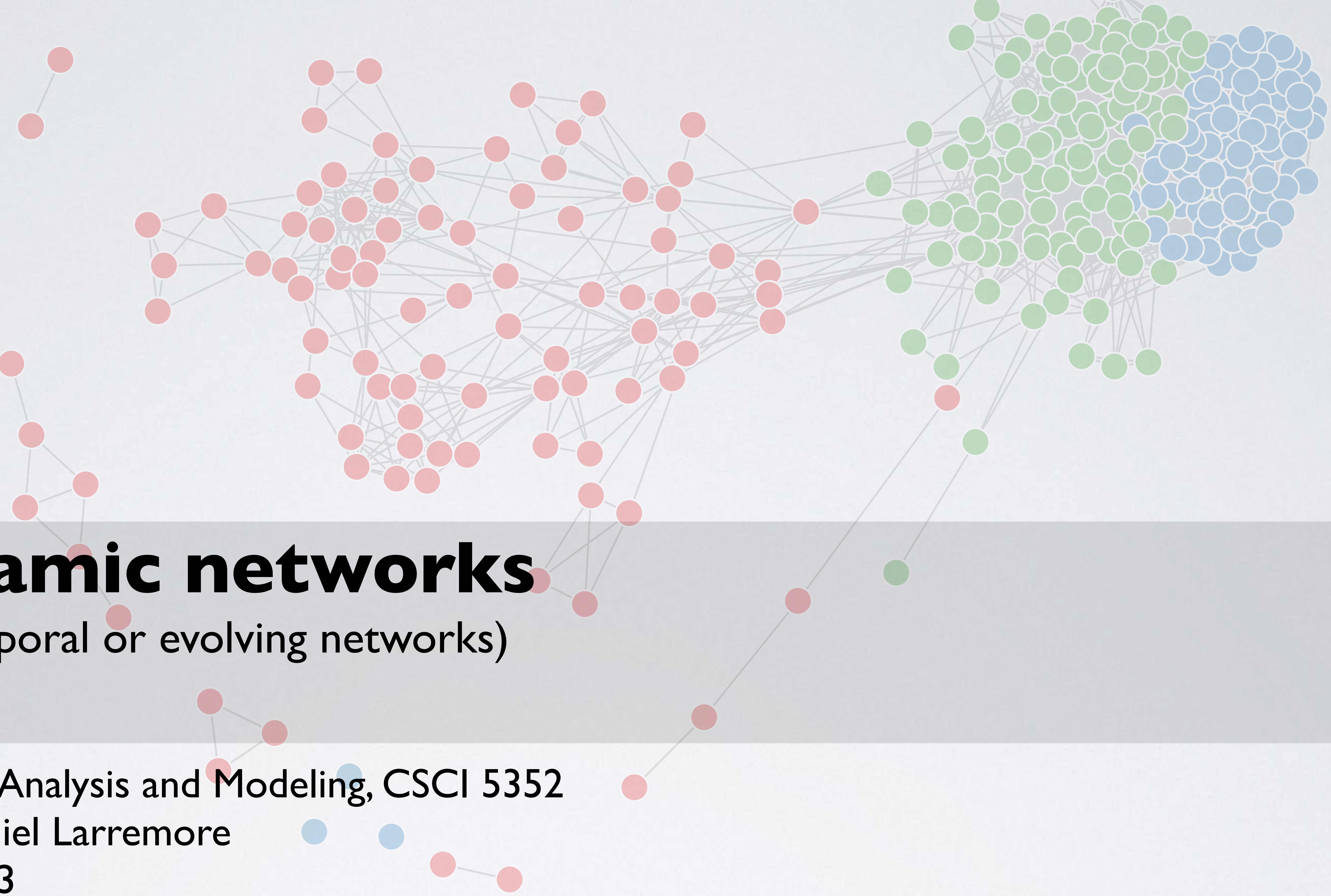


Dynamic networks

(aka temporal or evolving networks)

Network Analysis and Modeling, CSCI 5352
Prof. Daniel Larremore
Lecture 13

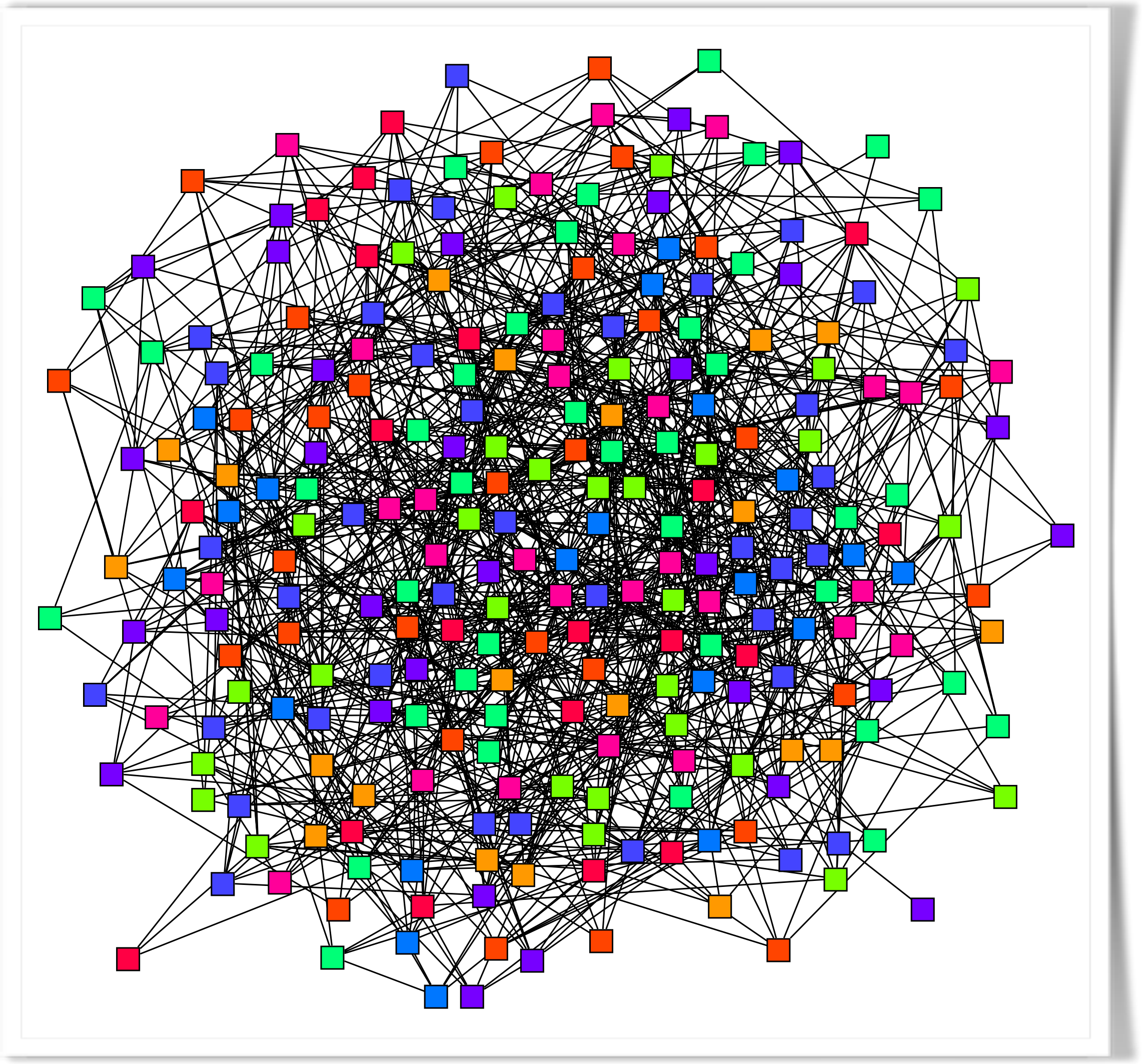
static network analysis

given network $G = (V, E)$

- centrality measures (degree-based, geometric, etc.)
- assortativity, transitivity, reciprocity
- distributions (degrees, distances, etc.)
- random walks on networks
- differences relative to configuration model
- community structure
- generative models
- etc.

static network analysis

G



empirical network

$$\xrightarrow{f(G)}$$

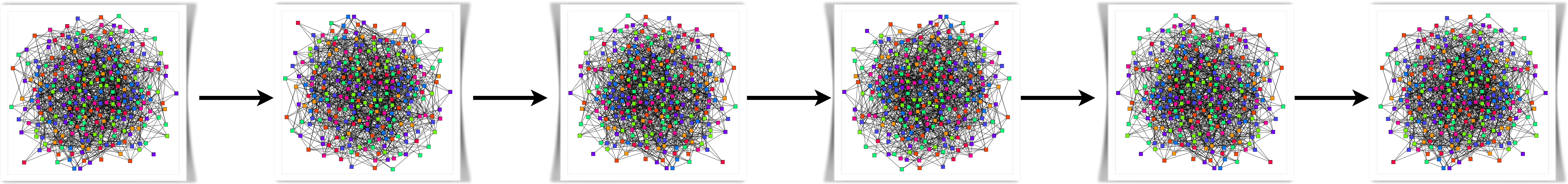
network
summary statistics

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}$$

temporal network analysis

idea 1:

empirical network sequence



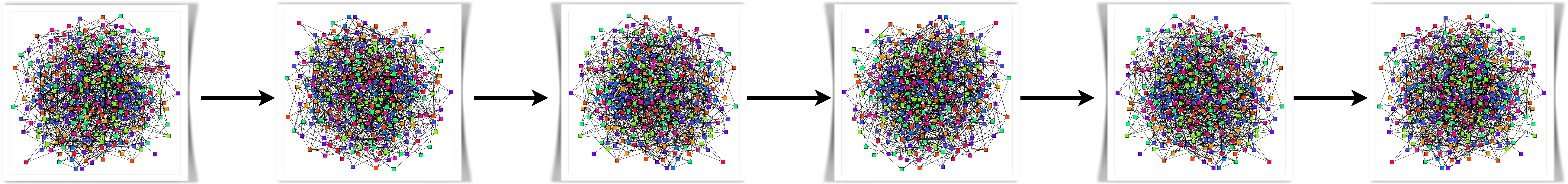
$G_{t=1}$ G_2 G_3 G_4 G_5

time-stamped interactions: $e = (i, j, t)$

temporal network analysis

idea 1:

empirical network sequence



$G_{t=1}$ $\quad G_2 \quad G_3 \quad G_4 \quad G_5$

time-stamped interactions: $e = (i, j, t)$

$$f(G_1) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{t=1} \longrightarrow f(G_2) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{2} \longrightarrow f(G_3) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{3} \longrightarrow f(G_4) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{4} \longrightarrow f(G_5) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_k \end{bmatrix}_{5}$$

temporal network analysis

idea 1:

given network sequence $G_t = (V, E_t)$

- compute statistics for each "snapshot" in sequence
- makes time series of scalar or vector values

$$\vec{x} = x_1, x_2, x_3, \ldots, x_T$$

- apply standard time series analysis tools
  - autocorrelation (periodicities)
  - change-point detection, non-stationarity
  - covariance of features
  - etc.

temporal network analysis

idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$

- durations of telephone calls
- time spent together
- etc.

temporal network analysis

idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$

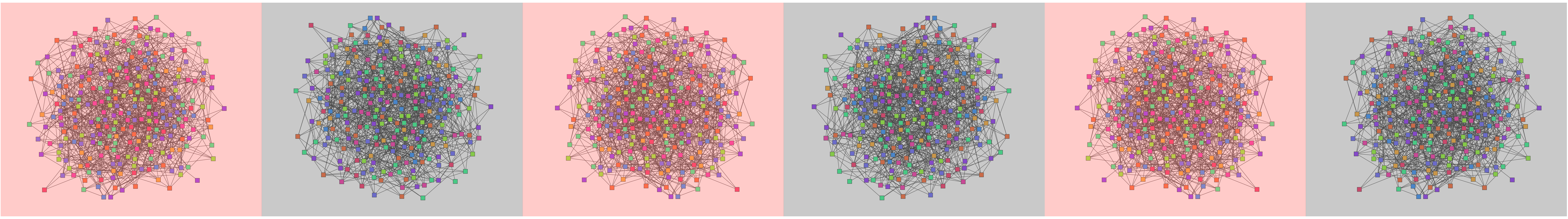
- durations of telephone calls
- time spent together
- etc.

discretize time and reduce to idea 1

temporal network analysis

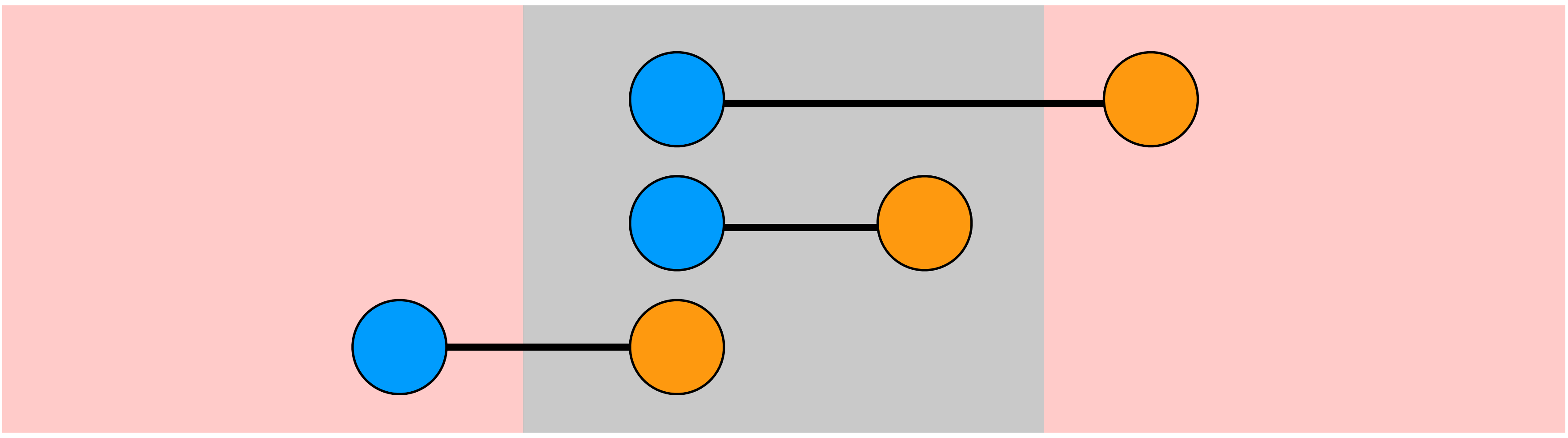
idea 2:

edges have durations $e = (i, j, t_s, \Delta t)$



$G_{t=1}$ G_2 G_3 G_4 G_5

edge in G_t if



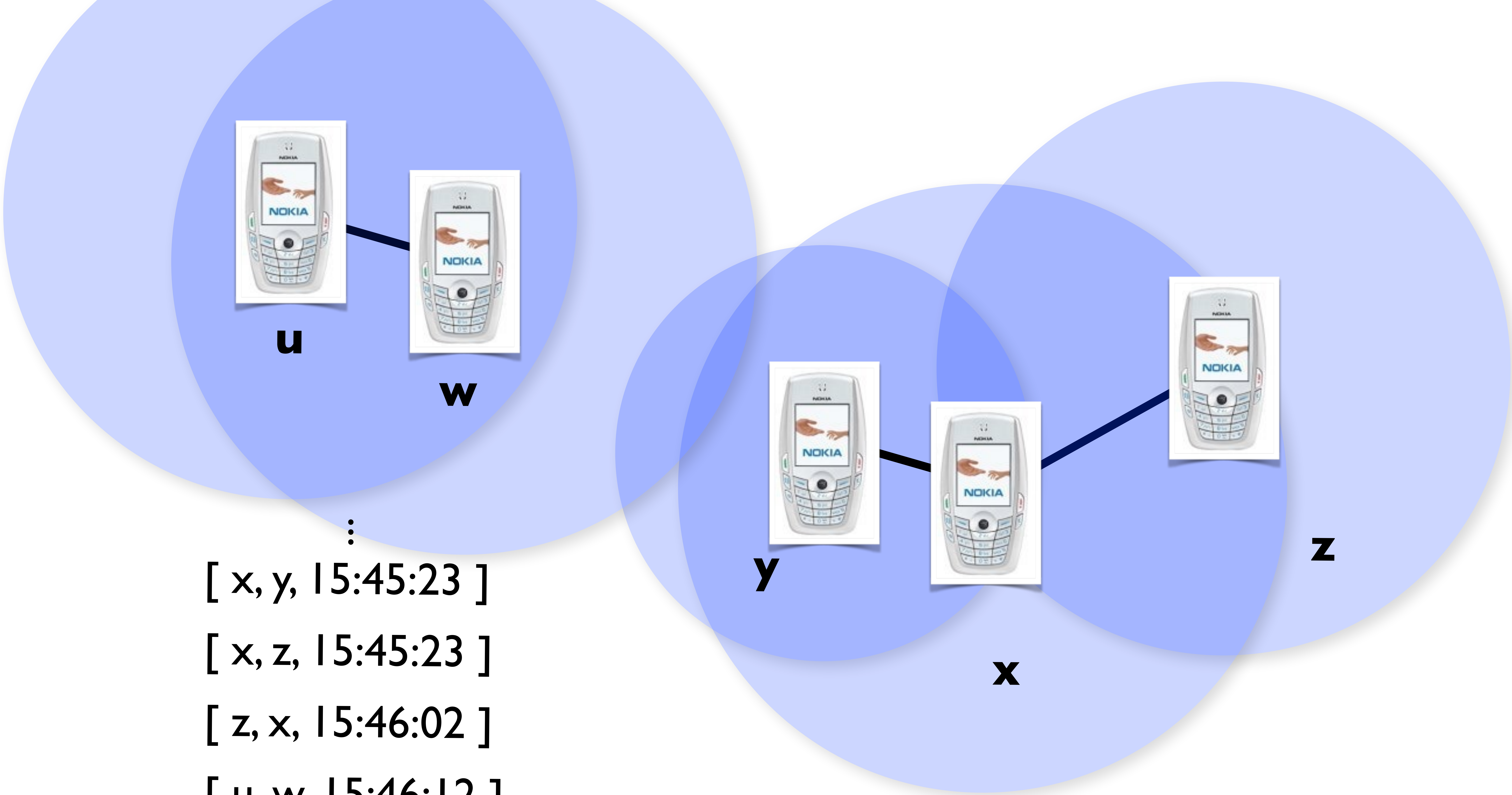
$t-1$ t $t+1$

dynamic proximity network

- MIT Reality Mining Project
- 100 mobile phones, 2 groups
- scan area with bluetooth
- every 5 minutes for 12 months (~100,000 minutes of data)
- record proximate devices (range: 5m)
- convert to dynamic proximity network (assume phone = person)

Clauset and Eagle, "Persistence and periodicity in a dynamic proximity network." DIMACS Workshop on Computational Methods for Dynamic Interaction Networks (2007) arxiv:1211.7343

http://realitycommons.media.mit.edu



⋮

[x, y, 15:45:23]

[x, z, 15:45:23]

[z, x, 15:46:02]

[u, w, 15:46:12]

⋮

proximity inference rule

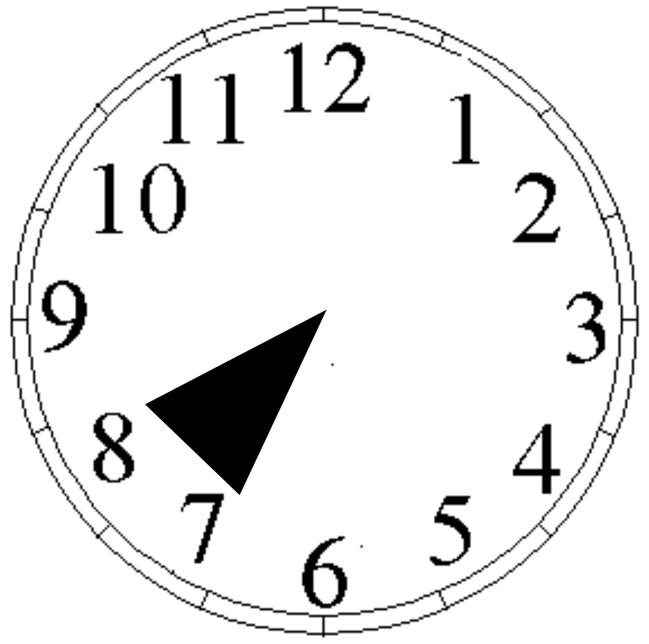
- proximities are time-stamped (i, j, t)
- we want to infer durations $(i, j, t_s, \Delta t)$
- proximities are noisy [some edges unobserved]
- high-resolution temporal sampling [every 5 mins]

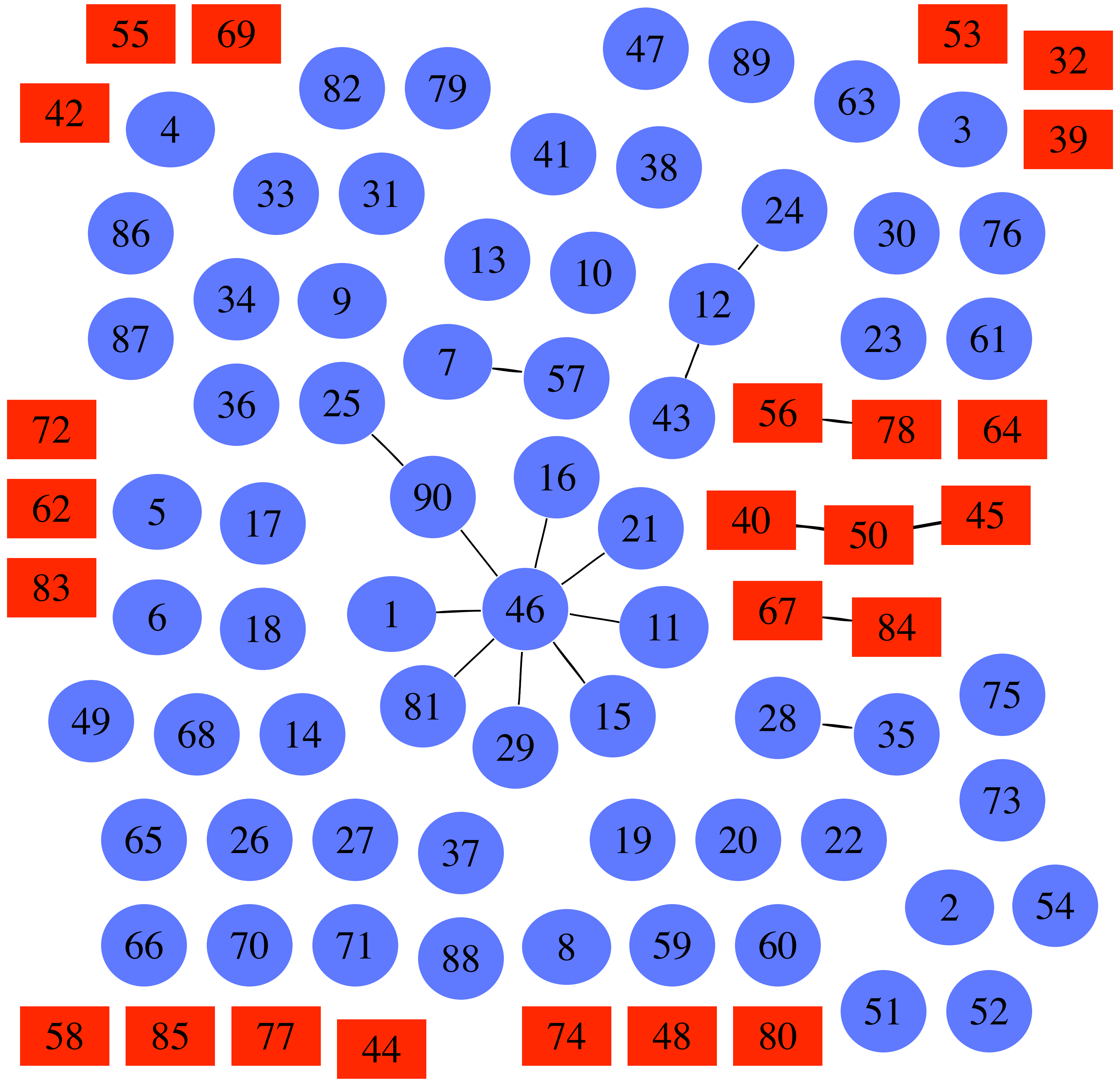
- rule:
  - define tolerance τ; if gap less than τ, assume continuous proximity

single day of proximities

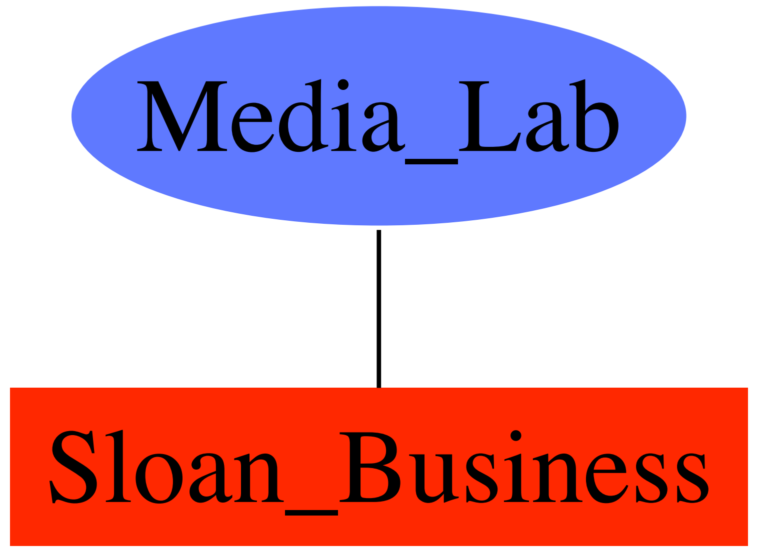
single day of proximities

Tuesday, 19 Oct 2004





very few connections



single day of proximities

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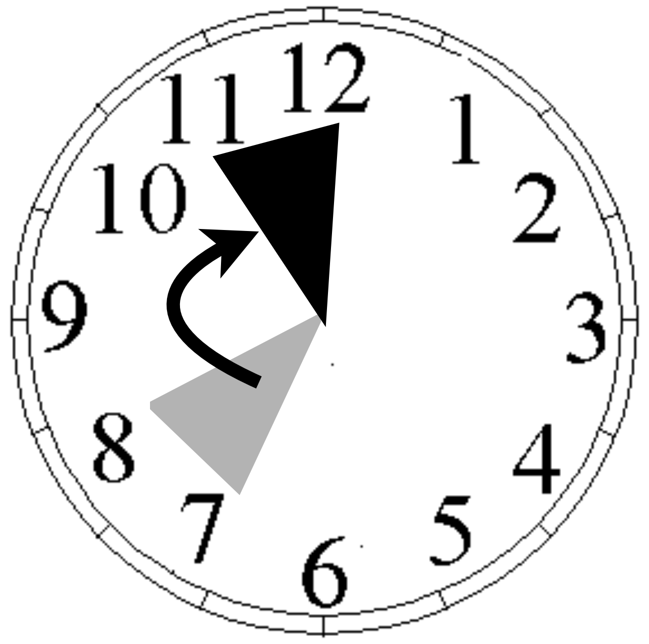
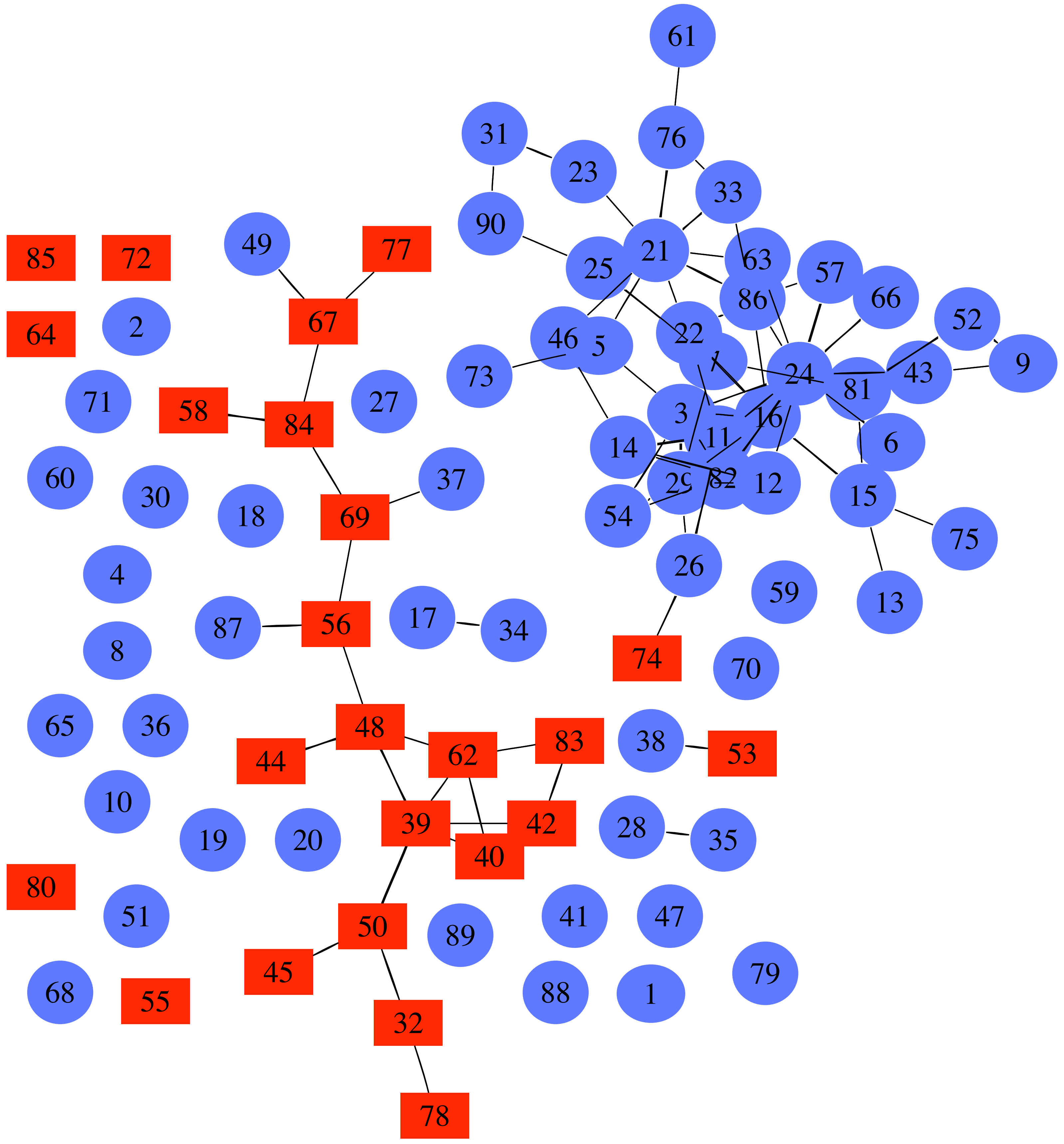
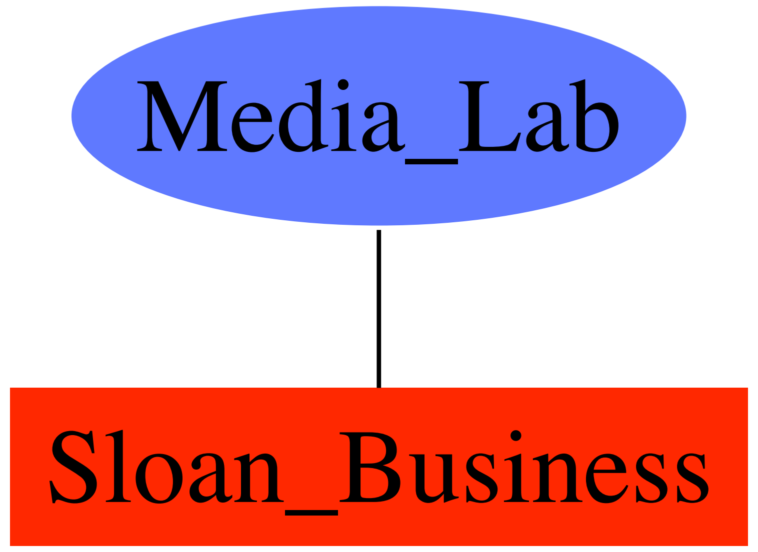




more connections



single day of proximities

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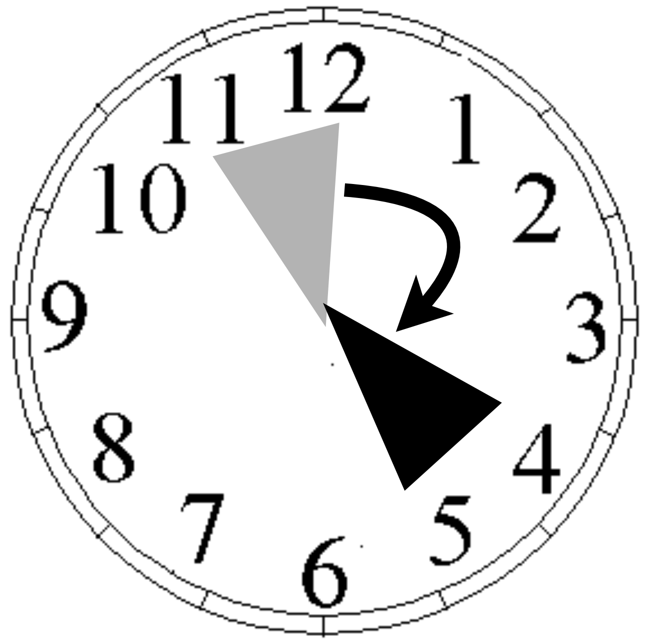
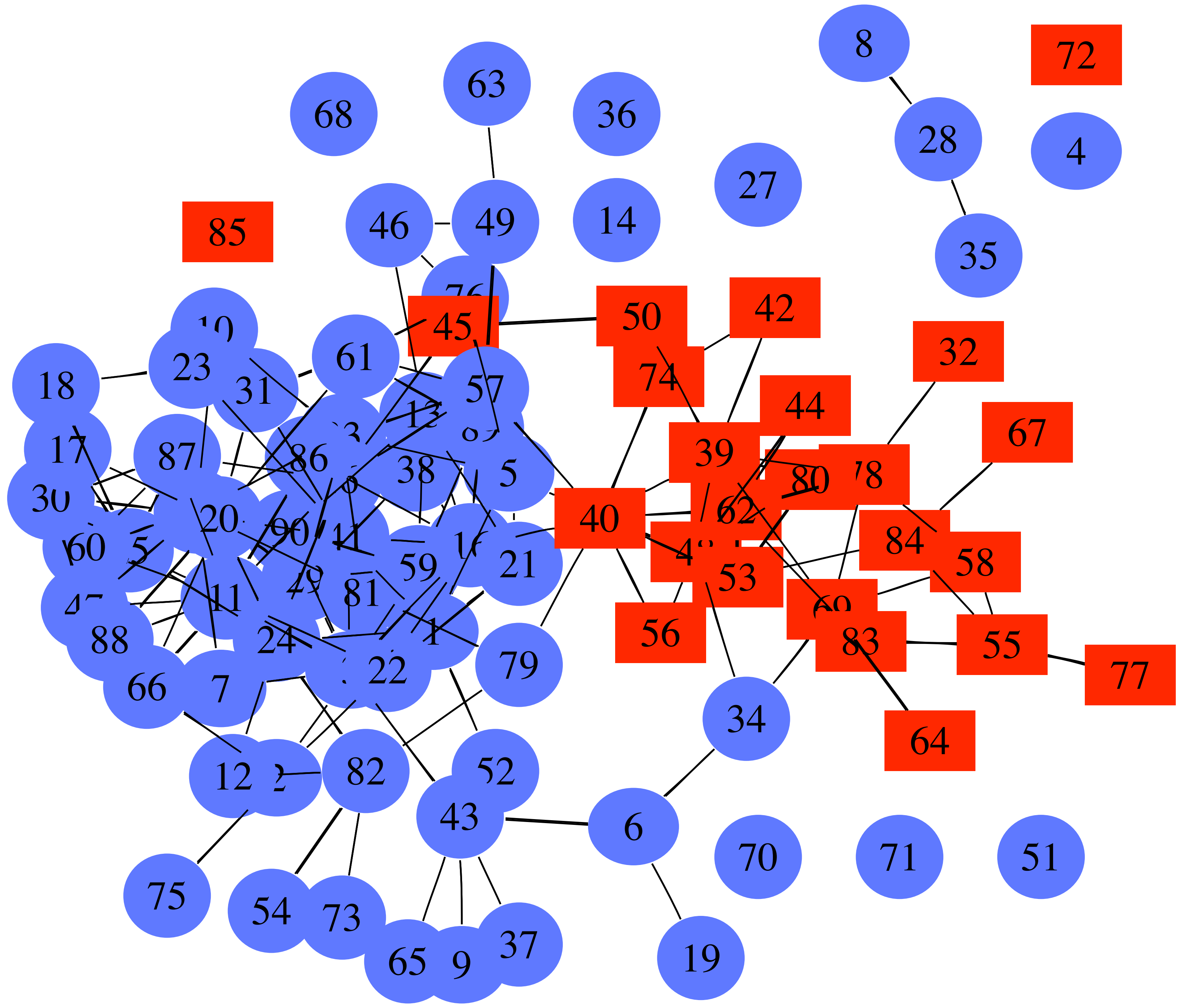
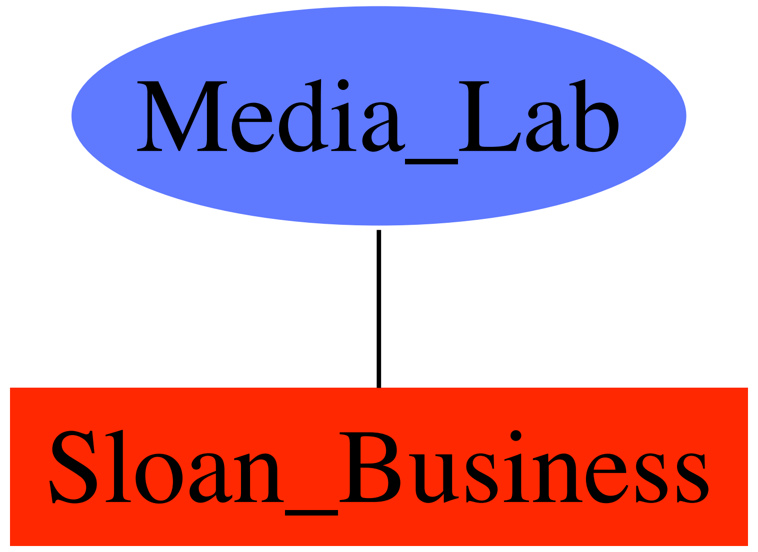




peak connections,
two communities



single day of proximities

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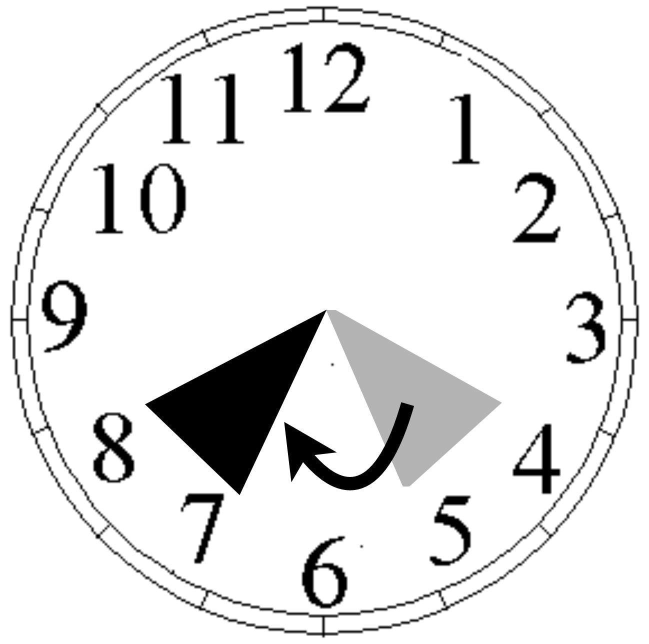
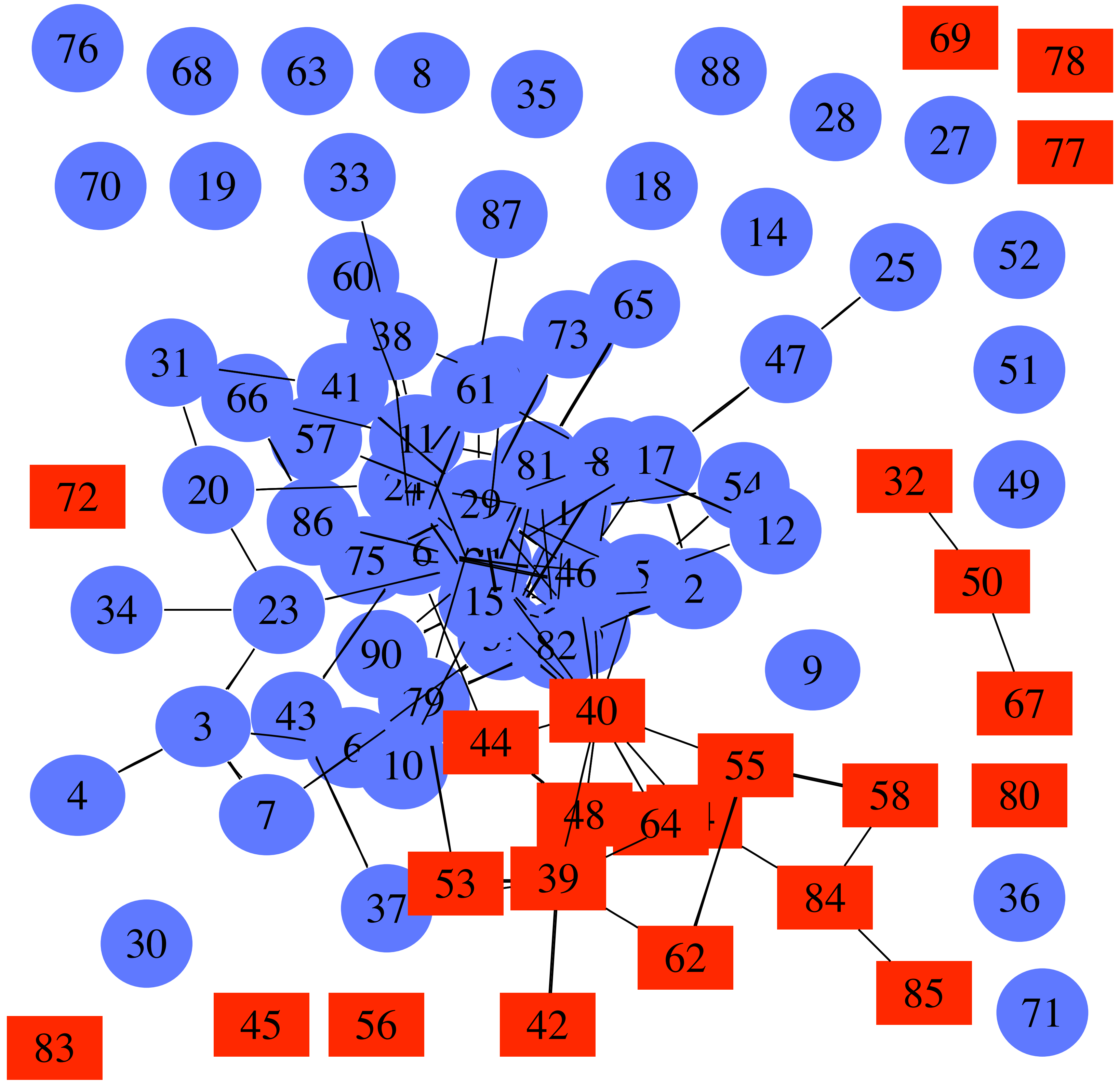
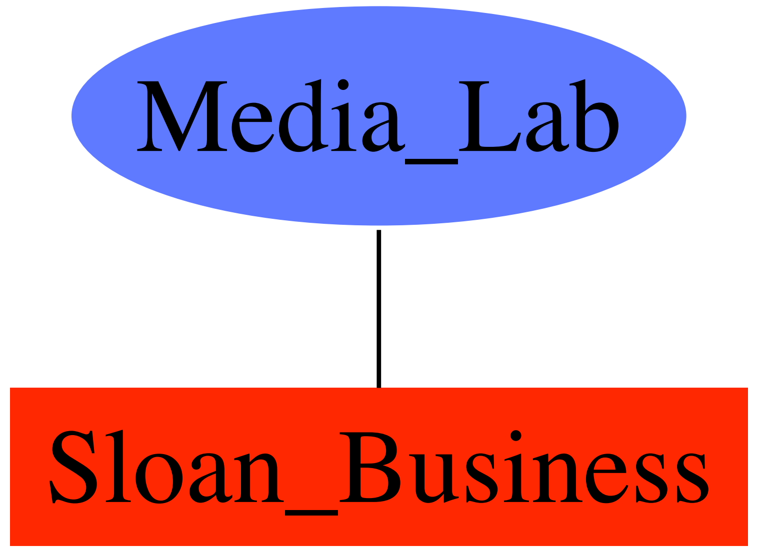


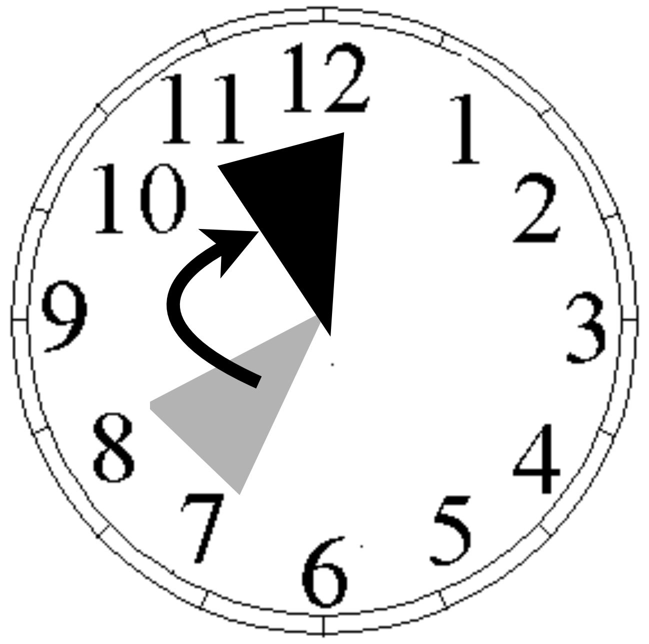


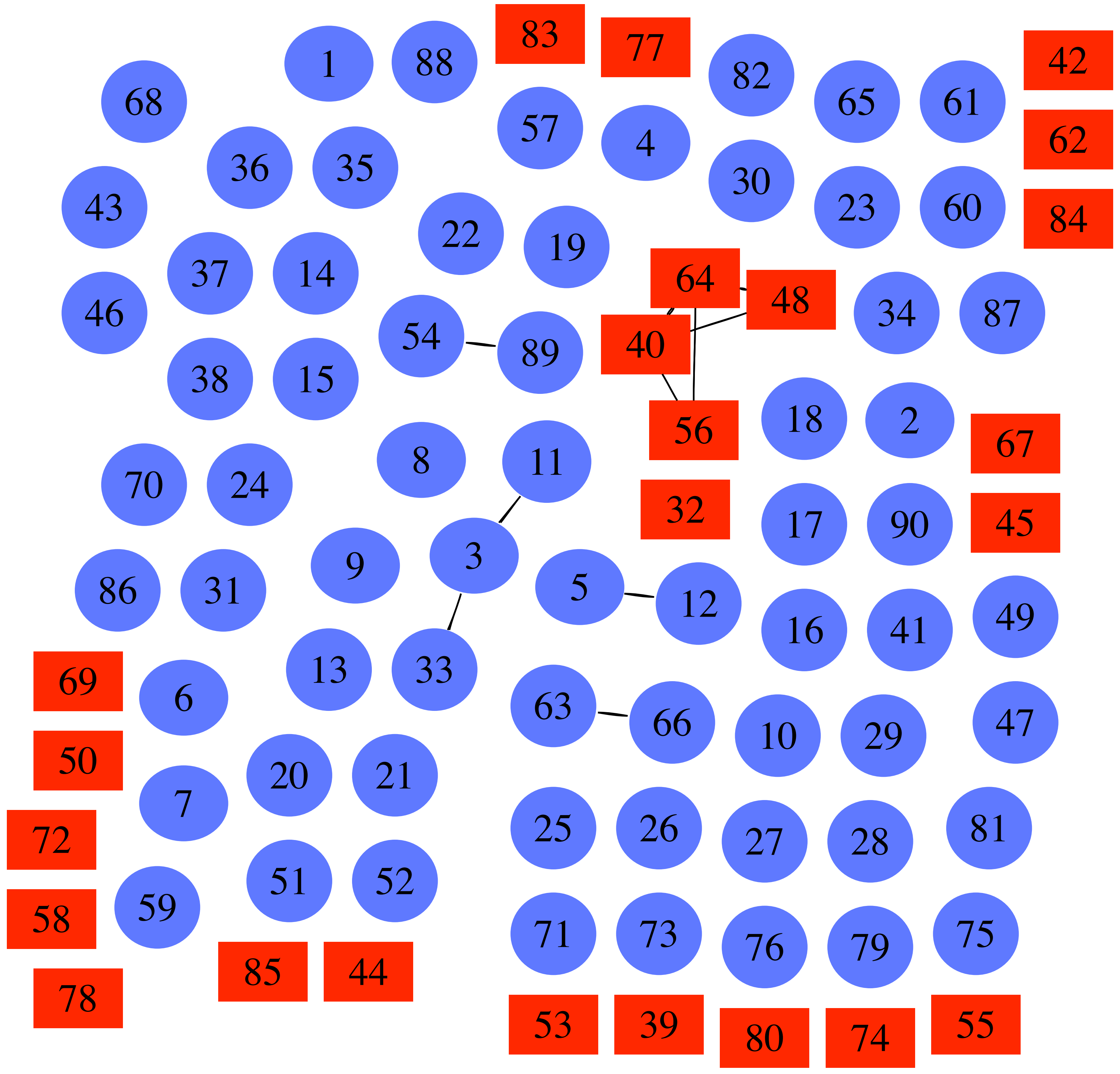
fewer connections



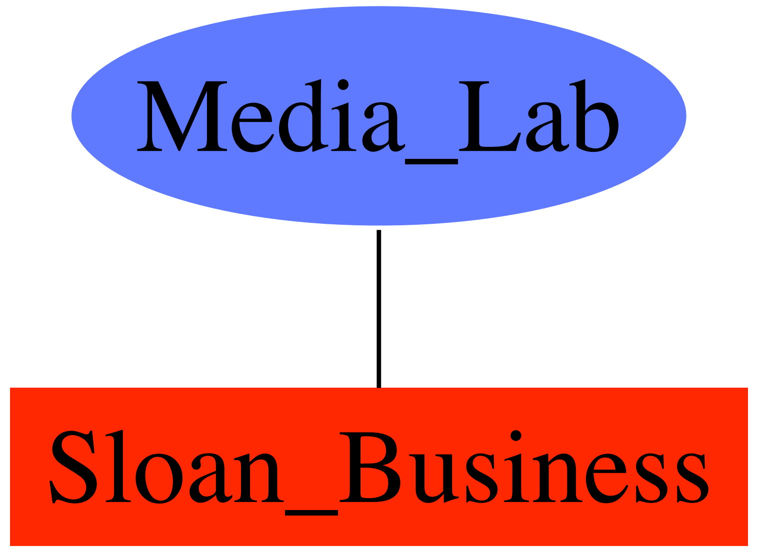
single day of proximities

Tuesday, 19 Oct 2004





very few connections



timing is everything?

- how long do edges last?
- how does structure vary over time?
- how stable is a local neighborhood?
- how does discrete time impact measures?

edge persistence

how long do edges last?

measure durations $\Pr(\Delta t)$

edge persistence

how long do edges last?

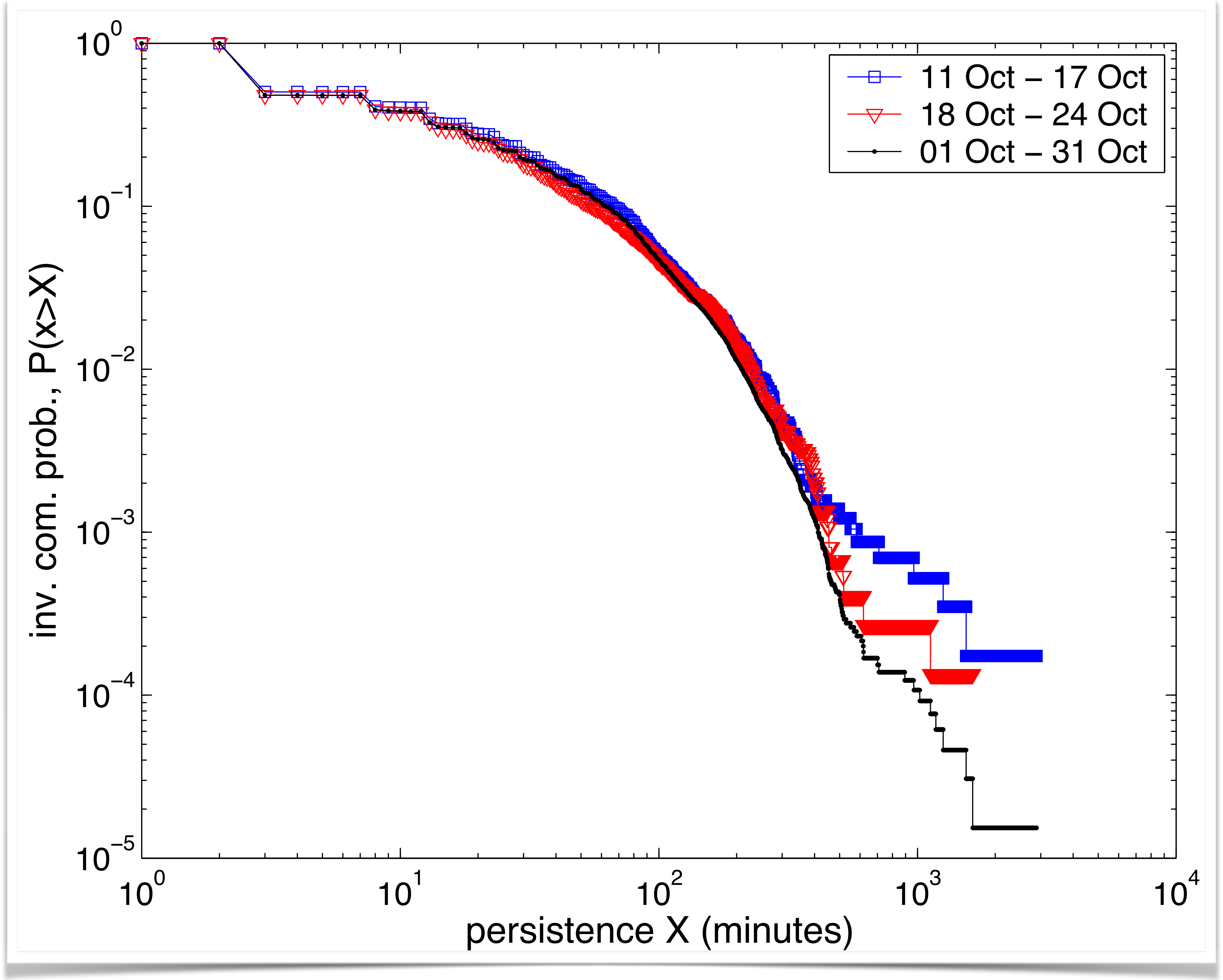
measure durations $\Pr(\Delta t)$

- month of October
- broad distribution

$\langle \Delta t \rangle = 22.8$ minutes

- changes at many time scales
- consistent up to

$\Delta t < 400$ minutes



network dynamics

how does structure vary over time?

vary aggregation window for snapshots

compute **mean degree** over time

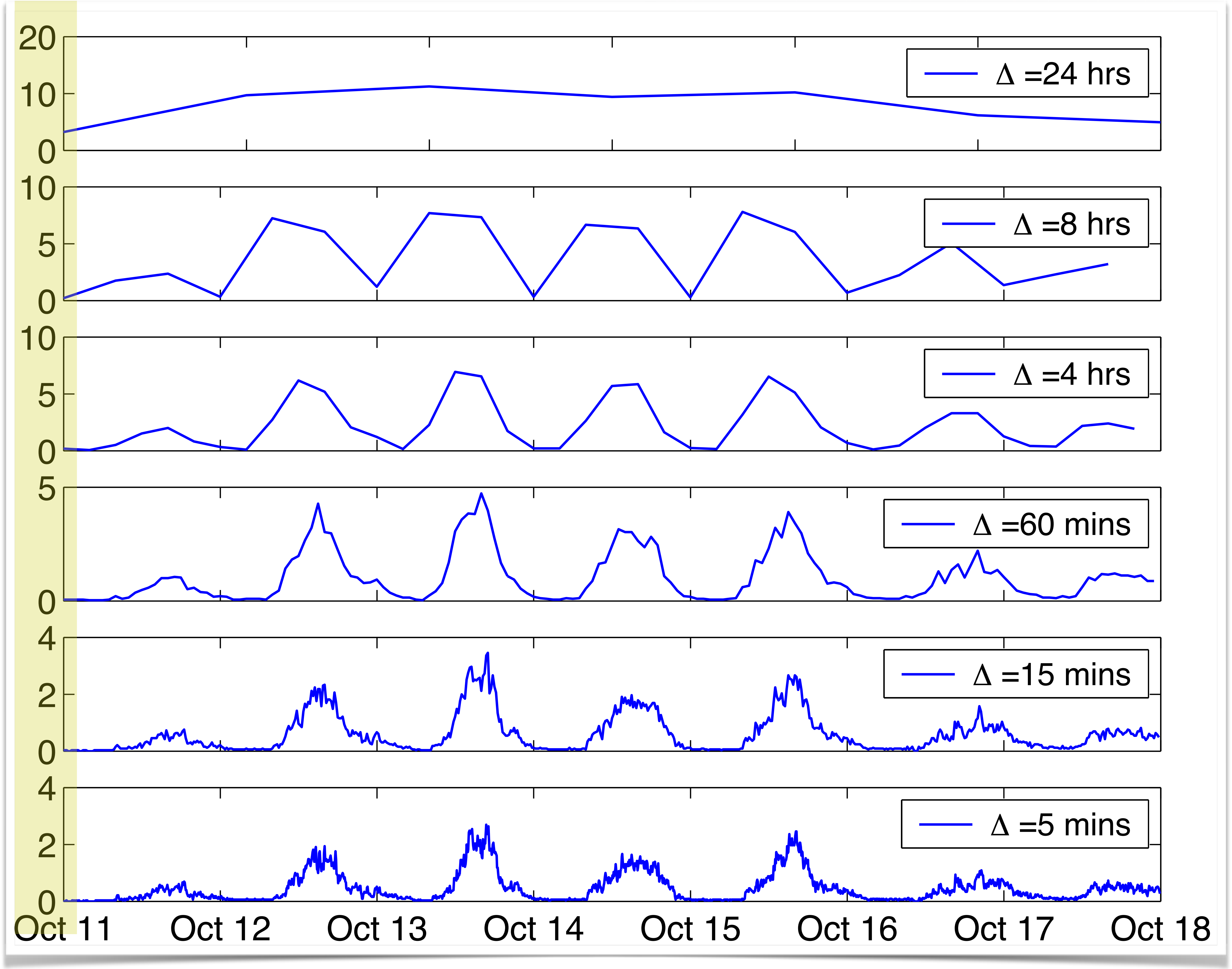
network dynamics

how does structure vary over time?

vary aggregation window for snapshots

compute **mean degree** over time

- one week of October
- highly periodic
- aggregation time matters



network dynamics

how stable are local neighborhoods?

vary aggregation window for snapshots

compute **adjacency correlation** over time

$$\gamma_j = \frac{\sum_{i \in N(j)} A_{ij}^{(x)} A_{ij}^{(y)}}{\sqrt{\sum_{i \in N(j)} A_{ij}^{(x)} \sum_{i \in N(j)} A_{ij}^{(y)}}}$$

for two adjacency matrices $A^{(x)}, A^{(y)}$

measures similarity among neighbors observed in either network

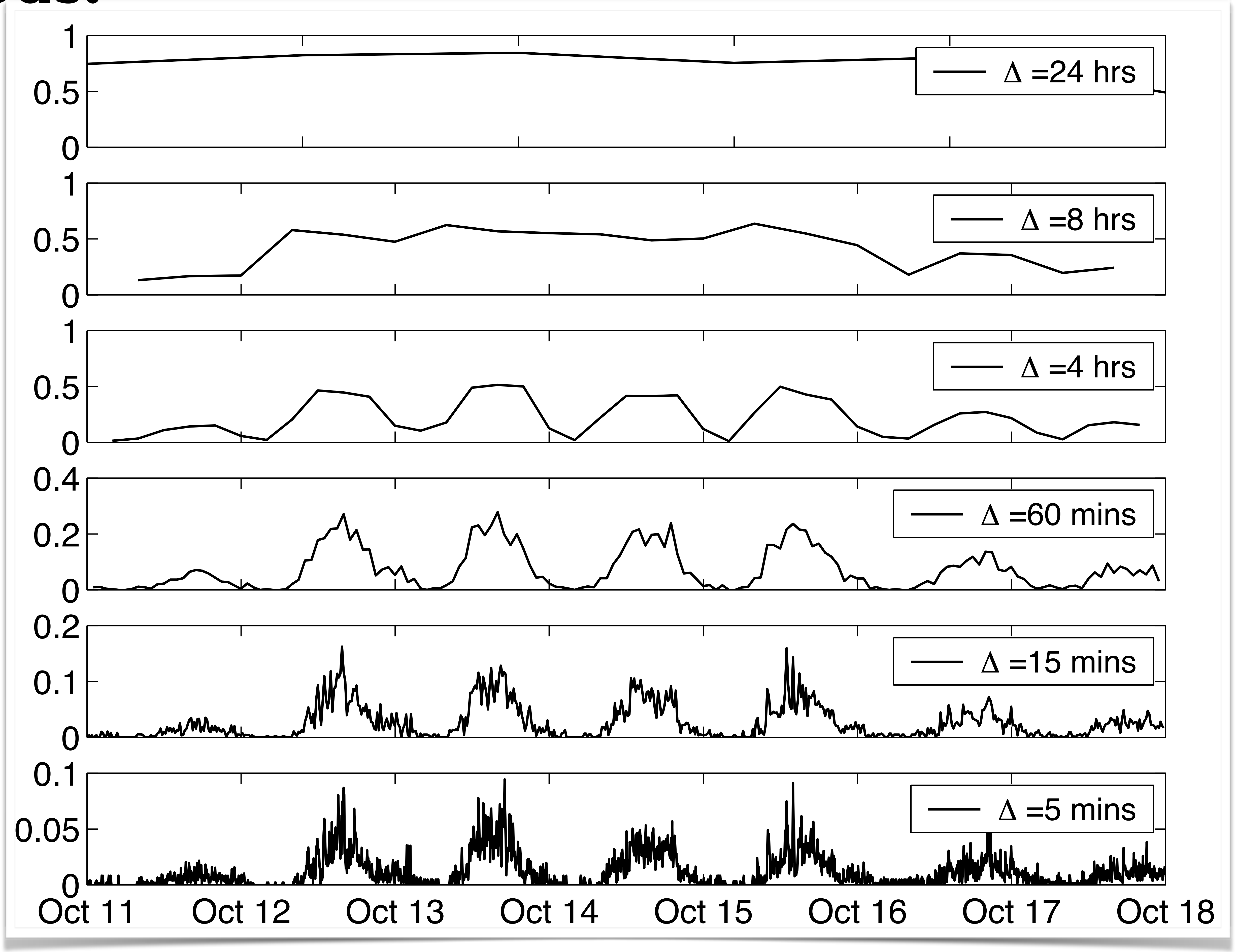
average overlap = mean value $\langle \gamma \rangle$

network dynamics

how stable are local neighborhoods?

vary aggregation window for snapshots

compute **adjacency correlation** over time



- one week of October
- highly consistent neighborhoods
- daily / weekly periodicity
- aggregation time matters

network dynamics

how does discrete time impact measures?

vary aggregation window for snapshots

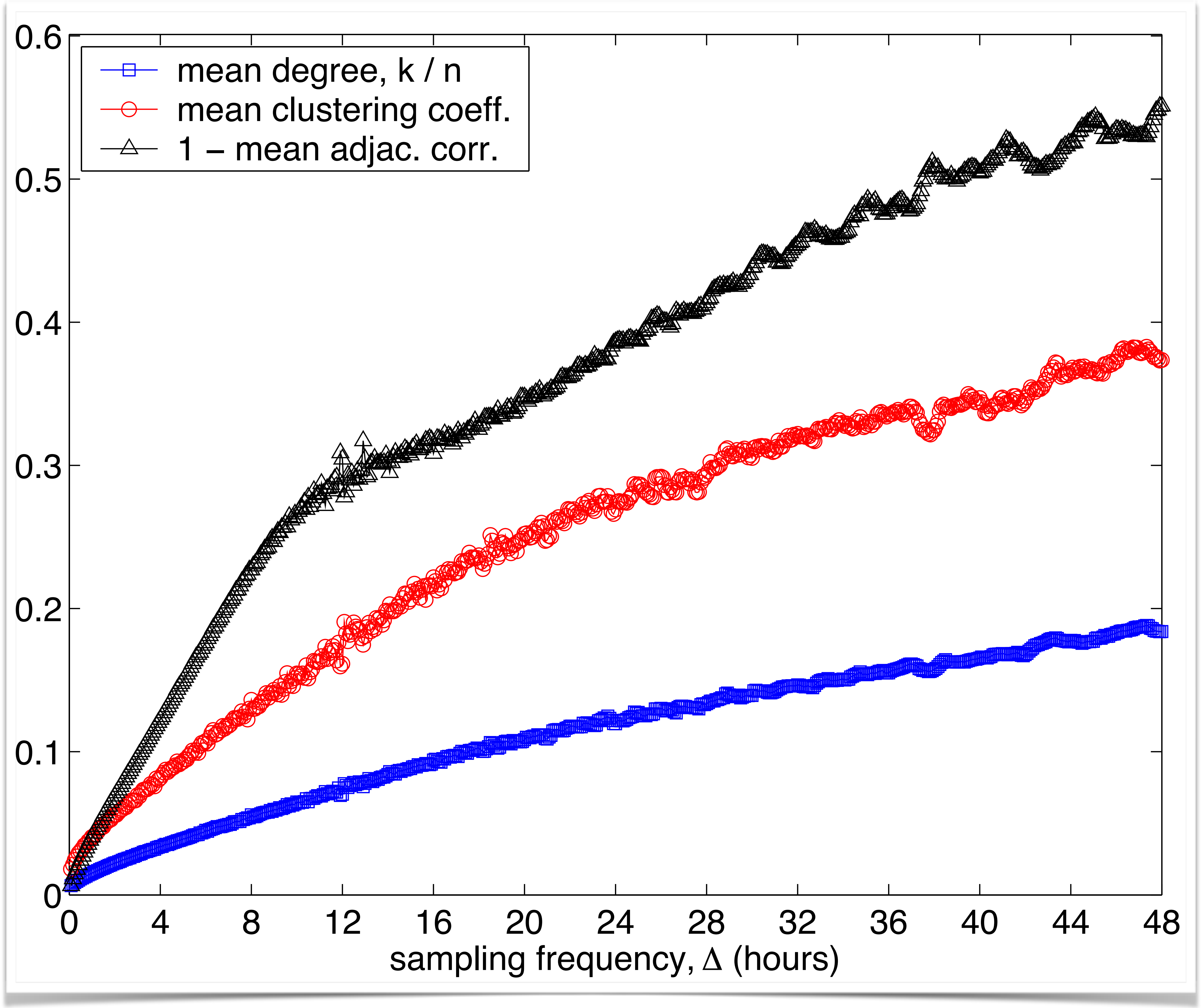
compute **summary statistics**

network dynamics

how does discrete time impact measures?

vary aggregation window for snapshots

compute **summary statistics**



- all statistics depend on aggregation duration
- choose a time scale = choose a statistical value

network dynamics

how to choose aggregation time?

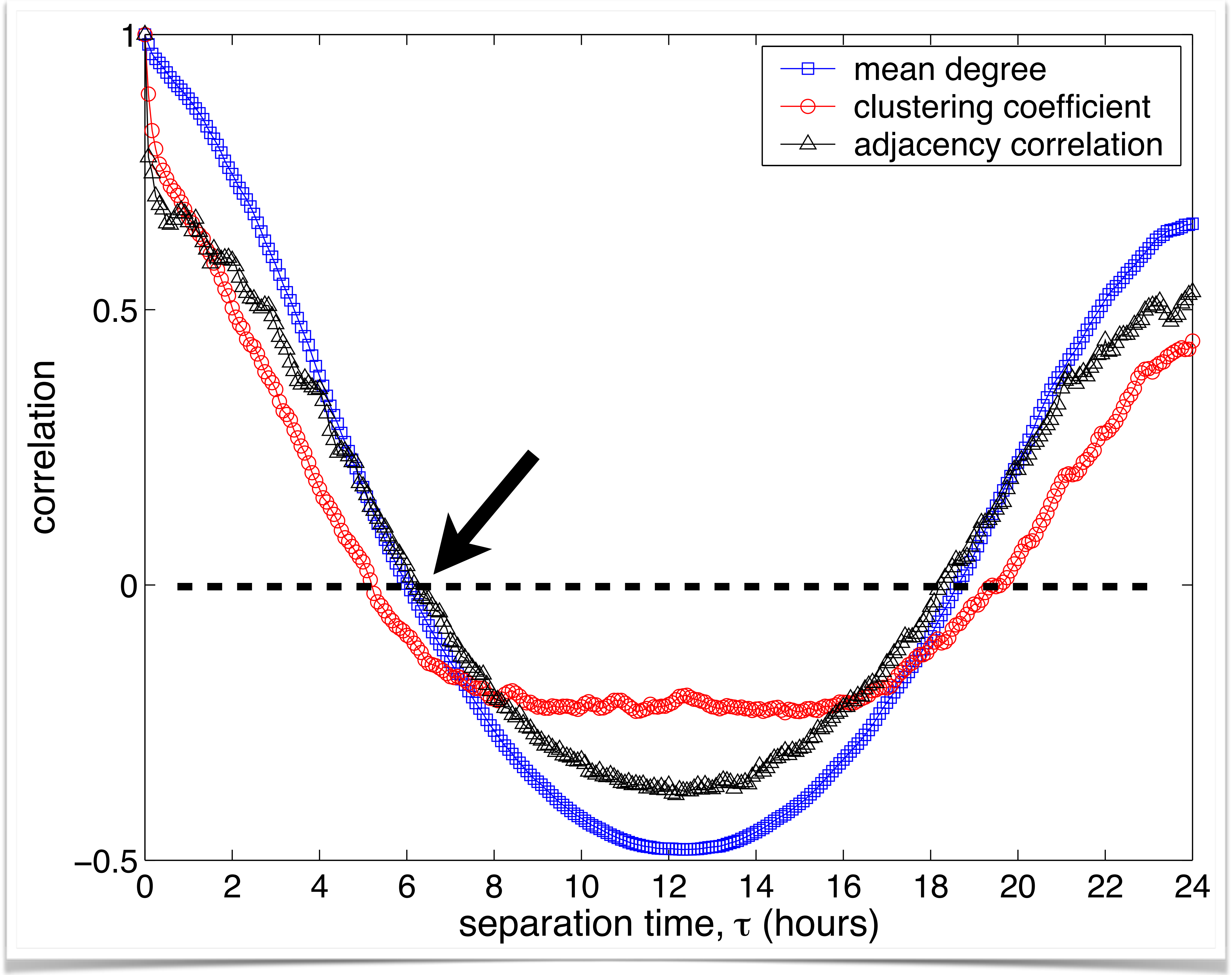
recall highly periodic dynamics

compute **autocorrelation function on network measures**

how to choose aggregation time?

recall highly periodic dynamics

compute **autocorrelation function on network measures**



network dynamics

how to choose aggregation time?

recall highly periodic dynamics

use frequency spectrum to choose sampling rate

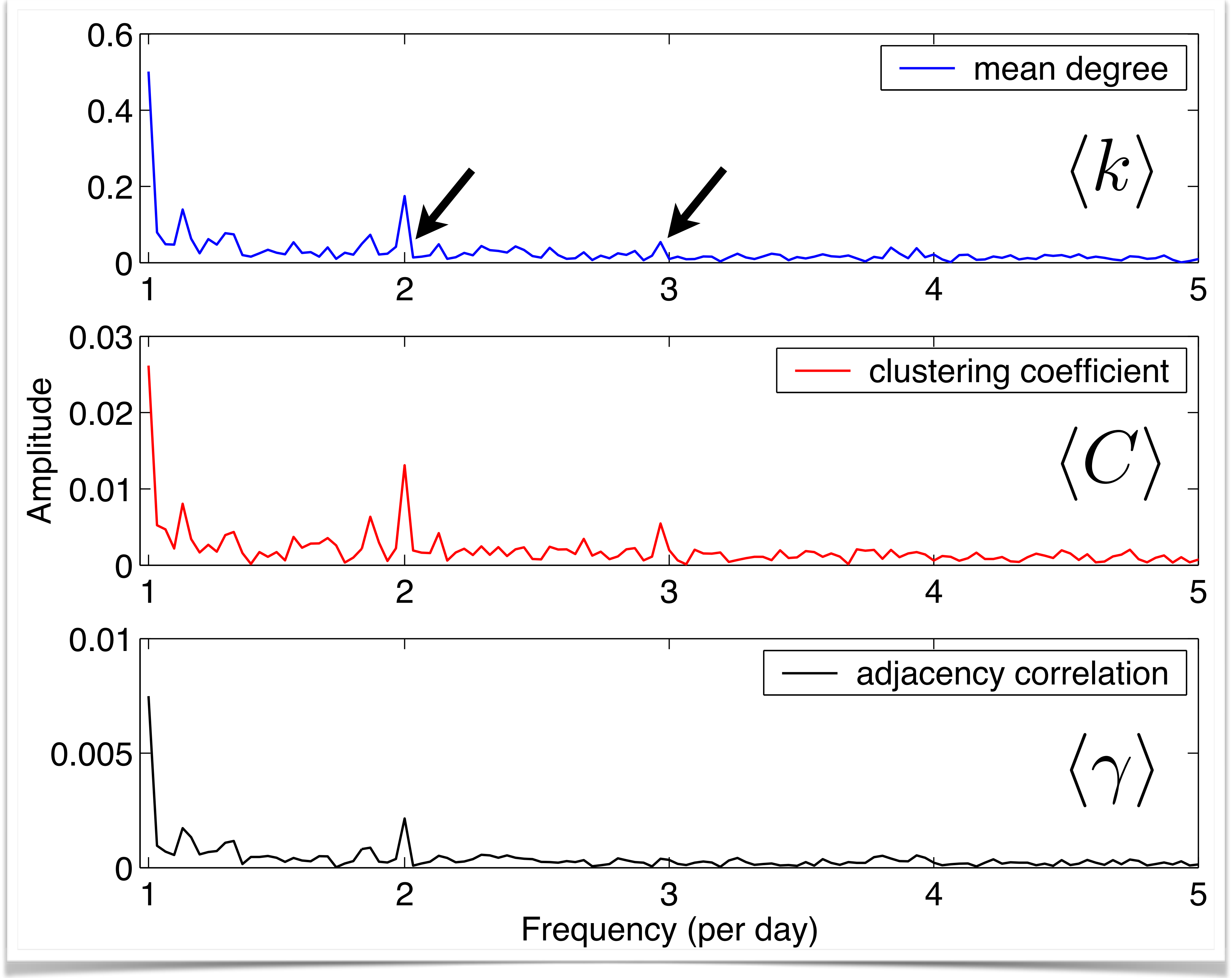
- periodicity at 1,2,3 samples per day
- Nyquist rate

$$\Delta_{\text{nat}} \simeq 4 \text{ hours}$$

degree $\langle k \rangle_{\text{nat}} = 2.24$

triangles $\langle C \rangle_{\text{nat}} = 0.084$

adj. corr. $\langle \gamma \rangle_{\text{nat}} = 0.88$



other ideas

- temporal "reachability" and continuous-time methods
- different parts evolving at different rates
- generative models?
- densification dynamics?
- temporal anomalies
- etc.

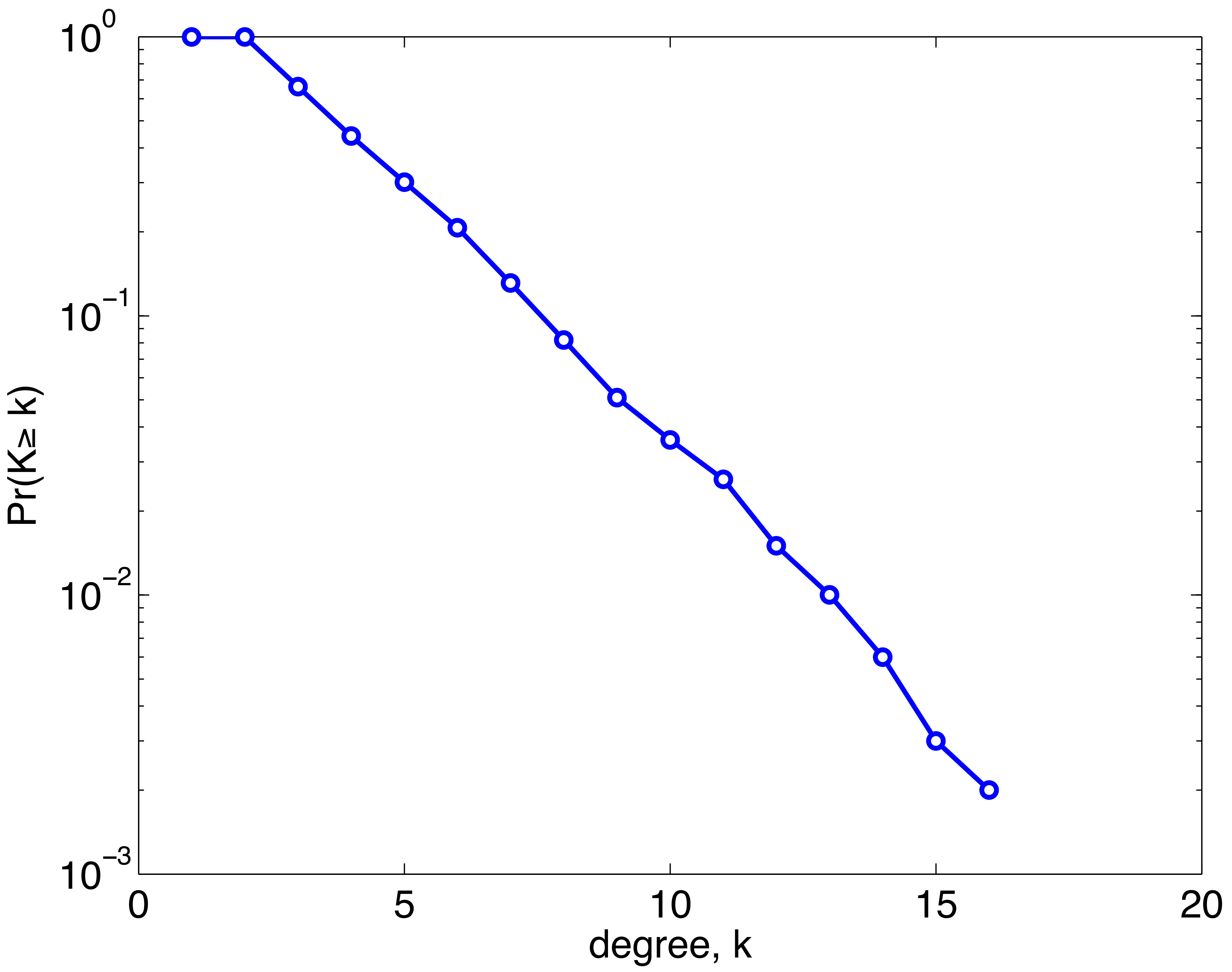
"densification laws"

- small-world phenomenon: diameter = $O(\log n)$
- how does diameter change in evolving networks?
- consider simple randomly-grown network:
  - at each time t, add vertex with degree c
  - attach each new edge via uniform attachment mechanism $\Pr(k_i \to k_i + 1) \propto \text{const.}$

- easy to simulate numerically

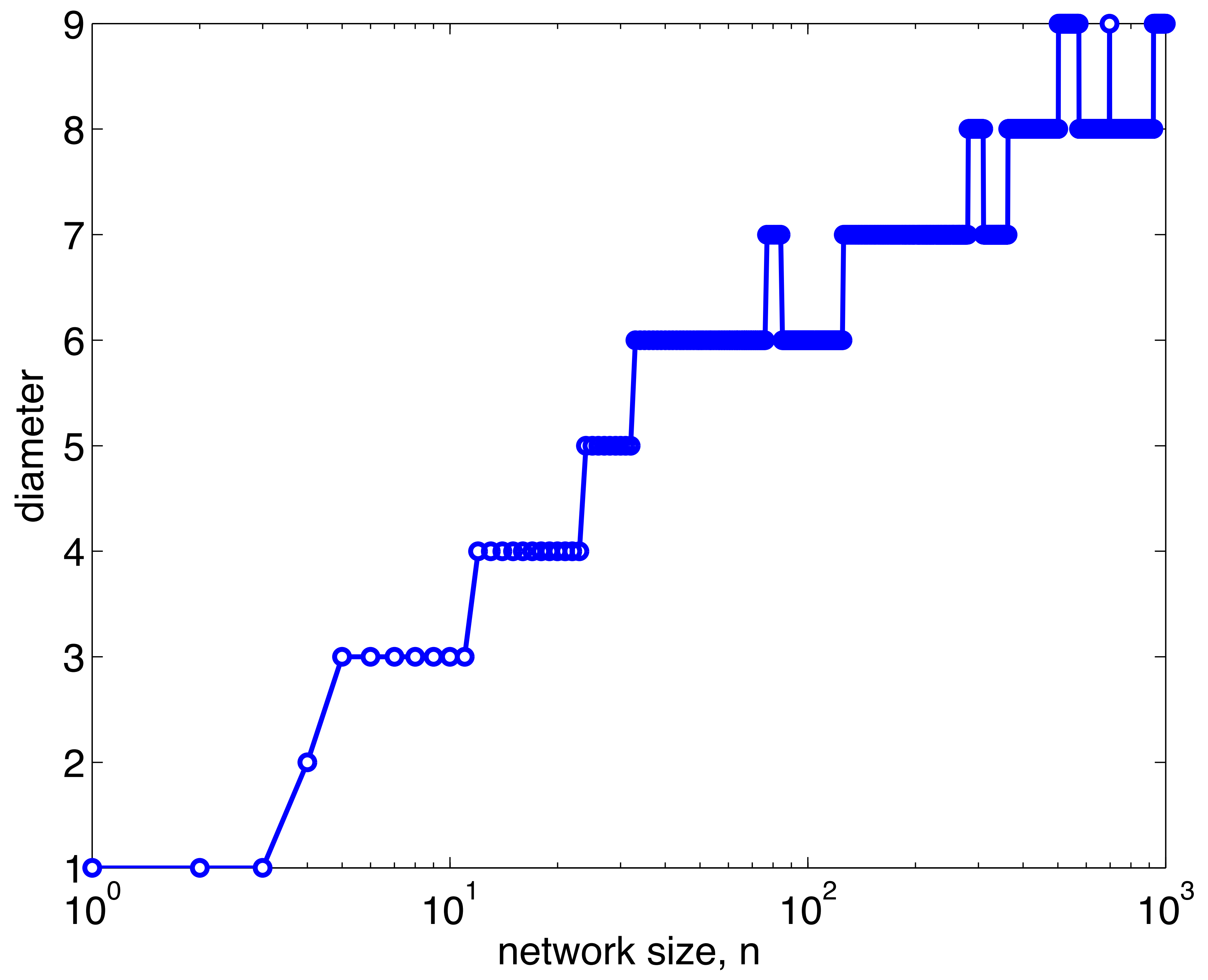
randomly grown networks

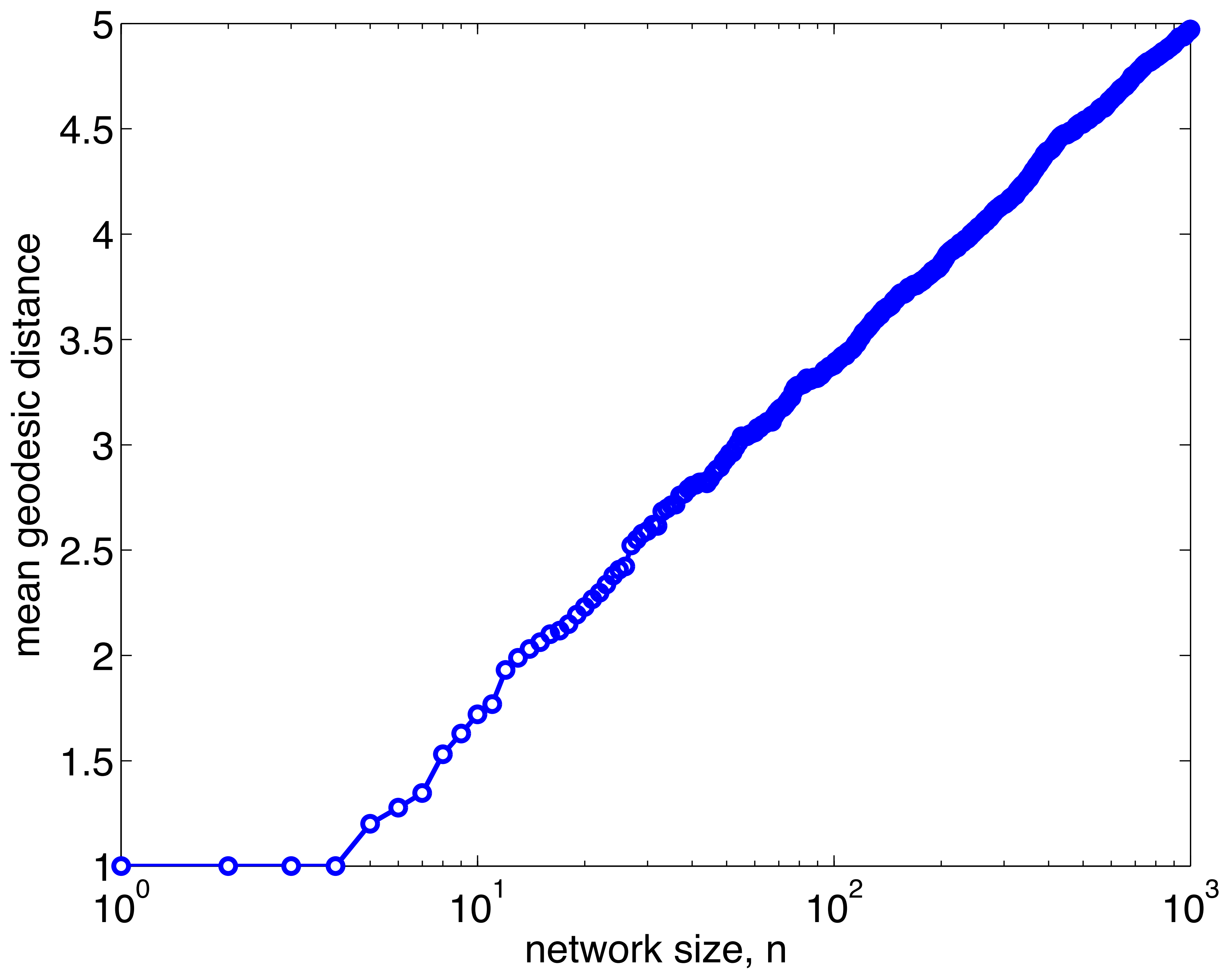
- choose $c = 2$ and $n = 10^3$



randomly grown networks

- choose $c = 2$ and $n = 10^3$ = increasing diameter $O(\log n)$





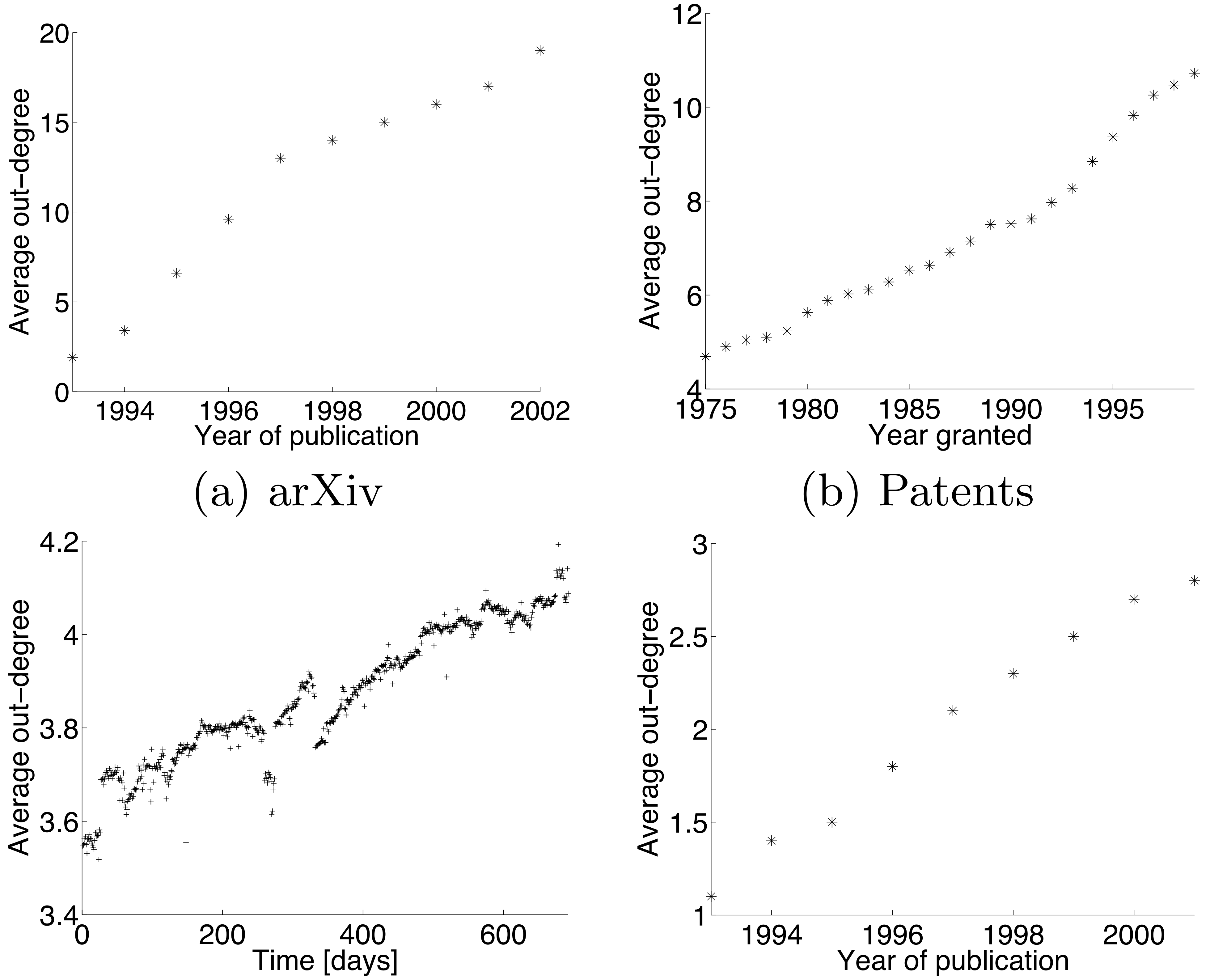
"densification laws"

- Leskovec, Kleinberg, Faloutsos (2005)
- examined 4 networks:
  - citation network (from arxiv.org),
  - US patents citation networks (from NBER)
  - Autonomous Systems (BGP) graph
  - author-paper bipartite network (from arxiv.org)

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations."
KDD (2005)

"densification laws"

- mean degree over time

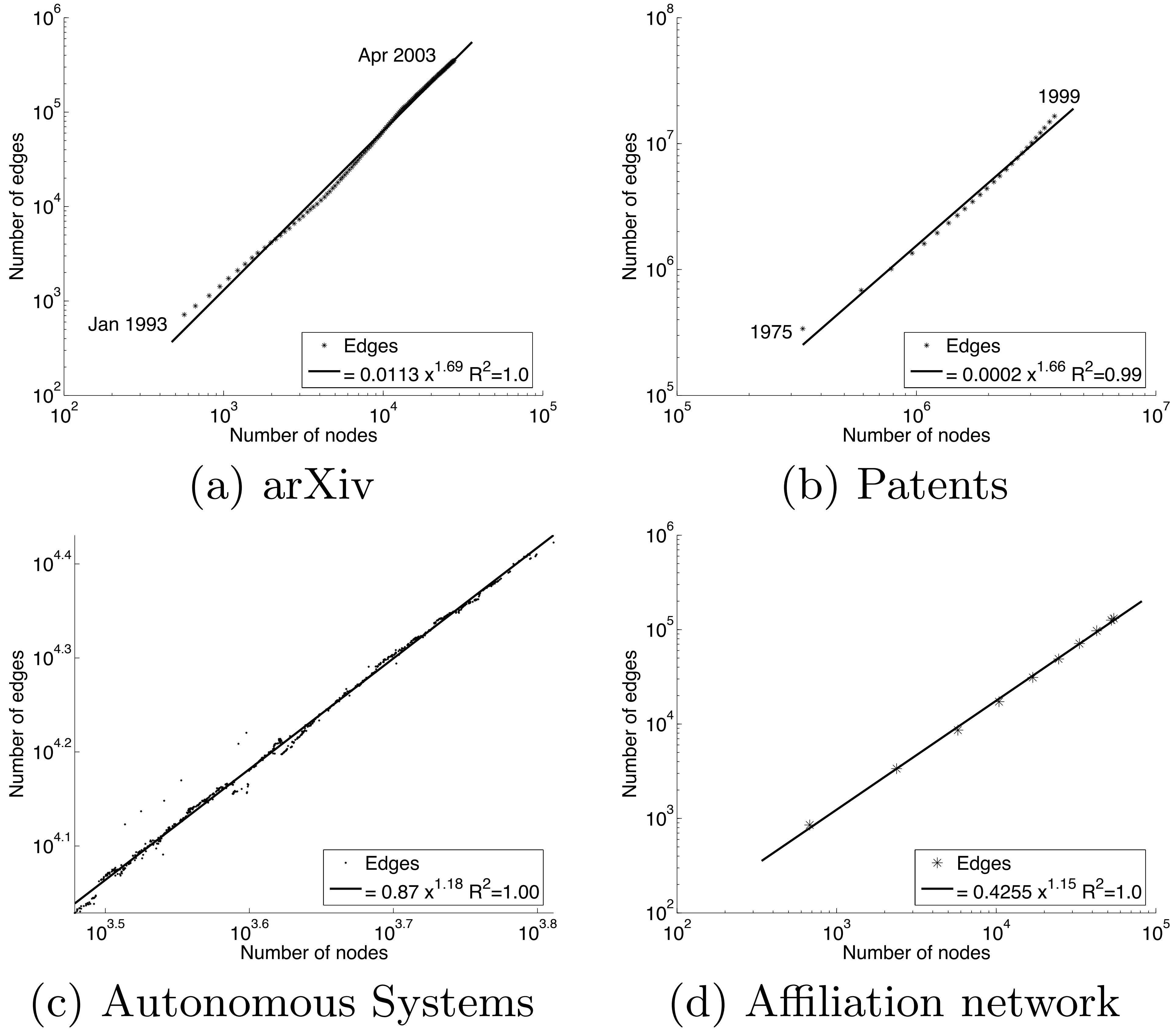


(a) arXiv (b) Patents (c) Autonomous Systems (d) Affiliation network

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations."
KDD (2005)

"densification laws"

- mean degree (again) over time



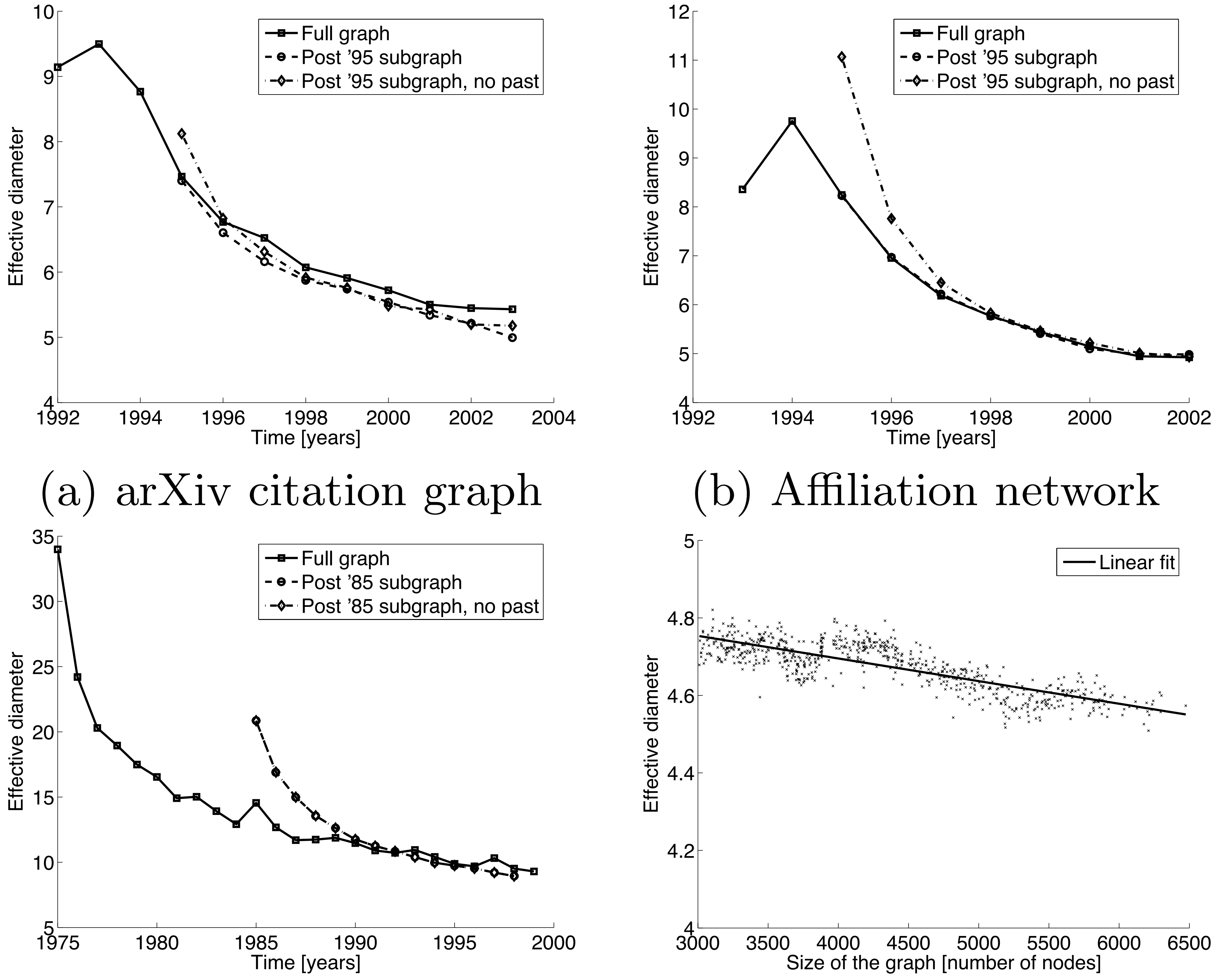
(a) arXiv (b) Patents

(c) Autonomous Systems (d) Affiliation network

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations." KDD (2005)

"densification laws"

- "effective" diameter over time (90% of node-pairs within this distance)



(a) arXiv citation graph (b) Affiliation network

(c) Patents (d) AS

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations." KDD (2005)

"densification laws"

- (effective) diameter is clearly shrinking in these networks
- mean degree is seems to be increasing, but slowly
- key questions:
  - is the mean degree really increasing? [need statistics]
  - can an increasing mean degree cause shrinking diameter in a growing network? [need a model]
  - how much does it need to increase? [more model]
  - what else could be going on with these networks? [need intuition, more data]

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations." KDD (2005)

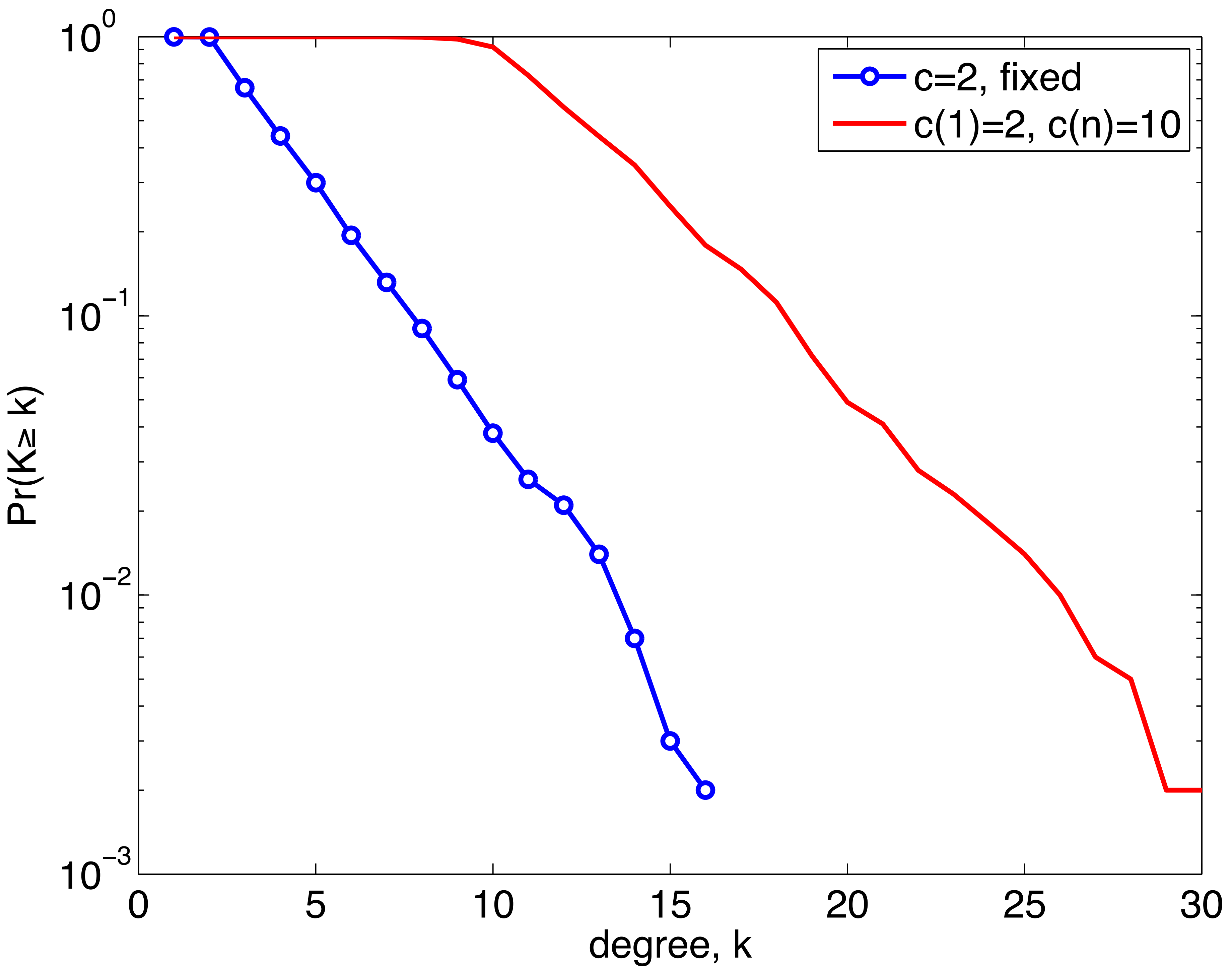
randomly grown networks (redux)

- how to get a shrinking diameter?
- idea: increase degree over time in our simple model
- simple randomly-grown network:
  - at each time t, add vertex with degree $c(t) \propto t$
  - attach each new edge via uniform attachment mechanism $\Pr(k_i \to k_i + 1) \propto \text{const.}$

- easy to simulate numerically

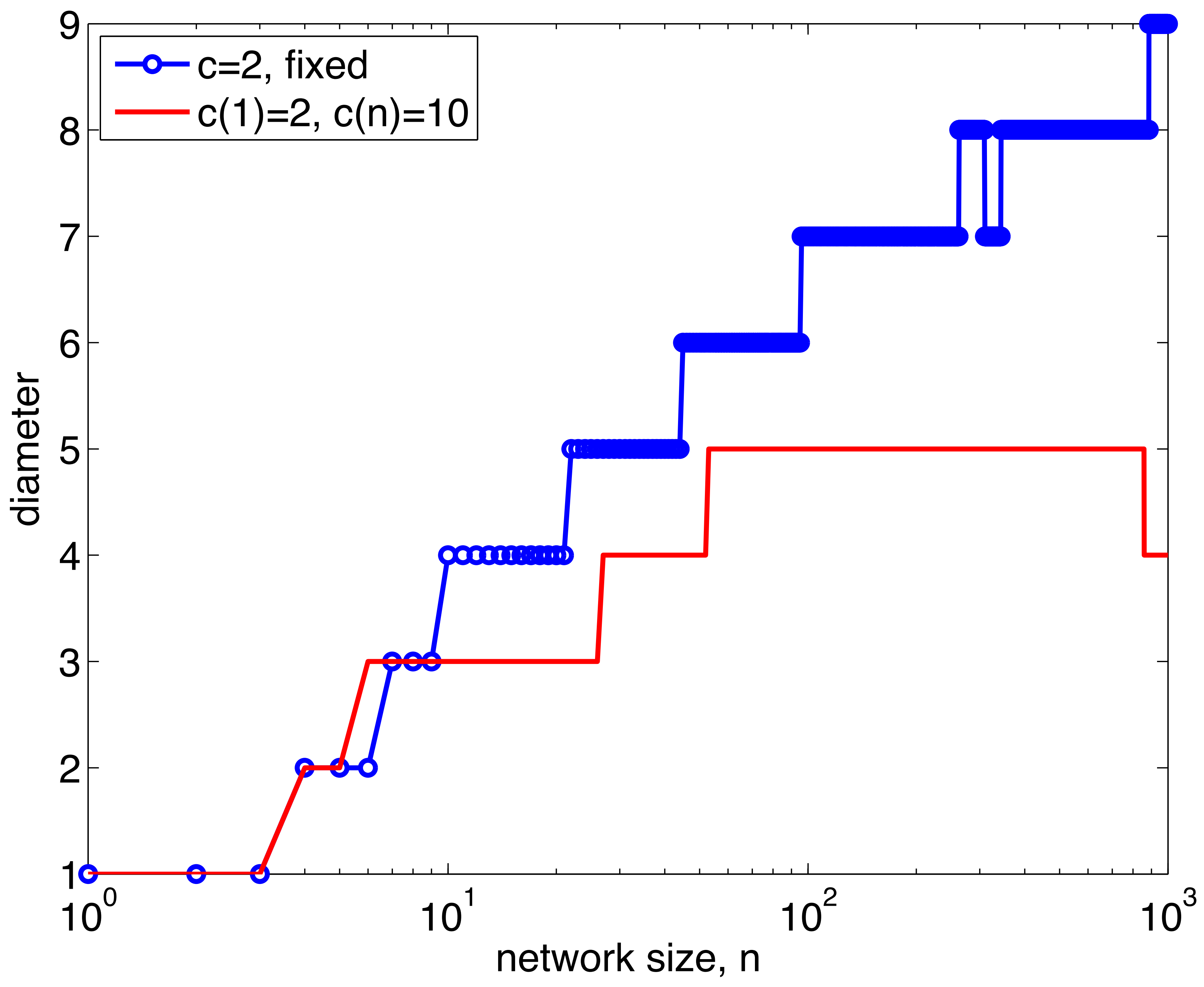
randomly grown networks (redux)

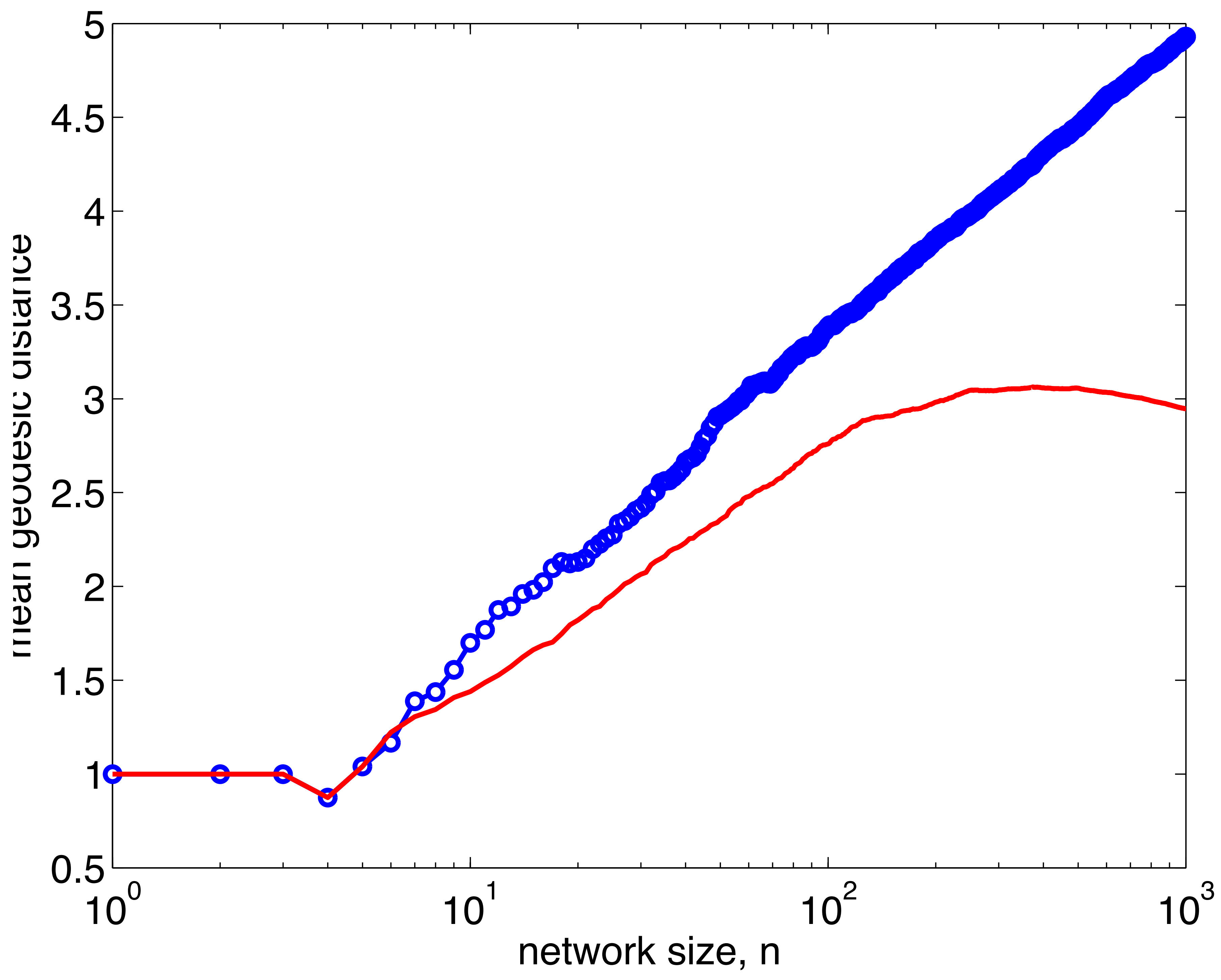
- choose $c(1) = 2$, growing linearly to $c(n) = 10$ for $n = 10^3$



randomly grown networks (redux)

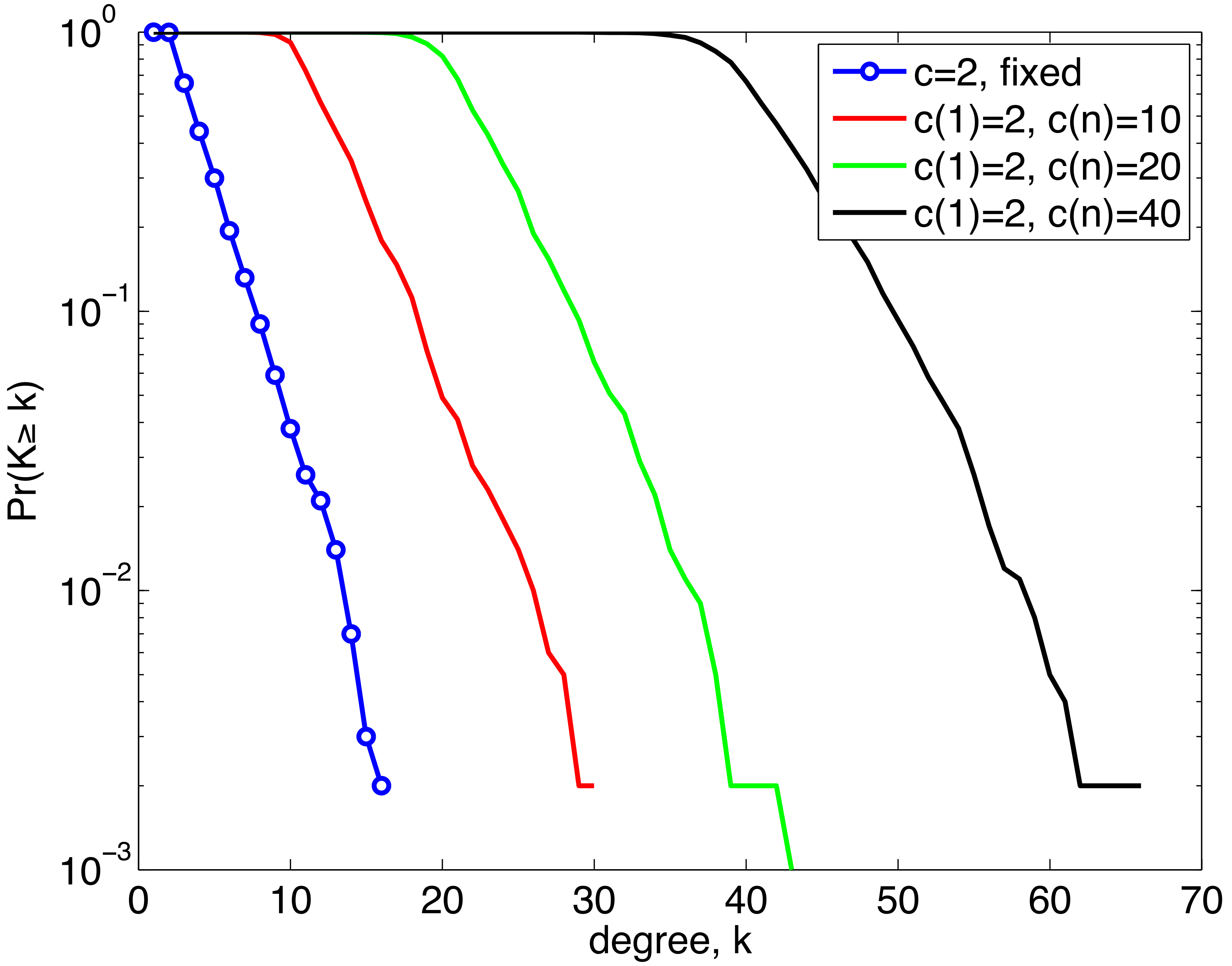
- choose $c(1) = 2$, growing linearly to $c(n) = 10$ for $n = 10^3$





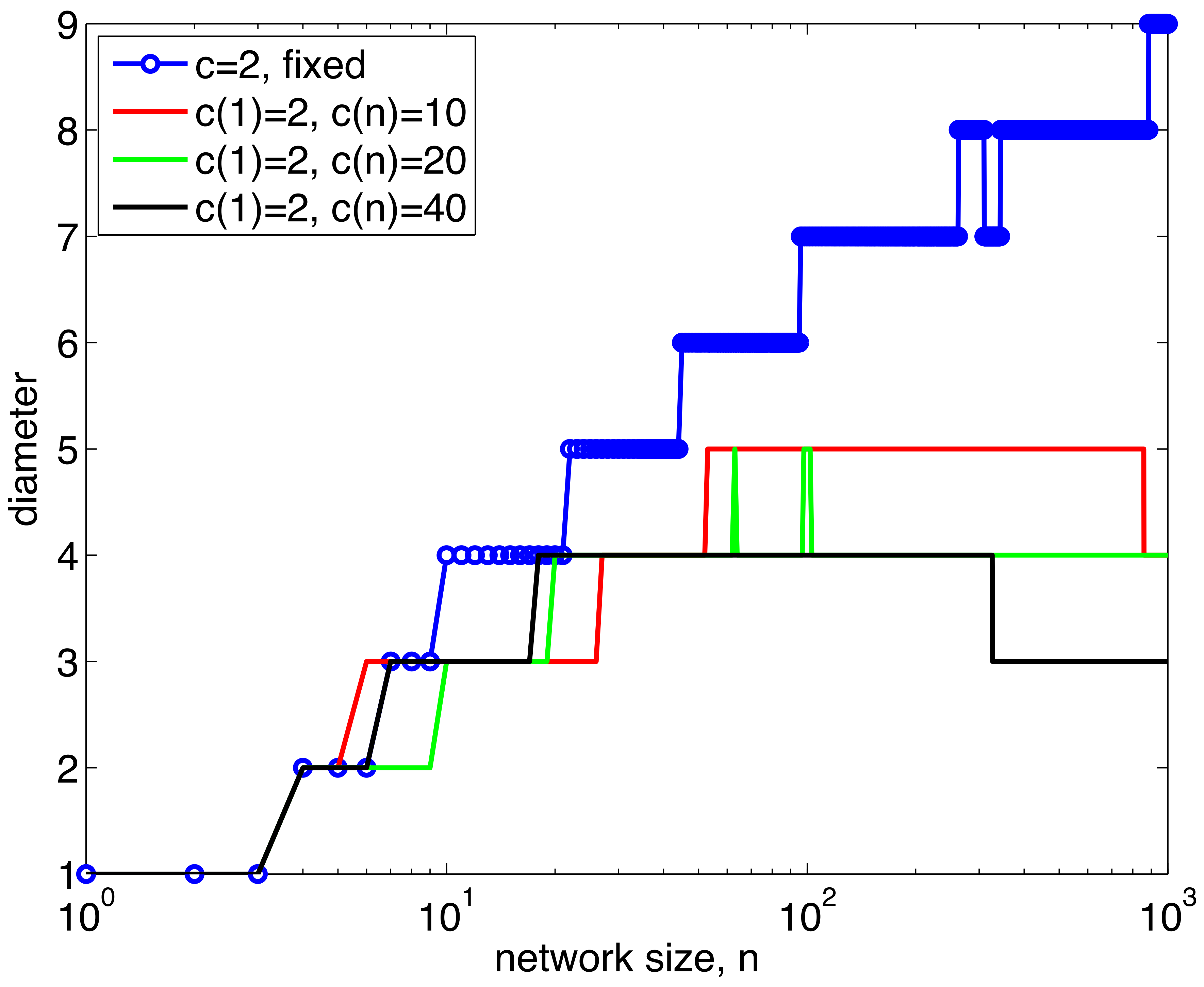
randomly grown networks (redux)

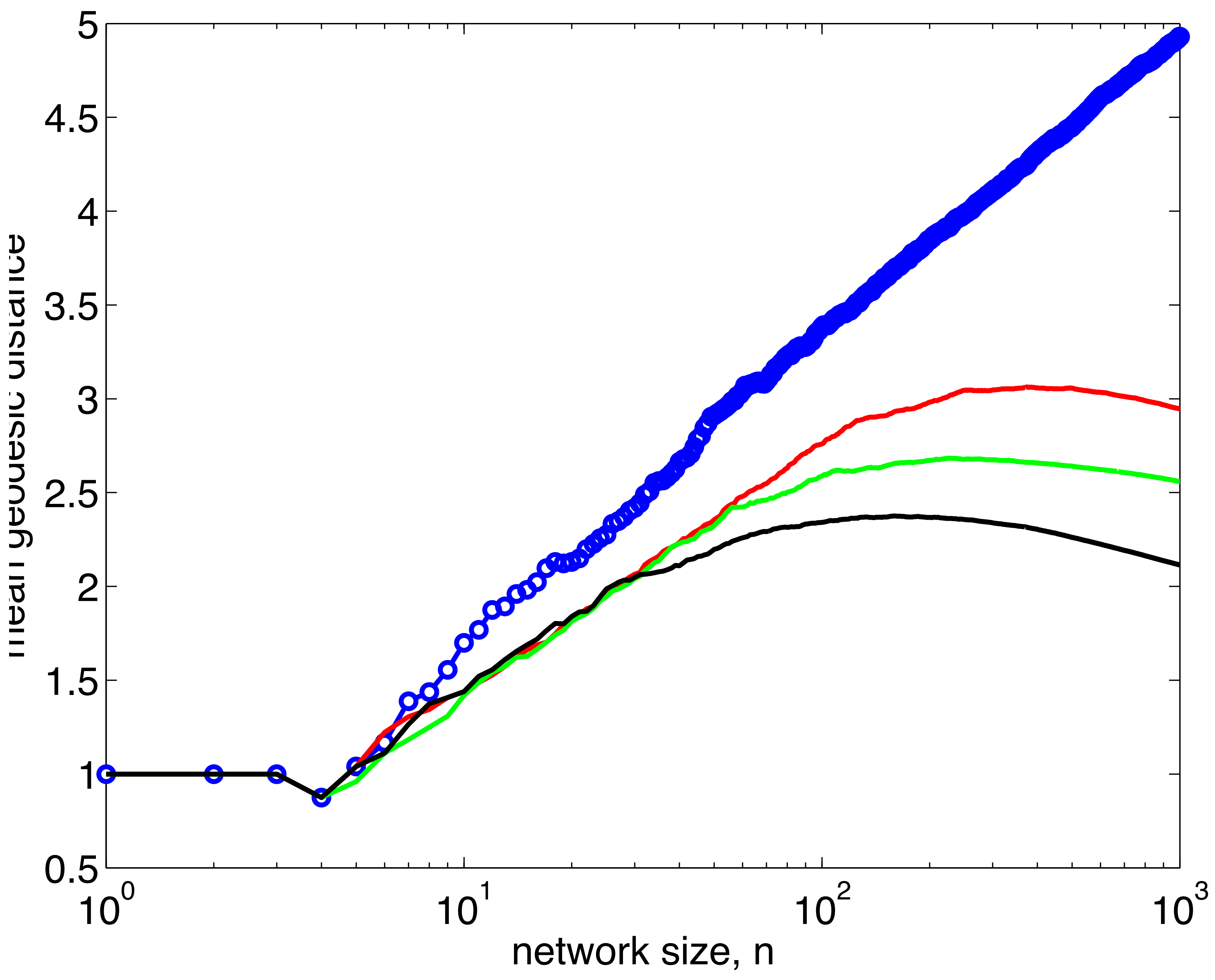
- choose $c(1) = 2$, growing linearly, for $n = 10^3$



randomly grown networks (redux)

- choose $c(1) = 2$, growing linearly, for $n = 10^3$





randomly grown networks (redux)

- very simple model!
- increasing mean degree can shrink the diameter
- but not *enough* shrinkage under this model
- how should we improve the model?
  - [what's "wrong" with our model?]

"densification laws"

- Leskovec, Kleinberg, Faloutsos propose
  - community guided attachment
    - [links form preferentially within a vertex's community]
  - "forest fire model" : a kind of recursive vertex-copy model

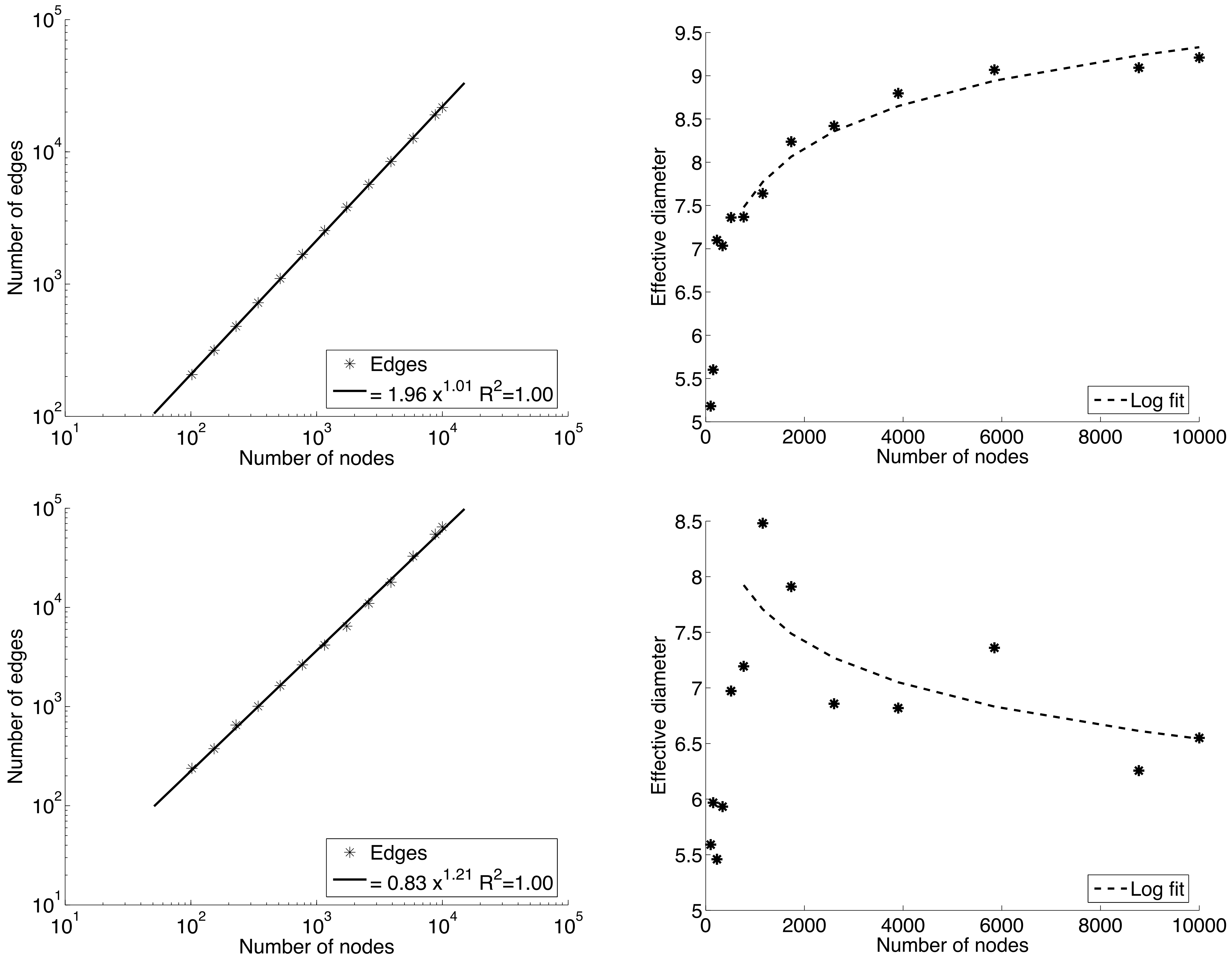
1. each new node u chooses uniformly random existing node v
2. links to each of v neighbors with probability p
3. repeat step 2 for each of linked* neighbor

Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations." KDD (2005)

* FF model is presented here as undirected. in LKF2005, FF model is directed, and in-links are selected r times less often than out-links in the burning

"densification laws"

- simulated results. FF model can produce both growing and shrinking diameters
- yields heavy-tailed degree distributions
- also makes lots of triangles



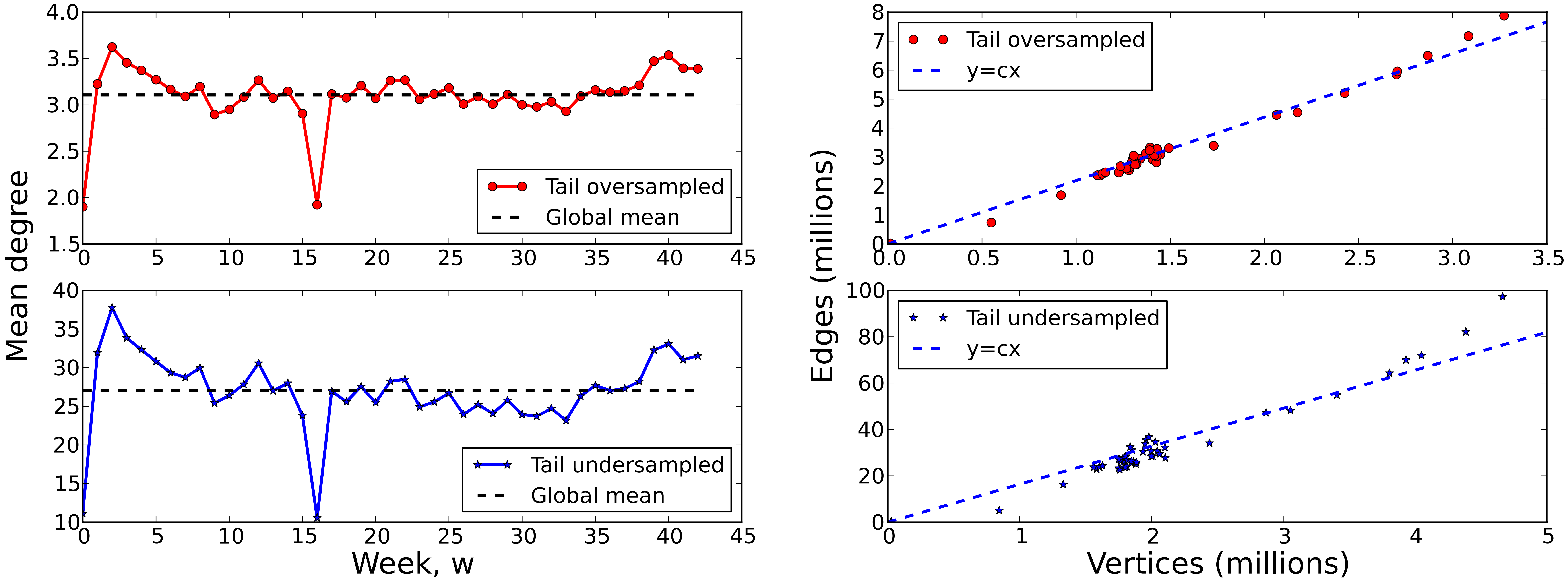
Leskovec, Kleinberg, Faloutsos, "Graphs over Time: Densification Laws, Shrinking Diameters, and Possible Explanations." KDD (2005)

are these results universal?

- densification "law"
- suggestion that this pattern occurs in all time-varying (growing?) networks

are these results universal?

- but it's not universal
- case in point: human friendship network (from Halo data)
- mean degree constant, so no densification behavior



are these results universal?

- but it's not universal
- case in point: human friendship network
  - (from Halo data)
- distances stable, so no shrinking diameter (despite ongoing turnover in vertices, edges)
- how can this be?
  - *idea: non-trivial edge maintenance costs mean degrees must remain bounded, no densification, no shrinking diameter*