

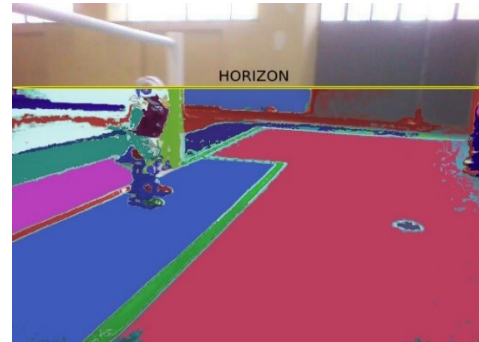
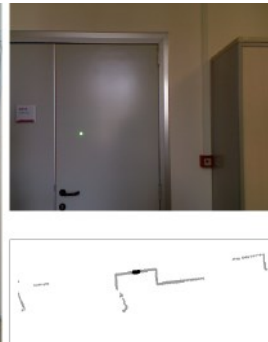
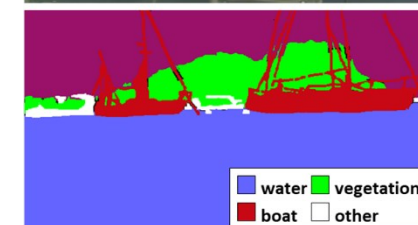
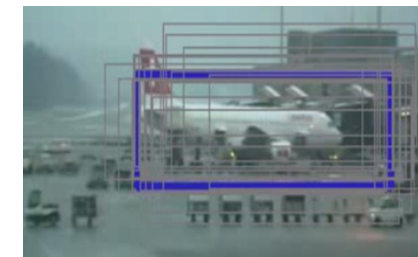


**UNIVERSITÀ DEGLI STUDI
DELLA BASILICATA**

Corso di Sistemi Informativi
A.A. 2018/19

Filtri

Docente
Domenico Daniele Bloisi



water vegetation
boat other

Aprile 2019

Immagine Digitale

- Una immagine digitale è una matrice di pixel
- Il termine pixel deriva da *picture element*
- Il pixel contiene l'informazione relativa alla rappresentazione della realtà che è stata catturata tramite uno scanner, una macchina fotografica o un frame grabber (per i video)



But the camera sees this:

194	210	201	212	199	213	215	195	178	158	182	209
180	189	190	221	209	205	191	167	147	115	129	163
114	126	140	188	176	165	152	140	170	106	78	88
87	103	115	154	143	142	149	153	173	101	57	57
102	112	106	131	122	138	152	147	128	84	58	66
94	95	79	104	105	124	129	113	107	87	69	67
68	71	69	98	89	92	98	95	89	88	76	67
41	56	68	99	63	45	60	82	58	76	74	65
20	41	69	75	56	41	51	73	55	70	63	44
50	50	57	69	75	75	73	74	53	68	59	37
72	59	53	66	84	92	84	74	57	72	63	42
67	61	58	65	75	78	76	73	59	75	69	50

Immagine come funzione

Possiamo pensare ad una immagine come ad una funzione f da \mathbb{R}^2 a \mathbb{R}

- $f(x, y)$ sarà l'intensità nella posizione (x, y)
- L'immagine sarà definita all'interno di un rettangolo e ogni elemento potrà assumere valori in range predefinito

$$f:[a,b] \times [c,d] \rightarrow [0,1]$$

Immagine a colori

Una immagine a colori potrà essere rappresentata come l'unione tra tre funzioni, una per ogni canale red, green, blue

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

Esempio

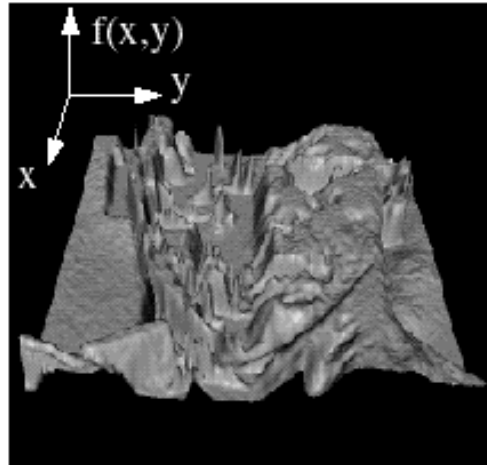
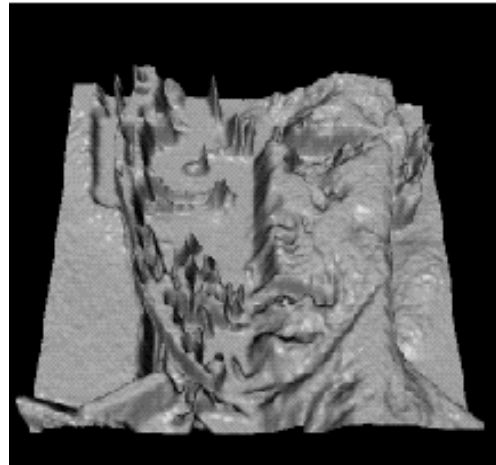
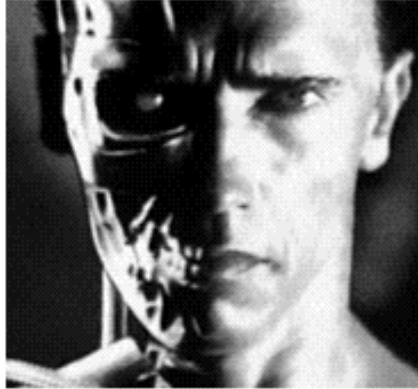
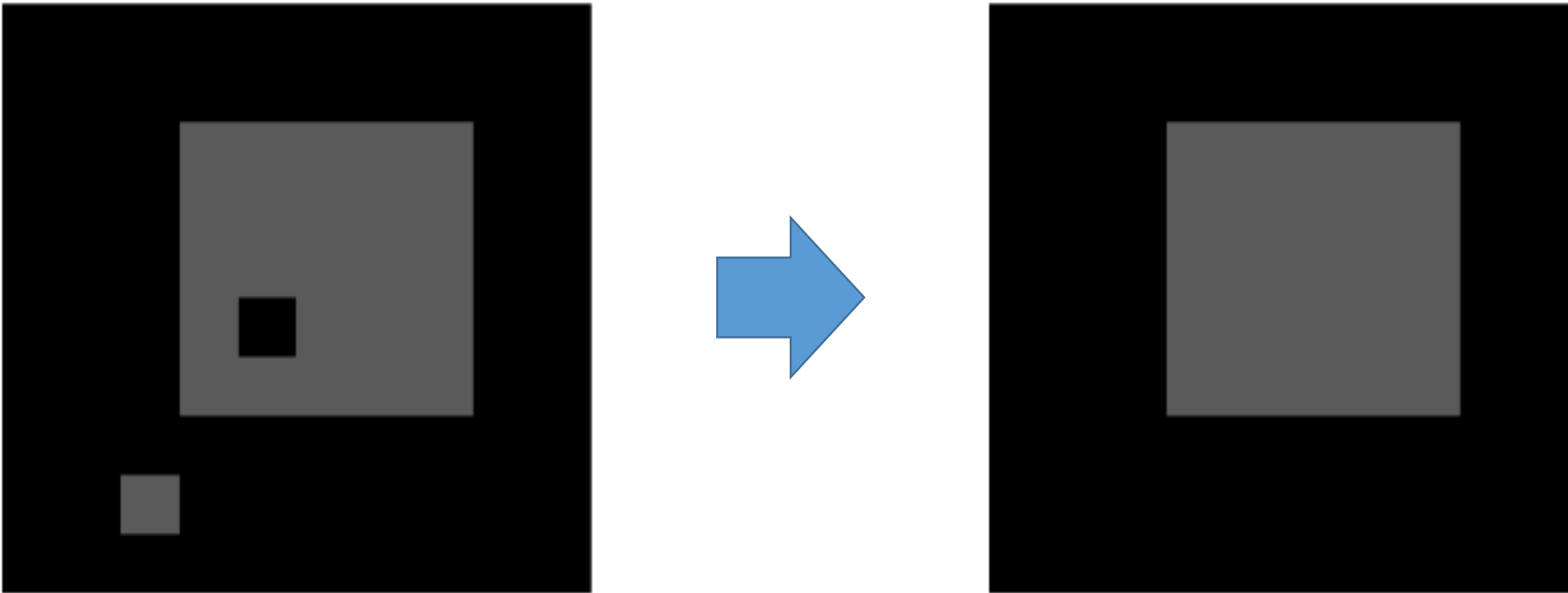


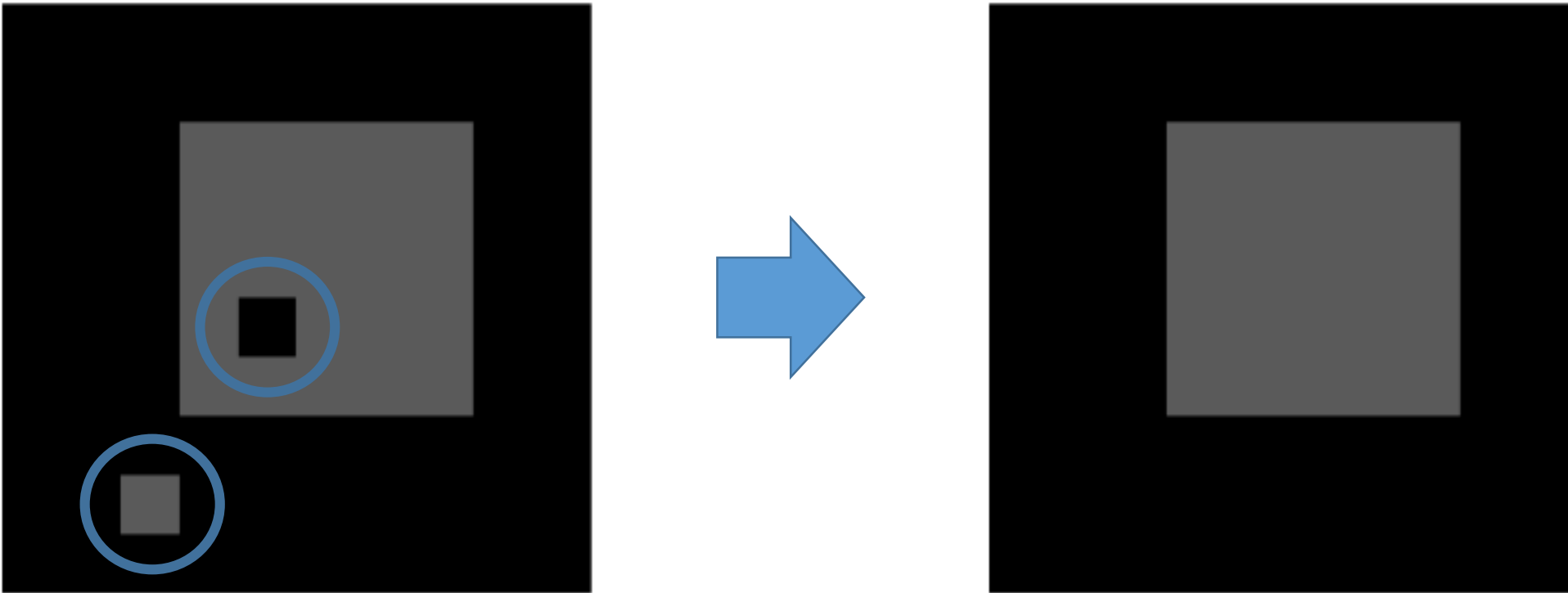
Image filtering

Vogliamo limitare il rumore presente nell'immagine sotto a sinistra per trasformarla in quella a destra



Smoothing

Una possibilità è quella di "smussare" l'immagine per ridurre il contrasto e nascondere gli outlier



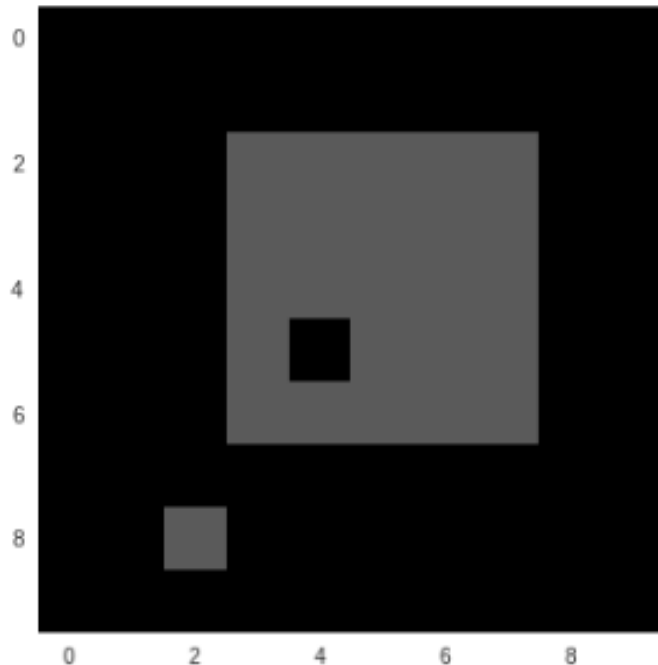
Creiamo l'immagine in Colab

```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image

numpy_img = np.ones([10,10,3], dtype=np.uint8)*0
numpy_img[2:7,3:8] = 90
numpy_img[5,4] = 0
numpy_img[8,2] = 90

img = Image.fromarray(numpy_img)
plt.grid(b=False)
plt.imshow(img)
```

<matplotlib.image.AxesImage at 0x7f0d51201d68>



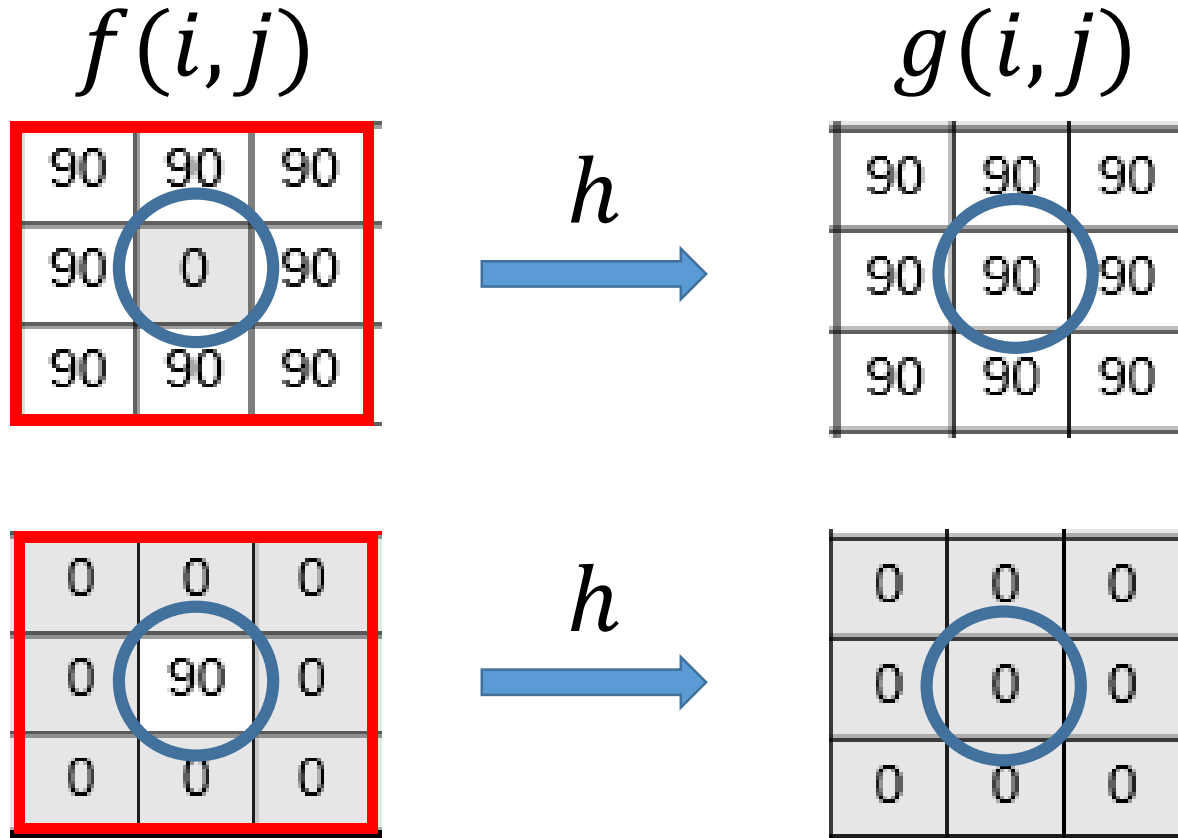
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Mean filter

Per smussare possiamo utilizzare una trasformazione h dell'immagine che modifichi i valori di intensità dei pixel in modo da renderli simili ai valori dei loro "vicini"

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Mean filtering



Il pixel $f(i, j)$ verrà modificato attraverso una trasformazione locale h che coinvolgerà un intorno di $f(i, j)$ per produrre il nuovo valore $g(i, j)$

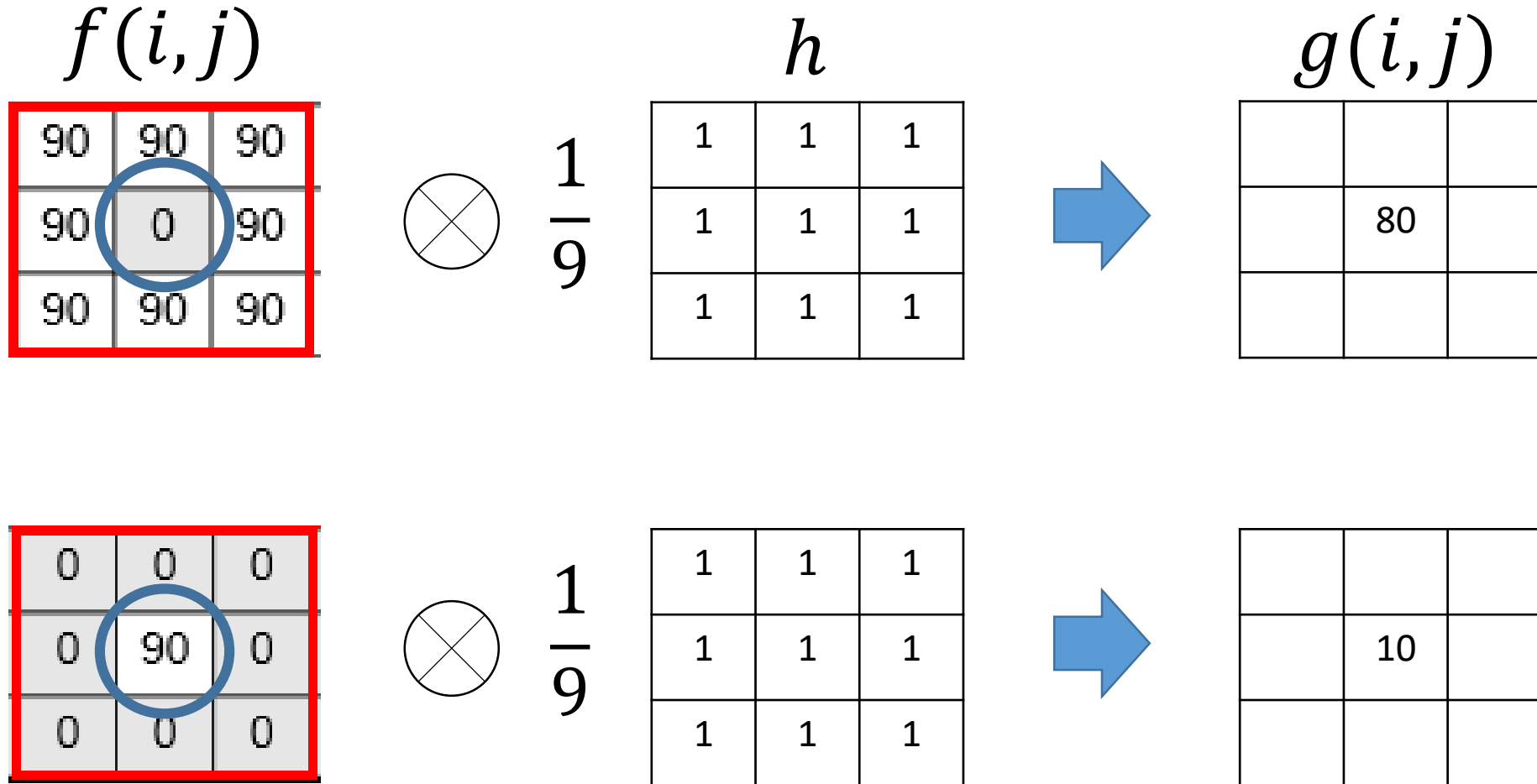
Linear filtering

L'operazione di trasformare i valori del pixel p utilizzando una combinazione lineare pesata dei valori dei pixel in un intorno (piccolo) di p prende il nome di linear filtering

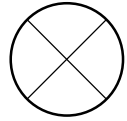
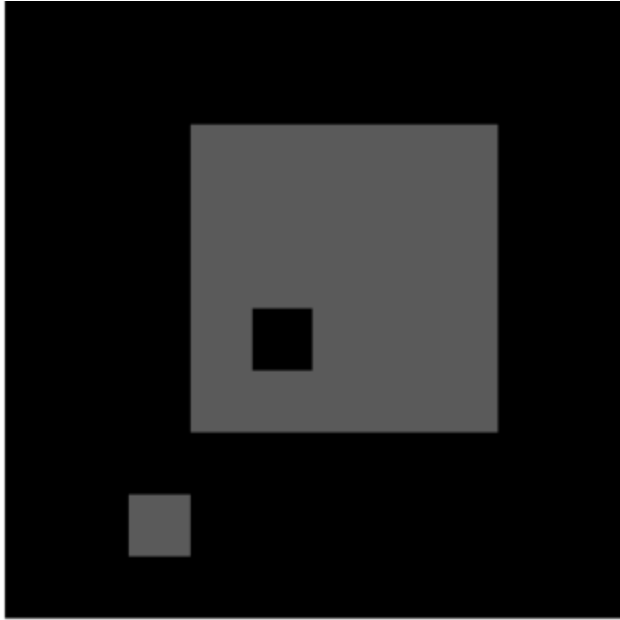
$$g(i, j) = \sum_{k, l} f(i + k, j + l) h(k, l)$$

- $h(k, l)$ prende il nome di kernel o maschera (mask)
- Questa trasformazione è detta **correlazione** e può essere denotata con $g = f \otimes h$

Mean kernel



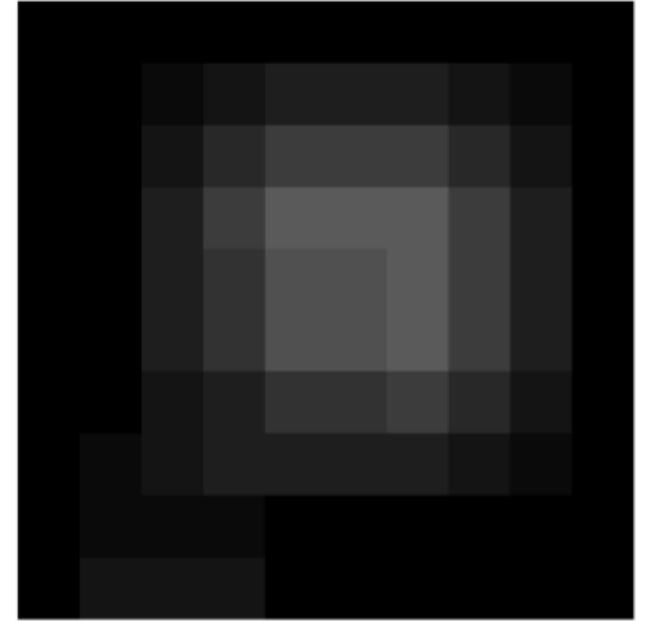
Mean filtering



$\frac{1}{9}$

h

1	1	1
1	1	1
1	1	1



Convoluzione

Una variante della formula di correlazione è la seguente

$$g(i, j) = \sum_{k, l} f(i - k, j - l) h(k, l) = \sum_{k, l} f(k, l) h(i - k, j - l)$$

dove il segno degli offset in f è stato invertito.

Questa operazione prende il nome di **convoluzione** ed è indicata come $g = f * h$

Convoluzione: esempio

45	60	98	127	132	133	137	133
46	65	98	123	126	128	131	133
47	65	96	115	119	123	135	137
47	63	91	107	113	122	138	134
50	59	80	97	110	123	133	134
49	53	68	83	97	113	128	133
50	50	58	70	84	102	116	126
50	50	52	58	69	86	101	120

$f(x,y)$

*

0.1	0.1	0.1
0.1	0.2	0.1
0.1	0.1	0.1

$h(x,y)$

=

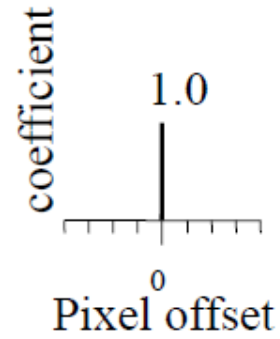
69	95	116	125	129	132
68	92	110	120	126	132
66	86	104	114	124	132
62	78	94	108	120	129
57	69	83	98	112	124
53	60	71	85	100	114

$g(x,y)$

Convoluzione



original



δ



Filtered
(no change)

$$f = f \otimes \delta$$

Convoluzione

La convoluzione è commutativa e associativa

$$a * b = b * a$$

$$a * (b * c) = (a * b) * c$$

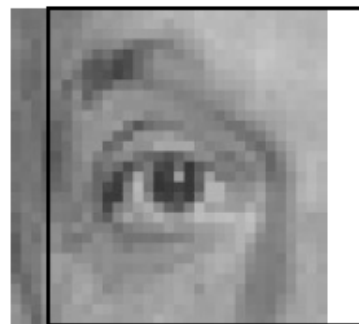
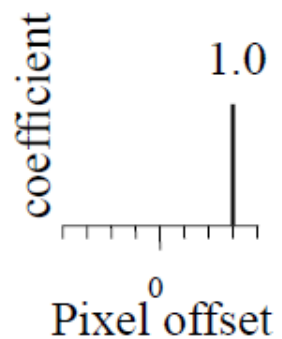
Inoltre,

$$(((a * b) * c) * d) = a * (b * c * d)$$

Shift



original

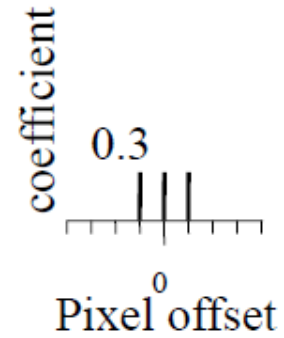


shifted

Blurring

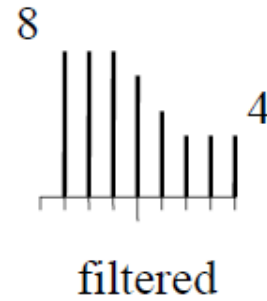
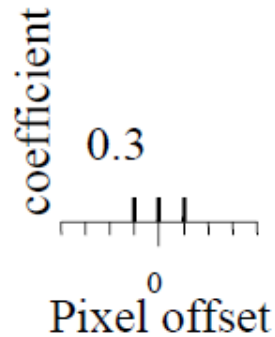
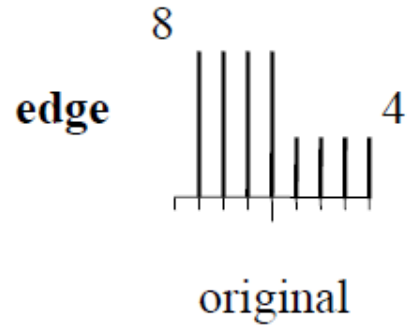
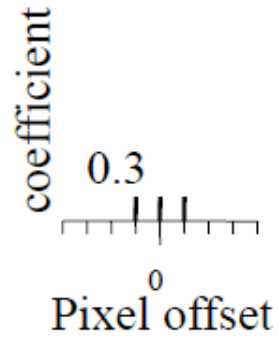
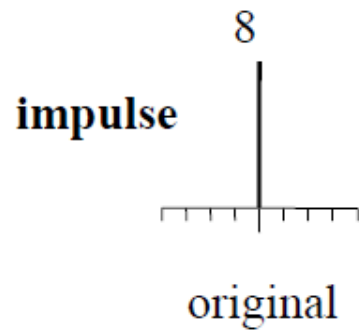


original



Blurred (filter applied in both dimensions).

Blur examples



How to read an image from url

```
from PIL import Image
import matplotlib.pyplot as plt

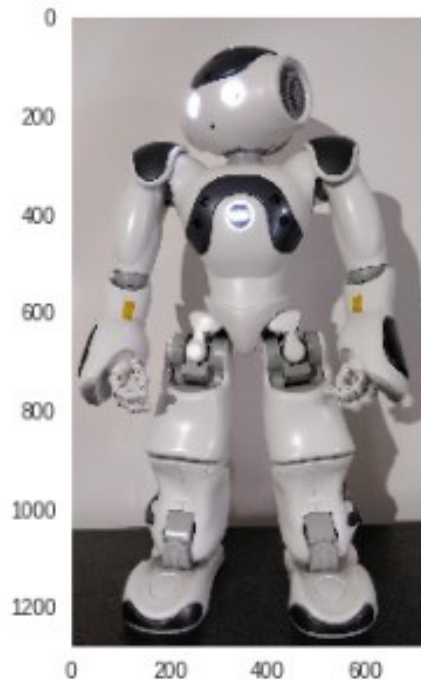
import urllib.request

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = Image.open(urllib.request.urlopen(url))

plt.grid(b=False)
plt.imshow(img)
```

<matplotlib.image.AxesImage at 0x7fd4f5933048>



Gaussian blurring



```
import matplotlib.pyplot as plt
import urllib.request

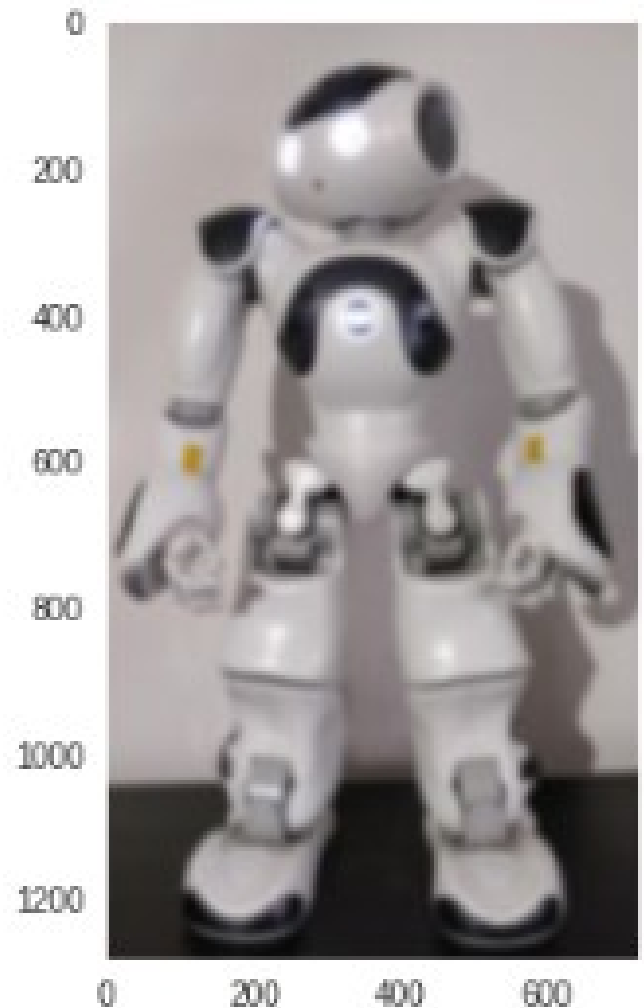
from PIL import Image
from PIL import ImageFilter

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = Image.open(urllib.request.urlopen(url))

blur_img = img.filter(ImageFilter.GaussianBlur(5))

plt.grid(b=False)
plt.imshow(blur_img)
```



Gaussian blurring

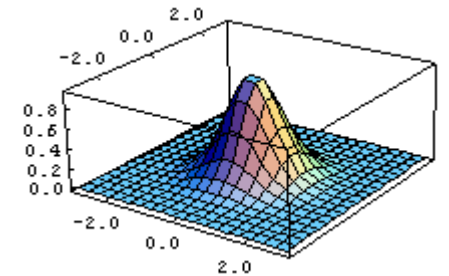
Un kernel Gaussiano dar  meno peso ai pixel distanti dal centro della finestra

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Questo kernel   una
approssimazione della
funzione Gaussiana:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

$$\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$



Gaussian blurring



```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
from PIL import ImageFilter

import cv2 as cv

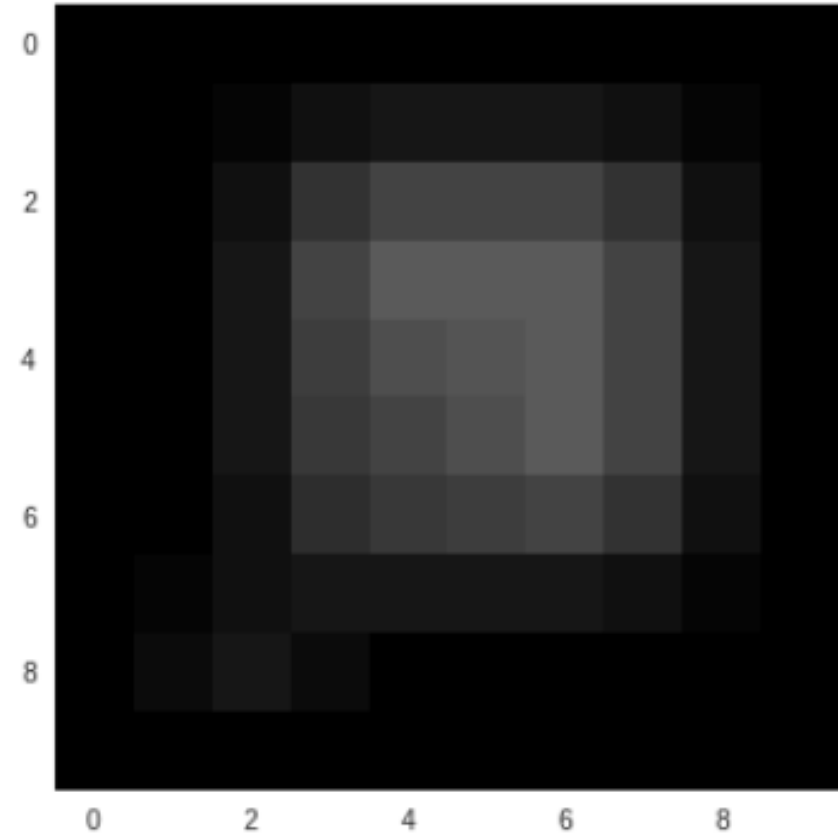
numpy_img = np.ones([10,10,3], dtype=np.uint8)*0
numpy_img[2:7,3:8] = 90
numpy_img[5,4] = 0
numpy_img[8,2] = 90

img = Image.fromarray(numpy_img)

kernel = 1/16 * np.array([1,2,1,2,4,2,1,2,1])

gaussian_blur = img.filter(ImageFilter.Kernel((3,3),kernel))

plt.grid(b=False)
plt.imshow(gaussian_blur)
```



Median filter



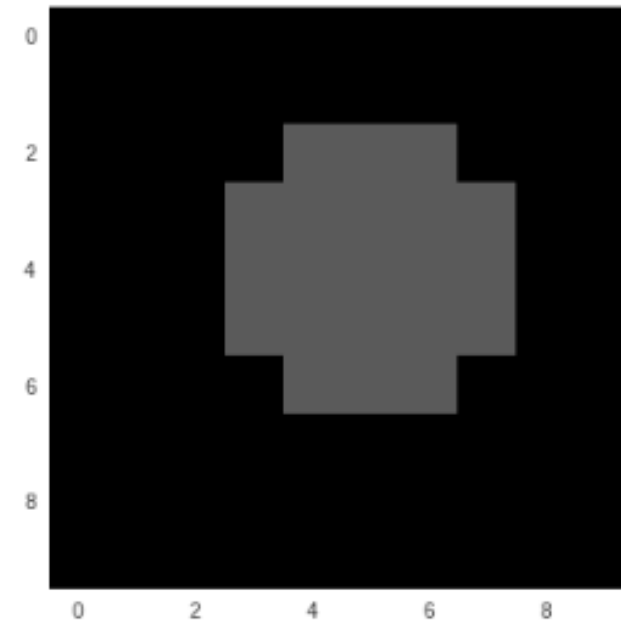
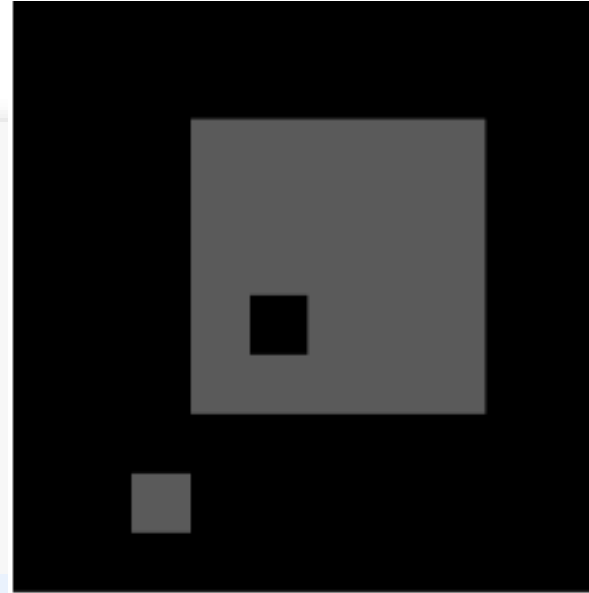
```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image

numpy_img = np.ones([10,10,3], dtype=np.uint8)*0
numpy_img[2:7,3:8] = 90
numpy_img[5,4] = 0
numpy_img[8,2] = 90

img = Image.fromarray(numpy_img)

blur_img = img.filter(ImageFilter.MedianFilter(3))

plt.grid(b=False)
plt.imshow(blur_img)
```



Median filter: esempio

Per realizzare il median filter andremo ad ordinare tutti i valori presenti nell'intorno del pixel su cui è applicato il filtro e poi prenderemo il valore che si trova nel mezzo del vettore ordinato

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Neighbourhood values:

115, 119, 120, 123, 124, 125, 126, 127, 150

Median value: 124

Median filter



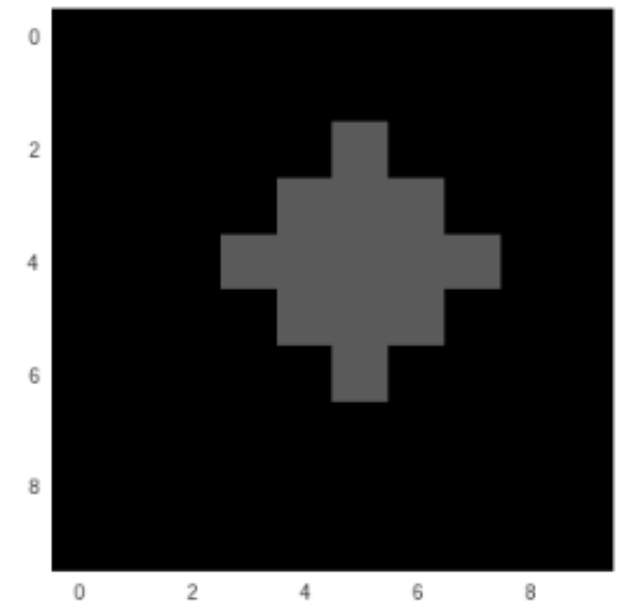
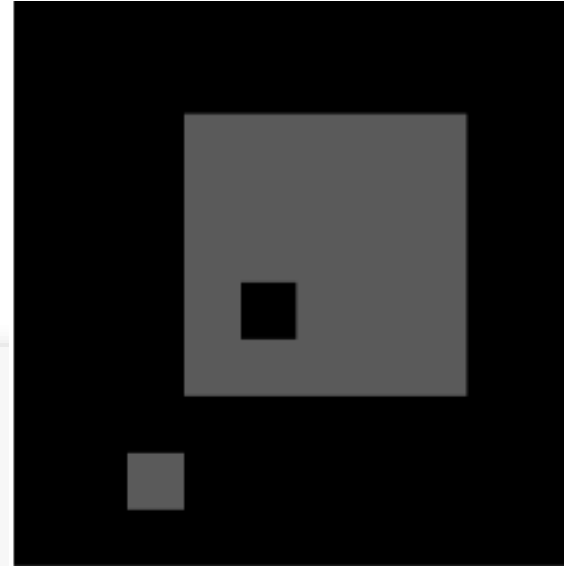
```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image

numpy_img = np.ones([10,10,3], dtype=np.uint8)*0
numpy_img[2:7,3:8] = 90
numpy_img[5,4] = 0
numpy_img[8,2] = 90

img = Image.fromarray(numpy_img)

blur_img = img.filter(ImageFilter.MedianFilter(5))

plt.grid(b=False)
plt.imshow(blur_img)
```



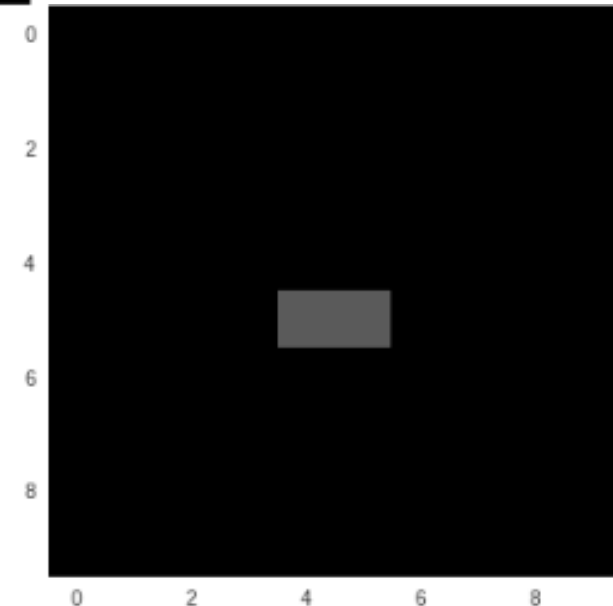
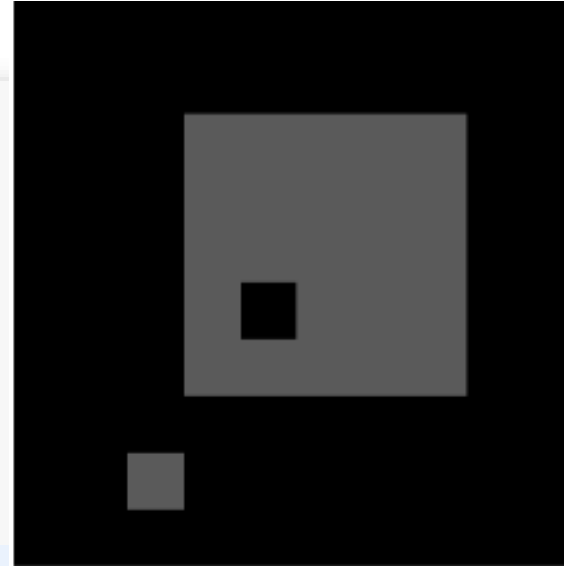
Median filter

```
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image

numpy_img = np.ones([10,10,3], dtype=np.uint8)*0
numpy_img[2:7,3:8] = 90
numpy_img[5,4] = 0
numpy_img[8,2] = 90

img = Image.fromarray(numpy_img)
blur_img = img.filter(ImageFilter.MedianFilter(7))

plt.grid(b=False)
plt.imshow(blur_img)
```



Median filter

```
import matplotlib.pyplot as plt
import urllib.request

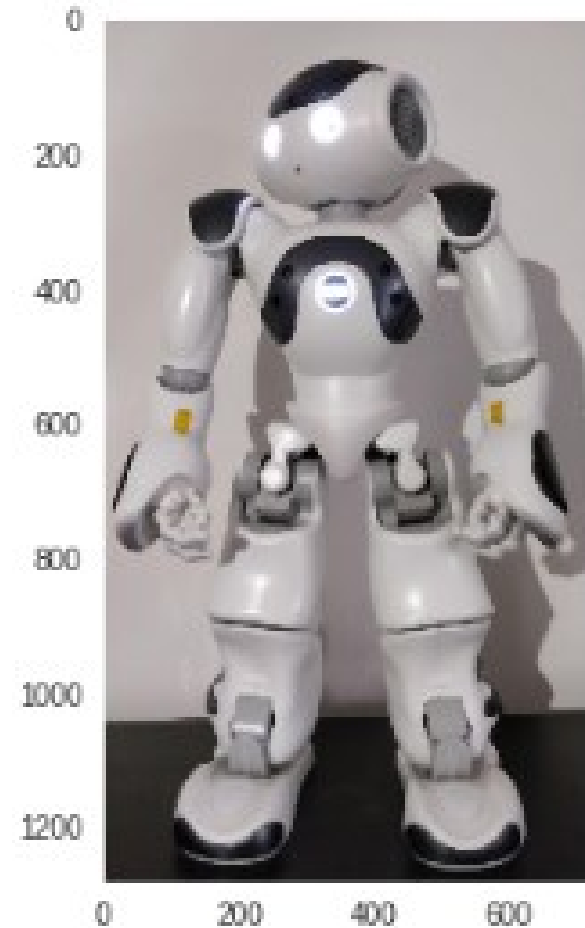
from PIL import Image
from PIL import ImageFilter

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = Image.open(urllib.request.urlopen(url))

blur_img = img.filter(ImageFilter.MedianFilter(7))

plt.grid(b=False)
plt.imshow(blur_img)
```



Custom filters



```
import matplotlib.pyplot as plt
import urllib.request

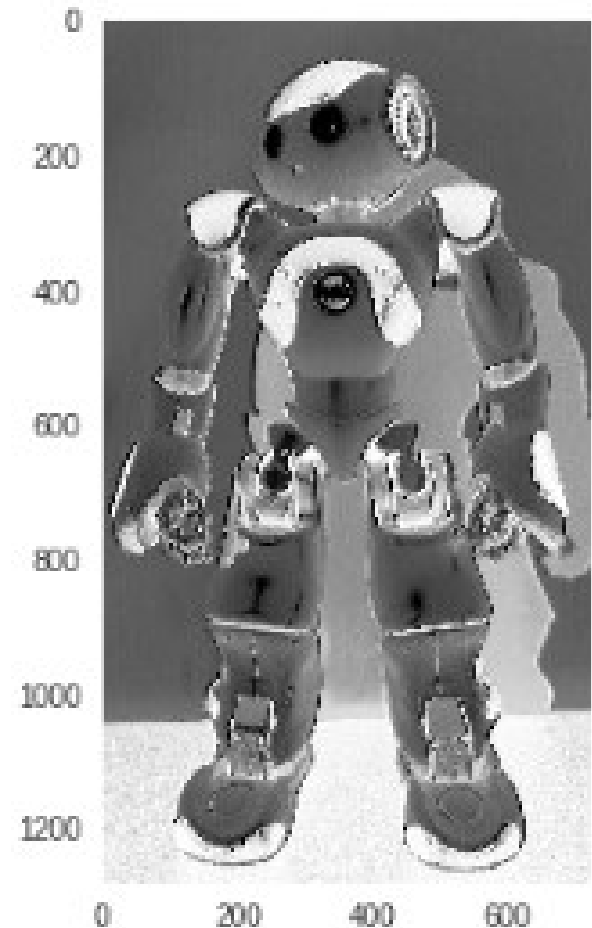
from PIL import Image
from PIL import ImageFilter

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = Image.open(urllib.request.urlopen(url))

img = img.convert("L")
new_img = img.filter(ImageFilter.Kernel((3,3),[1,0,-1,5,0,-5,1,0,1]))

plt.grid(b=False)
plt.imshow(new_img)
```



Gradiente

La derivata di una immagine è definita come la variazione nei valori di intensità dei pixel nell'immagine. Il tasso di variazione può essere calcolato come:

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

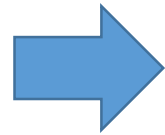
che diventa la differenza finita nel caso di immagini digitali

$$\frac{\partial f}{\partial x}[x, y] \approx F[x+1, y] - F[x, y]$$

Gradiente

Per una funzione 2D
 $f(x,y)$ avremo:

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \rightarrow 0} \left(\frac{f(x + \varepsilon, y) - f(x, y)}{\varepsilon} \right)$$

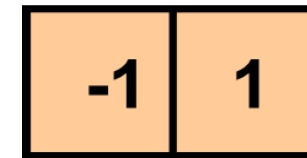


Possiamo utilizzare una
approssimazione

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

Si tratta di una
operazione lineare
invariante rispetto allo
shift (così come la
convoluzione)

che può essere calcolata
con una convoluzione



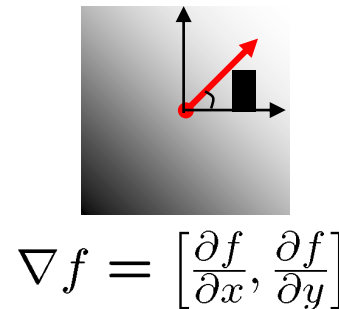
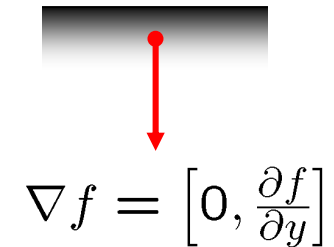
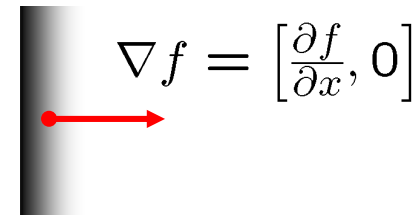
Gradiente

Gradiente dell'immagine: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$

$$\frac{\partial f}{\partial x}[i, j] \approx f[i + 1, j] - f[i, j]$$

$$\frac{\partial f}{\partial y}[i, j] \approx f[i, j + 1] - f[i, j]$$

Il gradiente punta nella direzione del più rapido cambio di intensità



Gradiente

La direzione del gradiente è data da:

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

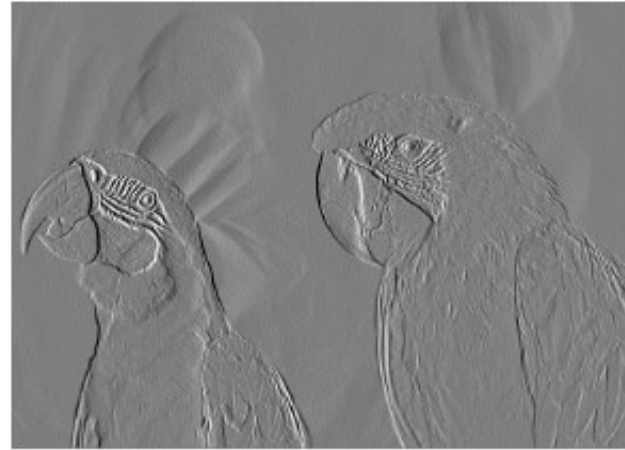
La forza di un bordo è data da:

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Gradiente



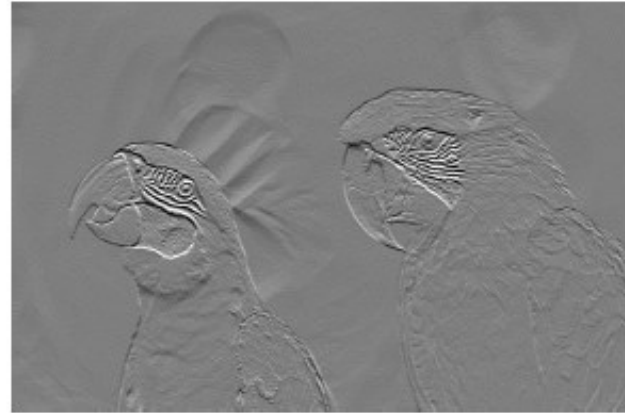
f



$\frac{\partial f}{\partial x}$



$||\nabla f||$



$\frac{\partial f}{\partial y}$

Sobel operator

$$S_x \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \quad S_y \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

L'operatore di Sobel fa uso di due kernel 3x3 per calcolare il valore approssimato delle derivate in direzione orizzontale e in direzione verticale

Skimage (scikit-image)

[←](#) [→](#) [↻](#) <https://scikit-image.org>



scikit-image
image processing in python

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0.14.2 - January 2019

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pre-0.15

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Image processing in Python

scikit-image is a collection of algorithms for image processing. It is available **free of charge and free of restriction**. We pride ourselves on high-quality, peer-reviewed code, written by an active **community of volunteers**.

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Stéfan van der Walt, Johannes L. Schönberger, Juan Nunez-Iglesias, François Boulogne, Joshua D. Warner, Neil Yager, Emmanuelle Gouillart, Tony Yu and the scikit-image contributors. **scikit-image: Image processing in Python**. PeerJ 2:e453 (2014) <https://doi.org/10.7717/peerj.453>

Sobel edge detection



```
import matplotlib.pyplot as plt
import urllib.request

from PIL import Image

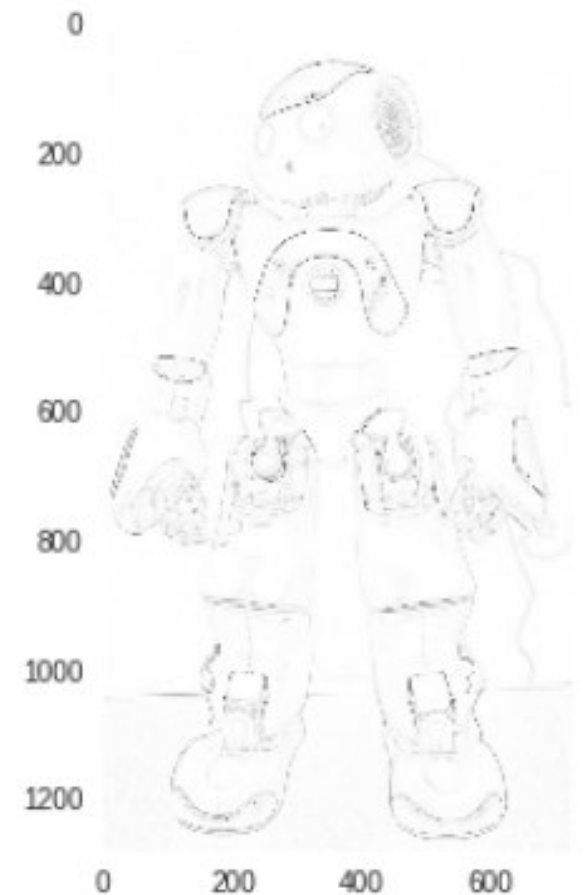
from skimage import io
from skimage import filters
from skimage import color

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = io.imread(urllib.request.urlopen(url))

img = color.rgb2gray(img)
edge = filters.sobel(img)

plt.grid(b=False)
plt.imshow(edge)
```



Canny edge detection

```
import matplotlib.pyplot as plt
import urllib.request

from PIL import Image

from skimage import io
from skimage import feature
from skimage import color

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = io.imread(urllib.request.urlopen(url))

img = color.rgb2gray(img)
edge = feature.canny(img,3)

plt.grid(b=False)
plt.imshow(edge)
```

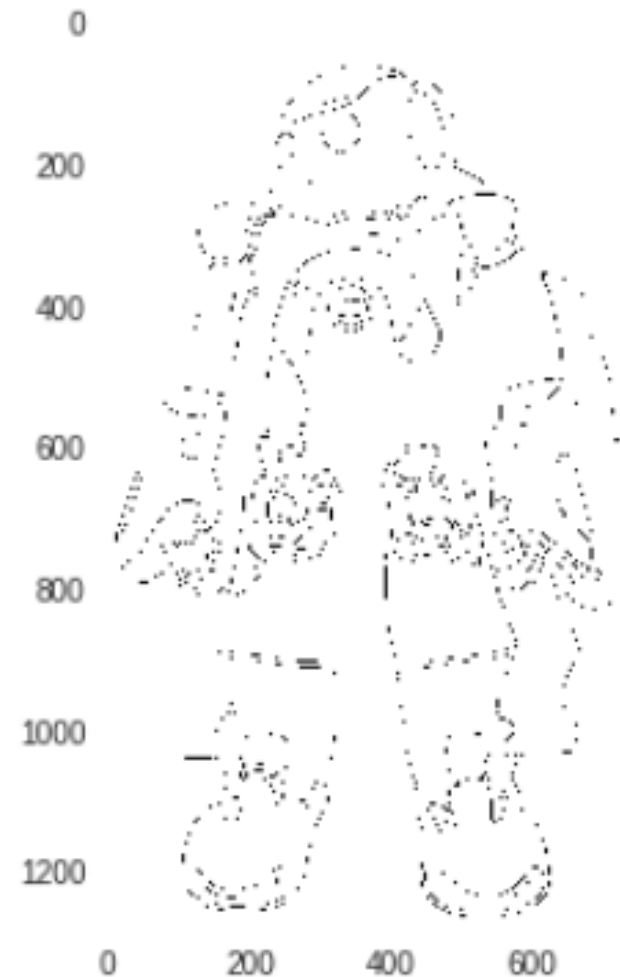


Image enhancement



```
import matplotlib.pyplot as plt
import urllib.request

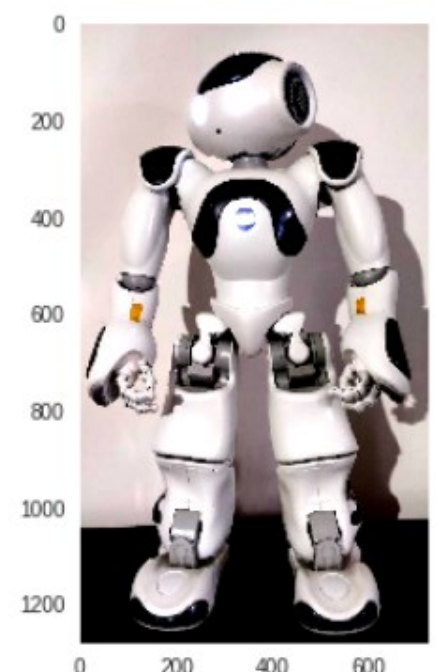
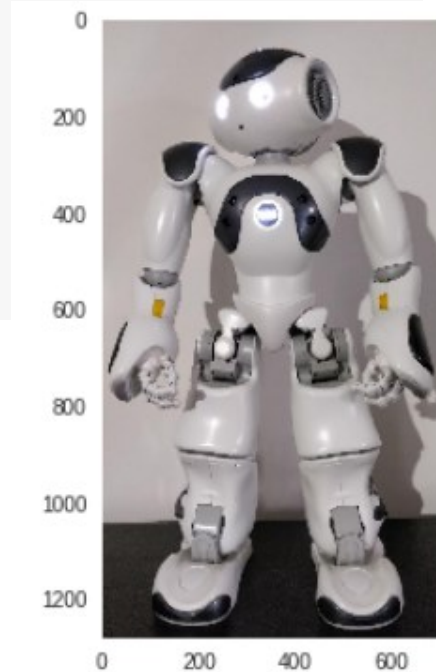
from PIL import Image
from PIL import ImageEnhance

url = "https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg"

img = Image.open(urllib.request.urlopen(url))

enhancer = ImageEnhance.Contrast(img)
new_img = enhancer.enhance(2)

plt.grid(b=False)
plt.imshow(new_img)
```



Esercizio

Applicare i filtri di Sobel e Canny sull'immagine

<https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg>

modificata tramite il contrast enhancement

Esercizio

Provare a modificare l'immagine

<https://dbloisi.github.io/corsi/images/nao-v6-spqr.jpg>

tramite cambio della **brightness**

<https://pillow.readthedocs.io/en/stable/reference/ImageEnhance.html#PIL.ImageEnhance.PIL.ImageEnhance.Brightness>

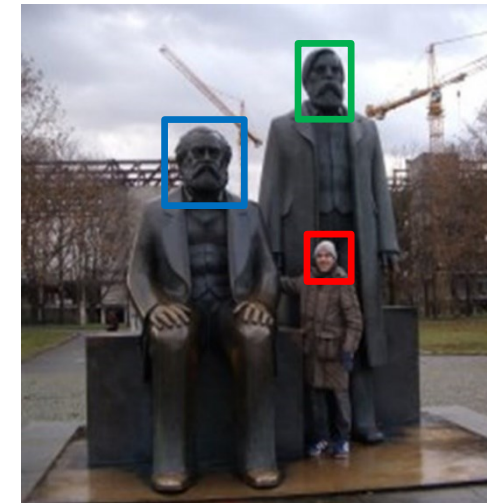
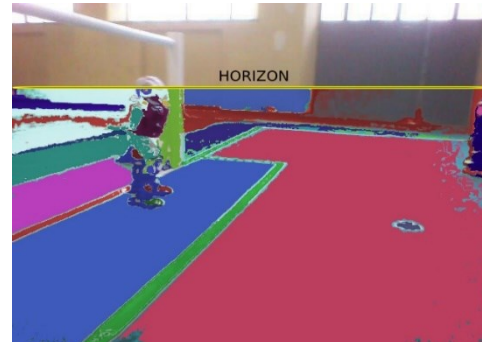
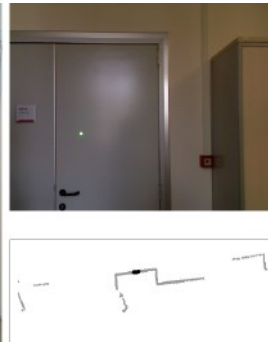
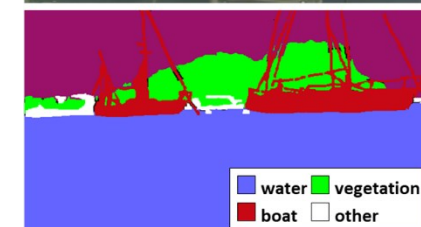
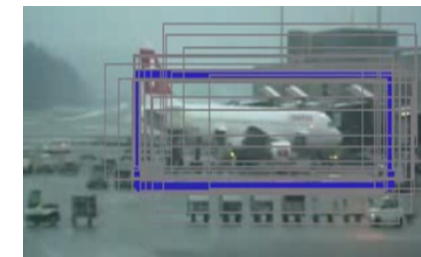


**UNIVERSITÀ DEGLI STUDI
DELLA BASILICATA**

Corso di Sistemi Informativi
A.A. 2018/19

Filtri

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