Futura Pendulum Project

Control System Design 482-01

by

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I. Introduction

The Furuta pendulum has been in use since 1992, when it was invented by Katsuhisa Furuta. The Furuta pendulum, or the rotational inverted pendulum consists of a driven arm rotating in the horizontal plane. In the following report is the design, implementation, and analyzation through state space representation of the Furuta pendulum, Figure 1 provides a depiction of a rotational inverted pendulum. The system will become balanced in real time using an engineered simulation, which is connected through a state space representation equation.

Figure 1: Furuta Pendulum

II. Modeling

The system consists of 2 masses, arm 1 *m* in the horizontal plane, and arm 2 the pendulum, which rotates freely in the vertical plane. In addition to a link connecting arm 1 and the pendulum, that is fixed, Figure 2 depicts a DC motor applying torque $\tau$to arm 1 *m*, which moves horizontally then controls the pendulum so it may remain in the upright position. The torque *t* is a critical component used in the feedback control system, which is done using a state-space model. The angular rotation from arm 1, $\theta_0$is measured in the horizontal plane, where it is assumed that the counter clockwise direction is considered to be positive. The angular rotation from the pendulum $\theta_1$is measured in the vertical plane, where it is also to be assumed that the counter clockwise direction is positive. The system is stable when the pendulum arm is in the upright vertical position $\theta_1 = n * 2\pi$.

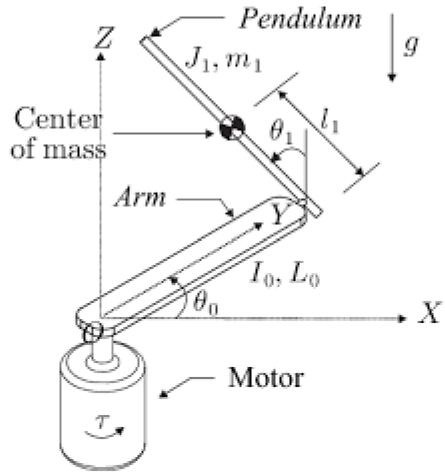

Figure 2: Diagram of Inverted Pendulum

Linear State-Space Model

When deriving the system dynamics, assumptions must be considered. Firstly, the motor and arm 1 are rigidly coupled and inelastic. In addition, the coordinate axes of arms 1 and 2 are the principal axes of inertia. However, the motor rotor inertia is also assumed to be negligible, so the viscous damping is the only one to be considered.

State-space equation is used as a mathematical model that represents the pendulum's physical system. The pendulum system is represented by a set of inputs, outputs, and state variables related by first-order differential equations. Eq 1 and Eq 2 can be found below representing the inputs *u* and outputs *x,* as well as the state space variable vector state *x* of the pendulum system.

$$x' = Ax + Bu \qquad \text{Eq(1).}$$

$$y = Cx + Du \qquad \text{Eq(2).}$$

While the state matrix *A*, input matrix *B*, output matrix *C*, and the forward matrix *D* can be solved using dynamic systems of equations found within Eq 1 and Eq 2. Table 1 below depicts the results for each state-space variable, in addition to the open loop pole coordinates.

Table 1: Parameter Results

Description	Value
State-Space Matrix A	$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -248.97 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.0869 & 0 \end{bmatrix}$

State-Space Matrix B	$\begin{bmatrix} 0 \\ 2121.71 \\ 0 \\ -422.97 \end{bmatrix}$
State-Space Matrix C	$\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}$
State-Space Matrix D	$\begin{bmatrix} 0 \end{bmatrix}$
Open-loop poles	0, 0 , 16.7417, and -16.7417

III. Sensor Calibration

The pendulum has an encoder which is traditionally used for calibration purposes. However, in the following report, Coppelia Sim, a computer application, is used as a live calibration device through Matlab code. The four state variables are directly read through the CoppeliaSim remote API, since the API provides functions to determine the angular position and velocity of any object in the scene.

IV. Controller Design and Simulations

Figure 3 provides a representation of the physical pendulum system model used as a constant calibration device. In order to keep the system in equilibrium real time calibrations are calculated constantly in Matlab. Eq 3 provides the equation used to calculate the pendulum stability, it is important to note that if the system's output is negative the system is stable. This is done by adding a new system matrix after including a full state feedback controller K. The new system matrix with the controller included is.

$$(\mathbf{A} - \mathbf{BK}) \qquad \text{Eq(3).}$$

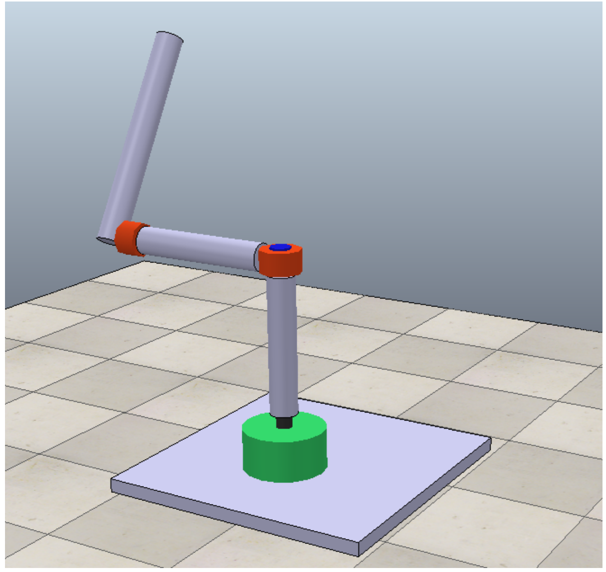

Figure 3: Coppelia Sim Pendulum Model

Table 2 provides the calculated parameters after adding the full state feedback controller K. The output parameters are negative, meaning the system is stable.

$$\begin{bmatrix} 0 & .001 & 0 & 0 \\ 1.233 & .5165 & 4.9507 & 0.4750 \\ 0 & 0 & 0 & .001 \\ -1.85 & -0.7747 & -7.3034 & -0.7126 \end{bmatrix}$$

Figure 4: Values of (**A-BK**)

The eigenvalues found from matrix A can be found depicted below with in Figure 4, it is shown going to zero, thus showing the systems stability.

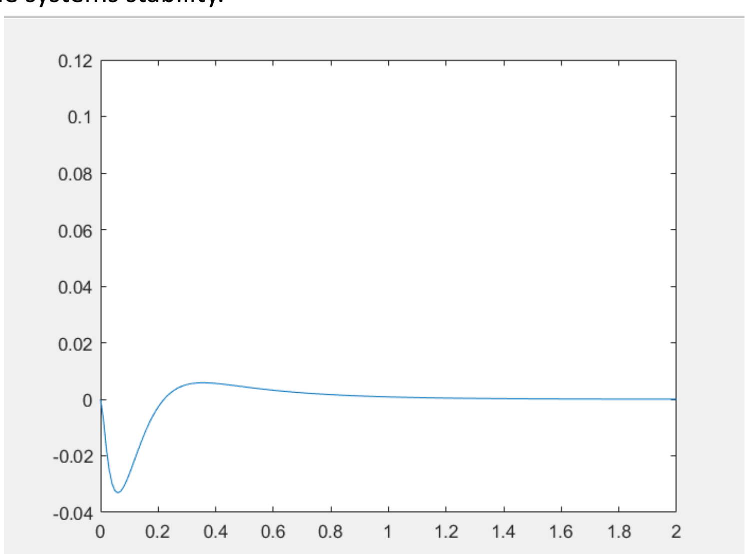

Figure 5: Pendulum stability Results

V. Checklist

Furuta pendulum github page

https://github.com/dboehler/furuta-pendulum

Youtube link

https://drive.google.com/file/d/1fQKJdQVxy26cqjUgUAHJwEGfyviB136H/view?usp=sharing

VI. Appendix

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Youtube link

VII. References

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