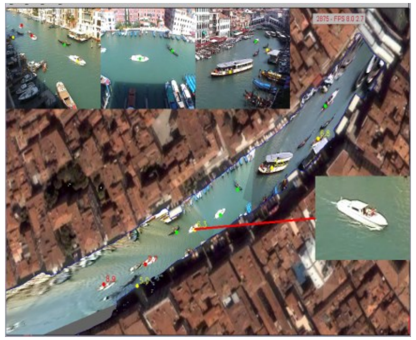
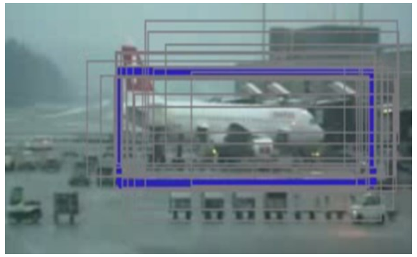
UNIVERSITÀ DEGLI STUDI
DELLA BASILICATA

Corso di Sistemi Informativi
A.A. 2018/19

Docente
Domenico Daniele Bloisi



Omografie



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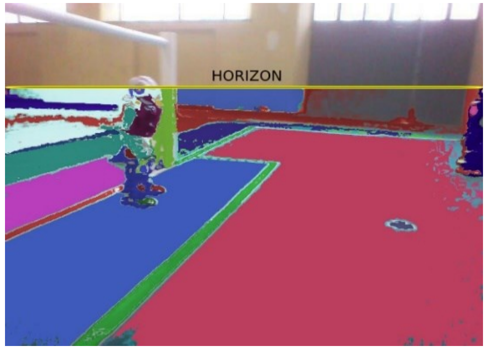
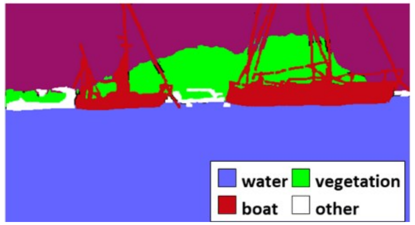




Riferimenti

- Queste slide sono adattate da
  Noah Snavely - CS5670: Computer Vision
  "Lecture 7: Transformations and warping"

- I contenuti fanno riferimento al capitolo 3 del libro
  "Computer Vision: Algorithms and Applications"
  di Richard Szeliski, disponibile al seguente indirizzo
  http://szeliski.org/Book/

Image alignment

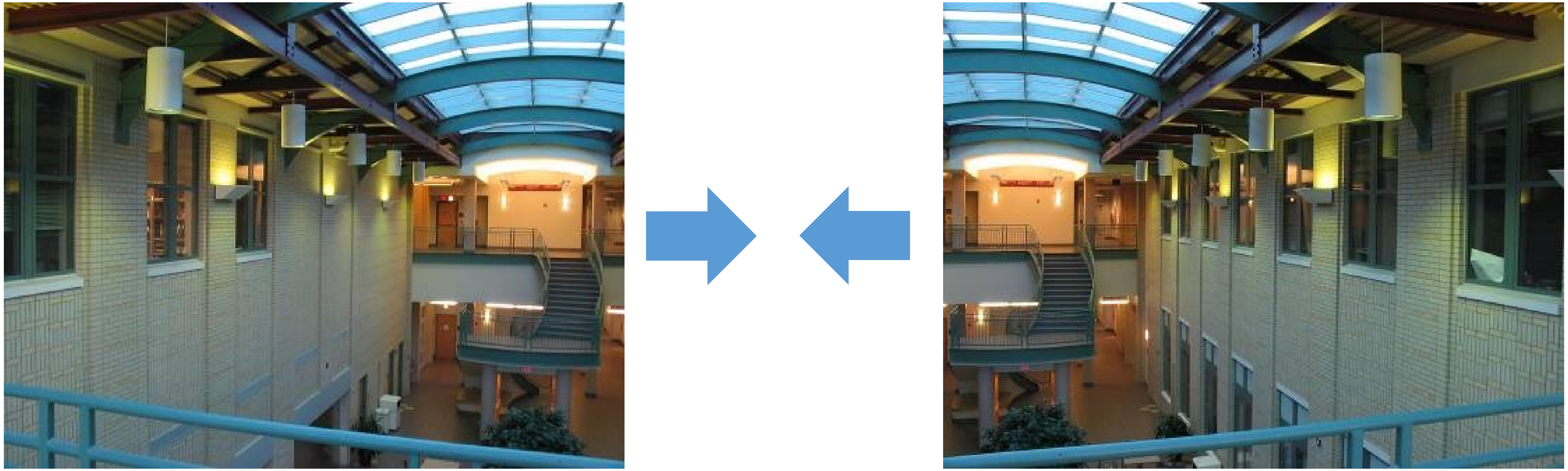


Image alignment

Similarity

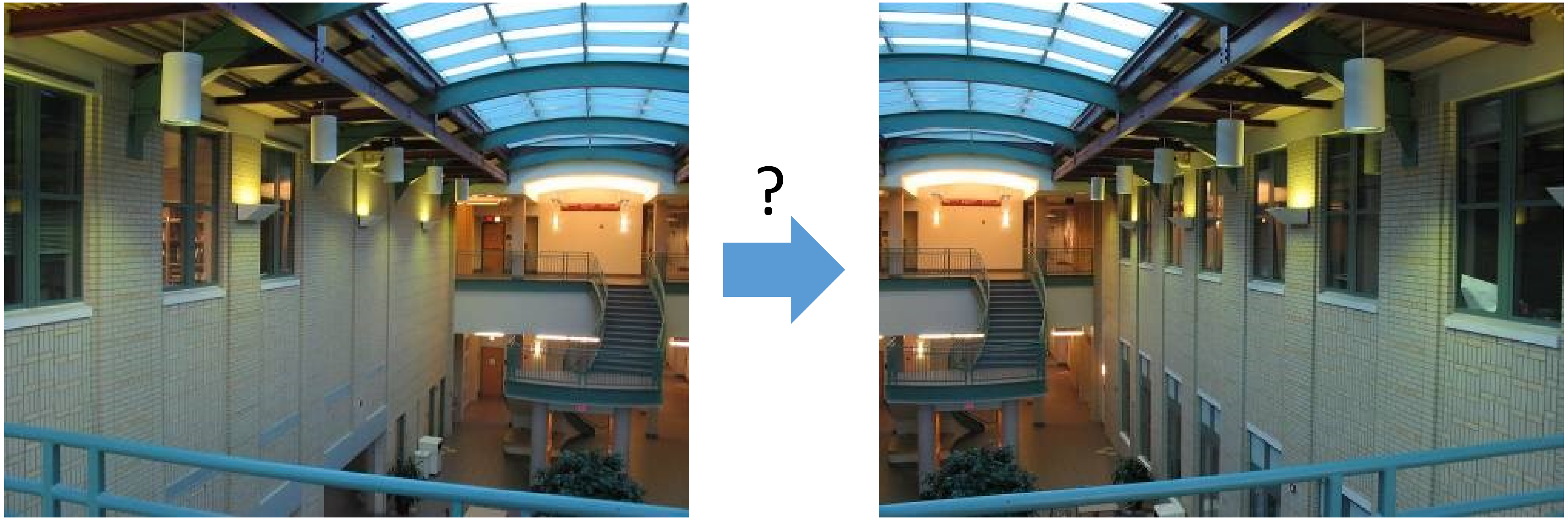
What is the geometric relationship between these two images?

?

Answer: Similarity transformation (translation, rotation, uniform scale)

Similarity?

What is the geometric relationship between these two images?



Similarity?

Very important for creating mosaics!

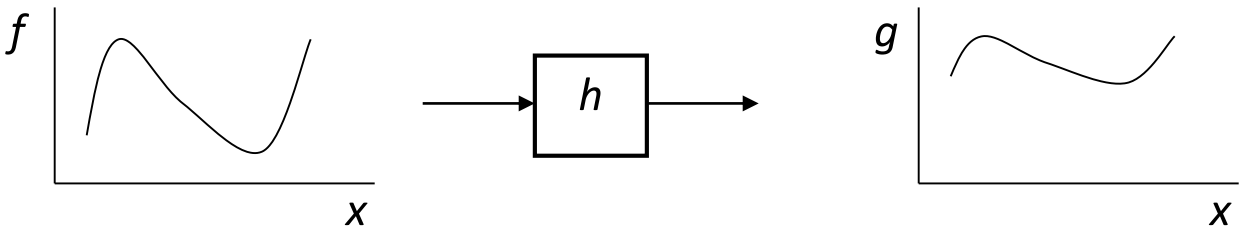
First, we need to know what this transformation is.

Second, we need to figure out how to compute it using feature matches.

Image Warping

- image filtering: change *range* of image

$$g(x) = h(f(x))$$



- image warping: change *domain* of image

$$g(x) = f(h(x))$$

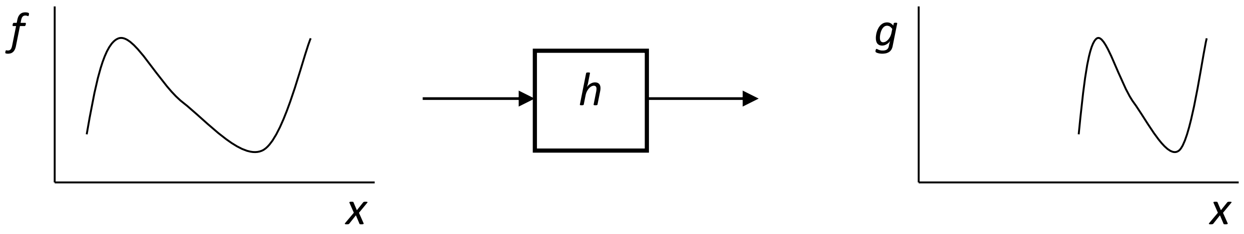


Image Warping

- image filtering: change *range* of image

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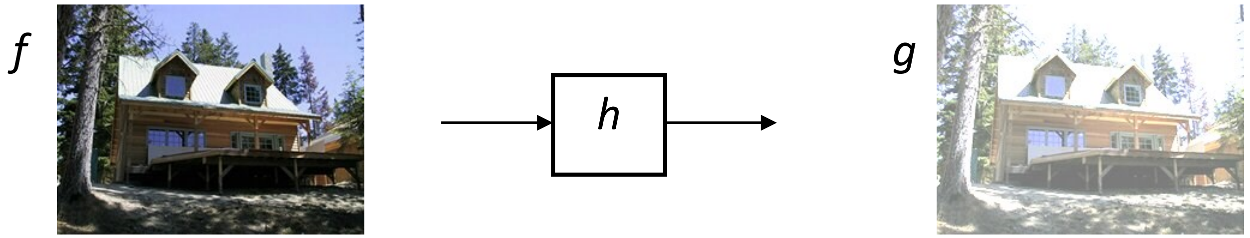
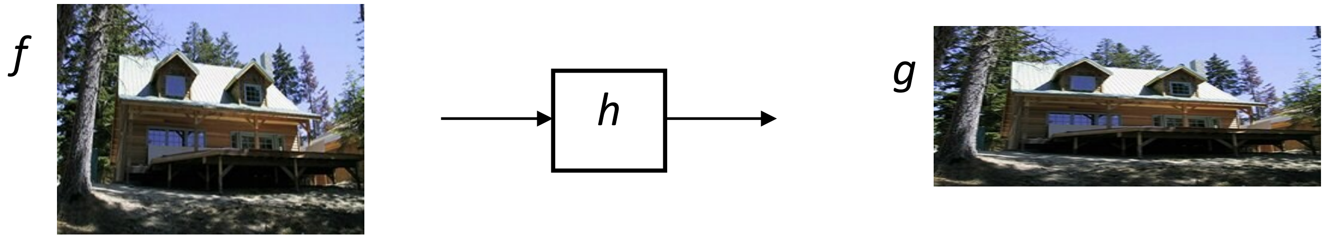


- image warping: change *domain* of image

$$g(x) = f(h(x))$$



Parametric (global) warping

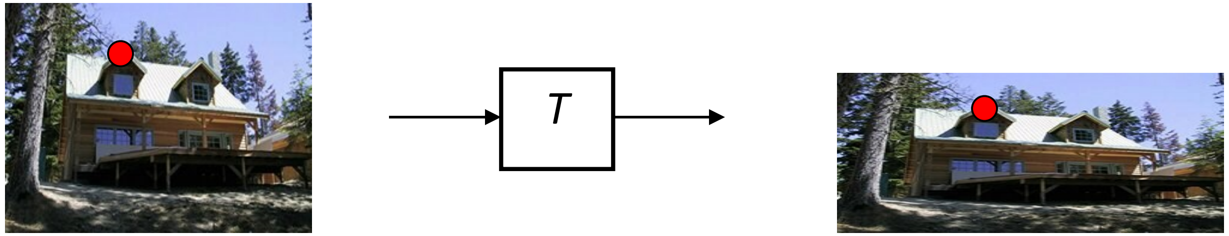
Examples of parametric warps:

translation

rotation

aspect

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is global?

- is the same for any point p
- can be described by just a few numbers (parameters)

Linear transforms

Let's consider *linear* transforms

(can be represented by a 2x2 matrix):

$$\mathbf{p}' = \mathbf{T}\mathbf{p}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling

Uniform scaling by *s*:

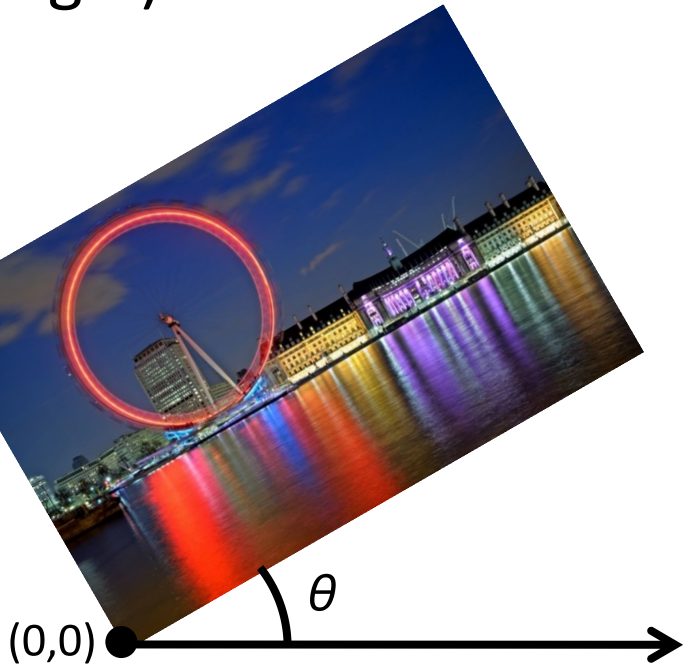
(0,0) 

(0,0) 

$$\mathbf{S} = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

Rotation

Rotation by angle θ (about the origin)



$$\mathbf{R} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D mirror about Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned} \qquad \mathbf{T} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

2D mirror across line y = x?

$$\begin{aligned} x' &= y \\ y' &= x \end{aligned} \qquad \mathbf{T} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$
\begin{aligned}
x' &= x + t_x \\
y' &= y + t_y
\end{aligned}
$$

NO!

Translation is not a linear operation on 2D coordinates

All 2D Linear Transformations

Linear transformations are combinations of …

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

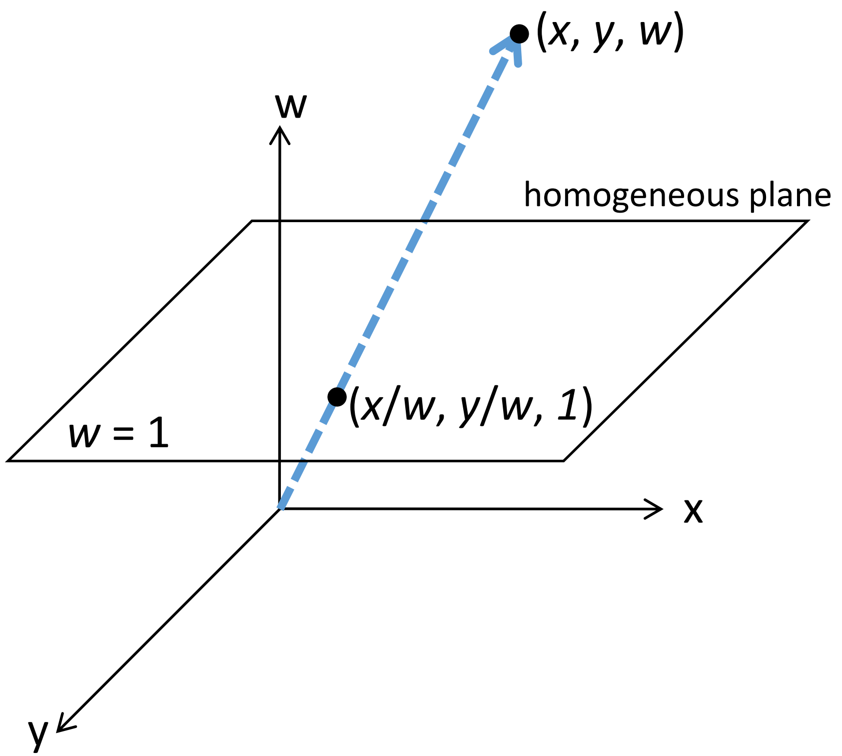
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous coordinates

Trick: add one more coordinate:

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates



Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Translation

Solution with homogeneous coordinates

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

Affine transformations

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ & & \end{bmatrix}$$

any transformation represented by a 3x3 matrix with last row [0 0 1] we call an *affine* transformation

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix}$$

Affine transformations

Affine transformations are combinations of …

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

Basic affine transformations

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{s_x} & 0 & 0 \\ 0 & \boldsymbol{s_y} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

2D *in-plane* rotation

$$\begin{bmatrix} \boldsymbol{x}' \\ \boldsymbol{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{sh_x} & 0 \\ \boldsymbol{sh_y} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ 1 \end{bmatrix}$$

Shear

É una trasformazione affine?

Omografie

$$\begin{bmatrix} a & b & c \\ d & e & f \\ & & \end{bmatrix}$$

affine transformation

$$\mathbf{H} = \begin{bmatrix} a & b & c \\ d & e & f \\ & & \end{bmatrix}$$

H is a *homography*
(or *planar perspective map*)

Punti all’infinito

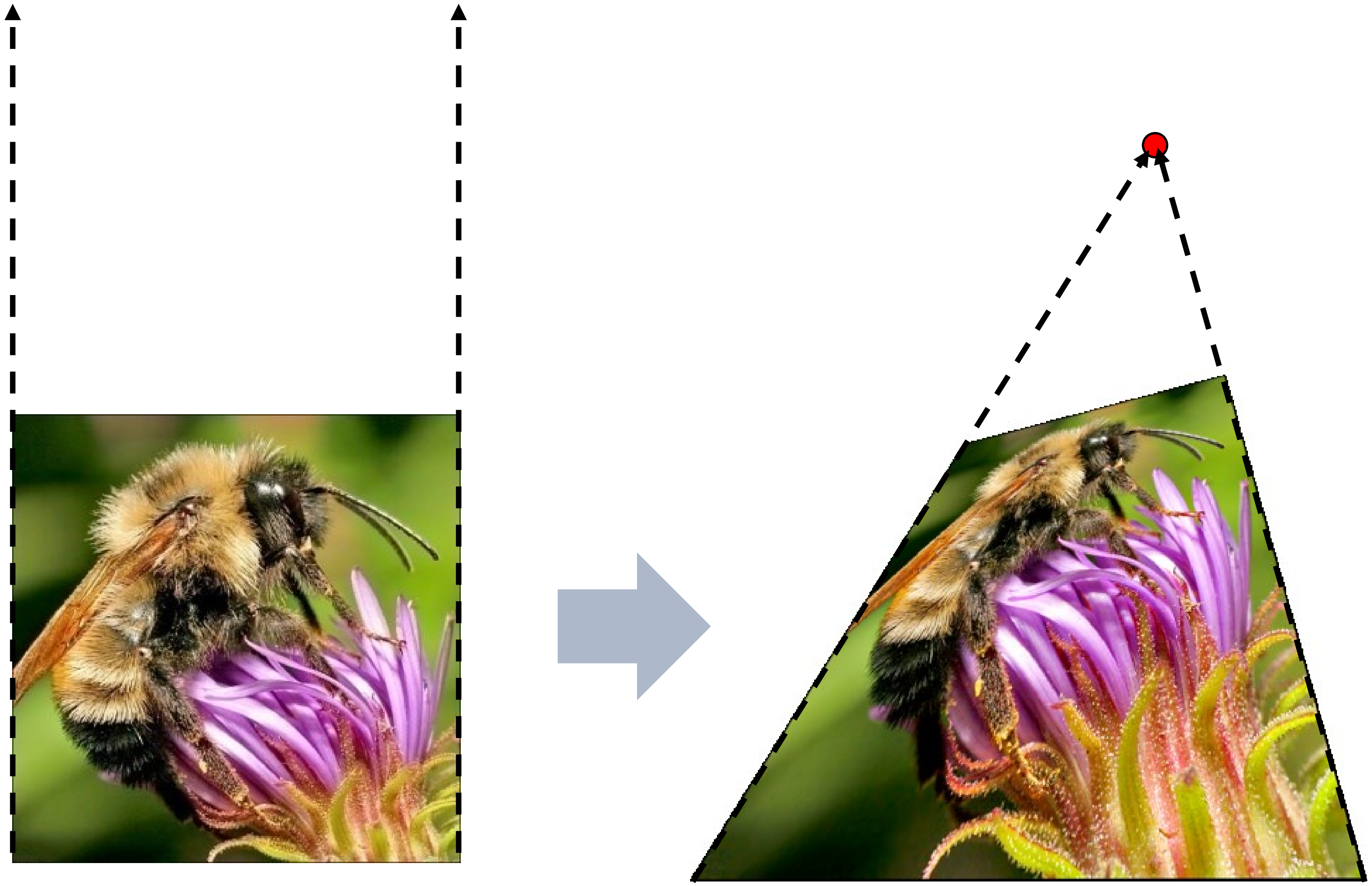


Image warping con omografie

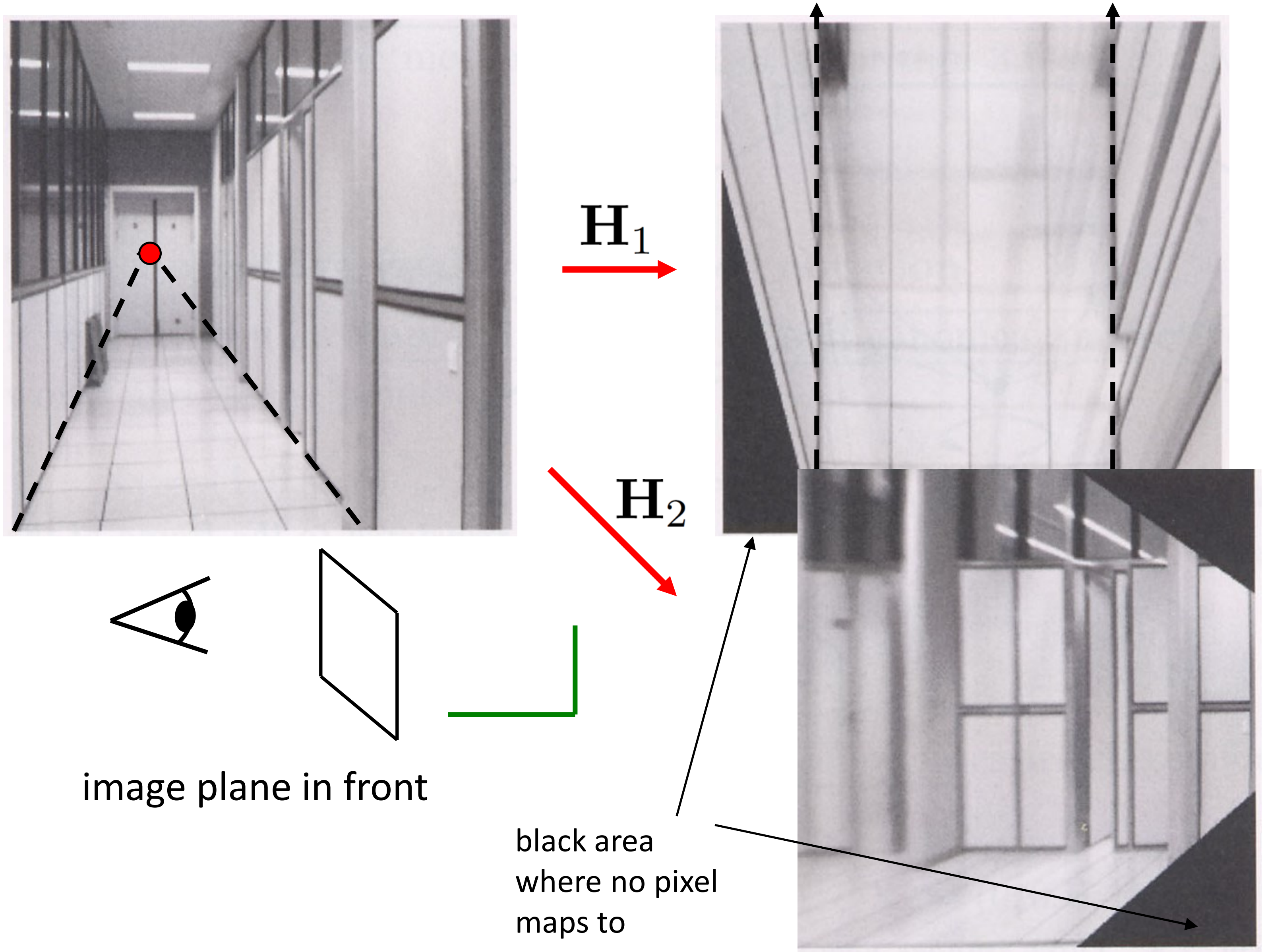
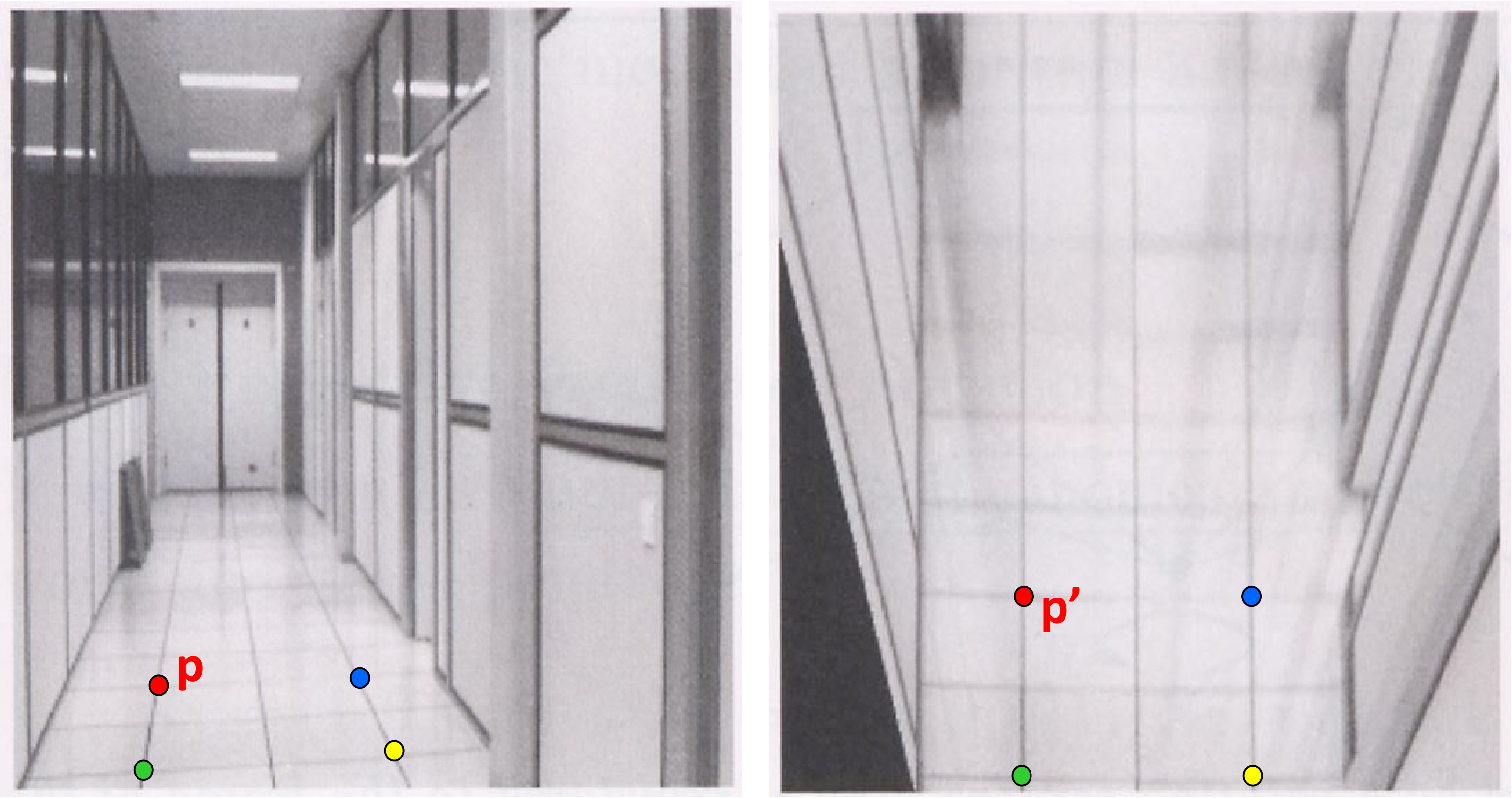


Image warping con omografie



Omografia

Omografie

Homographies …

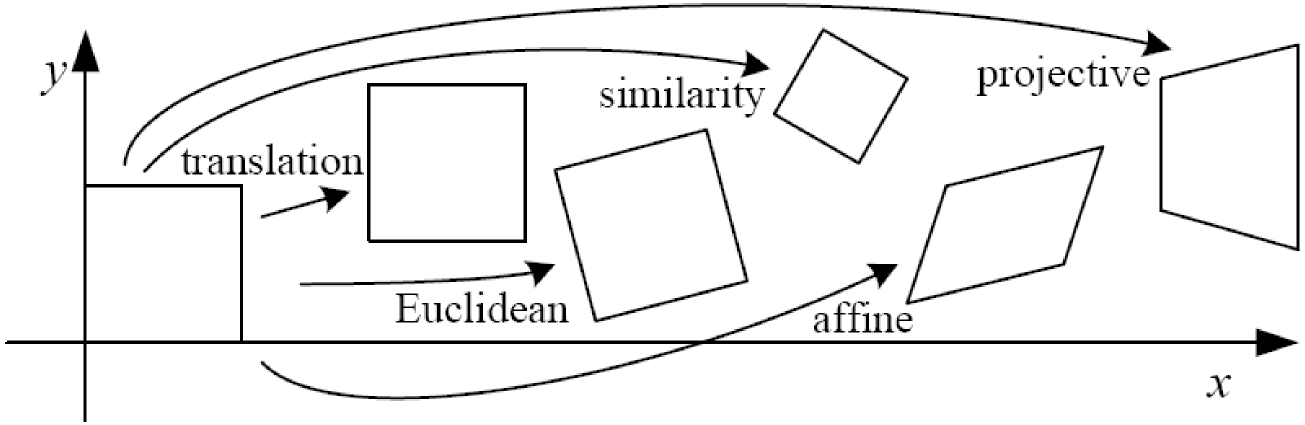
- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition

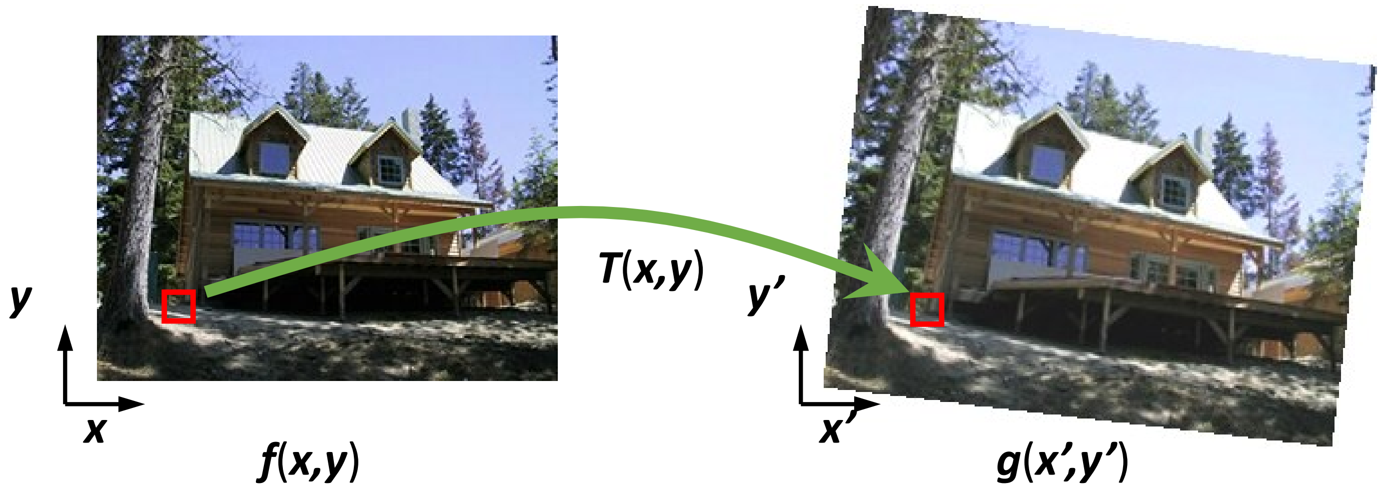
Ricapitolando



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\left[\begin{array}{c\|c} \boldsymbol{I} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\left[\begin{array}{c\|c} \boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\left[\begin{array}{c\|c} s\boldsymbol{R} & \boldsymbol{t} \end{array}\right]_{2\times 3}$	4	angles $+\cdots$	
affine	$\left[\begin{array}{c} \boldsymbol{A} \end{array}\right]_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\left[\begin{array}{c} \tilde{\boldsymbol{H}} \end{array}\right]_{3\times 3}$	8	straight lines	

Implementing image warping

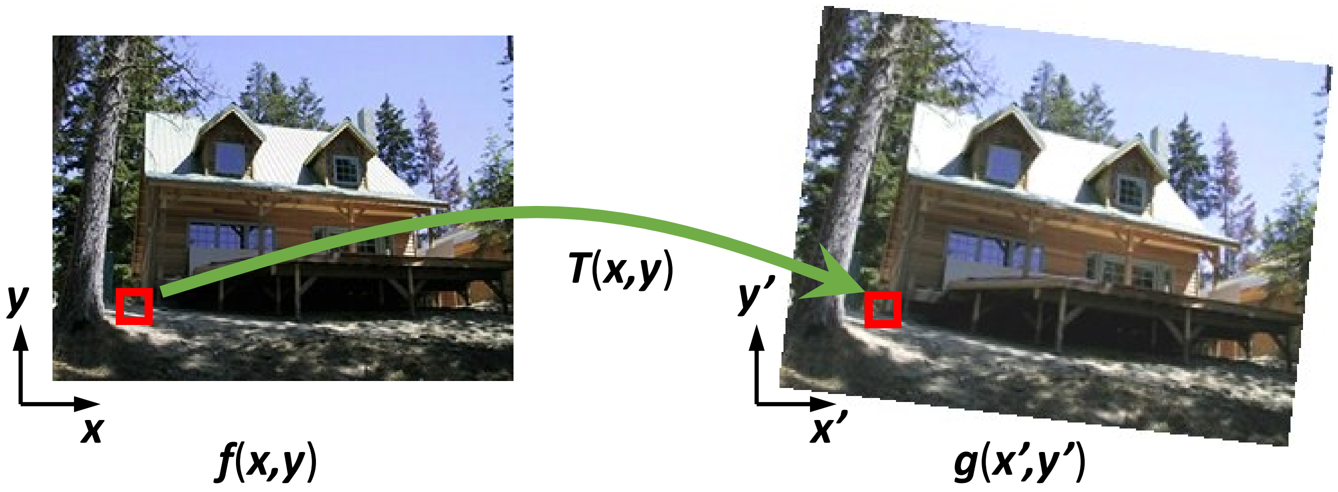
Given a coordinate transform $(\boldsymbol{x'},\boldsymbol{y'}) = \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$ and a source image $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$, how do we compute a transformed image $\boldsymbol{g}(\boldsymbol{x'},\boldsymbol{y'}) = \boldsymbol{f}(\boldsymbol{T}(\boldsymbol{x},\boldsymbol{y}))$?



Forward Warping

Send each pixel $\boldsymbol{f}(\boldsymbol{x})$ to its corresponding location $(\boldsymbol{x'},\boldsymbol{y'}) = \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$ in $\boldsymbol{g}(\boldsymbol{x'},\boldsymbol{y'})$

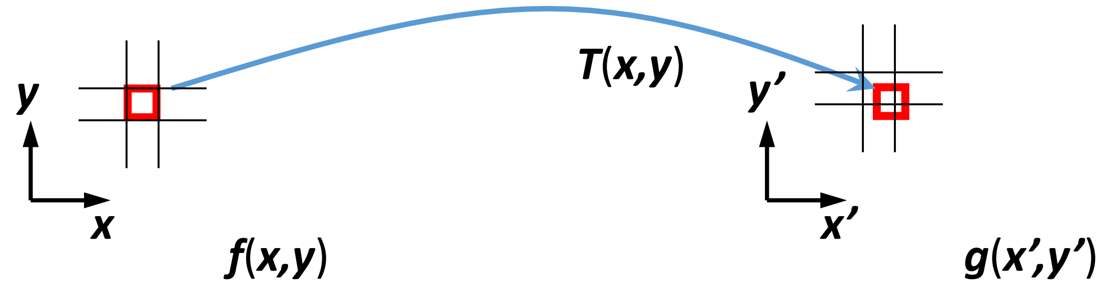
- What if pixel lands “between” two pixels?



Forward Warping

Send each pixel $\boldsymbol{f}(\boldsymbol{x})$ to its corresponding location
$(\boldsymbol{x'},\boldsymbol{y'}) = \boldsymbol{T}(\boldsymbol{x},\boldsymbol{y})$ in $\boldsymbol{g}(\boldsymbol{x'},\boldsymbol{y'})$

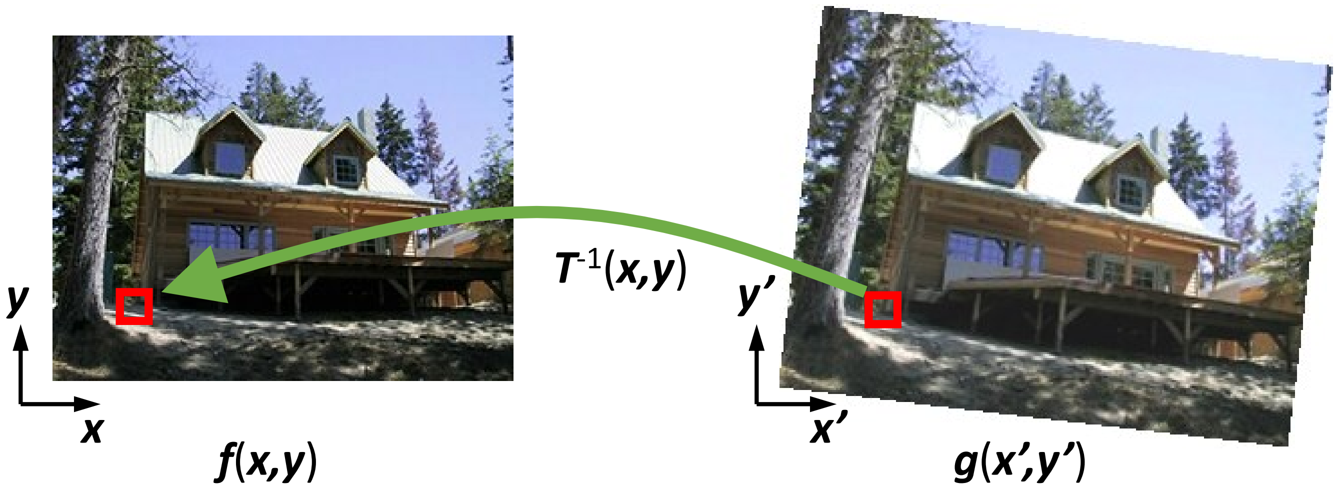
- What if pixel lands “between” two pixels?
- Answer: add “contribution” to several pixels, normalize later (splatting)
- Can still result in holes



Inverse Warping

Get each pixel $\boldsymbol{g}(\boldsymbol{x'},\boldsymbol{y'})$ from its corresponding location $(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{T}^{-1}(\boldsymbol{x},\boldsymbol{y})$ in $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{y})$

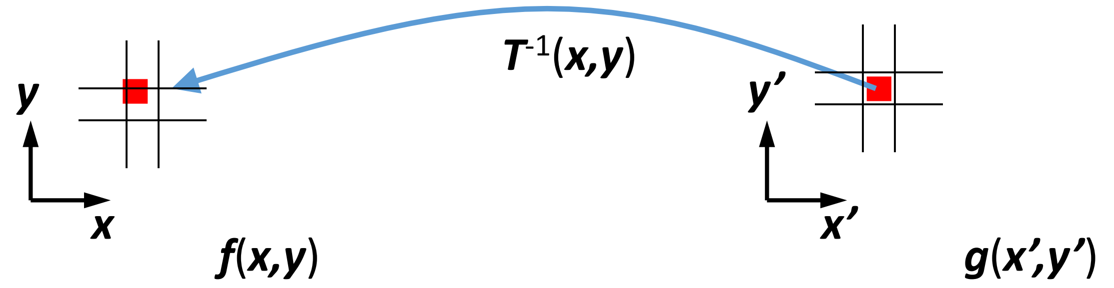
- Requires taking the inverse of the transform
- What if pixel comes from “between” two pixels?



Inverse Warping

Get each pixel $\boldsymbol{g}(\boldsymbol{x'})$ from its corresponding location $\boldsymbol{x'} = \boldsymbol{h}(\boldsymbol{x})$ in $\boldsymbol{f}(\boldsymbol{x})$

- What if pixel comes from "between" two pixels?
- Answer: *resample* color value from *interpolated* (*prefiltered*) source image



Interpolation

Possible interpolation filters:

- nearest neighbor
- bilinear
- bicubic
- sinc

Needed to prevent "jaggies" and "texture crawl"

(with prefiltering)

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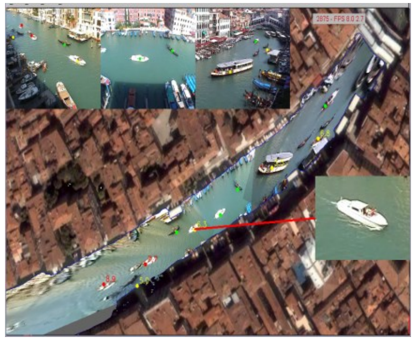
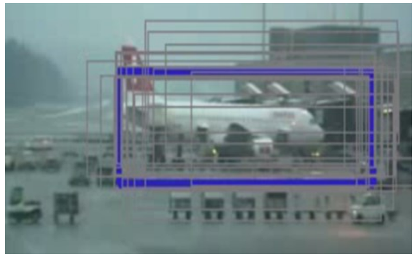
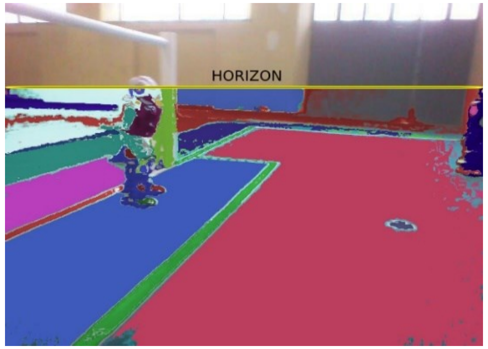
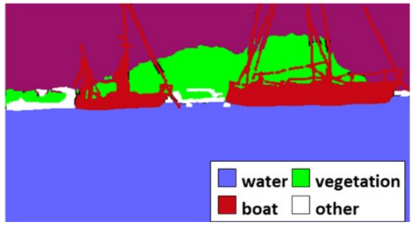



Omografie



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