

### The general problem:

We have information about a random variable's distribution (whether frequentist or Bayesian).

We want information about the distribution of a transformation of that variable.

For example, we have  $\text{var}(X)$ ; but we need  $\text{var}(g(X))$ .

### Our specific example:

We would like a rough estimate of the variance of the ODDS RATIO.

The **ODDS RATIO** is an important measure of association in 2 by 2 tables.

The odds ratio is :

<u>Population</u>	<u>Sample estimate</u>
$OR = \frac{\text{odds}(\text{row1}   \text{col1})}{\text{odds}(\text{row1}   \text{col2})}$	$\widehat{OR} = \frac{M_{11} / M_{21}}{M_{12} / M_{22}}$
$= \frac{\text{odds}(\text{col1}   \text{row1})}{\text{odds}(\text{col1}   \text{row2})}$	$= \frac{M_{11} / M_{12}}{M_{21} / M_{22}}$

A good rough estimate of the variance of  $\log \widehat{OR}$  is

$$\text{var}(\log \widehat{OR}) = \text{var}(\log M_{11} - \log M_{21} - \log M_{12} + \log M_{22}) \approx \frac{1}{M_{11}} + \frac{1}{M_{12}} + \frac{1}{M_{21}} + \frac{1}{M_{22}}$$

Example: In the table

	<b>G2 var</b>	<b>G2 wt</b>	
<b>G1 var</b>	<b>10</b>	<b>12</b>	<b>22</b>
<b>G1 wt</b>	<b>14</b>	<b>64</b>	<b>78</b>
	<b>24</b>	<b>76</b>	<b>100</b>

$$\widehat{OR} = (10 / 14) / (12 / 64) = 3.81$$

$$\text{var}(\log \widehat{OR}) \approx 10^{-1} + 14^{-1} + 12^{-1} + 64^{-1} = 0.27$$

So a rough confidence interval for  $\log OR$  is  $\log(3.81) \pm 1.96 * \sqrt{0.27}$ , which is the interval (0.318, 2.357), and a rough confidence interval for  $OR$  itself is  $(\exp(0.318), \exp(2.357)) = (1.37, 10.56)$ .

A so-called "exact" method uses the noncentral hypergeometric distribution, parametrized by  $\psi = \log(OR)$ .

From the R function `fisher.test()`, we get (1.197927, 11.801156).

**Lots of great stuff about 2x2 tables:**

**Agresti and Min, Simple improved confidence intervals for comparing matched proportions, Sta in Med 2005;**

**Agresti and Caffo, Teacher 's Corner Successes and Two Failures, American Statistician 2000;**

[Richard Darlington, Some New 2 x 2 Tests.](#)

### The principle behind the delta method

The delta method uses a linear approximation:

$$g(x) \approx g(E(x)) + g'(E(x)) \cdot (X - E(x))$$

then concludes

$$\text{var}(g(X)) \approx (g'(E(X)))^2 \cdot \text{var}(X).$$

An important example is the log of a random variable:

$$\text{var}(\log(X)) \approx \left( \frac{1}{E(X)} \right)^2 \text{var}(X) = (\text{coefficient of variation})^2.$$

Another important example is the “lognormal”  $Y$  where  $\log(Y) \sim N(0,1)$ . Here the  $\mathbf{g}$  function is  $\exp()$ , the inverse of the  $\log()$  function.