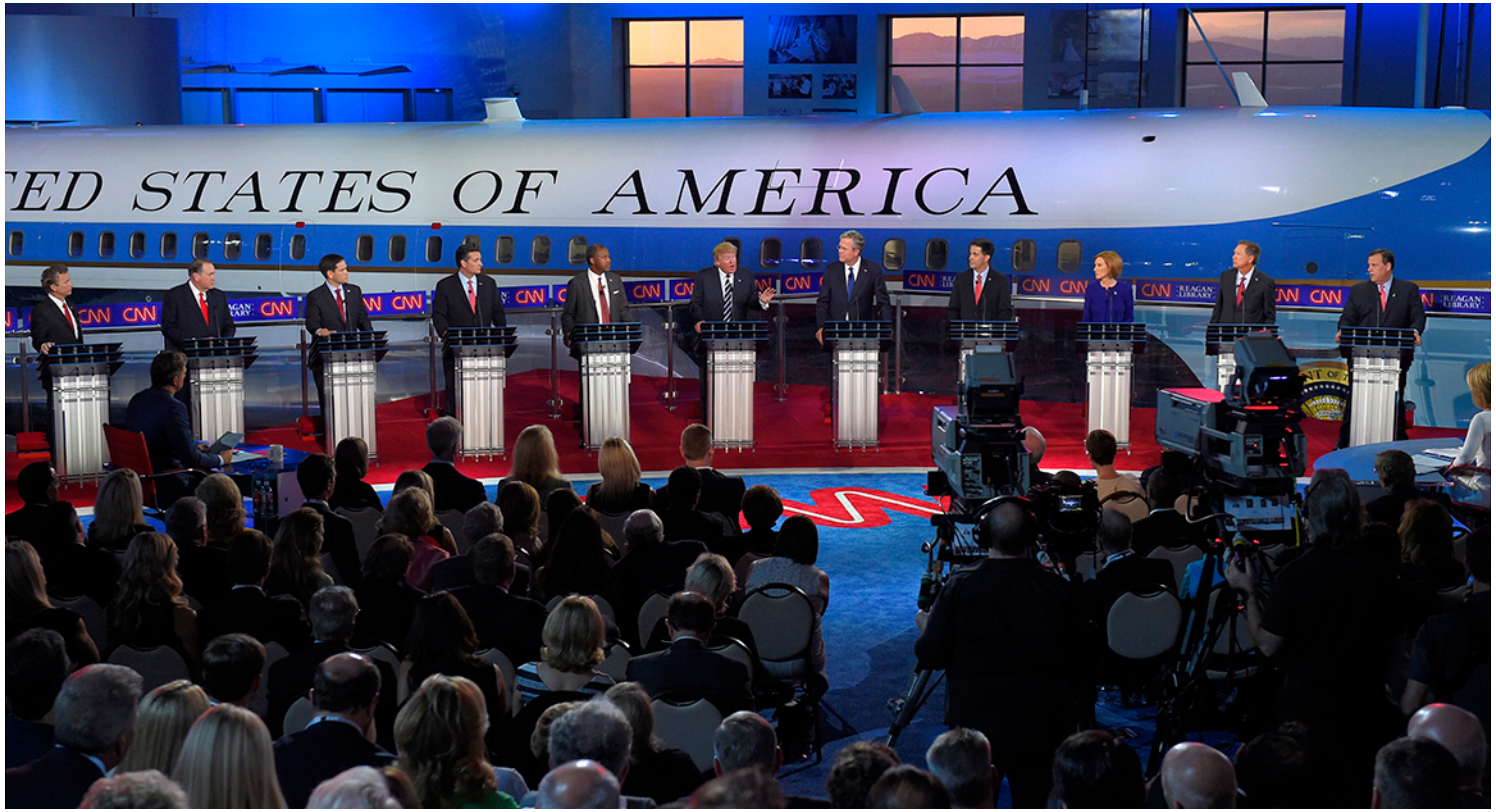


Pile-on Target the most recent loser. [kick 'em while they're down]

Pass-along Target lower-ranked after losing. [hurt people hurt people]

Hobson and DeDeo. *PLoS Computational Biology* (2015)

Complex models reveal complex behaviors



Pile-on Target the most recent loser. [kick 'em while they're down]

Pass-along Target lower-ranked after losing. [hurt people hurt people]

Opportunism Target a recent loser of higher rank. [now's my chance!]

Hobson and DeDeo. *PLoS Computational Biology* (2015)

Complex models reveal complex behaviors

Pile-on Target the most recent loser. [kick ‘em while they’re down]

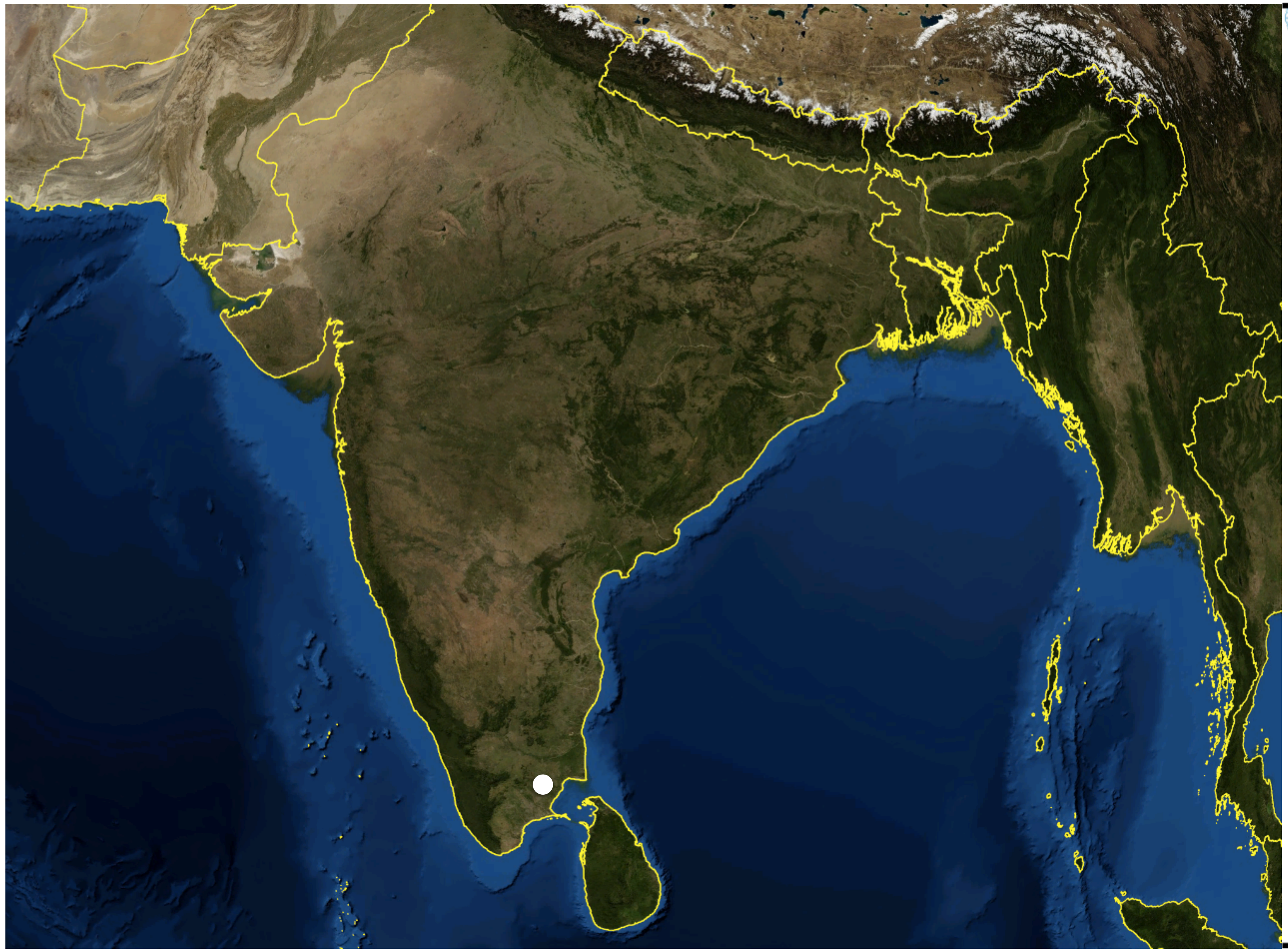
Pass-along Target lower-ranked after losing. [hurt people hurt people]

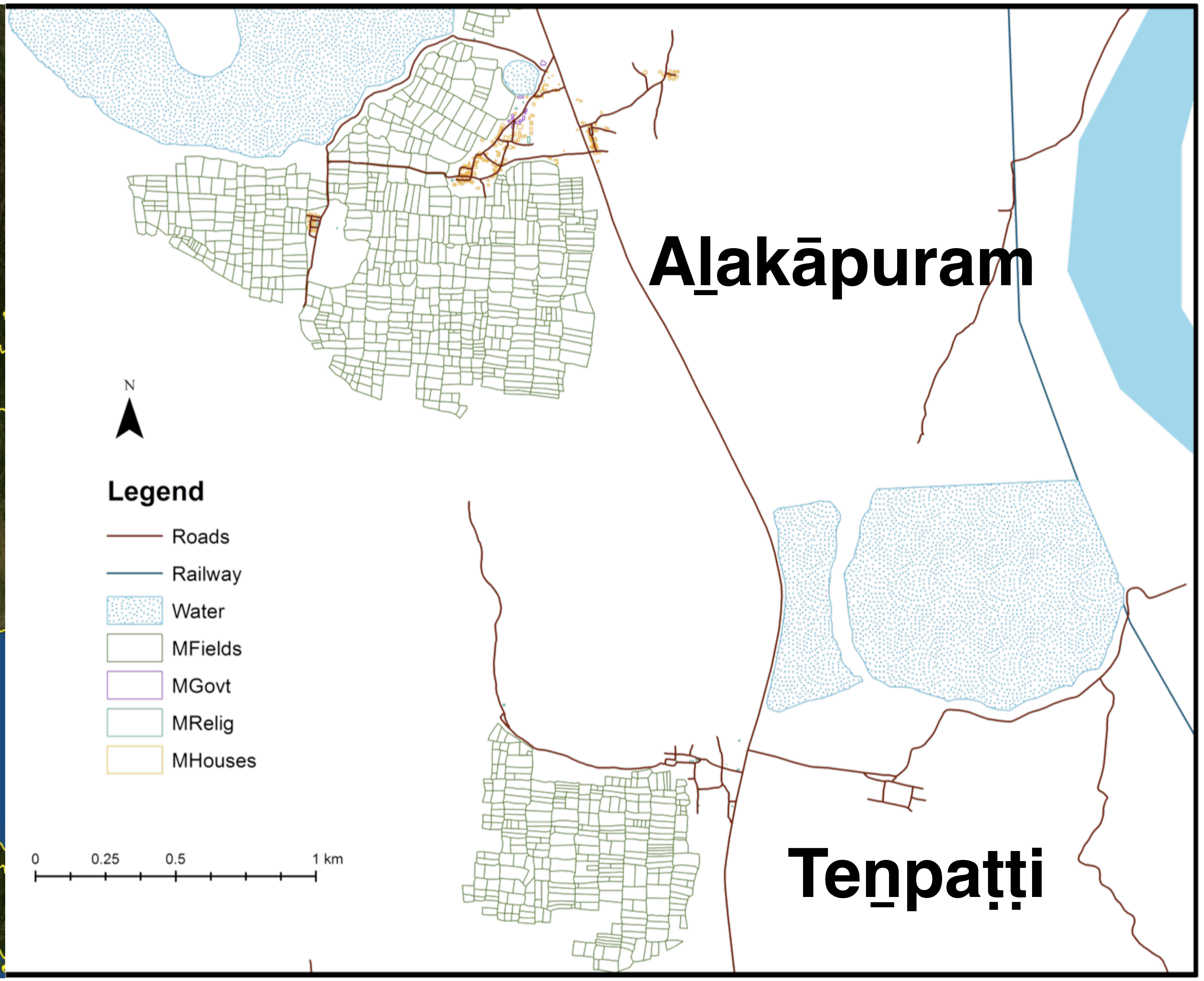
Opportunism Target a recent loser of higher rank. [now’s my chance!]

Hobson and DeDeo. *PLoS Computational Biology* (2015)


Groups and ordered structures

South Indian networks: Teṉpaṭṭi and Aḻakāpuram





1964 question of Srinavas and Béteille: beyond ethnographic investigations?

Ranked order quality, R

We propose to measure the **quality** of a ranked ordering by R

$$R = \frac{1}{m} \sum_{ij} (A_{ij} - E_{ij}) \, \mathbf{1}_{g_i \leq g_j}$$

links

network adjacency matrix

expectation (null model)

indicator
group of i ≤ group of j

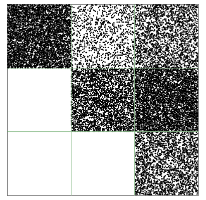
Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Ranked order quality, $\boldsymbol{R}$

We propose to measure the **quality** of a ranked ordering by $\boldsymbol{R}$

$$R = \frac{1}{m} \sum_{ij} (A_{ij} - E_{ij})\, \mathbf{1}_{g_i \leq g_j}$$

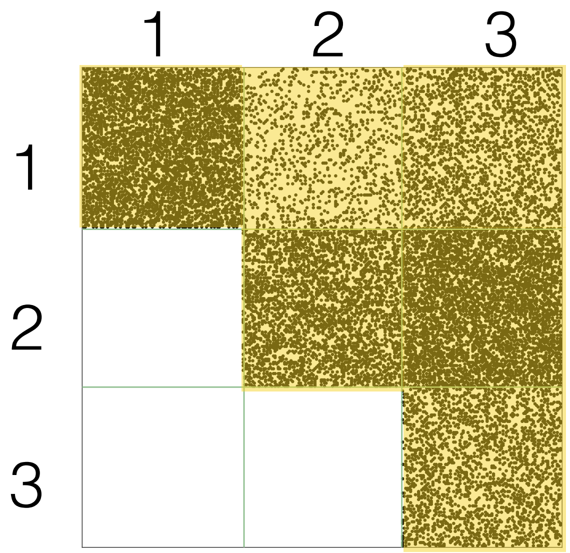
links



network adjacency matrix

expectation (null model)

indicator
group of i ≤ group of j



endorsement quality, R

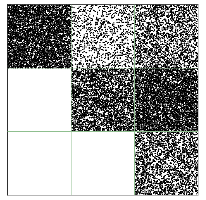
Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Ranked order quality, R

We propose to measure the **quality** of a ranked ordering by R

$$R = \frac{1}{m} \sum_{ij} (A_{ij} - E_{ij}) \, \mathbf{1}_{g_i \le g_j}$$

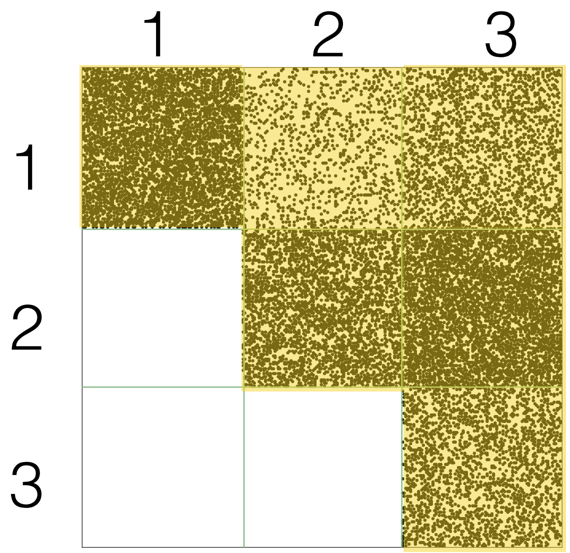
\# links



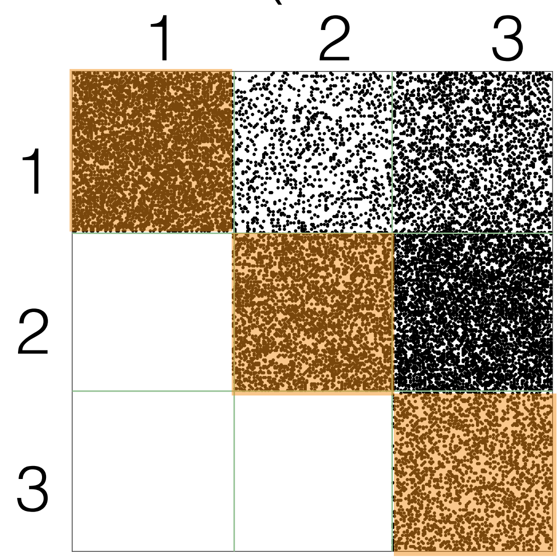
network adjacency matrix

expectation (null model)

indicator
group of i ≤ group of j



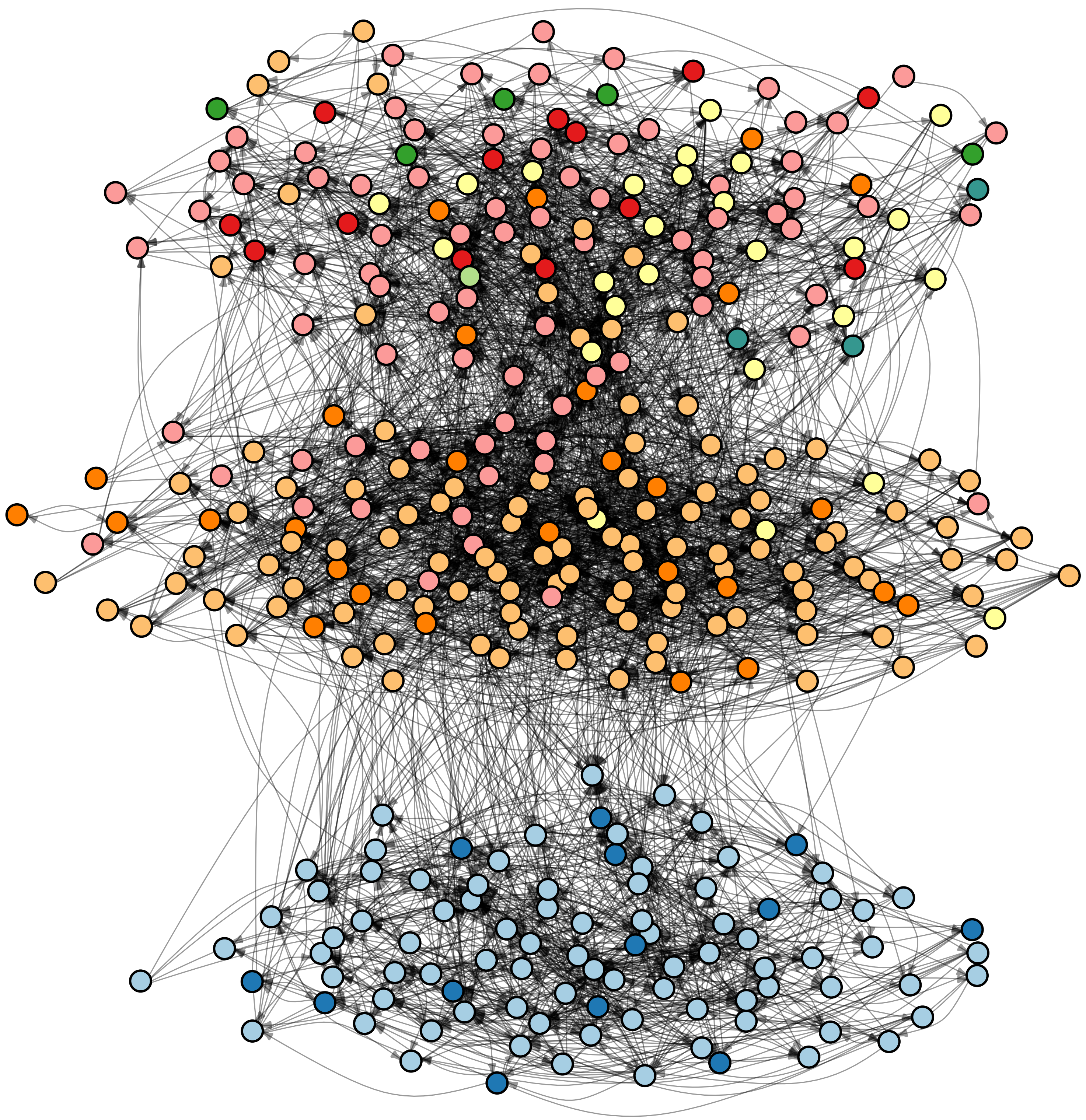
endorsement quality, R



modularity, Q

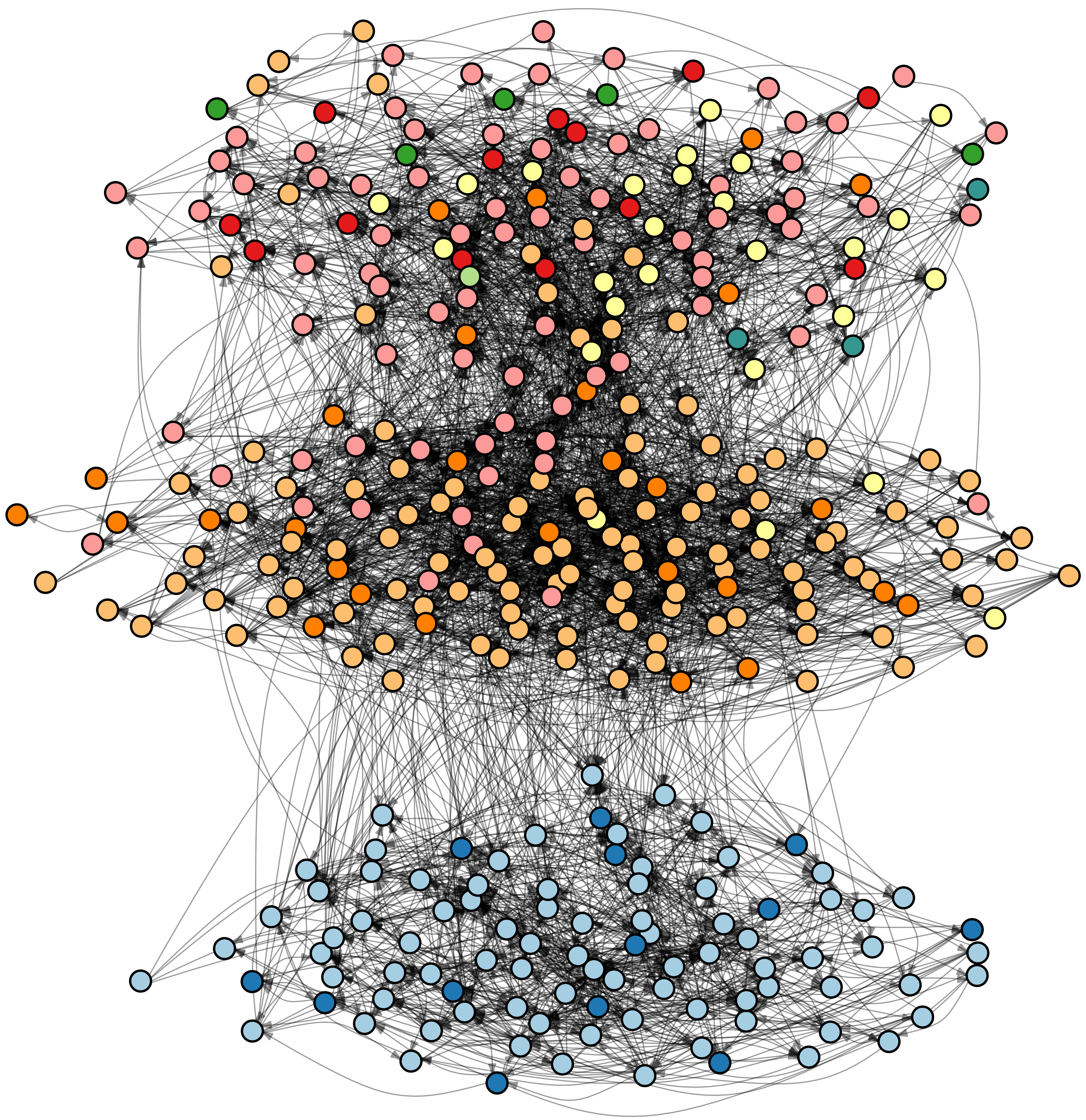
Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Teṉpaṭṭi



Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Teṉpaṭṭi

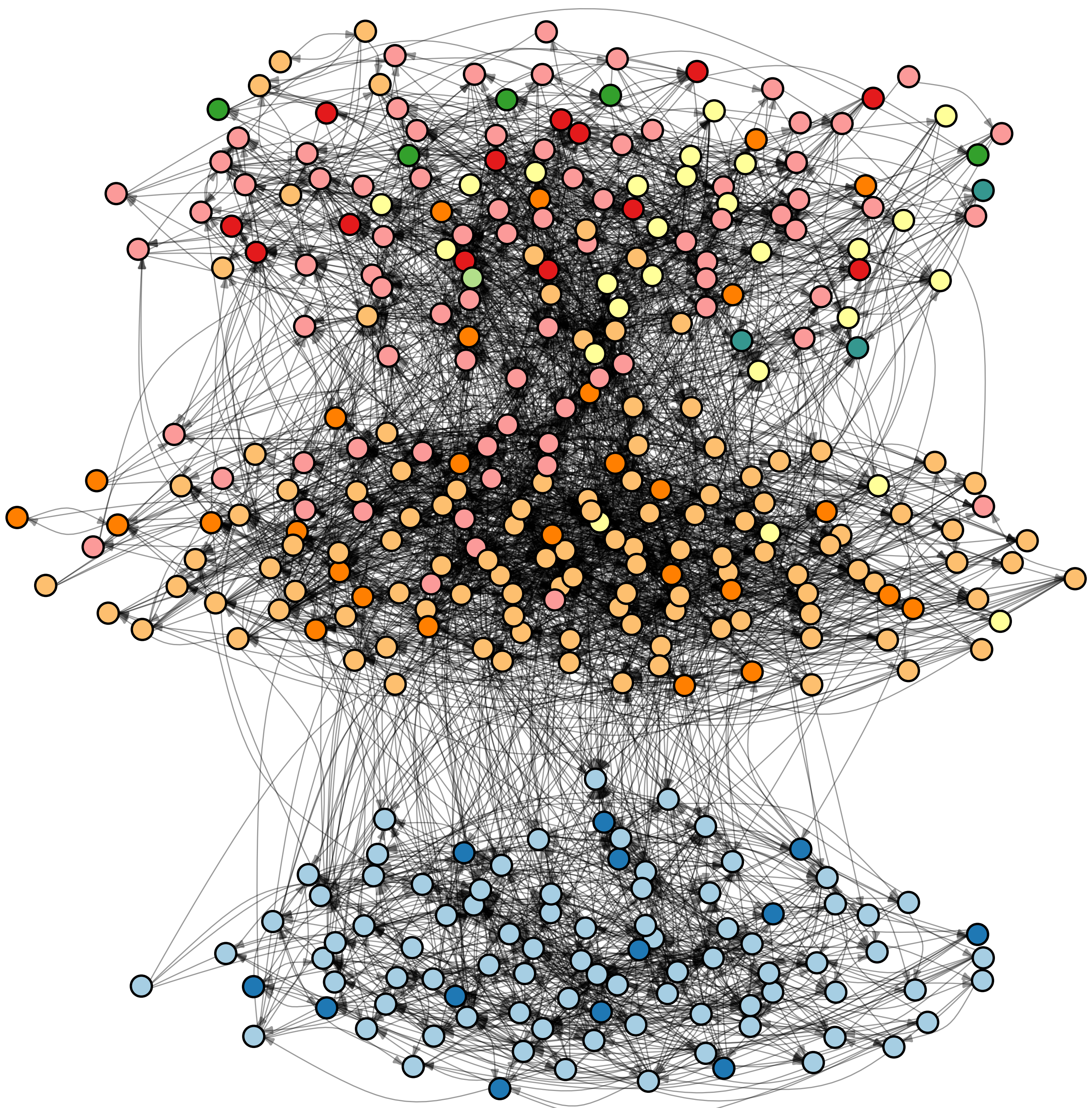


Caste

- Hindu Yaathavar
- Pallar
- Arundhathiyar
- Agamudaiyaan
- Aasaari
- Naayakkar
- RC Yaathavar
- Kallar
- Kulaalar

Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Teṉpaṭṭi

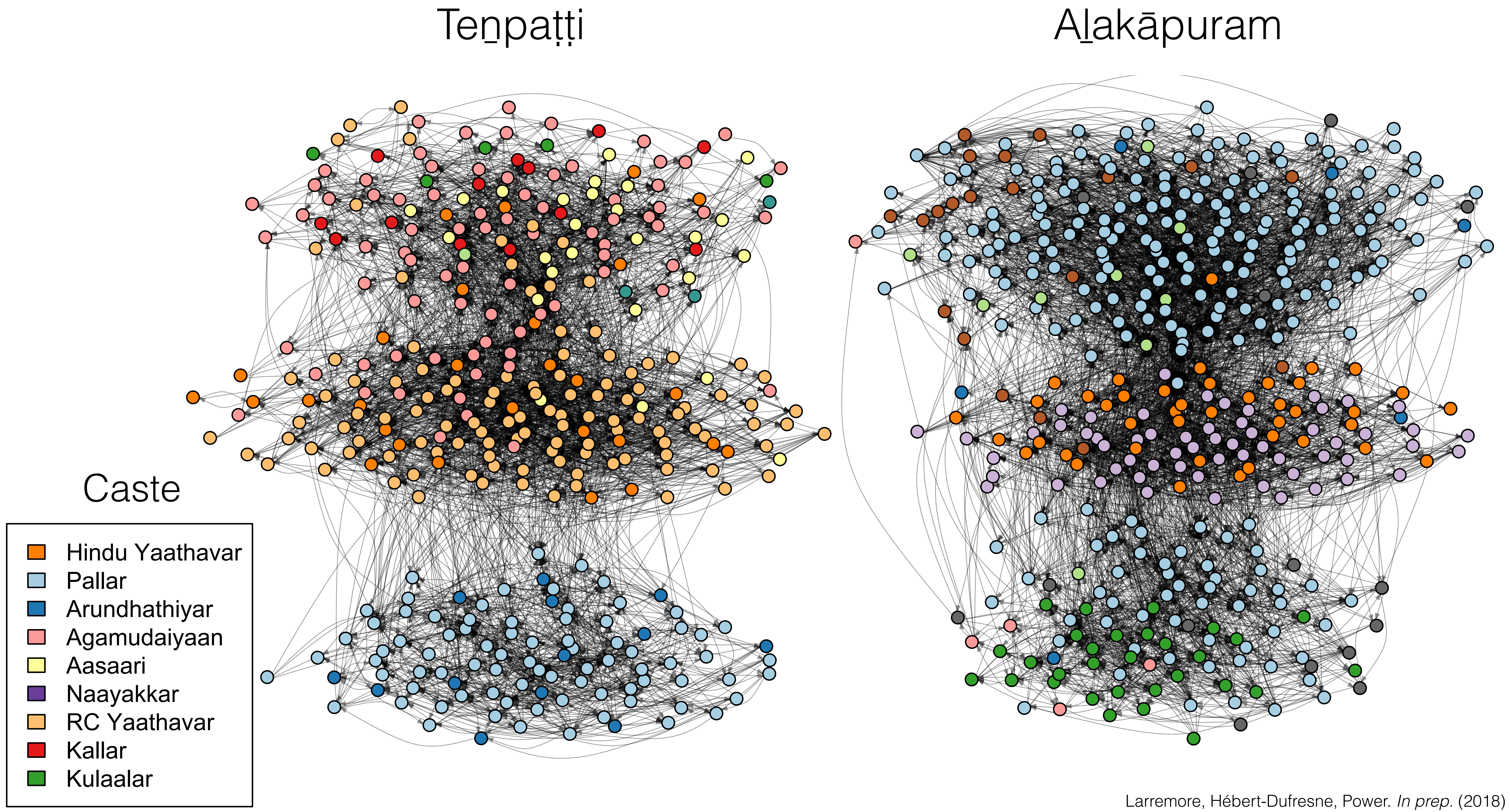


Caste

- Hindu Yaathavar
- Pallar
- Arundhathiyar
- Agamudaiyaan
- Aasaari
- Naayakkar
- RC Yaathavar
- Kallar
- Kulaalar

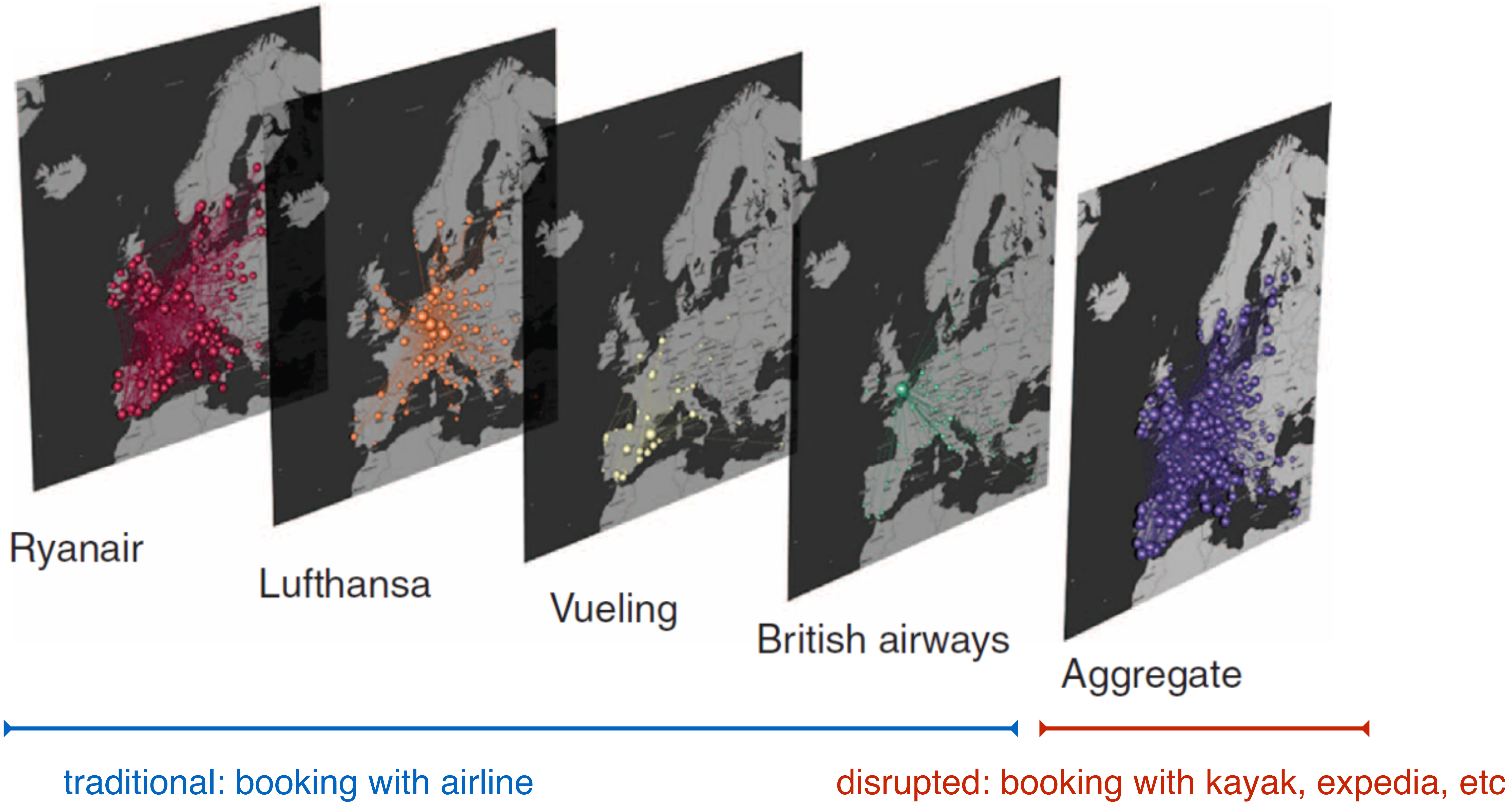
Scheduled castes—*dalit*
"untouchable"

Larremore, Hébert-Dufresne, Power. *In prep.* (2018)



Teṉpaṭṭi
Aḻakāpuram
Caste
Hindu Yaathavar
Pallar
Arundhathiyar
Agamudaiyaan
Aasaari
Naayakkar
RC Yaathavar
Kallar
Kulaalar
Larremore, Hébert-Dufresne, Power. *In prep.* (2018)

Multilayer network: air travel



Multilayer network: South Indian social support

There are separate layers for:

Who would you go to for **work**?
Who do you see has holding a good **position** in the community?

Who would you go to if you needed
- … someone to **babysit**?
- … a **loan**?
- … **advice** ?
- … to **talk**?
- … advice on an **important issue**?
- … someone to run an **errand**?
- … to **borrow** household stuff?
- … a little **cash** (smaller than a loan) ?

and more!

Layer interdependence

Are layers structurally similar? Complementary? Neither?

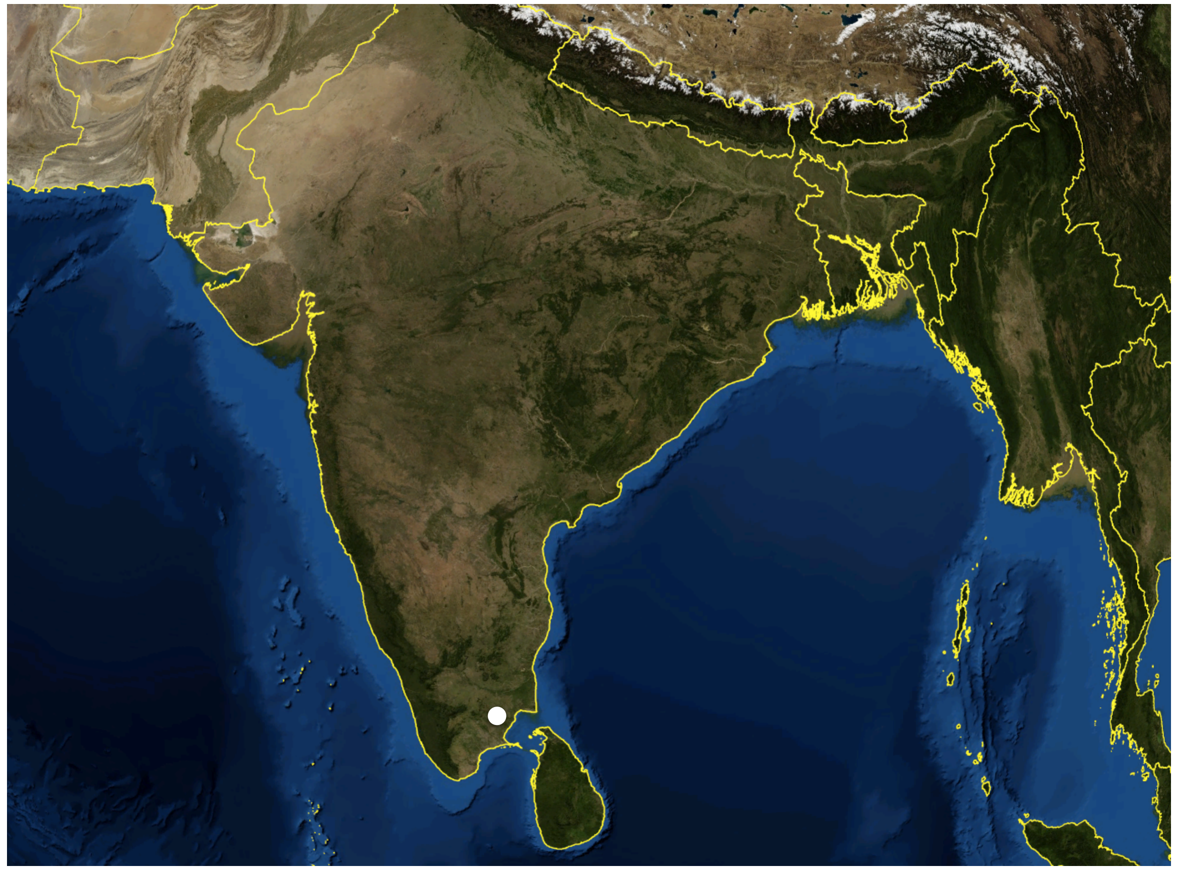
"Learn" a SBM from *m* layers; try to predict links of *m+1*.

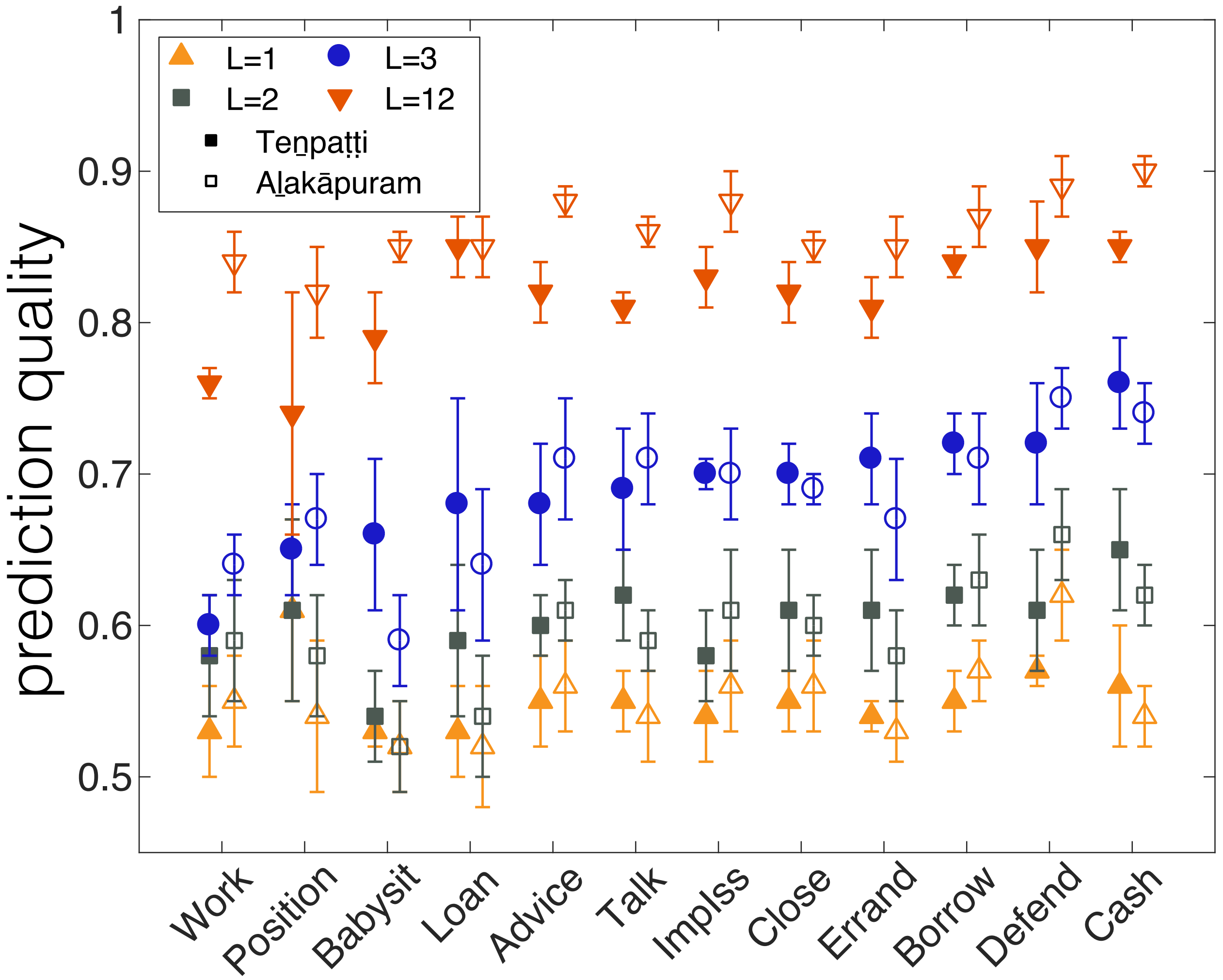
Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks" Physical Review E 95 042317

Layer interdependence

Are layers structurally similar? Complementary? Neither?

“Learn” a SBM from *m* layers; try to predict links of *m+1*.





12 layer social support network across 2 villages in South India.

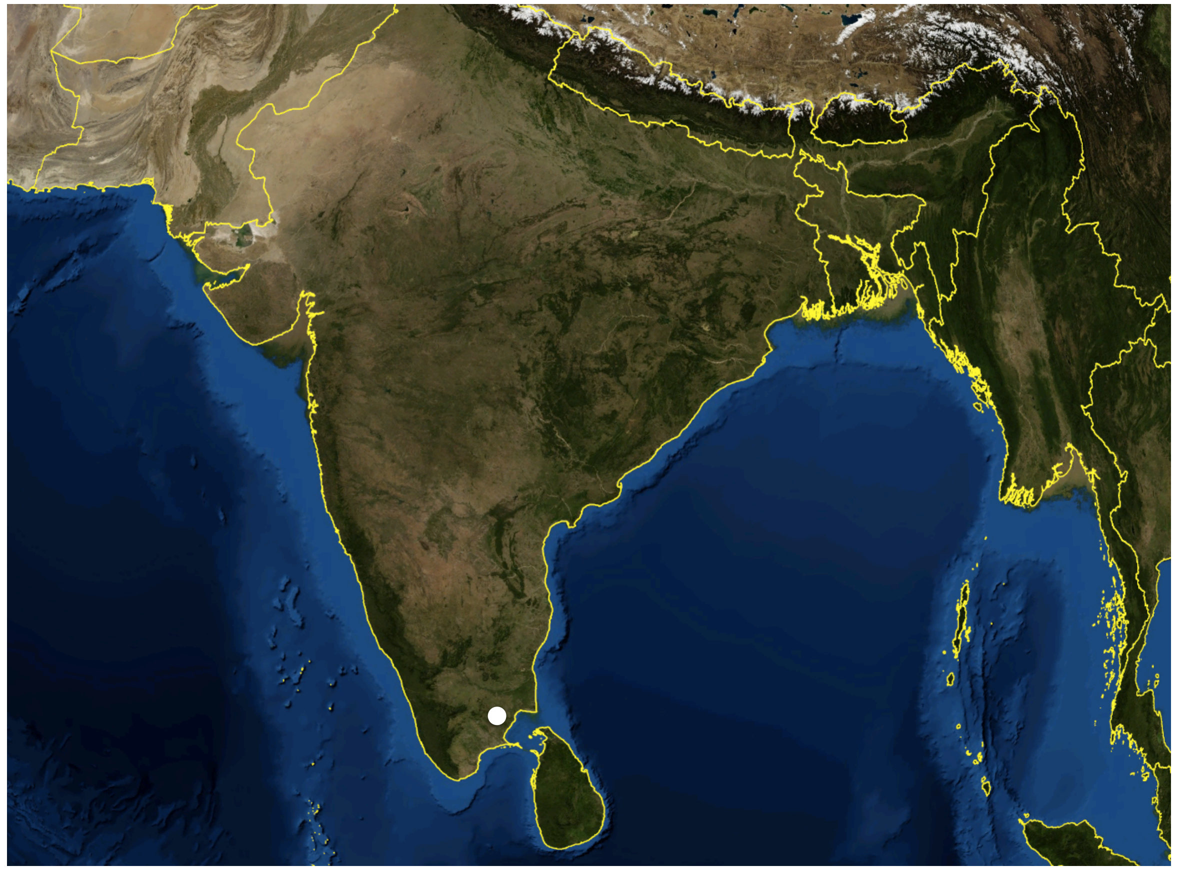
Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks” Physical Review E 95 042317

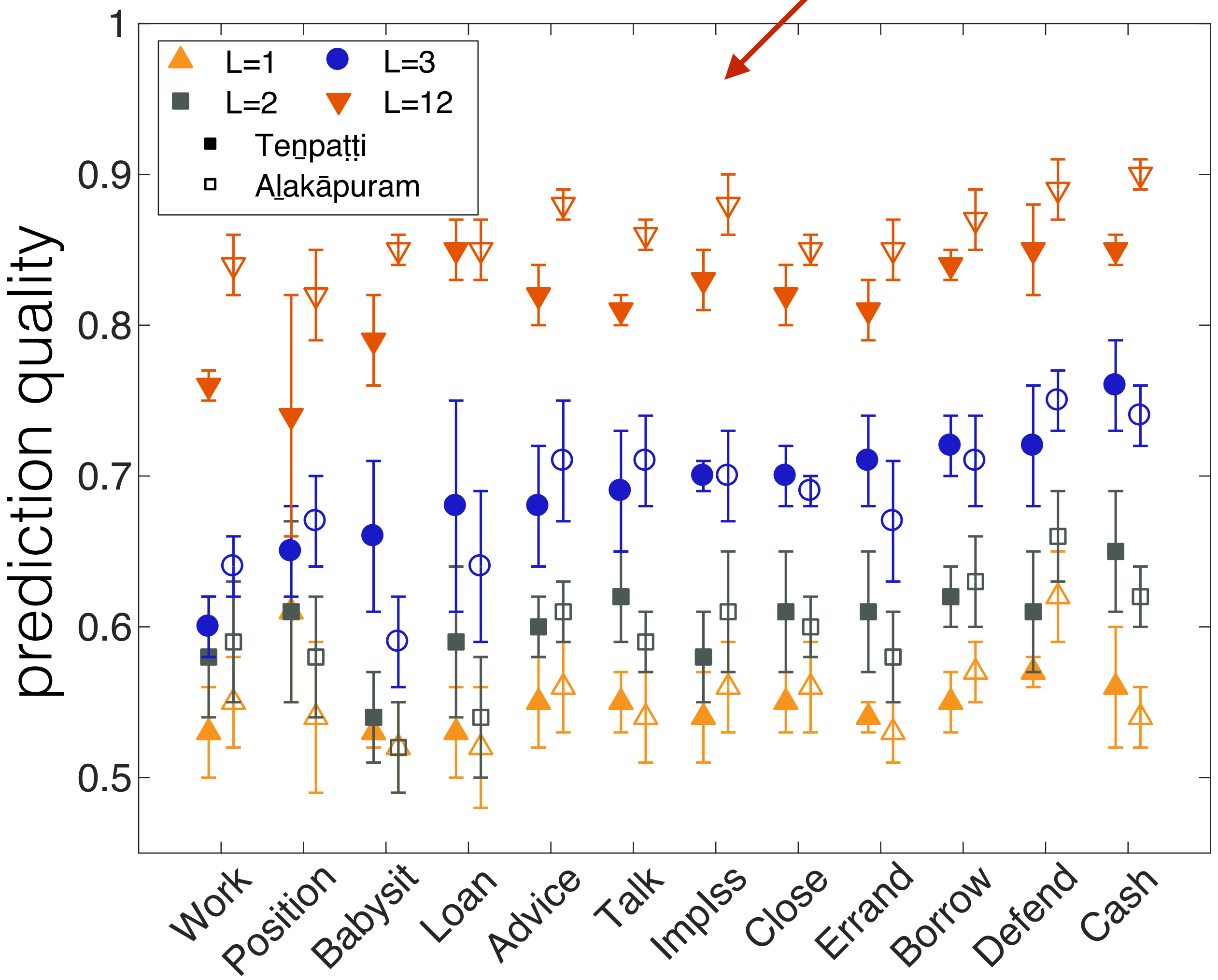
Layer interdependence

more layers = better performance
(layer structure generated by same social mech.)

Are layers structurally similar? Complementary? Neither?

"Learn" a SBM from *m* layers; try to predict links of *m+1*.

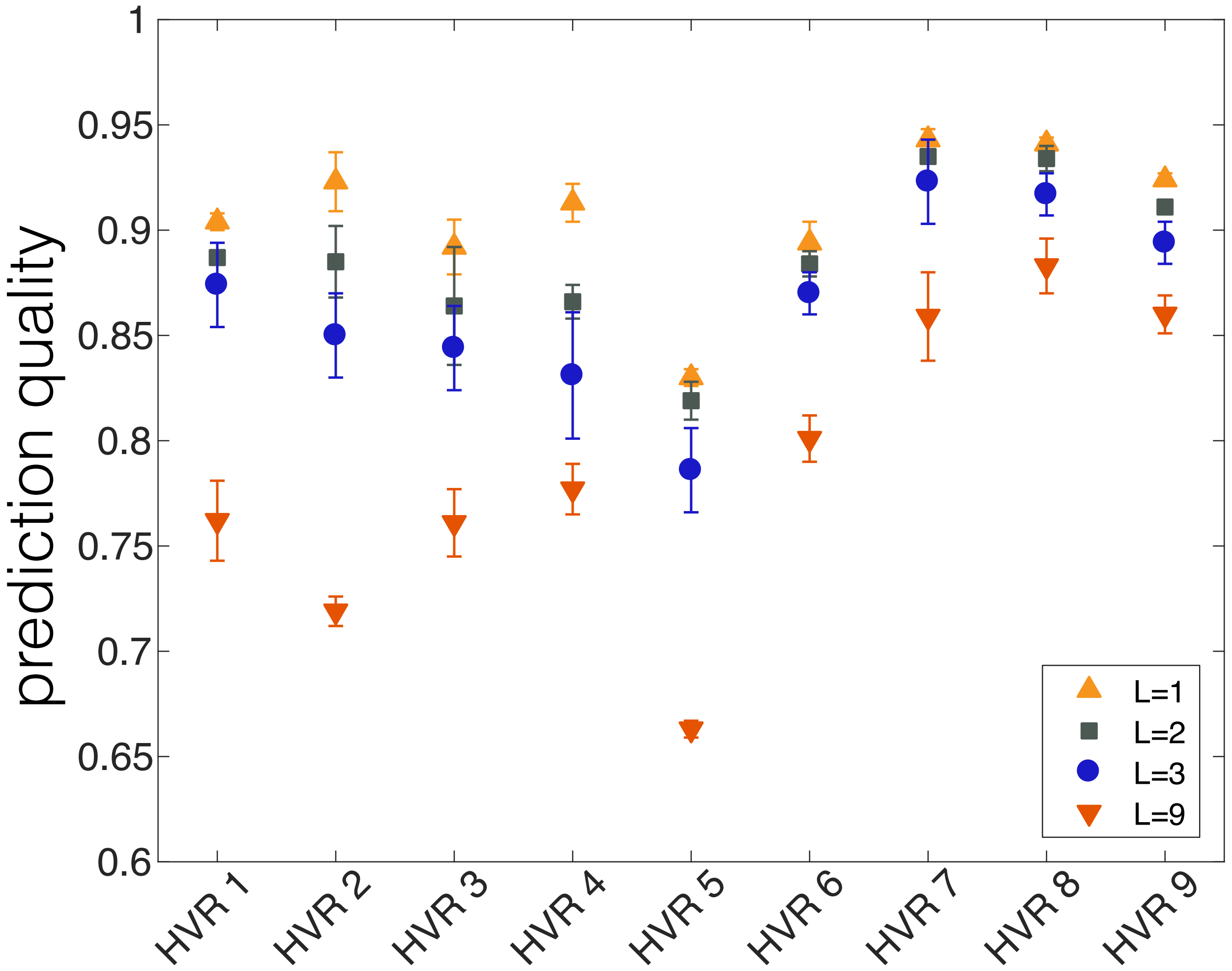




12 layer social support network across 2 villages in South India.

Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks" Physical Review E 95 042317

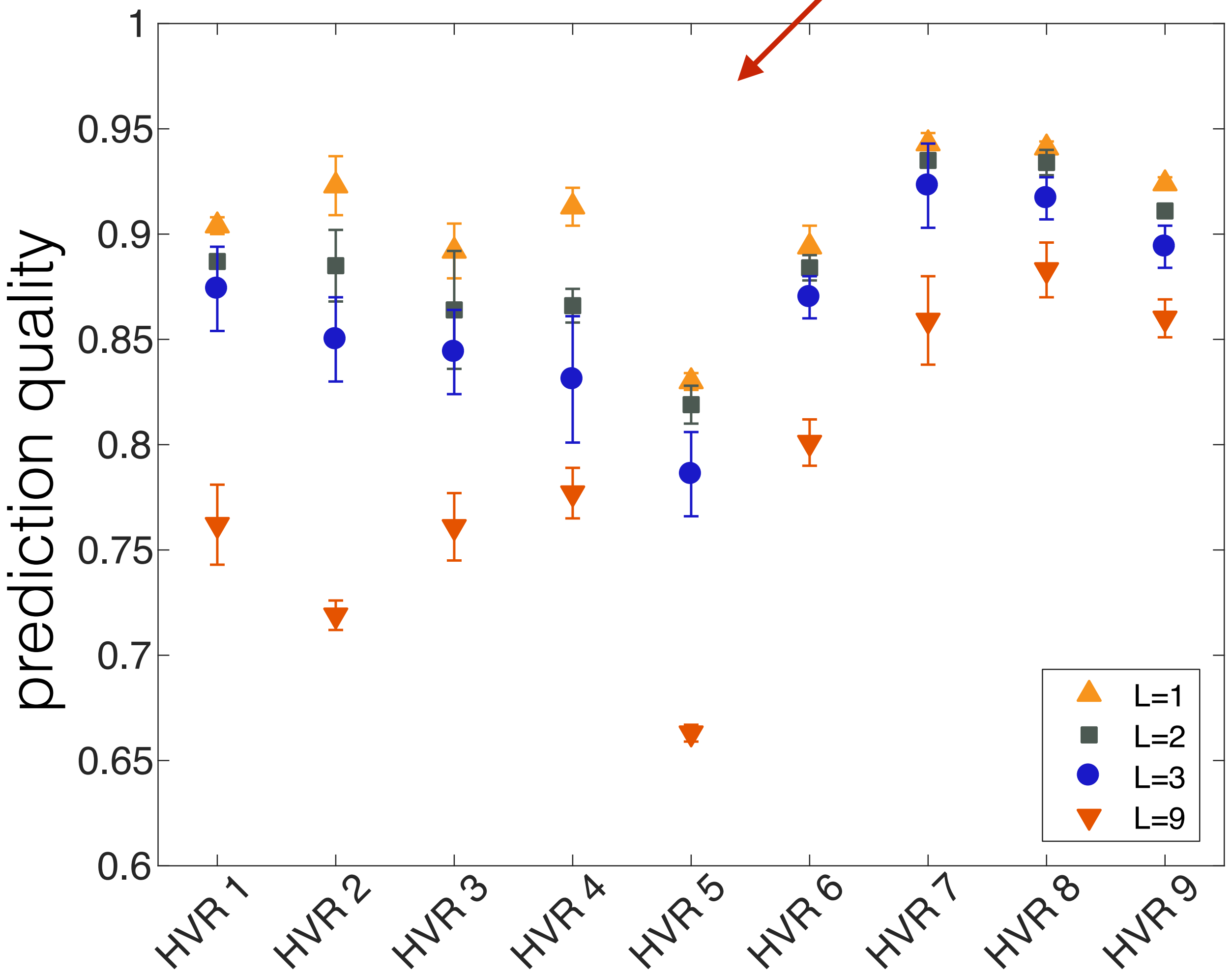
Layer interdependence - malaria



Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks" Physical Review E 95 042317

Layer interdependence - malaria

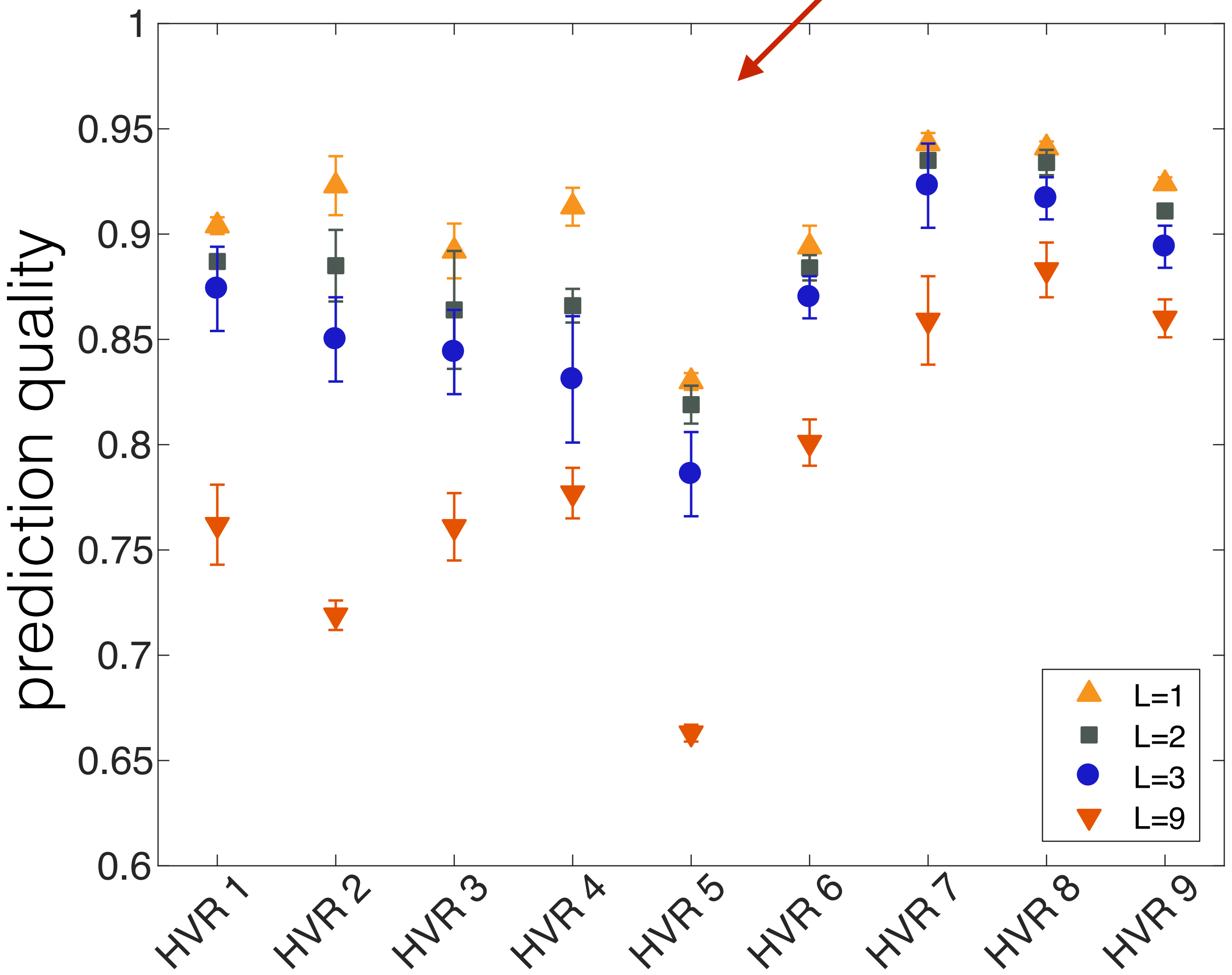
more layers = worse performance
(layer structure generated by different biol. mech.)



Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks" Physical Review E 95 042317

Layer interdependence - malaria

more layers = worse performance
(layer structure generated by different biol. mech.)

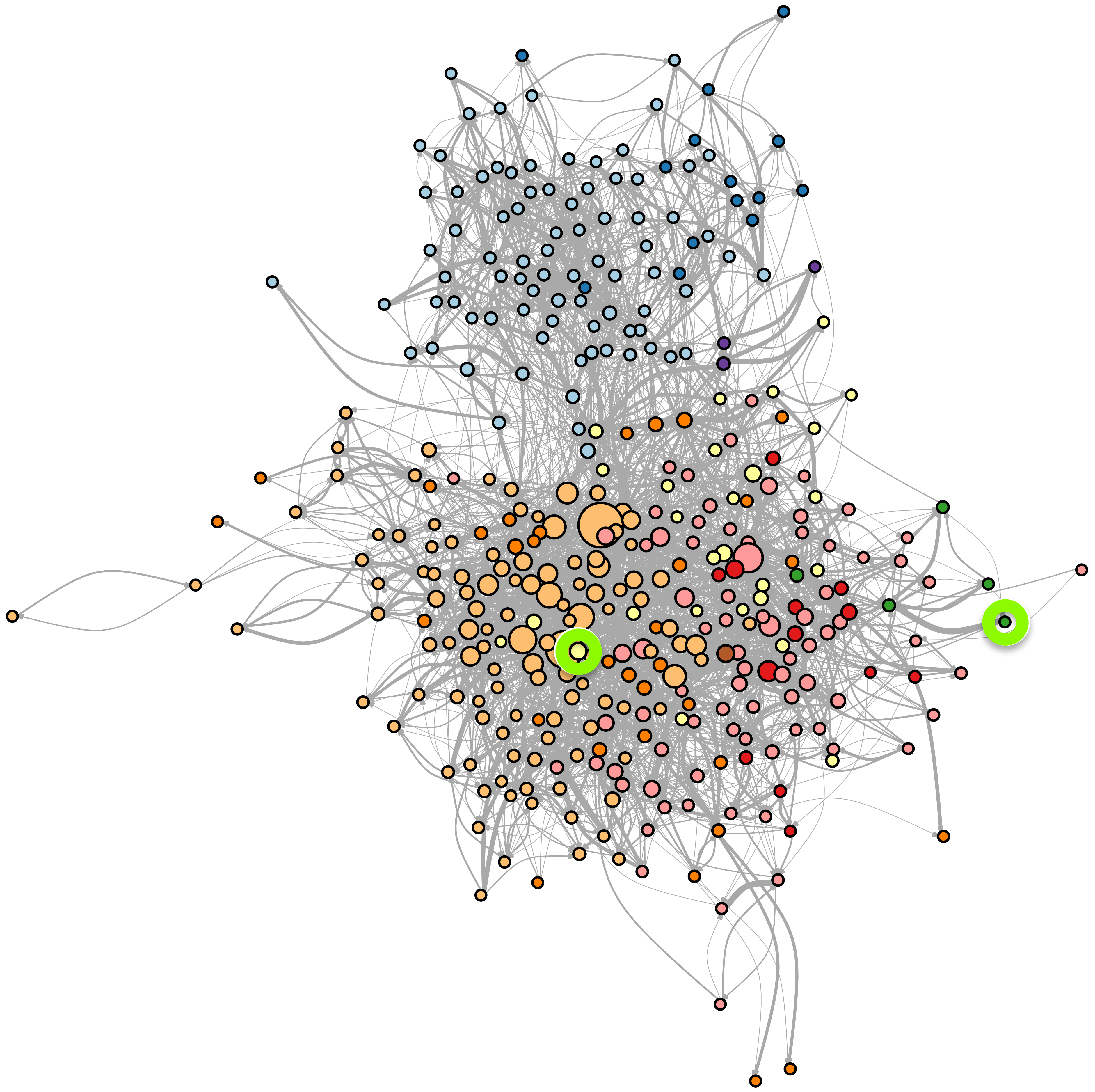


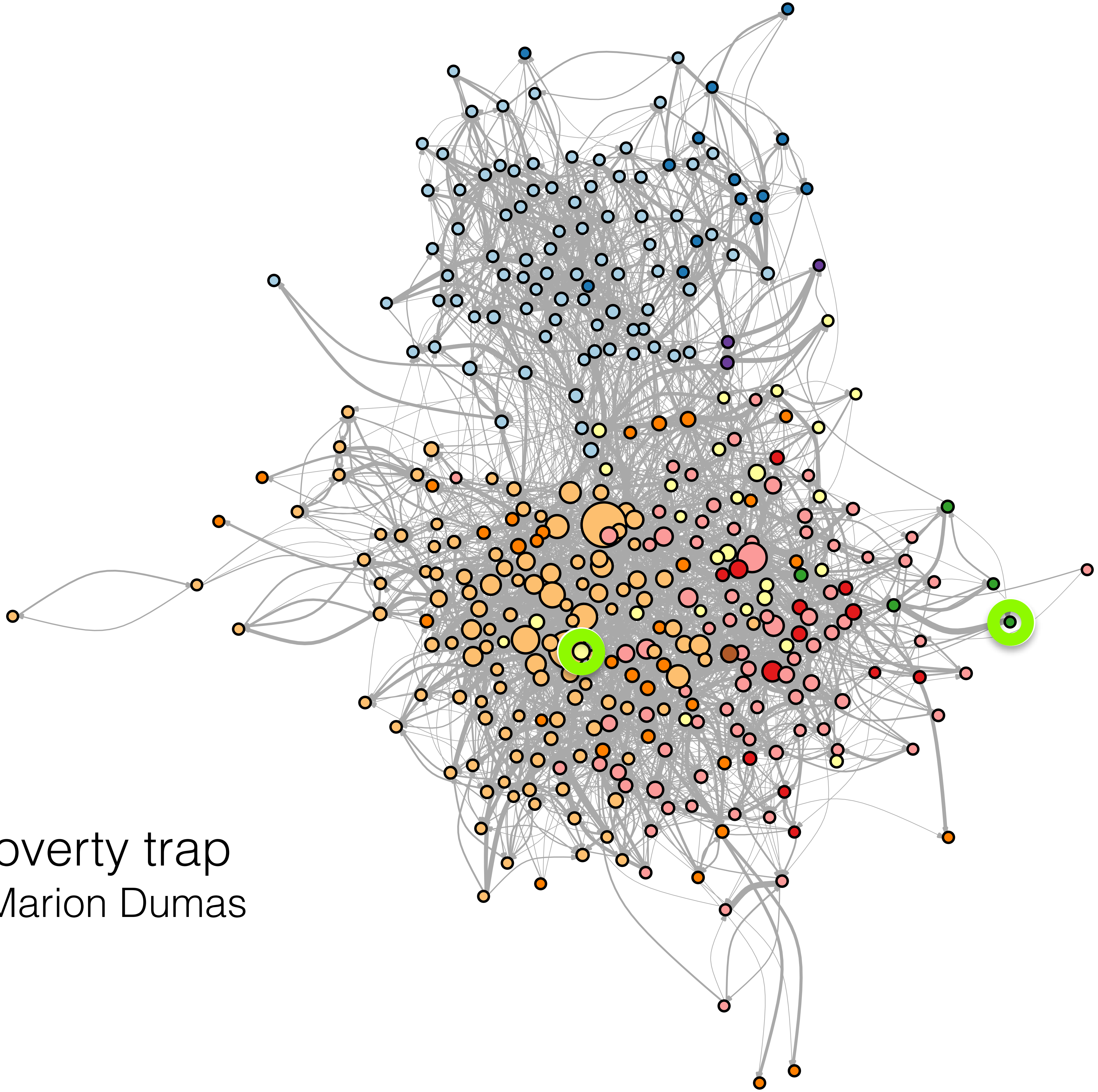
cannot predict the structure of one region in the immune-evasion genes by using other regions; layers are unrelated!

Caterina De Bacco, Eleanor A. Power, Daniel B. Larremore, and Cristopher Moore.
"Community detection, link prediction, and layer interdependence in multilayer networks" Physical Review E 95 042317


Photo: Eleanor A. Power

Photo: Eleanor A. Power





The reputational poverty trap

Eleanor Power & Marion Dumas

Many uses for models of large-scale structure

Treat the network like a system:

Extrapolation. Make predictions for as-yet unseen nodes (in "space" or time).
Interpolation. Identify missing links.
Generalization. Nodes of this type are like others of the same type.

Treat the network like an artifact:

Mechanisms. How did this network arise? What rules governed its assembly?
Explanations. Coarse-graining or compression.

Treat the network like a means to an end; an intermediate data structure:

Useful division. Need groups so that we can assign treatments in an A/B test.
Simplification. Downstream regression model needs ranks or groups.

intuition: compare this list with the list you would write for regression

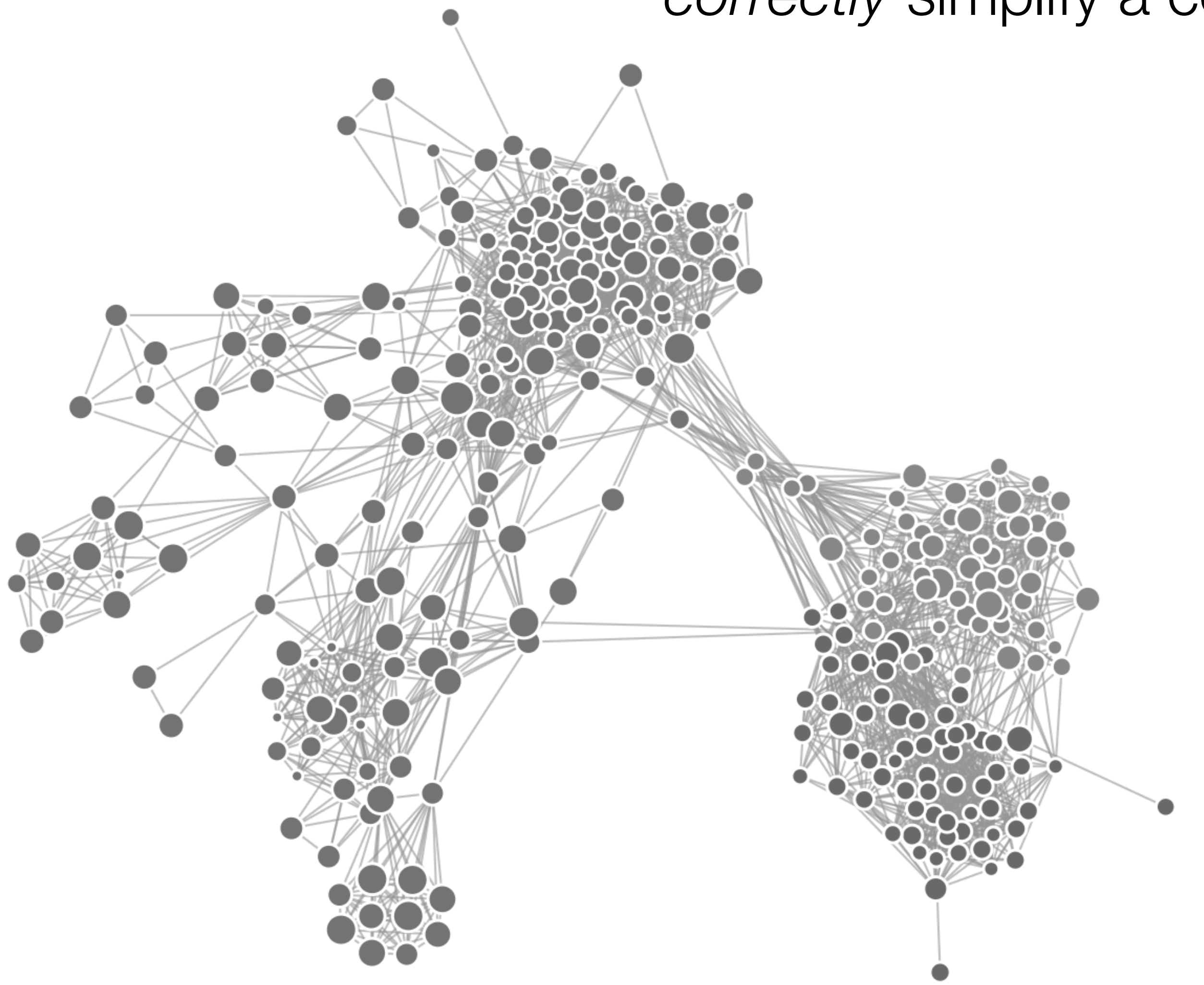
Simplicity is a great virtue but it requires hard work to achieve it and education to appreciate it. And to make matters worse: complexity sells better.

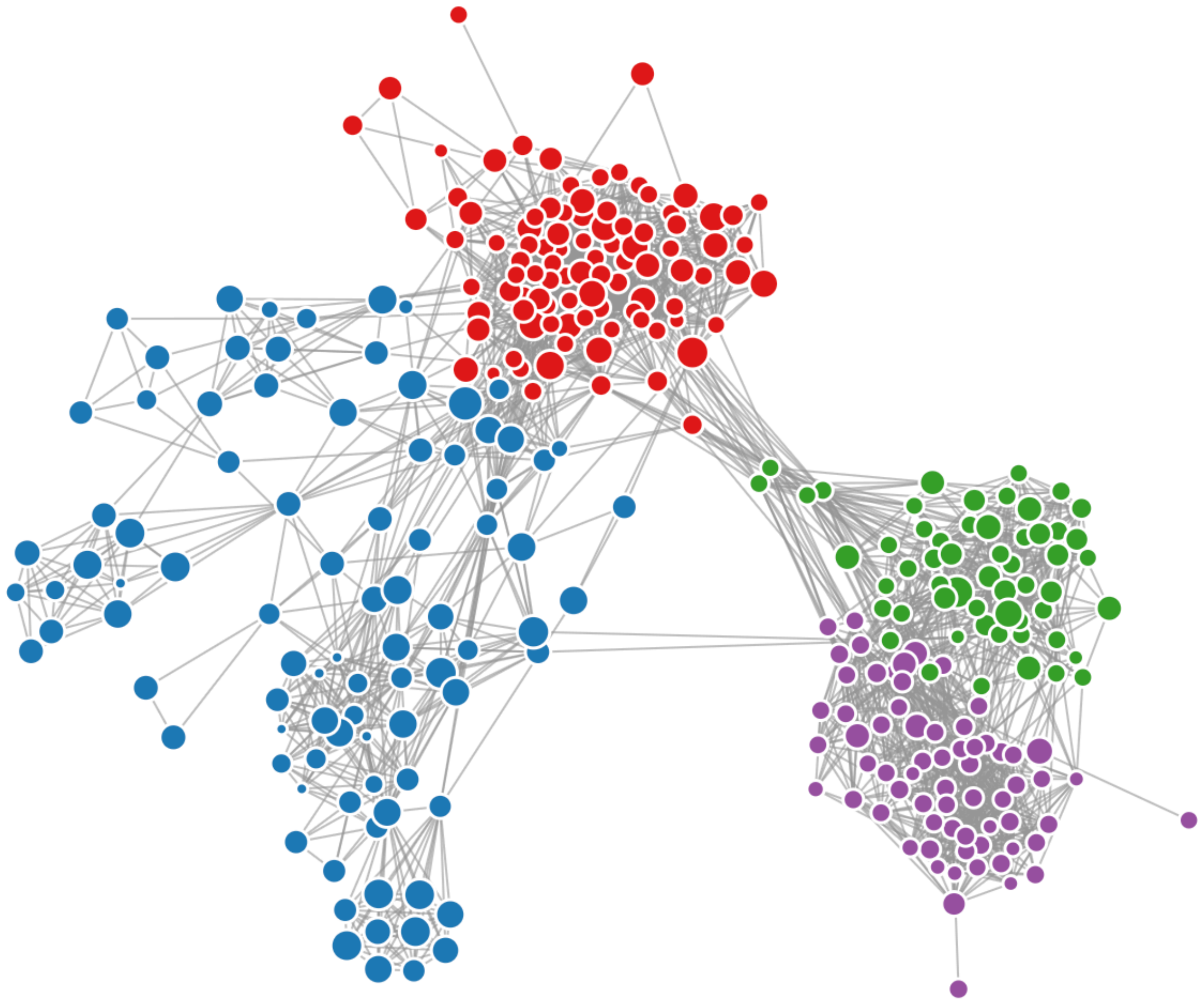
E. W. Dijkstra

We can interpret this in two ways:

The Cynic: Pictures of networks can be *really cool* but our goal is to do good science, not make pretty pictures.

The Scientist: The most beautiful science is when we *correctly* simplify a complex system.





X

We talked about 6 methods:

1. Minimum Violations Rankings & Agony
2. Random Utility Models (economics & marketing)
3. SpringRank (physics)
4. PageRank (random walks & the www)
5. Generative models (social science)
6. Niche models (ecology)

Beyond pictures: these things matter.
Inequalities, forecasting, cognition, courtship, & social organization.

And 5 applications:

1. Faculty hiring networks and prestige (computational social science)
2. Sales predictions (e-commerce)
3. Bird hierarchies and cognition (animal behavior)
4. Online dating & desirability (sociology)
5. Group-level social hierarchies (anthropology)

Thank you

@danlarremore
daniel.larremore@colorado.edu