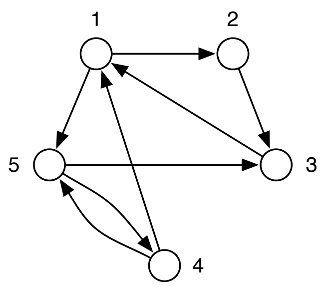
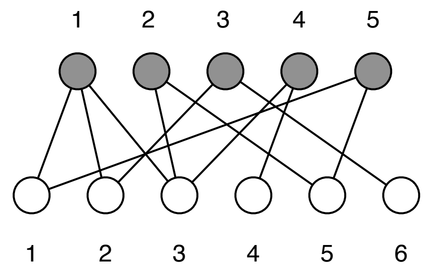
CSCI 5352 Network Analysis and Modeling Fall 2020
Prof. Daniel Larremore
Problem Set 1

1. (12 pts) Consider the following two networks:



(A)



(B)

   (a) (3 pts) Give the adjacency matrix for network (A).
   (b) (3 pts) Give adjacency list for network (A).
   (c) (6 pts) Give adjacency matrices for both one-mode projections of network (B).

2. (15 pts) Let $\mathbf{A}$ be the adjacency matrix of a simple graph (unweighted, undirected edges with no self-loops) and $\mathbf{1}$ be the column vector whose elements are all 1. In terms of these quantities, multiplicative constants and simple matrix operations like transpose and trace, write expressions for

   (a) (3 pts) the vector $\mathbf{k}$ whose elements are the degrees k_i of the vertices
   (b) (3 pts) the number m of edges in the network
   (c) (5 pts) the matrix $\mathbf{N}$ whose elements N_{ij} is equal to the number of common neighbors of vertices i and j
   (d) (4 pts) the total number of triangles in the network, where a triangle means three vertices, each connected by edges to both of the others.

3. (10 pts) Consider a bipartite network, with its two types of vertices, and suppose there are n_1 vertices of type 1 and n_2 vertices of type 2. Show that the mean degrees c_1 and c_2 of the two types are given by

$$c_2 = \frac{n_1}{n_2} c_1 \ .$$