8/28/2020 Week 1 simple structural measures

"degrees"

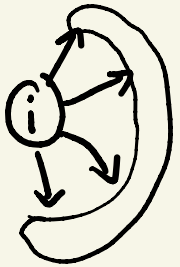
degree of a node: # of connections that a node has.

Let k_i be the degree of node i

$$k_i = \sum_j A_{ij}$$

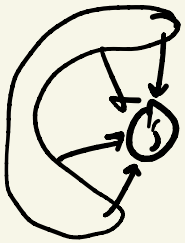
directed

A_{ij} $i \to j$



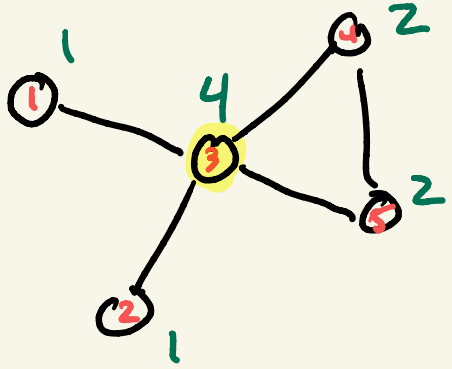
"out-degree"

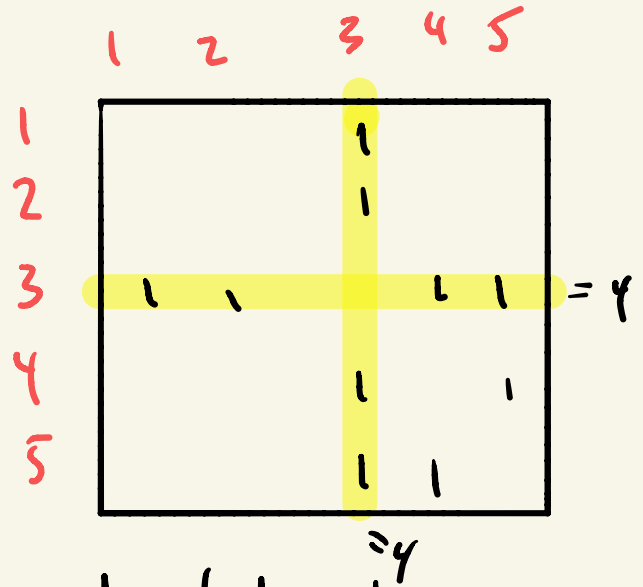
$$k_i^{out} = \sum_j A_{ij}$$



"in-degree"

$$k_i^{in} = \sum_i A_{ij}$$





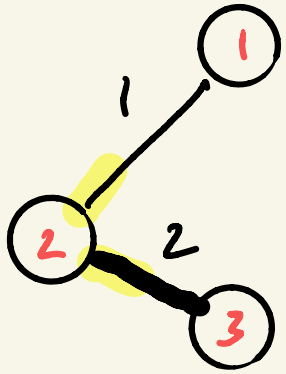
undirected net.

⇒

symmetric adj. mtx

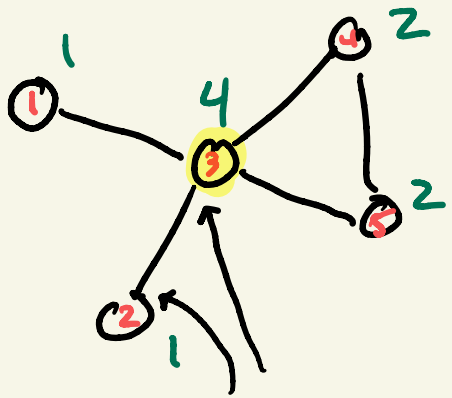
What about weighted?

	1	2	3
1	0	1	0
2	1	0	2
3	0	2	0



"weighted degree": 3 ← "strength" instead of degree.

"unweighted degree": 2



every edge has two ends

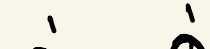
Let m = total # of edges in the network.

A_{ij} $i \to j$

directed only!

$$m = \sum_{i,j} A_{ij}$$

$$\sum_j \sum_i A_{ij} = \sum_i \underbrace{\sum_j A_{ij}}_{\text{adding up the whole matrix}} = \sum_i k_i = 2m$$



undirected only!

$$m = \frac{1}{2} \sum_{i,j} A_{ij}$$

Mean Degree

$$\langle k \rangle = \frac{1}{n}\sum_{i=1}^{n} k_i = \frac{2m}{n}$$

undirected.

(directed: $\frac{m}{n} = \langle k \rangle = \langle k^{in} \rangle = \langle k^{out} \rangle$)

typical average.

angle brackets.

out → in

mismatch!

\langle

\rangle

$$\sum_j k_j^{in} = \sum_j \sum_i A_{ij} = m = \sum_i \left(\sum_j A_{ij} \right) = \sum_i k_i^{out}$$

Density

$$\rho = \frac{\#\text{ edges}}{\#\text{ possible edges}} = \frac{m}{\binom{n}{2}} = \frac{2m}{n(n-1)} = \frac{\langle k \rangle}{(n-1)}$$

$\frac{n(n-1)}{2}$

binomial coeff.

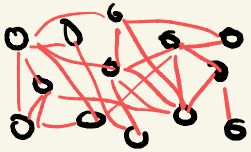
Binomial Coeff

$\binom{n}{k}$

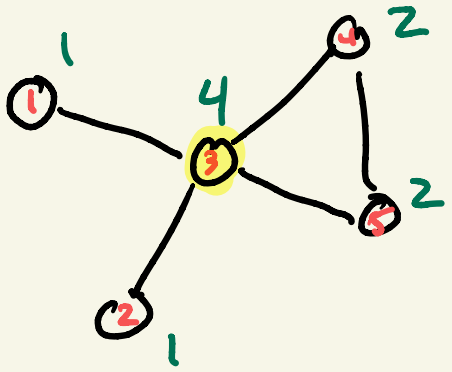
n choose k

How many ways can you choose k objects from n total.

n nodes



Degree Distribution = Prob. that a vertex, chosen unif. at random, has degree k.



k	$Pr(k)$
1	2/5
2	2/5
3	0
4	1/5

Empirical Networks:

generally { biological
social
technological ... }

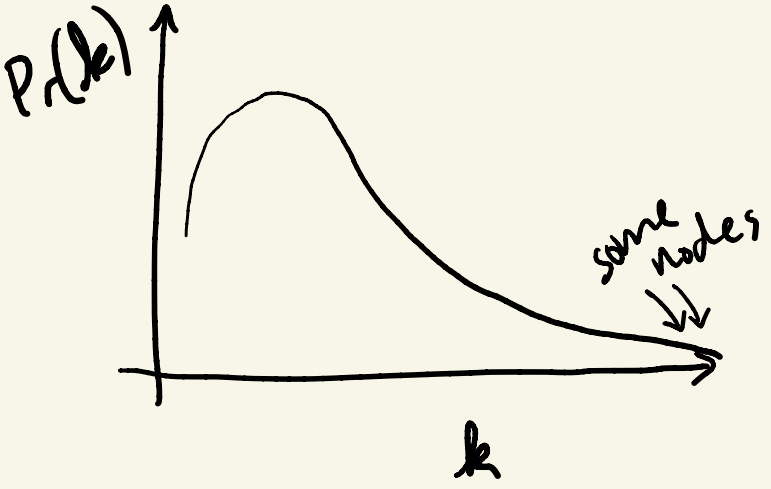
↓

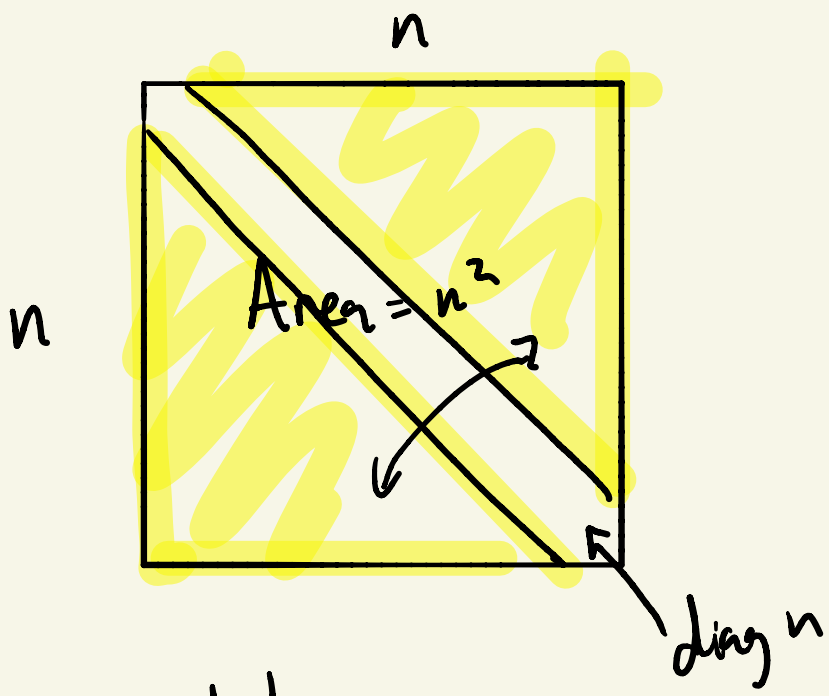
right-skewed
"heavy tailed"
degree distrib.

hubs, high-degree nodes!

[Lectures on Canvas.]

Page ↓ Zoom links.





n^2 total area

$$\frac{n^2 - n}{2}$$

$$\frac{n(n-1)}{2}$$